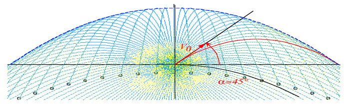
9/25/19 *Assignment 5 - due Wed Oct.2 - Chapters 9 - 12. "Families of orbits and contact envelopes."*



The atoms of NIST or volcanoes of Io

1. Suppose one of the volcanoes on Jupter's moon *Io* detonates in a constant gravity-$g(m{\cdot}s^{-2})$ vacuum sending equi-velocity $\pm v_0(m{\cdot}s^{-1})$ fragments off at initial elevation angles $\alpha=0°, 15°, 30°, 45°, 60°, 75°$, and $90°$ with the latter one going straight up to an altitude of $y=h_0=1$-*unit* on the attached plot 1 graph and then falling straight down.

(a.) That one distance unit has what *mks*-value in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$? h_0=____________( ).

(b.) Derive the parabolic time-coordinates $x(t)$= ____________, $y(t)$= ____________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$ and elevation angle α.

(c.) Derive the parabolic focus-locus coordinates x_{foc}= ____________, y_{foc}= ____________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1}$ and elevation angle α) for $h_0=1$ and construct its curve on plot 1. This curve has Thales geometry (subtended angle of circle diameter or rectangle diagonal) that relate to trajectories. Show it on plots 1 to 4.)

(d.) Derive the parabolic directrix coordinate y_{dir}= ____________ in terms of $h_0=1$ and elevation angle α and construct this directrix line on graph for the cases $\alpha=0°$ to $90°$ listed above. Plot directrix of envelope, too.

(e.) Give general parabolic trajectory curve function $y(x)$= _________________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$ and α for $h_0=1$.

Four plots for four different launch angles 90°, 60°, 45°, and 30°

2. For cases $\alpha=0°, 30°, 45°, 60°$, and $90°$ construct curve points, tangents, kites, and contacts for $\alpha=60°$ on an attached $\alpha=60°$ plot 2, for $\alpha=45°$ on $\alpha=45°$ plot 3, and for $\alpha=30°$ on $\alpha=30°$ plot 4. *(Separate plots for clarity.)*

(a.) Locate the envelope contact points for the cases $\alpha=0°, 30°, 45°, 60°$, and $90°$ and construct enough of the envelope points and tangents to accurately represent the envelope on each of plots 2 to 4. If a contact point lies off a graph indicate where. Deduce $y_{envelope}(x)$= _________________ in terms of $h_0=1$.

(b.) Each parabola trajectory has kite-like structure (See Fig. 9.4.) as does the envelope. Draw and relate them.

(c.) Do any of the α-trajectories have the same shape as the envelope? If so, tell which one.

3. Now consider time behavior implicit in problem 1. In a "snapshot" of each moment, volcano fragments lie on "blast-front" curve. A geometric time unit T_1 is the time for the $\alpha=90°$ fragment to reach its peak.

(a.) That one time unit has what *mks*-value in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$? T_1 =______________( ).

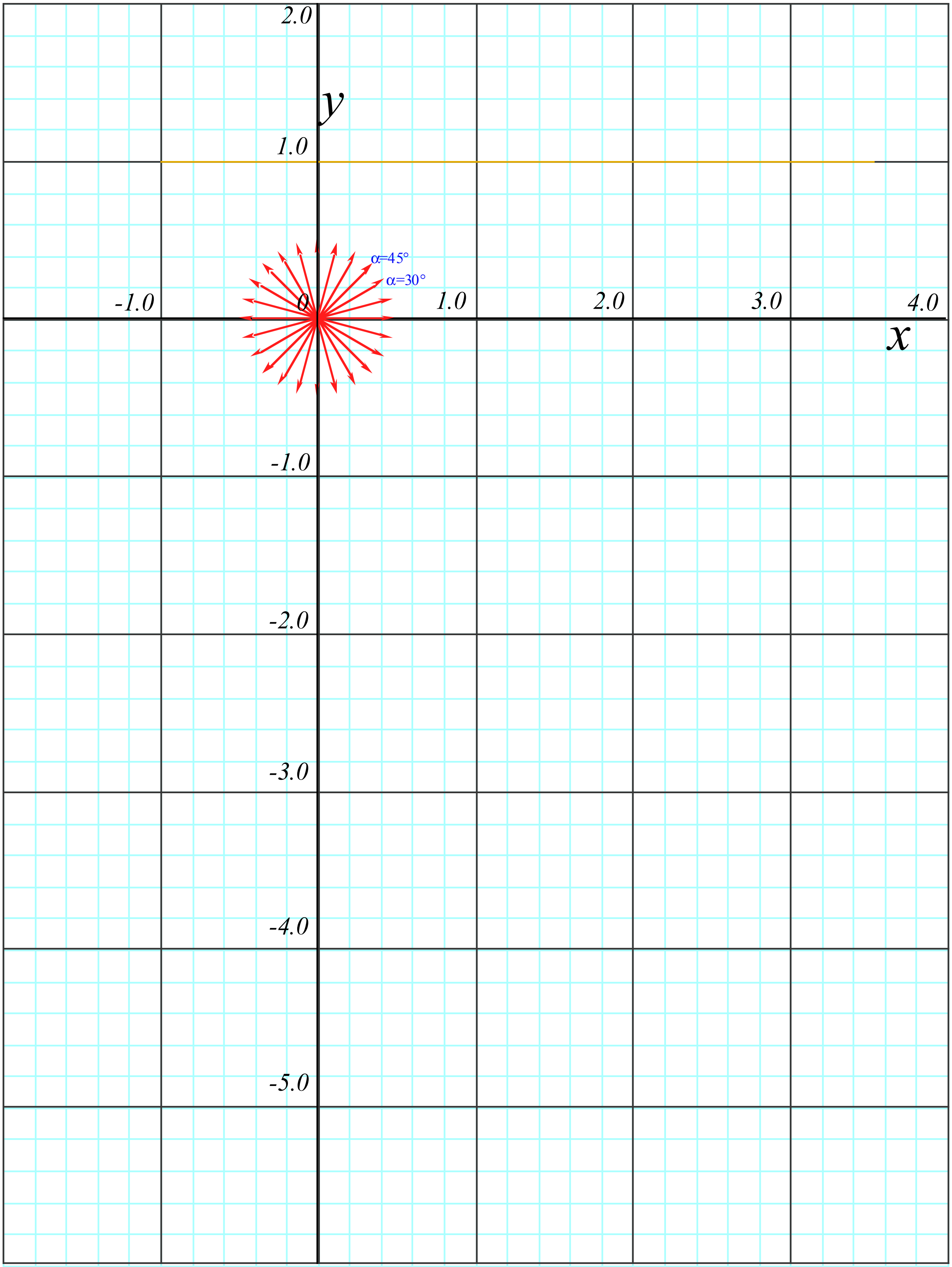
(b.) Give a brief explanation addressing why this "snapshot" curve or locus has to be (whichever): a parabola? ___ straight line?___ circle?____ ellipse?___(Check one and explain choice.)

(c.) Derive and/or construct the "blast-front" curve for the case $\alpha=90°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{90°}$=____ Find polar angle of contact normal.

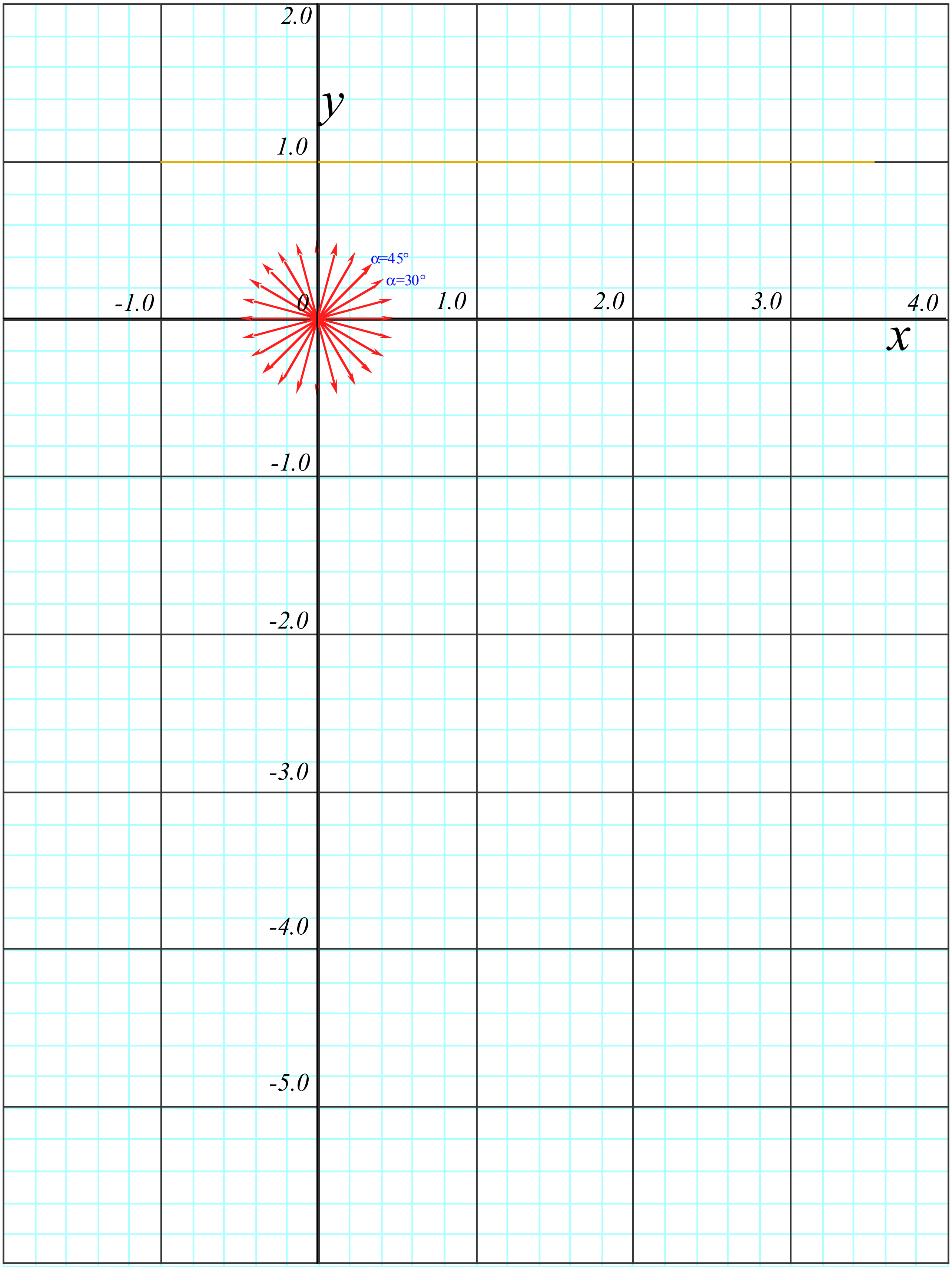
(d.) Derive and/or construct the "blast-front" curve for the case $\alpha=60°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{60°}$=____ Find polar angle of contact normal.

(e.) Derive and/or construct the "blast-front" curve for the case $\alpha=45°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{45°}$=____ Find polar angle of contact normal.

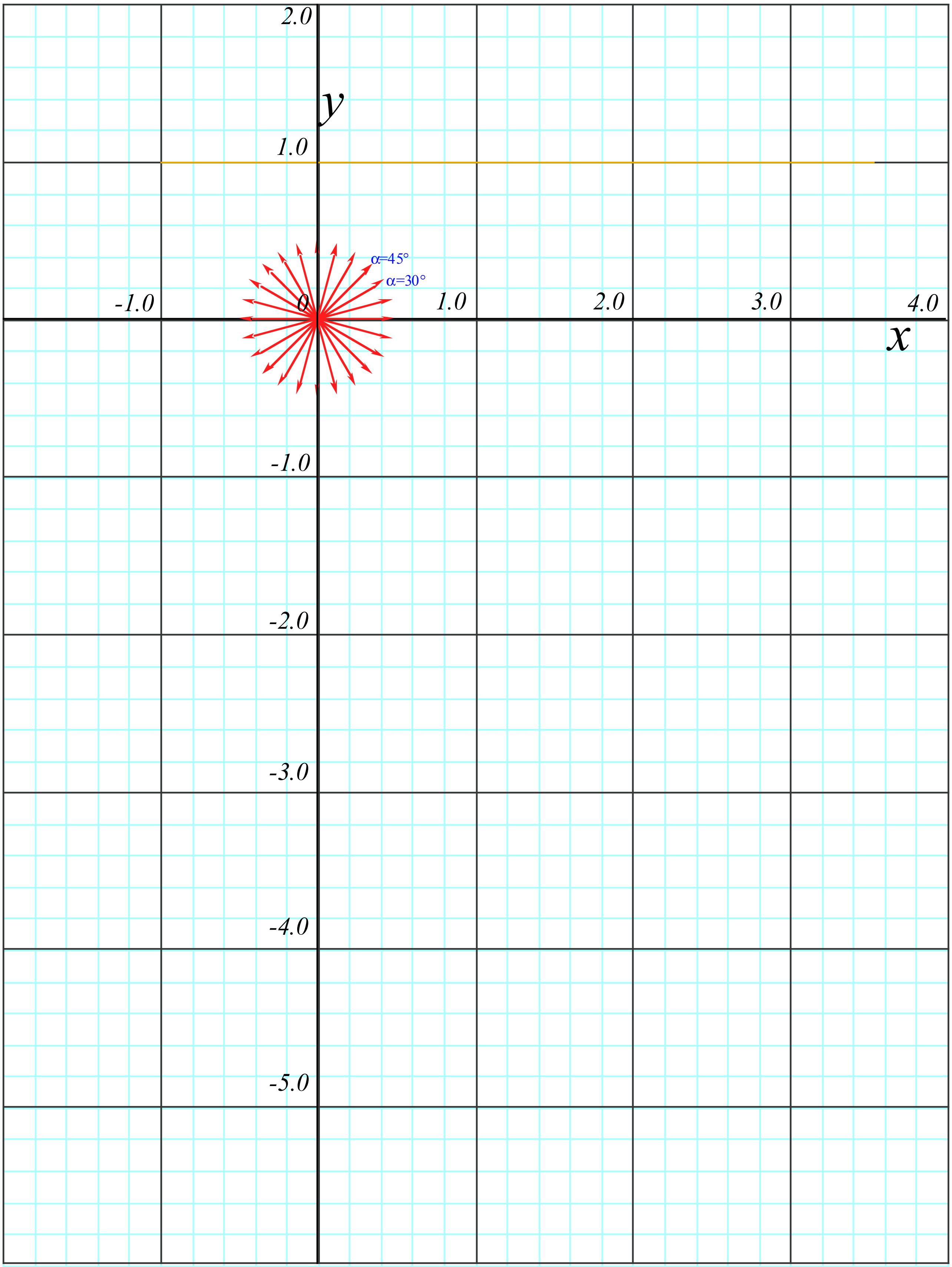
(f.) Derive and/or construct the "blast-front" curve for the case $\alpha=30°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{30°}$=____ Find polar angle of contact normal.



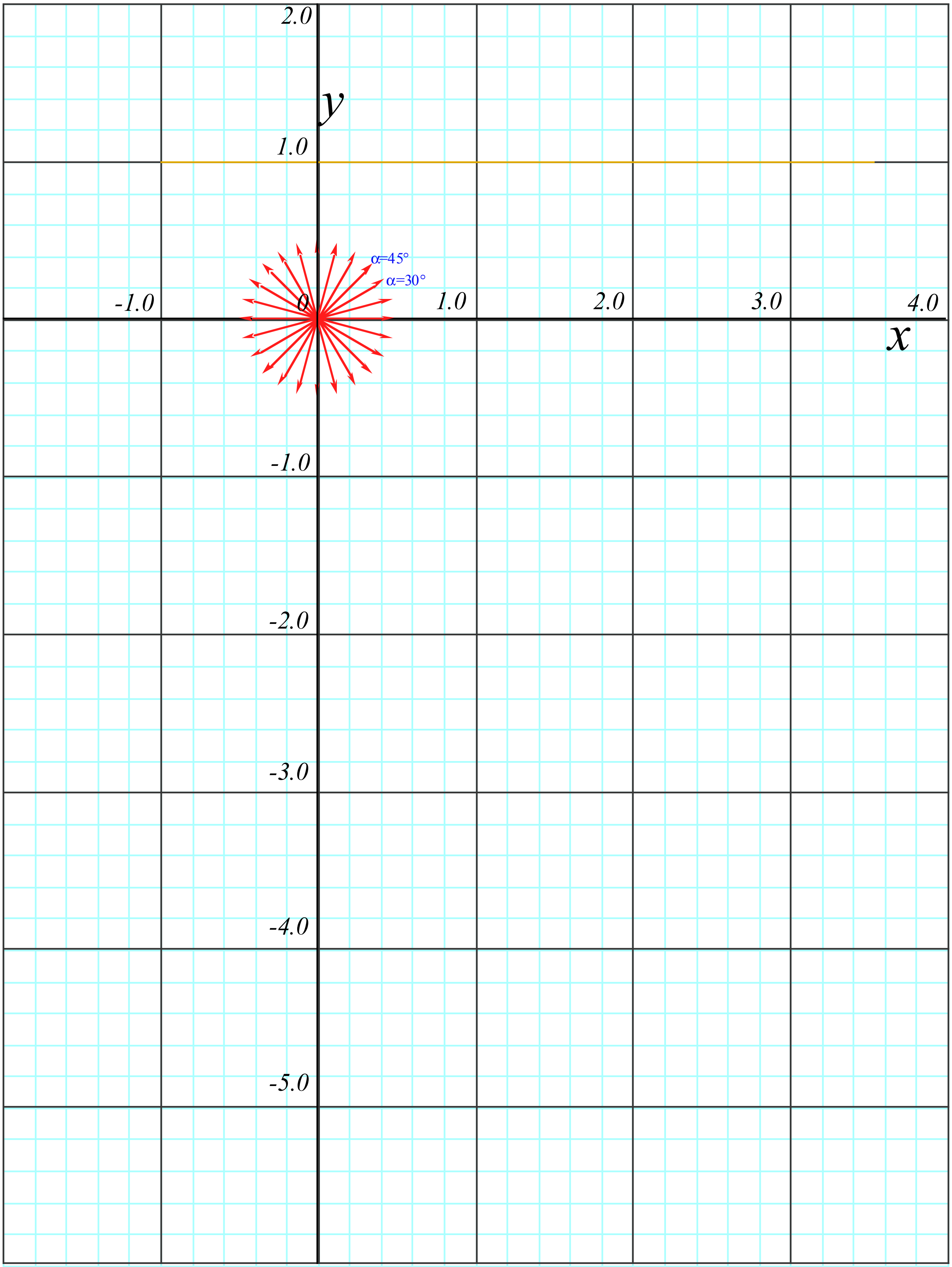
Plot 1: Geometry of α=90° path and α=0° path and where (if ever) they contact a “blast-front” .



2.Show geometry of α=60° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.

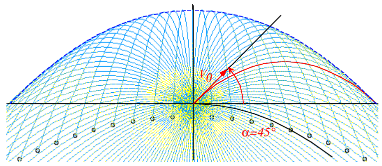


3.Show geometry of α=45° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.



4.Show geometry of α=30° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.

Solutions Assignment 5 (20186)



The volcanoes of Io

1. Suppose one of the volcanoes on Jupter's moon *Io* detonates in a constant gravity-$g(m\cdot s^{-2})$ vacuum sending equi-velocity $\pm v_0(m\cdot s^{-1})$ fragments off at initial elevation angles $\alpha=0°, 15°, 30°,..., 75°$, and $90°$ with the latter one going straight up to an altitude of $y=h_0=1$-*unit* in the attached graph and falling straight down.

a. That one distance unit has what *mks*-value in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$? $h_0= v_0^2/(2g)$ _(*meter*).

b. Derive the parabolic time-coordinates $x(t)= (v_0\cos\alpha)\cdot t$ ____, $y(t)= (v_0\sin\alpha)\cdot t - g\cdot t^2/2$ __ in terms of

c. Derive parab. focus-locus coordinates $x_{foc}=\frac{v_0^2}{2g}2\sin\alpha\cos\alpha = h_0\sin 2\alpha$, $y_{foc}=\frac{v_0^2}{2g}(\sin^2\alpha-\cos^2\alpha)=-h_0\cos 2\alpha$

d. Derive parabolic directrix coordinate $y_{dir}=\frac{v_0^2}{2g}(\sin^2\alpha+\cos^2\alpha)=h_0=1$ in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$

e. Give general parabolic trajectory curve function $y(x)=x\cdot\tan\alpha-\frac{g}{2v_0^2\cos^2\alpha}x^2 = x\cdot\tan\alpha-\frac{1}{4h_0\cos^2\alpha}x^2$ in

f. Locate the envelope contact points for the cases $\alpha=0°, 30°, 45°$, and $90°$ and construct enough of the envelope points and tangents to accurately represent the envelope on the graph. If a contact point lies off the graph indicate what happened. Deduce $y_{envelope}(x)=\frac{v_0^2}{2g}-\frac{g}{2v_0^2}x^2=h_0-\frac{x^2}{4h_0}$ in terms of $h_0=1$. g. Each trajectory has a kite structure. So does the envelope. Draw and relate the two.

h. Do any trajectories have same shape as envelope? Yes, the $\alpha=0°$ path parallels envelope h_0 below it.

2. Now consider time behavior implicit in problem 1. In a "snapshot" of each moment, volcano fragments lie on "blast-front" curve. A geometric time unit T_1 is the time for the $\alpha=90°$ fragment to reach its peak.

(a) That one time unit has what *mks*-value in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$? $T_1 = v_0/g$ _(*sec*).

(b) Give a brief explanation addressing why this "snapshot" curve or locus has to be (whichever): a parabola? ___ straight line?___ circle?_X_ ellipse? sort of (Check one and explain choice on graph.)

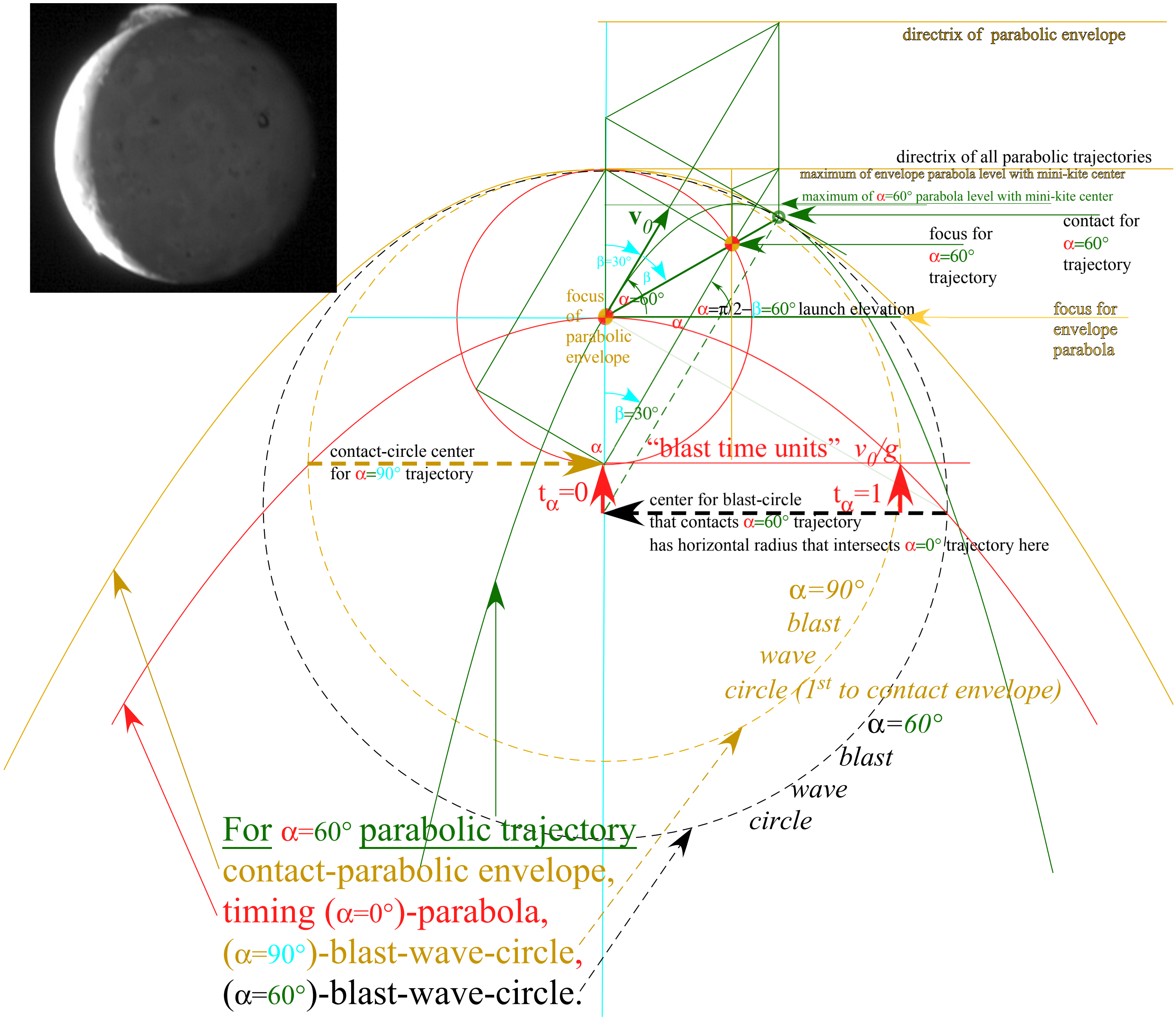
(c) Derive and/or construct the "blast-front" curve for the case $\alpha=90°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{90°}$=_1_ Show its center and contacts. Normal at $\alpha=90°$

(d) Derive and/or construct the "blast-front" curve for the case $\alpha=45°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{45°}$=_$\sqrt{2}$_ Show its center and contacts. Normal at $\alpha=45°$

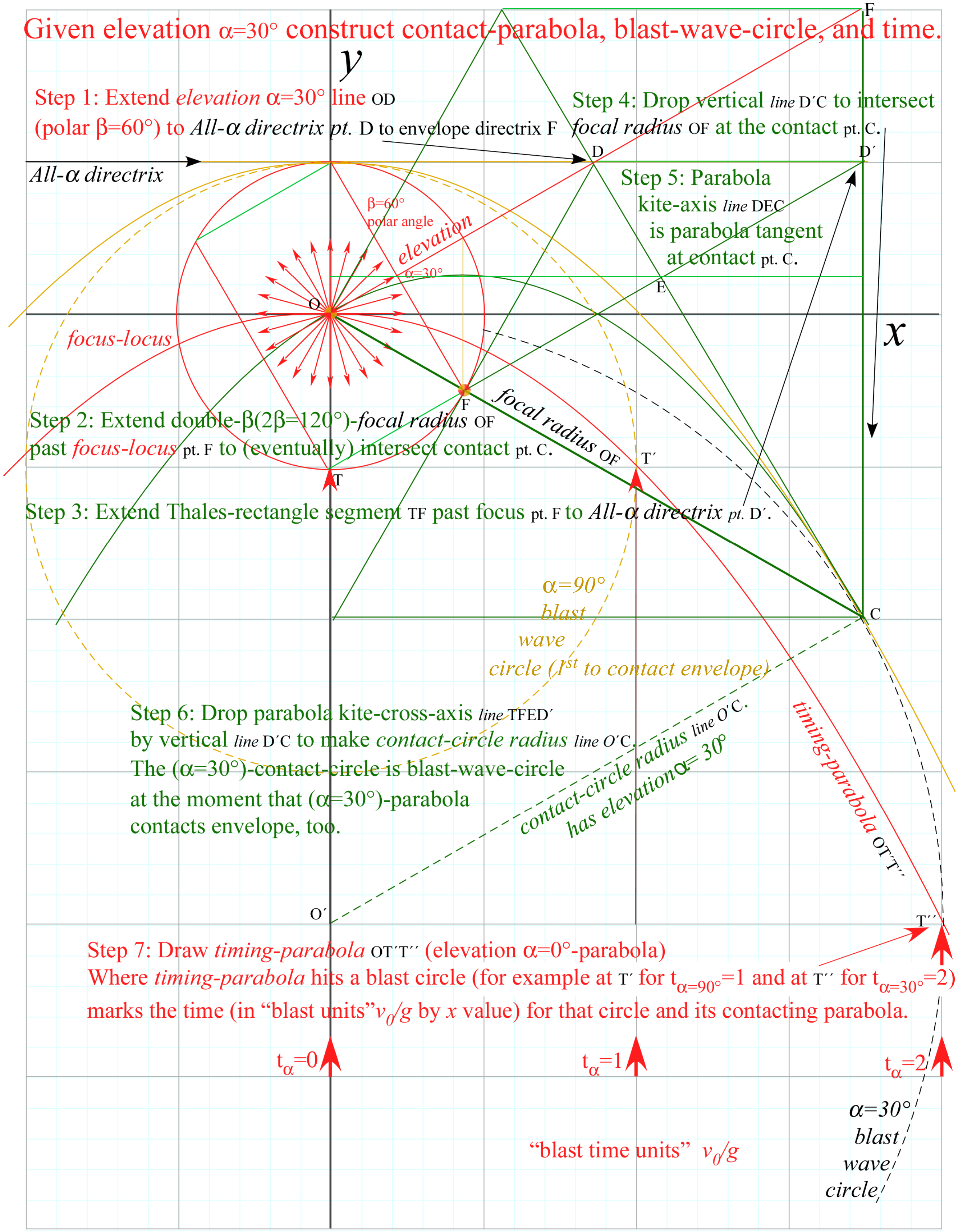
(e) Derive and/or construct the "blast-front" curve for the case $\alpha=30°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{30°}$=_2_ Show its center and contacts. Normal at $\alpha=30°$

3. Suppose fragments continue falling into a tunnel through moon-*Io* that has radius $R_{Io}=0.5\cdot 10^6\, h_0$. Estimate radius of tunnel at widest point assuming it just big enough to let all fragments orbit freely. R_{tunnel}=_$10^3(h_0)$

Ellipse minor radius is $b=\sqrt{a^2-(a-h_0)^2}=\sqrt{2ah_0-h_0^2}=\sqrt{2R_{Io}h_0-h_0^2}=h_0\sqrt{10^6-1}\approx 10^3$



directrix of parabolic envelope
directrix of all parabolic trajectories
maximum of envelope parabola level with mini-kite center
maximum of α=60° parabola level with mini-kite center
contact for α=60° trajectory
focus for α=60° trajectory
v_0
β=30°
β
α=60°
focus of parabolic envelope
α=π/2−β=60° launch elevation
α
focus for envelope parabola
β=30°
contact-circle center for α=90° trajectory
α
"blast time units" v_0/g
$t_\alpha=0$
$t_\alpha=1$
center for blast-circle that contacts α=60° trajectory has horizontal radius that intersects α=0° trajectory here
α=90° blast wave circle (1st to contact envelope)
α=60° blast wave circle
For α=60° parabolic trajectory
contact-parabolic envelope,
timing (α=0°)-parabola,
(α=90°)-blast-wave-circle,
(α=60°)-blast-wave-circle.


Given elevation α=30° construct contact-parabola, blast-wave-circle, and time.
Step 1: Extend elevation α=30° line OD (polar β=60°) to All-α directrix pt. D to envelope directrix F
Step 4: Drop vertical line D´C to intersect focal radius OF at the contact pt. C.
All-α directrix
Step 5: Parabola kite-axis line DEC is parabola tangent at contact pt. C.
β=60° polar angle
elevation α=30°
focus-locus
Step 2: Extend double-β(2β=120°)-focal radius OF past focus-locus pt. F to (eventually) intersect contact pt. C.
focal radius OF
Step 3: Extend Thales-rectangle segment TF past focus pt. F to All-α directrix pt. D´.
α=90° blast wave circle (1st to contact envelope)
Step 6: Drop parabola kite-cross-axis line TFED´ by vertical line D´C to make contact-circle radius line O´C. The (α=30°)-contact-circle is blast-wave-circle at the moment that (α=30°)-parabola contacts envelope, too.
contact-circle radius line O´C. has elevation α= 30°
timing-parabola OT´T´´
Step 7: Draw timing-parabola OT´T´´ (elevation α=0°-parabola)
Where timing-parabola hits a blast circle (for example at T´ for tα=90°=1 and at T´´ for tα=30°=2) marks the time (in "blast units" v0/g by x value) for that circle and its contacting parabola.
tα=0
tα=1
tα=2
"blast time units" v0/g
α=30° blast wave circle

3. Suppose fragments continue falling into a tunnel through moon-*Io* that has radius $R_{Io}=0.5\cdot 10^6\ h_0$. Estimate radius of tunnel at widest point if it just big enough to let all fragments orbit without hitting its walls. R_{tunnel}=_____(h_0) Note: For this problem the gravity is not uniform constant $g=9.8ms^{-2}$ except near surface. (Ellipse geometry.)