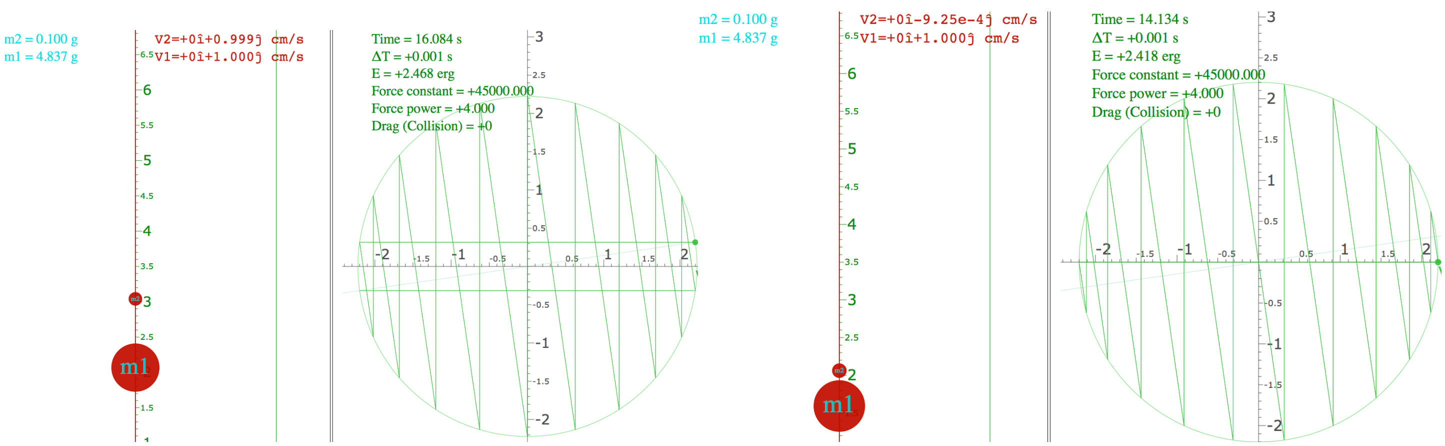
9/5 Assignment Set 3 - Read Unit 1 Ch. 3 thru Ch.7 Due Wed. *9/18/19* Name______________

Pseudo-Rotations for Independent Bounce Model

Exercise 1 Estrangian plot in Fig. 5.2 (Details on p.51-59 of Lecture 3) has mass ratio $M_1/m_2 = 49/1$ and has nearly periodic path plot. (Experiment using BounceIt link in Lect.3 p.74-77. https://modphys.hosted.uark.edu/markup/BounceItWeb.php?scenario=1014)
(Let small-mass be m_2=1 here.) Changing to M_1= 48.3.. gives more nearly periodic symmetry paths seen below.



(a) What order N=___ of C_N or D_N polygonal symmetry is appearing here?

(b) Give a closed formula for value of M_1= 48.3...(to 7 figures) that approaches *exactly* periodic behavior. Simplest formula should relate the tangent of a desired Estrangian rotation half-angle $\theta/2$ to mass M_1. Plot an Estrangian velocity path on the protractor graph-paper attached assuming initial velocity $V_1=1$ and $V_2=0$.

(c) Ceiling height (It is $y_{\max} = 7.0$ for cases above) may eventually affect or destroy periodicity. Use BounceIt to show cases that are affected. (Many have chaotic behavior.)

KE becomes PE

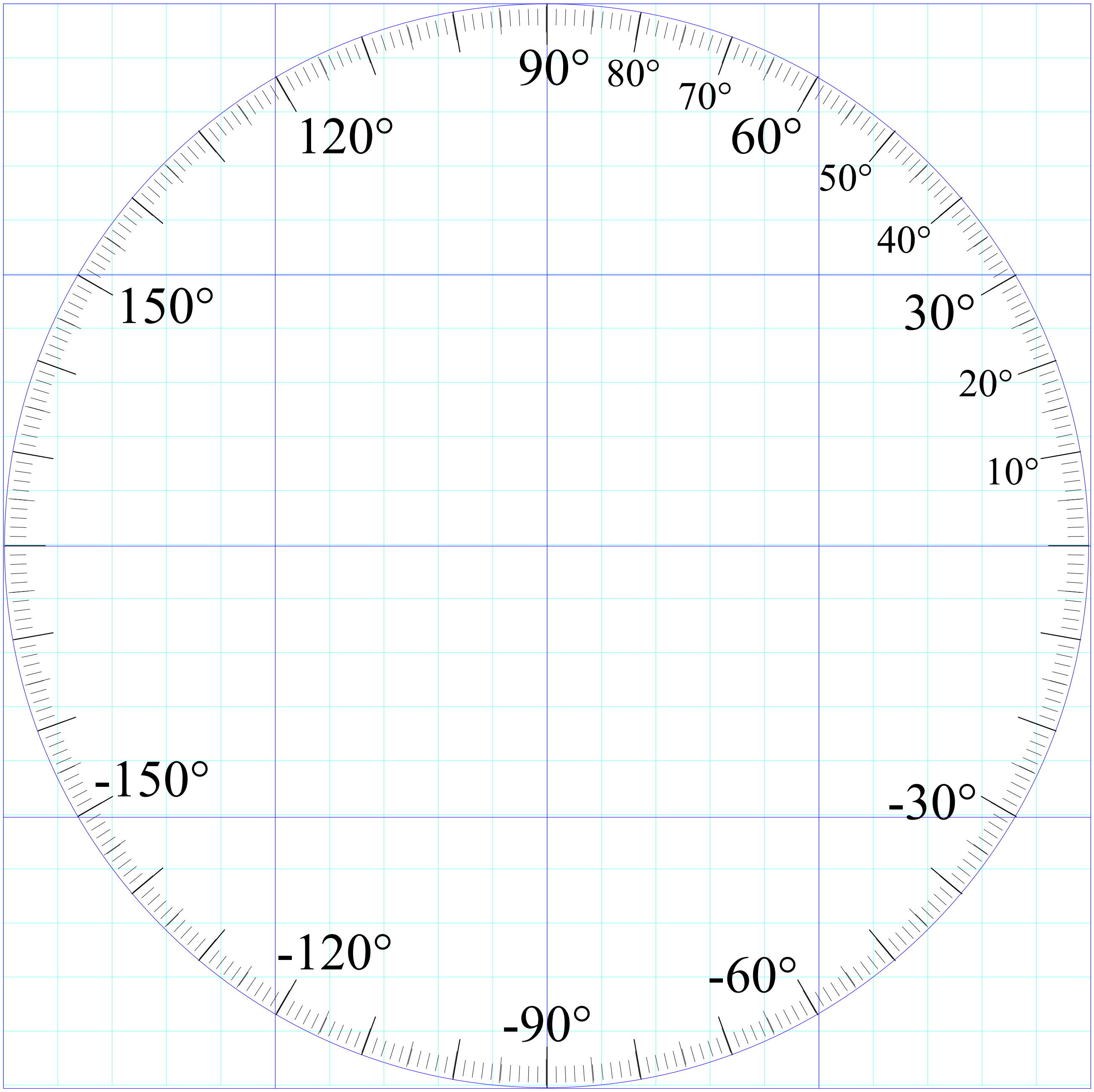
Exercise 2 A mass $m_1=1kg$ ball is trapped (like Fig. 6.3) between two smaller mass $m_2=1gm$ balls of high speed ($v_2(0)=1000m/s$ for $x=0$). Suppose this affects m_1 with an effective force law $F(x)$ of isothermal approximation (6.11). Assume m_1 motion is small and slow around $x=0$. ("Balls" idealize as point masses here.)

(a) A further approximation is the one-Dimensional Harmonic Oscillator (1D-HO) force and PE in (6.12). If each mass m_2 start in an interval $Y_0=1m$, derive approximate 1D-HO frequency and period for mass m_1.

(b) What if the adiabatic approximation is used instead? Does the frequency decrease, increase, or just become anharmonic? Compare isothermal and adiabatic quantitative results for $m_1=1kg$ ball being hit by two $m_2=1gm$ balls each having speed of $v_2(0)=1000m/s$ as each starts bouncing in a space of $Y_0=1m$ on either side of the equilibrium point $x=0$ for the $1kg$ ball.

(c) How does the frequency decrease or increase in isothermal case *versus* the adiabatic case if we shorten the run interval $Y_0=1m$ to one-quarter meter?...What if we reduce the mass ratio m_1/m_2 by one-quarter?

(d) Derive the adiabatic frequency and period for the case $M=50kg$ in adiabatic force of two $m=0.1kg$ masses of initial speed $v_0=20m/s$ and range $Y_0=3m$. Compare with Fig. 1.6.3c.


90°
80°
70°
60°
50°
40°
30°
20°
10°
120°
150°
-150°
-120°
-90°
-60°
-30°

Assignment 3- Solutions to Ex. 1

Solution to *Pseudo-Rotations for Independent Bounce Model*

Ex.1(a) Symmetry is that of 22-point polygon (22-agon), a cyclic group C_{22} or dihedral group D_{22}.

(b) Solving the mass ratio equation (5.10b) for m_1/m_2 in terms of angle θ that simplifies to:

$$\cos\theta \equiv \left(\frac{m_1 - m_2}{M}\right) \quad and: \ \sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{M}\right) \quad \cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} \quad and: \ \frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} = \cot^2\frac{\theta}{2} \quad or: \ \cot\frac{\theta}{2} = \sqrt{\frac{m_1}{m_2}}$$

Given angle $\theta = 2\pi/22 = \pi/11$ or $\theta/2 = \pi/22$ and $m_2=1$ predicts $m_1 = \cot^2\frac{\pi}{22} = 48.3741500787$

Assignment 3 Solutions to Ex. 2.

Ex.2 (a) A mass $m_1=1kg$ ball is trapped (like Fig. 6.3) between two smaller mass $m_2=1gm$ balls of high speed ($v_2(0)=1000m/s$ for $x=0$). Suppose this affects m_1 as effective force law $F(x)$ of isothermal approximation (6.11).

For isothermal we set: $v_2 = const. = v_2^{IN} = v_2(0)$ to give: $F^{isoth}(1wall\ Y)=\pm\frac{m_2 v_2^2}{Y} = \pm\frac{m_2 v_2^2(0)}{Y} = \pm\frac{const.}{Y} = \pm\frac{f}{Y}$

For $Y_0=10cm$ there is an isothermal spring constant $k \equiv 2f = 2\cdot\frac{m_2 v_2^2(0)}{Y_0^2}$ due to left (Y_0+x) AND right (Y_0-x) walls.

$$F^{isoth} = \frac{f}{Y_0+x} - \frac{f}{Y_0-x} = \frac{f}{Y_0}\left[1 - \frac{x}{Y_0} + ...\right] - \frac{f}{Y_0}\left[1 + \frac{x}{Y_0} + ...\right] \approx -2f\cdot\frac{x}{Y_0^2} = -2m_2 v_2^2(0)\cdot\frac{x}{Y_0^2} = -k\cdot x \quad \ldots$$

If $Y_0=1m$, $m_2=1gm=10^{-3}kg$ has speed $v_2=1000m/s$ so $k= 2m_2 v_2^2(0) =2(10^{-3})(1000)^2=2000$.

Then frequency of $m_1= 1$ kg is $\omega=\sqrt{(k/m_1)}=\sqrt{2000}=44.7$ rad/sec. Then $\upsilon=\sqrt{2000}/2\pi=\sqrt{500}/\pi=7.12$Hz with period $\pi=0.14$sec.

(b) What if the adiabatic approximation is used instead? Does the frequency decrease, increase, or just become anharmonic? Compare isothermal and adiabatic quantitative results for *Ex. 2(a)*.

For adiabatic v_2 is not constant but inversely dependent on Y so we set:

$$v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y_0}{Y}, \quad F^{adibad}(1\ wall)=\pm\frac{m_2 v_2^2}{Y} \simeq \pm m_2\frac{\left(v_2^{IN} Y_0\right)^2}{Y^3} = \pm\frac{g}{Y^3} \quad \text{where: } g=m_2\left(v_2^{IN} Y_0\right)^2 \sim f \quad (for: Y\sim 1).$$

$$F_{2-wall}^{adibad} = \frac{g}{(Y_0+x)^3} - \frac{g}{(Y_0-x)^3} = \frac{g}{Y^3_0}\left[\frac{g}{(1+\frac{x}{Y_0})^3} - \frac{g}{(1-\frac{x}{Y_0})^3}\right] = \frac{g}{Y^3_0}\left[1 - 3\frac{x}{Y_0} + ...\right] - \frac{g}{Y^3_0}\left[1 + 3\frac{x}{Y_0} - ...\right] = -\frac{6g}{Y^4_0}\cdot x + ...$$

$$= -\frac{6\cdot m_2 v_2^2(0) Y^2_0}{Y^4_0}\cdot x + ... = -\frac{6\cdot m_2 v_2^2(0)}{Y^2_0}\cdot x + ... \quad \text{(Effective spring constant increases by factor of 3)}$$

So 1kg mass angular frequency is $\omega=\sqrt{2000}\sqrt{3}=77.5$ rad/sec. (Increases by factor of $\sqrt{3}$ over the isothermal values.)

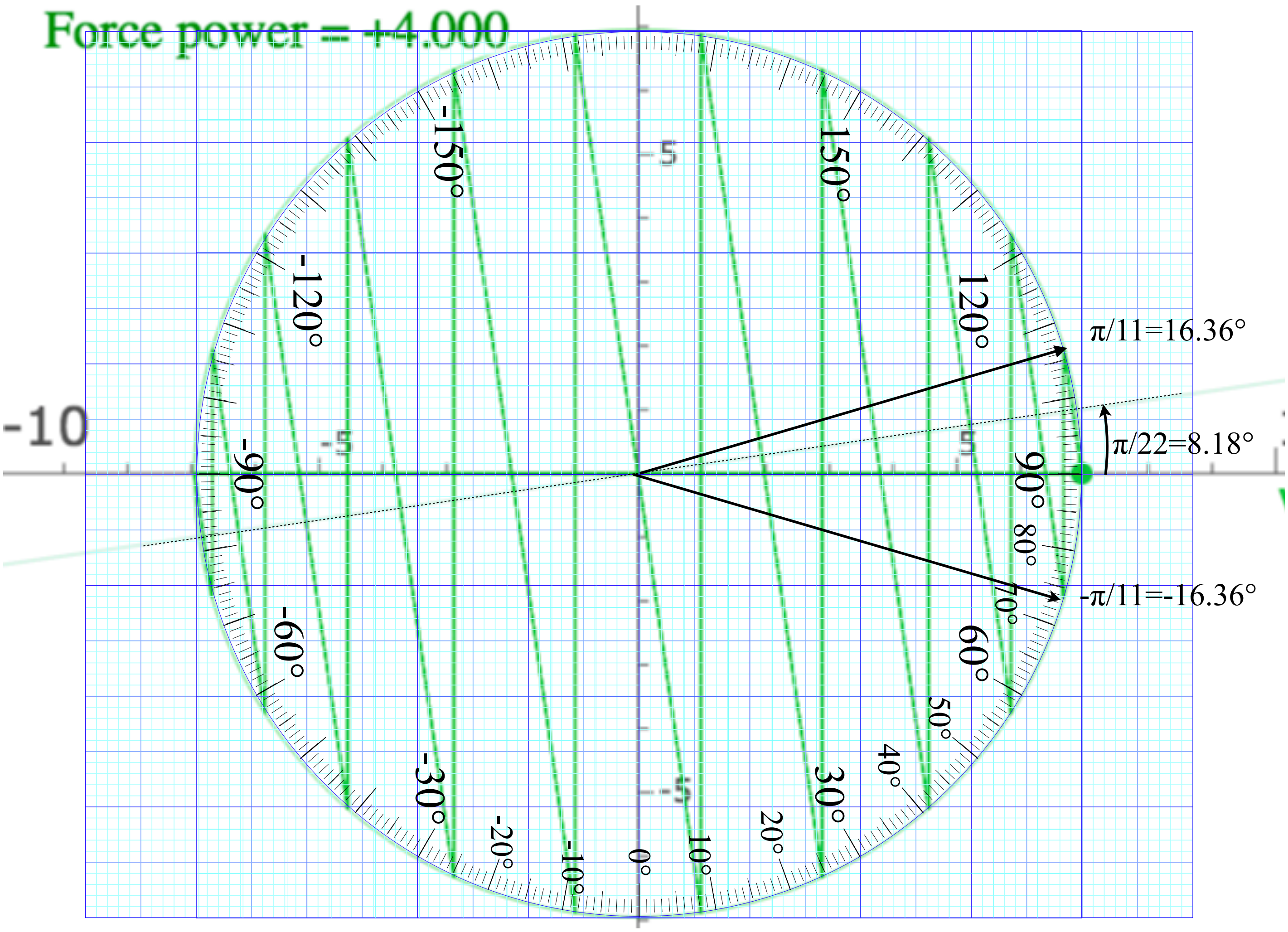
$\upsilon=\sqrt{2000}\sqrt{3}/2\pi=12.33$ Hz. $1/\upsilon=\tau=1/12.33=0.0833$ sec

(c) How does the frequency decrease or increase in isothermal case and in the adiabatic case if we shorten the run interval $Y_0=1m$ to one-quarter meter?……..if we reduce the mass ratio m_1/ m_2 by one-quarter?

(Both frequencies increase by factor of 4.) (Both frequencies increase by factor of 2.)

(d) Derive the adiabatic frequency and period for the case $M=50kg$ in adiabatic force of two $m_2=0.1kg$ masses of initial speed $v_0=20m/s$ and range $Y_0=3m$. Compare with Fig. 1.6.3c.

$$\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{6\cdot m_2 v_2^2(0)}{M\cdot Y^2_0}} = \frac{1}{2\pi}\sqrt{\frac{6\cdot 0.10\cdot 20^2}{50\cdot 3^2}} = \frac{1}{2\pi}\sqrt{\frac{24}{45}} = \frac{.73}{2\pi}$$ So period is $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{15}{8}} = 8.6\text{sec}$. (Close to Fig. 6.3)


Force power = +4.000
-10
-5
5
π/11=16.36°
π/22=8.18°
-π/11=-16.36°
-150°
150°
-120°
120°
-90°
90°
80°
70°
-60°
60°
50°
40°
-30°
30°
-20°
20°
-10°
10°
0°