Assignment Set 1 - 8.26.19 Read Unit 1 Chapters 1 thru Ch.3- Exercises due Wed. September 4

Exercise 1 ***Class exercise continued…***

Complete VW(10mph) vs. SUV(60mph) collision analysis and plot of IN and FIN velocity states done in class.
Extra ±80 by ±120 graphpaper attached is same as used in class
(a) For a totally inelastic *'ka-runch'* case derive final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1, V^{FIN}_2)$ from plot
(b) Derive and plot IN and FIN KE ellipses† and velocity vectors. (Use tensor algebra to clarify formulas.)
(c) For a totally elastic *'ka-bong'* case do the same. Compare IN and FIN KE values and ellipses† for the two cases.
(d) On the same plot draw ellipse(s) and velocity vectors as seen in the COM frame for both cases.

† At the end of Ch.1 is shown an easy ellipse construction given ellipse radii a and b. This should not be necessary for Exercise 2 but will come in handy for Exercise 1 and 3. Both use attached graph paper (also available online).

Exercise 2 ***Basic pool-shot (equal-mass) kinetics***

Use blank ±0.5 by ±1.0 graph paper (attached and available on-line).
Consider V_1 vs V_2 graphs for 1D-collisions between masses M_1 and M_2 described in Ch. 2 and Ch. 3.

(a) Draw a graph of a collision with initial velocities $\mathbf{V}^{IN} = (V^{IN}_1, V^{IN}_2) = (0.5, 0)$ for equal masses ($M_1 = 1 = M_2$).
(b) For a totally inelastic *'ka-runch'* case find final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1, V^{FIN}_2)$ from graph and plot KE ellipse†.
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Exercise 3 ***Head-on collision kinetics***

Solve using tensor algebraic methods and compare to geometric solution on ±0.5 by ±1.0 graph paper.

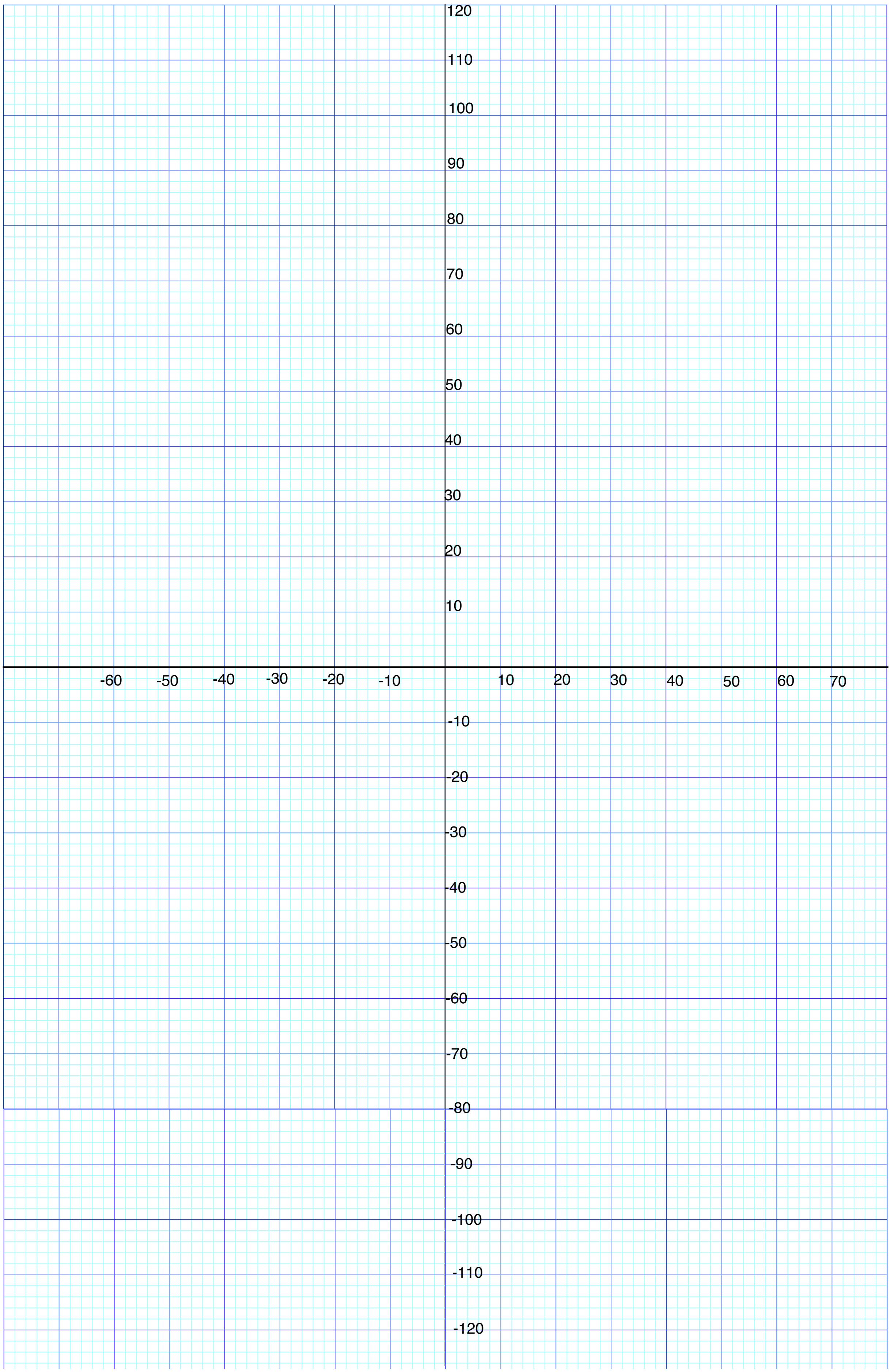
Analyze collisions for head-on initial velocities $\mathbf{V}^{IN} = (V^{IN}_1, V^{IN}_2) = (0.4, -0.2)$ for masses $M_1 = 5$ and $M_2 = 1$.
Derive final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1, V^{FIN}_2) = \mathbf{V}^{COM}$ for a totally inelastic *'ka-runch'* case.
Derive final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1, V^{FIN}_2)$ for totally elastic *'ka-bong'* case.
Derive KE=______, *KE-ellipse* radii $a_1=a$_____, $a_2=b$=______ for *ka-runch* case and construct its ellipse†.
Derive KE=______, *KE-ellipse* radii $a_1=a$_____, $a_2=b$=______ for *ka-bong* case and construct its ellipse†.

Derive KE=______, *KE-ellipse* radii $a_1=a$_____, $a_2=b$=______ for *ka-bong* case as viewed in COM frame.
Derive KE=______, *KE-ellipse* radii $a_1=a$_____, $a_2=b$=______ for *ka-runch* case as viewed in COM frame.

Construct resulting ellipse† for each case (if it exists).

Extra credit

Do the same algebra and geometric plot for non-head-on case $\mathbf{V}^{IN} = (V^{IN}_1, V^{IN}_2) = (0.4, +0.2)$ for same masses.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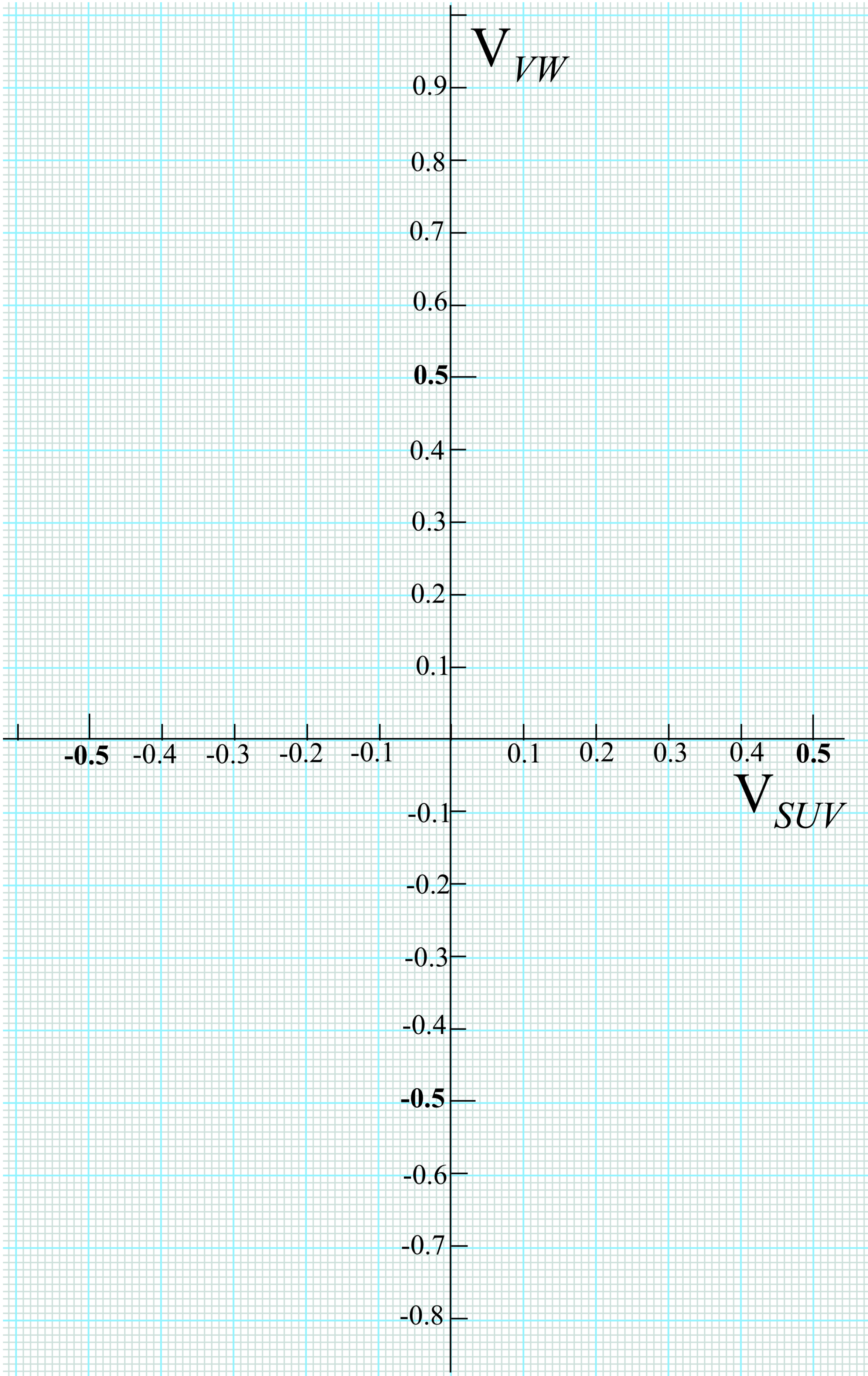
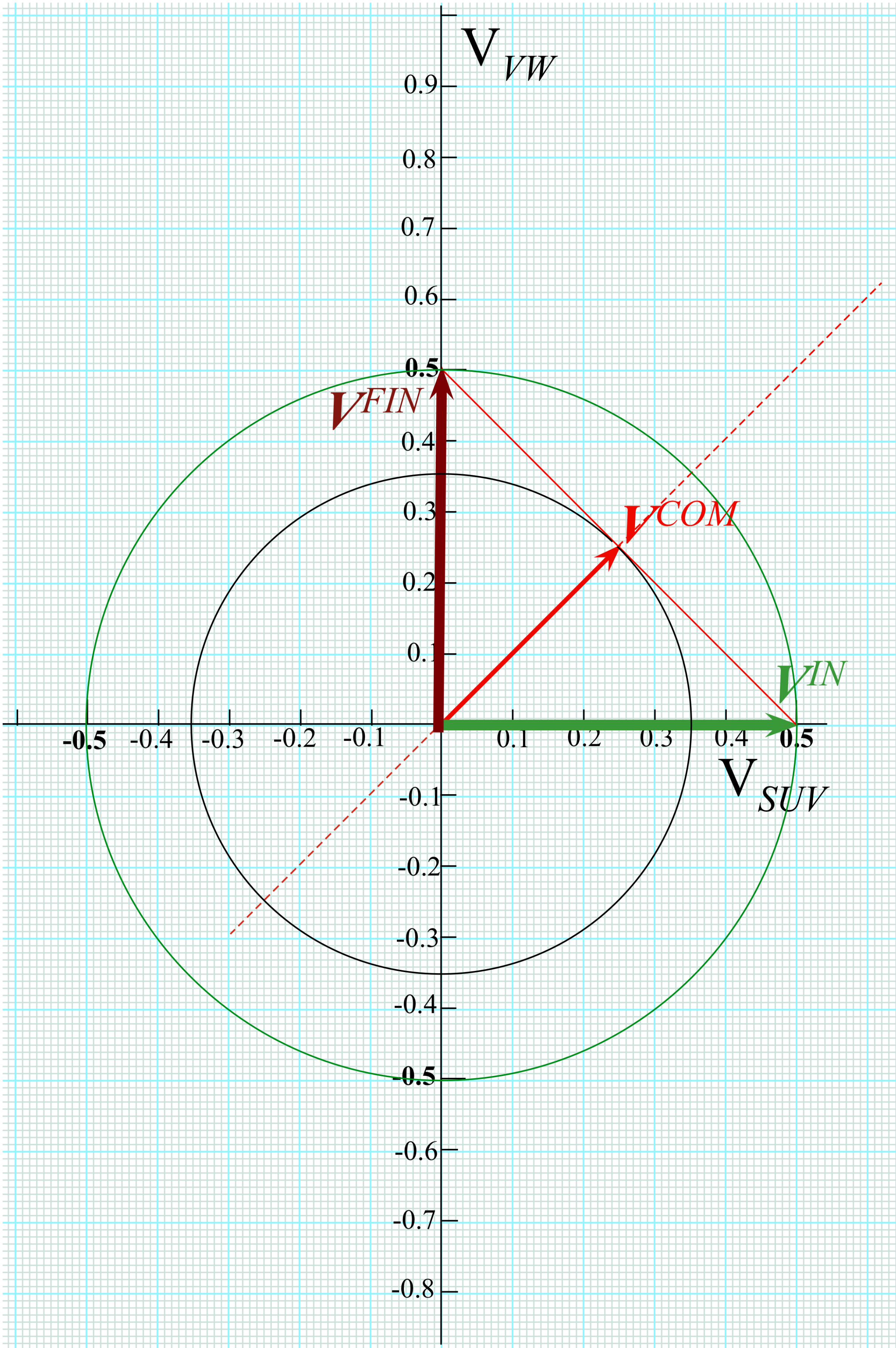


V_VW
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.5
-0.4
-0.3
-0.2
-0.1
0.1
0.2
0.3
0.4
0.5
V_SUV
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8

Set 1 Exercise 2 solution by geometry. (In-class Exercise 1 described in Text-Chapter 3 and Lecture 1.)



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Derive final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1 , V^{FIN}_2) = \mathbf{V}^{COM}$ for a totally inelastic *'ka-runch'* case.
Derive final velocities $\mathbf{V}^{FIN} = (V^{FIN}_1 , V^{FIN}_2)$ for totally elastic *'ka-bong'* case.
Derive KE=_0.27__, *KE-ellipse* radii a_1=0.3286a, $a_2=b$=0.7348a for *ka-runch* case and construct its ellipse†.
Derive KE=_0.42__, *KE-ellipse* radii a_1=*0.4099a*, $a_2=b$=0.9165a for *ka-bong* case and construct its ellipse†.

Derive KE=__0.15_, *KE-ellipse* radii a_1=0.2449, $a_2=b$=0.5477 for *ka-bong* case as viewed in COM frame.
Derive KE=__0.00_, *KE-ellipse* radii a_1=0.00 _, $a_2=b$=0.00 for *ka-runch* case as viewed in COM frame.

Construct resulting ellipse† for each case (if it exists).

Extra credit

Do the same algebra and geometric plot for non-head-on case $\mathbf{V}^{IN} = (V^{IN}_1 , V^{IN}_2)= (0.4, +0.2)$ for same massSet 1 Exercise 3 solutions. (Based upon ellipse plots on following page.)

Three key vectors pointing at momentum line are *initial* $\vec{\mathbf{V}}^{IN}=\begin{pmatrix}0.4\\-.2\end{pmatrix}$ and final *ka-bong* $\vec{\mathbf{V}}^{FIN}=\begin{pmatrix}.2\\.8\end{pmatrix}$ or *ka-runch* $\vec{\mathbf{V}}^{COM}=\begin{pmatrix}.3\\.3\end{pmatrix}$.

$$V^{COM}P_{Total}=\vec{\mathbf{V}}^{COM}\bullet\vec{\mathbf{P}}_{Total}=\vec{\mathbf{V}}^{COM}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}=\vec{\mathbf{V}}^{COM}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}=\vec{\mathbf{V}}^{COM}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM}=V^{COM}M_{Total}V^{COM}$$

$$0.3P_{Total}=\begin{pmatrix}.3 & .3\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.4\\-.2\end{pmatrix}=\begin{pmatrix}.3 & .3\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.2\\.8\end{pmatrix}=.3M_{Total}.3=.3\cdot6\cdot.3=.54$$

$$=.3\cdot5\cdot.4-.3\cdot1\cdot.2=.3\cdot5\cdot.2+.3\cdot1\cdot.8=.54\qquad P_{Total}=5\cdot.4-1\cdot.2=2-.2=1.8$$

Lop-sided tensor factors are equal: (True for any symmetric mass-matrix $\mathbf{M}^T=\mathbf{M}$.)

$$\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}=\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

$$\begin{pmatrix}.2 & .8\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.4\\-.2\end{pmatrix}=\begin{pmatrix}.4 & -.2\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.2\\.8\end{pmatrix}=0.24$$

$$=.2\cdot5\cdot.4-.8\cdot1\cdot.2=.4\cdot5\cdot.2-.2\cdot1\cdot.8$$

Diagonal tensor terms turn out to be equal, too. (And that proves KE conservation for T-symmetry!)

$$\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}=\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

$$\begin{pmatrix}.4 & -.2\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.4\\-.2\end{pmatrix}=\begin{pmatrix}.2 & .8\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.2\\.8\end{pmatrix}=0.84$$

$$=.4\cdot5\cdot.4+.2\cdot1\cdot.2=.2\cdot5\cdot.2+.8\cdot1\cdot.8$$

With T-symmetry: $\vec{\mathbf{V}}^{COM}=\frac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)=\begin{pmatrix}0.3\\0.3\end{pmatrix}$ in 1st equation (top of page) leads to KE conservation relation…

$$V^{COM}P_{Total}=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)\bullet\ddot{\mathbf{M}}\bullet\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)$$

$$.54\qquad=\qquad.54\qquad=\qquad.54$$

$$.3\cdot1.8=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)+\tfrac{1}{2}\left(\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}\right)+\tfrac{1}{2}\left(\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}\right)$$ factoring and...

$$.54=\tfrac{1}{2}0.24+\tfrac{1}{2}.84=\tfrac{1}{2}.84+\tfrac{1}{2}0.24$$ ...and checking sums...

Now subtract the lopsided tensor factor from each term to prove that KE is conserved under T-symmetry.

$$V^{COM}P_{Total}-\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}\right)=\tfrac{1}{4}\left(\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)+\tfrac{1}{4}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}\right)$$

$$V^{COM}P_{Total}-\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}\right)=\tfrac{1}{2}\left(\vec{\mathbf{V}}^{FIN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}\right)=KE_{Elastic}=KE_{ka-bong}$$

$$.3\cdot1.8-\frac{0.24}{2}=\tfrac{1}{2}\begin{pmatrix}.4 & -.2\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.4\\-.2\end{pmatrix}=\tfrac{1}{2}\begin{pmatrix}.2 & .8\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.2\\.8\end{pmatrix}=KE_{Elastic}=.42=KE_{IN}$$

$$.54-.12=\tfrac{1}{2}(.4\cdot5\cdot.4+.2\cdot1\cdot.2)=\tfrac{1}{2}(.2\cdot5\cdot.2+.8\cdot1\cdot.8)=.42=KE_{IN}$$

The *ka-bong* ellipse equation follows with x-radius: $a=\sqrt{\tfrac{2}{5}0.42}=.4099$ and y-radius: $b=\sqrt{2\cdot0.42}=.9165$.

$$\frac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}=\frac{1}{2}\begin{pmatrix}x & y\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}=KE_{Elastic}=.42=KE_{IN}=5\frac{x^2}{2}+1\frac{y^2}{2}\quad\text{or:}\quad1=\frac{x^2}{\frac{2}{5}0.42}+\frac{y^2}{2\cdot0.42}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

The *ka-runch* ellipse equation follows with x-radius: $a=\sqrt{\tfrac{2}{5}0.27}=.3286$ and y-radius: $b=\sqrt{2\cdot0.27}=.7348$.

$$\frac{1}{2}\vec{\mathbf{V}}^{COM}\bullet\ddot{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM}=\frac{1}{2}\begin{pmatrix}x & y\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}=KE_{Inelastic}=.27=5\frac{x^2}{2}+1\frac{y^2}{2}\quad\text{or:}\quad1=\frac{x^2}{\frac{2}{5}0.27}+\frac{y^2}{2\cdot0.27}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

$$\text{where: }KE_{ka-runch}=\frac{1}{2}\begin{pmatrix}.3 & .3\end{pmatrix}\cdot\begin{pmatrix}5&0\\0&1\end{pmatrix}\cdot\begin{pmatrix}.3\\.3\end{pmatrix}=\frac{1}{2}\cdot(.3\cdot5\cdot.3+.3\cdot1\cdot.3)=\frac{1}{2}\cdot(.54)=.27$$

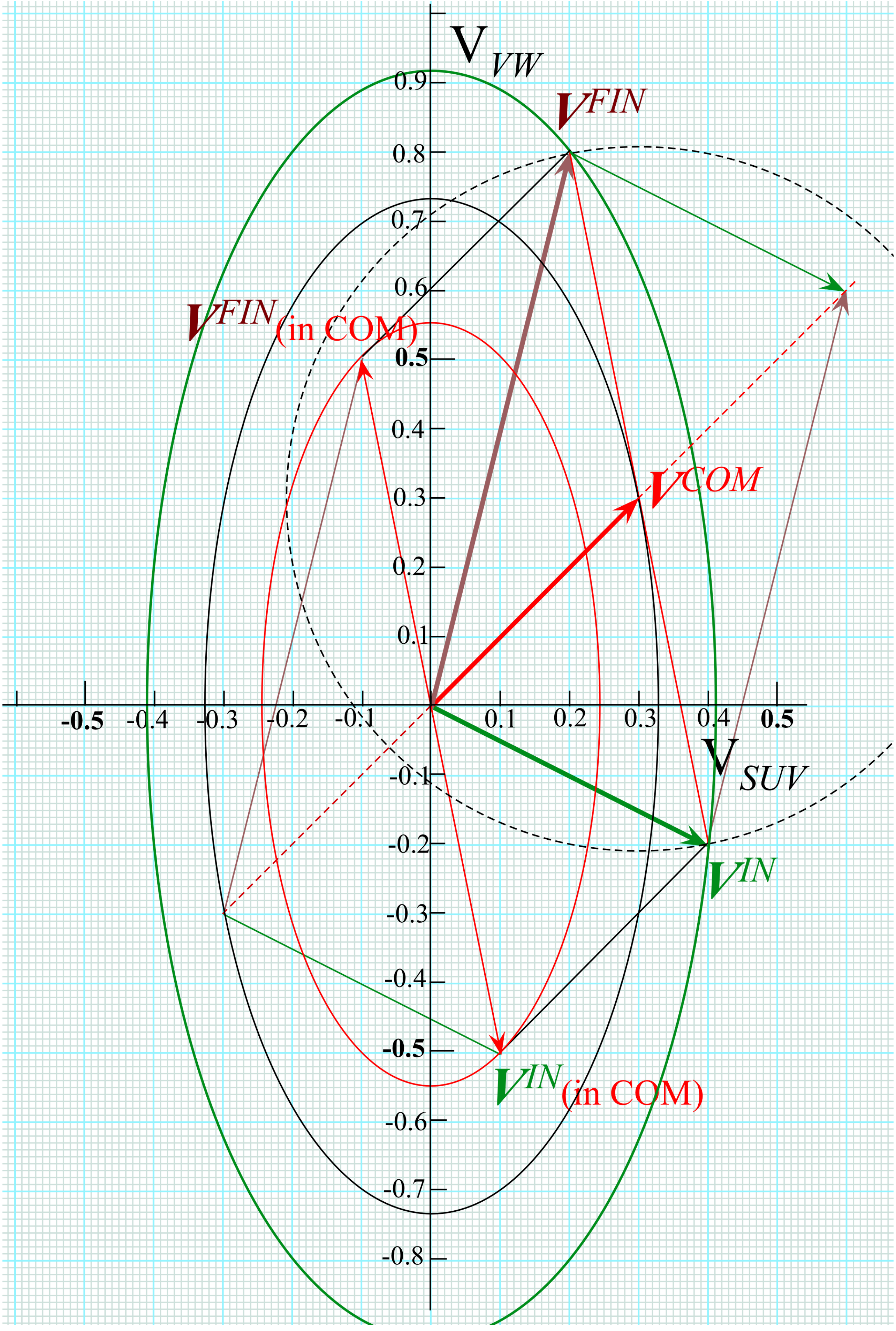
To convert equations to Center of Momentum (COM)frame we subtract $\vec{\mathbf{V}}^{COM}=\frac{1}{2}\left(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN}\right)=\begin{pmatrix}0.3\\0.3\end{pmatrix}$

$\vec{\mathbf{V}}^{IN}-\vec{\mathbf{V}}^{COM}=\frac{1}{2}\left(\vec{\mathbf{V}}^{IN}-\vec{\mathbf{V}}^{FIN}\right)=\begin{pmatrix}.4\\-.2\end{pmatrix}-\begin{pmatrix}0.3\\0.3\end{pmatrix}=\begin{pmatrix}0.1\\-.5\end{pmatrix}$ and: $\vec{\mathbf{V}}^{FIN}-\vec{\mathbf{V}}^{COM}=\frac{1}{2}\left(\vec{\mathbf{V}}^{FIN}-\vec{\mathbf{V}}^{IN}\right)=\begin{pmatrix}.2\\.8\end{pmatrix}-\begin{pmatrix}0.3\\0.3\end{pmatrix}=\begin{pmatrix}-0.1\\0.5\end{pmatrix}$

Elastic case KE$=\frac{1}{2}\begin{pmatrix}0.1, & -0.5\end{pmatrix}\begin{pmatrix}5&0\\0&1\end{pmatrix}\begin{pmatrix}0.1\\-0.5\end{pmatrix}=\frac{1}{2}(0.1\cdot5\cdot0.1+0.5\cdot1\cdot0.5)=\frac{1}{2}(0.05+0.25)=0.15$

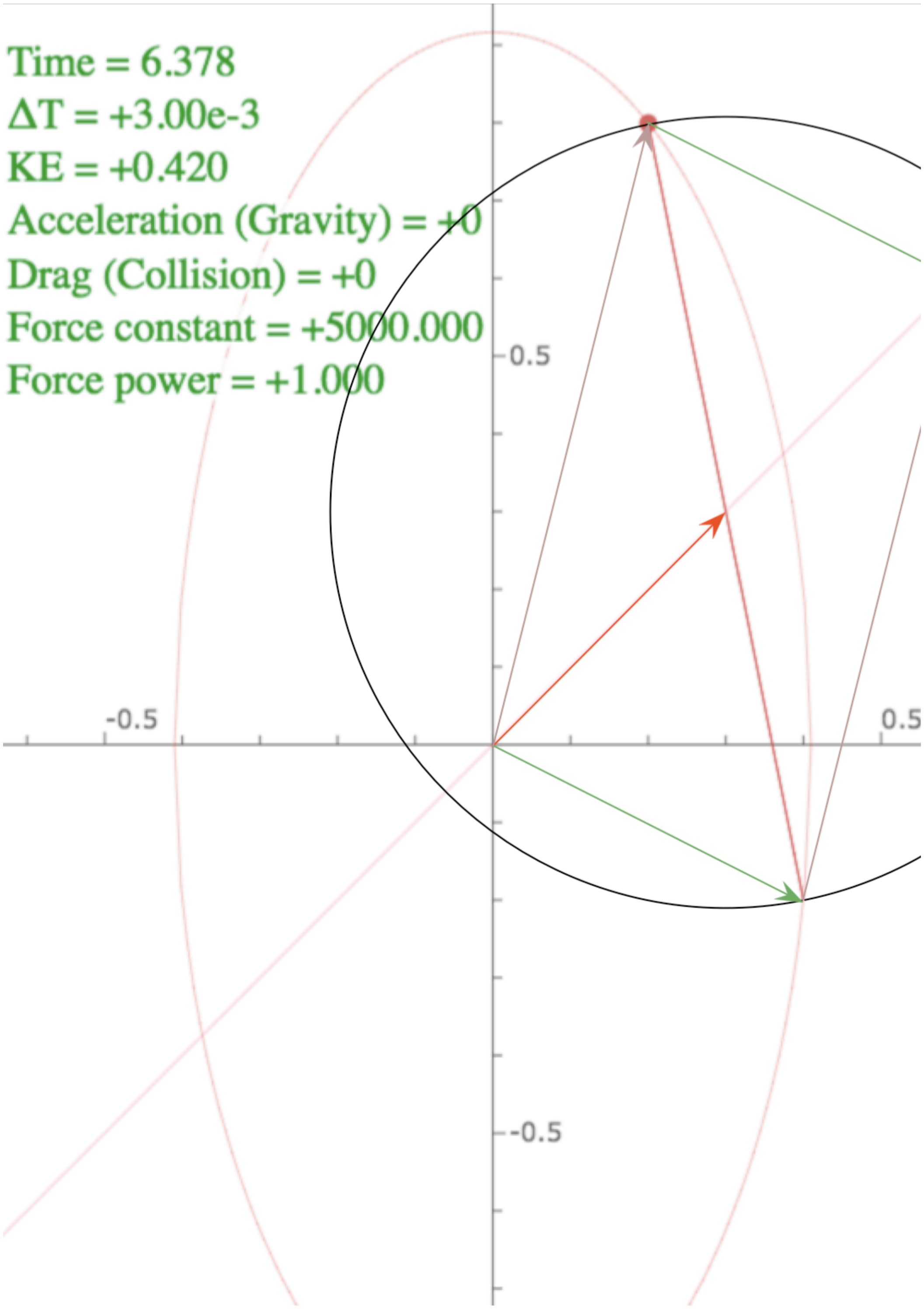
KE ellipse has x-radius: $a=\sqrt{\tfrac{2}{5}0.15}=.2449$ and y-radius: $b=\sqrt{2\cdot0.15}=.5477$

Inelastic case is all zeroes.



V VW
V SUV
V FIN
V FIN (in COM)
V COM
V IN
V IN (in COM)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.5
-0.4
-0.3
-0.2
-0.1
0.1
0.2
0.3
0.4
0.5

From *BounceIt* web app



Example with $(V^{IN}{}_1, V^{IN}{}_2)= (0.4, +0.2)$. Final velocities are closer so elastic and inelastic ellipses nearly overlap. The 45° Galilean lines are tangent to a much smaller COM ellipse.