Assignment 12 - **PHYS 5103**-11/13/17-Due Wed. Nov. 14 CMwBang! Ch 4.1 thru Ch.4.4. and Lectures 20-24

Ex.1 *The "standard" Lorentzian (Note: Review complex 2-pole potential $\phi(z)=1/z$ (10.42) in Unit 1-Ch. 10 Fig. 10.11.)*

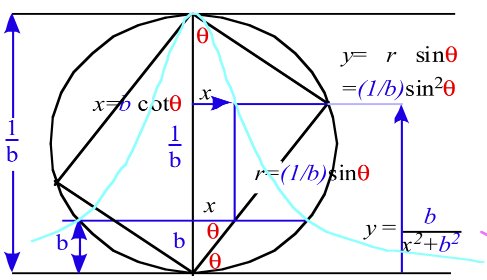
In physics literature, a standard Lorentzian function generally means a form $L(\Delta)=A/(\Delta^2+A^2)$ with constant A. In the *Near-Resonant Approximation* (NRA is (4.2.18)) the $L(\Delta)$ or its derivative is an approximation to exact G-equations (4.2.15).

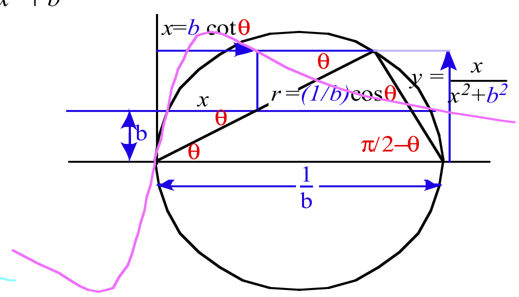
(a) Use NRA (4.2.18) to reduce (4.2.15a-d) and identify a standard Lorentzian function of the detuning parameter $\Delta=\omega_s-\omega_0$.

(b) Show that NRA for complex response $G=\text{Re}\,G+i\text{Im}\,G$ gives circular arcs in the complex $\omega=|\omega|e^{i\theta}=\Delta+i\Gamma$ plane for constant decay rate Γ and variable detuning or beat rate Δ. How does this circle deviate from what is almost a circle in Fig. 4.2.6? (Consider higher Γ values for which NRA breaks down such as Fig. 4.2.14.) What curve does fixed Δ with varying Γ give? Relate to dipole scalar-Φ and vector-A potential field lines for dipole field function $f(z)=1/z^2$ discussed on Ch. 10 of Unit 1.

(c) Do ruler-&-compass construction of NRA Lorentz functions following figures below for $b=1/2$ and for $b=1/3$.

$\text{Re}\,G_{\omega_0}(\omega_s)=y=\dfrac{b}{x^2+b^2}$, $\text{Im}\,G_{\omega_0}(\omega_s)=y=\dfrac{x}{x^2+b^2}$, and $|G_{\omega_0}(\omega_s)|$.(See p. 58-66 of Lect. 20.)





Ex.2 *Max and min G-values (Part (b-c) involves some derivative algebra!)*

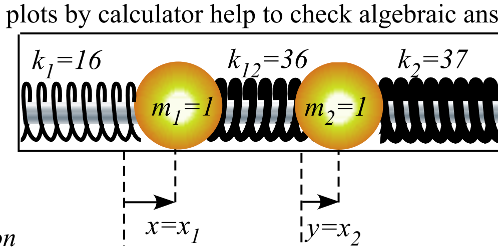
Derive equations for the extreme values for the Lorentz-Green response function or function related to G as asked below.

For part (a) *only* use *Near-Resonant Approximation* (*NRA*): See preceding **Ex.1**.

(a₁) Find values which give maxima for: $\text{Re}\,G_{\omega_0}(\omega_s)$, $\text{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming ω_0 is constant and ω_s varies.

(a₂) Find values which give maxima for: $\text{Re}\,G_{\omega_0}(\omega_s)$, $\text{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming ω_s is constant and ω_0 varies.

(b) Do **(a)** for *exact* $G_{\omega_0}(\omega_s)$. Exact plots by calculator help to check algebraic answers.



Ex.3 *Coupled oscillation by projection*

Two identical mass $M=1kg$ blocks slide friction-free on a rod and are connected by springs $k_1=16N\cdot m^{-1}$ and $k_2=37N\cdot m^{-1}$ to ends of a box and coupled to each other by spring $k_{12}=36N\cdot m^{-1}$.

(a) Write Lagrangian equations of motion and derive a **K**-matrix form of them.

(b) Solve for eigenmodes and eigenfrequencies of system and plot their directions on an X,Y-graph. Use spectral decomposition methods (Lect. 20 or Appendix 4.C) to derive eigensolution projectors and eigenvectors.

(c) Given initial conditions ($X(0)=1, Y(0)=0$), plot the resulting path in the XY-plane. Show it is a parabola.(*Tschebycheff* function)

(d) Use spectral decomposition (Lect. 20 or Appendix 4.C) to derive square-roots $\mathbf{H}=\sqrt{\mathbf{K}}$. (How many square-roots does **K** have?)

Ex.4 *U(2) view of AB-coupled oscillation*

(a) Rewrite the spring **K**-matrix for **Ex.3** into an **H**-matrix where $\mathbf{K}=\mathbf{H}^2$ as in (4.4.8).

(b) Give the resulting **H**-matrix as an *(A,B,C,D)* combination of **1**, σ_A, σ_B, and σ_C as in (4.4.9). (++ root of **K** results for **H**.)

(c) Sketch the resulting Ω-whirl vector or "crank" in real 3D *(A,B,C)*–space as in (4.4.10).

(d) For ($X(0)=1, Y(0)=0$) find initial **S**-state ("spin")vector in *(A,B,C)*–space as in (4.4.16). Show its evolution by Ω as in Fig. 4.4.2.

(e) Plot **H**-eigenvalues ($\varepsilon_1,\varepsilon_2$) as energy levels and indicate transition rate $\Omega=\varepsilon_1-\varepsilon_2$ spinor rate $\omega=(\varepsilon_1-\varepsilon_2)/2$ and phase rate $\Omega_0=(\varepsilon_1+\varepsilon_2)/2$.

Ex.5 *U(2) view of Bilaterally symmetric coupled oscillation*

Redo **Ex.3** and **Ex.4** for a *B*-symmetric version of the system having $k_1=4N\cdot m^{-1}=k_2$ and $k_{12}=30N\cdot m^{-1}$.

(**Ex.3(c)** should give a different *Tschebycheff* function)