

Lecture 5  
Tue. 9.05.2017

## *Dynamics of Potentials and Force Fields*

*(Ch. 7 and part of Ch. 8 of Unit 1)*

### *Potential energy dynamics of Superballs and related things*

*Thales geometry and “Sagittal approximation” to superball force law*

*Geometry and dynamics of single ball bounce*

*(a) Constant force  $F=-k$  (linear potential  $V=kx$ )*

*Some physics of dare-devil diving 80 ft. into kidee pool*

*(b) Linear force  $F=-kx$  (quadratic potential  $V=1/2kx^2$  (like balloon))*

*(c) Non-linear force (like superball-floor or ball-bearing-anvil)*

### *Geometry and potential dynamics of 2-ball bounce*

*A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)*

*A story of USC pre-meds visiting Whammo Manufacturing Co.*

### *Geometry and dynamics of n-ball bounces*

*Analogy with shockwave and acoustical horn amplifier*

*Advantages of a geometric  $m_1, m_2, m_3, \dots$  series*

*A story of Stirling Colgate (Palmolive) and core-collapse supernovae*

### *Many-body 1D collisions*

*Elastic examples: Western buckboard*

*Bouncing columns and Newton's cradle*

*Inelastic examples: “Zig-zag geometry” of freeway crashes*

*Super-elastic examples: This really is “Rocket-Science”*

# *Potential energy dynamics of Superballs and related things*

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## *Many-body 1D collisions*

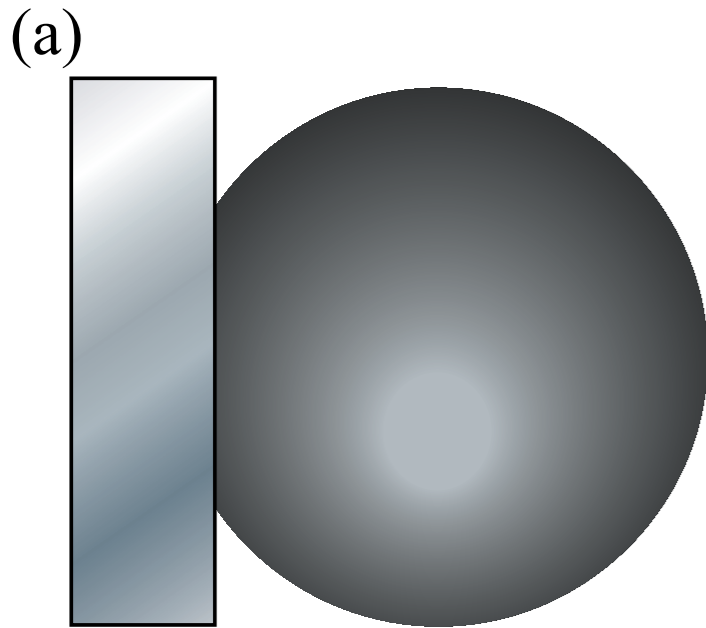
*Elastic examples: Western buckboard*

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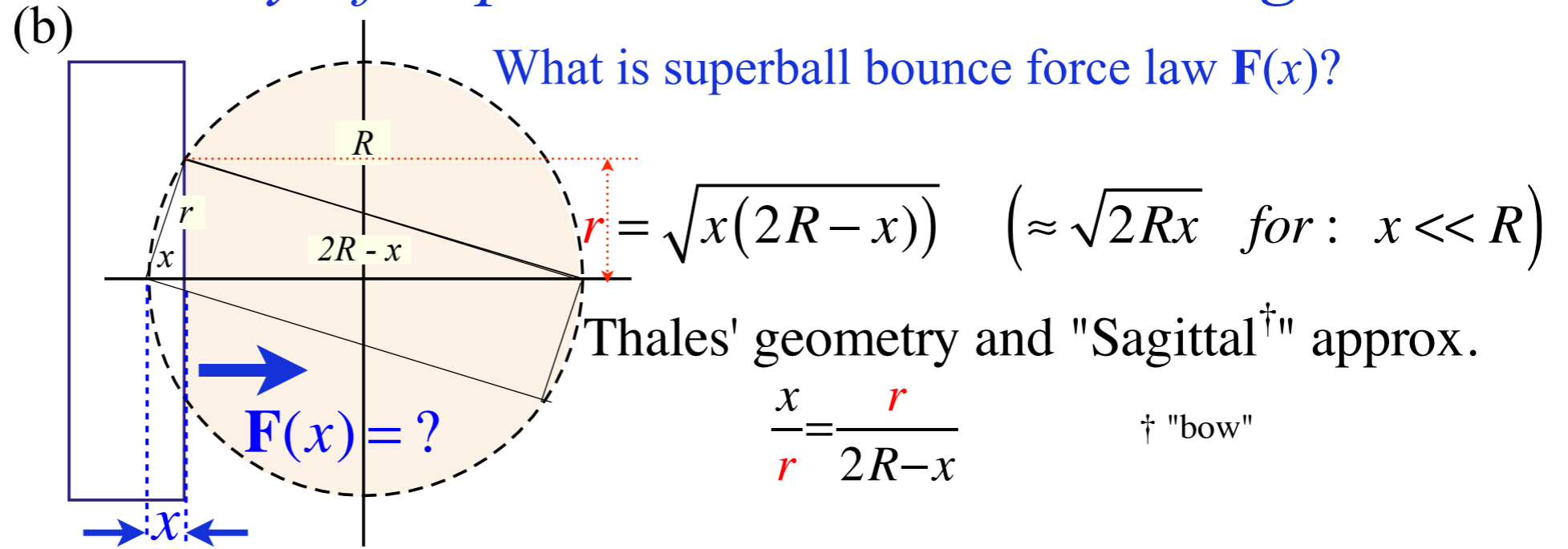
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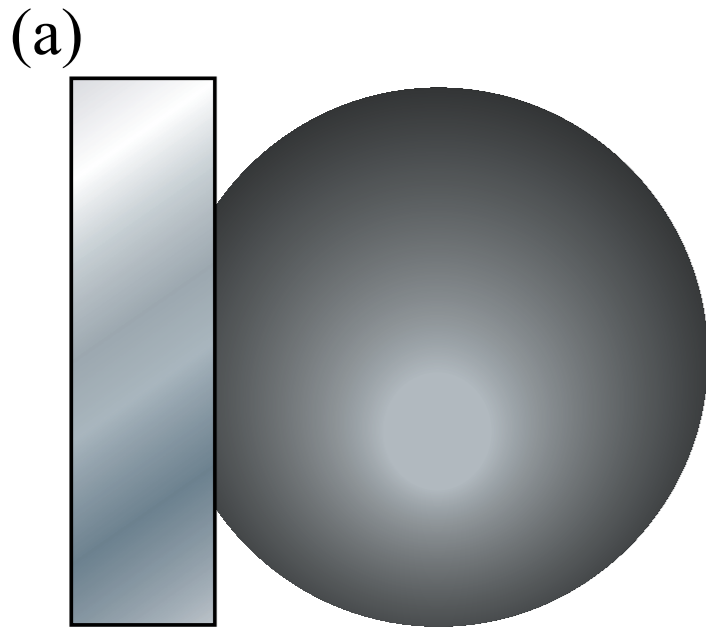
# Potential Energy Geometry of Superballs and Related things



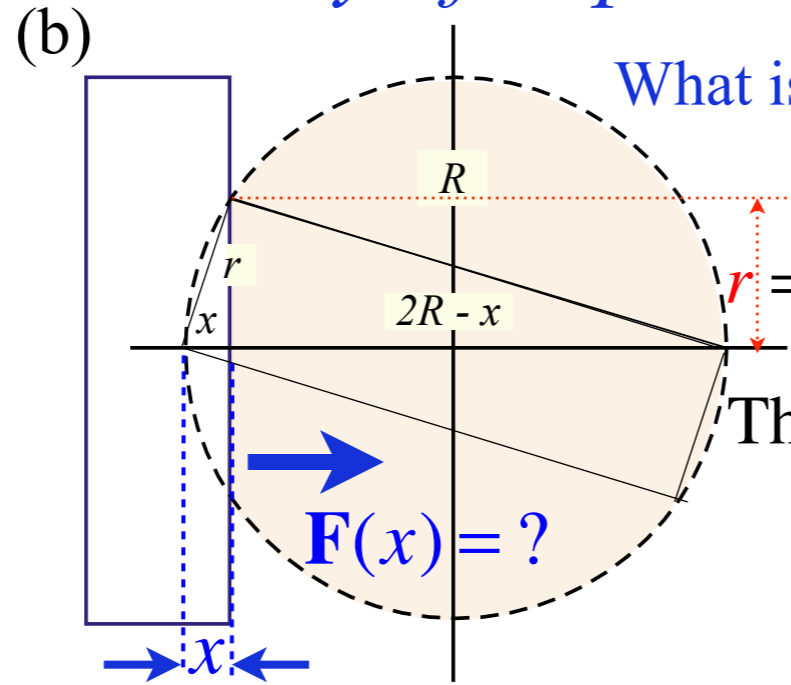
Unit 1  
Fig. 7.1  
(modified)



# Potential Energy Geometry of Superballs and Related things



Unit 1  
Fig. 7.1  
(modified)



What is superball bounce force law  $F(x)$ ?

$$r = \sqrt{x(2R - x)} \quad (\approx \sqrt{2Rx} \text{ for } x \ll R)$$

Thales' geometry and "Sagittal<sup>†</sup>" approx.

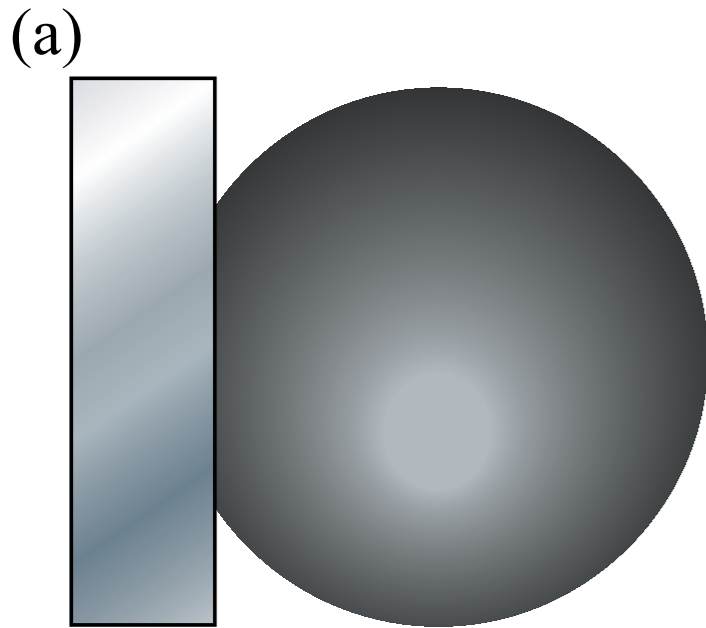
$$\frac{x}{r} = \frac{r}{2R - x}$$

† "bow"

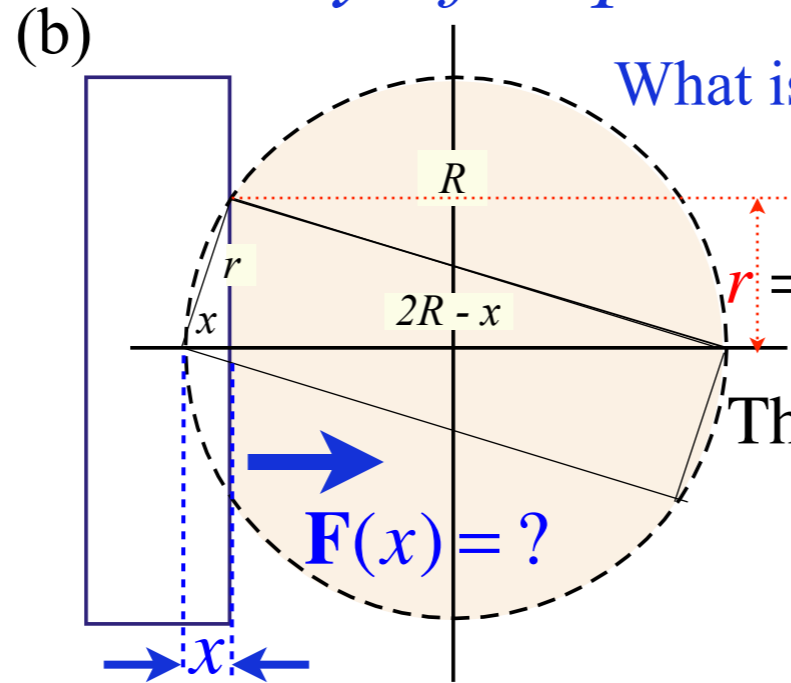
If superball was a balloon its bounce force law would be linear  $F = -k \cdot x$  (Hooke Law)

$$F_{\text{balloon}}(x) = P \cdot A = P \cdot \pi r^2 \approx P \cdot \pi 2Rx$$

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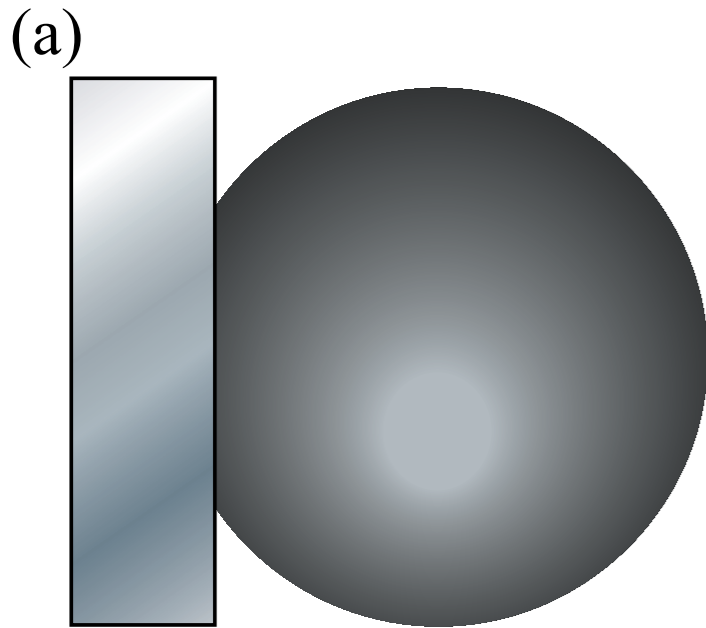
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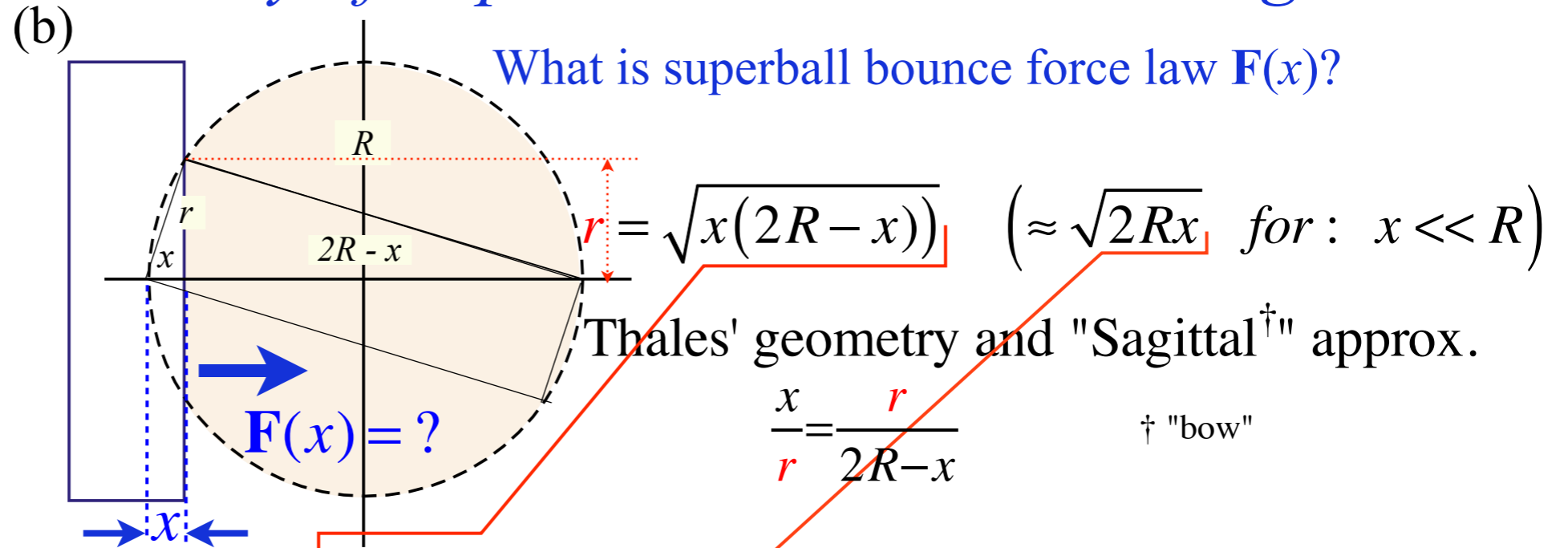
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$$\begin{aligned}
 F_{\text{balloon}}(x) &= P \cdot A = P \cdot \pi r^2 \\
 &\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx \quad (\text{Hooke spring constant } k) \\
 &= kx
 \end{aligned}$$

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Fig. 7.1  
(modified)



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(Pressure) · (Area)

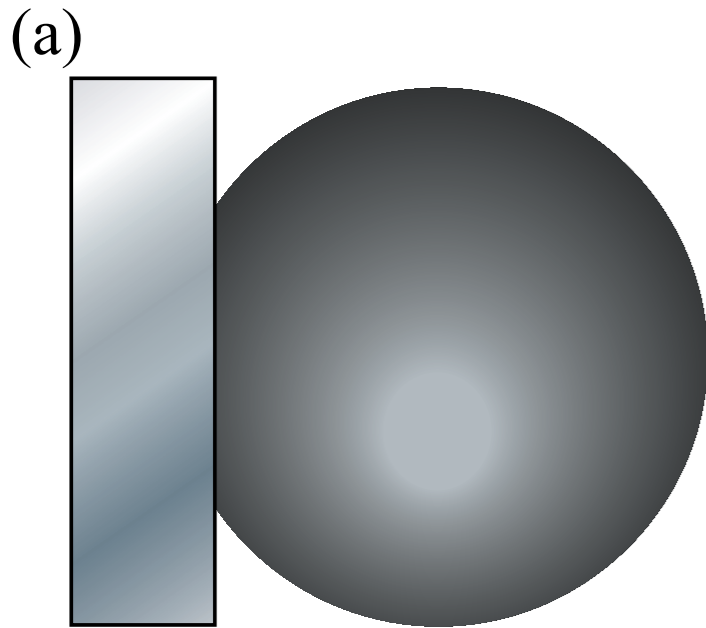
$$\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx \text{ (Hooke spring constant } k)$$

$$= kx$$

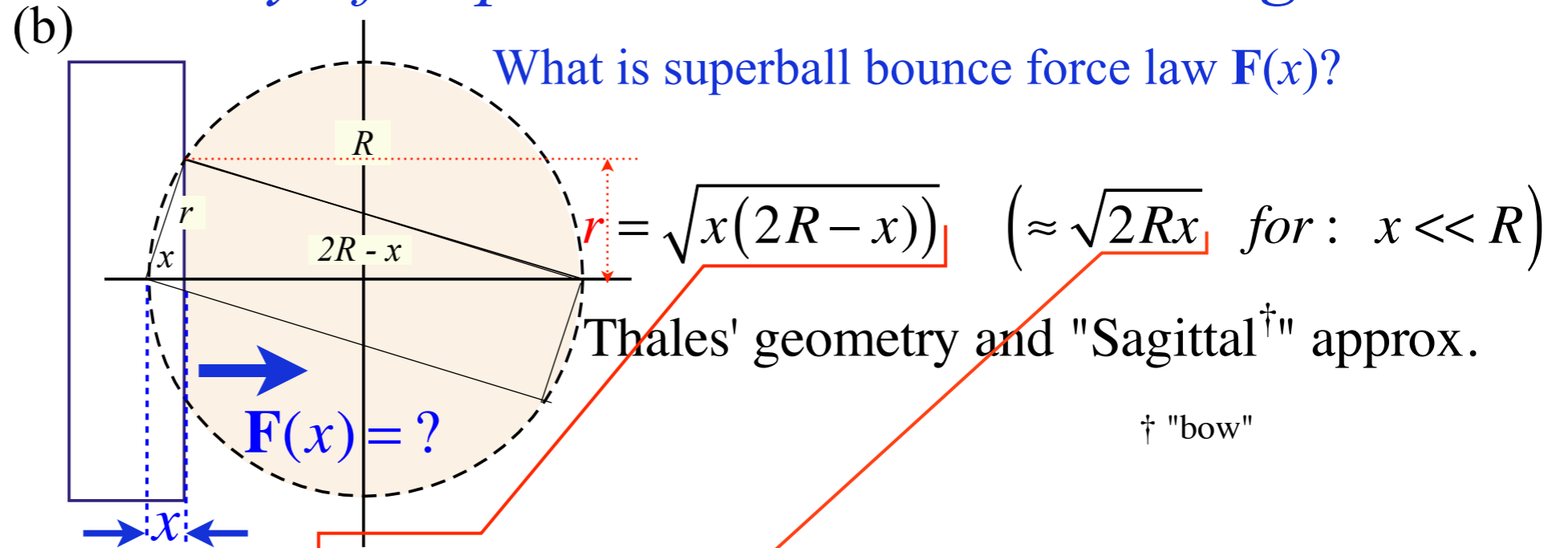
Instead superball force law depends on bulk *volume* modulus and is non-linear  $F \sim x^p + ?$  (Power Law?)

$$\text{Volume}(X) = \int_0^X \pi r^2 dx = \int_0^X \pi x(2R - x) dx$$

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(modified)



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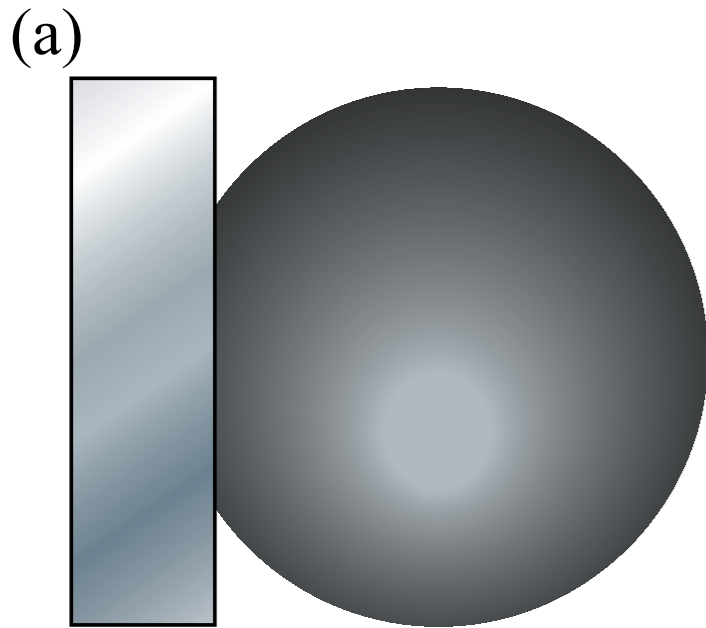
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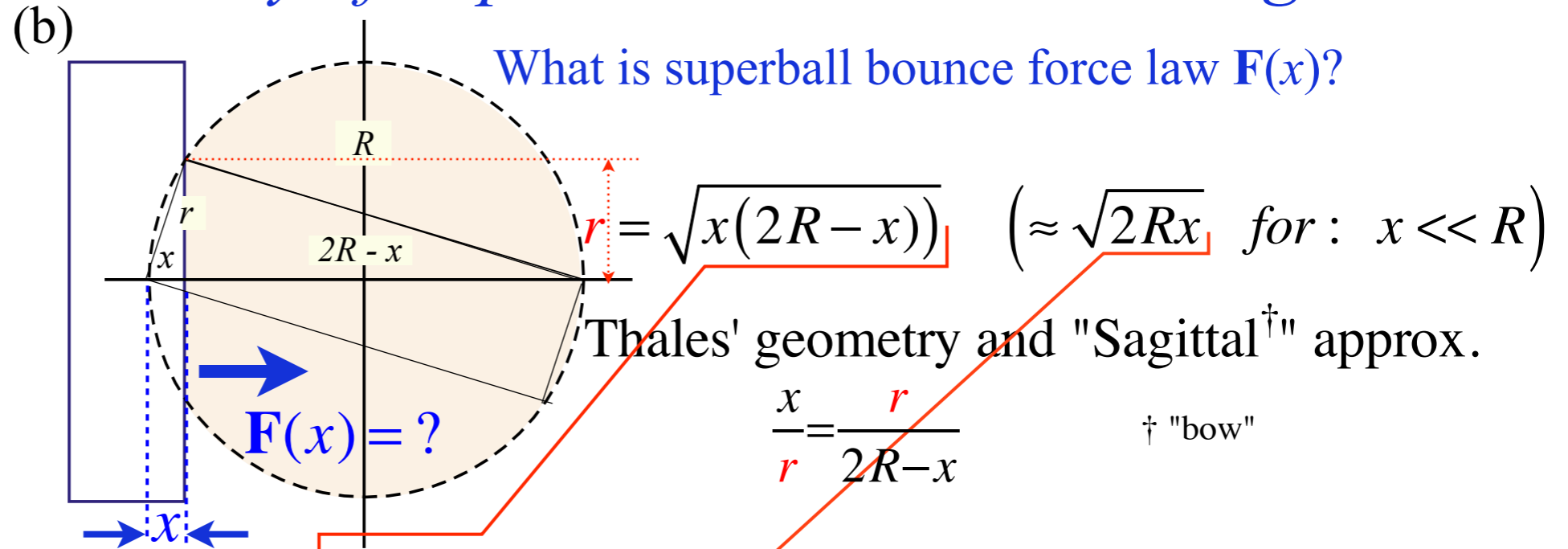
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$$Volume(X) = \int_0^X \pi r^2 dx = \int_0^X \pi x(2R - x) dx = \int_0^X 2R\pi x dx - \int_0^X \pi x^2 dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (\text{for } : X \ll R) \\ \frac{4}{3}\pi R^3 & (\text{for } : X = 2R) \end{cases}$$

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If superball was a balloon its bounce force law would be linear  $F = -k \cdot x$  (Hooke Law)

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It also depends on velocity  $\dot{x} = \frac{dx}{dt}$ . *Adiabatic* differs from *Isothermal* as shown by "Project-Ball\*"

\* [Am. J. Phys. 39, 656 \(1971\)](#)

(Discussed after p. 33)



# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

→ Geometry and dynamics of single ball bounce

→ General Non-linear force (like superball-floor or ball-bearing-anvil)

(Simulations)

Constant force  $F=-k$  (linear potential  $V=kx$ )

Some physics of dare-devil-diving 80 ft. into kidee pool

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Elastic examples: Western buckboard

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Super-elastic examples: This really is “Rocket-Science”

# Main Control Panel

Start Resume

This is the generic Bouncelt URL (or address):  
<http://www.uark.edu/ua/modphys/markup/BounceltWeb.html>

*Bouncelt Simulation: Force/Potential Plot  
 (Force power=4)*

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses

1 Balls

Acceleration of gravity

1.51 100x{cm/s^2}

*Sets gravity*

- Draw force vectors
- Pause (once) at top
- Constrain motion to Y-axis
- Plot v2 vs v1
- Plot p2 vs p1
- Plot V2 vs V1
- Plot Ellipses
- Plot Bisector Lines
- Old Color Scheme
- Show right panel information
- Show left panel information
- Set Initial positions

Collision friction (Viscosity)

0 x10^  0 {g}

Initial gap between balls

5.5 x10^  -1 {g}

Force Constant

*Usually need to decrease k for p = 1*

5 x10^  4 {g}

Force power law exponent

4

*This is non-linear F=-kx^4  
 (Set p = 1 for linear F=-kx^1)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

0.75

Initial x1 =  0.5 y Max =  7

Max x PE plot =  0.5 y Min =  0

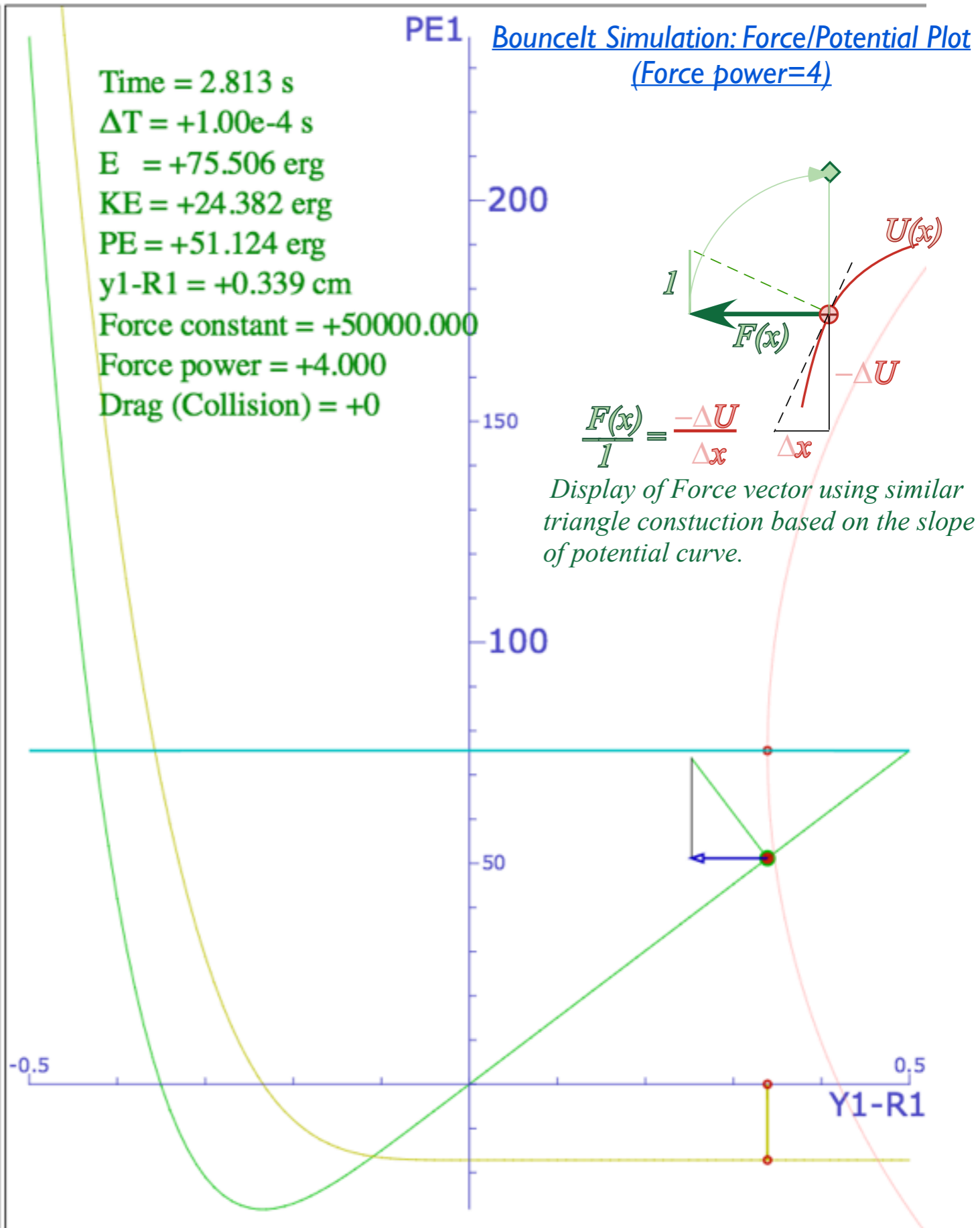
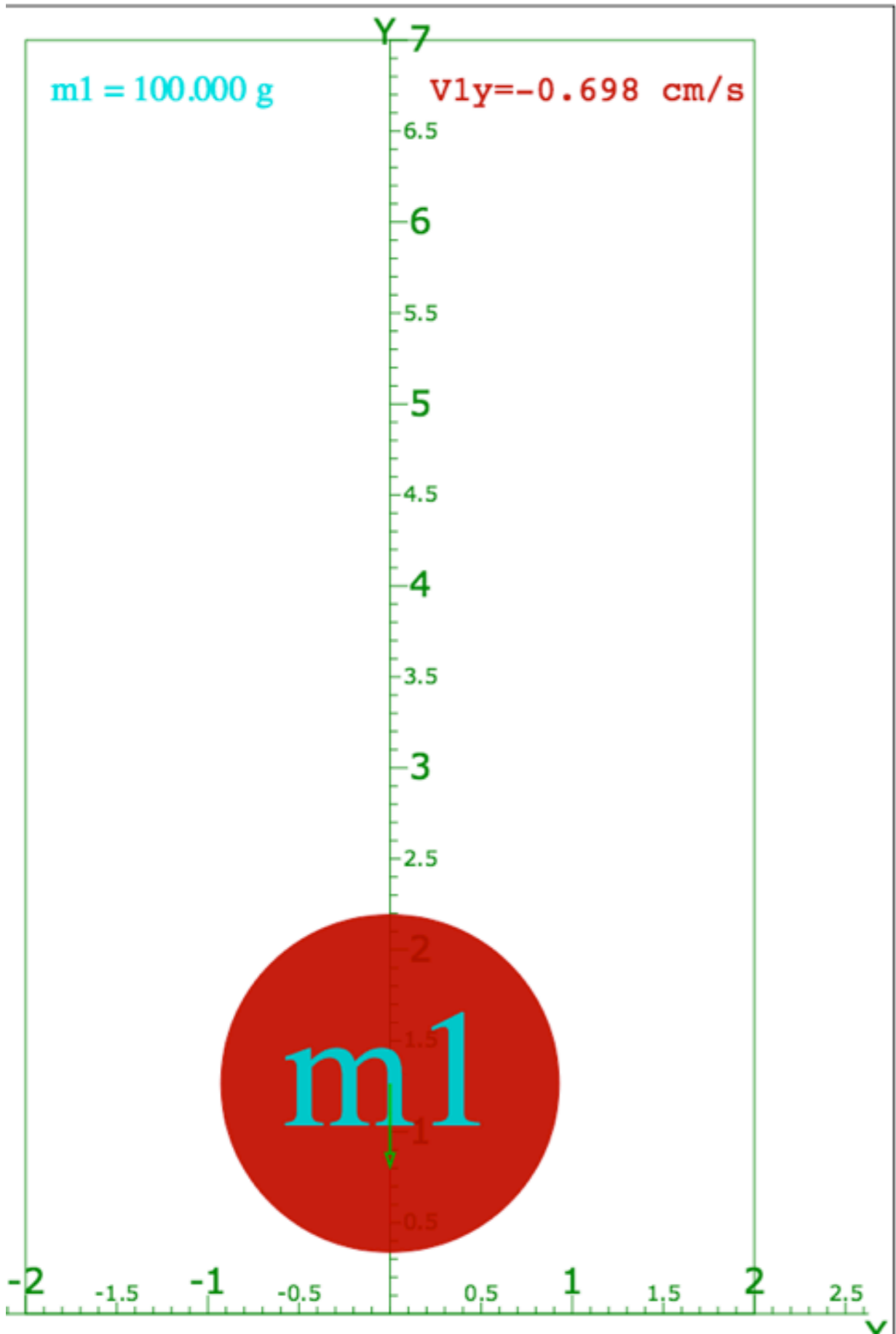
F-Vector scale =  0.003 V2y Max =  3.1112

Error step =  0.0000 V2y Min =  -3.111

Mass factor =  1

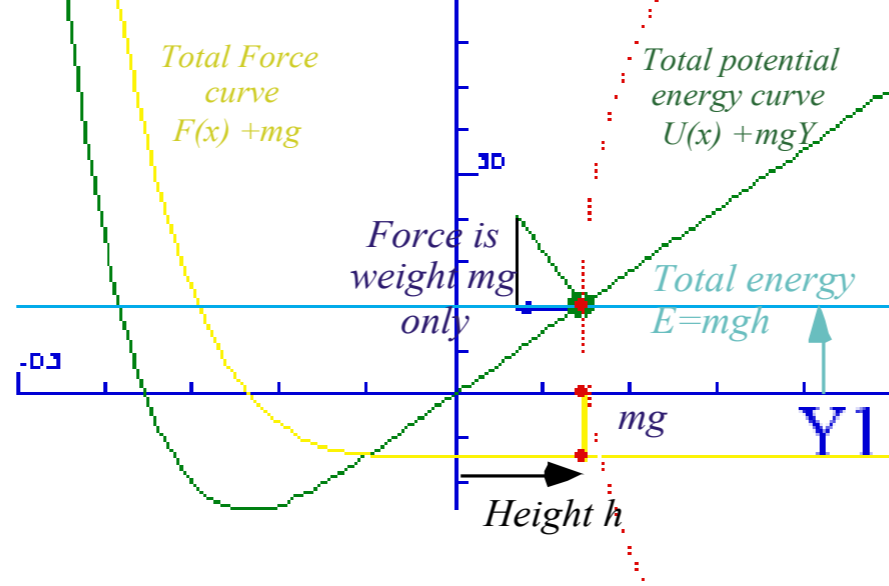
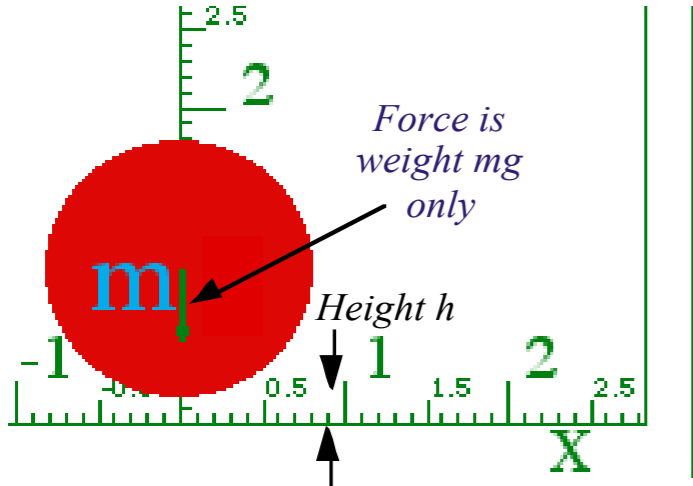
M<sub>1</sub> =  1 x10^  2 {g}

V<sub>1</sub> =  -1.1 x10^  -2 {cm/s}



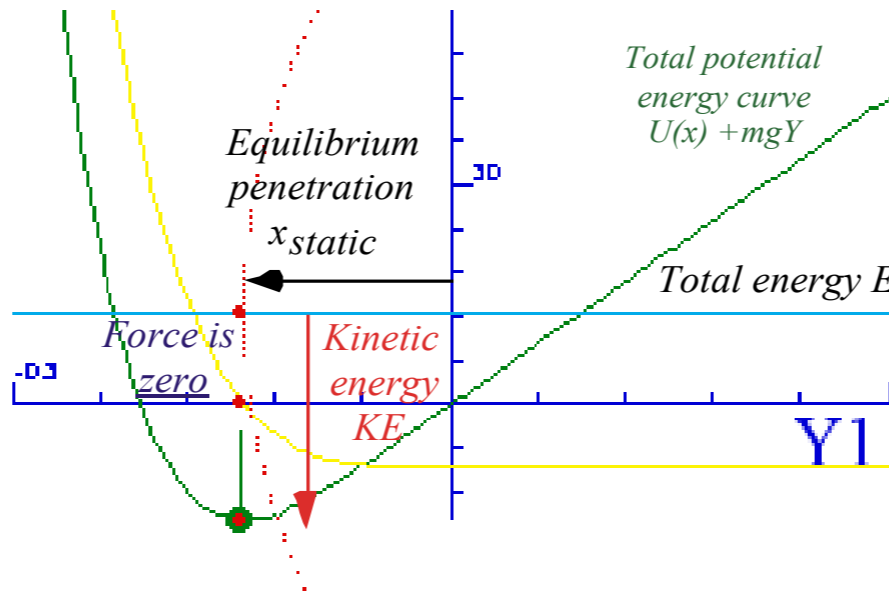
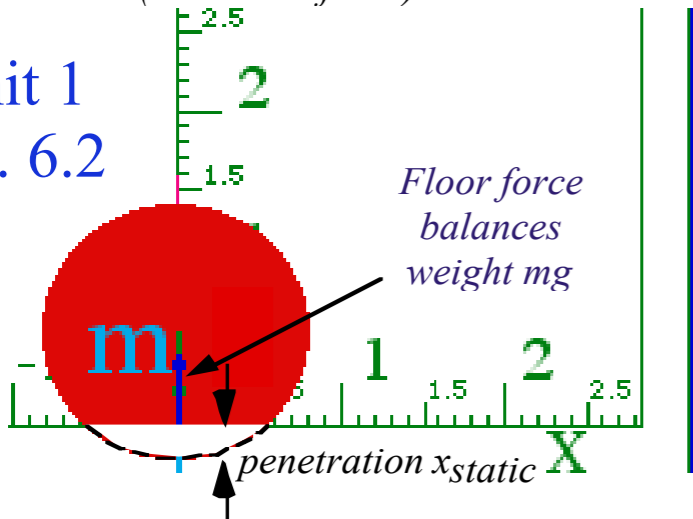
Details of each case follows using newer [BounceIt Web simulations](#)

(a) Drop height  
(Zero kinetic energy)

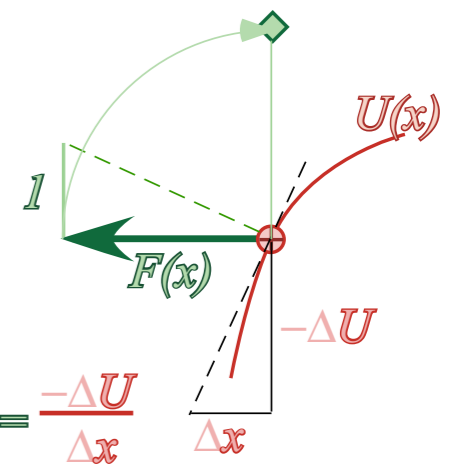
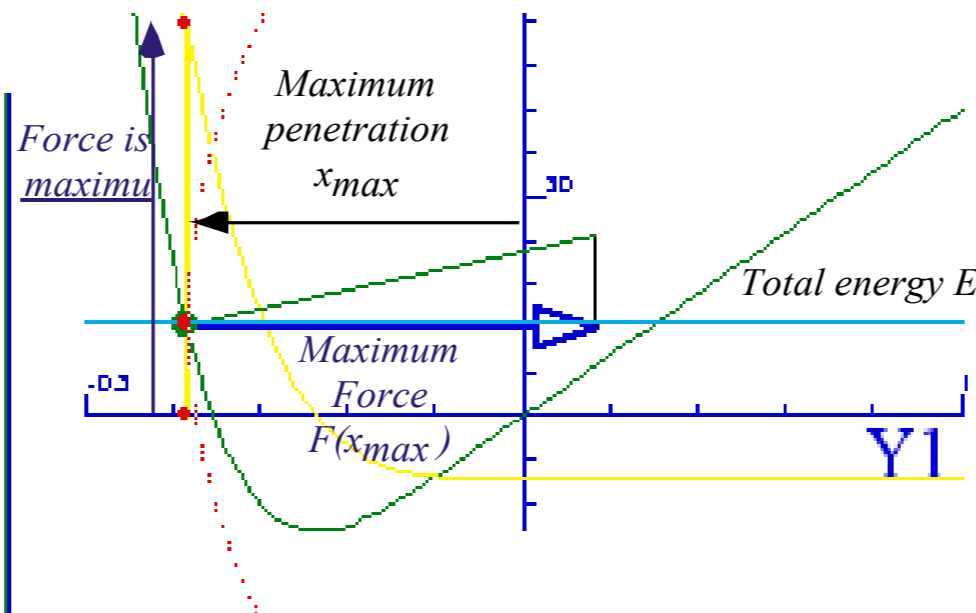
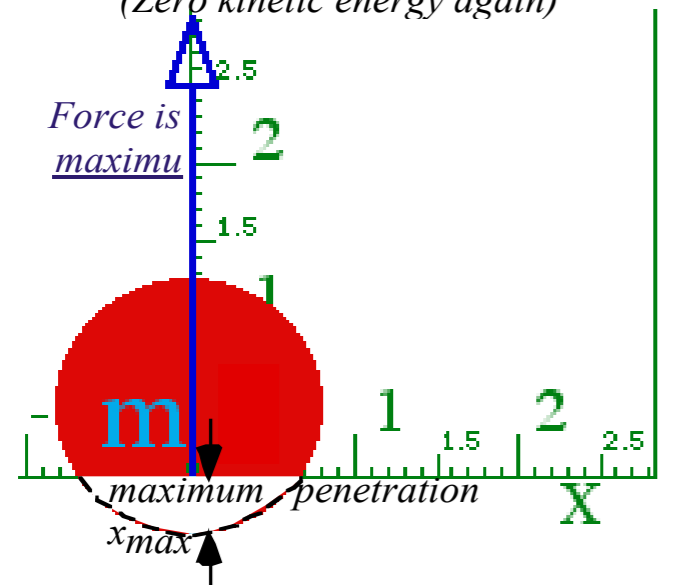


(b) Maximum kinetic energy  
(Zero total force)

Unit 1  
Fig. 6.2



(c) Maximum penetration  
(Zero kinetic energy again)



$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

Display of Force vector using similar triangle construction based on the slope of potential curve.

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*Sets gravity* →

Number of masses  Balls

Acceleration of gravity  100x{cm/s^2}

Draw force vectors

Pause (once) at top

Constrain motion to Y-axis

Plot v2 vs v1

Plot p2 vs p1

Plot V2 vs V1

Plot Ellipses

Plot Bisector Lines

Old Color Scheme

Collision friction (Viscosity)  x10^  {g}

Initial gap between balls  x10^  {g}

Force power law exponent  ← *This is linear setting (increase for non-linear)*

Force Constant

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

Initial x1 =  y Max =

Max x PE plot =  y Min =

F-Vector scale =  T Max =

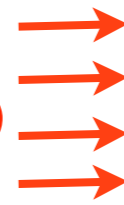
Error step =  V2y Max =

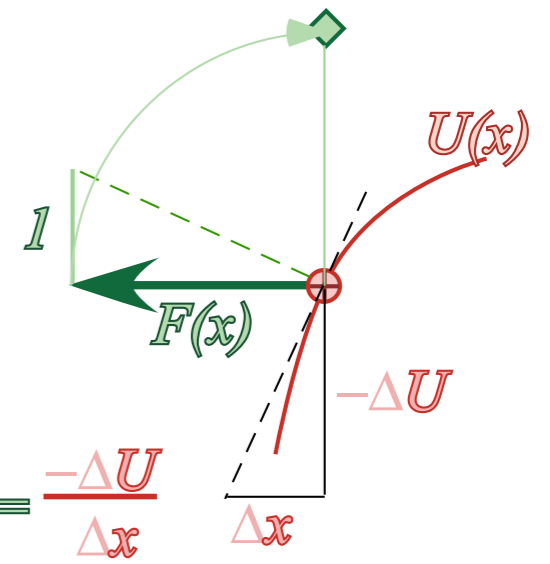
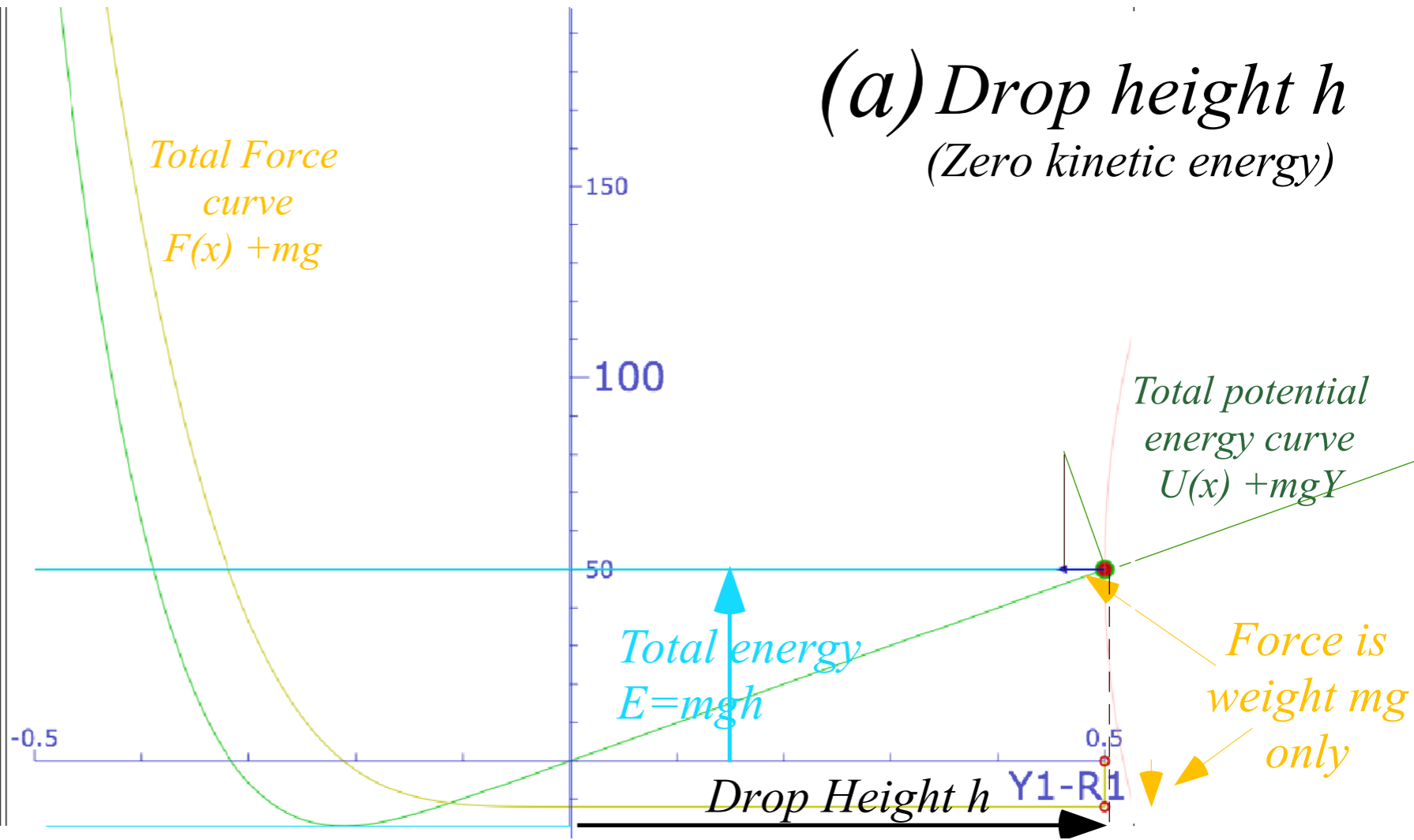
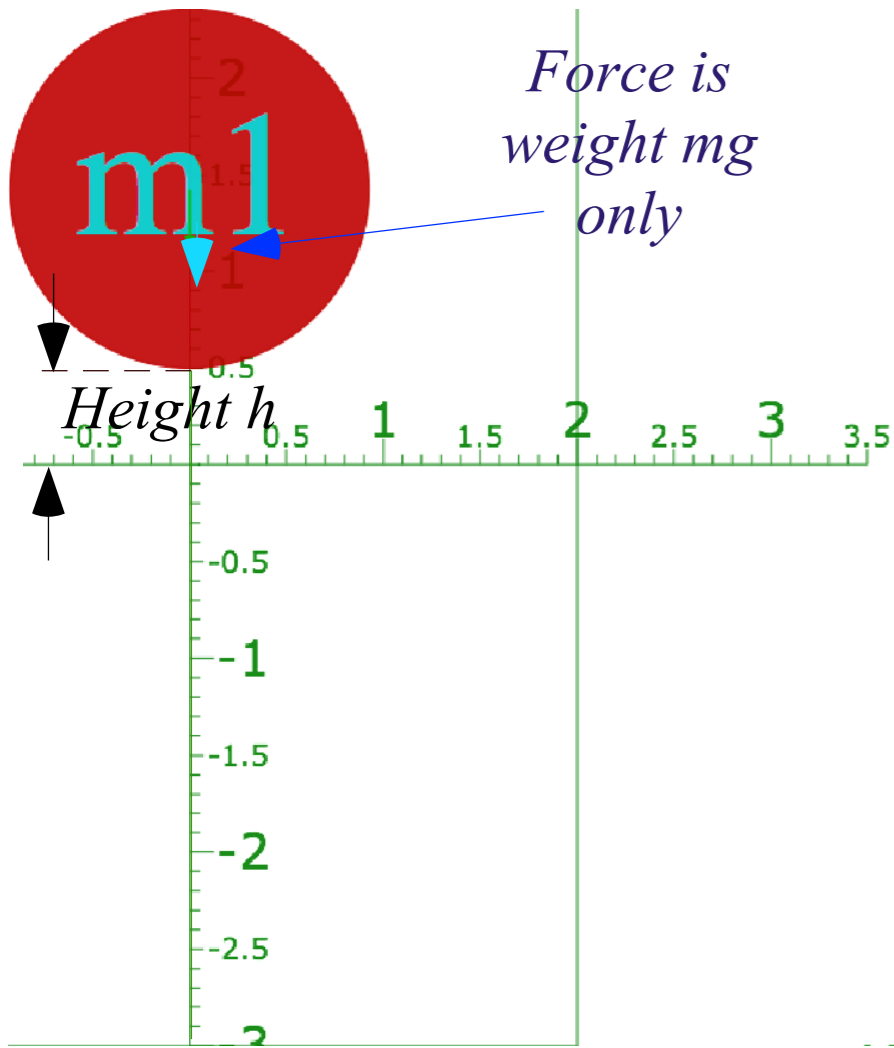
V2y Min =

m1 =  x10^  {g} V1<sub>0</sub> =  x10^  {cm/s}

- Zero Gap 2-Ball Collision (m1:m2 = 1:7)
- Linear 2-Ball Collision (m1:m2 = 1:7)
- Newton's Balls (Zero gap, Nonlinear force)
- Newton's Balls (Zero gap, Linear force)
- 3-Ball Tower
- 5-Ball Tower
- Potential Plot (1 Ball, Nonlinear force)
- Potential Plot (1 Ball, Linear force)
- Gravity Potential (1 Ball, Nonlinear force)
- Gravity Potential (1 Ball, Linear force)

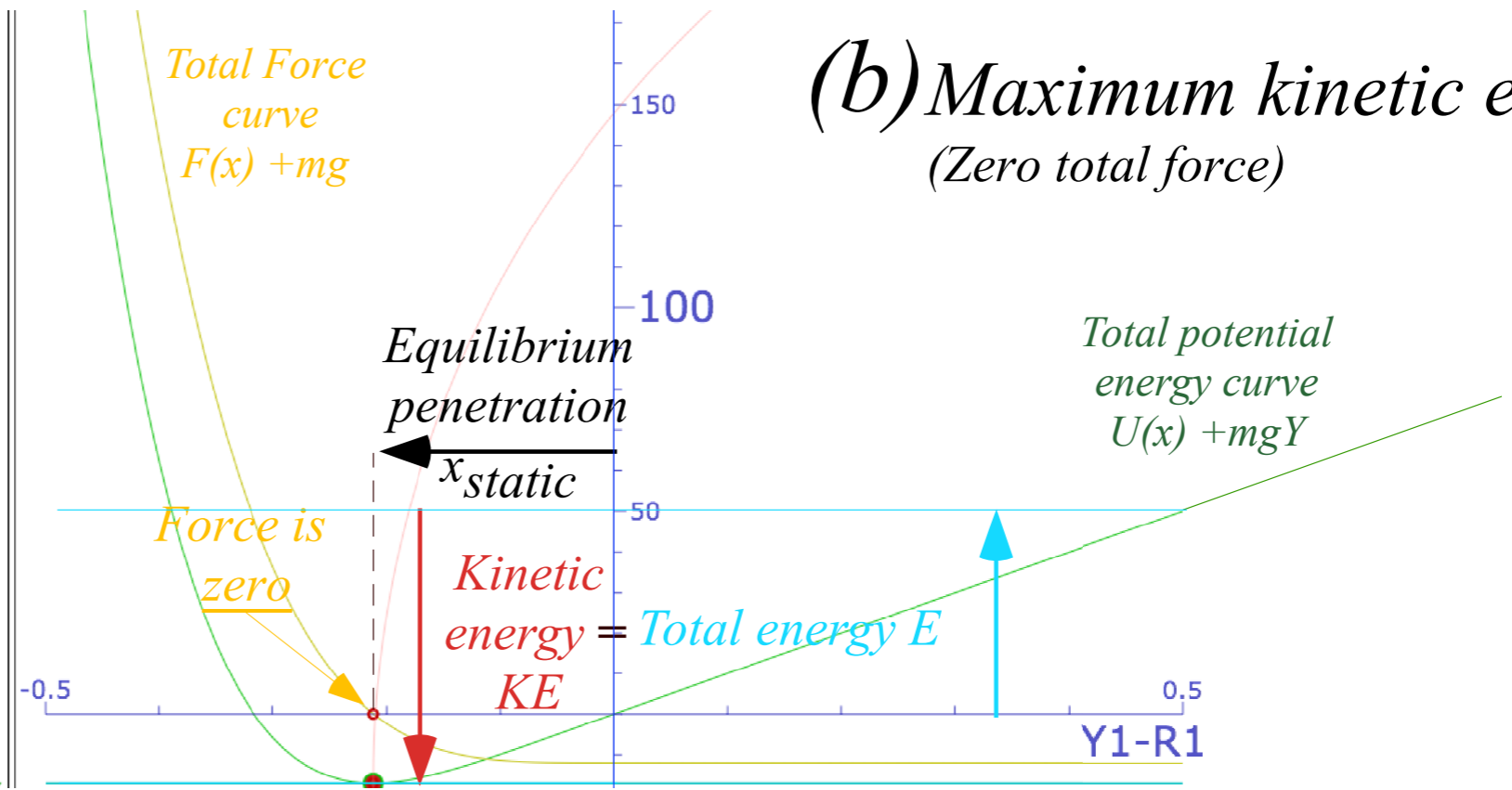
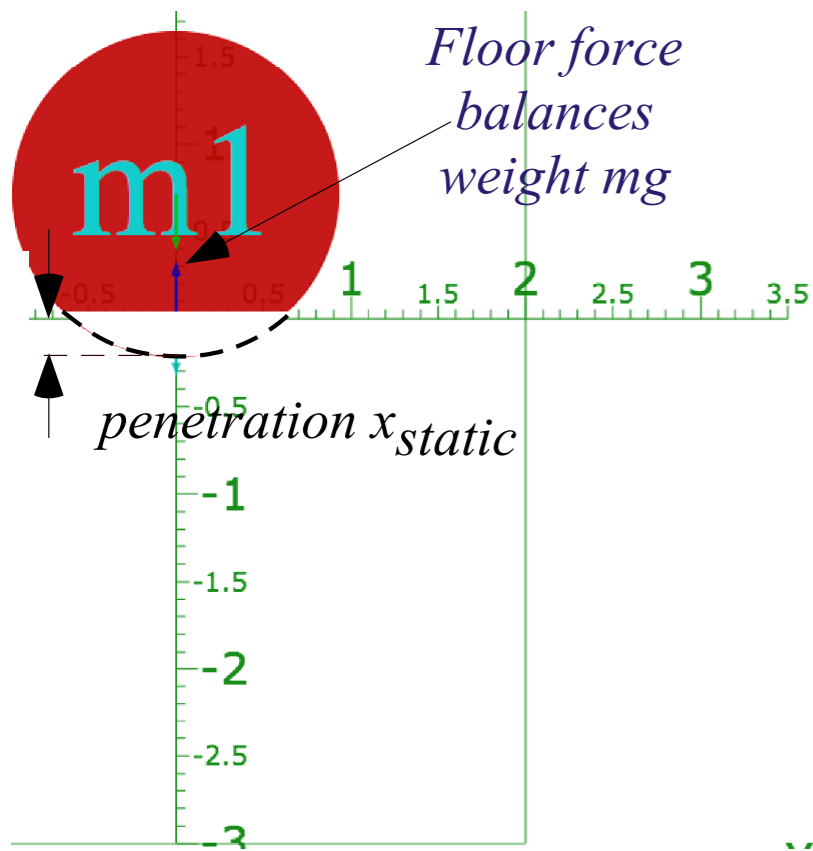
*(See Simulations)*



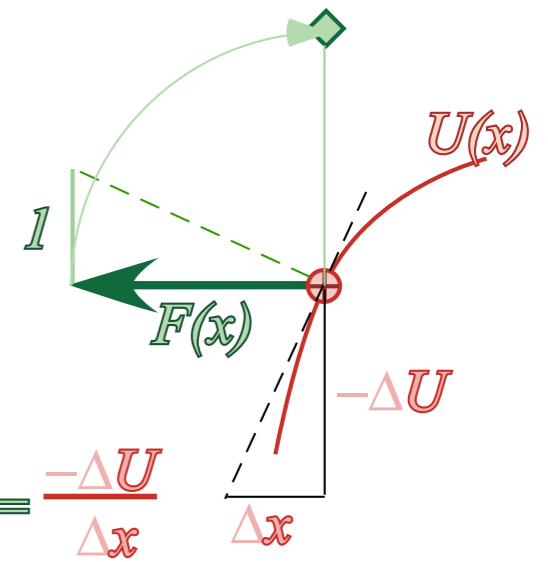


$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

Display of Force vector using similar triangle construction based on the slope of potential curve.



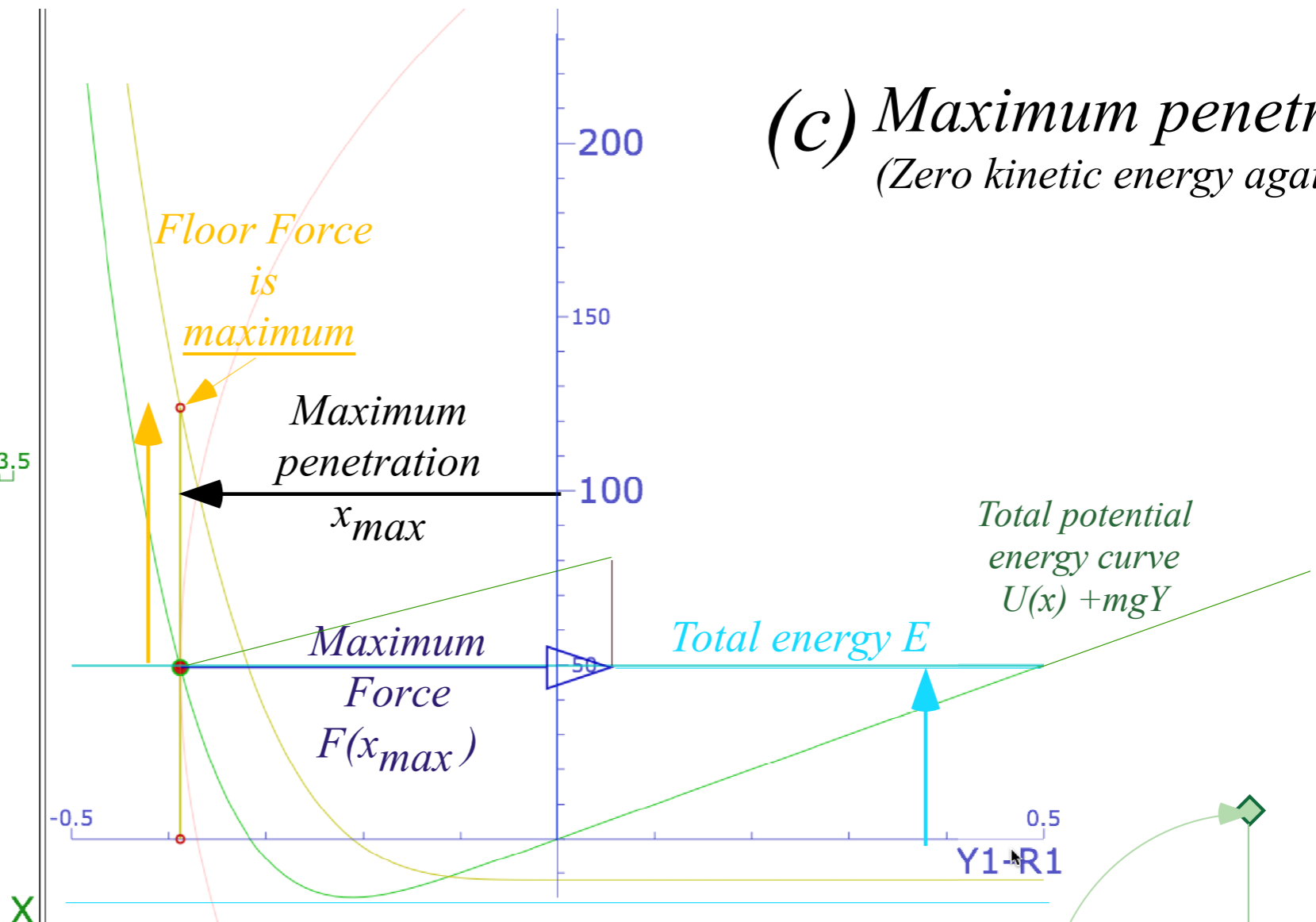
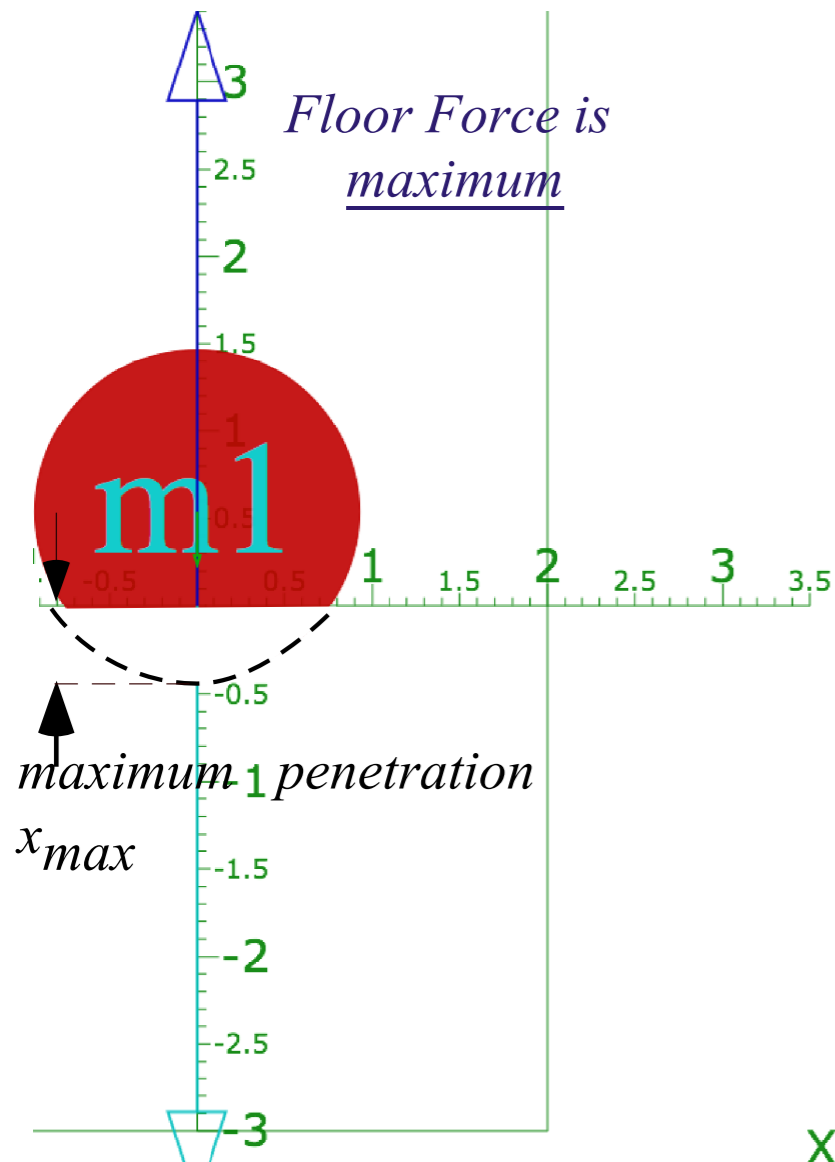
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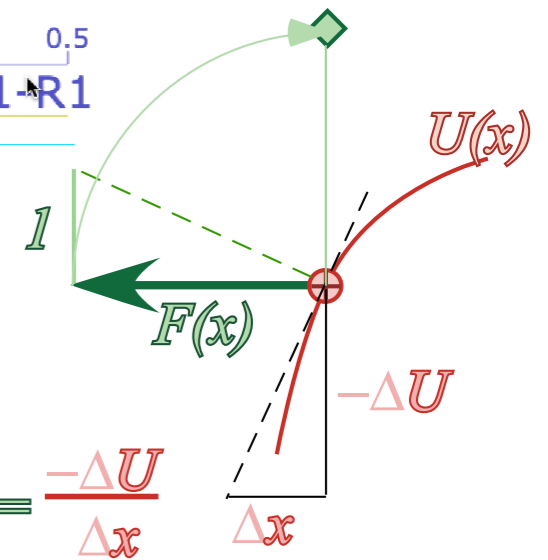
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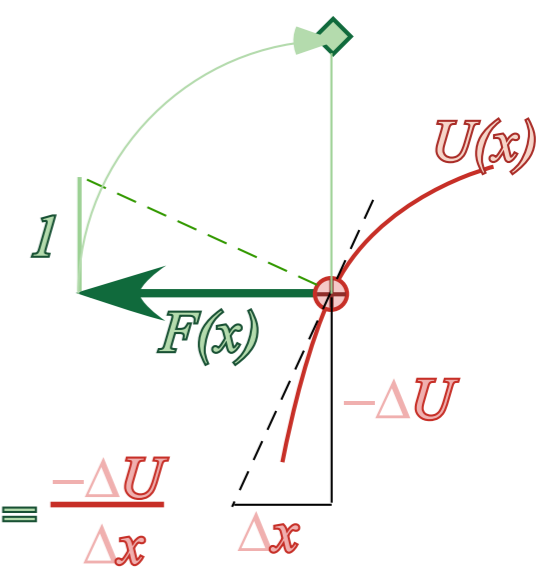
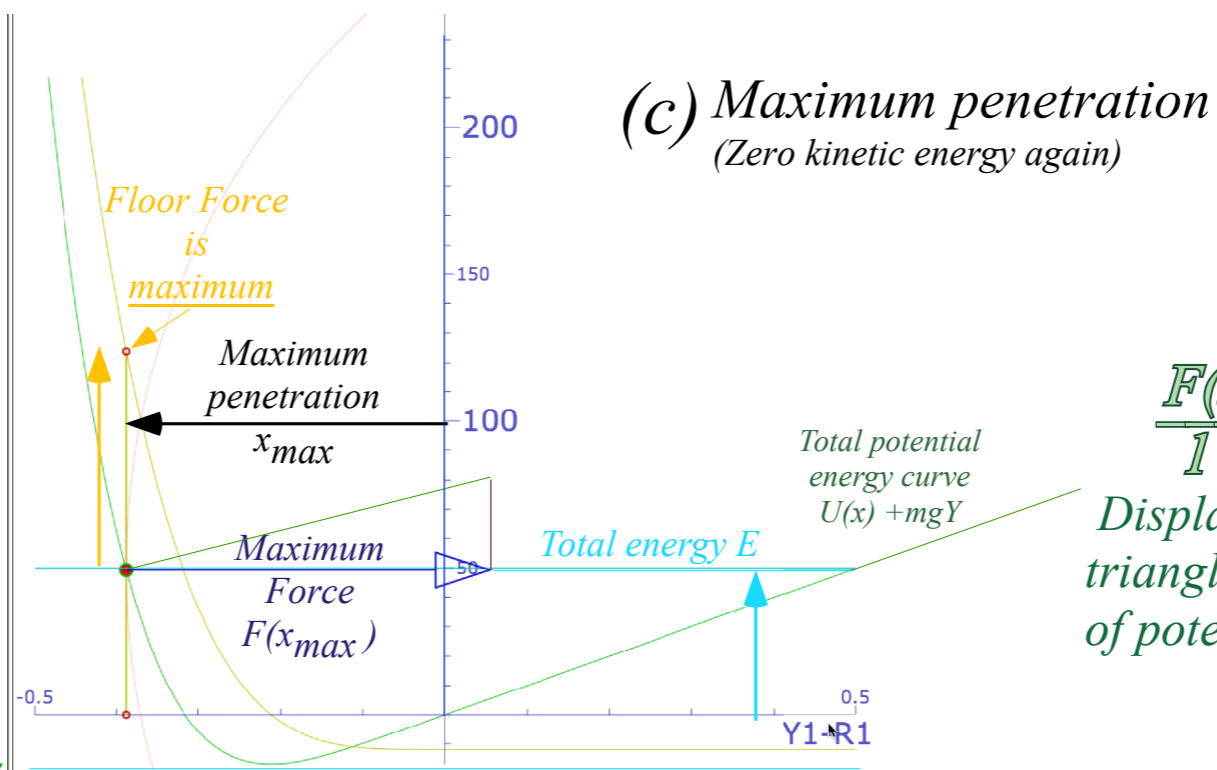
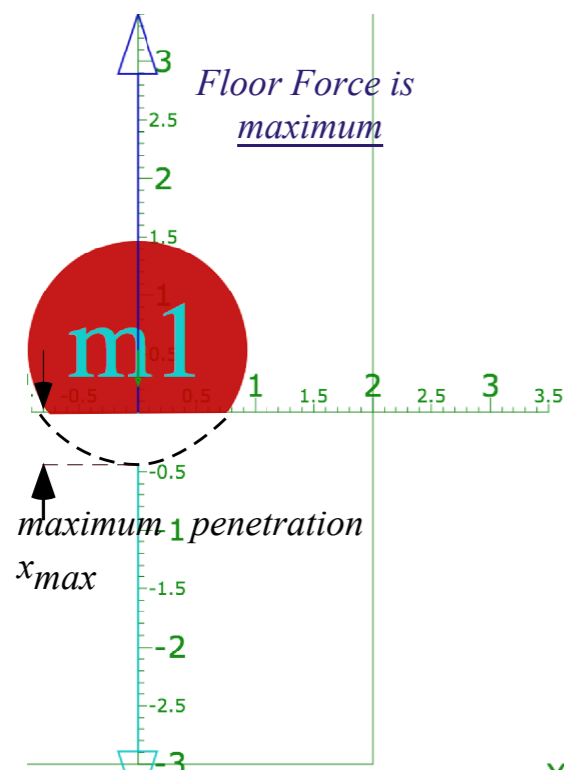
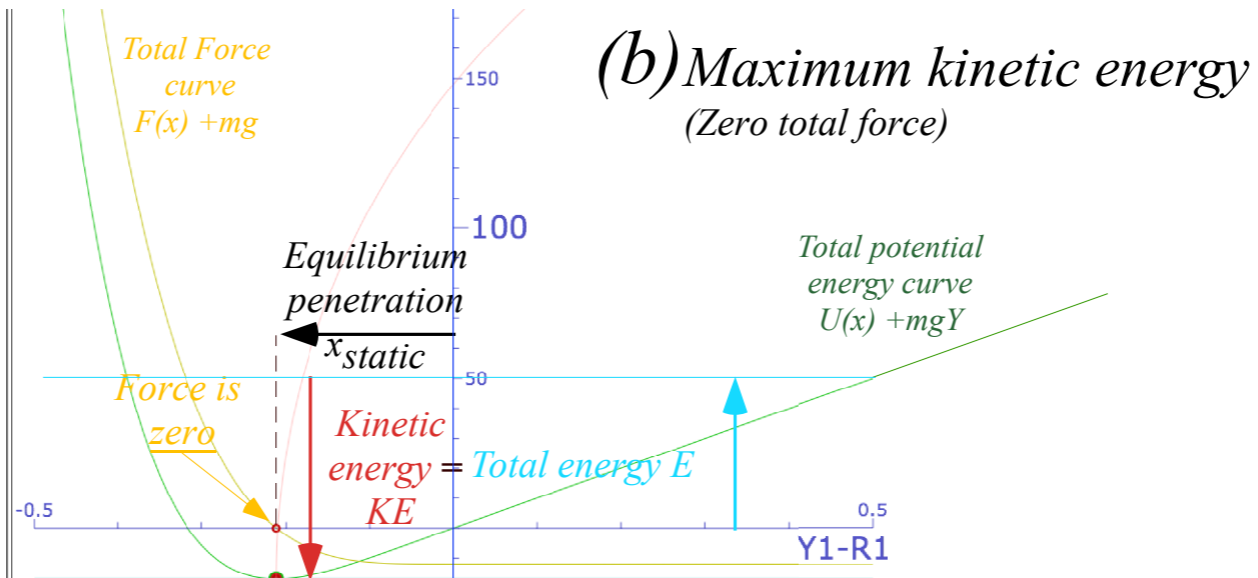
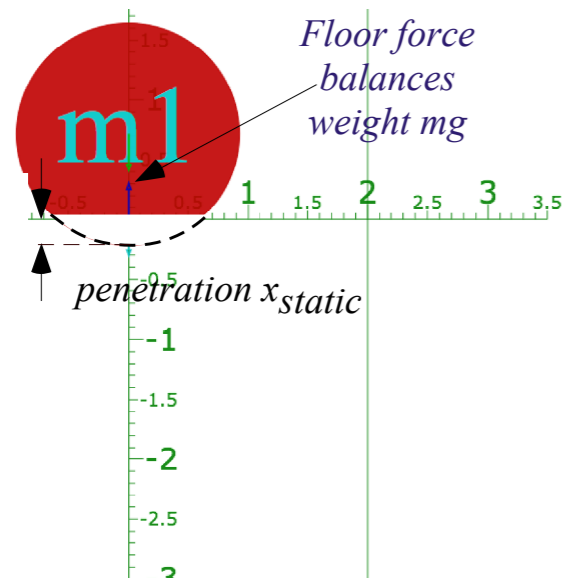
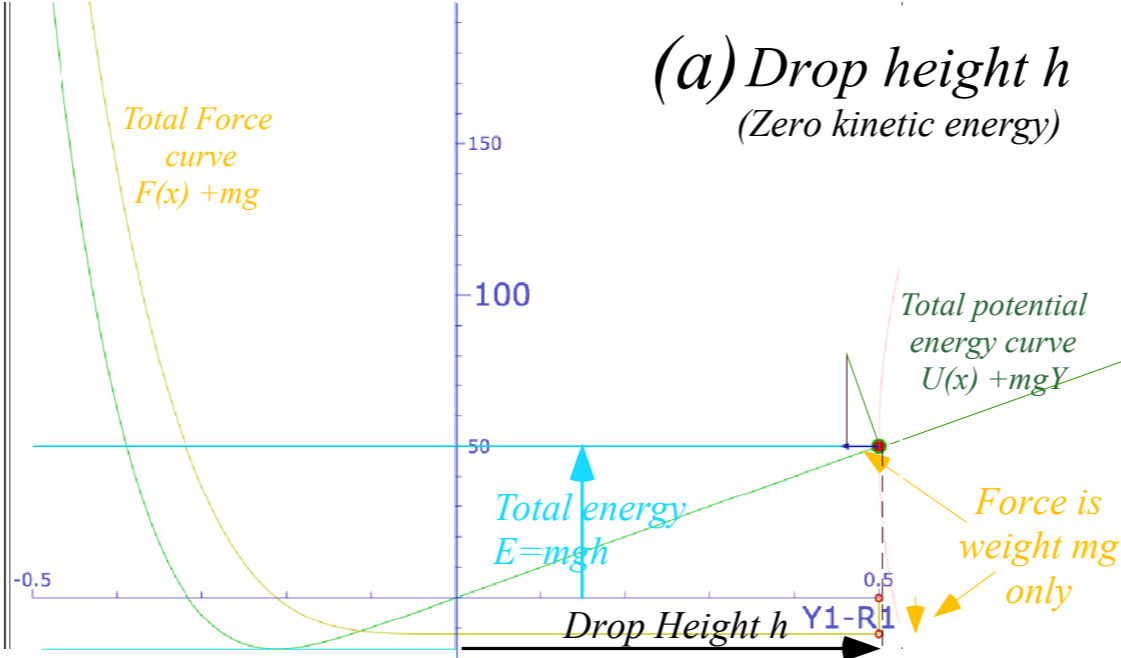
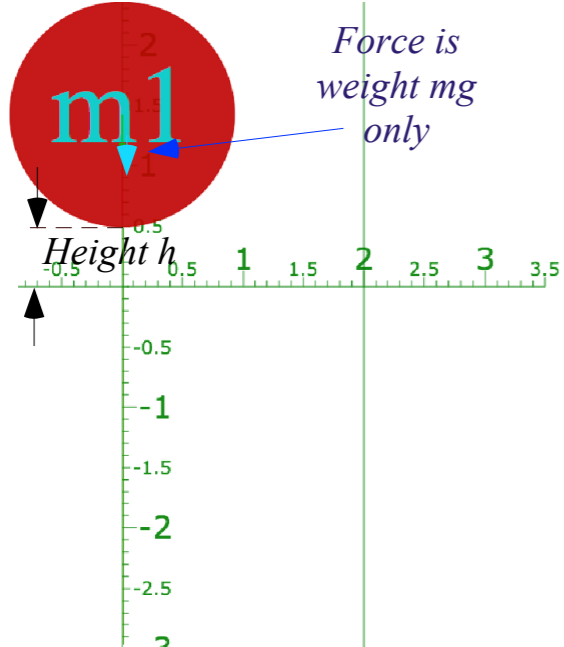


(c) Maximum penetration  
(Zero kinetic energy again)



Display of Force vector using similar triangle construction based on the slope of potential curve.





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Display of Force vector using similar triangle construction based on the slope of potential curve.

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→ General Non-linear force (like superball-floor or ball-bearing-anvil) (Calculus)

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## Many-body 1D collisions

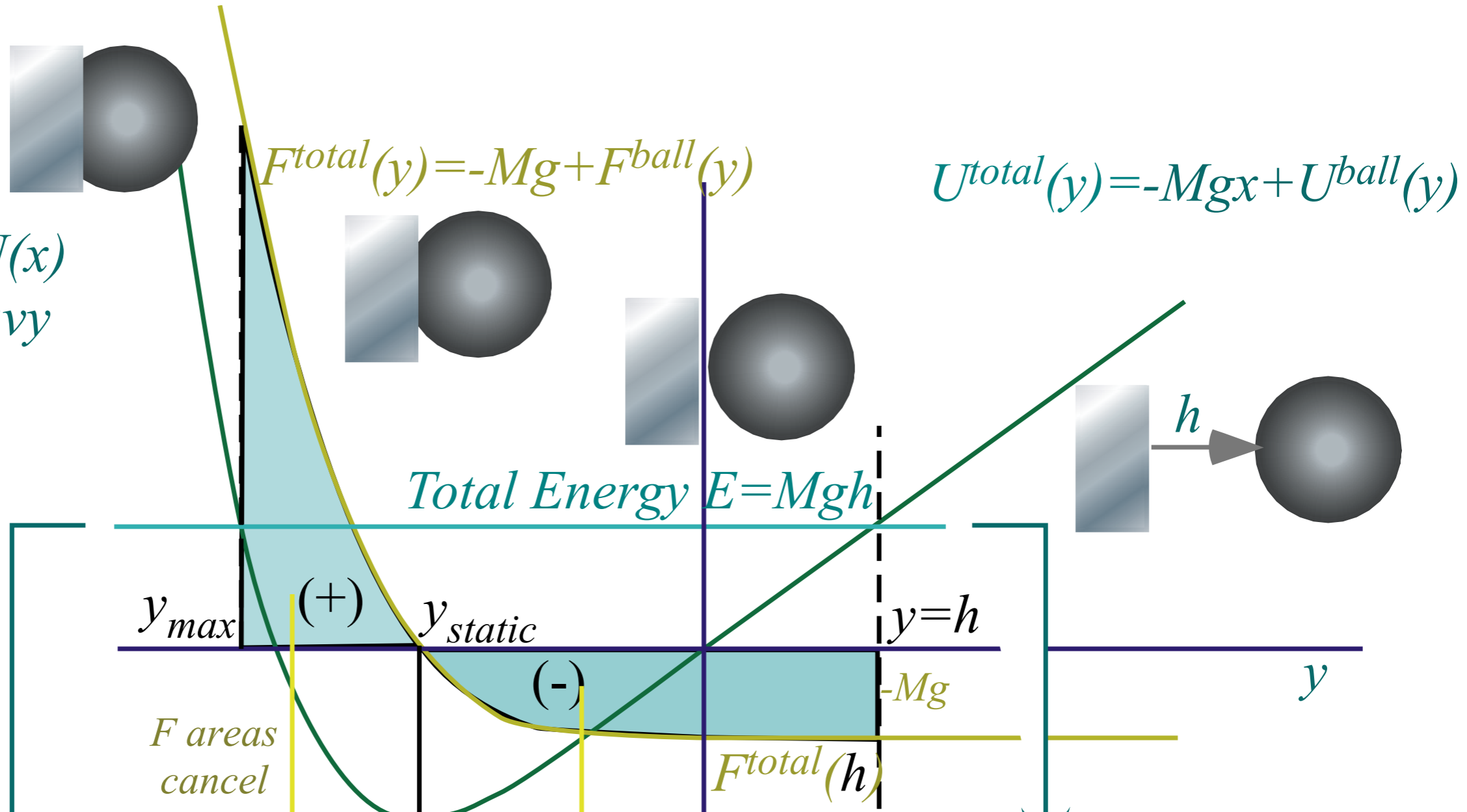
Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Force  $F(x)$   
and  
Potential  $U(x)$   
for soft heavy  
non-linear  
superball

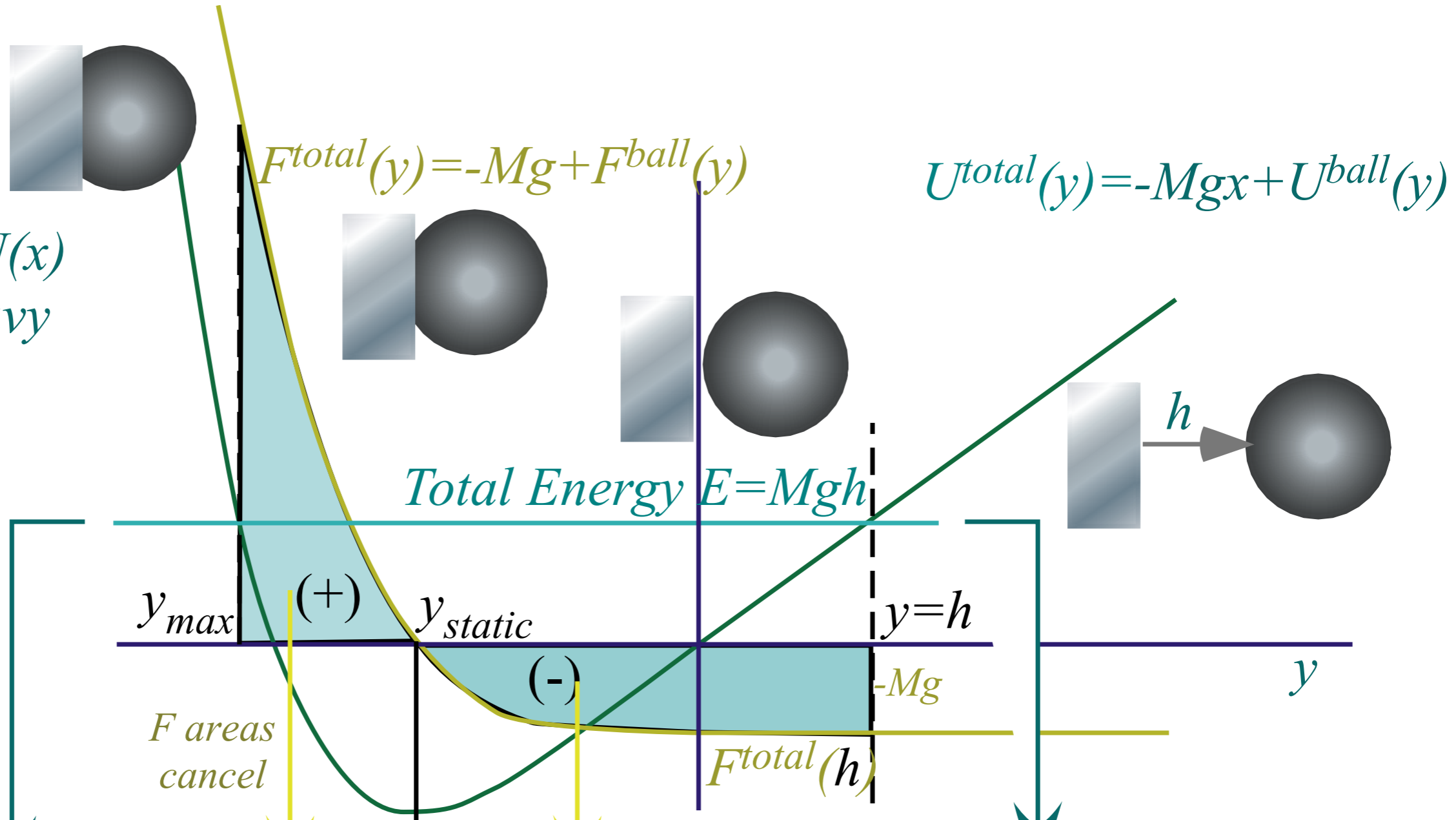


Unit 1  
Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y) dy + \int_{y=h}^{y_{static}} F^{total}(y) dy + U(h) = U(h) = E$$

$$F(x) = -\frac{dU(x)}{dx}$$

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and  
Potential  $U(x)$   
for soft heavy  
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superball



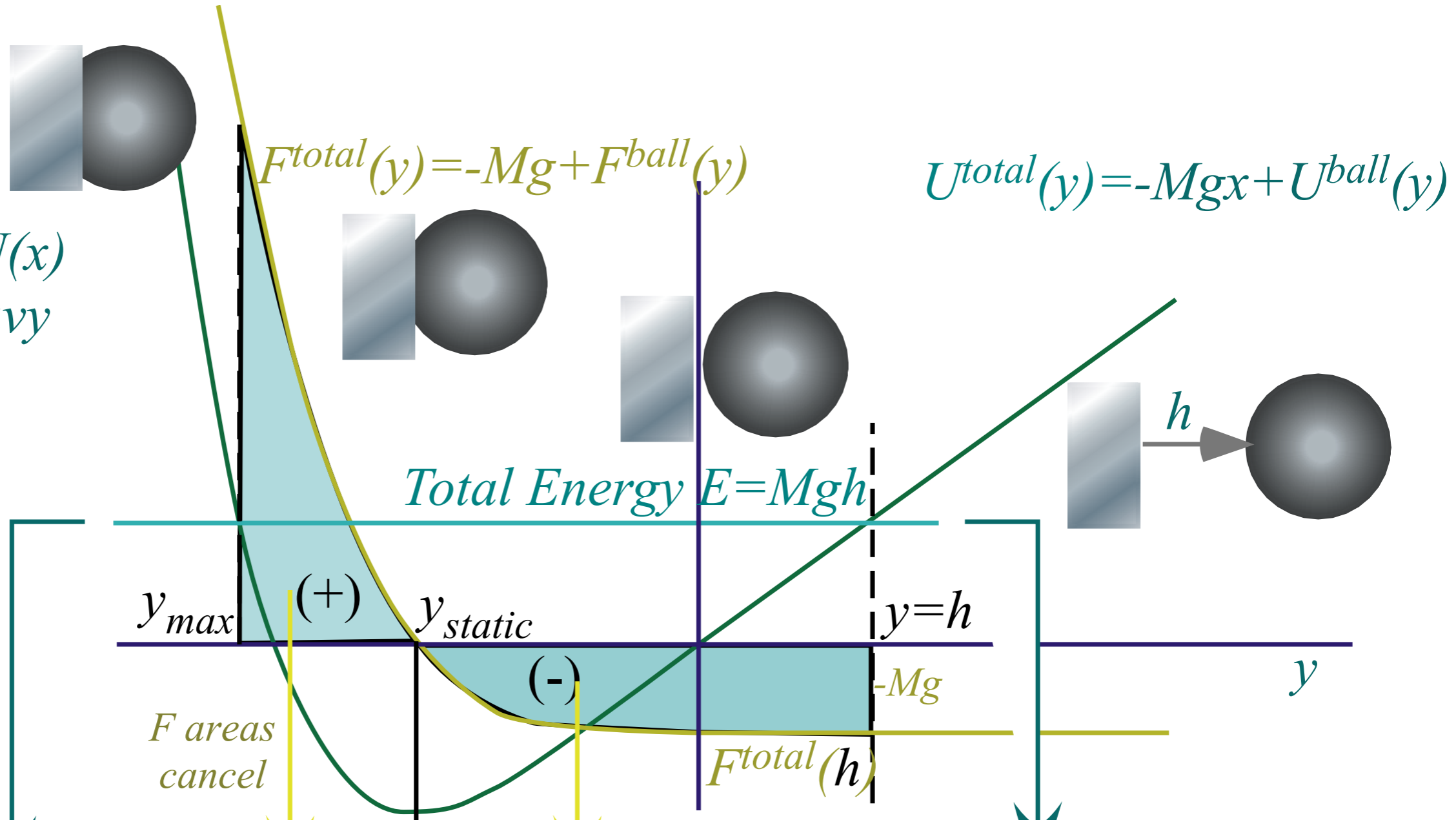
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Work =  $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Force  $F(x)$   
and  
Potential  $U(x)$   
for soft heavy  
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Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y) dy + \int_{y=h}^{y_{static}} F^{total}(y) dy + U(h) = U(h) = E$$

Work =  $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Impulse =  $P = \int F(t) dt = \text{Momentum acquired} = \text{Area of } F(t) = P(t)$

$$F(t) = \frac{dP(t)}{dt}$$

# *Potential energy dynamics of Superballs and related things*

*Thales geometry and “Sagittal approximation” to force law*

*Geometry and dynamics of single ball bounce*

*General Non-linear force (like superball-floor or ball-bearing-anvil)*

→ *Constant force  $F=-k$  (linear potential  $V=kx$ )*

→ *Some physics of dare-devil-diving 80 ft. into kidee pool (Calculus)*

*Linear force  $F=-kx$  (quadratic potential  $V=1/2kx^2$  (like balloon))*

## *Geometry and potential dynamics of 2-ball bounce*

*A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)*

*A story of USC pre-meds visiting Whammo Manufacturing Co.*

## *Geometry and dynamics of n-ball bounces*

*Analogy with shockwave and acoustical horn amplifier*

*Advantages of a geometric  $m_1, m_2, m_3, \dots$  series*

*A story of Stirling Colgate (Palmolive) and core-collapse supernovae*

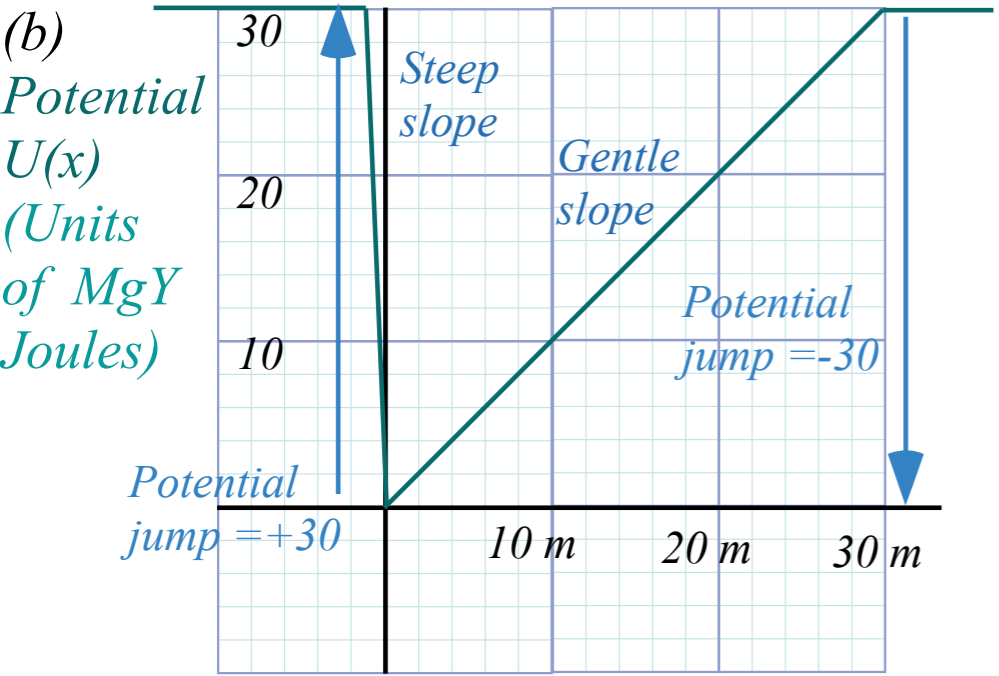
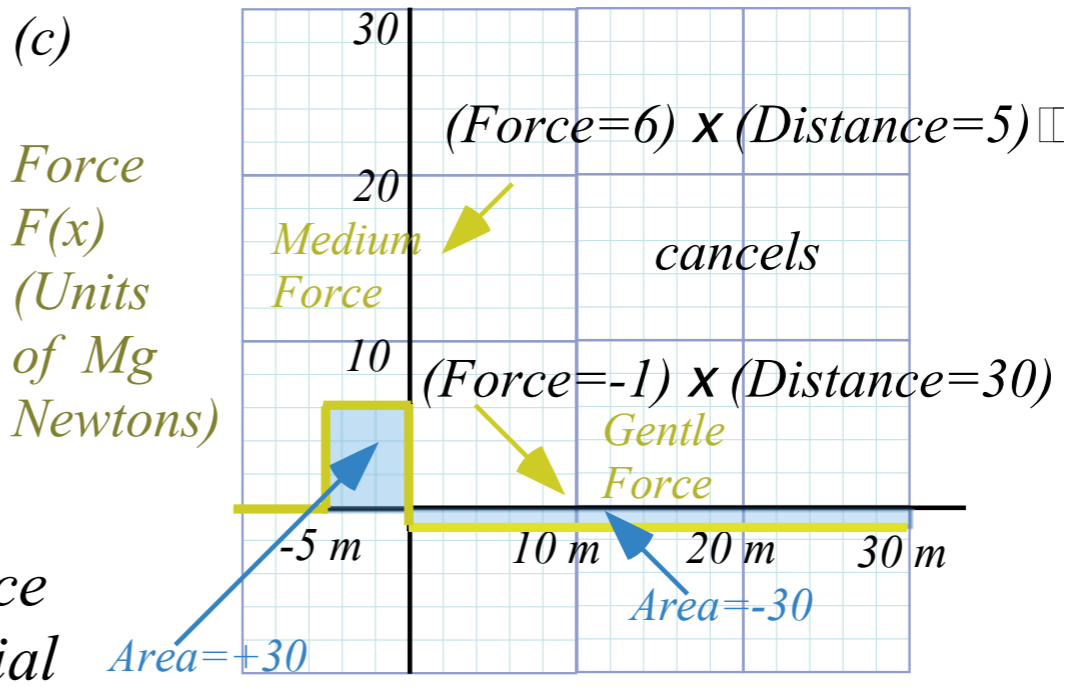
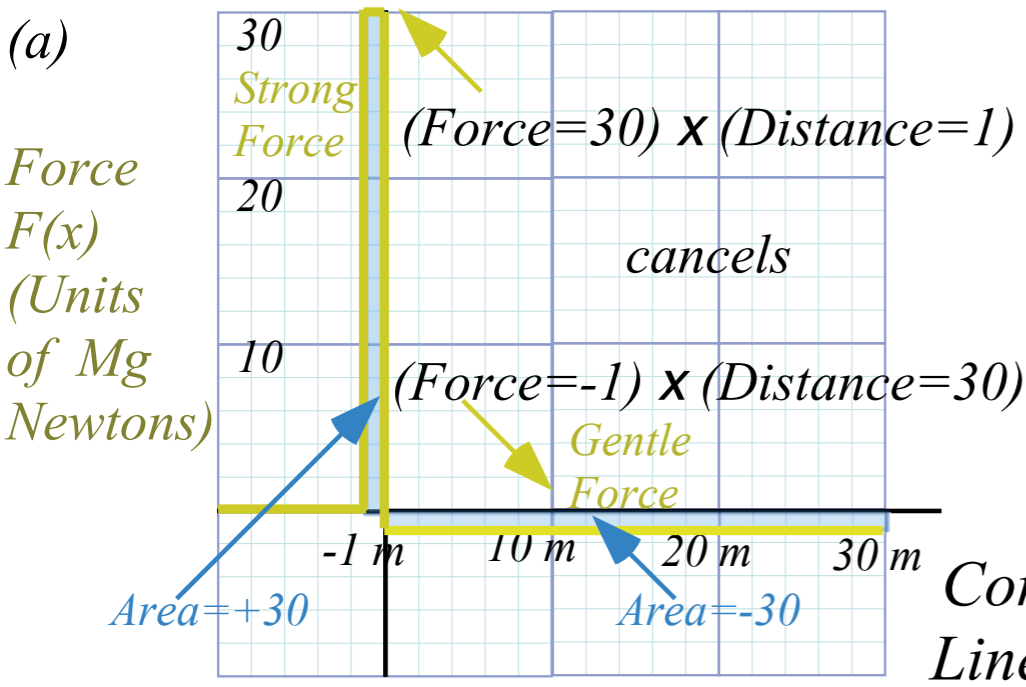
## *Many-body 1D collisions*

*Elastic examples: Western buckboard*

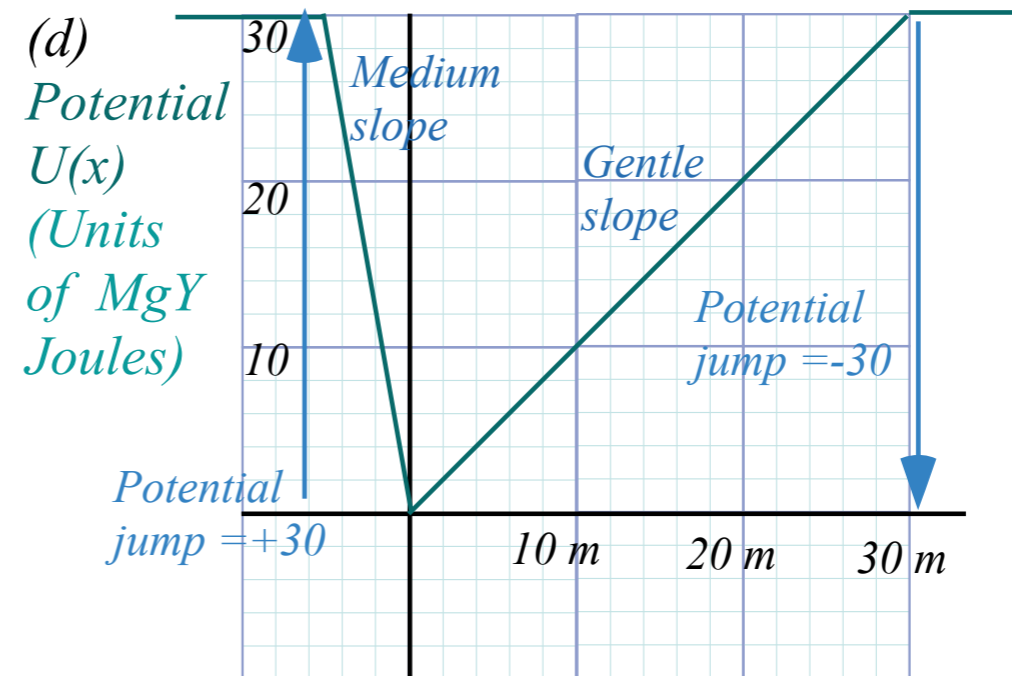
*Bouncing columns and Newton’s cradle*

*Inelastic examples: “Zig-zag geometry” of freeway crashes*

*Super-elastic examples: This really is “Rocket-Science”*



Models:  
 $F(x) = k$ ,  
 $U(x) = -kx$



Unit 1  
 Fig. 7.3

$Work = W = \int F(x) dx = Energy\ acquired = Area\ of\ F(x) = -U(x)$

$F(x) = -\frac{dU(x)}{dx}$

$Impulse = P = \int F(t) dt = Momentum\ acquired = Area\ of\ F(t) = P(t)$

$F(t) = \frac{dP(t)}{dt}$

# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

→ Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force  $F=-k$  (linear potential  $V=kx$ )

Some physics of dare-devil-diving 80 ft. into kidee pool

→ Linear force  $F=-kx$  (quadratic potential  $V=1/2kx^2$  (like balloon))

(Simulations)

## Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

## Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric  $m_1, m_2, m_3, \dots$  series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

## Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”



Start Resume

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses

1 Balls

Acceleration of gravity

0.5 100x{cm/s<sup>2</sup>}

*Sets gravity*

- Draw force vectors
- Pause (once) at top
- Constrain motion to Y-axis
- Plot v2 vs v1
- Plot p2 vs p1
- Plot V2 vs V1
- Plot Ellipses
- Plot Bisector Lines
- Old Color Scheme
- Show right panel information
- Show left panel information
- Set Initial positions

Collision friction (Viscosity)

0 x10<sup>^</sup>  0 {g}

Initial gap between balls

5.5 x10<sup>^</sup>  -1 {g}

Force Constant *Usually need to increase k for p > 1*

5 x10<sup>^</sup>  2 {g}

Force power law exponent

1

*This is linear F=-kx<sup>1</sup>  
(increase p > 1  
for non-linear F=-kx<sup>p</sup>)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

0.75

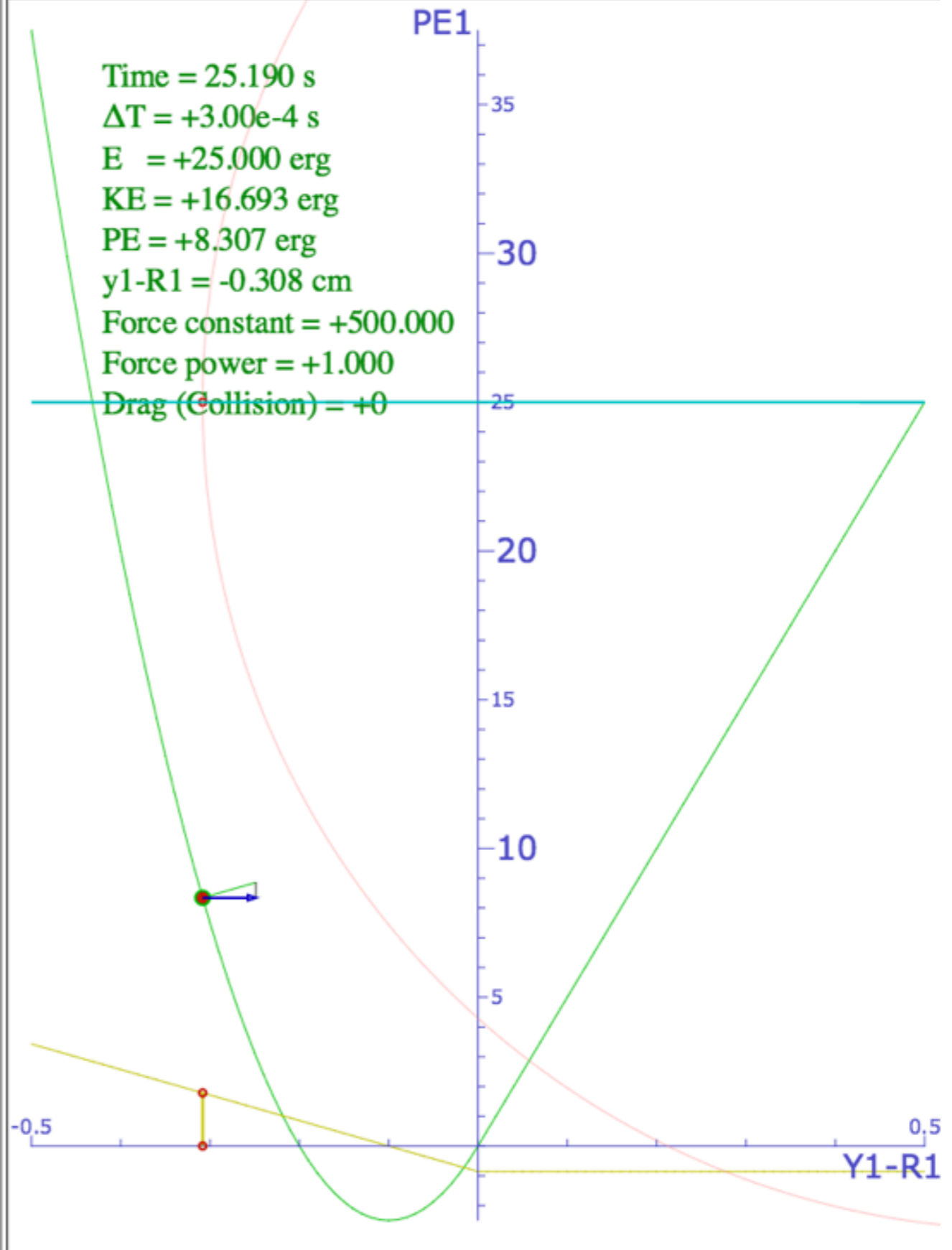
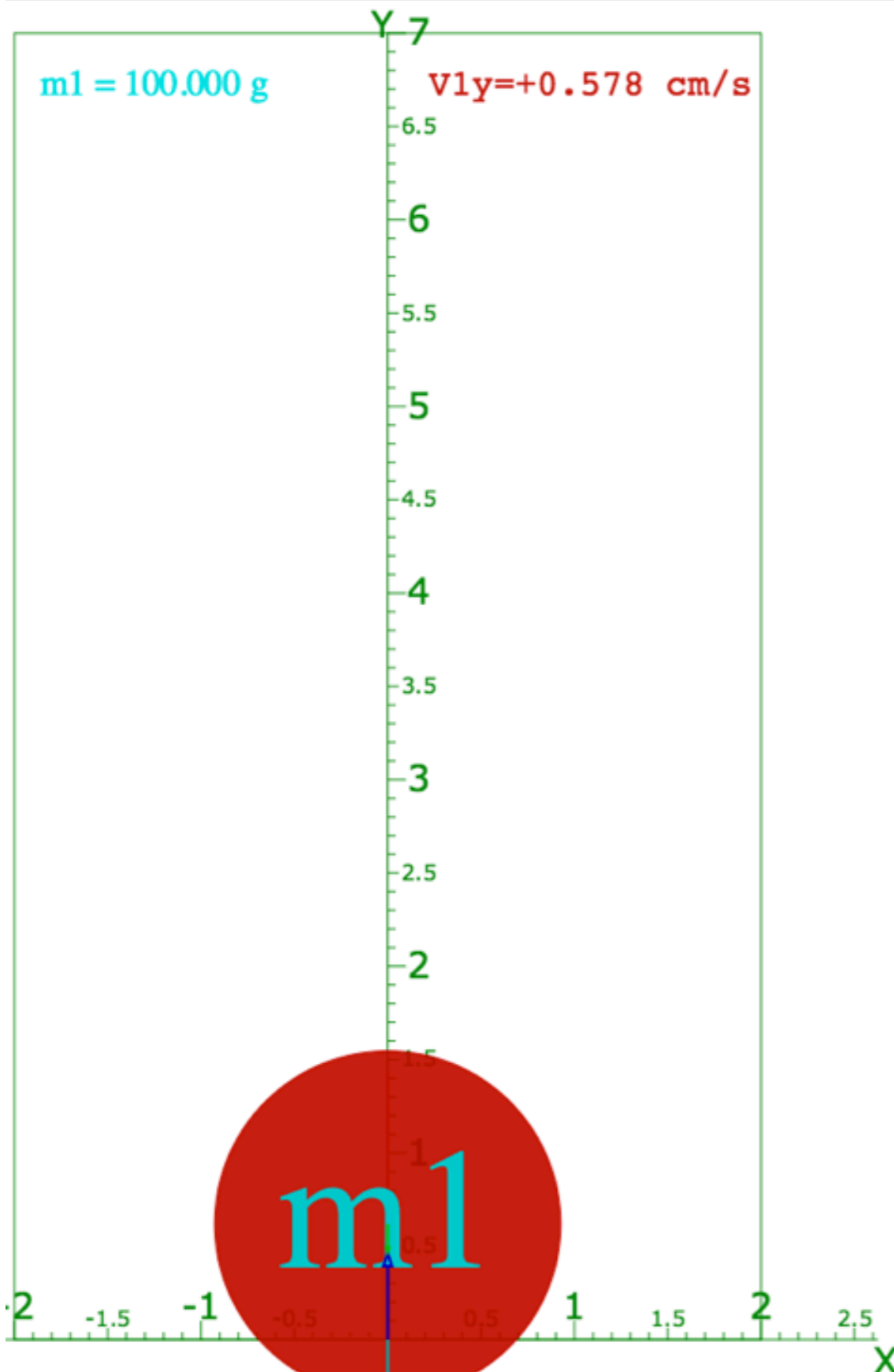
|                  |                       |        |           |                       |        |
|------------------|-----------------------|--------|-----------|-----------------------|--------|
| Initial x1 =     | <input type="range"/> | 0.5    | y Max =   | <input type="range"/> | 7      |
| Max x PE plot =  | <input type="range"/> | 0.5    | y Min =   | <input type="range"/> | 0      |
| F-Vector scale = | <input type="range"/> | 0.003  | T Max =   | <input type="range"/> | 6      |
| Error step =     | <input type="range"/> | 0.0000 | V2y Max = | <input type="range"/> | 3.1112 |
|                  |                       |        | V2y Min = | <input type="range"/> | -3.111 |

Mass factor =  1

M<sub>1</sub> =  1 x10<sup>^</sup>  2 {g}

V<sub>1</sub> =  0 x10<sup>^</sup>  0 {cm/s}

Controls & Scenarios Resume Reset T=0 Reset & Pause@Edge Erase Paths Speed  $\times 10^{\wedge}$



# *Potential energy dynamics of Superballs and related things*

*Thales geometry and “Sagittal approximation” to force law*

*Geometry and dynamics of single ball bounce*

*General Non-linear force (like superball-floor or ball-bearing-anvil)*

*Constant force  $F=-k$  (linear potential  $V=kx$ )*

*Some physics of dare-devil-diving 80 ft. into kidee pool*

 *Linear force  $F=-kx$  (quadratic potential  $V=1/2kx^2$  (like balloon))* *(Calculations)*

## *Geometry and potential dynamics of 2-ball bounce*

*A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)*

*A story of USC pre-meds visiting Whammo Manufacturing Co.*

## *Geometry and dynamics of n-ball bounces*

*Analogy with shockwave and acoustical horn amplifier*

*Advantages of a geometric  $m_1, m_2, m_3, \dots$  series*

*A story of Stirling Colgate (Palmolive) and core-collapse supernovae*

## *Many-body 1D collisions*

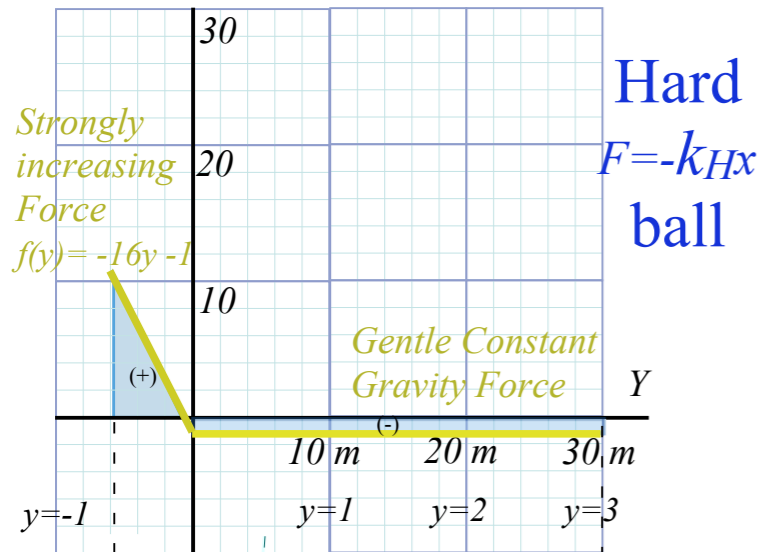
*Elastic examples: Western buckboard*

*Bouncing columns and Newton's cradle*

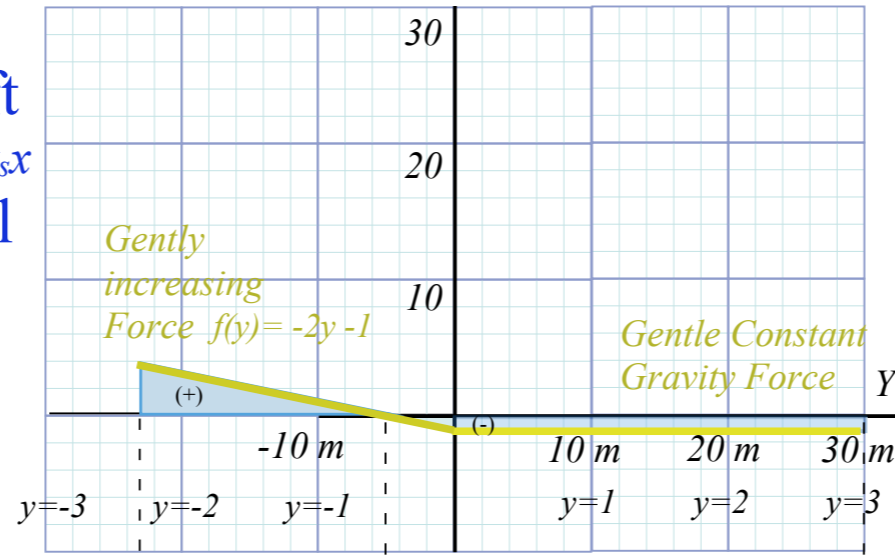
*Inelastic examples: “Zig-zag geometry” of freeway crashes*

*Super-elastic examples: This really is “Rocket-Science”*

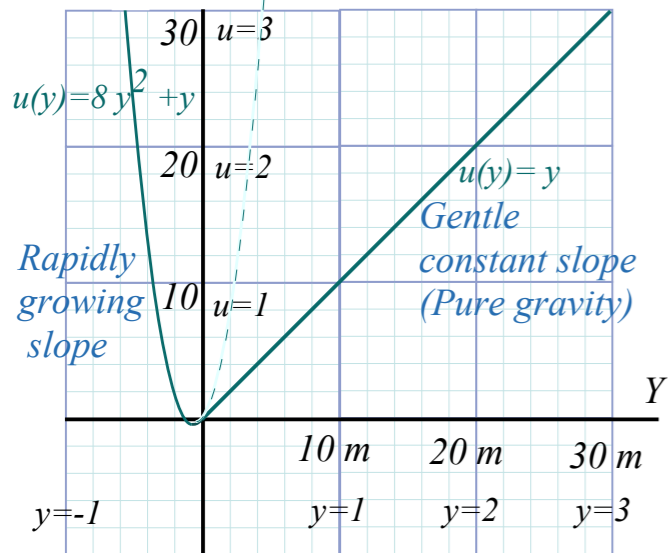
(a) Force  $F(Y)$  Units  $Mg$  (N)



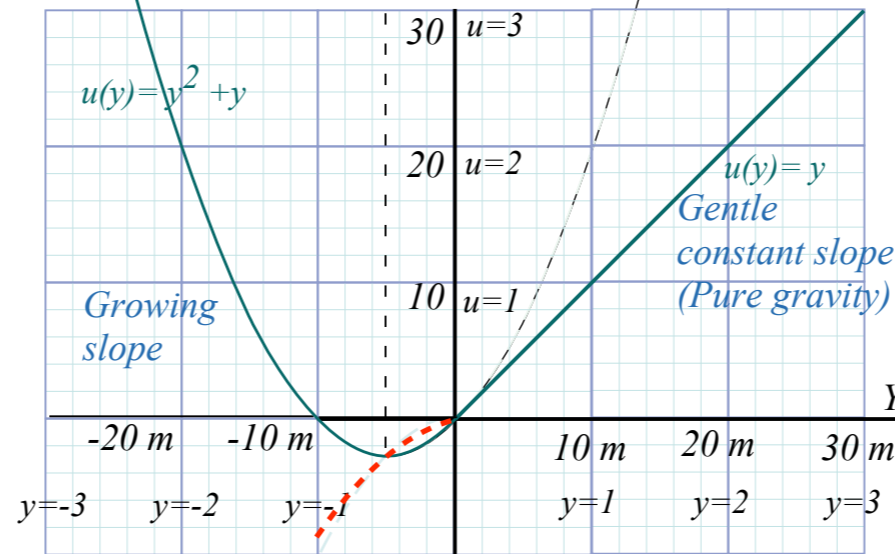
(c) Force  $F(Y)$  Units  $Mg$  (N)



(b) Potential  $U(Y)$  Units of  $MgY$  (J)



(d) Potential  $U(Y)$  Units of  $MgY$  (J)



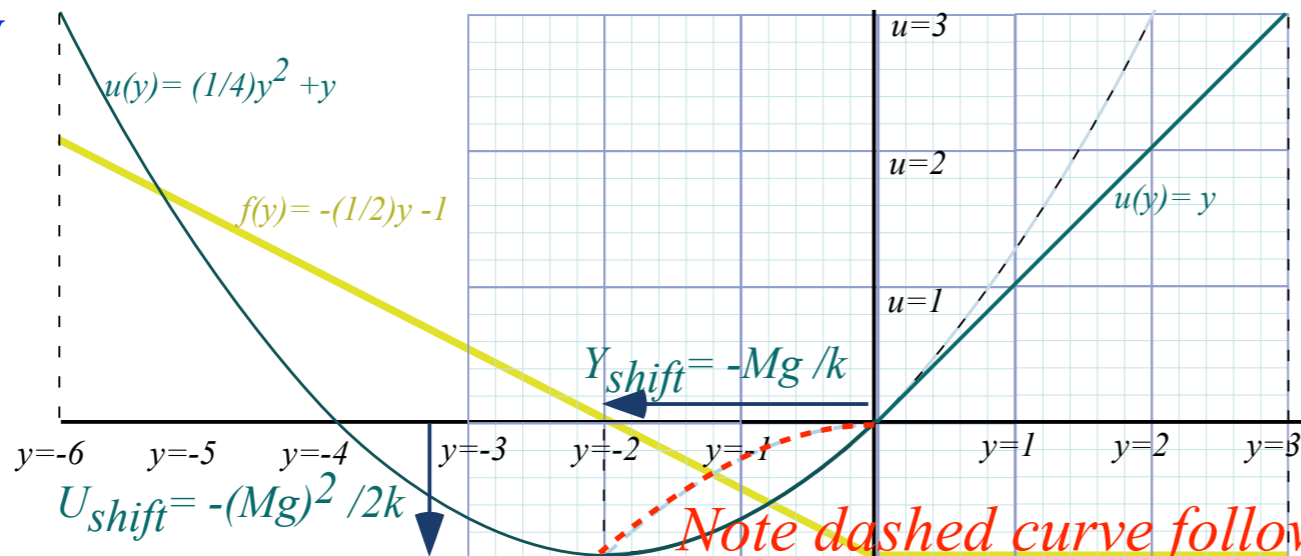
Unit 1  
Fig. 7.4

(e) Geometry of Linear Force with Constant  $Mg$  and Quadratic Potential

$$F(Y) = -kY - Mg$$

$$U(Y) = (1/2)kY^2 + MgY$$

Close view of Soft ball  $F = -k_s x$



$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg y & (y \geq 0) \\ Mg y + \frac{1}{2} ky^2 & (y < 0) \end{cases}$$

Note dashed curve followed by PE minimum. Parabola? What?

# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

→ → General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force  $F=-k$  (linear potential  $V=kx$ )

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force  $F=-kx$  (quadratic potential  $V=1/2kx^2$  (like balloon))

(Reviewing  
calculations  
and noticing  
“gap” effect)

## Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

## Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric  $m_1, m_2, m_3, \dots$  series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

## Many-body 1D collisions

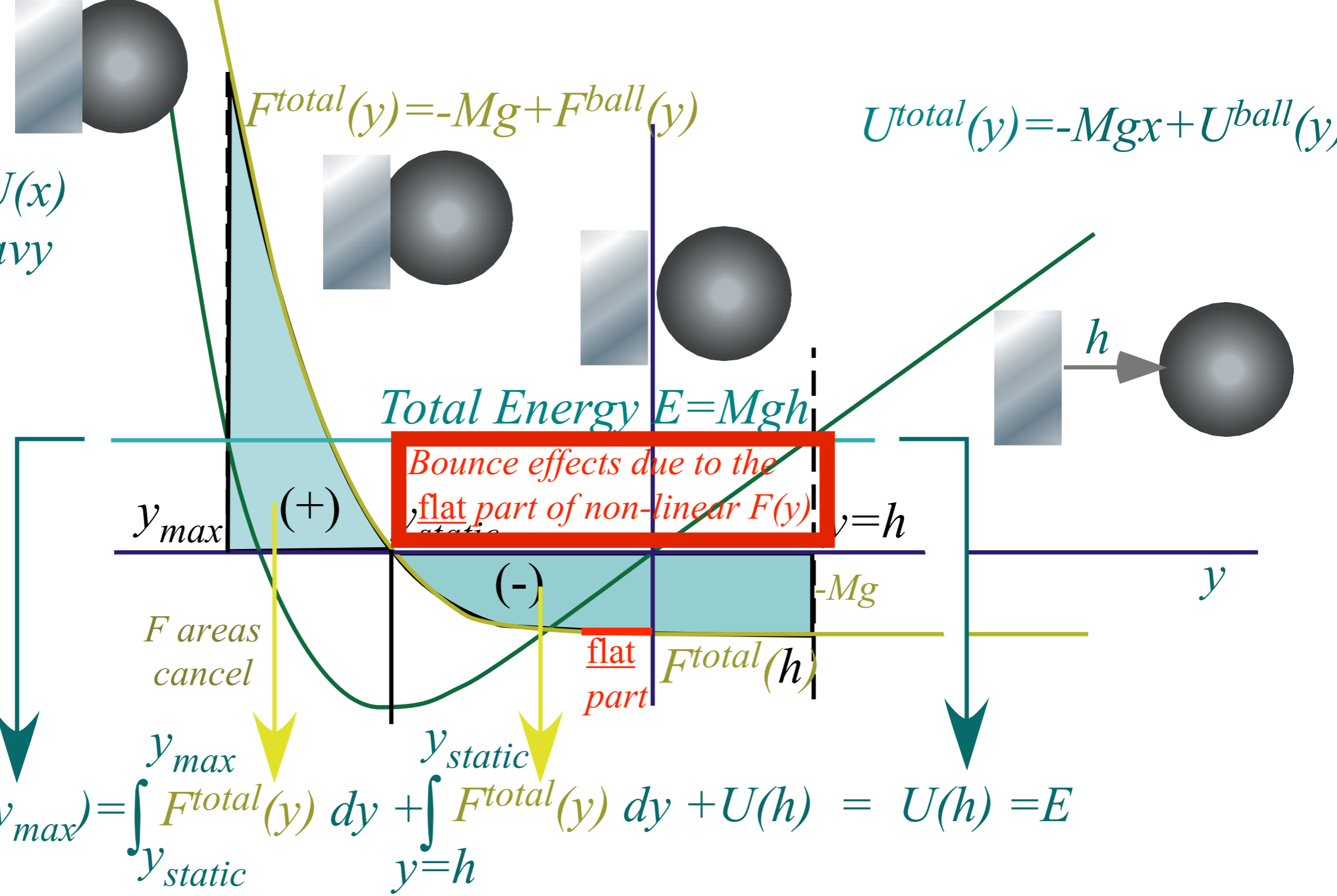
Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Force  $F(x)$   
and  
Potential  $U(x)$   
for soft heavy  
non-linear  
superball



Unit 1  
Fig. 7.5

Work =  $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Impulse =  $P = \int F(t) dt = \text{Momentum acquired} = \text{Area of } F(t) = P(t)$

$$F(t) = \frac{dP(t)}{dt}$$

# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force  $F=-k$  (linear potential  $V=kx$ )

(Simulations)

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force  $F=-kx$  (quadratic potential  $V=\frac{1}{2}kx^2$  (like balloon))

## Geometry and potential dynamics of 2-ball bounce

A parable of **RumpCo.** vs **CrapCorp.** (introducing 3-mass potential-driven dynamics)

**Parable allegory for Los Alamos**  
Cheap&practical “seat-of-the pants” approach

**Parable allegory for Livermore**  
Fancy&overpriced “political” approach

Advantages of a geometric  $m_1, m_2, m_3, \dots$  series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

## Many-body 1D collisions

Elastic examples: Western buckboard

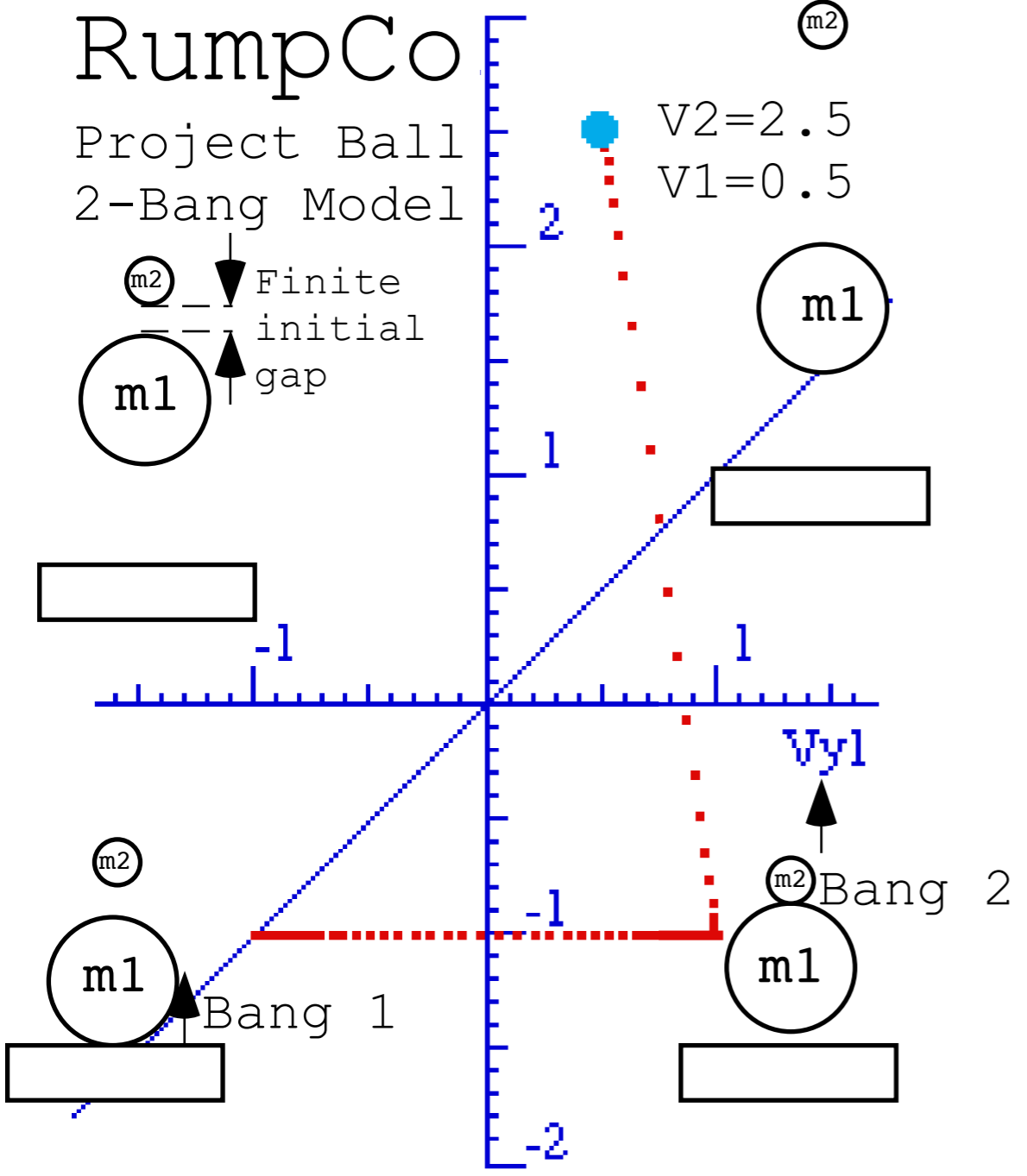
Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

*Parable allegory for Los Alamos*

*Cheap&practical "seat-of-the pants" approach*

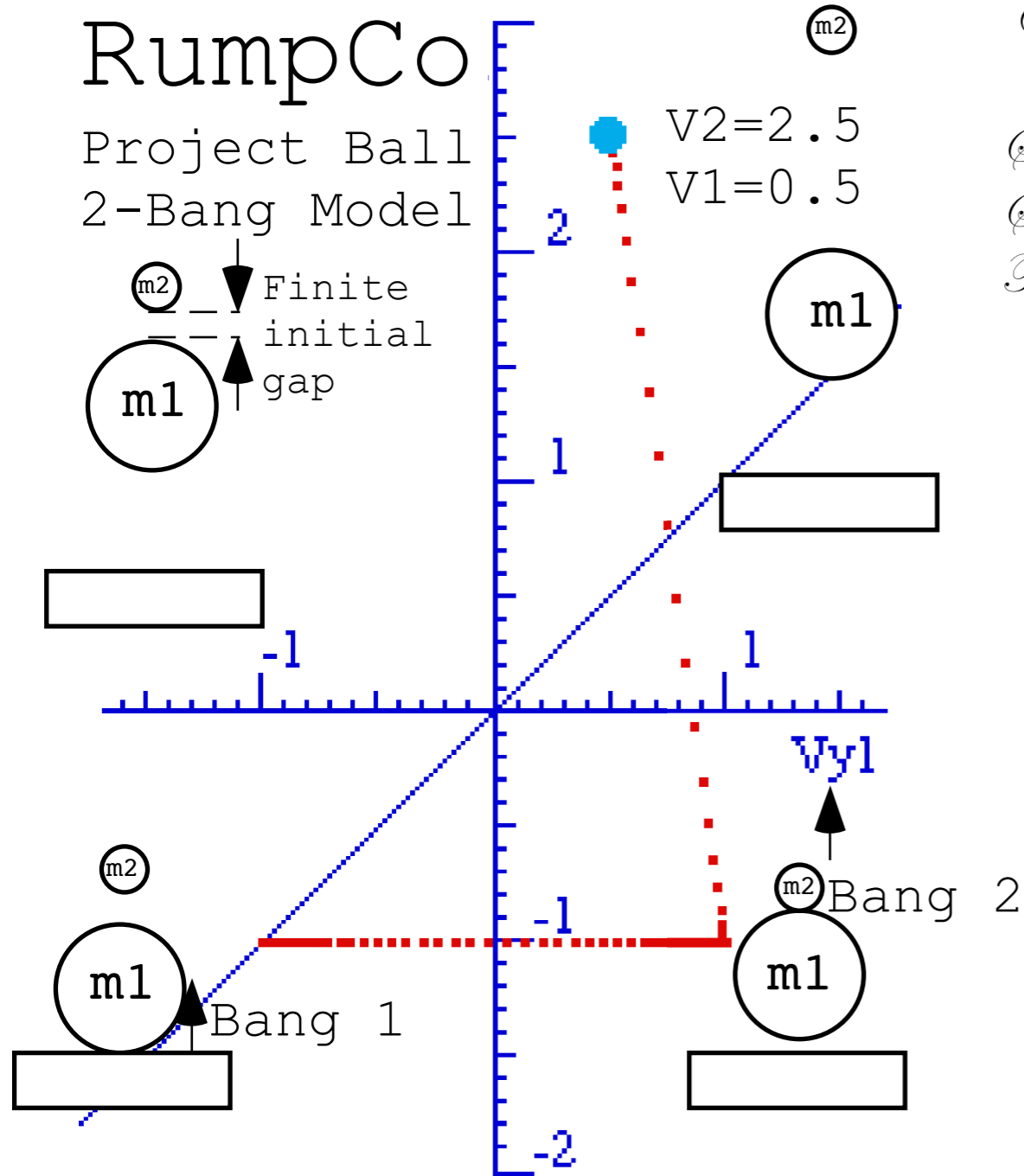


*Velocity amplification  
or "throw" factor = 2.5*



*Parable allegory for Los Alamos*

*Cheap&practical "seat-of-the pants" approach*



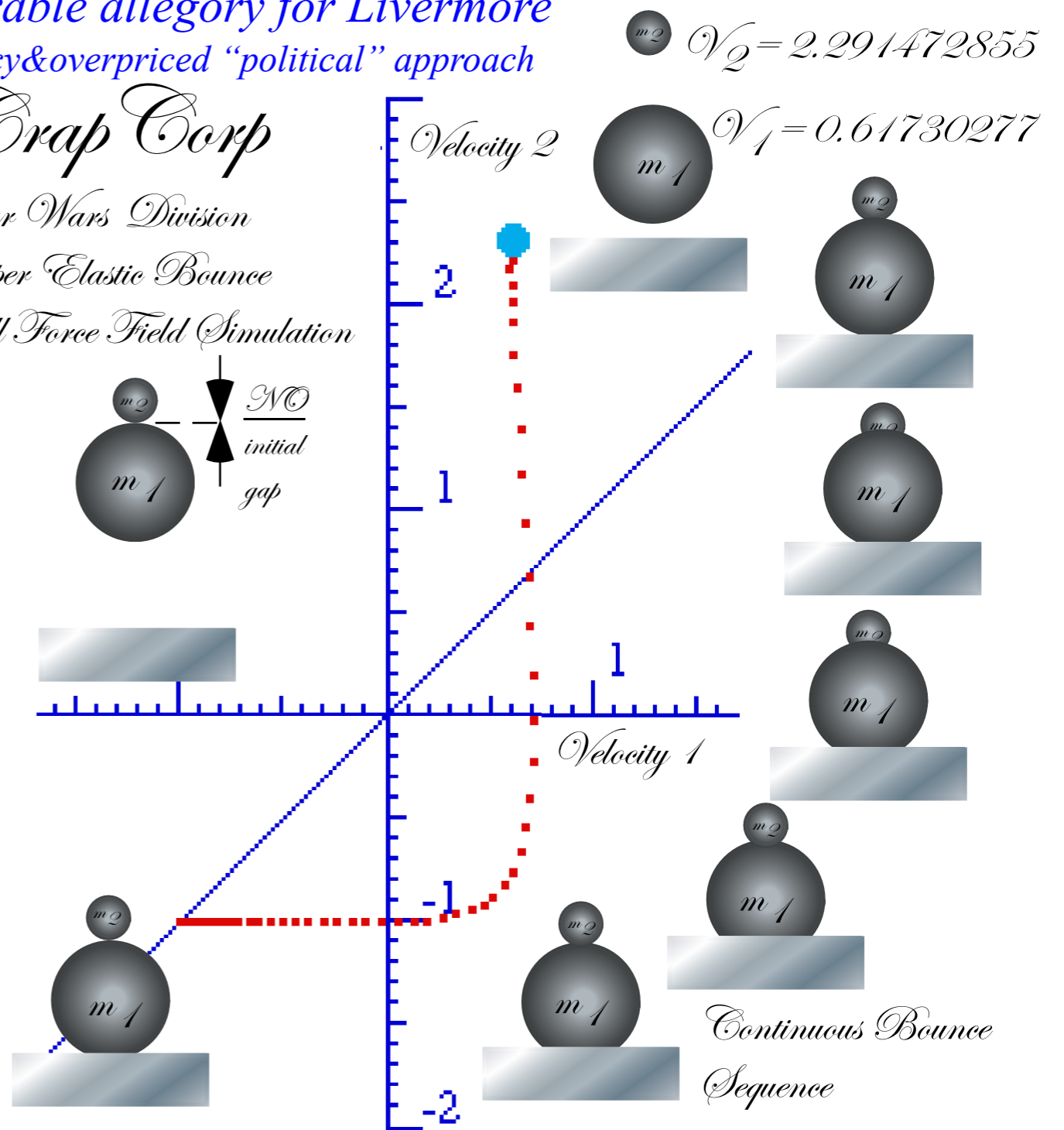
*Velocity amplification  
or "throw" factor = 2.5*

*Parable allegory for Livermore*

*Fancy&overpriced "political" approach*

*Crap Corp*

*Star Wars Division  
Super Elastic Bounce  
Full Force Field Simulation*



*Velocity amplification  
or "throw" factor = 2.3  
(about equal to RumpCo  
finite gap experiment)*

Unit 1  
Fig. 7.6

Bouncelt Simulation: Zero Gap  
(Force power=4)

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths *Sets gravity*
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses  
 2 Balls

Acceleration of gravity  
 0 100x{cm/s^2}

Draw force vectors

Pause (once) at top

Constrain motion to Y-axis

Plot v2 vs v1

Plot p2 vs p1

Plot V2 vs V1

Plot Ellipses

Plot Bisector Lines

Old Color Scheme

Show right panel information

Show left panel information

Set Initial positions

Collision friction (Viscosity)  
 0 x10^  0 {g}

Initial gap between balls *zero-gap*  
 0 x10^  0 {g}

Force Constant *Usually need to decrease k for p = 1*  
 5 x10^  4 {g}

Force power law exponent  
 4 *This is non-linear F=-kx^4 (Set p = 1 for linear F=-kx^1)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0  
 0.75

Initial x1 =  1.09

Max x PE plot =  0.5

F-Vector scale =  0.003

Error step =  0.0000

y Max =  7

y Min =  0

T Max =  6

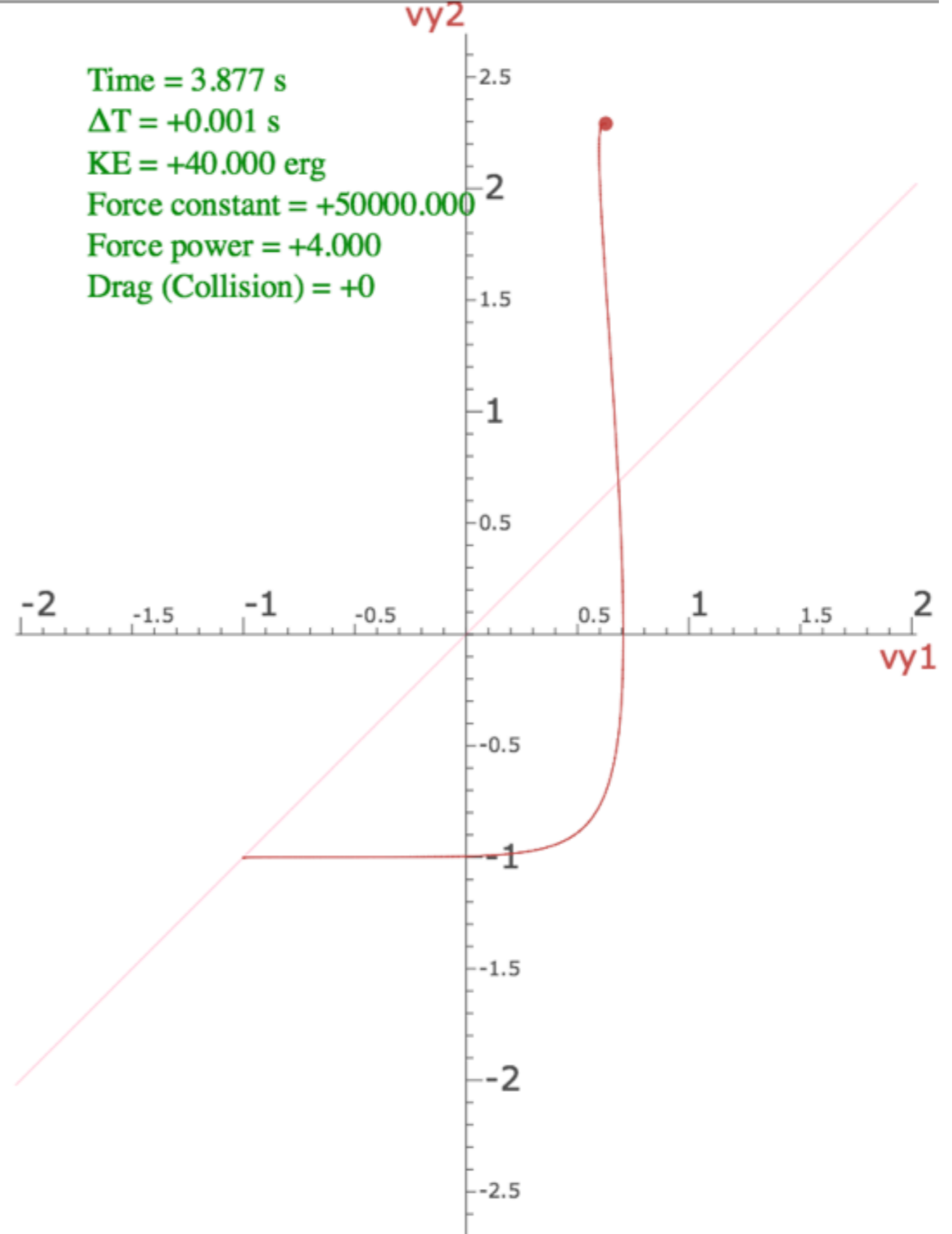
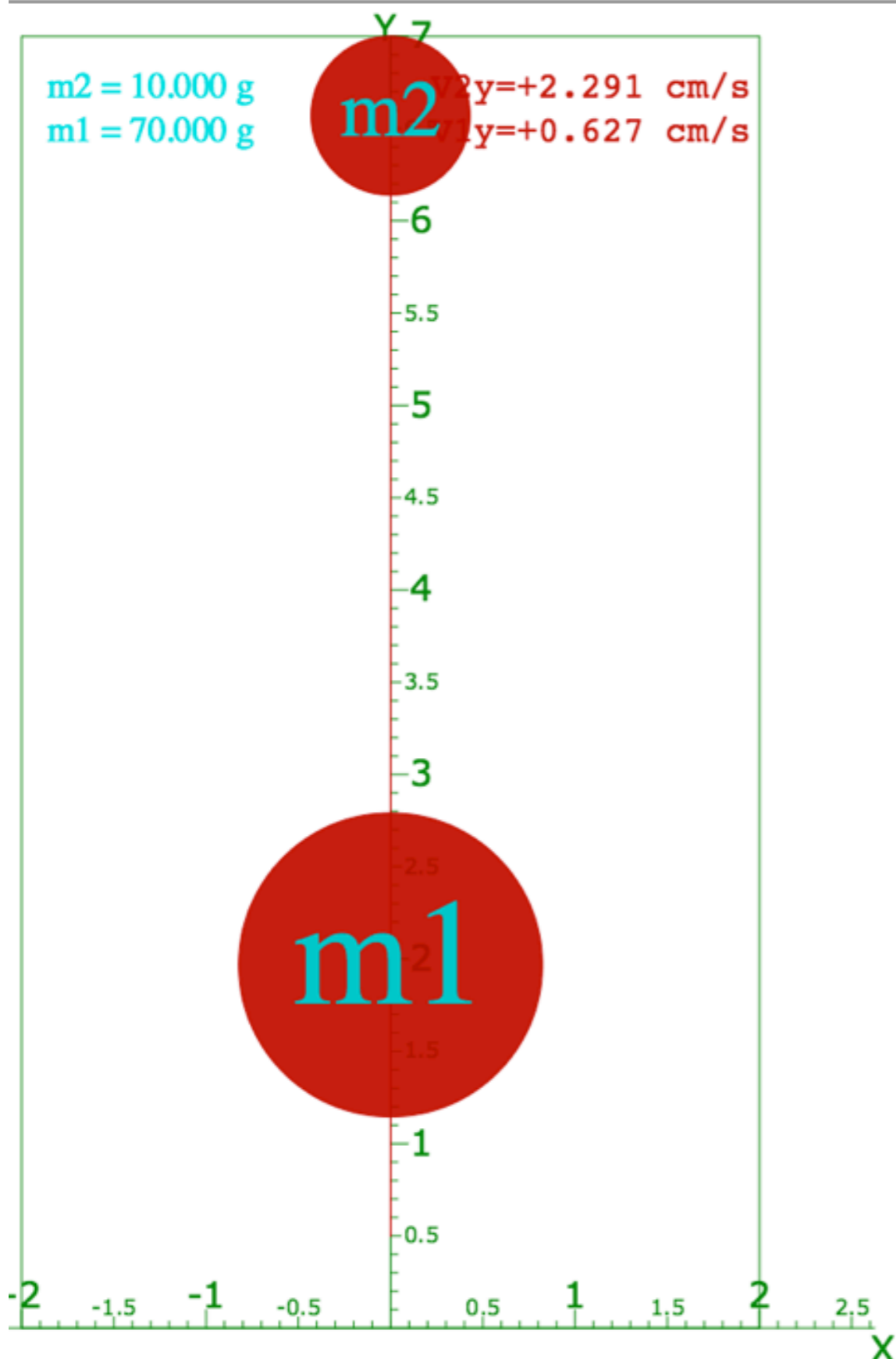
V2y Max =  2.6944

V2y Min =  -2.694

Mass factor =  1

M<sub>1</sub> =  7 x10^  1 {g}    V<sub>1</sub> =  -1 x10^  0 {cm/s}

M<sub>2</sub> =  1 x10^  1 {g}    V<sub>2</sub> =  -1 x10^  0 {cm/s}



*Bouncelt Simulation: Zero Gap (Force power=4)*

Bouncelt Simulation: Zero Gap  
(Force power=1)

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses

2 Balls

Acceleration of gravity

0 100x{cm/s<sup>2</sup>}

- Draw force vectors
- Pause (once) at top
- Constrain motion to Y-axis
- Plot v2 vs v1
- Plot p2 vs p1
- Plot V2 vs V1
- Plot Ellipses
- Plot Bisector Lines
- Old Color Scheme
- Show right panel information
- Show left panel information
- Set Initial positions

Collision friction (Viscosity)

0 x10<sup>^</sup>  0 {g}

Initial gap between balls

0 x10<sup>^</sup>  0 {g} *zero-gap*

Force Constant

5 x10<sup>^</sup>  2 {g} *Usually need to increase k for p > 1*

Force power law exponent

1 *This is linear F=-kx<sup>1</sup> (increase p > 1 for non-linear F=-kx<sup>p</sup>)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

0.75

Initial x1 =  1.09 y Max =  7

Max x PE plot =  0.5 y Min =  0

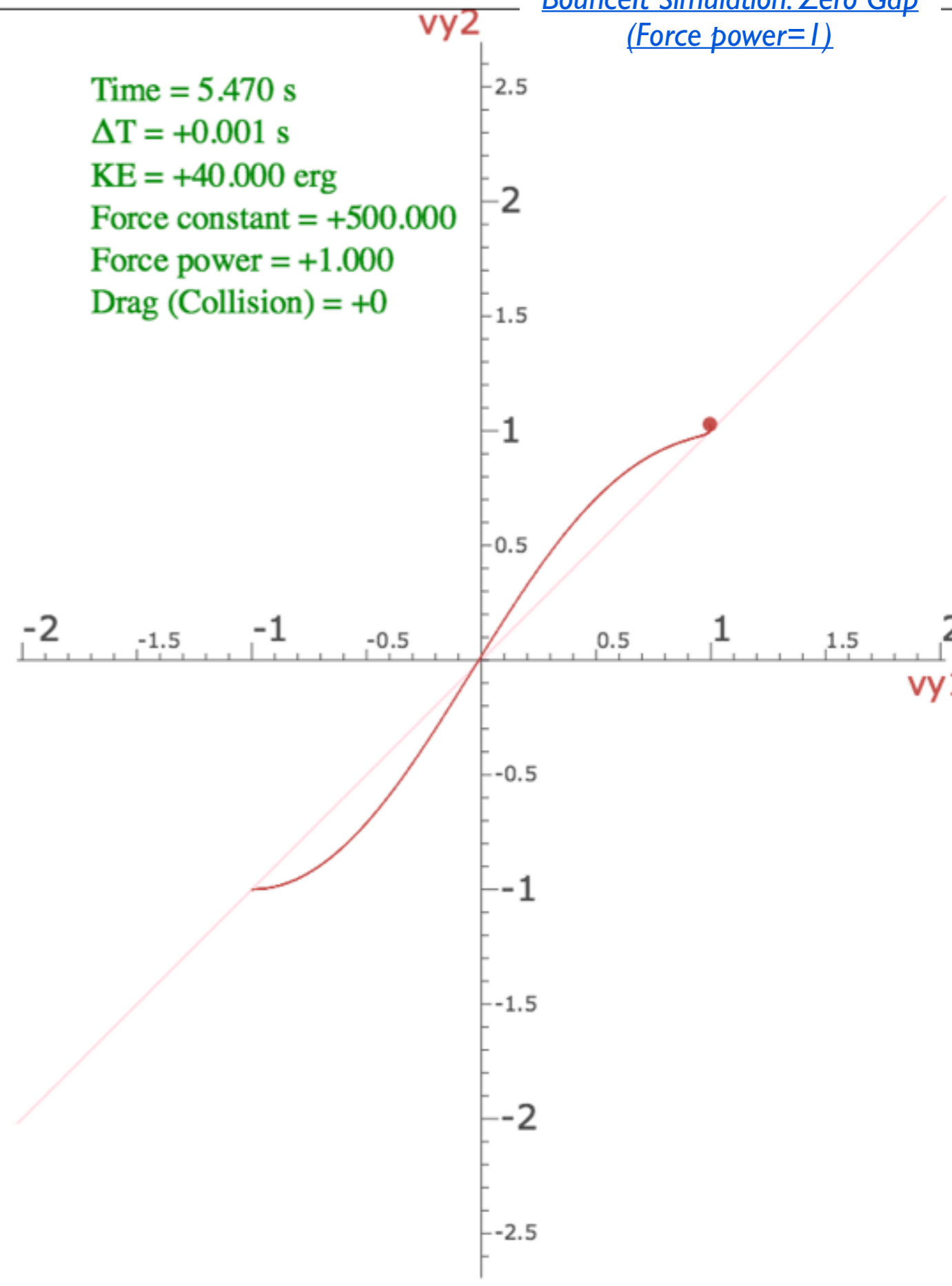
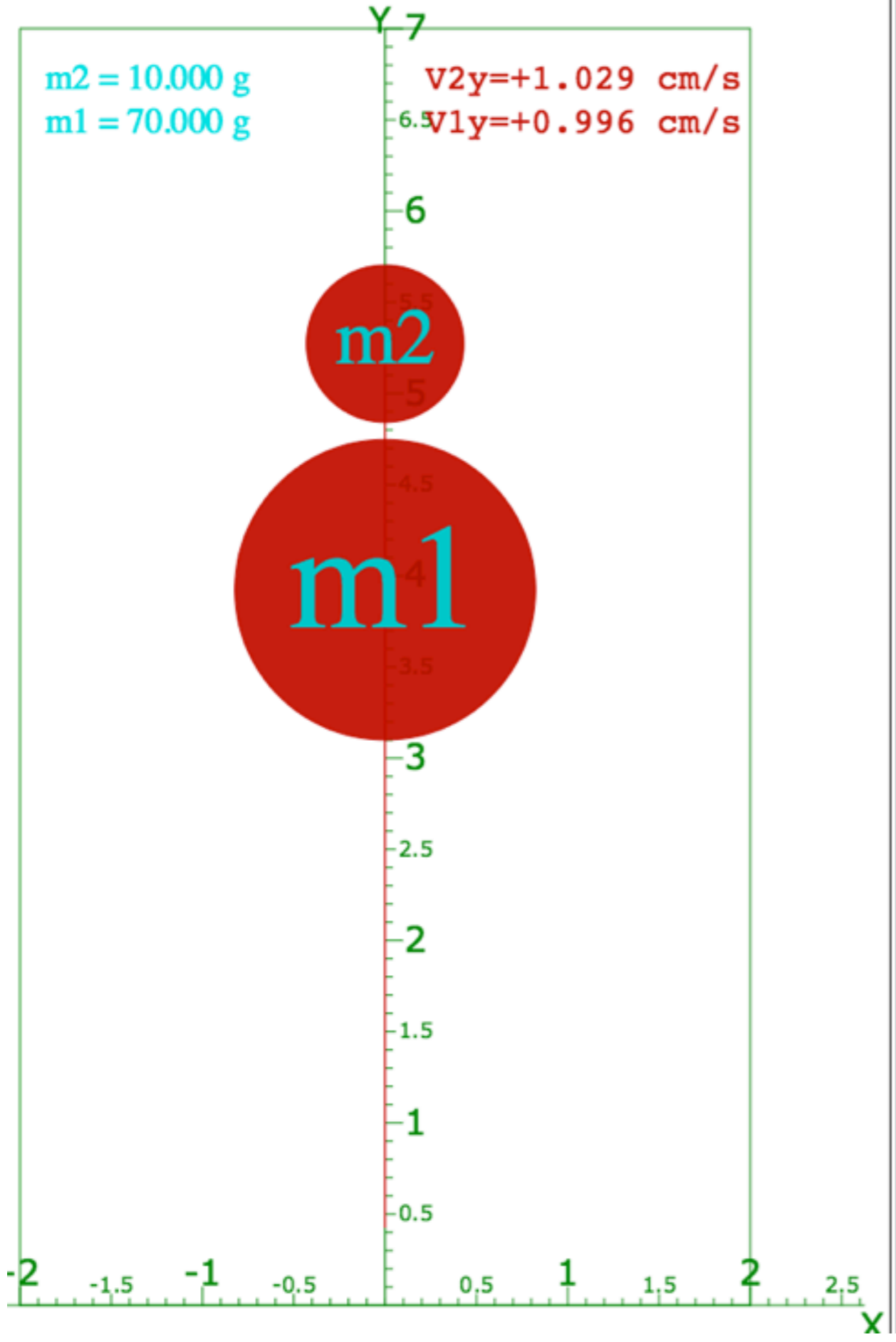
F-Vector scale =  0.003 V2y Max =  2.6944

Error step =  0.0000 V2y Min =  -2.694

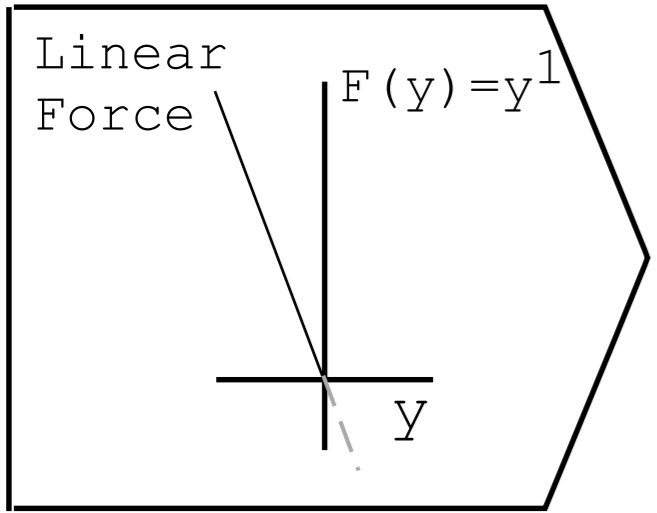
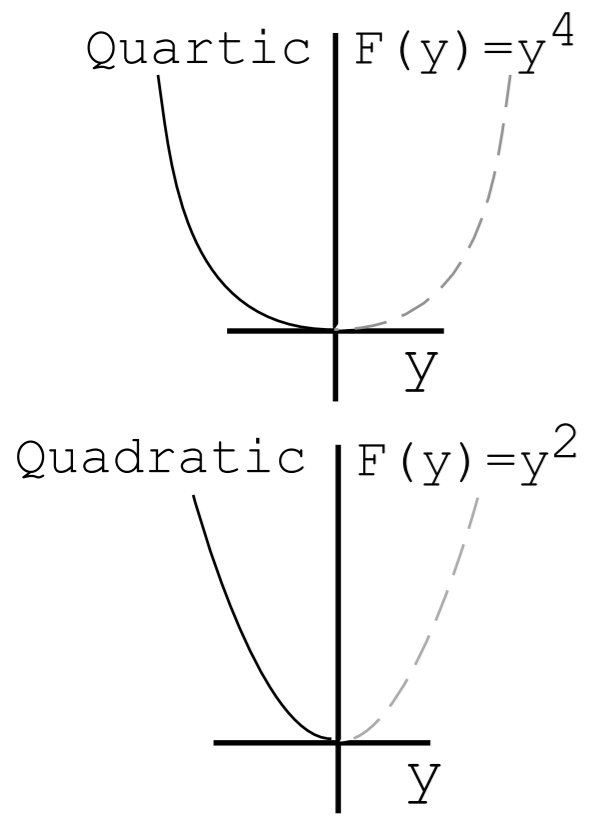
Mass factor =  1

M<sub>1</sub> =  7 x10<sup>^</sup>  1 {g} V<sub>1</sub> =  -1 x10<sup>^</sup>  0 {cm/s}

M<sub>2</sub> =  1 x10<sup>^</sup>  1 {g} V<sub>2</sub> =  -1 x10<sup>^</sup>  0 {cm/s}



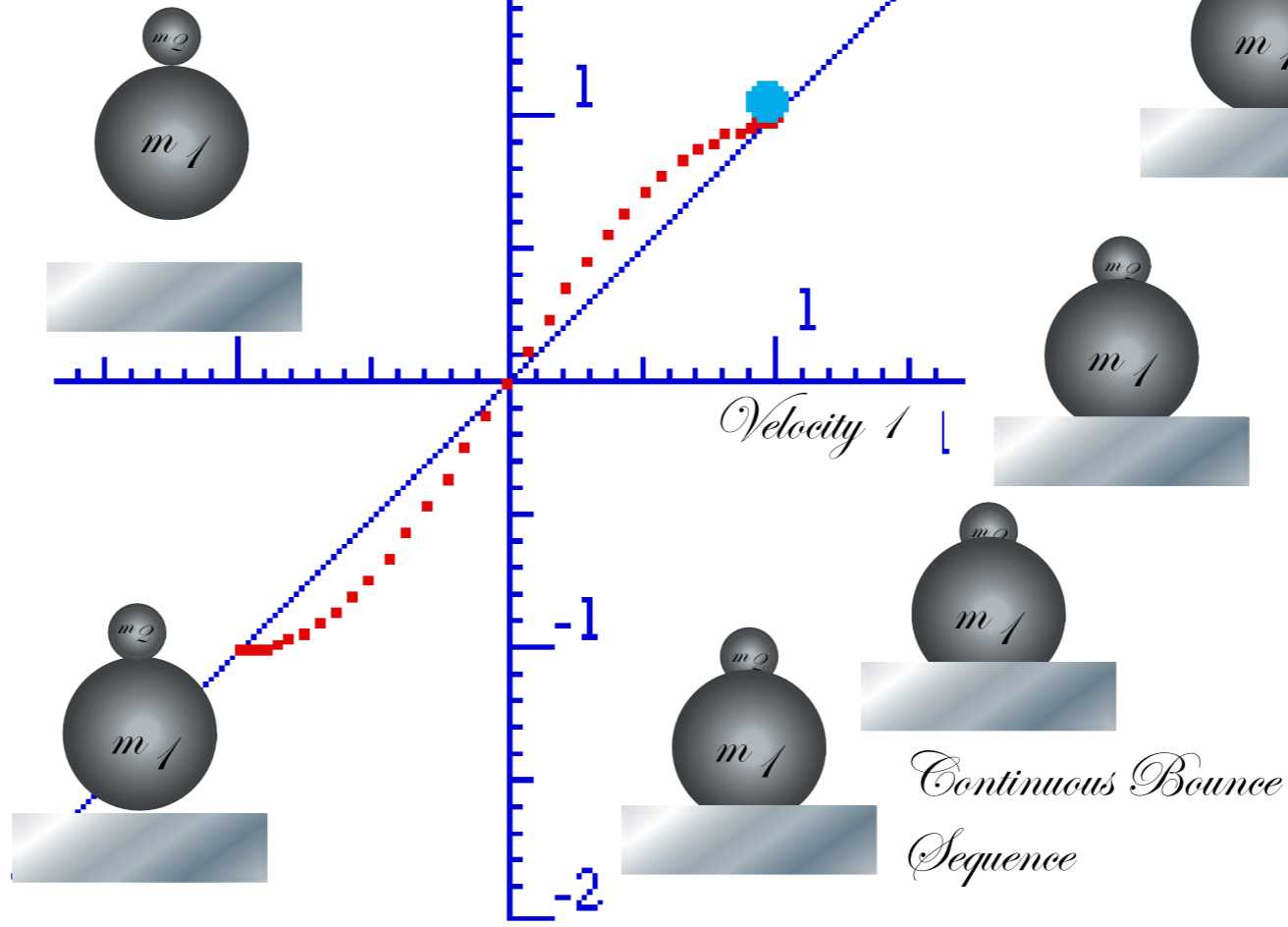
Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"



*Cra Rumpny Ltd* 3

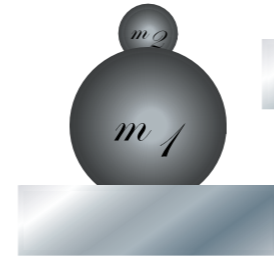
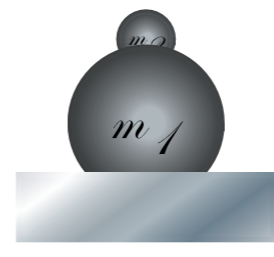
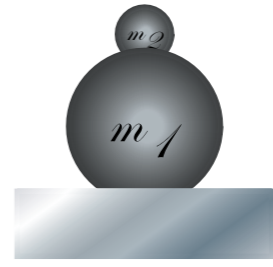
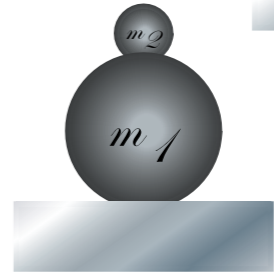
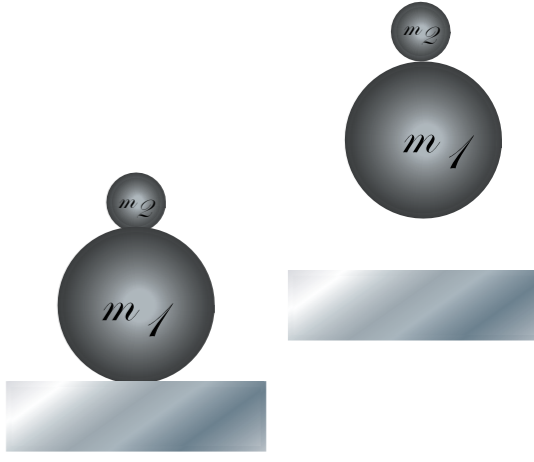
*Linear Force Field Simulation*

Quite surprising "non-effect" !  
Why?



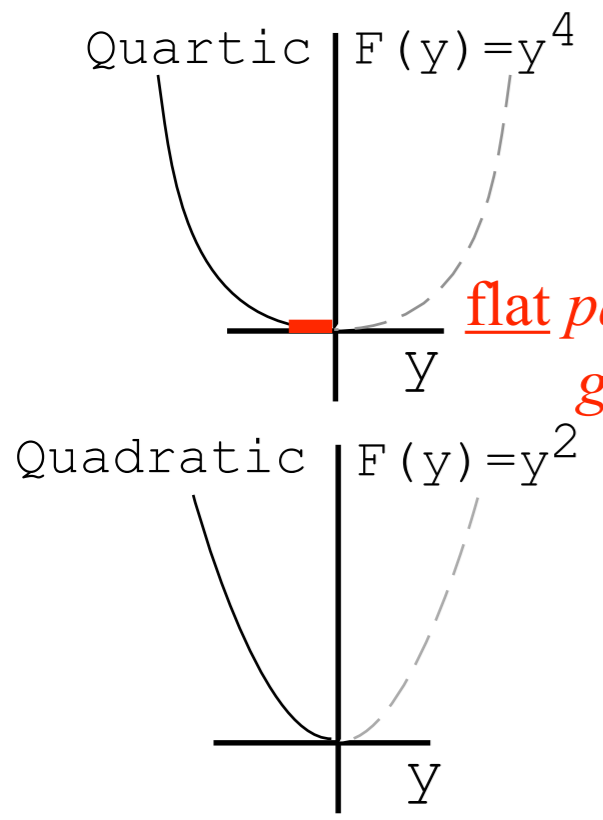
$V_2 = 1.03$

$V_1 = 0.996$



Unit 1  
Fig. 7.7

Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

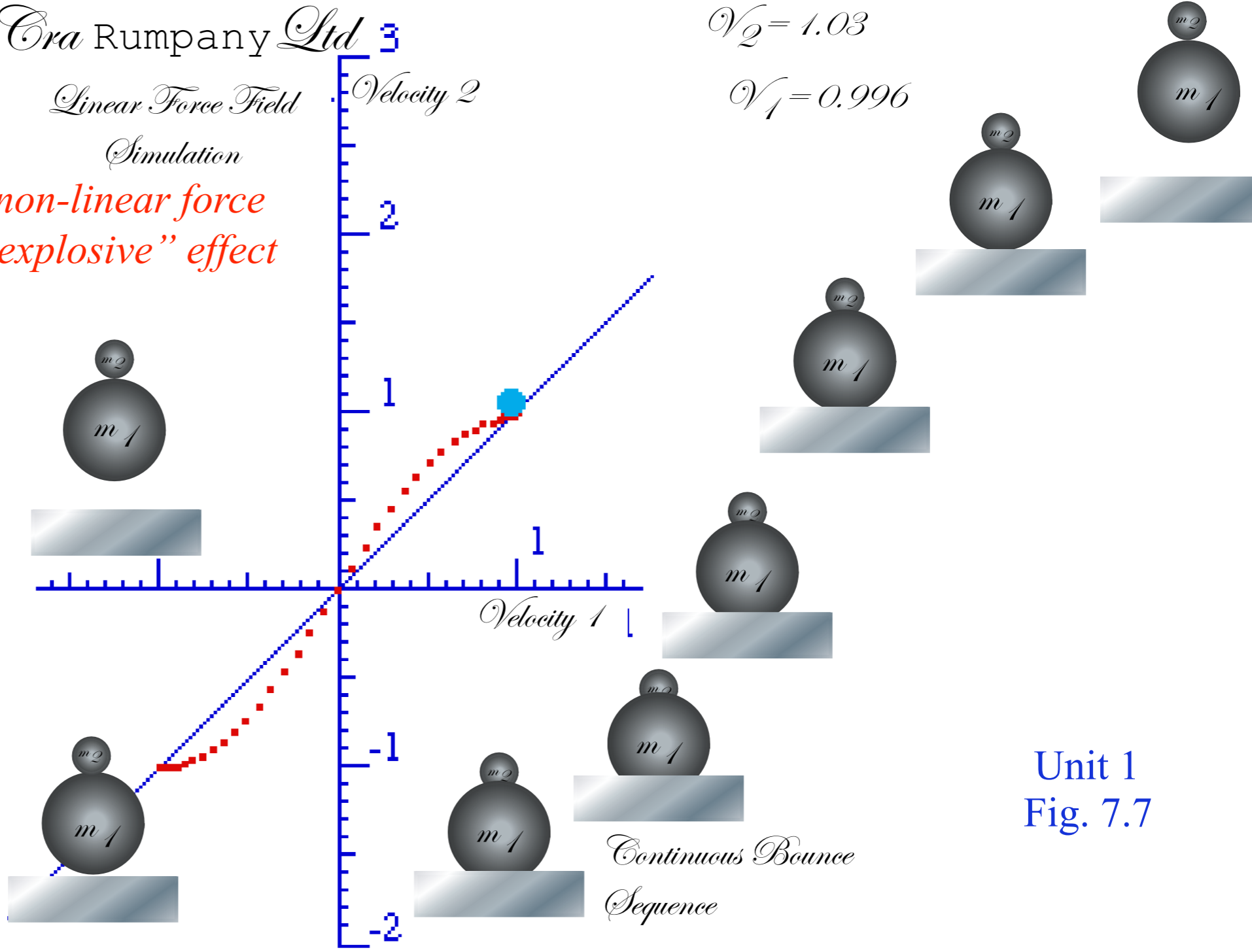
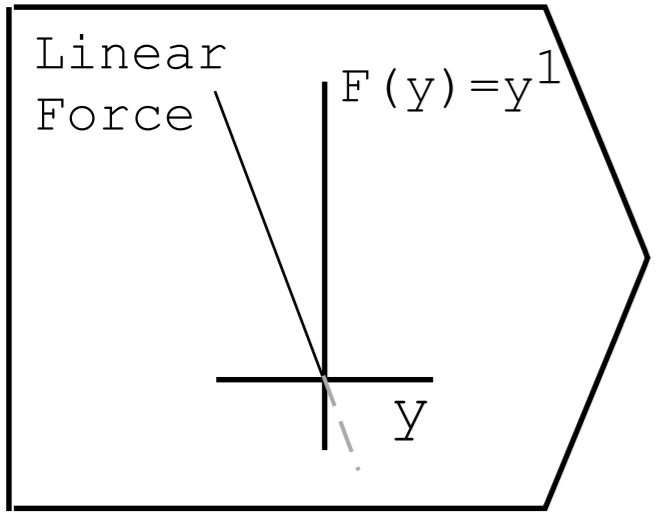


*flat part of non-linear force gives "explosive" effect*

*Cra Rumpany Ltd 3*

*Linear Force Field Simulation*

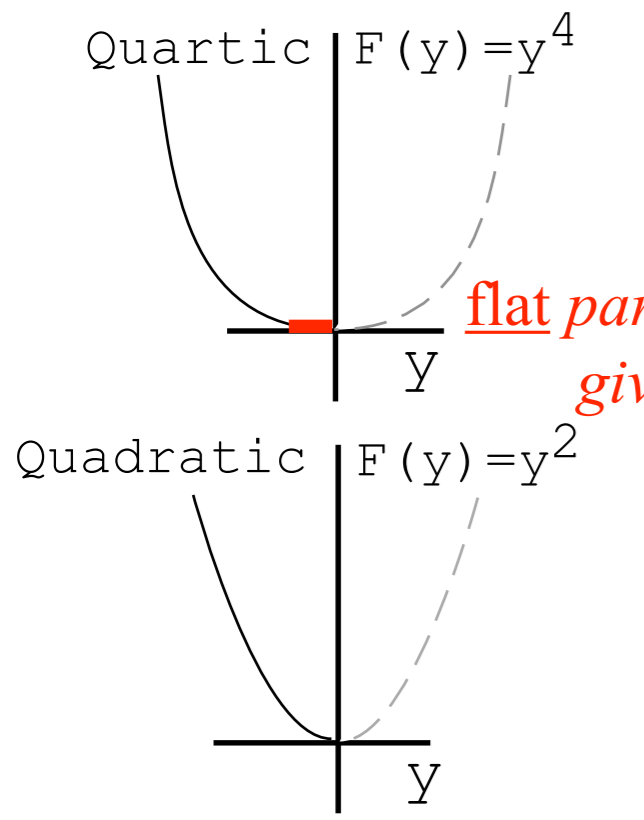
$V_2 = 1.03$   
 $V_1 = 0.996$



*Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force  $F(y) = ky$*

Unit 1  
Fig. 7.7

Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

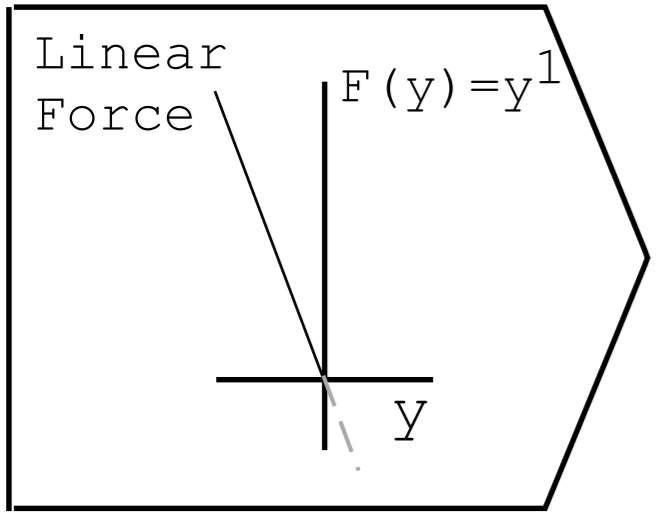
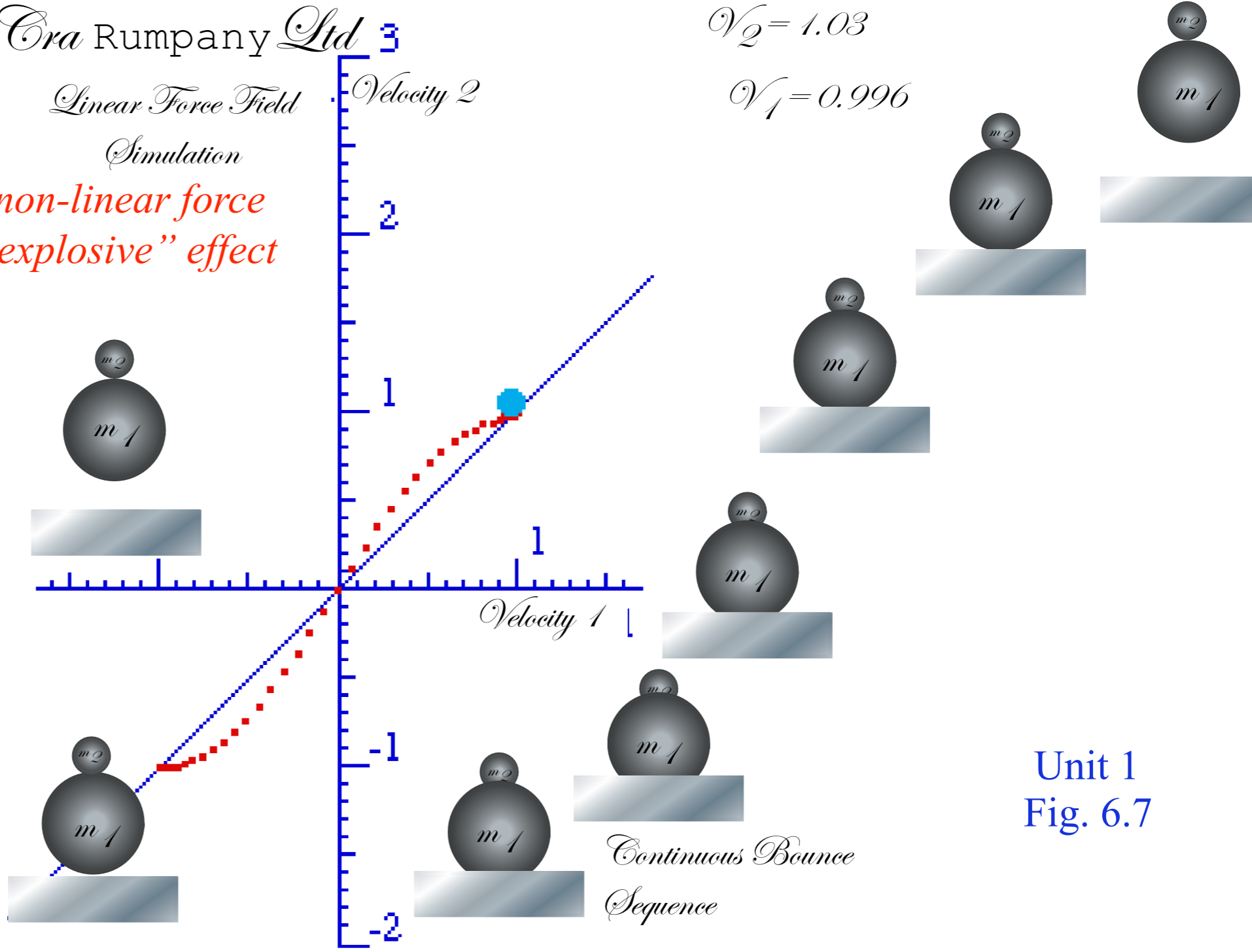


*flat part of non-linear force gives "explosive" effect*

*Cra Rumpany Ltd 3*

*Linear Force Field Simulation*

$V_2 = 1.03$   
 $V_1 = 0.996$



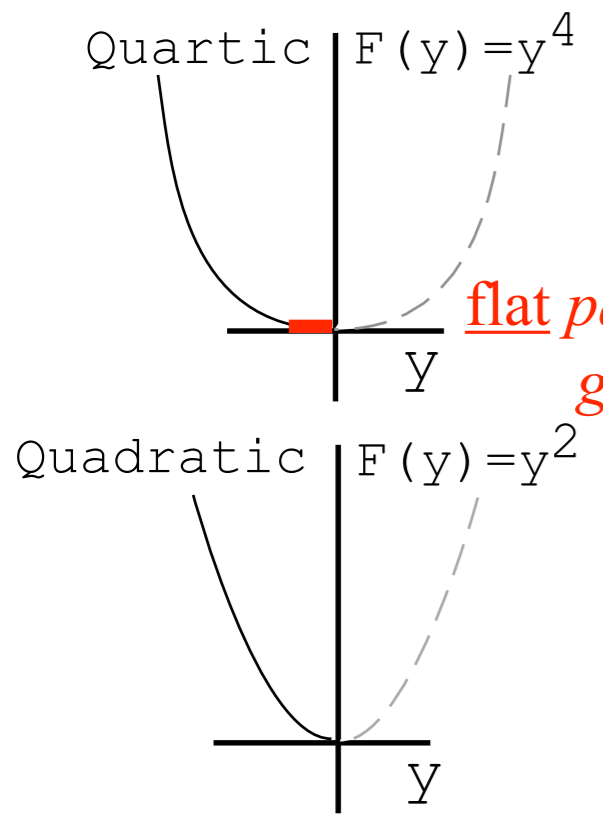
*Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force  $F(y) = ky$*

*Lesson: Fasten your seatbelt*

Unit 1  
Fig. 6.7



Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

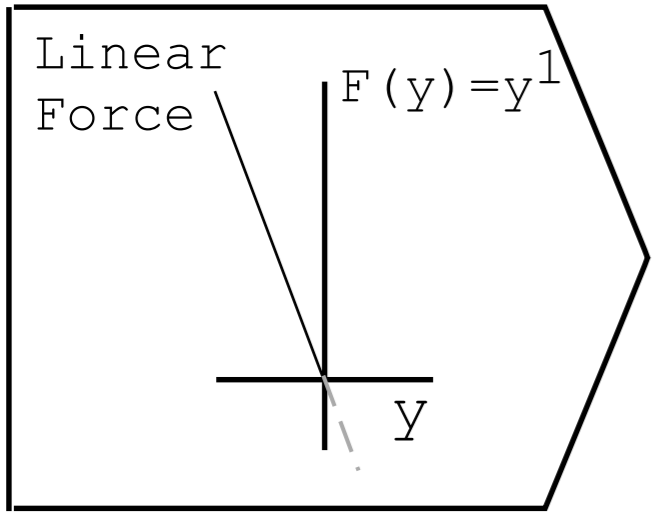
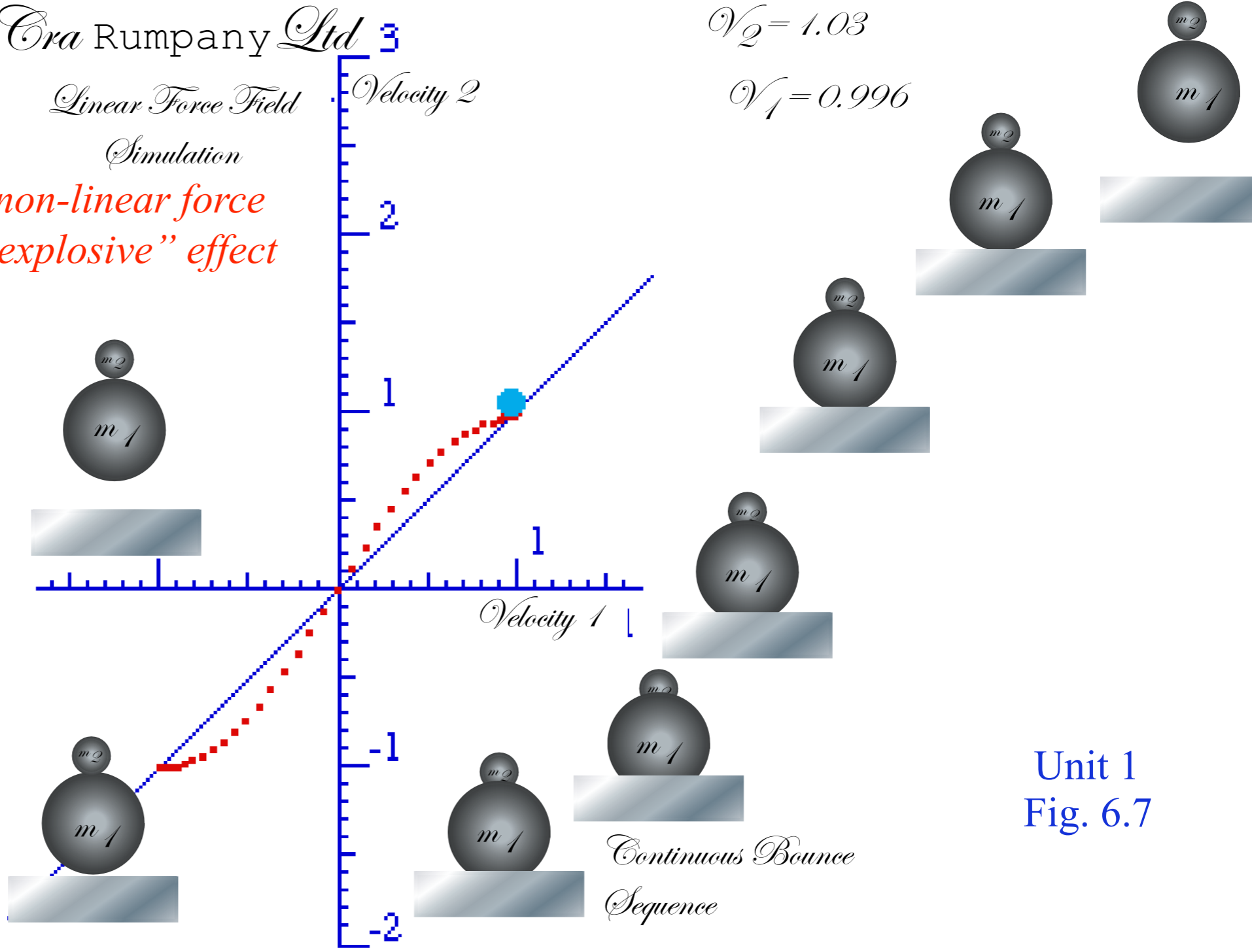


*flat part of non-linear force gives "explosive" effect*

*Cra Rumpany Ltd 3*

*Linear Force Field Simulation*

$V_2 = 1.03$   
 $V_1 = 0.996$



*Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force  $F(y) = ky$*

*Lesson: Fasten your seatbelt TIGHTLY!*

Unit 1  
Fig. 6.7

# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force  $F=-k$  (linear potential  $V=kx$ )

Some physics of dare-devil-diving 80 ft. into kidee pool

(Simulations)

(b) Linear force  $F=-kx$  (quadratic potential  $V=\frac{1}{2}kx^2$  (like balloon))

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potential analysis of

2, 3, and 4 body towers)

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric  $m_1, m_2, m_3, \dots$  series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

## Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Velocity Amplification in Collision Experiments Involving Superballs <Link>

CLASS OF WILLIAM G. HARTER\*  
 University of Southern California  
 Los Angeles, California 90007  
 (Received 25 September 1969; revised 25 September 1970)

*If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.*

\* The members of the class of Dr. William G. Harter included: Calvin W. Gray, Jr., Robert C. Frickman, Brian P. Harney, Steven H. Hendrickson, Scott T. Jacks, David F. Judy, William D. Koltun, Sam C. Kaplan, Morton J. Kern, Edmund H. Kwan, Wayne E. Long, Michael E. Mason, William D. Moore, Willard W. Mosier, Gary P. Rudolf, Henry G. Rosenthal, William F. Skinner, Jay L. Stearn, Michael Weinberg, Mark Weiner, Frank J. Wilkinson, and David Willner.

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**INTRODUCTION**

Shortly after the well-known Superball<sup>1</sup> appeared on the market, one of the authors quite accidentally discovered a surprising effect.<sup>2</sup> The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

<sup>1</sup> Trade name of product by Whammo Manufacturing Co., San Gabriel, Calif.

*Class of W. G. Harter*

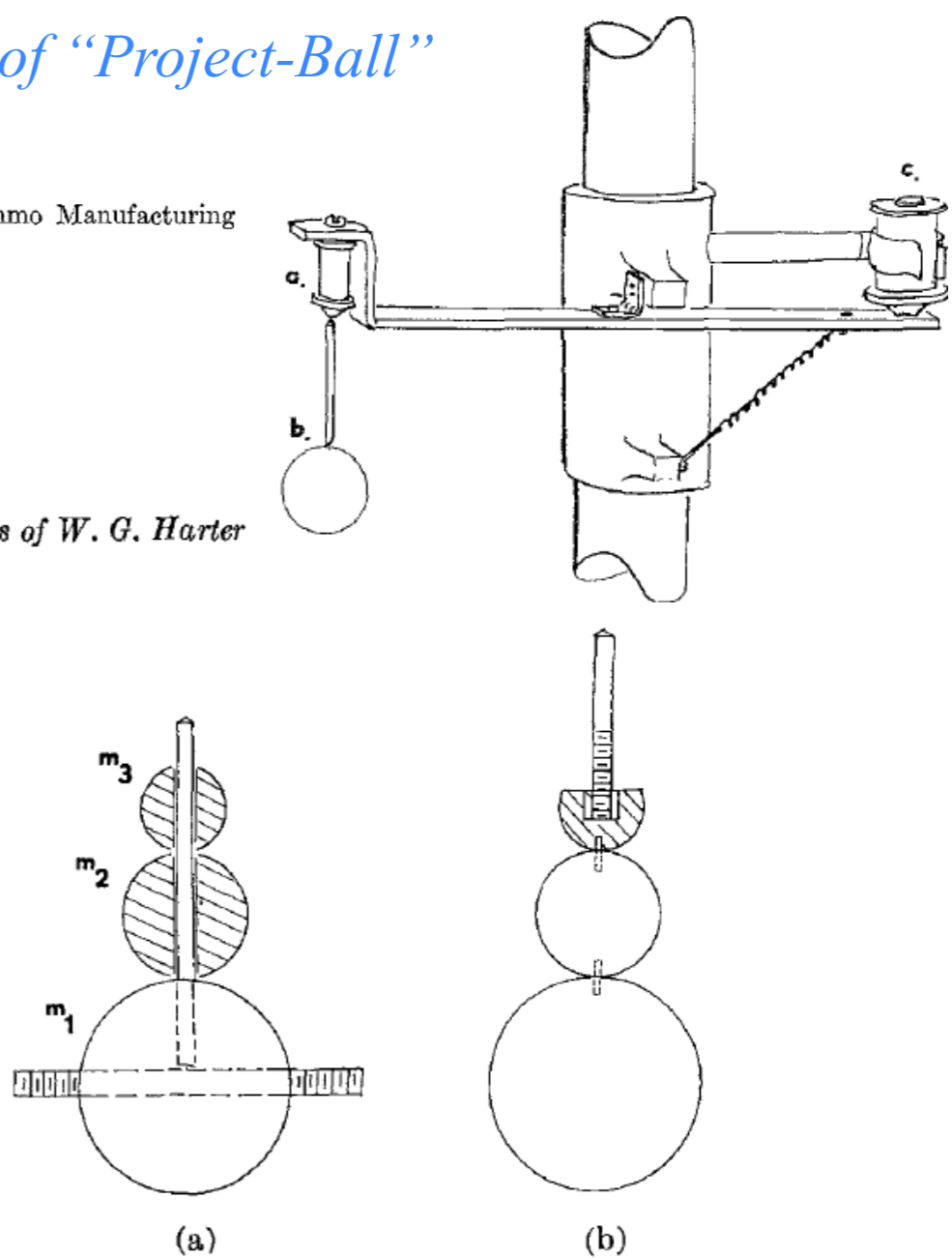


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

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*Much later....*

*Lots of profs try this out...*

*...including the unfortunate Harvard professor M. Tinkham...*

*( Still trying to find the video of the Tinkham incident...)*

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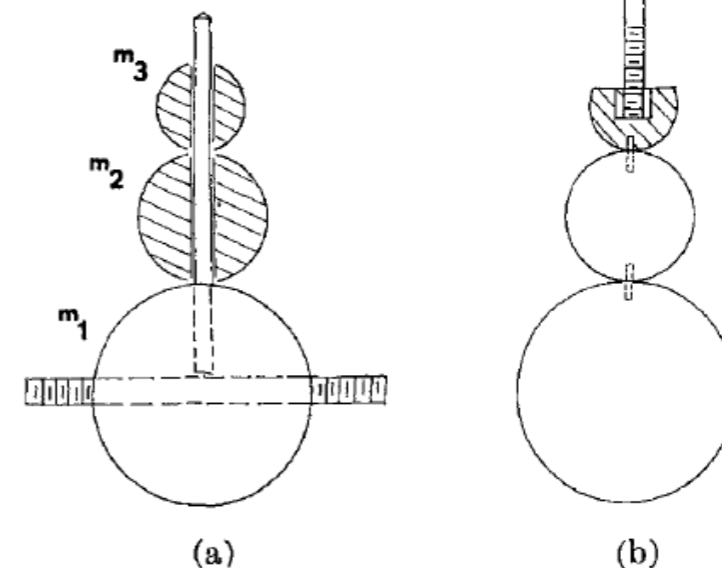


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

### Basketball and Tennis Ball

Dropping a tennis ball on top of a basketball causes the tennis ball to bounce very high.

Source: [8.01 Physics I: Classical Mechanics, Fall 1999](#)  
Prof. Walter Lewin

Course Material Related to This Topic:

- Watch [video clip from Lecture 17 \(21:30 - 24:08\)](#)

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*...and some results of “Project-Ball”*

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*Somebody drops a box of balls that immediately bounce into the wet paint.*

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*The rest is history.*

*Little paint spots on floor show what was wrong with our fancy-pants computer theory*

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The rest is history.

Little paint spots on floor show what was wrong with our fancy-pants computer theory.

The engineering curves were isothermal not adiabatic.

Need latter. Can do latter by dropping dyed balls and measuring spot-size.

Measuring spot-size  $d$  gives energy vs. height.  
Slope of  $E(x)$  gives force  $F(x)$  and  $G(x)$ .

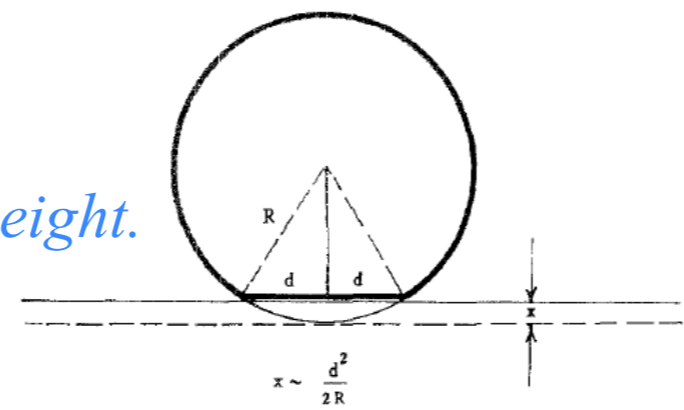
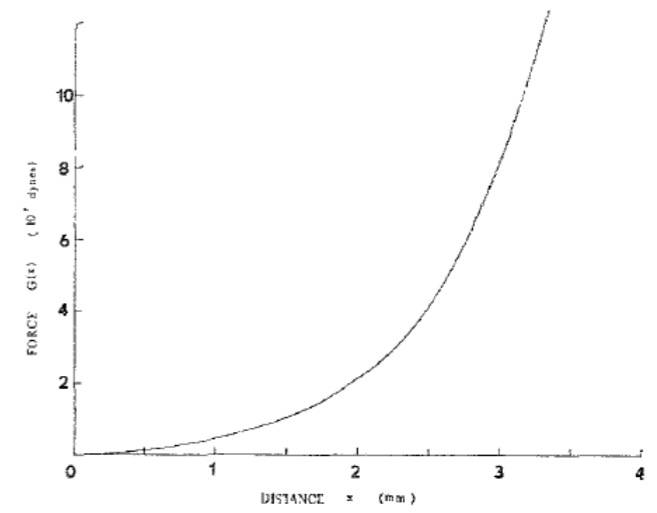


FIG. 10. Sagittal formula.

Collisions Involving Superballs



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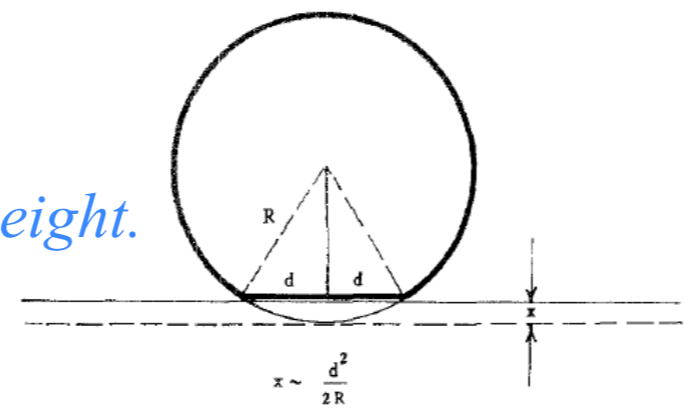


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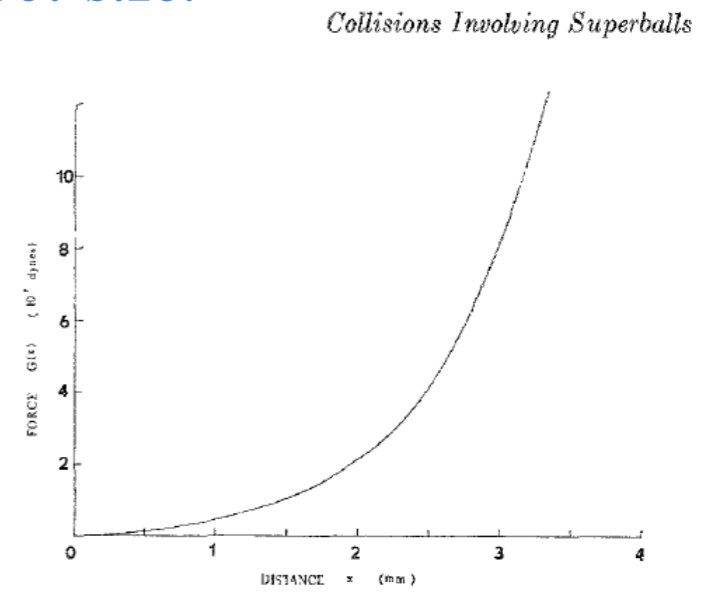


FIG. 12. Adiabatic force function  $G(x)$ .

If  $F(x)$  and  $G(x)$  were linear for all  $x$ , then the

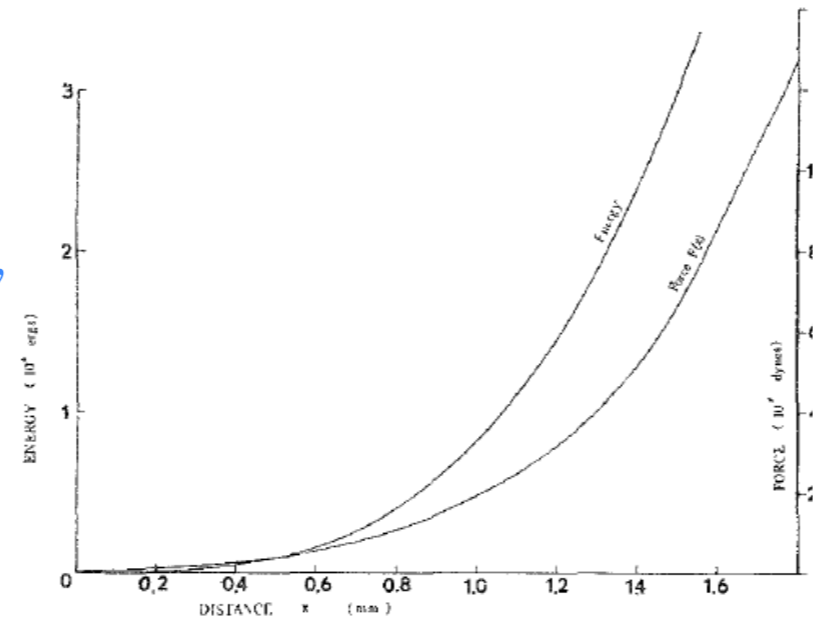


FIG. 11. Adiabatic force  $F(x)$  and energy curves for Superball.

*Then fancy-pants computer theory can predict N-ball tower bounce*

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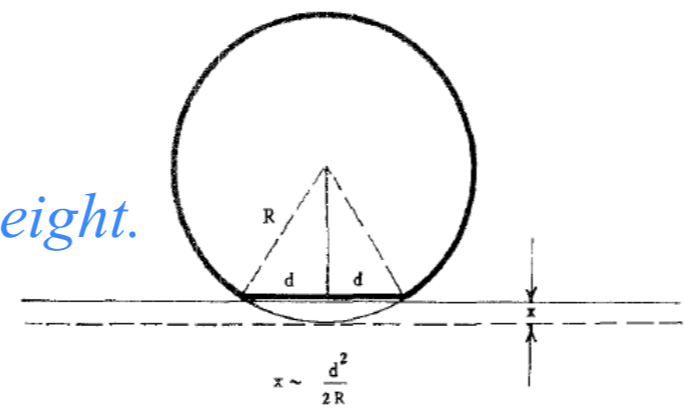


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Collisions Involving Superballs

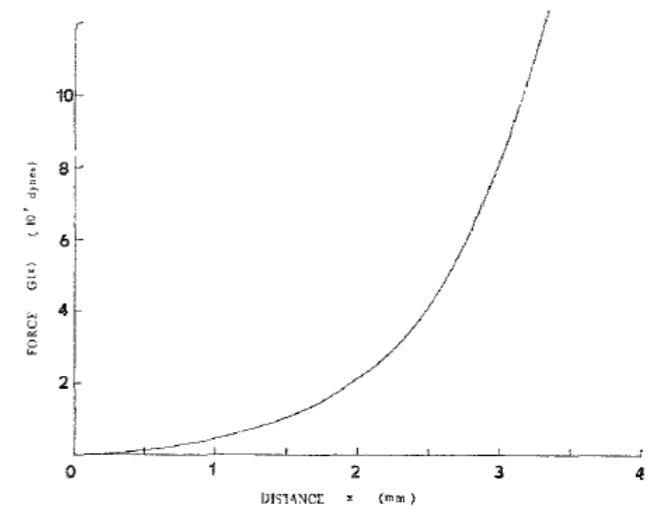


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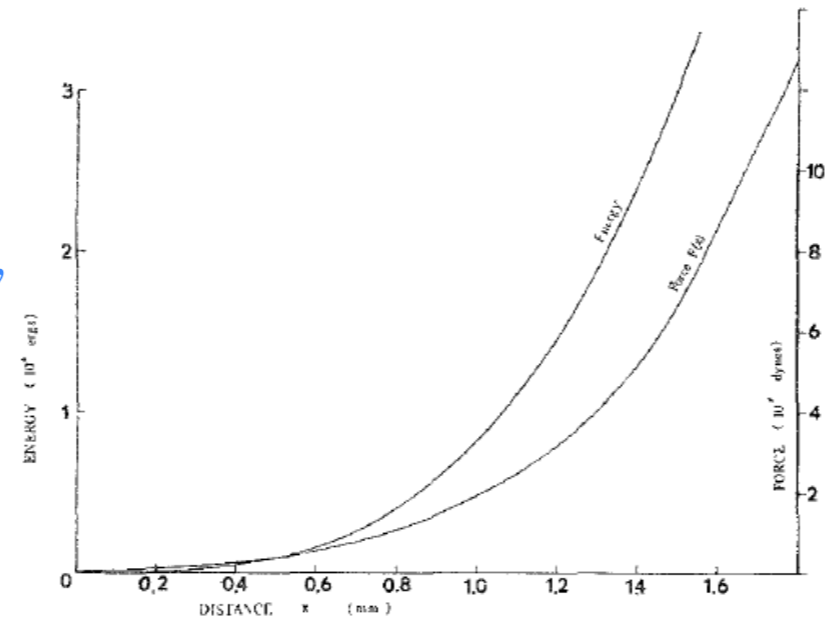


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Functions  $F(x)$  and  $G(x)$  were then placed on the function generators of the analog computer.

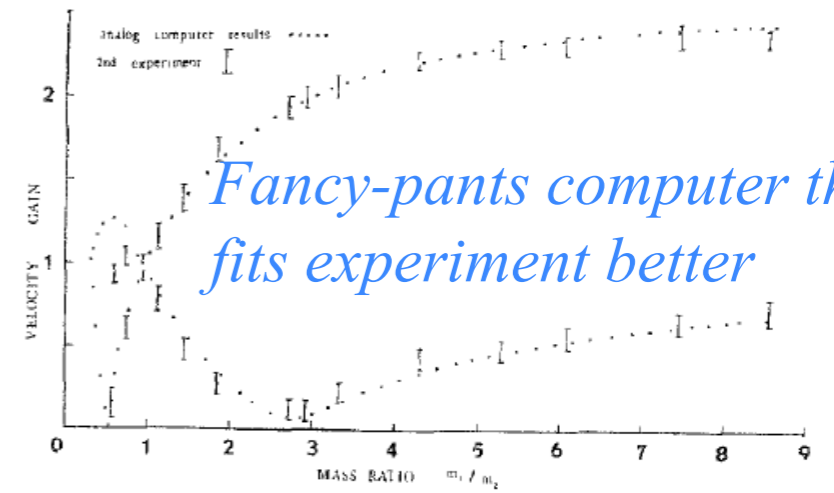


FIG. 13. Comparison between analog computer gain curves and second experiment.

*Then fancy-pants computer theory can predict N-ball tower bounce*

*Fancy-pants computer theory fits experiment better*

*\* Am. J. Phys. 39, 656 (1971)*

*Then fancy-pants computer theory can predict N-ball tower bounces*

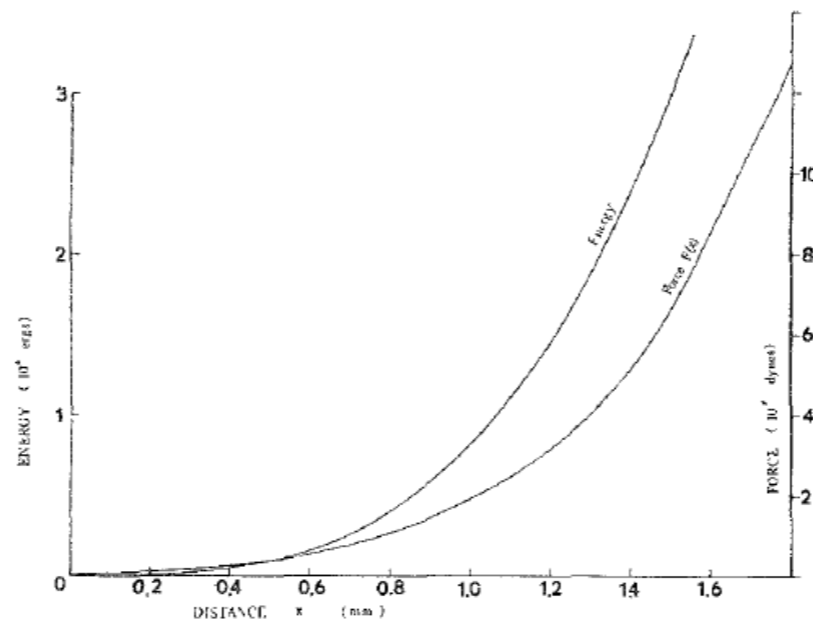
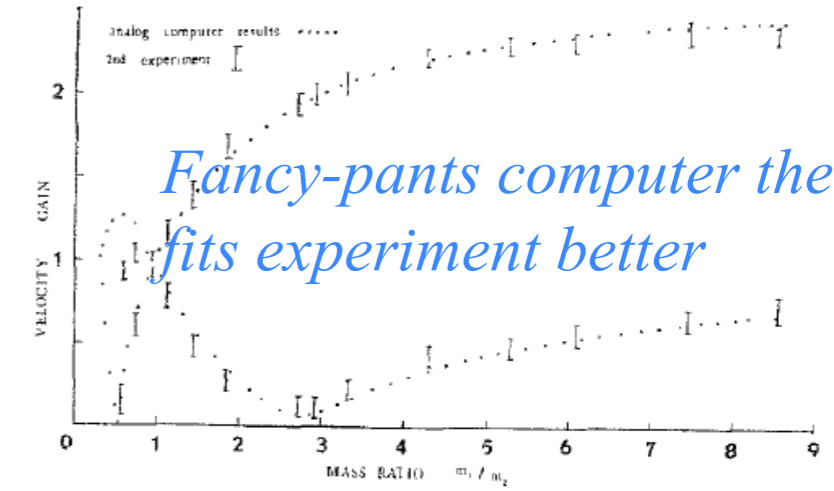


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*Fancy-pants computer theory fits experiment better*

FIG. 13. Comparison between analog computer gain curves and second experiment.

*Here are some 3-ball tower bounce predictions*

*AJP Volume 39 / 661*

*Class of W. G. Harter*

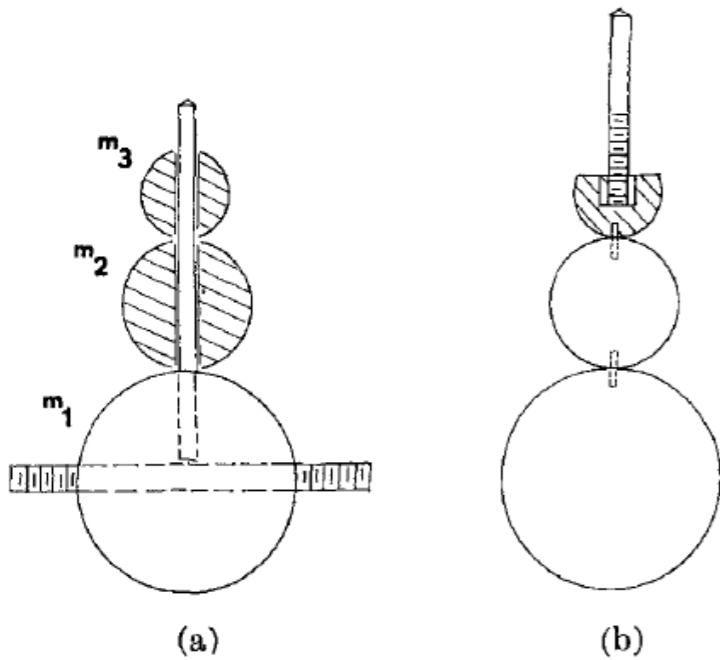


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

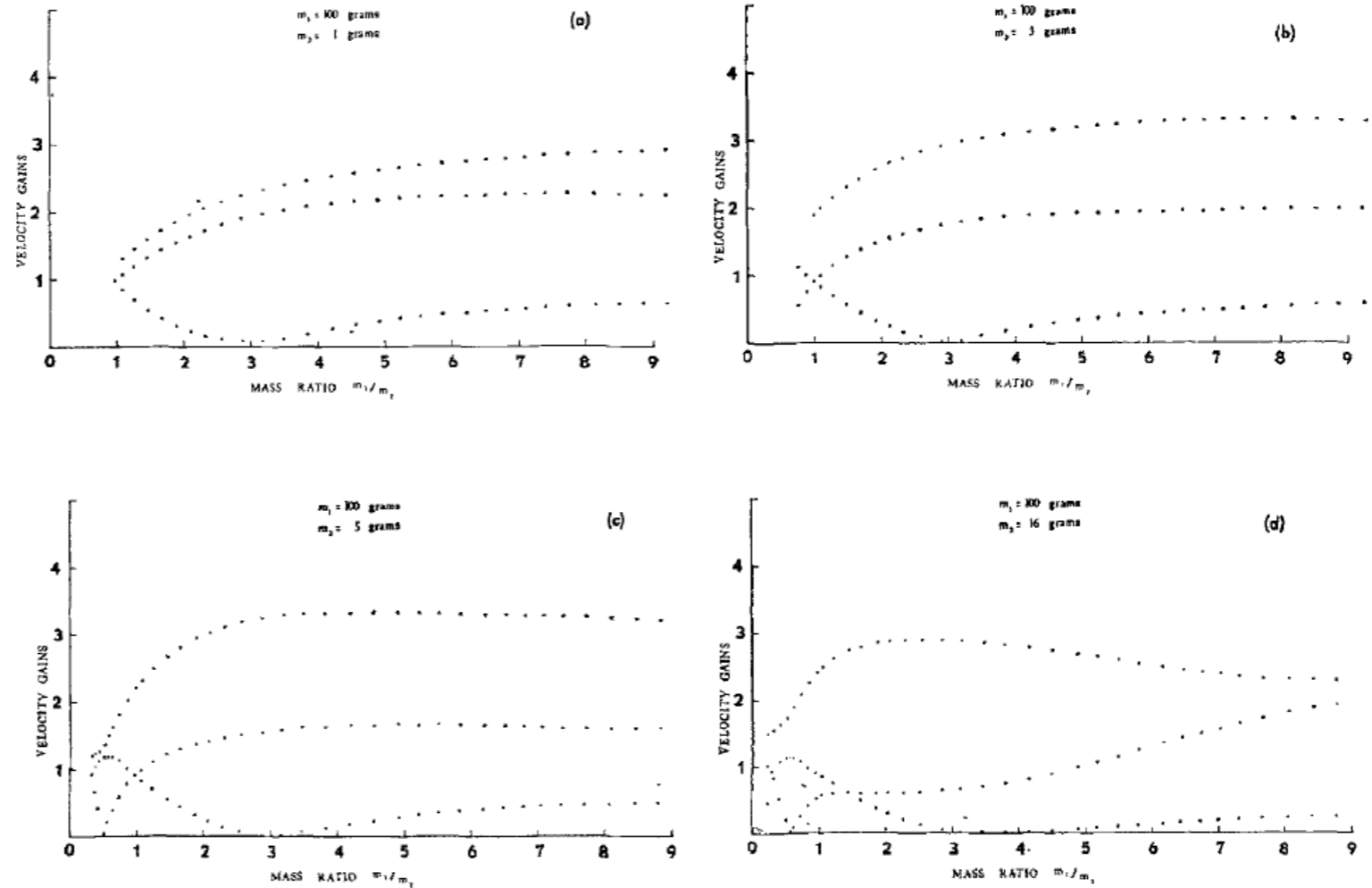


FIG. 15. (a)–(d) Analog computer output for velocity gains of three-ball system.



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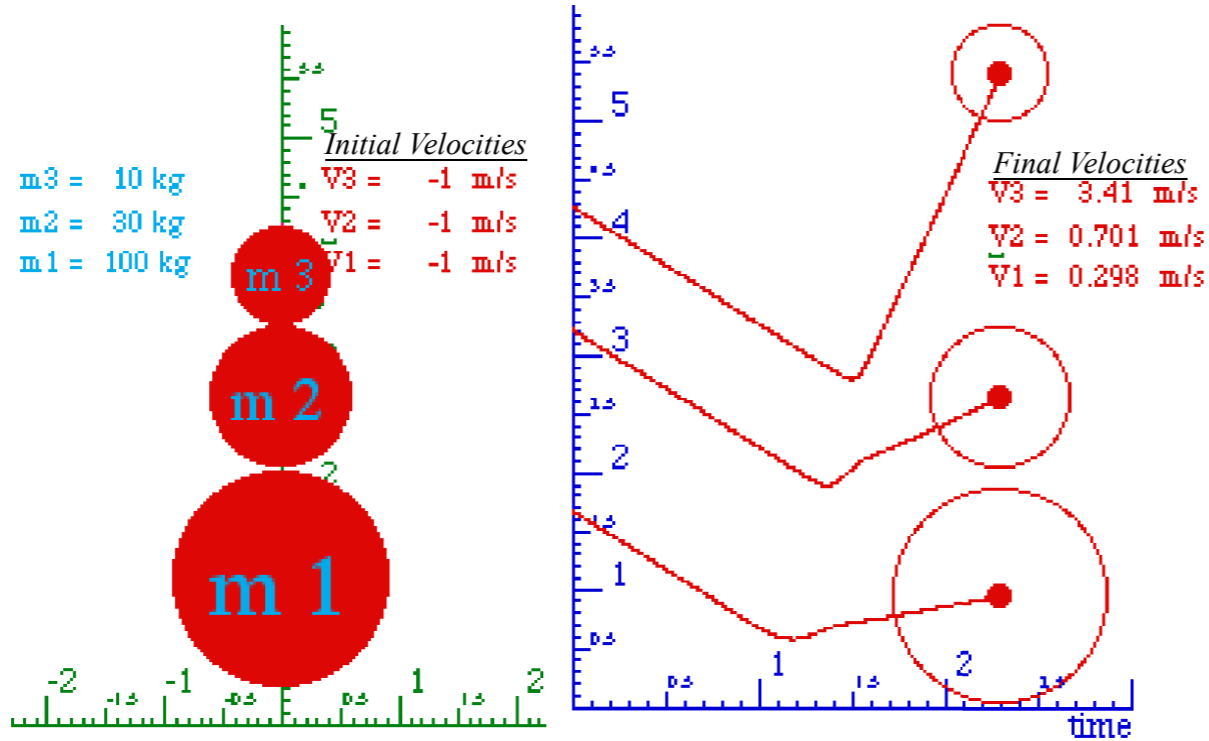
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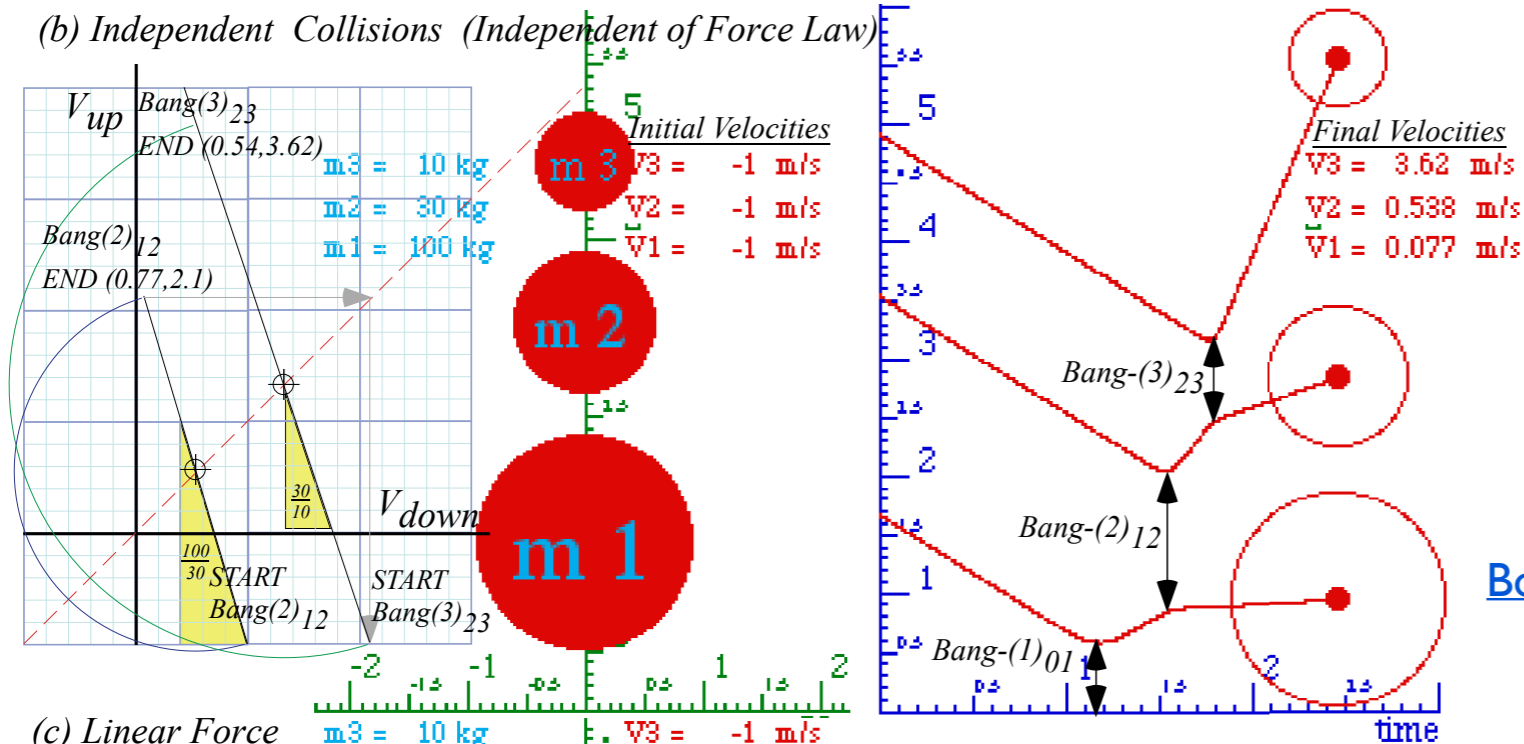
(a) *Quartic Force*  
 $F(y) = k y^4$



Unit 1  
 Fig. 8.1a-c  
*Independent Bang Model*  
 (IBM)  
*3-Body Geometry*

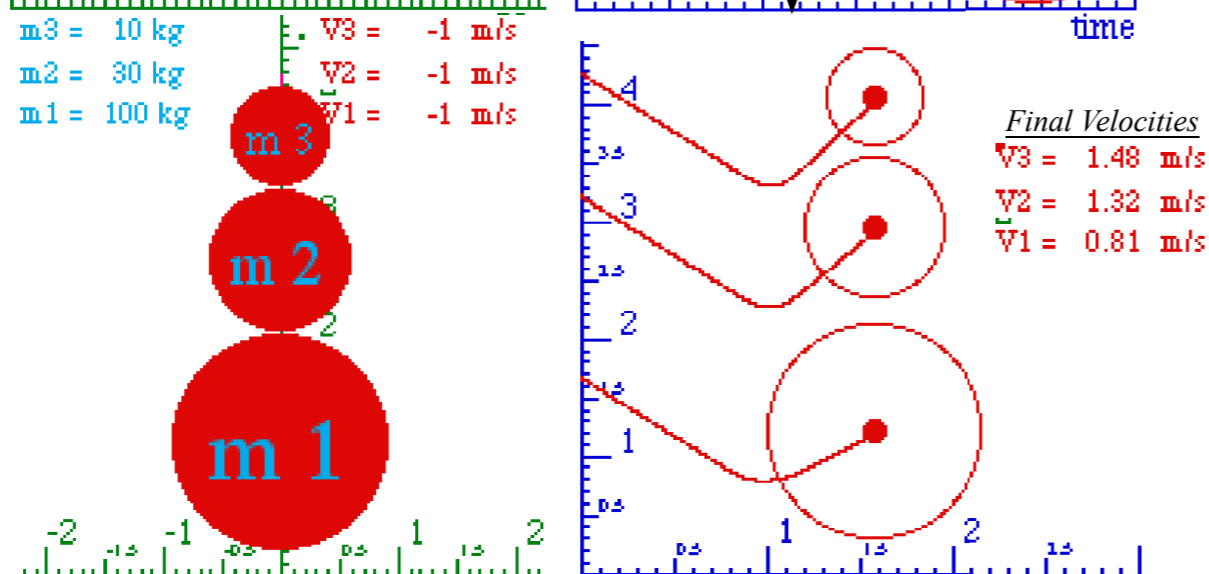
Bouncelt Simulation: 3-Ball Tower w/ Quartic Force

(b) *Independent Collisions (Independent of Force Law)*



Bouncelt Simulation: 3-Ball (Gapped) Tower w/ Linear Force

(c) *Linear Force*  
 $F(y) = k y$



Bouncelt Simulation: 3-Ball Tower w/ Linear Force

Unit 1  
Fig. 8.1b

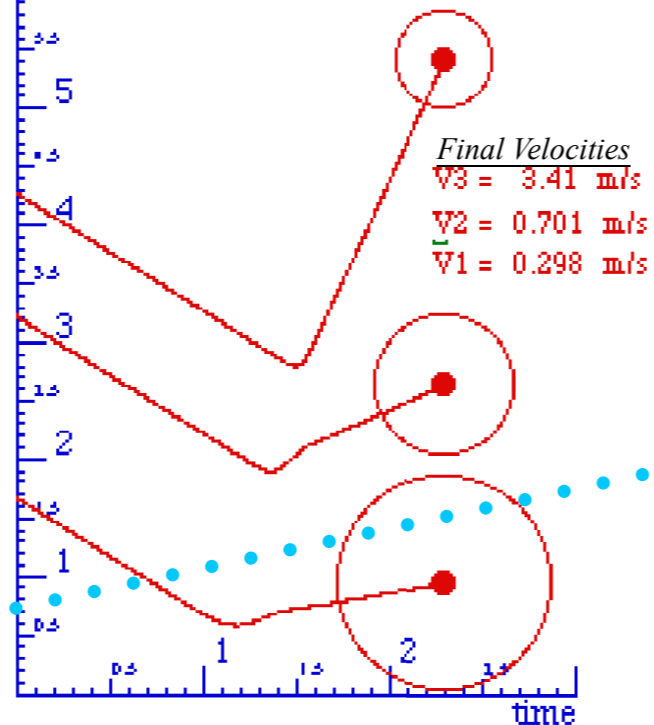
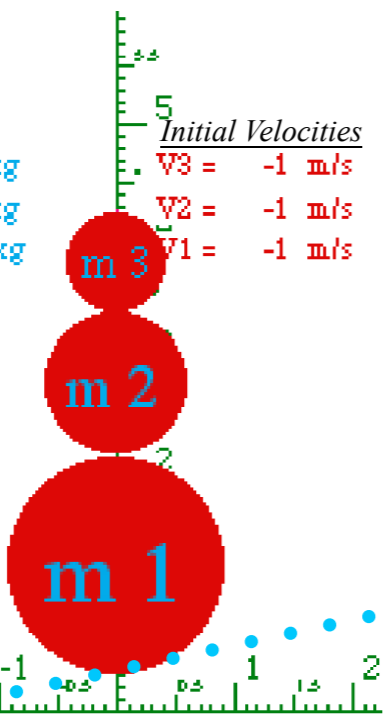
*Independent Bang Model  
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(a) *Quartic Force*  
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m3 = 10 kg  
m2 = 30 kg  
m1 = 100 kg

*Initial Velocities*  
V3 = -1 m/s  
V2 = -1 m/s  
V1 = -1 m/s

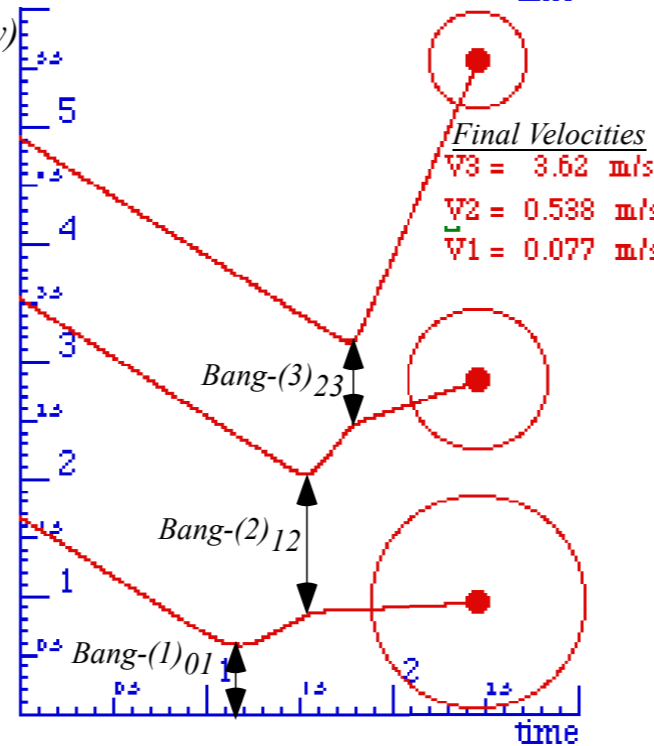
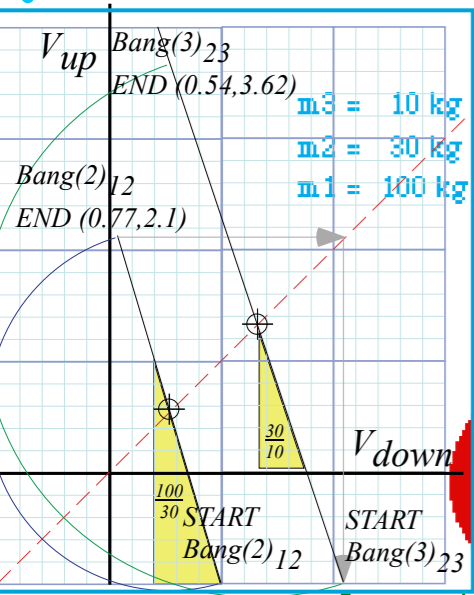
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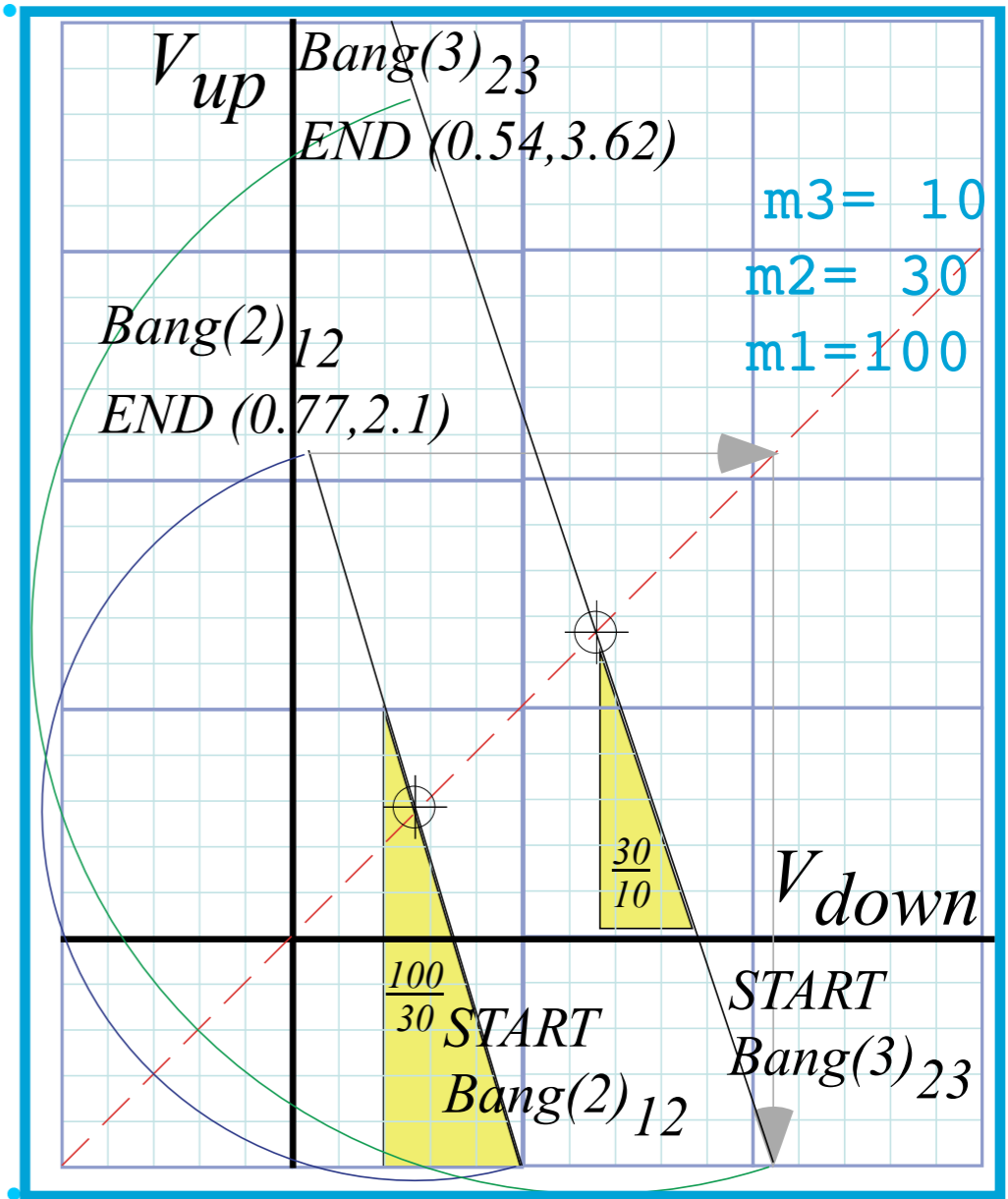
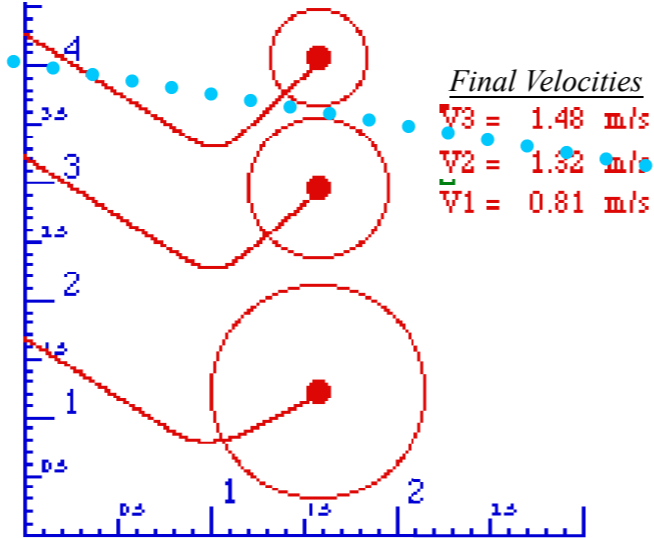
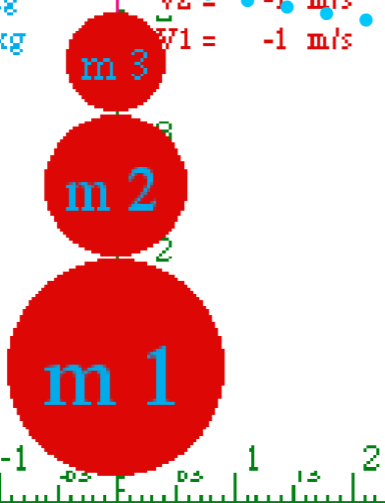
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(c) *Linear Force*  
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m2 = 30 kg  
m1 = 100 kg

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Bouncelt Simulation: 3-Ball Tower w/ Linear Force

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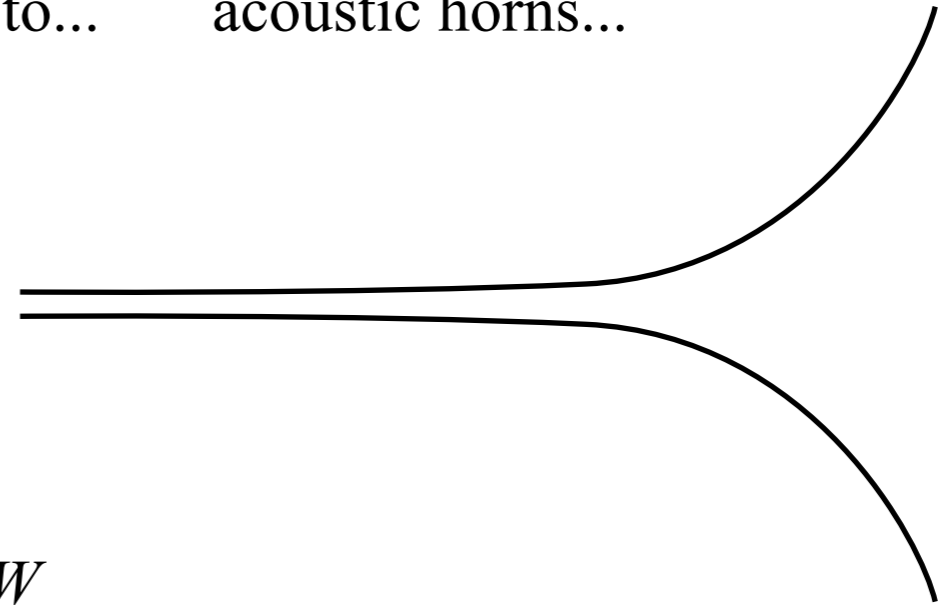
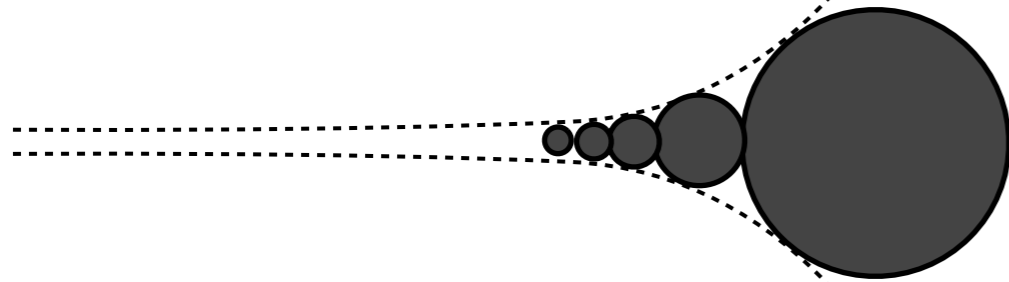
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Superball towers...

analogous to...

acoustic horns...



*small&fast... impedance matched to... BIG&SLOW*

<sup>6</sup>J. B. Hart and R. B. Herrmann, Amer. J. Phys. **36**,  
46 (1968).

[https://www.researchgate.net/publication/243487193\\_Energy\\_Transfer\\_in\\_One-Dimensional\\_Collisions\\_of\\_Many\\_Objects](https://www.researchgate.net/publication/243487193_Energy_Transfer_in_One-Dimensional_Collisions_of_Many_Objects)

### 1.8.3 *The optimal idler (An algebra/calculus problem)*

To get highest final  $v_3$  of mass  $m_3$  find optimum mass  $m_2$  in terms of masses  $m_1$  and  $m_3$  that does that.

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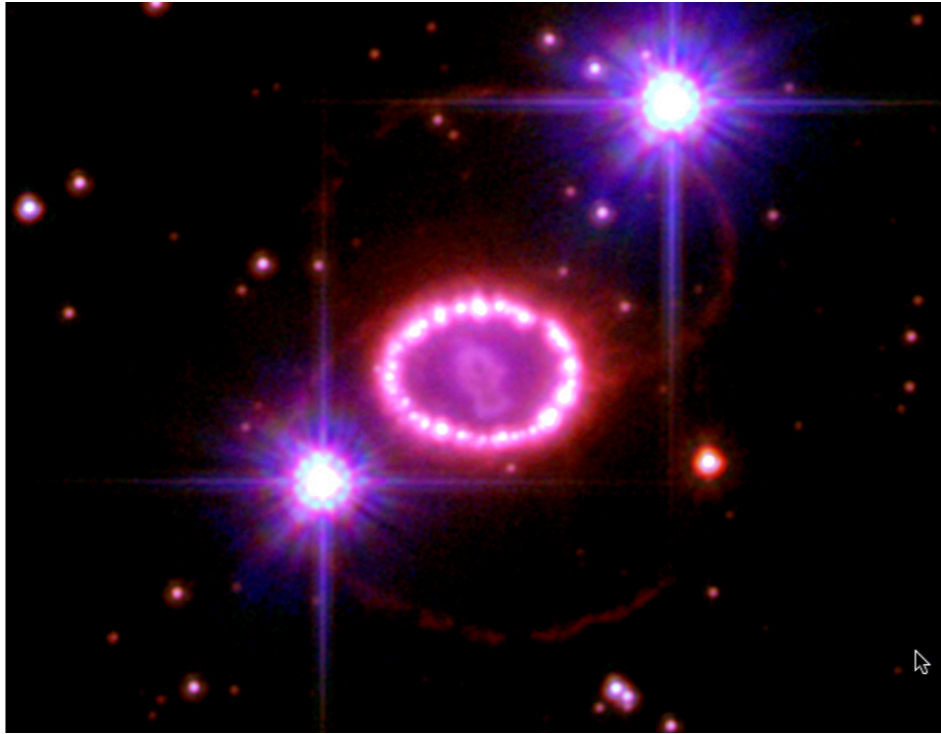
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# A story of Stirling Colgate (Palmolive) and core-collapse supernovae

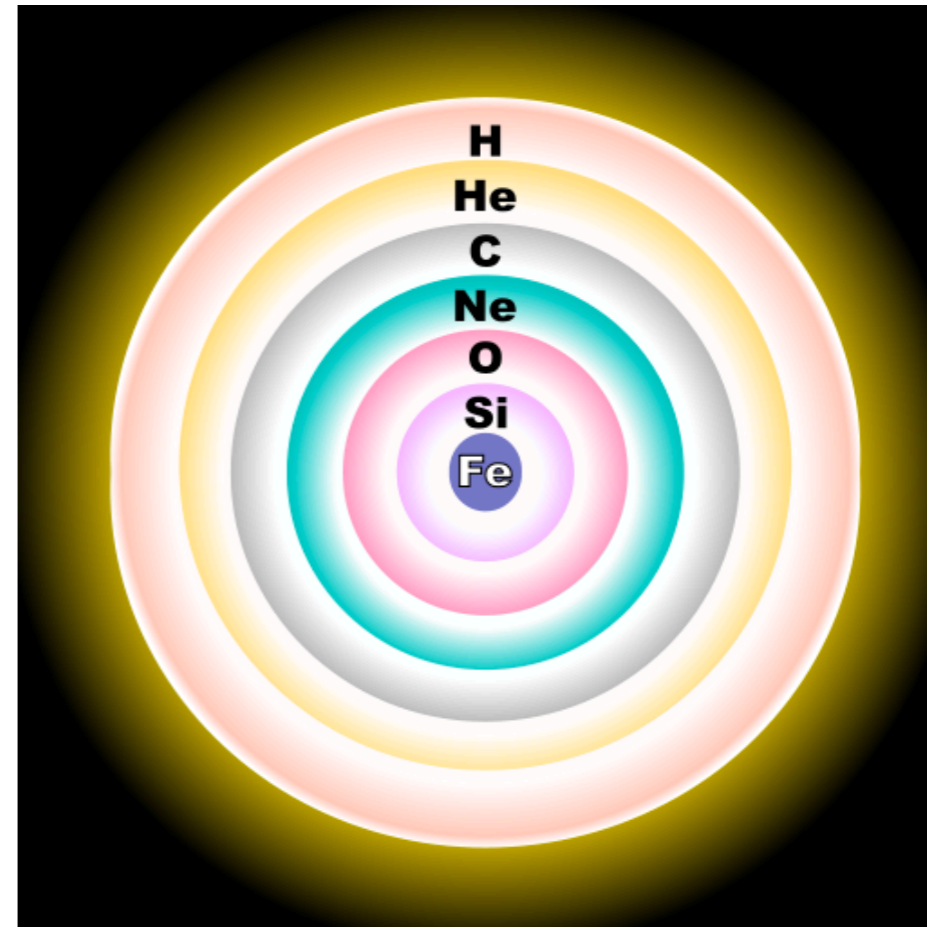


Source

<http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/>

Author

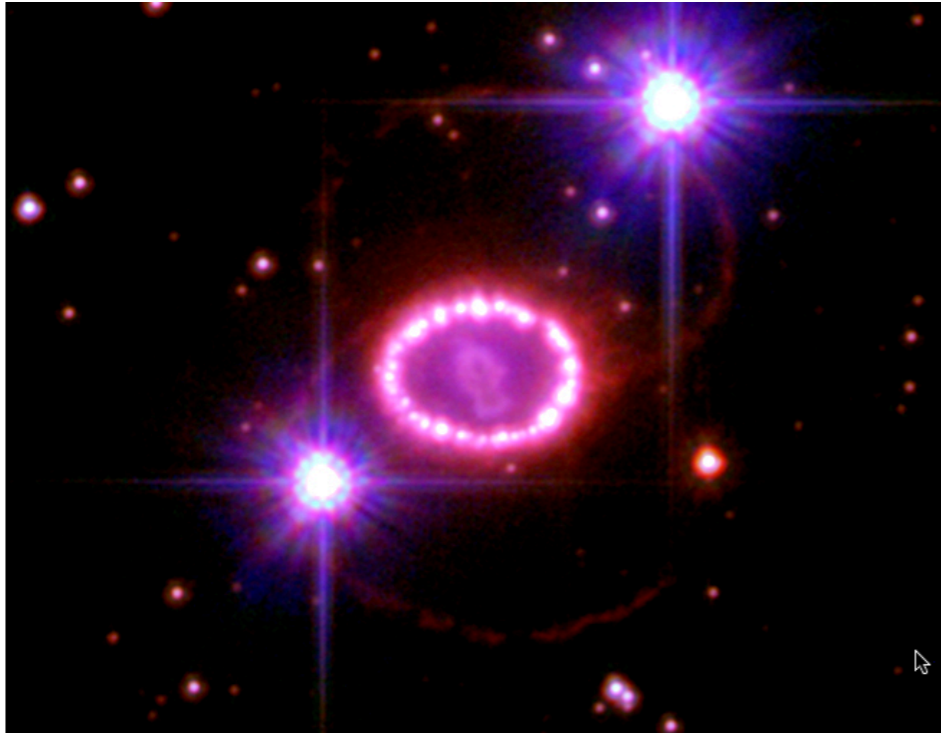
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

| Process                 | Main fuel | Main products             | 25 M <sub>☉</sub> star <sup>[6]</sup> |                              |                       |
|-------------------------|-----------|---------------------------|---------------------------------------|------------------------------|-----------------------|
|                         |           |                           | Temperature (Kelvin)                  | Density (g/cm <sup>3</sup> ) | Duration              |
| hydrogen burning        | hydrogen  | helium                    | 7×10 <sup>7</sup>                     | 10                           | 10 <sup>7</sup> years |
| triple-alpha process    | helium    | carbon, oxygen            | 2×10 <sup>8</sup>                     | 2000                         | 10 <sup>6</sup> years |
| carbon burning process  | carbon    | Ne, Na, Mg, Al            | 8×10 <sup>8</sup>                     | 10 <sup>6</sup>              | 10 <sup>3</sup> years |
| neon burning process    | neon      | O, Mg                     | 1.6×10 <sup>9</sup>                   | 10 <sup>7</sup>              | 3 years               |
| oxygen burning process  | oxygen    | Si, S, Ar, Ca             | 1.8×10 <sup>9</sup>                   | 10 <sup>7</sup>              | 0.3 years             |
| silicon burning process | silicon   | nickel (decays into iron) | 2.5×10 <sup>9</sup>                   | 10 <sup>8</sup>              | 5 days                |

# A story of Stirling Colgate (Palmolive) and core-collapse supernovae

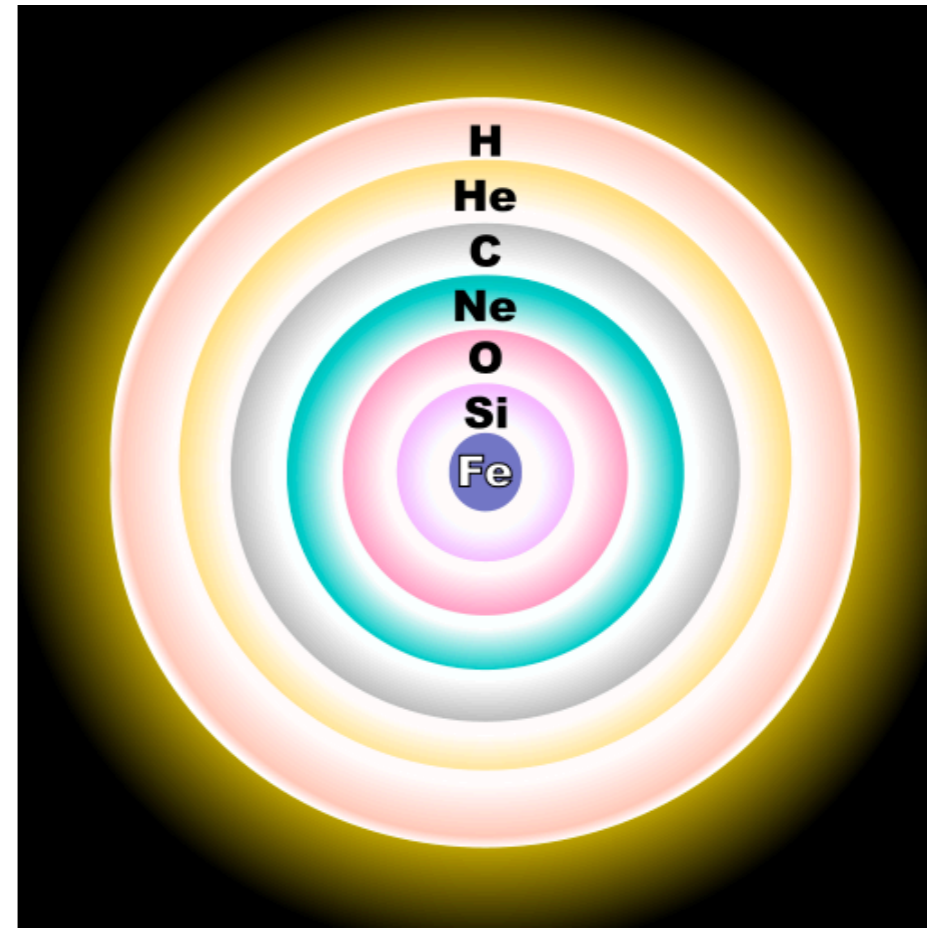


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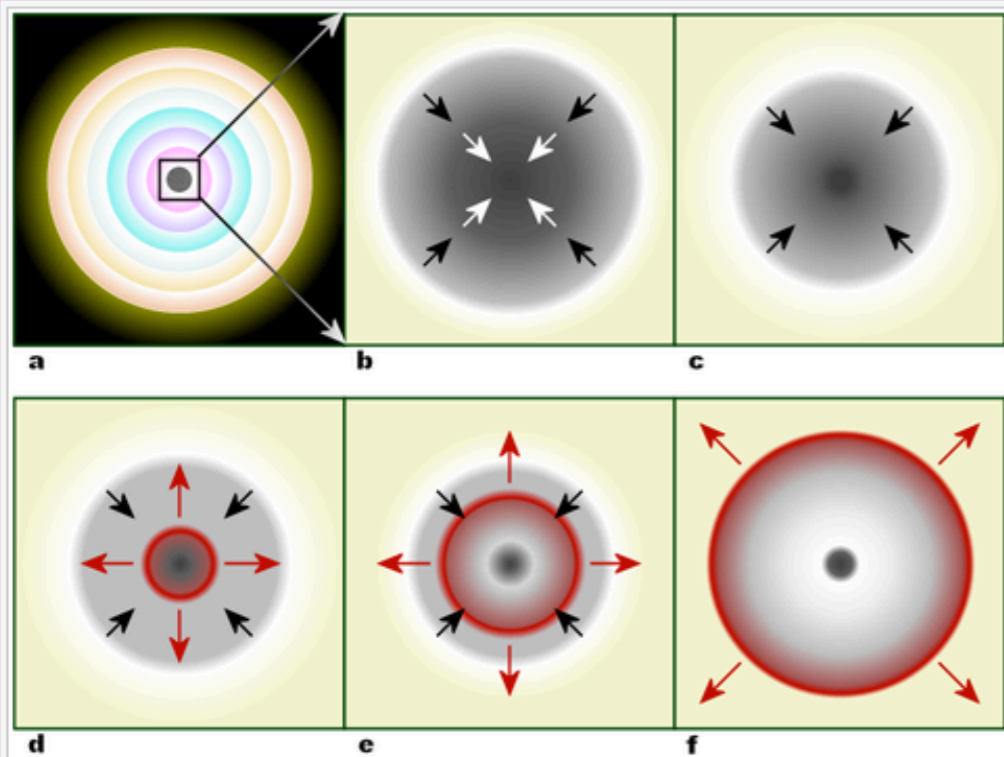
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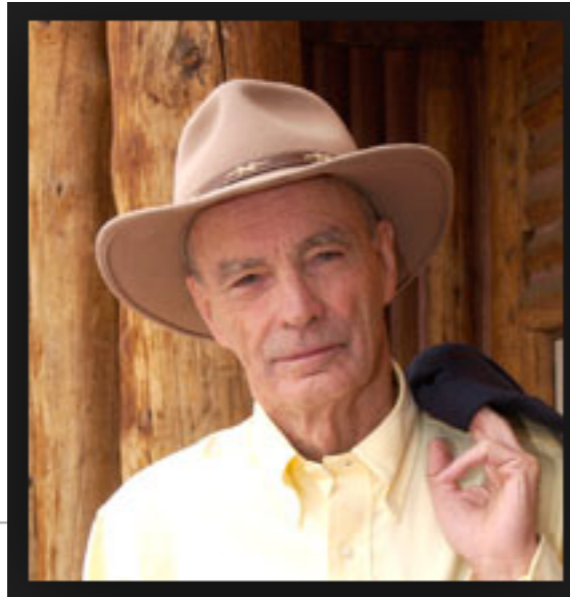
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Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.





## Stirling Colgate

From Wikipedia, the free encyclopedia

**Stirling Auchincloss Colgate** (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech),<sup>[1]</sup> and an heir to the Colgate toothpaste family fortune.<sup>[2]</sup> He was America's premier<sup>[citation needed]</sup> diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics.<sup>[3]</sup> He was born in New York City in 1925, to Henry Auchincloss and Jeanette Thurber (née Pruyn) Colgate.<sup>[4]</sup>



*..an amusing off-color aside  
story of Stirling Colgate's NMIMT resignation...*

*(Not told in Wikipedia!)*

### Quote

- "I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs."

# Multiple-collision accelerator assembly

US 5256071 A

## ABSTRACT

A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics--the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

|   |  |
|---|--|
| Publication number  | US5256071 A  |
| Publication type  | Grant  |
| Application number  | US 07/748,804  |
| Publication date  | Oct 26, 1993   |
| Filing date   | Aug 22, 1991   |
| Priority date <sup>?</sup>  | Aug 22, 1991   |
| Fee status <sup>?</sup>   | Paid   |
| Inventors   | Edward W. Hones, William G. Hones, Stirling A. Colgate |
| Original Assignee   | Hones Edward W, Hones William G, Colgate Stirling A    |
| Export Citation   | BiBTeX, EndNote, RefMan                                |
| <a href="#">Patent Citations (3)</a> , <a href="#">Referenced by (4)</a> , <a href="#">Classifications (7)</a> , <a href="#">Legal Events (7)</a> |  |
| <b>External Links:</b> <a href="#">USPTO</a> , <a href="#">USPTO Assignment</a> , <a href="#">Espacenet</a>                                       |  |

*(Point allowing patent over previous 1973 proposal (4))*

*1st publication describing theory and experiment of this device 20 years before.*

## Velocity Amplification in Collision Experiments Involving Superballs

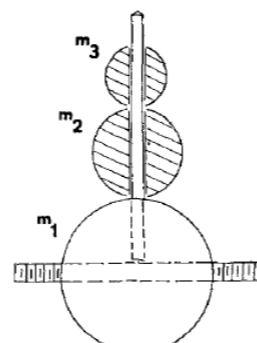
William G. Harter<sup>1</sup> (class of WGH)

— HIDE AFFILIATIONS

<sup>1</sup> University of Southern California, Los Angeles, California 90007

View the Scitation page for University of Southern California (USC).

Am. J. Phys. **39**, 656 (1971); <http://dx.doi.org/10.1119/1.1986253>



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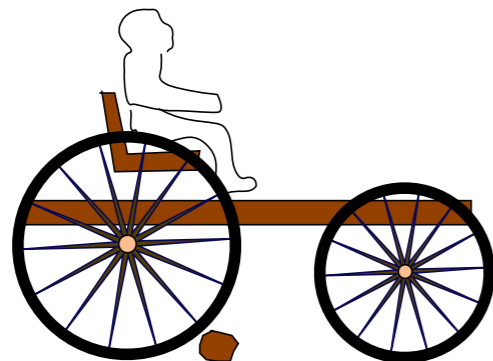
Super-elastic examples: This really is “Rocket-Science”

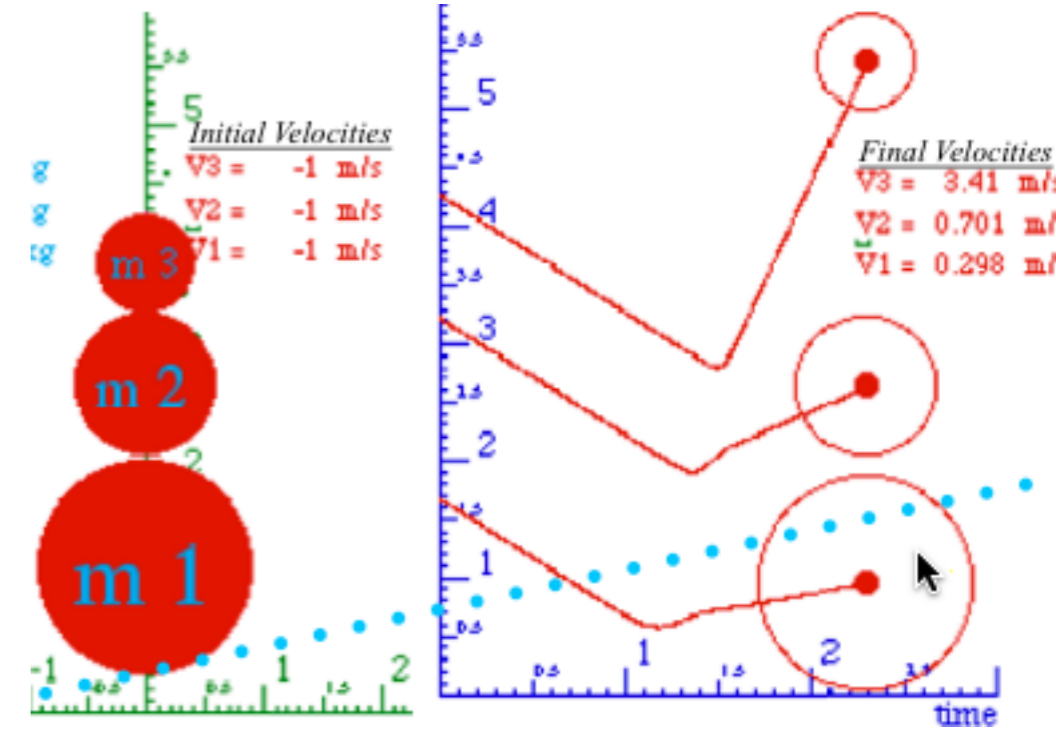


*Western buckboard* = ??????

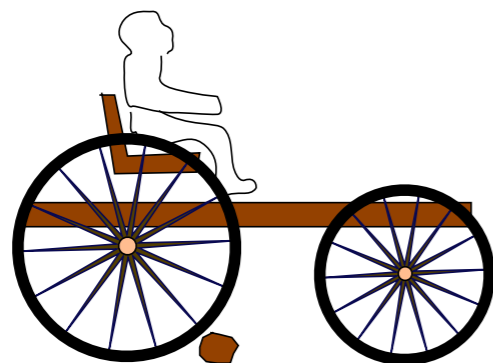


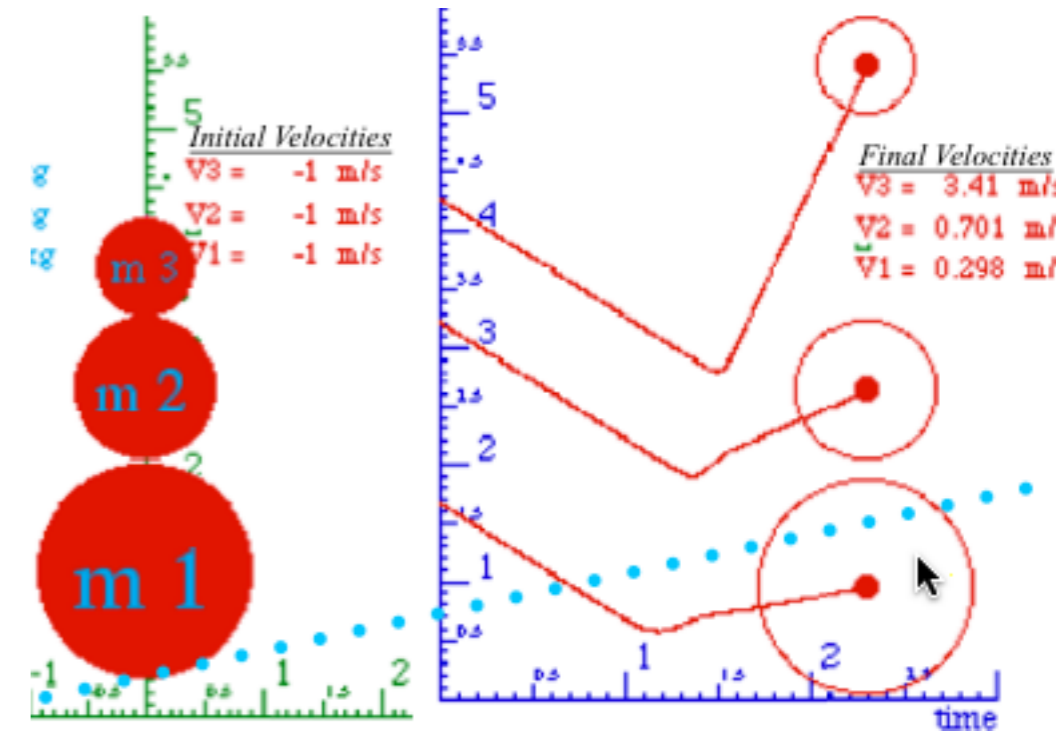
*Western buckboard* = ??????



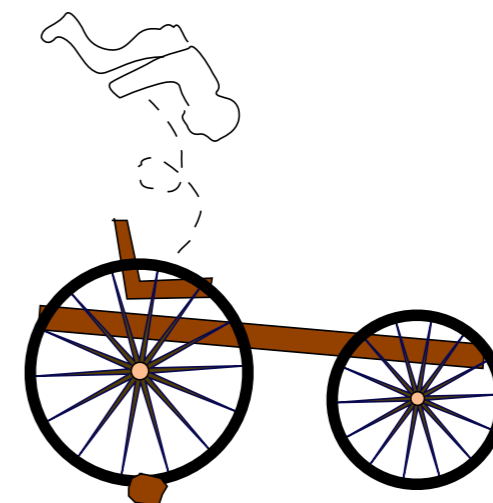
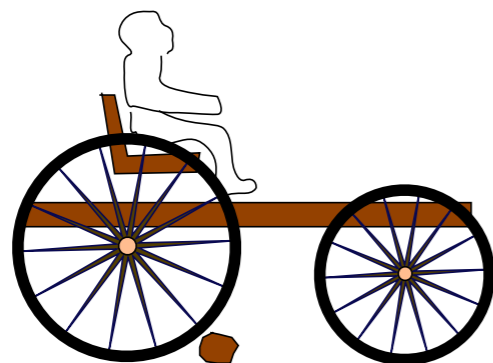


*Western buckboard = 3-ball analogy*





*Western buckboard = 3-ball analogy Disaster!*



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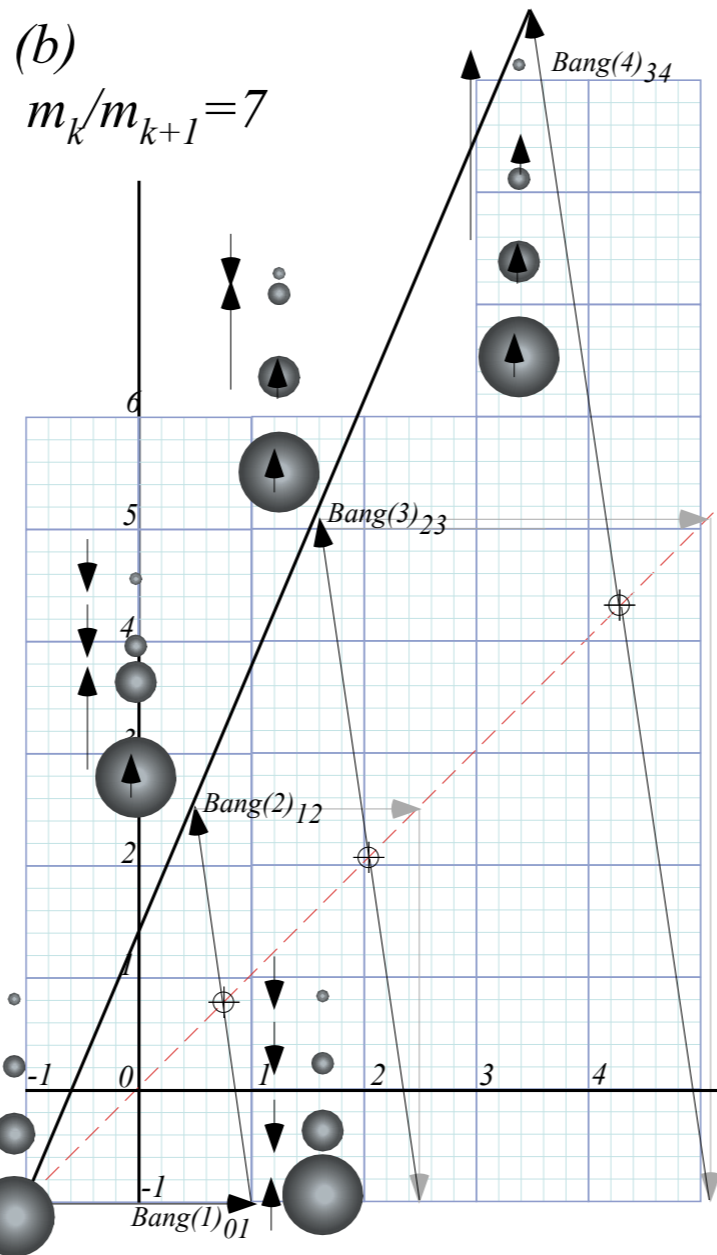
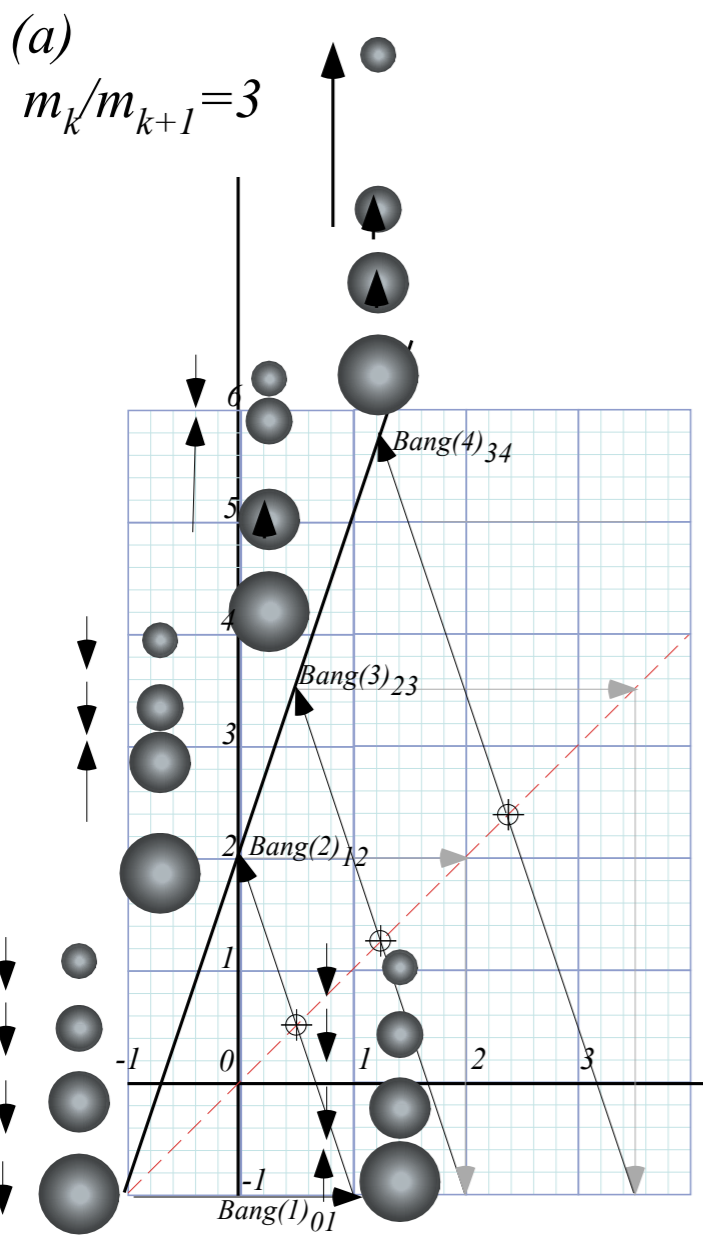


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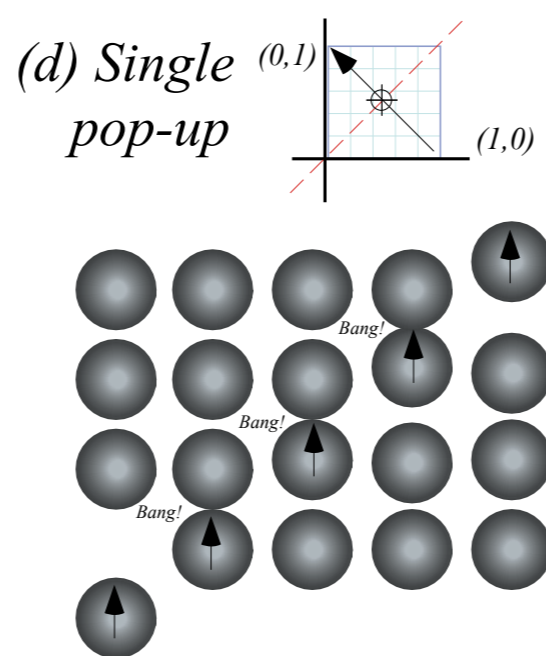
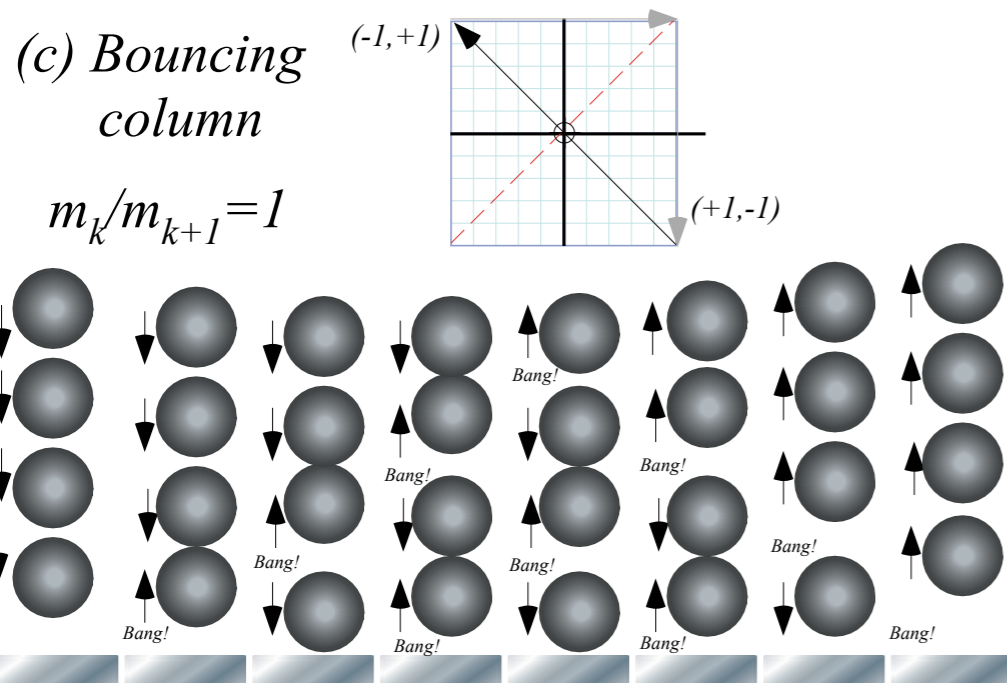
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Unit 1  
 Fig. 8.2a-b  
 4-Body IBM Geometry  
 Fig. 8.2c-d  
 4-Equal-Body Geometry

Bouncelt Simulation: 4-Ball Tower w/  $m_k/m_{k+1} = 3$



4-Equal-Body  
 "Shockwave" or pulse wave  
 Dynamics

Opposite of continuous wave dynamics  
 introduced in Unit 2

Bouncelt Simulation: 4-Ball Tower w/  $m_k/m_{k+1} = 1$

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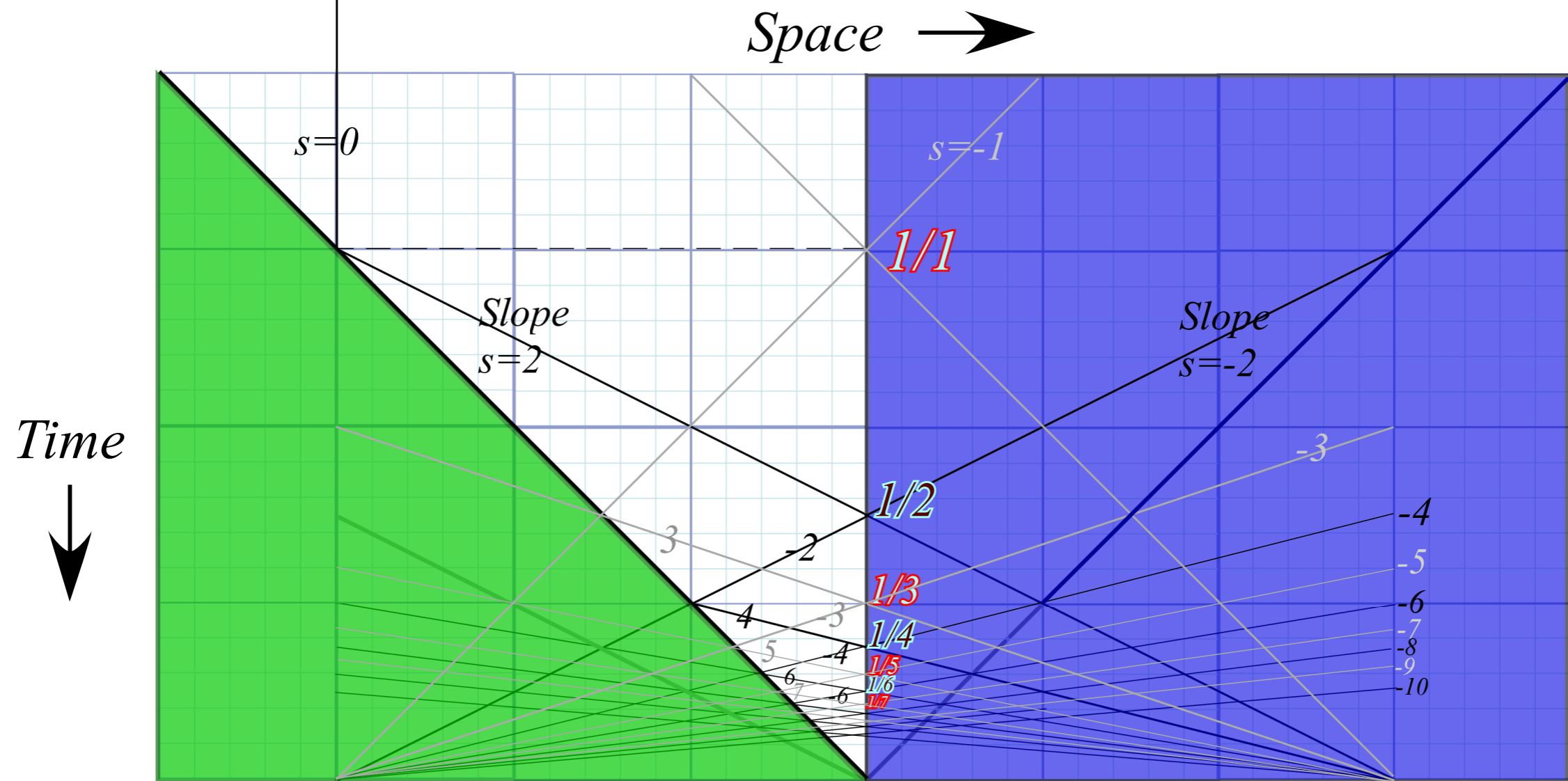
→ Inelastic examples: “Zig-zag geometry” of freeway crashes

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# Inelastic examples: “Zig-zag geometry” of freeway crashes

First recall “zig-zag” fractions of “Monster Mash” in Lect. 4

Trajectory geometry exposed (Harmonic series  $1/1, 1/2, 1/3, 1/4, \dots$ )



*Speeding car* and five stationary cars

$(V_{M(0)}=60, V_{m(1)}=0)$

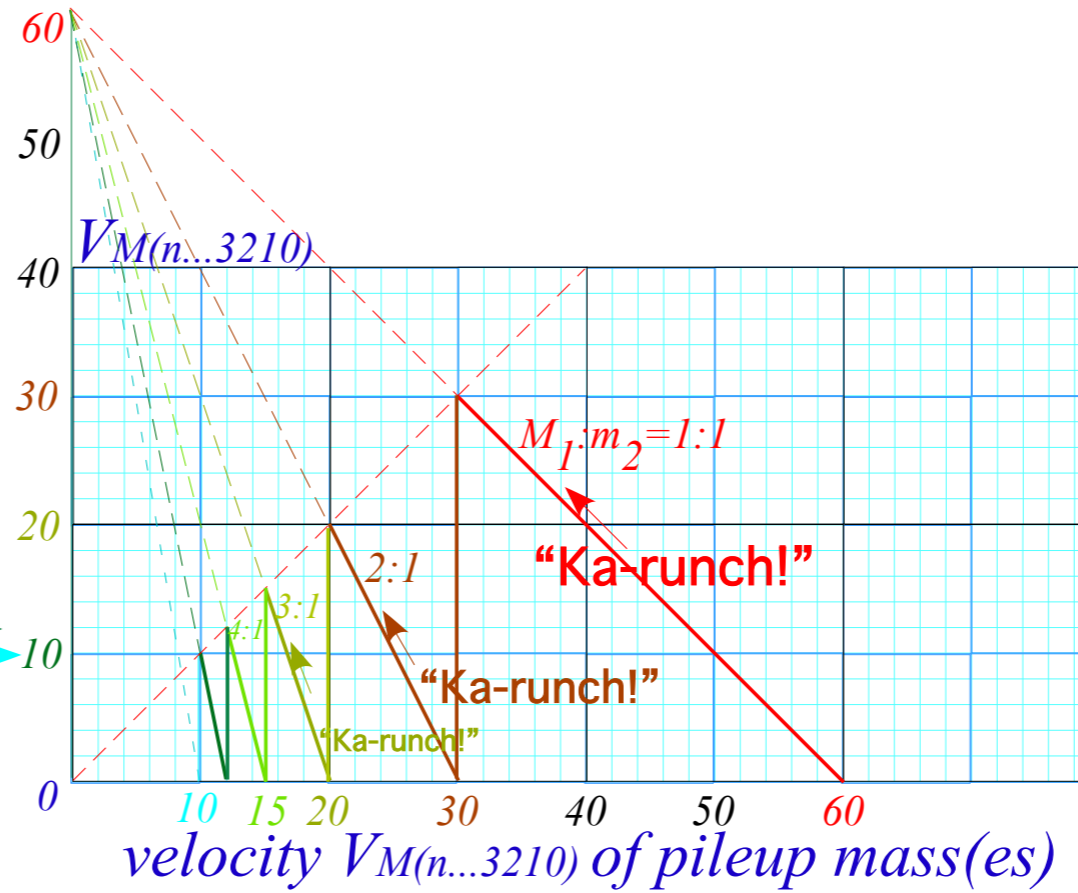
$V_{M(01)}=30$

$V_{M(012)}=20$

$V_{M(0123)}=15$

$V_{M(01234)}=12$

$V_{M(01235)}=10$



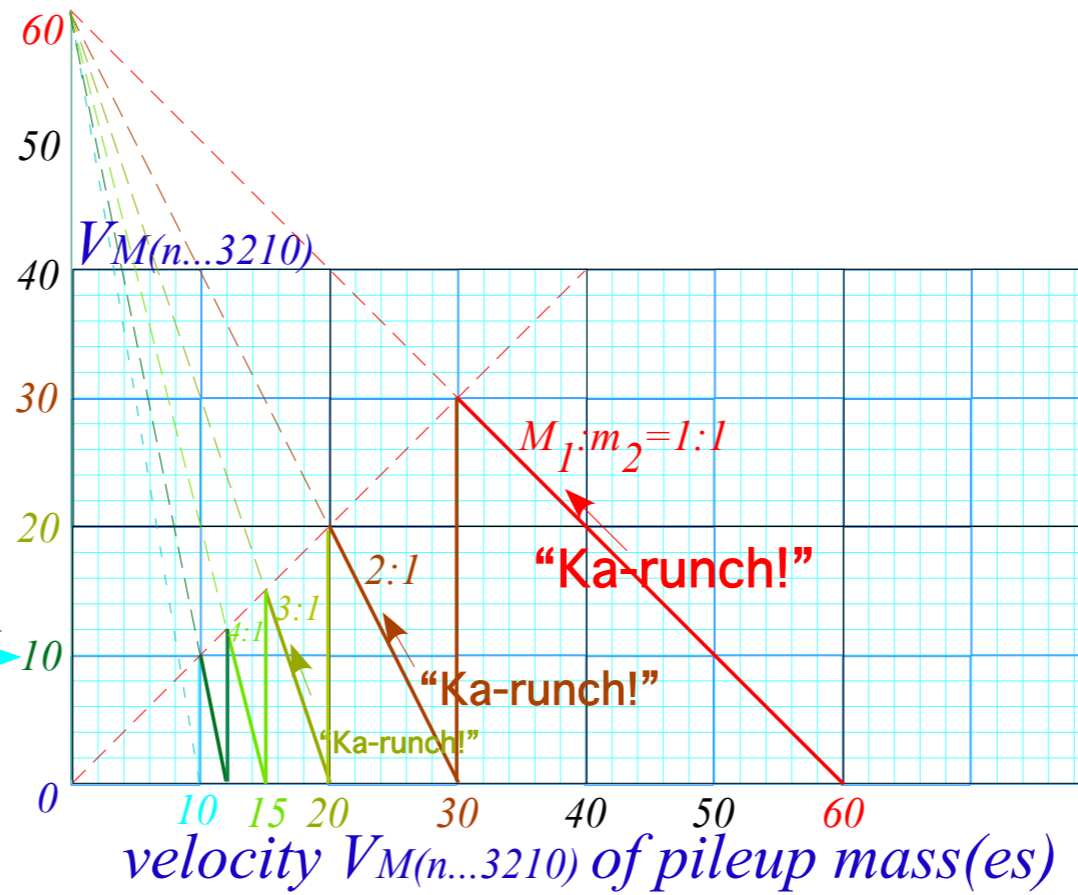
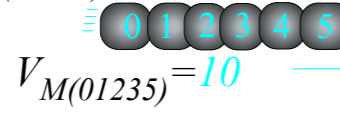
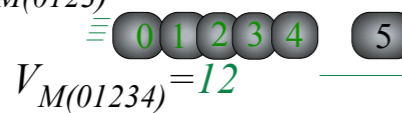
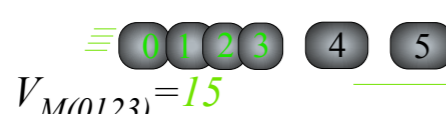
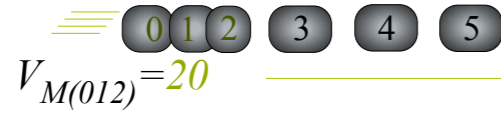
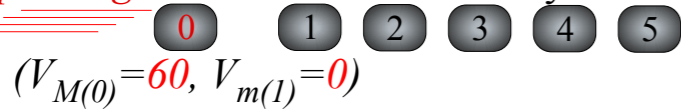
Unit 1  
Fig. 8.5  
Pile-up:  
One 60mph car  
hits  
five standing cars

Bouncelt Simulation: One ball hits 5 stationary balls (y vs x) and ( $x_i$  vs t)

These graphs plot User determined quantities.  
Choose and select from a context menu via right click on target axis,  
like the following set to  $V_{1x}$  and  $V_{2x}$

Bouncelt Simulation: One ball hits 5 stationary balls (y vs x) and ( $V_{i+1x}$  vs  $V_{ix}$ )

*Speeding car* and five stationary cars



Unit 1

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Pile-up:  
One 60mph car  
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five standing cars

Five speeding cars and a stationary car

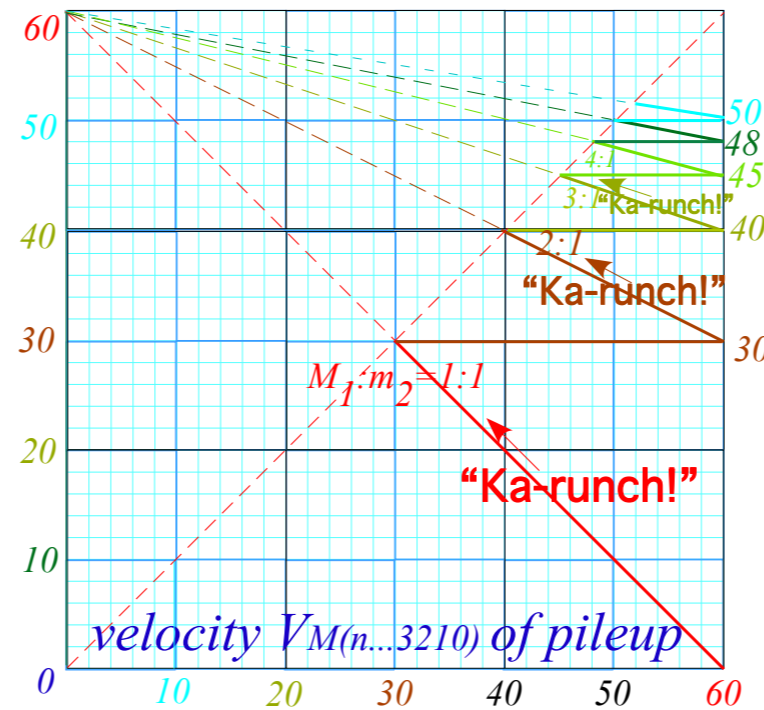
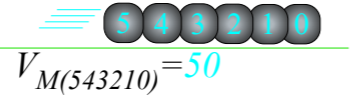
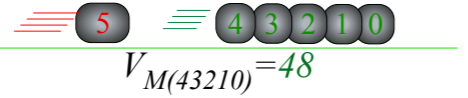
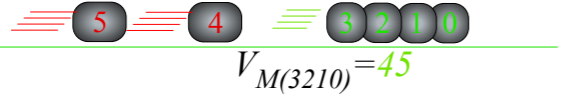
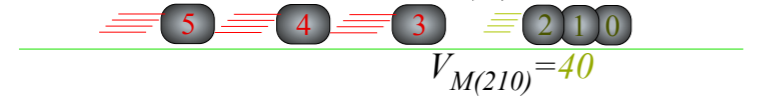
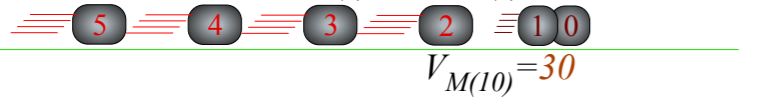
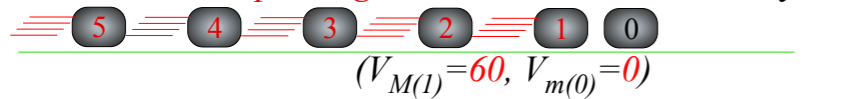
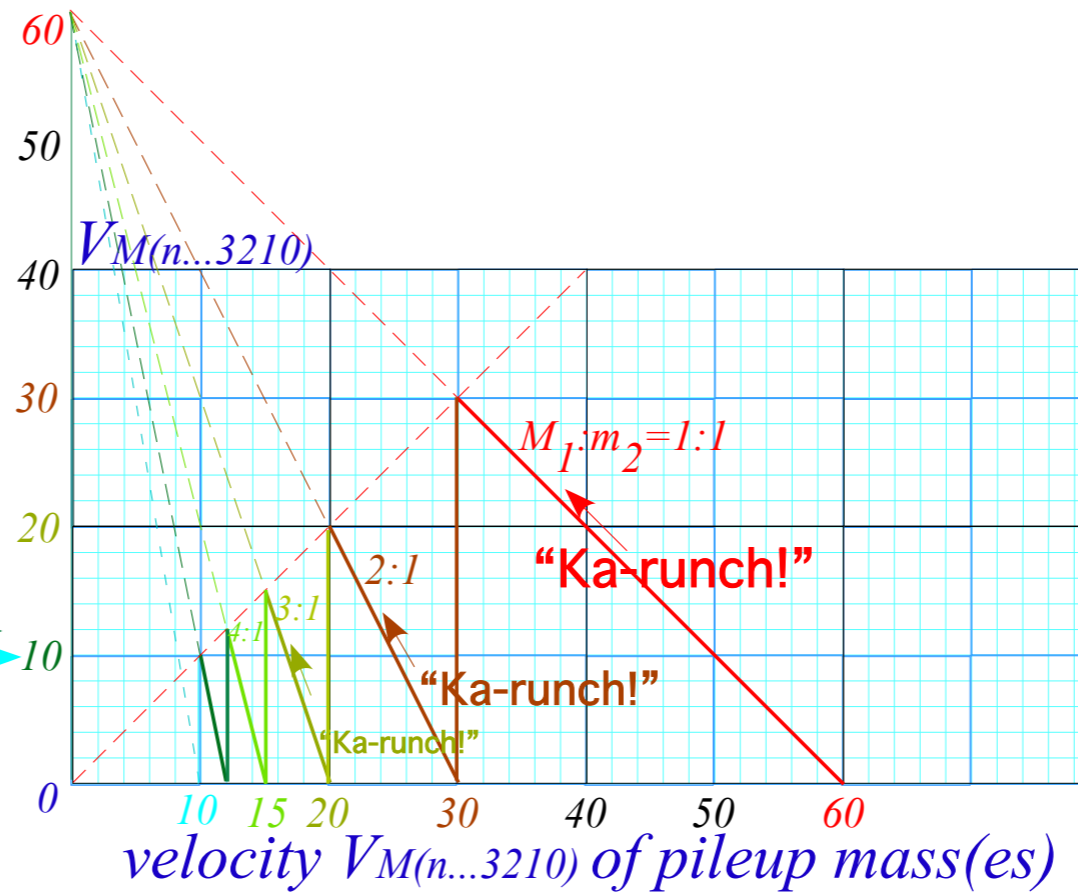
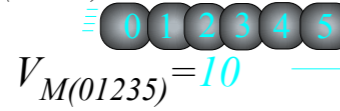
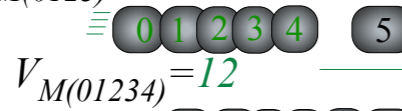
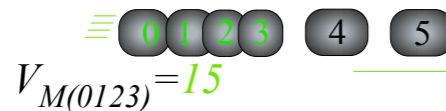
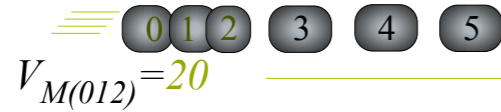
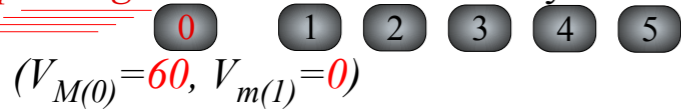


Fig. 8.6  
Pile-up:  
Five 60mph cars  
hit  
one standing cars

Bouncelt Simulation: 5 balls hit 1 stationary ball (y vs x) and (v<sub>6x</sub> vs v<sub>5x</sub>)

*Speeding car* and five stationary cars



Unit 1

Fig. 8.5  
Pile-up:  
One 60mph car  
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*Five speeding cars* and a stationary car

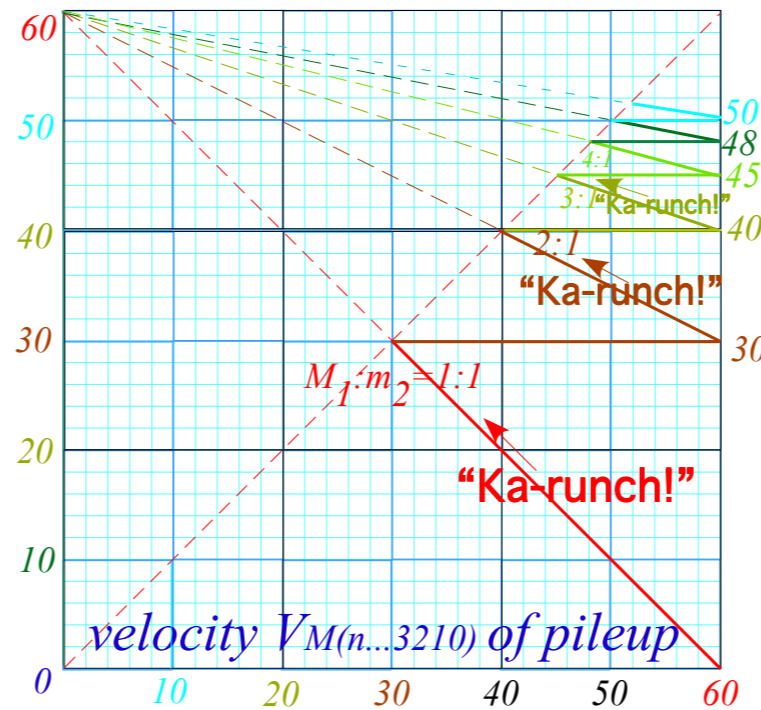
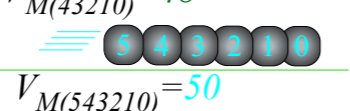
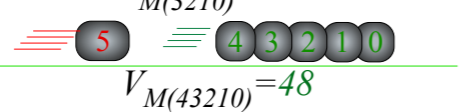
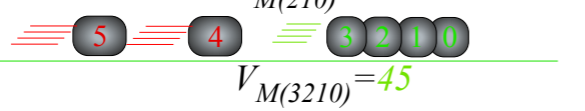
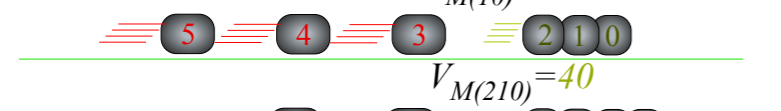
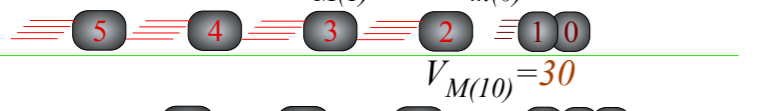
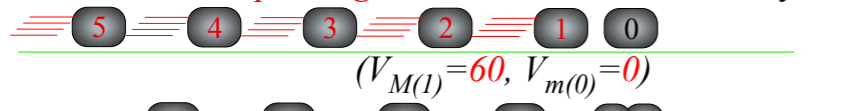


Fig. 8.6  
Pile-up:  
Five 60mph cars  
hit  
one standing cars

*Five speeding cars* and five stationary cars

*(Fug-gedda-aboud-dit!!)*

(Many possible scenarios depending on initial positions!)

Fig. 8.7  
Pile-up:  
Five 60mph cars  
hit  
five standing cars

# Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force  $F=-k$  (linear potential  $V=kx$ )

Some physics of dare-devil-diving 80 ft. into kidee pool

(Simulations)

(b) Linear force  $F=-kx$  (quadratic potential  $V=\frac{1}{2}kx^2$  (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

## Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

(Leads to Sagittal

## Geometry and dynamics of $n$ -ball bounces

potential analysis of

Analogy with shockwave and acoustical horn amplifier

2, 3, and 4 body towers)

Advantages of a geometric  $m_1, m_2, m_3, \dots$  series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

## Many-body 1D collisions

Elastic examples: Western buckboard

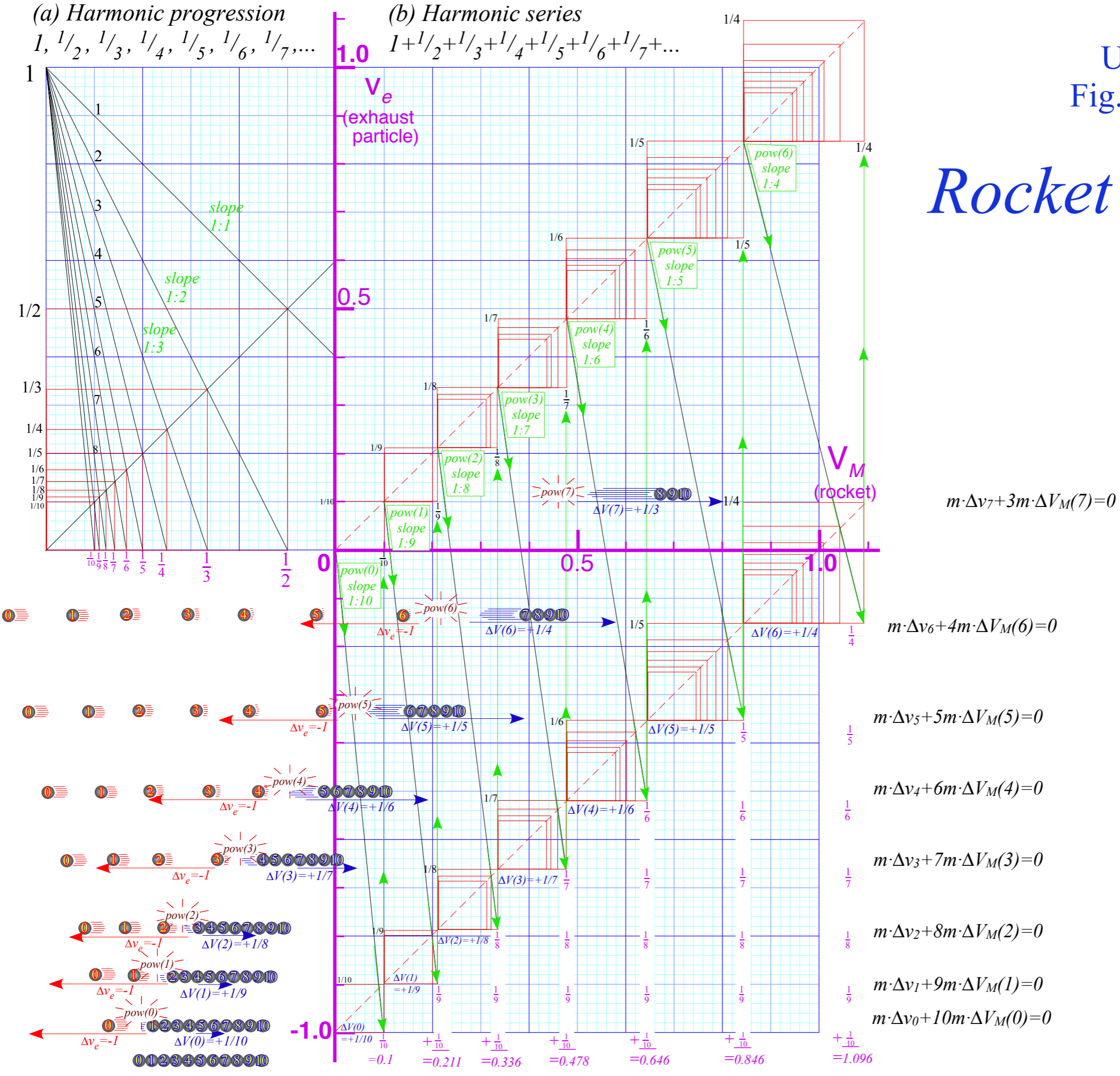
Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

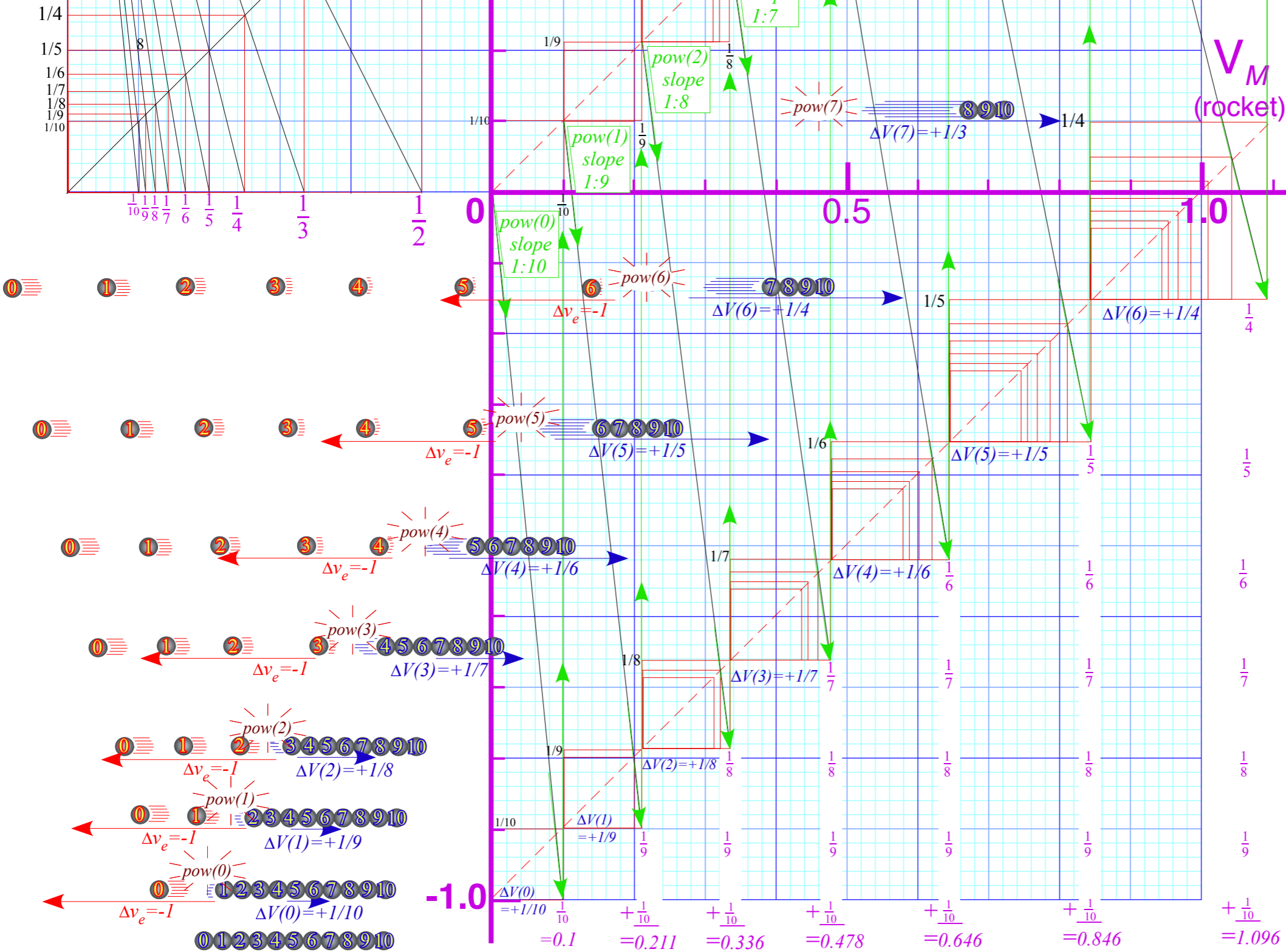
→ Super-elastic examples: This really is “Rocket-Science”

Unit 1  
Fig. 8.8a-b

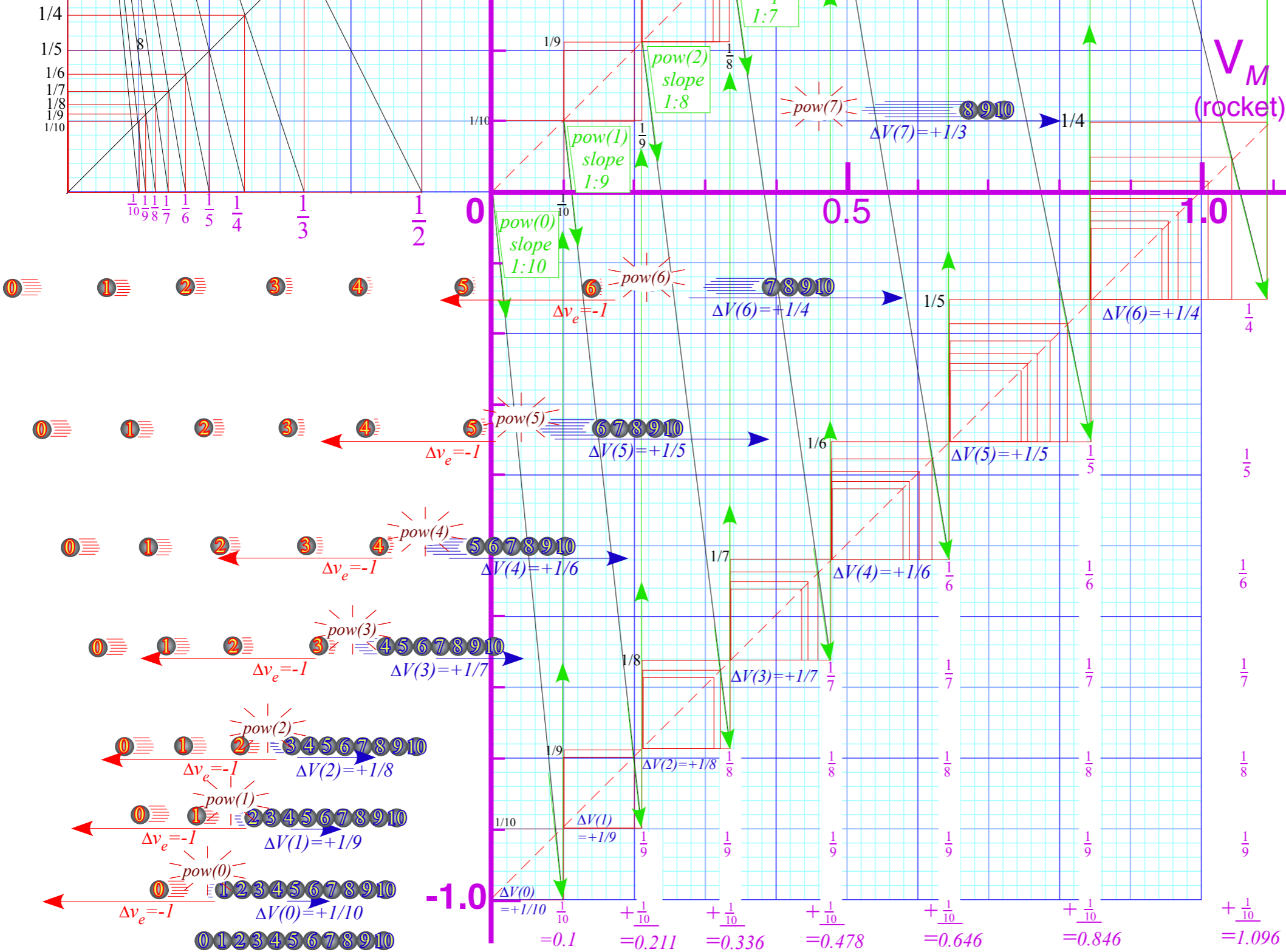
# Rocket Science!







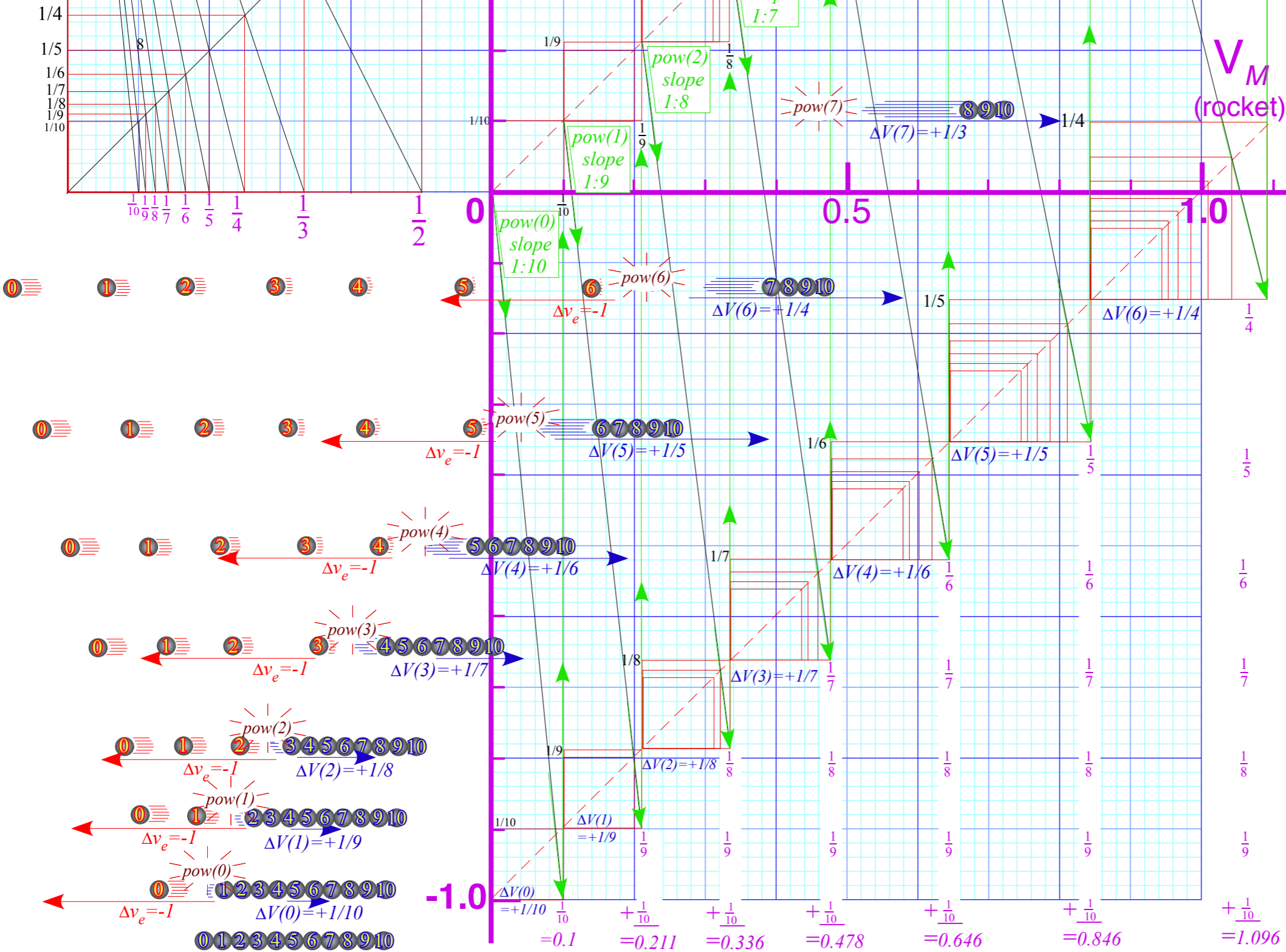
$0^{th}: V(0) = 1/10 = 0.1$        $1^{st}: V(1) = 1/10 + 1/9 = 0.211$        $2^{nd}: V(2) = 1/10 + 1/9 + 1/8 = 0.336$   
 $3^{rd}: V(3) = V(2) + 1/7 = 0.478$        $4^{th}: V(4) = V(3) + 1/6 = 0.646$        $5^{th}: V(5) = V(4) + 1/5 = 0.846$   
 $6^{th}: V(6) = V(5) + 1/4 = 1.096$        $7^{th}: V(7) = V(6) + 1/3 = 1.429$        $8^{th}: V(8) = V(7) + 1/2 = 1.929$



- 0<sup>th</sup>:  $V(0) = 1/10 = 0.1$
- 1<sup>st</sup>:  $V(1) = 1/10 + 1/9 = 0.211$
- 2<sup>nd</sup>:  $V(2) = 1/10 + 1/9 + 1/8 = 0.336$
- 3<sup>rd</sup>:  $V(3) = V(2) + 1/7 = 0.478$
- 4<sup>th</sup>:  $V(4) = V(3) + 1/6 = 0.646$
- 5<sup>th</sup>:  $V(5) = V(4) + 1/5 = 0.846$
- 6<sup>th</sup>:  $V(6) = V(5) + 1/4 = 1.096$
- 7<sup>th</sup>:  $V(7) = V(6) + 1/3 = 1.429$
- 8<sup>th</sup>:  $V(8) = V(7) + 1/2 = 1.929$

$v_e$  known as  
“Specific Impulse”

By calculus:  $M \cdot \Delta V = -v_e \cdot \Delta M$  or:  $dV = -v_e \frac{dM}{M}$  Integrate:  $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$



$V_M$   
(rocket)

0<sup>th</sup>:  $V(0) = 1/10 = 0.1$

3<sup>rd</sup>:  $V(3) = V(2) + 1/7 = 0.478$

6<sup>th</sup>:  $V(6) = V(5) + 1/4 = 1.096$

1<sup>st</sup>:  $V(1) = 1/10 + 1/9 = 0.211$

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8<sup>th</sup>:  $V(8) = V(7) + 1/2 = 1.929$

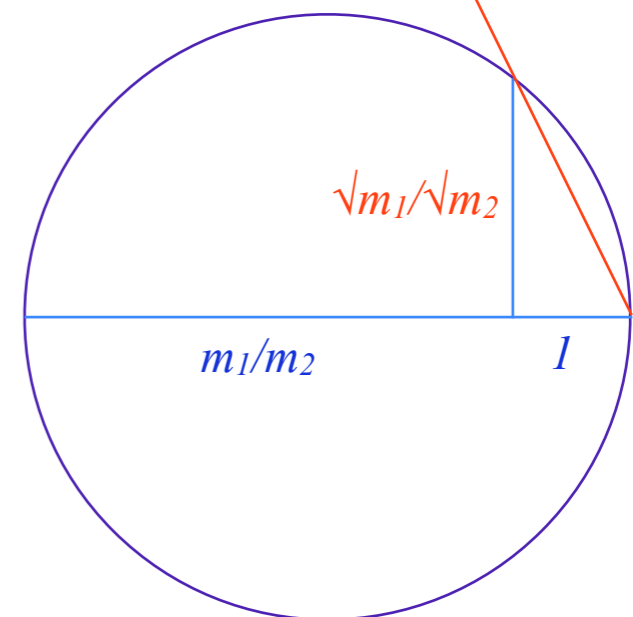
$v_e$  known as  
"Specific Impulse"

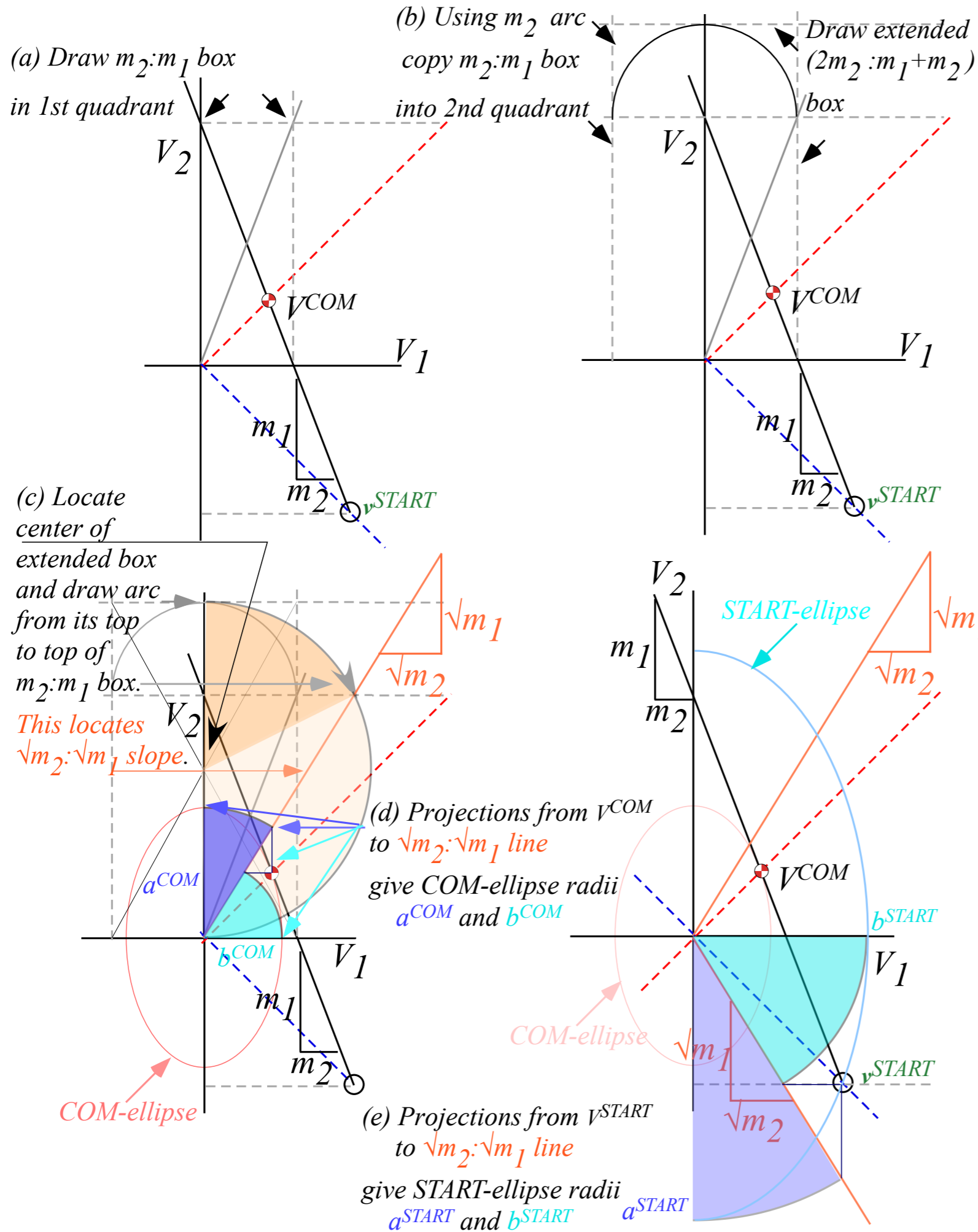
By calculus:  $M \cdot \Delta V = -v_e \cdot \Delta M$  or:  $dV = -v_e \frac{dM}{M}$  Integrate:  $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$

The Rocket Equation:  $V_{FIN} - V_{IN} = -v_e [\ln M_{FIN} - \ln M_{IN}] = v_e \left[ \ln \frac{M_{IN}}{M_{FIN}} \right]$

## *A Thales construction for momentum-energy*

(Made obsolete by Estrangian scaling to circular  $(V_1, V_2)$  plots. Still, one has to construct  $\sqrt{m_1}/\sqrt{m_2}$  slopes. )





Unit 1  
Fig. 8.4a-d

*This is a detailed construction of the energy ellipse in a Largangian  $(v_1, v_2)$  plot given the initial  $(v_1, v_2)$ .*

*The Estrangian  $(V_1, V_2)$  plot makes the  $(v_1, v_2)$  plot and this construction obsolete.*

*(Easier to just draw circle through initial  $(V_1, V_2)$ .)*