

Lecture 6
Thur. 9.07.2017

Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

Introducing 2D IHO orbits and phasor geometry

Phasor “clock” geometry

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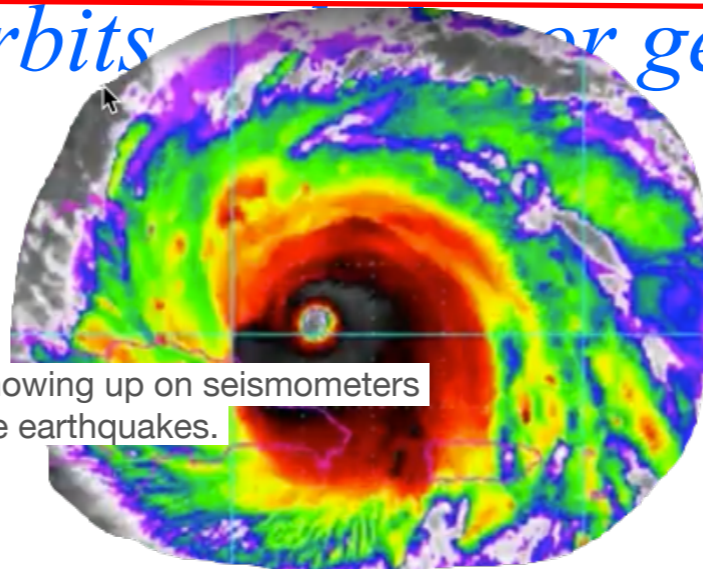
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Hurricane Irma is so strong it's showing up on seismometers
— equipment designed to measure earthquakes.

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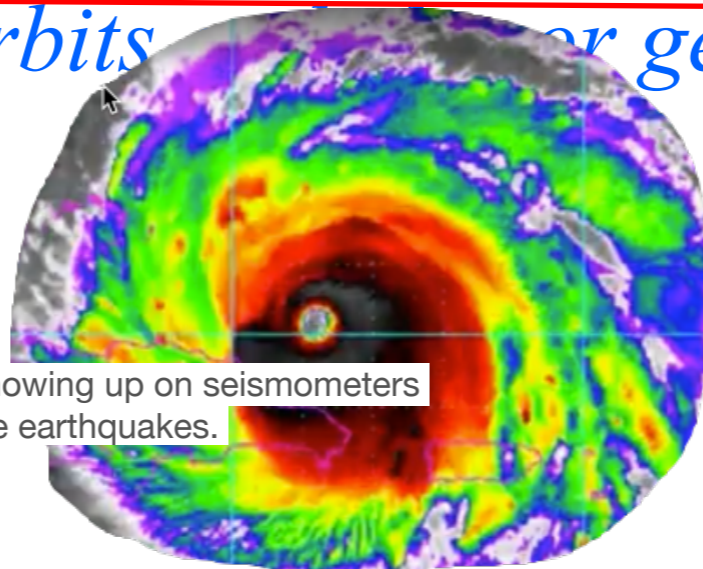
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Climate change is Chinese Hoax!



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“Zig-Zag” geometry of a power sequence

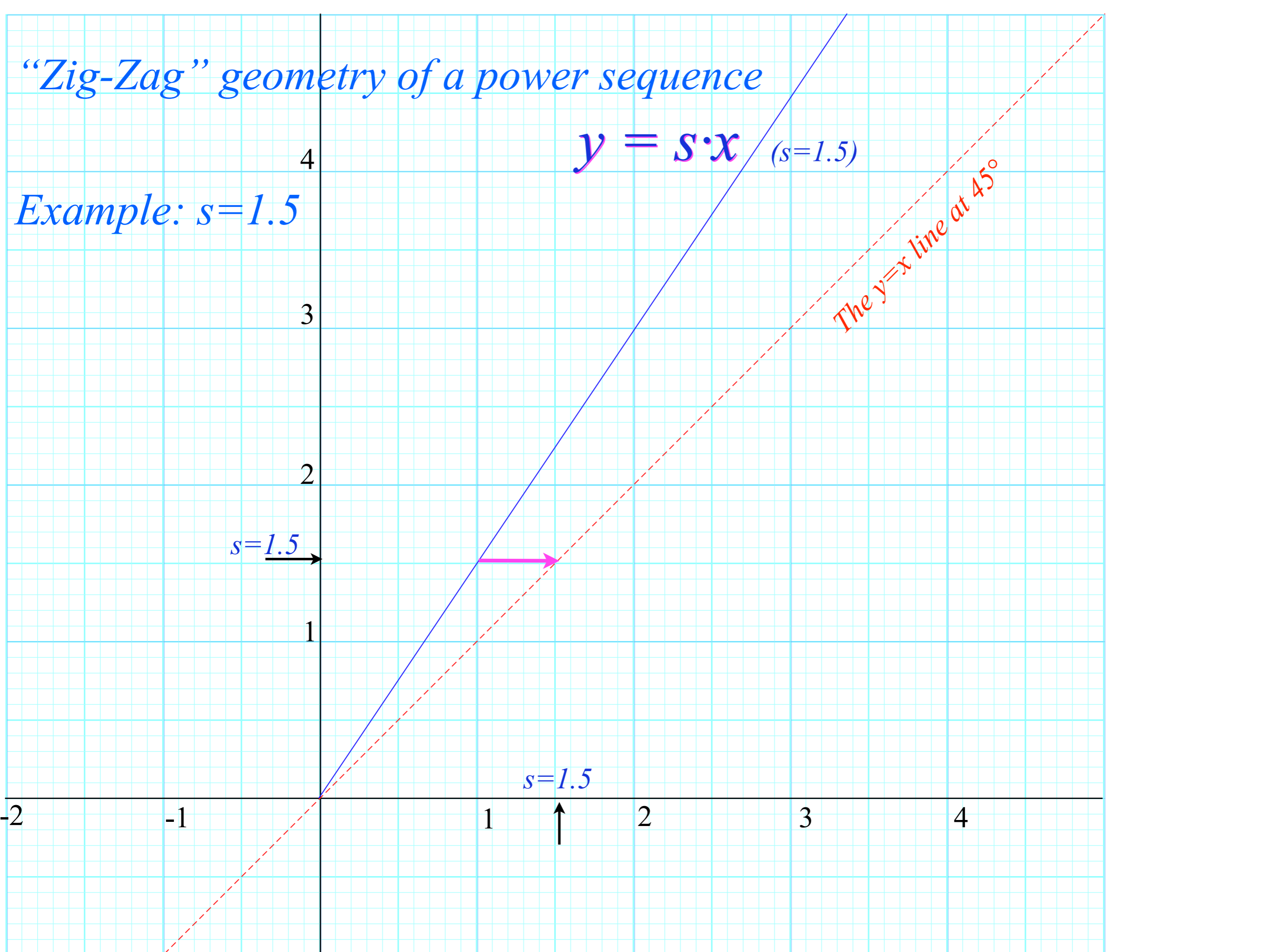
$$y = s \cdot x \quad (s=1.5)$$

Example: $s=1.5$

The $y=x$ line at 45°

$s=1.5$

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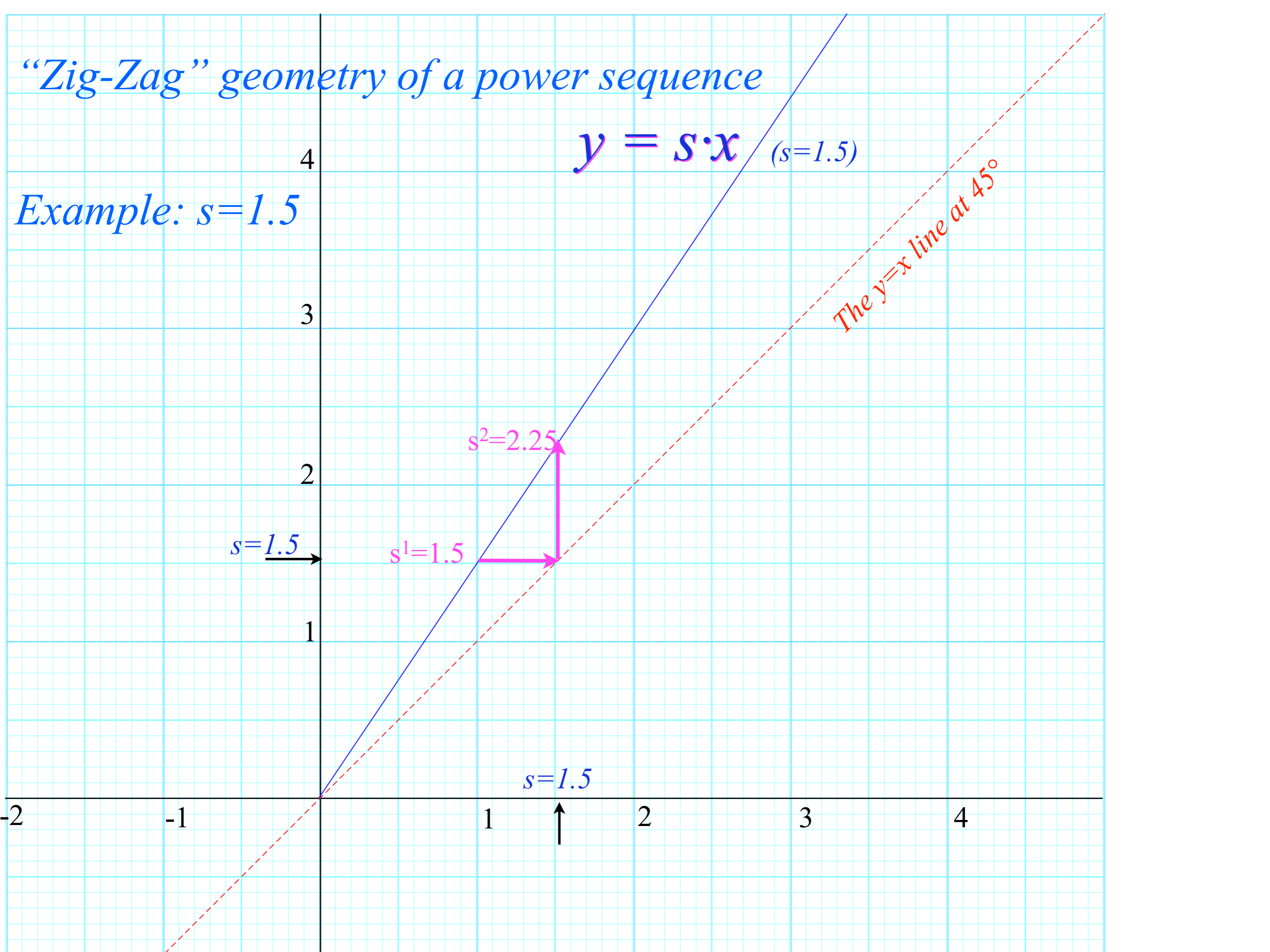


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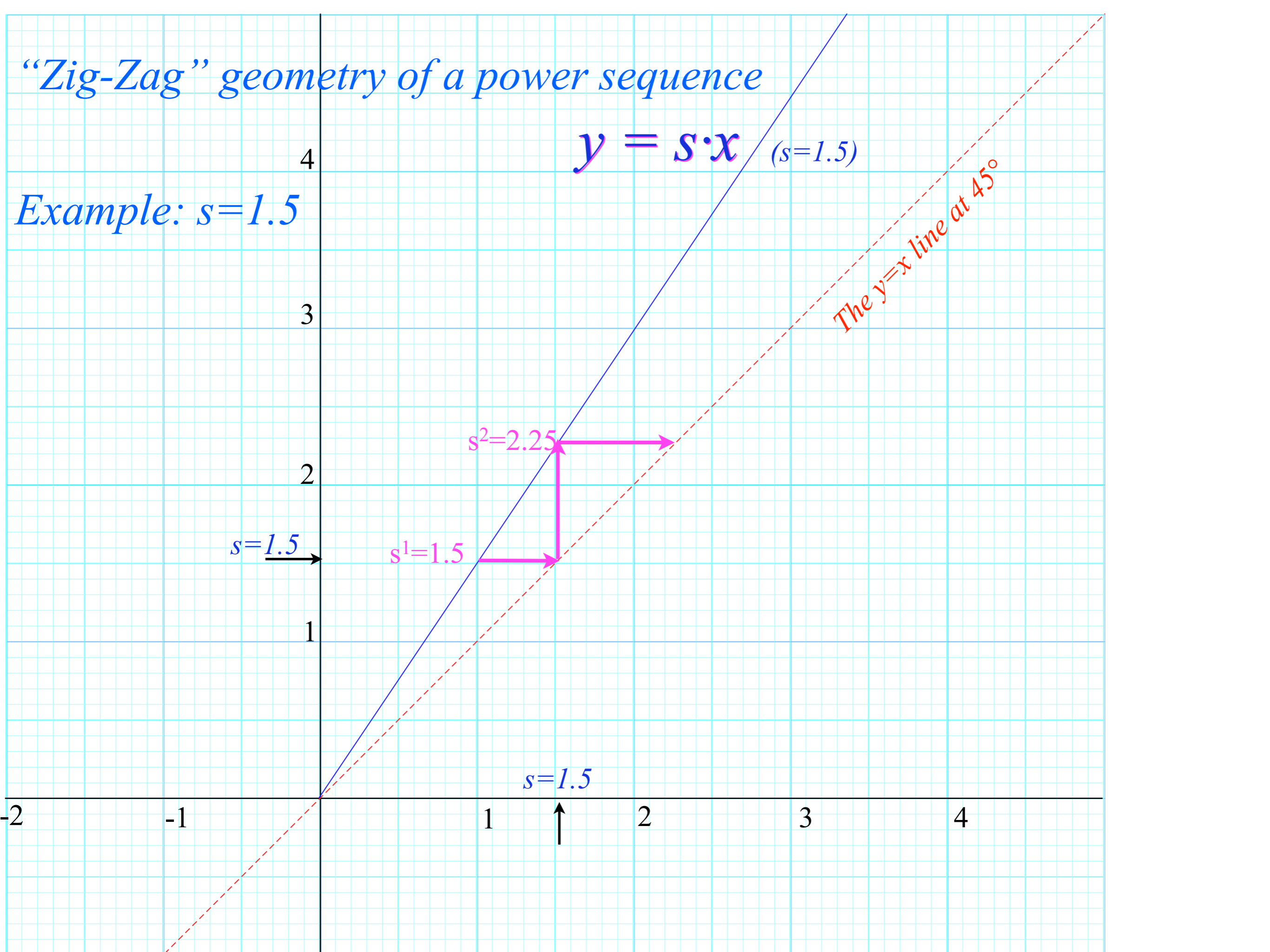


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Example: $s=1.5$

The $y=x$ line at 45°



$s=1.5$

$s^1=1.5$

$s^2=2.25$

$s=1.5$

-2

-1

1

2

3

4

4

3

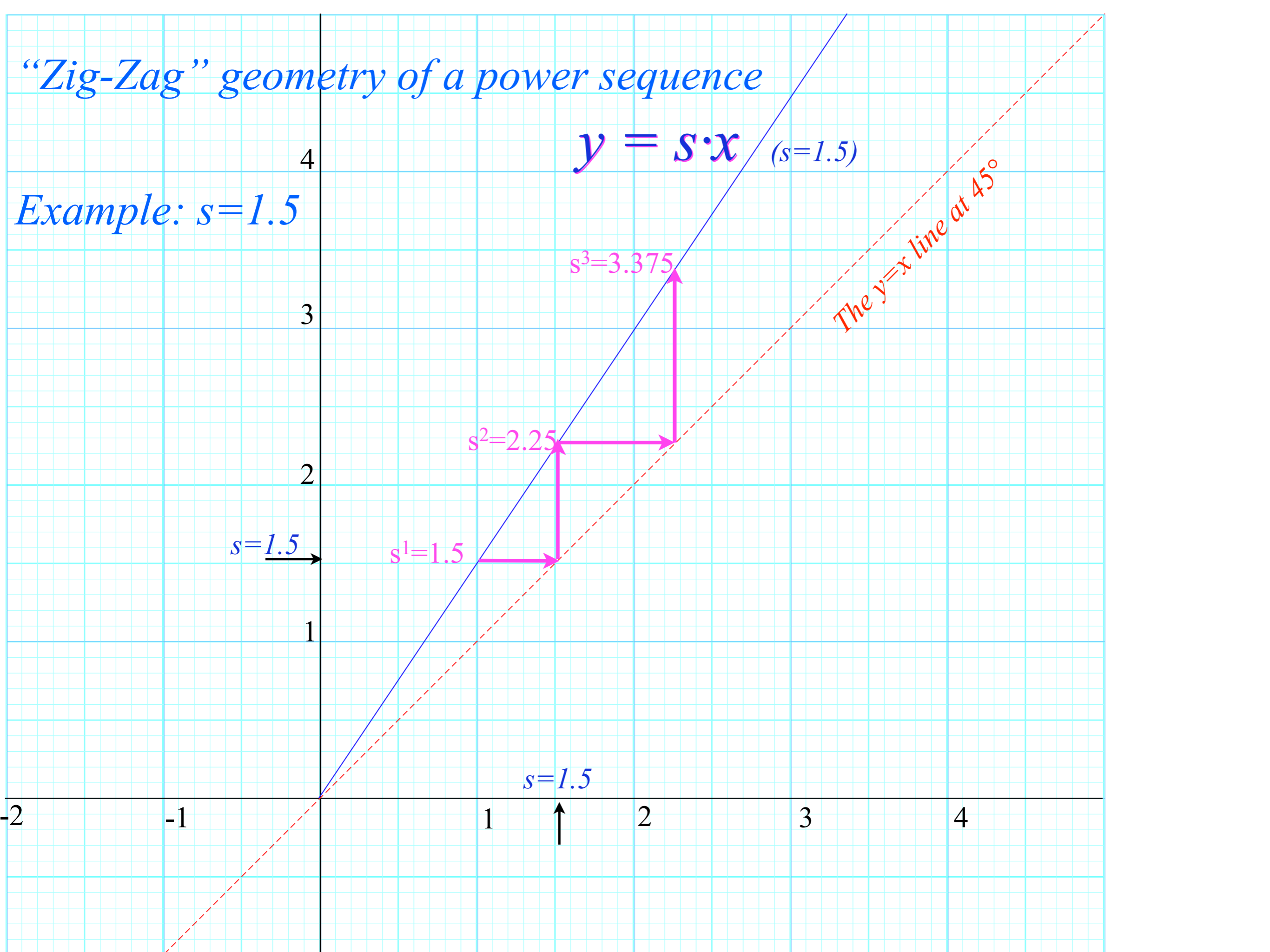
2

1

“Zig-Zag” geometry of a power sequence

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Example: $s=1.5$



The $y=x$ line at 45°

$s=1.5$

$s^1=1.5$

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$s^3=3.375$

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-2

-1

1

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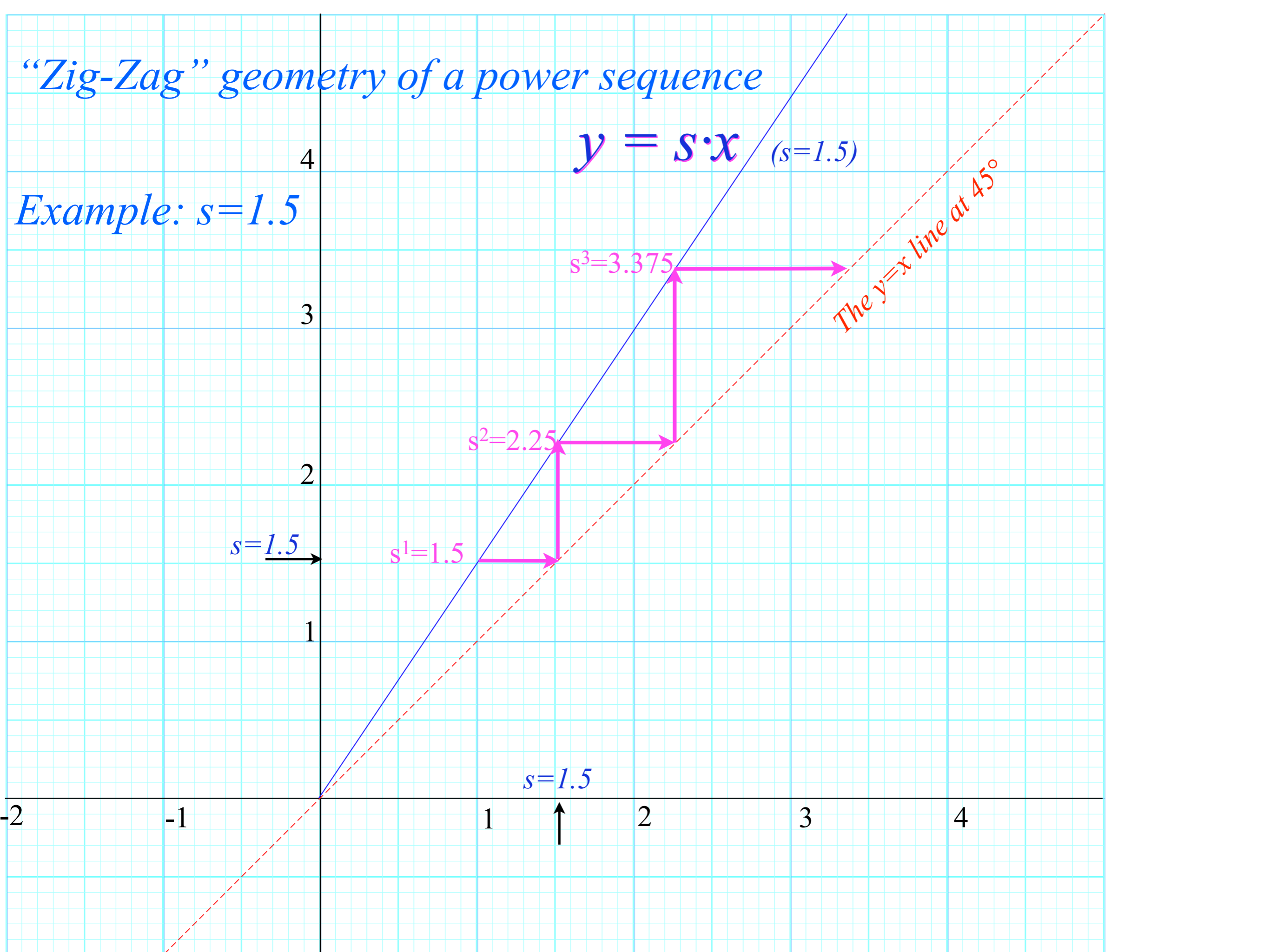
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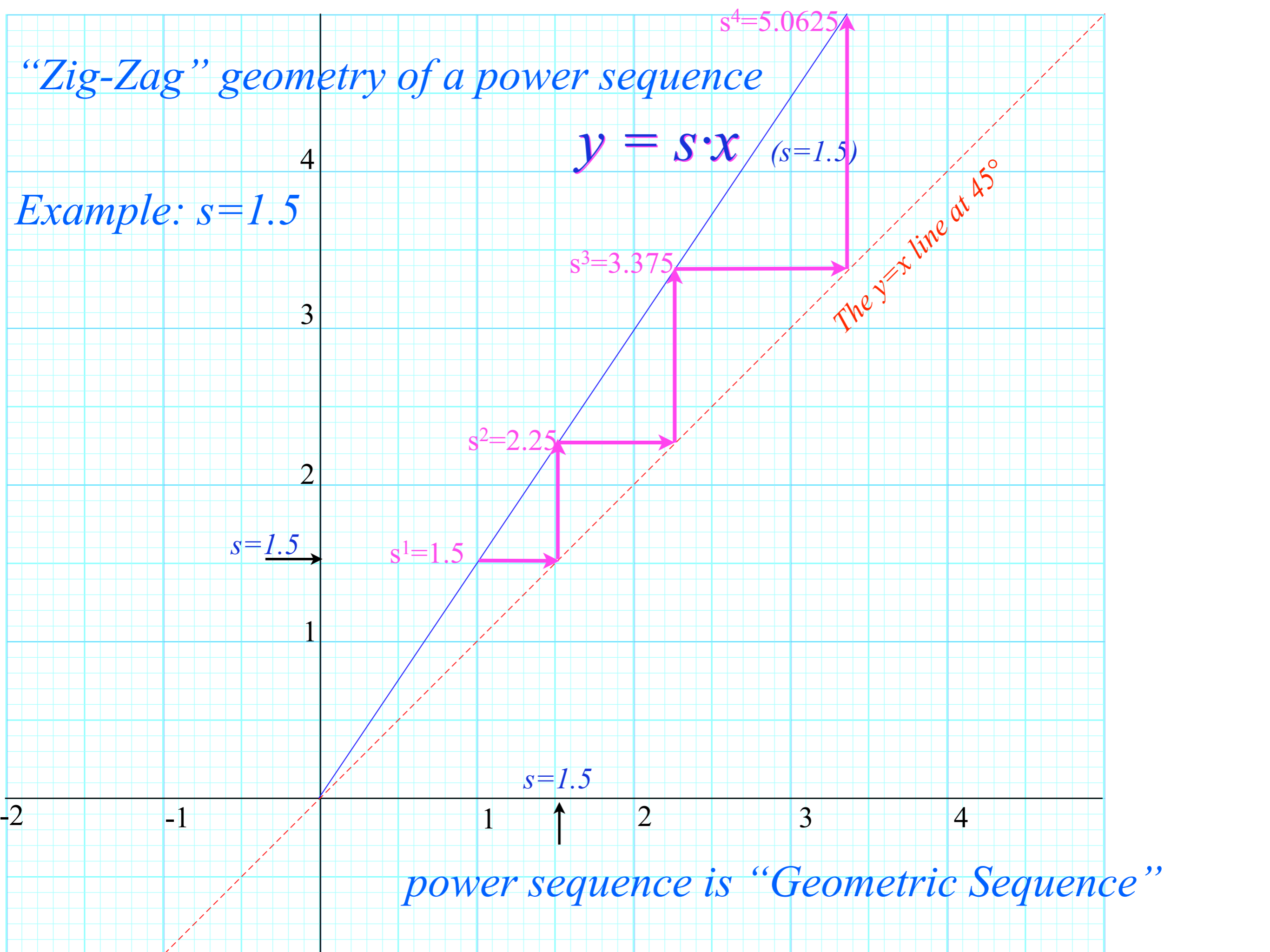


“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

$y = s \cdot x$ ($s=1.5$)

The $y=x$ line at 45°



power sequence is “Geometric Sequence”

“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

$$y = s \cdot x \quad (s=1.5)$$

The $y=x$ line at 45°

$s=1.5$

$s^1=1.5$

$s^2=2.25$

$s^3=3.375$

$s^4=5.0625$

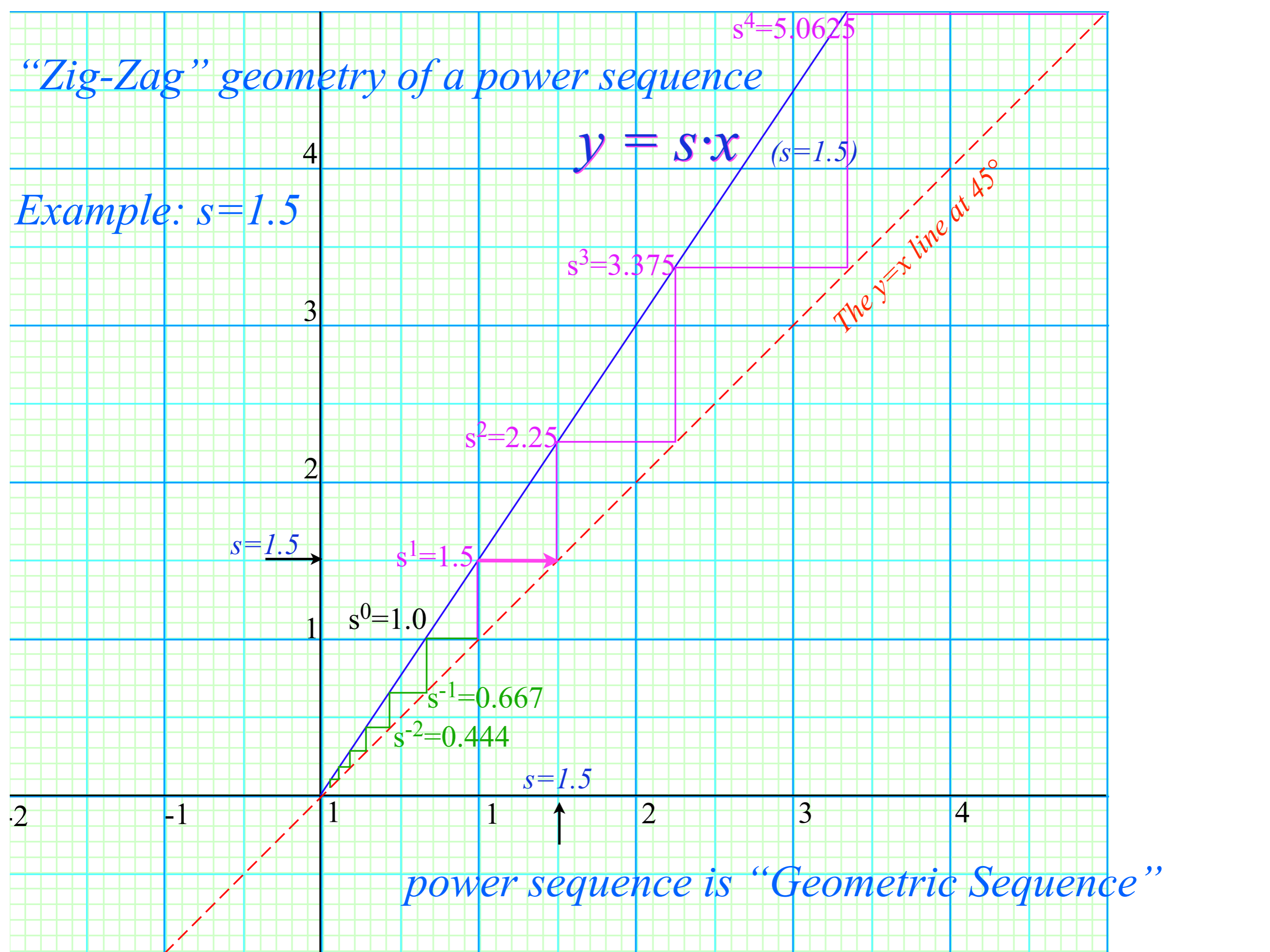
$s^0=1.0$

$s^{-1}=0.667$

$s^{-2}=0.444$

$s=1.5$

power sequence is “Geometric Sequence”

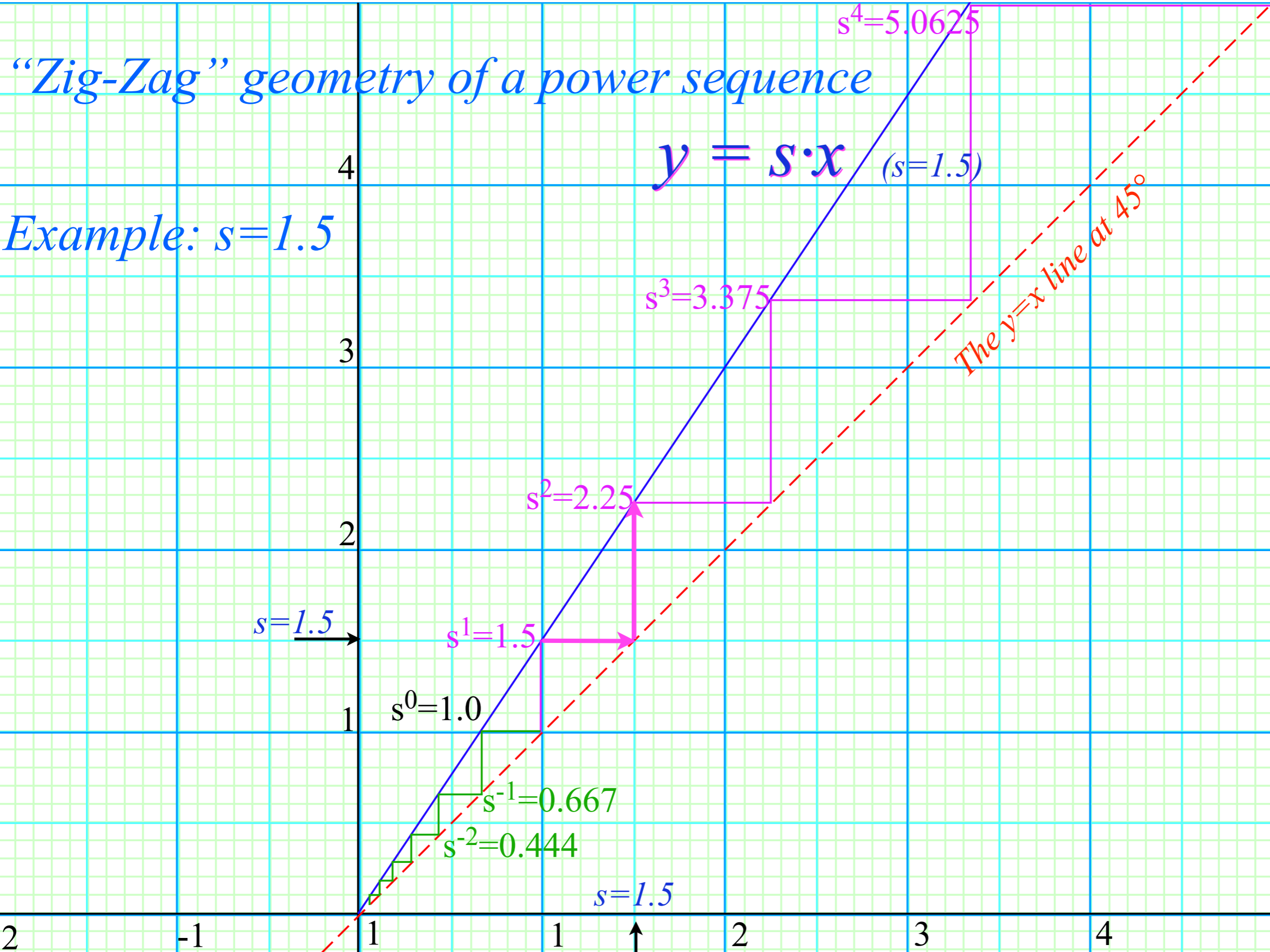


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power sequence is “Geometric Sequence”

“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

$$y = s \cdot x \quad (s=1.5)$$

The $y=x$ line at 45°

$$s^2 = 1.5^2$$

$$s = 1.5$$

$$s^0 = 1.0$$

$$s^1 = 1.5$$

$$s^2 = 2.25$$

$$s^3 = 3.375$$

$$s^4 = 5.0625$$

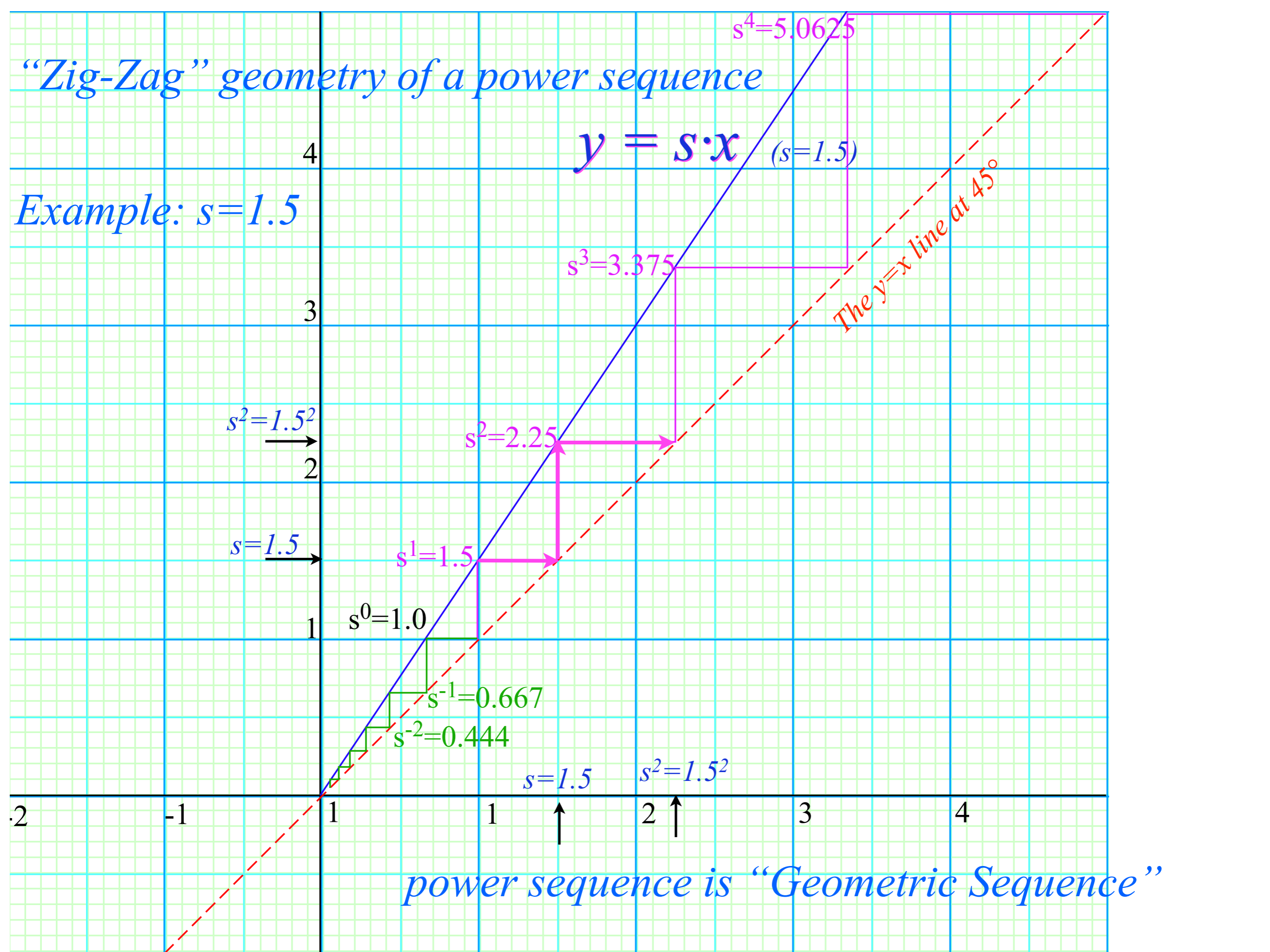
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Example: $s=1.5$

$$s^3=1.5^3$$

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$$y = s \cdot x \quad (s=1.5)$$

$$s^4=5.0625$$

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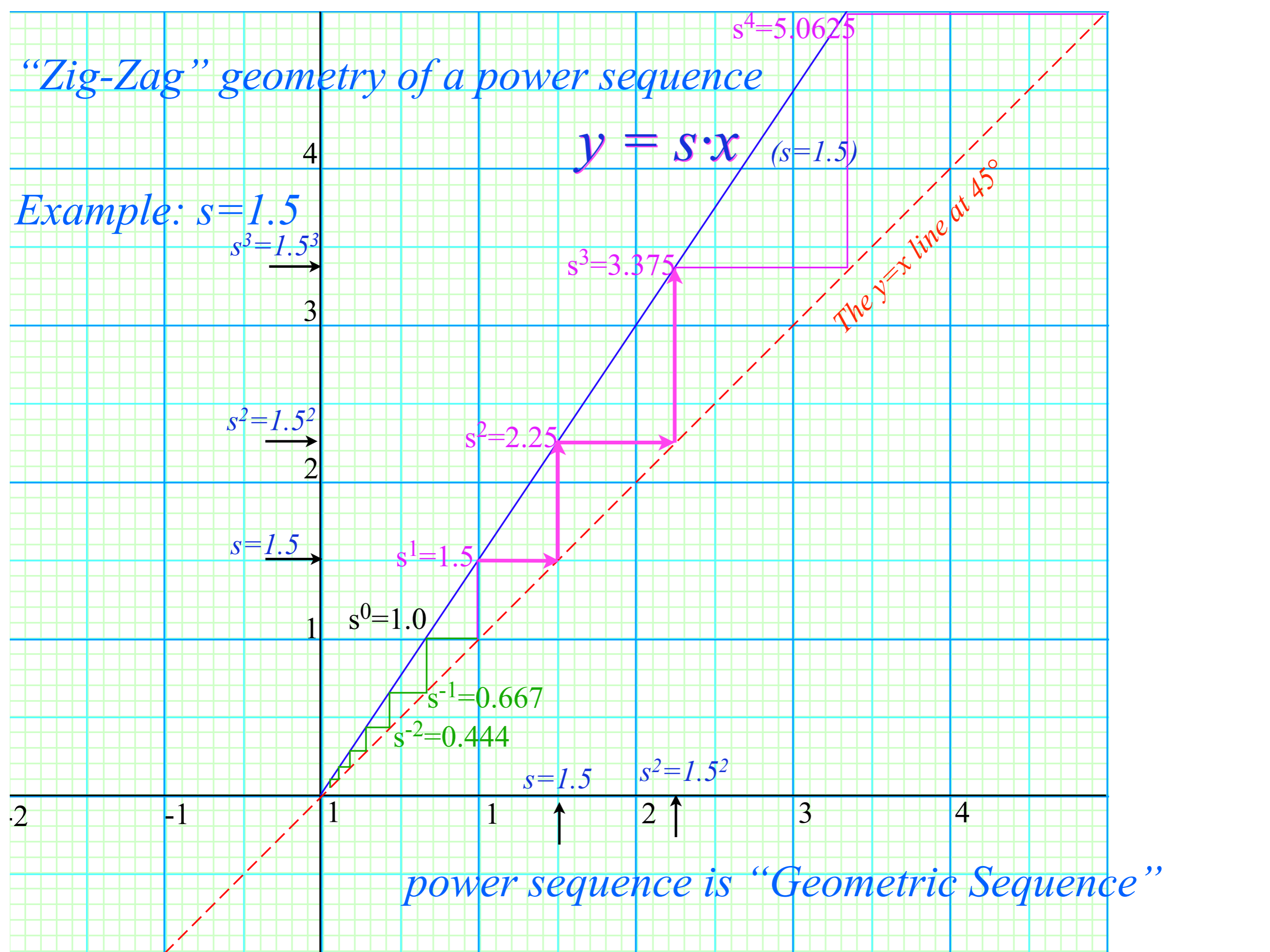
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The $y=x$ line at 45°

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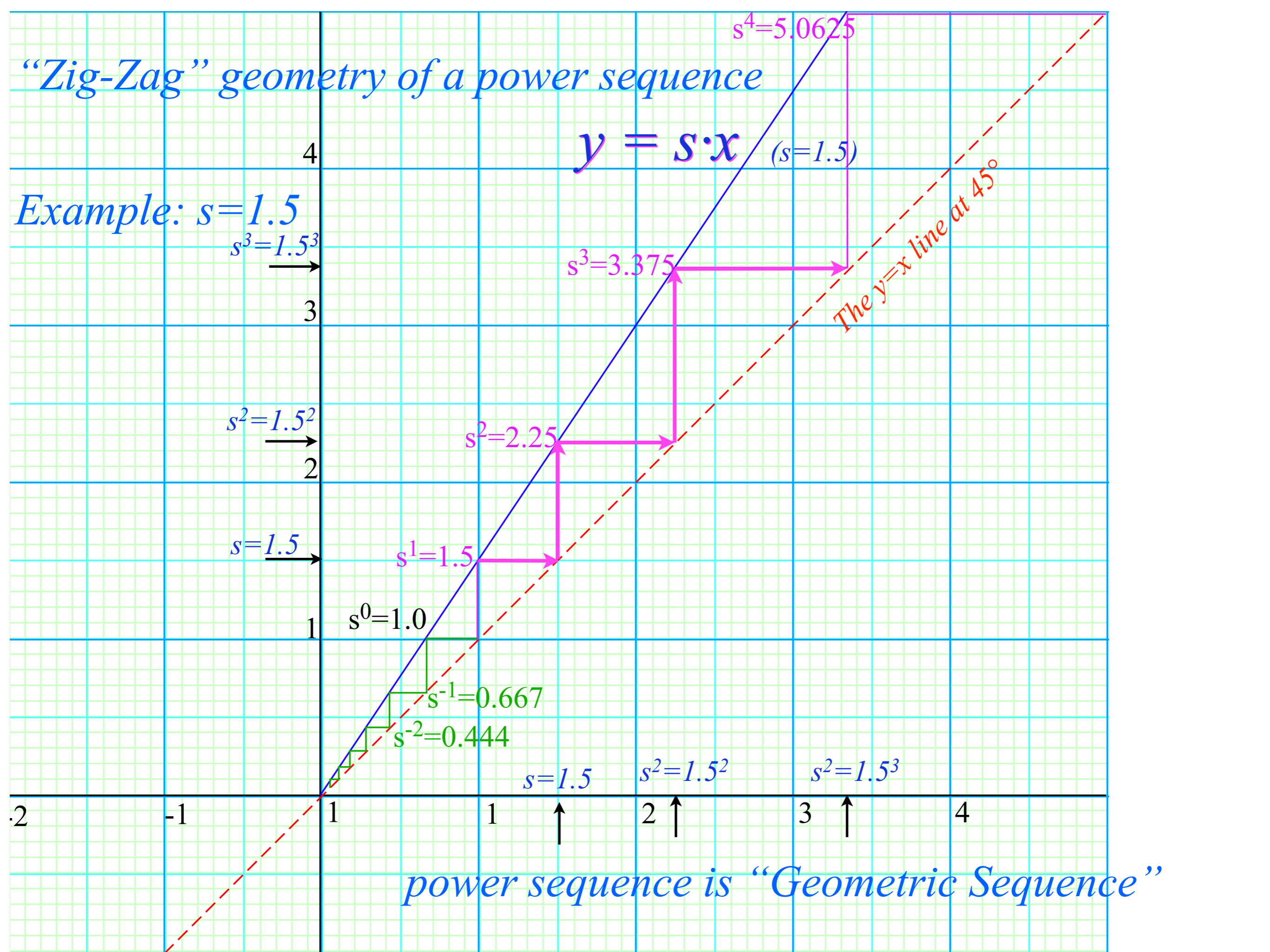
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$$s = 1.5$$

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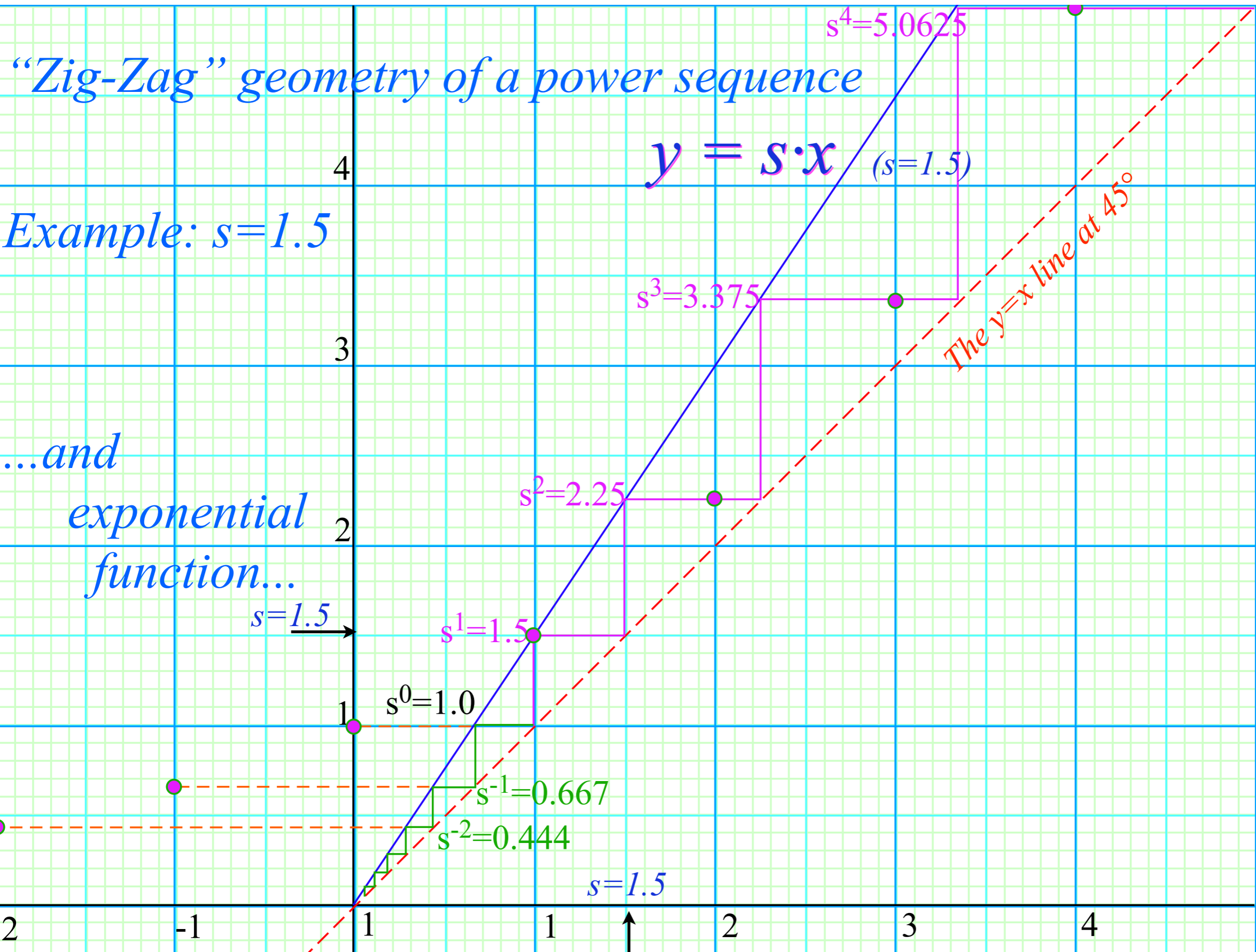
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“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

...and exponential function...



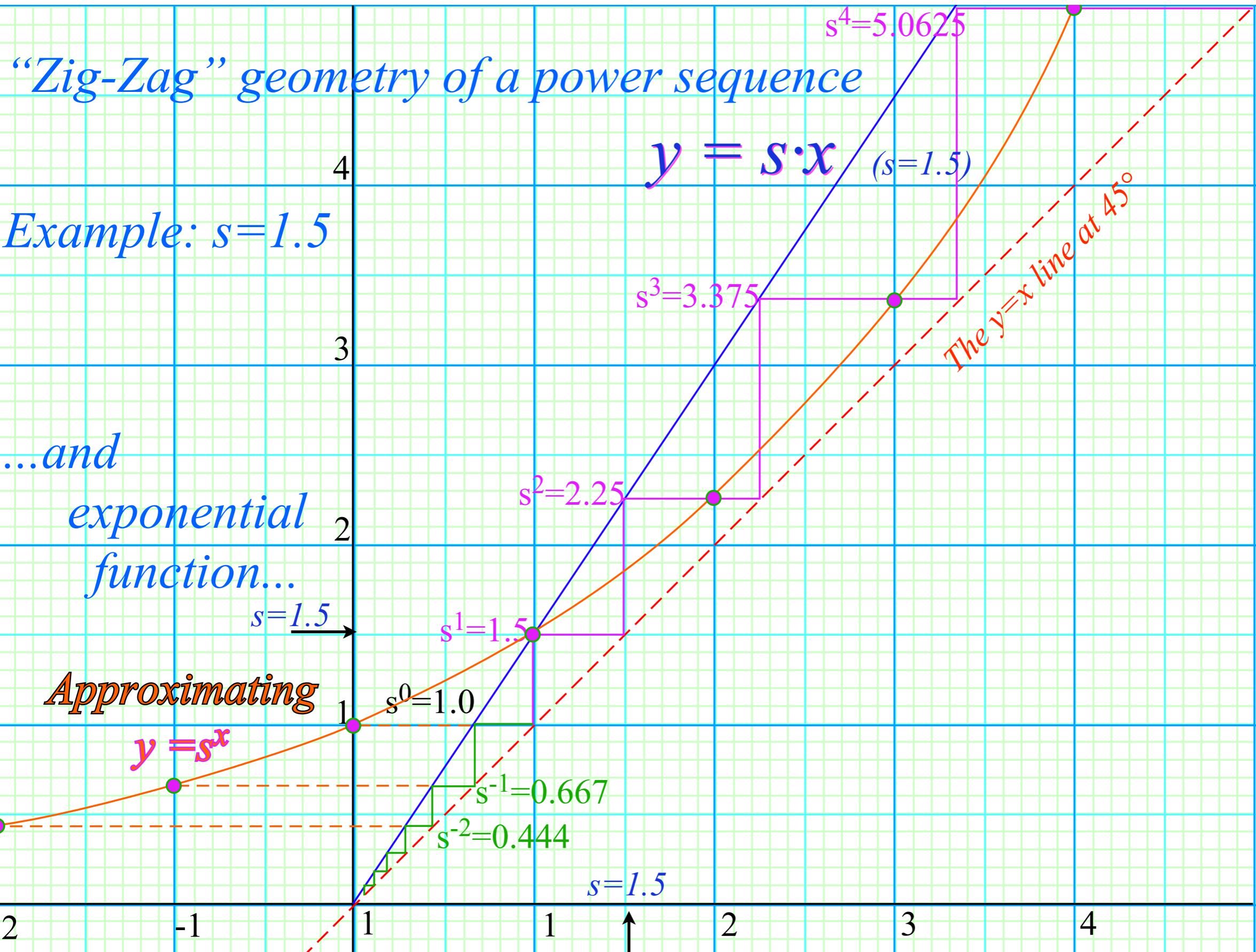
power sequence is “Geometric Sequence”

“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

...and exponential function...

Approximating



$y = s \cdot x$ ($s=1.5$)

The $y=x$ line at 45°

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power sequence is “Geometric Sequence”

“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

...and exponential function...

...and logarithm function...

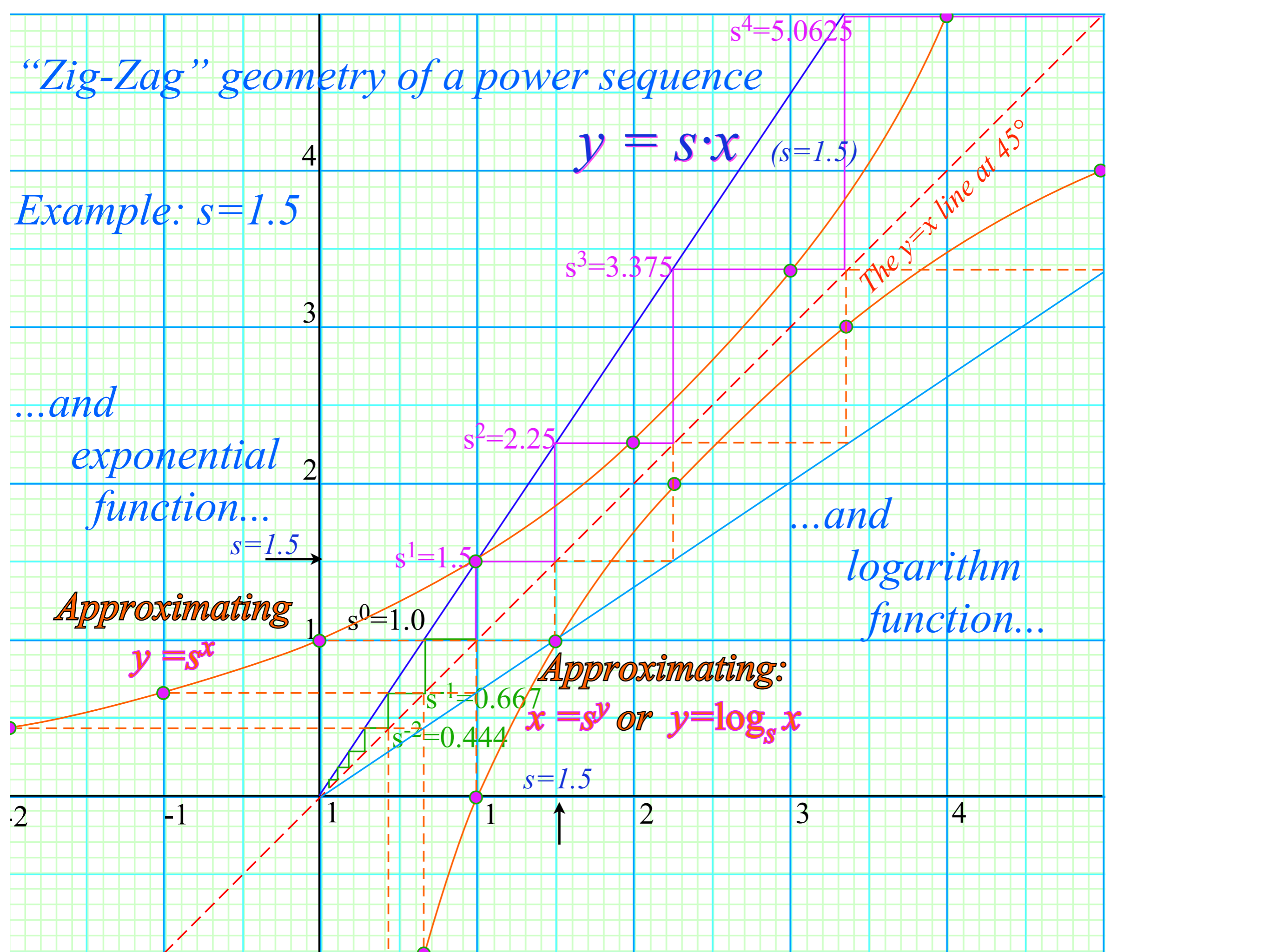
$$y = s \cdot x \quad (s=1.5)$$

The $y=x$ line at 45°

Approximating

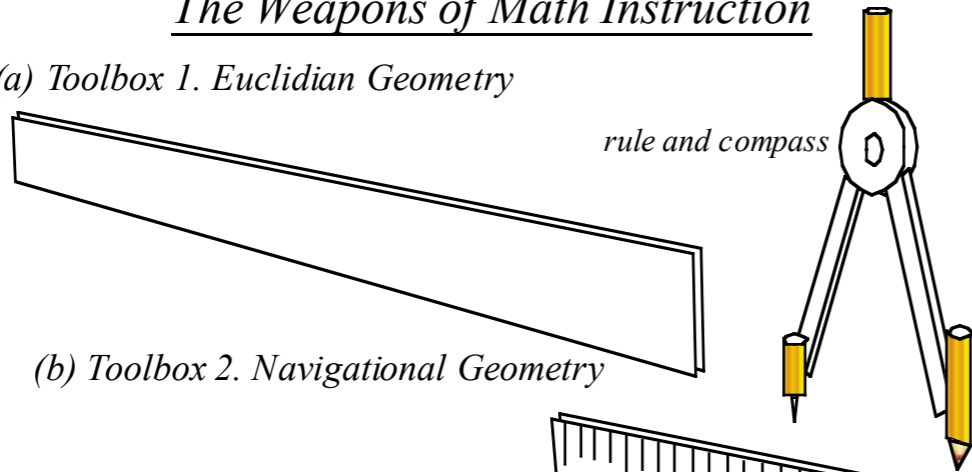
Approximating:

$$x = s^y \quad \text{or} \quad y = \log_s x$$

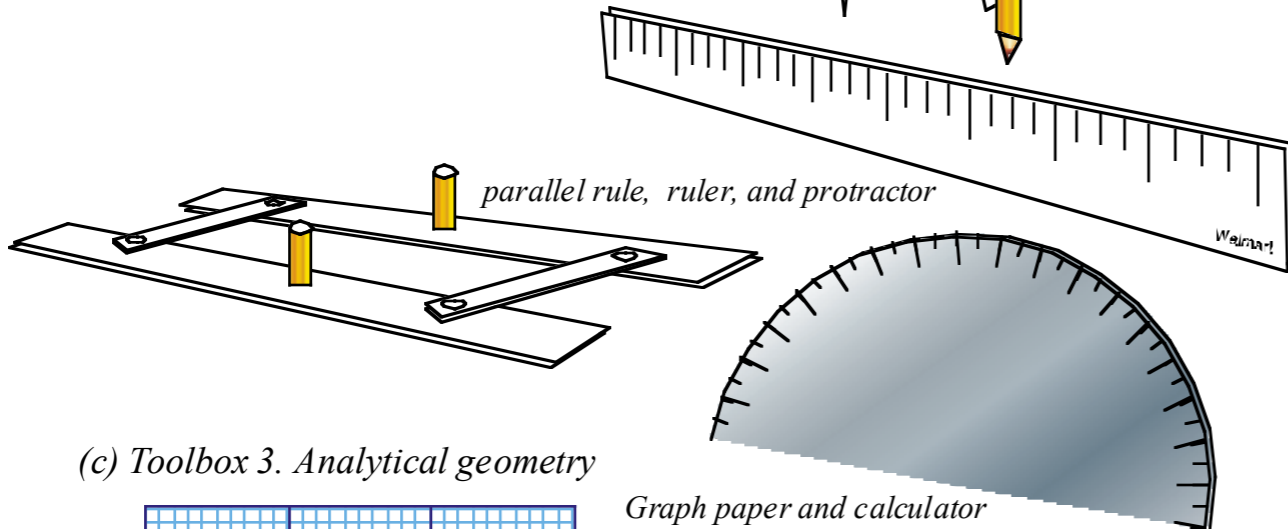


The Weapons of Math Instruction

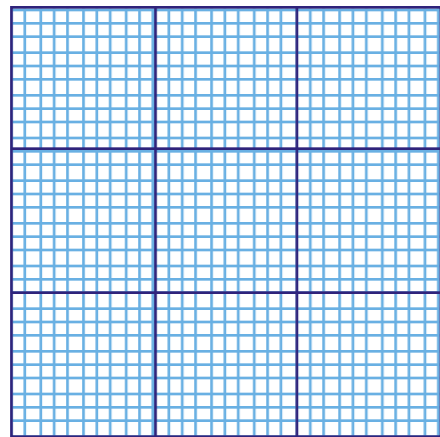
(a) Toolbox 1. Euclidian Geometry



(b) Toolbox 2. Navigational Geometry



(c) Toolbox 3. Analytical geometry

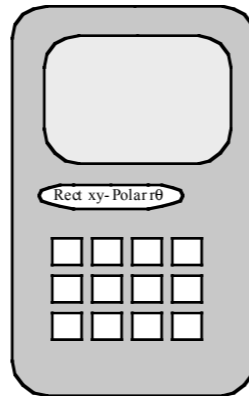


Graph paper and calculator

Complex algebra and calculus

$$1/z = r^{-1} e^{-i\theta}$$

$$\int 1/z dz = \ln z$$



So far we mostly use
Toolbox (a-b)

What follows uses
Toolbox (c) ...

...and Toolbox (d)

(d) Toolbox 4. Computer geometry...Anything goes!



Facelt



Bandlt



Bohrlt



Bouncelt



Colorlt U2



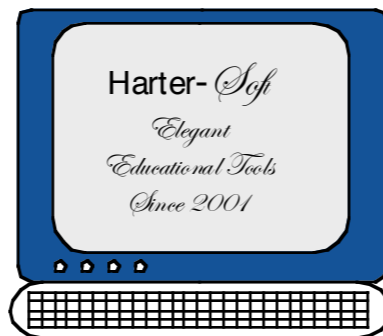
Oscillt

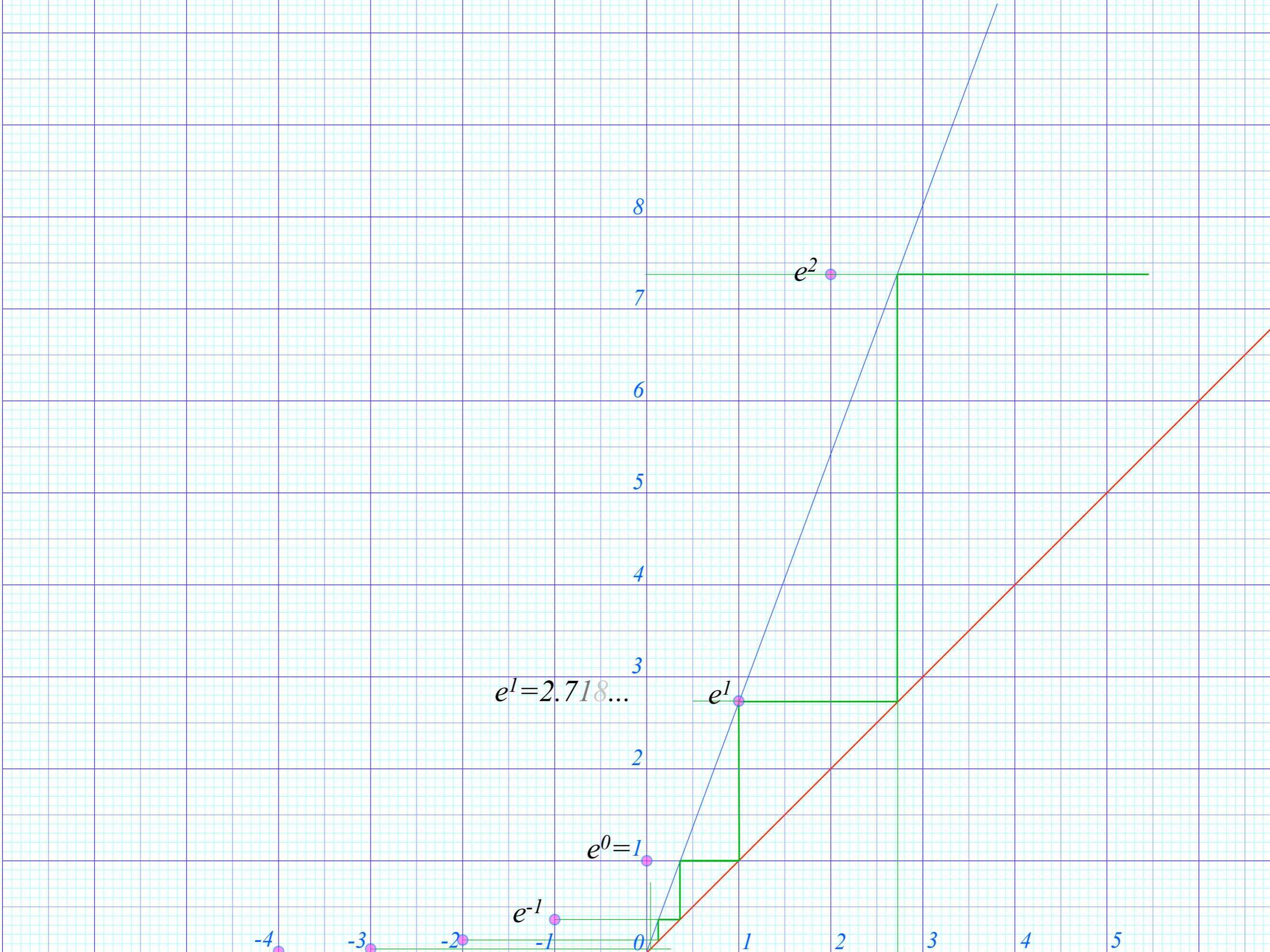


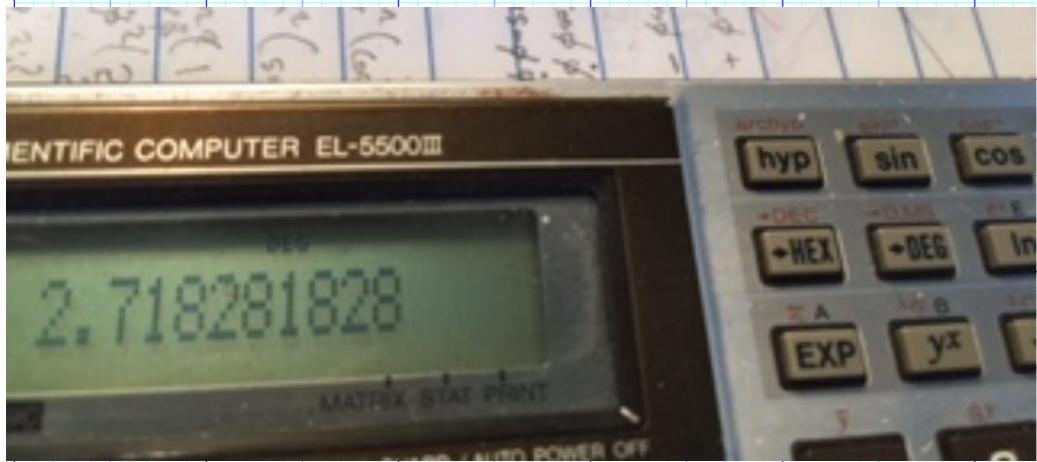
Relativt



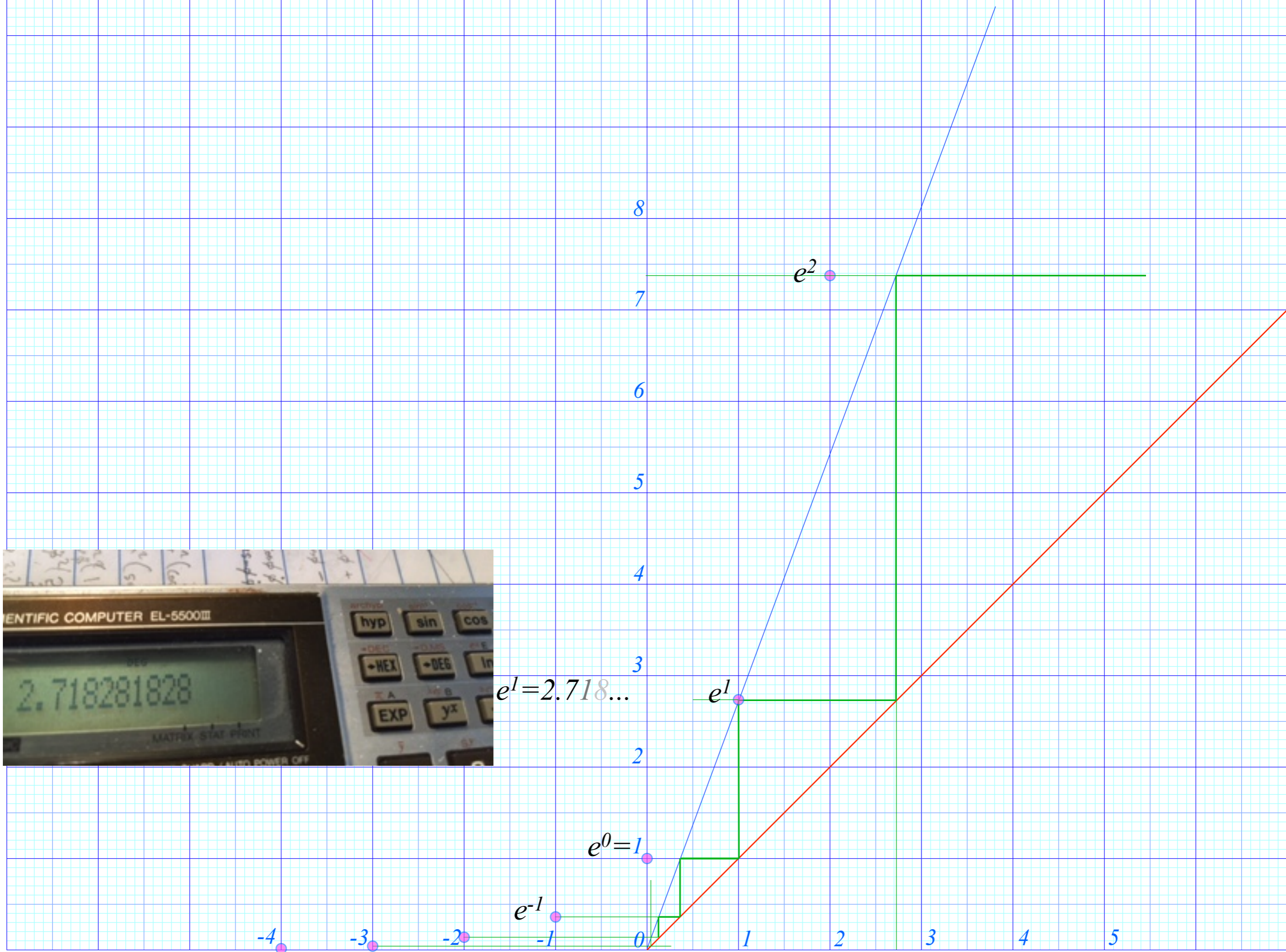
Wavelt







$e^1 = 2.718\dots$



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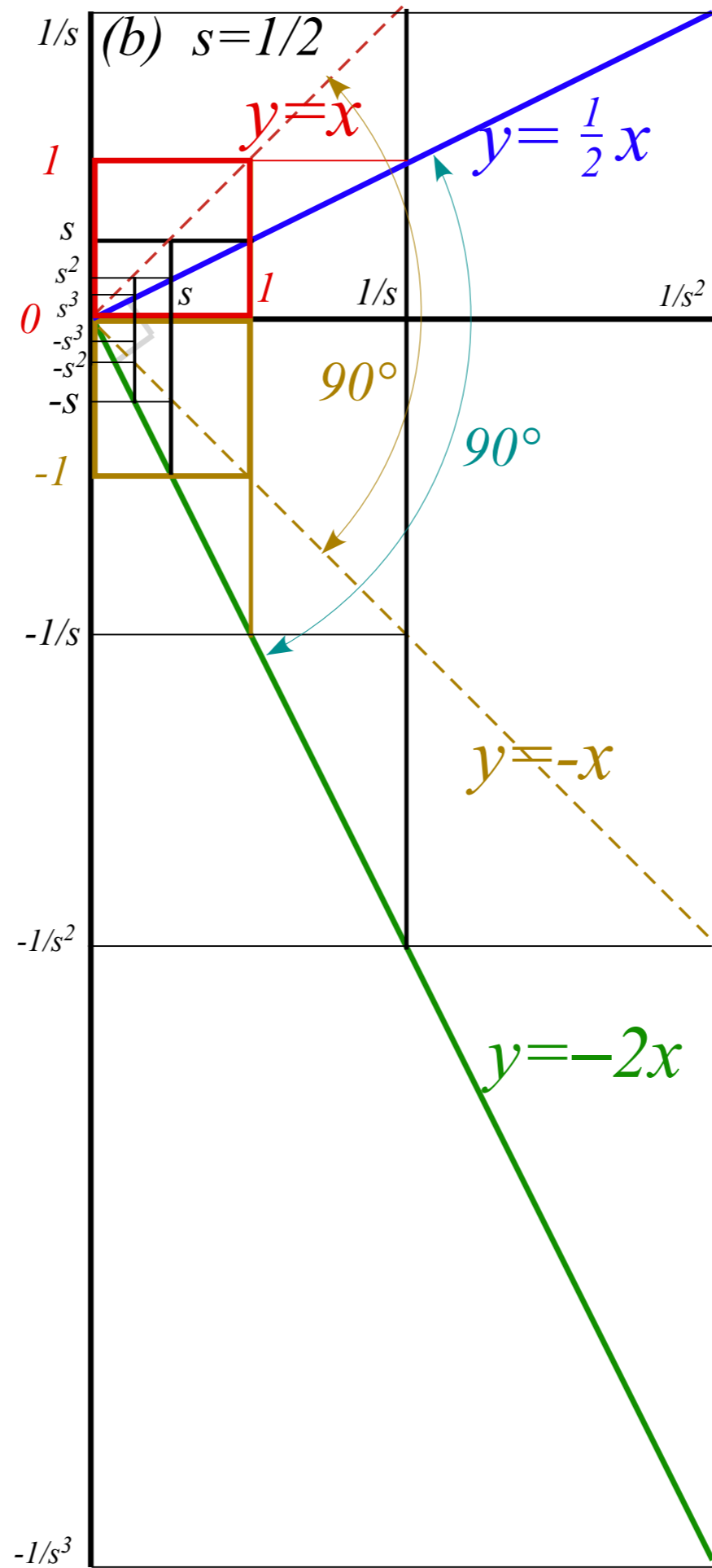
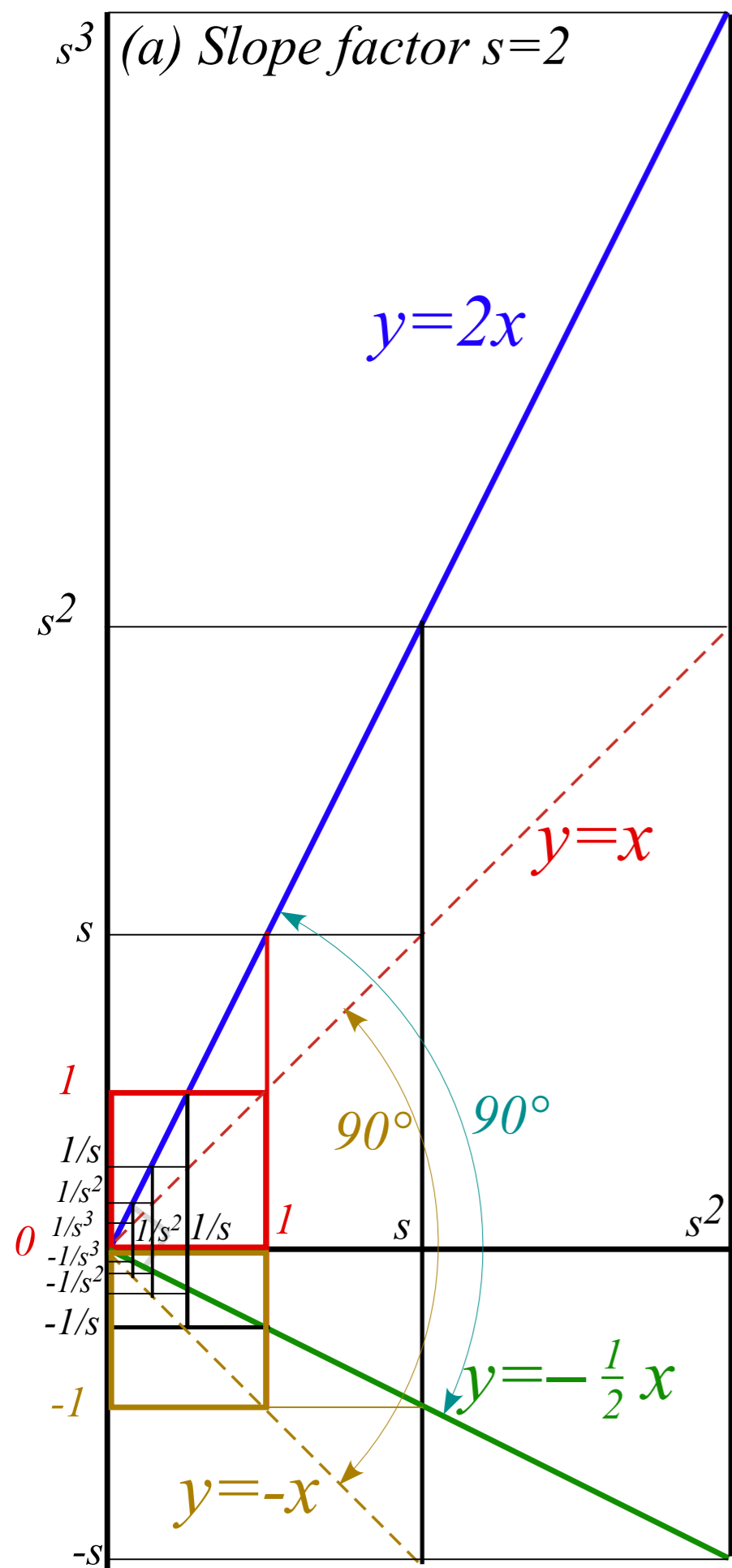


Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

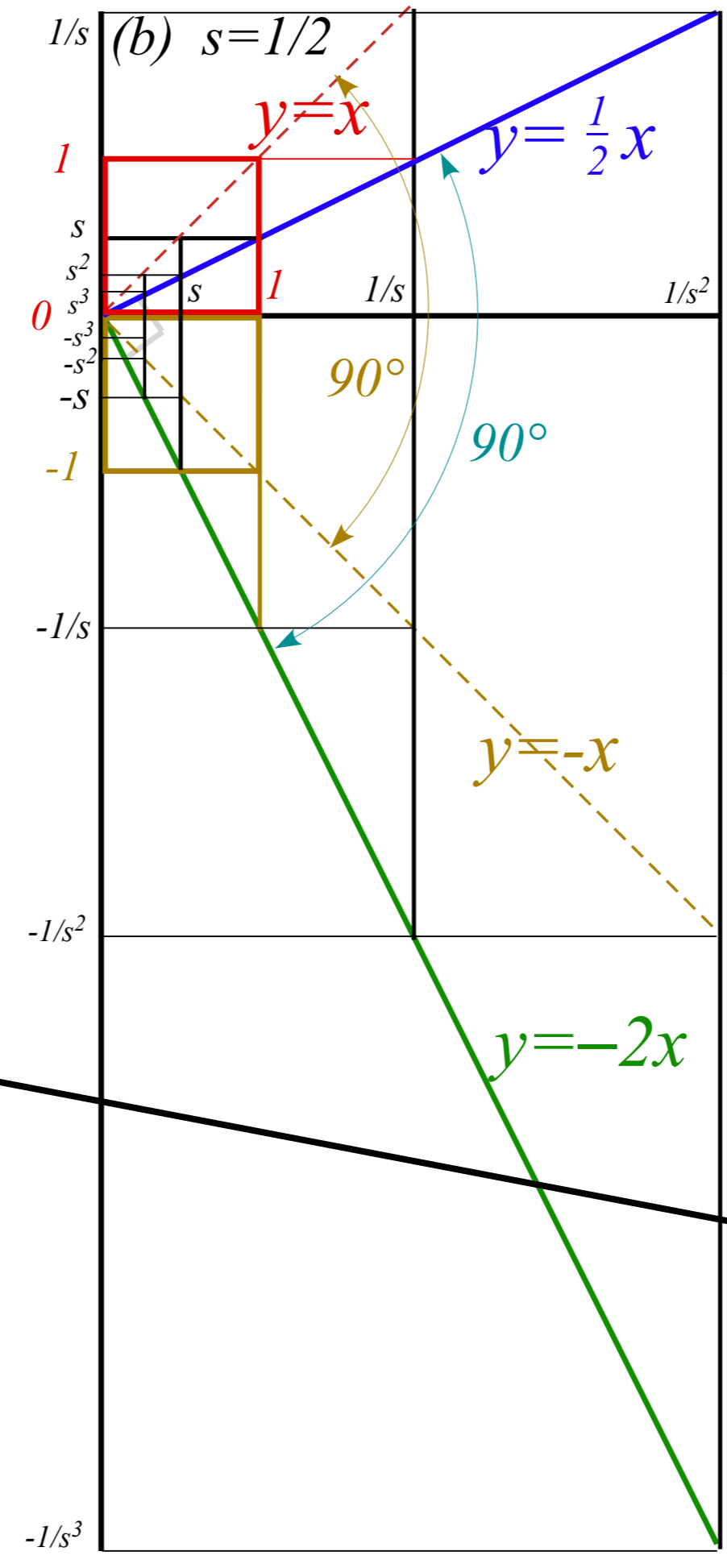
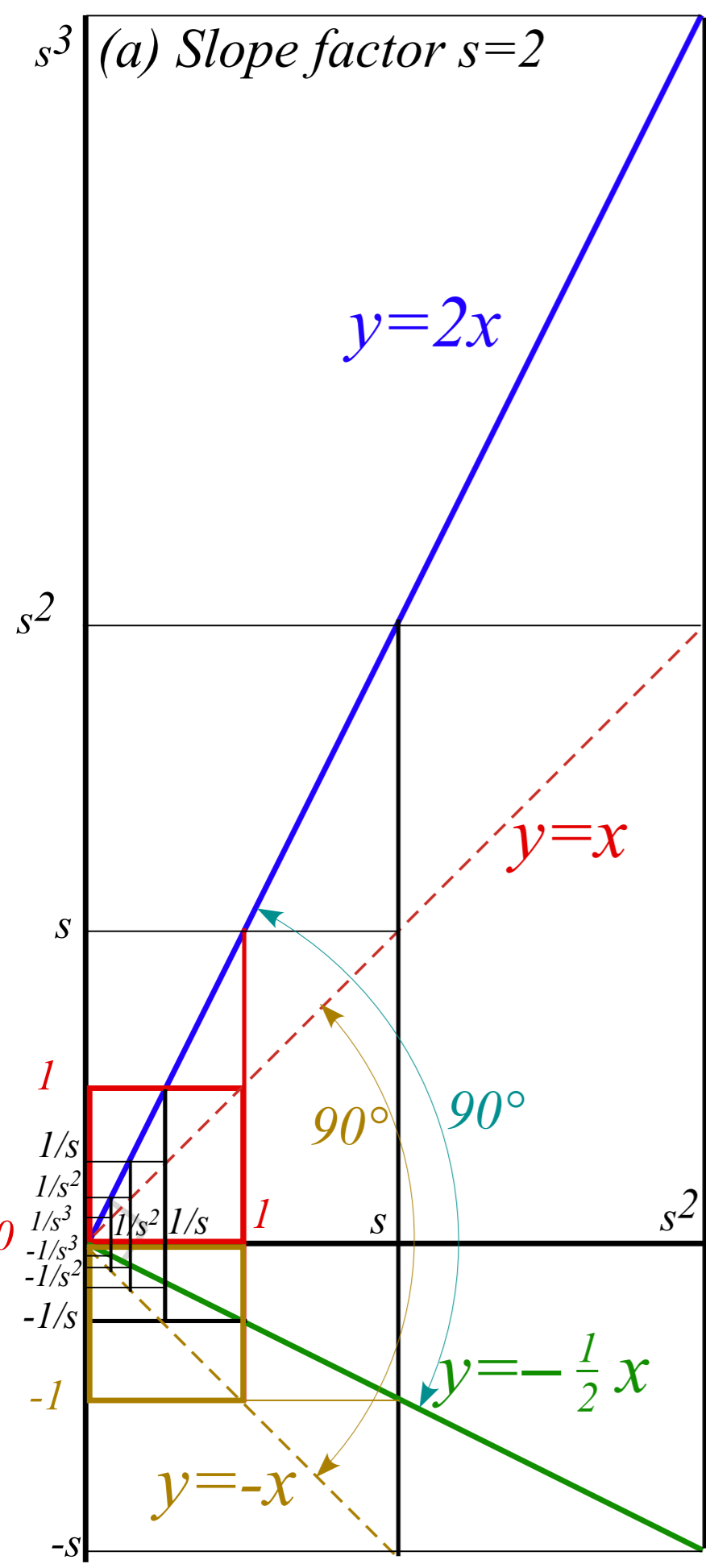
Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

Compare mks units of Coulomb Electrostatic vs. Gravity



“Zig-Zags” give perspective geometry (1D-vanishing point)

Unit 1
Fig. 9.2



“Zig-Zags” give perspective geometry (1D-vanishing point)

Unit 1
Fig. 9.2

1st-day-of-school perspective of 12th-grader

1st-day-of-school perspective of 1st-grader

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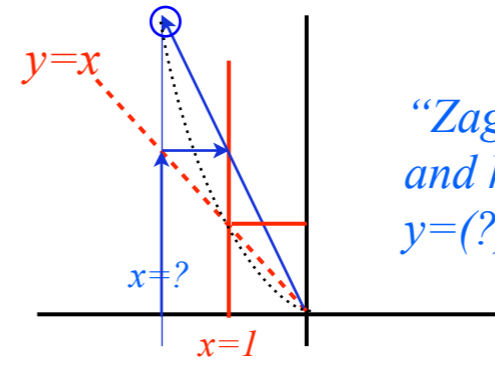
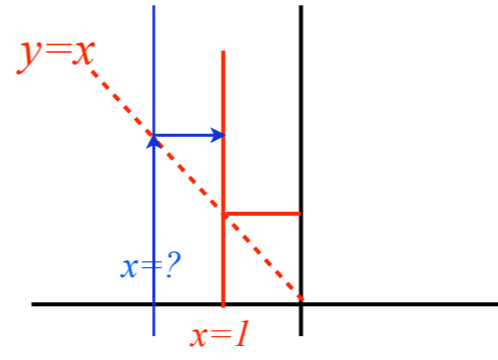
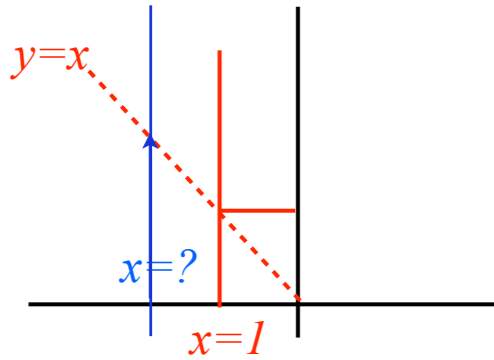
Compare mks units of Coulomb Electrostatic vs. Gravity

Each $y=x^2$ parabola point found by just one “Zig-Zag”

1. Pick an $(x=?)$ -line

2. “Zig” from its $y=x$ intersection to $x=1$ line

3. “Zag” from origin back to $(x=?)$ -line



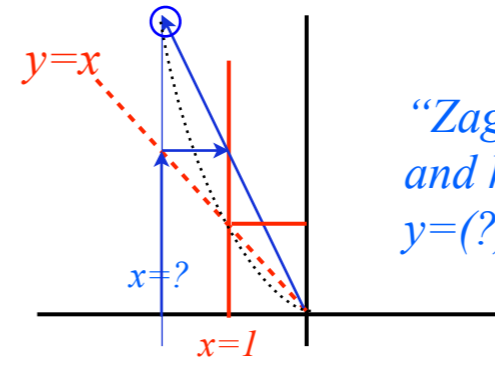
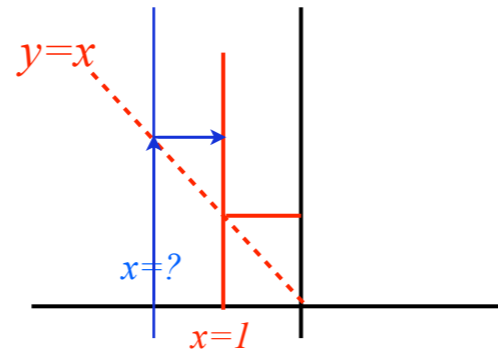
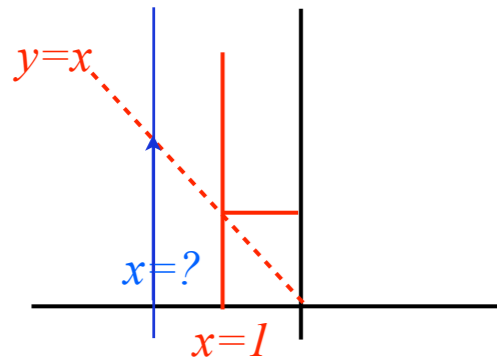
“Zag” line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$

Each $y=x^2$ parabola point found by just one “Zig-Zag”

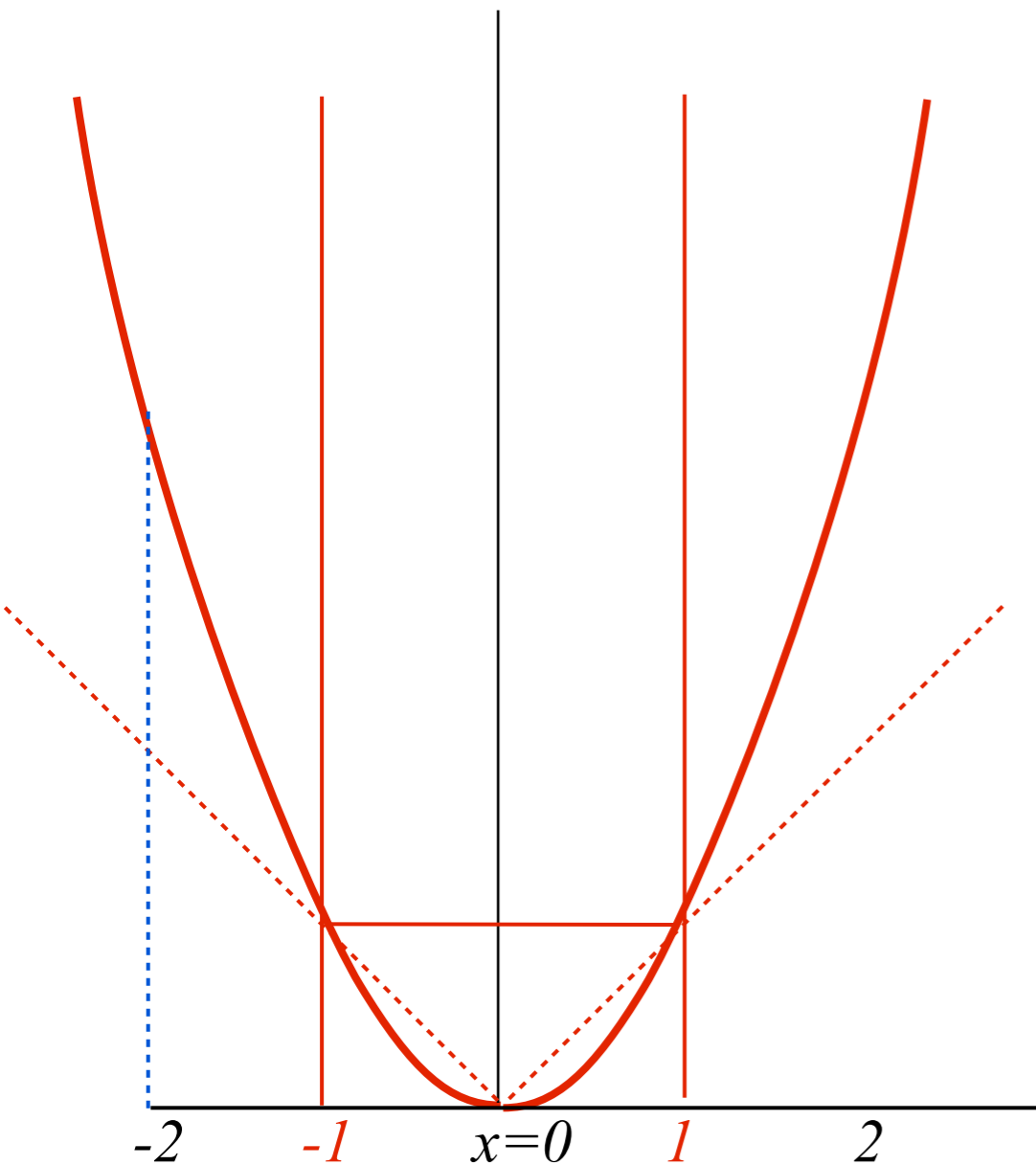
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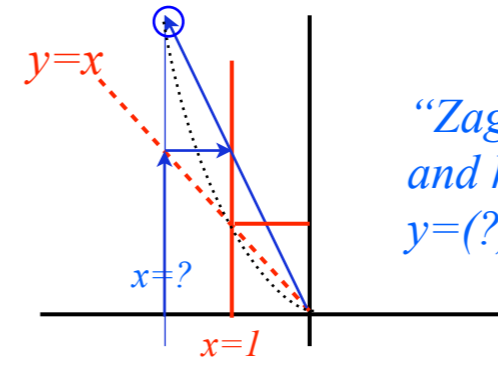
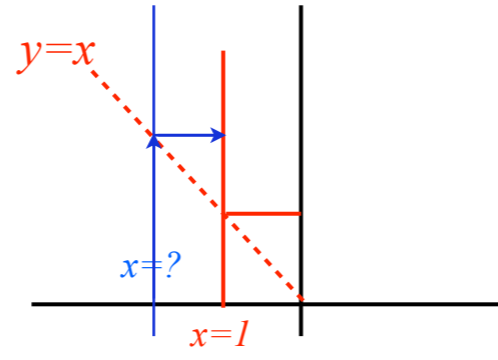
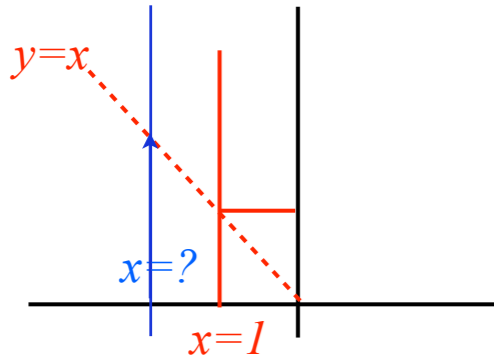
Unit 1
Fig. 9.1

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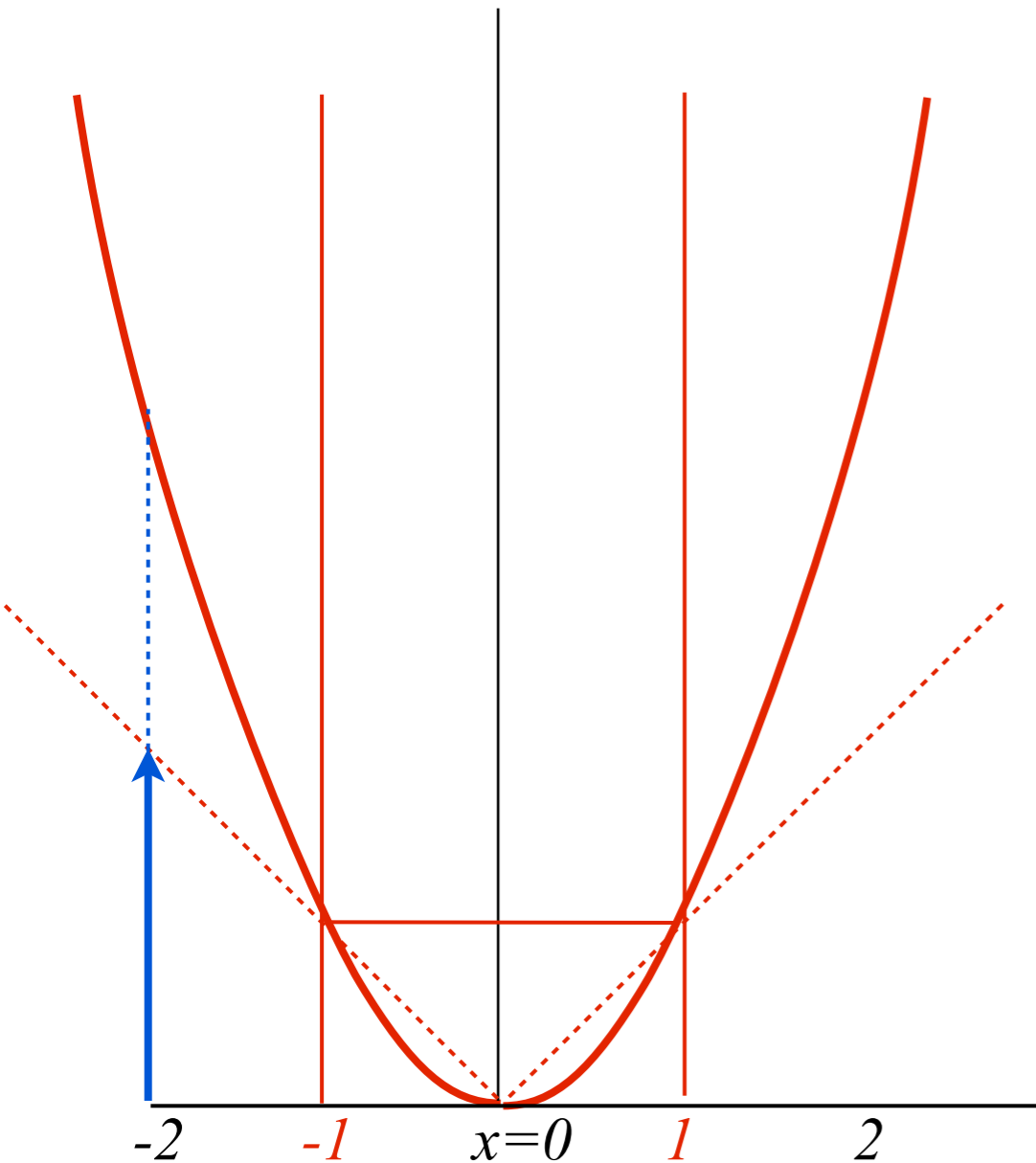
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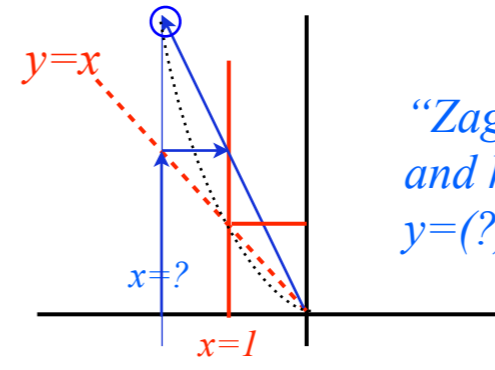
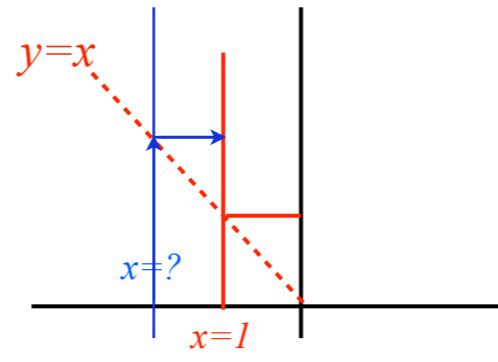
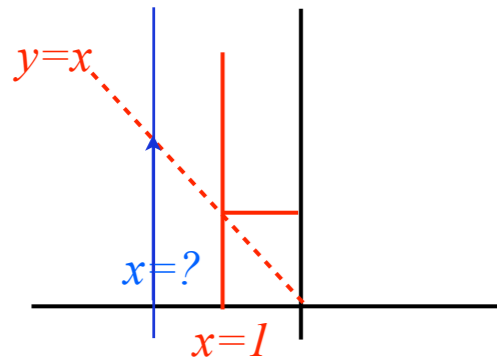
Unit 1
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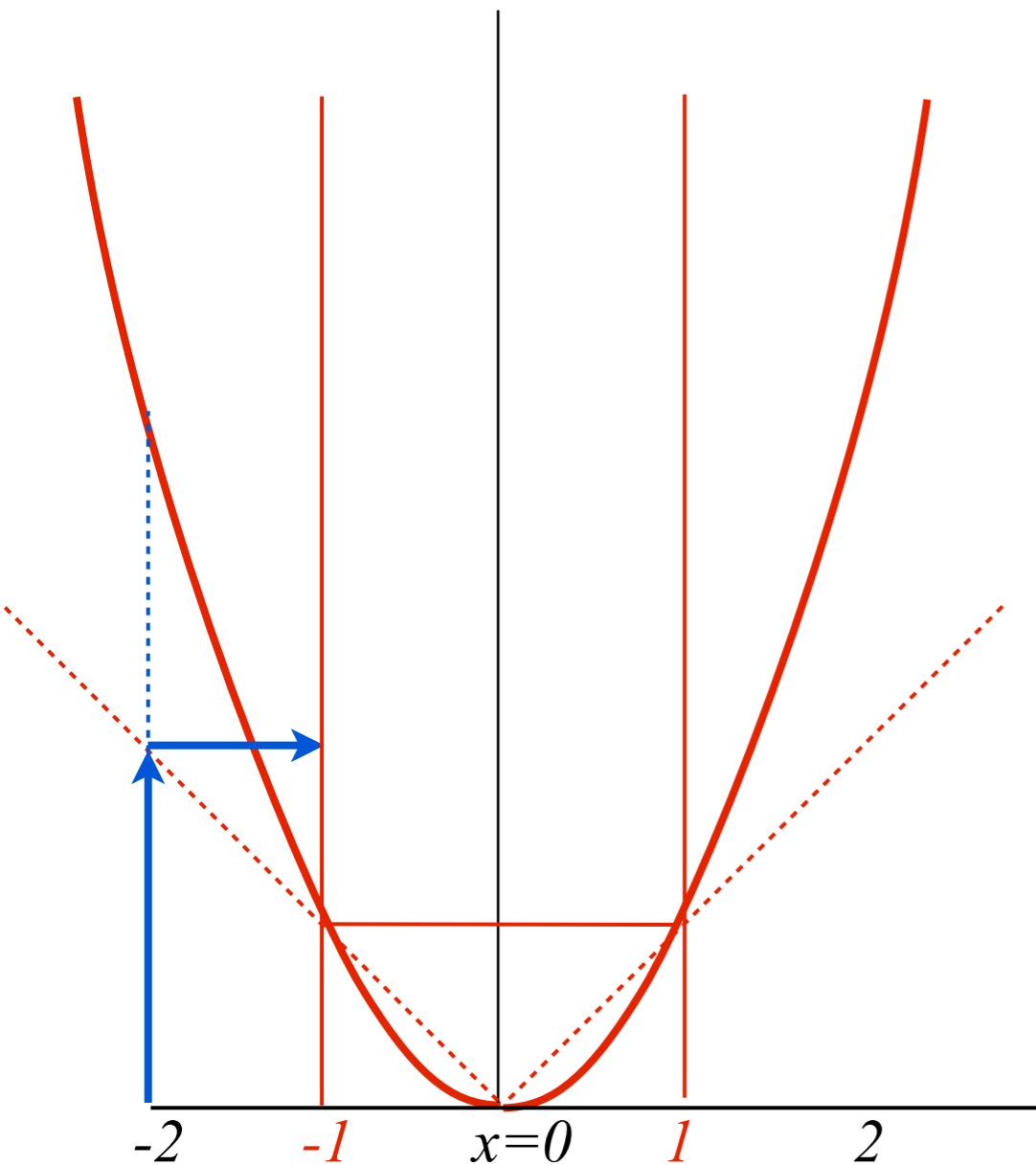
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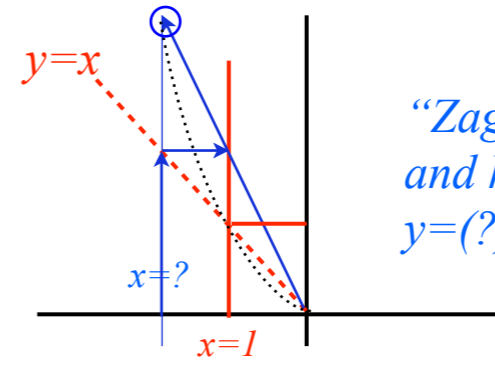
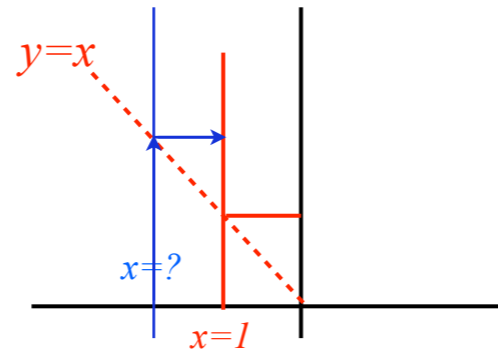
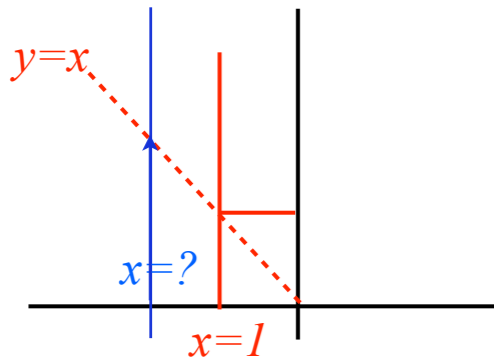
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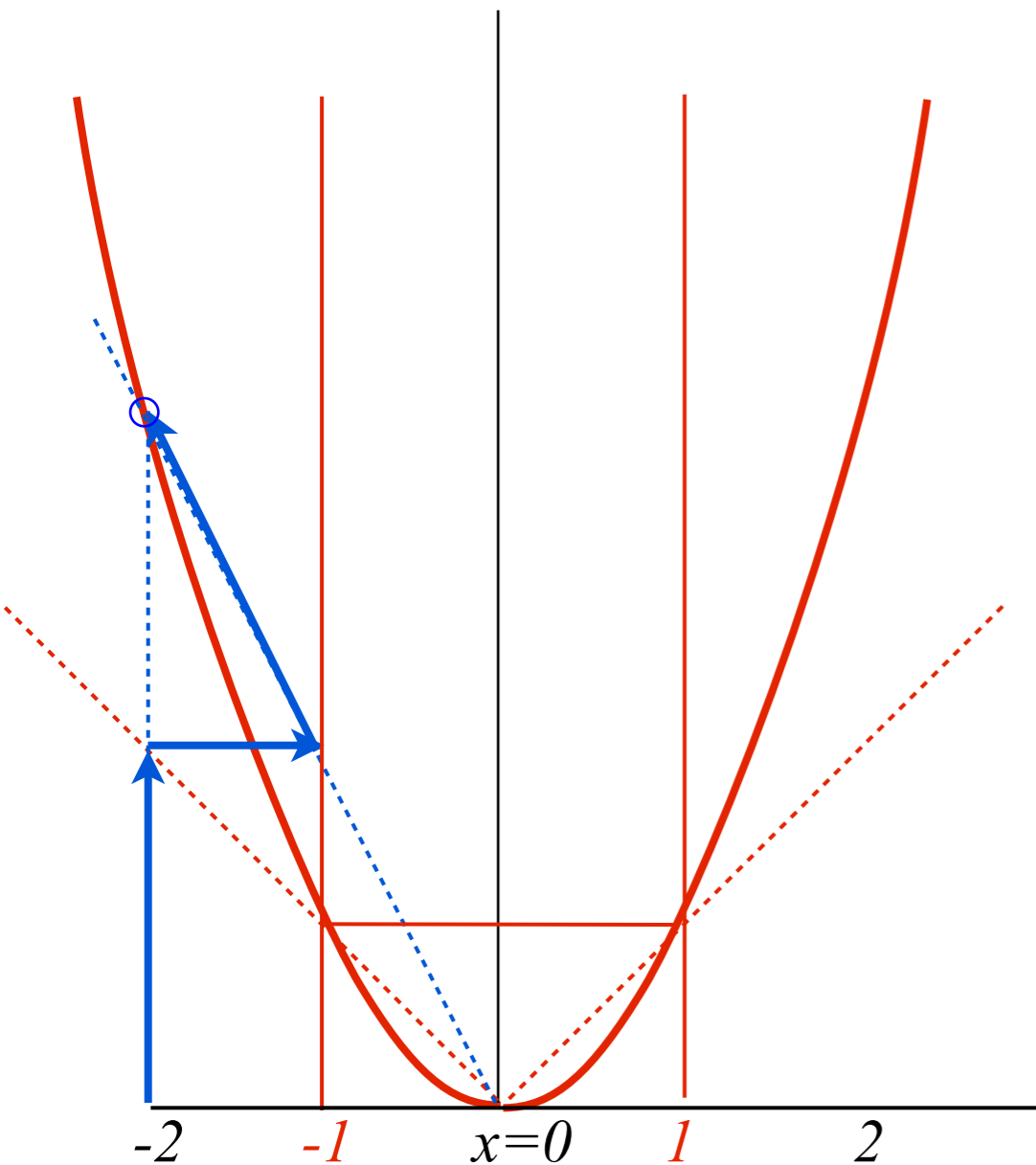
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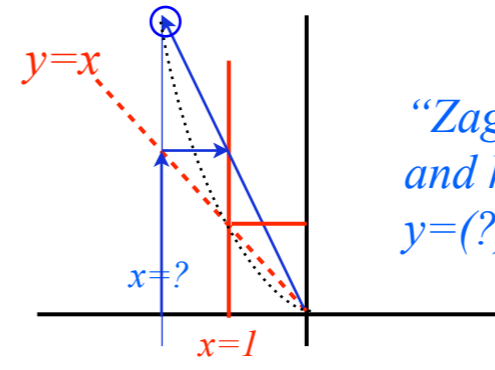
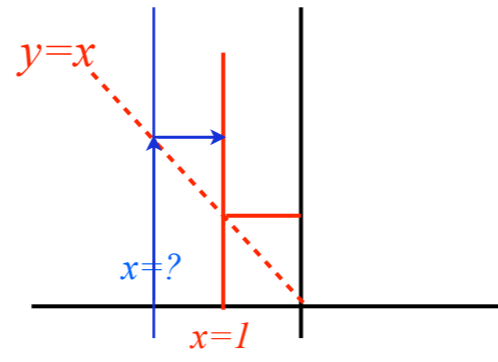
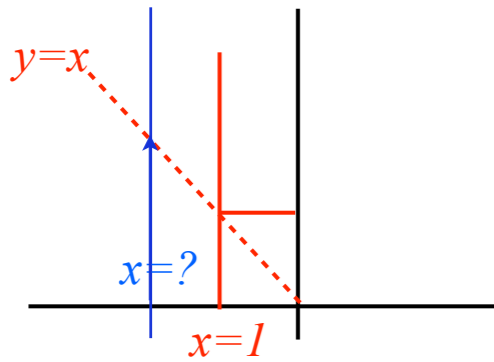
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one “Zig-Zag”

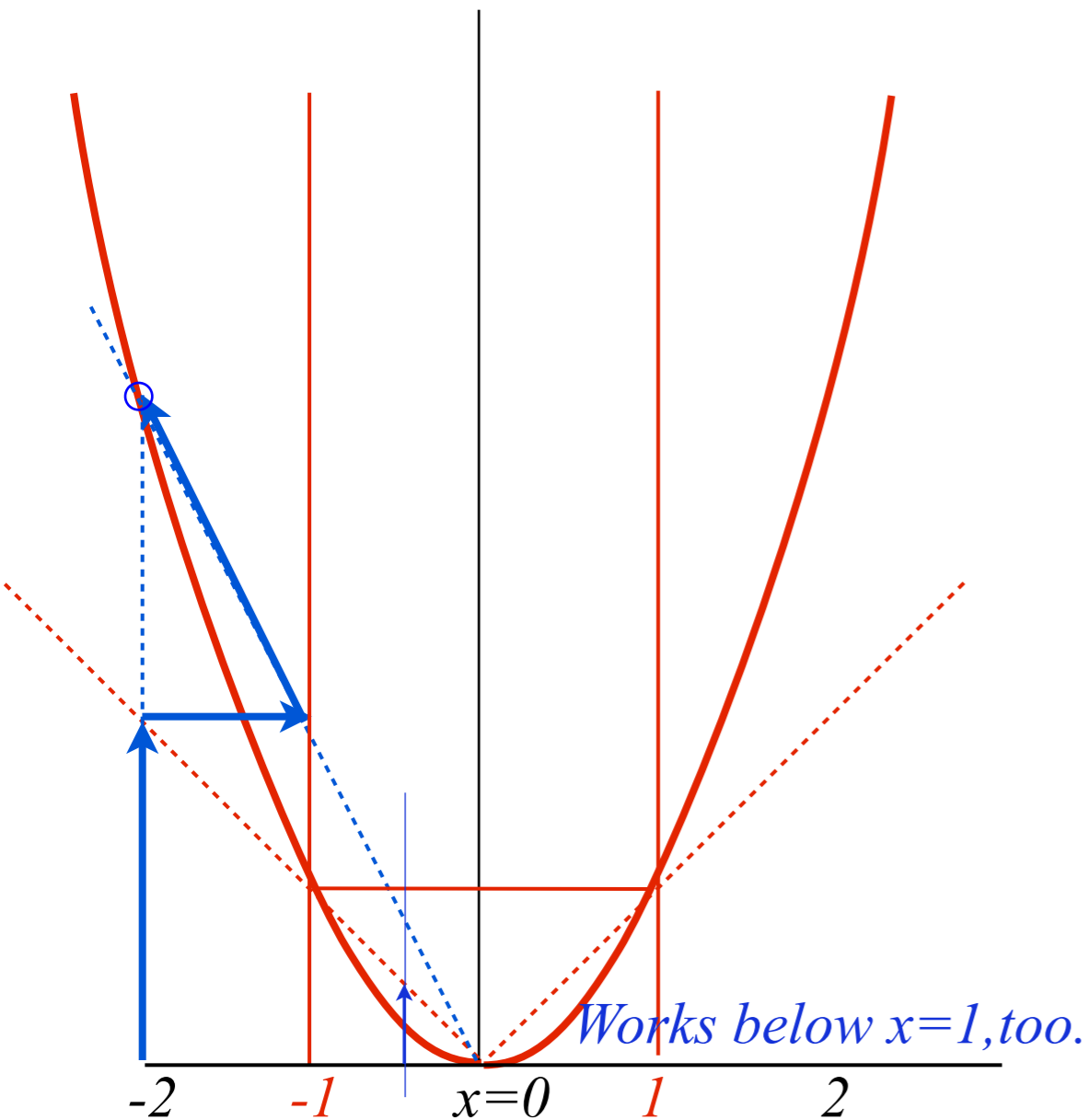
1. Pick an $(x=?)$ -line

2. “Zig” from its $y=x$ intersection to $x=1$ line

3. “Zag” from origin back to $(x=?)$ -line



“Zag” line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$

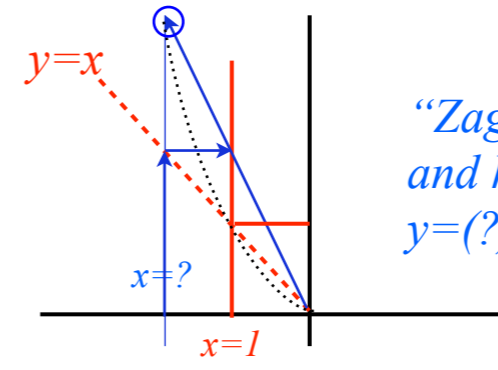
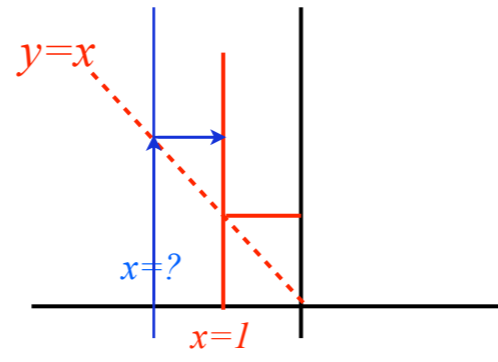
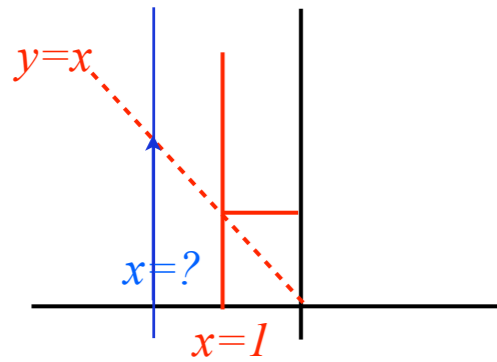


Each $y=x^2$ parabola point found by just one “Zig-Zag”

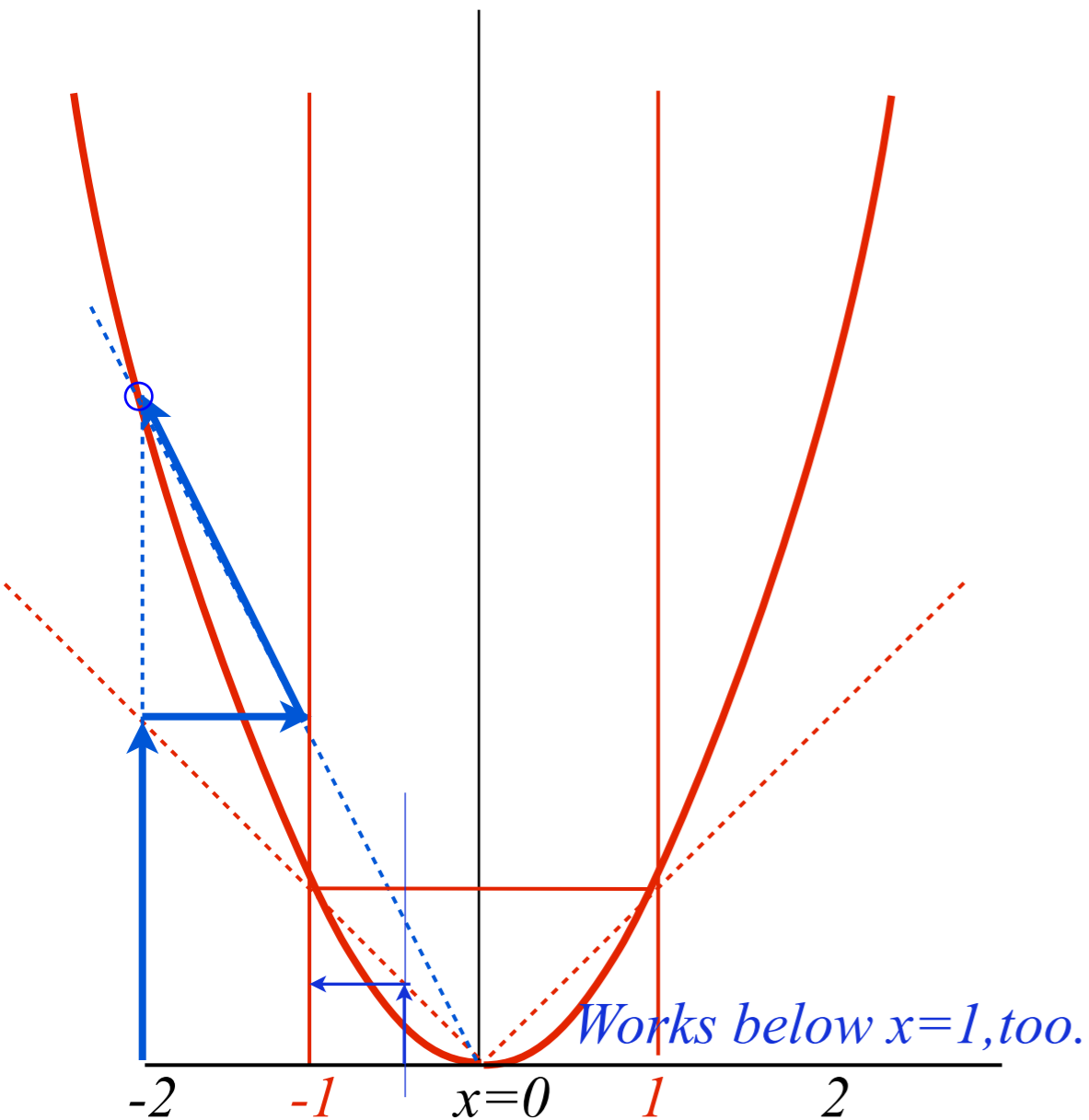
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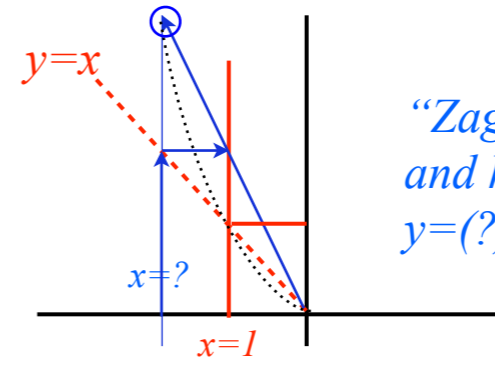
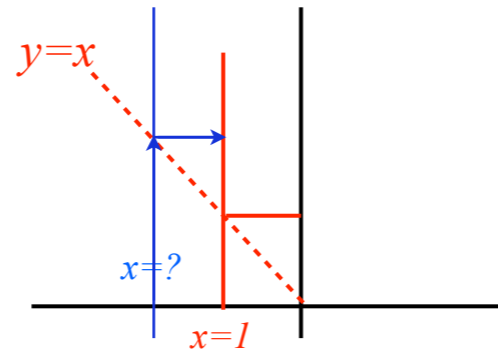
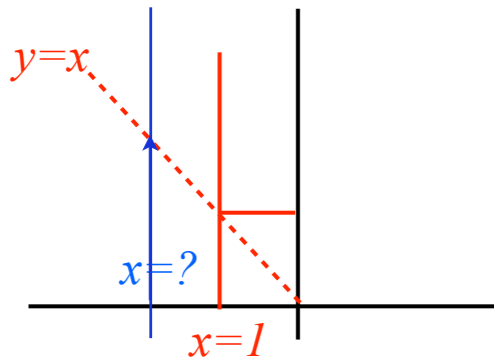


Each $y=x^2$ parabola point found by just one “Zig-Zag”

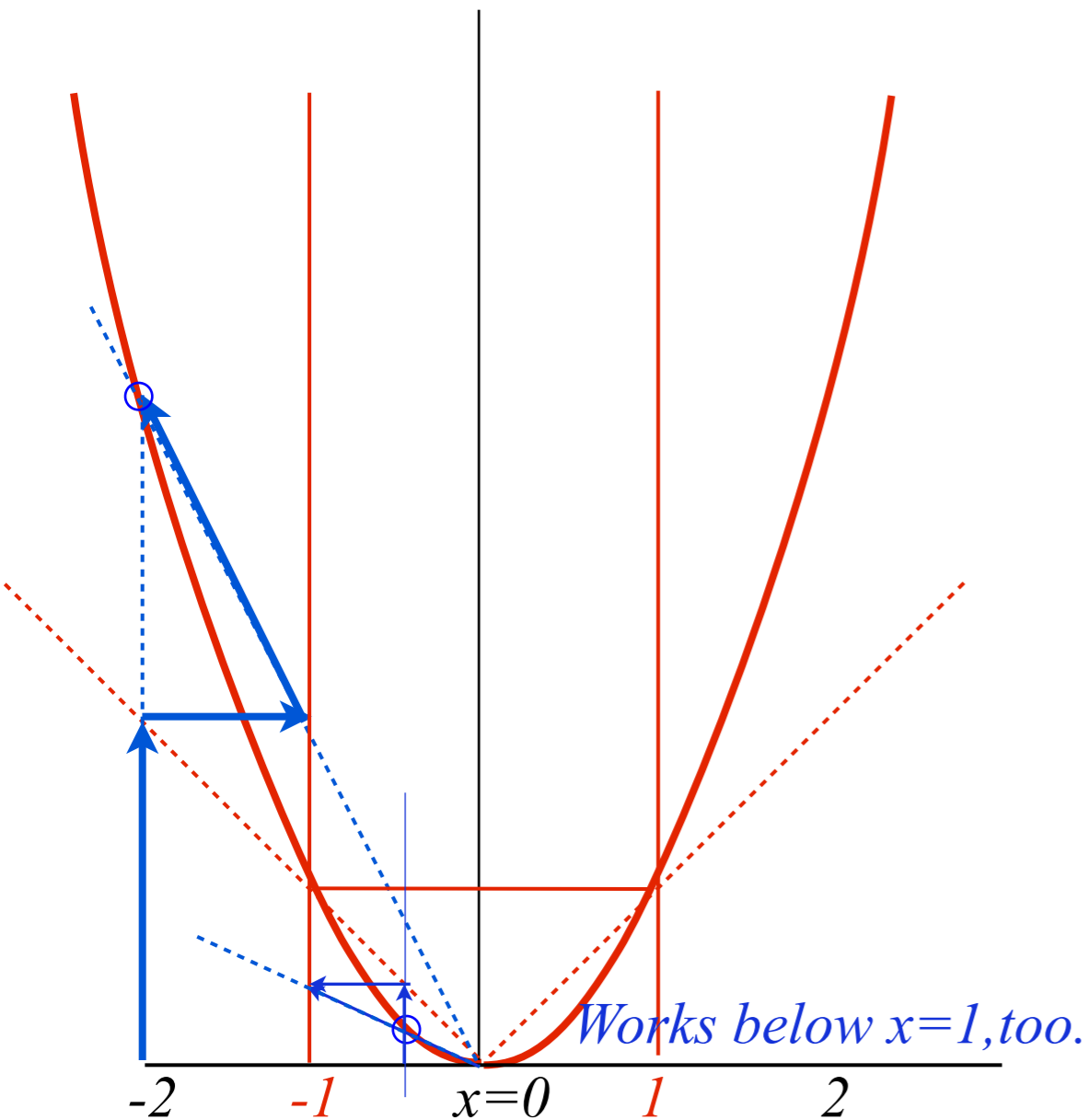
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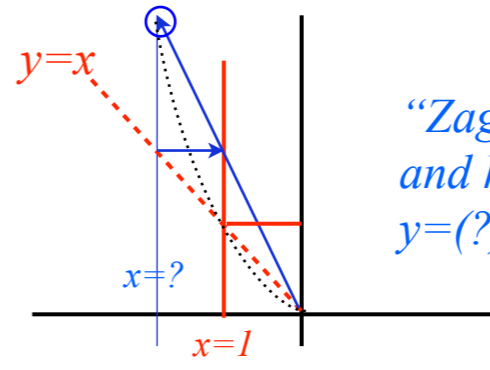
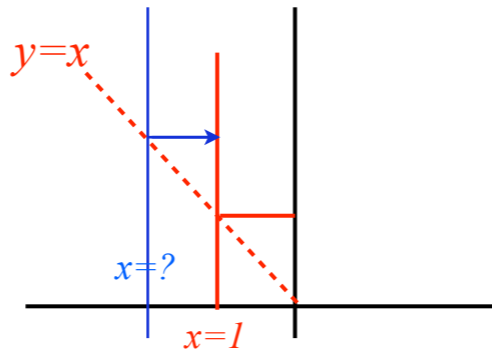
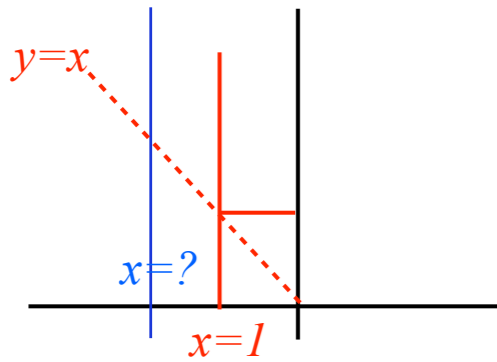
“Zag” line is $y=(?)\cdot x$
and hits $(x=?)$ -line at
 $y=(?)\cdot(?)=(?)^2$



Unit 1
Fig. 9.1

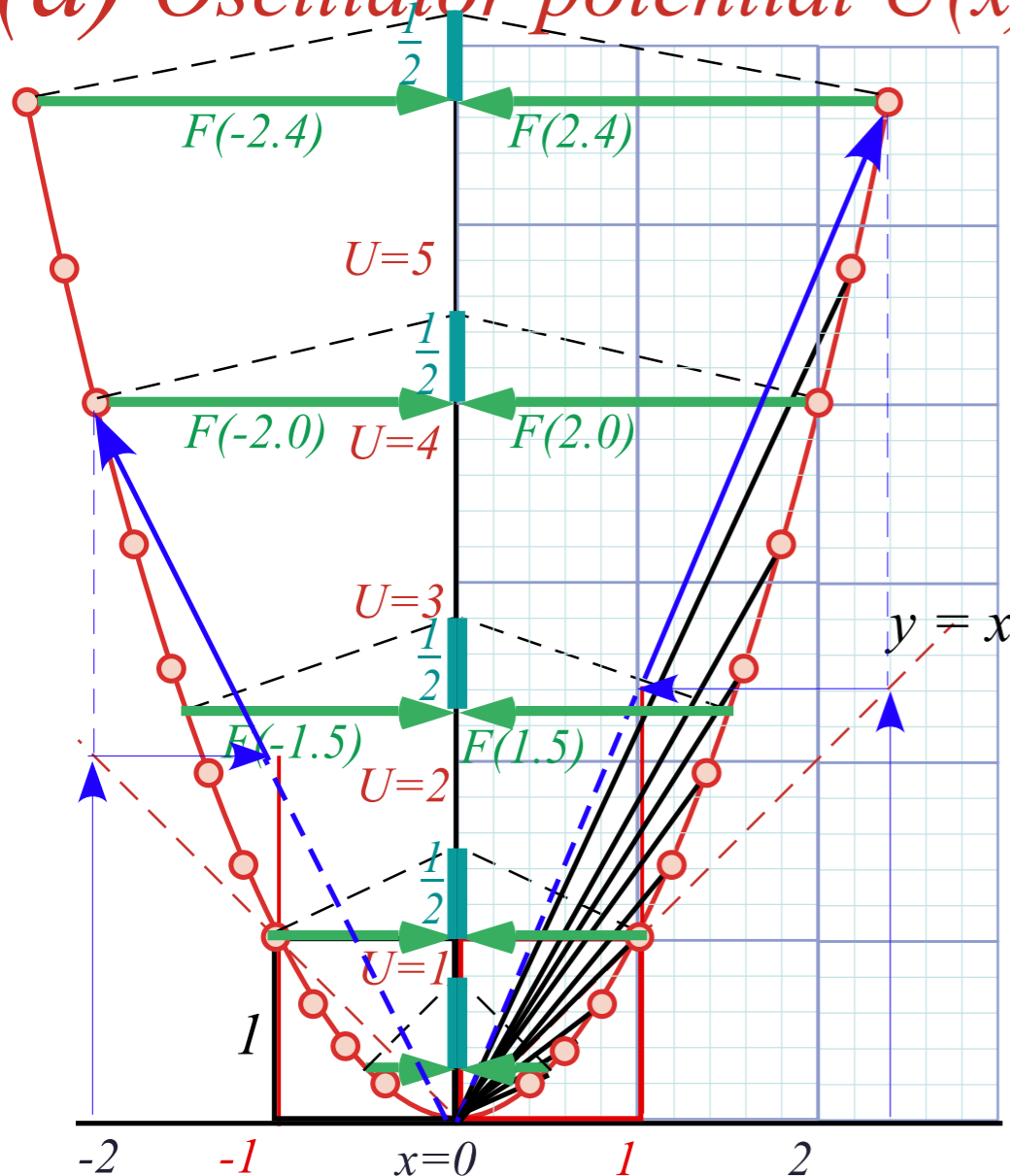
Each $y=x^2$ parabola point found by just one "Zig-Zag"

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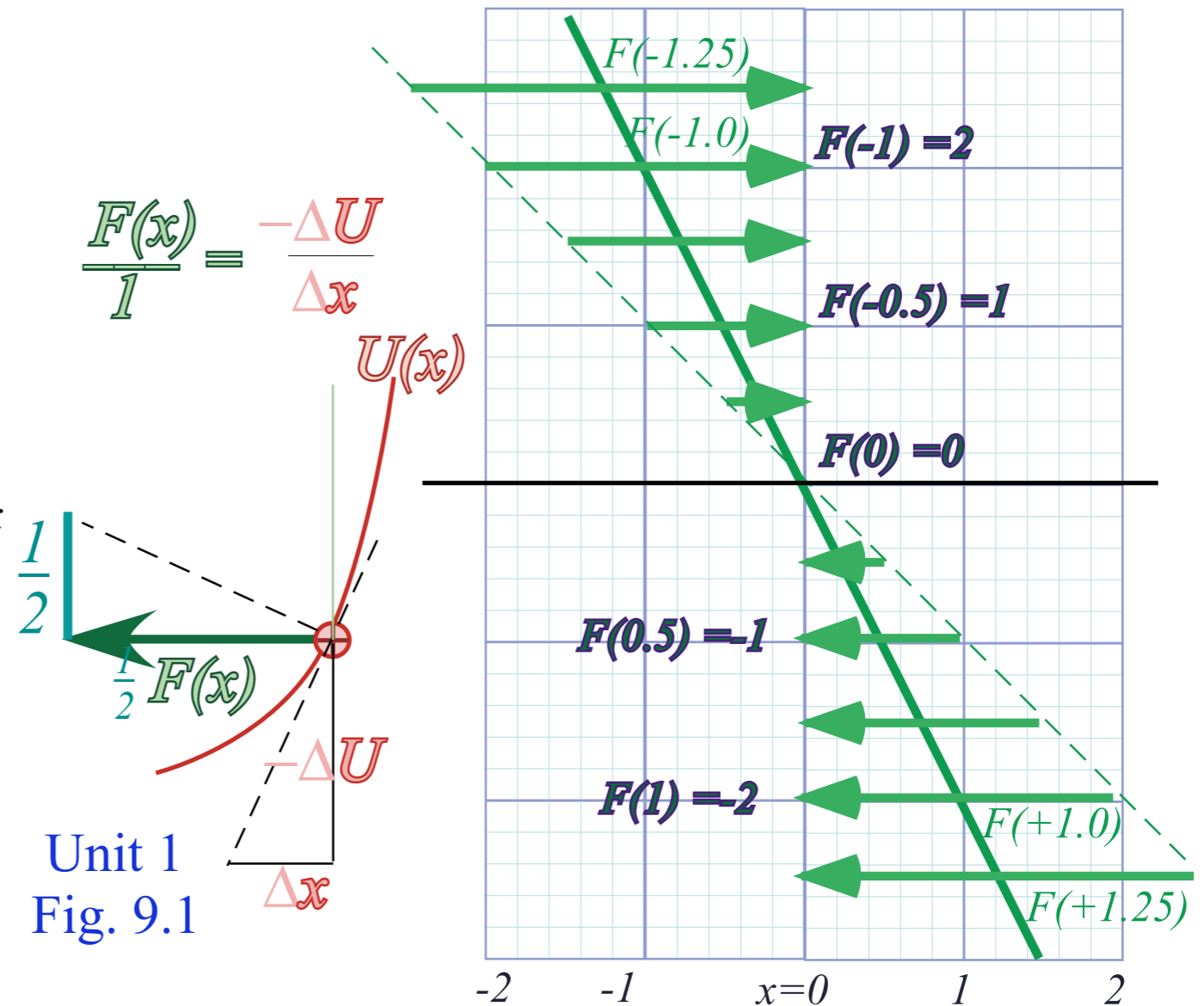


"Zag" line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$

(a) Oscillator potential $U(x)=x^2$



(b) Hooke-Law Force $F(x) = -2x$

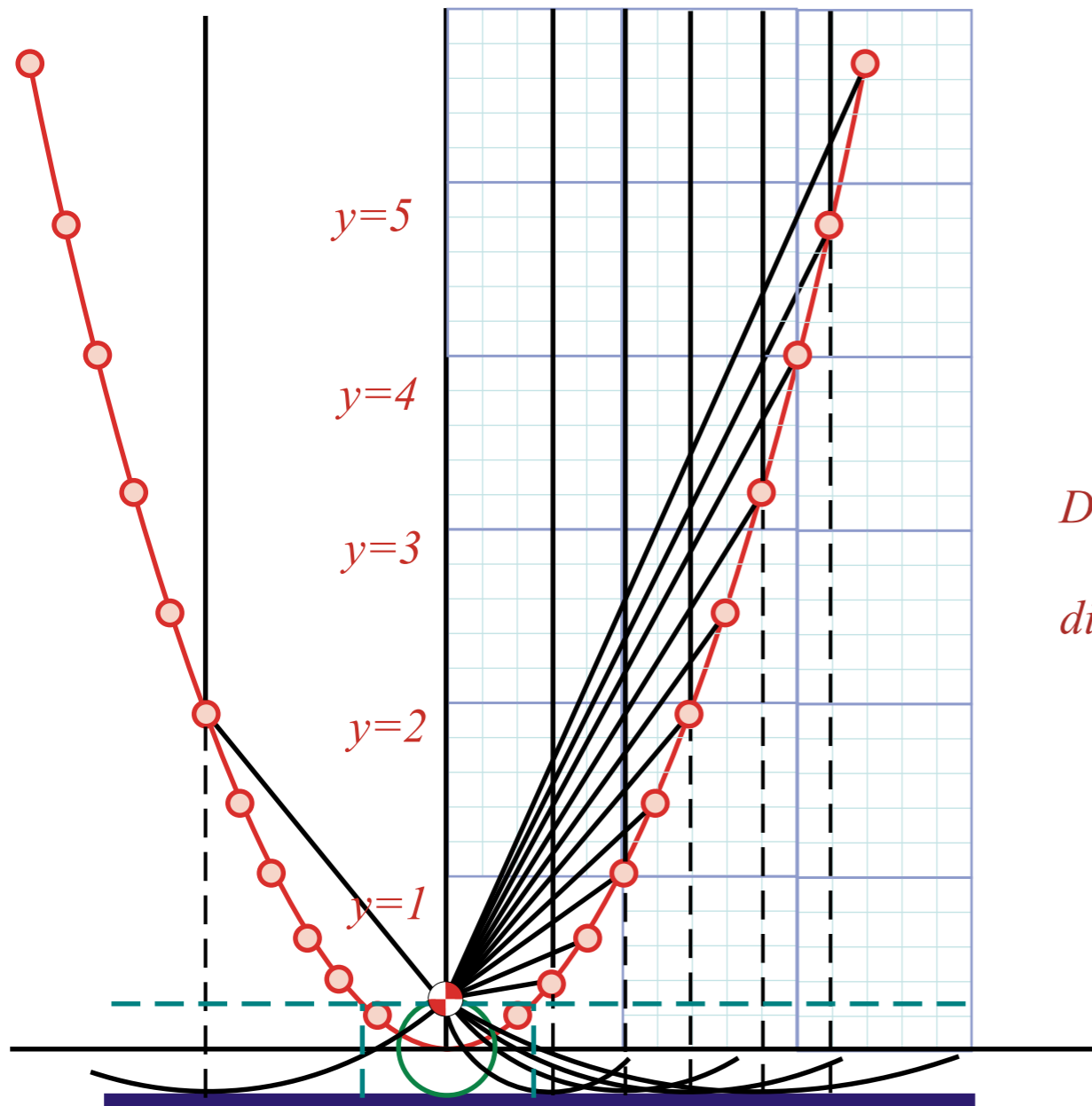


$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

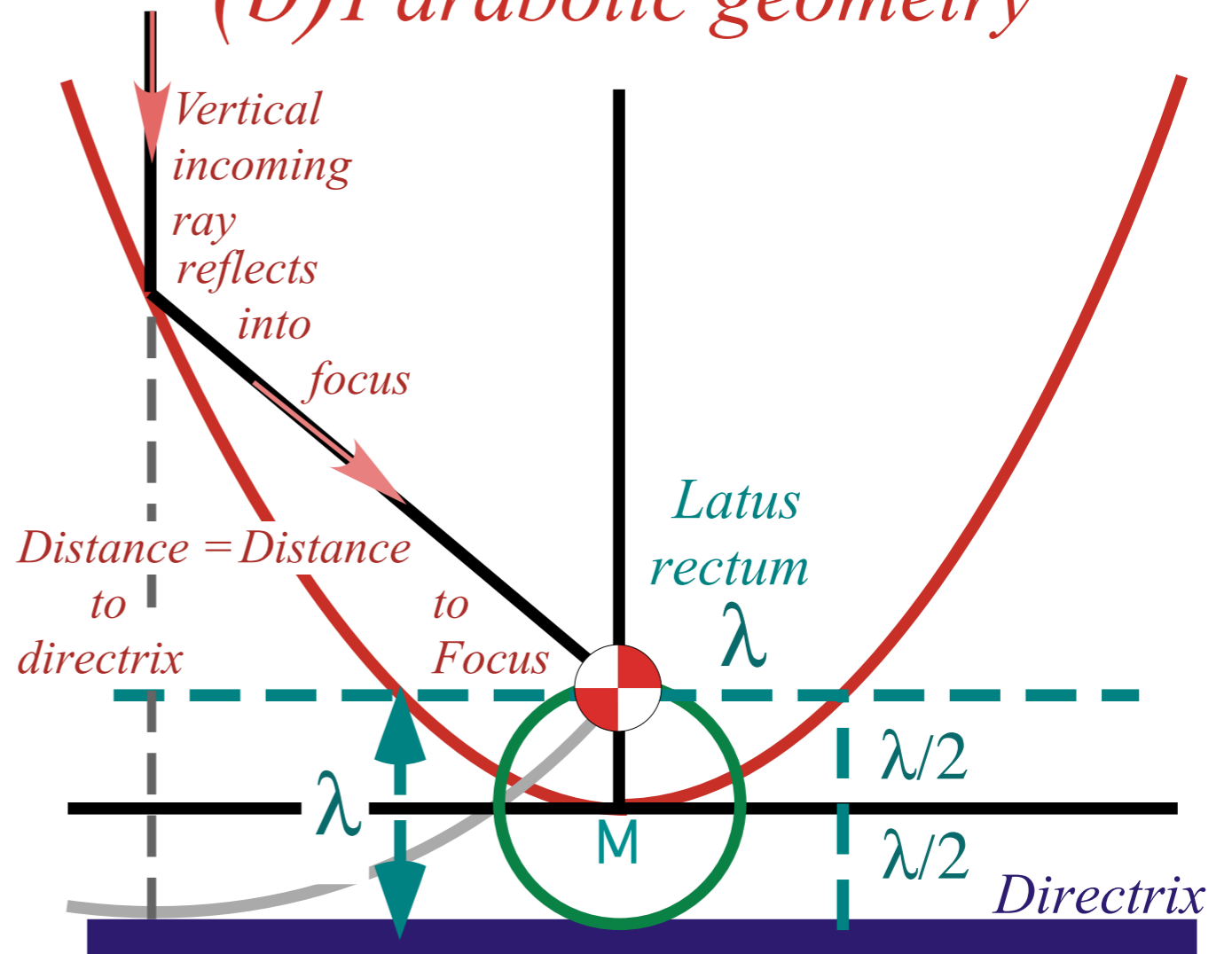
Unit 1
Fig. 9.1

A more conventional parabolic geometry... (uses focal point)

(a) Parabolic Reflector $y=x^2$



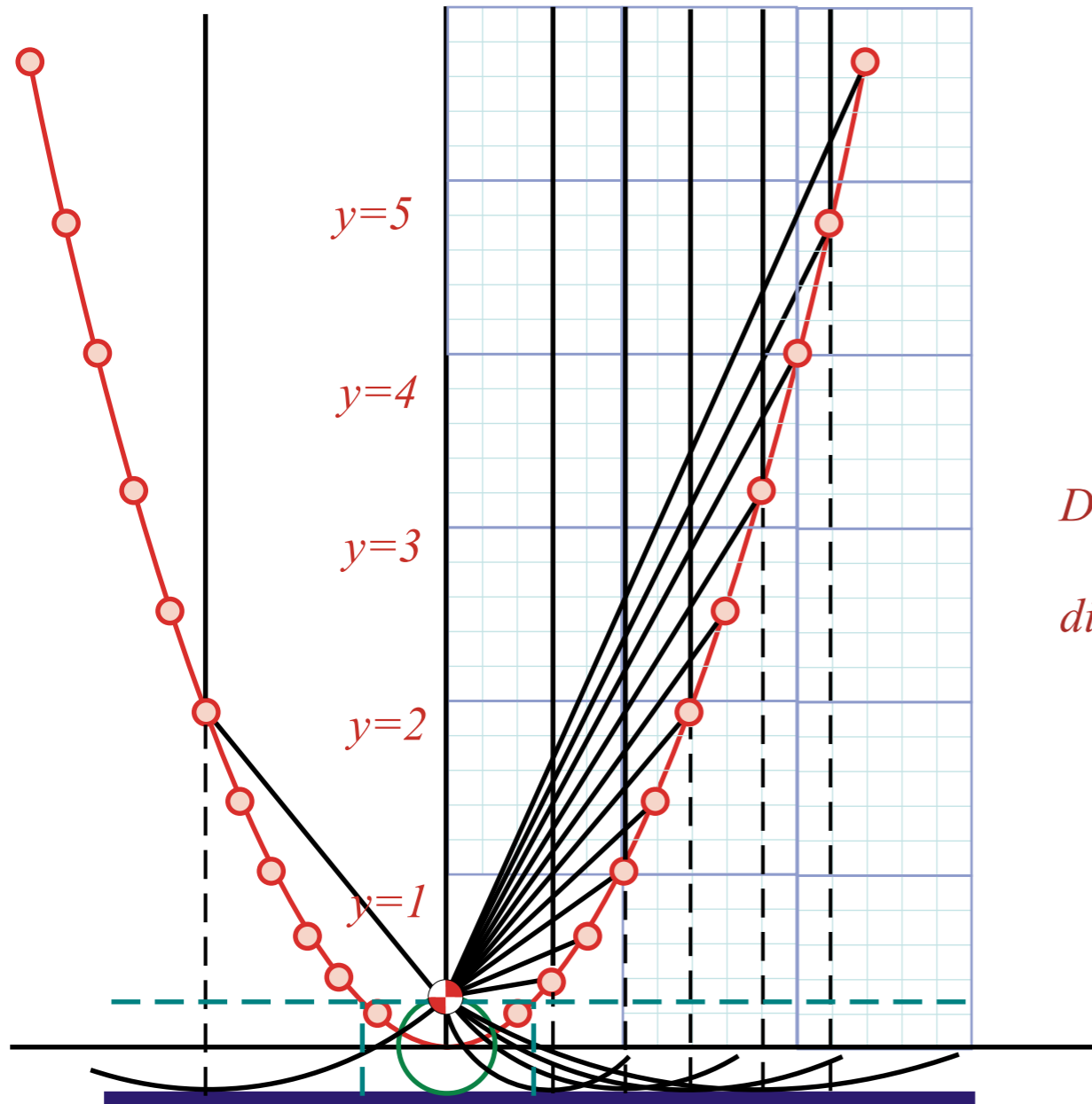
(b) Parabolic geometry



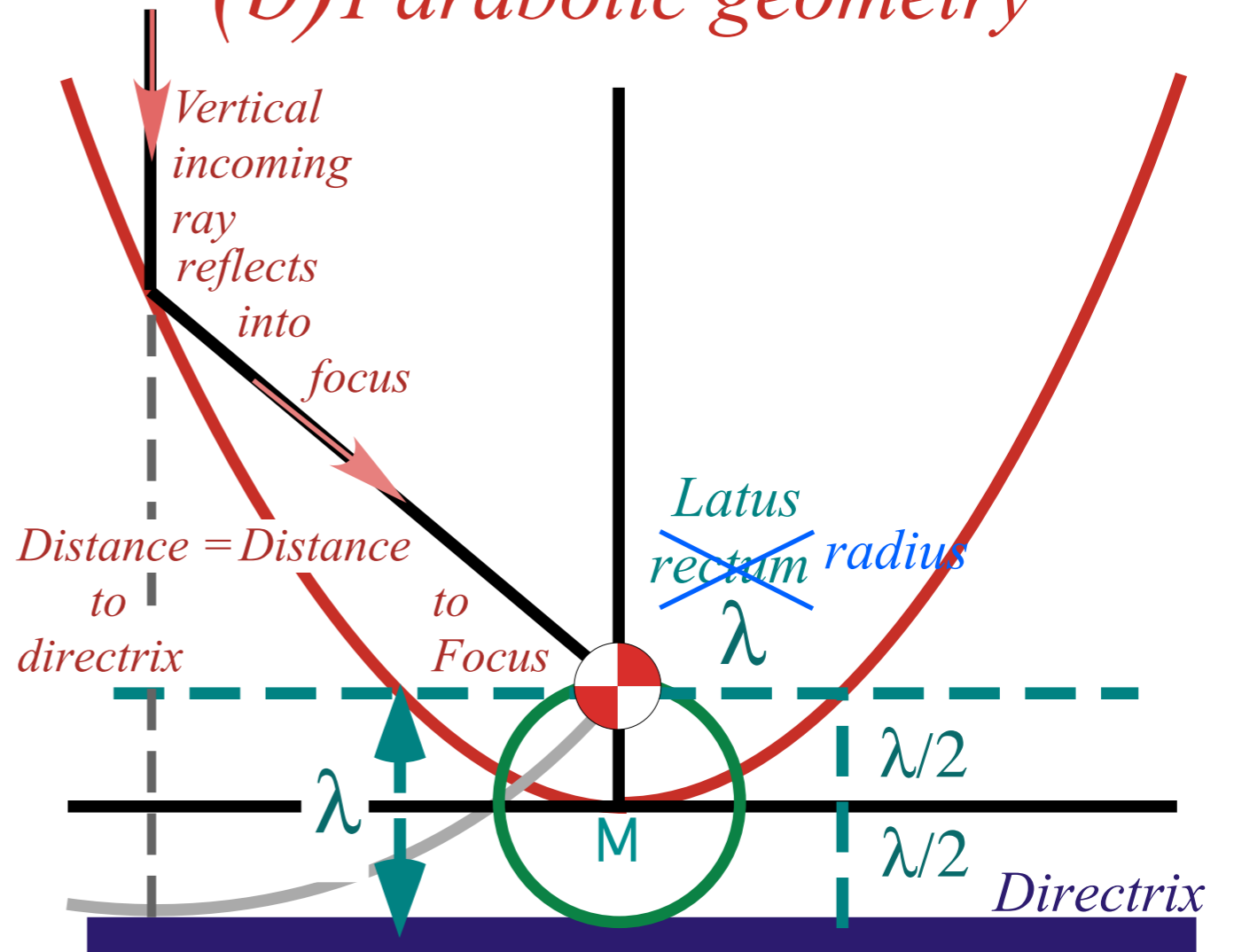
Unit 1
Fig. 9.3

A more conventional parabolic geometry...

(a) Parabolic Reflector $y=x^2$



(b) Parabolic geometry

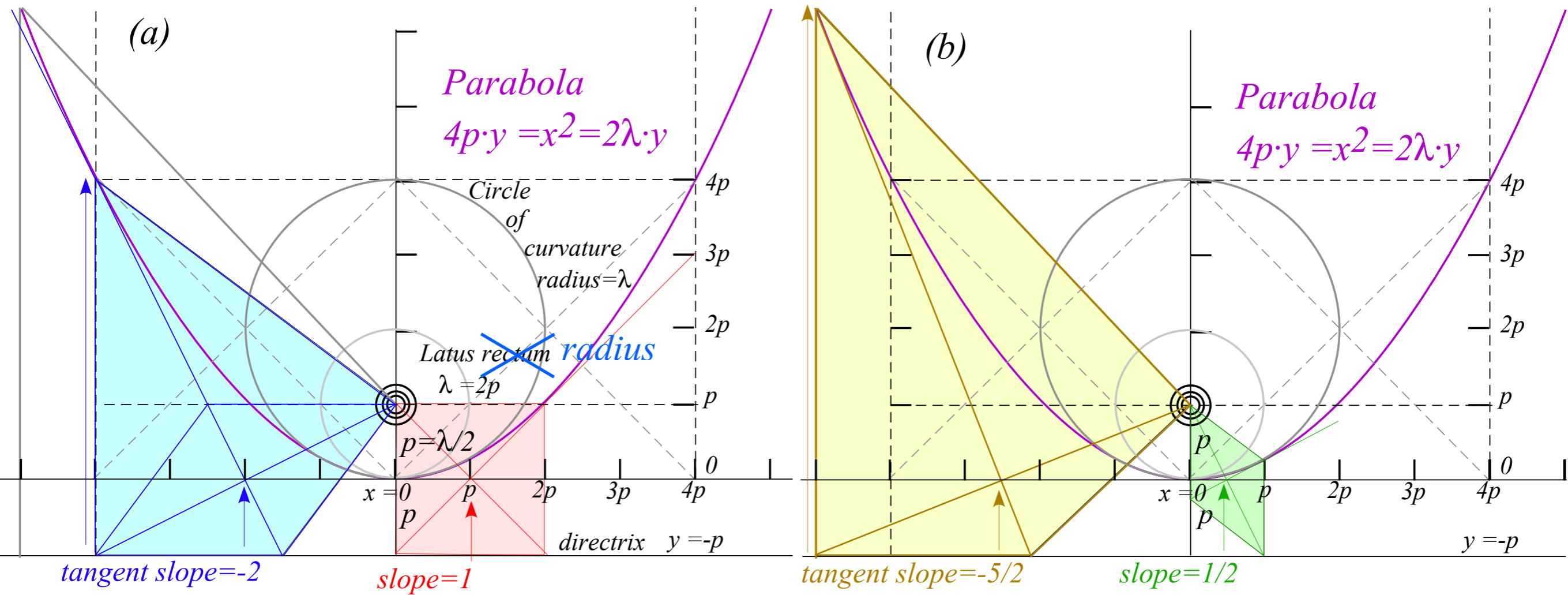


Better name† for λ : *latus radius*

Unit 1
Fig. 9.3

† Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

...conventional parabolic geometry...carried to extremes...



Unit 1
Fig. 9.4


Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

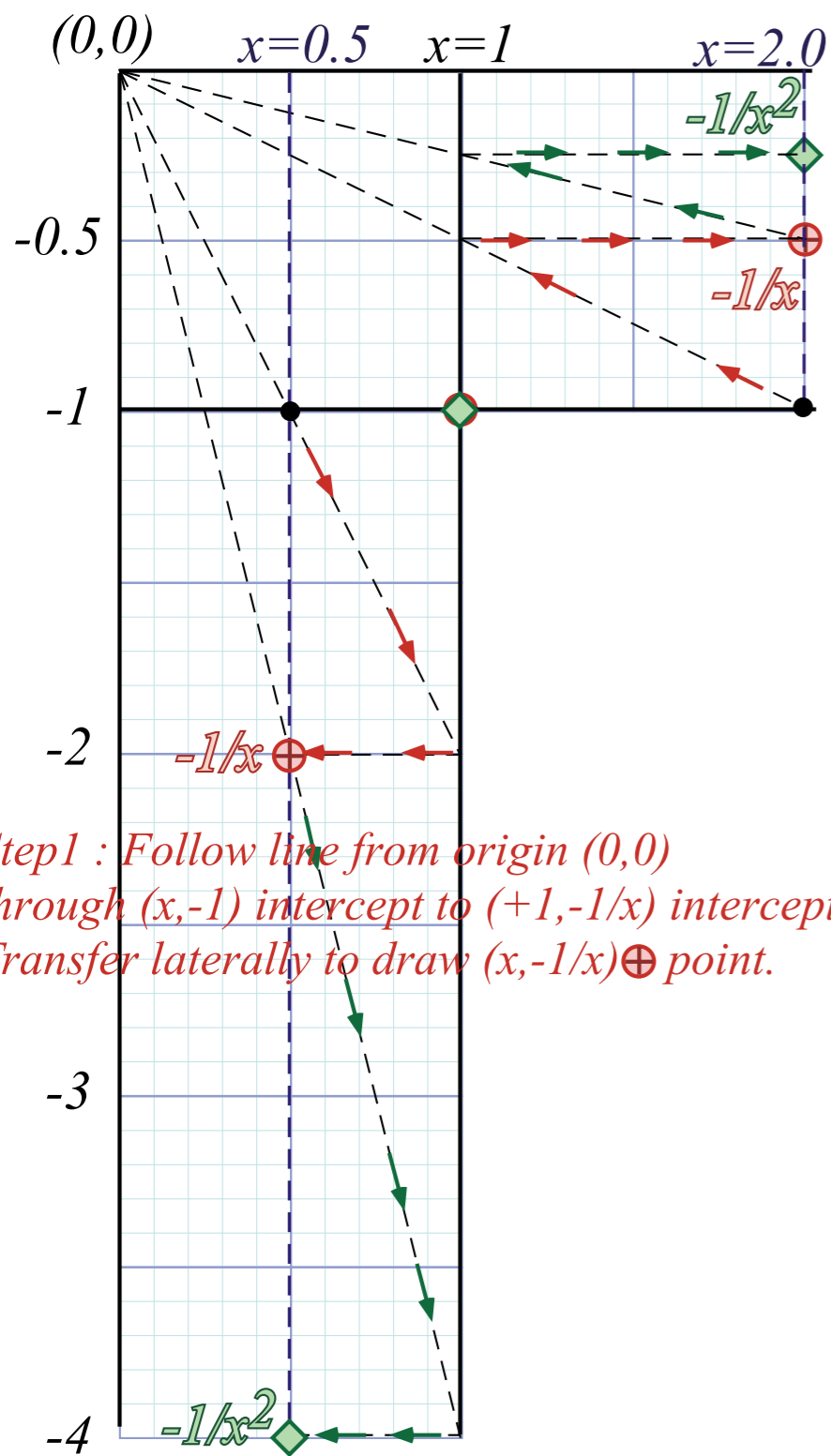
Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

 *Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields*

Compare mks units of Coulomb Electrostatic vs. Gravity

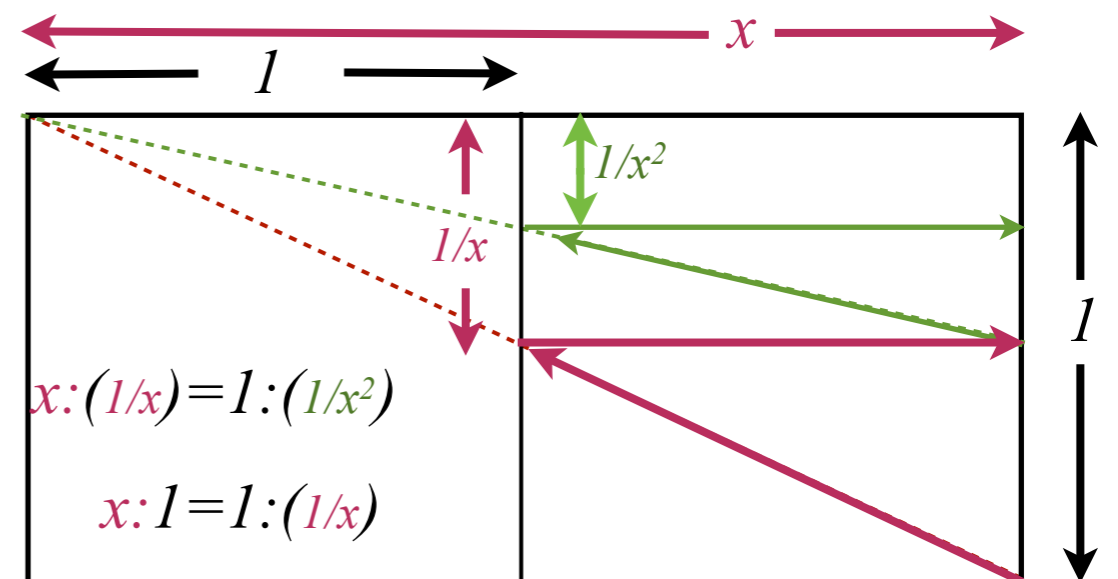
Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
 $F(x) = -1/r^2$ $U(x) = -1/r$



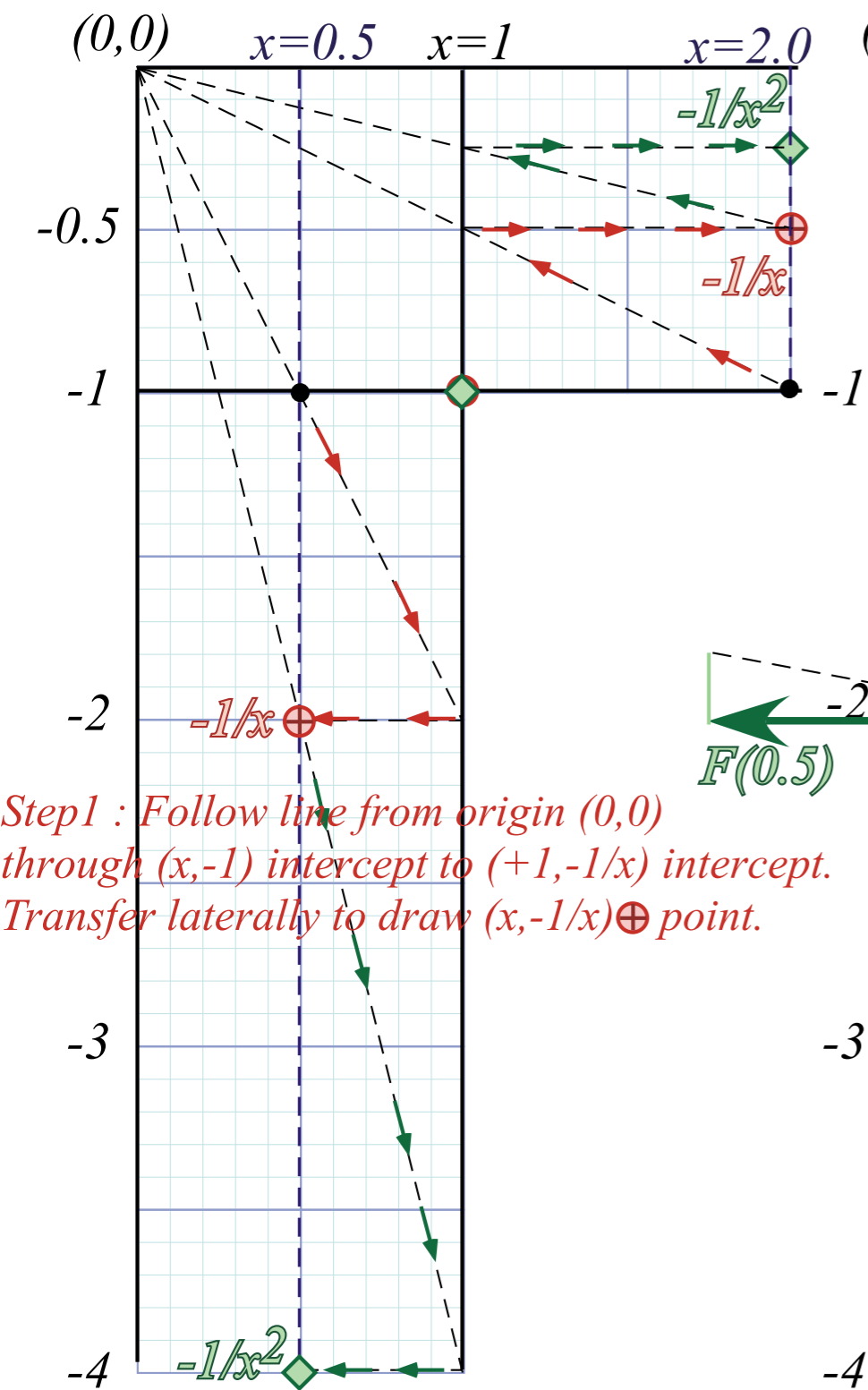
Step 1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step 2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x^2) intercept. Transfer laterally to draw (x,-1/x^2)◇ point.



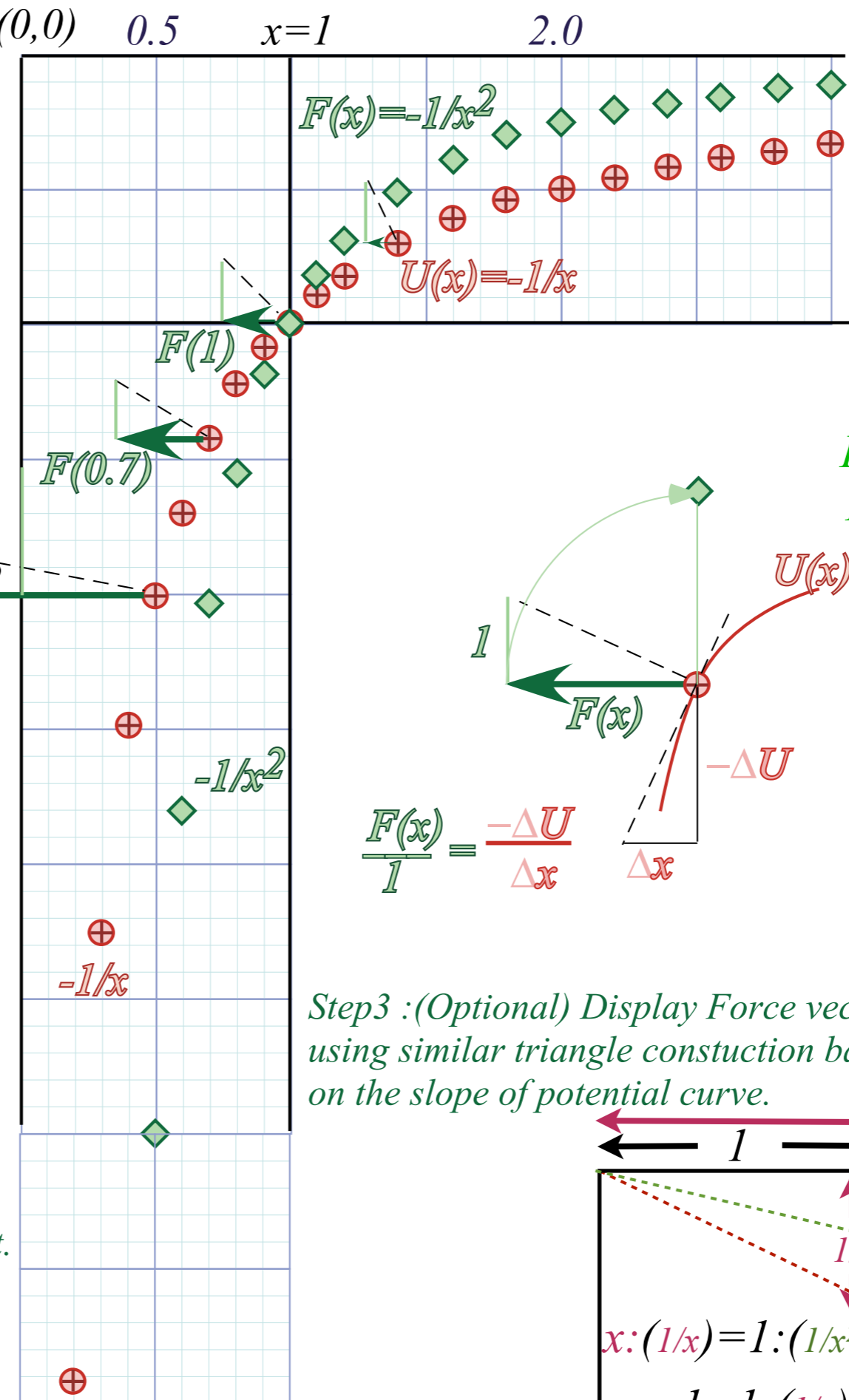
Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
 $F(x) = -1/r^2$ $U(x) = -1/r$

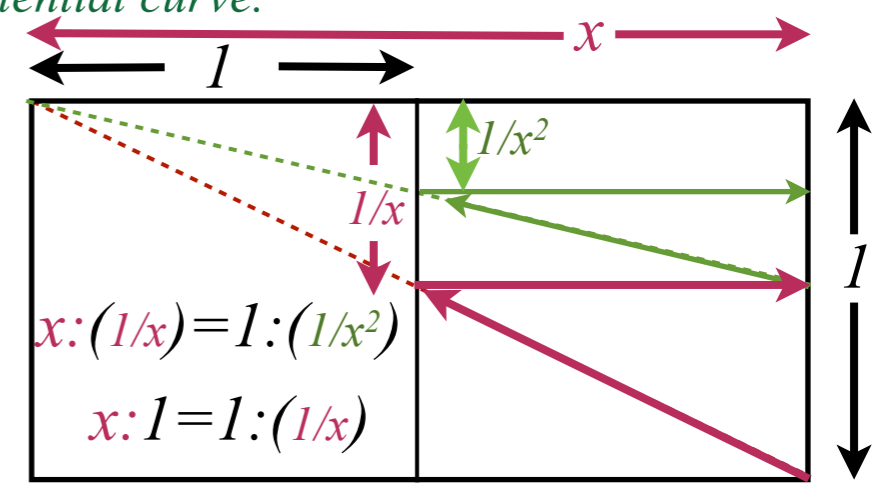


Step 1 : Follow line from origin (0,0) through (x,-1/x) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x) ⊕ point.

Step 2 : Follow line from origin (0,0) through (x,-1/x) point ⊕ to (+1,-1/x²) intercept. Transfer laterally to draw (x,-1/x²) ◇ point.

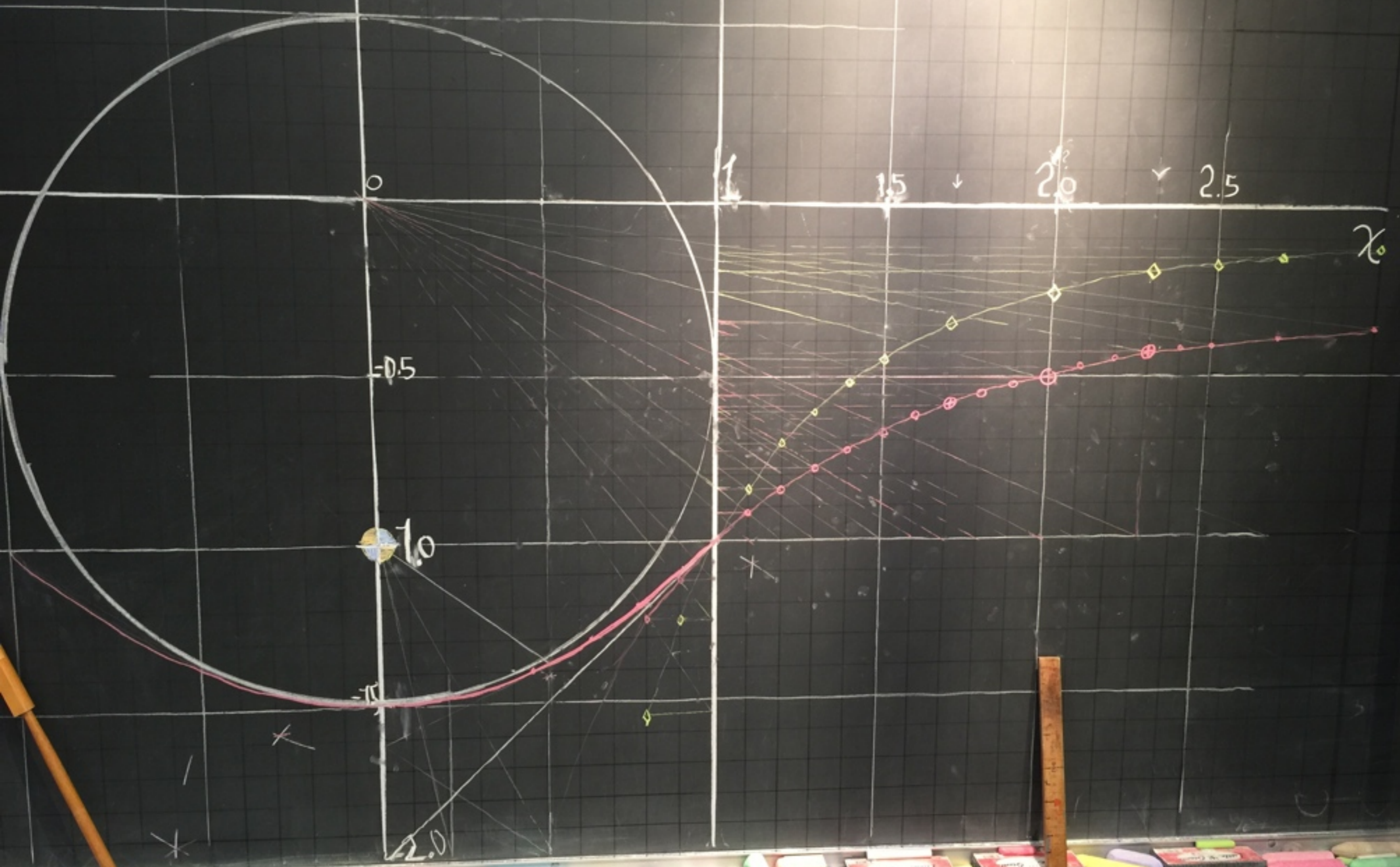


Step 3 : (Optional) Display Force vector using similar triangle construction based on the slope of potential curve.



$$V(x) = \frac{1}{x}$$

$$F(x) = \frac{1}{x^2}$$



Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

 *Compare mks units of Coulomb Electrostatic vs. Gravity*

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong \boxed{?.?.10^?} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong \overset{\sim 9E9 \sim 10^{10}}{9,000,000,000} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

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More precise value for electrostatic constant : $1/4\pi\epsilon_0 = 8.987,551 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19} \text{ Coulomb}$



Repulsive (+)(+) or (-)(-)

Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

Compare mks units for Coulomb fields

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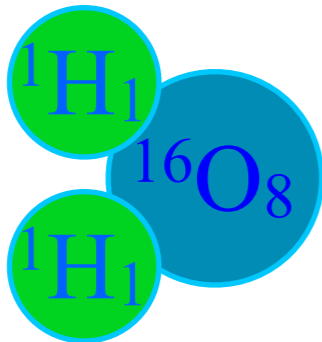
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 8 notes that 1 (cm)³ = 1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules

Avogadro's
Number

$\sim 0.3 \cdot 10^{23}$

H₂O Molecular weight ~18



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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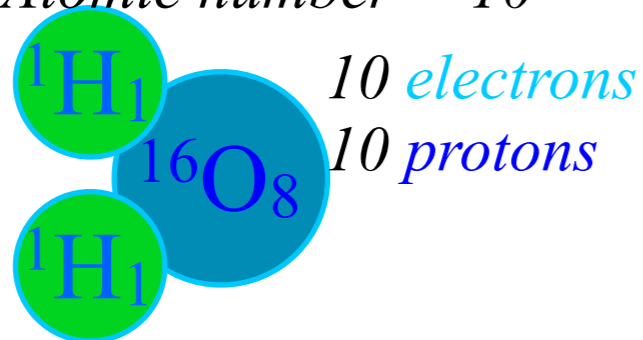
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“Fingertip Physics” of Ch. 9 notes that 1 (cm)³ = 1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or $\sim 3 \cdot 10^{23}$ electrons
 $\sim 0.3 \cdot 10^{23}$ and $\sim 3 \cdot 10^{23}$ protons.

H₂O Molecular weight ~ 18

Atomic number = 10



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

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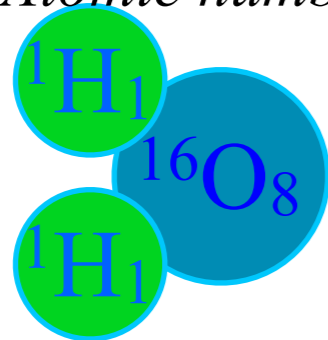
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 $\sim 0.3 \cdot 10^{23}$ and $\sim 3 \cdot 10^{23}$ protons.

H₂O Molecular weight ~ 18

Atomic number = 10



10 electrons That is $\sim -3 \cdot 10^{23} \cdot 1.6022 \cdot 10^{-19}$ Coulomb or about $-0.5 \cdot 10^{+5}$ C or - 50,000 Coulomb
 10 protons plus $\sim +3 \cdot 10^{23} \cdot 1.6022 \cdot 10^{-19}$ Coulomb or about $+0.5 \cdot 10^{+5}$ C or +50,000 Coulomb

Equals zero total charge

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb



Repulsive (+)(+) or (-)(-)

Attractive (+)(-) or (-)(+)

vs

Always Attractive (so far)

↑ COMPARE! ↓

BIG

vs
small



2. Gravitational force between m (kilograms) and M (kg.)

$$F^{grav.}(r) = -G \frac{mM}{r^2} \quad \text{where: } G = \boxed{?.?.? \cdot 10^?} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\sim 9E9 \sim 10^{10} \text{ Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

More precise value for electrostatic constant : $1/4\pi\epsilon_0 = 8.987,551 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19} \text{ Coulomb}$



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Always Attractive (so far)

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BIG

vs
small



2. Gravitational force between m (kilograms) and M (kg.) !!!!

$$F^{grav.}(r) = -G \frac{mM}{r^2} \quad \text{where: } G = 0.000,000,000,067 \frac{\sim 2/3 10^{-10} \sim 10^{-10} \text{ Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3) 10^{-10}$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}} \quad \text{!!!!}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

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Discussion of repulsive force and PE in Ch. 9...

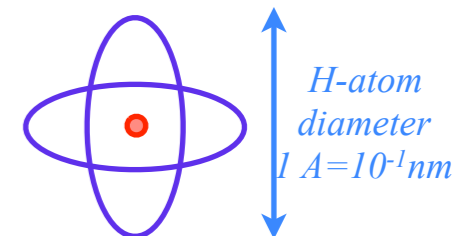
1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

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Nuclear size $\sim 10^{-15}$ m = 1 femtometer = 1fm

Atomic size ~ 1 Angstrom = 10^{-10} m



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$



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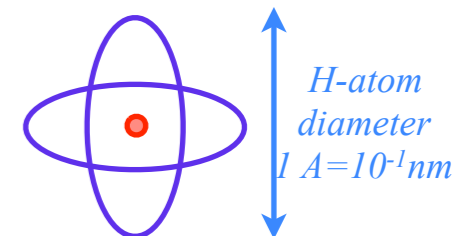
$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$



Nuclear size $\sim 10^{-15} \text{ m} = 1 \text{ femtometer} = 1 \text{ fm}$

Atomic size $\sim 1 \text{ Angstrom} = 10^{-10} \text{ m}$

Big molecule $\sim 10 \text{ Angstrom} = 10^{-9} \text{ m} = 1 \text{ nanometer} = 1 \text{ nm}$



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

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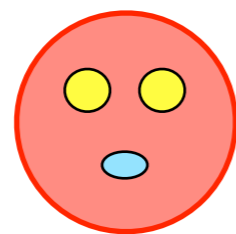
$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$

Nuclear size $\sim 10^{-15} \text{ m} = 1 \text{ femtometer} = 1 \text{ fm}$

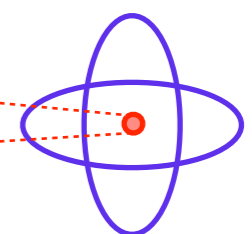
 Atomic size $\sim 1 \text{ Angstrom} = 10^{-10} \text{ m}$

Big molecule $\sim 10 \text{ Angstrom} = 10^{-9} \text{ m} = 1 \text{ nanometer} = 1 \text{ nm}$

also: $1 \text{ fm} = 10^{-13} \text{ cm} = 1 \text{ Fermi} = 1 \text{ Fm}$



1 Fermi



H-atom diameter
 $1 \text{ A} = 10^{-1} \text{ nm}$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$



Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

quantum of charge: $|e|=1.6022 \cdot 10^{-19}$ Coulomb

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$

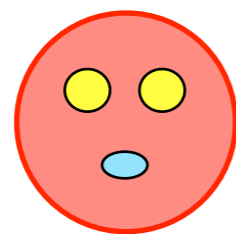


Atomic size ~ 1 Angstrom = 10^{-10} m

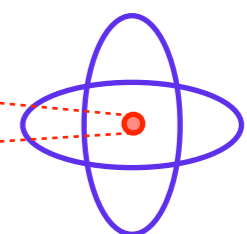
Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm

Nuclear size $\sim 10^{-15}$ m = 1 femtometer = 1fm

also: 1fm = 10^{-13} cm = 1Fermi
= 1Fm



1 Fermi



H-atom
diameter
1 A = 10^{-1} nm

nuclear radii are 100,000 to 1,000,000 times smaller than atomic/chemical radii

...so nuclear qQ/r energy 100,000 to 1,000,000 times **bigger** that of atomic/chemical...

Geometry of idealized “Sophomore-physics Earth”

→ *Coulomb field outside Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

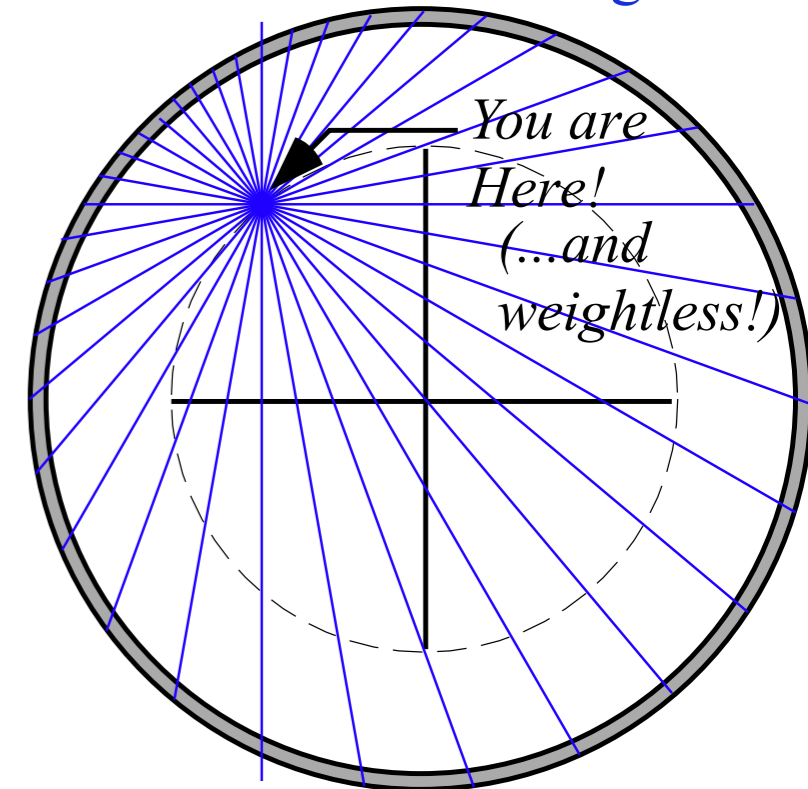
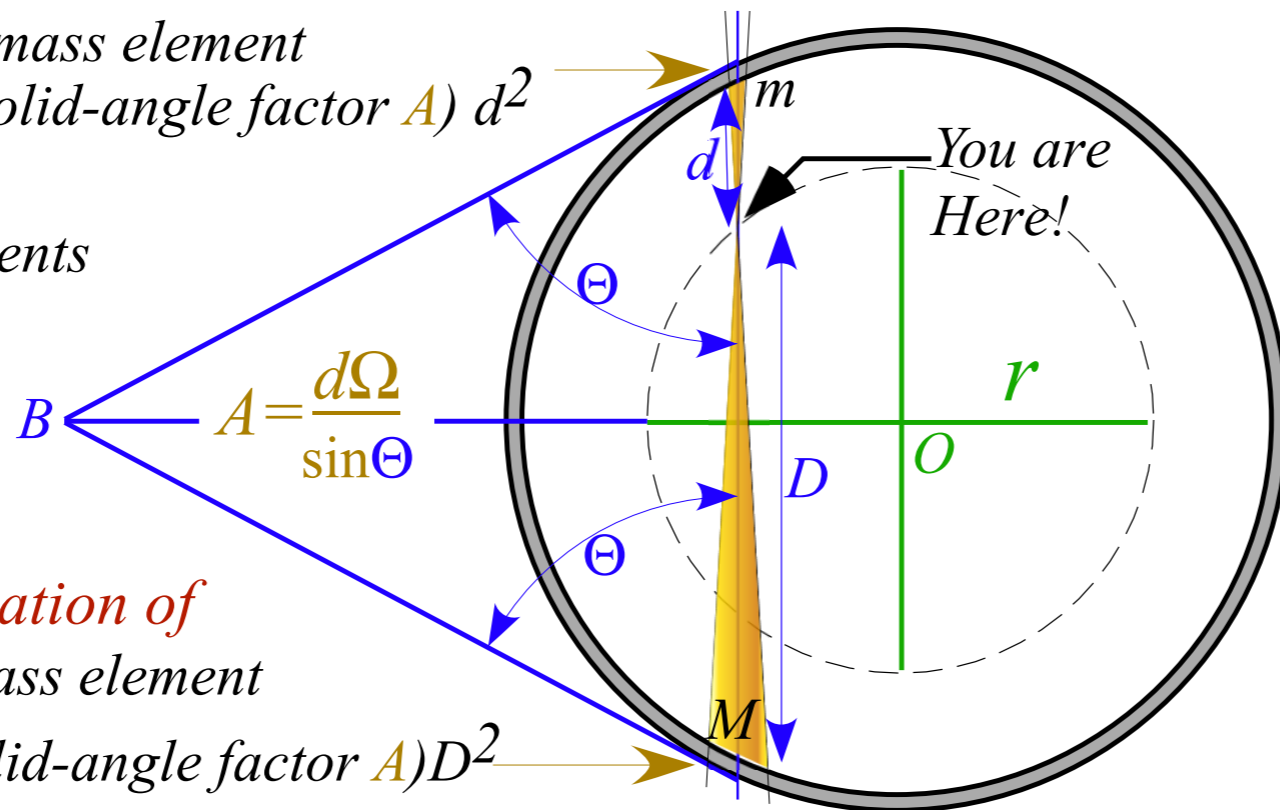
Gravity at r
due to shell mass elements

$$\frac{GM}{D^2} - \frac{Gm}{d^2} =$$

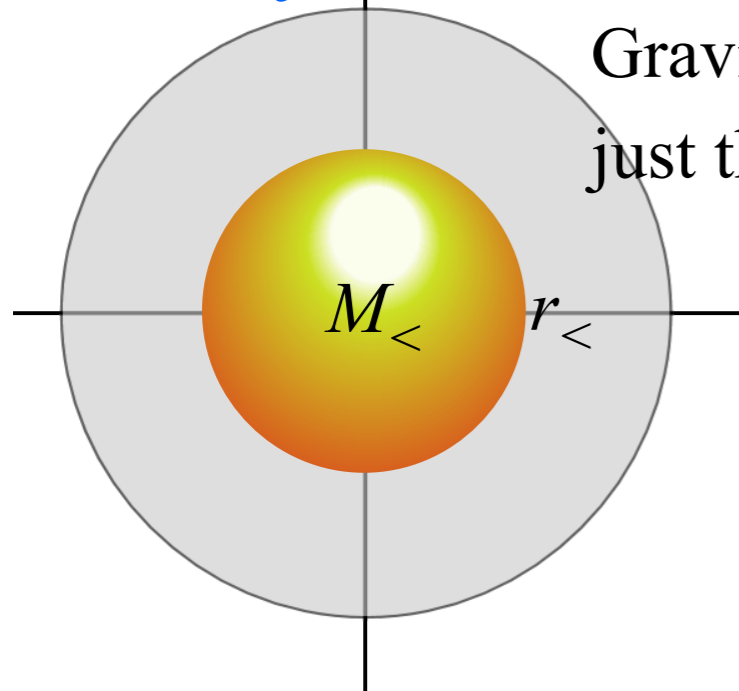
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of
Shell mass element

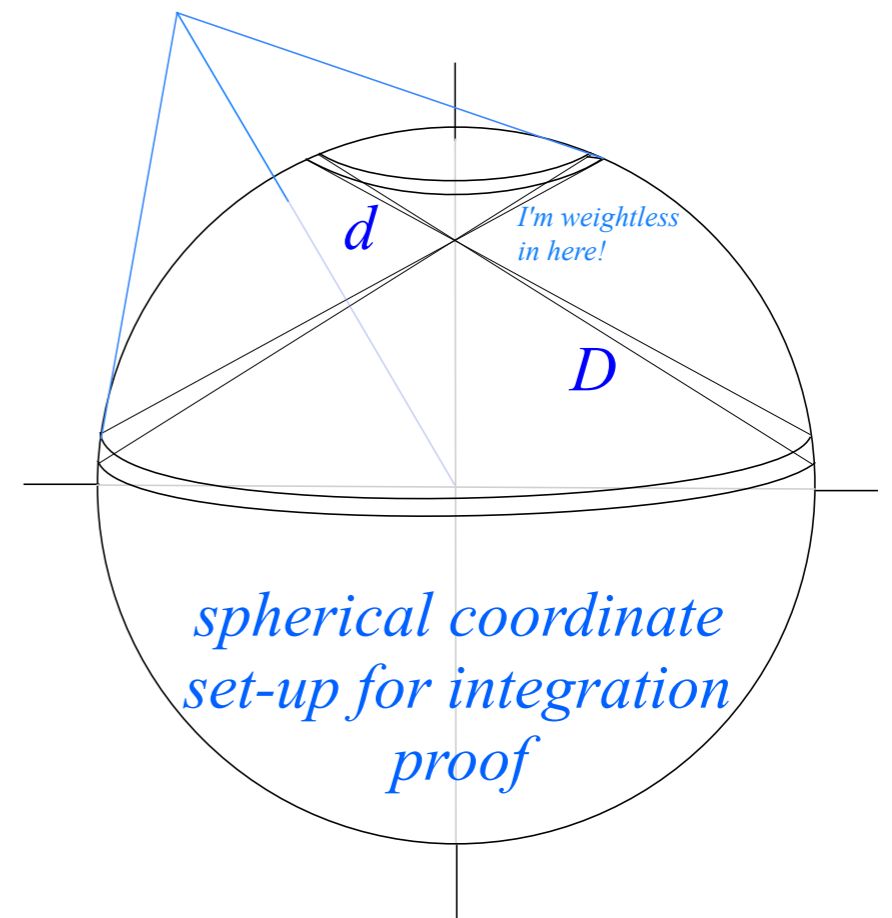
$M = (\text{solid-angle factor } A)D^2$



Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet $M_<$ below $r_<$



spherical coordinate
set-up for integration
proof

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

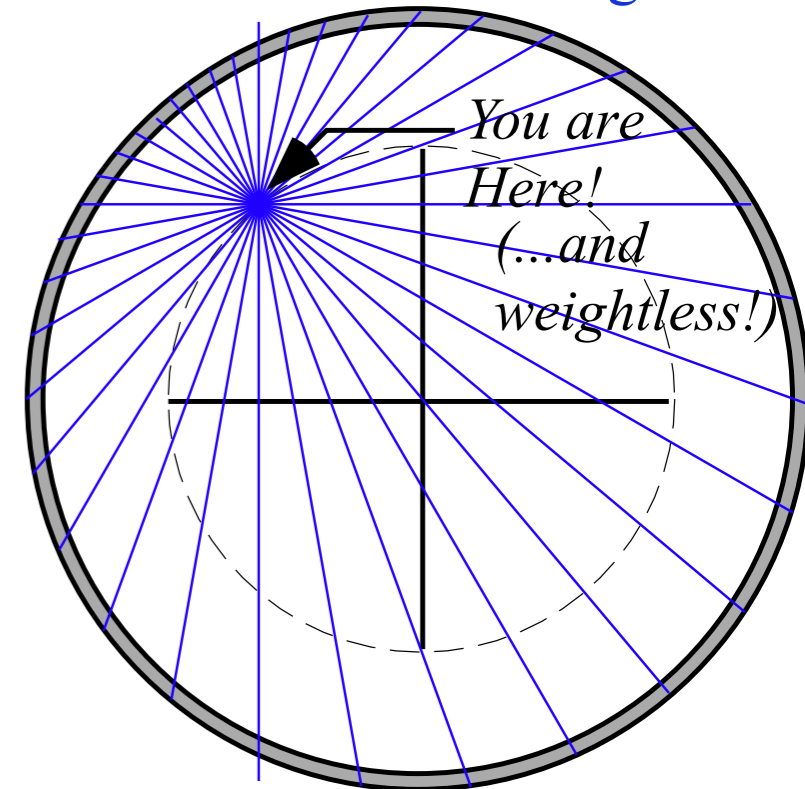
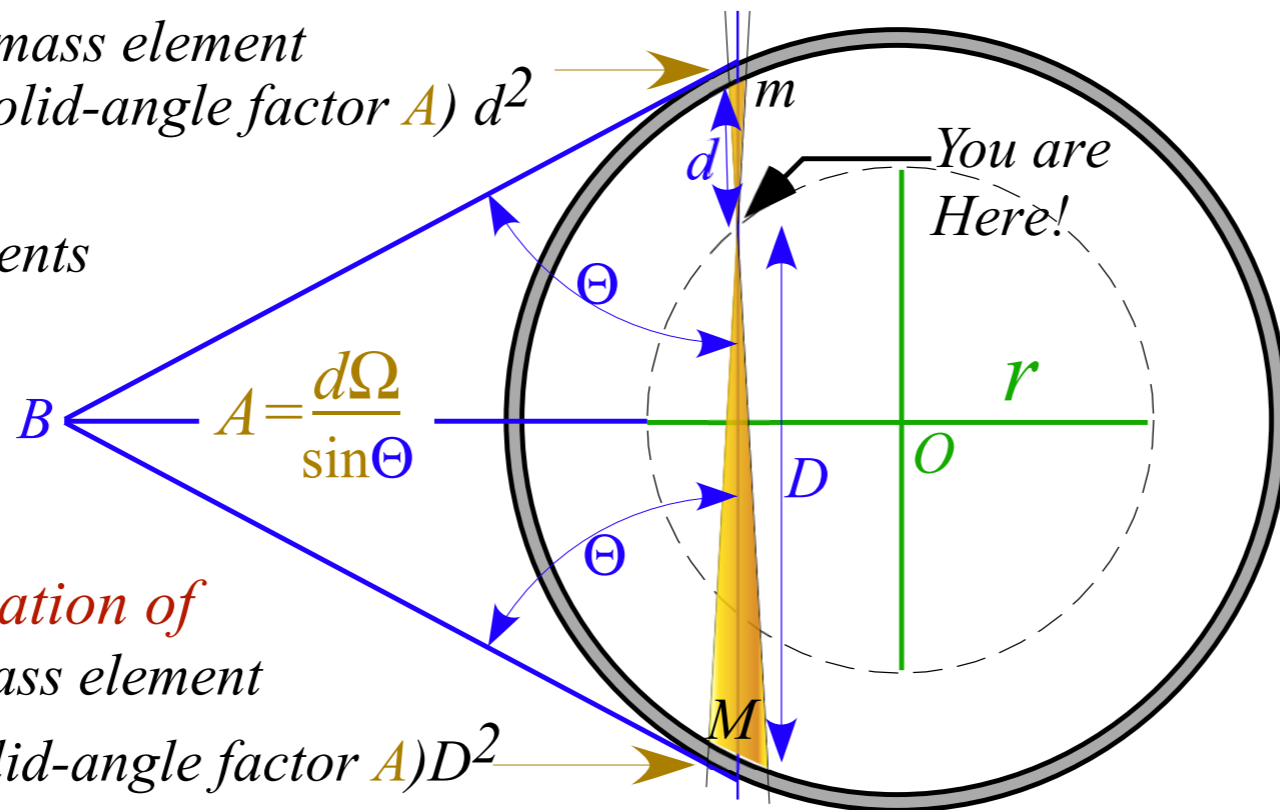
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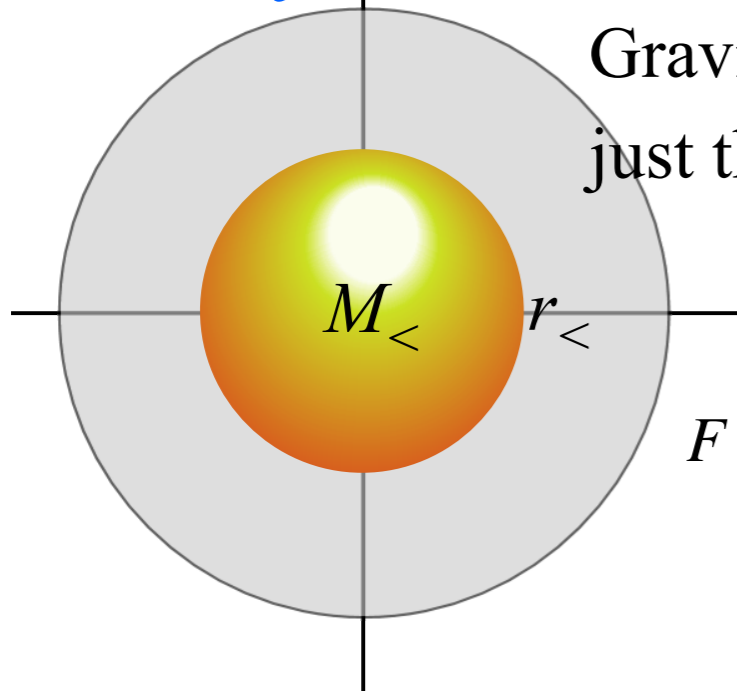
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of
Shell mass element

$M = (\text{solid-angle factor } A)D^2$



Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet $M_<$ below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G \frac{mM_<}{r_<^2} = Gm \frac{4\pi}{3} \frac{M_<}{\frac{4\pi}{3} r_<^3} r_< = Gm \frac{4\pi}{3} \rho_{\oplus} r_< = mg \frac{r_<}{R_{\oplus}} \equiv mg \cdot x$$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

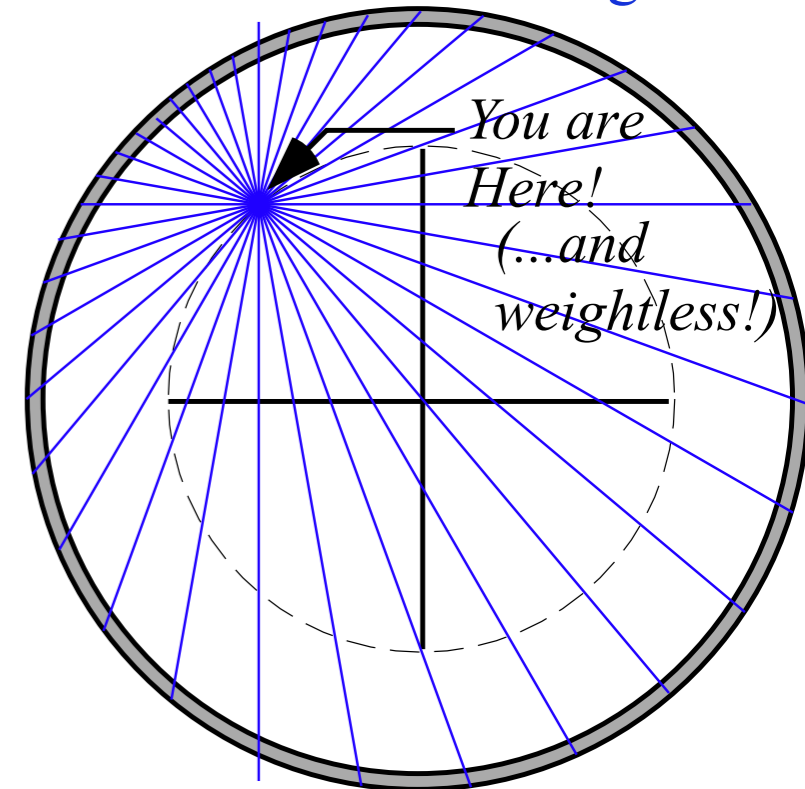
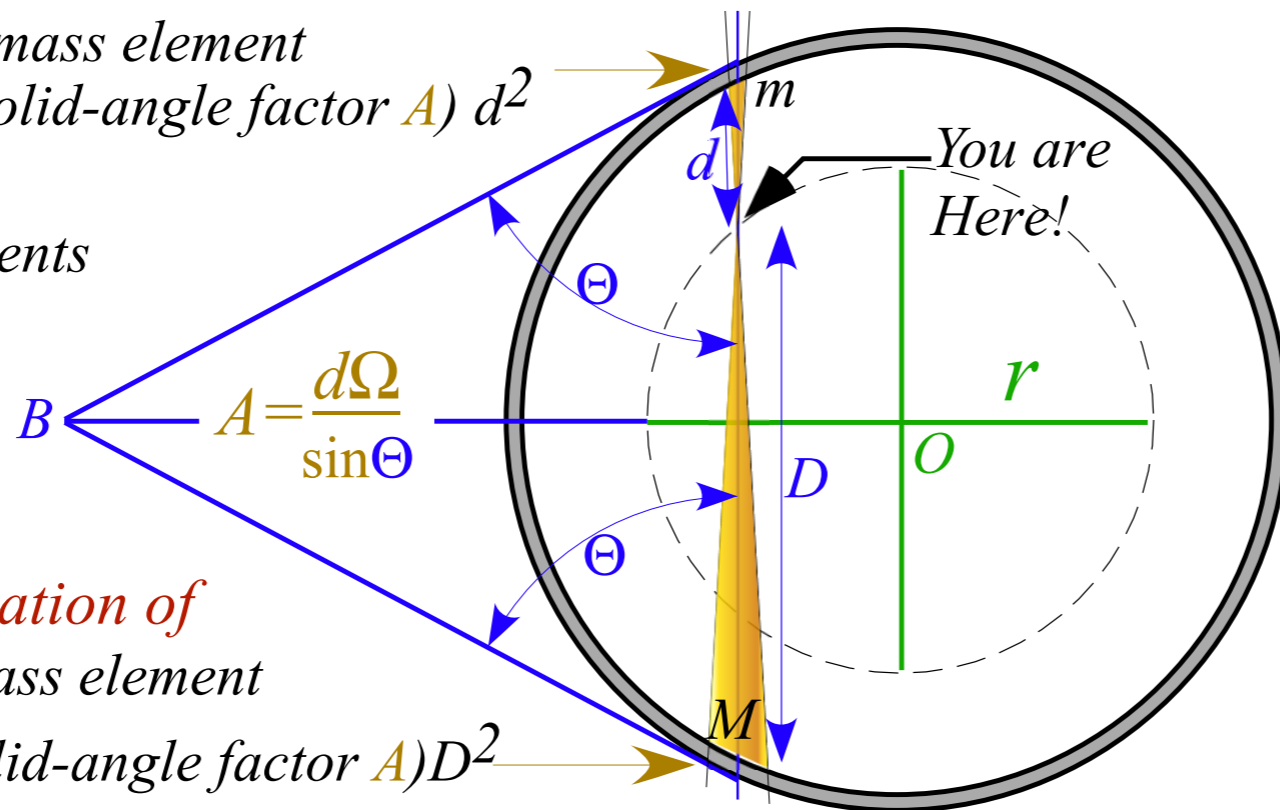
Gravity at r
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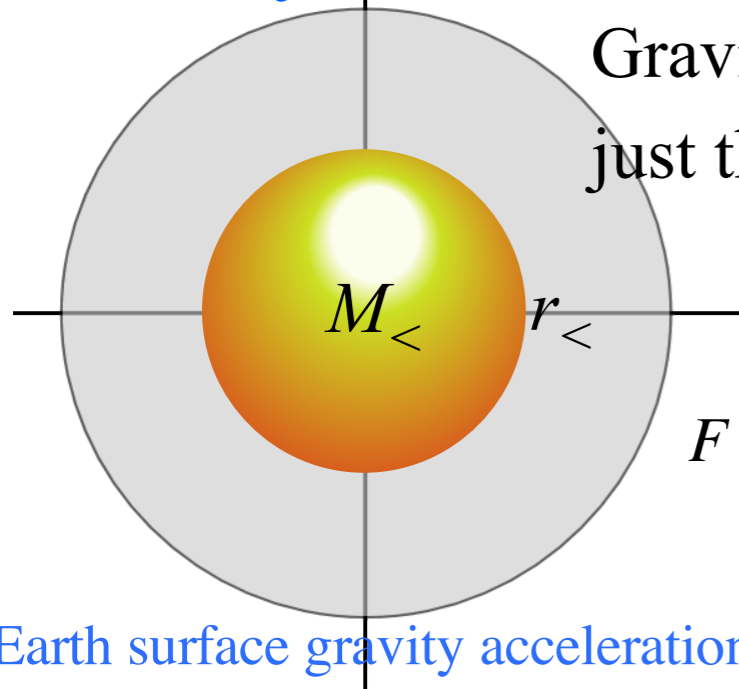
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Earth surface gravity acceleration: $g = G \frac{M_{\oplus}}{R_{\oplus}^2} = G \frac{M_{\oplus}}{R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \frac{M_{\oplus}}{4\pi R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \rho_{\oplus} R_{\oplus} = 9.8 m/s^2$

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3) 10^{-10}$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

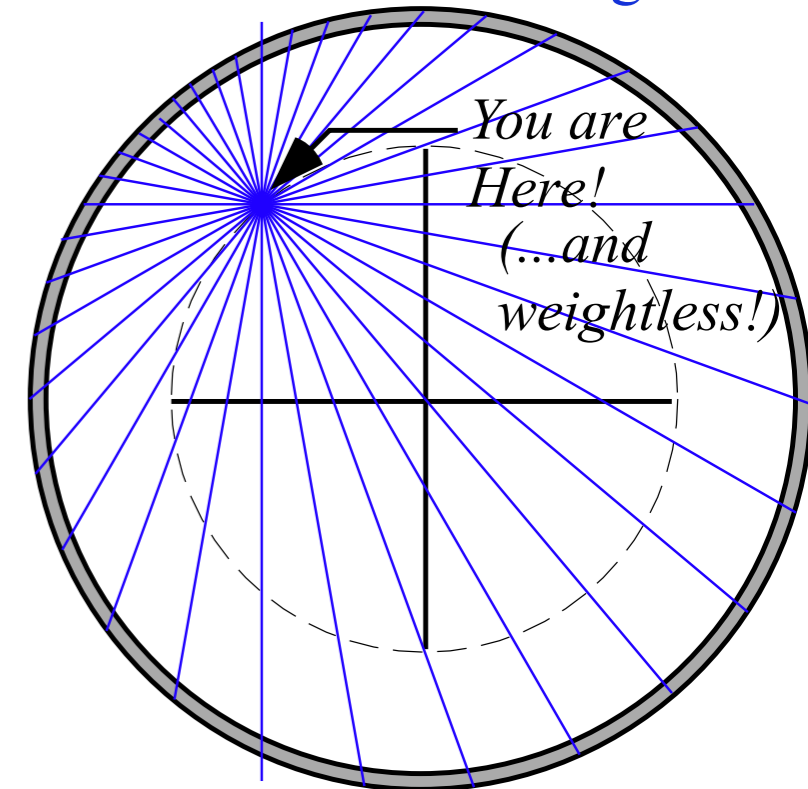
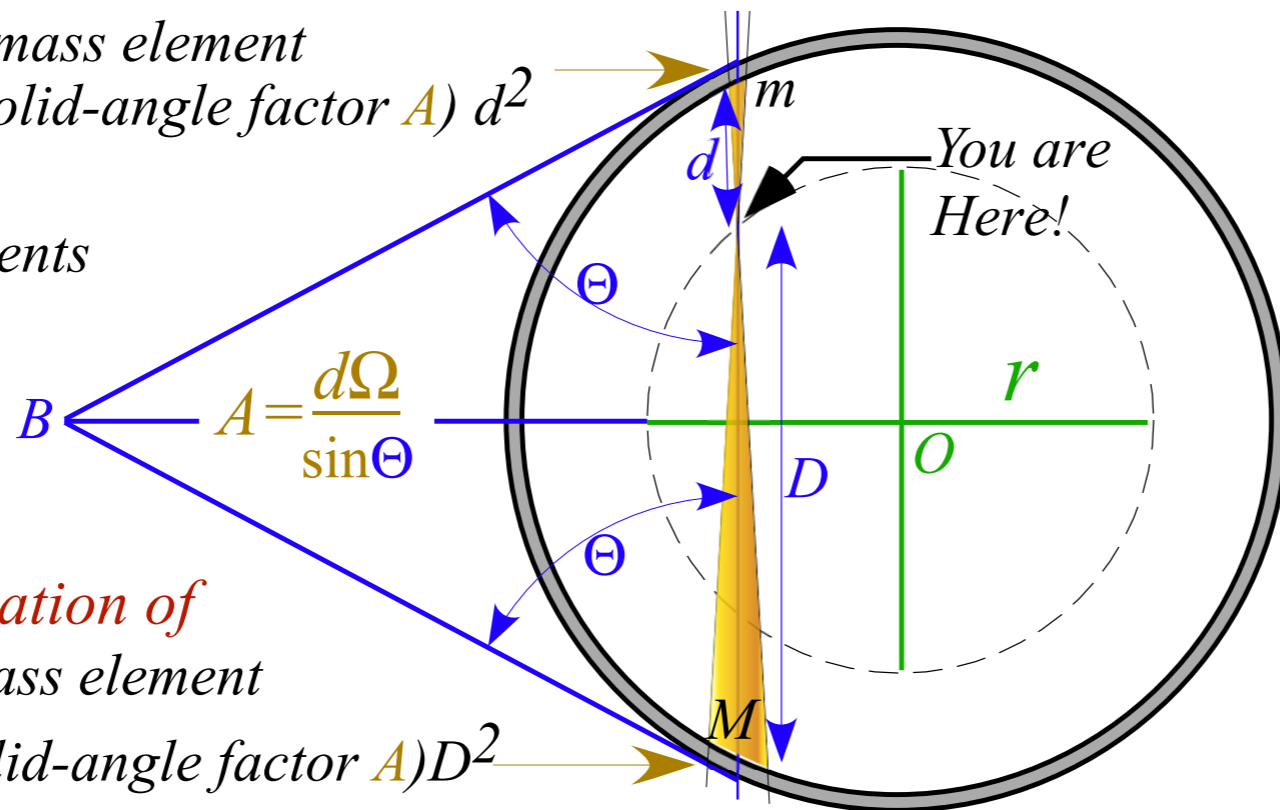
Gravity at r
due to shell mass elements

$$\frac{GM}{D^2} - \frac{Gm}{d^2} =$$

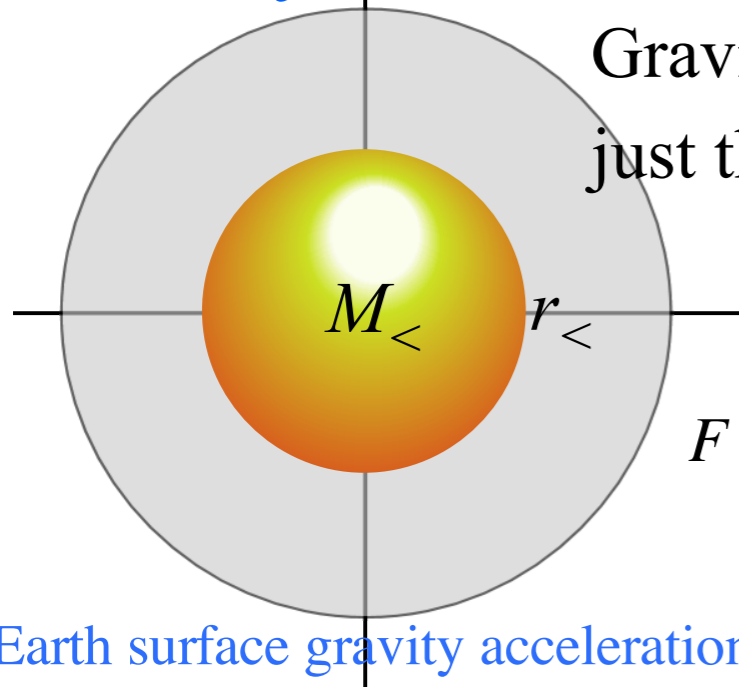
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of
Shell mass element

$M = (\text{solid-angle factor } A)D^2$



Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
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Earth surface gravity acceleration: $g = G \frac{M_{\oplus}}{R_{\oplus}^2} = G \frac{M_{\oplus}}{R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \frac{M_{\oplus}}{4\pi R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \rho_{\oplus} R_{\oplus} = 9.8 \text{ m/s}^2$

$G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3) 10^{-10}$

Earth radius: $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$

Earth mass: $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}$

Solar radius: $R_{\odot} = 6.955 \times 10^8 \text{ m} \approx 7.0 \cdot 10^8 \text{ m}$

Solar mass: $M_{\odot} = 1.9889 \times 10^{30} \text{ kg} \approx 2.0 \cdot 10^{30} \text{ kg}$

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

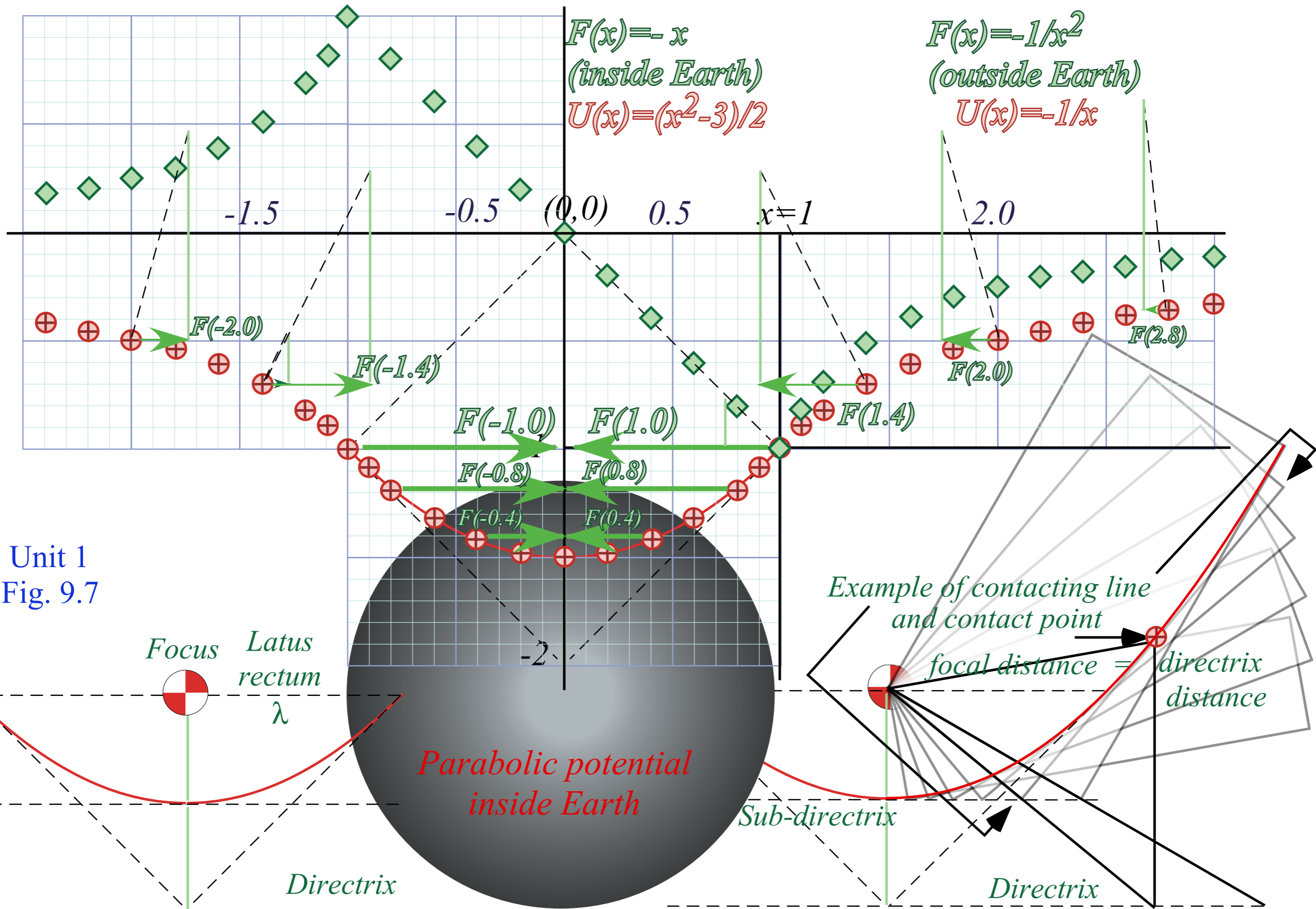
→ *Contact-geometry of potential curve(s)*

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

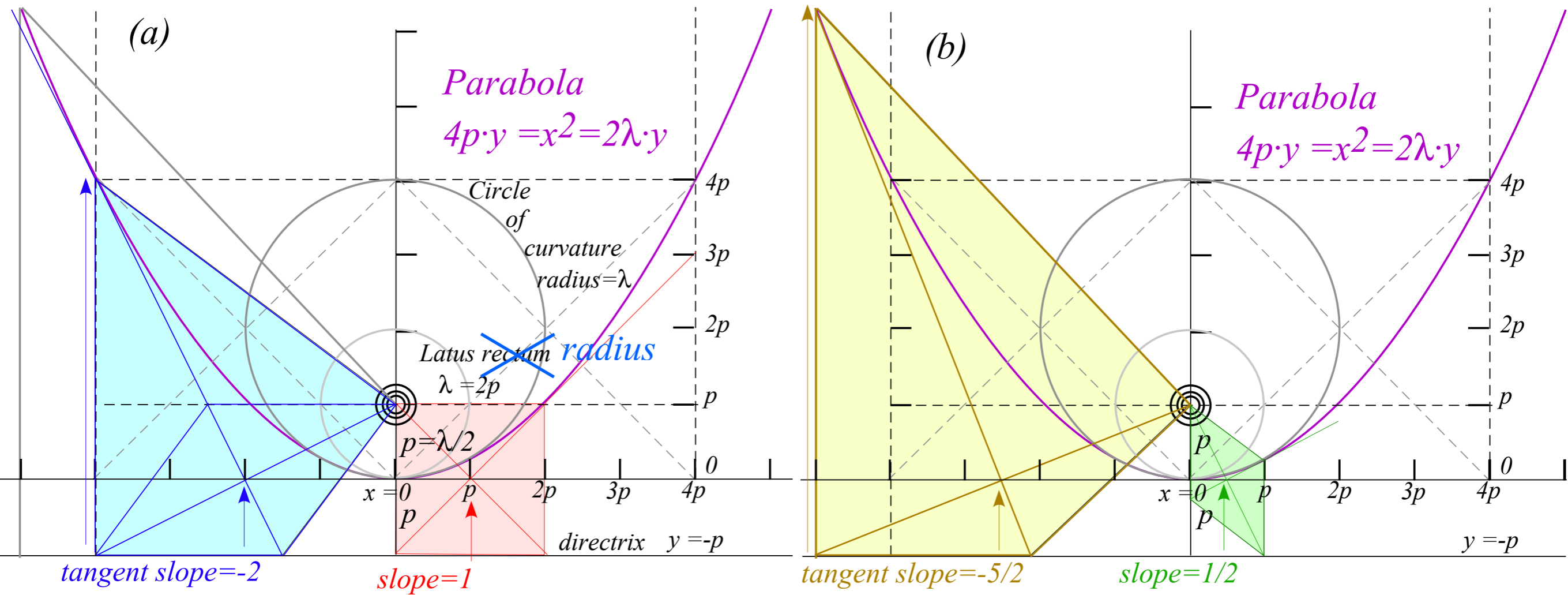
*Introducing the “neutron starlet” and **“Black-Hole-Earth”***

The ideal "Sophomore-Physics-Earth" model of geo-gravity



...conventional parabolic geometry...carried to extremes...

(From p.18)



Unit 1
Fig. 9.4

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

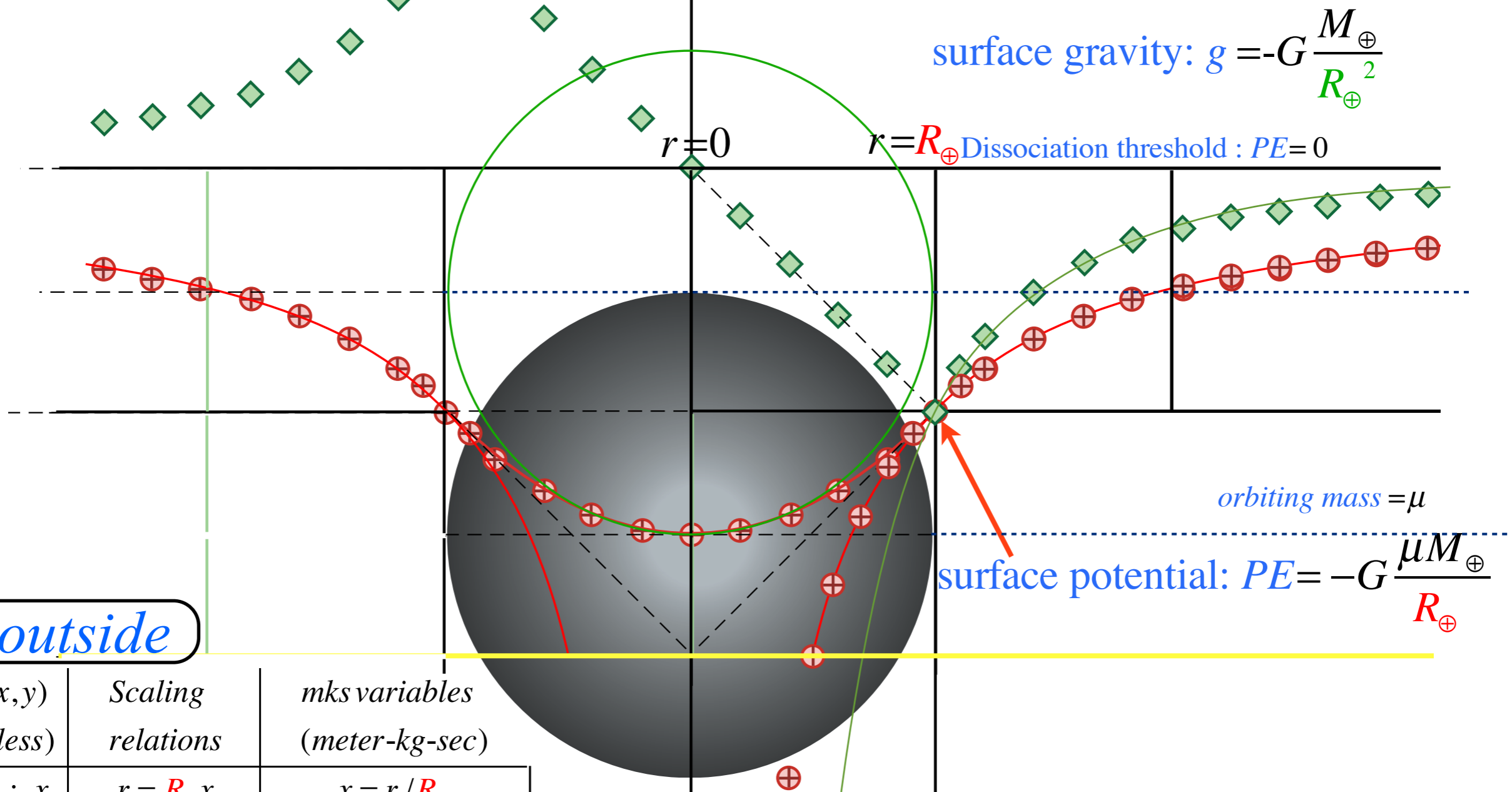
Contact-geometry of potential curve(s)

 *“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”*

Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and **“Black-Hole-Earth”***

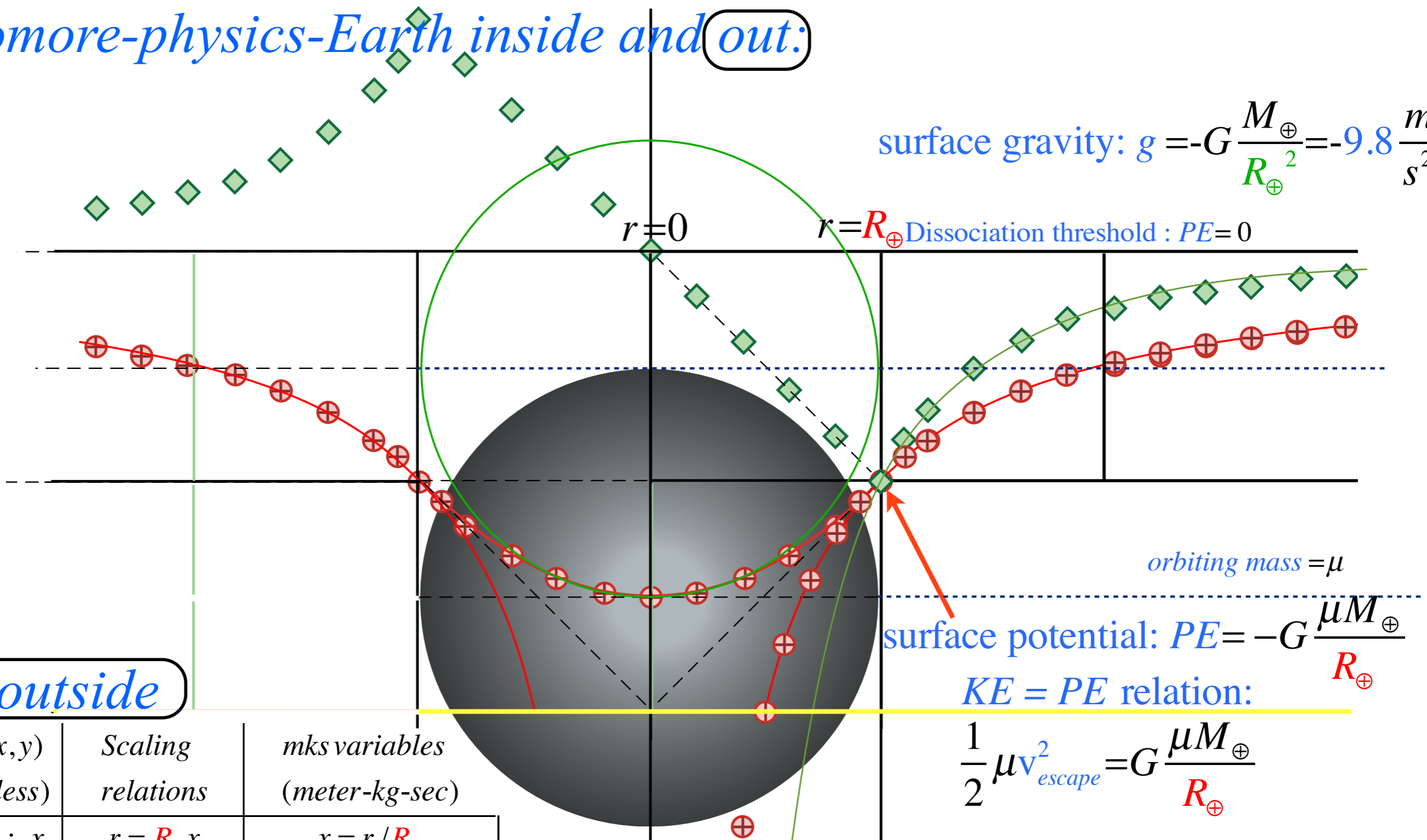
Sophomore-physics-Earth inside and out:



Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$

Sophomore-physics-Earth inside and out:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$



orbiting mass = μ

surface potential: $PE = -G \frac{\mu M_{\oplus}}{R_{\oplus}}$

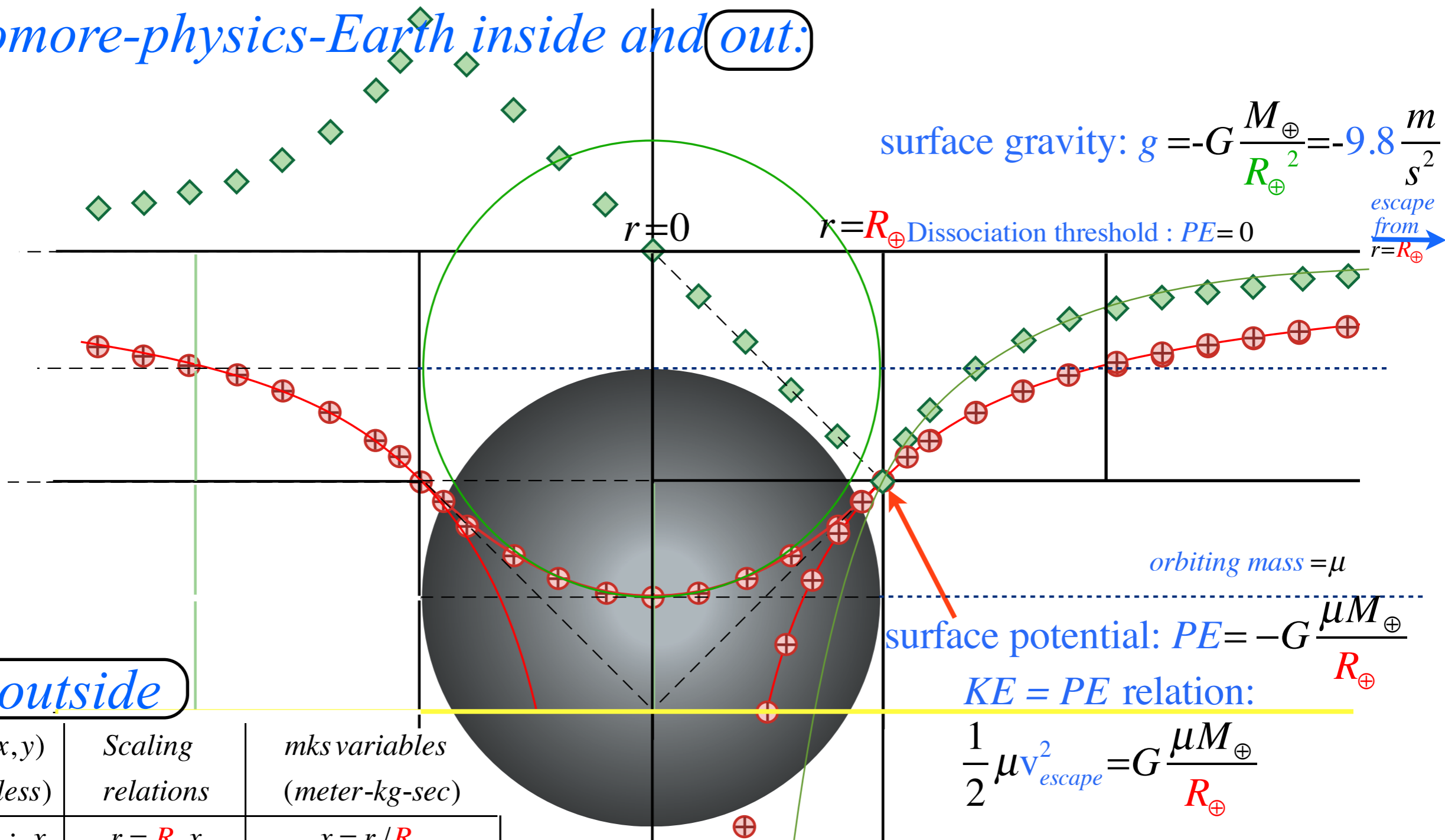
KE = PE relation:

$$\frac{1}{2} \mu v_{\text{escape}}^2 = G \frac{\mu M_{\oplus}}{R_{\oplus}}$$

outside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$

Sophomore-physics-Earth inside and out:



outside

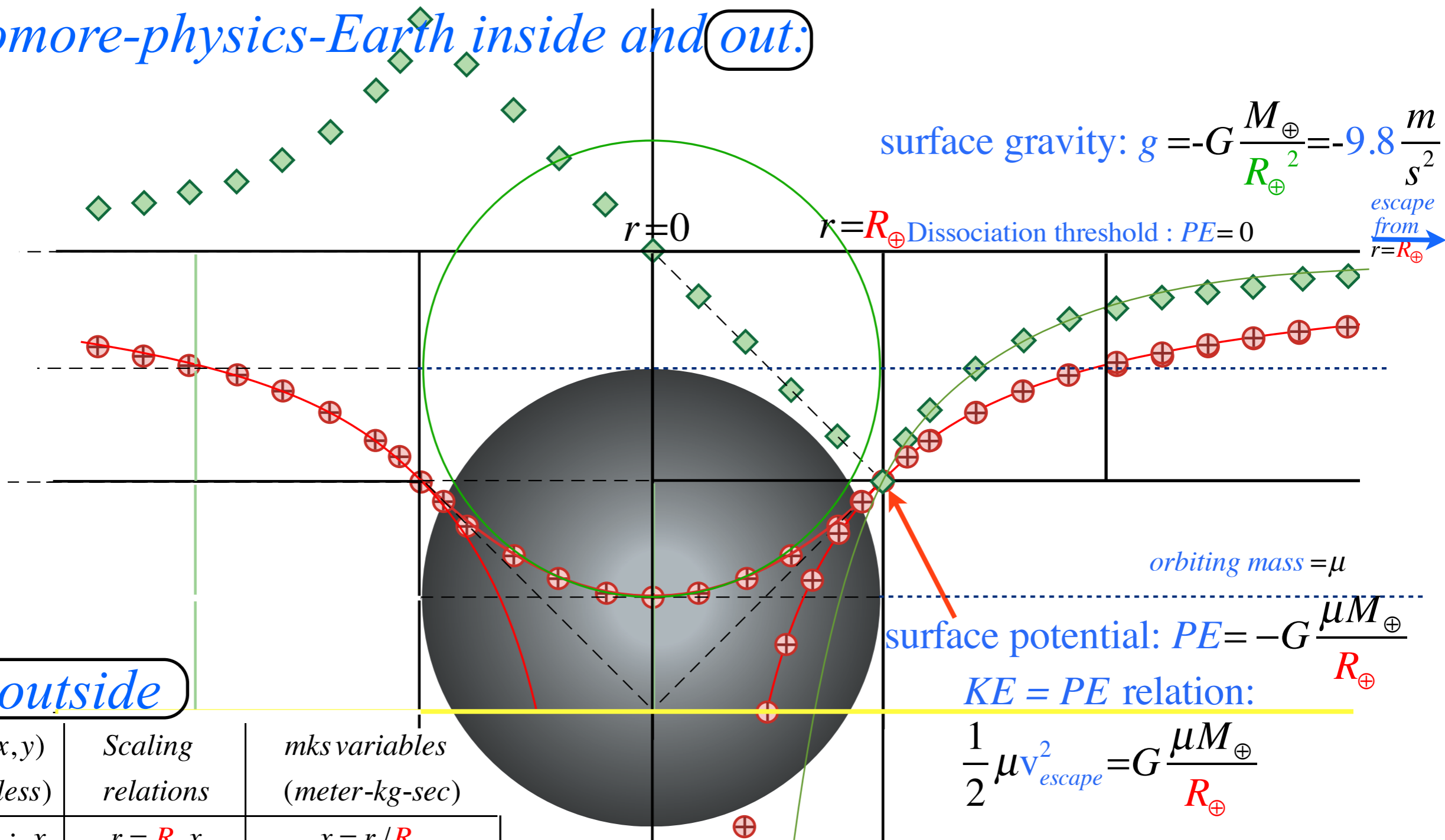
$$\frac{1}{2} \mu v_{\text{escape}}^2 = G \frac{\mu M_{\oplus}}{R_{\oplus}}$$

R_{\oplus} -escape-velocity

$$v_{\text{escape}} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

$r=R_{\oplus}$ Dissociation threshold : $PE=0$ escape from $r=R_{\oplus}$

orbiting mass = μ

surface potential: $PE = -G \frac{\mu M_{\oplus}}{R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{\text{escape}}^2 = G \frac{\mu M_{\oplus}}{R_{\oplus}}$$

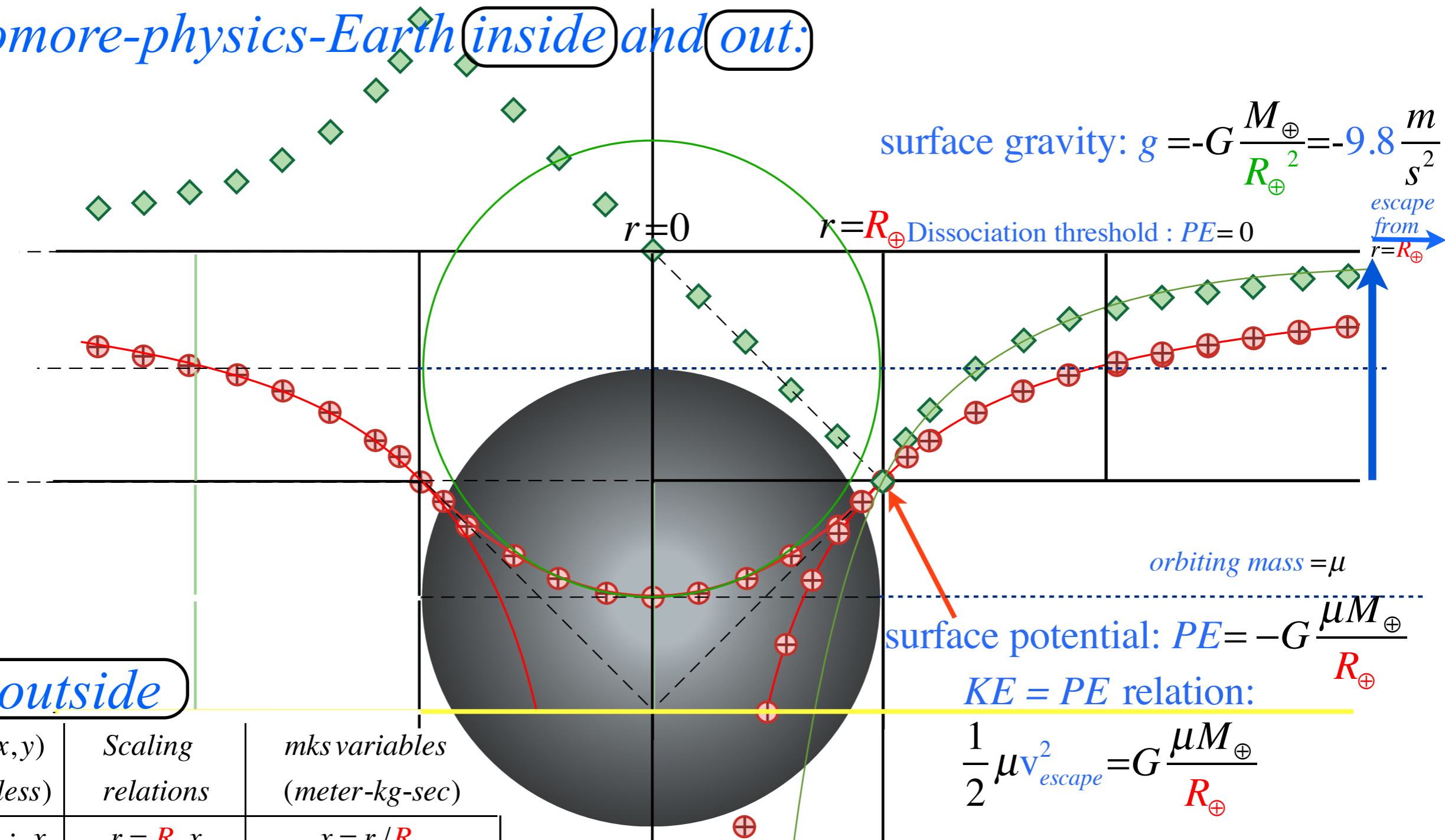
R_{\oplus} -escape-velocity

$$v_{\text{escape}} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$$

outside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$

Sophomore-physics-Earth (inside) and (out):



outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
$Force$ for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$
PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$		
$Force$ for $ x < 1$: $y^{Force} = -x$		

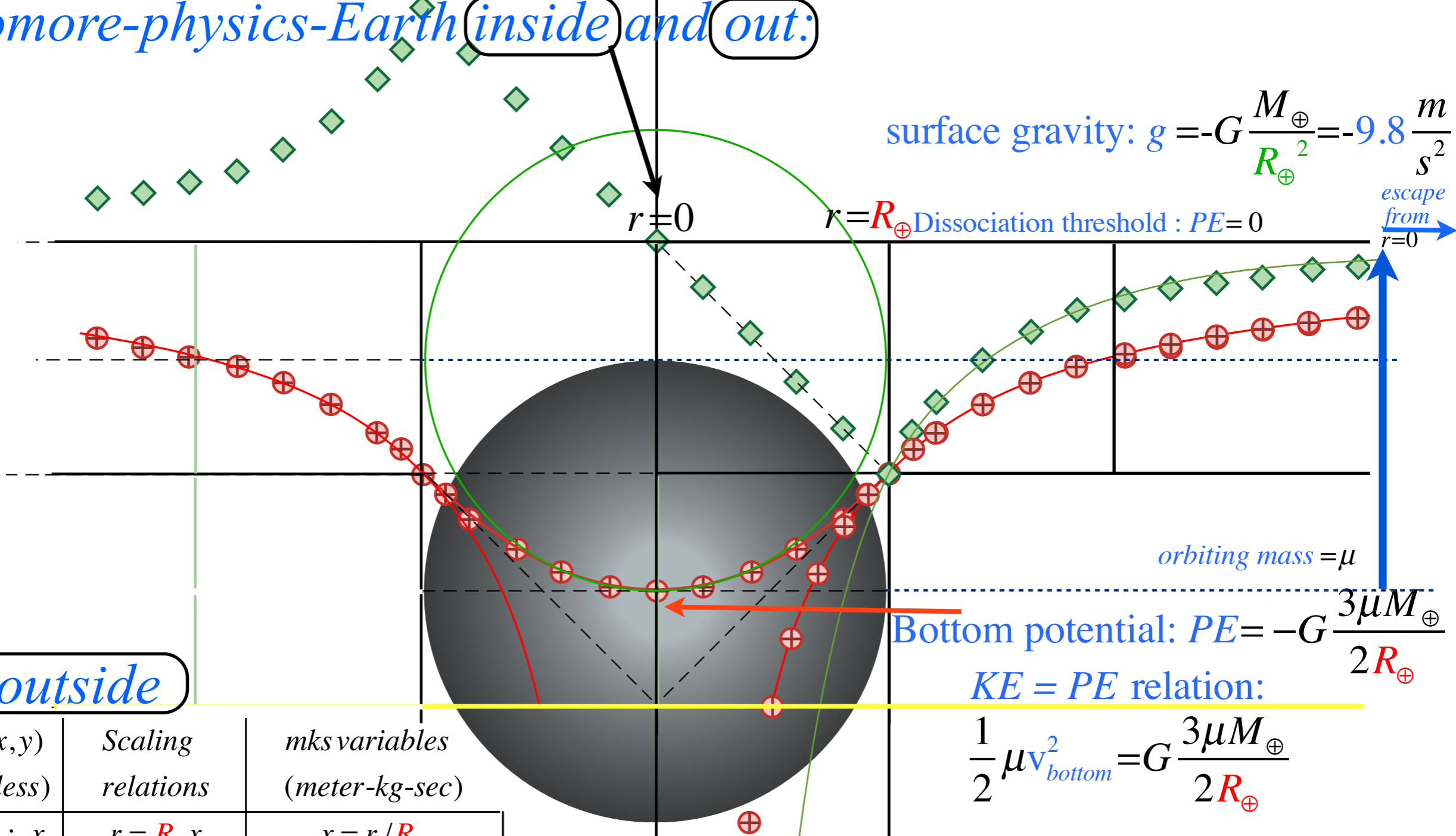
R_{\oplus} -escape-velocity

$v_{\text{escape}} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$

11.1 km/sec

Sophomore-physics-Earth inside and out:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$



$r=R_{\oplus}$ Dissociation threshold : $PE=0$

orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

outside

inside

$(r=0)$ -escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

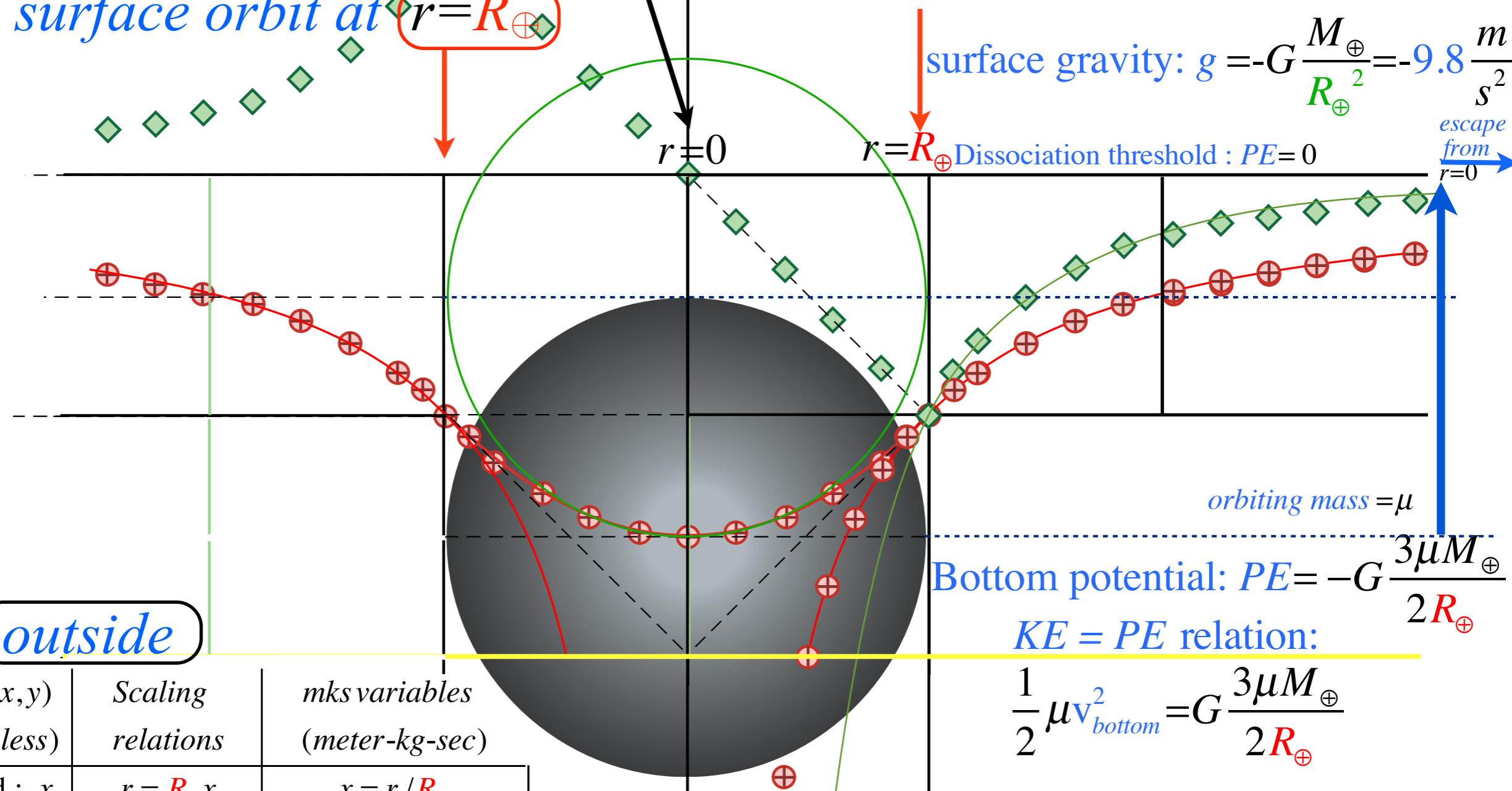
13.7km/sec

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
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PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$
Force for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$



outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
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Force for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$	

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

(r=0)-escape-velocity

$$v_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

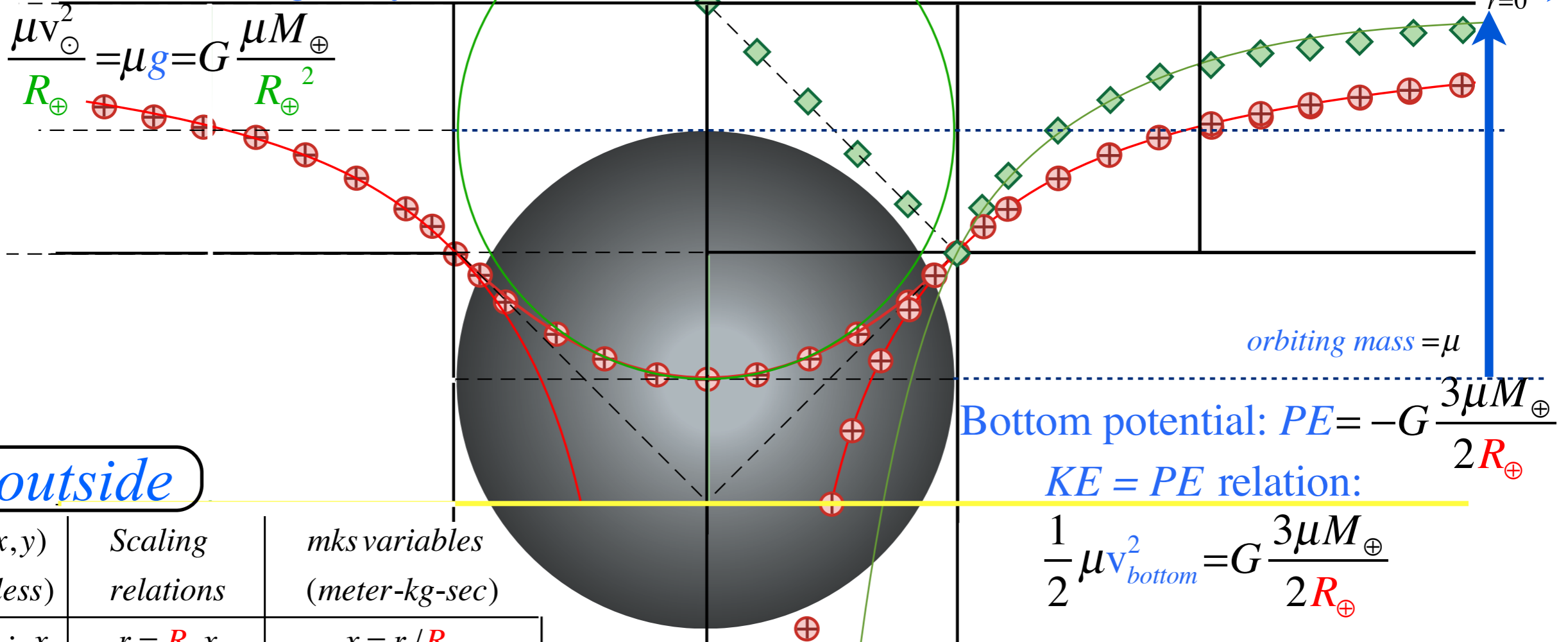
surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

Centrifugal force = surface gravity:

$$\frac{\mu v_{\oplus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Dissociation threshold : $PE=0$

escape from $r=0$



orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

outside

inside

($r=0$)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)	
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$	
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Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$	Force for $ x < 1$: $y^{Force} = -x$
			$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

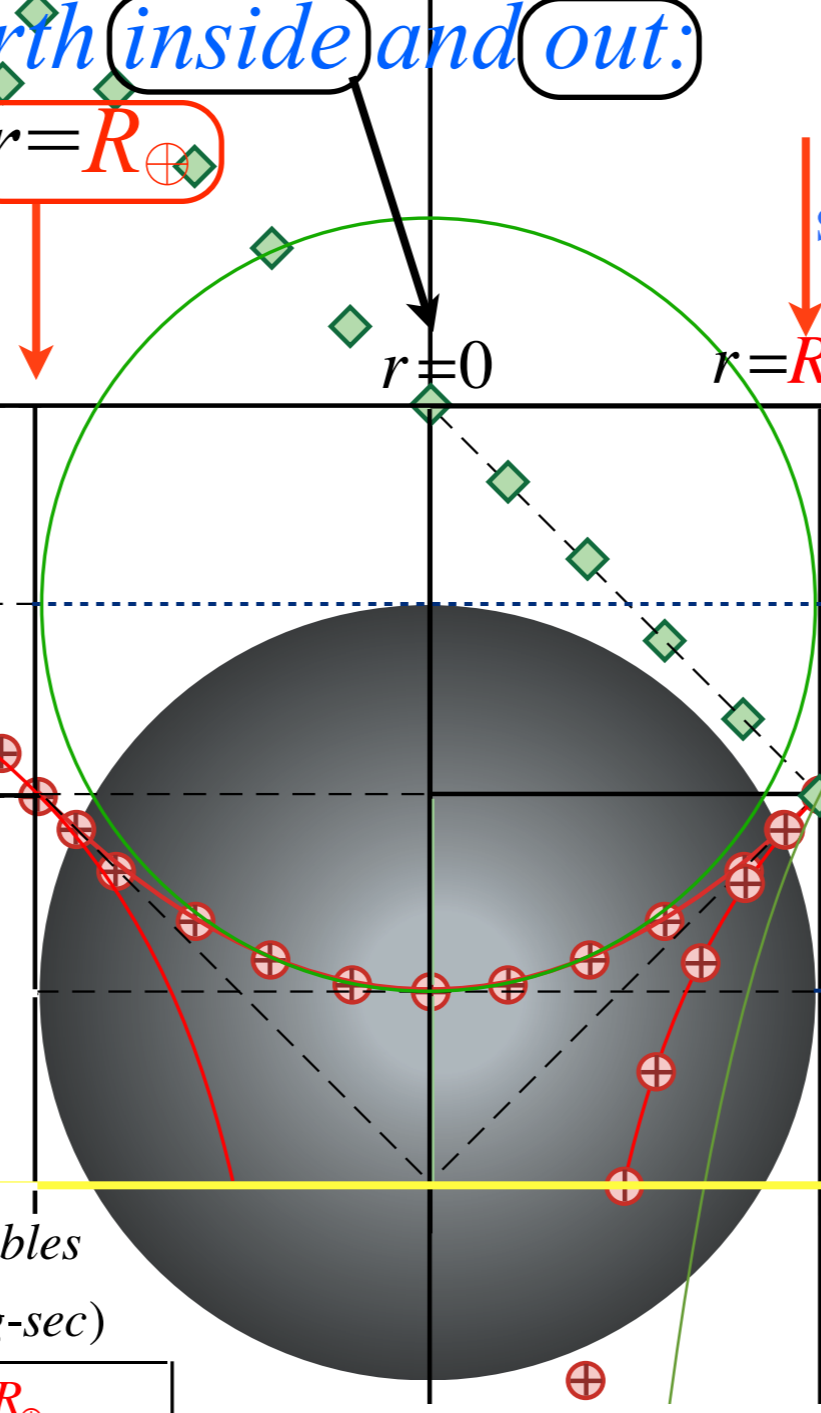
surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

Centrifugal force = surface gravity:

$$\frac{\mu v_{\oplus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Orbit KE = $\frac{1}{2} \mu v_{\oplus}^2 = G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Dissociation threshold : $PE=0$ escape from $r=0$



orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

outside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
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inside

PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$
Force for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

($r=0$)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

Centrifugal force = surface gravity:

$$\frac{\mu v_{\oplus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Orbit KE = $\frac{1}{2} \mu v_{\oplus}^2 = G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Orbit $E_{\oplus}^{Total} = \frac{1}{2} \mu v_{\oplus}^2 - G \frac{\mu M_{\oplus}}{R_{\oplus}} = -G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Dissociation threshold : $PE=0$ escape from $r=0$

orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

($r=0$)-escape-velocity

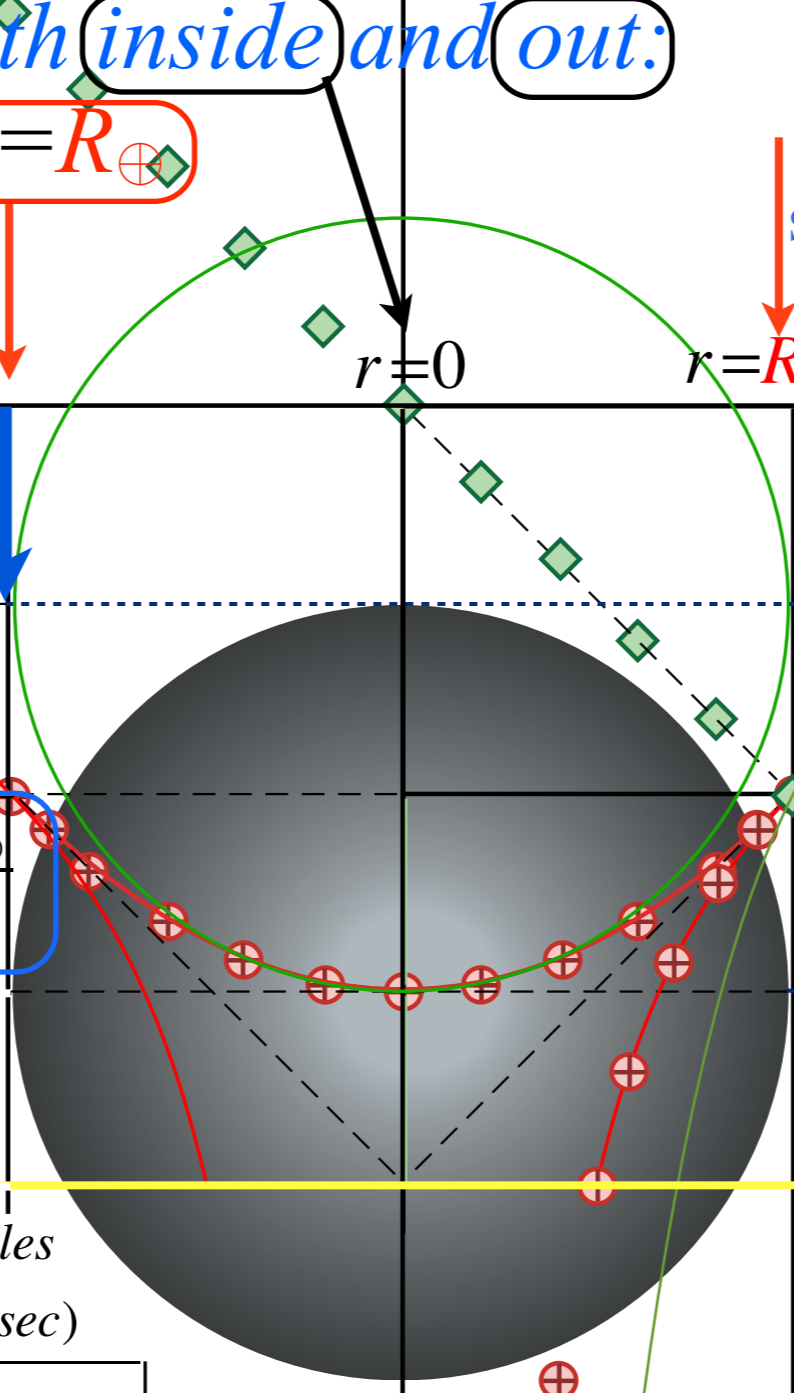
$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}} = 13.7 \text{ km/sec}$$

outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
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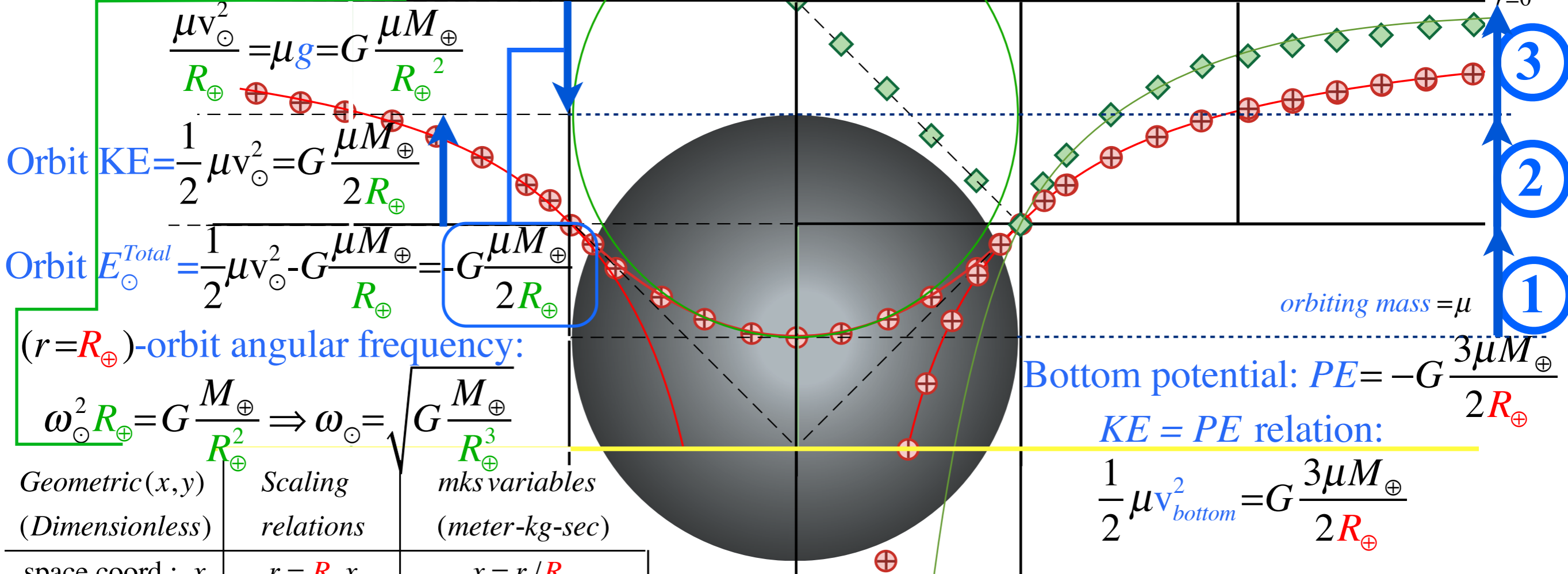


Sophomore-physics-Earth (inside and out): "3-steps out of (or into) Hell"

...and surface orbit at $r=R_{\oplus}$

Centrifugal force = surface gravity:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$



Orbit $KE = \frac{1}{2} \mu v_{\oplus}^2 = G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

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$(r=R_{\oplus})$ -orbit angular frequency:

$\omega_{\oplus}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\oplus} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation: $\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)		
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inside

$(r=0)$ -escape-velocity

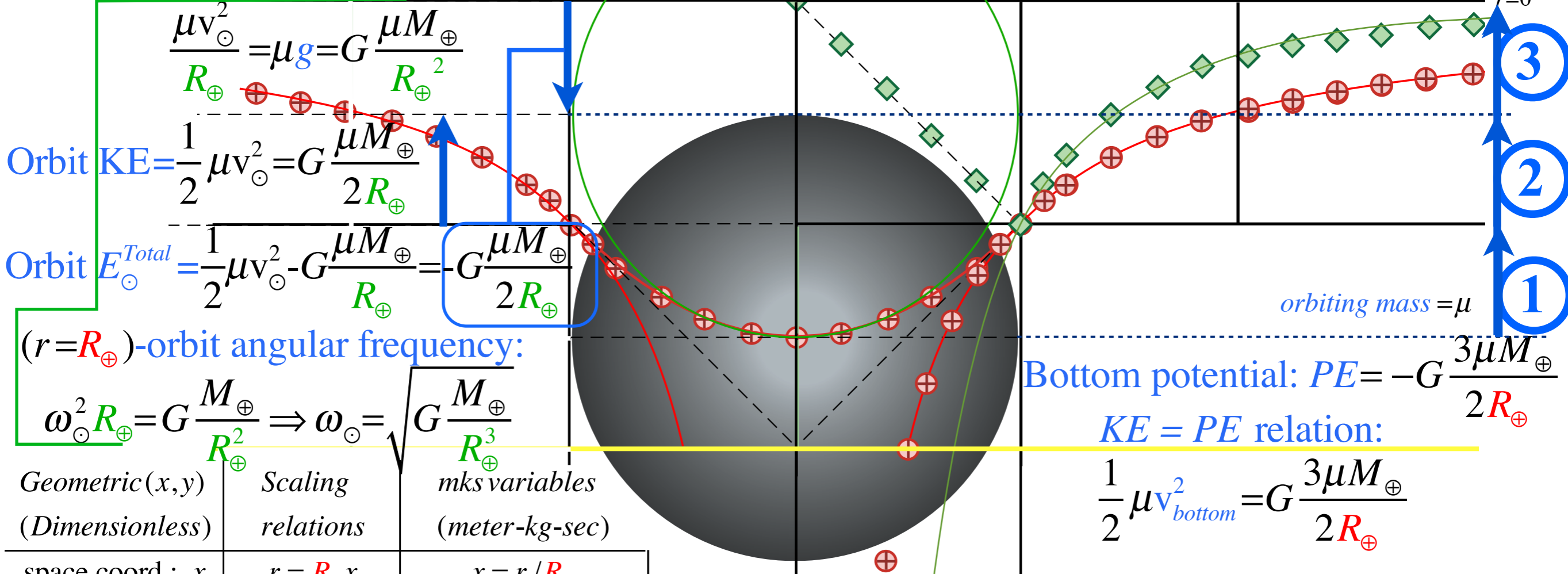
$v_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$
13.7km/sec

Sophomore-physics-Earth (inside and out): "3-steps out of (or into) Hell"

...and surface orbit at $r=R_{\oplus}$

Centrifugal force = surface gravity:

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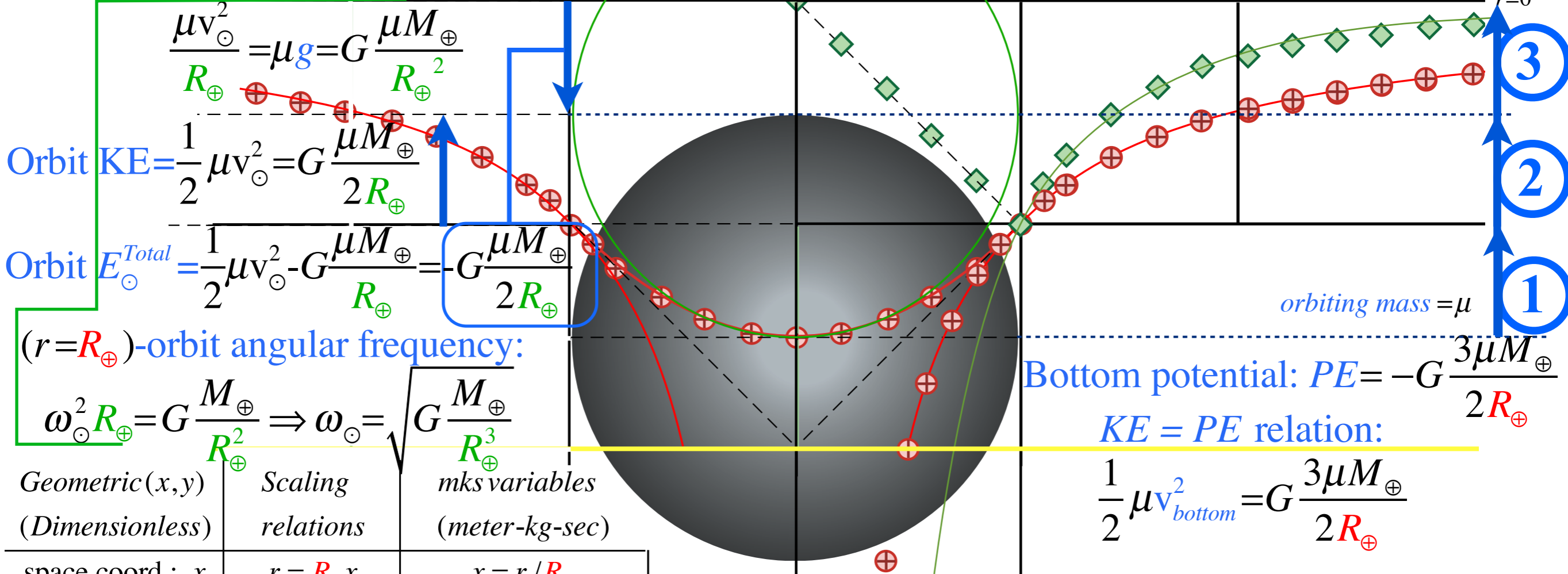
Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)			(r=0)-escape-velocity
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Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$	Force for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$	$(r=R_{\oplus})$ -orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}} = \sqrt{gR_{\oplus}}$ 7.9km/sec

Sophomore-physics-Earth (inside and out): "3-steps out of (or into) Hell"

...and surface orbit at $r=R_{\oplus}$

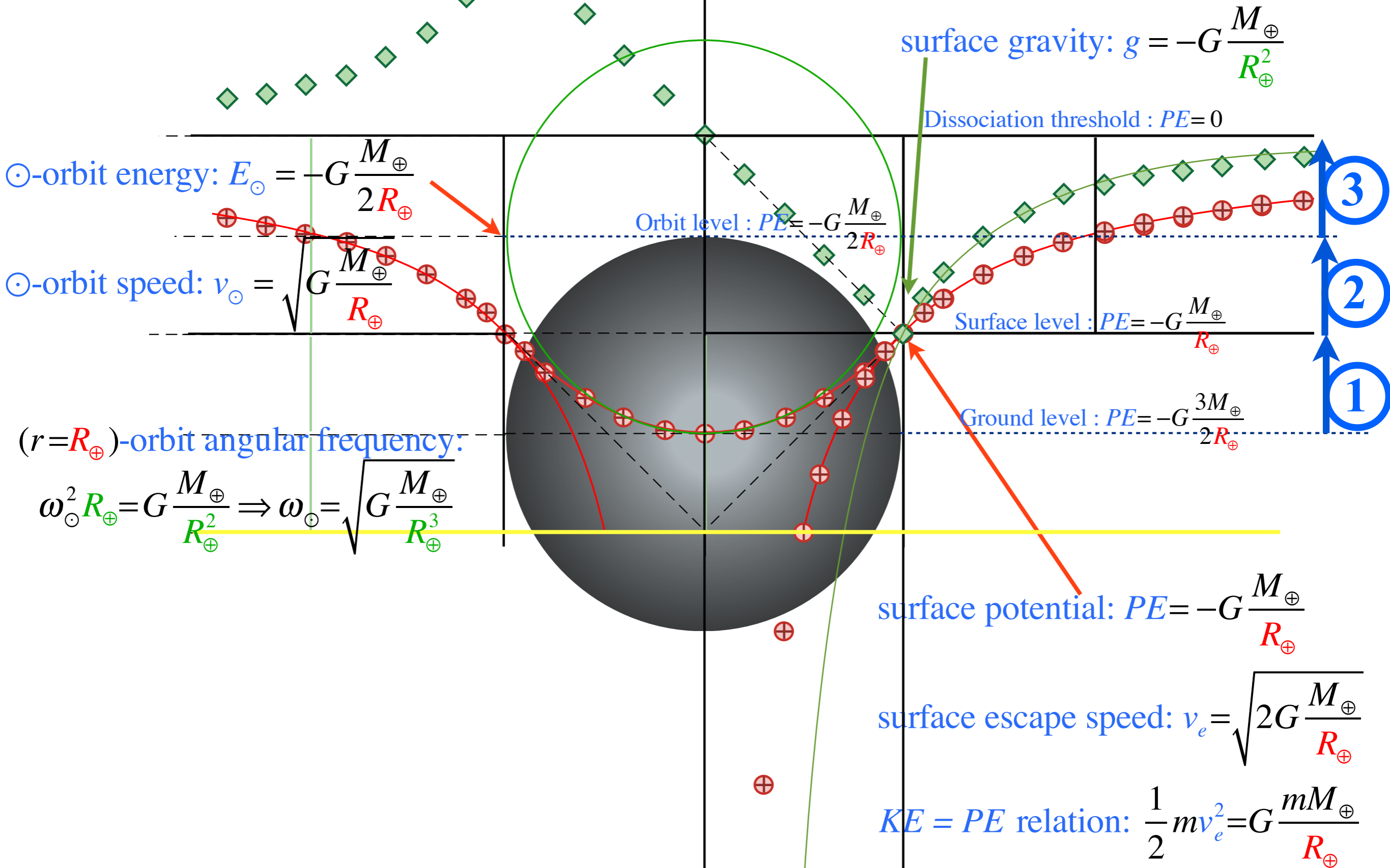
Centrifugal force = surface gravity:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

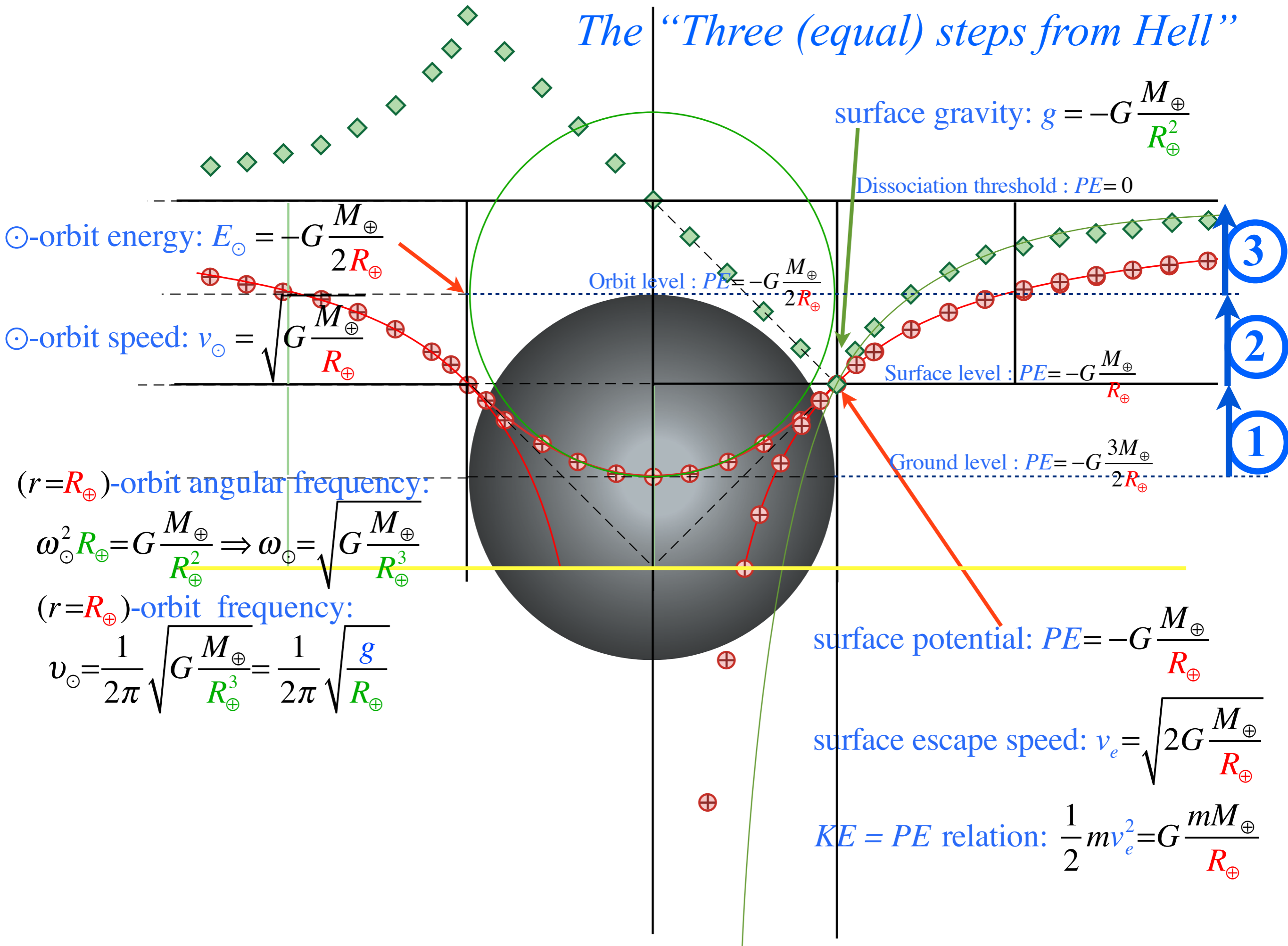


Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)			(r=0)-escape-velocity
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$			
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Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$	Force for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$	(r=R_earth)-escape velocity: $v_{escape} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$ 11.1km/sec

The "Three (equal) steps from Hell"



The "Three (equal) steps from Hell"



surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2}$

Dissociation threshold : $PE=0$

☉-orbit energy: $E_{\odot} = -G \frac{M_{\oplus}}{2R_{\oplus}}$

Orbit level : $PE = -G \frac{M_{\oplus}}{2R_{\oplus}}$

☉-orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}}$

Surface level : $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

$(r=R_{\oplus})$ -orbit angular frequency:

$$\omega_{\odot}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$$

Ground level : $PE = -G \frac{3M_{\oplus}}{2R_{\oplus}}$

$(r=R_{\oplus})$ -orbit frequency:

$$v_{\odot} = \frac{1}{2\pi} \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}} = \frac{1}{2\pi} \sqrt{\frac{g}{R_{\oplus}}}$$

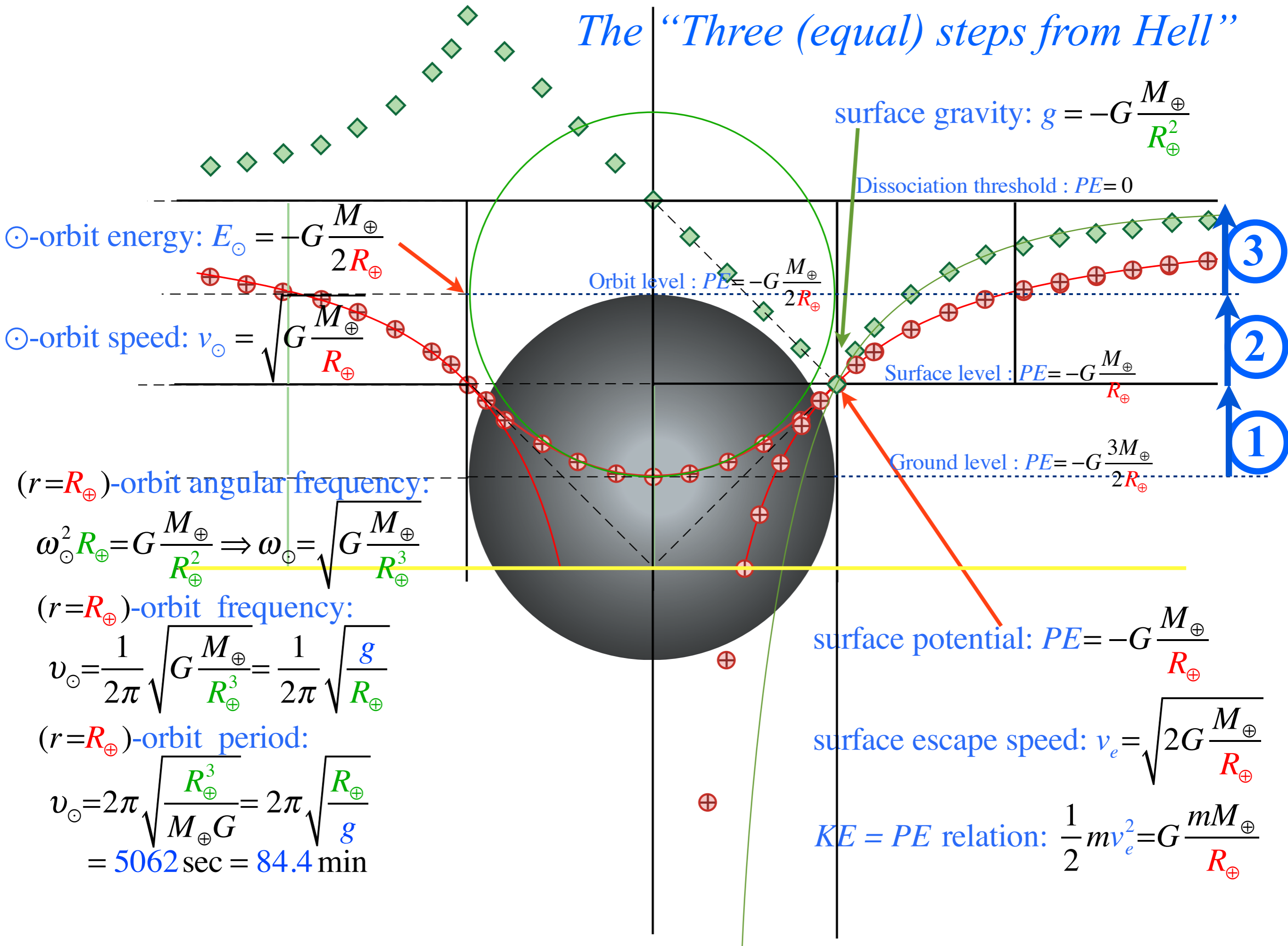
surface potential: $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

surface escape speed: $v_e = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$

KE = PE relation: $\frac{1}{2} m v_e^2 = G \frac{m M_{\oplus}}{R_{\oplus}}$

- 3
- 2
- 1

The "Three (equal) steps from Hell"



surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2}$

Dissociation threshold : $PE = 0$

⊙-orbit energy: $E_{\odot} = -G \frac{M_{\oplus}}{2R_{\oplus}}$

⊙-orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}}$

Orbit level : $PE = -G \frac{M_{\oplus}}{2R_{\oplus}}$

Surface level : $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

Ground level : $PE = -G \frac{3M_{\oplus}}{2R_{\oplus}}$

$(r = R_{\oplus})$ -orbit angular frequency:

$$\omega_{\odot}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$$

$(r = R_{\oplus})$ -orbit frequency:

$$v_{\odot} = \frac{1}{2\pi} \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}} = \frac{1}{2\pi} \sqrt{\frac{g}{R_{\oplus}}}$$

$(r = R_{\oplus})$ -orbit period:

$$v_{\odot} = 2\pi \sqrt{\frac{R_{\oplus}^3}{M_{\oplus} G}} = 2\pi \sqrt{\frac{R_{\oplus}}{g}} = 5062 \text{ sec} = 84.4 \text{ min}$$

surface potential: $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

surface escape speed: $v_e = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$

KE = PE relation: $\frac{1}{2} m v_e^2 = G \frac{m M_{\oplus}}{R_{\oplus}}$

- 3
- 2
- 1

Suppose Earth radius crushed to 1/2: ($R_{\oplus} = 6.4 \cdot 10^6 m$ crushed to $R_{\oplus}/2 = 3.2 \cdot 10^6 m$)

All formulas identical to ones derived on p.63 to 78.

Imagine reducing R_{\oplus} to $R_{\oplus}/2$

- 3
- 2
- 1

⊙ - Orbit level : $PE = -G \frac{M_{\oplus}}{2R_{\oplus}}$

2 times ⊙-orbit energy: $E_{\odot} = -G \frac{M_{\oplus}}{2R_{\oplus}}$

$\sqrt{2}$ times ⊙-orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}}$

2 times the surface potential: $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

$\sqrt{2}$ times surface escape speed: $v_e = \sqrt{G \frac{2M_{\oplus}}{R_{\oplus}}}$

4 times the surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2}$

2x Crushed Earth

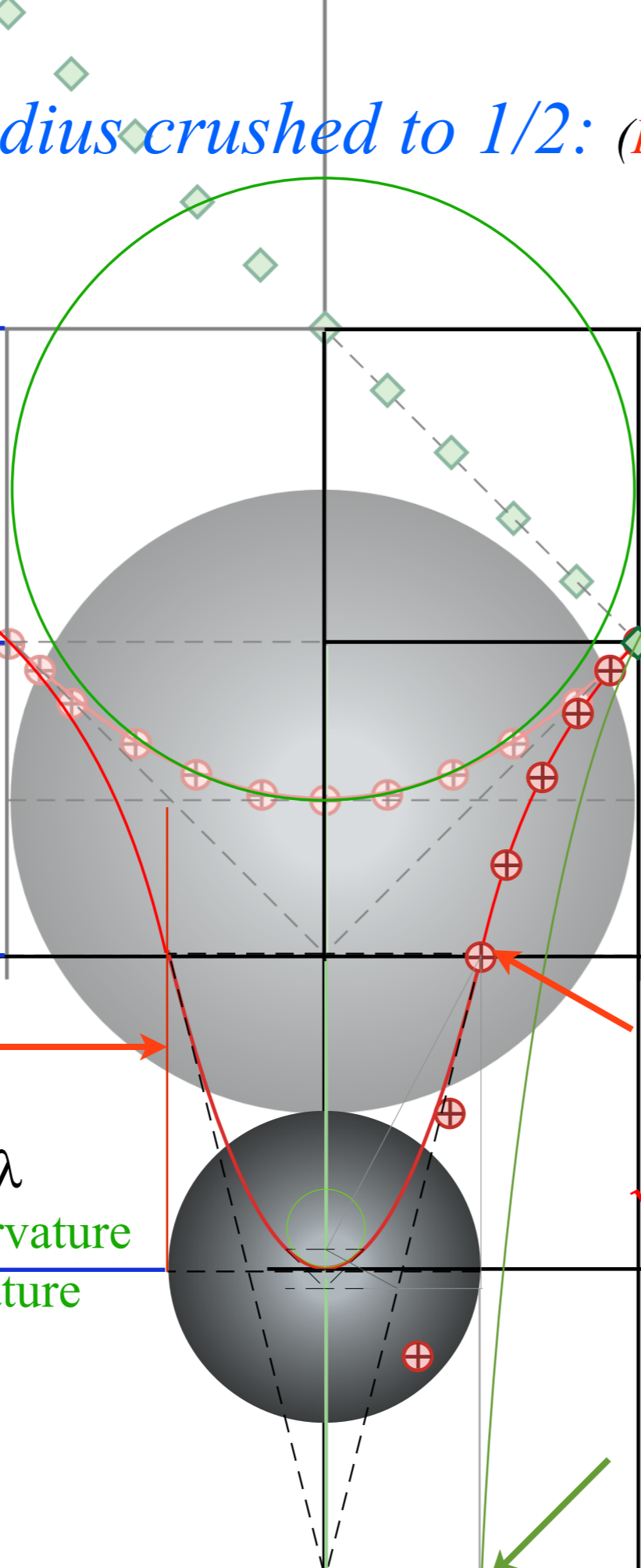
1/2 radius

8 times as dense

1/8 focal distance or λ

1/8 minimum radius of curvature

8 times maximum curvature



Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

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Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:



*Introducing the “neutron starlet” and **“Black-Hole-Earth”***

Examples of “crushed” matter

Earth matter Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \simeq 6.0 \cdot 10^{24} \text{ kg}$. Density $\rho_{\oplus} = ??$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \simeq 6.4 \cdot 10^6 \text{ m}$ Earth volume : $(4\pi / 3) R_{\oplus}^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21} \text{ m}^3$

$(6.4)^3 \sim 262$ and $(4\pi/3)260 = 1098 \sim 10^3$

Examples of “crushed” matter

$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}^3}$$

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$$(6.4)^3 \sim 262 \text{ and } (4\pi/3)260 = 1089 \sim 10^3$$

Density of solid Fe = $7.9 \cdot 10^3 \text{ kg/m}^3$

Density of liquid Fe = $6.9 \cdot 10^3 \text{ kg/m}^3$

Examples of “crushed” matter

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Nuclear matter Nucleon mass = $1.67 \cdot 10^{-27} \text{ kg} \sim 2 \cdot 10^{-27} \text{ kg}$. (“fingertip physics”)

Say a nucleus of atomic weight 50 has a radius of 3 fm, or 50 nucleons each with a mass $2 \cdot 10^{-27} \text{ kg}$.

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$$\frac{4\pi}{3}3^3 = 36\pi = 113 \sim 10^2$$

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 $\frac{4\pi}{3}3^3 = 36\pi = 113 \sim 10^2$

Nuclear density is $10^{-25+43} = 10^{18} \text{ kg/m}^3$ or a trillion (10^{12}) kilograms in a **fingertip** $(1 \text{ cm})^3$.

$$(1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$$

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Nuclear density is $10^{-25+43} = 10^{18} \text{ kg/m}^3$ or a trillion (10^{12}) kilograms in a **fingertip** (1 cm)³.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{\text{crush}\oplus} \approx 300 \text{ m}$ would approach neutron-star density. $(1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$



Geometry and algebra of idealized “Sophomore-physics Earth” fields

Coulomb field outside

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 *Introducing the “neutron starlet”*

*Fantasizing a “**Black-Hole-Earth**”*

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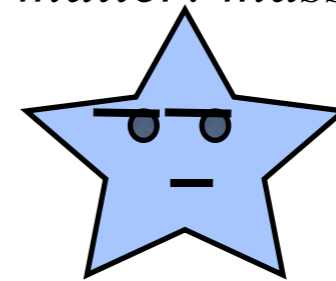
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Introducing the “Neutron starlet” 1 cm^3 of nuclear matter: mass = 10^{12} kg .



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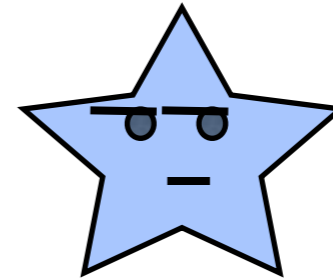
Say a nucleus of atomic weight 50 has a radius of 3 fm, or 50 nucleons each with a mass $2 \cdot 10^{-27} \text{ kg}$.

That's $100 \cdot 10^{-27} = 10^{-25} \text{ kg}$ packed into a volume of $4\pi/3 r^3 = 4\pi/3 (3 \cdot 10^{-15})^3 \text{ m}^3$ or about 10^{-43} m^3 .

Nuclear density is $10^{-25+43} = 10^{18} \text{ kg/m}^3$ or a trillion (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{\text{crush}\oplus} \approx 300 \text{ m}$ would approach neutron-star density.

Introducing the “Neutron starlet” 1 cm^3 of nuclear matter: mass = 10^{12} kg .



Fantasizing the “Black Hole Earth” Suppose Earth is crushed so that its

surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 \text{ m/s}$.

$$V_{\text{escape}} = \sqrt{(2GM/R_{\oplus})}$$

(from p. 65, 66, ..., 82)

$$G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3) 10^{-10}$$

(from p. 61)

$$c \equiv 299,792,458 \text{ m/s (EXACTLY)}$$

$$\text{Earth radius : } R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$$

$$\text{Earth mass : } M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}$$

Examples of “crushed” matter

$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}^3}$$

Earth matter Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}$. Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 \text{ kg/m}^3$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$ Earth volume : $(4\pi/3)R_{\oplus}^3 \approx 4 \cdot 260 \cdot 10^{18} \sim 10^{21} \text{ m}^3$

Nuclear matter Nucleon mass = $1.67 \cdot 10^{-27} \text{ kg} \sim 2 \cdot 10^{-27} \text{ kg}$. (“fingertip physics”)

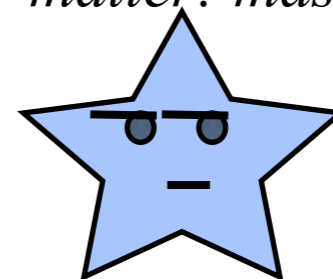
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$$G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3)10^{-10}$$

(from p. 61)

$$R_{\oplus} = 2GM/c^2 = 8.9 \text{ mm} \sim 1 \text{ cm}$$

(fingertip size!)



→ *Introducing 2D IHO orbits and phasor geometry*
Phasor “clock” geometry

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

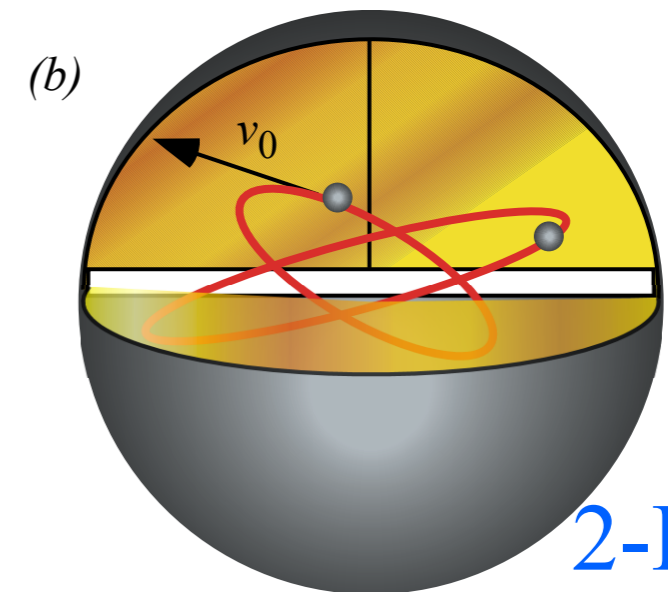
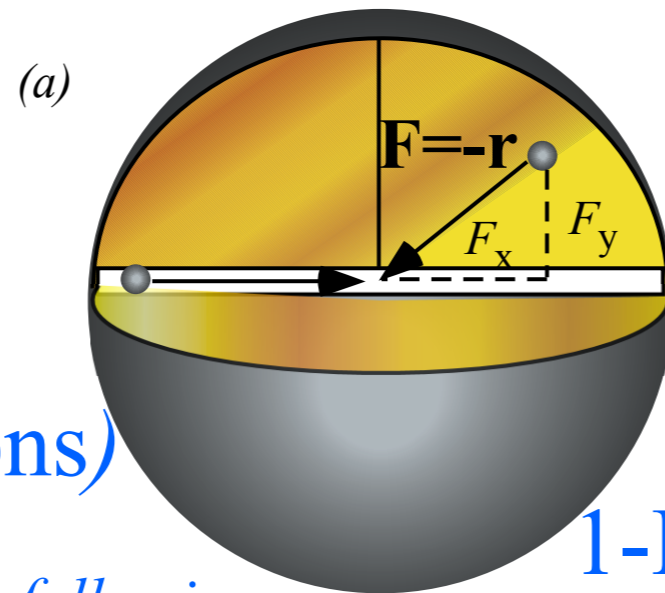
I.H.O. Force law

$$F = -x \quad (1\text{-Dimension})$$

$$\mathbf{F} = -\mathbf{r} \quad (2 \text{ or } 3\text{-Dimensions})$$

Each dimension x , y , or z obeys the following:

$$\text{Total } E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const.}$$



Unit 1
Fig. 9.10

(Paths are *always*
2-D ellipses if
viewed right!)

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

I.H.O. Force law

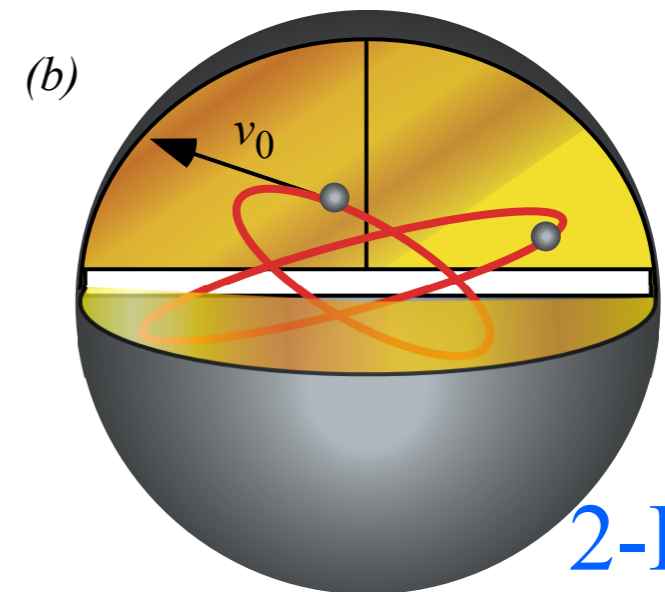
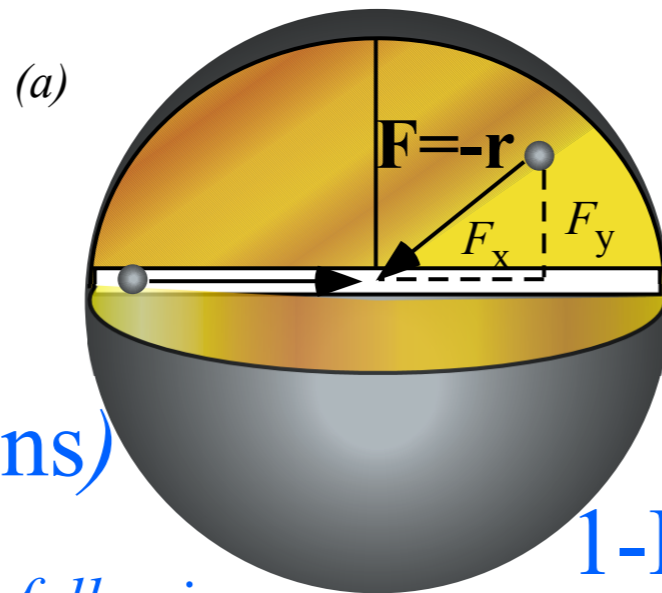
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Equations for x -motion
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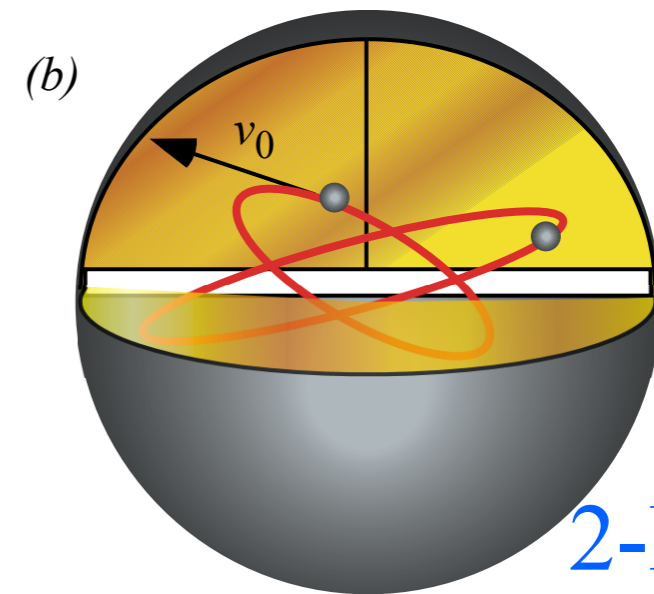
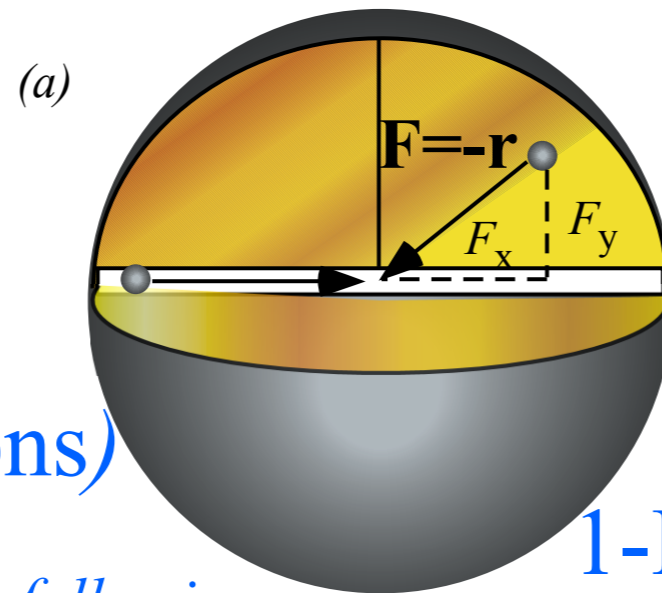


Unit 1
Fig. 9.10

(Paths are *always* 2-D
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Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



2-D or 3-D
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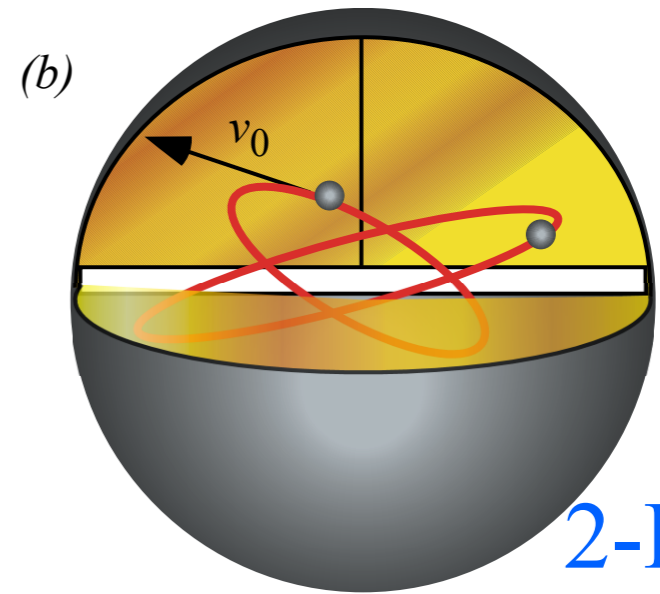
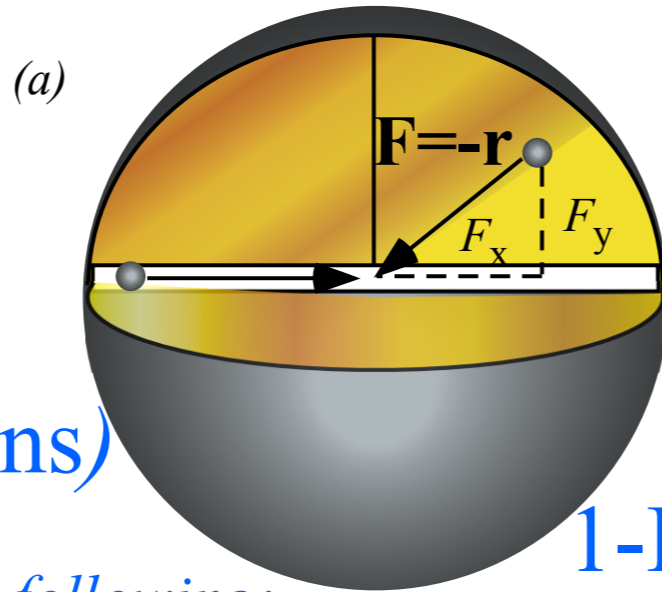
$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old “scale-a-circle” trick...

Let : **(1)** *velocity:* $v = \sqrt{2E/m} \cos\theta$, and : **(2)** *position:* $x = \sqrt{2E/k} \sin\theta$

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



2-D or 3-D
(Paths are *always* 2-D ellipses if viewed right!)

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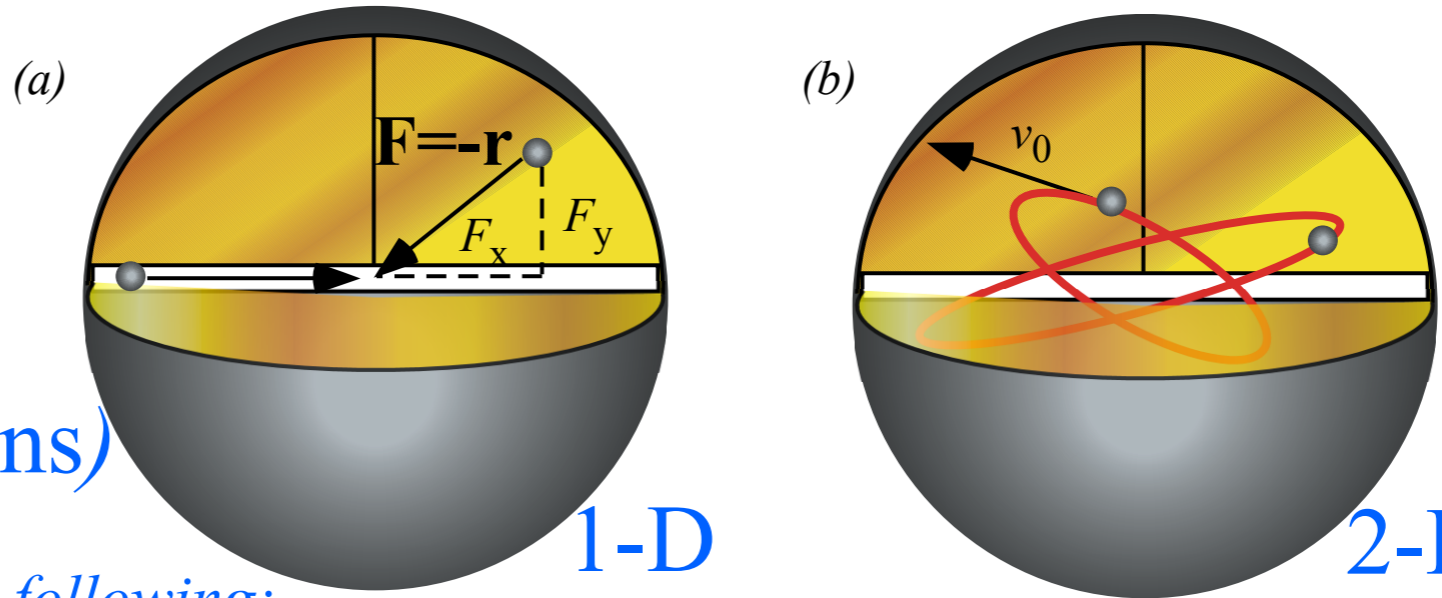
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Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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velocity:

position:

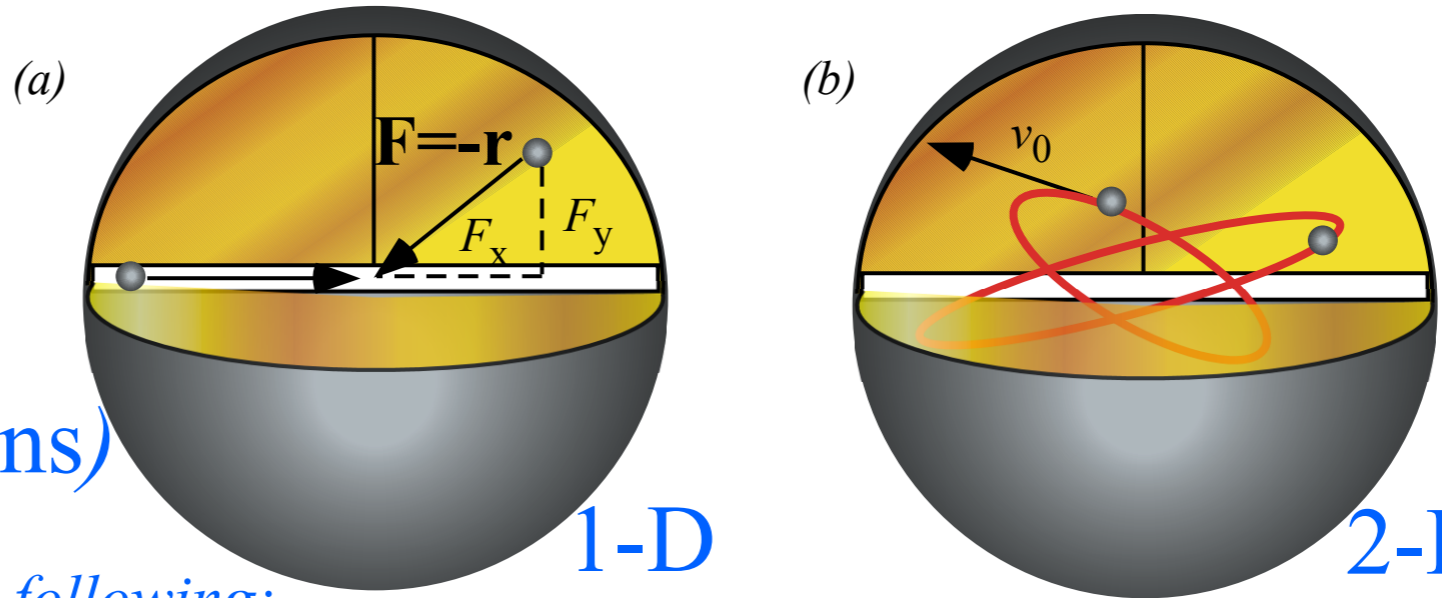
angular velocity:

$$\sqrt{\frac{2E}{m}} \cos\theta = v = \frac{dx}{dt}$$

by (1)

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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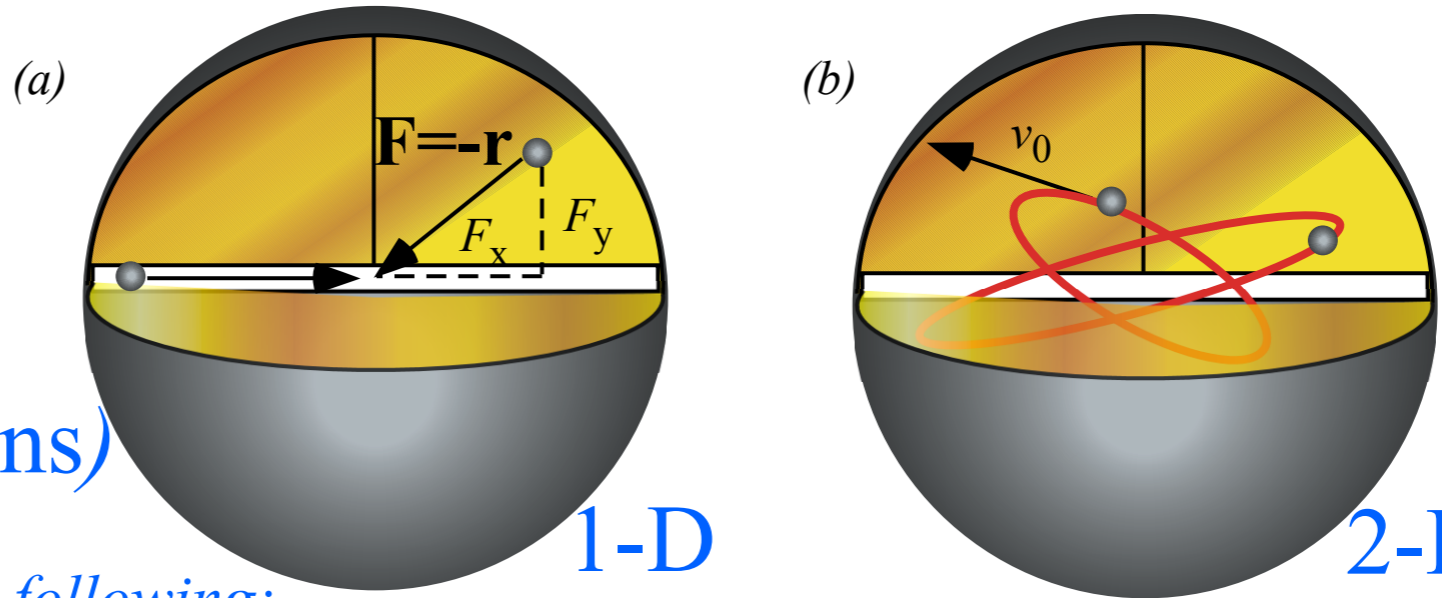
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Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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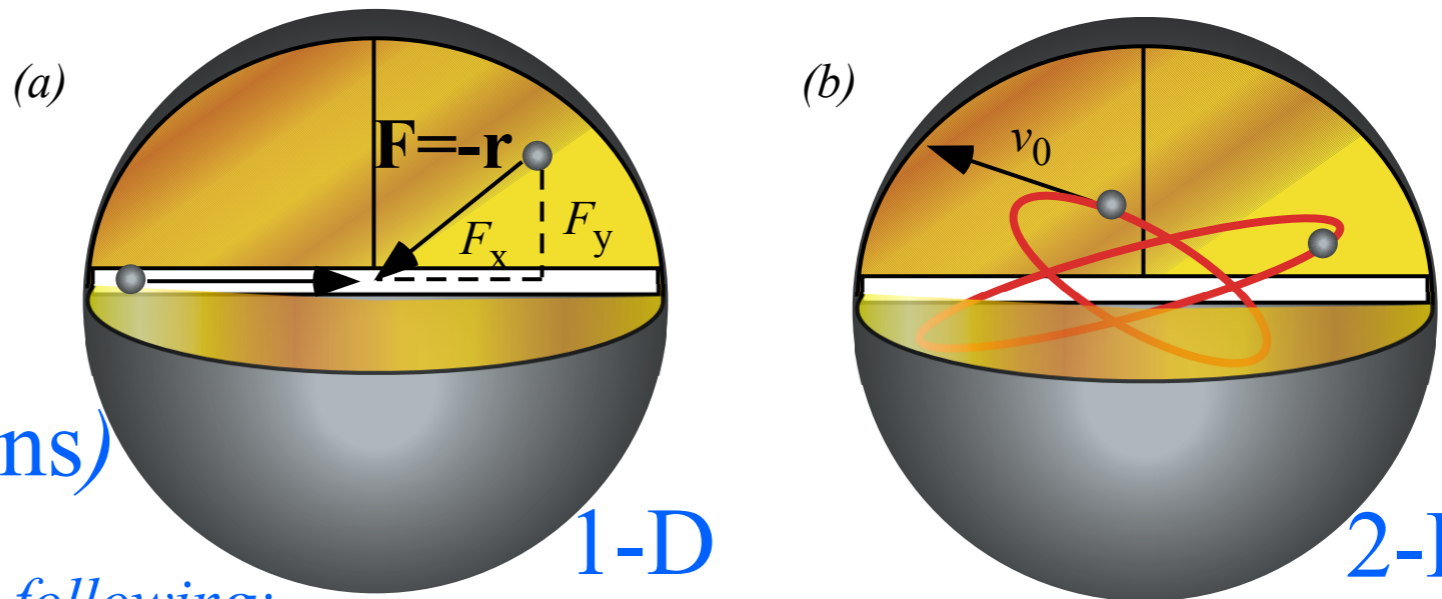
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Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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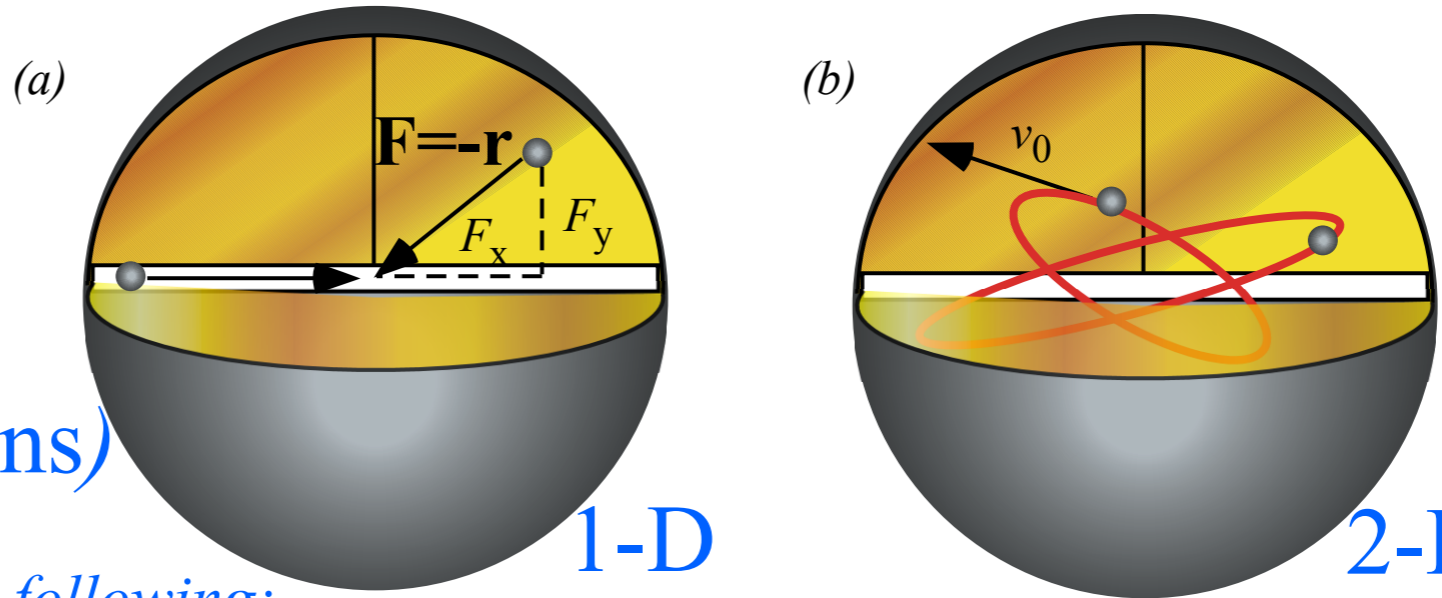
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Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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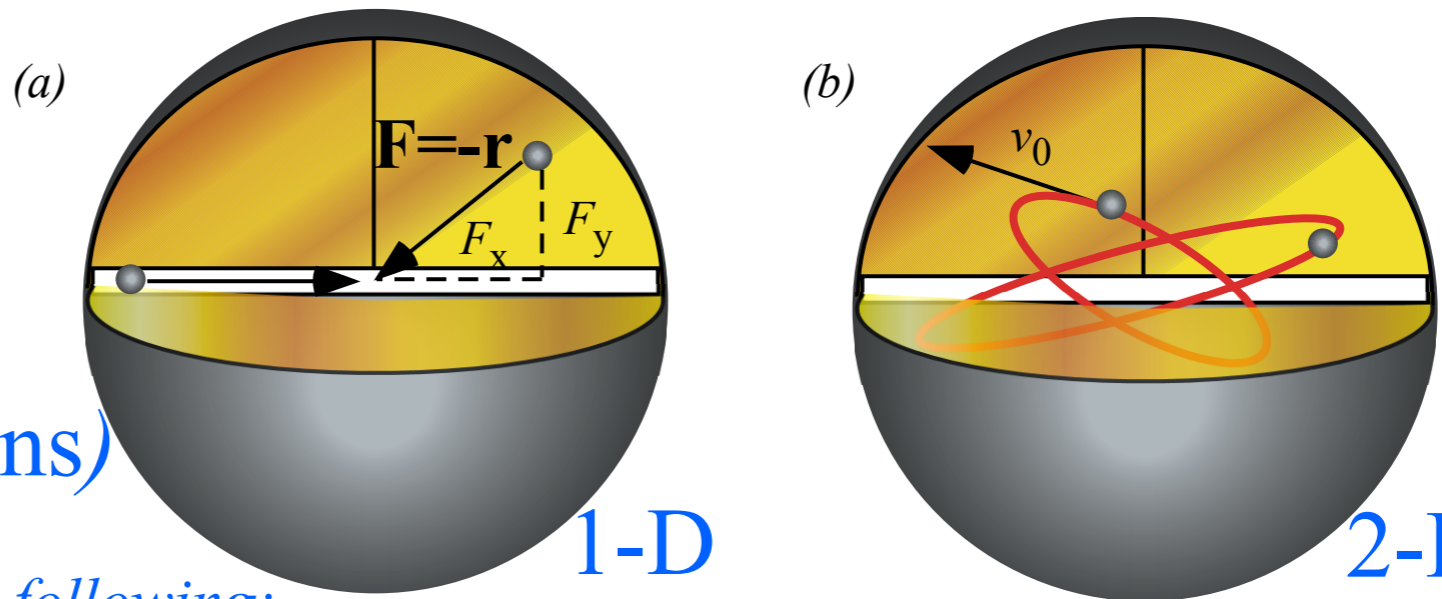
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by def. (3)

$$\omega = \frac{d\theta}{dt}$$

Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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$$\omega = \frac{d\theta}{dt} \stackrel{\text{by def. (3)}}{=} \frac{\sqrt{\frac{2E}{m}} \cos\theta}{\sqrt{\frac{2E}{k}} \cos\theta} \stackrel{\text{divide (1)}}{=} \sqrt{\frac{k}{m}}$$

by (2) derivative

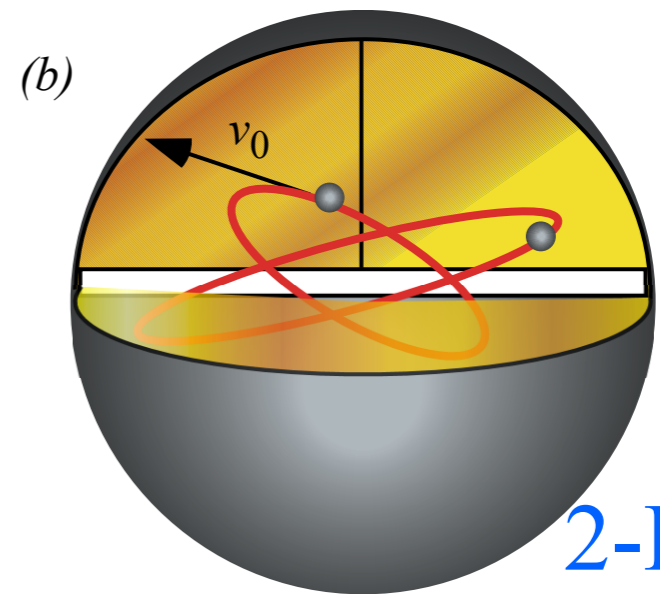
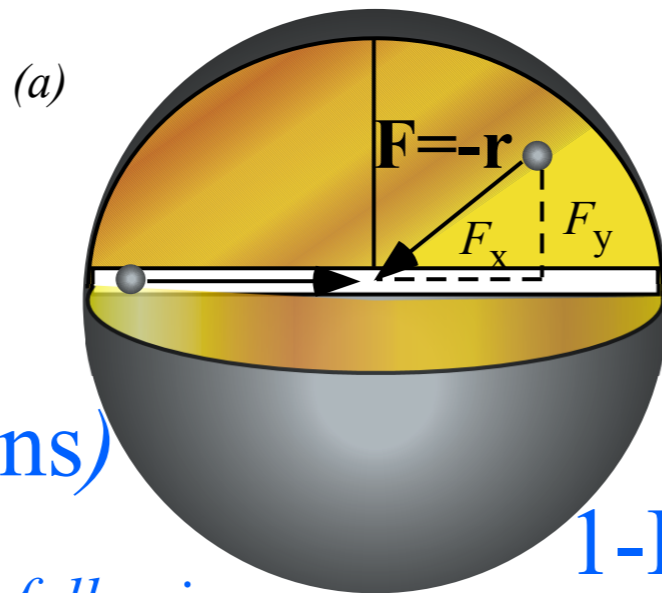
Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10

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2-D or 3-D
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$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{divide (1)}}{}}{\sqrt{\frac{2E}{k}} \cos\theta \stackrel{\text{by (2) derivative}}{}}} = \sqrt{\frac{k}{m}}$$

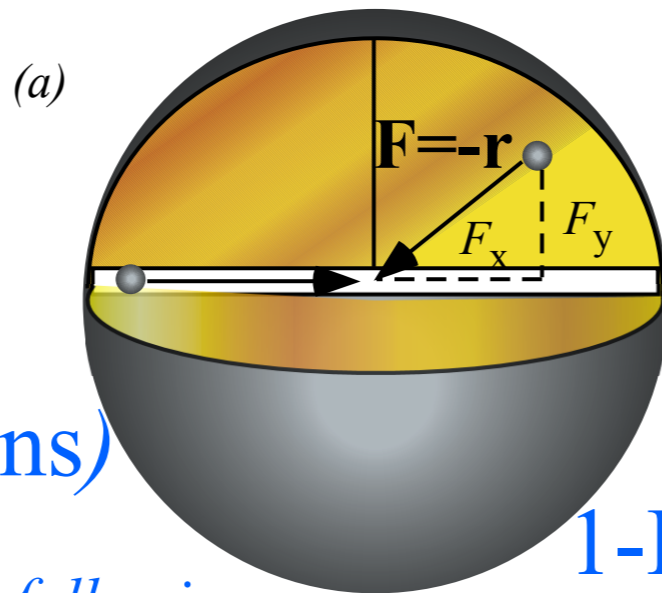
Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10

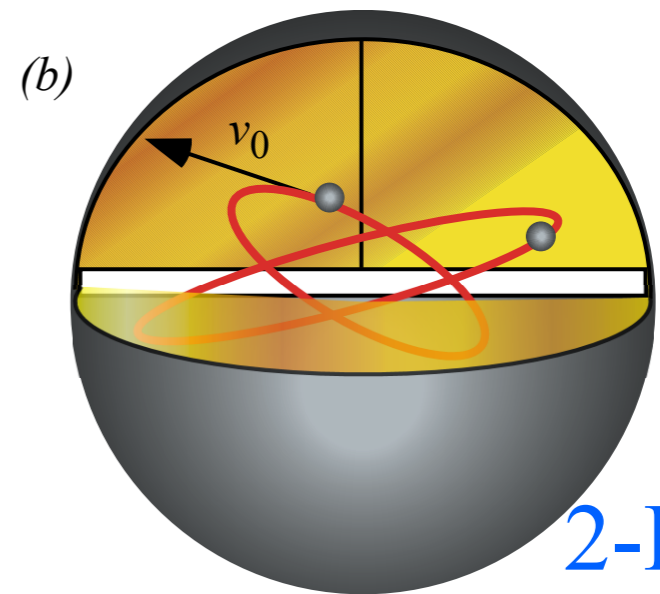
I.H.O. Force law

$F = -x$ (1-Dimension)

$F = -\mathbf{r}$ (2 or 3-Dimensions)



1-D



2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x , y , or z obeys the following:

$$\text{Total } E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const.}$$

Equations for x -motion

$[x(t)$ and $v_x=v(t)]$ are given first. They apply as well to dimensions

$[y(t)$ and $v_y=v(t)]$ and $[z(t)$ and $v_z=v(t)]$ in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}} \right)^2 + \left(\frac{x}{\sqrt{2E/k}} \right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old “scale-a-circle” trick...

Let : **(1)** $v = \sqrt{2E/m} \cos\theta$, and : **(2)** $x = \sqrt{2E/k} \sin\theta$ def. **(3)** $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{by (1)}}{=} v = \frac{dx}{dt} \stackrel{\text{by def. (3)}}{=} \frac{d\theta}{dt} \frac{dx}{d\theta} \stackrel{\text{by (2)}}{=} \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos\theta$$

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

by integration given constant ω :

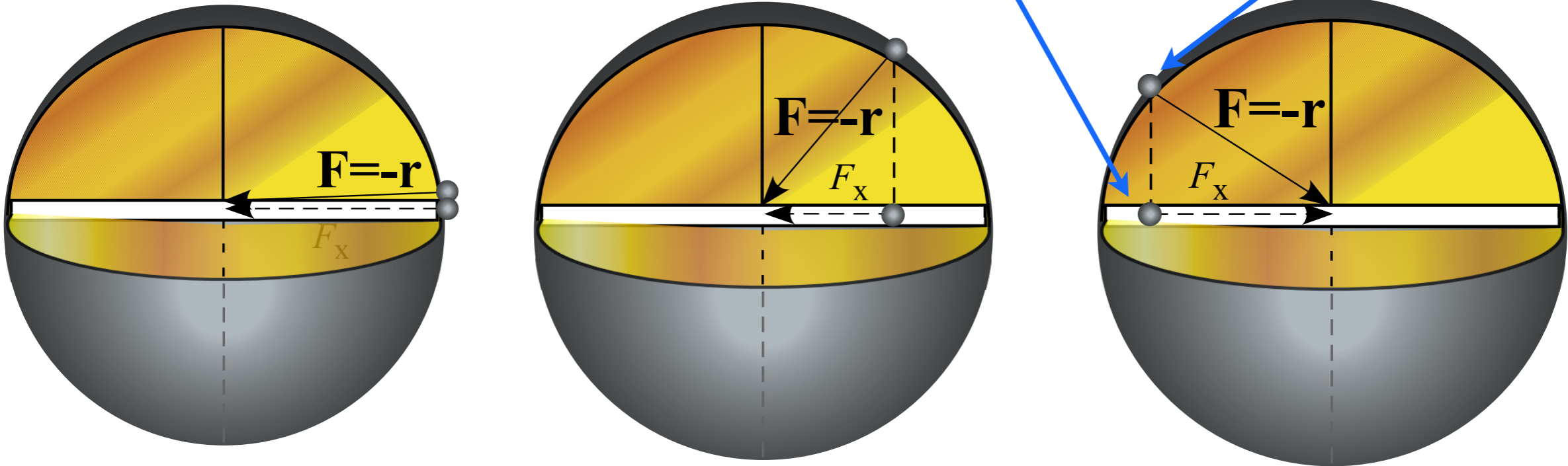
$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$



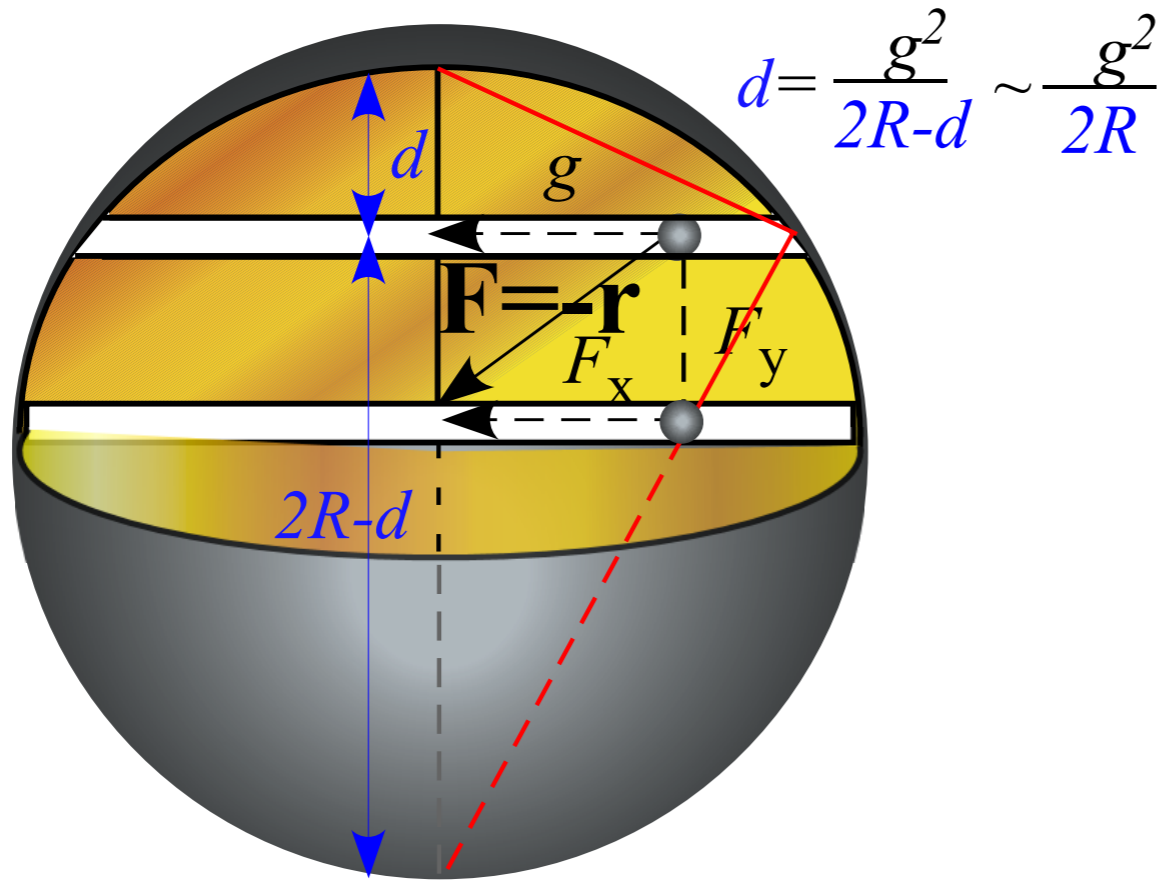
Introducing 2D IHO orbits and phasor geometry

Phasor “clock” geometry

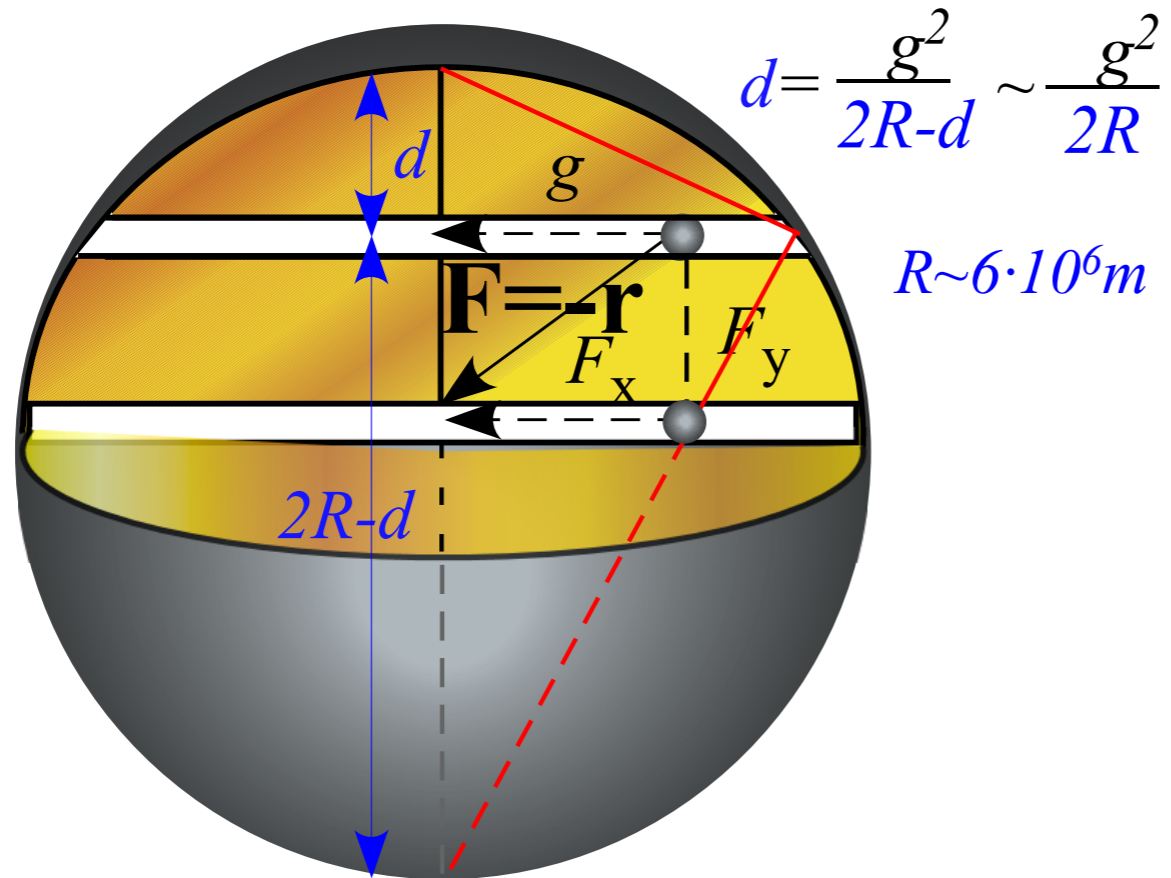
Isotropic Harmonic Oscillator makes tunneling ball track orbiting ball



Isotropic Harmonic Oscillator makes balls in parallel tunnel track each other



Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...

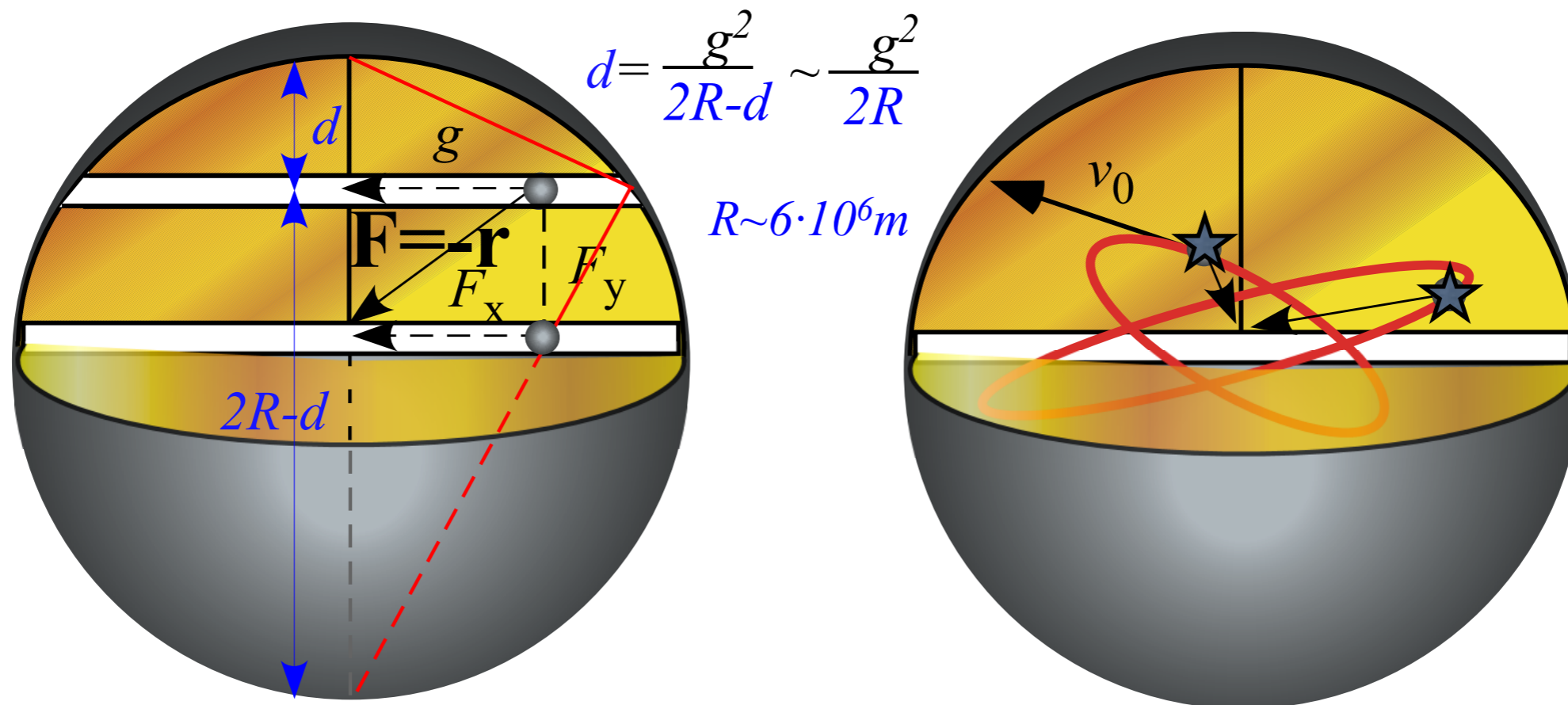


$$d \sim \frac{1}{2R}$$

...even if track length is just $g = 1m$ so $d \sim (1/12)$ micron

They all take about 84 minutes to go from right to left and back, again.

Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...



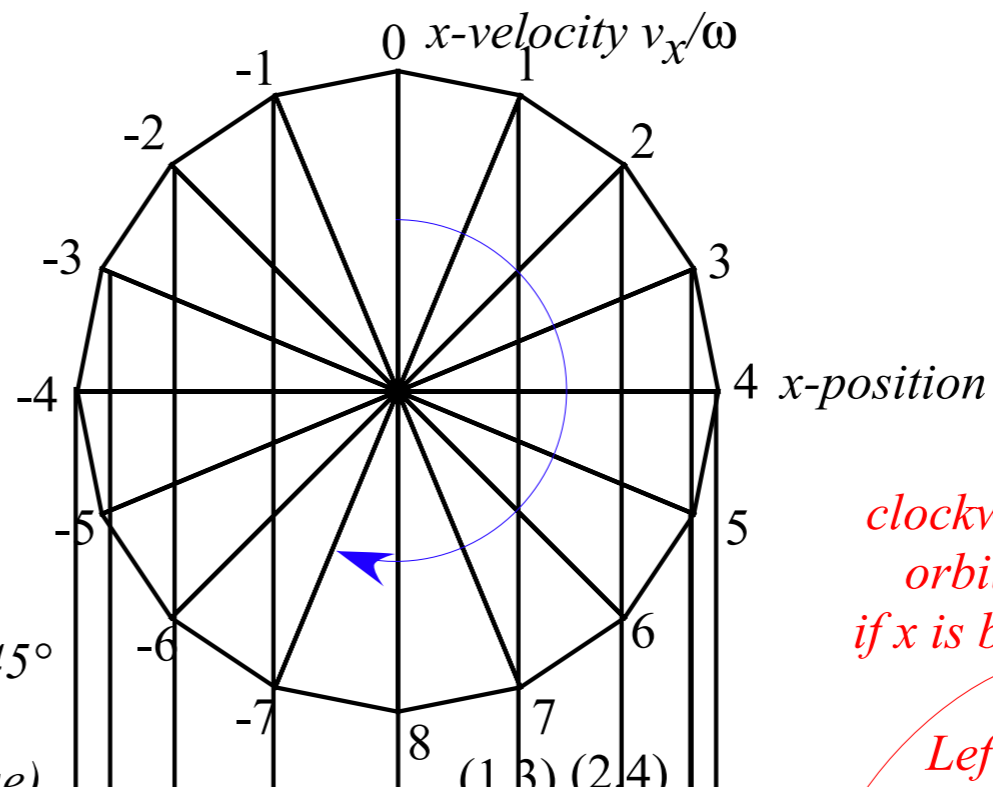
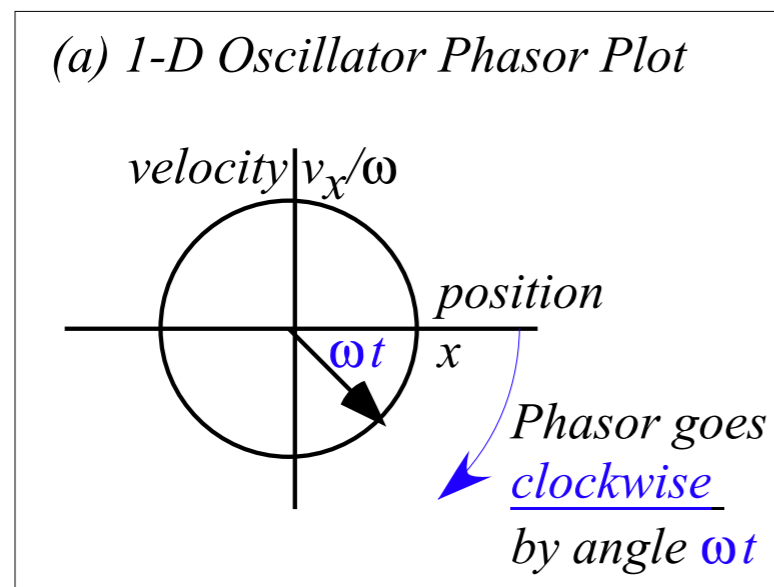
...even if track length is just $g = 1m$ so $d = (1/12)micron$

The all take about 84 minutes to go from right to left and back, again.

Most neutron starlet (★) orbits are centered ellipses

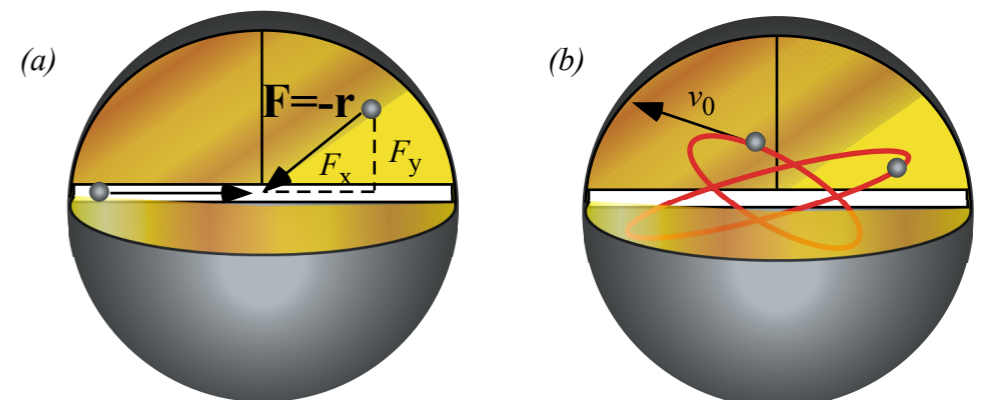
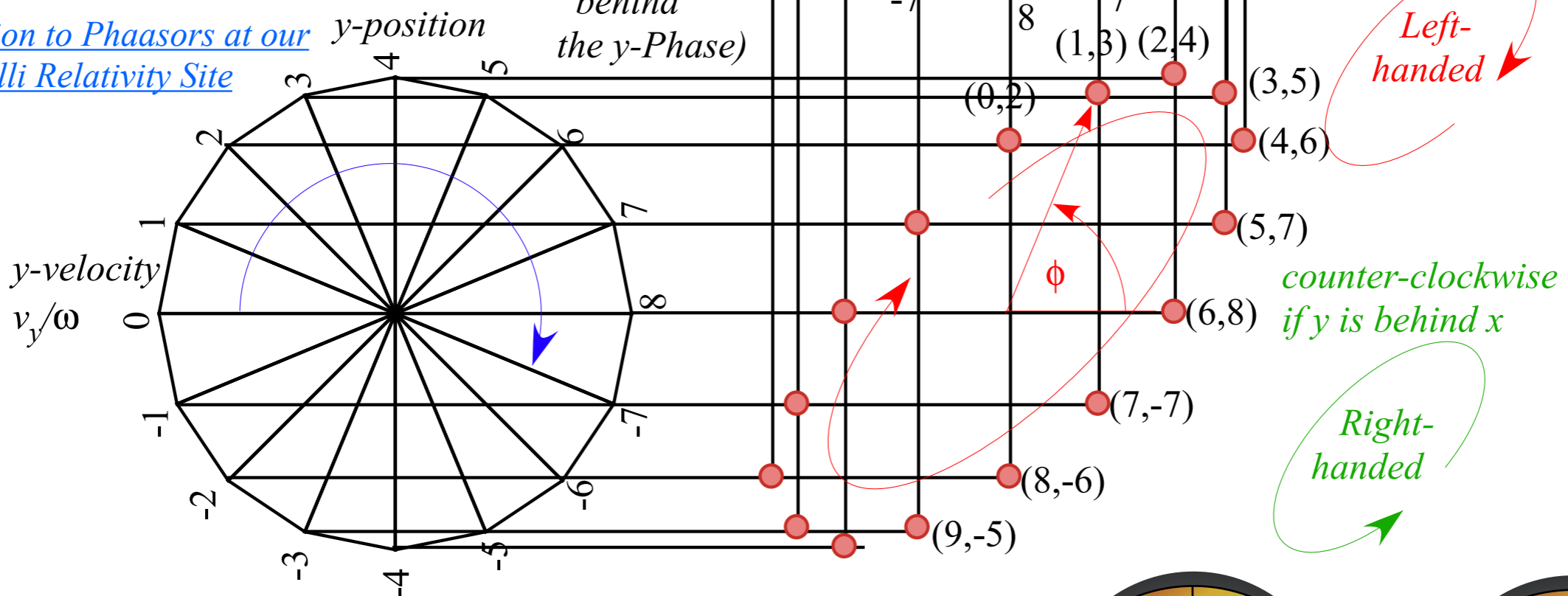
Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



(b) 2-D Oscillator Phasor Plot

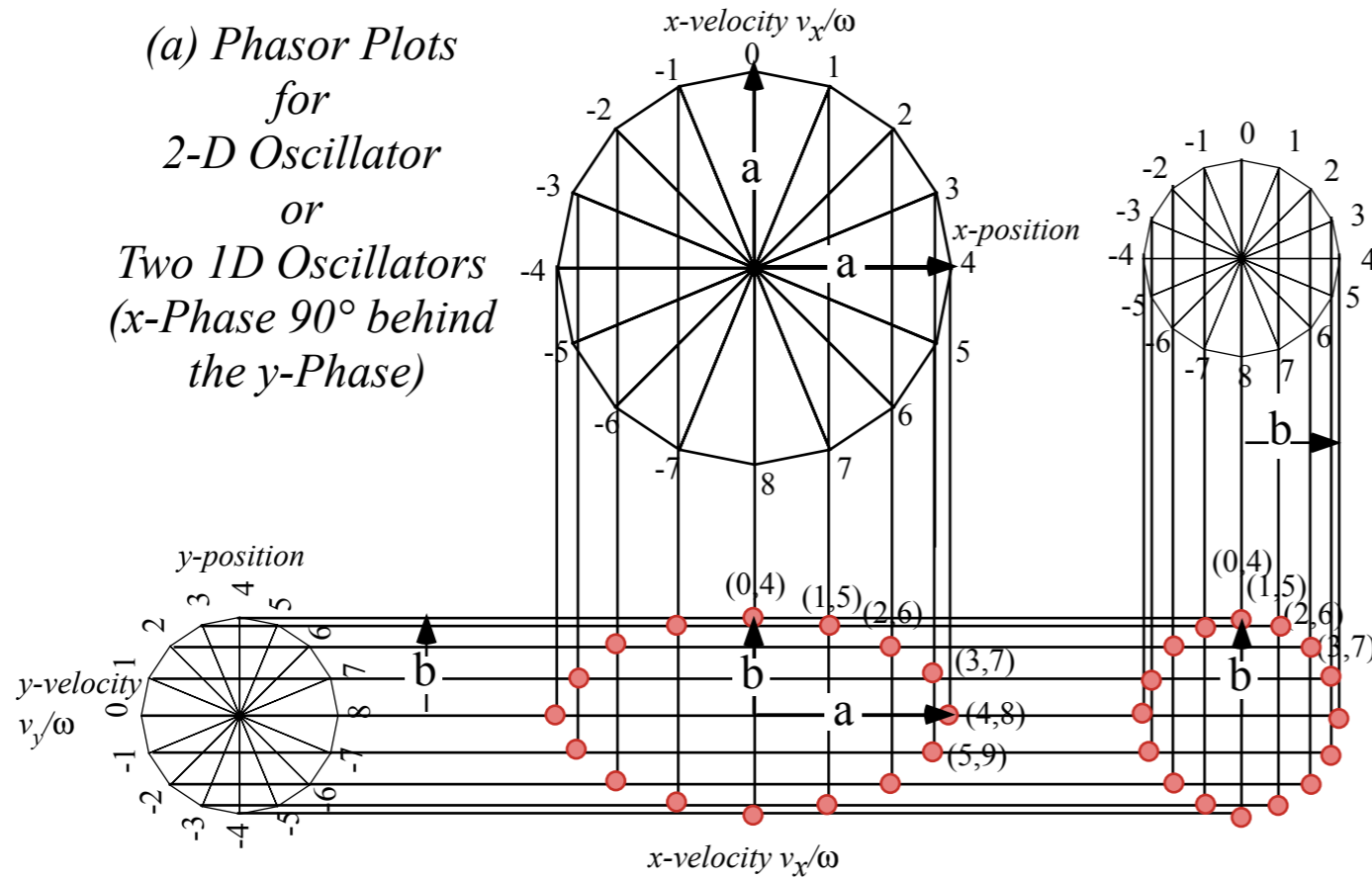
[Introduction to Phasors at our Pirelli Relativity Site](#)



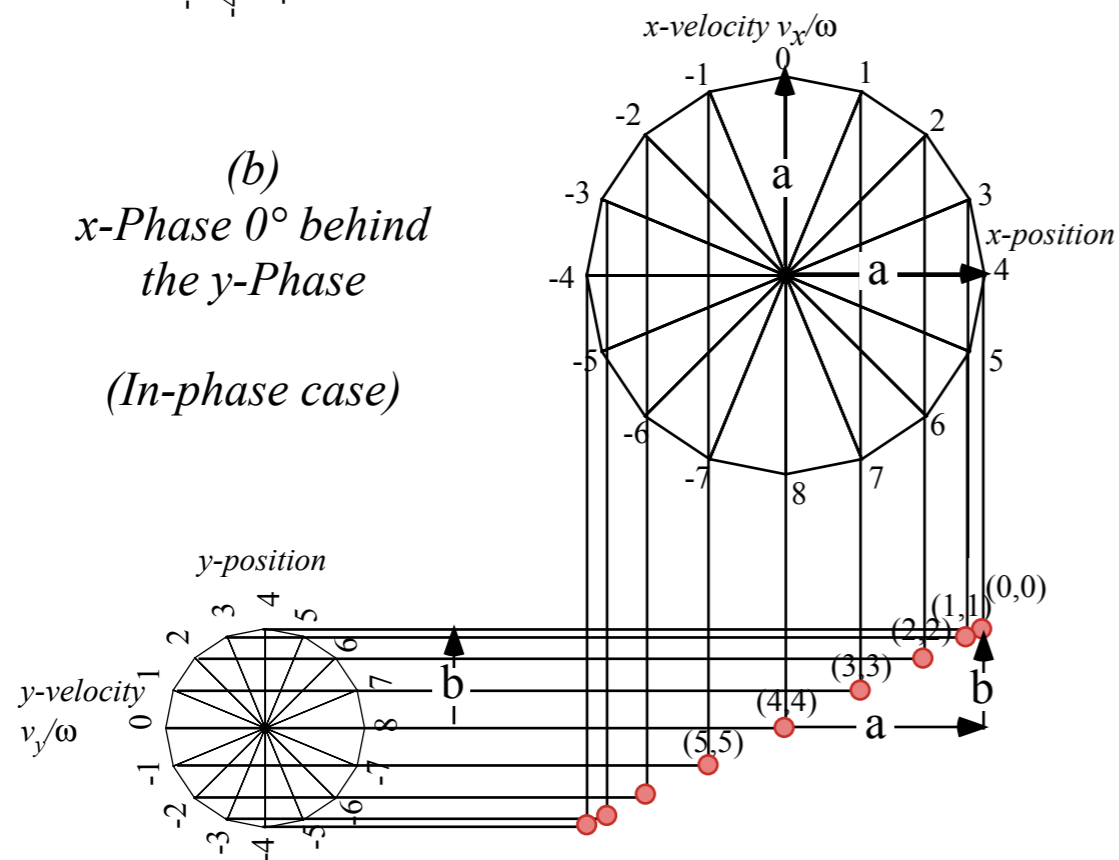
[BoxIt web simulation - With \$y\$ -Phasor is on other side of \$xy\$ plot](#)

[RelaWavity web simulation - Contact ellipsometry](#)

(a) Phasor Plots
for
2-D Oscillator
or
Two 1D Oscillators
(x -Phase 90° behind
the y -Phase)



(b)
 x -Phase 0° behind
the y -Phase
(In-phase case)



*These are more generic examples
with radius of x -phasor differing
from that of the y -phasor.*