Lecture 4
Thur. 8.31.2017

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ collision dynamics *High mass ratio M_1/m_2 =49*

Force "field" or "pressure" due to many small bounces

- *Force defined as momentum transfer rate*
- *The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3*

Potential field due to many small bounces

- *Example of 1D-Adiabatic potential $U(y)$=const./y^2*
  - *Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$*
- *Example of 1D-Isothermal potential $U(y)$=const. ln(y)*

"Monster Mash" classical segue to Heisenberg action relations

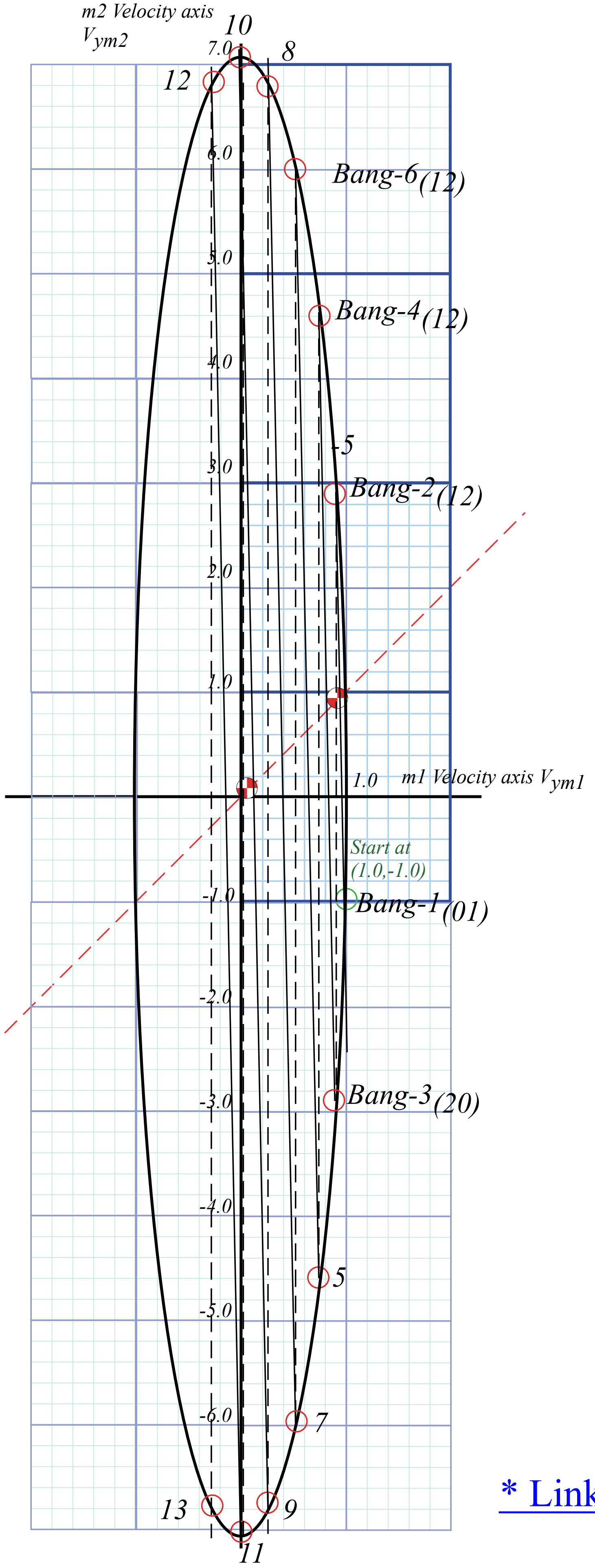
- *Example of very very large M_1 ball-wall(s) crushing a poor little m_2*
  - *How m_2 keeps its action*
- *An interesting wave analogy: The "Tiny-Big-Bang"* [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]
- *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015)]

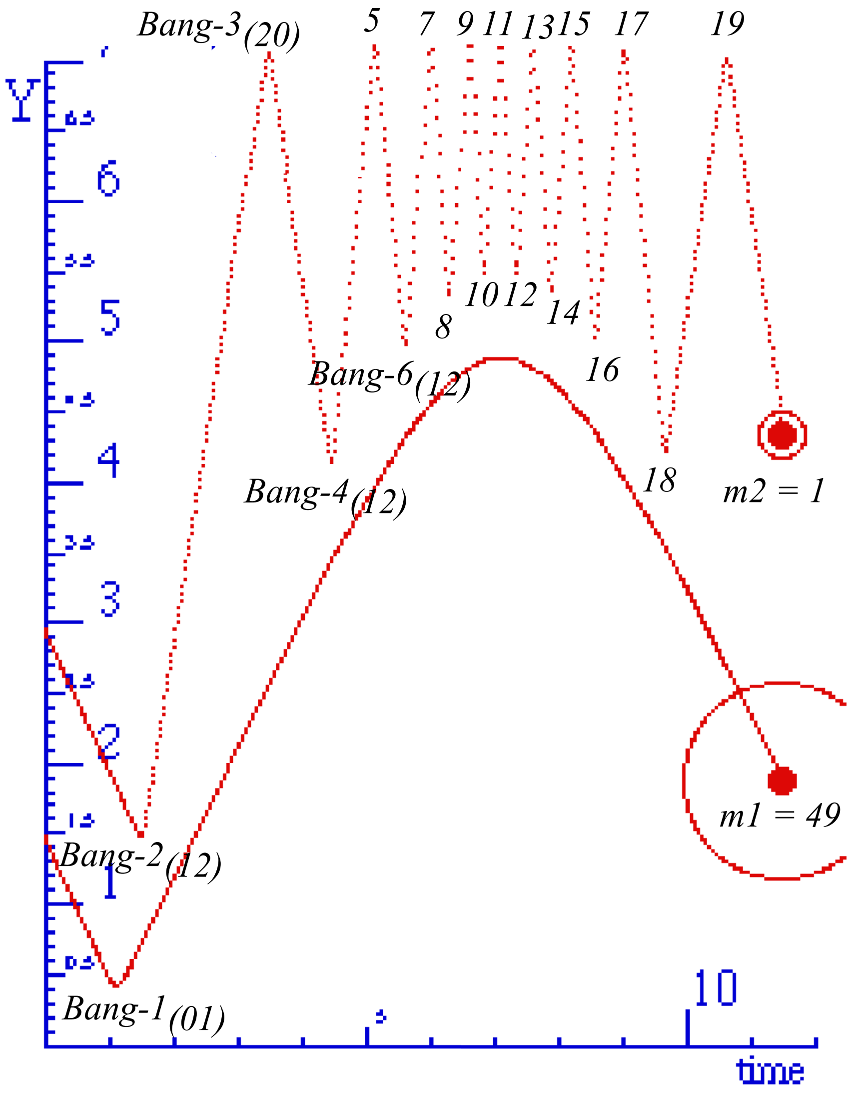
Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

→ High mass ratio $M_1/m_2 = 49$

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: $m_1=49$ and $m_2=1$



Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

* Link to BounceIt: $Y_i(t)$ animation

Fig. 5.1
in Unit 1

* Link to BounceIt: V_{y2} vs V_{y1} animation

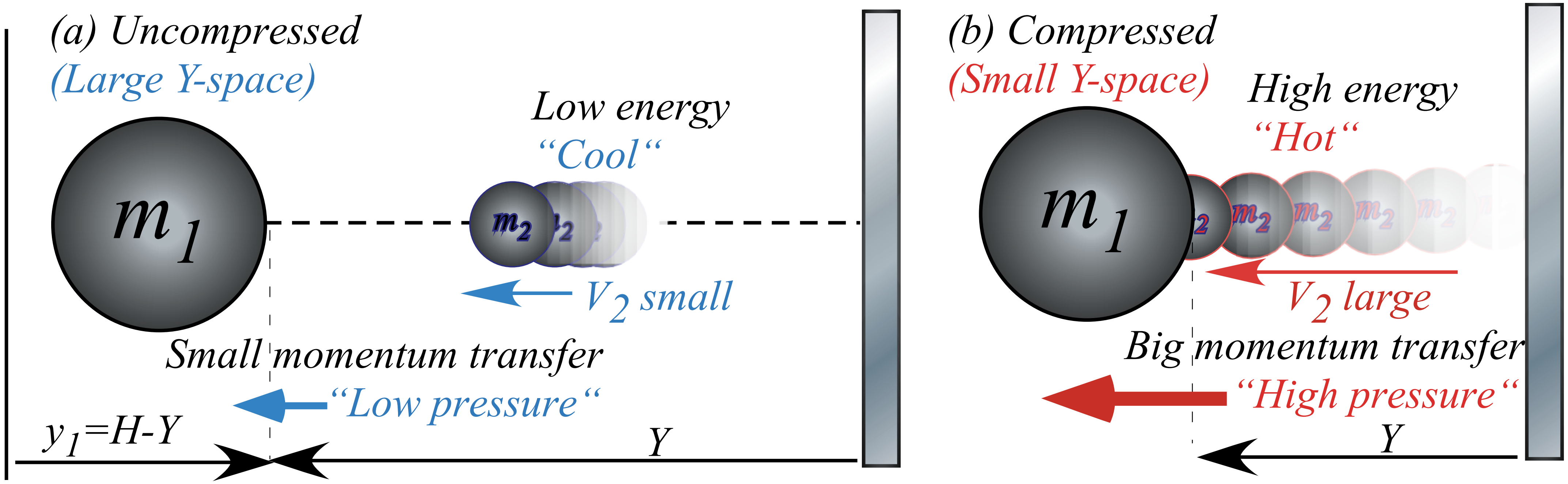
Force "field" or "pressure" due to many small bounces

→ *Force defined as momentum transfer rate*

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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Unit 1
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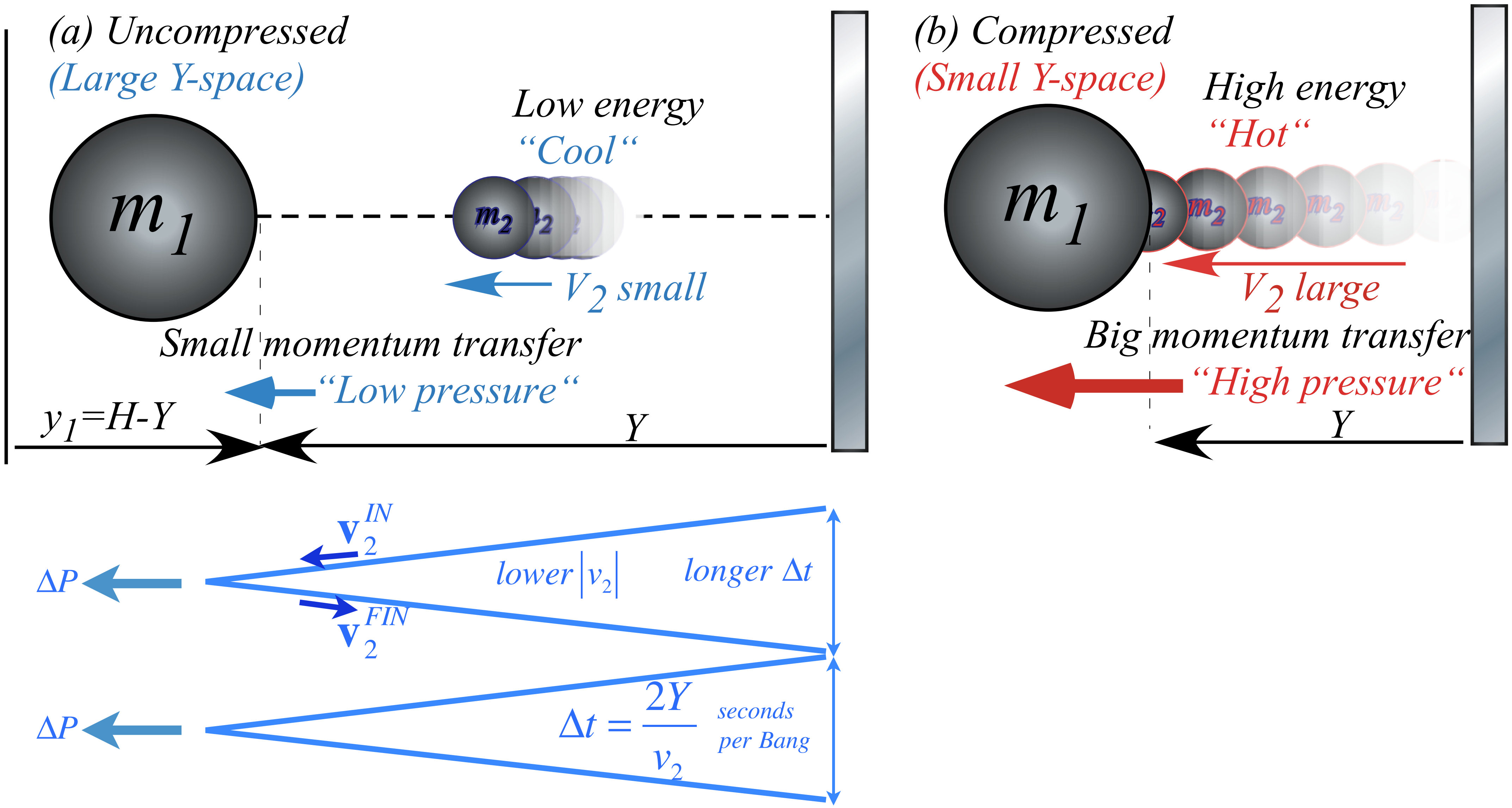

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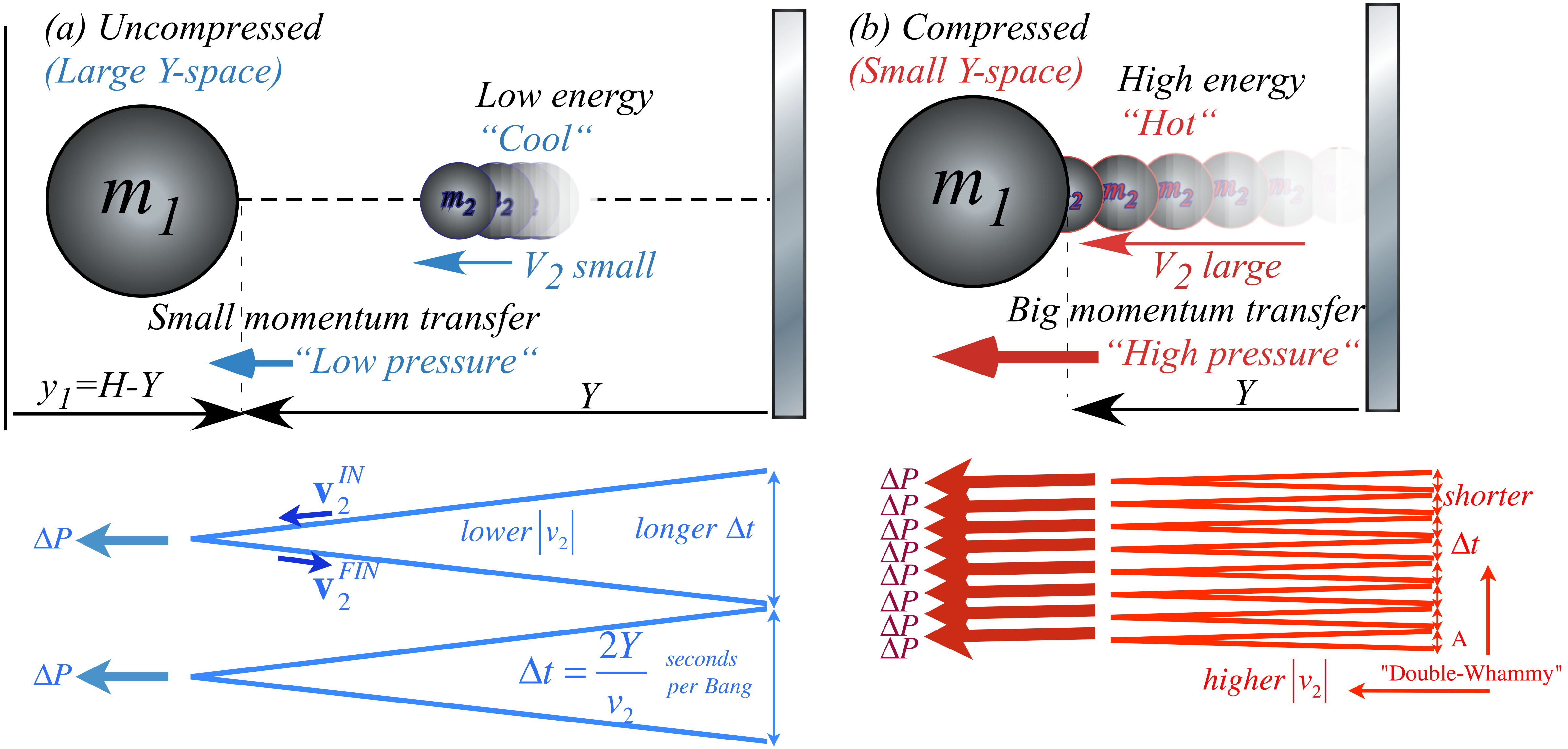

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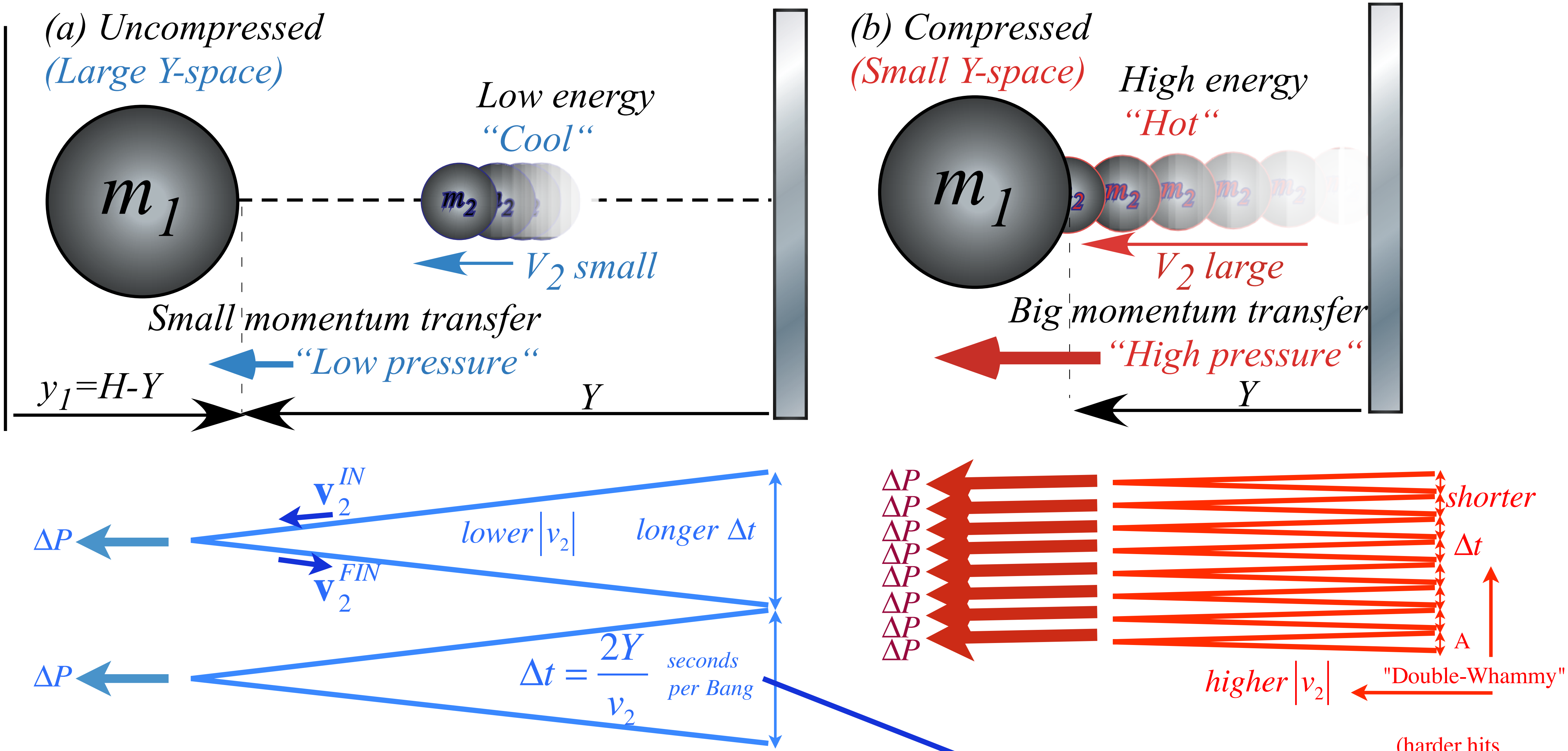


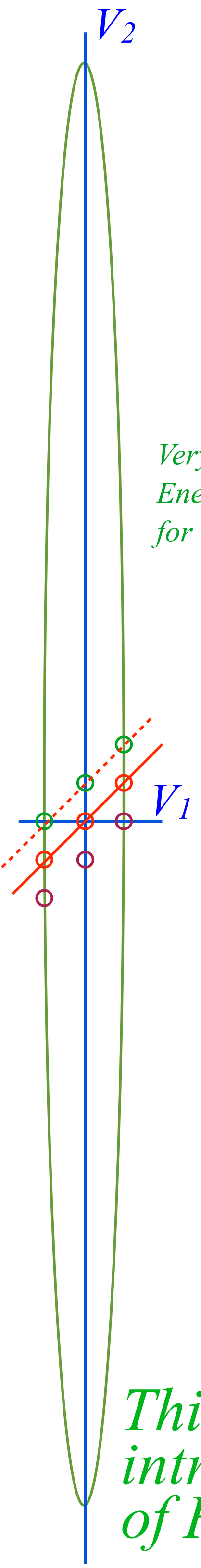
(harder hits
and
more hits/sec.)

This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

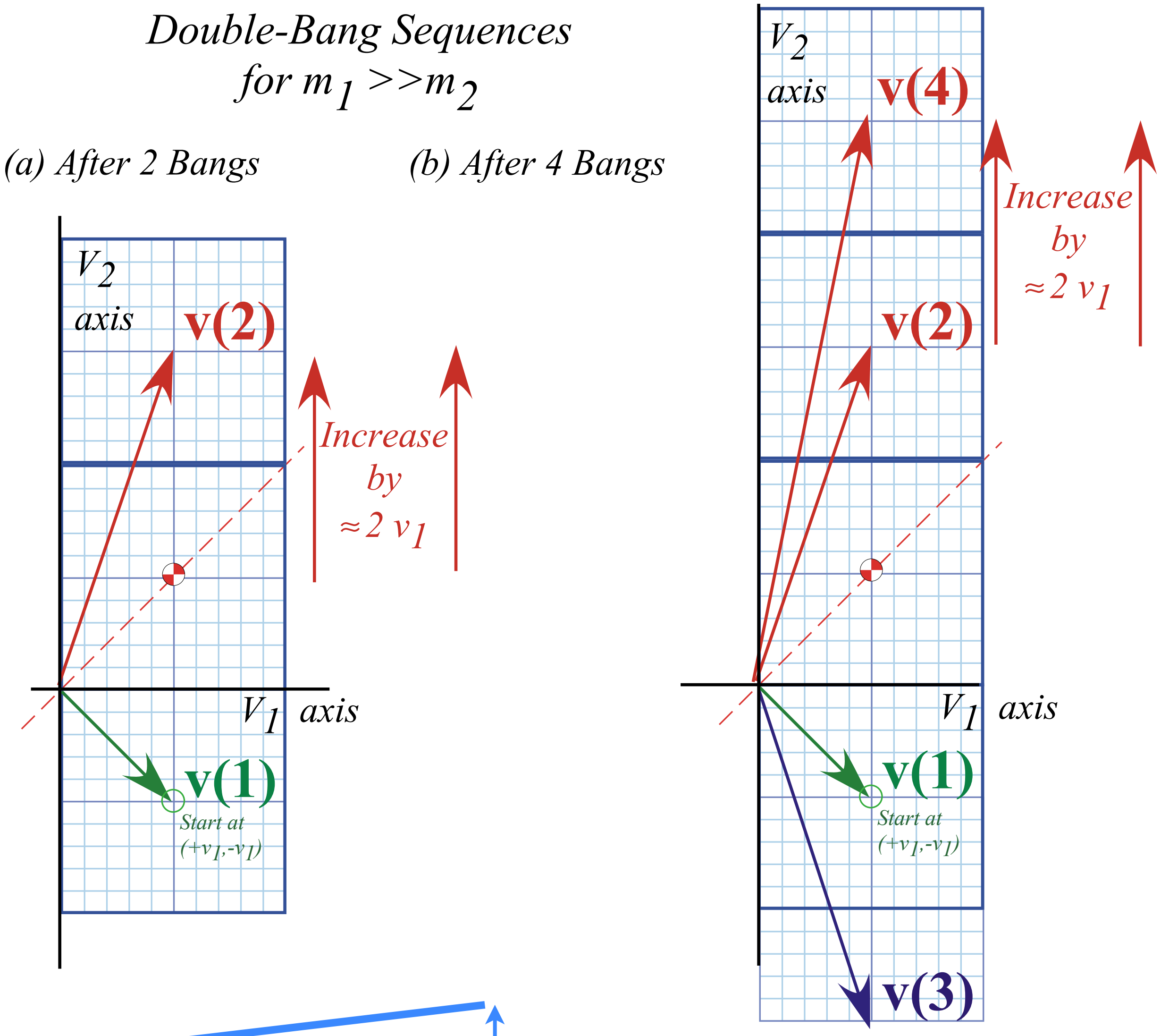
Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

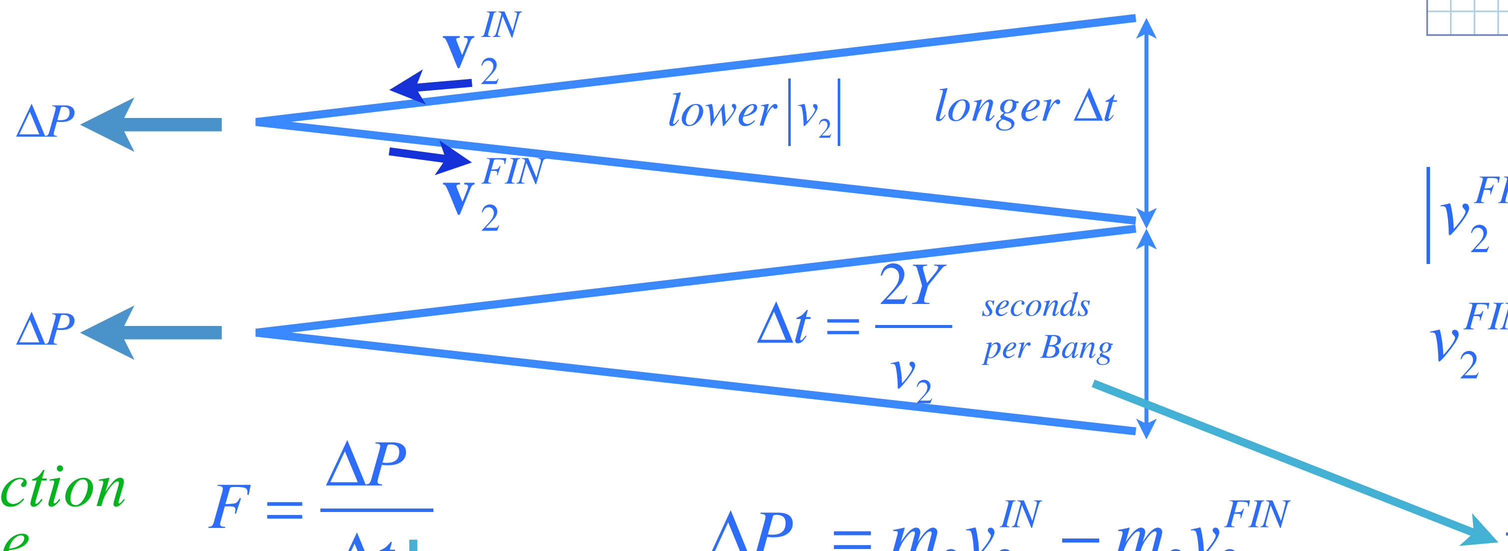


Very skinny Energy ellipse for $m_1>>m_2$

Double-Bang Sequences for $m_1>>m_2$


(a) After 2 Bangs (b) After 4 Bangs

Unit 1
Fig. 6.2



$$\Delta t = \frac{2Y}{v_2}$$ seconds per Bang

$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

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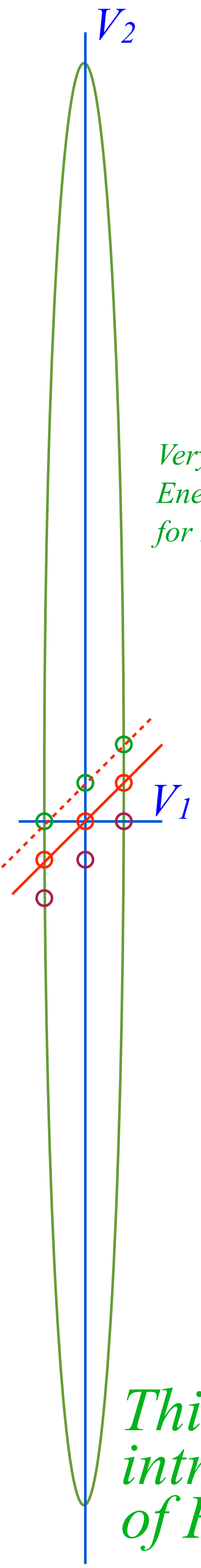
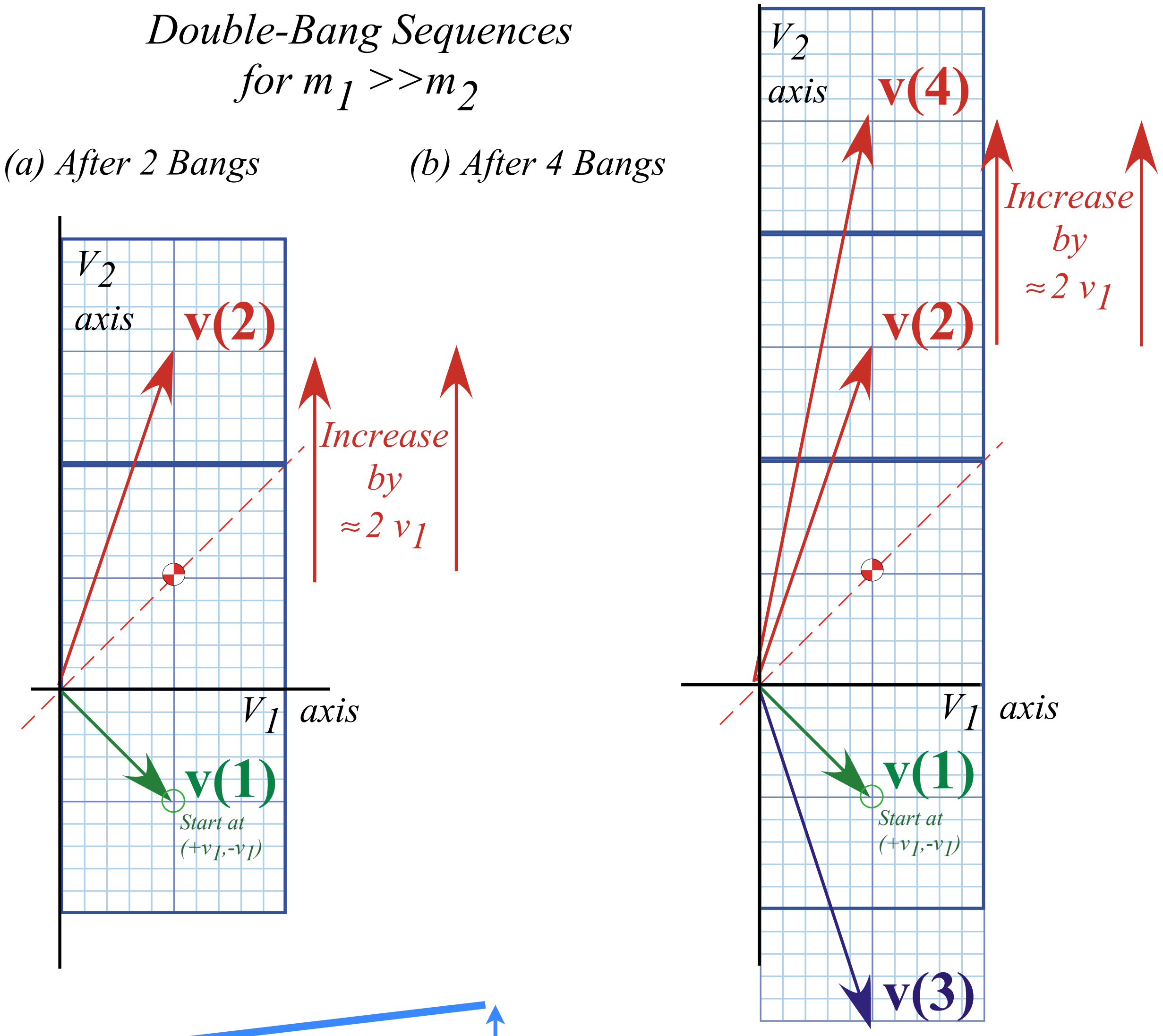
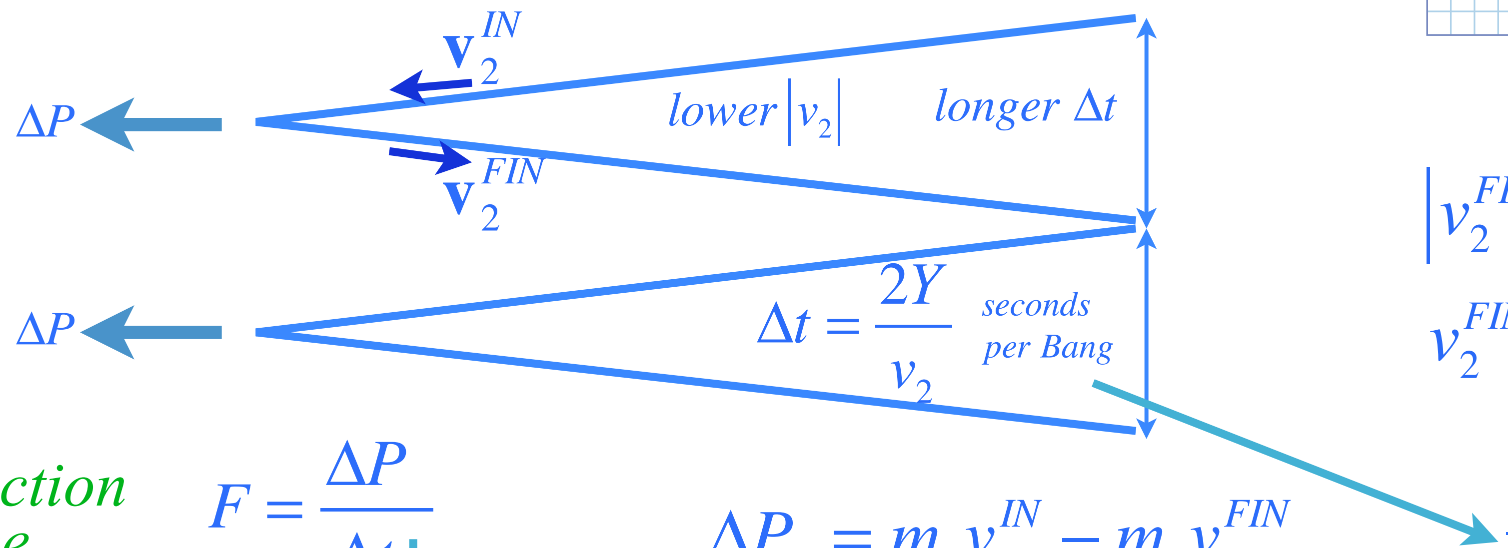


Double-Bang Sequences
for $m_1 >> m_2$



Unit 1
Fig. 6.2



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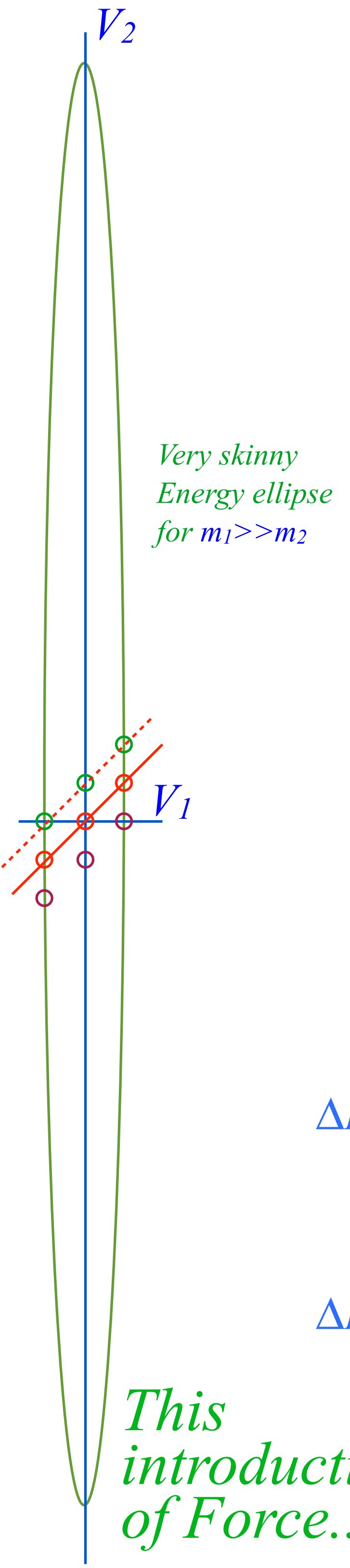
Force F on m₁ = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2 \left(-v_2^{IN} - 2v_1\right) = 2m_2 v_2^{IN} + 2m_2 v_1$$

...is more of a *definition* *than another* *axiom*



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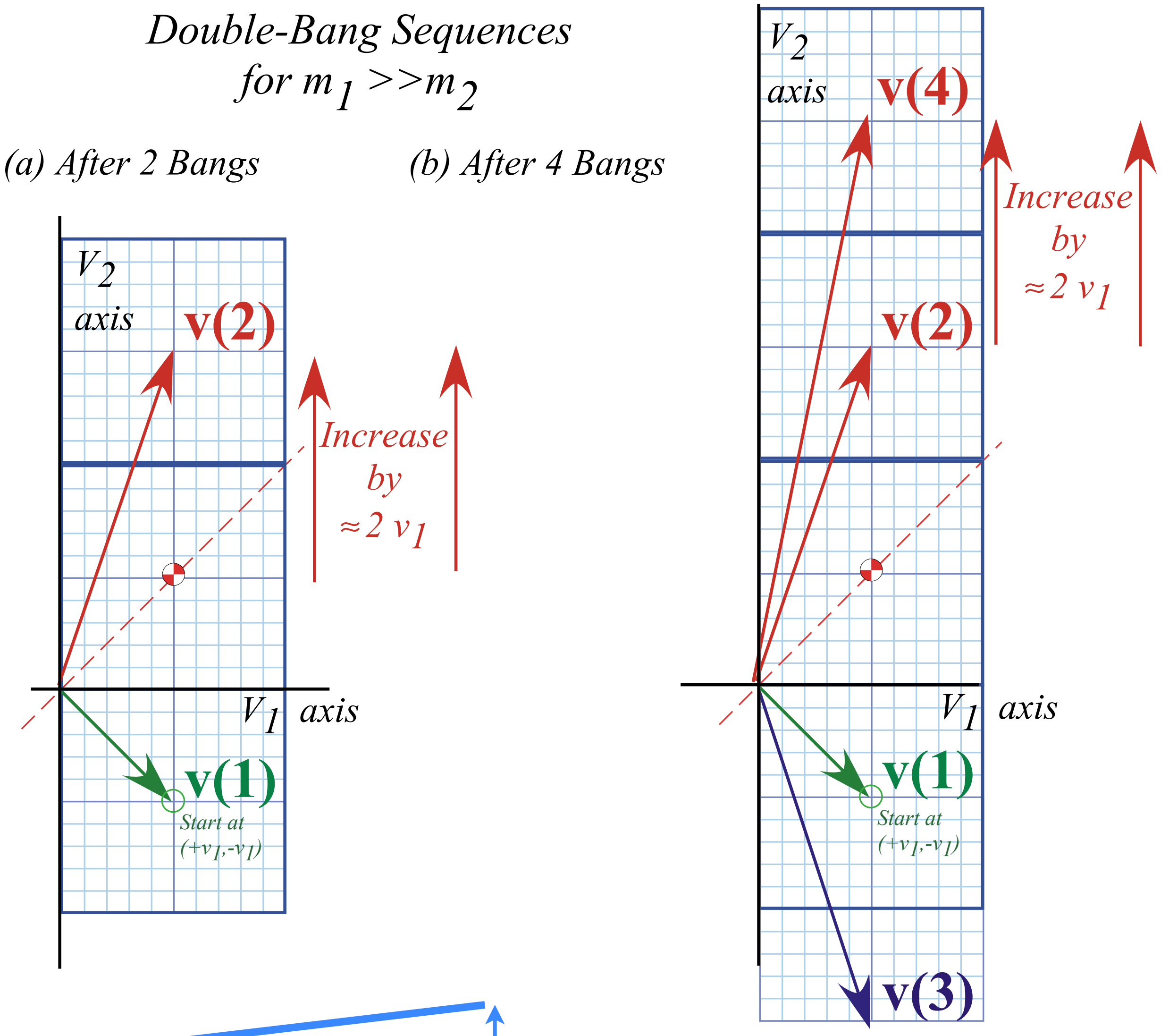
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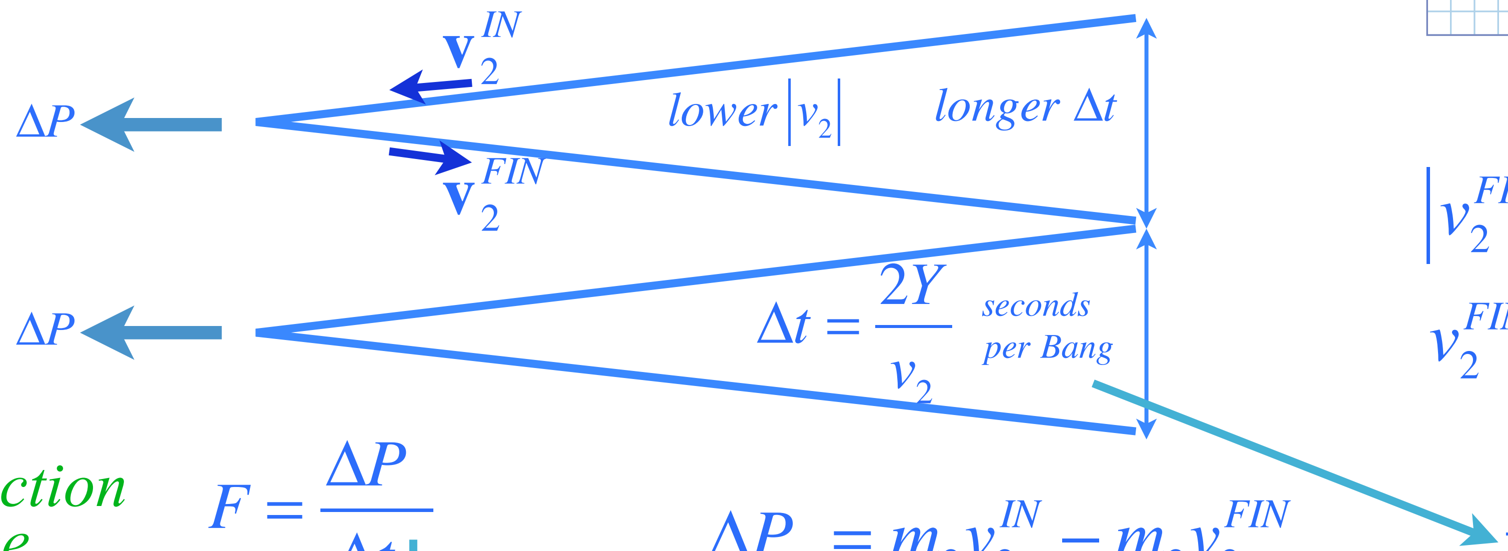

$$\Delta t = \frac{2Y}{v_2}$$

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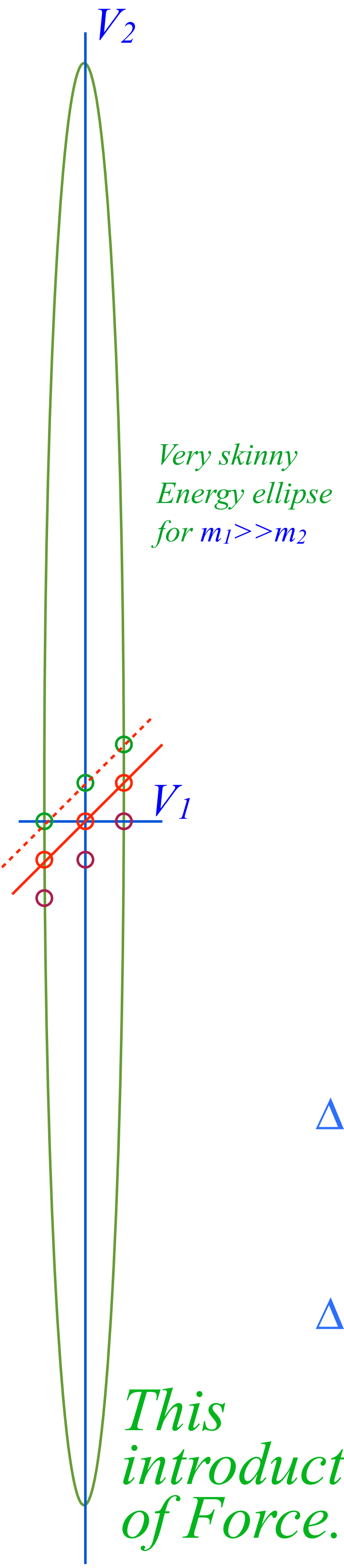
Force F on m₁ = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

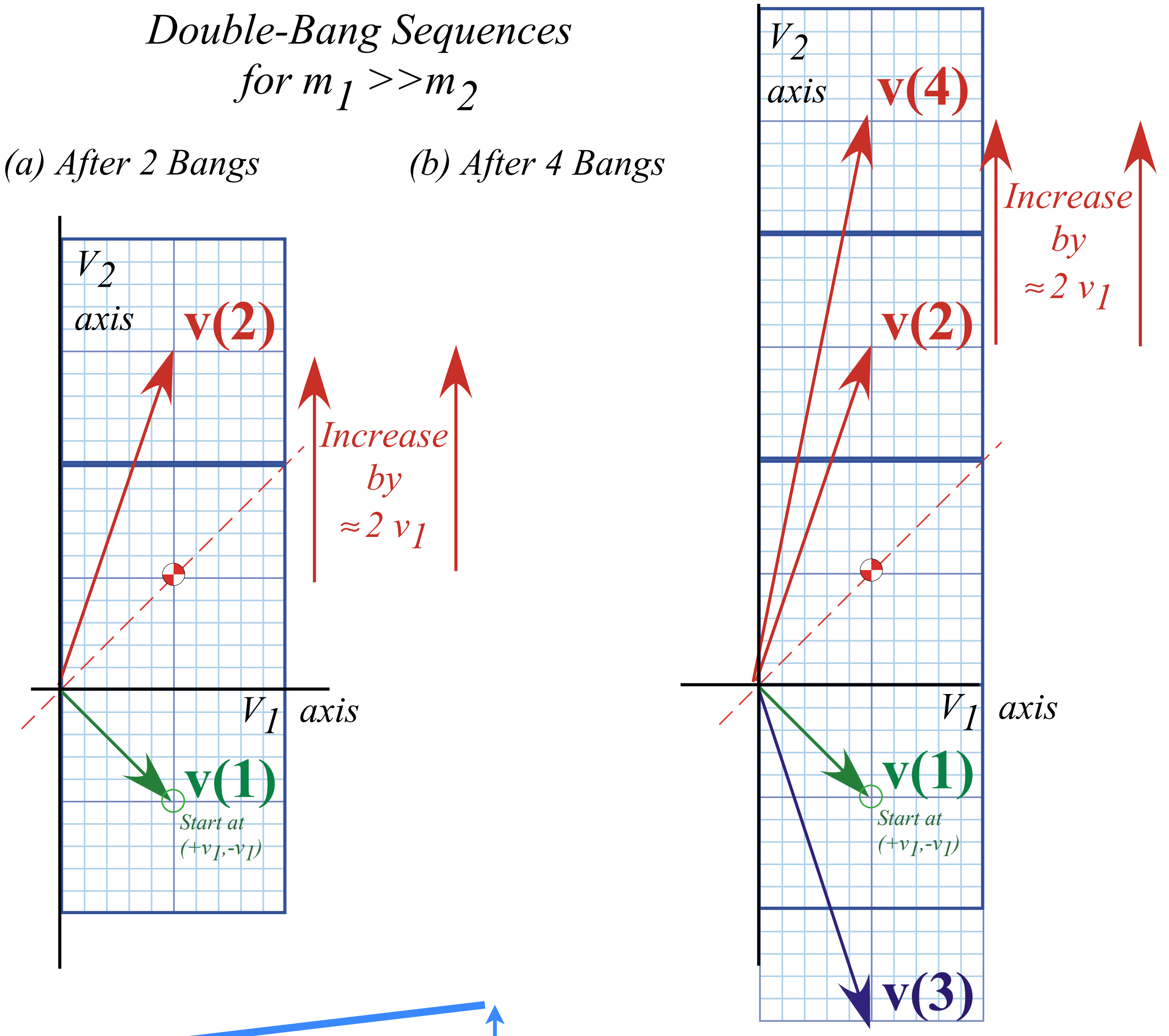
$$\Delta P = m_2 v_2^{IN} - m_2 (-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

...is more of a definition than another axiom

Assuming slow m_1 : $v_1 \ll v_2$



Double-Bang Sequences for $m_1>>m_2$



Unit 1
Fig. 6.2

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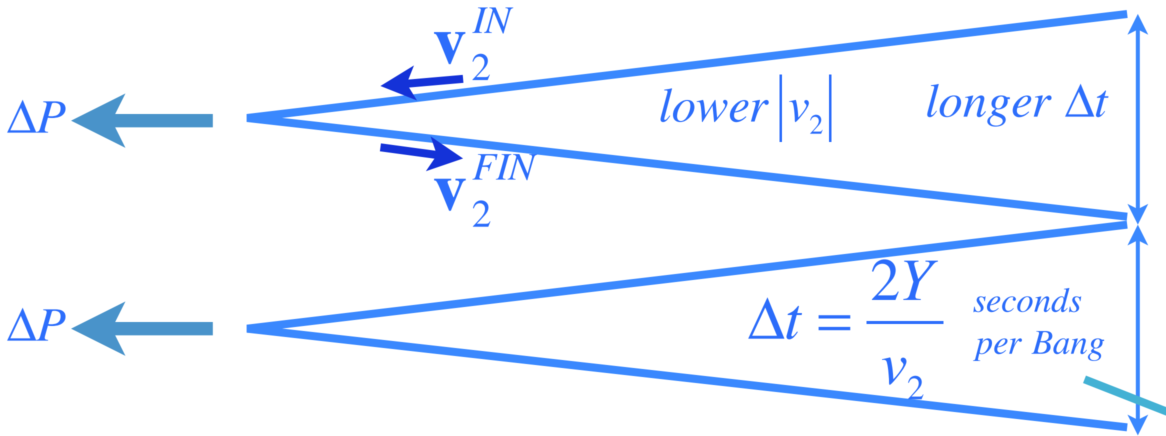

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$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Assuming slow m_1 : $v_1 \ll v_2$

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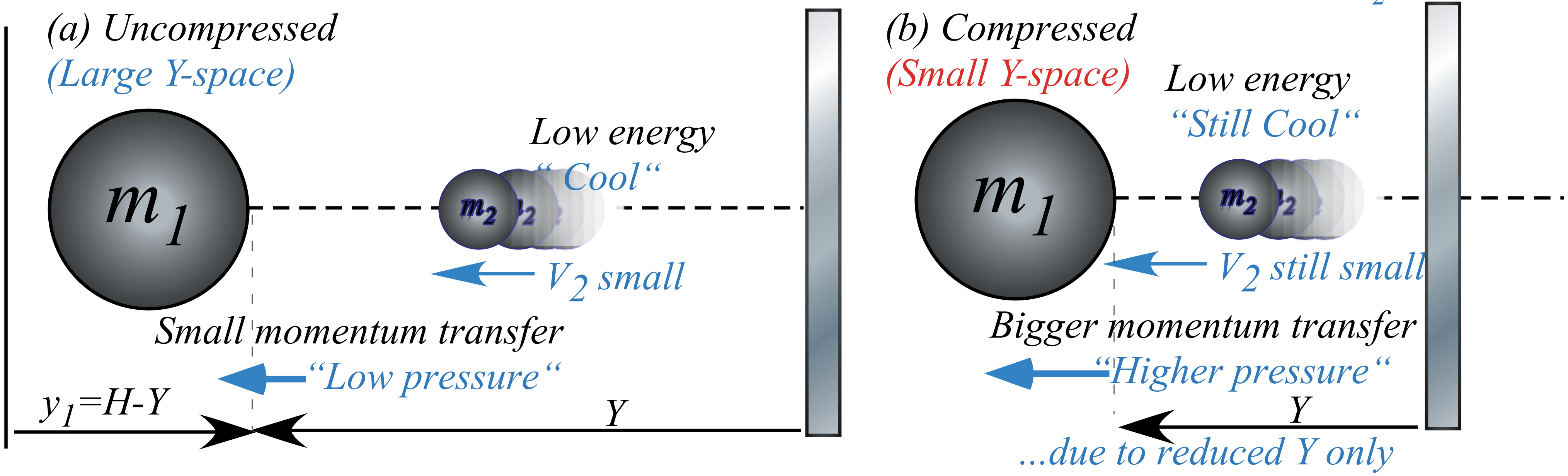
Not a "Double-Whammy"...

...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

Isothermal expansion or contraction: Wall serves as thermal bath to keep m_2 cool



(a) Uncompressed
(Large Y-space)

m_1

m_2

Low energy
"Cool"

V_2 small

Small momentum transfer
"Low pressure"

y_1=H-Y

Y

(b) Compressed
(Small Y-space)

m_1

m_2

Low energy
"Still Cool"

V_2 still small

Bigger momentum transfer
"Higher pressure"

Y

...due to reduced Y only

Force “field” or “pressure” due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ *and the 1D-Adiabatic force field* $F(y)=const./y^3$

$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

Not a "Double-Whammy"...
...only a "Single-Whammy"

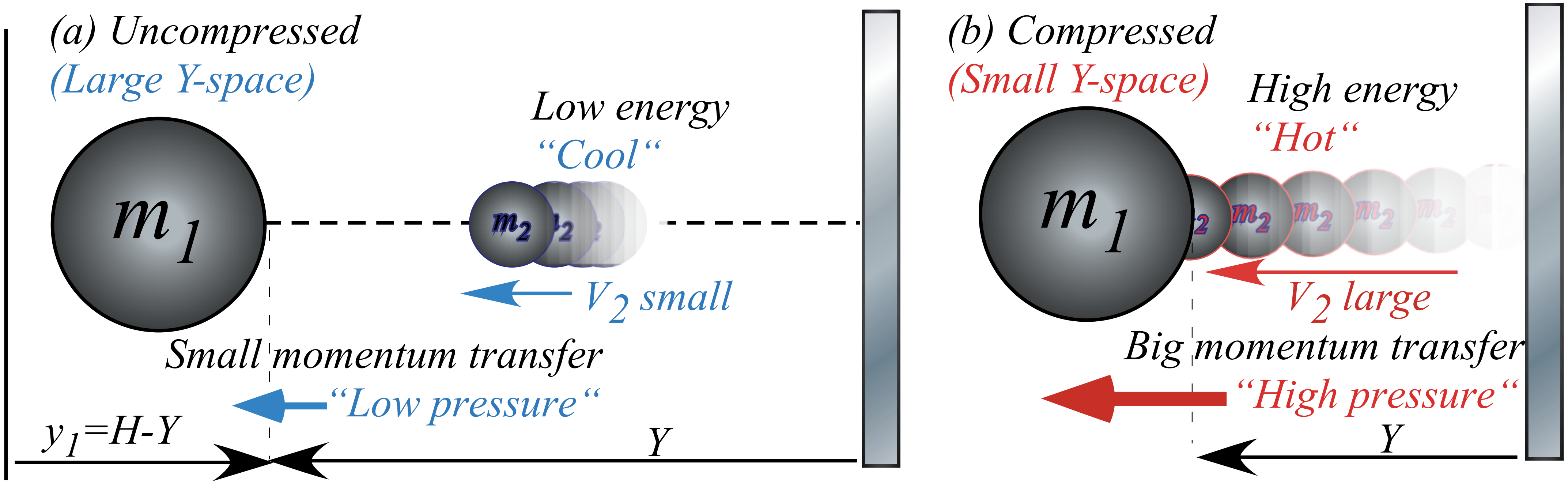
1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y}$$

Wall not given time to give or take KE



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simplifies to:

$$\frac{dv_2}{v_2}=-\frac{dY}{Y}$$

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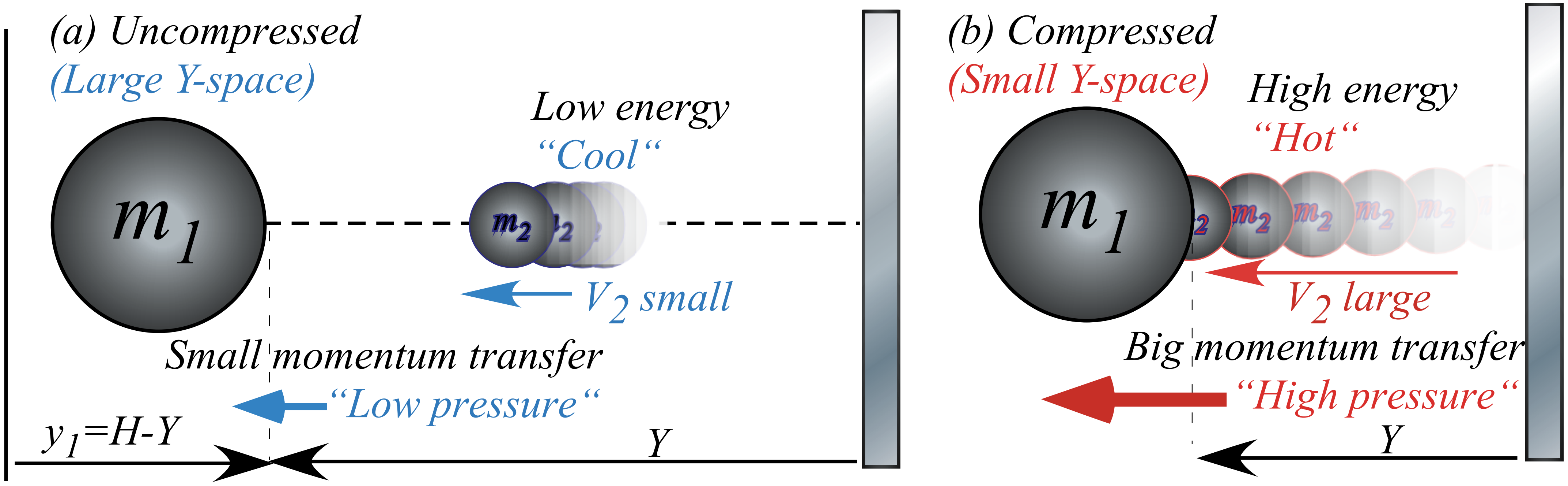

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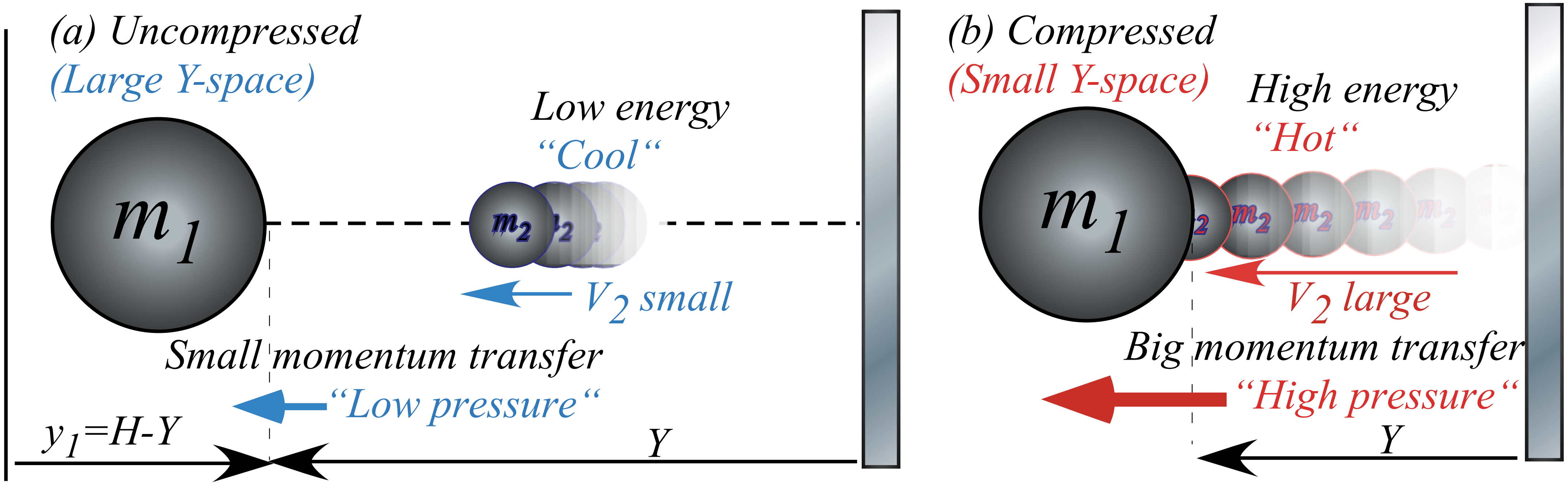
Differential equation results and has logarithmic integral. $\int\frac{dx}{x}=\ln x+C=\log_e x+\log_e e^C=\log_e(e^Cx)$

$$\frac{dv_2}{v_2}=-\frac{dY}{Y}\quad\text{integrates to: }\ln v_2=-\ln Y+C\quad\text{or:}\quad\ln v_2=\ln\frac{const.}{Y}\quad\text{or:}\quad v_2=\frac{const.}{Y}$$

Wall not given time to give or take KE



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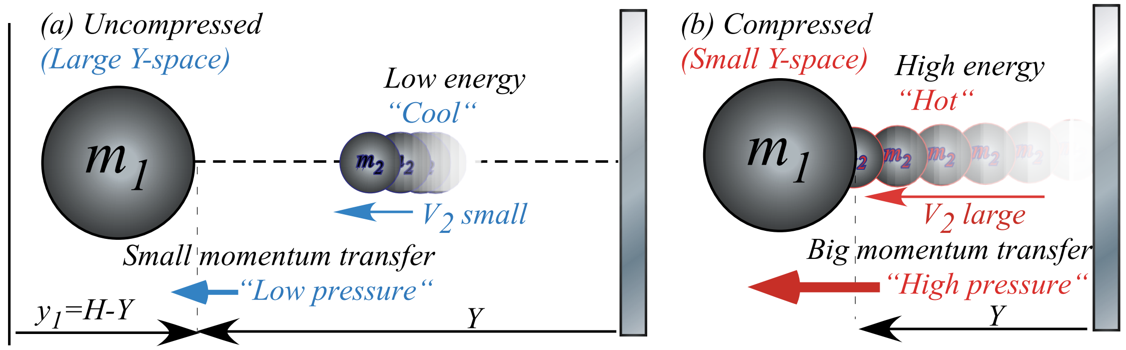
$$\frac{dv_2}{v_2}=-\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2=-\ln Y+C \quad \text{or:} \quad \ln v_2=\ln\frac{const.}{Y} \quad \text{or:} \quad v_2=\frac{const.}{Y}$$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2=\frac{const.}{Y}=\frac{v_2^{IN}Y(t=0)}{Y}$ *):* $\boxed{F=\frac{m_2\left(v_2^{IN}Y(t=0)\right)^2}{Y^3}=\frac{const.}{Y^3}}$



A
"Double-Whammy"

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ \ln(y)$

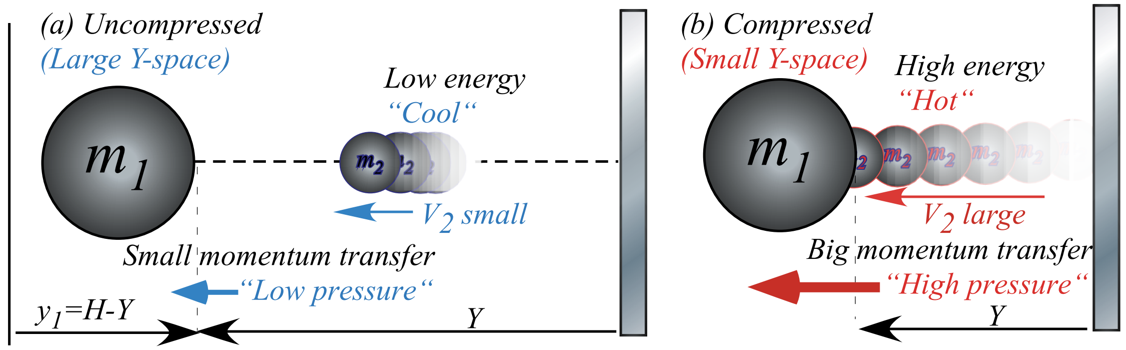
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



A
"Double-Whammy"
system

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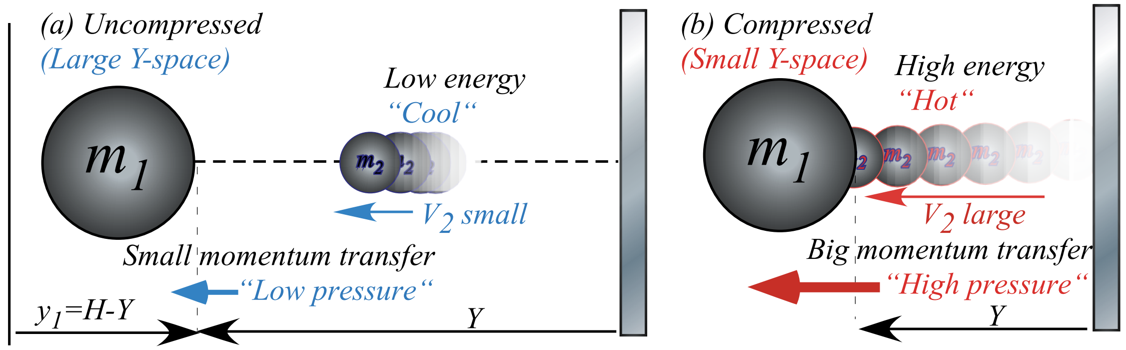
Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations F·dY=±dU*



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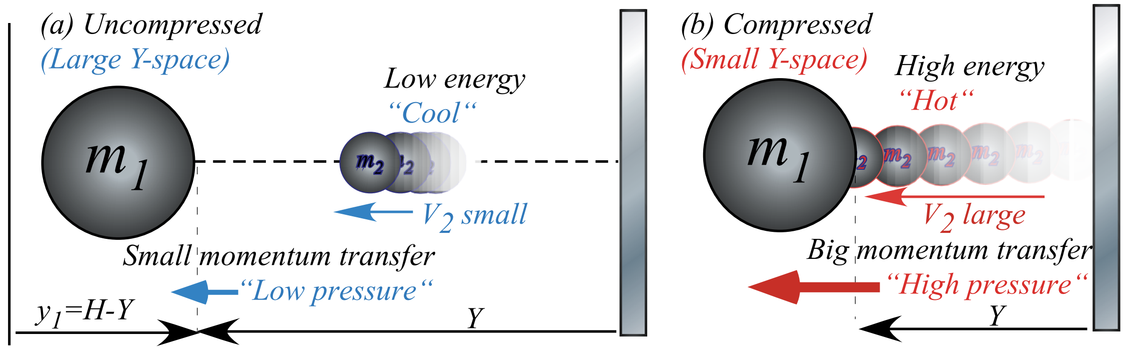
Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?*



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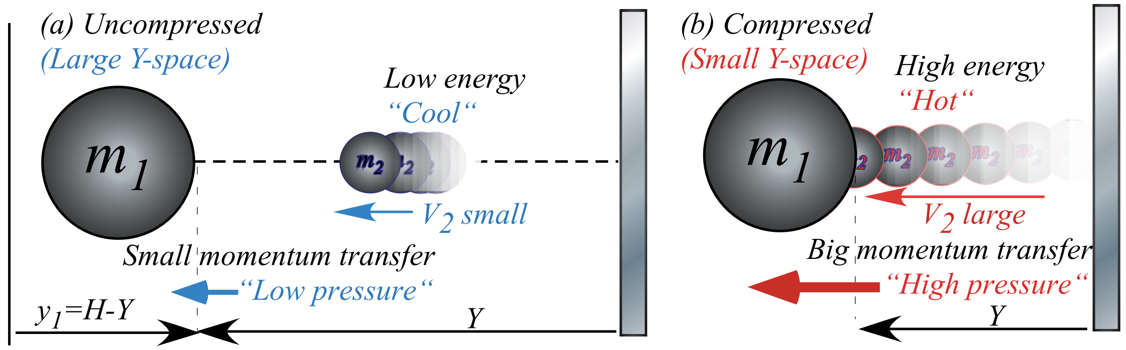
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \boxed{\frac{1}{2}m_1v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.



A
"Double-Whammy"
system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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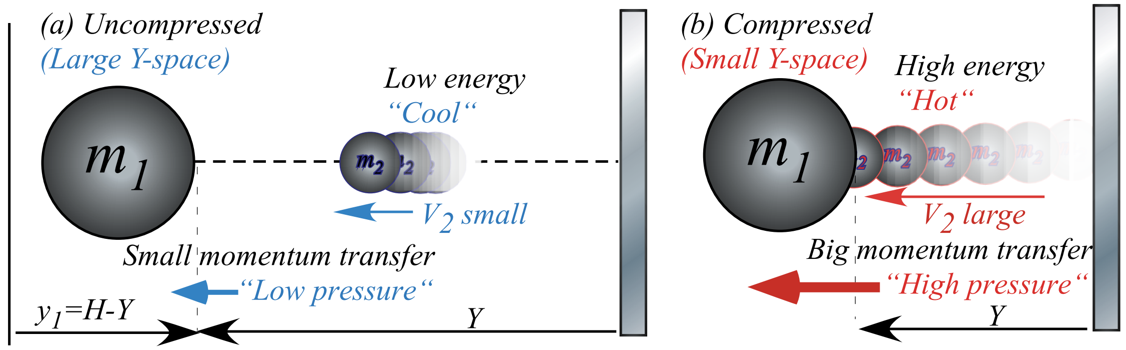
(Here: $V = v_2$)



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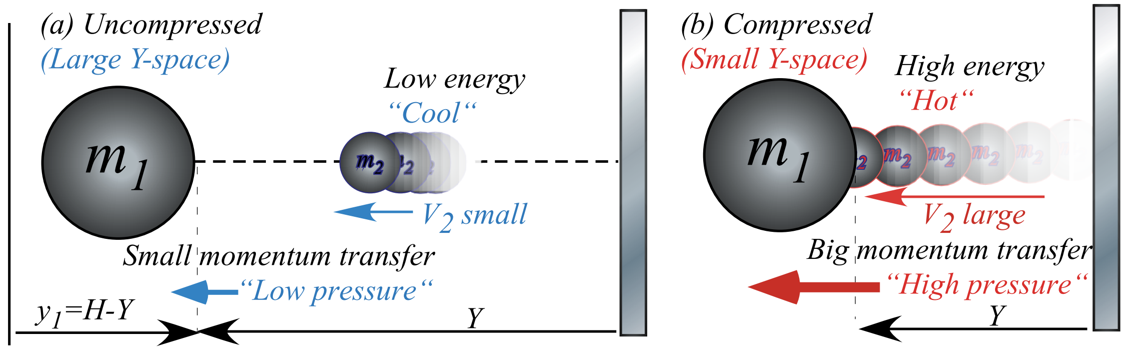
or else :

$$F\cdot\frac{dY}{dt} = \frac{dp}{dt}\cdot V = \frac{d(mV)}{dt}\cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$$

(Here: $V = v_2$)



A
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Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ *vs. Mathematician's Definition* $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

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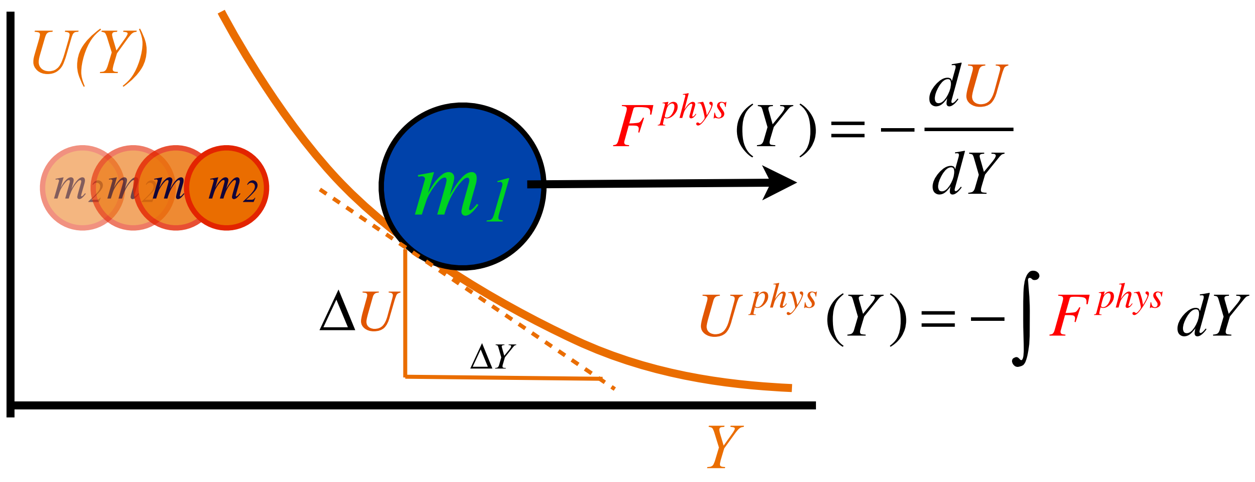
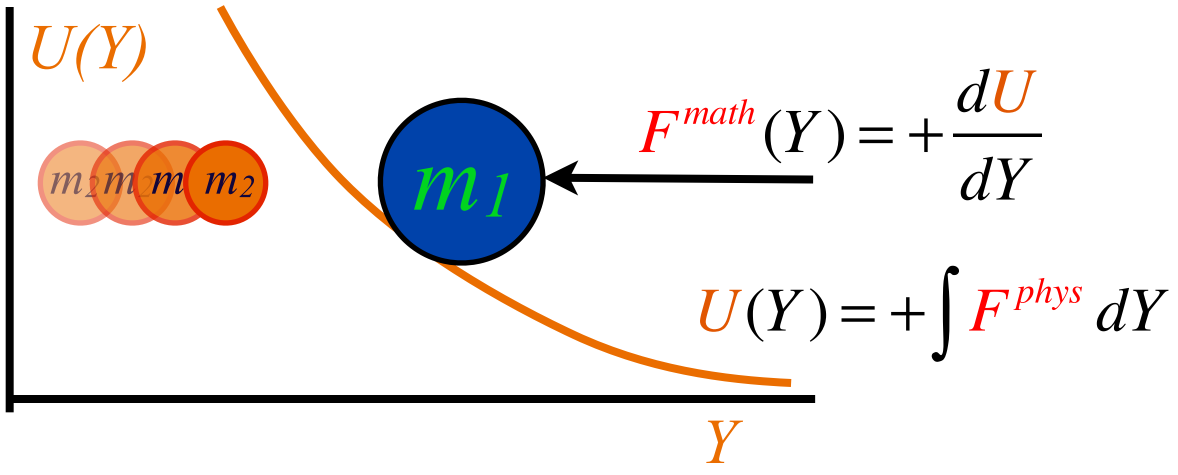
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

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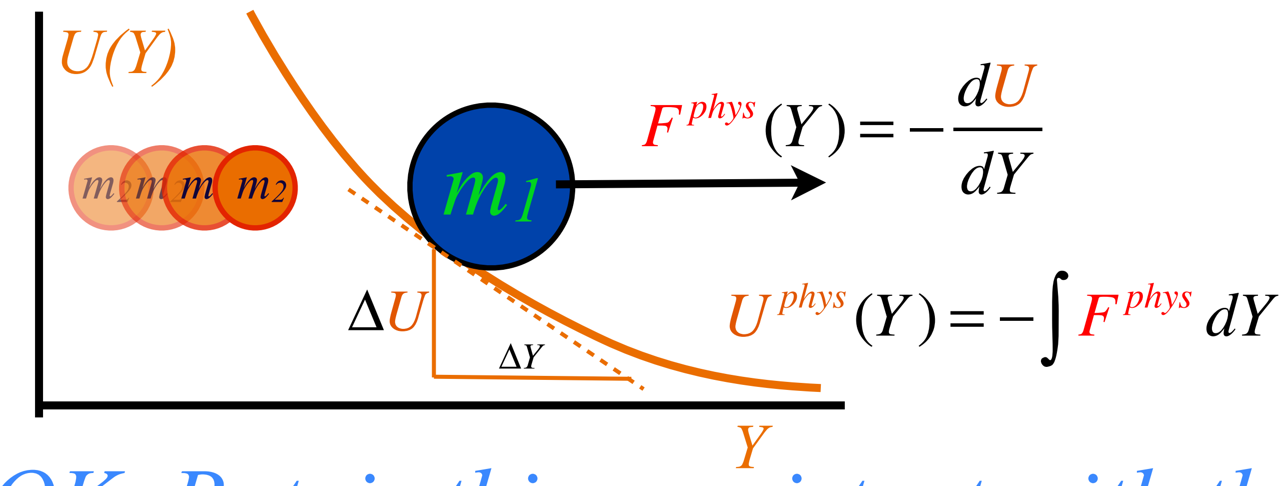

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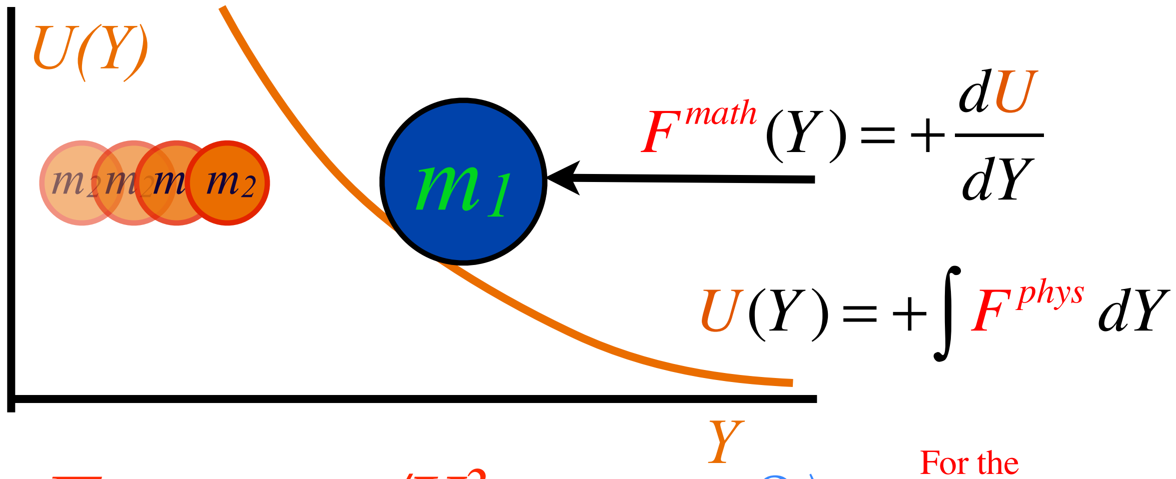

$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, is this consistent with the $F=(const.)^2/Y^3$ *(on p.18)?)*

For the "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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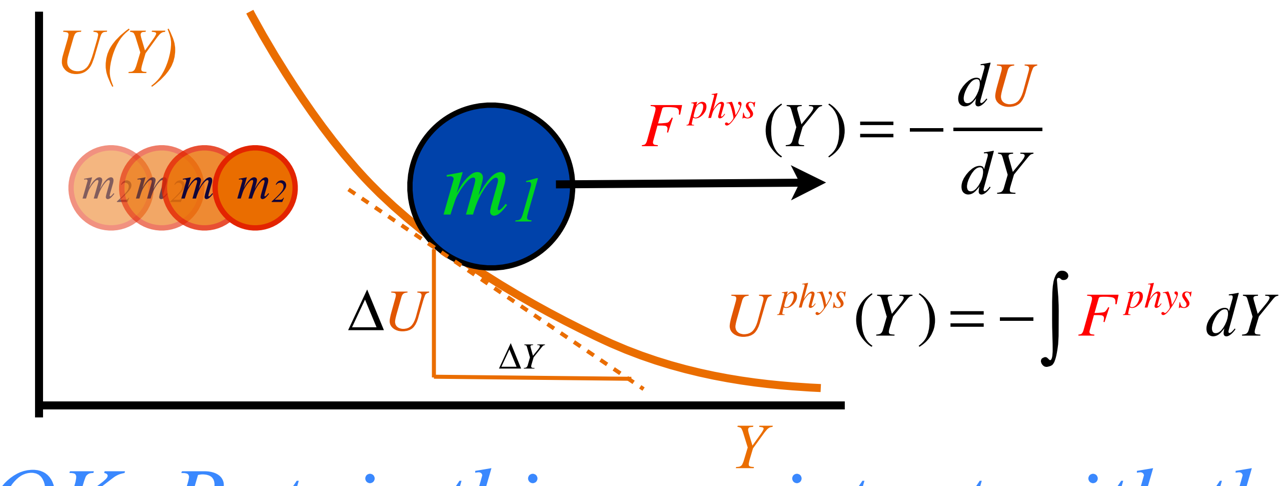

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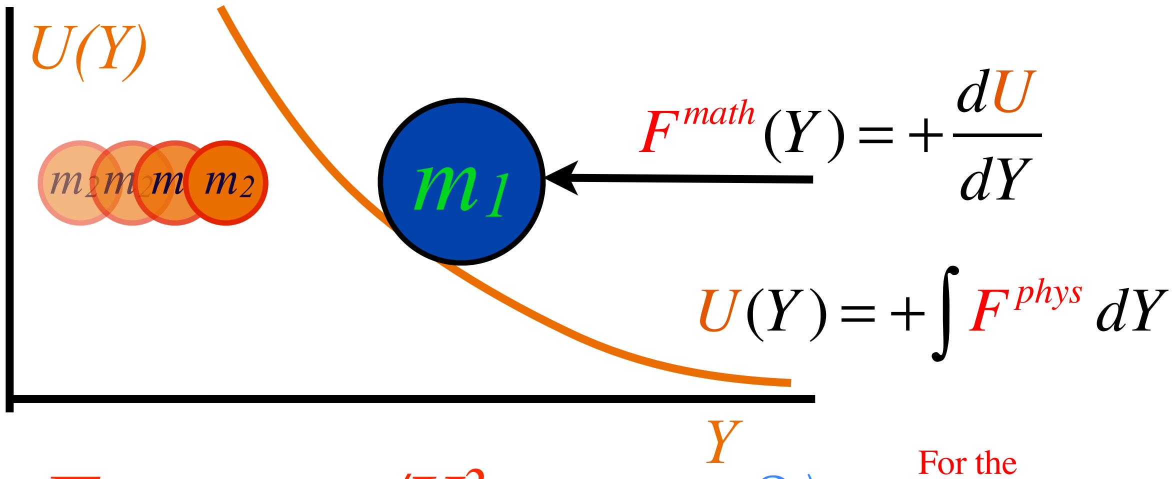

(OK, But, is this consistent with the $F=(const.)^2/Y^3$ (on p.18)?)

For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ *vs. Mathematician's Definition* $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential* $U(y)=const.\ ln(y)$

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

Not a

"Double-Whammy"...

...only a

"Single-Whammy"

$$F^{phys} = \frac{m_2 v_2{}^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U(Y) = \int -F^{phys} dY = \int -\frac{m_2 v_2{}^2}{Y} dY = -m_2 v_2{}^2 \ln(Y)$$

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Not a "Double-Whammy"... ...only a "Single-Whammy"

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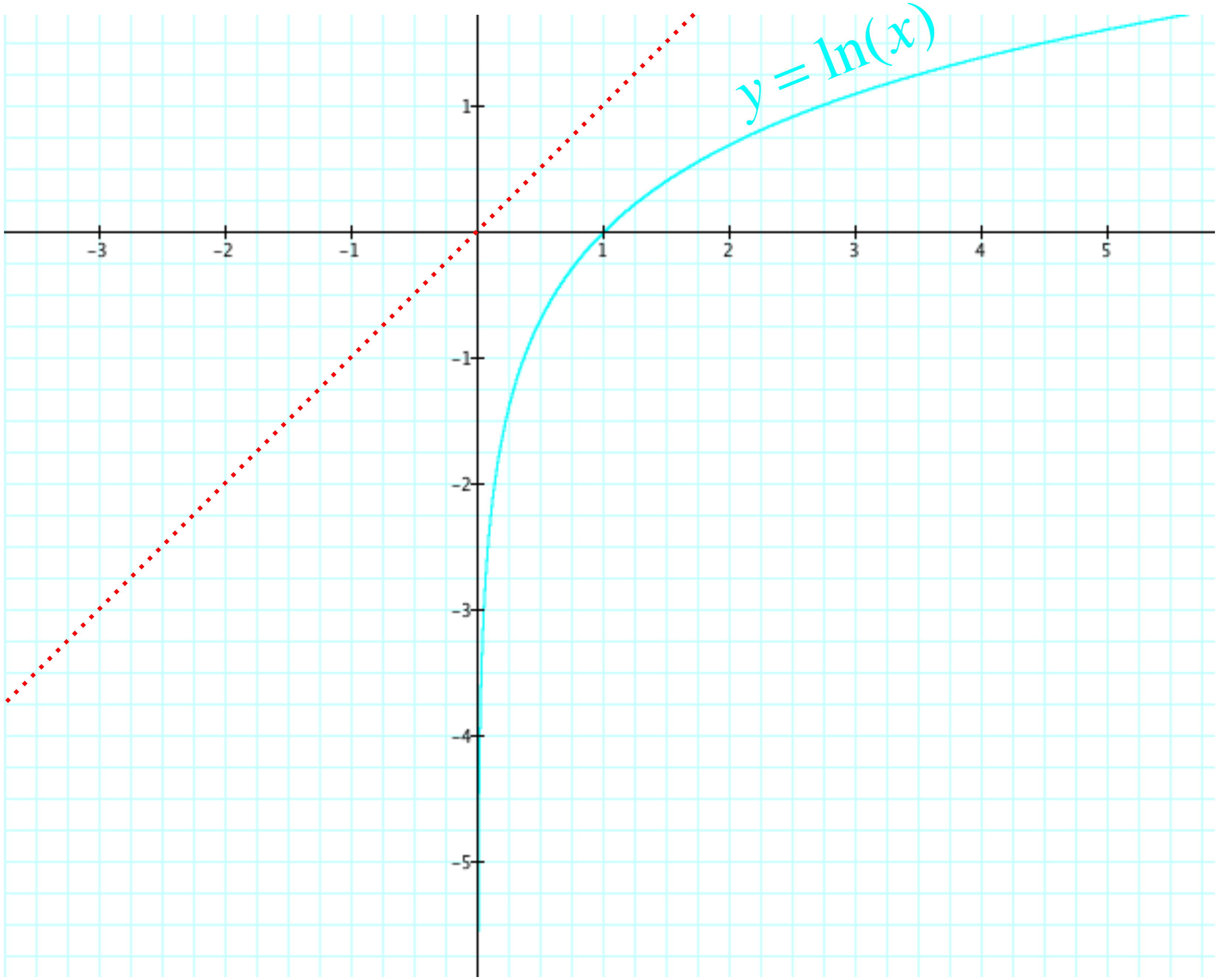

Notice how tightly $\ln(x)$ hugs y-axis...

It's the backside of exponential $y=e^x$...

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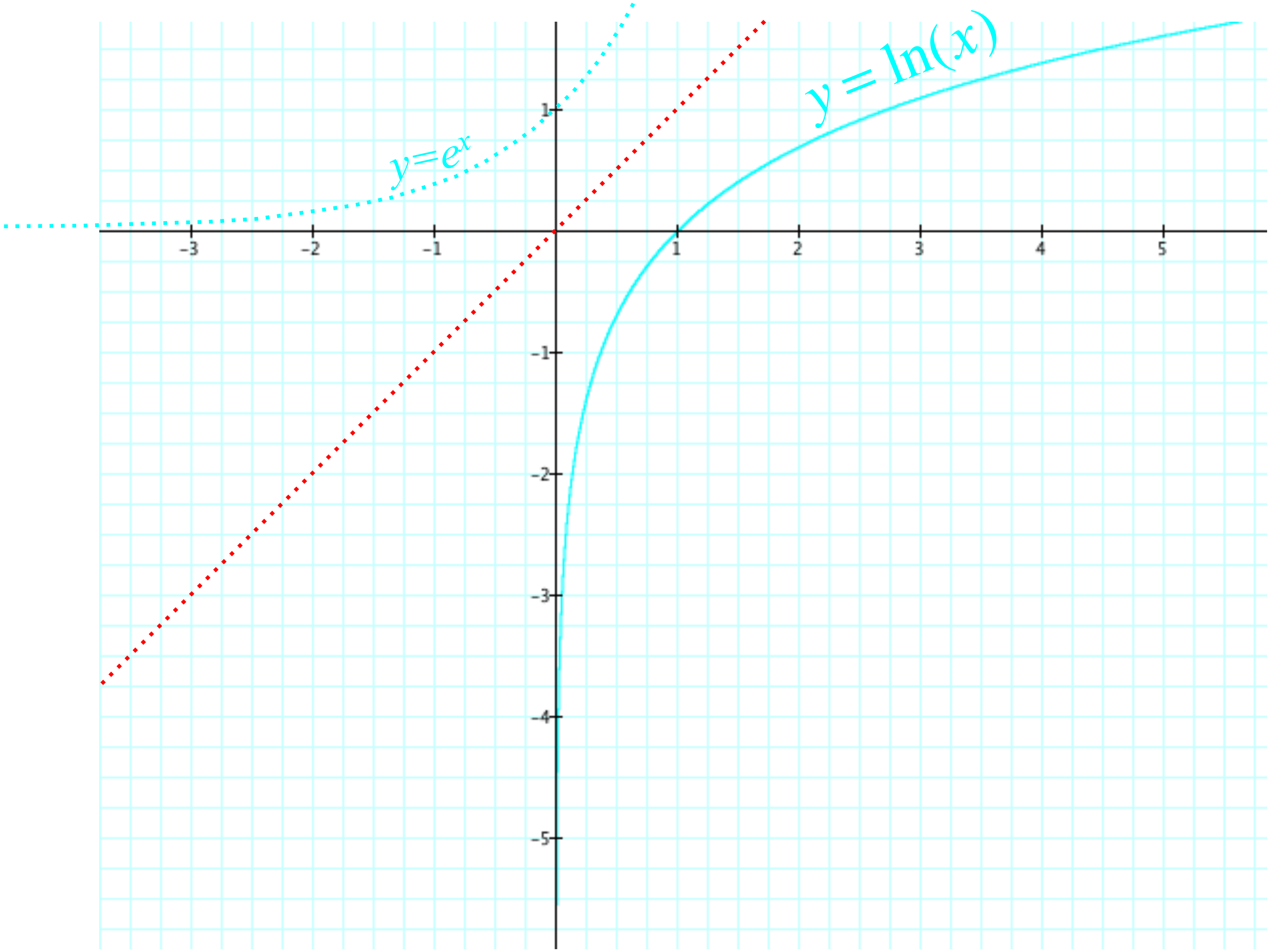

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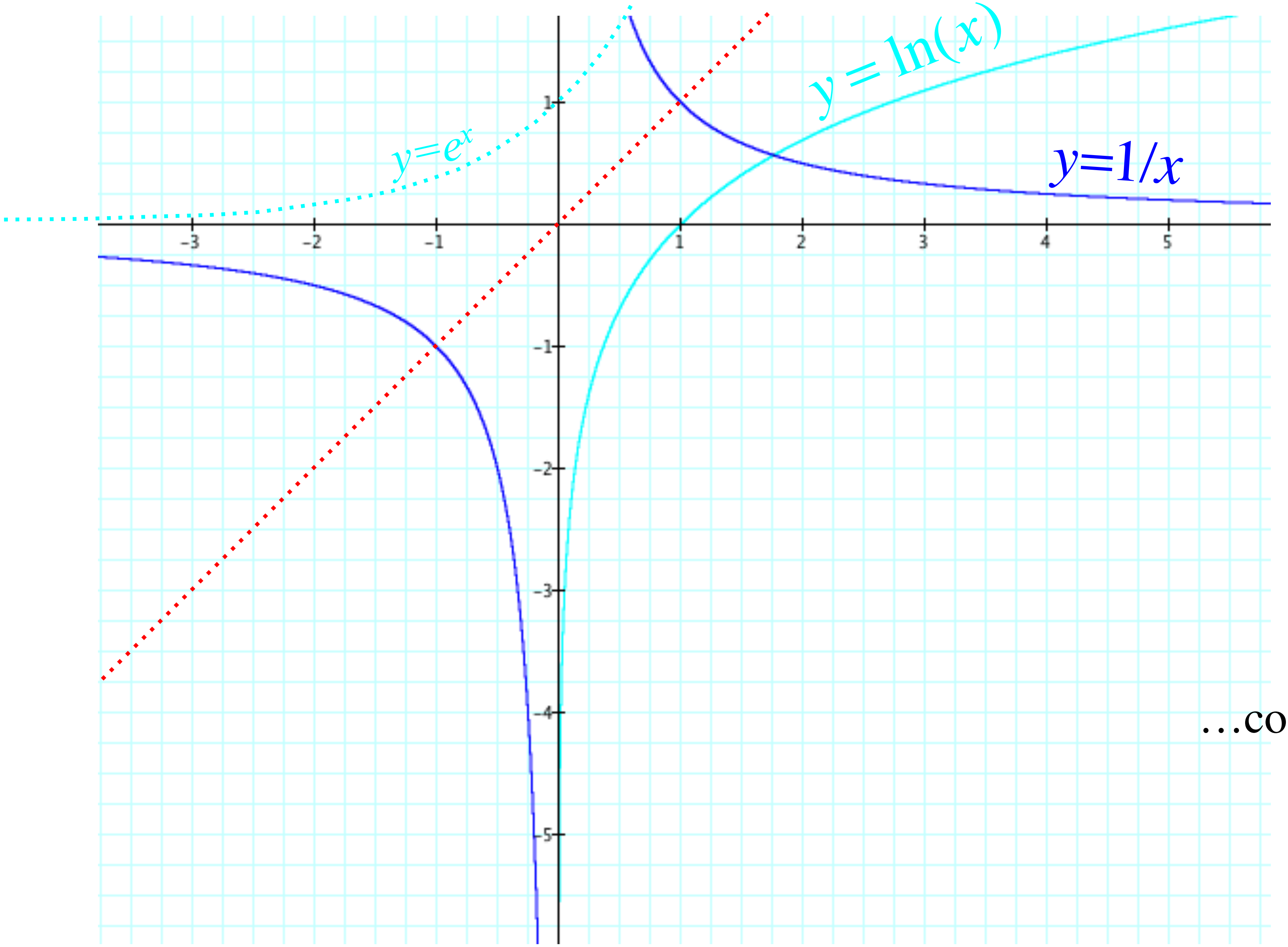

Notice how tightly
ln(x) hugs y-axis…
It's the backside of exponential $y=e^x$…

…compared to $y=1/x$ *or* $x=1/y$

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

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...only a "Single-Whammy"

$$F^{phys}=\frac{m_2v_2{}^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U(Y)=\int -F^{phys}dY=\int -\frac{m_2v_2{}^2}{Y}dY=-m_2v_2{}^2\ln(Y)$$

$$const.=E=\frac{1}{2}m_1v_1{}^2+U(Y) \quad where: U(Y)=-m_2v_2{}^2\ln(Y)$$

Define for big mass m_1 : *Kinetic energy KE(v_1) vs Potential energy PE(Y)=U(Y)*

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1D-Isothermal Force Law (assume v_2 *is constant for all Y):* $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

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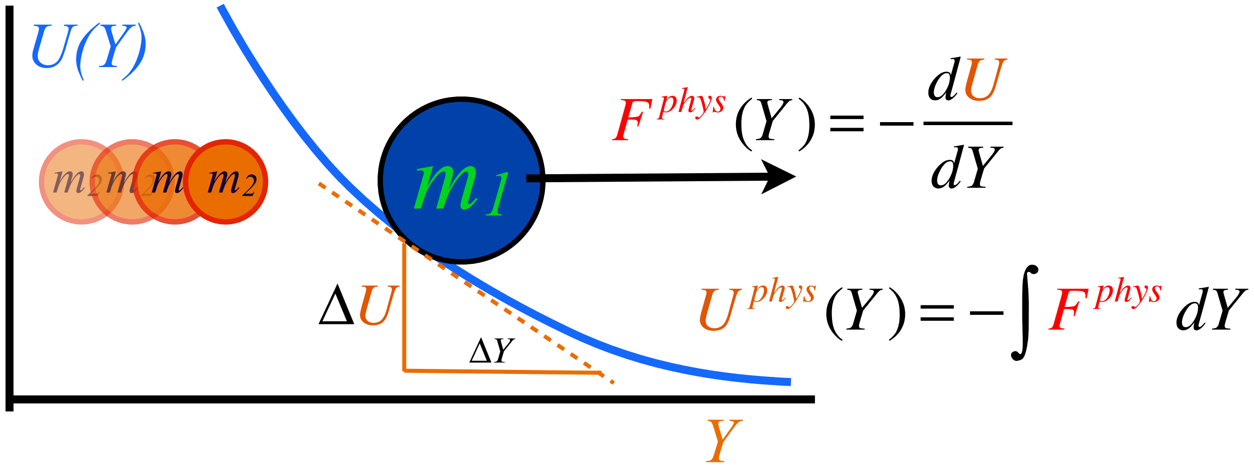
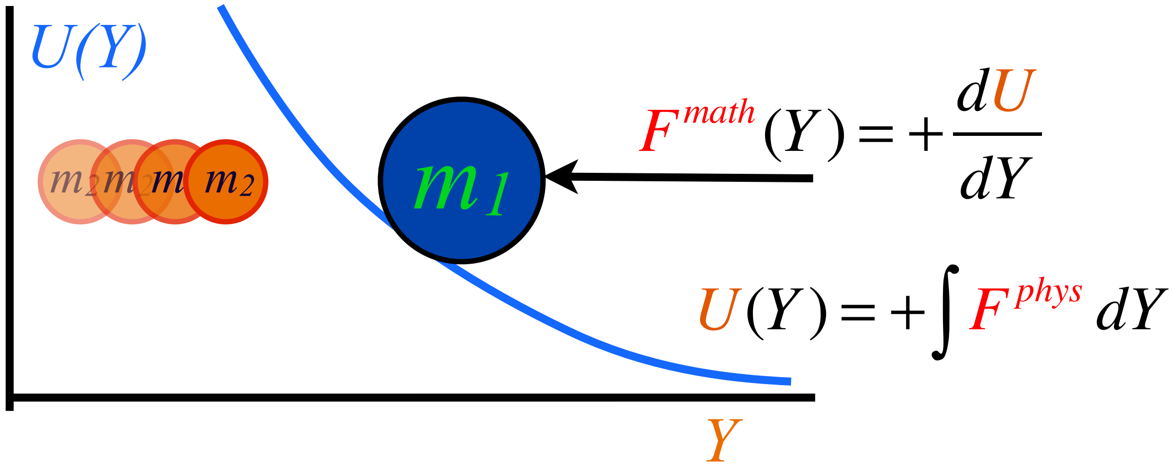
The "Physicist" View of Force



The "Mathematician" View of Force



1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

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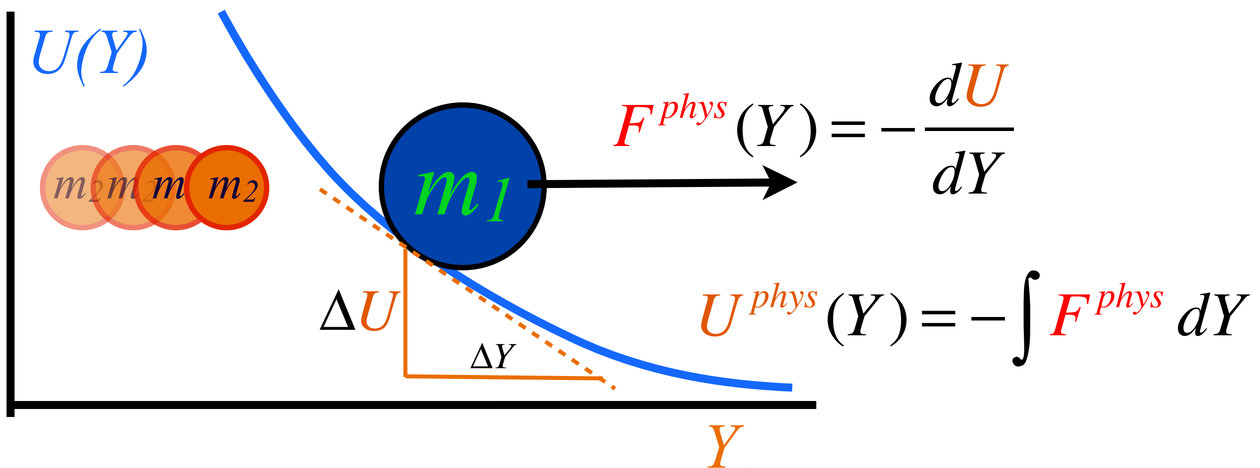
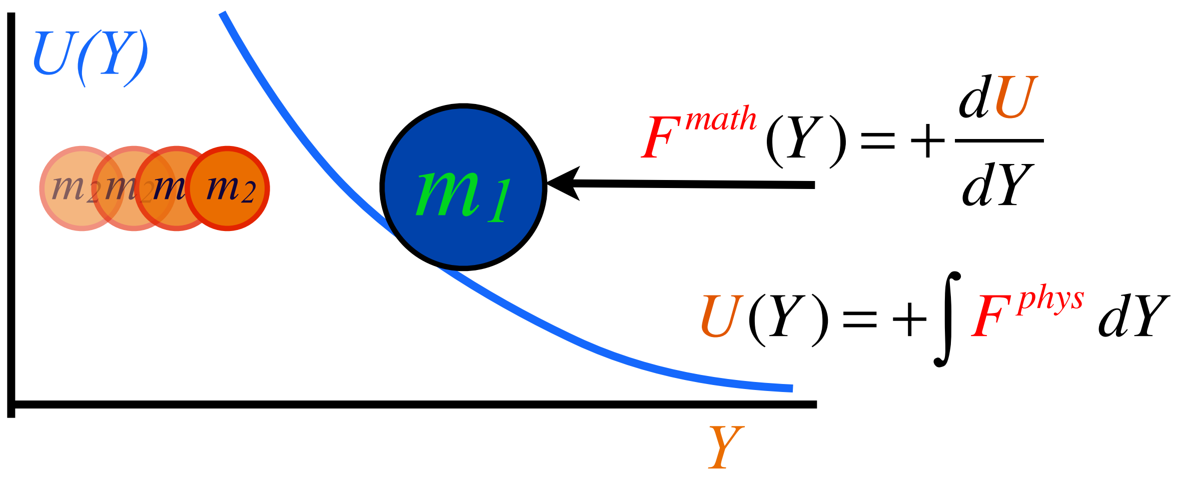


$$F^{phys}(Y)=-\frac{dU}{dY}$$

$$U^{phys}(Y)=-\int F^{phys}dY$$

The "Mathematician" View of Force



$$F^{math}(Y)=+\frac{dU}{dY}$$

$$U(Y)=+\int F^{phys}dY$$

(Same integral/differential relations)

$$F^{phys}=\frac{m_2v_2^2}{Y}=\frac{const.}{Y} \quad consistent\ with: \quad F^{phys}=-\frac{\Delta U}{\Delta Y}=-\frac{d}{dY}\left(-const.\ln(Y)\right)=\frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

→ Example of oscillator with opposing Isothermal potentials

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

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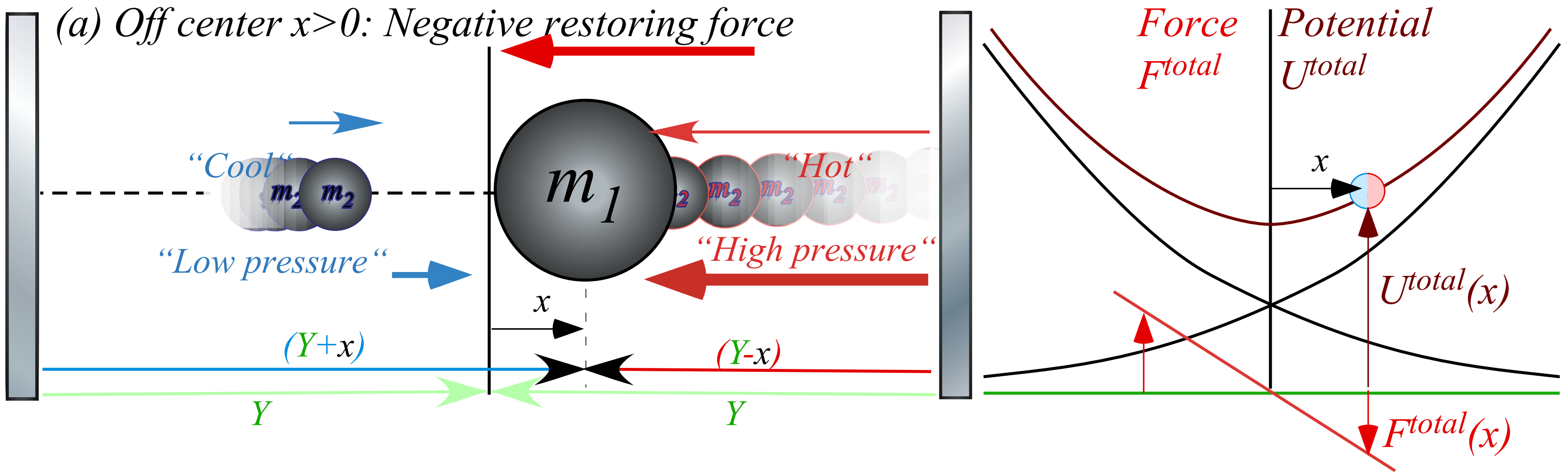
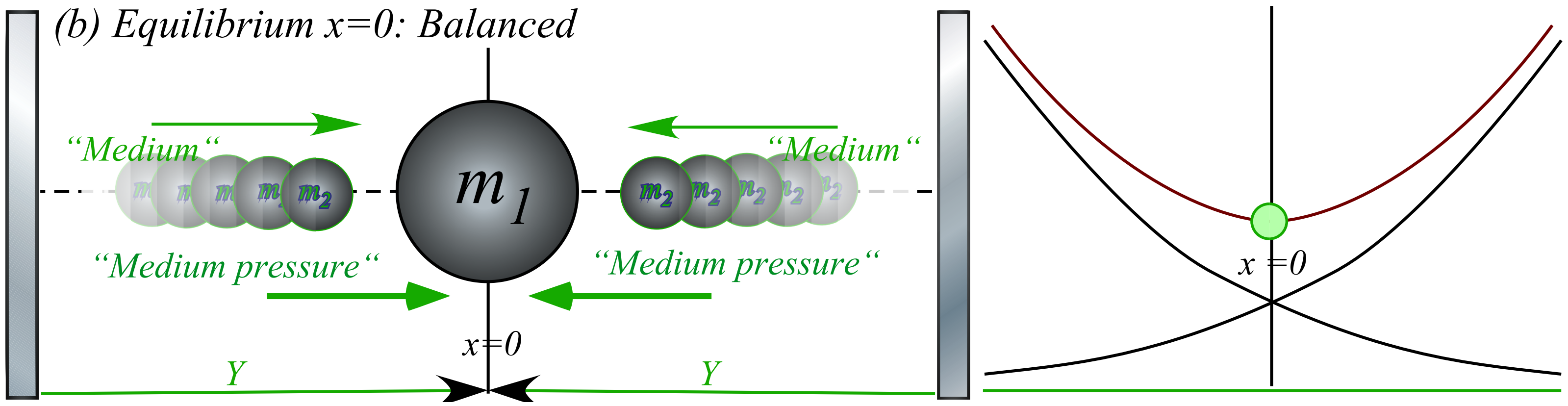


(a) Off center x>0: Negative restoring force



(b) Equilibrium x=0: Balanced

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f\left[1 - x + x^2 - x^3 \ldots\right] - f\left[1 + x + x^2 + x^3 \ldots\right] = -2f\cdot x - 2f\cdot x^3 - \ldots$$

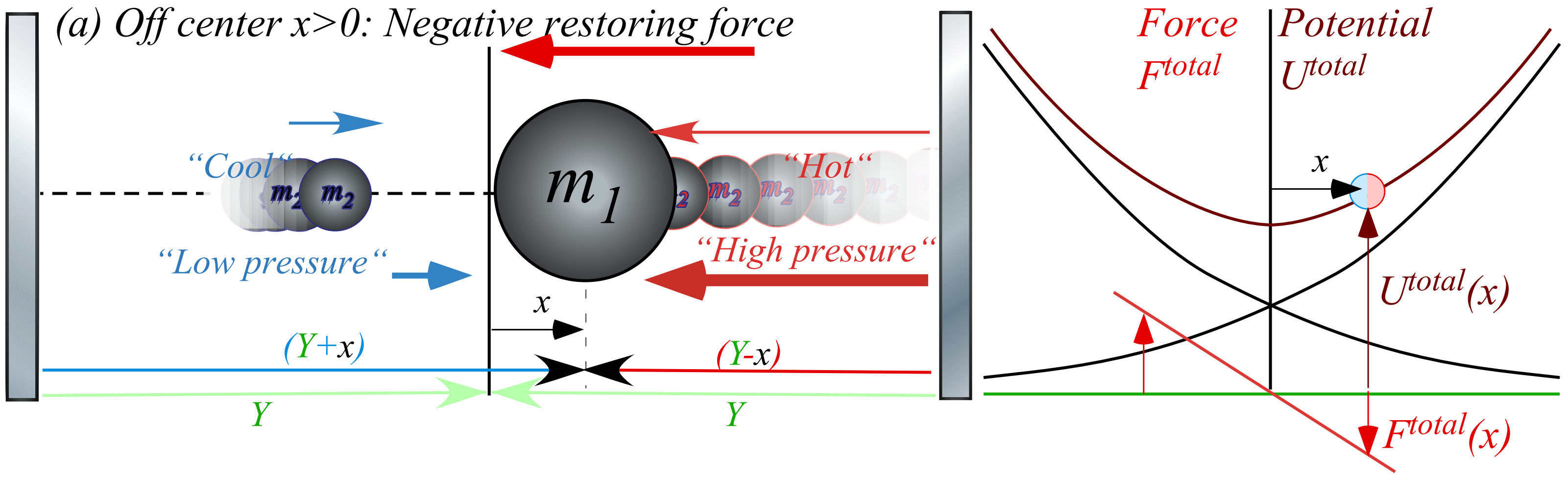
$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x} \qquad U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx \qquad \text{HO ∡frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\upsilon$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U=-m_2 v_2^2 \ln(Y)$$



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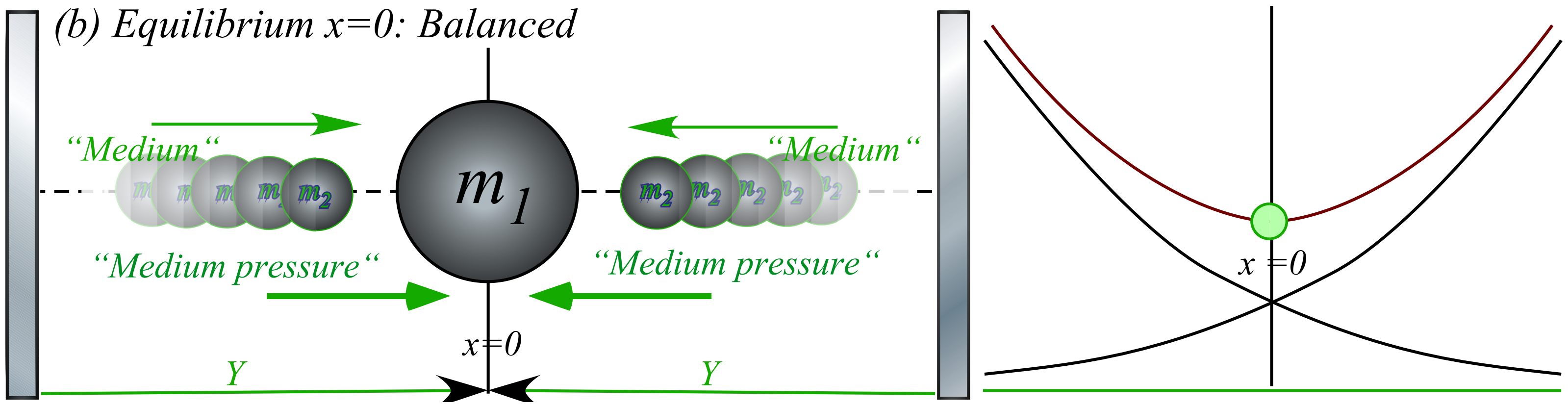

(b) Equilibrium x=0: Balanced

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}$$

Example of oscillator with opposing *Isothermal potentials*

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

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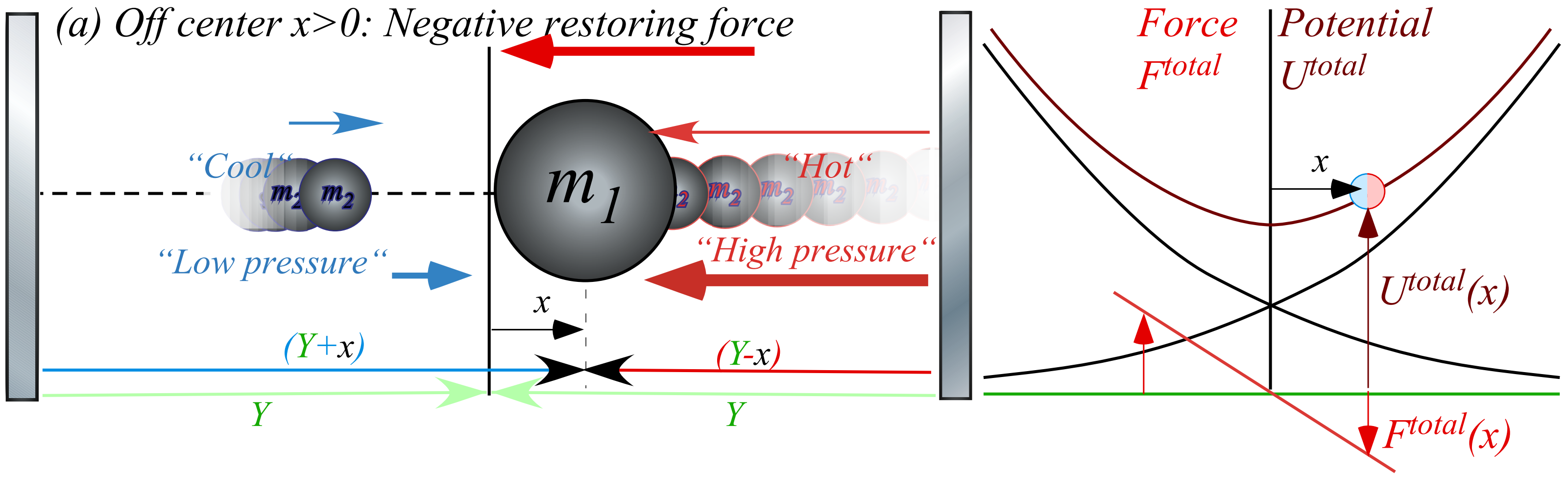
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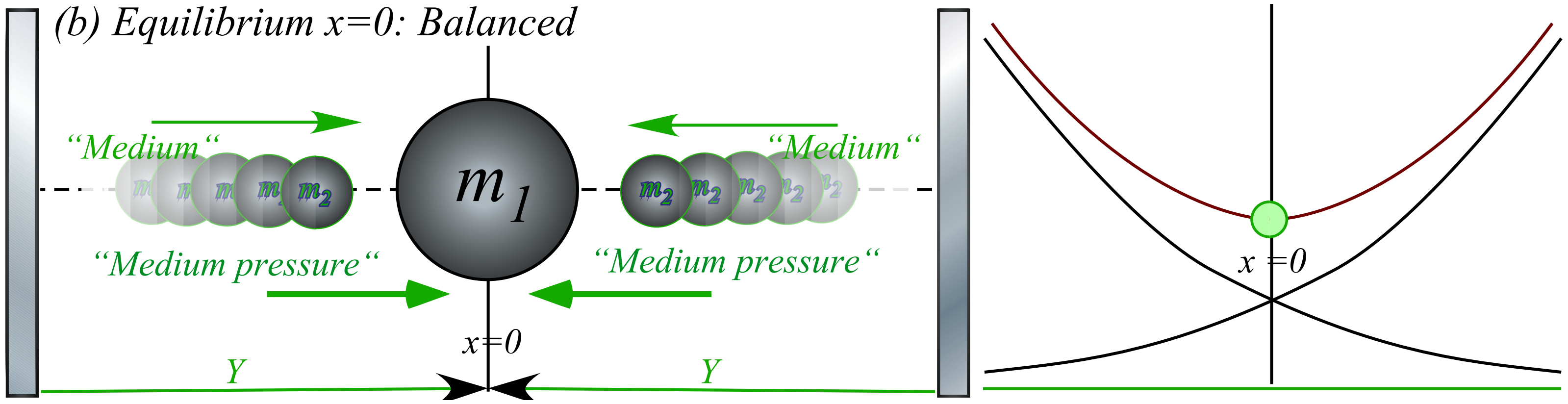

(b) Equilibrium x=0: Balanced

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2Y_0^{-3} - x^3Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n-1)}{2}Y_0^{n-2}x^2 + \frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4 \ldots$$

Binomial Theorem

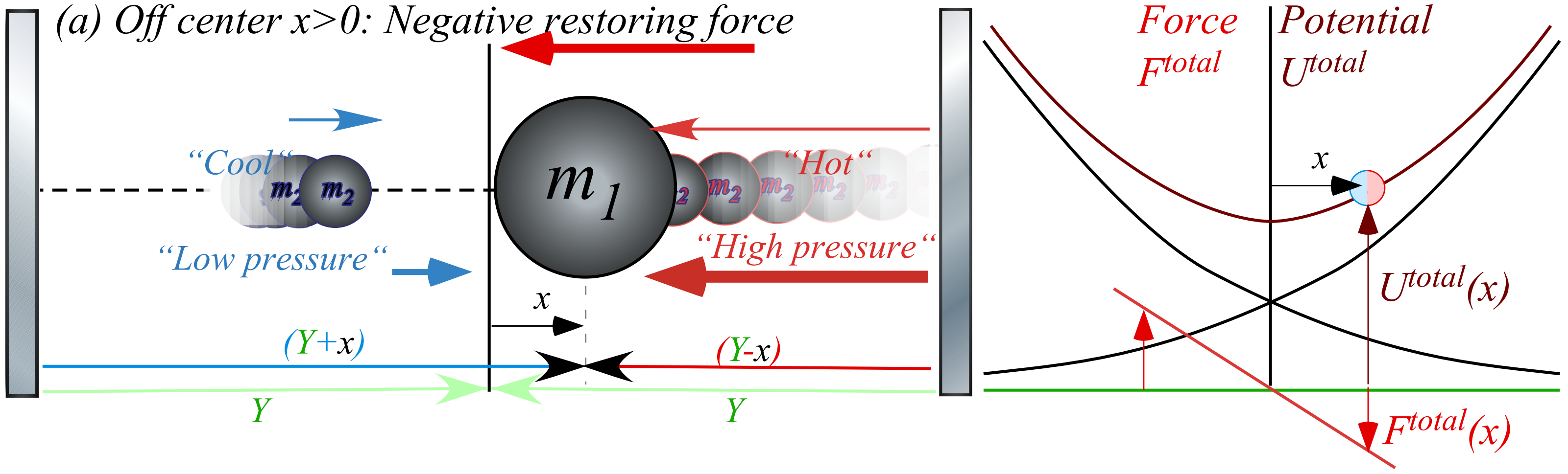
Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$

(a) Off center x>0: Negative restoring force



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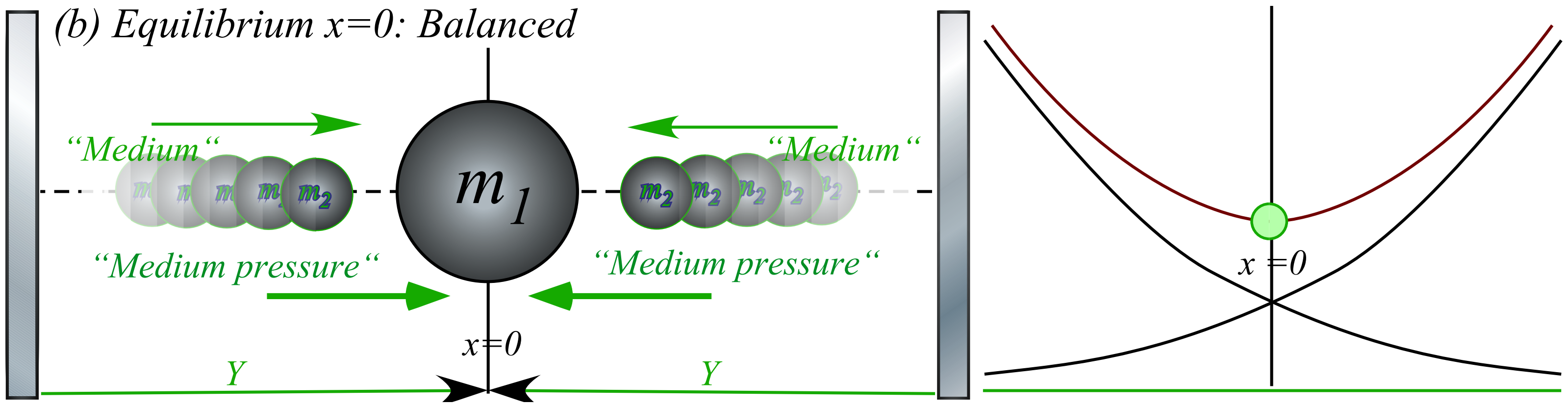


Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - \ldots$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots$$

$$\boxed{(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n\text{-}1)}{2} Y_0^{n-2}x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2\cdot 3} Y_0^{n-3}x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2\cdot 3\cdot 4} Y_0^{n-4}x^4 \ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

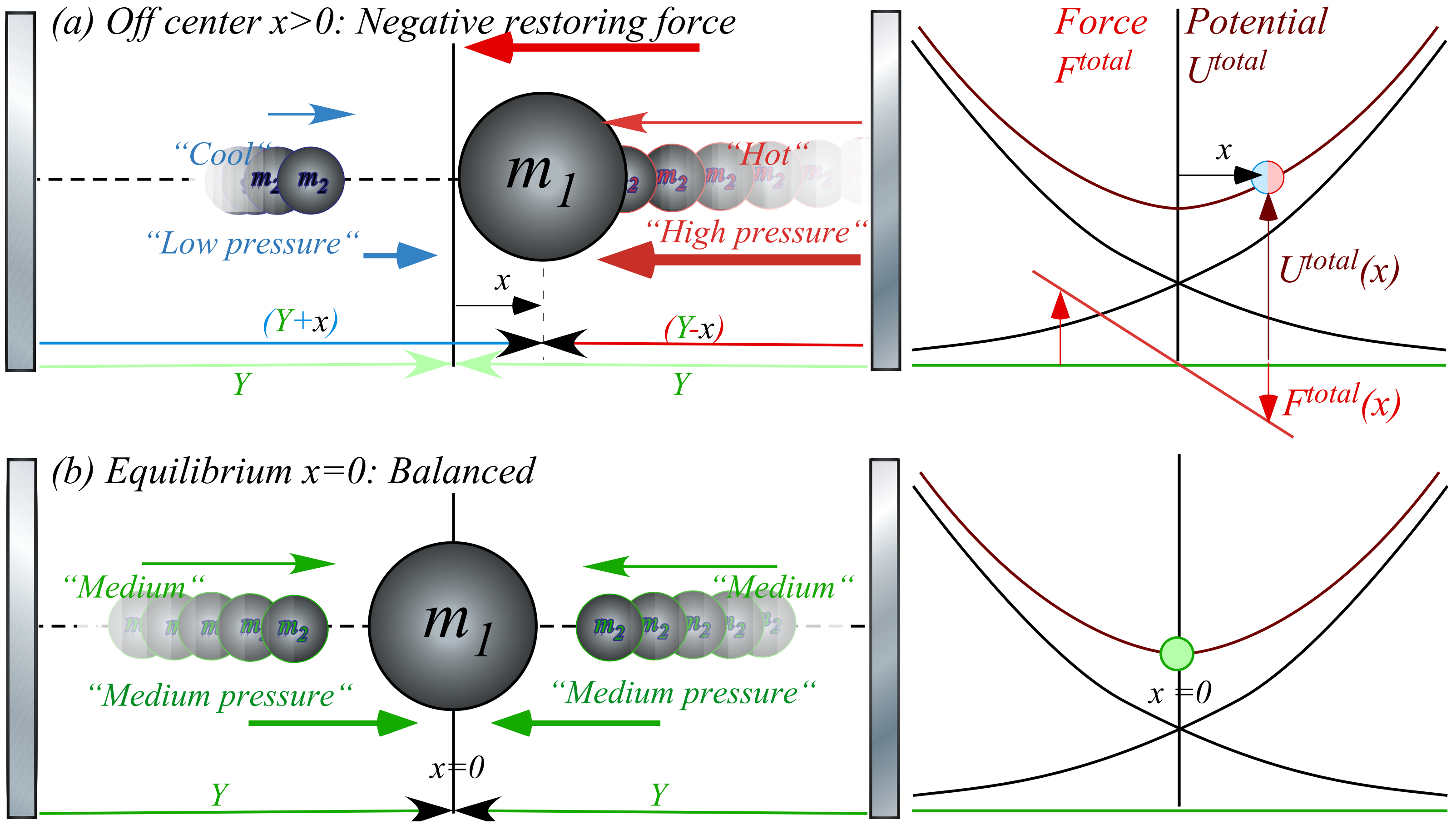
1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y}$$

implies :

$$U=-m_2 v_2^2\ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\frac{1}{Y_0}+\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}+\frac{x^3}{Y_0^4}+\cdots\right]$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots$$

$$(Y_0-x)^{-1}=Y_0^{-1}+xY_0^{-2}+x^2Y_0^{-3}+x^3Y_0^{-4}\ldots$$

$$(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots$$

Binomial Theorem

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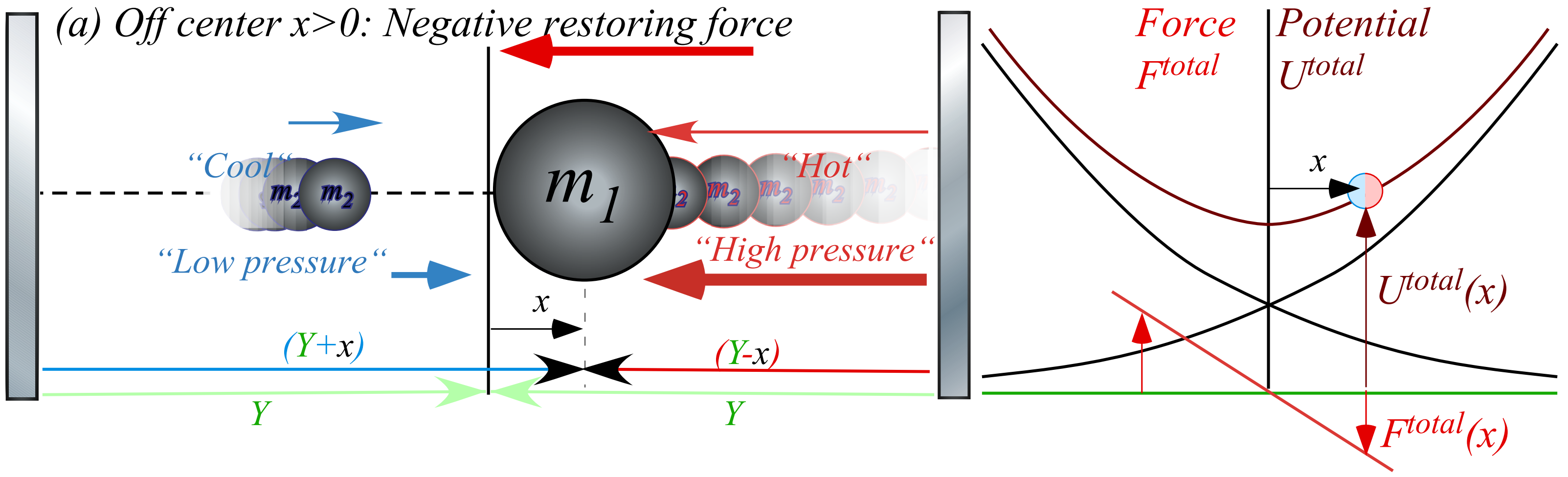
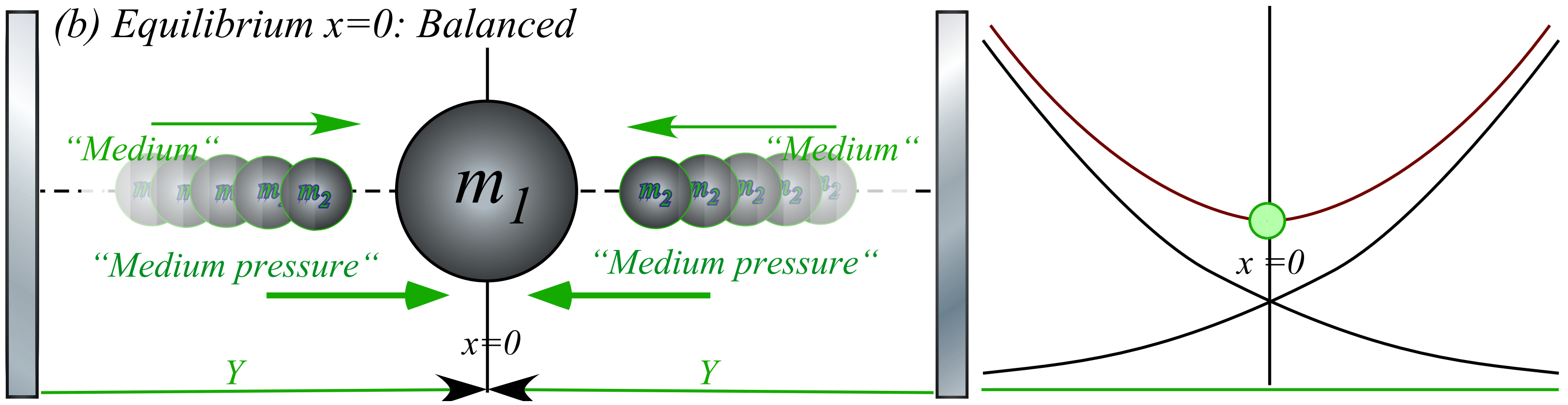
1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



(a) Off center x>0: Negative restoring force



(b) Equilibrium x=0: Balanced

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0+x} - \frac{f}{Y_0-x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right]$$

$$(Y_0+x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2Y_0^{-3} - x^3Y_0^{-4} \ldots \qquad (Y_0-x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2Y_0^{-3} + x^3Y_0^{-4} \ldots$$

$$(Y_0+x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n\text{-}1)}{2}Y_0^{n-2}x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4 \ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

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$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

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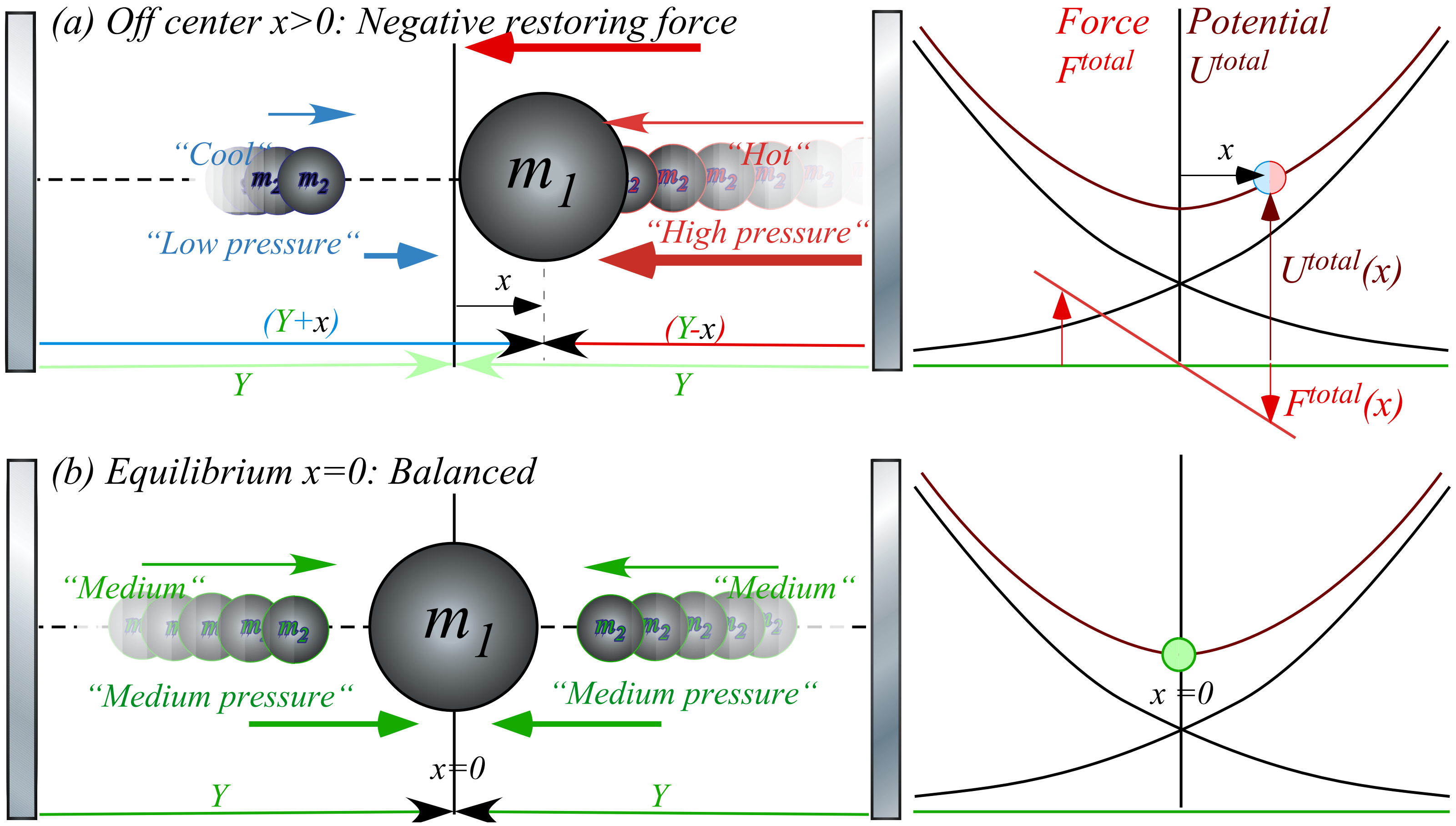


Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{Y_0+x} - \frac{f}{Y_0-x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right] = \overbrace{-2f\frac{x}{Y_0^2}} - 2f\frac{x^3}{Y_0^4} - \ldots$$

$$(Y_0+x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2Y_0^{-3} - x^3Y_0^{-4}\ldots \qquad (Y_0-x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2Y_0^{-3} + x^3Y_0^{-4}\ldots$$

$$\boxed{(Y_0+x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n-1)}{2}Y_0^{n-2}x^2 + \frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

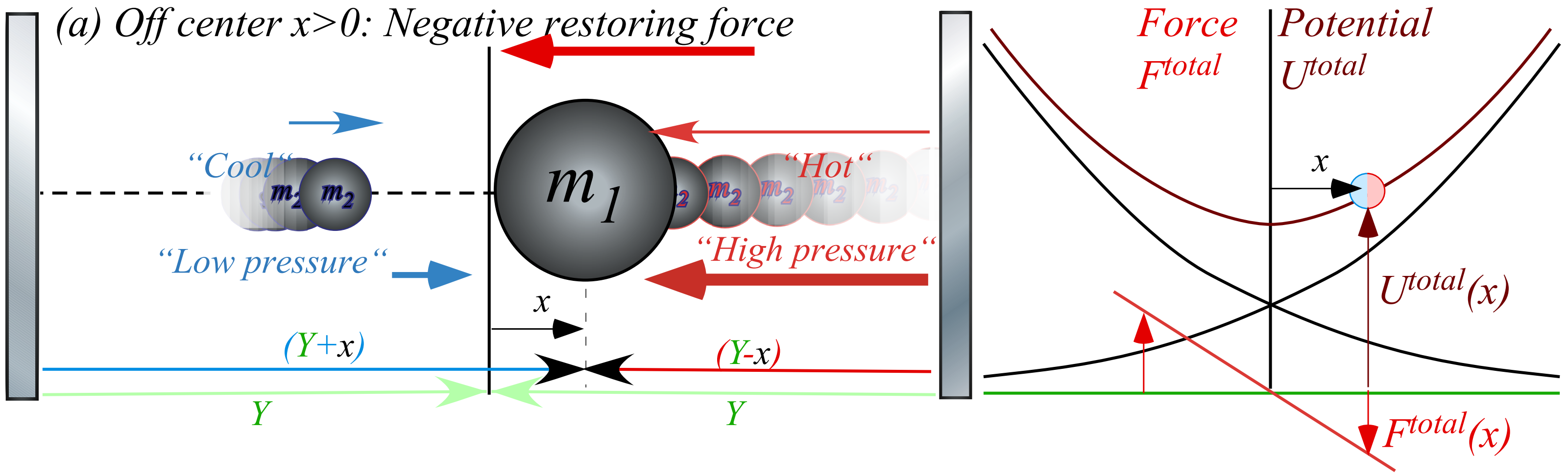
1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$$

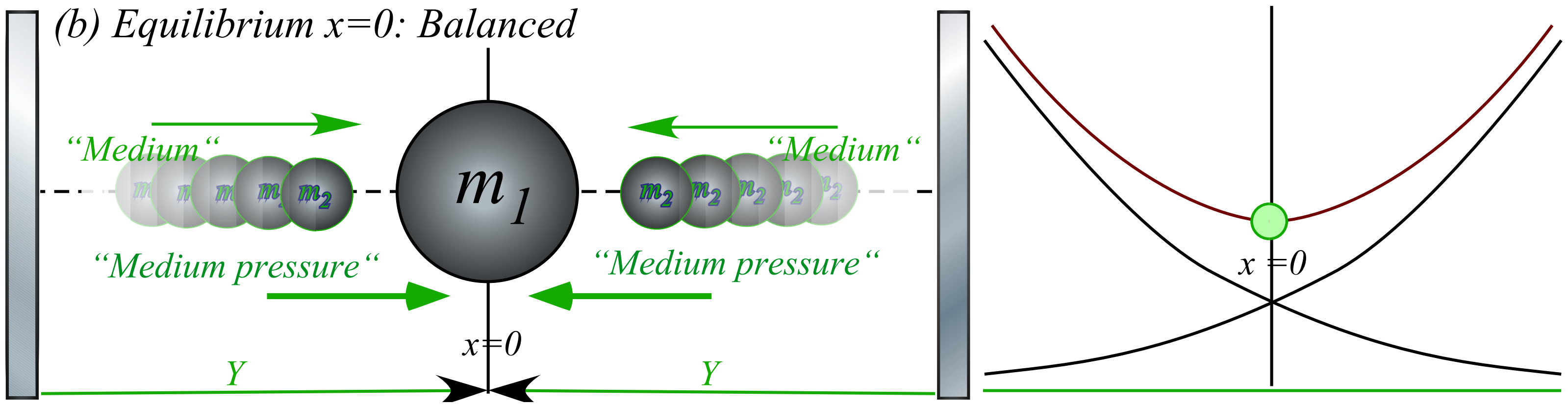
$$F^{phys}=\frac{m_2v_2^2}{Y}=-\frac{\Delta U}{\Delta Y}$$

implies :

$$U=-m_2v_2^2\ln(Y)$$



(a) Off center x>0: Negative restoring force



(b) Equilibrium x=0: Balanced

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=\overbrace{-2f\frac{x}{Y_0^2}}^{\text{Harmonic oscillator term}}\overbrace{-2f\frac{x^3}{Y_0^4}-\dots}^{\text{Anharmonic oscillator terms...}}$$

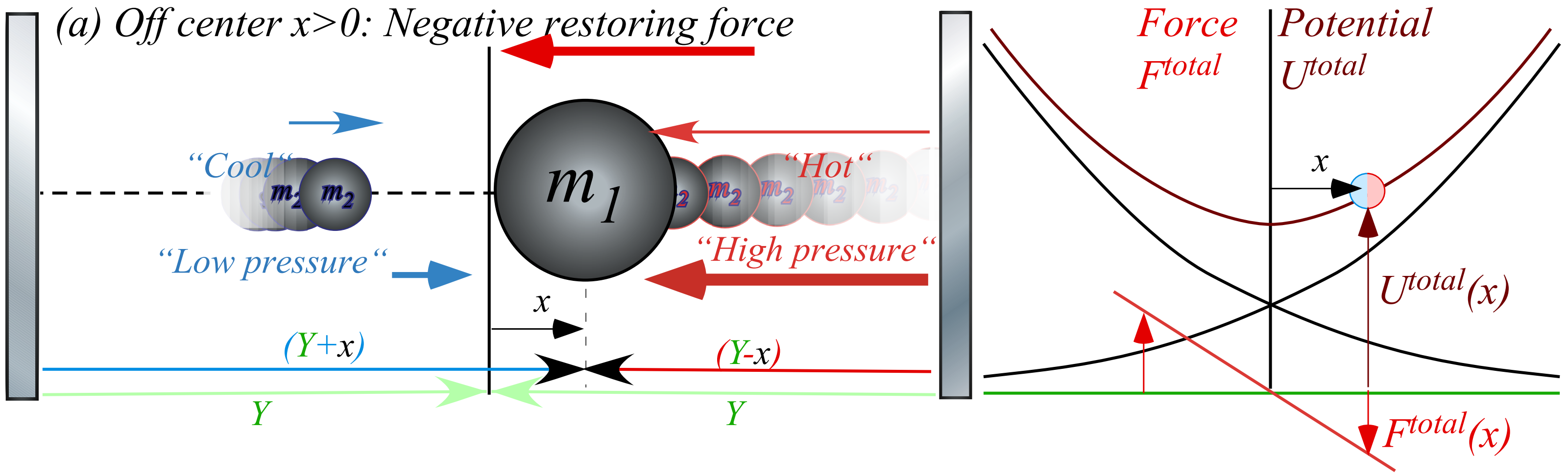
Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Example of oscillator with opposing Isothermal potentials

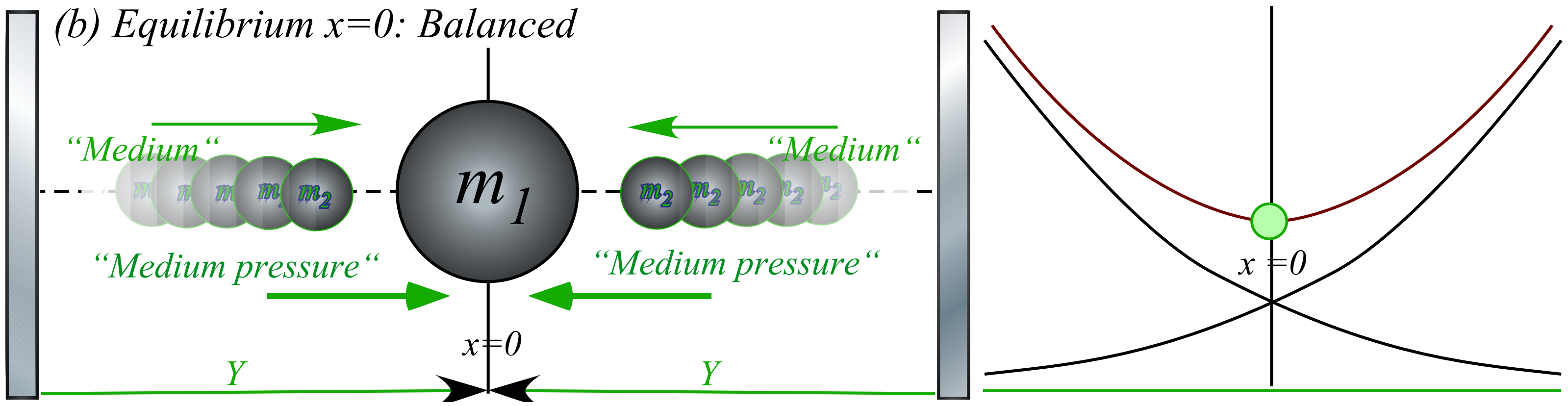
1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U=-m_2 v_2^2 \ln(Y)$$

(a) Off center $x>0$: Negative restoring force



(b) Equilibrium $x=0$: Balanced



Unit 1
Fig. 6.2

Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=\overbrace{-2f\frac{x}{Y_0^2}}-2f\frac{x^3}{Y_0^4}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

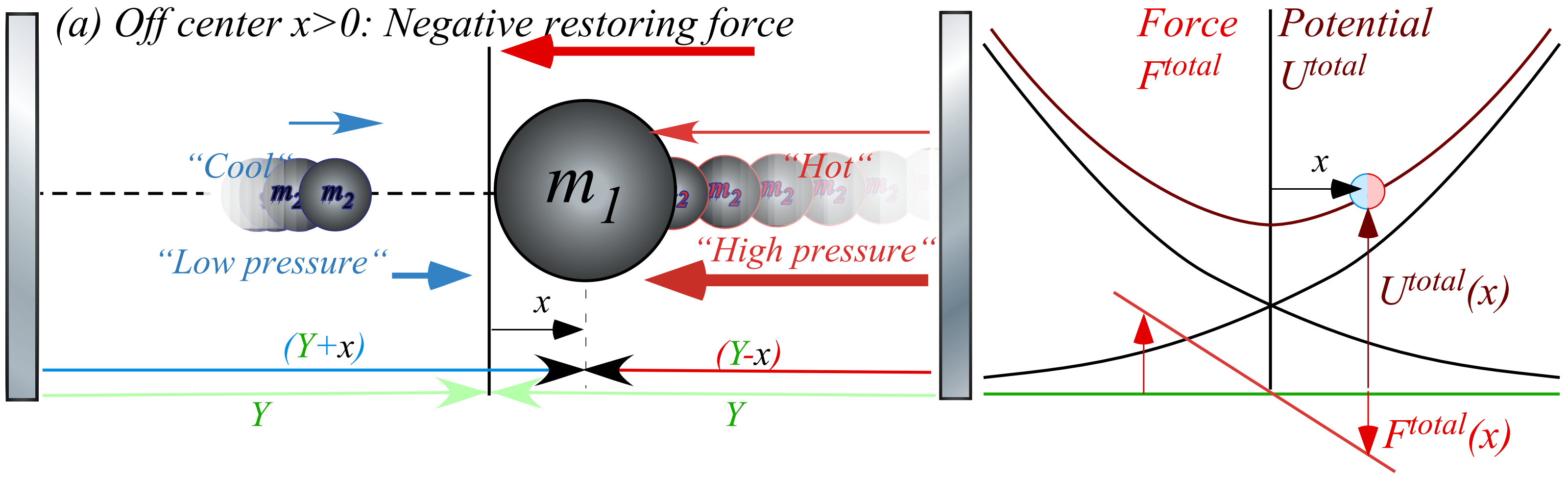
Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

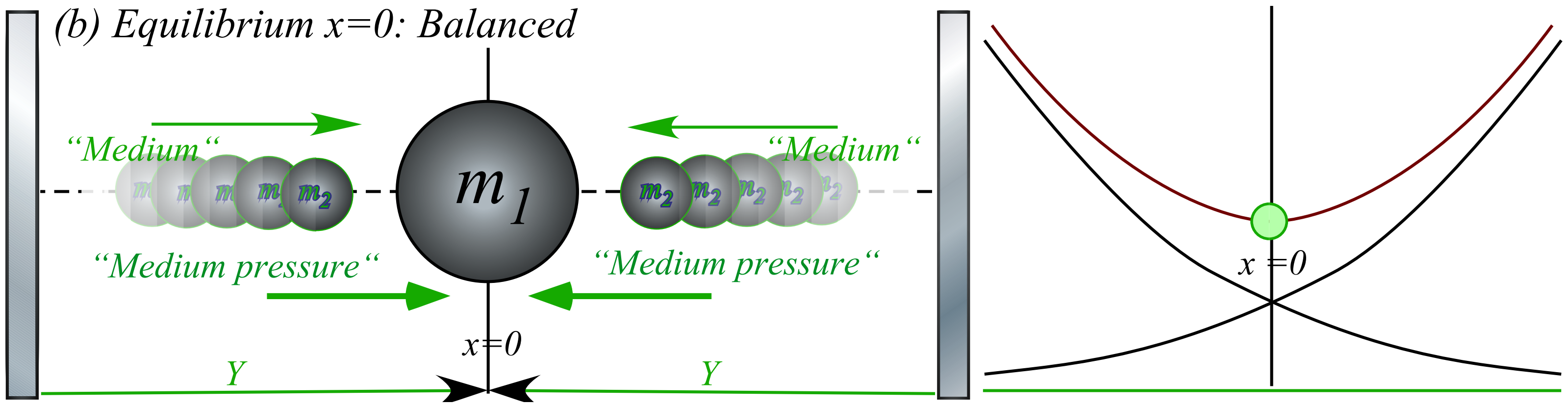
$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad implies: \quad U=-m_2 v_2^2 \ln(Y)$$

(a) Off center x>0: Negative restoring force



(b) Equilibrium x=0: Balanced



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\,\overbrace{\frac{x}{\underbrace{Y_0^2}}}\;-2f\,\frac{x^3}{Y_0^{-4}}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

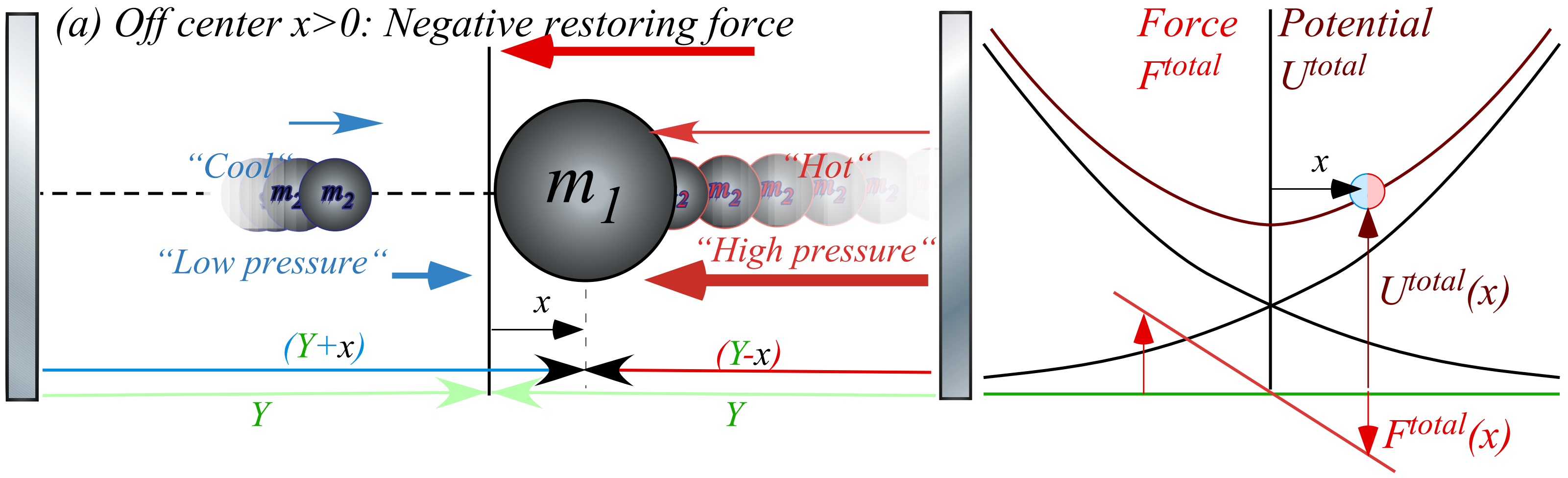
Potential

$$U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx$$

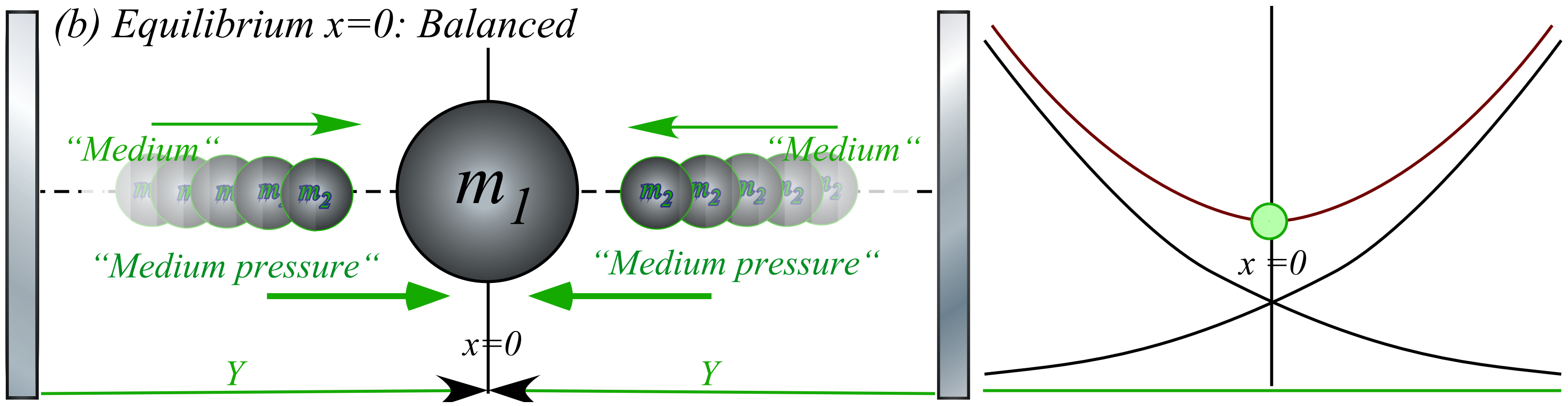
Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):
$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y}\qquad \text{implies :}\qquad U=-m_2 v_2^2\ln(Y)$$


(a) Off center x>0: Negative restoring force

Unit 1
Fig. 6.2


(b) Equilibrium x=0: Balanced

Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\,\overbrace{\underbrace{\frac{x}{Y_0^2}}}\;-2f\,\frac{x^3}{Y_0^4}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force
$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Potential
$$U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx$$

Frequency
$$\text{HO ∡frequency: }\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}}=2\pi\upsilon$$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$$Y = A\sin\omega t \quad \text{with velocity} \quad V = A\omega\cos\omega t$$

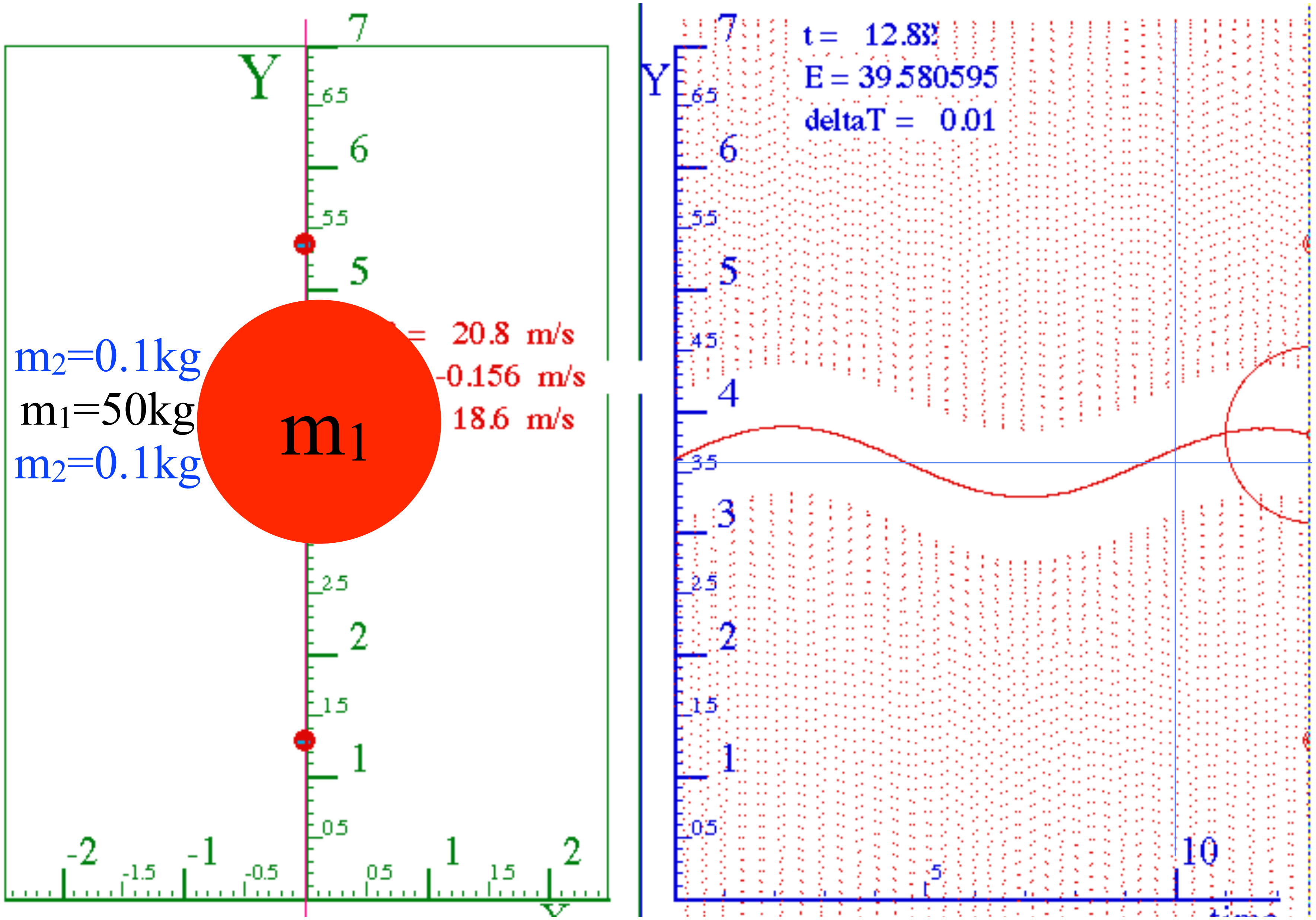
Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2 A^2 \left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2 \left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2 A^2 \qquad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

Switch
$m_1=m_3$
with
m_2
to match
formula



m_2=0.1kg
m_1=50kg
m_2=0.1kg

Unit 1
Fig. 6.3

Simulation of

the adiabatic case

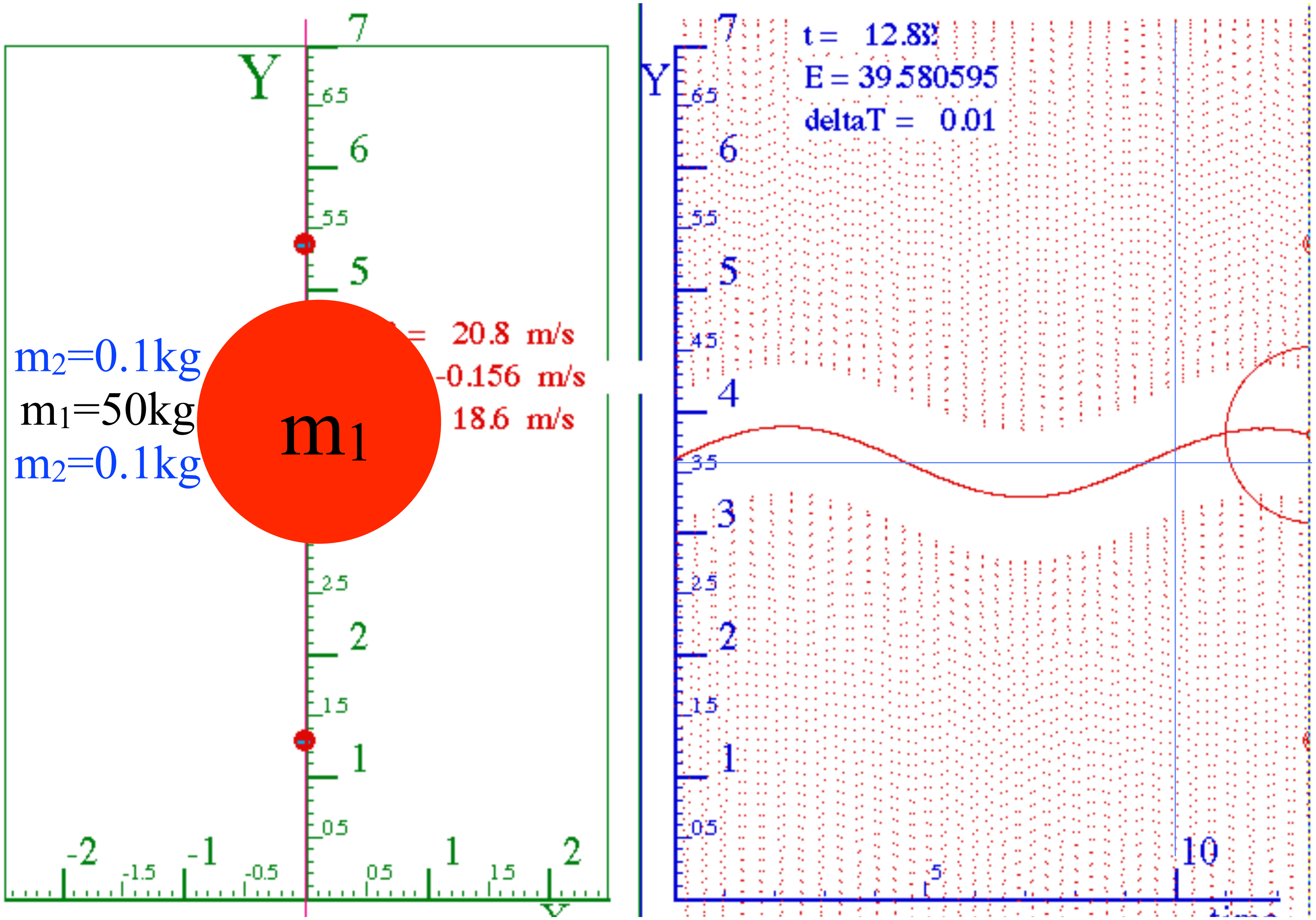
BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

Sample problem: *Compute isothermal frequency and/or period*

Frequency

HO ∡frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}}\frac{v_2}{Y_0}=2\pi\upsilon$

Switch
$m_1=m_3$
with
m_2
to match
formula



t = 12.83
E = 39.580595
deltaT = 0.01

= 20.8 m/s
-0.156 m/s
18.6 m/s

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

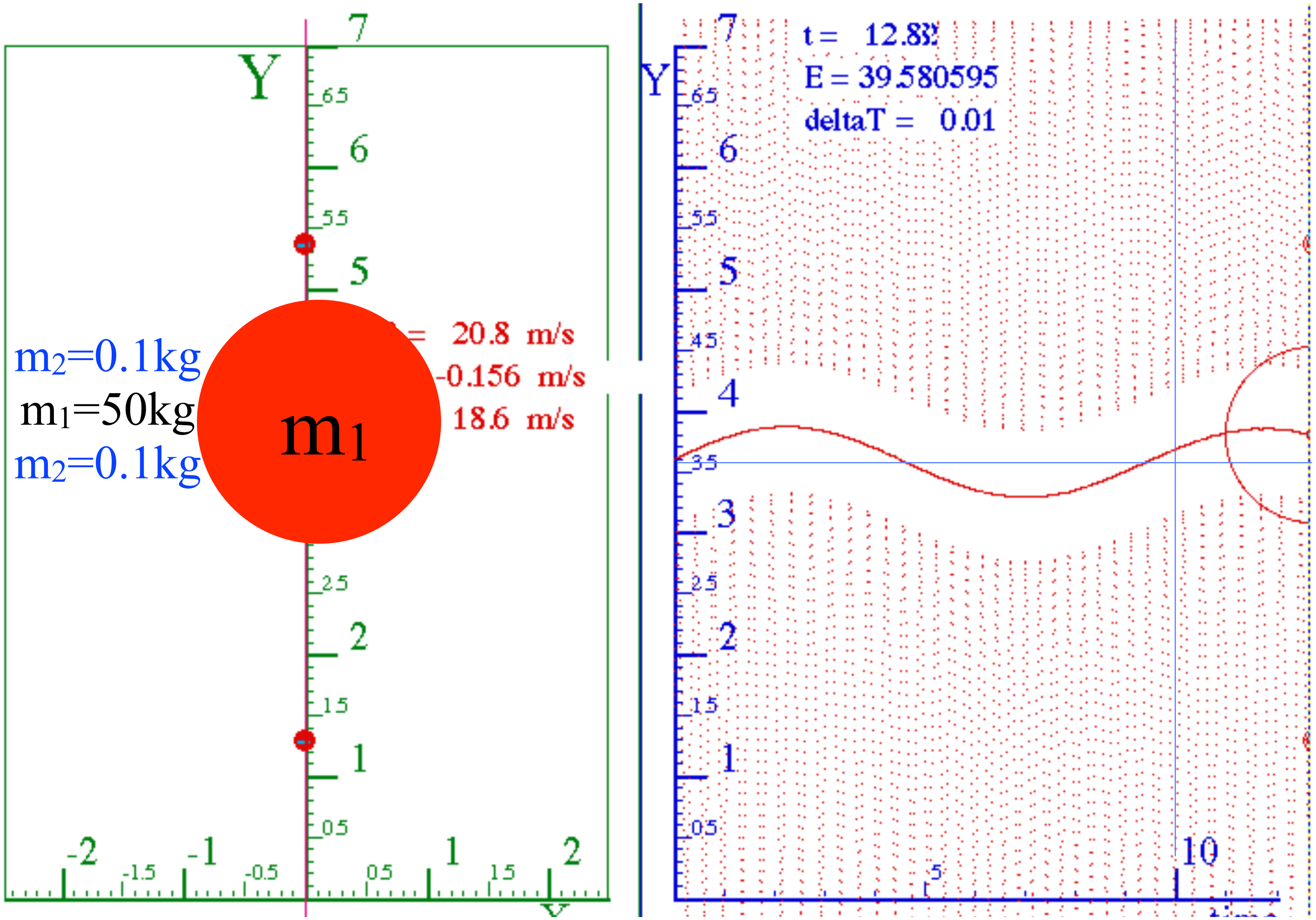
Sample problem: *Compute isothermal frequency and/or period*

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$

Switch
m_1=m_3
with
m_2
to match
formula



BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

Sample problem: *Compute isothermal period given* m_1=50, m_2=0.1=m_3, v_2=20, Y_0=3.5

Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}}\frac{Y_0}{v_2} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}}\frac{3.5}{20}$$

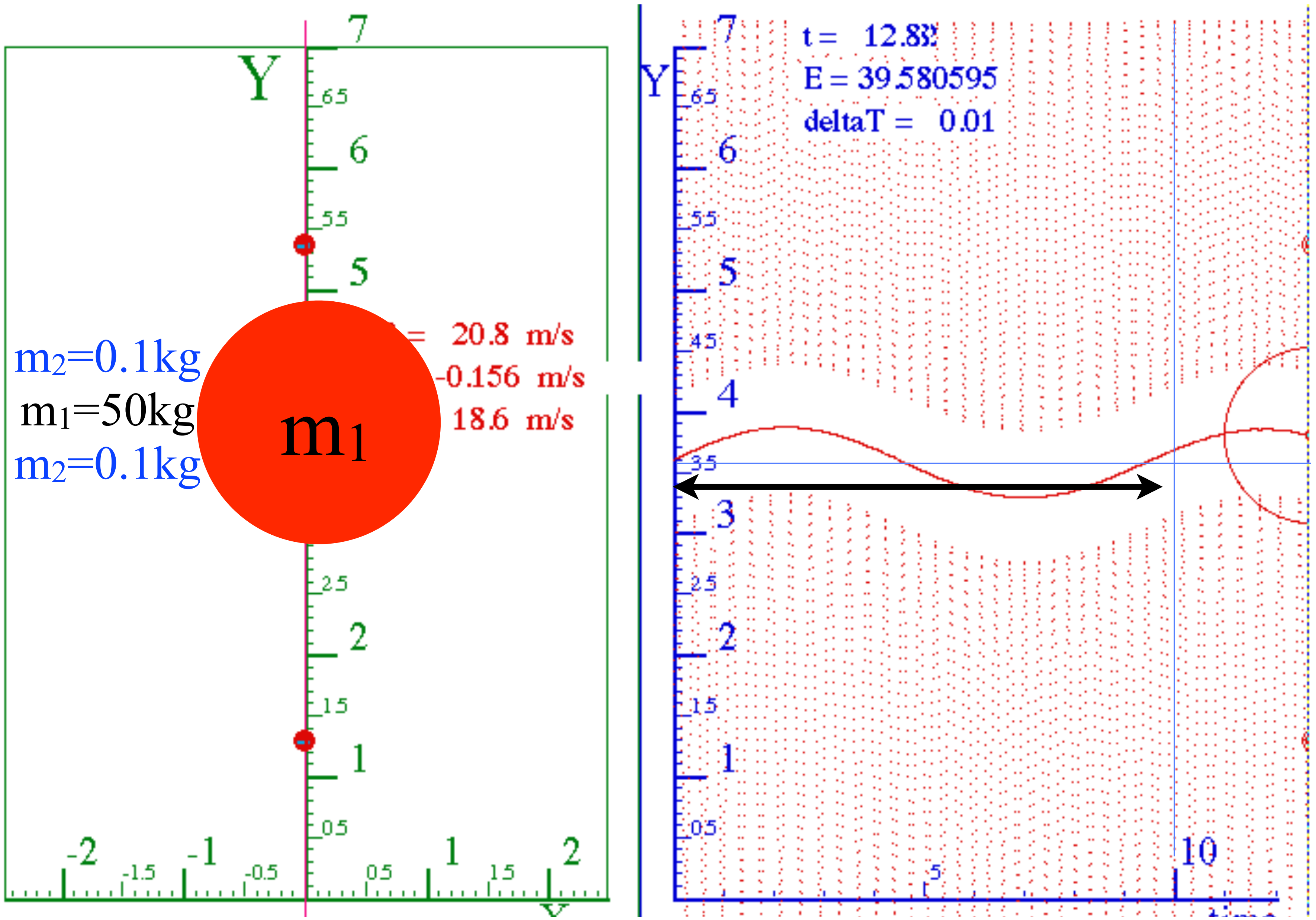
$$=17.38$$

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}}\frac{Y_0}{v_2}$

Frequency

HO $\measuredangle$frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}}\frac{v_2}{Y_0} = 2\pi\upsilon$

Switch
m_1=m_3
with
m_2
to match
formula



m_2=0.1kg
m_1=50kg
m_2=0.1kg

Simulation of

the adiabatic case

BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

Sample problem: *Compute isothermal period given* $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

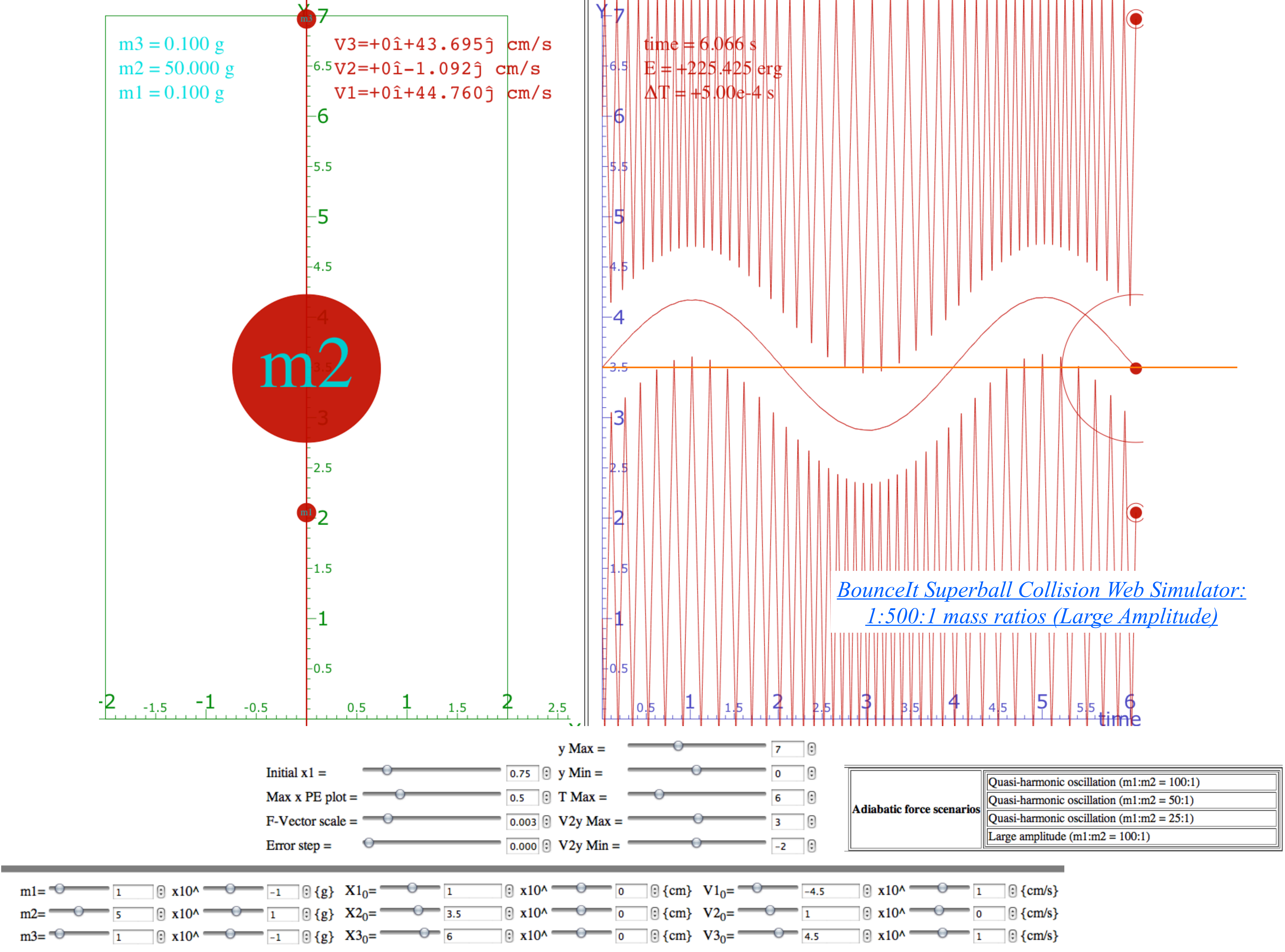
Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$=17.38$ *That's about √3 times too big!*

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$



m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s

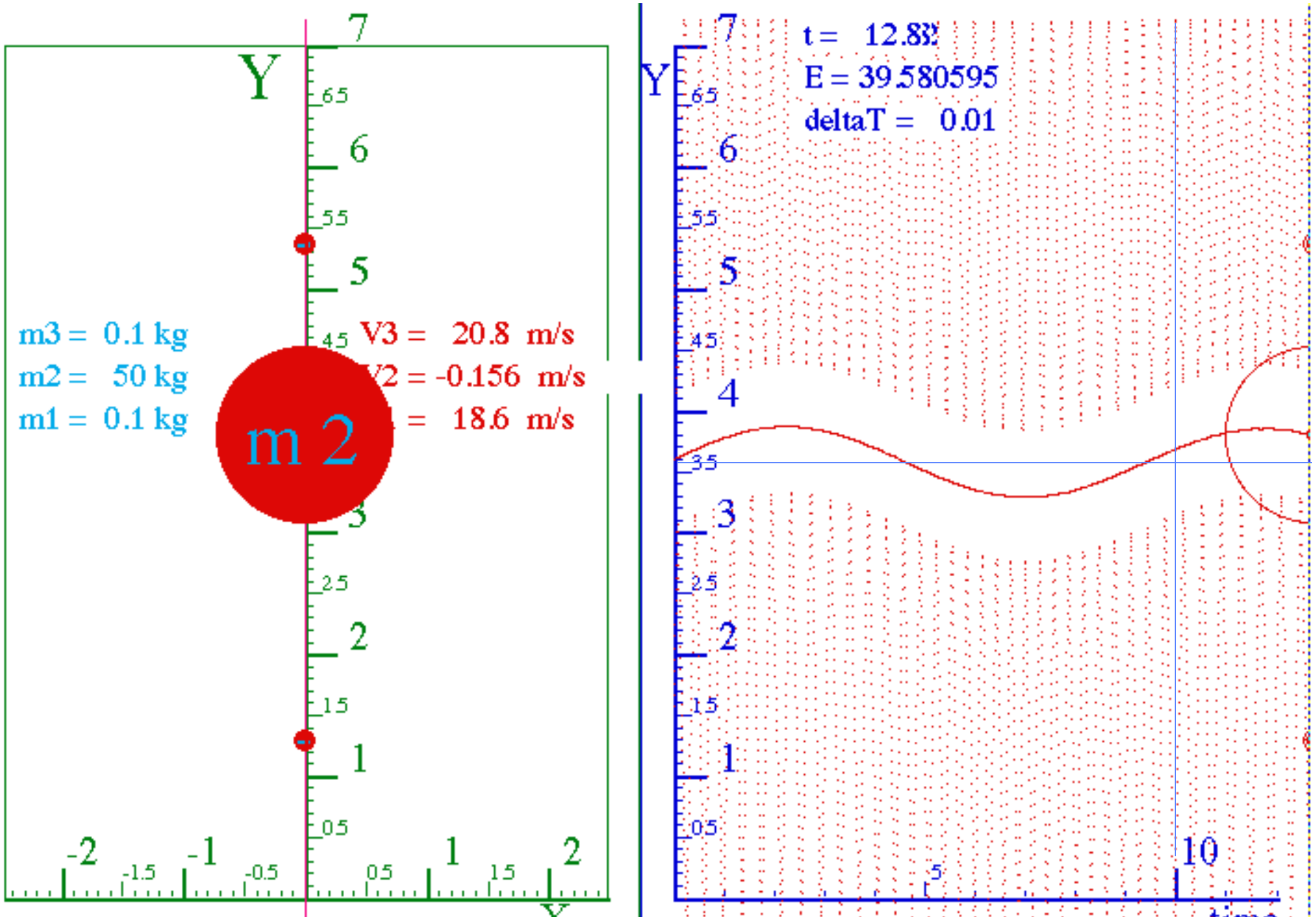
time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s

BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Large Amplitude)

y Max = 7
Initial x1 = 0.75
y Min = 0
Max x PE plot = 0.5
T Max = 6
F-Vector scale = 0.003
V2y Max = 3
Error step = 0.000
V2y Min = -2

Adiabatic force scenarios	Quasi-harmonic oscillation (m1:m2 = 100:1) Quasi-harmonic oscillation (m1:m2 = 50:1) Quasi-harmonic oscillation (m1:m2 = 25:1) Large amplitude (m1:m2 = 100:1)

m1= 1 x10^ -1 {g} X1$_0$= 1 x10^ 0 {cm} V1$_0$= -4.5 x10^ 1 {cm/s}
m2= 5 x10^ 1 {g} X2$_0$= 3.5 x10^ 0 {cm} V2$_0$= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} X3$_0$= 6 x10^ 0 {cm} V3$_0$= 4.5 x10^ 1 {cm/s}



m3 = 0.1 kg
m2 = 50 kg
m1 = 0.1 kg

V3 = 20.8 m/s
V2 = -0.156 m/s
= 18.6 m/s

t = 12.83
E = 39.580595
deltaT = 0.01

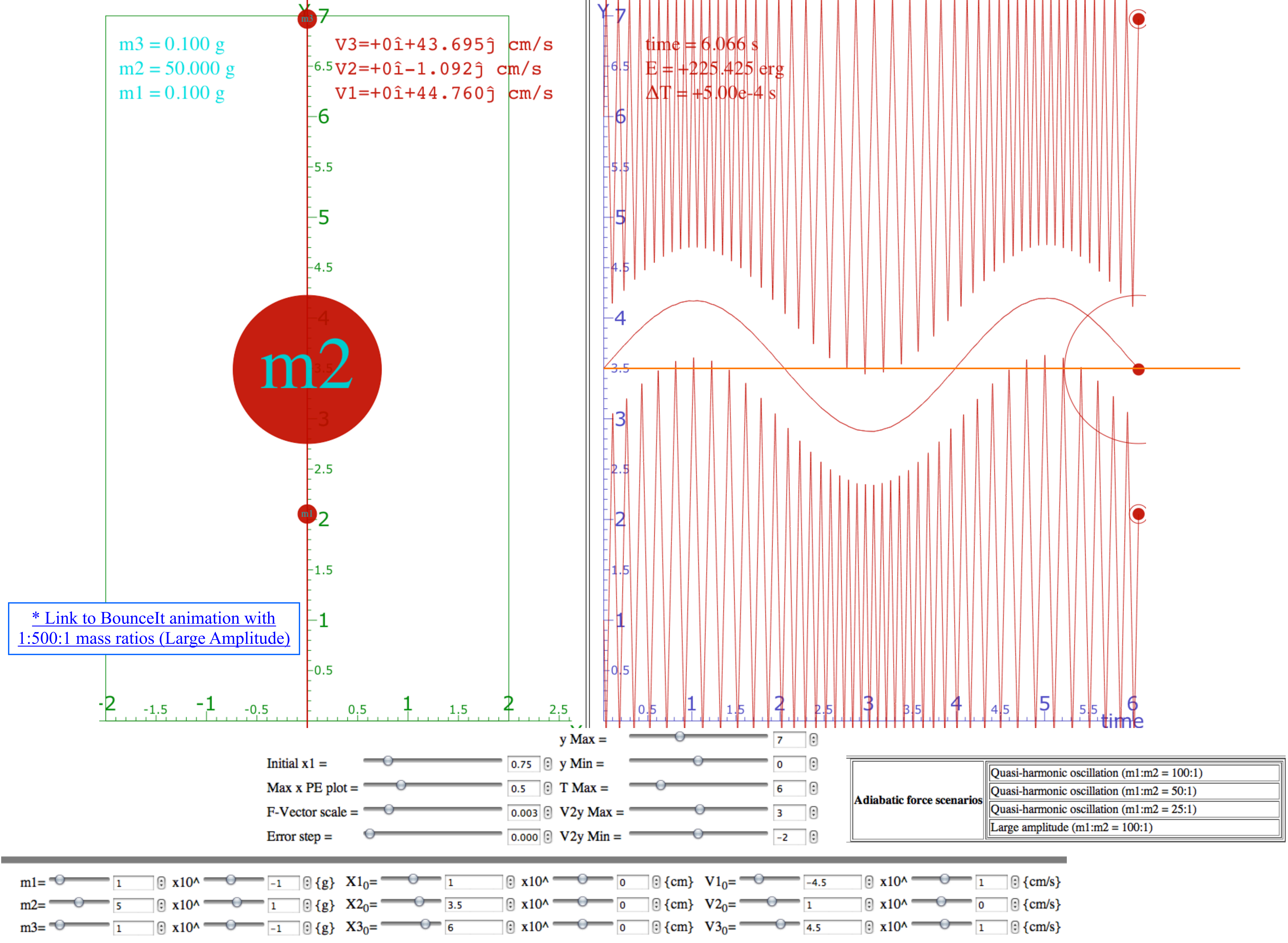
Unit 1
Fig. 6.3

Simulation of

the adiabatic case

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]

See Homework problem 1.6.5: *Compute frequency and/or period for both isoT and adiabatic cases*



m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s

time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s

* Link to BounceIt animation with 1:500:1 mass ratios (Large Amplitude)

y Max = 7

Initial x1 = 0.75
y Min = 0

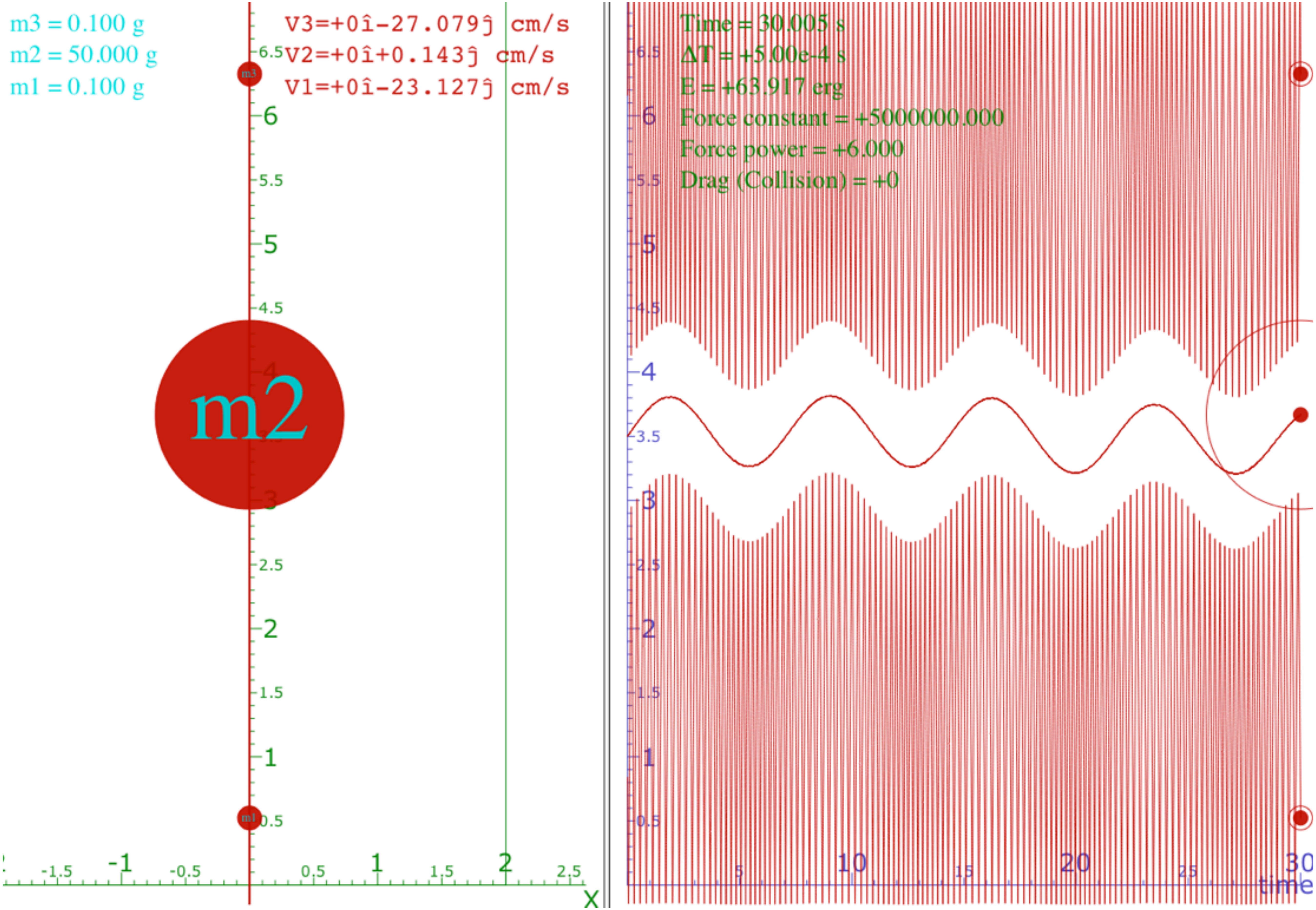
Max x PE plot = 0.5
T Max = 6

F-Vector scale = 0.003
V2y Max = 3

Error step = 0.000
V2y Min = -2

Adiabatic force scenarios	Quasi-harmonic oscillation (m1:m2 = 100:1) Quasi-harmonic oscillation (m1:m2 = 50:1) Quasi-harmonic oscillation (m1:m2 = 25:1) Large amplitude (m1:m2 = 100:1)

m1= 1 x10^ -1 {g} $X1_0$= 1 x10^ 0 {cm} $V1_0$= -4.5 x10^ 1 {cm/s}
m2= 5 x10^ 1 {g} $X2_0$= 3.5 x10^ 0 {cm} $V2_0$= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} $X3_0$= 6 x10^ 0 {cm} $V3_0$= 4.5 x10^ 1 {cm/s}

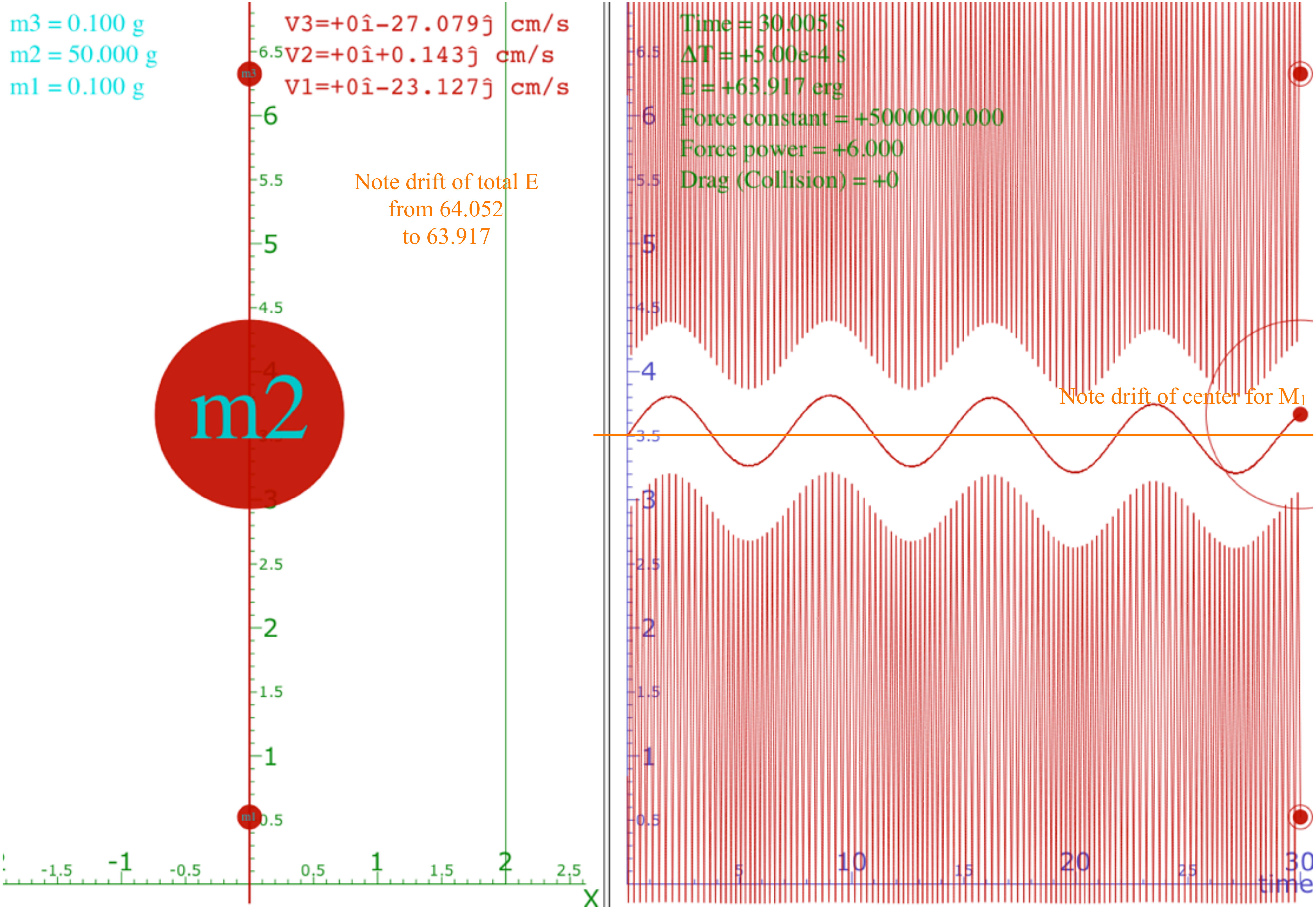


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

V3=+0î-27.079ĵ cm/s
V2=+0î+0.143ĵ cm/s
V1=+0î-23.127ĵ cm/s

Time = 30.005 s
ΔT = +5.00e-4 s
E = +63.917 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

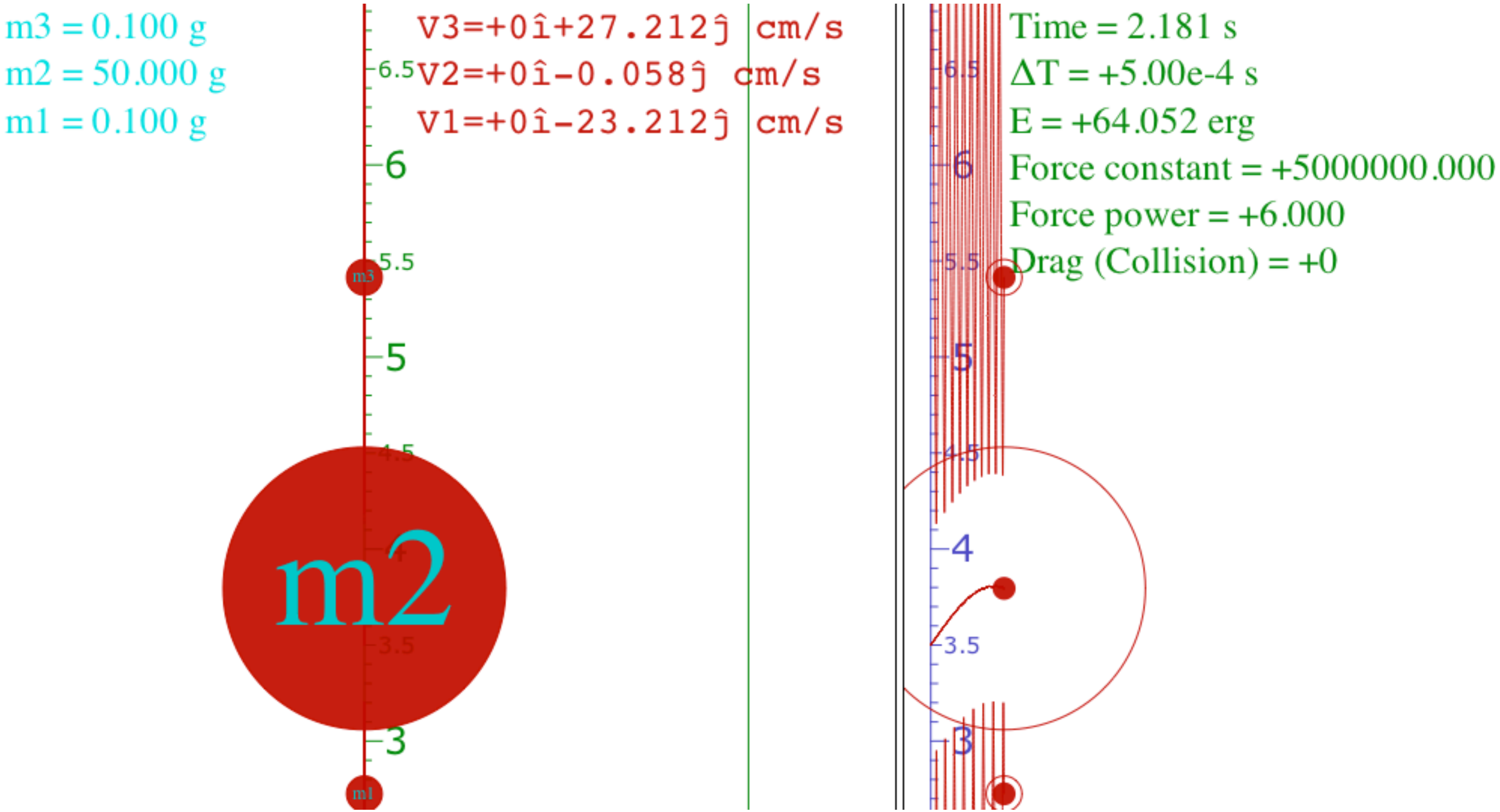
V3=+0î-27.079ĵ cm/s
V2=+0î+0.143ĵ cm/s
V1=+0î-23.127ĵ cm/s

Time = 30.005 s
ΔT = +5.00e-4 s
E = +63.917 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0

Note drift of total E
from 64.052
to 63.917

Note drift of center for M_1

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

V3=+0î+27.212ĵ cm/s
V2=+0î-0.058ĵ cm/s
V1=+0î-23.212ĵ cm/s

Time = 2.181 s
ΔT = +5.00e-4 s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0

“Monster Mash”classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

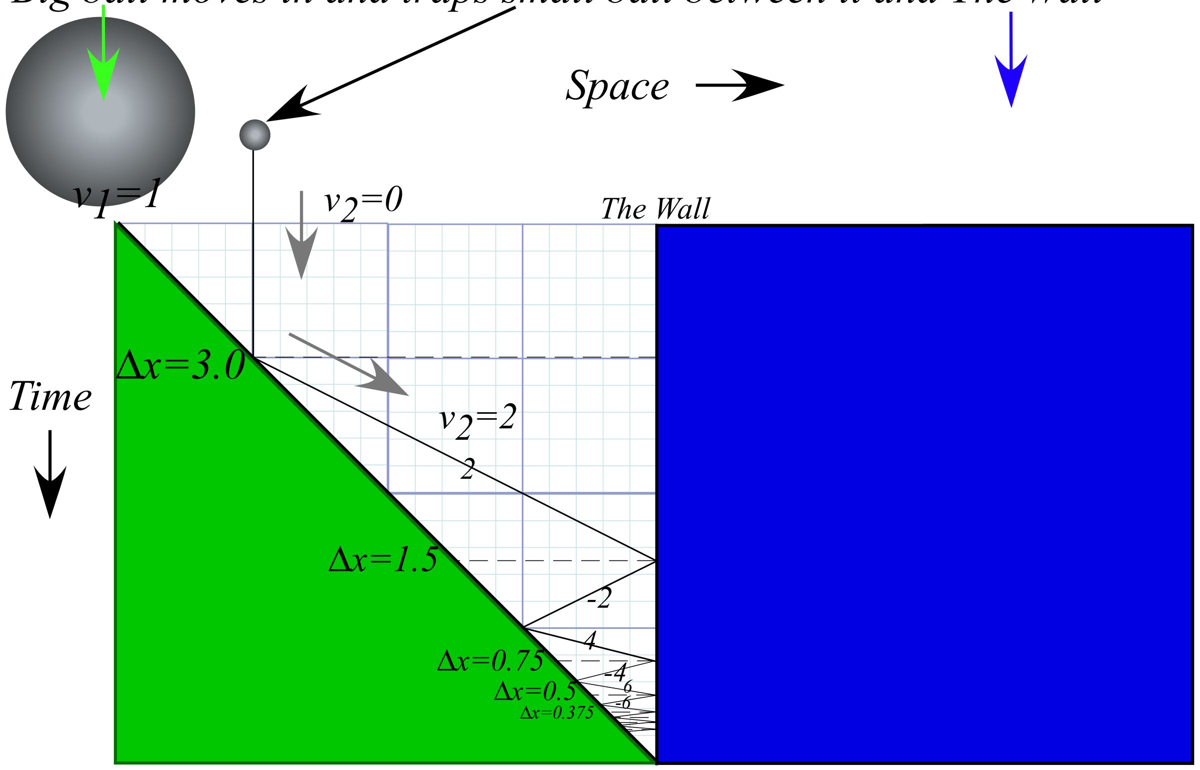
How m_2 keeps its action

An interesting wave analogy: The “Tiny-Big-Bang” [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

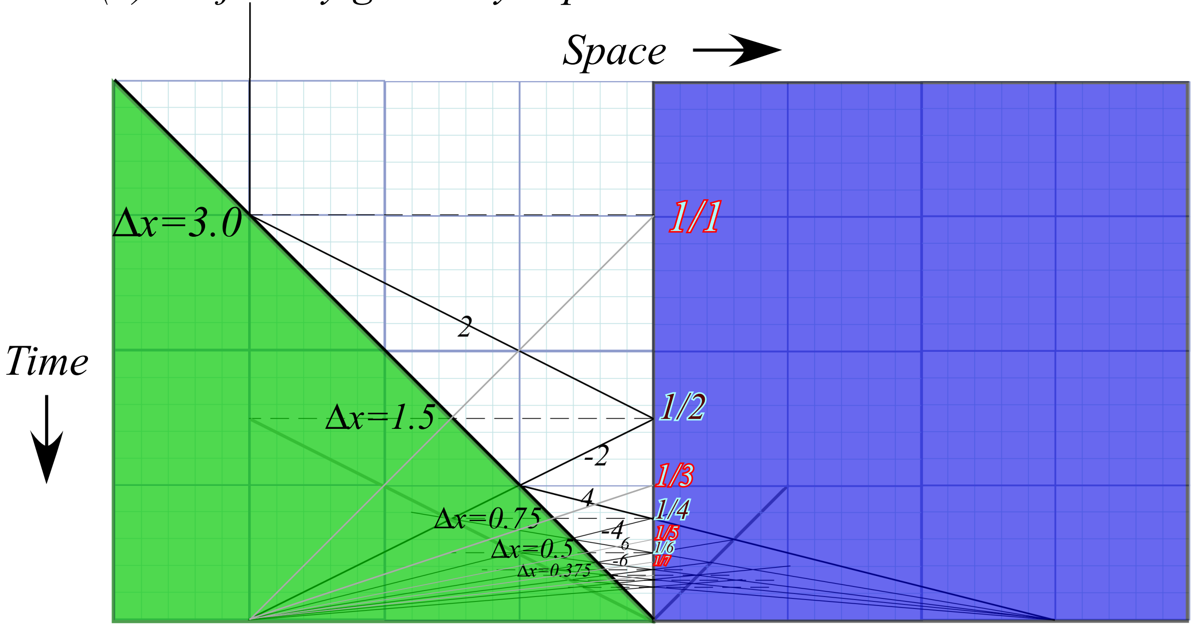
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

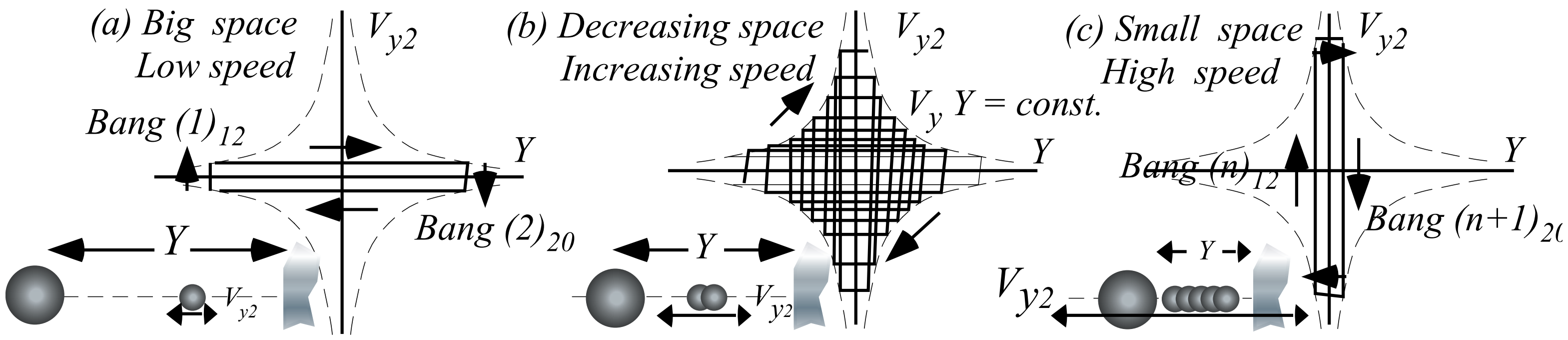
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed





The Classical “Monster Mash”

Classical introduction to

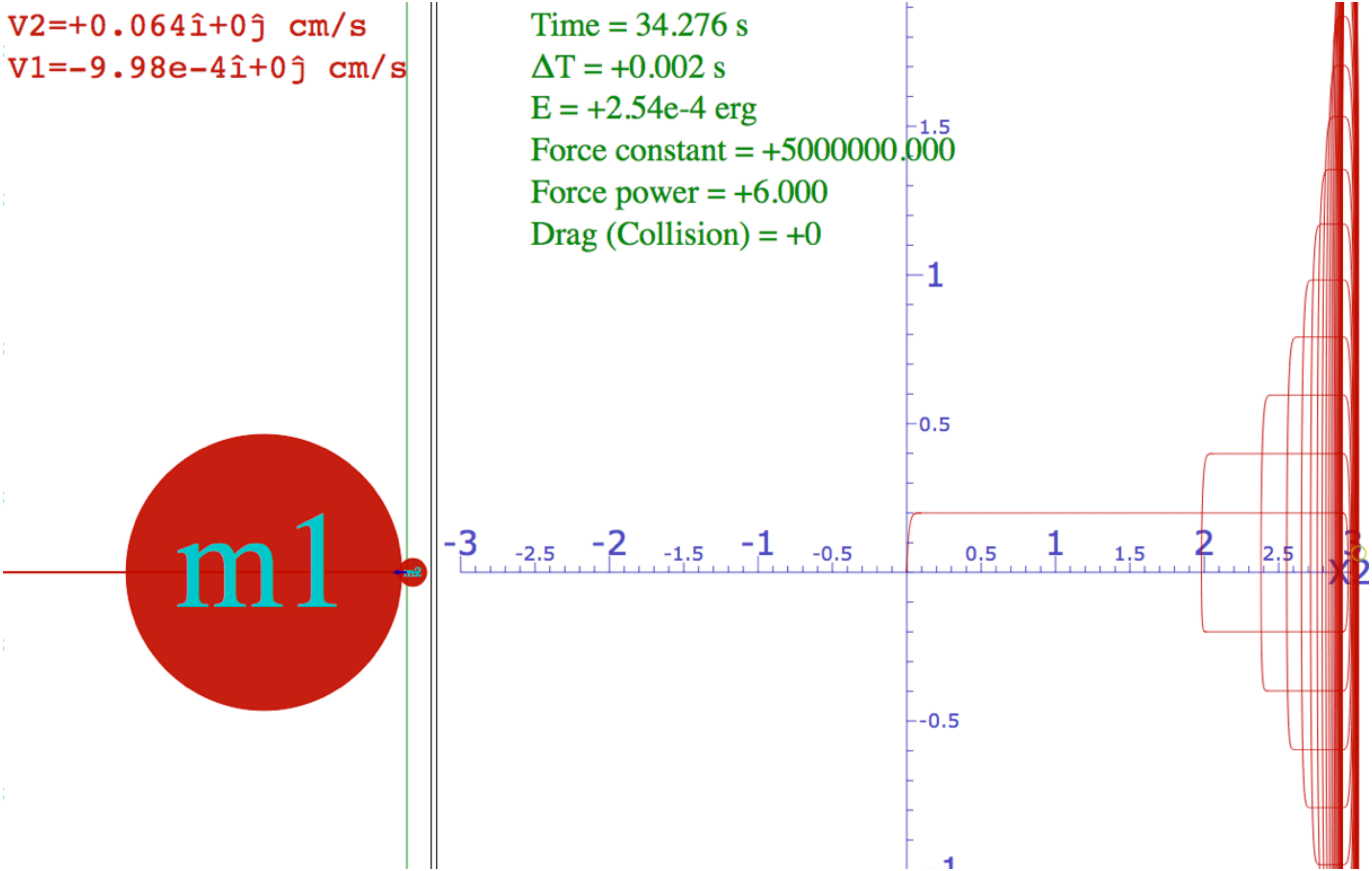
Heisenberg “Uncertainty” Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

Unit 1
Fig. 6.4

* Link to BounceIt “*Monster Mash*” $x_2(t)$ animation
(Note: Time sense is inverted)



```
V2=+0.064î+0ĵ cm/s
V1=-9.98e-4î+0ĵ cm/s
```

Time = 34.276 s

ΔT = +0.002 s

E = +2.54e-4 erg

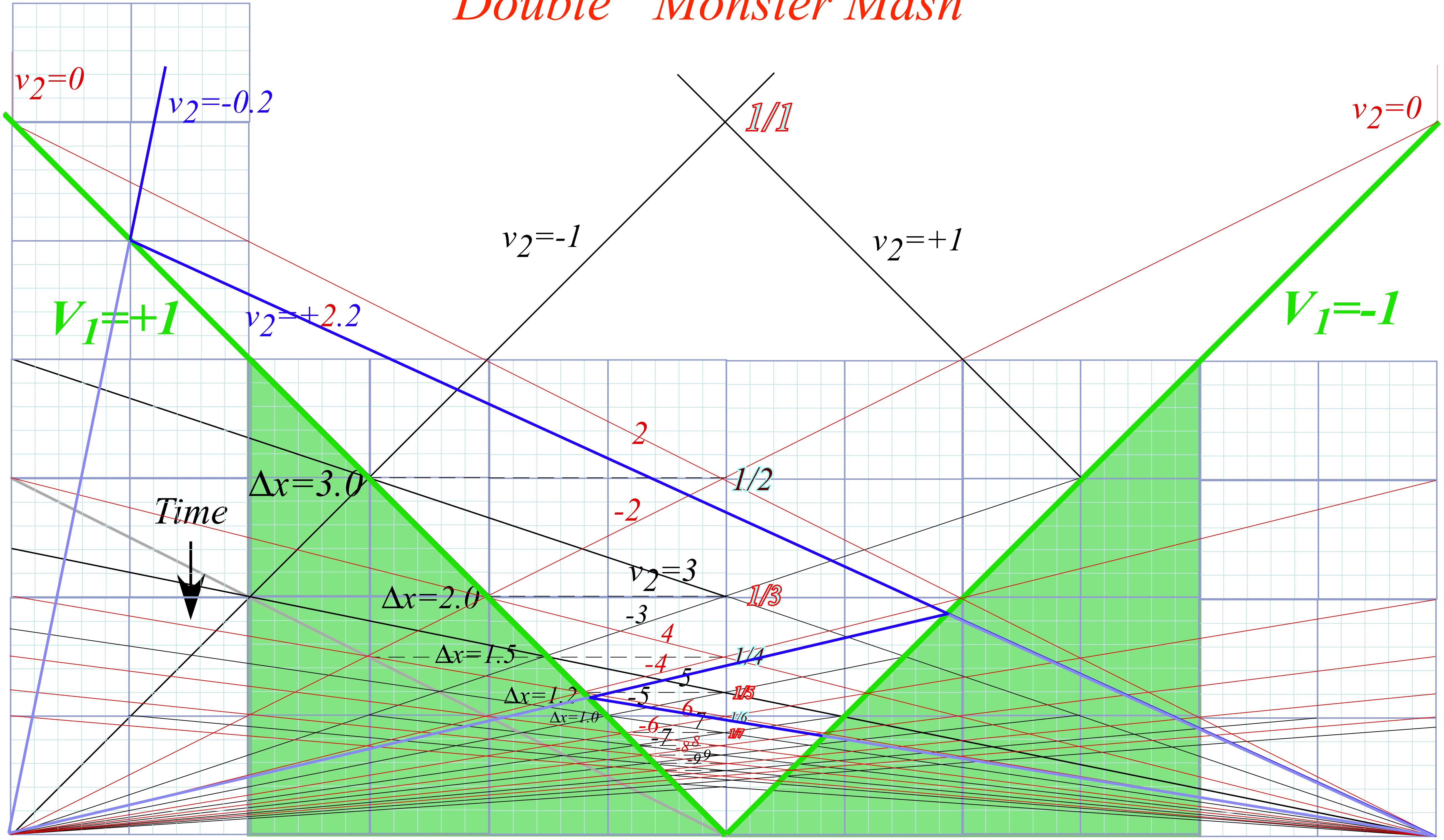
Force constant = +5000000.000

Force power = +6.000

Drag (Collision) = +0

* Link to BounceIt “*Monster Mash*” Vx_2 vs x_2 animation

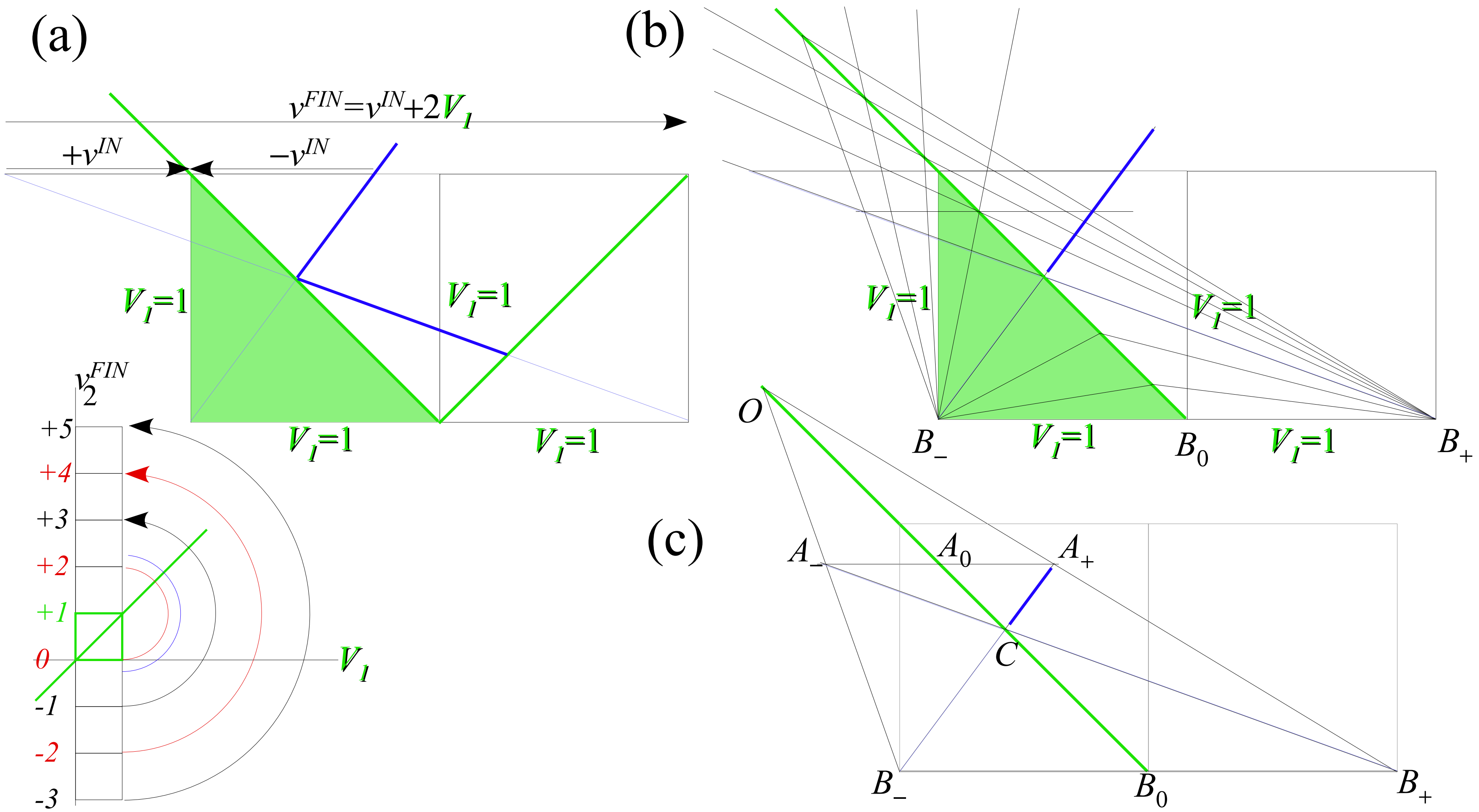
Double "Monster Mash"

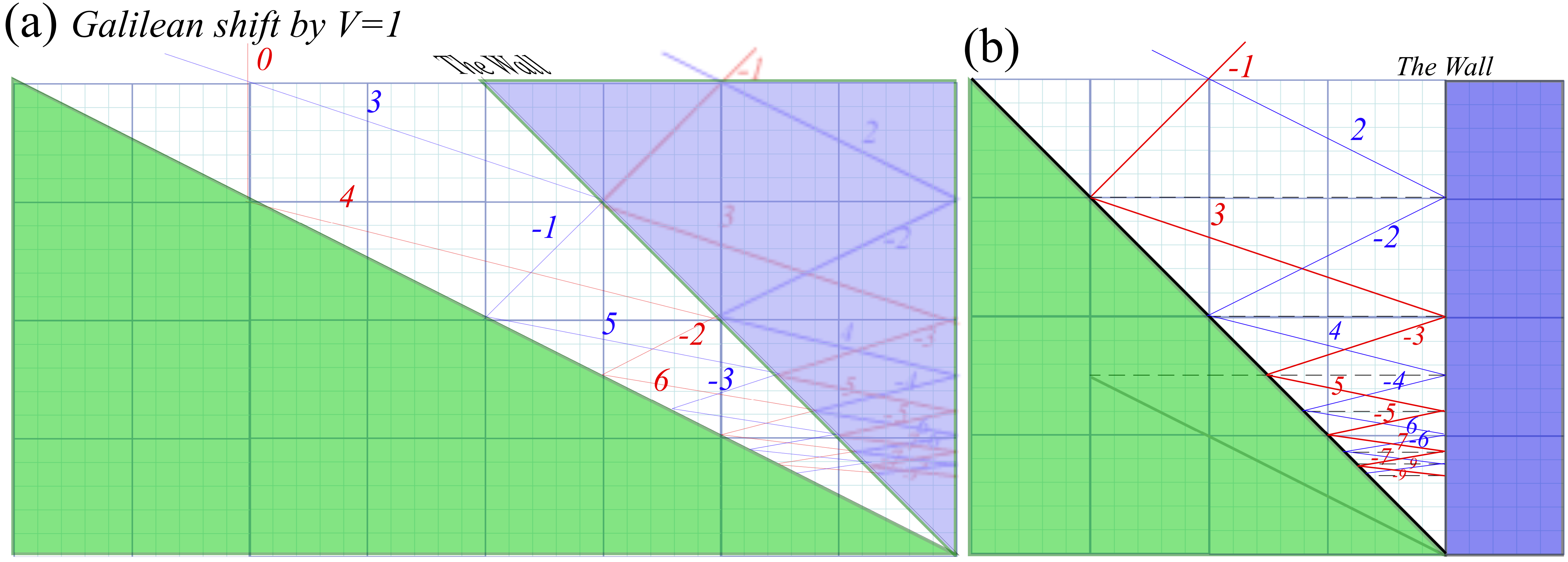


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





“Monster Mash”classical segue to Heisenberg action relations

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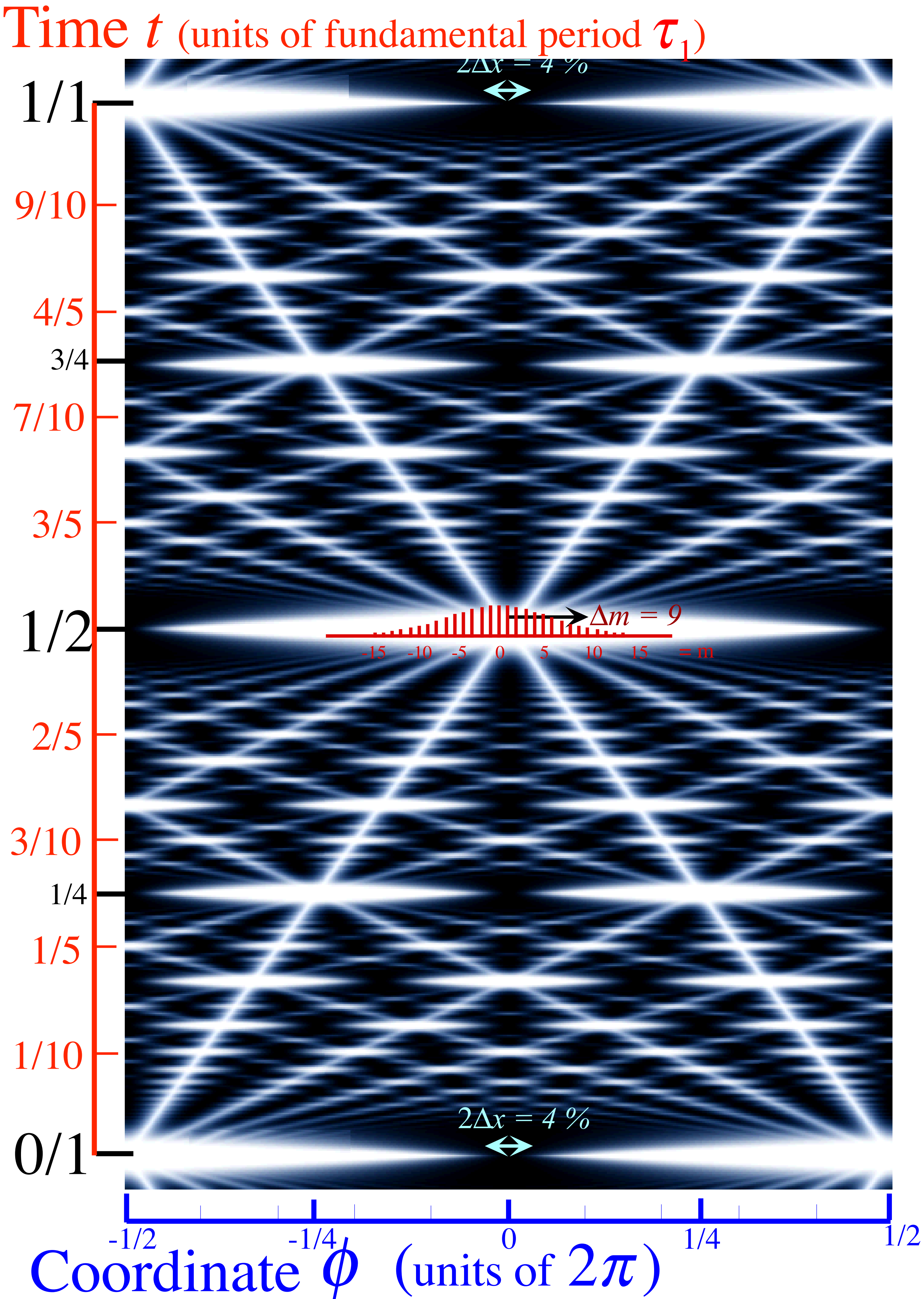
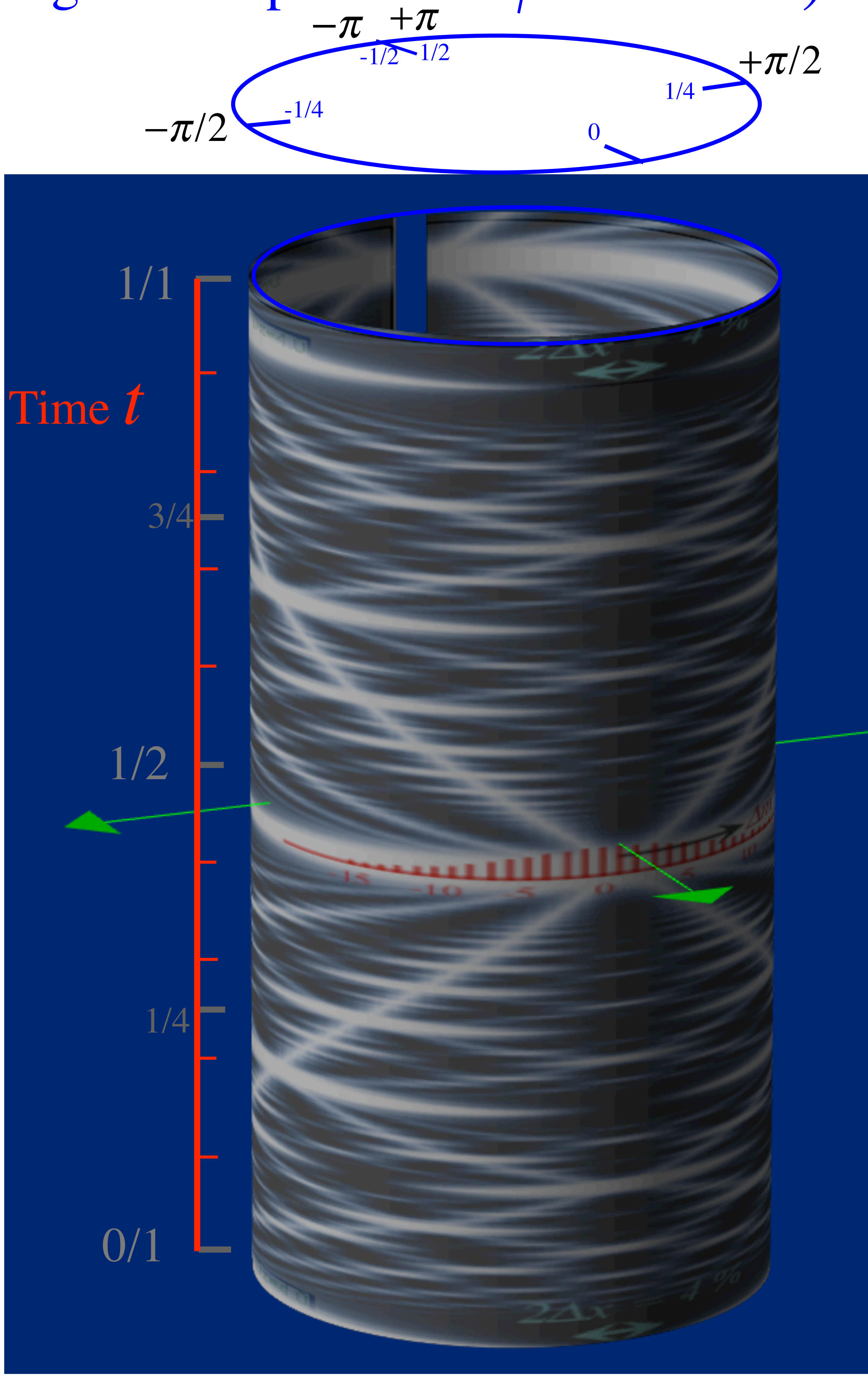


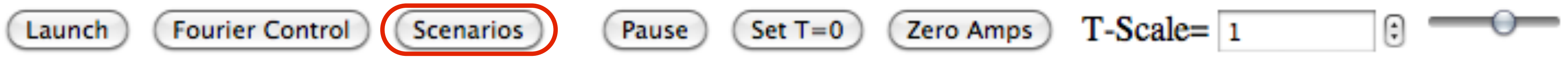
(Imagine "wrap-around" ϕ-coordinate)



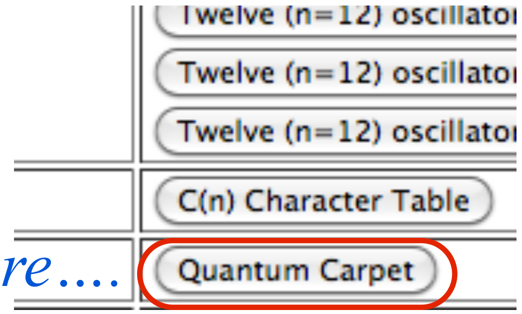
WaveIt Web simulation

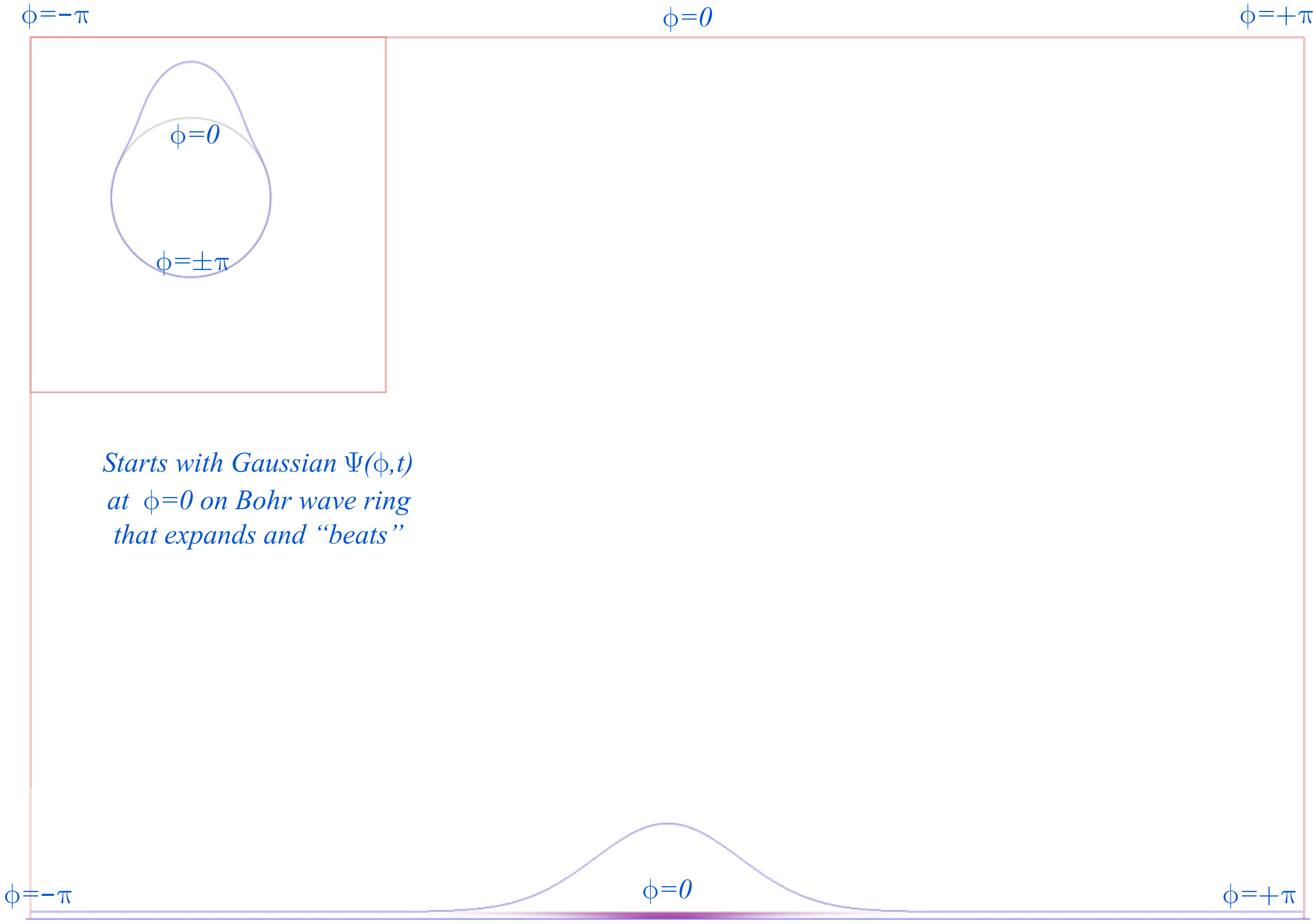
http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

Click here....



..then here....





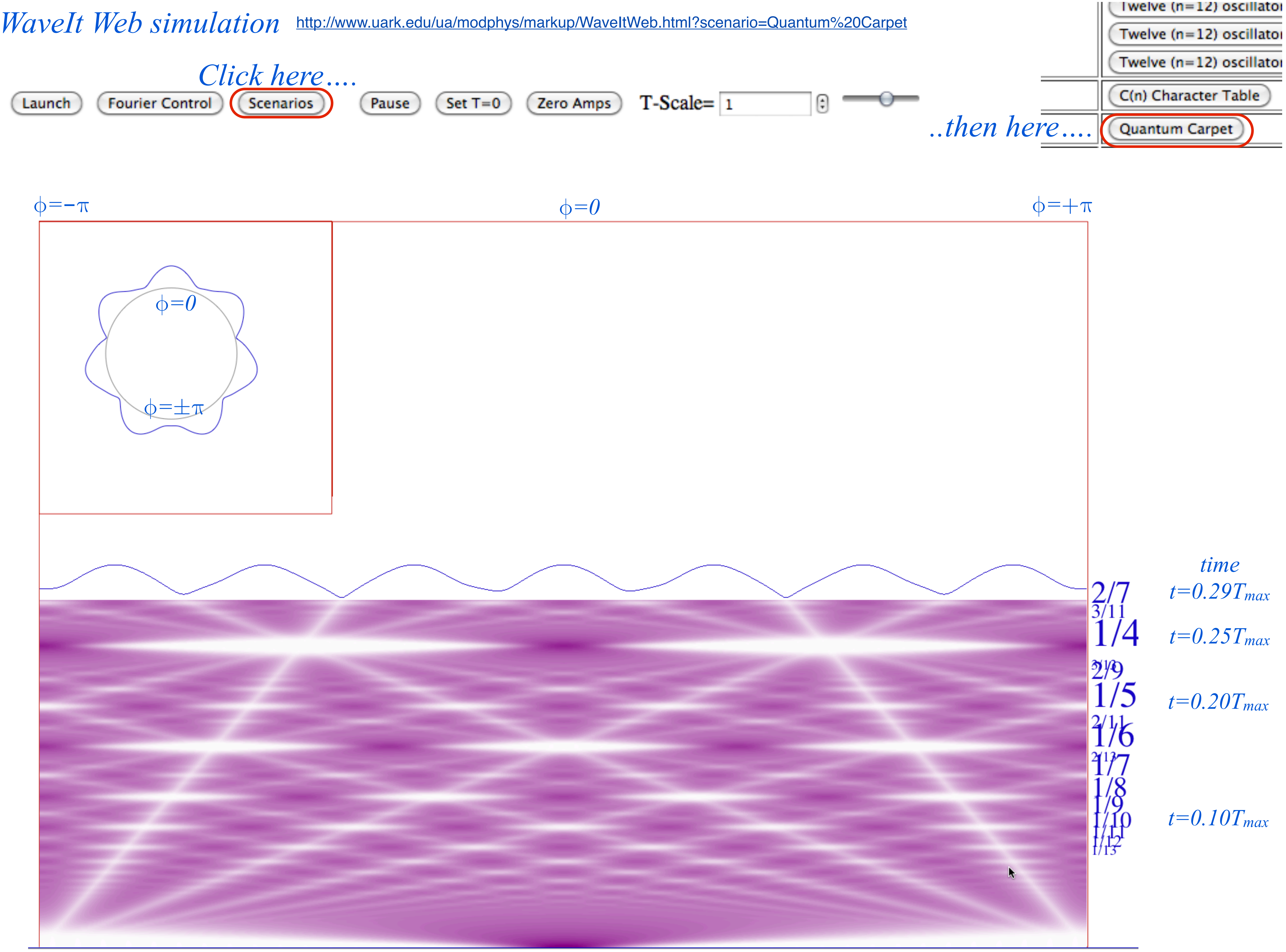
Starts with Gaussian $\Psi(\phi,t)$
at $\phi=0$ on Bohr wave ring
that expands and "beats"

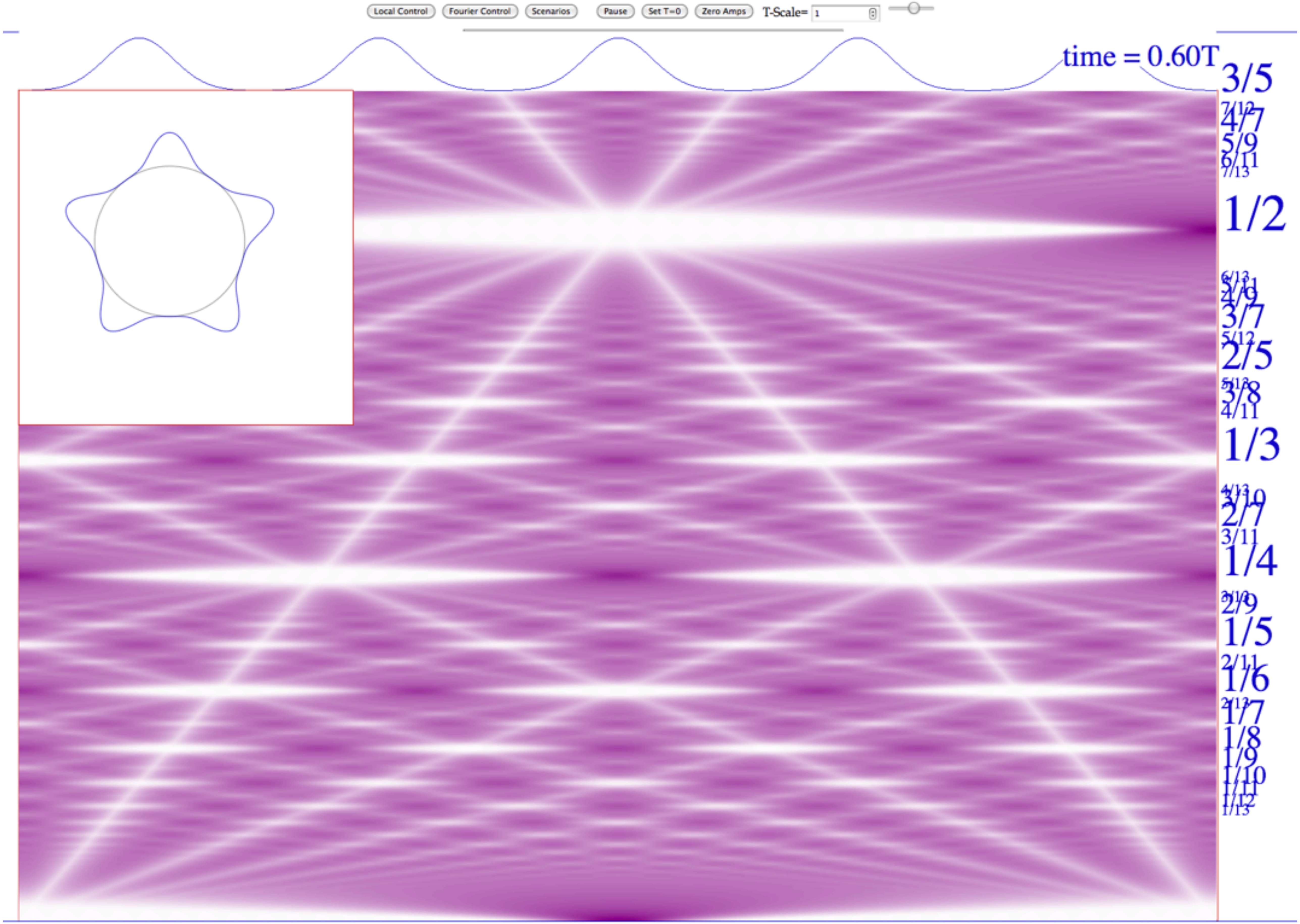
WaveIt Web simulation

http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

Click here....

..then here....




Local Control
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale= 1
time = 0.60T
3/5
7/12
4/7
5/9
6/11
7/13
1/2
6/13
5/11
4/9
3/7
5/12
2/5
5/13
3/8
4/11
1/3
4/13
3/10
2/7
3/11
1/4
3/13
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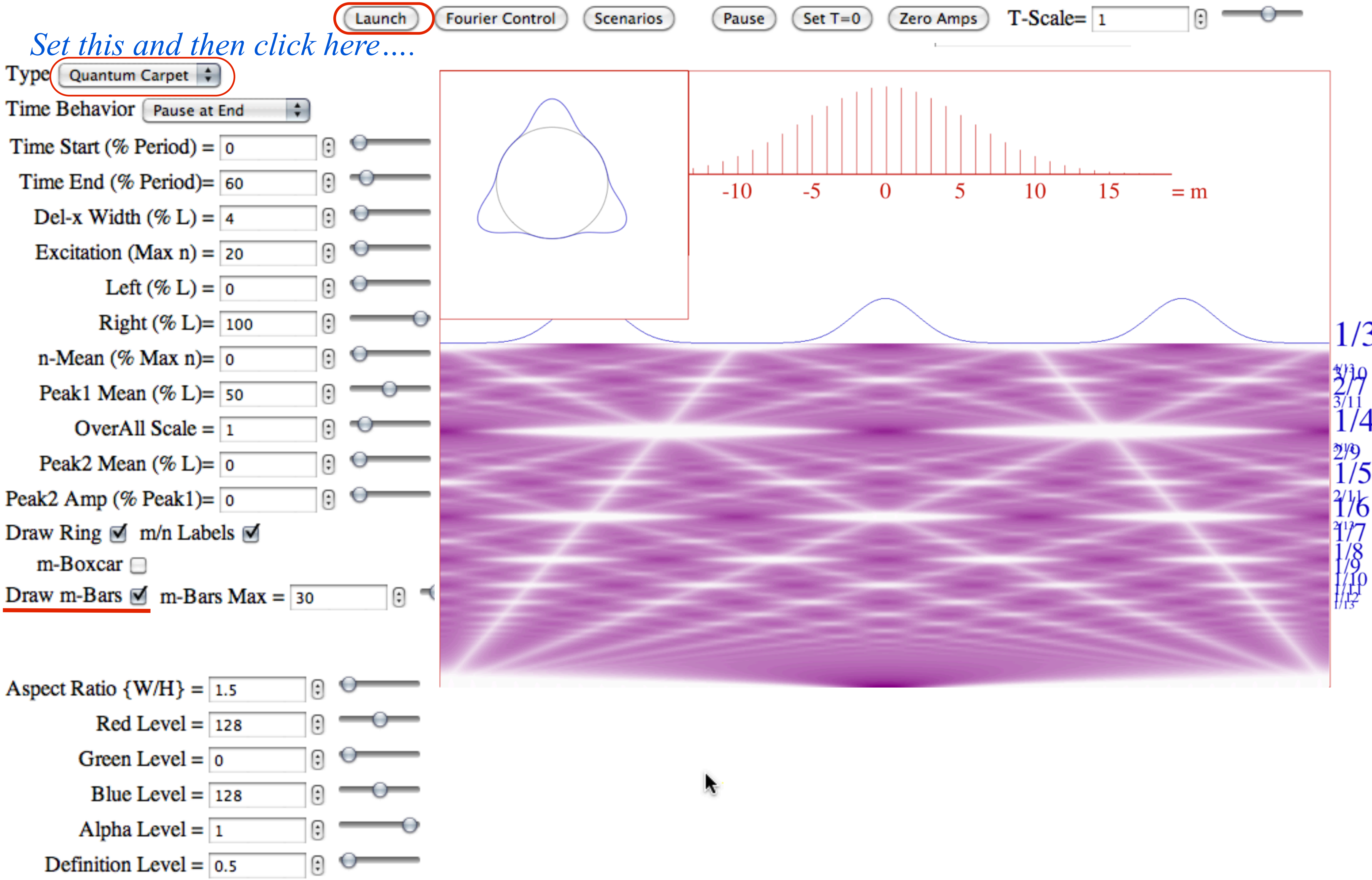
1/10
1/11
1/12
1/13



Set this and then click here....

Type: Quantum Carpet

Time Behavior: Pause at End

T-Scale= 1

Time Start (% Period) = 0

Time End (% Period)= 60

Del-x Width (% L) = 4

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100

n-Mean (% Max n)= 0

Peak1 Mean (% L)= 50

OverAll Scale = 1

Peak2 Mean (% L)= 0

Peak2 Amp (% Peak1)= 0

Draw Ring ☑ m/n Labels ☑

m-Boxcar ☐

Draw m-Bars ☑ m-Bars Max = 30

Aspect Ratio {W/H} = 1.5

Red Level = 128

Green Level = 0

Blue Level = 128

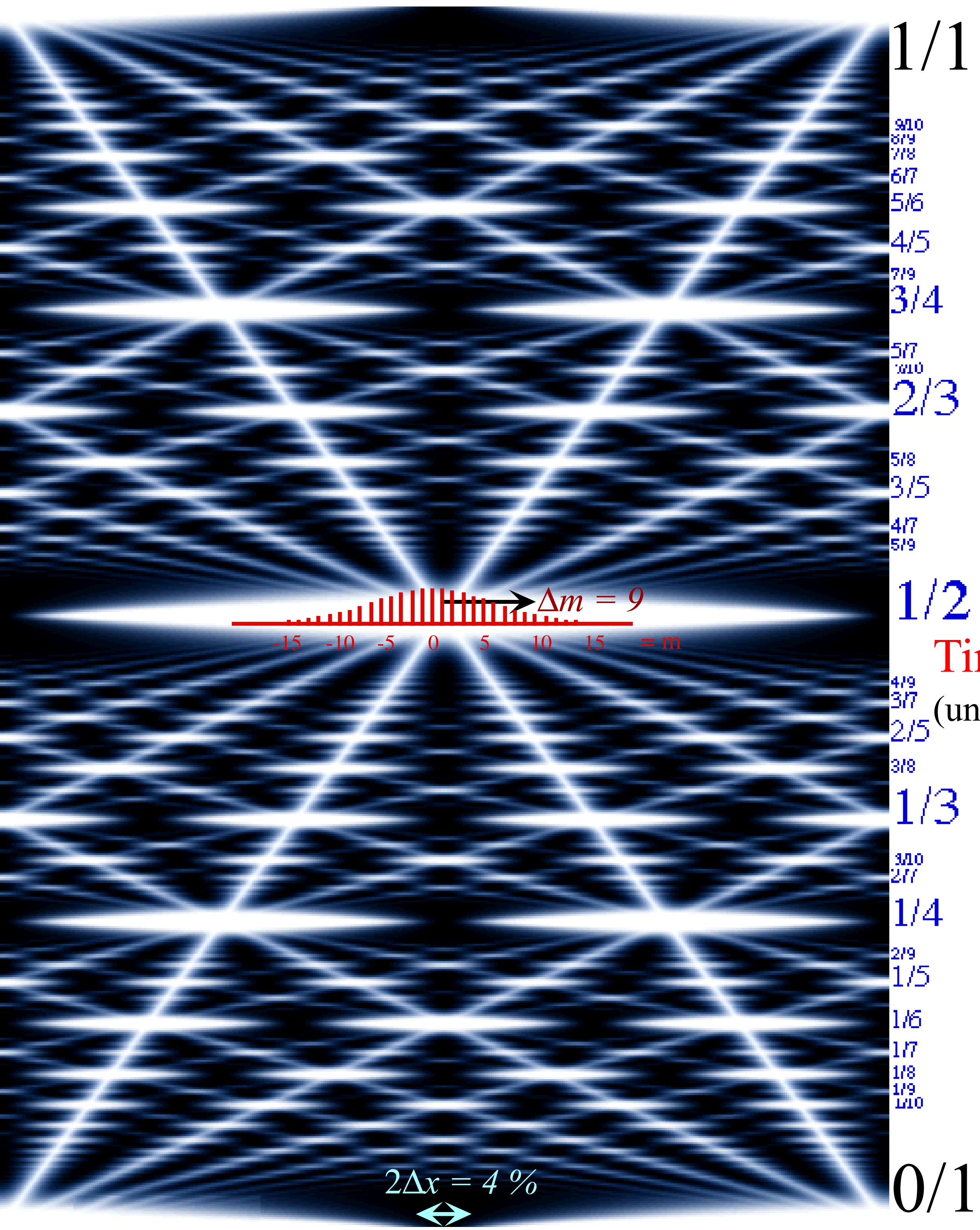
Alpha Level = 1

Definition Level = 0.5

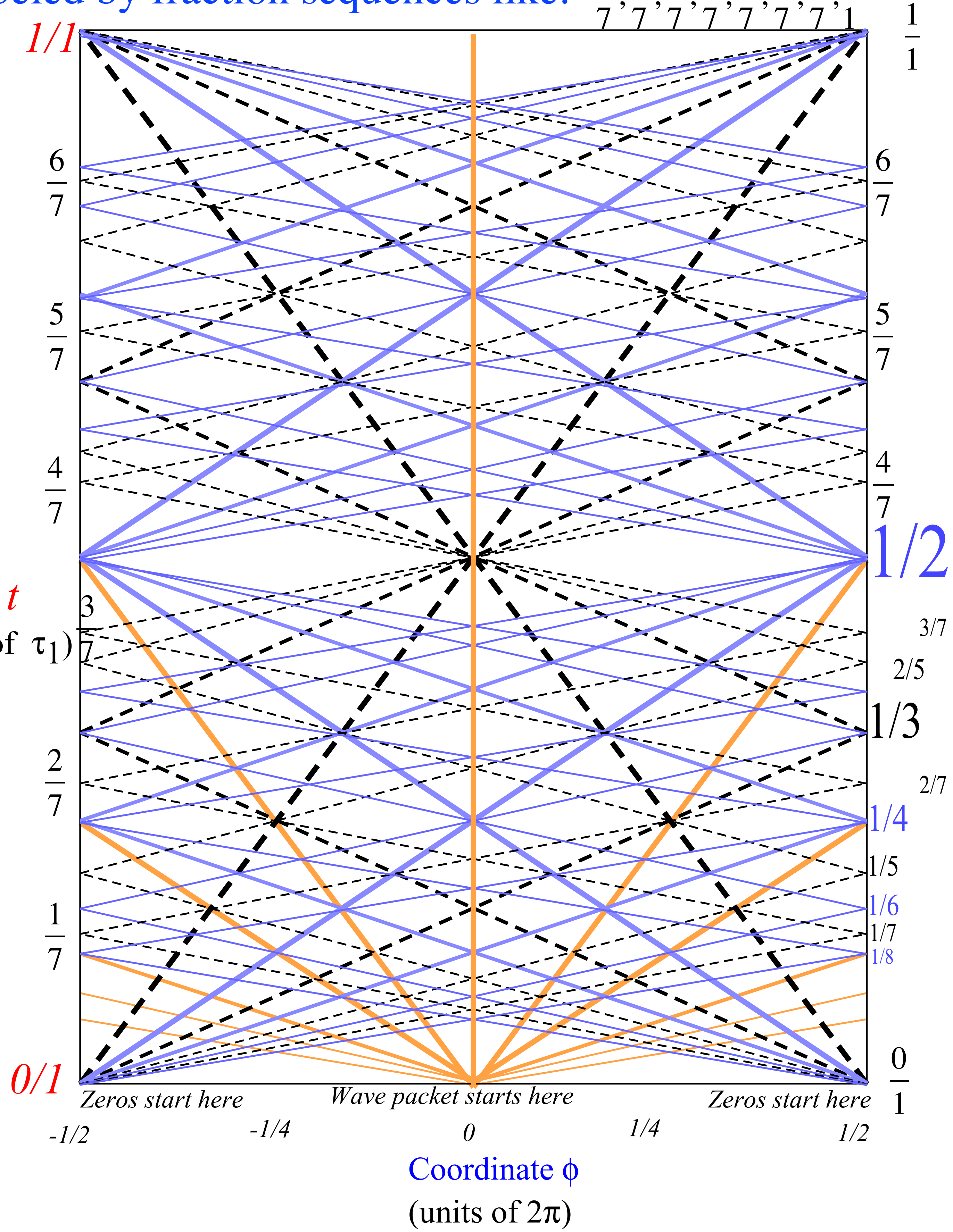
N-level-system and revival-beat wave dynamics

(9 or10-levels $(0, \pm 1, \pm 2, \pm 3, \pm 4, ..., \pm 9, \pm 10, \pm 11 ...)$ excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$

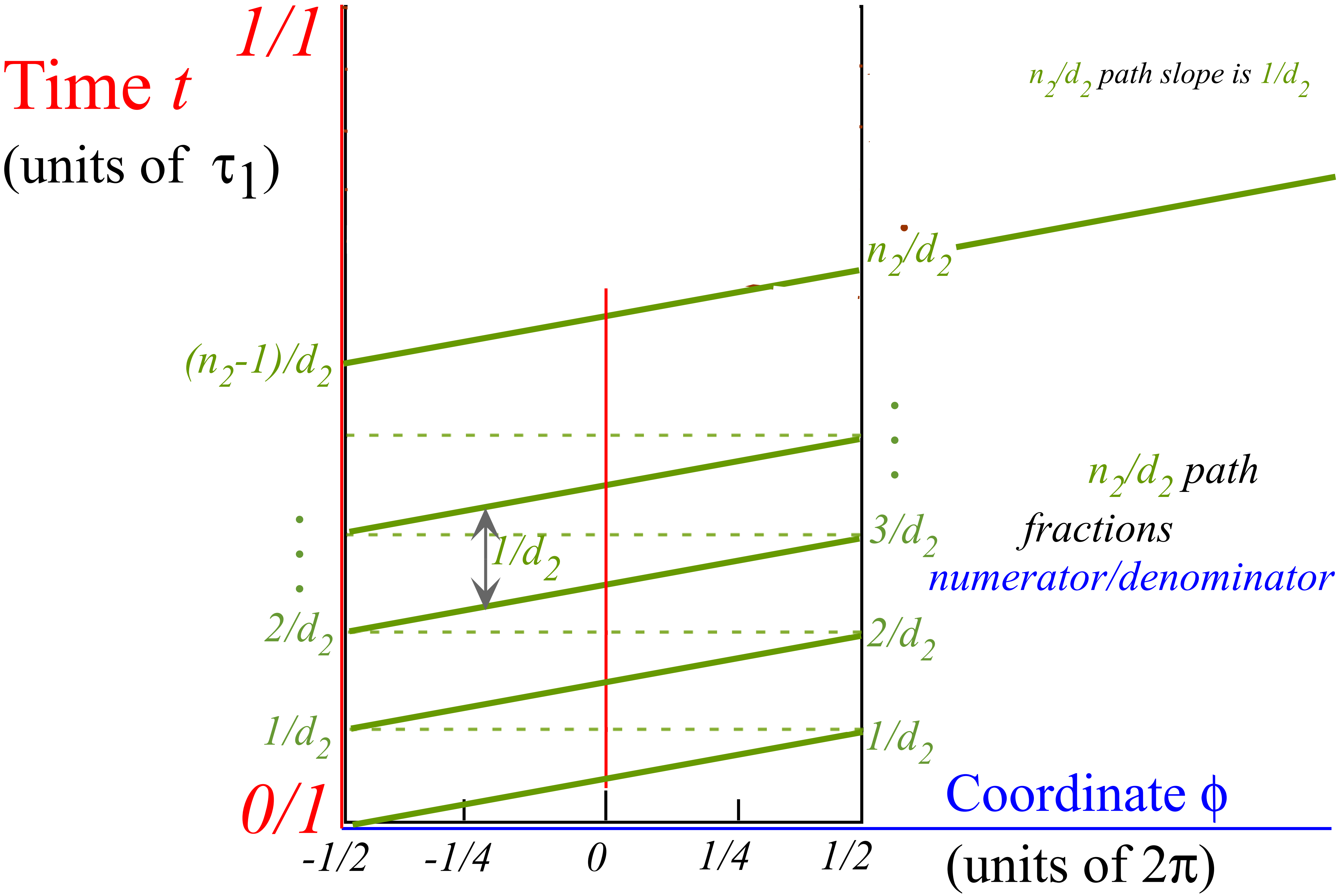


Time *t* (units of τ_1)



Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



Farey Sum algebra of revival-beat wave dynamics

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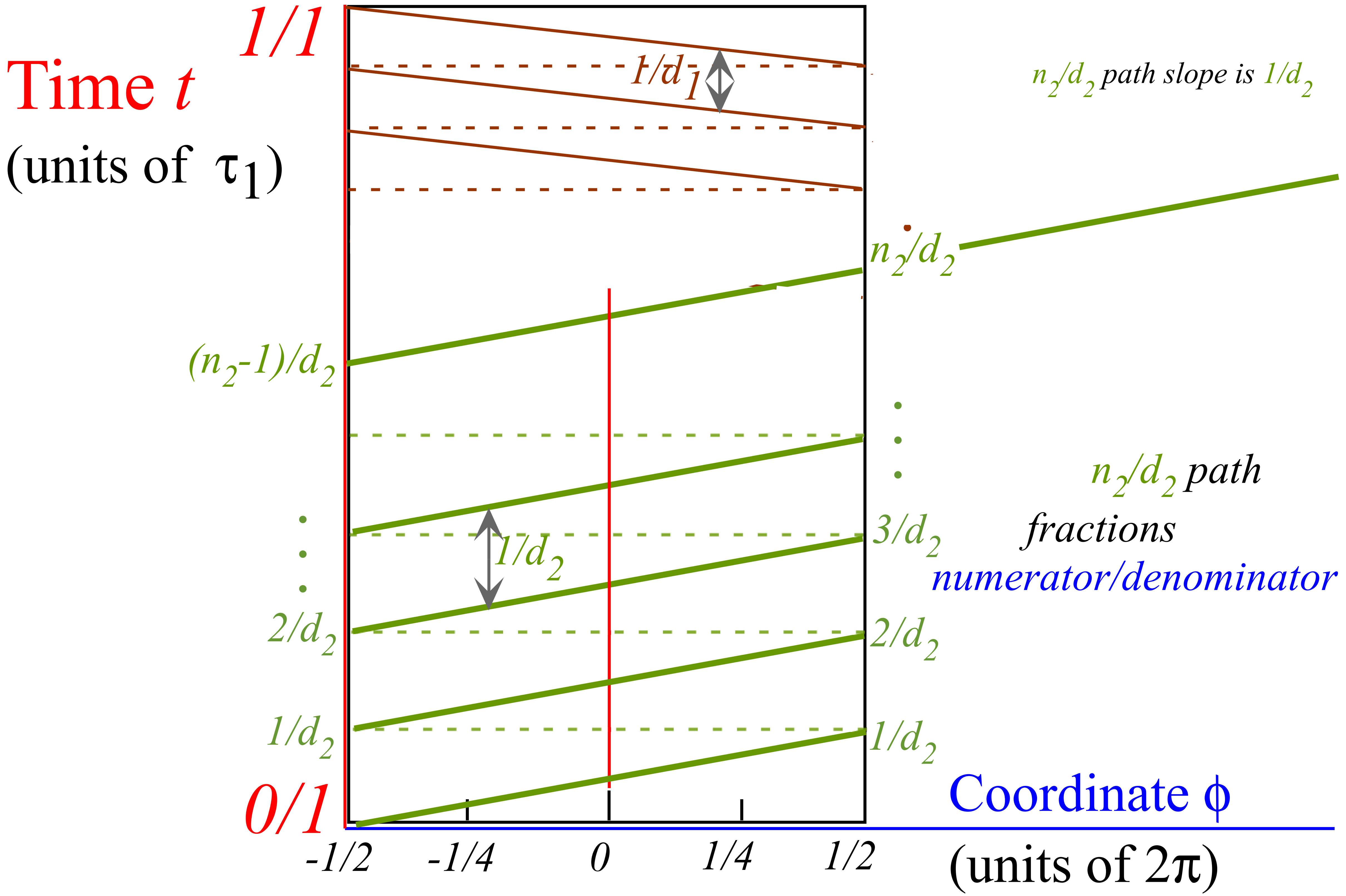
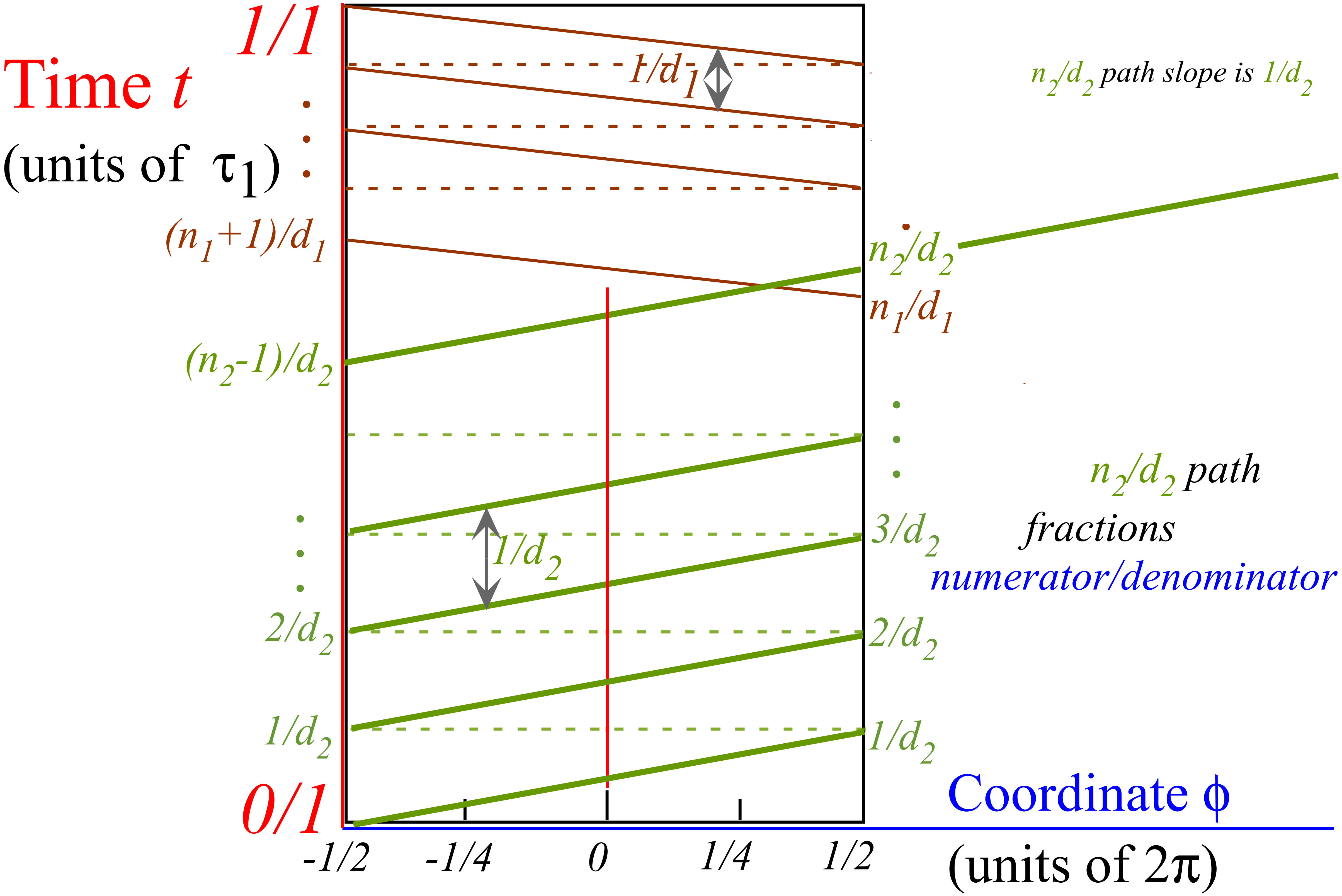


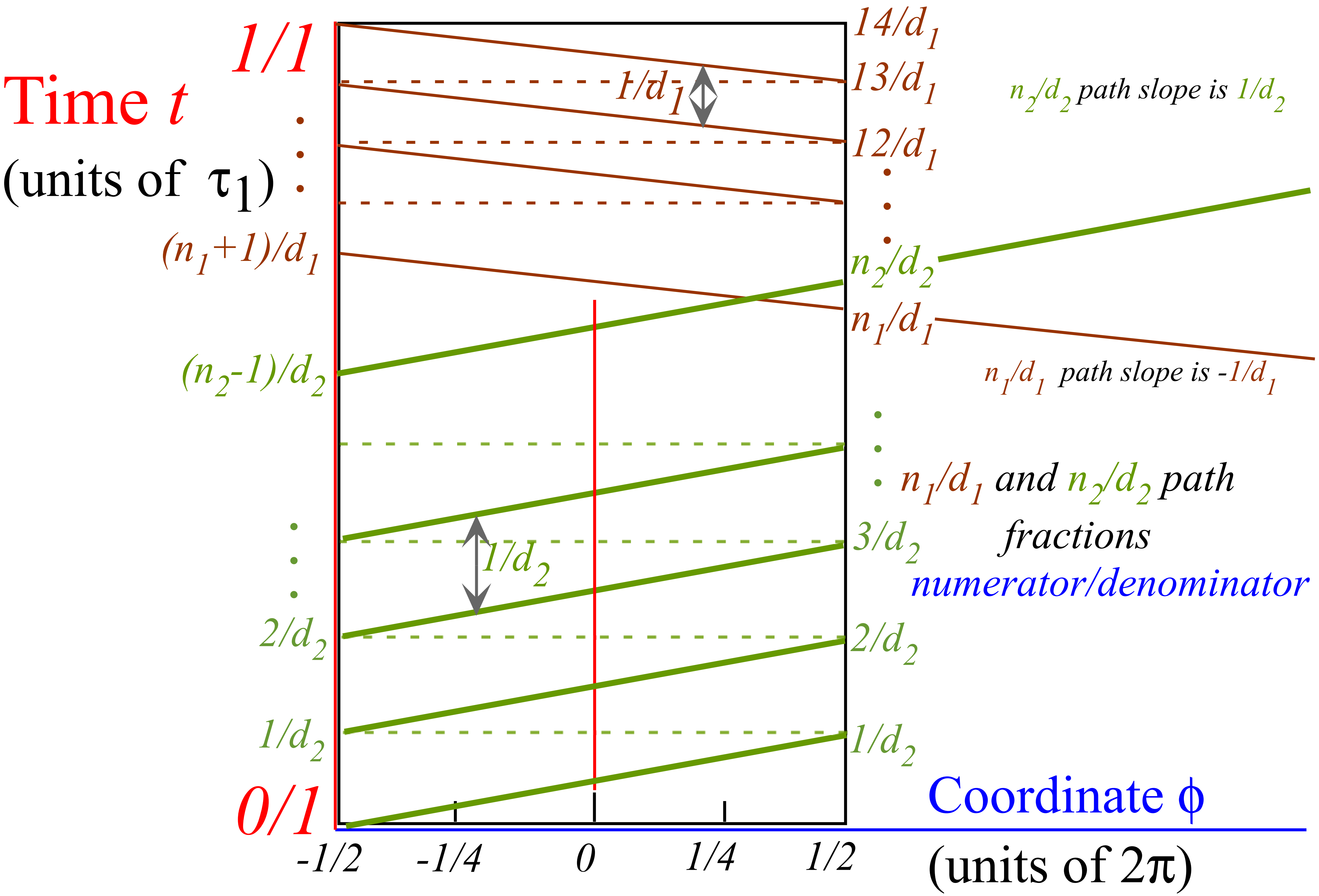
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



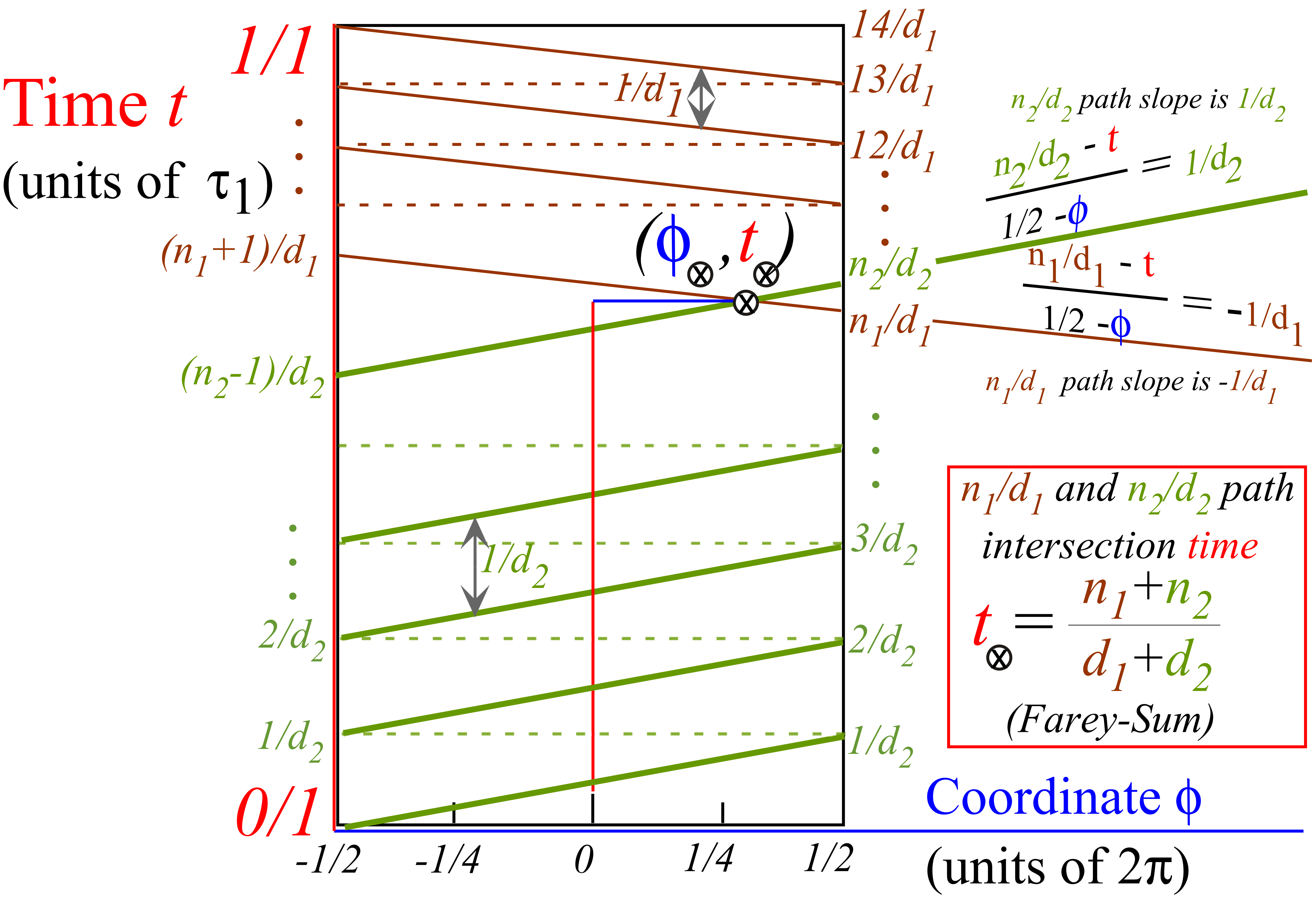
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



n_1/d_1 and n_2/d_2 path intersection time

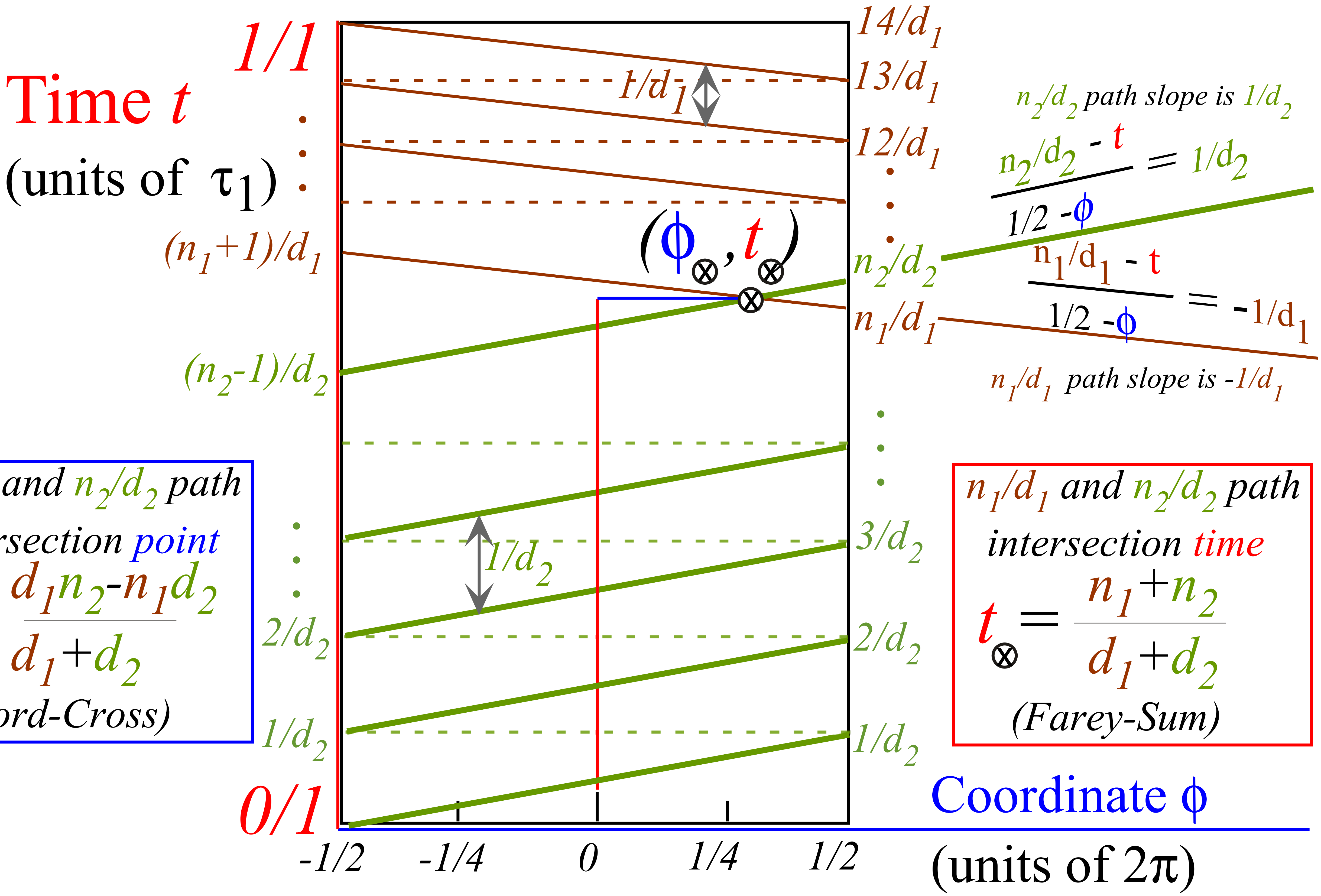
$$t_\otimes = \frac{n_1+n_2}{d_1+d_2}$$

(Farey-Sum)

[John Farey, Phil. Mag.(1816)]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*]

[*John Farey, Phil. Mag.(1816)*]

"Monster Mash" classical segue to Heisenberg action relations

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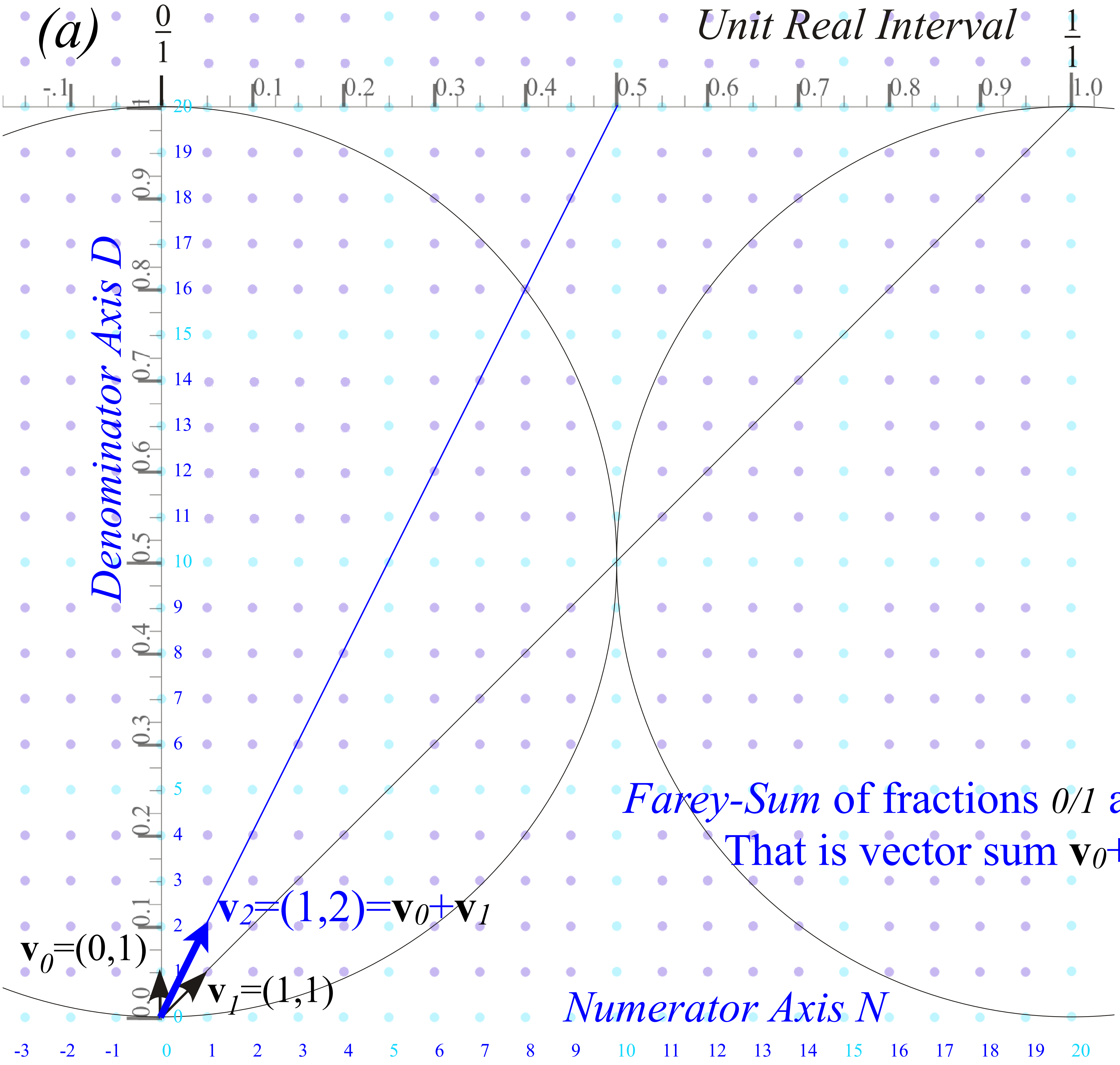
→ *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

[Lester. R. Ford, Am. Math. Monthly 45,586(1938) [John Farey, Phil. Mag.(1816)]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



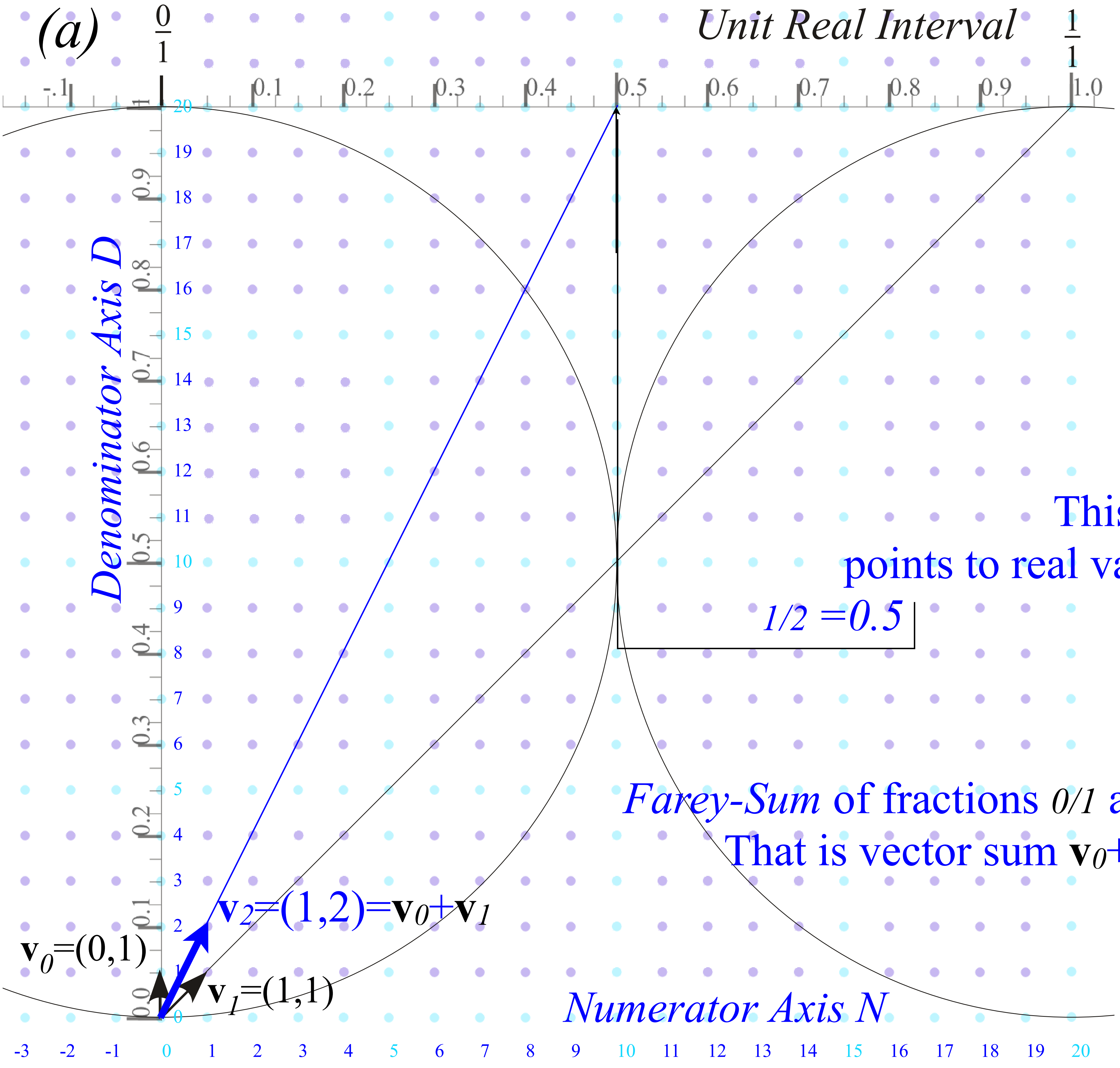
Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[*Li, Harter, Chem.Phys.Letters (2015)*]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

Farey-Sum of fractions *0/1* and *1/1* is *1/2*
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[Li, Harter, Chem.Phys.Letters (2015)]

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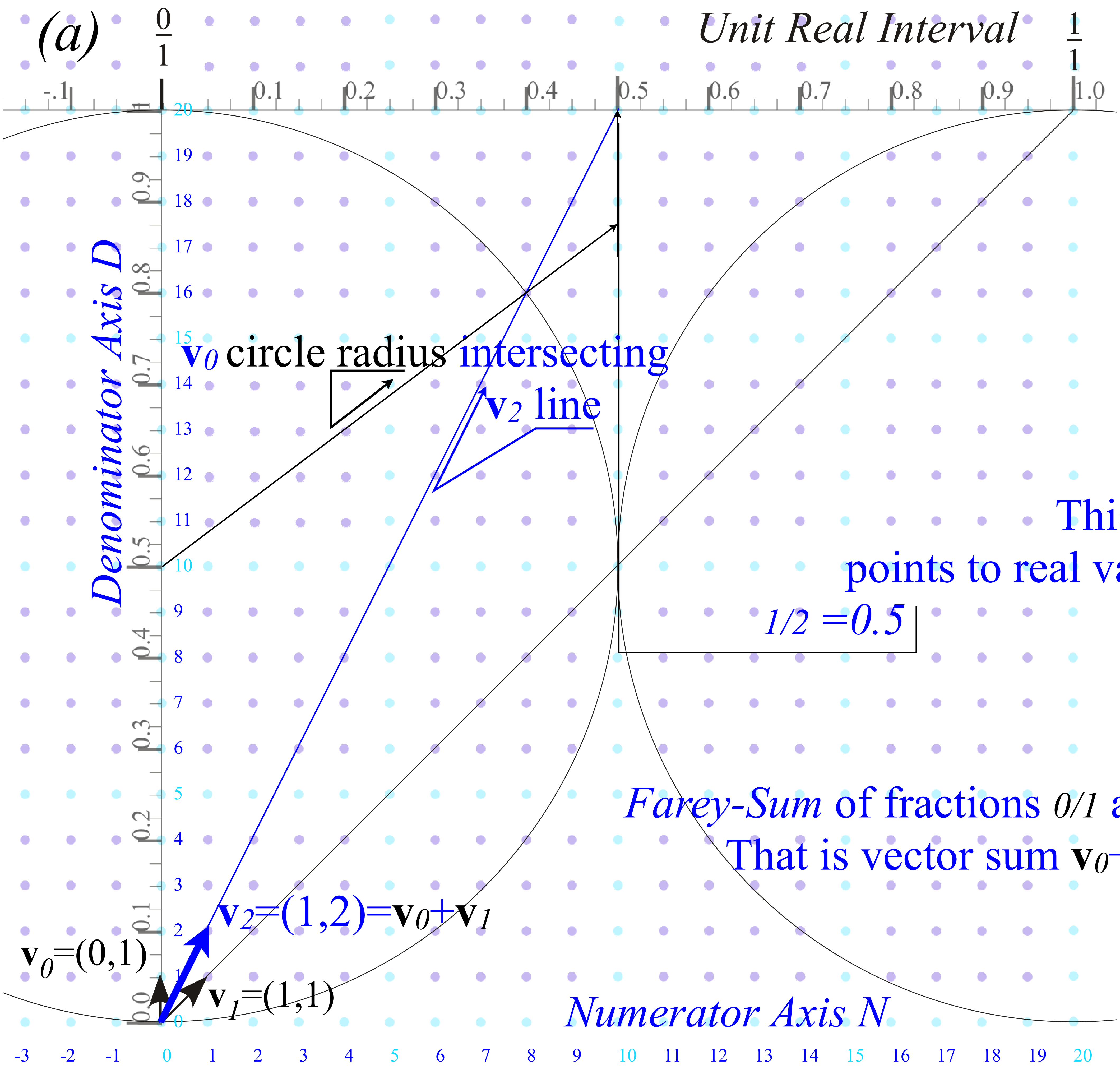
1/1-circle has diameter *1*



$\mathbf{v}_0$ circle radius intersecting $\mathbf{v}_2$ line

$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$

$\mathbf{v}_0=(0,1)$

$\mathbf{v}_1=(1,1)$

This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

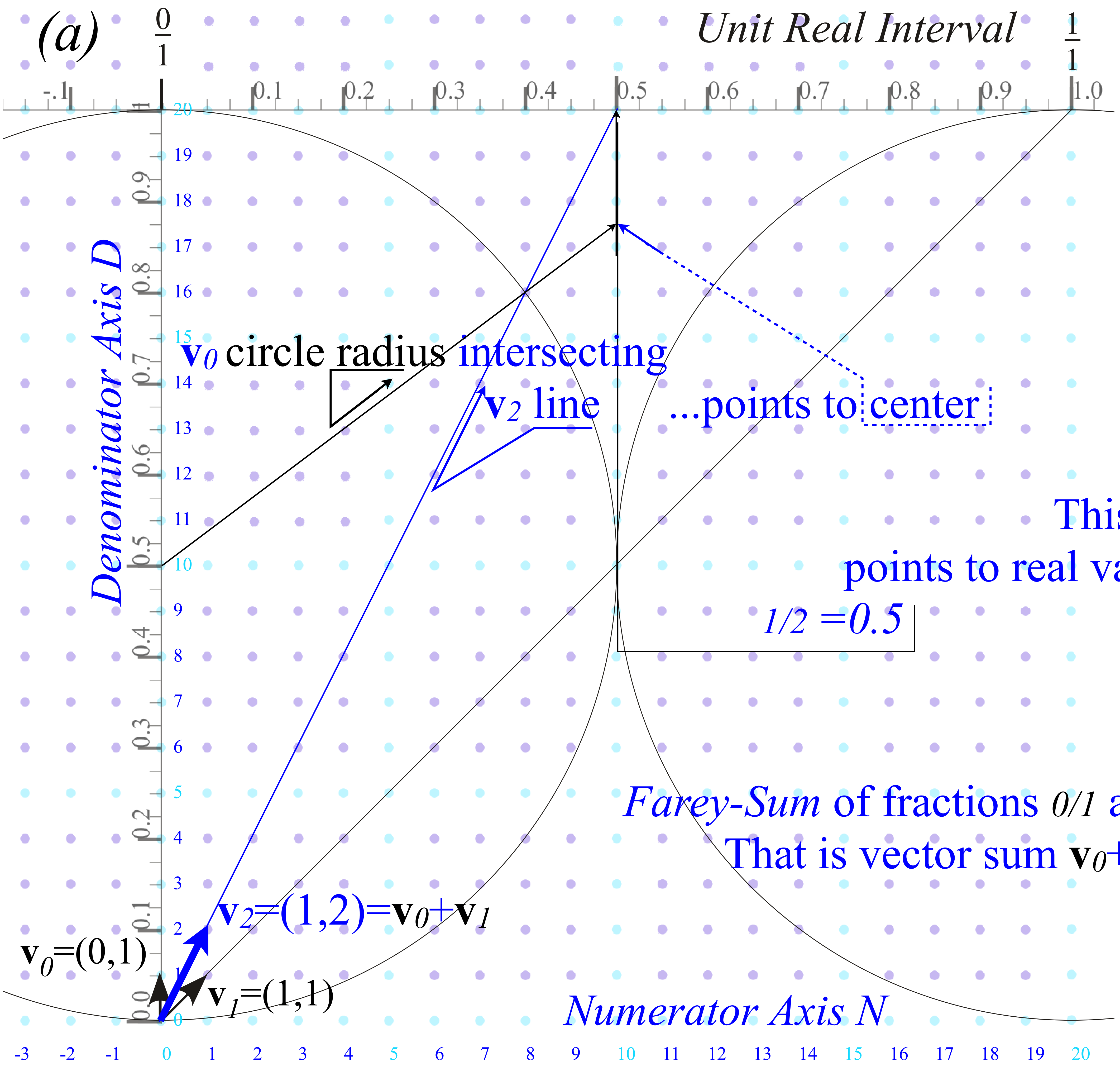
Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1=(1,2)=\mathbf{v}_2$

[Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



$\mathbf{v}_0$ circle radius intersecting $\mathbf{v}_2$ line

...points to center

This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

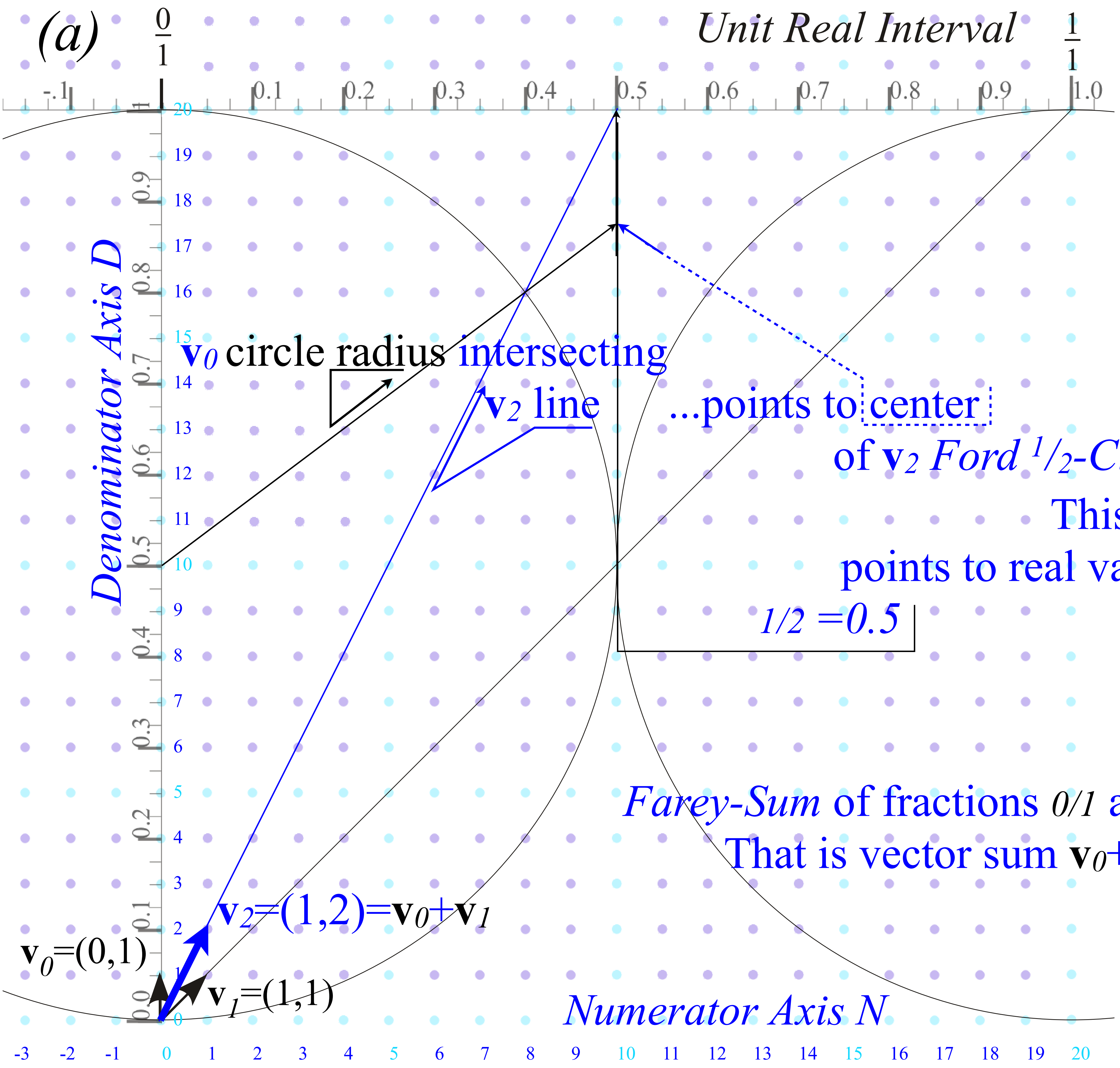
Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[*Li, Harter, Chem.Phys.Letters (2015)*]

Farey Sum related to vector sum and Ford Circles

1/1-circle has diameter *1*



$\mathbf{v}_0$ circle radius intersecting $\mathbf{v}_2$ line

...points to center of $\mathbf{v}_2$ *Ford* $^1/_2$*-Circle*

This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

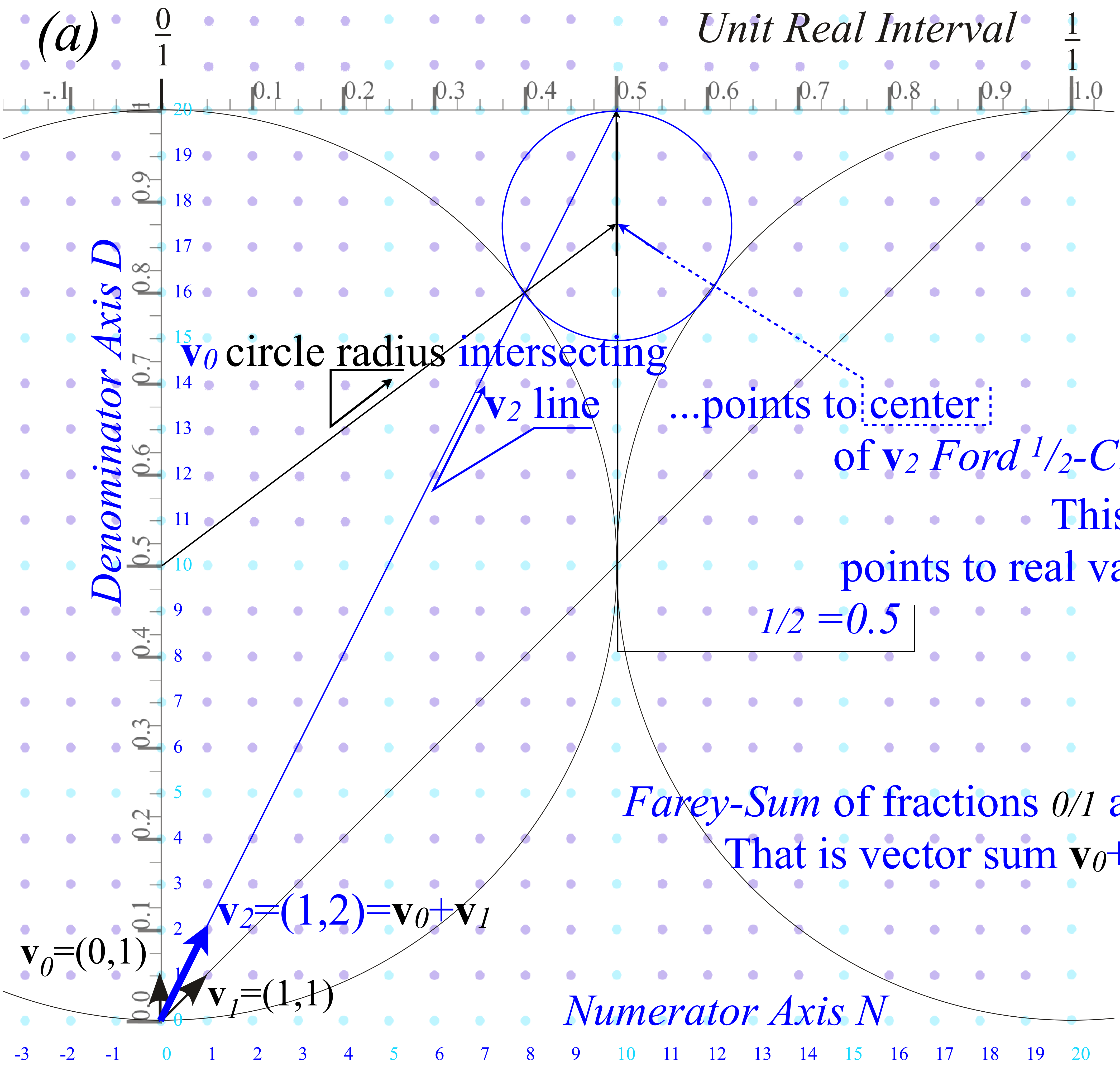
Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[*Li, Harter, Chem.Phys.Letters (2015)*]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



$\mathbf{v}_0$ circle radius intersecting $\mathbf{v}_2$ line

...points to center of $\mathbf{v}_2$ *Ford* $^1/_2$*-Circle*

This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[Li, Harter, Chem.Phys.Letters (2015)]

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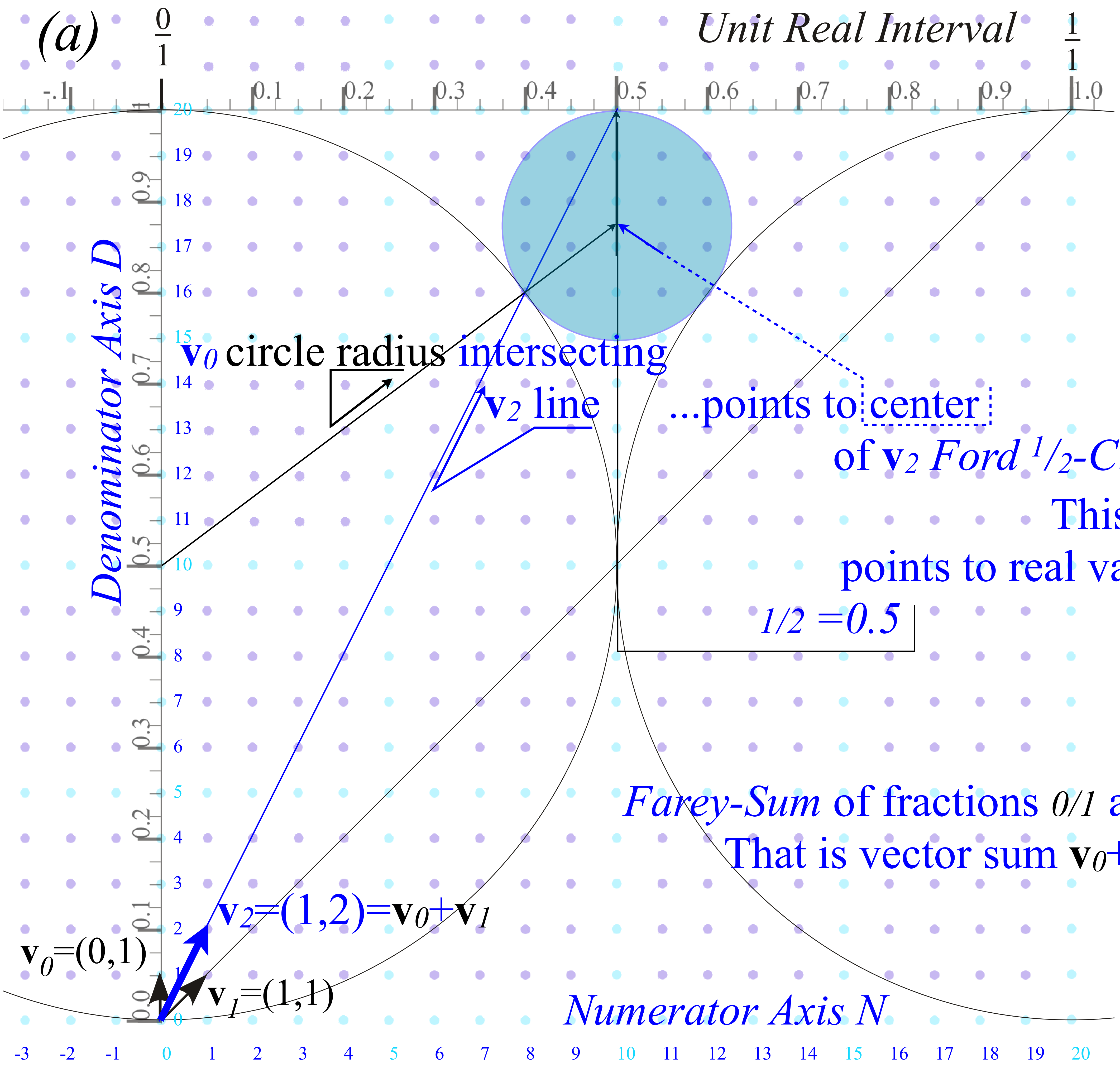
1/1-circle has diameter *1*



$\mathbf{v}_0$ circle radius intersecting $\mathbf{v}_2$ line

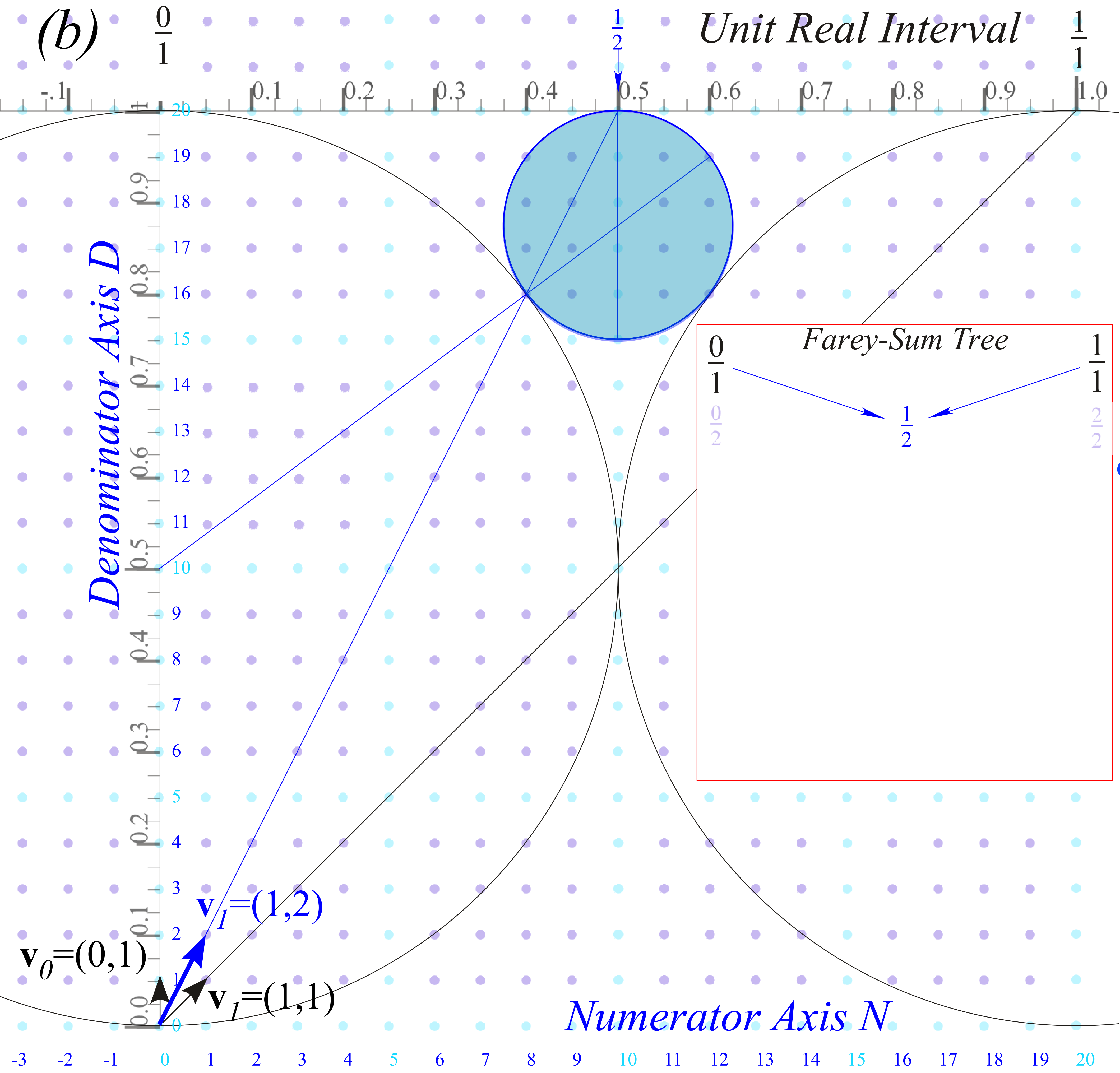
...points to center of $\mathbf{v}_2$ *Ford* ${}^1/_2$*-Circle*

This vector $\mathbf{v}_2$ points to real value *1/2 =0.5*

$\mathbf{v}_2$=(1,2)=$\mathbf{v}_0$+$\mathbf{v}_1$

Farey-Sum of fractions *0/1* and *1/1* is *1/2*
That is vector sum $\mathbf{v}_0+\mathbf{v}_1$ = (1,2) = $\mathbf{v}_2$

[Li, Harter, Chem.Phys.Letters (2015)]



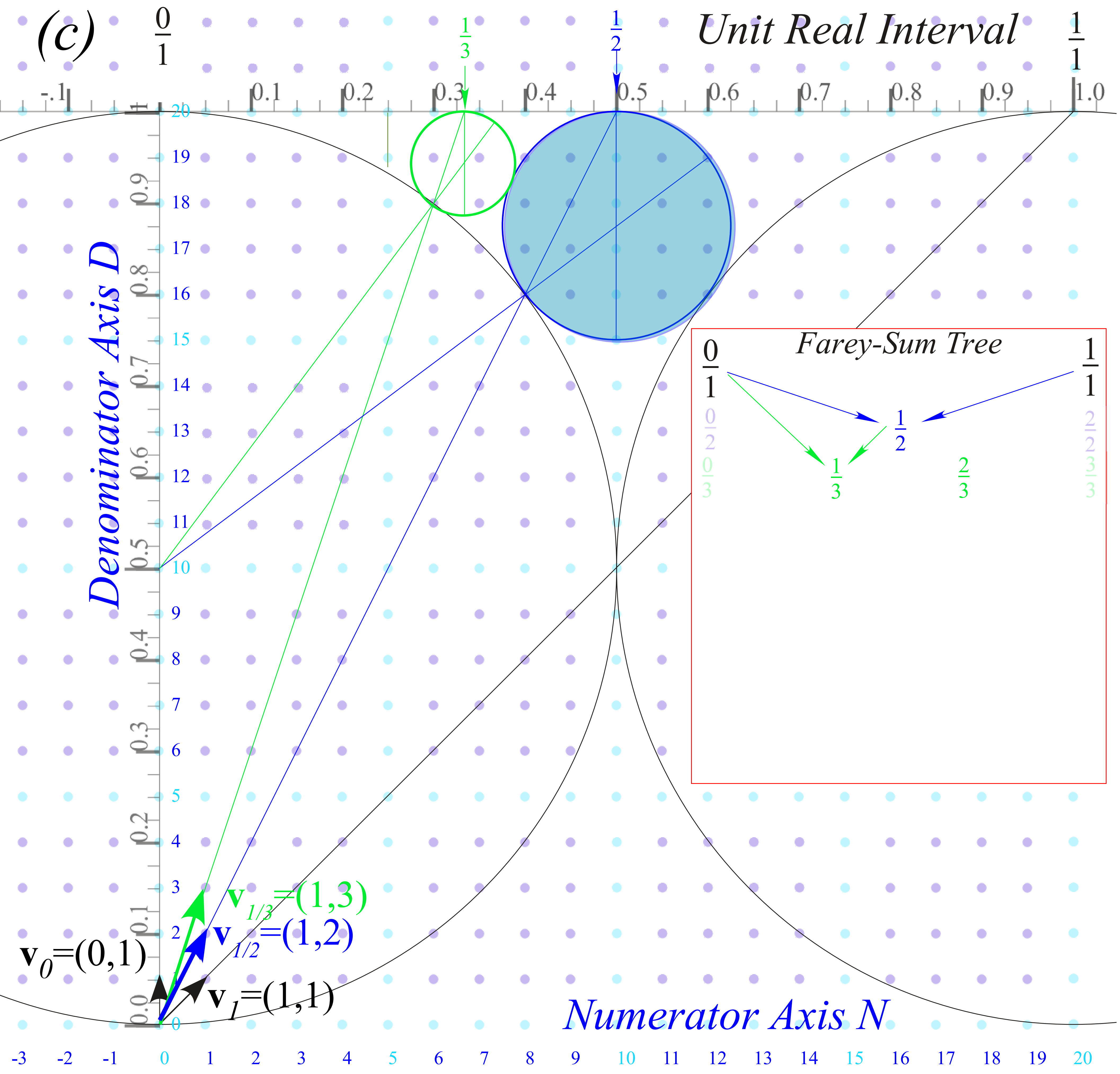
Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

[Li, Harter, Chem.Phys.Letters (2015)]

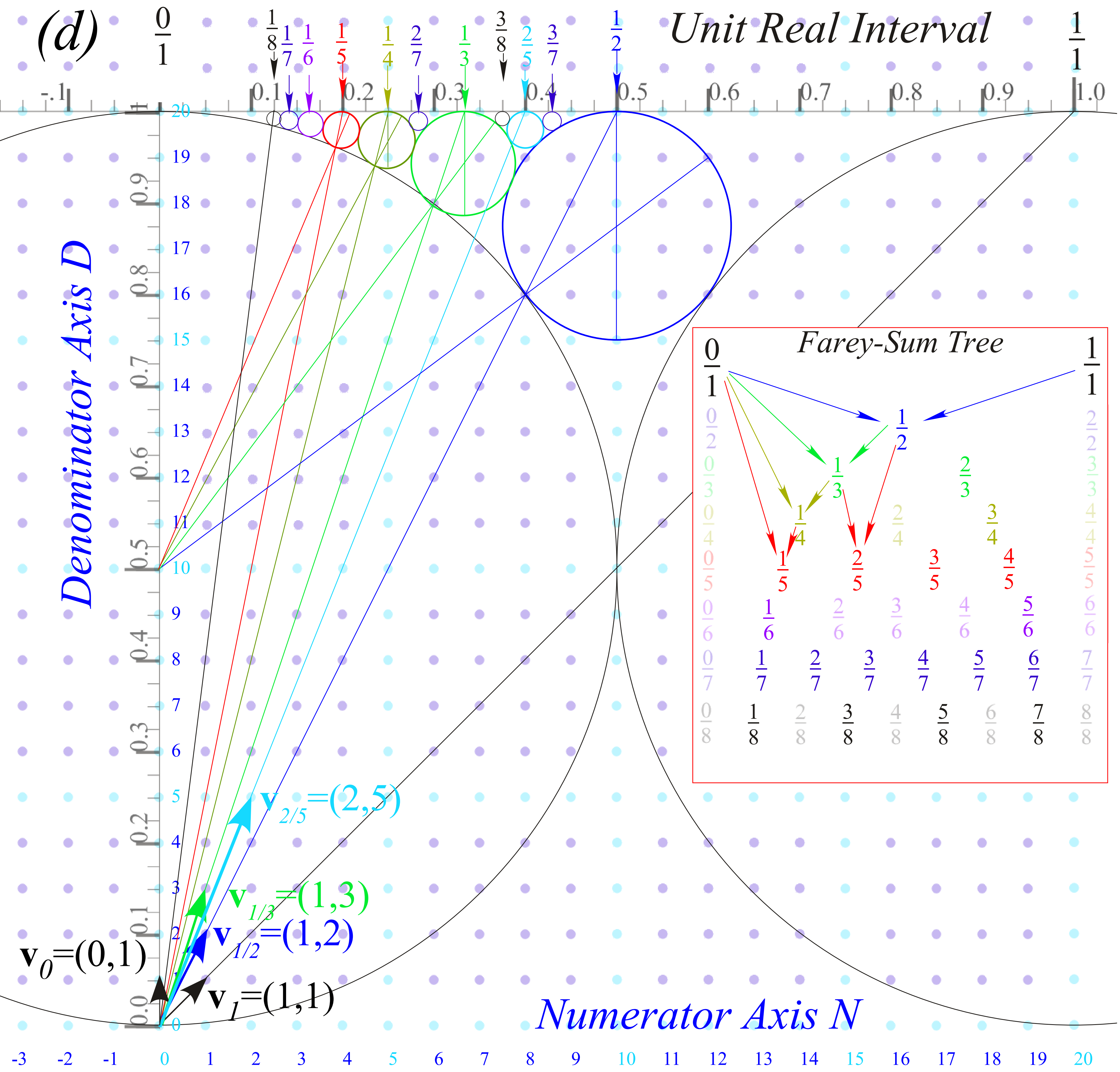
Farey Sum related to vector sum and *Ford Circles*



1/2-circle has diameter *1/2²=1/4*

1/3-circles have diameter *1/3²=1/9*

[Li, Harter, Chem.Phys.Letters (2015)]



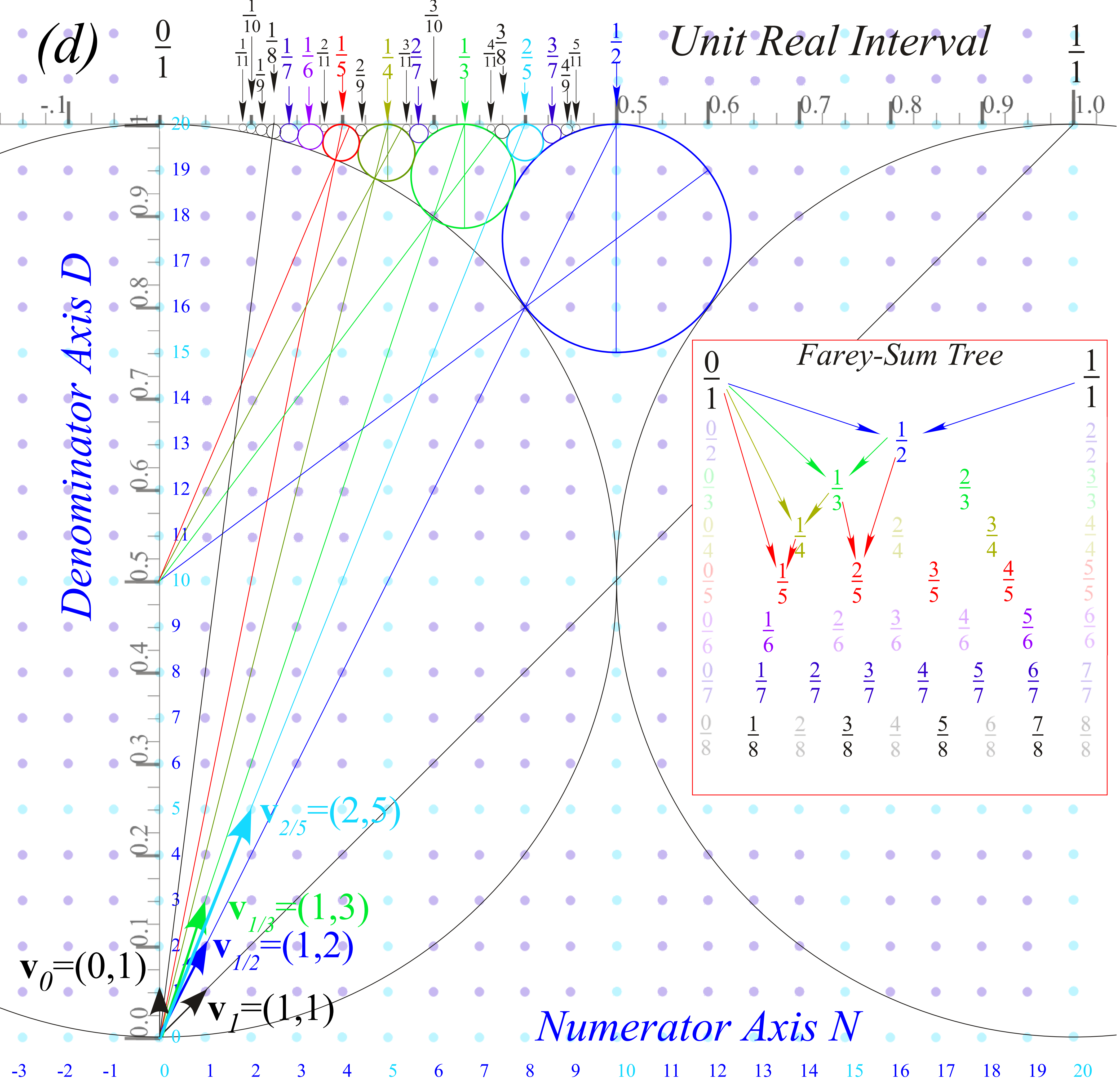
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]



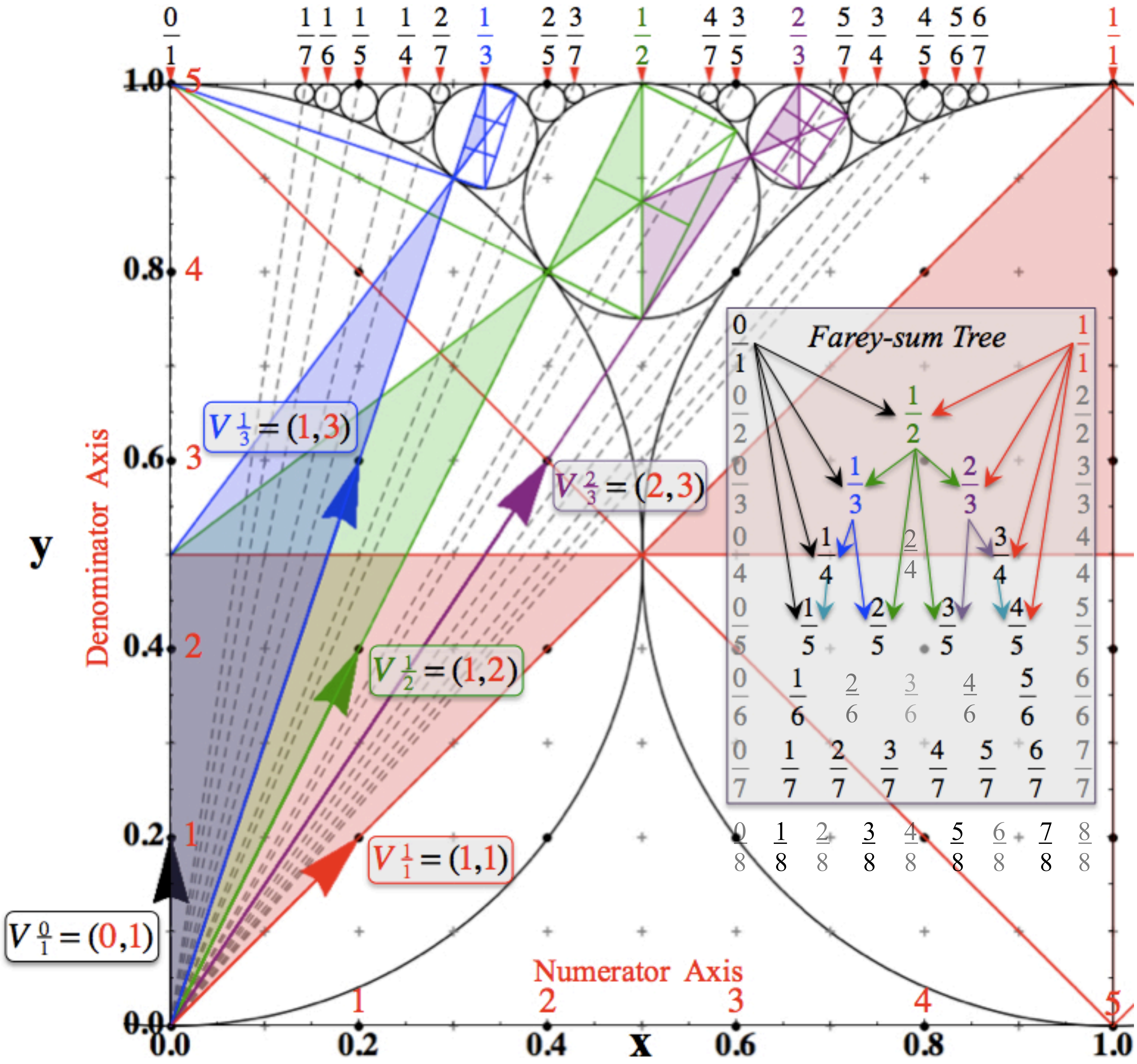
[Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter *1/2²=1/4*

1/3-circles have diameter *1/3²=1/9*

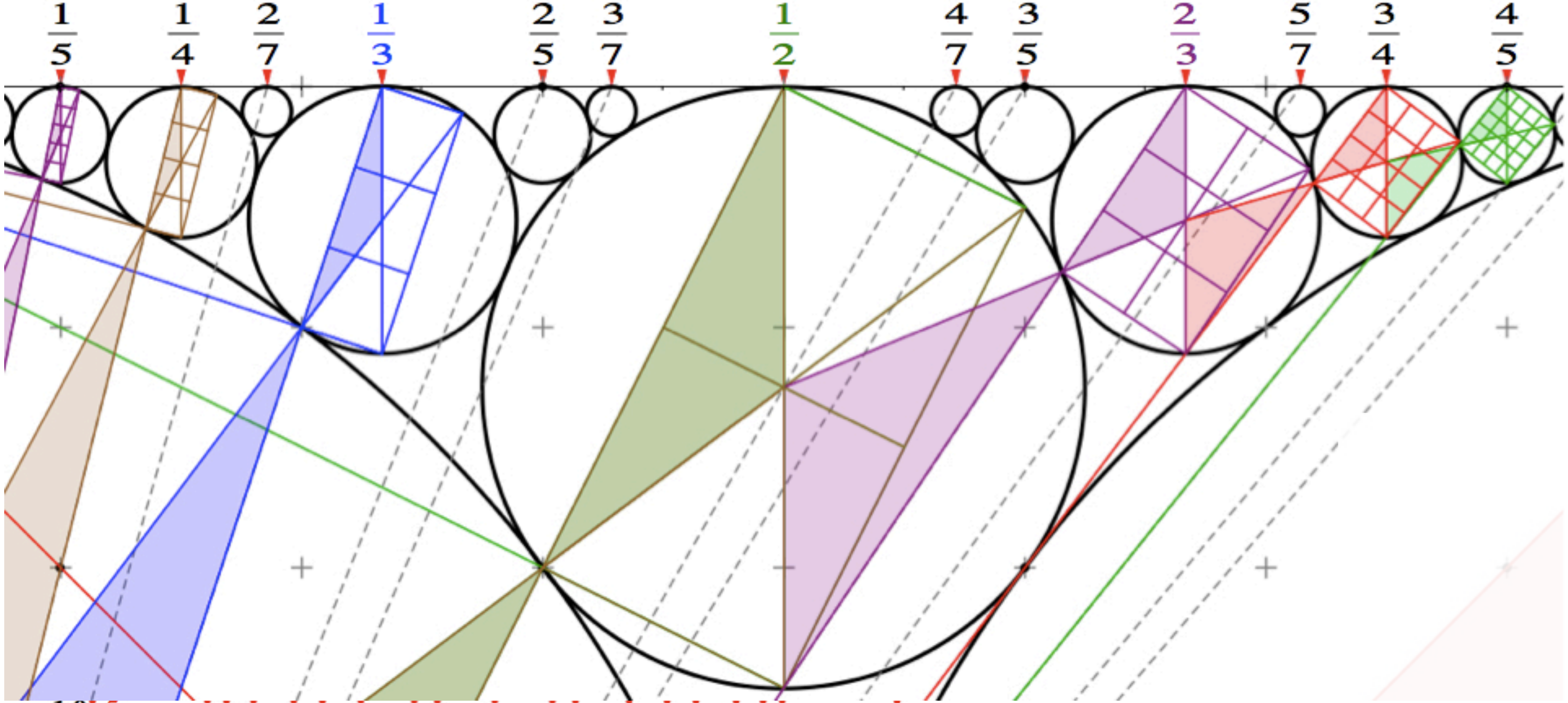
n/d-circles have diameter *1/d²*



Thales Rectangles

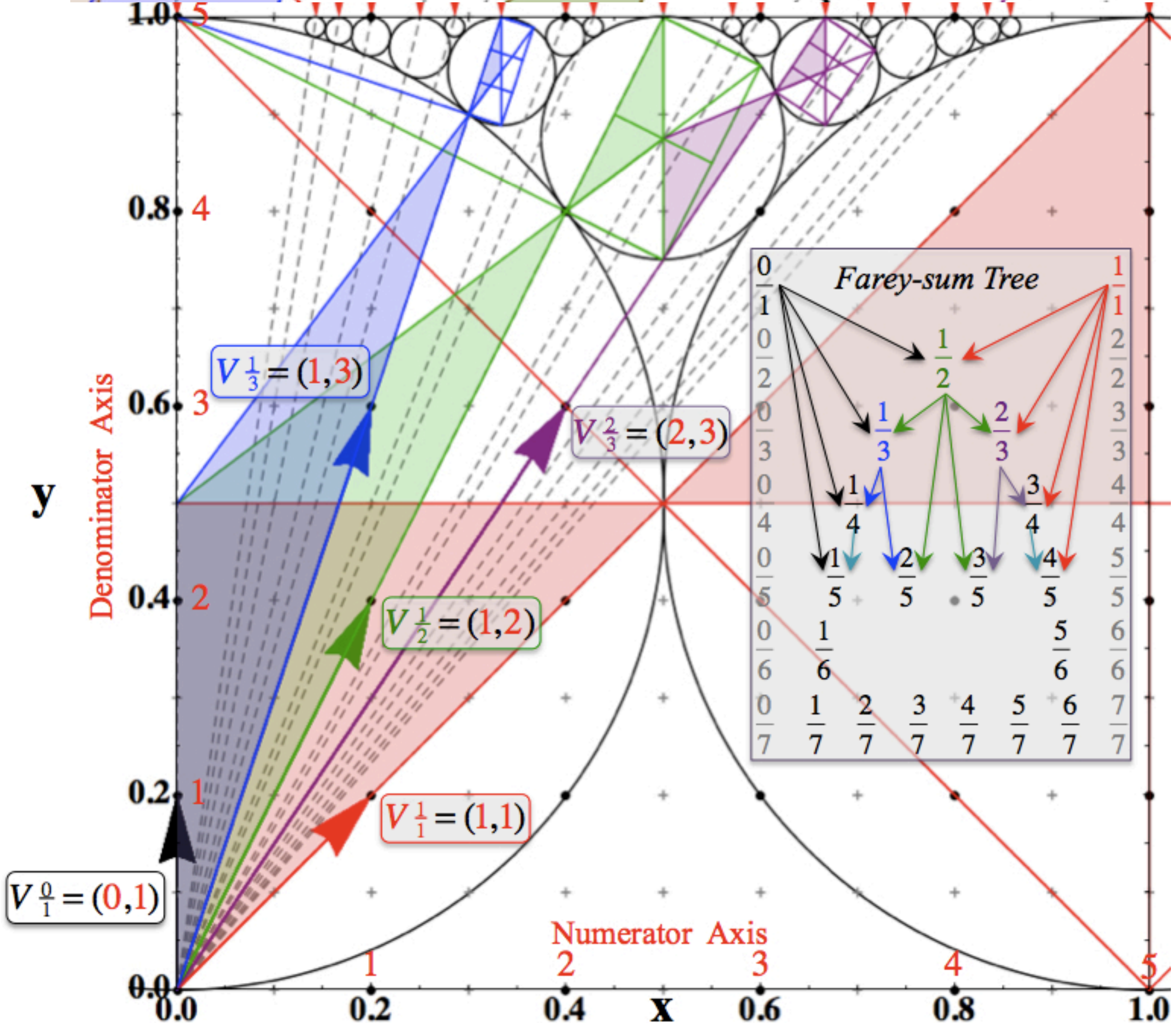
provide
analytic geometry
of
fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



“Quantized” Thales Rectangles

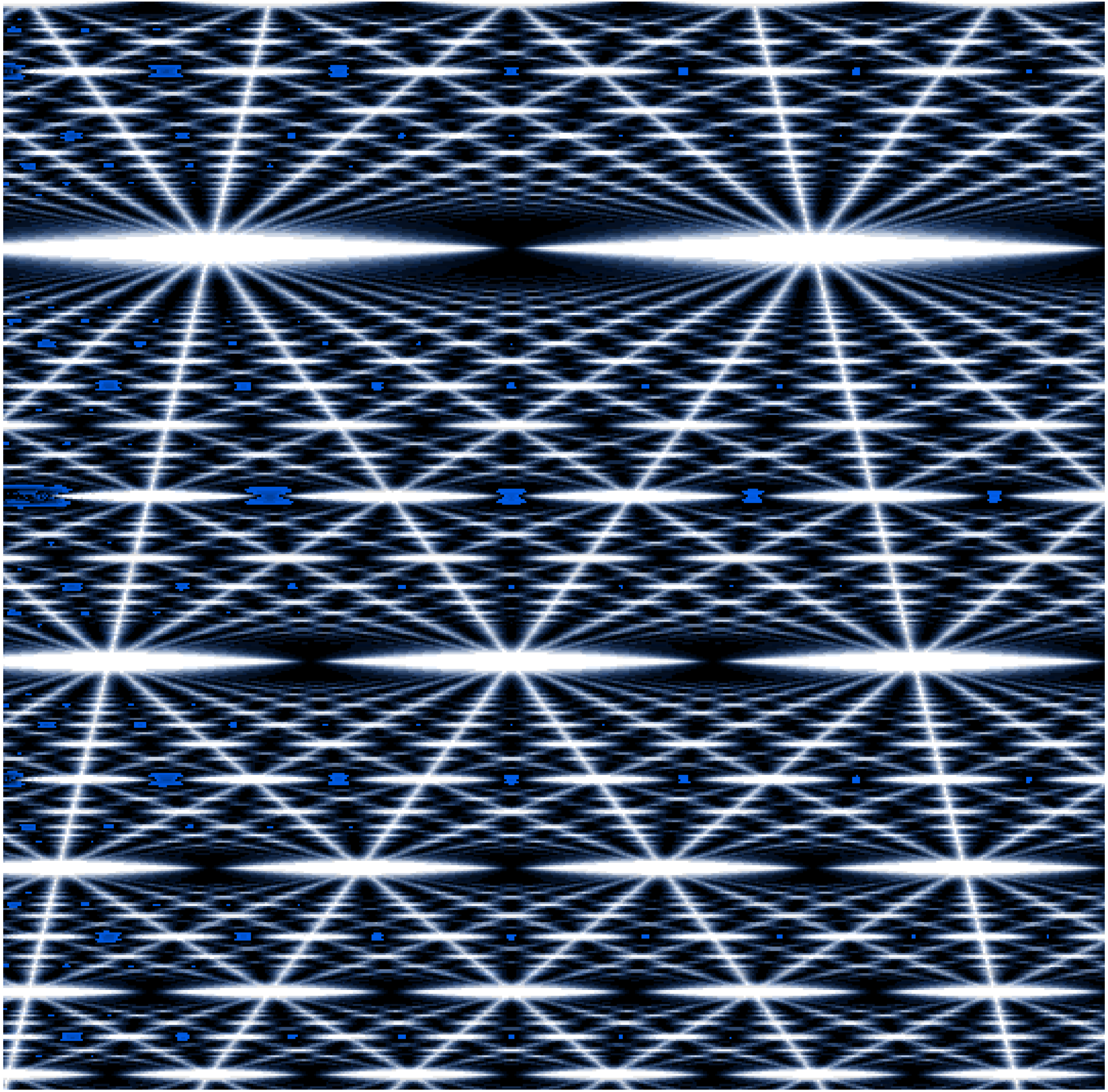
provide
analytic geometry
of
fractal structure



[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep “3D-Magic-Eye” picture



Geometric "Integration" (Converting Velocity data to Spacetime)

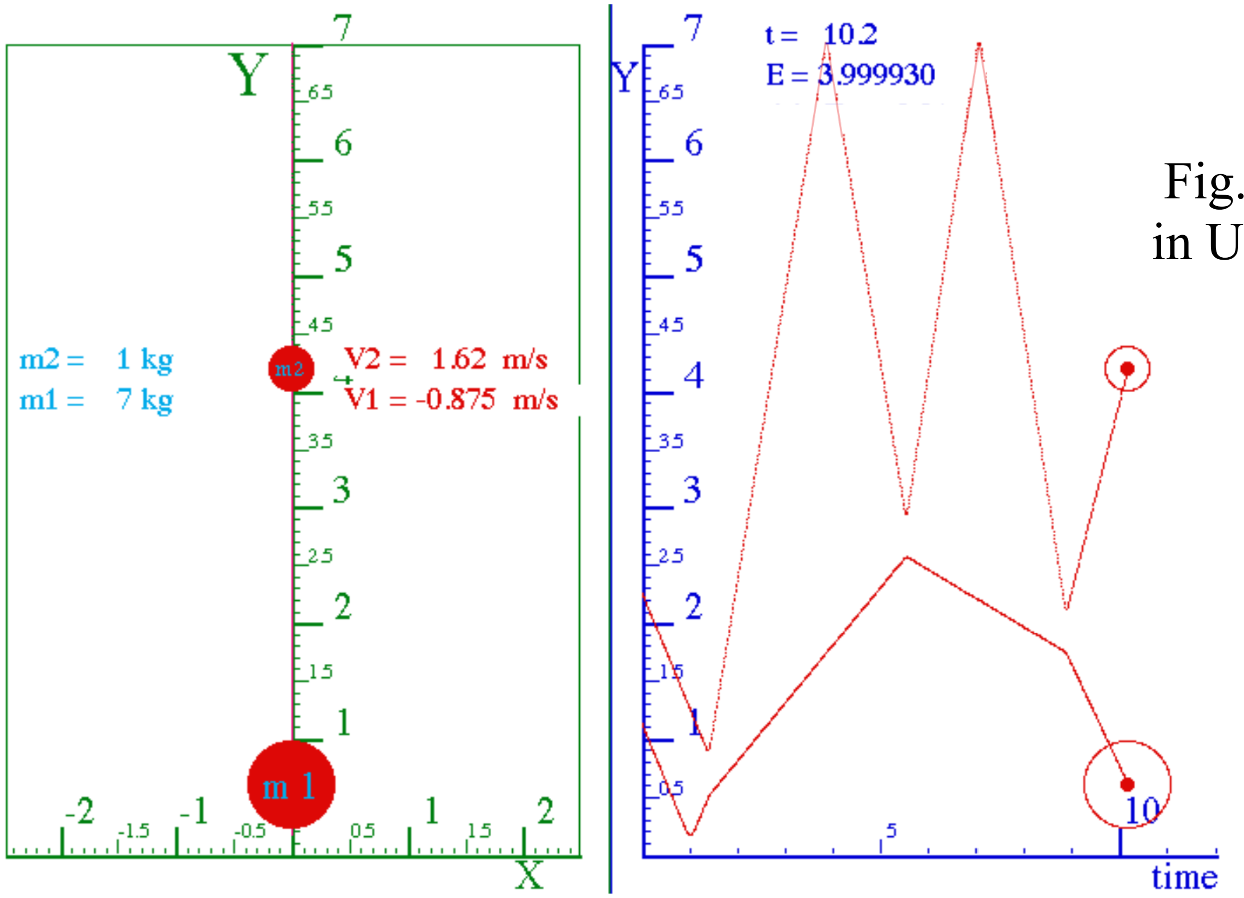


Fig. 4.8
in Unit 1

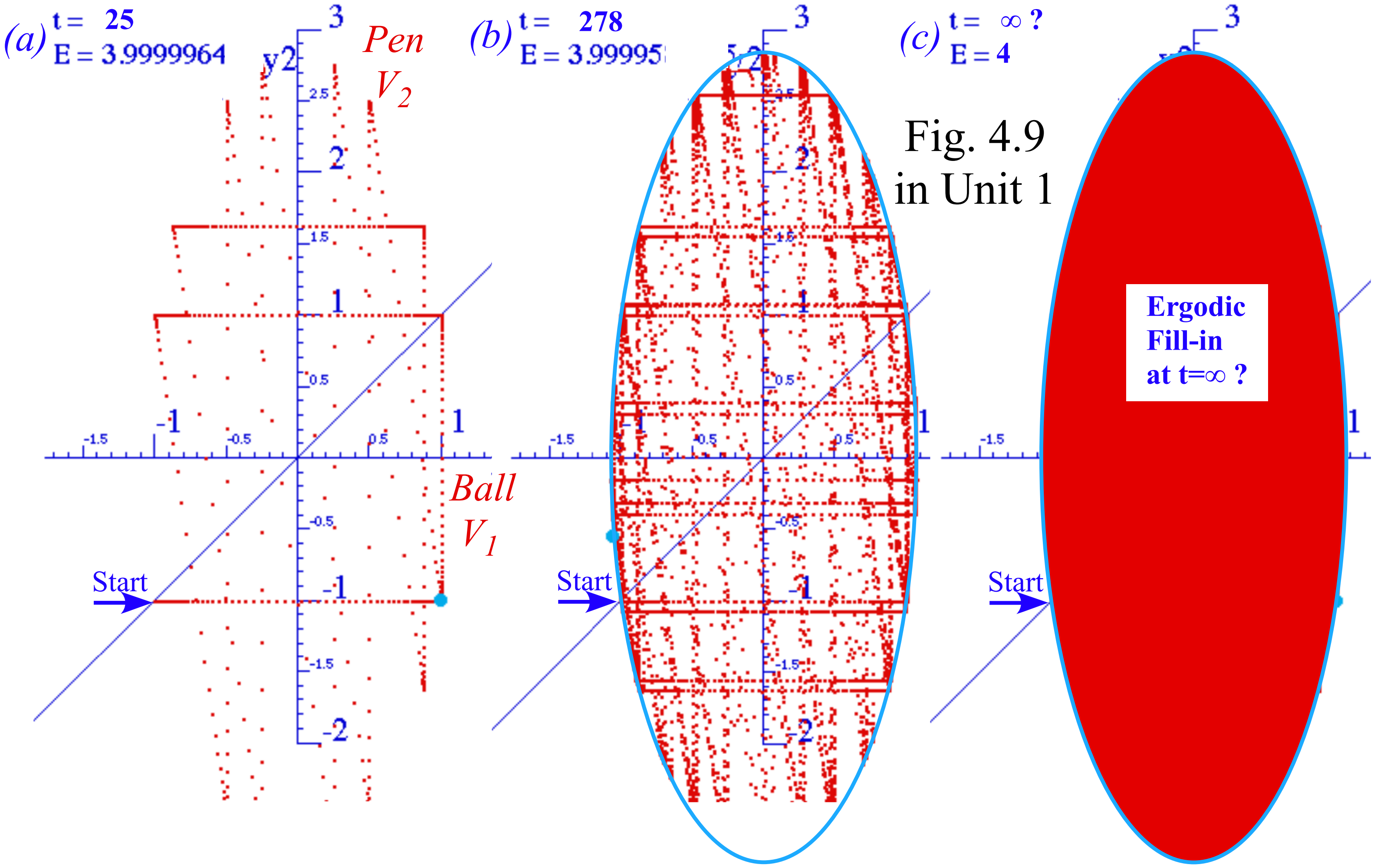
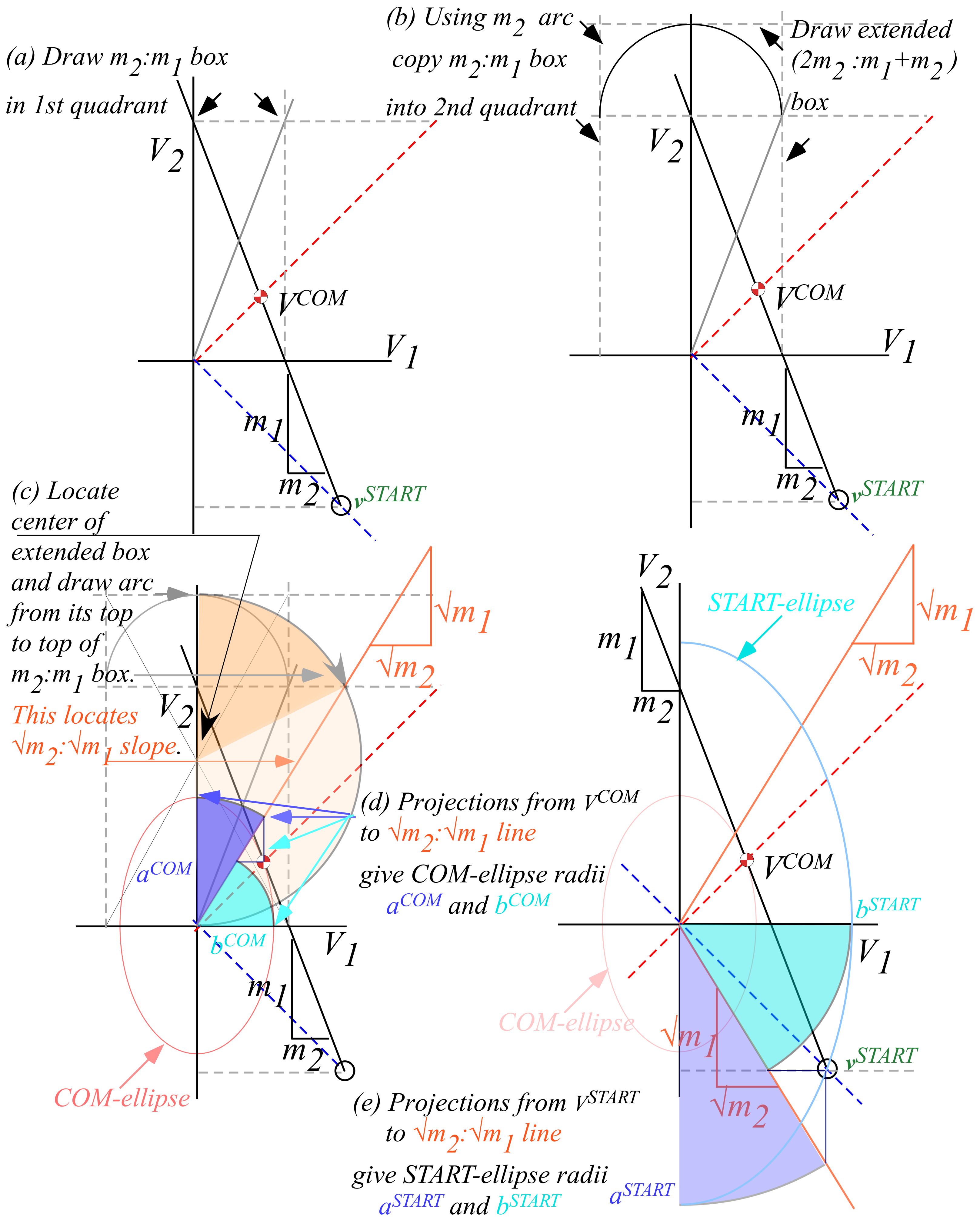


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)

Still, if you know a simpler construction, we'd like to hear about it!