Lecture 3
Thur 8.28.2017

Analysis of 1D 2-Body Collisions: Reflection groups

(Ch. 2 to Ch. 4 of Unit 1)

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)

Matrix and tensor algebra of 1-D 2-body collisions: "Mass-bang" matrix **M**,
"Floor-bang" matrix **F**, *"Ceiling-bang" matrix* **C**, *and group product*: **R**= **C•M**

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle
How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics in Ch.12
Reflections in the clothing store: "It's all done with mirrors!"
Introducing hexagonal symmetry D_6~C_{6v} (Resulting for m_1/m_2=3)
Group multiplication and product table
Classical collision paths with D_6~C_{6v} (Resulting from m_1/m_2=3)
Other not-so-symmetric examples: m_1/m_2=4 and m_1/m_2=7 and (M_1=100, M_2=1)

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)

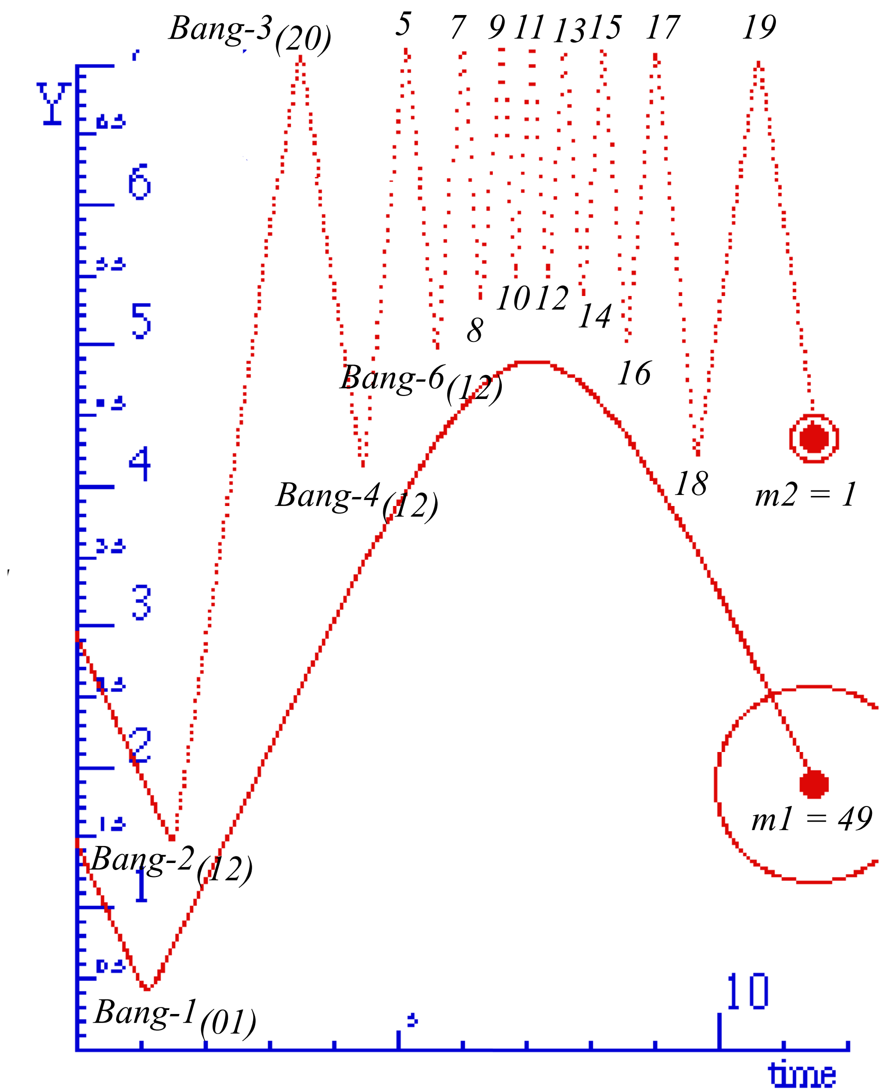
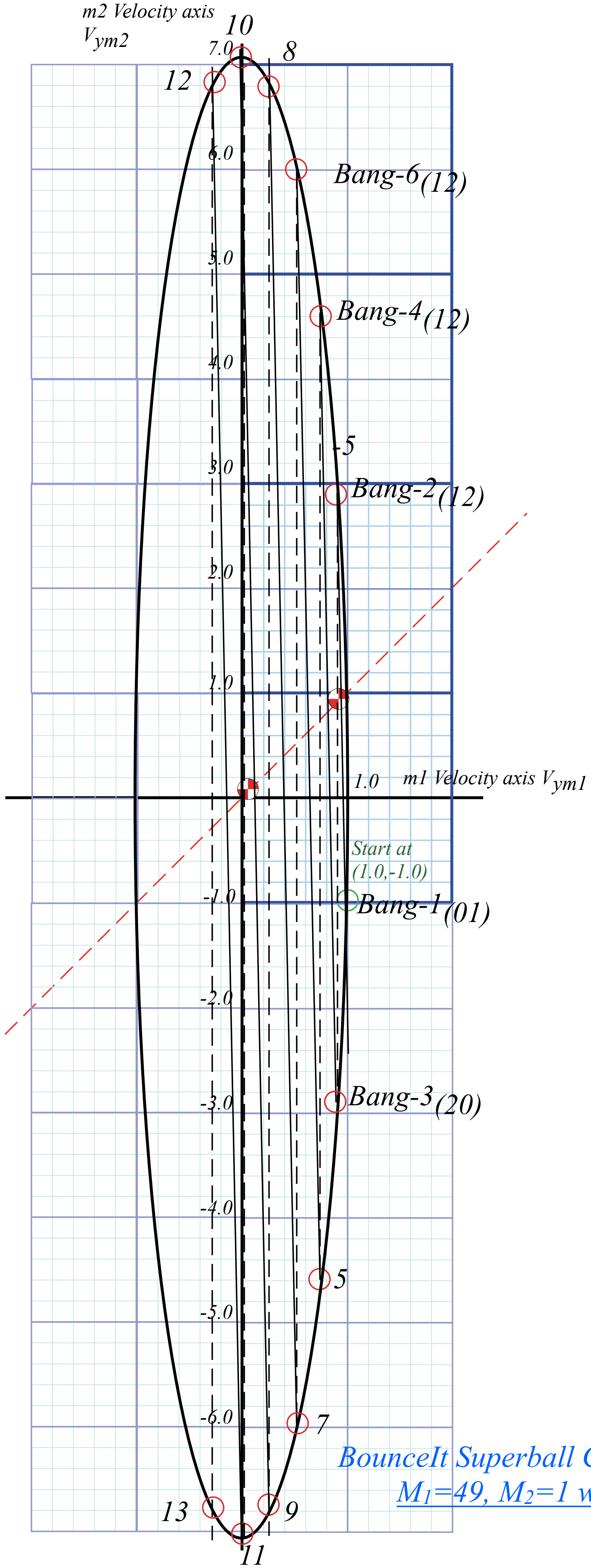


Fig. 5.1 in Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with Newtonian time plot]

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with V_2 vs V_1 plot]

Review: *Geometry of 1-D 2-body collisions* (*Example with masses:* $m_1=49$ *and* $m_2=1$)



BounceIt Superball Collision Web Simulator:
$M_1=49$, $M_2=1$ with V_2 vs V_1 plot

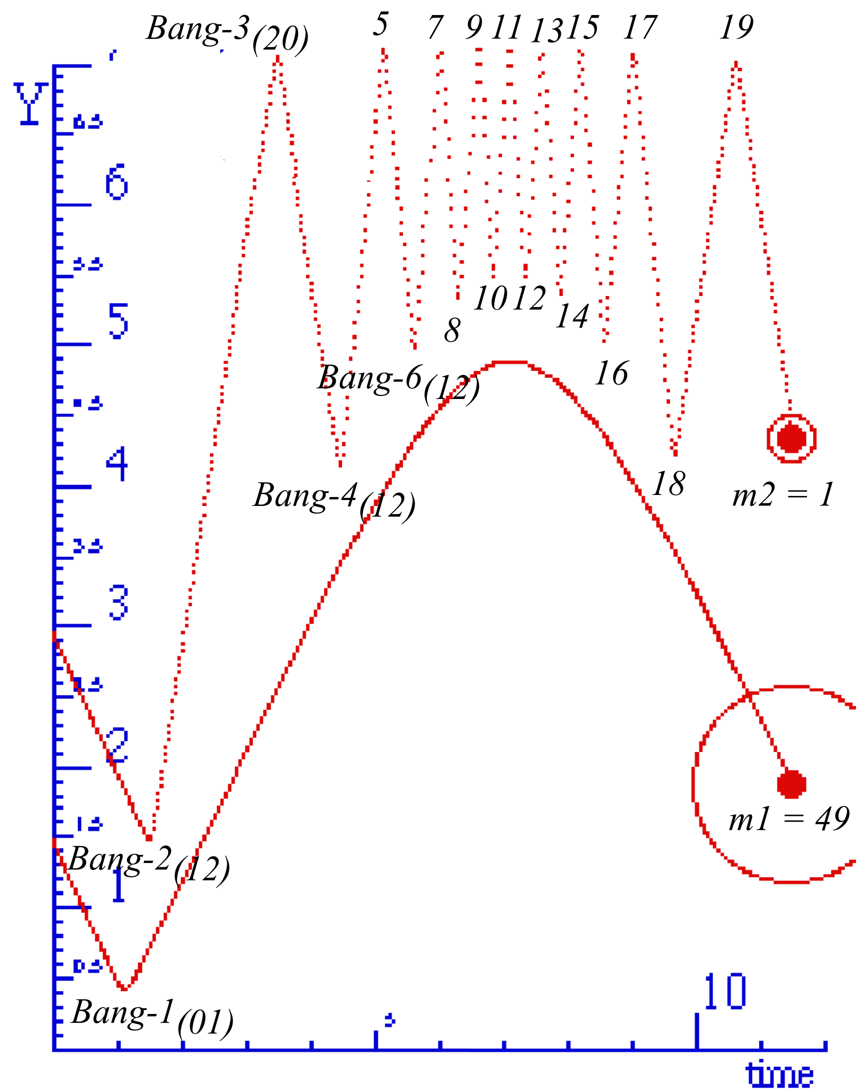
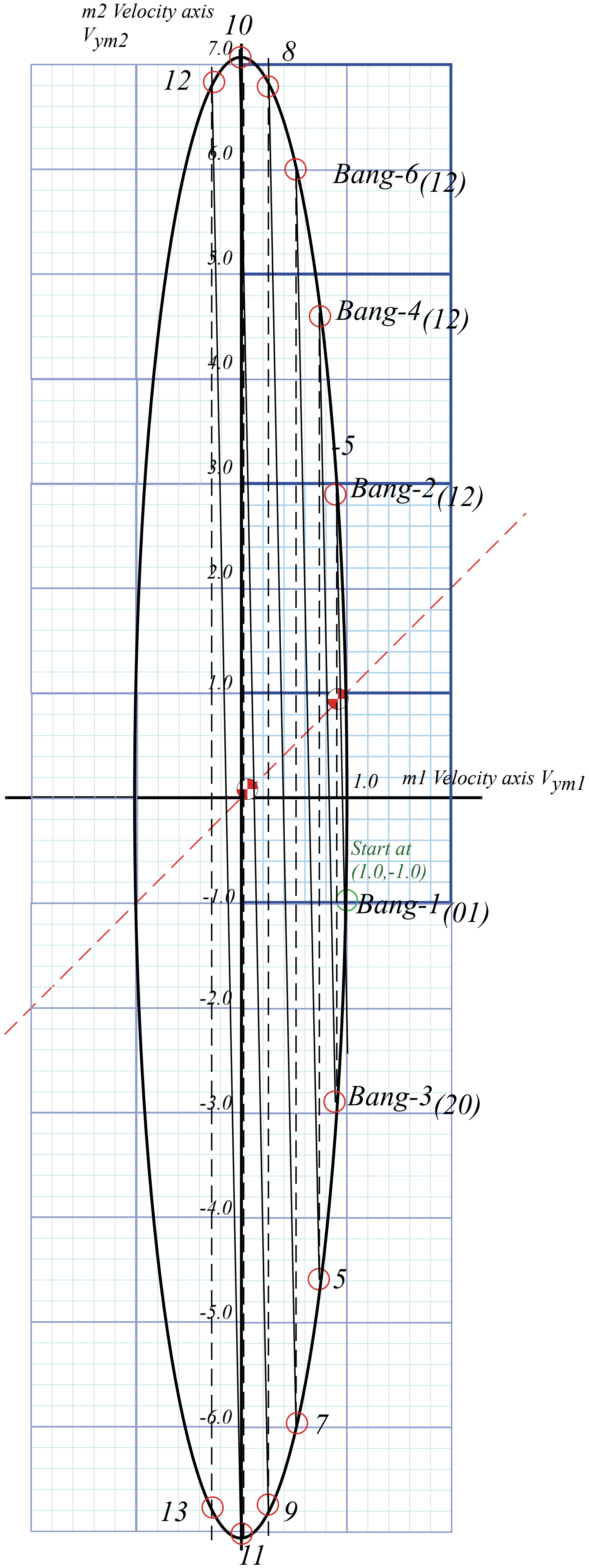


Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
$M_1=49$, $M_2=1$ with Newtonian time plot

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)



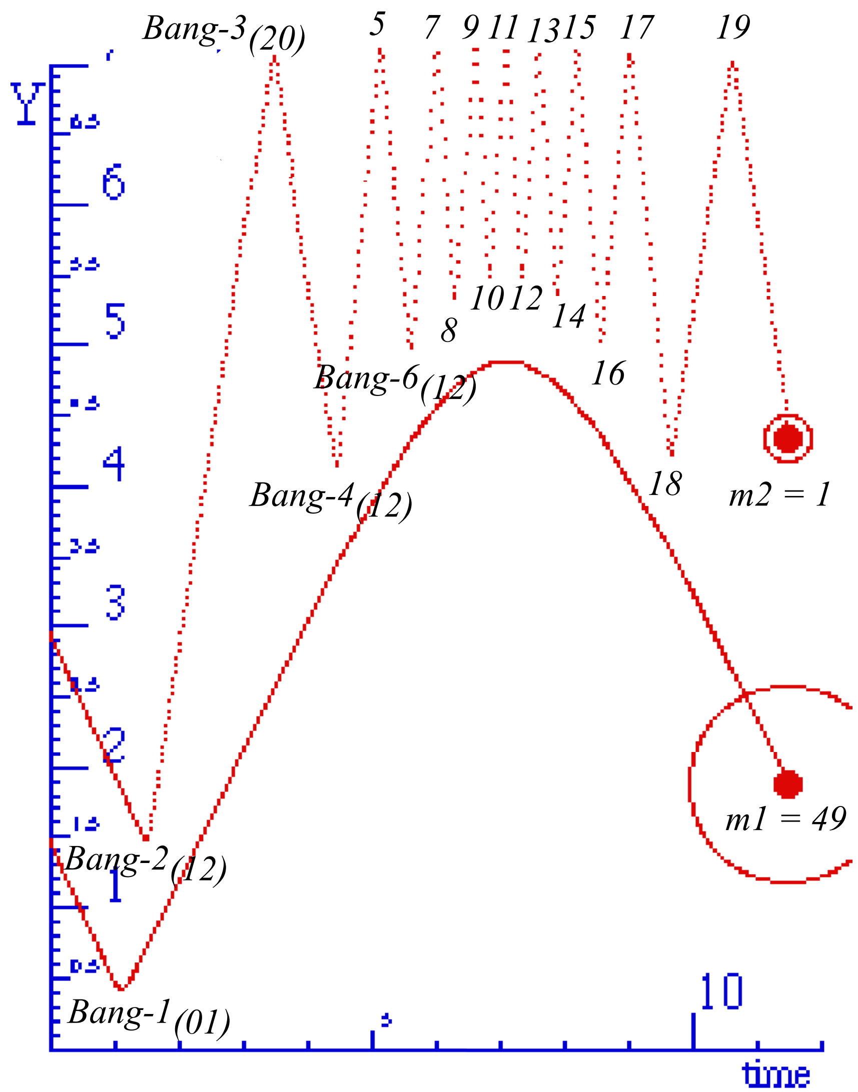


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1	*Ellipse radius 2*
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/m_2}$
$= \sqrt{2KE/49}$	$= \sqrt{2KE/1}$
$= \sqrt{50/49}$	$= \sqrt{50/1}$
$= 1.01$	$= 7.07$

Review: *Multiple collisions calculated by matrix operator products*

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**·**M**

Review: Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M}\bullet\mathbf{v}^{IN}$.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ *and* $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Review:Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN}+\mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1+m_2}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix} = \tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M}\bullet\mathbf{v}^{IN}$.

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} = \begin{pmatrix}2V^{COM}-v_1^{IN}\\2V^{COM}-v_2^{IN}\end{pmatrix} = \begin{pmatrix}2\frac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_1^{IN}\\2\frac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_2^{IN}\end{pmatrix} = \frac{\begin{pmatrix}m_1v_1^{IN}-m_2v_1^{IN}+2m_2v_2^{IN}\\2m_1v_1^{IN}+m_2v_2^{IN}-m_1v_2^{IN}\end{pmatrix}}{m_1+m_2} = \frac{\begin{pmatrix}m_1-m_2 & 2m_2\\2m_1 & m_2-m_1\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}}{m_1+m_2}$$

$$= \begin{pmatrix}\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_1^{IN}-m_2v_1^{IN}}{m_1+m_2}\\\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_2^{IN}-m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} = \begin{pmatrix}\frac{m_1-m_2}{m_1+m_2} & \frac{2m_2}{m_1+m_2}\\\frac{2m_1}{m_1+m_2} & \frac{m_2-m_1}{m_1+m_2}\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix}\frac{m_1-m_2}{m_1+m_2} & \frac{2m_2}{m_1+m_2}\\\frac{2m_1}{m_1+m_2} & \frac{m_2-m_1}{m_1+m_2}\end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

Let: $m_1=49$ *and* $m_2=1$ $$\mathbf{M} = \begin{pmatrix}0.96 & 0.04\\1.96 & -0.96\end{pmatrix}$$

Define *"ellipse-Rotation"* **R** as *group product*: $$\mathbf{R} = \mathbf{C}\bullet\mathbf{M} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\cdot\begin{pmatrix}0.96 & 0.04\\1.96 & -0.96\end{pmatrix} = \begin{pmatrix}0.96 & 0.04\\-1.96 & 0.96\end{pmatrix}$$

$$\left|FIN^9\right\rangle = \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$\left|FIN^9\right\rangle = \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

"ellipse-Rotation" group product: $\mathbf{R} = \mathbf{C} \bullet \mathbf{M}$

$$\left|FIN^9\right\rangle = \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{C} \bullet \mathbf{M} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$\left|FIN^9\right\rangle = \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{R} \bullet \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: $\mathbf{R} = \mathbf{C} \bullet \mathbf{M}$

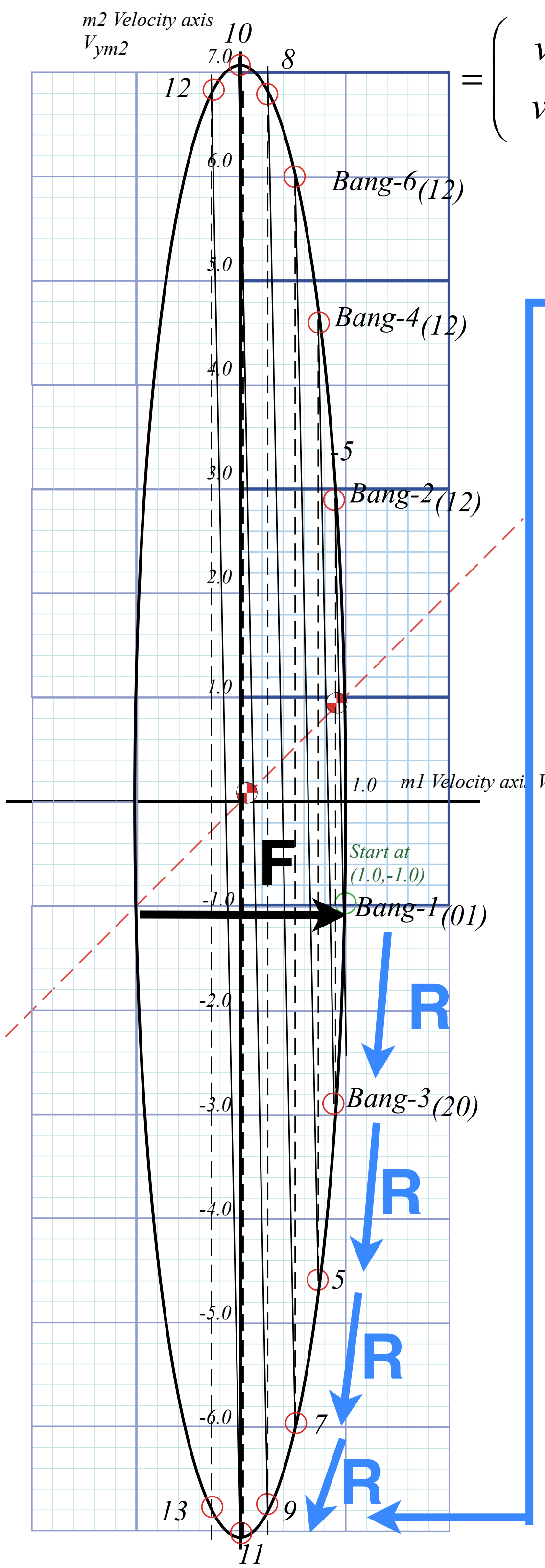
$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL }(0))}$$

$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{(\text{after Bang-9})}$$



"ellipse-Rotation" group product: **R**= **C•M**

Collisions for mass ratio m_1:m_2= 49:1

Fig. 5.1a (revised)

$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

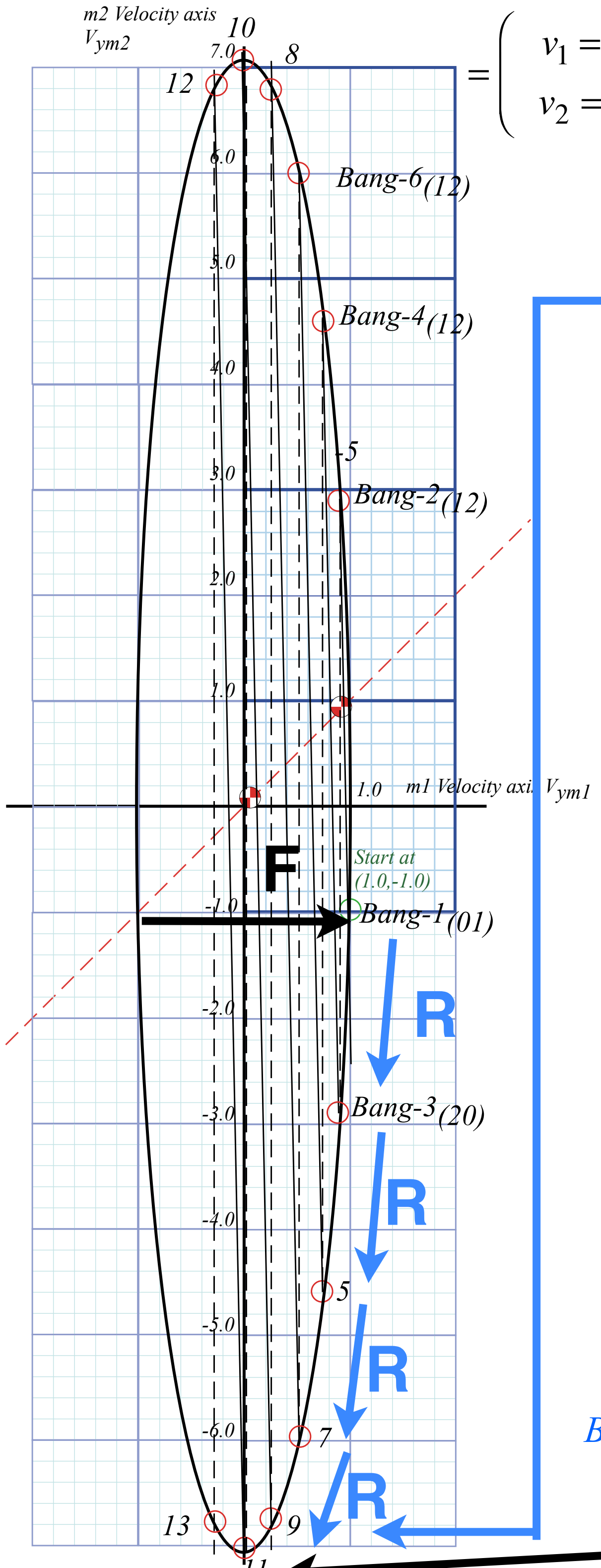
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

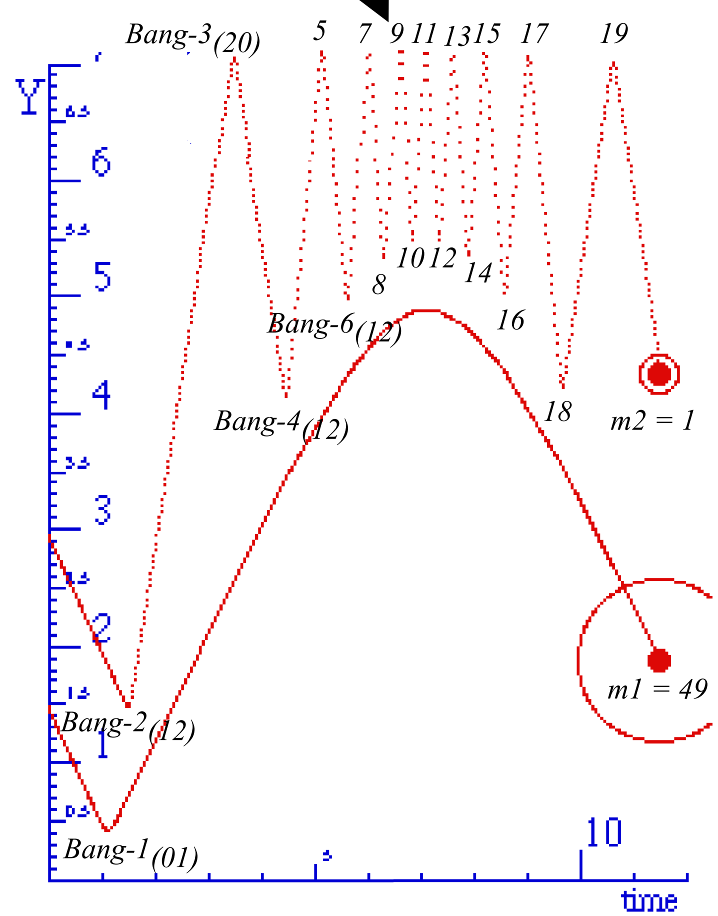
$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: $\mathbf{R} = \mathbf{C}\cdot\mathbf{M}$





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{\text{(after Bang-11)}}$$

Collisions for mass ratio $m_1 : m_2 = 49:1$

<<Under Construction>>
Matrix Collision Web Simulator:
M_1=49, M_2=1 V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with V_2 vs V_1 plot

Fig. 5.1a-b
(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

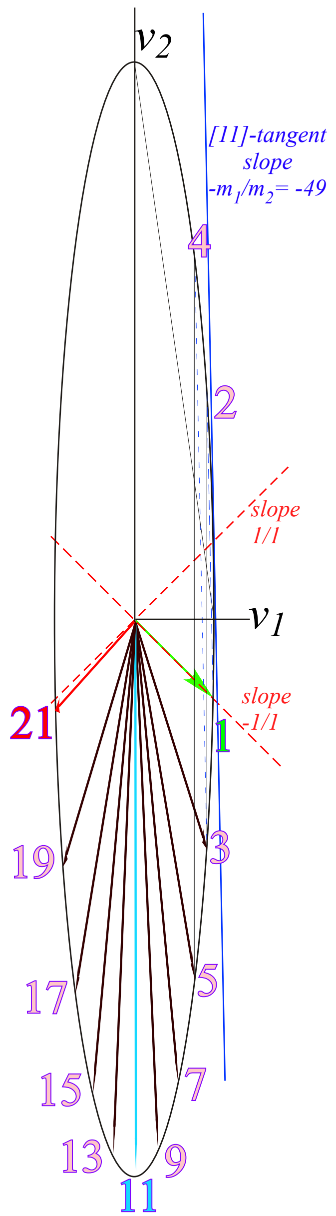
Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$



Collisions for mass ratio $m_1:m_2 = 49:1$

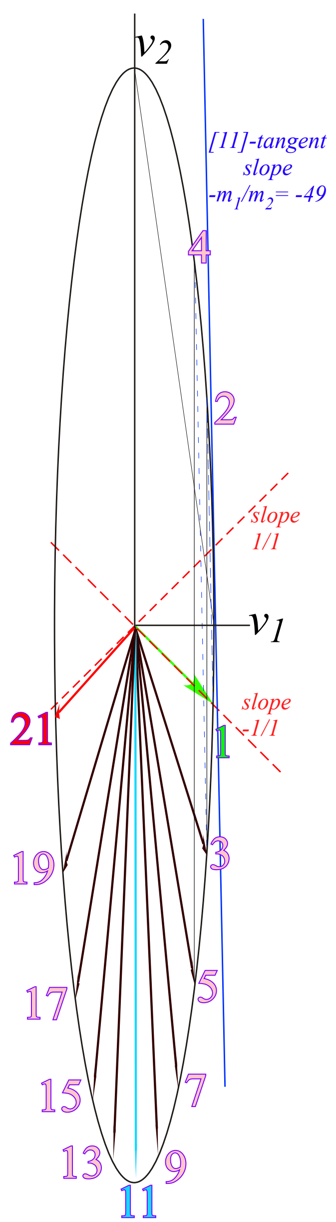
Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$



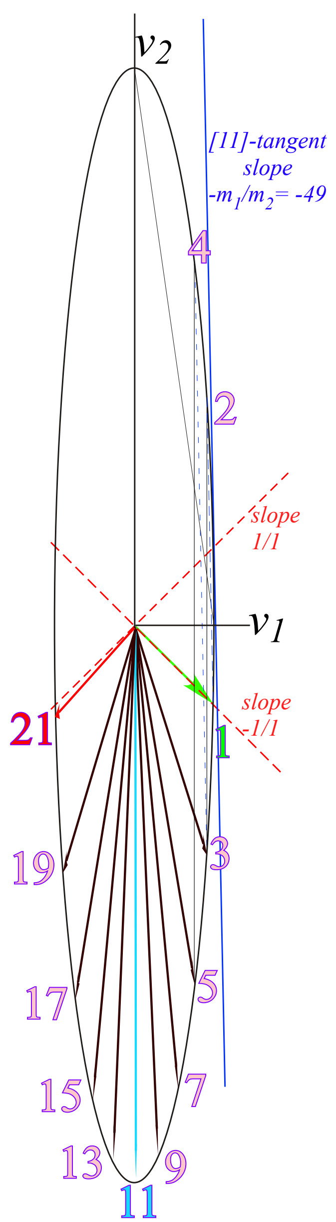
Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \bullet \vec{\mathsf{V}} \ , \quad \text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \bullet \mathsf{M} \bullet \vec{\mathsf{V}}$$



Collisions for mass ratio $m_1 : m_2 = 49:1$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

or:

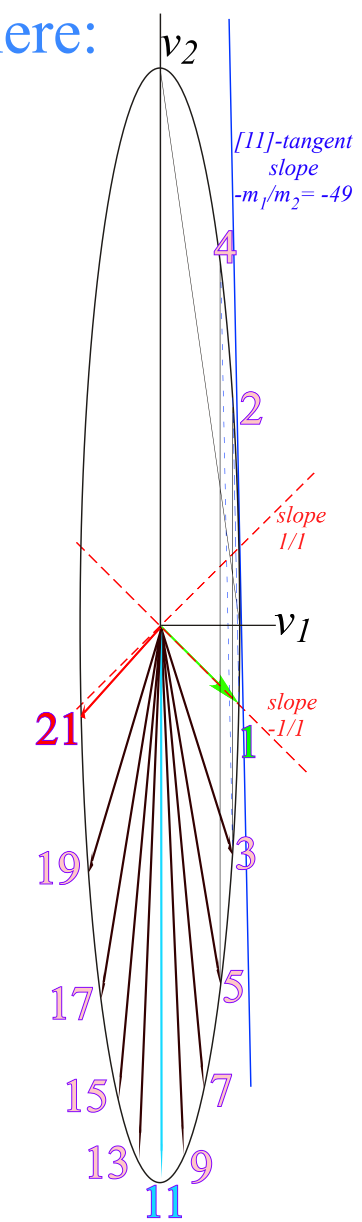
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \bullet \vec{\mathsf{V}}$$

or:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \bullet \mathsf{M} \bullet \vec{\mathsf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

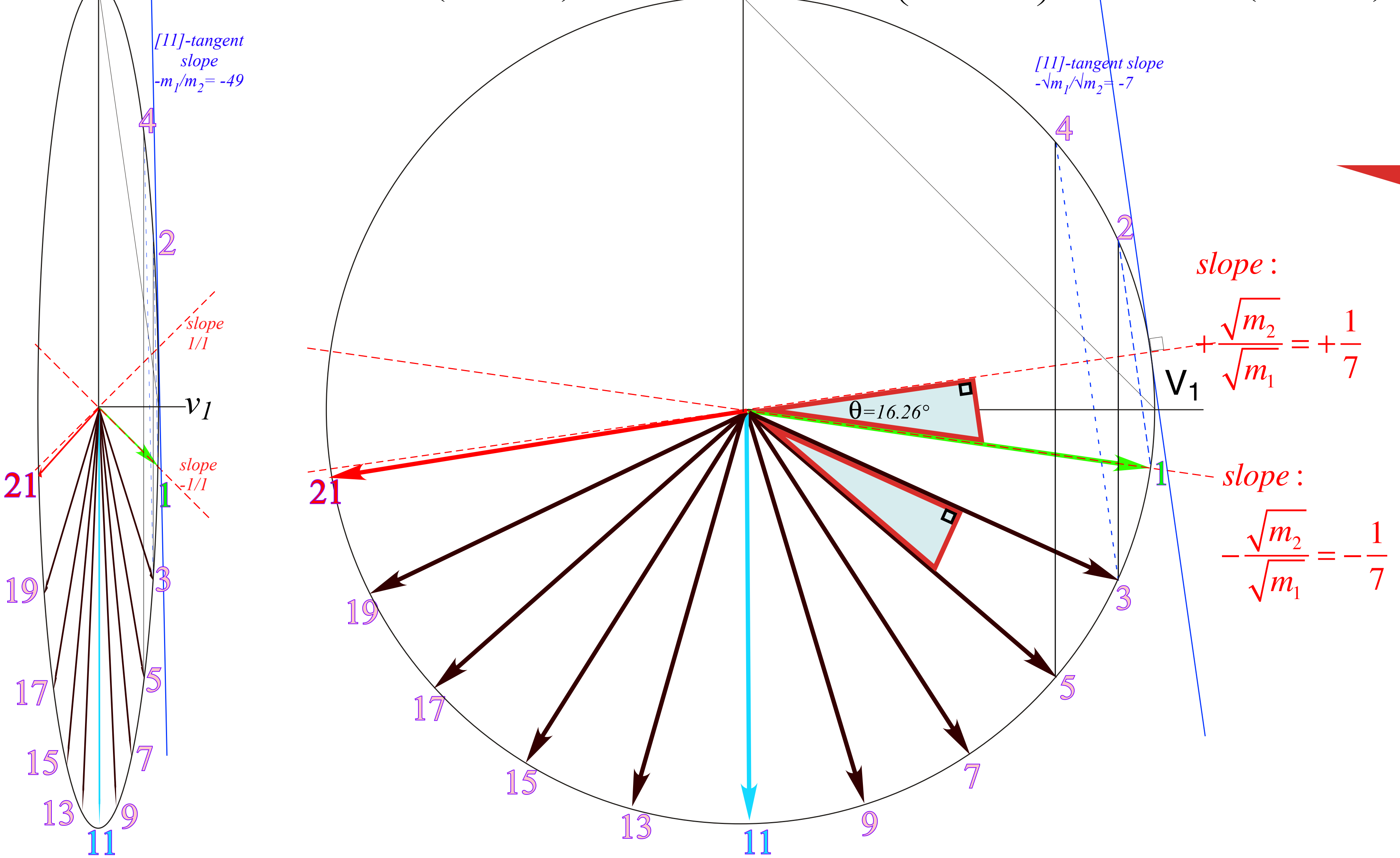
Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\mathsf{V}_1^2 + \frac{1}{2}\mathsf{V}_2^2$

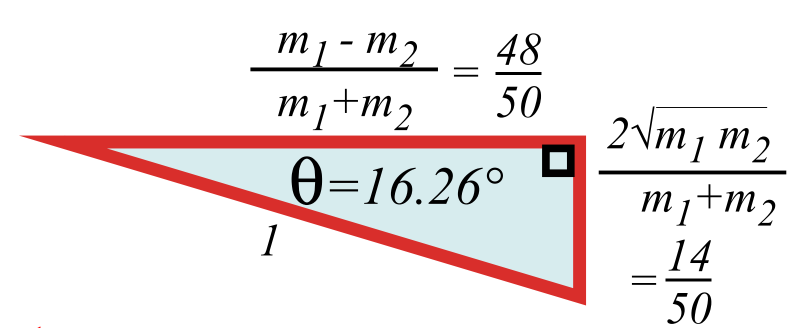
$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathsf{V}_1^{FIN_1}/\sqrt{m_1} \\ \mathsf{V}_2^{FIN_1}/\sqrt{m_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1/\sqrt{m_1} \\ \mathsf{V}_2/\sqrt{m_2} \end{pmatrix}$$

or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{M}\bullet\vec{\mathsf{V}}$$, or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{C}\bullet\mathbf{M}\bullet\vec{\mathsf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$





Collisions for mass ratio $m_1:m_2 = 49:1$

Fig. 5.2a-c

(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\mathsf{V}_1^2 + \frac{1}{2}\mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

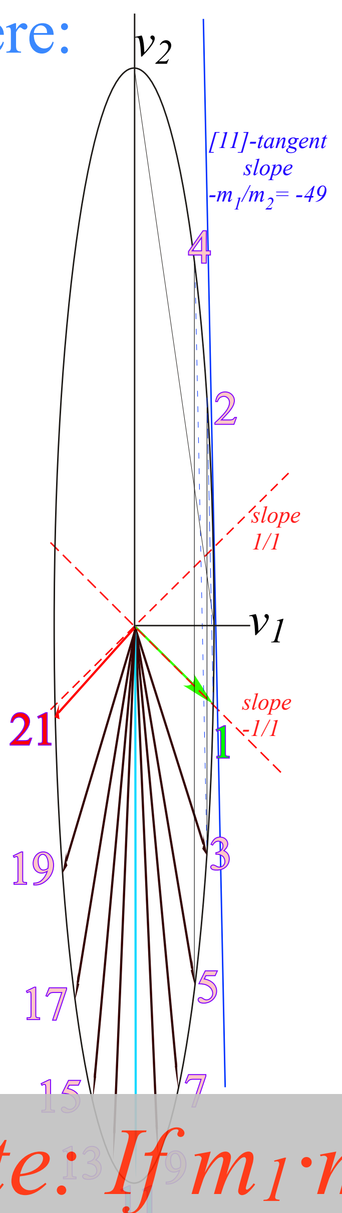
becomes:

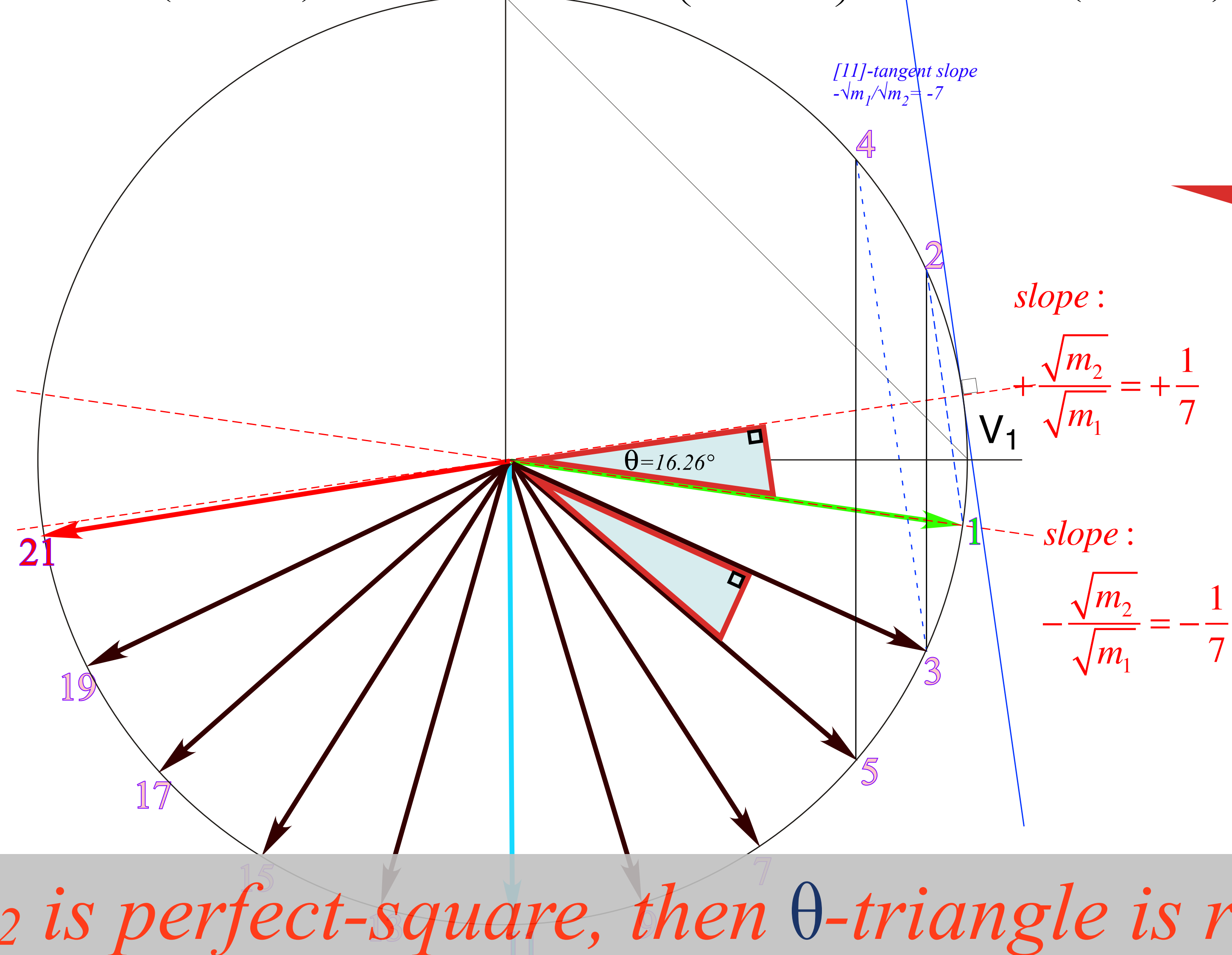
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1}/\sqrt{m_1} \\ \mathsf{V}_2^{FIN_1}/\sqrt{m_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1/\sqrt{m_1} \\ \mathsf{V}_2/\sqrt{m_2} \end{pmatrix}$$

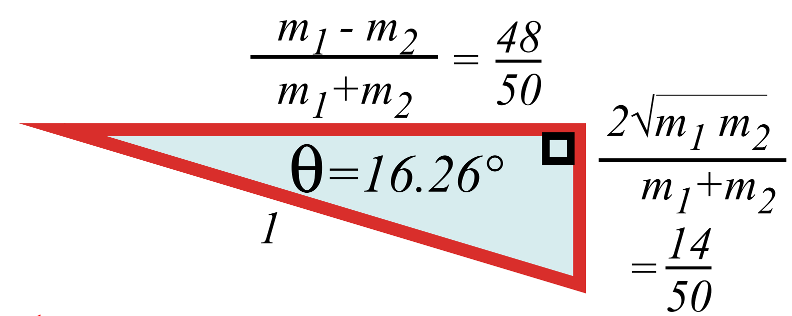
or: $\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{M}\bullet\vec{\mathsf{V}}$, or: $\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{C}\bullet\mathbf{M}\bullet\vec{\mathsf{V}}$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 5.2a-c

(revised)

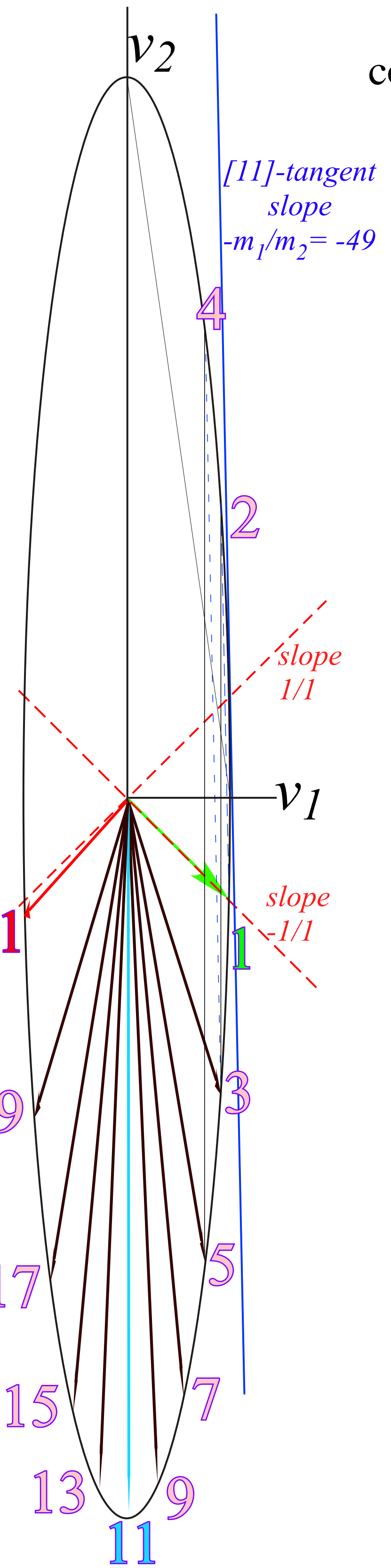
Note: If $m_1 \cdot m_2$ is perfect-square, then θ-triangle is rational ($3^2+4^2=5^2$, etc.)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$



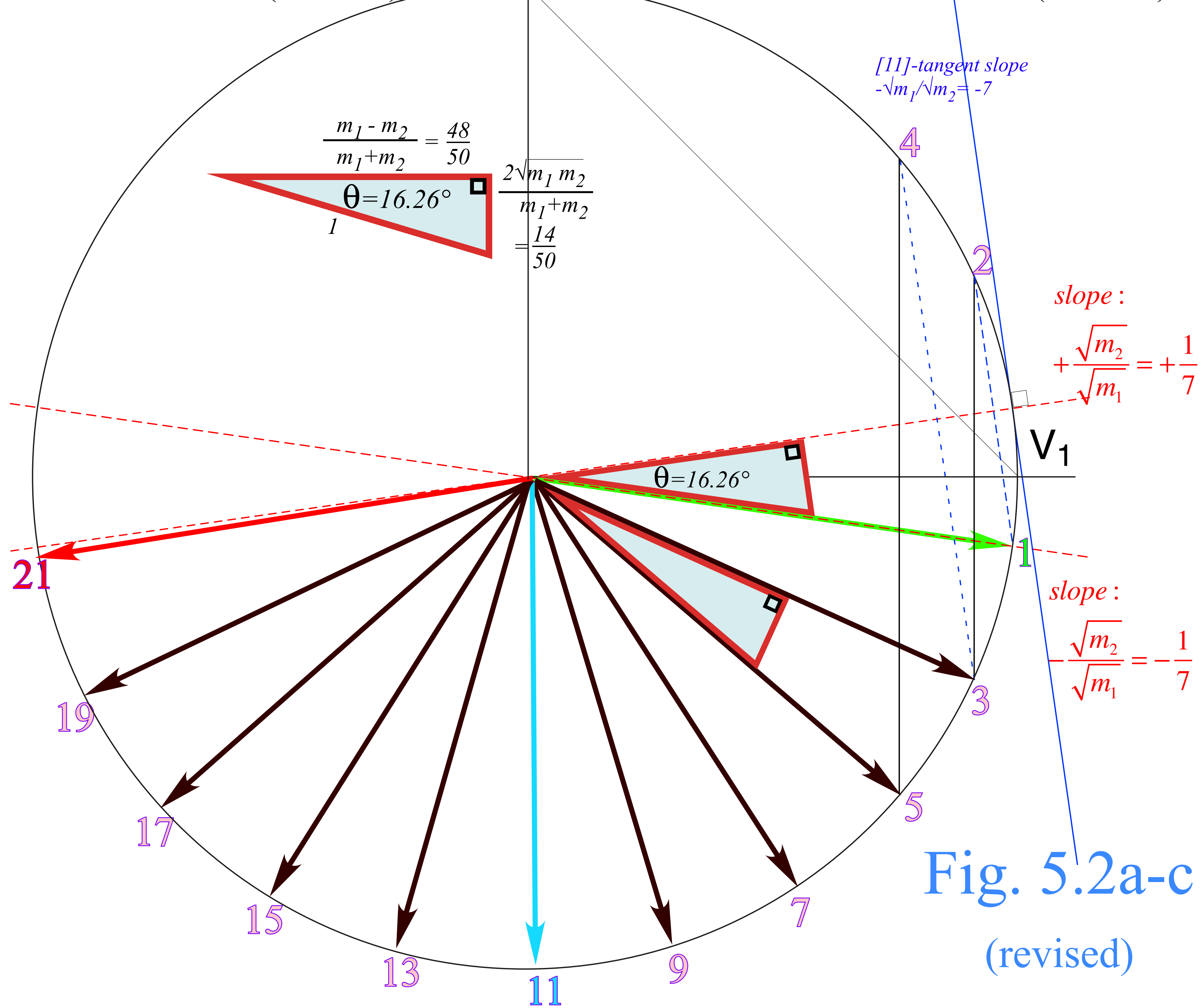


Fig. 5.2a-c

(revised)

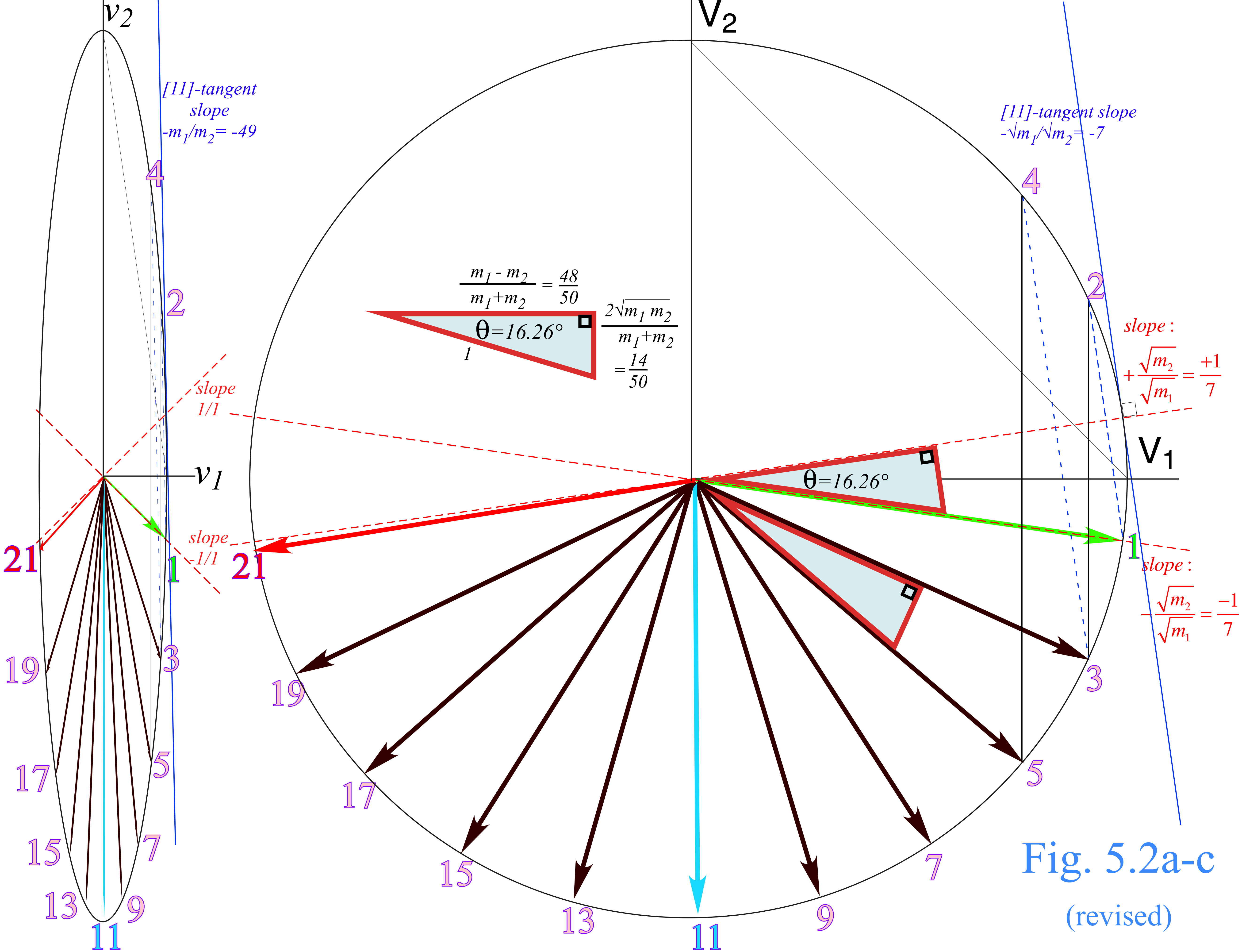


Fig. 5.2a-c

(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

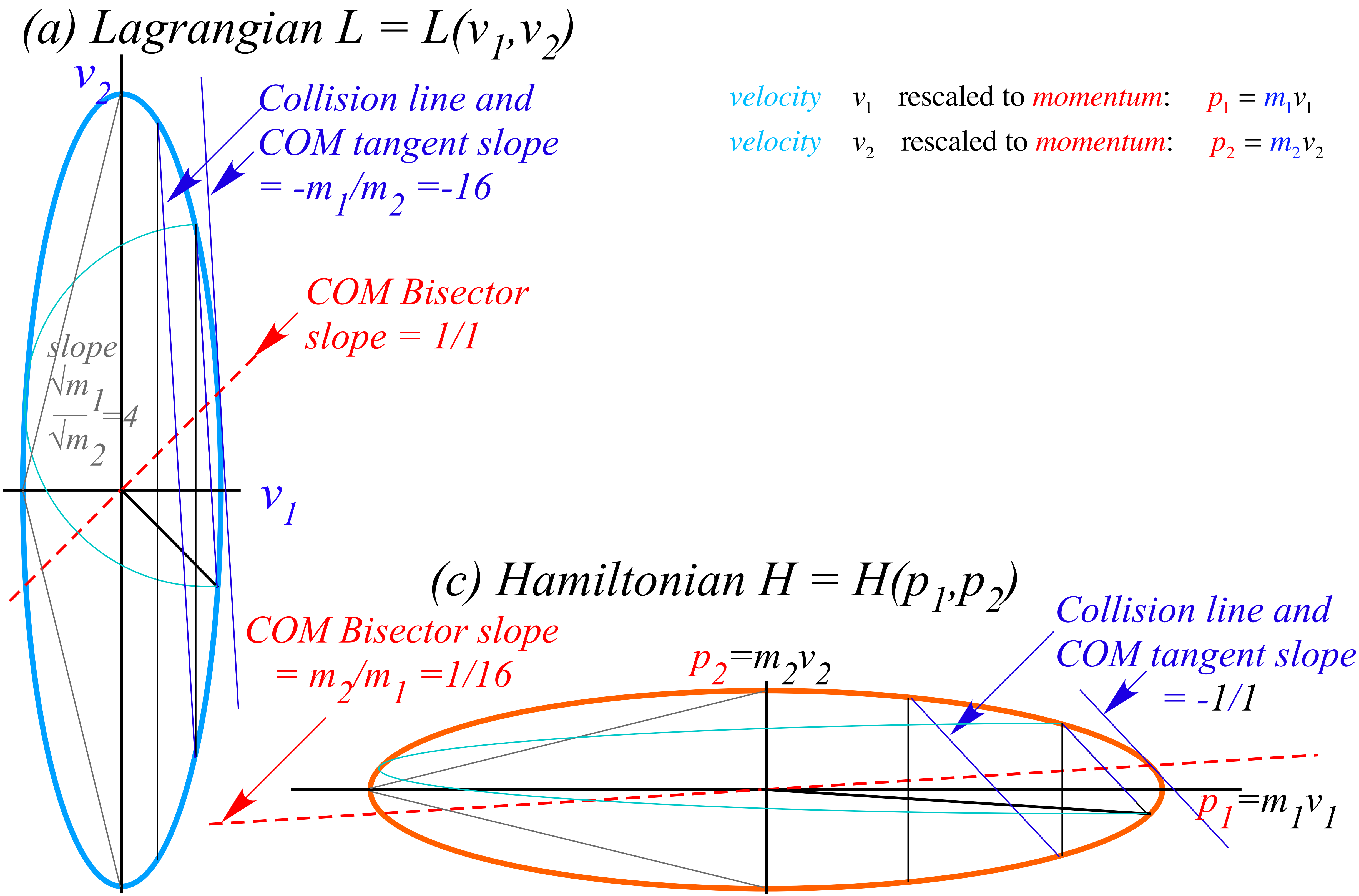
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

What ellipse rescaling leads to...

How this relates to Lagrangian, and Hamiltonian mechanics later on

velocity v_1 rescaled to momentum: $p_1 = m_1 v_1$

velocity v_2 rescaled to momentum: $p_2 = m_2 v_2$

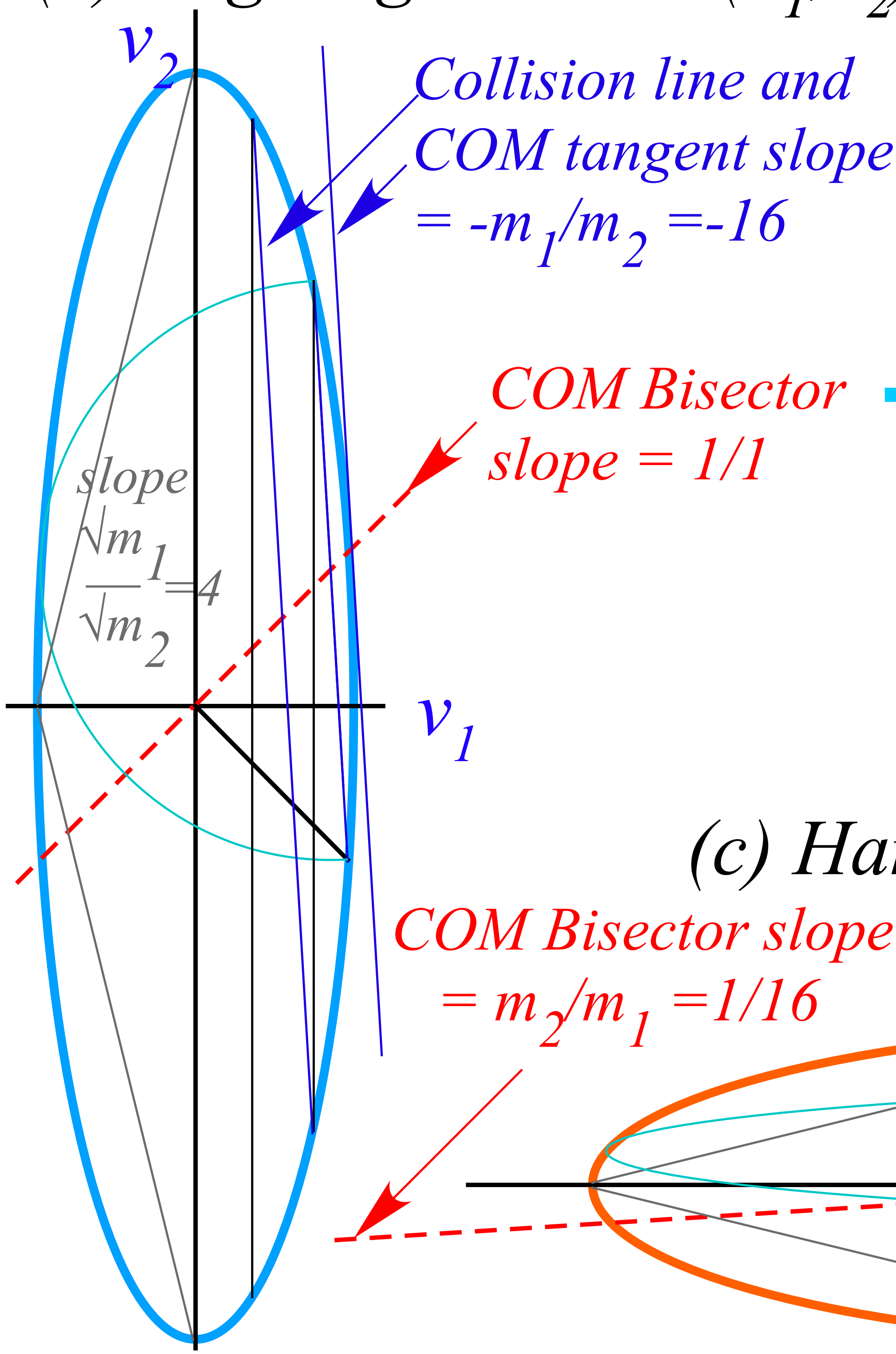


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

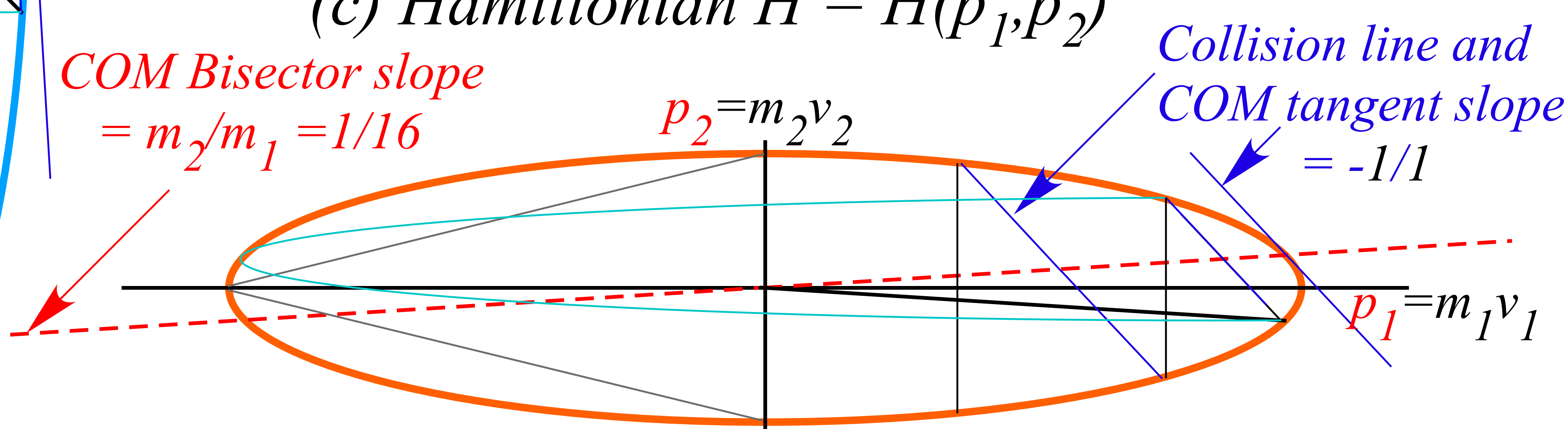
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

(c) Hamiltonian $H = H(p_1, p_2)$

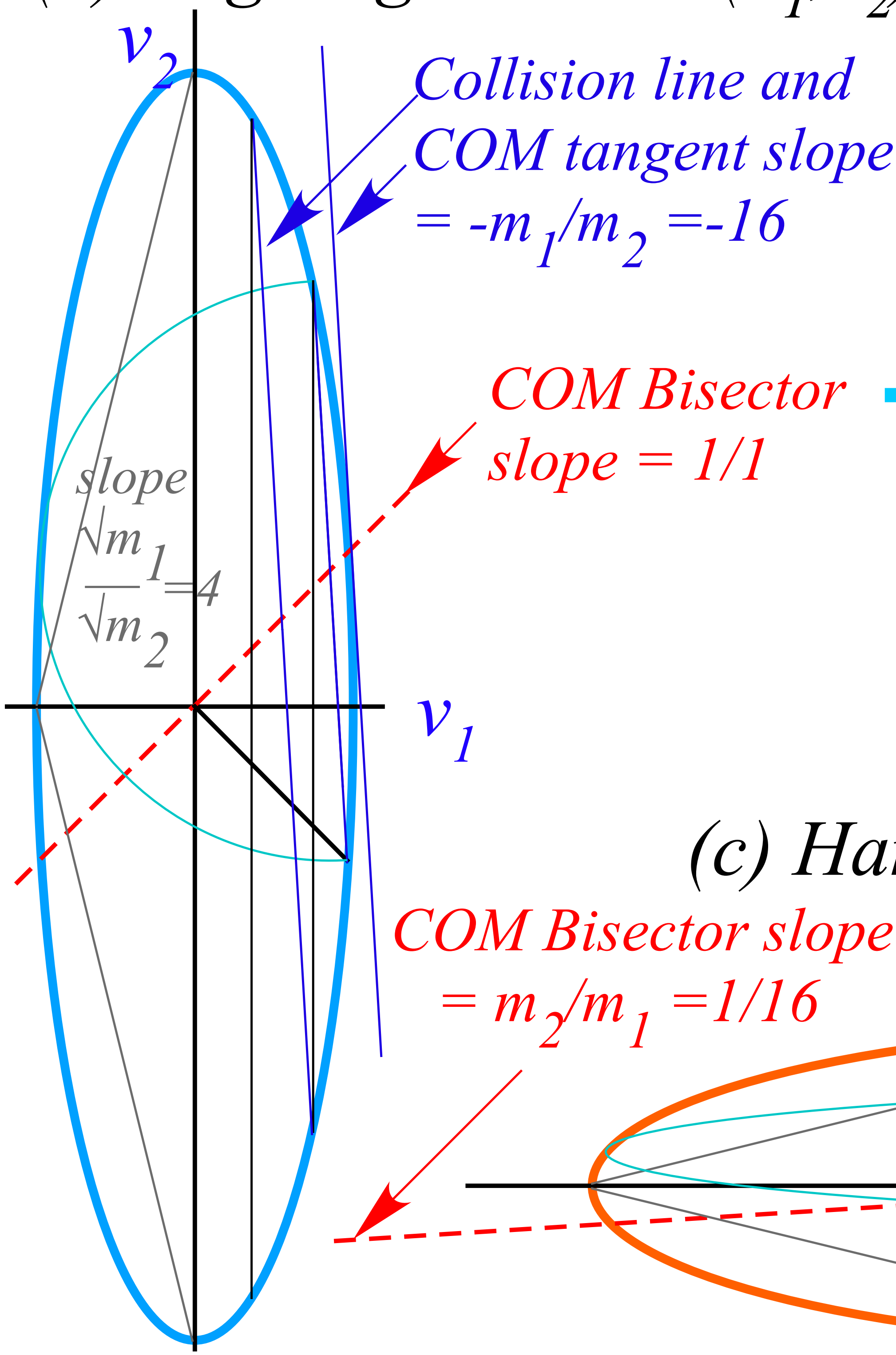


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on

(a) Lagrangian $L = L(v_1,v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

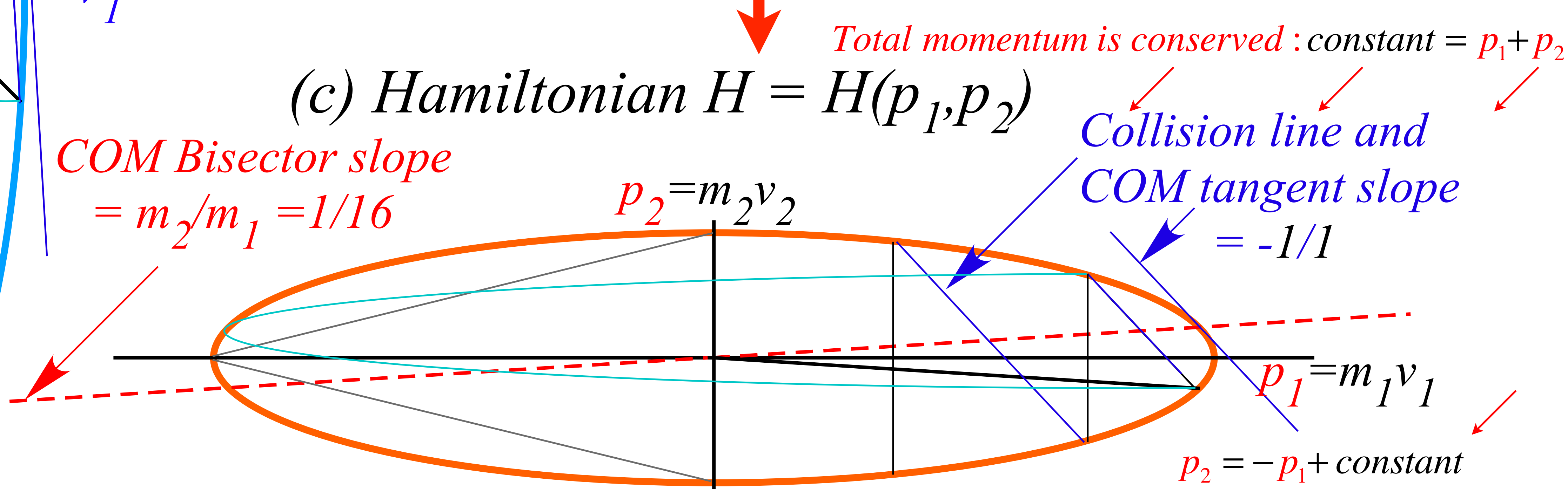
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1,v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1,p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

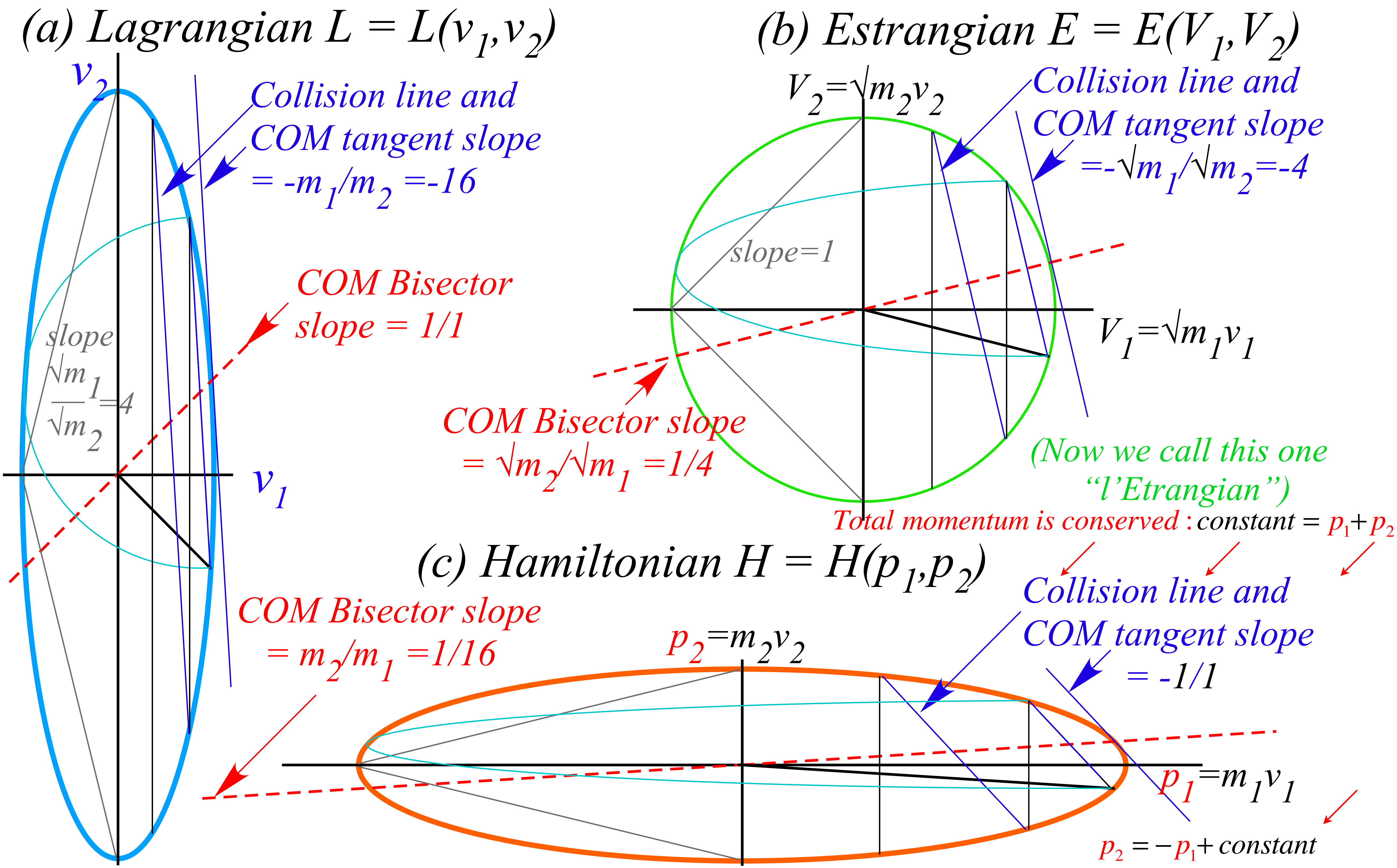
(c) Hamiltonian $H = H(p_1,p_2)$



What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

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Reflections in clothing store mirrors

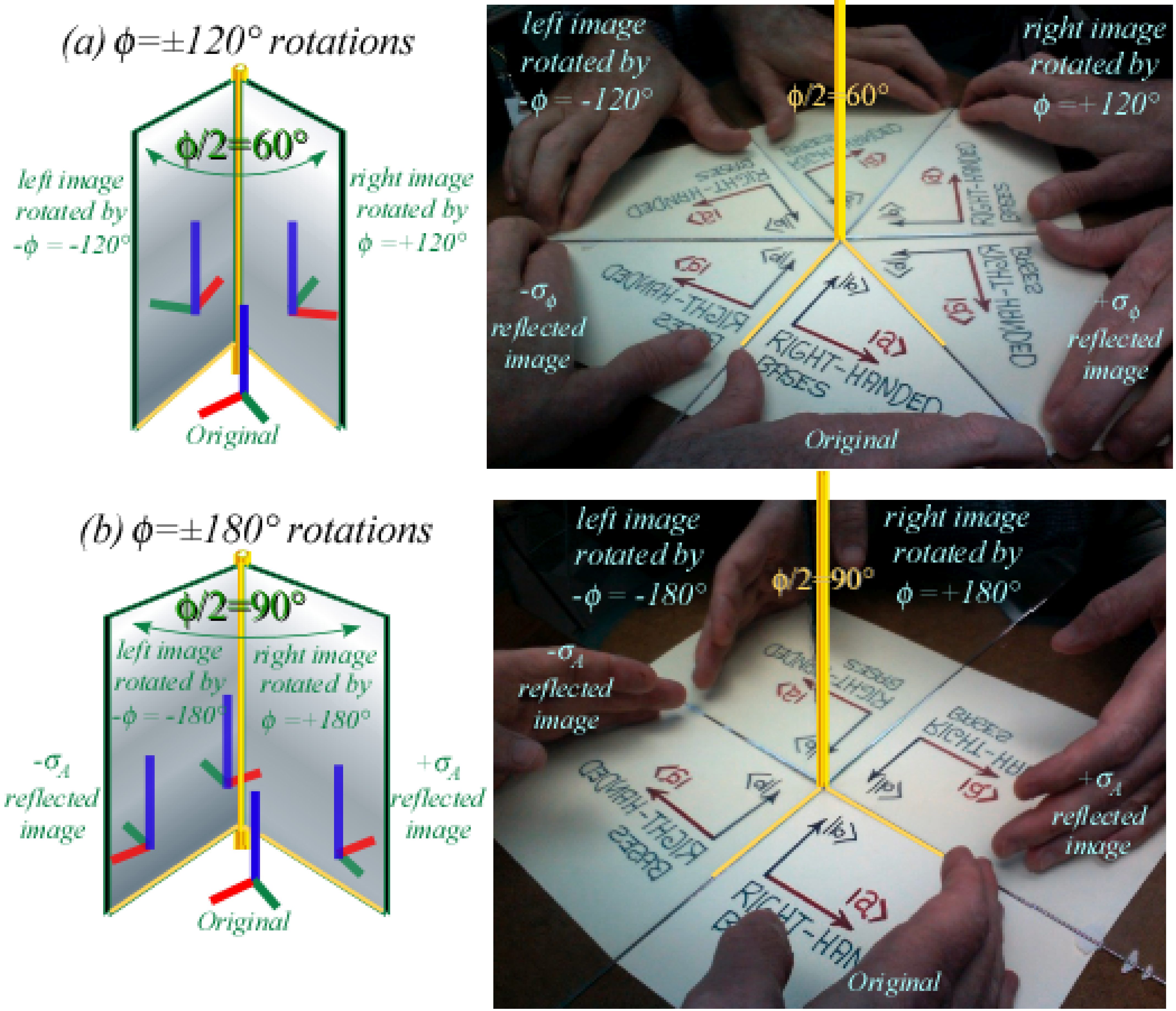

Fig. 5.4a-b

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

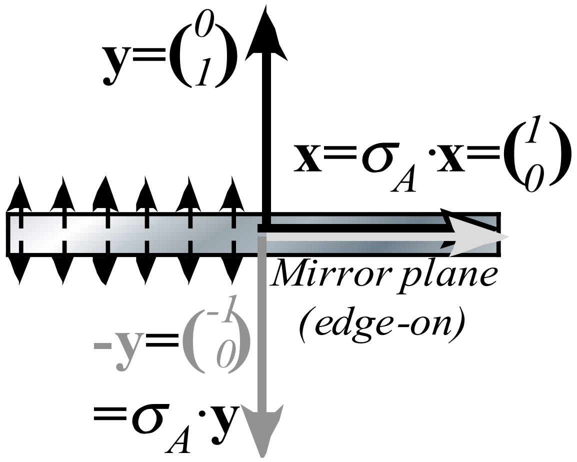

Fig.
5.3a-e

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

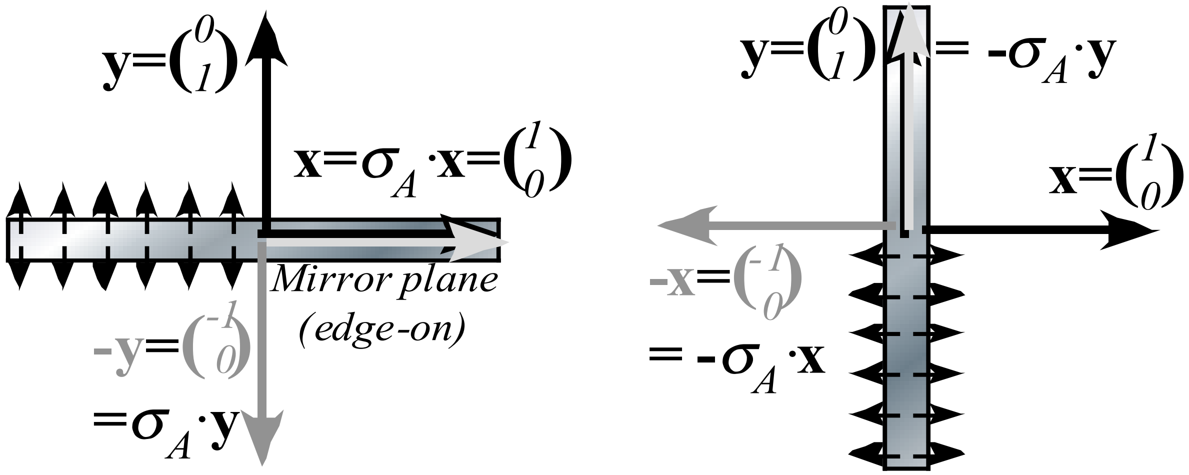

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ *(b)Reflections* $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

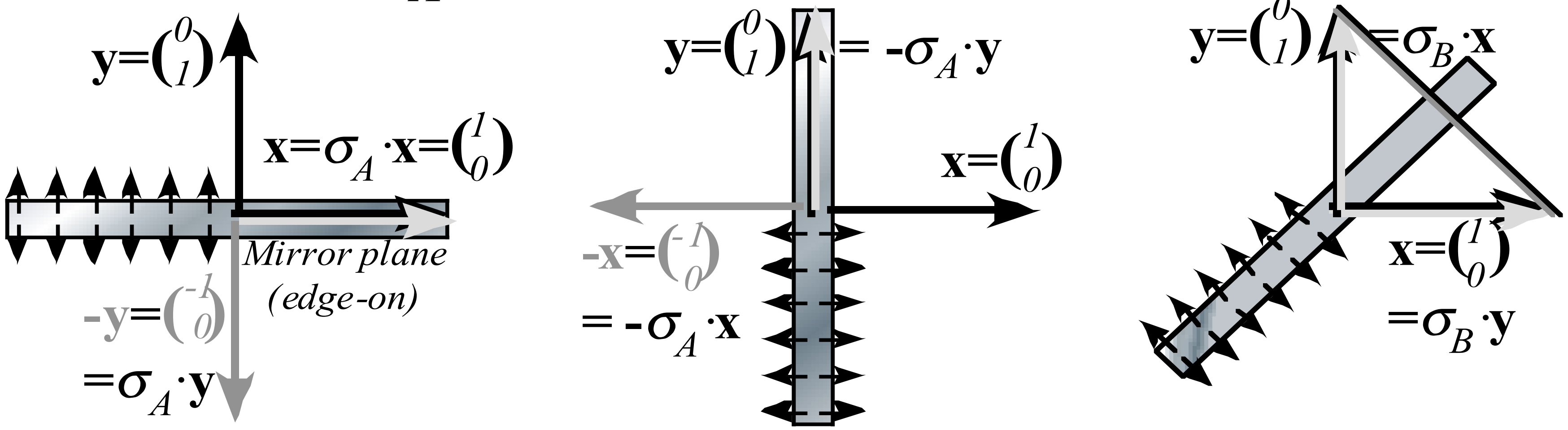

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

(a) Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

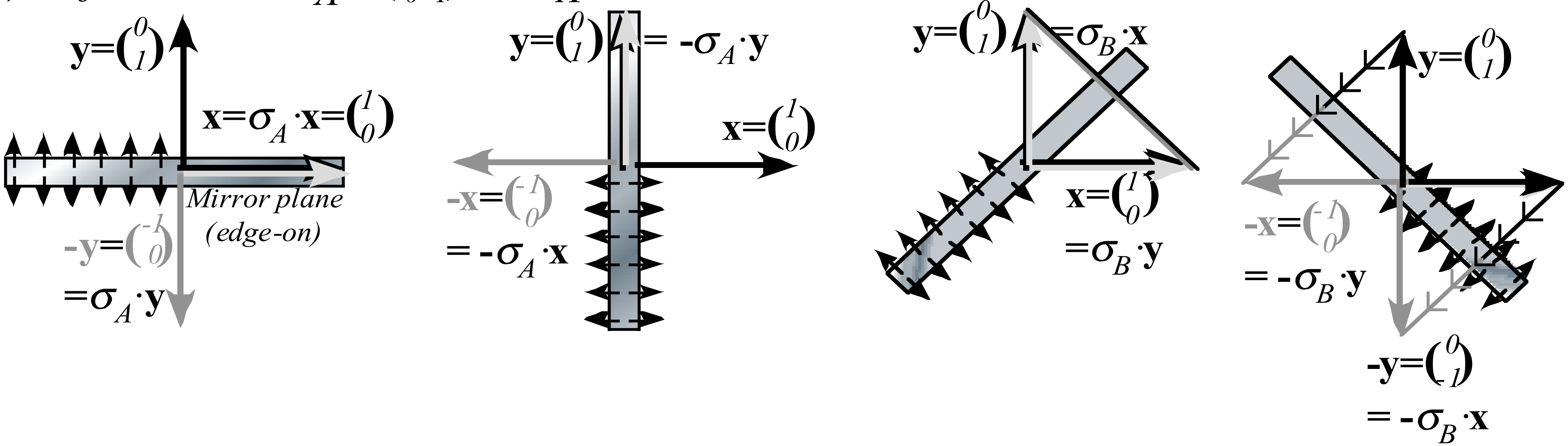

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

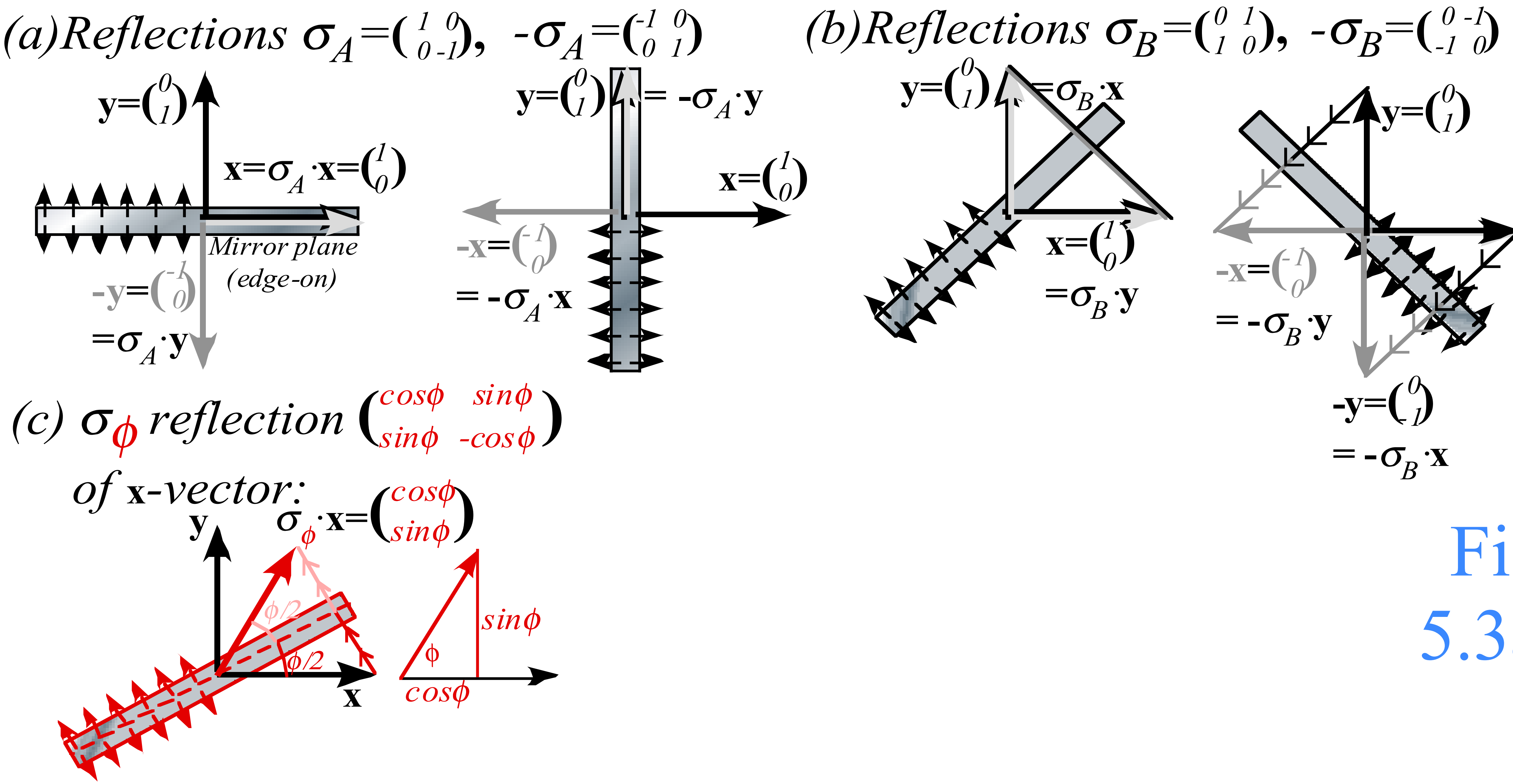

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

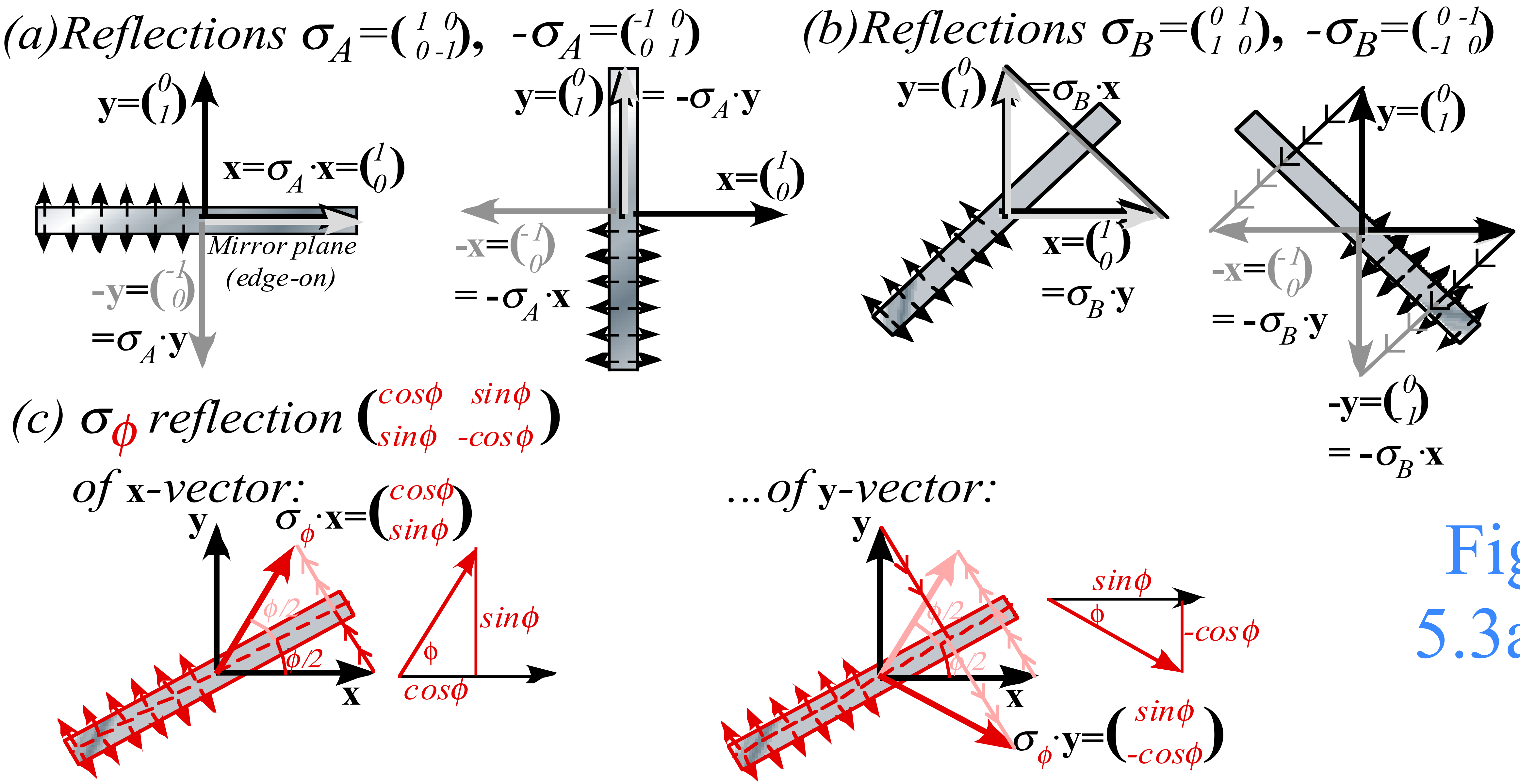

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

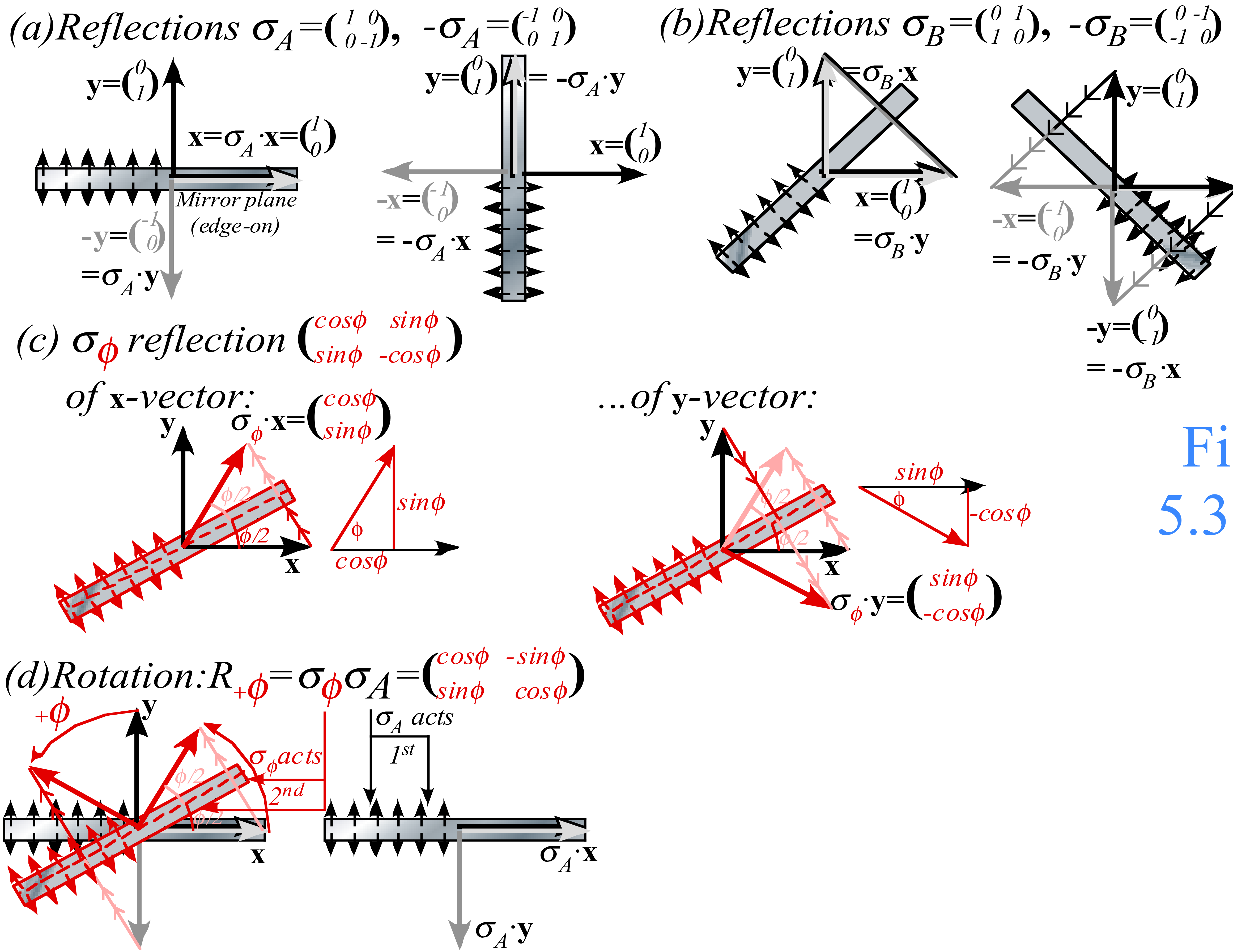

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

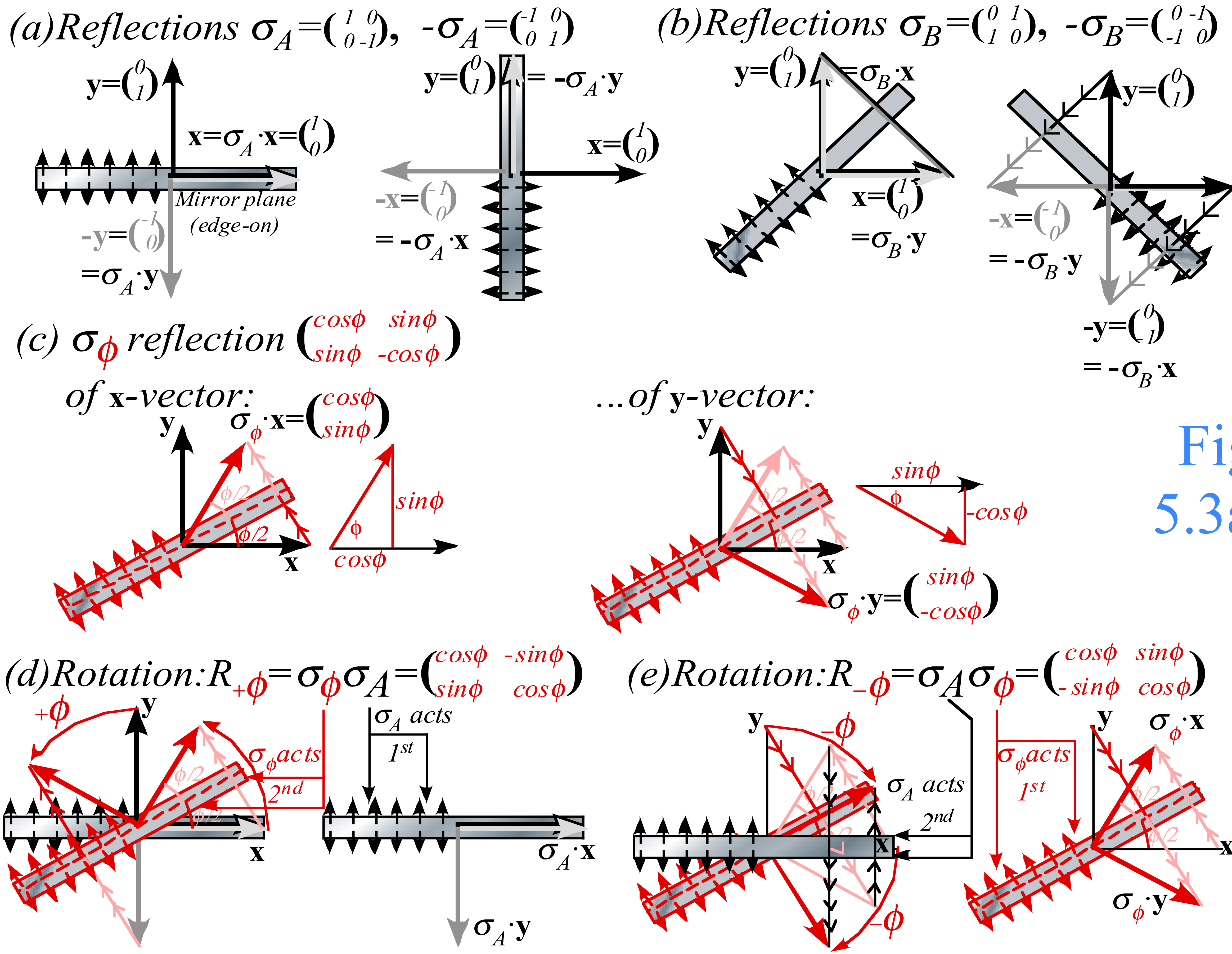

Fig. 5.3a-e

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a reflection

... even number of reflections is a rotation (or unit-op **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a *reflection*

... even number of reflections is a *rotation* *(or unit-op* **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and *rotate* *easily*
Waves patterns are non-rigid and *reflect* *easily*

∴ *...wave reflections underlie modern physics*

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

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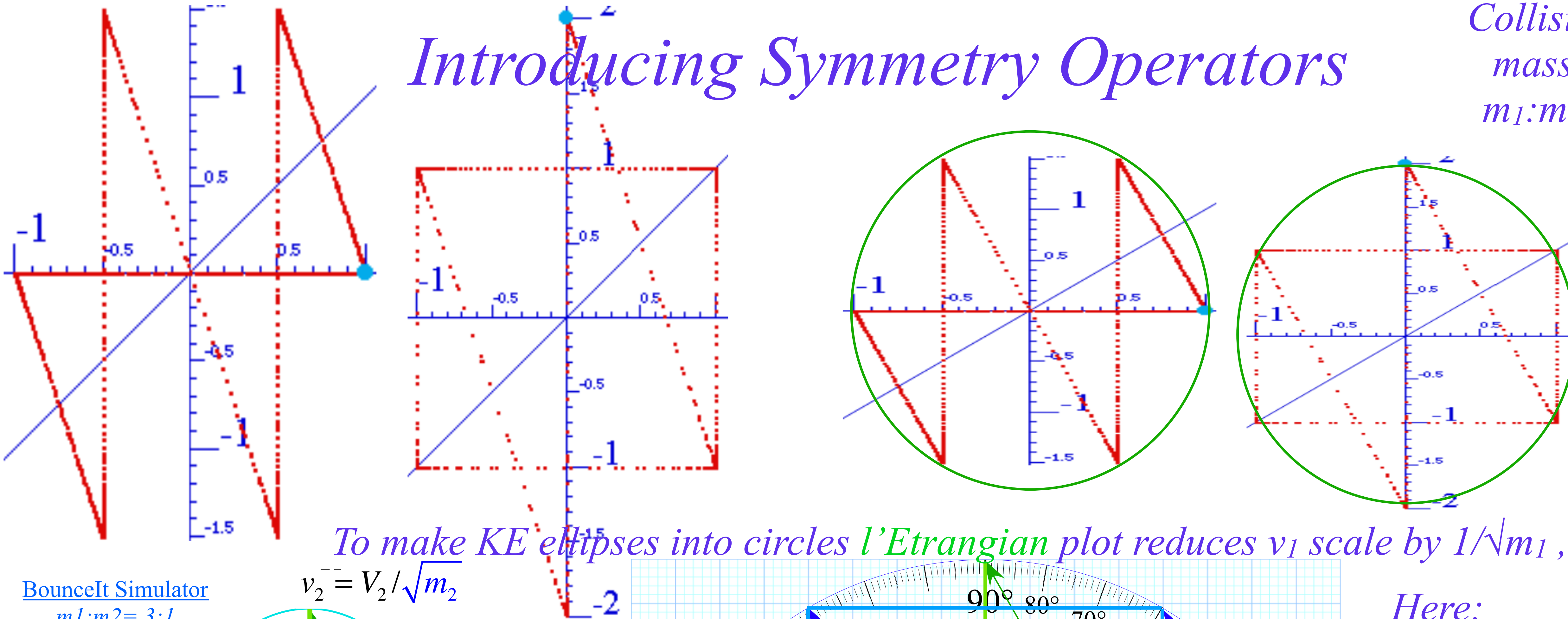
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

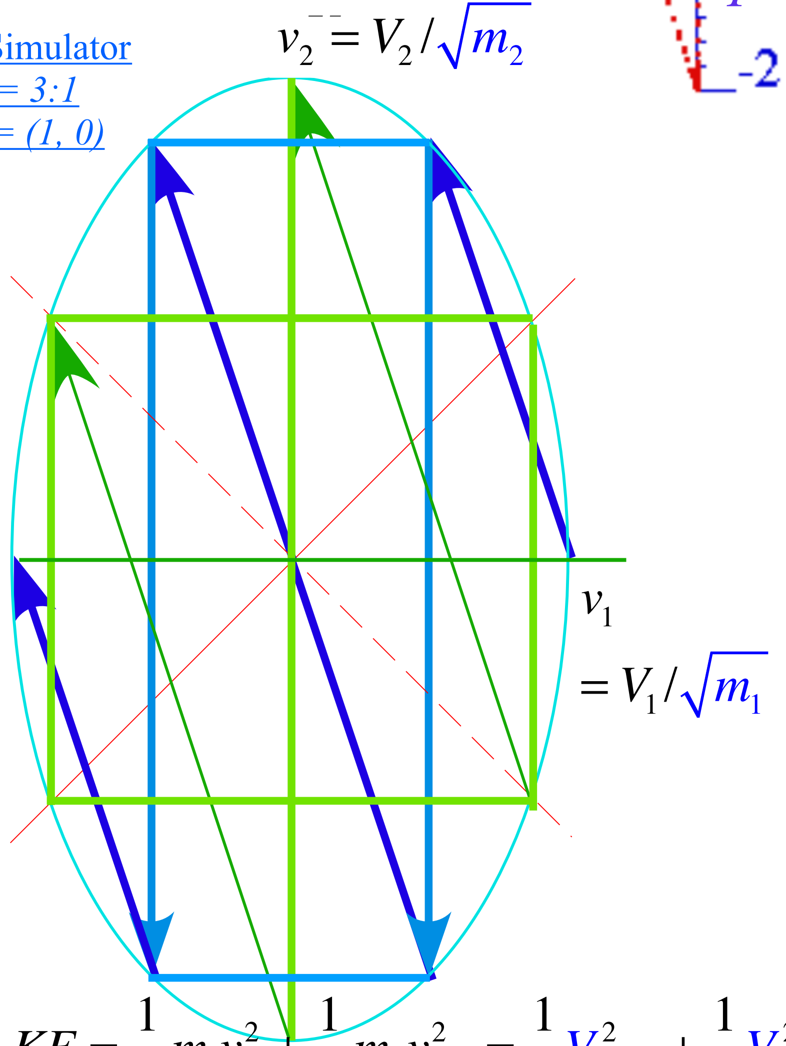
Introducing Symmetry Operators

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$

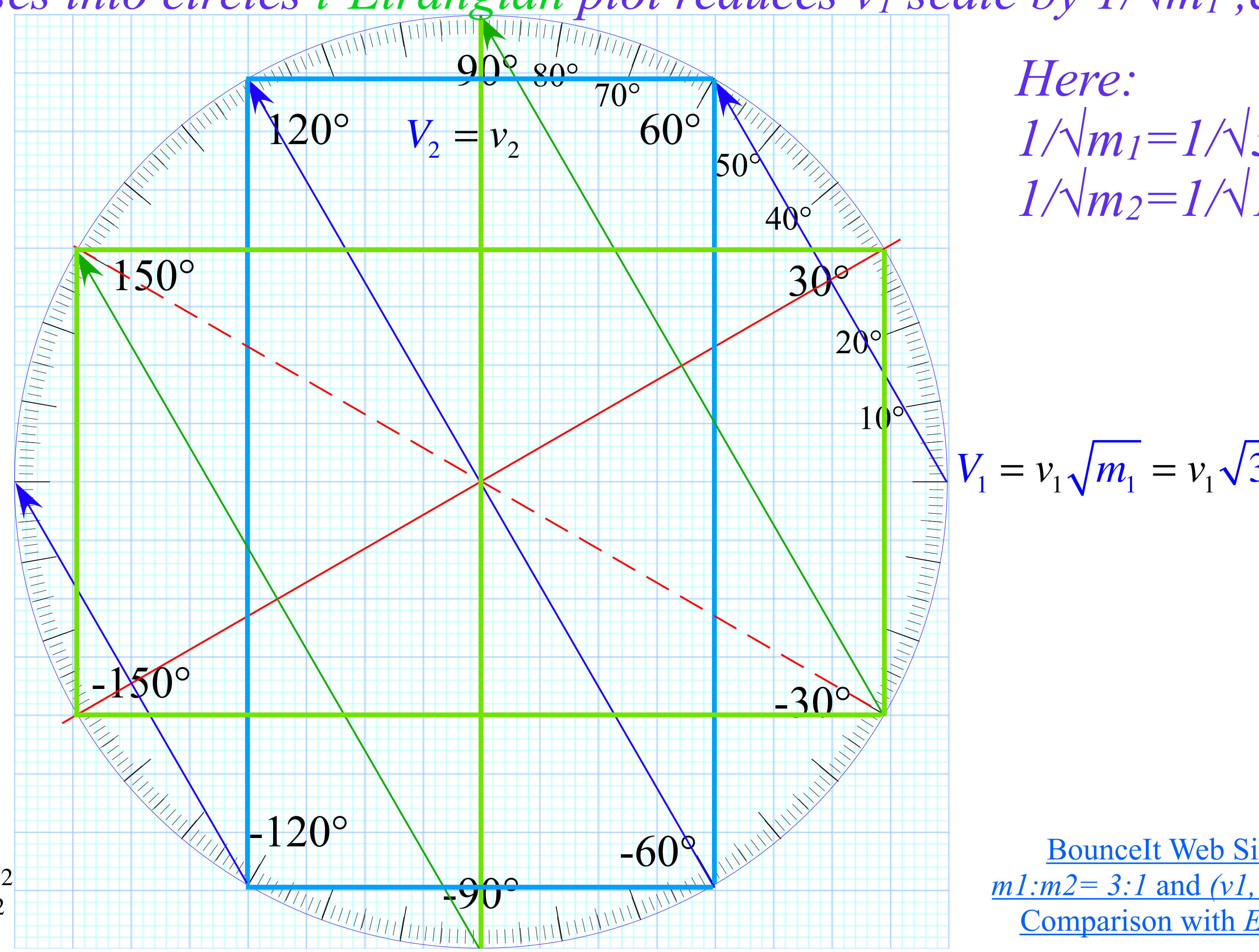


To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

BounceIt Simulator
$m1{:}m2 = 3{:}1$
$(v1, v2) = (1, 0)$



$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$

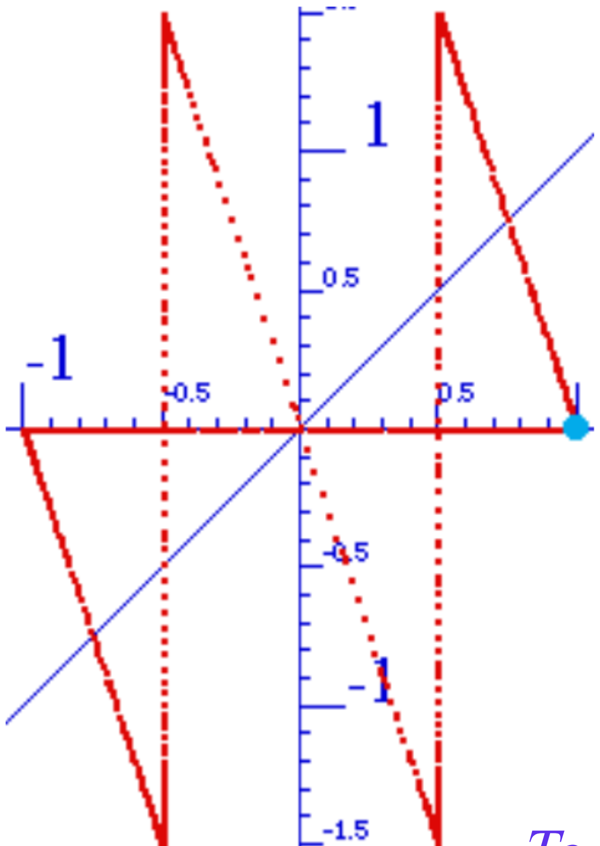


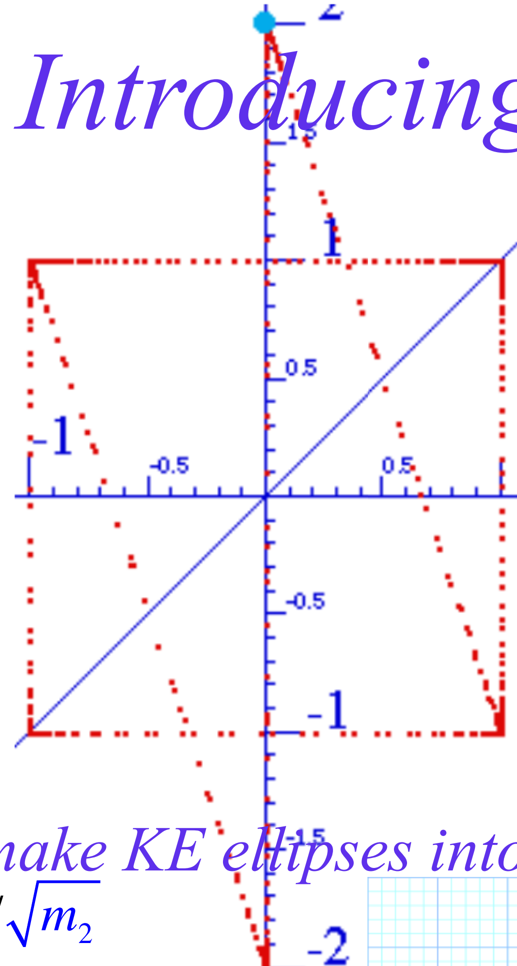
Here:
$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$
$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

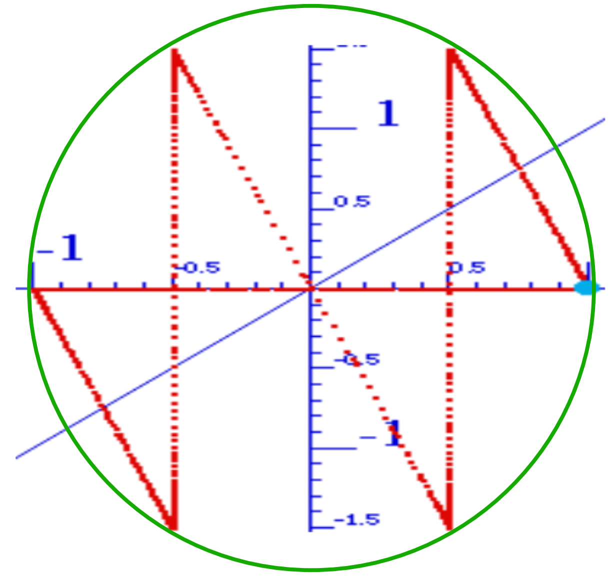
BounceIt Web Simulator
$m1{:}m2 = 3{:}1$ and $(v1, v2) = (1, 0)$
Comparison with *Estrangian*

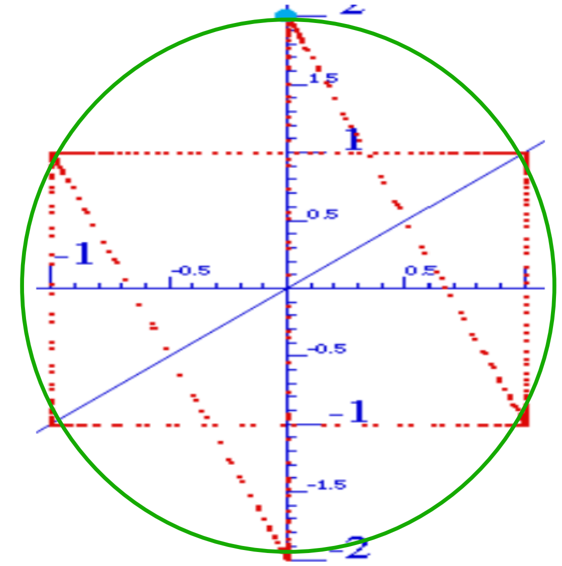
Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1



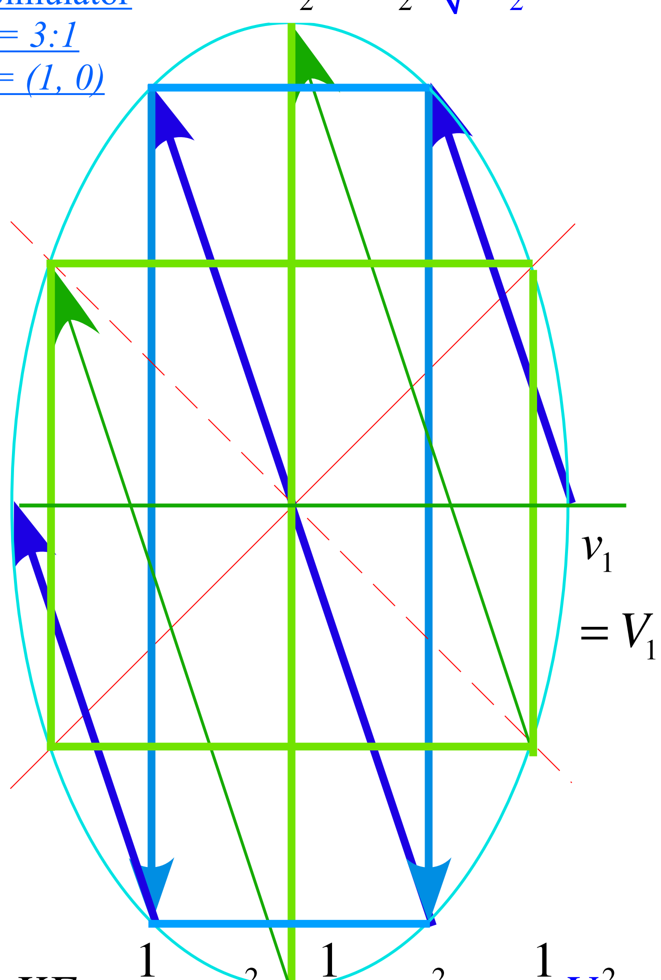




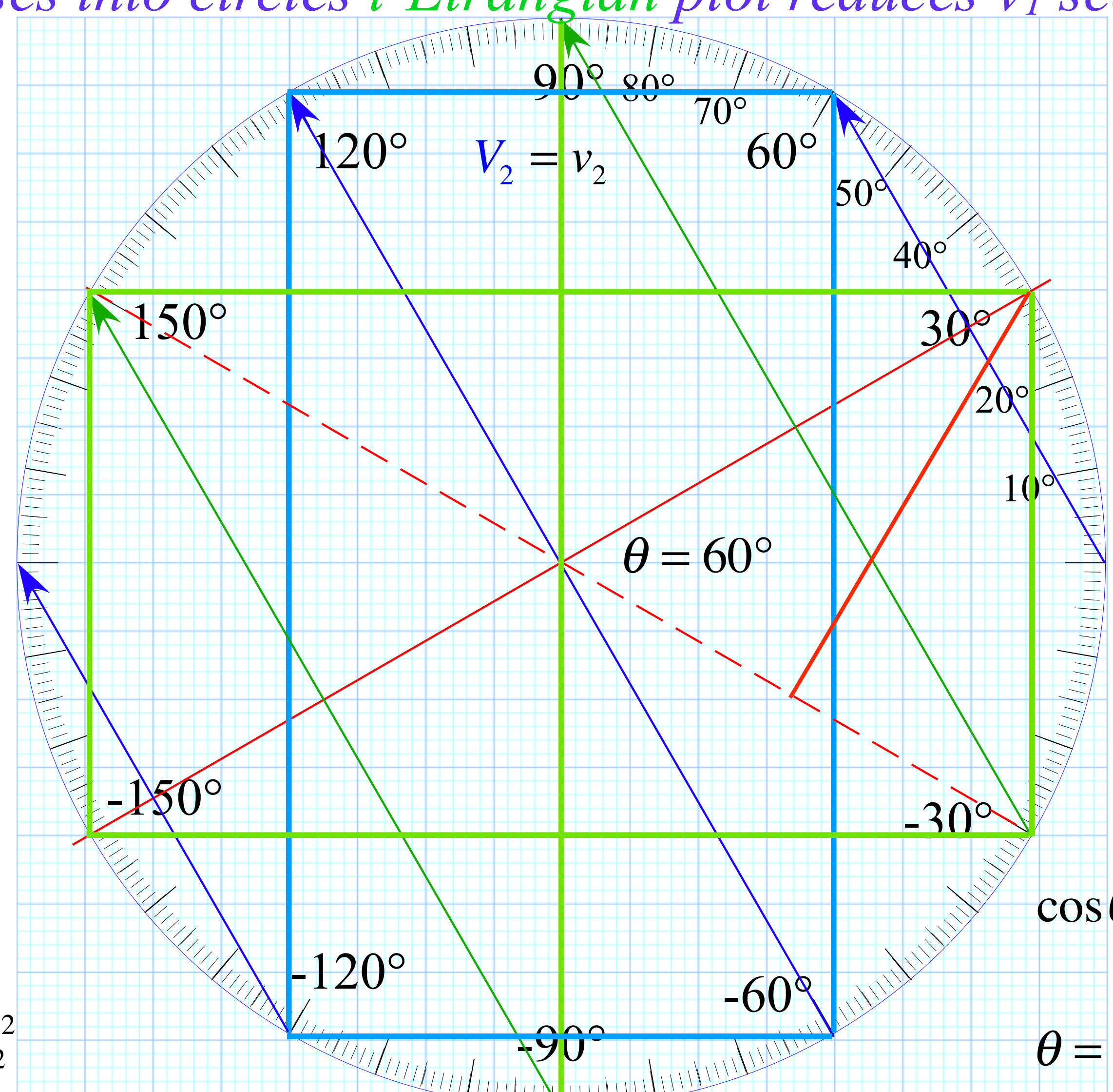


To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.

[BounceIt Simulator](#)
[*m1:m2= 3:1*](#)
[*(v1, v2) = (1, 0)*](#)



$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$



Here:
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$1/\sqrt{m_2}=1/\sqrt{1}=1.0$

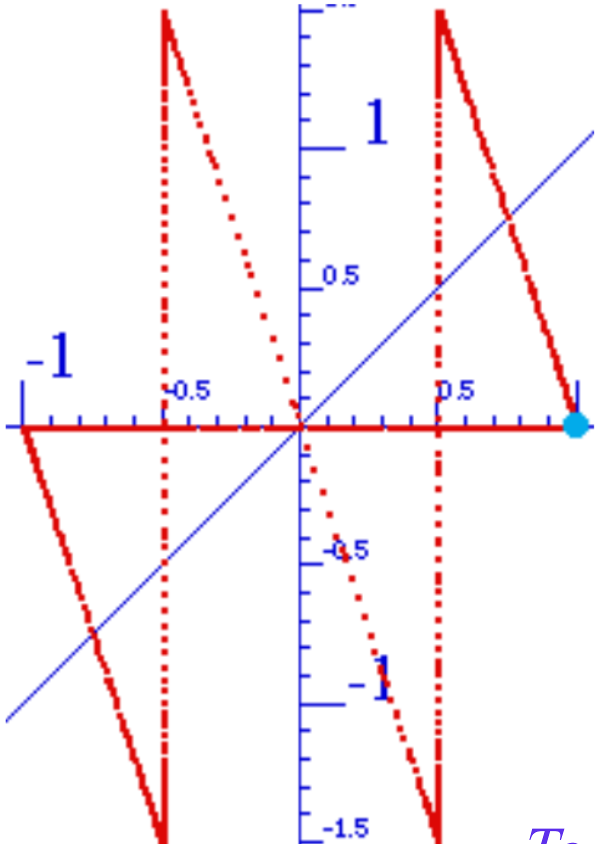
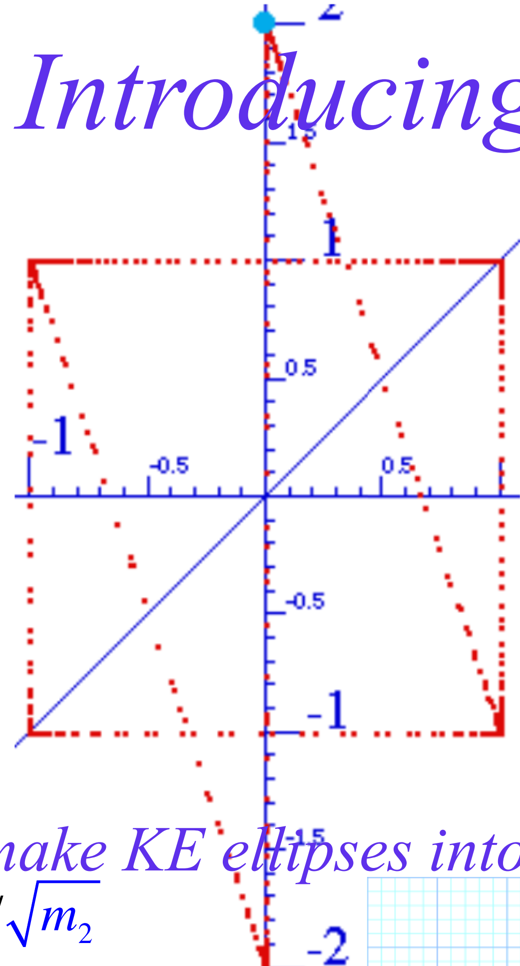
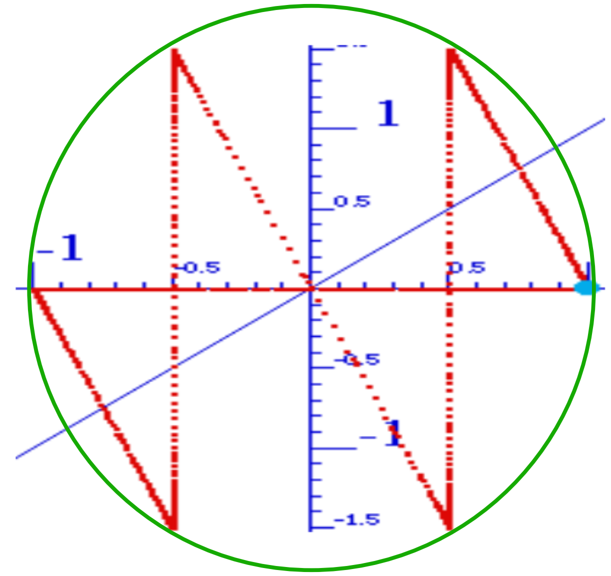
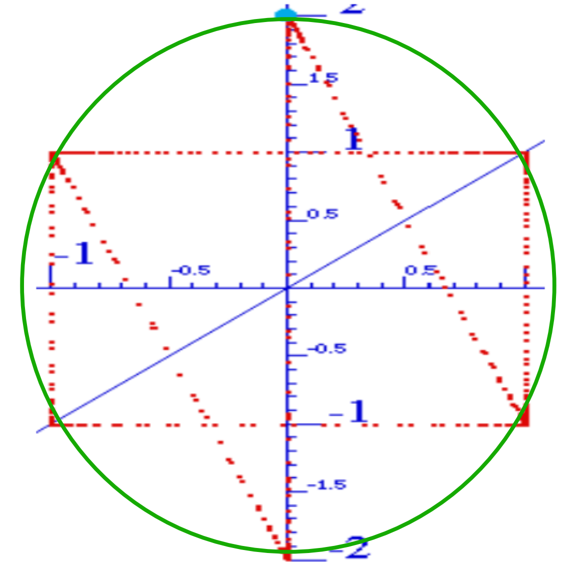
$$\cos\theta = \frac{m_1-m_2}{m_1+m_2} = \frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1} = \frac{2}{4} = \frac{1}{2}$$

$\theta = 60°$

[BounceIt Web Simulator](#)
[Comparison with *Estrangian*](#)

Introducing Symmetry Operators

Collisions for mass ratio m_1:m_2= 3:1

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BounceIt Simulator
m1:m2= 3:1
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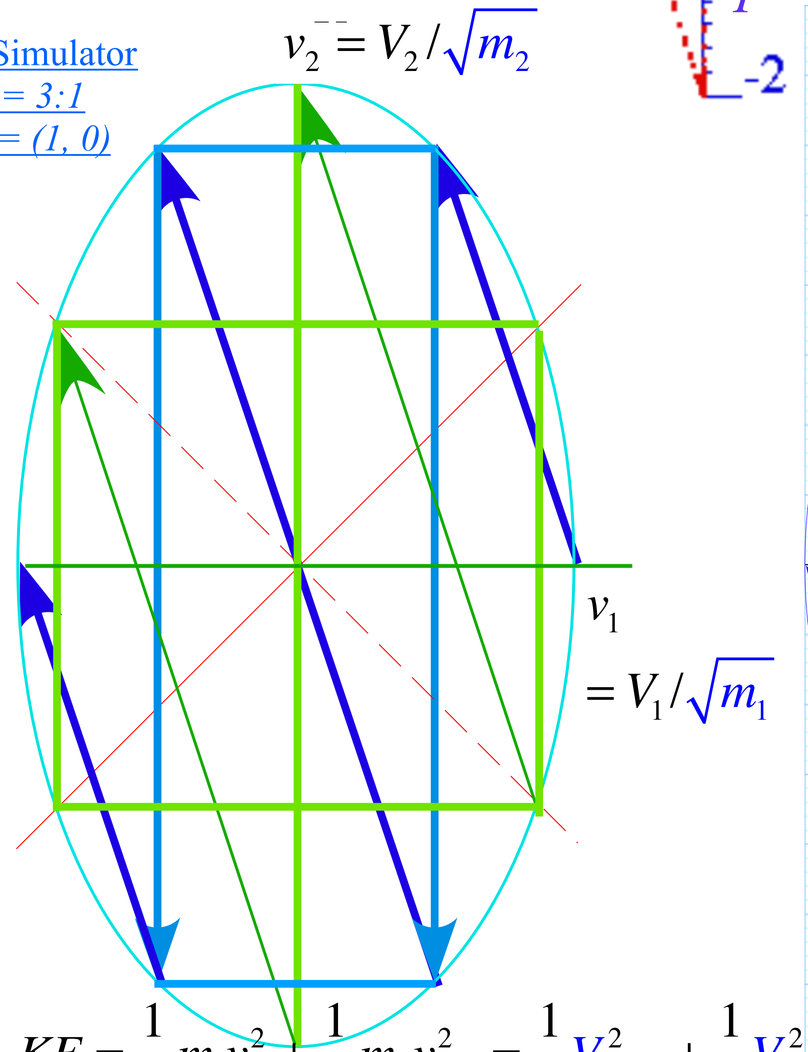

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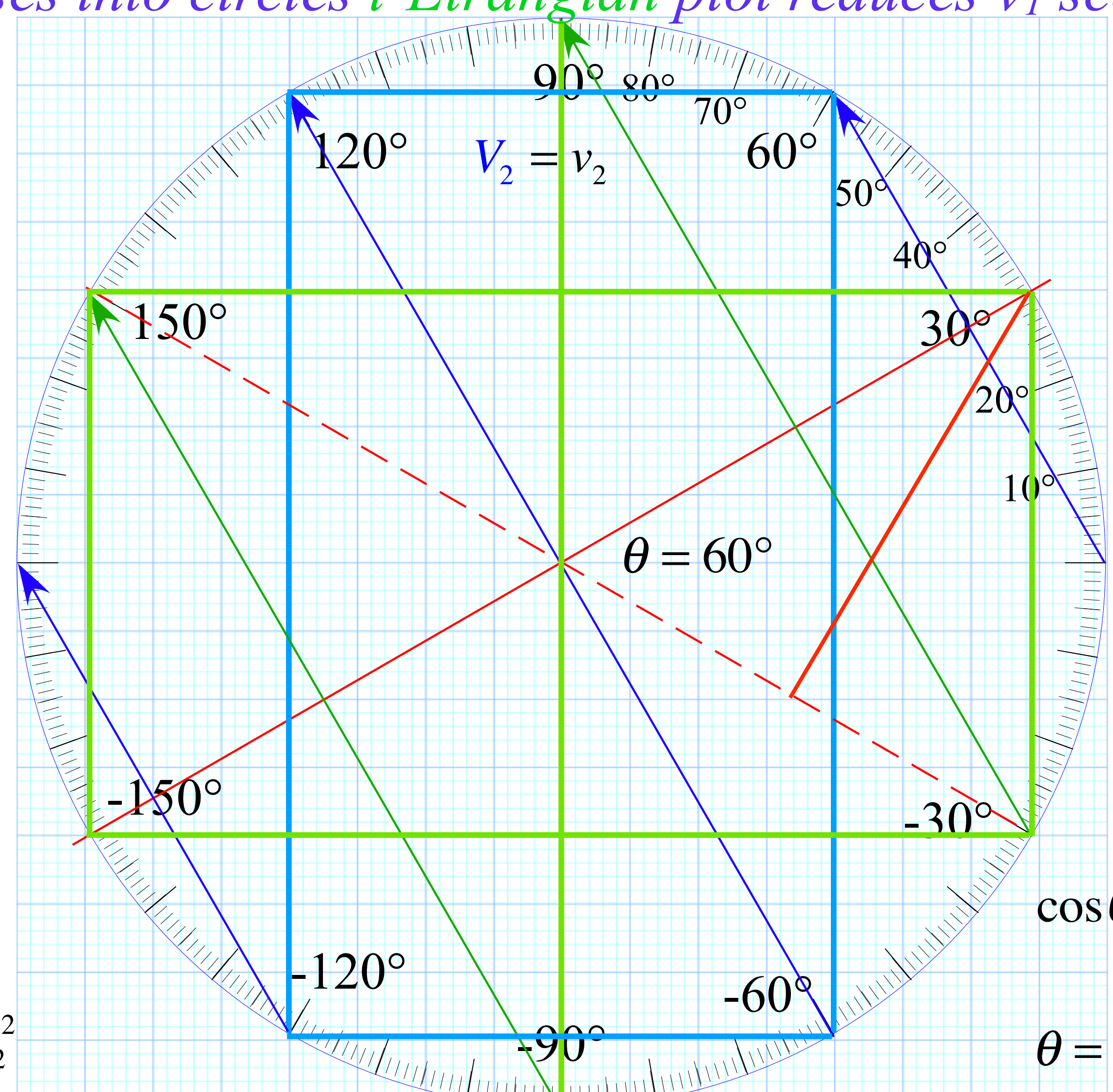

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$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

$$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$$

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

$\theta = 60°$

BounceIt Web Simulator
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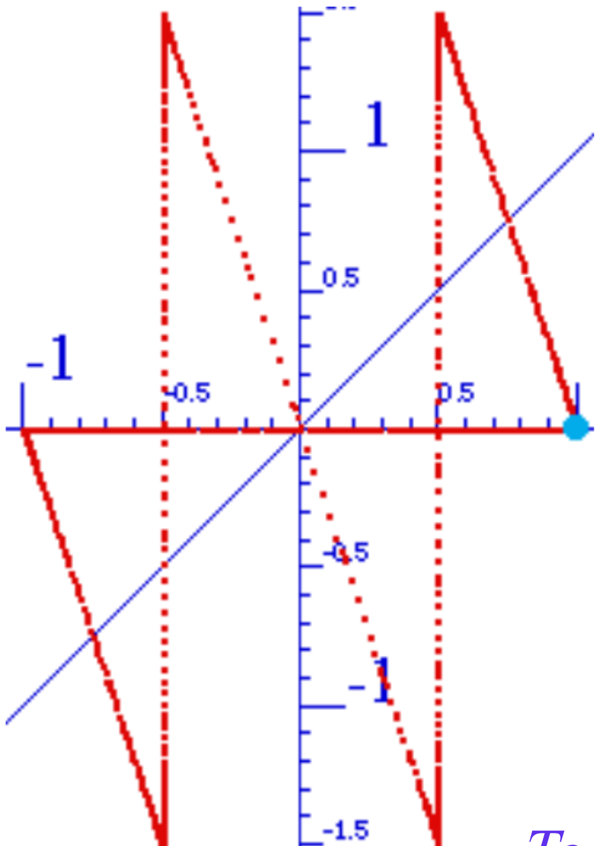
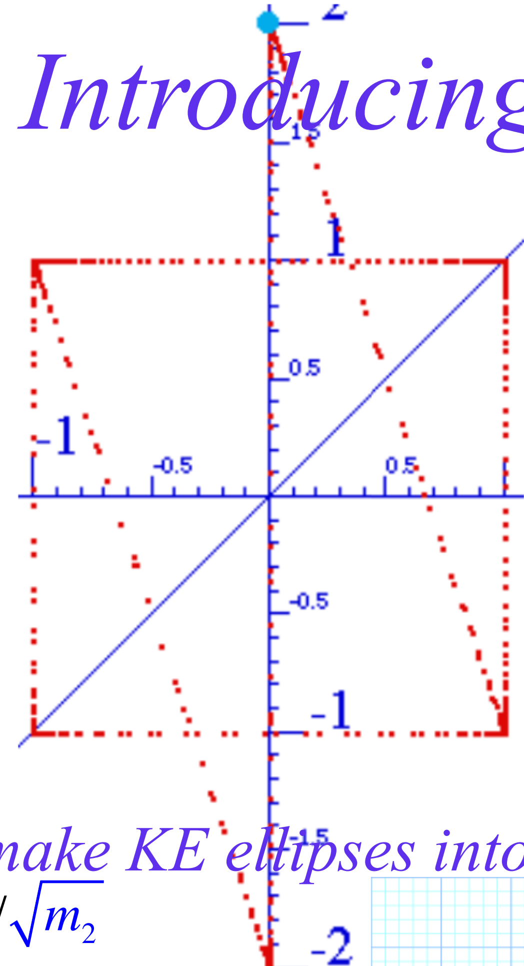
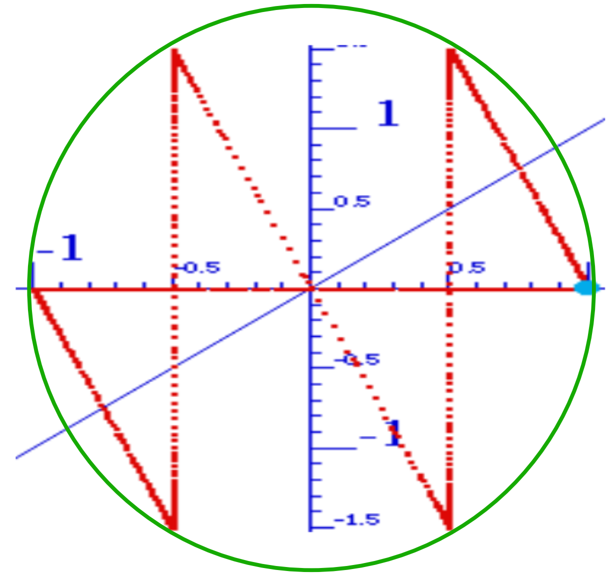
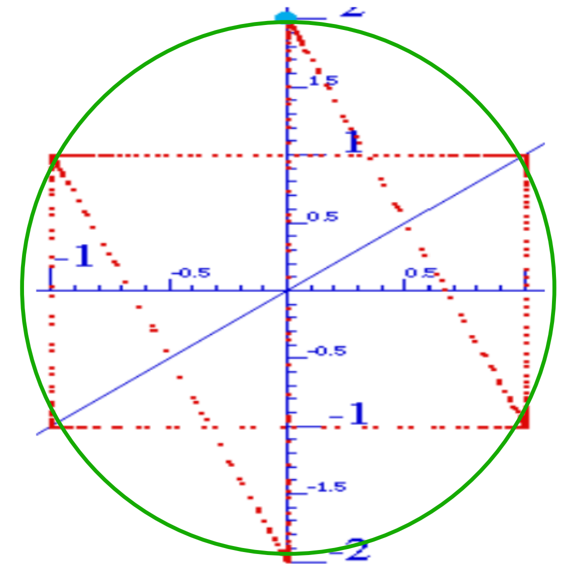

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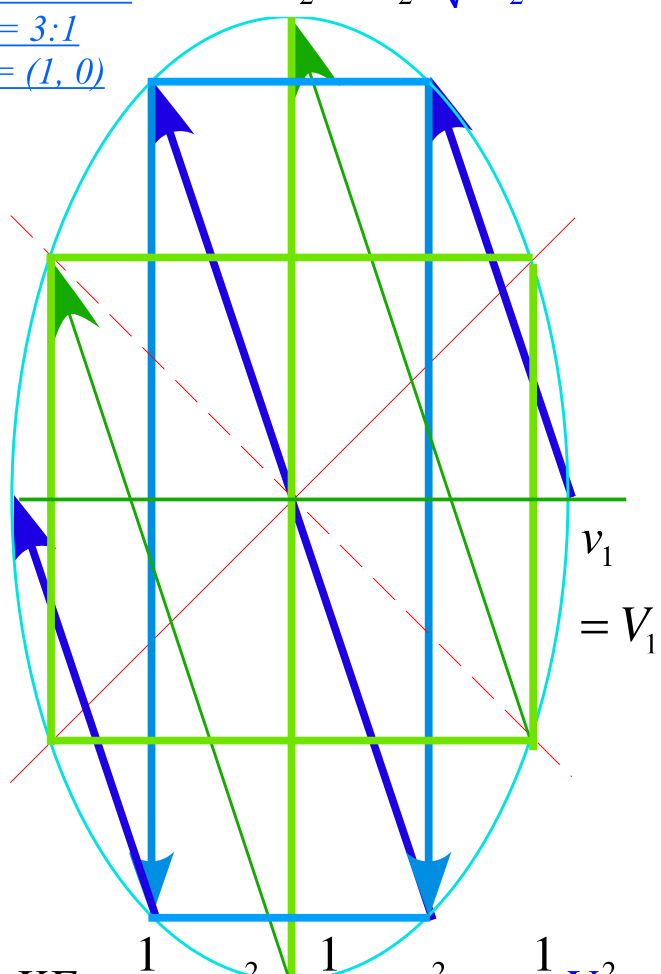
[BounceIt Simulator
m1:m2= 3:1
(v1, v2) = (1, 0)]

$v_2 = V_2/\sqrt{m_2}$



$v_1 = V_1/\sqrt{m_1}$

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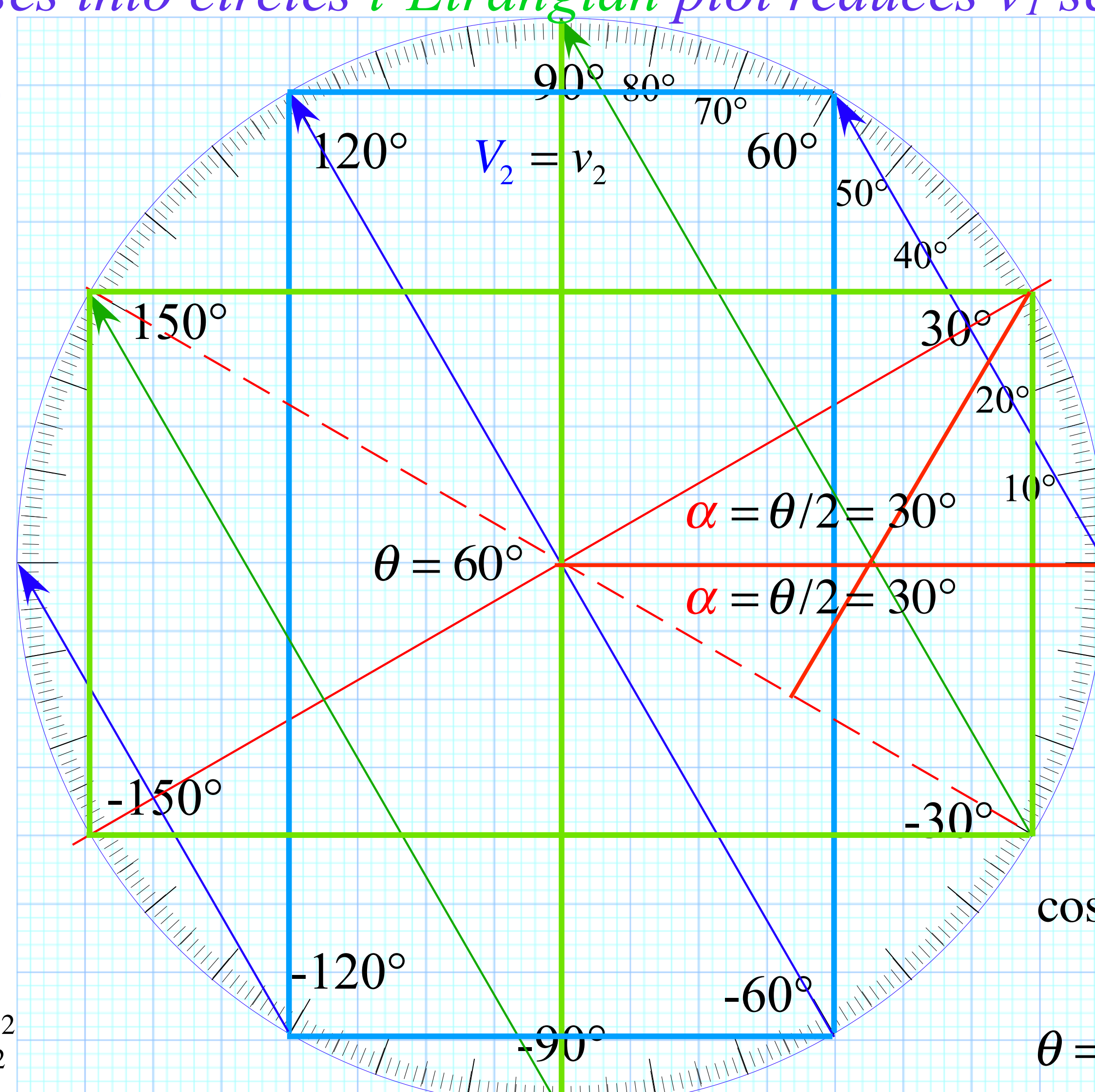

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$$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

$$\alpha = \theta/2$$

$$\cos\theta = \frac{m_1-m_2}{m_1+m_2} = \frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1} = \frac{2}{4} = \frac{1}{2}$$

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[BounceIt Web Simulator
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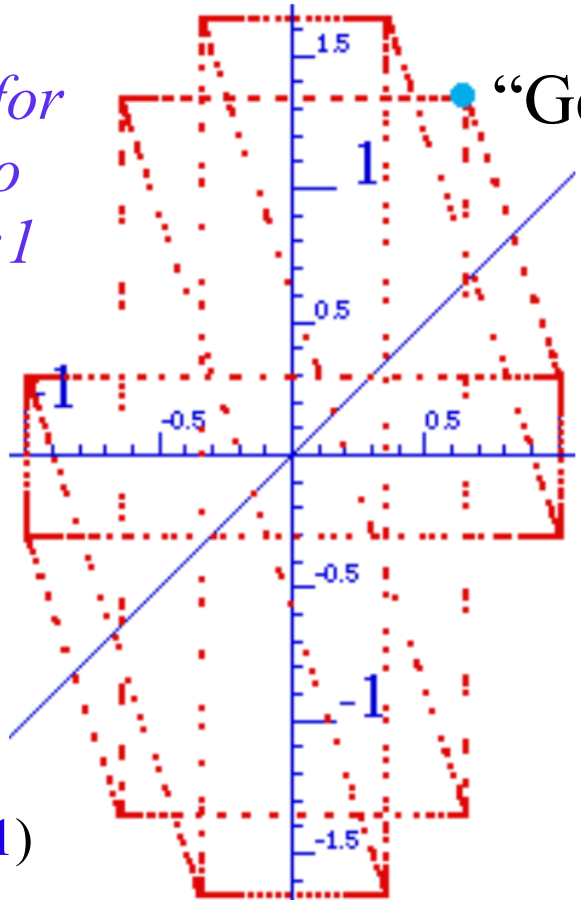


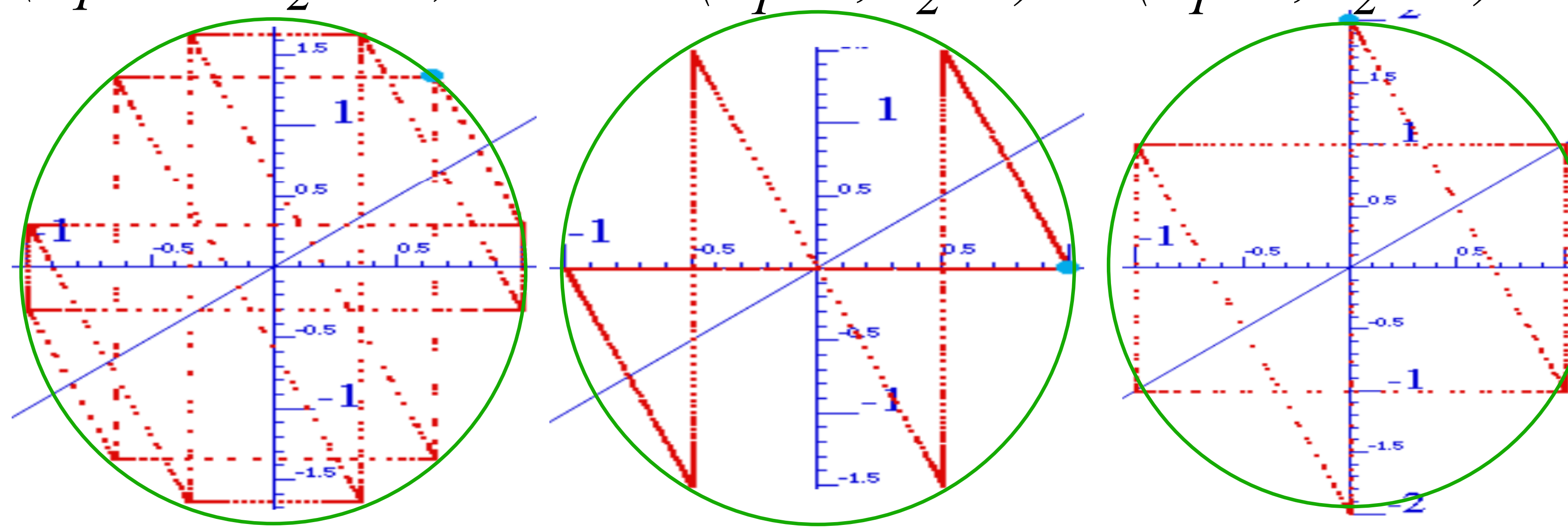
m_1/m_2=(3)/(1)

“Generic” initial velocity

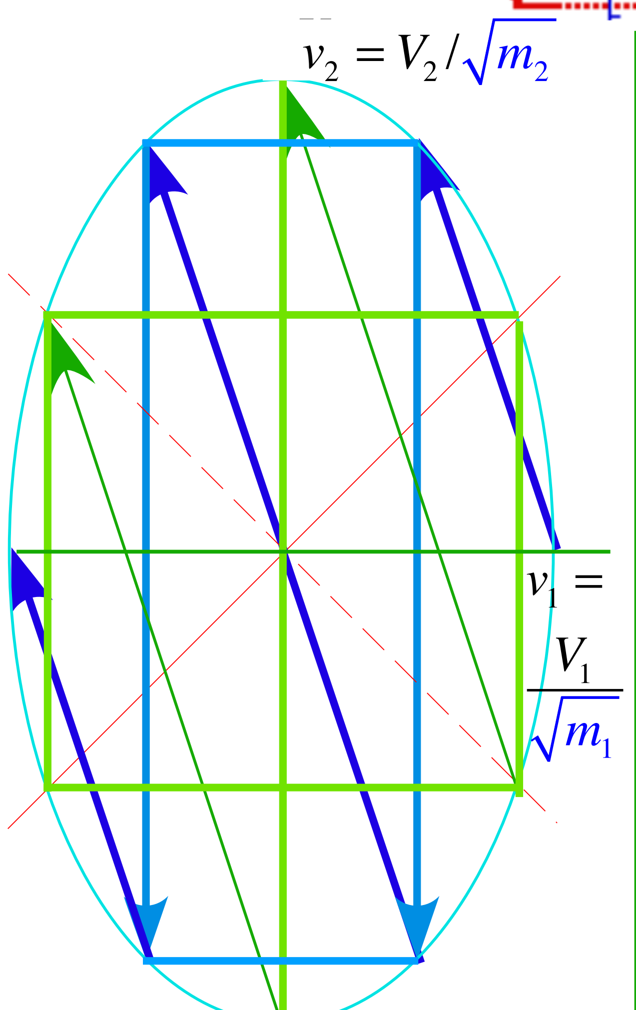
(v_1=1.0, v_2=0.1)

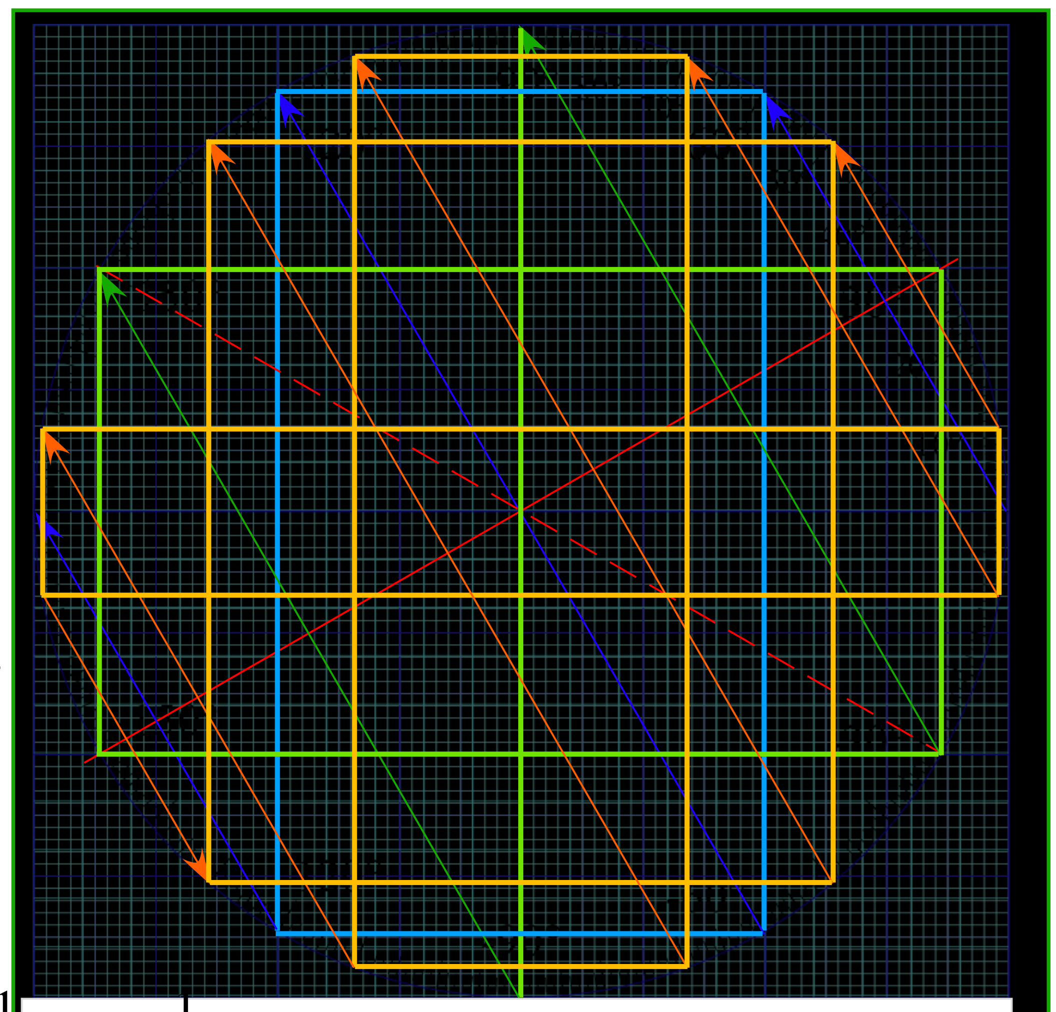
“Symmetric” initial velocity

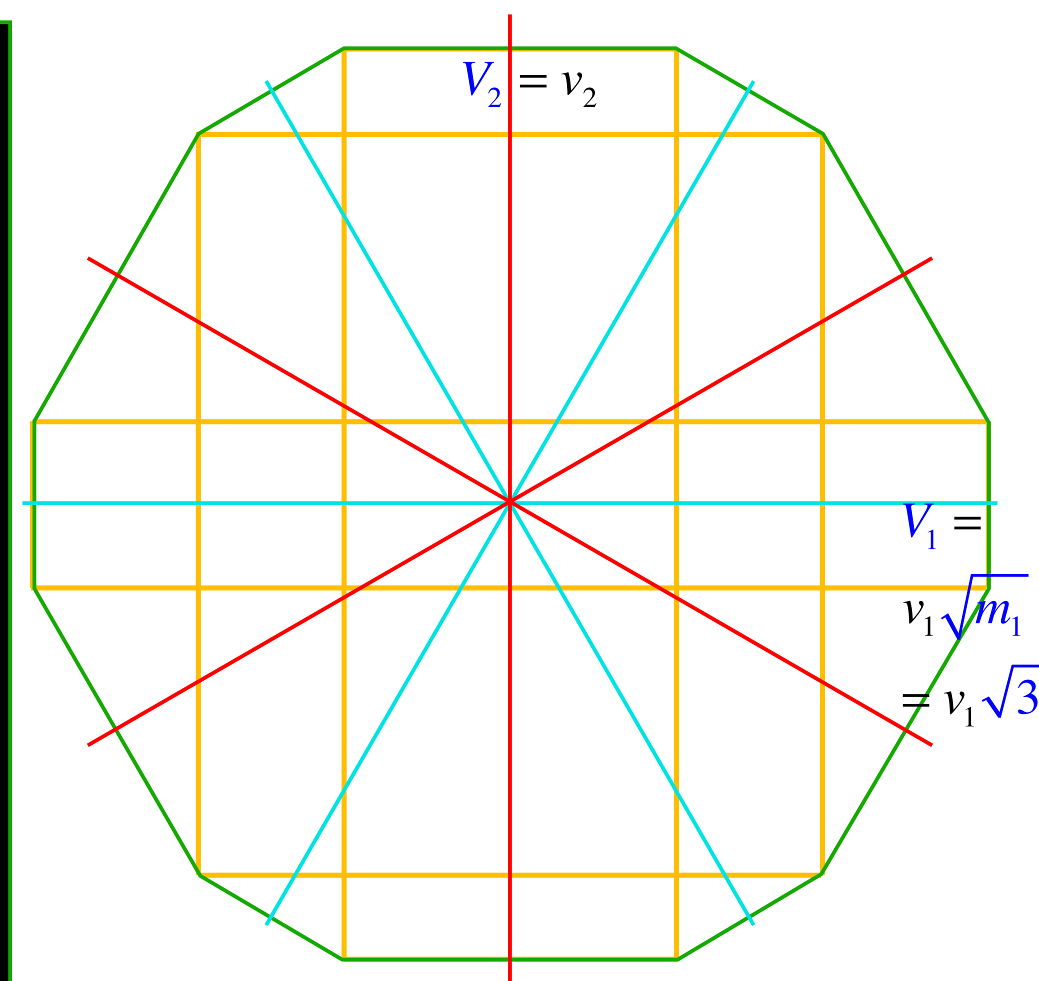
(v_1=1, v_2=0) or (v_1=1, v_2=-1)



reduce v_1 scale by $1/\sqrt{m_1} = 1/\sqrt{3}$=0.577







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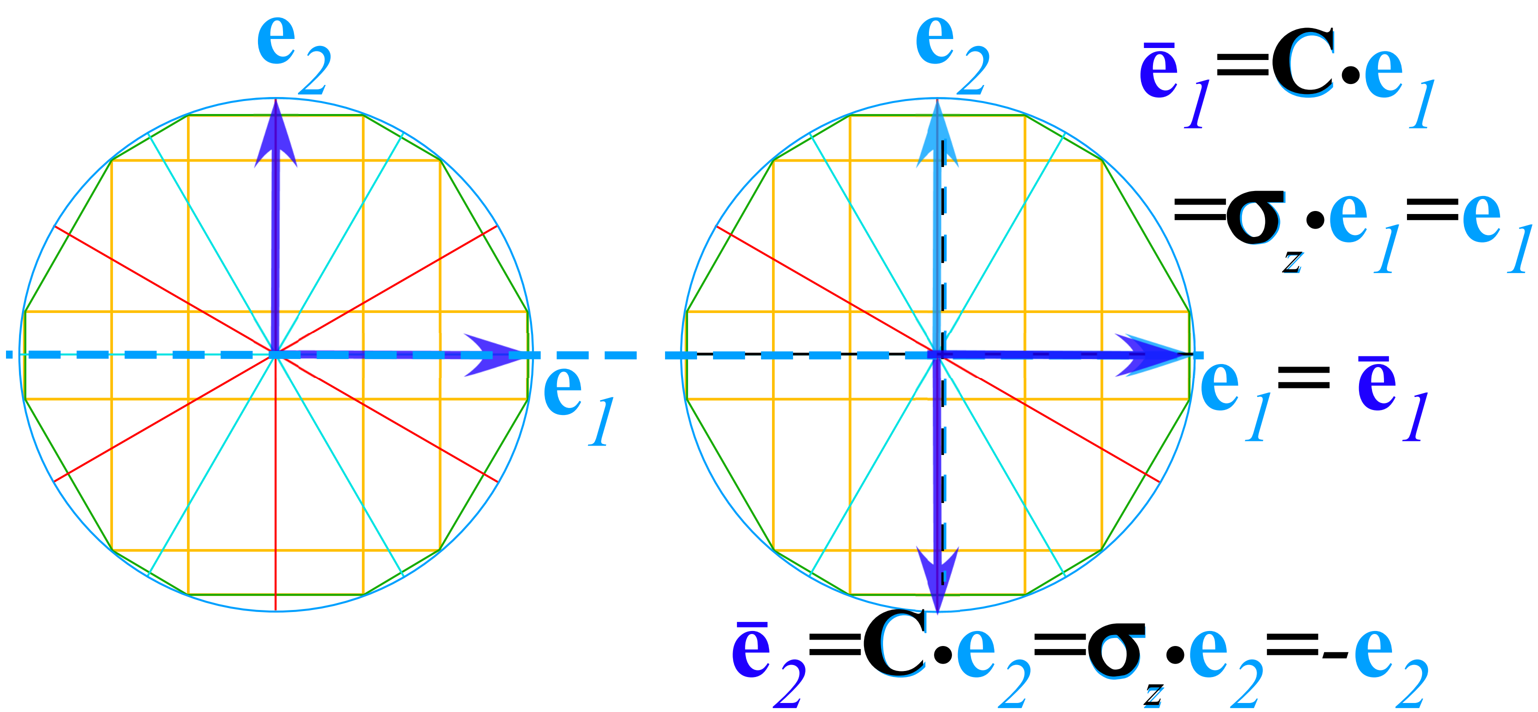
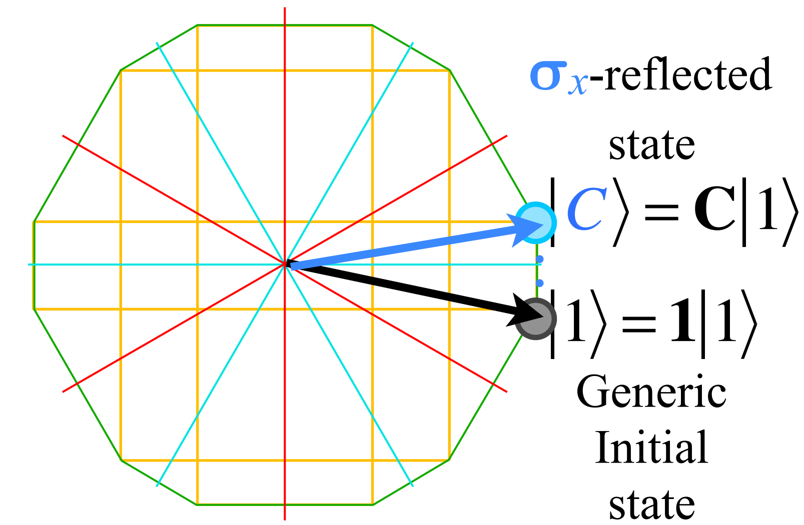
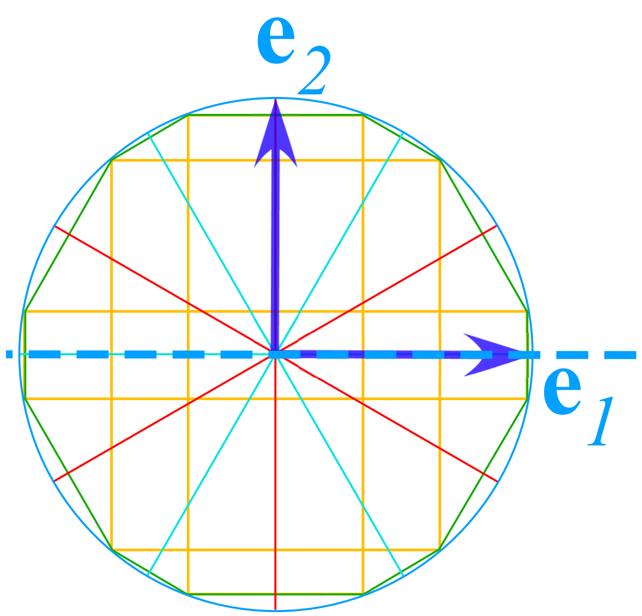
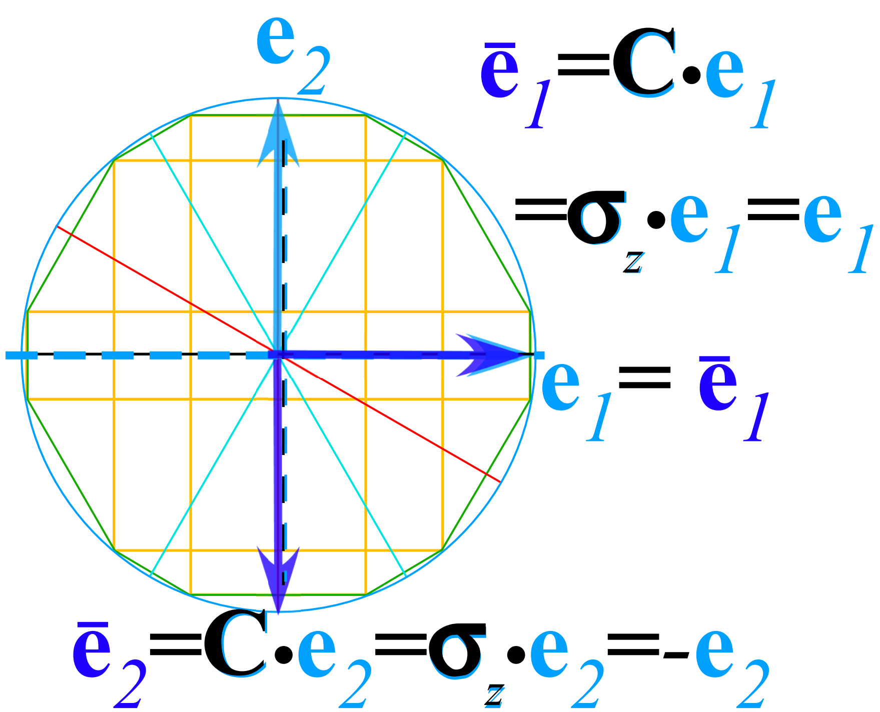
Effects of Ceiling Bang Matrix $\mathbf{C}=\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1\bullet\mathbf{C}\bullet\mathbf{e}_1 & \mathbf{e}_1\bullet\mathbf{C}\bullet\mathbf{e}_2 \\ \mathbf{e}_2\bullet\mathbf{C}\bullet\mathbf{e}_1 & \mathbf{e}_2\bullet\mathbf{C}\bullet\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_1\bullet\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_2\bullet\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

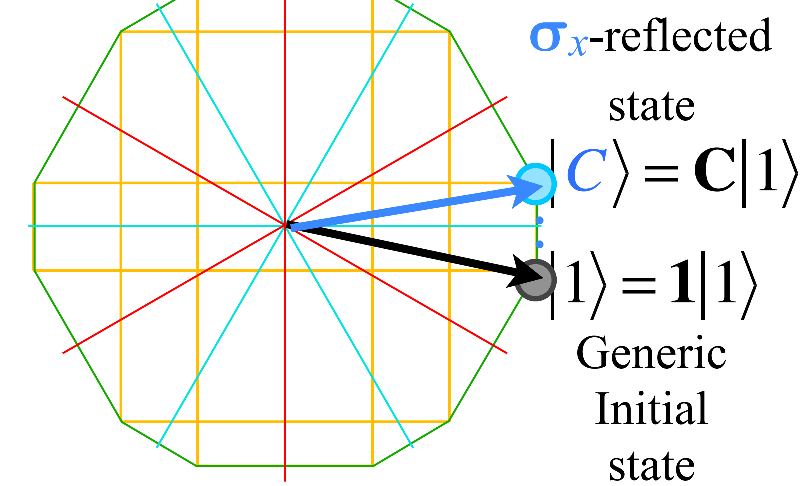




Effects of Ceiling Bang Matrix $\mathbf{C}=\sigma_z = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

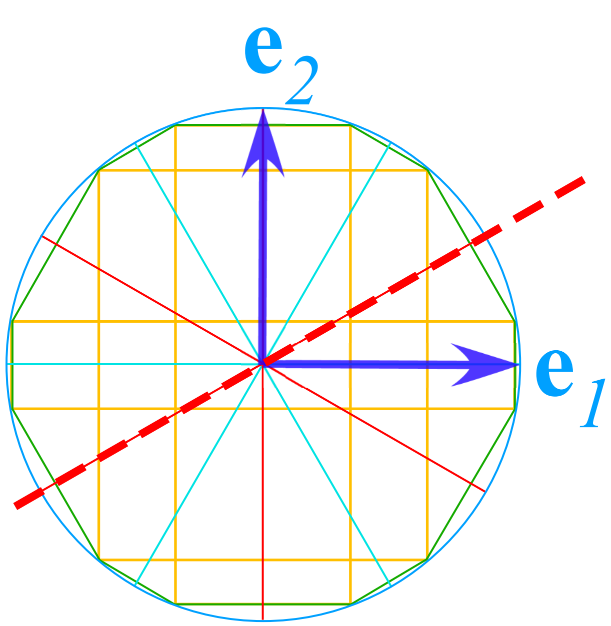


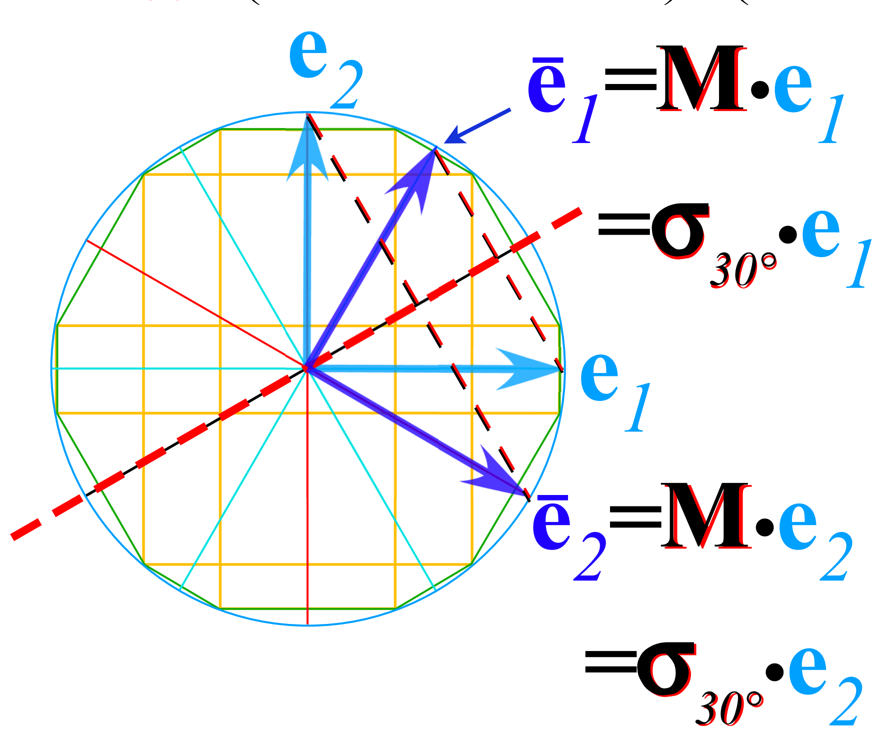


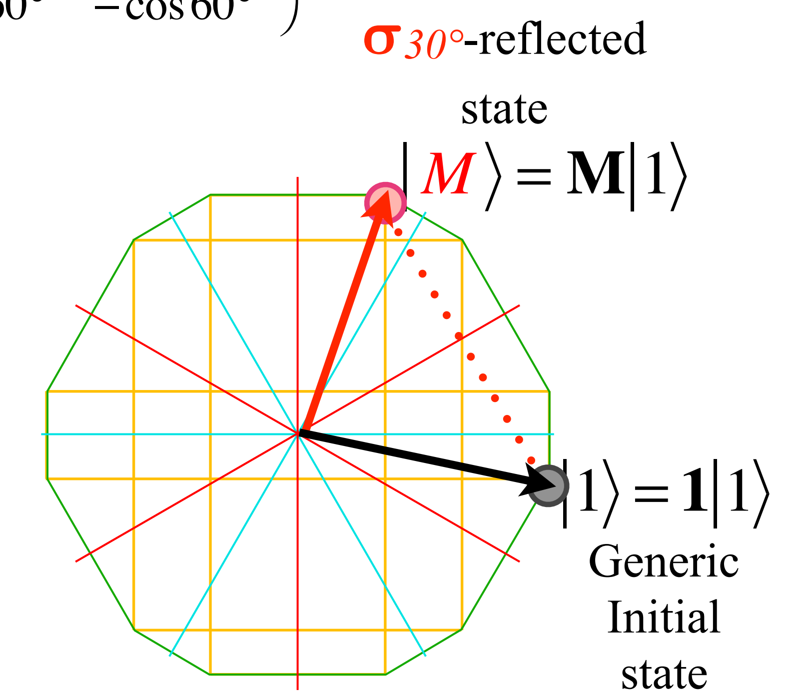


Known as *matrix elements* or *components* — Known as *relative direction cosines*

Effects of Mass Bang Matrix $\mathbf{M}=\sigma_{30°} = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{M}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{M}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{M}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{M}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{pmatrix}$

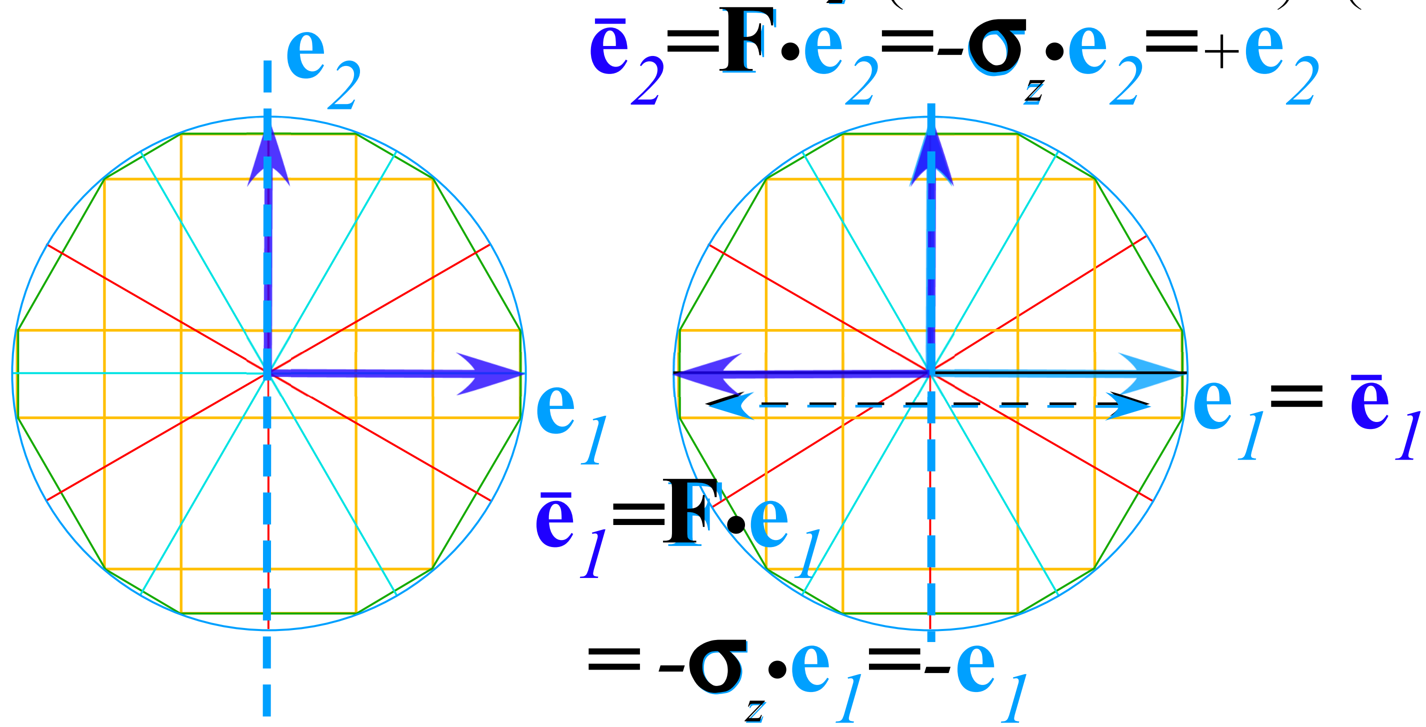


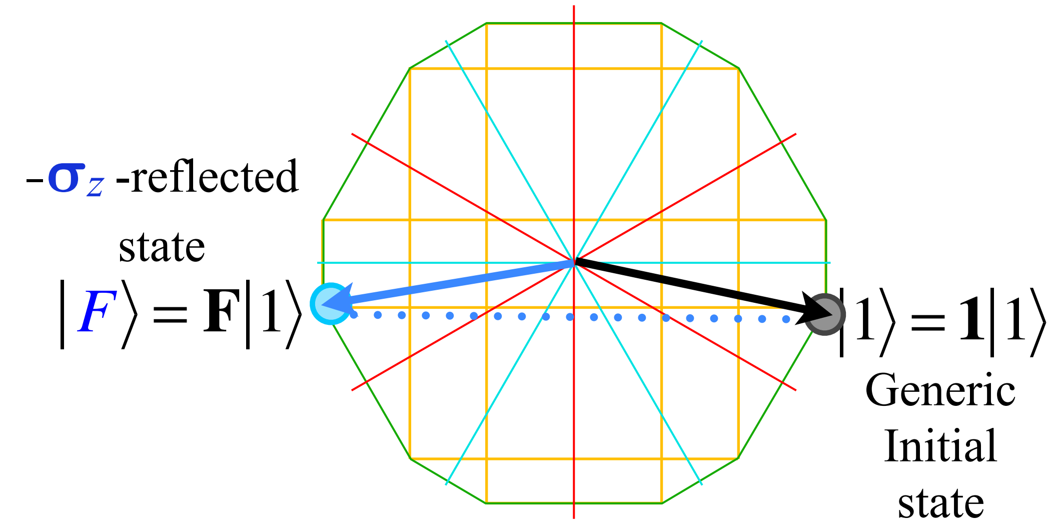




Effects of Floor Bang Matrix

$$\mathbf{F} = -\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1\bullet\mathbf{F}\bullet\mathbf{e}_1 & \mathbf{e}_1\bullet\mathbf{F}\bullet\mathbf{e}_2 \\ \mathbf{e}_2\bullet\mathbf{F}\bullet\mathbf{e}_1 & \mathbf{e}_2\bullet\mathbf{F}\bullet\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_1\bullet\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_2\bullet\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$





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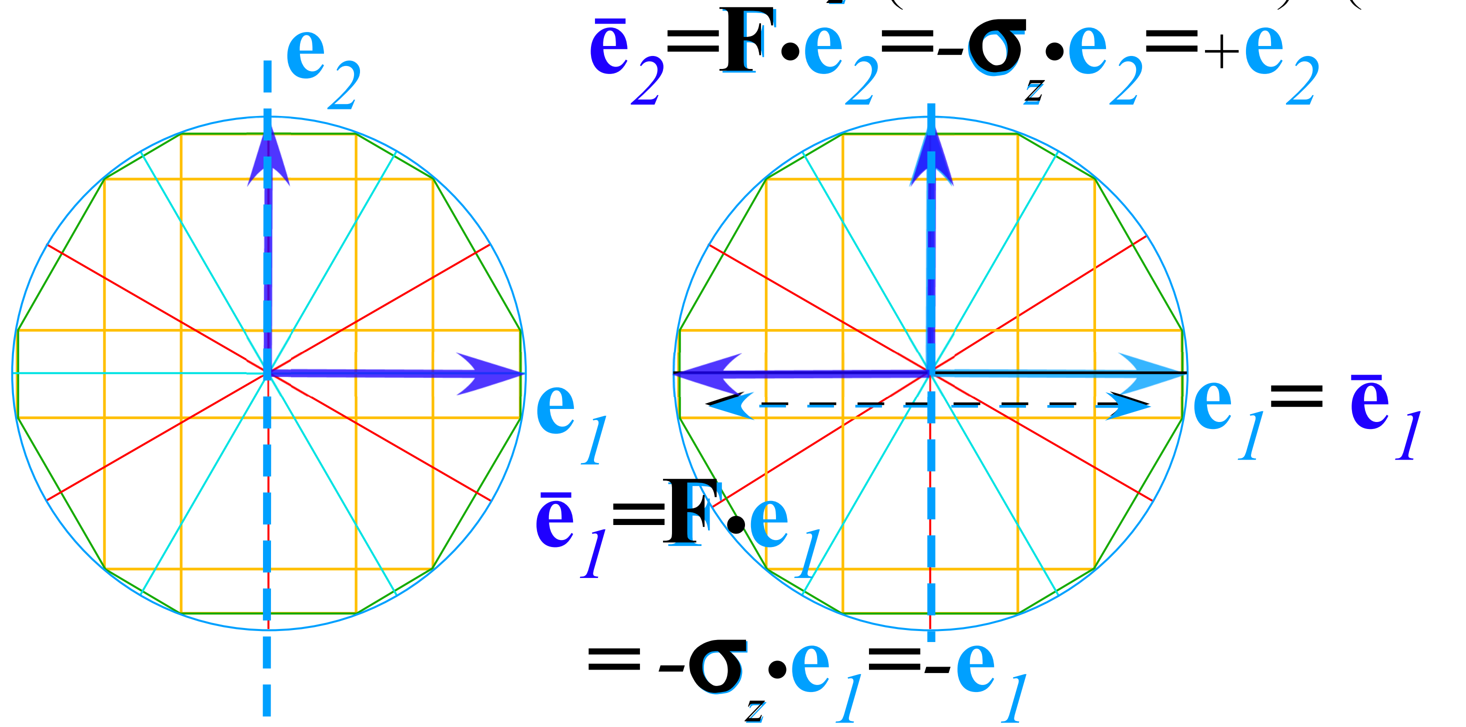
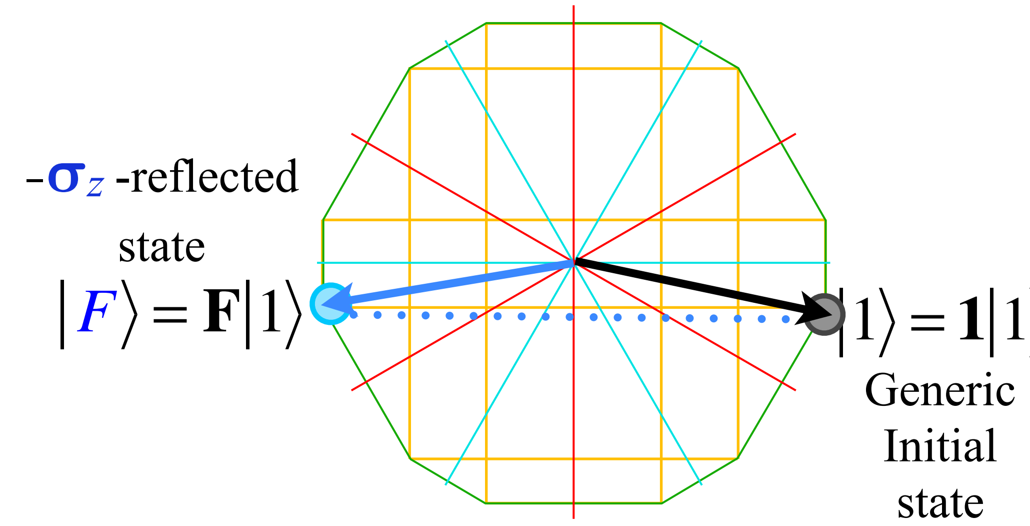
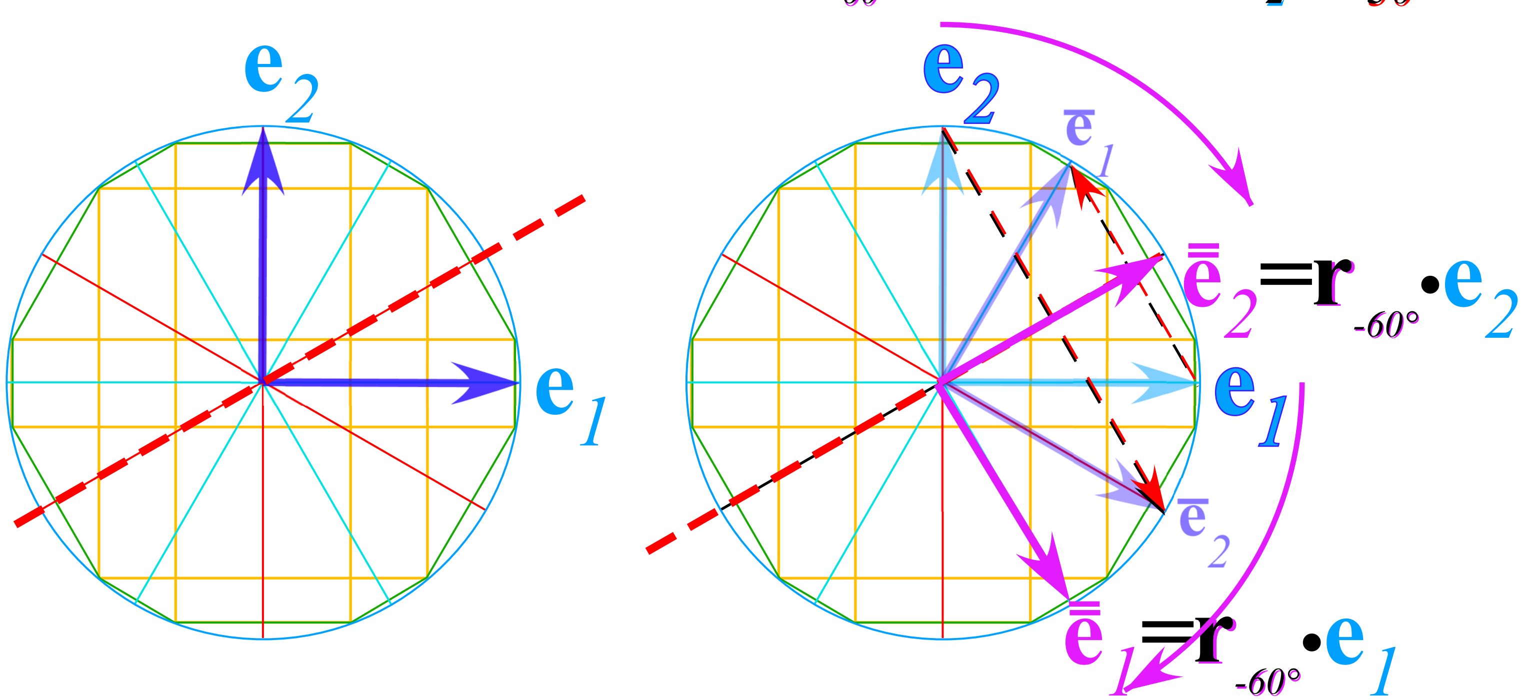
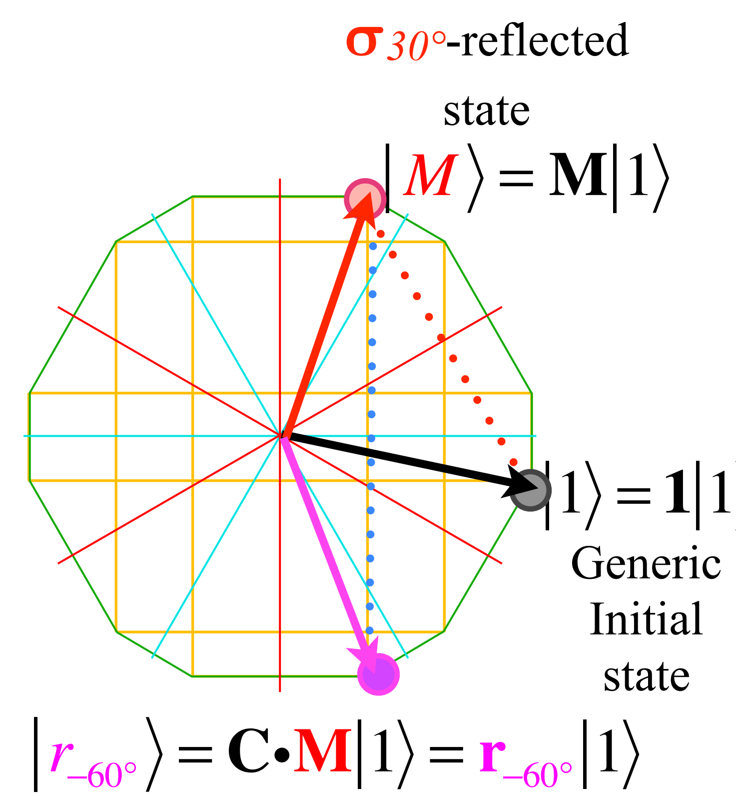




Effects of Ceiling $\mathbf{C}$ after Bang $\mathbf{M}$: $\mathbf{r}_{-60°}=\mathbf{C}\bullet\mathbf{M}=\boldsymbol{\sigma}_z\bullet\boldsymbol{\sigma}_{30°}$





$\boldsymbol{\sigma}_{30°}$ $\boldsymbol{\sigma}_{30°}$-reflected state is a $\mathbf{r}_{-60°}$-rotated state

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→ *Group multiplication and product table* ←

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

D_6	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
1	1											
$\bar{r}_{120}$		1										
r_{120}			1									
σ_{60}				1								
$\bar{\sigma}_{60}$					1							
σ_z						1					$\bar{r}_{60}$	
I							1					
r_{60}								1				
$\bar{r}_{60}$									1			
$\bar{\sigma}_{30}$										1		
σ_{30}						r_{60}					1	
$\bar{\sigma}_z$												1

Note: $\bar{r}_{60} = Ir_{120} = r_{120}I = r_{-60}$ and: $I = r_{\pm 180}$

$\bar{r}_{120} = Ir_{60} = r_{60}I = r_{-120}$ and: $I^2 = 1$

$\sigma_{60} = I\bar{\sigma}_{30} = \bar{\sigma}_{30}I$

$\bar{\sigma}_{60} = I\sigma_{30} = \sigma_{30}I$

$\bar{\sigma}_z = I\sigma_z = \sigma_z I$

Easy to make hexagonal (D_6) symmetry group table:

Example 1: Find $\boldsymbol{\sigma}_{30°}\bullet\boldsymbol{\sigma}_{-60°}$ = _____?

Solution: Find $\boldsymbol{\sigma}_{30°}$-plane and state-$|\boldsymbol{\sigma}_{-60°}\rangle$

Operate former on latter to get: $\boldsymbol{\sigma}_{30°}|\boldsymbol{\sigma}_{-60°}\rangle = |\mathbf{I}\rangle$

That gives answer: $\boldsymbol{\sigma}_{30°}\bullet\boldsymbol{\sigma}_{-60°} = \mathbf{I}$.

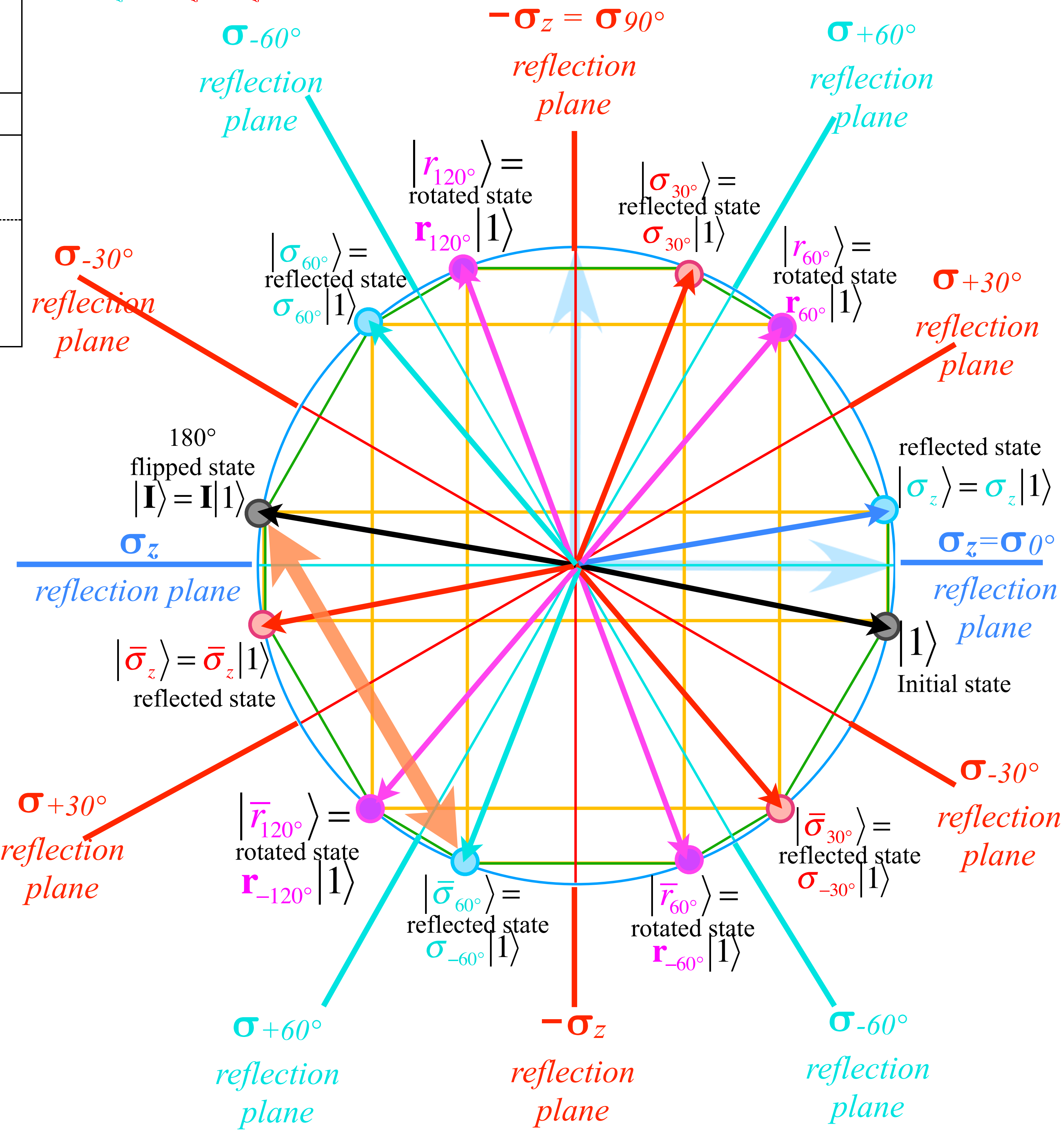
Rest of $\boldsymbol{\sigma}_{30°}$ row follows:

11^{th} row	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
σ_{30}	σ_{30}	$\bar{\sigma}_{30}$	$\bar{\sigma}_z$	$\bar{r}_{60}$	I	r_{60}	$\bar{\sigma}_{60}$	σ_{60}	σ_z	r_{120}	1	$\bar{r}_{120}$

Example 2: Find $\mathbf{r}_{60°}\bullet\boldsymbol{\sigma}_{-60°}$ = _____?

Solution: Do $\mathbf{r}_{60°}$-rotation $\mathbf{r}_{60°}|\boldsymbol{\sigma}_{-60°}\rangle = |\boldsymbol{\sigma}_{-30°}\rangle$

That gives answer: $\mathbf{r}_{60°}\bullet\boldsymbol{\sigma}_{-60°} = \boldsymbol{\sigma}_{-30°}$



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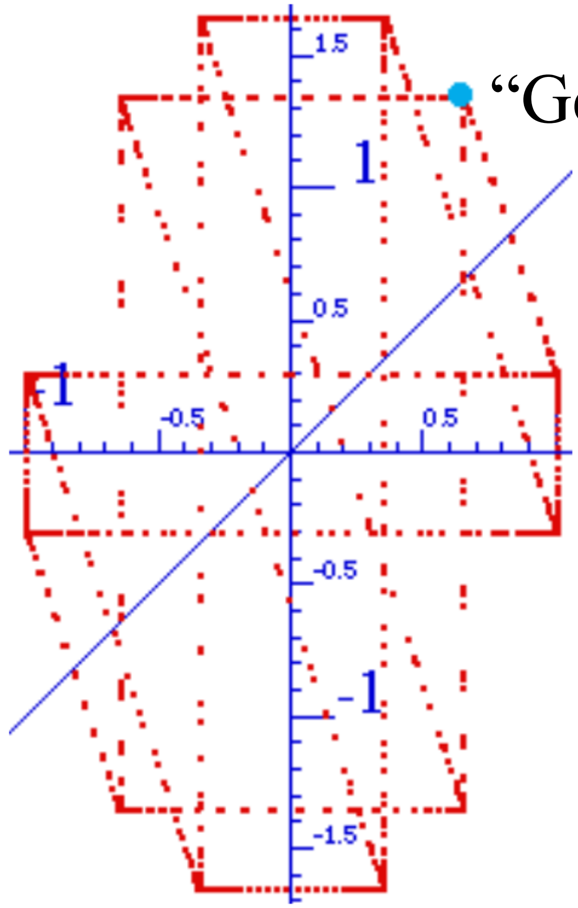
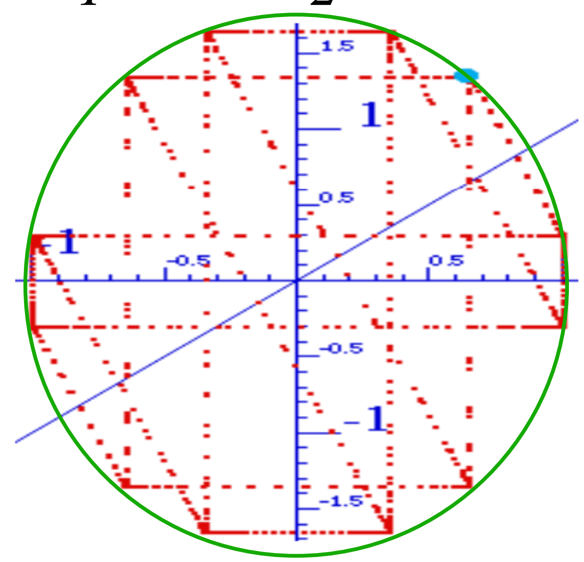
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Collisions for mass ratio m_1:m_2= 3:1

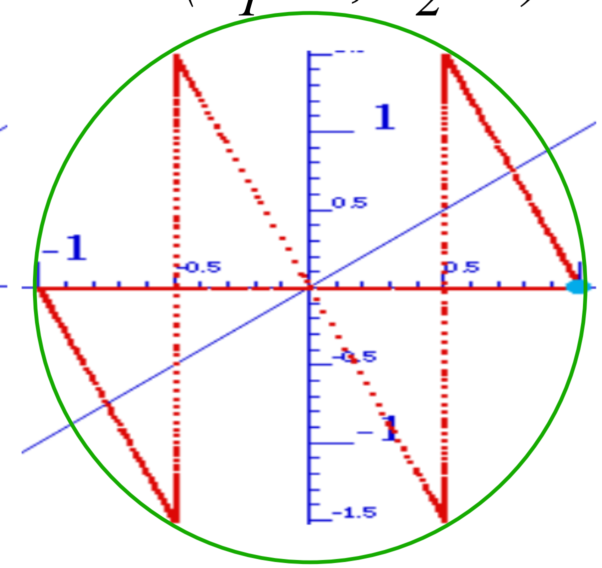


“Generic” initial velocity
$(v_1=1.0, v_2=0.1)$

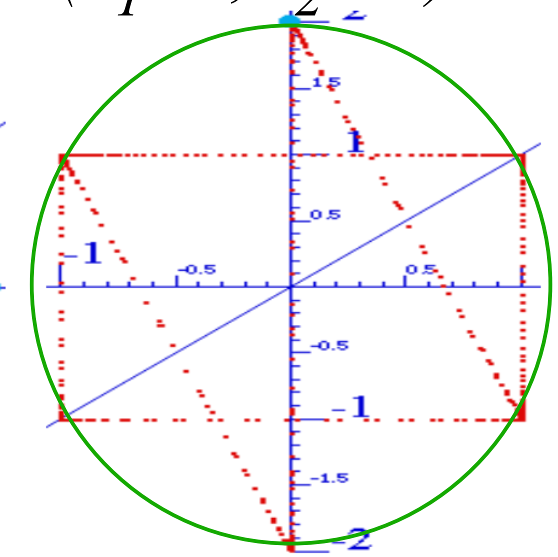
“Symmetric” initial velocity
$(v_1=1, v_2=0)$ or $(v_1=1, v_2=-1)$



$(v_1, v_2)=(1, 0.1)$



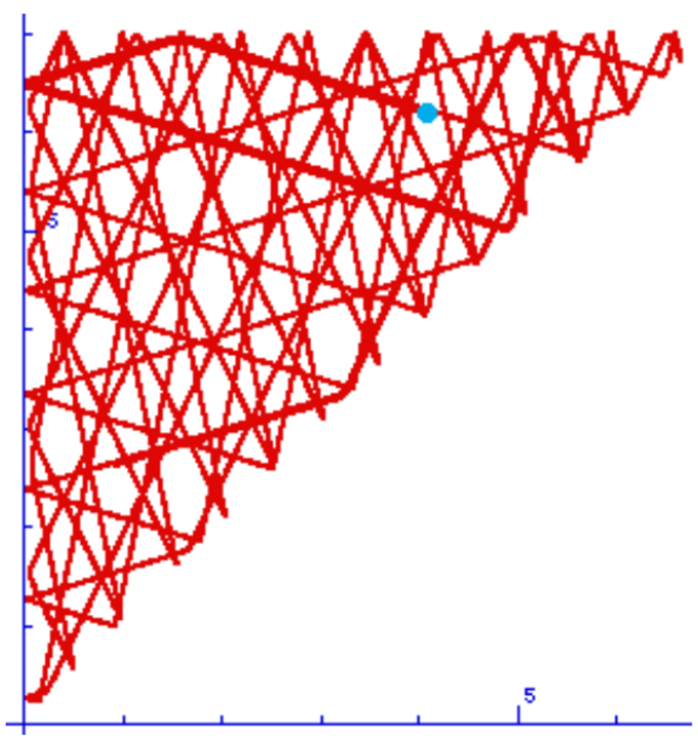
$(v_1, v_2)=(1, 0)$



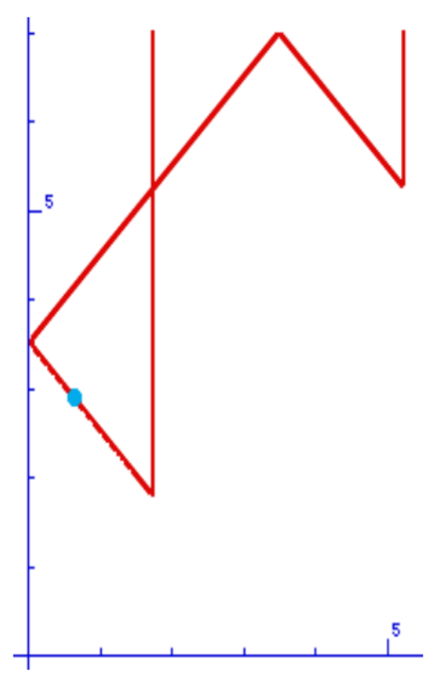
$(v_1, v_2)=(1, -1)$

BounceIt
$m_1:m_2 = 3:1$
Dual plots
v_2 vs v_1 and V_2 vs V_1

Corresponding space-space (y_1, y_2) paths



$(v_1, v_2)=(1, 0.1)$



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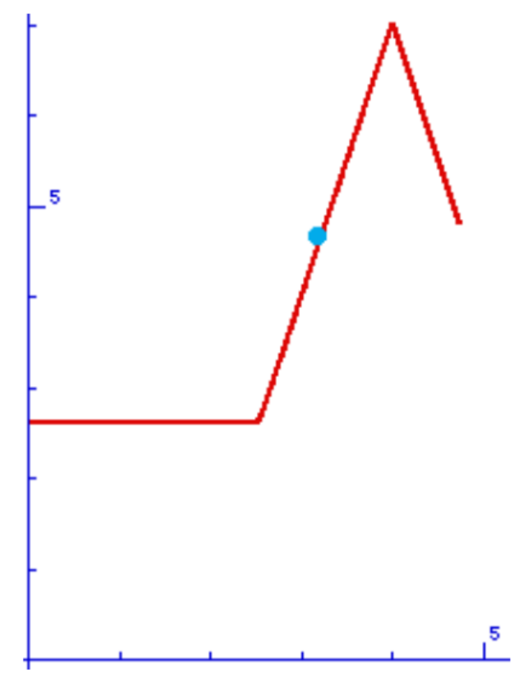
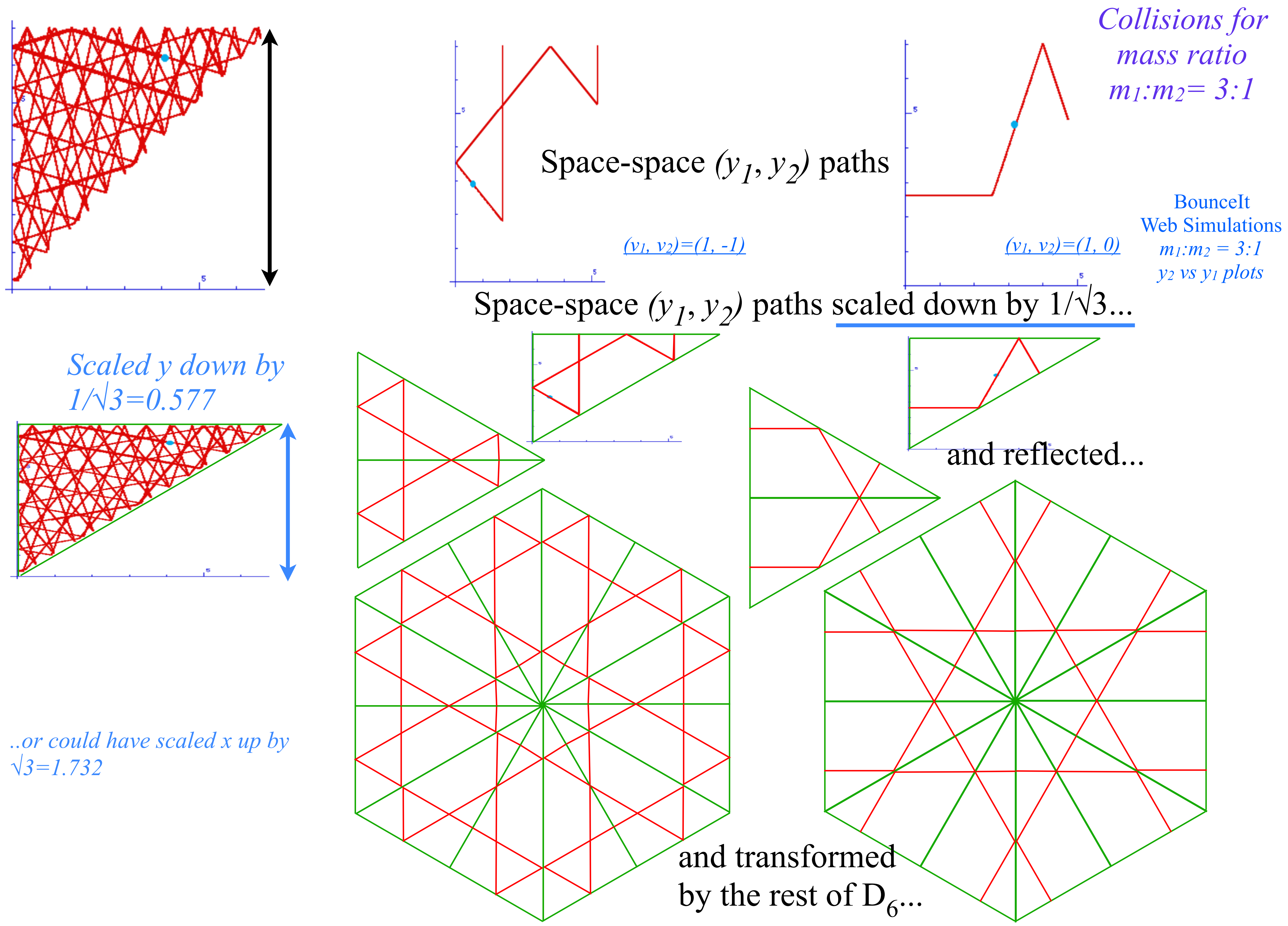
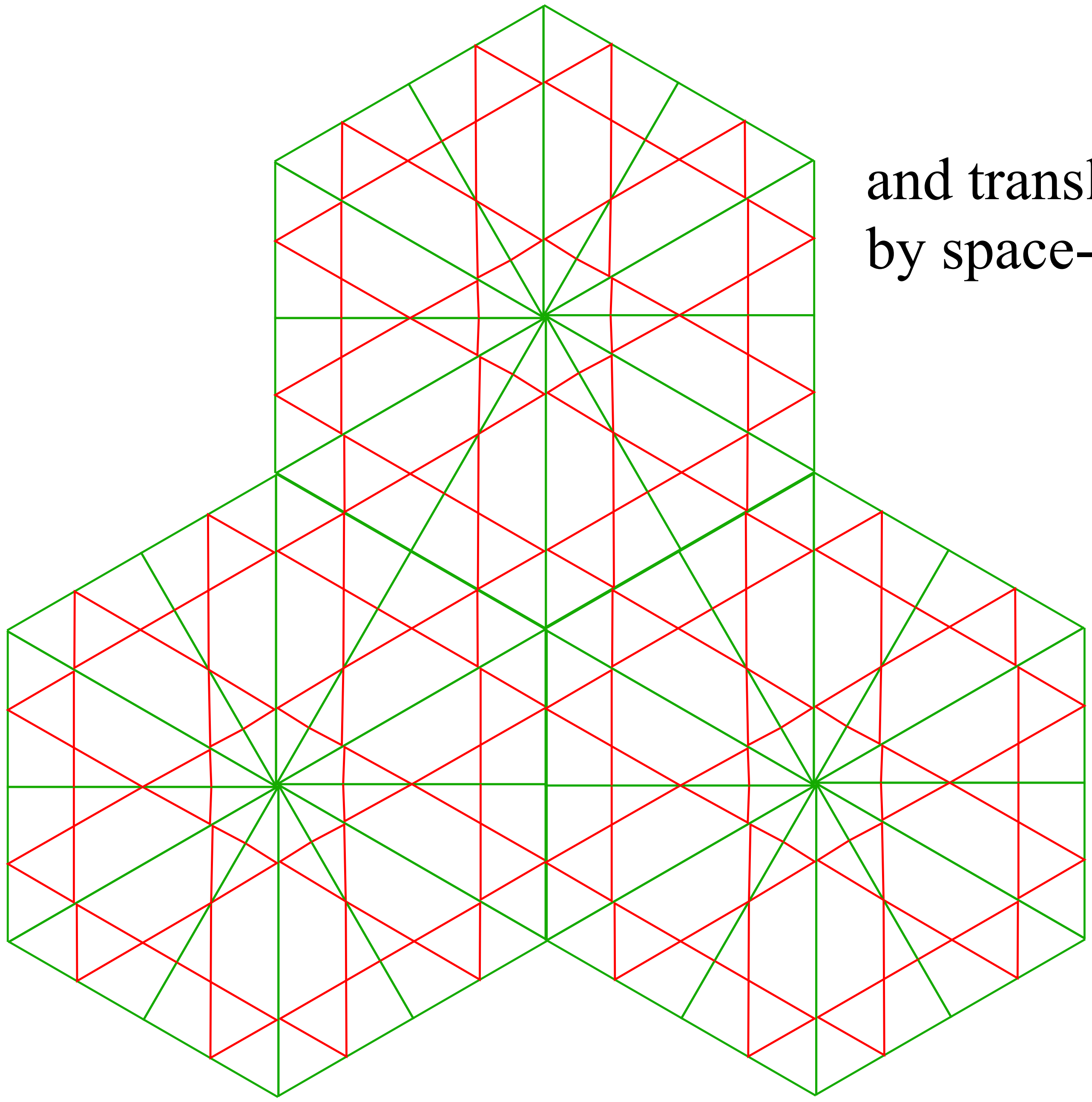


$(v_1, v_2)=(1, 0)$

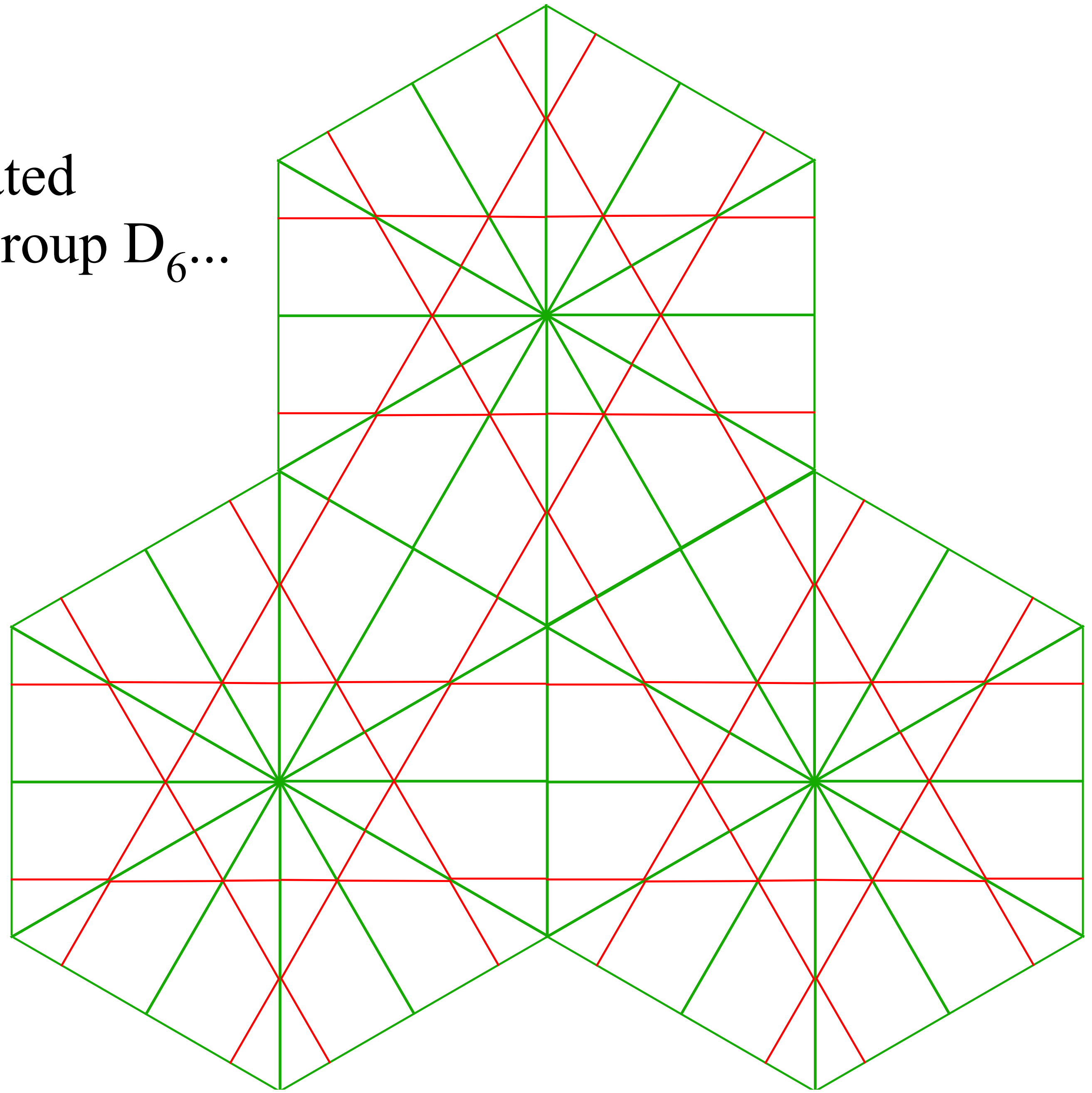
BounceIt
$m_1:m_2 = 3:1$
y_2 vs y_1 plots


Collisions for mass ratio $m_1{:}m_2= 3{:}1$
Space-space (y_1, y_2) paths
$(v_1, v_2)=(1, -1)$
$(v_1, v_2)=(1, 0)$
BounceIt Web Simulations
$m_1{:}m_2 = 3{:}1$
y_2 vs y_1 plots
Space-space (y_1, y_2) paths scaled down by $1/\sqrt{3}$...
Scaled y down by $1/\sqrt{3}=0.577$
and reflected...
..or could have scaled x up by $\sqrt{3}=1.732$
and transformed by the rest of D_6...

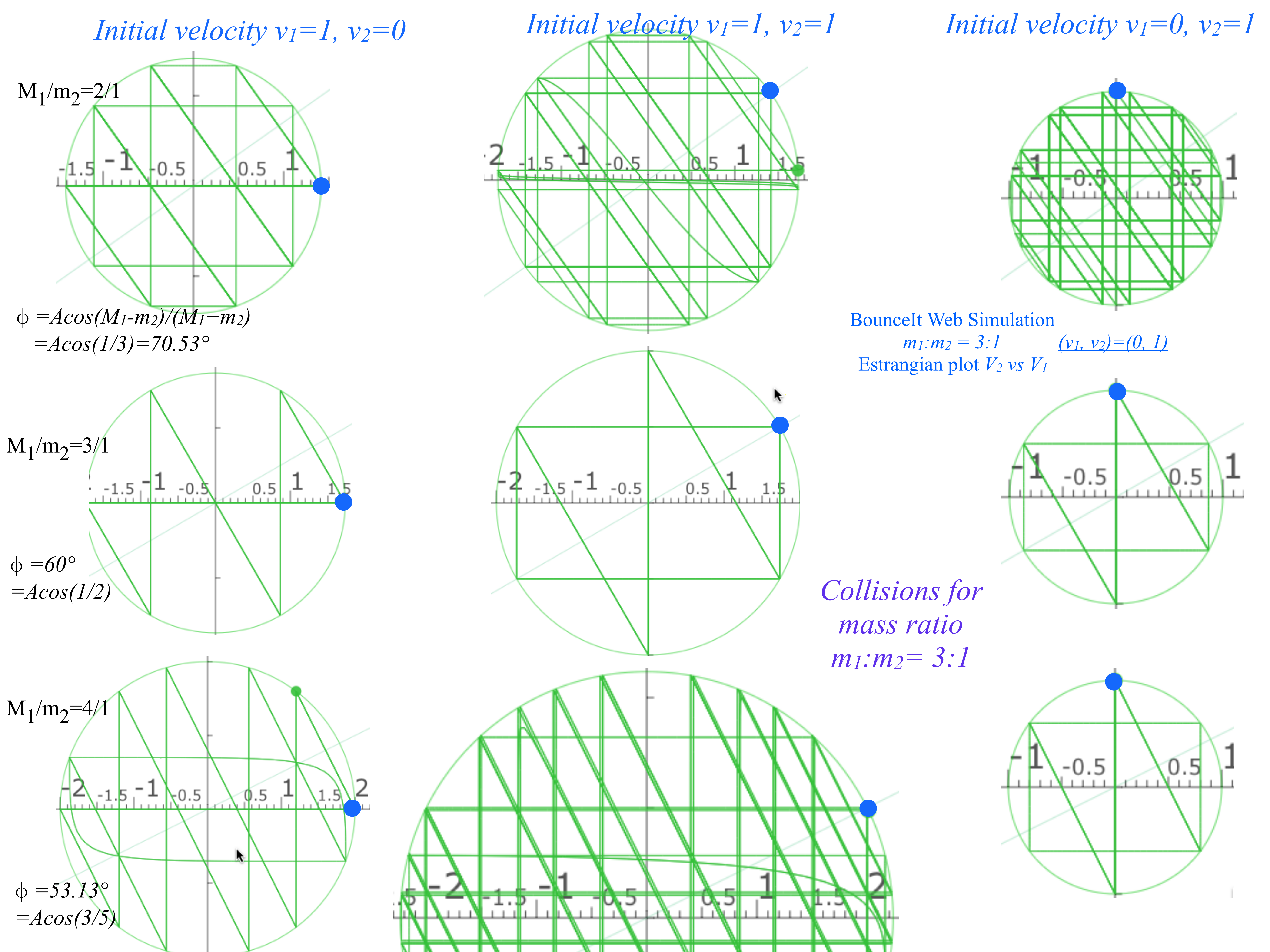
Collisions for mass ratio $m_1:m_2 = 3:1$



and translated by space-group D_6...



...they're just straight lines going forever.



Initial velocity v1=1, v2=0
Initial velocity v1=1, v2=1
Initial velocity v1=0, v2=1
M1/m2=2/1
ϕ =Acos(M1-m2)/(M1+m2)
=Acos(1/3)=70.53°
M1/m2=3/1
ϕ =60°
=Acos(1/2)
M1/m2=4/1
ϕ =53.13°
=Acos(3/5)
BounceIt Web Simulation
m1:m2 = 3:1
(v1, v2)=(0, 1)
Estrangian plot V2 vs V1
Collisions for
mass ratio
m1:m2= 3:1

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

→ *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$* ←

Geometric "Integration" (Converting Velocity data to Spacetime)

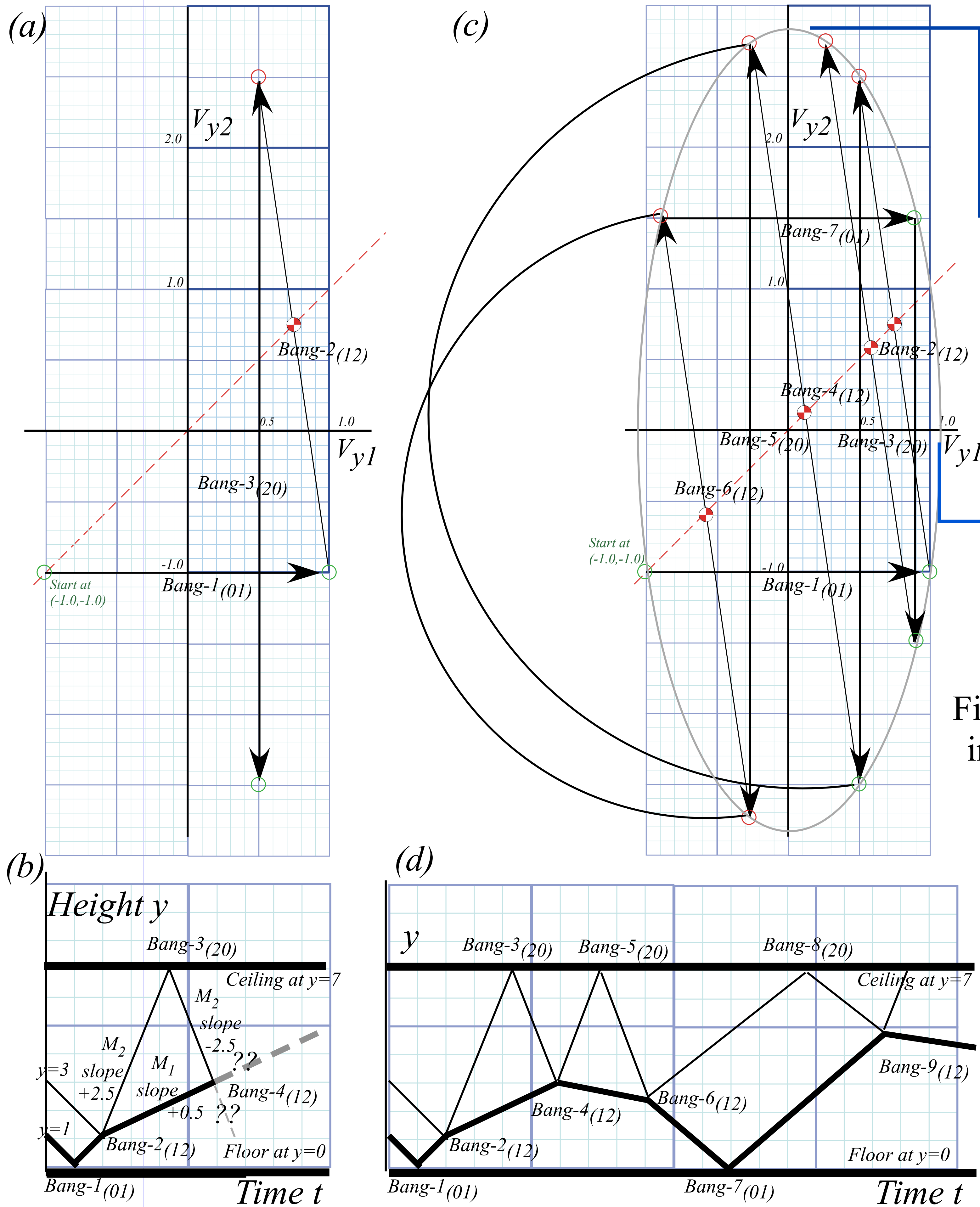


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

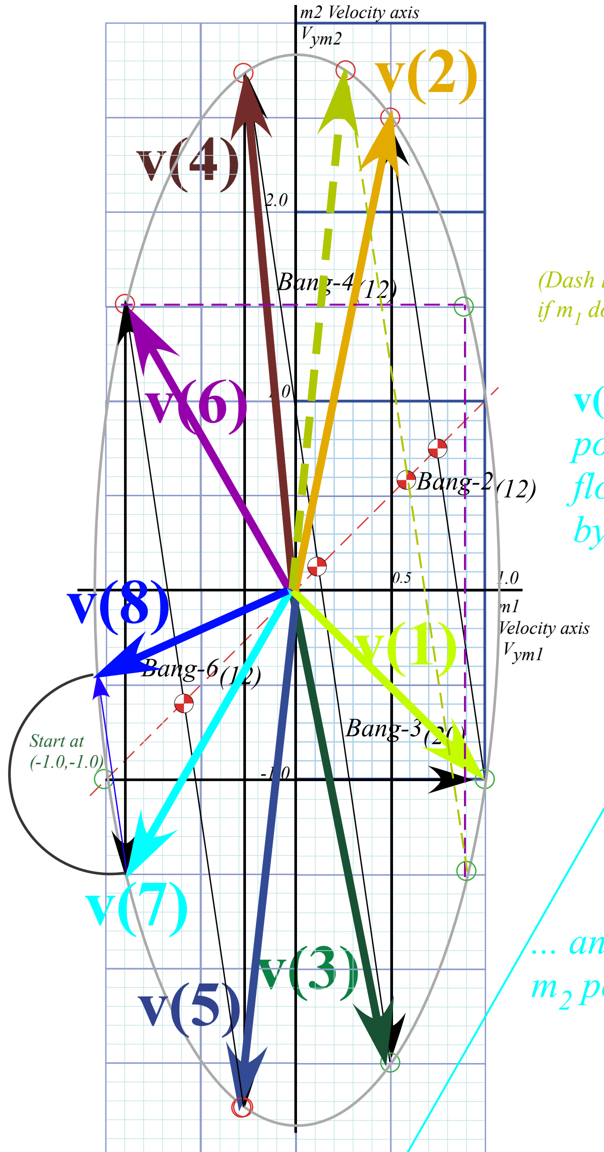
Ellipse radius 2

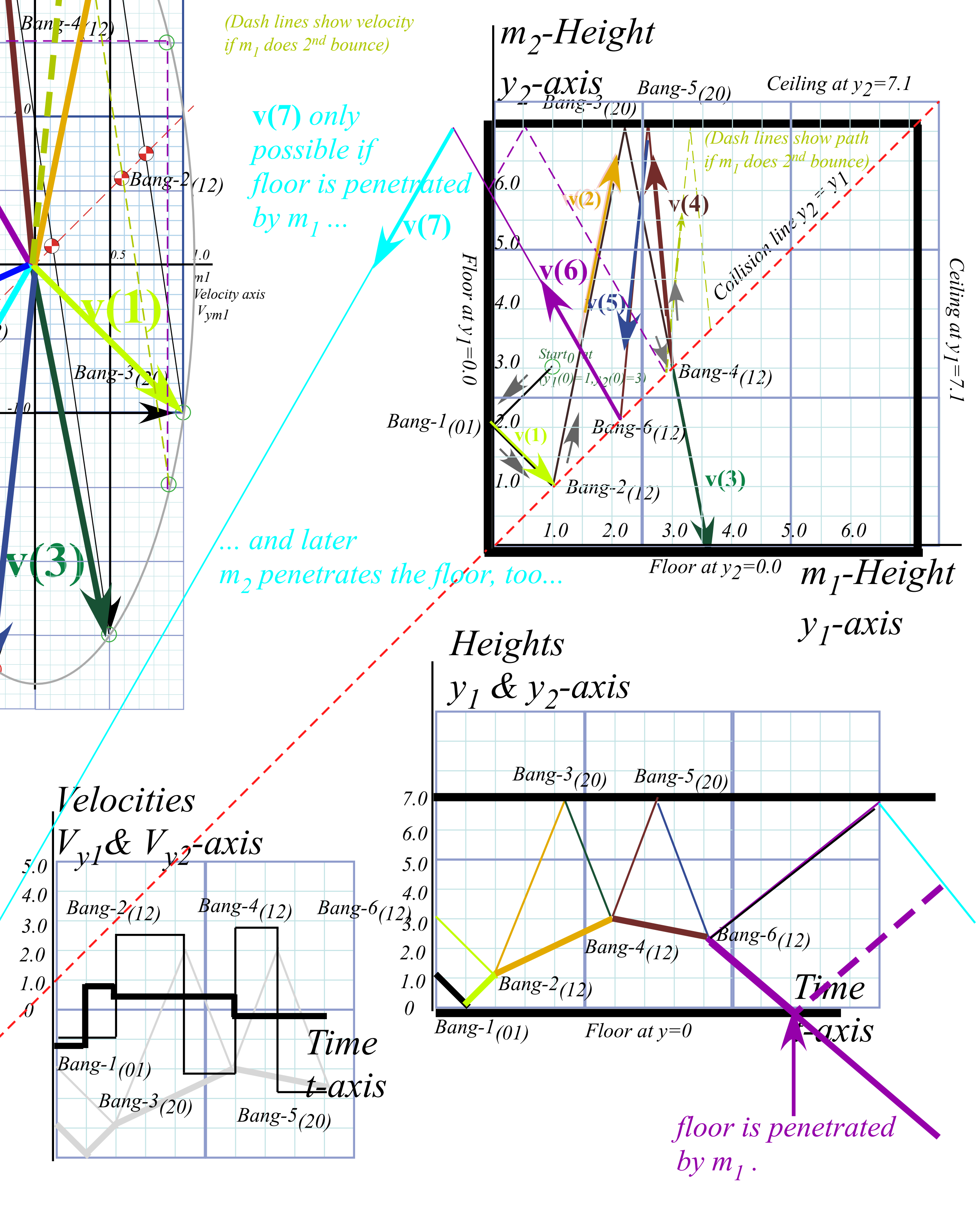
$$a_2 = \sqrt{2KE/M_2} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Collisions for mass ratio $m_1{:}m_2 = 7{:}1$

Collisions for mass ratio m_1:m_2= 7:1

Step-2: Extend **v**(2) line to ceiling point **y**(3)=(?,7.1) and draw Bang-3$_{(20)}$ velocity **v**(3)=(1,-1) line. (Find **v**(3) using V-V plot.)
Step-3: Extend **v**(3) line to collision point **y**(4)=(?,?) and draw Bang-4$_{(12)}$ velocity **v**(4)=(0.5,2.5). (Find **v**(4) using V-V plot.)
Step-4: Extend **v**(4) line to ceiling point **y**(4)=(?,7.1) and draw Bang-5$_{(20)}$ velocity **v**(5)=(1,-1) line. (Find **v**(5) using V-V plot.)
Step-5: Extend **v**(5) line to collision point **y**(6)=(?,?) and draw Bang-6$_{(12)}$ velocity **v**(6)=(0.5,2.5). (Find **v**(6) using V-V plot.)





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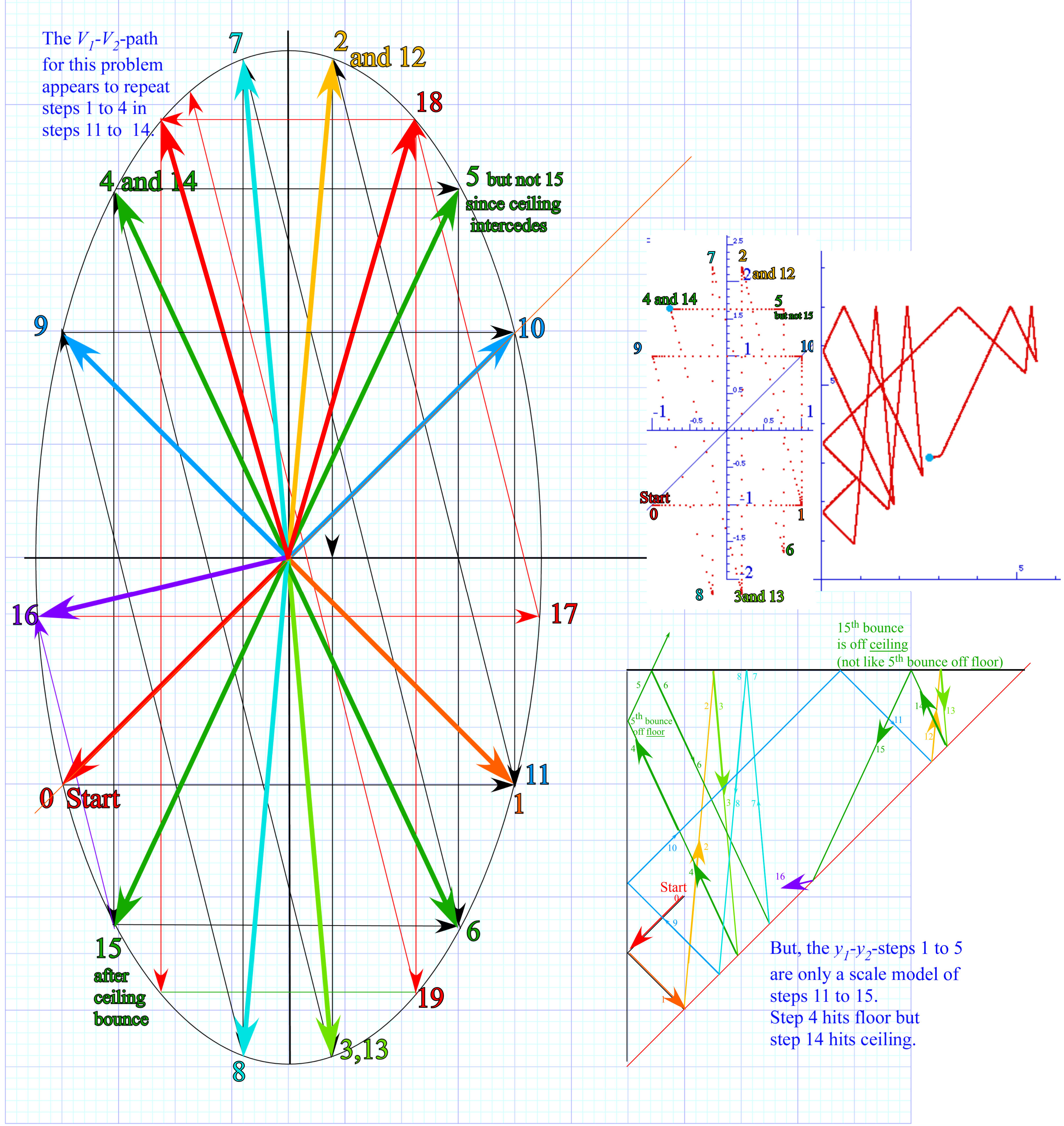
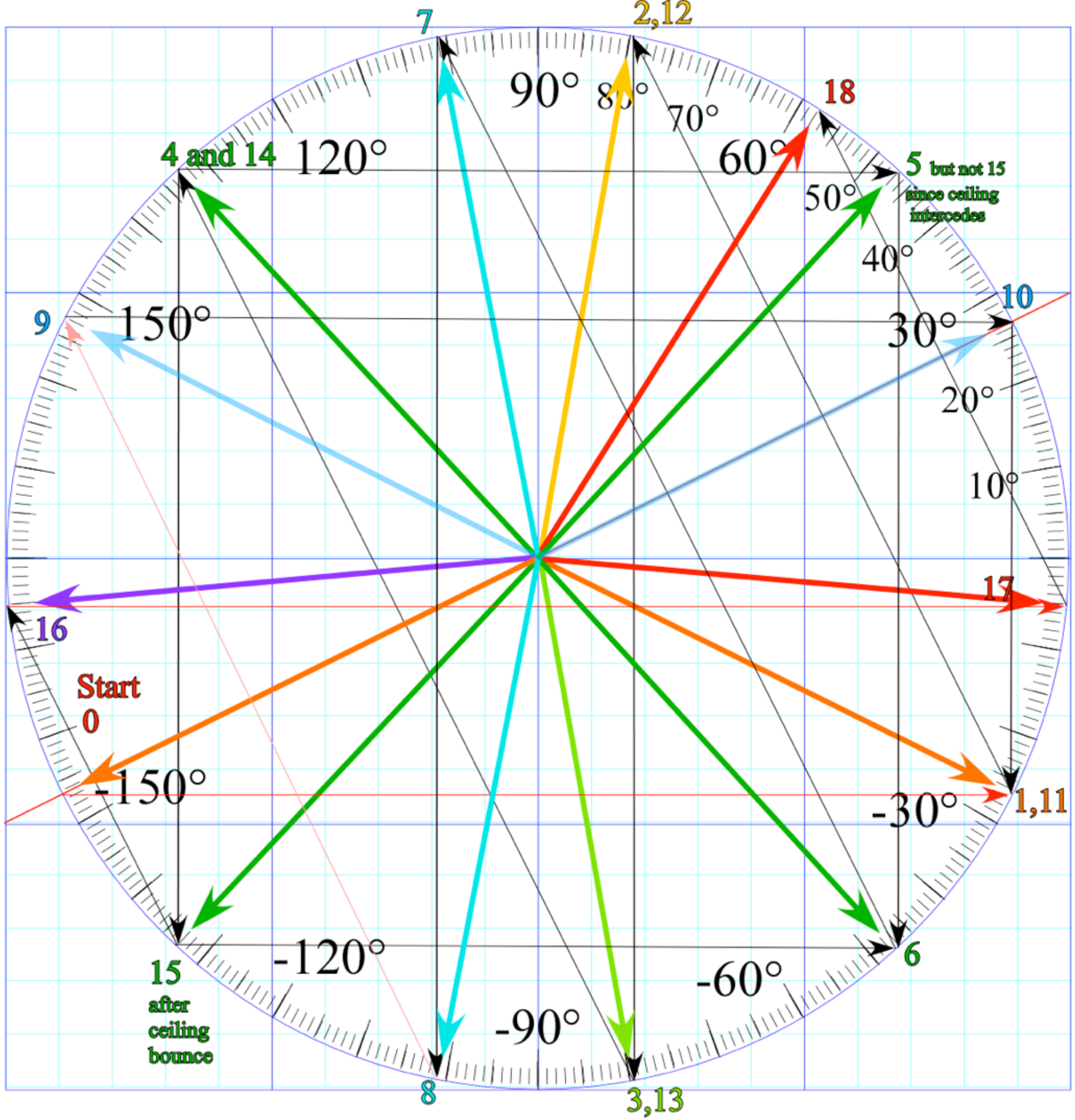
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

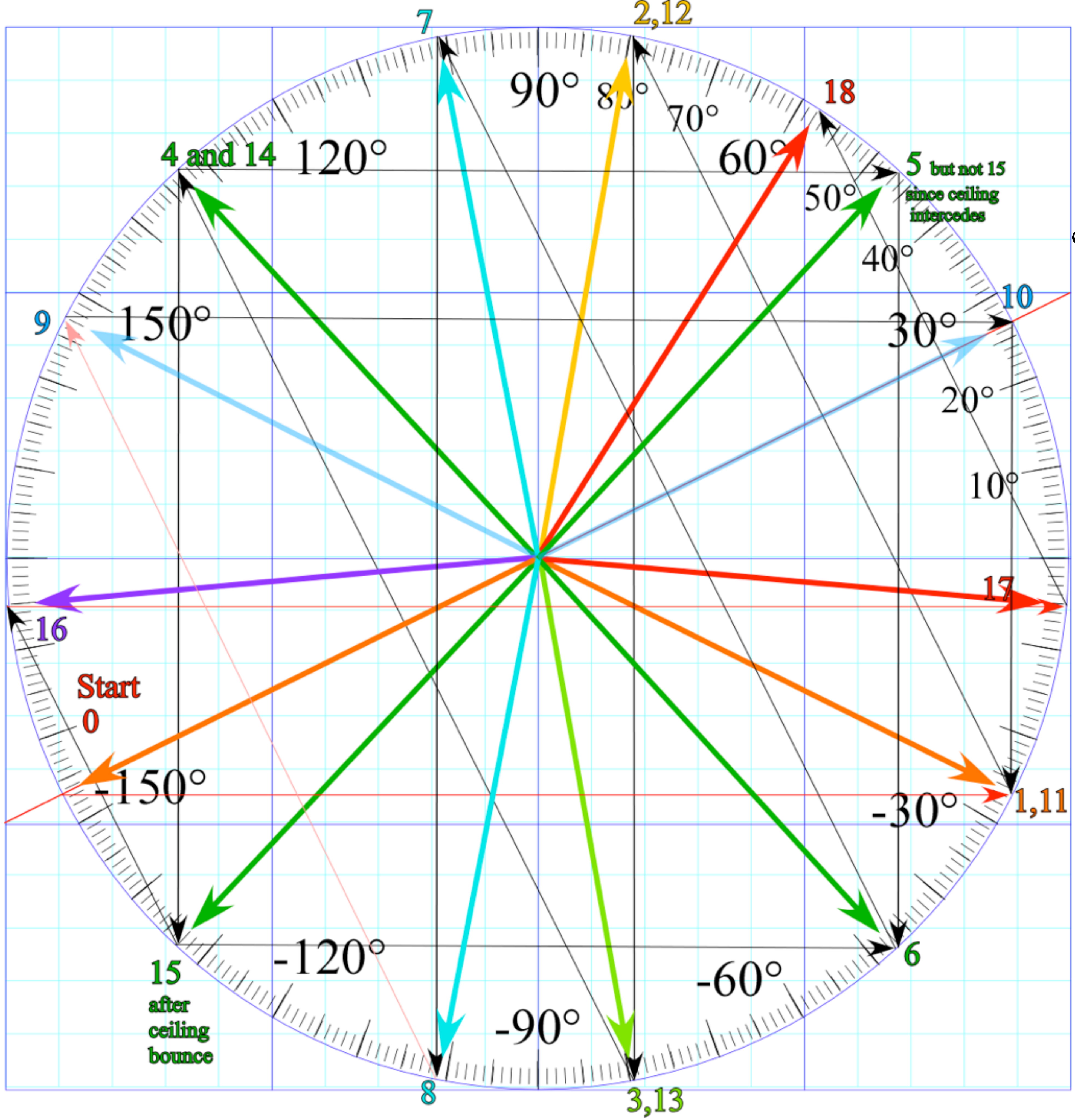
Collisions for mass ratio m_1:m_2= 4:1

First part of Exercise 1.4.1 has pen-ball initial values $v_1(0)$=-1=$v_2(0)$





Collisions for mass ratio $m_1:m_2= 4:1$



Collisions for mass ratio m_1:m_2= 4:1

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{3}{5} = 0.6$$

$$\theta = 53.13°$$

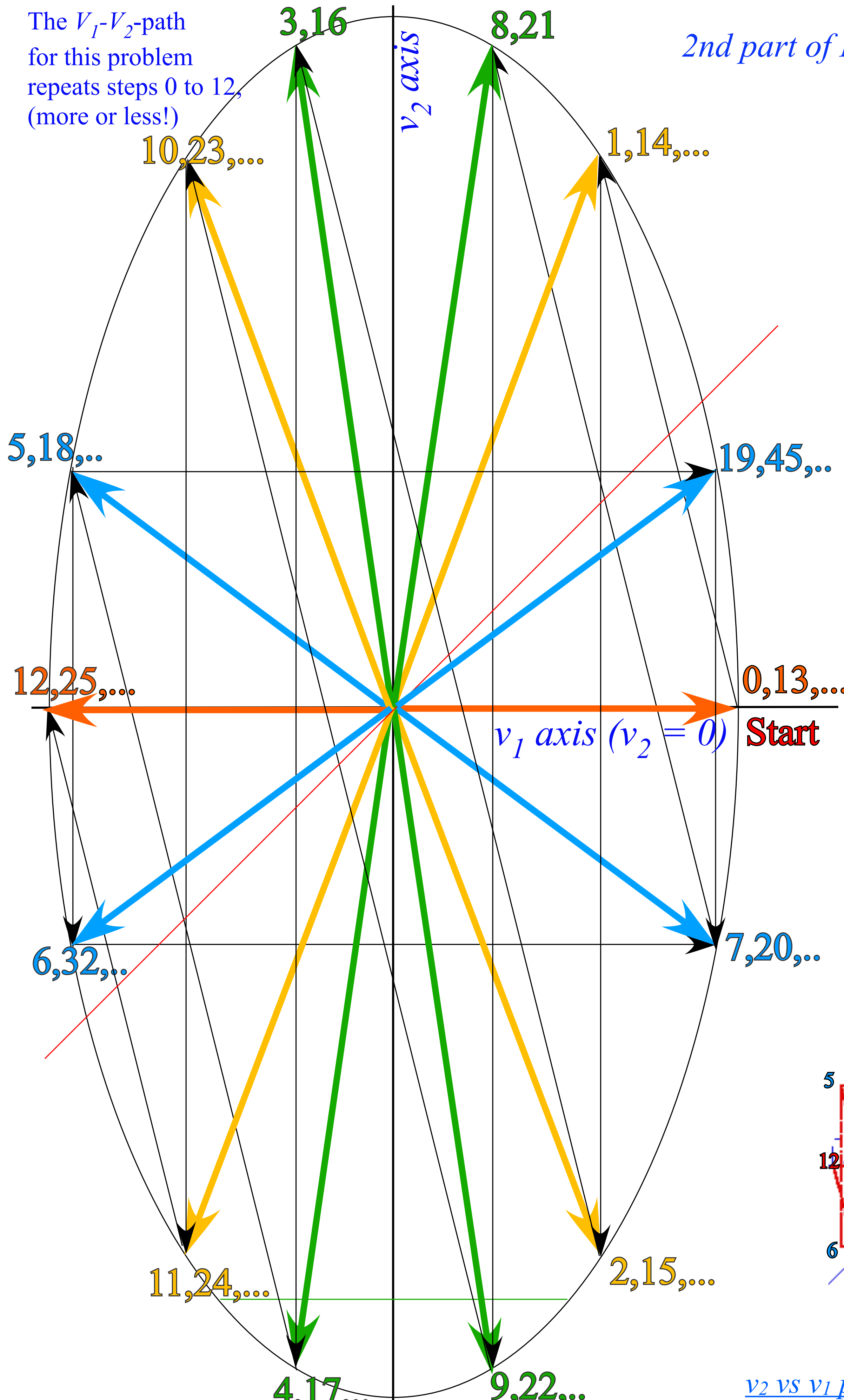
$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

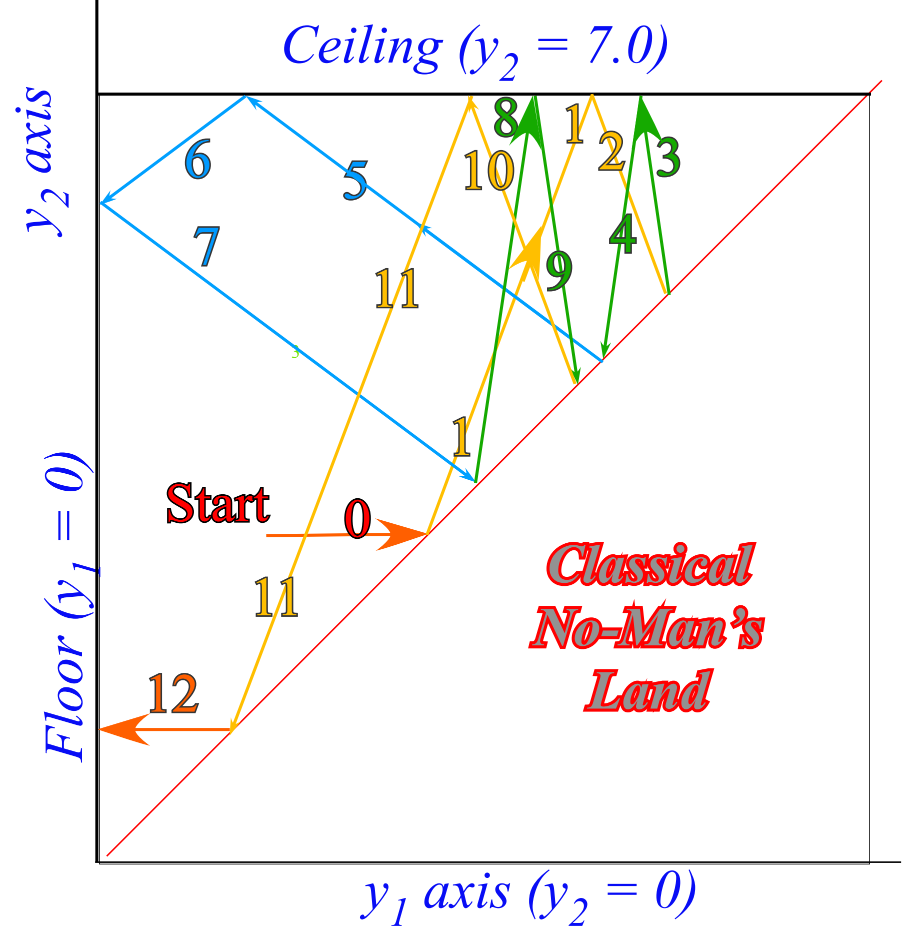
$$\alpha = \theta/2 = 26.565°$$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{8/5}{2/5} = 4$$

2nd part of Exercise 1.4.1 has pen-ball initial velocity values $v_1(0)=1$ and $v_2(0)=0$

at: $x_1(0)=1.5$ and $x_2(0)=3.0$

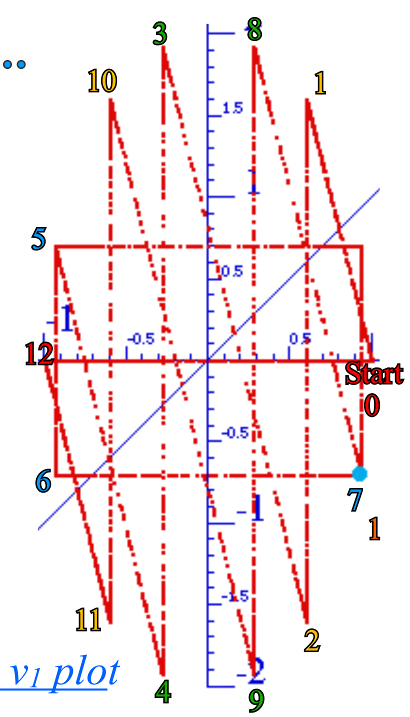




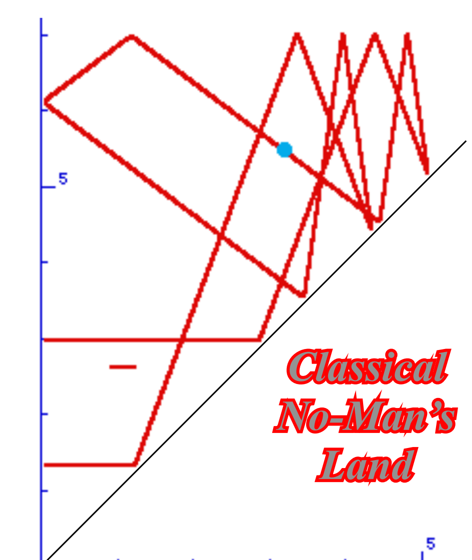
Collisions for mass ratio $m_1:m_2 = 4:1$

BounceIt Web Simulations

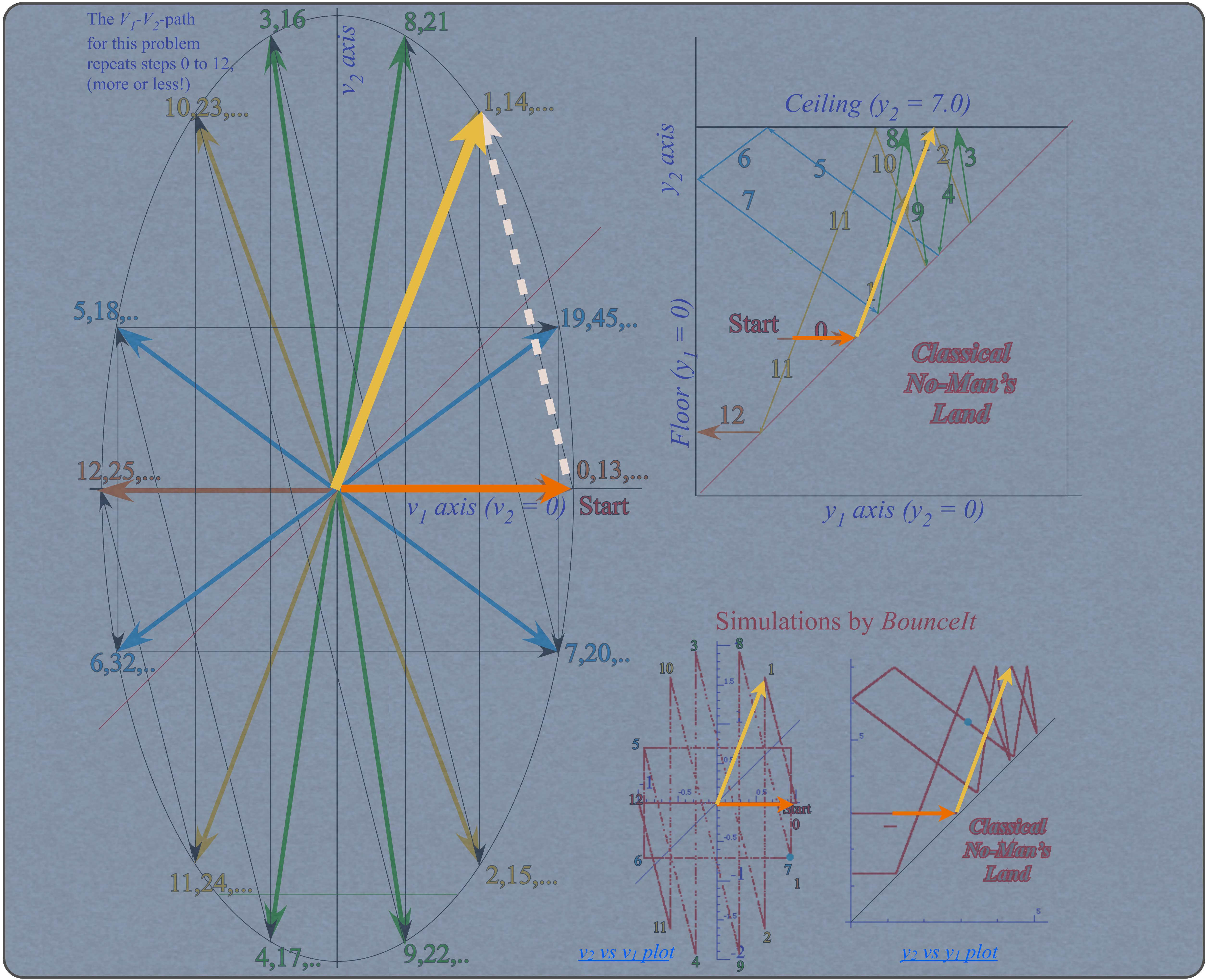
$m_1:m_2 = 4:1$ $(v_1, v_2)=(1, 0)$



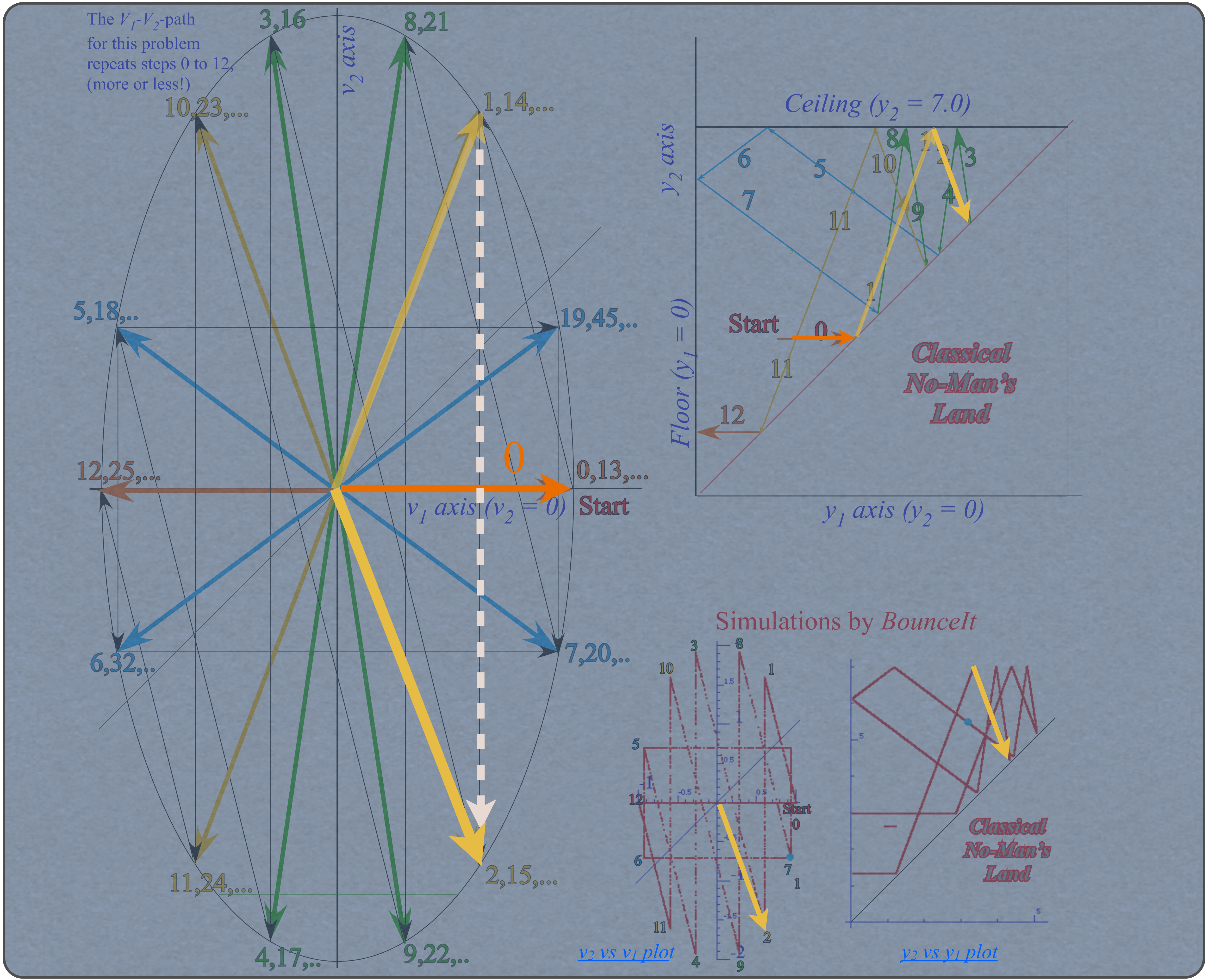
v_2 vs v_1 plot



y_2 vs y_1 plot



The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical No-Man's Land
Simulations by *BounceIt*
Start
Classical No-Man's Land
v_2 vs v_1 plot
y_2 vs y_1 plot



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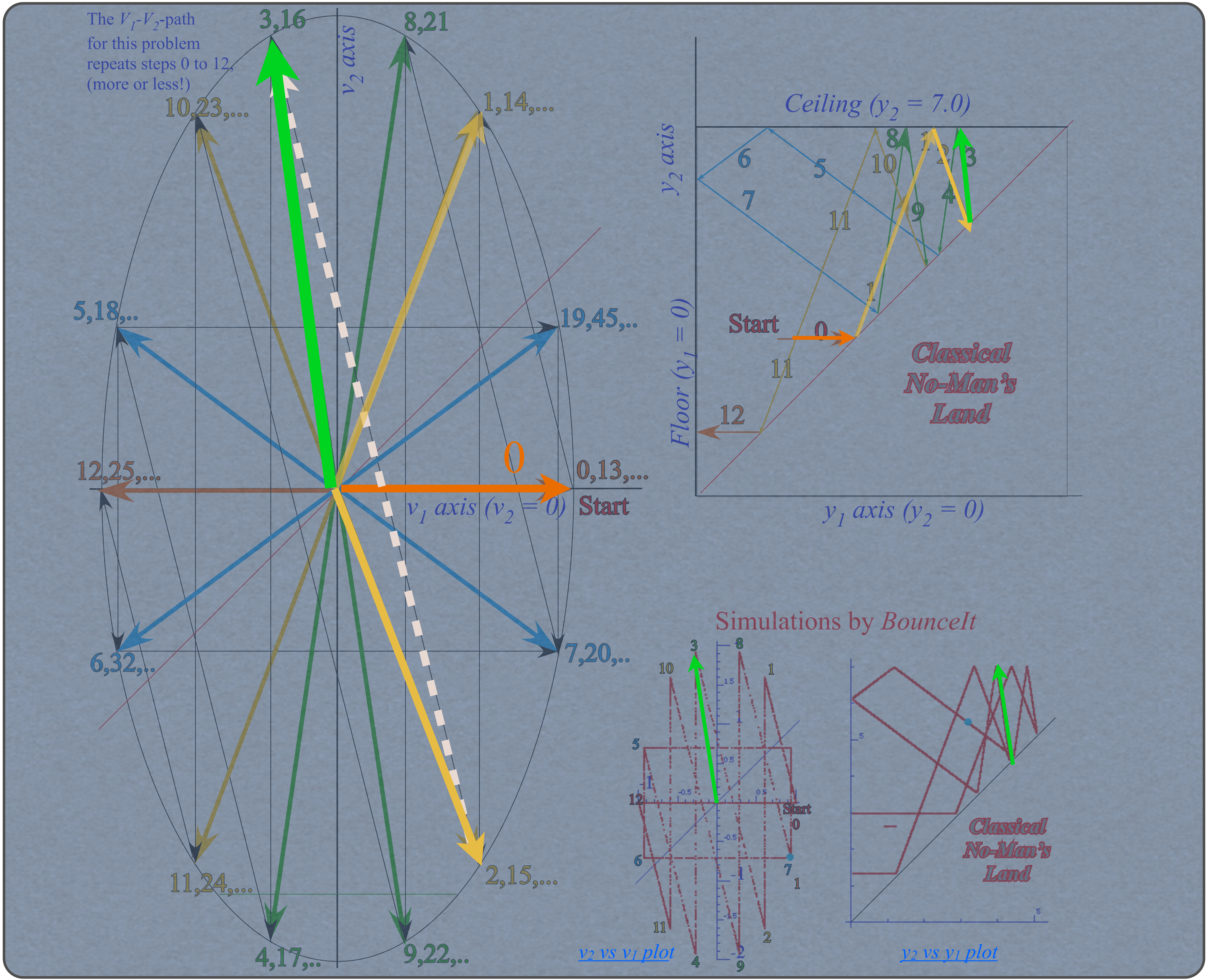
Classical No-Man's Land
12
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical No-Man's Land
v2 vs v1 plot
y2 vs y1 plot



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4
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11
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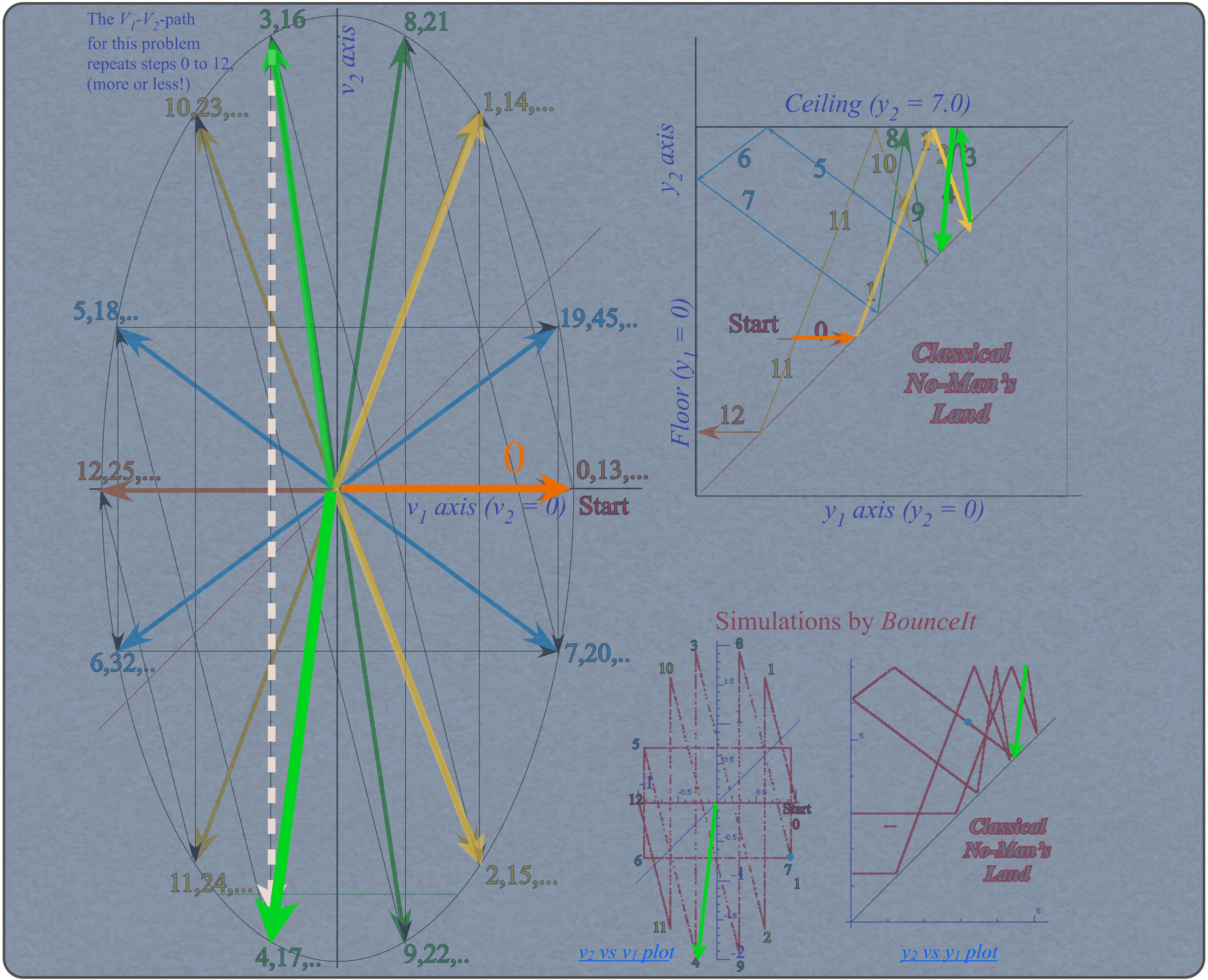

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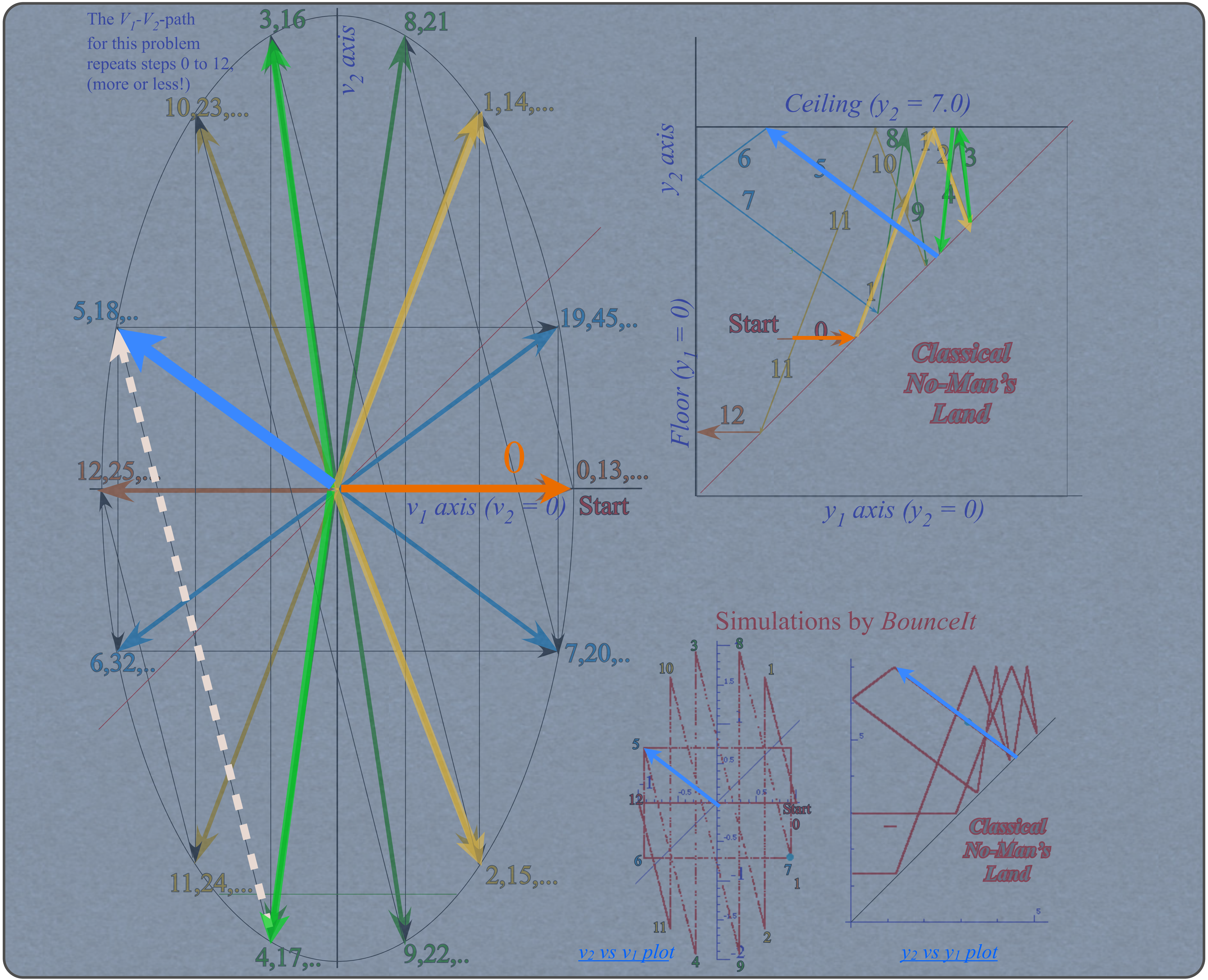
[v2 vs v1 plot]
[y2 vs y1 plot]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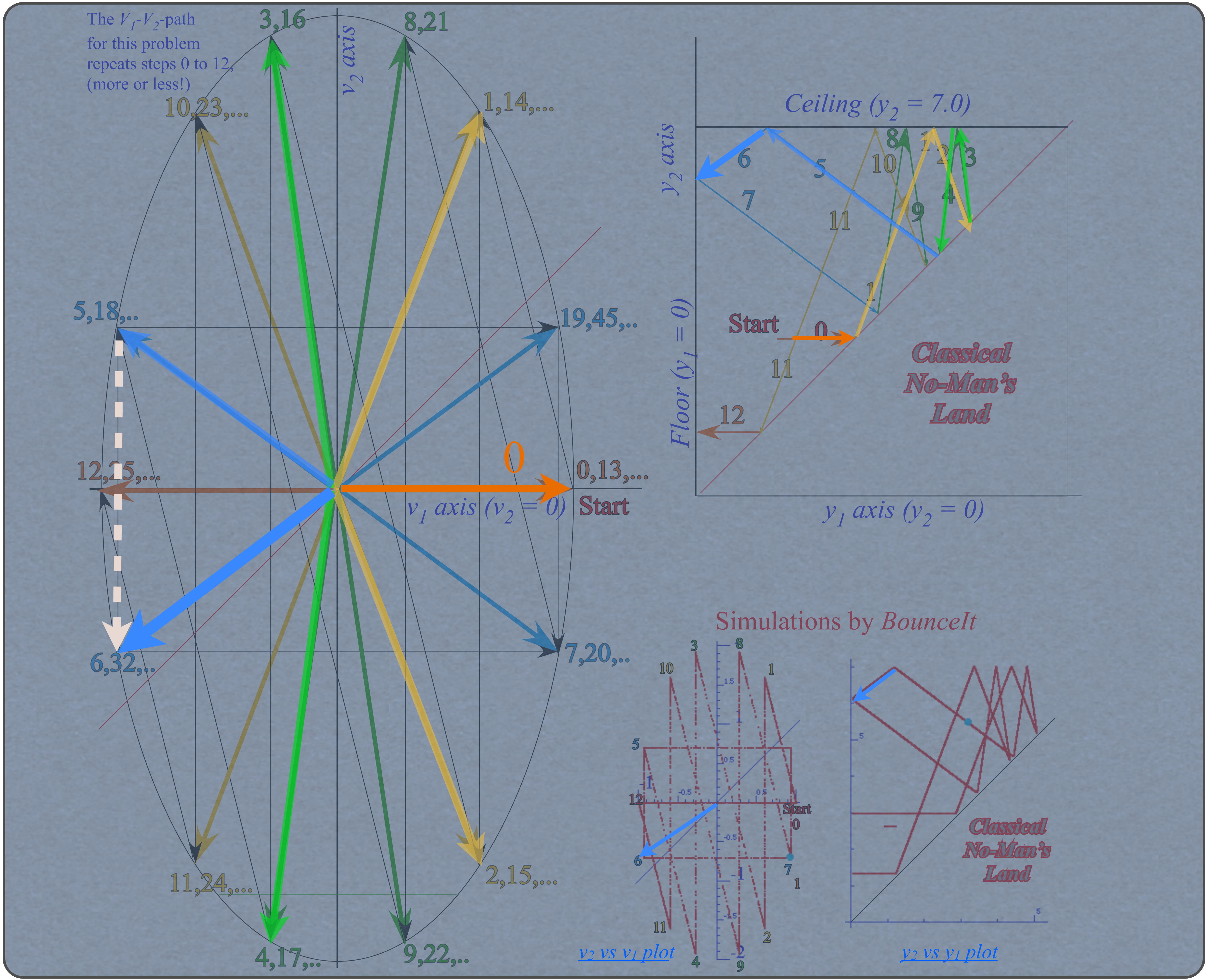
y_2 vs y_1 plot



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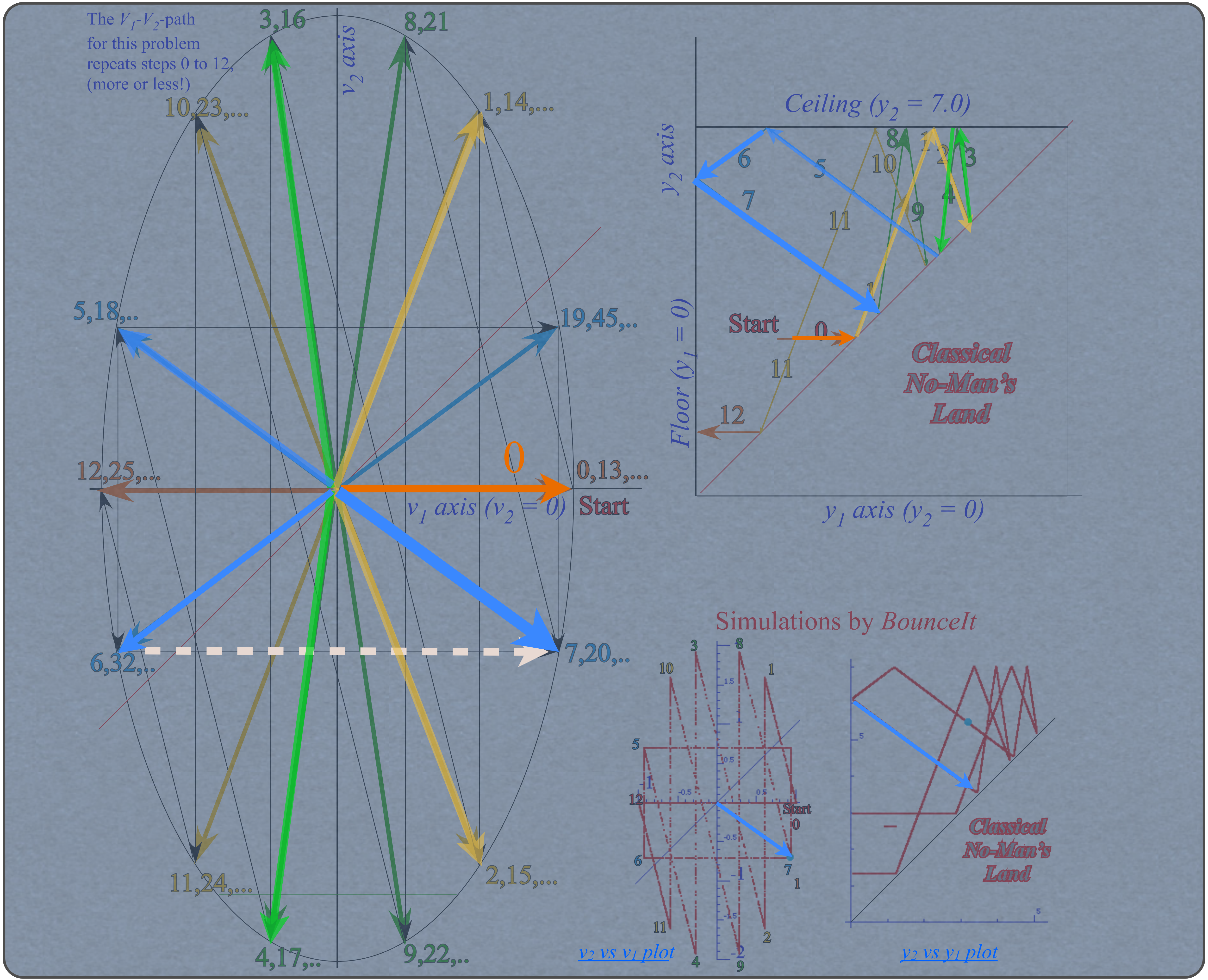
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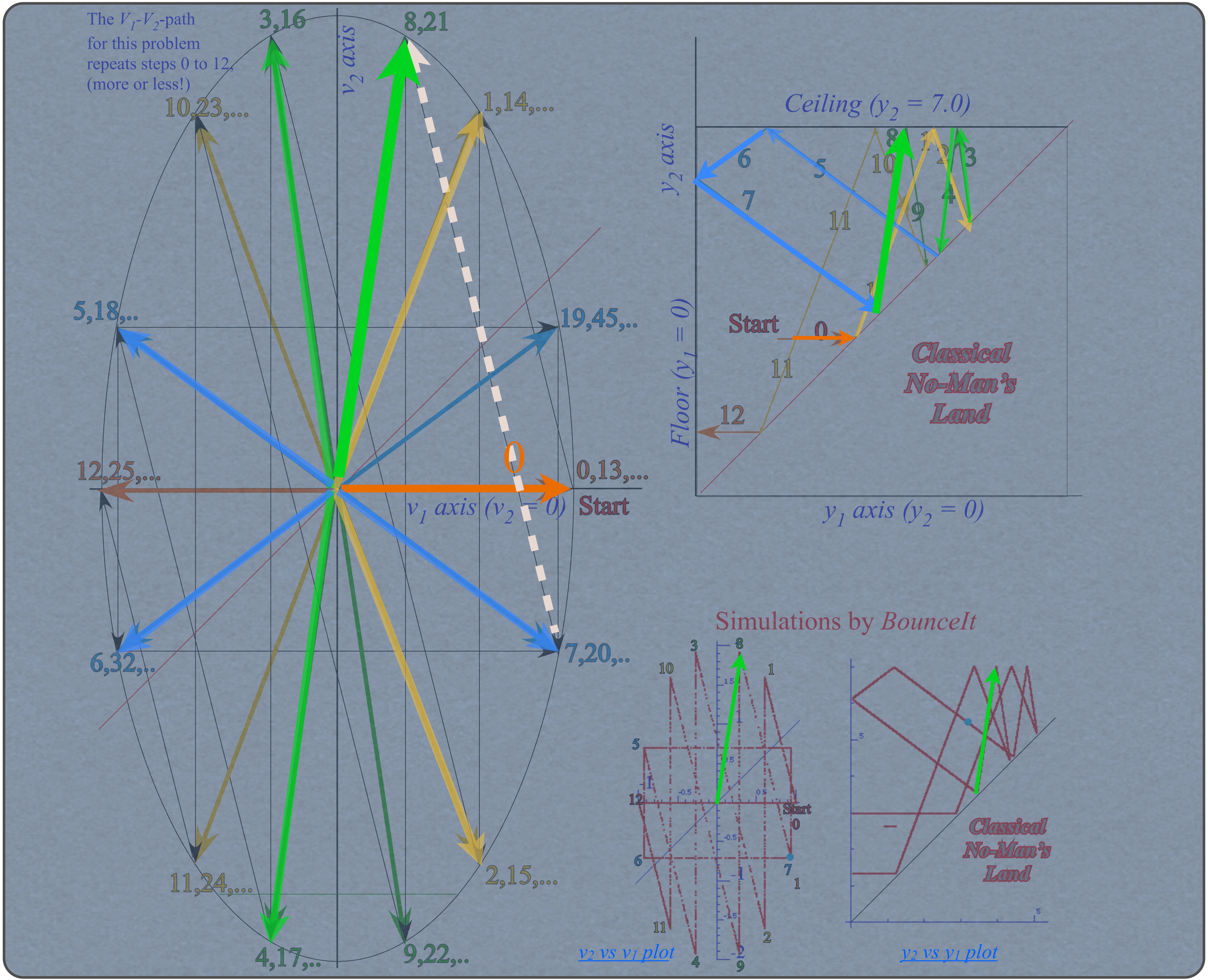

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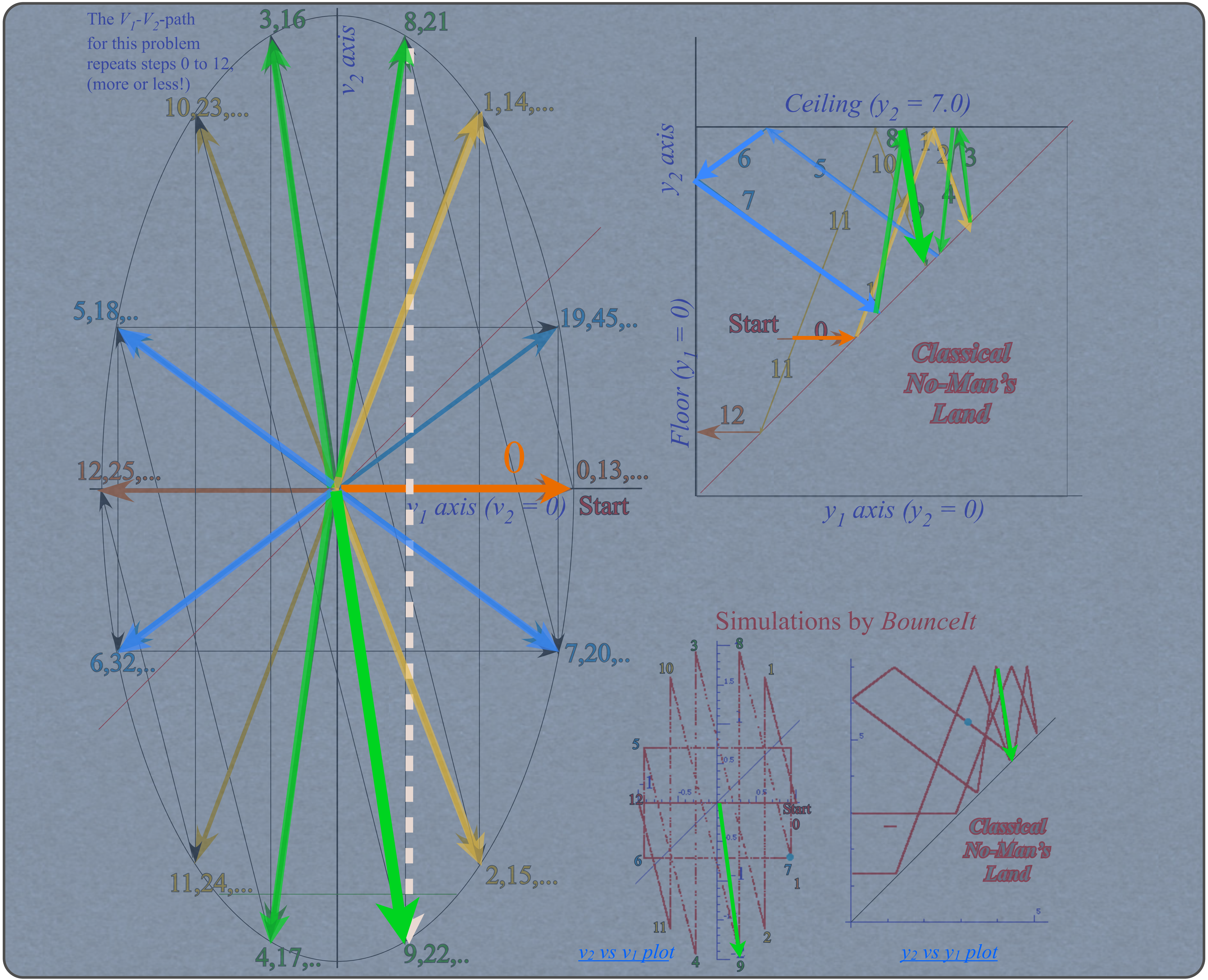

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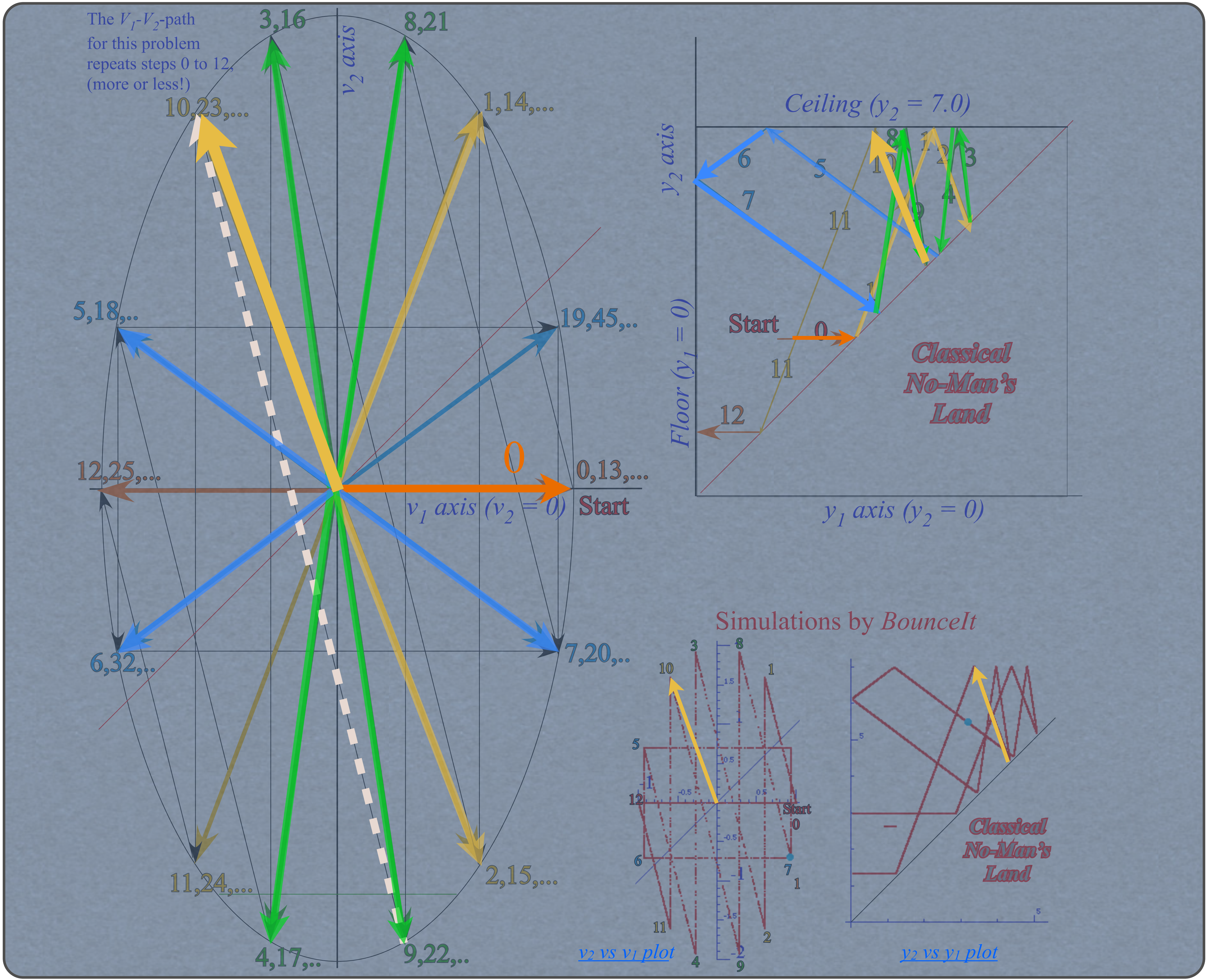

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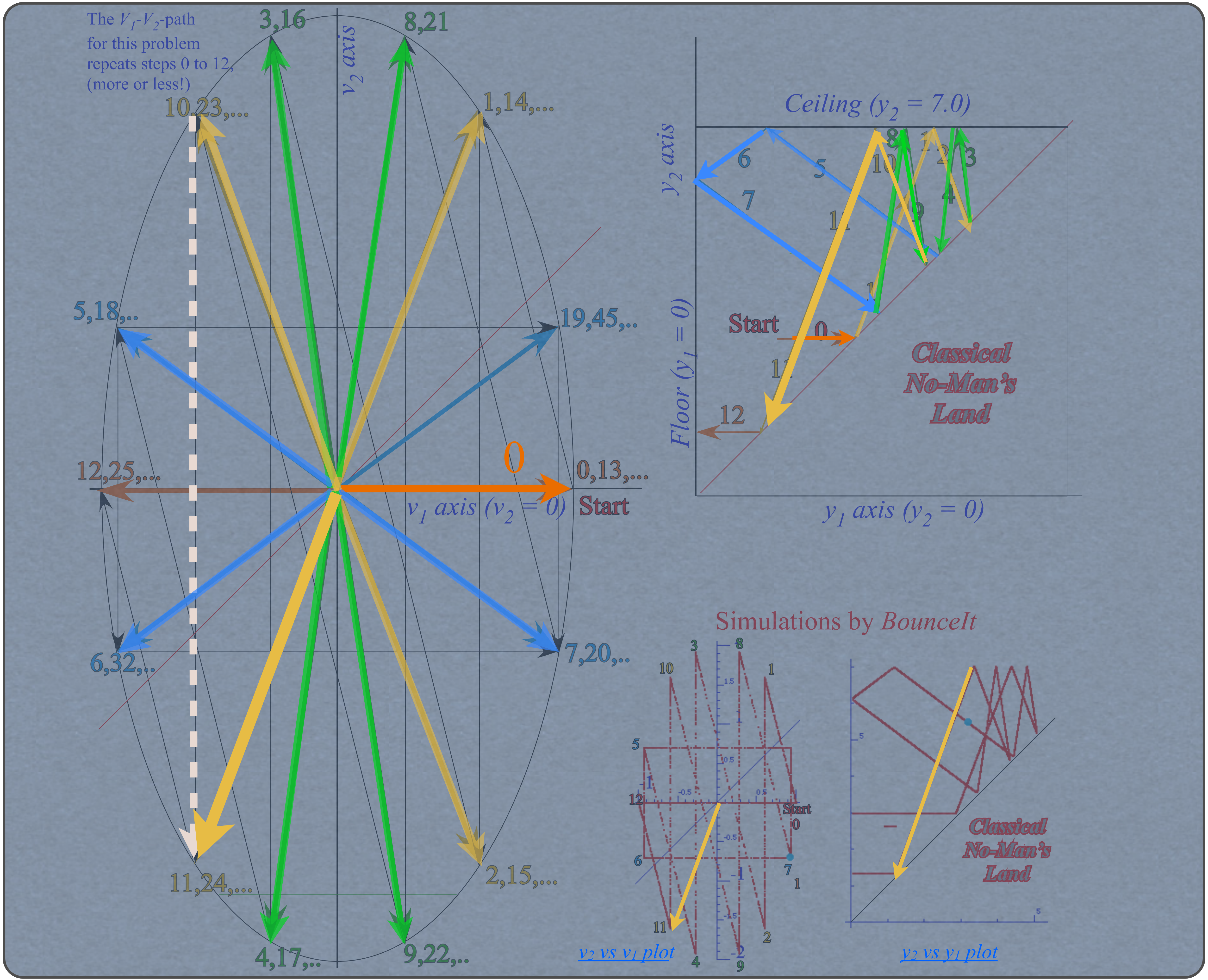

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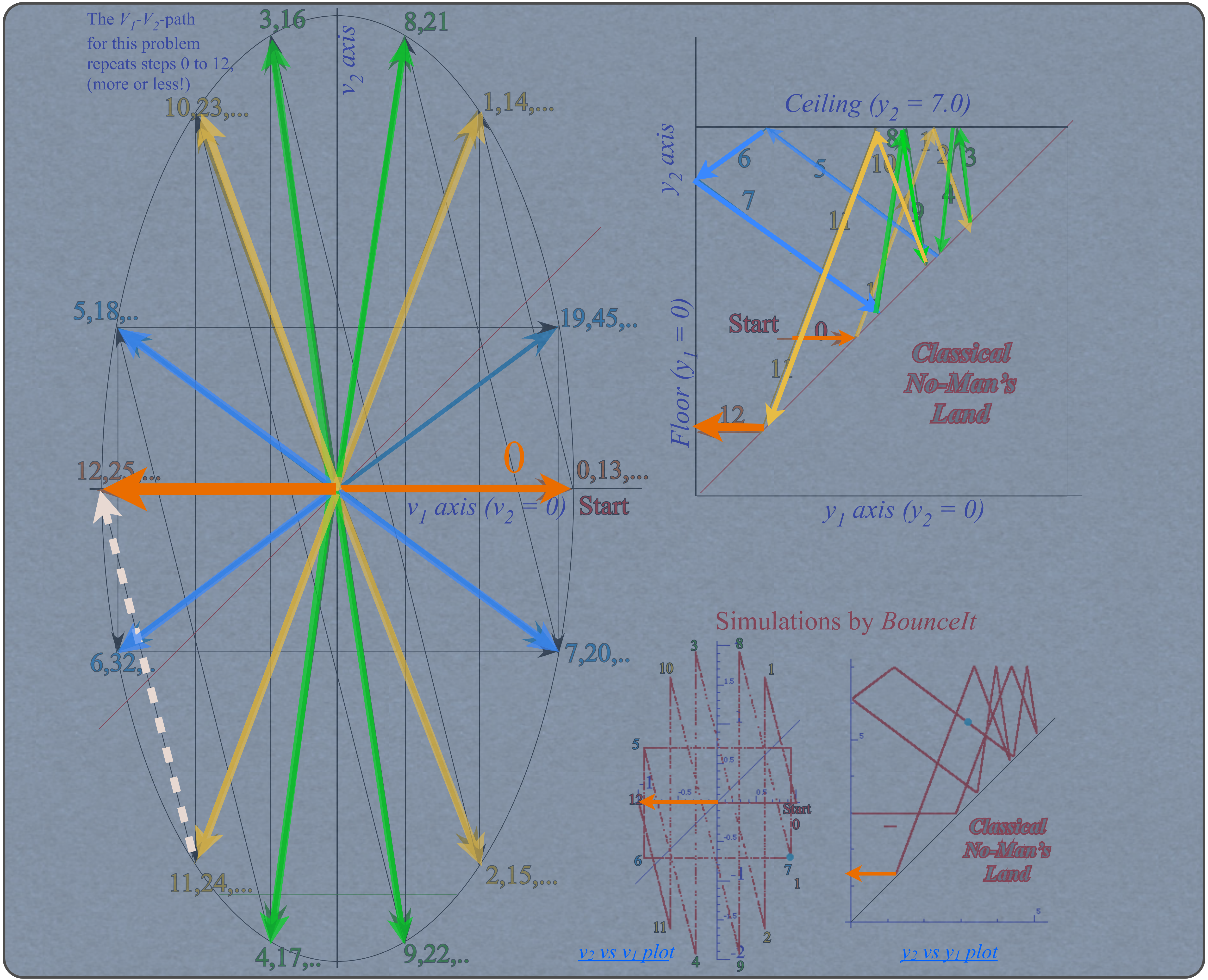

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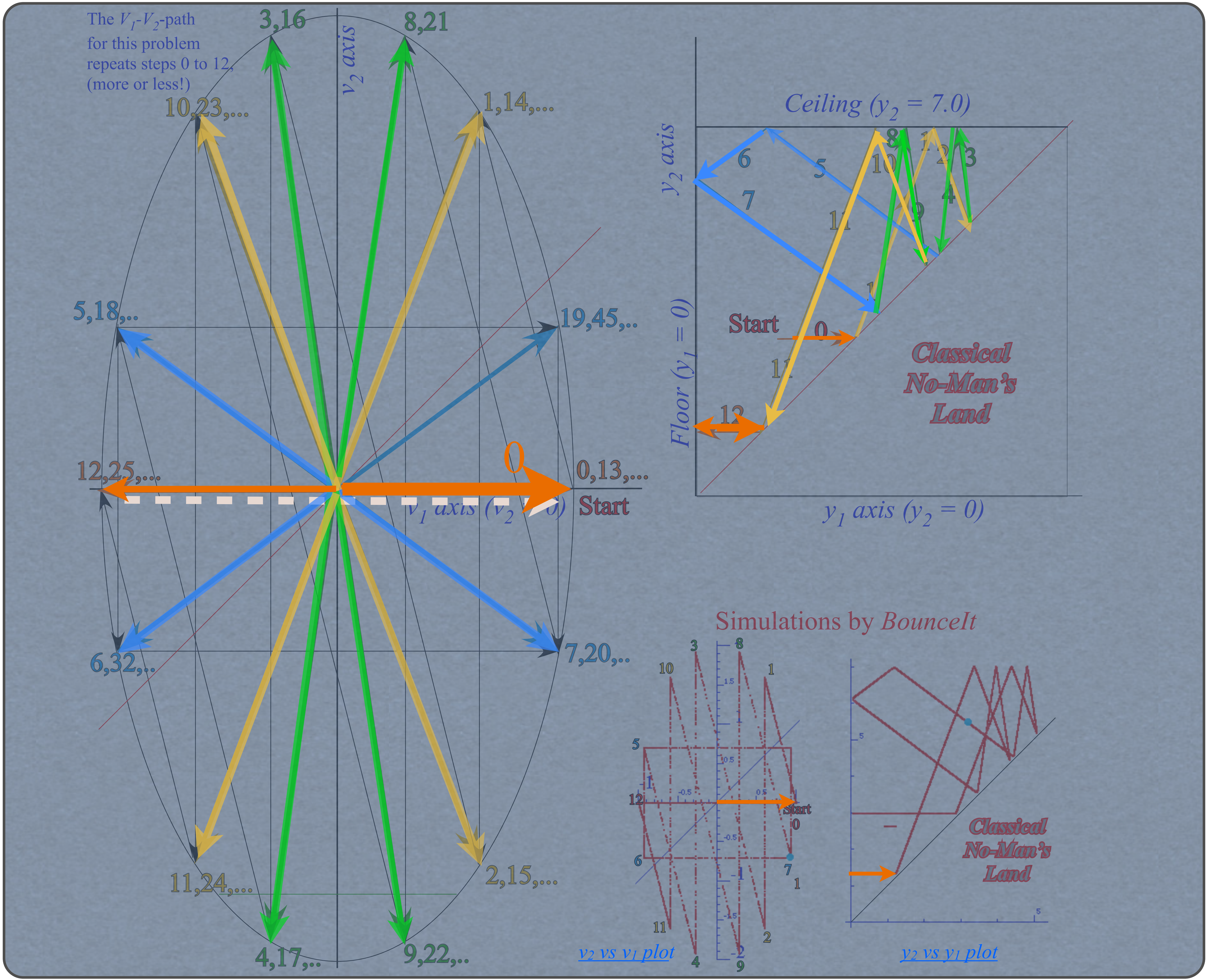
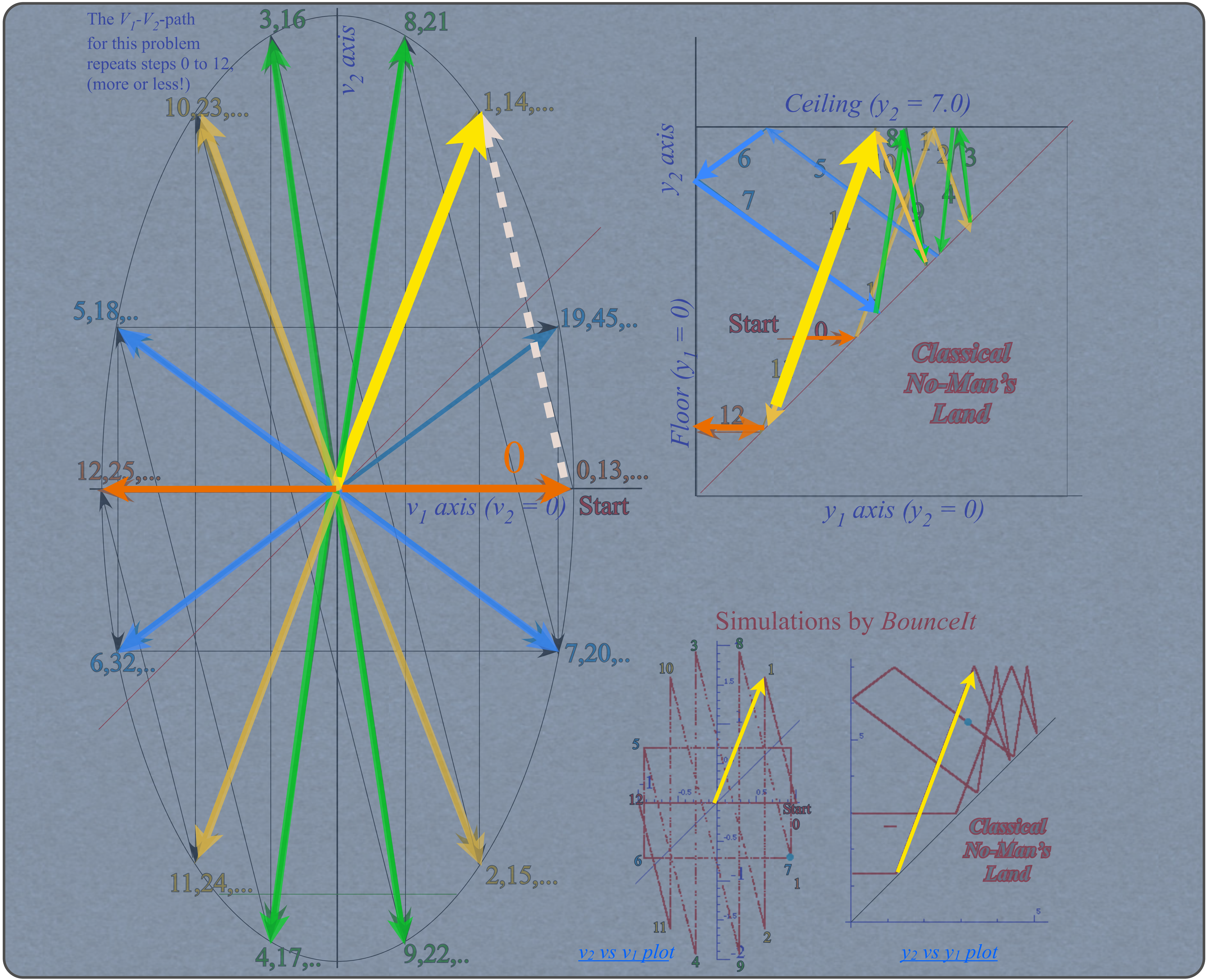
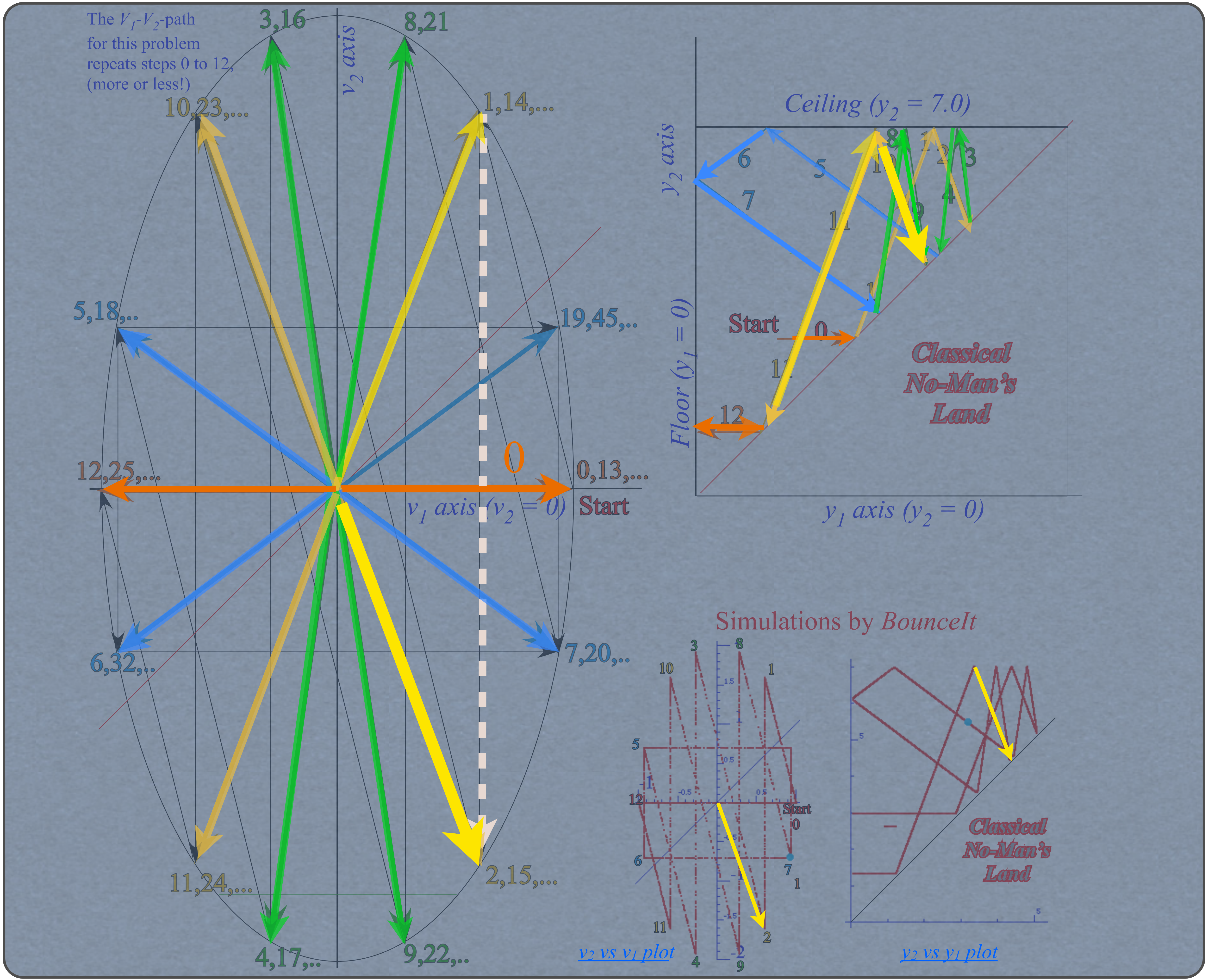
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

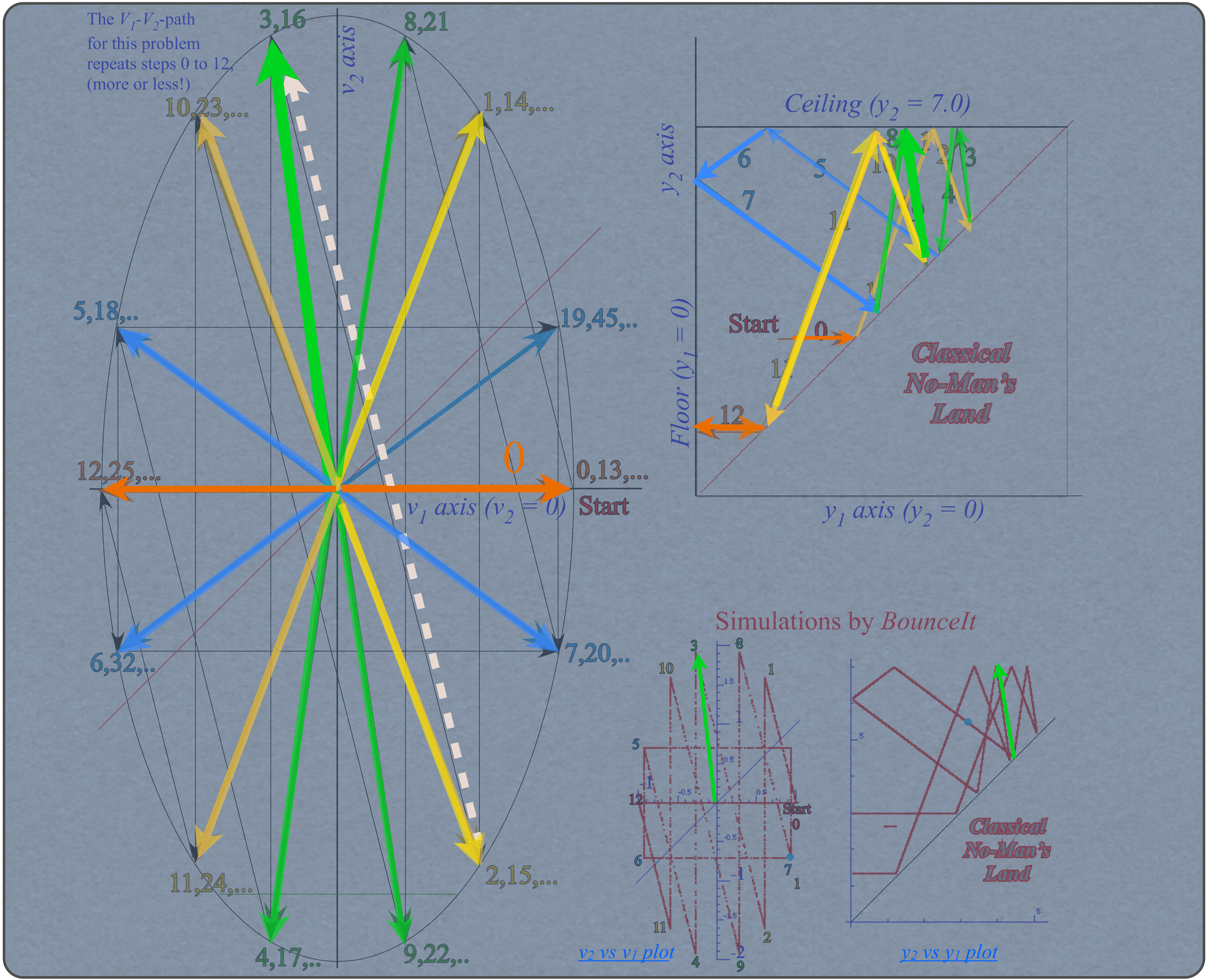


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
Start
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

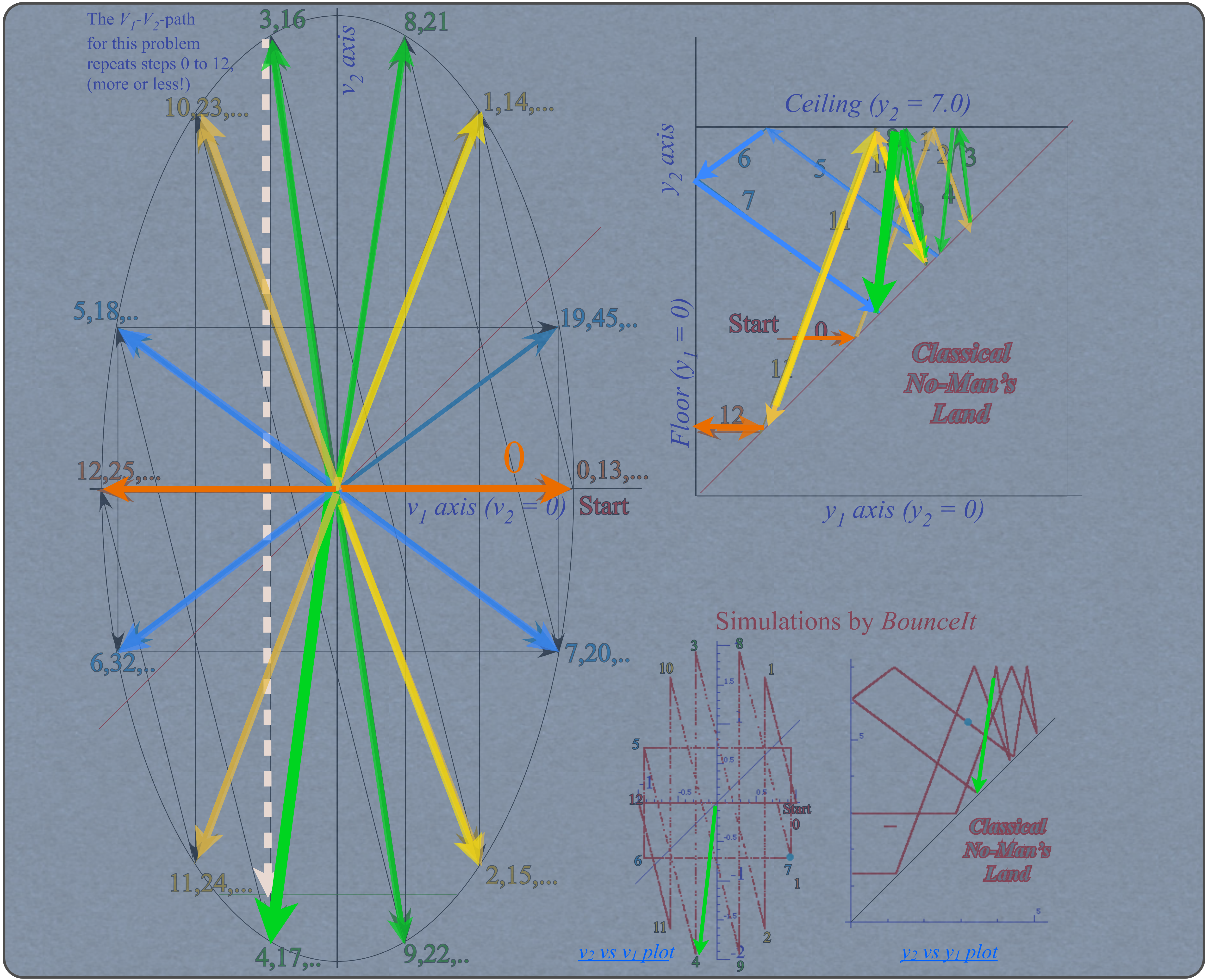


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
12
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

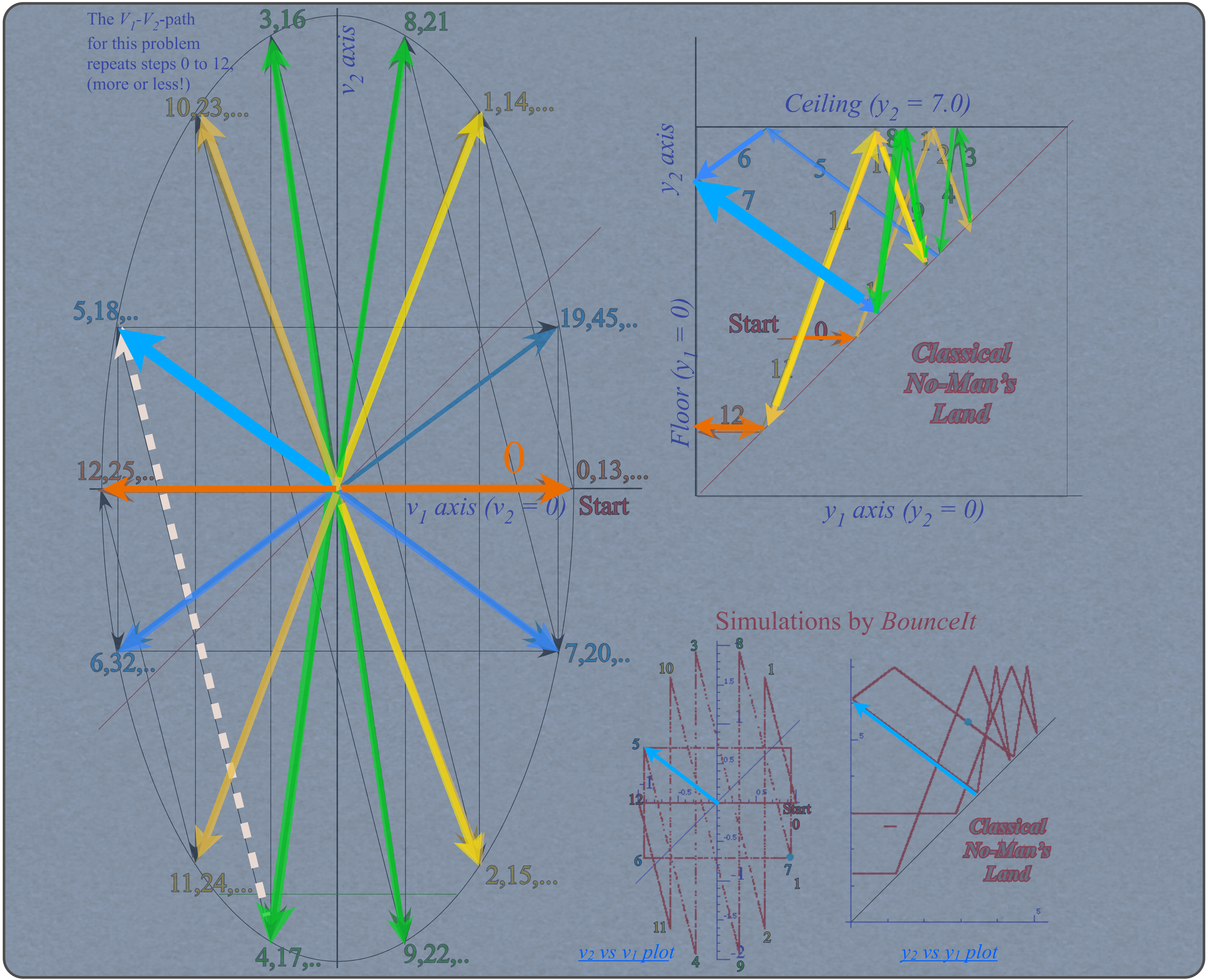

The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot

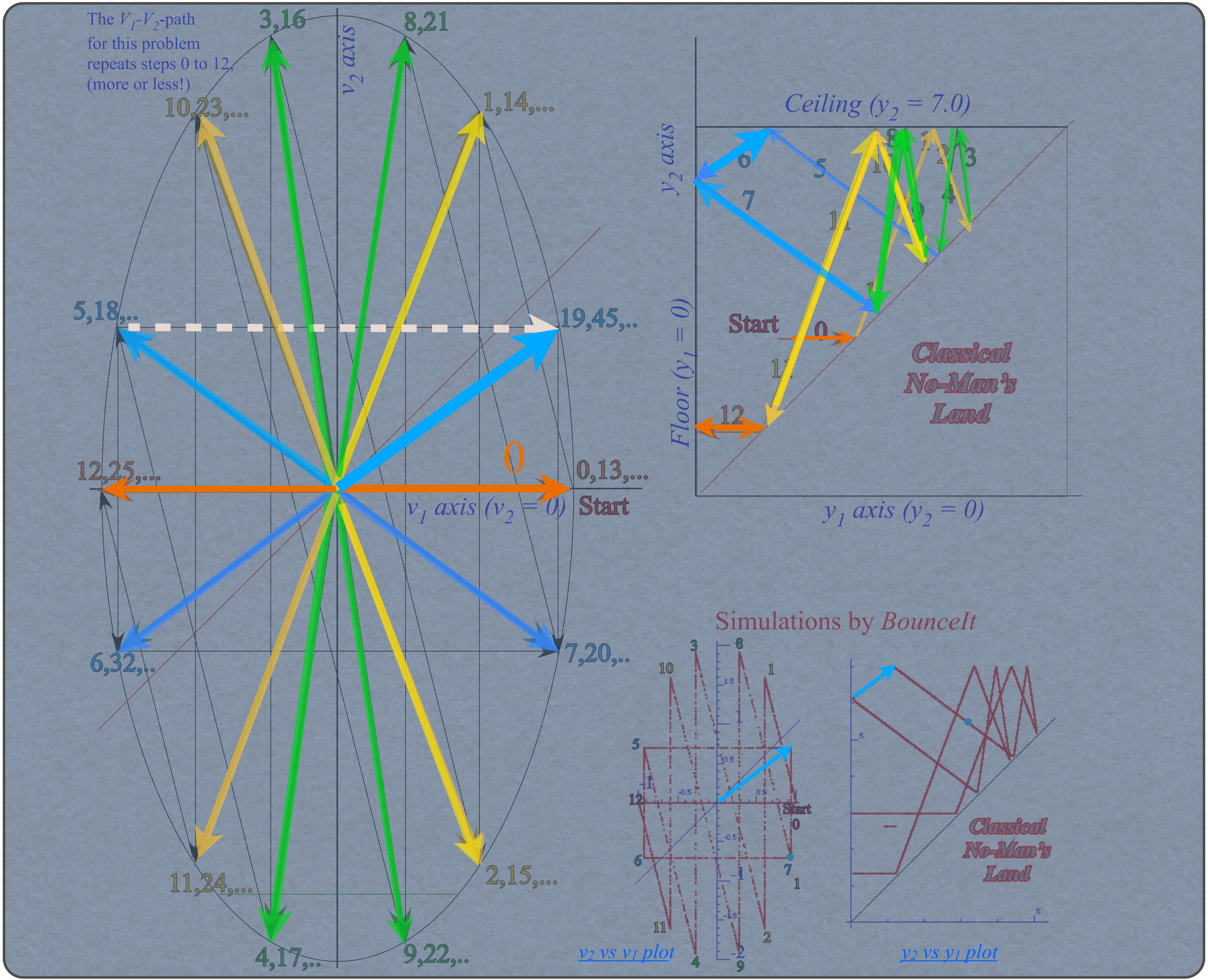


The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
0
12
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot

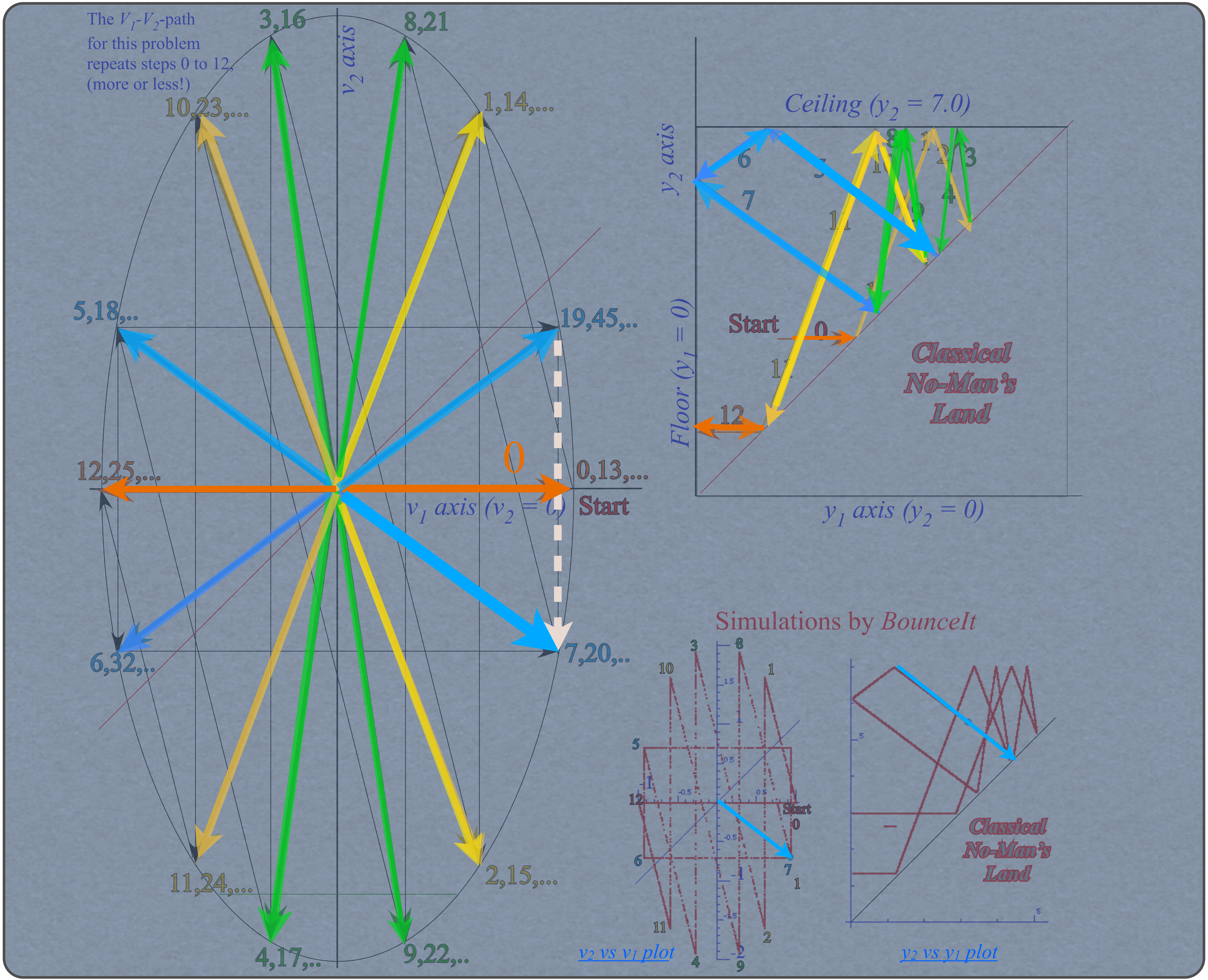


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis ($v_2 = 0$)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling ($y_2 = 7.0$)
y_2 axis
Floor ($y_1 = 0$)
Start
Classical
No-Man's
Land
y_1 axis ($y_2 = 0$)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land

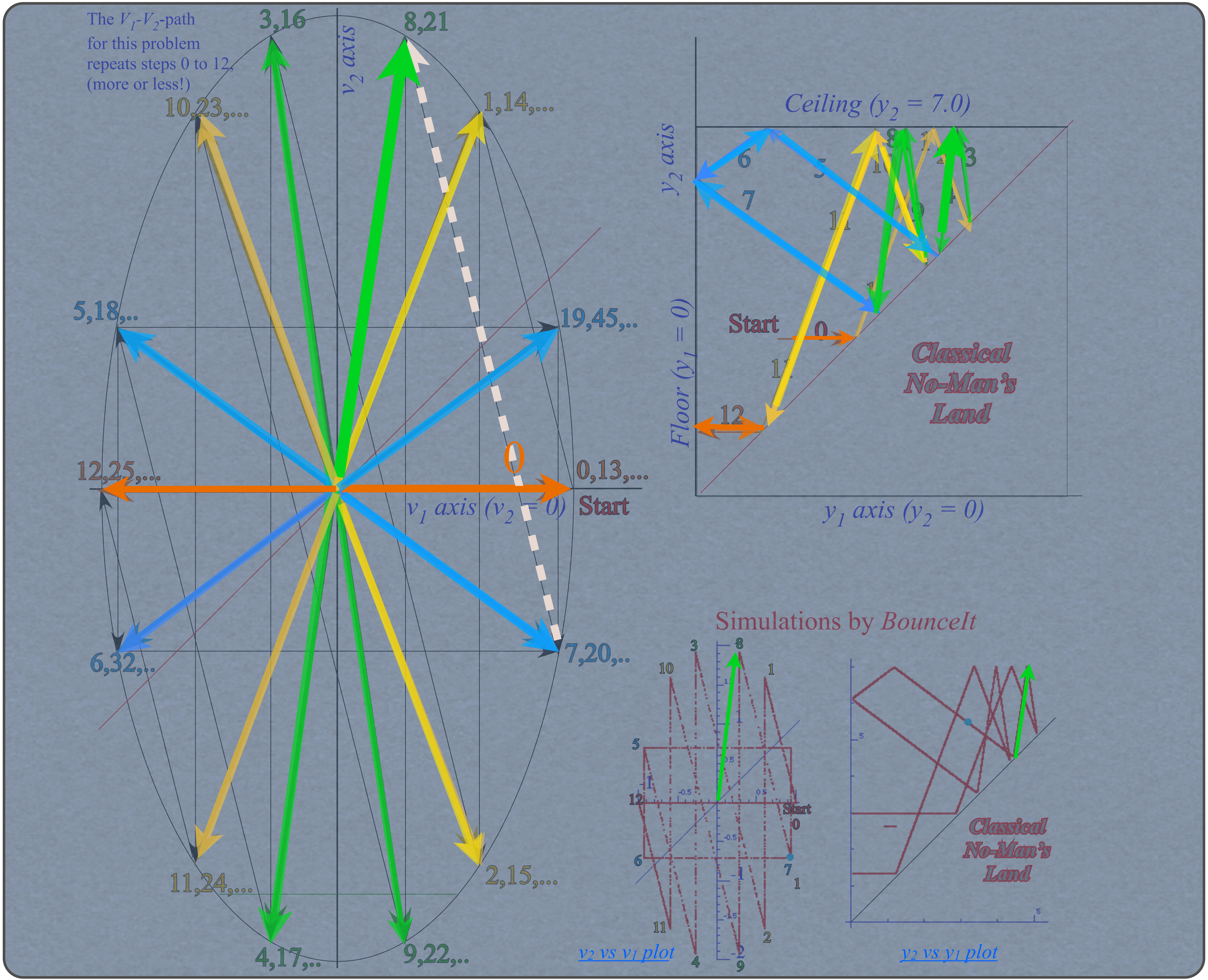

The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
0
12,25,..
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land



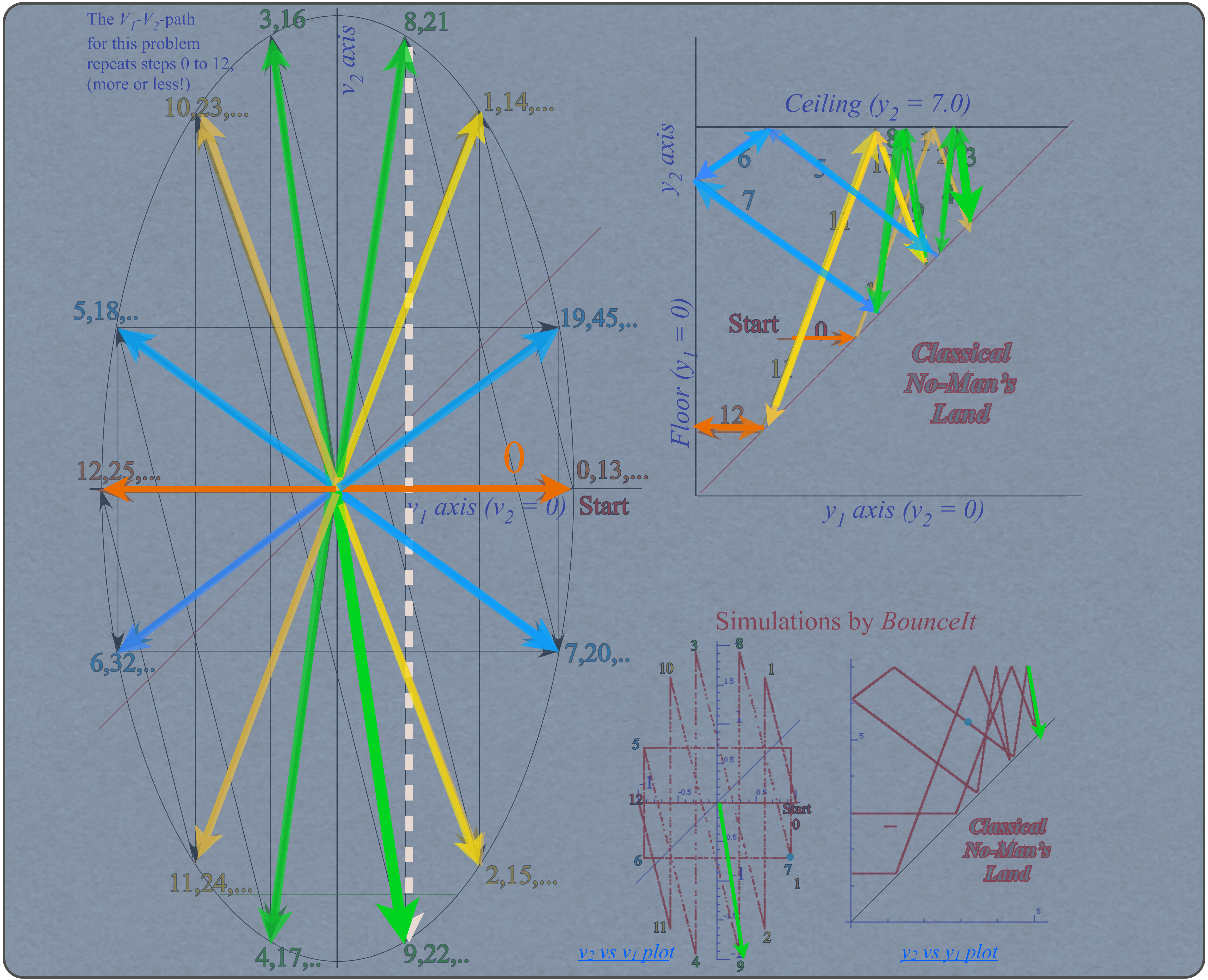
The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land



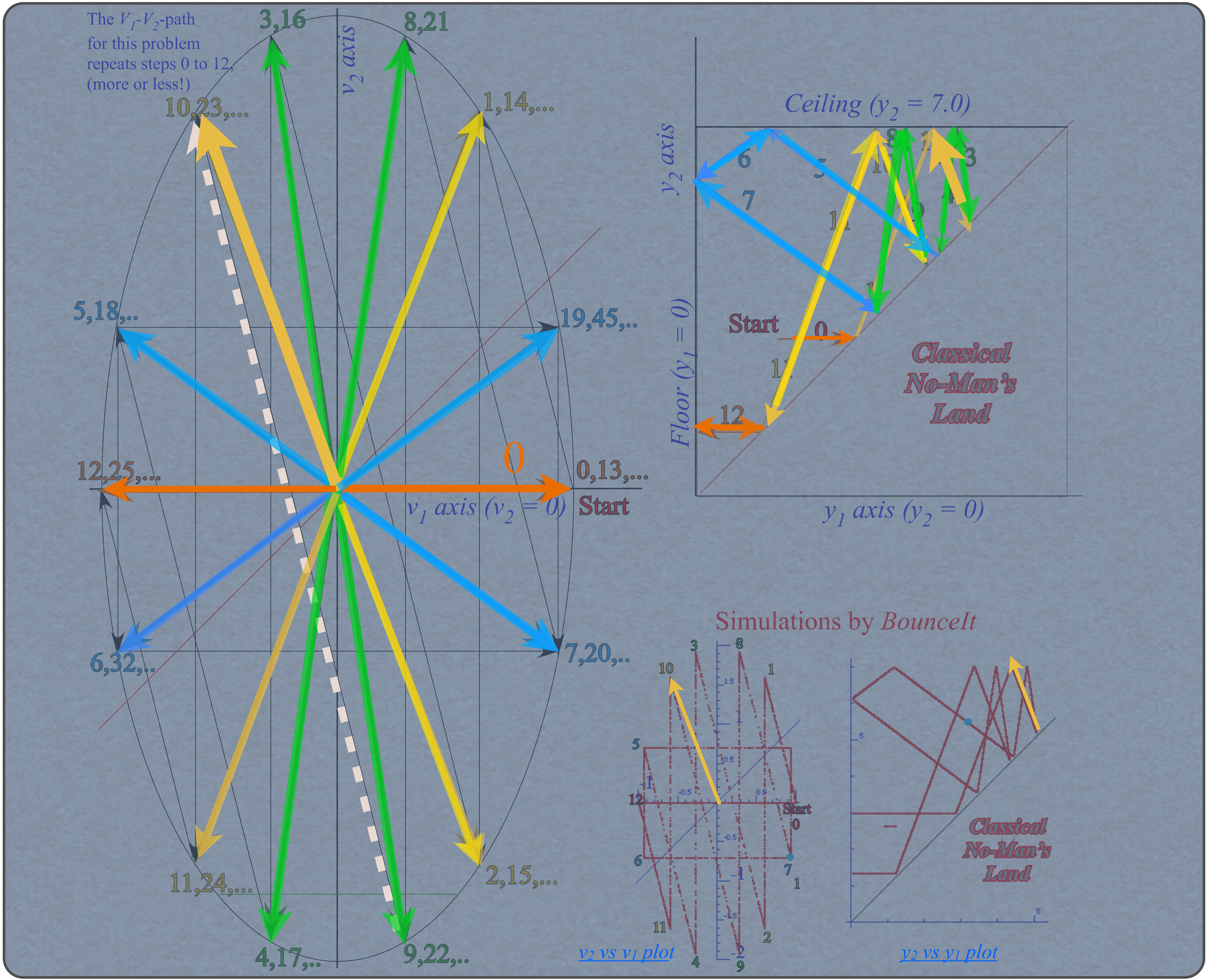
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
v_2 axis
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
0
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical No-Man's Land



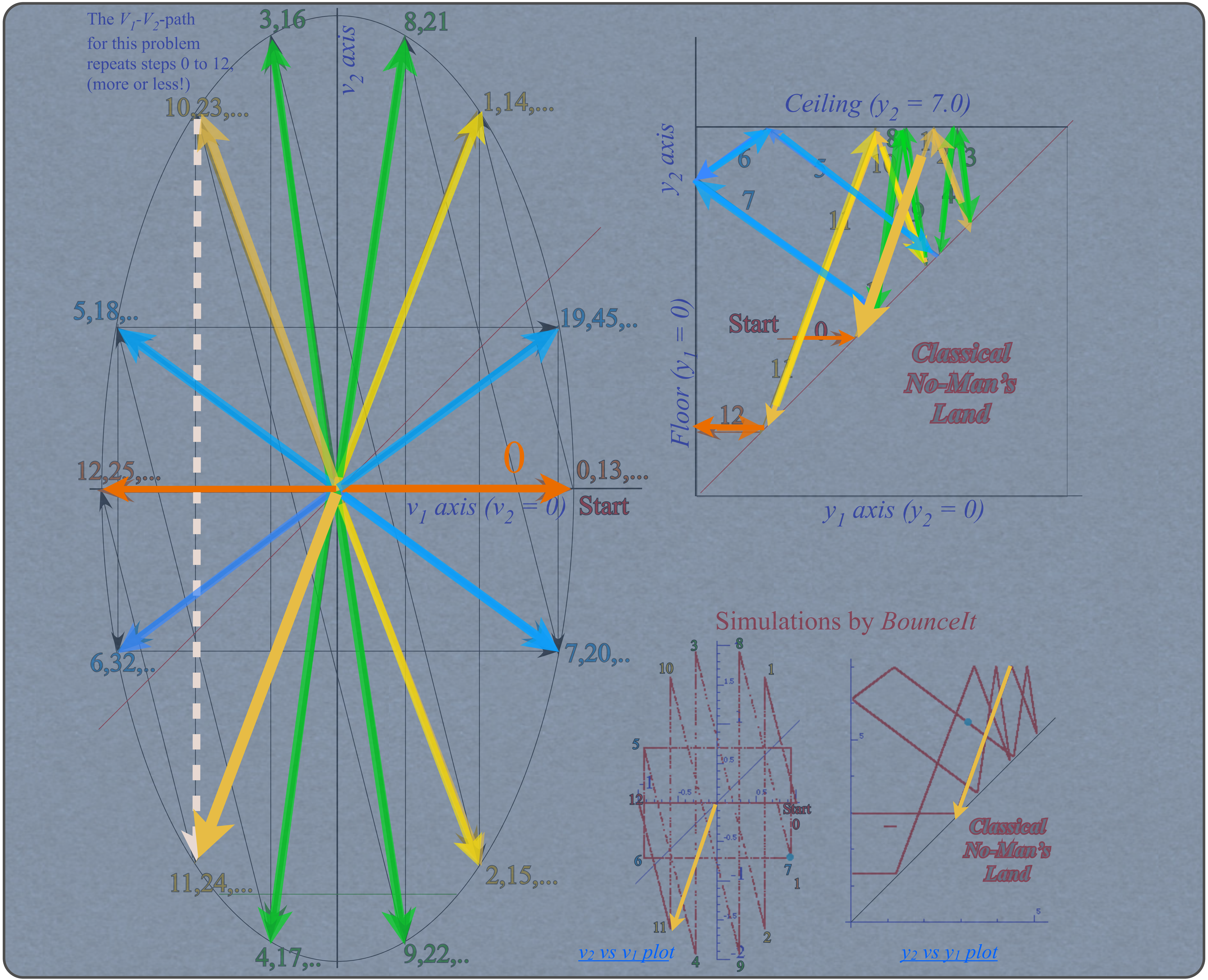
The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
10,23,...
1,14,...
v_2 axis
5,18,..
19,45,..
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Start
Floor (y_1 = 0)
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land



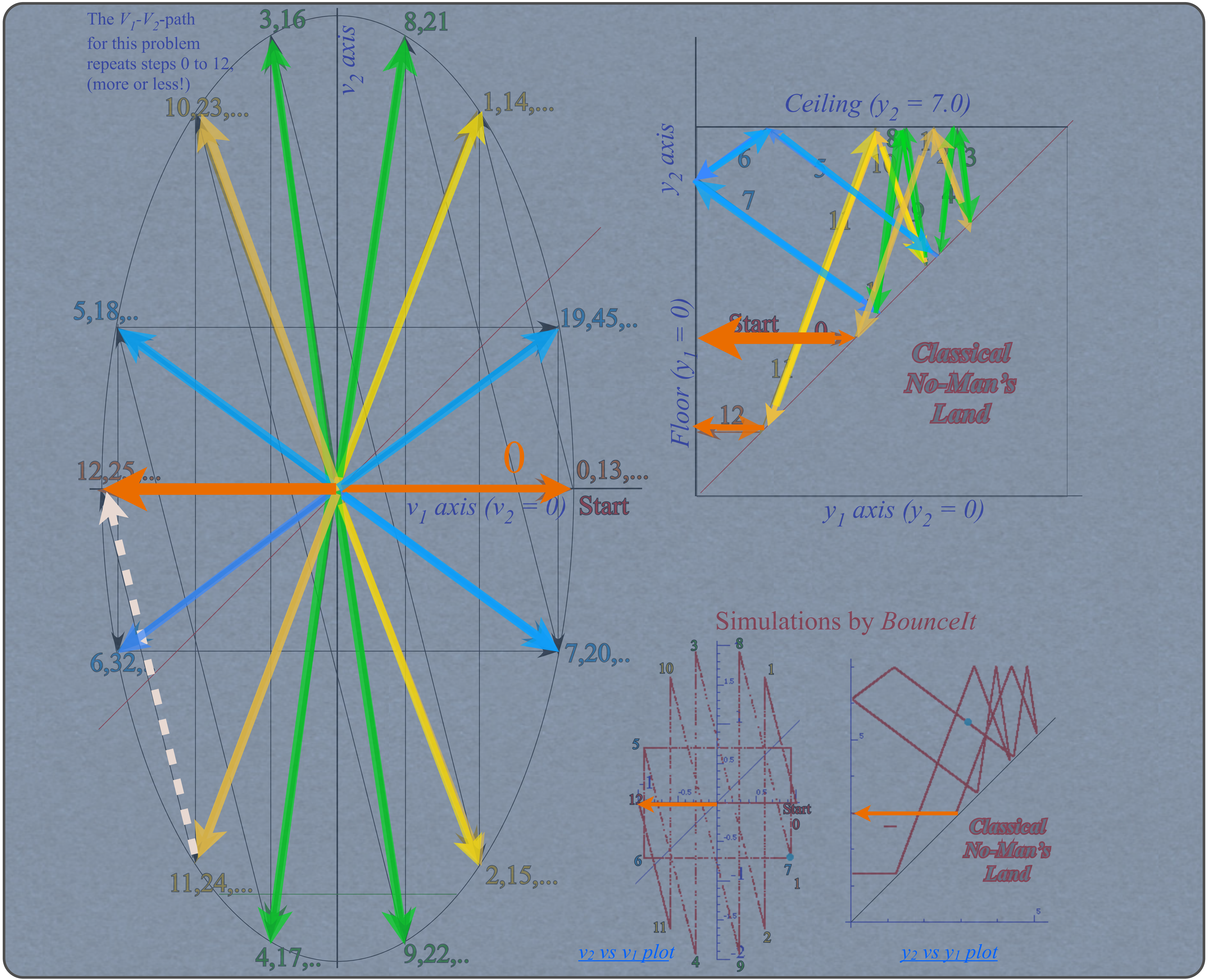
The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
y_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
0
12
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land



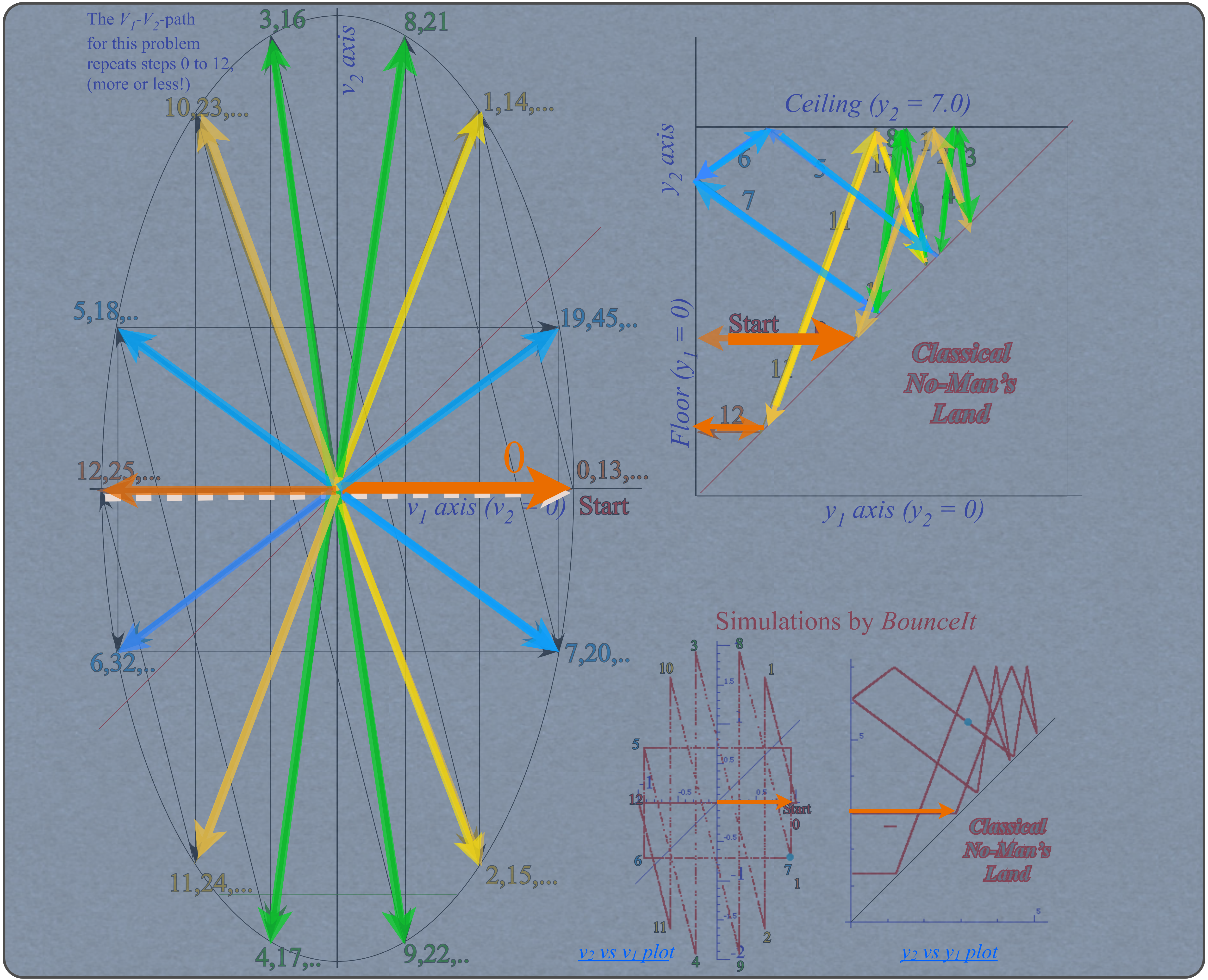
The V1-V2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical No-Man's Land
y1 axis (y2 = 0)
Simulations by BounceIt
Classical No-Man's Land
v2 vs v1 plot
y2 vs y1 plot



The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v2 vs v1 plot
y2 vs y1 plot



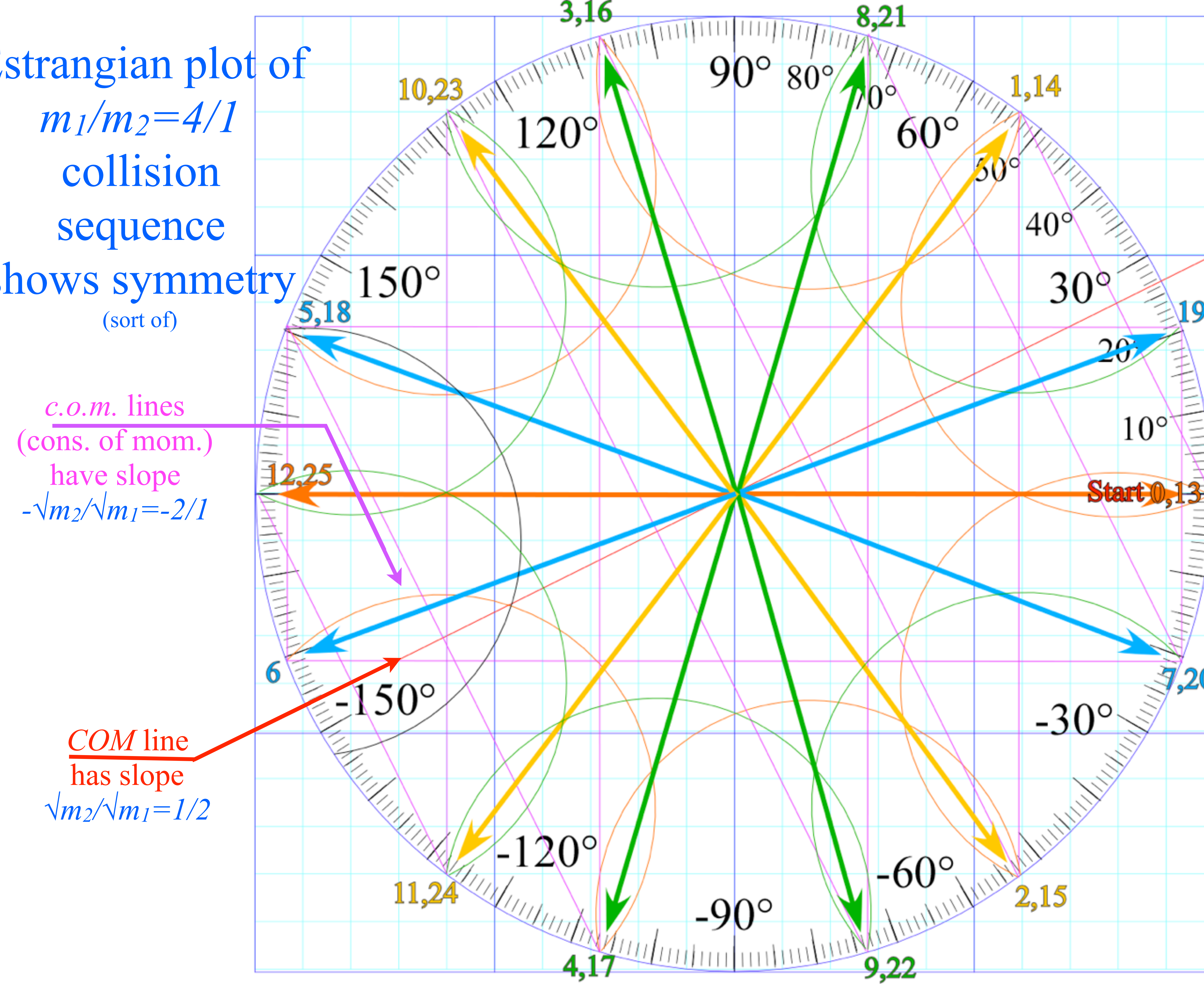
The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Start
Floor (y1 = 0)
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Start
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot



The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
v_2 axis
3,16
8,21
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
12
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical No-Man's Land
v_2 vs v_1 plot
y_2 vs y_1 plot

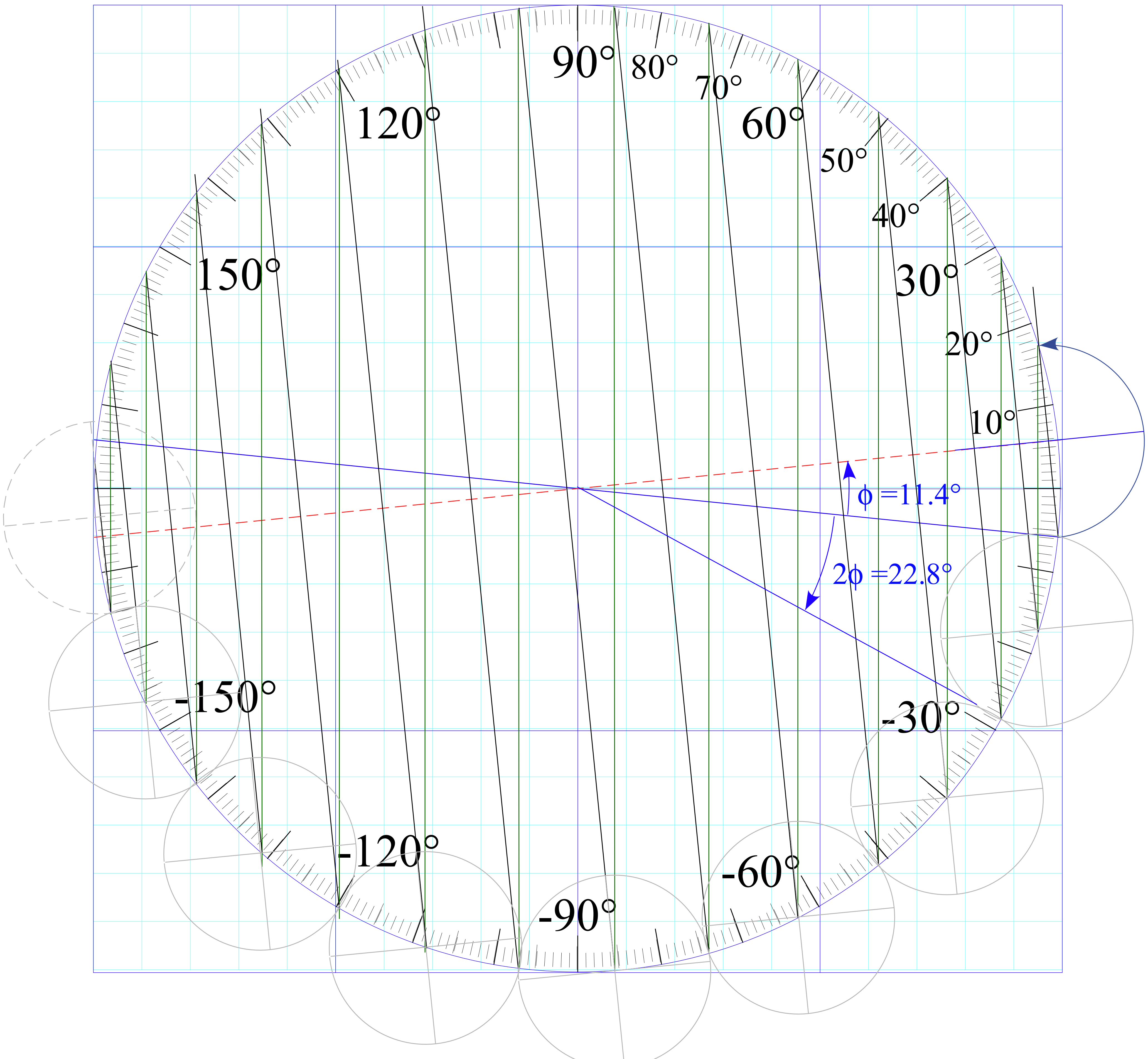
Estrangian plot of $m_1/m_2=4/1$ collision sequence shows symmetry

(sort of)

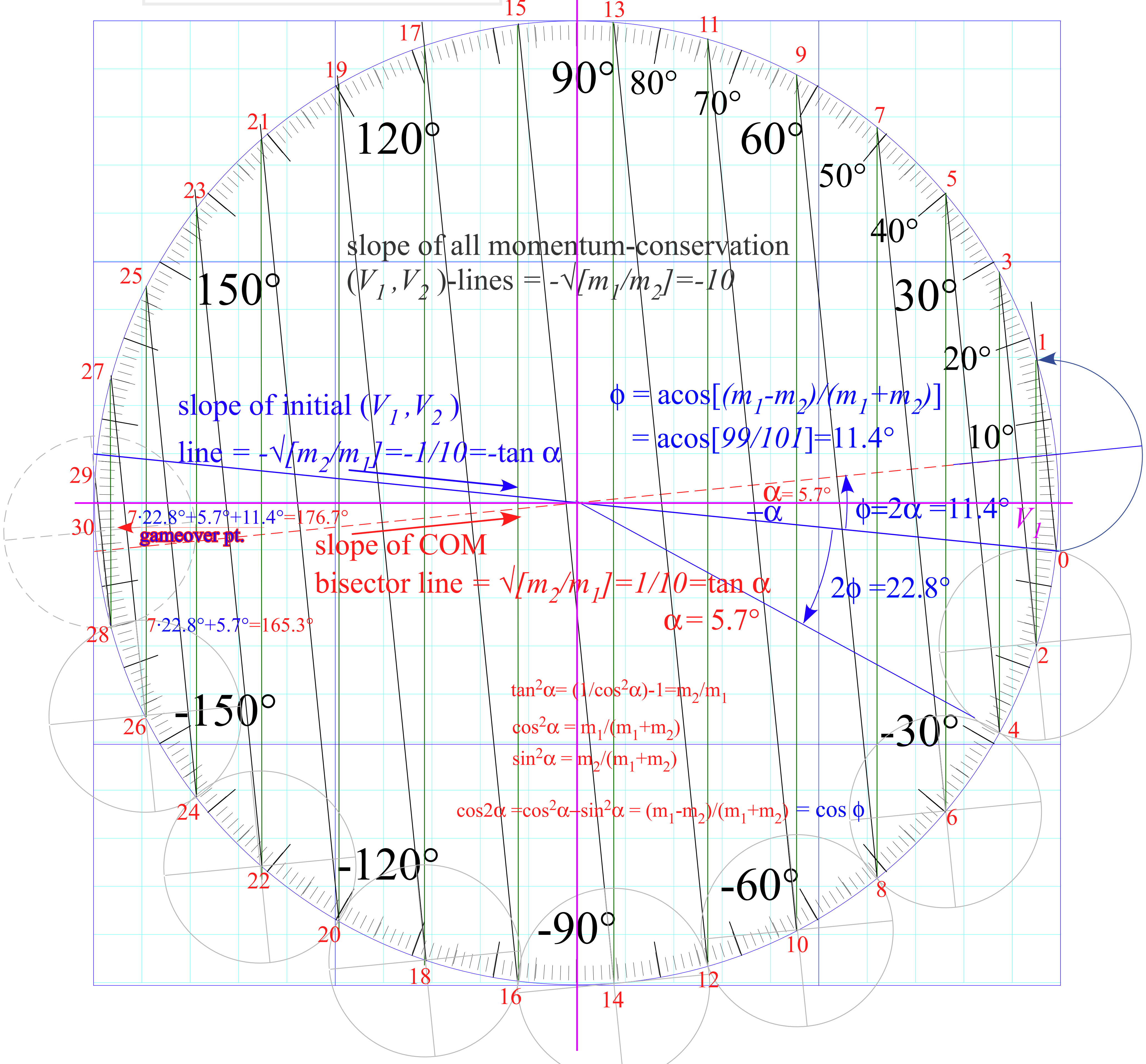


$$\phi = \mathrm{acos}[(m_1 - m_2)/(m_1 + m_2)] = \mathrm{acos}[99/101] = 11.4°$$

Collisions for mass ratio $m_1 : m_2 = 100:1$



Solutions to Exercise 1.5.2

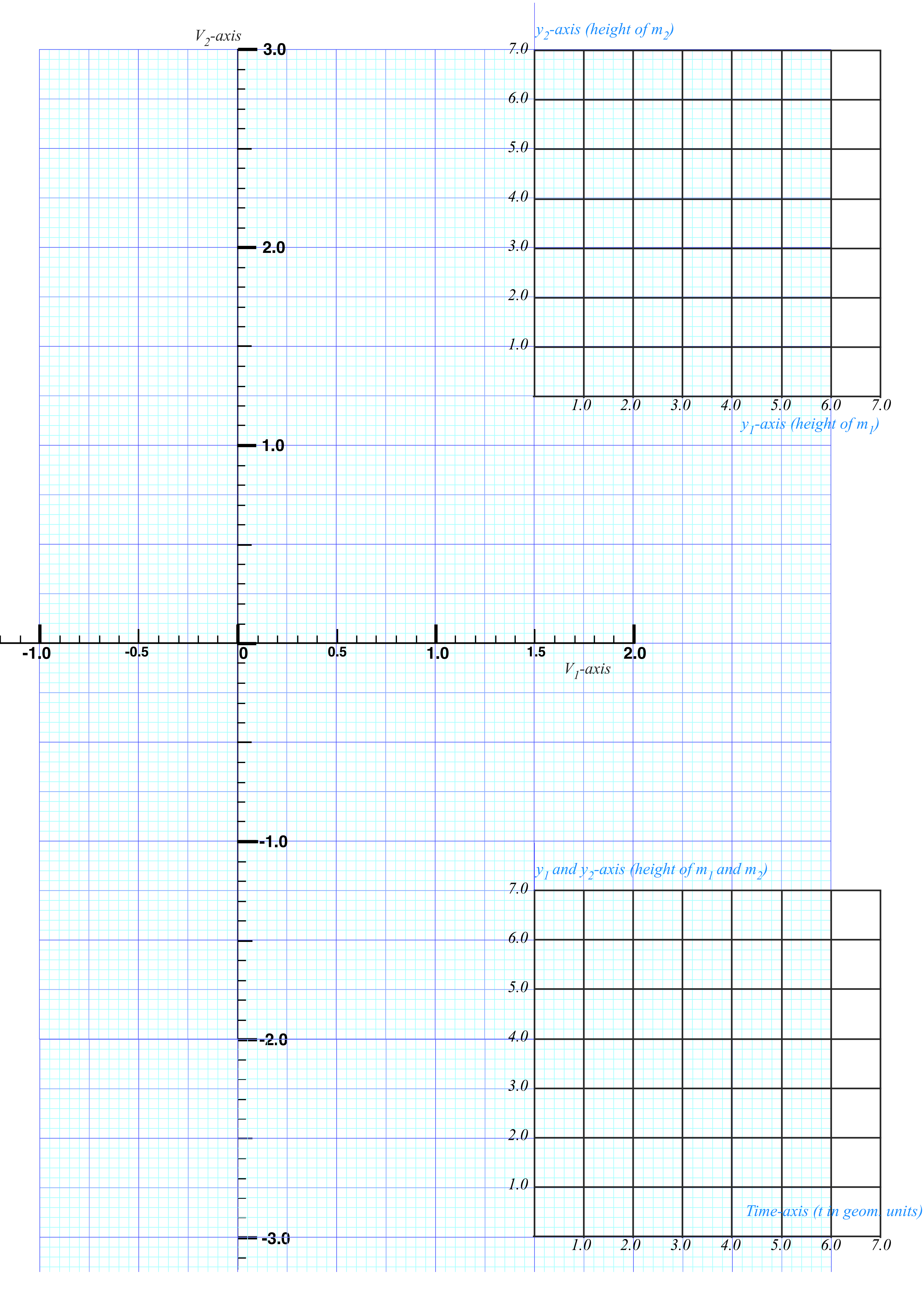


Collisions for mass ratio $m_1:m_2= 100:1$

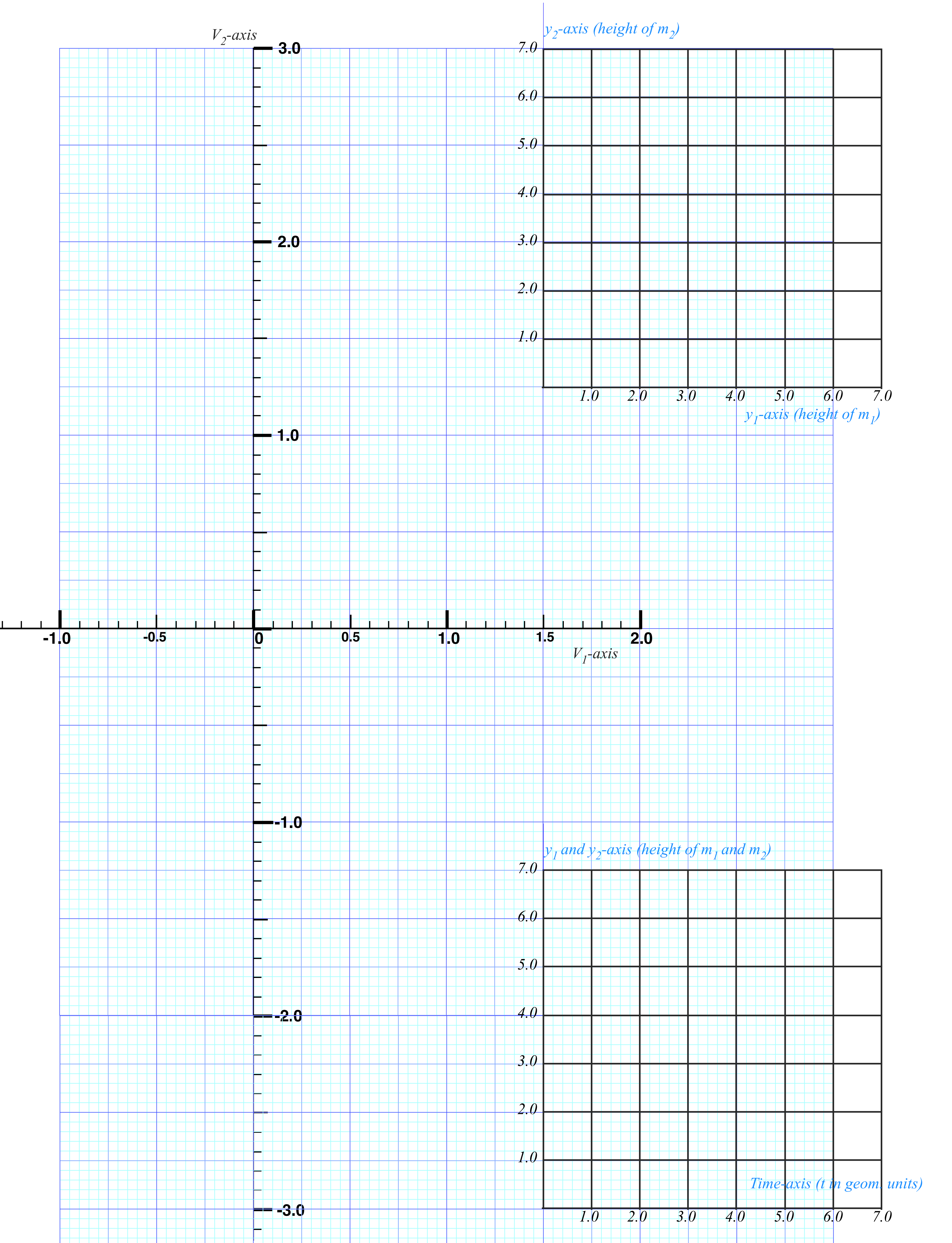
BounceIt Web Simulations
$m_1:m_2 = 100:1$ $(v_1, v_2)=(1, 0)$

V_2 vs V_1 Estrangian plot

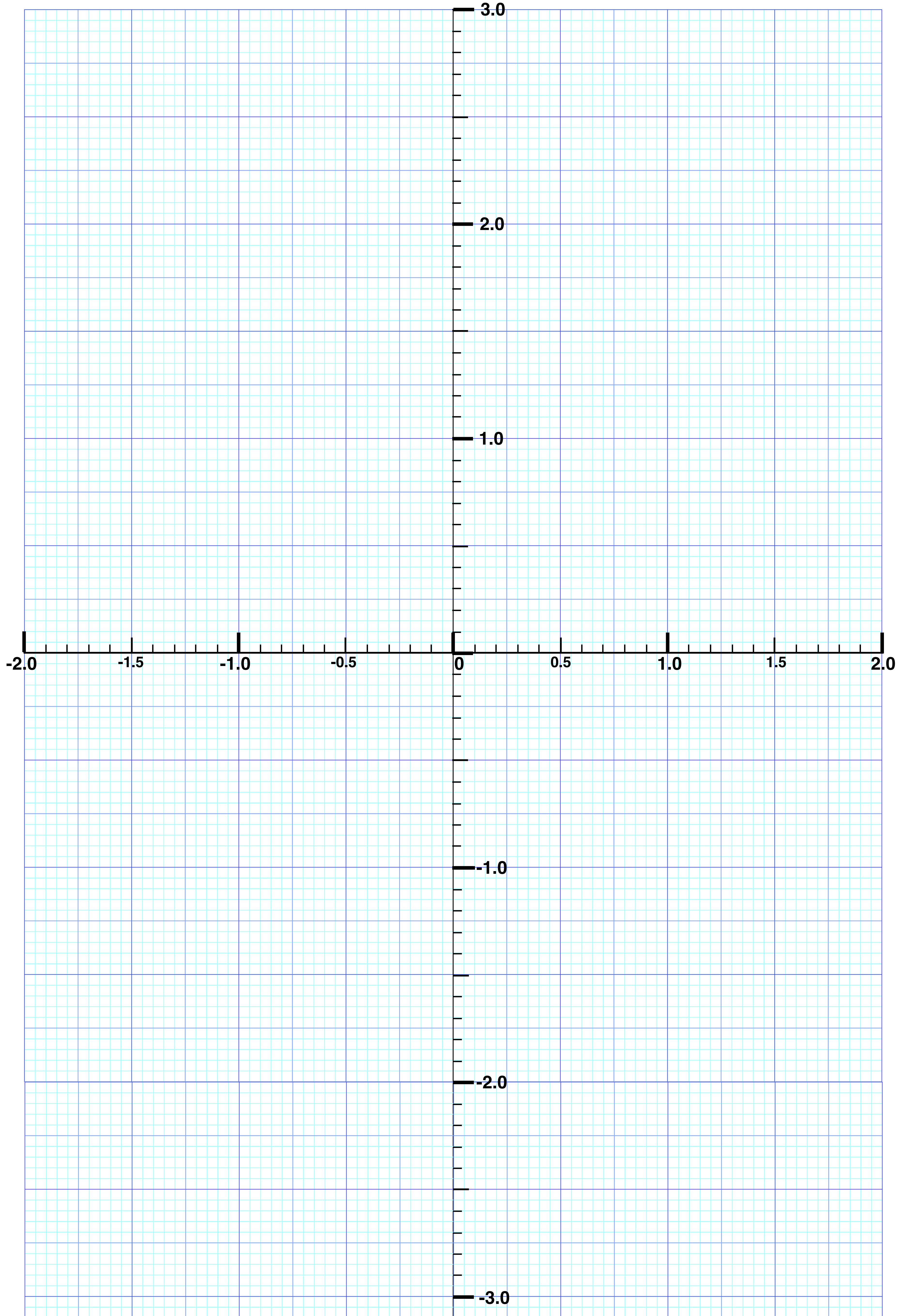
Supplementary: y_2 vs y_1 plot



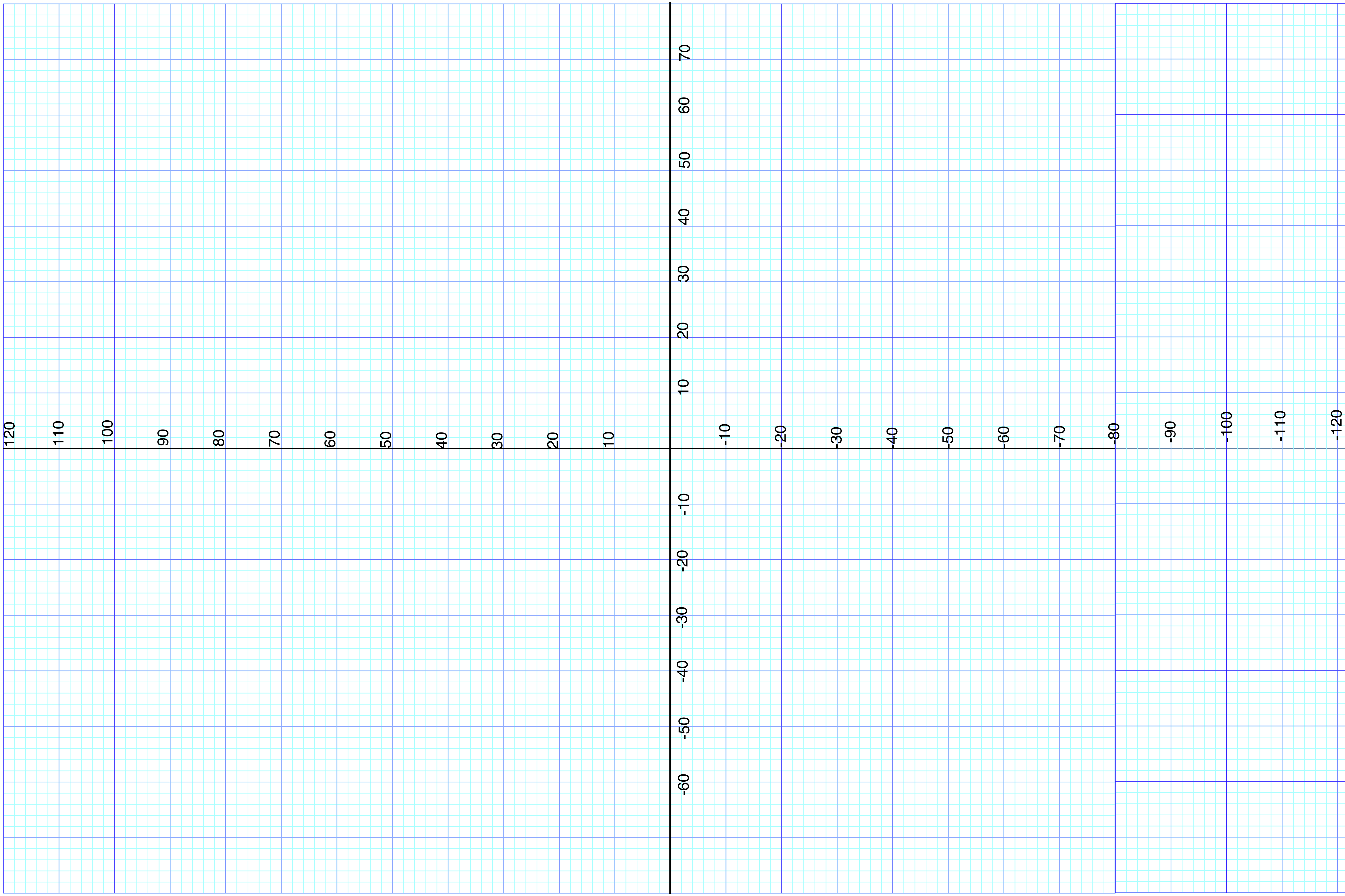
V_2-axis
V_1-axis
y_2-axis (height of m_2)
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)



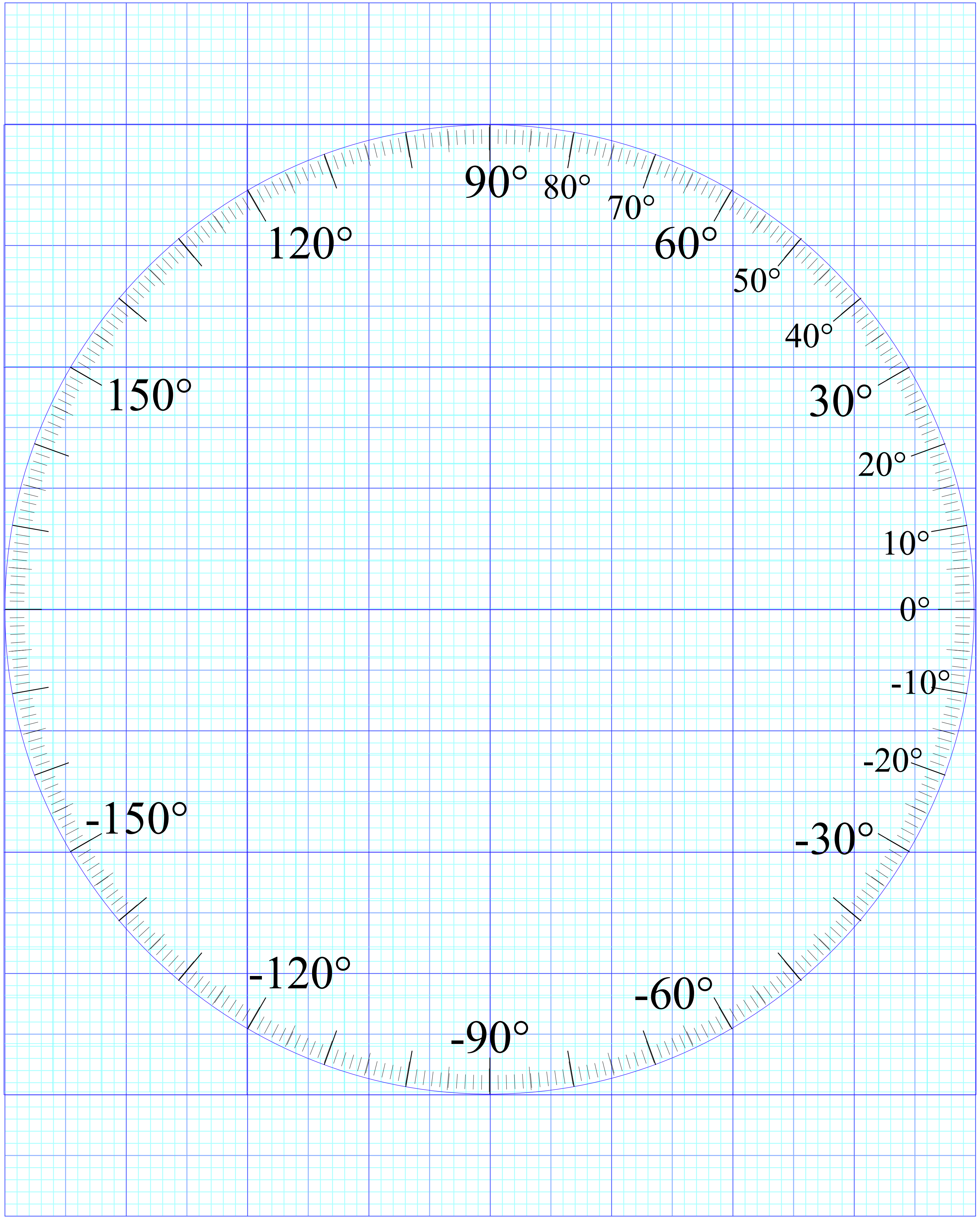
V_2-axis
3.0
2.0
1.0
-1.0
-2.0
-3.0
-1.0
-0.5
0
0.5
1.0
1.5
2.0
V_1-axis
y_2-axis (height of m_2)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1.0 2.0 3.0 4.0 5.0 6.0 7.0
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1.0 2.0 3.0 4.0 5.0 6.0 7.0
Time-axis (t in geom. units)



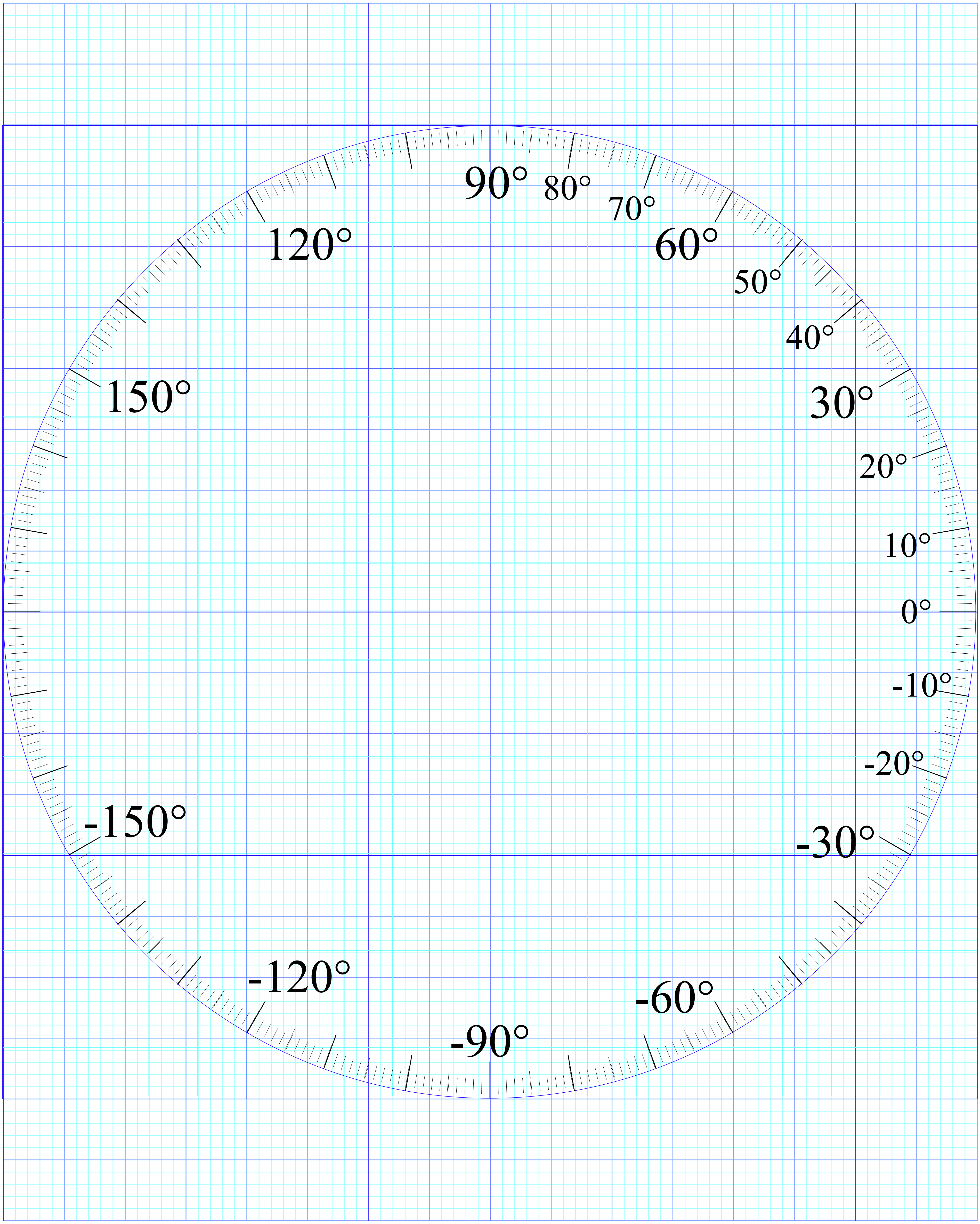
3.0
2.0
1.0
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0
1.5
2.0
-1.0
-2.0
-3.0



120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60



90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
-10°
-20°
-30°
-60°
-90°
-120°
-150°
150°
120°



90°
80°
70°
120°
60°
50°
40°
150°
30°
20°
10°
0°
-10°
-20°
-150°
-30°
-120°
-60°
-90°