

Lecture 2

Thur 8.24.2017

Analysis of 1D 2-Body Collisions

(Ch. 2 to Ch. 4 of Unit 1)

Review: COM Momentum line, elastic vs inelastic kinetic energy ellipse geometry

The X2 Superball pen launcher

Perfectly elastic “ka-bong” velocity amplification effects (Faux-Flubber)

Geometry of X2 launcher bouncing in box (gravity-free)

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R**= **C**•**M***

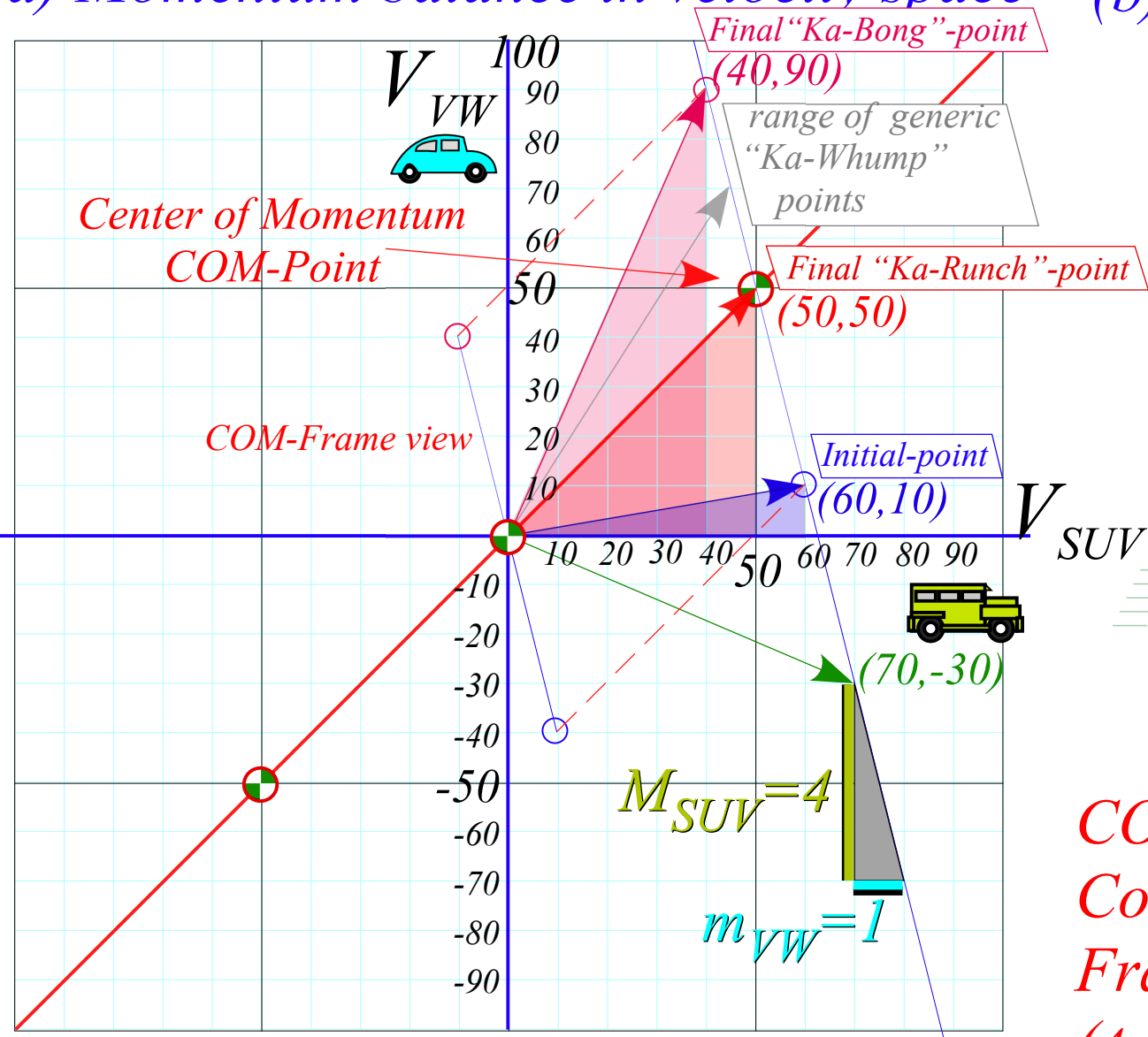
Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics in Ch. 12

Review of Momentum line and COM point geometry

(a) Momentum balance in velocity space



(b) Momentum balance in coordinate space

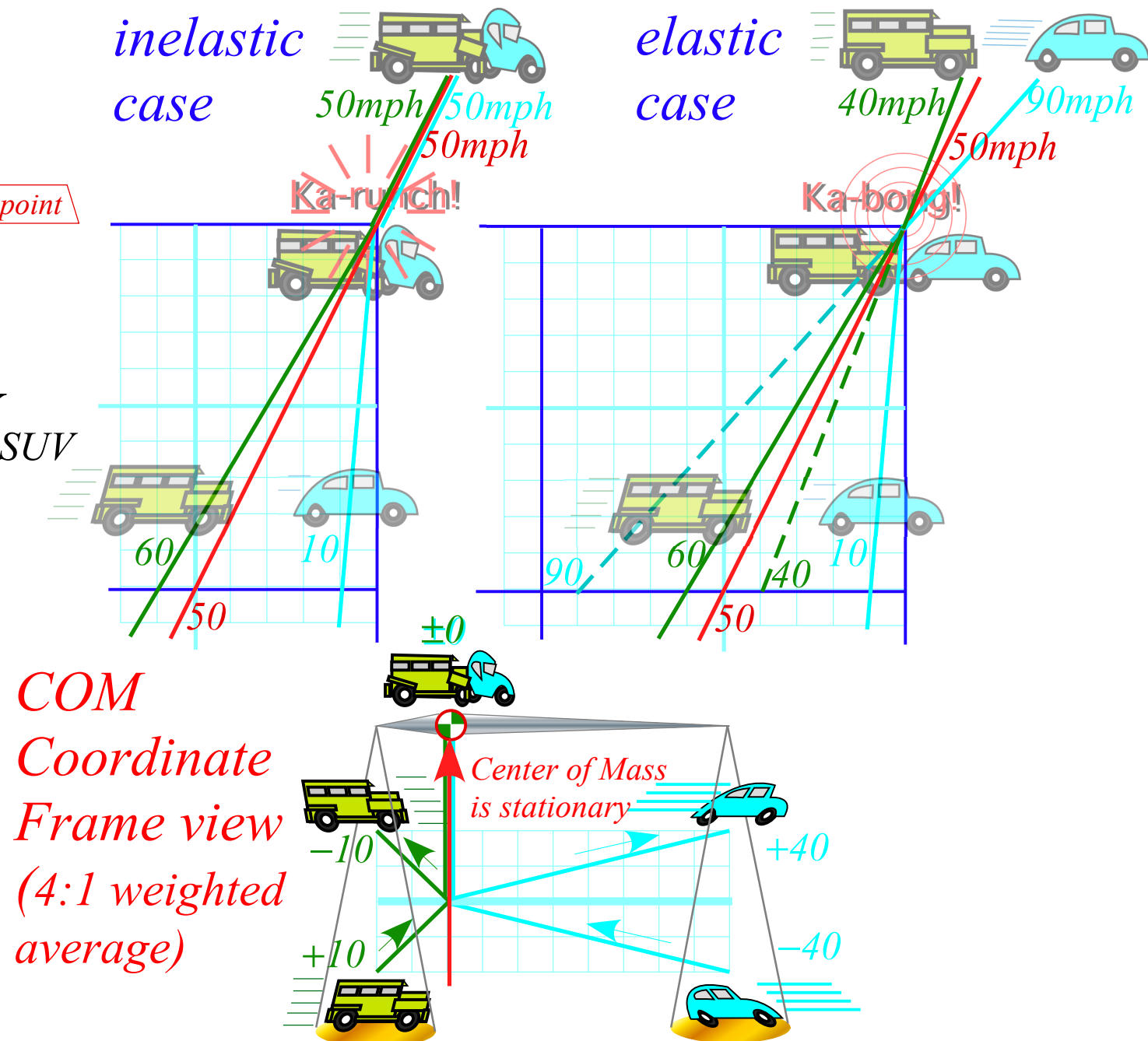


Fig. 1.2.6ab
(Unit 1)

Review of Kinetic Energy ellipse geometry

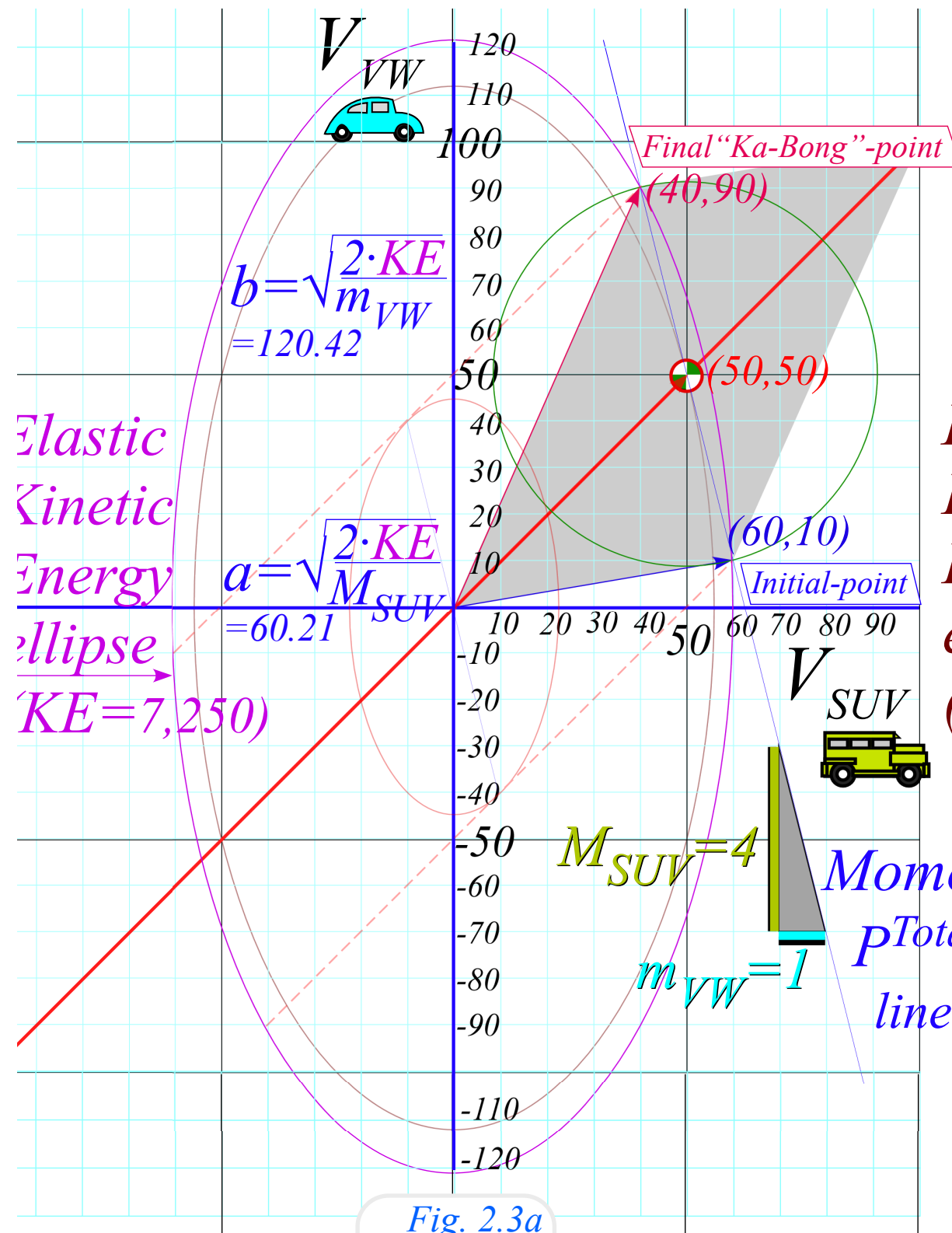


Fig. 2.3a
(Unit 1)

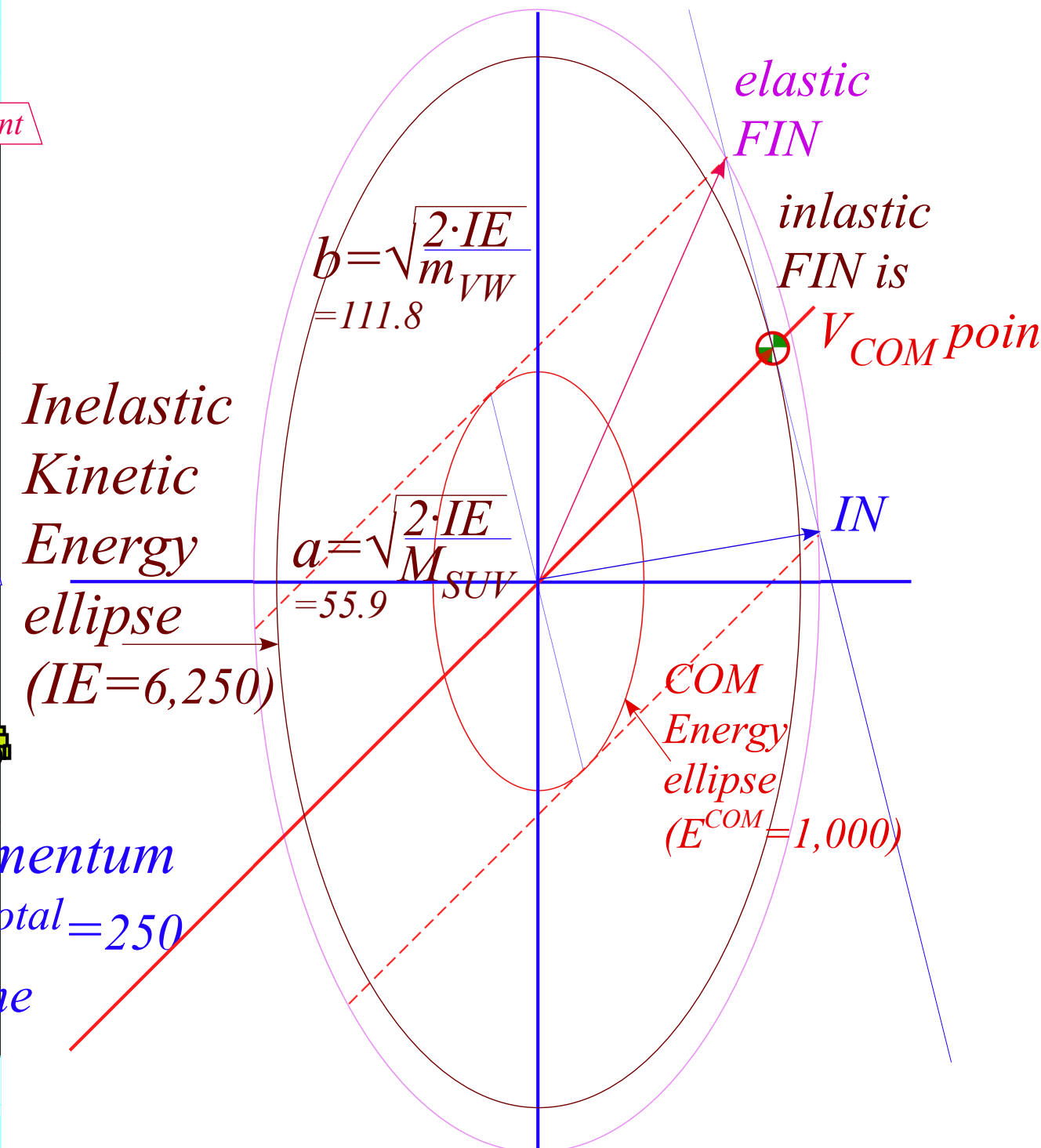
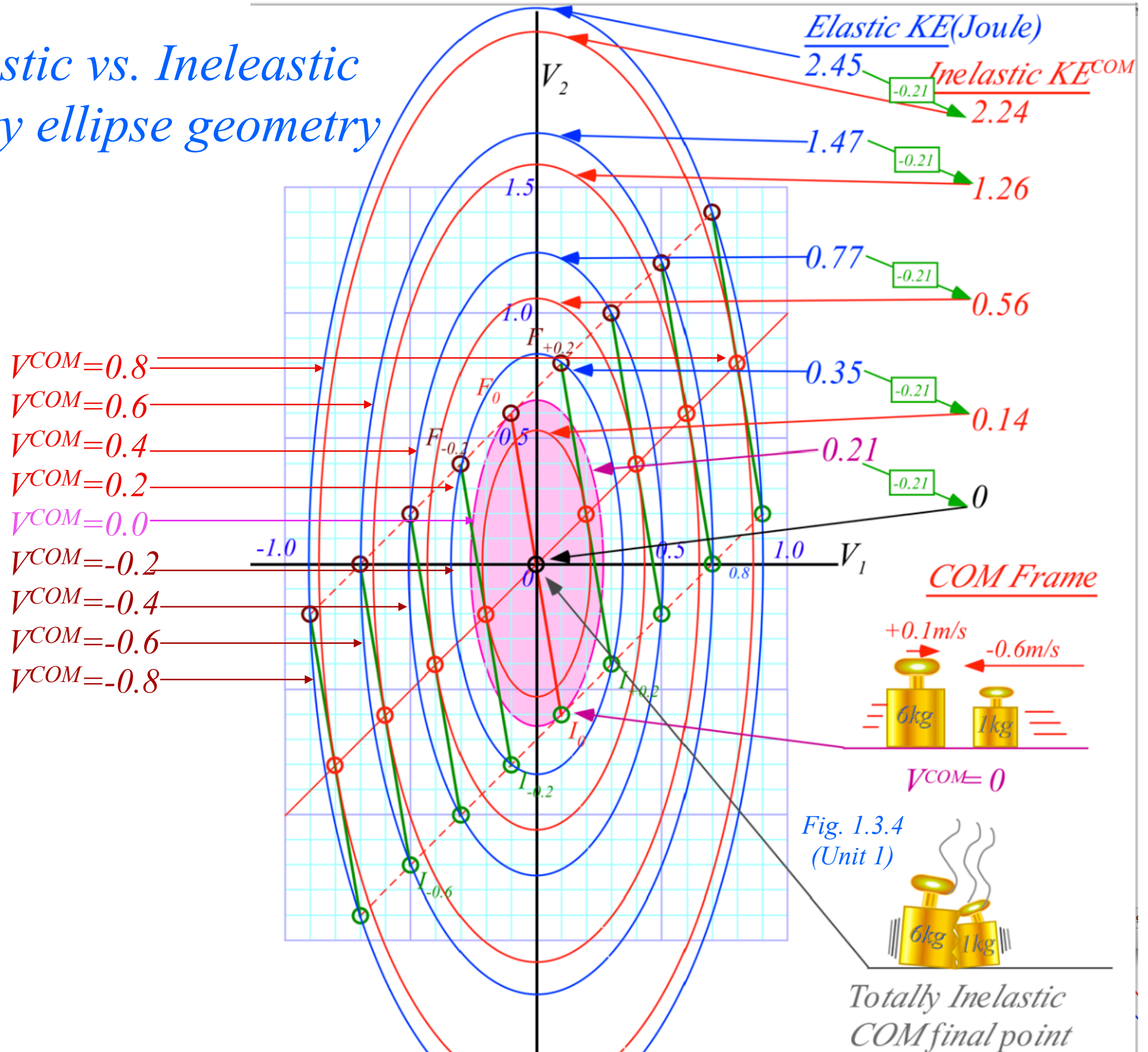


Fig. 2.3b
(Unit 1)

*Review of Elastic vs. Inelastic
Kinetic Energy ellipse geometry*

*Same collision
viewed from nine
different COM
reference frames*



Geometry of X2 launcher bouncing in box (gravity-free)

 *Independent Bounce Model (IBM)*

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

Integration of (V_1, V_2) data to space-space plots (y_1, y_2)

The X-2 Pen launcher and Superball Collision Simulator*

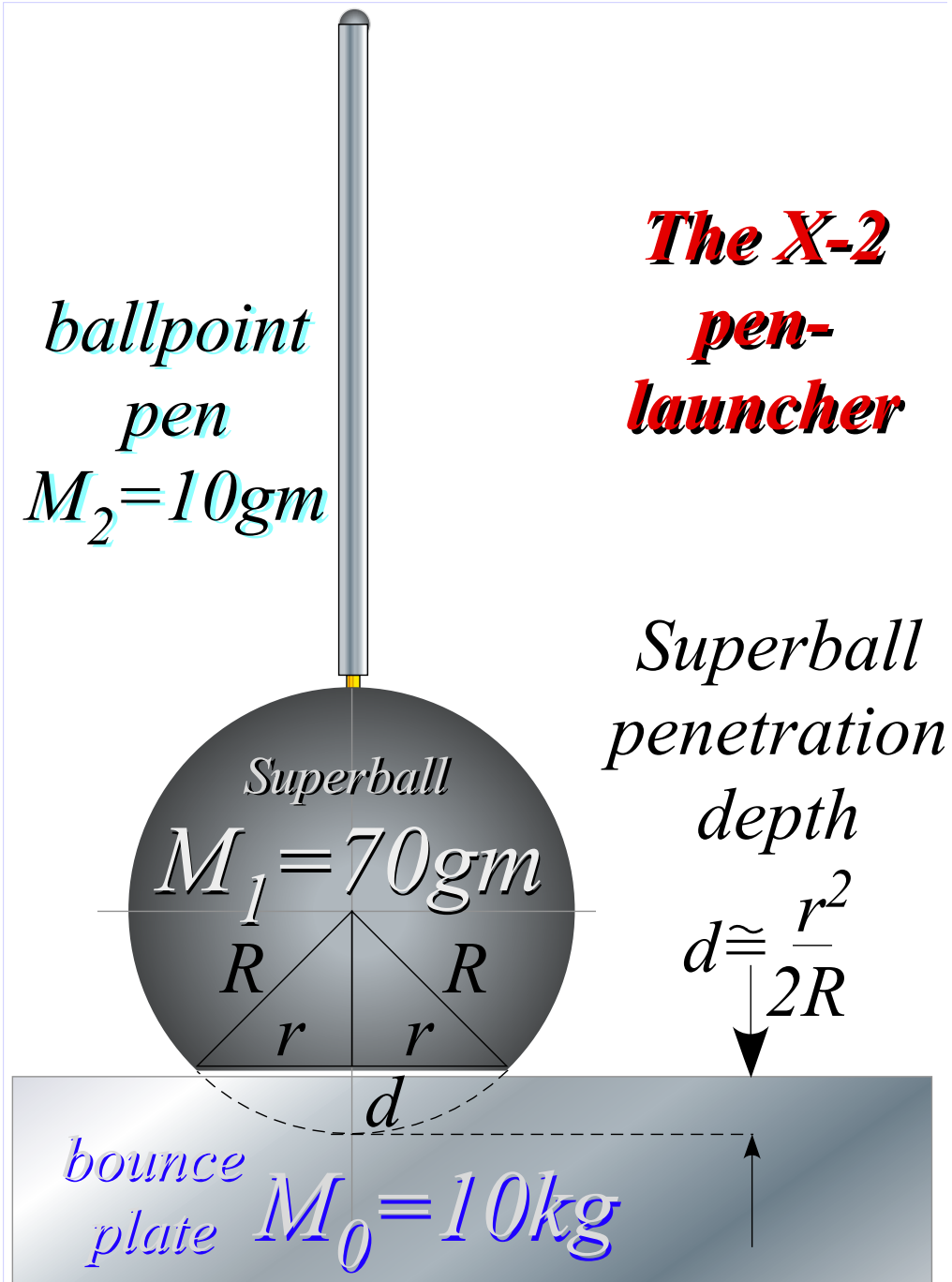
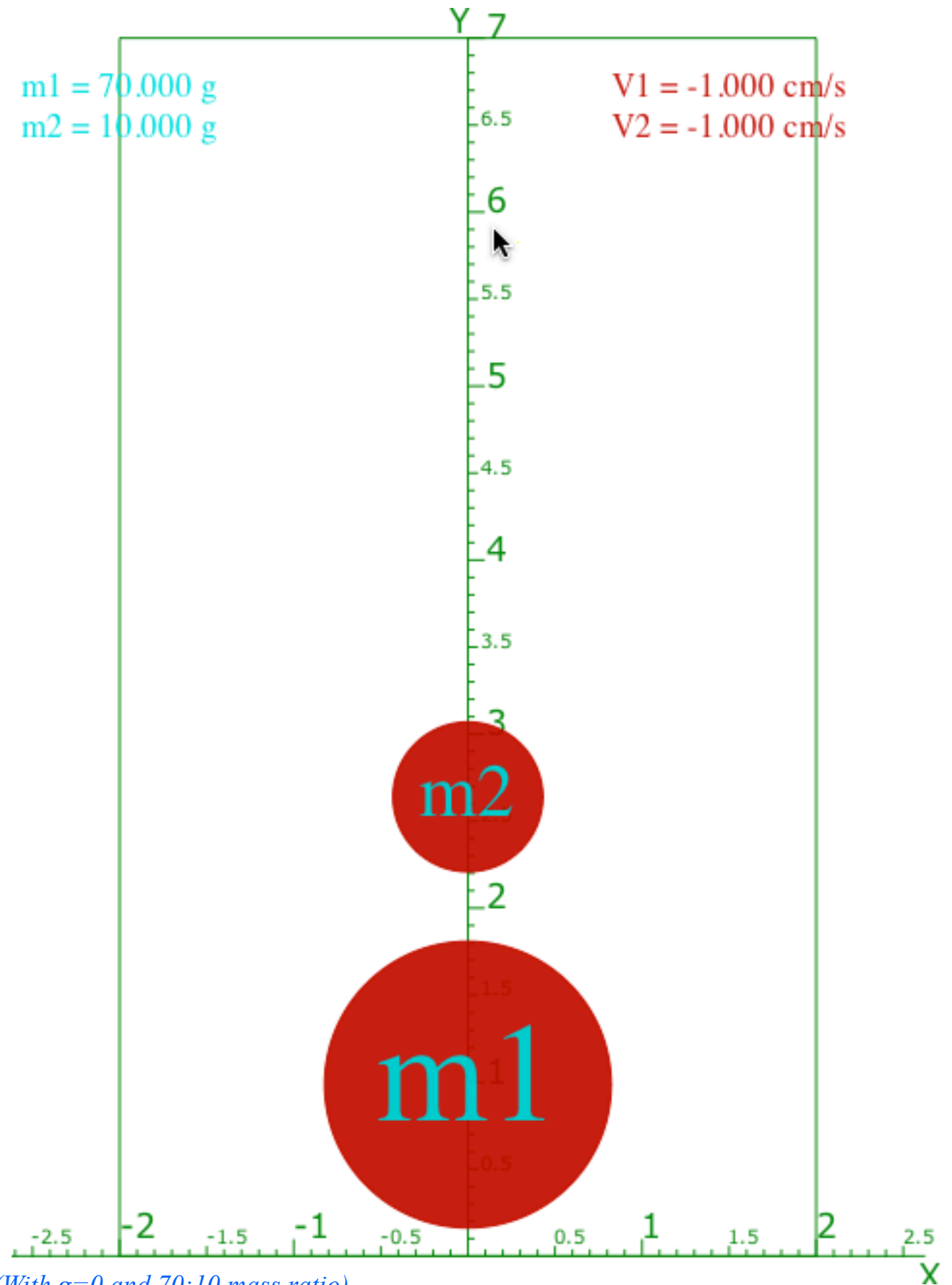


Fig. 3.1
(Unit 1)



(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

The X-2 Pen launcher and Superball Collision Simulator*

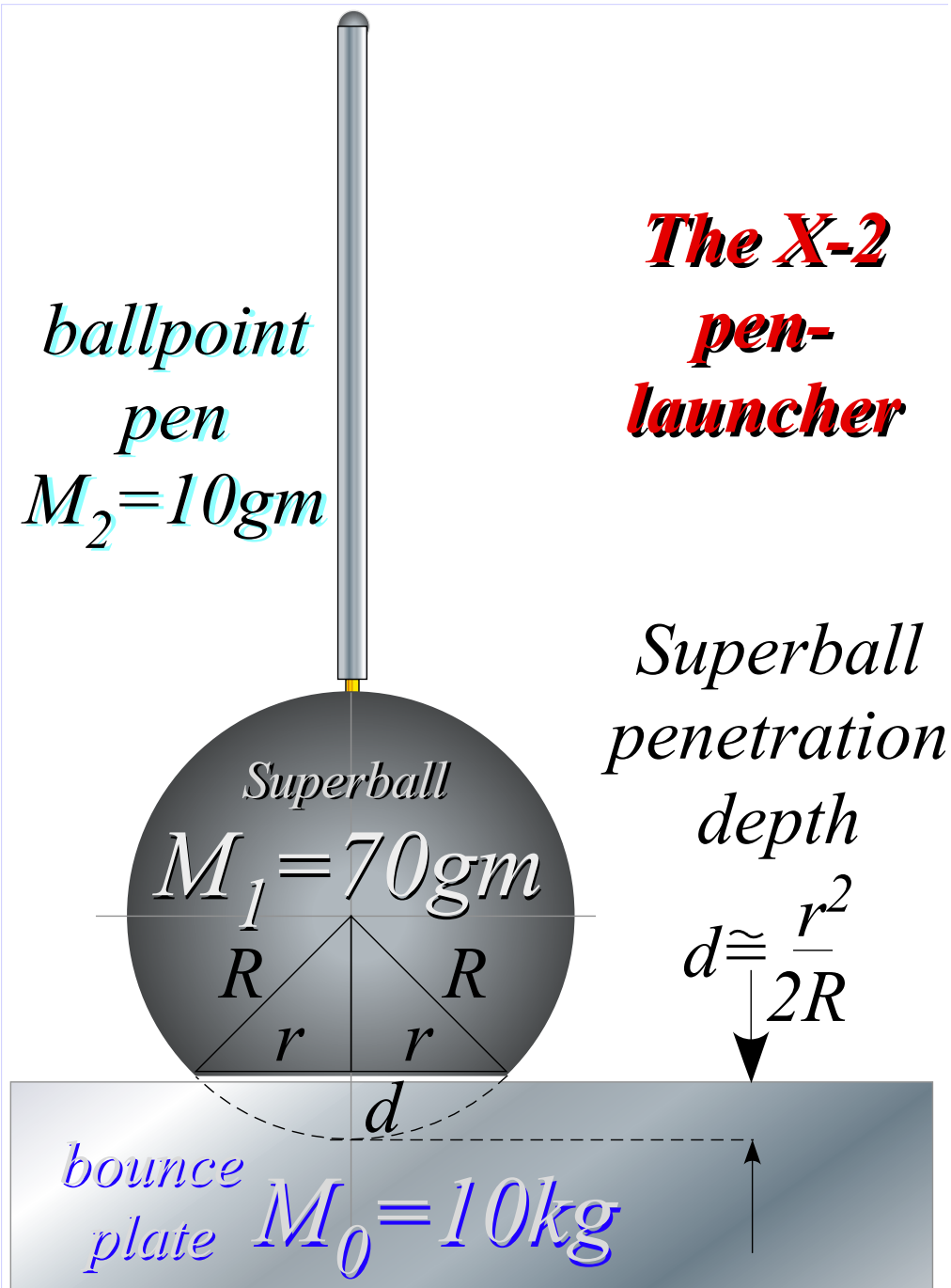


Fig. 3.1
(Unit 1)

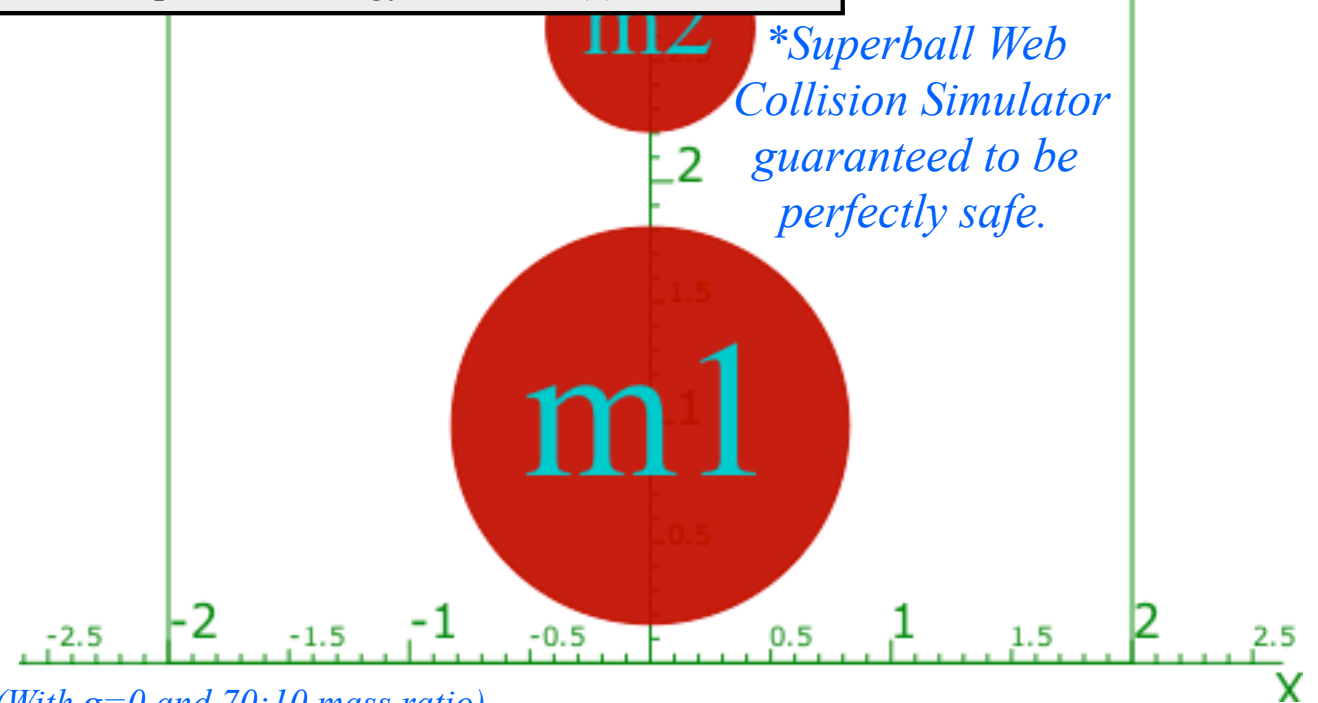
Caution: Product Liability Disclaimer

This ballpoint pen could be hazardous to your health! The experiments which are the subject of this discussion are both spectacular and potentially dangerous, and care to protect one's eyes should be taken. The simplest experiment involves sticking a ball point pen into a superball or other hard rubber ball and dropping the two onto a hard floor. If done correctly the pen will eject the ball with such force it may stick in the ceiling of the room. Obviously you want to be careful with this weapon. And, this goes doubly and triply for the more advanced models that may be developed in the course of studying this stuff. It is recommended that experimenters wear safety glasses when doing these experiments with pens. (We could just say don't use pens, but that's an easy way to do this experiment and probably the way most people will go about it.) Some of the tangential experiments associated with this development are less hazardous. To measure the potential force function of a ball one may simply paint the ball and measure the spot size as a function of drop height h .

The saggital approximation $d = r^2 / 2R$ allows one to quickly convert spot radius r to penetration depth x for a superball of radius R as shown in the figure. Equating this to Mgh gives the ball potential energy function $V(x)$.

$$V_1 = -1.000 \text{ cm/s}$$

$$V_2 = -1.000 \text{ cm/s}$$



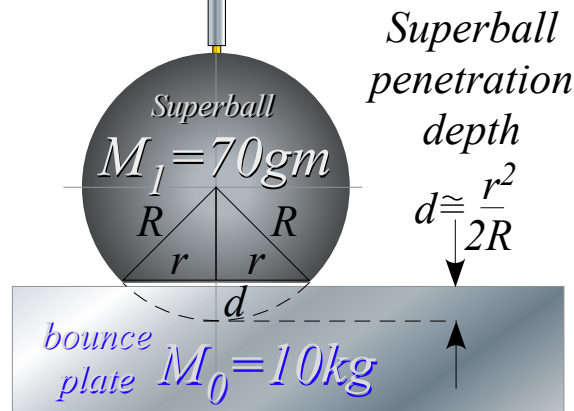
(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

ballpoint
pen
 $M_2=10gm$

**The X-2
pen-
launcher**



(a)
Bang-1
 (01)
1st bang:
mass (M_0) vs. mass (M_1)

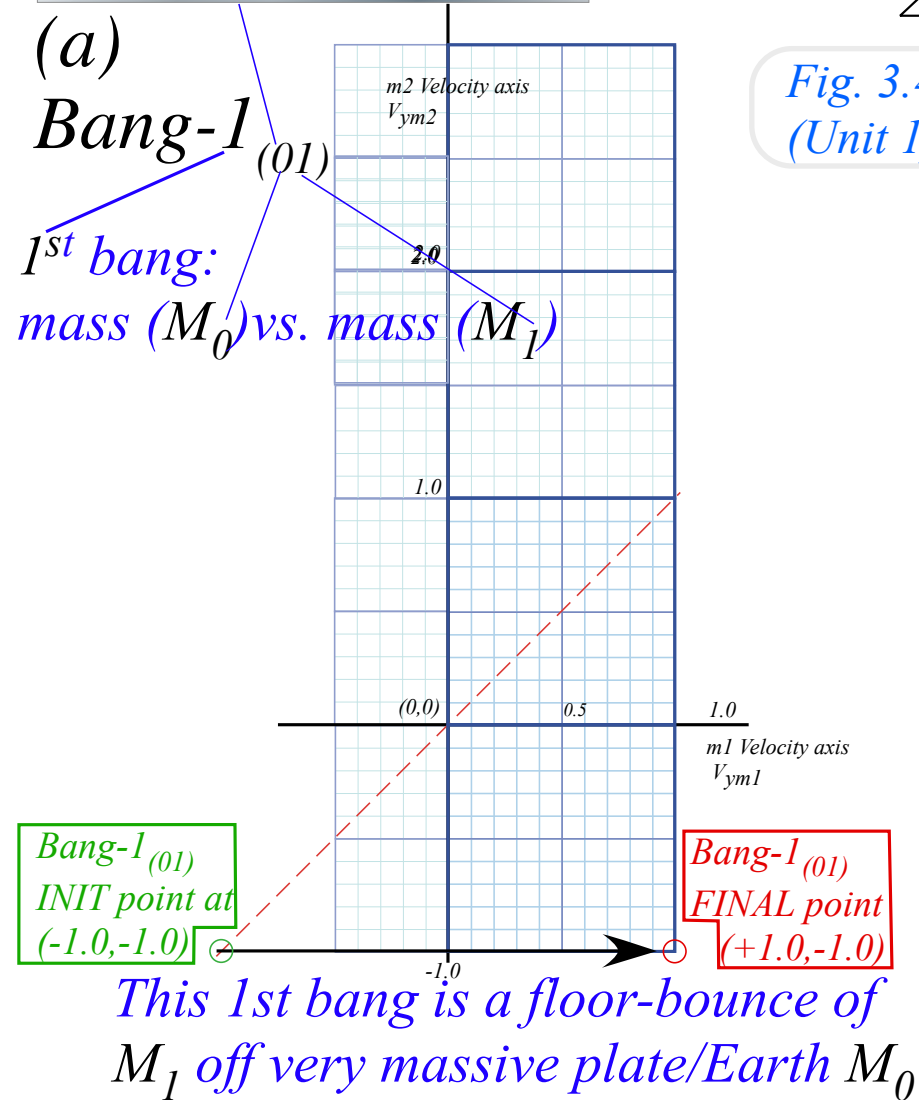


Fig. 3.3
(Unit 1)

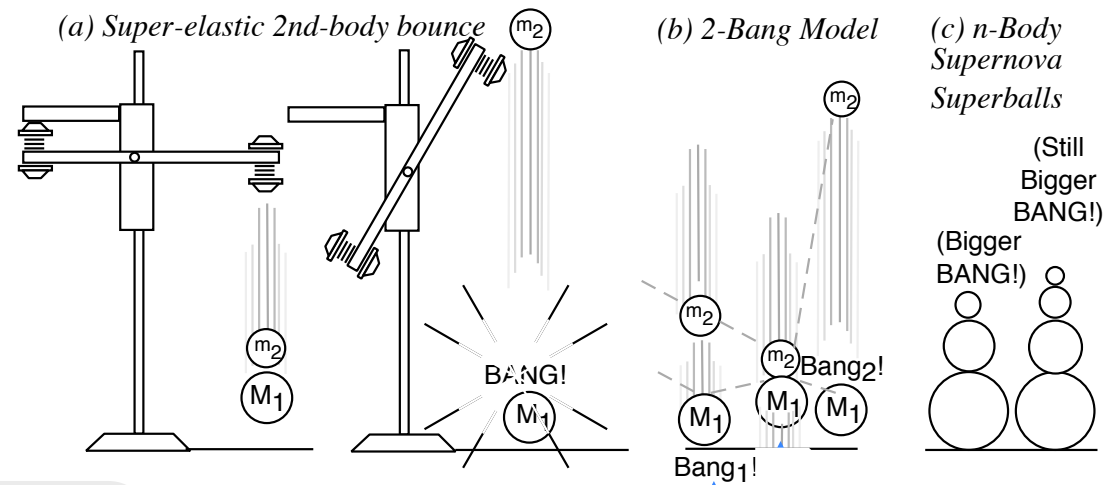


Fig. 3.4
(Unit 1)

1st bang:
 M_1 off floor

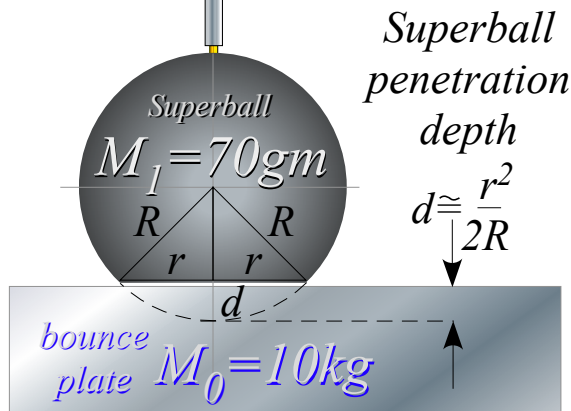
(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html>

ballpoint
pen
 $M_2=10\text{gm}$

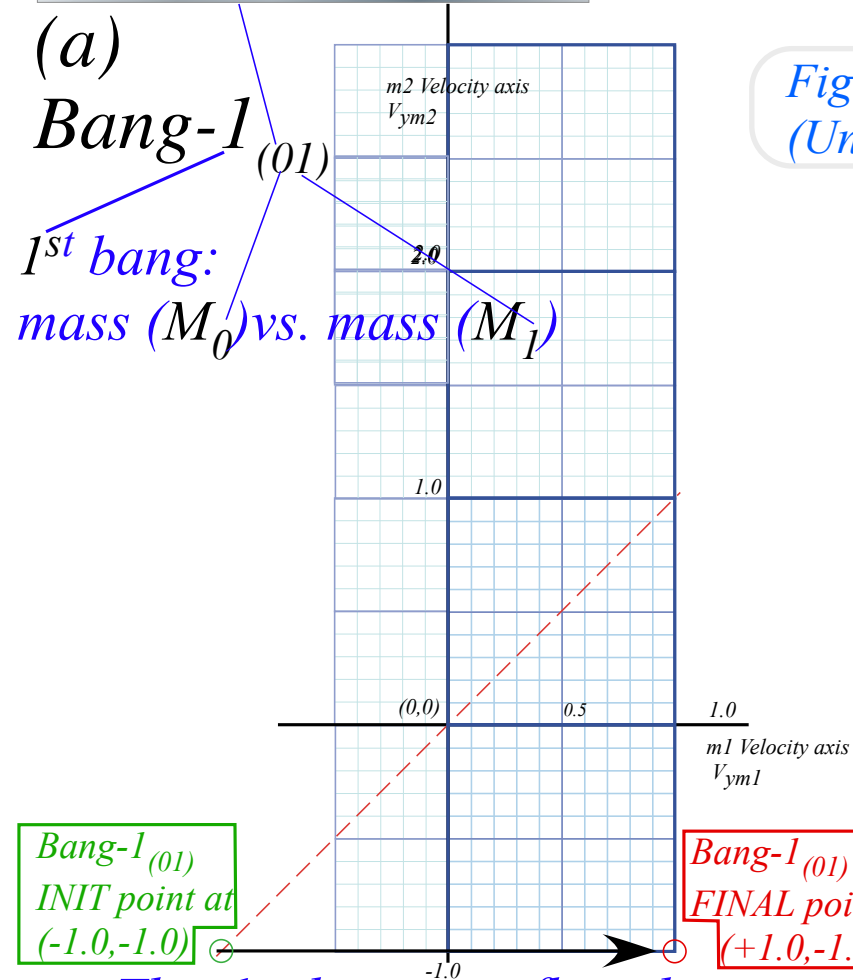
**The X-2
pen-
launcher**



(a)

Bang-1

1st bang:
mass (M_0) vs. mass (M_1)



This 1st bang is a floor-bounce of
 M_1 off very massive plate/Earth M_0

Fig. 3.3
(Unit 1)

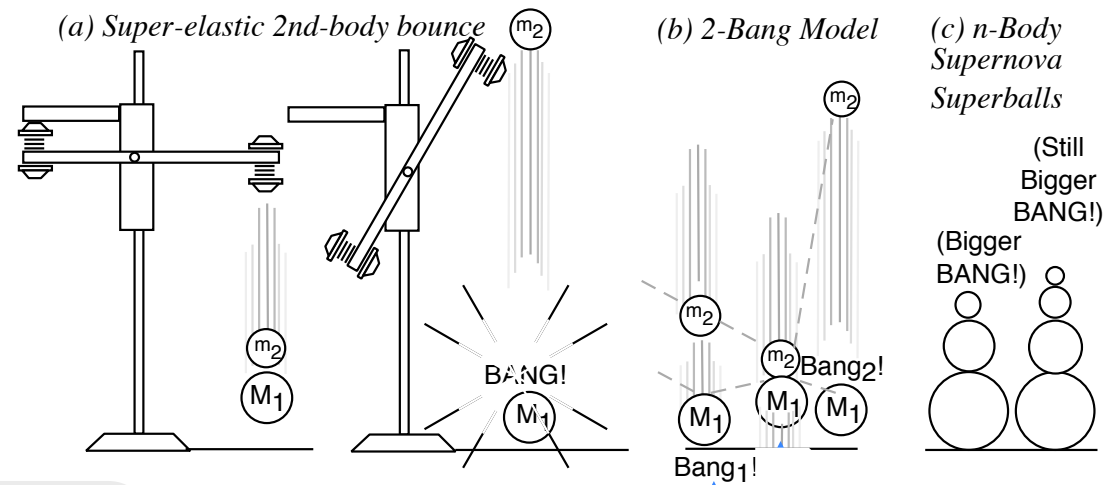
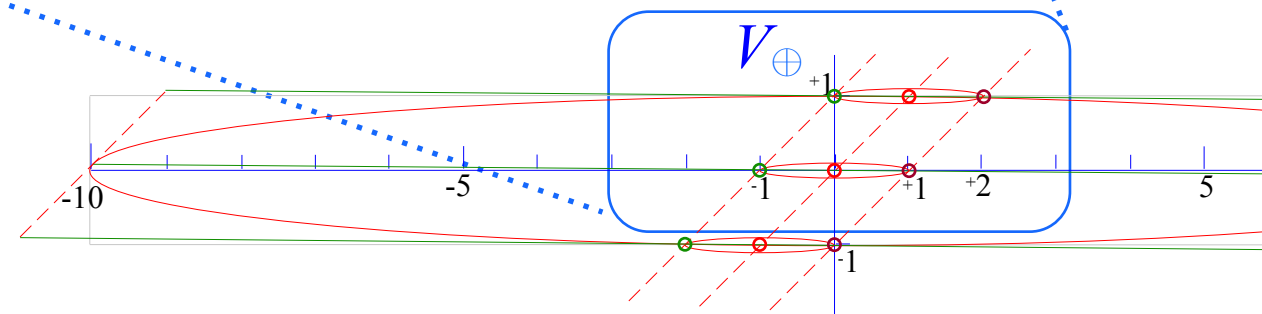
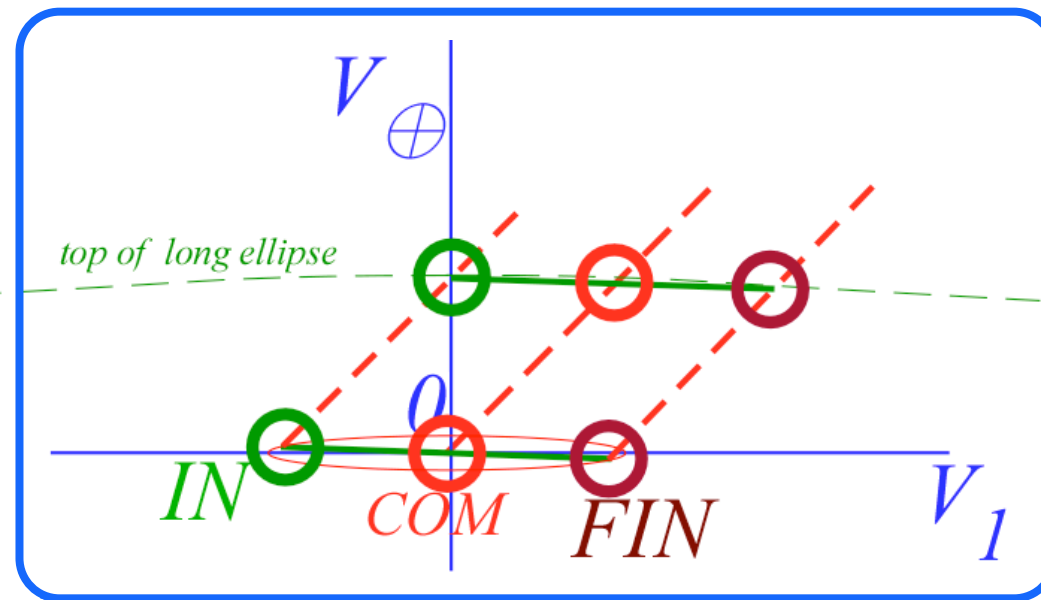


Fig. 3.4
(Unit 1)

1st bang:
 M_1 off floor



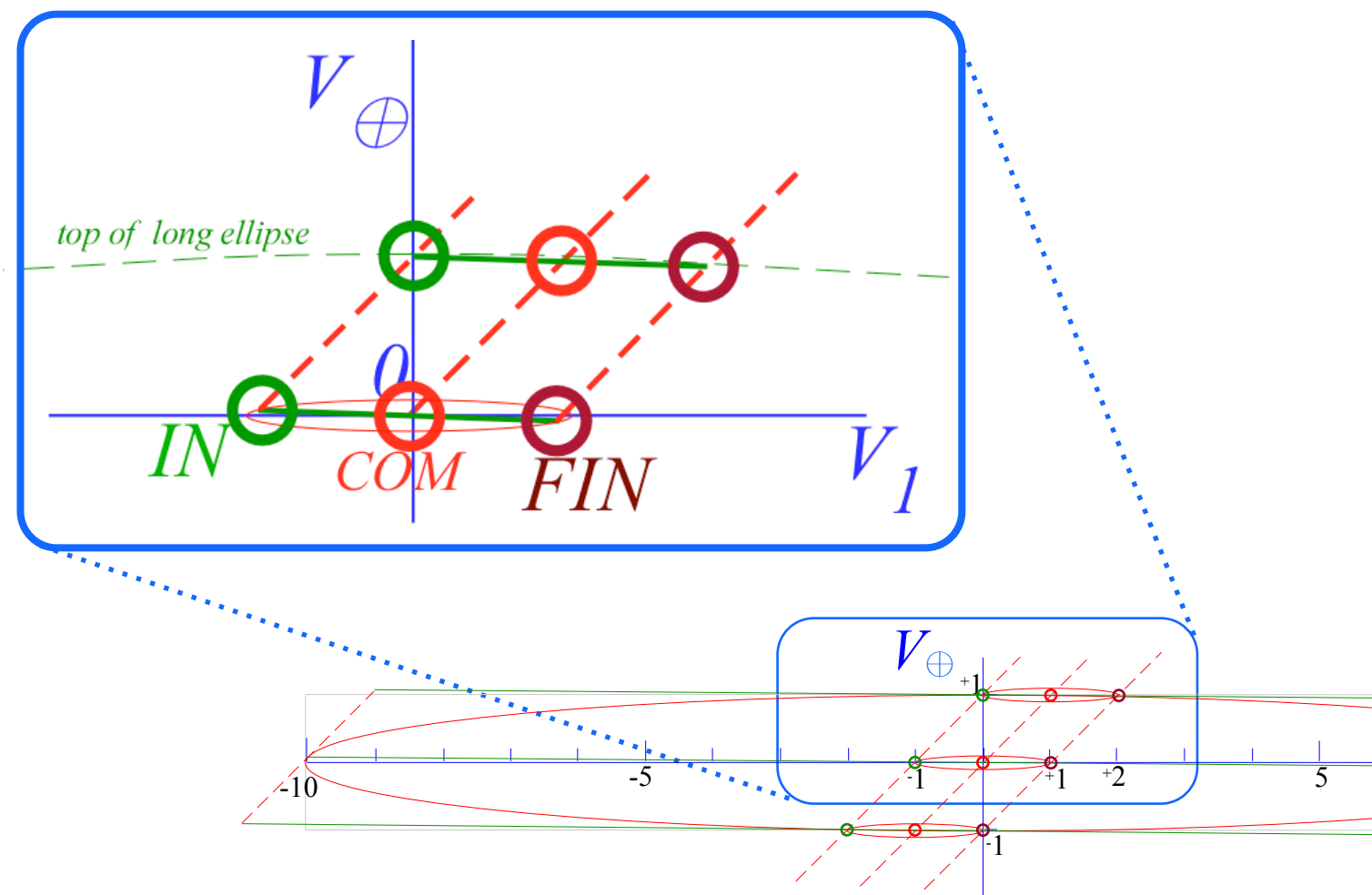
(With $g=0$ and 70:10 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)

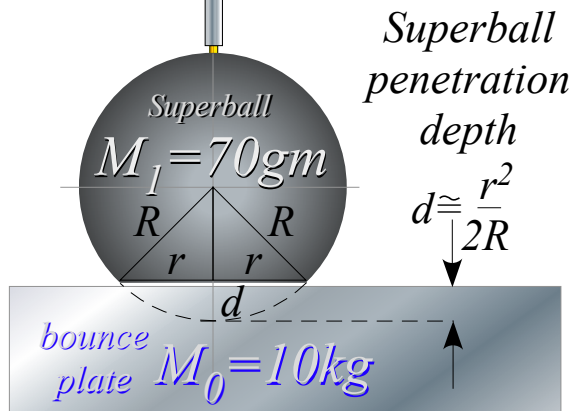
<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=6300>

(a) 1st bang of M_1 off
 floor plate $M_{\oplus} = 100 M_1$ along
 (V_1, V_{\oplus}) -momentum line of slope
 $-M_1/M_{\oplus} = -1/100$
 from IN-end to COM to FIN-end
 of $(a/b = \sqrt{M_{\oplus}}/\sqrt{M_1} = 10)$ ellipse



ballpoint
pen
 $M_2=10\text{gm}$

**The X-2
pen-
launcher**



(a)

Bang-1
(01)

1st bang:
mass (M_0) vs. mass (M_1)

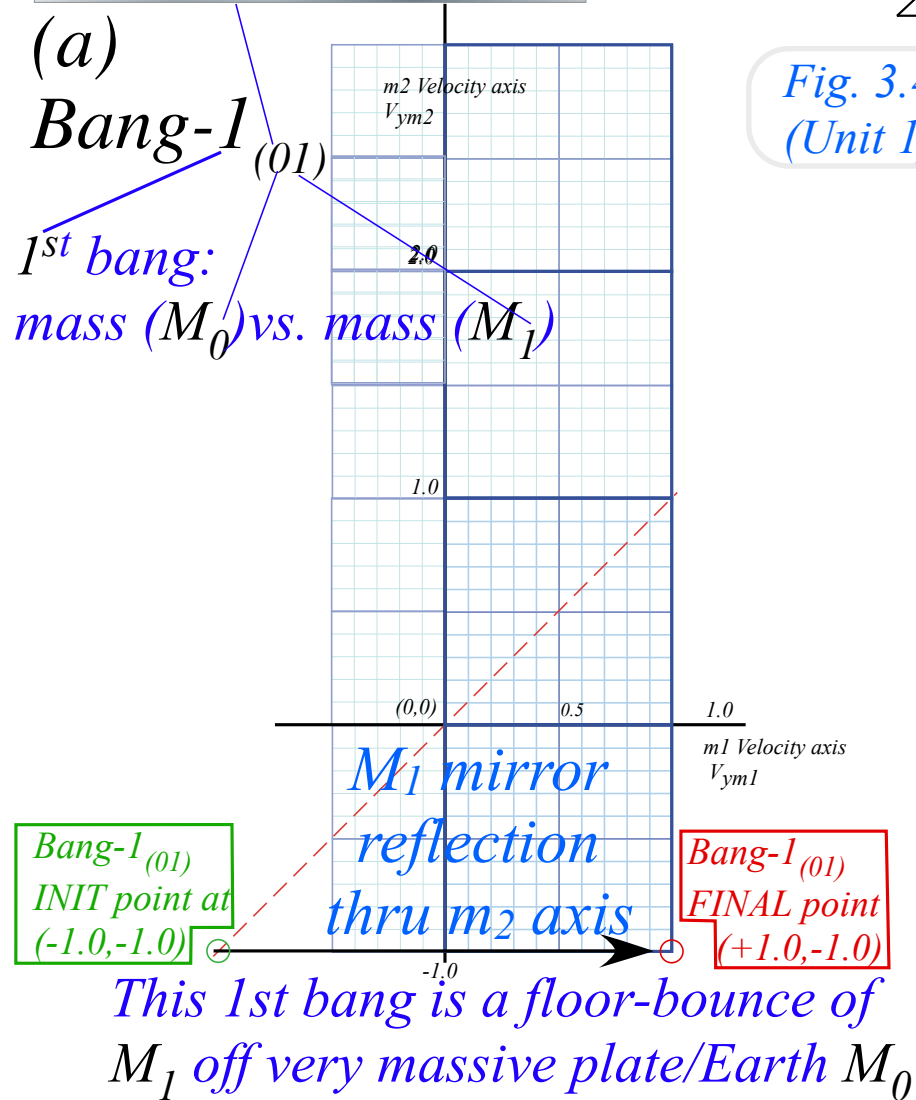


Fig. 3.3
(Unit 1)

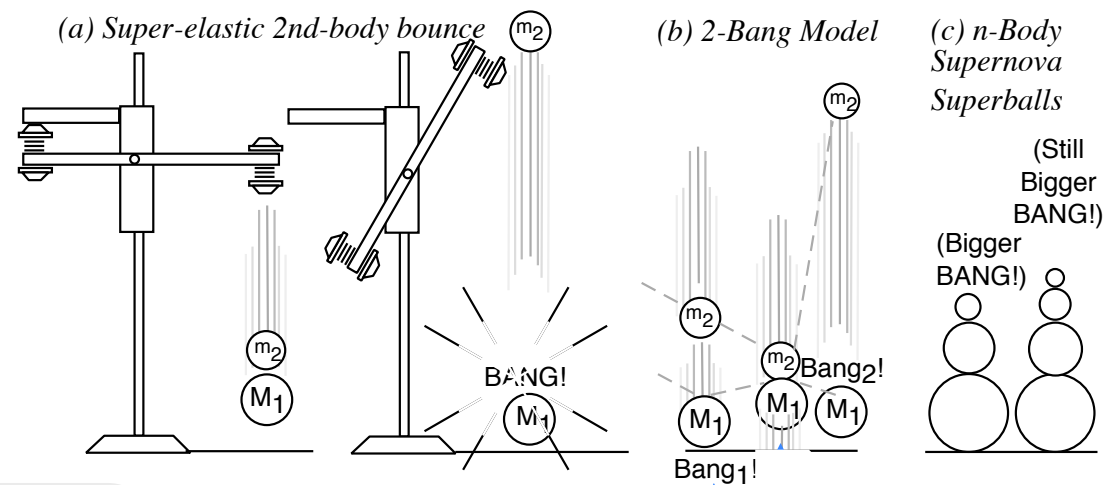


Fig. 3.4
(Unit 1)

1st bang:
 M_1 off floor

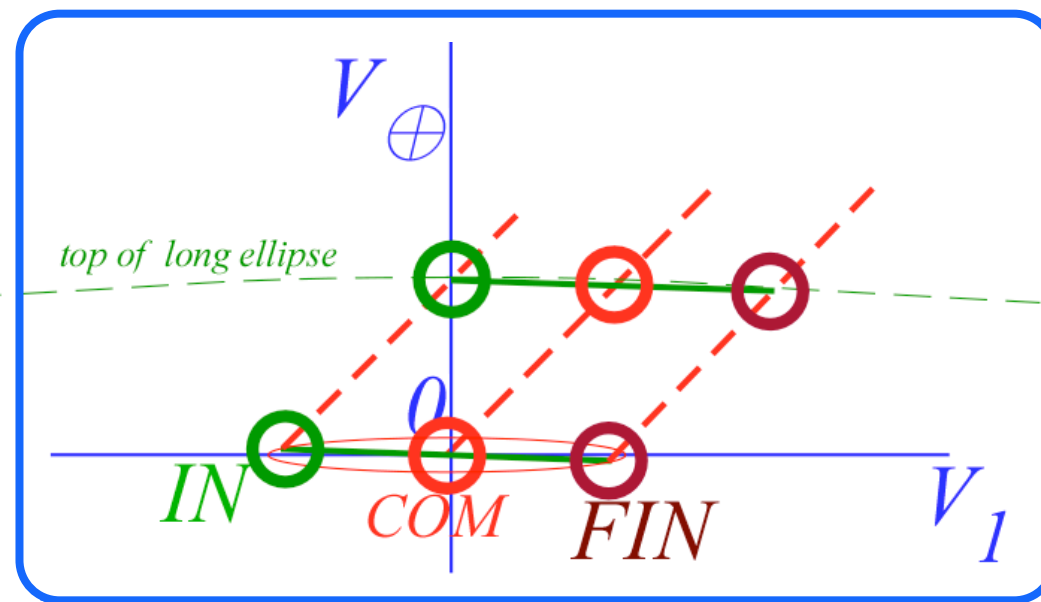
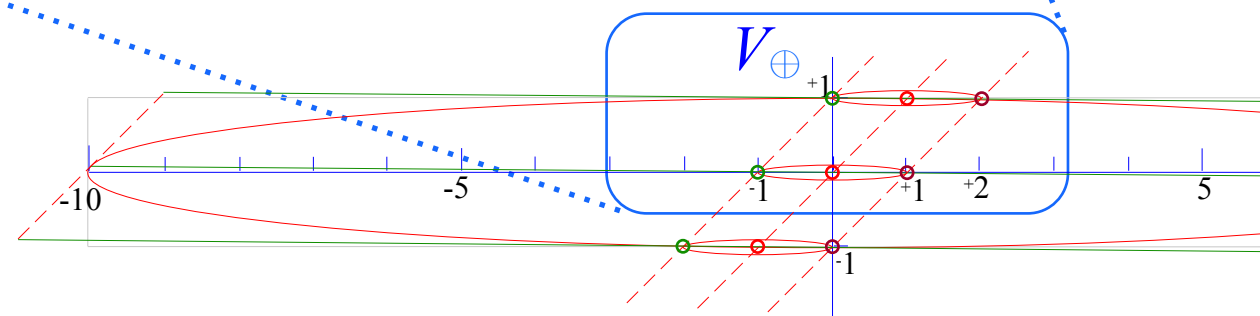
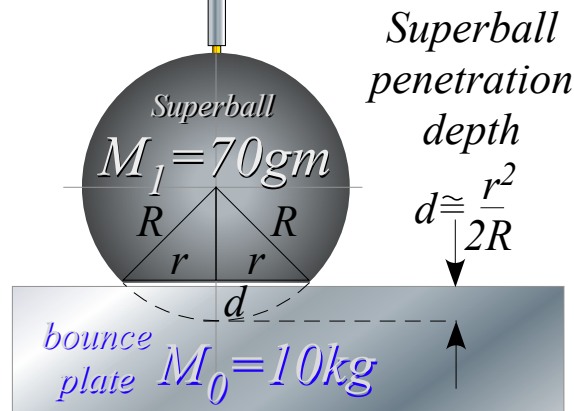


Fig. 3.2a
(Unit 1)



ballpoint
pen
 $M_2=10\text{gm}$

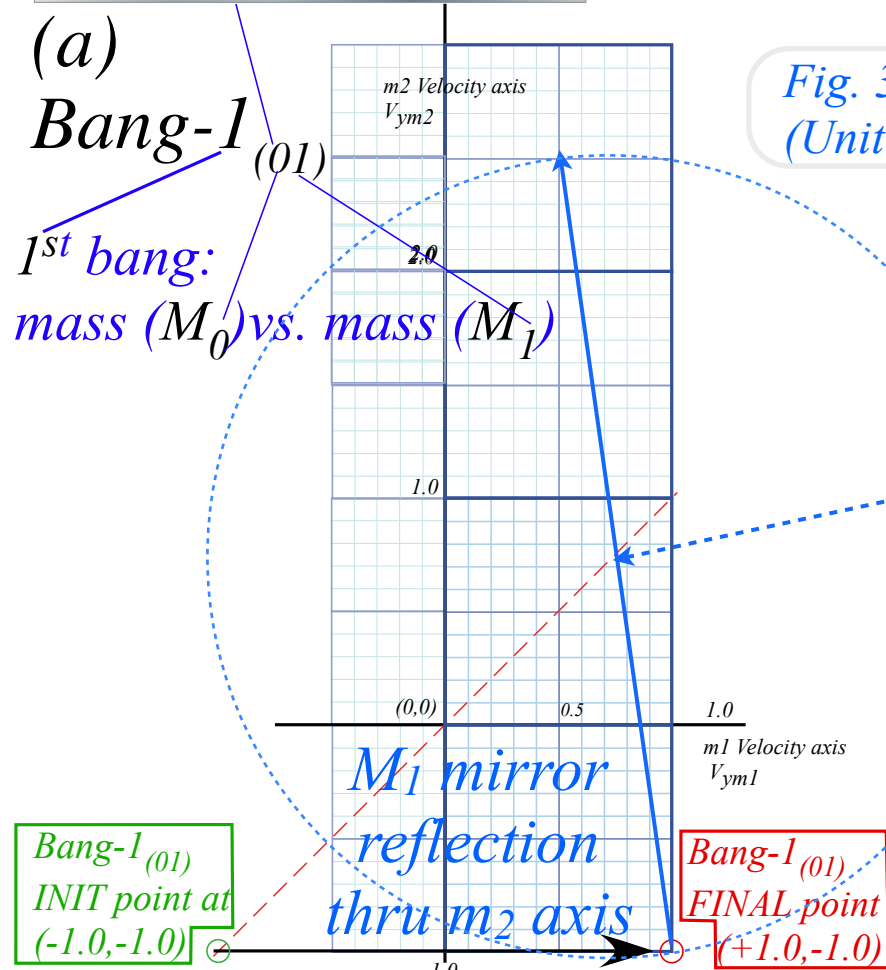
**The X-2
pen-
launcher**



(a)

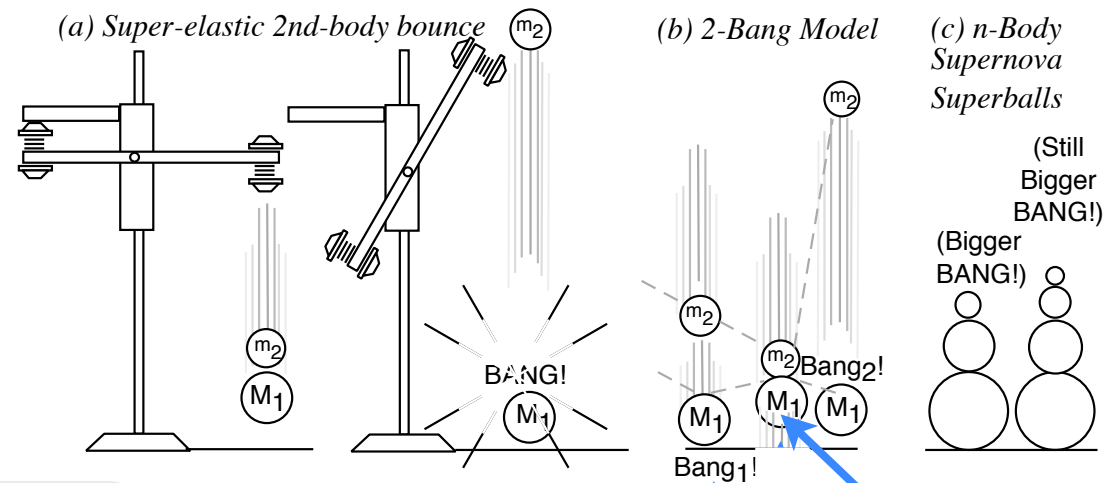
Bang-1
(01)

1st bang:
mass (M_0) vs. mass (M_1)



This 1st bang is a floor-bounce of
 M_1 off very massive plate/Earth M_0

Fig. 3.3
(Unit 1)

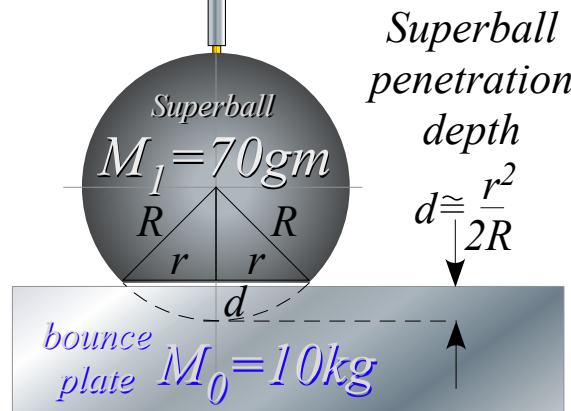


1st bang:
 M_1 off floor

2nd bang:
 m_2 off M_1

ballpoint
pen
 $M_2=10gm$

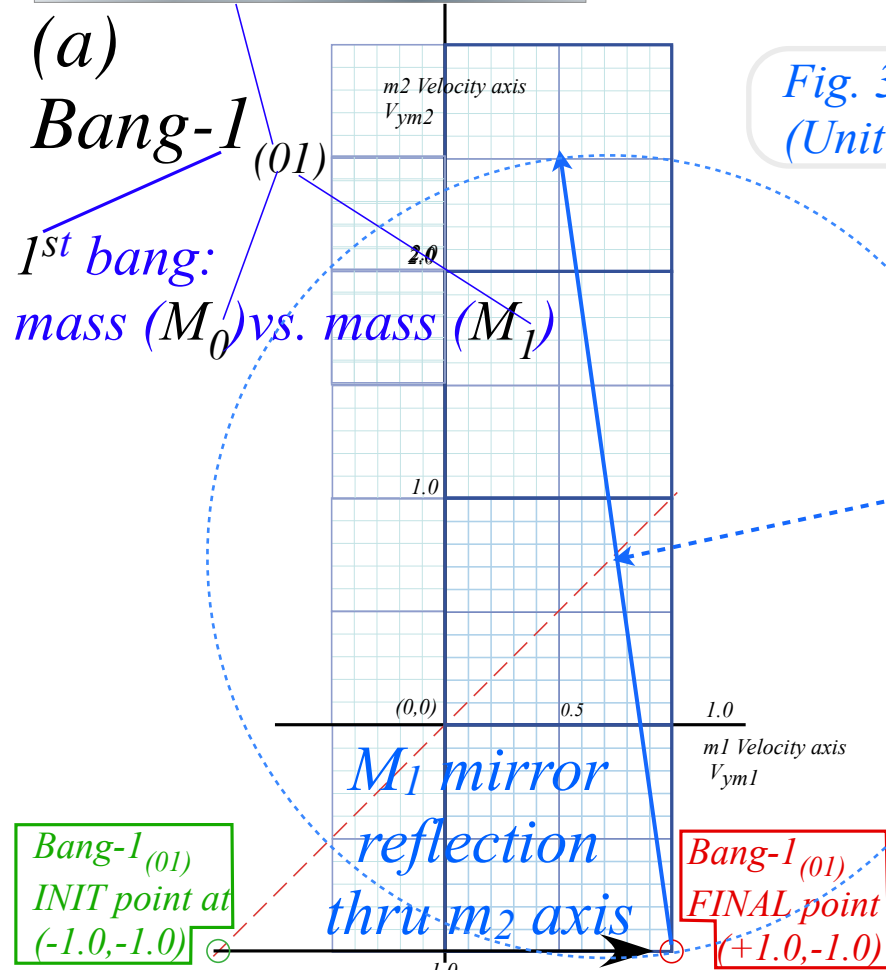
**The X-2
pen-
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(a)

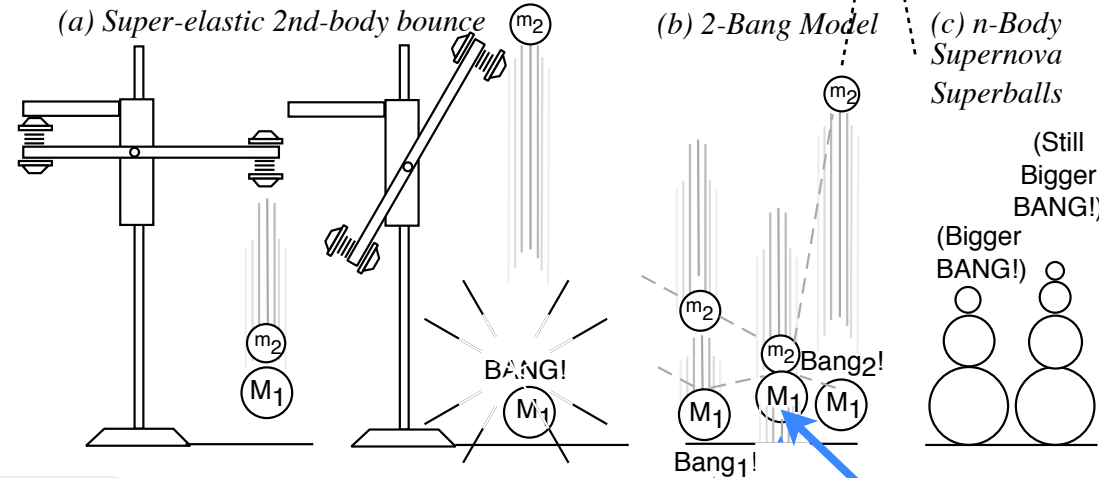
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1st bang:
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Fig. 3.3
(Unit 1)



1st bang:
 M_1 off floor

2nd bang:
 m_2 off M_1

3rd bang:
 m_2 off ceiling

Fig. 3.4
(Unit 1)

Geometry of X2 launcher bouncing in box (gravity-free)

 *Independent Bounce Model (IBM)*

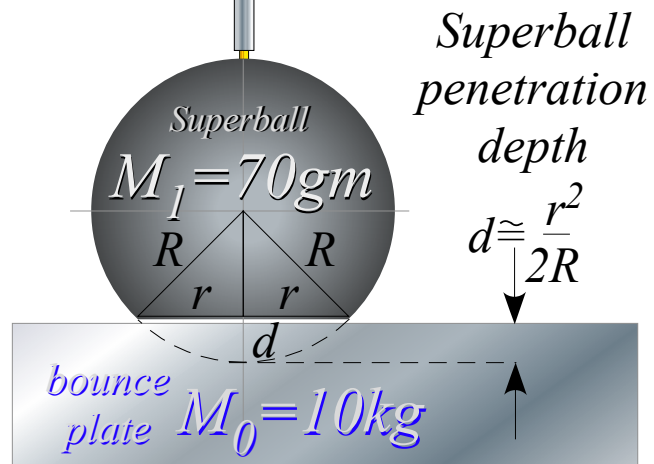
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Integration of (V_1, V_2) data to space-space plots (y_1, y_2)

ballpoint
pen
 $M_2=10\text{gm}$

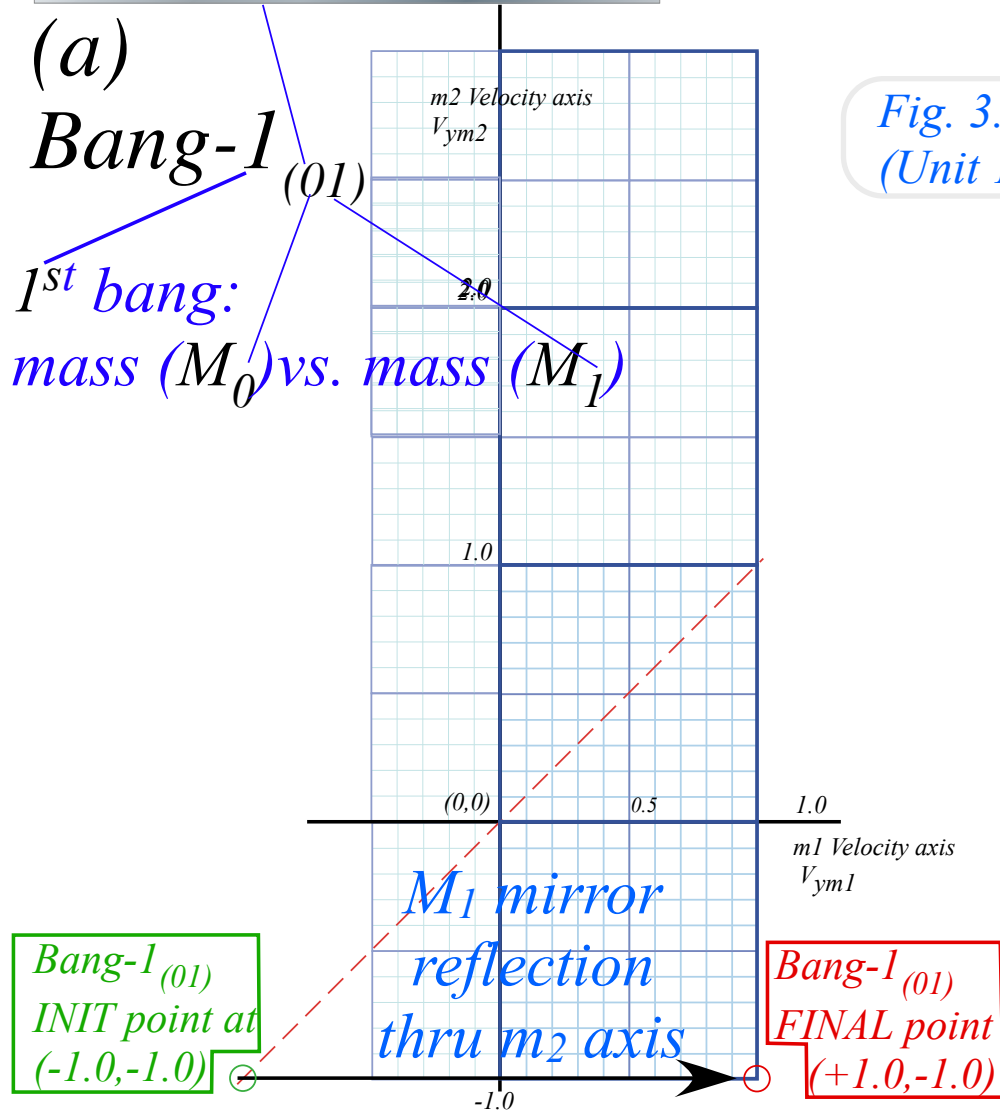
The X-2 pen- launcher



(a)

Bang-1

1st bang:
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Fig. 3.3
(Unit 1)

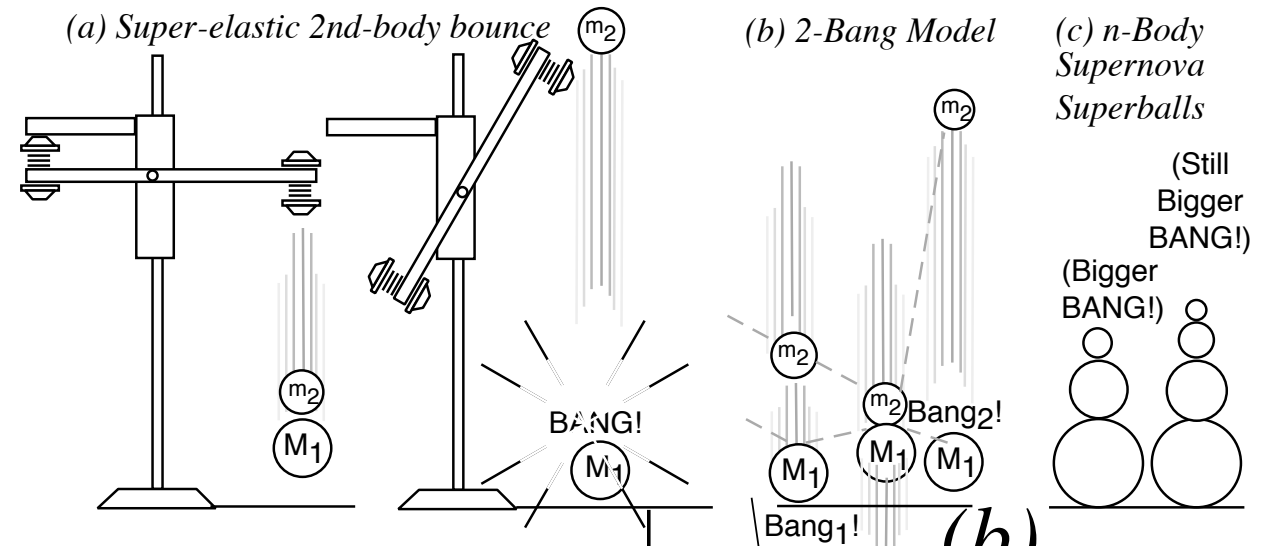
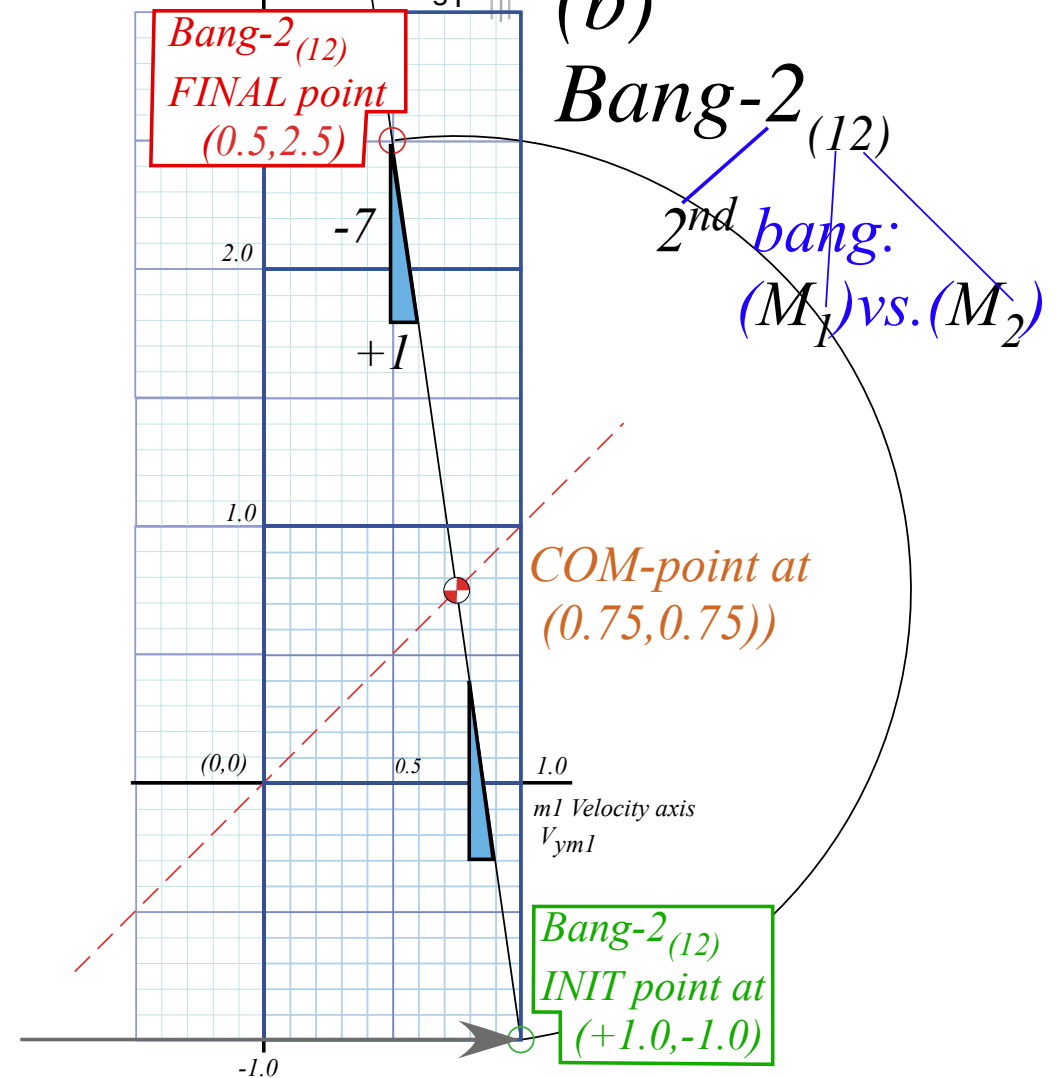


Fig. 3.4
(Unit 1)



Geometry of X2 launcher bouncing in box (gravity-free)

Independent Bounce Model (IBM)

 *Geometric optimization and range-of-motion calculation(s)*

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

Integration of (V_1, V_2) data to space-space plots (y_1, y_2)

ballpoint
pen
 $M_2=10\text{gm}$

**The X-2
pen-
launcher**

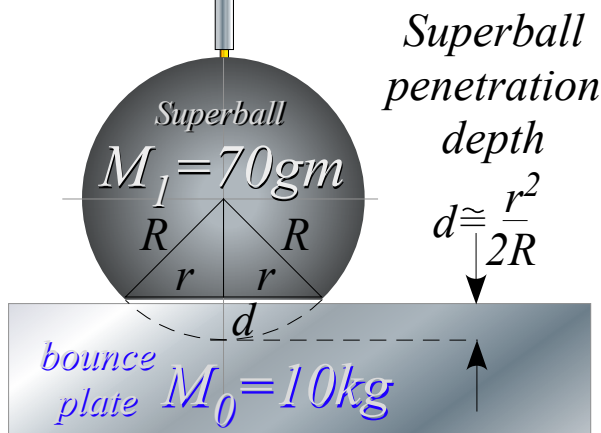
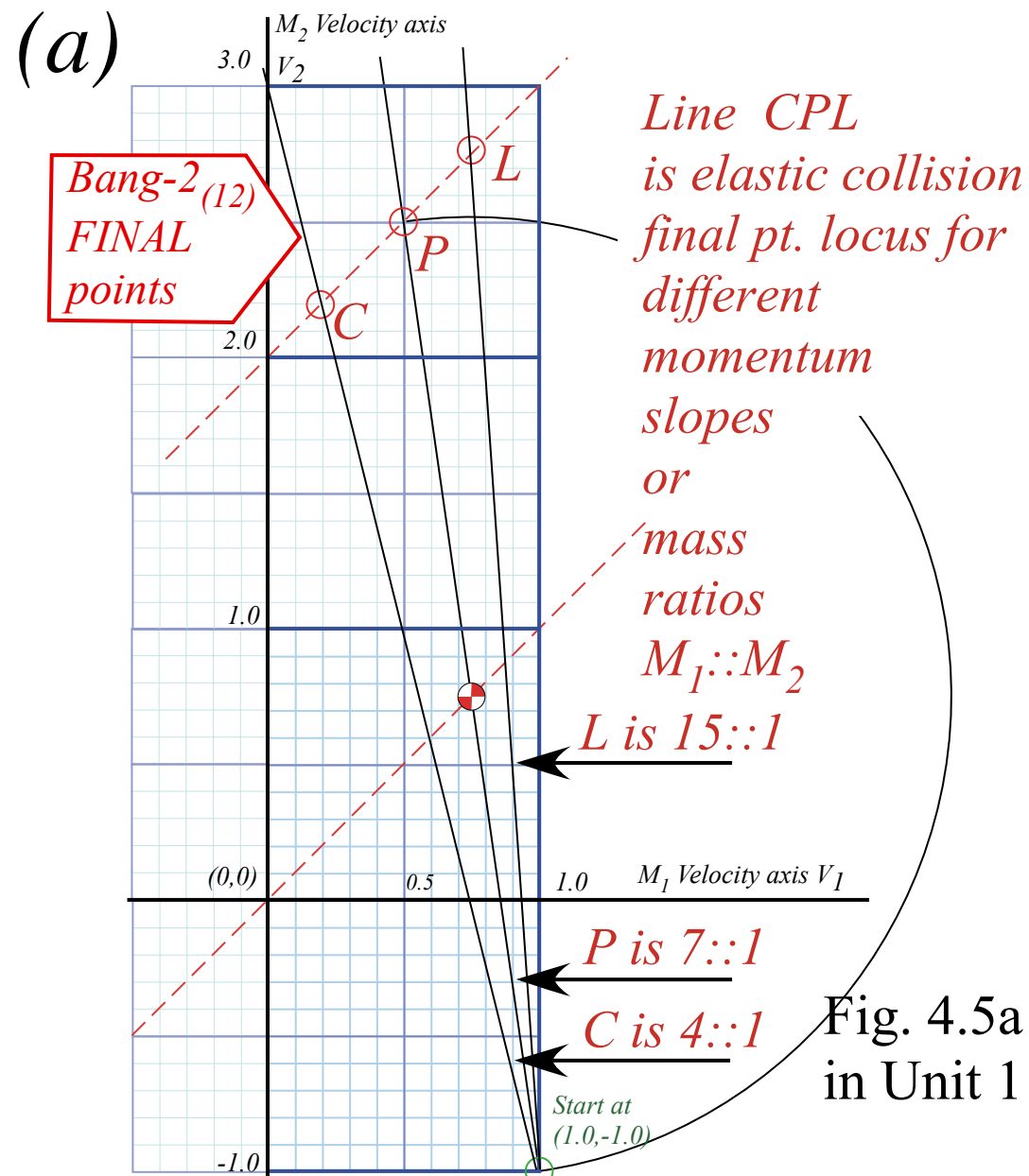
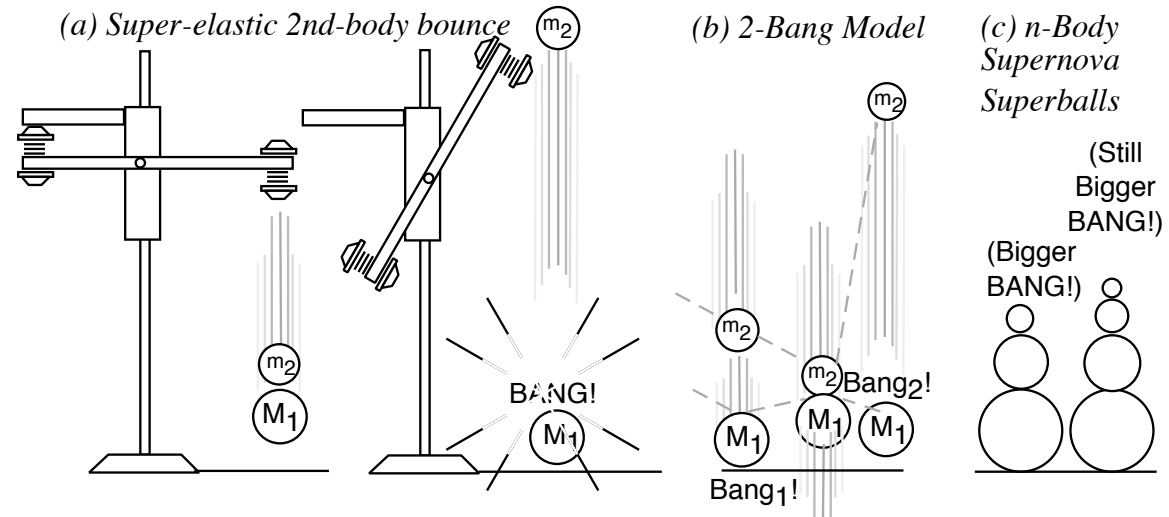


Fig. 3.3
(Unit 1)

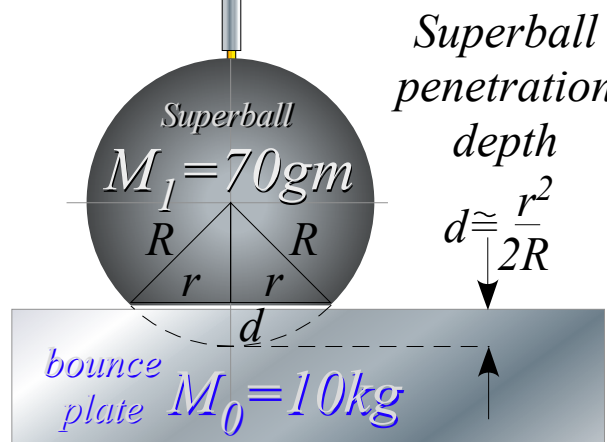


(With $g=0$ and 70:10 mass ratio)
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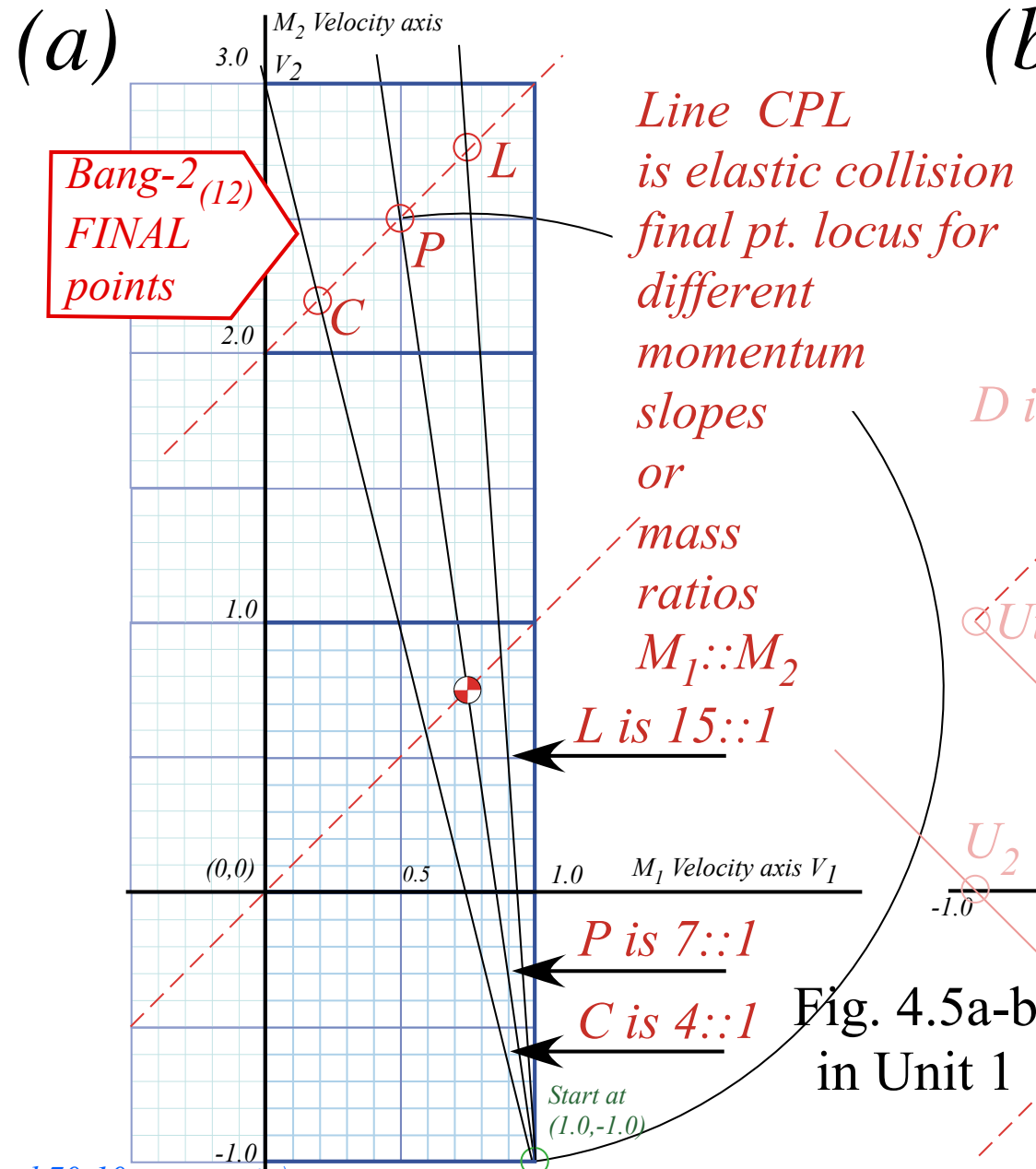
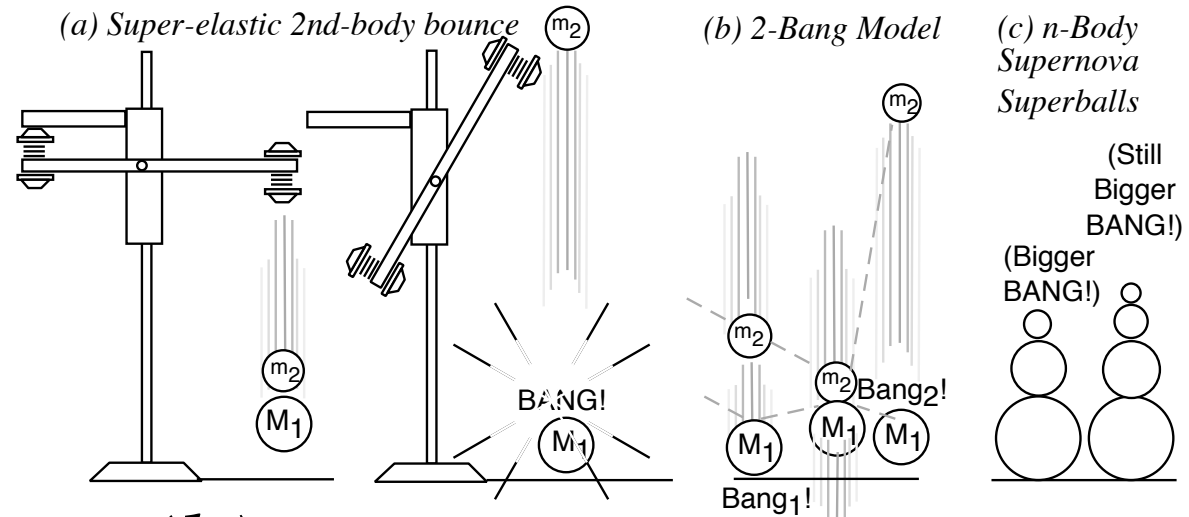
ballpoint
pen
 $M_2=10\text{gm}$

**The X-2
pen-
launcher**

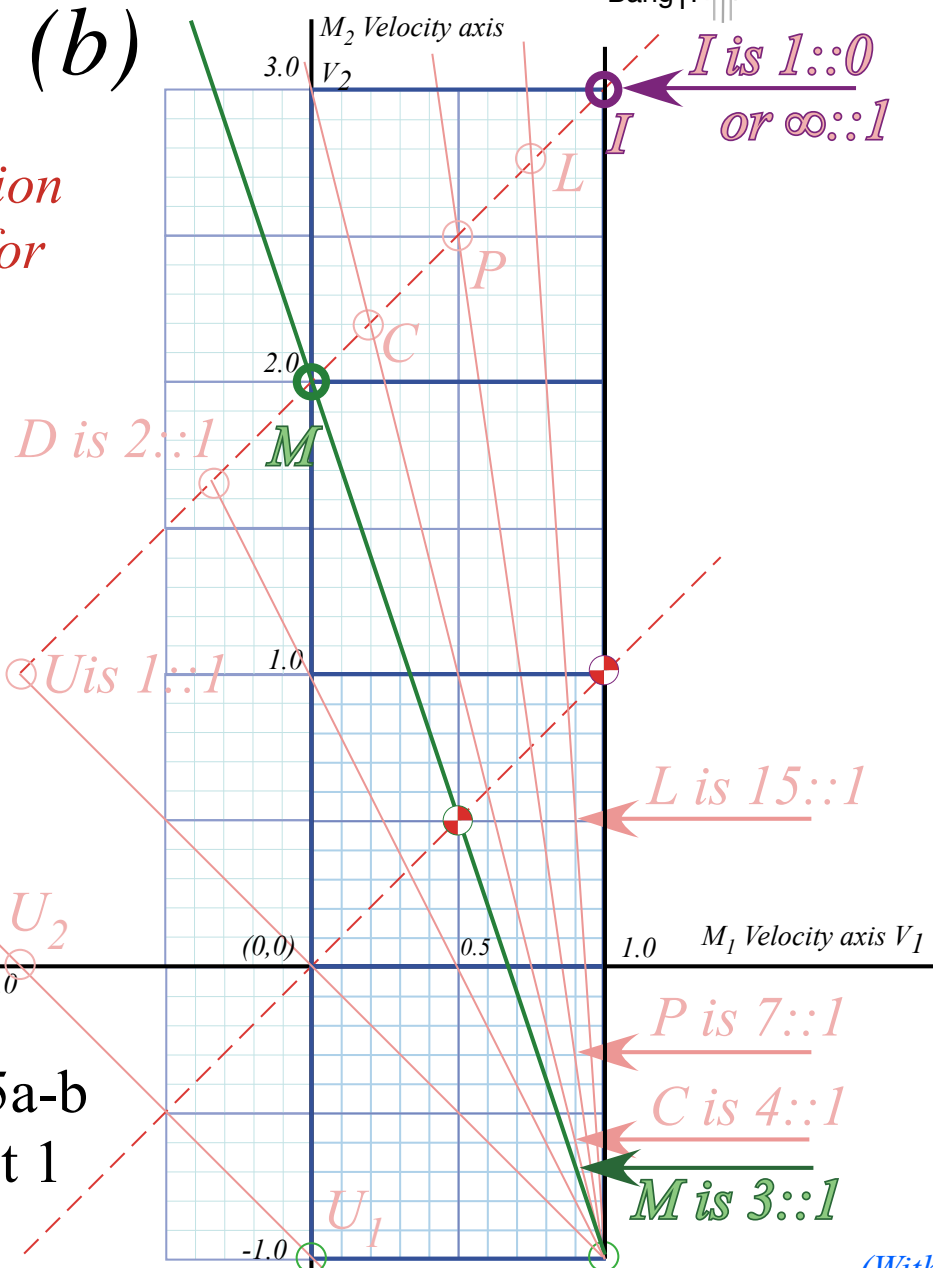


**Fig. 3.3
(Unit 1)**

BounceIt Simulation (passing massRatio in URL):
With $g=0$ and 30:10 mass ratio



**Fig. 4.5a-b
in Unit 1**



(With $g=0$ and 70:10 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=6300>

Geometry of X2 launcher bouncing in box (gravity-free)

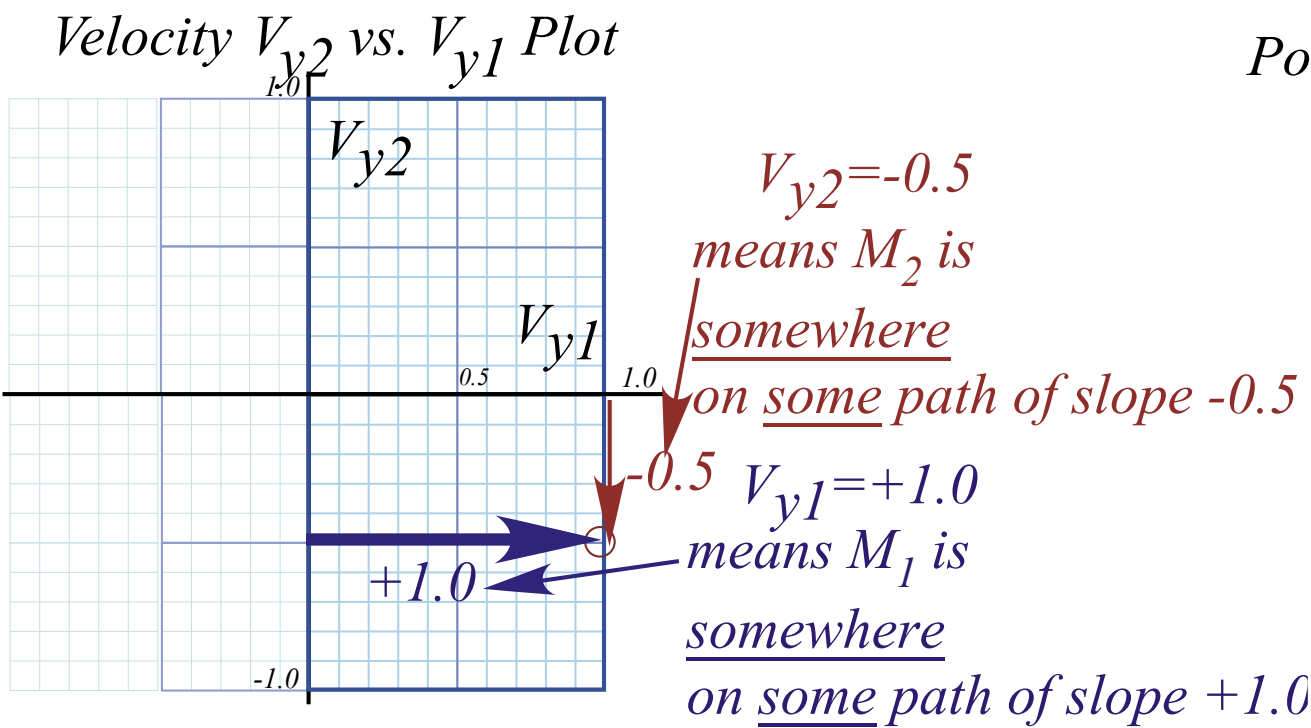
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Geometric optimization and range-of-motion calculation(s)

 *Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots*

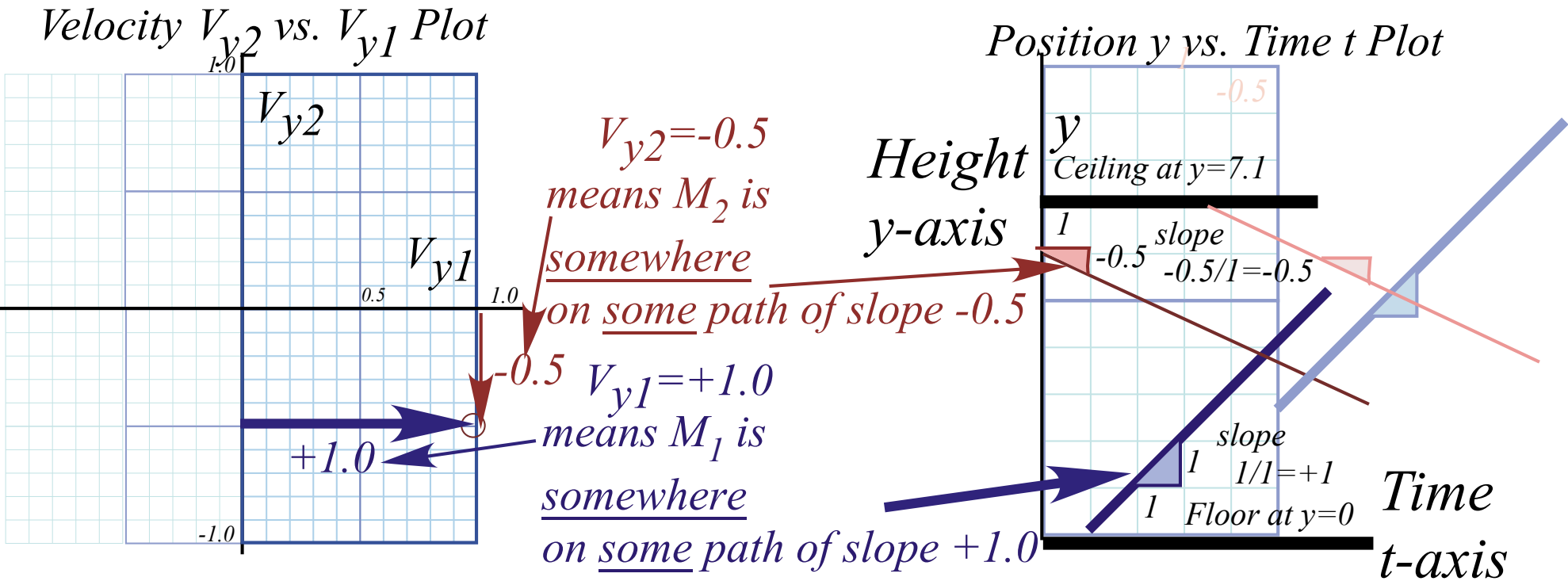
Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$

Geometric “Integration” (Converting Velocity data to Spacetime)

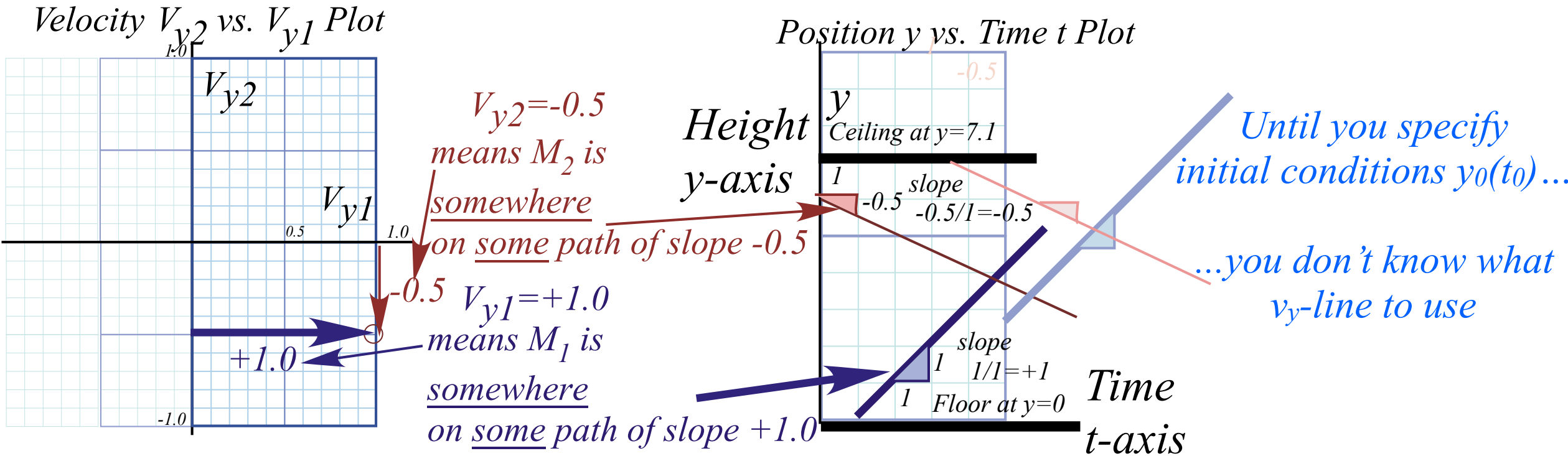


Position y vs. Time t Plot

Geometric “Integration” (Converting Velocity data to Spacetime)



Geometric “Integration” (Converting Velocity data to Spacetime)



Geometric “Integration” (Converting Velocity data to Spacetime)

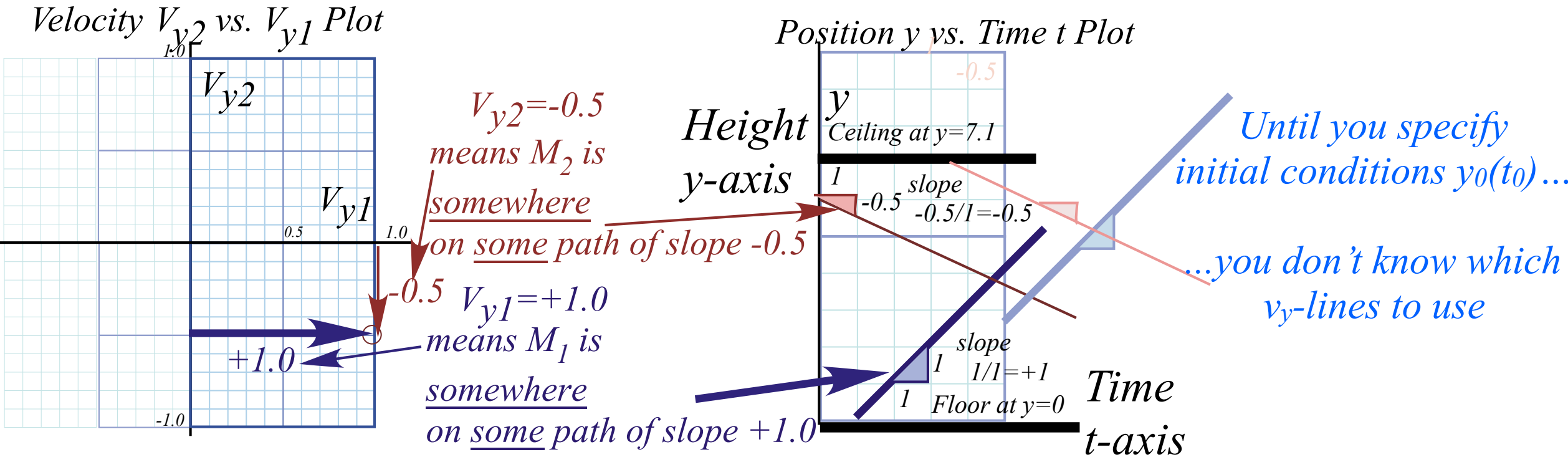
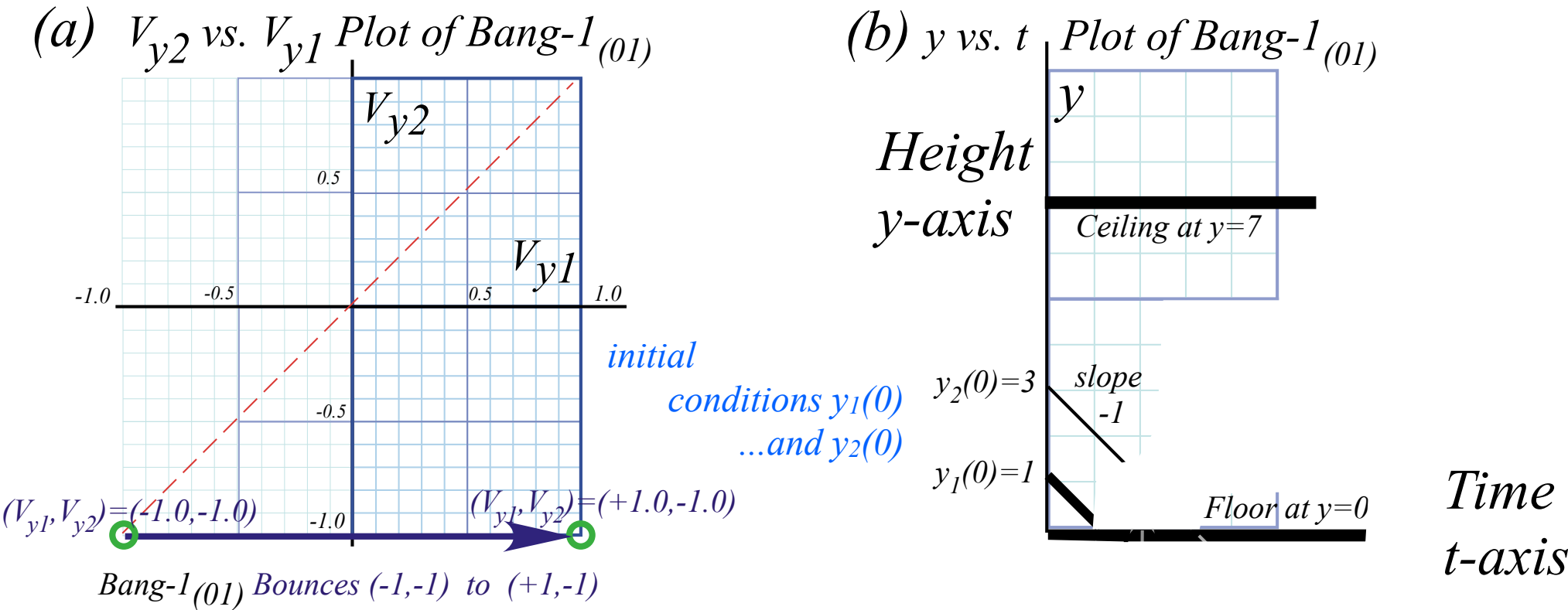


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)

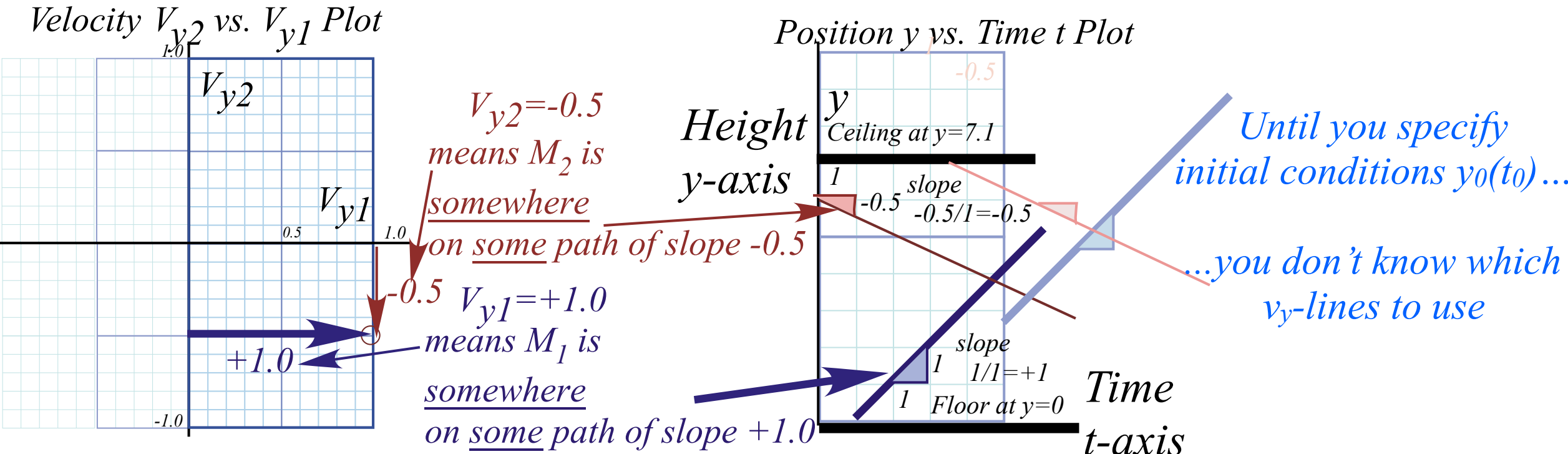
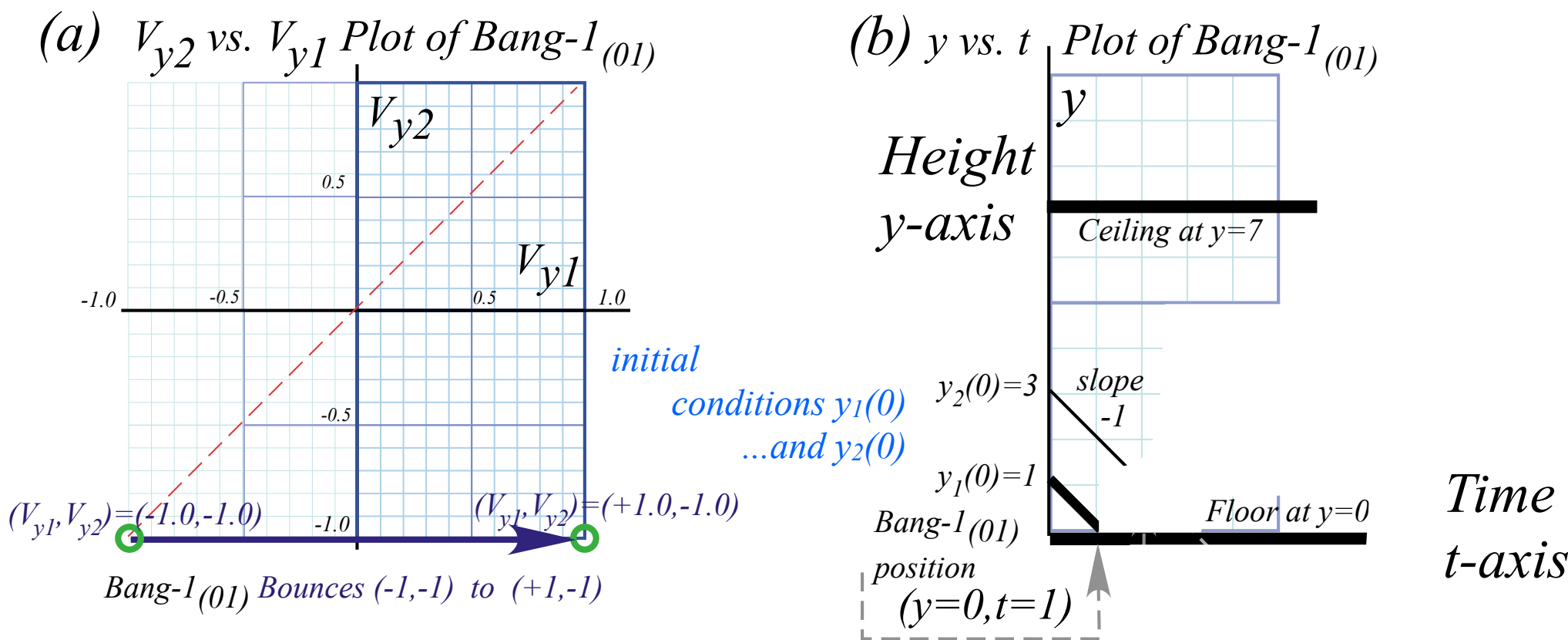


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)

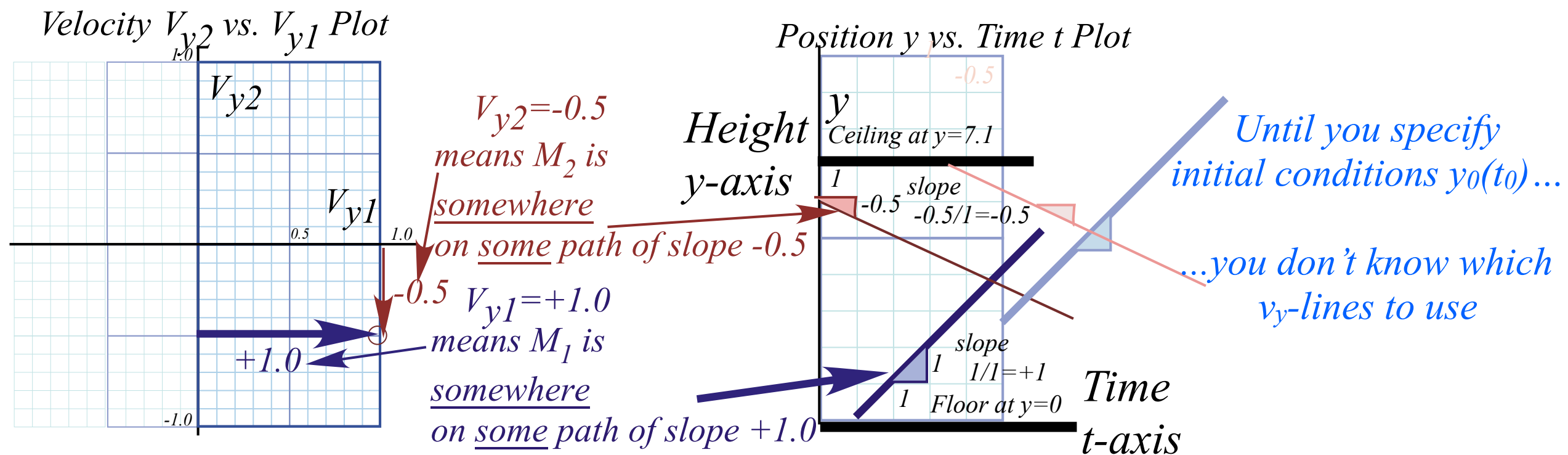
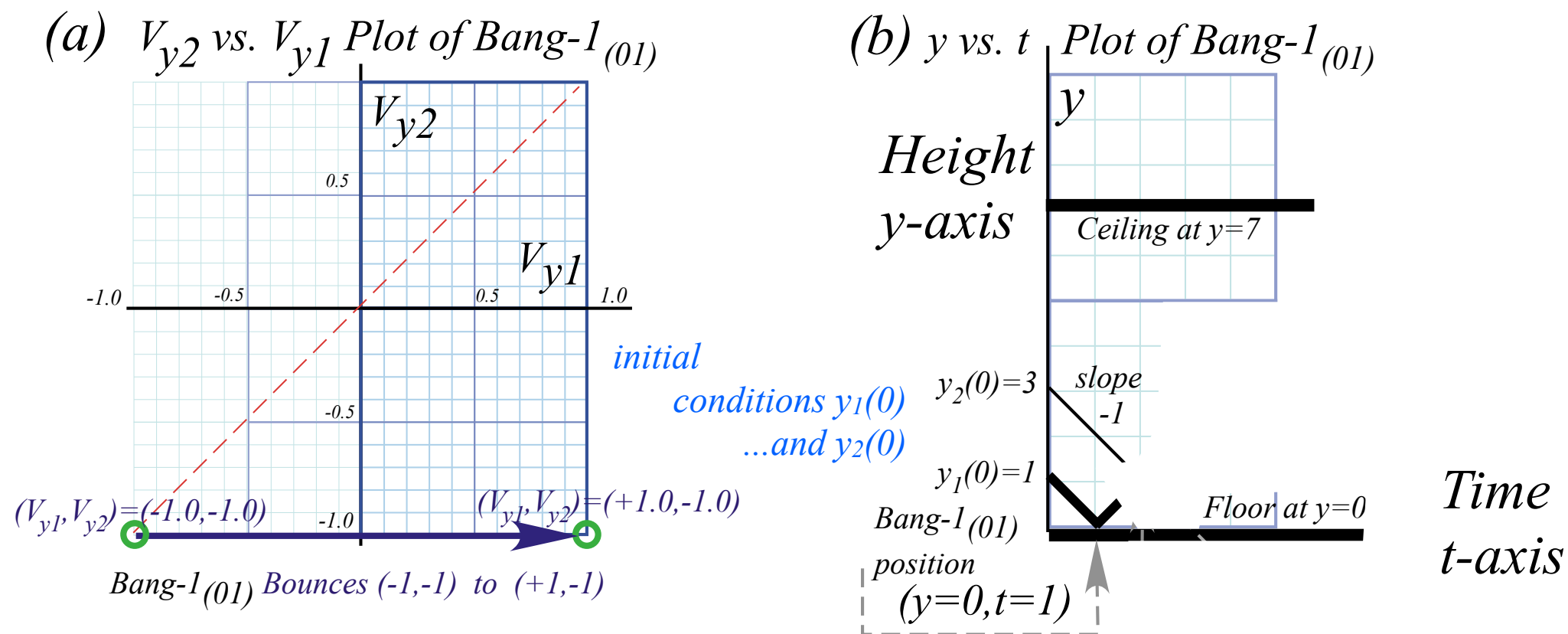


Fig. 4.6a-b
in Unit 1



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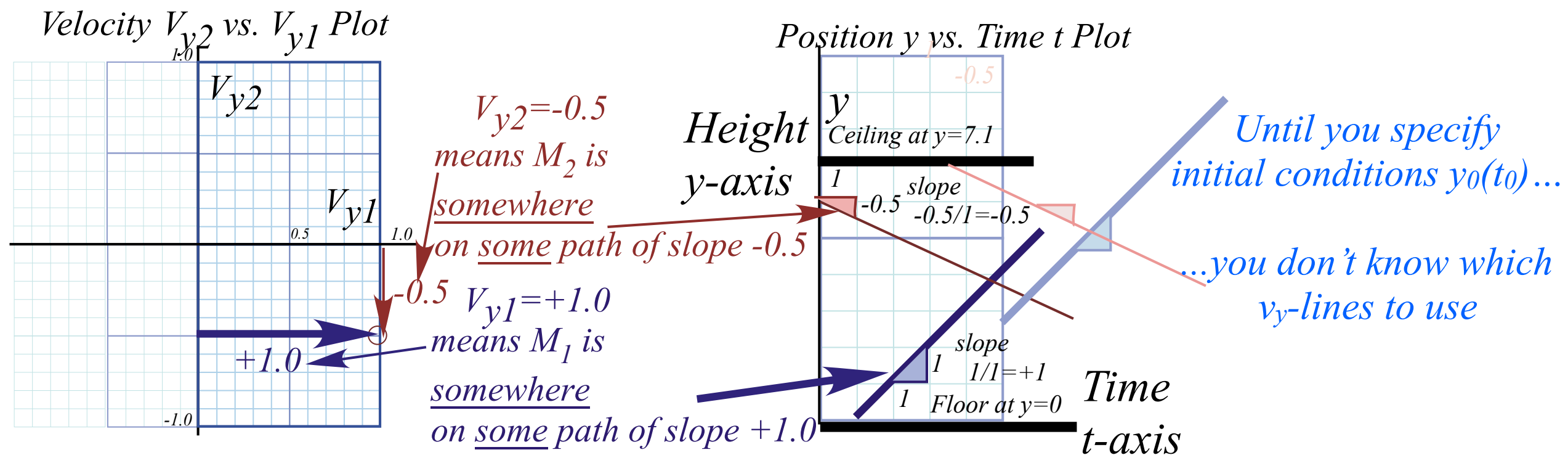
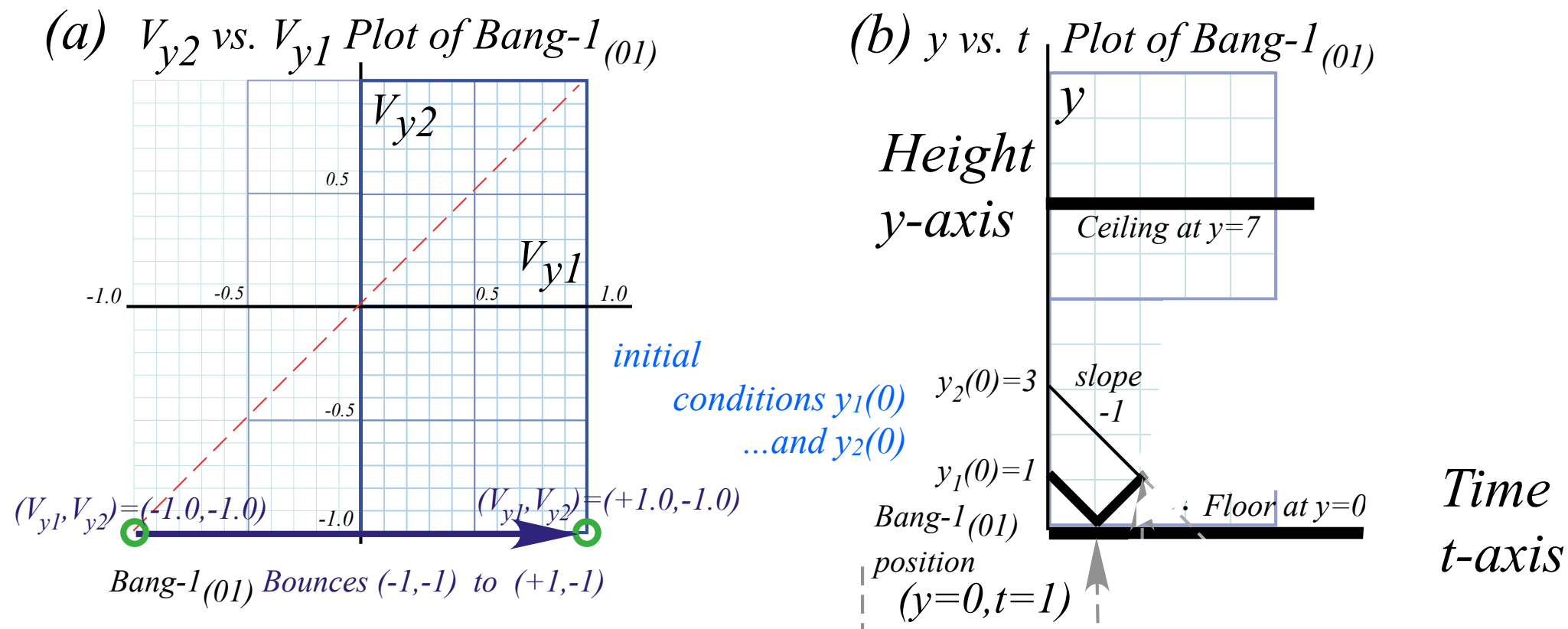


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)

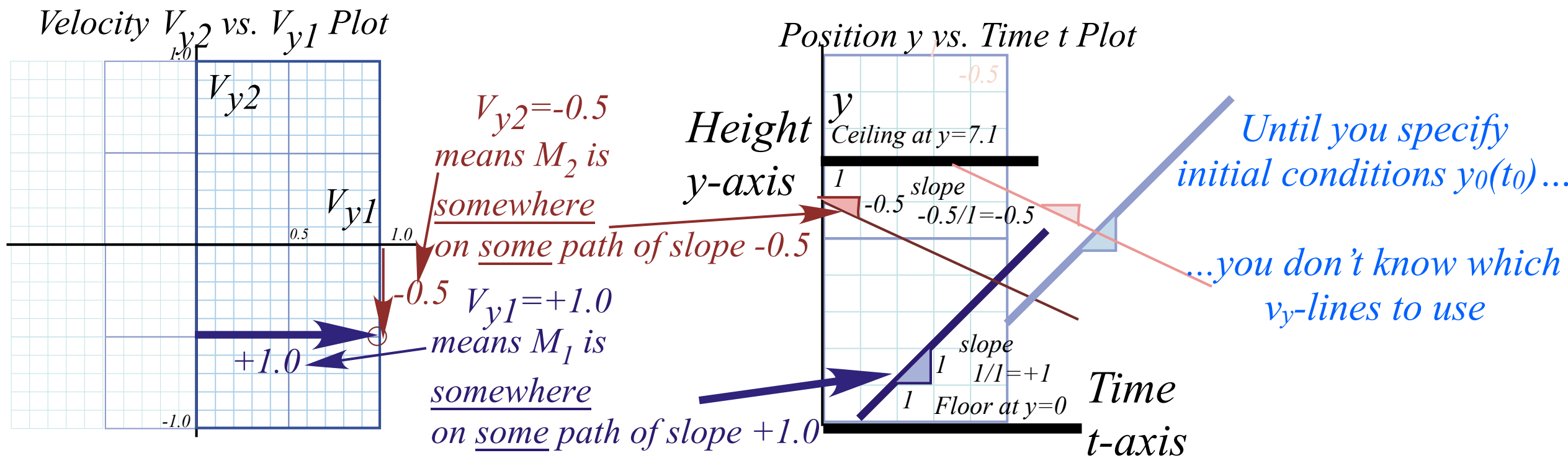
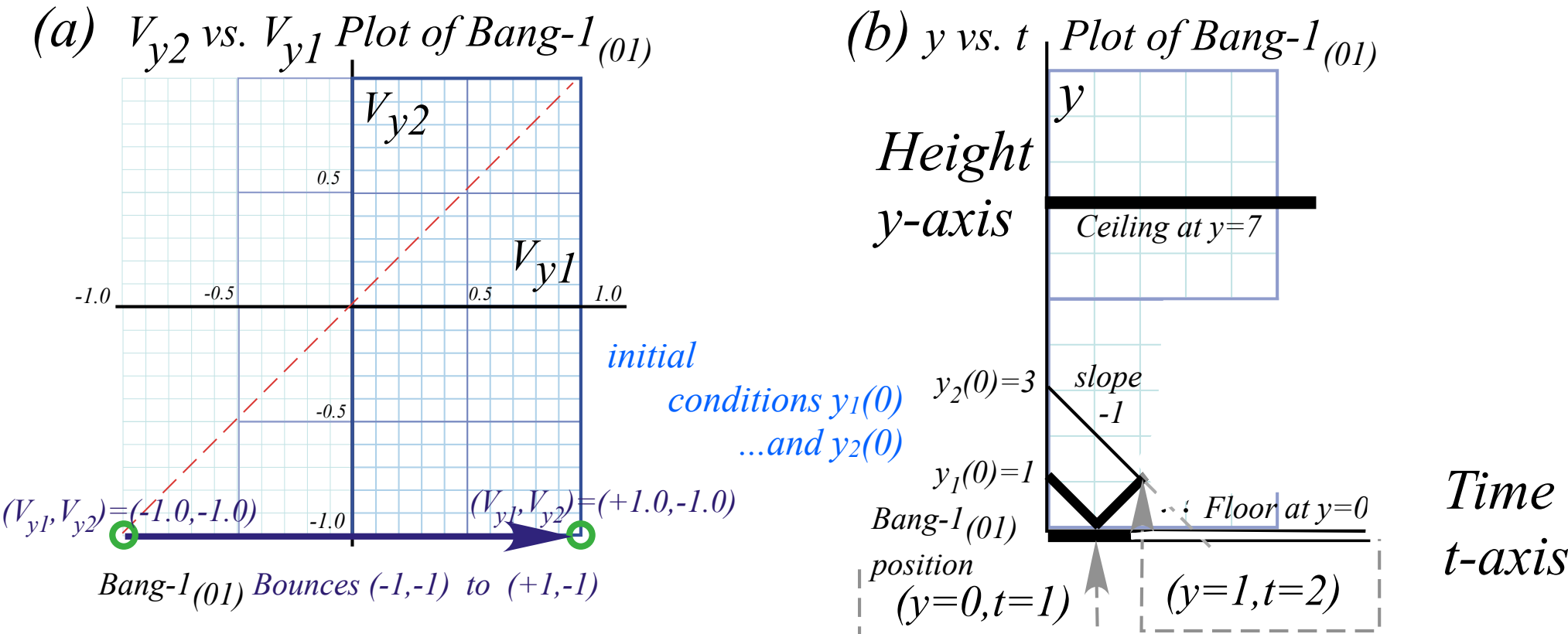


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)

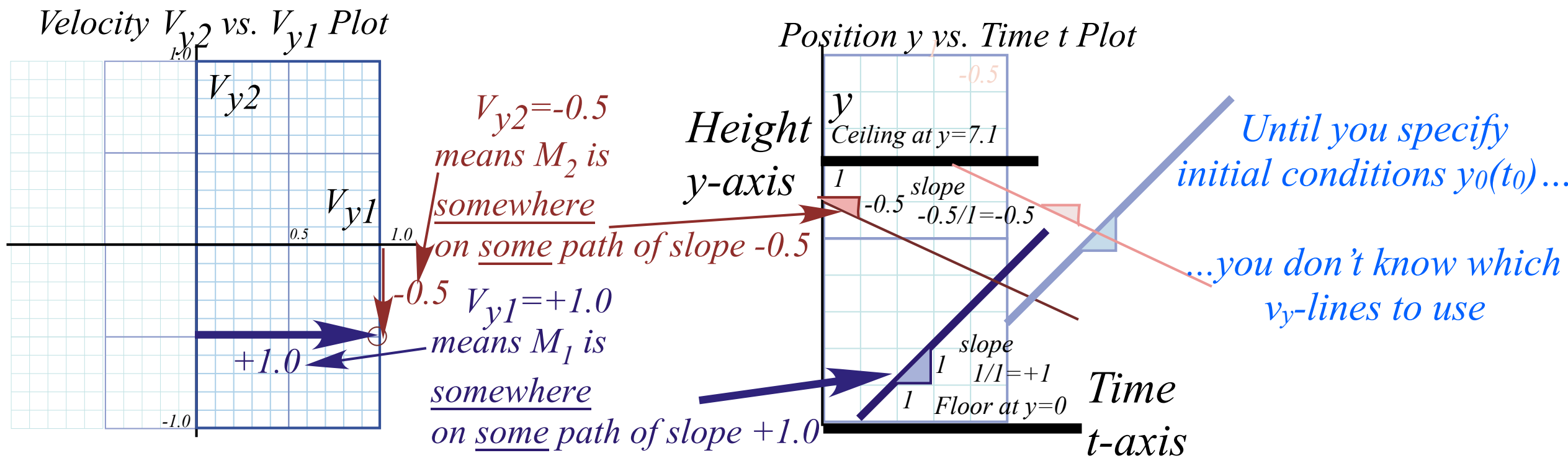
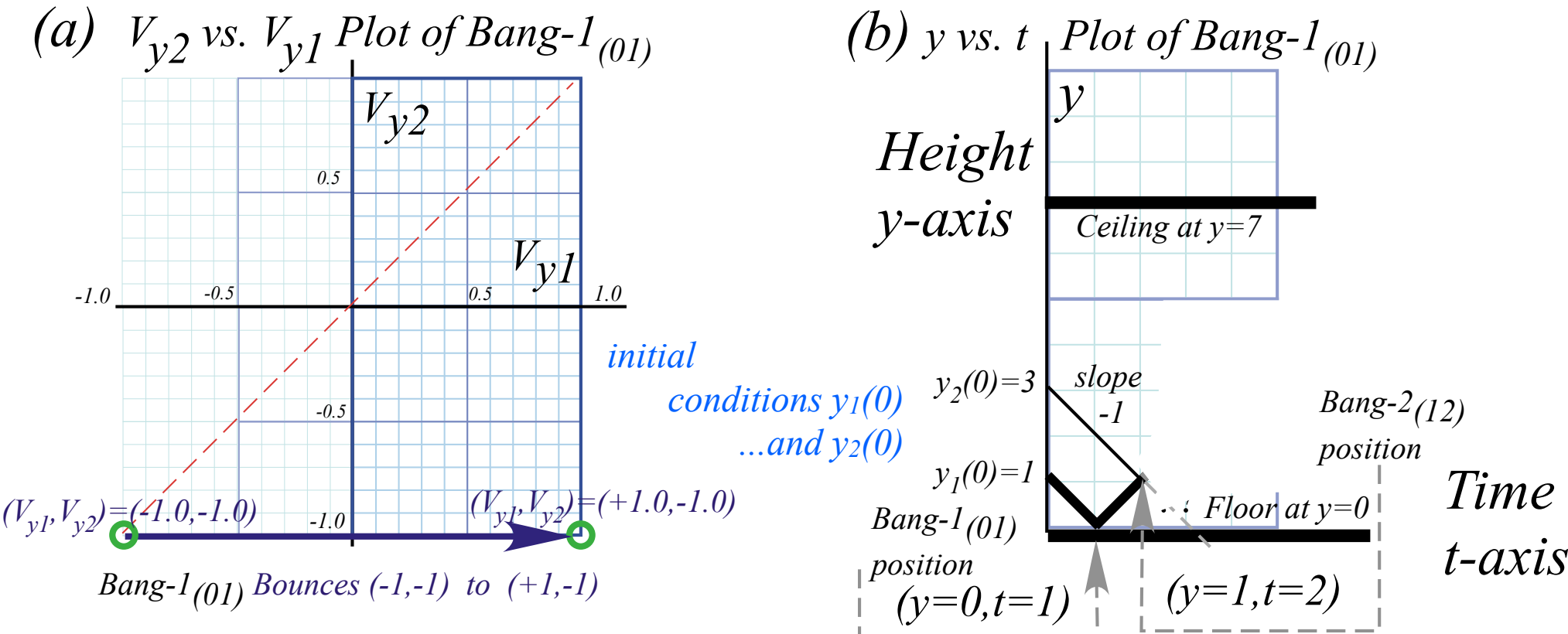


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)

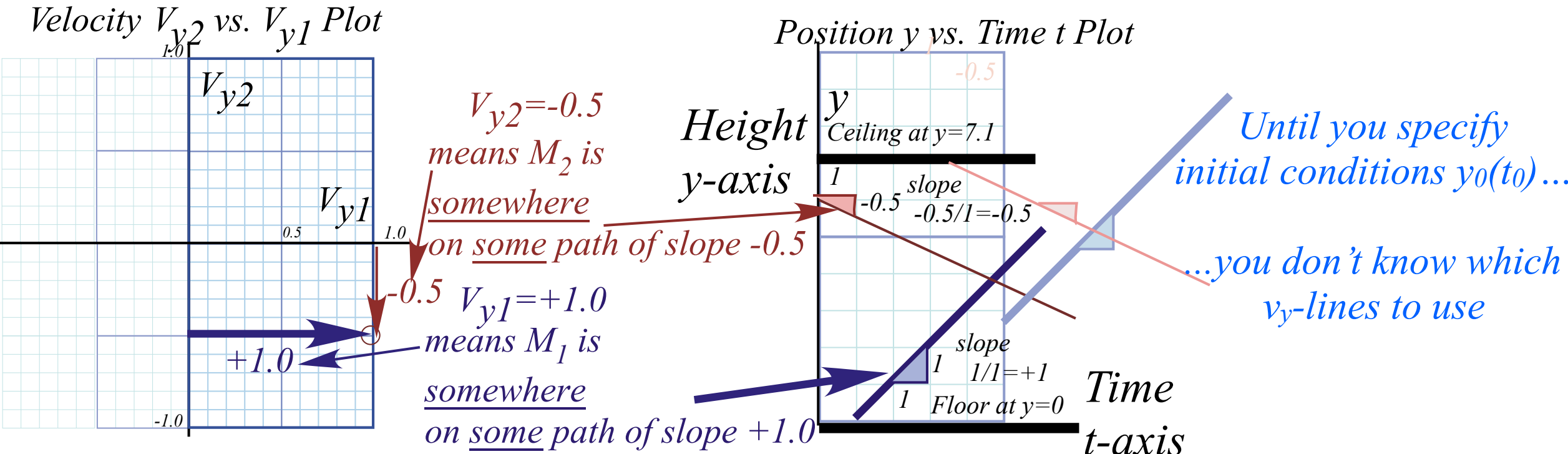
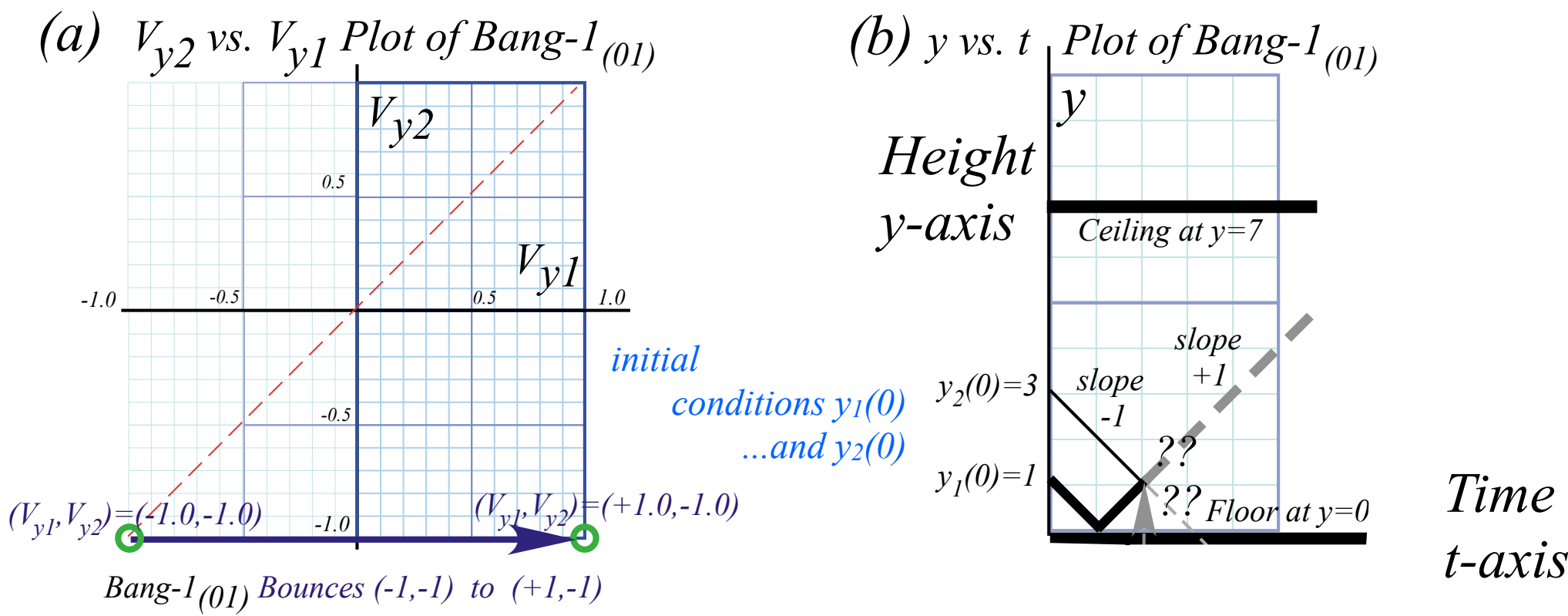


Fig. 4.6a-b
in Unit 1



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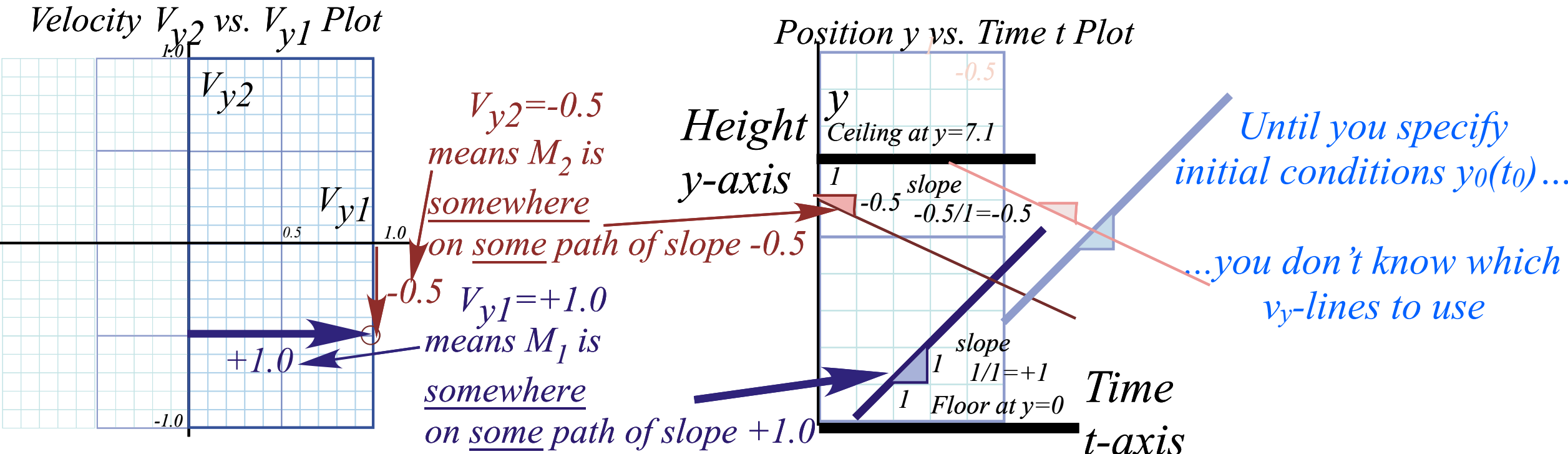
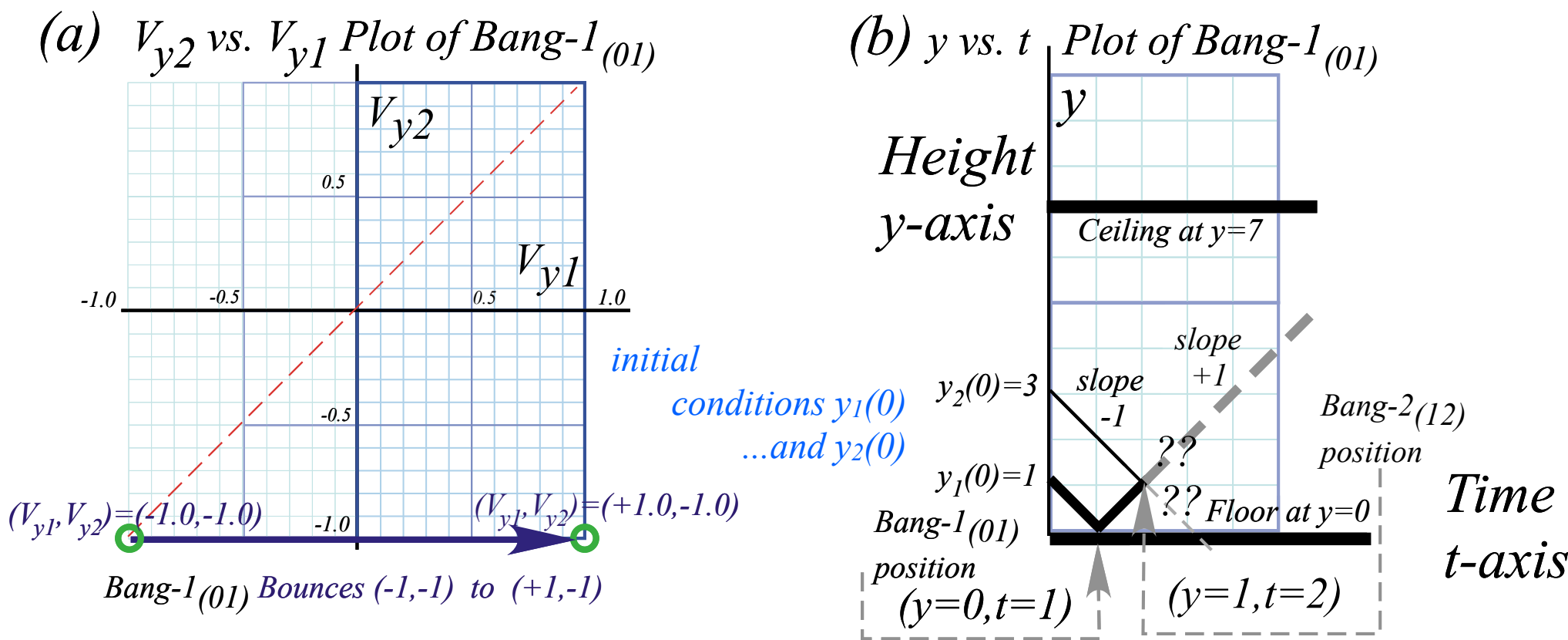
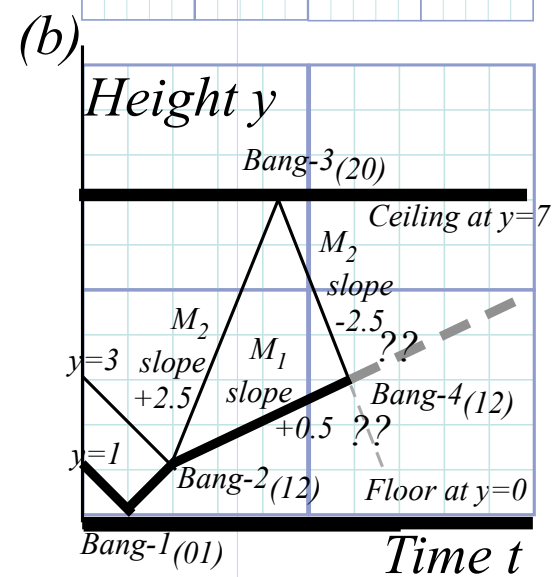
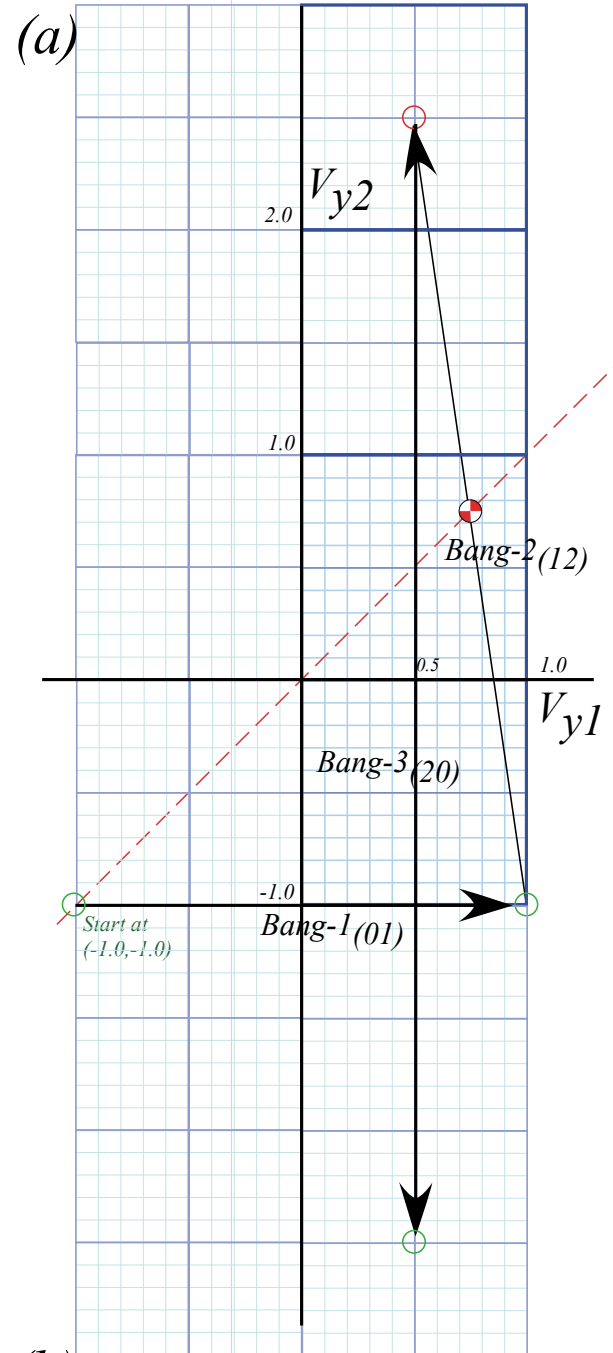


Fig. 4.6a-b
in Unit 1



Geometric “Integration” (Converting Velocity data to Spacetime)



Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

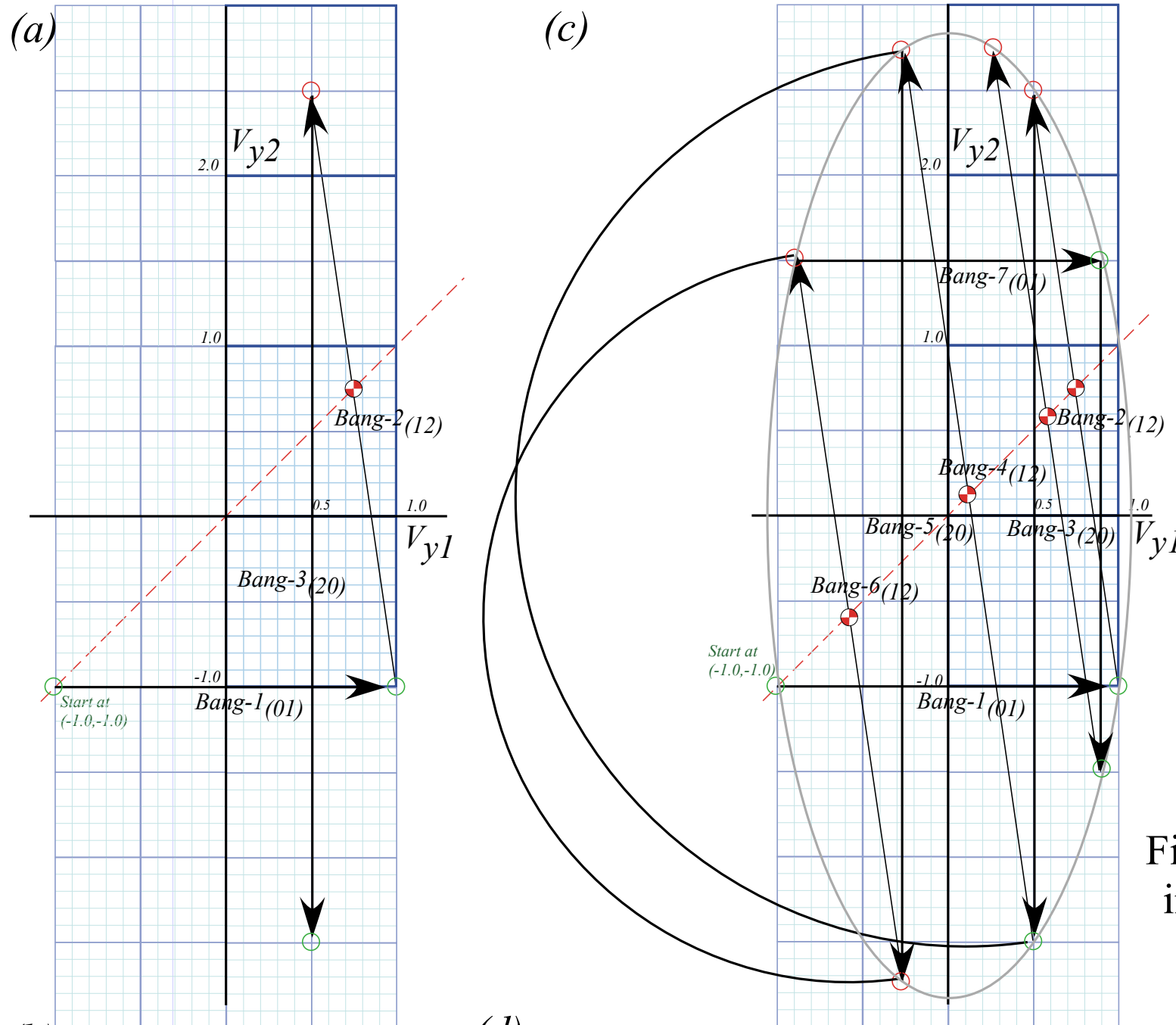


Fig. 4.7a-d
in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE / M_1}$$

Ellipse radius 2

$$a_2 = \sqrt{2KE / M_2}$$

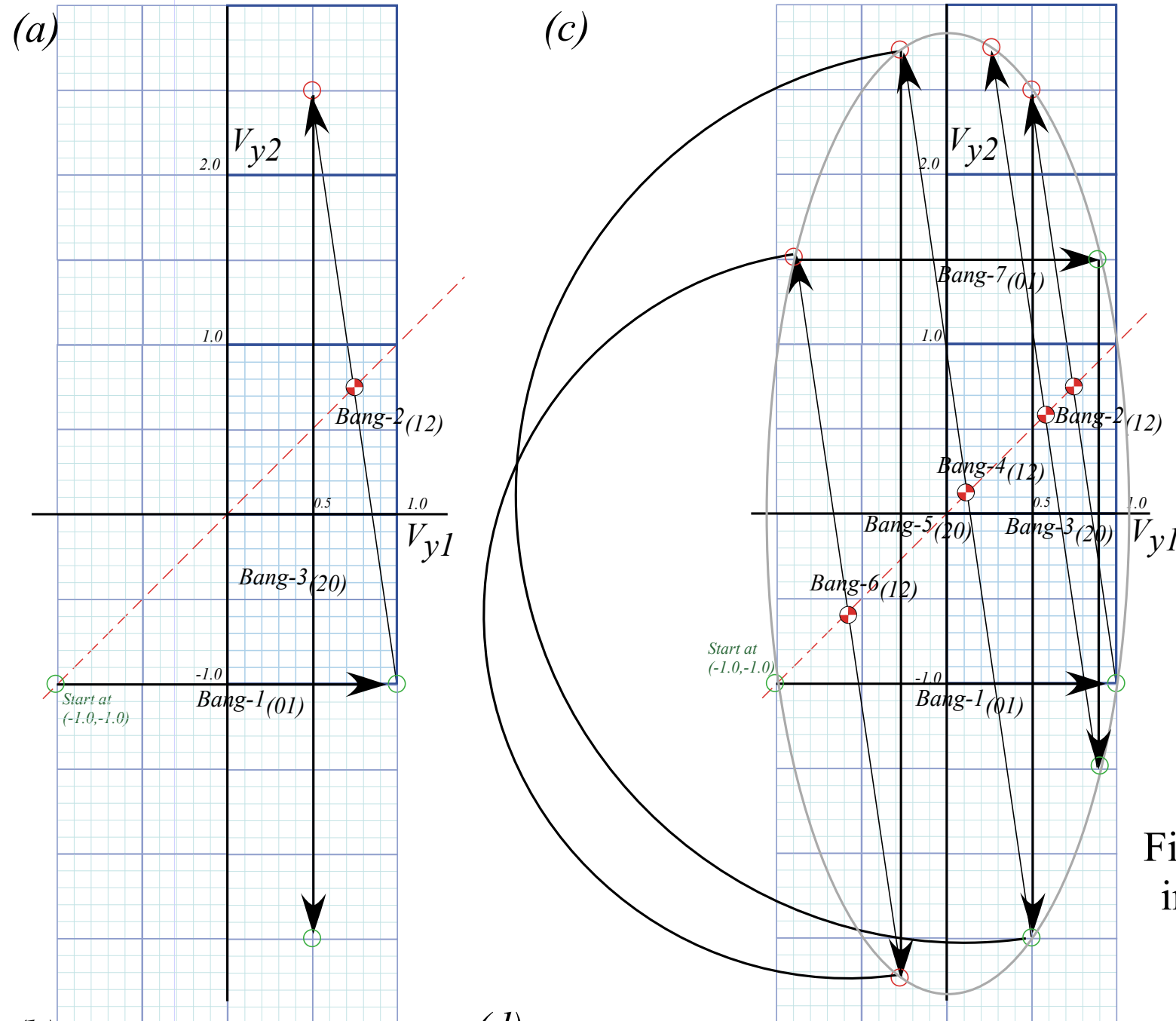


Fig. 4.7a-d
in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$\begin{aligned} a_1 &= \sqrt{2KE / M_1} \\ &= \sqrt{2KE / 7} \\ &= \sqrt{8/7} \\ &= 1.07 \end{aligned}$$

Ellipse radius 2

$$\begin{aligned} a_2 &= \sqrt{2KE / M_2} \\ &= \sqrt{2KE / 1} \\ &= \sqrt{8/1} \\ &= 2.83 \end{aligned}$$

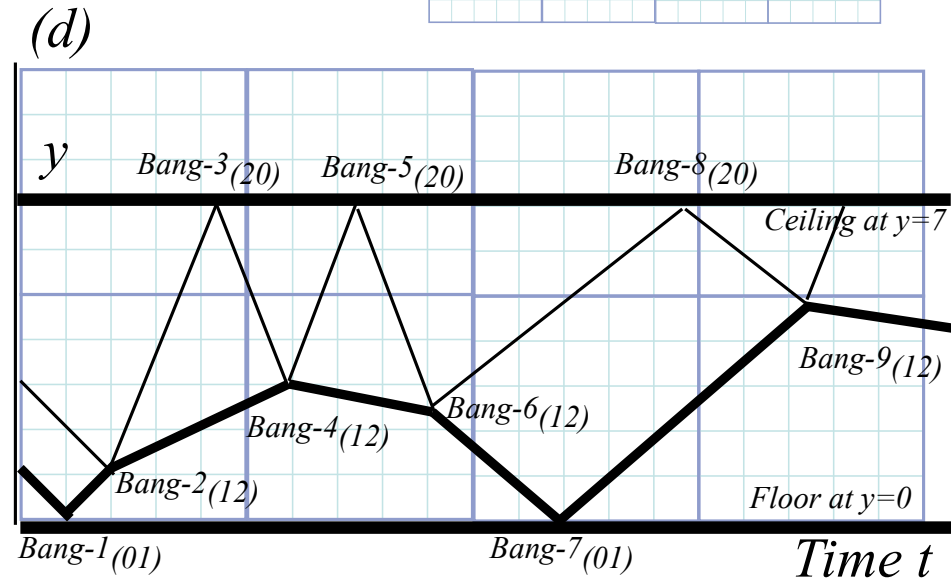
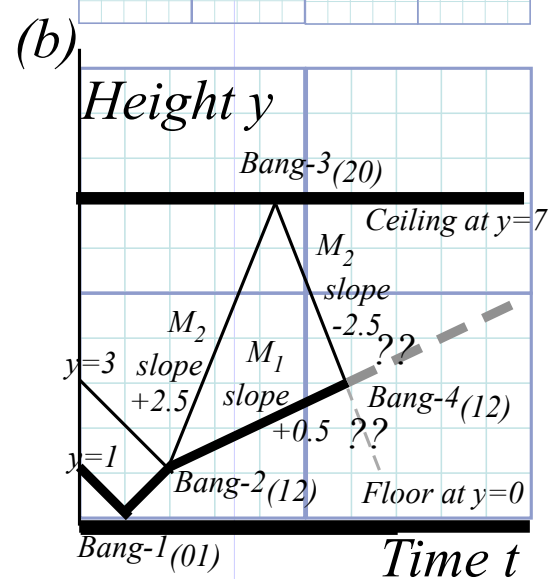
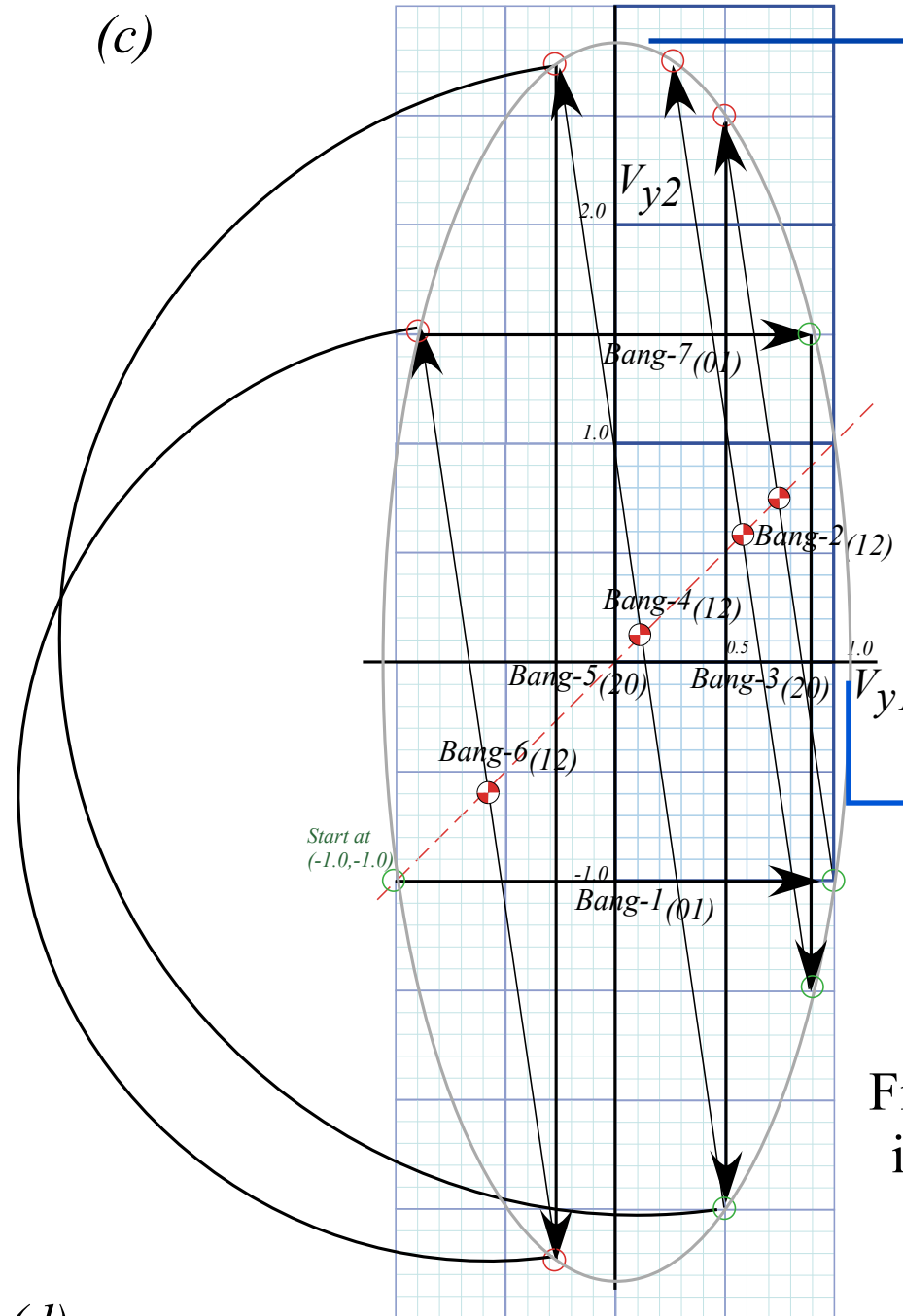
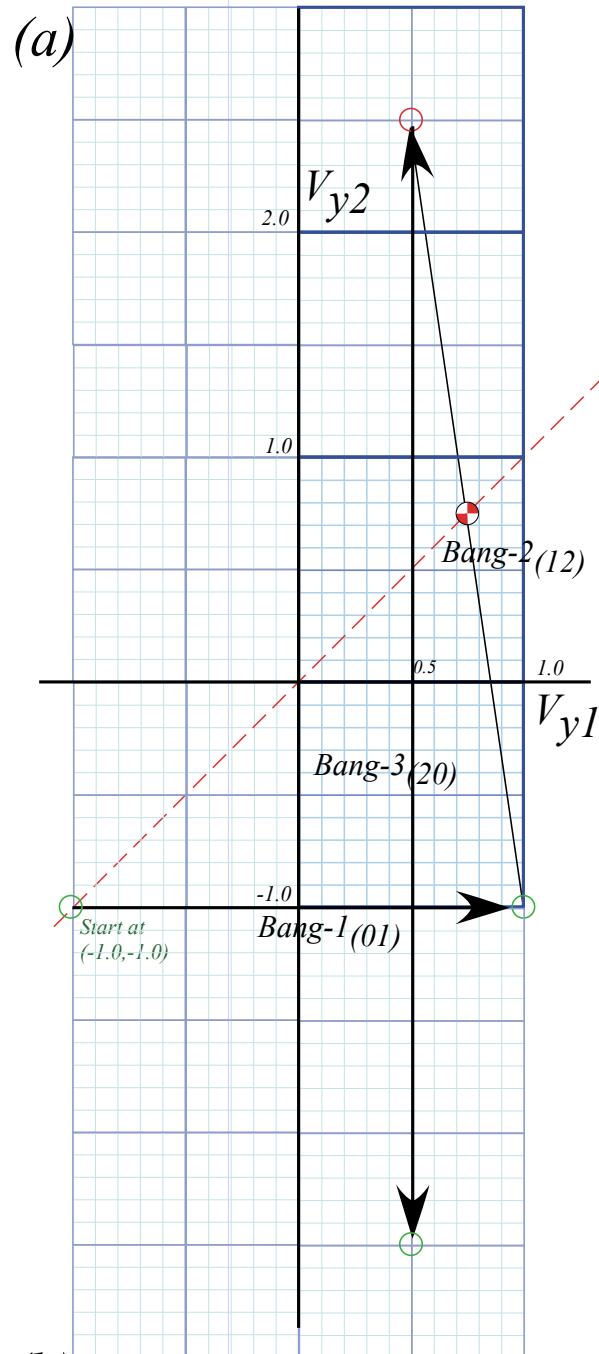
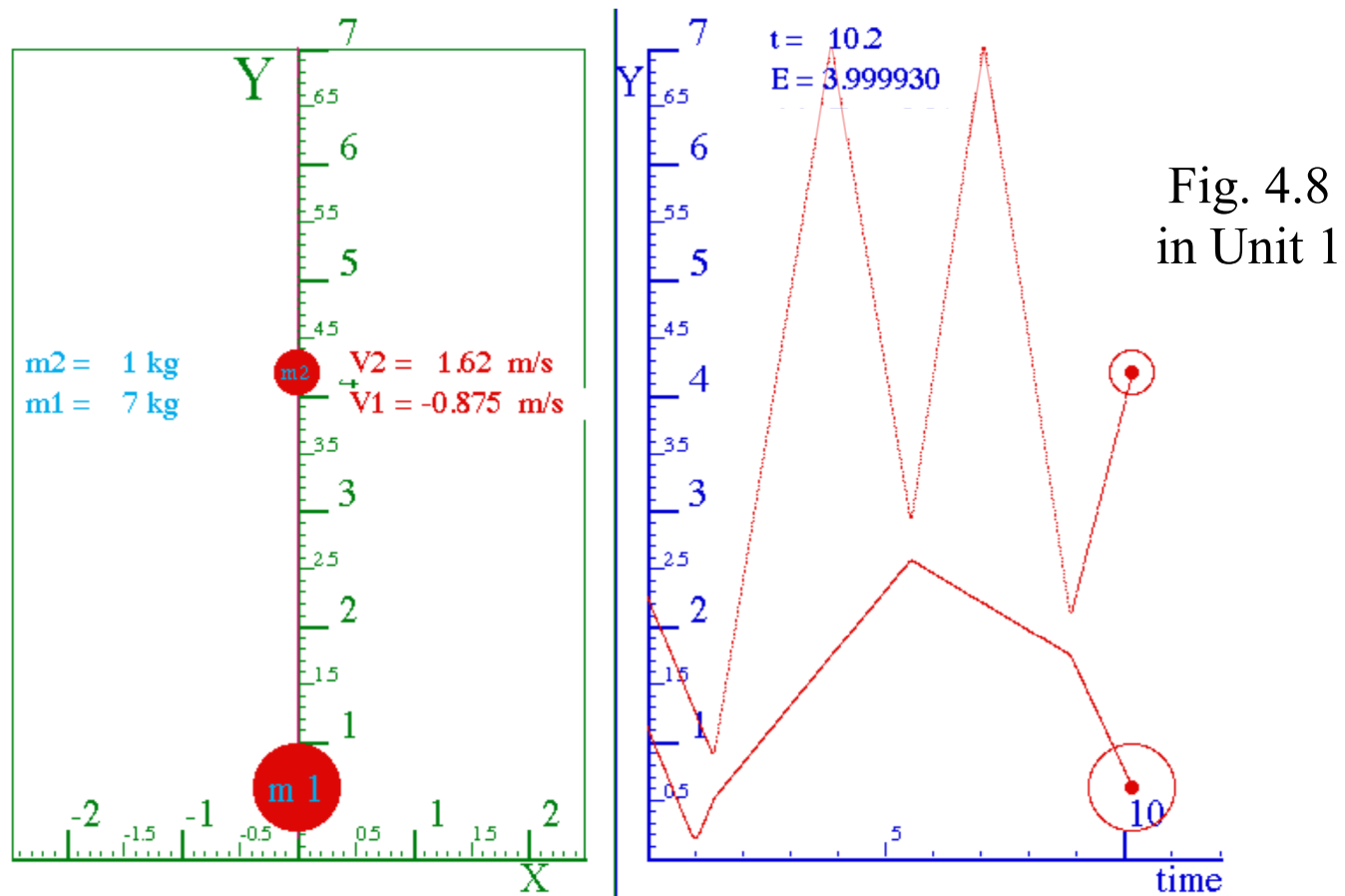
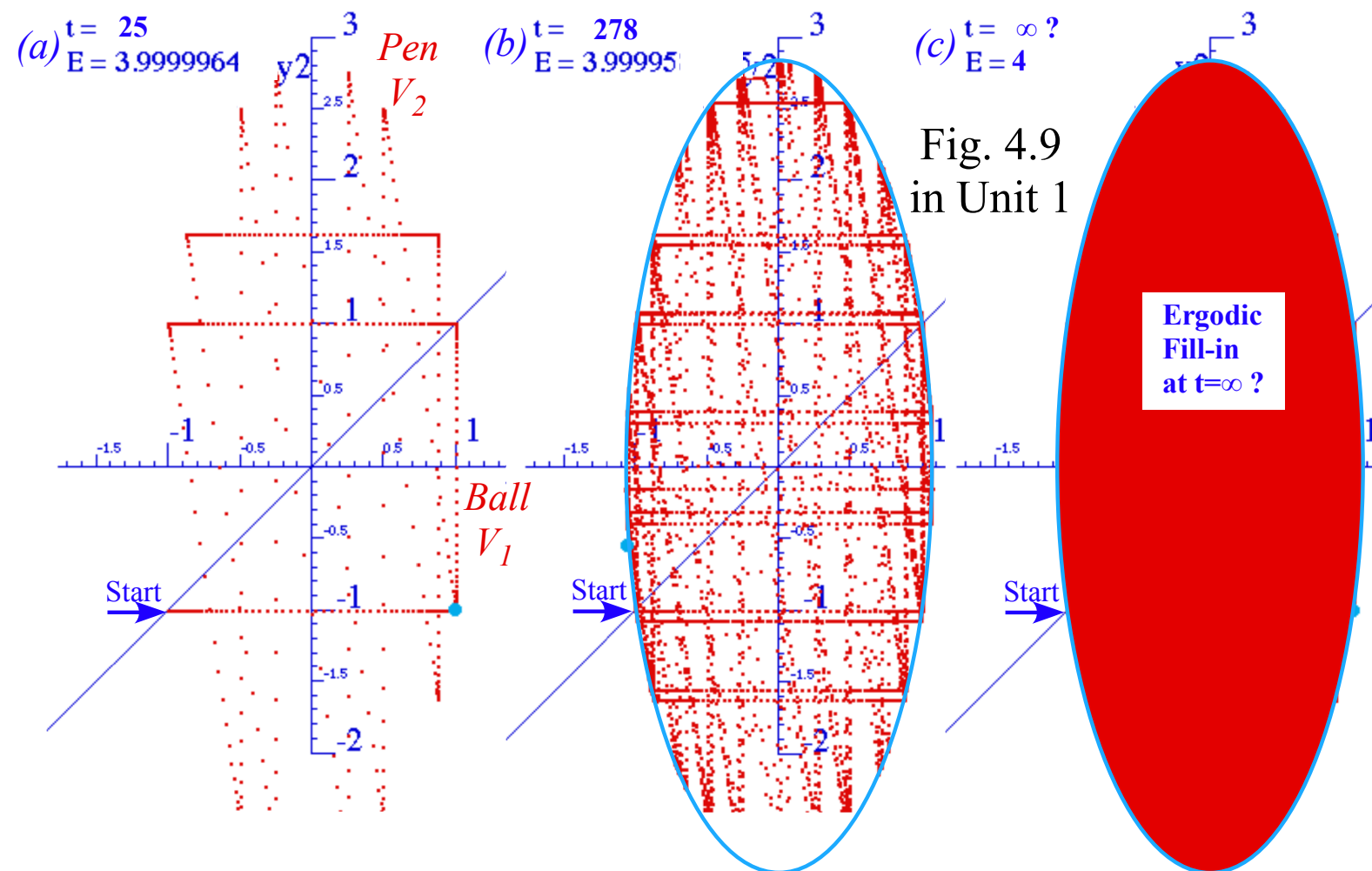


Fig. 4.7a-d
in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)



BounceIt Superball Collision Web Simulator:
 $M_1=70$, $M_2=10$ with Newtonian time plot



BounceIt Superball Collision Web Simulator:
 $M_1=70$, $M_2=10$ with V_2 vs V_1 plot

Geometry of X2 launcher bouncing in box (gravity-free)

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

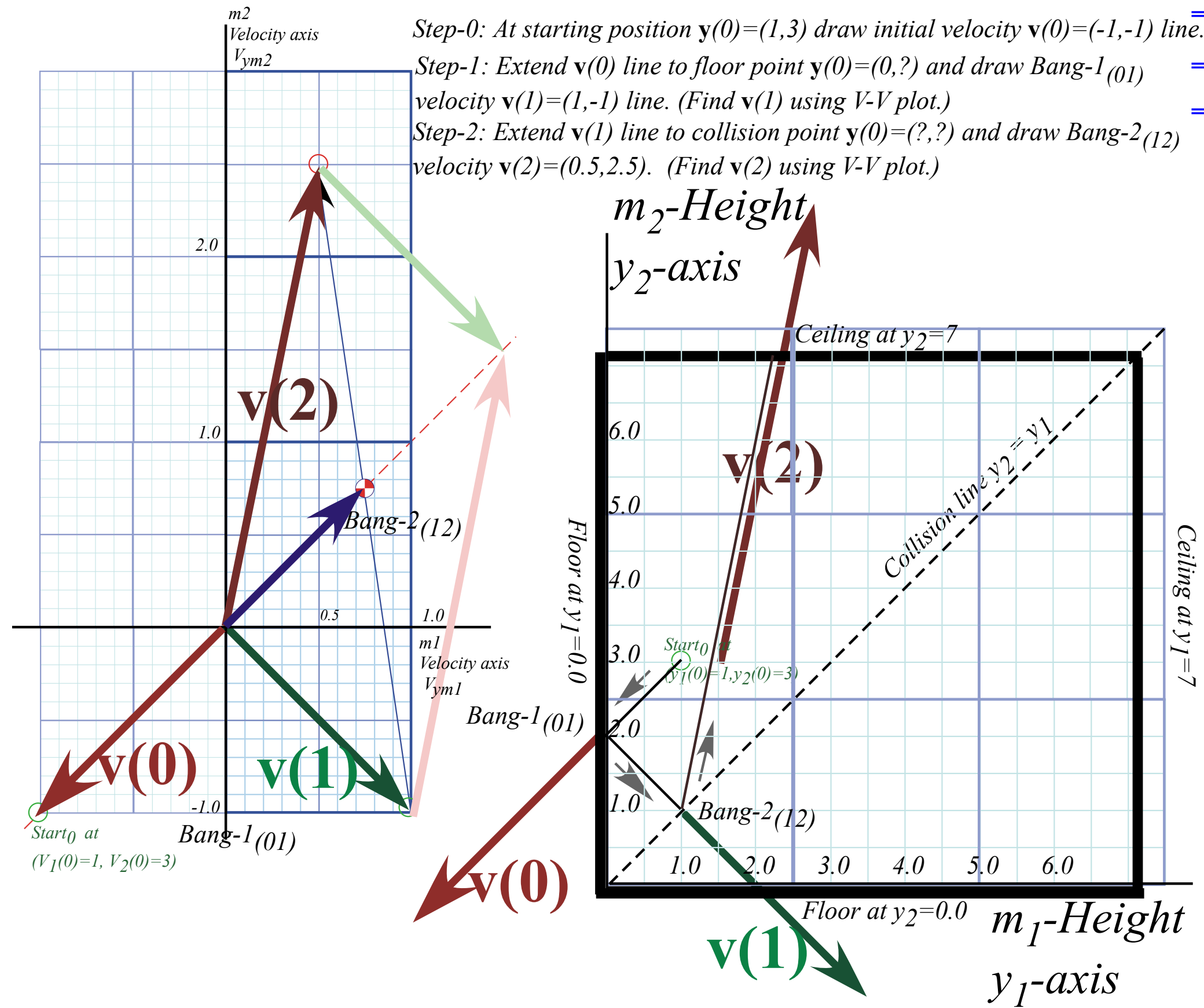
 *Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

Geometric “Integration” (Converting Velocity data to Space-space trajectory)

Fig. 4.11
in Unit 1

Step-0: At starting position $\mathbf{y}(0)=(1,3)$ draw initial velocity $\mathbf{v}(0)=(-1,-1)$ line.
Step-1: Extend $\mathbf{v}(0)$ line to floor point $\mathbf{y}(0)=(0,?)$ and draw Bang-1₍₀₁₎ velocity $\mathbf{v}(1)=(1,-1)$ line. (Find $\mathbf{v}(1)$ using V-V plot.)
Step-2: Extend $\mathbf{v}(1)$ line to collision point $\mathbf{y}(0)=(?,?)$ and draw Bang-2₍₁₂₎ velocity $\mathbf{v}(2)=(0.5,2.5)$. (Find $\mathbf{v}(2)$ using V-V plot.)

Ellipse radius 1	Ellipse radius 2
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_2}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

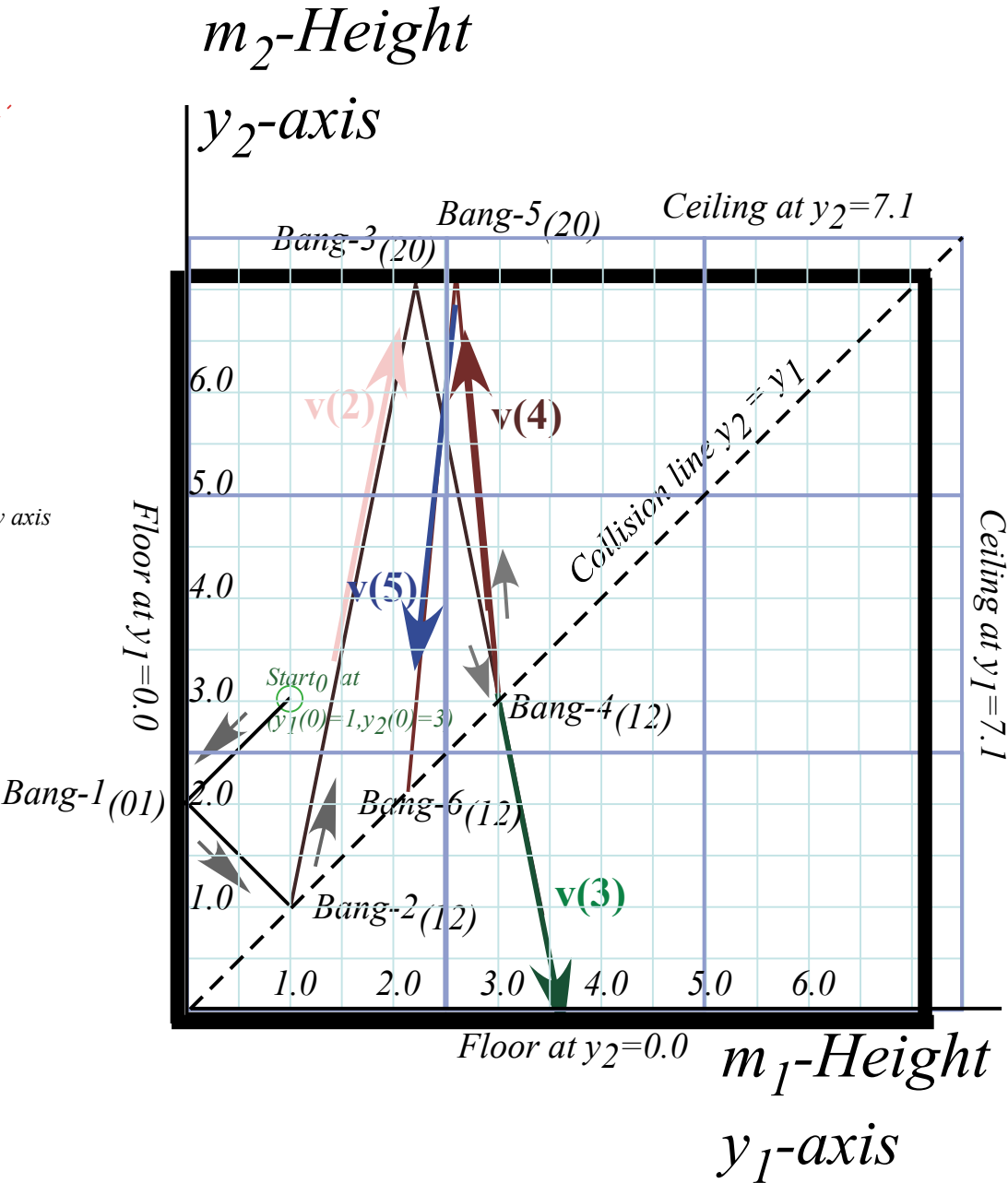
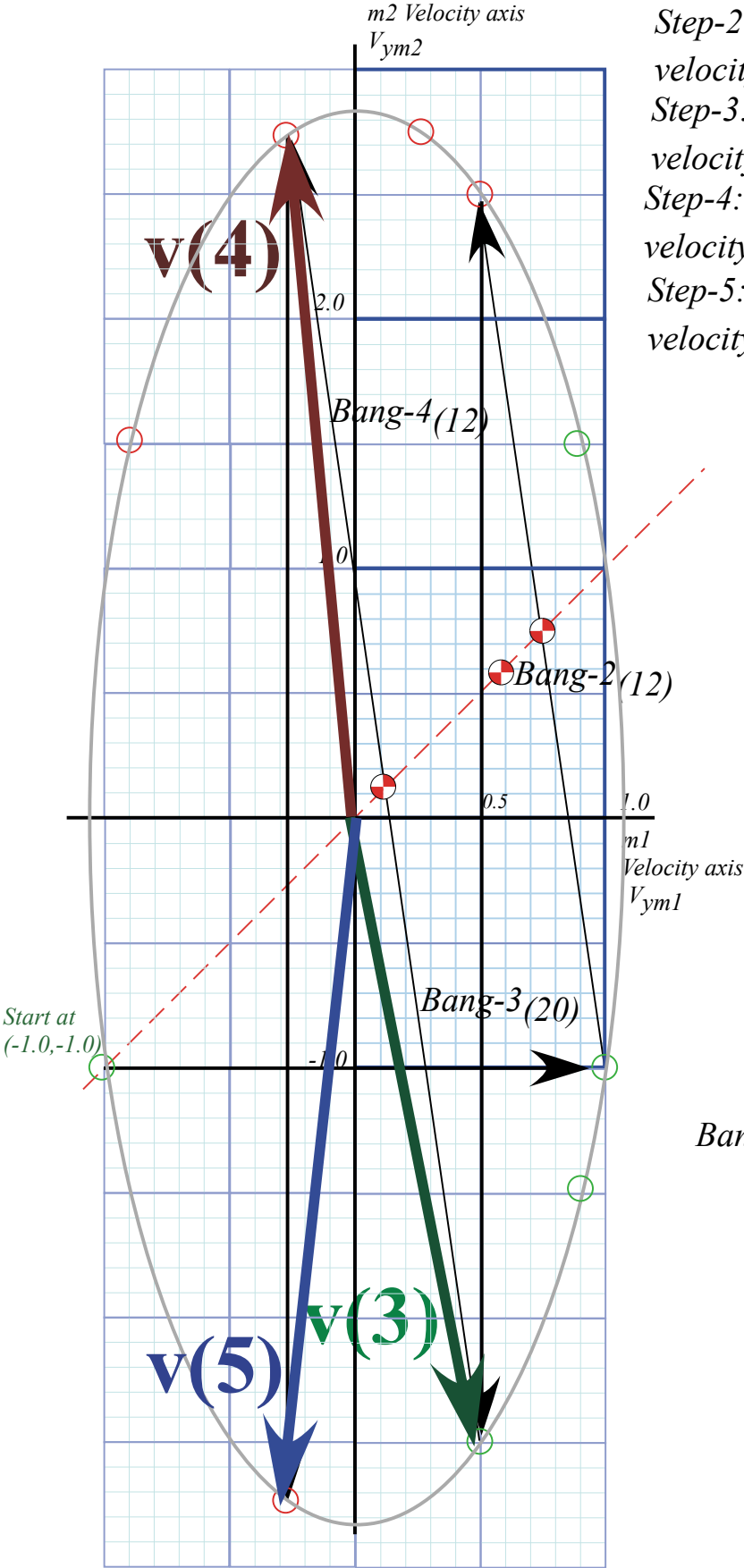


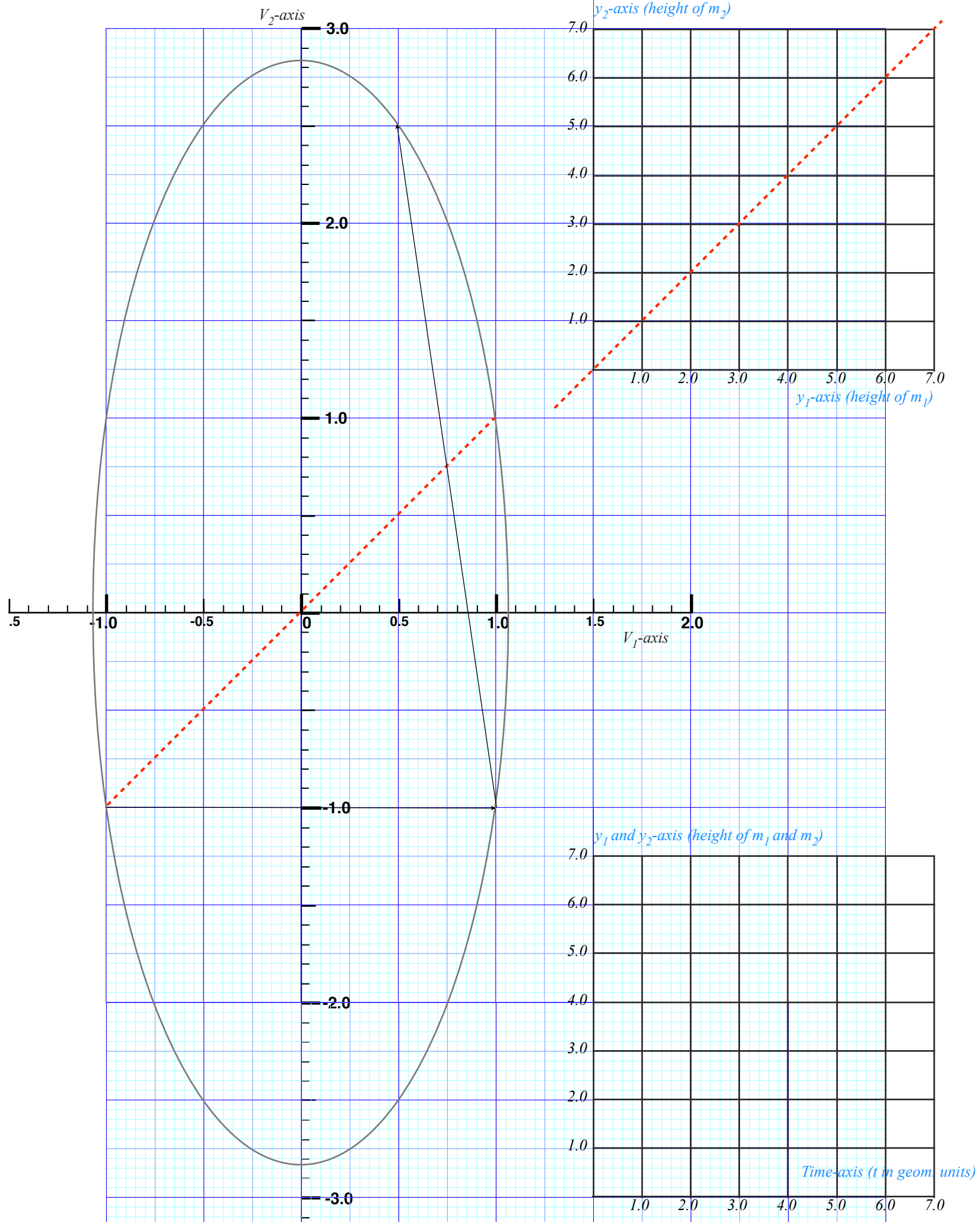
Geometric “Integration” (Converting Velocity data to Space-space trajectory)

Fig. 4.11
in Unit 1

Step-2: Extend $\mathbf{v}(2)$ line to ceiling point $\mathbf{y}(3)=(?,7.1)$ and draw Bang-3(20) velocity $\mathbf{v}(3)=(1,-1)$ line. (Find $\mathbf{v}(3)$ using V-V plot.)
Step-3: Extend $\mathbf{v}(3)$ line to collision point $\mathbf{y}(4)=(?,?)$ and draw Bang-4(12) velocity $\mathbf{v}(4)=(0.5,2.5)$. (Find $\mathbf{v}(4)$ using V-V plot.)
Step-4: Extend $\mathbf{v}(4)$ line to ceiling point $\mathbf{y}(4)=(?,7.1)$ and draw Bang-5(20) velocity $\mathbf{v}(5)=(1,-1)$ line. (Find $\mathbf{v}(5)$ using V-V plot.)
Step-5: Extend $\mathbf{v}(5)$ line to collision point $\mathbf{y}(6)=(?,?)$ and draw Bang-6(12) velocity $\mathbf{v}(6)=(0.5,2.5)$. (Find $\mathbf{v}(6)$ using V-V plot.)

Ellipse radius 1	Ellipse radius 2
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_2}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$





<u>Ellipse radius 1</u>	<u>Ellipse radius 2</u>
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_2}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)

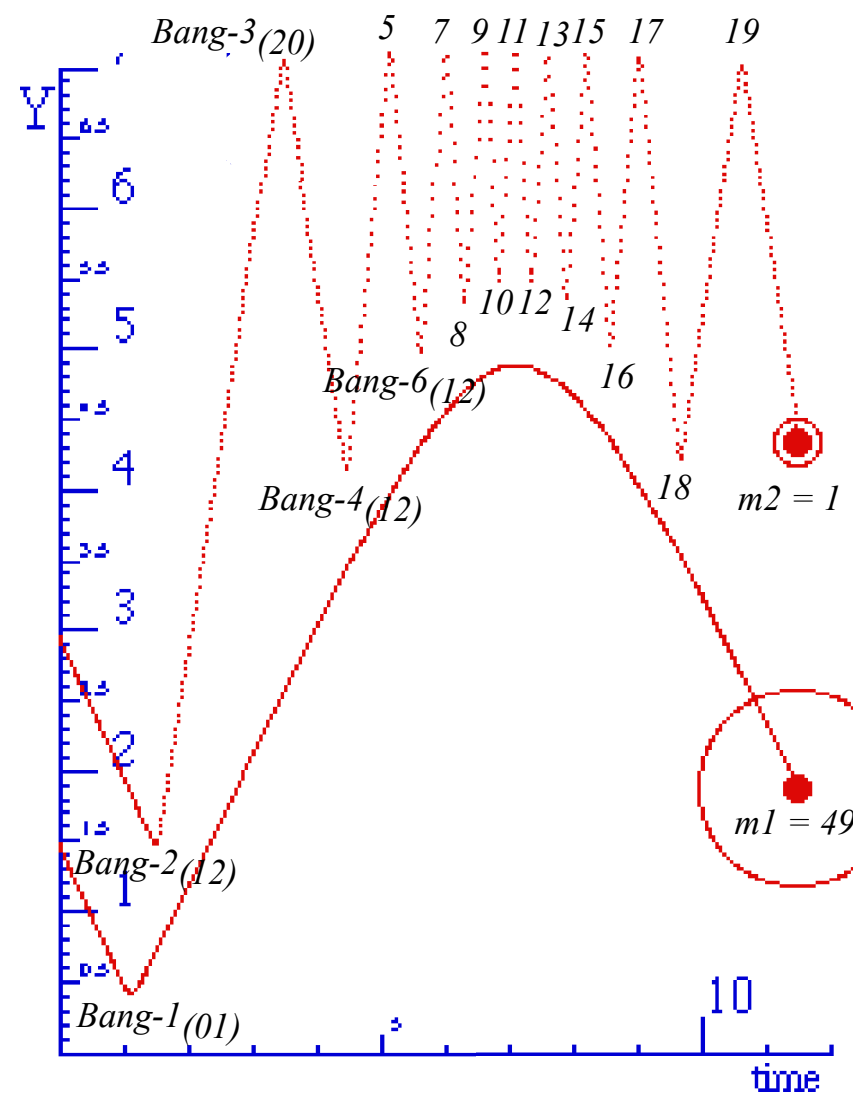


Fig. 5.1 [BounceIt Superball Collision Web Simulator:](#)
 $M_1=49, M_2=1$ with Newtonian time plot
 in Unit 1

[BounceIt Superball Collision Web Simulator:](#)
 $M_1=49, M_2=1$ with V_2 vs V_1 plot

Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)

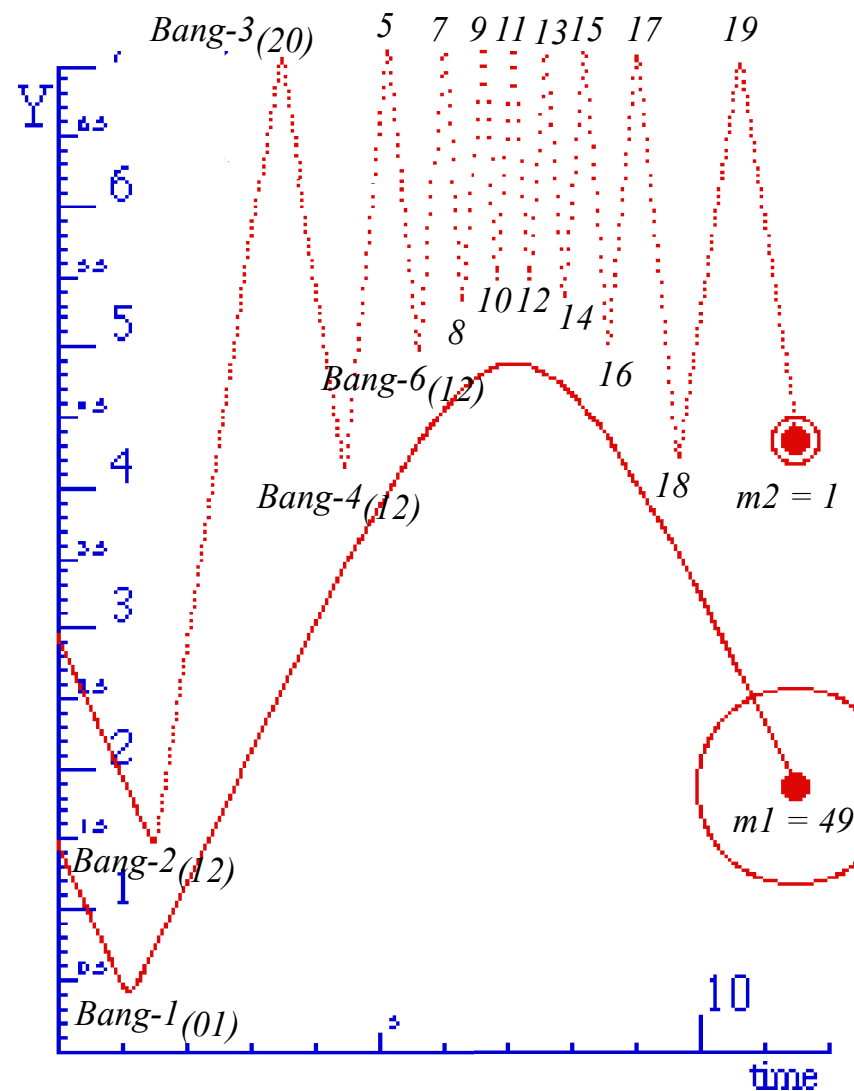
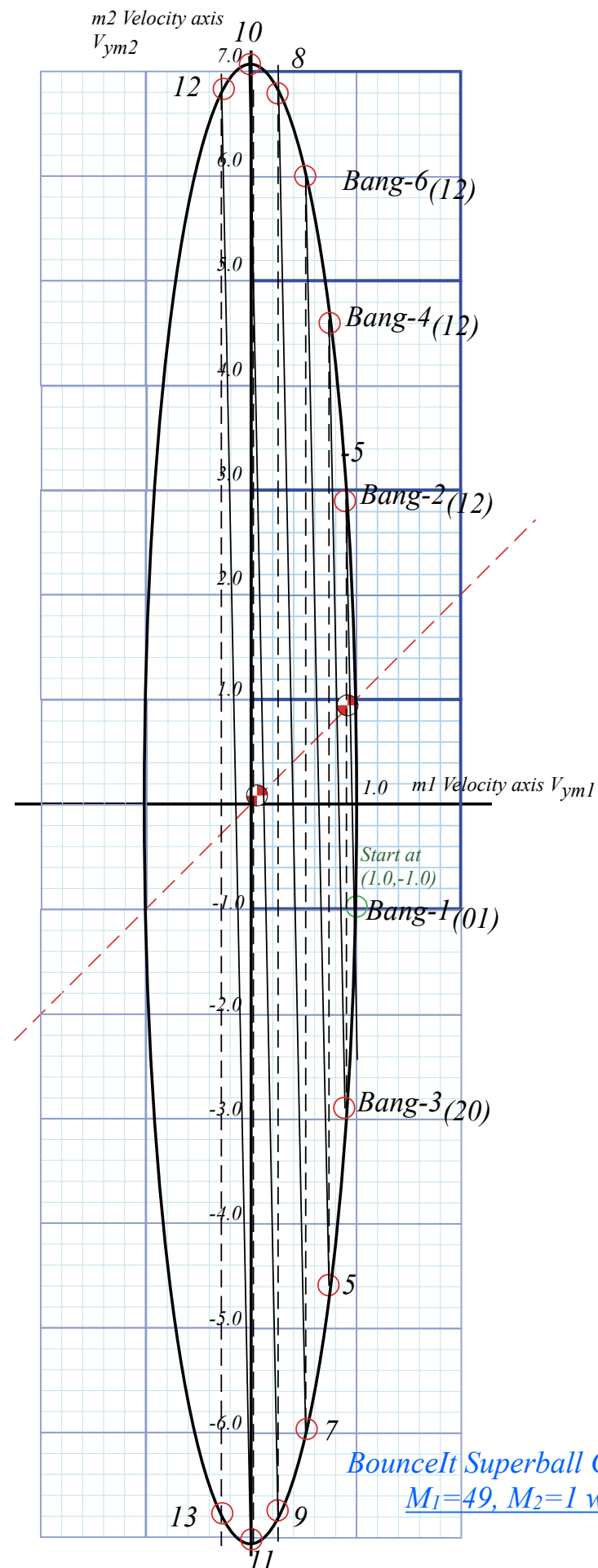
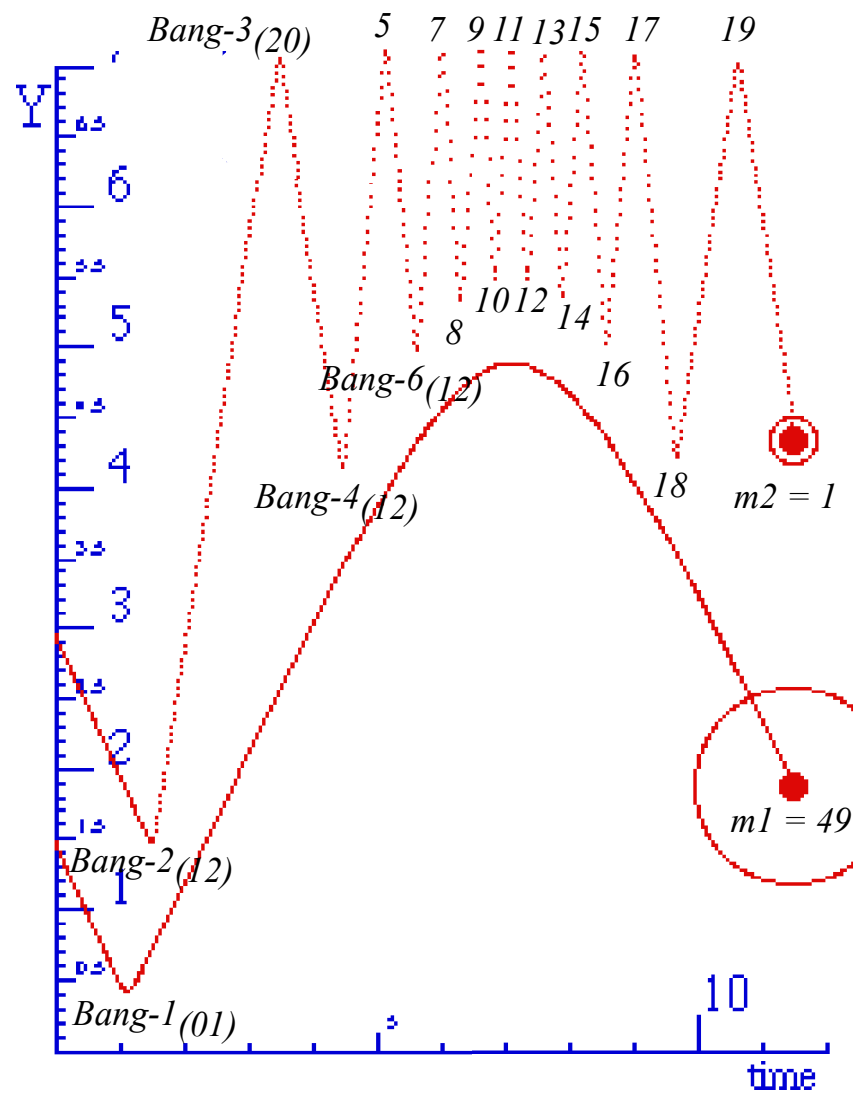
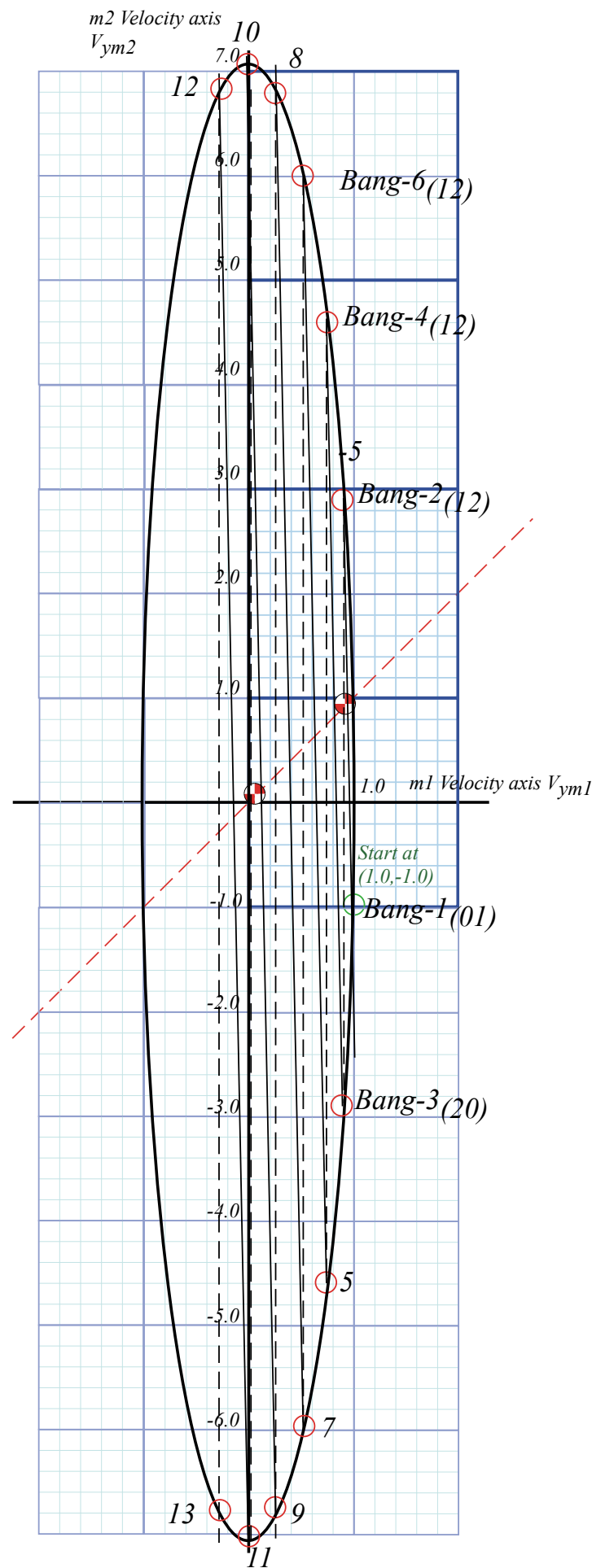


Fig. 5.1 [BounceIt Superball Collision Web Simulator: \$M_1=49, M_2=1\$ with Newtonian time plot](#)

[BounceIt Superball Collision Web Simulator: \$M_1=49, M_2=1\$ with \$V_2\$ vs \$V_1\$ plot](#)

Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$\begin{aligned} a_1 &= \sqrt{2KE/M_1} \\ &= \sqrt{2KE/49} \\ &= \sqrt{50/49} \\ &= 1.01 \end{aligned}$$

Ellipse radius 2

$$\begin{aligned} a_2 &= \sqrt{2KE/m_2} \\ &= \sqrt{2KE/1} \\ &= \sqrt{50/1} \\ &= 7.07 \end{aligned}$$

Fig. 5.1
in Unit 1

Multiple collisions calculated by matrix operator products

 *Matrix or tensor algebra of 1-D 2-body collisions*

What about that 2nd quadratic solution?

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R= C•M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} :$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \frac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \frac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$.


$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \frac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \frac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

 *What about that 2nd quadratic solution?*

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R= C•M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

What about that 2nd quadratic solution?

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$ gives just *one* solution to quadratic collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

What about that 2nd quadratic solution?

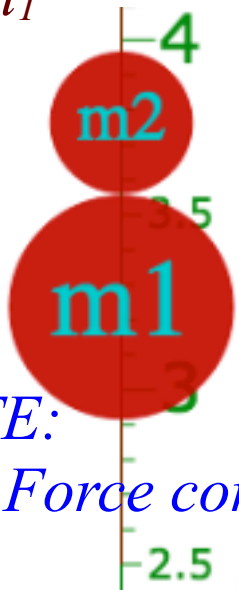
Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$ gives just *one* solution to quadratic collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

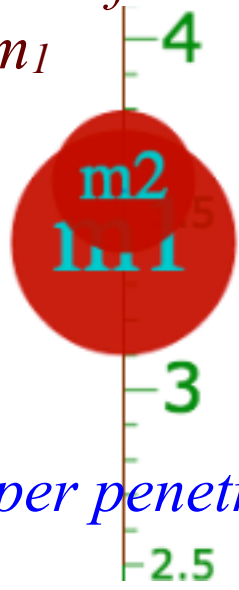
Q: What is the *second* solution and to what simple process would it correspond?

Example with friction

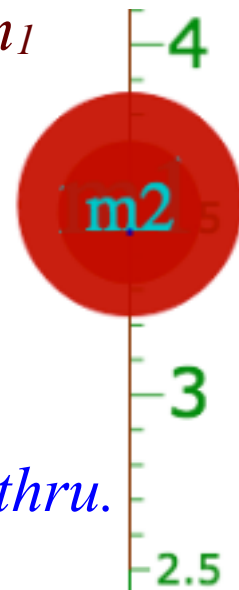
m_2
enters
 m_1



center of m_2
approaches
center of
 m_1



center of m_2
just past
center of
 m_1



...and quickly
accelerates
downward...

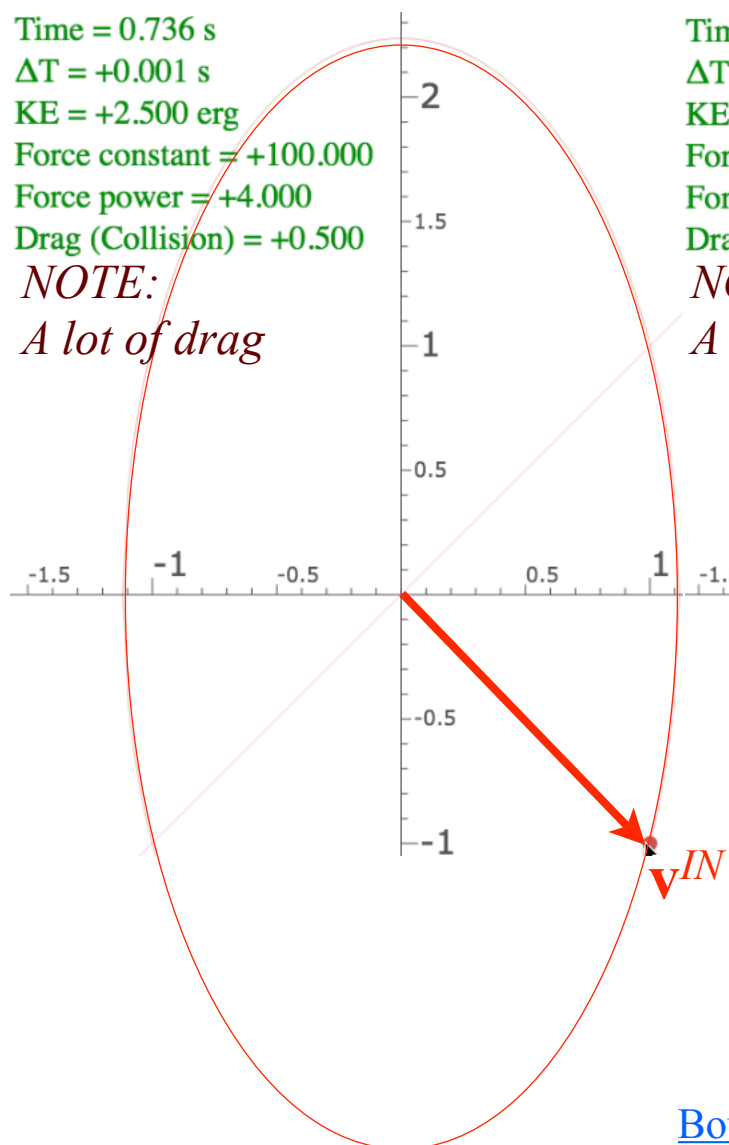


...thru drag until emerging from
 m_1 with $|\mathbf{v}^{FIN}|$ less than $|\mathbf{v}^{IN}|$

NOTE:
Low Force constant allows deeper penetration and pass-thru.

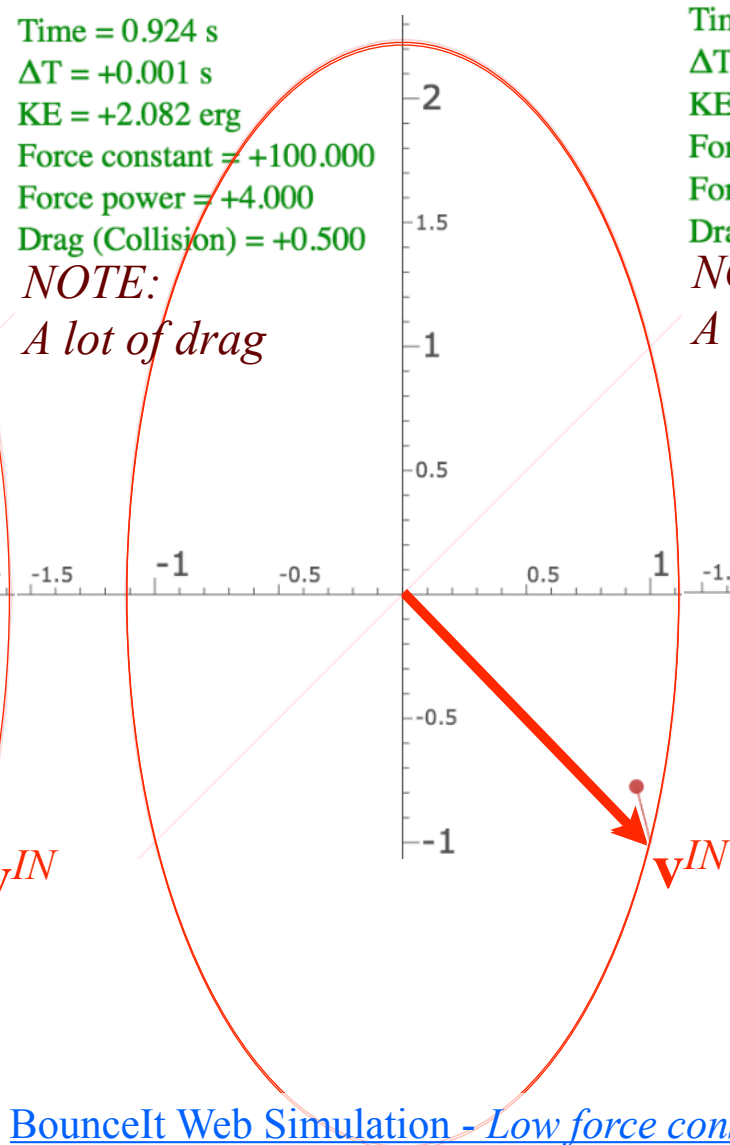
Time = 0.736 s
 $\Delta T = +0.001$ s
KE = +2.500 erg
Force constant = +100.000
Force power = +4.000
Drag (Collision) = +0.500

NOTE:
A lot of drag



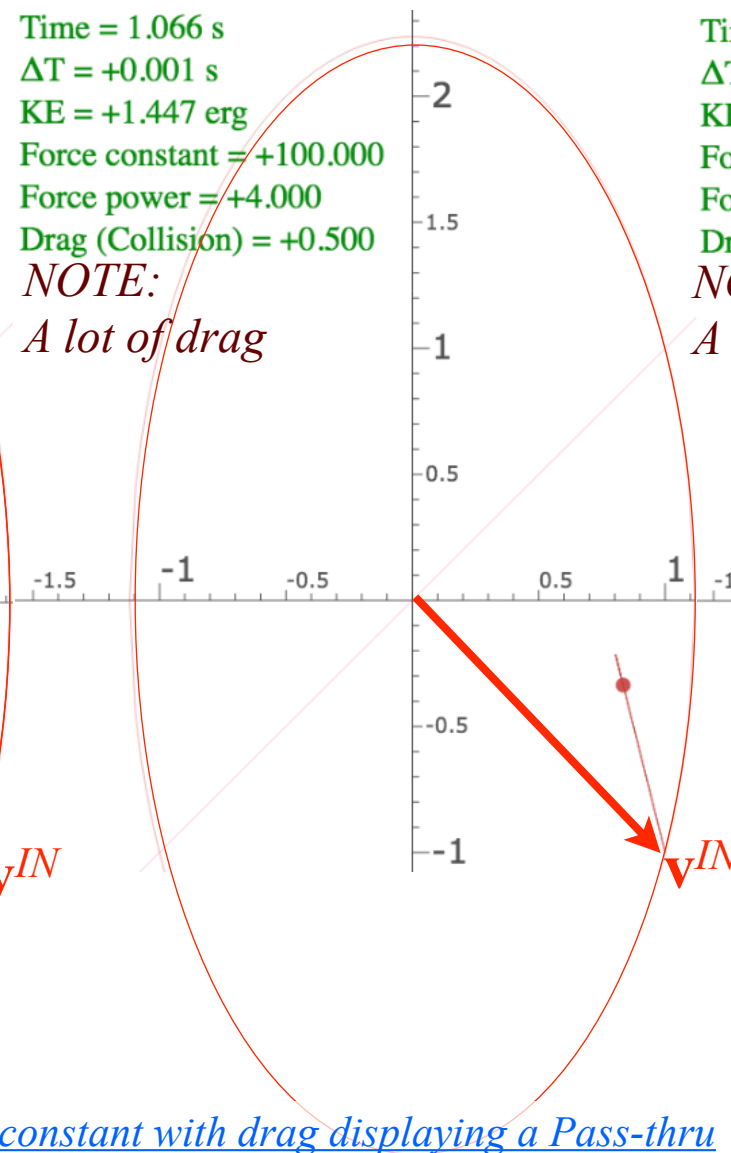
Time = 0.924 s
 $\Delta T = +0.001$ s
KE = +2.082 erg
Force constant = +100.000
Force power = +4.000
Drag (Collision) = +0.500

NOTE:
A lot of drag



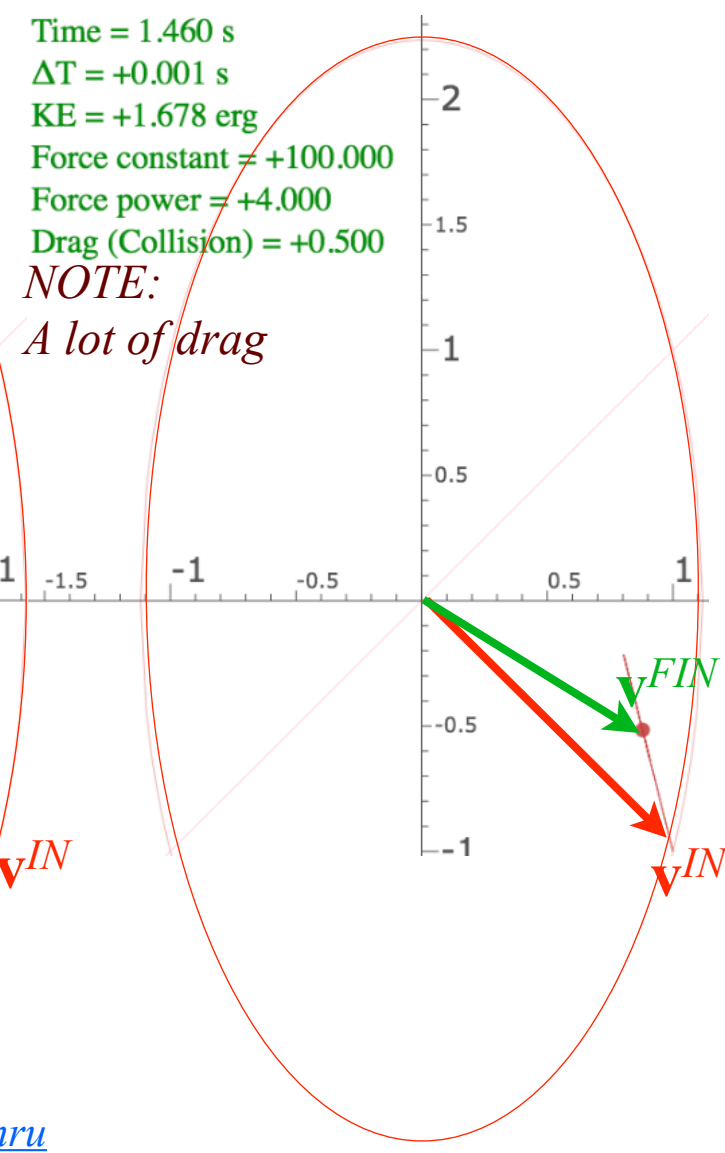
Time = 1.066 s
 $\Delta T = +0.001$ s
KE = +1.447 erg
Force constant = +100.000
Force power = +4.000
Drag (Collision) = +0.500

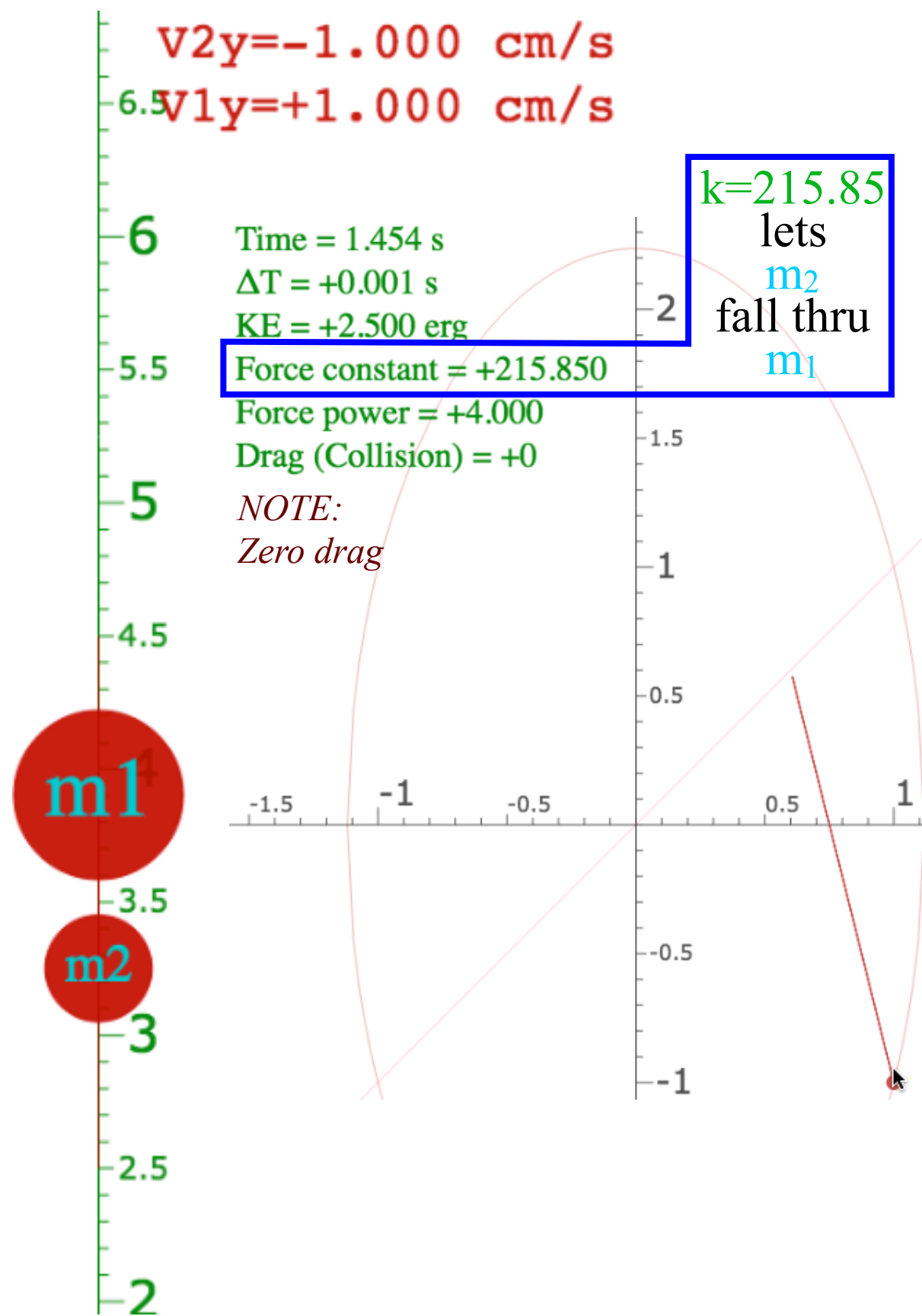
NOTE:
A lot of drag



Time = 1.460 s
 $\Delta T = +0.001$ s
KE = +1.678 erg
Force constant = +100.000
Force power = +4.000
Drag (Collision) = +0.500

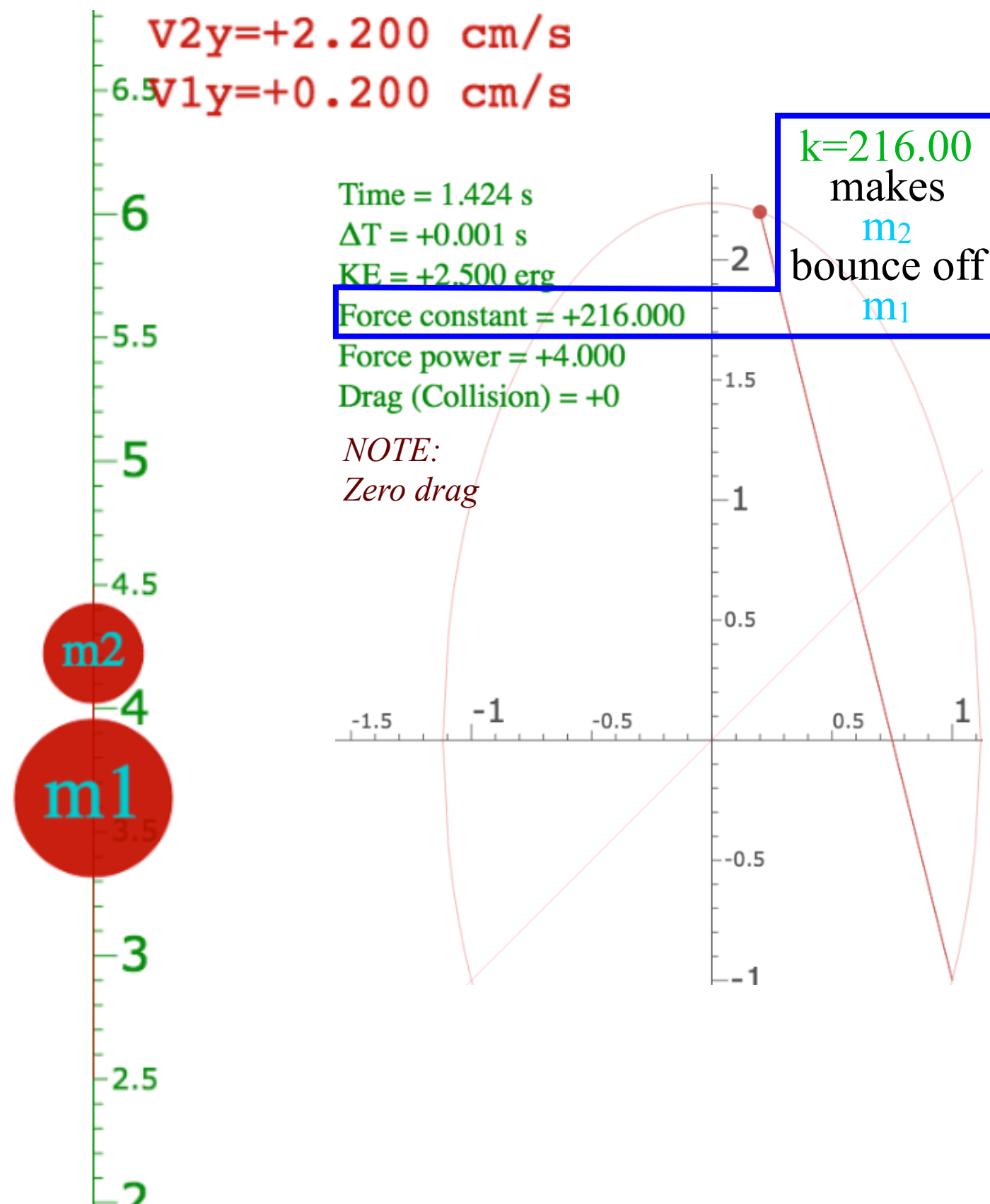
NOTE:
A lot of drag





Fall-Thru

BounceIt Superball Collision Web Simulations
with Low force constant & Zero drag



Bounce-Off

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

 *“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R**= **C**•**M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_I :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

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$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

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Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

 *Geometry and algebra of “ellipse-Rotation” group product: **R**= **C**•**M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define “ellipse-Rotation” \mathbf{R} as group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

$$\begin{pmatrix} |FIN^9\rangle \\ v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \textcolor{red}{\mathbf{C}} \cdot \textcolor{blue}{\mathbf{M}} \cdot \textcolor{red}{\mathbf{C}} \cdot \textcolor{blue}{\mathbf{M}} \cdot \textcolor{red}{\mathbf{C}} \cdot \textcolor{blue}{\mathbf{M}} \cdot \textcolor{red}{\mathbf{C}} \cdot \textcolor{blue}{\mathbf{M}} \cdot \textcolor{red}{\mathbf{F}} \begin{pmatrix} |IN^0\rangle \\ v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL (0)})}$$

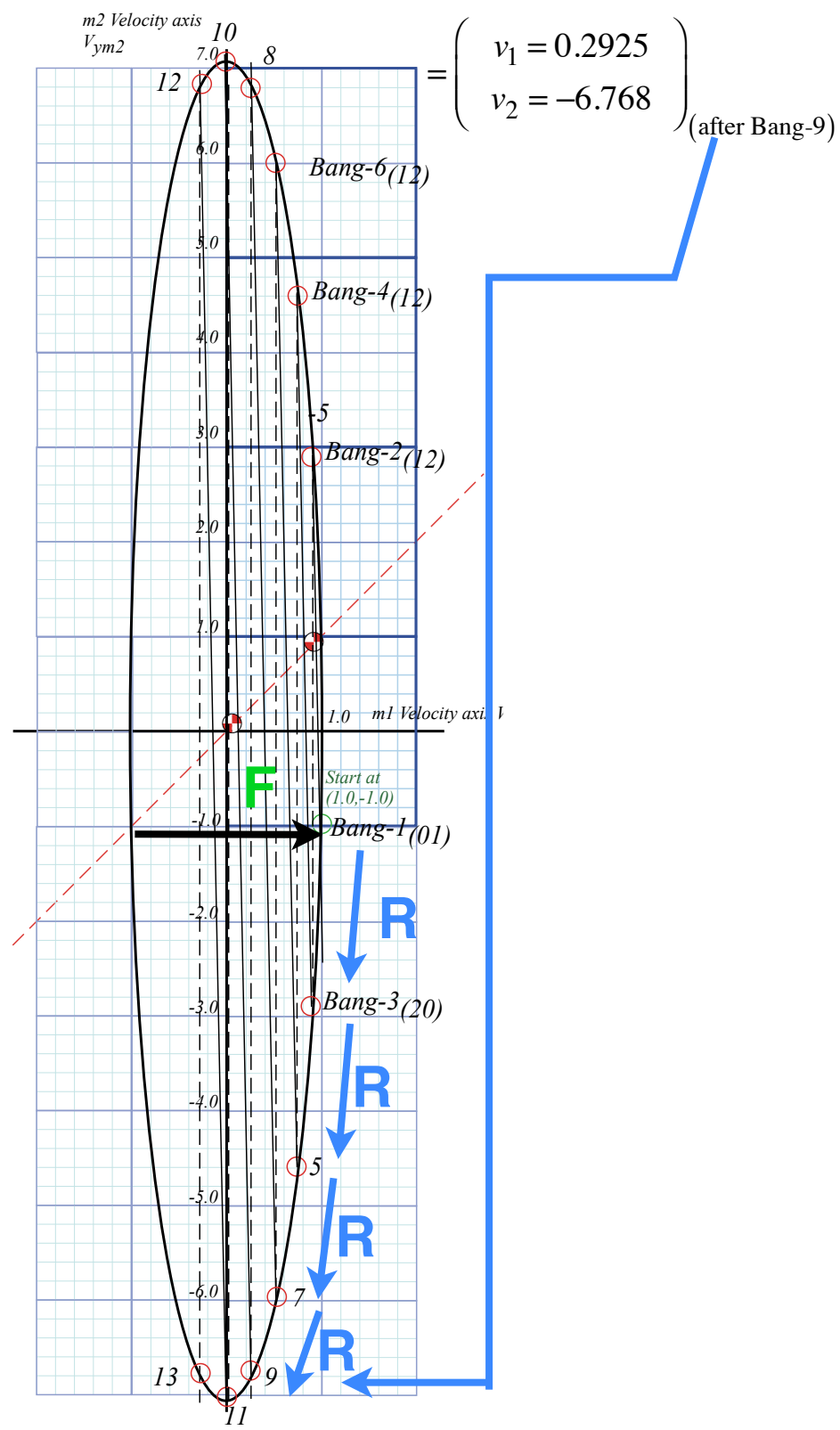
$$\begin{aligned}
|FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} |IN^0\rangle \\
\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL (0)})} \\
|FIN^9\rangle &= \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle \\
\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}
\end{aligned}$$

“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$\begin{aligned}
\left| FIN^9 \right\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \left| IN^0 \right\rangle \\
\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL (0)})} \\
\left| FIN^9 \right\rangle &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \mathbf{F} \left| IN^0 \right\rangle \\
\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})} \\
&= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{(\text{after Bang-9})}
\end{aligned}$$

“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$\begin{aligned}
 |FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL (0)})} \\
 |FIN^9\rangle &= \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}
 \end{aligned}$$

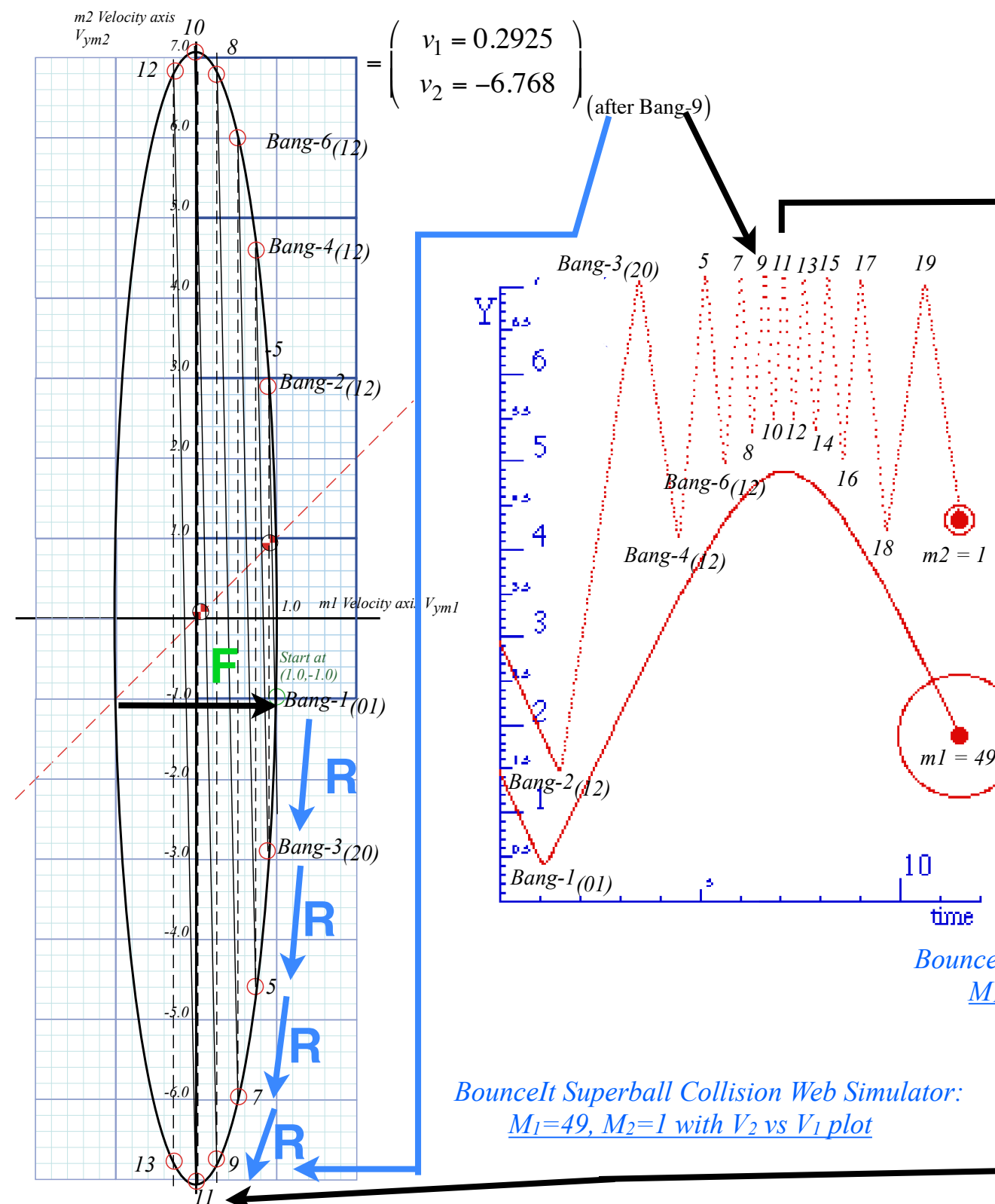


“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

Collisions for
mass ratio
 $m_1:m_2 = 49:1$

Fig. 5.1a
(revised)

$$\begin{aligned}
 |FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix} \quad (\text{INITIAL (0)}) \\
 |FIN^9\rangle &= \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix} \quad (\text{after Bang-1})
 \end{aligned}$$



“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$\begin{aligned}
 \begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} \\
 &= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix} \quad (\text{after Bang-11})
 \end{aligned}$$

Collisions for
mass ratio
 $m_1:m_2 = 49:1$

BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with Newtonian time plot

<<Under Construction>>
Matrix Collision Web Simulator:
 $M_1=49, M_2=1$ V_2 vs V_1 plot

Fig. 5.1a-b
(revised)

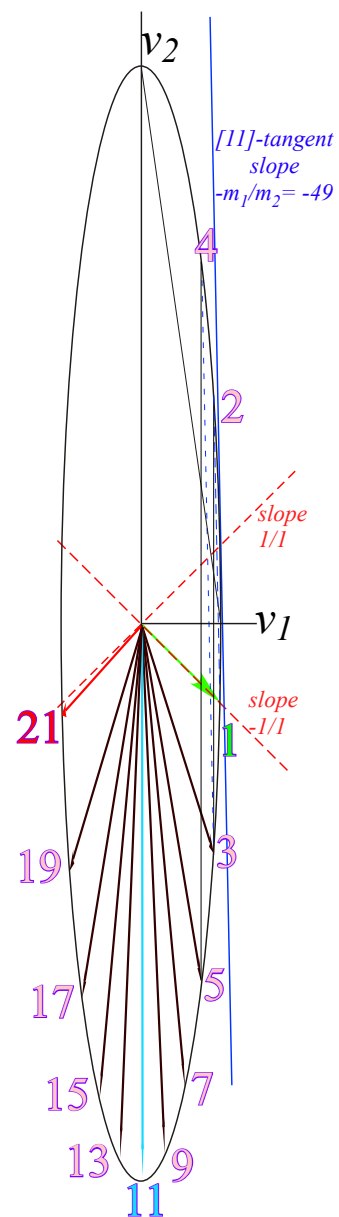
Ellipse rescaling-geometry and reflection-symmetry analysis

 *Rescaling KE ellipse to circle*

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

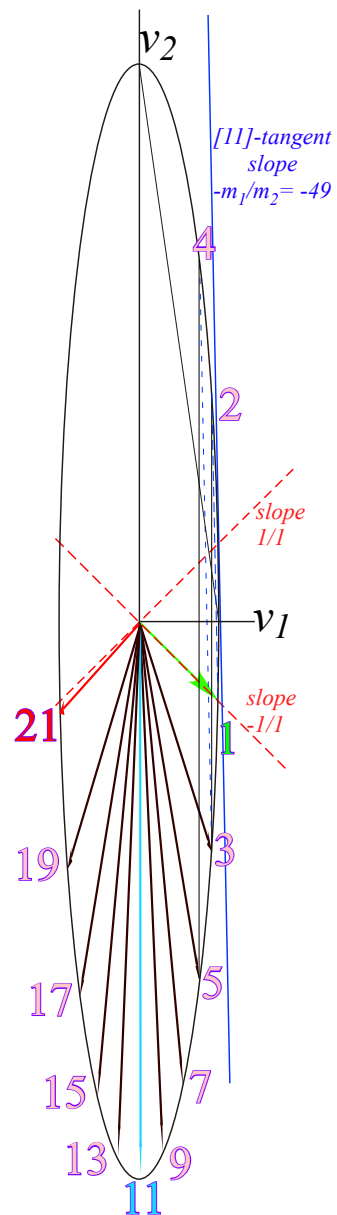


*Collisions for
mass ratio
 $m_1:m_2 = 49:1$*

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathbf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathbf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 / \sqrt{m_1} \\ \mathbf{V}_2 / \sqrt{m_2} \end{pmatrix}$$



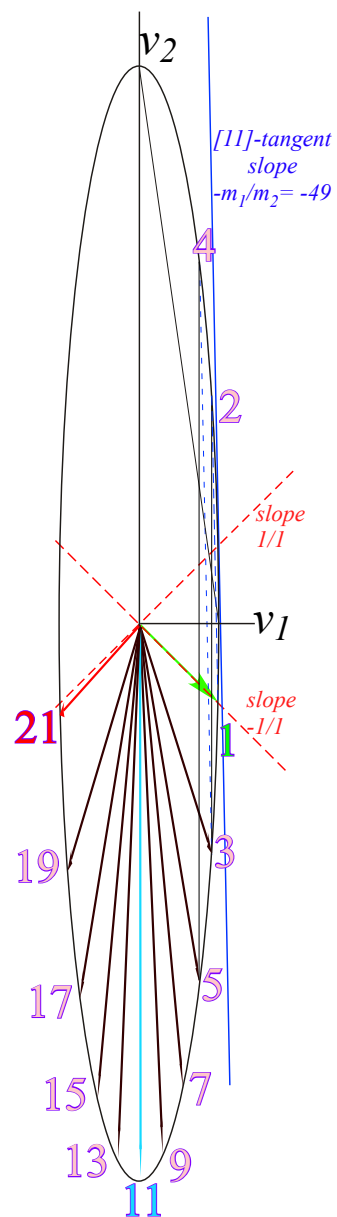
*Collisions for
mass ratio
 $m_1:m_2=49:1$*

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or: } \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V} \quad , \quad \text{or: } \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$



Collisions for
mass ratio
 $m_1:m_2 = 49:1$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathbf{V}_1 = \mathbf{v}_1 \cdot \sqrt{m_1}$, $\mathbf{V}_2 = \mathbf{v}_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathbf{V}_1^2 + \frac{1}{2} \mathbf{V}_2^2$

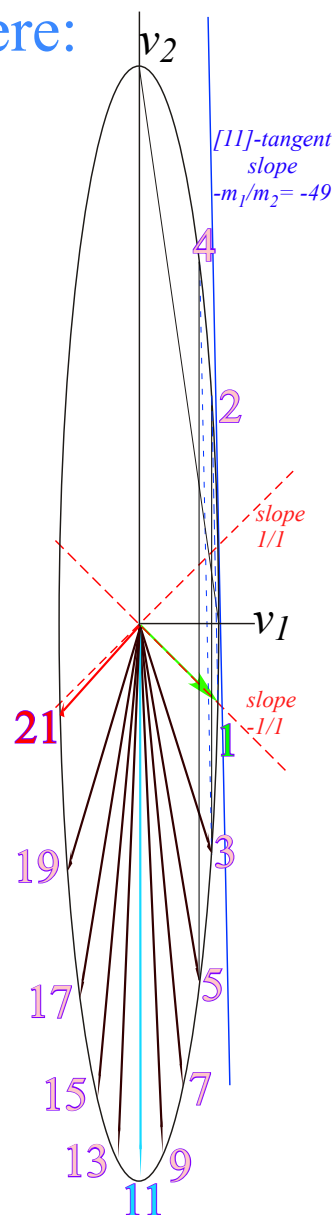
$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathbf{V}_1^{FIN1} / \sqrt{m_1} \\ \mathbf{V}_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 / \sqrt{m_1} \\ \mathbf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or: } \begin{pmatrix} \mathbf{V}_1^{FIN1} \\ \mathbf{V}_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{pmatrix} = \mathbf{M} \cdot \vec{\mathbf{V}} \quad , \quad \text{or: } \begin{pmatrix} \mathbf{V}_1^{FIN2} \\ \mathbf{V}_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{\mathbf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where:

$$\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad \text{and:} \quad \sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right) \quad \text{with:} \quad \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$$



Collisions for
mass ratio
 $m_1:m_2 = 49:1$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

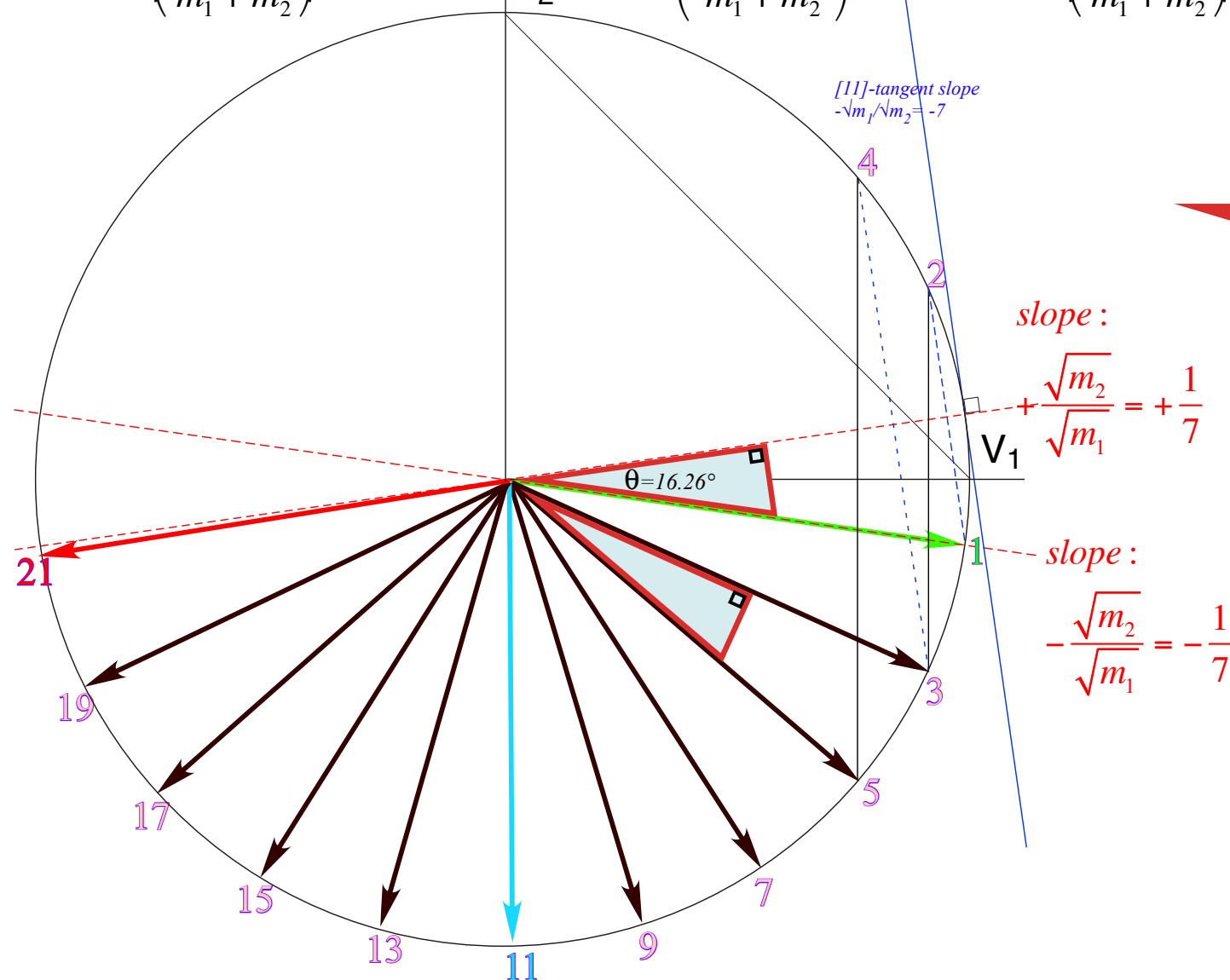
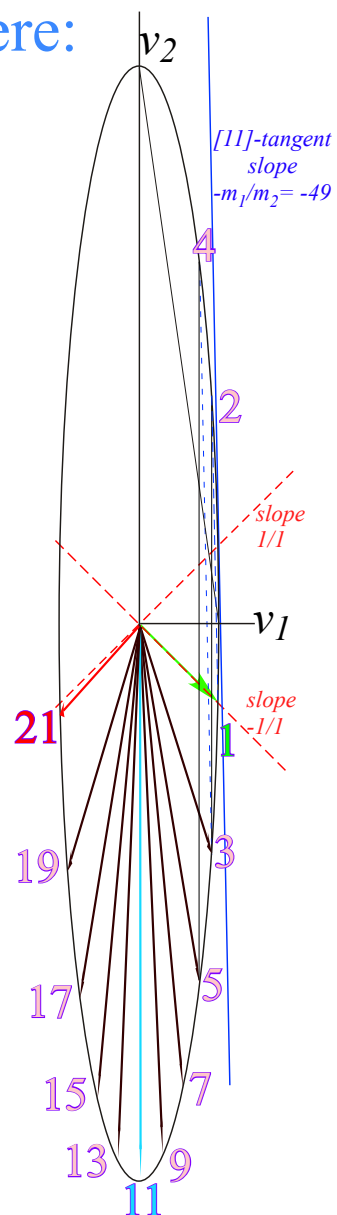
$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

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Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where:

$$\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad \text{and:} \quad V_2 \sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right) \quad \text{with:} \quad \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$$



$$\frac{m_1 - m_2}{m_1 + m_2} = \frac{48}{50} \quad \frac{2\sqrt{m_1 m_2}}{m_1 + m_2} = \frac{14}{50} \quad \theta = 16.26^\circ$$

Collisions for
mass ratio
 $m_1:m_2 = 49:1$

Fig. 5.2a-c
(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

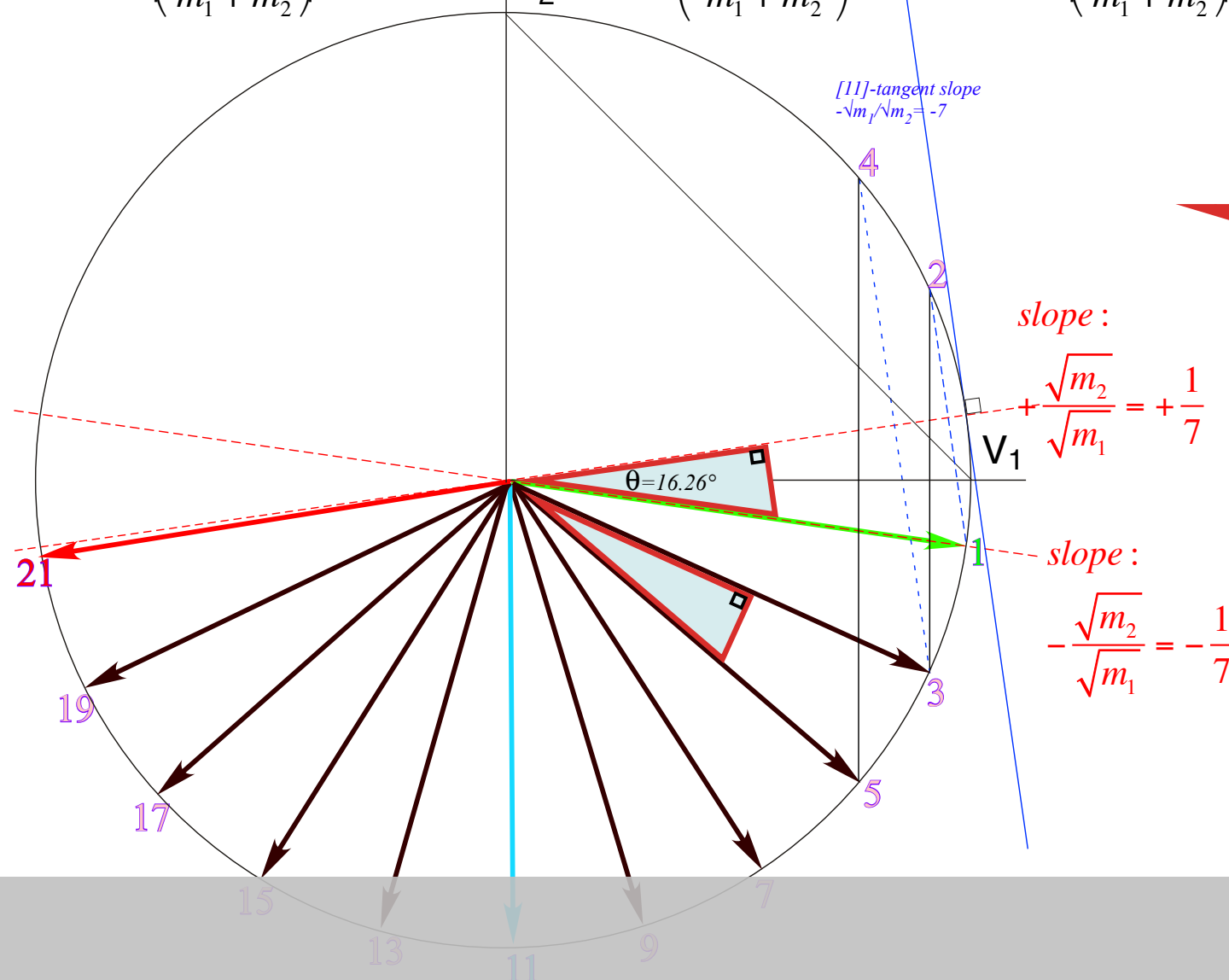
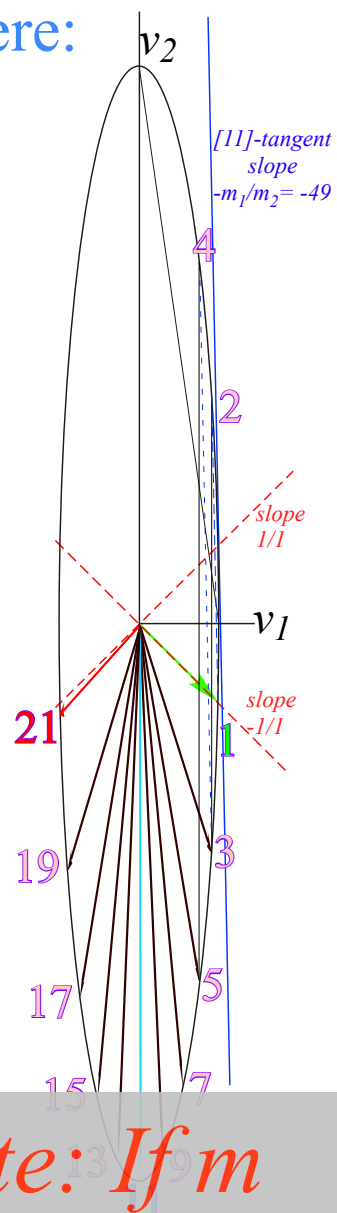
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$$\text{or: } \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V} \quad , \quad \text{or: } \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

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Collisions for
mass ratio
 $m_1:m_2 = 49:1$

Fig. 5.2a-c
(revised)

Note: If m

Ellipse rescaling geometry and reflection symmetry analysis

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with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

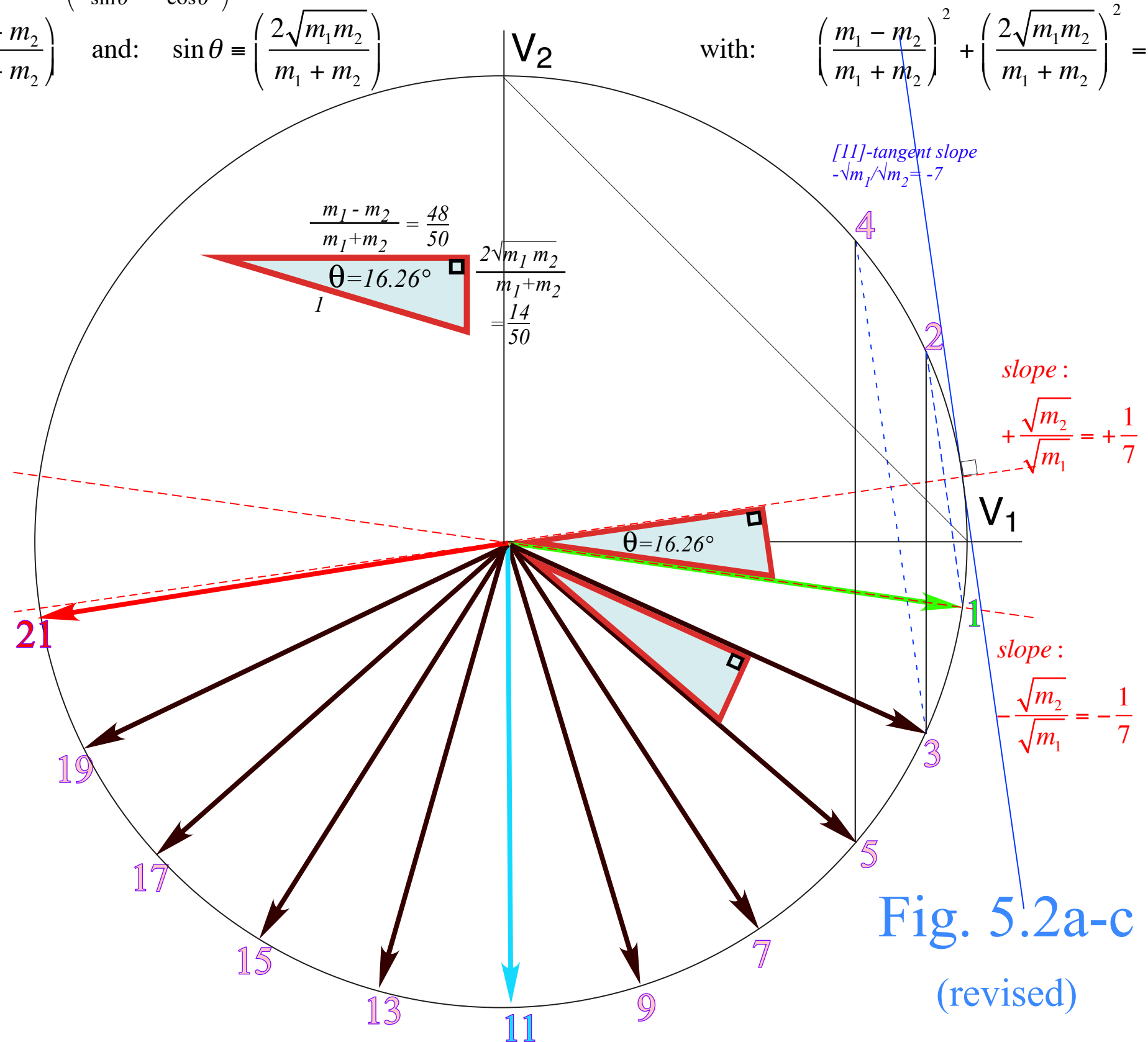
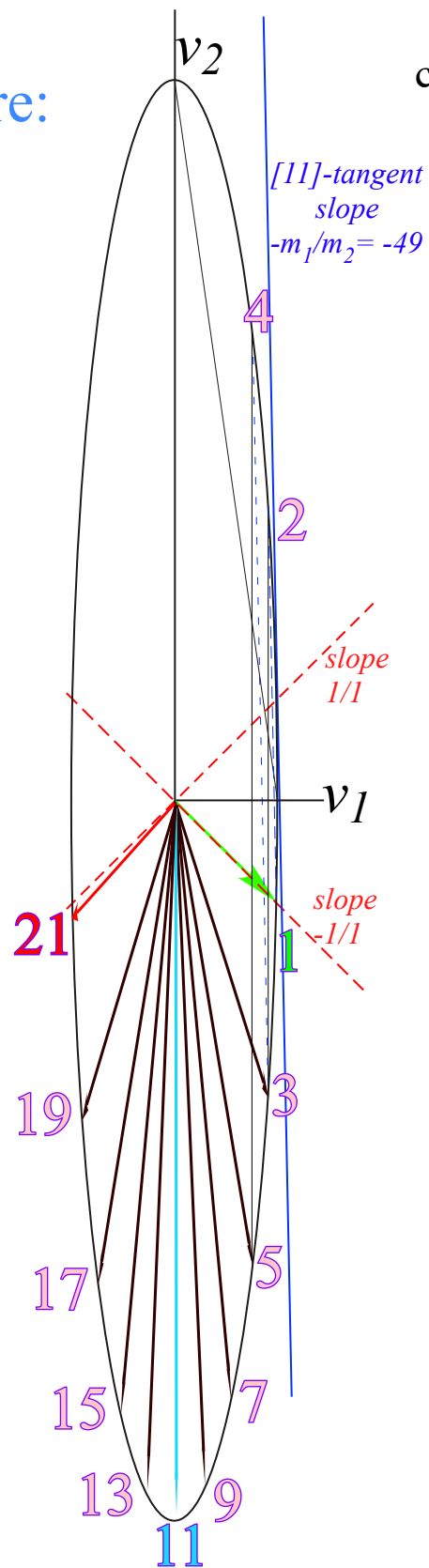
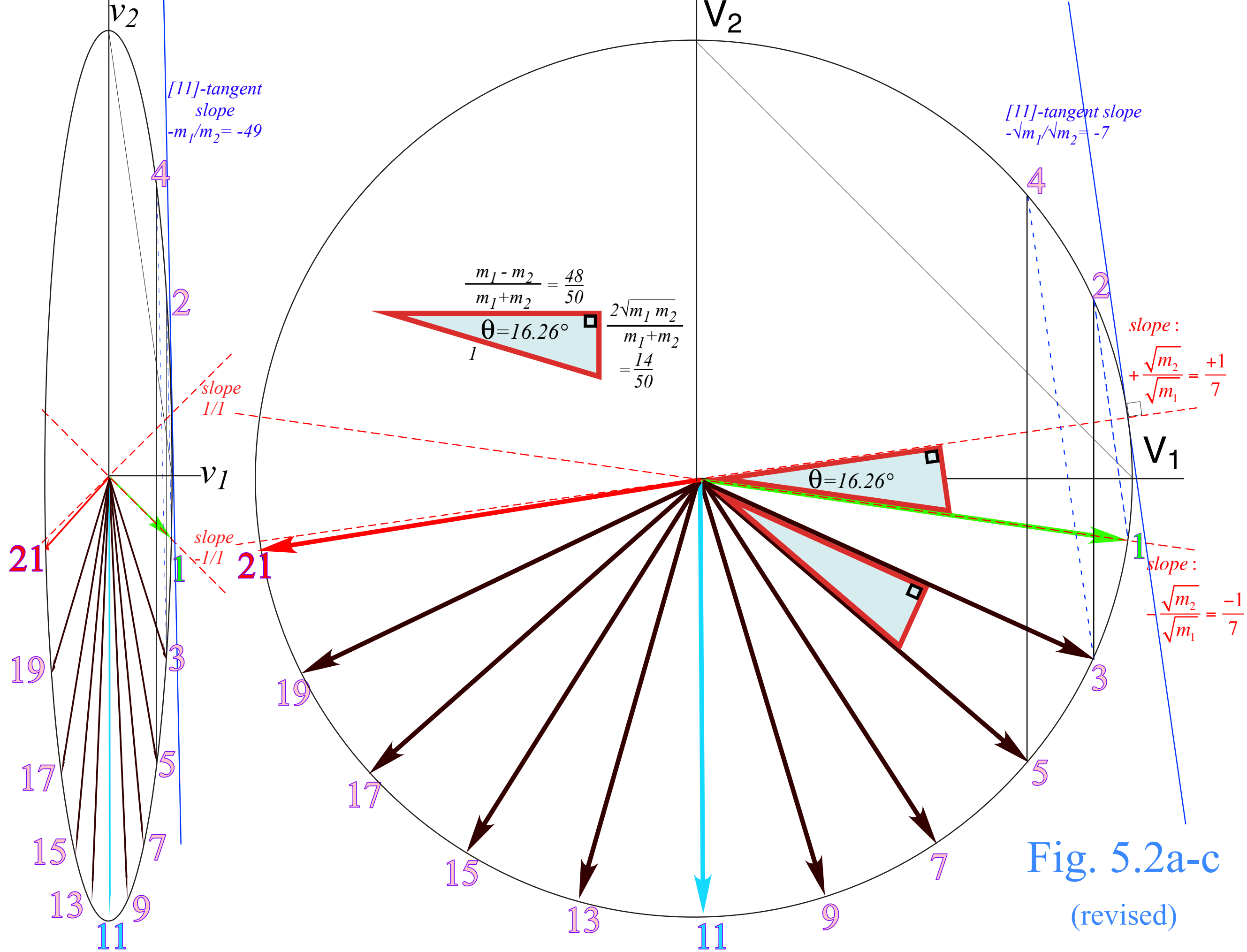


Fig. 5.2a-c
(revised)



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

 *How this relates to **Lagrangian**, **l'Etrangian**, and **Hamiltonian** mechanics later on*

Reflections in the clothing store: “It’s all done with mirrors!”

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

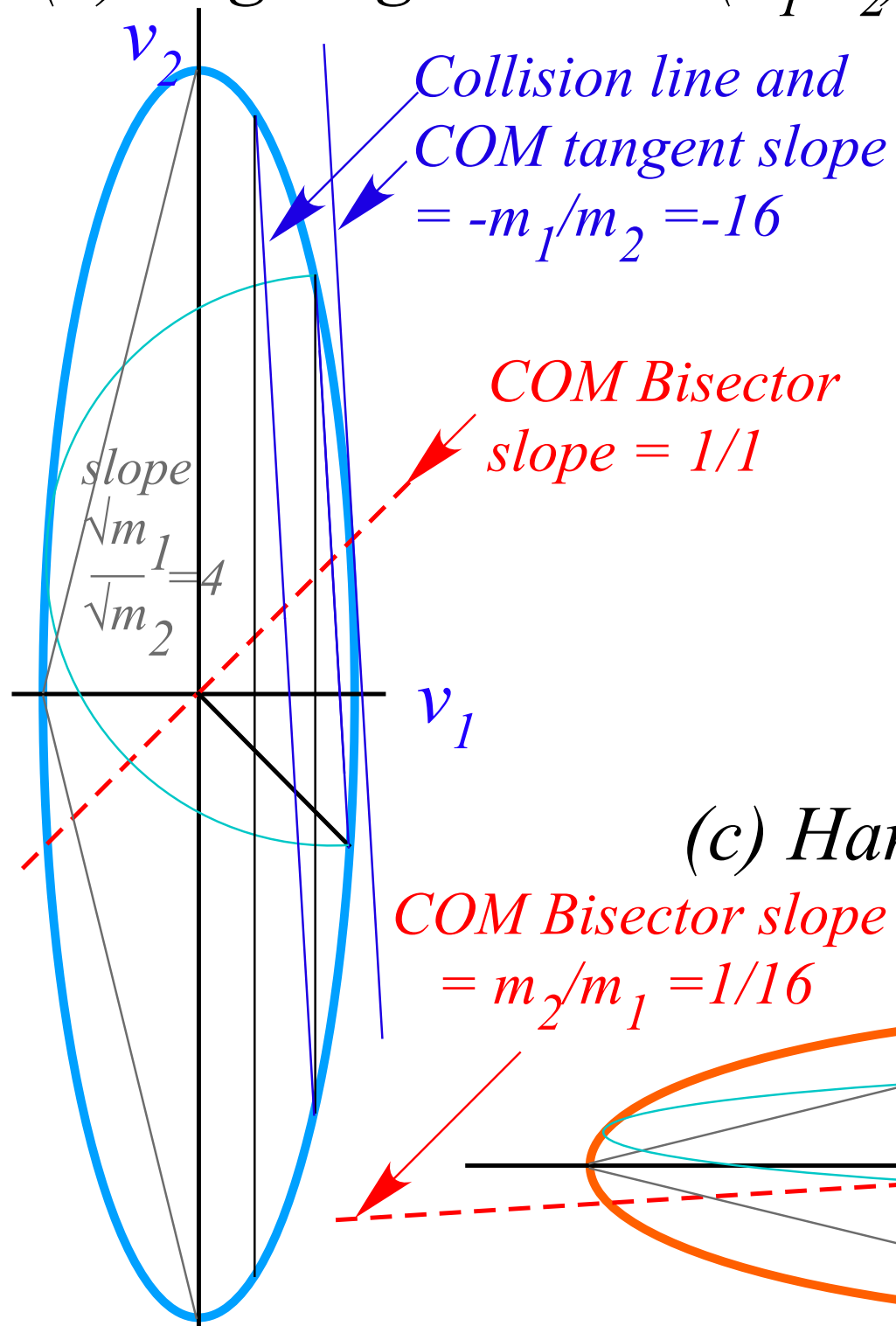
Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

What ellipse rescaling leads to...

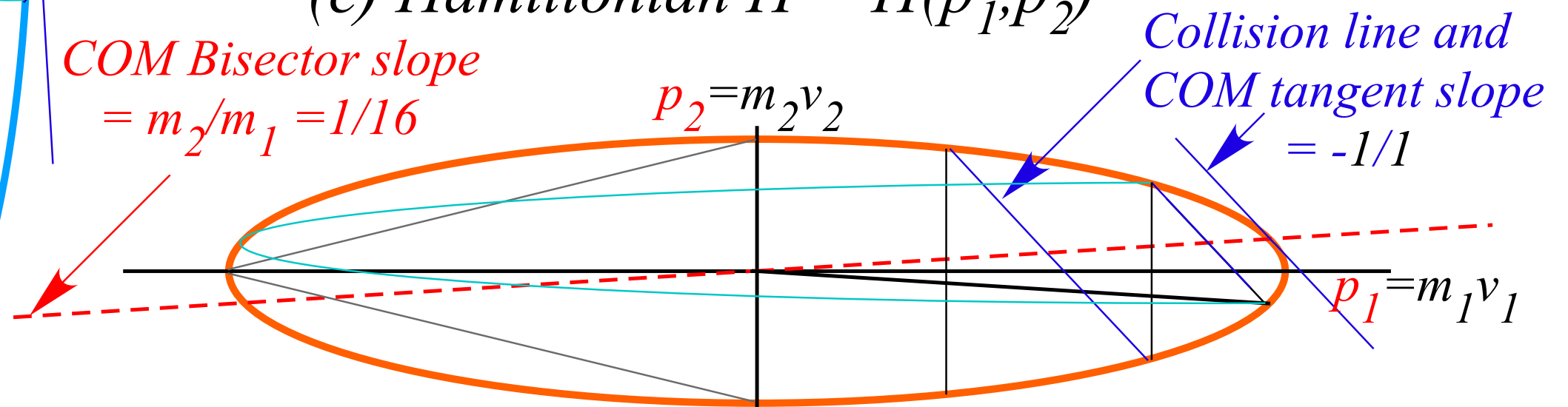
How this relates to *Lagrangian*, and *Hamiltonian* mechanics later on

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$
 velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

(c) Hamiltonian $H = H(p_1, p_2)$

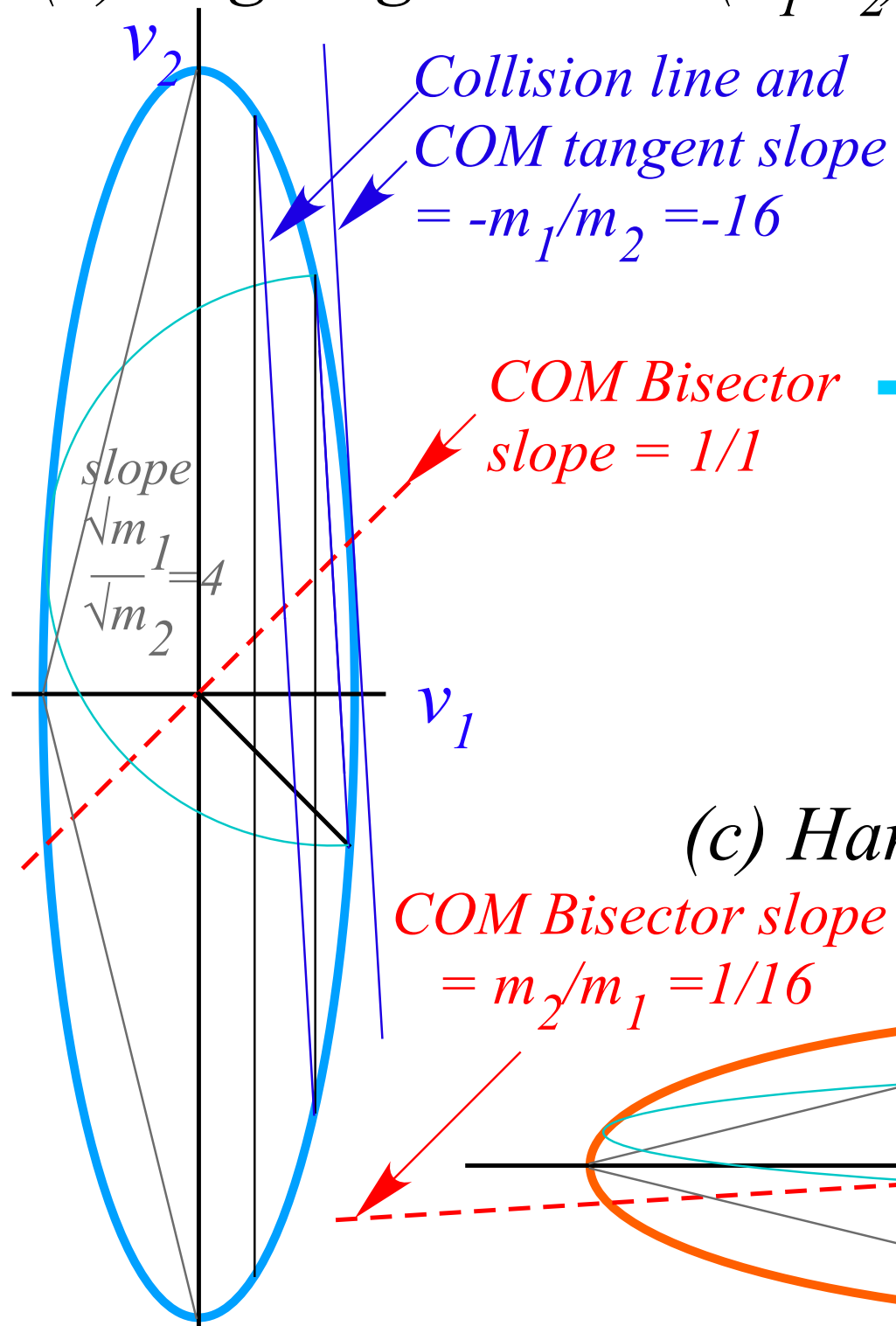


What ellipse rescaling leads to...

Fig. 12.1
(Unit 1)

How this relates to *Lagrangian*, and *Hamiltonian* mechanics later on

(a) Lagrangian $L = L(v_1, v_2)$

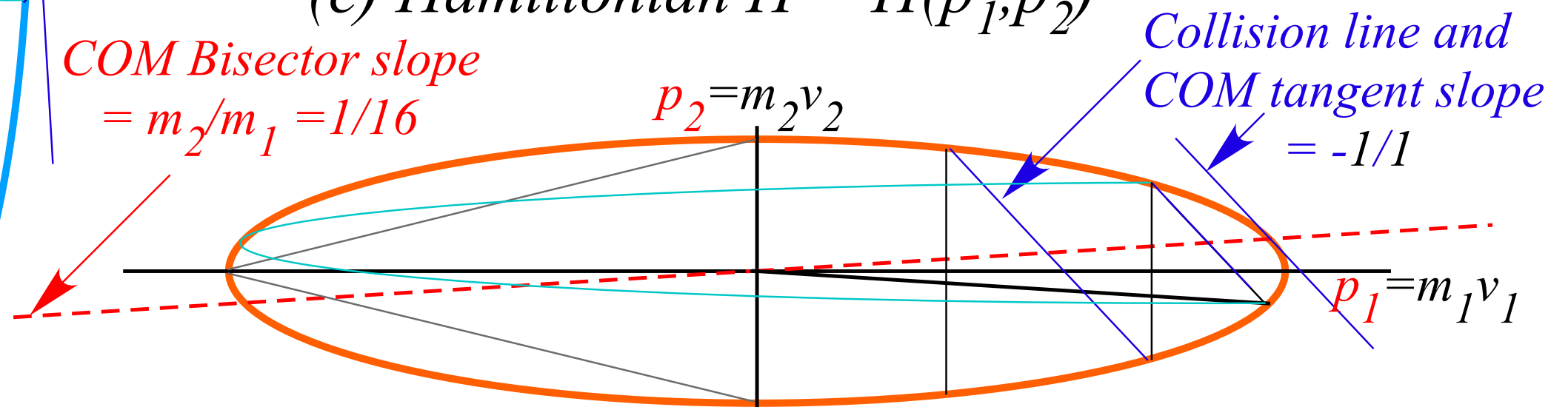


velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$
rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

(c) Hamiltonian $H = H(p_1, p_2)$

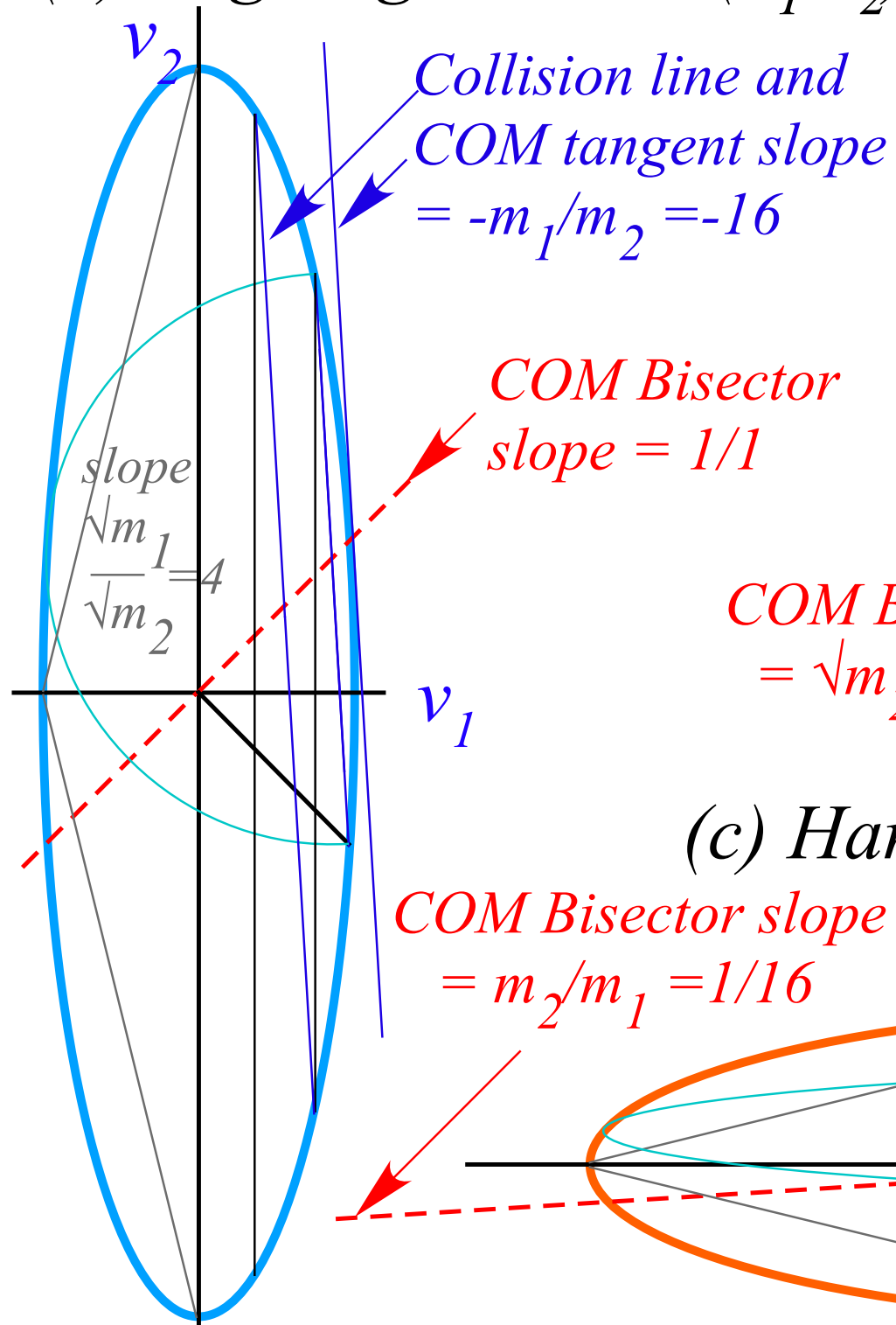


What ellipse rescaling leads to...

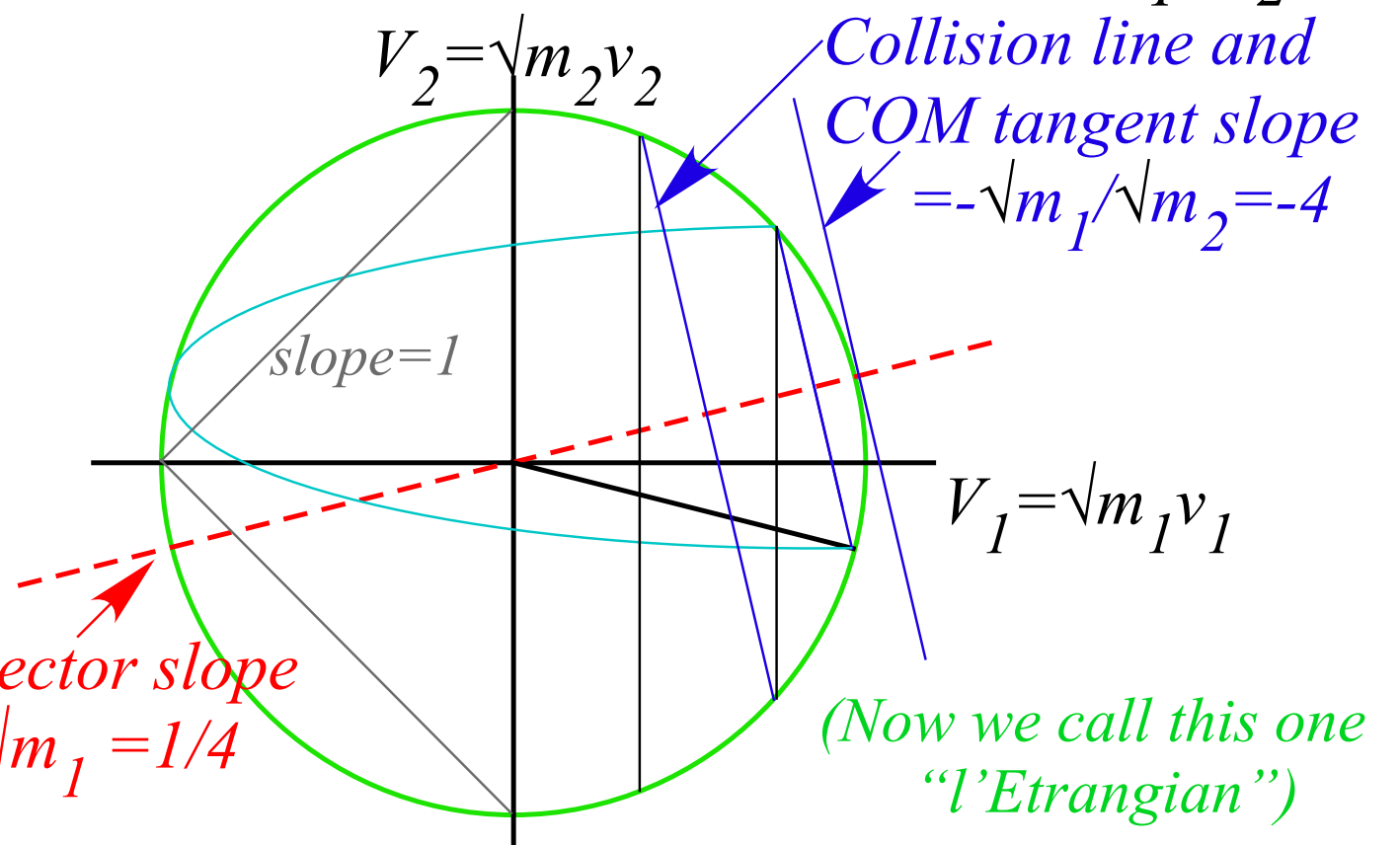
Fig. 12.1
(Unit 1)

How this relates to *Lagrangian*, *l'Etrangian*, and *Hamiltonian* mechanics later on

(a) Lagrangian $L = L(v_1, v_2)$



(b) Estrangian $E = E(V_1, V_2)$



(c) Hamiltonian $H = H(p_1, p_2)$

