# Lecture 29
# Tue. 12.14.2017

## *Relawavity : a novel introduction to relativistic mechanics I.*

*(Unit 8 12.14.07)*

*Why* ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

Wave velocity formulas

Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* $\rho$

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation

*Rapidity* $\rho$ related to *stellar aberration angle* $\sigma$ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry

"Occams Sword" and geometry of 16 parameter functions of $\rho$ and $\sigma$

Application to TE-Waveguide modes and synchrotron beam relativity

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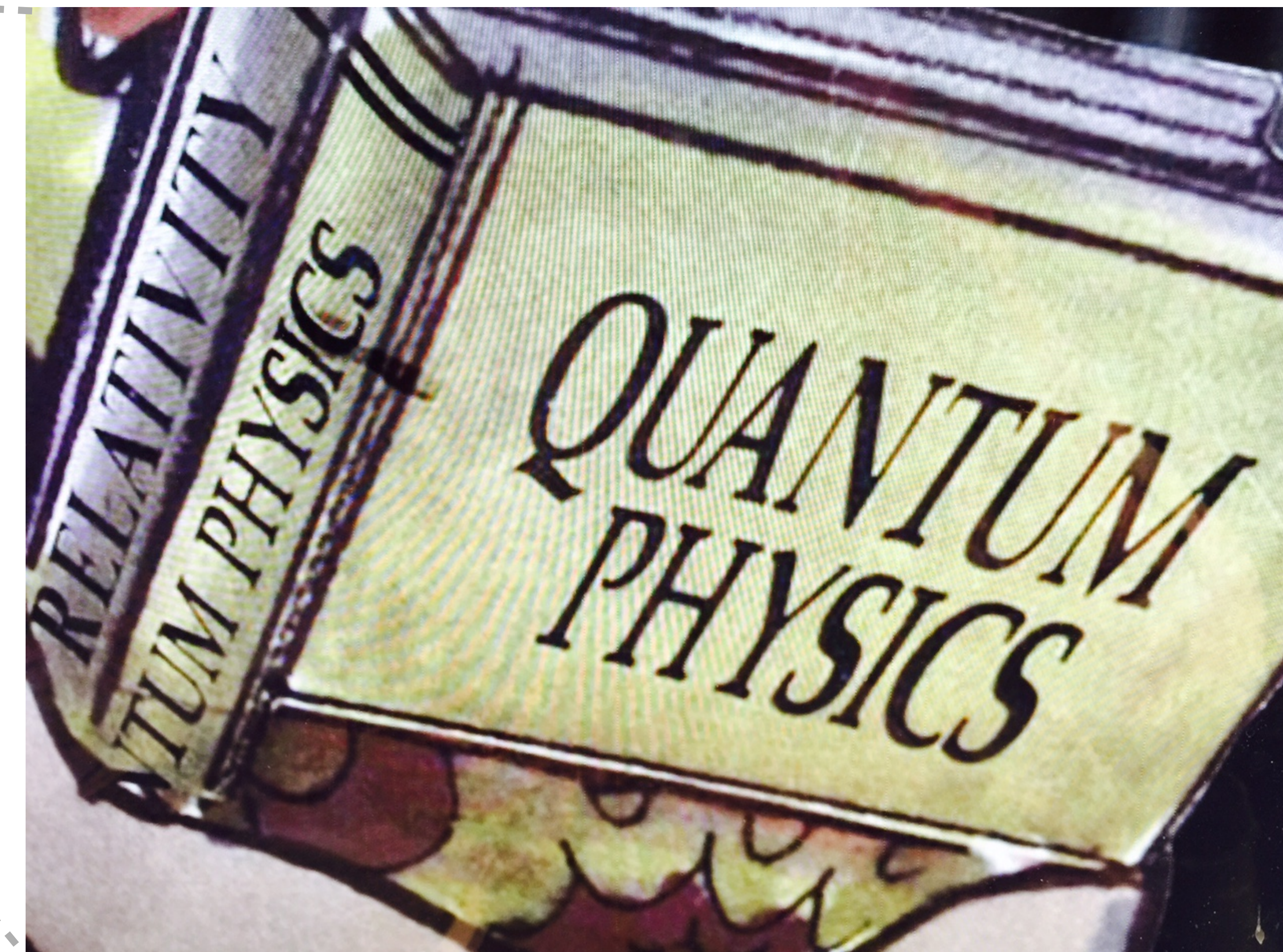
*For an introductory, web based development of this and other concepts in special relativity see our entrant in the* 2005 Pirelli Challenge*:*

*A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers*

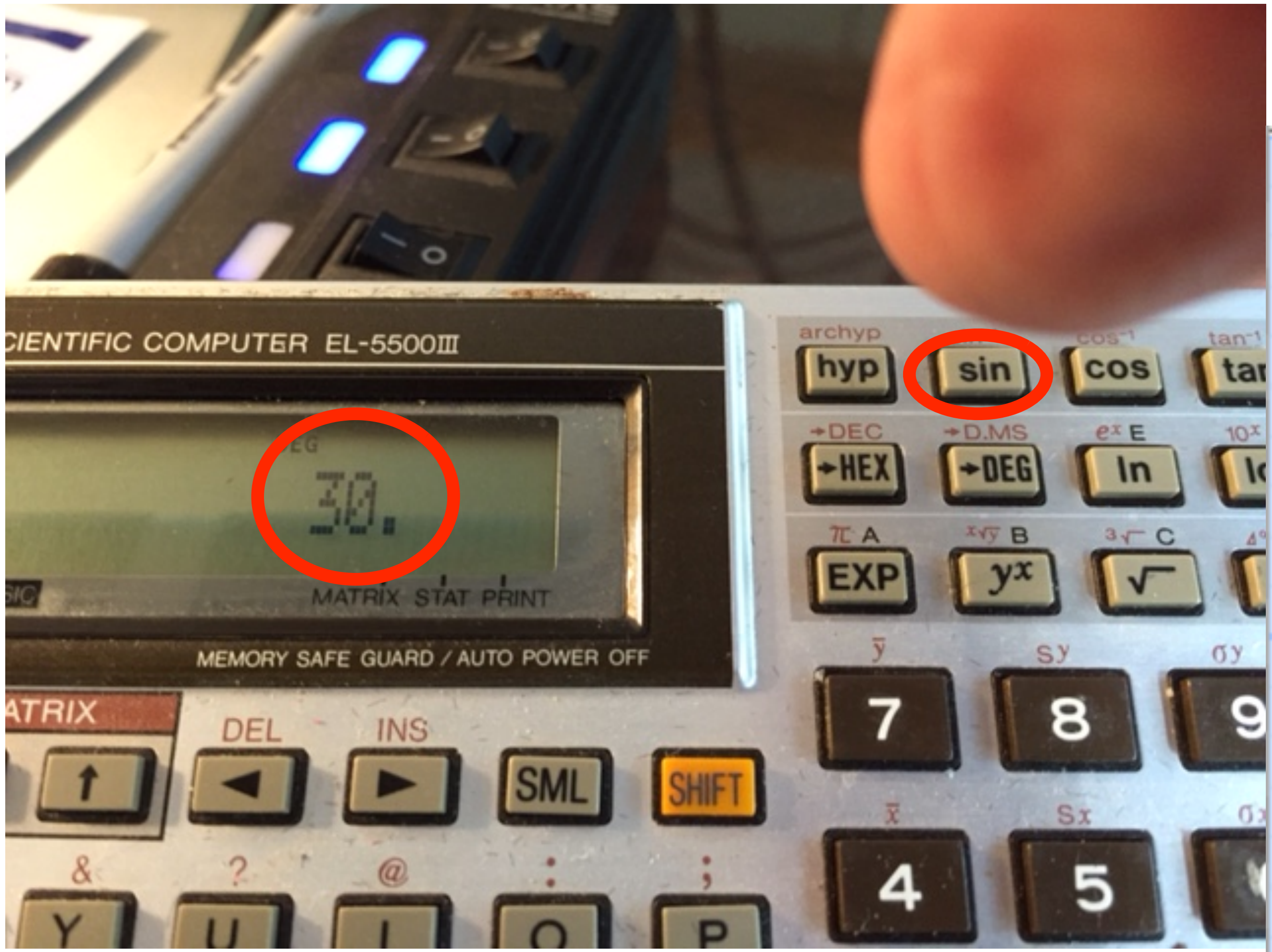

(Why a ***Men In Black*** candidate shot little Suzy)

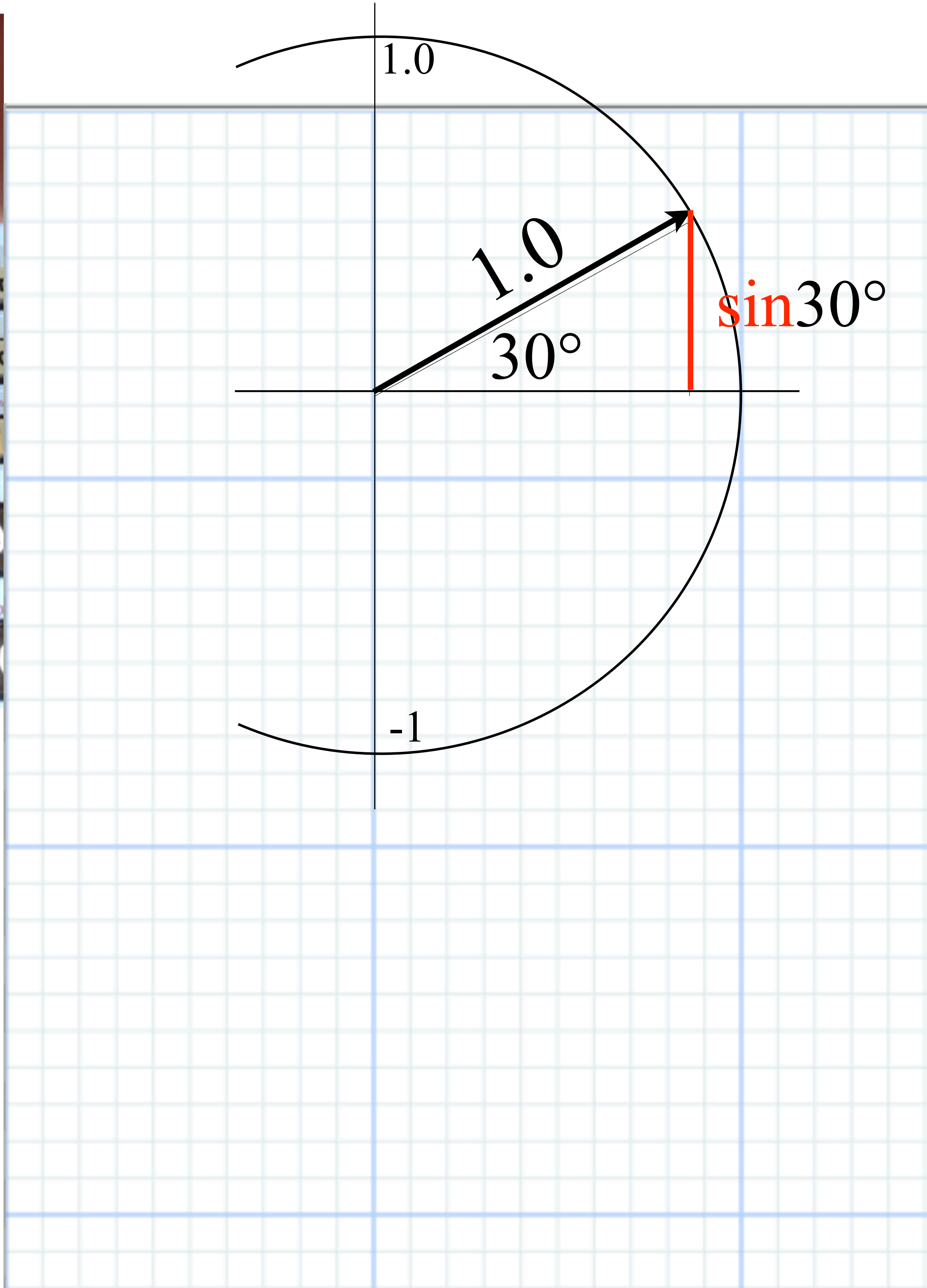



*Bad Suzy!*
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

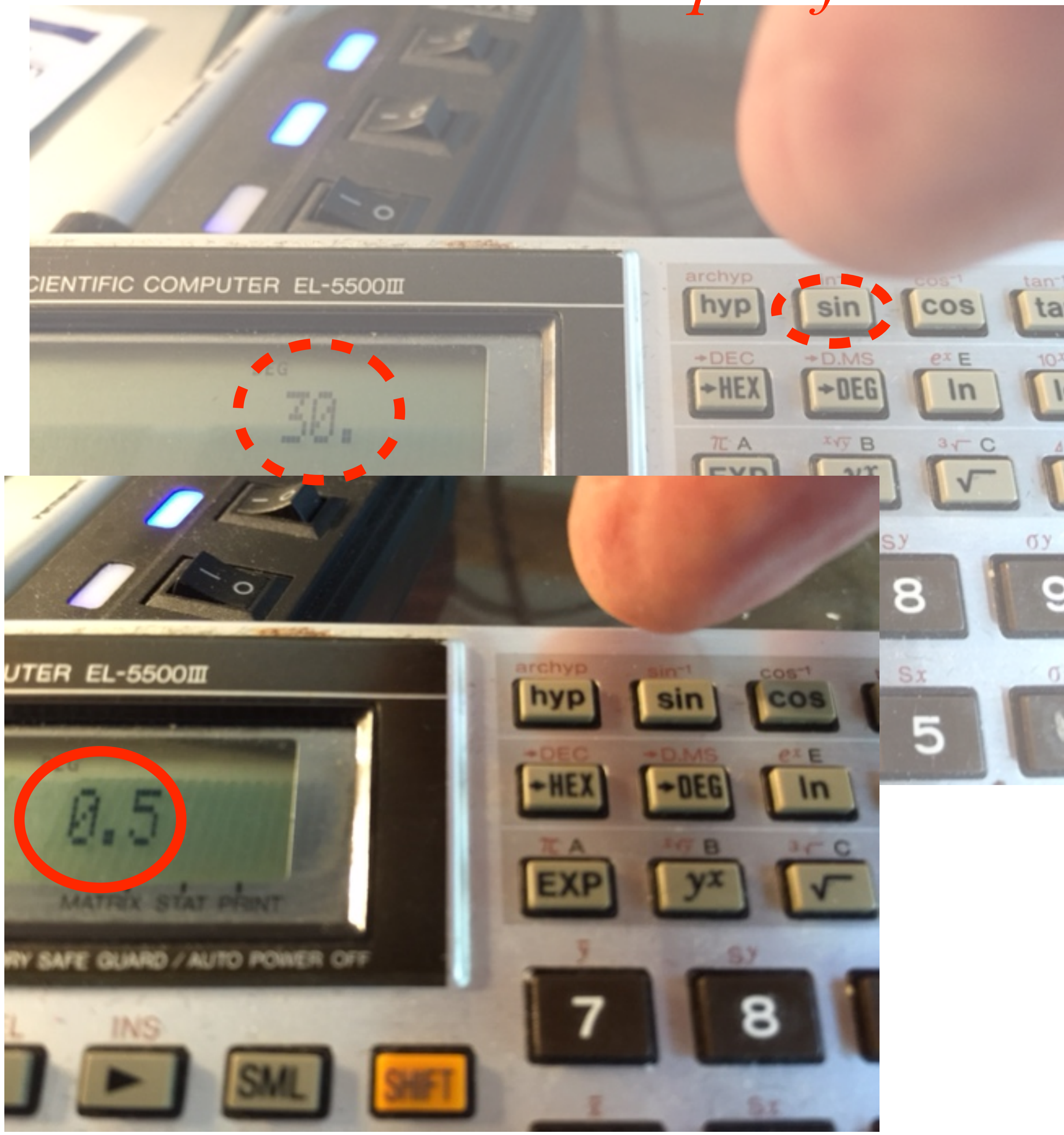
We call that a ***Relawavity*** *book.*
(It's a *lot* lighter!)

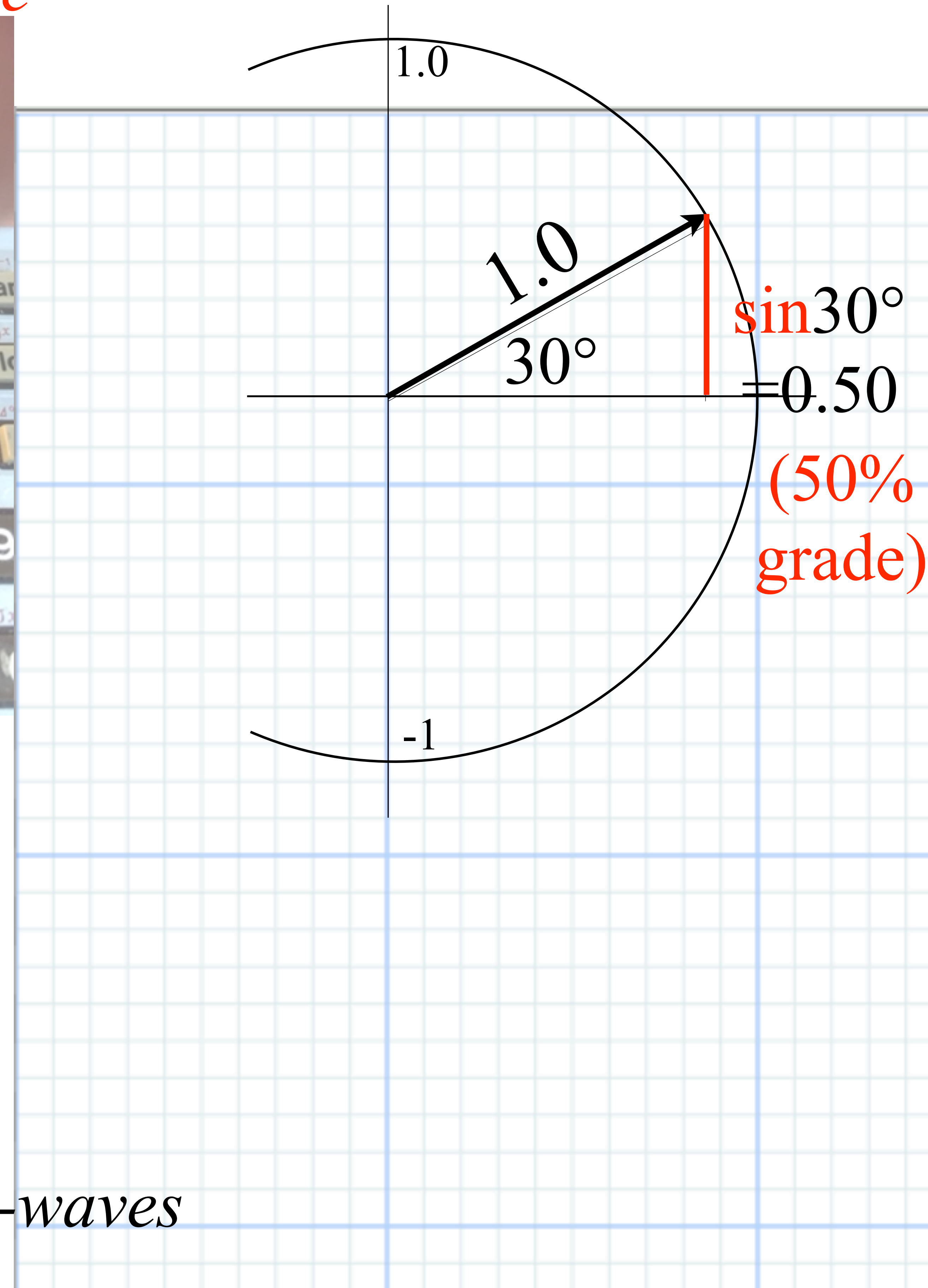
# Learning about SIN



Learning about SIN
*"Slope of INcline"*

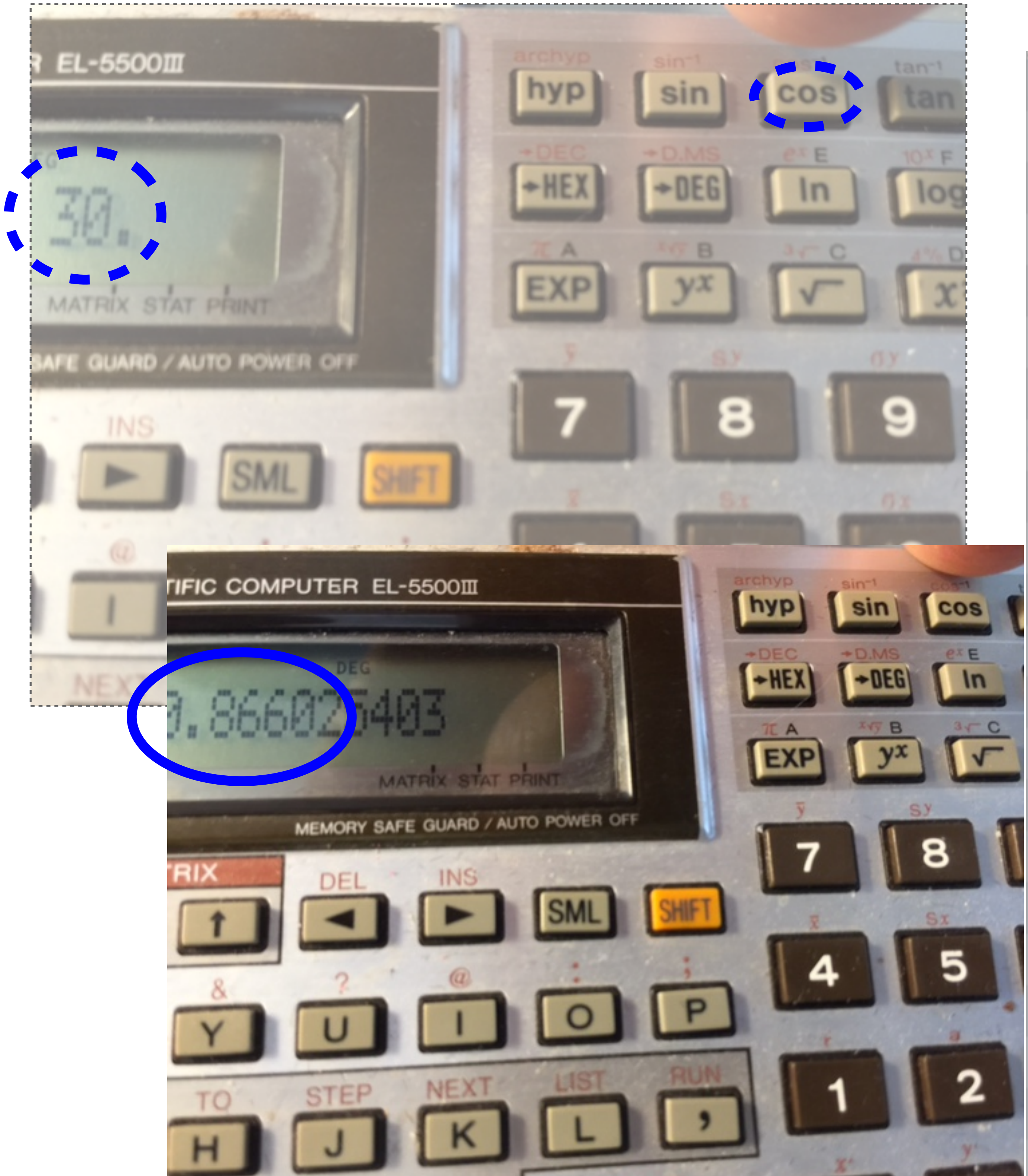


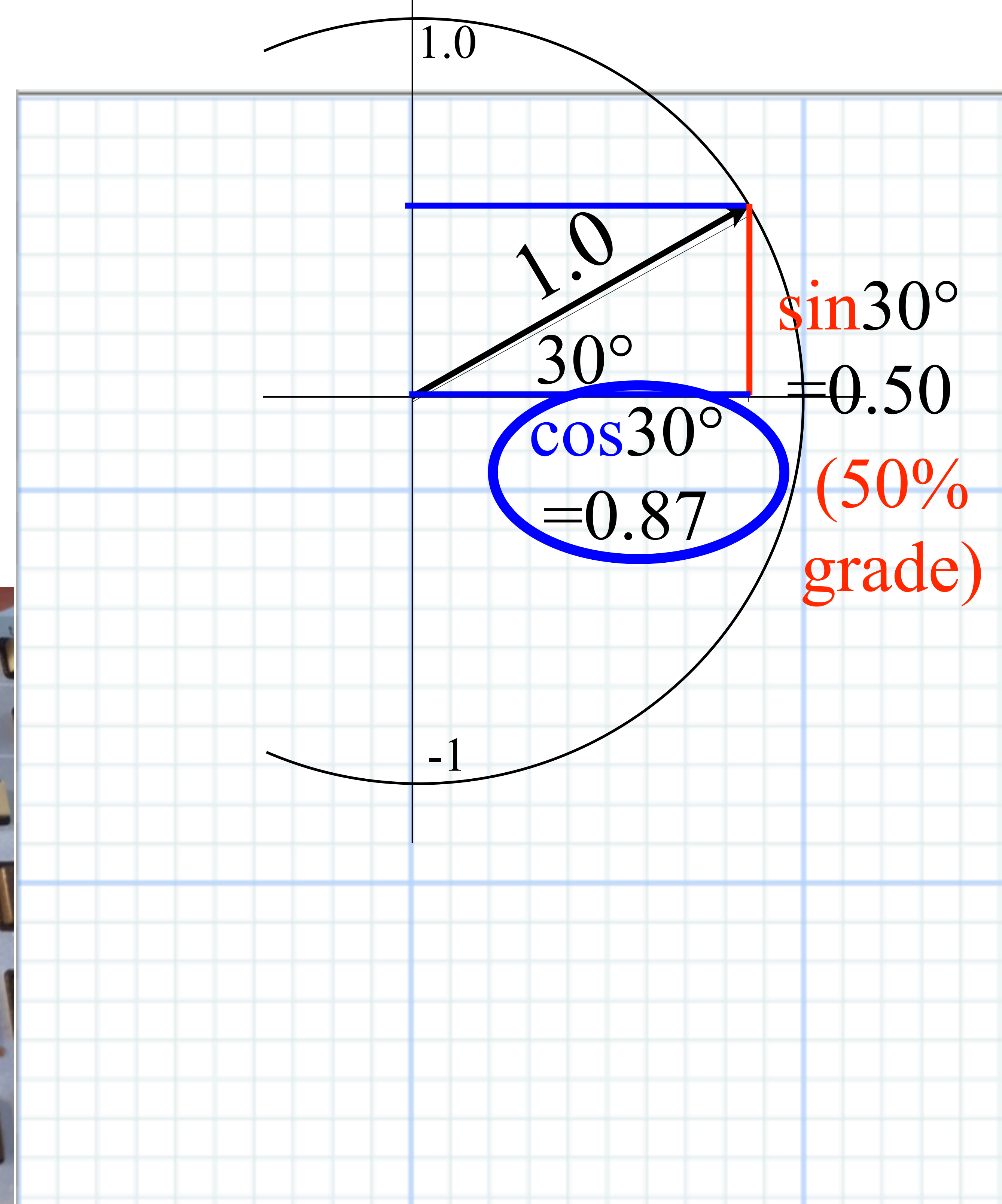




It's mostly about triangles *and sine-waves*

# Learning about SIN and the COSin

*"Slope of INcline"* "COmplimentarySlope"

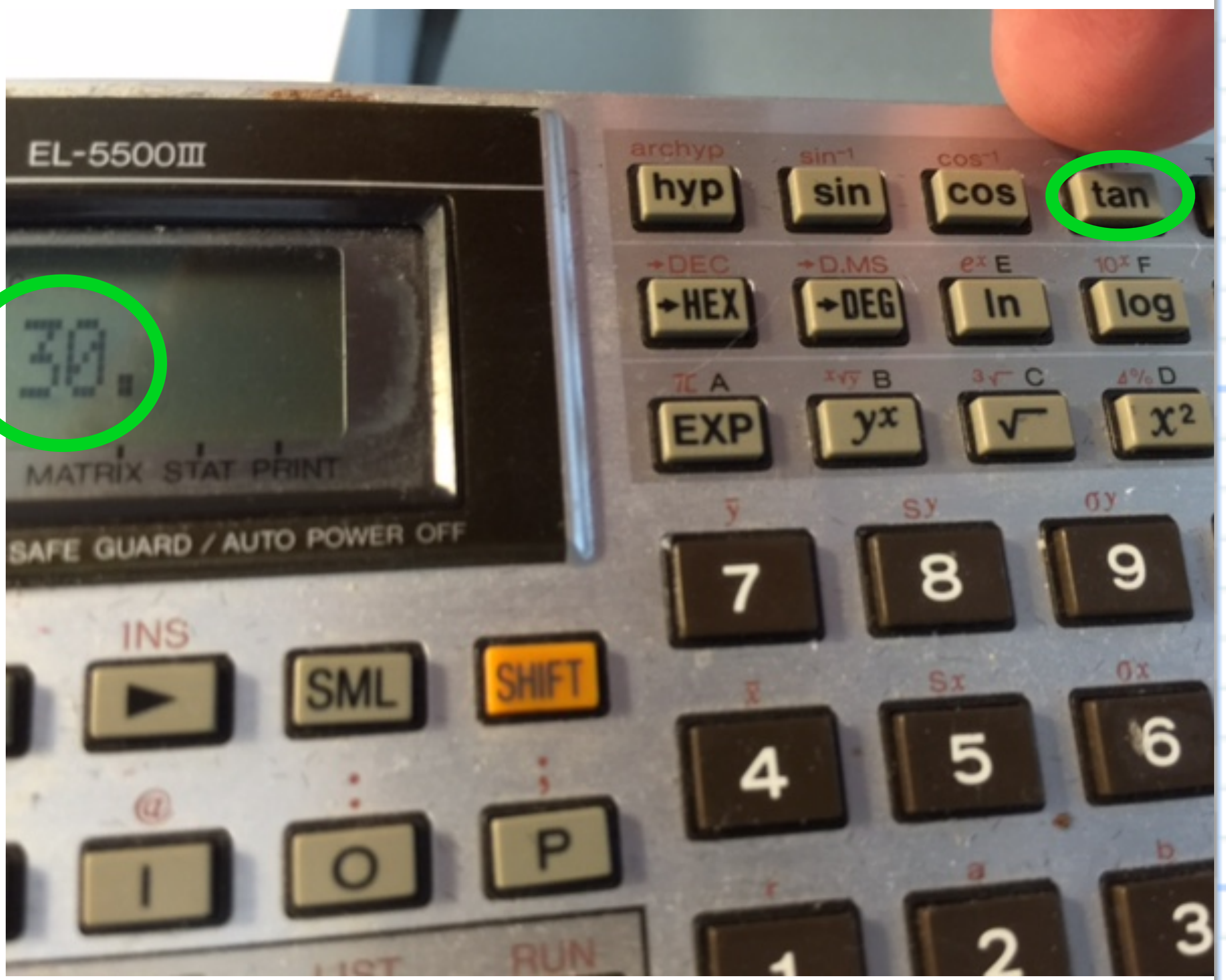


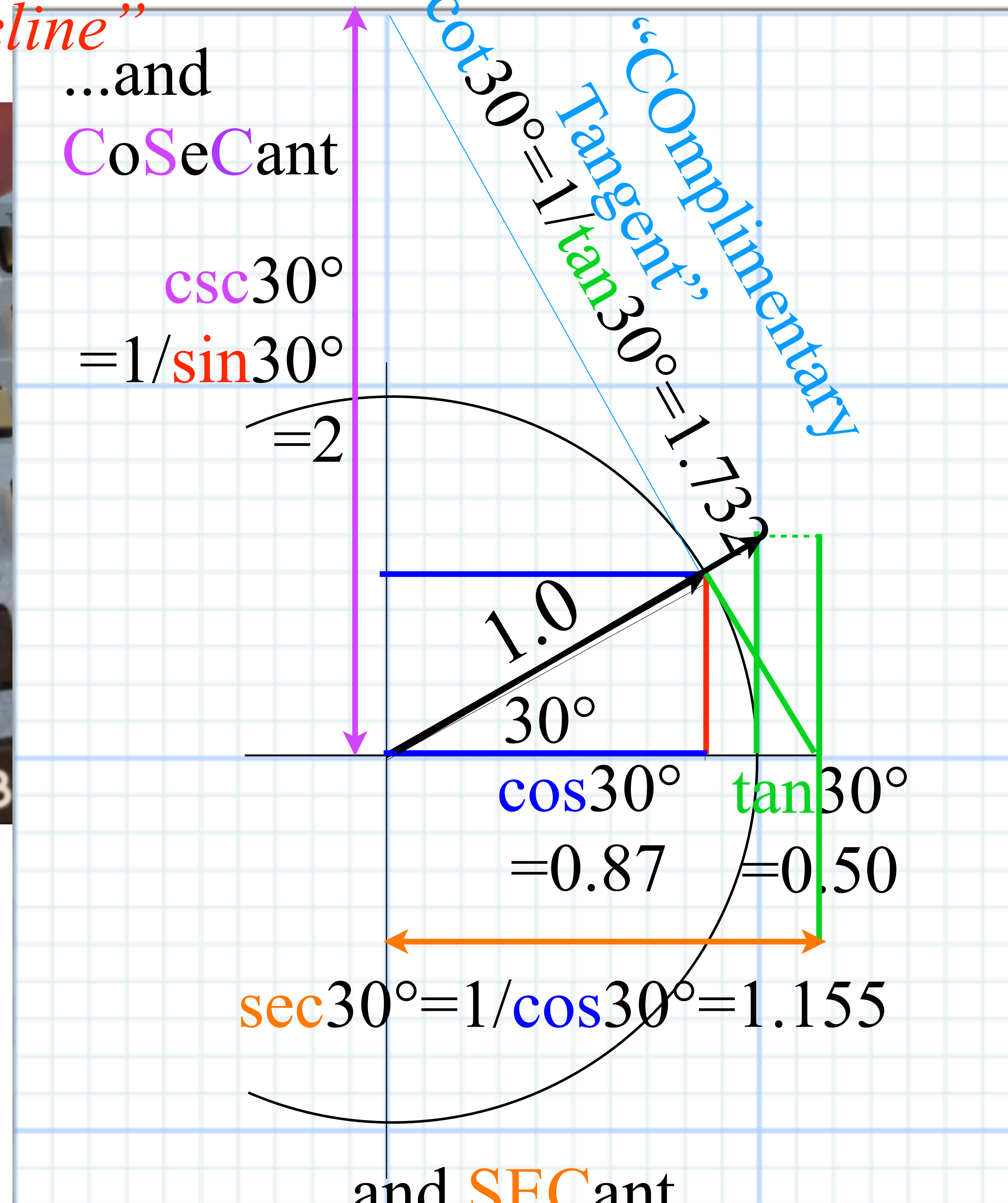




It's mostly about triangles *and sine-waves and cosine-waves*

# Learning about SIN and the COSin and TANgent and COTangent

*"Slope of INcline"*

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*Why* ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

➨ Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra
1CW wavefunctions and phasors

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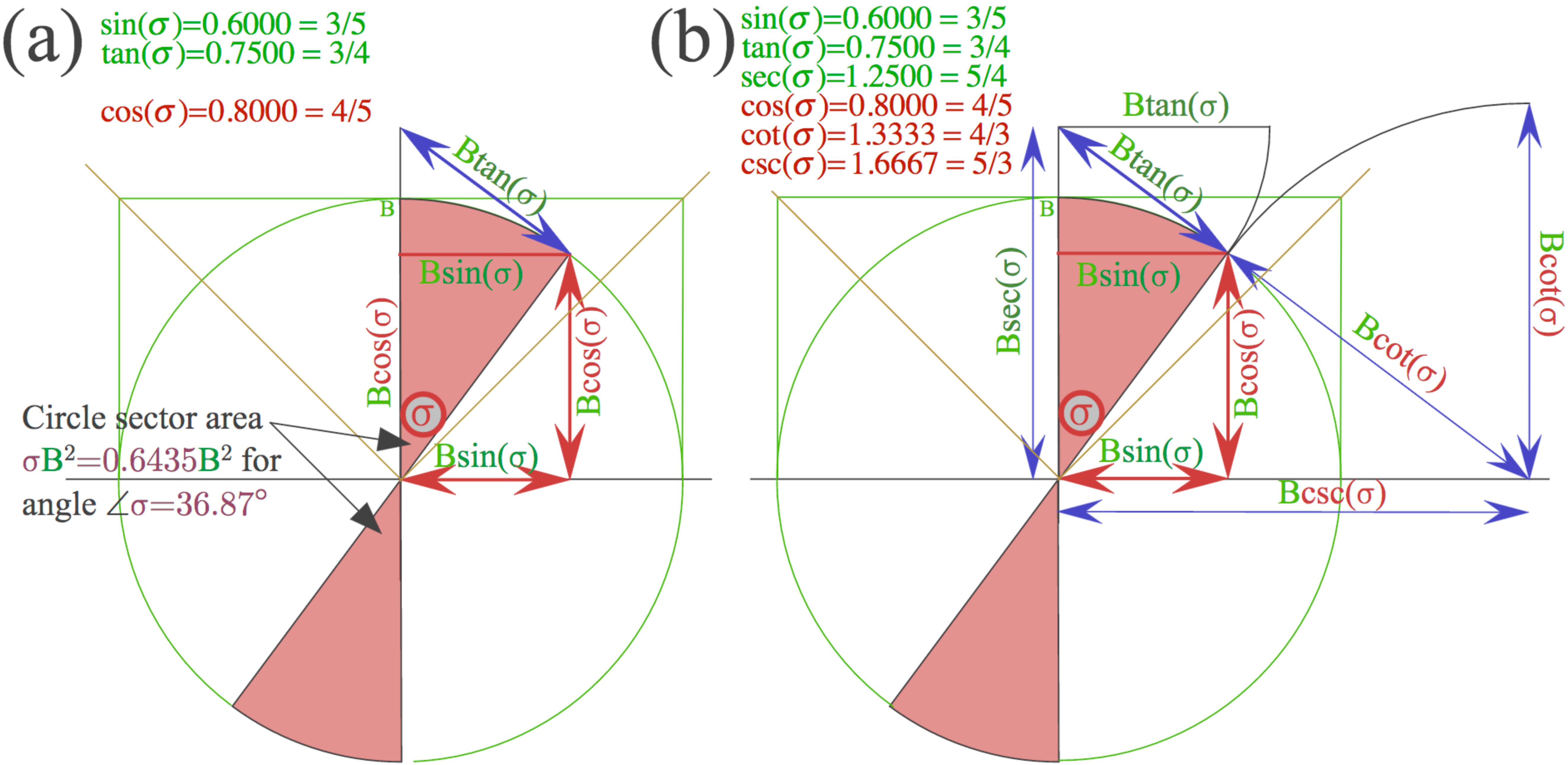
# *Trigonometric road maps*




*All this physics of relativity is mostly simple trigonometry of optical wave interference!*

*And, it derives fundamentals of quantum theory, too!*

RelaWavity Web Simulation
Relations between
Hypergeometric and
Hypergeometric functions

# *Trigonometric road maps become hyperbolic trig maps…*

RelaWavity Web Simulation
Relations between
Hypergeometric and
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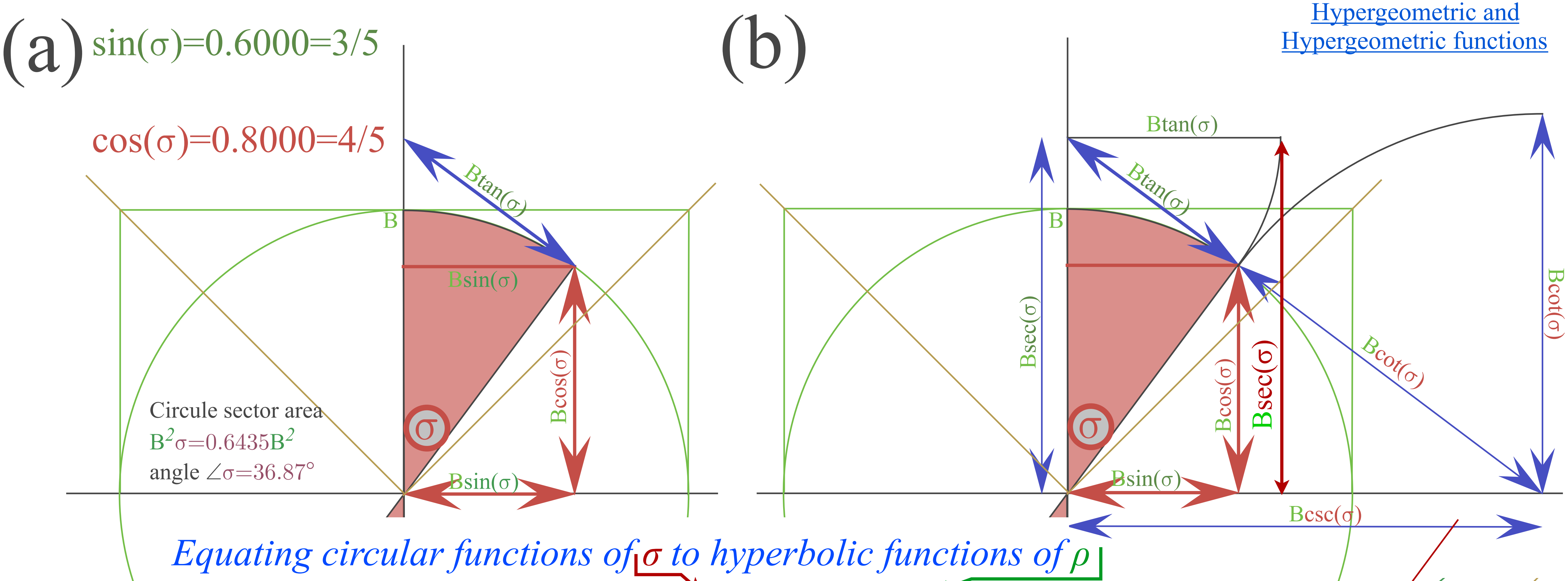


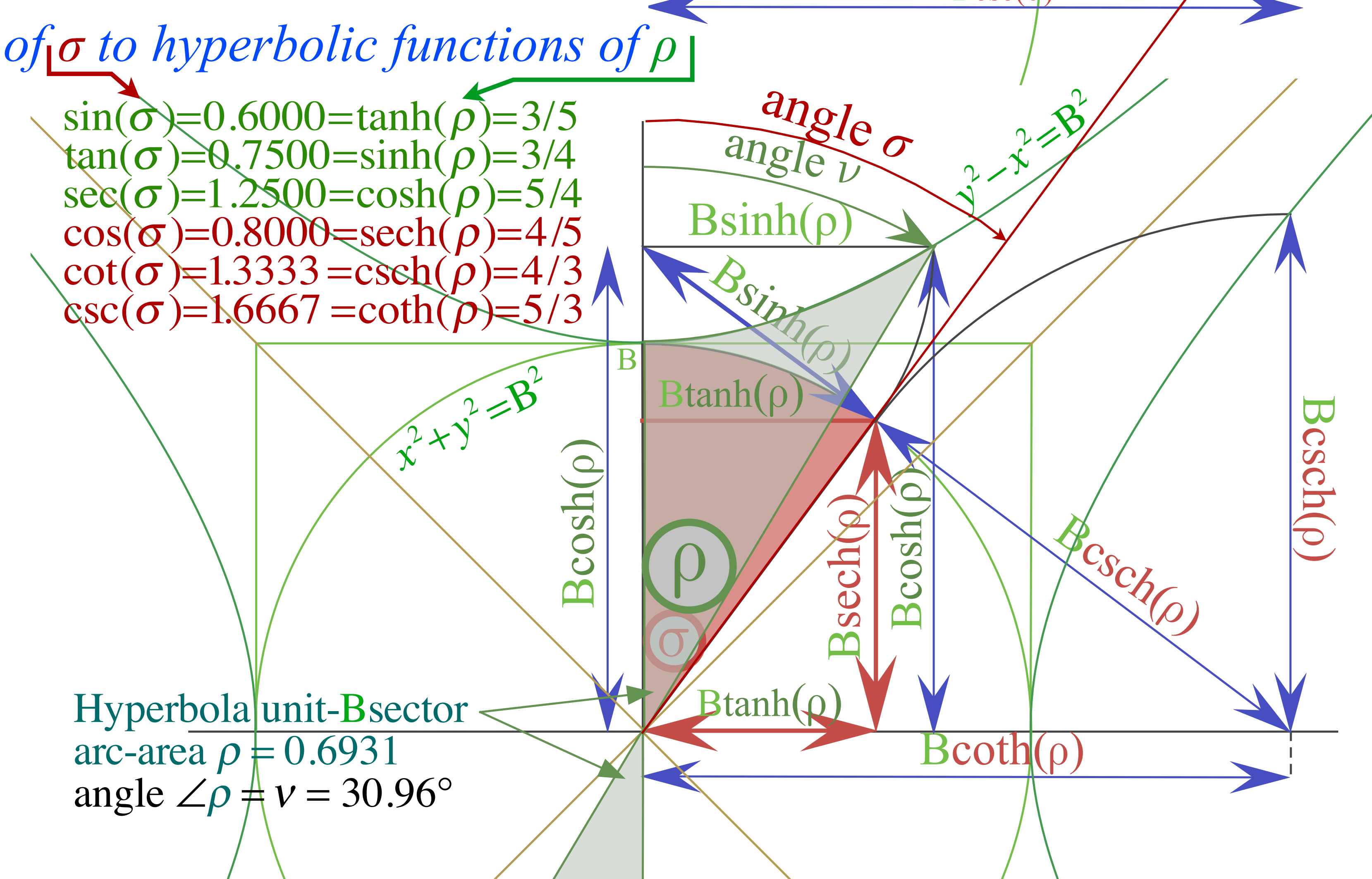


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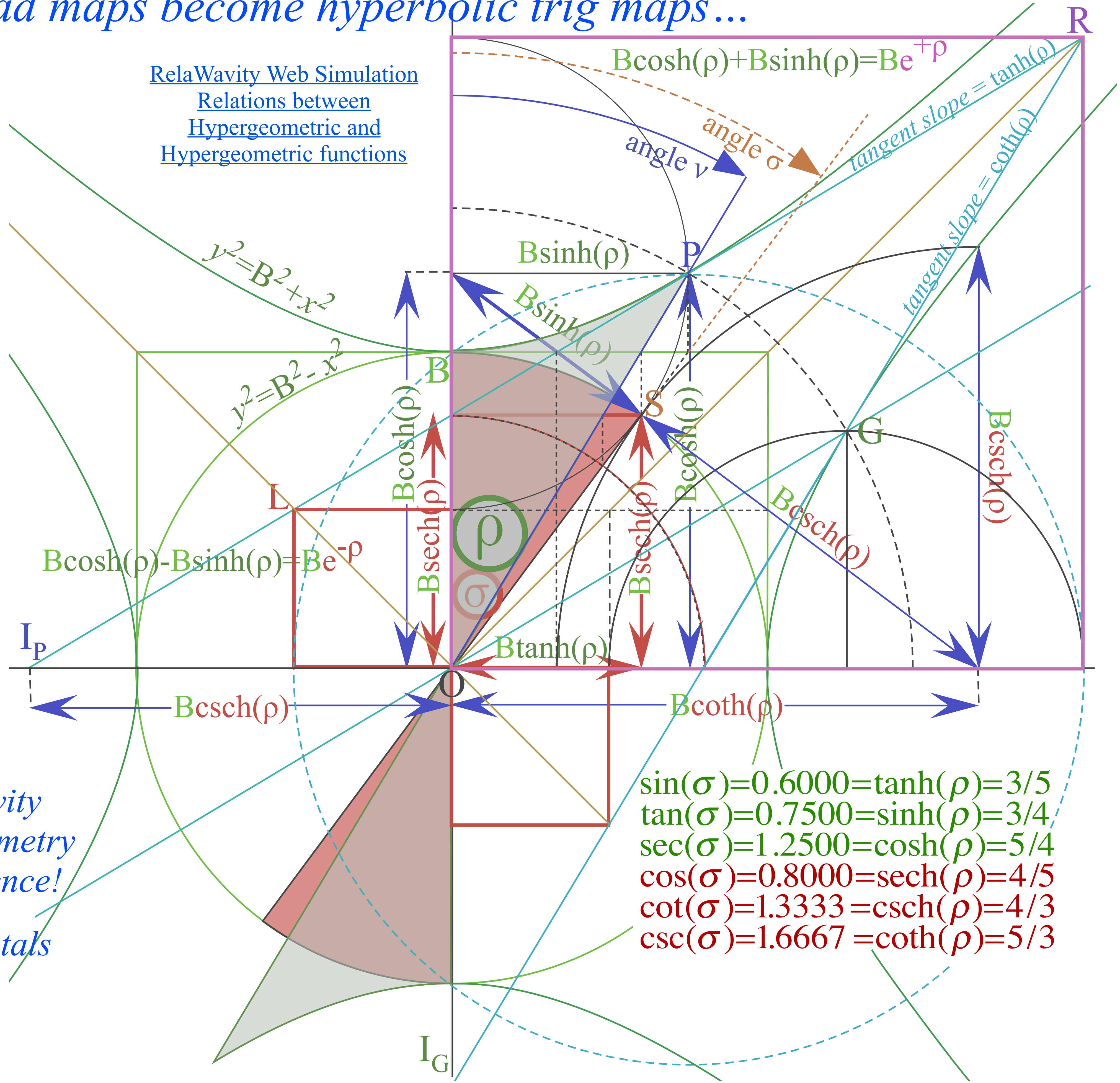
# *Trigonometric road maps become hyperbolic trig maps…*

*Need to see how trig road map matches the physical map on following page.*

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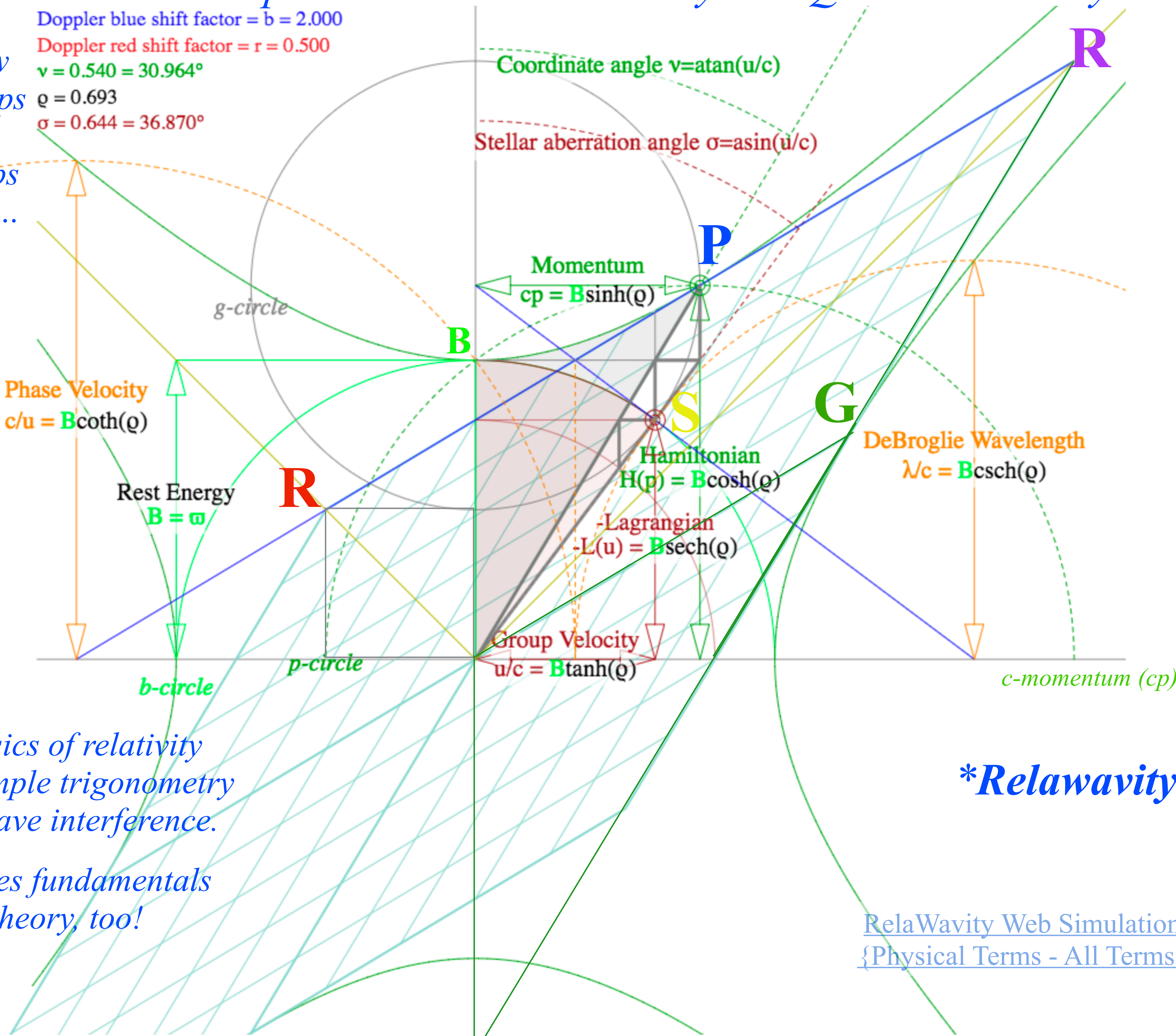
# *Trigonometric road maps…. to Relativity and Quantum Theory**

*Need to show trig road maps can match physical maps like this one…*

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***Relawavity**

RelaWavity Web Simulation
{Physical Terms - All Terms}

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Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra ⬅

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# *Hyper*-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

Exponential derived by infinite-$n$-compounding limit of the interest rate-$r$ formula.

$$e^{rt} = \lim_{n\to\infty}\left(1+\frac{rt}{n}\right)^n$$

Infinite-$n$ limit of binomial series is an exponential power-$p$ series of $(rt)^p$ with $1/p!$ coefficients.

$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots$$

$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots$$

Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine (cosh($rt$)) and sine (sinh($rt$)).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt)$$

$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

*Hyper*-Trig

cosh$\rho$ and sinh$\rho$

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$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

*Hyper*-Trig

cosh$\rho$ and sinh$\rho$

Replace rate $r$ with imaginary rate $ir$ and $i=\sqrt{-1}$ powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,…

Then *hyper*-sine-cosine becomes the *circular*-sine-cosine.

$$\frac{e^{+i\,rt}+e^{-i\,rt}}{2} = 1 - \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cos rt$$

$$\frac{e^{+i\,rt}-e^{-i\,rt}}{2} = i\,rt - i\frac{(rt)^3}{2\cdot 3} + i\frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} - \ldots = i\sin rt$$

*Circular*-Trig

cos$\sigma$ and sin$\sigma$

Sum and difference of this pair gives the Euler-DeMoivre relations of exponentials vs trig-functions.

$$e^{+i\sigma} = \cos\sigma + i\sin\sigma\ , \qquad e^{-i\sigma} = \cos\sigma - i\sin\sigma .$$

$$e^{+\rho} = \cosh\rho + \sinh\rho\ , \qquad e^{-\sigma} = \cosh\rho - \sinh\rho .$$

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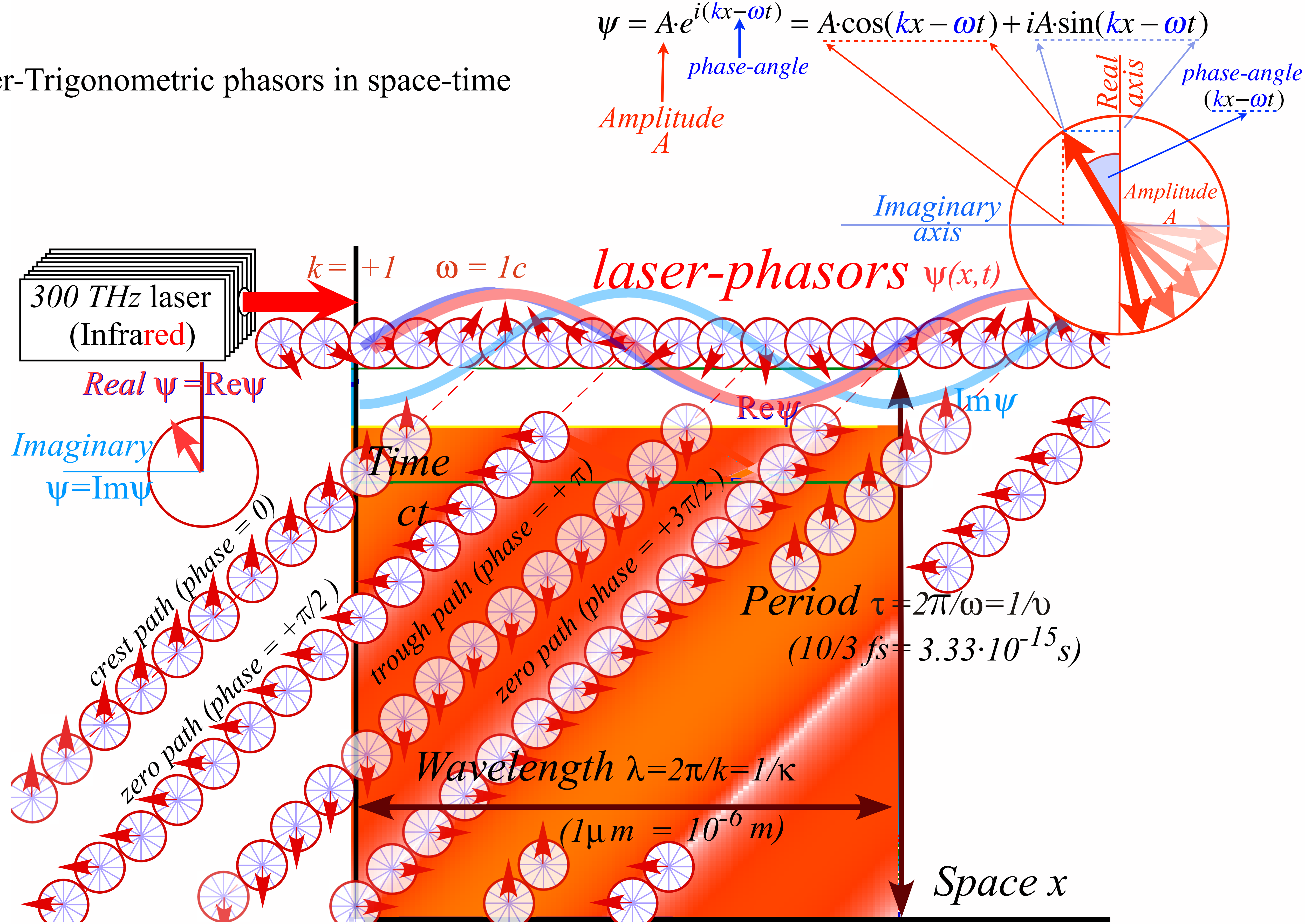
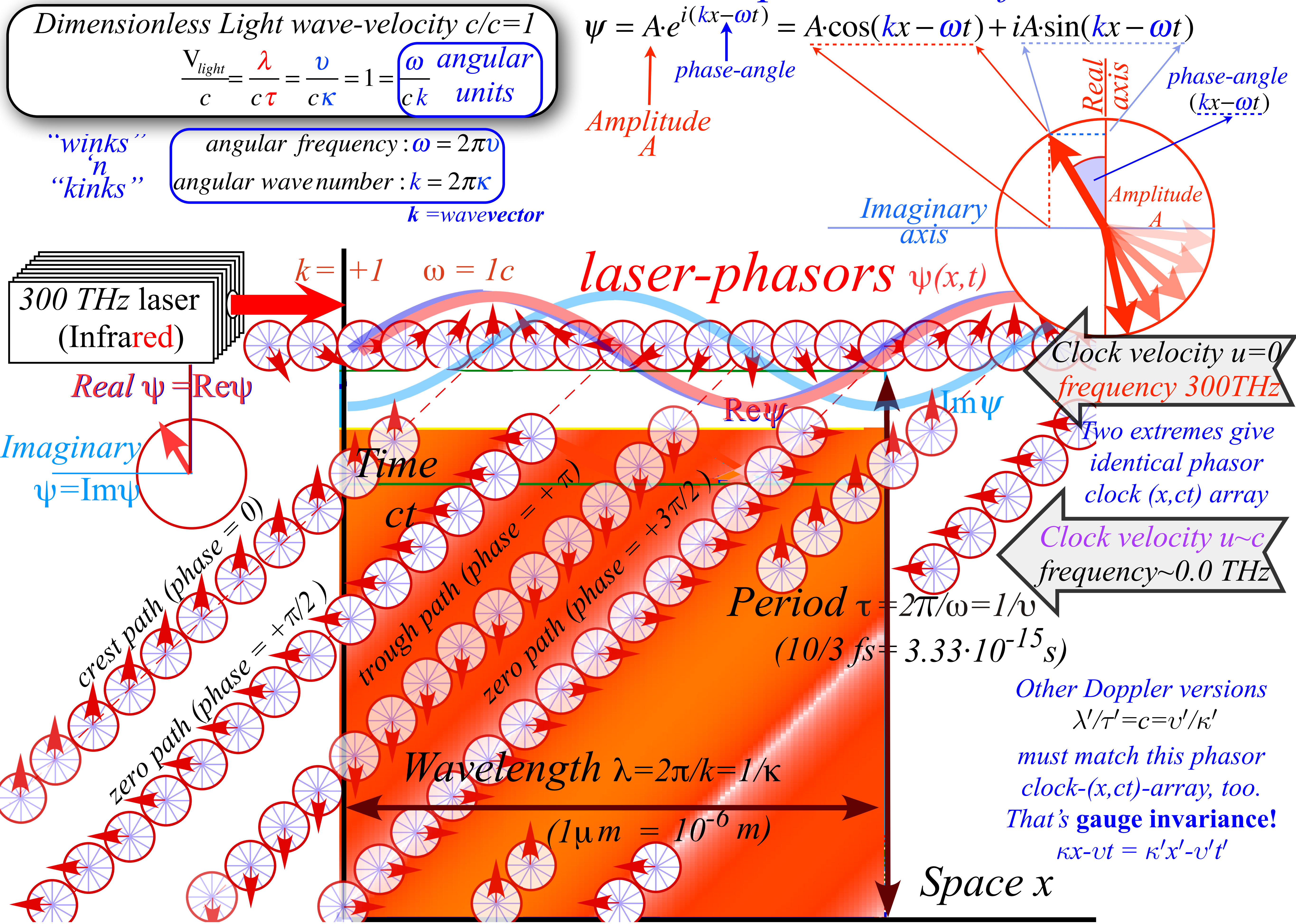
# 1CW Laser-phasor wave function

Hyper-Trigonometric phasors in space-time




(a) Single-phasor plot of wave-function at $(x,ct)$. (b) Array of phasors at many $(x,ct)$-points.

# 1CW Laser-phasor wave function



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The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

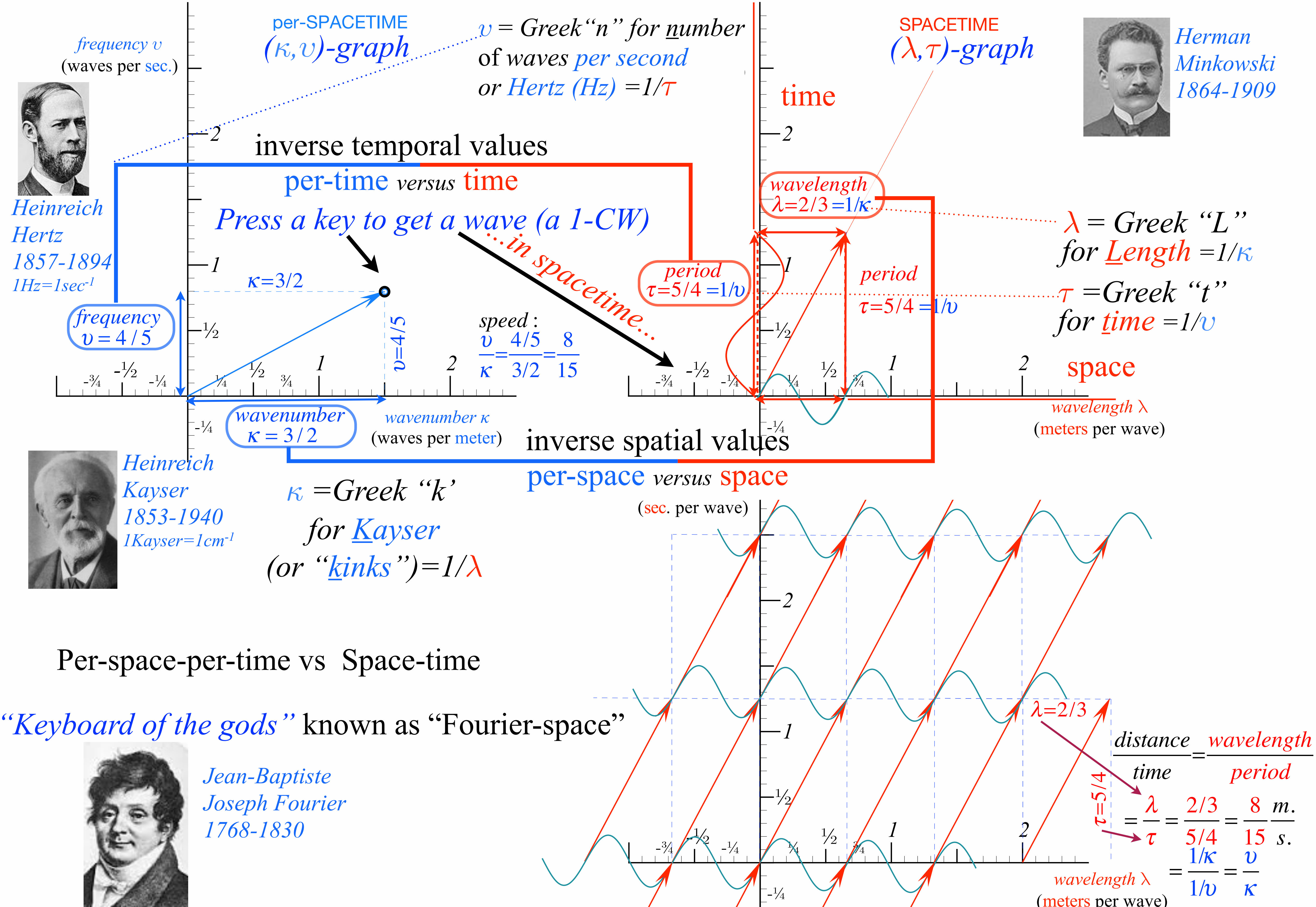
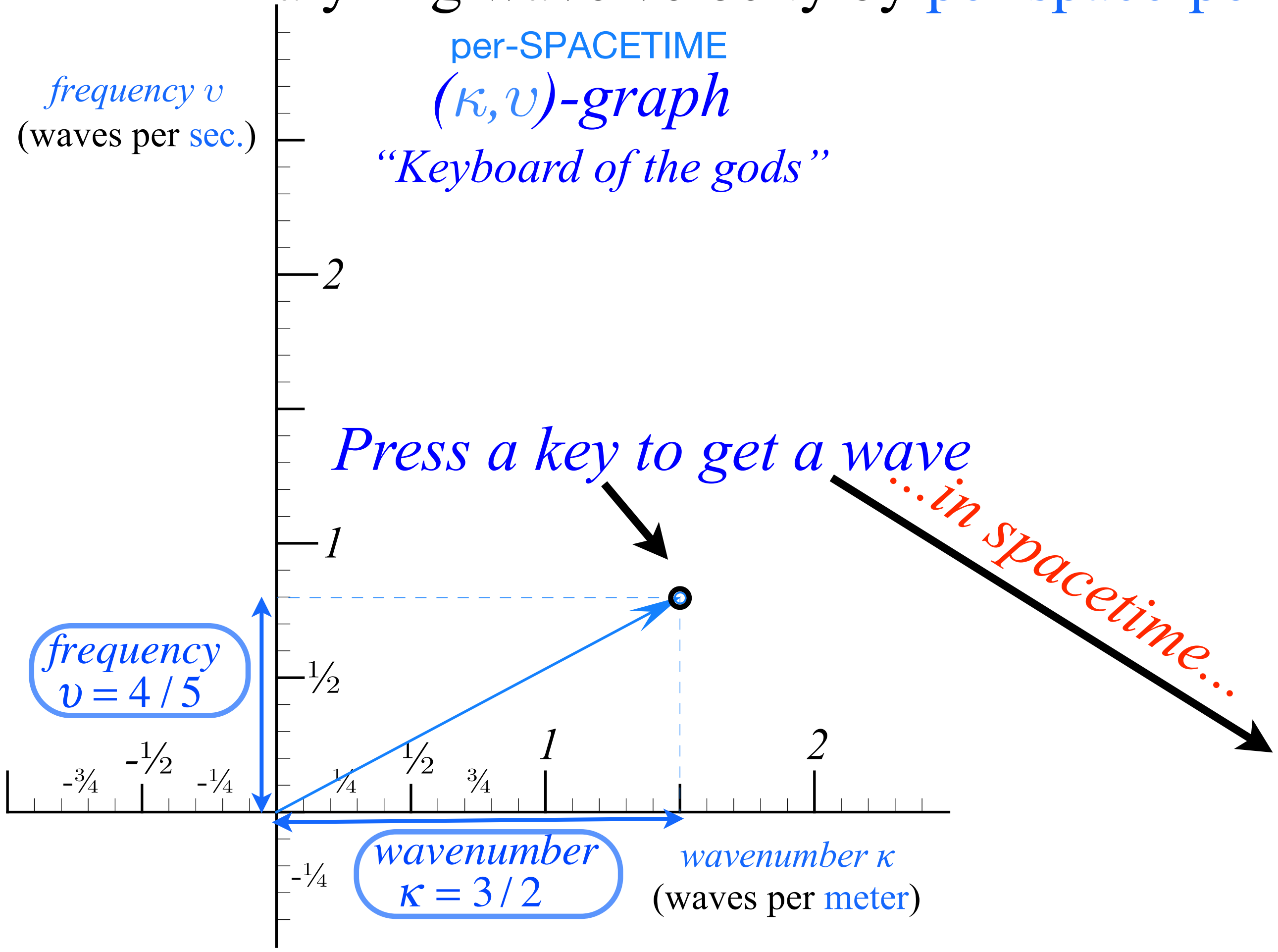
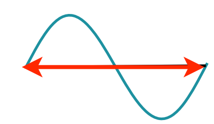



Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

# Analyzing wave velocity by per-space-per-time *and* space-time graphs




SPACETIME
*(λ,τ)-graph*



*"Keyboard of the gods"* is known as "Fourier-space"

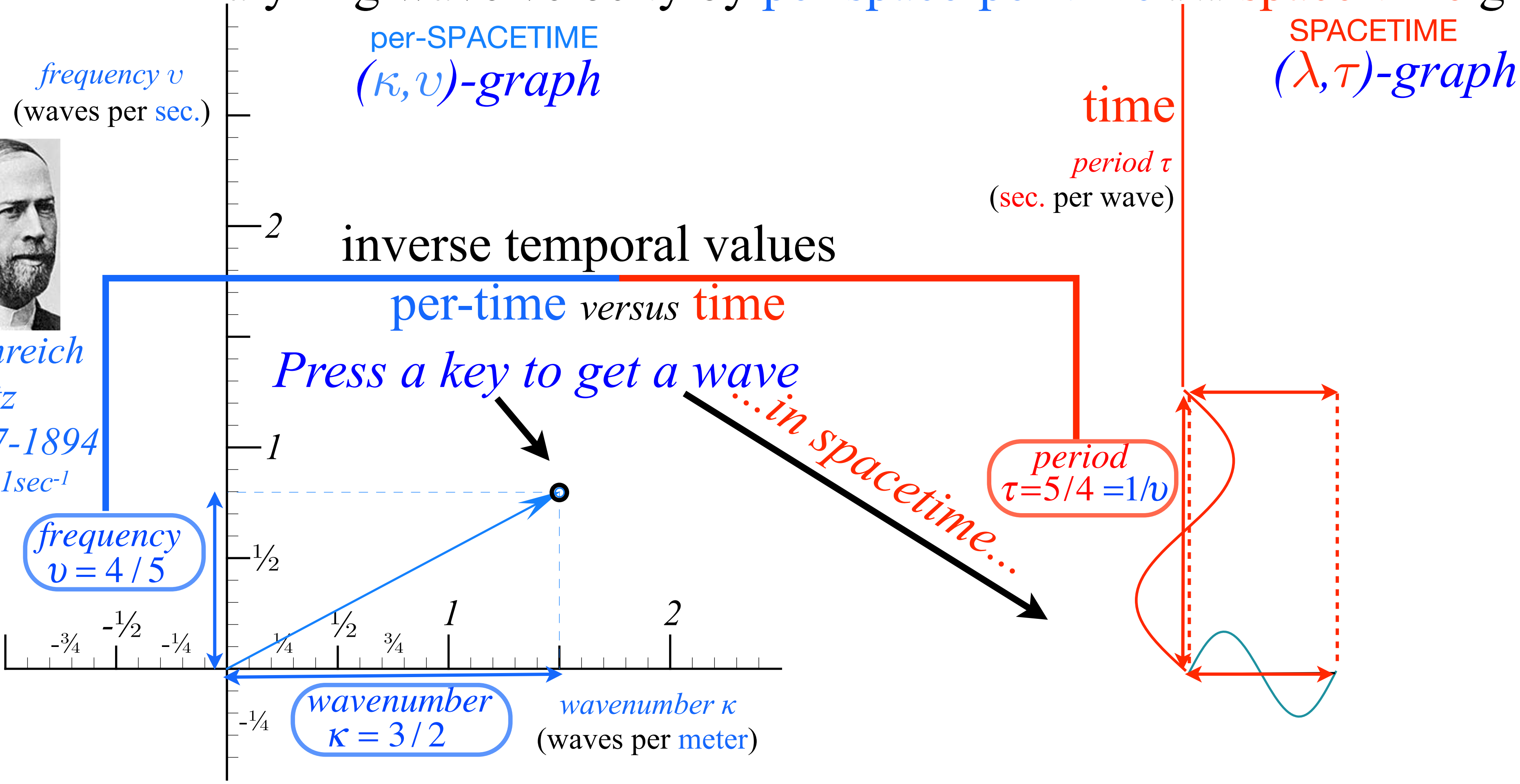
*Jean-Baptiste*
*Joseph Fourier*
*1768-1830*

*•How to understand waves*
*and*
*wave velocity $V_{wave}$*

RelaWavity Web Simulation
*Keyboard of the Gods*
(per-Time *vs* per-Space)

# Analyzing wave velocity by per-space-per-time *and* space-time graphs

Heinreich Hertz 1857-1894
1Hz=1sec⁻¹

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*•How to understand waves and wave velocity $V_{wave}$*

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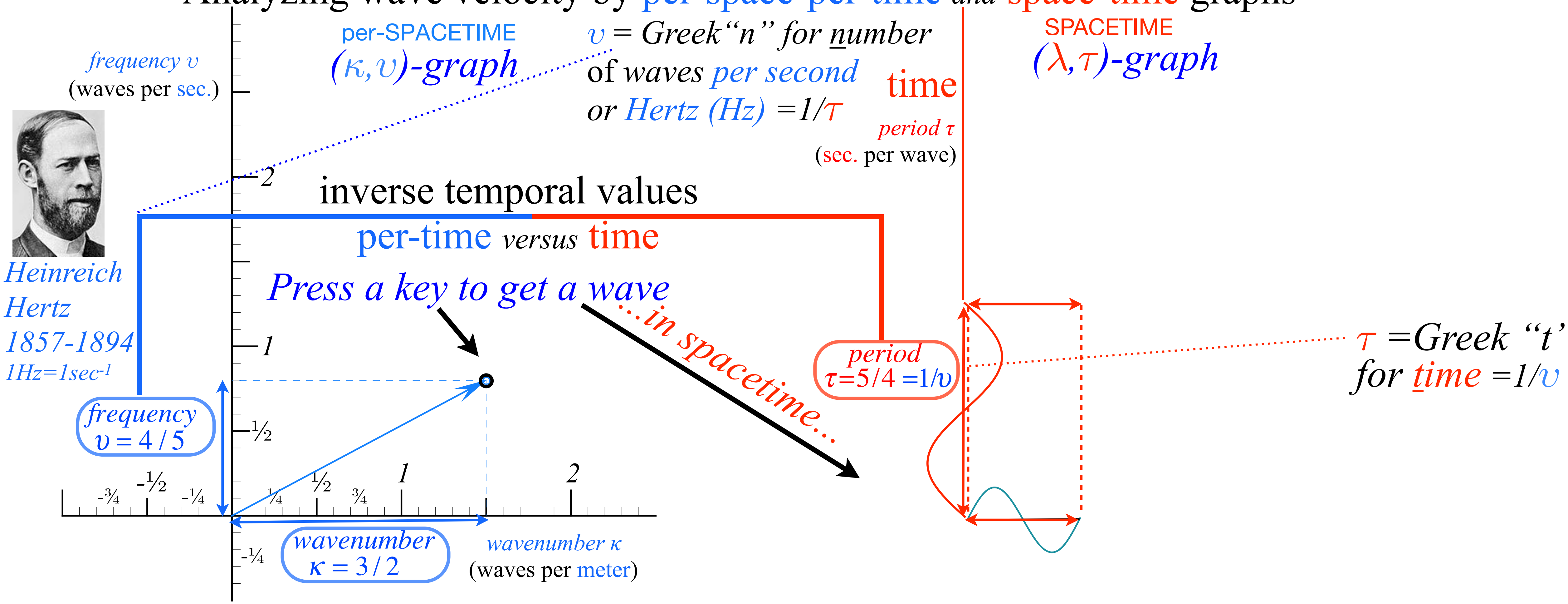


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RelaWavity Web Simulation
*Keyboard of the Gods*
(Dual Plot)

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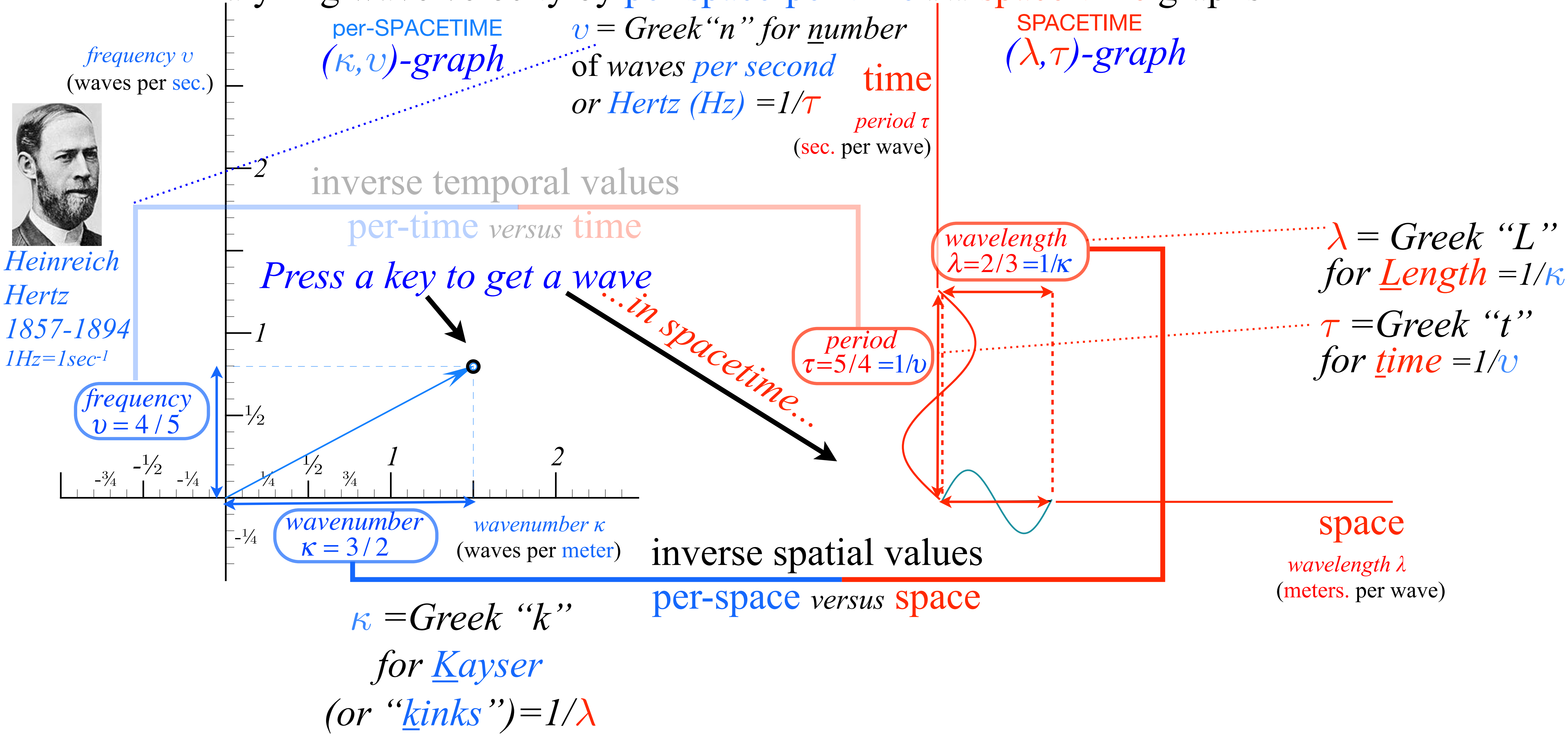


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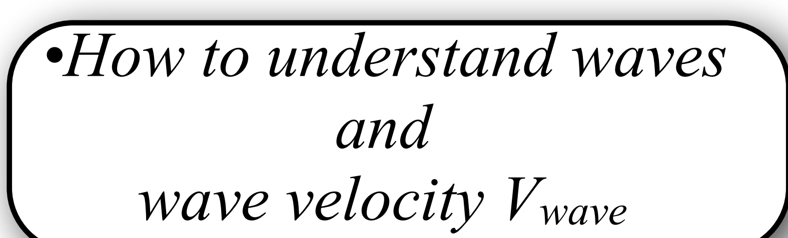
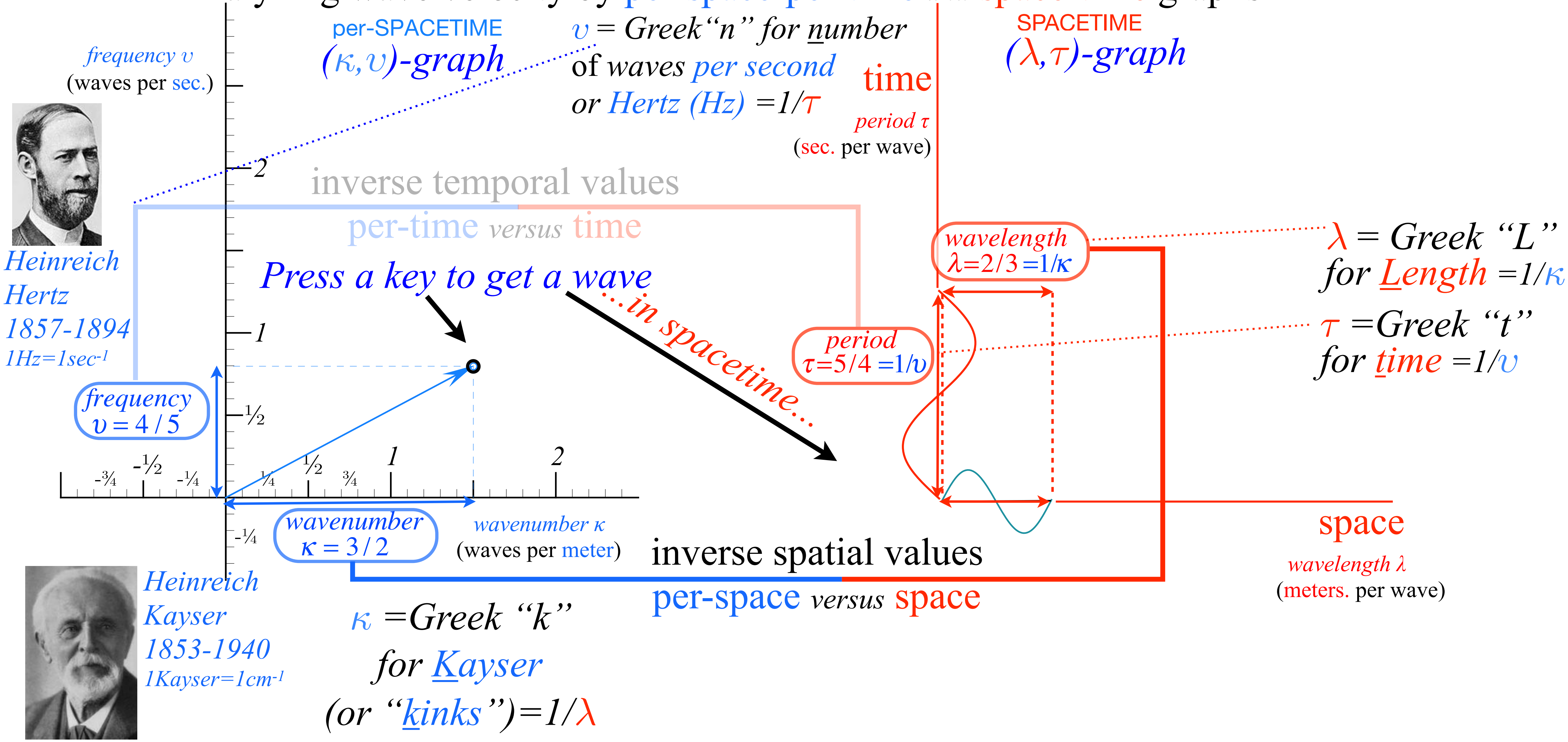


# Analyzing wave velocity by per-space-per-time *and* space-time graphs

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[RelaWavity Web Simulation *Keyboard of the Gods* (Dual Plot)]

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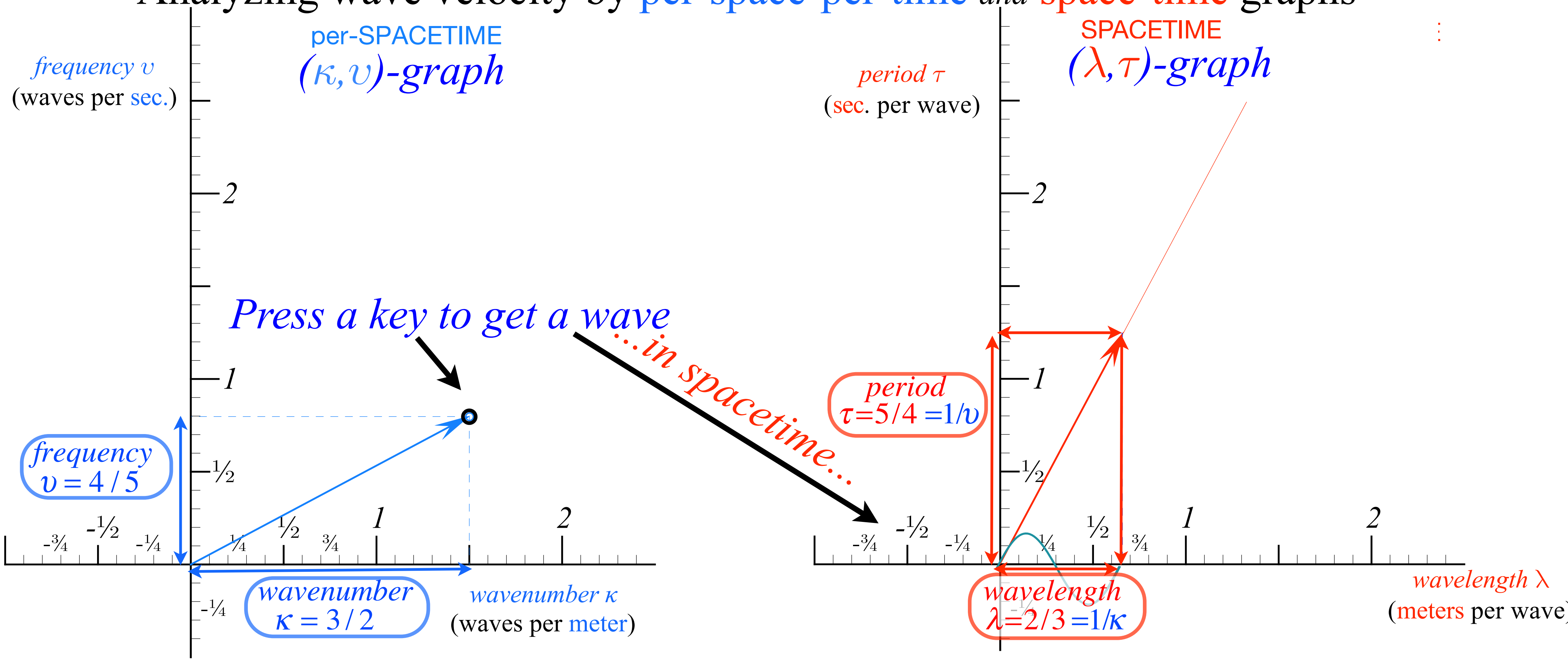


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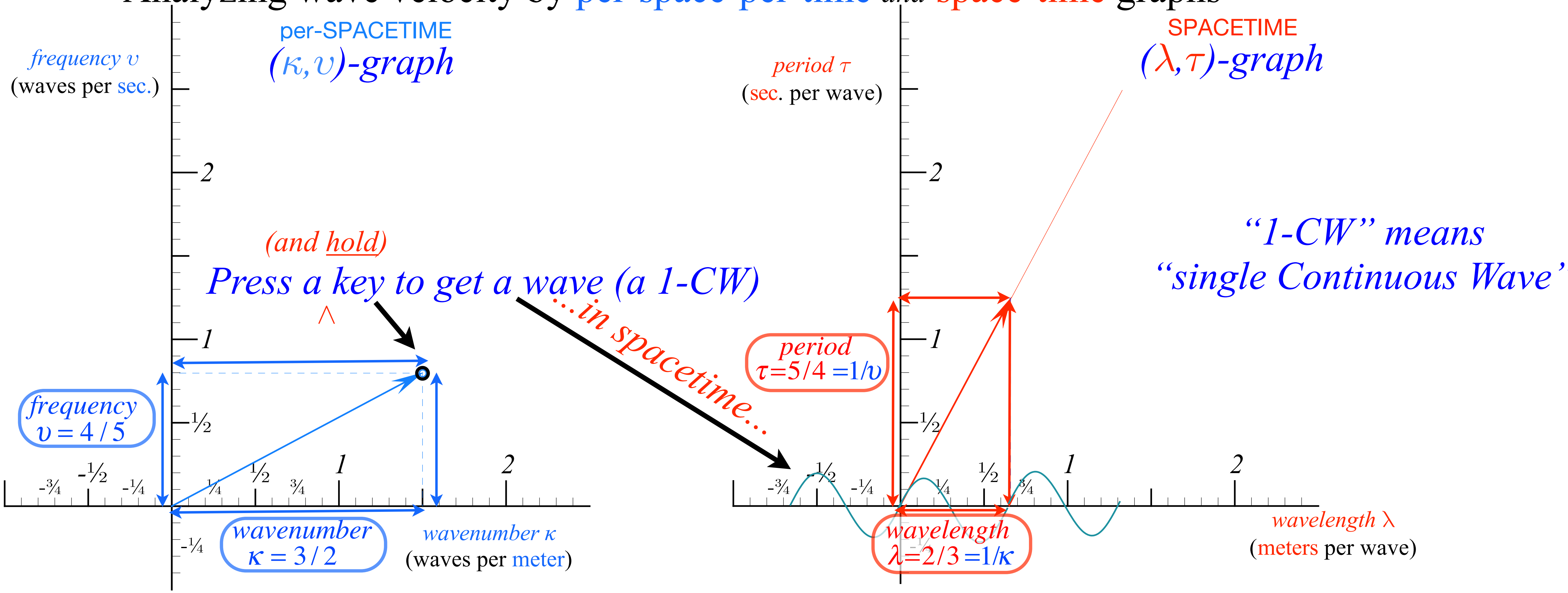


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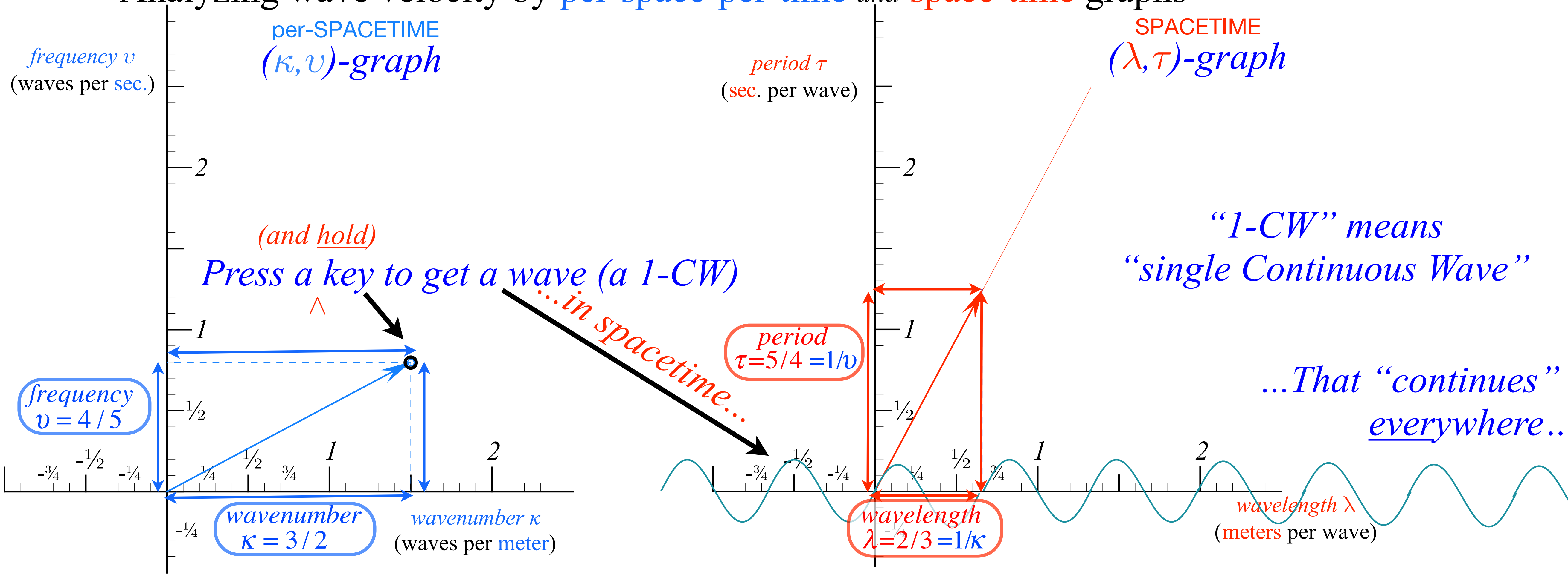
RelaWavity Web Simulation
*Keyboard of the Gods*
*(Dual Plot)*

Analyzing wave velocity by per-space-per-time *and* space-time graphs

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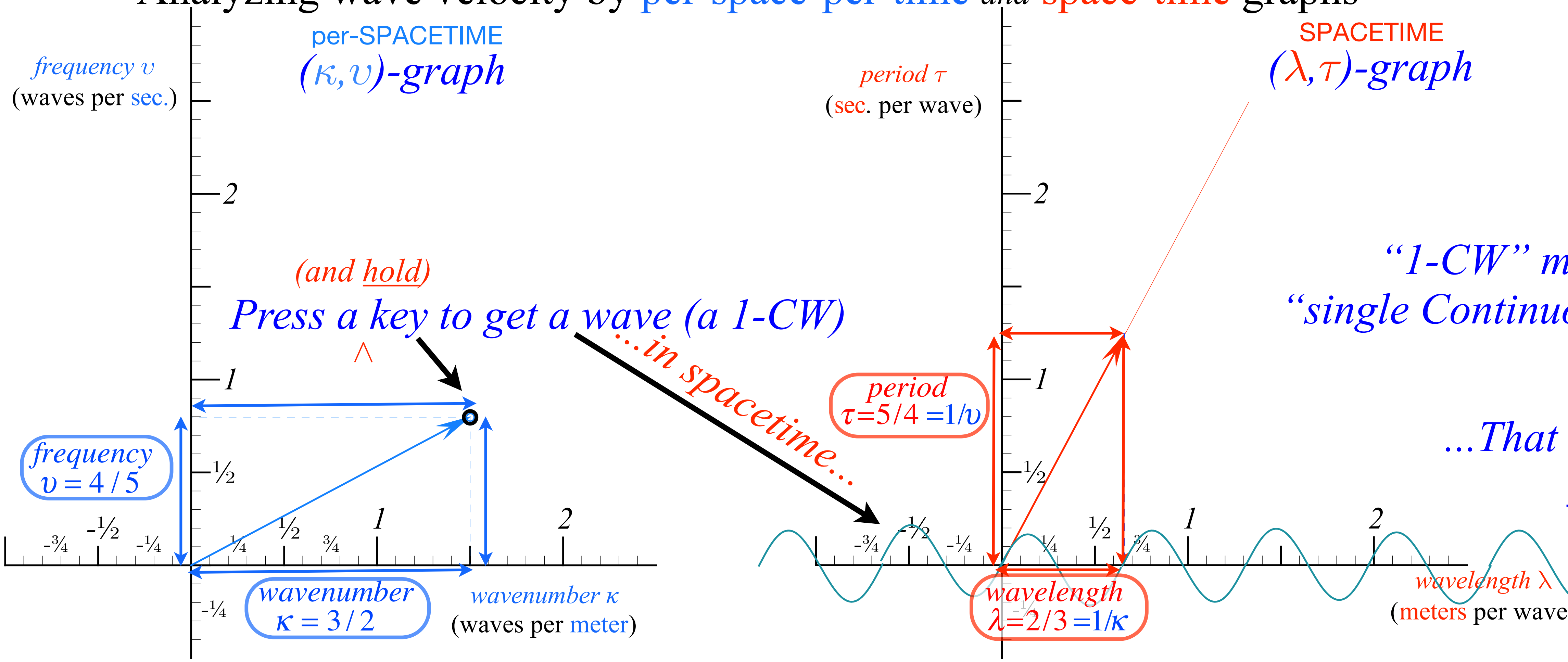
[RelaWavity Web Simulation](#)
[*Keyboard of the Gods*](#)
[*(Dual Plot)*](#)

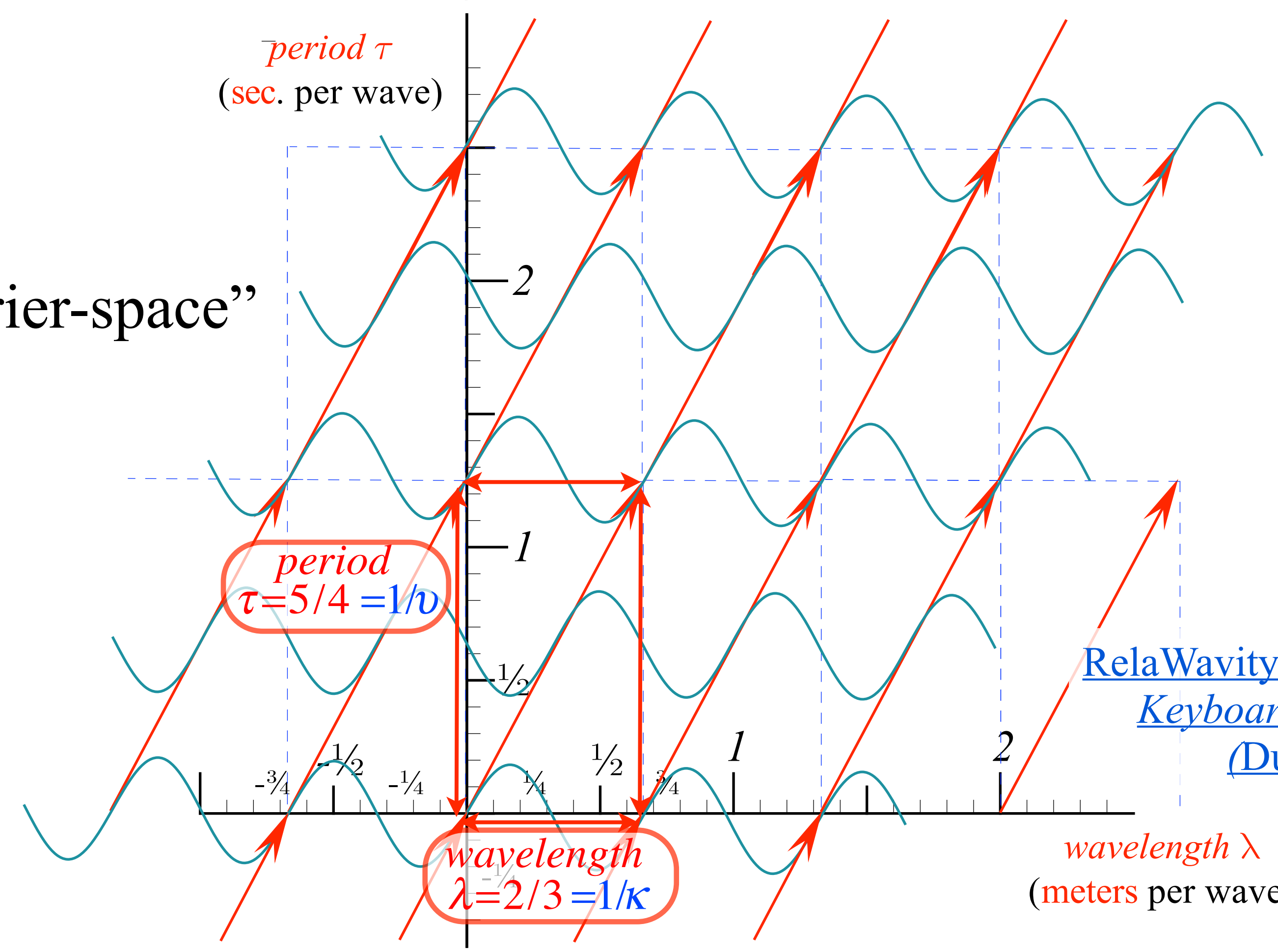
# Analyzing wave velocity by per-space-per-time *and* space-time graphs




*"1-CW" means "single Continuous Wave"*

*...That "continues" everywhere...*

*...for all time...*

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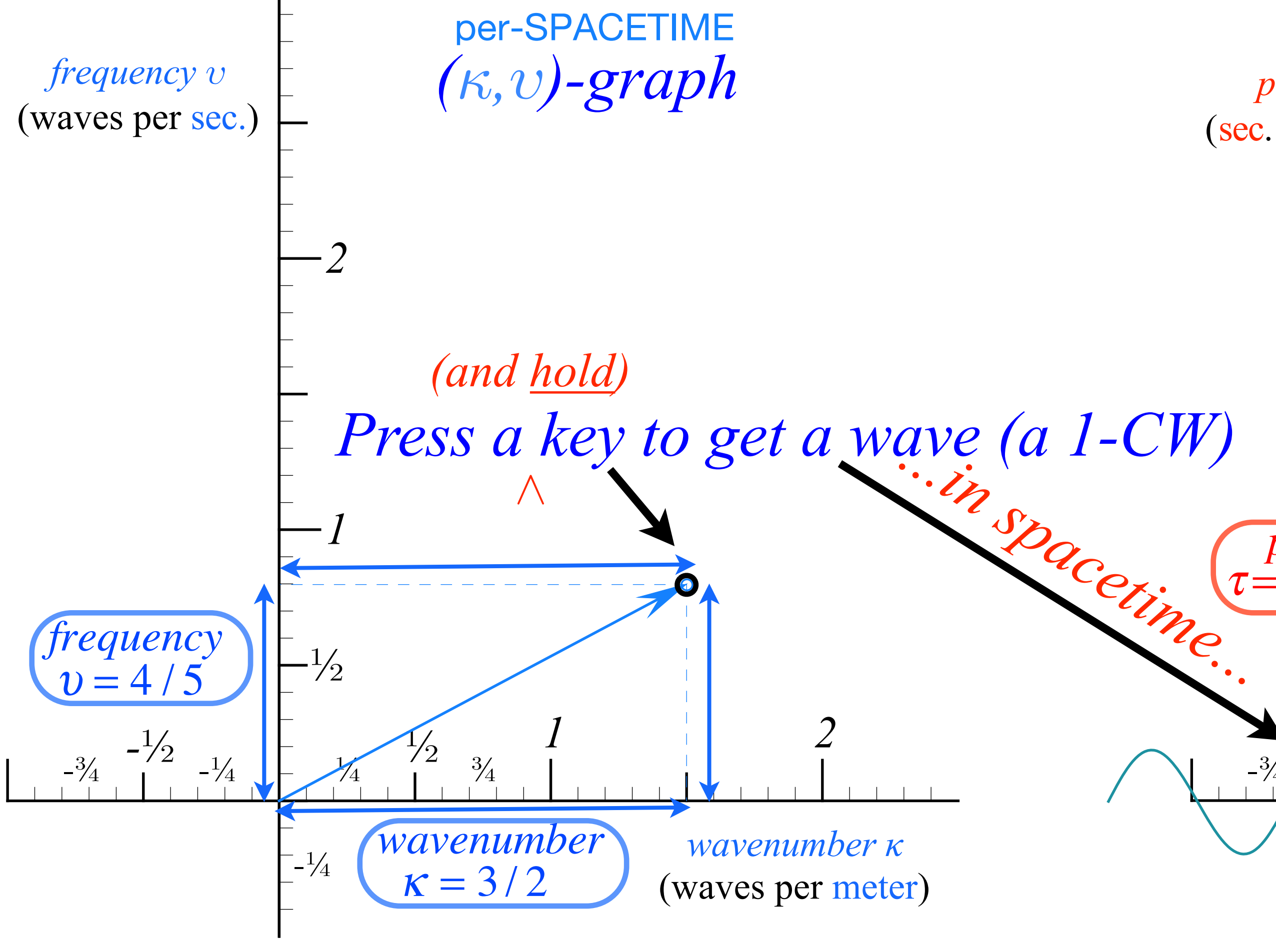
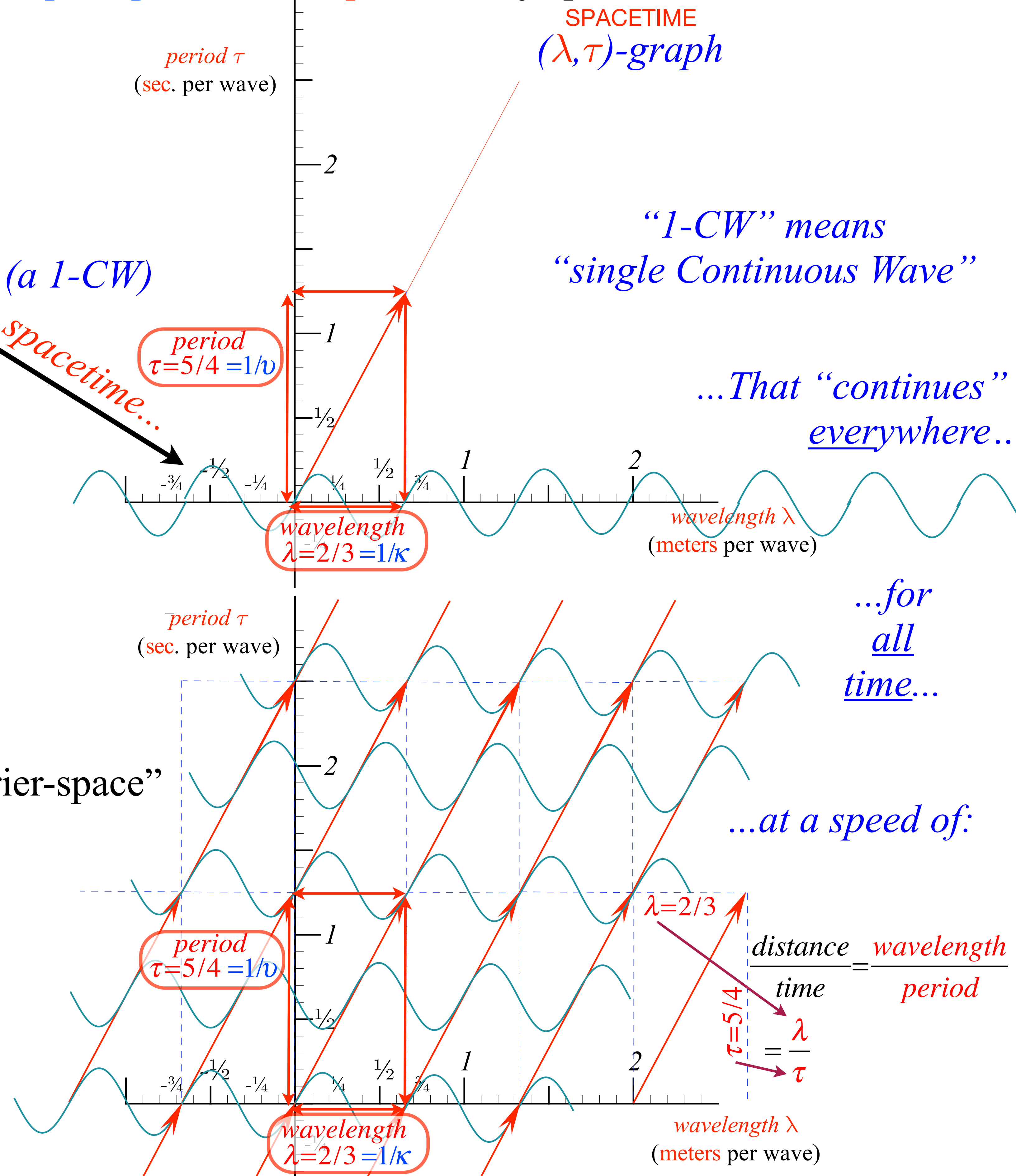
# Analyzing wave velocity by per-space-per-time *and* space-time graphs



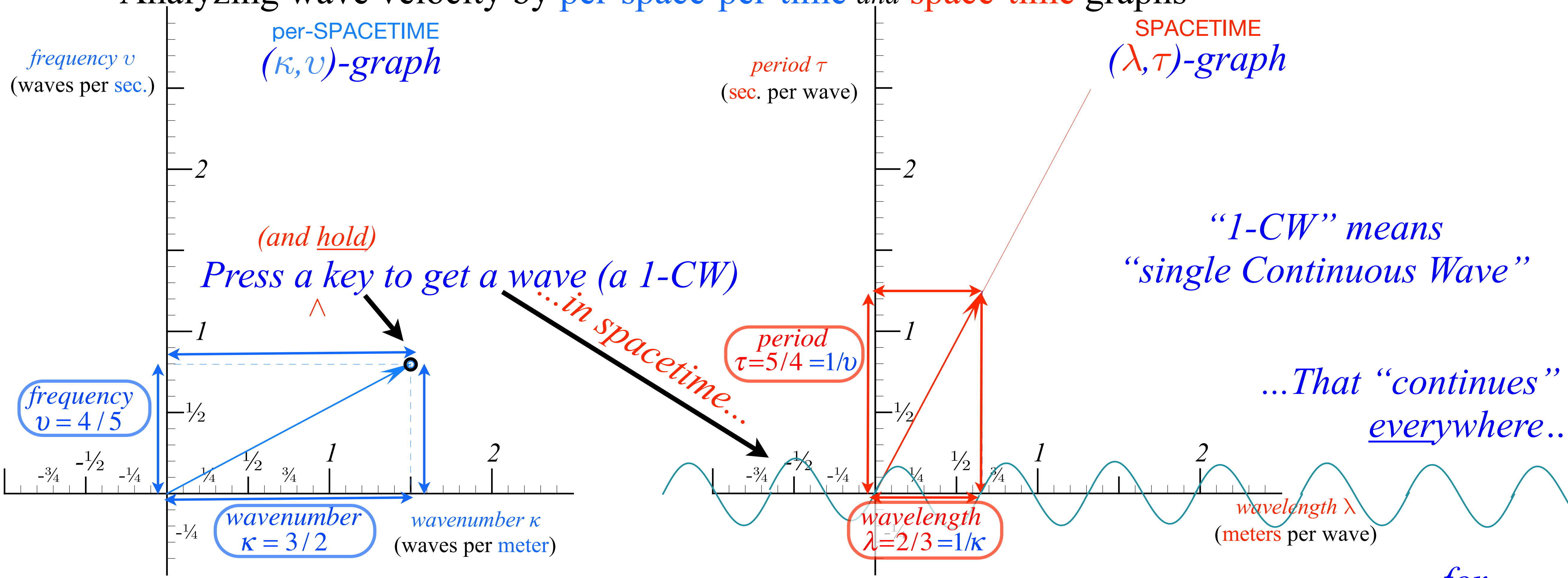



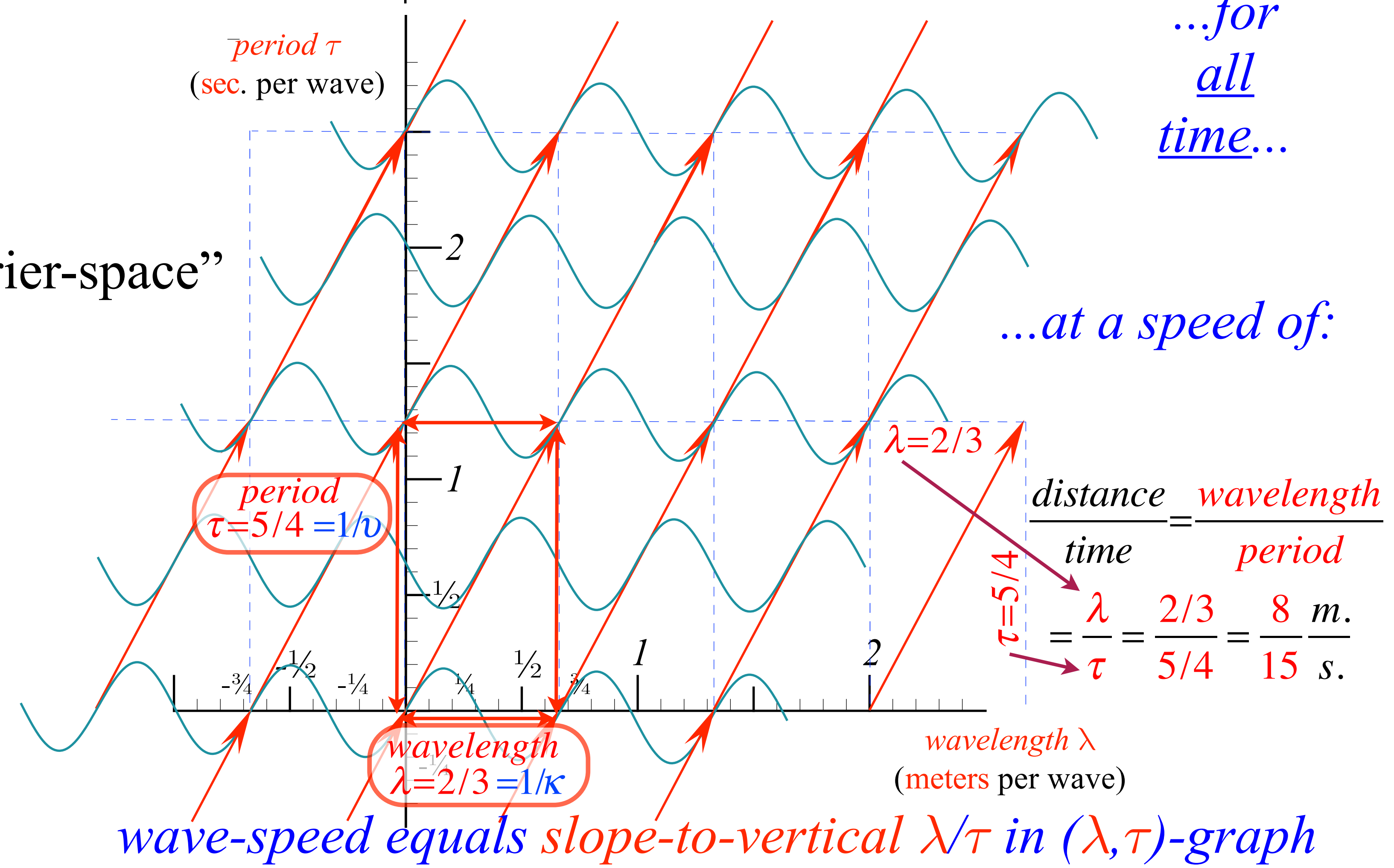

"*Keyboard of the gods*" is known as "Fourier-space"

*Jean-Baptiste*
*Joseph Fourier*
*1768-1830*

*•How to understand waves*
*and*
*wave velocity $V_{wave}$*

# Analyzing wave velocity by per-space-per-time *and* space-time graphs

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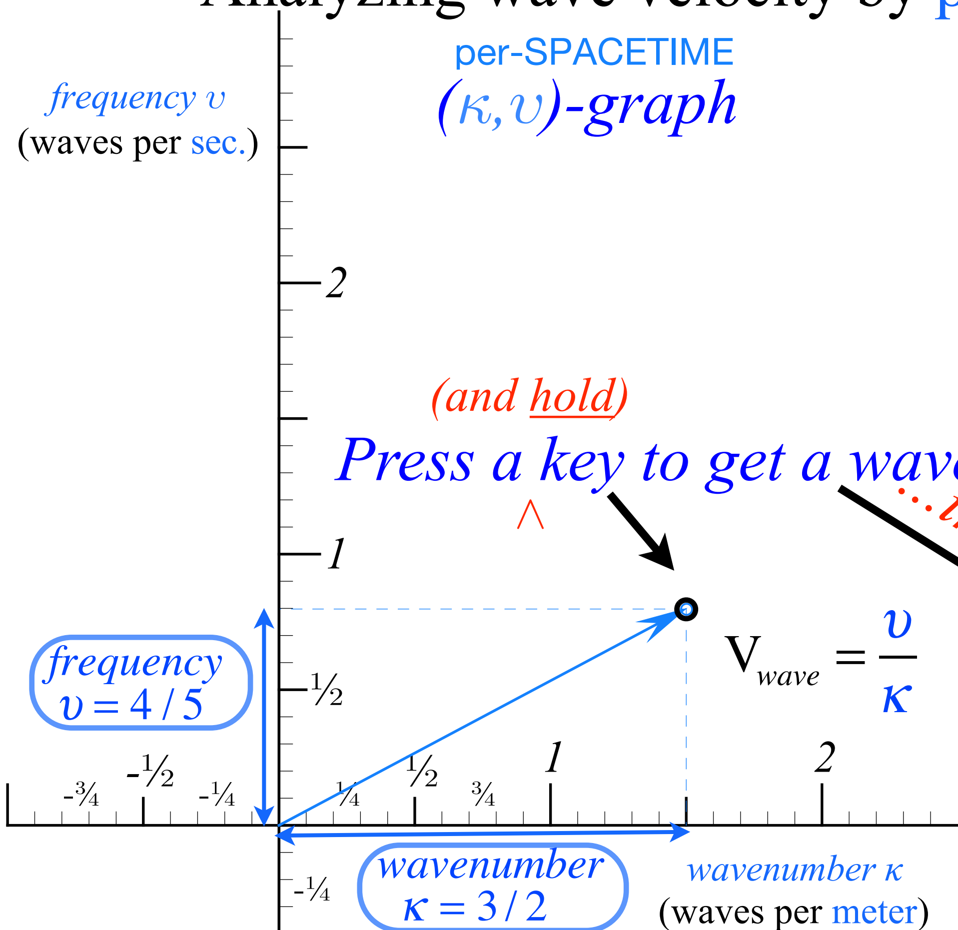
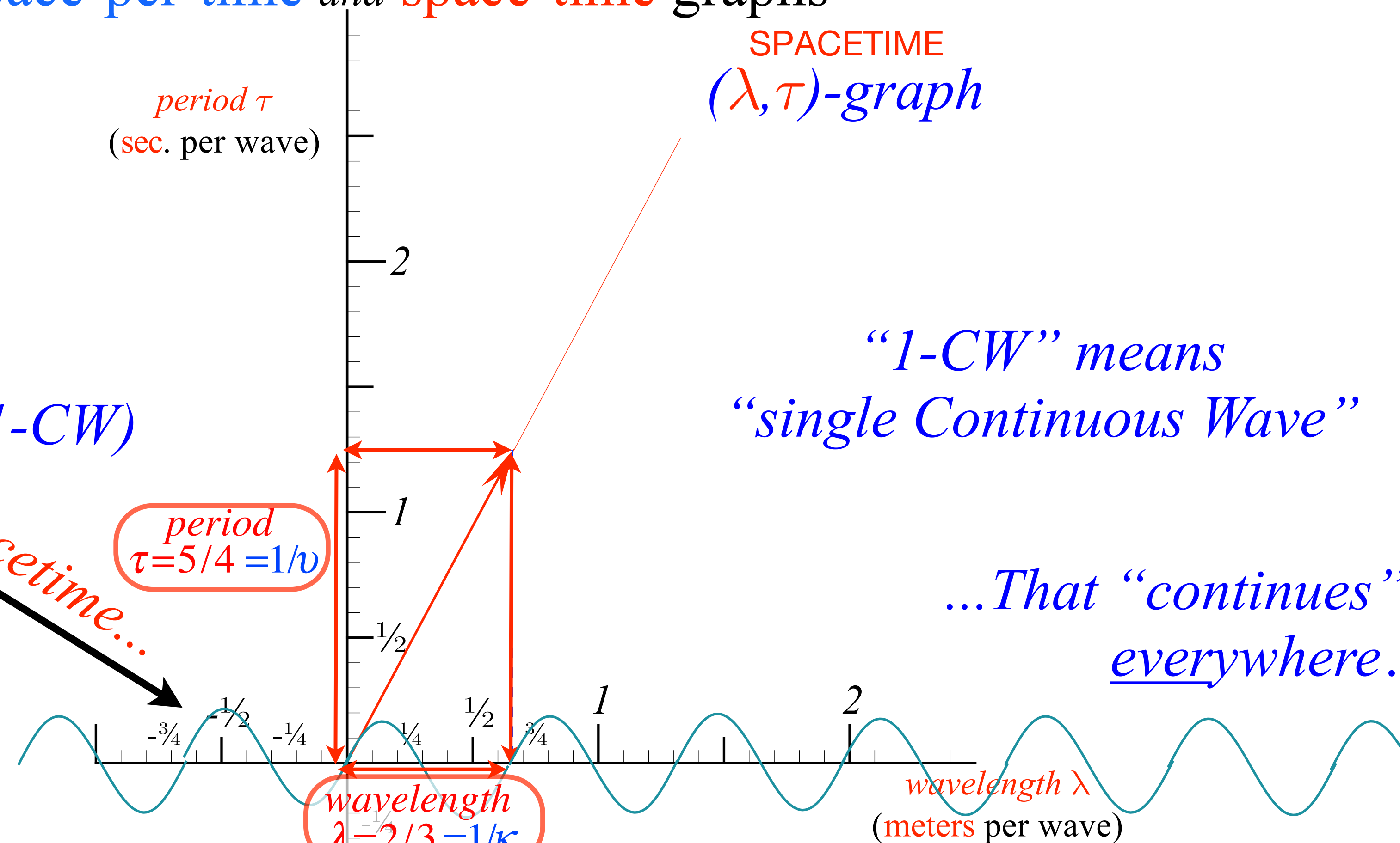


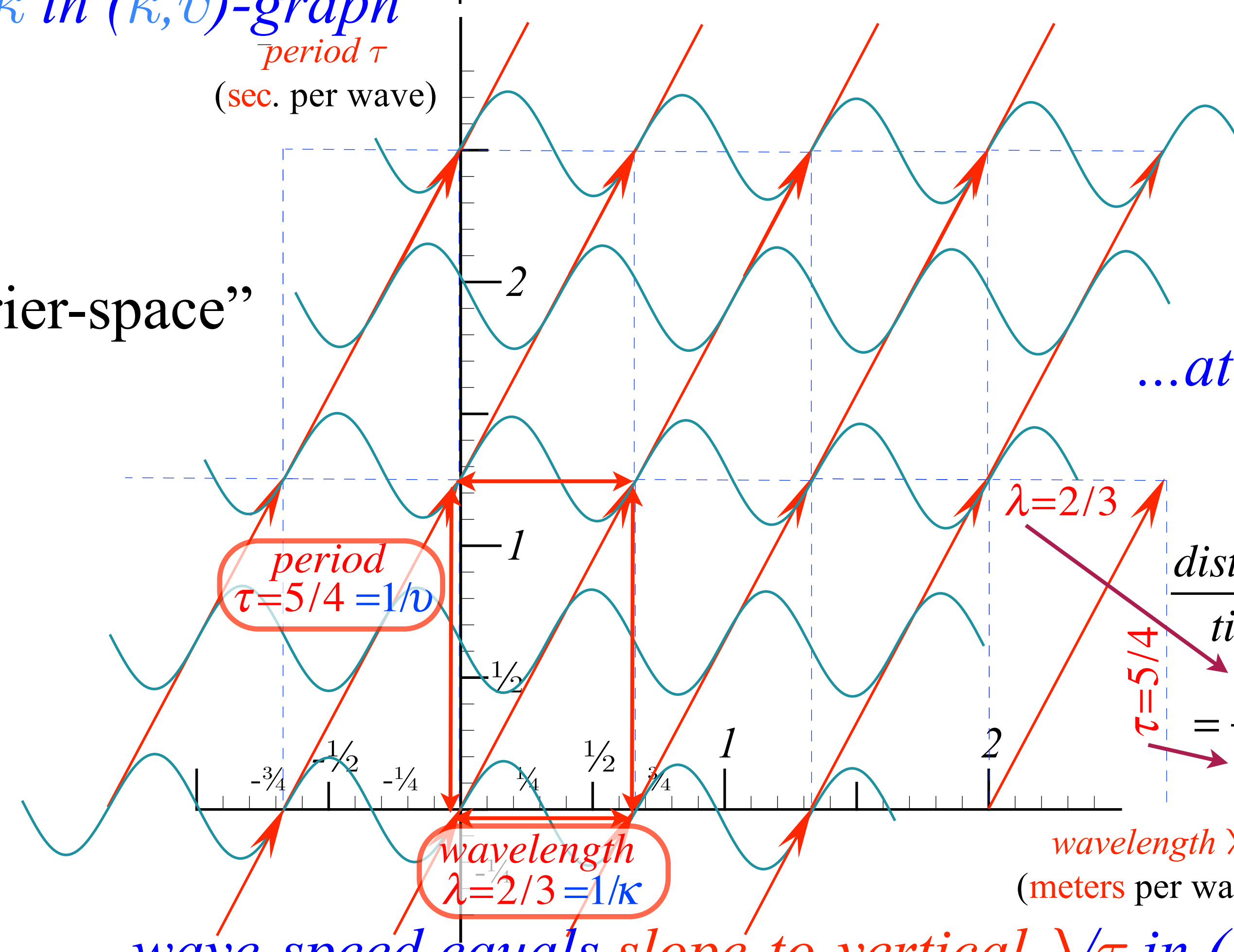




*"1-CW" means "single Continuous Wave"*

*...That "continues" everywhere...*

*wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph*

*...for all time...*




*...at a speed of:*

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

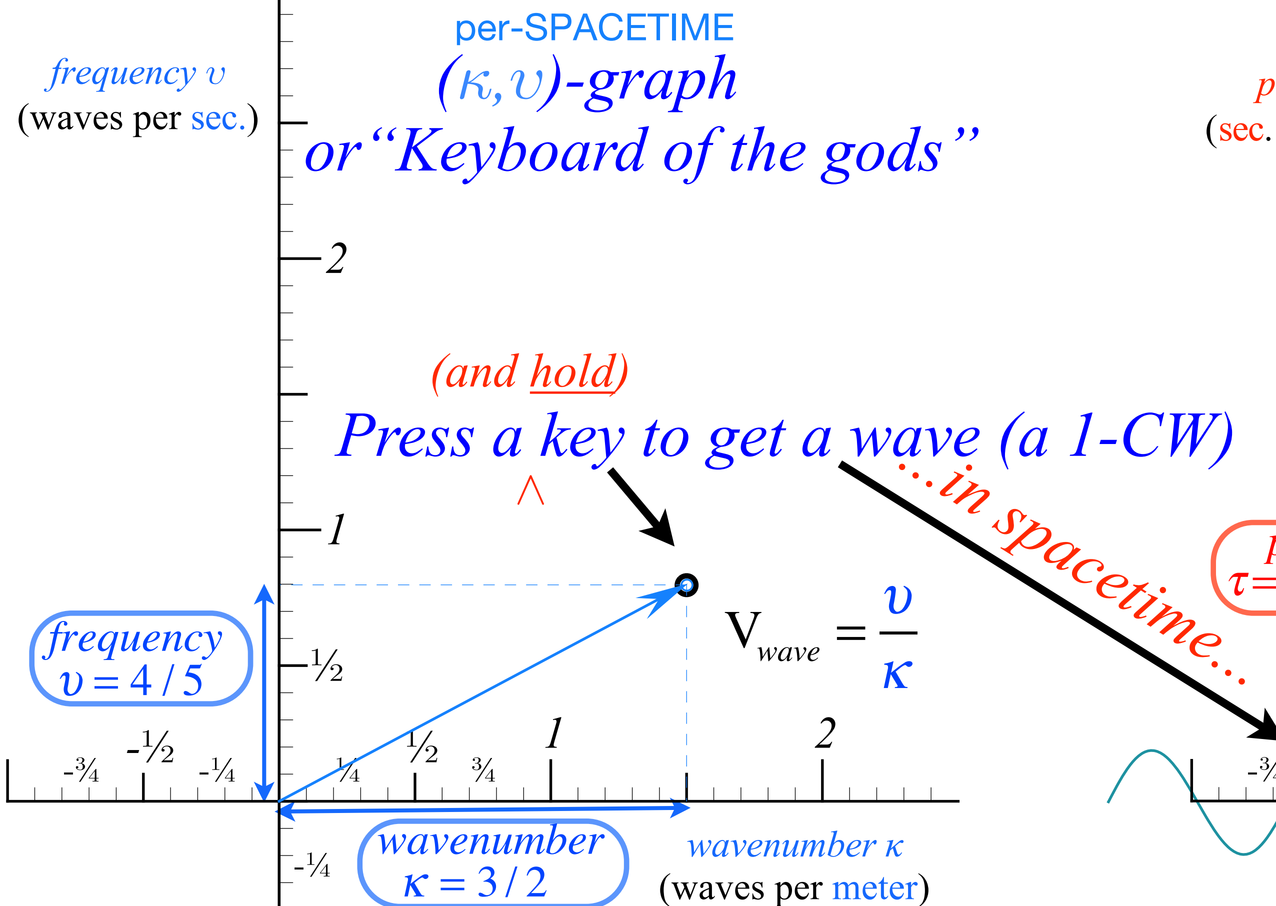
*"Keyboard of the gods"* is known as "Fourier-space"

*Jean-Baptiste Joseph Fourier 1768-1830*

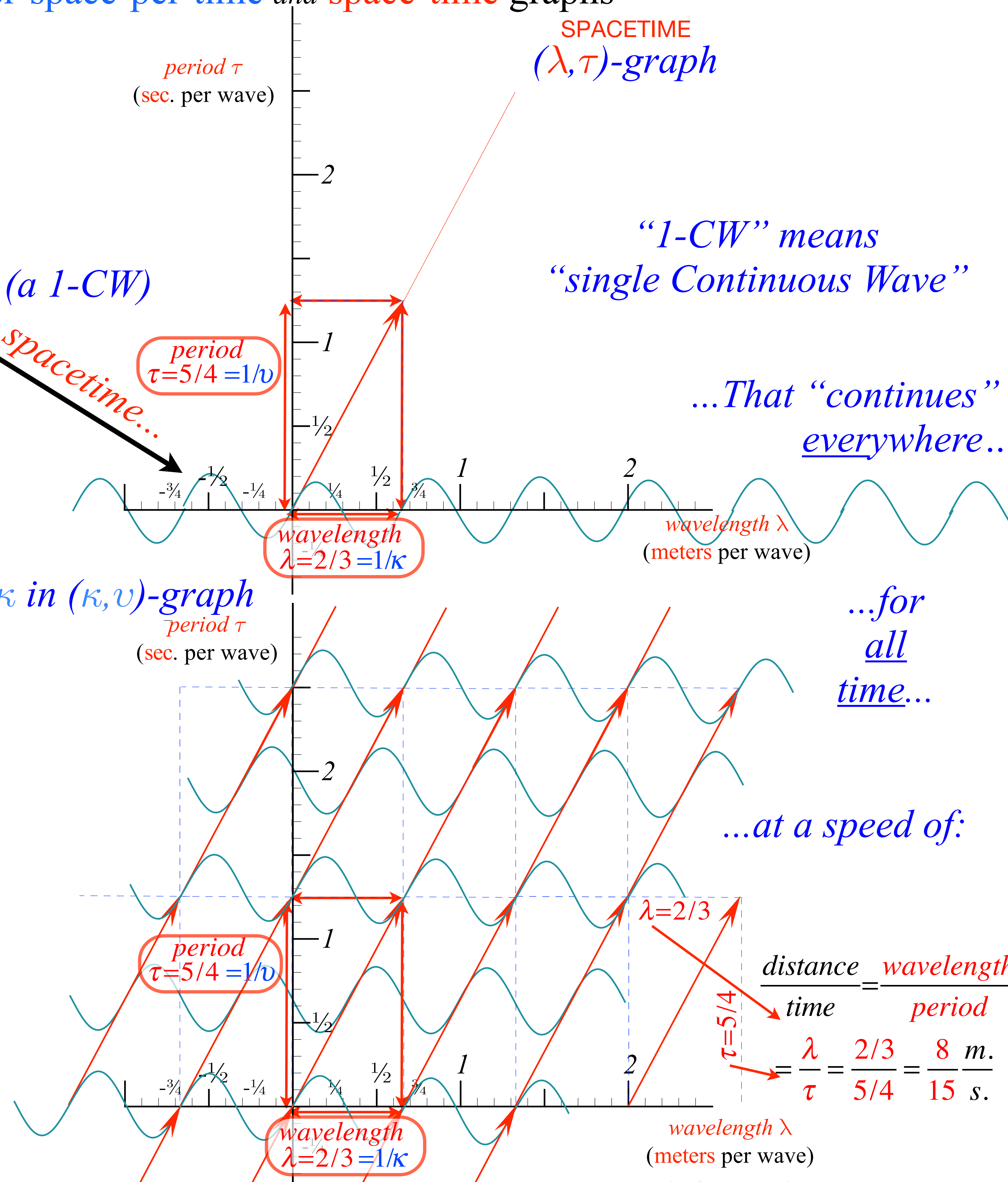
*•How to understand waves and wave velocity $V_{wave}$*

*wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph*

# Analyzing wave velocity by per-space-per-time *and* space-time graphs




*wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph*




## *wave-velocity formulas*

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}}$$

$$= \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

*wave arithmetic is simpler to explain using fractions*

•*How to understand waves*
*and*
*"1st quantization"*

*wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph*

*Why* ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

Wave velocity formulas

➔ Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* $\rho$

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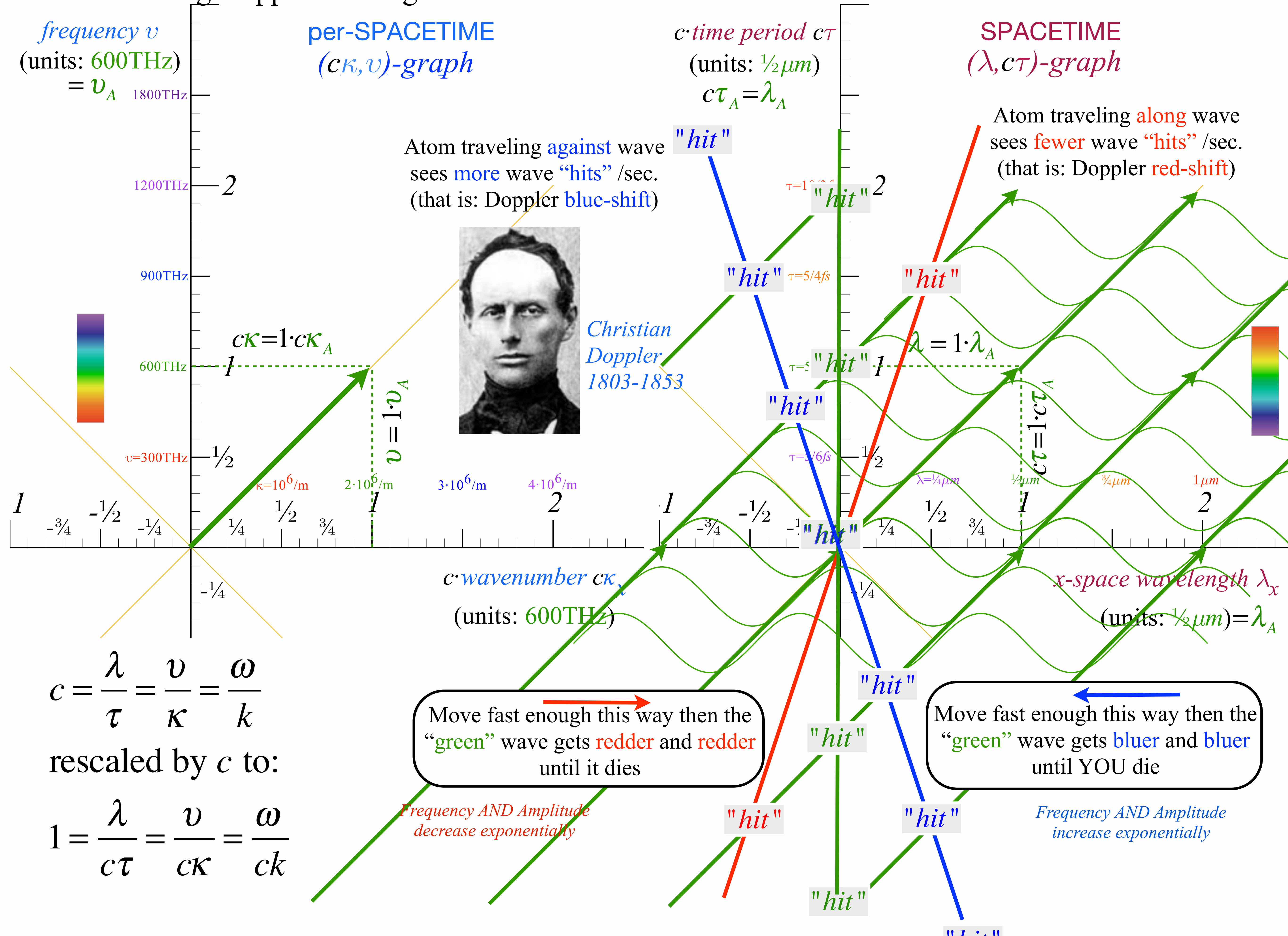
# Introducing Doppler shifting




$$c = \frac{\lambda}{\tau} = \frac{\upsilon}{\kappa} = \frac{\omega}{k}$$

rescaled by $c$ to:

$$1 = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = \frac{\omega}{ck}$$

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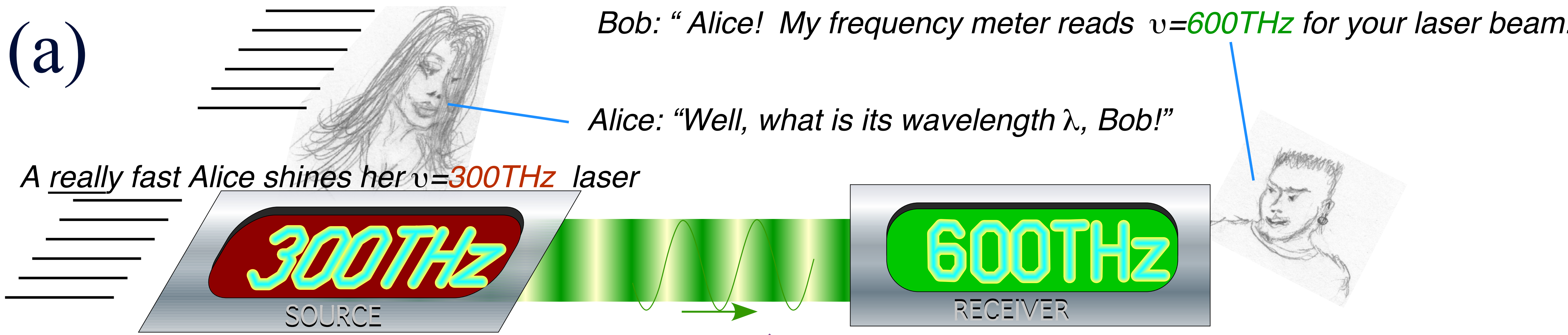
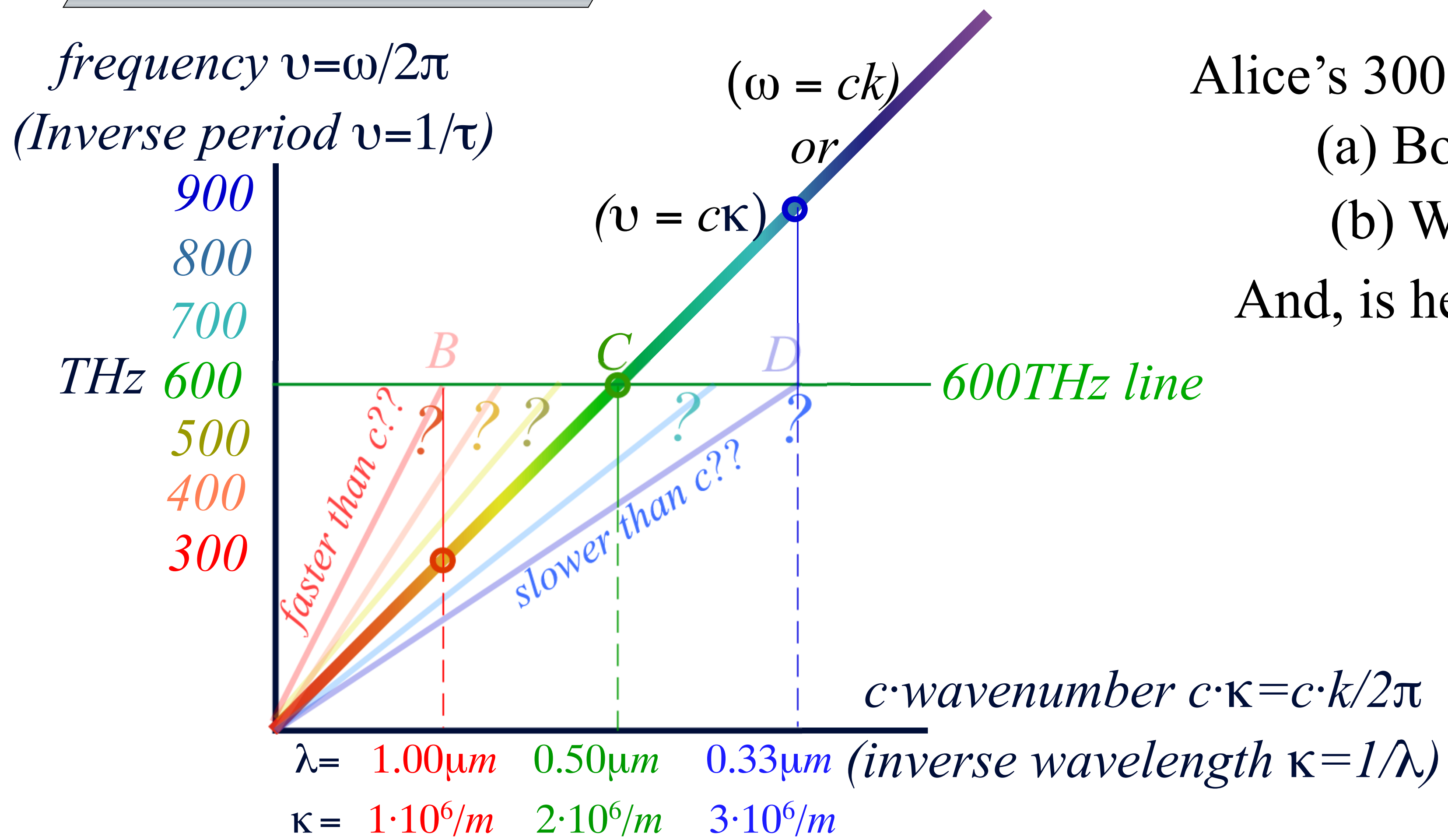
Introducing Doppler shifting and why *c* is *so* constant (and *so slow*)

(a)




(b)



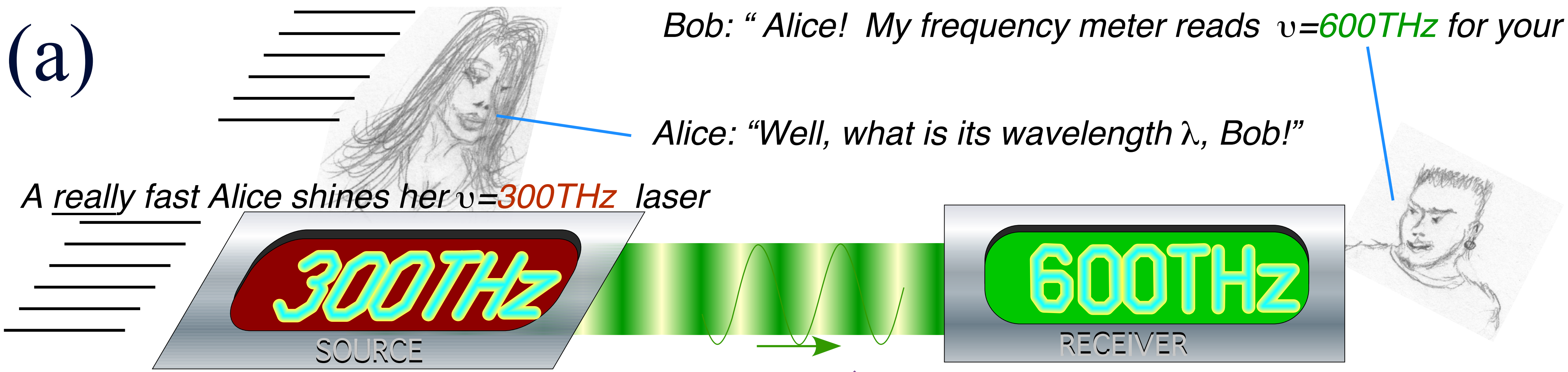

Alice's 300THz laser approaches Bob.

(a) Bob sees $\upsilon$=600THz.

(b) What $\lambda = 1/\kappa$ does Bob measure?

And, is he seeing a '*phony*' *green*?

Introducing Doppler shifting and why $c$ is constant



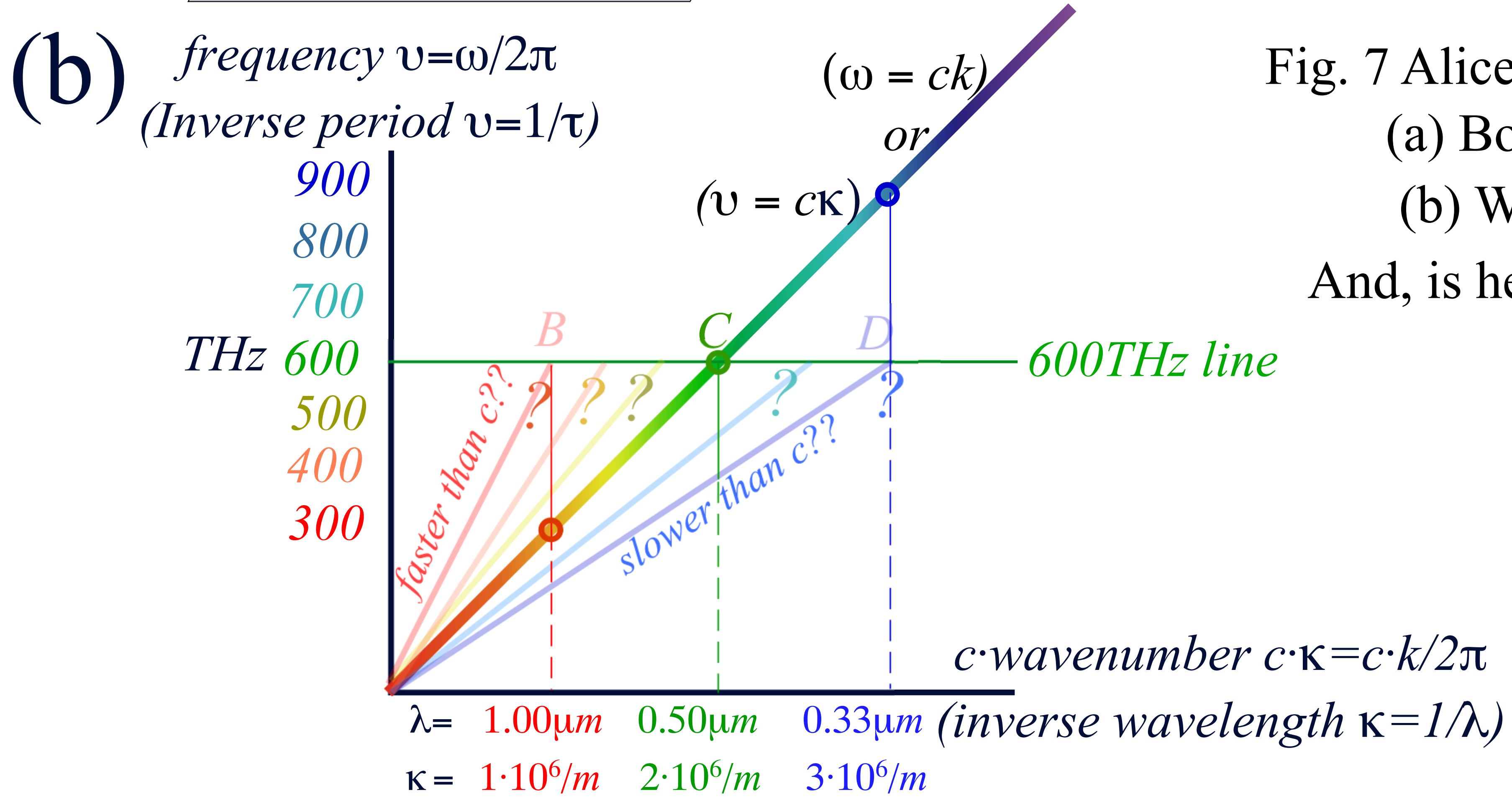
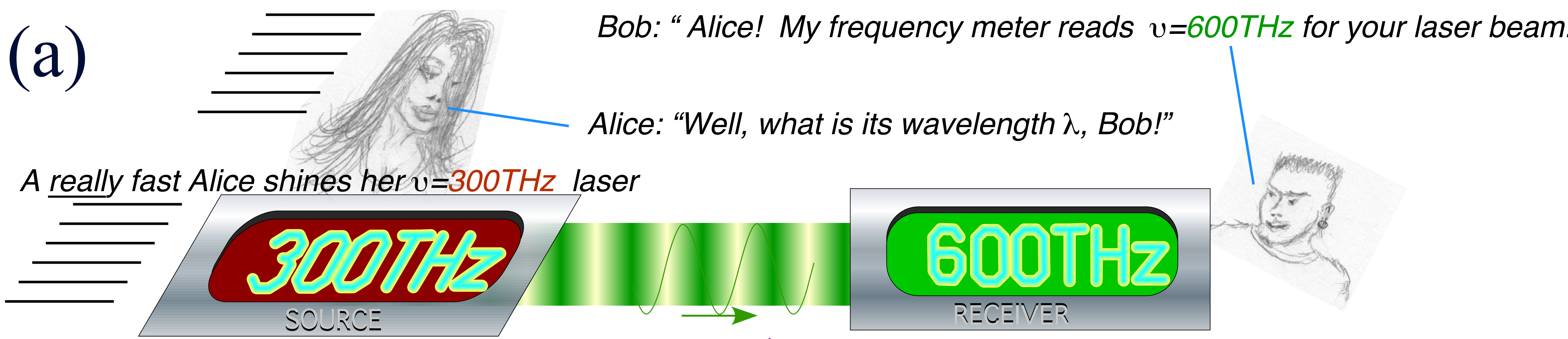



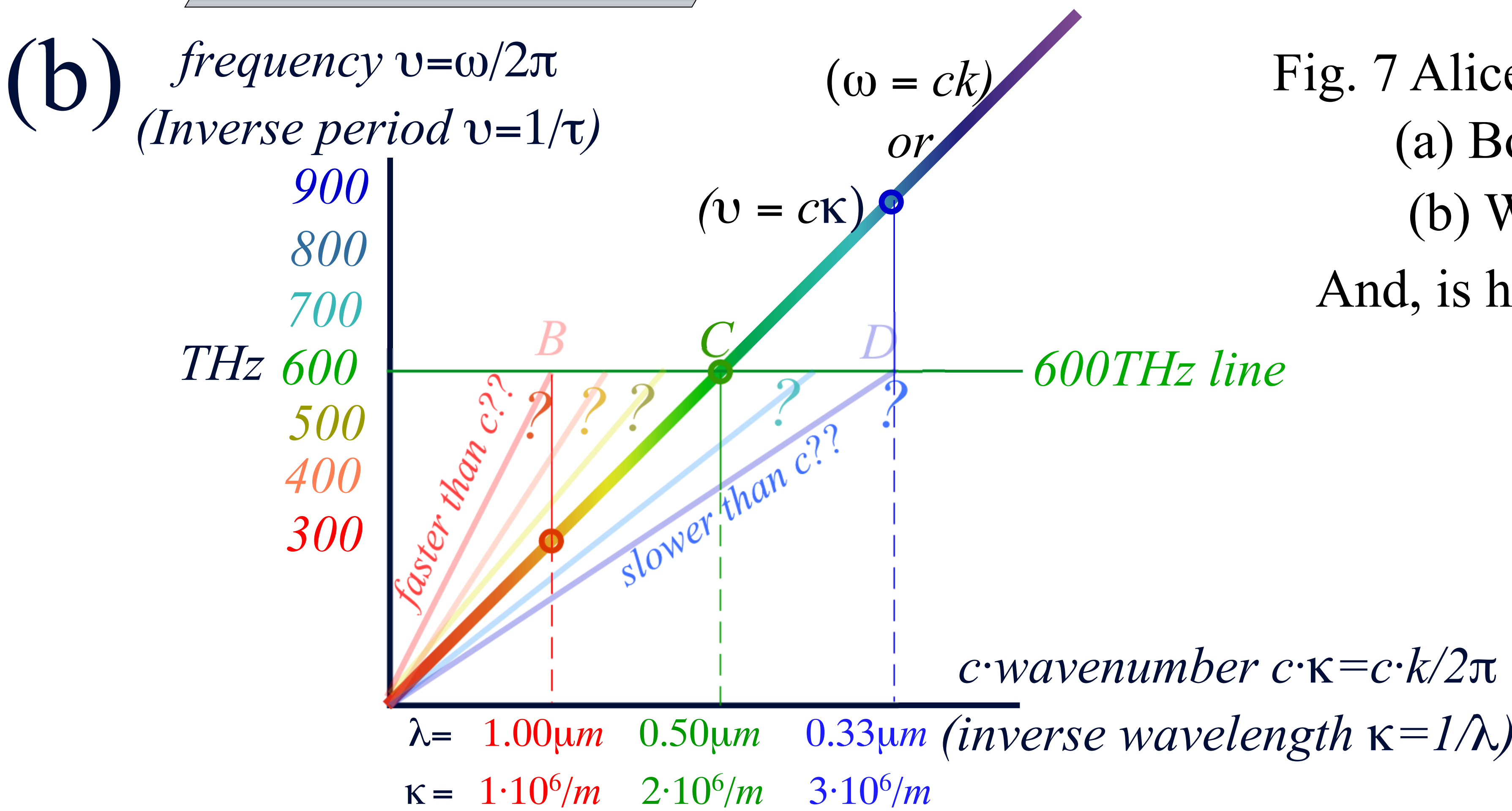

Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees $\upsilon$=600THz.
(b) What $\lambda=1/\kappa$ does Bob measure?
And, is he seeing a '*phony' green*?

Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength $\lambda$=0.5*micron.*

The only choice is C.

# Introducing Doppler shifting and why c is constant

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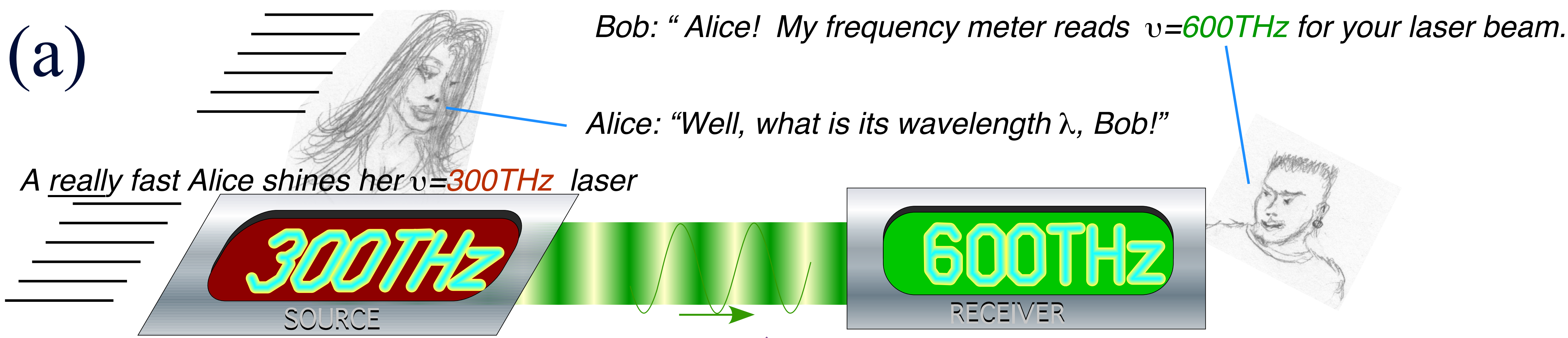
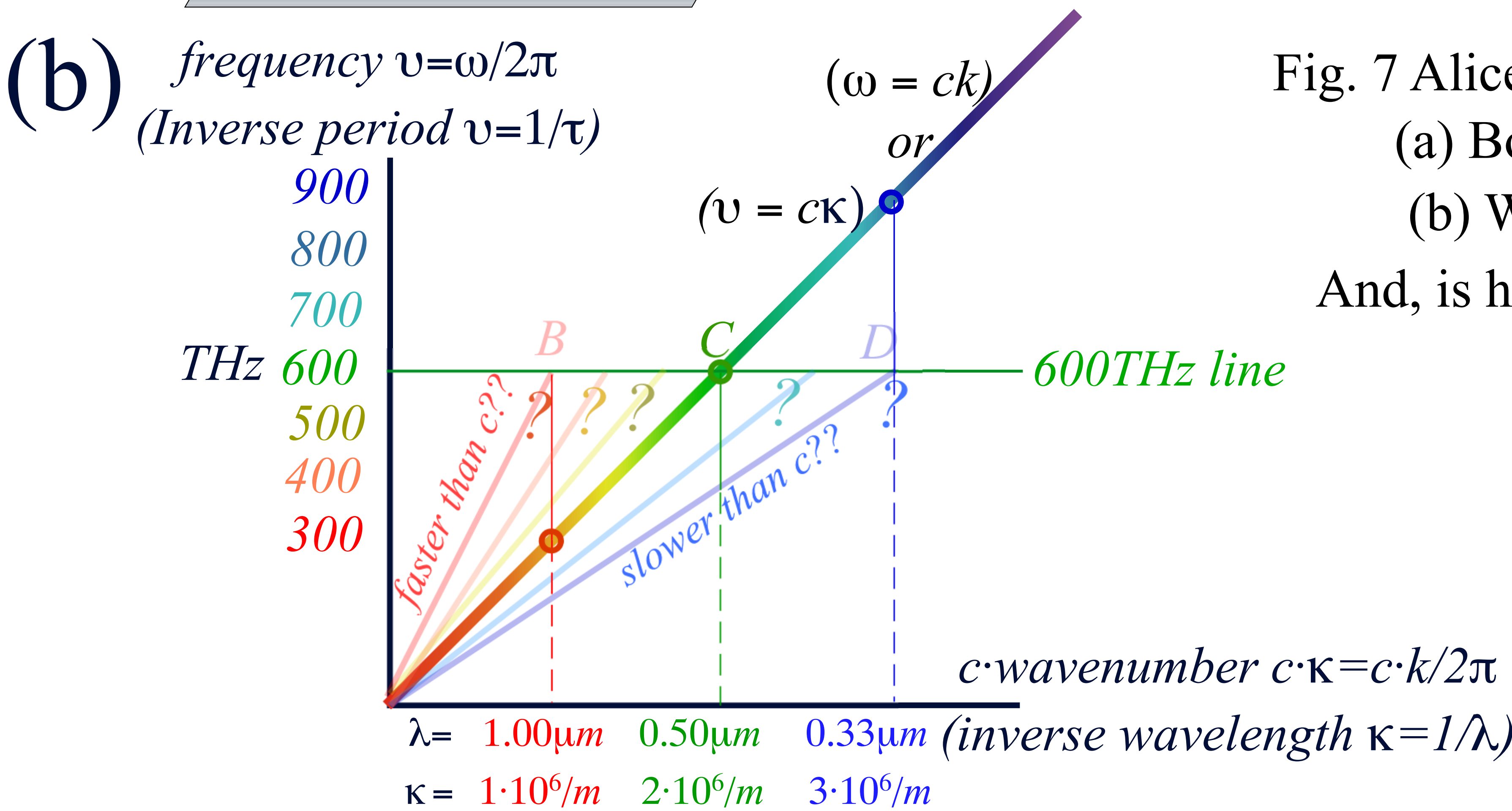
Introducing Doppler shifting and why c is constant

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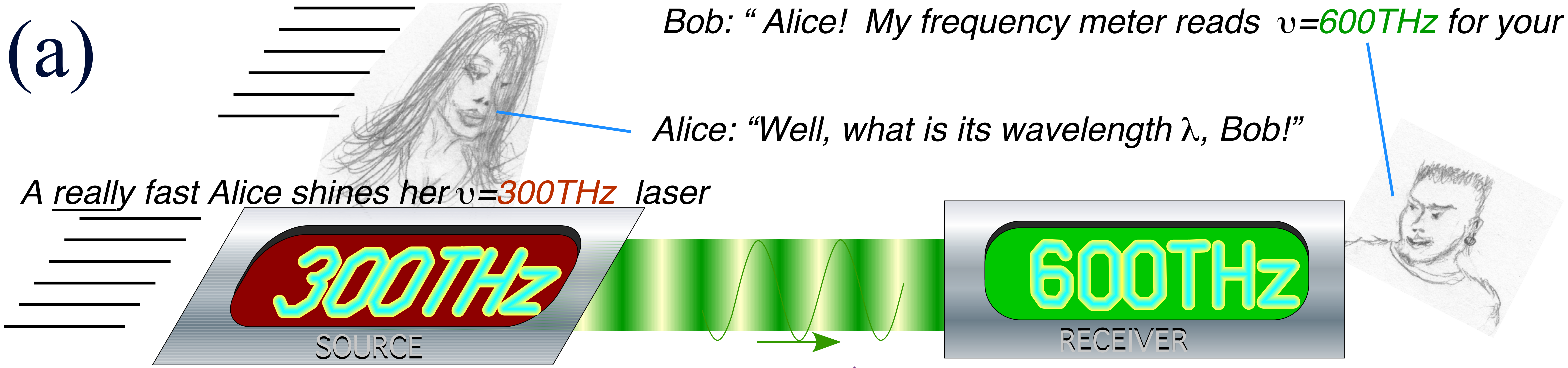
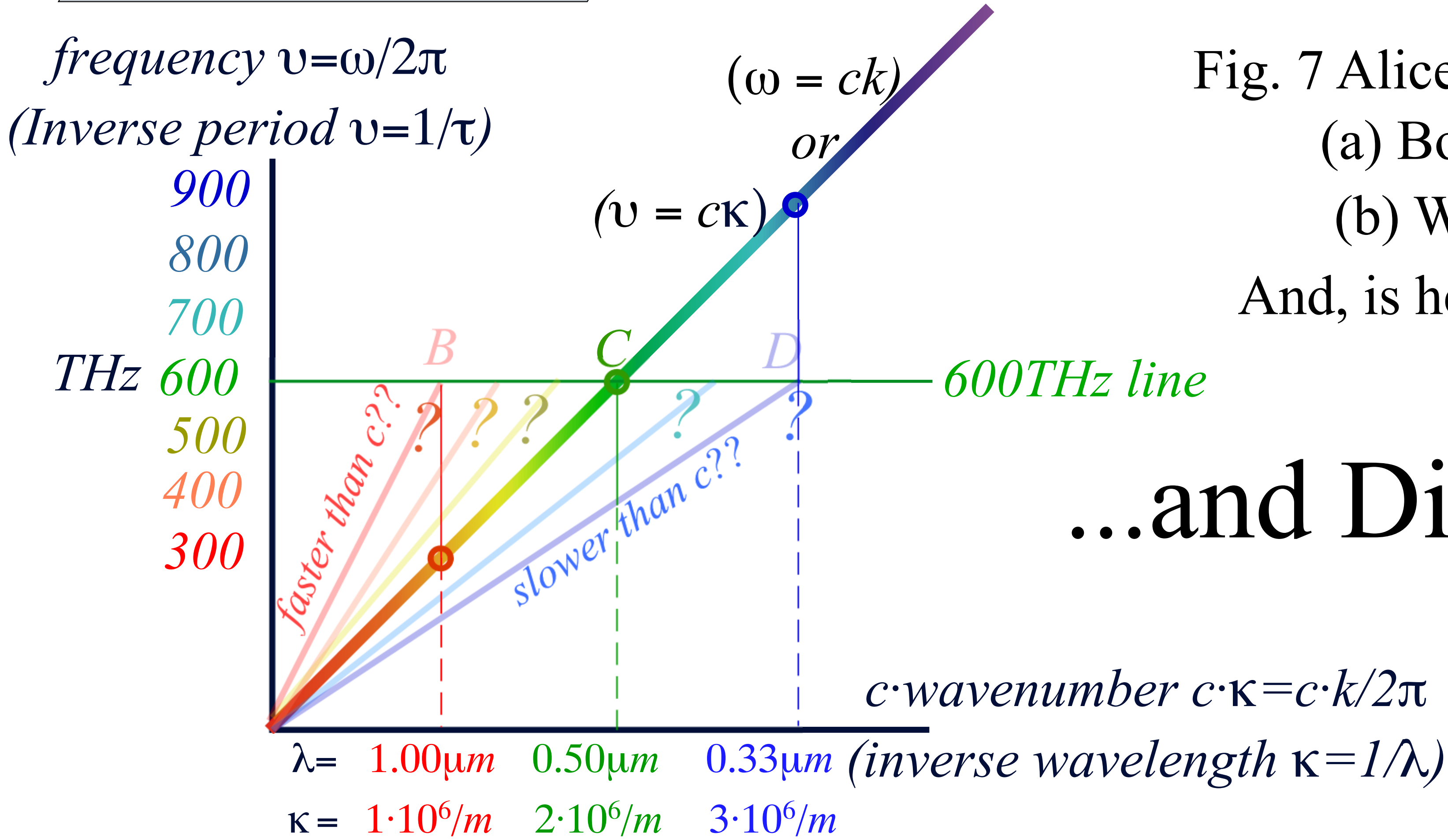
Actually: $2.99792458\cdot 10^{8} m\cdot s^{-1}$

# Introducing Doppler shifting and why c is constant

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# ...and Dispersion-Free!

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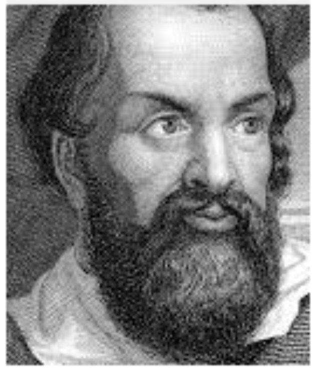
Galileo Galilei



1564-1642

**Galileo's Revenge (part 1)**
*Rapidity adds just like Galilean velocity*

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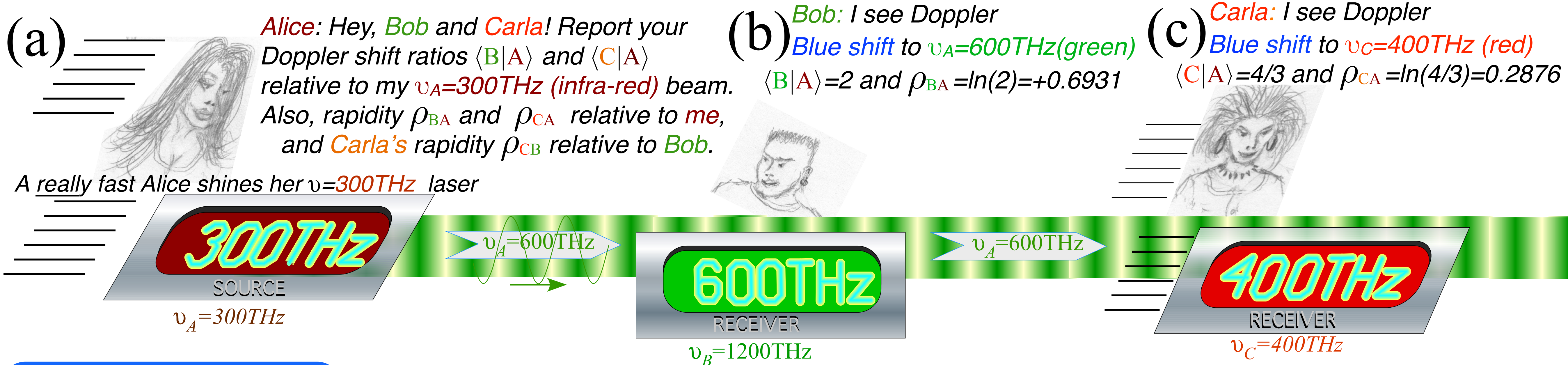



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

*Definition of Rapidity* $\rho_{RS}$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

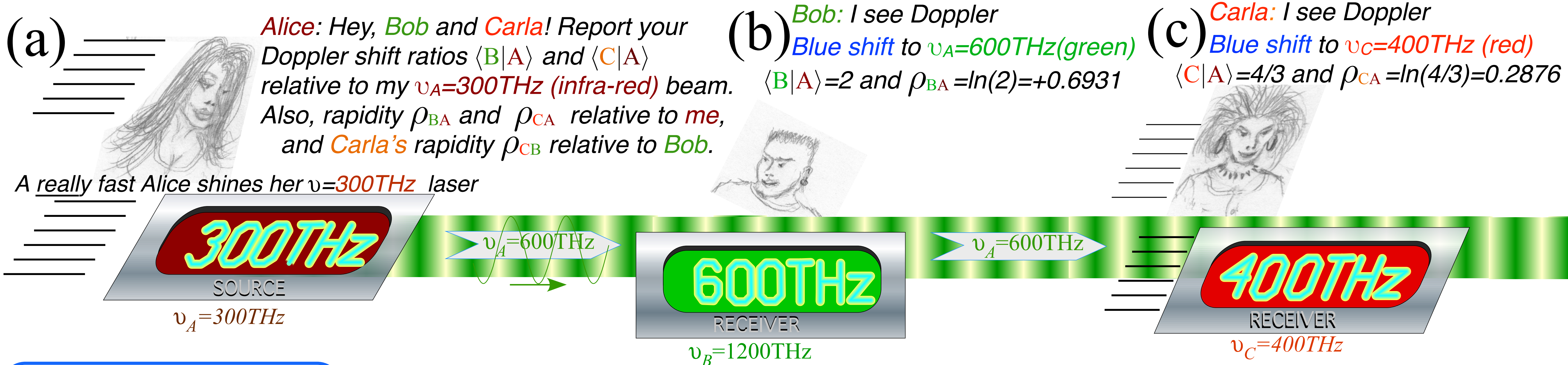
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ

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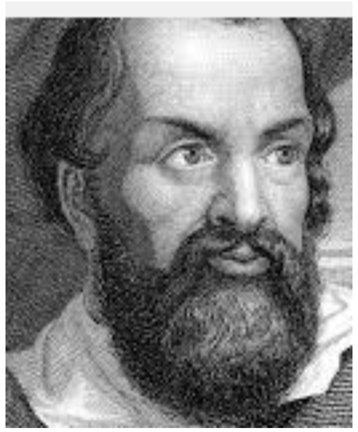
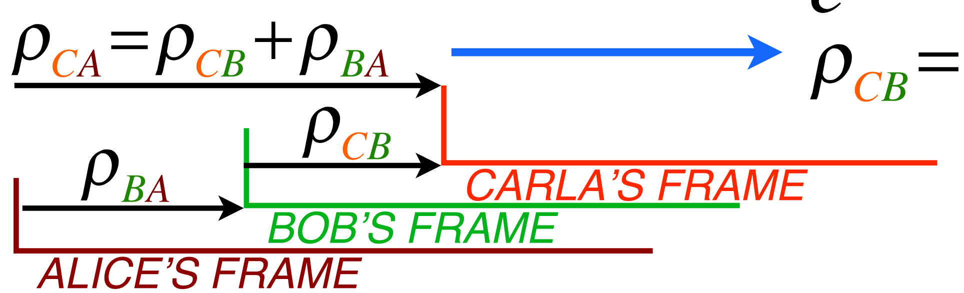
*Galileo Galilei*



*1564-1642*

***Galileo's Revenge (part 1)***

*Rapidity adds just like Galilean velocity*




Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

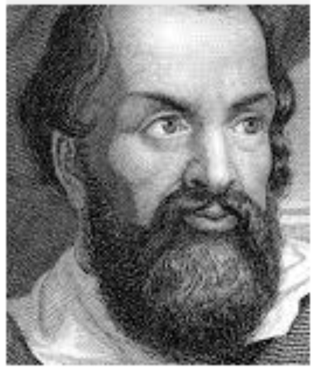
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$

*Galileo Galilei*



*1564-1642*

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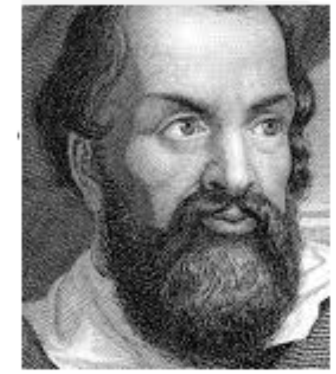

***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
*Galilean velocity*

➔ Optical interference "baseball-diamond" displays *phase* and *group* velocity
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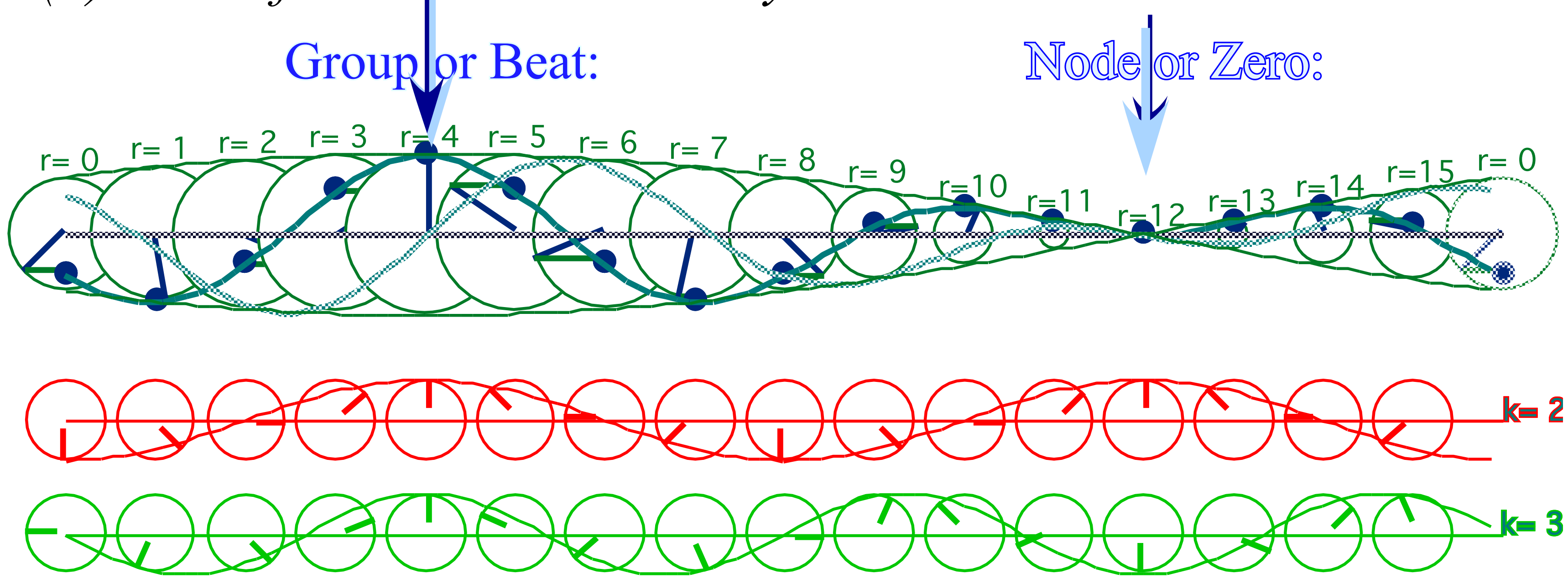
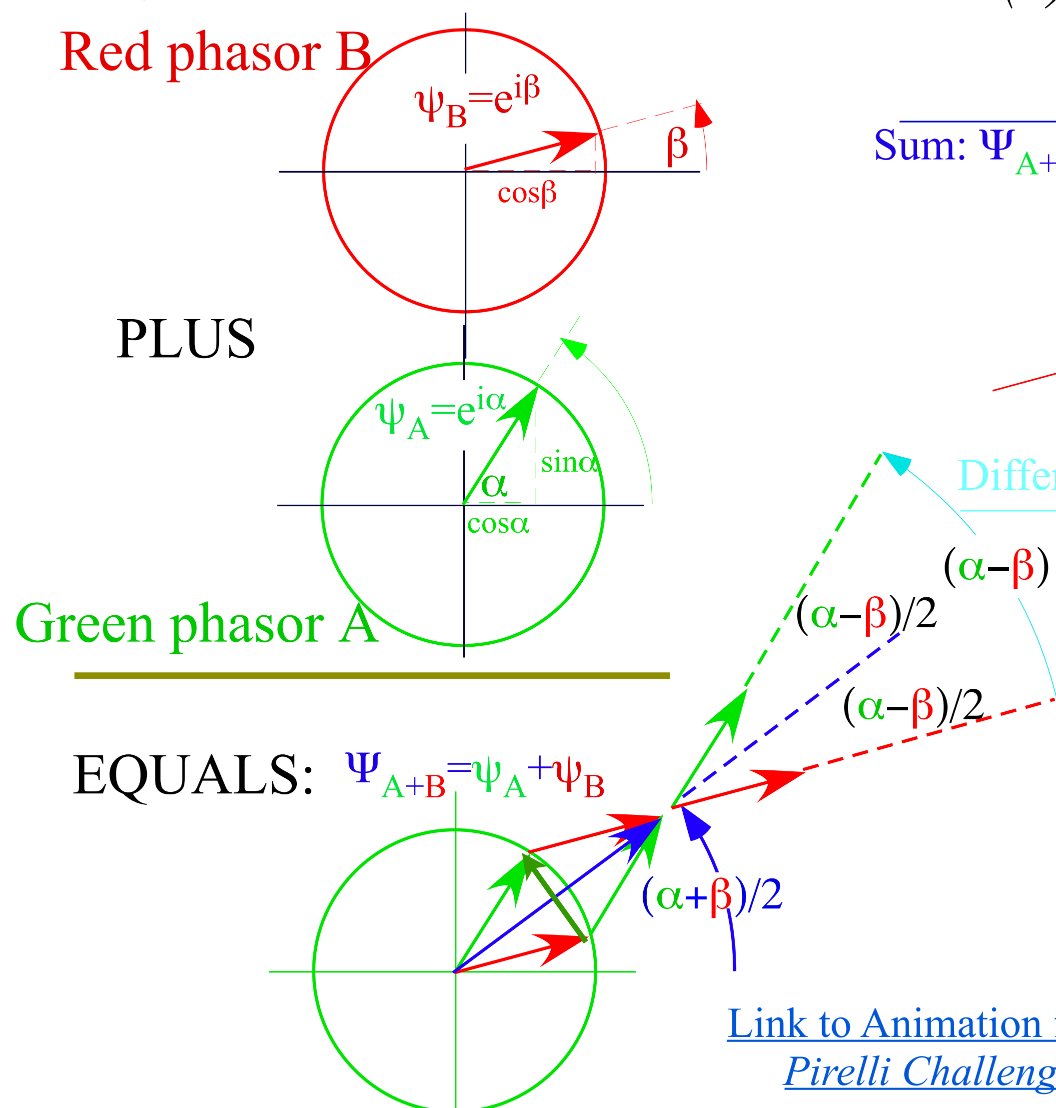
## (a) Sum of Wave Phasor Array

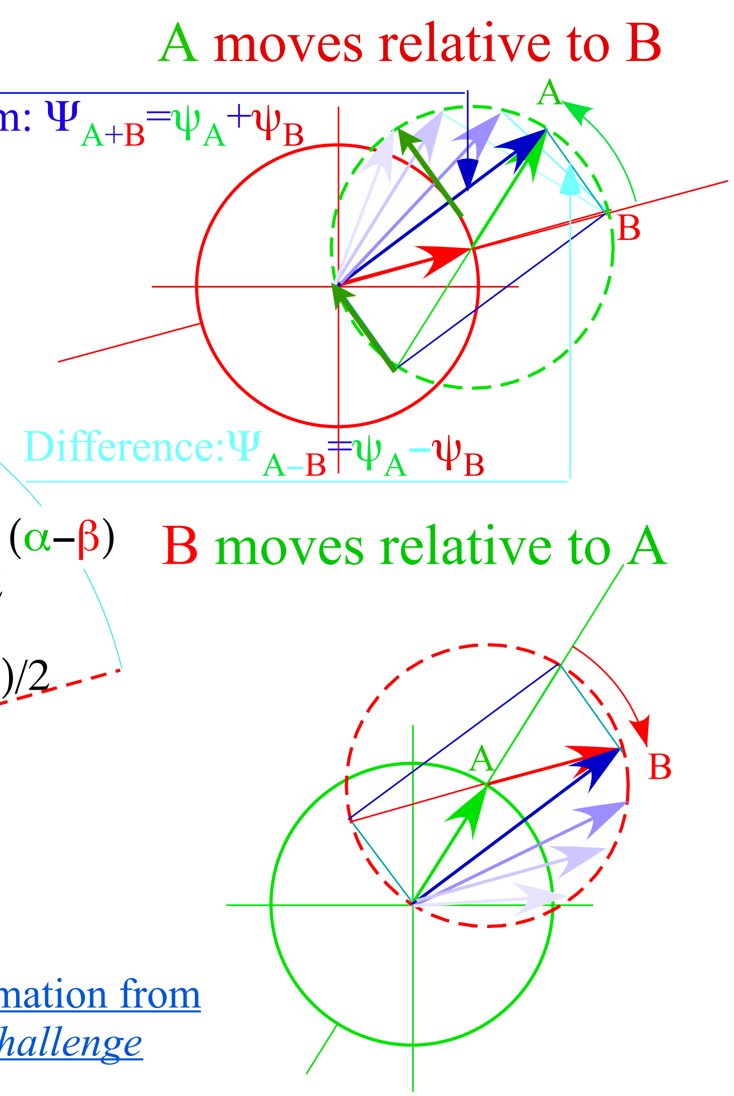


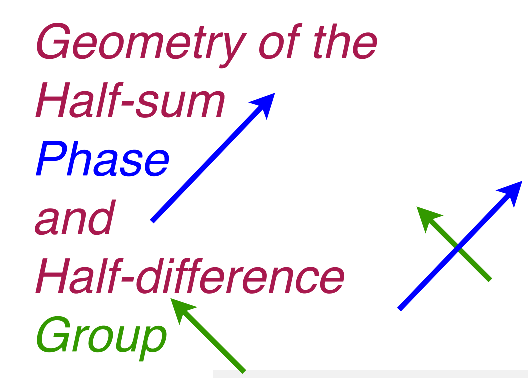

## (b) Typical Phasor Sum:



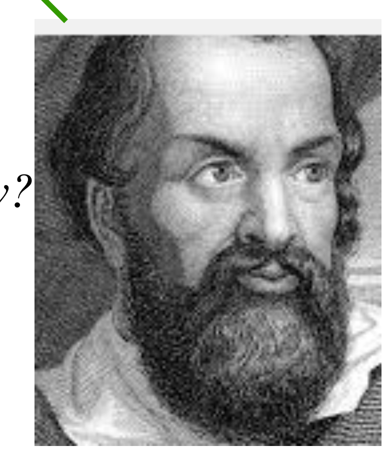

[Link to Animation from *Pirelli Challenge*]

## (c) Phasor-relative views




*Happy now?*



***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
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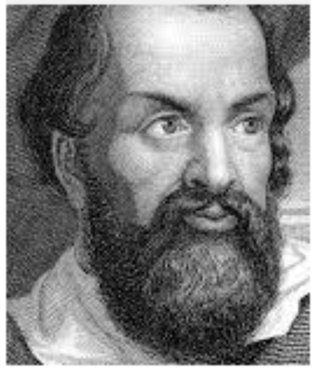
*Galileo Galilei*



*1564-1642*

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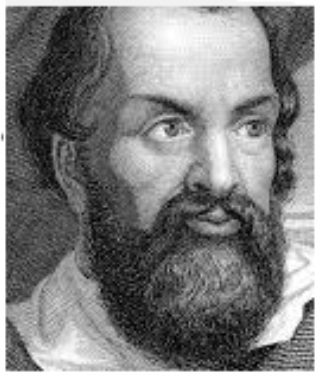

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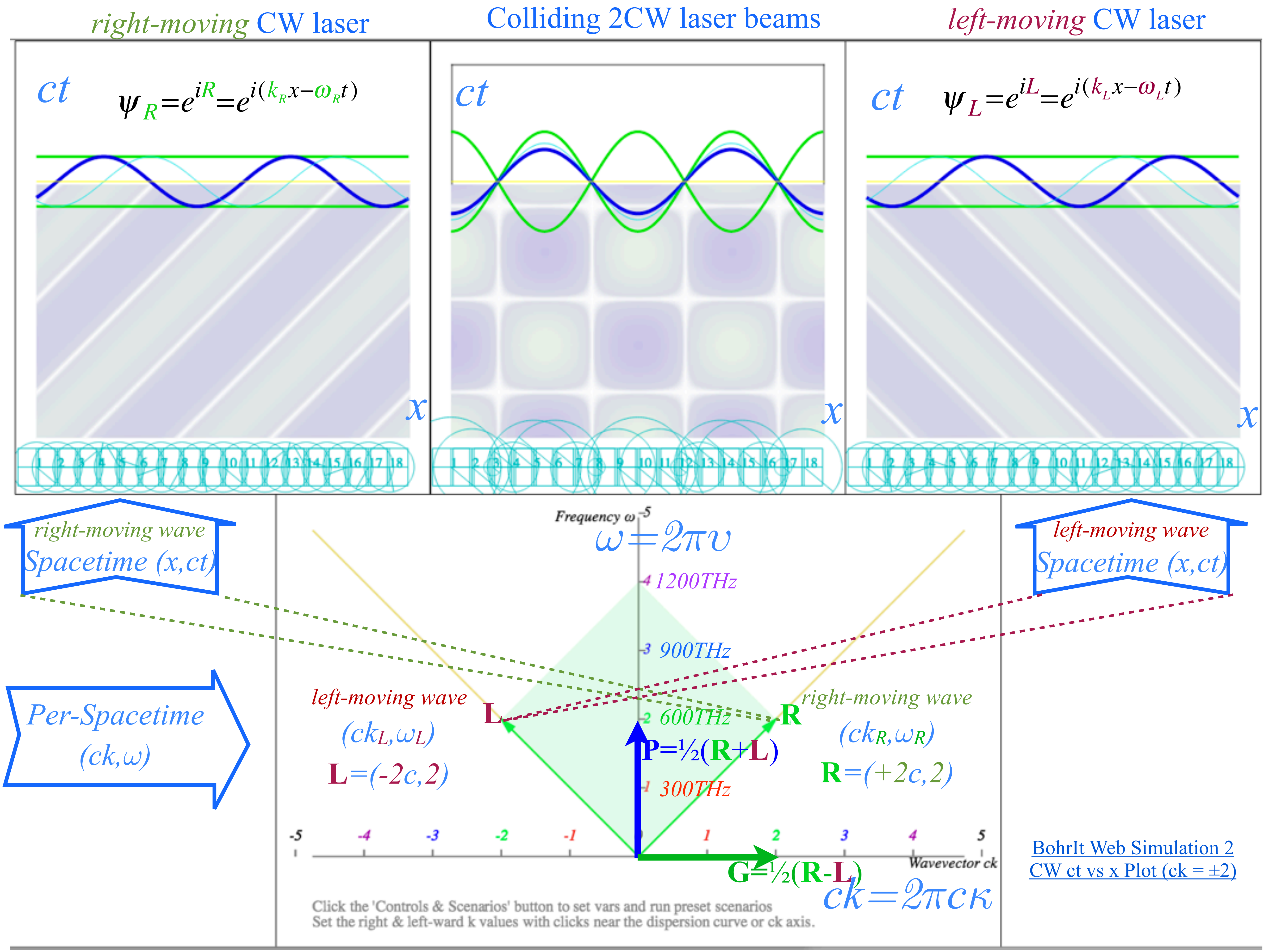
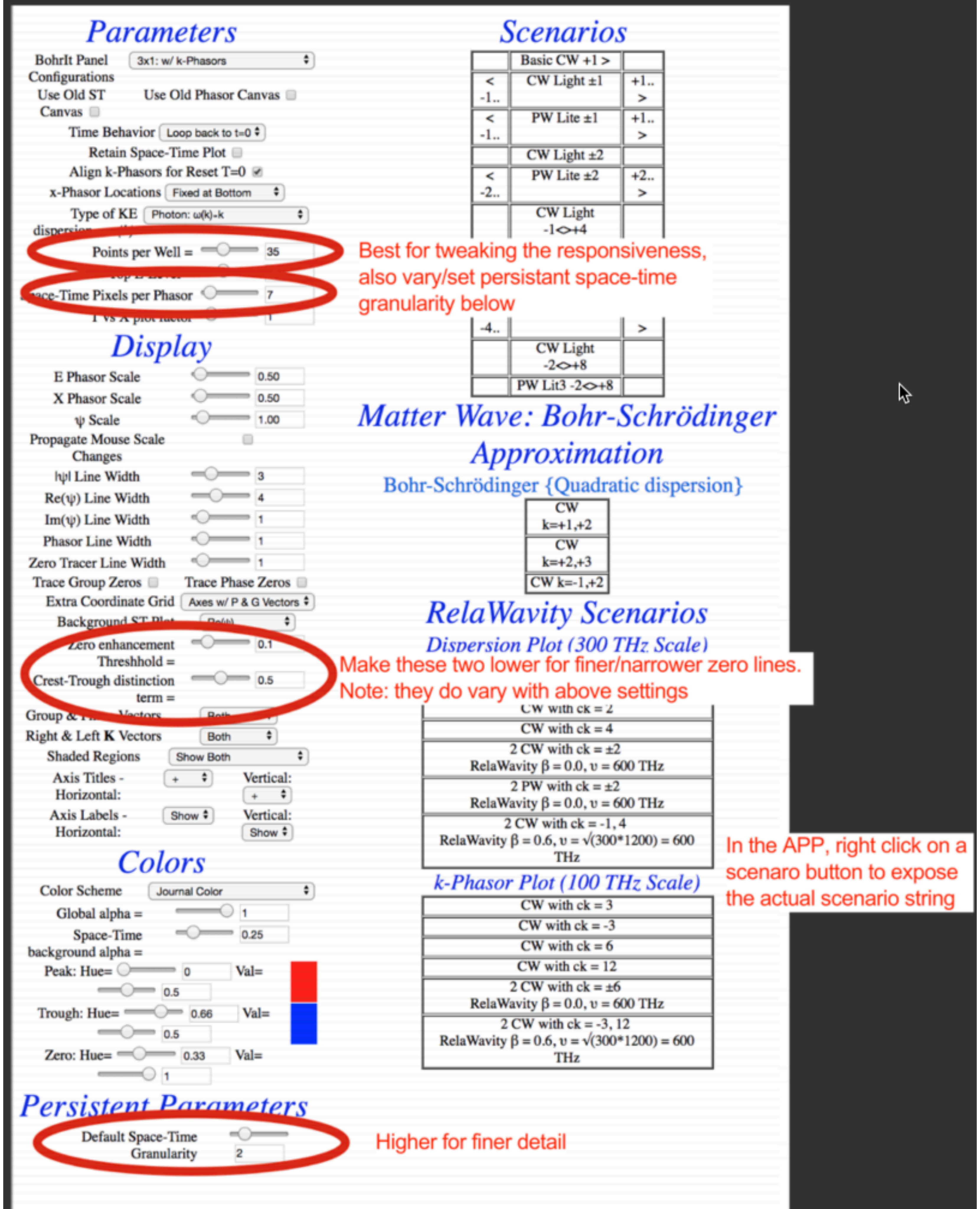
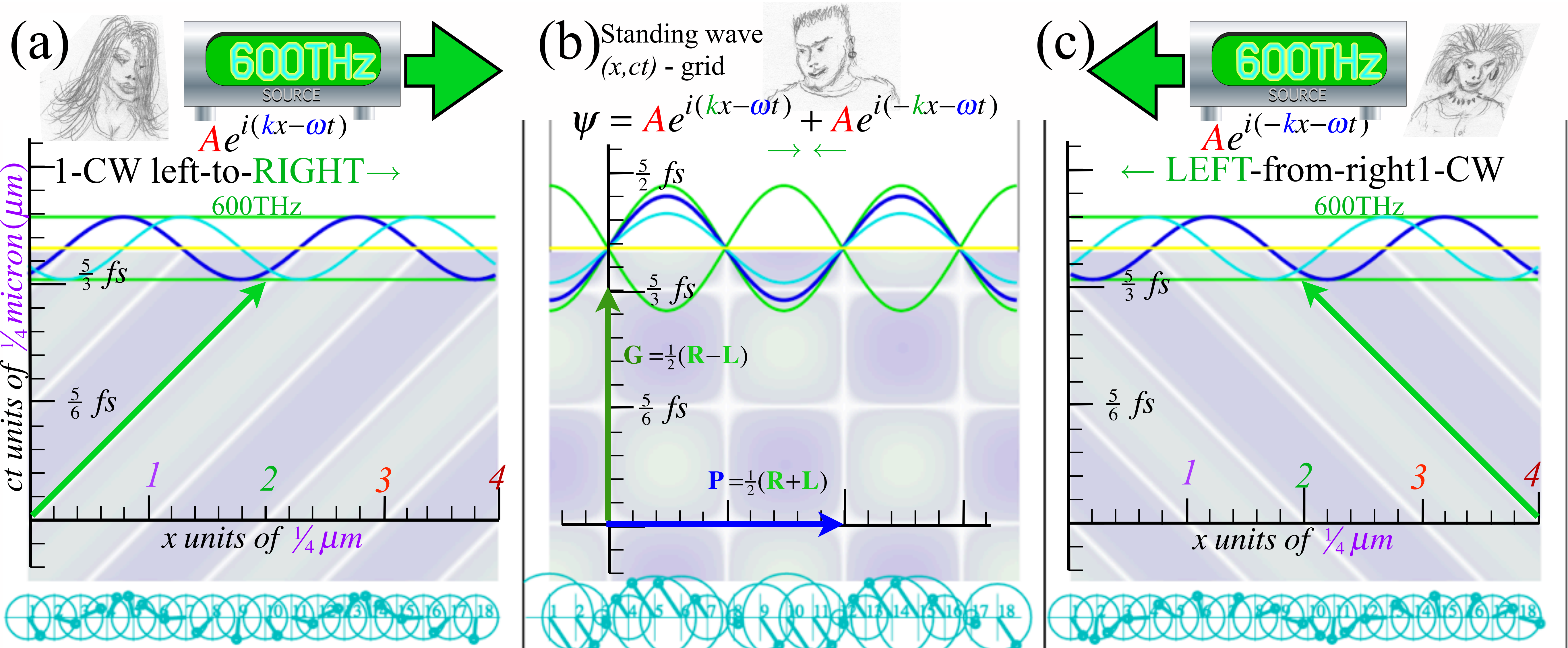


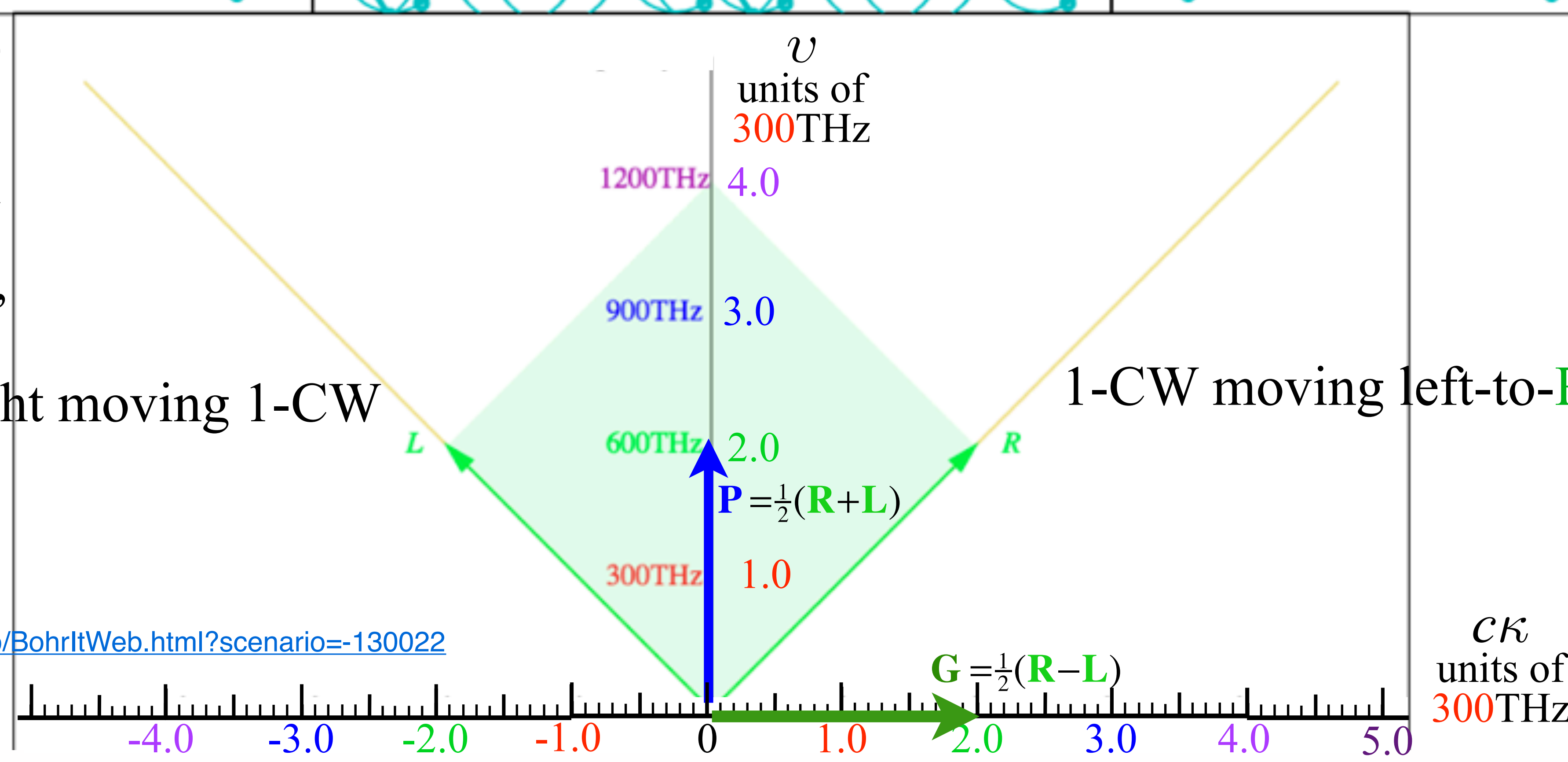
right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
x
ct
x
ct
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
$\omega = 2\pi\upsilon$
1200THz
900THz
600THz
300THz
left-moving wave
$(ck_L, \omega_L)$
L=(-2c,2)
right-moving wave
$(ck_R, \omega_R)$
R=(+2c,2)
L
R
P=½(R+L)
G=½(R-L)
Wavevector ck
$ck = 2\pi c\kappa$
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)


Parameters
BohrIt Panel Configurations
3x1: w/ k-Phasors
Use Old ST Canvas
Use Old Phasor Canvas
Time Behavior
Loop back to t=0
Retain Space-Time Plot
Align k-Phasors for Reset T=0
x-Phasor Locations
Fixed at Bottom
Type of KE dispersion
Photon: ω(k)-k
Points per Well =
35
Space-Time Pixels per Phasor
7
Best for tweaking the responsiveness, also vary/set persistant space-time granularity below
Display
E Phasor Scale
0.50
X Phasor Scale
0.50
ψ Scale
1.00
Propagate Mouse Scale Changes
|ψ| Line Width
3
Re(ψ) Line Width
4
Im(ψ) Line Width
1
Phasor Line Width
1
Zero Tracer Line Width
1
Trace Group Zeros
Trace Phase Zeros
Extra Coordinate Grid
Axes w/ P & G Vectors
Zero enhancement Threshhold =
0.1
Crest-Trough distinction term =
0.5
Make these two lower for finer/narrower zero lines. Note: they do vary with above settings
Right & Left K Vectors
Both
Shaded Regions
Show Both
Axis Titles - Horizontal:
Vertical:
Axis Labels - Horizontal:
Show
Vertical:
Show
Colors
Color Scheme
Journal Color
Global alpha =
1
Space-Time background alpha =
0.25
Peak: Hue=
0
Val=
0.5
Trough: Hue=
0.66
Val=
0.5
Zero: Hue=
0.33
Val=
1
Persistent Parameters
Default Space-Time Granularity
2
Higher for finer detail
Scenarios
Basic CW +1 >
< -1..
CW Light ±1
+1.. >
PW Lite ±1
CW Light ±2
< -2..
PW Lite ±2
+2.. >
CW Light -1<>+4
-4..
CW Light -2<>+8
PW Lit3 -2<>+8
Matter Wave: Bohr-Schrödinger Approximation
Bohr-Schrödinger {Quadratic dispersion}
CW k=+1,+2
CW k=+2,+3
CW k=-1,+2
RelaWavity Scenarios
Dispersion Plot (300 THz Scale)
CW with ck = 4
2 CW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 PW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -1, 4 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
In the APP, right click on a scenaro button to expose the actual scenario string
k-Phasor Plot (100 THz Scale)
CW with ck = 3
CW with ck = -3
CW with ck = 6
CW with ck = 12
2 CW with ck = ±6 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -3, 12 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz



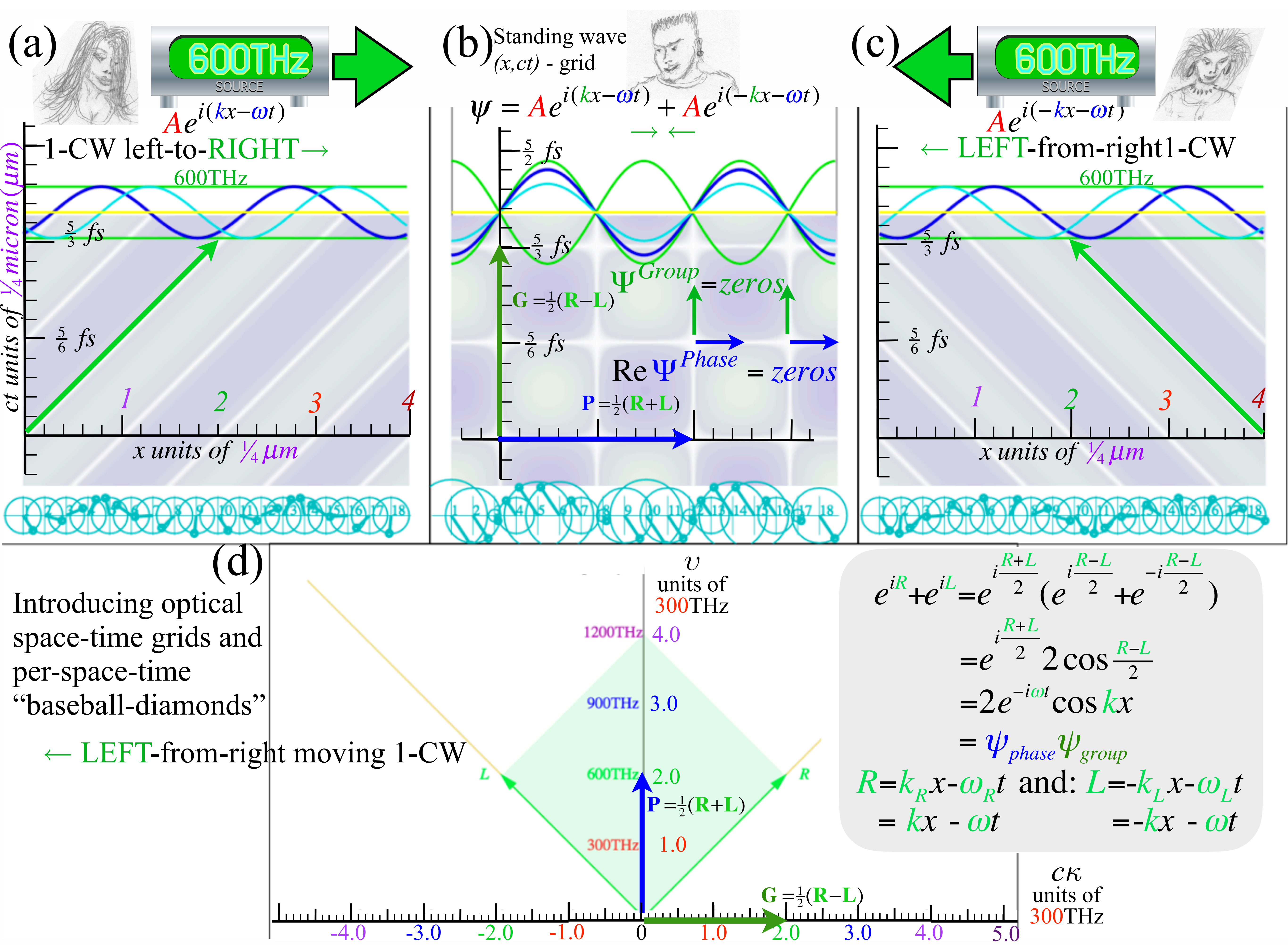

(d)

Introducing optical space-time grids and per-space-time "baseball-diamonds"

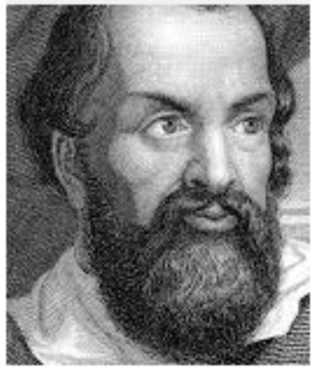



(ck = ±2)

www.uark.edu/ua/modphys/markup/BohrItWeb.html?scenario=-130022

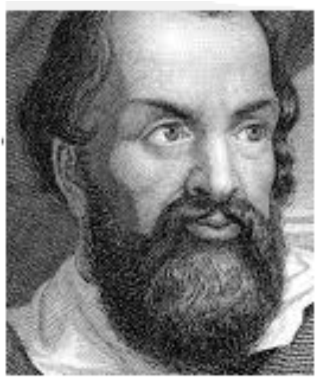

(a)
600THz
SOURCE
$Ae^{i(kx-\omega t)}$
1-CW left-to-RIGHT→
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
ct units of ¼ micron (μm)
x units of ¼ μm
1 2 3 4
(b) Standing wave (x,ct) - grid
$\psi = Ae^{i(kx-\omega t)} + Ae^{i(-kx-\omega t)}$
→ ←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\Psi^{Group}$ = zeros
$\mathrm{Re}\,\Psi^{Phase}$ = zeros
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
600THz
SOURCE
$Ae^{i(-kx-\omega t)}$
← LEFT-from-right1-CW
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(d)
Introducing optical space-time grids and per-space-time “baseball-diamonds”
← LEFT-from-right moving 1-CW
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
L
R
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
cκ units of 300THz
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0 5.0
$e^{iR}+e^{iL}=e^{i\frac{R+L}{2}}(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}})$
$=e^{i\frac{R+L}{2}}\,2\cos\frac{R-L}{2}$
$=2e^{-i\omega t}\cos kx$
$=\psi_{phase}\psi_{group}$
$R=k_R x-\omega_R t$ and: $L=-k_L x-\omega_L t$
$= kx - \omega t$ $= -kx - \omega t$

Galileo Galilei



1564-1642

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*



***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
*Galilean velocity*

Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
➔ Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
Analyzing wave velocity by per-space-per-time *and* space-time graphs
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Thales geometry of Lorentz transformation
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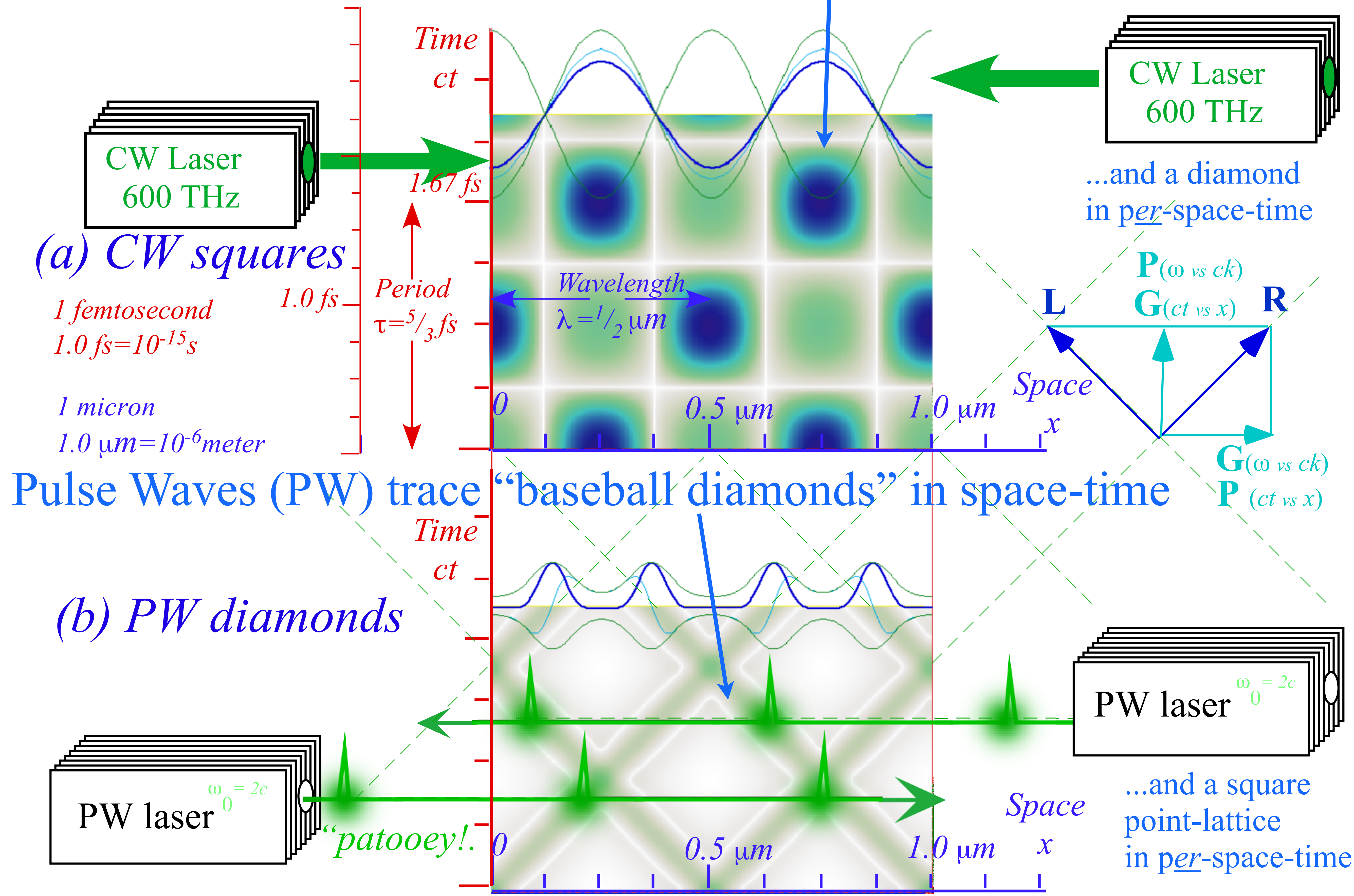
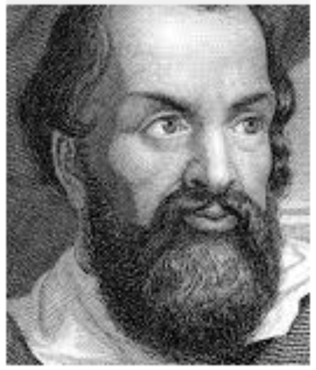
# Continuous Waves (CW) trace "Cartesian squares" in space-time




BohrIt Web Simulation: 2 PW *ct vs x* Plot

*Galileo Galilei*



*1564-1642*

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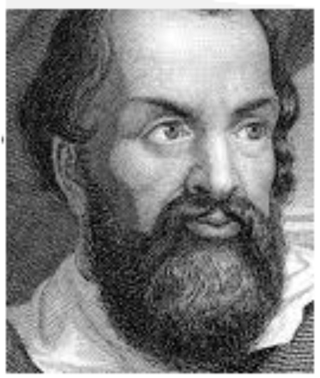

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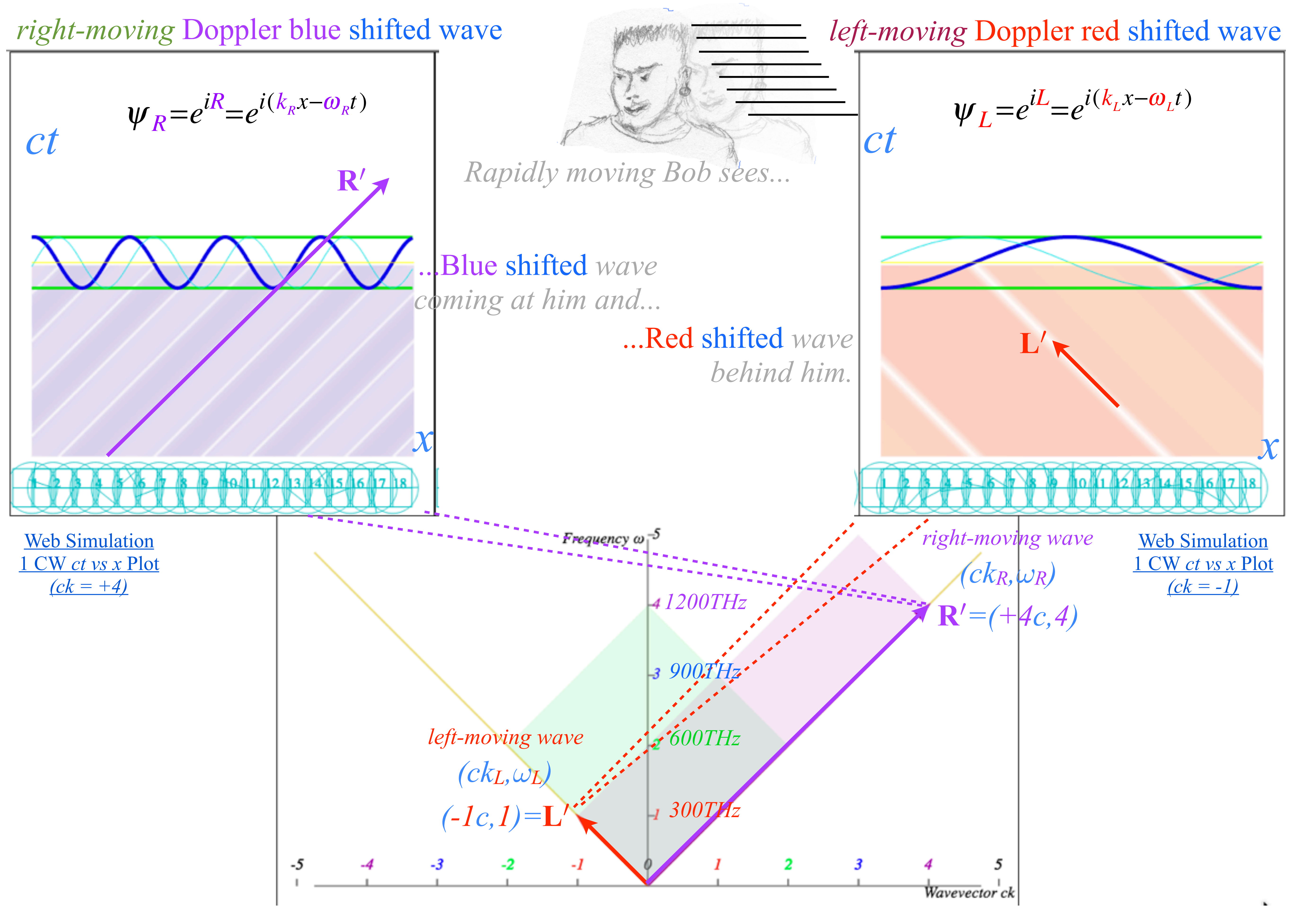
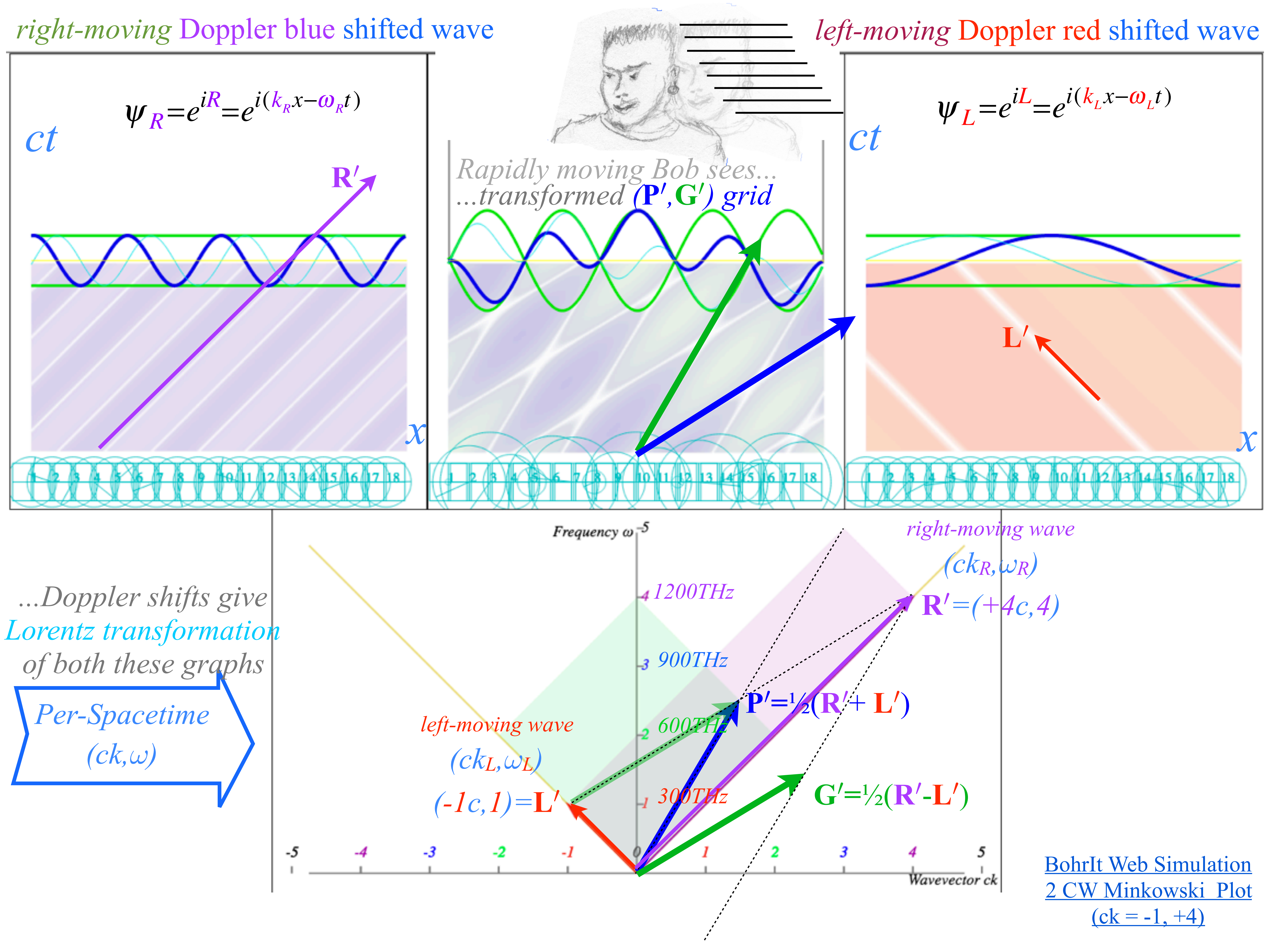
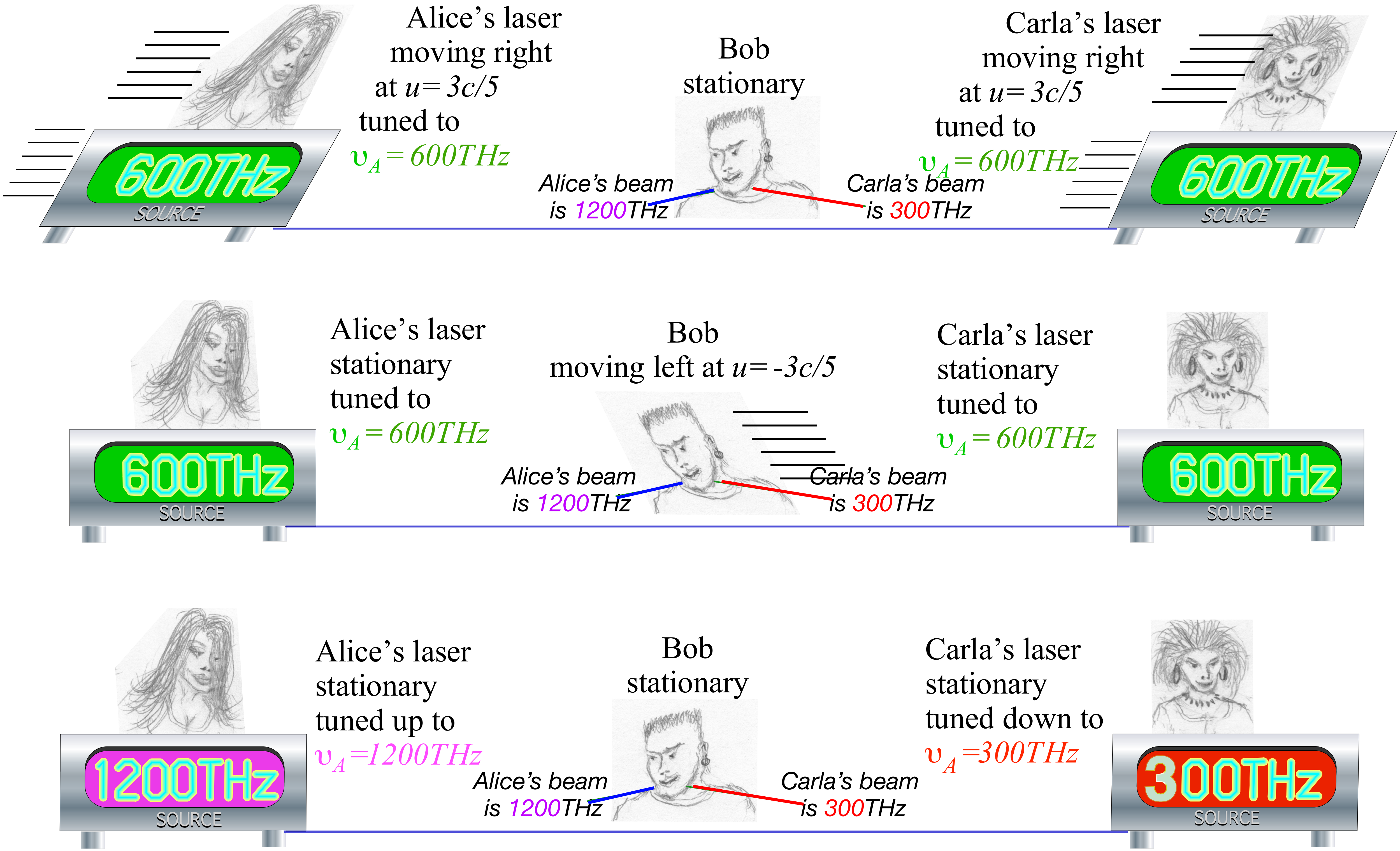


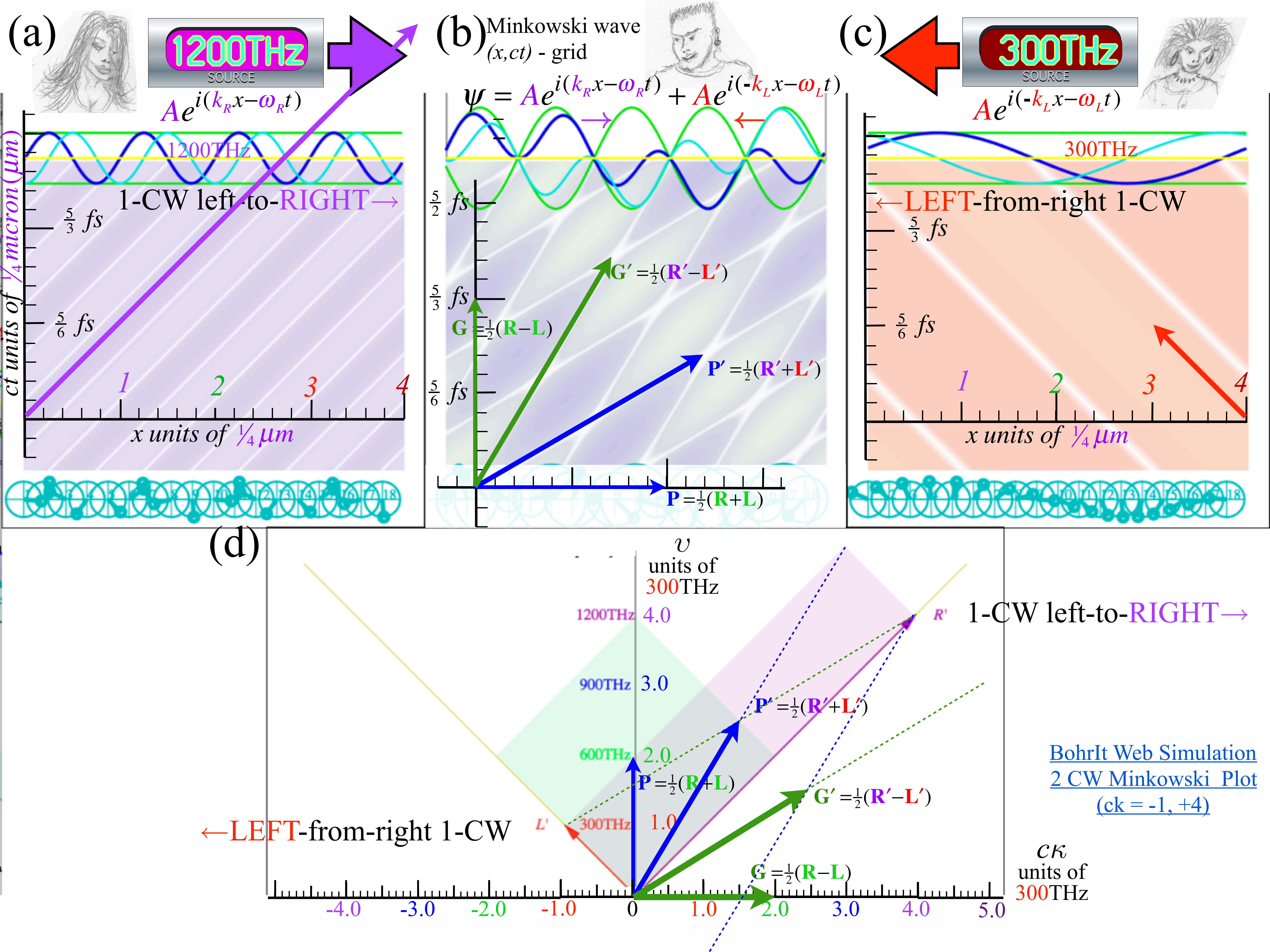
right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
ct
R′
x
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
L′
x
Rapidly moving Bob sees...
...Blue shifted wave coming at him and...
...Red shifted wave behind him.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Web Simulation
1 CW ct vs x Plot
(ck = +4)
Web Simulation
1 CW ct vs x Plot
(ck = -1)
Frequency ω
5
4 1200THz
3 900THz
2 600THz
1 300THz
right-moving wave
$(ck_R, \omega_R)$
$R' = (+4c, 4)$
left-moving wave
$(ck_L, \omega_L)$
$(-1c, 1) = L'$
-5 -4 -3 -2 -1 0 1 2 3 4 5
Wavevector ck


right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...transformed (P′,G′) grid
...Doppler shifts give
Lorentz transformation
of both these graphs
Per-Spacetime
(ck,ω)
Frequency ω
1200THz
900THz
600THz
300THz
Wavevector ck
right-moving wave
$(ck_R, \omega_R)$
$R' = (+4c, 4)$
$P' = \frac{1}{2}(R' + L')$
$G' = \frac{1}{2}(R' - L')$
left-moving wave
$(ck_L, \omega_L)$
$(-1c, 1) = L'$
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

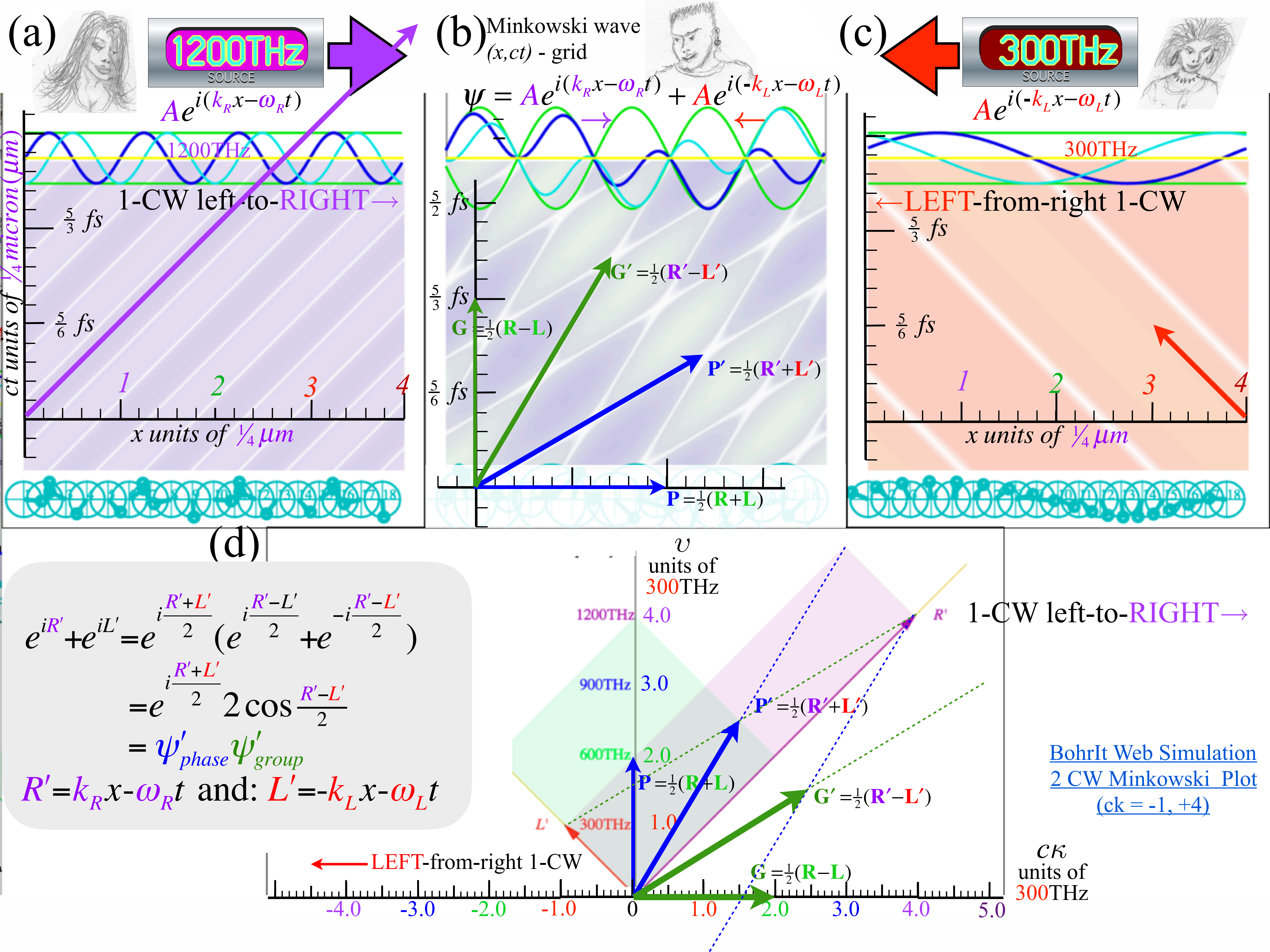
# Three scenarios that look the same to Bob




*Much cheaper (and safer) to do the 3rd scenario!$!*

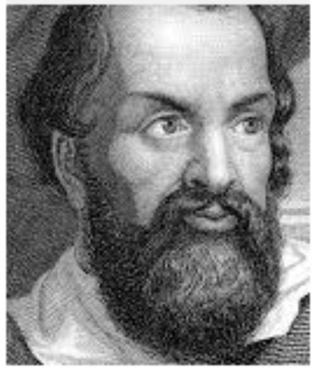



[BohrIt Web Simulation 2 CW Minkowski Plot (ck = -1, +4)]



(a)
1200THz
SOURCE
Ae^{i(k_R x−ω_R t)}
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
5/3 fs
5/6 fs
1
2
3
4
x units of ¼ μm
(b)
Minkowski wave (x,ct) - grid
ψ = Ae^{i(k_R x−ω_R t)} + Ae^{i(-k_L x−ω_L t)}
5/2 fs
5/3 fs
5/6 fs
G′ = ½(R′−L′)
G = ½(R−L)
P′ = ½(R′+L′)
P = ½(R+L)
(c)
300THz
SOURCE
Ae^{i(-k_L x−ω_L t)}
300THz
←LEFT-from-right 1-CW
5/3 fs
5/6 fs
1
2
3
4
x units of ¼ μm
(d)
e^{iR′}+e^{iL′} = e^{i(R′+L′)/2}(e^{i(R′−L′)/2}+e^{−i(R′−L′)/2})
= e^{i(R′+L′)/2} 2cos((R′−L′)/2)
= ψ′_phase ψ′_group
R′=k_R x-ω_R t and: L′=-k_L x-ω_L t
υ
units of
300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R′
L′
1-CW left-to-RIGHT→
P′ = ½(R′+L′)
P = ½(R+L)
G′ = ½(R′−L′)
G = ½(R−L)
←LEFT-from-right 1-CW
cκ
units of
300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

Galileo Galilei



1564-1642

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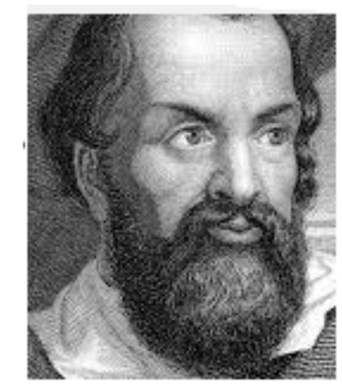
*Galilean velocity*



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***Phasor angular velocity***
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*Galilean velocity*

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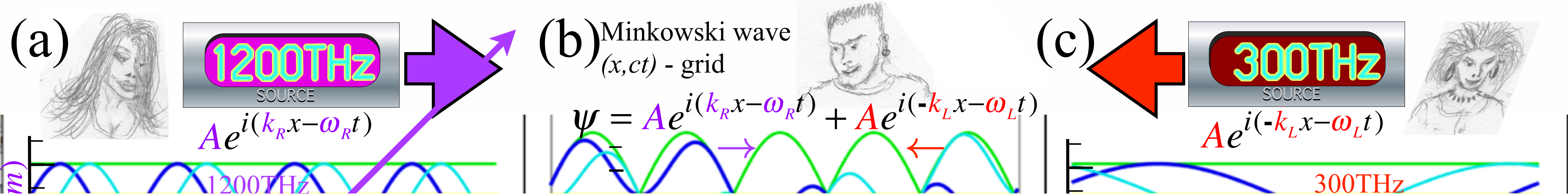


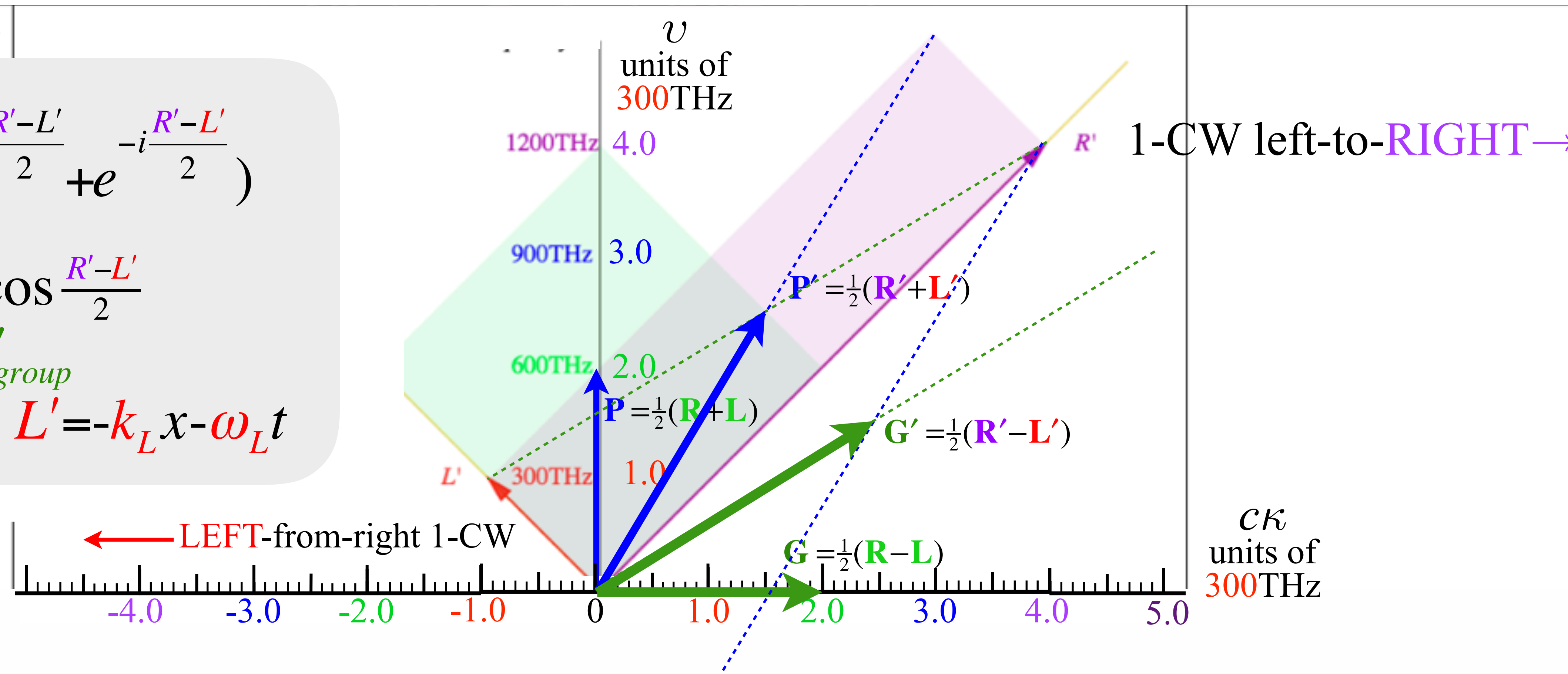

$$\mathbf{P}' = \begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'+\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}+e^{-\rho}) \\ \frac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}_{\substack{Bob's\\View}} \text{ or: } \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\substack{Alice's\\View}}$$

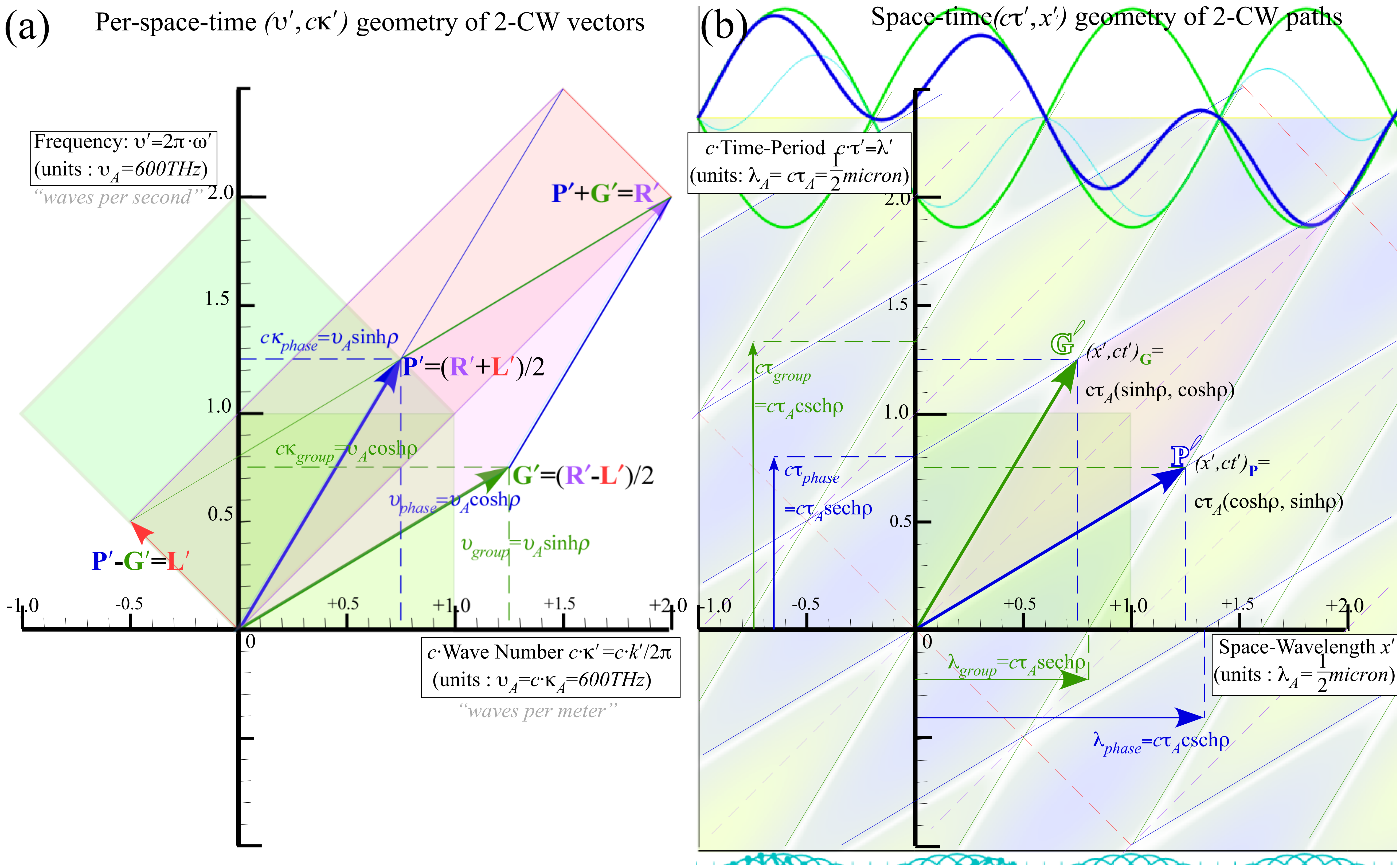
$$\mathbf{G}' = \begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'-\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}-e^{-\rho}) \\ \frac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{3}{2} \\ \frac{5}{2} \end{pmatrix}_{\substack{Bob's\\View}} \text{ or: } \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\substack{Alice's\\View}}$$

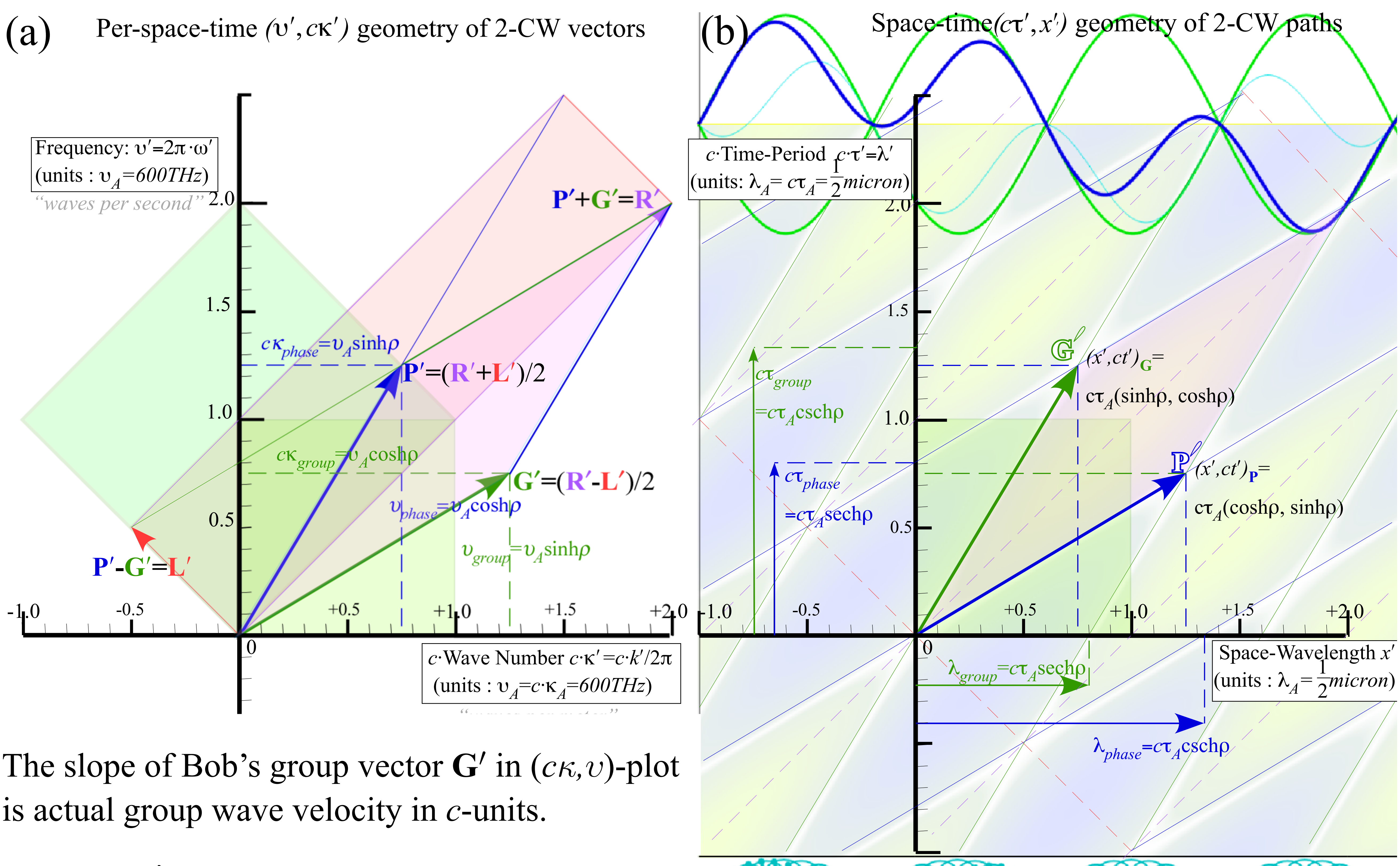
(d)

$$e^{iR'}+e^{iL'} = e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$= e^{i\frac{R'+L'}{2}}\, 2\cos\frac{R'-L'}{2}$$
$$= \psi'_{phase}\psi'_{group}$$

$R' = k_R x - \omega_R t$ and: $L' = -k_L x - \omega_L t$




(a) Per-space-time (υ′, cκ′) geometry of 2-CW vectors
Frequency: υ′=2π·ω′
(units : υ_A=600THz)
"waves per second"
P′+G′=R′
cκ_phase=υ_A sinhρ
P′=(R′+L′)/2
cκ_group=υ_A coshρ
G′=(R′-L′)/2
υ_phase=υ_A coshρ
υ_group=υ_A sinhρ
P′-G′=L′
c·Wave Number c·κ′=c·k′/2π
(units : υ_A=c·κ_A=600THz)
"waves per meter"
(b) Space-time (cτ′, x′) geometry of 2-CW paths
c·Time-Period c·τ′=λ′
(units: λ_A= cτ_A=1/2 micron)
cτ_group
=cτ_A cschρ
cτ_phase
=cτ_A sechρ
G′ (x′,ct′)_G=
cτ_A(sinhρ, coshρ)
P′ (x′,ct′)_P=
cτ_A(coshρ, sinhρ)
λ_group=cτ_A sechρ
λ_phase=cτ_A cschρ
Space-Wavelength x′
(units : λ_A=1/2 micron)

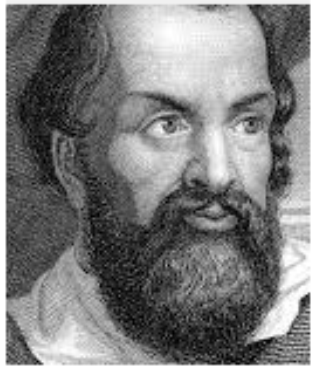



The slope of Bob's group vector **G′** in $(c\kappa,\upsilon)$-plot is actual group wave velocity in $c$-units.

$$\frac{V^{group}}{c} = \frac{\upsilon'_{group}}{c\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5} \equiv \frac{u}{c} \equiv \beta$$

Group vector $\mathbb{G}'$ in $(x,ct)$-plot has 3/5 slope relative to time axis

*Galileo Galilei*



*1564-1642*

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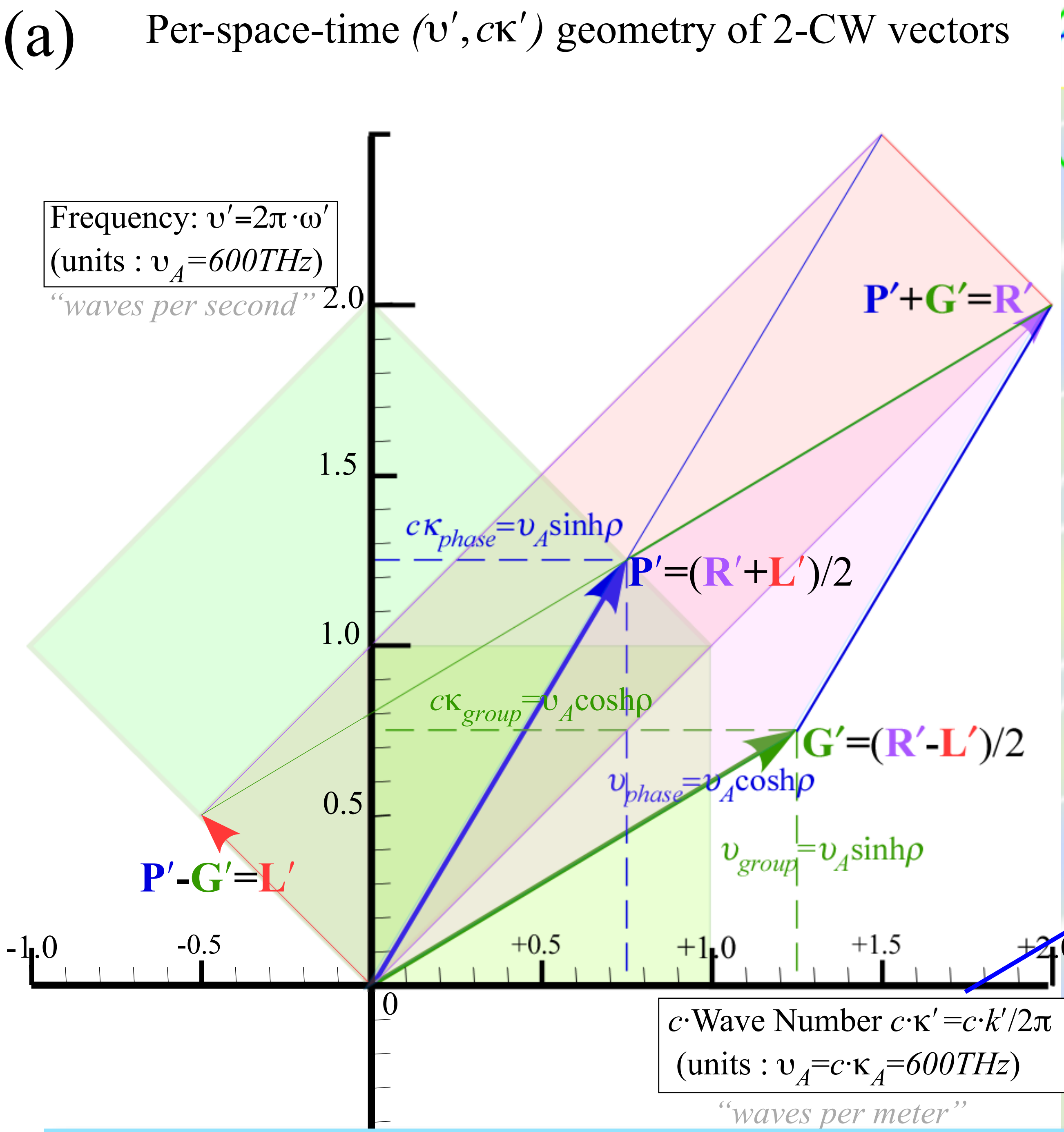
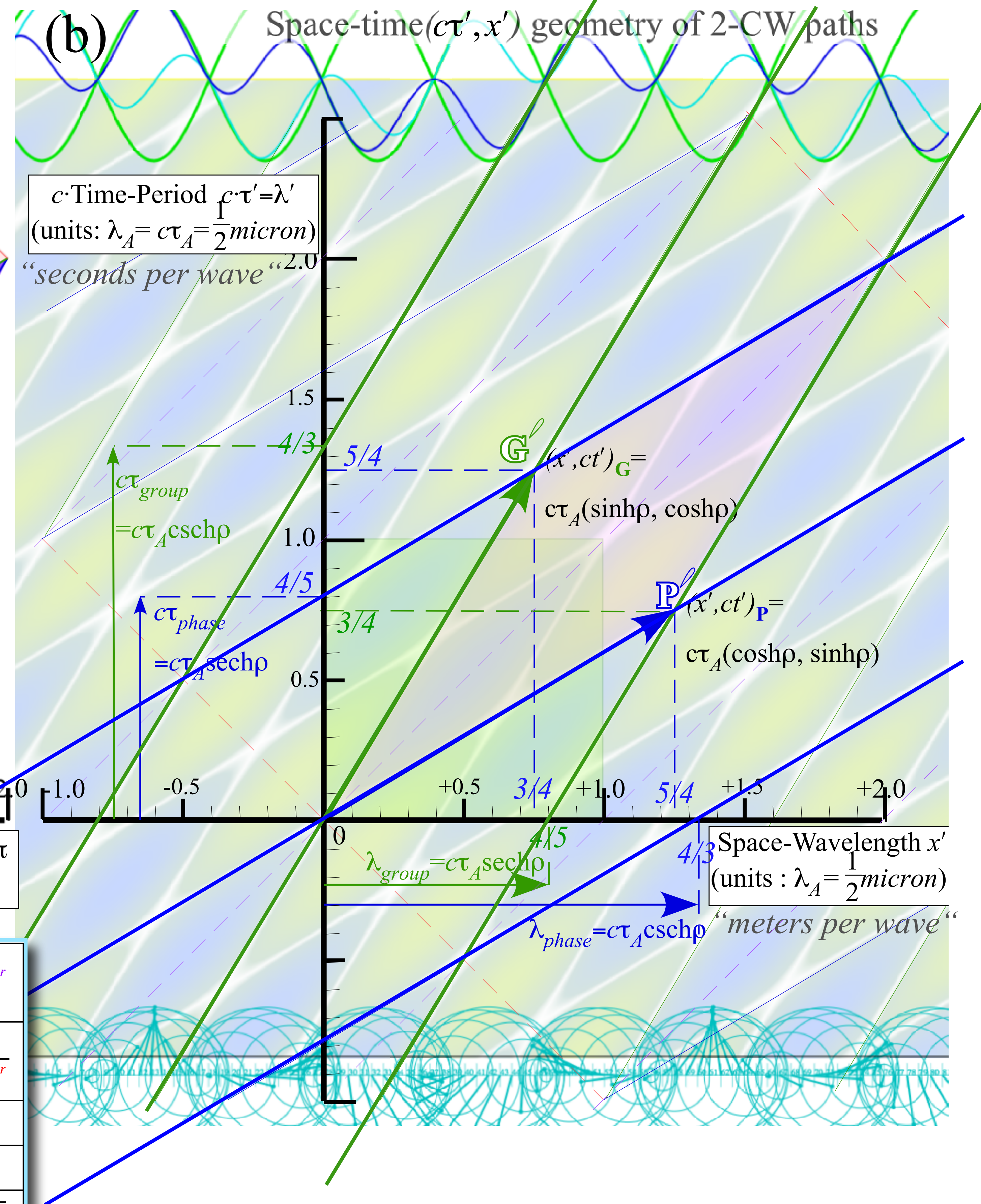



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *group* | $b^{Doppler}_{RED}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $b^{Doppler}_{BLUE}$ |
| *phase* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\mathrm{sech}\,\rho$ | $\cosh\rho$ | $\mathrm{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *stellar* $\forall$ *angle* $\sigma$ | $1/e^{+\rho}$ | $\sin\sigma$ | $\tan\sigma$ | $\cos\sigma$ | $\sec\sigma$ | $\cot\sigma$ | $\csc\sigma$ | $1/e^{-\rho}$ |
| $\beta\equiv\frac{u}{c}$ | $\sqrt{\frac{1-\beta}{1+\beta}}$ | $\frac{\beta}{1}$ | $\frac{1}{\sqrt{\beta^{-2}-1}}$ | $\frac{\sqrt{1-\beta^2}}{1}$ | $\frac{1}{\sqrt{1-\beta^2}}$ | $\frac{\sqrt{\beta^{-2}-1}}{1}$ | $\frac{1}{\beta}$ | $\sqrt{\frac{1+\beta}{1-\beta}}$ |
| *value for* $\beta=3/5$ | $\frac{1}{2}=0.5$ | $\frac{3}{5}=0.6$ | $\frac{3}{4}=0.75$ | $\frac{4}{5}=0.80$ | $\frac{5}{4}=1.25$ | $\frac{4}{3}=1.33$ | $\frac{5}{3}=1.67$ | $\frac{2}{1}=2.0$ |

*Fig. 11 in text* *Relawavity...*

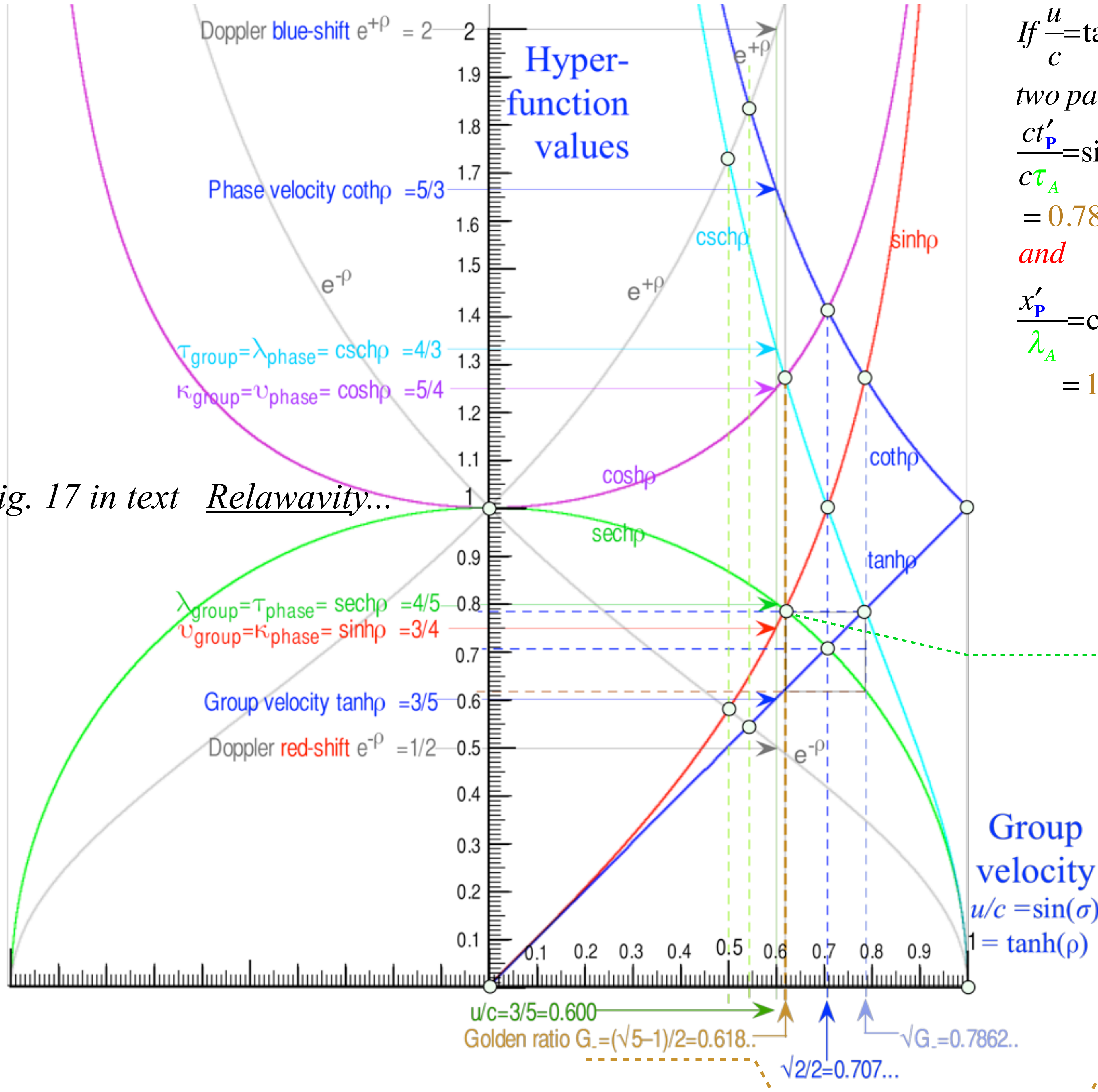



*Fig. 17 in text* *Relawavity...*

*If* $\frac{u}{c} = \tanh\rho = 0.618...$ (*Golden-Mean* $G_{\_}$)

*two parameters become exactly equal* :

$$\frac{ct'_P}{c\tau_A} = \sinh\rho = \frac{\lambda_{group}}{\lambda_A} = \frac{\tau_{phase}}{\tau_A} = \mathrm{sech}\,\rho$$

$$= 0.786.. = \sqrt{G_{\_}} \qquad = 0.786..$$

*and*

$$\frac{x'_P}{\lambda_A} = \cosh\rho = \frac{\lambda_{phase}}{\lambda_A} = \frac{\tau_{group}}{\tau_A} = \mathrm{csch}\,\rho$$

$$= 1.272.. = 1/\sqrt{G_{\_}} \qquad = 1.272..$$

*Solve* :

$\mathrm{sech}\,\rho = \sinh\rho$

or:

$\sinh\rho\cosh\rho = 1$

or:

$\sinh 2\rho = 2$

$\rho = \frac{1}{2}\sinh^{-1}2 = 0.7218...$

$\tanh\rho = 0.618... = \frac{\sqrt{5}-1}{2}$

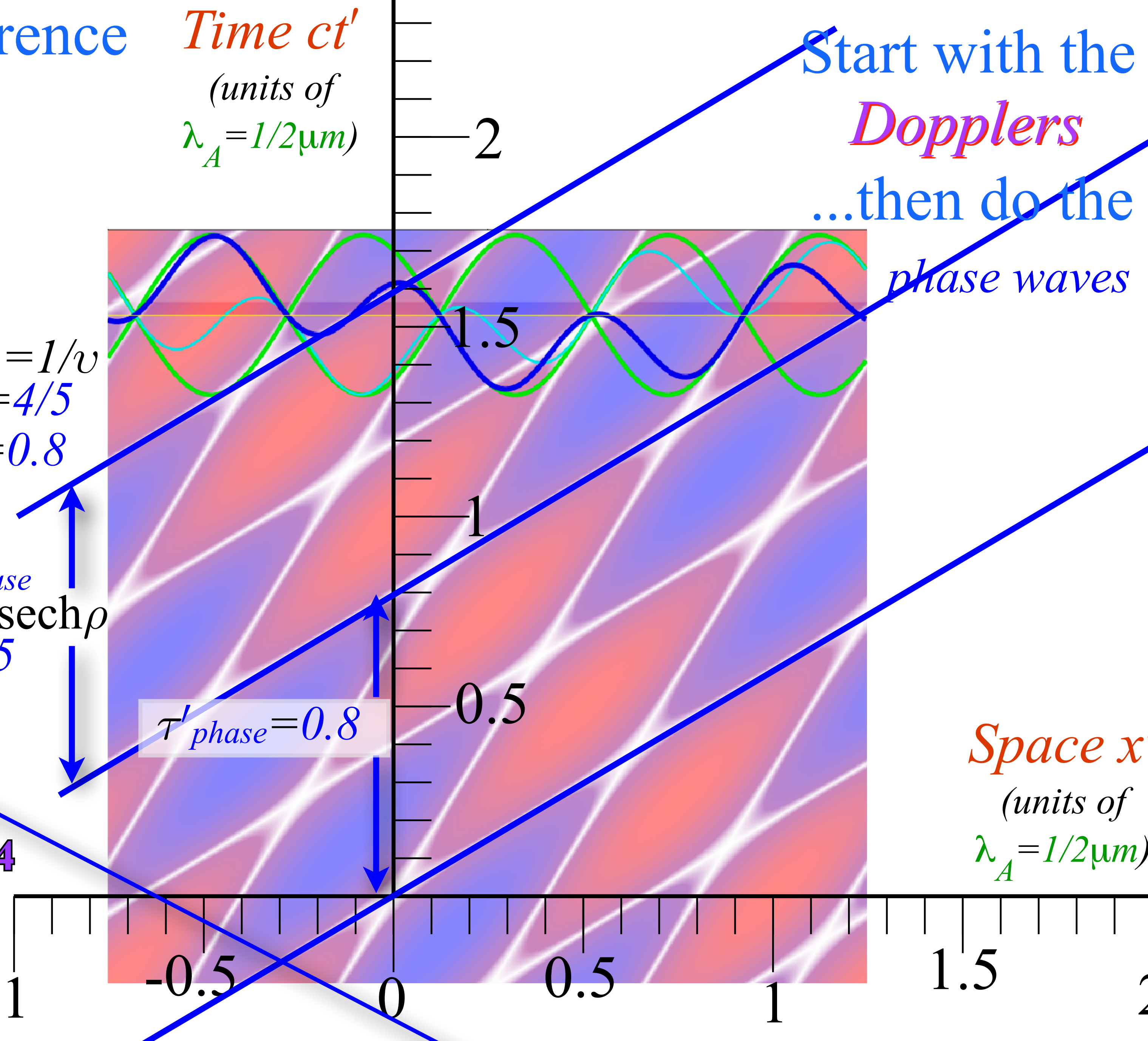
# The 16 dimensions of 2CW interference

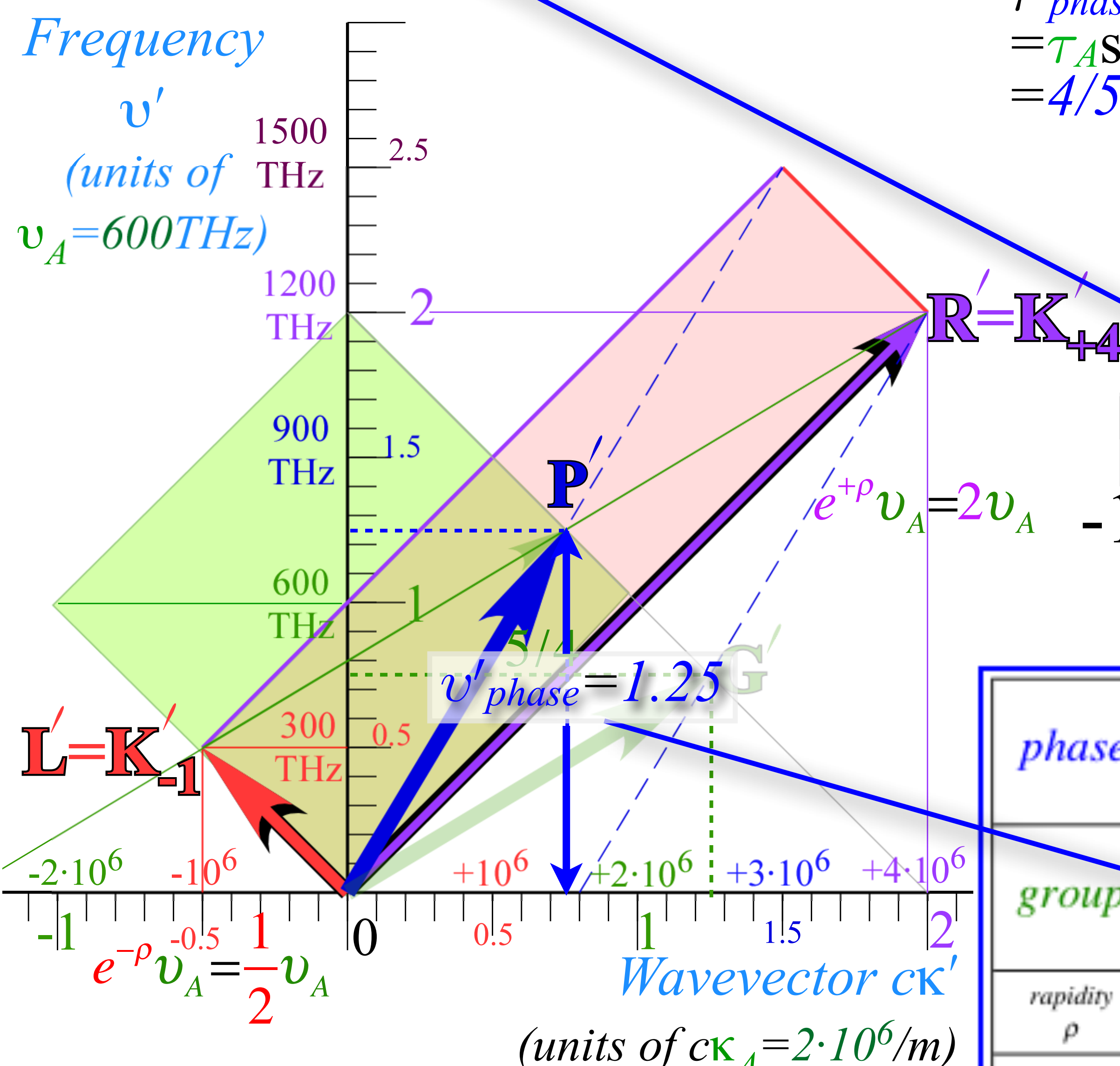
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

*Phase frequency* $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

*Phase period* $\tau = 1/\upsilon$: $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$




| phase | $b_{RED}^{Doppler}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b_{BLUE}^{Doppler}$ |
|---|---|---|---|---|---|---|---|---|
| group | $\frac{1}{b_{BLUE}^{Doppler}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b_{RED}^{Doppler}}$ |
| rapidity $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\operatorname{sech}\rho$ | $\cosh\rho$ | $\operatorname{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| value for $\beta=3/5$ | $\frac{1}{2}=0.5$ | $\frac{3}{5}=0.6$ | $\frac{3}{4}=0.75$ | $\frac{4}{5}=0.80$ | $\frac{5}{4}=1.25$ | $\frac{4}{3}=1.33$ | $\frac{5}{3}=1.67$ | $\frac{2}{1}=2.0$ |

# The 16 dimensions of 2CW interference

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

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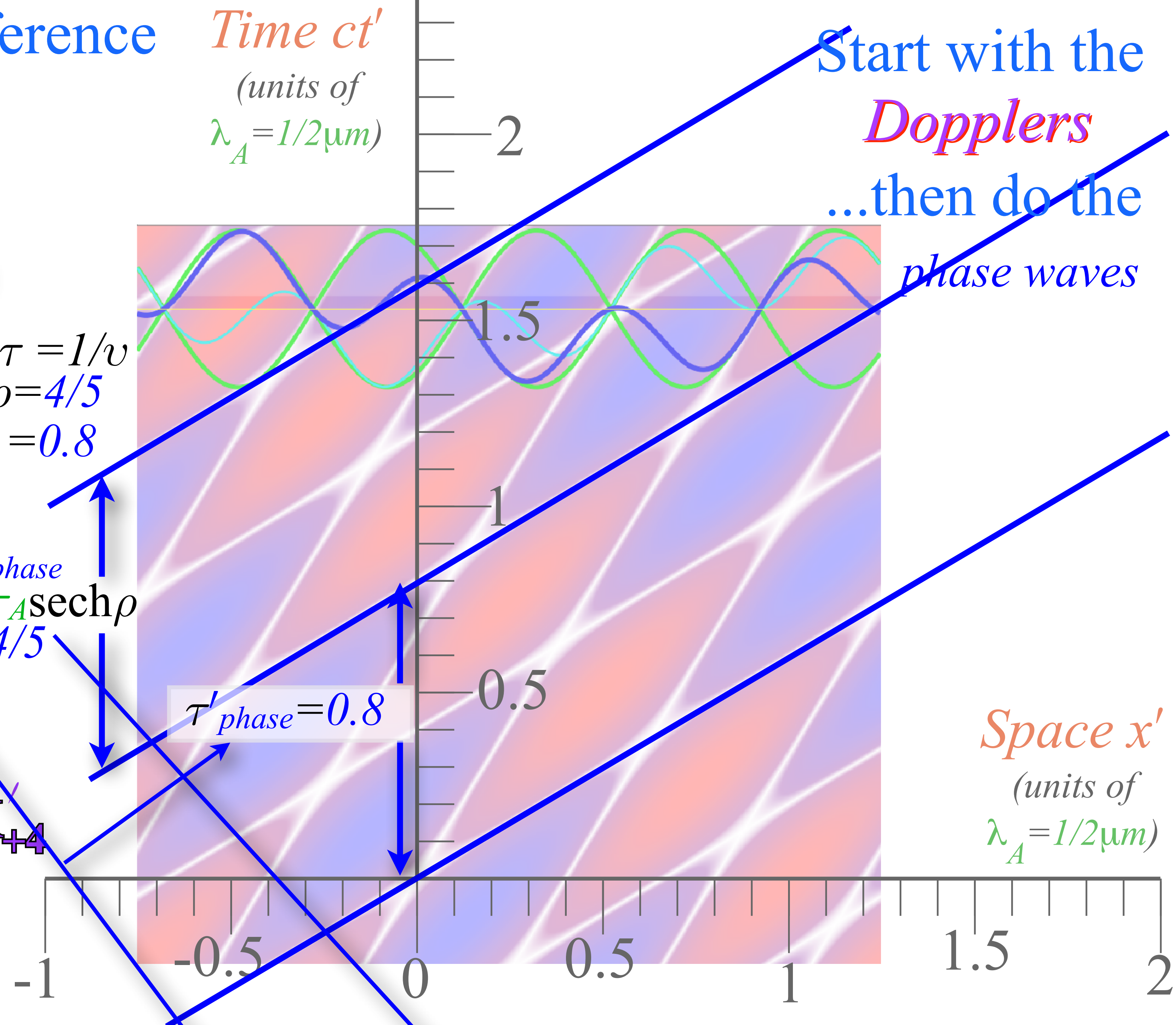
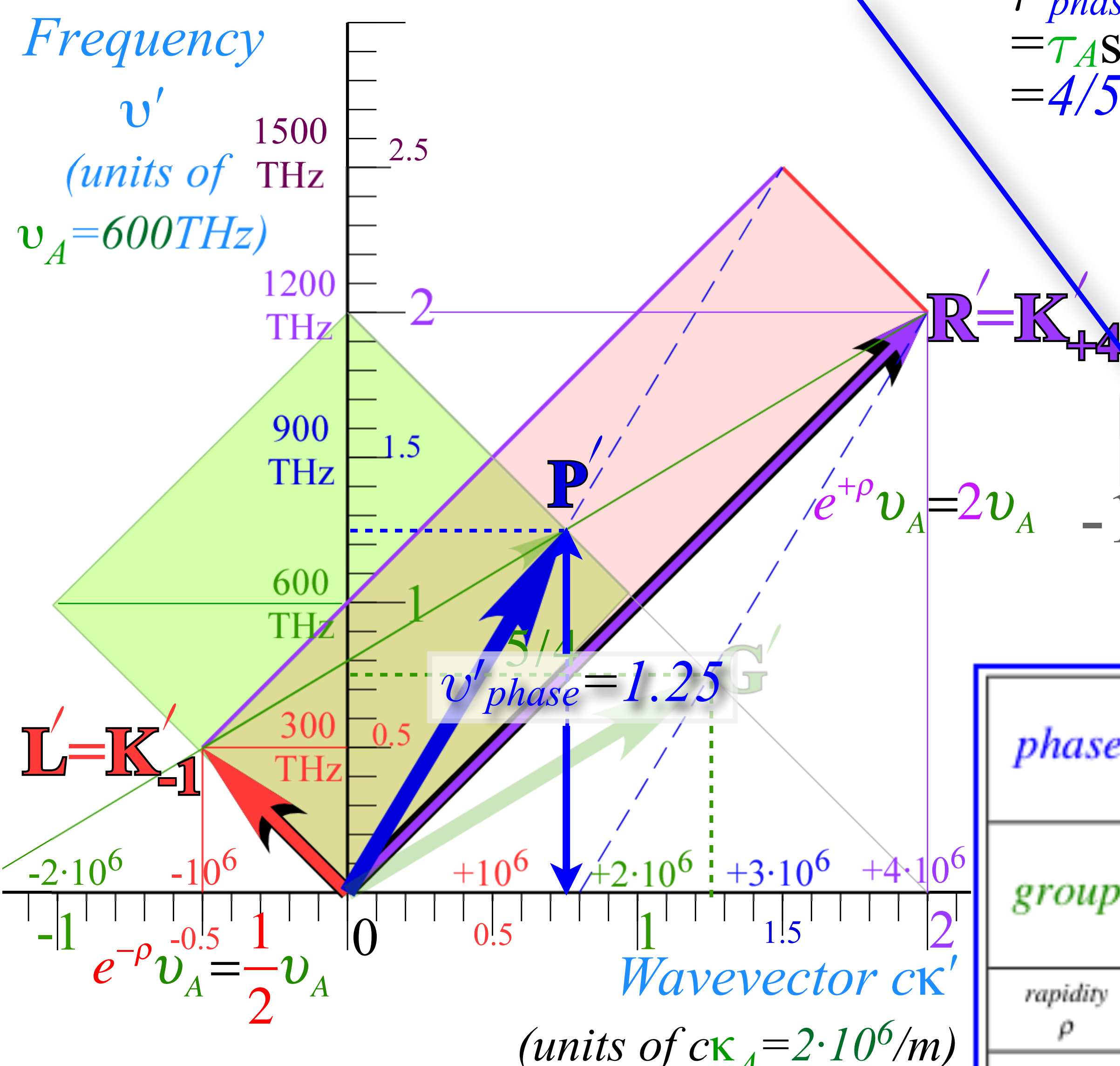



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *phase* | $b^{Doppler}_{RED}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b^{Doppler}_{BLUE}$ |
| *group* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\operatorname{sech}\rho$ | $\cosh\rho$ | $\operatorname{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *value for* $\beta=3/5$ | $\frac{1}{2}=0.5$ | $\frac{3}{5}=0.6$ | $\frac{3}{4}=0.75$ | $\frac{4}{5}=0.80$ | $\frac{5}{4}=1.25$ | $\frac{4}{3}=1.33$ | $\frac{5}{3}=1.67$ | $\frac{2}{1}=2.0$ |

*Phase wavenumber* flips to *Phase wavelength* $\lambda = 1/\kappa$

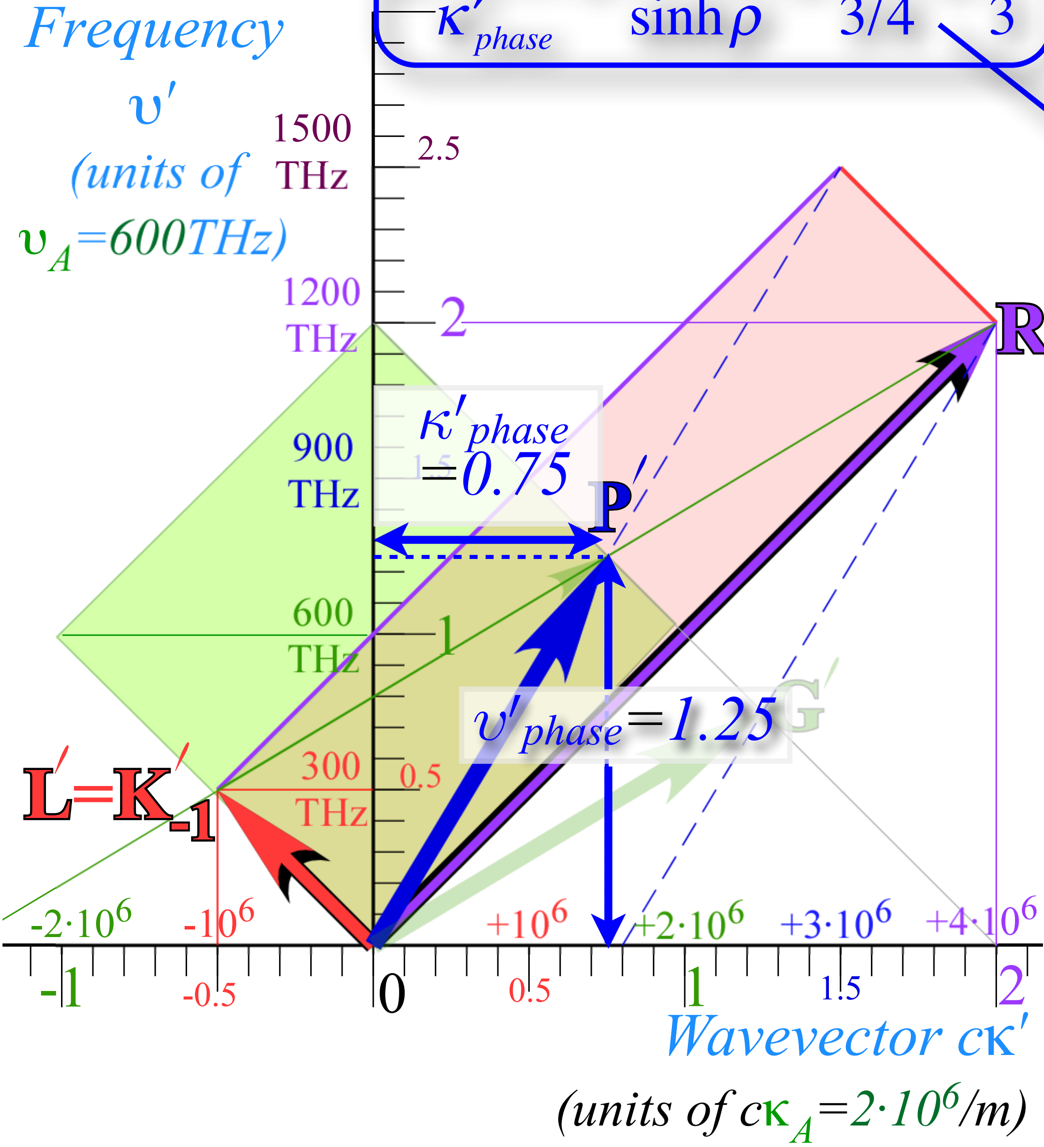
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

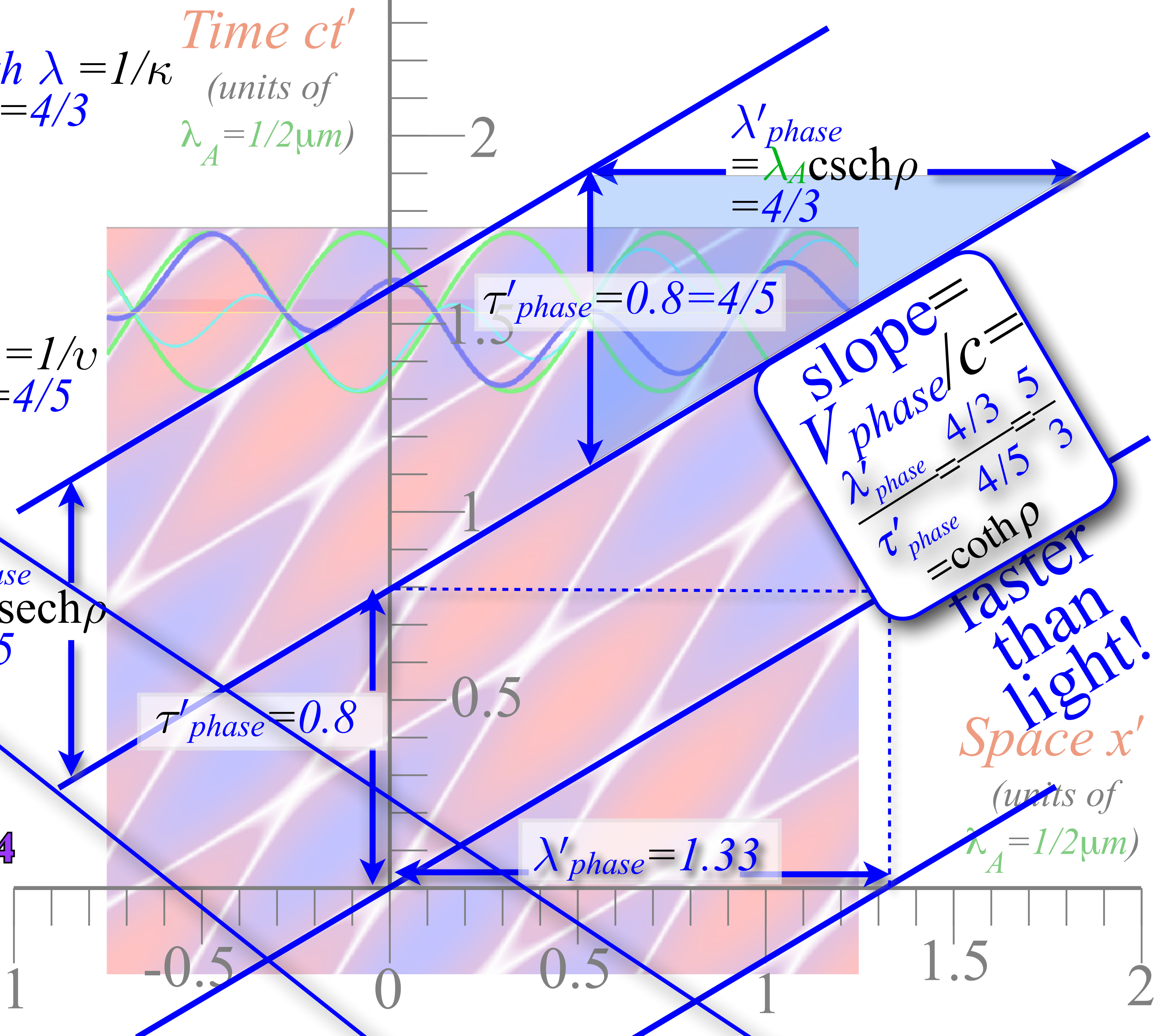
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

*Phase frequency* flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$




| phase | $b^{Doppler}_{RED}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b^{Doppler}_{BLUE}$ |
|---|---|---|---|---|---|---|---|---|
| group | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| rapidity $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\operatorname{sech}\rho$ | $\cosh\rho$ | $\operatorname{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
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*Group wavenumber* $\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

*Group wavelength* $\lambda = 1/\kappa$: $\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

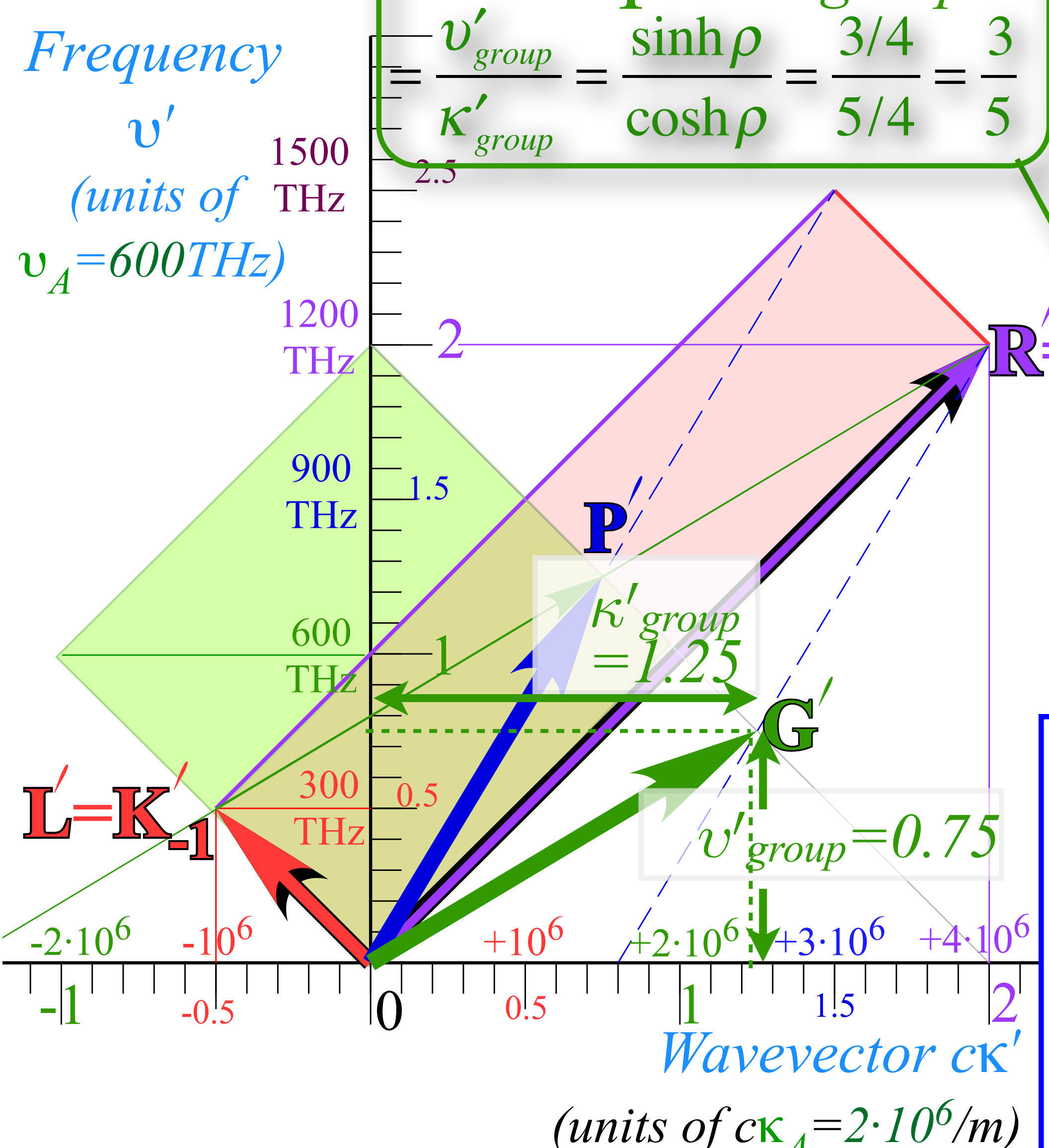
*Group frequency* $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

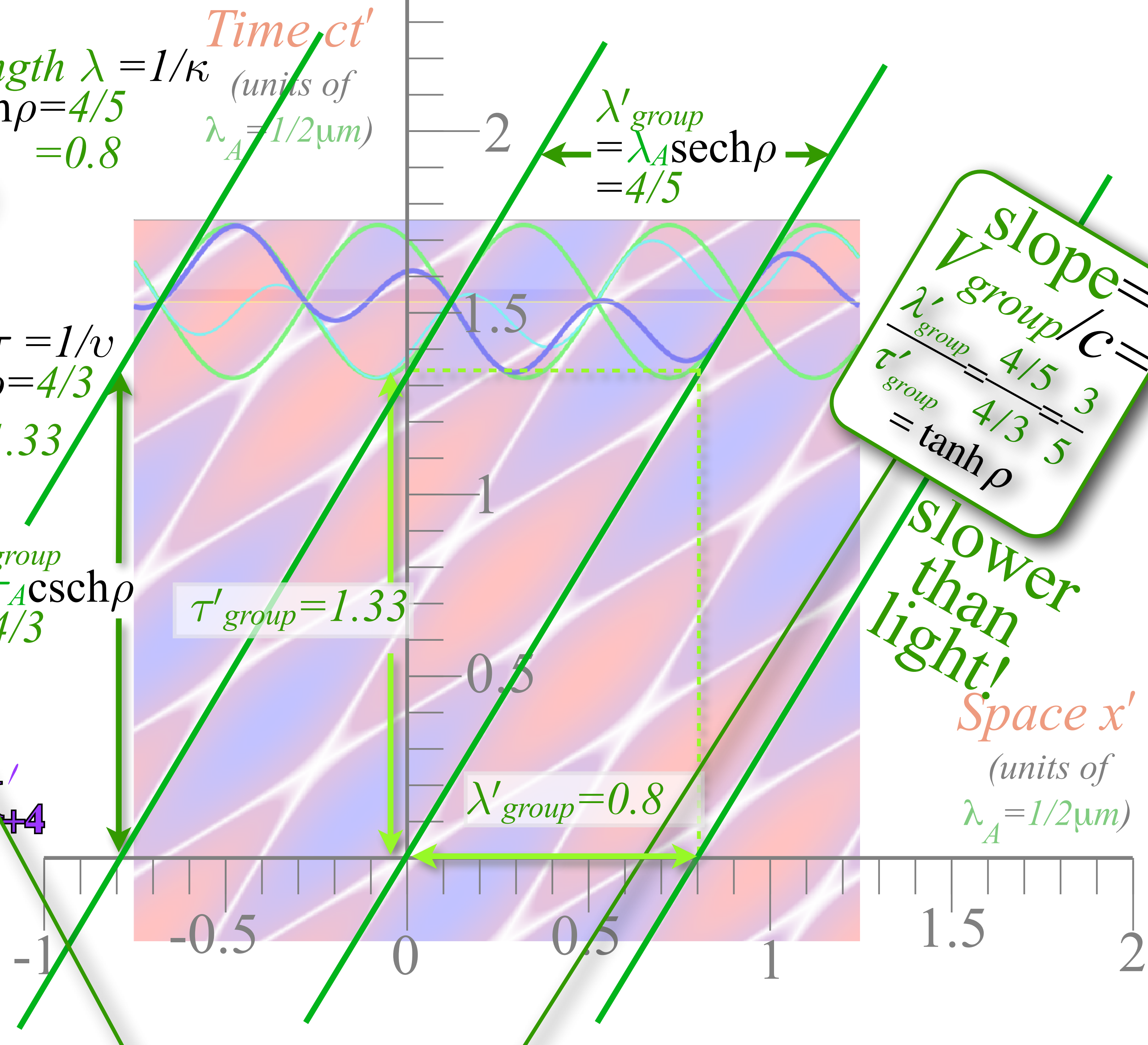
flips to

*Group period* $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

$$\mathbf{G}\text{-slope} = V_{group}/c = \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$




| phase | $b^{Doppler}_{RED}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b^{Doppler}_{BLUE}$ |
|---|---|---|---|---|---|---|---|---|
| group | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| rapidity $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\operatorname{sech}\rho$ | $\cosh\rho$ | $\operatorname{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
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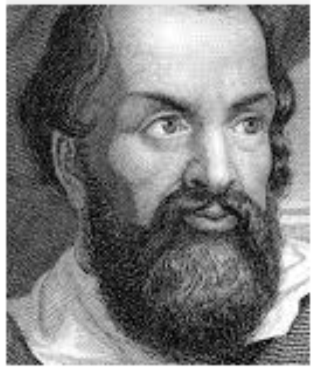
*Galileo Galilei*



*1564-1642*

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*

***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
*Galilean velocity*

Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
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16 coefficients of relativistic 2CW interference
➔ Two "famous-name" coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
*Rapidity* $\rho$ related to *stellar aberration angle* $\sigma$ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
"Occams Sword" and geometry of 16 parameter functions of $\rho$ and $\sigma$
Application to TE-Waveguide modes and synchrotron beam relativity

# Lorentz transformations...

*write* **G′** *and* **P′** *in terms of* **G** *and* **P** *using* cosh$\rho$ *and* sinh$\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

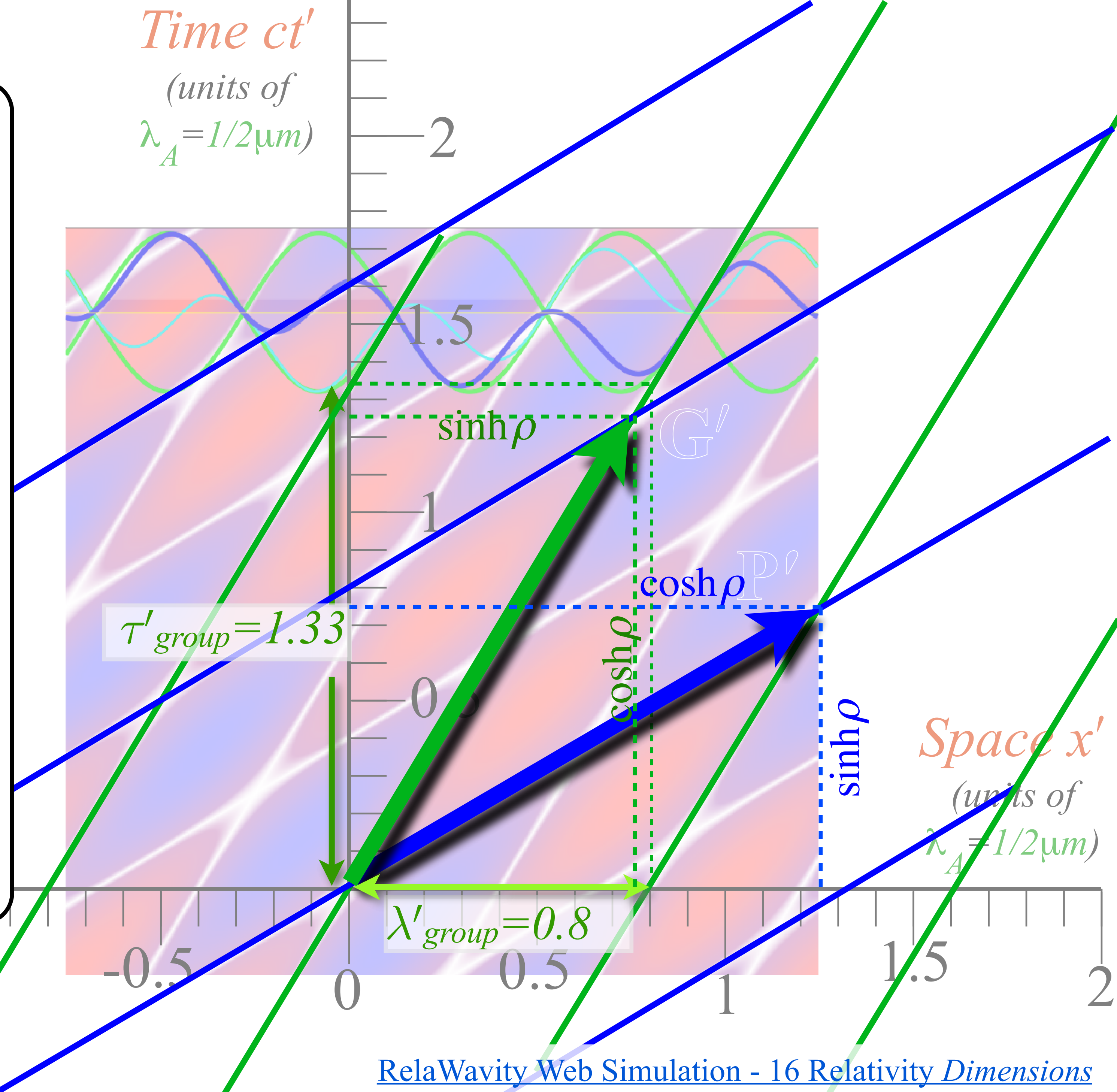
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

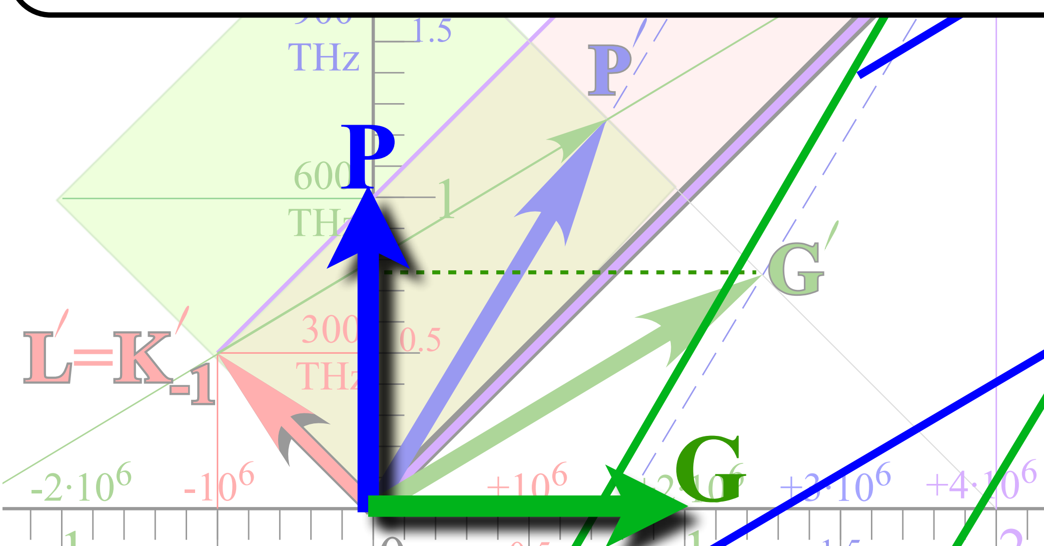
$$\mathbf{G}' = \mathbf{G}\cosh\rho + \mathbf{P}\sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G}\sinh\rho + \mathbf{P}\cosh\rho$$




$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$

Lorentz transform matrix

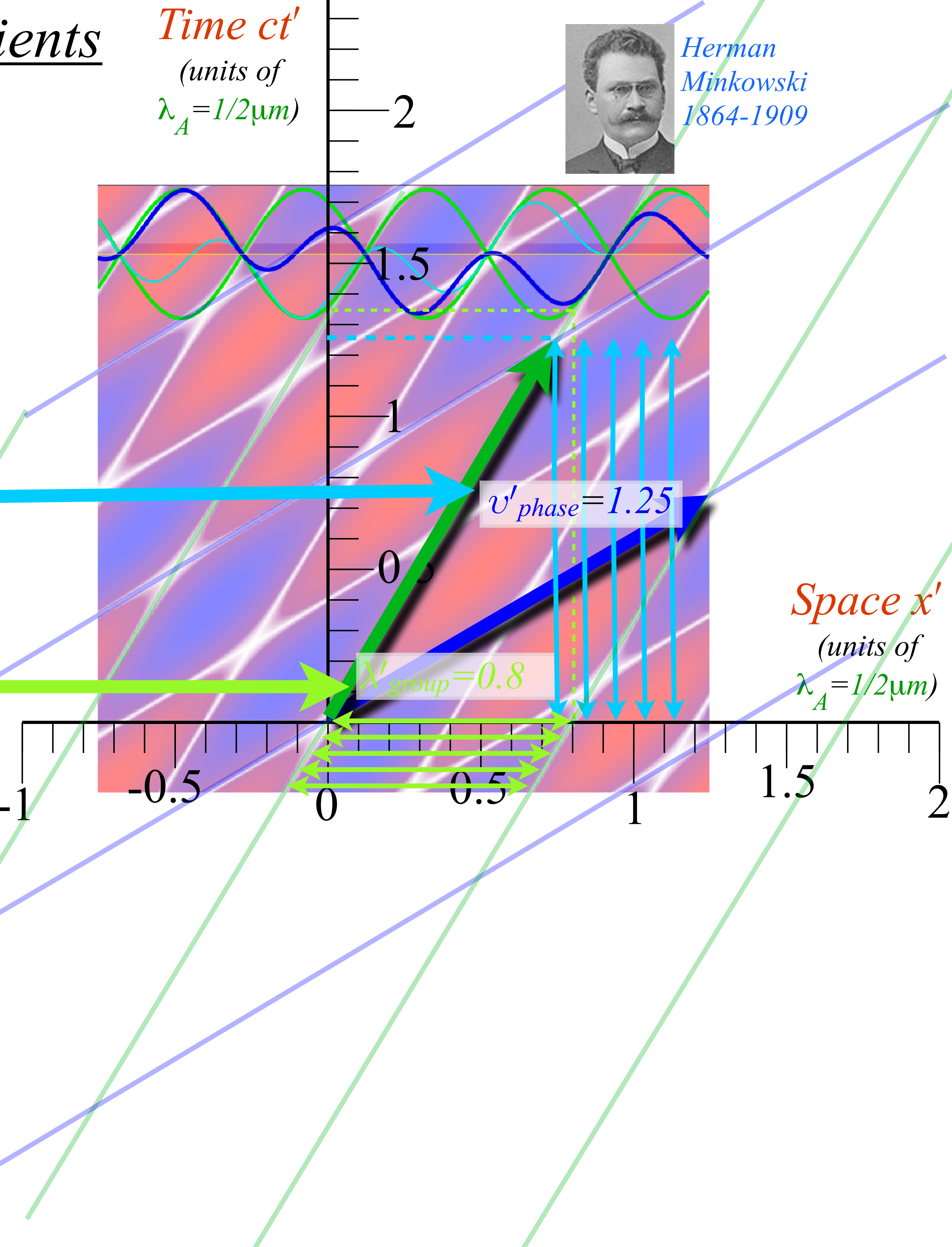
RelaWavity Web Simulation - 16 Relativity *Dimensions*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *phase* | $b^{Doppler}_{RED}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b^{Doppler}_{BLUE}$ |
| *group* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\text{sech}\,\rho$ | $\cosh\rho$ | $\text{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *value for* $\beta$=3/5 | $\frac{1}{2}$= 0.5 | $\frac{3}{5}$=0.6 | $\frac{3}{4}$=0.75 | $\frac{4}{5}$=0.80 | $\frac{5}{4}$=1.25 | $\frac{4}{3}$=1.33 | $\frac{5}{3}$=1.67 | $\frac{2}{1}$=2.0 |

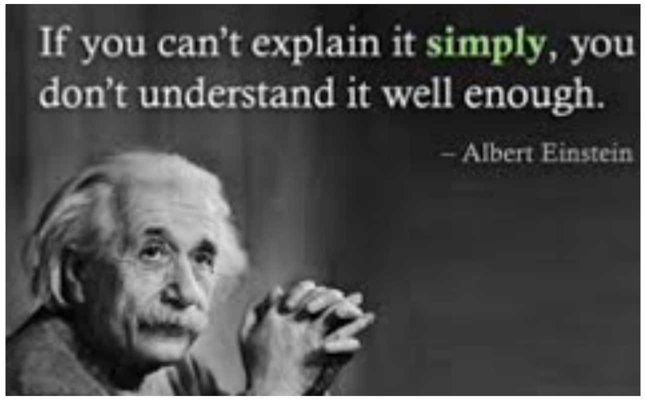
# Two Famous-Name Coefficients

*This number*
*is called an:* ***Einstein time-dilation***
*(dilated by 25% here)*

*This number*
*is called a:* ***Lorentz length-contraction***
*(contracted by 20% here)*

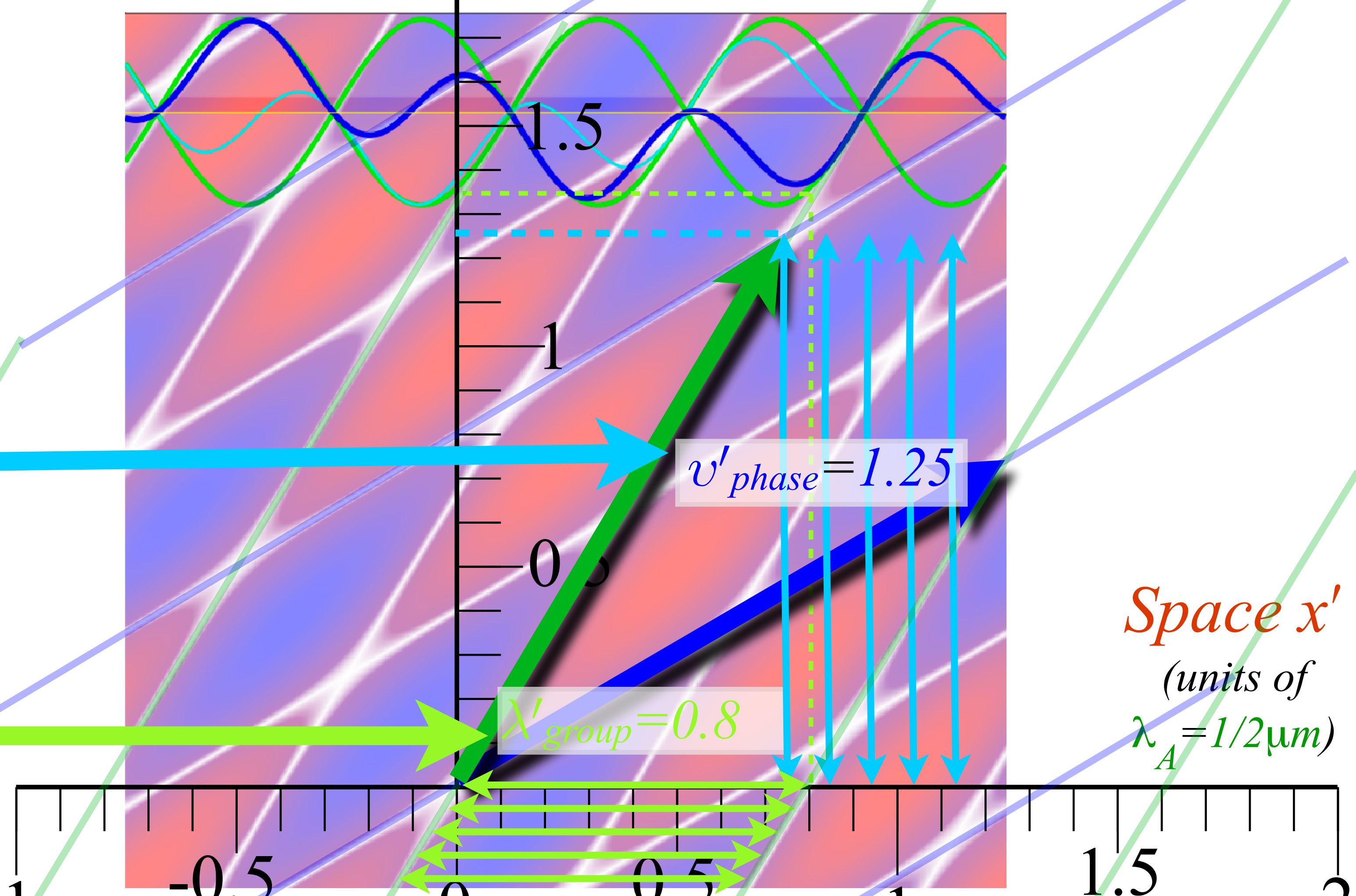


# Two Famous-Name Coefficients




*Albert Einstein 1859-1955*

*Herman Minkowski 1864-1909*

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*This number is called a:* **Lorentz length-contraction**
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*Hendrik A. Lorentz 1853-1928*

## Old-Fashioned Notation

[RelaWavity Web Simulation - Relativistic *Terms* (Expanded Table)]

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *phase* | $b^{Doppler}_{RED}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $b^{Doppler}_{BLUE}$ |
| *group* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\text{sech}\,\rho$ | $\cosh\rho$ | $\text{csch}\,\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| $\beta\equiv\frac{u}{c}$ | $\sqrt{\frac{1-\beta}{1+\beta}}$ | $\frac{\beta}{1}$ | $\frac{1}{\sqrt{\beta^{-2}-1}}$ | $\frac{\sqrt{1-\beta^2}}{1}$ | $\frac{1}{\sqrt{1-\beta^2}}$ | $\frac{\sqrt{\beta^{-2}-1}}{1}$ | $\frac{1}{\beta}$ | $\sqrt{\frac{1+\beta}{1-\beta}}$ |
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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *group* | $b_{RED}^{Doppler}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{V_{phase}}{c}$ | $b_{BLUE}^{Doppler}$ |
| *phase* | $\frac{1}{b_{BLUE}^{Doppler}}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{c}{V_{group}}$ | $\frac{1}{b_{RED}^{Doppler}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\operatorname{sech}\rho$ | $\cosh\rho$ | $\operatorname{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *stellar* $\forall$ *angle* $\sigma$ | $1/e^{+\rho}$ | $\sin\sigma$ | $\tan\sigma$ | $\cos\sigma$ | $\sec\sigma$ | $\cot\sigma$ | $\csc\sigma$ | $1/e^{-\rho}$ |
| $\beta\equiv\frac{u}{c}$ | $\sqrt{\frac{1-\beta}{1+\beta}}$ | $\frac{\beta}{1}$ | $\frac{1}{\sqrt{\beta^{-2}-1}}$ | $\frac{\sqrt{1-\beta^2}}{1}$ | $\frac{1}{\sqrt{1-\beta^2}}$ | $\frac{\sqrt{\beta^{-2}-1}}{1}$ | $\frac{1}{\beta}$ | $\sqrt{\frac{1+\beta}{1-\beta}}$ |
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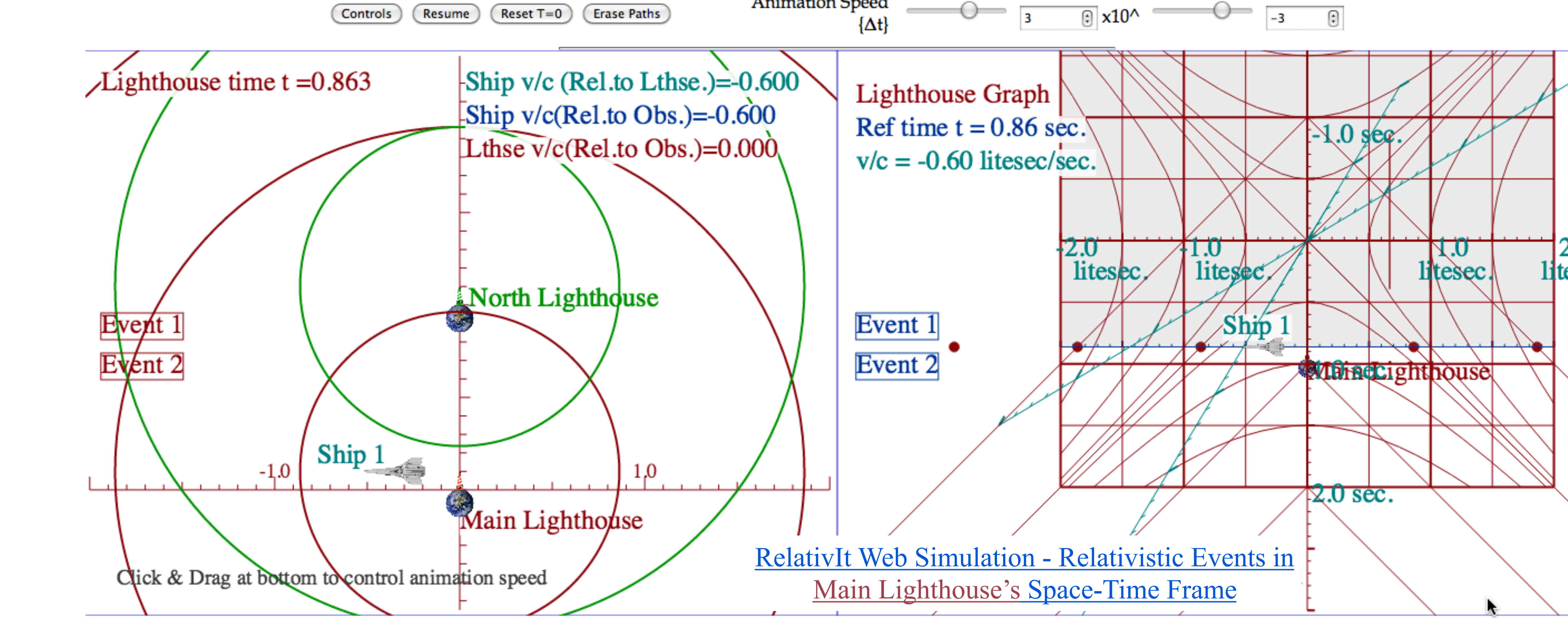
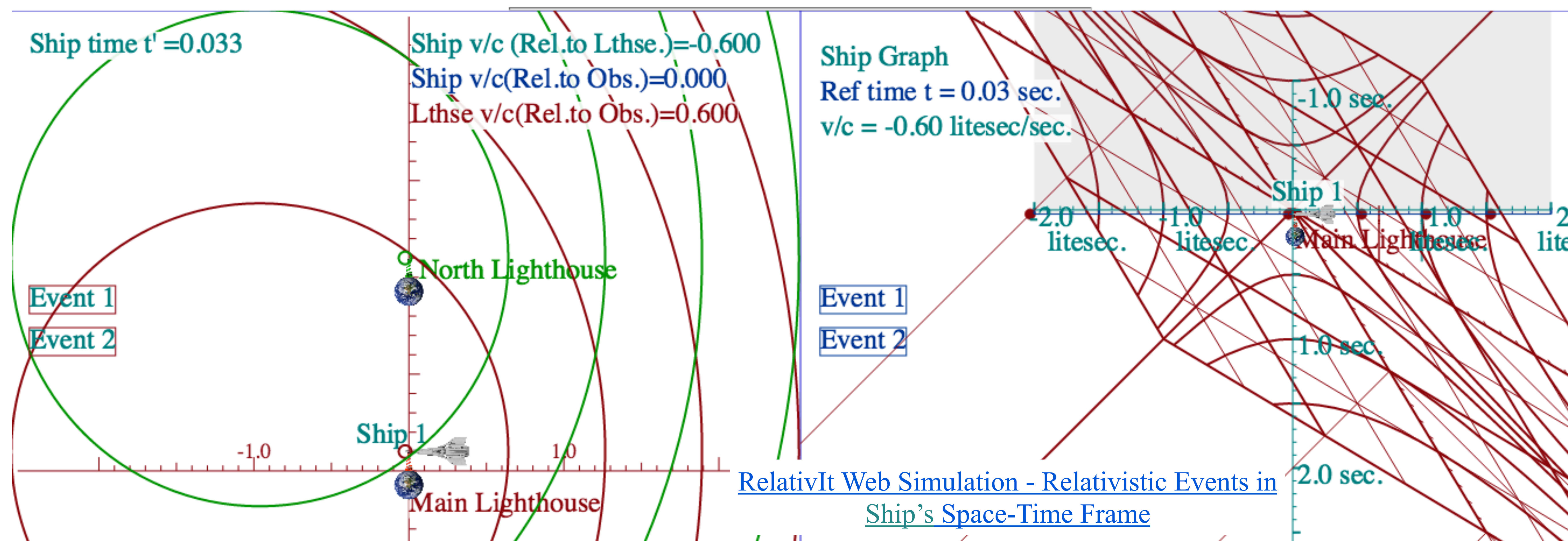
| *effects* | $b_{RED}^{Doppler}$ | $V_{group}$ | *past-future asymmetry* (*off-diagonal Lorentz-transform*) | *x-contraction*$^{(Lorentz)}$ $\tau_{phase}$*-contraction* | *t-dilation*$^{(Einstein)}$ $\upsilon_{phase}$*-dilation* (*on-diagonal Lorentz-transform*) | *inverse asymmetry* | $V_{phase}$ | $b_{BLUE}^{Doppler}$ |

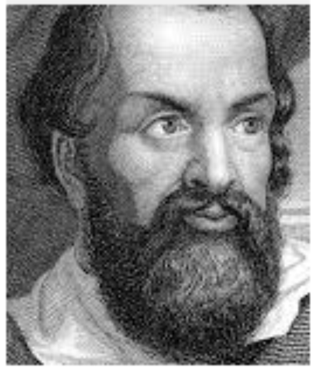


RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame



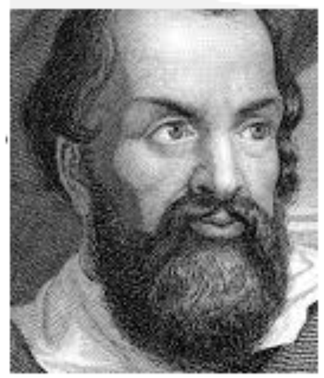
RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

*Galileo Galilei*



*1564-1642*

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*



***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
*Galilean velocity*

Optical interference "baseball-diamond" displays *phase* and *group* velocity
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Application to TE-Waveguide modes and synchrotron beam relativity

# Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

(2.) What is that frequency $\omega_E$ ?

# Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

(2.) What is that frequency $\omega_E$ ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

# Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

(2.) What is that frequency $\omega_E$ ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

(2.) What is that frequency $\omega_E$ ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Query (2.) similarly: What $\omega_E$ is blue-shift $b\omega_L$ of $\omega_L$ *and* red-shift $\omega_R/b$ of $\omega_R$ ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

(2.) What is that frequency $\omega_E$ ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$\frac{300}{900}$$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\,\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\,\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\,\frac{600-300}{600+300} = \frac{1}{3}c$$

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$$\sqrt{6\cdot 3} = 3\sqrt{2} = 4.24$$

$$\omega_E = b\,\omega_L = \omega_R/b \quad\Rightarrow\quad b = \sqrt{\omega_R/\omega_L} \quad\Rightarrow\quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

*Geometric mean*

Doppler Jeopardy


(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\omega_E$?

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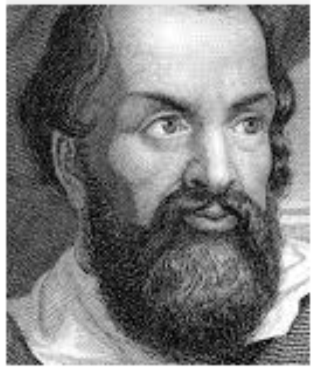
$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

$V_{group}/c$ is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$ . Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

*Geometric mean*

*Galileo Galilei*

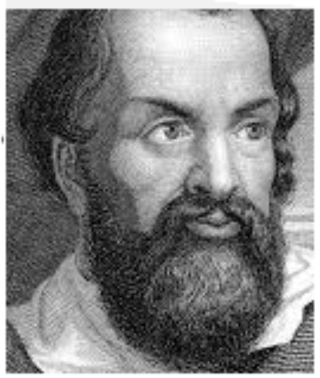


*1564-1642*

***Galileo's Revenge (part 1)***

*Rapidity adds just like*

*Galilean velocity*



***Galileo's Revenge (part 2)***

***Phasor angular velocity***

*adds just like*

*Galilean velocity*

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

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Doppler shifted "baseball-diamond" displays Lorentz frame transformation

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16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

➔ Thales geometric mean υ geometry of Lorentz transformation

*Rapidity ρ* related to *stellar aberration angle σ* and L. C. Epstein's approach to relativity

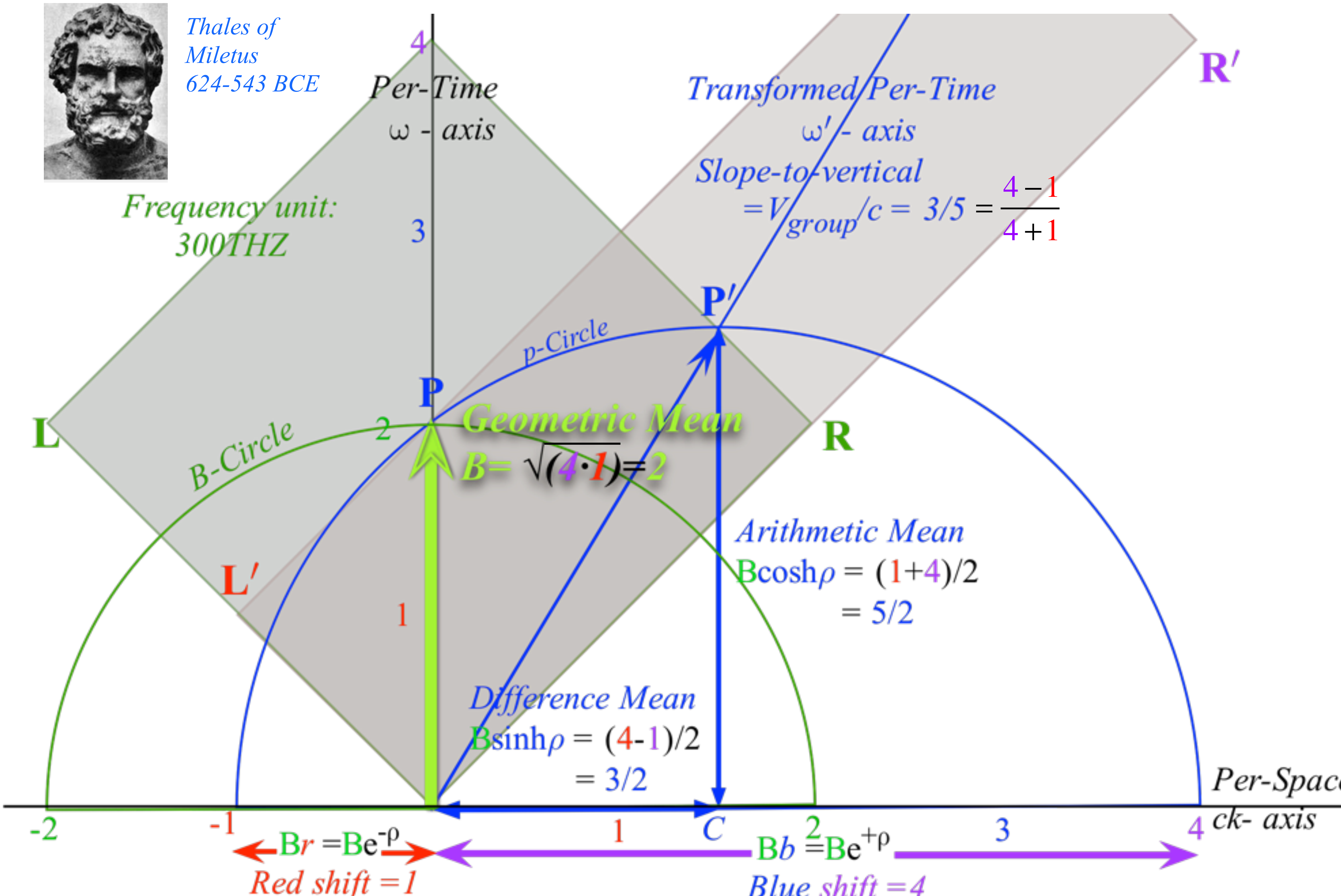
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes and synchrotron beam relativity

# Thales Mean Geometry (600BCE)

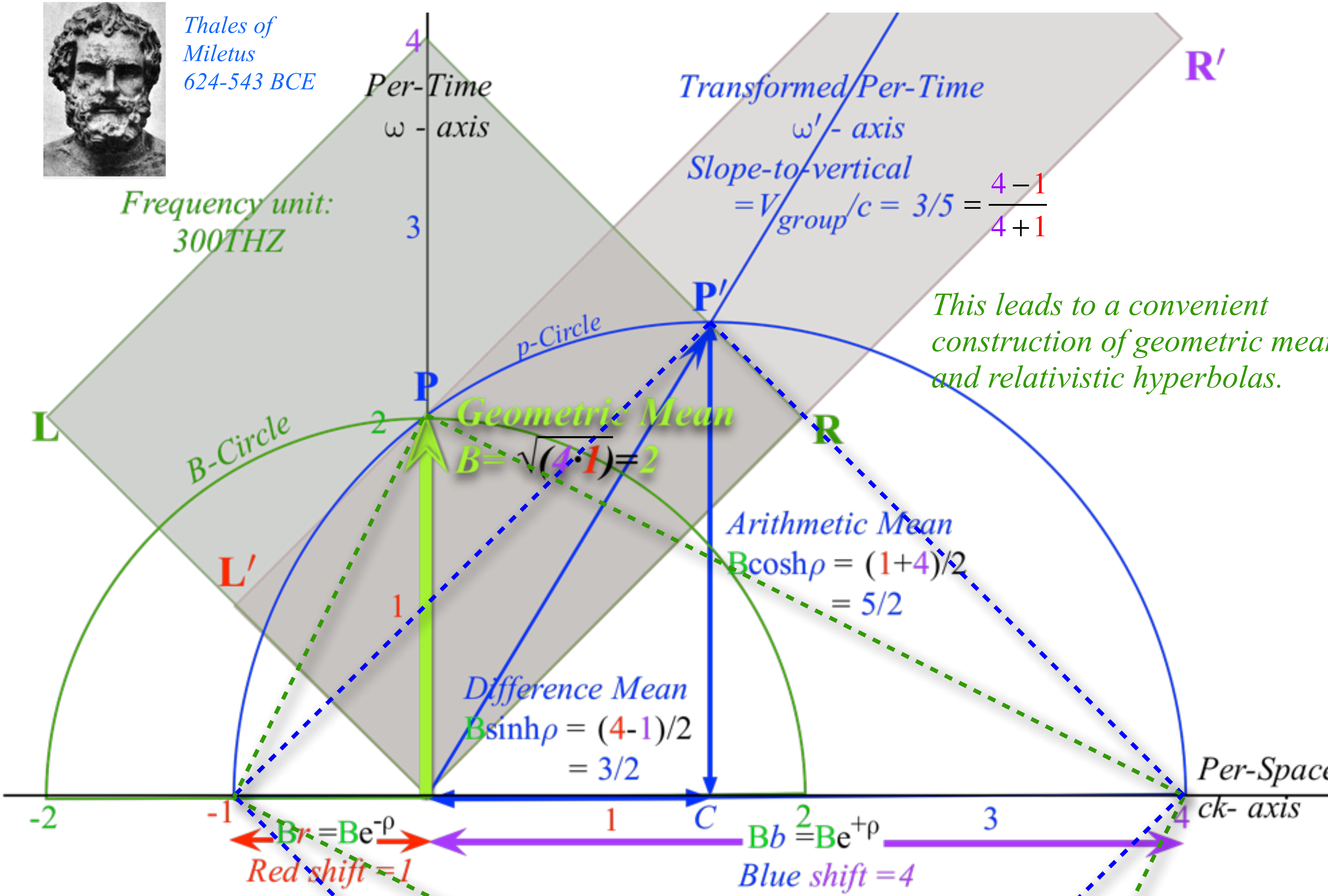
helps "Relawavity"



# Thales Mean Geometry (600BCE)

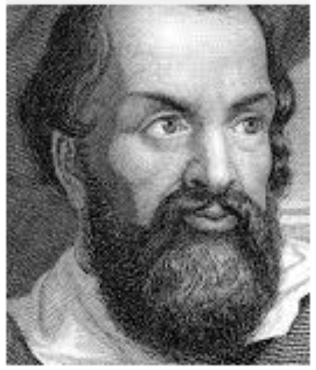
helps “Relawavity” *Thales showed a circle diameter subtends a right angle with any circle point P*

*Thales of Miletus 624-543 BCE*




*This leads to a convenient construction of geometric means and relativistic hyperbolas.*

*Galileo Galilei*



*1564-1642*

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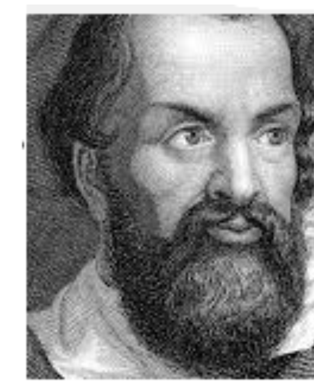

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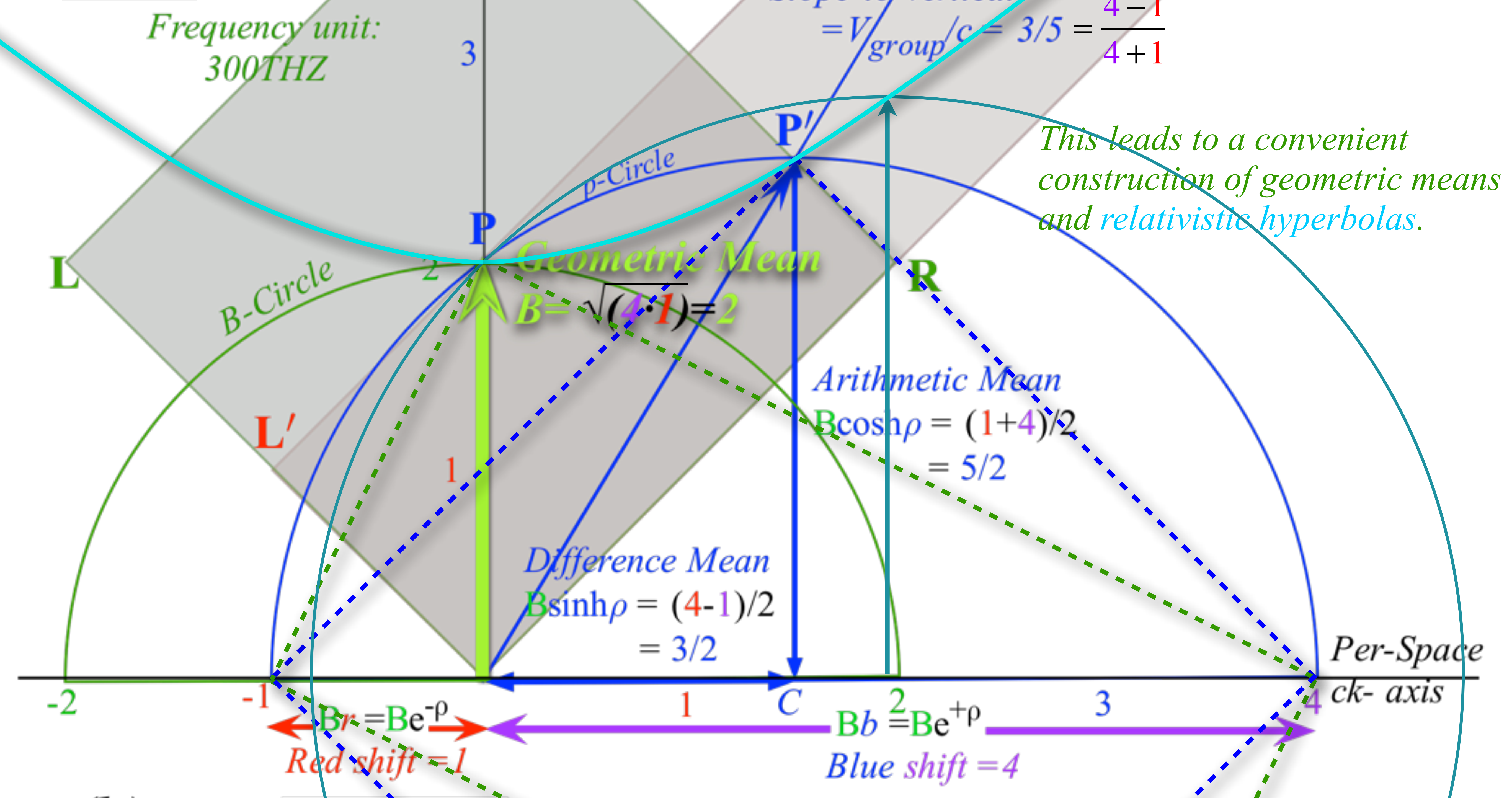

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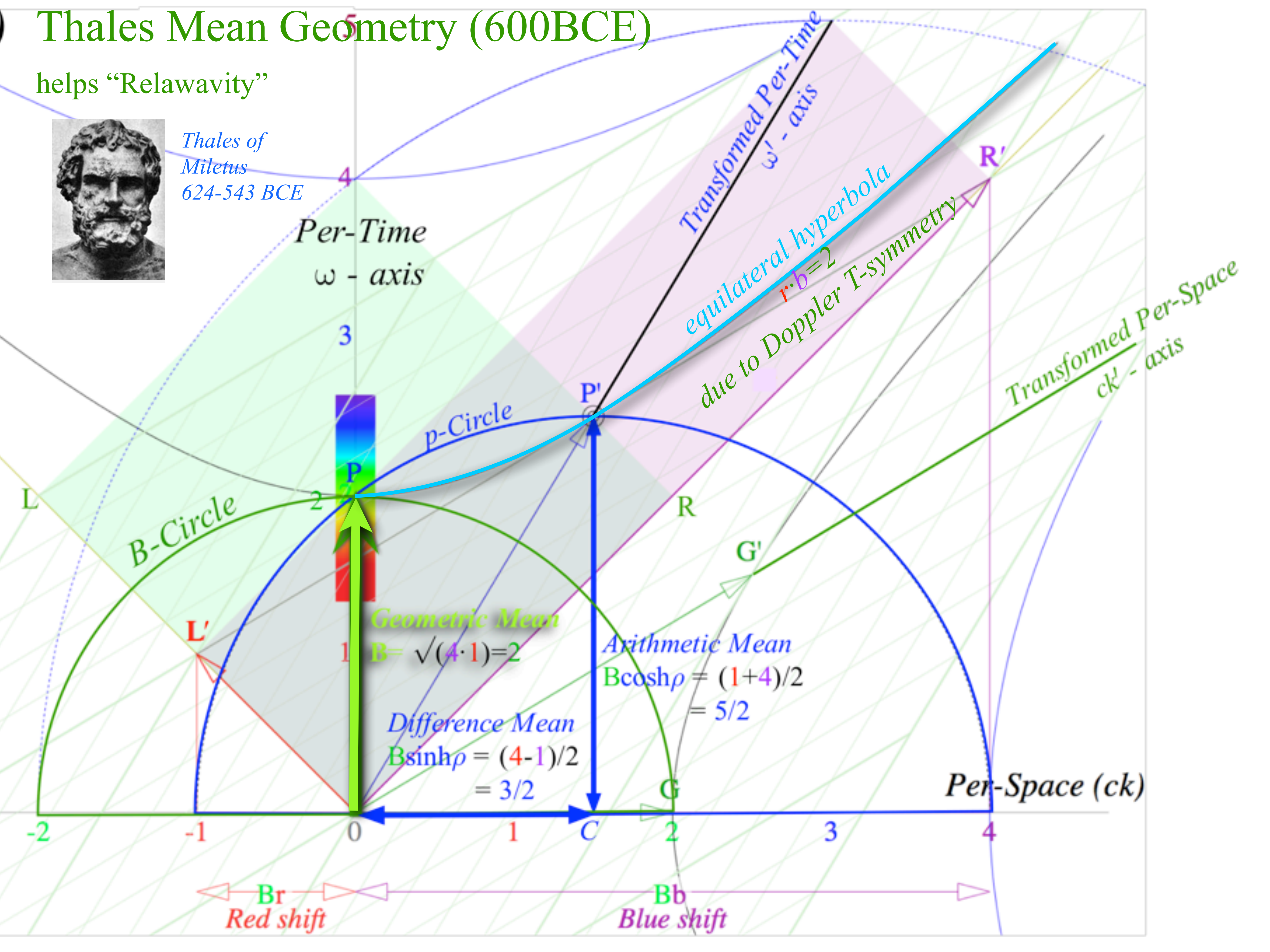
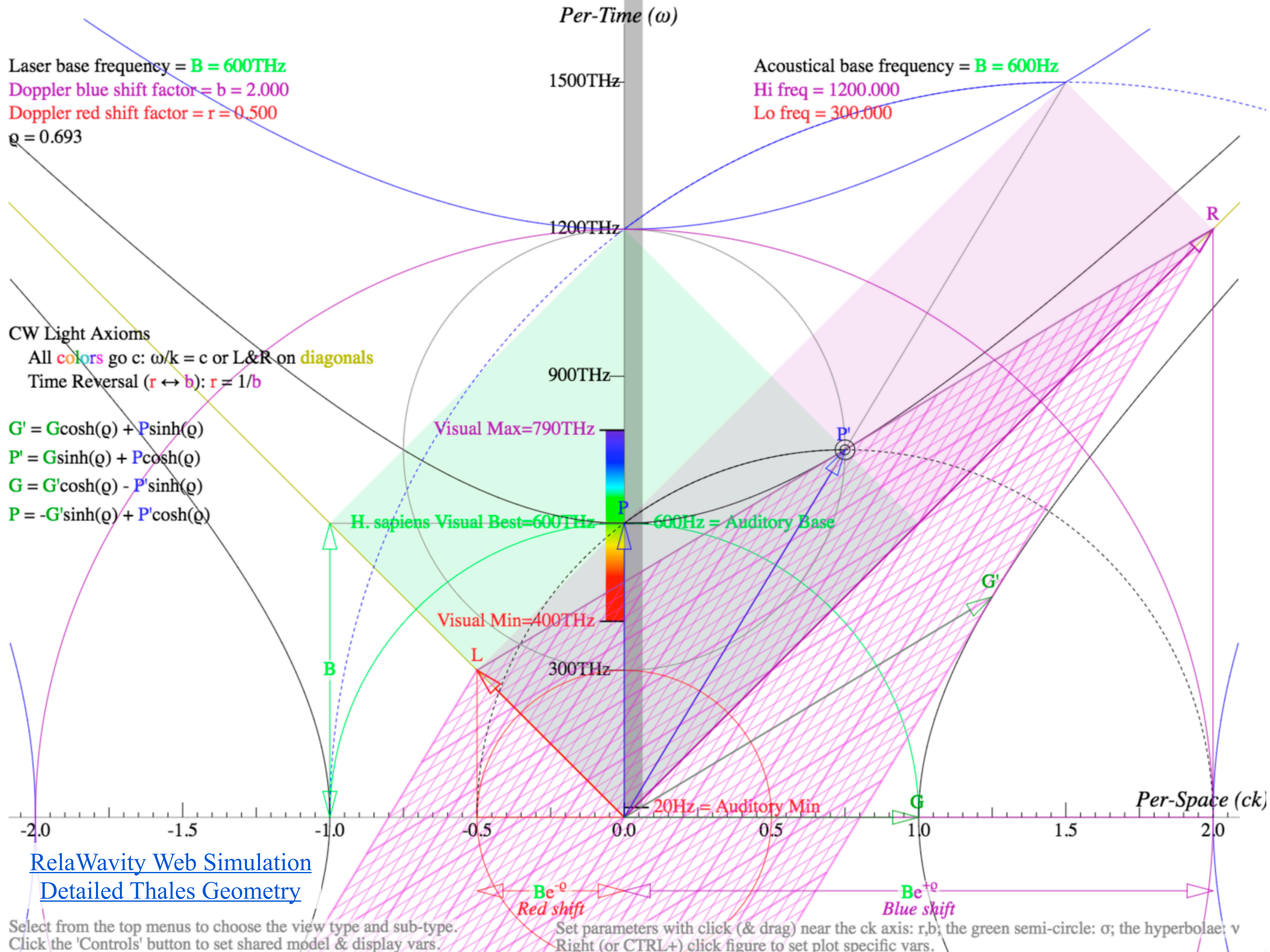
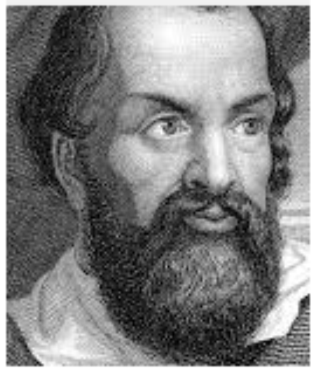
helps “Relawavity”



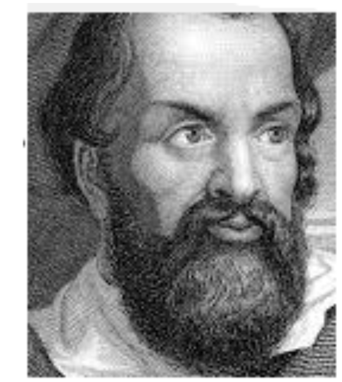

Per-Time (ω)
Laser base frequency = B = 600THz
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ϱ = 0.693
Acoustical base frequency = B = 600Hz
Hi freq = 1200.000
Lo freq = 300.000
1500THz
1200THz
900THz
300THz
CW Light Axioms
All colors go c: ω/k = c or L&R on diagonals
Time Reversal (r ↔ b): r = 1/b
G' = Gcosh(ϱ) + Psinh(ϱ)
P' = Gsinh(ϱ) + Pcosh(ϱ)
G = G'cosh(ϱ) - P'sinh(ϱ)
P = -G'sinh(ϱ) + P'cosh(ϱ)
Visual Max=790THz
H. sapiens Visual Best=600THz
600Hz = Auditory Base
Visual Min=400THz
20Hz = Auditory Min
P
P'
G
G'
L
R
B
Per-Space (ck)
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
RelaWavity Web Simulation
Detailed Thales Geometry
Be^-ϱ
Red shift
Be^+ϱ
Blue shift
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: ν
Right (or CTRL+) click figure to set plot specific vars.

*Galileo Galilei*



*1564-1642*

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*



***Galileo's Revenge (part 2)***
***Phasor angular velocity***
***adds just like***
***Galilean velocity***

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# Comparing Longitudinal relativity parameter: Rapidity $\rho$ = $\log_e$(Doppler Shift)

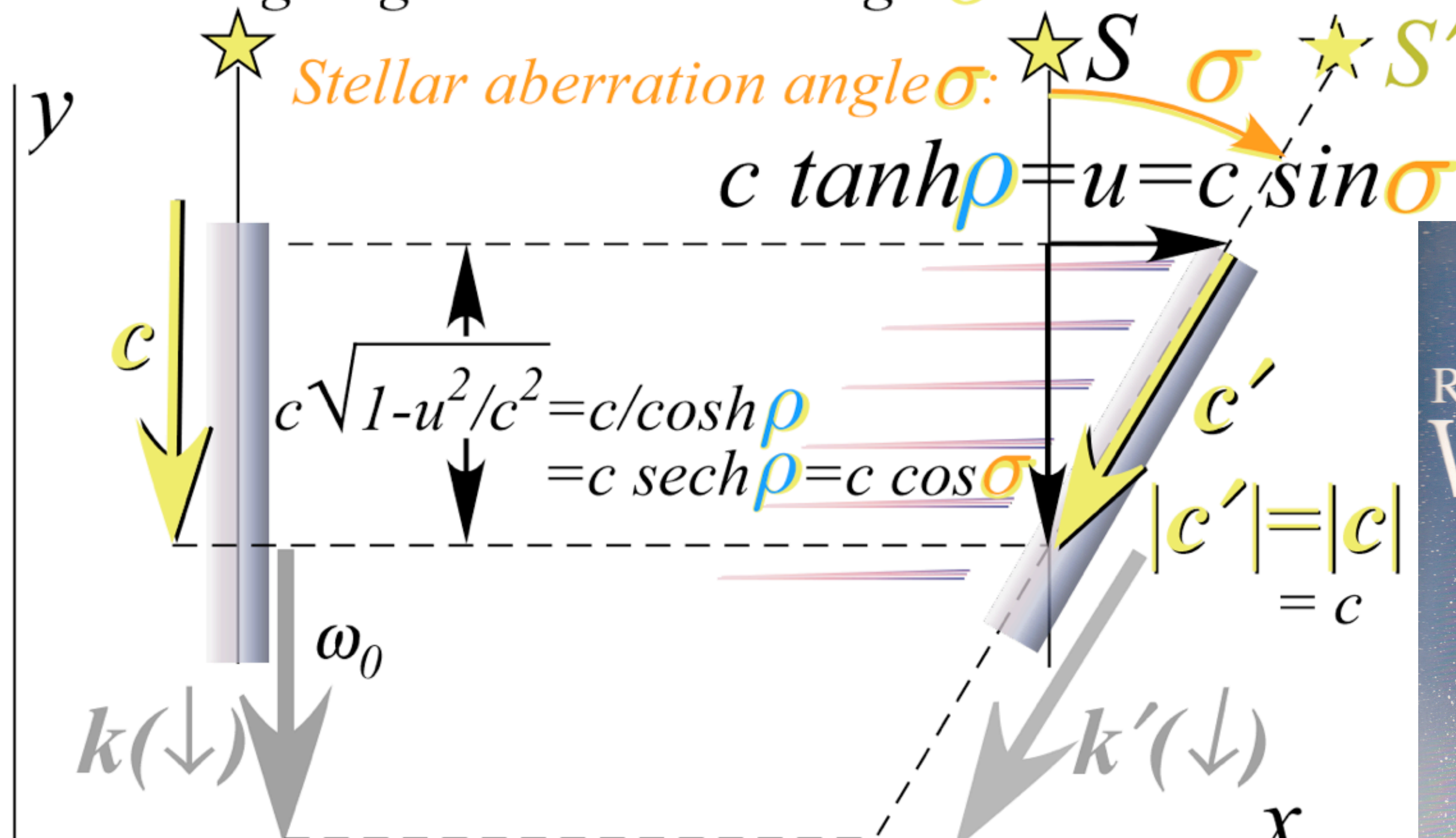
# to a Transverse*relativity parameter: Stellar aberration angle $\sigma$

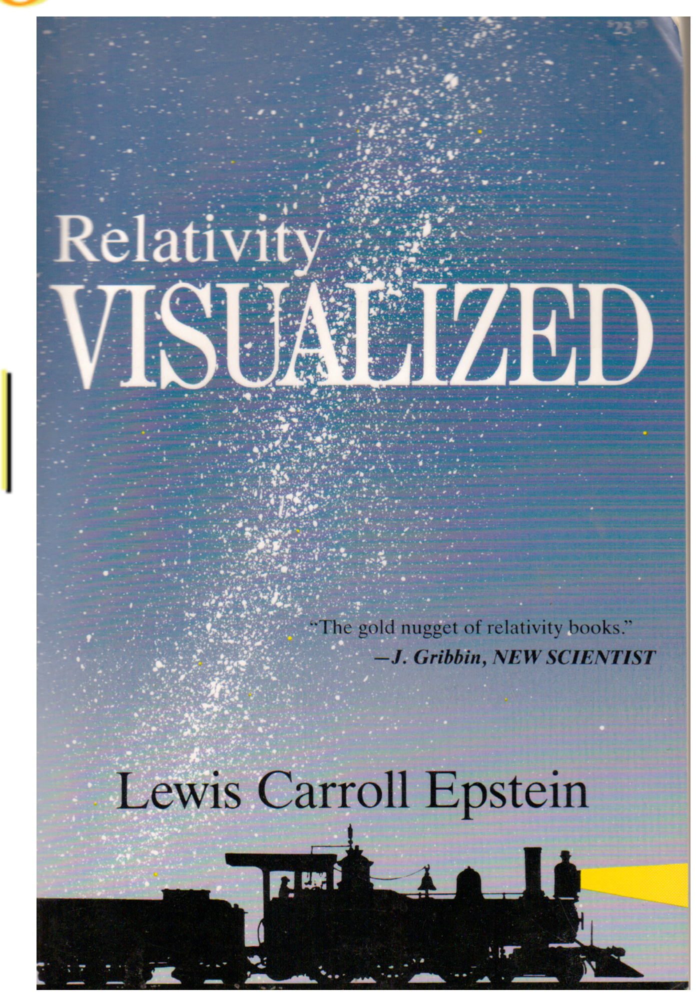
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

*Observer fixed below star sees it directly overhead.*

*Observer going* ***u*** *sees star at angle* $\sigma$ *in* ***u*** *direction.*

We used notion $\sigma$ for stellar-ab-angle, (a "flipped-out" $\rho$ ). Epstein not interested in $\rho$ analysis or in relation of $\sigma$ and $\rho$.




*Purchase at:* AllBookstores.com amazon

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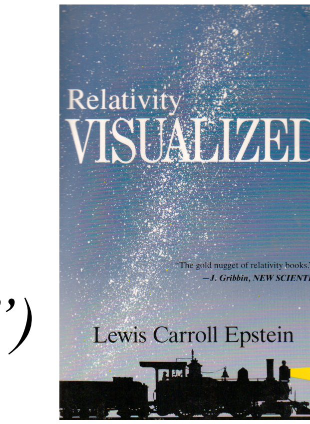
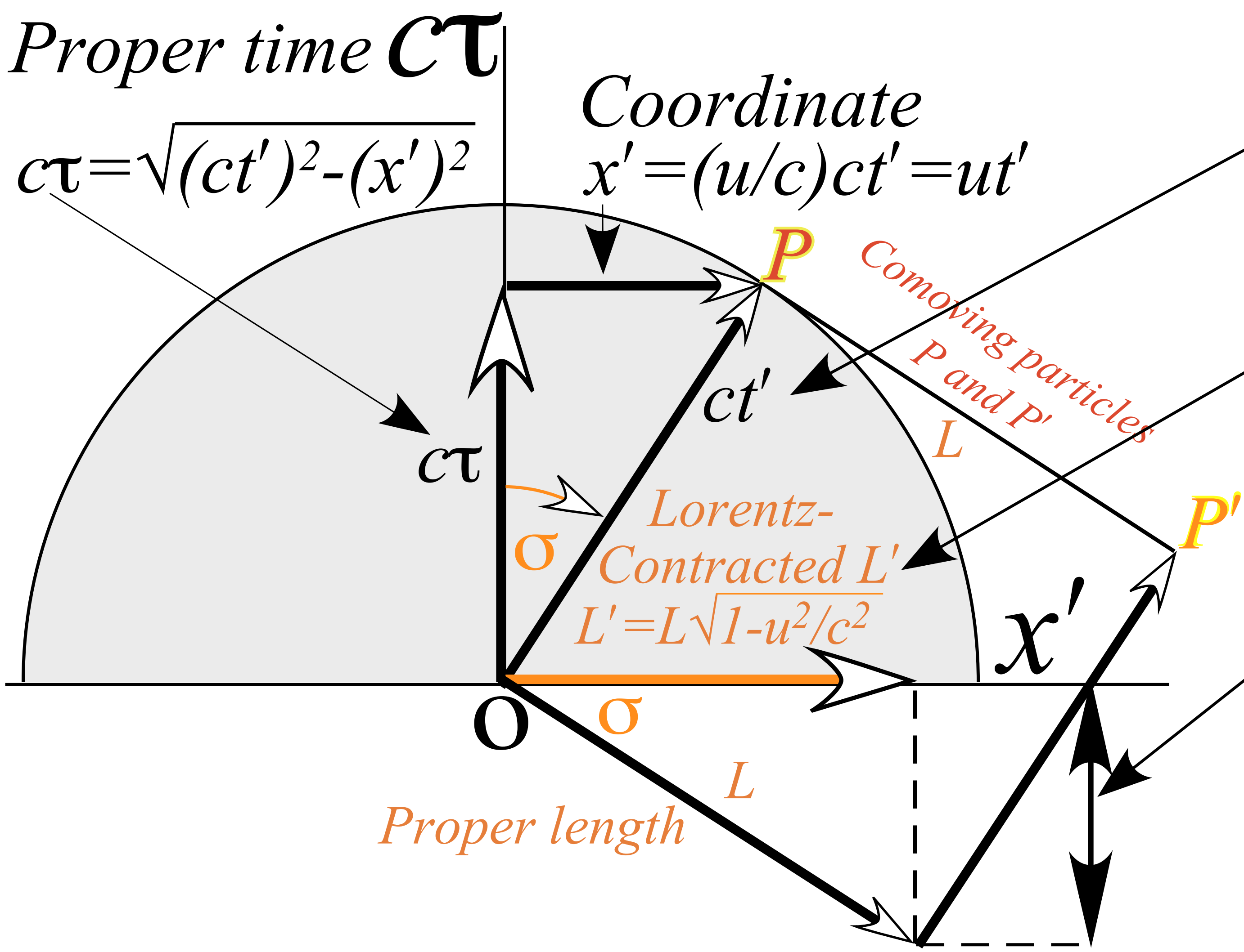



*Proper time* $c\tau$ *vs. coordinate space x* - *(L. C. Epstein's "Cosmic Speedometer")*

*Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)*




*Einstein time dilation:*

$$ct' = c\tau\ \sec\sigma = c\tau\ \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

*Lorentz length contraction:*

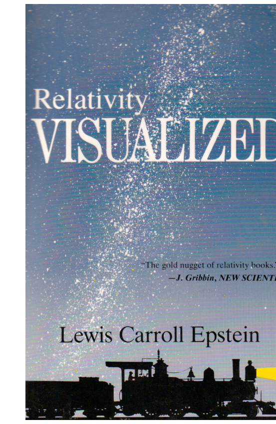
$$L' = L\ \mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

*Proper Time asimultaneity:*

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

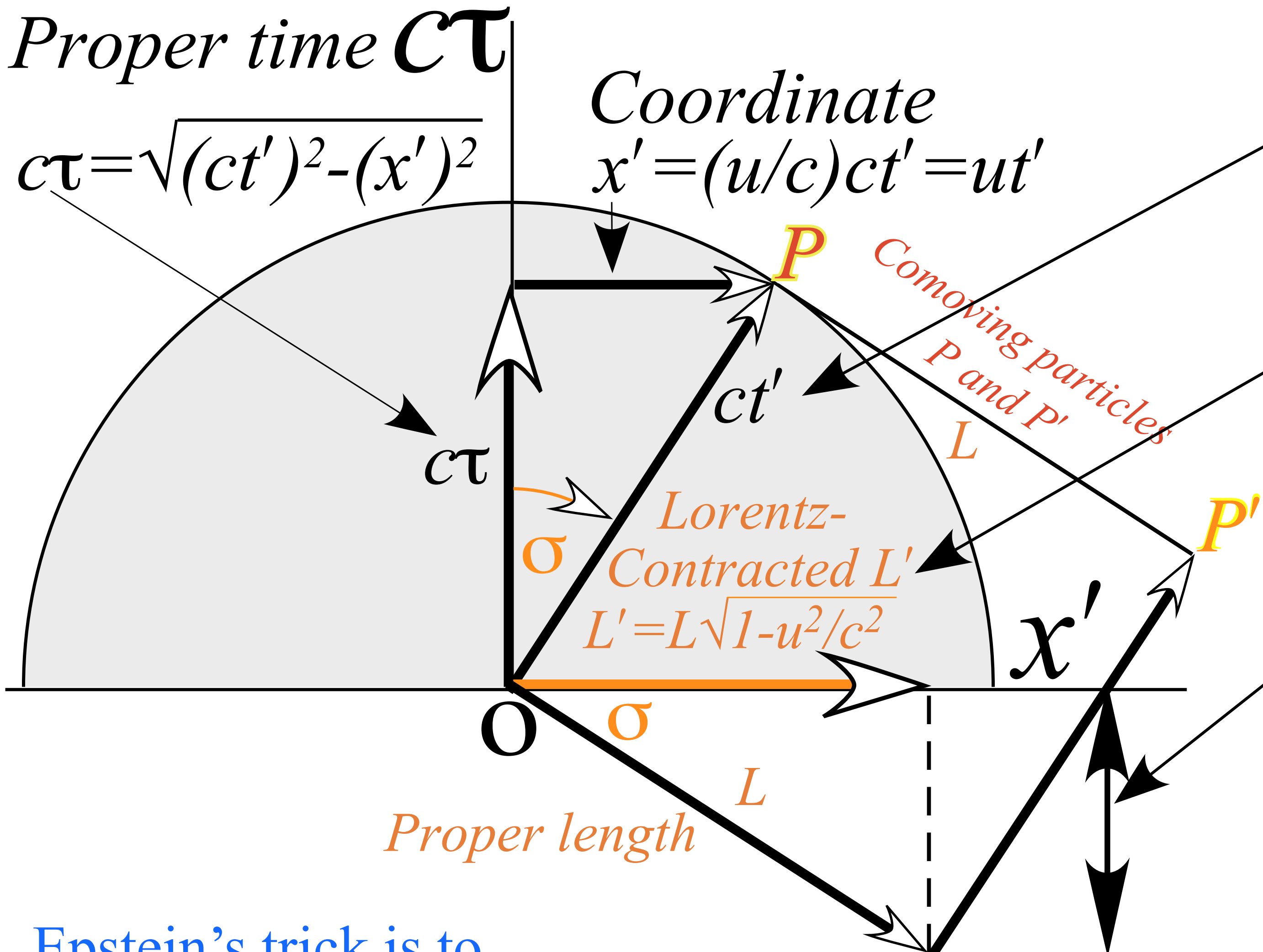
# Comparing Longitudinal relativity parameter: Rapidity $\rho$ = log$_e$(Doppler Shift)
# to a Transverse*relativity parameter: Stellar aberration angle $\sigma$

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-




*Proper time $c\tau$ vs. coordinate space $x$ - (L. C. Epstein's "Cosmic Speedometer")*

*Particles P and P′ have speed $u$ in $(x', ct')$ and speed $c$ in $(x, c\tau)$*




*Einstein time dilation:*

$$ct' = c\tau\sec\sigma = c\tau\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

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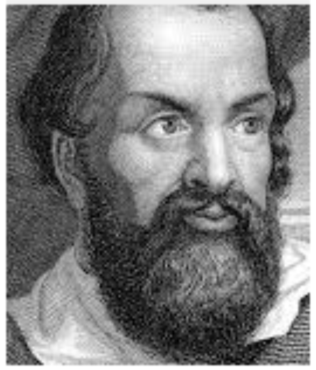
$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

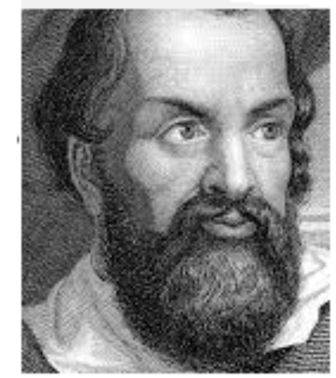
Then everything (and everybody) always goes speed $c$ through $(x', c\tau)$ space!

Galileo Galilei



1564-1642

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*



***Galileo's Revenge (part 2)***
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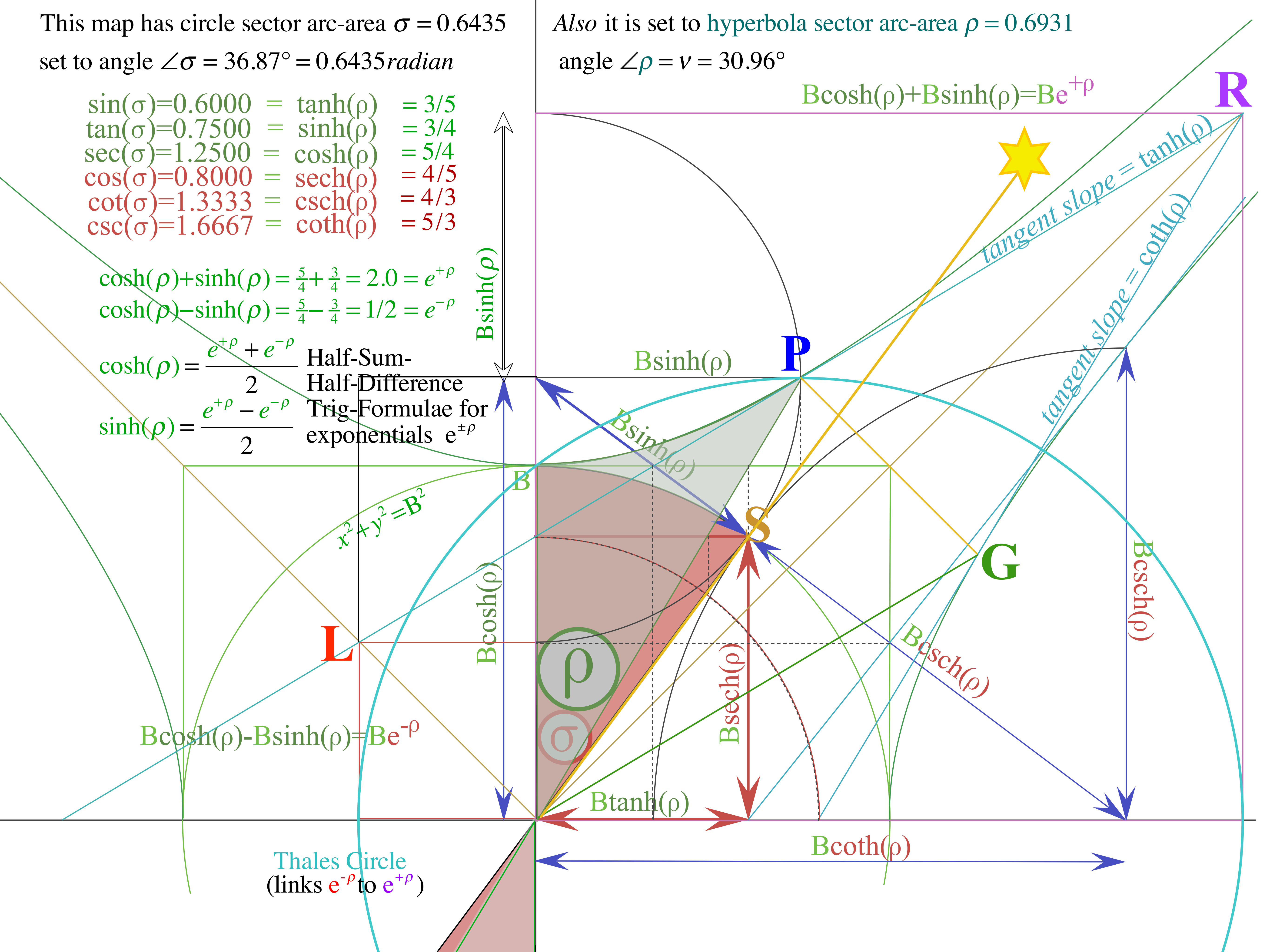
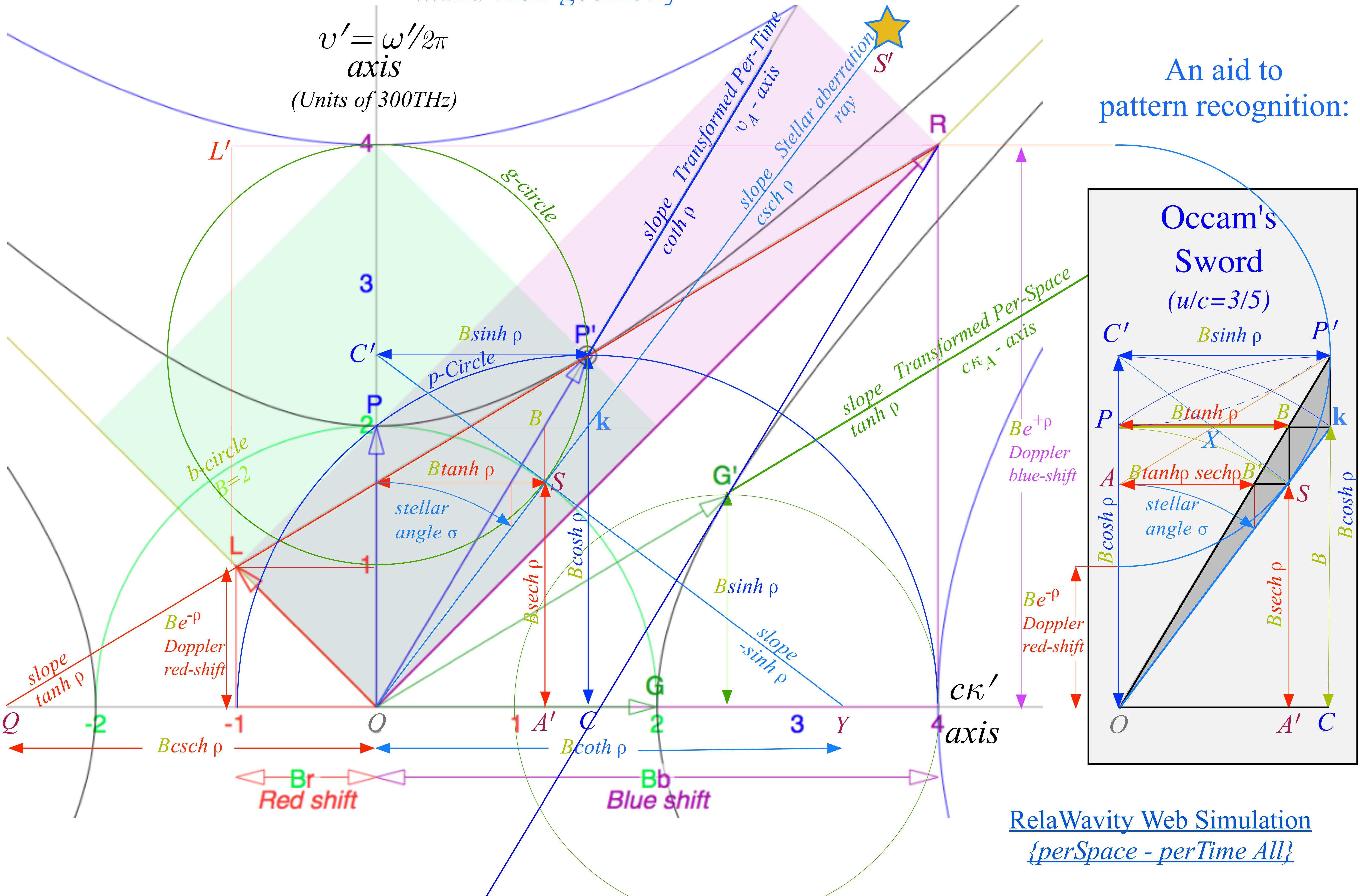

This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435radian
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
Bsinh(ρ)
P
Bsinh(ρ)
B
x²+y²=B²
S
G
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})

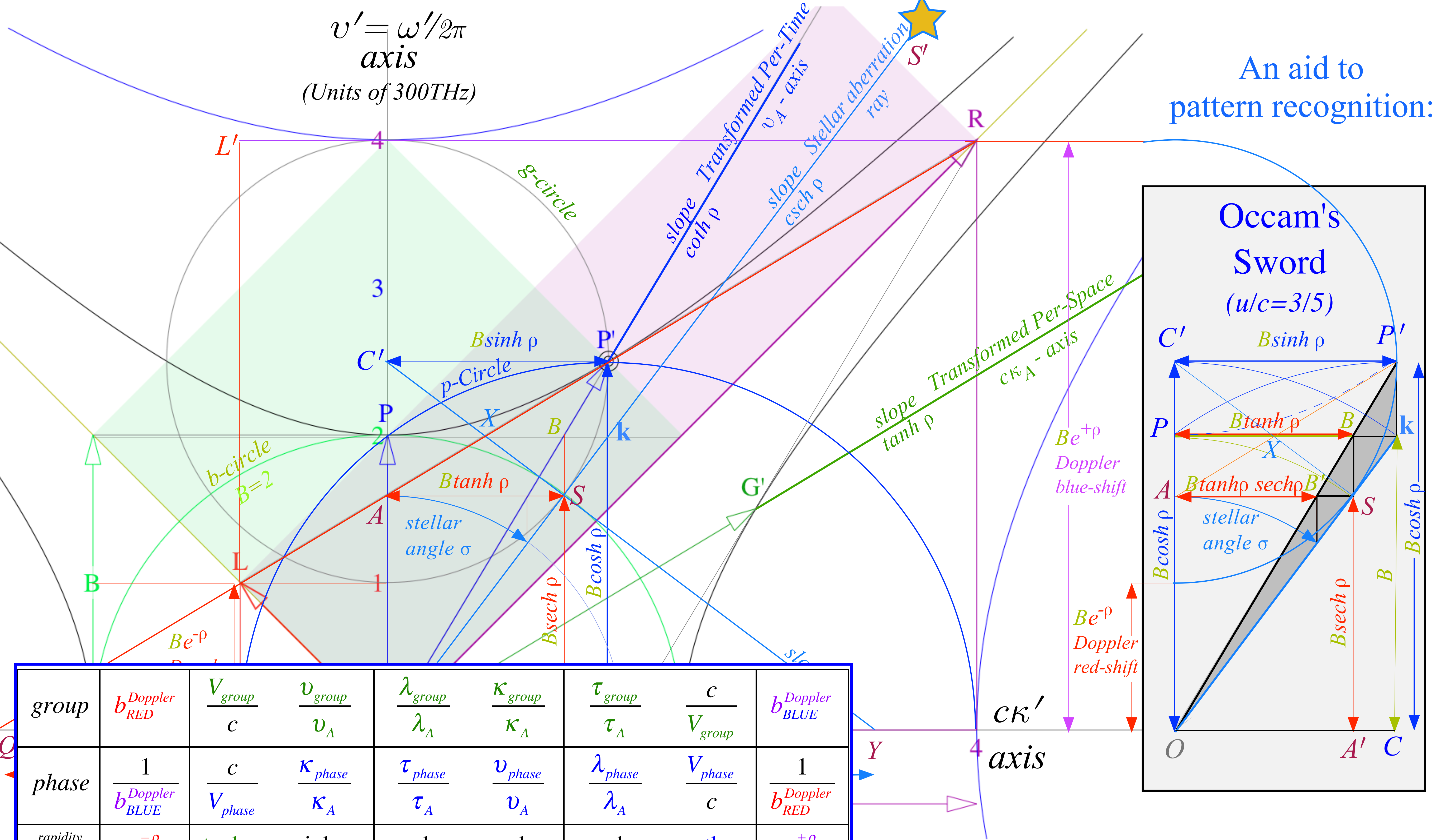
# Summary of optical wave parameters for relativity and QM

...and their geometry




[RelaWavity Web Simulation](#)
[{perSpace - perTime All}](#)

An aid to pattern recognition:




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *group* | $b^{Doppler}_{RED}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $b^{Doppler}_{BLUE}$ |
| *phase* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\mathrm{sech}\,\rho$ | $\cosh\rho$ | $\mathrm{csch}\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *stellar* ∀ *angle* $\sigma$ | $1/e^{+\rho}$ | $\sin\sigma$ | $\tan\sigma$ | $\cos\sigma$ | $\sec\sigma$ | $\cot\sigma$ | $\csc\sigma$ | $1/e^{-\rho}$ |
| $\beta\equiv\frac{u}{c}$ | $\sqrt{\frac{1-\beta}{1+\beta}}$ | $\frac{\beta}{1}$ | $\frac{1}{\sqrt{\beta^{-2}-1}}$ | $\frac{\sqrt{1-\beta^2}}{1}$ | $\frac{1}{\sqrt{1-\beta^2}}$ | $\frac{\sqrt{\beta^{-2}-1}}{1}$ | $\frac{1}{\beta}$ | $\sqrt{\frac{1+\beta}{1-\beta}}$ |
| *value for* $\beta$=3/5 | $\frac{1}{2}$= 0.5 | $\frac{3}{5}$=0.6 | $\frac{3}{4}$=0.75 | $\frac{4}{5}$=0.80 | $\frac{5}{4}$=1.25 | $\frac{4}{3}$=1.33 | $\frac{5}{3}$=1.67 | $\frac{2}{1}$=2.0 |

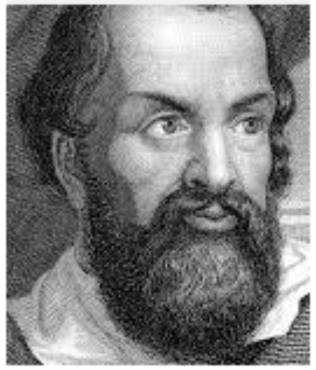
# Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation

Expanded Relativistic Relations

*Galileo Galilei*



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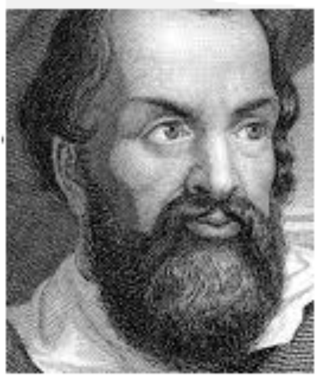

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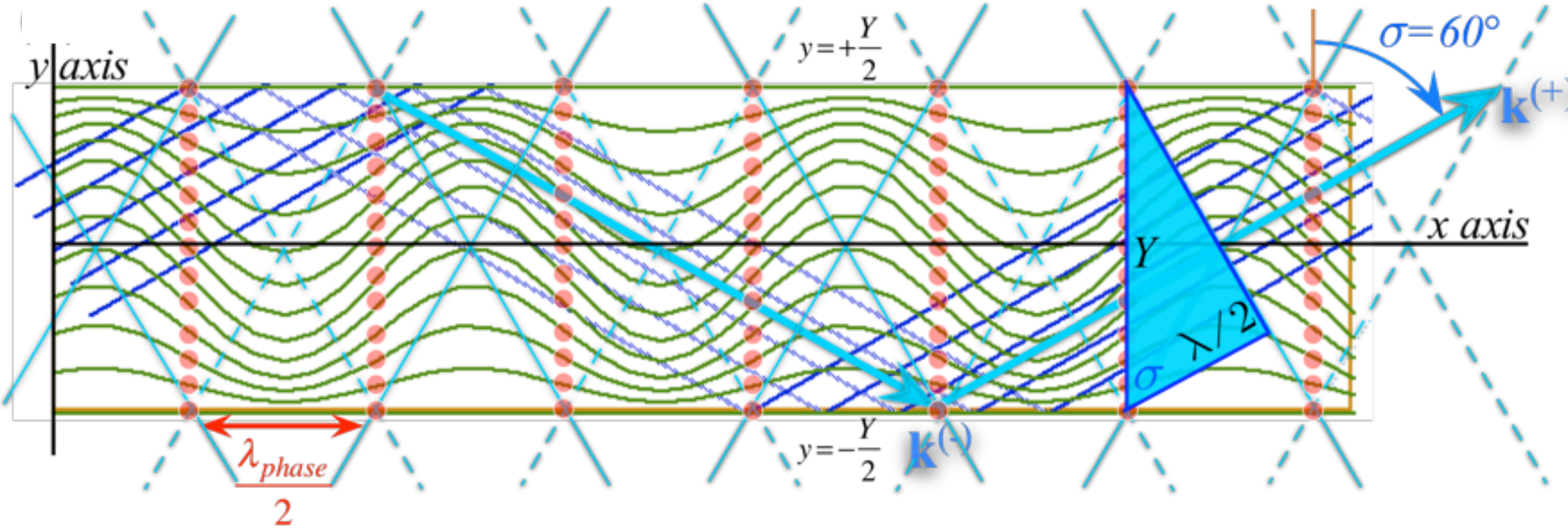
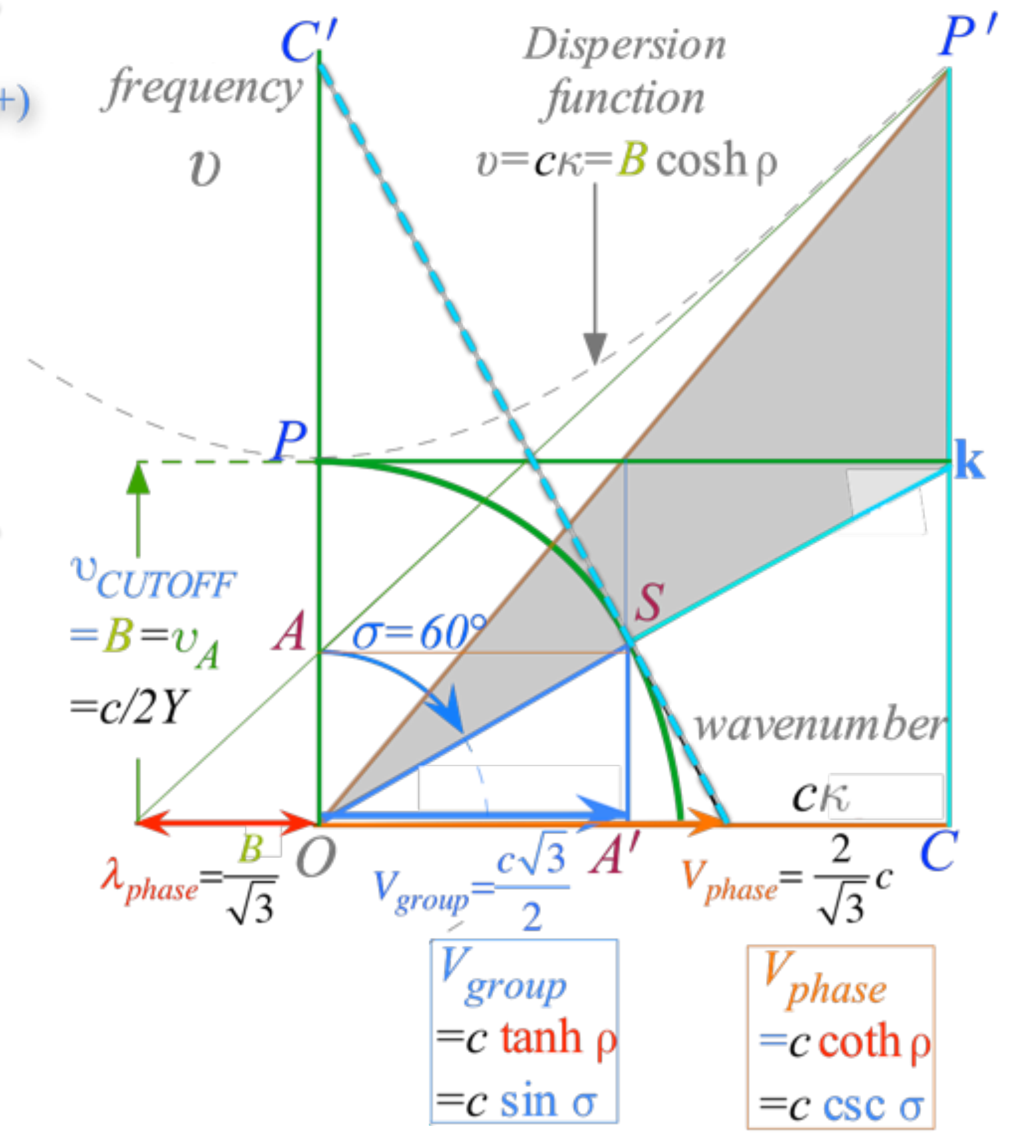
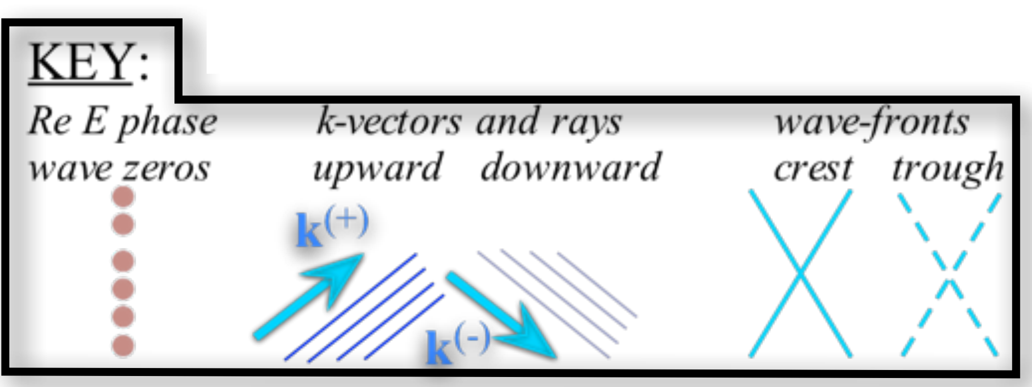
# Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to $(x,y)$ space-space
to $(k_x,k_y)$per-space-per-space
to $(x,ct)$ space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$ . (Low dispersion.)

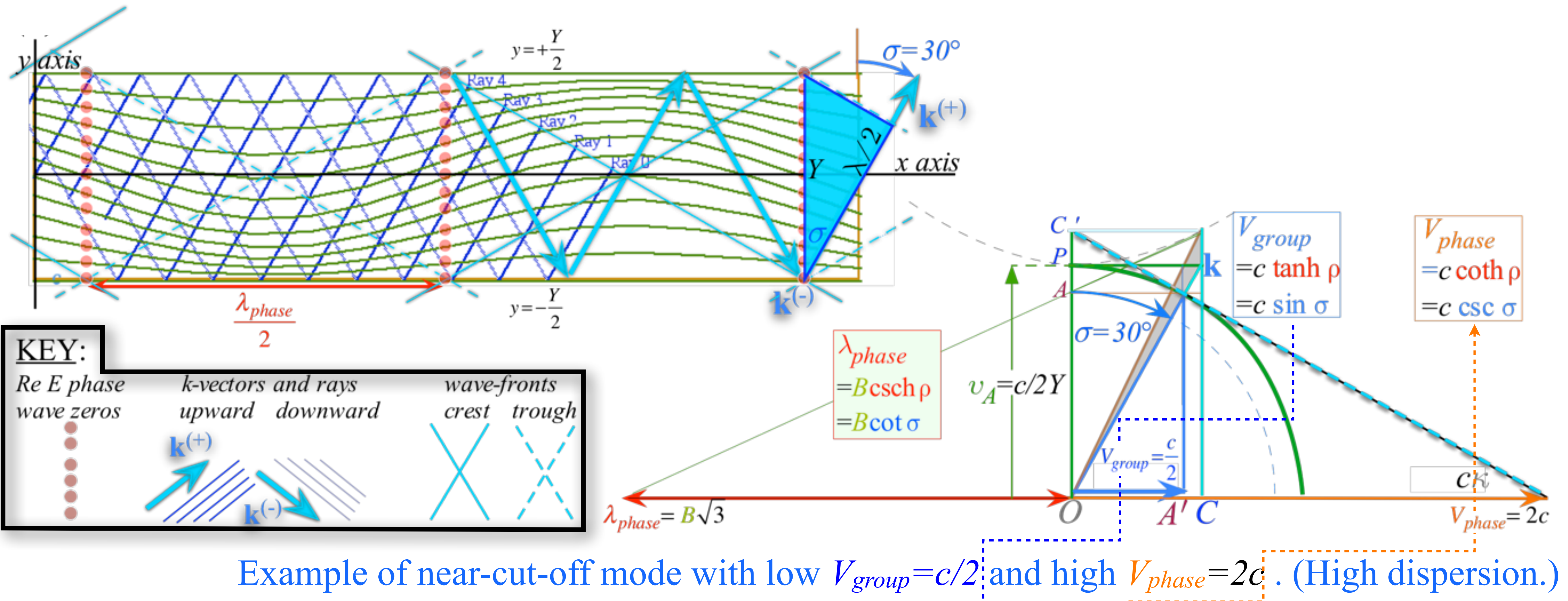


# Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to $(x,y)$ space-space
to $(k_x,k_y)$per-space-per-space
to $(x,ct)$ space-time

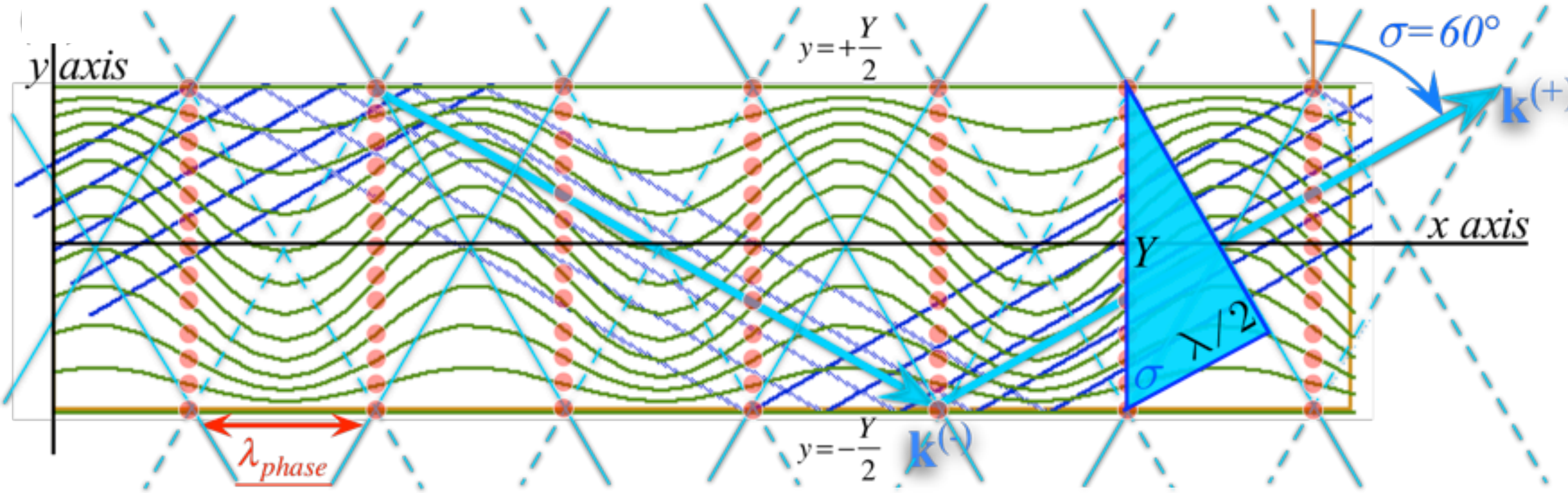



Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$ . (High dispersion.)

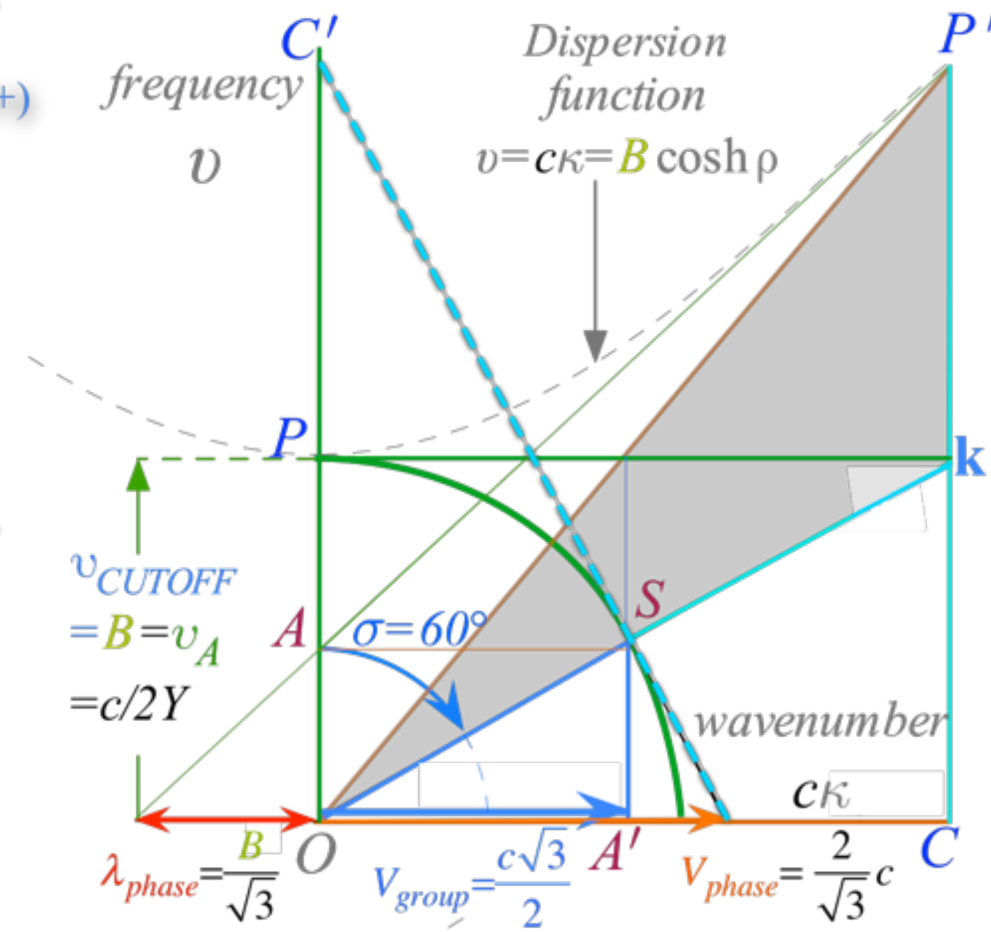
# Optical wave guide relativistic geometry aided by Occam's Sword

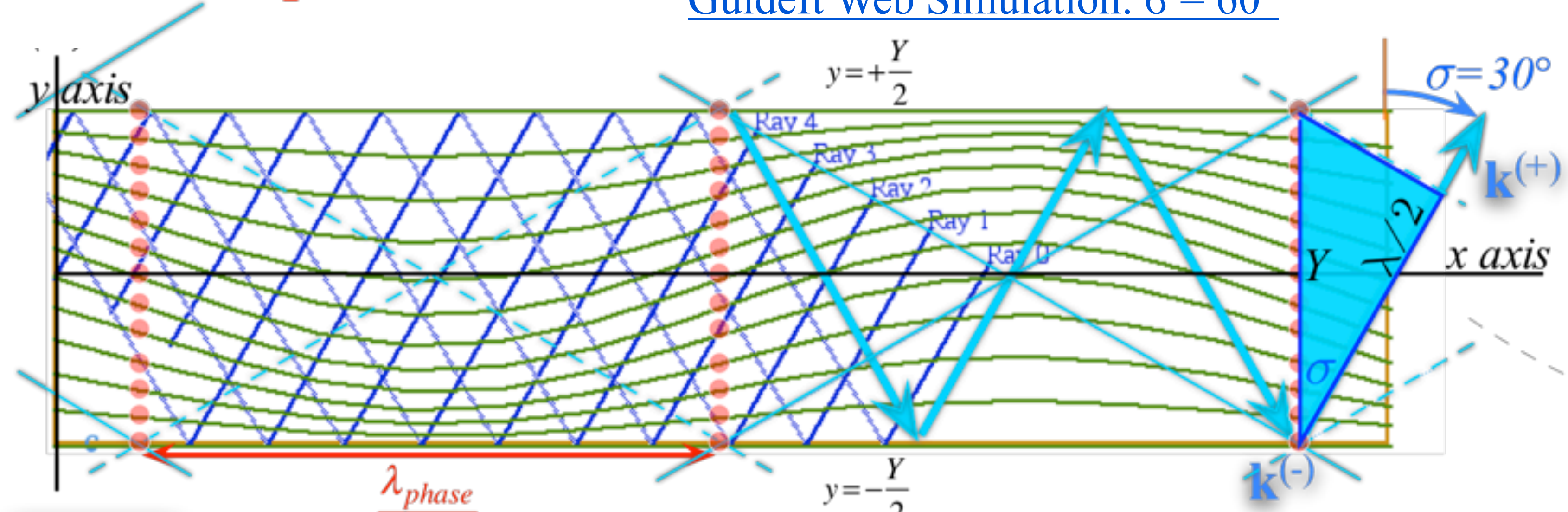
geometry applies to $(x,y)$ space-space
to $(k_x,k_y)$per-space-per-space
to $(x,ct)$ space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$ . (Low dispersion.)

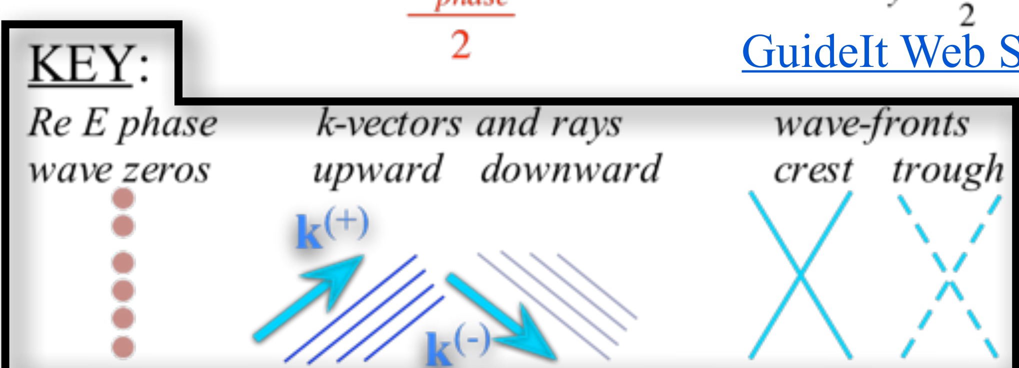


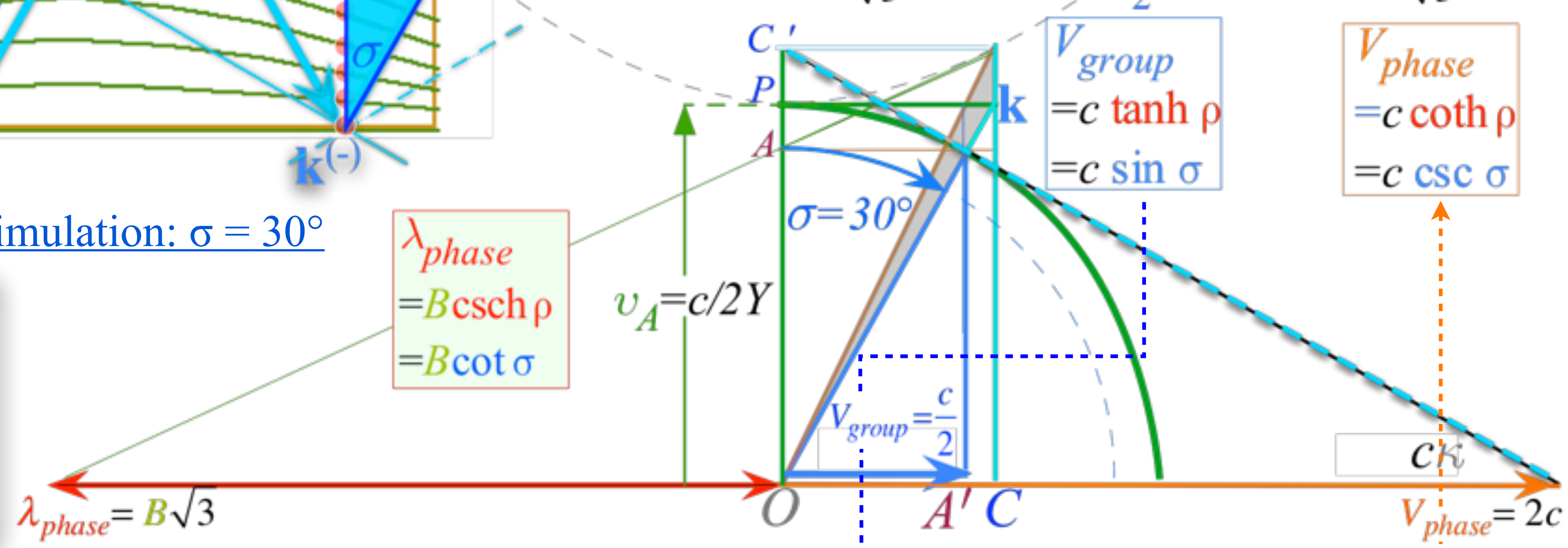

GuideIt Web Simulation: σ = 60°



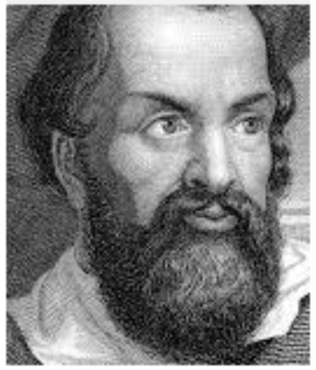

GuideIt Web Simulation: σ = 30°




Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$ . (High dispersion.)

*Galileo Galilei*



*1564-1642*

***Galileo's Revenge (part 1)***
*Rapidity adds just like*
*Galilean velocity*

***Galileo's Revenge (part 2)***
***Phasor angular velocity***
*adds just like*
*Galilean velocity*

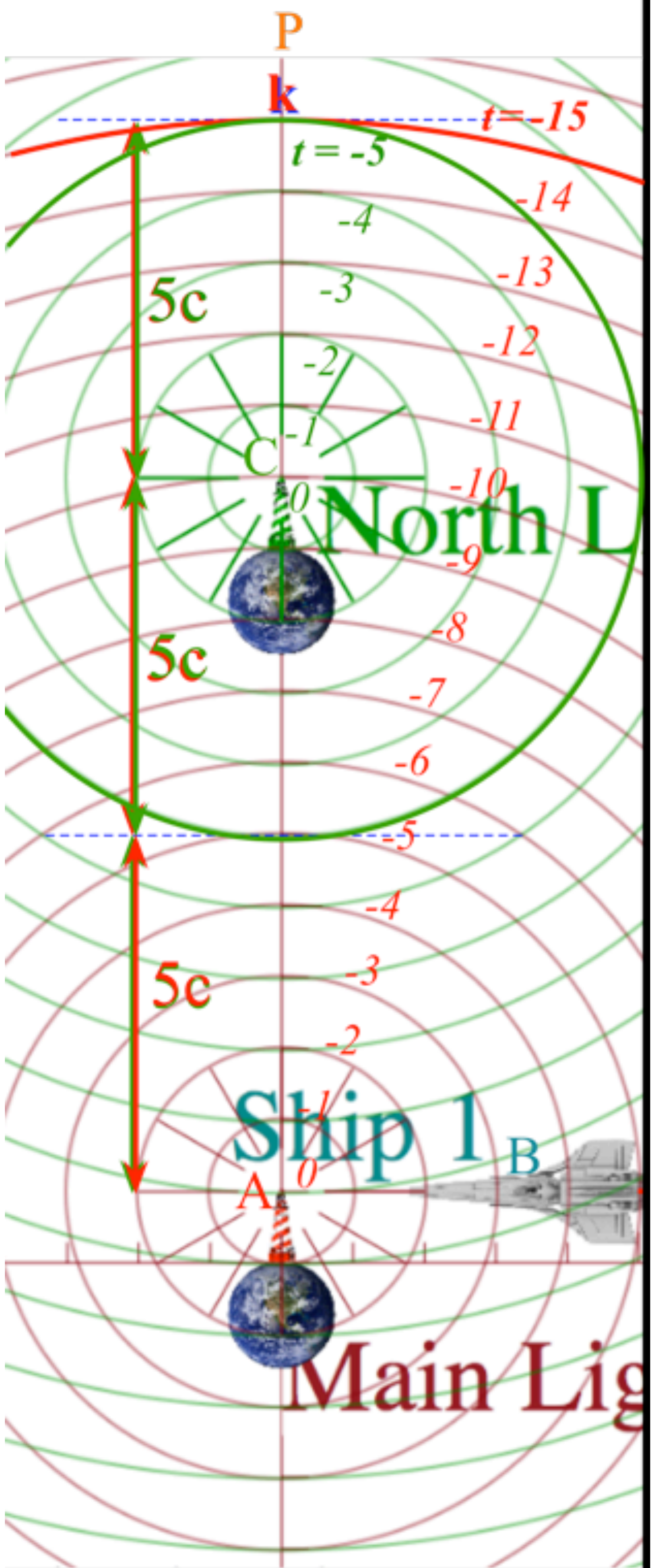
Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
Analyzing wave velocity by per-space-per-time *and* space-time graphs
16 coefficients of relativistic 2CW interference
Two "famous-name" coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation …and invariant hyperbolas
*Rapidity* $\rho$ related to *stellar aberration angle* $\sigma$ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
"Occams Sword" and geometry of 16 parameter functions of $\rho$ and $\sigma$
Application to TE-Waveguide modes and synchrotron beam relativity

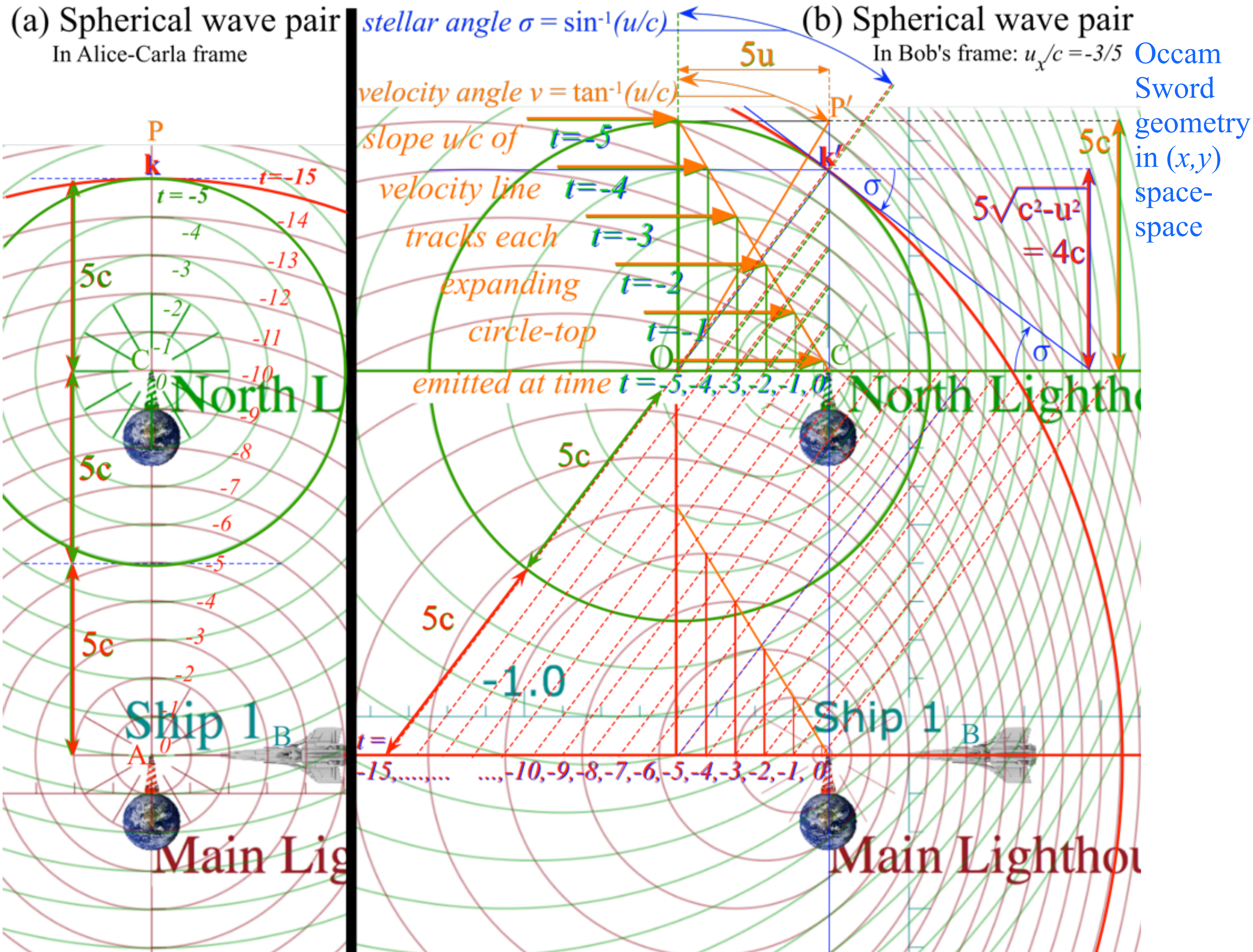
# Spherical wave relativistic geometry

Also, aided by Occam's Sword

(a) Spherical wave pair

In Alice-Carla frame

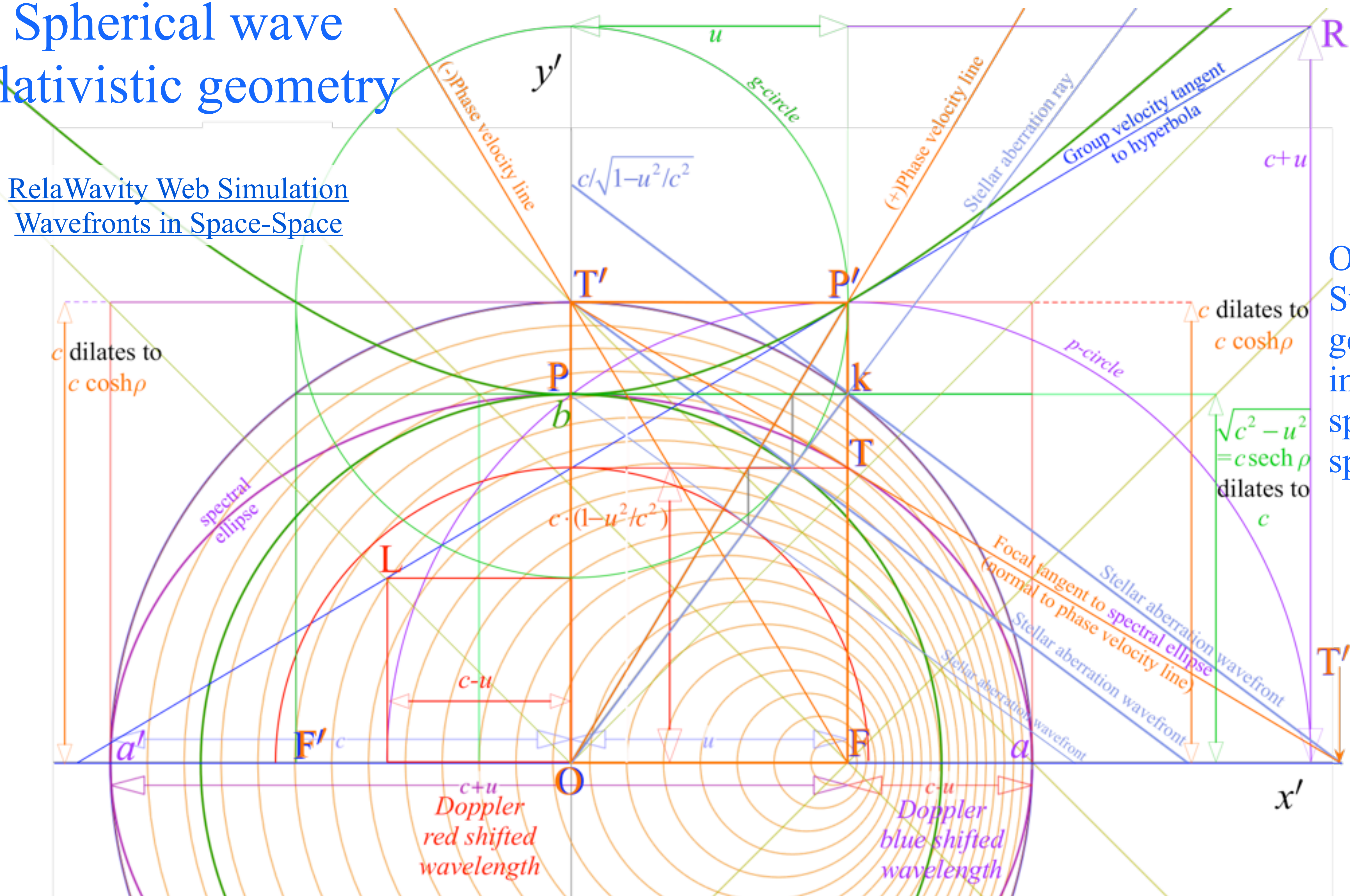




(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
Occam Sword geometry in $(x,y)$ space-space
stellar angle $\sigma = \sin^{-1}(u/c)$
5u
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of velocity line tracks each expanding circle-top emitted at time $t = -5, -4, -3, -2, -1, 0$
t=-5
t=-4
t=-3
t=-2
t=-1
P
k
P′
k′
t=-15
5c
$5\sqrt{c^2-u^2} = 4c$
σ
O
C
North Lighthouse
Ship 1
A
B
-1.0
t = -15,......, ...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0
Main Lighthouse

# Spherical wave relativistic geometry

[RelaWavity Web Simulation Wavefronts in Space-Space]




Occam Sword geometry in $(x,y)$ space-space

Doppler Red $\lambda$=c+u

dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius $a$=**OF**$a$=$c$

dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:

$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$

dilates to: $u\cosh\rho = c\sinh\rho$

ellipse latus radius **FT**=$c(1-u^2/c^2)$

dilates to: $c(1-u^2/c^2)\cosh\rho$

$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Doppler Blue $\lambda$=$c-u$

dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

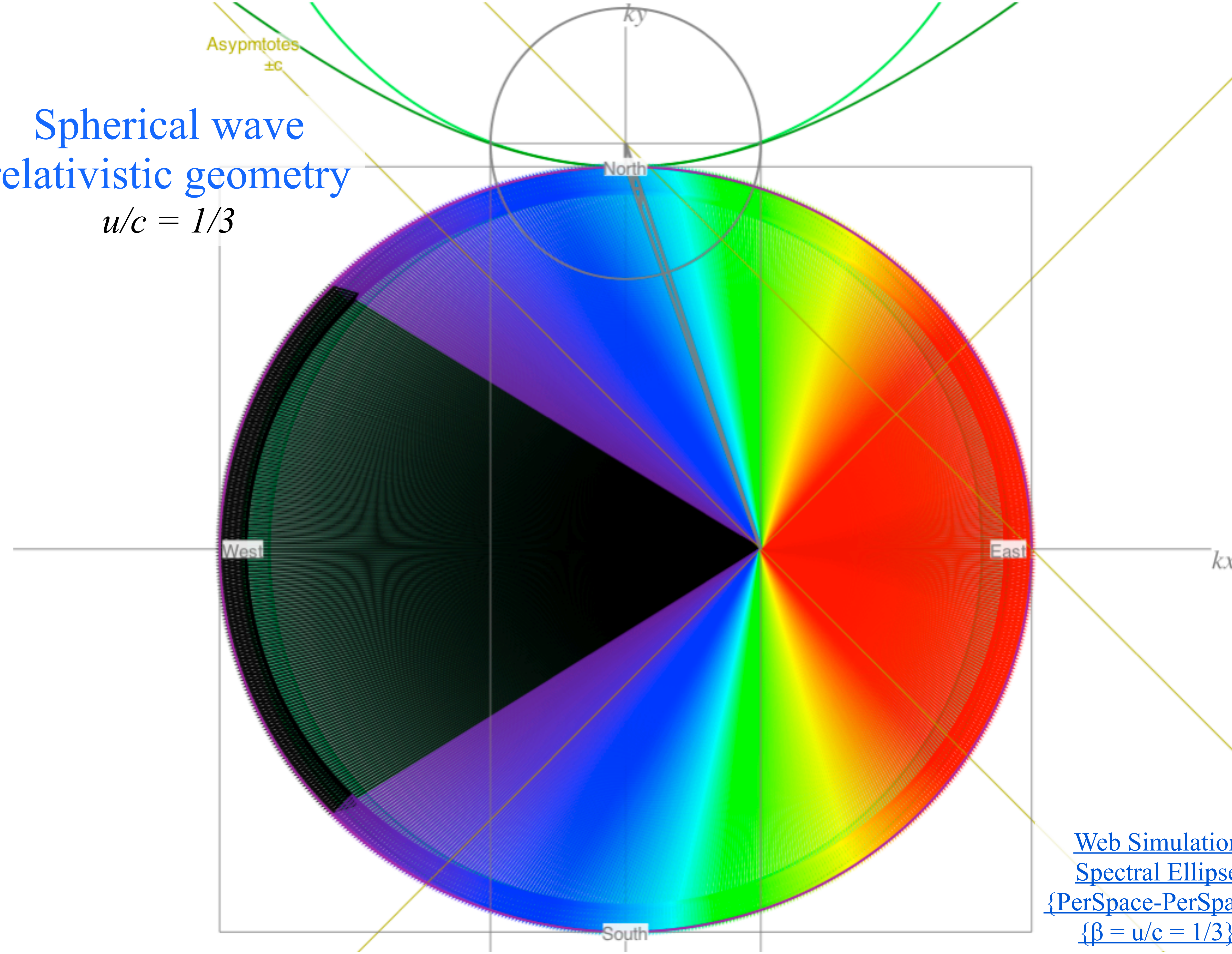
Base height **FTk**=$\sqrt{c^2-u^2}$

dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$

(equal to ellipse minor radius $b$)

# Spherical wave relativistic geometry

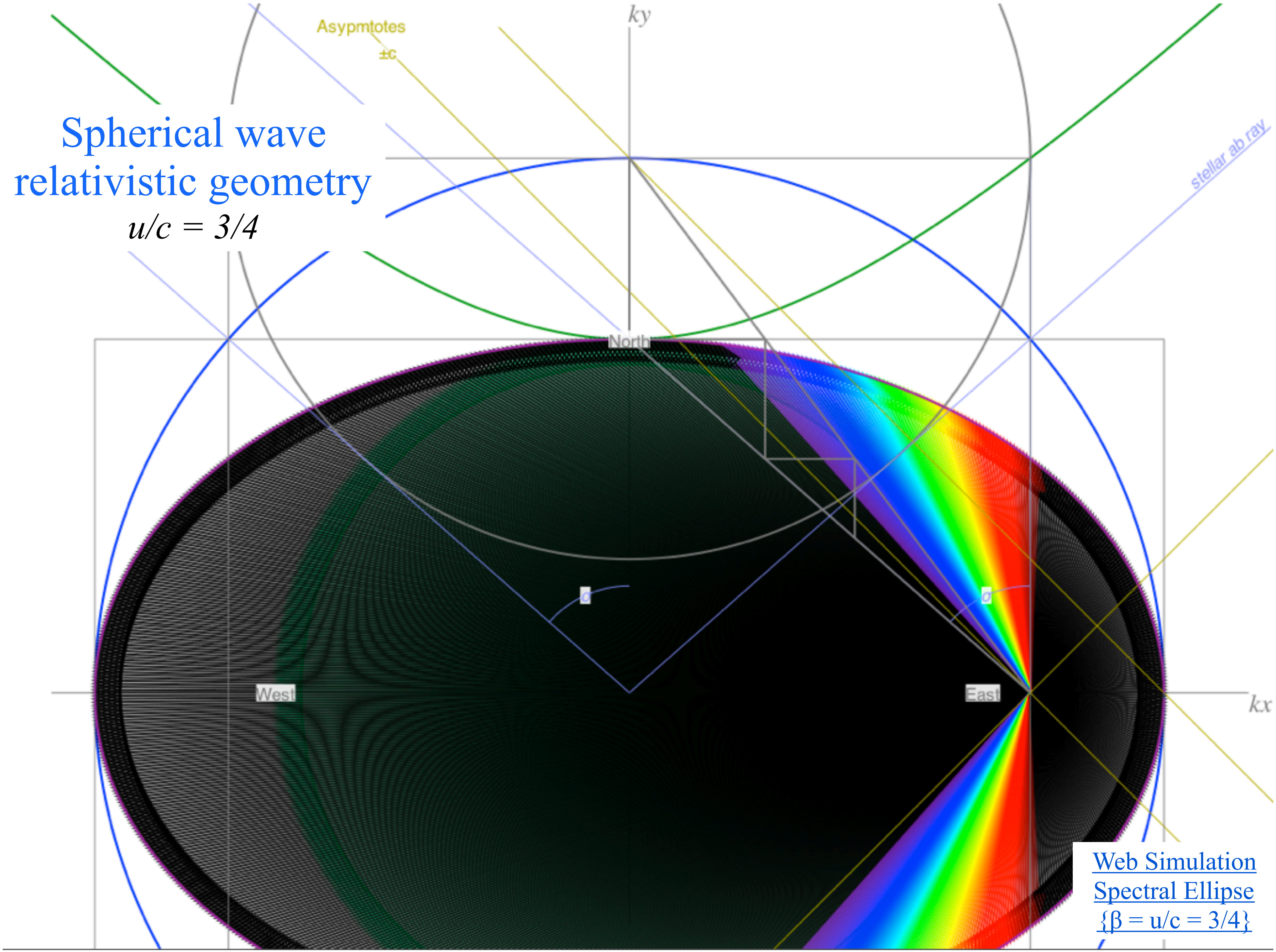
*u/c = 1/3*



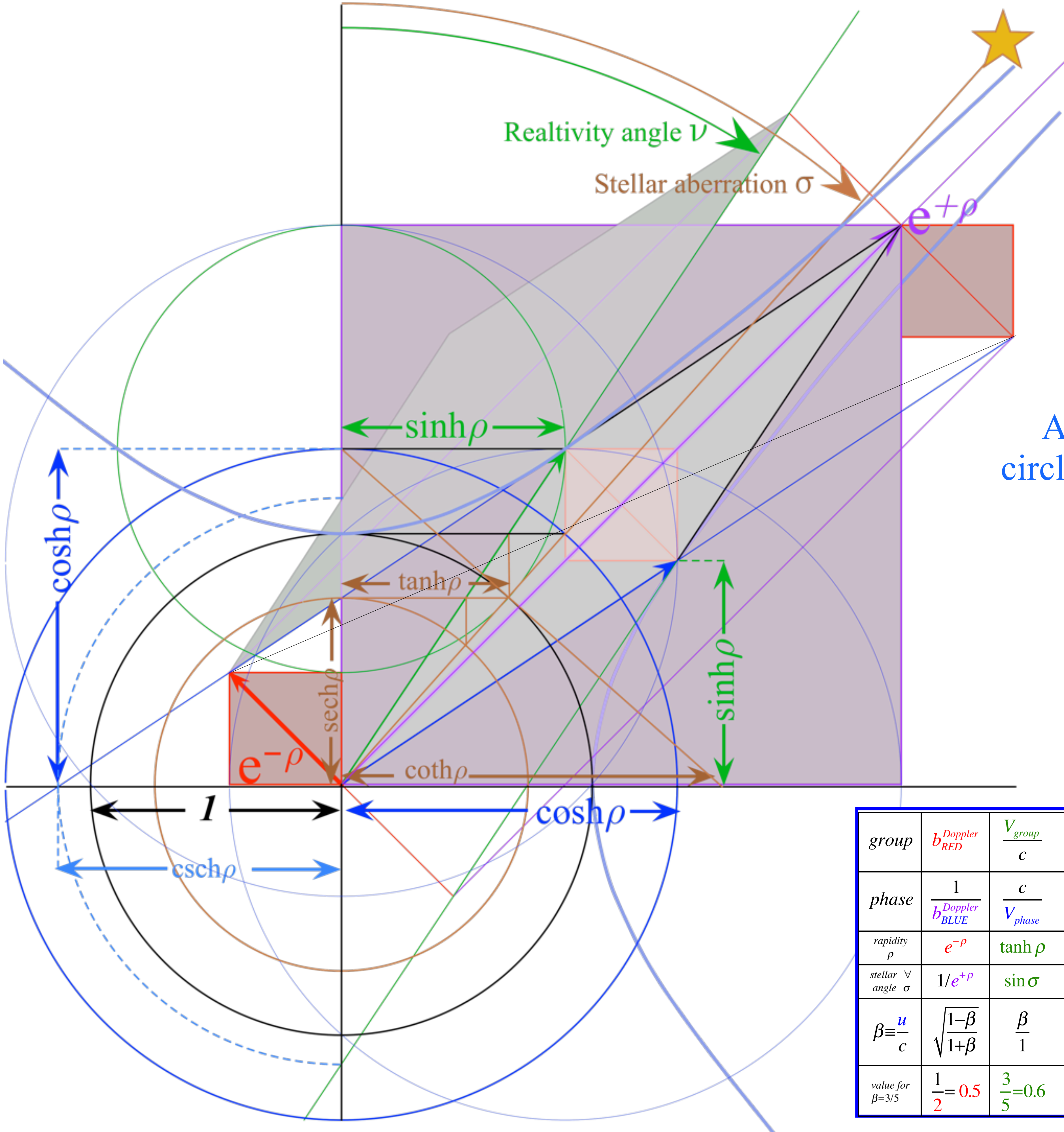

[Web Simulation
Spectral Ellipse
{PerSpace-PerSpace}
{β = u/c = 1/3}]

# Spherical wave relativistic geometry

*u/c = 3/4*




Web Simulation
Spectral Ellipse
{β = u/c = 3/4}




# A more compact circle-based geometry

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *group* | $b^{Doppler}_{RED}$ | $\frac{V_{group}}{c}$ | $\frac{\upsilon_{group}}{\upsilon_A}$ | $\frac{\lambda_{group}}{\lambda_A}$ | $\frac{\kappa_{group}}{\kappa_A}$ | $\frac{\tau_{group}}{\tau_A}$ | $\frac{c}{V_{group}}$ | $b^{Doppler}_{BLUE}$ |
| *phase* | $\frac{1}{b^{Doppler}_{BLUE}}$ | $\frac{c}{V_{phase}}$ | $\frac{\kappa_{phase}}{\kappa_A}$ | $\frac{\tau_{phase}}{\tau_A}$ | $\frac{\upsilon_{phase}}{\upsilon_A}$ | $\frac{\lambda_{phase}}{\lambda_A}$ | $\frac{V_{phase}}{c}$ | $\frac{1}{b^{Doppler}_{RED}}$ |
| *rapidity* $\rho$ | $e^{-\rho}$ | $\tanh\rho$ | $\sinh\rho$ | $\mathrm{sech}\,\rho$ | $\cosh\rho$ | $\mathrm{csch}\,\rho$ | $\coth\rho$ | $e^{+\rho}$ |
| *stellar angle* $\forall\sigma$ | $1/e^{+\rho}$ | $\sin\sigma$ | $\tan\sigma$ | $\cos\sigma$ | $\sec\sigma$ | $\cot\sigma$ | $\csc\sigma$ | $1/e^{-\rho}$ |
| $\beta \equiv \frac{u}{c}$ | $\sqrt{\frac{1-\beta}{1+\beta}}$ | $\frac{\beta}{1}$ | $\frac{1}{\sqrt{\beta^{-2}-1}}$ | $\frac{\sqrt{1-\beta^2}}{1}$ | $\frac{1}{\sqrt{1-\beta^2}}$ | $\frac{\sqrt{\beta^{-2}-1}}{1}$ | $\frac{1}{\beta}$ | $\sqrt{\frac{1+\beta}{1-\beta}}$ |
| *value for* $\beta=3/5$ | $\frac{1}{2}=0.5$ | $\frac{3}{5}=0.6$ | $\frac{3}{4}=0.75$ | $\frac{4}{5}=0.80$ | $\frac{5}{4}=1.25$ | $\frac{4}{3}=1.33$ | $\frac{5}{3}=1.67$ | $\frac{2}{1}=2.0$ |