Assignment 8 Oct 10 Due Tuesday Oct 17: Based on Unit 2 Chapter 1-3 and Unit 3 Chapter 1-3.

Well-known Coordinates (OCC) *NOTE: Save copy of solution to this Ex.***1** *for next Assignment 9*

1. Find Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric tensors g_{mn} and g^{mn} for OCC (a) and (b). Do (b) first. Then reduce to (a).

(a) Cylindrical coordinates $\{q^1=\rho, q^2=\phi, q^3=z\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$, $z=x^3$.

(b) Spherical coordinates: $\{q^1=r, q^2=\theta, q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\sin\phi$, $z=x^3=r\cos\theta$.

"Plopped" Parabolic Coordinates (GCC) *(In attached figure)*

2. Consider the GCC(Cartesian) definition: $q^1 = (x)^2 + y \quad q^2 = (y)^2 - x$

(a) Does an analytic Cartesian coordinate definition $x^j = x^j(q^m)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors at the point $(x=1, y=1)$ (Arrow) and at the point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there singularities of this GCC.

"Sliding" Parabolic Coordinates (GCC) *(In attached figure)*

3. Consider the Cartesian(GCC) definition: $x = 0.4\,(q^1)^2 - q^2\,,\quad y = q^1 - 0.4\,(q^2)^2$

(a) Does an analytic GCC coordinate definition $q^m = q^m(x^j)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors near point $(x=1, y=1)$ (Arrow) and near point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there singularities of this GCC.

4. *Covariant vs Contravariant Geometry* *(In attached figure)*

GCC components of a vector **V** in attached figure are realized by line segments OA, BV, etc. Give each segment length by single terms of the form V_m or V^m times $(\sqrt{g_{mm}})^{+1}$, $(\sqrt{g_{mm}})^{-1}$, $(\sqrt{g^{mm}})^{+1}$, or $(\sqrt{g^{mm}})^{-1}$ with the correct $m=1$ or 2. Also label each unitary vector as $\mathbf{E}_1$, $\mathbf{E}^1$, $\mathbf{E}_2$, or $\mathbf{E}^2$, whichever it is.

You should be able to do this quickly without looking at the text figures.

"Unprofessional" Paraboloidal Coordinates (GCC) *(In attached figure)*

5. The surface $z = f(x,y) = \frac{1}{2}x^2 + y^2$ (See *xyz*-plot) introduces 3D partial derivative chain rules. It is the $(q^3=0)$-surface in a 3D GCC coordinate grid $q^1=x,\ q^2=y,\ q^3=\frac{1}{2}x^2+y^2-z$. It contains a projection of an orthogonal (x,y) Cartesian coordinate grid on the surface that is obviously *not* orthogonal most places.

a. Derive the *3-by-3* Jacobian $J(x,y,z)$ and Kajobian $K(x,y,z)$ for $(q^3=0)$.

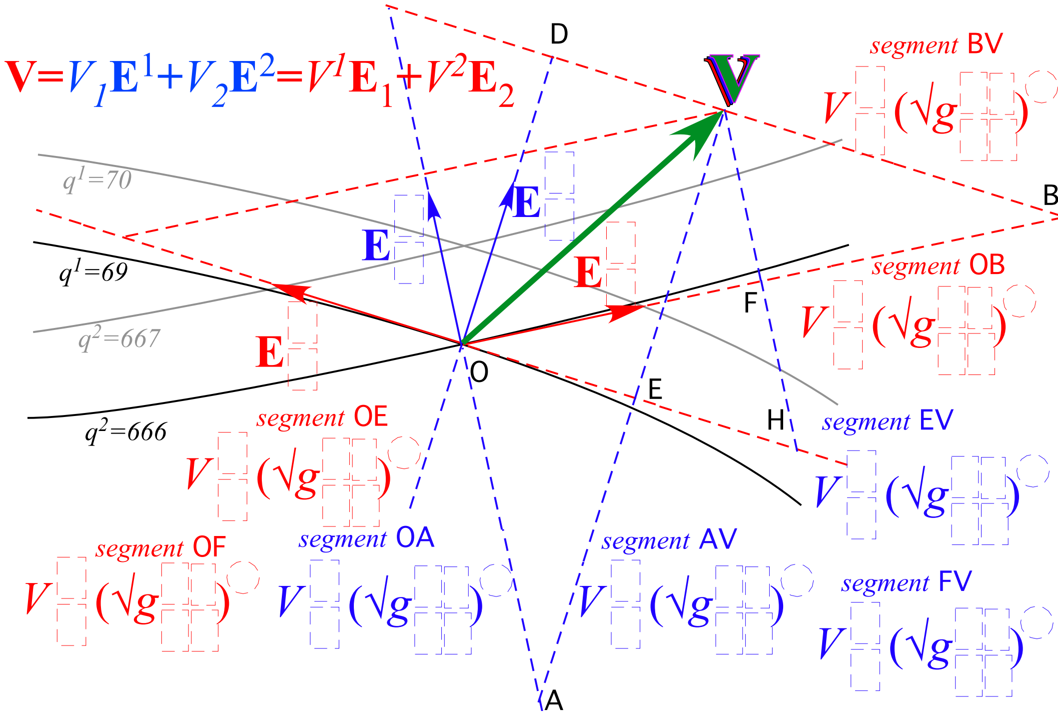
b. Extract covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ and contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ vectors represented in Cartesian (x,y,z) basis.

c. Derive the *3-by-3* covariant metric $g_{\upsilon\nu}(x,y)$ and contravariant metric $g^{\upsilon\nu}(x,y)$ for $(q^3=0)$ and tell which if any points on the surface have grids that are locally *orthogonal* and which if any are locally *orthonormal*.

*(Graph provided separately for Ex.***5***d and Ex.***5***e.*

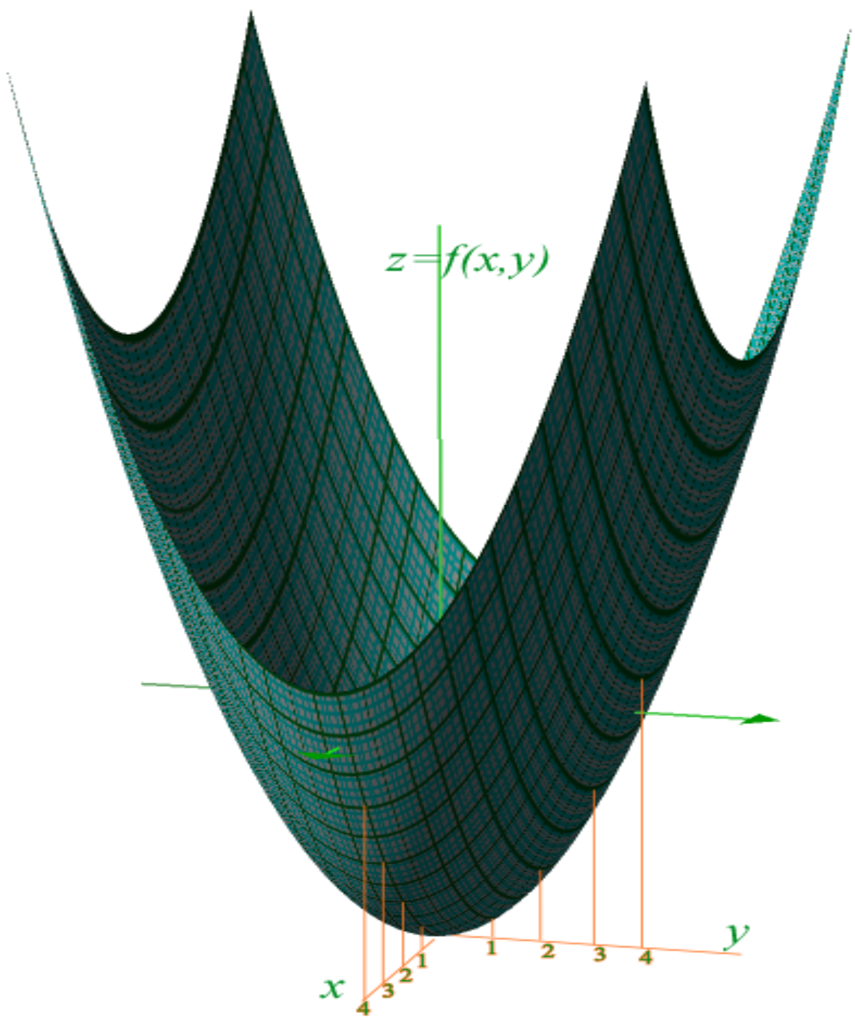
d. Calculate and sketch covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ on $(q^3=0)$ surface where $(x=4, y=-2)$ and where $(x=3, y=+2)$.

e. Calculate and sketch contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ on $(q^3=0)$ surface where $(x=4, y=+2)$ and where $(x=0, y=+4)$.

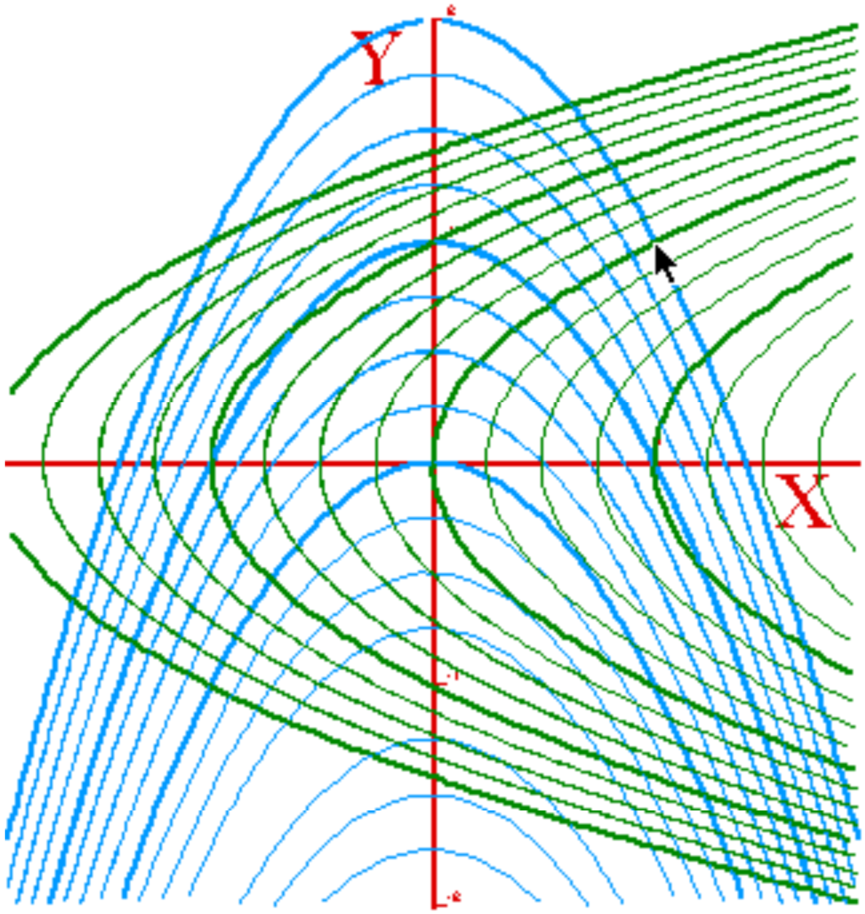


Above is 2D plot for *Ex*. **4**

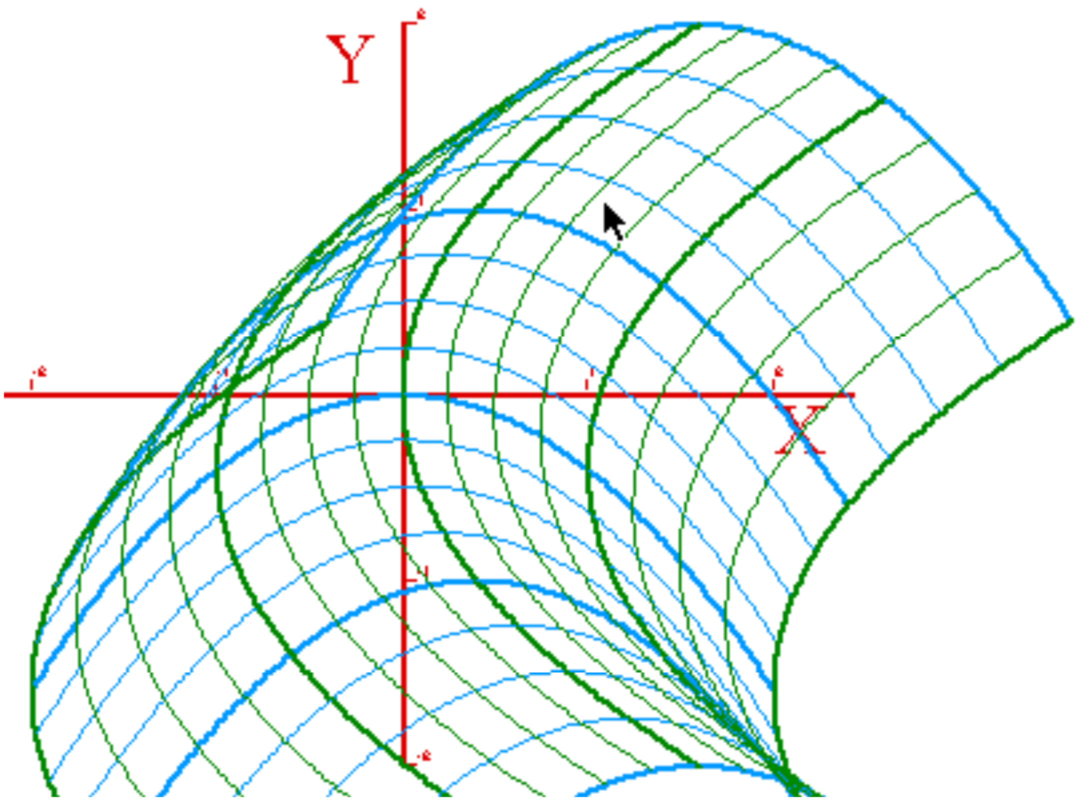
*Below is 3D (xyz) plot for Ex.***5***.*



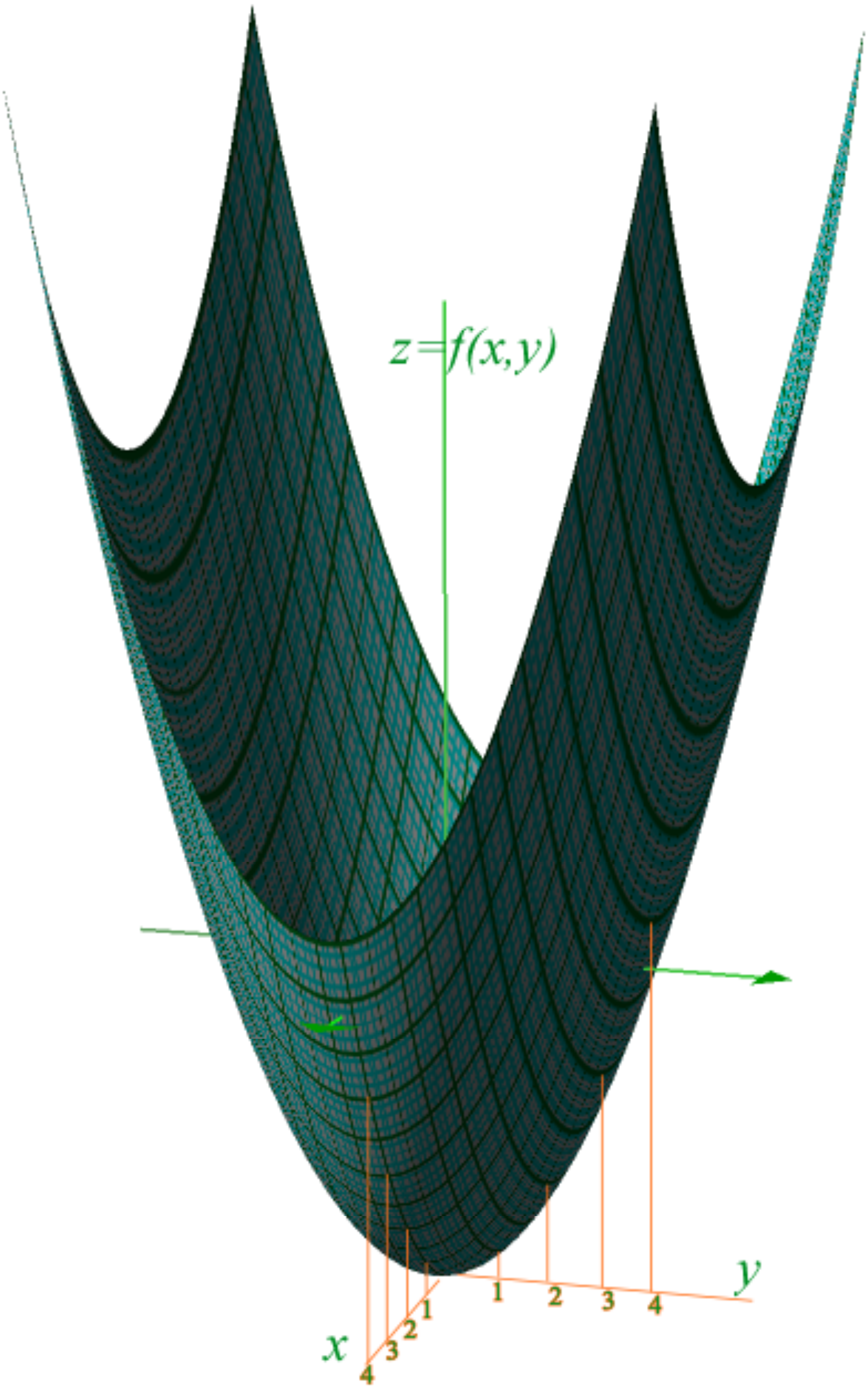
*"Plopped" and"Sliding" Parabolic Coordinates are 2D (xy) plots for Ex.***2** *and Ex.***3** *(despite 3D appearance of latter.)*



*"Plopped" Parabolic Coordinates for Ex.***2**



*"Sliding" Parabolic Coordinates for Ex.***3**



Assignment 8(contd.) - Ex. **5**
"Unprofessional" Paraboloidal Coordinates