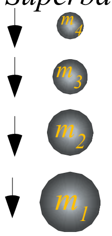
Assignment Set 4 - Read Unit 1 Ch. 3 thru Ch.8 Due *9/19/17* Name______________

The following is to acquaint you with of some lesser known properties of all-important parabolic PE functions

1. A most important mechanics problems is that of atomic oscillators affected by electric fields since it is basic to all spectroscopy. A useful approximate model is potential $V^{atom}(x)=k\,x^2/2$ function of center x of charge Q where k is a spring constant of atomic polarizability. A uniform electric field E is assumed to apply a force $F=Q\cdot E$ to the charge by adding a potential $V^E(x)$ to $V^{atom}(x)$. (Give $V^E(x)$=______ and $F^E(x)$=_____)
Consider the resulting potential $V^{total}(x)$ for an atom for unit constants $k=1$ and $Q=1$. Derive and plot the new values for equilibrium position $x^{equil}(E)$, energy $V^{equil}(E)$, dipole moment $p^{equil}(E)=Q\cdot x^{equil}$. Plot $V^{total}(x)$ for field values of $E=-3,-2,-1,\ 0,\ 1,\ 2,$ *and* 3. Does oscillation frequency $\omega^{equil}(E)$ vary with field E? If so, how?

Superball tower IBM model constructions (Independent Bang Model with initial $V_k=-1$*)*



The 100% energy transfer limit (IBM values are $v_1^{IN}=1$ and $-1=v_2^{IN}=v_3^{IN}=v_4^{IN}=\ldots$ *after 1st floor bang.)*

2. Suppose each m_k has just the right mass ratio m_k/m_{k+1} with the m_{k+1} above it to pass on all its energy to m_{k+1} so the top ball-N, a *1gm* pellet, goes off with the total energy. Construct velocity-velocity diagrams, indicate velocity at each stage, and derive the required intermediate mass values for (a) $N=2$, (b) $N=3$, (c) $N=4$.
(d) Give algebraic formula for this *Maximum Amplified Velocity* factor in terms of N *(MAV(N)=*___________*?).*
(e) Give algebraic formula neighbor-mass ratios $R=M_{N-1}/M_N$ in terms of N *(R(N)=*_________*?).*

N-Ball tower ∞-limits

3. Suppose each m_k is very much larger than m_{k+1} above it so that final v_{k+1} approaches its upper limit. Then top m_N goes off with nearly the highest velocity v_N attainable. Construct the velocity-velocity diagrams. Indicate each intermediate velocity limit value at each stage and the limiting top value for (a) $N=2$, (b) $N=3$, (c) $N=4$.
(d) Give algebraic formula for *Absolute Maximum Amplified Velocity* factor in terms of N *(AMAV(N)=*_______*?).*

The optimal idler (An algebra/calculus vs. *geometry problem)*

4.(a) To get highest final v_3 of mass m_3 find optimum mass m_2 in terms of masses m_1 and m_3 that will do that.
(b) Consider this problem in Galileo-shifted frame with: $v_1^{IN}=2$ and $0=v_2^{IN}=v_3^{IN}$ (Algebra simplifies for this.)
(c) Do V-V Graph† for case m_1=4 and m_3=1 (where m_2=____?) …for non-optimal case m_1=4, m_2=3, m_3=1.
(d) Give formula for optimal top mass final velocity in terms of m_1, m_2, and m_3 and compare to result of 4(a).

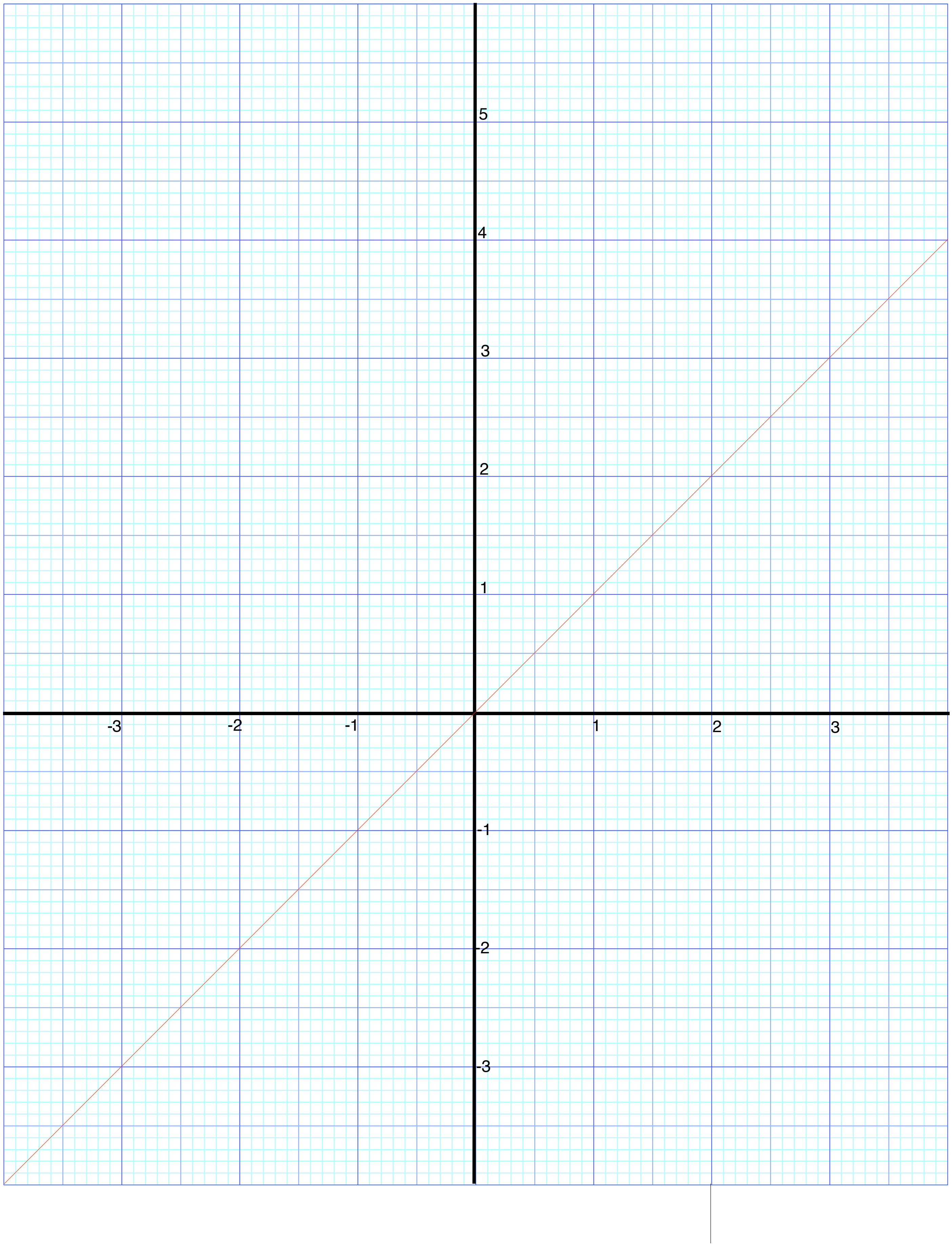
Rocket science: The backsides of exponentials

5. Compare discrete-blast rocketry in eq.(8.7) or Fig. 8.8 with continuous-blast "rocket science" of eq.(8.8) and study logarithmic-exponential geometry of the latter.
(a) In particular, at what point are blasted exhaust particles not being sent *backwards* but end up going in the *same* direction as the rocket in the initial (lab) frame where the rocket starts out with zero velocity? Compare discrete case in Fig. 8.8 with continuous limit derived by eq.(8.8).

The following is to acquaint you with of some lesser known properties of exponentials and logarithms

(b) Do plots of exponential $y=e^x$ and $y=\log_e x$ functions on the same graph and draw any tangent-triangle whose hypotenuse is tangent to one of the curves and intercepts the x or y axis at integers -2, -1, 0, 1, 2,..
(c) As a roller-coaster car moves down a track $y=e^x$ it shines one laser beam along the track and another beam vertically down so both makes spots on baseline $y=0$. Find the distance between spots as function of x.

† See Fig. 1.8.1(b) p.103 of Unit 1 in *CMwithBANG!*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