Aprox. constants for exam purposes: $M_\oplus = 6{\cdot}10^{24}\,kg$, $R_\oplus = 6{\cdot}10^{6}\,m$, $G = \frac{2}{3}{\cdot}10^{-10}\,Nm^2kg^{-2}$, $c = 3{\cdot}10^{8}\,ms^{-1}$

Plotting graph copies provided. Use multiple copies if necessary. Most problems are solved more easily by geometry.

EXAM SHEET 1:First do this exam with closed book in 2 hours. EXAM SHEET 2:Re-do this exam open source no time limit.

Both parts of this Assignment 15 due by Thursday Dec.14

Io returns (With a vengeance!)

1. Suppose *Io* volcano **L** ejects lava at such a speed $v_{IN\,45°}^{hit\,90°}$ that ejecta with initial angle $\alpha=45°$ have range ρ=90° to hit equatorial target $\mathbf{T}_{90°}$. (On graph of *Io* assume mass $M_\oplus=6{\cdot}10^{24}kg$ and radius $R_\oplus=6{\cdot}10^{6}m$ near to that of Earth. Use $R_\oplus$ as a geometric unit. This *Io* has an $R_\oplus$-circular-orbit of period $t_\oplus$ =_____min.)

a. First find and label "focus-locus" line for fixed $\alpha=45°$ and varying v_{IN} and locate 2nd focal point (or points) corresponding to pole-to-equator flight.

b. Construct this $\mathbf{T}_{90°}$ orbit while indicating major & minor axes, foci, and enough points to show its shape. Plot *KE/PE* ratio *R*-scale and show eccentricity vector ε and give its magnitude $|\varepsilon|=\varepsilon=$_____.

c. From this estimate its initial *KE/PE* ratio *R*=_____ and initial $v_{IN\,45°}^{Hit\,90°}=$ _____ *km/s*.

d. Estimate the time to impact $t_{\mathbf{L}\text{-to-}\mathbf{T}_{90°}}$ ($\alpha=45°$)=___minutes. *(Derive and discuss geometrically using Kepler laws.)*

e. Construct and label the "focus-locus" circle for fixed $v_0=v_{IN\,45°}^{Hit\,90°}$ and varying α. Does another initial path with the same $v_0=v_{IN\,?°}^{Hit\,90°}$ but different angle α=_____? also hit $\mathbf{T}_{90°}$? If so find or construct its orbital geometry (major & minor axes, foci, and ε), value of *KE/PE* ratio *R*=_____ , and time to impact $t_{\mathbf{L}\text{-to-}\mathbf{T}_{90°}}$ =_______.

f. What v_0-trajectory has minimum time to impact $t_{MIN\text{-}\mathbf{L}\text{-to-}\mathbf{T}_{90°}}$(Give α_{MIN}=_____ ° and $t_{MIN\text{-}\mathbf{L}\text{-to-}\mathbf{T}_{90°}}$=___min.)

More difficult problems:

g. Construct an envelope contacting all trajectories with the <u>same</u> initial speed v_0 but various launch angles α. Indicate the contact points of the envelope with ellipse(s) drawn so far.

h. Tell if targets $\mathbf{T}_{30°}$__?, $\mathbf{T}_{60°}$__?, $\mathbf{T}_{120°}$__?, $\mathbf{T}_{150°}$__?, $\mathbf{T}_{180°}$__?, are also within range. *(Yes, No, or maybe)*

i. Can missiles launched at $\alpha=45°$ with higher $v_{IN}>v_0$ reach beyond $\mathbf{T}_{90°}$ to $\mathbf{T}_{120°}$_?, to $\mathbf{T}_{180°}$_?, beyond $\mathbf{T}_{180°}$_?

j. D*iscuss* what ellipse, parabola, or hyperbola limits $\alpha=45°$-path(s) to a maximum range $(\rho_{max})°$= _______(what?)

k. Construct points of that maximal trajectory visible on the graph (include path below-$R_\oplus=6{\cdot}10^6m$) while we now imagine actual mass of *Io* has been compressed to a Schwarzchild black hole of radius $r_{\oplus S}$ at **C**.

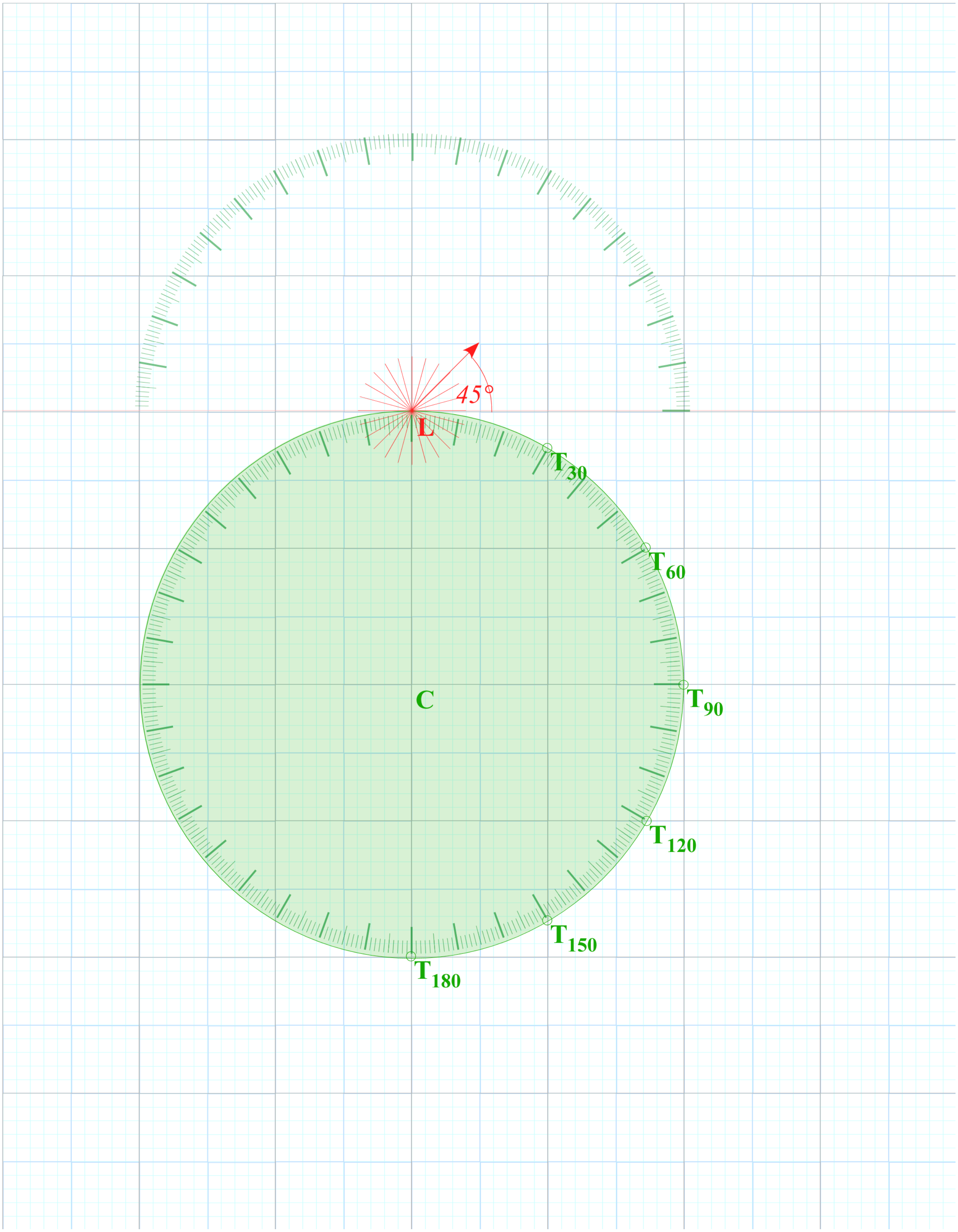
(*Xtra cred:* If instead it is a neutron starlet plowing thru *Io*, would it deviate above or below the compressed *Io*-path? Discuss)

l. About how big is a Schwarzchild-limited Earth radius? $r_{\oplus S}$=______cm. Plot $r_{\oplus S}$ if possible.

Still more difficult problems:

m. Find $v_{IN}<v_0$-trajectory of minimum *KE/PE* ratio R_{MIN} to hit $\mathbf{T}_{90°}$(Give α_{MINR}=___° , ε=___, and R_{MIN}___°) Sketch or construct orbit. Derive algebraic formulae if possible.

n. Give radius r_R of circular fixed-*R* "focus-locus" in terms of initial radius r_o and ratio *R*. r_R =_____________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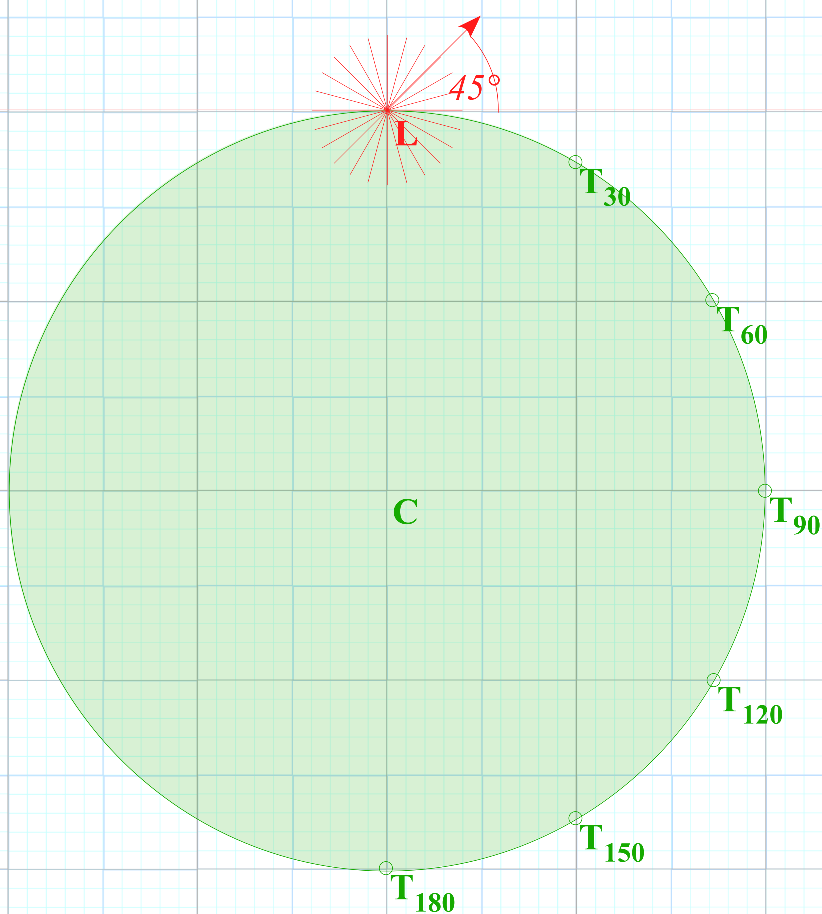
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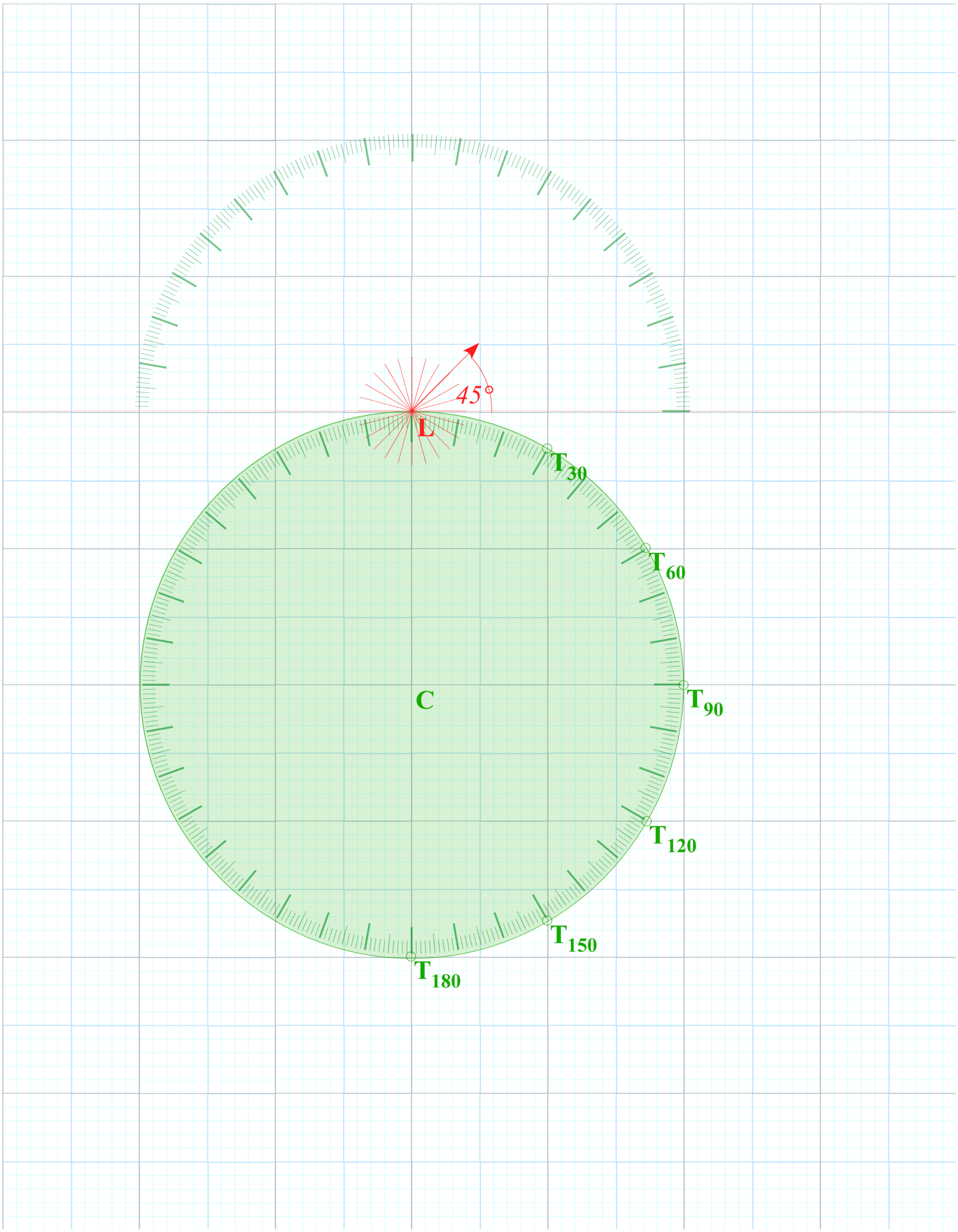
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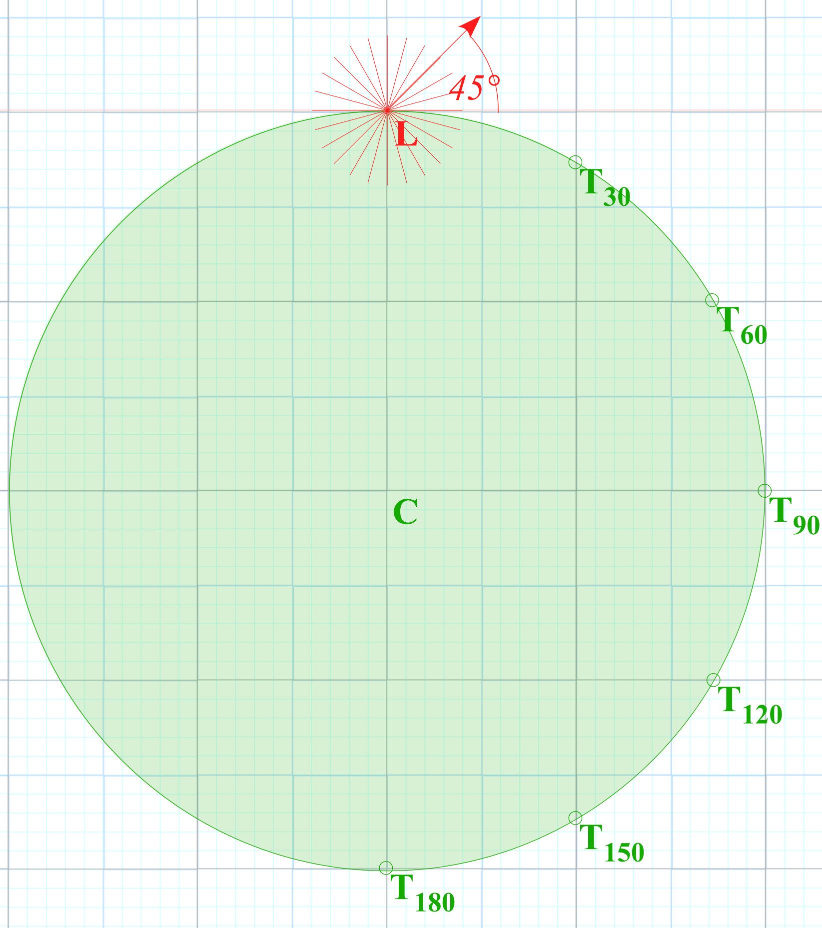
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