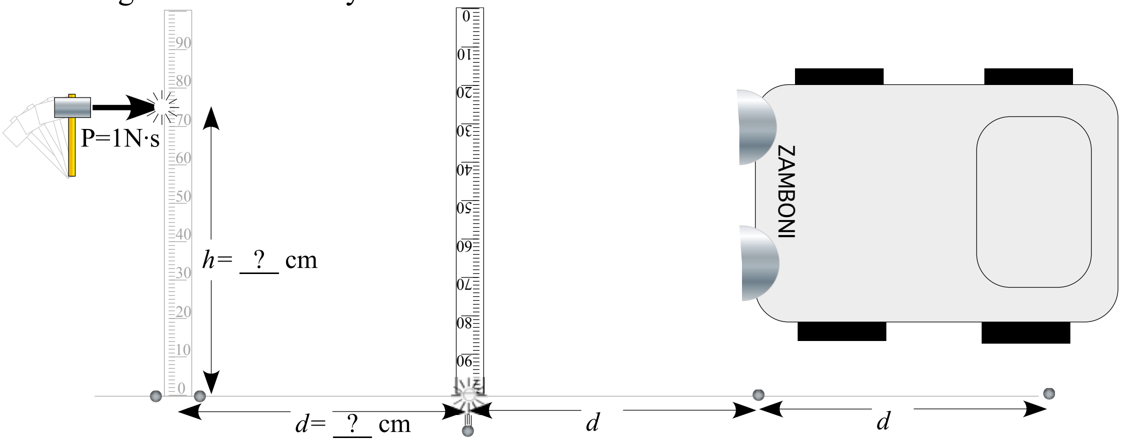
Assignment 11 - **Classical Mechanics 5103** 11/07/167 Due Tue Nov. 14

Main Reading: In *Class. Mech. with a BANG!* Unit 2 thru 2.9. Unit 3 thru 3.8. - Lect.19 p.73-74 & p.102-110

An icy cycloid problem

Ex.1 (a) A *1kg*. meter stick lies on a smooth icy hockey rink surface with two marbles sitting at its end on either side of the 0.0cm mark. (See figure) A hammer give impulse $\mathbf{P}=(1\text{N}\cdot\text{s})\mathbf{e_x}$ to the stick at the *h*-cm. mark.
What height *h* is *least* likely to disturb the marbles.



(b) Now assume *h*-value from (a) and friction-free "icy" surface. At what distances *d, 2d, 3d, …* along *x*-axis should the *3rd, 4th, 5th*,…marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time Δt interval snapshots of the stick as it flips by 180° and then to 360°. What is Δt for a 1kg stick?

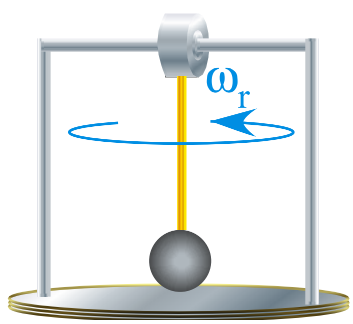
Electromagnetic cycloids

Ex.2 A unit mass $m=1$ *kg* and charge $Q=1$ *Coul.* (Dangerous!) starts at $(x=0=y)$ on a frictionless (x,y)-surface in vertical Earth gravity (Say $g_y=-10m/s^2$) and in a strong *z*-axial magnetic field $\mathbf{B}_z=(0,0,B_z)$ normal to surface.

(a) What field B_z (in *Tesla*) causes the mass with zero initial velocity $(v_x(0),v_y(0))=(0,0)$ to follow a cycloid of 0.5 meter radius along -*x* axis? What *x*-axis points does it hit? Are these hit points different for different $\mathbf{v}(0)$?

(b) What initial $\mathbf{v}(0)$ would cause the mass to fly a straight line along the -*x*-axis? … along the +*x*-axis?

(c) Describe and plot the resulting trajectory if instead the mass is thrown down with $(v_x(0),v_y(0))=(0,-2m/s)$.



Pendulum on turntable (Soft-mode resonance)

Ex.3 Suppose a pendulum supported by a circular ball bearing may swing without friction in the vertical plane of the bearing. The bearing plane is secured to a turntable that rotates at a constant angular frequency ω_r. The pendulum consists of a mass *m* at the end of a rod of length $\ell=1m$ and negligible mass with natural frequency of small θ-angle motion at zero-ω_r in gravity acceleration (Say $g=10m/s^2$) given by $\omega_0(\omega_r=0)$=___.

(a) Derive the Lagrangian and Hamiltonian using spherical coordinates in the rotating frame.

(b) Derive the θ-equilibrium points and small-oscillation frequency as a function of the frequency ω_r and ω_0. Overlay plots of effective θ-potential for several key values of ω_r. What ω_r value makes $\theta=0$ angle unstable?