Lecture 4
Thur. 9.01.2015

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations *High mass ratio $M_1/m_2 = 49$*

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)$=const. $\ln(y)$

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]

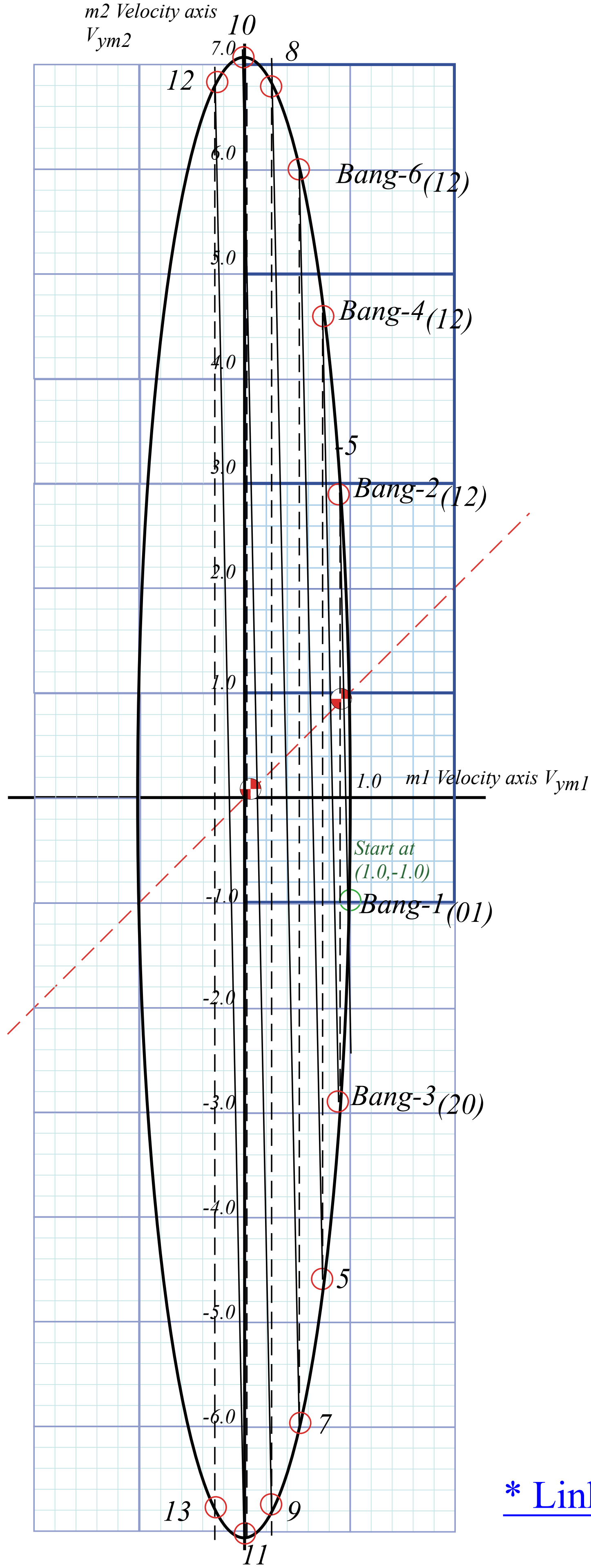
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015)]

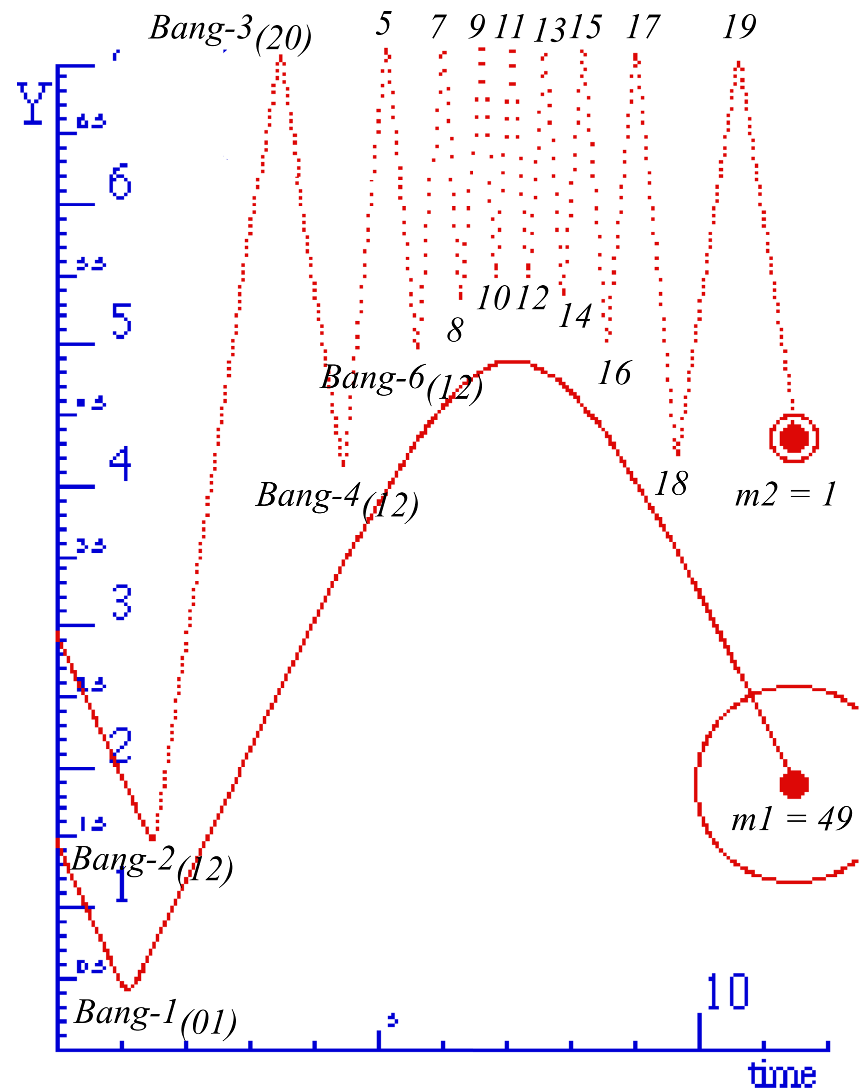
Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

→ *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1



Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Fig. 5.1
in Unit 1

* Link to BounceIt: $Y_i(t)$ animation

* Link to BounceIt: V_{y2} vs V_{y1} animation

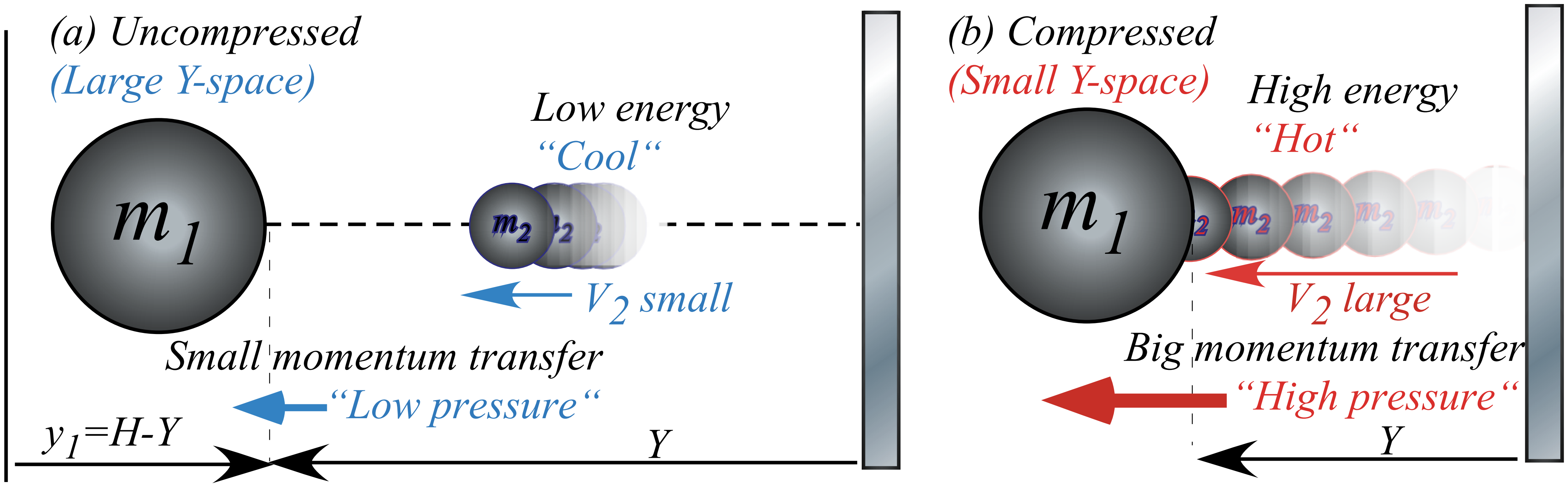
Force "field" or "pressure" due to many small bounces

→ *Force defined as momentum transfer rate*

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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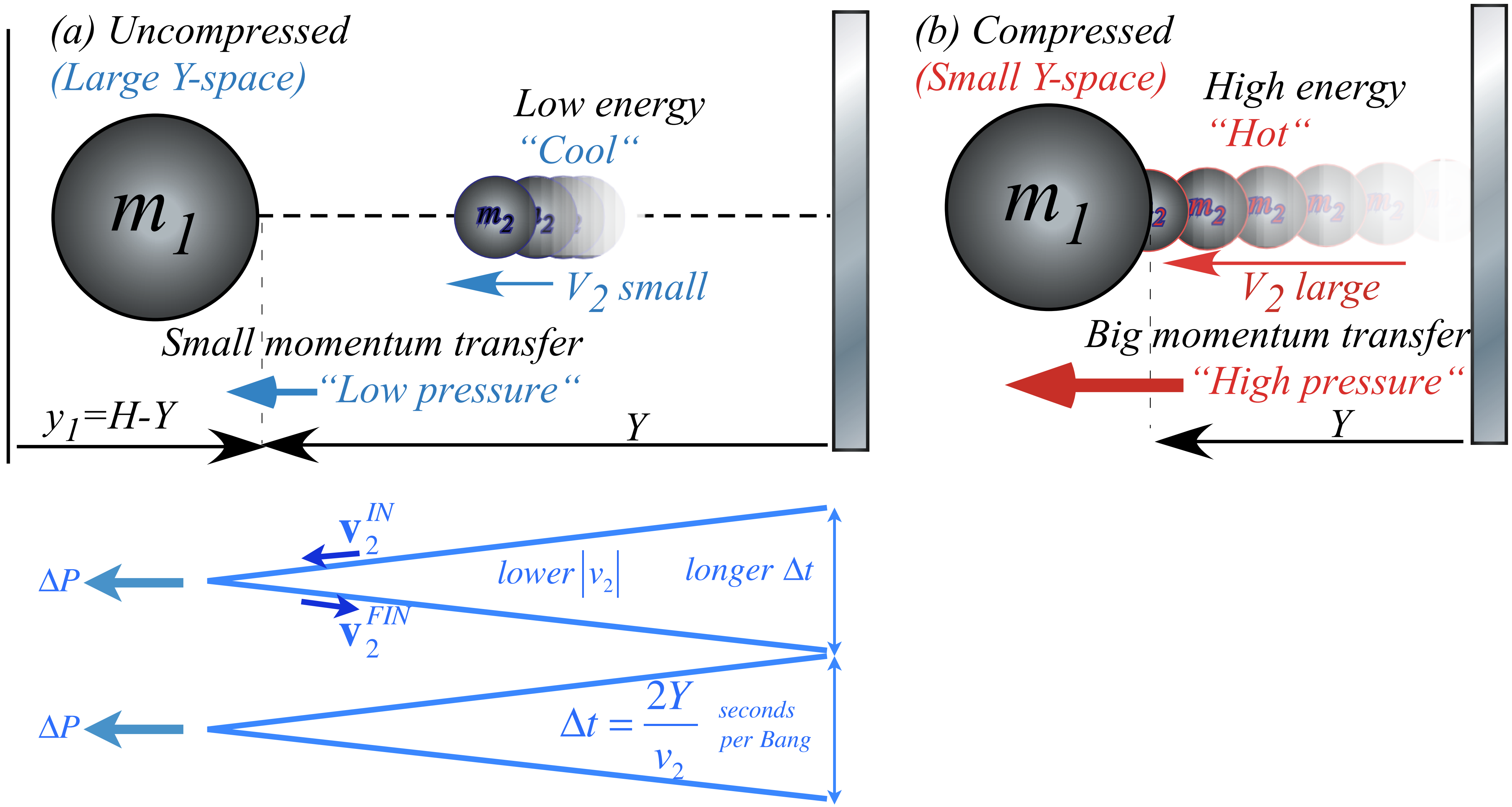

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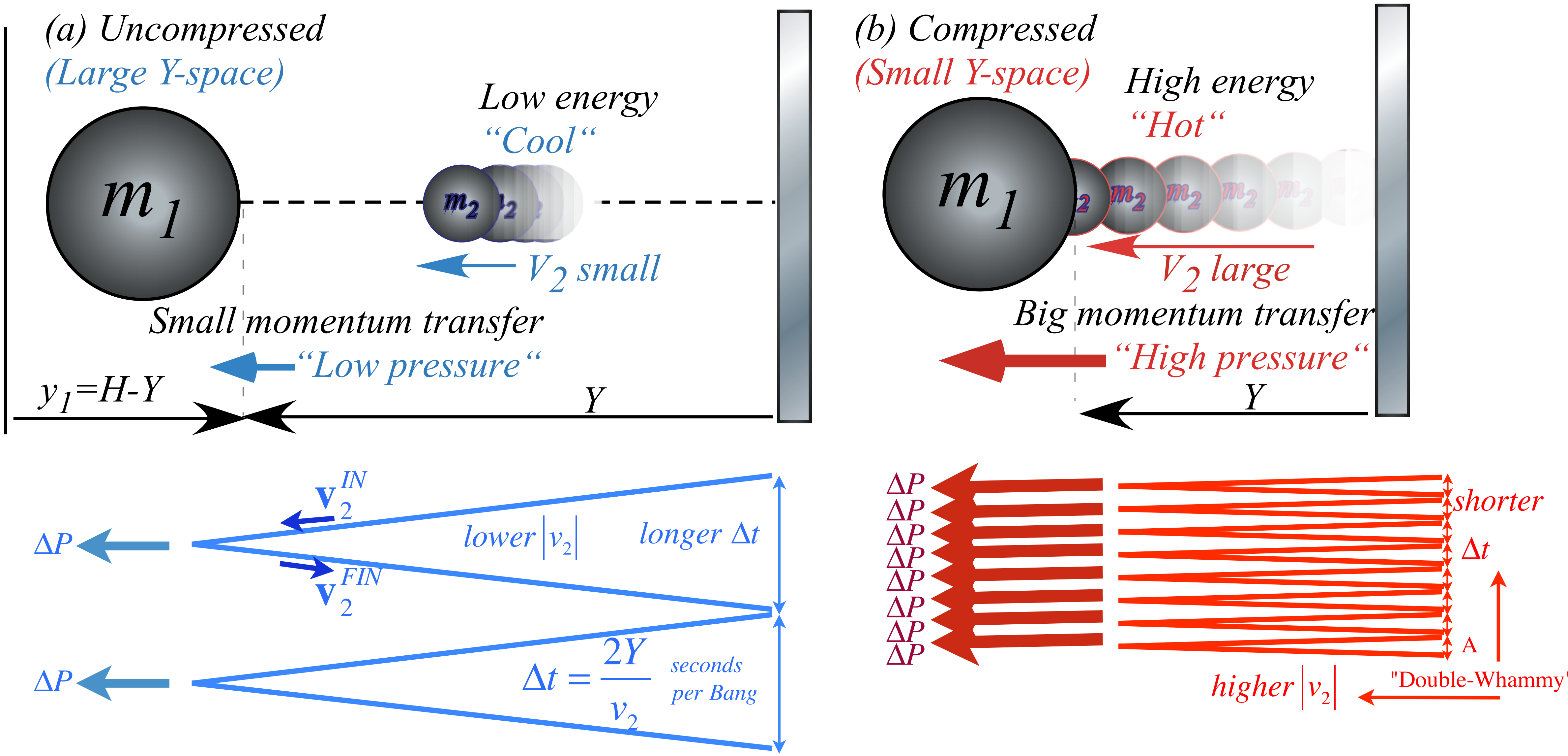

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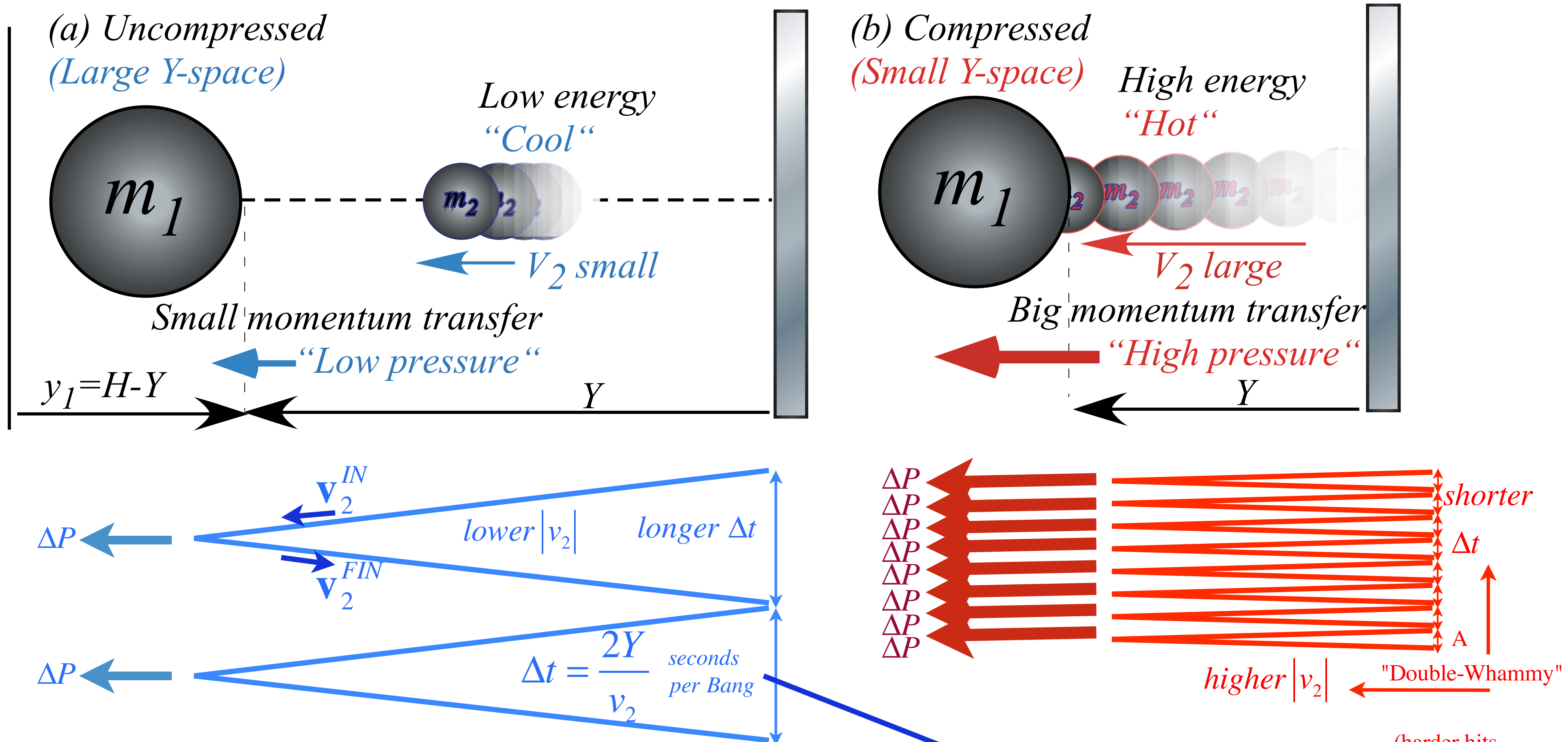


$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

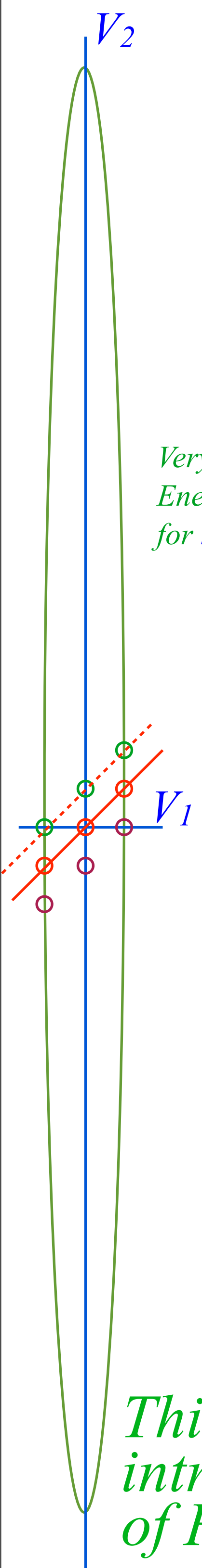
This introduction of Force...

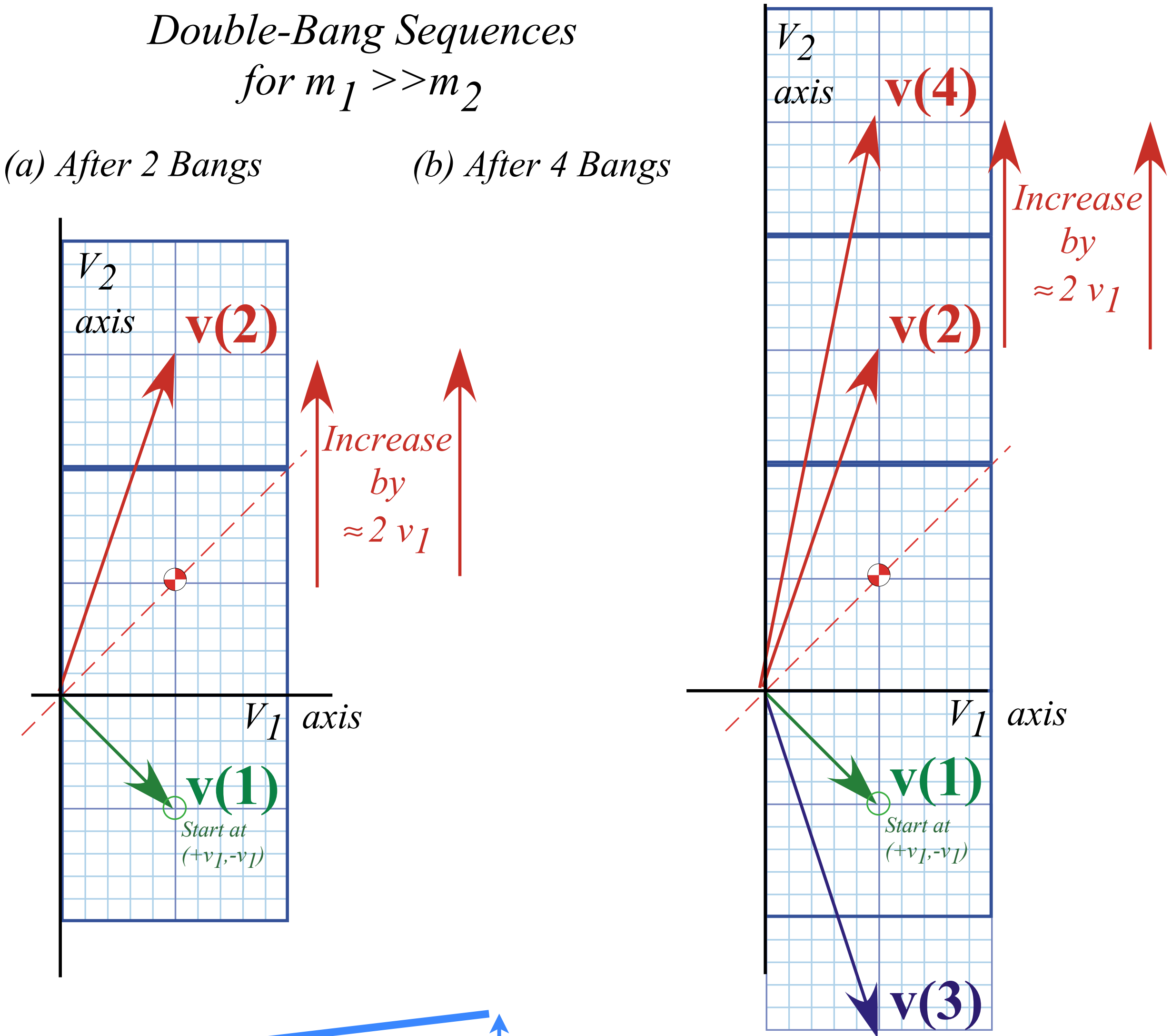
Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

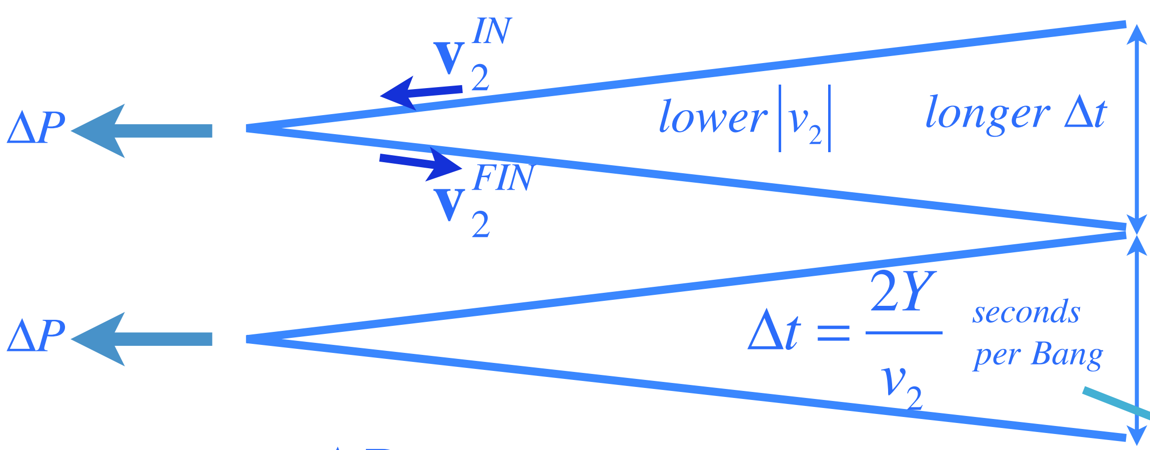
Double-Bang Sequences for $m_1 >> m_2$

(a) After 2 Bangs (b) After 4 Bangs





Unit 1
Fig. 6.2



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

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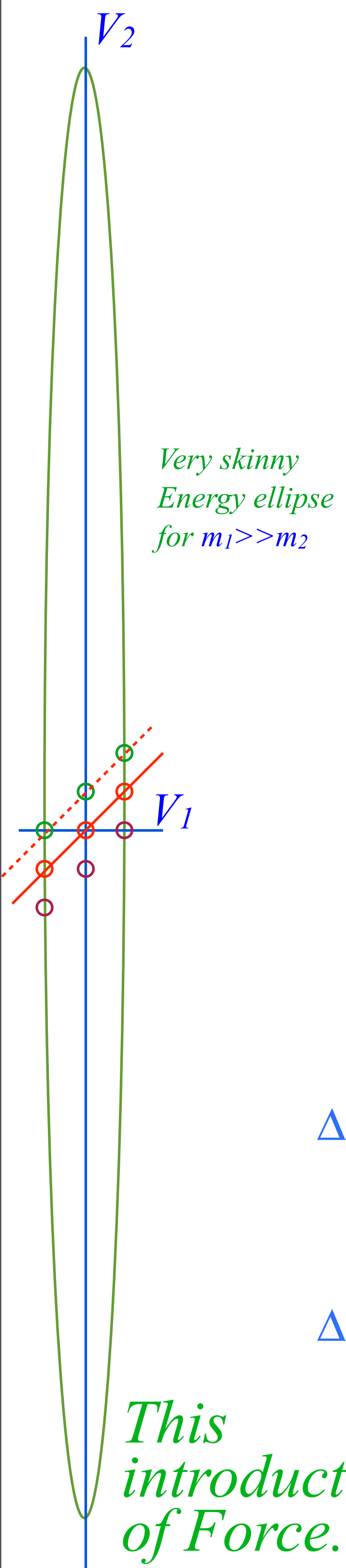

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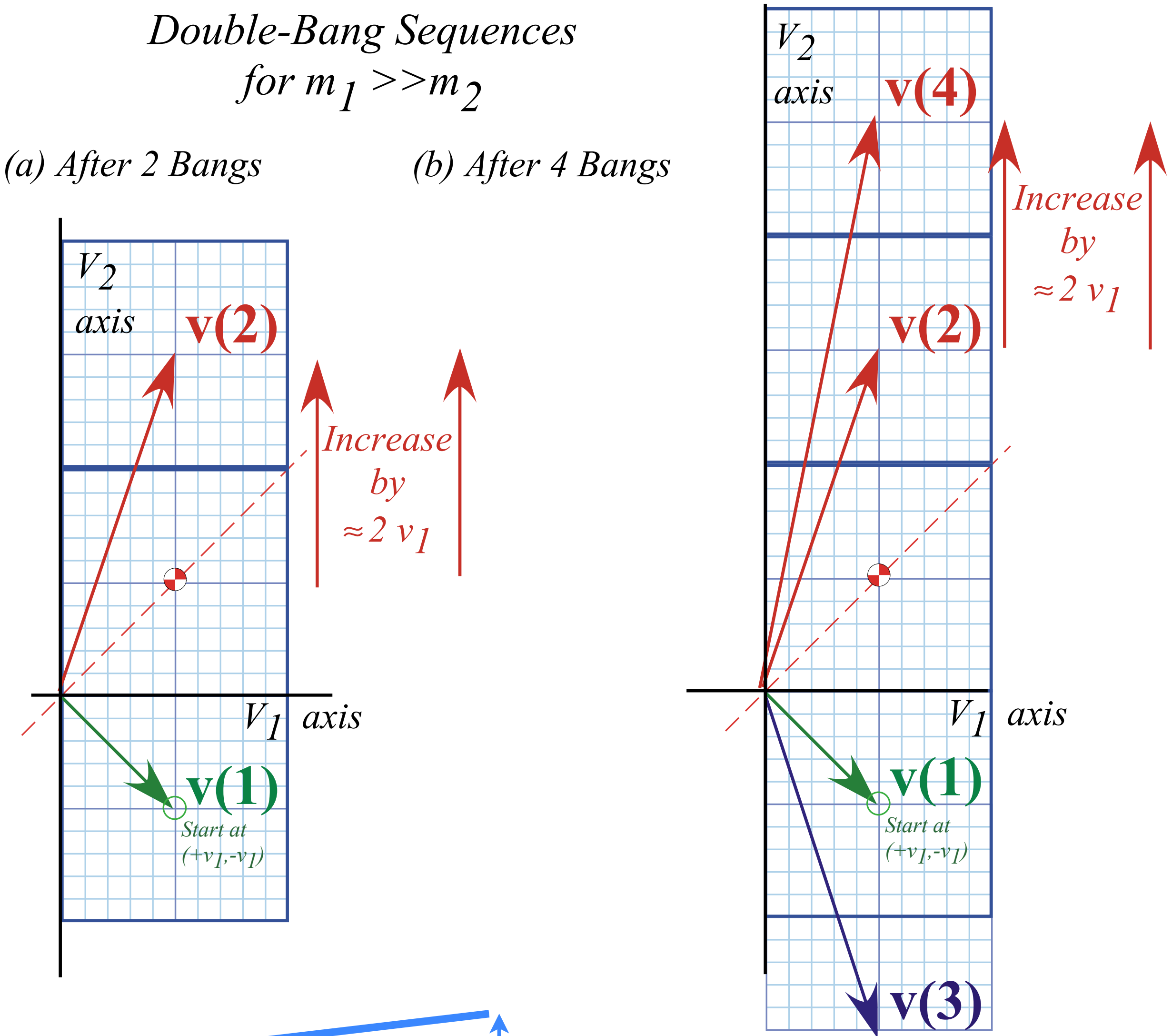

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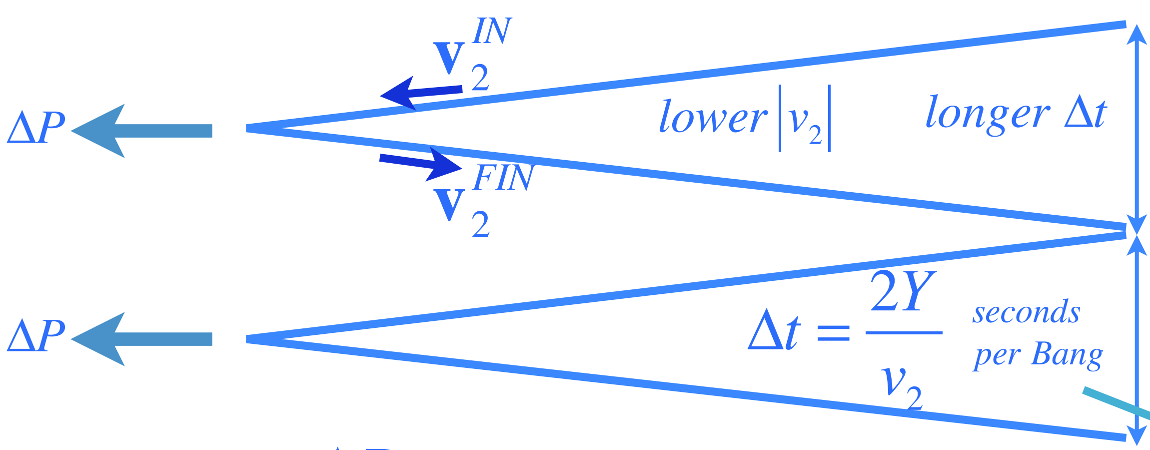

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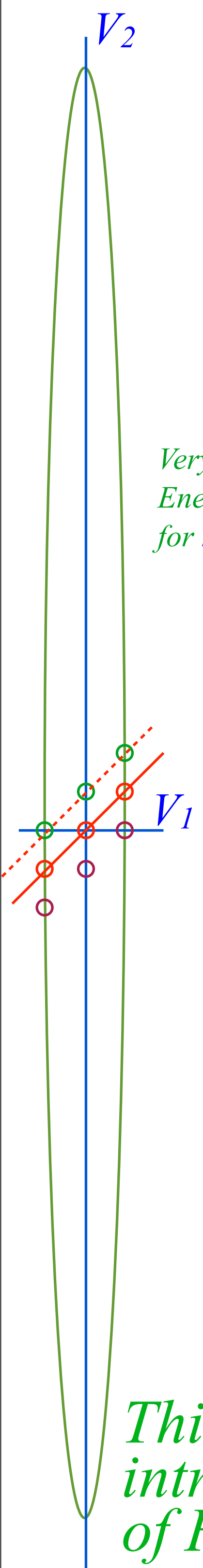
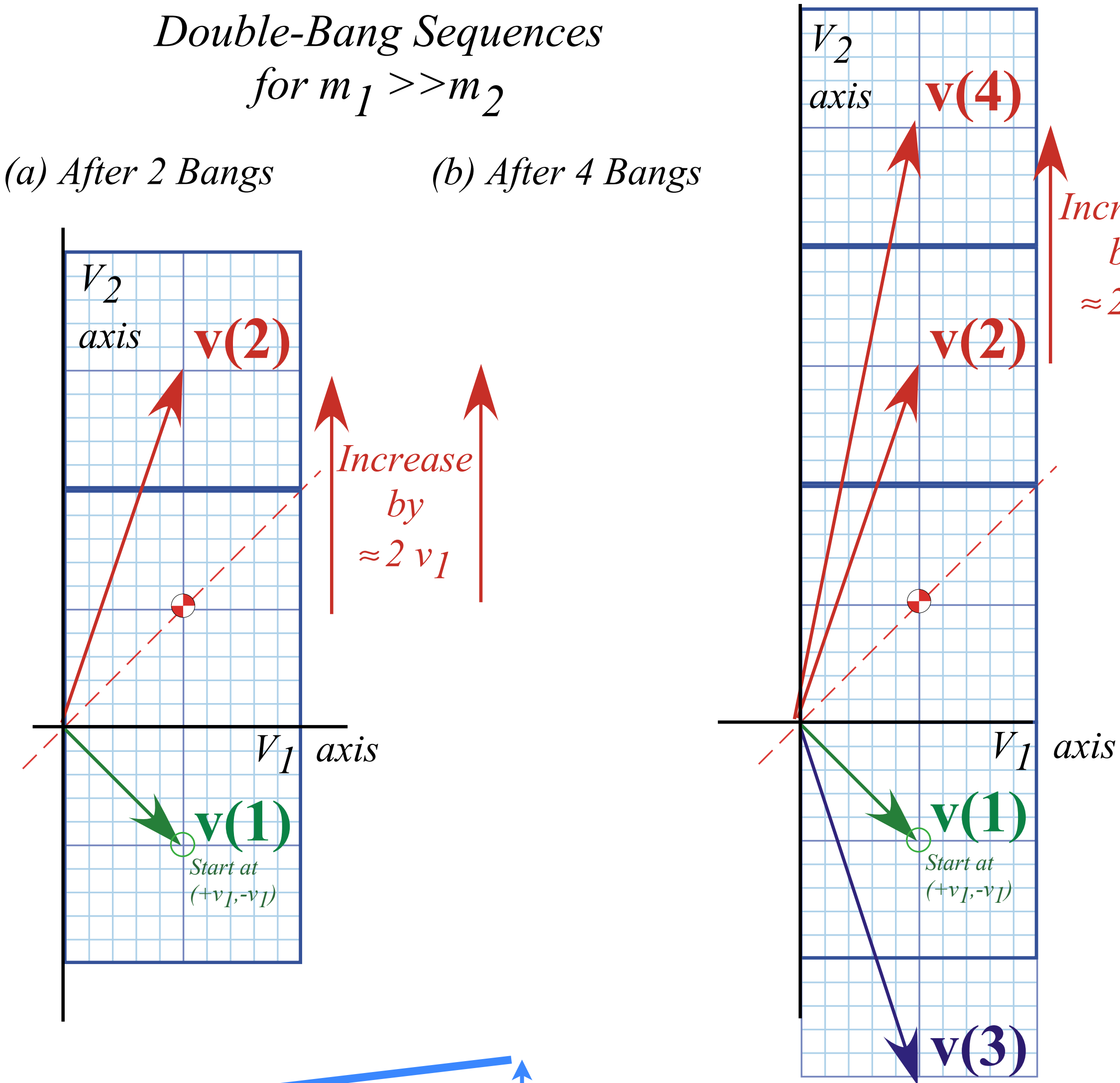


Very skinny Energy ellipse for $m_1>>m_2$



Unit 1
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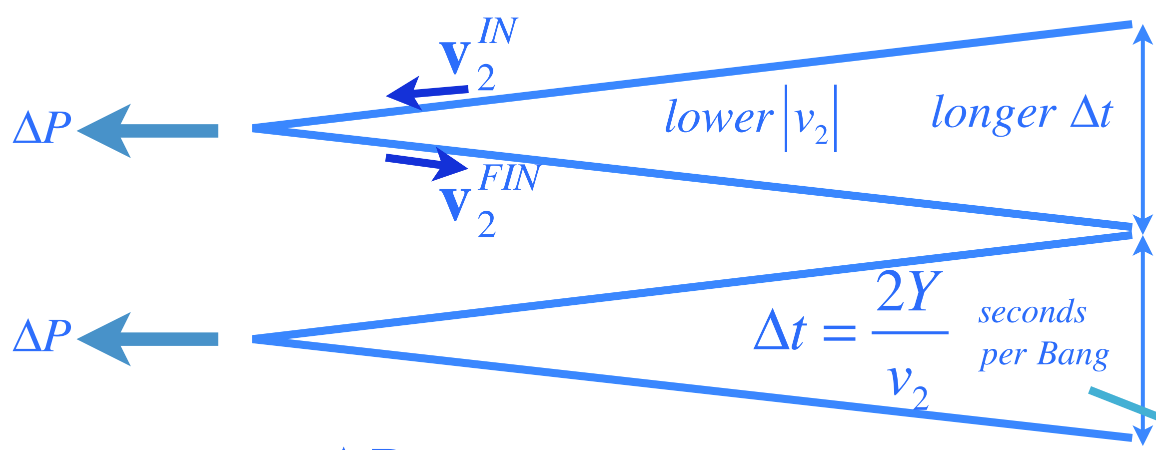

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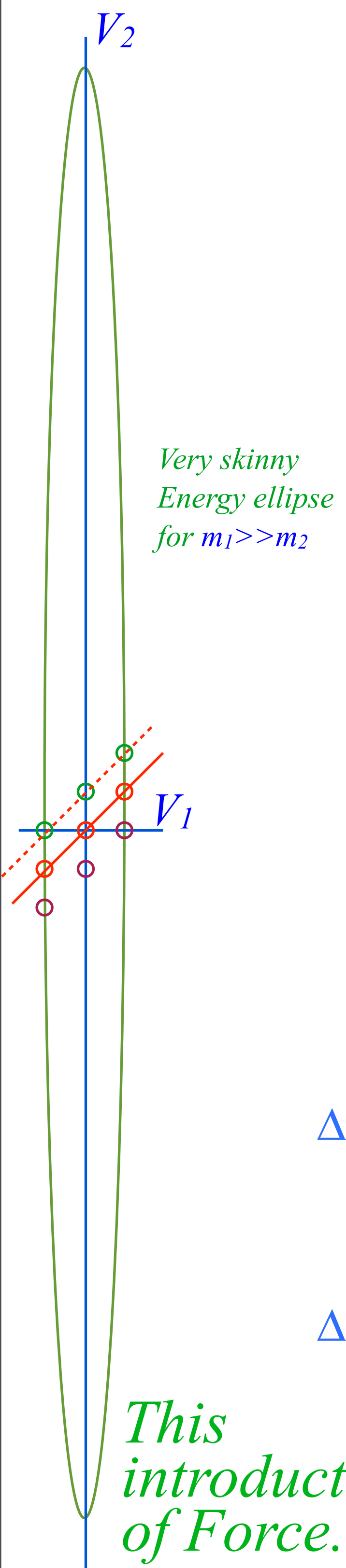
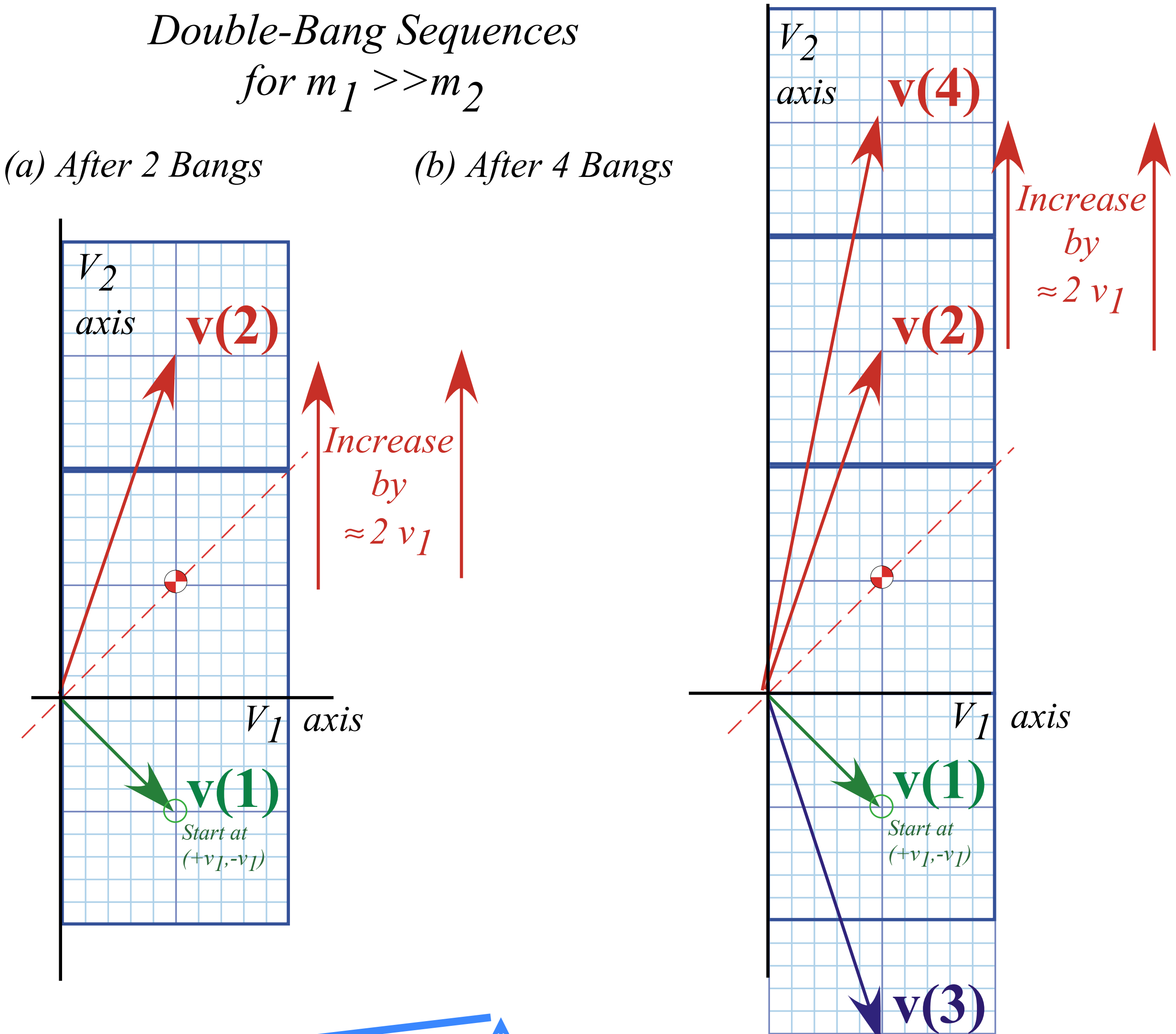
Assuming slow m_1 : $v_1 \ll v_2$





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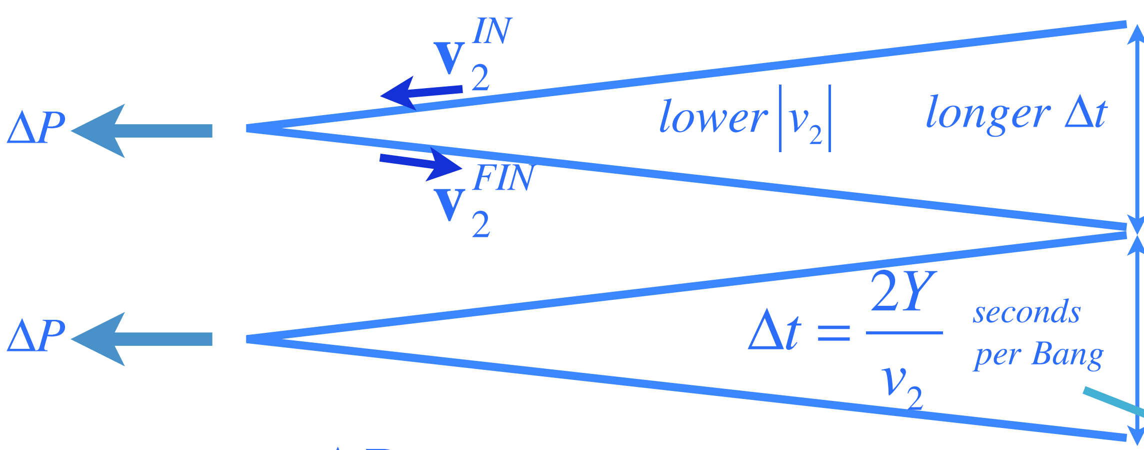

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$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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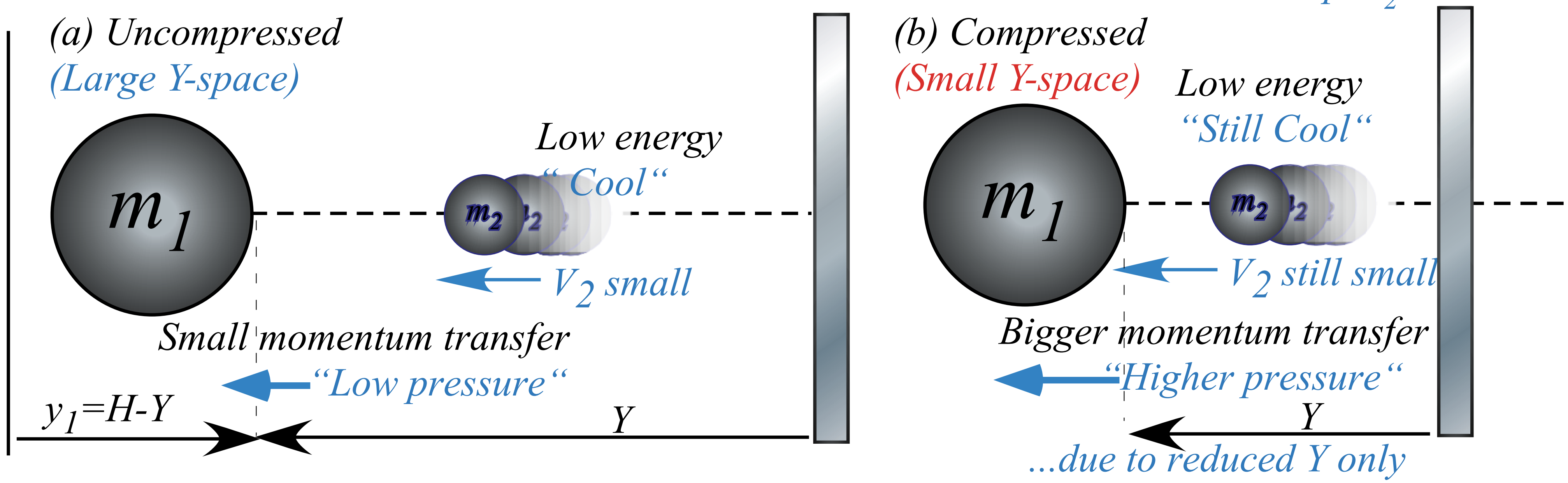
Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}$

Isothermal expansion or contraction: *Wall serves as thermal bath to keep m_2 cool*



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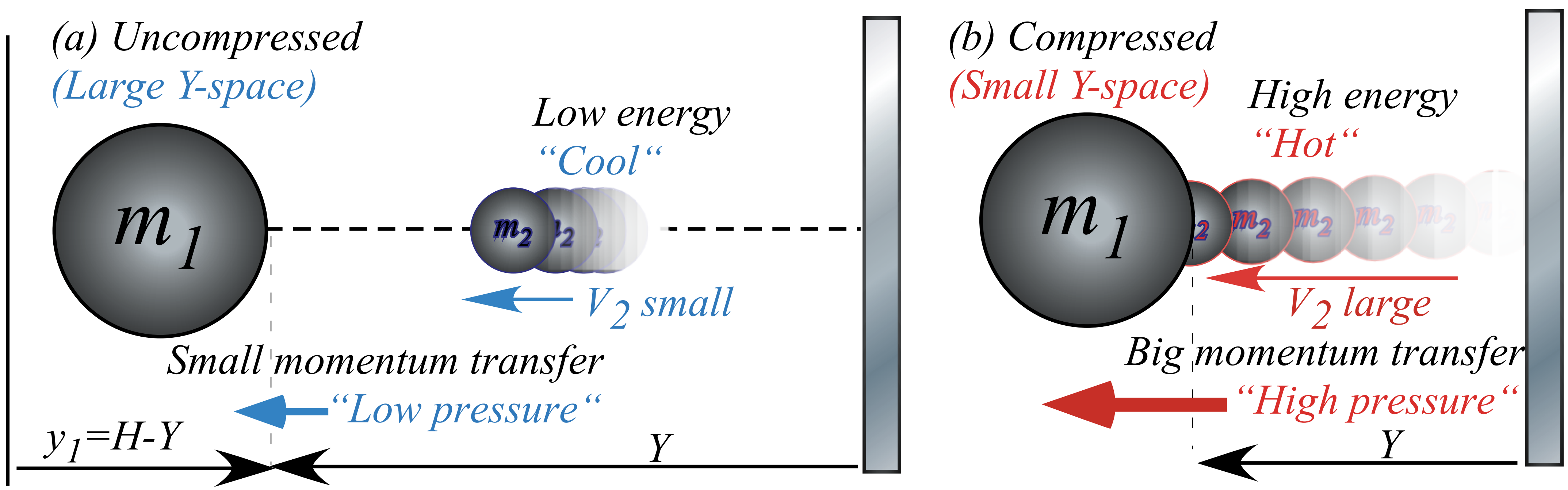
However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y}$$

Wall not given time to give or take KE



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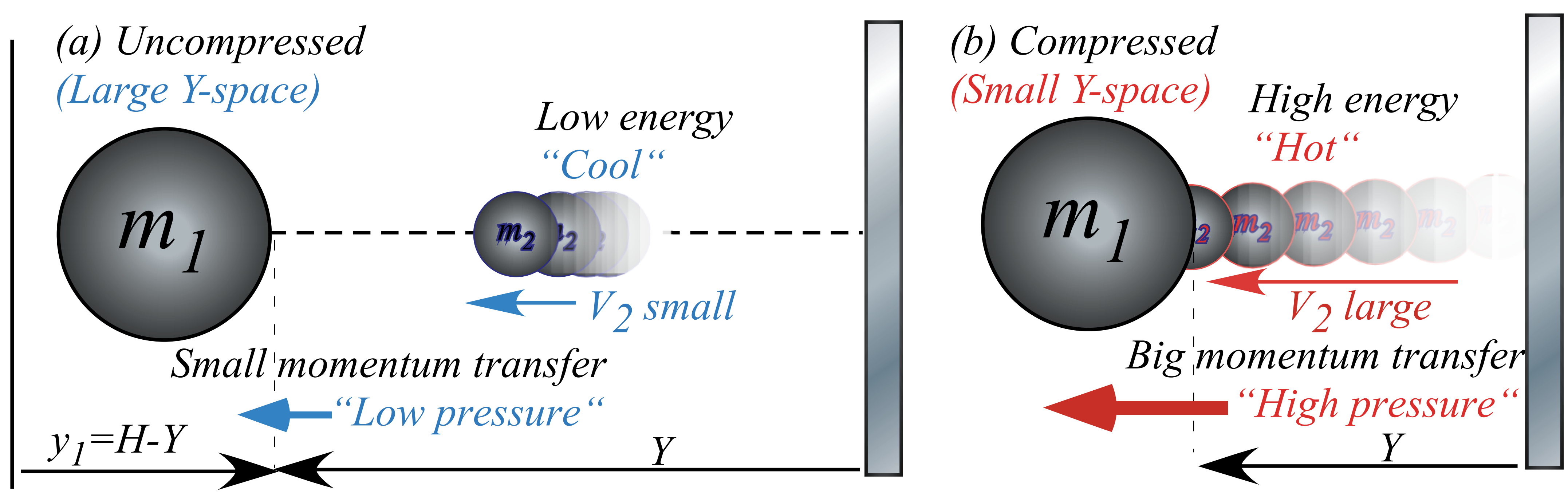

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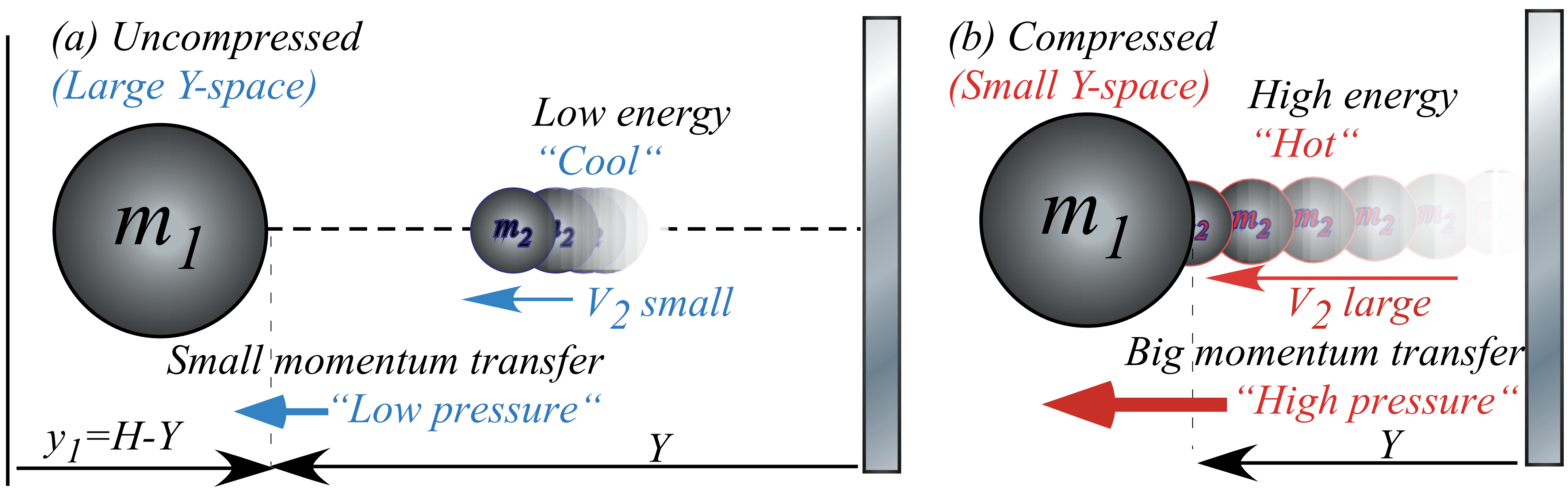
Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e (e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln \frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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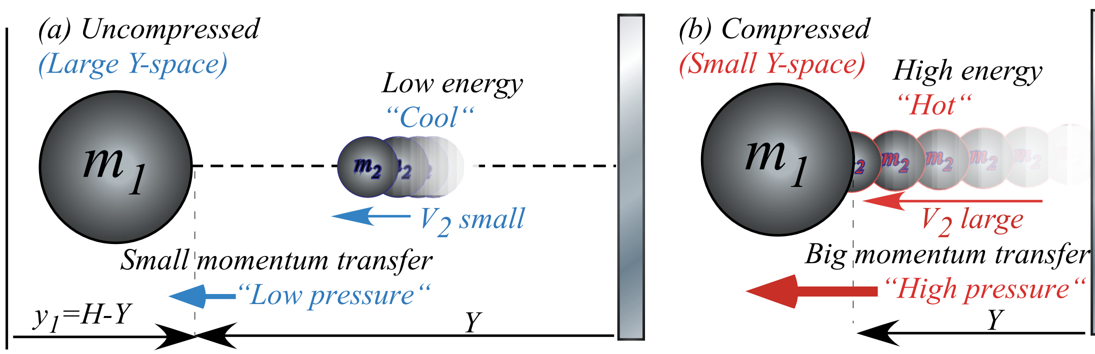
Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$*):* $F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}$



A "Double-Whammy"

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)$=const. ln(y)

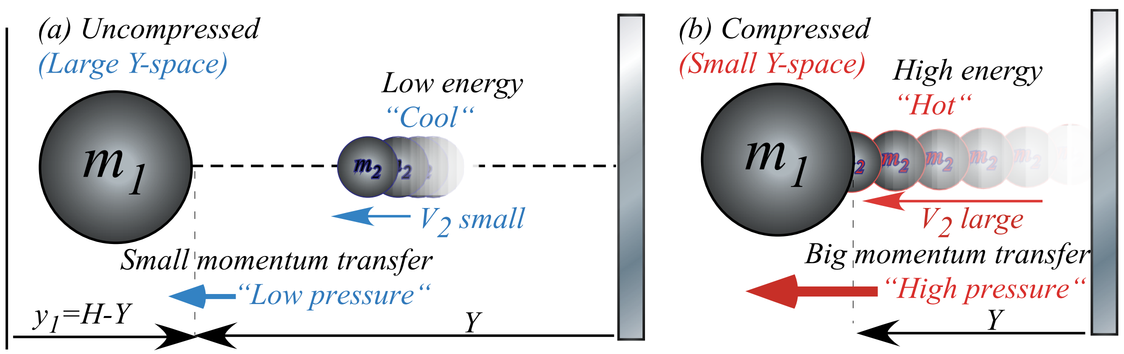
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



A "Double-Whammy" system

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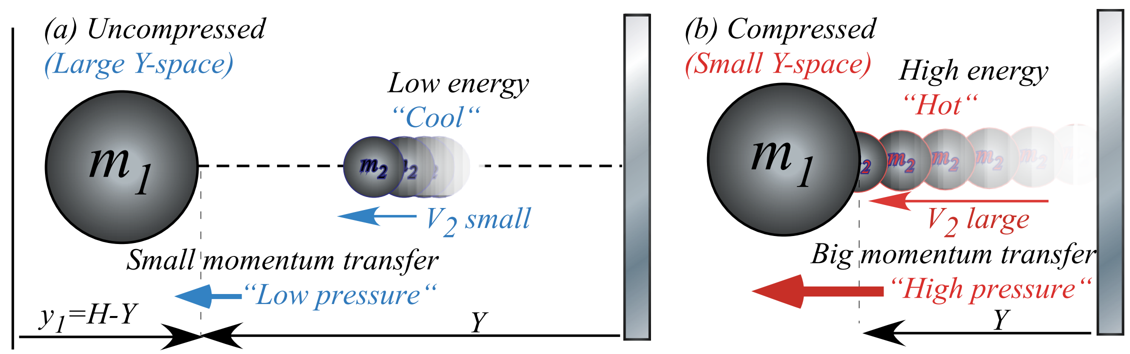
Define for big mass m_1 : *Kinetic energy* *KE(v_1)* *vs* *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)$=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY = \pm dU$



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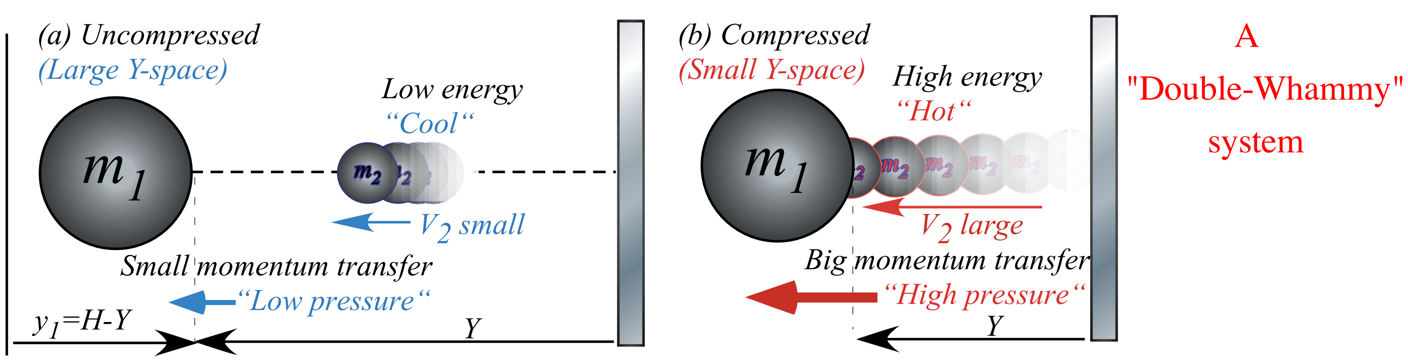
Q*?Another axiom?*



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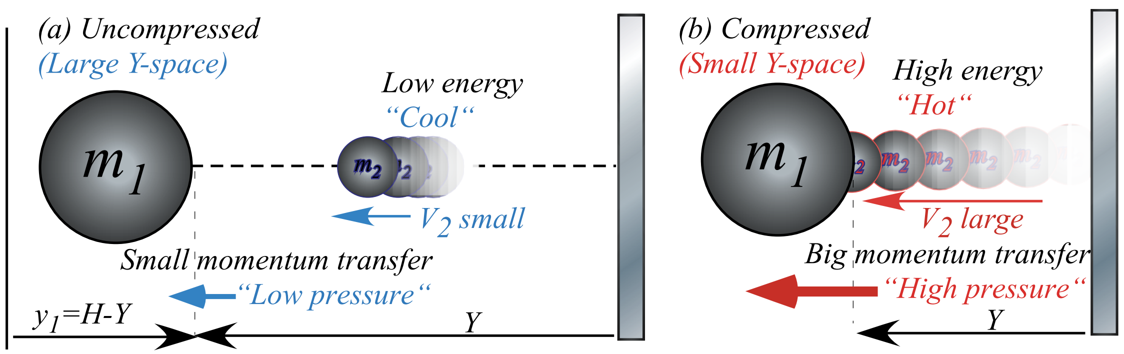
Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \dfrac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* *KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

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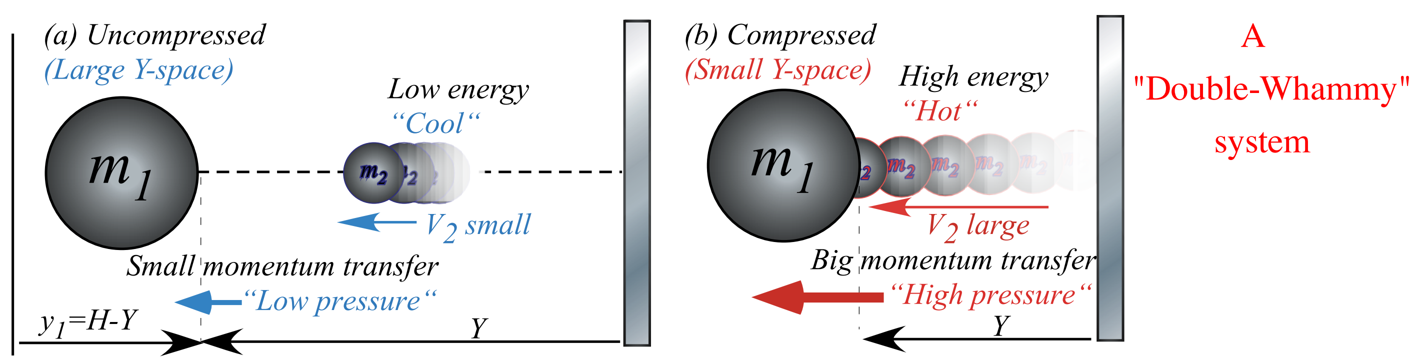
Q*?Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$

(Here: $V = v_2$)



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Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q? *Another axiom?* A: No.

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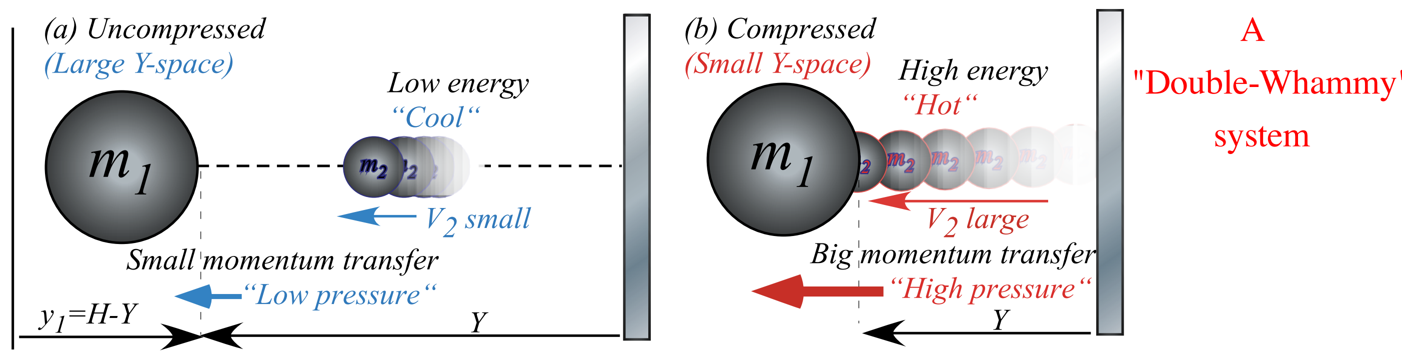
or else : $$F\cdot\frac{dY}{dt} = \frac{dp}{dt}\cdot V = \frac{d(mV)}{dt}\cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$$ (Here: $V = v_2$)



Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

→ *Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$*

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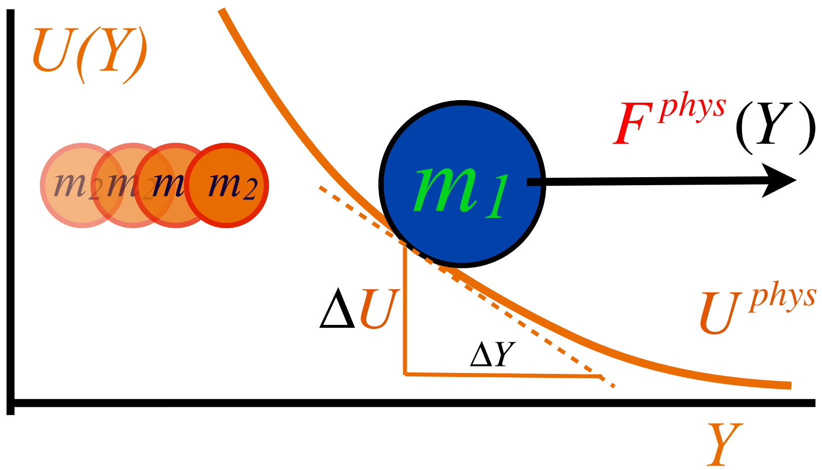
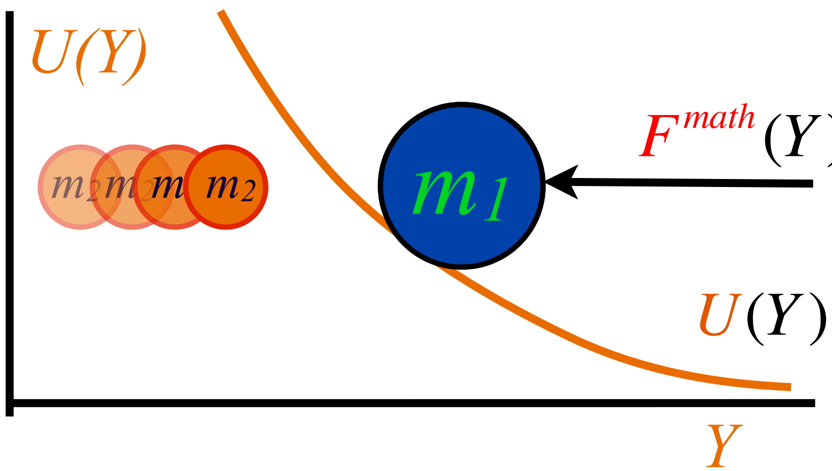
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

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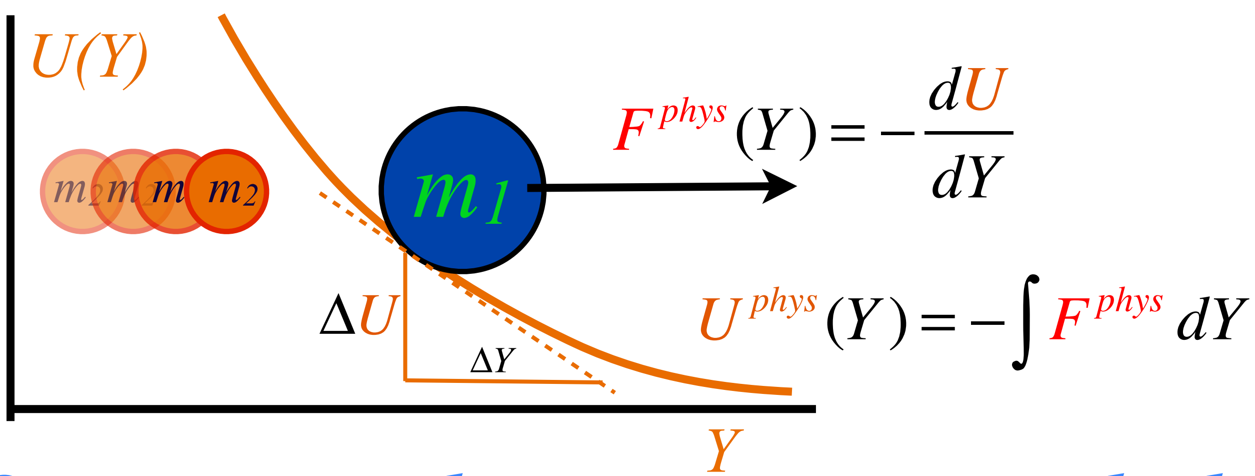
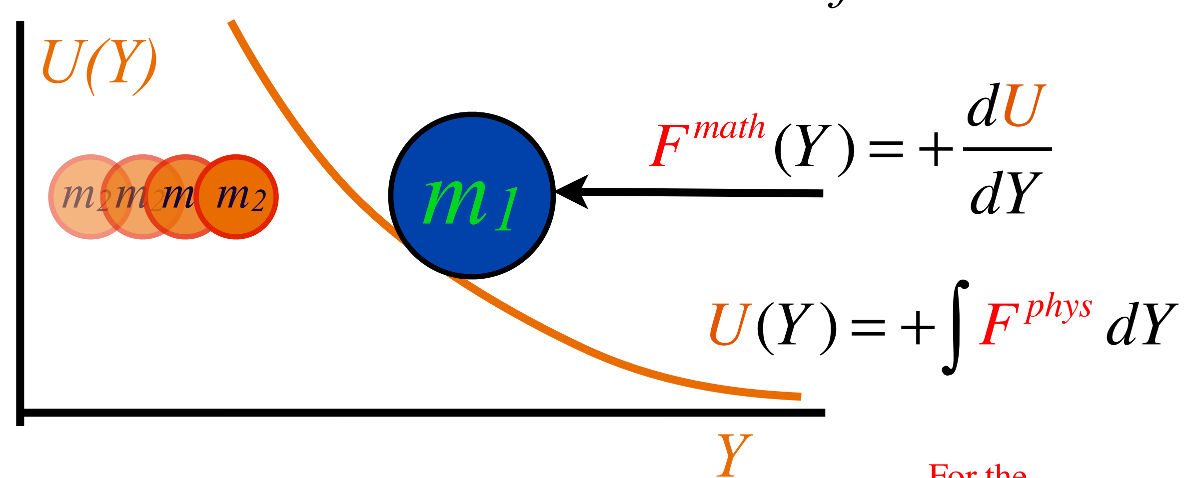
The "Physicist" View of Force



The "Mathematician" View of Force



(OK, But, is this consistent with the $F=(const.)^2/Y^3$ *(on p.18)**?)*

For the "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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Define for big mass m_1: *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

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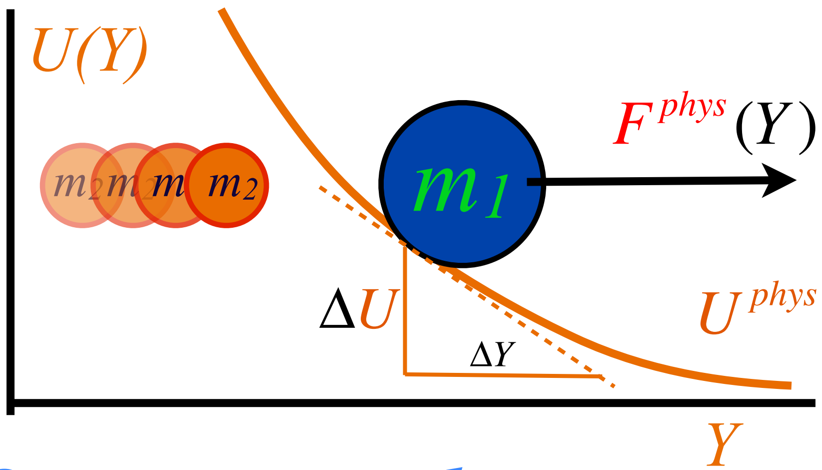
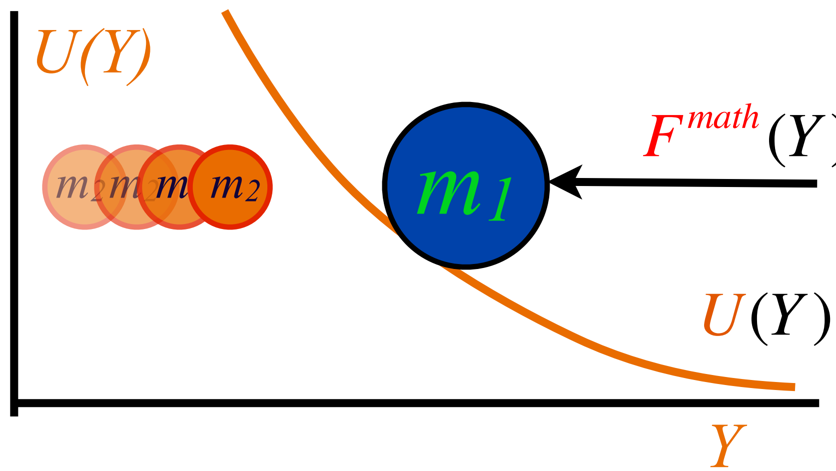
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, is this consistent with the $F=(const.)^2/Y^3$ (on p.18)?)

For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ *vs. Mathematician's Definition* $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential* $U(y)=const.\ ln(y)$

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \dfrac{m_2 v_2^2}{Y} = \dfrac{const.}{Y}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

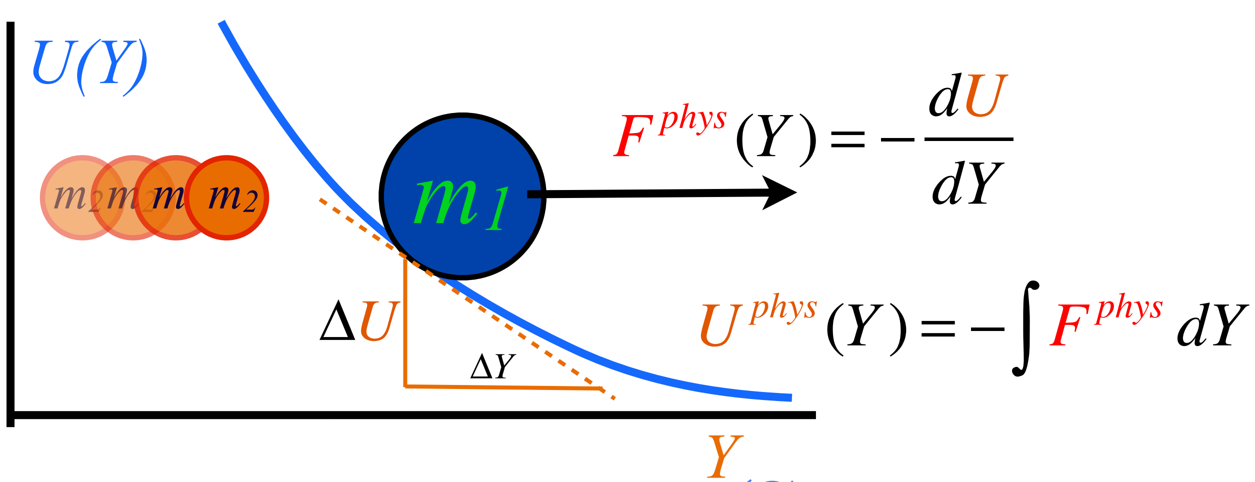
$$F^{phys} = \frac{m_2 {v_2}^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad implies: \quad U(Y) = \int -F^{phys} dY = \int -\frac{m_2 {v_2}^2}{Y} dY = -m_2 {v_2}^2 \ln(Y)$$

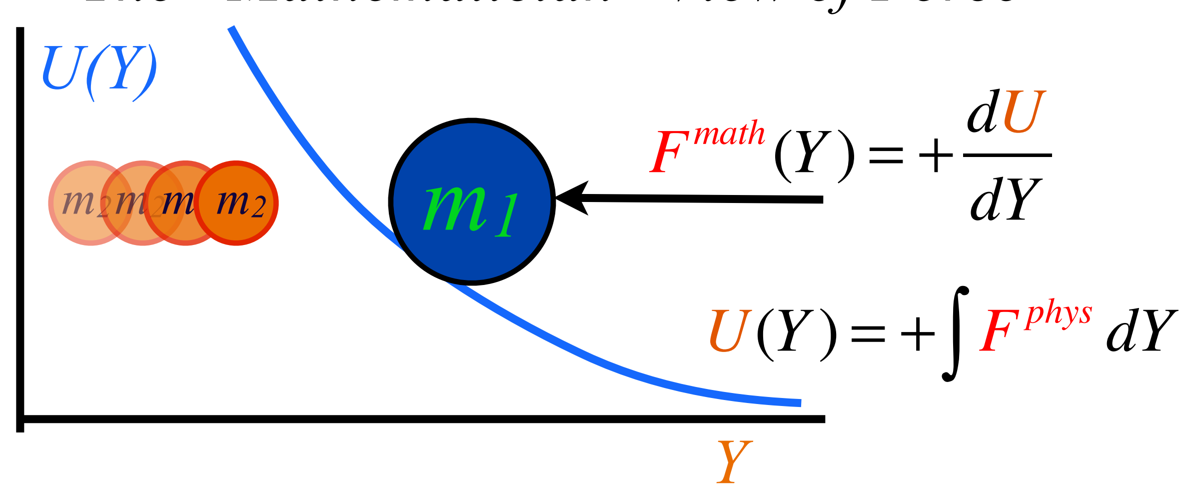
$$const. = E = \frac{1}{2} m_1 {v_1}^2 + U(Y) \quad where: U(Y) = -m_2 {v_2}^2 \ln(Y)$$

Define for big mass m_1: *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$-m_2 {v_2}^2 \ln(Y)$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$

The "Physicist" View of Force | *The "Mathematician" View of Force*





(Same integral/differential relations)

$$F^{phys} = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y} \quad \text{consistent with:} \quad F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\left(-const.\ln(Y)\right) = \frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

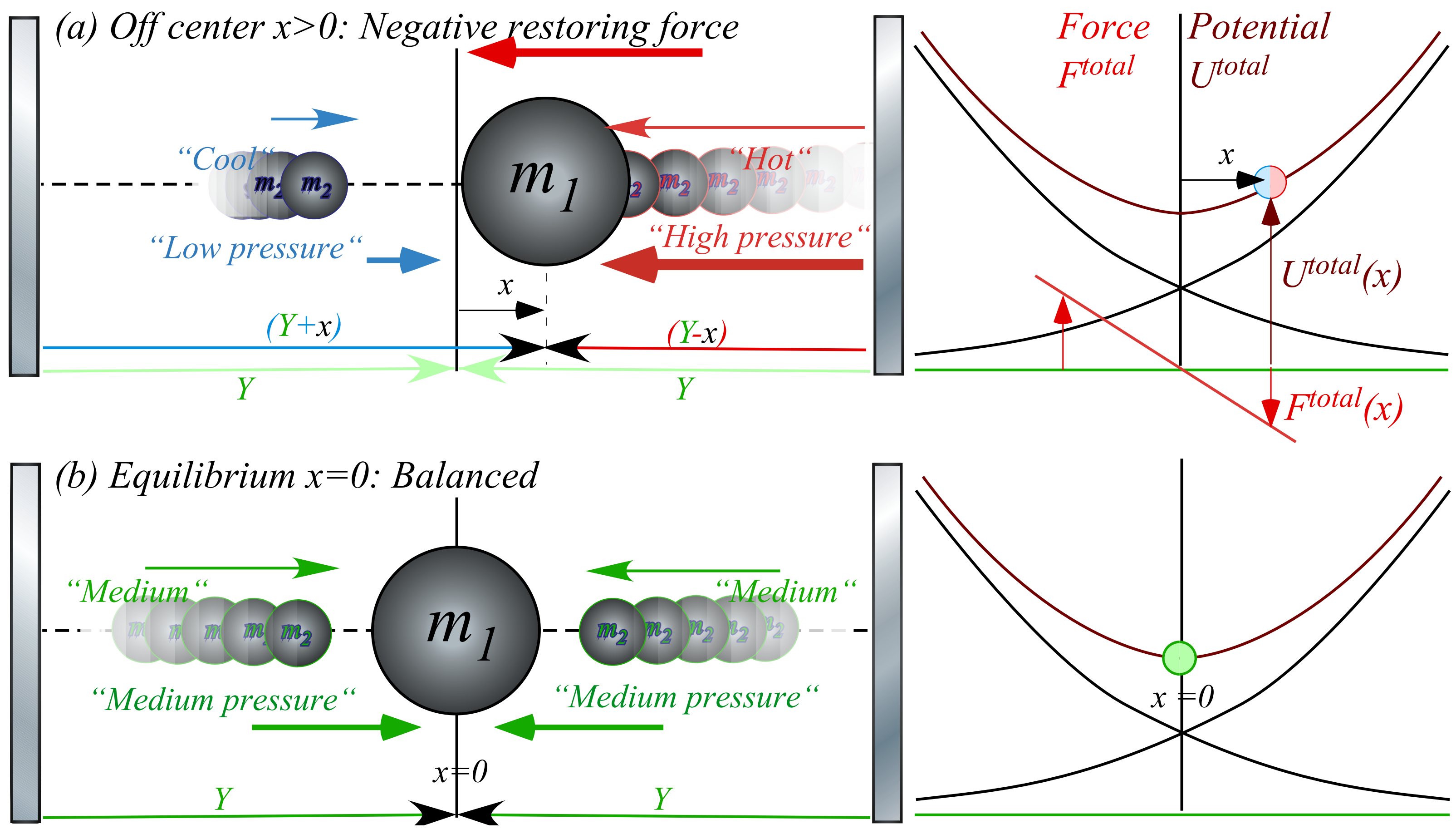
→ *Example of oscillator with opposing Isothermal potentials*

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f\left[1 - x + x^2 - x^3 \ldots\right] - f\left[1 + x + x^2 + x^3 \ldots\right] = -2f \cdot x - 2f \cdot x^3 - \ldots$$

$$F^{HO} = -k \cdot x = -\frac{\partial U^{HO}}{\partial x} \qquad U^{HO} = \frac{1}{2} k \cdot x^2 = -\int F^{HO}\, dx \qquad \text{HO } \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\upsilon$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

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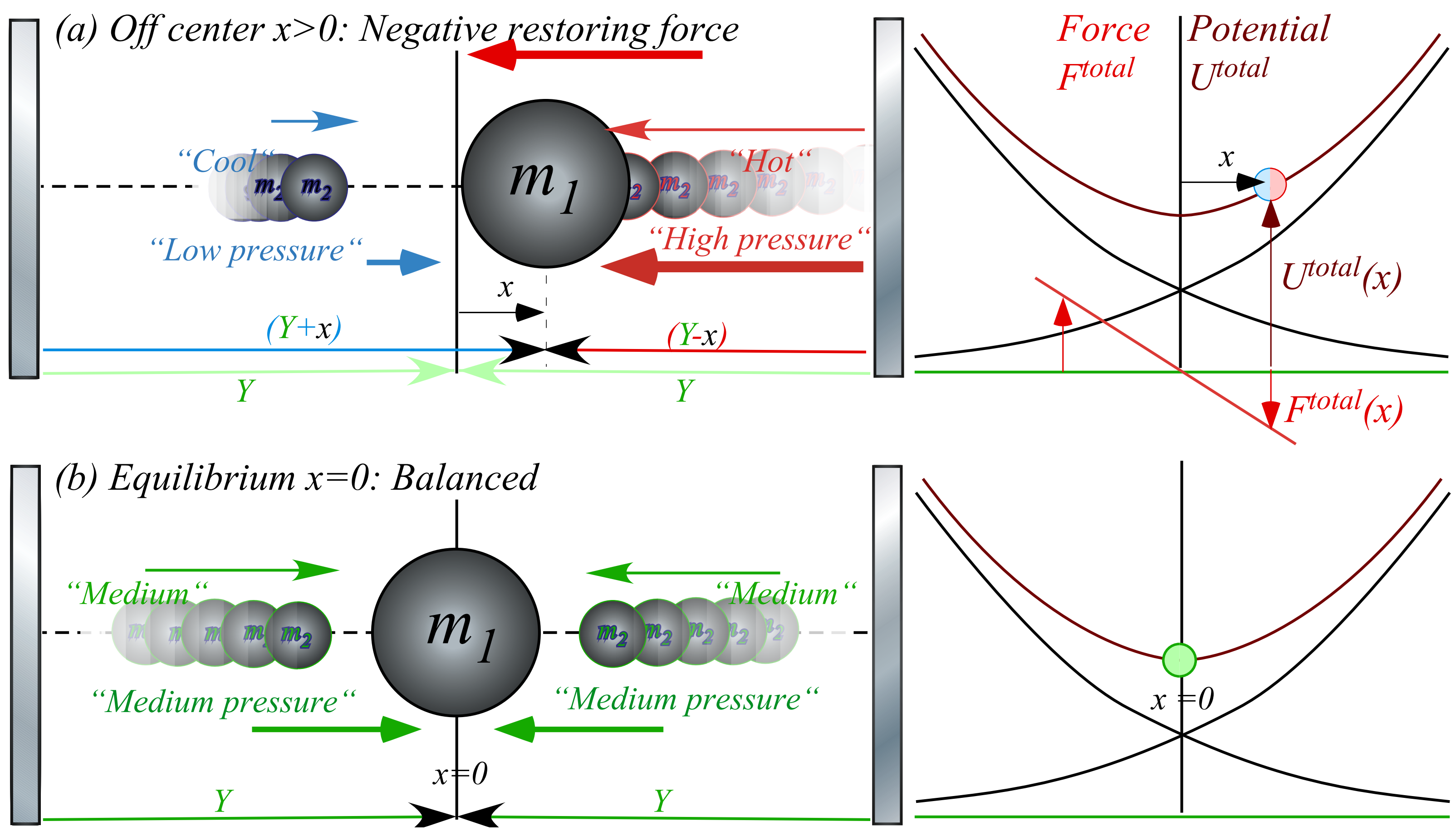

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

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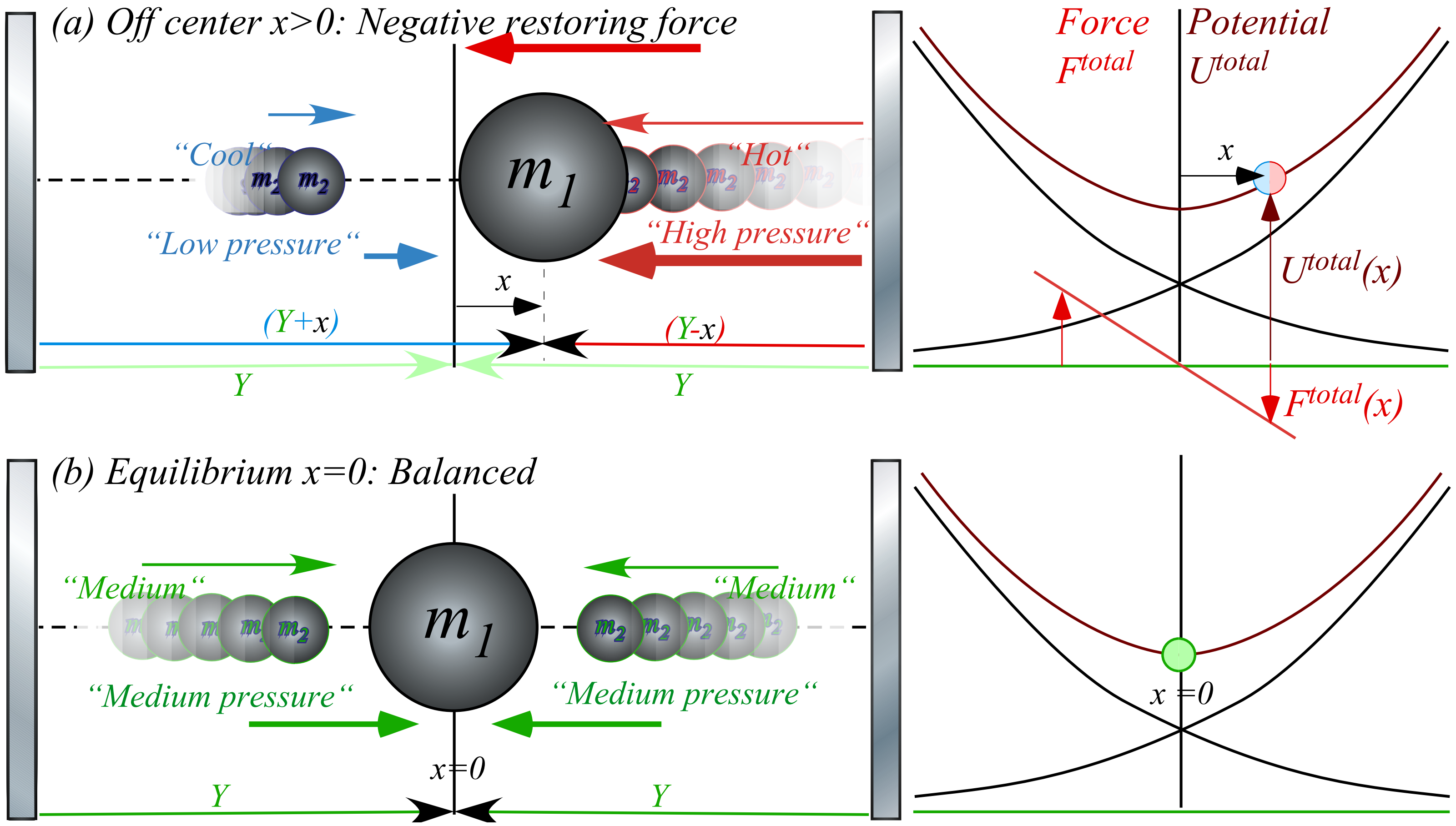

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n\text{-}1)}{2} Y_0^{n-2} x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2 \cdot 3} Y_0^{n-3} x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2 \cdot 3 \cdot 4} Y_0^{n-4} x^4 \ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

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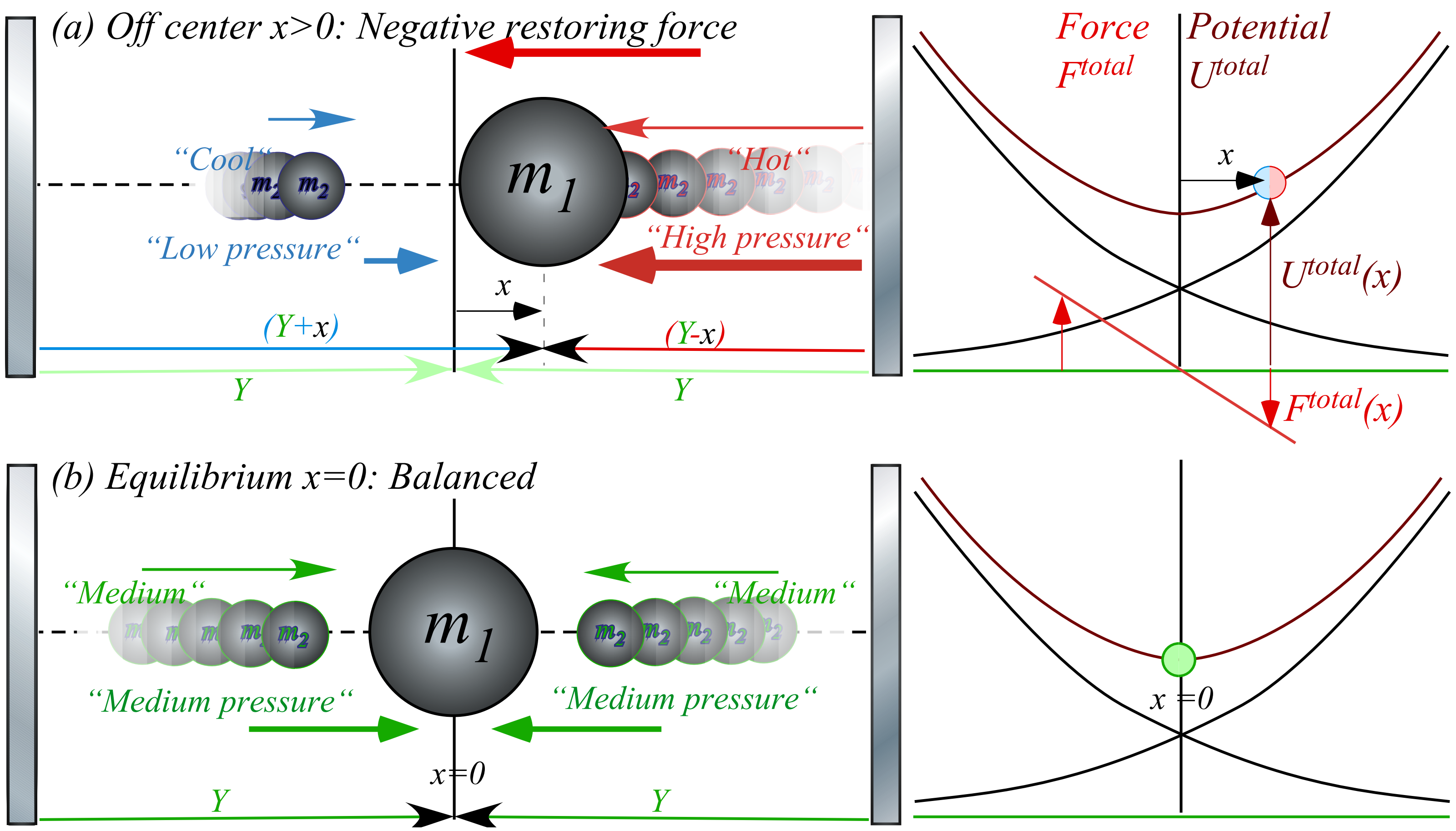

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - \ldots$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n-1)}{2}Y_0^{n-2}x^2 + \frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4 \ldots$$

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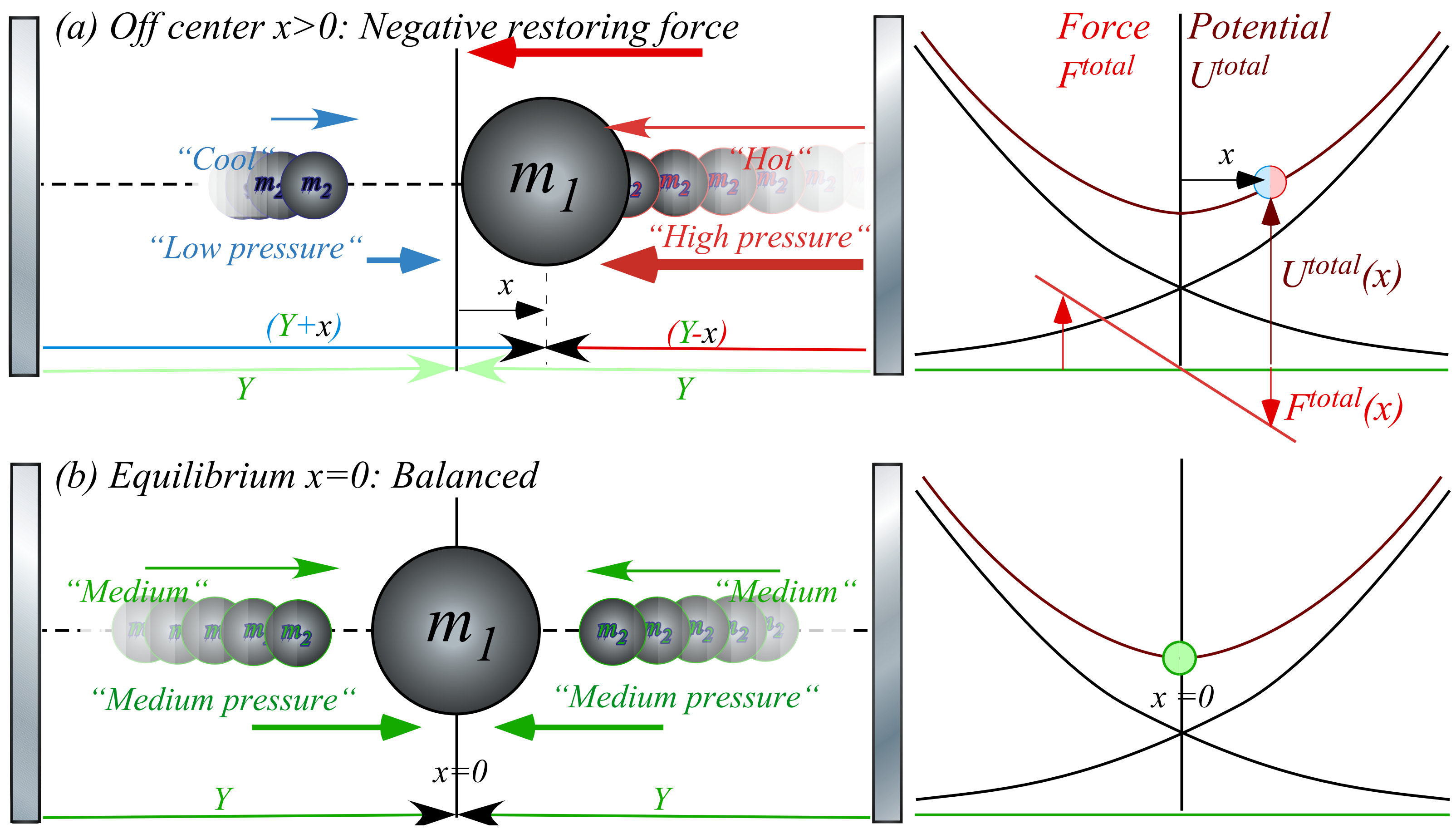

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

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$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

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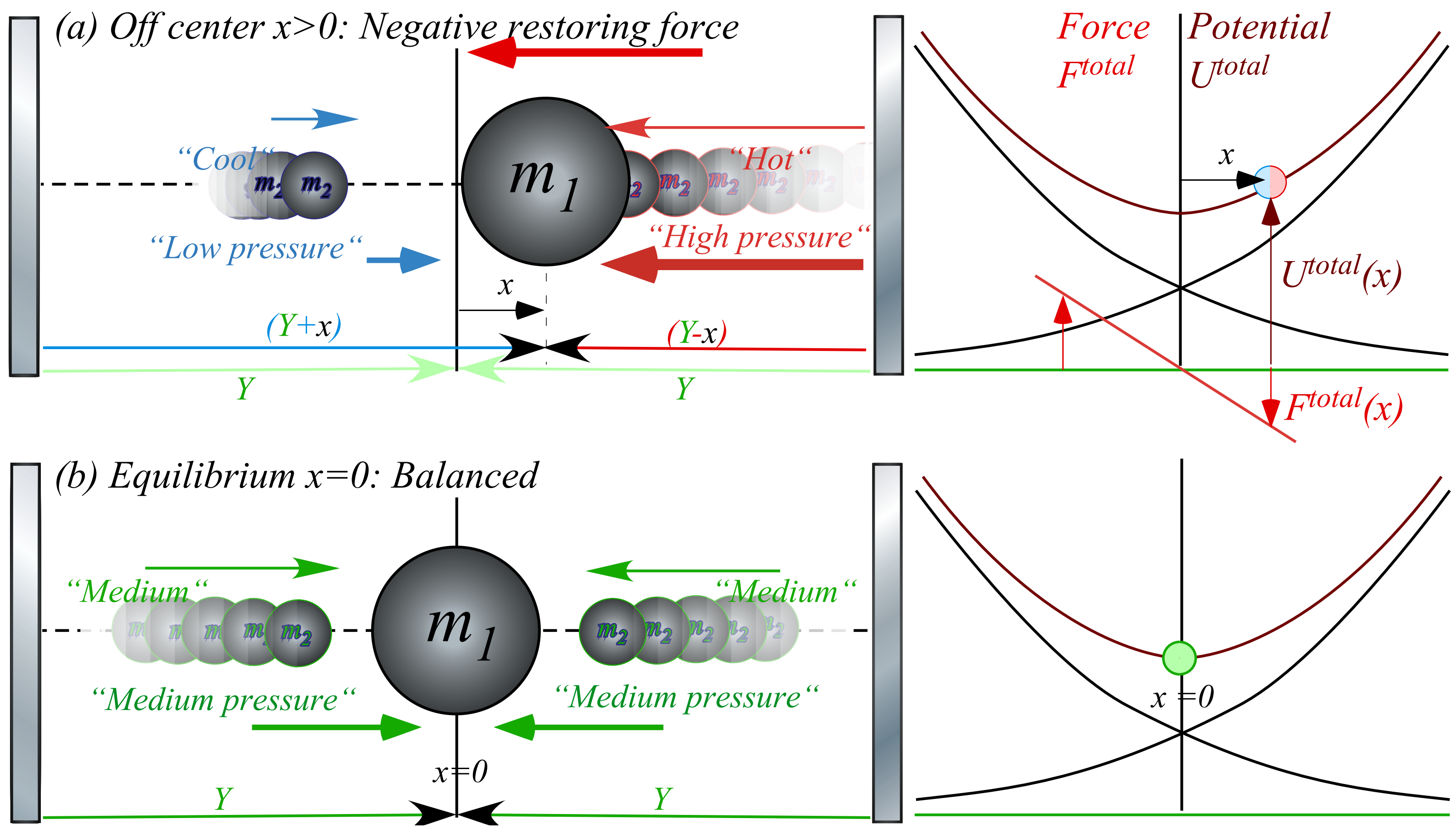

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

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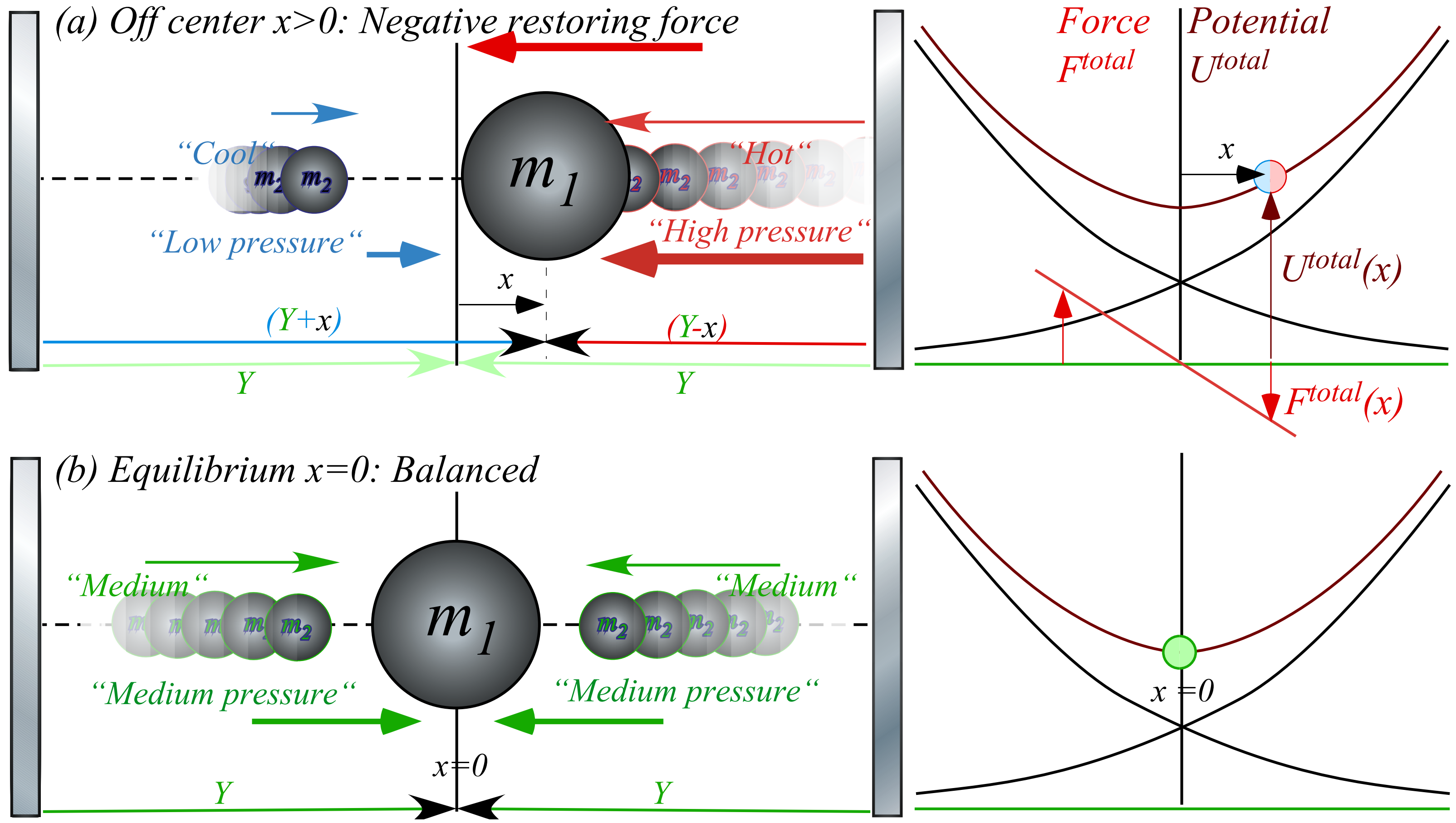

Unit 1
Fig. 6.2

Harmonic oscillator term

Anharmonic oscillator terms...

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\frac{1}{Y_0} + \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

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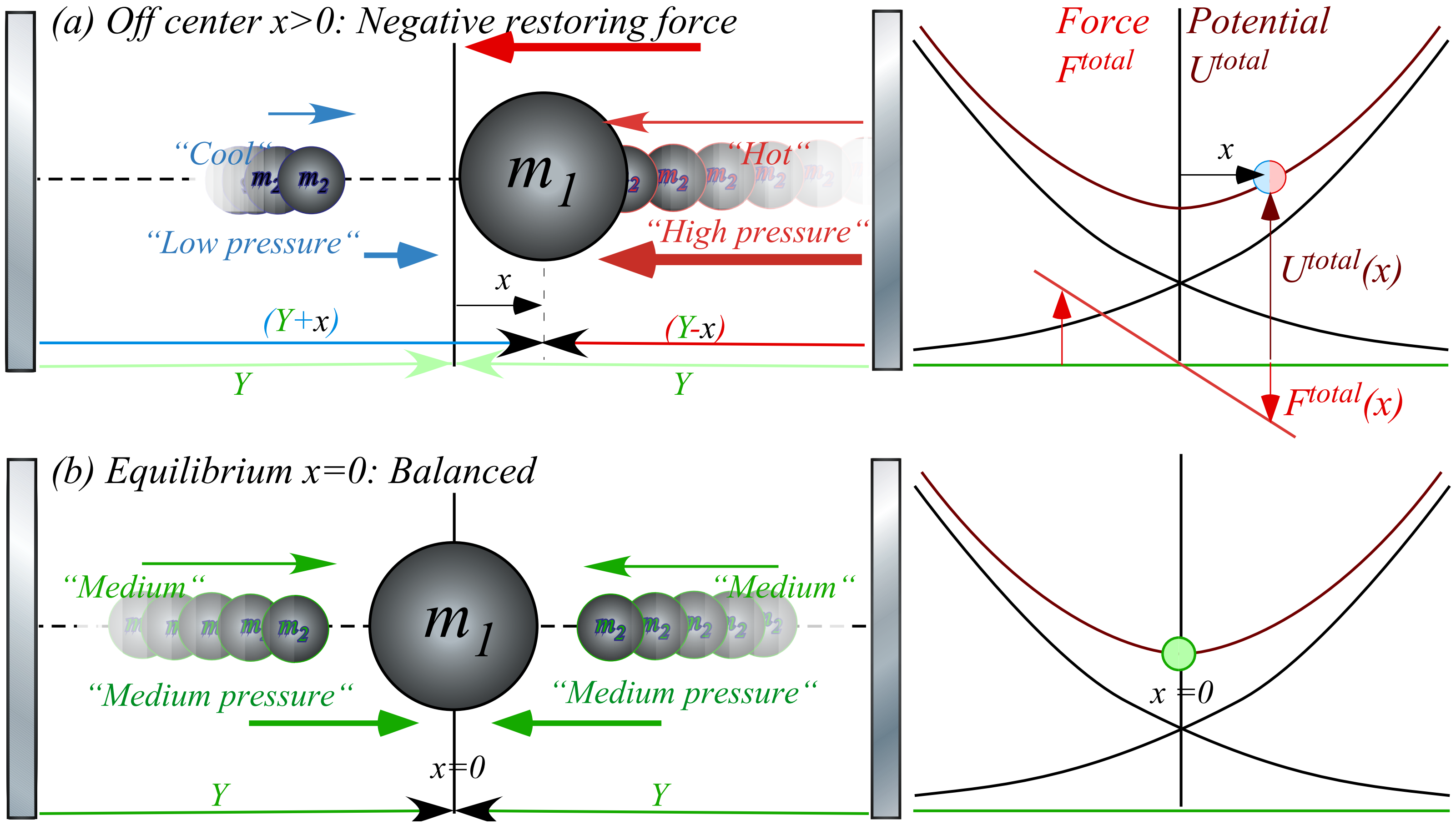
Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

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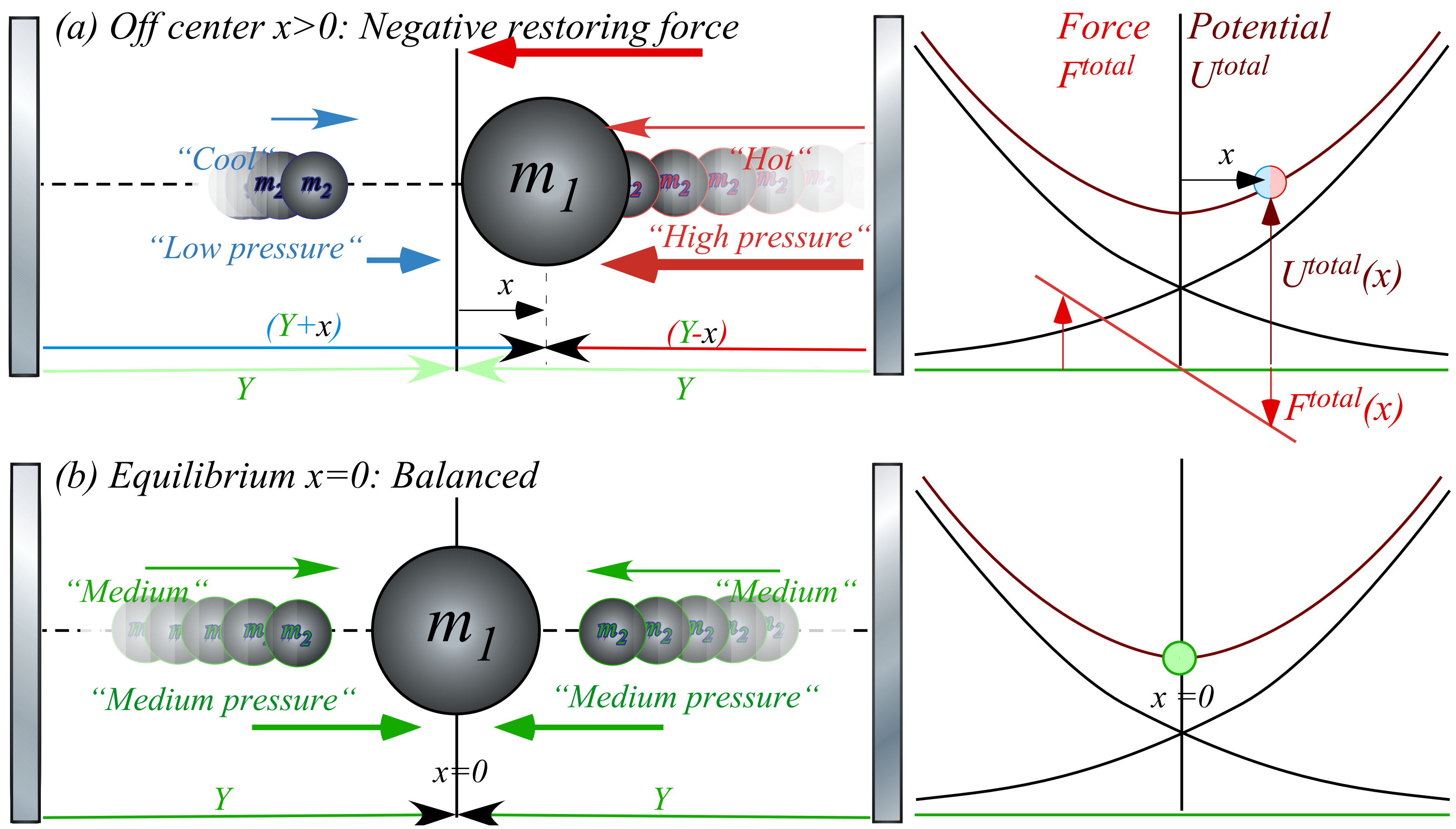
Harmonic oscillator force constant : $k = 2f/Y_0^2 = 2m_2 v_2^2/Y_0^2$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{Y_0+x} - \frac{f}{Y_0-x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

Harmonic oscillator force constant : $k = 2f/Y_0^2 = 2m_2 v_2^2/Y_0^2$

Harmonic Oscillator Force

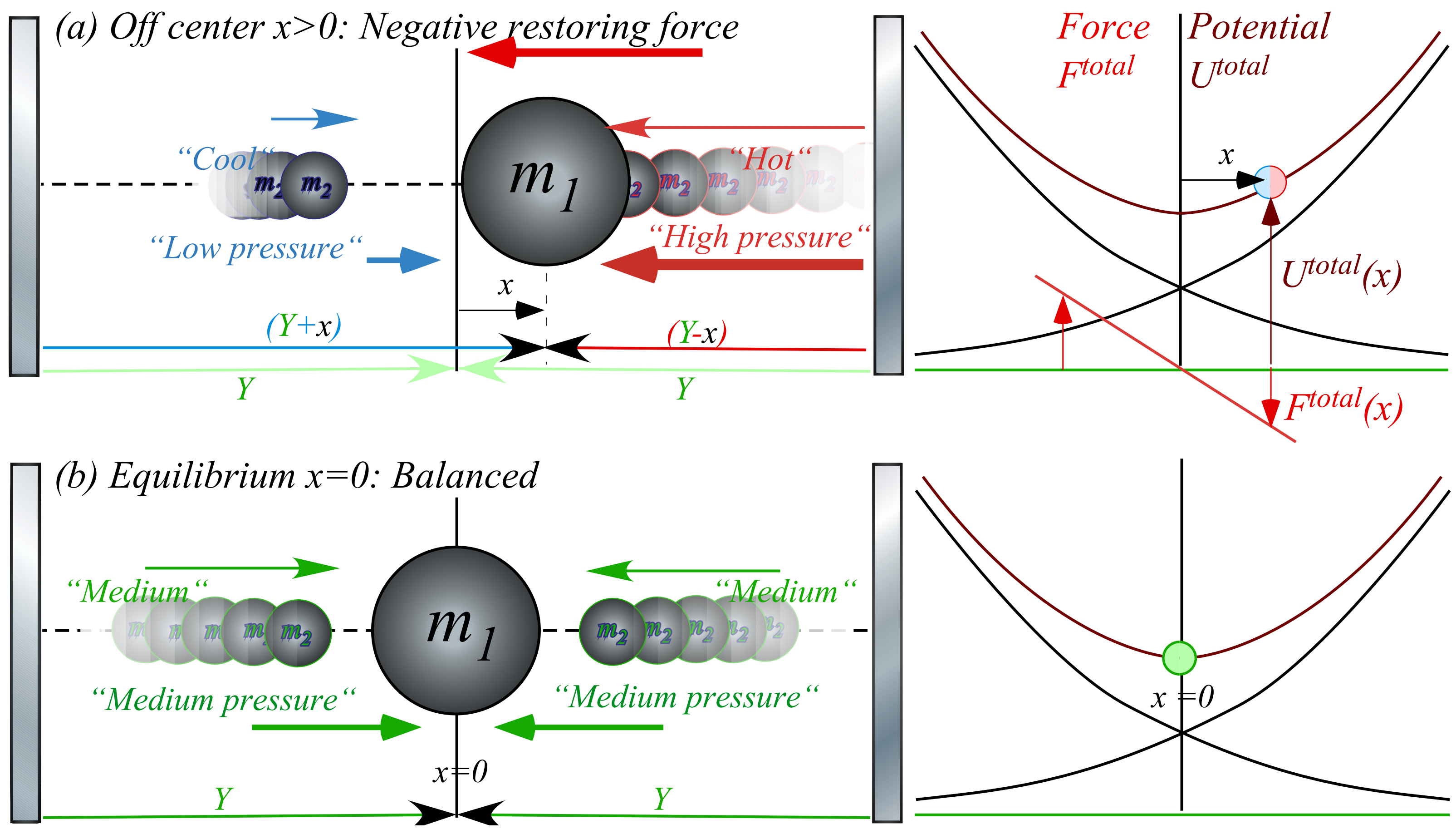
$$F^{HO} = -k \cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

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$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\frac{1}{Y_0} + \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^{-4}} - \ldots$$

Harmonic oscillator force constant : $k = 2f/Y_0^2 = 2m_2 v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Potential

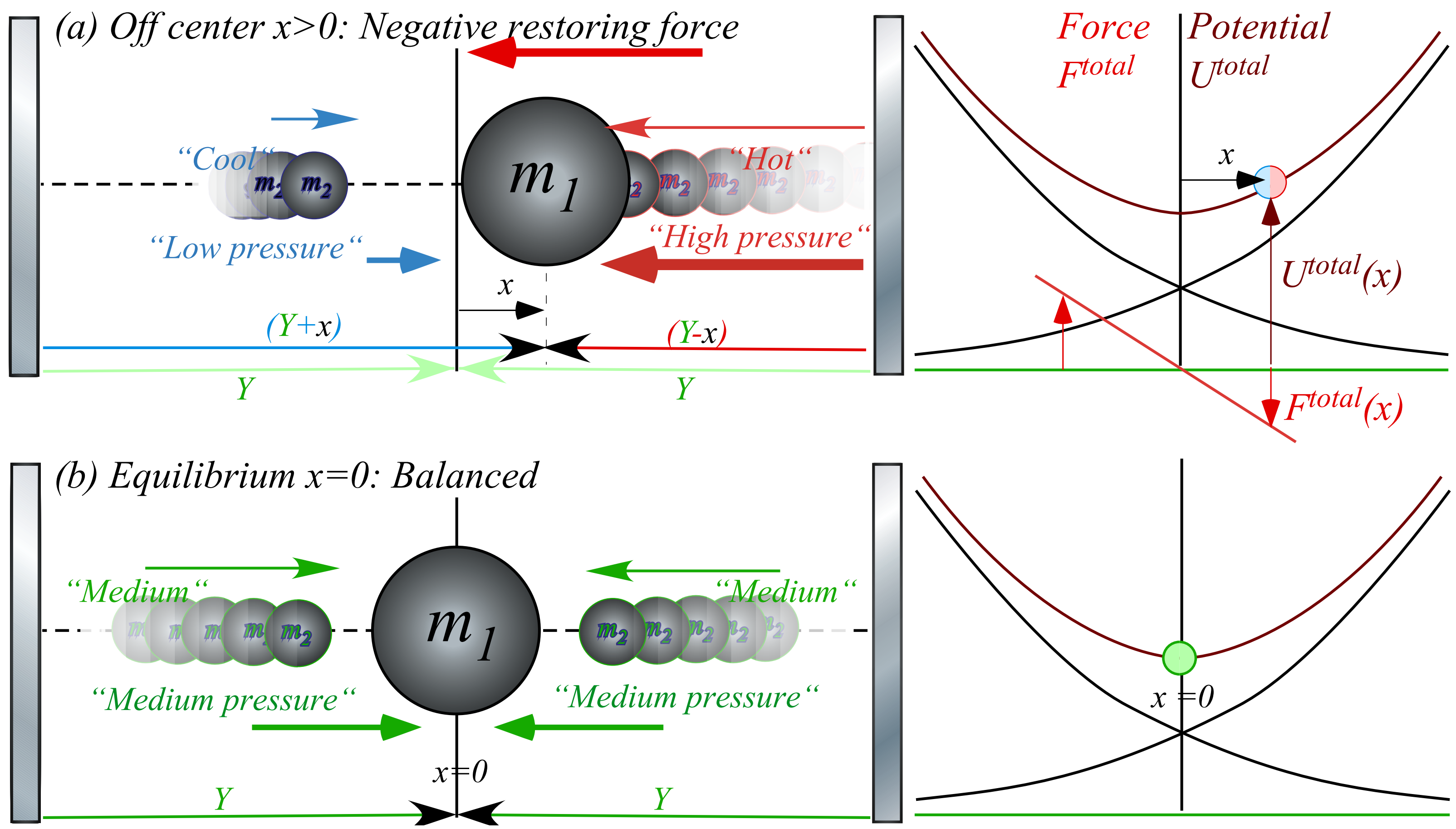
$$U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

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$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

Harmonic oscillator force constant : $k = 2f/Y_0^2 = 2m_2 v_2^2 / Y_0^2$

Harmonic Oscillator Force

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\, dx$$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}}\frac{v_2}{Y_0} = 2\pi\upsilon$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

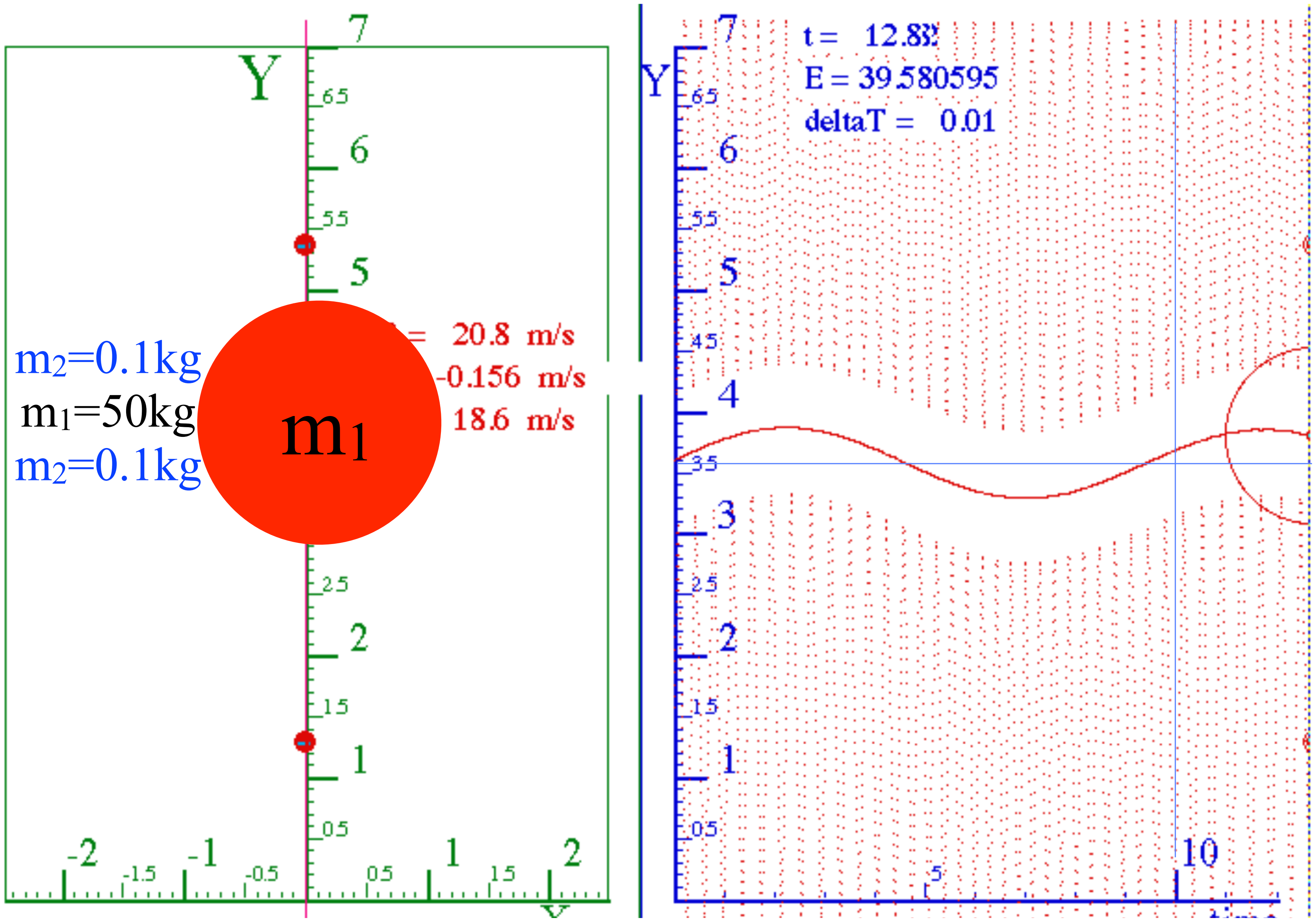
Because then: $E = \frac{1}{2}m(A\omega\cos\omega t)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2 A^2(\cos\omega t)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2(\cos^2\omega t + \sin^2\omega t)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2 A^2 \quad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

Switch
m_1=m_3
with
m_2
to match
formula



[BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)]

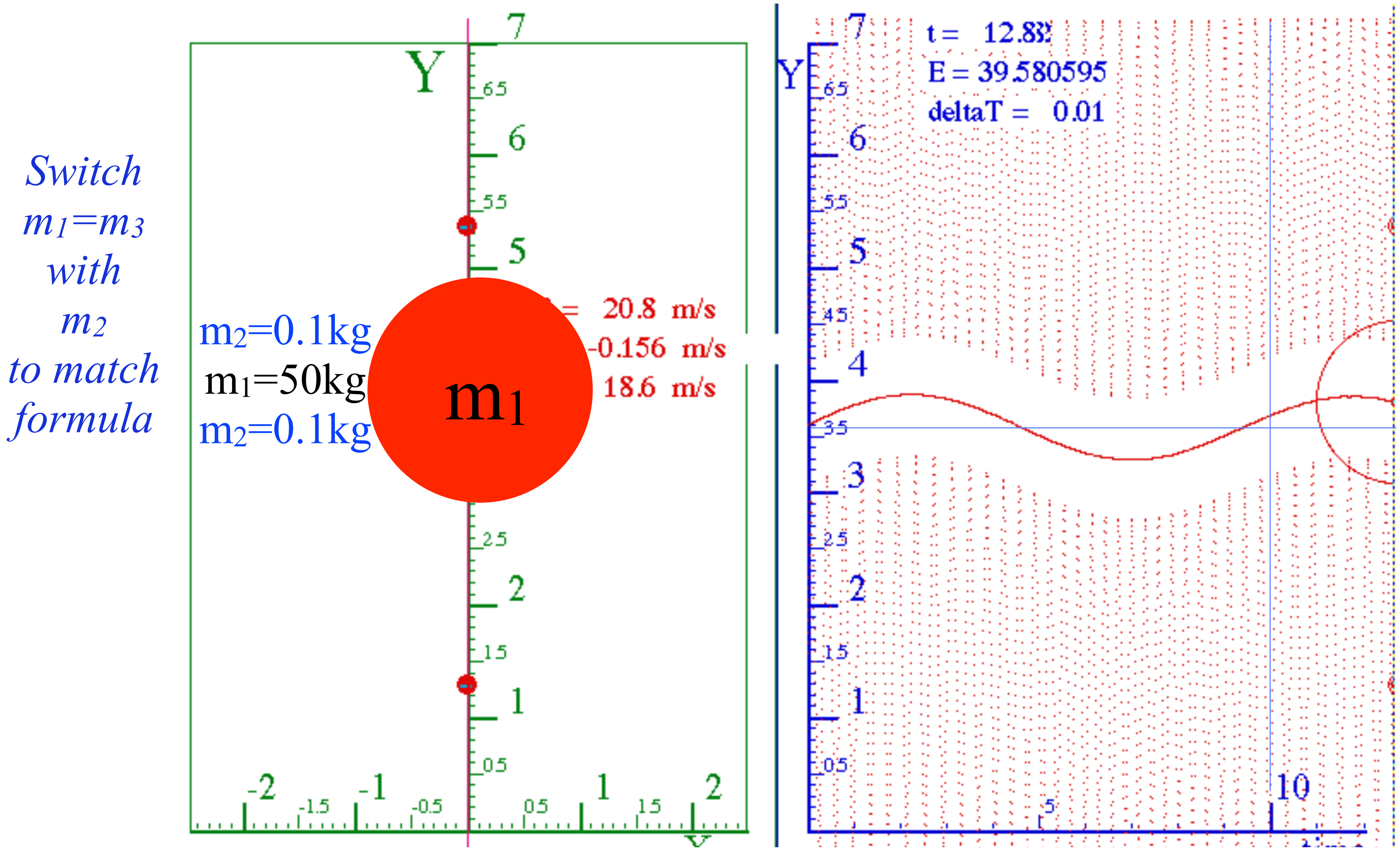
Unit 1
Fig. 6.3

Simulation of
the adiabatic case

Sample problem: *Compute isothermal frequency and/or period*

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$



[BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)]

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

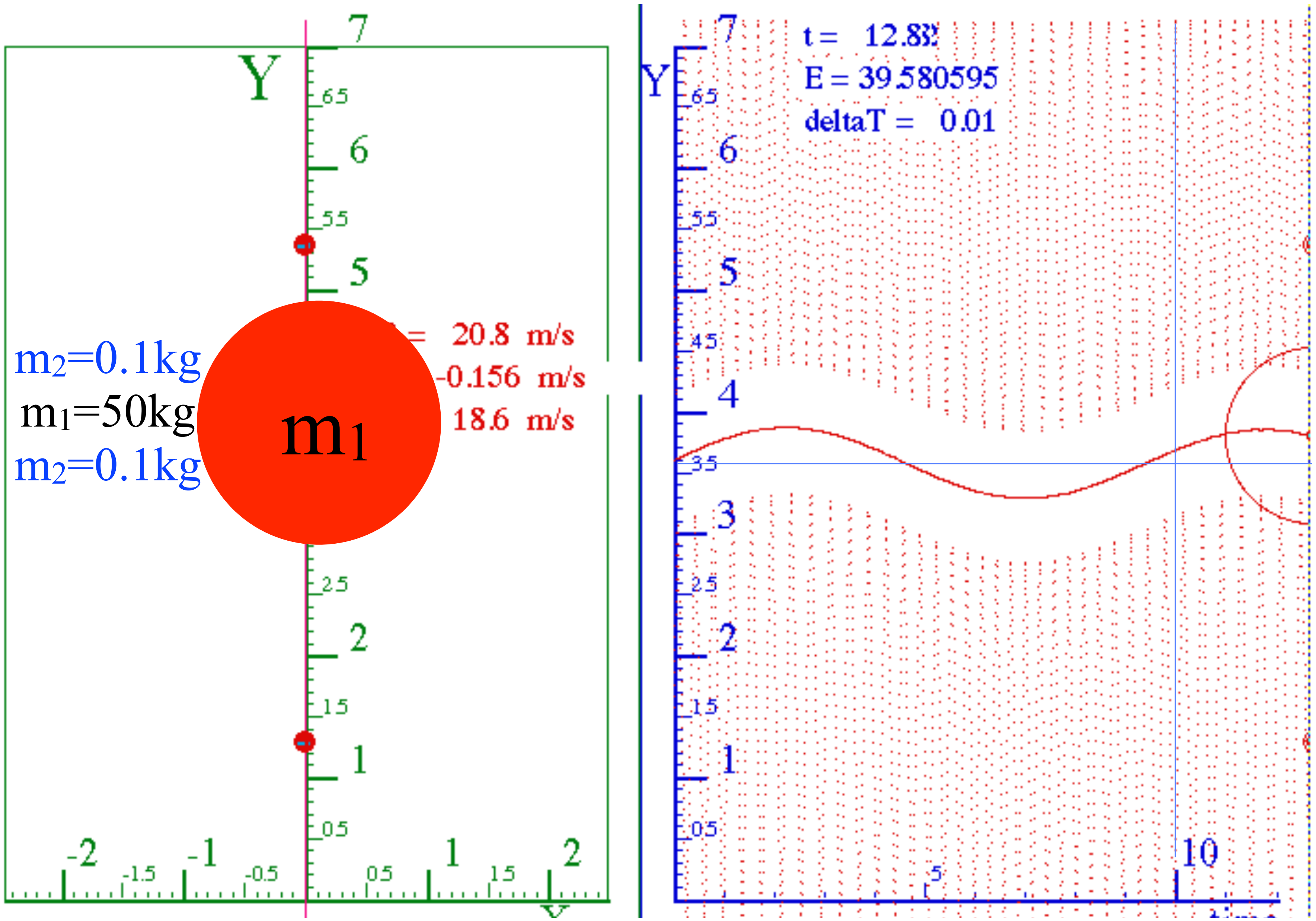
Sample problem: *Compute isothermal frequency and/or period*

$$\text{Period}: \quad \tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$$

Frequency

$$\text{HO } \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$$

Switch $m_1 = m_3$ with m_2 to match formula



Unit 1
Fig. 6.3

Simulation of
the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]()

Sample problem: *Compute isothermal period given m_1=50, m_2=0.1=m_3, v_2=20, Y_0=3.5*

Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

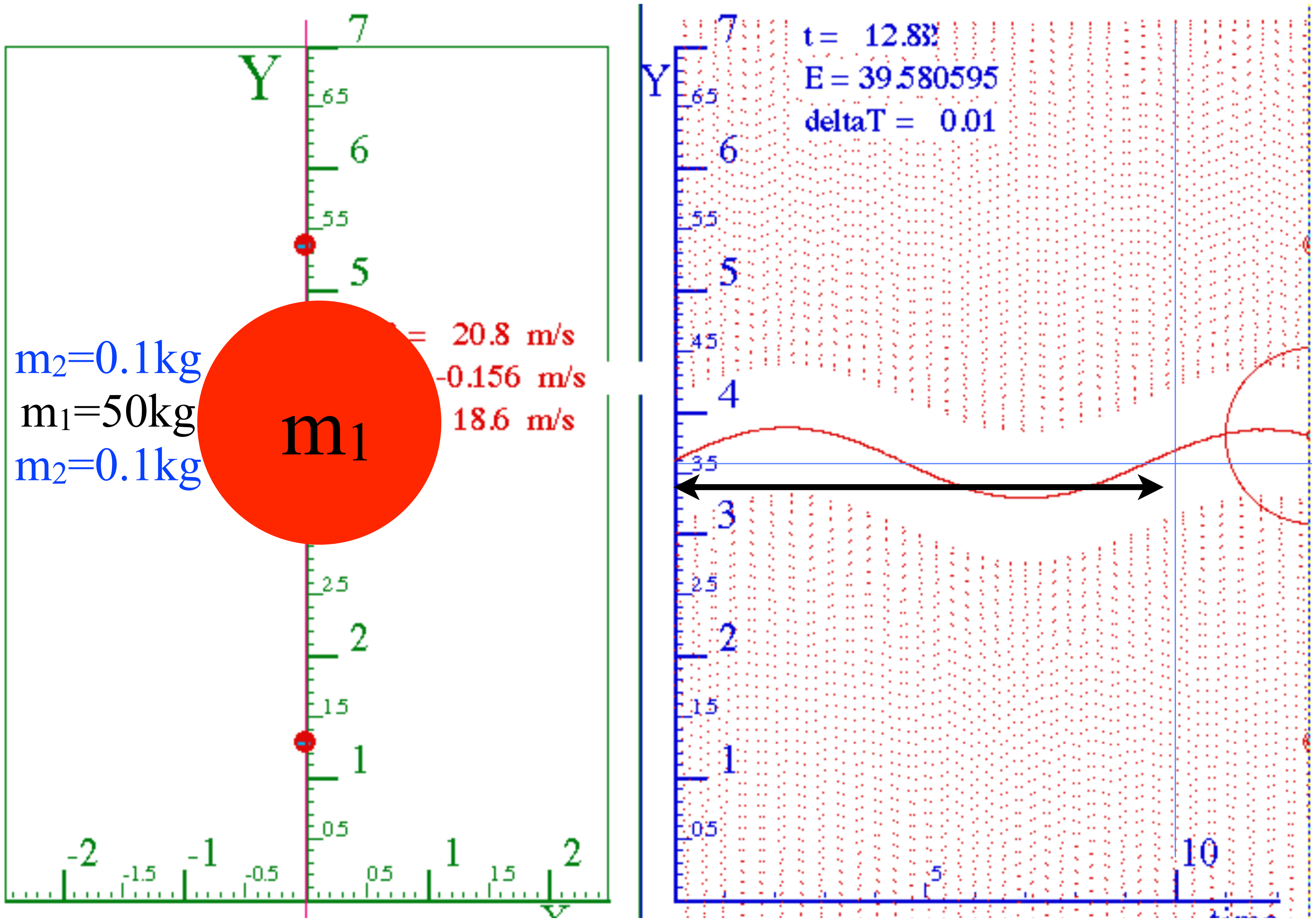
$$=17.38$$

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$

Switch
$m_1=m_3$
with
m_2
to match
formula



Simulation of

the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute isothermal period given $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$*

Period :

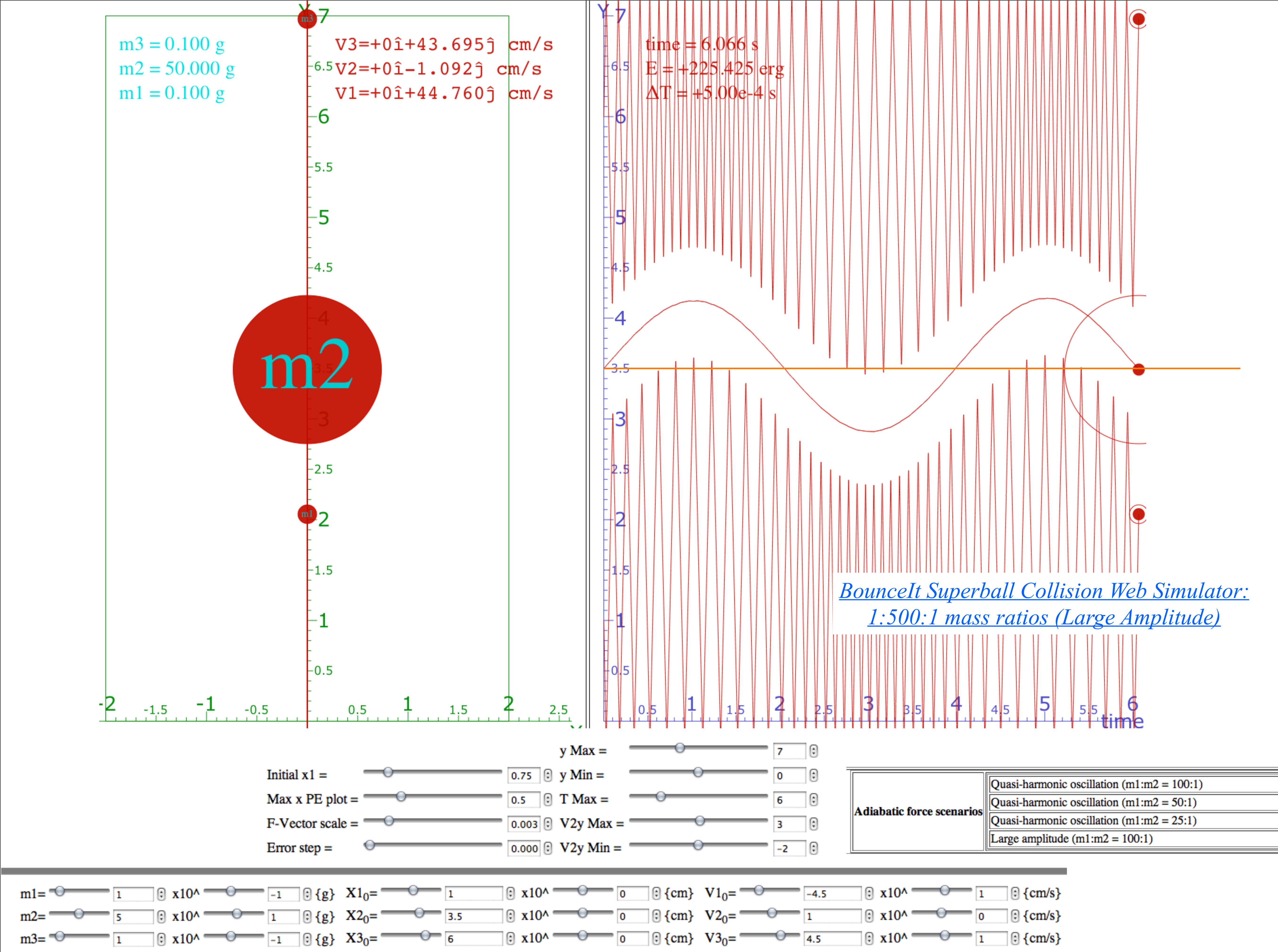
$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

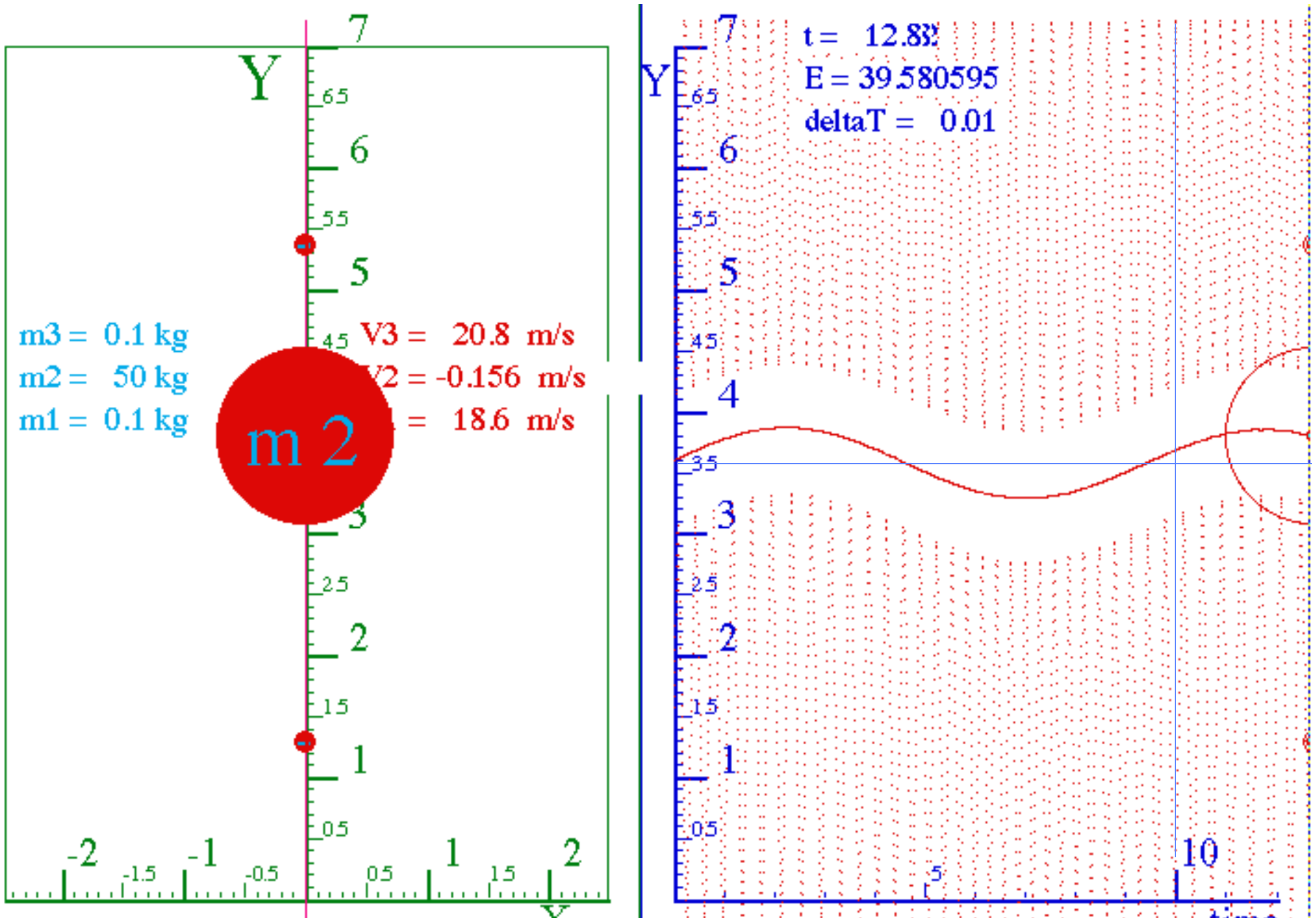
$=17.38$ *That's about √3 times too big!*

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s
m2
time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s
BounceIt Superball Collision Web Simulator:
1:500:1 mass ratios (Large Amplitude)
time
y Max = 7
Initial x1 = 0.75
y Min = 0
Max x PE plot = 0.5
T Max = 6
F-Vector scale = 0.003
V2y Max = 3
Error step = 0.000
V2y Min = -2
Adiabatic force scenarios
Quasi-harmonic oscillation (m1:m2 = 100:1)
Quasi-harmonic oscillation (m1:m2 = 50:1)
Quasi-harmonic oscillation (m1:m2 = 25:1)
Large amplitude (m1:m2 = 100:1)
m1= 1 x10^ -1 {g} X1₀= 1 x10^ 0 {cm} V1₀= -4.5 x10^ 1 {cm/s}
m2= 5 x10^ 1 {g} X2₀= 3.5 x10^ 0 {cm} V2₀= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} X3₀= 6 x10^ 0 {cm} V3₀= 4.5 x10^ 1 {cm/s}

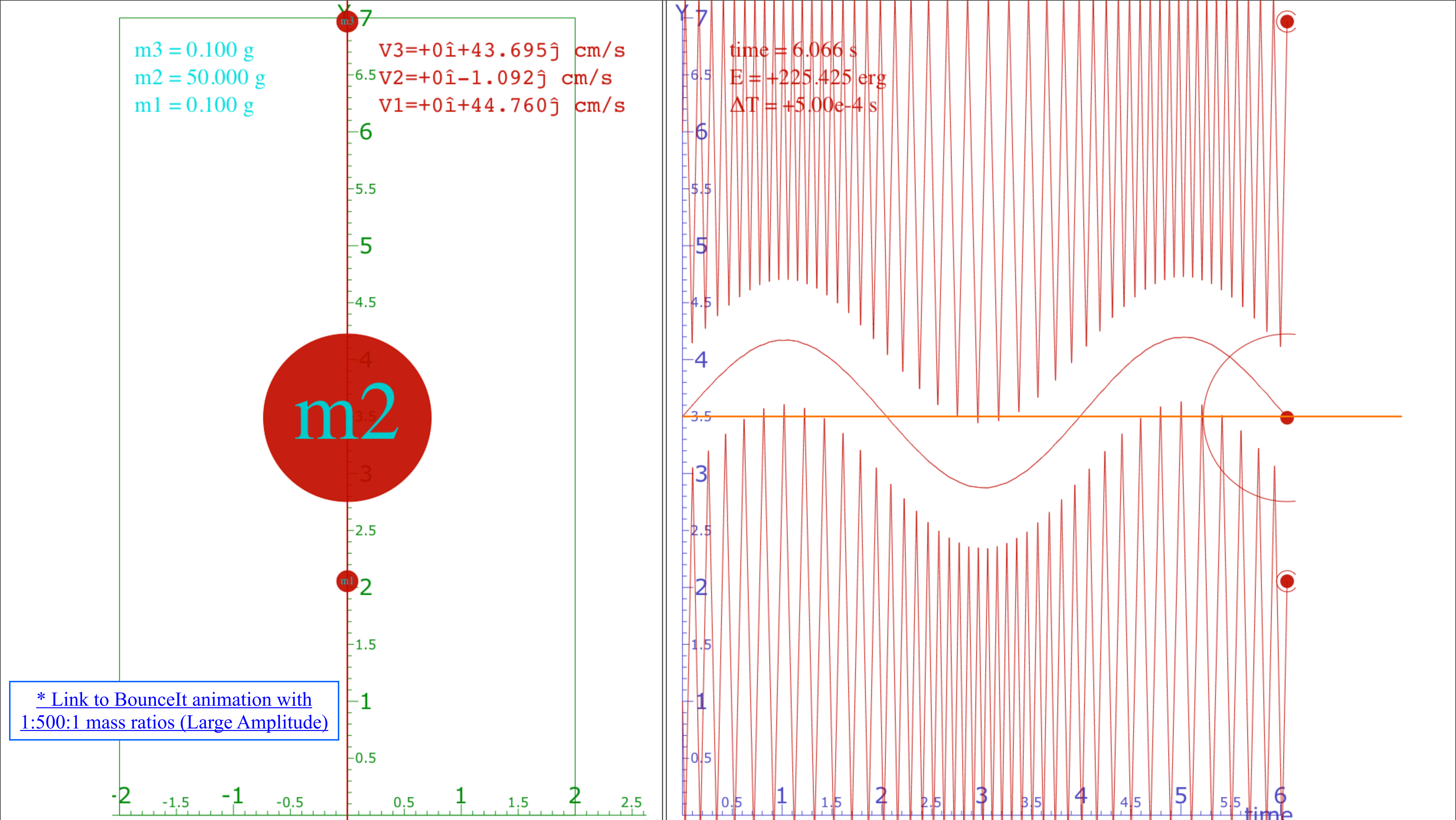


Unit 1
Fig. 6.3

Simulation of
the adiabatic case

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]

See Homework problem 1.6.5: *Compute frequency and/or period for both isoT and adiabatic cases*



y Max = 7

Initial x1 = 0.75
y Min = 0

Max x PE plot = 0.5
T Max = 6

F-Vector scale = 0.003
V2y Max = 3

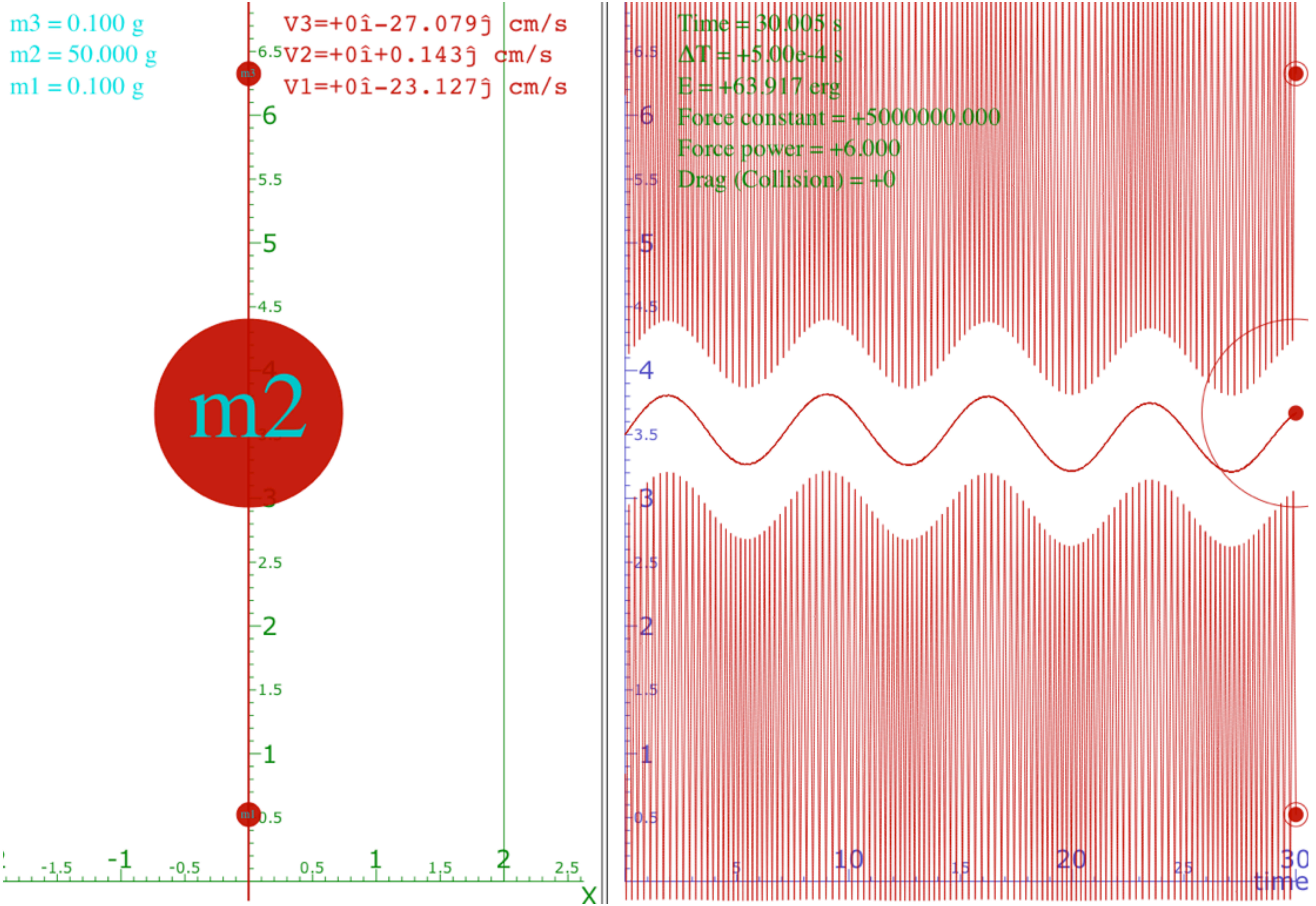
Error step = 0.000
V2y Min = -2

Adiabatic force scenarios	
	Quasi-harmonic oscillation (m1:m2 = 100:1)
	Quasi-harmonic oscillation (m1:m2 = 50:1)
	Quasi-harmonic oscillation (m1:m2 = 25:1)
	Large amplitude (m1:m2 = 100:1)

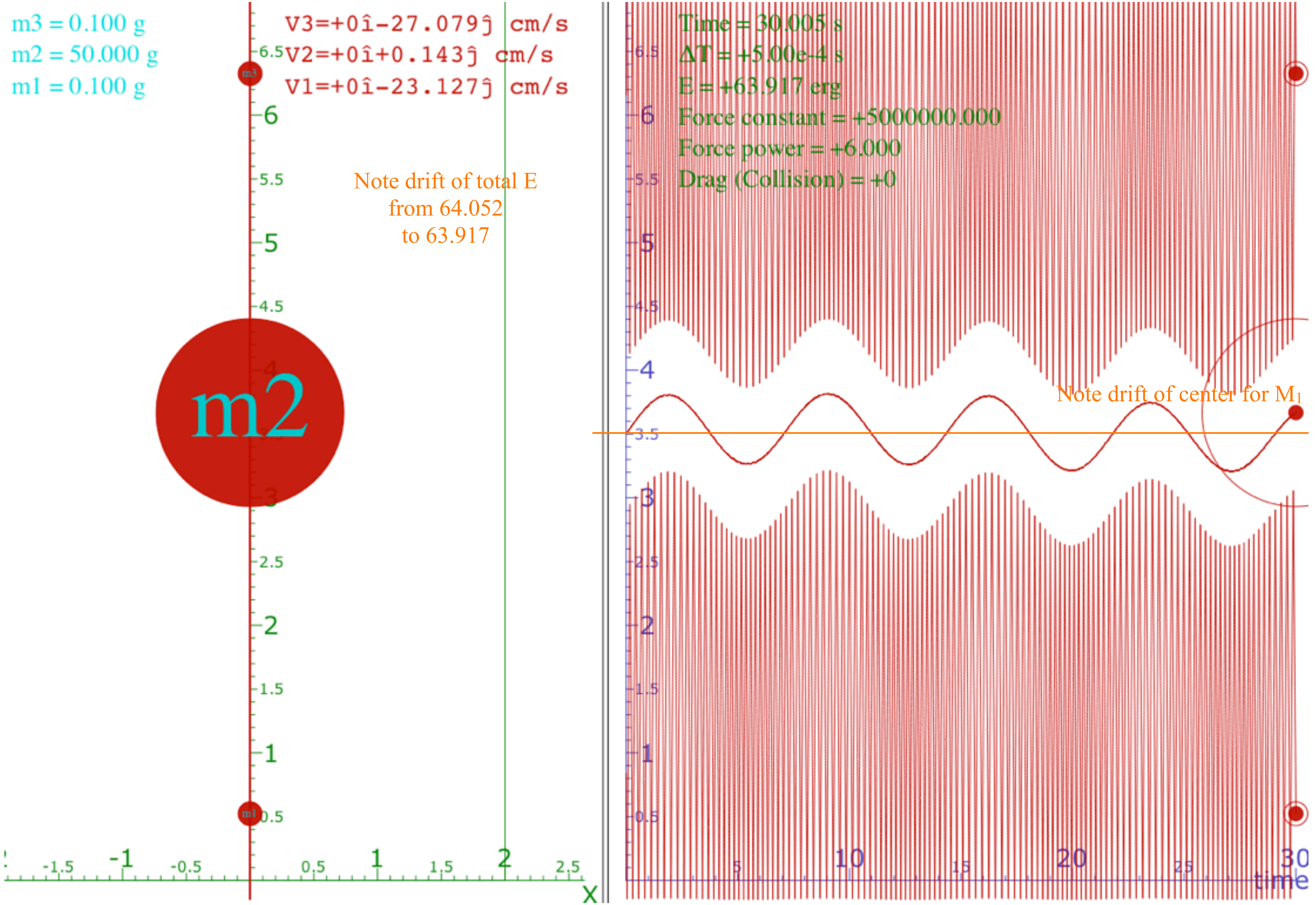
m1= 1 x10^ -1 {g} $X1_0$= 1 x10^ 0 {cm} $V1_0$= -4.5 x10^ 1 {cm/s}

m2= 5 x10^ 1 {g} $X2_0$= 3.5 x10^ 0 {cm} $V2_0$= 1 x10^ 0 {cm/s}

m3= 1 x10^ -1 {g} $X3_0$= 6 x10^ 0 {cm} $V3_0$= 4.5 x10^ 1 {cm/s}



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



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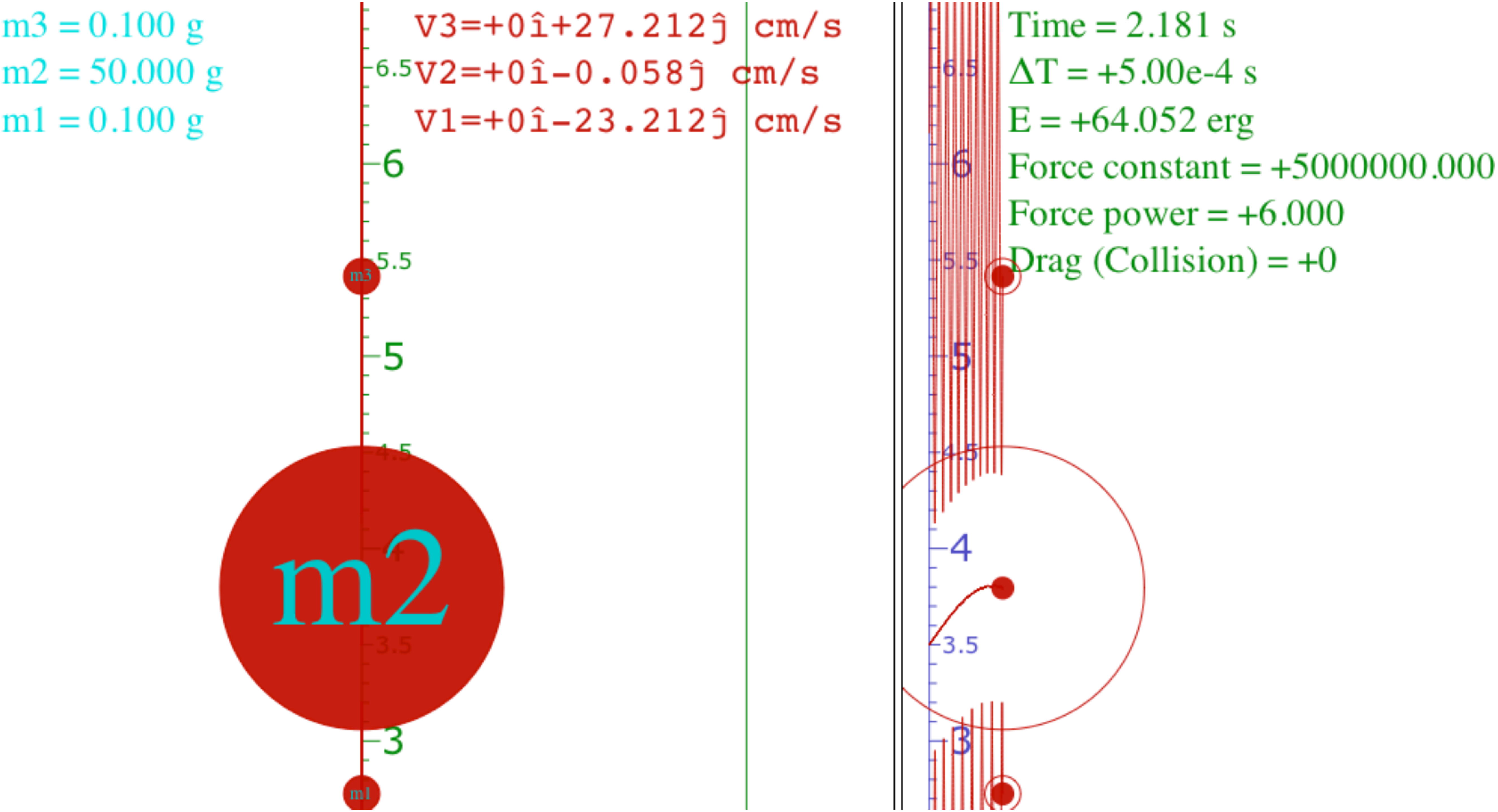

m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+27.212ĵ cm/s
V2=+0î-0.058ĵ cm/s
V1=+0î-23.212ĵ cm/s
Time = 2.181 s
ΔT = +5.00e-4 s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0
m2
m3
m1
6.5
6
5.5
5
4.5
4
3.5
3

"Monster Mash" classical segue to Heisenberg action relations

→ *Example of very very large M_1 ball-walls crushing a poor little m_2*

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

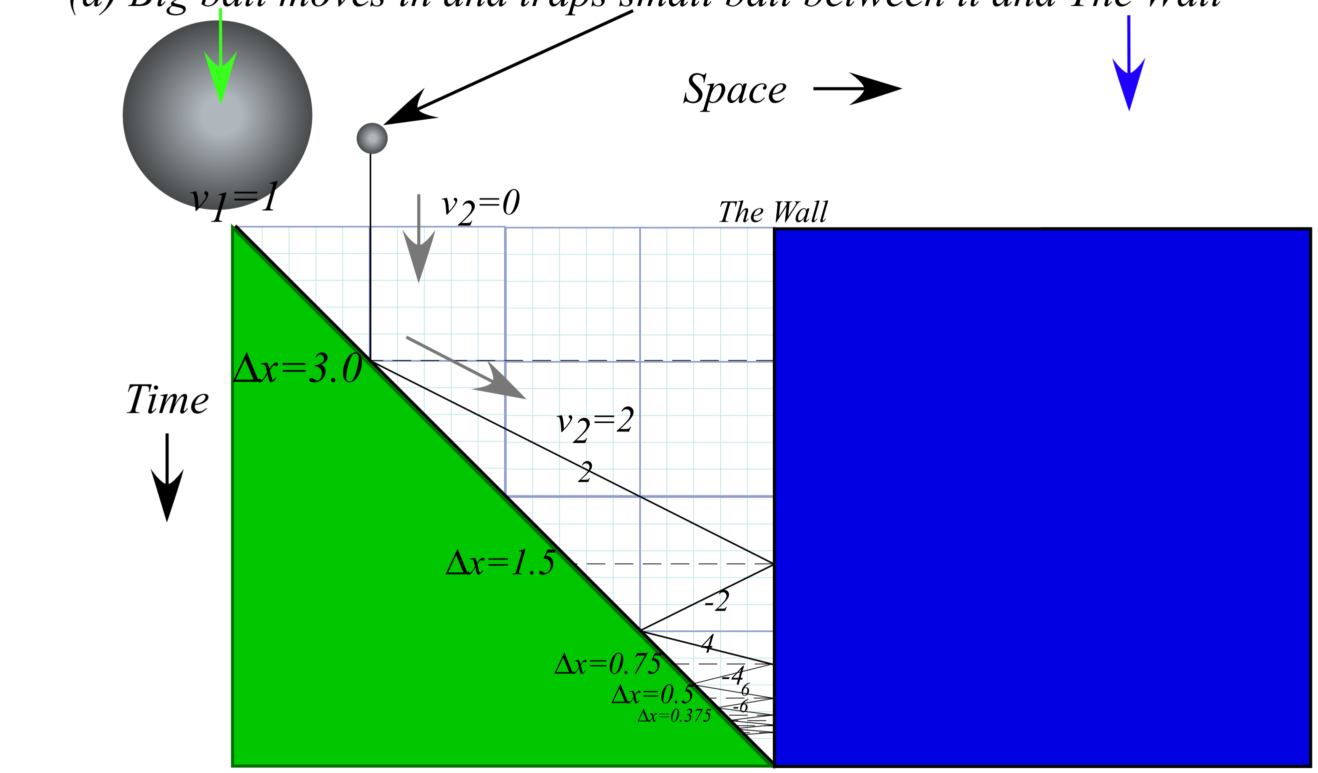
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

The Classical "Monster Mash"

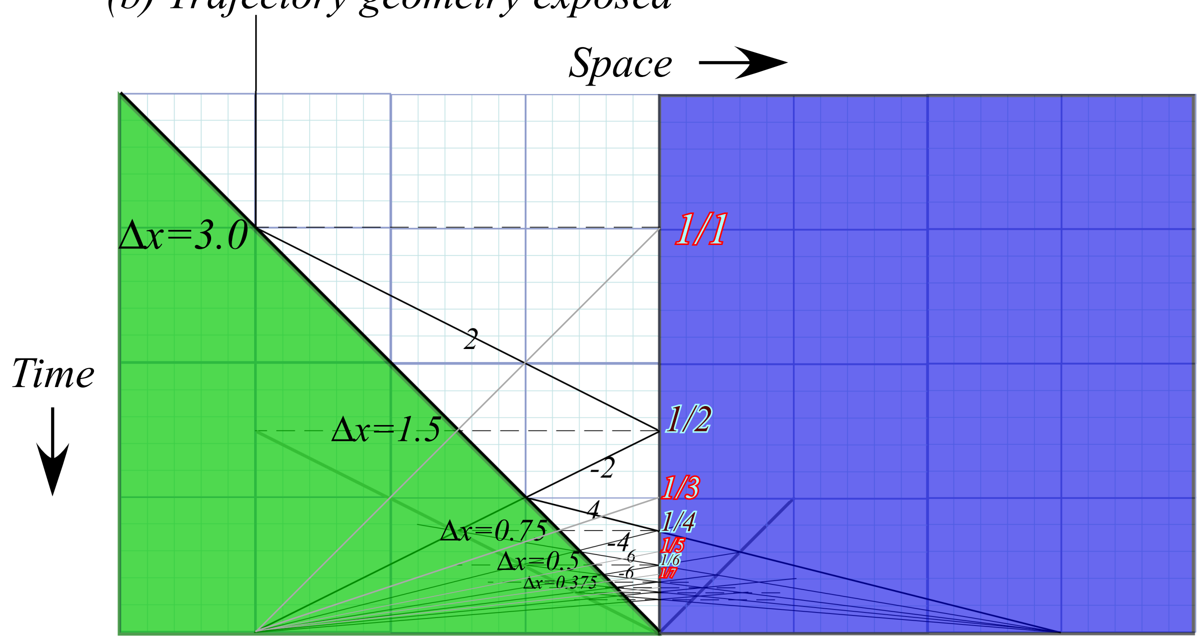
Classical introduction to

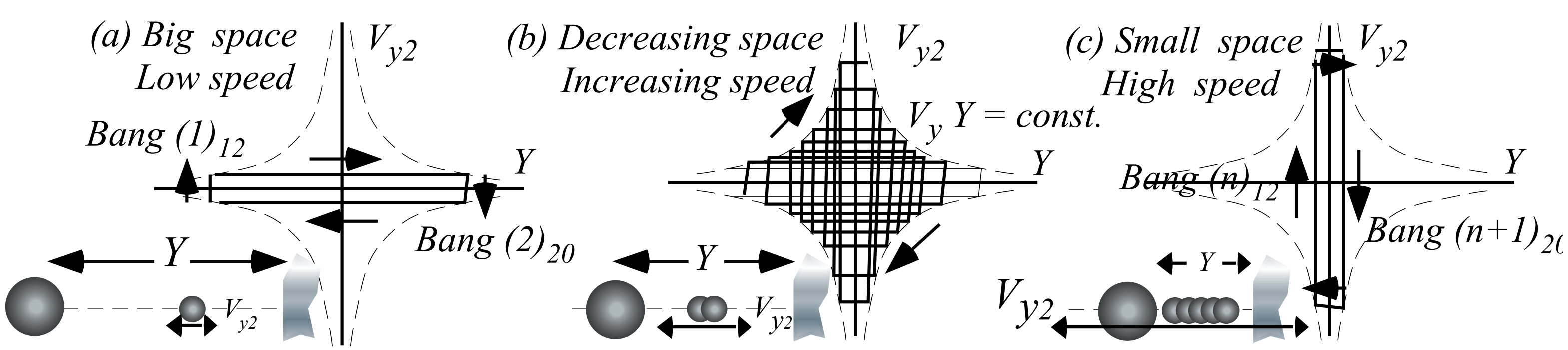
Heisenberg "Uncertainty" Relations

(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed



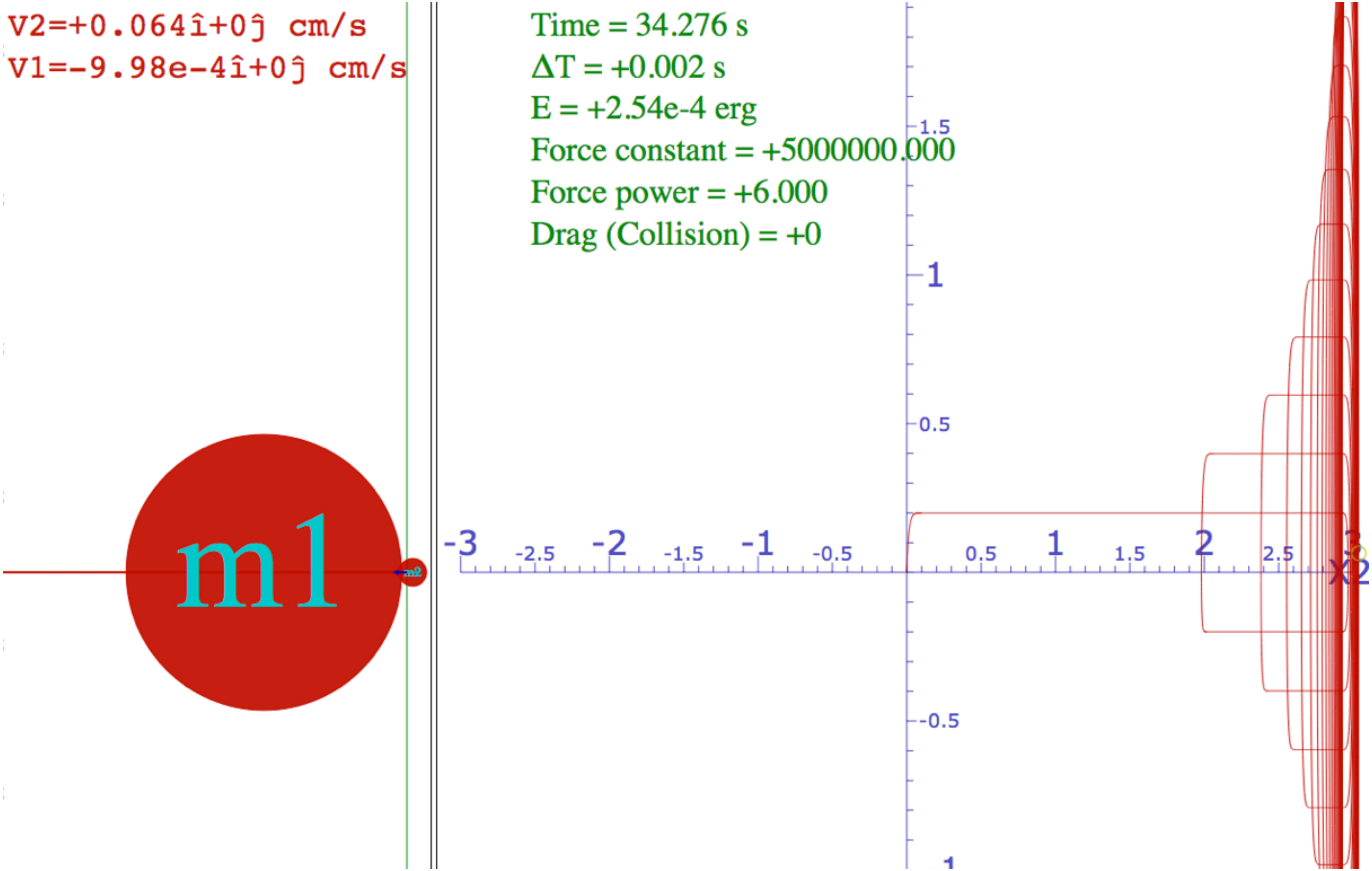


$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

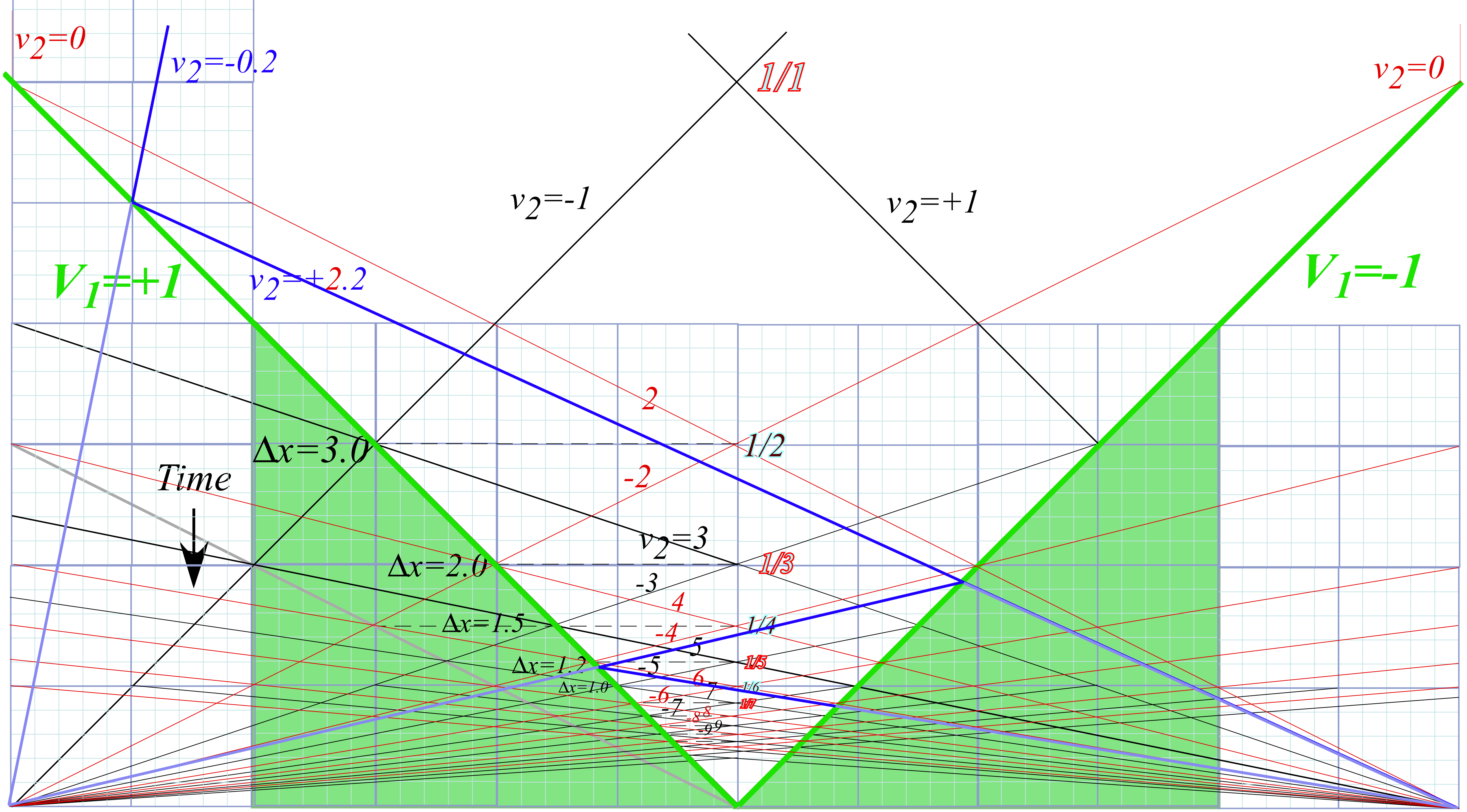
Unit 1
Fig. 6.4

[* Link to BounceIt "*Monster Mash*" $x_2(t)$ animation](#)
[(Note: Time sense is inverted)](#)



* Link to BounceIt "*Monster Mash*" Vx_2 vs x_2 animation

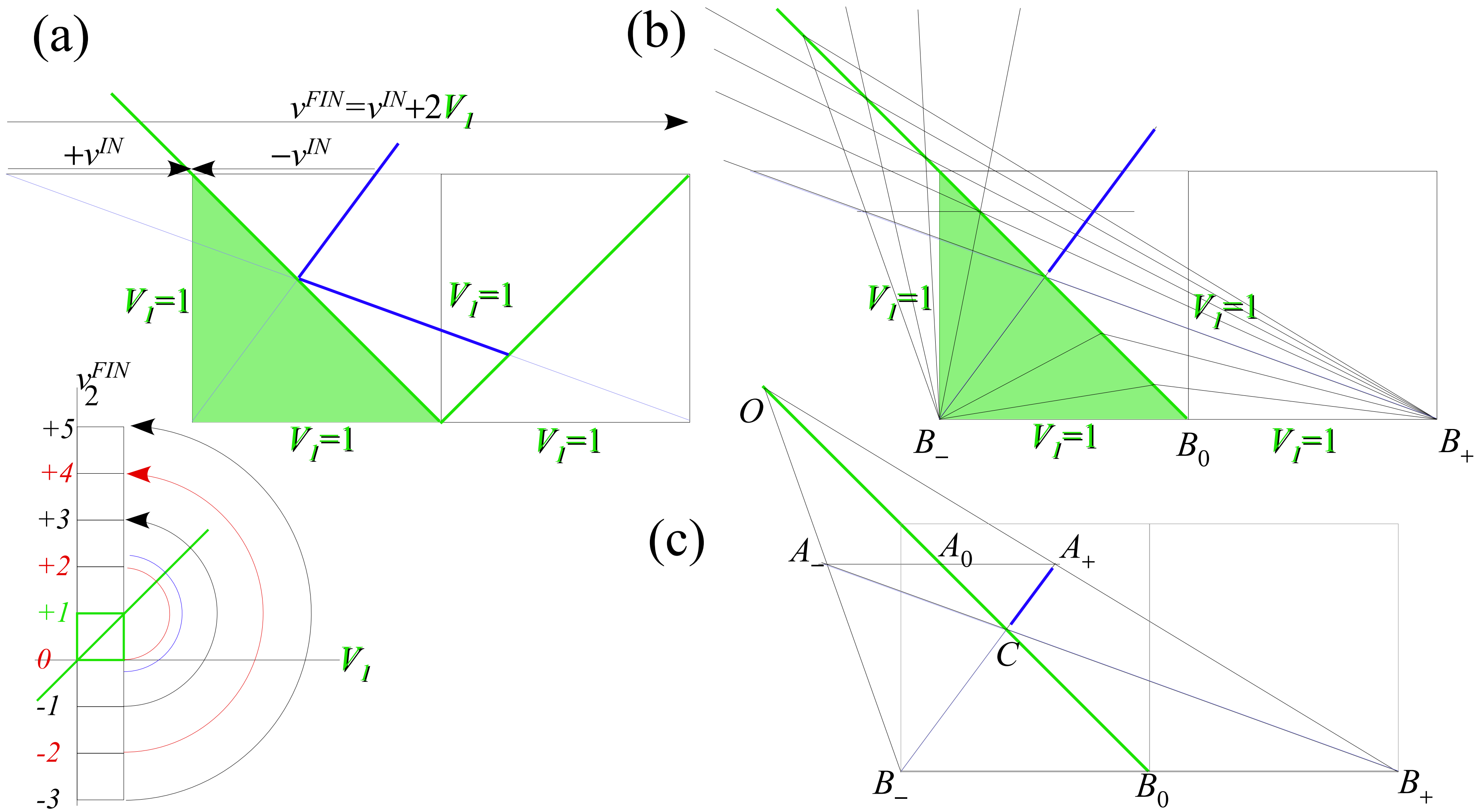
Double "Monster Mash"

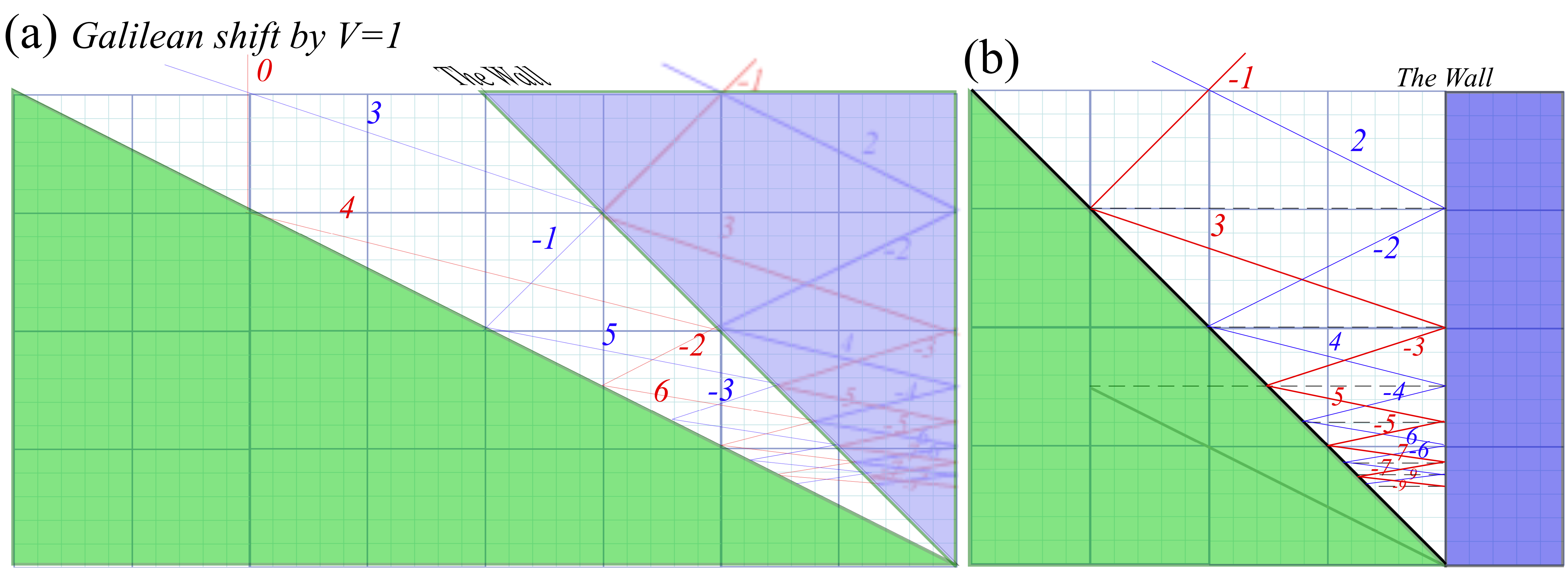


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

→ *An interesting wave analogy: The "Tiny-Big-Bang"* [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

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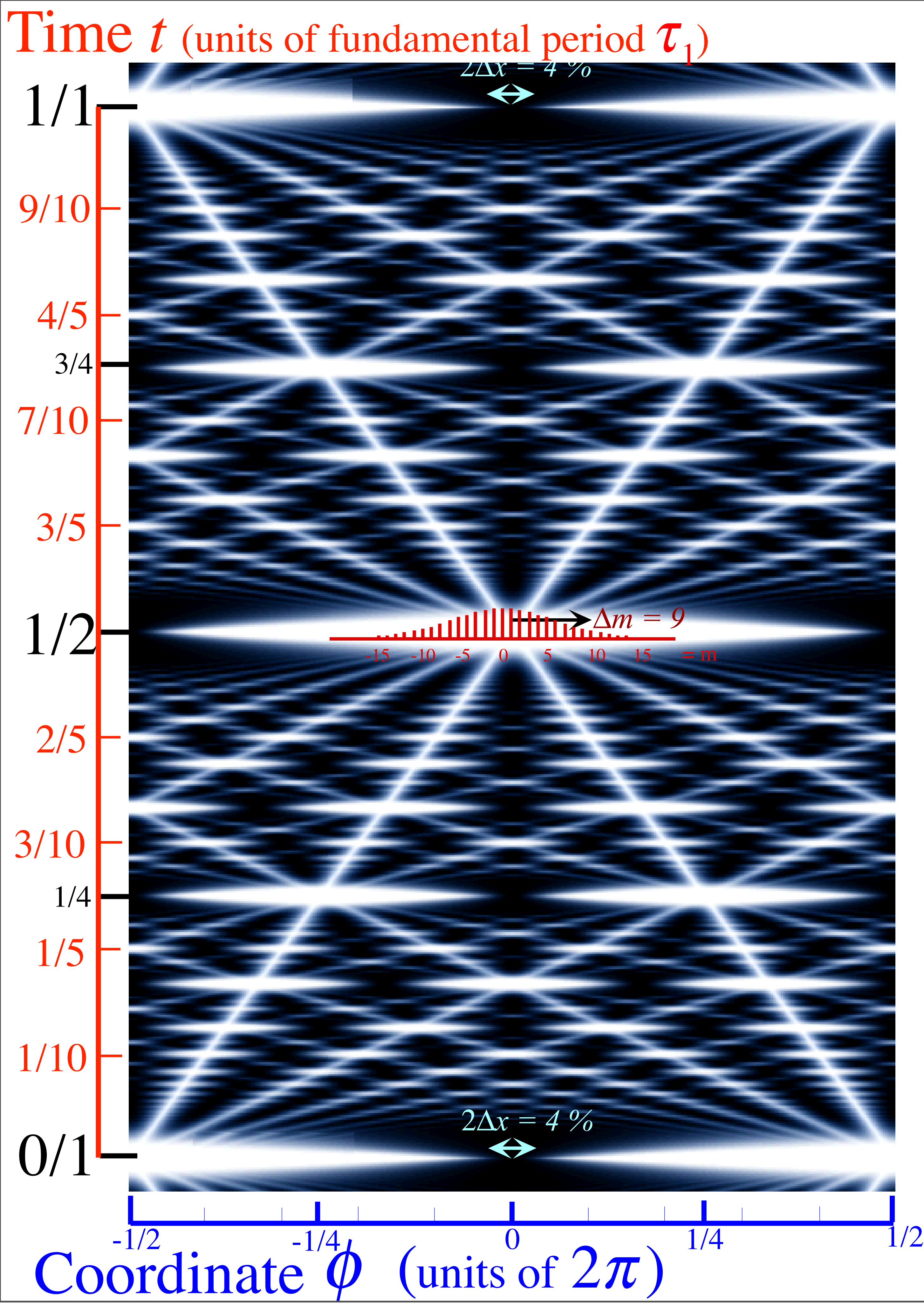
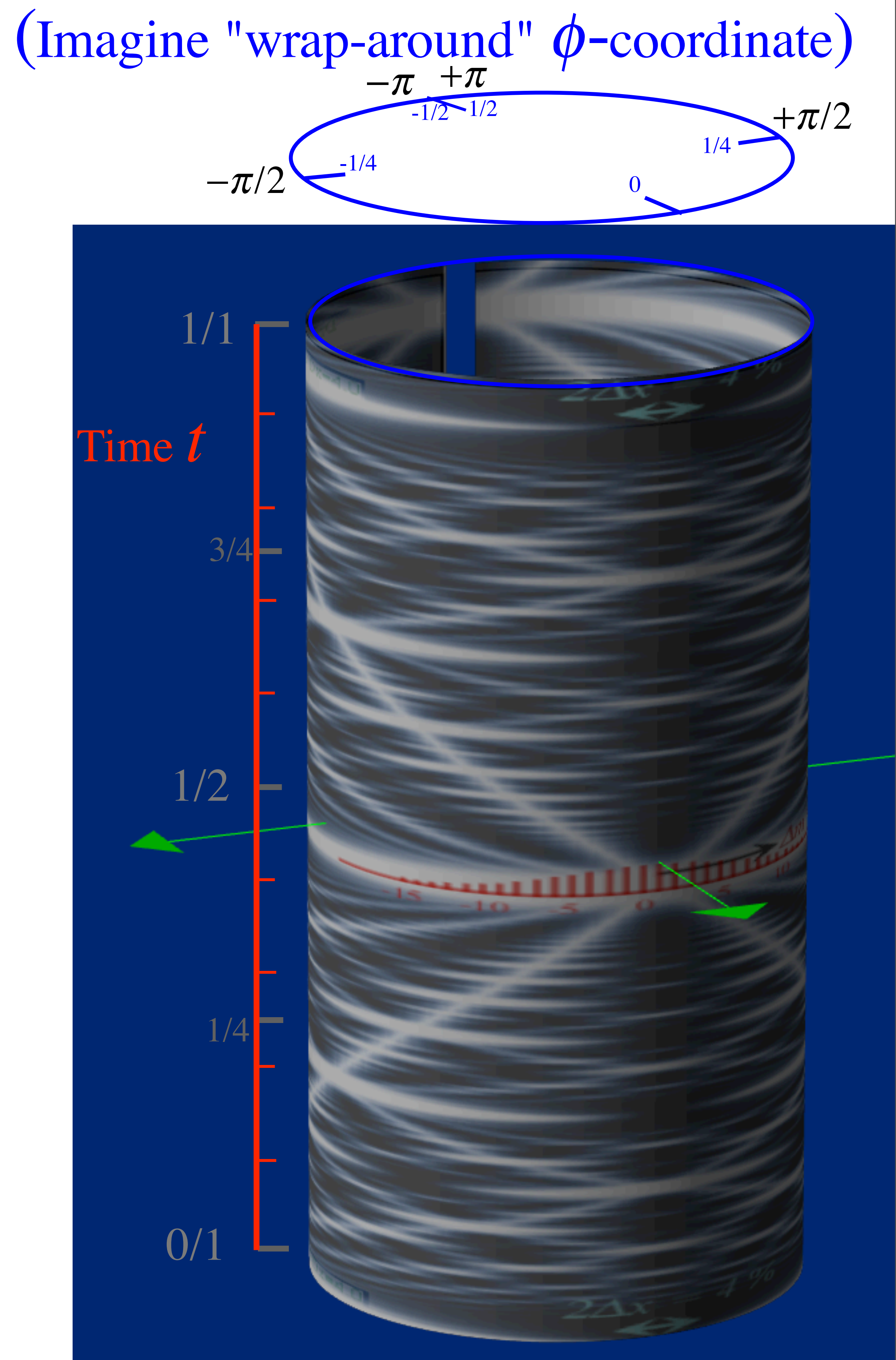


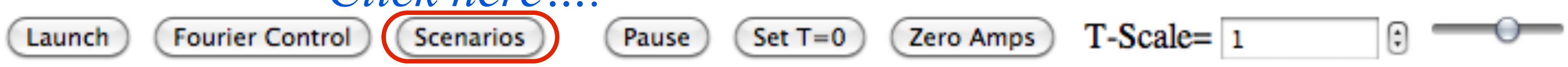
Time t (units of fundamental period τ_1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4\ \%$
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
-1/2
-1/4
0
1/4
1/2
Coordinate ϕ (units of 2π)



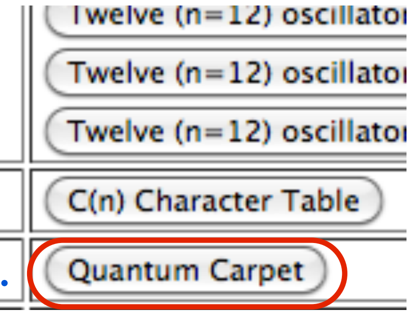
(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
$-\pi/2$
-1/4
0
Time t
1/1
3/4
1/2
1/4
0/1

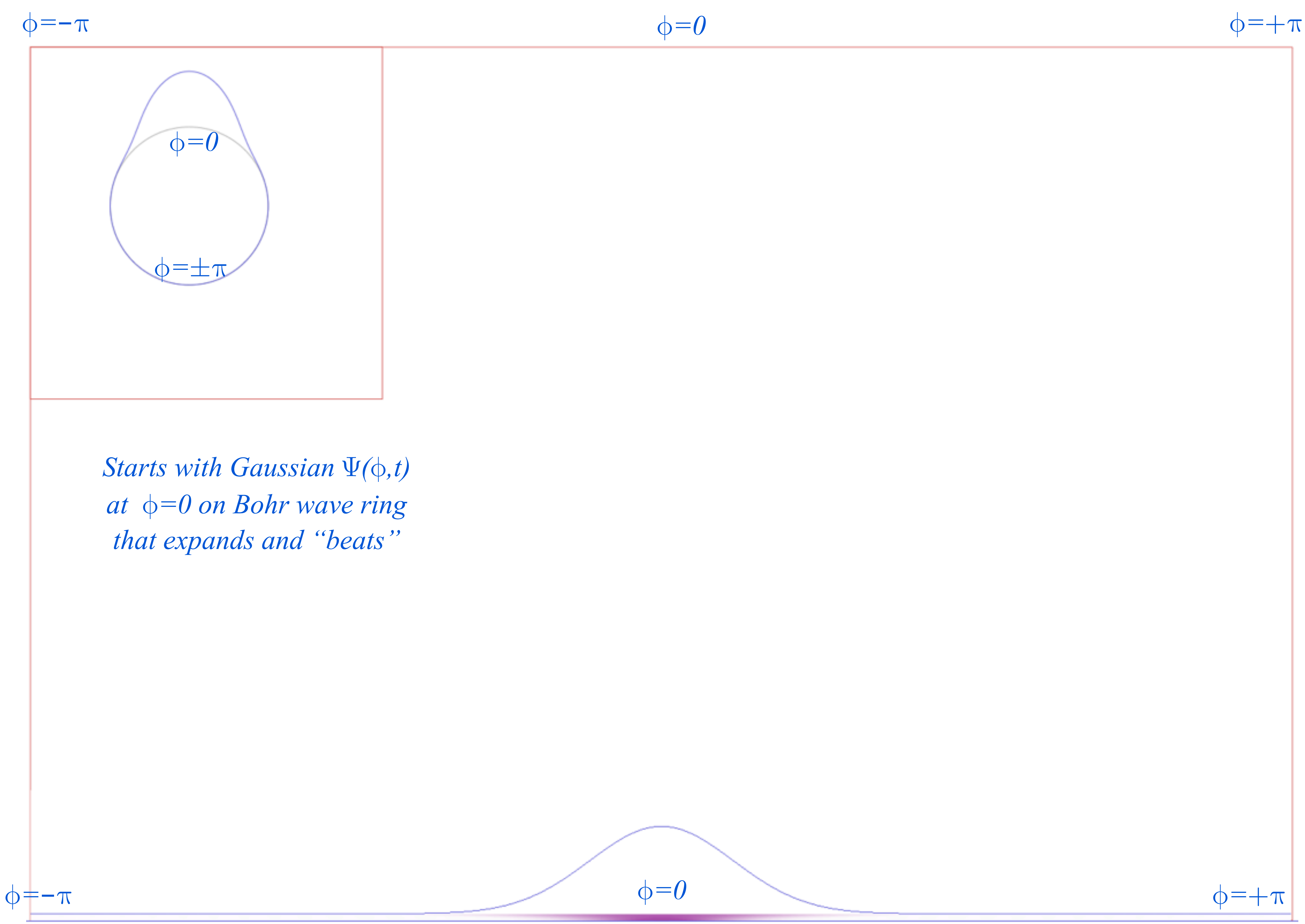
WaveIt Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

Click here....



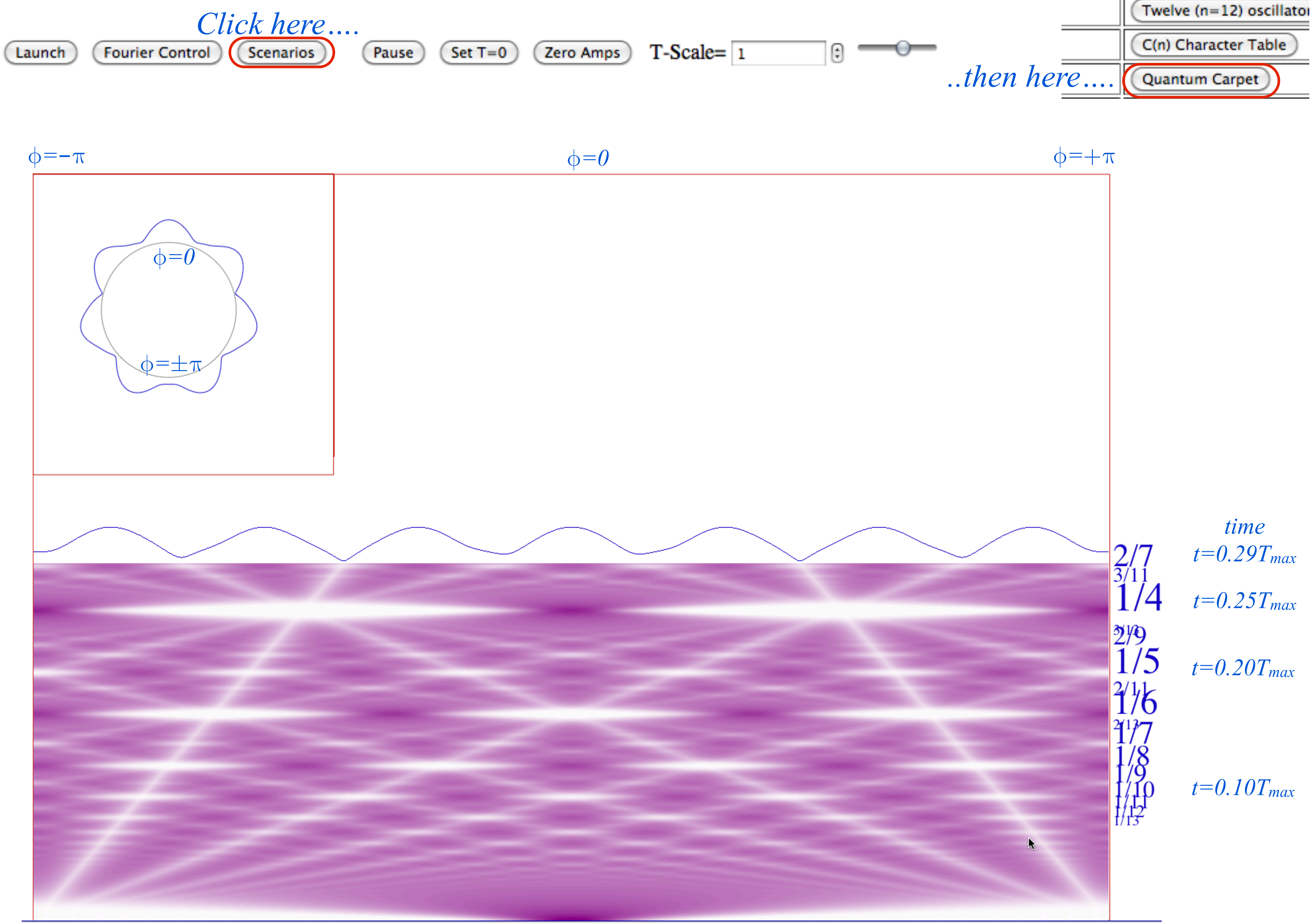
..then here....

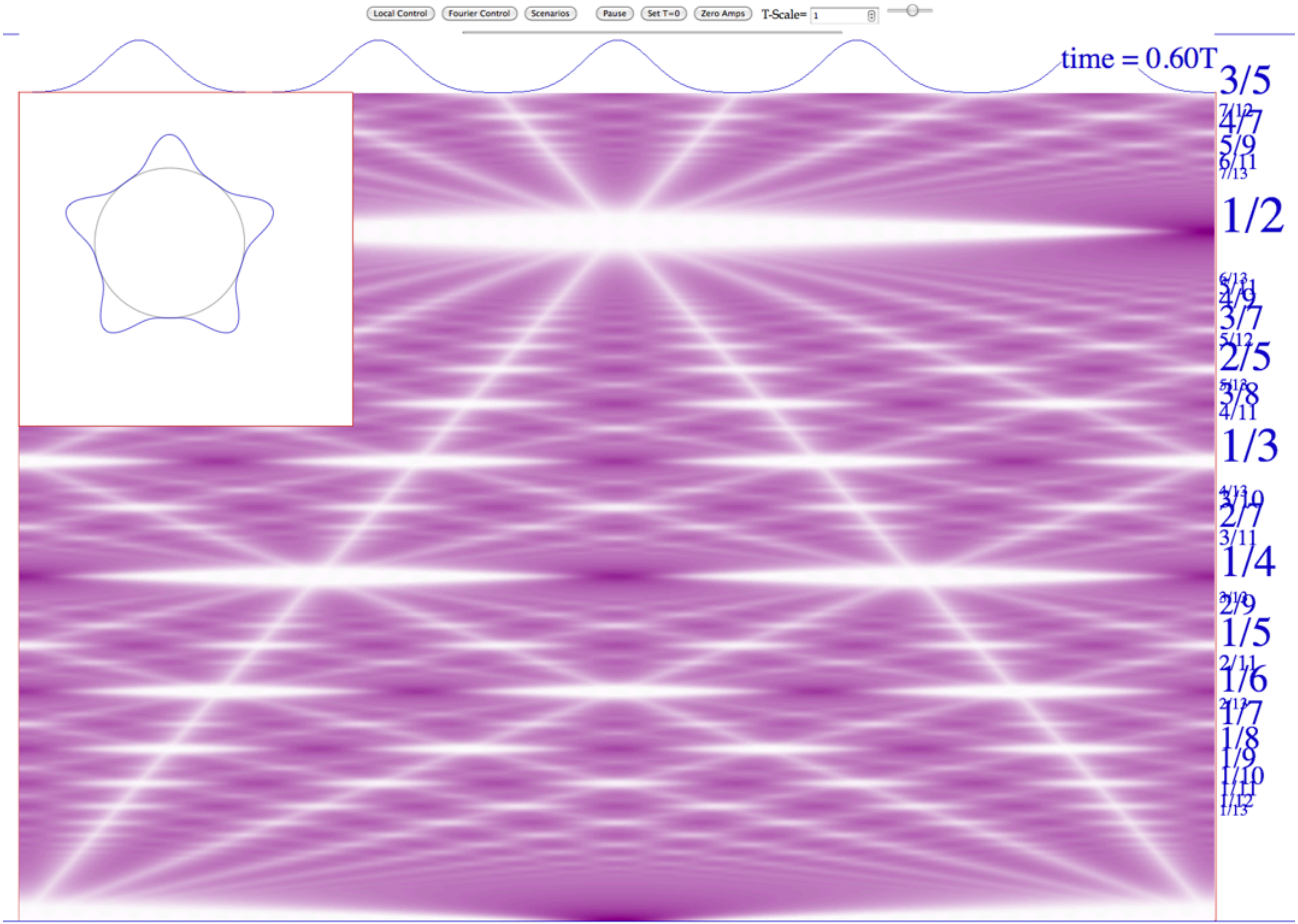




Starts with Gaussian Ψ(ϕ,t)
at ϕ=0 on Bohr wave ring
that expands and "beats"

WaveIt Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet




Local Control
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale= 1
time = 0.60T
3/5
7/12
4/7
5/9
6/11
7/13
1/2
6/13
5/11
4/9
3/7
5/12
2/5
5/13
3/8
4/11
1/3
4/13
3/10
2/7
3/11
1/4
3/13
2/9
1/5
2/11
1/6
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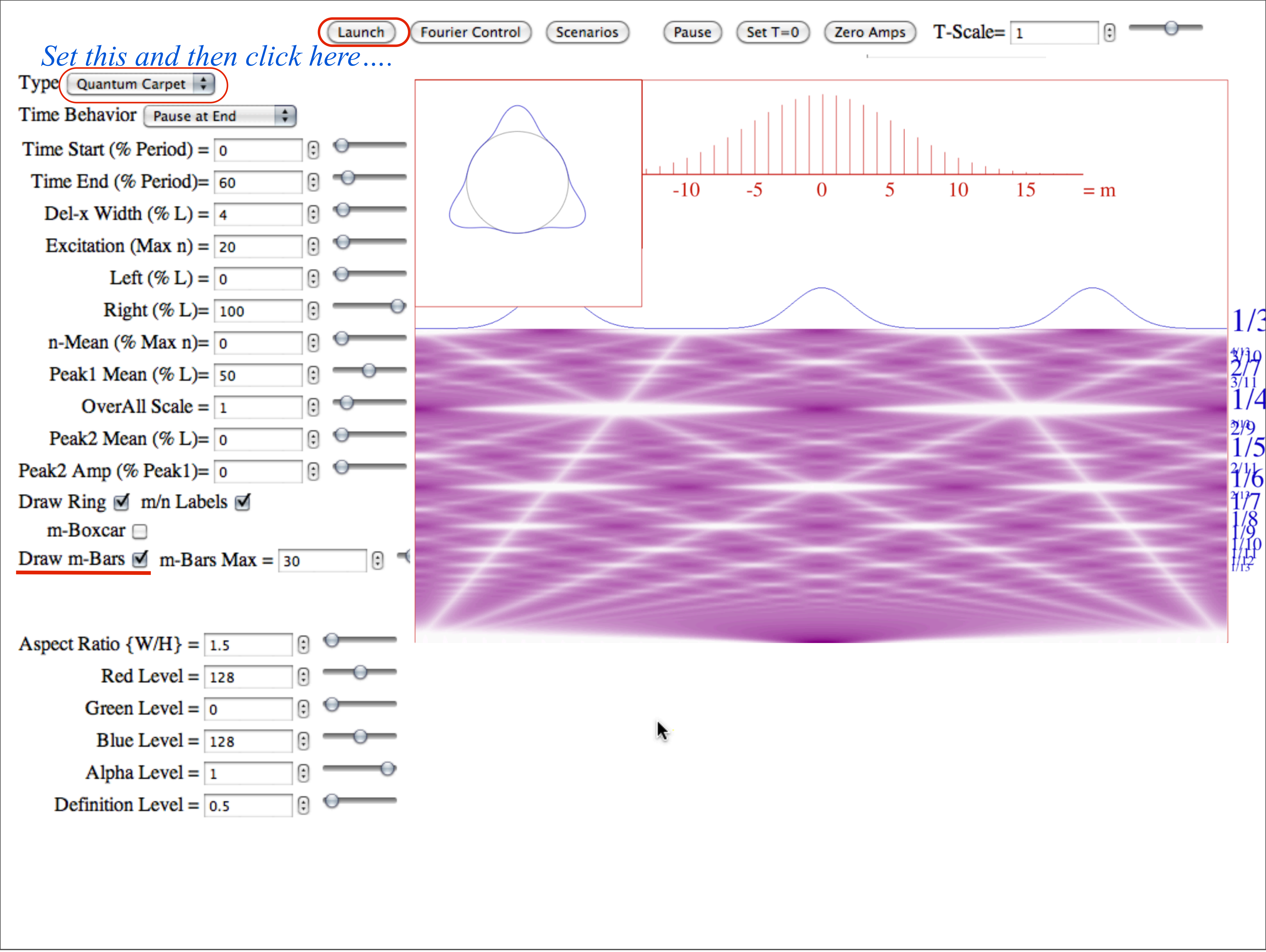
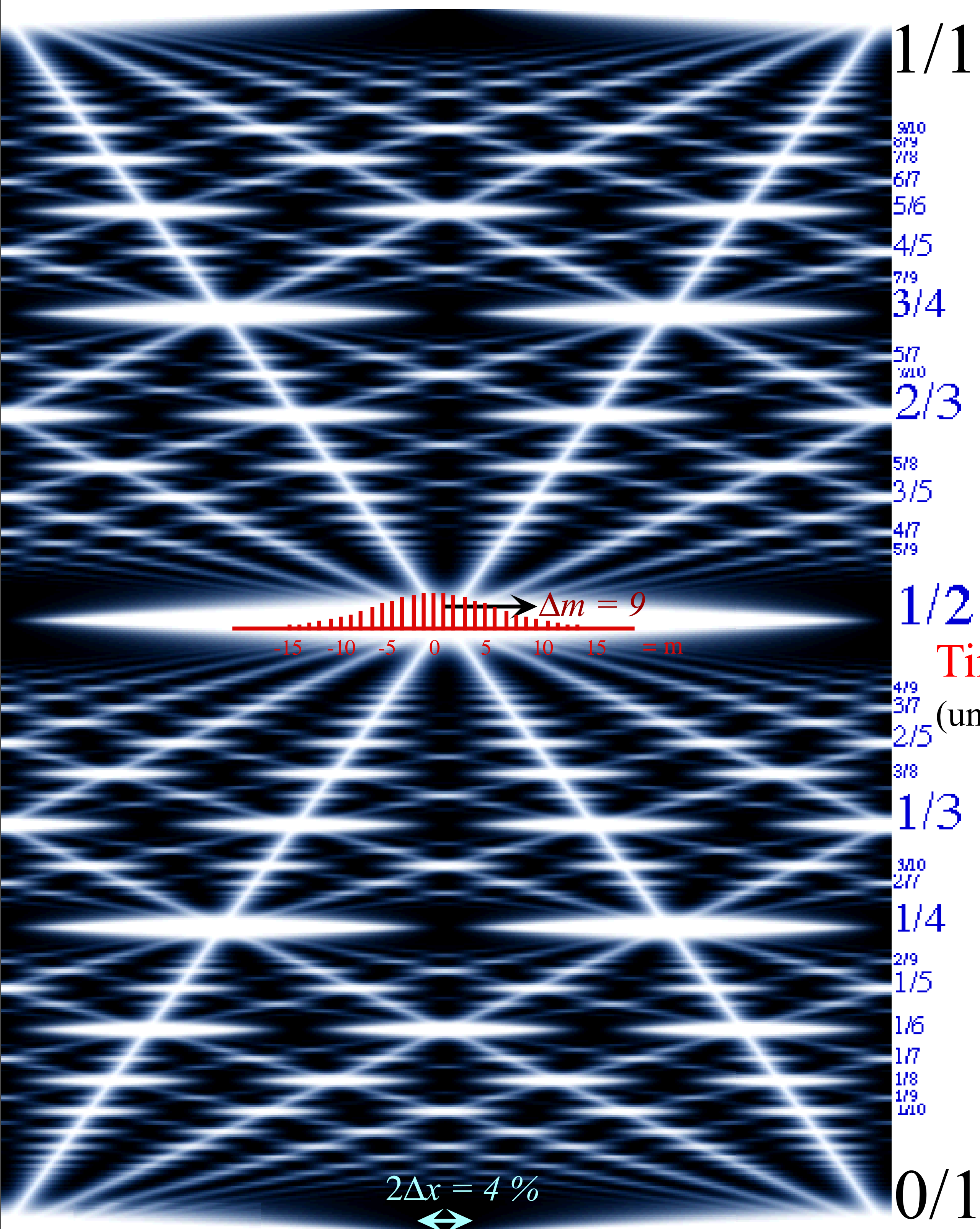
1/11
1/12
1/13

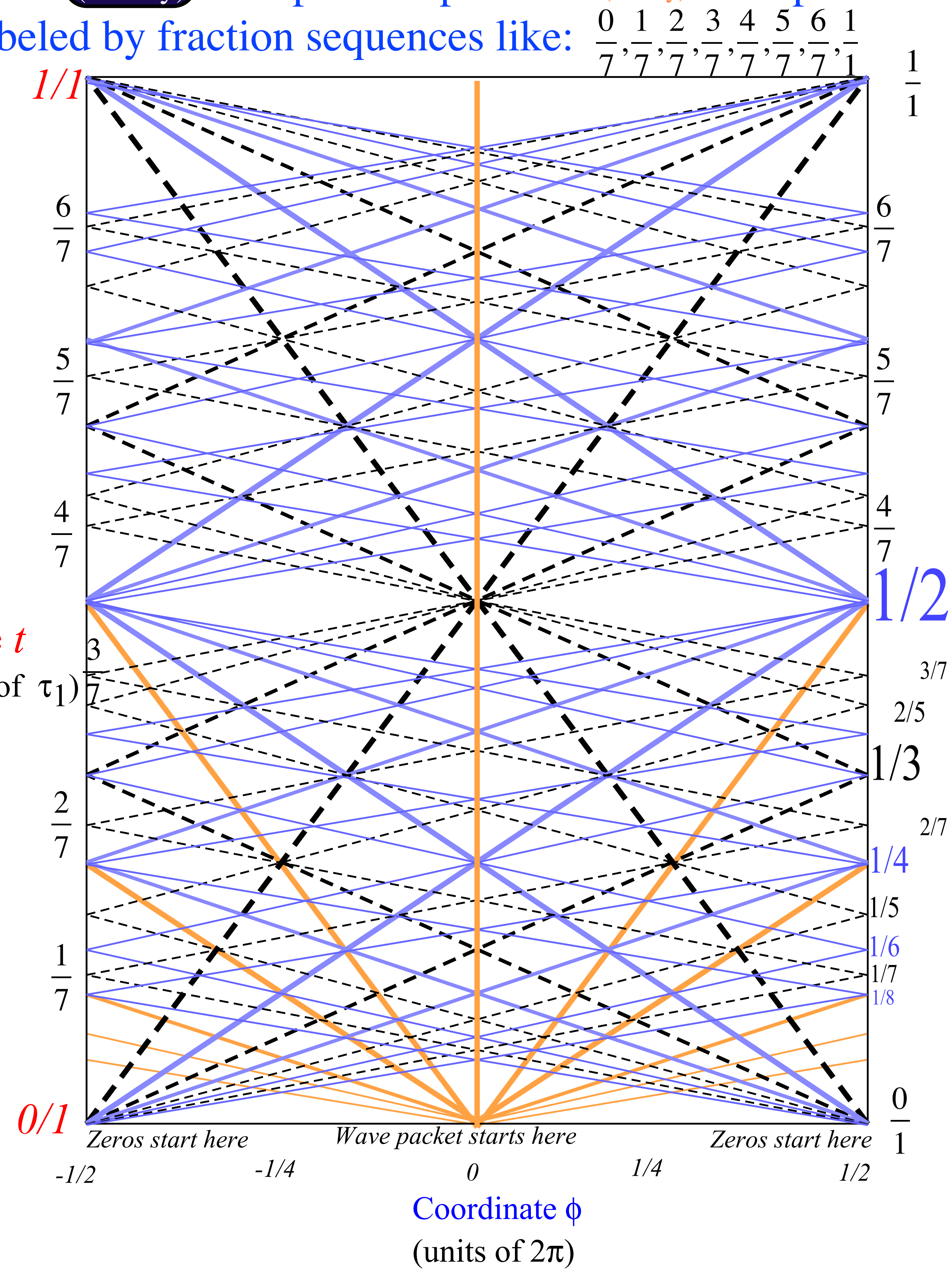

Set this and then click here....
Launch
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale= 1
Type Quantum Carpet
Time Behavior Pause at End
Time Start (% Period) = 0
Time End (% Period)= 60
Del-x Width (% L) = 4
Excitation (Max n) = 20
Left (% L) = 0
Right (% L)= 100
n-Mean (% Max n)= 0
Peak1 Mean (% L)= 50
OverAll Scale = 1
Peak2 Mean (% L)= 0
Peak2 Amp (% Peak1)= 0
Draw Ring m/n Labels
m-Boxcar
Draw m-Bars m-Bars Max = 30
Aspect Ratio {W/H} = 1.5
Red Level = 128
Green Level = 0
Blue Level = 128
Alpha Level = 1
Definition Level = 0.5
-10
-5
0
5
10
15
= m
1/3
2/7
1/4
2/9
1/5
1/6
1/7
1/8
1/9
1/10

N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

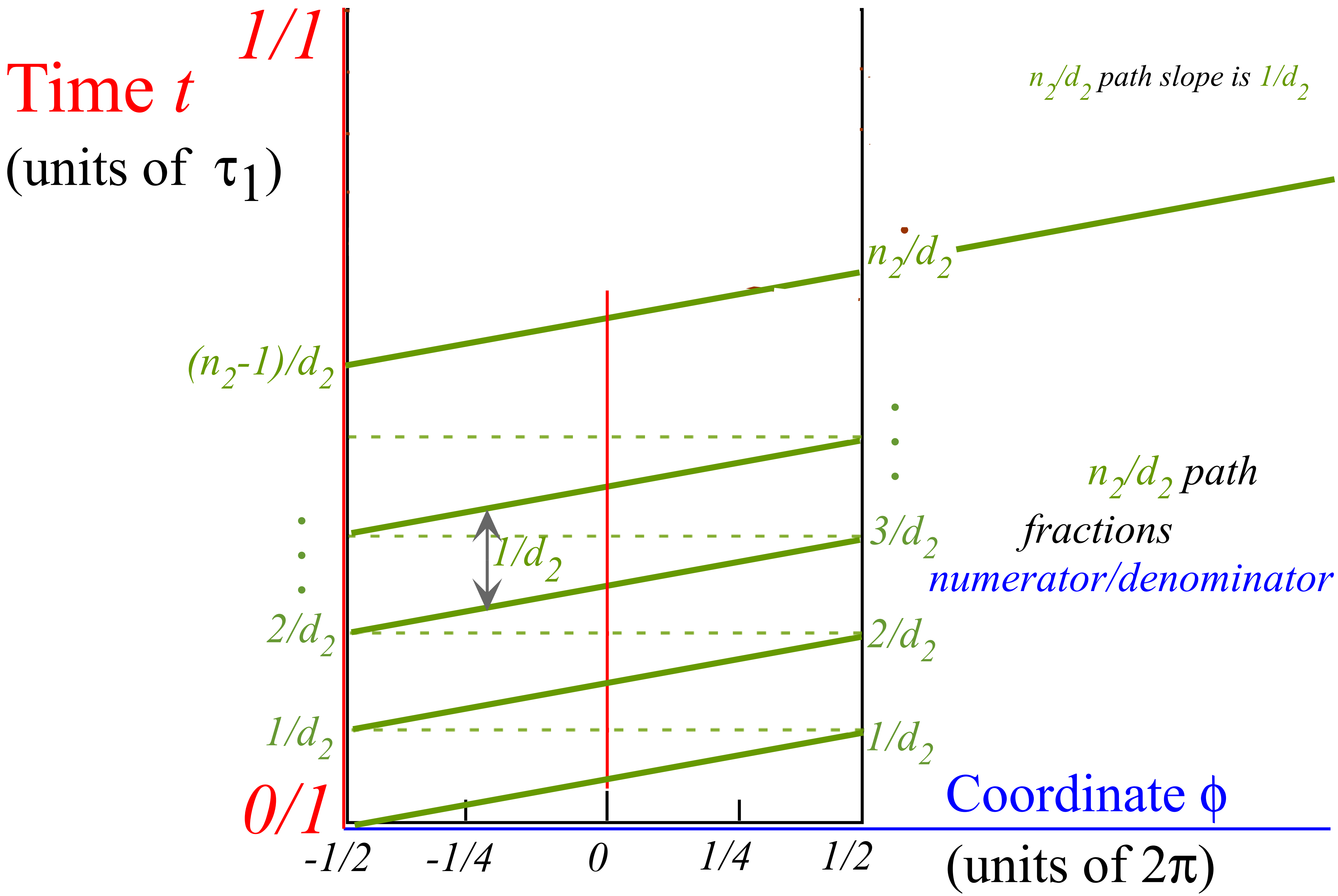
Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$





Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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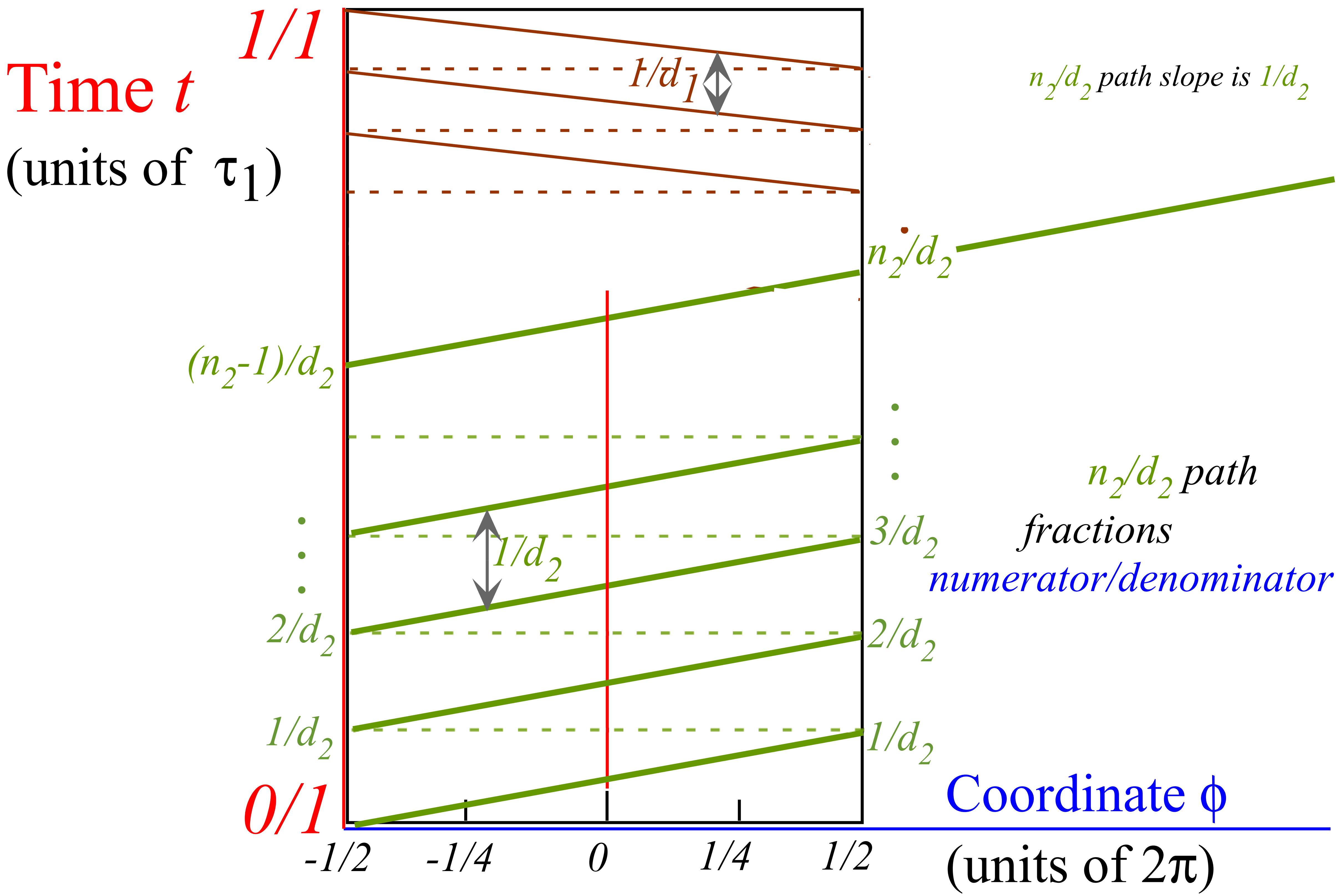

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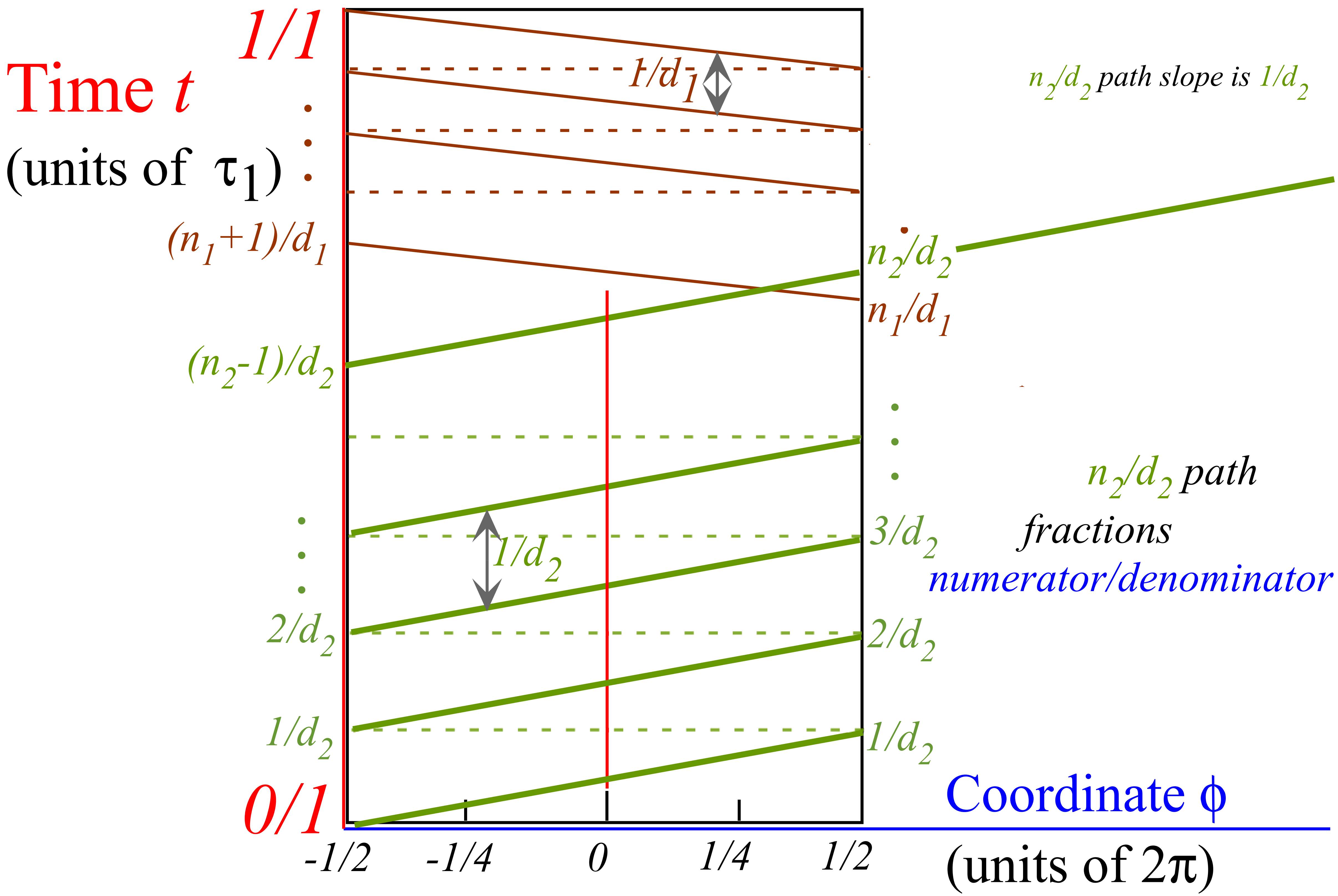

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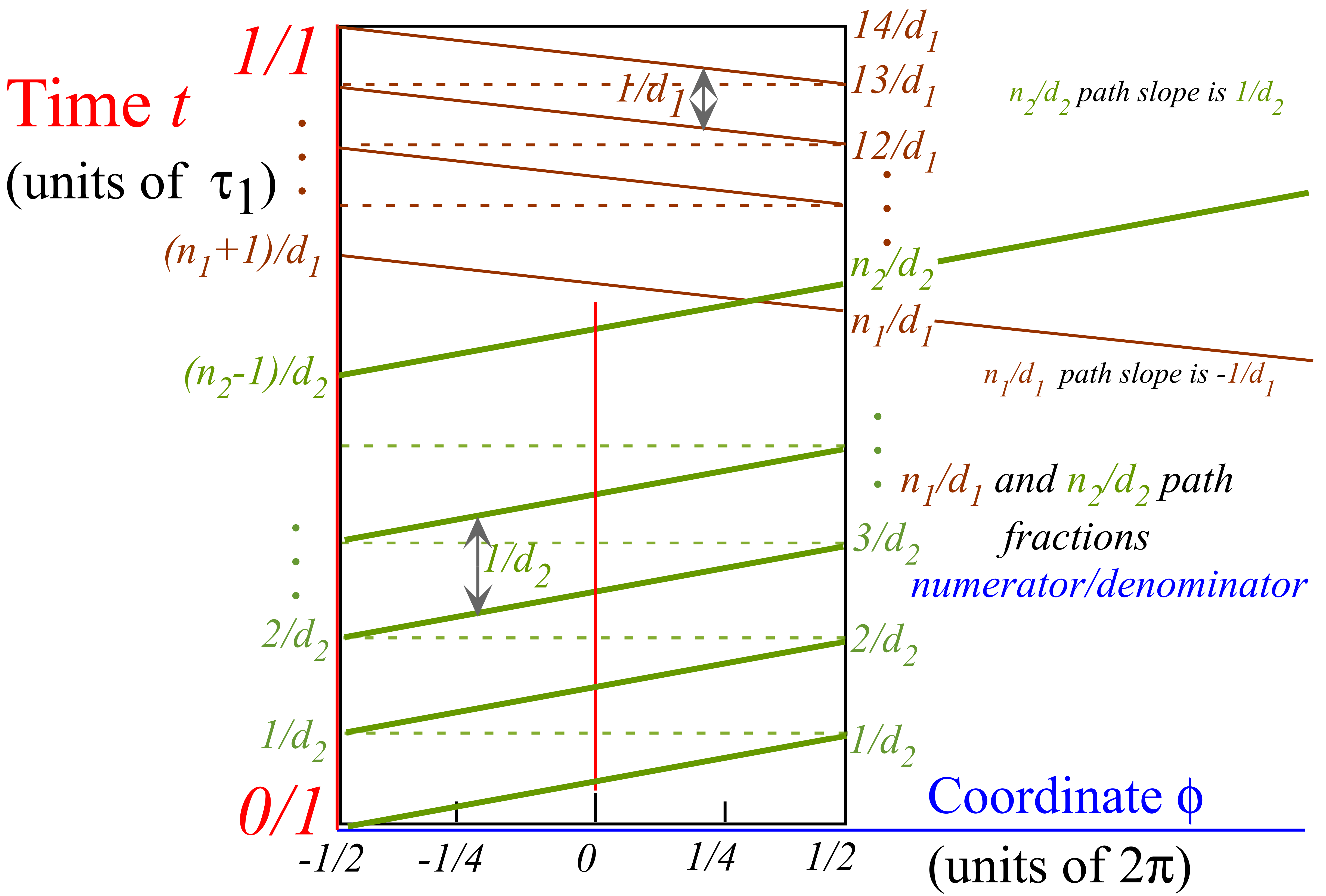

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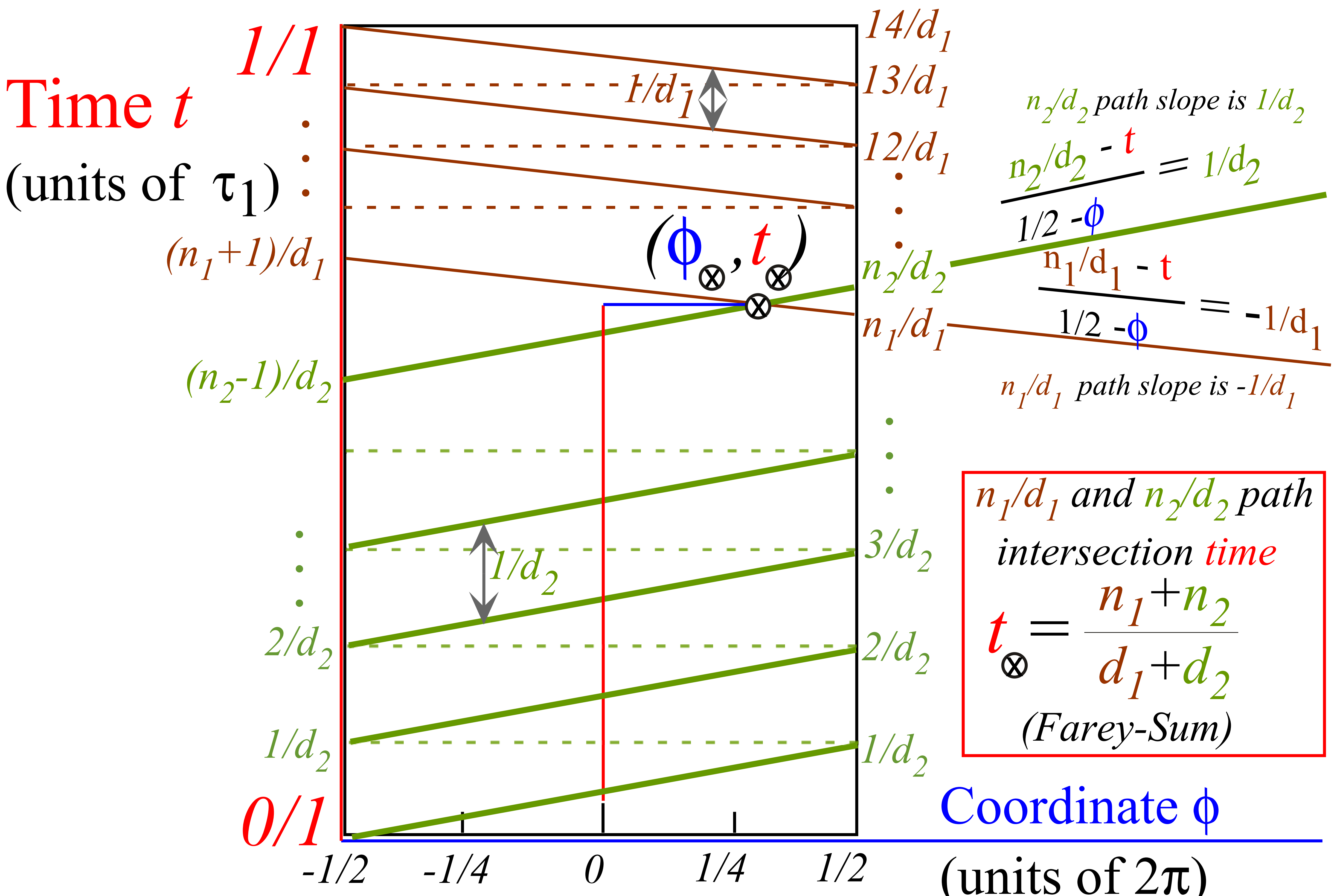

[John Farey, Phil. Mag.(1816)]

Farey Sum algebra of revival-beat wave dynamics

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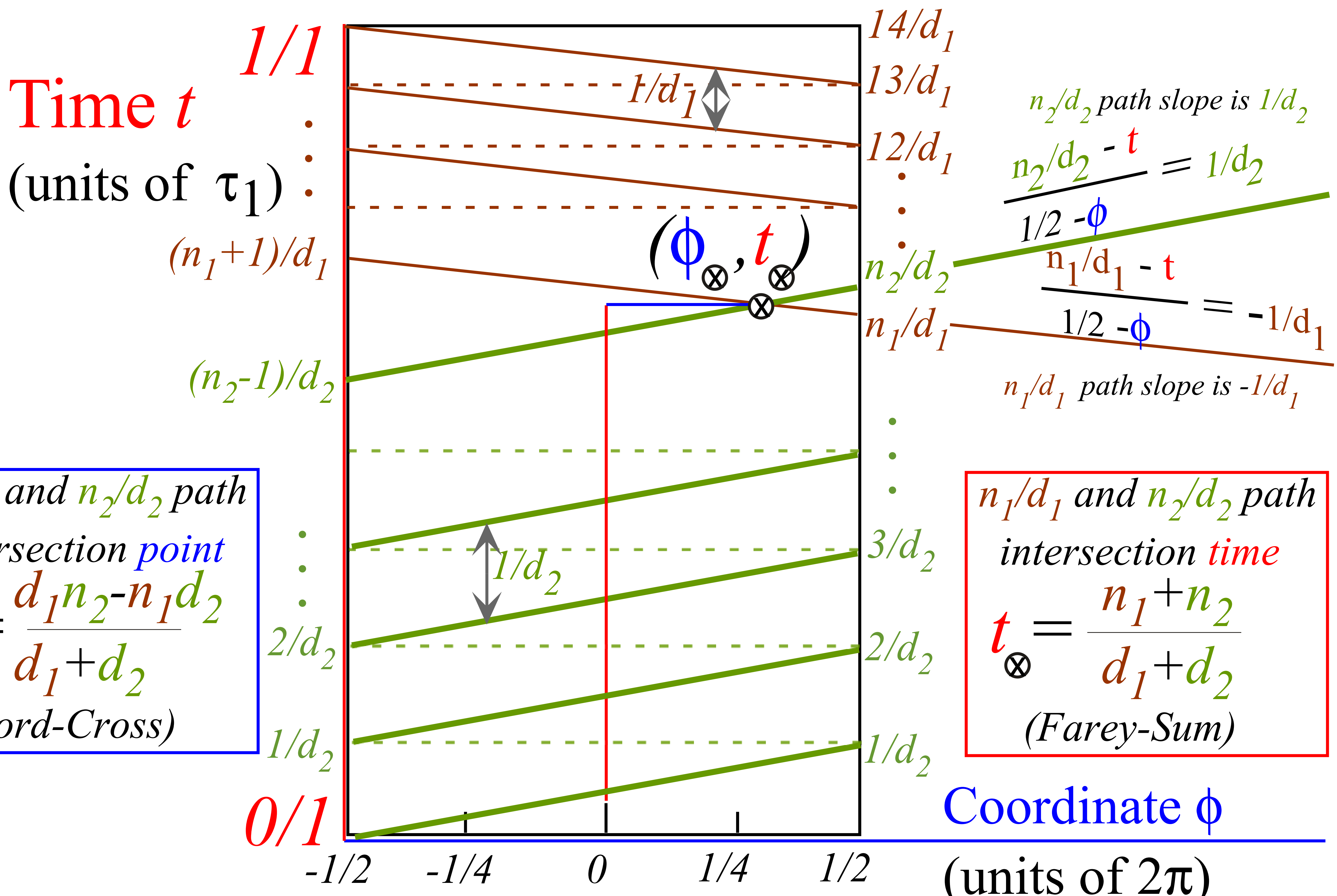


n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*]

[*John Farey, Phil. Mag.(1816)*]

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

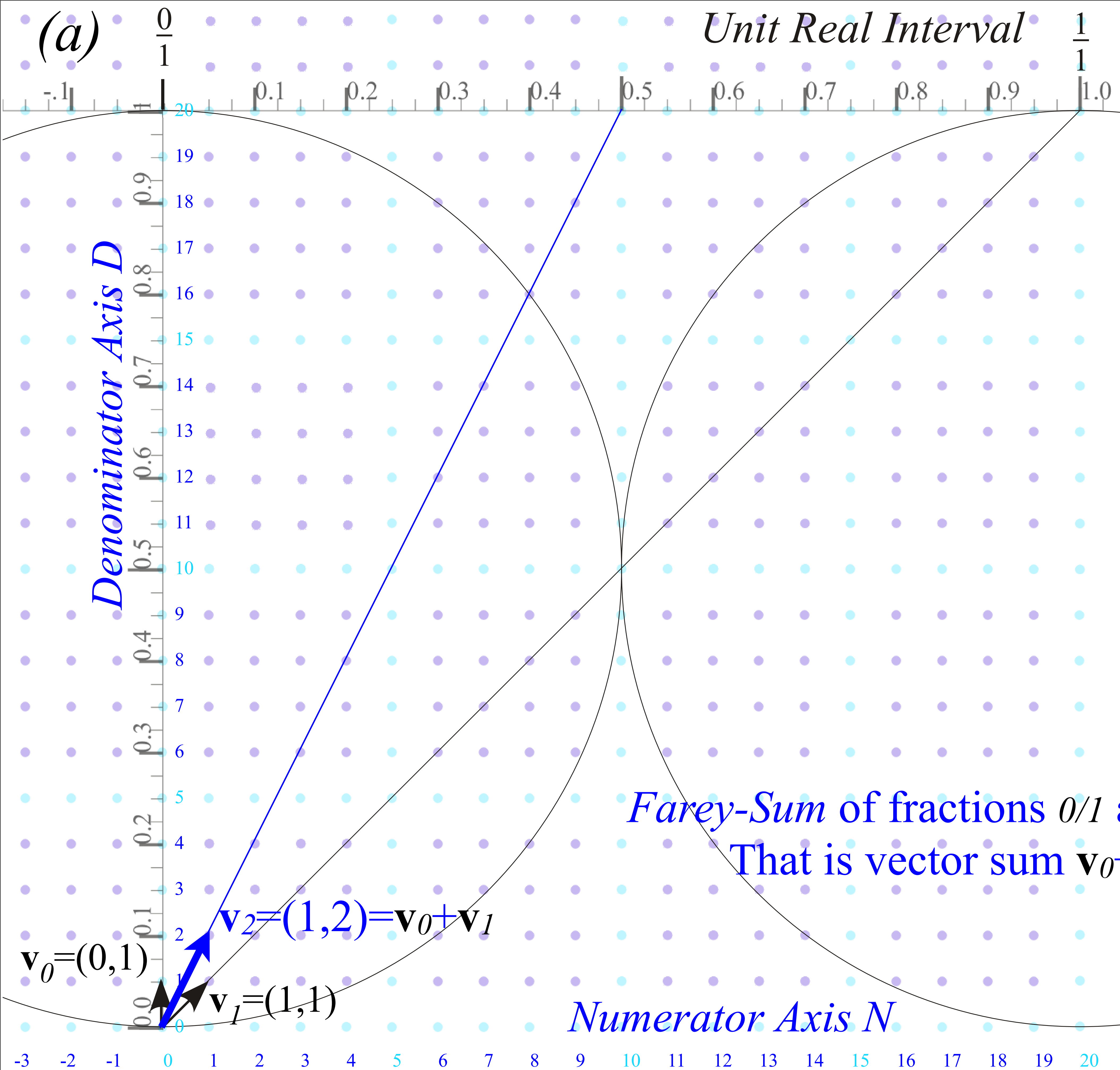
An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

→ *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

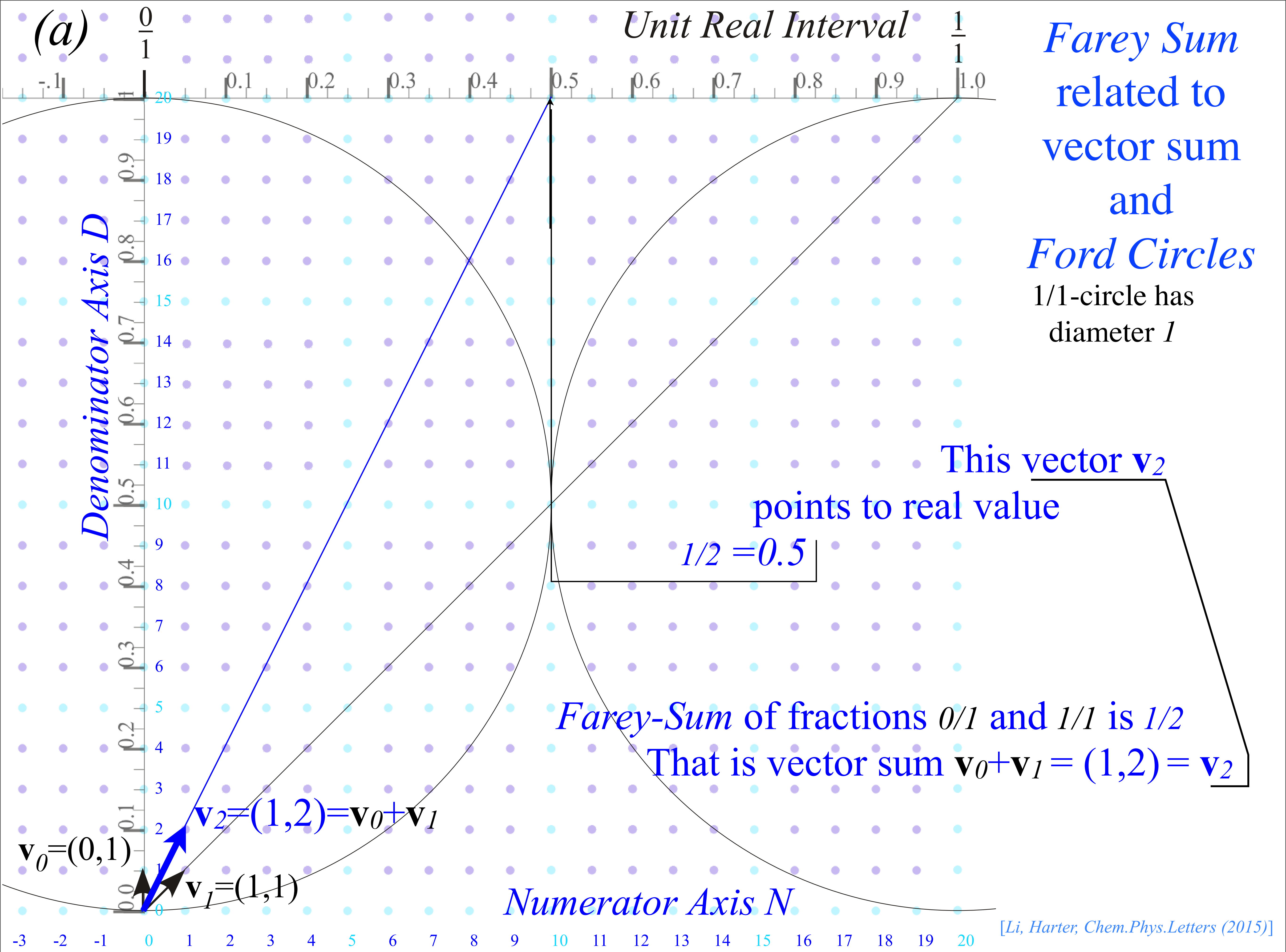
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

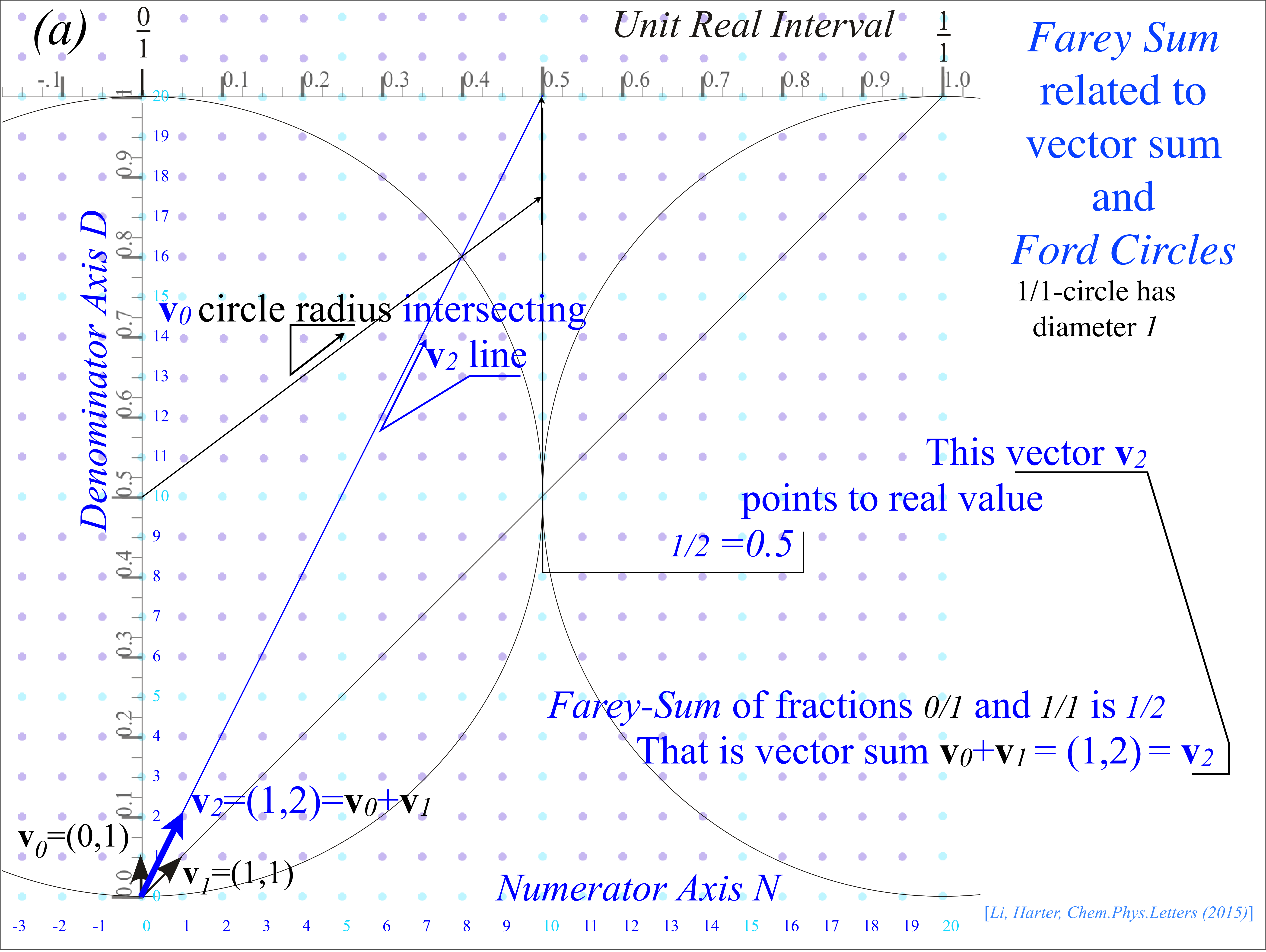
Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



[Li, Harter, Chem.Phys.Letters (2015)]


(a)
0/1
Unit Real Interval
1/1
Farey Sum
related to
vector sum
and
Ford Circles
1/1-circle has
diameter 1
Denominator Axis D
This vector v2
points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]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v0 circle radius intersecting
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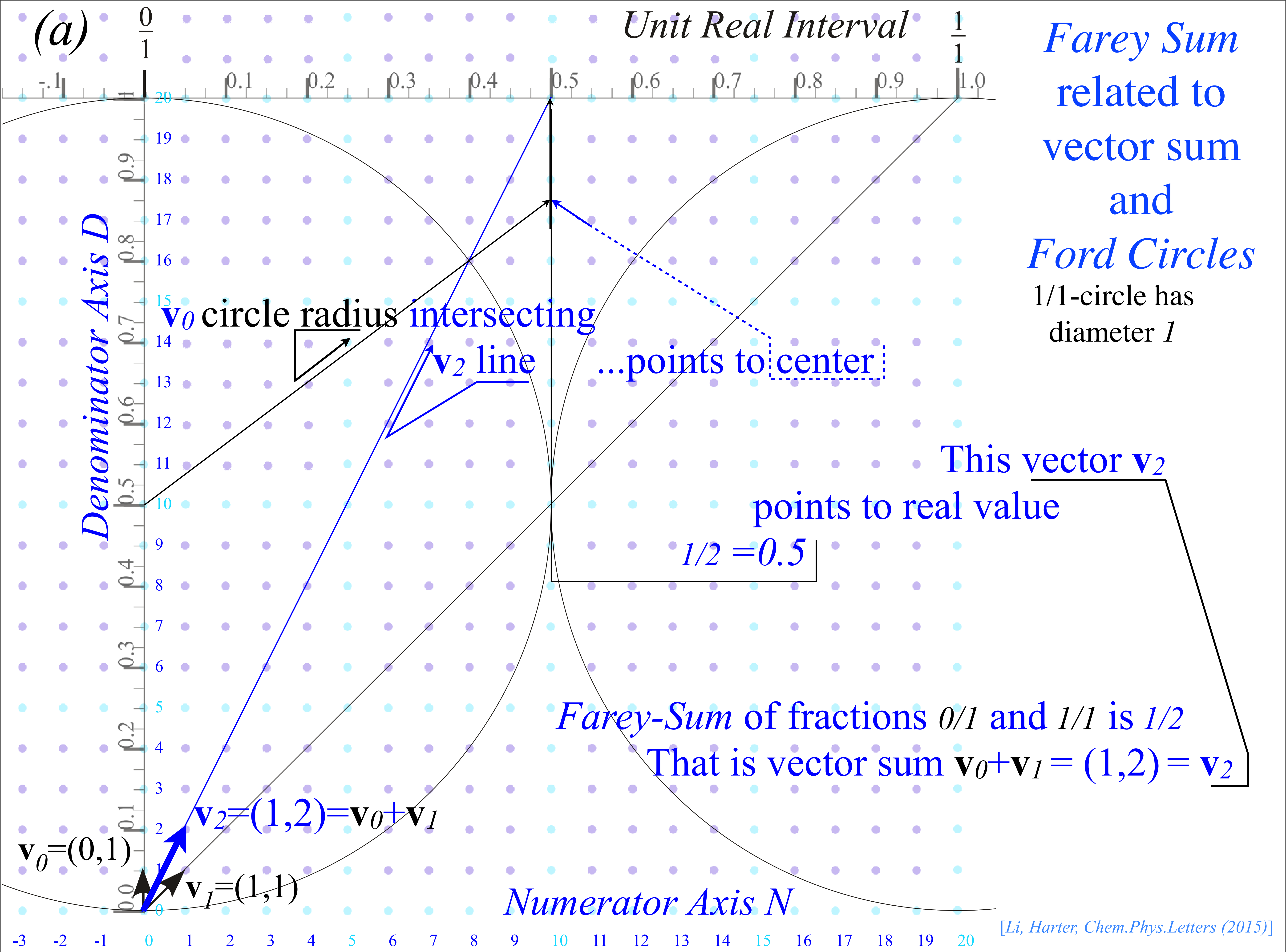
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1/1-circle has diameter 1
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v0 circle radius intersecting
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...points to center
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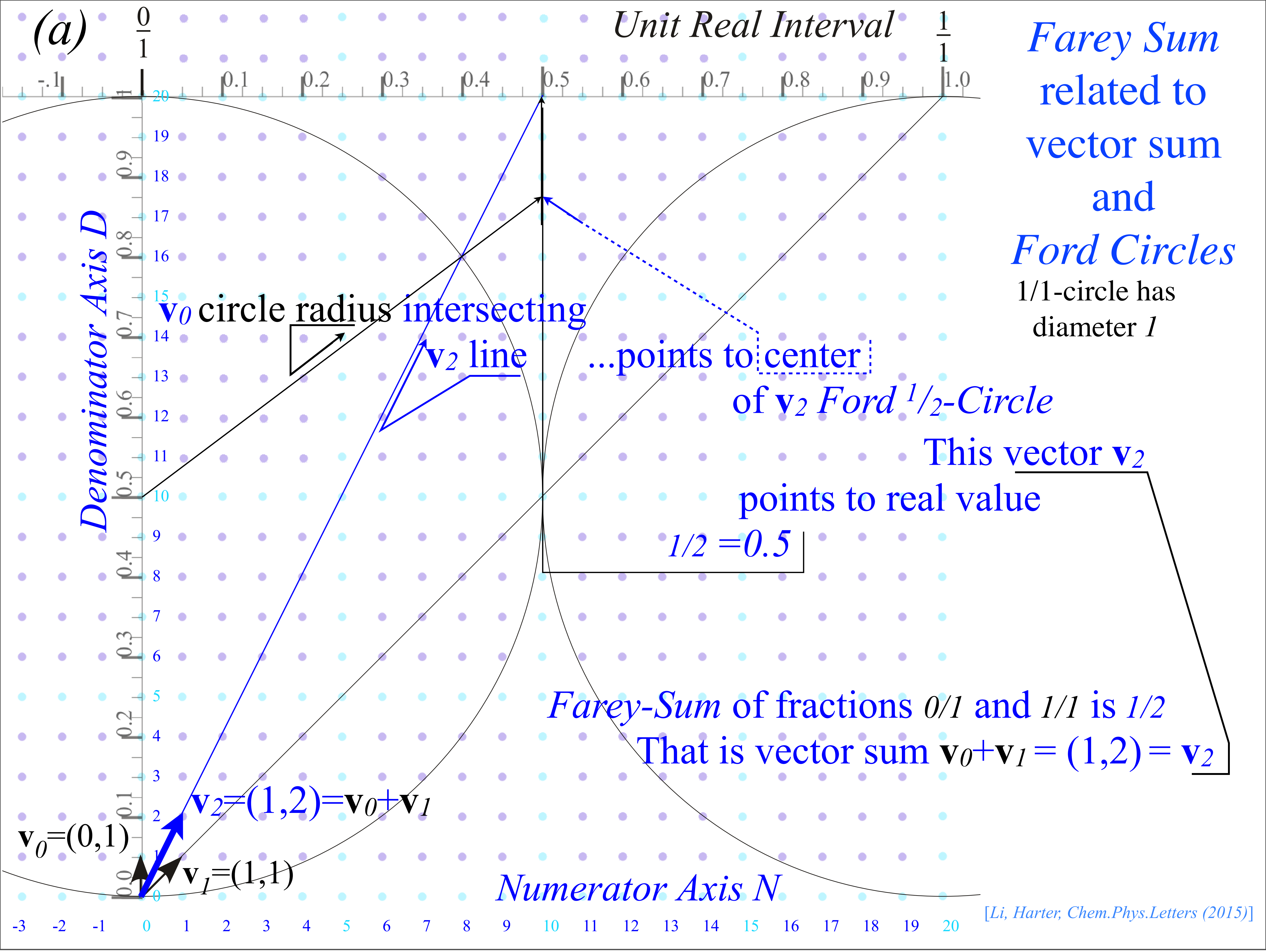
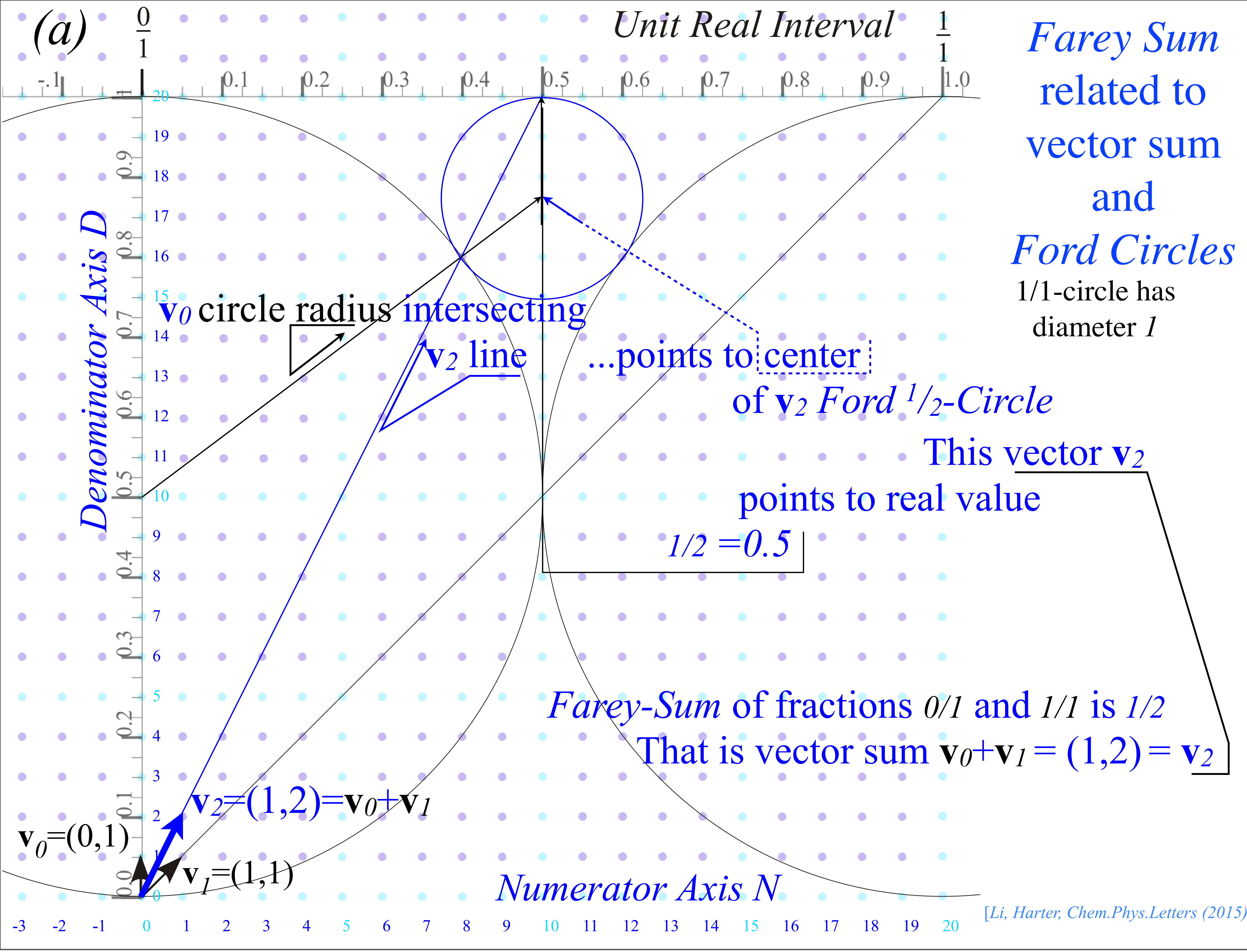
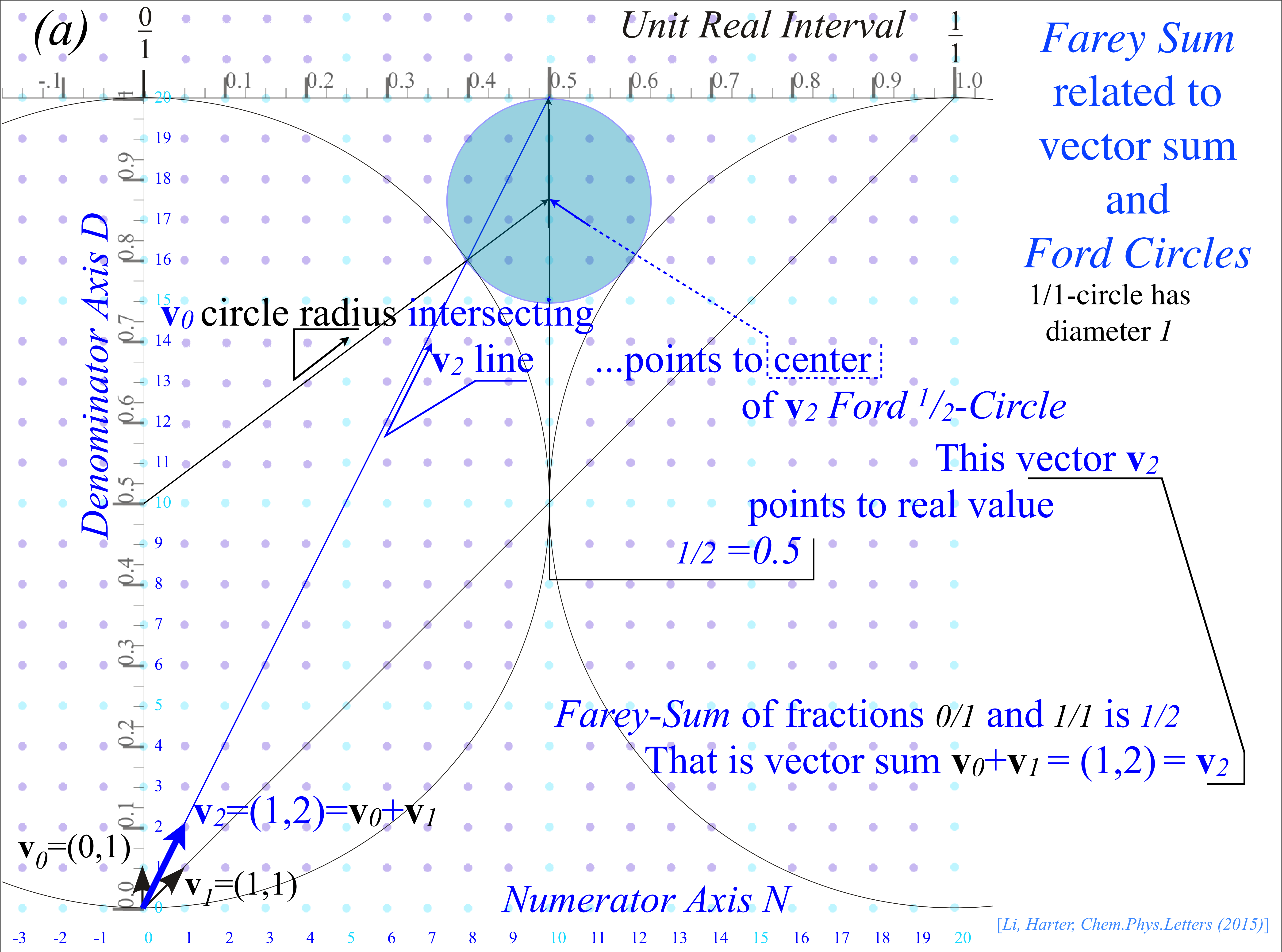

(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
$\mathbf{v}_0$ circle radius intersecting
$\mathbf{v}_2$ line
...points to center of $\mathbf{v}_2$ Ford $^1/_2$-Circle
This vector $\mathbf{v}_2$ points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$
$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$
$\mathbf{v}_0=(0,1)$
$\mathbf{v}_1=(1,1)$
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum related to vector sum and *Ford Circles*

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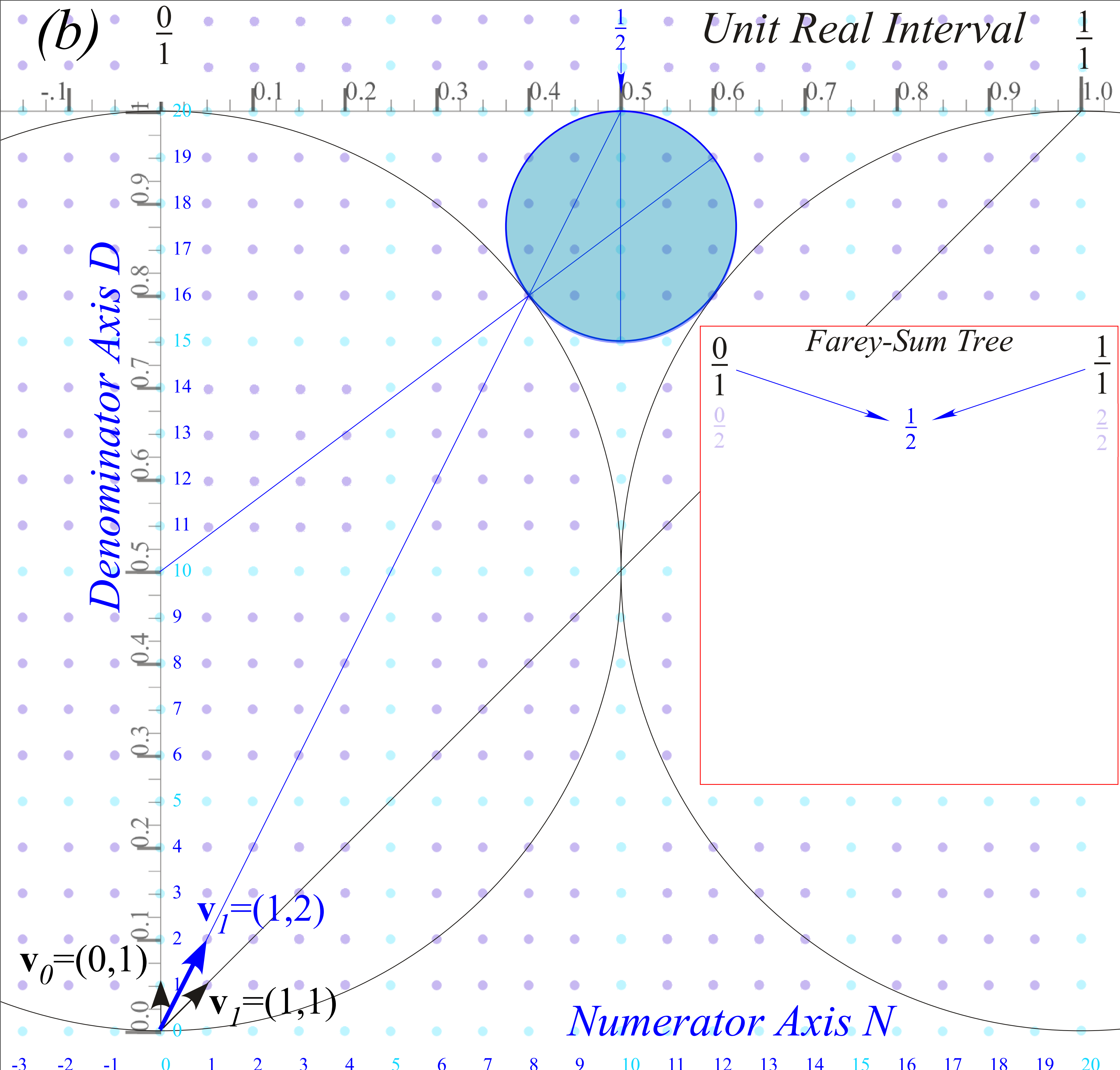
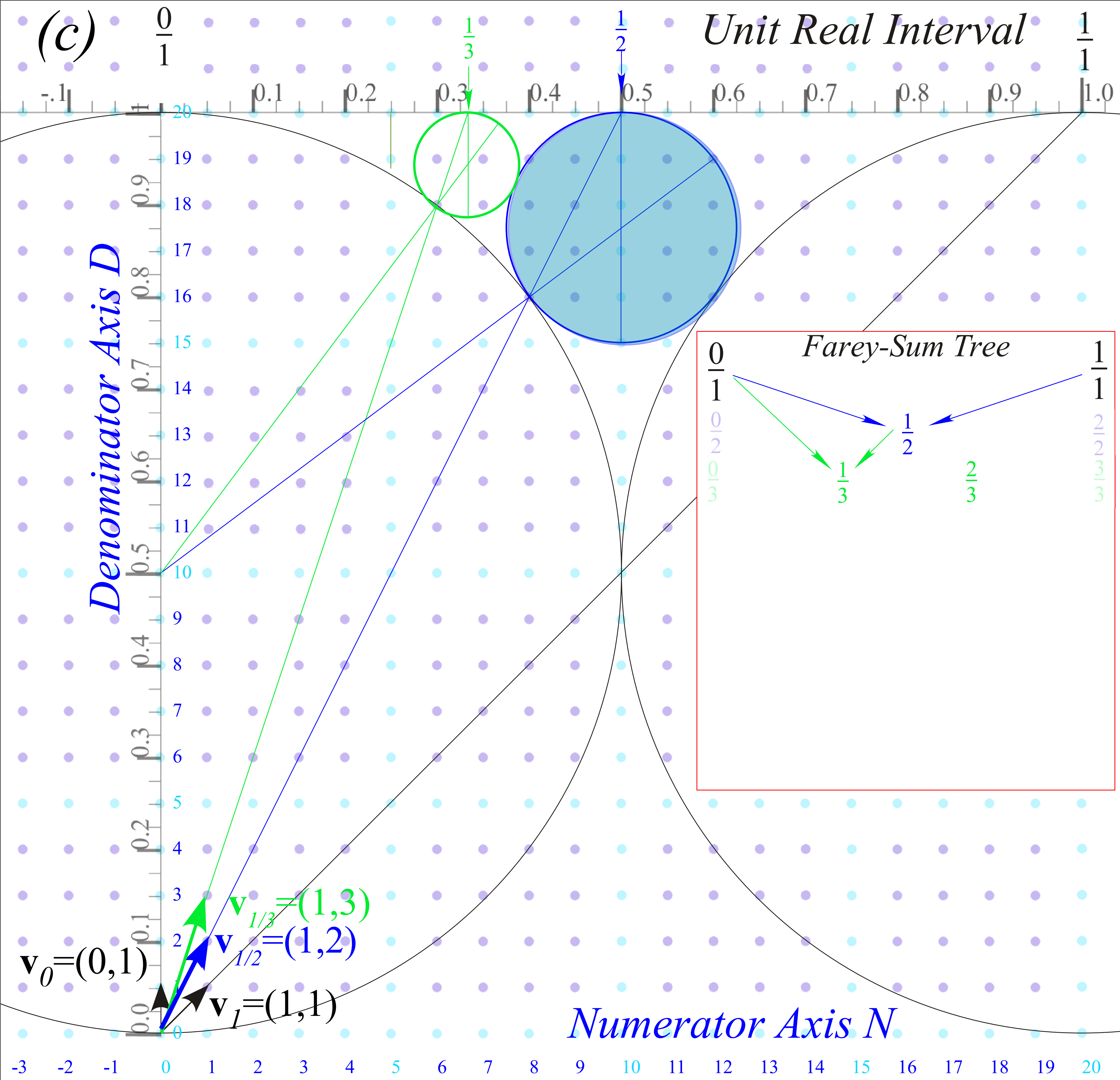


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

[Li, Harter, Chem.Phys.Letters (2015)]

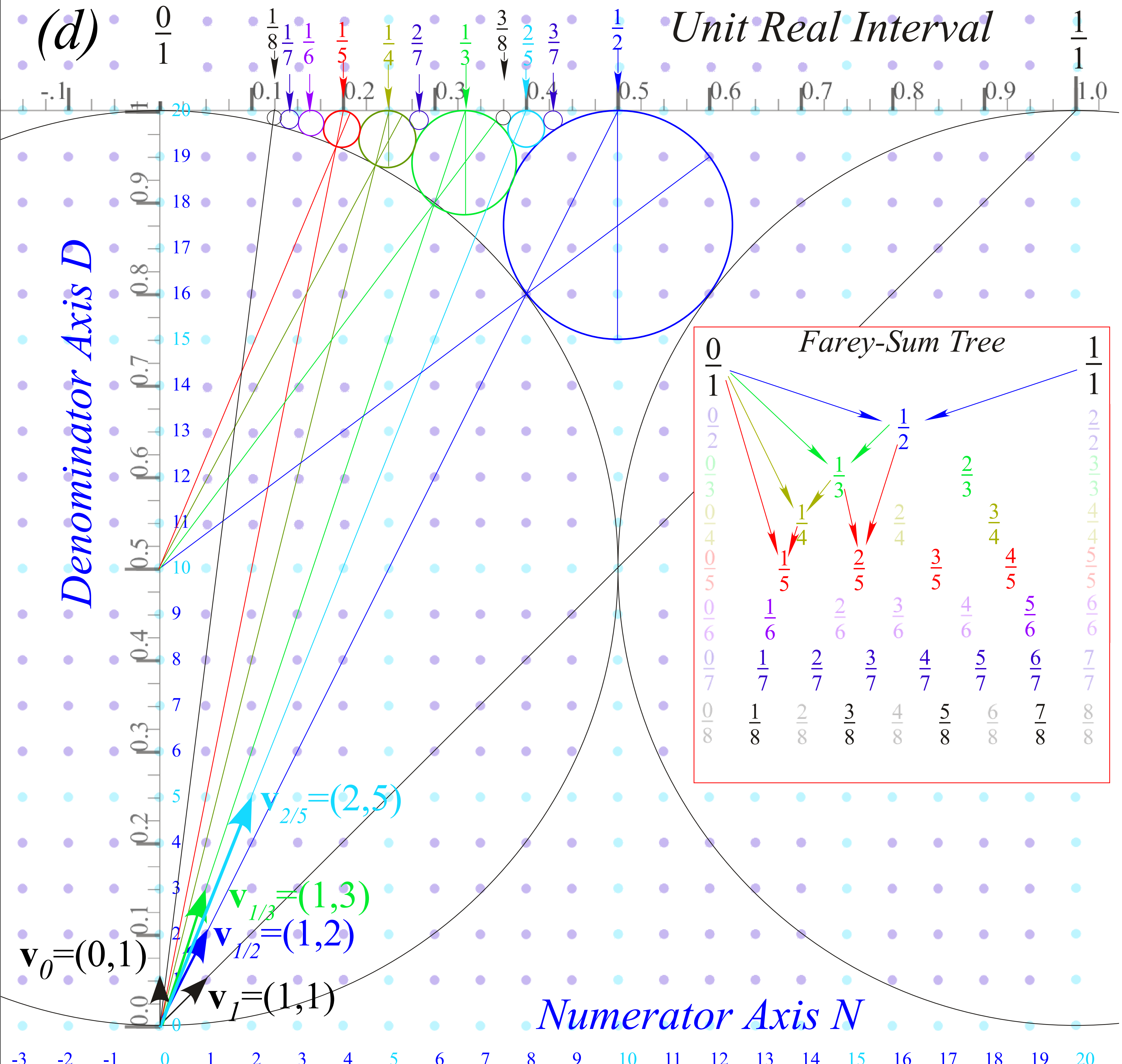


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

[*Li, Harter, Chem.Phys.Letters (2015)*]



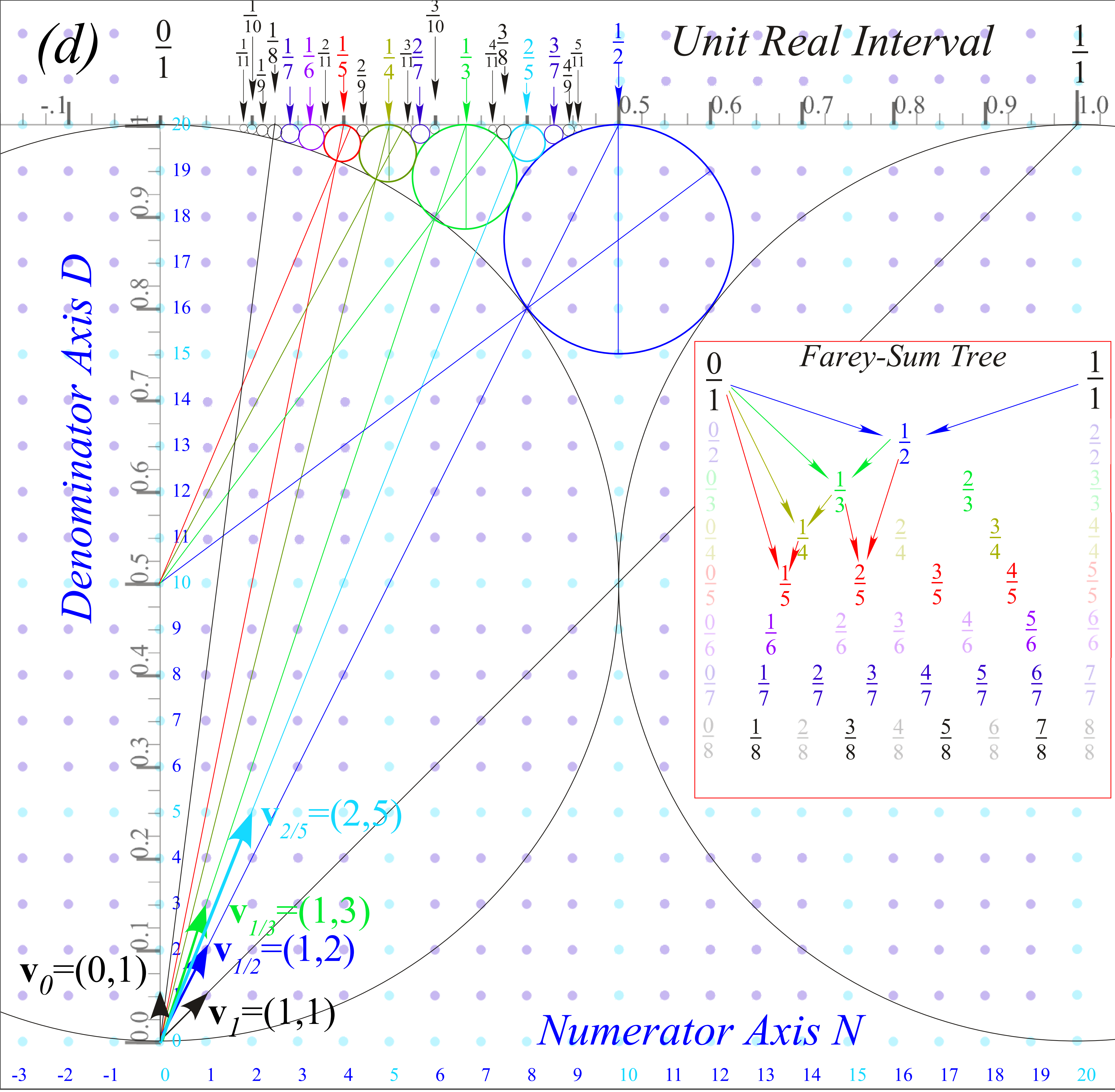
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

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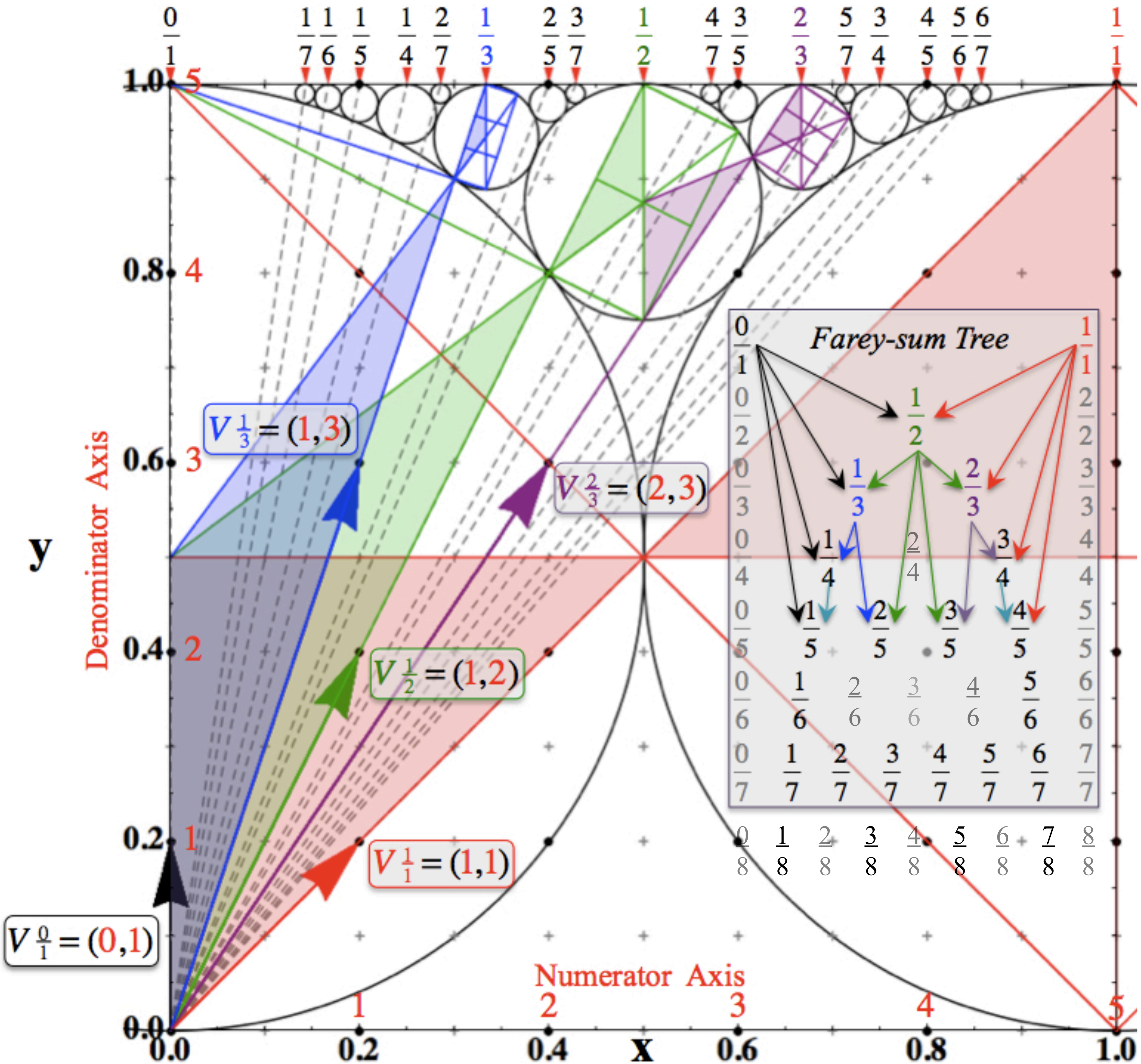
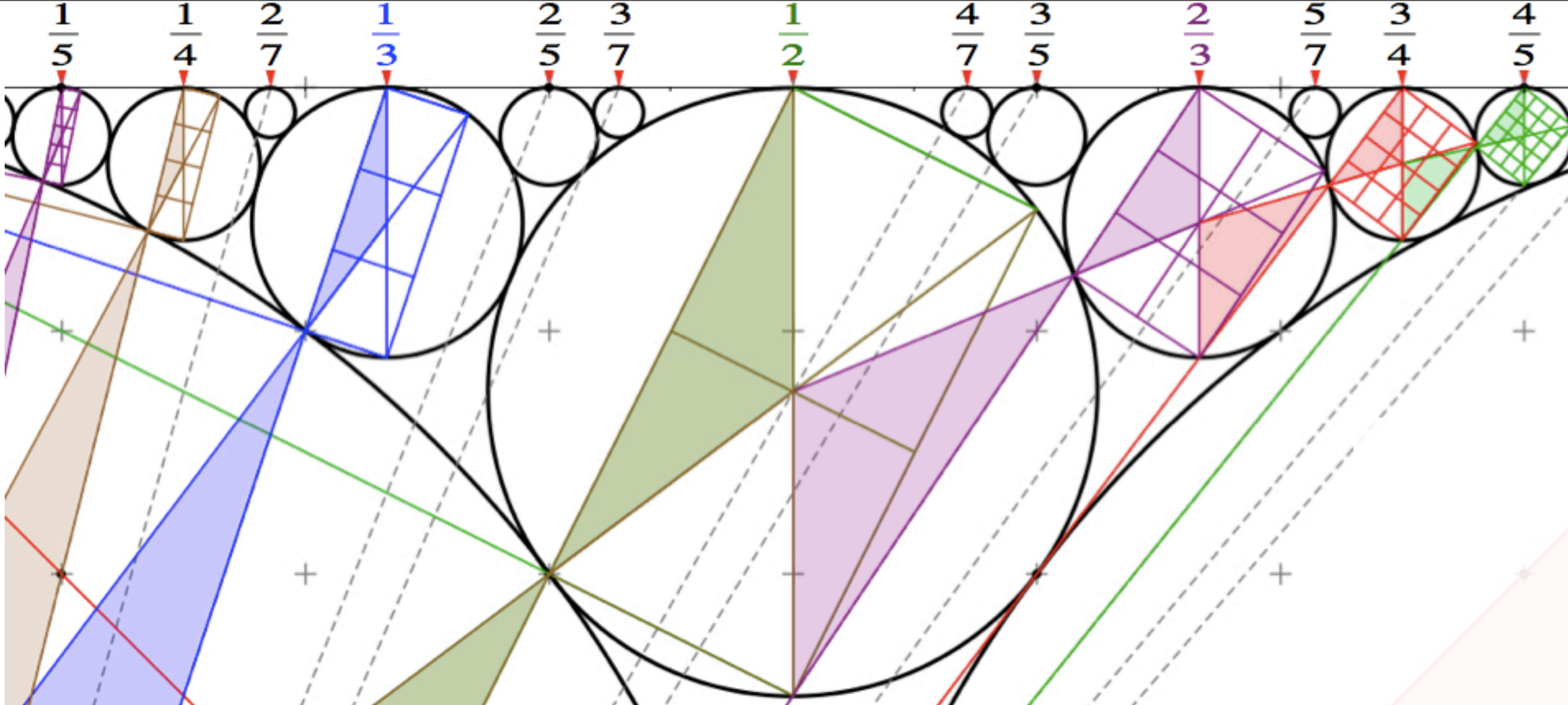


Thales Rectangles

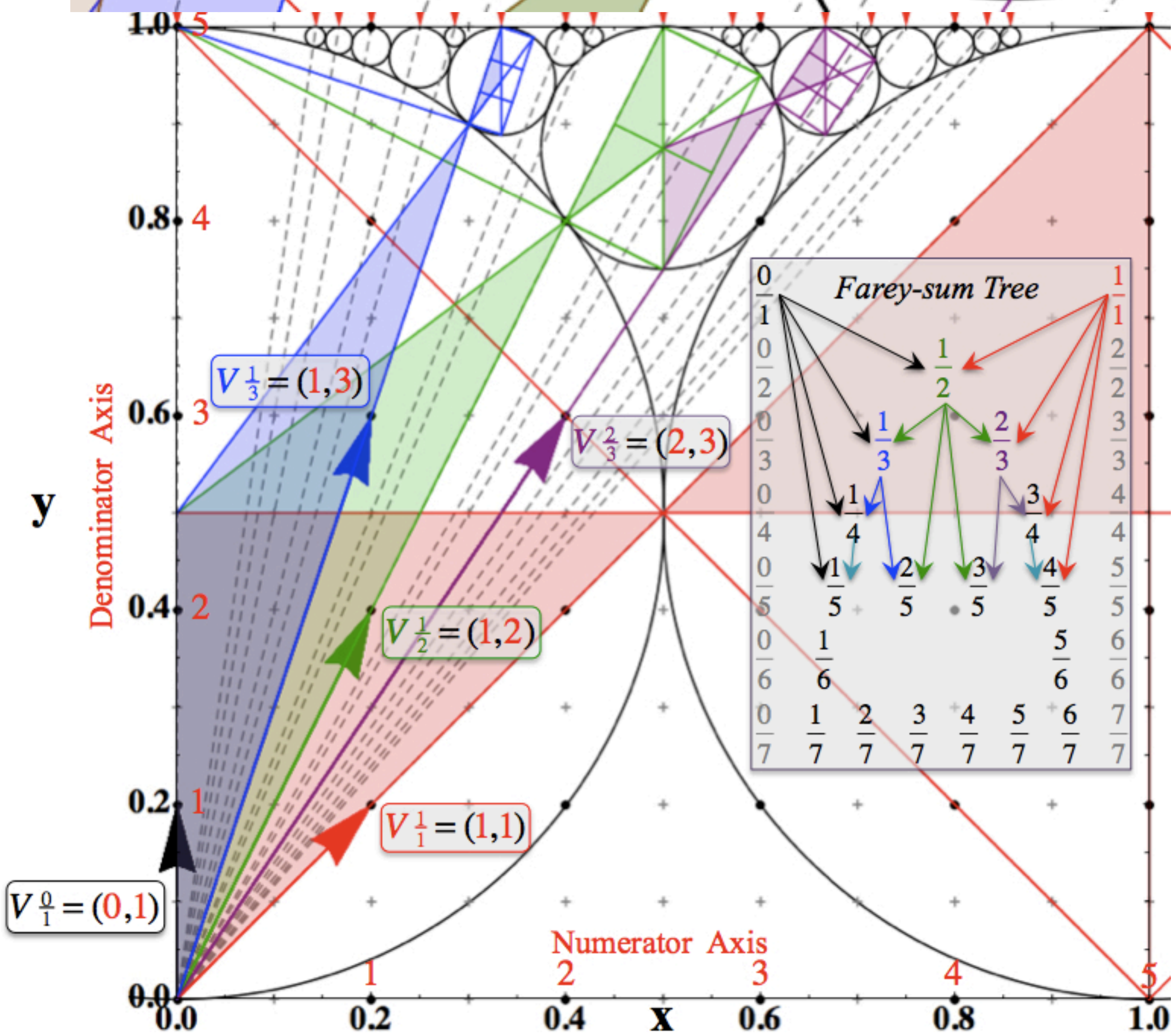
provide
analytic geometry
of
fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



“Quantized” Thales Rectangles

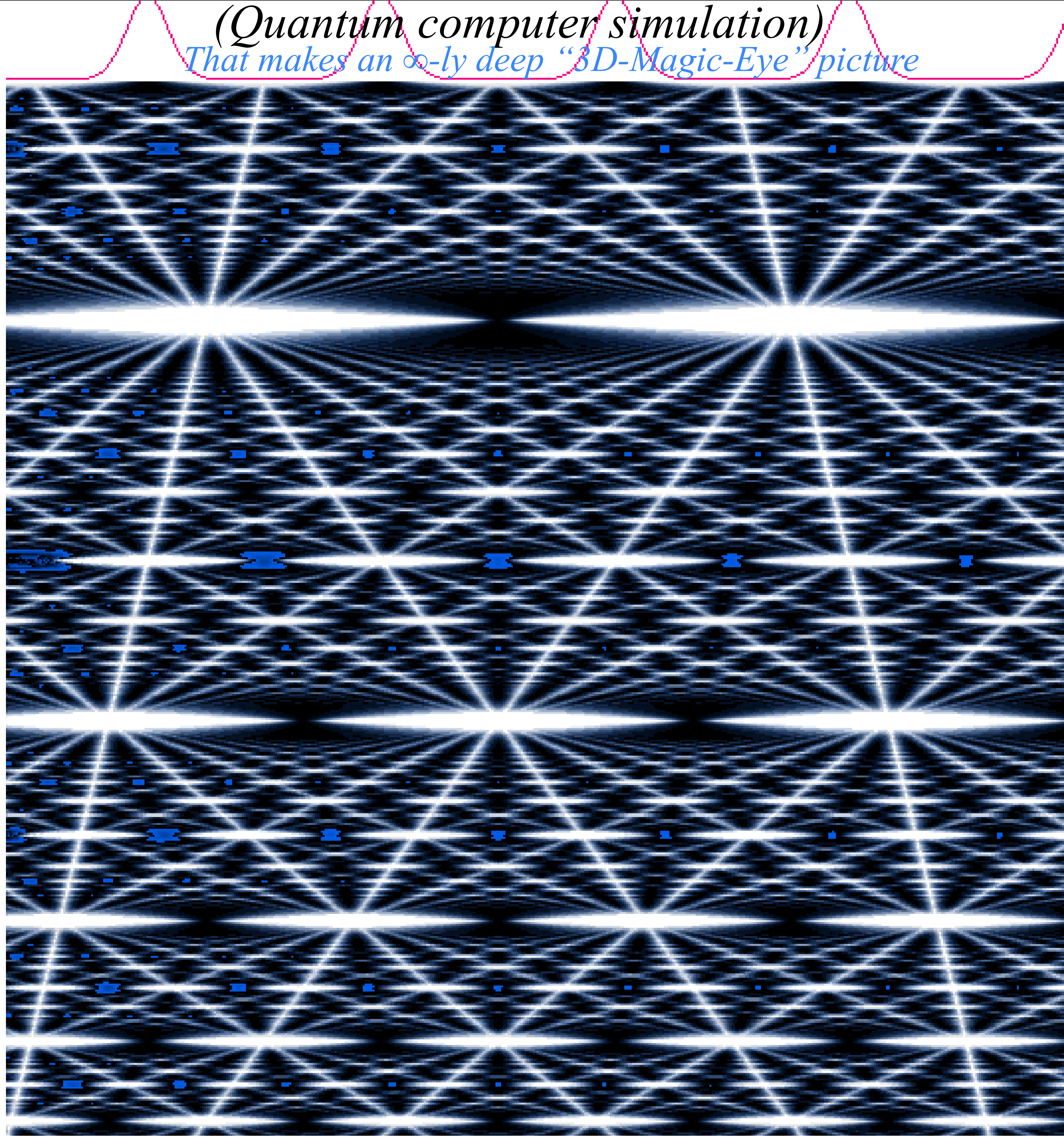
provide analytic geometry of fractal structure



[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



Geometric "Integration" (Converting Velocity data to Spacetime)

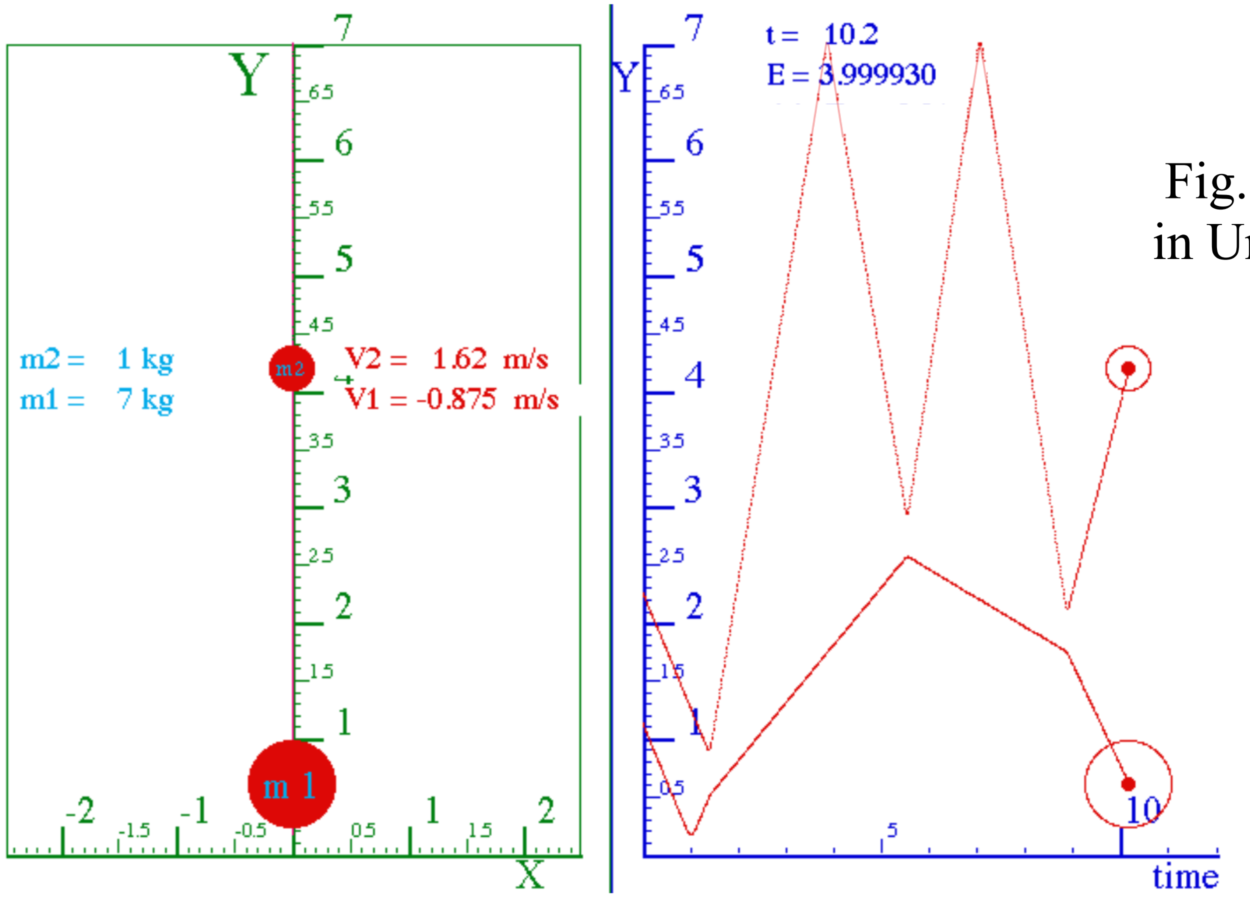


Fig. 4.8
in Unit 1

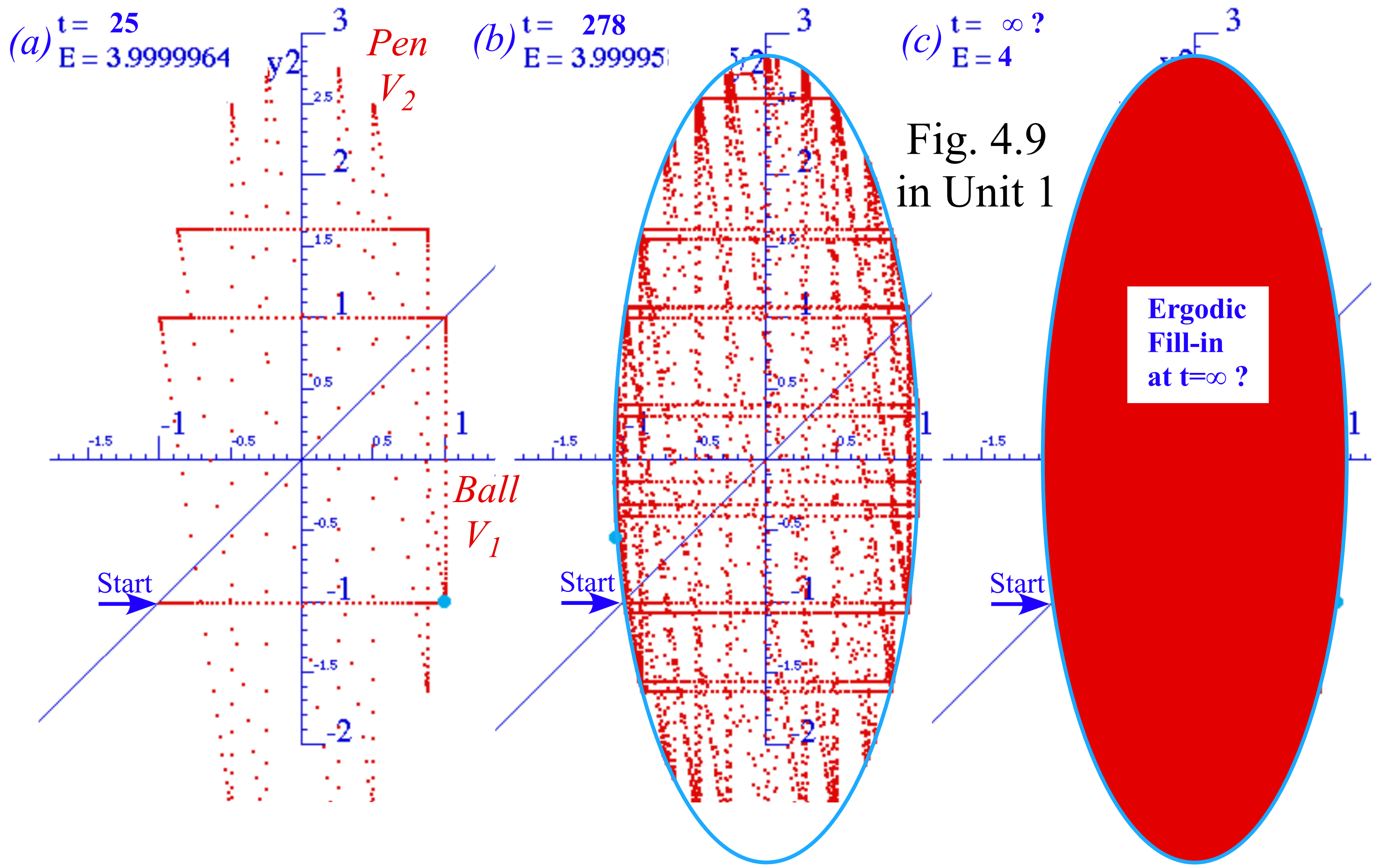
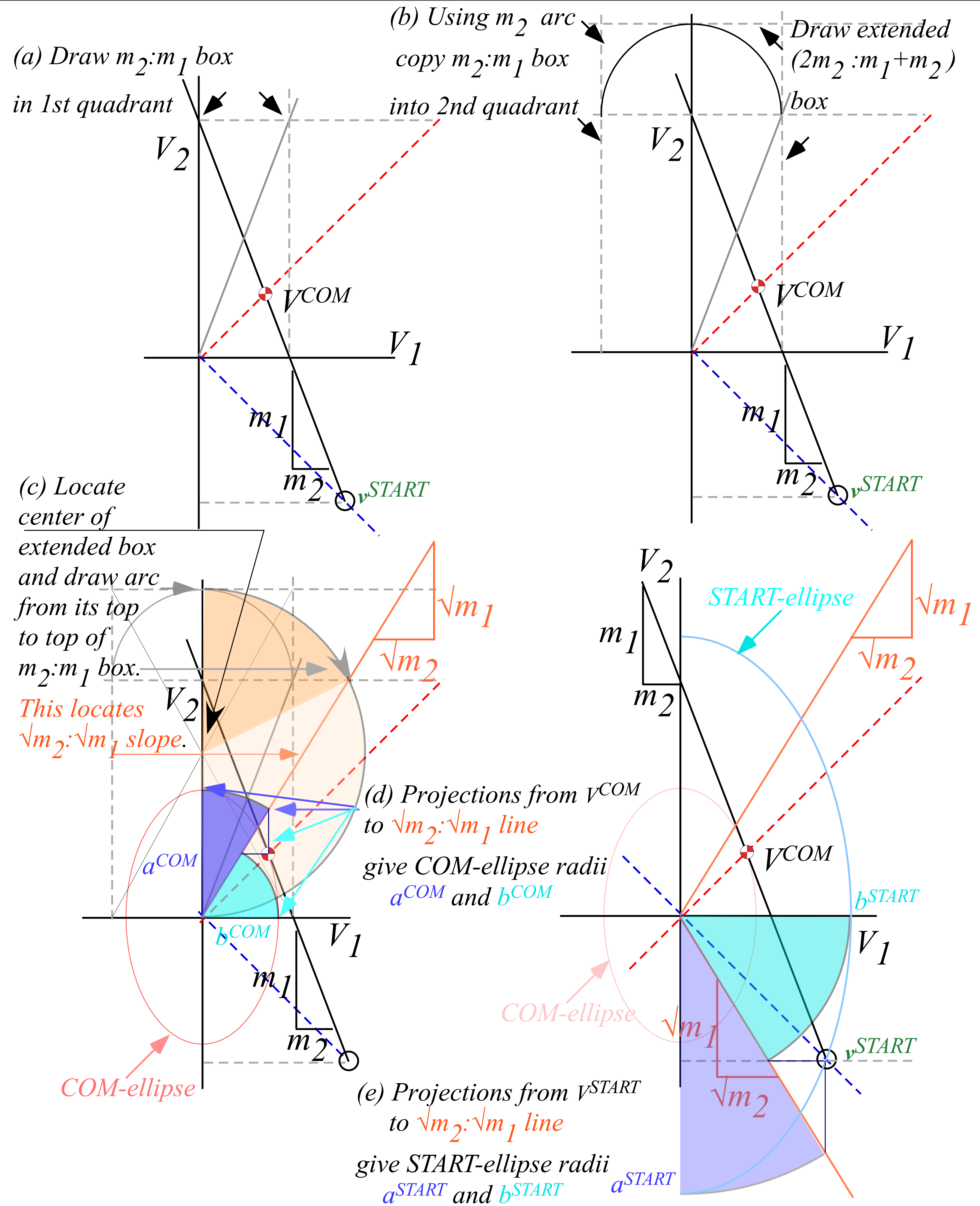


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)