Lecture 3
Thur 8.30.2016

Analysis of 1D 2-Body Collisions: Reflection groups

(Ch. 2 to Ch. 4 of Unit 1)

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

"Mass-bang" matrix **M**, "Floor-bang" matrix **F**, "Ceiling-bang" matrix **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

What about that 2nd quadratic solution?

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$ and (M_1=100, M_2=1)

Review: *Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)*

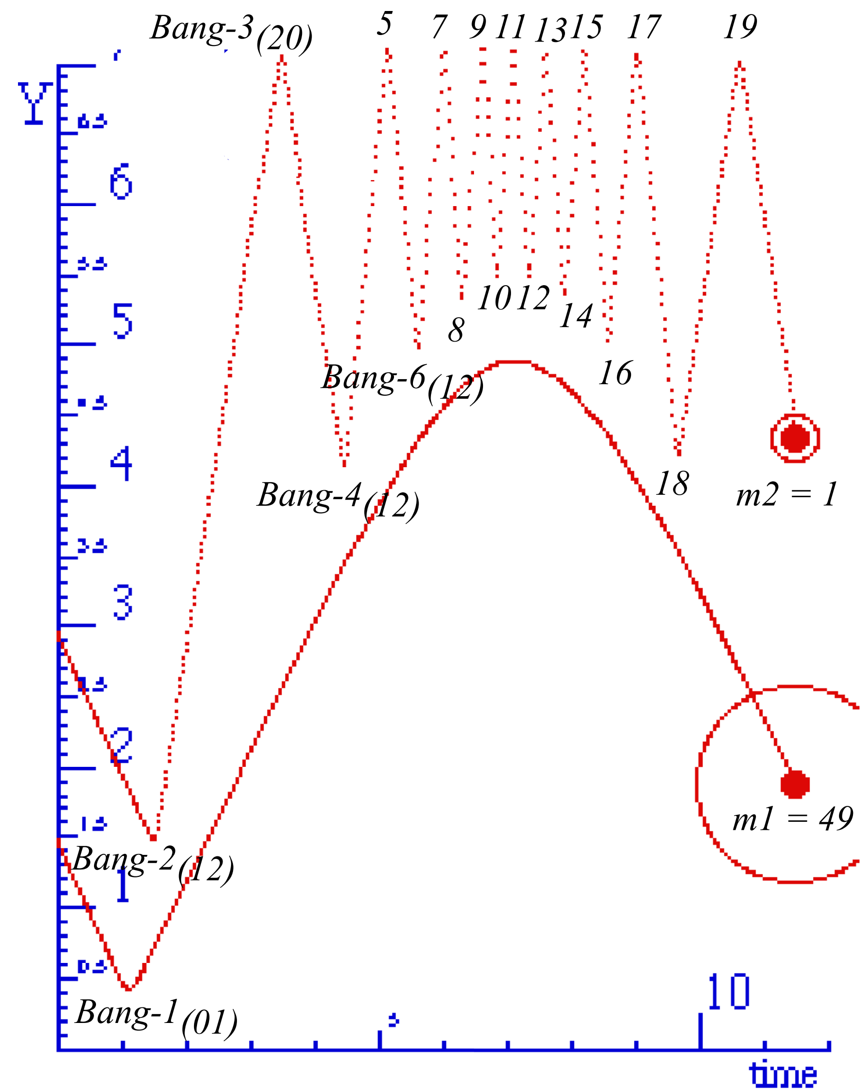
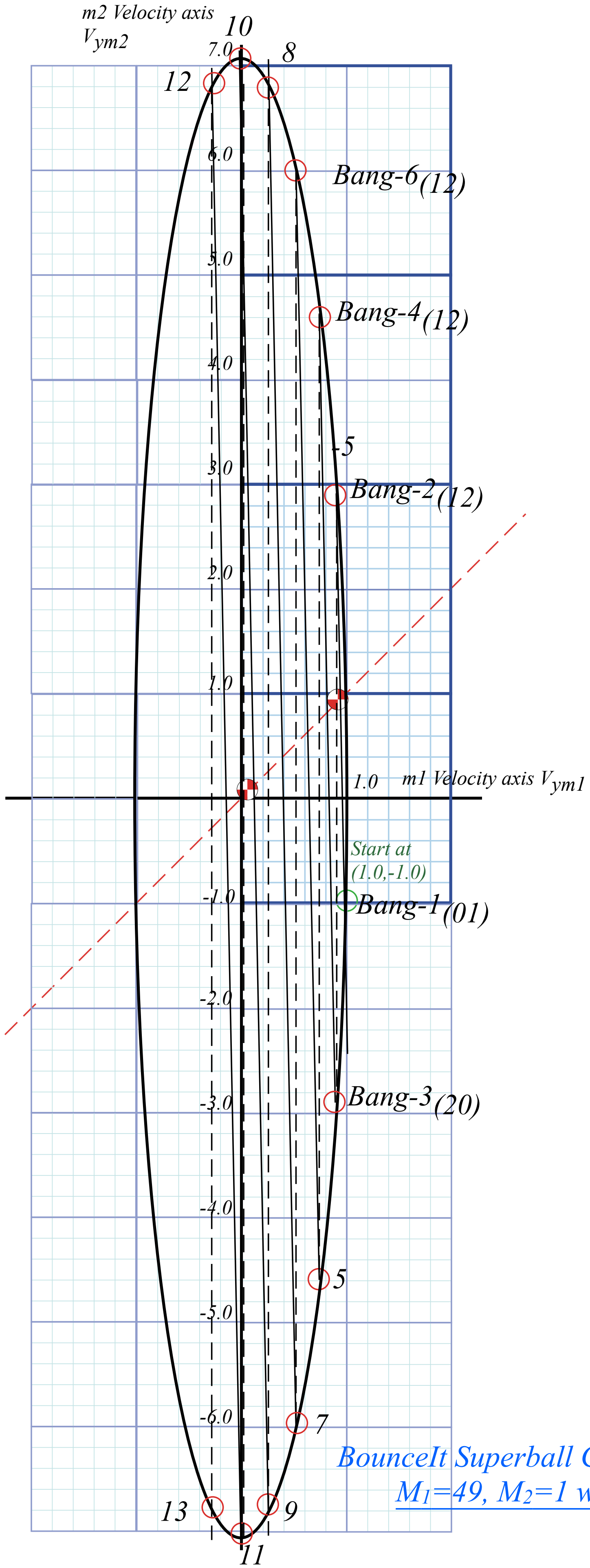


Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
$M_1=49$, $M_2=1$ with Newtonian time plot

BounceIt Superball Collision Web Simulator:
$M_1=49$, $M_2=1$ with V_2 vs V_1 plot

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)



[BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with V_2 vs V_1 plot]()

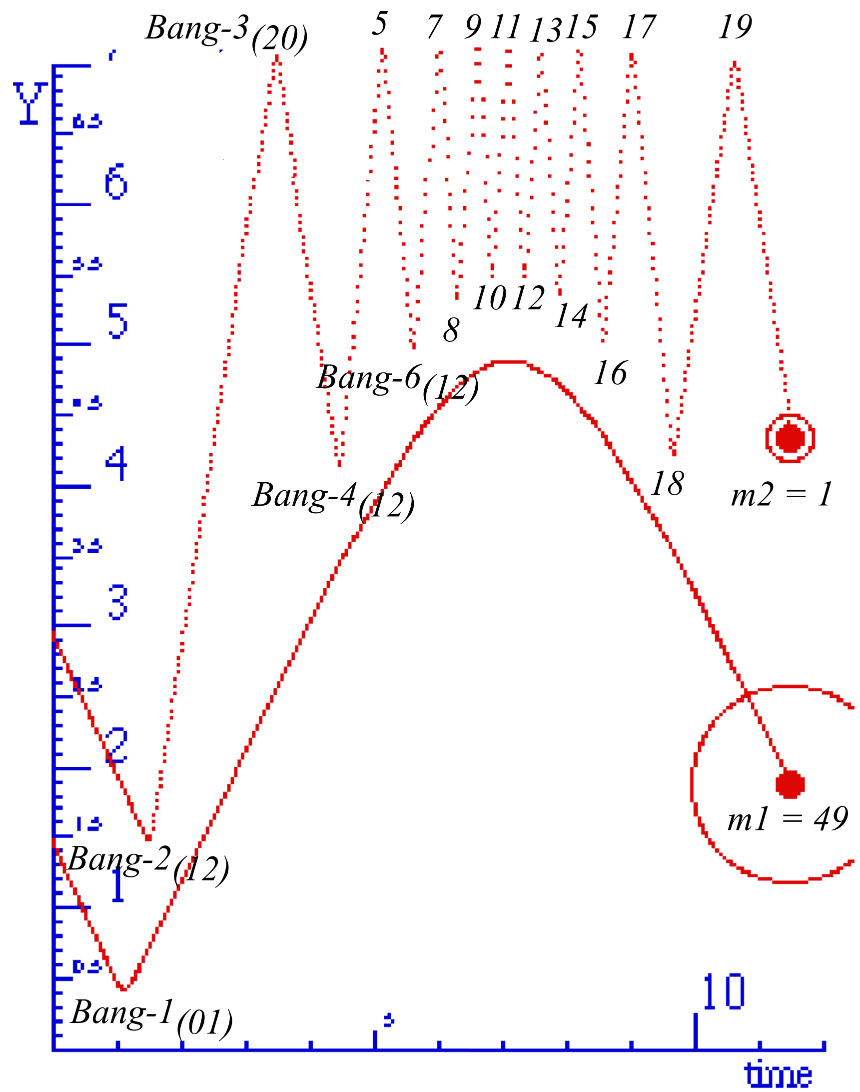
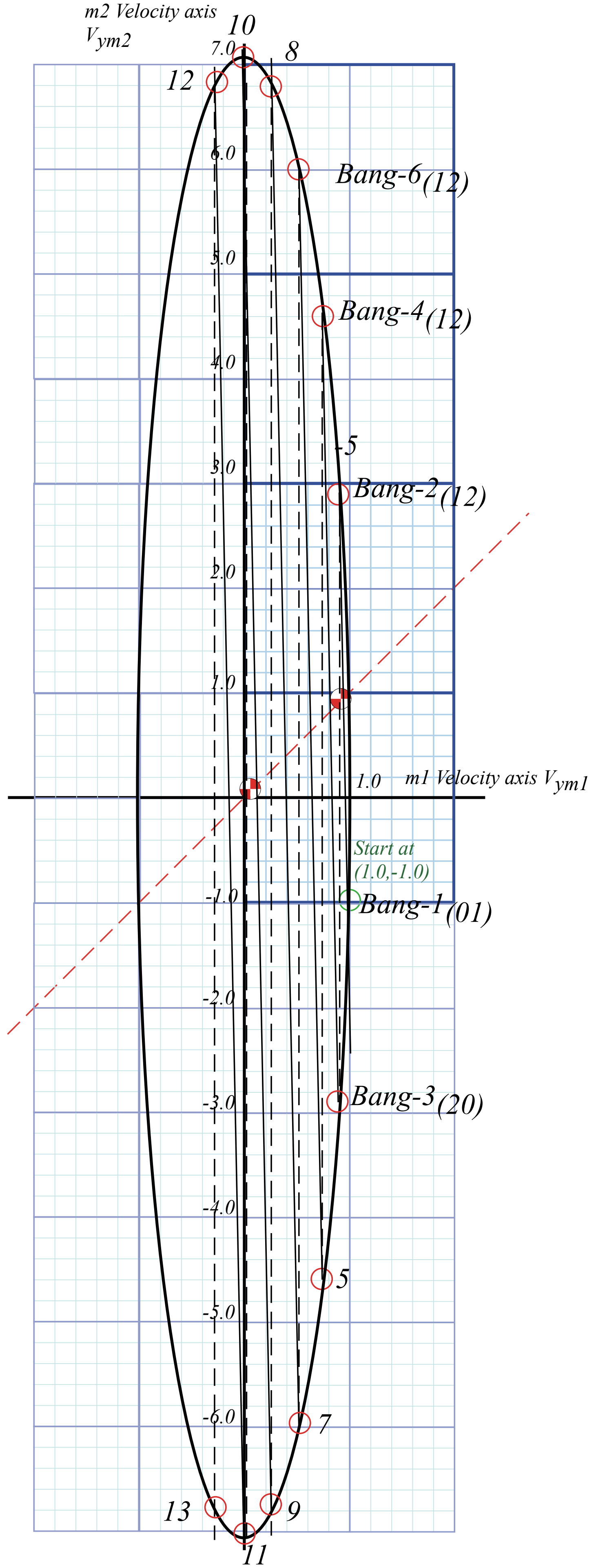


Fig. 5.1
in Unit 1

[BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with Newtonian time plot]()

Review: *Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)*



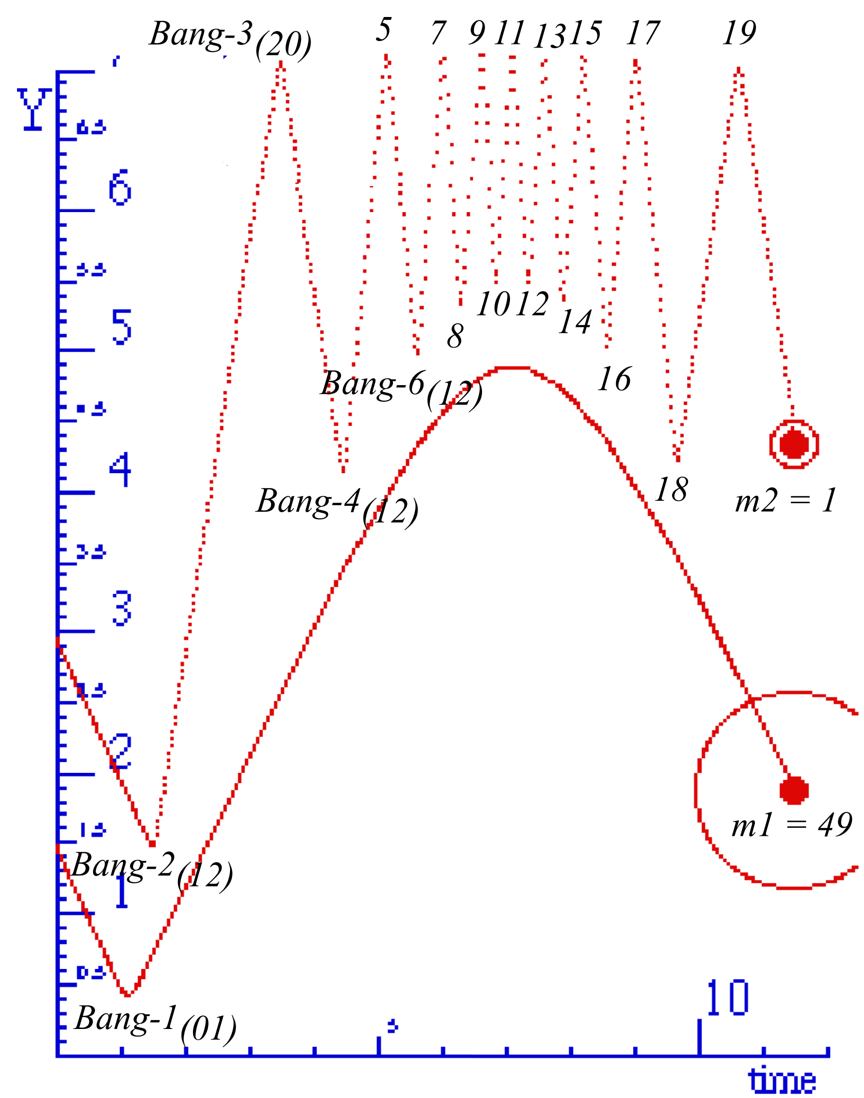


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1	*Ellipse radius 2*
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/m_2}$
$= \sqrt{2KE/49}$	$= \sqrt{2KE/1}$
$= \sqrt{50/49}$	$= \sqrt{50/1}$
$= 1.01$	$= 7.07$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Multiple Collisions by Matrix Operator Products

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Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$

Multiple Collisions by Matrix Operator Products

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$$\mathbf{V}^{COM}=\frac{\mathbf{V}^{FIN}+\mathbf{V}^{IN}}{2}=\frac{m_1 v_1+m_2 v_2}{m_1+m_2}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=\tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}+\tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}=\begin{pmatrix}2V^{COM}-v_1^{IN}\\2V^{COM}-v_2^{IN}\end{pmatrix}=\begin{pmatrix}2\frac{m_1 v_1^{IN}+m_2 v_2^{IN}}{m_1+m_2}-v_1^{IN}\\2\frac{m_1 v_1^{IN}+m_2 v_2^{IN}}{m_1+m_2}-v_2^{IN}\end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

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Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN}=\mathbf{M}\bullet\mathbf{v}^{IN}$...

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}=\begin{pmatrix}2V^{COM}-v_1^{IN}\\2V^{COM}-v_2^{IN}\end{pmatrix}=\begin{pmatrix}2\frac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_1^{IN}\\2\frac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_2^{IN}\end{pmatrix}=\frac{\begin{pmatrix}m_1v_1^{IN}-m_2v_1^{IN}+2m_2v_2^{IN}\\2m_1v_1^{IN}+m_2v_2^{IN}-m_1v_2^{IN}\end{pmatrix}}{m_1+m_2}=\frac{\begin{pmatrix}m_1-m_2 & 2m_2\\2m_1 & m_2-m_1\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}}{m_1+m_2}$$

$$=\begin{pmatrix}\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_1^{IN}-m_2v_1^{IN}}{m_1+m_2}\\\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_2^{IN}-m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM}=\frac{\mathbf{V}^{FIN}+\mathbf{V}^{IN}}{2}=\frac{m_1v_1+m_2v_2}{m_1+m_2}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=\tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}+\tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

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$$=\begin{pmatrix}\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_1^{IN}-m_2v_1^{IN}}{m_1+m_2}\\\frac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_2^{IN}-m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}=\begin{pmatrix}\frac{m_1-m_2}{m_1+m_2}&\frac{2m_2}{m_1+m_2}\\\frac{2m_1}{m_1+m_2}&\frac{m_2-m_1}{m_1+m_2}\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

→ *What about that 2^{nd} quadratic solution?*

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

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$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

What about that 2nd quadratic solution?

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Multiple Collisions by Matrix Operator Products

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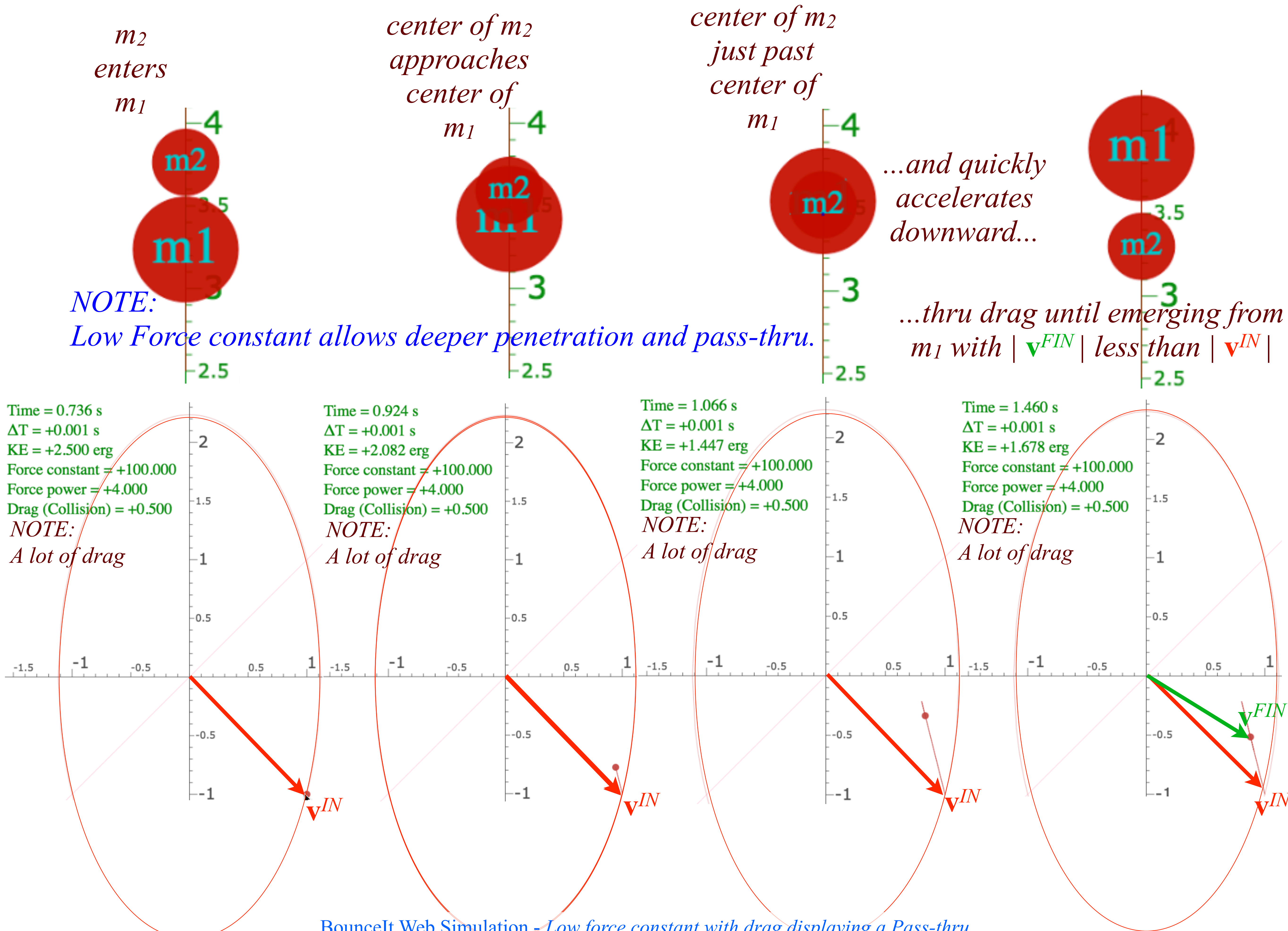
What about that 2nd quadratic solution?

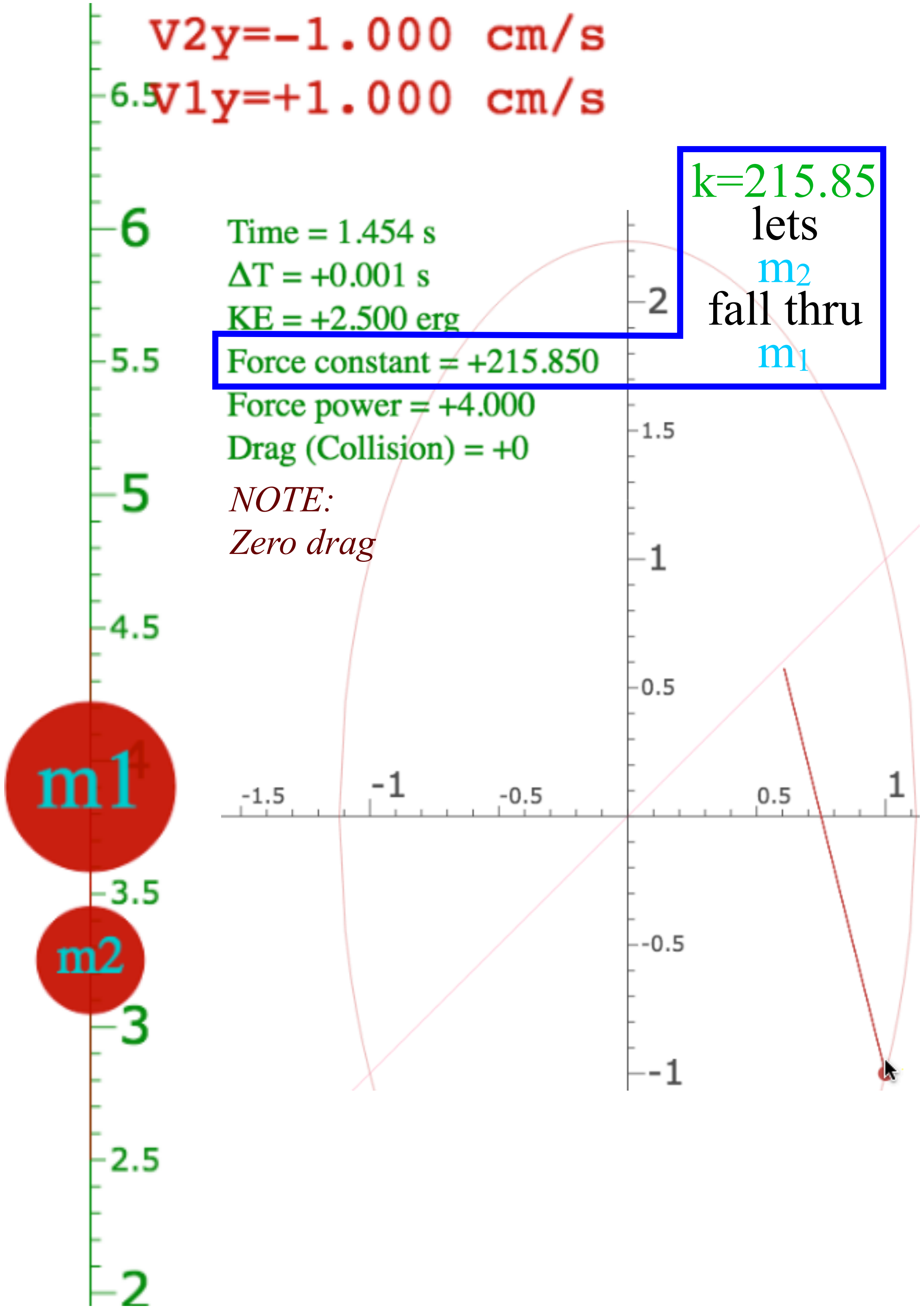
Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Q: What is the *second* solution and to what simple process would it correspond?

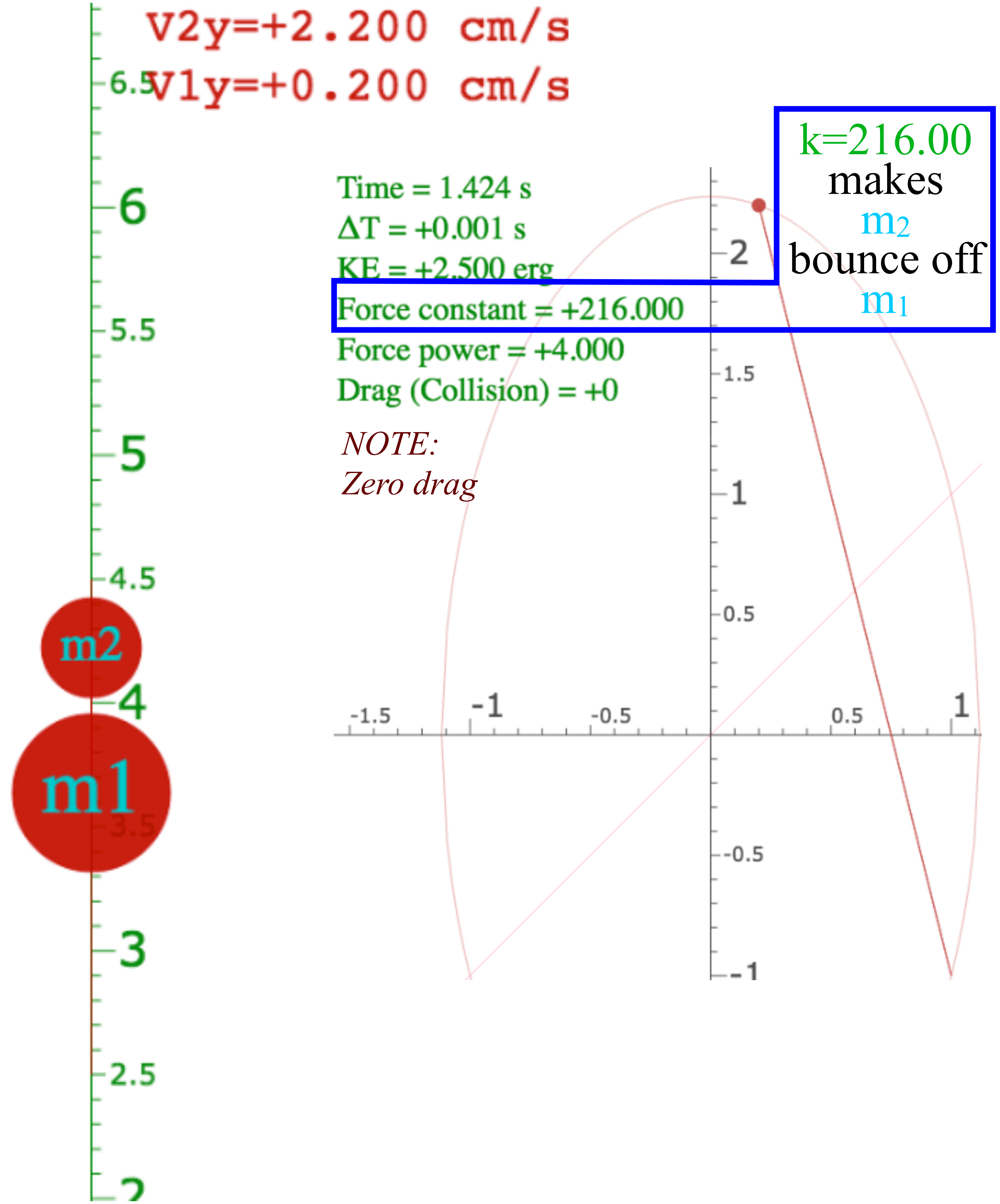
Example with friction



[BounceIt Web Simulation - *Low force constant with drag displaying a Pass-thru*]



Fall-Thru



Bounce-Off

BounceIt Superball Collision Web Simulations
with *Low force constant & Zero drag*

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

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Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$= \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1 = 49$ *and* $m_2 = 1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

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→ *Geometry and algebra of "ellipse-Rotation" group product*: **R**= **C•M**

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: m_1=49 and m_2=1

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define *"ellipse-Rotation"* **R** as *group product*: $\mathbf{R} = \mathbf{C} \bullet \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

$$\left|FIN^9\right\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \; |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F}|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

"ellipse-Rotation" group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \; |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R= C•M**

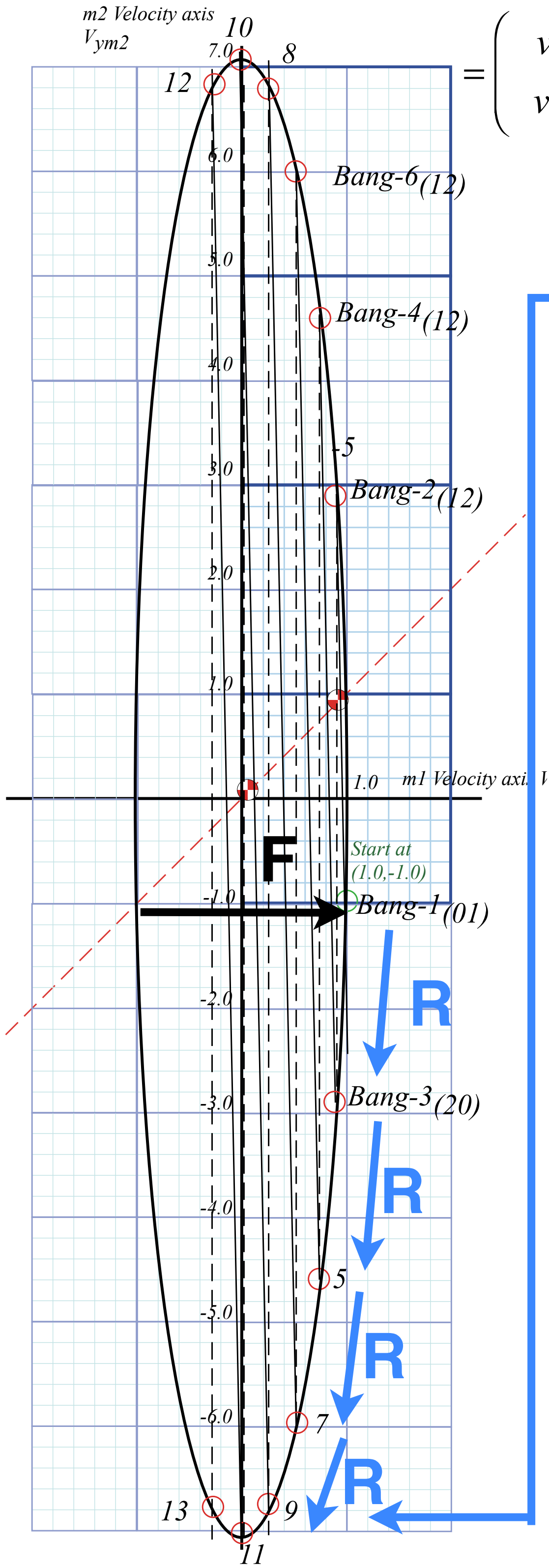
$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F}\, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F}\, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$



"ellipse-Rotation" group product: **R**= **C**•**M**

Collisions for
mass ratio
m_1:m_2= 49:1

Fig. 5.1a
(revised)

$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

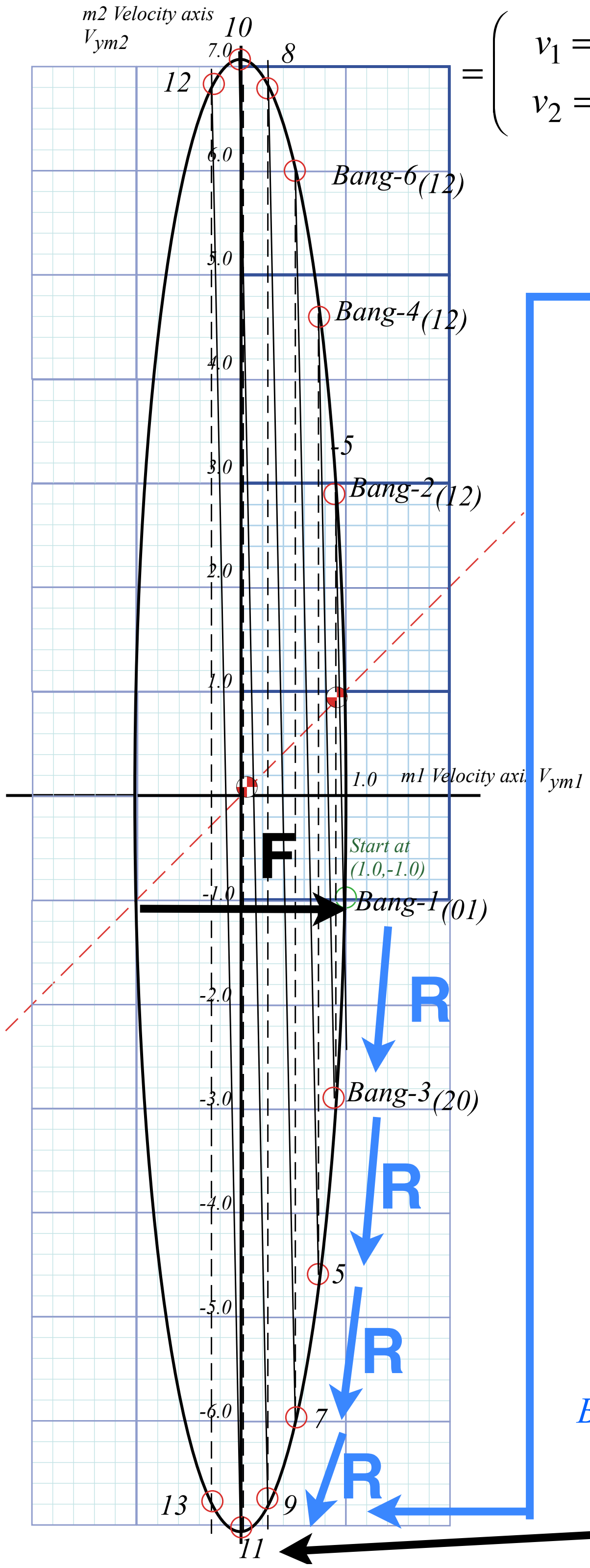
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

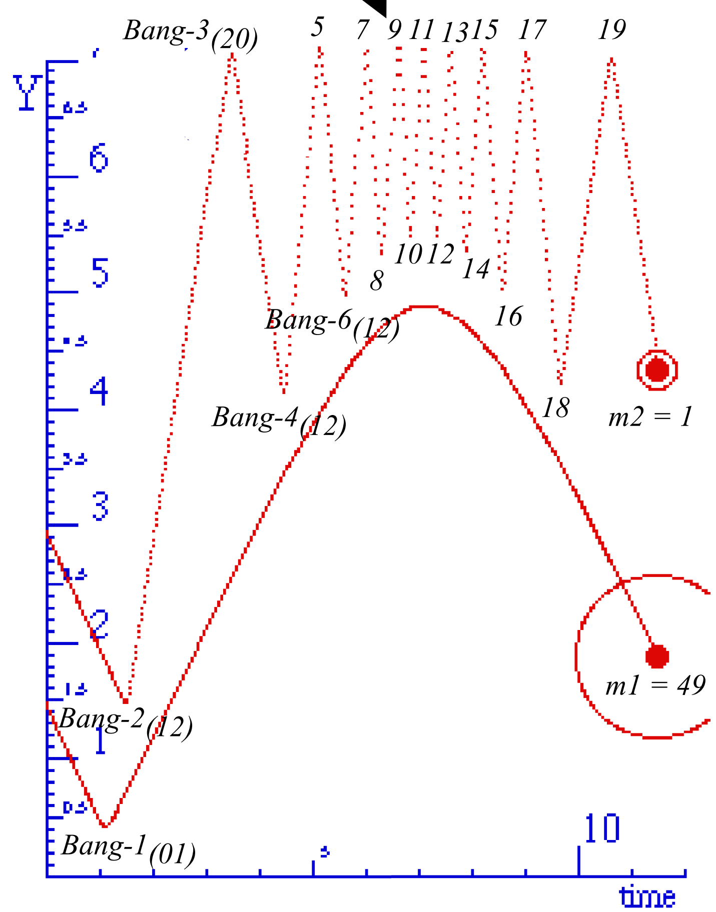
$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1=1 \\ v_2=-1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R**= **C**·**M**





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{\text{(after Bang-11)}}$$

Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

BounceIt Superball Collision Web Simulator: $M_1=49$, $M_2=1$ with V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator: $M_1=49$, $M_2=1$ with Newtonian time plot

<<Under Construction>>
Matrix Collision Web Simulator: $M_1=49$, $M_2=1$ V_2 vs V_1 plot

Fig. 5.1a-b
(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

→ *Rescaling KE ellipse to circle*

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

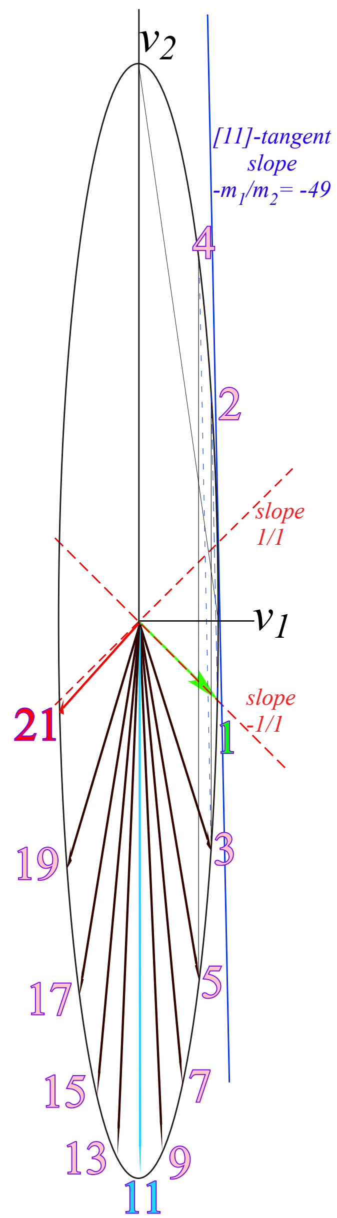
Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$



Collisions for mass ratio m_1:m_2= 49:1

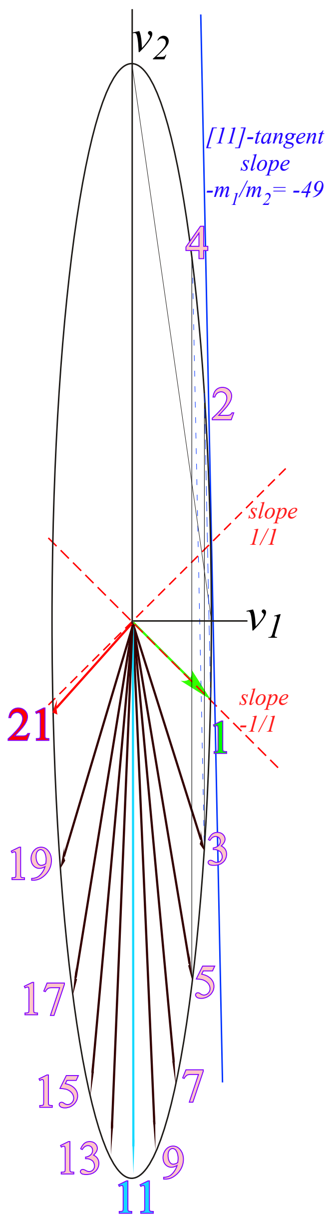
Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

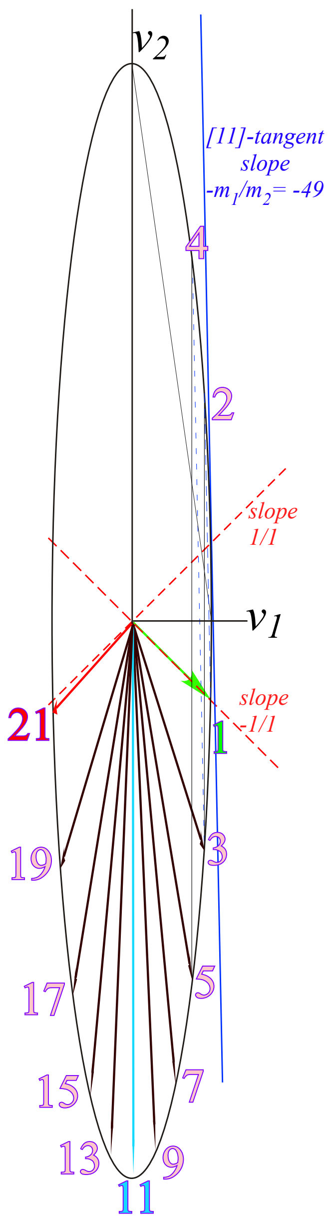
$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \cdot \vec{\mathsf{V}}$$,

or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \cdot \mathsf{M} \cdot \vec{\mathsf{V}}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

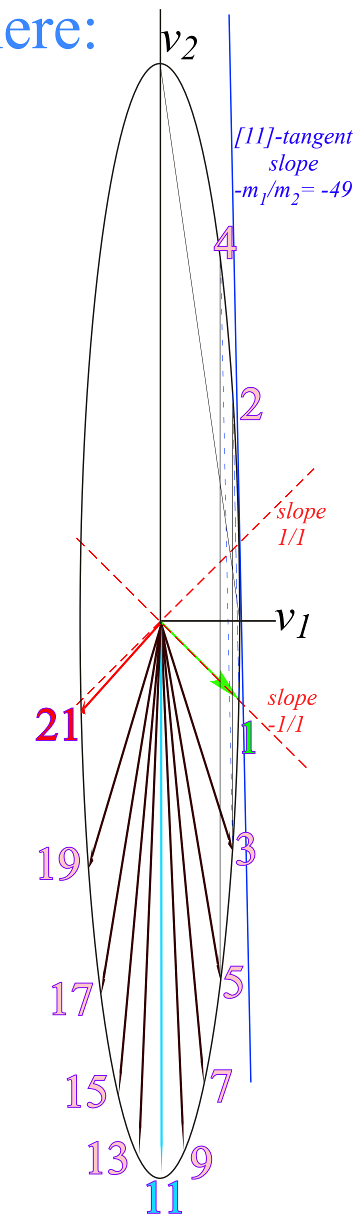
Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\mathsf{V}_1^2 + \frac{1}{2}\mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} \mathsf{V}_1^{FIN_1}/\sqrt{m_1} \\ \mathsf{V}_2^{FIN_1}/\sqrt{m_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1/\sqrt{m_1} \\ \mathsf{V}_2/\sqrt{m_2} \end{pmatrix}$$

$$\text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M}\bullet\vec{\mathsf{V}} \ , \quad \text{or:} \begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C}\bullet\mathsf{M}\bullet\vec{\mathsf{V}}$$

Then collisions become ***reflections*** $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become ***rotations*** $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

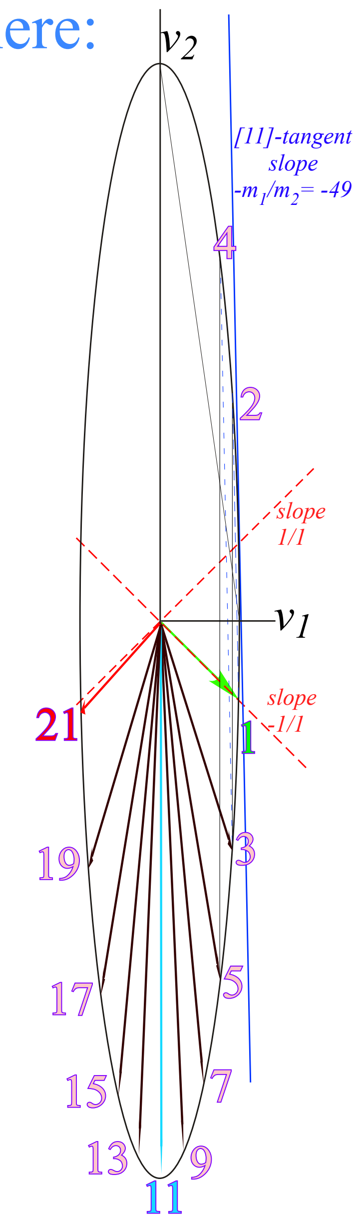
$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

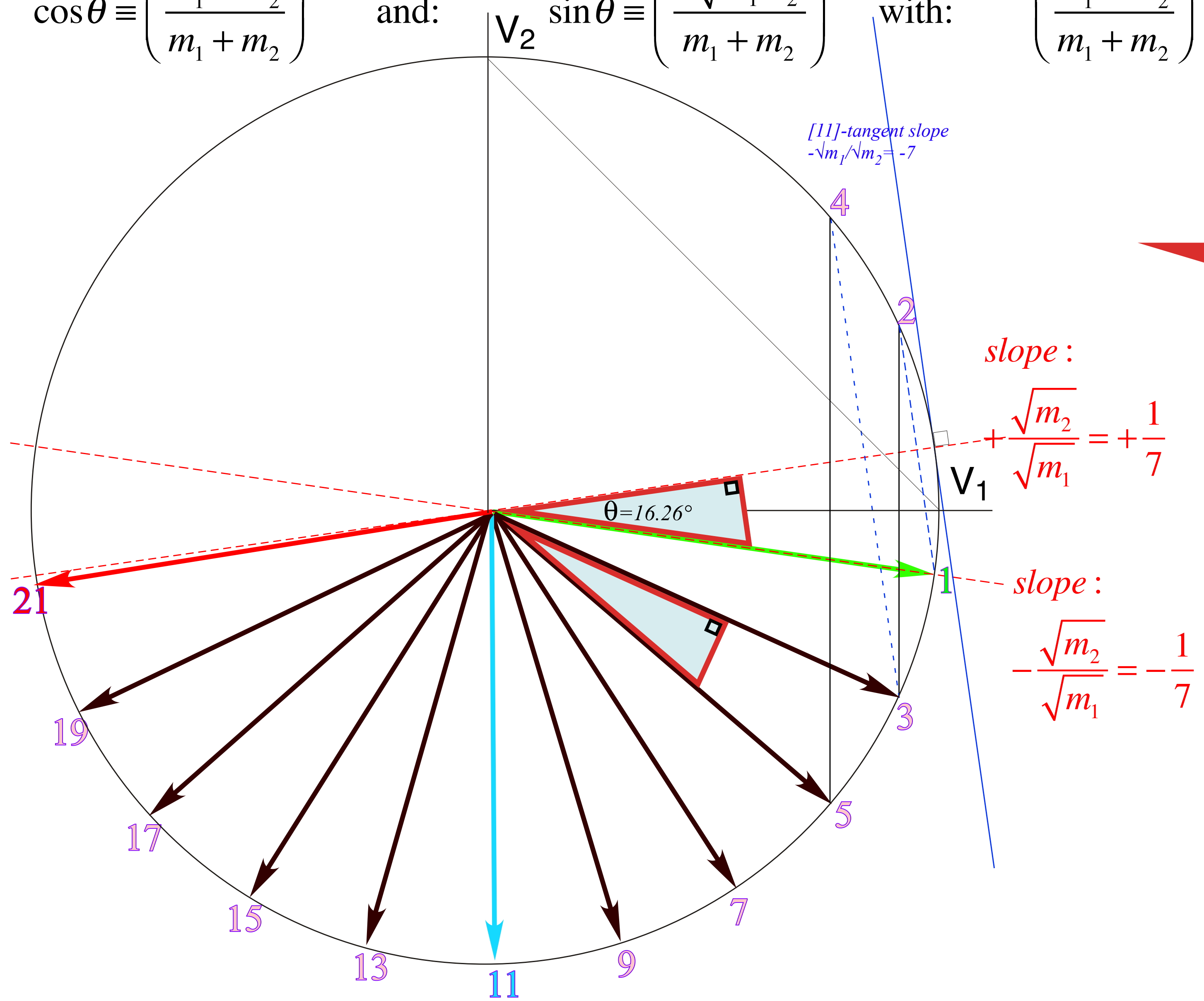
or: $$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \bullet \vec{V}$$,

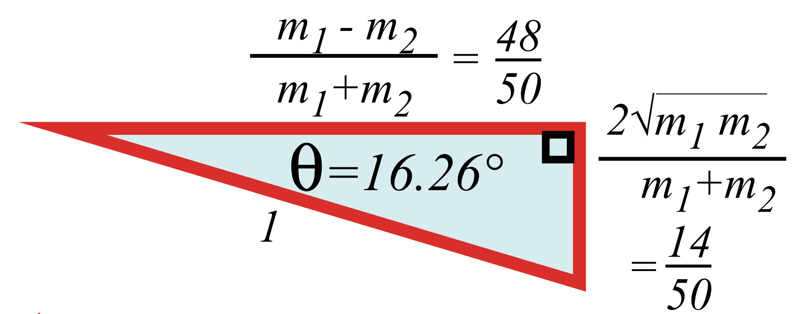
or: $$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 5.2a-c

(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

or:

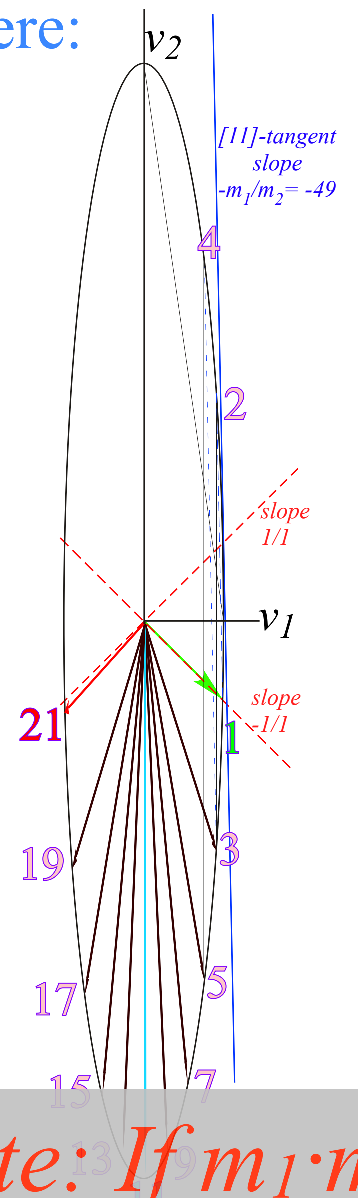
$$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \bullet \vec{V} ,$$

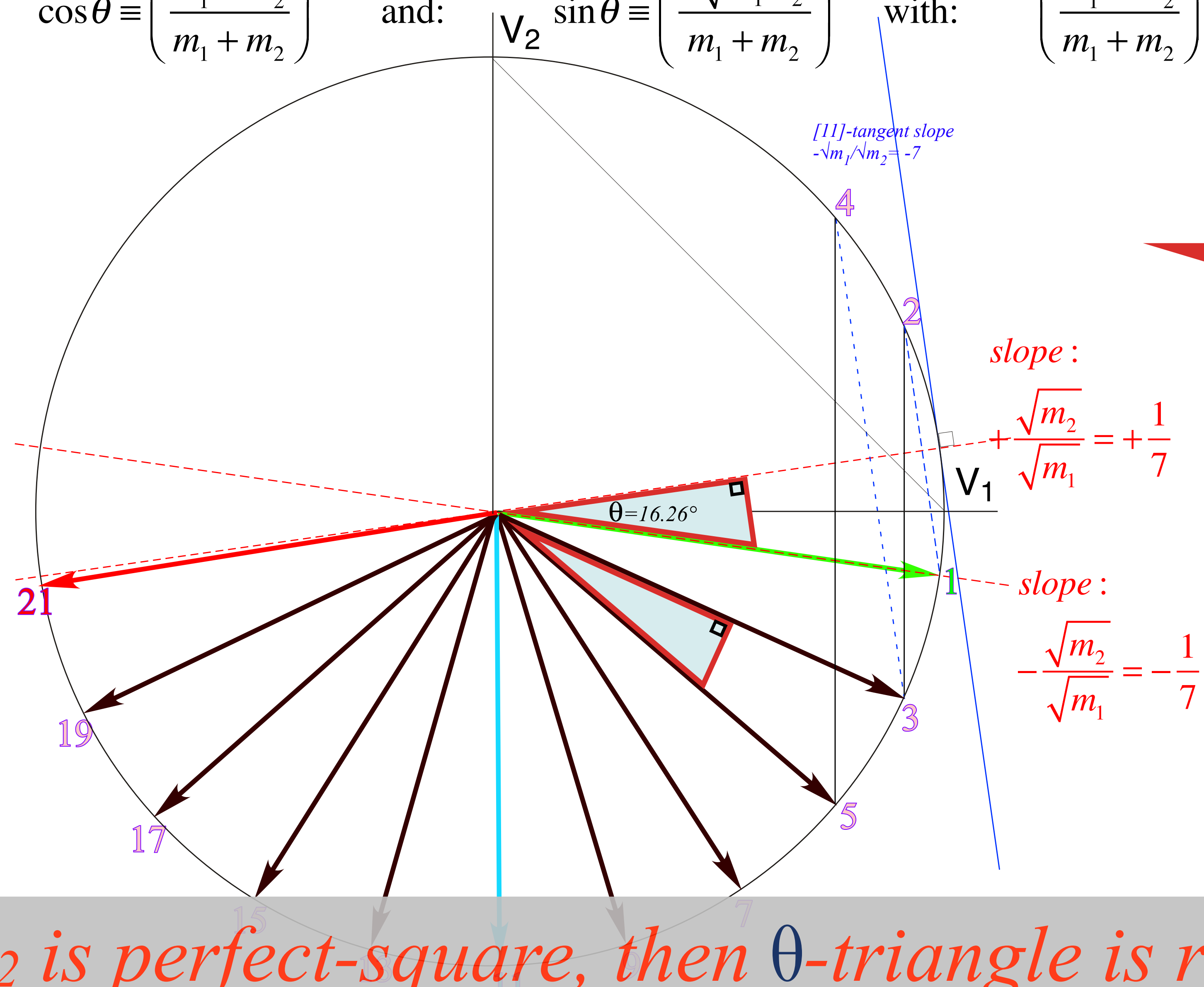
or:

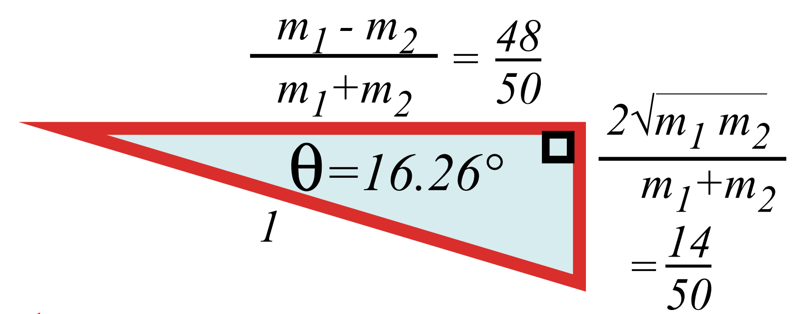
$$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{V}$$

Then collisions become ***reflections*** $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become ***rotations*** $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

Fig. 5.2a-c

(revised)

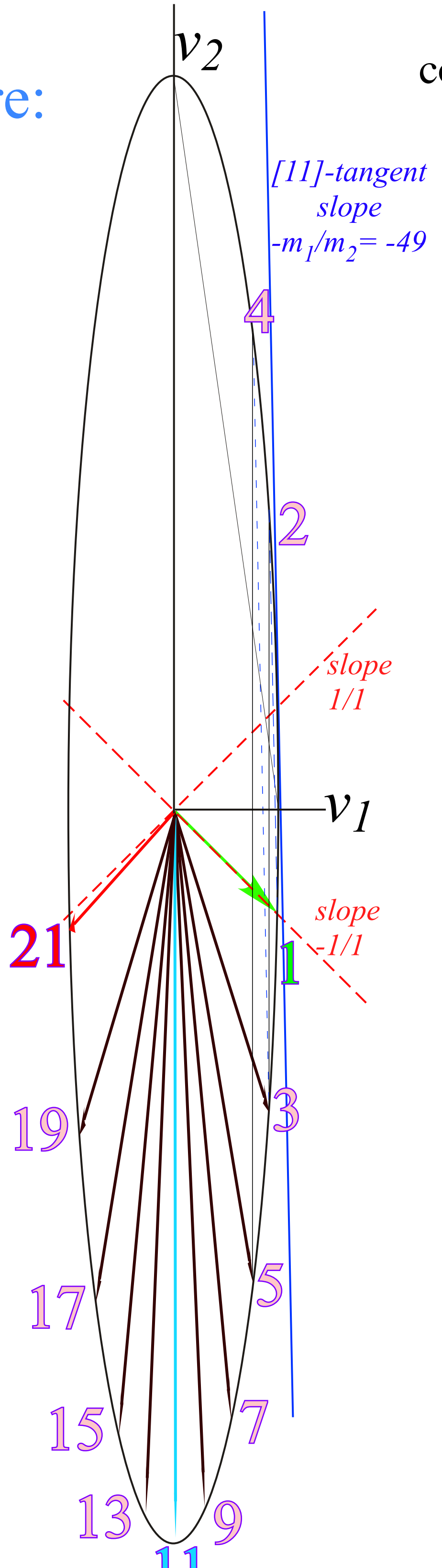
Note: If $m_1 \cdot m_2$ is perfect-square, then θ-triangle is rational ($3^2+4^2=5^2$, etc.)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$



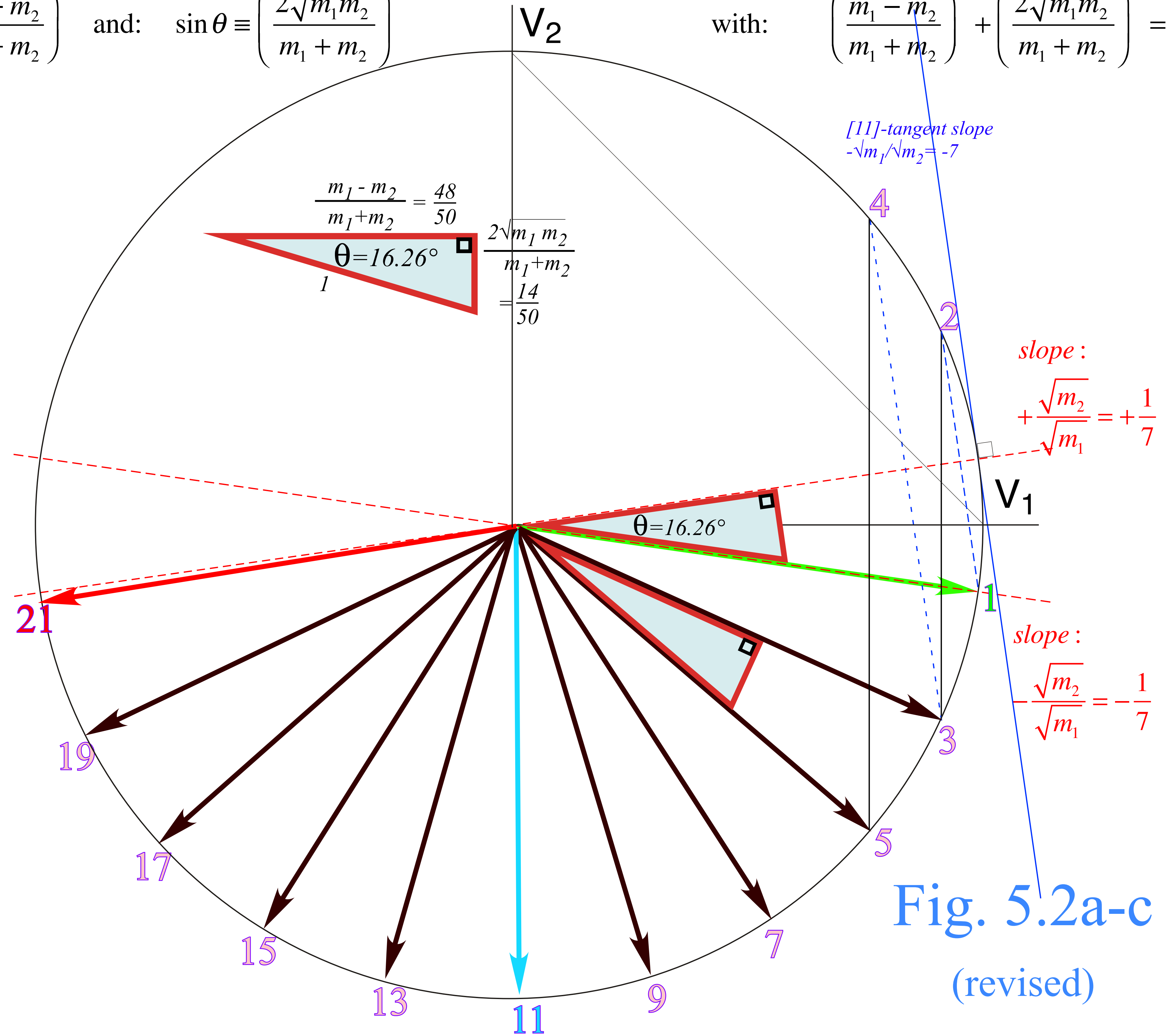


Fig. 5.2a-c

(revised)

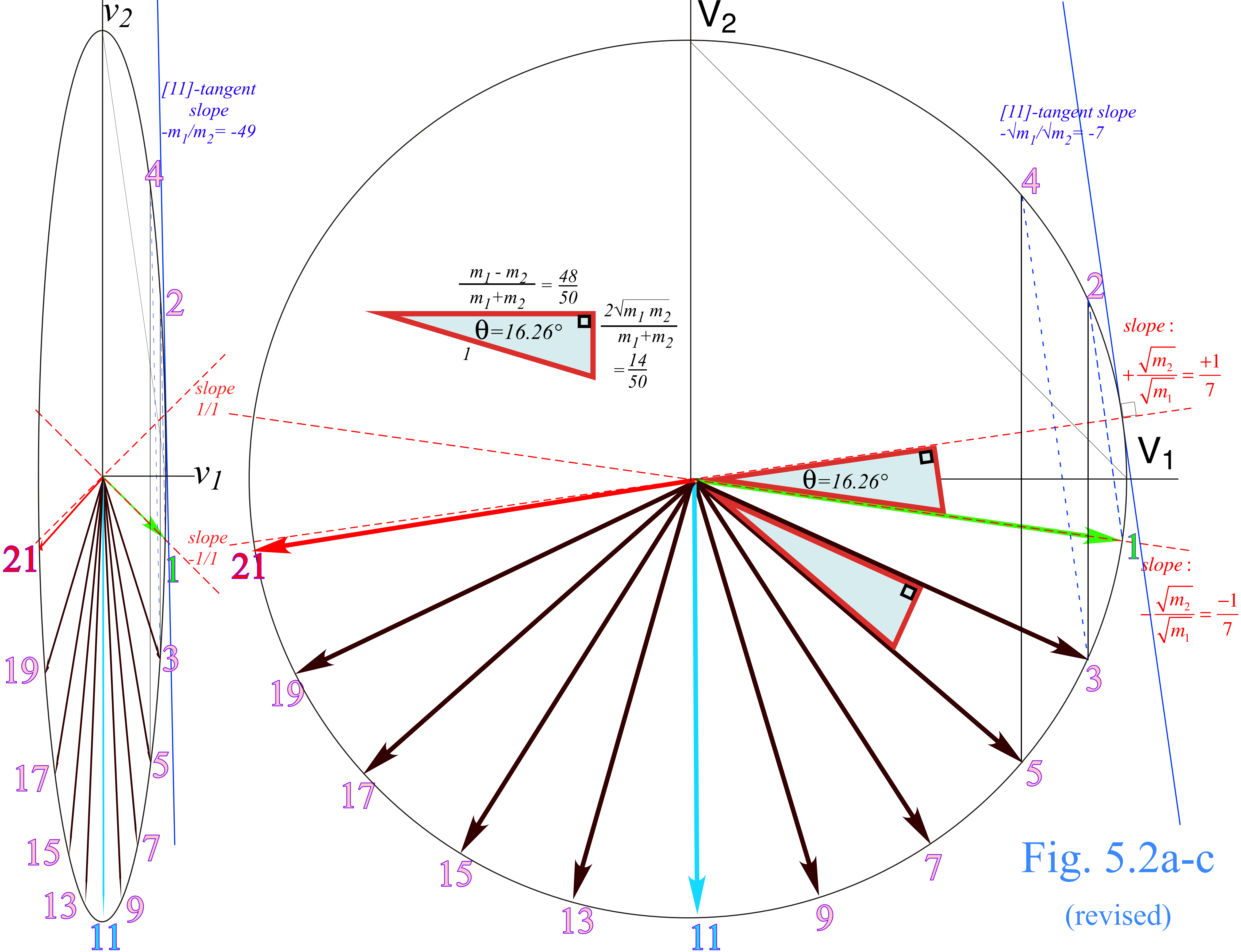


Fig. 5.2a-c

(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

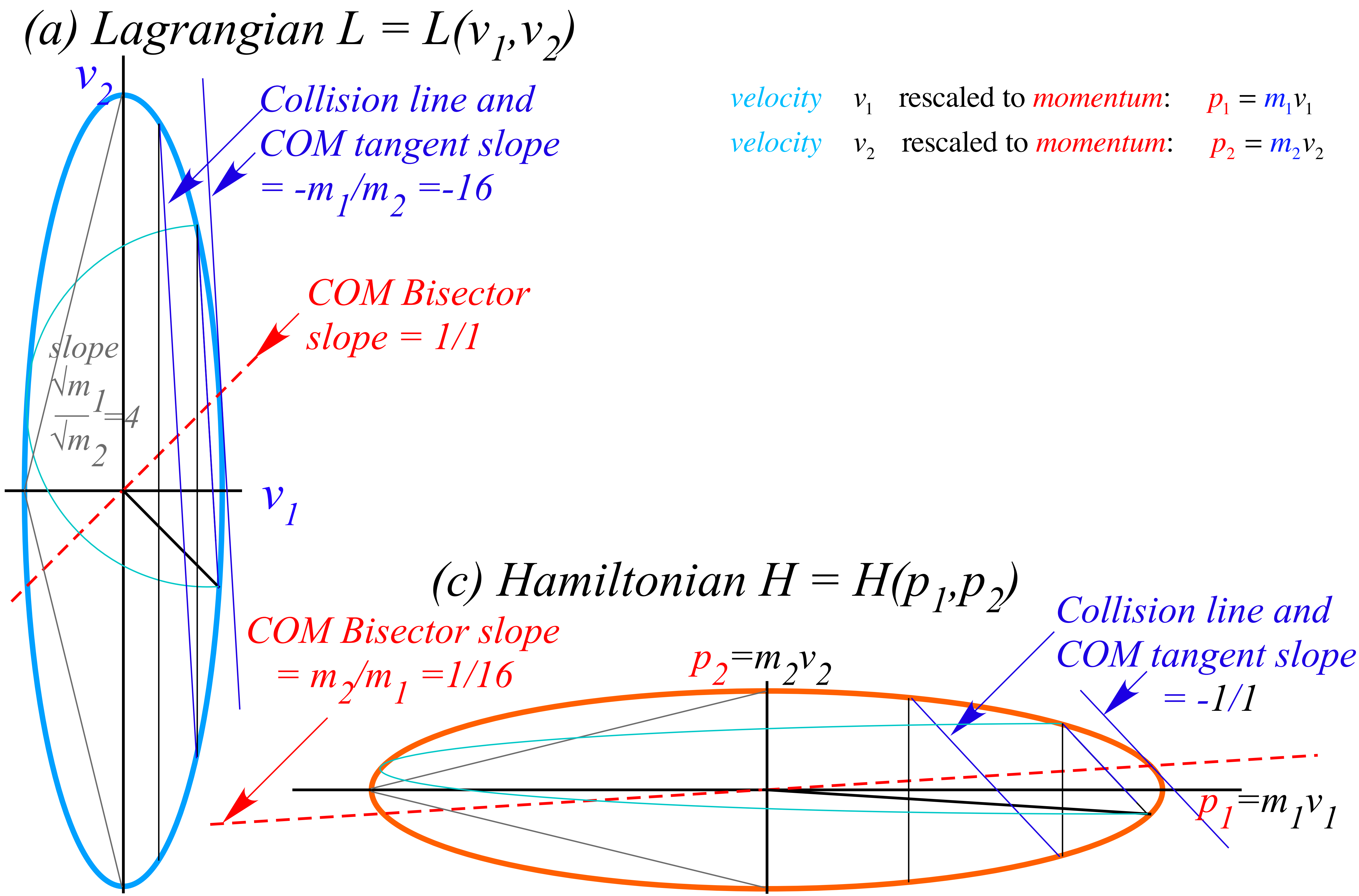
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

What ellipse rescaling leads to...

How this relates to Lagrangian, and Hamiltonian mechanics later on

velocity v_1 rescaled to momentum: $p_1 = m_1 v_1$

velocity v_2 rescaled to momentum: $p_2 = m_2 v_2$

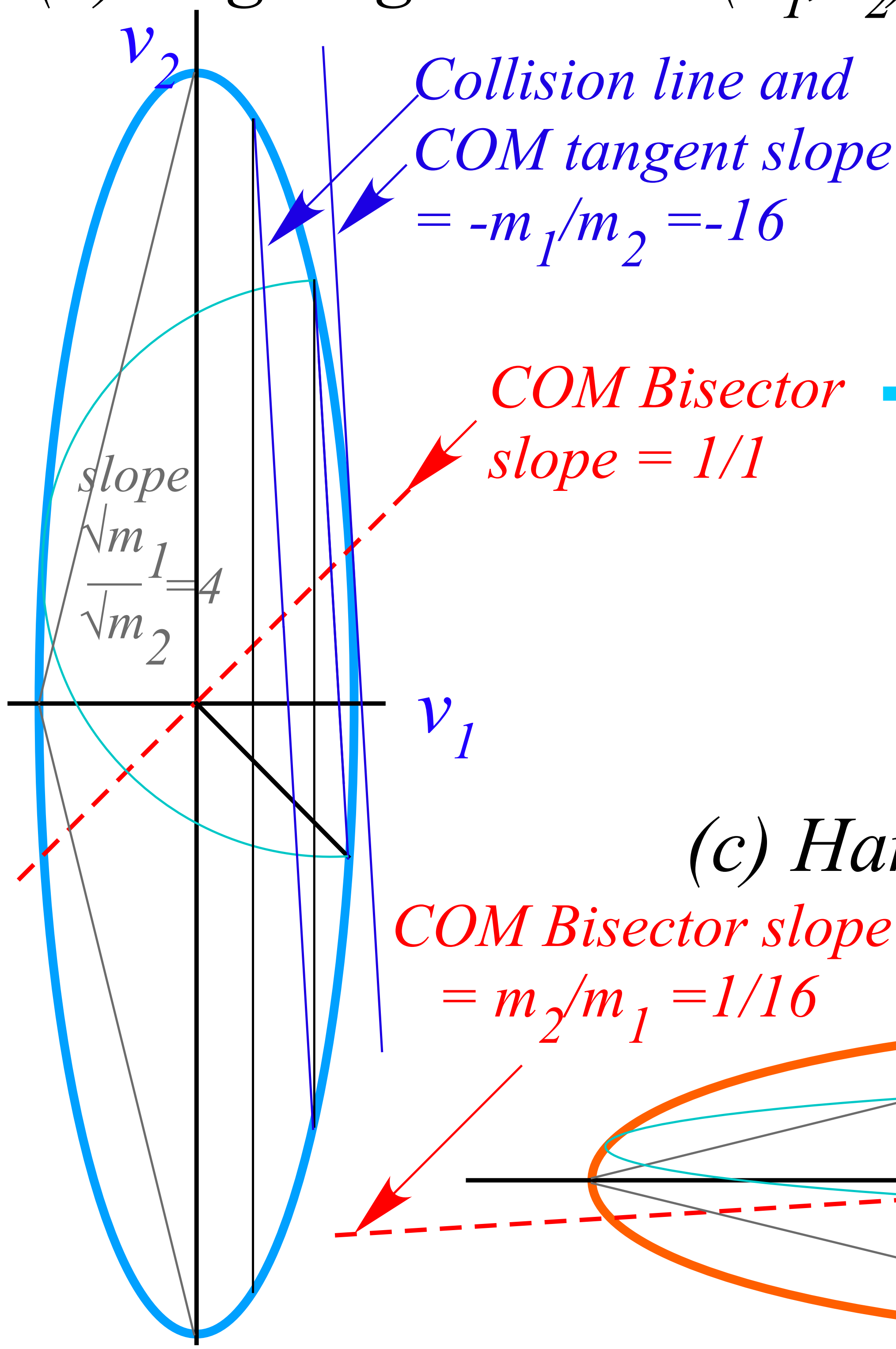


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on

(a) Lagrangian $L = L(v_1,v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

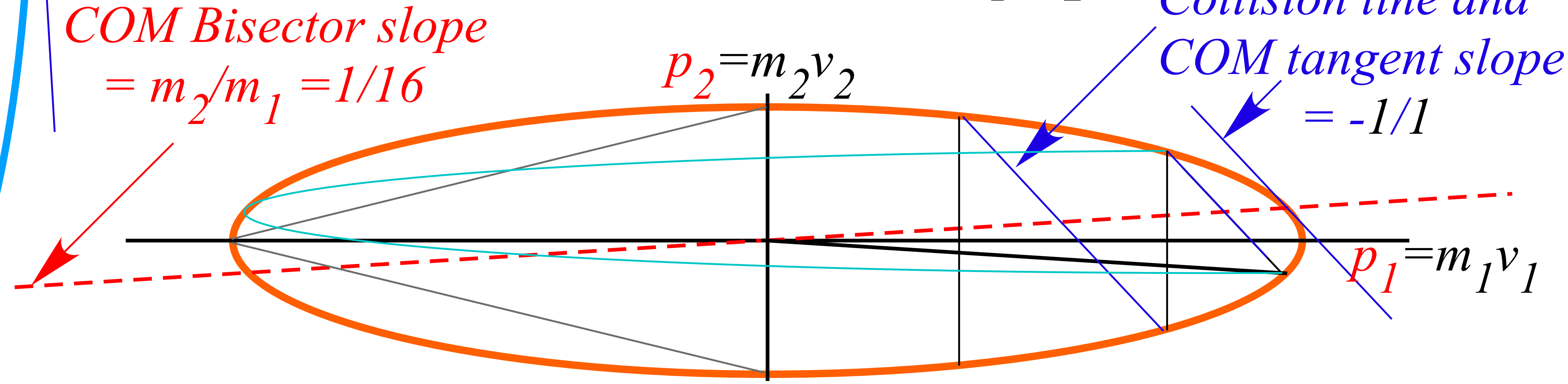
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1,v_2) = KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1,p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

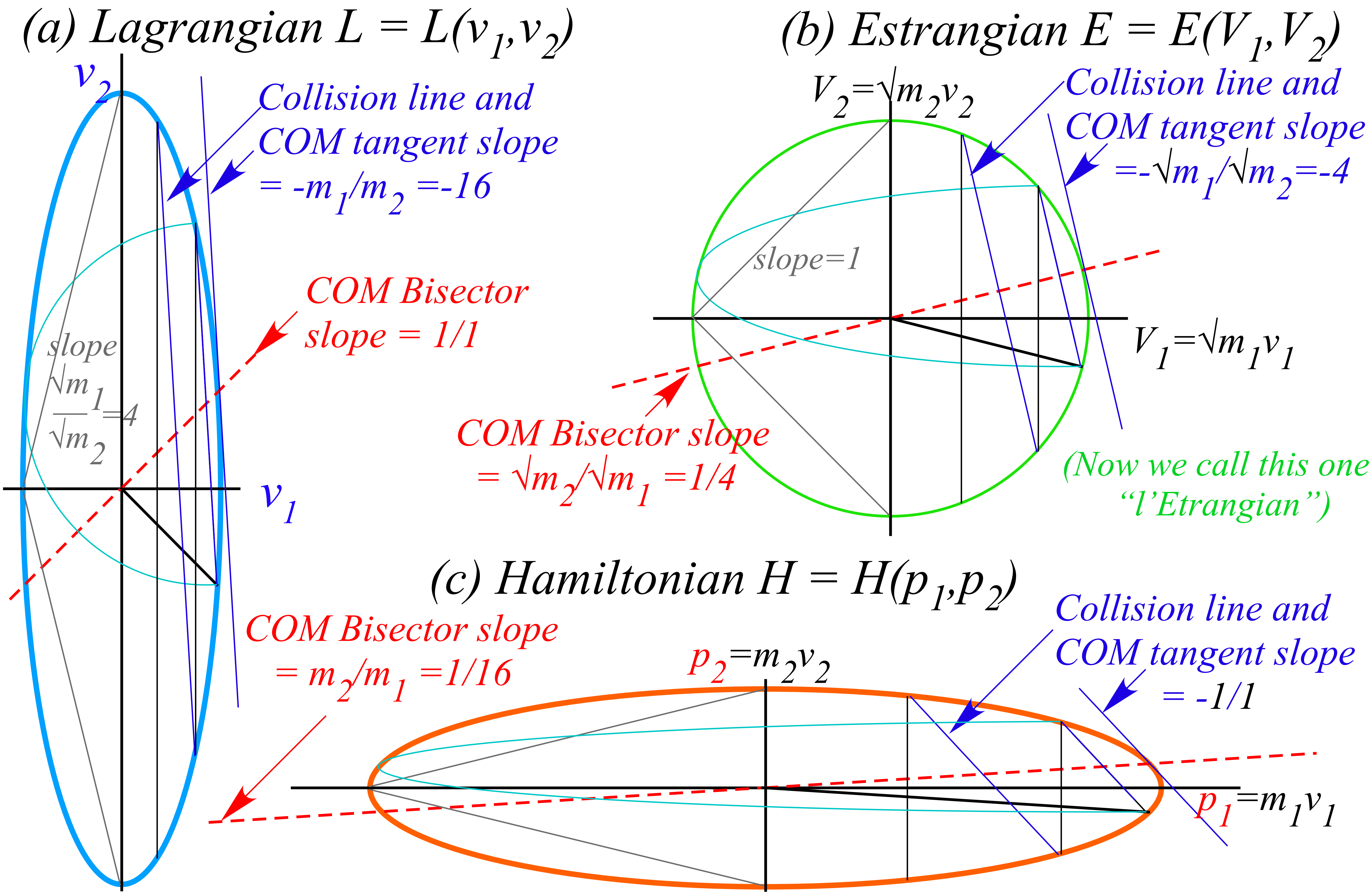
(c) Hamiltonian $H = H(p_1,p_2)$



What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

→ *Reflections in the clothing store: "It's all done with mirrors!"*

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Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Reflections in clothing store mirrors

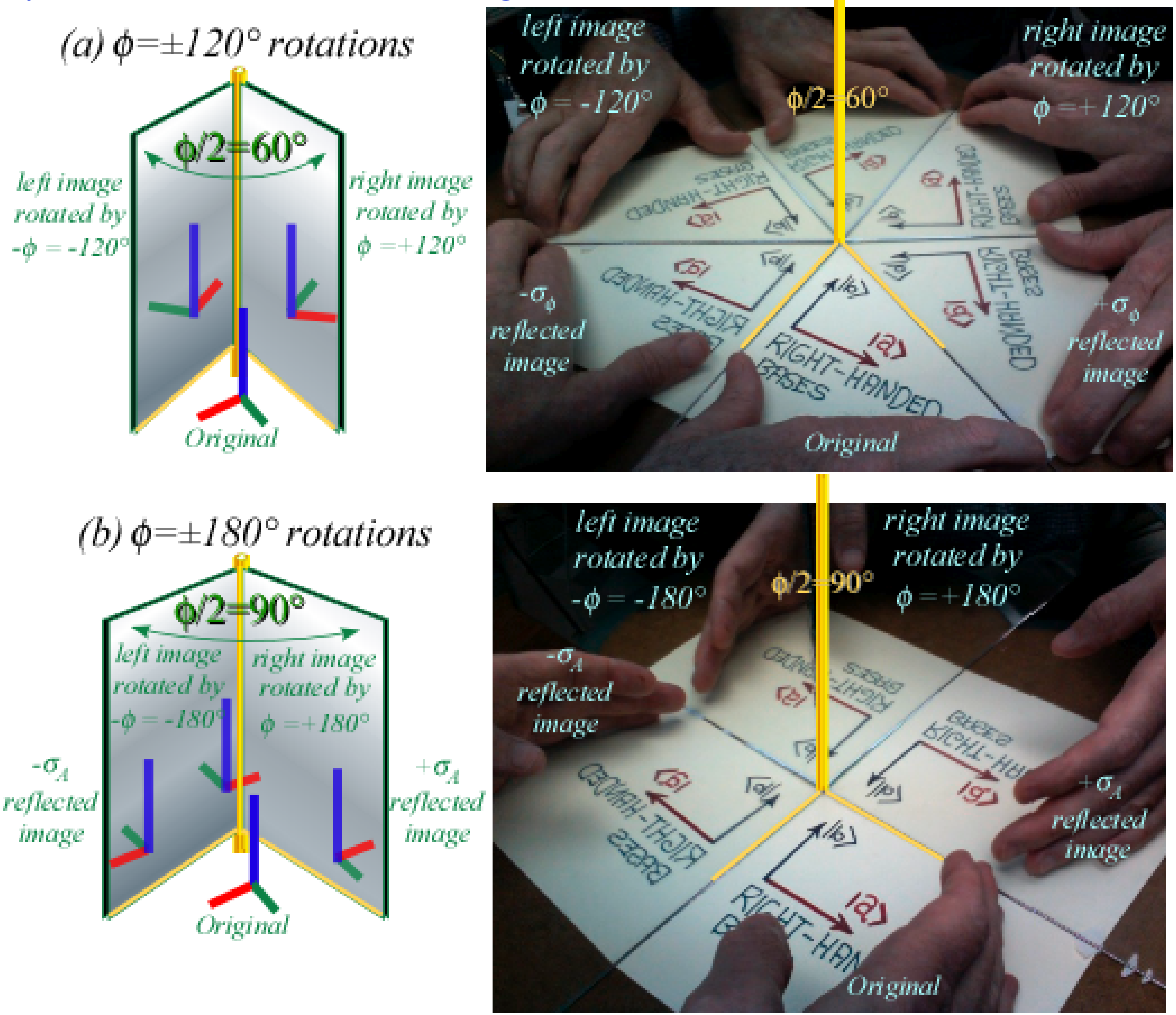

Fig. 5.4a-b

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $-\sigma_A=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ *(b)Reflections* $\sigma_B=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $-\sigma_B=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

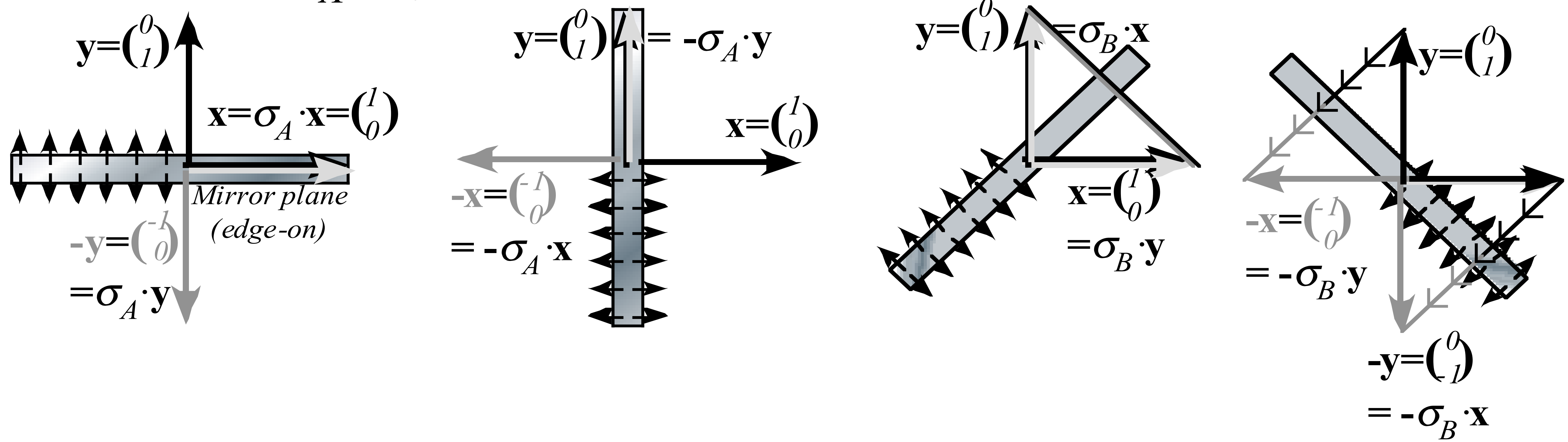

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

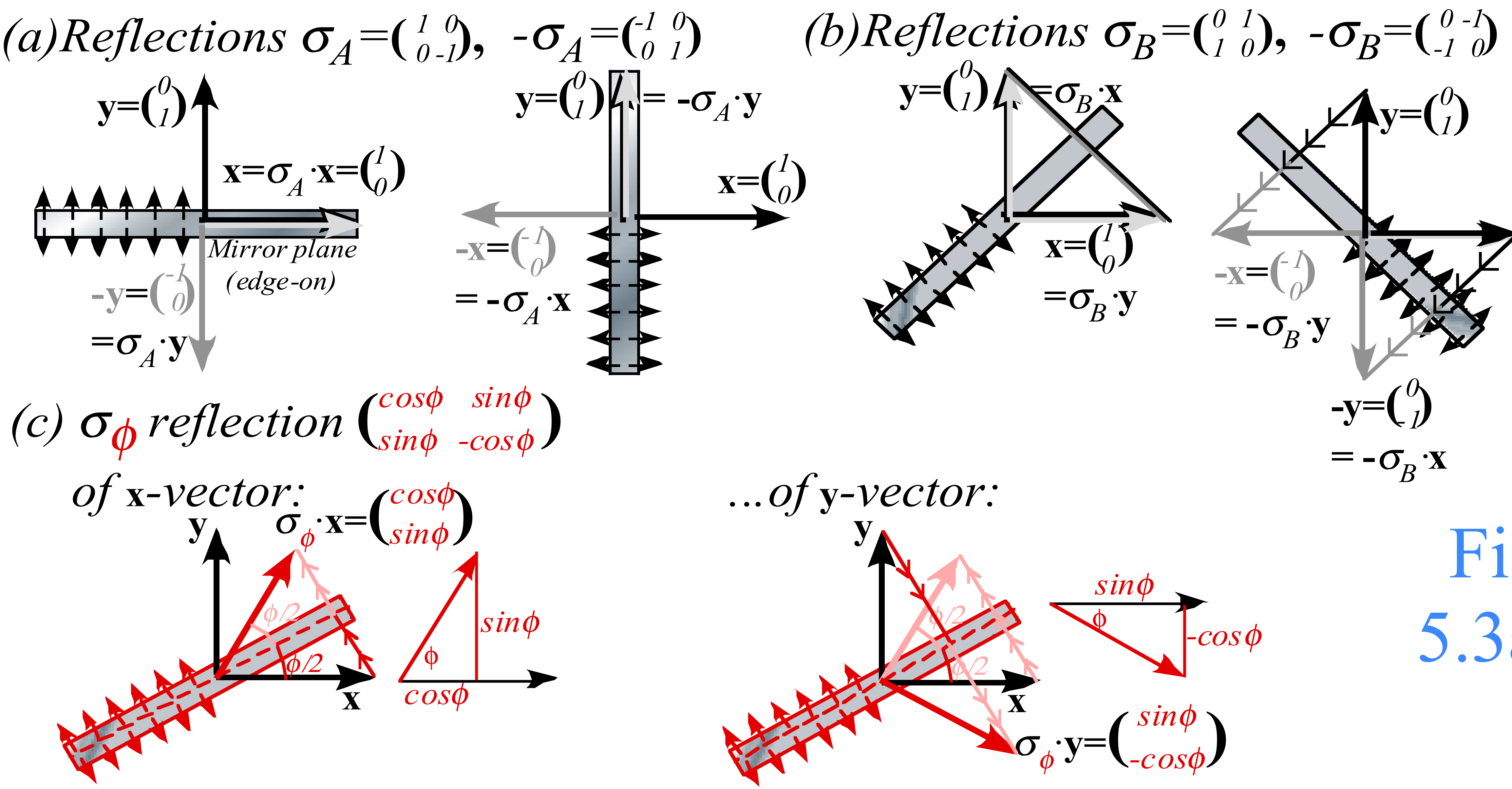

Fig. 5.3a-e

Symmetry: It's all done with mirrors!

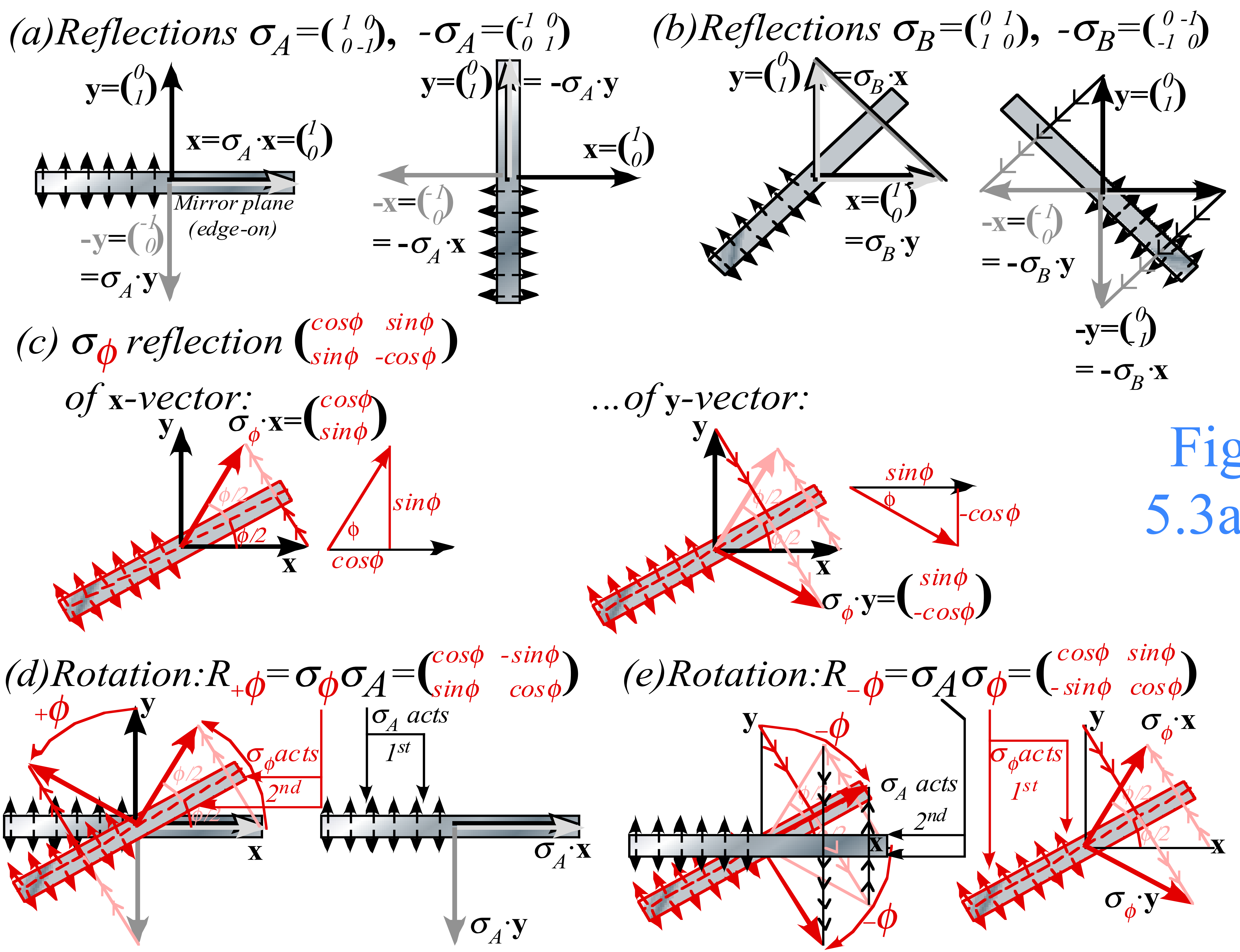

Fig. 5.3a-e

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a reflection

... even number of reflections is a rotation (or unit-op **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a *<u>reflection</u>*

... even number of reflections is a *<u>rotation</u>* *(or unit-op* **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and *<u>rotate</u>* *easily*
Waves patterns are non-rigid and *<u>reflect</u>* *easily*

∴ *...wave reflections underlie modern physics*

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

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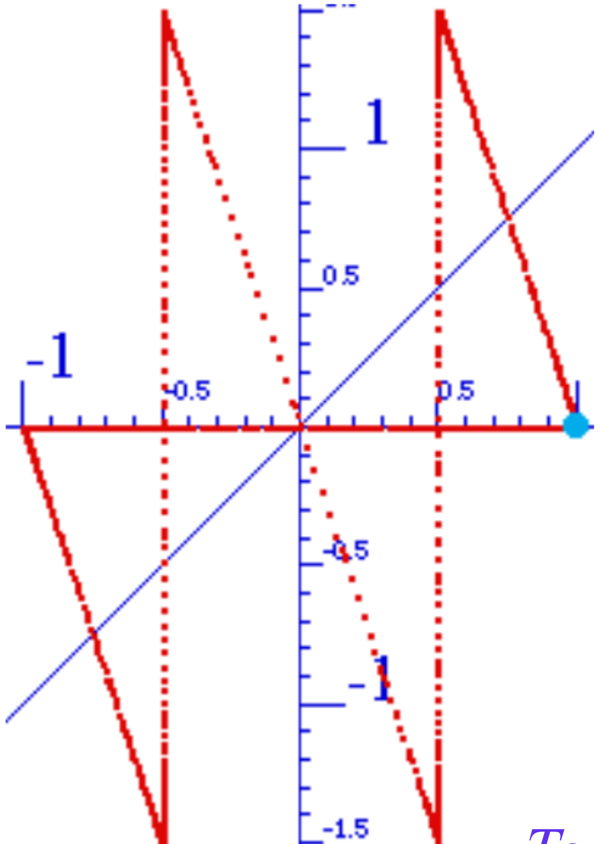
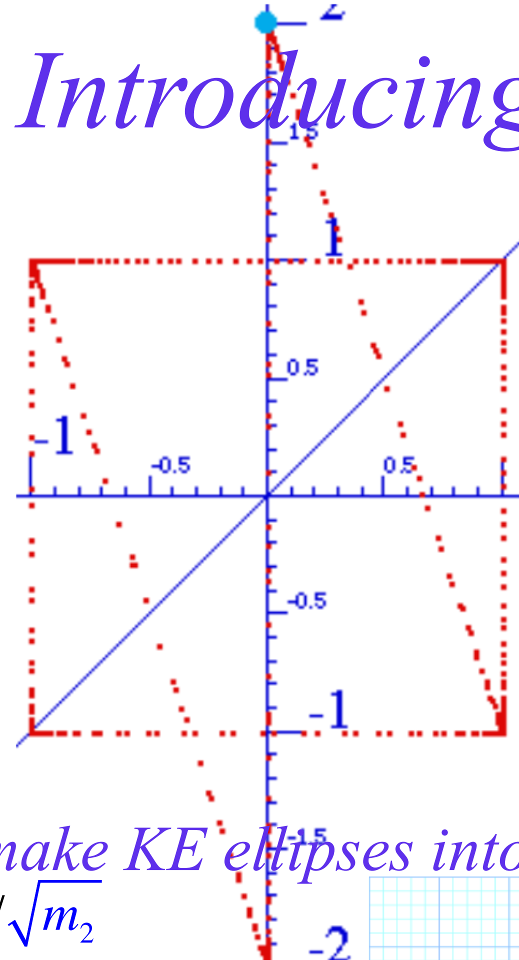
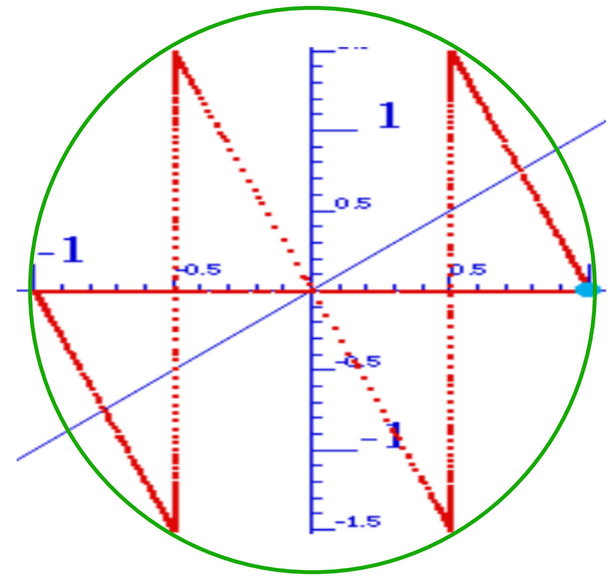
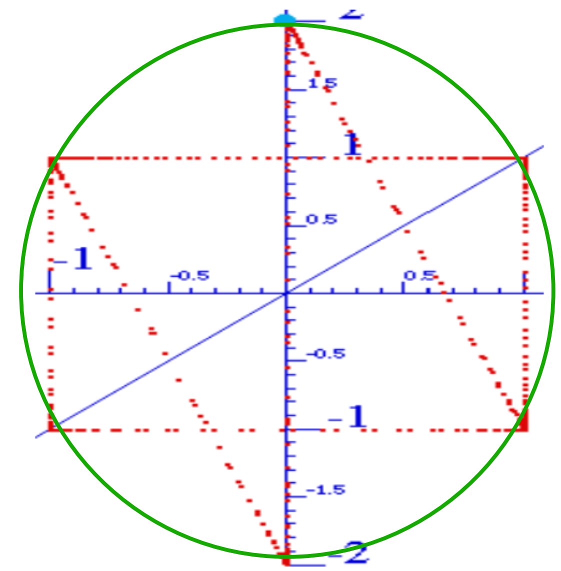
Introducing Symmetry Operators

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$



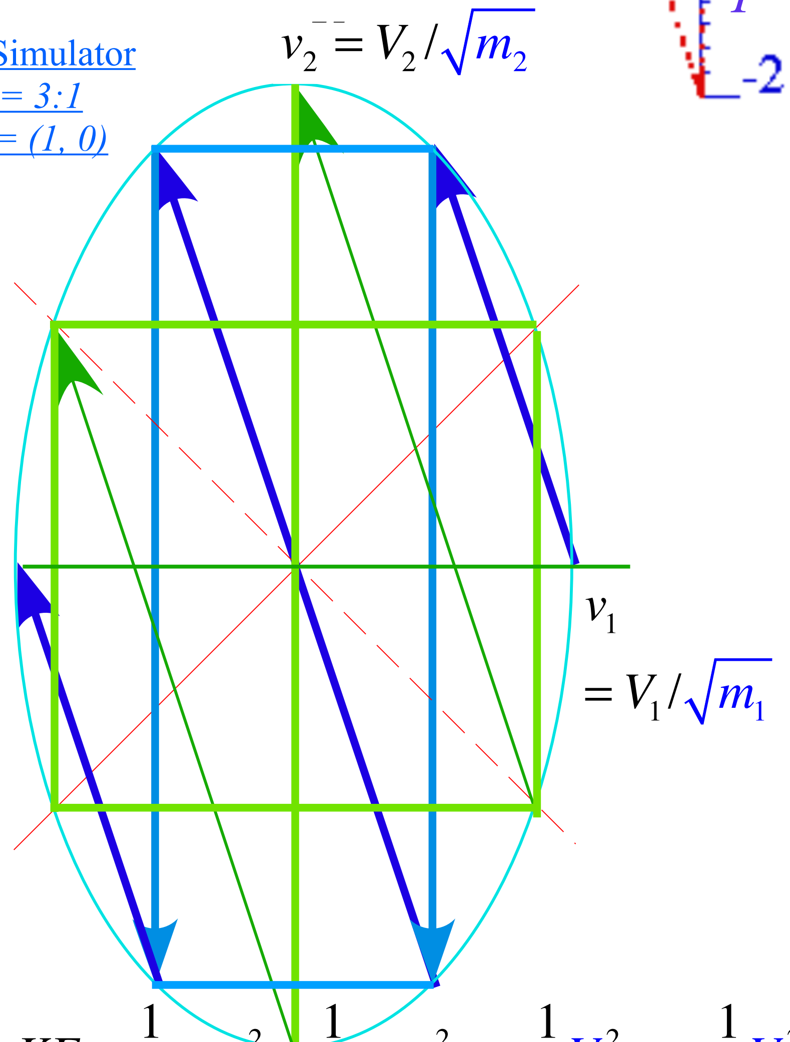


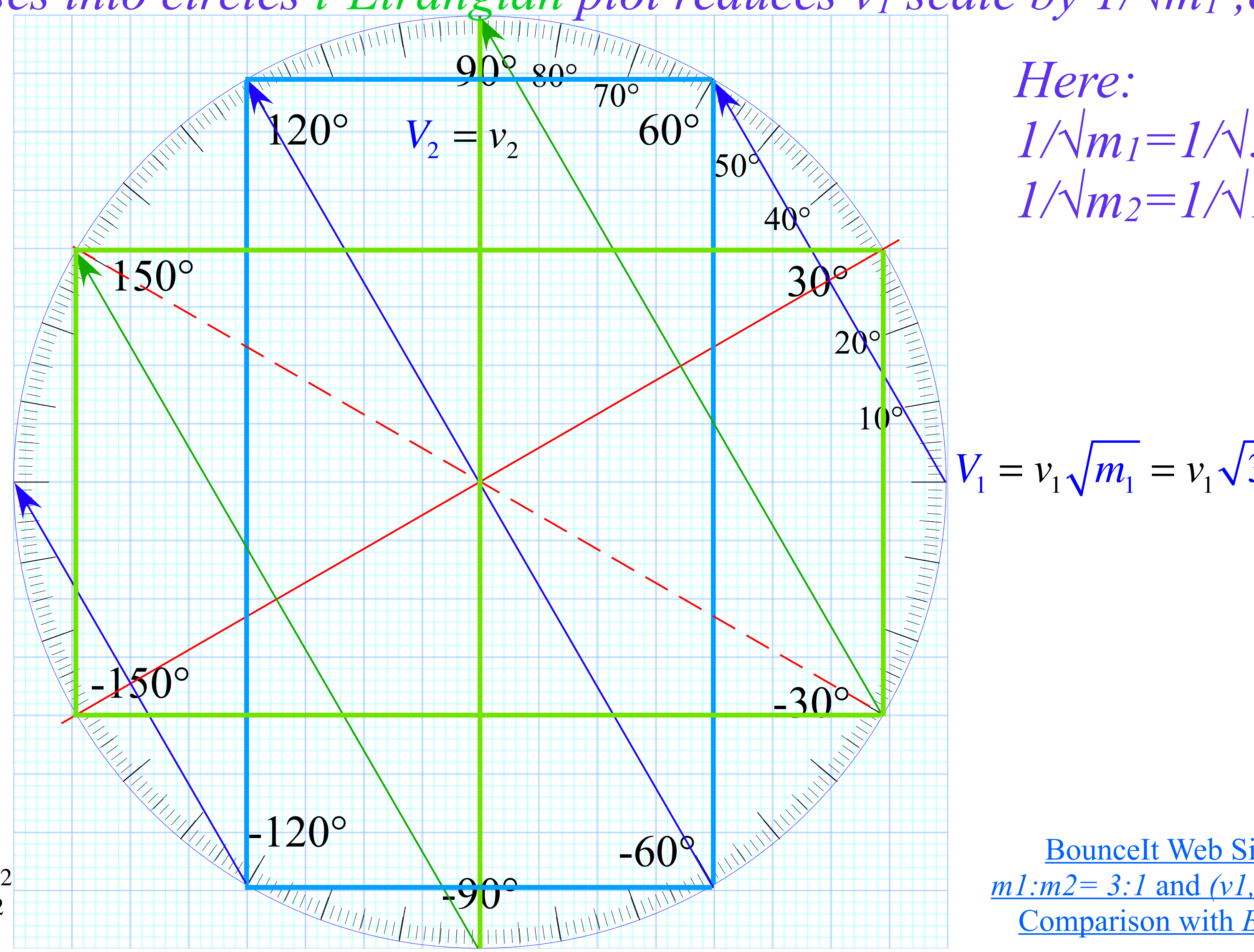




To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

[BounceIt Simulator
m1:m2= 3:1
(v1, v2) = (1, 0)]





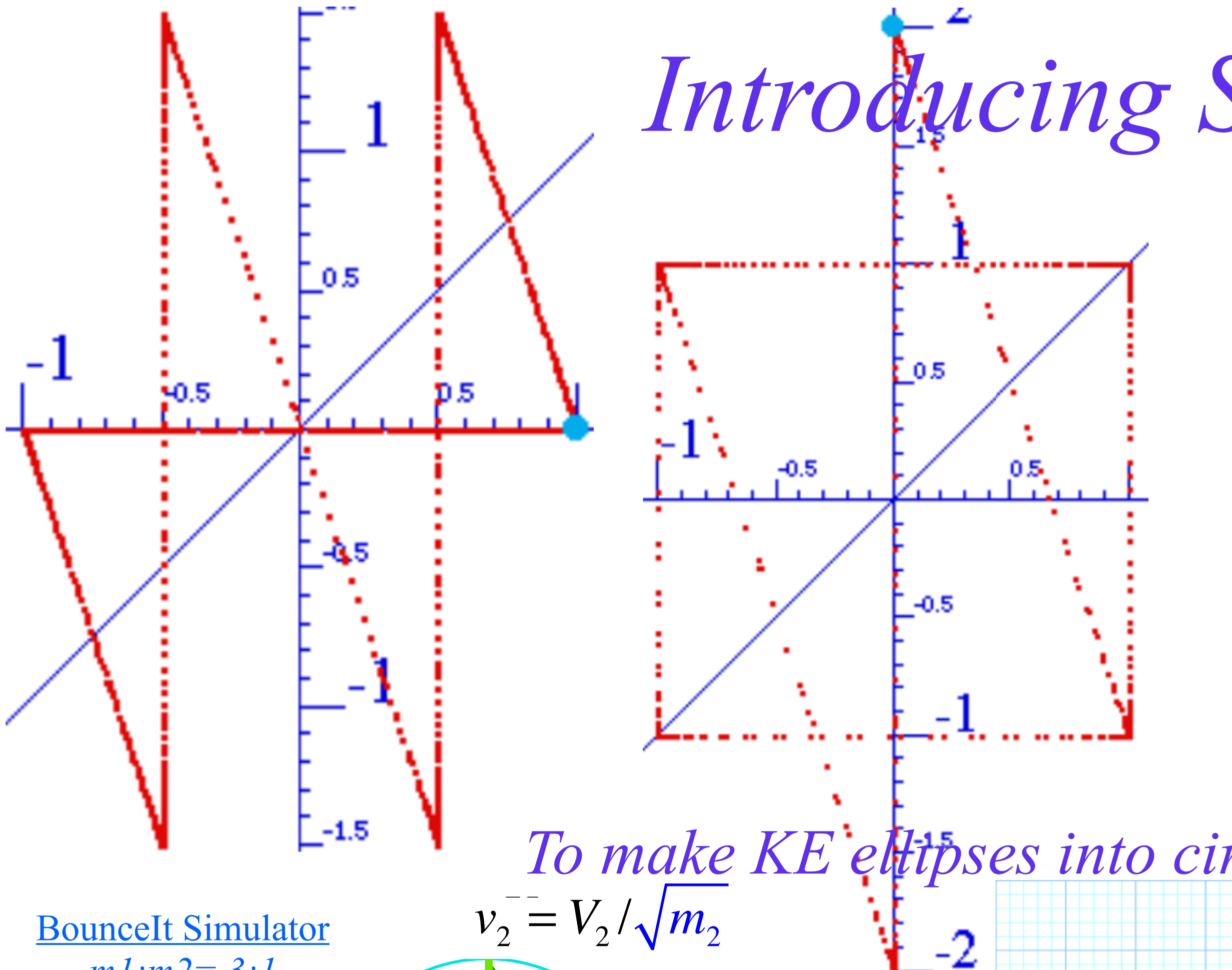
Here:
$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$
$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

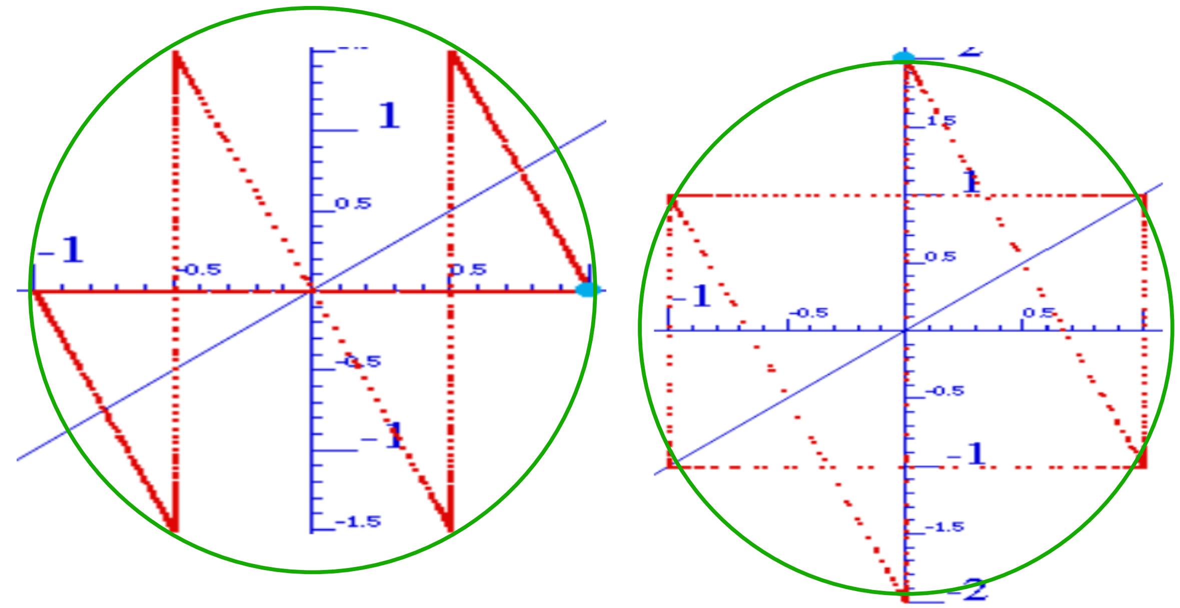
$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$

[BounceIt Web Simulator
m1:m2= 3:1 and *(v1, v2) = (1, 0)*
Comparison with *Estrangian*]

Introducing Symmetry Operators

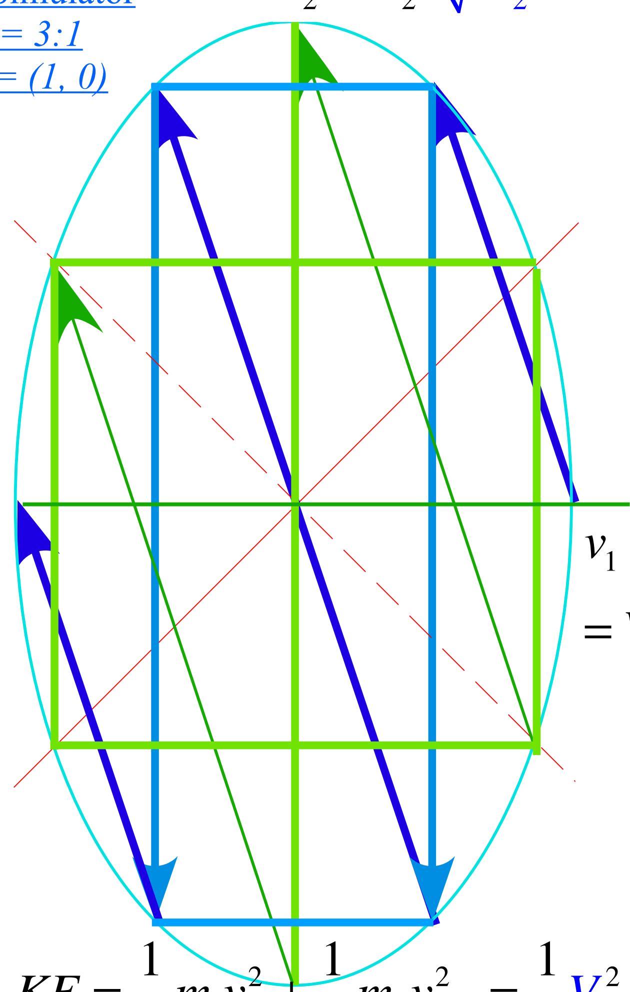
Collisions for mass ratio m_1:m_2= 3:1



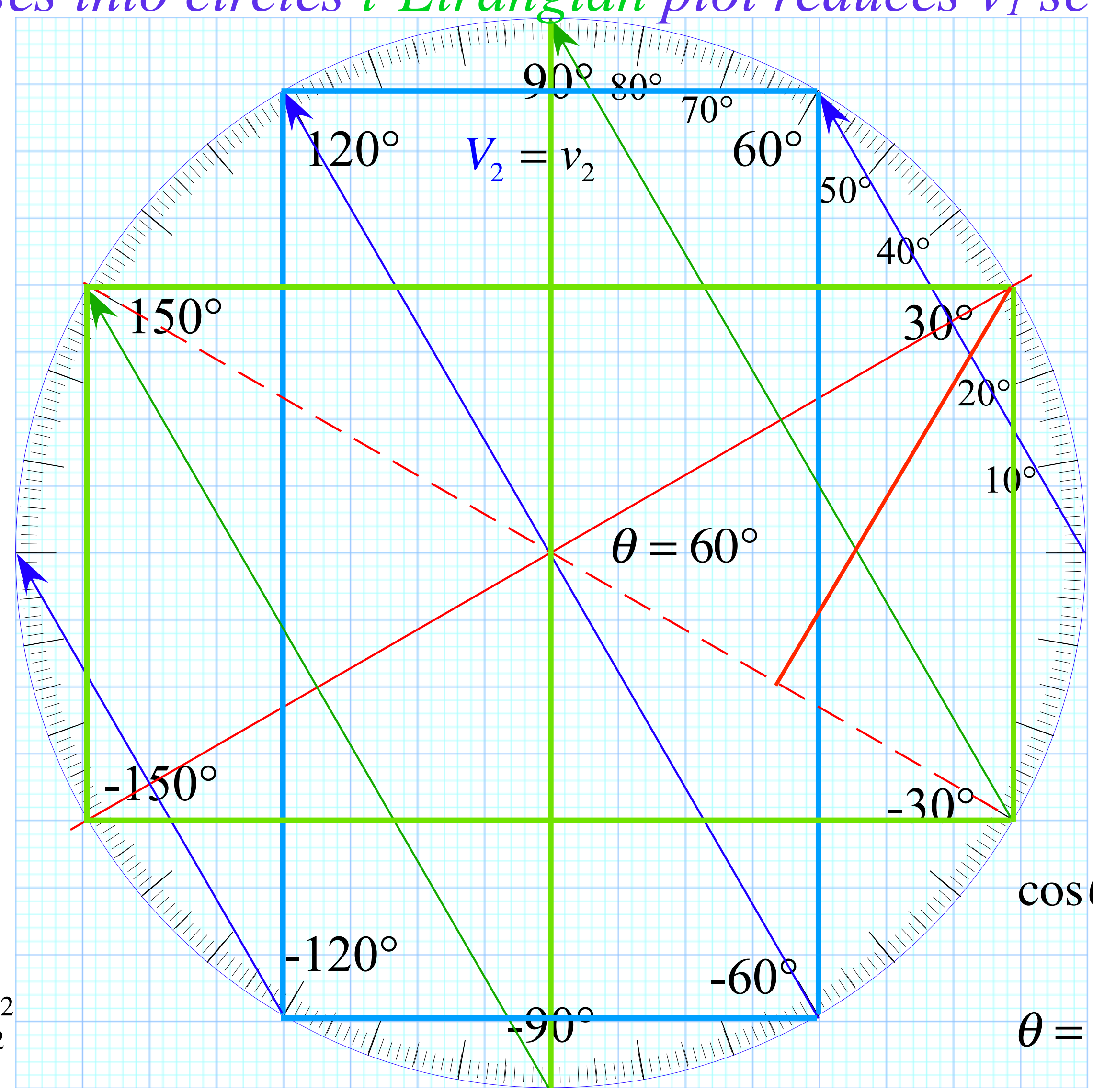


To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

BounceIt Simulator
m1:m2= 3:1
(v1, v2) = (1, 0)



$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$



$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

Here:
$1/\sqrt{m_1}=1/\sqrt{3}=0.577$
$1/\sqrt{m_2}=1/\sqrt{1}=1.0$

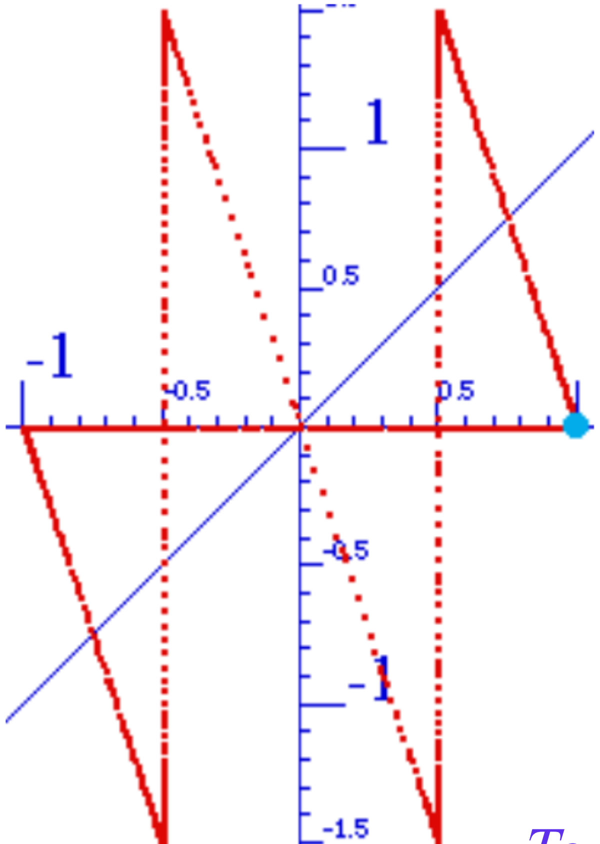
$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

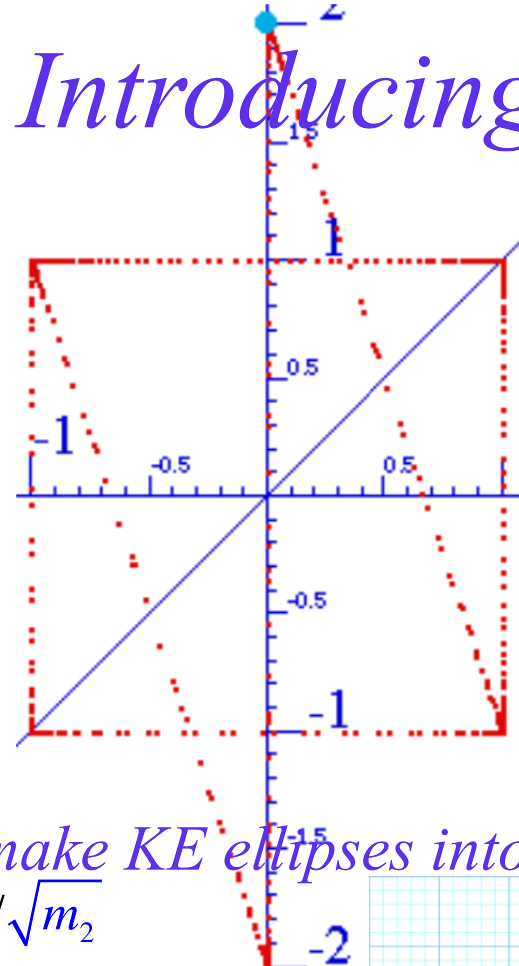
$\theta = 60°$

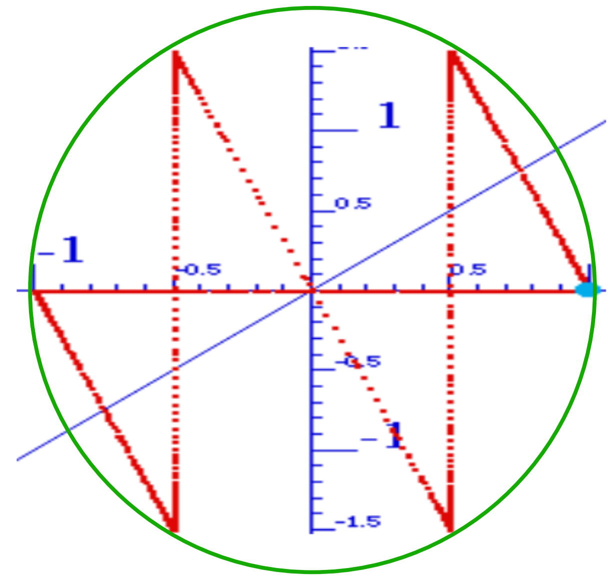
BounceIt Web Simulator
Comparison with *Estrangian*

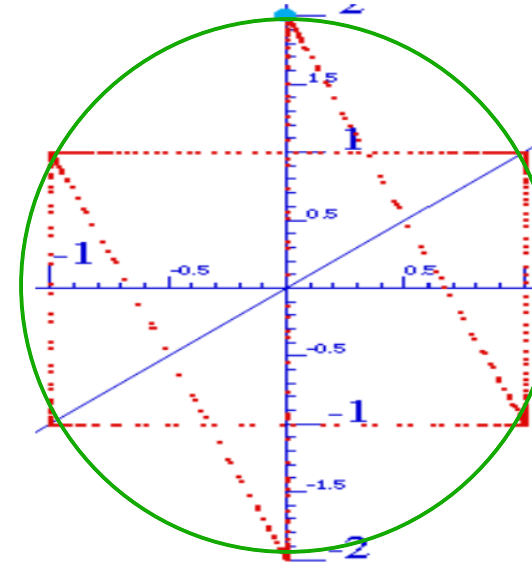
Introducing Symmetry Operators

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$





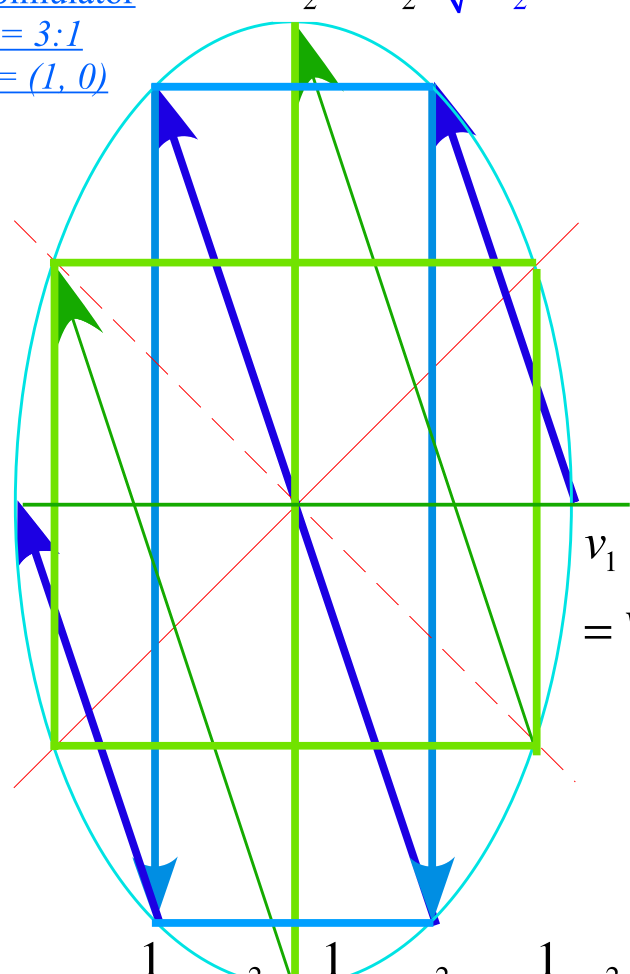




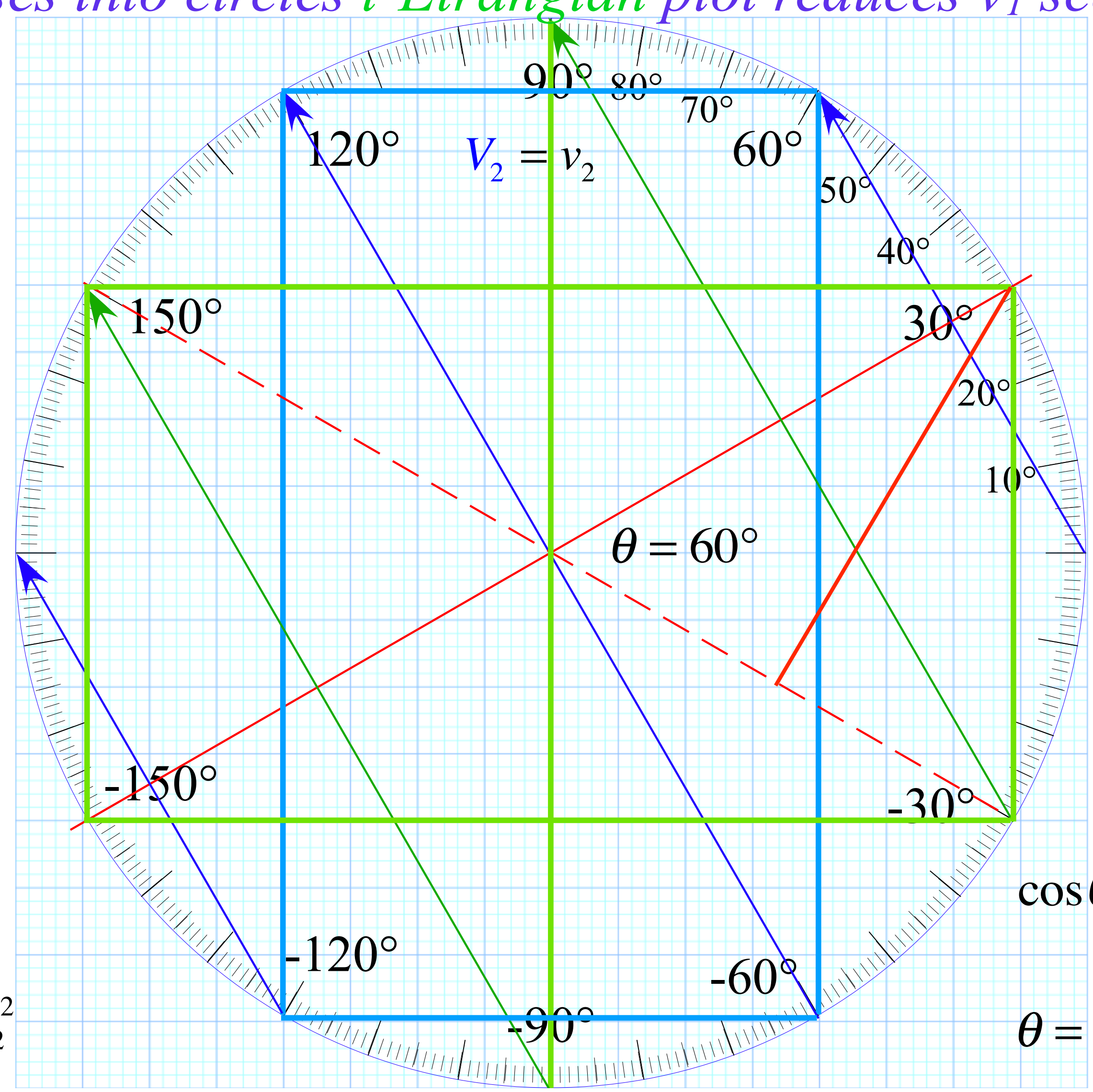
To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$,etc.

BounceIt Simulator
m1:m2= 3:1
(v1, v2) = (1, 0)

$v_2 = V_2/\sqrt{m_2}$



$v_1 = V_1/\sqrt{m_1}$



$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

Here:
$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$
$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

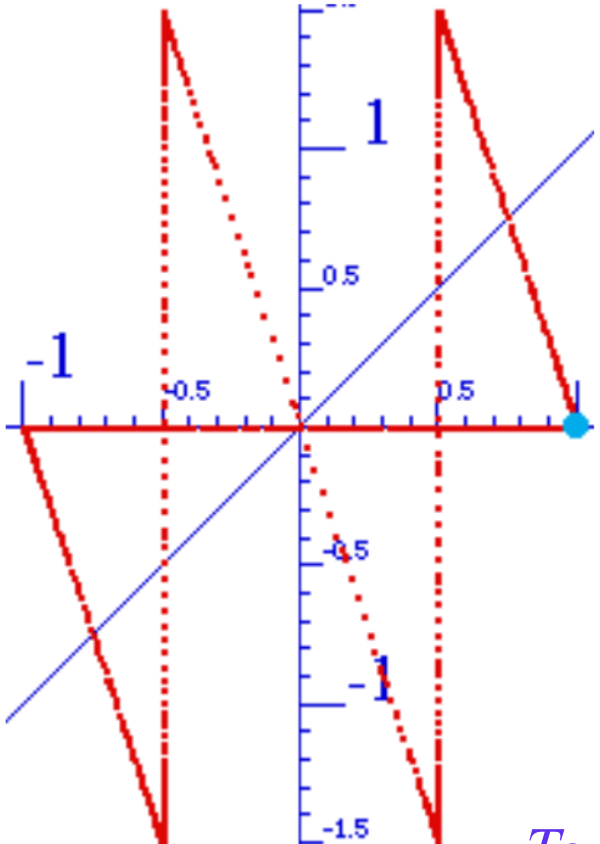
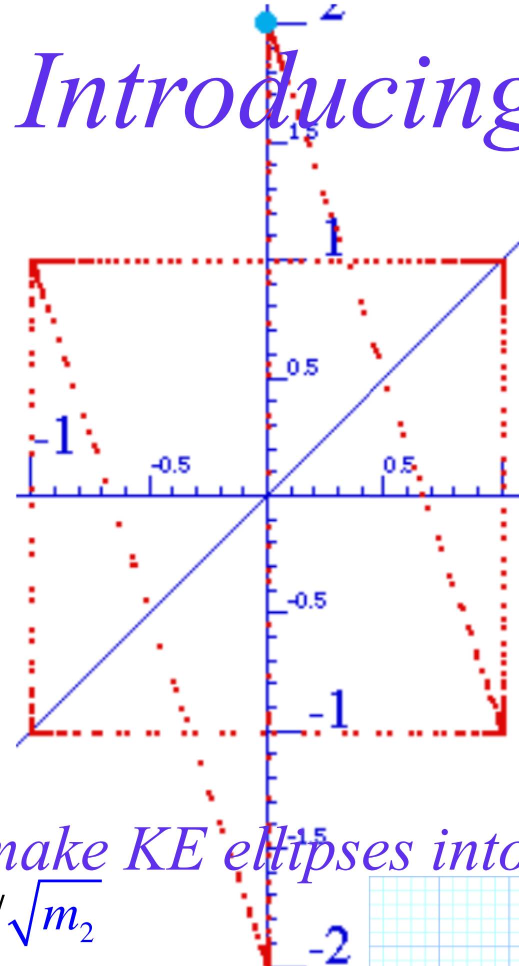
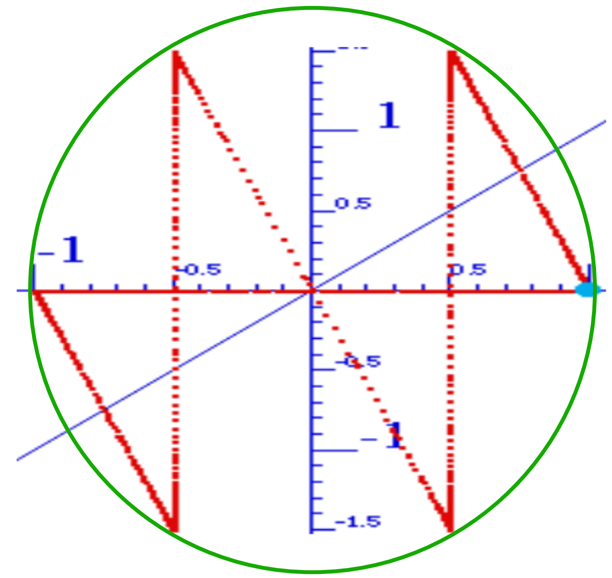
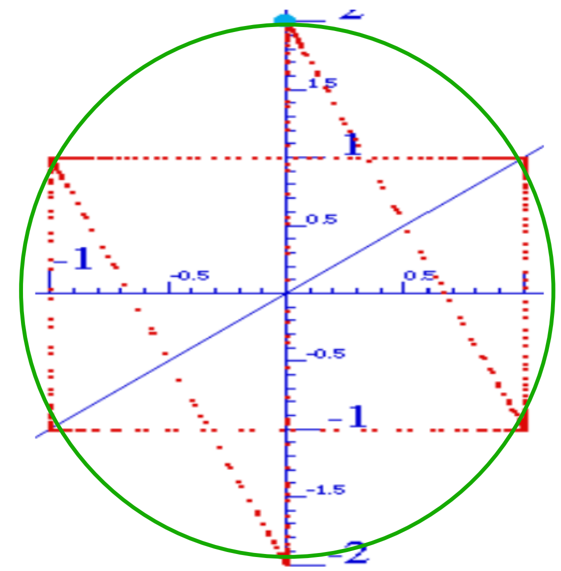
$\theta = 60°$

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$

BounceIt Web Simulator
Comparison with *Estrangian*

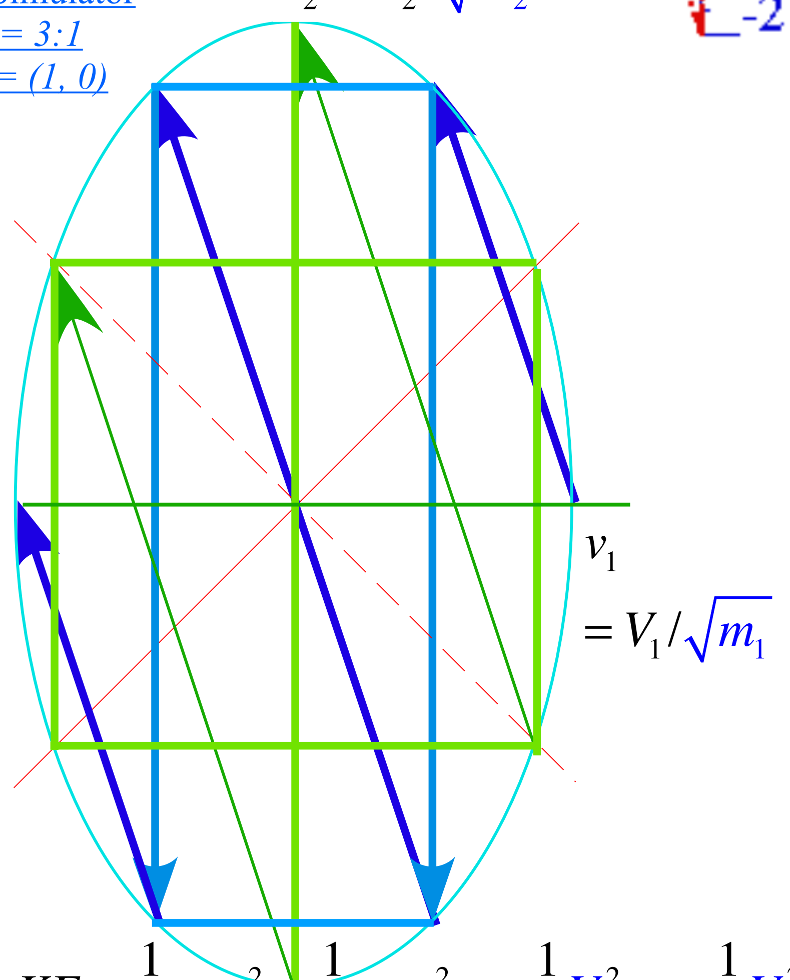
Introducing Symmetry Operators

Collisions for mass ratio $m_1:m_2 = 3:1$

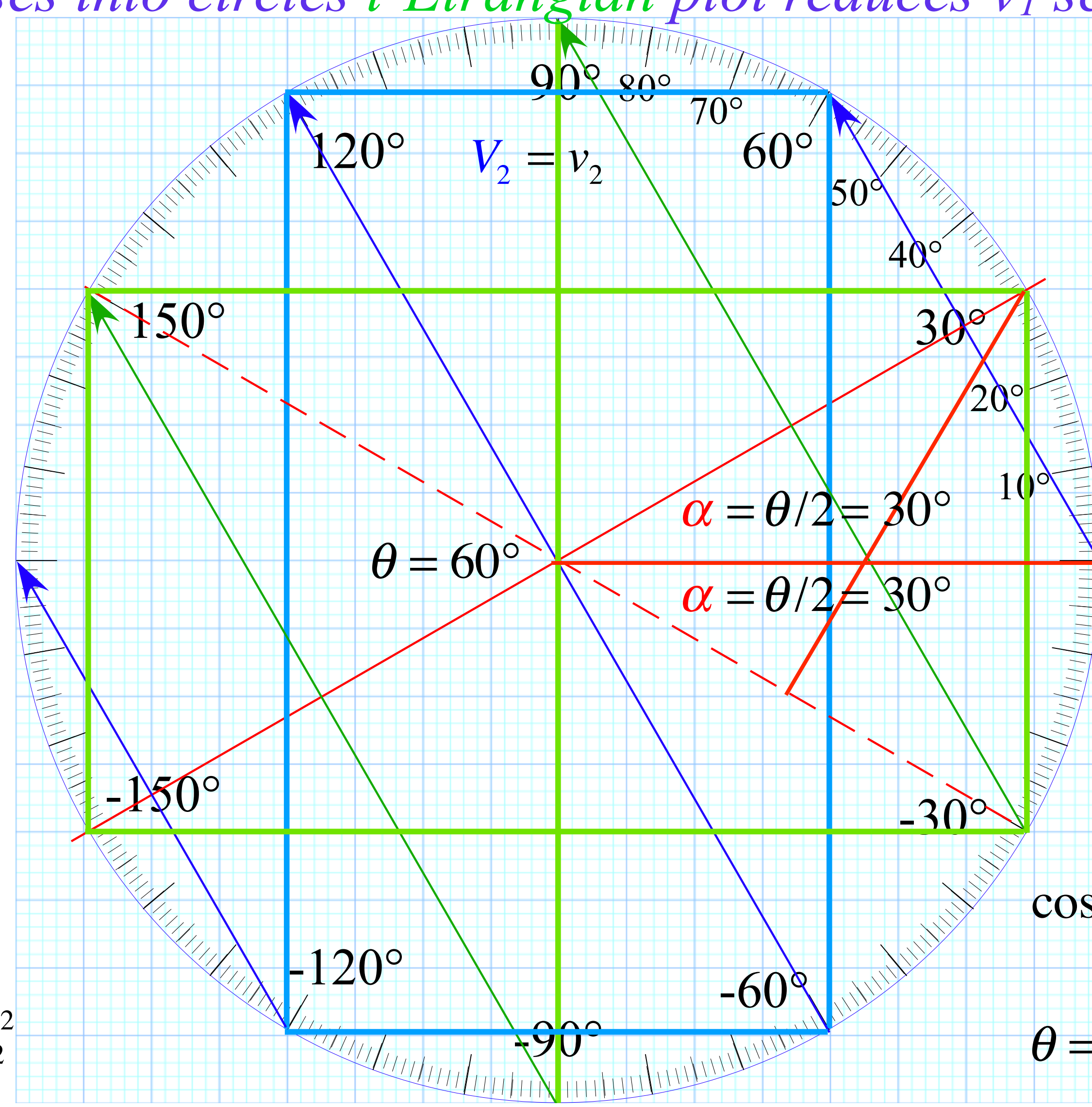





To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

[BounceIt Simulator](#)
[m1:m2= 3:1](#)
[(v1, v2) = (1, 0)](#)



$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$



$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

Here:
$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$
$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

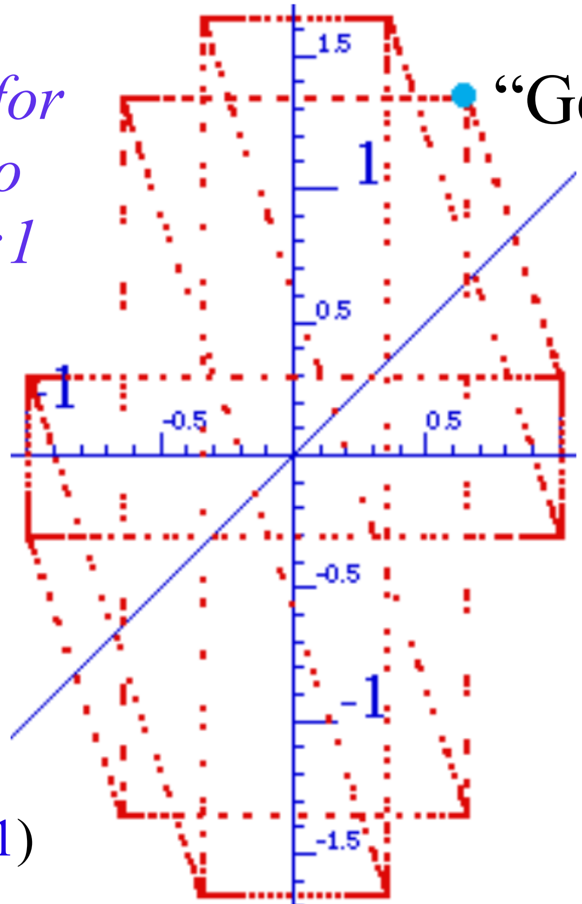
$$\alpha = \theta/2$$

$$\cos\theta = \frac{m_1-m_2}{m_1+m_2} = \frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 60°$$

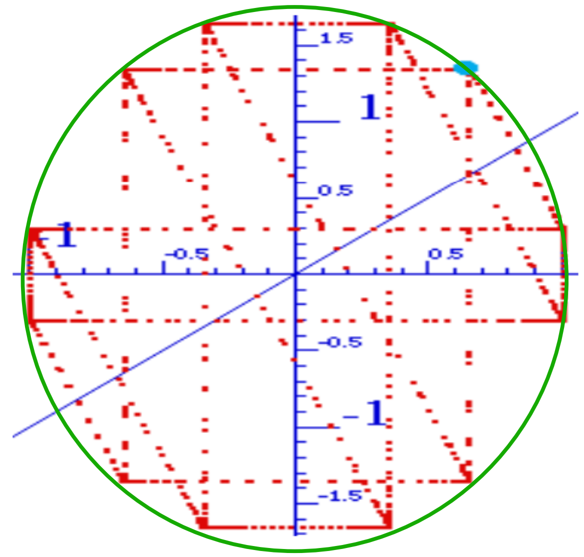
[BounceIt Web Simulator](#)
[Comparison with *Estrangian*](#)

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$

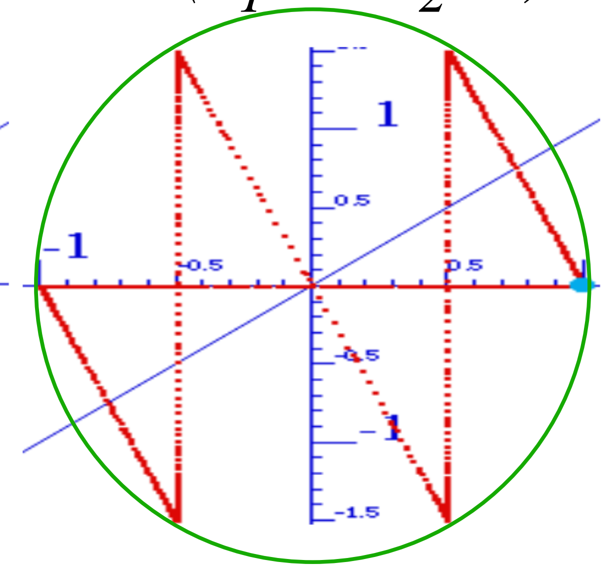


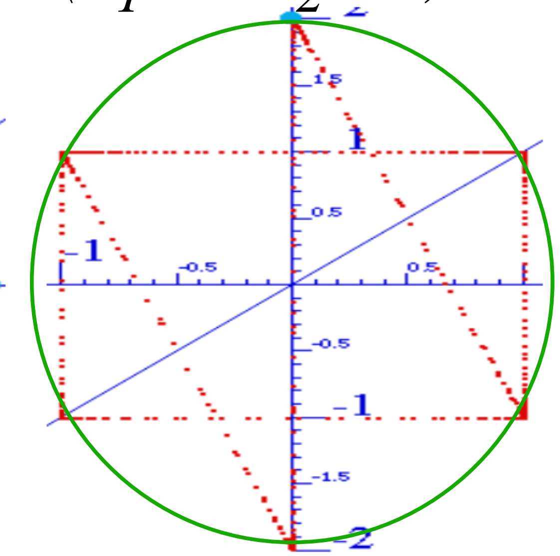
$m_1/m_2=(3)/(1)$

"Generic" initial velocity
(v_1=1.0, v_2=0.1)



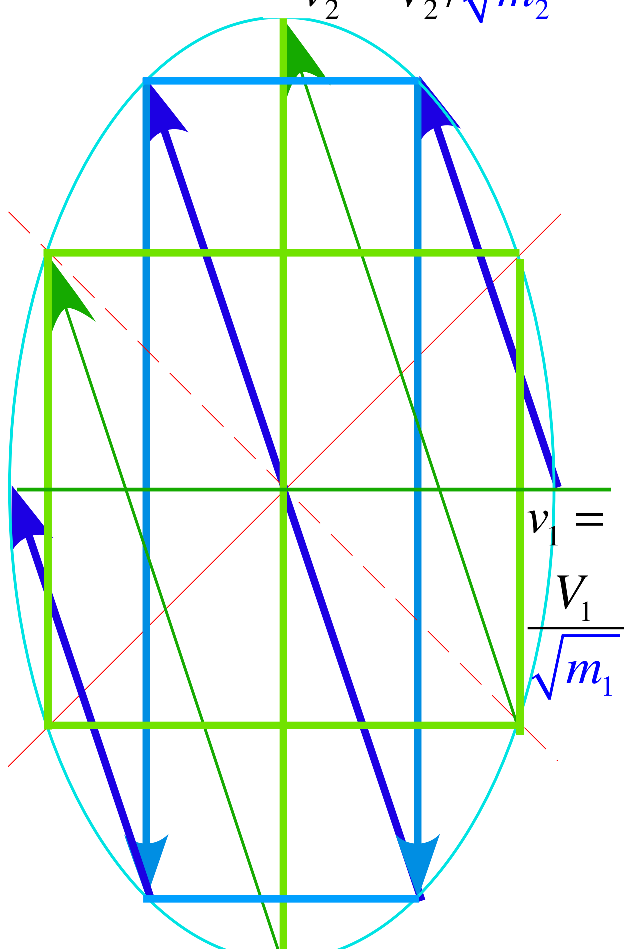
"Symmetric" initial velocity
(v_1=1, v_2=0) or (v_1=1, v_2=-1)



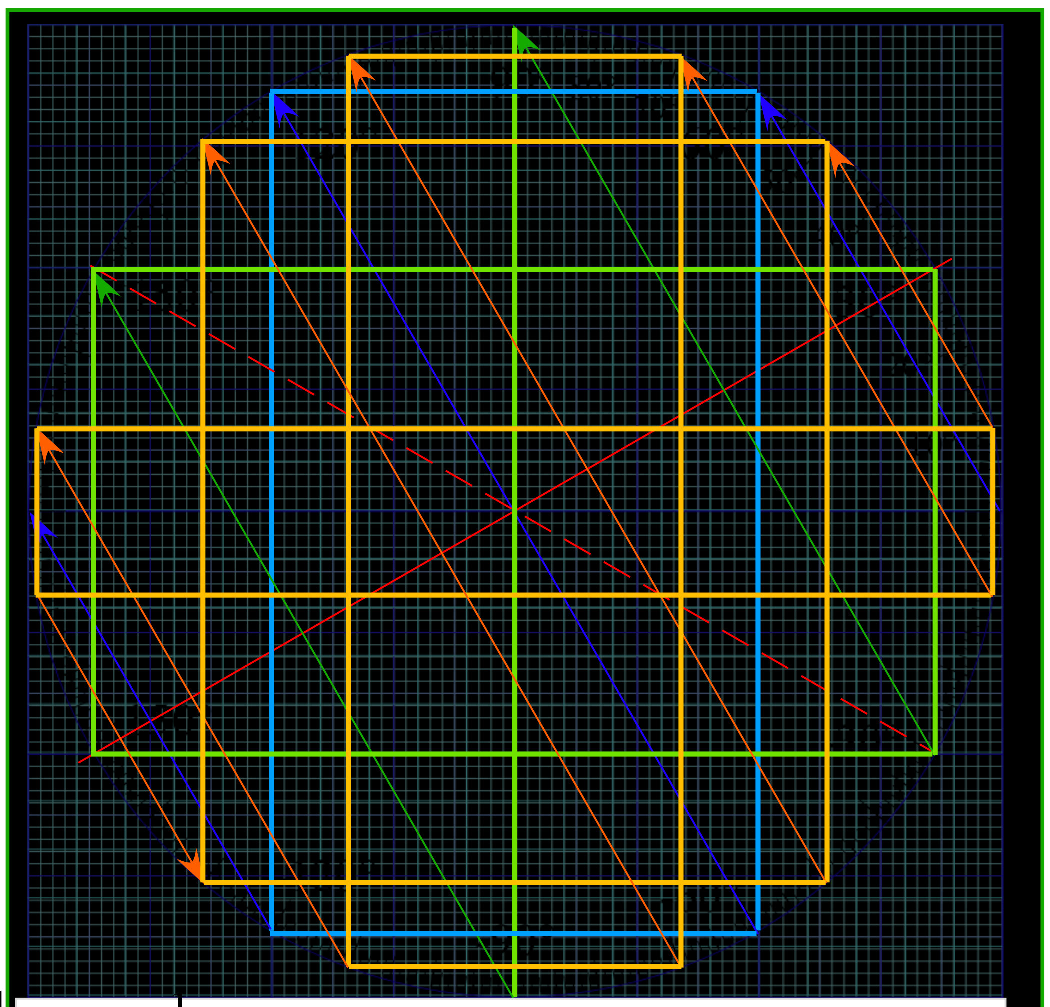


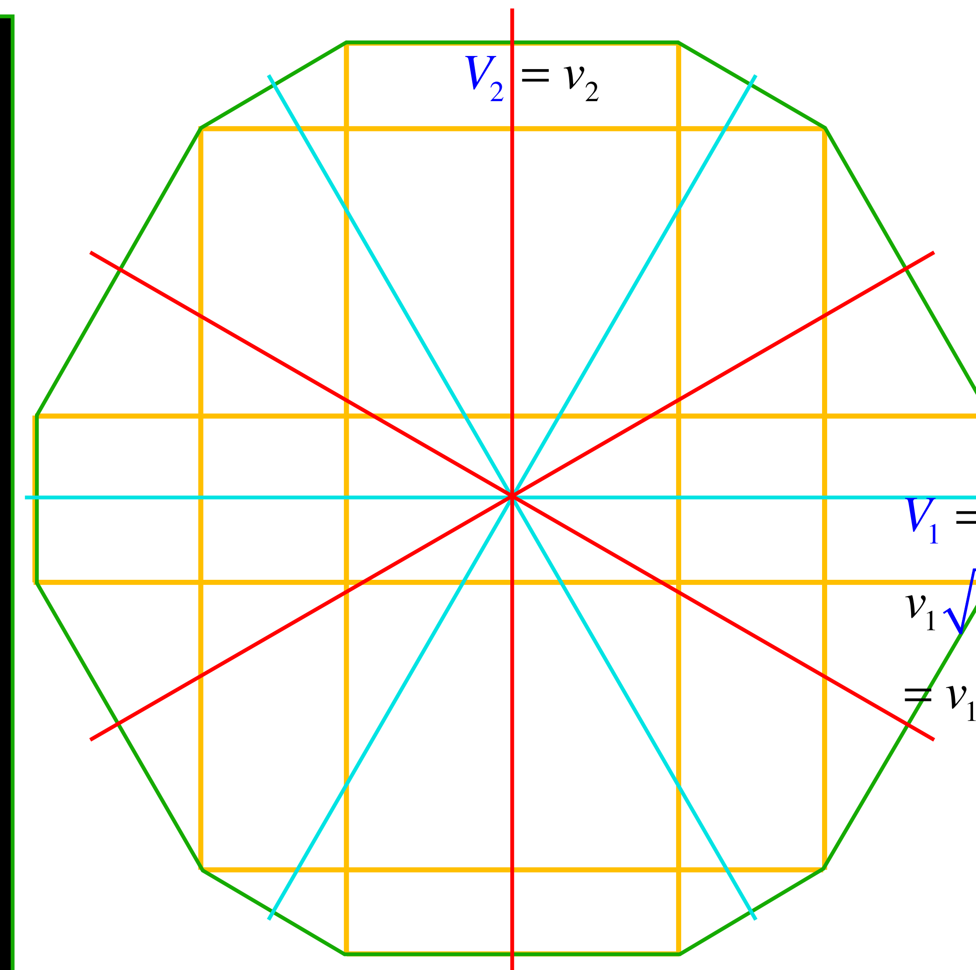
reduce v_1 scale by $1/\sqrt{m_1} = 1/\sqrt{3}=0.577$

$v_2 = V_2/\sqrt{m_2}$



$v_1 = \dfrac{V_1}{\sqrt{m_1}}$





$V_2 = v_2$

$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

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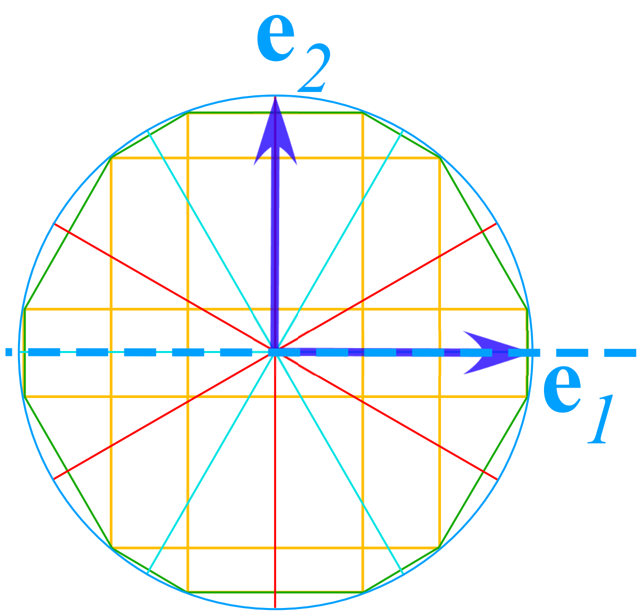
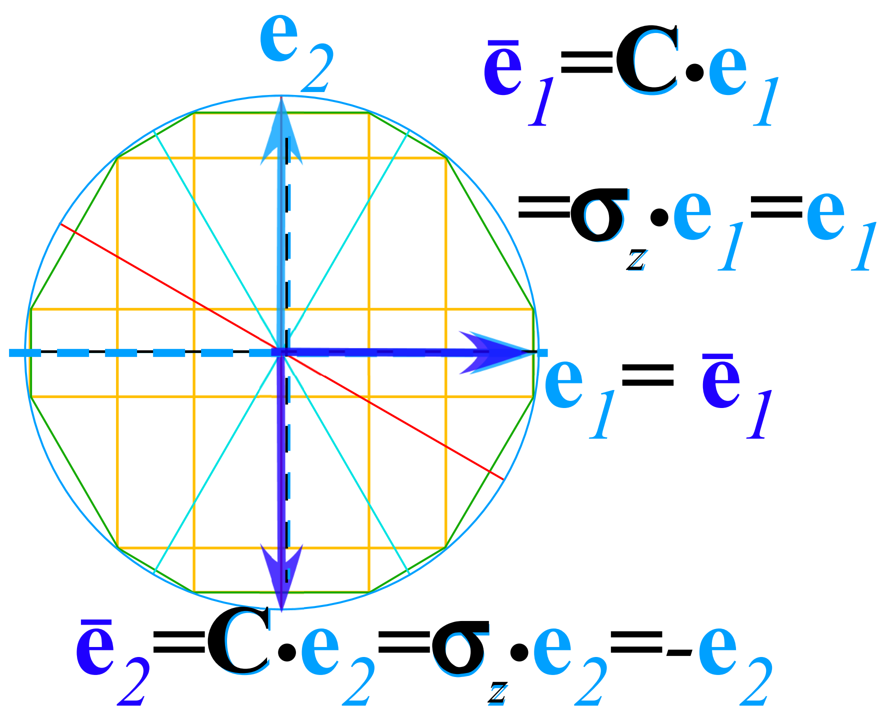
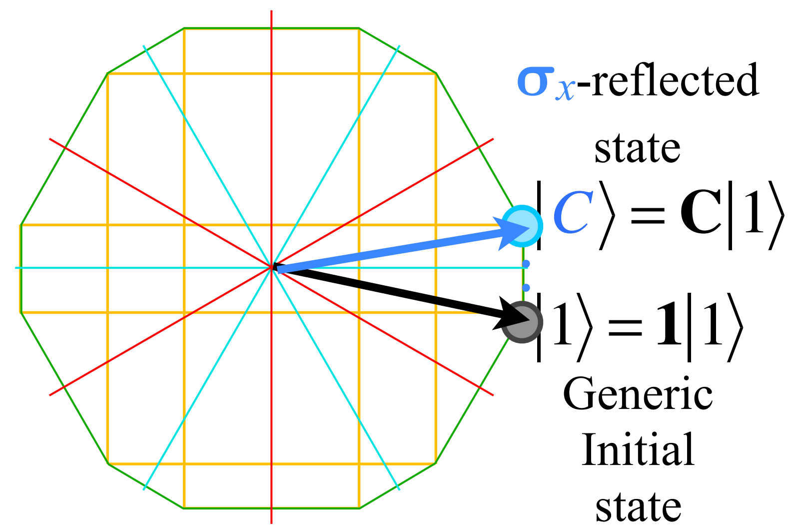
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

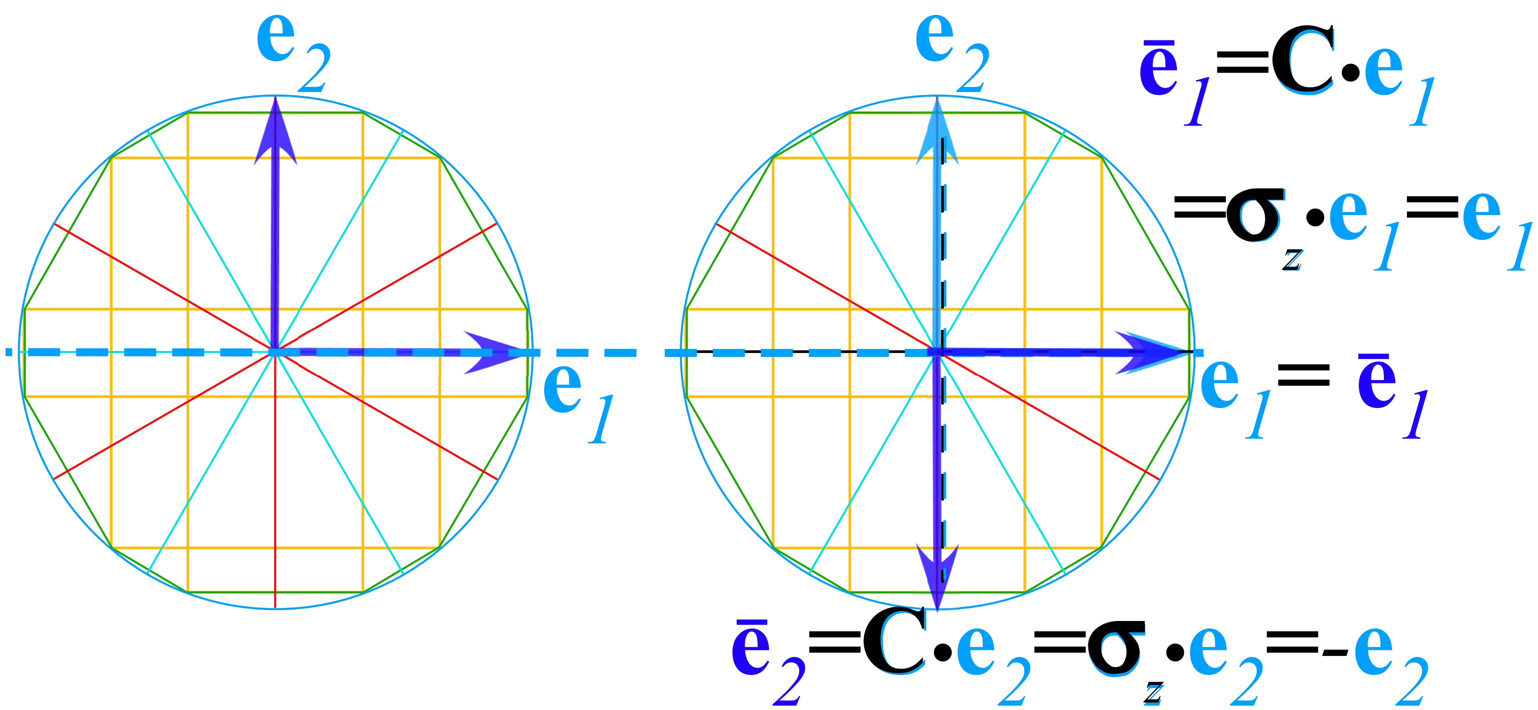
Effects of Ceiling Bang Matrix $\mathbf{C}=\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

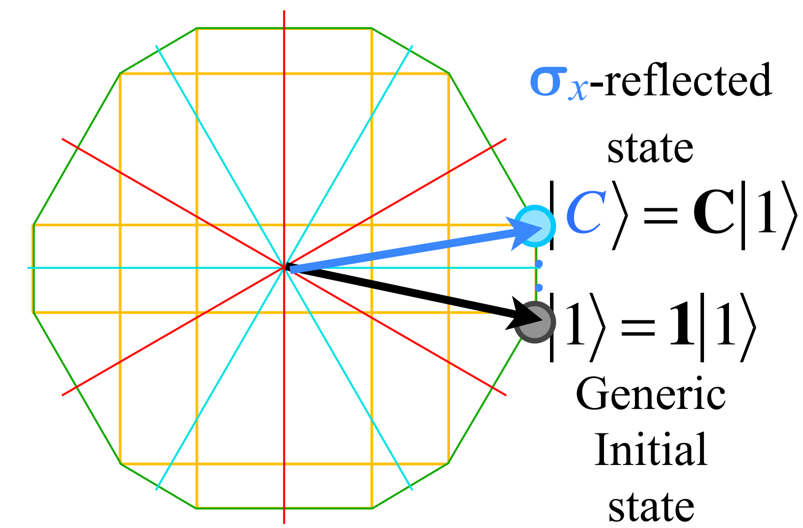






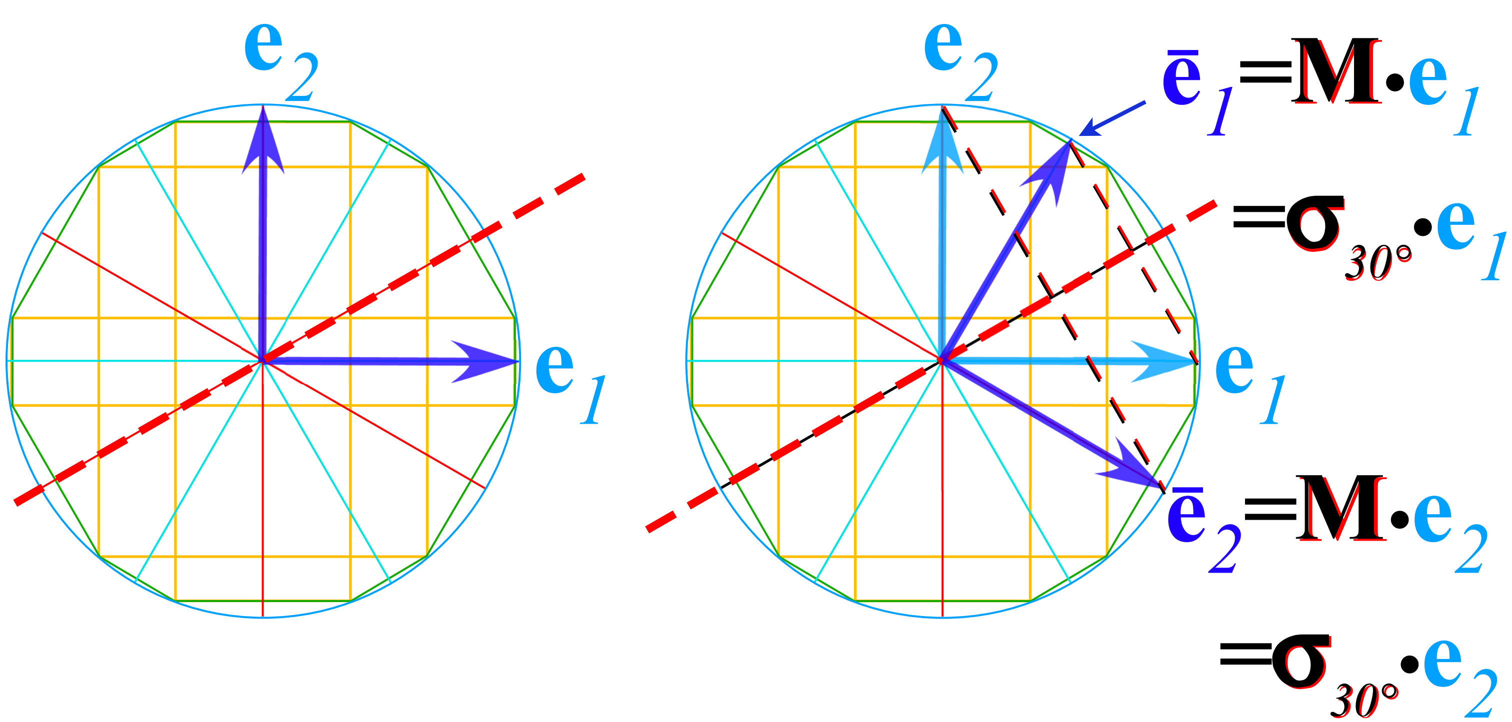
Effects of Ceiling Bang Matrix $\mathbf{C}=\sigma_z = \begin{pmatrix} \mathbf{e}_1\bullet\mathbf{C}\bullet\mathbf{e}_1 & \mathbf{e}_1\bullet\mathbf{C}\bullet\mathbf{e}_2 \\ \mathbf{e}_2\bullet\mathbf{C}\bullet\mathbf{e}_1 & \mathbf{e}_2\bullet\mathbf{C}\bullet\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_1\bullet\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_2\bullet\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

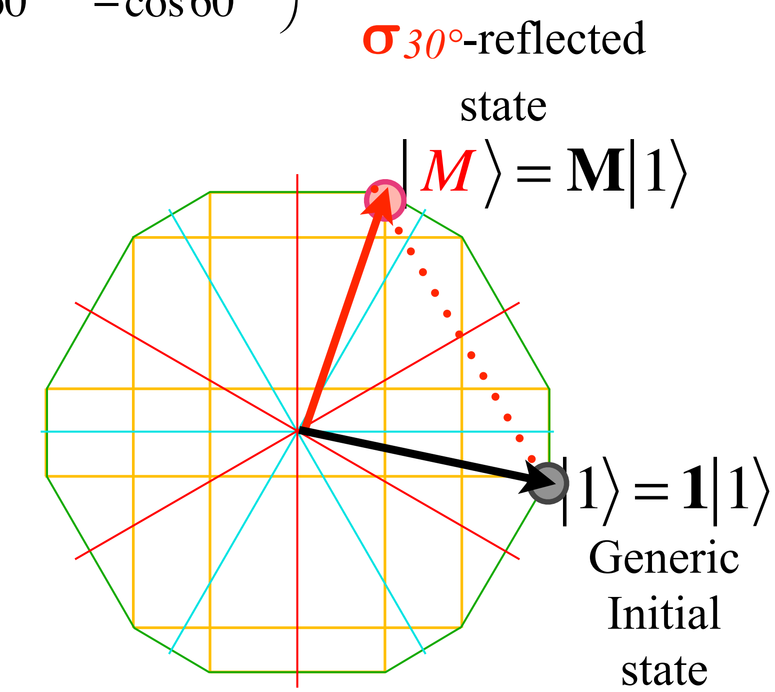




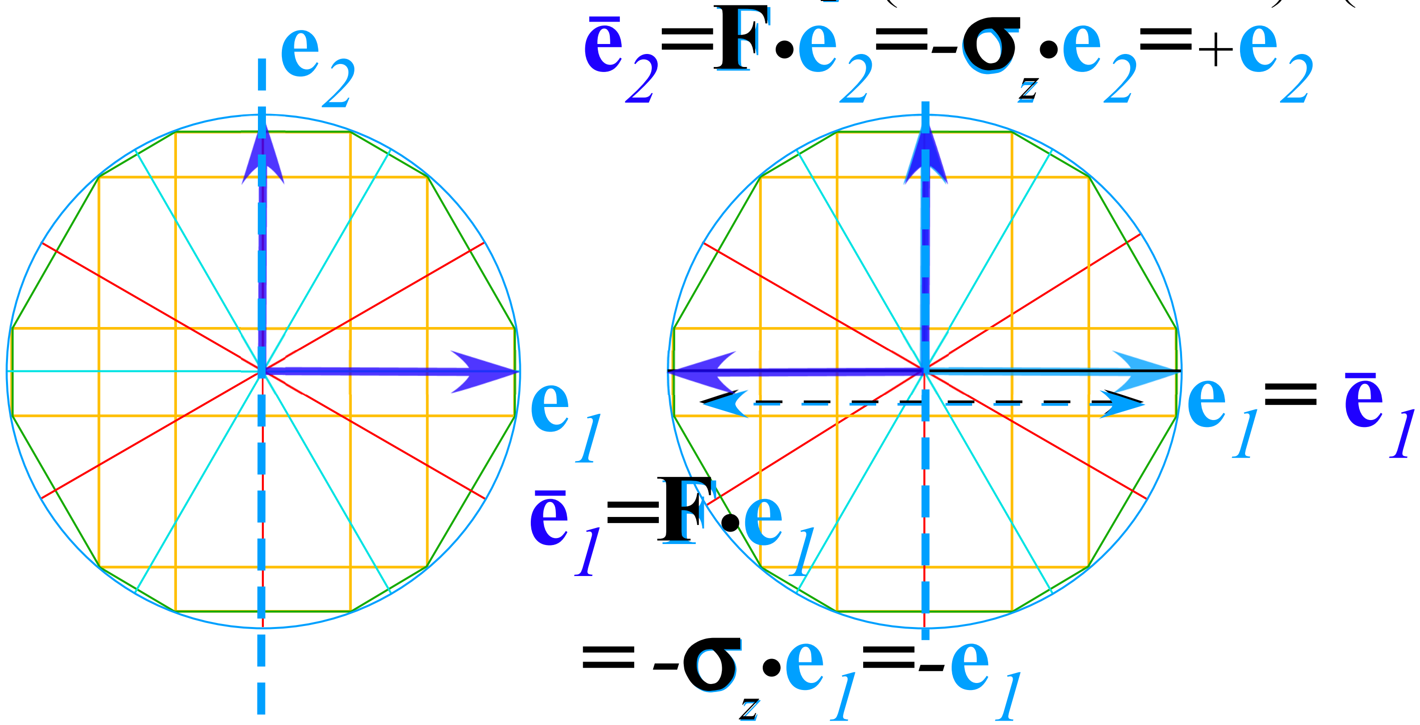
Known as *matrix elements* or *components* — Known as *relative direction cosines*

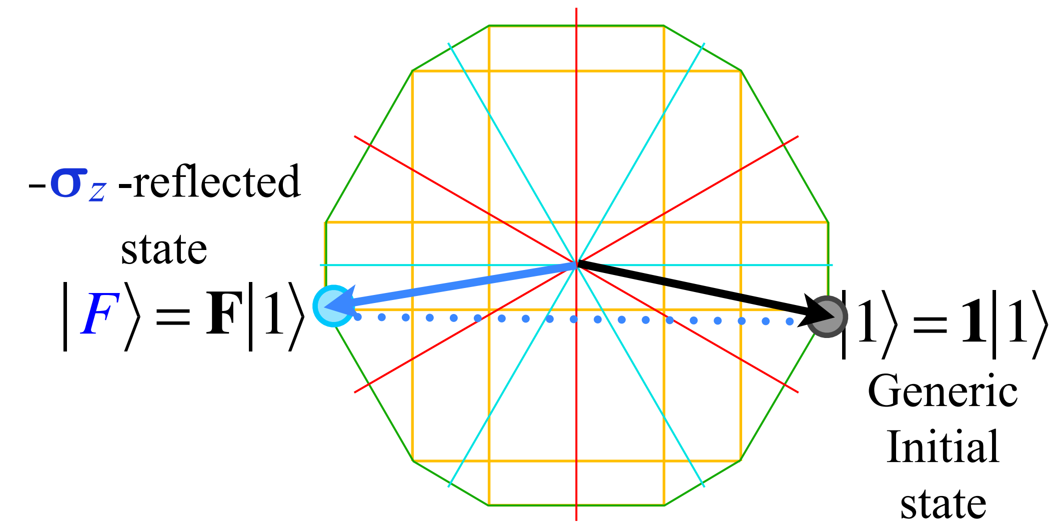
Effects of Mass Bang Matrix $\mathbf{M}=\sigma_{30°} = \begin{pmatrix} \mathbf{e}_1\bullet\mathbf{M}\bullet\mathbf{e}_1 & \mathbf{e}_1\bullet\mathbf{M}\bullet\mathbf{e}_2 \\ \mathbf{e}_2\bullet\mathbf{M}\bullet\mathbf{e}_1 & \mathbf{e}_2\bullet\mathbf{M}\bullet\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_1\bullet\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_2\bullet\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{pmatrix}$



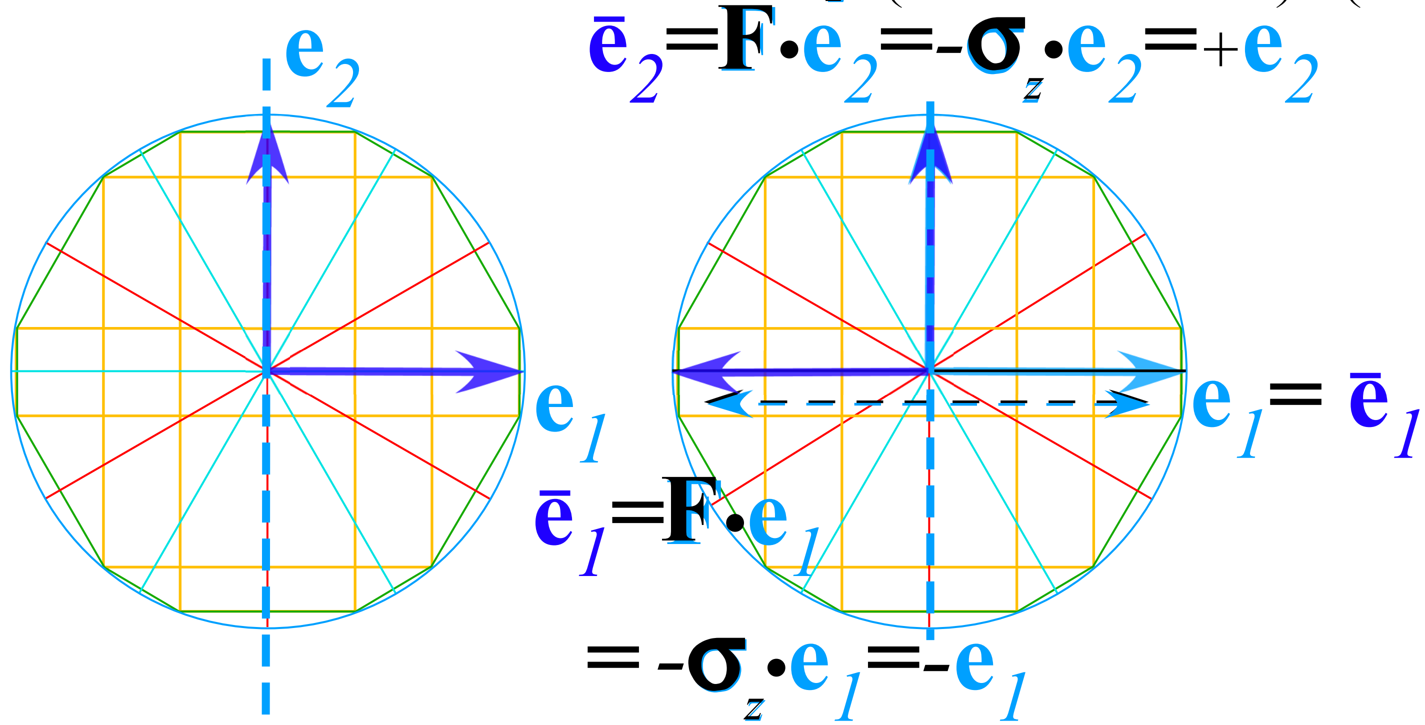


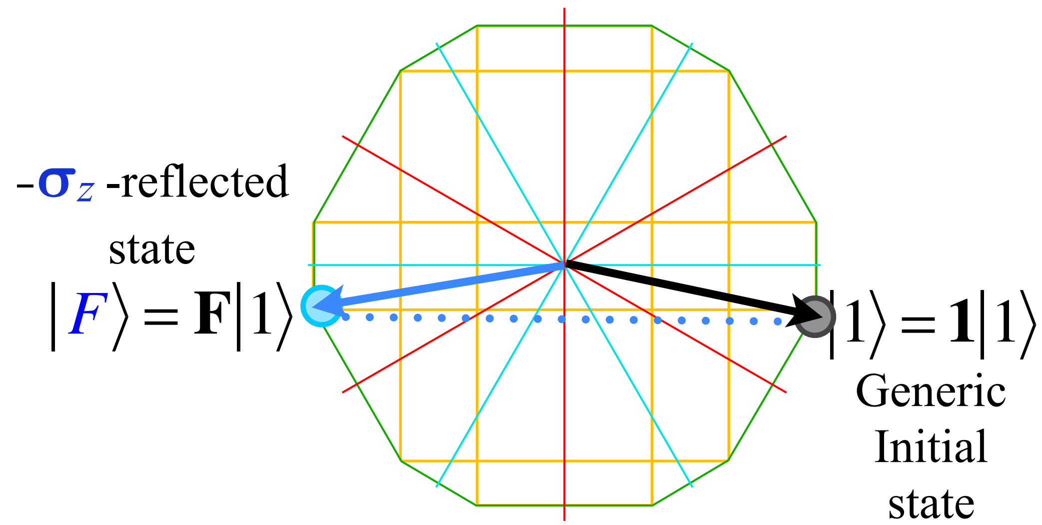
Effects of Floor Bang Matrix $\mathbf{F} = -\sigma_z = \begin{pmatrix} \mathbf{e}_1 \bullet \mathbf{F} \bullet \mathbf{e}_1 & \mathbf{e}_1 \bullet \mathbf{F} \bullet \mathbf{e}_2 \\ \mathbf{e}_2 \bullet \mathbf{F} \bullet \mathbf{e}_1 & \mathbf{e}_2 \bullet \mathbf{F} \bullet \mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1 \bullet \bar{\mathbf{e}}_1 & \mathbf{e}_1 \bullet \bar{\mathbf{e}}_2 \\ \mathbf{e}_2 \bullet \bar{\mathbf{e}}_1 & \mathbf{e}_2 \bullet \bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$



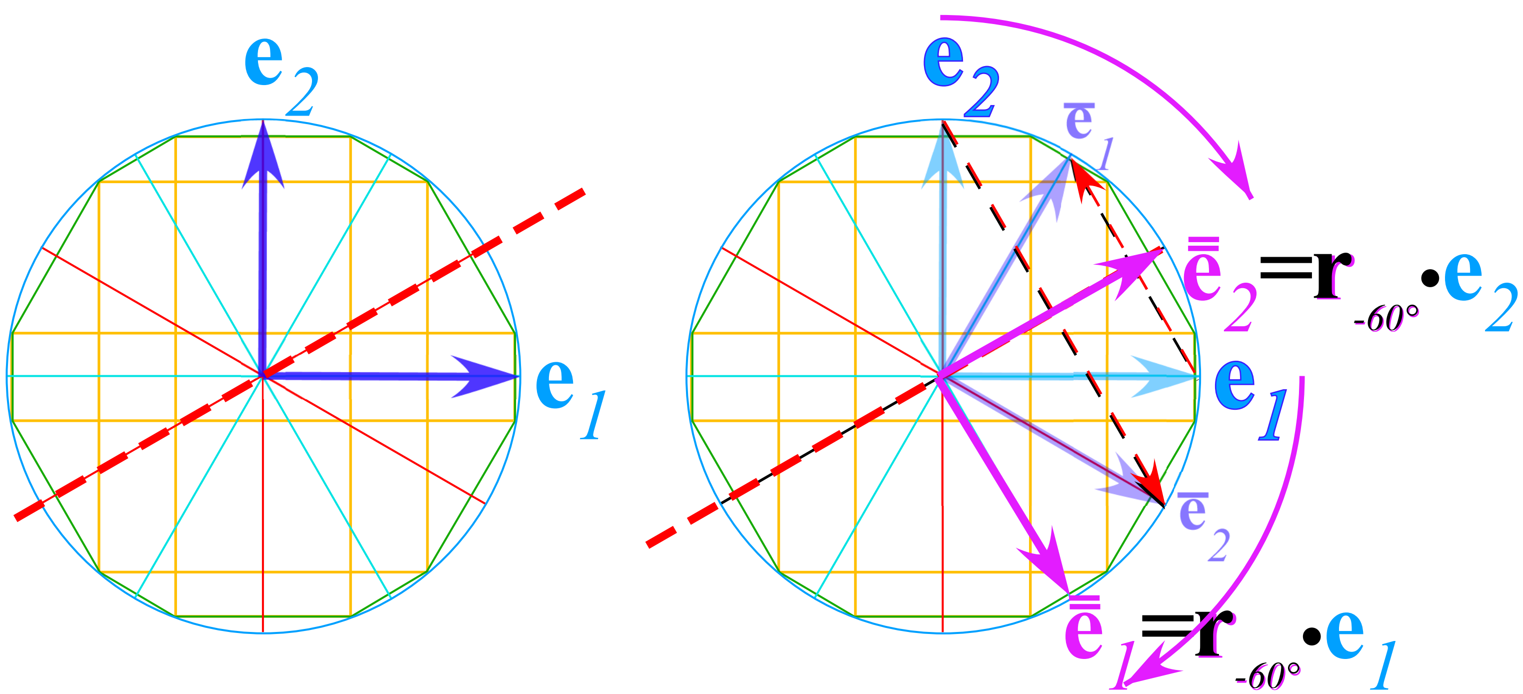


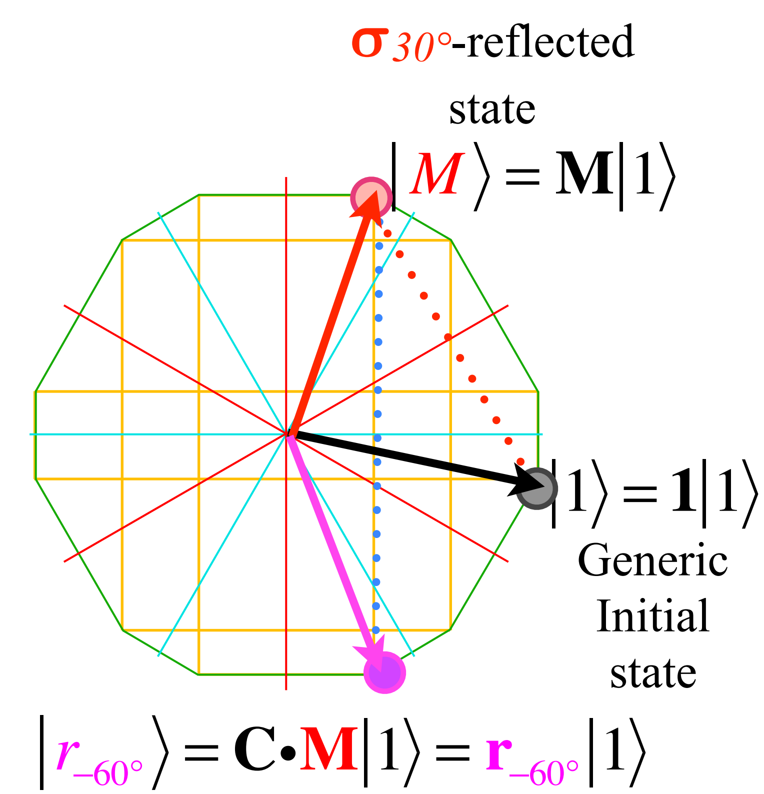
Effects of Floor Bang Matrix $\mathbf{F}=-\boldsymbol{\sigma}_z=\begin{pmatrix} \mathbf{e}_1\bullet\mathbf{F}\bullet\mathbf{e}_1 & \mathbf{e}_1\bullet\mathbf{F}\bullet\mathbf{e}_2 \\ \mathbf{e}_2\bullet\mathbf{F}\bullet\mathbf{e}_1 & \mathbf{e}_2\bullet\mathbf{F}\bullet\mathbf{e}_2 \end{pmatrix}=\begin{pmatrix} \mathbf{e}_1\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_1\bullet\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\bullet\bar{\mathbf{e}}_1 & \mathbf{e}_2\bullet\bar{\mathbf{e}}_2 \end{pmatrix}=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$





Effects of Ceiling $\mathbf{C}$ after Bang $\mathbf{M}$: $\mathbf{r}_{-60°}=\mathbf{C}\bullet\mathbf{M}=\boldsymbol{\sigma}_z\bullet\boldsymbol{\sigma}_{30°}$





$\boldsymbol{\sigma}_{30°}$ $\boldsymbol{\sigma}_{30°}$-reflected state is a $\mathbf{r}_{-60°}$-rotated state

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→ *Group multiplication and product table* ←

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

D_6	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
1	1											
$\bar{r}_{120}$		1										
r_{120}			1									
σ_{60}				1								
$\bar{\sigma}_{60}$					1							
σ_z						1					$\bar{r}_{60}$	
I							1					
r_{60}								1				
$\bar{r}_{60}$									1			
$\bar{\sigma}_{30}$										1		
σ_{30}						r_{60}					1	
$\bar{\sigma}_z$												1

Note: $\bar{r}_{60} = Ir_{120} = r_{120}I = r_{-60}$ and: $I = r_{\pm 180}$

$\bar{r}_{120} = Ir_{60} = r_{60}I = r_{-120}$ and: $I^2 = 1$

$\sigma_{60} = I\bar{\sigma}_{30} = \bar{\sigma}_{30}I$

$\bar{\sigma}_{60} = I\sigma_{30} = \sigma_{30}I$

$\bar{\sigma}_z = I\sigma_z = \sigma_z I$

Easy to make hexagonal (D_6) symmetry group table:

Example 1: Find $\sigma_{30°}\cdot\sigma_{-60°}$ = ______?

Solution: Find $\sigma_{30°}$*-plane and state-*$|\sigma_{-60°}\rangle$

Operate former on latter to get: $\sigma_{30°}|\sigma_{-60°}\rangle = |\mathbf{I}\rangle$

That gives answer: $\sigma_{30°}\cdot\sigma_{-60°} = \mathbf{I}$.

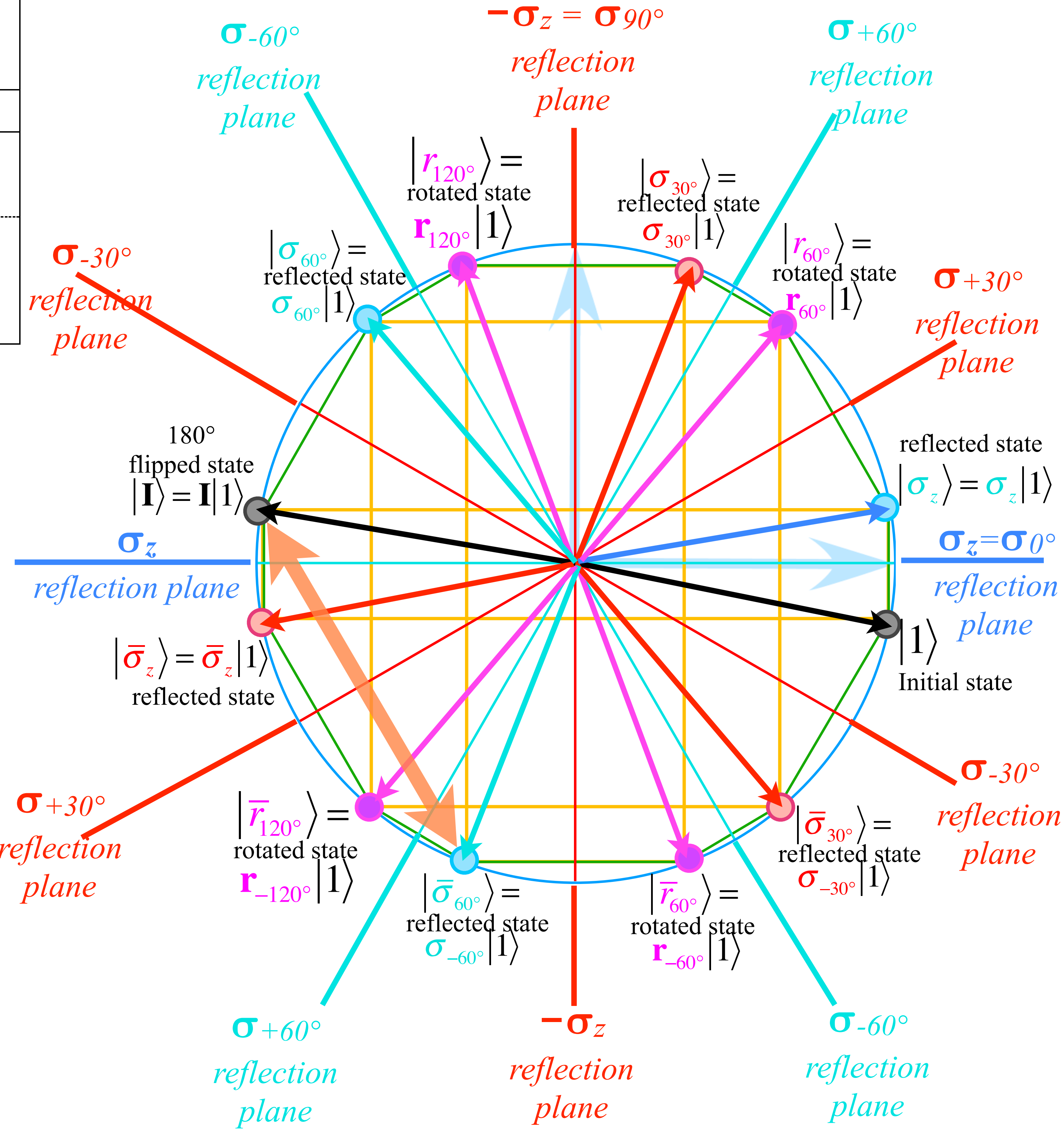
Rest of $\sigma_{30°}$ *row follows:*

11^{th} row	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
σ_{30}	σ_{30}	$\bar{\sigma}_{30}$	$\bar{\sigma}_z$	$\bar{r}_{60}$	I	r_{60}	$\bar{\sigma}_{60}$	σ_{60}	σ_z	r_{120}	1	$\bar{r}_{120}$

Example 2: Find $\mathbf{r}_{60°}\cdot\sigma_{-60°}$ = ______?

Solution: Do $\mathbf{r}_{60°}$*-rotation* $\mathbf{r}_{60°}|\sigma_{-60°}\rangle = |\sigma_{-30°}\rangle$

That gives answer: $\mathbf{r}_{60°}\cdot\sigma_{-60°} = \sigma_{-30°}$



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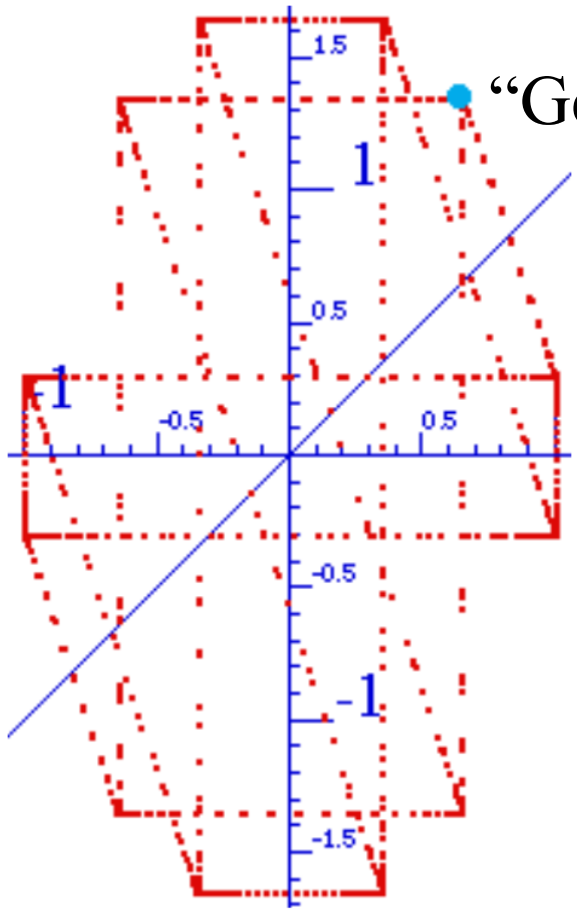
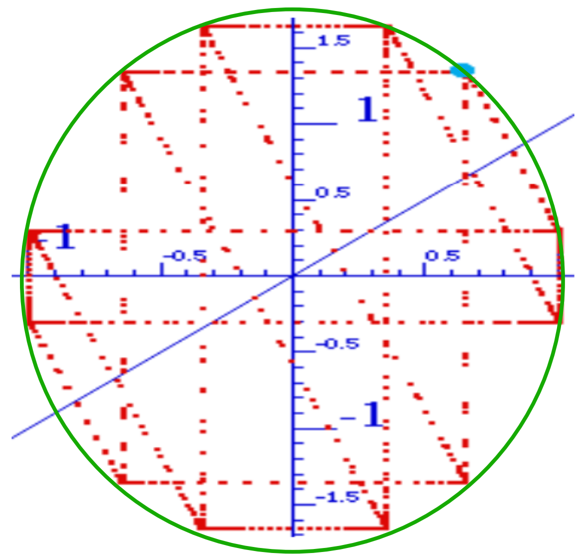
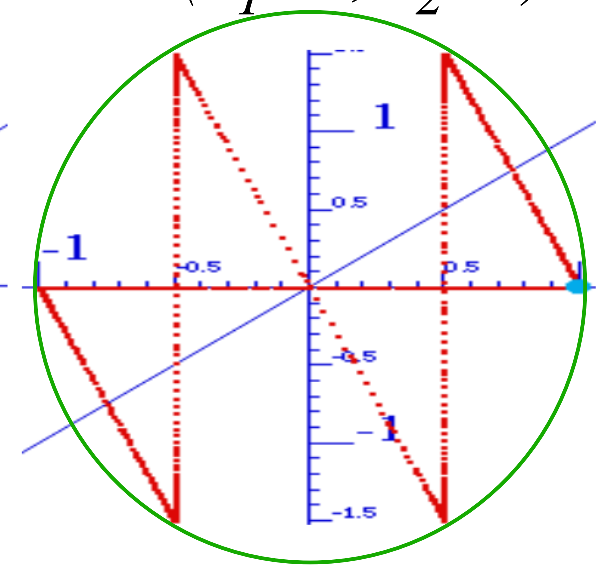


“Generic” initial velocity
$(v_1=1.0, v_2=0.1)$

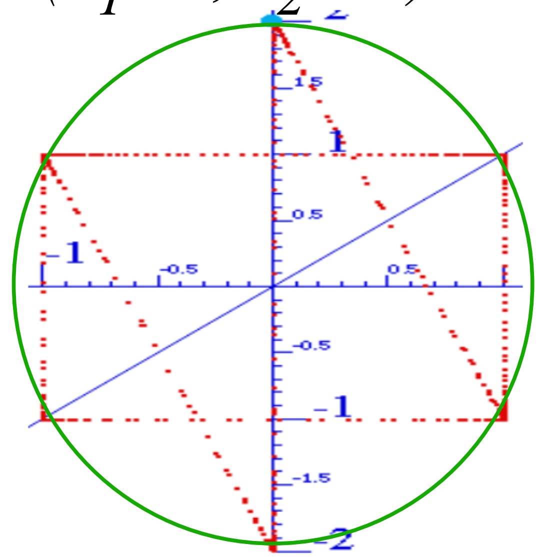
“Symmetric” initial velocity
$(v_1=1, v_2=0)$ or $(v_1=1, v_2=-1)$



$(v_1, v_2)=(1, 0.1)$



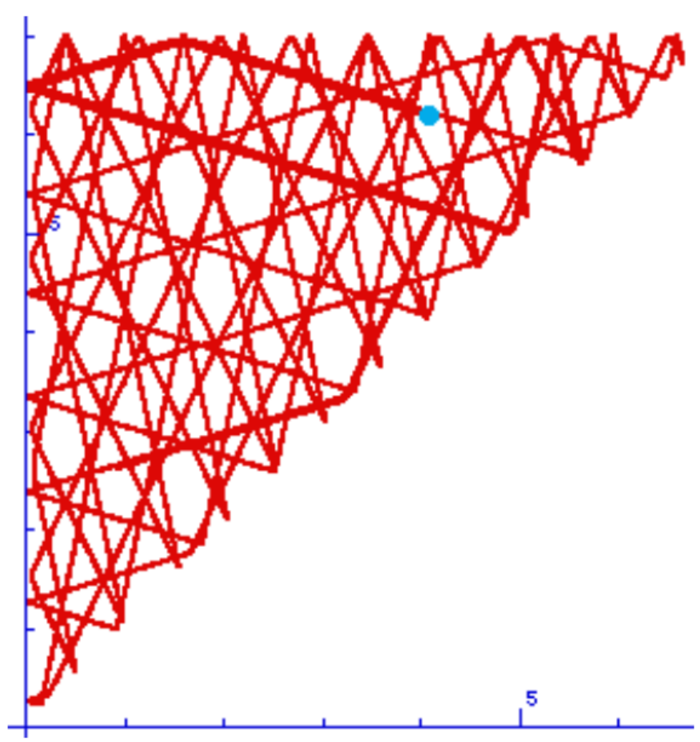
$(v_1, v_2)=(1, 0)$



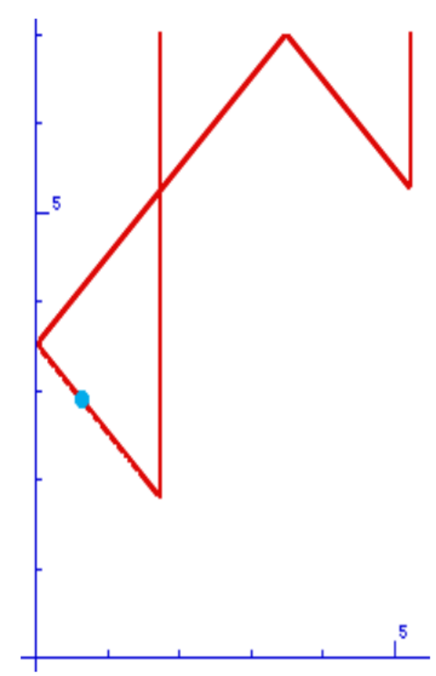
$(v_1, v_2)=(1, -1)$

BounceIt
m_1:m_2 = 3:1
Dual plots
v_2 vs v_1 and V_2 vs V_1

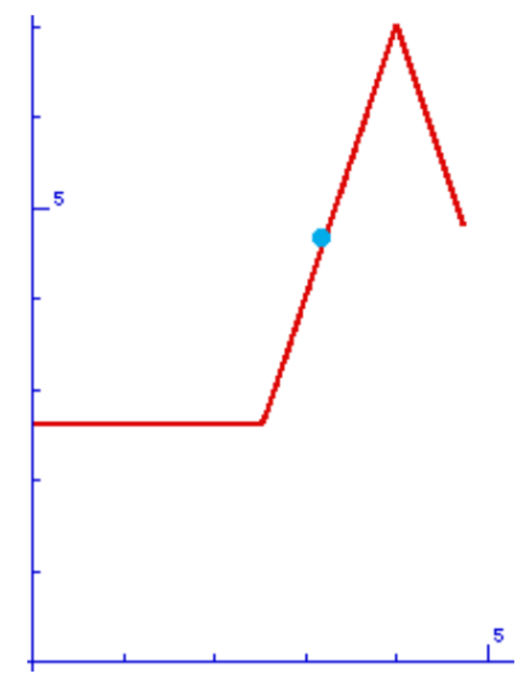
Corresponding space-space (y_1, y_2) paths



$(v_1, v_2)=(1, 0.1)$



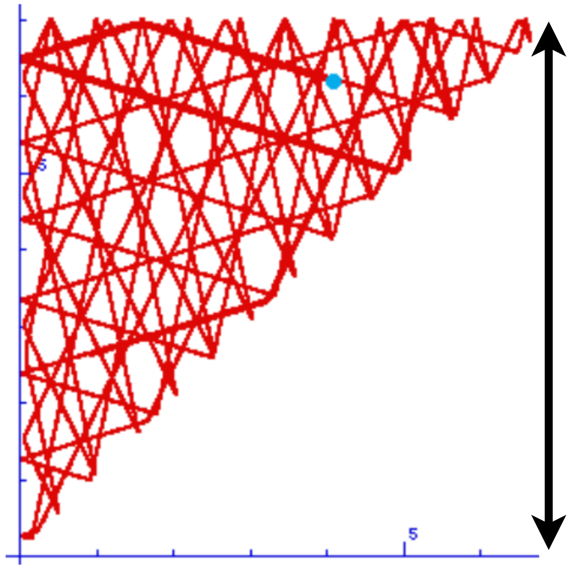
$(v_1, v_2)=(1, -1)$

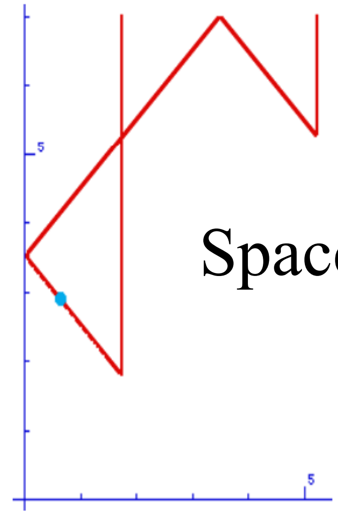


$(v_1, v_2)=(1, 0)$

BounceIt
m_1:m_2 = 3:1
y_2 vs y_1 plots

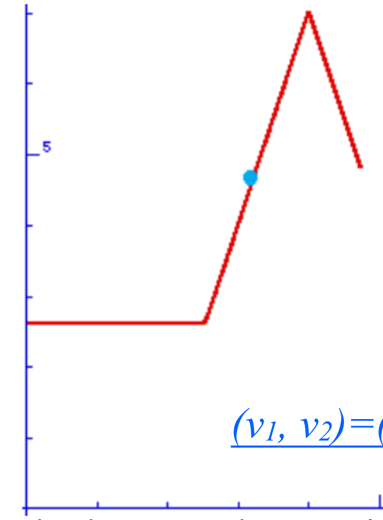
Collisions for mass ratio m_1:m_2= 3:1





Space-space (y_1, y_2) paths

$(v_1, v_2)=(1, -1)$

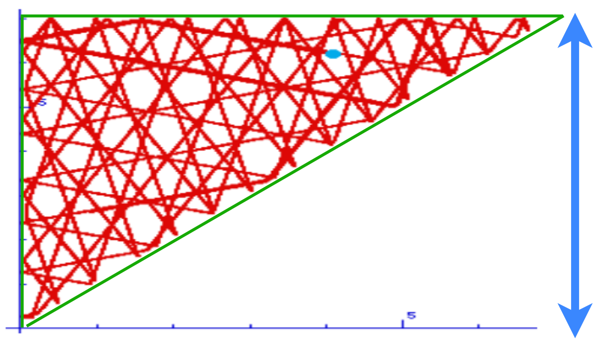


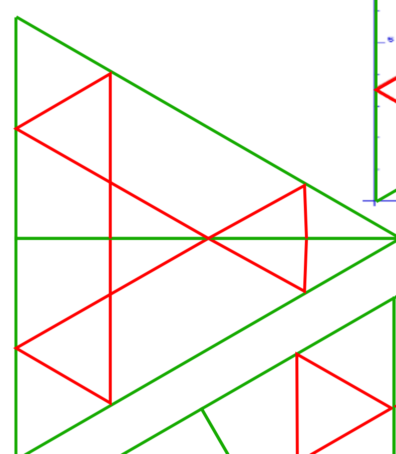
$(v_1, v_2)=(1, 0)$

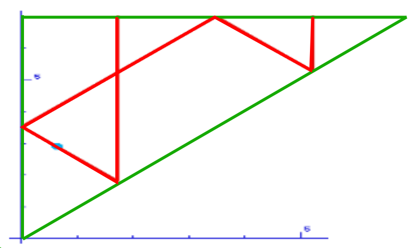
BounceIt Web Simulations
m_1:m_2 = 3:1
y_2 vs y_1 plots

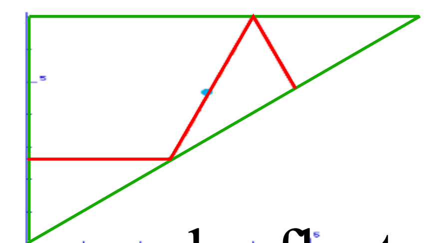
Space-space (y_1, y_2) paths scaled down by 1/√3...

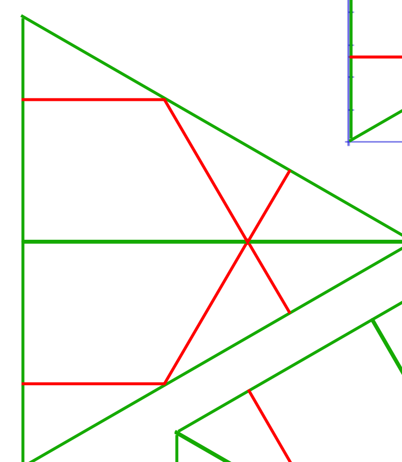
Scaled y down by 1/√3=0.577



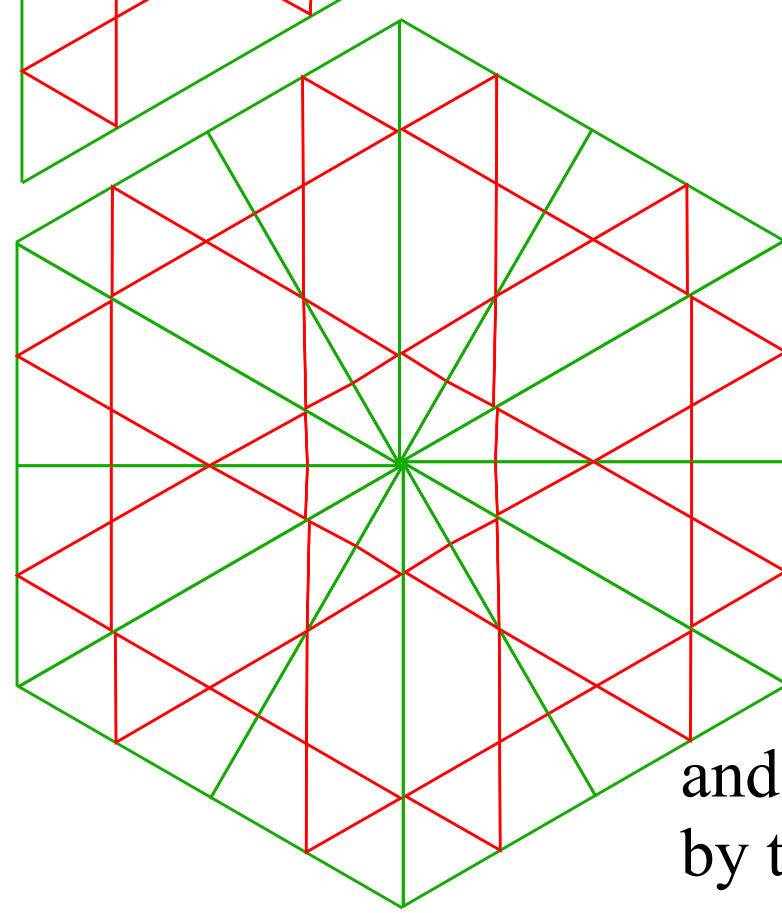


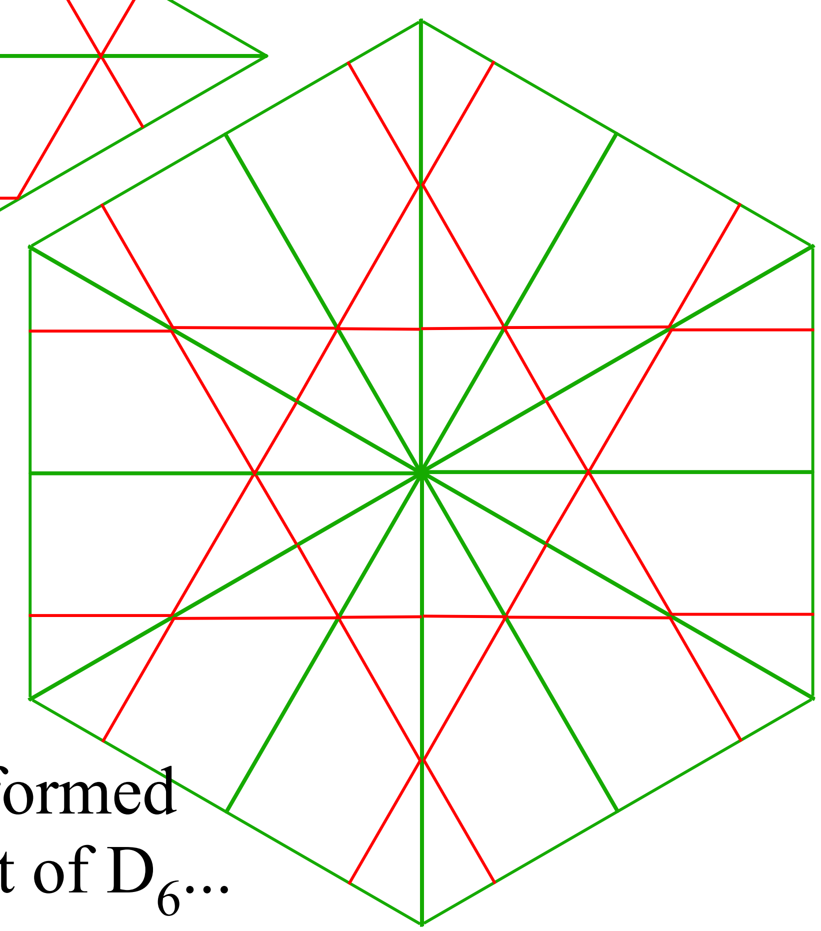






and reflected...



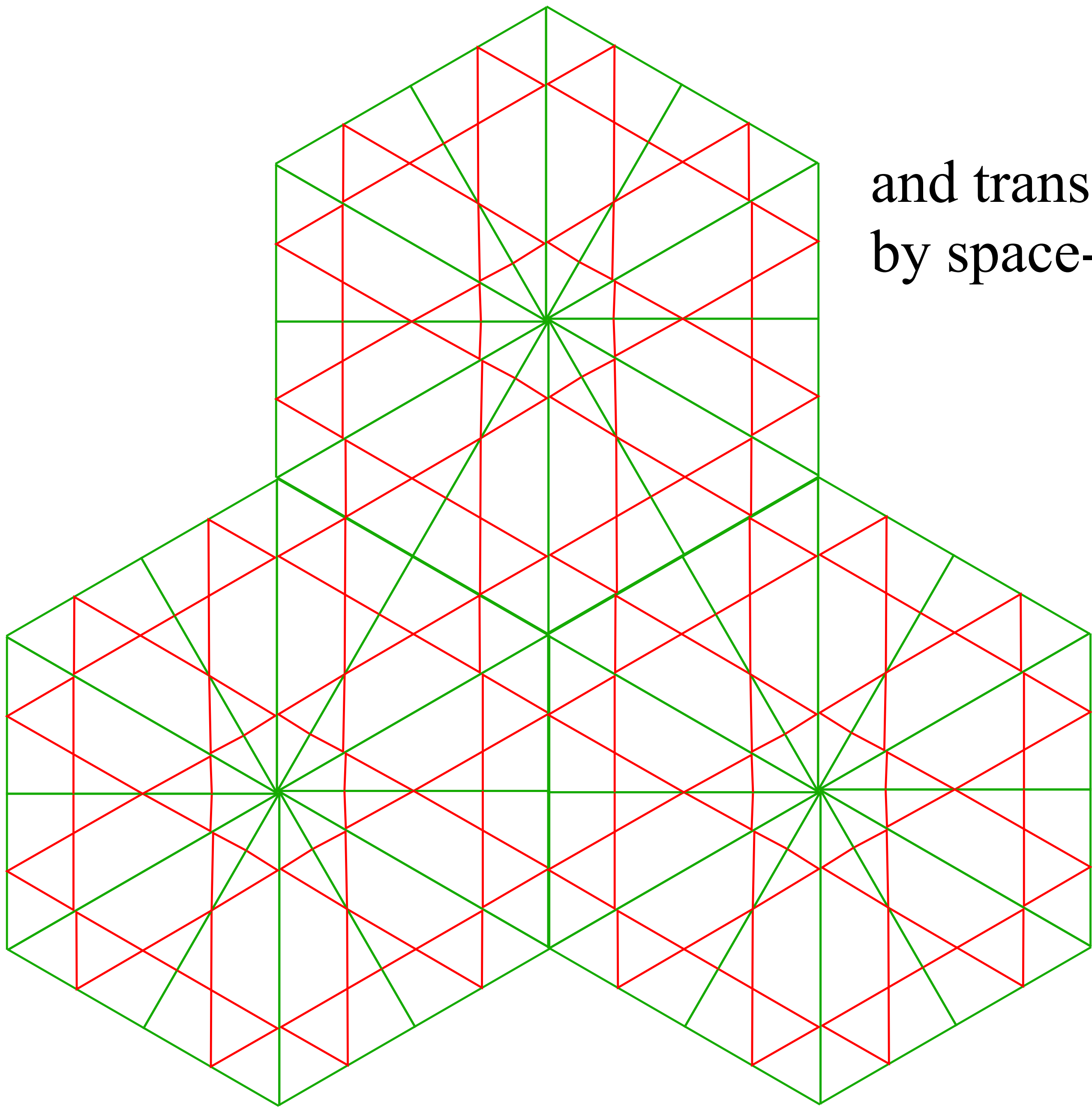


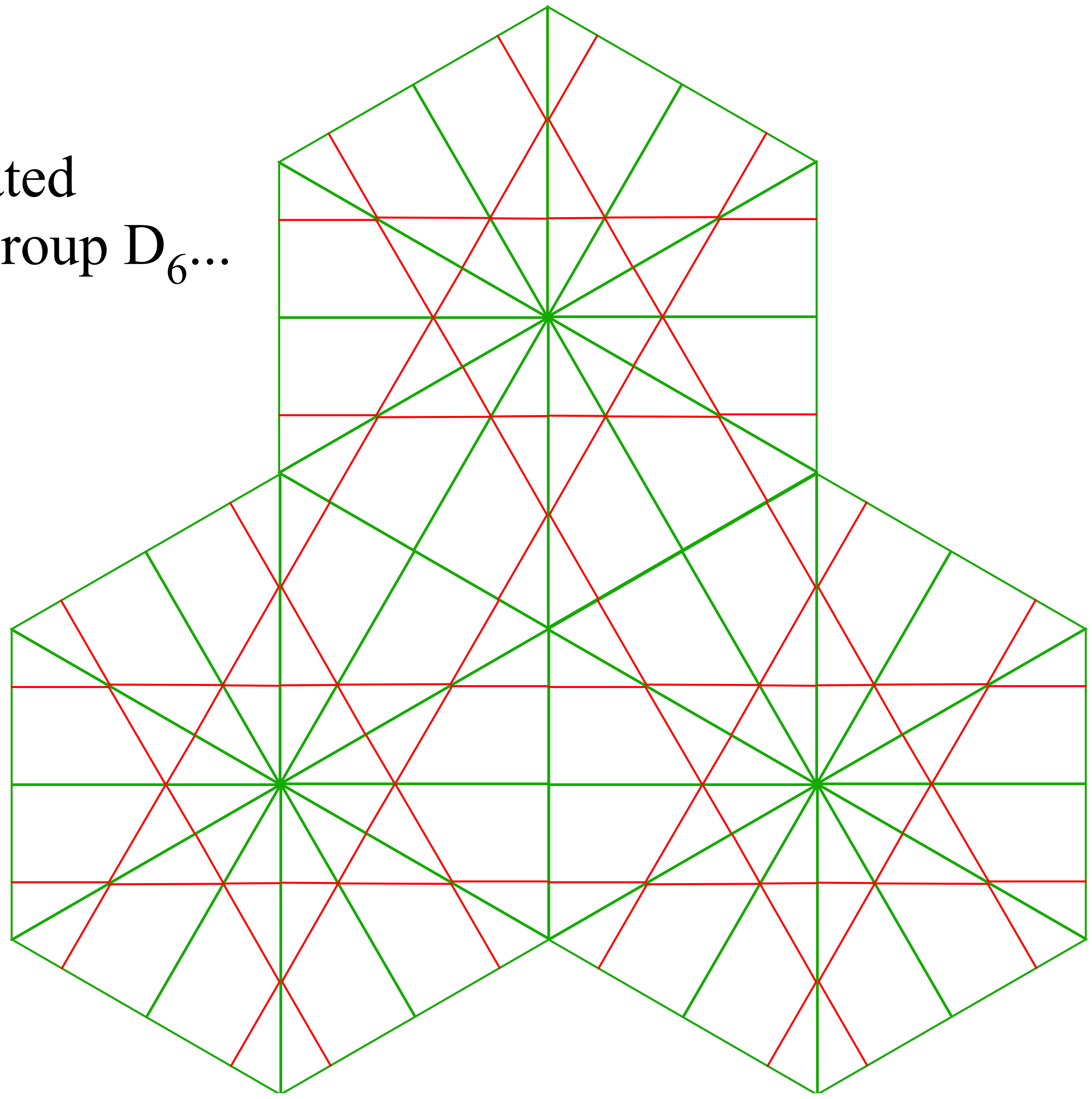
..or could have scaled x up by √3=1.732

and transformed by the rest of D_6...

Collisions for mass ratio m_1:m_2= 3:1

and translated
by space-group D_6...

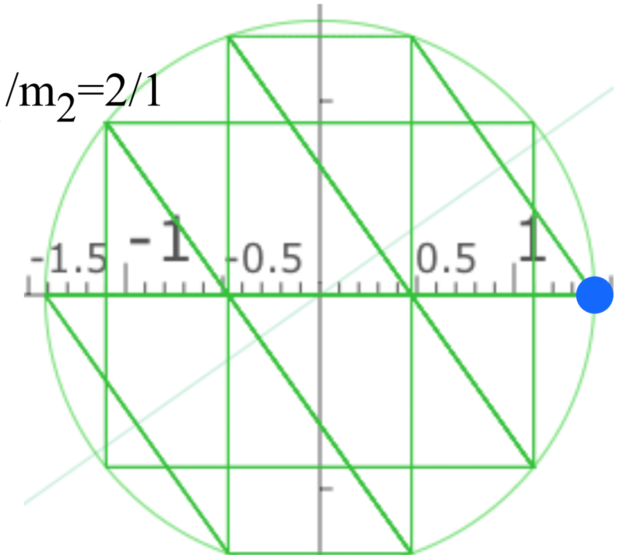




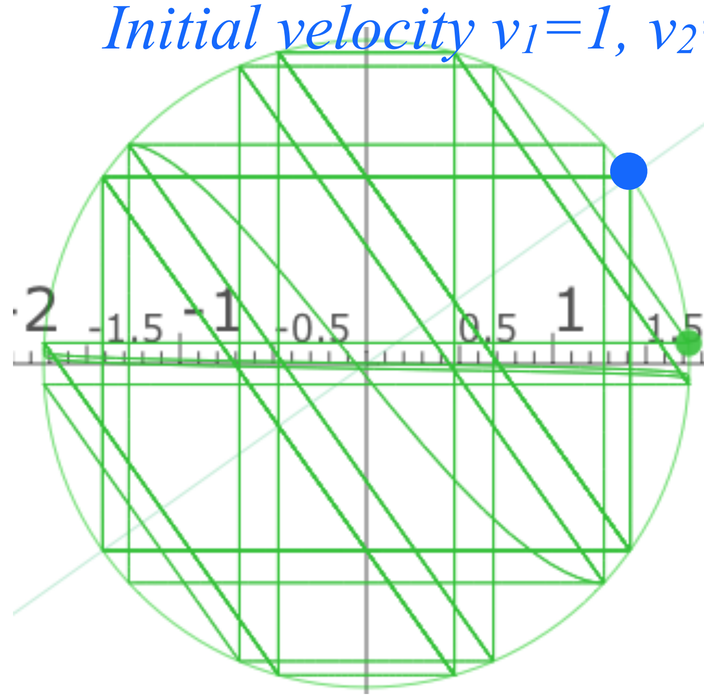
...they're just straight lines going forever.

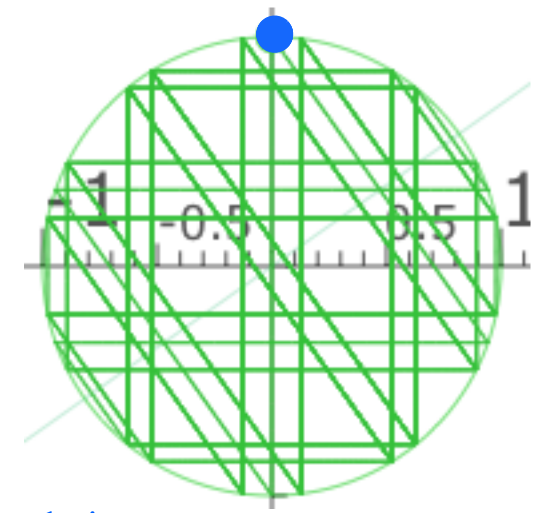
Initial velocity $v_1=1$, $v_2=0$ | *Initial velocity $v_1=1$, $v_2=1$* | *Initial velocity $v_1=0$, $v_2=1$*

$M_1/m_2=2/1$



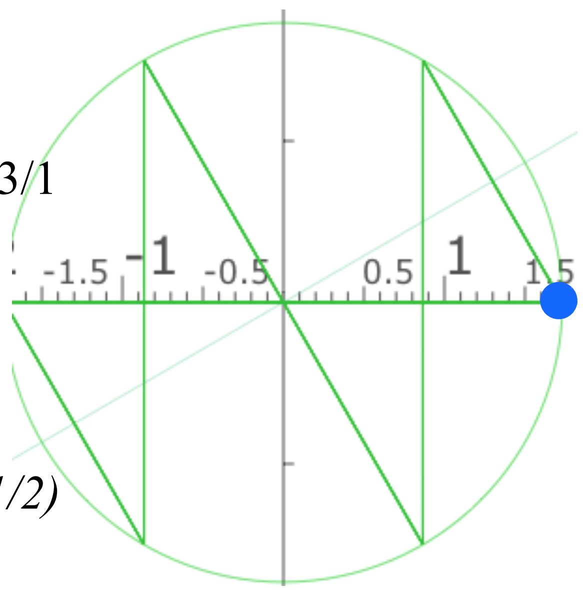
$\phi = Acos(M_1-m_2)/(M_1+m_2)$
$= Acos(1/3) = 70.53°$



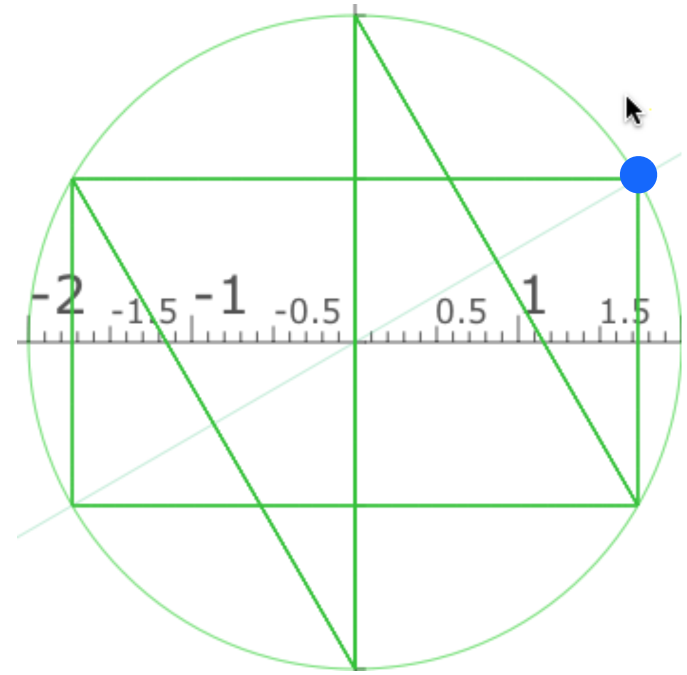


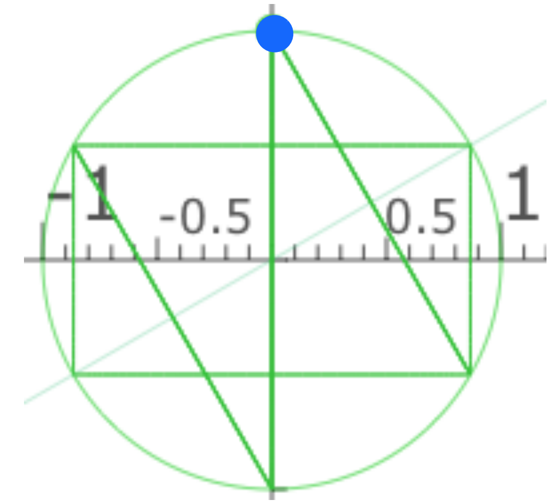
BounceIt Web Simulation
$m_1:m_2 = 3:1$ $(v_1, v_2)=(0, 1)$
Estrangian plot V_2 vs V_1

$M_1/m_2=3/1$



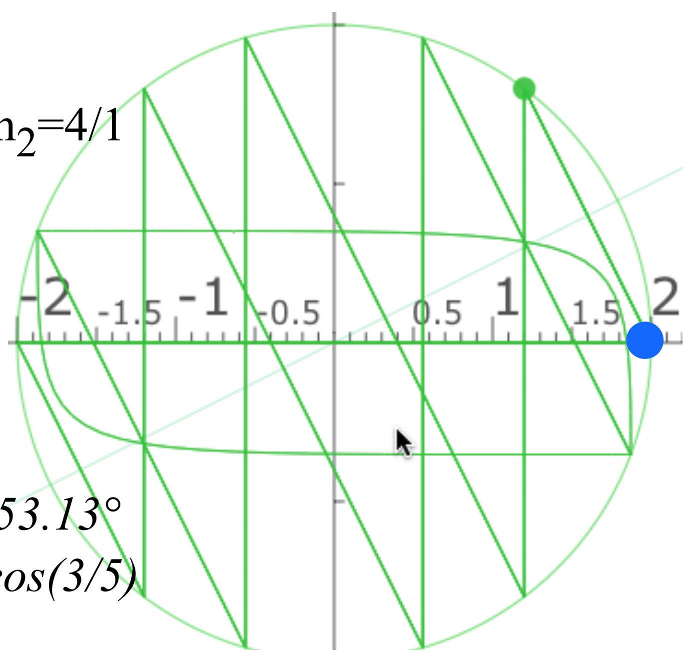
$\phi = 60°$
$= Acos(1/2)$



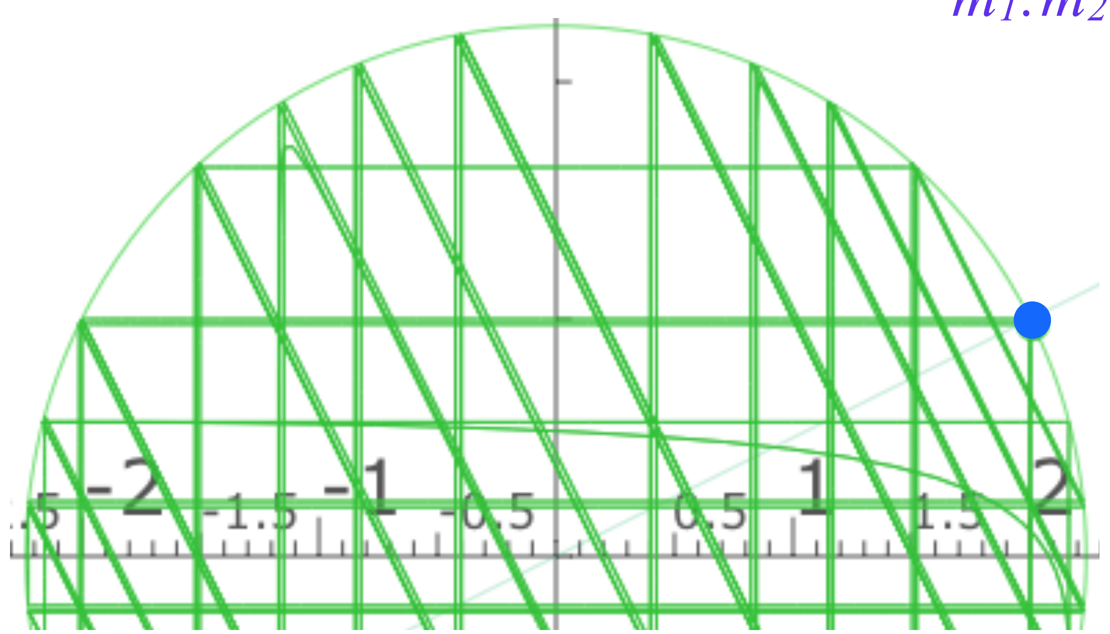


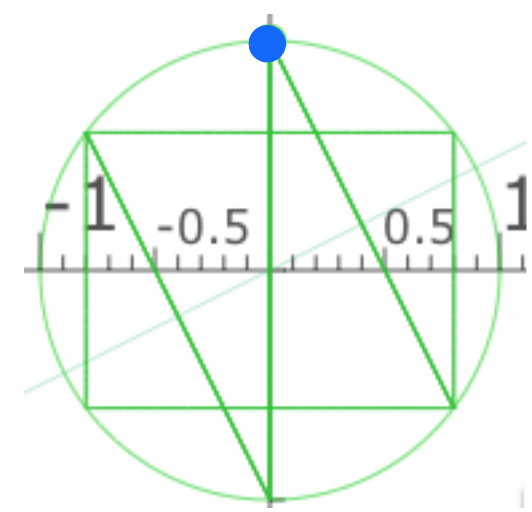
Collisions for mass ratio $m_1:m_2= 3:1$

$M_1/m_2=4/1$



$\phi = 53.13°$
$= Acos(3/5)$





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→ *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$* ←

Geometric "Integration" (Converting Velocity data to Spacetime)

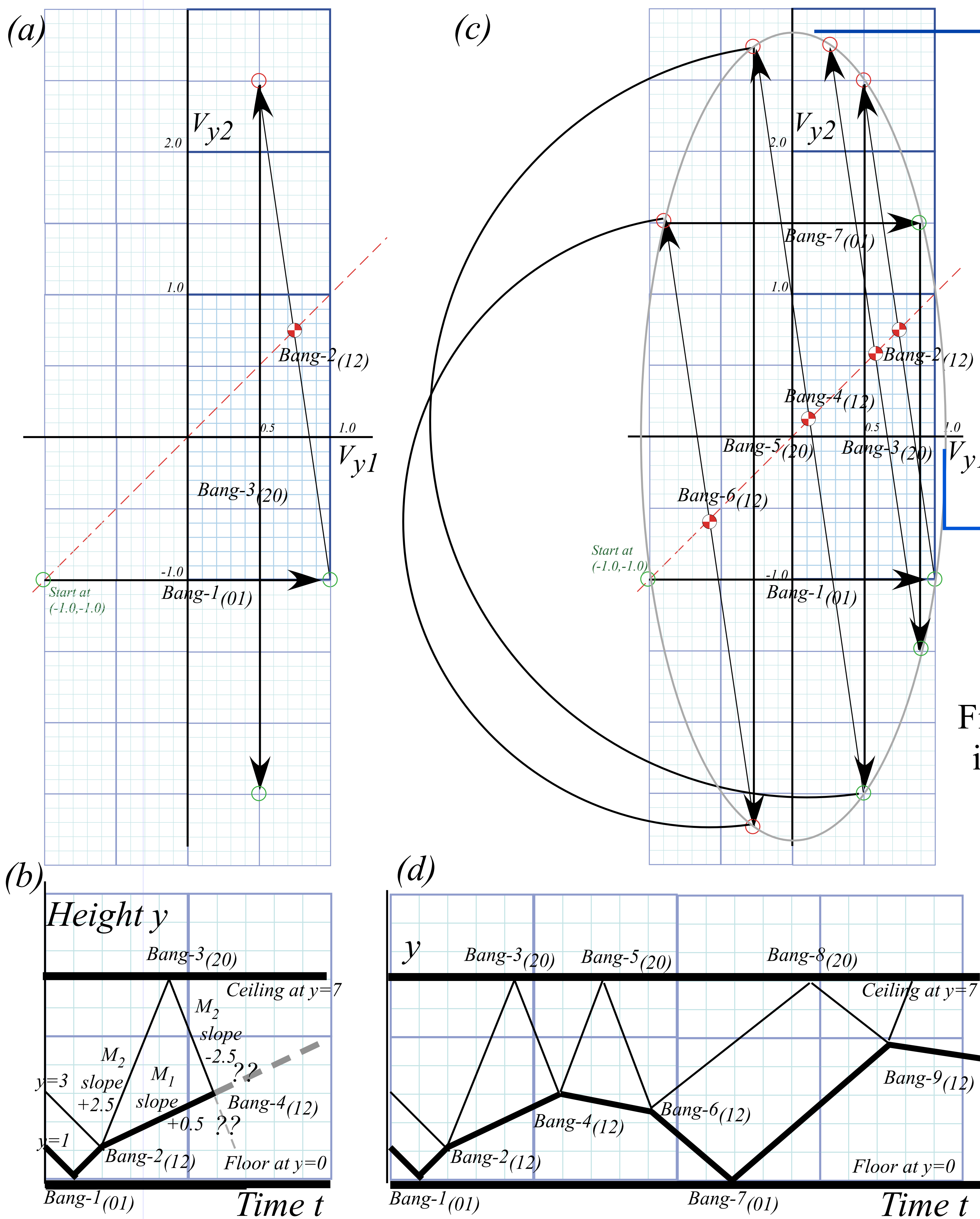


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

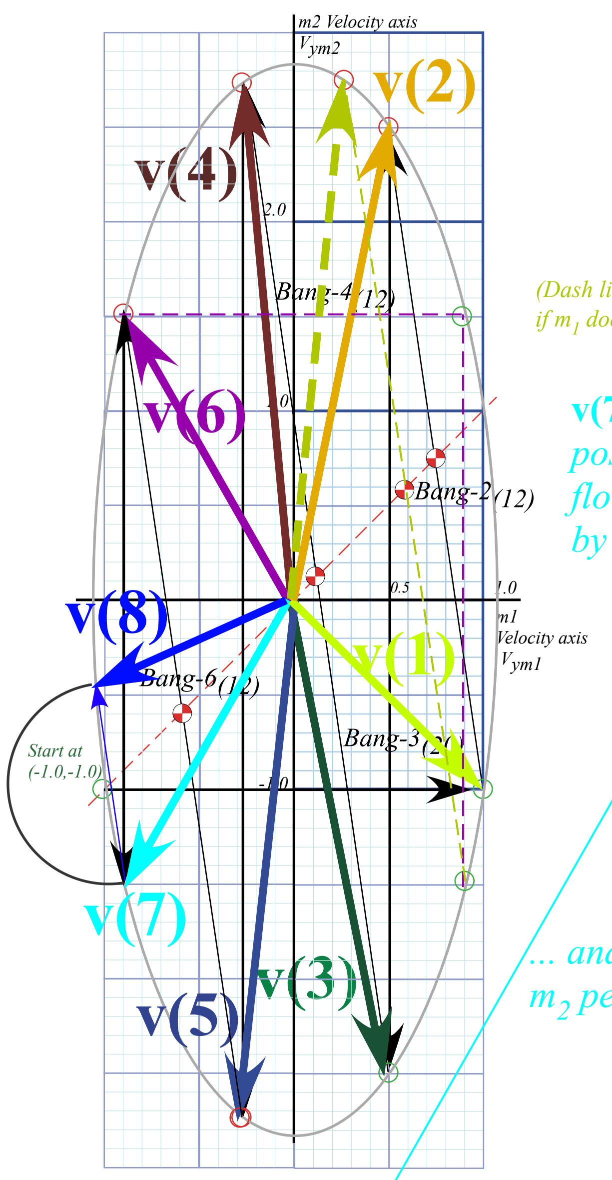
$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

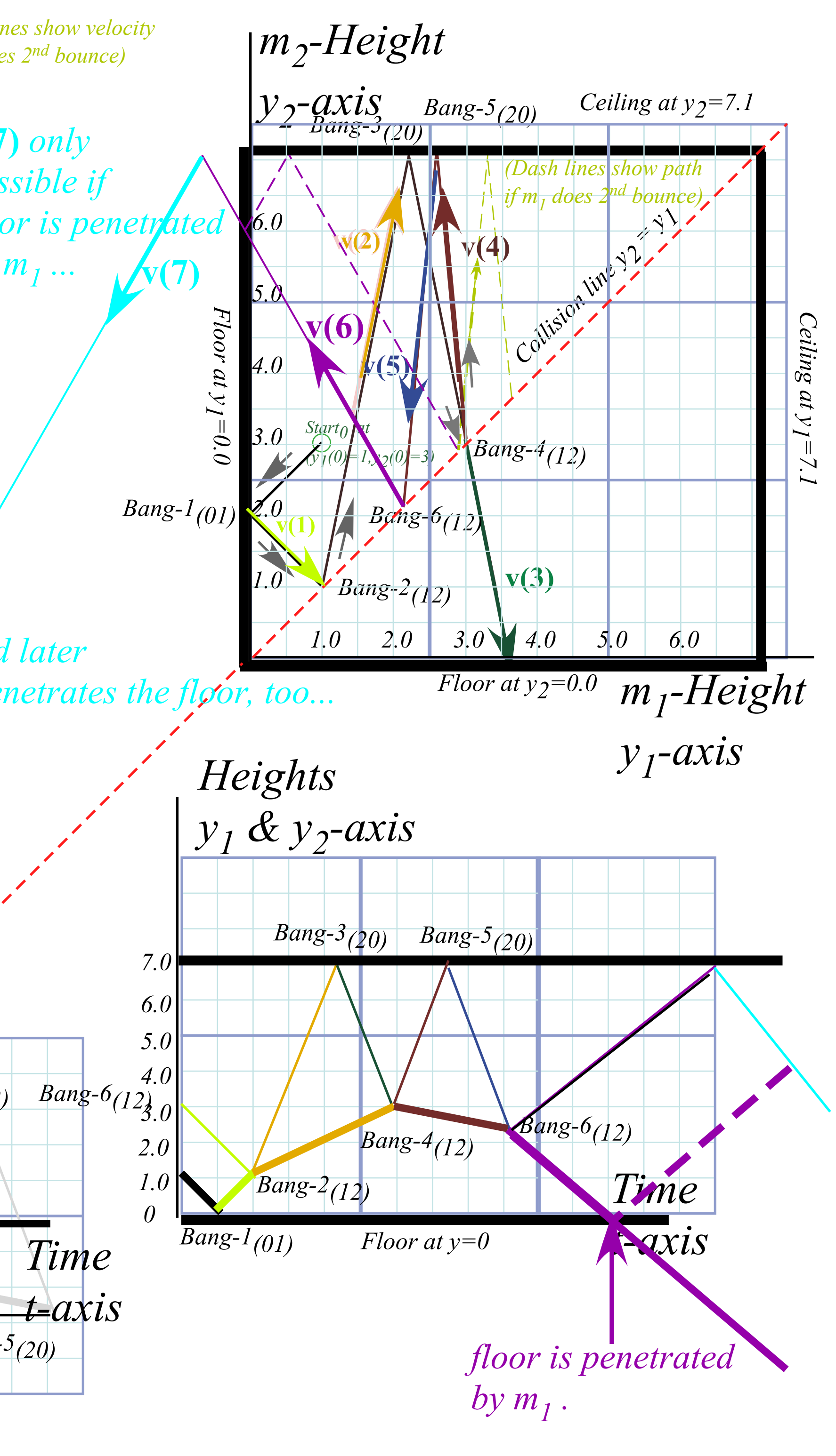
Ellipse radius 2

$$a_2 = \sqrt{2KE/M_2} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Collisions for mass ratio $m_1{:}m_2 = 7{:}1$

Step-2: Extend **v**(2) *line to ceiling point* **y**(3)=(?,7.1) *and draw Bang-3*$_{(20)}$ *velocity* **v**(3)=(1,-1) *line. (Find* **v**(3) *using V-V plot.)*
Step-3: Extend **v**(3) *line to collision point* **y**(4)=(?,?) *and draw Bang-4*$_{(12)}$ *velocity* **v**(4)=(0.5,2.5). *(Find* **v**(4) *using V-V plot.)*
Step-4: Extend **v**(4) *line to ceiling point* **y**(4)=(?,7.1) *and draw Bang-5*$_{(20)}$ *velocity* **v**(5)=(1,-1) *line. (Find* **v**(5) *using V-V plot.)*
Step-5: Extend **v**(5) *line to collision point* **y**(6)=(?,?) *and draw Bang-6*$_{(12)}$ *velocity* **v**(6)=(0.5,2.5). *(Find* **v**(6) *using V-V plot.)*





Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

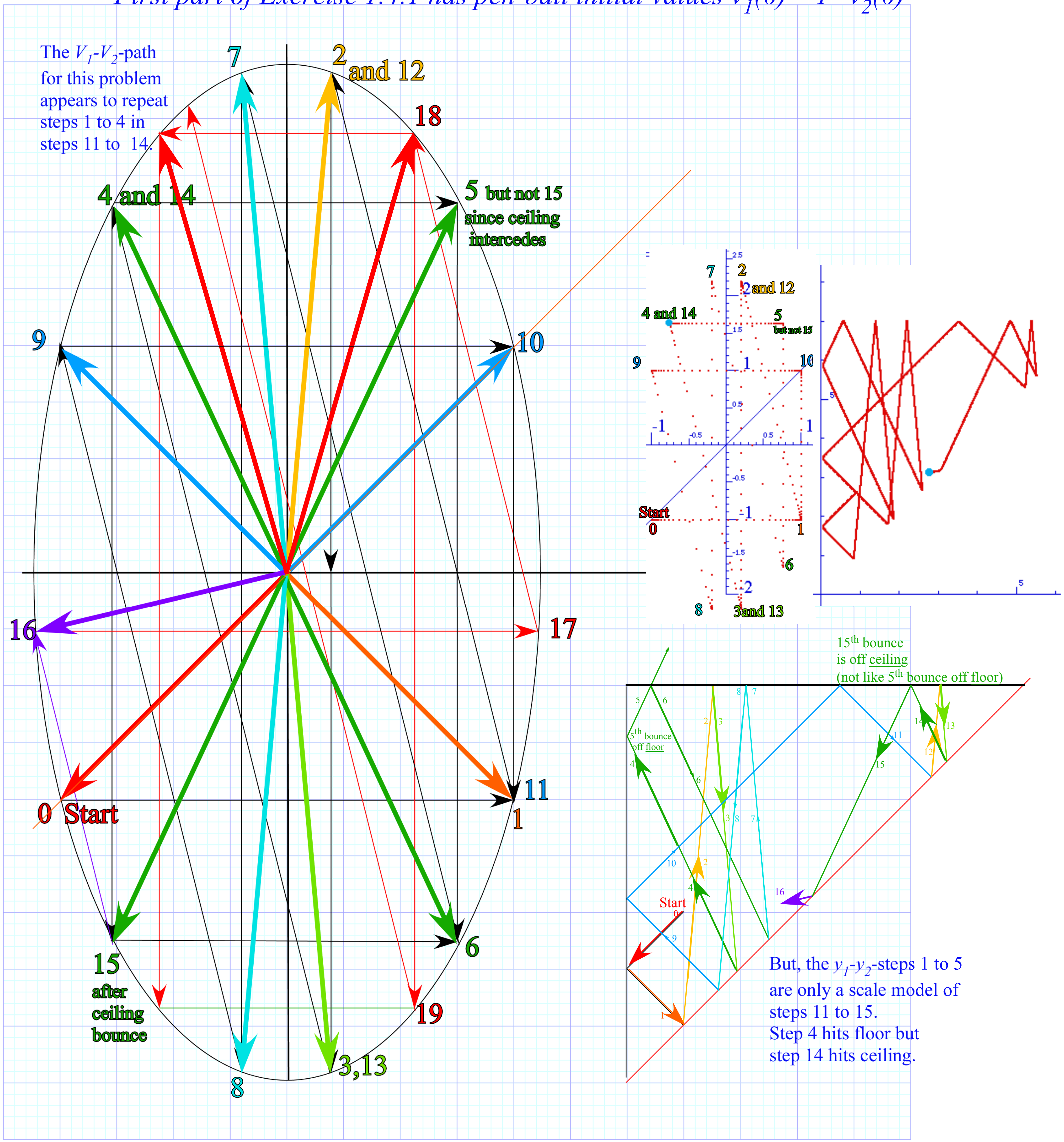
Introducing hexagonal symmetry D_6~C_{6v} (Resulting for $m_1/m_2=3$)

Group multiplication and product table

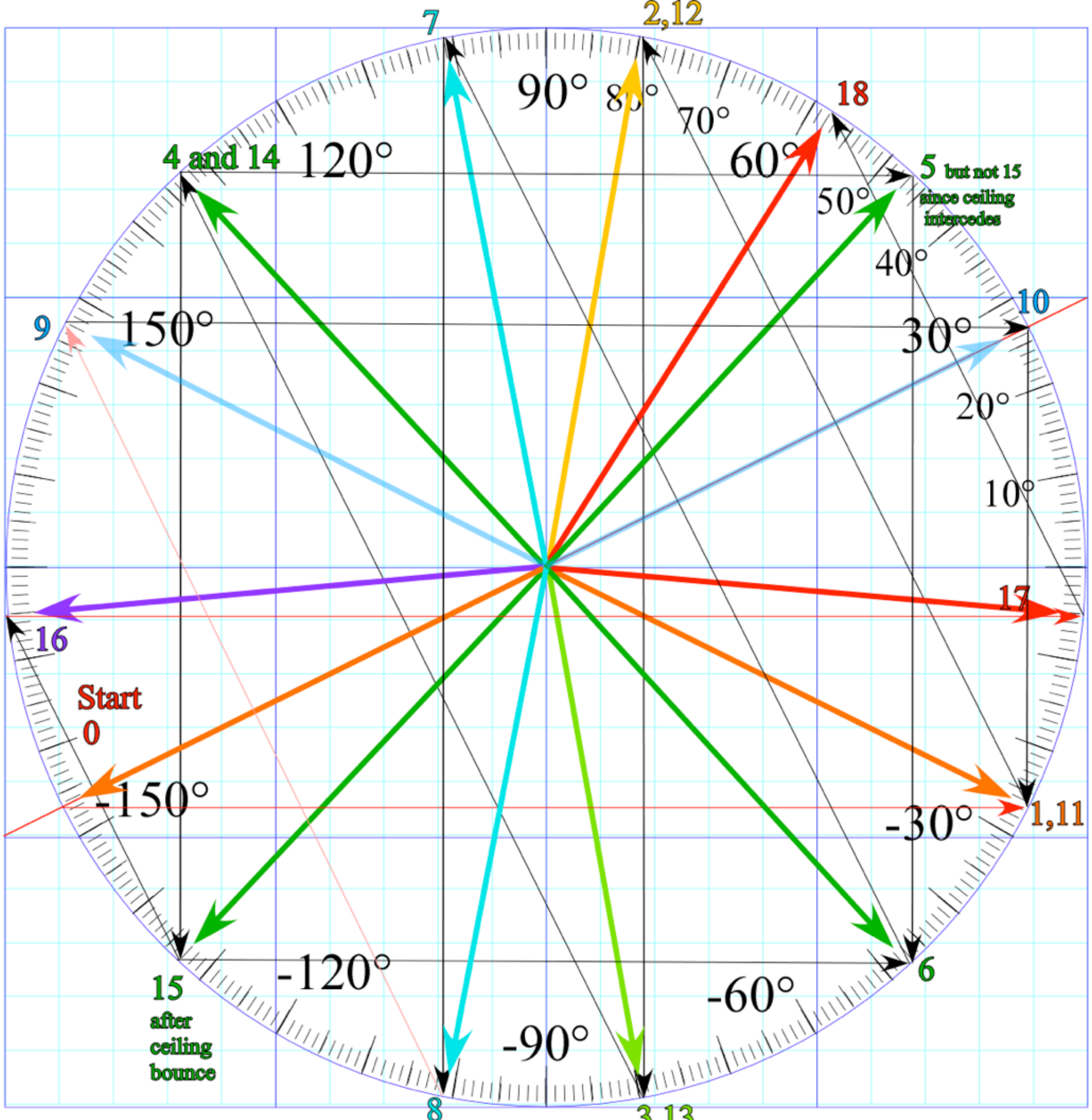
Classical collision paths with D_6~C_{6v} (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

First part of Exercise 1.4.1 has pen-ball initial values $v_1(0)=-1=v_2(0)$

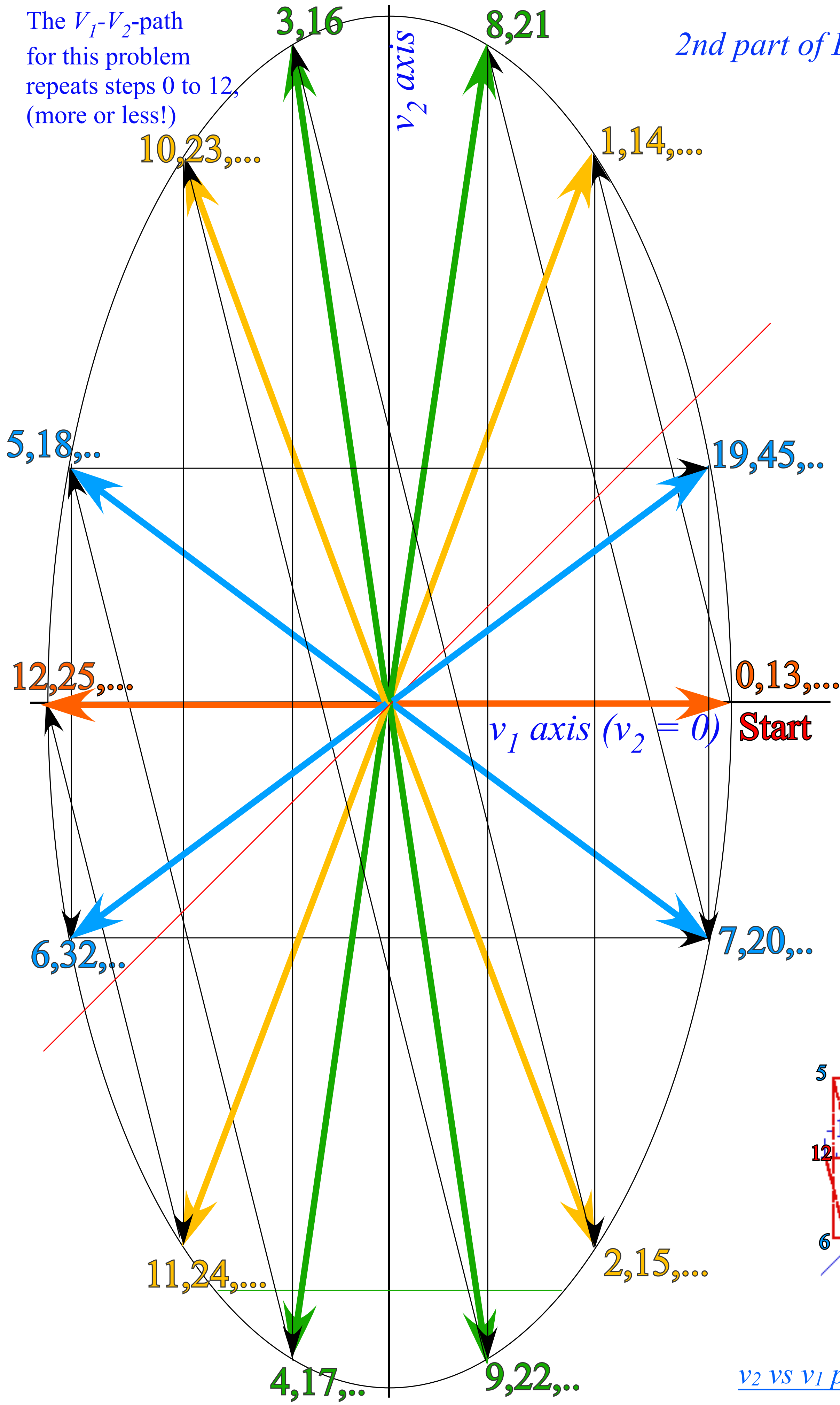


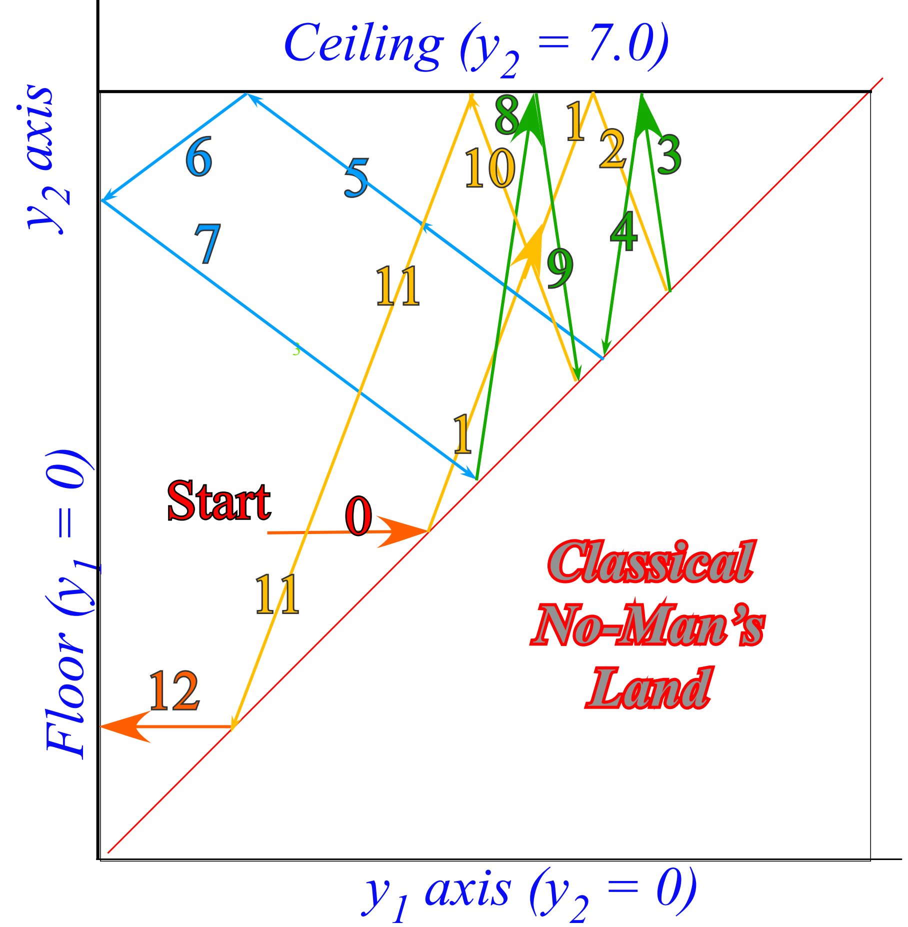
Collisions for mass ratio m_1:m_2= 4:1



2nd part of Exercise 1.4.1 has pen-ball initial velocity values $v_1(0)=1$ and $v_2(0)=0$
at: $x_1(0)=1.5$ and $x_2(0)=3.0$

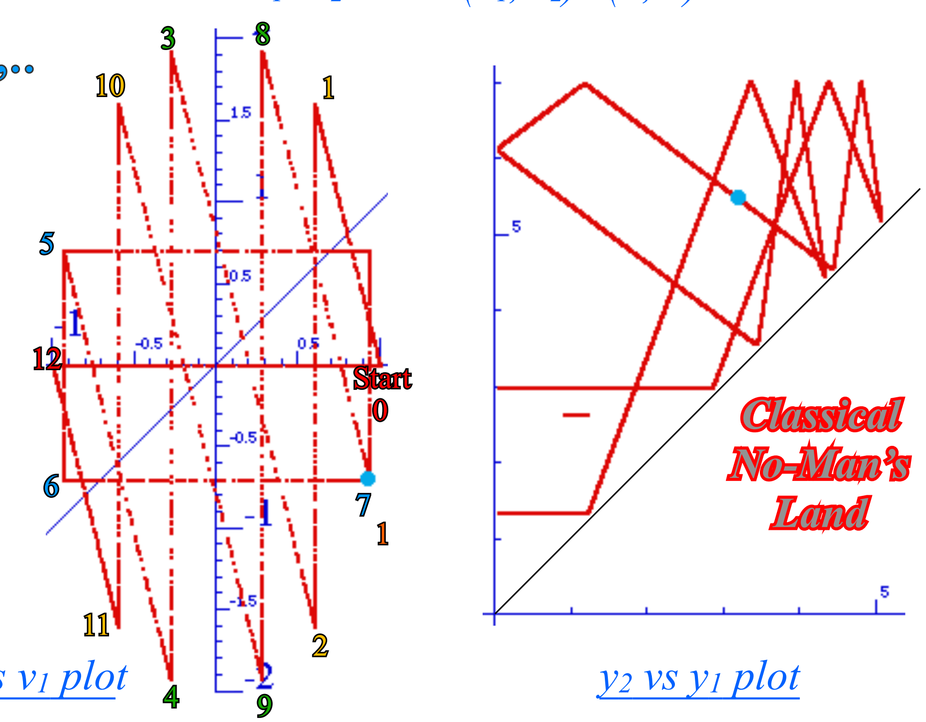
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)



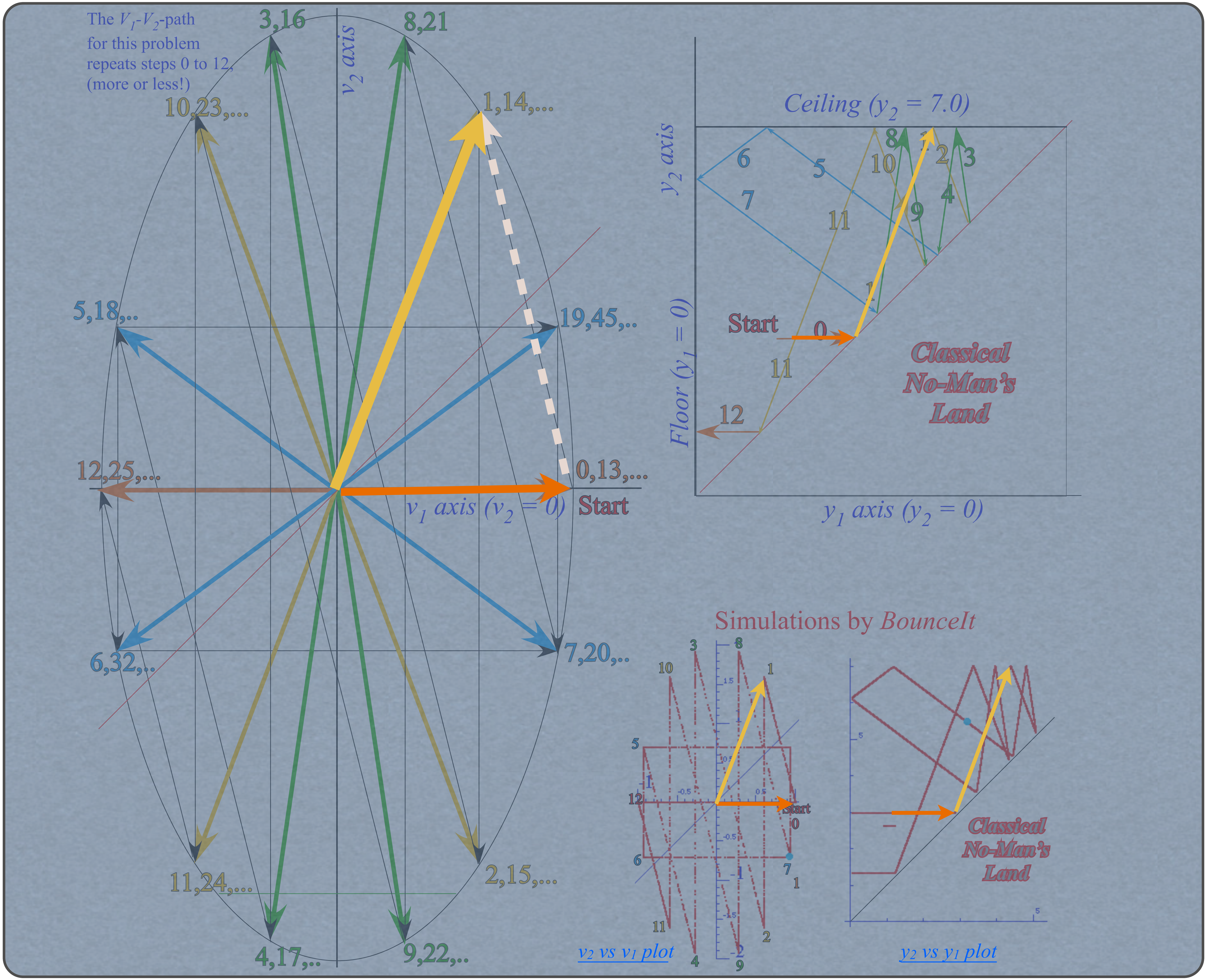


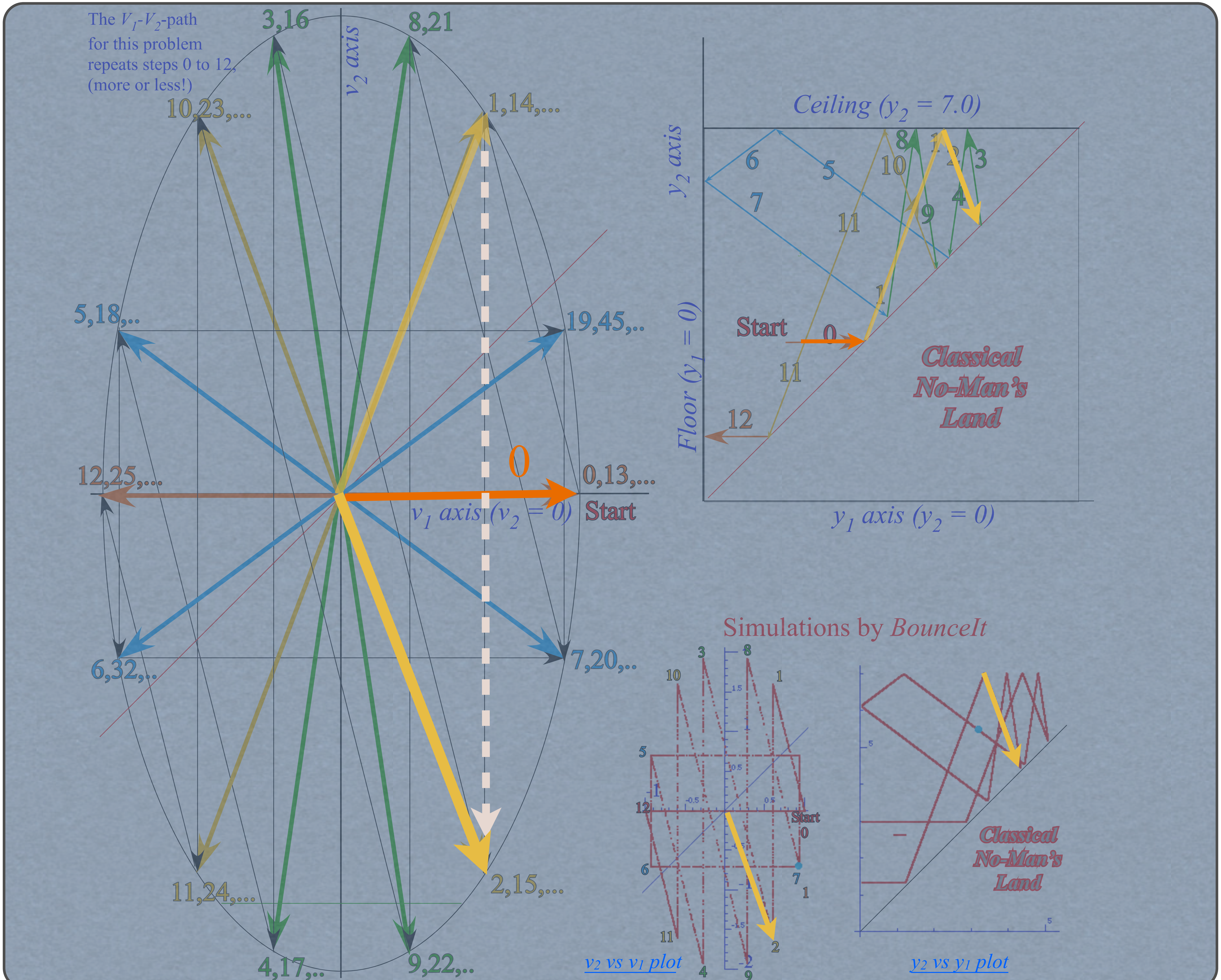
Collisions for mass ratio $m_1:m_2= 4:1$

BounceIt Web Simulations
$m_1:m_2 = 4:1$ $(v_1, v_2)=(1, 0)$



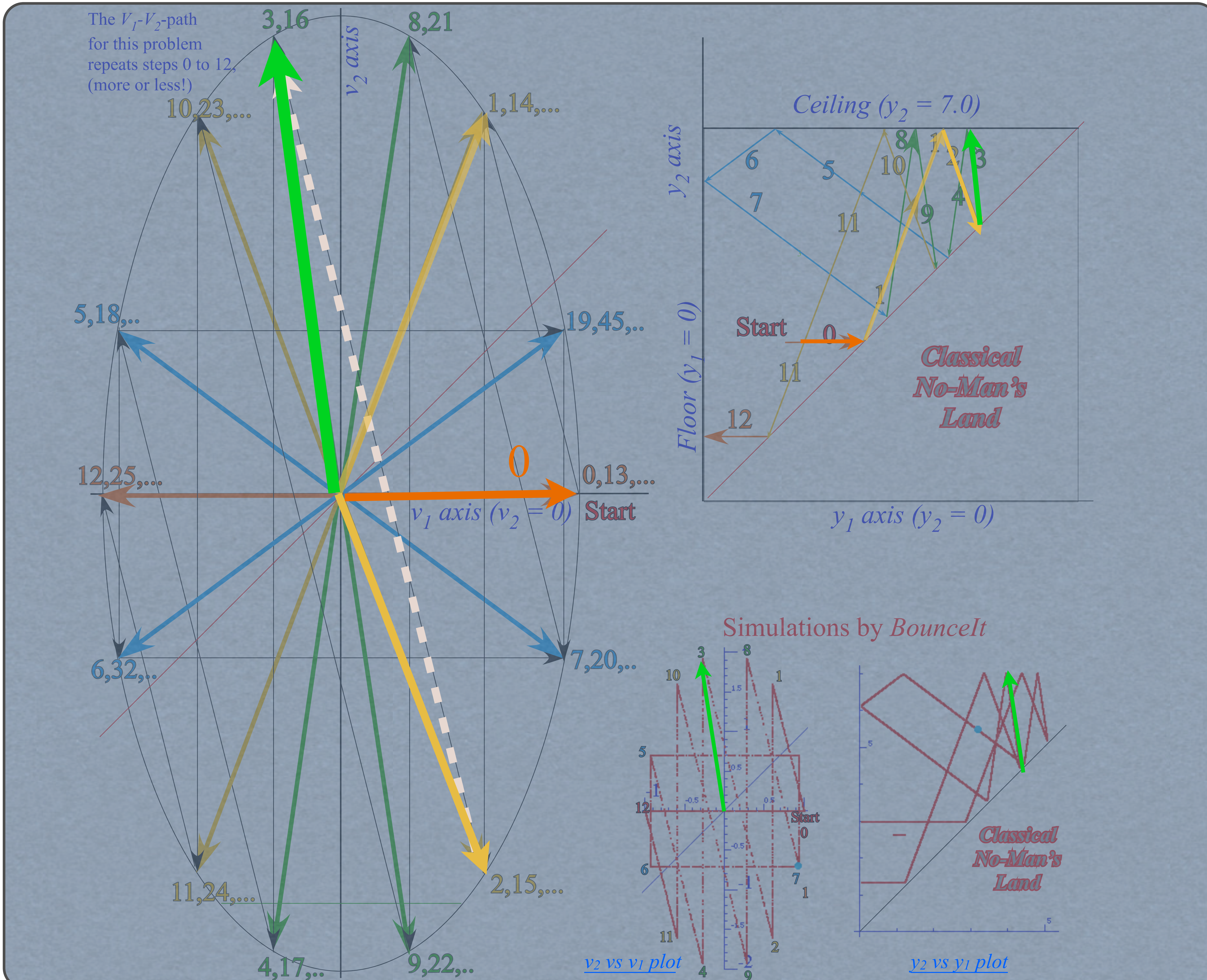
v2 vs v1 plot   y2 vs y1 plot


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
v_2 axis
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical No-Man's Land
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical No-Man's Land



[v_2 vs v_1 plot]

[y_2 vs y_1 plot]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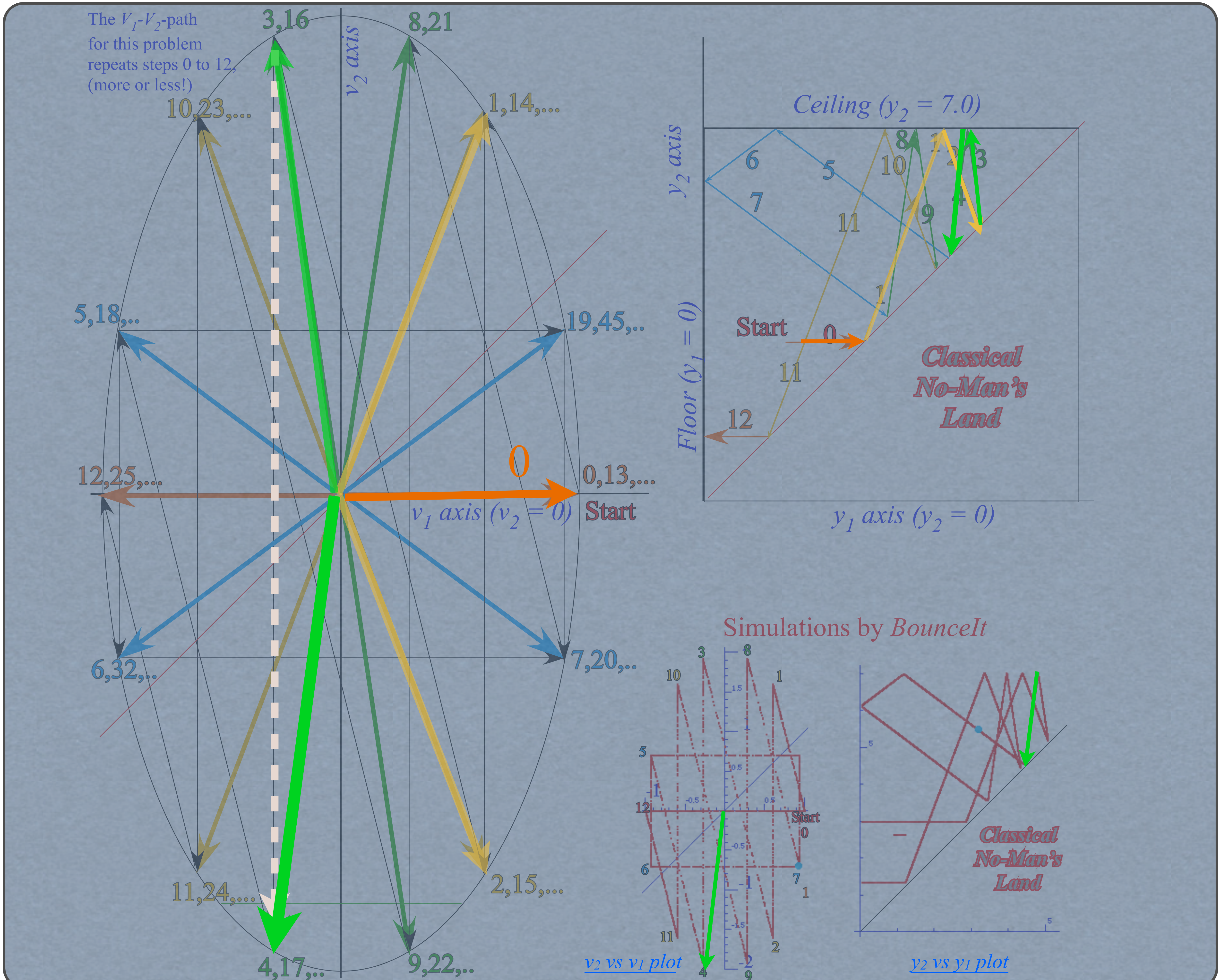
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot



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y_2 axis
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y_1 axis (y_2 = 0)
Start
Classical
No-Man's
Land
Simulations by BounceIt
Start
Classical
No-Man's
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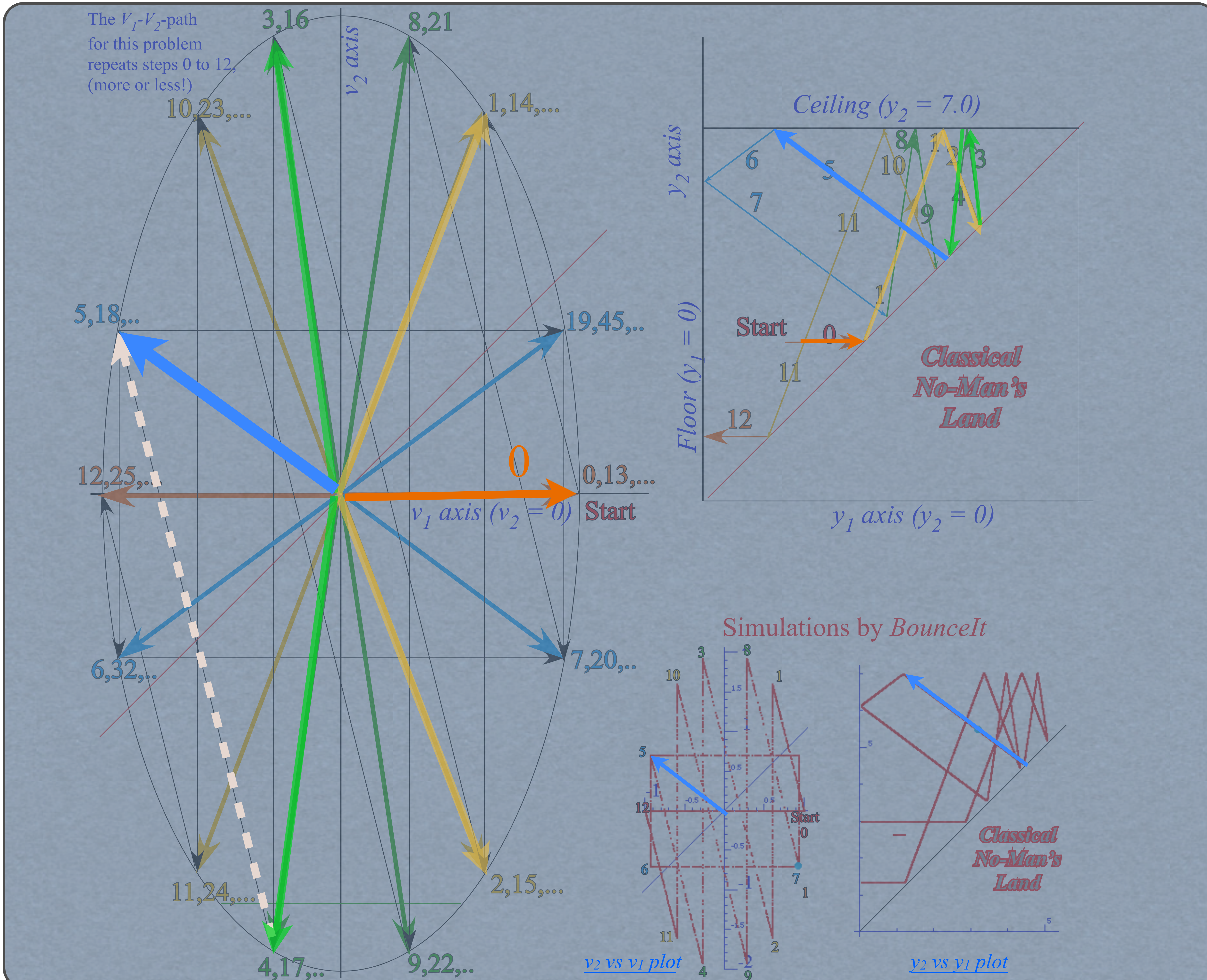
v_2 vs v_1 plot
y_2 vs y_1 plot



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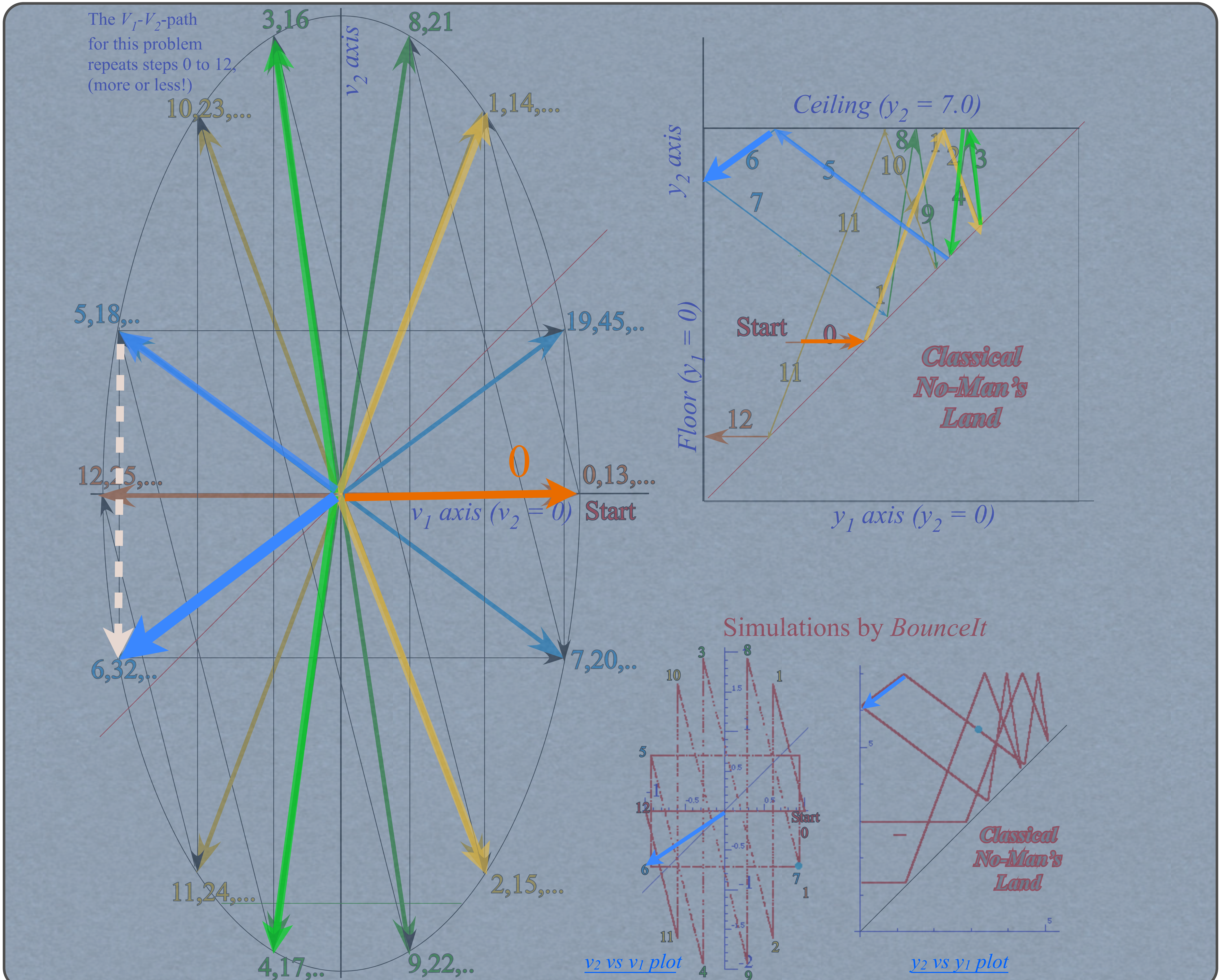
Ceiling ($y_2 = 7.0$)
y_2 axis
Floor ($y_1 = 0$)
Start
0
Classical
No-Man's
Land
12
y_1 axis ($y_2 = 0$)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land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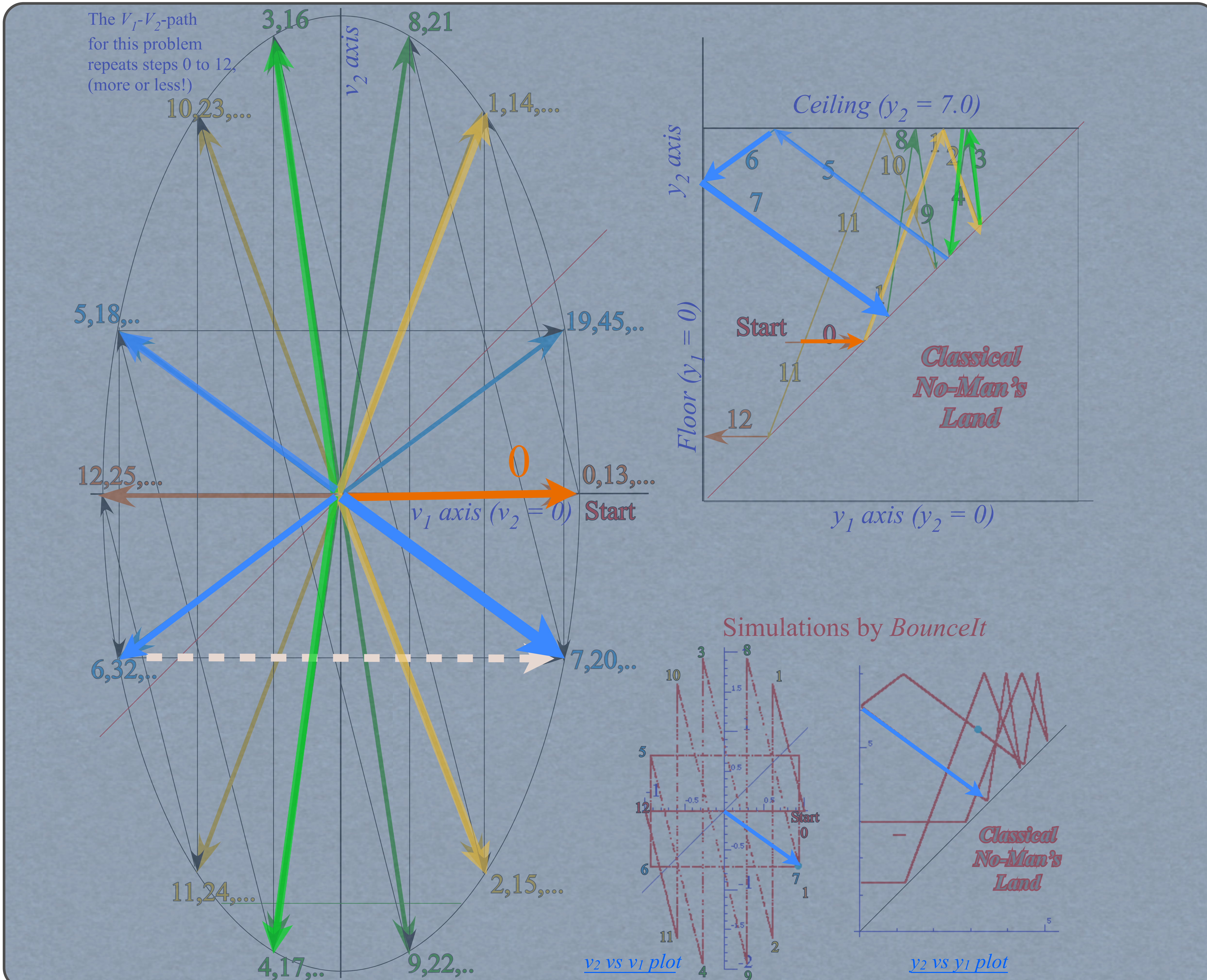
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot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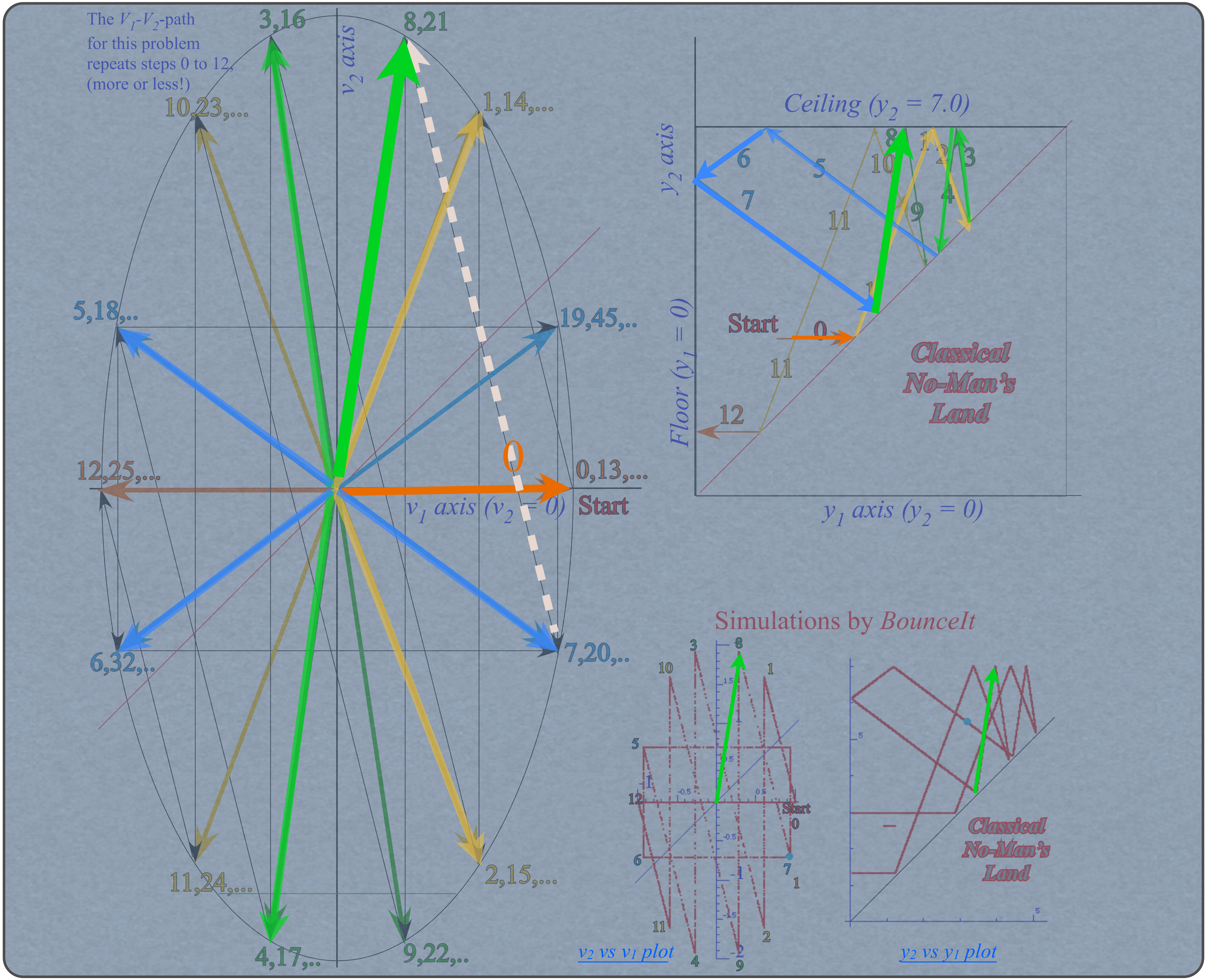
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No-Man's
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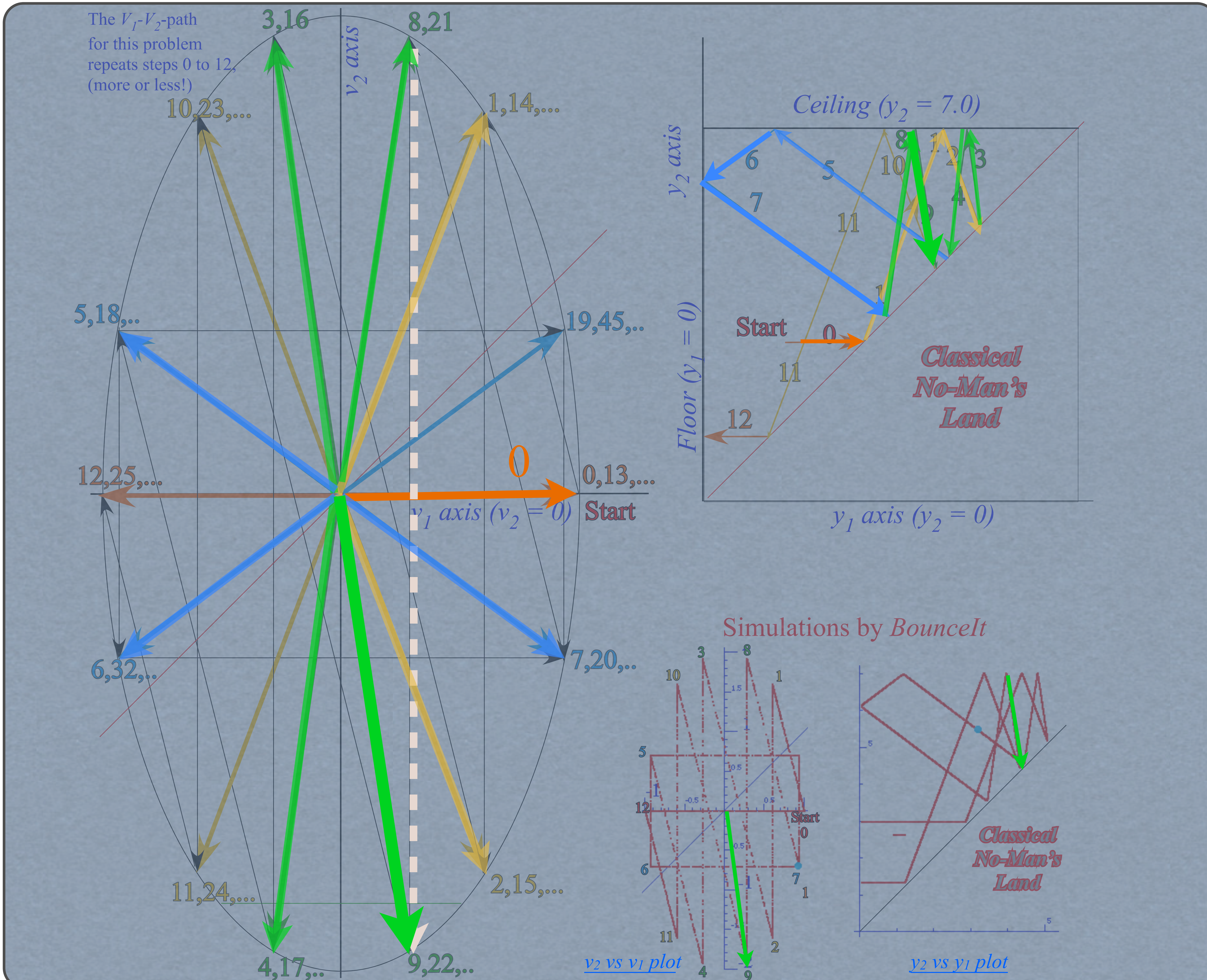
v_2 vs v_1 plot
y_2 vs y_1 plot



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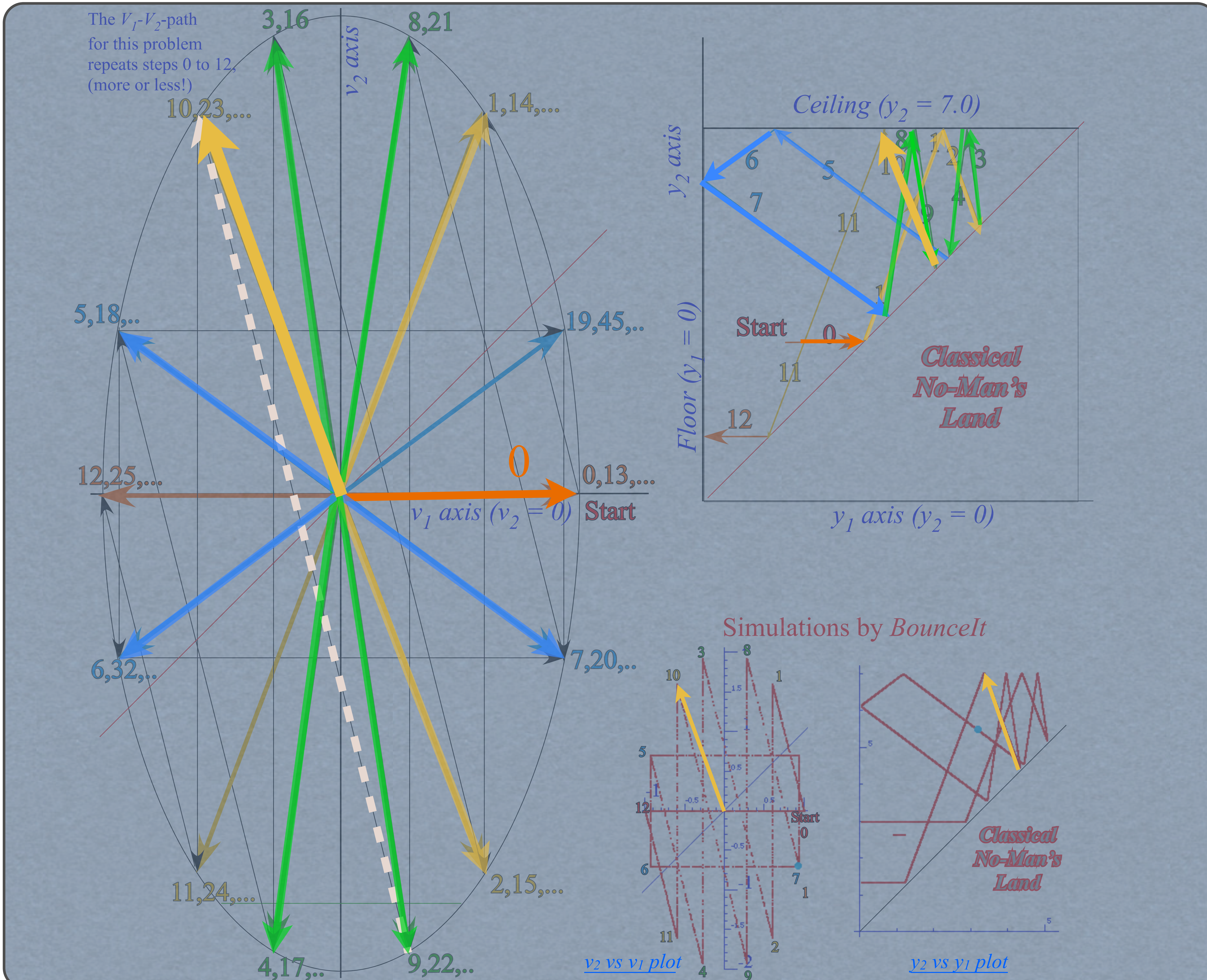
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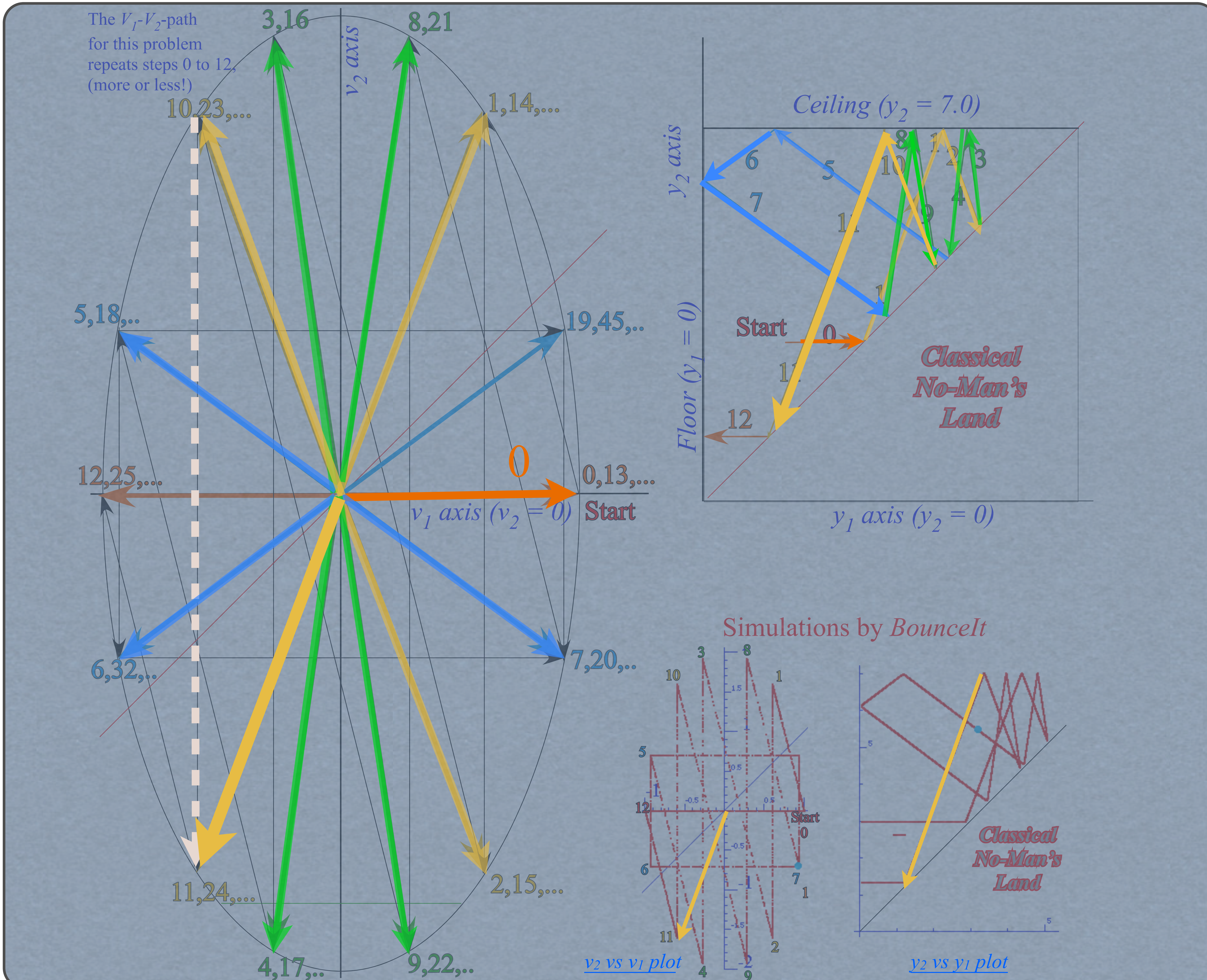
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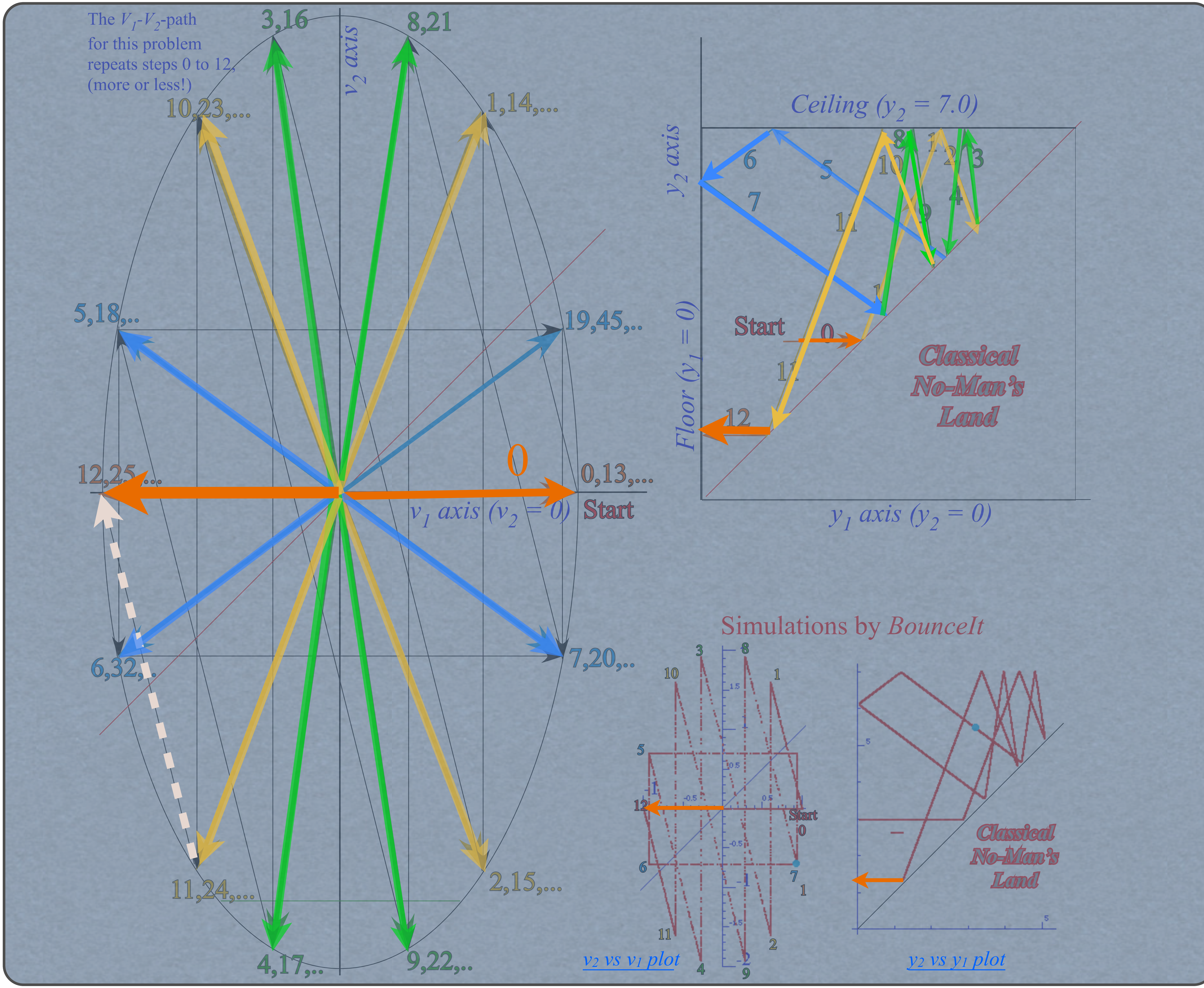
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y_2 vs y_1 plot
Classical No-Man's Land

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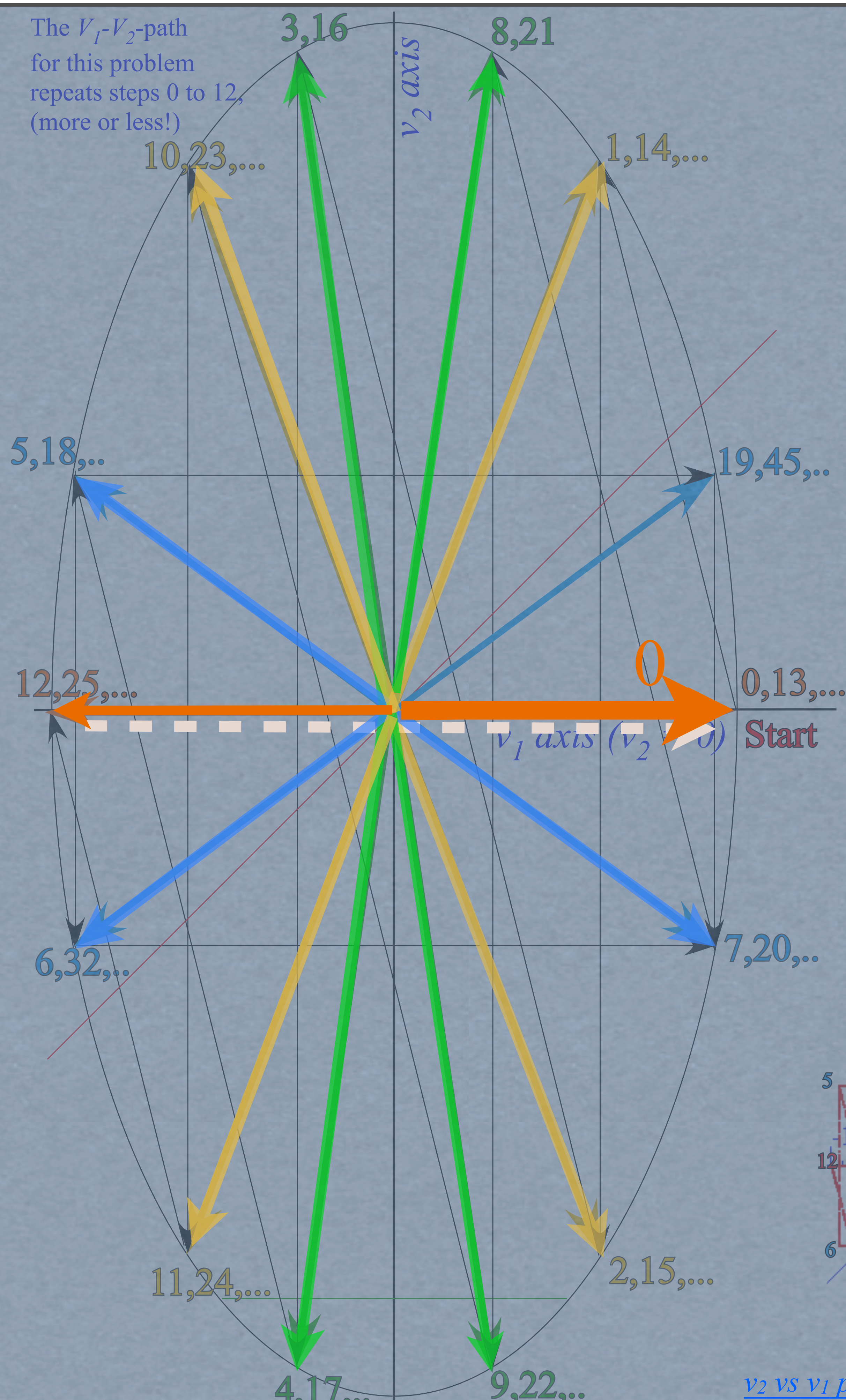
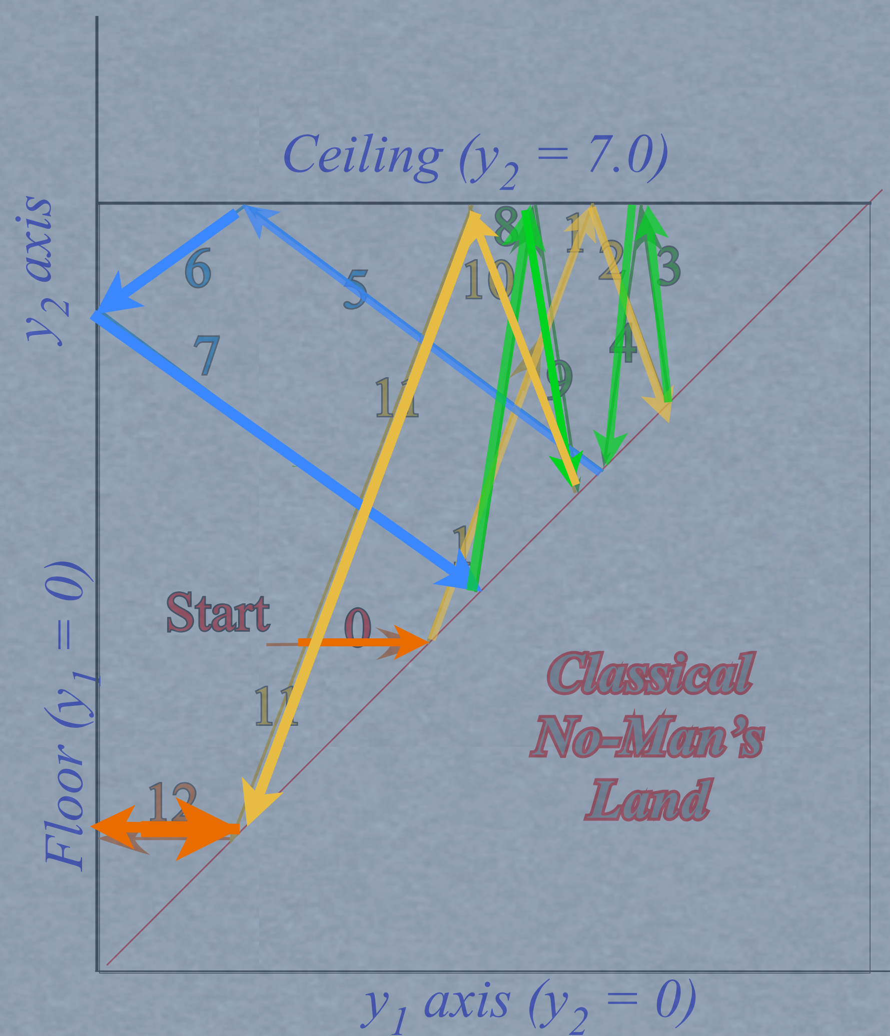
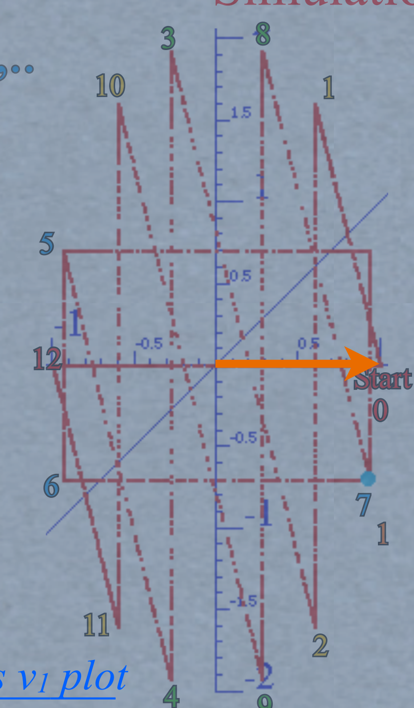
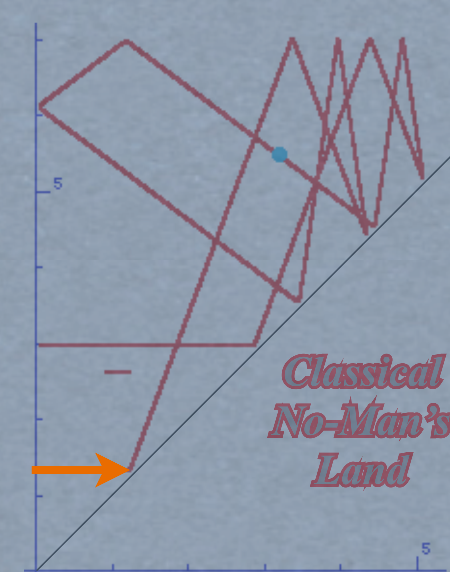




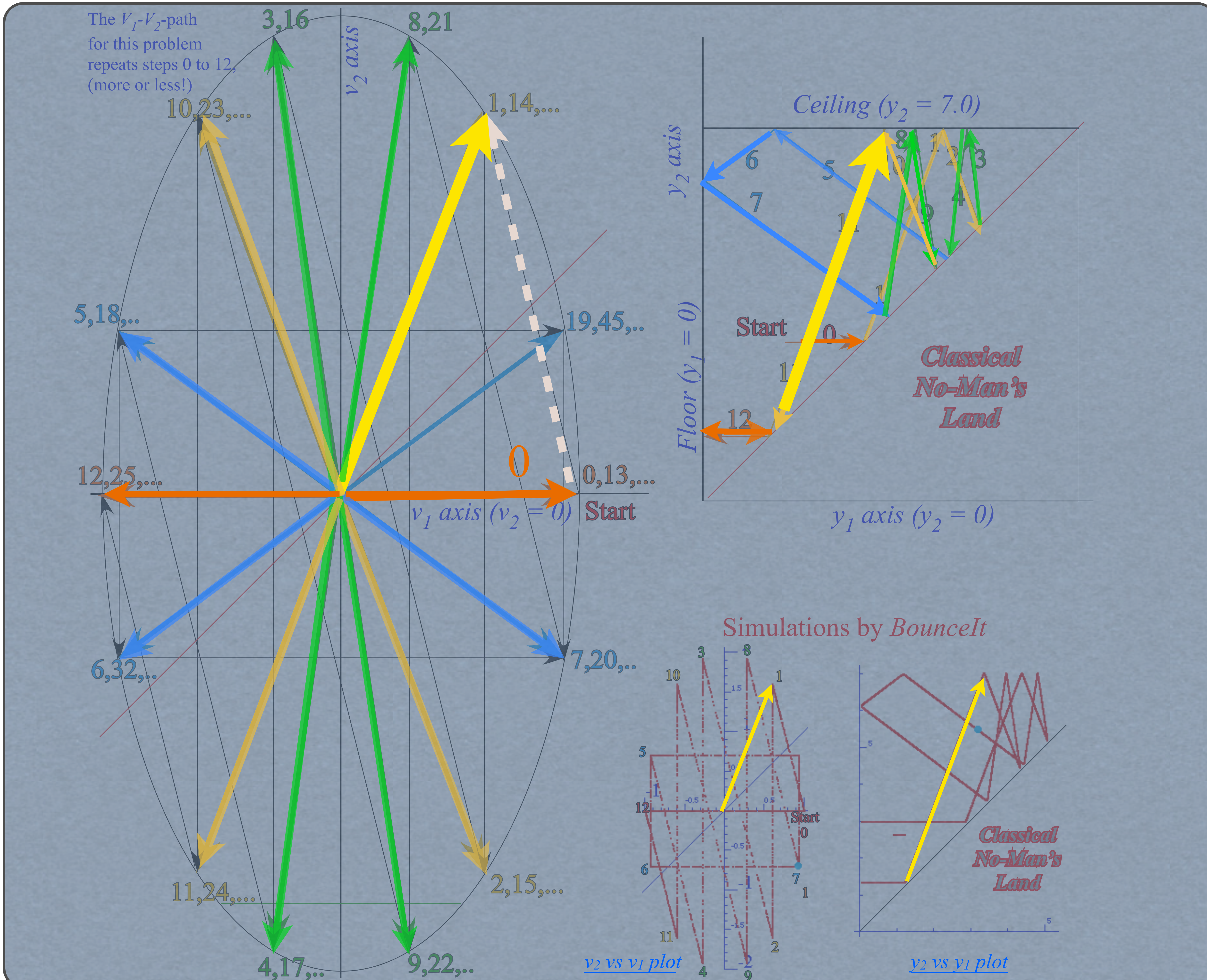
Simulations by *BounceIt*



[v_2 vs v_1 plot]



[y_2 vs y_1 plot]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Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Start
Classical
No-Man's
Land
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
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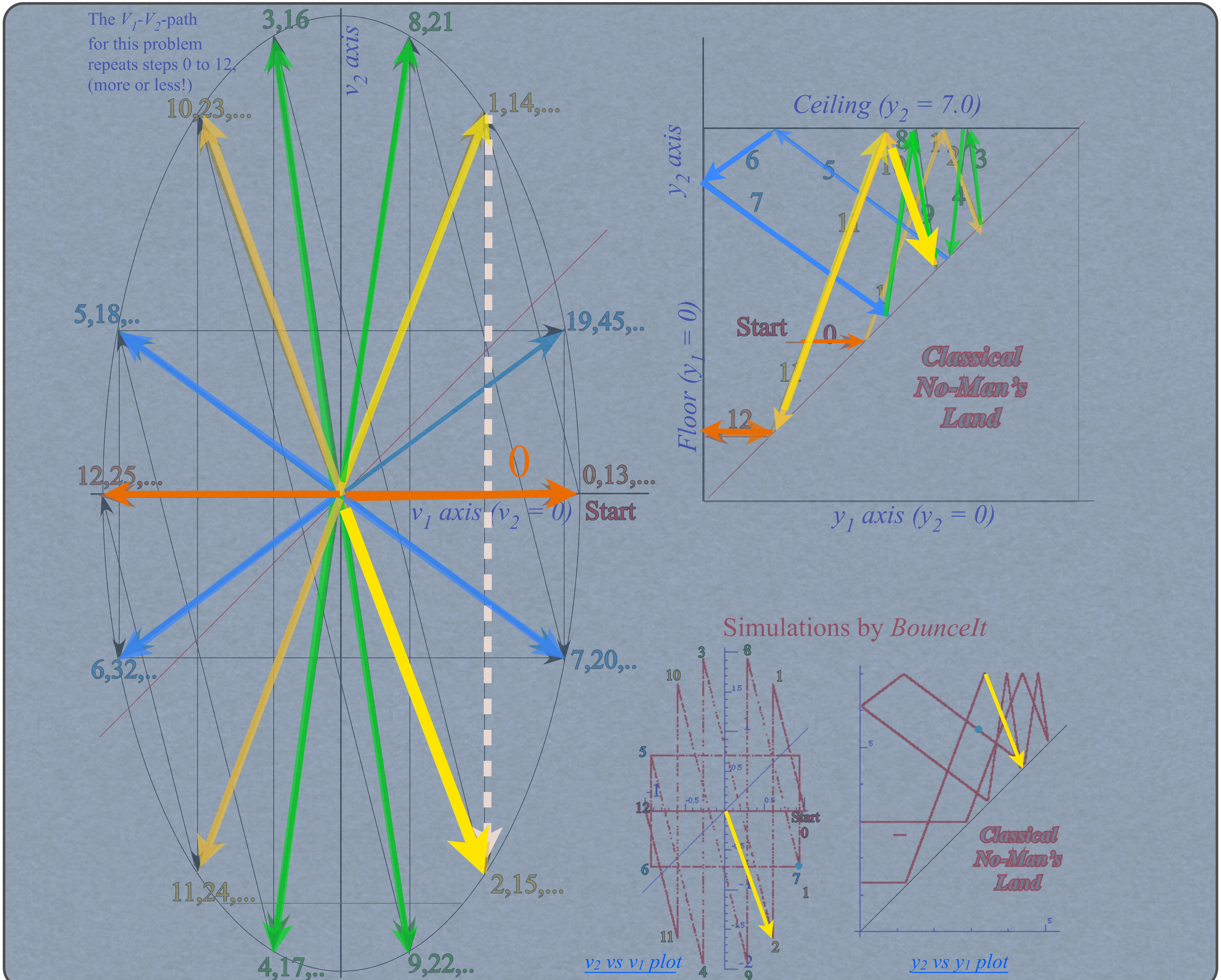
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y_1 axis $(y_2 = 0)$
Simulations by *BounceIt*
v_2 vs v_1 plot
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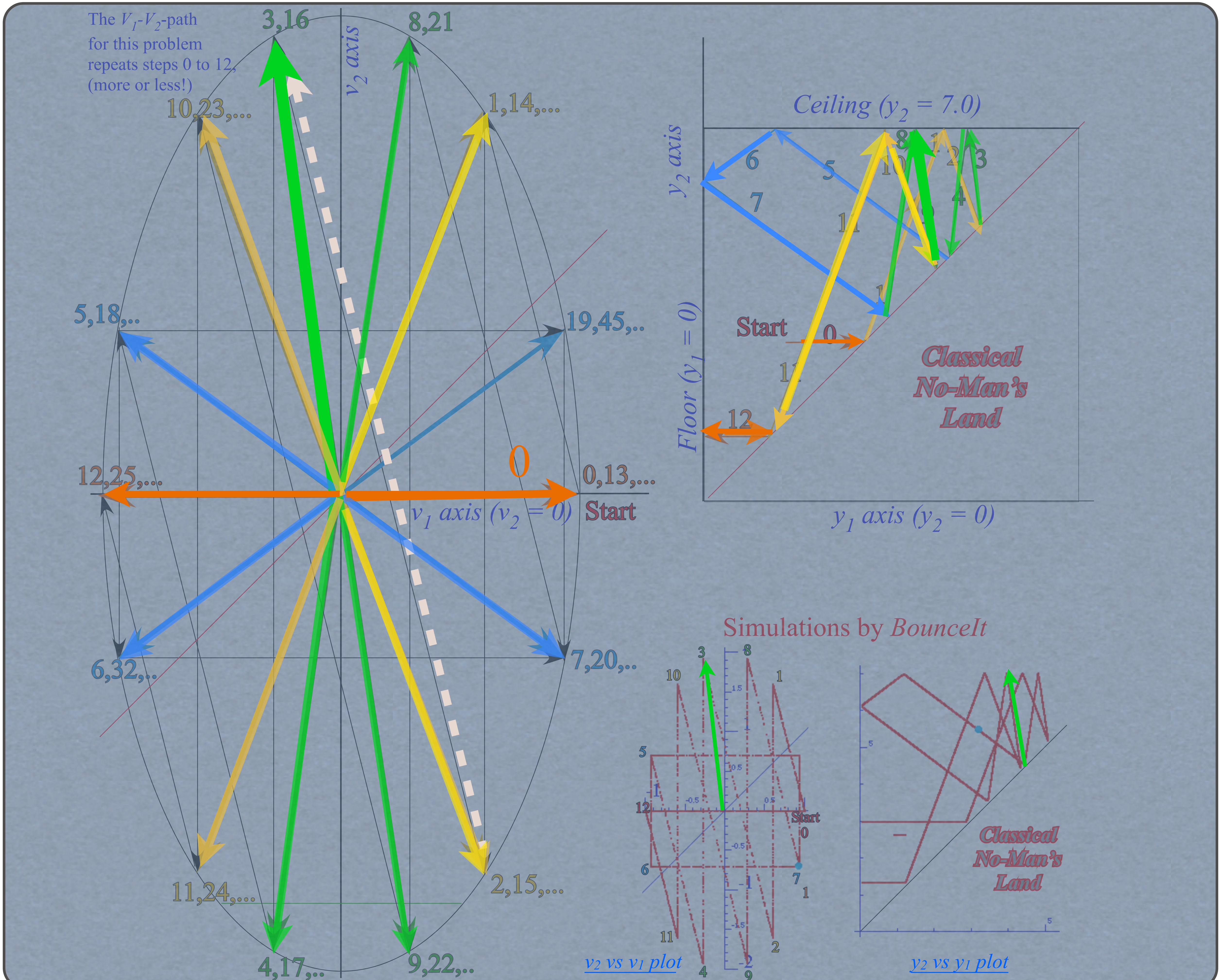

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v_2 vs v_1 plot
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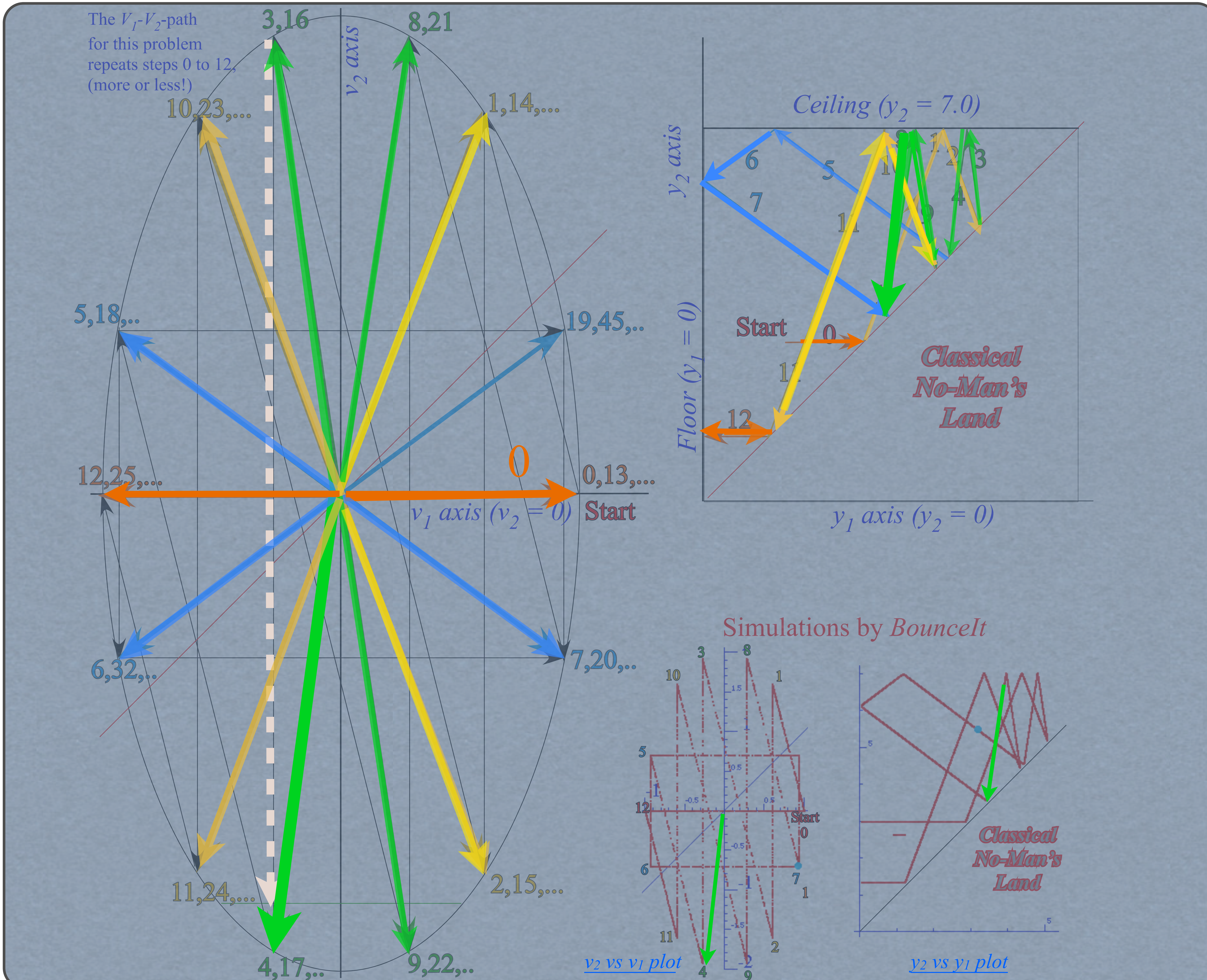

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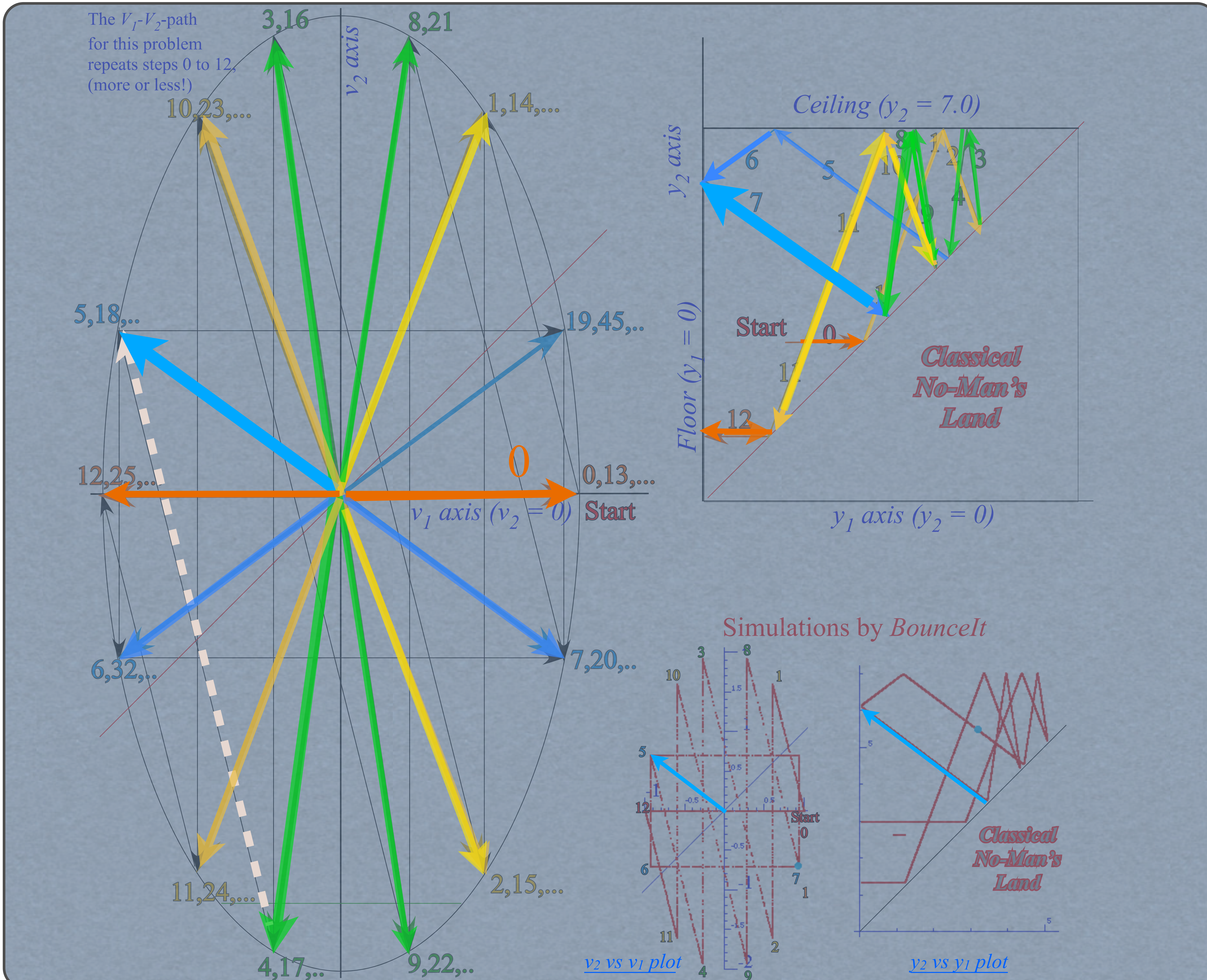
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9,22,..
Ceiling (y2 = 7.0)
y2 axis
6
5
7
Start
0
Floor (y1 = 0)
12
Classical
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y1 axis (y2 = 0)
Simulations by BounceIt
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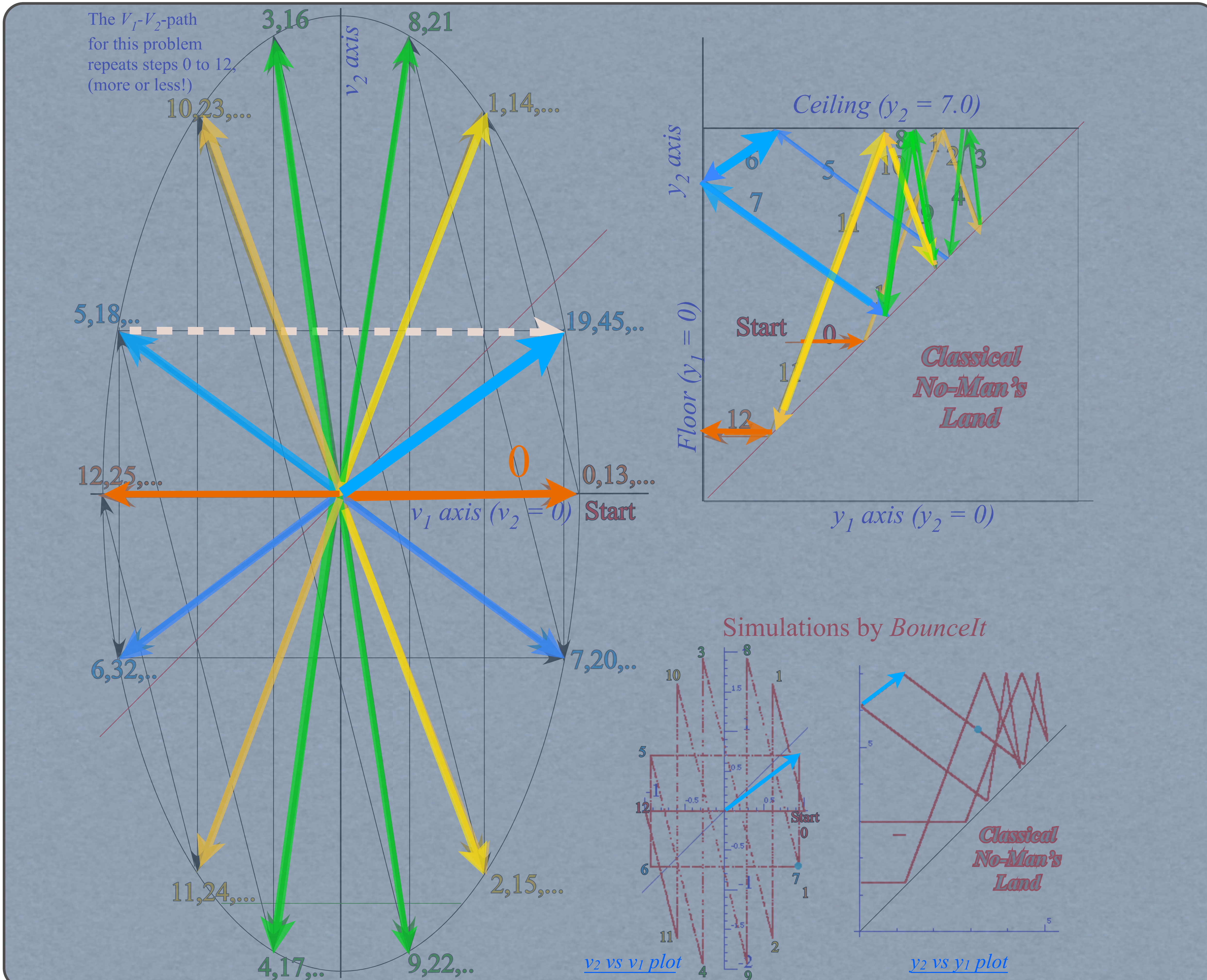
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot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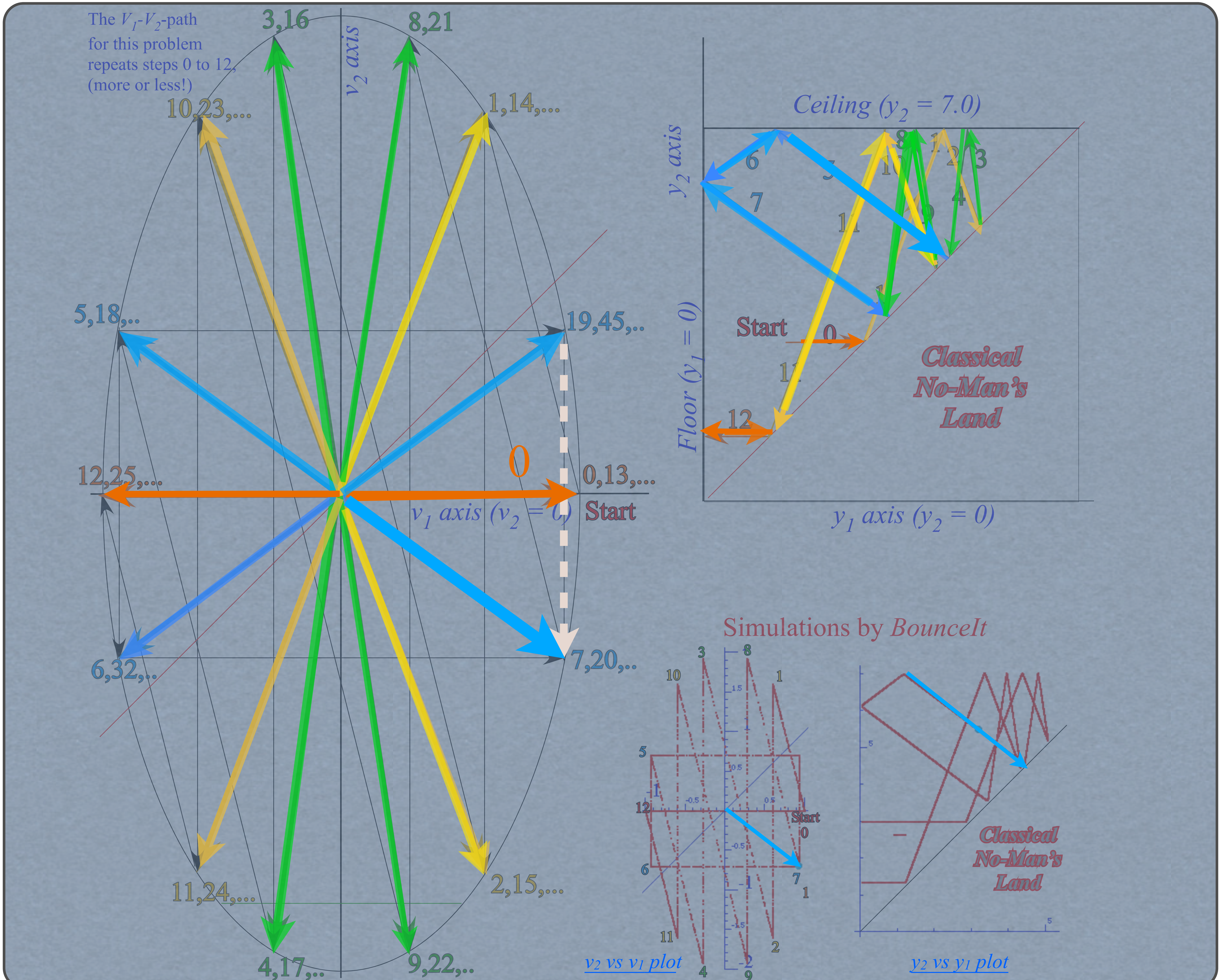

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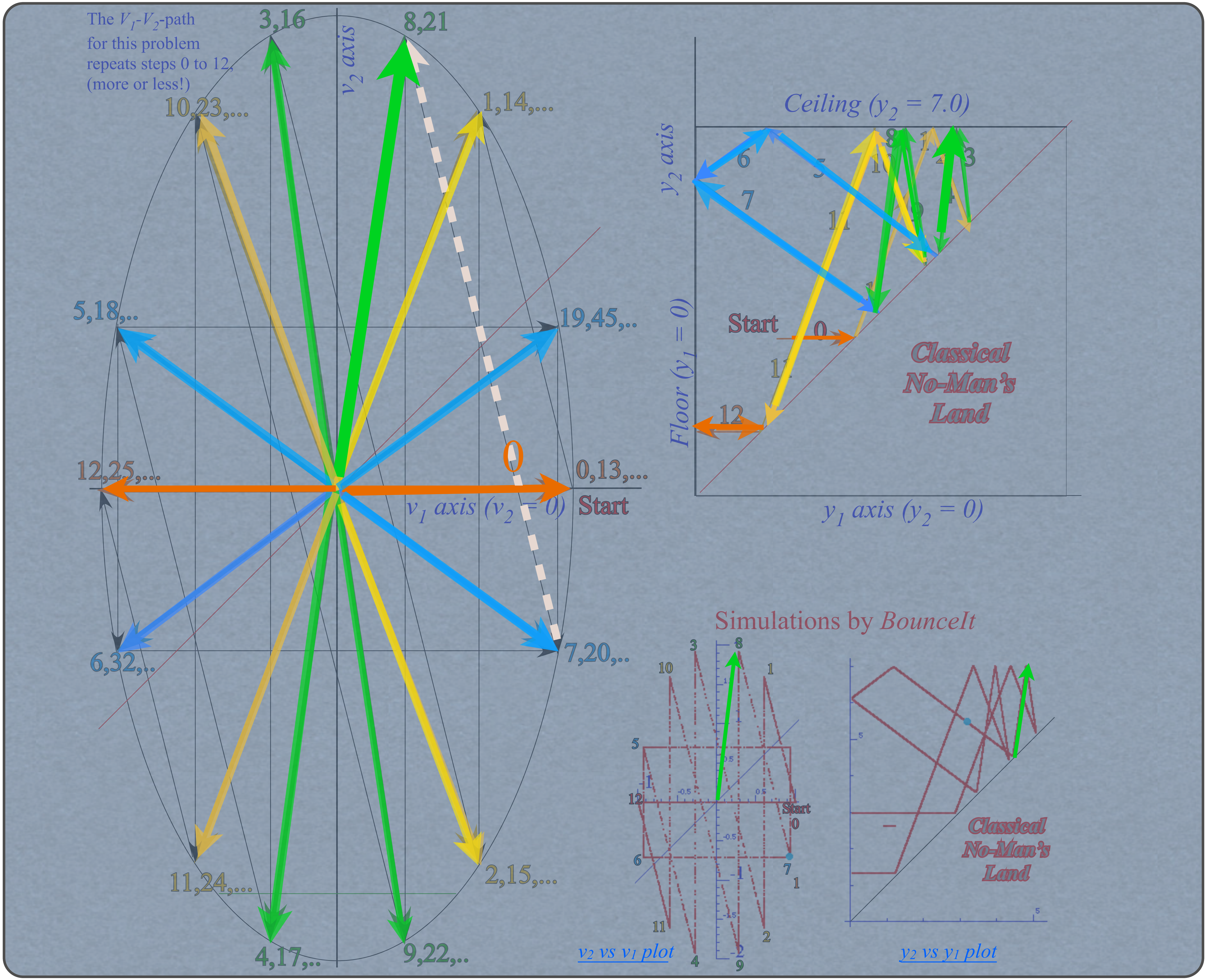
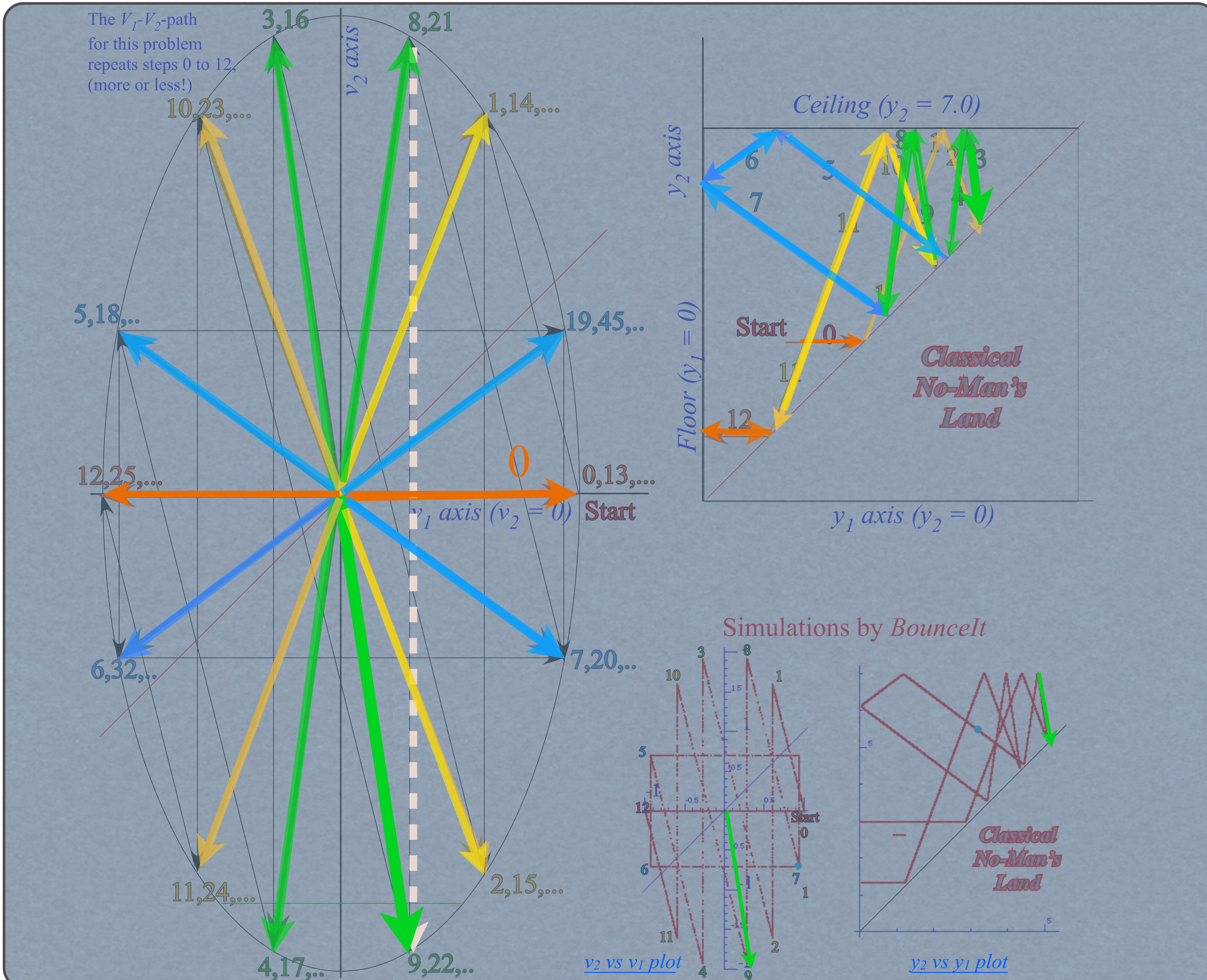
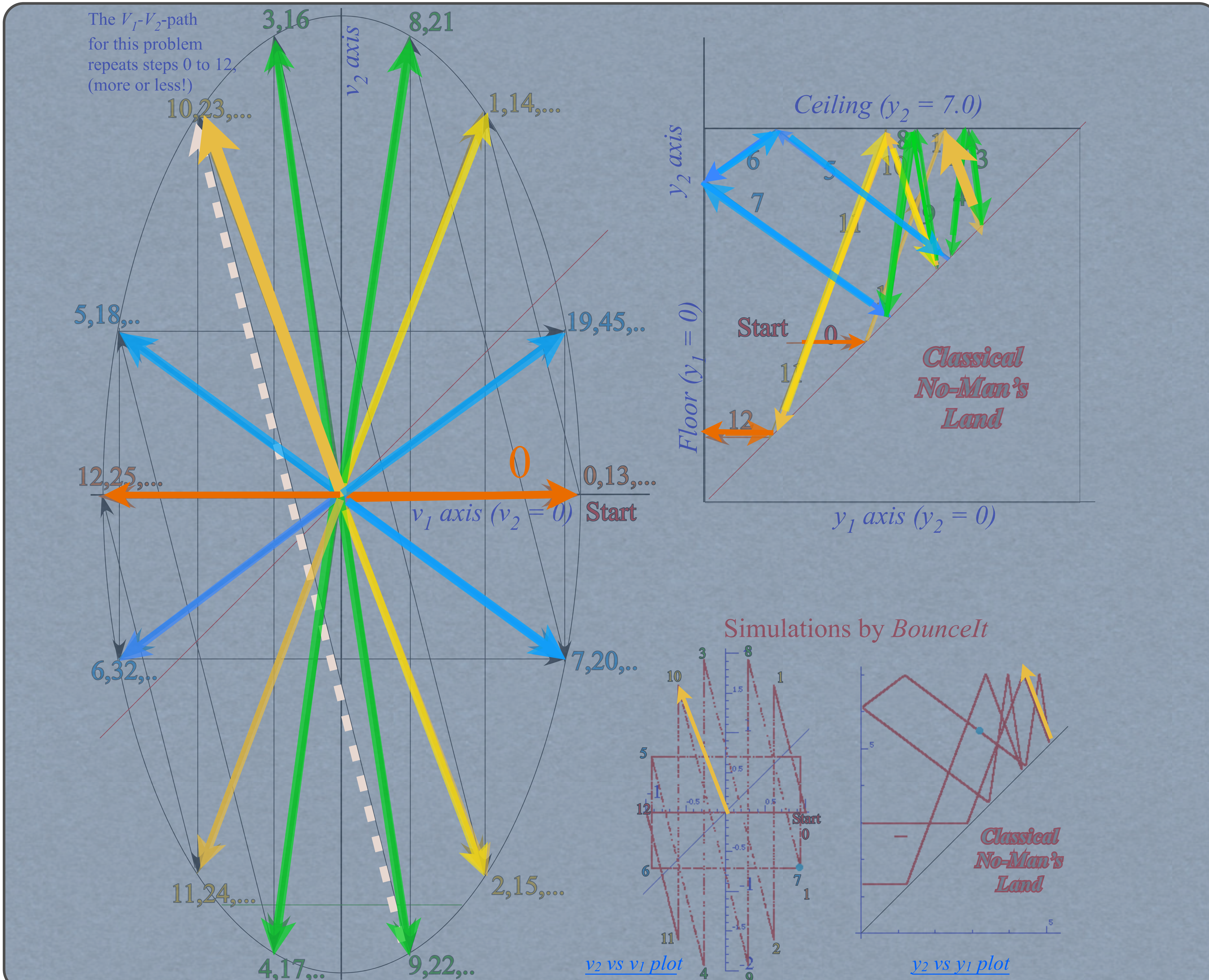
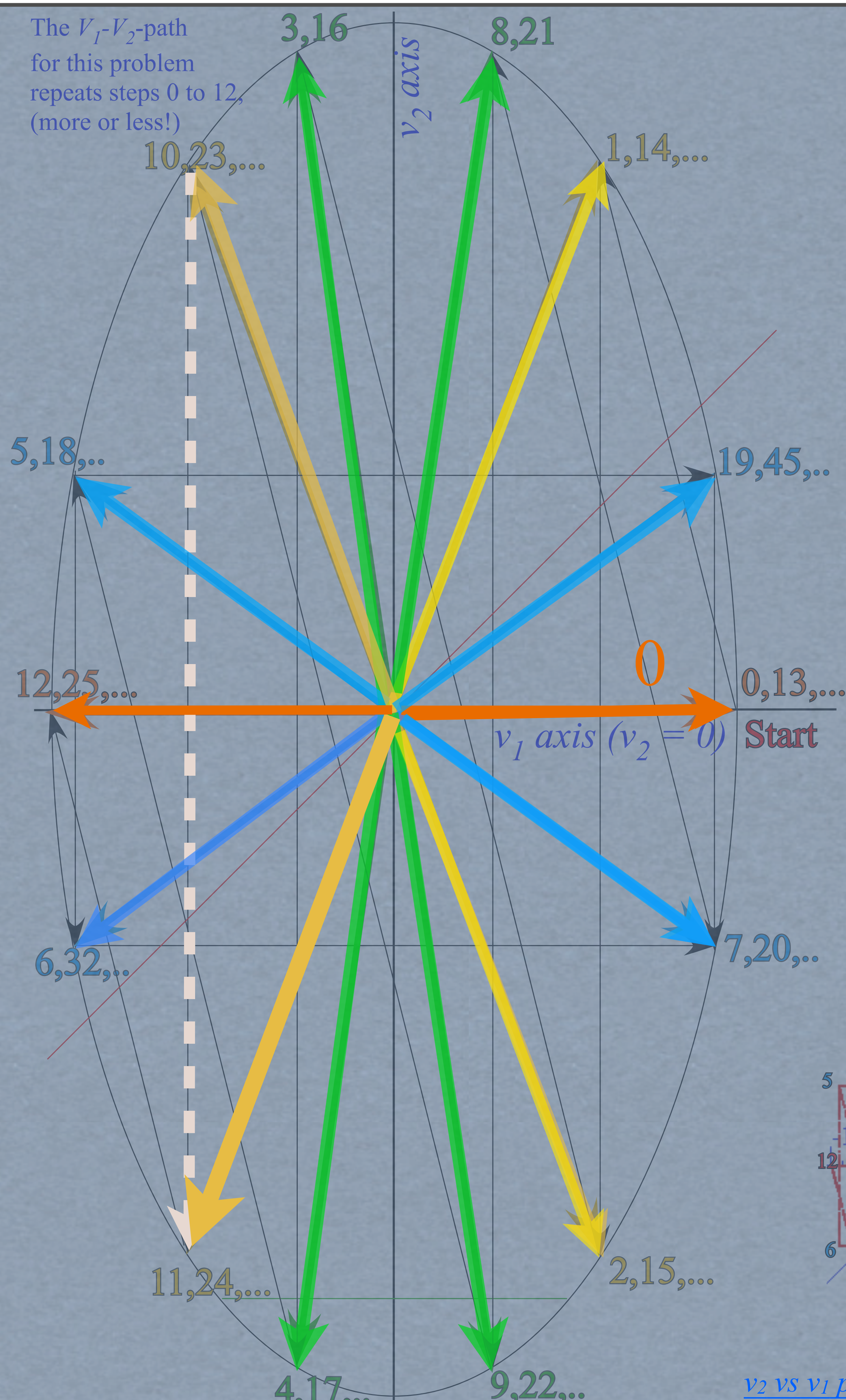
No-Man's
Land
Simulations by BounceIt
v2 vs v1 plot
y2 vs y1 plot
Classical
No-Man's
Land

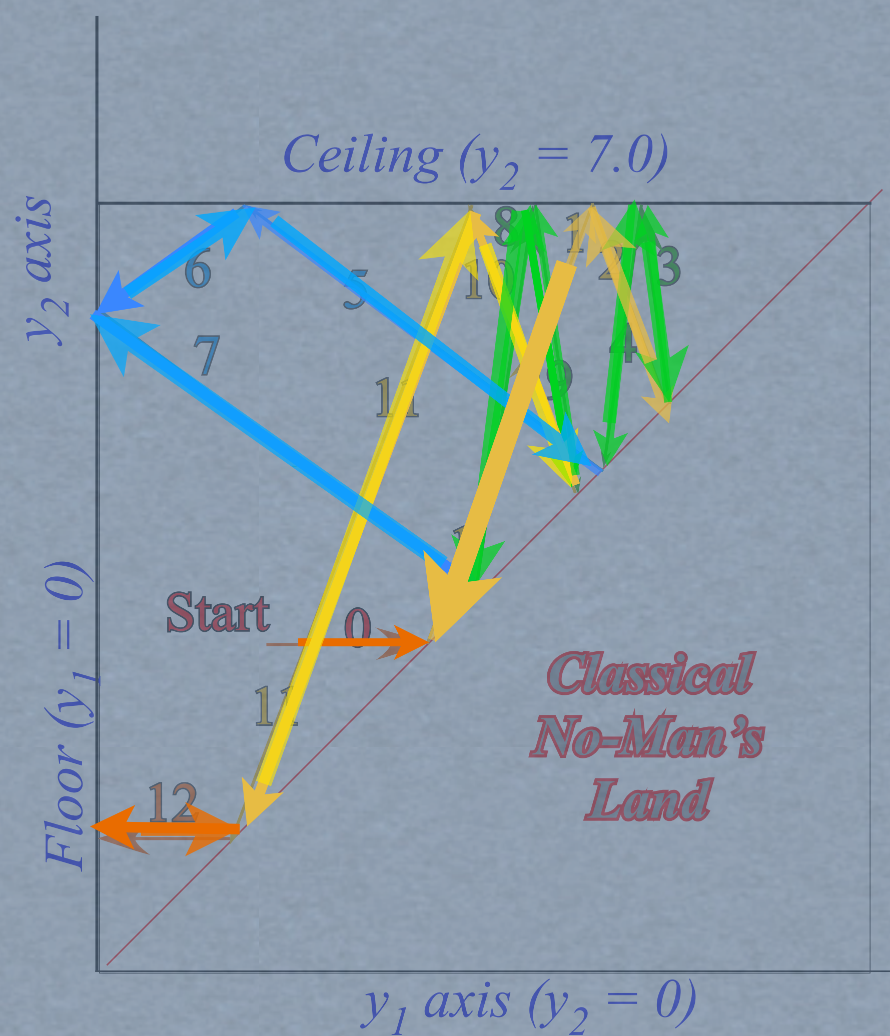

The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
v_2 axis
3,16
8,21
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
y_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Start
Floor (y_1 = 0)
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

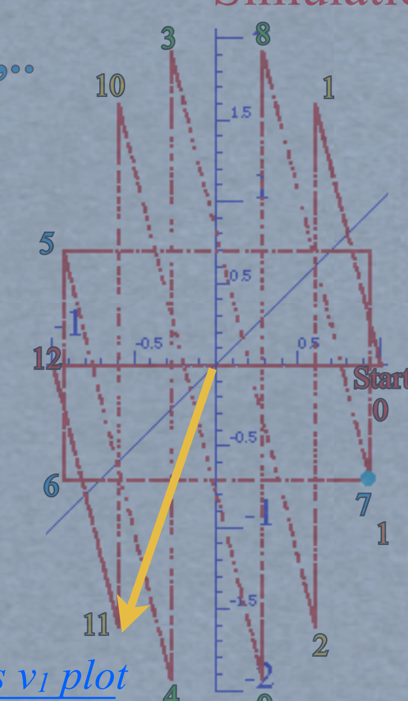


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
y_1 axis (y_2 = 0)
Classical No-Man's Land
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot

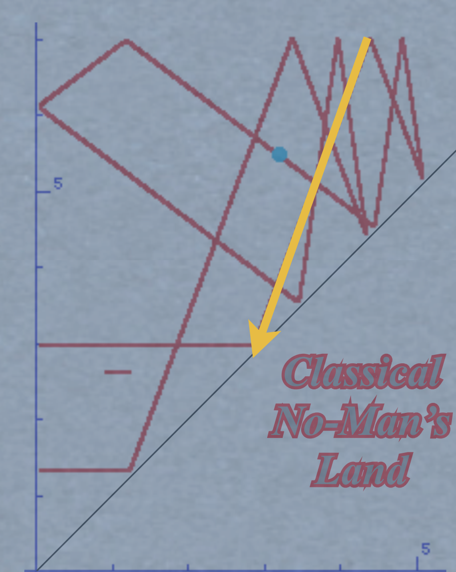




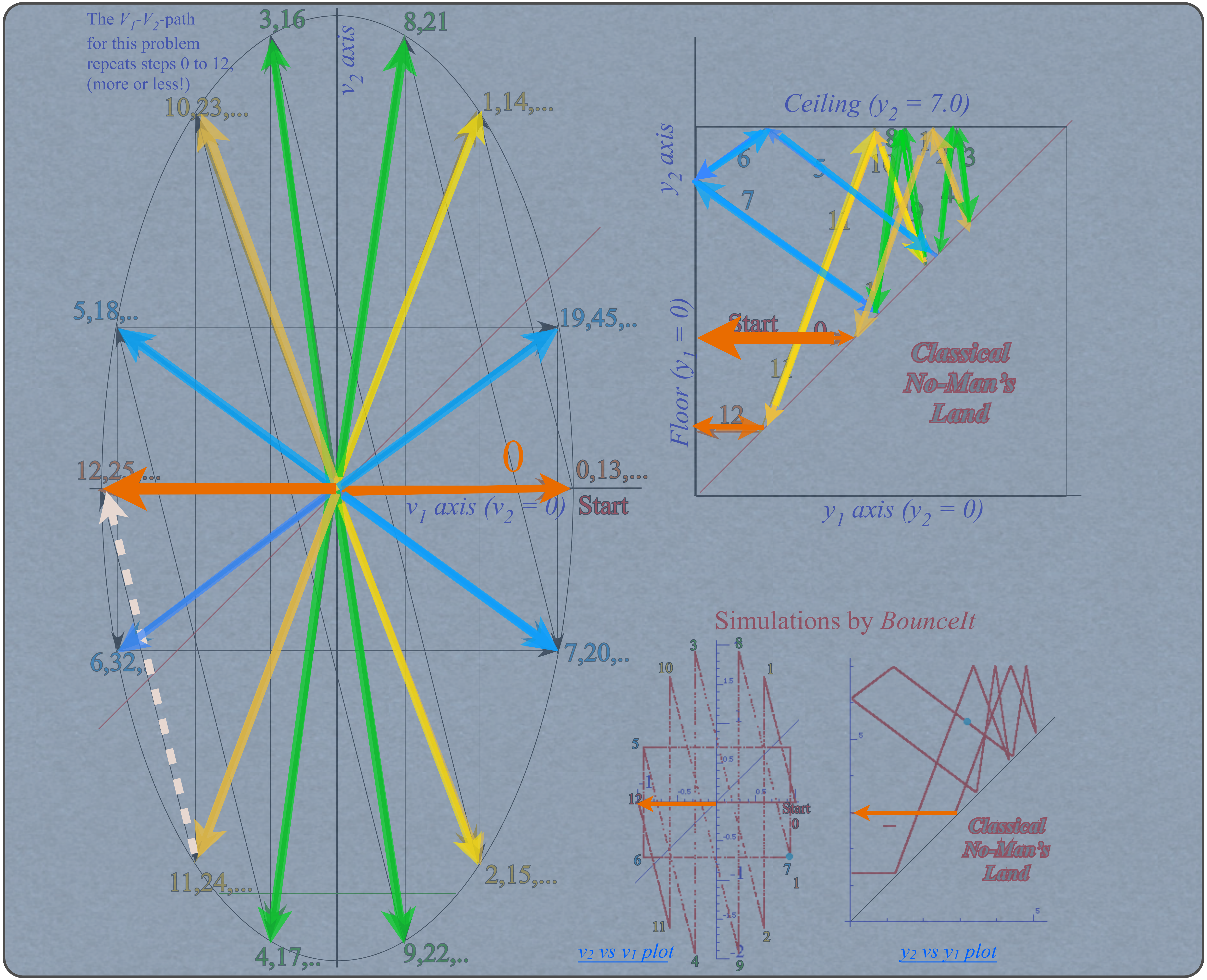
Simulations by *BounceIt*

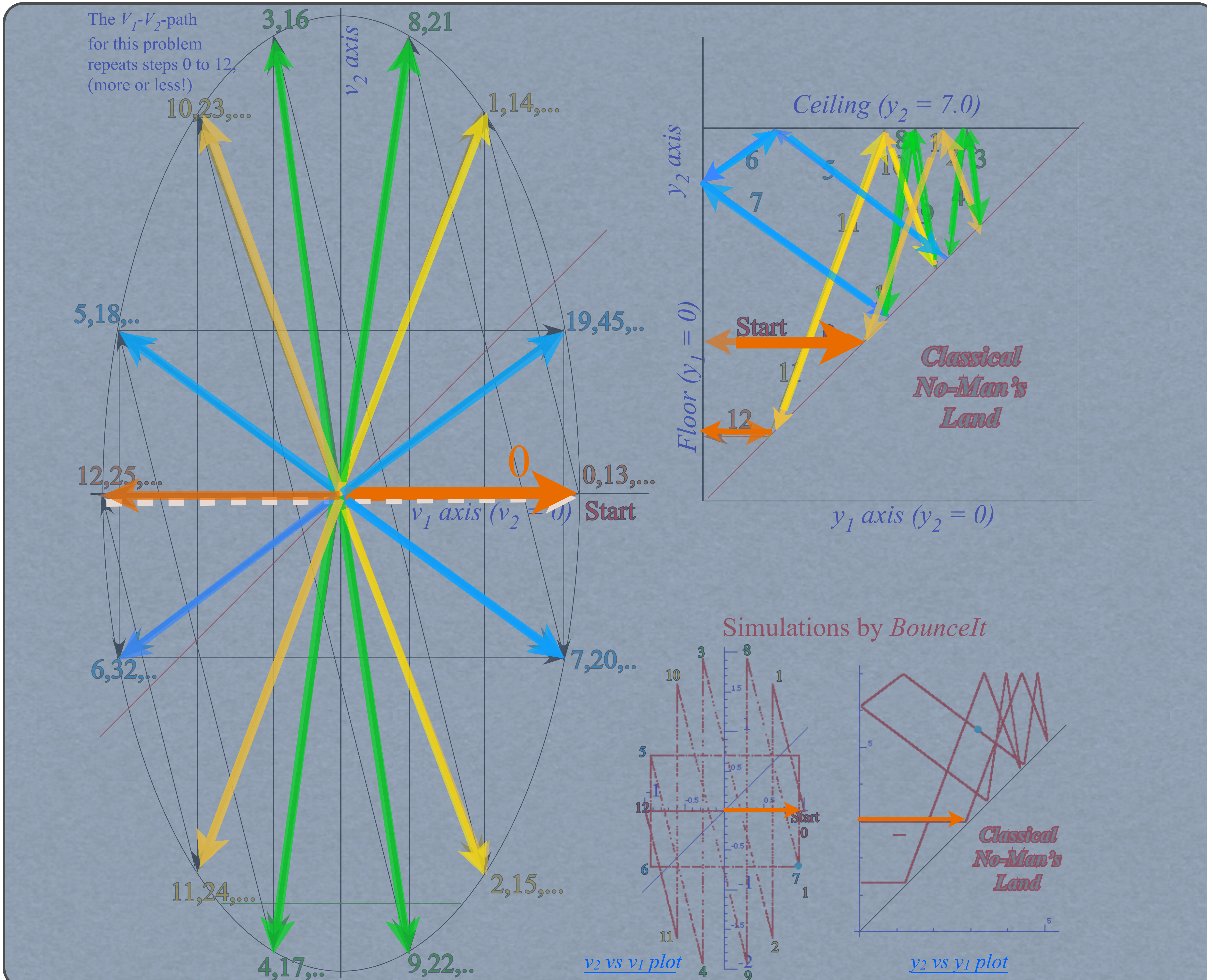


v_2 vs v_1 plot

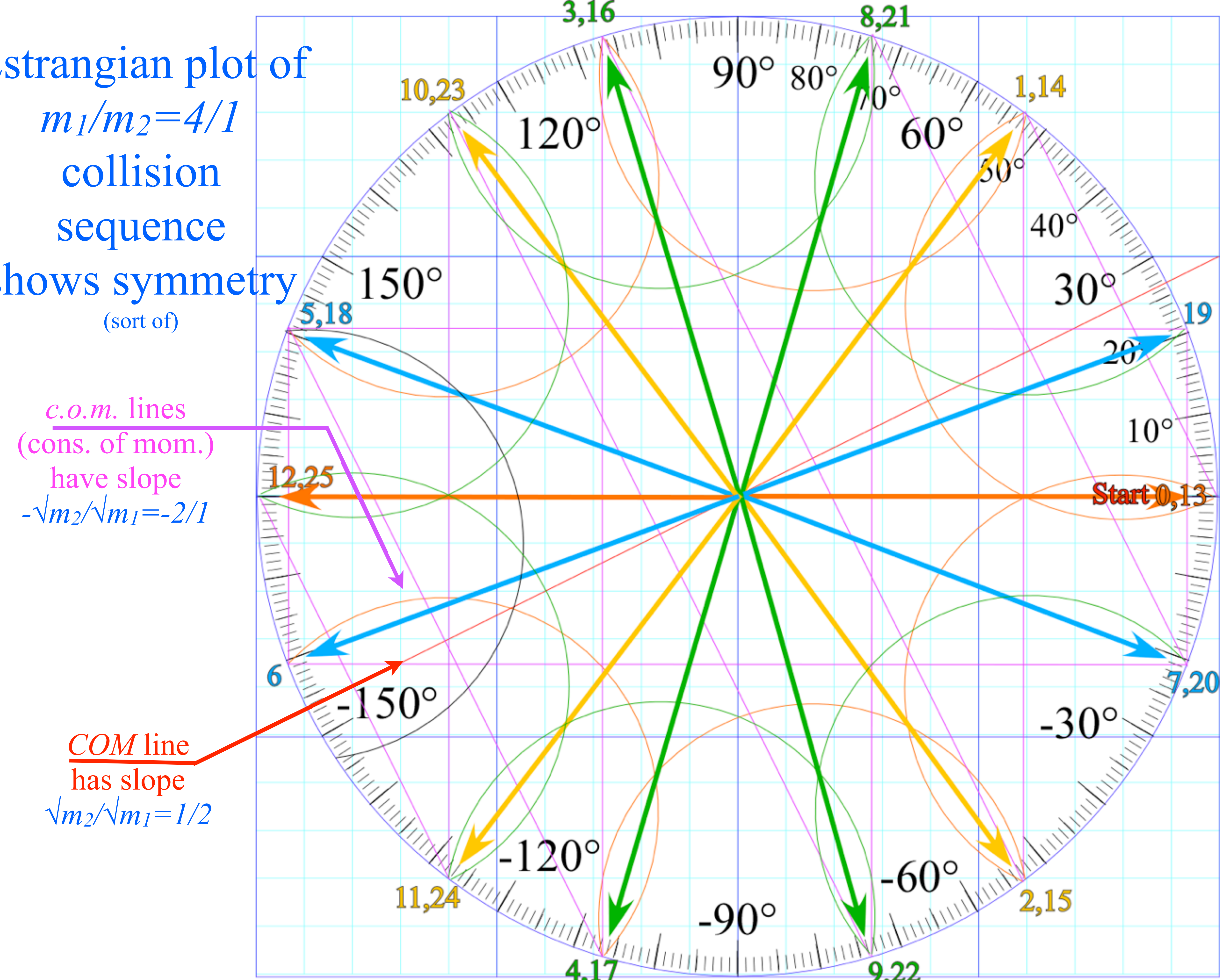


y_2 vs y_1 plot


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

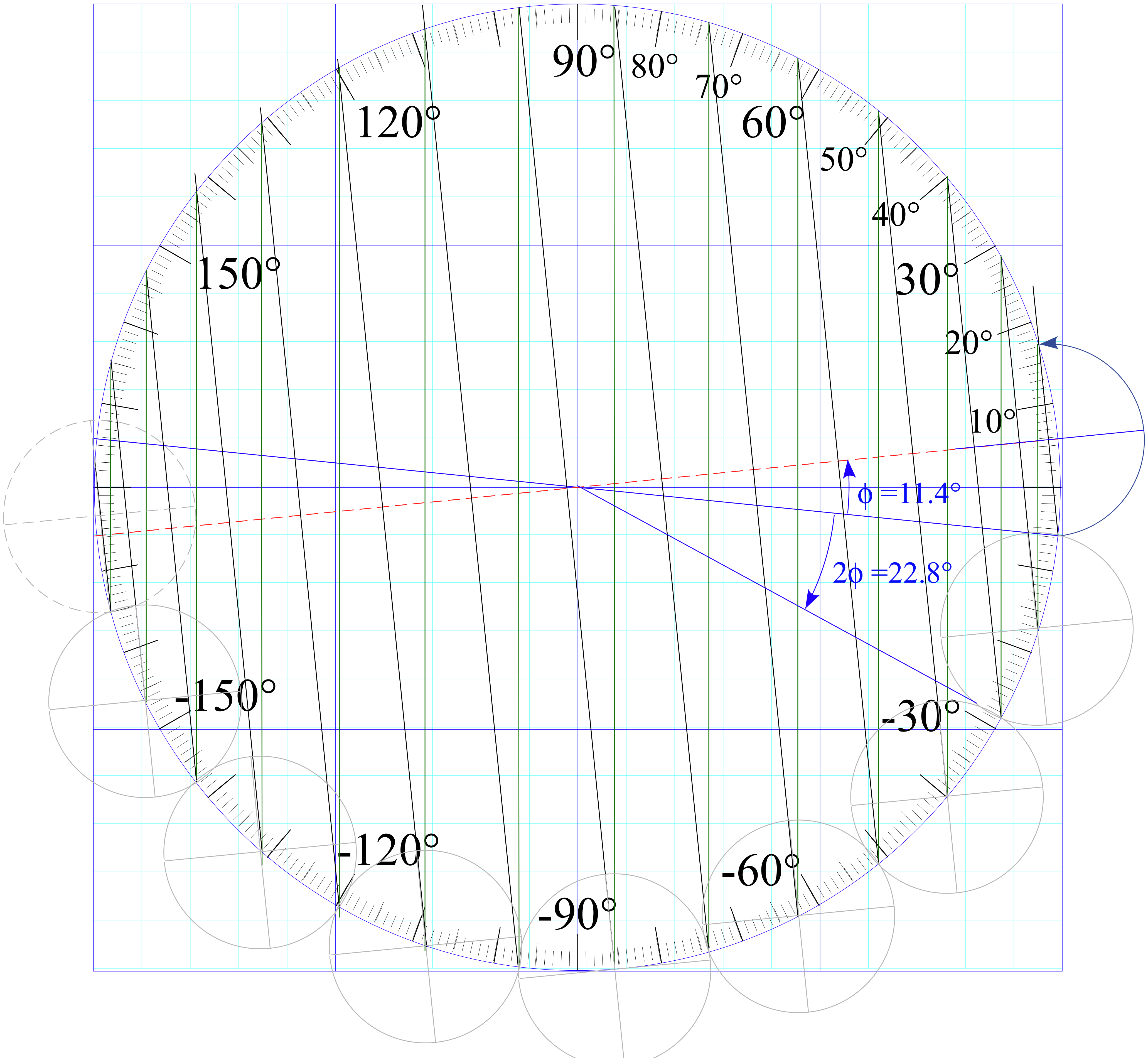

The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
Start
v_1 axis (v_2 = 0)
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
12
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
Start
Classical
No-Man's
Land
v_2 vs v_1 plot
y_2 vs y_1 plot

Estrangian plot of $m_1/m_2=4/1$ collision sequence shows symmetry
(sort of)

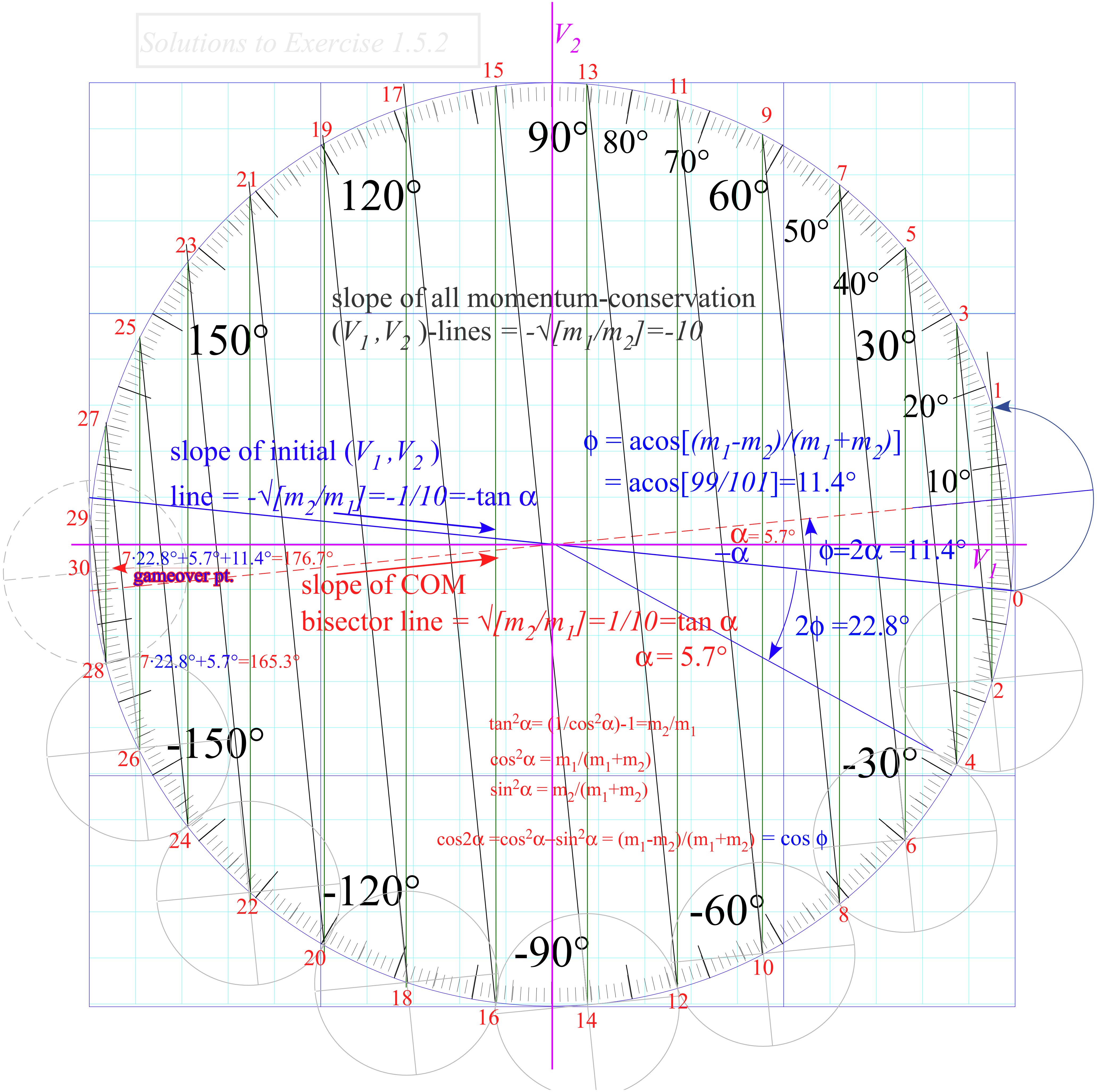


$$\phi = \text{acos}[(m_1 - m_2)/(m_1 + m_2)] = \text{acos}[99/101] = 11.4°$$

Collisions for mass ratio $m_1 : m_2 = 100:1$



Solutions to Exercise 1.5.2

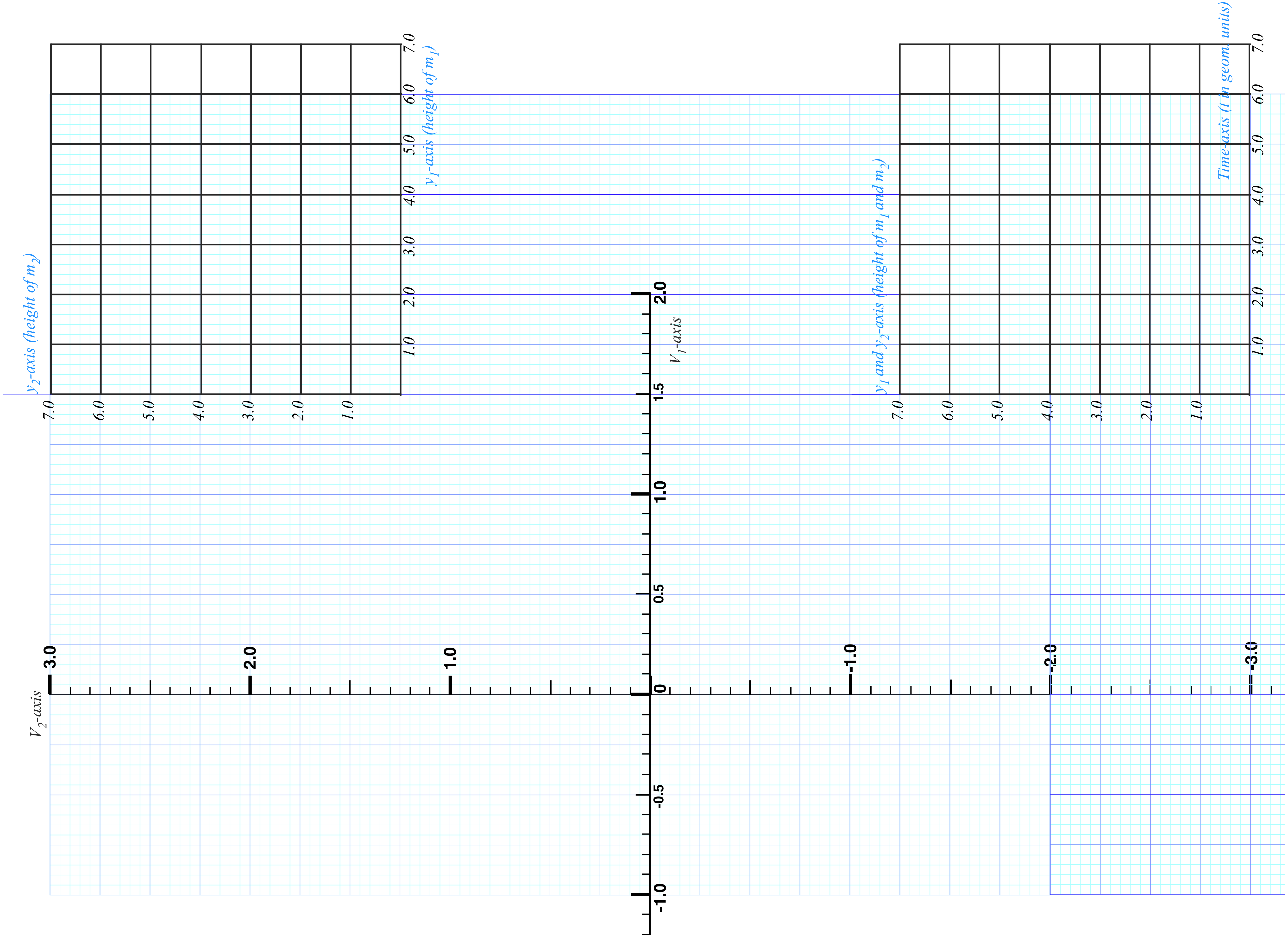


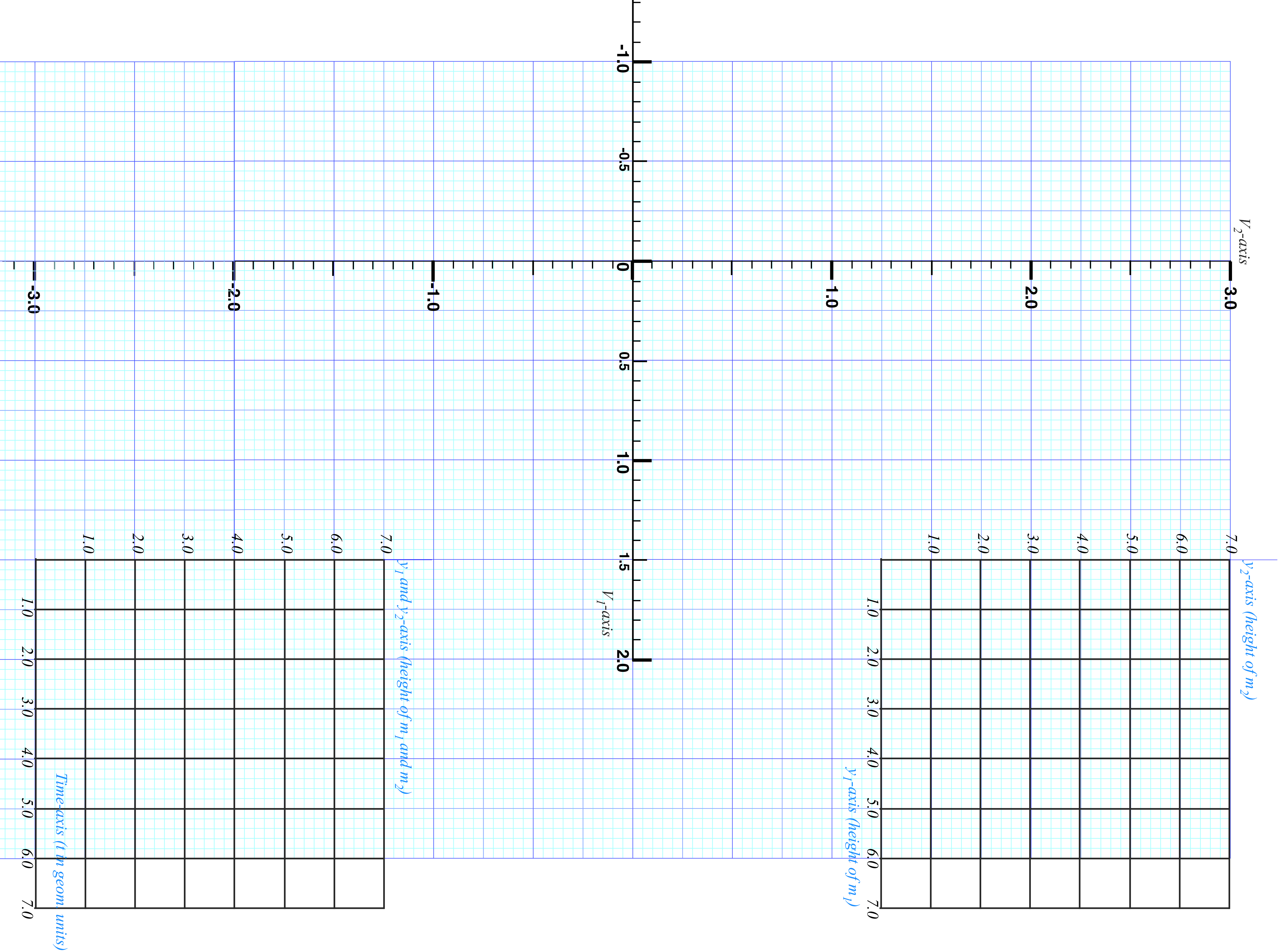
Collisions for mass ratio $m_1:m_2= 100:1$

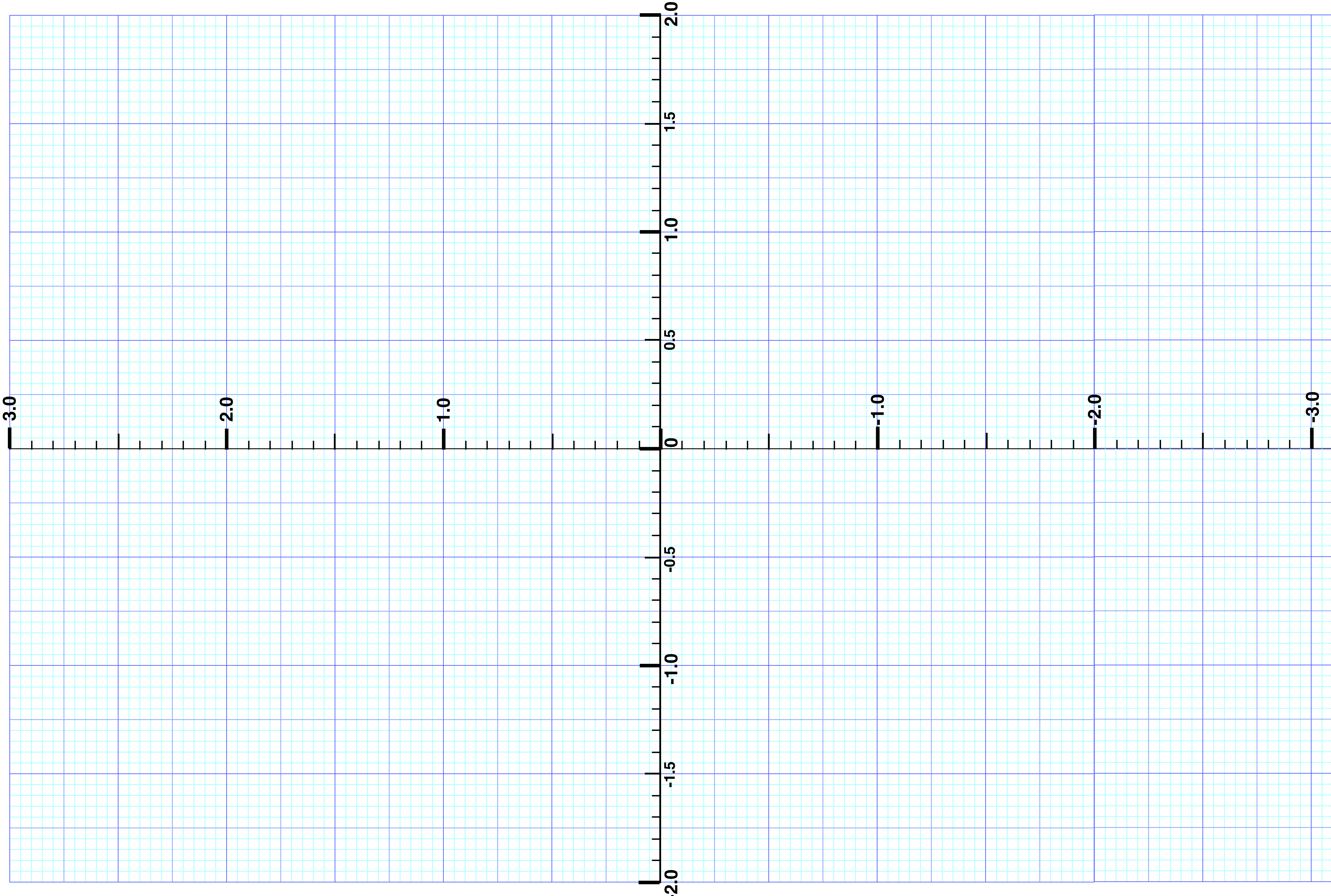
BounceIt Web Simulations
$m_1:m_2 = 100:1$ $(v_1, v_2)=(1, 0)$

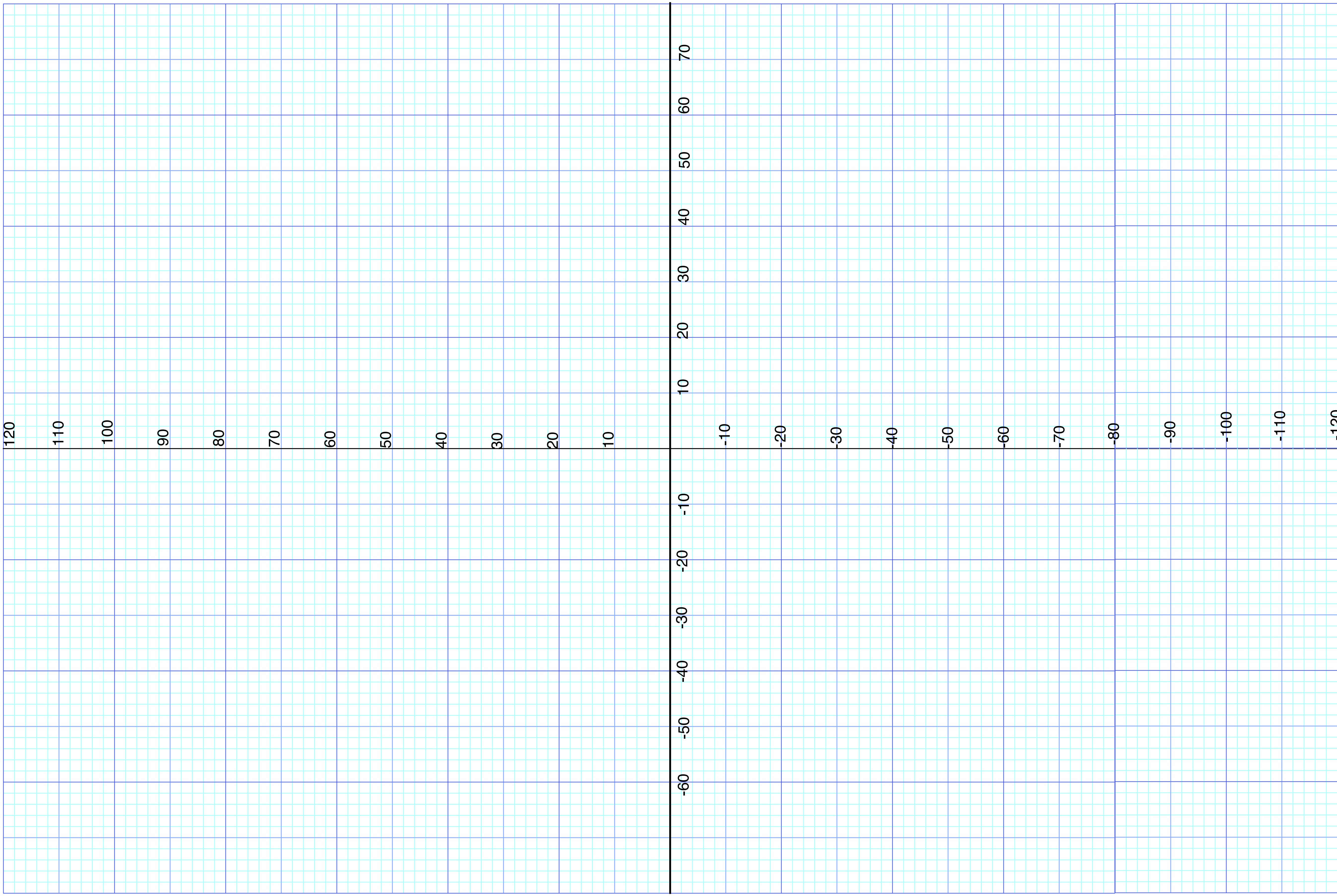
V_2 vs V_1 Estrangian plot

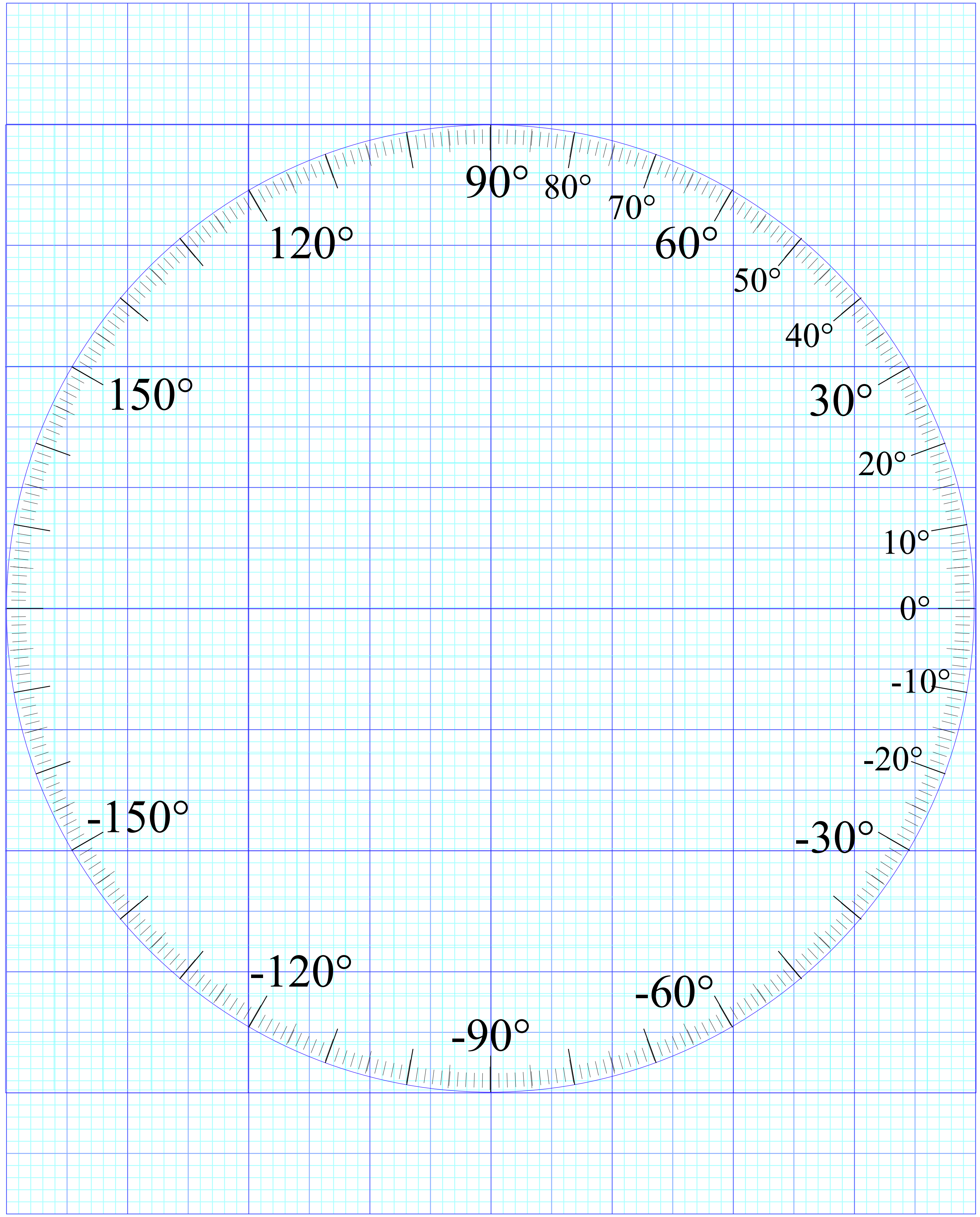
Supplementary: y_2 vs y_1 plot

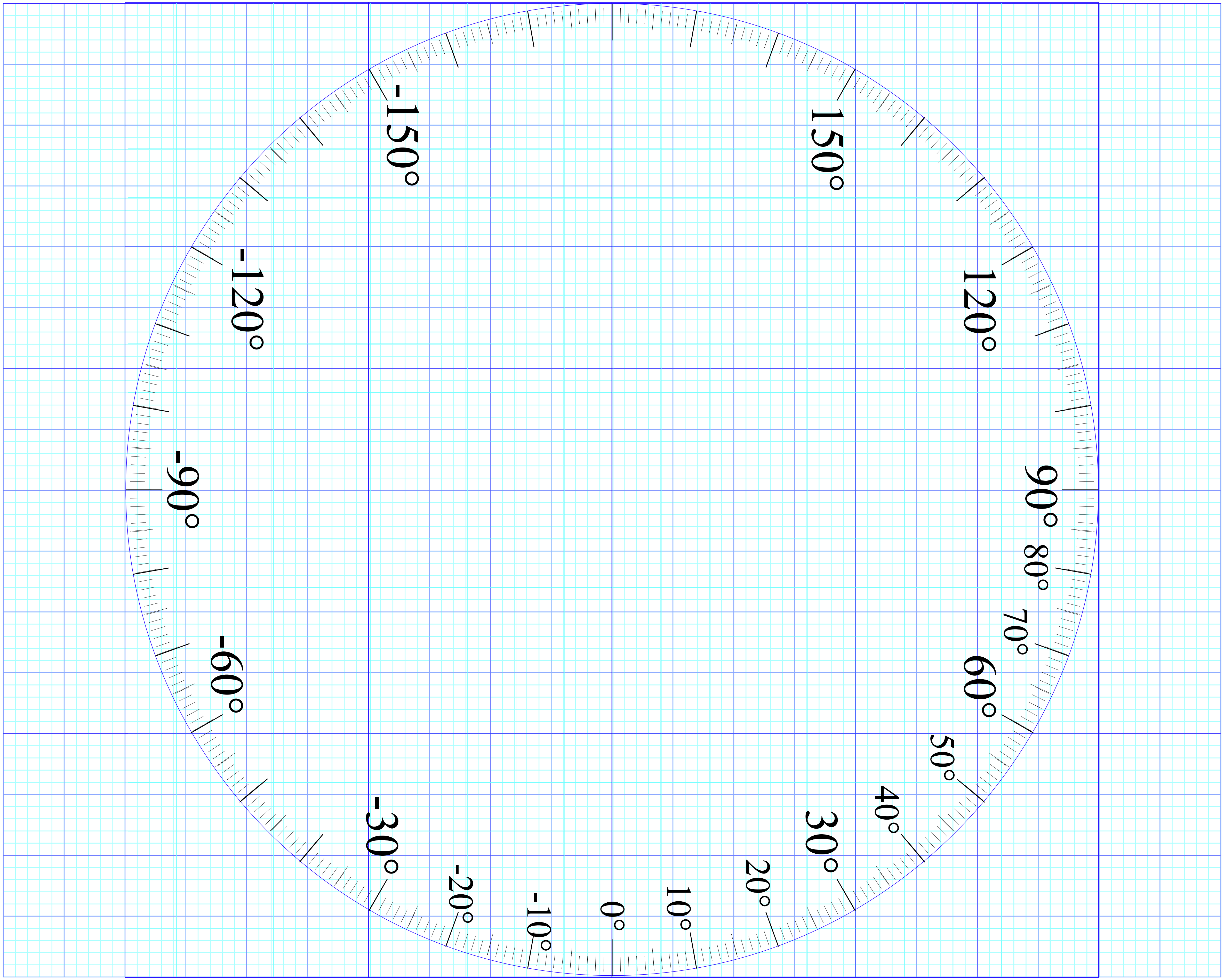

y₁-axis (height of m₁)
y₂-axis (height of m₂)
y₁ and y₂-axis (height of m₁ and m₂)
Time-axis (t in geom. units)
V₁-axis
V₂-axis
1.0 2.0 3.0 4.0 5.0 6.0 7.0
-1.0 -0.5 0 0.5 1.0 1.5 2.0
3.0 2.0 1.0 0 -1.0 -2.0 -3.0


v_1-axis
v_2-axis
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)
y_2-axis (height of m_2)
y_1-axis (height of m_1)


2.0
1.5
1.0
0.5
3.0
2.0
1.0
0
-1.0
-2.0
-3.0
-0.5
-1.0
-1.5
-2.0


120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60


90°
80°
70°
120°
60°
50°
40°
150°
30°
20°
10°
0°
-10°
-20°
-150°
-30°
-120°
-60°
-90°


0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
120°
150°
-10°
-20°
-30°
-60°
-90°
-120°
-150°