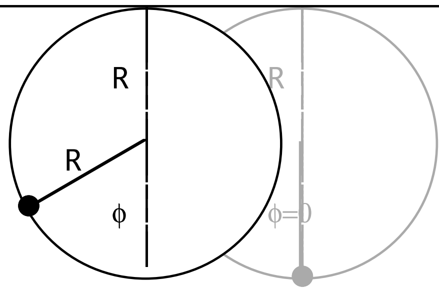
Assignment 8 - **Classical Mechanics 5103** 10/20/16 *Due Thur. Oct. 27*

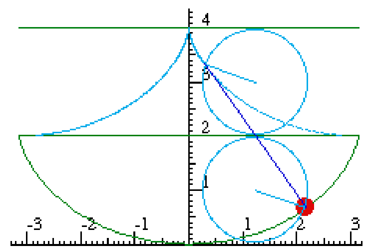
Main Reading: In new text (*Classical Mechanics with a BANG!*) Unit 2 thru 2.8 and Unit 3 thru 3.8.

Christoffel not Christawful

3.7.1 Derivatives of spherical coordinate metric (prob. 3.6.1) give Christoffel coefficients $\Gamma_{ij;k}$ (1st kind) and $\Gamma_{ij;}^{k}$ (2nd kind) by eq. (3.6.10). It is easier to use Lagrange derivative equations (2.4.1) or covariant force Lagrange equations (3.5.10) as in Sec. 3.7 to derive both kinds of Christoffel Γ coefficients. Do this and discuss what "ficticious" forces or accelerations the spherical coordinate $\Gamma_{ij;k}$ or $\Gamma_{ij;}^{k}$ give. (You might redo cylindrical $\Gamma_{ij;k}$ without looking at 3.7.)

Huygen's problem. For 40 years Christian Huygens worked to improve harmonicity of pendulums and only solved the problem you are about to do, just before he died. Let's hope it doesn't take <u>*you*</u> *as long!* *(cackle! cackle!)*





A scary roller coaster

3.8.1. A mass *m* is constrained to slide friction-free along a cycloid of radius *R* in gravity *g*. Or else, a "tin can" or hoop-wheel is held up by magnetic force to roll with no slip on ceiling as sketched above.

(o) First construct *24pt* normal cycloids geometrically using ruler-compass on graph paper (or other means). (As shown in class, divide a 6´´*x-axis* into 24 intervals and "roll" circles of radii 3/π´´ by *nπ/12* above and below and π out of phase. Check geometry with algebra for some key points.))

(a) Write parametric equations $x(\phi)$, $y(\phi)$ for a circumference point mass on a wheel rolling on the ceiling where ϕ is the wheel-center-relative angle the mass has rotated from its lowest point at lab origin $(x=0,y=0)$. (Imagine a magnetized steel can or hoop hanging from a horizontal steel beam with a lead weight of mass *M* fixed to its rim. Let the hoop roll without slipping or rolling friction and have negligible mass $m<<M$.)

(b) Derive Lagrangian $L(d\phi/dt,\phi)$ and find canonical momentum p_ϕ and equation of motion.

(c) Derive total energy and Hamiltonian function *H*. Are any of these *(L, p_ϕ, E, or H) ever* constants of motion?

(d) Derive an expression for the arc length $s(\phi)$ that *m* travels to ϕ from the lowest point $\phi=0$. How long is a string wrapped around a cycloid from $\phi=0$ to $\phi=\pi$.

(e) Show the period of oscillation of this mass kicked from $\phi=0$ is independent of initial velocity for velocity less than_______? (What is that max $v_{\max}=\dot{x}_{\max}$? … and max $\omega_{\max}=\dot{\phi}_{\max}$?)

(f) Derive $\phi(t)$ for free oscillation. Can the hoop roll across the ceiling with constant speed $v_{const}=\dot{x}_{const}$ for some initial conditions ? Discuss. If the hoop rolls on the floor does such a uniform velocity state exist?

An even scarier roller coaster

3.8.2 A hypo-cycloid is traced by a point on a unit ($r=1$)-circle rolling without slipping inside a larger ($R=3$) circle.

(a) Derive the $(x(\phi),y(\phi))$ coordinates of this point as a function of the polar angle ϕ of contact point P of unit circle-*r* inside *R*-circle. Let ϕ be relative to fixed vertical *y*-axis of stationary *R*-circle. Sketch or plot this cycloid.

(b) Let the $(x(\phi),y(\phi))$-cycloid be a friction-free subway tunnel is a Sophomore-Physics-Earth of radius $R_\oplus$. Derive the Hamiltonian and equations of motion for a car of mass *m* in a $r=R_\oplus/3$ tunnel in terms of polar coordinates (ρ,ϕ).

(c) If the car starts at radius $\rho=R$ at rest near *y*-axis ($\phi=0$), what is its time of travel to the other end of the tunnel? Compare time-of-orbit $\tau_{(cycloid)}$ or frequency $\upsilon_{(cycloid)}$ to the $\tau_\oplus$ or $\upsilon_\oplus$ of a straight subway with the same terminals.

(d) Do the same comparison with the optimal-bend V-shape subway in previous problem.