Assignment 7 Oct 13 Due Tuesday Oct 20: Based on Unit 2 Chapter 1-3 and Unit 3 Chapter 1-3.

Well-known Coordinates (OCC)

3.6.1 Find Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric tensors g_{mn} and g^{mn} for OCC (a) and (b). Do (b) and reduce it to (a).

(a) Cylindrical coordinates $\{q^1=\rho, q^2=\phi, q^3=z\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$, $z=x^3$.

(b) Spherical coordinates: $\{q^1=r, q^2=\theta, q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\cos\phi$, $z=x^3=r\cos\theta$.

"Sliding" Parabolic Coordinates (GCC)

3.6.2 Consider the Cartesian(GCC) definition: $x = 0.4\,(q^1)^2 - q^2$, $y = q^1 - 0.4\,(q^2)^2$

(a) Does an analytic GCC coordinate definition $q^m = q^m(x^j)$ exist? If so find it.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the <u>appropriate</u> graph on attached pages sketch the unitary vectors near point $(x=1, y=1)$ (Arrow) and near point $(x=1, y=0)$. Where, if anywhere, is the grid an <u>O</u>CC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there singularities of this GCC.

"Plopped" Parabolic Coordinates (GCC)

3.6.3 Consider the GCC(Cartesian) definition: $q^1 = (x)^2 + y$ $q^2 = (y)^2 - x$

(a) Does an analytic Cartesian coordinate definition $x^j = x^j(q^m)$ exist? If so find it.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

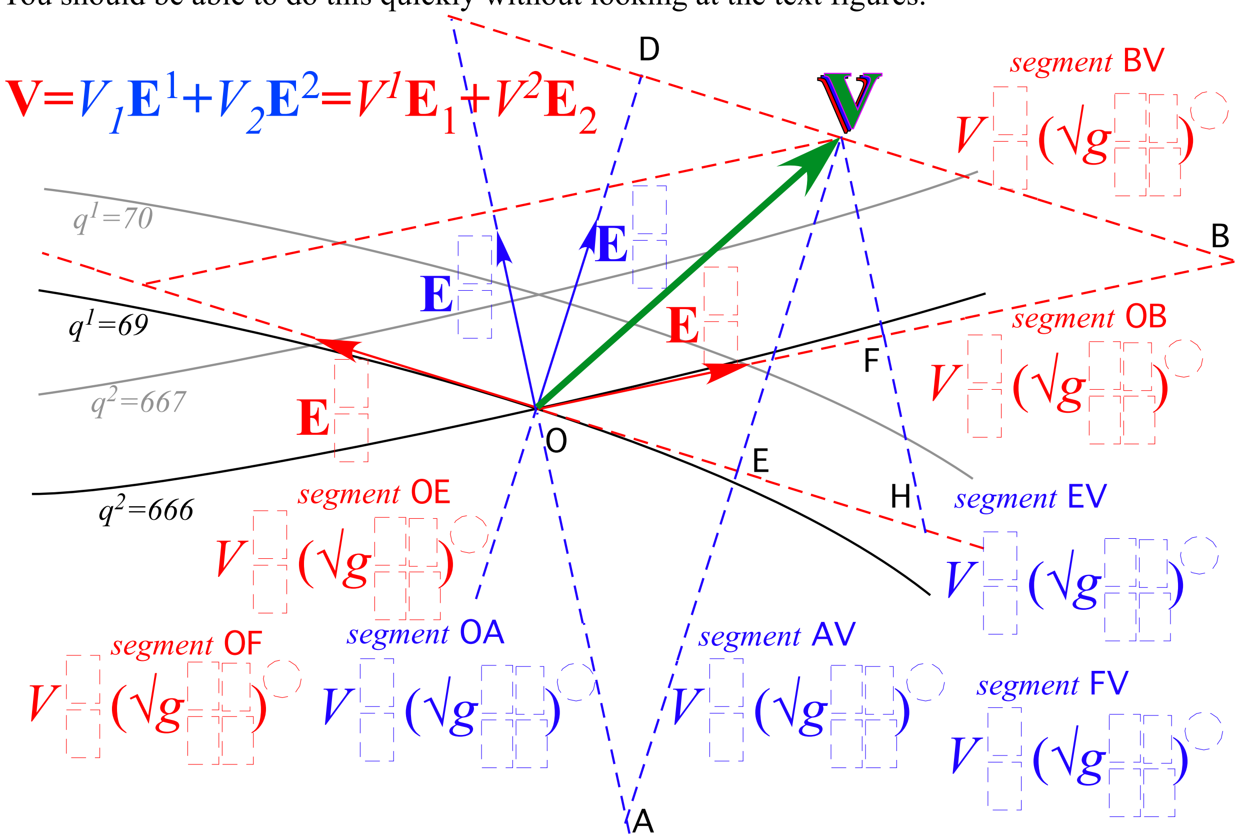
(c) On the <u>appropriate</u> graph on attached pages sketch the unitary vectors at the point $(x=1, y=1)$ (Arrow) and at the point $(x=1, y=0)$. Where, if anywhere, is the grid an <u>O</u>CC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there singularities of this GCC.

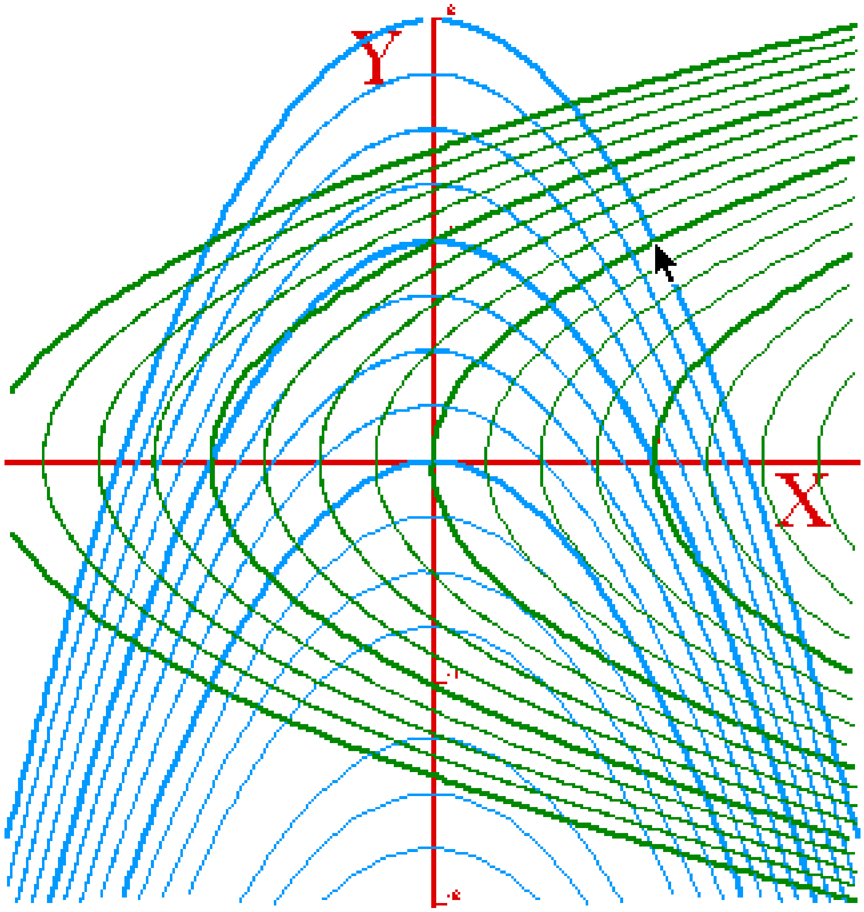
3.3.3 *Covariant vs Contravariant Geometry.*

GCC components of a vector **V** in the figure below are realized by line segments OA, BV, etc. Give each segment length by single terms of the form V_m or V^m times $(\sqrt{g_{mm}})^{+1}$, $(\sqrt{g_{mm}})^{-1}$, $(\sqrt{g^{mm}})^{+1}$, or $(\sqrt{g^{mm}})^{-1}$ with the correct $m=1$ or 2. Also label each unitary vector as $\mathbf{E}_1$, $\mathbf{E}^1$, $\mathbf{E}_2$, or $\mathbf{E}^2$, whichever it is.

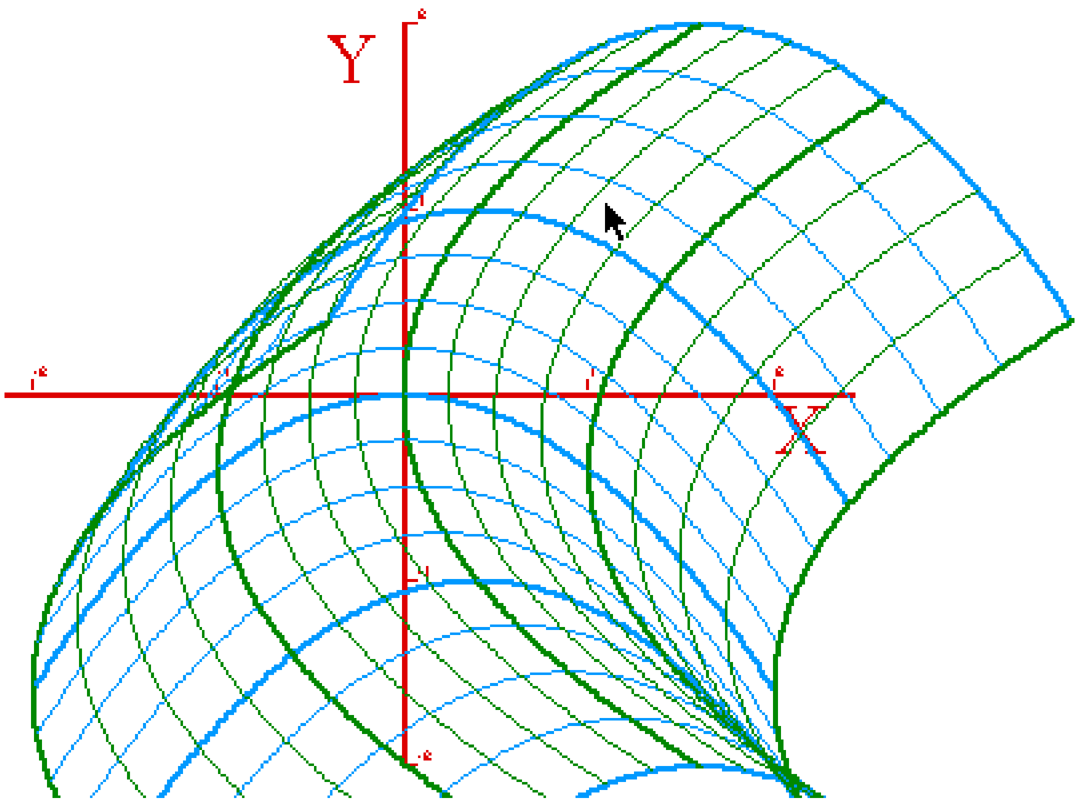
You should be able to do this quickly without looking at the text figures.



"Plopped" and"Sliding" Parabolic Coordinates are 2D (xy) plots (despite 3D appearance of latter.)



"Plopped" Parabolic Coordinates



"Sliding" Parabolic Coordinates