*Assignment 11* - **Classical Mechanics 5103** 11/10/16: Unit 4 thru Ch. 4.2.

*Due Thur. Nov. 17*

### *Exercise 4.2.4 DC G-response of FDHO*

DC response $G_{\omega_0}(0)$ is the amplitude caused by a unit acceleration (a=1) at zero stimulus frequency. This may be confusing since there is no acceleration at zero frequency. A clearer definition of $G_{\omega_0}(0)$ is response to a static force of magnitude $F = ma = m$ acting on mass $m$ . Show this $G_{\omega_0}(0)$ is consistent with the spring force equation (4.2.2) and Hooke's Law.

### *Exercise 4.2.5 Resonant G-response of FDHO*

Resonant response $G_{\omega_0}(\omega_0)$ is the amplitude caused by a unit acceleration amplitude $\left(\left|a(t)\right| = 1\right)$ or force amplitude $\left(\left|F(t)\right| = m\right)$ at resonance$\left(\omega_s = \omega_0\right)$. Compute $G_{\omega_0}(\omega_0)$ and show it is consistent with drag force (4.2.3). Work done by this goes where?

### *Exercise 4.2.6 The "standard" Lorentzian (Note: Review complex 2-pole potemtial $\phi(z)=1/z$ (10.42) in Unit 1-Ch. 10 Fig. 10.11.)*

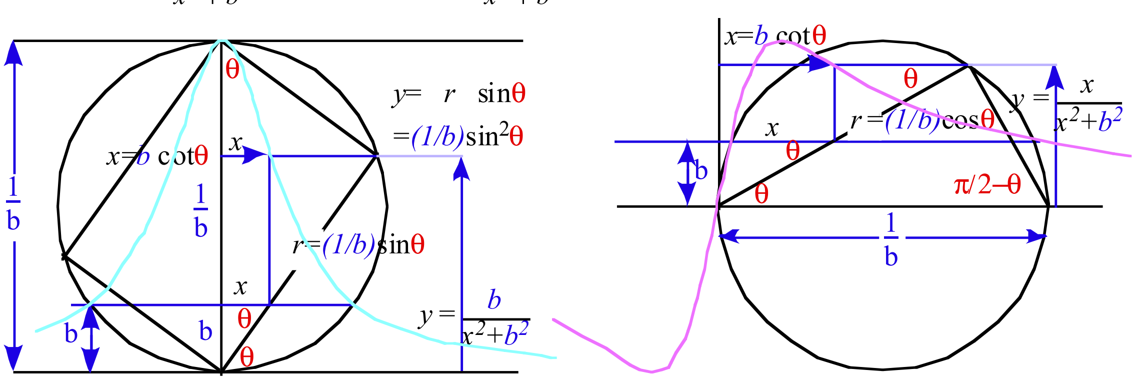
In physics literature, a standard Lorentzian function generally means a form $L(\Delta) = A/(\Delta^2 + A^2)$ with constant $A$. In the Near-Resonant Approximation (NRA is (4.2.18)) the $L(\Delta)$ or its derivative is an approximation to exact $G$-equations (4.2.15).

(a) Use NRA (4.2.18) to reduce (4.2.15a-d) and identify a standard Lorentzian function of the detuning parameter $\Delta=\omega_s-\omega_0$.

(b) Show that NRA for complex response $G$=Re $G$ +iIm $G$ gives circular arcs in the complex $\omega=|\omega|e^{i\theta}=\Delta+i\Gamma$ plane for constant decay rate $\Gamma$ and variable detuning or beat rate $\Delta$. How does this circle deviate from what is almost a circle in Fig. 4.2.6? (Consider higher $\Gamma$ values for which NRA breaks down such as Fig. 4.2.14.) Fixed $\Delta$ and varying $\Gamma$ give what curve?

(c) Verify and do ruler-and-compass constructions of NRA Lorentz-Green functions following the figures below.

$\mathrm{Re}\,G_{\omega_0}(\omega_s) = y = \frac{b}{x^2+b^2}$, $\mathrm{Im}\,G_{\omega_0}(\omega_s) = y = \frac{x}{x^2+b^2}$, and $|G_{\omega_0}(\omega_s)|$ for $b=1/2$ and $b=1/3$. What happens for b>>1?




### *Exercise 4.2.7 Max and min G-values (Part (b-c) involves some algebra!)*

Derive equations for the extreme values for the response function or function related to G as asked below.

For part (a) *only* use *Near-Resonant Approximation* (*NRA*): See preceding Ex. 4.2.6.

($a_1$) Find values which give maxima for: $\mathrm{Re}\,G_{\omega_0}(\omega_s)$, $\mathrm{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming $\omega_0$ is constant and $\omega_s$ varies.

($a_2$) Find values which give maxima for: $\mathrm{Re}\,G_{\omega_0}(\omega_s)$, $\mathrm{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming $\omega_s$ is constant and $\omega_0$ varies.

(b) Do (a) for *exact* $G_{\omega_0}(\omega_s)$ . Exact plots by calculator help check these answers.

(c) Find exact value to maximize peak KE of responding oscillator.(1st show total $KE=\frac{1}{2}m\omega^2A^2$ for oscillation of amplitude $A$.)

*Exercise 4.2.8 Lifetimes*

*Compare the number of heartbeats in your lifetime (assuming you live to a ripe old age of 100 years) to the number of oscillations* in atomic and molecular lifetimes given below. (First, estimate your own angular quality factor $q$.)

Typical atomic energy decay time is $t_{5\%} = 3/\Gamma = 3.4 \times 10^{-8}\, s$ for a green spectral 600Thz line. Compute atomic $q$ and $Q$.

Optional *Exercise 4.2.9 Initializing*

Derive the initial transient components $\mathrm{Re}\, A$ *and* $\mathrm{Im}\, A$ in terms of initial values $x_0 = x(0)$, $v_0 = \dot{x}(0)$ of stimulated FDHO, response magnitude $R = |G_{\omega_0}(\omega_s)|$, initial stimulus $a(0) = |a|e^{i\alpha}$, $\Gamma$, $\omega_\Gamma$, and $\rho$.

(Check that your total solution (4.2.25) does satisfy the initial conditions.)

*Exercise 4.2.10 Wiggling Old-Main lamp posts (Don't do a lot of algebra for this one!)*

Let a static force of *10N* on a lamp post cause it to bend *1cm*. Upon release it vibrates at $\omega_\Gamma$ for a minute before its amplitude dies to less than 5%. Estimate $\omega_0$, $\Gamma$, and $q$ and how much it bends 1 minute after a 1 Hz oscillating force of ±1N starts. What is the bending after 2 minutes? Do this quickly by reasoning using $q$-factor properties and 5% mnemonics. (Points-off for too much algebra!)

*Exercise 4.2.11 Timing is everything! (A formula to remember)*

(a) Let oscillating force $F(t)=F_s\cos\omega t$ act on a mass whose response $x(t)=G\cos(\omega t-\rho)$ also is frequency $\omega$ but with a amplitude $G$ and a phase lag of $\rho$. Derive a formula for the work loop integral $W=\oint F\,dx$ for exactly one period of oscillation. Discuss how result relates to work done against friction in a FDHO. (Recall Ex. 4.2.5.)

(b) Let oscillations $x_1$ and $x_2$ have amplitude $A$ but $x_1$ lags $x_2$ by phase $\rho$. Show by geometry (below) an $x_1$ vs. $x_2$ path is an ellipse of major axis $a=A\sqrt{2}\cos(\rho/2)$ and $b=A\sqrt{2}\sin(\rho/2)$ and area $W$=____. Compare this $W$ to loop work derived in part (a). Construct example for $\rho=60°$.