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16 coefficients of relativistic 2CW interference
Two “famous-name” coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
Relawavity and a novel introduction to relativistic mechanics II.

(Ch. 6-8 of Unit 8  12.10.15)

Rapidity $\rho$ related to stellar aberration angle $\sigma$ and Epstein’s approach to relativity

- Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
- Applications to optical waveguide, spherical waves, accelerator radiation
- Derivation of relativistic quantum mechanics
- What’s the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations

Relativistic optical transitions and Compton recoil formulae

- Feynman diagram geometry
- Compton recoil related to rocket velocity formula
- Relation of 2nd quantization amplitude “photon” $N$ and 1st quantization wavenumber

Relawavity in accelerated frames
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Two “famous-name” coefficients and the Lorentz-Thales geometry of Lorentz transformation

For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge:

A *Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers*
right-moving wave

Spacetime \((x, ct)\)

Per-Spacetime \((ck, \omega)\)

left-moving wave

Colliding 2CW laser beams

BohrIt Web Simulation

2 CW \(ct\) vs \(x\) Plot

\(ck = \pm 2\)
**Per-Spacetime**

**right-moving wave**

Spacetime \((x, ct)\)

\[
\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}
\]

**left-moving wave**

Spacetime \((x, ct)\)

\[
\psi_L = e^{iL} = e^{i(k_L x + \omega_L t)}
\]

**Wave-sum**

\[
\psi_R + \psi_L = e^{iR} + e^{iL} = e^{i(k_R x - \omega_R t)} + e^{i(k_L x + \omega_L t)}
\]

factored:

\[
= e^{i(k_R x - \omega_R t)} (e^{2i\omega_L t} + e^{-2i\omega_L t})
= e^{i(k_R x - \omega_R t)} \psi_{Phase} \cdot \psi_{Group}
\]

**Phase**

\[
\psi_{Phase} = \frac{\psi_R + \psi_L}{\psi_{Group}}
\]

**Group**

\[
\psi_{Group} = e^{i(k_{R+L} x - \omega_{R+L} t)}
\]

**Re**

\[
\text{Re} \psi_{Phase} = \text{zero}
\]

**Frequency**

\[
\omega = 2\pi \nu
\]

**Frequency values**

- 300THz
- 600THz
- 900THz
- 1200THz
- 1500THz
- 1800THz
- 2100THz
- 2400THz
- 2700THz
- 3000THz
- 3300THz
- 3600THz
- 3900THz
- 4200THz
- 4500THz
- 4800THz
- 5100THz
- 5400THz
- 5700THz
- 6000THz
- 6300THz
- 6600THz
- 6900THz
- 7200THz
- 7500THz
- 7800THz
- 8100THz
- 8400THz
- 8700THz
- 9000THz
- 9300THz
- 9600THz
- 9900THz
- 10200THz
- 10500THz
- 10800THz
- 11100THz
- 11400THz
- 11700THz
- 12000THz

**Right-moving wave**

\((ck_R, \omega_R)\)

\[R = (+2c, 2)\]

**Left-moving wave**

\((ck_L, \omega_L)\)

\[L = (-2c, 2)\]

Bohr-It Web Simulation

2 CW ct vs x Plot

\((ck = \pm 2)\)
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Thales geometry of Lorentz transformation
right-moving Doppler blue shifted wave

$$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$$

Rapidly moving Bob sees...

...Blue shifted wave coming at him and...

left-moving Doppler red shifted wave

$$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$$

...Red shifted wave behind him.

Web Simulation 1 CW ct vs x Plot (ck = +4)

Web Simulation 1 CW ct vs x Plot (ck = -1)

Rapidly moving Bob sees...

...Blue shifted wave coming at him and...

...Red shifted wave behind him.

Rapidly moving Bob sees...

...Blue shifted wave coming at him and...

...Red shifted wave behind him.
right-moving Doppler blue shifted wave

\[ \psi_R = e^{iR} = e^{i(k_R x - \omega_R t)} \]

Rapidly moving Bob sees...
...transformed \((P', G')\) grid

left-moving Doppler red shifted wave

\[ \psi_L = e^{iL} = e^{i(k_L x - \omega_L t)} \]

...Doppler shifts give Lorentz transformation of both these graphs

Per-Spacetime \((c k, \omega)\)
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Some have forgotten... Special relativity and quantum mechanics are very much a story of the geometry of light-wave motion.

Need to review...

• Where Galilean relativity fails for light waves, ...and where it doesn’t.

and then see...

• How to make sense of light-wave axiom(s)

$$c = 299,792,458 \text{ m/s}$$

Good approximation: $c \approx 300 \text{ million m/s}$

300 Megameter/s

(We’ll use frequencies divisible by 3)
Some have forgotten... Special relativity and quantum mechanics are very much a story of the geometry of light-wave motion.

Need to review...

• Where Galilean relativity fails for light waves, ...and where it doesn’t.

and then see...

• How to make sense of light-wave axiom(s) by comparing *Einstein Pulse Wave (PW)* axiom with *Evenson Continuous Wave (CW)* axiom in space-time and inverse space-time.

**SPEED LIMIT**

\[ c = 299,792,458 \text{ m/s} \]

Good approximation: \( c \cong 300 \text{ million m/s} \)

300 Megameter/s

(We’ll use frequencies divisible by 3)

Link to ⇒ Speed of Light From Direct Frequency and Wavelength Measurements


In 2005 the Nobel Prize in physics was awarded to Glauber, Hall, and Hensch‡‡ for laser optics and metrology.

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\begin{itemize}
  \item Einstein’s PW (Pulse-Wave) Axiom
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• How do you make sense of light-wave axiom(s)?

And, HE-eee-rRE’S Albert!

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

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• How do you make sense of light-wave axiom(s)?

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A “road-runner” axiom is a “show-stopper”

Is cartoon physics a reality?!
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is $c$.
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• How do you make sense of light-wave axiom(s)?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is \( c \)

**Using Occam’s Razor**

(And Evenson’s lasers)

**Evenson Continuous Wave (CW) Axiom:** CW speed for all colors is \( c \)

Cut a PW to just one Continuous Wave
• How do you make sense of light-wave axiom(s)?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is c

PW peaks precisely locate places where wave is.

**Continuous Wave (CW) axiom:** CW speed for all colors is c

CW zeros precisely locate places where wave is not.

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Cut a PW to just one Continuous Wave (1CW) that changes Color if you accelerate!
• How do you make sense of light-wave axiom(s)?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is \( c \)

**Using Occam’s Razor**
- Using Simplified PW is more self-evident and productive

**Evenson Continuous Wave (CW) Axiom:** CW speed for all **colors** is \( c \)

- \( 1^{\text{st}} \)-order Doppler *Blue* shifts \( b = e^{+\rho} \)
- *Red* shifts \( r = e^{-\rho} \) of frequency \( \nu \) and wavenumber \( \kappa \)

**Cut a PW to just one Continuous Wave (1CW)** that changes **Color** if you accelerate!

**Continuous wave (CW) train**
- PW peaks precisely locate places where wave is
- CW zeros precisely locate places where wave is not
- Simpler CW coherence; It’s “Zen-like”

**Standing wave**
- \( A_1 \cos \omega t \)

**Standing wave**
- \( A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots \)

---

SPEED LIMIT
\( c = 299,792,458 \text{ m/s} \)

---

1. Albert Einstein (1879-1955)
2. William of Ockham (1285-1349)
• How do you make sense of light-wave axiom(s)?

Using Occam’s Razor

Evenson Continuous Wave (CW) axiom: CW speed for all colors is  

Cut a PW to just one Continuous Wave (1CW) that changes Color if you accelerate!

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A major objection to relativity/QM theory:
It’s the only major theoretical development that starts with 2nd-order (and quite mysterious!) effects.

It’s going c.
It looks blue!

It’s going c.
It looks green.
(Of course)

1CW is affected by 1st-order Doppler Blue shifts  

Blue shifts  

and Red shifts  

of frequency  

and wavenumber  

(1CW)

It’s going c.
It looks red!

Simpler 1CW coherence
It’s “Zen-like”

Can be made more self-evident and productive

PW peaks precisely locate places where wave is.

Continuous wave (CW) train

CW zeros precisely locate places where wave is not.

SPEED LIMIT
C = 299,792,458 m/s

Albert Einstein
1879-1955

William of Ockham
1285-1349

Kenneth Evenson
1932-2002

1CW also stands for “Cosine Wave” or “Coherent Wave” or “Colored Wave” (all helpful things!)
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Press a key to get a wave

...in spacetime...

"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier
1768-1830

• How to understand waves and wave velocity $V_{wave}$
Analyzing wave velocity by per-space-per-time and space-time graphs

Press a key to get a wave...

...in spacetime...

period \( \tau = 5/4 = 1/\nu \)

Heinreich Hertz
1857-1894
1Hz = 1sec\(^{-1}\)

Jean-Baptiste Joseph Fourier
1768-1830

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Press a key to get a wave... in spacetime...

Press a key to get a wave... in spacetime...

Heinrich Hertz
1857-1894
1Hz = 1 sec⁻¹

Jean-Baptiste Joseph Fourier
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How to understand waves and wave velocity \( V_{\text{wave}} \)
Analyzing wave velocity by per-space-per-time and space-time graphs

per-SPACETIME (κ, ν)-graph

ν = Greek "n" for number of waves per second or Hertz (Hz) = 1/τ

wavenumber κ = Greek "k" for Kayser (or "kinks") = 1/λ

Press a key to get a wave...

...in spacetime...

wavelength λ = 2/3 = 1/κ

period τ = 5/4 = 1/ν

Relativity Web Simulation
Keyboard of the Gods (Dual Plot)

How to understand waves and wave velocity $V_{wave}$
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• How to understand waves and wave velocity $V_{wave}$
Analyzing wave velocity by per-space-per-time and space-time graphs

Press a key to get a wave (a 1-CW)

...in spacetime...

“1-CW” means “single Continuous Wave”

― Keyboard of the gods ― is known as “Fourier-space”

Jean-Baptiste Joseph Fourier
1768-1830

How to understand waves and wave velocity $V_\text{wave}$

RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot)
Analyzing wave velocity by per-space-per-time and space-time graphs

- Press a key to get a wave (a 1-CW)
- "1-CW" means "single Continuous Wave"
- "continues" everywhere...

"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

• How to understand waves and wave velocity $V_{wave}$
Analyzing wave velocity by per-space-per-time and space-time graphs

Press a key to get a wave (a 1-CW)...
in spacetime...

...That “continues” everywhere...

...for all time...

“1-CW” means “single Continuous Wave”

“Keyboard of the gods” is known as “Fourier-space”

Jean-Baptiste Joseph Fourier
1768-1830

Keyboard of the gods

RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot)

•How to understand waves and wave velocity $V_{wave}$
Analyzing wave velocity by per-space-per-time and space-time graphs

Press a key to get a wave (a 1-CW)

...in spacetime...

...That “continues” everywhere...

...for all time...

...at a speed of:

“Keyboard of the gods” is known as “Fourier-space”

Jean-Baptiste Joseph Fourier 1768-1830

“Keyboard of the gods” is known as “Fourier-space”

•How to understand waves and wave velocity $V_{\text{wave}}$
Analyzing wave velocity by per-space-per-time and space-time graphs

Press a key to get a wave (a 1-CW)

...in spacetime...

"1-CW" means "single Continuous Wave"

...That “continues” everywhere...

...for all time...

...at a speed of:

"Keyboard of the gods" is known as “Fourier-space"

Jean-Baptiste Joseph Fourier
1768-1830

“How to understand waves and wave velocity $V_{\text{wave}}$"
Analyzing wave velocity by per-space-per-time and space-time graphs

**Per-SPACETIME (κ, υ)-graph**

- Frequency υ (waves per sec.)
- Wavenumber κ (waves per meter)
- Wave-speed equals slope-to-horizontal υ/κ in (κ, υ)-graph

**SPACETIME (λ, τ)-graph**

- Frequency υ (waves per sec.)
- Period τ (sec. per wave)
- Wavelength λ (meters per wave)
- Wave-speed equals slope-to-vertical λ/τ in (λ, τ)-graph

“1-CW” means “single Continuous Wave”...

...That “continues” everywhere...

...for all time...

...at a speed of:

\[
\frac{λ}{τ} = \frac{2/3}{5/4} = \frac{1}{15} \text{ m} \text{ per sec.}
\]

**Jean-Baptiste Joseph Fourier**

1768-1830

“Keyboard of the gods” is known as “Fourier-space”

**How to understand waves and wave velocity V_wave**

- Press a key to get a wave (a 1-CW)
- (and hold)

- Wave-speed equals slope-to-vertical λ/τ in (λ, τ)-graph

- Wave-speed equals slope-to-horizontal υ/κ in (κ, υ)-graph

- Wave-speed

- Period κ = 3/2

- Wavelength λ = 2/3

- Frequency υ = 4/5

- Wave-speed equals slope-to-horizontal υ/κ in (κ, υ)-graph

- Wave-speed equals slope-to-vertical λ/τ in (λ, τ)-graph
Wave arithmetic is simpler to explain using fractions.

**How to understand waves and “1st quantization”**

Analyzing wave velocity by per-space-per-time and space-time graphs.

- Frequency $\nu$ (waves per sec.): $V_{wave} = \frac{\nu}{\kappa}$
- Period $\tau$ (sec. per wave): $V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\nu} = \frac{1/\nu \cdot \kappa}{1/\tau}$

Wavelength $\lambda$: $\lambda = \frac{2}{3} \quad \lambda = \frac{4}{5} \quad \lambda = \frac{8}{15}$ meters.

Wave-speed equals slope-to-horizontal $\nu/\kappa$ in $(\kappa, \nu)$-graph.

Wave-speed equals slope-to-vertical $\lambda/\tau$ in $(\lambda, \tau)$-graph.

"1-CW" means "single Continuous Wave"...That "continues" everywhere...

...for all time...

...at a speed of:

**Wave-speed formulas**

- Distance = Wavelength = Frequency
  - Time = Period = Wavenumber
  - $V_{wave} = \frac{\lambda}{\tau} = \frac{1}{\kappa}$
  - $V_{wave} = \frac{\nu}{\kappa} = \frac{1}{\tau}$
  - $V_{wave} = \frac{4}{5}$
  - $V_{wave} = \frac{2}{3}$
  - $V_{wave} = \frac{8}{15}$

*Hold*...
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**Frequency** $\nu$ (units: $600$ THz)

- $\nu_A = 1800$ THz
- $\nu = 300$ THz
- $\nu = 1/2 \nu_A$
- $\nu = 1/4 \nu_A$

**Time period** $c\tau$ (units: $1/2 \mu m$)

- $c\tau_A = \lambda_A$

**Wavenumber** $c\kappa$ (units: $600$ THz)

- $c\kappa = 1 \cdot c\kappa_A$

**Space wavelength** $\lambda_x$ (units: $1/2 \mu m$)

- $\lambda_A = \lambda_x$

**SPACETIME** $(\lambda, c\tau)$-**graph**

Atom traveling along wave sees less wave “hits”/sec. (that is: Doppler red-shift)

Move fast enough this way then the “green” wave gets redder and redder until it dies.

**Moving along a 600 THz 1CW could Doppler red shift it to 300 THz**
Analyzing wave velocity using per-space-per-time graphs and space-time graphs

**frequency** $\nu$

(units: 600THz)

$\nu = \nu_A$

1800THz

$\nu = 300THz$

$\nu = 2\cdot10^6m$

$\nu = 4\cdot10^6m$

$\nu = 1800THz$

$\nu = 1200THz$

$\nu = 900THz$

$\nu = 600THz$

$\nu = 300THz$

$\nu = 180THz$

Atom traveling along wave sees less wave “hits” /sec. (that is: Doppler red-shift)

Atom traveling against wave sees more wave “hits” /sec. (that is: Doppler blue-shift)

Christian Doppler 1803-1853

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Frequency AND Amplitude increase exponentially

Frequency AND Amplitude decrease exponentially

Move fast enough this way then the “green” wave gets bluer and bluer until YOU die

Move fast enough this way then the “green” wave gets redder and redder until it dies

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz
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Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads \( v = 600\text{THz} \) for your laser beam.

Alice: “Well, what is its wavelength \( \lambda \), Bob!”

A really fast Alice shines her \( v = 300\text{THz} \) laser.
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Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

Q2: If so, what “phony” $\lambda$ does Bob see?

---

**frequency** $\nu=\omega/2\pi$

(Inverse period $\nu=1/\tau$)

<table>
<thead>
<tr>
<th>THz</th>
<th>Check it out in per-spacetime</th>
<th>Is it A, B, C, or D? etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**wavenumber** $\kappa=k/2\pi$

<table>
<thead>
<tr>
<th>THz</th>
<th>wavenumber $\kappa$</th>
<th>inverse wavelength $\kappa=1/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td></td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

$\lambda=1.00\mu m$, $0.50\mu m$, $0.33\mu m$ (inverse wavelength $\kappa=1/\lambda$)

$\kappa=1\cdot10^6/m$, $2\cdot10^6/m$, $3\cdot10^6/m$
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

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A really fast Alice shines her $\nu=300\text{THz}$ laser

---

Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

Q2: If so, what “phony” $\lambda$ does Bob see?

Answer to Q2 is C, the one with slope $\nu/\kappa=\nu\cdot\lambda=c$.

If he sees Green 600THz then he measures $\lambda=0.5\mu m$.
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads ν=600THz for your laser beam.

Alice: “Well, what is its wavelength λ, Bob!”

A really fast Alice shines her ν=300THz laser

Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

Q2: If so, what “phony” λ does Bob see?

Answer to Q2 is C, the one with slope ν/κ=ν⋅λ= c.

If he sees Green 600THz then he measures λ=0.5µm.
If he sees Red 300THz then he measures λ=1.0µm.

frequency ν
(Inverse period ν=1/τ)

THz
900
800
700
600
500
400
300

Only ONE kind of RED allowed (ONE that goes c)

wavenumber κ
(inline wavelengt h κ=1/λ)

κ= 1·10⁶/m  2·10⁶/m  3·10⁶/m

λ= 1.00µm  0.50µm  0.33µm

Saturday, December 12, 2015
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads \( \nu = 600 \text{THz} \) for your laser beam.

Alice: “Well, what is its wavelength \( \lambda \), Bob!”

A really fast Alice shines her \( \nu = 300 \text{THz} \) laser

---

**Q1:** Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

**Q2:** If so, what “phony” \( \lambda \) does Bob see?

Answer to Q2 is C, the one with slope \( \nu/\kappa = \nu \cdot \lambda = c \). If he sees Green 600THz then he measures \( \lambda = 0.5 \mu\text{m} \). If he sees Red 300THz then he measures \( \lambda = 1.0 \mu\text{m} \).

Answer to Q1 is NO!

CW Light carries no birth-certificate!

Vacuum only makes one \( \lambda \) for each \( \nu \).*

“All colors go \( c = \lambda \nu = \nu/\kappa \)”

Then Evenson’s axiom holds:

---

*for each beam and polarization orientation
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

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A really fast Alice shines her $\nu=300\text{THz}$ laser

---

**frequency $\nu$**
(Inverse period $\nu=1/\tau$)

**THz**

900
800
700
600
500
400
300

**λ = 1.00μm**
**0.50μm**
**0.33μm** (inverse wavelength $\kappa=1/\lambda$)

**κ = 1·10⁹/m**
**2·10⁹/m**
**3·10⁹/m**

**Only ONE kind of RED allowed (ONE that goes c)**

**600THz line**

**More evidence supporting Evenson’s axiom**

If bluer waves were even 0.1% faster (or slower) than redder ones then each flash from a 5-billion light-year distant galaxy shows up dispersed over 5-million years. (Goodbye galactic astronomy!)

**Evenson’s axiom:** $\nu = c\kappa$

Vacuum only makes one $\lambda$ for each $\nu$.*

“All colors go $c = \lambda\nu = \nu/\kappa$”

Then Evenson’s axiom holds:

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Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A \rangle$ and $\langle C|A \rangle$ to my 600THz beam.

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

I got $\langle B|A \rangle = 2$,

I got $\langle C|A \rangle = \frac{2}{3}$,

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A \rangle$ and $\langle C|A \rangle$ to my 600THz beam.

Bob-Alice Doppler ratio:

$\langle B|A \rangle = \frac{\nu_B}{\nu_A} = \frac{1200}{600} = \frac{2}{1}$

Carla-Alice Doppler ratio:

$\langle C|A \rangle = \frac{\nu_C}{\nu_A} = \frac{400}{600} = \frac{2}{3}$

Doppler ratio:

$\langle R|S \rangle = \frac{\nu_{RECEIVER}}{\nu_{SOURCE}}$

ALICE’S LASER GAUNTLET

600THz

$\nu_A = 600$THz

1200THz

$\nu_B = 1200$THz

400THz

$\nu_C = 400$THz

$\nu_A = 600$THz
Easy Doppler-shift and Rapidity calculation

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B | A \rangle \) and \( \langle C | A \rangle \) to my 600THz beam.

Bob: I see Doppler Blue shift to 1200THz

\[ \langle B | A \rangle = \frac{\nu_B}{\nu_A} = \frac{1200}{600} = \frac{2}{1} \]

Carla: I see Doppler Red shift to 400THz

\[ \langle C | A \rangle = \frac{\nu_C}{\nu_A} = \frac{400}{600} = \frac{2}{3} \]

IMPORTANT POINT:

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so all frequencies Doppler shift in same geometric proportion \( \langle R | S \rangle \).

\[ u_{RS} = -\frac{c}{4} \quad 0 \quad +\frac{c}{4} \]
Easy Doppler-shift and Rapidity calculation

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A \rangle$ and $\langle C|A \rangle$ to my 600THz beam.

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I got $\langle B|A \rangle = 2$,

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IMPORTANT POINT:

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so all frequencies Doppler shift in same geometric proportion $\langle R|S \rangle$.

If Alice sends $v_A = 600$THz

Bob sees: $v_B = \langle B|A \rangle v_A = 1200$THz

If Alice sends $v_A = 60$ THz

Bob sees: $v_B = \langle B|A \rangle v_A = 120$THz

If Alice sends $v_A = 6$ Hz

Bob sees: $v_B = \langle B|A \rangle v_A = 12$ Hz

$\langle B|A \rangle = 2$ for any frequency Alice and Bob use while they maintain their relative velocity.

$u_{RS} = -\frac{c}{4}$

0

$+\frac{c}{4}$

Saturday, December 12, 2015
Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B|A \rangle \) and \( \langle C|A \rangle \) to my 600THz beam. Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

I got \( \langle B|A \rangle = 2 \),

I got \( \langle C|A \rangle = 2/3 \),

**Easy Doppler-shift and Rapidity calculation**

**ALICE’S LASER GAUNTLET**

**600THz SOURCE**

\( \nu_A = 600\text{THz} \)

**1200THz RECEIVER**

\( \nu_B = 1200\text{THz} \)

**400THz RECEIVER**

\( \nu_C = 400\text{THz} \)

**Bob-Alice Doppler ratio:**

\[
\langle B|A \rangle = \frac{\nu_B}{\nu_A} = \frac{1200}{600} = \frac{2}{1}
\]

**Carla-Alice Doppler ratio:**

\[
\langle C|A \rangle = \frac{\nu_C}{\nu_A} = \frac{400}{600} = \frac{2}{3}
\]

**Rapidity:**

\[
\rho_{RS} = \log_e \langle R|S \rangle
\]

**Definition of Rapidity**

Rapidity is most convenient!

1 TeV proton has

\( u = 0.999995598\cdot c \) (Pain in the A)

or: \( \langle R|S \rangle = 2131.6 \) (Better)

or: \( \rho_{RS} = 7.6646 \) (Best)

For low velocity \( u \ll c \) rapidity \( \rho_{RS} \) approaches \( u/c \)

**IMPORTANT POINTS:**

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so all frequencies Doppler shift in same geometric proportion \( \langle R|S \rangle \).

Geometric phenomena tend to involve logarithmic/exponential functionality!
Bob-Alice Doppler ratio:
\[
\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}
\]

Bob-Alice rapidity:
\[
\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1}
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Carla-Alice Doppler ratio:
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\]

Definition of Rapidity
\[
\rho_{RS} = \log_e \langle R|S \rangle
\]

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \langle B|A \rangle and \langle C|A \rangle to my 600THz beam.

Also, rapidity \(\rho_{BA}\) and \(\rho_{CA}\) relative to me.

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

I got \(\langle B|A \rangle = 2\), and \(\rho_{BA} = \ln 2\)

I got \(\langle C|A \rangle = 2/3\),
Easy Doppler-shift and Rapidity calculation

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B|A \rangle \) and \( \langle C|A \rangle \) to my 600THz beam.
Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Bob: I see Doppler Blue shift to 1200THz
I got \( \langle B|A \rangle = 2 \), and \( \rho_{BA} = \ln(2) \)

Carla: I see Doppler Red shift to 400THz
I got \( \langle C|A \rangle = 2/3 \), and \( \rho_{CA} = \ln(2/3) \)

Doppler ratio:
\[
\langle R|S \rangle = \frac{v_{RECEIVER}}{v_{SOURCE}}
\]

rapidity:
\[
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Carla-Alice Doppler ratio:
\[
\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}
\]

Carla-Alice rapidity:
\[
\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3}
\]

Definition of Rapidity
**Easy Doppler-shift and Rapidity calculation**

**ALICE’S LASER GAUNTLET**

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A \rangle$ and $\langle C|A \rangle$ to my 600THz beam.

Also, rapidity $\rho_{BA}$ and $\rho_{CA}$ relative to me.

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

---

Bob: I got $\langle B|A \rangle = 2$, and $\rho_{BA} = \ln(2)$ = +0.69

Carla: I got $\langle C|A \rangle = 2/3$, and $\rho_{CA} = \ln(2/3) = -0.41$

---

**Bob-Alice Doppler ratio:**

$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

**Carla-Alice Doppler ratio:**

$$\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

---

**Bob-Alice Rapidity:**

$$\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1} = 0.69$$

(\text{time-reversed})

**Carla-Alice Rapidity:**

$$\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3} = -0.41$$

---

### Definition of Rapidity

$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{2}{1}$$

is time-reversal of:

$$\langle A|B \rangle = \frac{v_A}{v_B} = \frac{1}{2}$$

**Mnemonic:** You can think of rapidity $\rho_{BA}$ as “R” for “Romance”… (+) positive on approach, (-) negative on reproach.

---

**Do the stars hate us?**

Saturday, December 12, 2015
Easy Doppler-shift and Rapidity calculation

**ALICE’S LASER GAUNTLET**

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B|A \rangle \) and \( \langle C|A \rangle \) to my 600THz beam.

Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Bob: I see Doppler Blue shift to 1200THz

\[ \langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1} \]

\( \rho_{BA} = 0.69 \) (time-reversed)

\( \rho_{BA} = 0.69 \) (so: \( \rho_{AB} = -0.69 \))

Carla: I see Doppler Red shift to 400THz

\[ \langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3} \]

\[ \rho_{CA} = -0.41 \]

Now, Carla, what’s your rapidity \( \rho_{CB} \) relative to Bob?

**Rapidity**

\[ \rho_{RS} = \log_e \langle R|S \rangle \]

**Definition of Rapidity**

\[ \langle B|A \rangle = \frac{v_B}{v_A} = \frac{2}{1} \]

is time-reversal of:

\[ \langle A|B \rangle = \frac{v_A}{v_B} = \frac{1}{2} \]

Carla-Bob Doppler ratio:

\[ \langle C|B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \cdot \frac{v_A}{v_B} = \langle C|A \rangle \langle A|B \rangle \]

**Mnemonic:** You can think of rapidity \( \rho_{BA} \) as “R” for “Romance”… (+) positive on approach, (-) negative on reproach

More at Pirelli Challenge page: Time Reversal Symmetry
Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B|A \rangle \) and \( \langle C|A \rangle \) to my 600THz beam. Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

Now, Carla, what’s your rapidity \( \rho_{CB} \) relative to Bob?

\[
\begin{align*}
\text{Bob-Alice Doppler ratio:} & \quad \langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1} \\
\text{Bob-Alice rapidity:} & \quad \rho_{BA} = \log e \langle B|A \rangle = \log e \frac{2}{1} = 0.69 \quad (so: \rho_{AB} = -0.69) \\
\text{Carla-Alice Doppler ratio:} & \quad \langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3} \\
\text{Carla-Alice rapidity:} & \quad \rho_{CA} = \log e \langle C|A \rangle = \log e \frac{2}{3} = -0.41 \\
\text{Carla-Bob: rapidity:} & \quad e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}
\end{align*}
\]
**Easy Doppler-shift and Rapidity calculation**

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B | A \rangle \) and \( \langle C | A \rangle \) to my 600THz beam.

Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

**Bob-Alice Doppler ratio:**
\[
\langle B | A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}
\]

**Bob-Alice rapidity:**
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**Carla-Alice Doppler ratio:**
\[
\langle C | A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}
\]

**Carla-Alice rapidity:**
\[
\rho_{CA} = \log_e \langle C | A \rangle = \log_e \frac{2}{3} = -0.41
\]

**Carla-Bob Doppler ratio:**
\[
\langle C | B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \times \frac{v_A}{v_B} = \langle C | A \rangle \langle A | B \rangle
\]

**Carla-Bob rapidity:**
\[
e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies: } \rho_{CB} = \rho_{CA} + \rho_{AB}
\]
\[
e^{\rho_{CB}} = e^{\rho_{CA} + \rho_{AB}} = -0.41 - 0.69 = -1.10
\]
Easy Doppler-shift and Rapidity calculation

ALICE’S LASER GAUNTLET

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios \( \langle B \mid A \rangle \) and \( \langle C \mid A \rangle \) to my 600THz beam.

Also, rapidity \( \rho_{BA} \) and \( \rho_{CA} \) relative to me.

Now, Carla, what’s your rapidity \( \rho_{CB} \) relative to Bob?

Bob-Alice Doppler ratio:
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Carla-Alice rapidity:
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\rho_{CA} = \log_e \langle C \mid A \rangle = \log_e \frac{2}{3}
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\langle C \mid B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \frac{v_A}{v_B} = \langle C \mid A \rangle \langle A \mid B \rangle
\]

Carla-Bob rapidity:
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e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies: } \rho_{CB} = \rho_{CA} + \rho_{AB}
\]

\[
= -0.41 - 0.69 = -1.10
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I got \( \langle B \mid A \rangle = 2 \), and \( \rho_{BA} = \ln(2) = +0.69 \)

Carla: I see Doppler Red shift to 400THz

I got \( \langle C \mid A \rangle = 2/3 \), and \( \rho_{CA} = \ln(2/3) = -0.41 \)

We’re in Splitsville!

Bob: I got \( \langle C \mid A \rangle = 2/3 \), and \( \rho_{CA} = \ln(2/3) = -0.41 \)

Galileo’s Revenge (part I)

Rapidity adds just like Galilean velocity

Galileo’s Revenge (part I)

Rapidity adds just like Galilean velocity

\[
\rho_{CB} = \rho_{CA} + \rho_{AB}
\]

\[
= -0.41 - 0.69 = -1.10
\]

Happy now, Galileo?
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- Details of 1CW wavefunctions and phasors
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Galileo’s Revenge (part2): Galilean addition of phasor angular velocity
Structure of rest frame “baseball-diamonds”

Details of 2CW wavefunctions of moving frame velocities of phase and group waves
16 coefficients of relativistic 2CW interference
Two “famous-name” coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
Dimensionless Light wave-velocity $c/c=1$

\[
\frac{V_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{v}{c \tau} = 1 = \frac{\omega}{c \kappa} \quad \text{units}
\]

“winks”

“kinks”

angular frequency: $\omega = 2\pi \nu$

angular wavenumber: $k = 2\pi \kappa$

$k =$ wavevector

300 THz laser (Infrared)

$\psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t)$

$\psi(x,t)$

$\psi =$ Real $\psi$ = Re$\psi$

$\psi =$ Imaginary $\psi$ = Im$\psi$

Wavelength $\lambda = 2\pi / k = 1 / \kappa$

$(1\mu m = 10^{-6} m)$
**ICW Laser-phasor wave function**

\[
\psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t)
\]

- **Amplitude** $A$
- **Real** $Re$ \(\psi\) = Re$\psi$
- **Imaginary** $Im$ \(\psi\) = Im$\psi$

**Wavelength** $\lambda = 2\pi/k = 1/\kappa$

$1 \mu m = 10^{-6} m$

**Dimensionless Light wave-velocity** $c/c = 1$

\[
V_{\text{light}} = \frac{\lambda}{c} = \frac{\nu}{c} = \frac{\omega}{c} = \frac{\kappa}{k} = 1 = \text{angular units}
\]

- “winks”
- “kinks”

**300 THz laser** (Infrared)

**Mantra for most of the US publicly traded corporations**

"winks" 'n "kinks"

**Mantra for most of the US publicly traded corporations**

Imagination precedes Reality by exactly One Quarter!
\[ \psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t) \]

**Dimensionless Light wave-velocity** $c/c = 1$

\[ \frac{V_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{\nu}{c \kappa} = 1 = \frac{\omega}{\omega \text{ angular units}} \]

**Q:** Where is phase $= (kx - \omega t) = 0$?

**A:** It is wherever this is:

\[ x = \frac{\omega}{t} = \frac{k}{\kappa} \]

**300 THz laser (Infrared)**

Real $\psi = \text{Re} \psi$

Imaginary $\psi = \text{Im} \psi$

**BohrIt Web Simulation**

**1 CW ct vs x Plot**

\( ck = +1 \)

**Wavelength** $\lambda = 2\pi/k = 1/\kappa$

\( (1\mu m = 10^{-6} m) \)

**Period** $\tau = 2\pi/\omega = 1/\nu$

\( (10/3 \text{ fs} = 3.33 \cdot 10^{-15} \text{ s}) \)
$\psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t)$

**Real \( \psi \) = \text{Re} \psi**

**Imaginary \( \psi \) = \text{Im} \psi**

**Phase-angle**

**Amplitude**

**Real axis**

**Imaginary axis**

**Wavelength** $\lambda = 2\pi / k = 1 / \kappa$

**Period** $\tau = 2\pi / \omega = 1 / \nu$

**Dimensionless Light wave-velocity** $c / c = 1$

$$V_{\text{light}} = \frac{\lambda}{ct} = \frac{\nu}{c\kappa} = 1 = \frac{\omega \text{ angular}}{c k \text{ units}}$$

**“winks”**

**“kinks”**

**Angular frequency** $\omega = 2\pi \nu$

**Angular wavenumber** $k = 2\pi \kappa$

**300 THz laser** (Infrared)

**Period (1 fs) = 3.33 $10^{-15}$ s**

**Wavelength (1 \( \mu \text{m} \)) = 10^{-6} \text{ m}**

**Space \( x \)**

**Time \( ct \)**

**(10/3 fs) = 3.33 $10^{-15}$ s**

**Phase-path (phase = 0)**

**Crest path (phase = $+\pi/2$)**

**Zero path (phase = $+\pi/2$)**

**Trough path (phase = $+\pi$)**
Dimensionless Light wave-velocity \( c/c = 1 \)

\[
\frac{V_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{\nu}{c} = \frac{1}{\omega} \text{ angular units}
\]

“winks”

“kinks”

angular frequency: \( \omega = 2\pi\nu \)

angular wavenumber: \( k = 2\pi\kappa \)

\( k = \text{wavevector} \)

300 THz laser (Infrared)

\( \psi = A \cdot e^{i(kx - \omega t)} = A \cdot \cos(kx - \omega t) + iA \cdot \sin(kx - \omega t) \)

1CW Laser-phasor wave function

Clock velocity \( u=0 \)

frequency \( 300 \text{THz} \)

Two extremes give identical phasor clock \((x,ct)\) array

Clock velocity \( u \approx c \)

frequency \( \approx 0.0 \text{ THz} \)

Amplitude \( A \)

Phase-angle \( \lambda \)

Period \( \tau = 2\pi/\omega = 1/\nu \)

(\(10/3 \text{ fs} = 3.33 \times 10^{-15} \text{ s}\))

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)

(1\(\mu\)m = 10\(^{-6}\) m)

Space \( x \)

Time \( ct \)

Real \( \psi = \text{Re} \psi \)

Imaginary \( \psi = \text{Im} \psi \)

Crest path (phase = 0)

Trough path (phase = +\( \pi \))

Zero path (phase = +\( \pi/2 \))

"winks"

"kinks"

"winks"

"kinks"
1CW Laser-phasor wave function

$$\psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t)$$

Amplitude $A$

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

300 THz laser (Infrared)

Clock velocity $u = 0$

frequency 300THz

Two extremes give identical phasor clock-$(x,ct)$-array, too.

That's gauge invariance!

$\lambda/\tau = c = \nu'/\kappa'$

must match this phasor clock-$(x,ct)$-array, too.

Other Doppler versions

$\kappa - \nu t = \kappa' x - \nu' t'$

Dimensionless Light wave-velocity $c/c = 1$

$$V_{light} = \frac{\lambda}{c} = \frac{\nu}{c \tau} = 1 = \left(\frac{\omega \text{ angular}}{c \text{ units}}\right)$$

“winks” “kinks”

angular frequency: $\omega = 2\pi \nu$

angular wavenumber: $k = 2\pi \kappa$

$k = \text{wavevector}$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

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$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

$\psi = \Re \psi$

$\psi = \Im \psi$

$300 \text{ THz}$ laser (Infrared)

Real $\psi = \Re \psi$

Imaginary $\psi = \Im \psi$

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Colliding 2CW laser beams

Right-moving wave \( e^{i(kx-\omega t)} \)

- \( k = +2 \)
- \( \omega = 2c \)

Alice's laser

- CW Dye-laser
- 600 THz

Left-moving wave \( e^{i(-kx-\omega t)} \)

- \( k = -2 \)
- \( \omega = 2c \)

Carla's laser

- CW Dye-laser
- 600 THz

Time \( ct \)

Space \( x \)

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)

\[ (1/2\mu m = 0.5 \cdot 10^{-6} m) \]

Period \( \tau = 2\pi/\omega = 1/\nu \)

\[ (5/3fs = 1.67 \cdot 10^{-15}s) \]
Carla: Easy!
You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Bob: Cool!
You guys made me a space-time graph out of real zeros.

How'd it do that?

BohrIt Web Simulation
2 CW ct vs x Plot
($ck = \pm 2$)
Right-moving CW $e^{i(kx-\omega t)}$

Left-moving CW $e^{i(-kx-\omega t)}$

$Wavelength \, \lambda=2\pi/k=1/\kappa$

$(1/2\mu m=0.5\times10^{-6}m)$

$Period \, \tau=2\pi/\omega=1/\nu$

$(5/3fs=1.67\times10^{-15}s)$

Cool!

You guys made me a space-time graph out of real zeros.

How’d it do that?

**Bob:**

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into phase and group parts.

**Carla:**

Easy!

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

**Bob:**

Cool!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into phase and group parts.

**Carla:**

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into phase and group parts.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.
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More at Pirelli Challenge page: 'Un Grande Affare’ - Light Meets Light
Right-moving CW $e^{i(kx-\omega t)}$

- CW Dye-laser
- $k = 2, \omega = 2c$

Left-moving CW $e^{i(-kx-\omega t)}$

- CW Dye-laser
- $k = -2, \omega = 2c$

Carla:

Easy!
You get zeros of any wave-sum $e^{ia} + e^{ib}$
by factoring it into phase and group parts.

Remember your algebra? Exponents of
products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and
half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!
You factor $e^{ia} + e^{ib}$ into $e^{\frac{i(a+b)}{2}} \left(e^{\frac{i(a-b)}{2}} + e^{-\frac{i(a-b)}{2}}\right)$.

Red phasor B

Typical Phasor Sum:

Green phasor A

EQUALS: $\Psi_{A+B} = \Psi_A + \Psi_B$
(a) Sum of Wave Phasor Array

(b) Typical Phasor Sum:

Red phasor B

\[ \psi_B = e^{i\beta} \]

\[ \cos\beta \]

\[ \sin\alpha \]

PLUS

Green phasor A

\[ \psi_A = e^{i\alpha} \]

\[ \cos\alpha \]

EQUALS:

\[ \psi_{A+B} = \psi_A + \psi_B \]

\[ \frac{\alpha + \beta}{2} \]

\[ \frac{\alpha - \beta}{2} \]

(c) Phasor-relative views

A moves relative to B

Sum:

\[ \Psi_{A+B} = \psi_A + \psi_B \]

B moves relative to A

Difference:

\[ \Psi_{A-B} = \psi_A - \psi_B \]

\[ \frac{\alpha - \beta}{2} \]

Happy now?

Link to Animation from Pirelli Challenge

Galileo's Revenge (part 2)
Phasor angular velocity adds just like Galilean velocity

Geometry of the Half-sum Phase and Half-difference Group

Saturday, December 12, 2015
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➡ Structure of rest frame “baseball-diamonds”
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16 coefficients of relativistic 2CW interference
Two “famous-name” coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
Right-moving CW $e^{i(kx-\omega t)}$

Left-moving CW $e^{i(-kx-\omega t)}$

Carla:

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$

by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$

and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!

You factor $e^{ia}+e^{ib}$ into $e^{\frac{a+b}{2}} \left( e^{\frac{a-b}{2}} + e^{-\frac{a-b}{2}} \right)$

Alice 1CW phase: $a = kx - \omega t$

Carla 1CW phase: $b = -kx - \omega t$

Cool!

You guys made me a space-time graph out of real zeros.

How'd it do that?
You factor \( e^{ia} + e^{ib} \) into \( e^{i \frac{a+b}{2}} \left( e^{i \frac{a-b}{2}} + e^{-i \frac{a-b}{2}} \right) \).

Bob's 2CW Group-phase: \( k = \frac{a-b}{2} \)

Group wave: \( e^{-ikx} + e^{-ikx} = 2\cos{kx} \)

is standing wave (does not vary with time \( t \))

Remember your algebra? Exponents of products add.

So, half-sum \( \frac{a+b}{2} \) plus half-diff \( \frac{a-b}{2} \) gives \( a \), and half-sum \( \frac{a+b}{2} \) minus half-diff \( \frac{a-b}{2} \) gives \( b \).

Presto! You get zeros of any wave-sum \( e^{ia} + e^{ib} \) by factoring it into phase and group parts.
Right-moving CW \( e^{i(kx-\omega t)} \)

Left-moving CW \( e^{i(-kx-\omega t)} \)

Bob: Let’s plot this in per-spacetime?!

Cool! You guys made me a space-time graph out of real zeros.

How’d it do that?

Carla: Easy!

You get zeros of any wave-sum \( e^{ia} + e^{ib} \) by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum \( \frac{a+b}{2} \) plus half-diff \( \frac{a-b}{2} \) gives \( a \), and half-sum \( \frac{a+b}{2} \) minus half-diff \( \frac{a-b}{2} \) gives \( b \).

Presto!

You factor \( e^{ia} + e^{ib} \) into \( e^{i+a-b/2} \left( e^{a-b/2} + e^{-i(a-b)/2} \right) \).

Alice 1CW phase: \( a = kx - \omega t \)

Carla 1CW phase: \( b = -kx - \omega t \)

Bob’s 2CW Group-phase: \( +k = \frac{a-b}{2} \)

Group wave: \( e^{-ikx} + e^{-ikx} = 2\cos kx \)

is standing wave (does not vary with time \( t \)).

Bob’s 2CW Phase-phase: \( -\omega = \frac{a+b}{2} \)

Phase wave real part: \( \text{Re}(e^{-i\omega t}) = \cos(\omega t) \)

is “instanton” wave (does not vary in space \( x \)).
Standing 2CW in per-space-time

Frequency

\( \omega = 2\pi \nu \)

Wavelength \( \lambda = \frac{2\pi}{k} = \frac{1}{\kappa} \)

\((0.5\mu m = 0.5 \cdot 10^{-6} m)\)

Period \( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \)

\((1.67 fs = 0.16 \cdot 10^{-15} s)\)

\(\Psi(x,t) = (e^{-i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}\)

Bob:
The \( P \) and \( G \) vectors are scale models of zero-grid lattice vectors (but \( P \) and \( G \) switch places)

Carla:
OK, Bob!
It looks like a baseball diamond
with \( P \) at Pitcher's mound and \( G \) at the Grandstand*. I'm on 1st base! (R)

*Thanks, Woody!

**Carla:**
OK, Bob!
It looks like a baseball diamond with P at Pitcher’s mound and G at the Grandstand*. I’m on 1st base! (R)

*Thanks, Woody!
The (ψ,κ) "Baseball Diamond"

Standing 2CW in per-space-time

Frequency ω=2πν

Wavelength \( \lambda = \frac{2\pi}{k} = \frac{1}{\kappa} \)
(0.5μm=0.5×10⁻⁶m)

Period \( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \)
(1.67fs=0.167×10⁻¹⁵s)

\( \Psi(x,t) = (e^{-i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{-i(kx-\omega t)} \)

Phase vector
1/2-sum:
\( \mathbf{K}_{\text{phase}} = \mathbf{P} = \frac{\mathbf{R} + \mathbf{L}}{2} \)

Group vector
1/2-difference
\( \mathbf{K}_{\text{group}} = \mathbf{G} = \frac{\mathbf{R} - \mathbf{L}}{2} \)

Bob: The \( \mathbf{P} \) and \( \mathbf{G} \) vectors are scale models of zero-grid lattice vectors (but \( \mathbf{P} \) and \( \mathbf{G} \) switch places)

Carla: OK, Bob!
It looks like a baseball diamond with
\( \mathbf{P} \) at Pitcher's mound and
\( \mathbf{G} \) at the Grandstand*.
Ok, I'm on 3rd base \( \mathbf{L} \).

*Thanks, Woody!

RelaWaveity Site - Phase and Group Vectors in per-Time vs per-Space

G slope-to-vertical is zero
P slope-to-vertical is \( \infty \)
Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares
1 femtosecond
1.0 fs = 10^{-15} s
1 micron
1.0 µm = 10^{-6} meter

Pulse Waves (PW) trace “baseball diamonds” in space-time

(b) PW diamonds

CW Laser 600 THz

P (ω vs ck)
P (ct vs x)

Pulse Waves (PW)

...and a diamond in per-space-time

BohrIt Web Simulation: 2 PW ct vs x Plot

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Alice: Now our 600THz lasers move left-to-right. My 600THz laser is going so fast its beam blasts you with UV 1200THz.

Carla’s 600THz laser is going away so you get a nice infrared 300THz.

Bob: That UV burns! I need to put on my sunglasses.

\[ \nu = 1200\text{THZ} \text{ or } \lambda = 1/4 \text{ } \mu\text{m} \]

\[ \nu = 300\text{THZ} \text{ or } \lambda = 1 \text{ } \mu\text{m} \]
Evenson axiom says, "Stay on your baseline!"

Bob: Sunglasses help. Wow! Your 1st baseline $R'$ is Doppler blued up by $v'_s = e^{-\nu_4}$. My UV 1200THz $R'$ is a lot nicer!

You’ll need glasses to see $P'$ and $G'$ lines or coordinates.
Alice: OK. My UV 1200THz vector is fierce! You'll need glasses to see \( \mathbf{P}' \) and \( \mathbf{G}' \) lines or coordinates.

Carla: My UV 300THz is a lot nicer! (and half as long.)

Bob: Sunglasses help. Wow! Your 1st baseline \( \mathbf{R}' \) is Doppler blued up by \( e^{\mu} = 2 \).

But, Carla’s 3rd baseline \( \mathbf{L}' \) is Doppler red shifted by \( e^{\mu} = \frac{1}{2} \).

**Wavevector \( \mathbf{cK} \)**
- (units of \( c\kappa_A = 2 \cdot 10^6 / \text{m} \))
- \( \nu_C = e^{-\mu} \nu_A = \frac{1}{2} \nu_A = 300 \text{THz} \)
- \( e^{-\mu} = \frac{1}{2} \)
- \( -e^{-\mu} = -\frac{1}{2} \)
- \( +10^6 \)
- \( +2 \cdot 10^6 \)
- \( +3 \cdot 10^6 \)
- \( +4 \cdot 10^6 \)

**Frequency \( \nu' \)**
- (units of \( \nu_A = 600 \text{THz} \))
- \( \nu' = e^{\mu} \nu_A = 2 \nu_A = 1200 \text{THz} \)
- \( \nu' = \frac{1}{2} \nu_A = 600 \text{THz} \)

**2CW per-Spacetime Plot**

- New 1st base (Alice)
- New 3rd base (Carla)

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)
- \( (1/4\mu m = 0.25 \cdot 10^{-6} \text{m}) \)
- \( (1 \mu m = 10^{-6} \text{m}) \)
Alice: OK.

My UV 1200THz \( R' \) vector is fierce!

You'll need glasses to see \( P' \) and \( G' \) lines or coordinates.

Carla: My UV 300THz \( L' \) 3rd baseline is a lot nicer! (and half as long.)

**Wavevector \( c \kappa' \)**

\[
\begin{align*}
\kappa_A' &= e^{+p} \kappa_A = 2 \kappa_A \\
\kappa_C' &= e^{-p} \kappa_C = 2 \kappa_C \\
\kappa' &= e^{-p} \kappa + e^{+p} \kappa_C \\
&= 2 \kappa_A + 2 \kappa_C
\end{align*}
\]

**Frequency \( \nu' \)** (units of \( \nu_A = 600 \text{THz} \))

\[
\begin{align*}
\nu_A' &= e^{+p} \nu_A = 2 \nu_A \\
\nu_C' &= e^{-p} \nu_A = \frac{1}{2} \nu_A = 300 \text{THz}
\end{align*}
\]

**RelaWavity Simulation: Shifted (b=2) Phase and Group Vectors in per-Time vs per-Space**
Alice: OK. My UV 1200 THz vector is fierce! You'll need glasses to see $P'$ and $G'$ lines or coordinates.

Carla: My UV 300 THz is a lot nicer! (and half as long.)

Bob: Sunglasses help. Wow! Your 1st baseline is Doppler blued up by $e^{+\rho}=2$;

But, Carla's 3rd baseline is Doppler red shifted by $e^{-\rho}=1/2$:

New "Pitcher-mound" $P'$ (Phase pt.) is 1/2-sum $(R'+L')/2$:

$$v' = \frac{v_A}{2} \left( e^{+\rho} + e^{-\rho} \right) + \frac{v_A}{2} \left( e^{+\rho} - e^{-\rho} \right) = v_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = v_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$v'_c = e^{-\rho} v_A = \frac{1}{2} v_A = 300 \text{ THz}$$
Alice: OK.

My UV 1200THz \( \mathbf{R}' \) vector is fierce! You'll need glasses to see \( \mathbf{P}' \) and \( \mathbf{G}' \) lines or coordinates.

Carla: My UV 300THz \( \mathbf{L}' \) 3rd baseline is a lot nicer! (and half as long.)

Bob: Sunglasses help.
Wow! Your 1st baseline is Doppler blued up by \( e^{\nu} = 2 \):

\[
\begin{align*}
K'_{\text{phase}} &= \mathbf{P}' - \frac{\mathbf{R}' + \mathbf{L}'}{2} \\
K'_{\text{phase}} &= \begin{pmatrix}
-1/2 \\
2 - 1/2 \\
2 + 1/2 \\
2
\end{pmatrix}
\end{align*}
\]

New “Pitcher-mound” \( \mathbf{P}' \) (Phase pt.) is 1/2-sum \( (\mathbf{R}' + \mathbf{L}')/2 \):

\[
\begin{align*}
\mathbf{v}'_{\text{phase}} &= \begin{pmatrix} 2 \\ 2 \\ 2 + 1/2 \end{pmatrix} \\
\mathbf{v}'_{\text{phase}} &= \begin{pmatrix} -1/2 \\ 2 \\ 2 \end{pmatrix}
\end{align*}
\]

New “Grandstand” \( \mathbf{G}' \) (Group pt.) is 1/2-difference \( (\mathbf{R}' - \mathbf{L}')/2 \):

\[
\begin{align*}
\mathbf{v}'_{\text{phase}} &= \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix} \\
\mathbf{v}'_{\text{phase}} &= \begin{pmatrix} -1/2 \\ 2 \\ 2 \end{pmatrix}
\end{align*}
\]
My UV 1200THz vector is fierce! You’ll need glasses to see $P'$ and $G'$ lines or coordinates.

Alice: OK.

My UV 300THz vector is fierce! You’ll need glasses to see $P'$ and $G'$ lines or coordinates.

Carla: My UV 300THz vector is fierce! You’ll need glasses to see $P'$ and $G'$ lines or coordinates.
Frequency $\nu'$ (units of $\nu_A = 600$ THz)

Wavelength $\lambda = 2\pi/k = 1/\kappa$

$1/4\mu m = 0.25 \times 10^{-6} m$

$1 \mu m = 10^{-6} m$

$\nu' = \nu_A$

$\mu_m' = \mu_m$

$\kappa' = \kappa$

Wavevector $ck'$

(units of $ck_A = 2 \times 10^6 / m$)

Phase vector $P$

1/2-sum vector

$K_{phase}' = P = \frac{R' + L'}{2}$

Group vector $G$

1/2-diff vector

$K_{group}' = G = \frac{R' - L'}{2}$

Time $ct'$

(units of $\lambda_A = 1/2 \mu m$)

Bob: The spacetime wave-zeros replicate the same pattern.

$2CW$ per-Spacetime Plot

$2CW$ Minkowski-Spacetime Grid
Frequency $\nu'$
(units of $\nu_A=600\text{THz}$)

$\nu' = \sqrt{\frac{c}{\lambda}}$

$\lambda = \frac{2\pi}{\nu_A}$

$\nu_A = 600\text{THz}$

Wave vector $c k'$
(units of $c k_A = 2 \cdot 10^6 / \text{m}$)

$P' = \frac{R' + L'}{2}$

$K_{phase}' = P' = \frac{R' + L'}{2}$

$G' = \frac{R' - L'}{2}$

$K_{group}' = \frac{R' - L'}{2}$

$2\text{CW Minkowski-Spacetime Grid}$

$2\text{CW Minkowski-spacetime group}$

Bob: The spacetime wave-zeros replicate the same pattern.

(Except $P'$-phase and $G'$-group indicators get switched again.)

Let's measure these in careful detail!
How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
Optical interference “baseball-diamond” displays phase and group velocity
Doppler shifted “baseball-diamond” displays Lorentz frame transformation

Developing Axioms to update Galileo’s relativity: A critical look at $c$-axioms
Einstein’s PW (Pulse-Wave) Axiom
Evenson’s CW (Continuous Wave) Axiom and Occam’s Razor
Analyzing wave velocity by per-space-per-time and space-time graphs
Introducing optical Doppler effects
Clarifying Evenson’s CW Axiom using Doppler effects
  *Galileo’s Revenge (part1):* Galilean Doppler-shift arithmetic using rapidity $\rho$

Developing optical “baseball-diamond” and relativistic $\rho$-functions and transformations
Details of 1CW wavefunctions and phasors
Details of 2CW wavefunctions in rest frame
  *Galileo’s Revenge (part2):* Galilean addition of phasor angular velocity
Structure of rest frame “baseball-diamonds”
Details of 2CW wavefunctions of moving frame velocities of phase and group waves
  * ➤ 16 coefficients of relativistic 2CW interference*
Two “famous-name” coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
The 16 dimensions of 2CW interference

Frequency

\( \nu' \)
(units of
\( \nu_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

RelaWavity Web Simulation - 16 Relativity Dimensions

Start with the Dopplers

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu \text{m} \))

\[ c' = 2 \nu_A \]

\[ c = 2 \nu_A \]

\[ \nu_A = 600 \text{THz} \]

\[ \lambda_A = 1/2 \mu \text{m} \]
The 16 dimensions of 2CW interference

\[
\mathbf{P'} = \begin{pmatrix} c \kappa'_{\text{phase}} \\ v'_{\text{phase}} \end{pmatrix} = \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( v'_{\text{phase}} = v_A \cosh \rho = 5/4 = 1.25 \)

Phase period \( \tau'_{\text{phase}} = \tau_A \sech \rho = 4/5 \)

Frequency \( v' \)
(units of \( v_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu \text{m} \))

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( c' )</th>
<th>( \kappa' )</th>
<th>( \tau'_{\text{phase}} )</th>
<th>( \frac{v'_{\text{phase}}}{v_A} )</th>
<th>( \frac{\lambda_A}{\kappa_A} )</th>
<th>( \frac{\tau_A}{\tau'_{\text{phase}}} )</th>
<th>( \frac{v_A}{v'_{\text{phase}}} )</th>
<th>( b_{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>1</td>
<td>( \frac{1}{B_{\text{Doppler RED}}} )</td>
<td>( \frac{1}{B_{\text{Doppler BLUE}}} )</td>
<td>( \frac{1}{\lambda_A} )</td>
<td>( \frac{1}{\kappa_A} )</td>
<td>( \frac{\tau_A}{\tau'_{\text{phase}}} )</td>
<td>( \frac{v_A}{v'_{\text{phase}}} )</td>
<td>( \frac{1}{B_{\text{Doppler RED}}} )</td>
<td>( \frac{1}{B_{\text{Doppler BLUE}}} )</td>
</tr>
<tr>
<td>rapidity ( \rho )</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \sech \rho )</td>
<td>( \cosh \rho )</td>
<td>( \csch \rho )</td>
<td>( \coth \rho )</td>
<td>( e^+ \rho )</td>
<td></td>
</tr>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>1/2 = 0.5</td>
<td>3/5 = 0.6</td>
<td>3/4 = 0.75</td>
<td>4/5 = 0.80</td>
<td>5/4 = 1.25</td>
<td>4/3 = 1.33</td>
<td>5/3 = 1.67</td>
<td>2/1 = 2.0</td>
<td></td>
</tr>
</tbody>
</table>

Start with the Dopplers... then do the phase waves.

\( \tau'_{\text{phase}} = 0.8 \)

Time \( ct' \)
(units of \( \lambda_A = 1/2 \mu \text{m} \))

Saturday, December 12, 2015
The 16 dimensions of 2CW interference

Phase frequency
\[ \frac{\nu'_A \cosh \rho}{\nu_A} = \frac{5}{4} \]
Phase period
\[ \tau'_{\text{phase}} = \tau_A \sech \rho = \frac{4}{5} \]

Frequency
\[ \nu' = \frac{\nu_A}{600 \text{THz}} \]

Wavevector
\[ c \kappa' = \nu'_A \left( \begin{array}{c} \sinh \rho \\ \cosh \rho \end{array} \right) = \frac{3}{4} \nu_A \]

Space
\[ x' = \frac{\lambda_A}{1/2 \mu m} \]

Time
\[ ct' = \frac{1}{\nu_A} \]

Group
\[ \begin{array}{cccccccc}
\text{phase} & b_{\text{RED}}^{\text{Doppler}} & c & \kappa & \frac{\tau_{\text{phase}}}{\tau_A} & \frac{\nu_{\text{phase}}}{\nu_A} & \frac{\lambda_A}{\lambda_0} & \frac{V_{\text{group}}}{c} & b_{\text{BLUE}}^{\text{Doppler}} \\
\text{rapidity} \ \rho & e^{-\rho} & \tanh \rho & \sinh \rho & \frac{\sech \rho}{\cosh \rho} & \cosh \rho & \frac{\sech \rho}{\cosh \rho} & \frac{\cosh \rho}{\sech \rho} & e^{+\rho} \\
\text{value for } \beta = \frac{3}{5} & \frac{1}{2} = 0.5 & \frac{3}{5} = 0.6 & \frac{3}{4} = 0.75 & \frac{4}{5} = 0.80 & \frac{5}{4} = 1.25 & \frac{4}{3} = 1.33 & \frac{5}{3} = 1.67 & \frac{2}{1} = 2.0
\end{array} \]
Phase wavenumber
\[ \kappa_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \]
flips to
Phase wavelength
\[ \lambda'_{\text{phase}} = \lambda_A \csch \rho = 4/3 \]

Phase frequency
\[ \upsilon'_{\text{phase}} = \upsilon_A \cosh \rho = 5/4 \]
flips to
Phase period
\[ \tau'_{\text{phase}} = \tau_A \sech \rho = 4/5 \]

\[
P' = \begin{pmatrix} \cosh \rho & 0 \\ 0 & \sinh \rho \end{pmatrix}
\]
\[
\begin{pmatrix} \upsilon'_{\text{phase}} \\ \upsilon'_{\text{phase}} \end{pmatrix} = \begin{pmatrix} \cosh \rho & 0 \\ 0 & \sinh \rho \end{pmatrix} \begin{pmatrix} \upsilon_A \\ \upsilon_A \end{pmatrix}
\]

Frequency
\[ \upsilon' = \frac{\upsilon_A}{n} \]
(units of \( \upsilon_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

Time \( c't' \)
(units of \( \lambda_A = 1/2 \mu \text{m} \))

Space \( x' \)
(units of \( \kappa_A = 1/2 \mu \text{m} \))

\[ \upsilon'_{\text{phase}} = 1.25 \]

\[ \kappa'_{\text{phase}} = 0.75 \]

\[ \lambda'_{\text{phase}} = 1.33 \]

\[ \tau'_{\text{phase}} = 0.8 \]

\[ \rho \]

\[ e^{-\rho} \]

\[ \tanh \rho \]

\[ \sinh \rho \]

\[ \sech \rho \]

\[ \cosh \rho \]

\[ \csch \rho \]

\[ \coth \rho \]

\[ e^{\rho} \]

\[ \frac{1}{2} = 0.5 \]

\[ \frac{3}{5} = 0.6 \]

\[ \frac{3}{4} = 0.75 \]

\[ \frac{4}{5} = 0.80 \]

\[ \frac{5}{4} = 1.25 \]

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\[ \frac{4}{3} = 1.33 \]

\[ \frac{3}{5} = 0.6 \]

\[ \frac{2}{1} = 0.2 \]

2015-12-06
Phase wavenumber
\[ \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \]
Phase frequency
\[ \upsilon'_{\text{phase}} = \upsilon_A \cosh \rho = 5/4 \]
Phase wavelength
\[ \lambda'_{\text{phase}} = \lambda_A \text{sech} \rho = 4/3 \]
Phase period
\[ \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \]

Time \( c' \)
(units of \( \lambda_A = 1/2 \mu m \))

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu m \))

\[ \text{Phase wavenumber} \]
\[ \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \]

\[ \text{Phase frequency} \]
\[ \upsilon'_{\text{phase}} = \upsilon_A \cosh \rho = 5/4 \]

\[ \text{Phase wavelength} \]
\[ \lambda'_{\text{phase}} = \lambda_A \text{sech} \rho = 4/3 \]

\[ \text{Phase period} \]
\[ \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \]

**Diagram:**
- Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \)
- Phase frequency \( \upsilon'_{\text{phase}} = \upsilon_A \cosh \rho = 5/4 \)
- Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \text{sech} \rho = 4/3 \)
- Phase period \( \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \)

**Table:**

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( \upsilon'_{\text{phase}} )</th>
<th>( \lambda'_{\text{phase}} )</th>
<th>( \tau'_{\text{phase}} )</th>
<th>( \upsilon_A )</th>
<th>( \lambda_A )</th>
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<th>( \cosh \rho )</th>
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<td>( \frac{3}{4} = 0.75 )</td>
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<td>( \frac{2}{1} = 2.0 )</td>
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</table>
The 16 dimensions of 2CW interference

\[ G' = \left( \begin{array}{ll} \frac{c \kappa'}{\nu_{\text{group}}} \\ \nu'_{\text{group}} \end{array} \right) = \nu_A \left( \begin{array}{ll} \cosh \rho \\ \sinh \rho \end{array} \right) = \nu_A \left( \begin{array}{ll} \frac{5}{4} \\ \frac{3}{4} \end{array} \right) \]

Group frequency

\[ \nu'_{\text{group}} = \nu_A \sinh \rho = \frac{3}{4} \]

flips to

Group period

\[ \tau'_{\text{group}} = \tau_A \text{csch} \rho = \frac{4}{3} \]

\[ = 1.33 \]

Frequency

\[ \nu' \]

(units of \( \nu_A = 600 \text{THz} \))

Wavevector

\[ c \kappa' \]

(units of \( c \kappa_A = 2 \times 10^6 \text{m} \))

\[ L = K - 1 \]

\[ = 0.75 \]

\[ = 0.0 \]

\[ = +10^6 \]

\[ +2 \times 10^6 \]

\[ +3 \times 10^6 \]

\[ +4 \times 10^6 \]

\[ \rho \]

\[ \text{value for } \beta = 3/5 \]

\[ \frac{1}{2} = 0.5 \]

\[ \frac{3}{5} = 0.6 \]

\[ \frac{3}{4} = 0.75 \]

\[ \frac{4}{5} = 0.80 \]

\[ \frac{5}{4} = 1.25 \]

\[ \frac{4}{3} = 1.33 \]

\[ \frac{5}{3} = 1.67 \]

\[ \frac{2}{1} = 2.0 \]

\[ \text{Dopplers} \]

...then do the

phase waves

...then the

group waves

\[ \text{Space } x' \]

(units of \( \lambda_A = 1/2 \mu \text{m} \))

\[ \text{Time } c t' \]

(units of \( \lambda_A = 1/2 \mu \text{m} \))

Start with the

The 16 dimensions of 2CW interference
The 16 dimensions of 2CW interference

\[
\begin{align*}
G' &= \left( \begin{array}{c}
ck'_{\text{group}} \\
\nu'_{\text{group}}
\end{array} \right) = \nu_A \begin{pmatrix} 
cosh \rho \\
\sinh \rho
\end{pmatrix} = \nu_A \begin{pmatrix} 
5/4 \\
3/4
\end{pmatrix}
\end{align*}
\]

Group frequency
\[
\nu'_{\text{group}} = \nu_A \sinh \rho = 3/4
\]

Group period
\[
\tau'_{\text{group}} = \tau_A \text{csch} \rho = 4/3
\]

\[
\begin{align*}
\tau'_{\text{group}} &= \tau_A \text{csch} \rho = 4/3 \\
\tau'_{\text{group}} &= 1.33
\end{align*}
\]

Frequency
\[
\nu' = \nu_A = 600 \text{THz}
\]

Wavevector \(ck'\)
\[
\begin{align*}
L &= K_{-1} \\
R &= K_{+4}
\end{align*}
\]

Space \(x'\)
\[
\begin{align*}
\text{Start with the Dopplers} \\
\text{...then do the phase waves} \\
\text{...then the group waves}
\end{align*}
\]

**Group frequency**
\[
\nu'_{\text{group}} = \nu_A \sinh \rho = 3/4
= 0.75
\]

**Group period**
\[
\tau'_{\text{group}} = \tau_A \text{csch} \rho = 4/3
\]

**Starting with**
\[
\begin{align*}
\nu'_{\text{group}} &= 0.75 \\
\tau'_{\text{group}} &= 1.33
\end{align*}
\]

**Phase waves**
\[
\begin{align*}
\nu'_{\text{phase}} &= \nu_A \sinh \rho = 3/4 \\
\tau'_{\text{phase}} &= \tau_A \text{csch} \rho = 4/3
\end{align*}
\]

**Group waves**
\[
\begin{align*}
\nu'_{\text{group}} &= 0.75 \\
\tau'_{\text{group}} &= 1.33
\end{align*}
\]

**Table**

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{phase} & b_{\text{Doppler}}^{\text{RED}} & c & \kappa_{\text{phase}} & \tau_{\text{phase}} & \nu_{\text{phase}} & \lambda_{\text{phase}} & V_{\text{phase}} & b_{\text{Doppler}}^{\text{BLUE}} \\
\hline
\text{group} & b_{\text{Doppler}}^{\text{BLUE}} & \nu_A & \nu'_{\text{group}} & \kappa_{\text{group}} & \tau_A & \nu_A & \lambda_A & V_A \\
\hline
\text{rapidity} \ \rho & e^{-\rho} & \tanh \rho & \sinh \rho & \sech \rho & \cosh \rho & \csch \rho & \coth \rho & e^{+\rho} \\
\hline
\text{value for} \ \beta = 3/5 & 1/2 = 0.5 & 3/5 = 0.6 & 3/4 = 0.75 & 4/5 = 0.80 & 5/4 = 1.25 & 4/3 = 1.33 & 5/3 = 1.67 & 2/1 = 2.0 \\
\hline
\end{array}
\]

Saturday, December 12, 2015
**Group wavenumber**

\[ \kappa'_{\text{group}} = \kappa_A \cosh \rho = \frac{5}{4} \]

\[ \kappa'_{\text{group}} = 1.25 \]

**Group frequency**

\[ v'_{\text{group}} = v_A \sinh \rho = \frac{3}{4} \]

\[ v'_{\text{group}} = 0.75 \]

**Group wavelength**

\[ \lambda'_{\text{group}} = \lambda_A \text{sech} \rho = \frac{4}{5} \]

\[ \lambda'_{\text{group}} = 0.8 \]

**Frequency**

\[ v' \]

(units of \[ v_A = 600 \text{THz} \])

- 1500 THz
- 1200 THz
- 900 THz
- 600 THz
- 300 THz
- 150 THz
- 60 THz
- 30 THz
- 0 THz
- -30 THz
- -60 THz
- -90 THz
- -120 THz
- -150 THz

**Wavevector**

\[ \mathbf{c} \cdot \mathbf{k} = 2 \times 10^6 / \text{m} \]

**Time**

\[ ct' \]

(units of \[ \lambda_A = 1/2 \mu \text{m} \])

Start with the Dopplers... then do the phase waves... then the group waves.

**Space**

\[ x' \]

(units of \[ \lambda_A = 1/2 \mu \text{m} \])

**Table**

<table>
<thead>
<tr>
<th>group</th>
<th>( b'_{\text{Doppler RED}} )</th>
<th>( v_{\text{group}} / c )</th>
<th>( \kappa_{\text{phase}} / \kappa_A )</th>
<th>( \tau_{\text{phase}} / \tau_A )</th>
<th>( v_{\text{phase}} / v_A )</th>
<th>( \lambda_{\text{phase}} / \lambda_A )</th>
<th>( V_{\text{phase}} / V_A )</th>
<th>( b'_{\text{Doppler BLUE}} )</th>
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</thead>
<tbody>
<tr>
<td>phase</td>
<td>( b'_{\text{Doppler RED}} )</td>
<td>( v_{\text{group}} / c )</td>
<td>( \kappa_{\text{phase}} / \kappa_A )</td>
<td>( \tau_{\text{phase}} / \tau_A )</td>
<td>( v_{\text{phase}} / v_A )</td>
<td>( \lambda_{\text{phase}} / \lambda_A )</td>
<td>( V_{\text{phase}} / V_A )</td>
<td>( b'_{\text{Doppler BLUE}} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>rapidity ( \rho )</th>
<th>( e^{-\rho} )</th>
<th>( \tanh \rho )</th>
<th>( \sinh \rho )</th>
<th>( \sec h \rho )</th>
<th>( \cosh \rho )</th>
<th>( \text{csch} \rho )</th>
<th>( \coth \rho )</th>
<th>( e^{+\rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
</tr>
</tbody>
</table>
Lorentz transformations...

write \( \mathbf{G}' \) and \( \mathbf{P}' \) in terms of \( \mathbf{G} \) and \( \mathbf{P} \) using \( \cosh \rho \) and \( \sinh \rho \)

\[
\mathbf{G}' = \begin{pmatrix}
 c \kappa'_{\text{group}} \\
 \nu'_{\text{group}}
\end{pmatrix} = \nu_A \begin{pmatrix}
 \cosh \rho \\
 \sinh \rho
\end{pmatrix} = \nu_A \begin{pmatrix}
 5/4 \\
 3/4
\end{pmatrix}
\]

\[
\mathbf{P}' = \mathbf{G} \sinh \rho + \mathbf{P} \cosh \rho
\]

\[
\mathbf{G}' = \begin{pmatrix}
 c \kappa'_{\text{phase}} \\
 \nu'_{\text{phase}}
\end{pmatrix} = \nu_A \begin{pmatrix}
 \sinh \rho \\
 \cosh \rho
\end{pmatrix} = \nu_A \begin{pmatrix}
 3/4 \\
 5/4
\end{pmatrix}
\]

\[
\mathbf{P}' = \mathbf{G} \sinh \rho + \mathbf{P} \cosh \rho
\]
How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations

Optical interference “baseball-diamond” displays phase and group velocity
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Details of 2CW wavefunctions in rest frame

Galileo’s Revenge (part2): Galilean addition of phasor angular velocity
Structure of rest frame “baseball-diamonds”

Details of 2CW wavefunctions of moving frame velocities of phase and group waves
16 coefficients of relativistic 2CW interference

Two “famous-name” coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
Two Famous-Name Coefficients

Albert Einstein
1859-1955

Hendrik A. Lorentz
1853-1928

This number is called a: Einstein time-dilation
(dilated by 25% here)

This number is called a: Lorentz length-contraction
(contracted by 20% here)

Old-Fashioned Notation

RelaWavity Web Simulation - Relativistic Terms (Expanded Table)

<table>
<thead>
<tr>
<th>group</th>
<th>(b_{Doppler}^{RED})</th>
<th>(\frac{c}{V_{phase}})</th>
<th>(\frac{\kappa_{phase}}{\kappa_A})</th>
<th>(\frac{\tau_{phase}}{\tau_A})</th>
<th>(\frac{\nu_{phase}}{\nu_A})</th>
<th>(\frac{\lambda_{phase}}{\lambda_A})</th>
<th>(\frac{V_{phase}}{c})</th>
<th>(b_{Doppler}^{BLUE})</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase group</td>
<td>1</td>
<td>(\frac{c}{V_{phase}})</td>
<td>(\frac{\kappa_{phase}}{\kappa_A})</td>
<td>(\frac{\tau_{phase}}{\tau_A})</td>
<td>(\frac{\nu_{phase}}{\nu_A})</td>
<td>(\frac{\lambda_{phase}}{\lambda_A})</td>
<td>(\frac{V_{phase}}{c})</td>
<td>1</td>
</tr>
<tr>
<td>rapidity</td>
<td>(e^{-\rho})</td>
<td>tanh (\rho)</td>
<td>sinh (\rho)</td>
<td>sech (\rho)</td>
<td>cosh (\rho)</td>
<td>csch (\rho)</td>
<td>coth (\rho)</td>
<td>(e^{+\rho})</td>
</tr>
<tr>
<td>(\beta=\frac{u}{c})</td>
<td>(\sqrt{1-\beta^2})</td>
<td>(\frac{\beta}{1})</td>
<td>(\frac{1}{\sqrt{1+\beta^2}})</td>
<td>(\frac{\sqrt{1-\beta^2}}{1})</td>
<td>(\sqrt{\beta^2-1})</td>
<td>(\frac{1}{\sqrt{1-\beta^2}})</td>
<td>(\frac{1}{\beta})</td>
<td>(\sqrt{1+\beta^2})</td>
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<td>(\frac{2}{1}=2.0)</td>
</tr>
</tbody>
</table>
RelativIt Web Simulation - Relativistic Events in Main Lighthouse’s Space-Time Frame

Lighthouse time $t = 0.863$

Ship v/$c$ (Rel. to Lthse.) = -0.600
Ship v/$c$(Rel. to Obs.) = 0.600
Lthse v/$c$(Rel. to Obs.) = 0.000

Click & Drag at bottom to control animation speed

RelativIt Web Simulation - Relativistic Events in Ship’s Space-Time Frame

Event 1
Event 2

Lighthouse Graph
Ref time $t = 0.86$ sec.
$v/c = -0.60$ litesec/sec.

Ship Graph
Ref time $t = 0.03$ sec.
$v/c = -0.60$ litesec/sec.
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Thales geometry of Lorentz transformation
Thales Mean Geometry (600BCE) helps “Relawavity”

Thales of Miletus
624-543 BCE

Frequency unit:
300THZ

Per-Time
ω - axis

Transformed Per-Time
ω' - axis
Slope-to-vertical
= \frac{V_{\text{group}}}{c} = \frac{3}{5} = \frac{4 - 1}{4 + 1}

Geometric Mean
B = \sqrt{(4 \cdot 1)} = 2

Arithmetic Mean
B_{\text{cosh} \rho} = \frac{(1+4)}{2} = \frac{5}{2}

Difference Mean
B_{\text{sinh} \rho} = \frac{(4-1)}{2} = \frac{3}{2}

Br = e^{-\rho}
Red shift = 1

Bb = e^{+\rho}
Blue shift = 4

Per-Space
ck - axis
Thales Mean Geometry (600BCE)

Thales of Miletus
624-543 BCE

Thales showed a circle diameter subtends a right angle with any circle point $P$.

This leads to a convenient construction of geometric means and relativistic hyperbolas.

\[
\frac{4 - 1}{4 + 1} = \frac{3}{5}
\]

Frequency unit: 300THZ
Thales Mean Geometry (600BCE)

Thales showed a circle diameter subtends a right angle with any circle point $P$

This leads to a convenient construction of geometric means and relativistic hyperbolas.

\[
\begin{align*}
    r \cdot b &= 2 \\
    4 - 1 &= 4 + 1 \\
    B \cos \rho &= (1+4)/2 \\
    B \sinh \rho &= (4-1)/2 \\
    Bb &= 2e^{+\rho} \\
    B &= \sqrt{(1+1)} = 2
\end{align*}
\]
Thales Mean Geometry (600 BCE) helps “Relawavity”

Thales of Miletus, 624-543 BCE

\[ r \cdot b = 2 \]

due to Doppler T-symmetry

Geometric Mean
\[ B = \sqrt{4 \cdot 1} = 2 \]

Arithmetic Mean
\[ B_{cosh} \rho = (1+4)/2 = 5/2 \]

Difference Mean
\[ B_{sins} \rho = (4-1)/2 = 3/2 \]

Transformed Per-Time
\( \omega' \)-axis
equilateral hyperbola

Transformed Per-Space
\( ck' \)-axis

Red shift
Blue shift
Laser frequency = $B = 2 = 600 \text{THz}$
Doppler blue shift factor = $b = 1.983$
Doppler red shift factor = $r = 0.504$
$p = 0.685$

CW Light Axioms
All colors go $c$: $\omega/k = c$ or L&R on diagonals
Time Reversal ($r \leftrightarrow b$): $r = 1/b$

$G' = G \cosh(p) + P \sinh(p)$
$P' = G \sinh(p) + P \cosh(p)$
$G = G' \cosh(p) - P' \sinh(p)$
$P = -G' \sinh(p) + P' \cosh(p)$

RelaWavity Web Simulation
Detailed Thales Geometry