Lecture 12
Thur. 10.8.2015

Poincare, Lagrange, Hamiltonian, and Jacobi mechanics

(Unit 1 Ch. 12, Unit 2 Ch. 2-7, Unit 3 Ch. 1-3, Unit 7 Ch. 1-2)

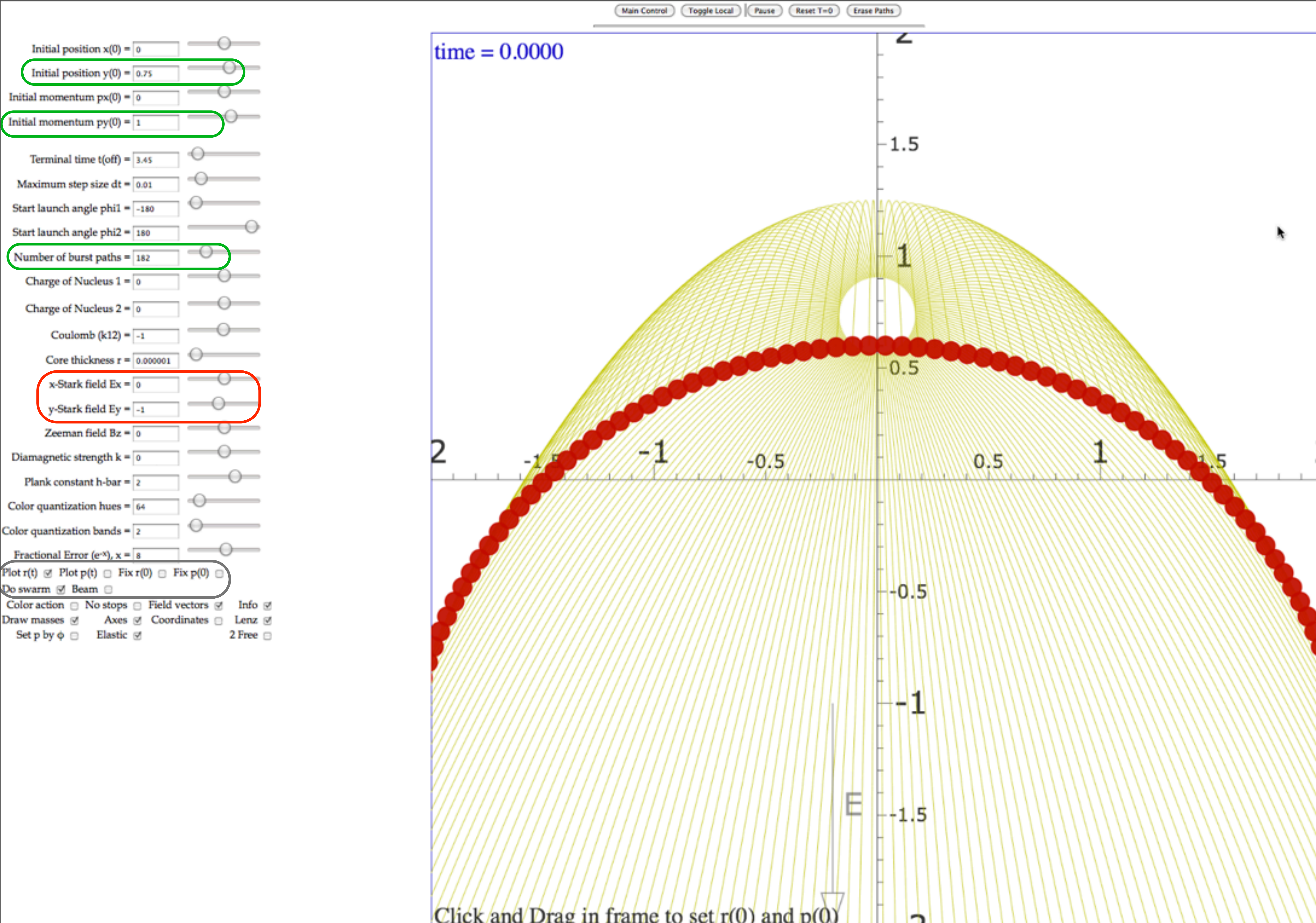
Parabolic and 2D-IHO orbital envelopes (Review of Lecture 9 p.56-81 and a generalization.)
Clues for future assignments (Web Simulation: CoulIt)

Examples of Hamiltonian mechanics in phase plots
1D Pendulum and phase plot (Web Simulations: Pendulum, Cycloidulum, JerkIt (Vert Driven Pendulum))
1D-HO phase-space control (Web Simulation of "Catcher in the Eye")

Exploring phase space and Lagrangian mechanics more deeply
A weird "derivation" of Lagrange's equations
Poincare identity and Action, Jacobi-Hamilton *equations*
How Classicists might have "derived" quantum equations
Huygen's contact transformations enforce minimum action
How to do quantum mechanics if you only know classical mechanics
("Color-Quantization" simulations: Davis-Heller "Color-Quantization" or "Classical Chromodynamics"*)*

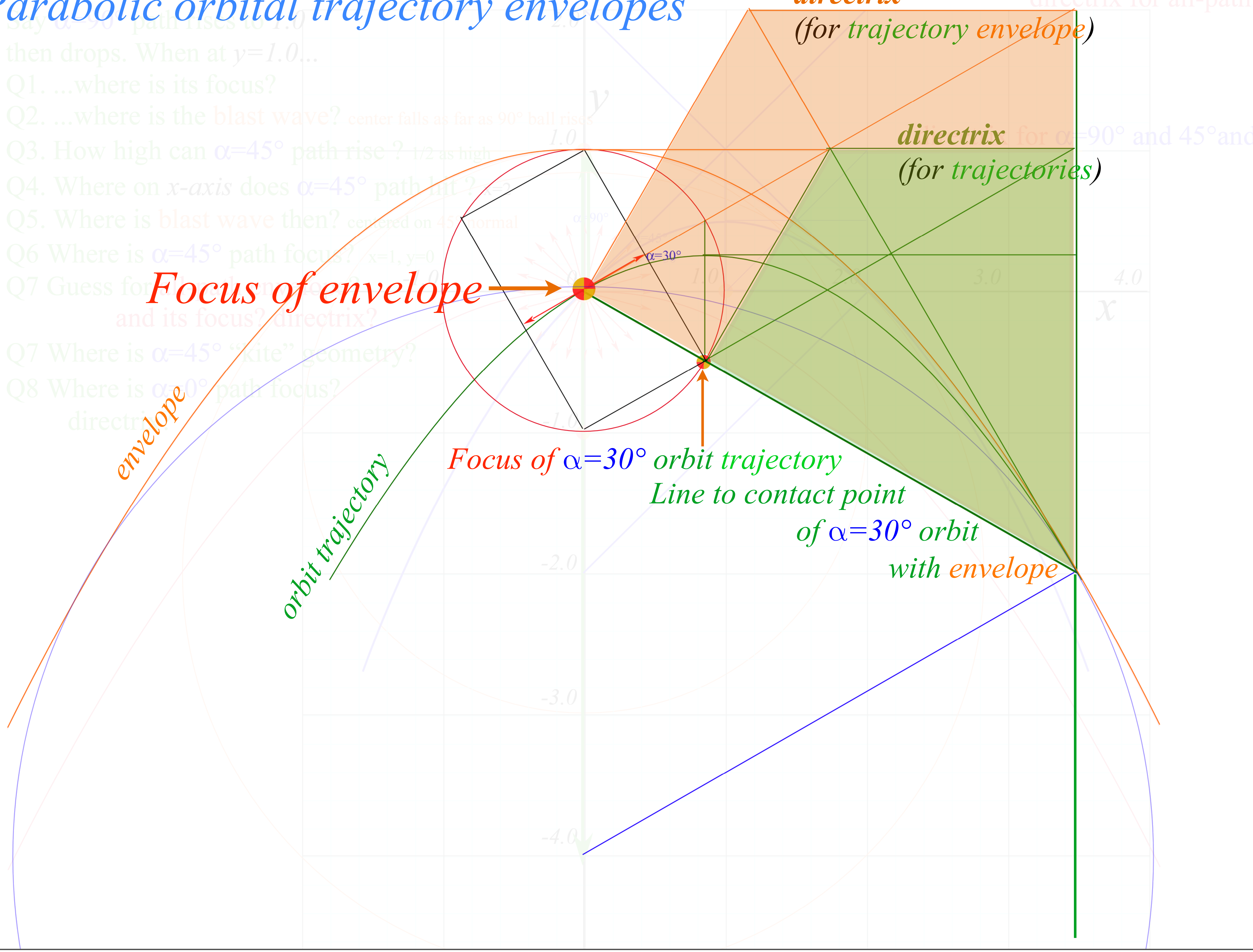
Parabolic and 2D-IHO orbital envelopes (Review of Lecture 9 p.56-81 and a generalization.)

Some clues for future assignments *(Web Simulation: CoulIt)*



Web Simulation: CoulIt - Volcanoes Of Io

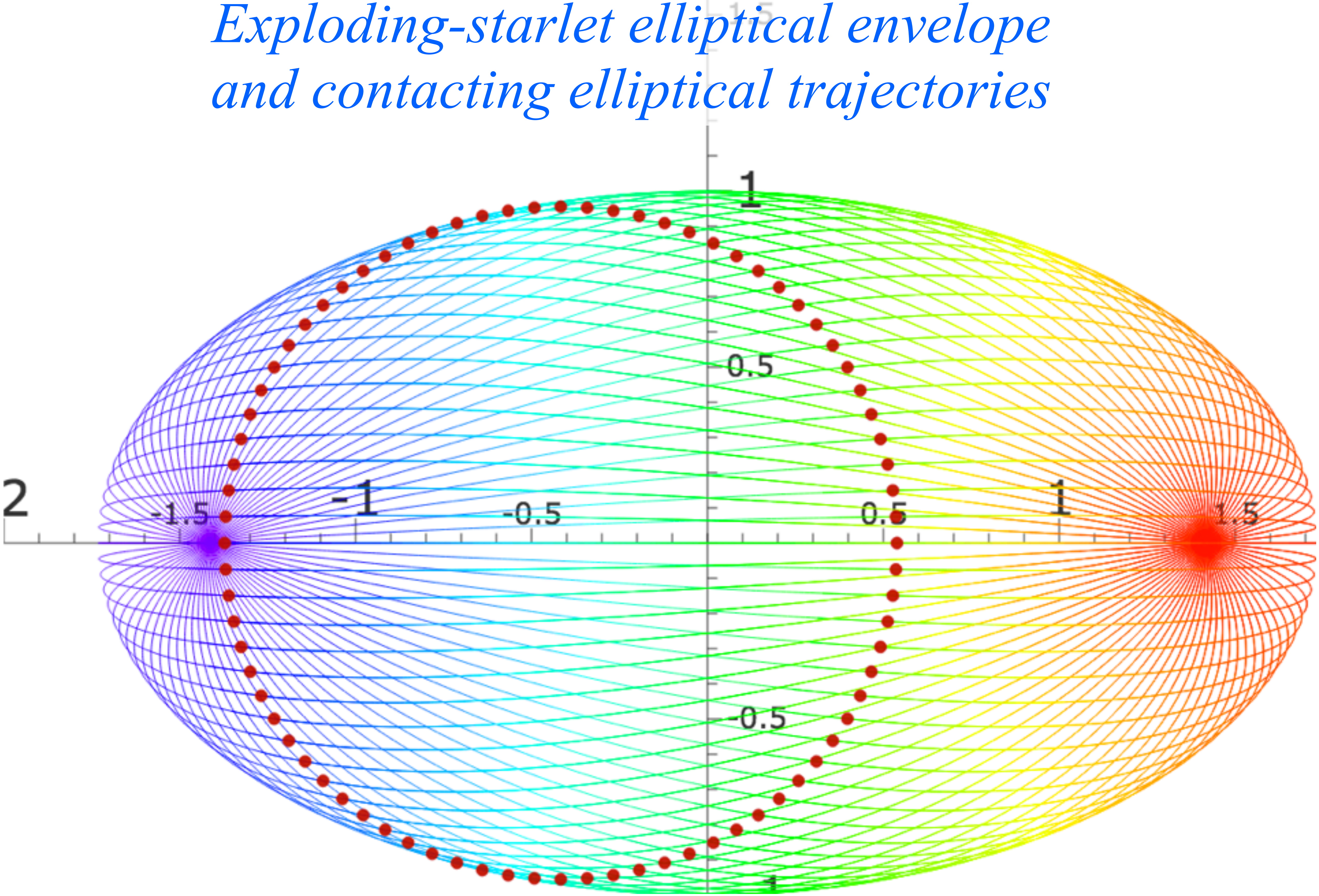
Parabolic orbital trajectory envelopes



Parabolic and 2D-IHO orbital envelopes (Review of Lecture 9 p.56-81 and a generalization.)

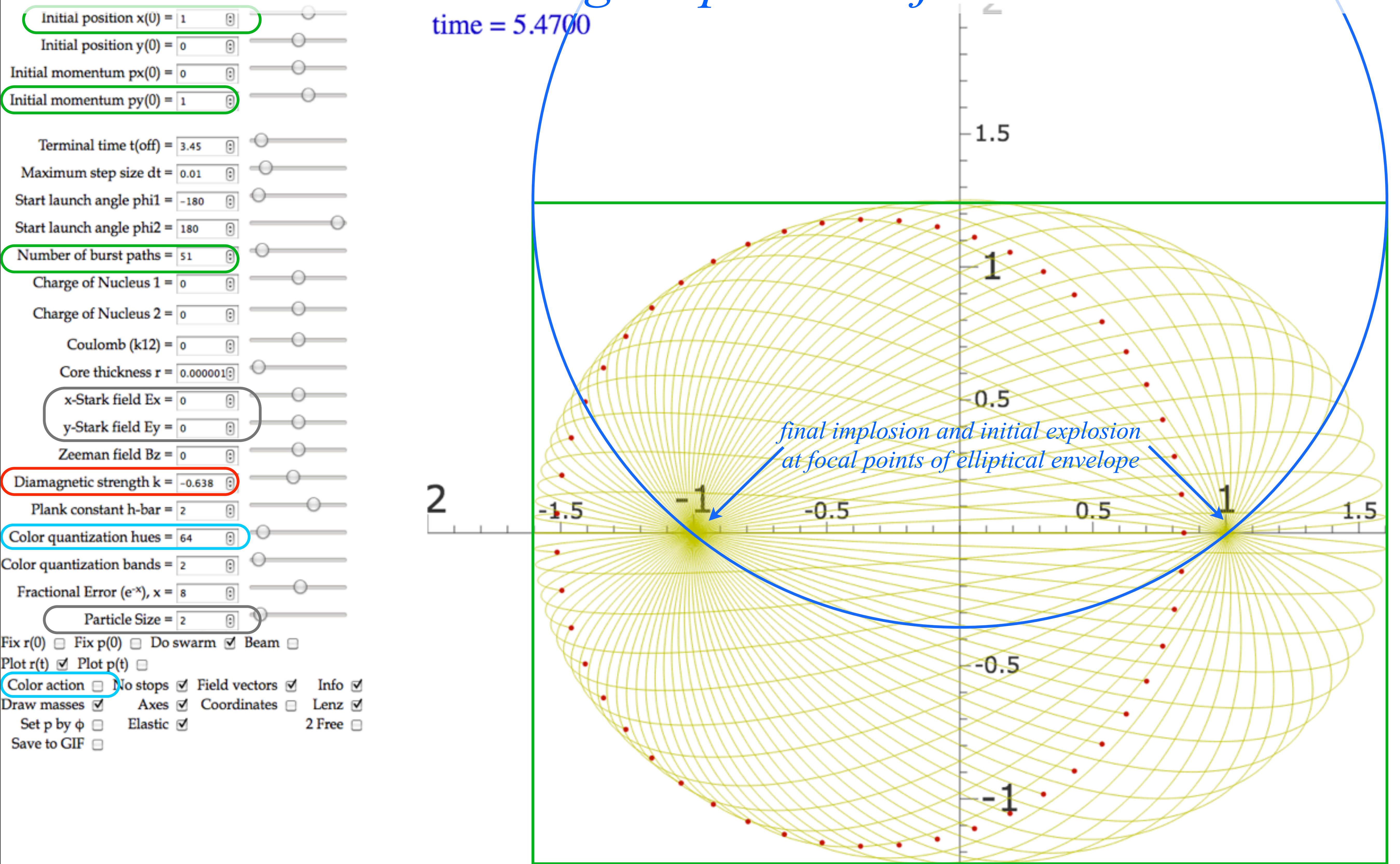
Some clues for future assignments *(Web Simulation: CoulIt)*

Exploding-starlet elliptical envelope and contacting elliptical trajectories



*Web Simulation: CouIIt - Exploding*Starlet (IHO Potential)*

Exploding-starlet elliptical envelope and contacting elliptical trajectories



Web Simulation: CoulIt - Exploding*Starlet (IHO Potential)

Examples of Hamiltonian mechanics in phase plots

1D Pendulum and phase plot *(Web Simulations: Pendulum, Cycloidulum, JerkIt (Vert Driven Pendulum))*

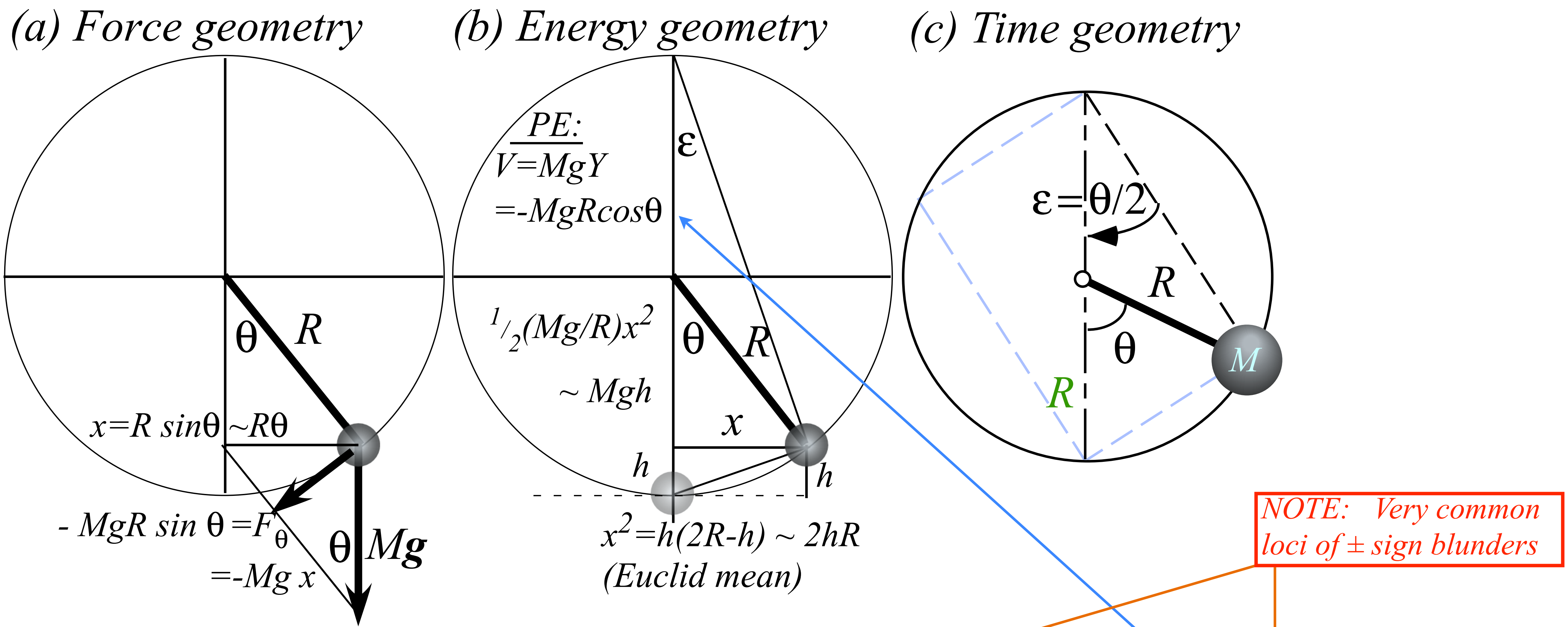
Circular pendulum dynamics and elliptic functions

Cycloid pendulum dynamics and "sawtooth" functions

1D-HO phase-space control *(Web Simulation of "Catcher in the Eye")*

1D Pendulum and phase plot

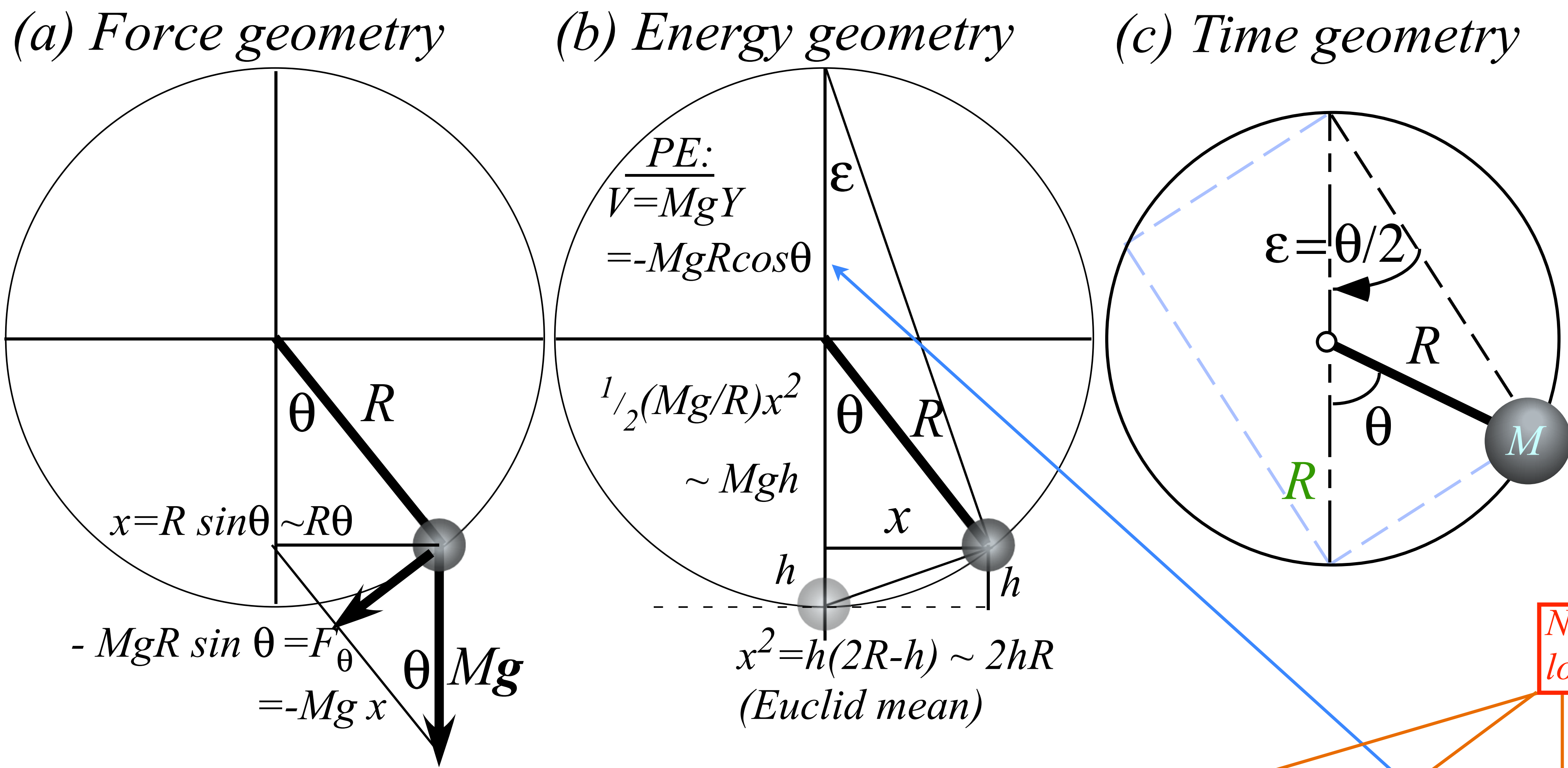
(Unit 2 Chapter 7 Fig. 2)



Lagrangian function L= KE - PE = T - U where potential energy is U(θ) = $-MgR\cos\theta$

$$L(\dot{\theta},\theta) = \frac{1}{2} I\dot{\theta}^2 - U(\theta) = \frac{1}{2} I\dot{\theta}^2 + MgR\cos\theta$$

1D Pendulum and phase plot



NOTE: Very common loci of ± sign blunders

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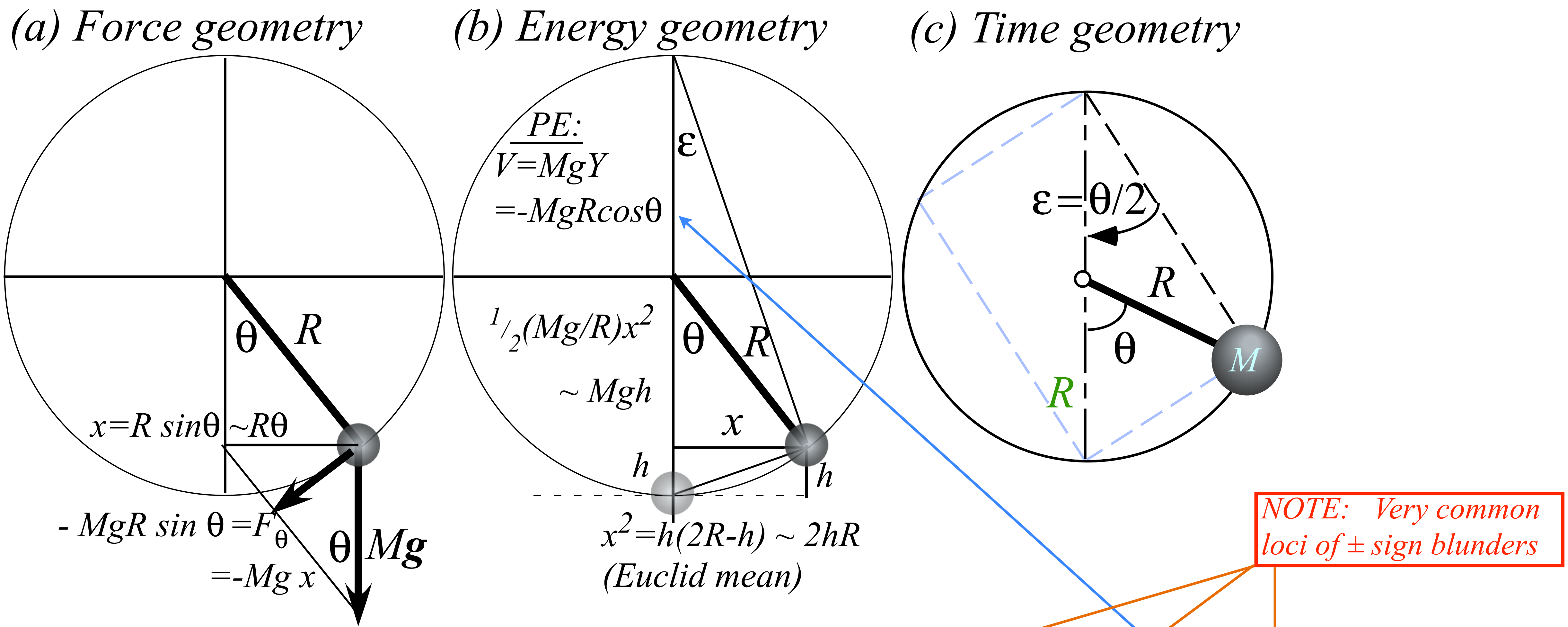
$$L(\dot{\theta},\theta)=\frac{1}{2}I\dot{\theta}^2-U(\theta)=\frac{1}{2}I\dot{\theta}^2+MgR\cos\theta$$

Hamiltonian function H= KE + PE = T +U where potential energy is U(θ) = $-MgR\cos\theta$

$$H(p_\theta,\theta)=\frac{1}{2I}p_\theta{}^2+U(\theta)=\frac{1}{2I}p_\theta{}^2-MgR\cos\theta \;=E \quad =const.$$

1D Pendulum and phase plot



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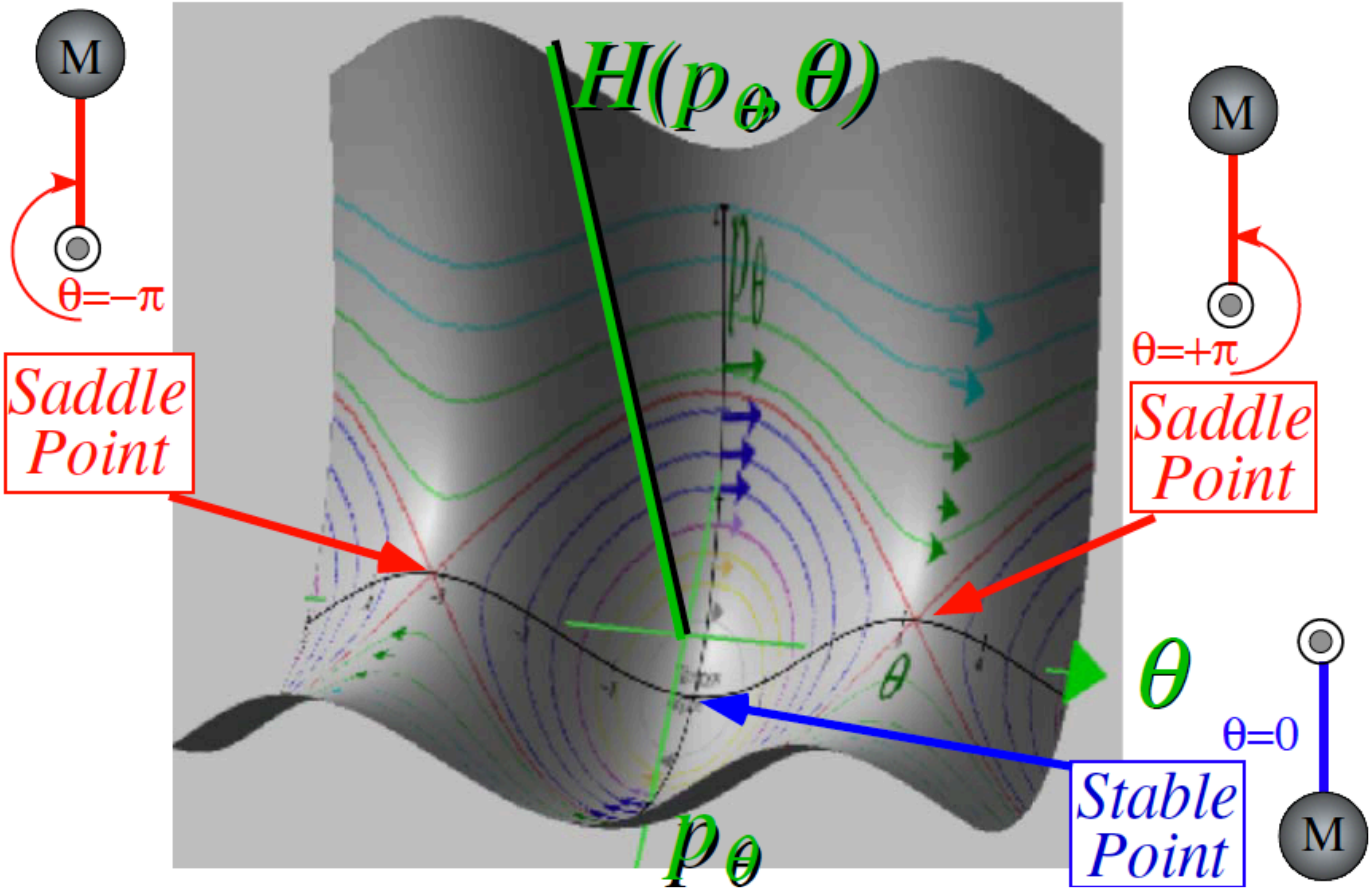
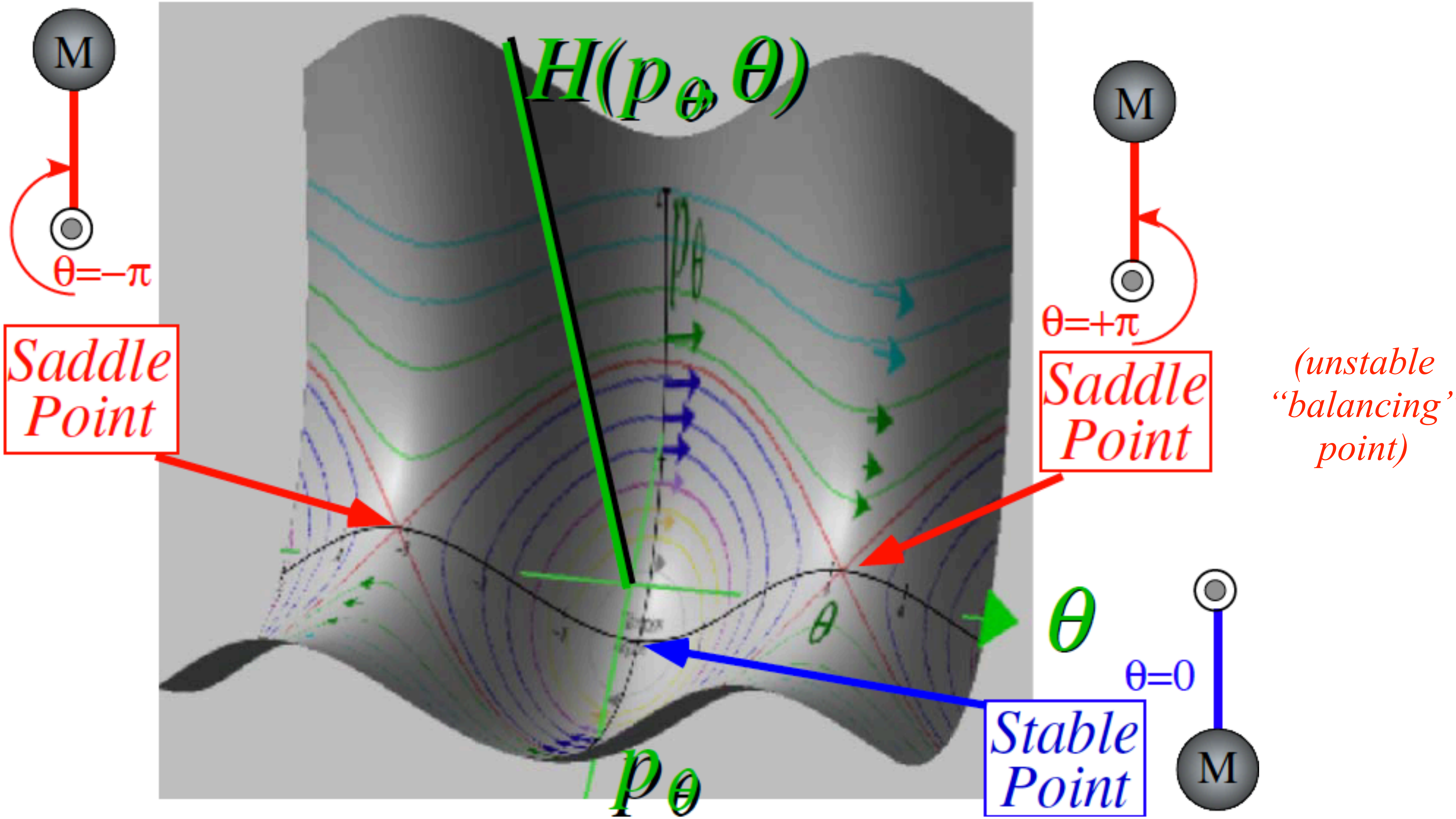
implies: $p_\theta = \sqrt{2I\left(E + MgR\cos\theta\right)}$



Example of plot of Hamilton for 1D-solid pendulum in its Phase Space (θ,p_θ)

$$H(p_\theta,\theta) = E = \frac{1}{2I}p_\theta{}^2 - MgR\cos\theta\ , \quad \text{or:} \quad p_\theta = \sqrt{2I\left(E + MgR\cos\theta\right)}$$



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Funny way to look at Hamilton's equations:

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} \partial_p H \\ -\partial_q H \end{pmatrix} = \mathbf{e_H} \times (-\nabla H) = \overrightarrow{(\text{H-axis})} \times \overrightarrow{(\text{fall line})}, \text{ where:} \begin{cases} \overrightarrow{(\text{H-axis})} = \mathbf{e_H} = \mathbf{e_q} \times \mathbf{e_p} \\ \overrightarrow{(\text{fall line})} = -\nabla H \end{cases}$$

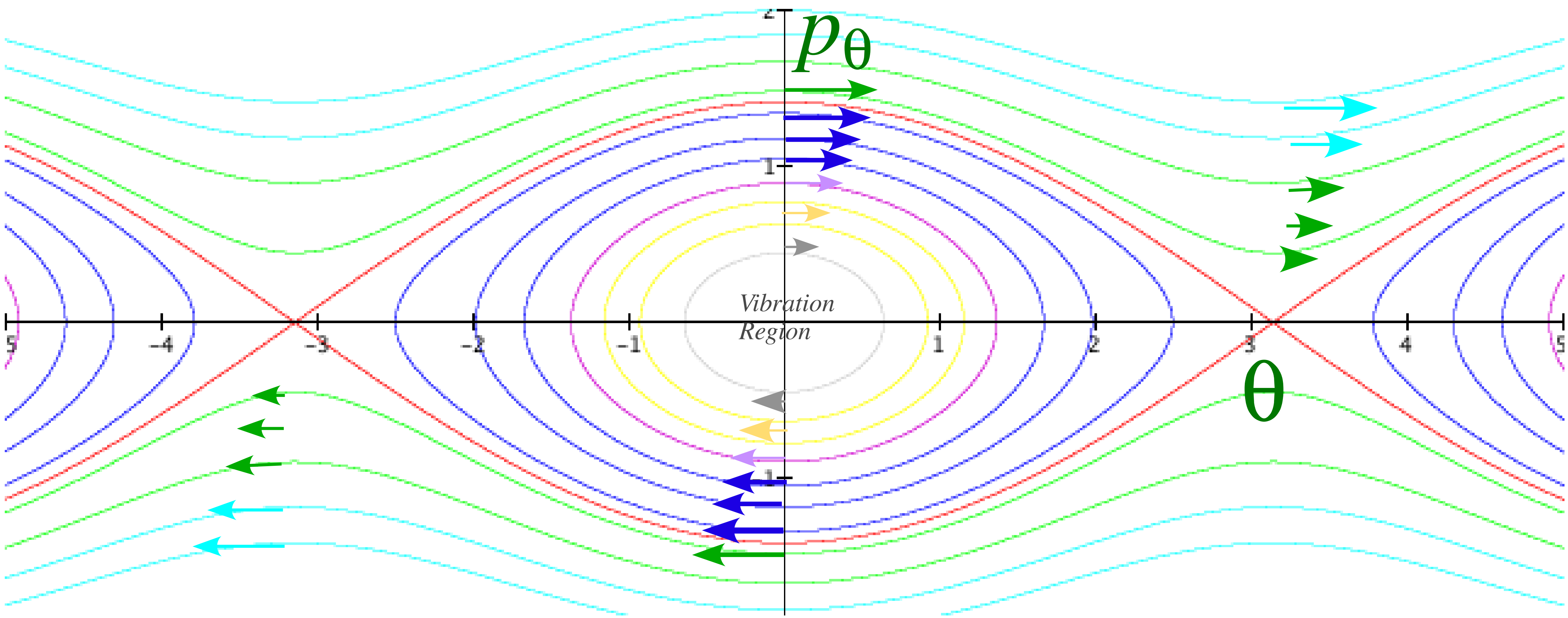

Fig. 2.7.2 Phase portrait or topography map for simple pendulum

(Unit 2 Chapter 7 Fig. 2)

Examples of Hamiltonian mechanics in phase plots

1D Pendulum and phase plot *(Web Simulations: Pendulum, Cycloidulum, JerkIt (Vert Driven Pendulum))*

→ *Circular pendulum dynamics and elliptic functions*

Cycloid pendulum dynamics and "sawtooth" functions

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Circular pendulum dynamics and elliptic functions

Hamiltonian function H= KE + PE = T +U where potential energy is $U(\theta) = -MgR\cos\theta$

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Quadrature integral gives quarter-period $\tau_{1/4}$*:*

$$\sqrt{\frac{I}{2MgR}}\int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = \int_0^{\theta_0} dt = \left(\text{Travel time 0 to } \theta_0\right) = \tau_{1/4}$$

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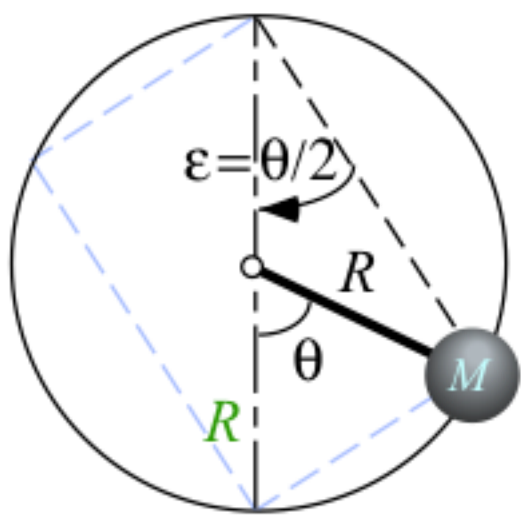

Uses a half-angle coordinate $\varepsilon = \theta/2$

$$\cos\theta = 1 - 2\sin^2\frac{\theta}{2} = 1 - 2\sin^2\varepsilon,$$

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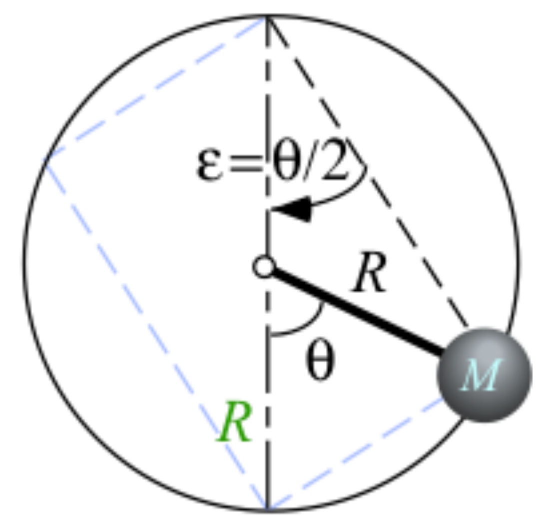

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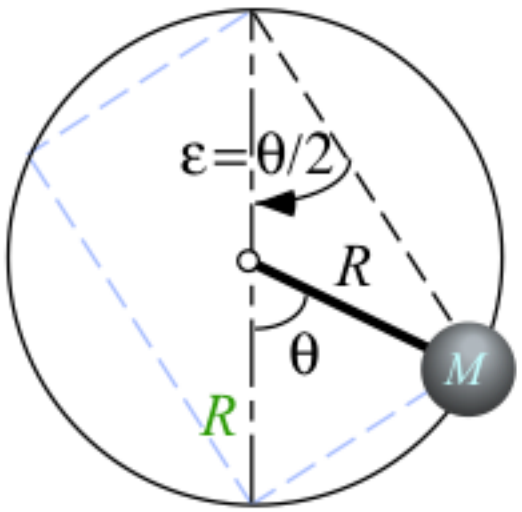

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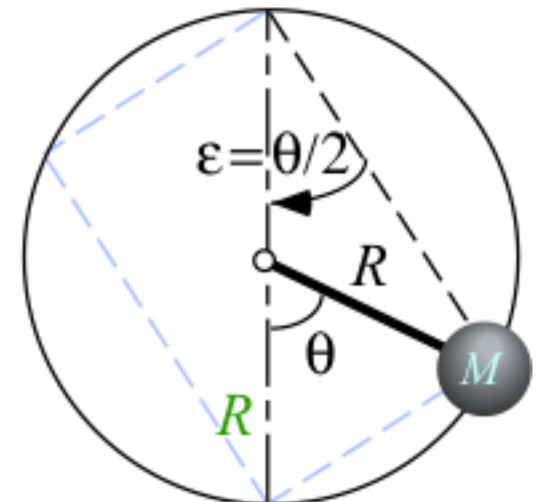

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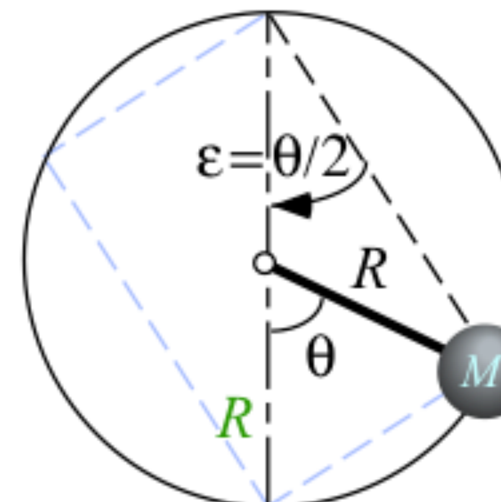

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The integral is an *elliptic integral of the first kind:* $F(k,\varepsilon_0)=am^{-1}$ or the "inverse amu" function.

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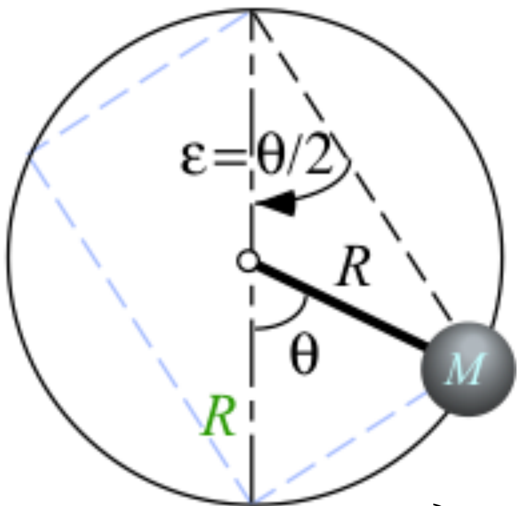

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For low amplitude $\varepsilon \ll 1$: $\sin\varepsilon_0 \simeq \varepsilon_0$ reduces $\tau_{1/4}$ to $\tau\frac{2\pi}{4}$

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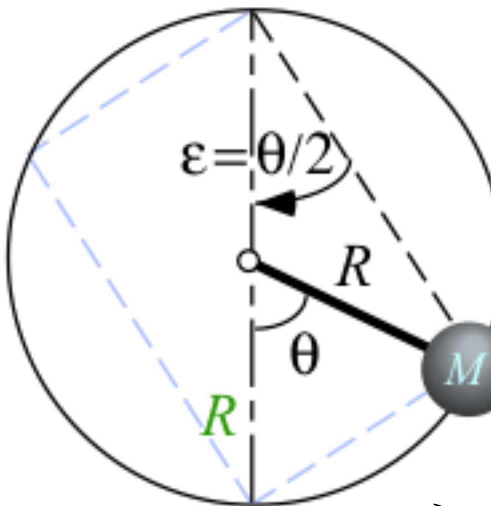


Uses a half-angle coordinate $\varepsilon=\theta/2$

$$\cos\theta = 1 - 2\sin^2\frac{\theta}{2} = 1 - 2\sin^2\varepsilon, \qquad \cos\theta - \cos\theta_0 = 2\sin^2\varepsilon_0 - 2\sin^2\varepsilon$$

$$\tau_{1/4} = \sqrt{\frac{I}{MgR}}\int_0^{\varepsilon_0}\frac{d\varepsilon}{\sqrt{\sin^2\varepsilon_0 - \sin^2\varepsilon}} = \sqrt{\frac{R}{g}}\int_0^{\varepsilon_0}\frac{k\,d\varepsilon}{\sqrt{1 - k^2\sin^2\varepsilon}} \text{, where:} \left\{ \begin{array}{c} 1/k = \sin\varepsilon_0 = \sin\frac{\theta_0}{2} \\ I = MR^2 \end{array} \right\}$$

The integral is an *elliptic integral of the first kind:* $F(k,\varepsilon_0)=am^{-1}$ or the "inverse amu" function.

$$F(k,\varepsilon_0) \equiv \int_0^{\varepsilon_0}\frac{d\varepsilon}{\sqrt{1 - k^2\sin^2\varepsilon}} \equiv am^{-1}(k,\varepsilon_0)$$

$$\tau_{1/4} = \sqrt{\frac{R}{g}}\int_0^{\varepsilon_0}\frac{d\varepsilon}{\sqrt{\varepsilon_0^{\;2} - \varepsilon^2}} = \sqrt{\frac{R}{g}}\sin^{-1}\frac{\varepsilon}{\varepsilon_0}\Big|_0^{\varepsilon_0} = \sqrt{\frac{R}{g}}\frac{\pi}{2} = \tau\frac{2\pi}{4}$$

low $\varepsilon \ll 1$: $t = \sqrt{\frac{R}{g}}\int_0^{\varepsilon(t)}\frac{d\varepsilon}{\sqrt{\varepsilon_0^2 - \varepsilon^2}} = \sqrt{\frac{R}{g}}\sin^{-1}\frac{\varepsilon}{\varepsilon_0}\Big|_0^{\varepsilon(t)} = \sqrt{\frac{R}{g}}\sin^{-1}\frac{\varepsilon(t)}{\varepsilon_0}$

For low amplitude $\varepsilon \ll 1$: $\sin\varepsilon_0 \simeq \varepsilon_0$ reduces $\tau_{1/4}$ to $\tau\frac{2\pi}{4}$

Circular pendulum dynamics and elliptic functions

Hamiltonian function H= KE + PE = T +U where potential energy is $U(\theta) = -MgR\cos\theta$

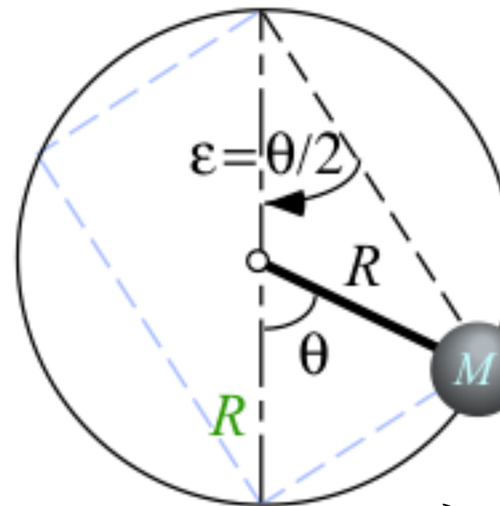
$$H(p_\theta,\theta) = \frac{1}{2I}p_\theta{}^2 + U(\theta) = \frac{1}{2I}p_\theta{}^2 - MgR\cos\theta = E \quad = const. \qquad \text{implies: } p_\theta = \sqrt{2I(E + MgR\cos\theta)}$$

Let $E=MgY=-MgR\cos\theta_0$ *be potential energy where* $KE=0$ *or* $p_\theta=0$

$$\frac{\partial H}{\partial p_\theta} = \dot{\theta} = \frac{d\theta}{dt} = p_\theta / I = \sqrt{2I(E+MgR\cos\theta)}/I \quad \text{where: } I = MR^2 \quad \text{or : } dt = \frac{d\theta}{\sqrt{2(E+MgR\cos\theta)/I}}$$

Quadrature integral gives quarter-period $\tau_{1/4}$*:*

$$\sqrt{\frac{I}{2MgR}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta-\cos\theta_0}} = \int_0^{\theta_0} dt = (\text{Travel time 0 to } \theta_0) = \tau_{1/4}$$



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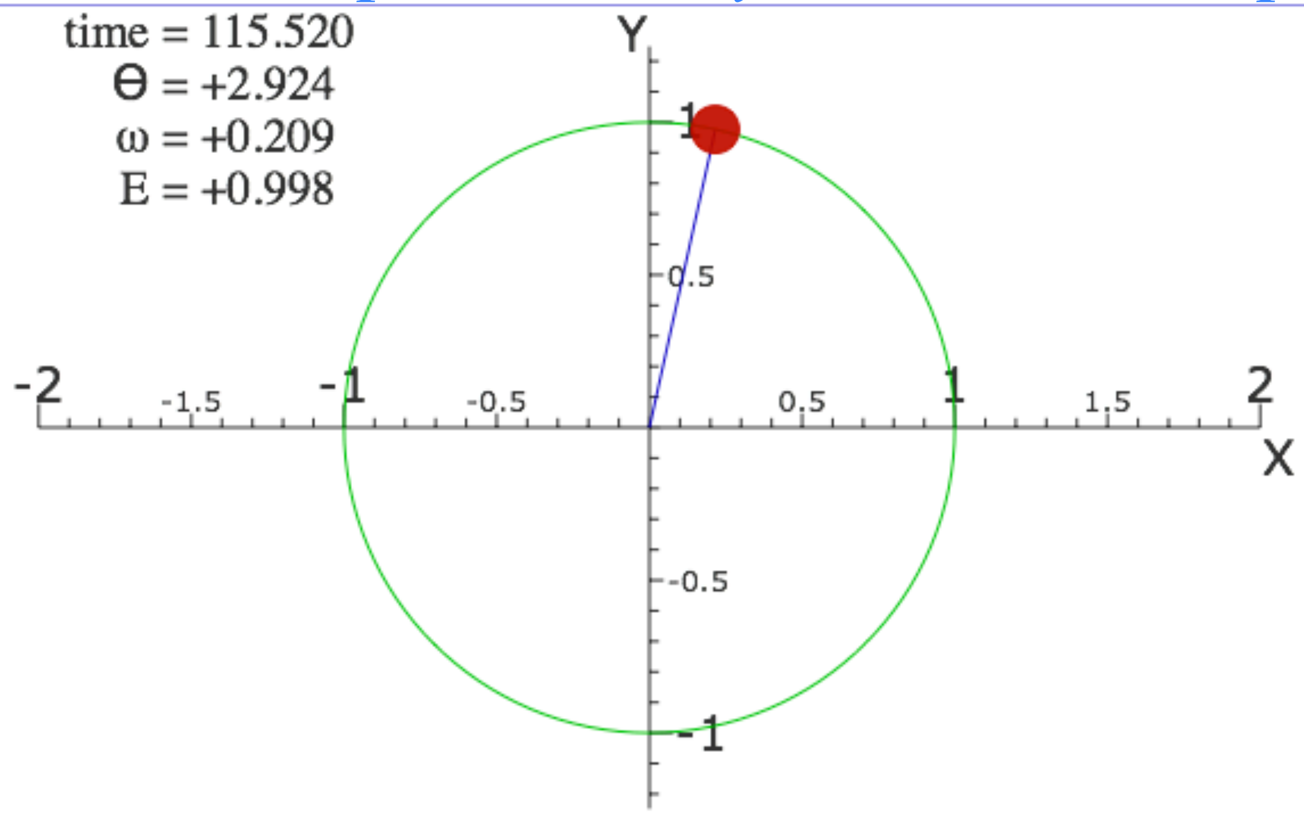
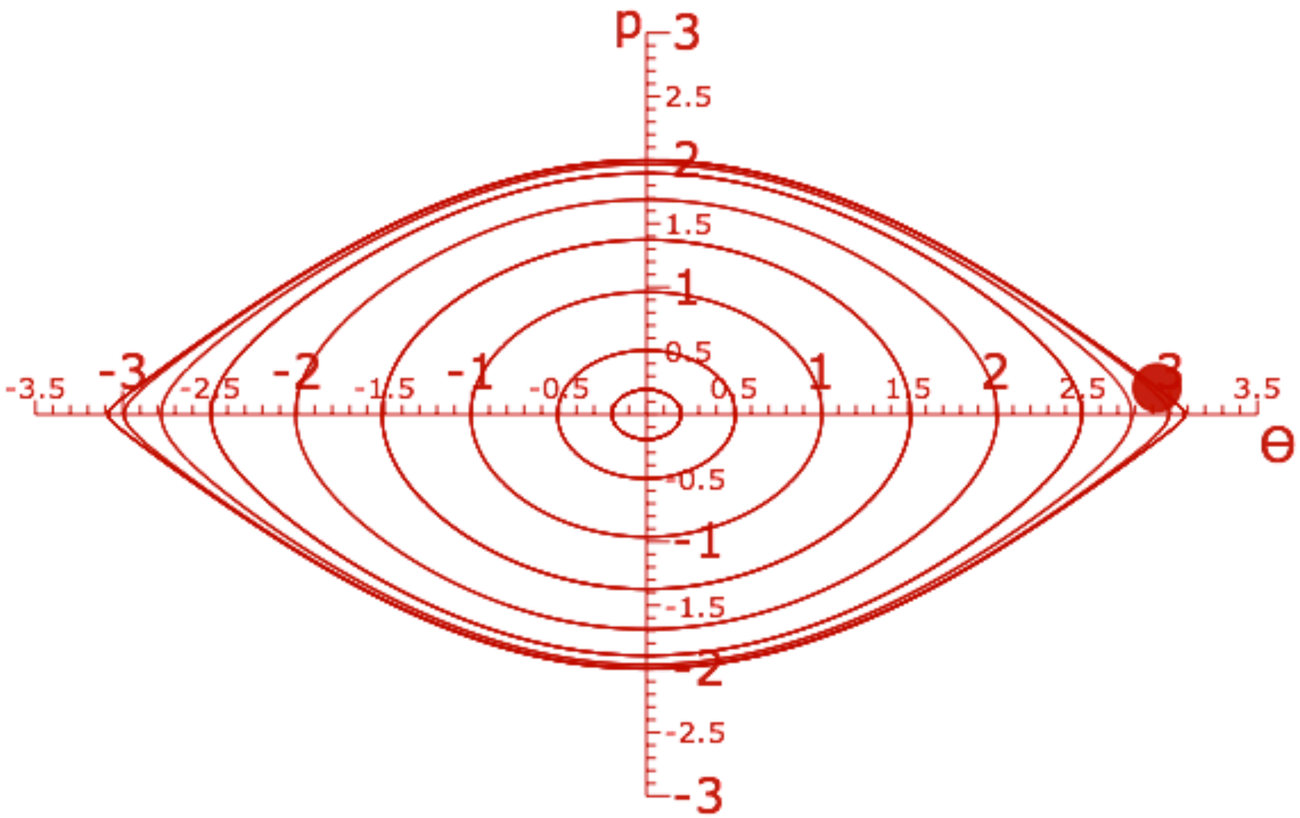
..reduces to sine...

$$\varepsilon(t) = \varepsilon_0\sin\sqrt{\frac{g}{R}}t = \varepsilon_0\sin\omega t, \text{ where: } \omega = \sqrt{\frac{g}{R}}$$

For low amplitude $\varepsilon \ll 1$: $\sin\varepsilon_0 \simeq \varepsilon_0$ reduces $\tau_{1/4}$ to $\tau\frac{2\pi}{4}$

Circular pendulum dynamics and elliptic functions

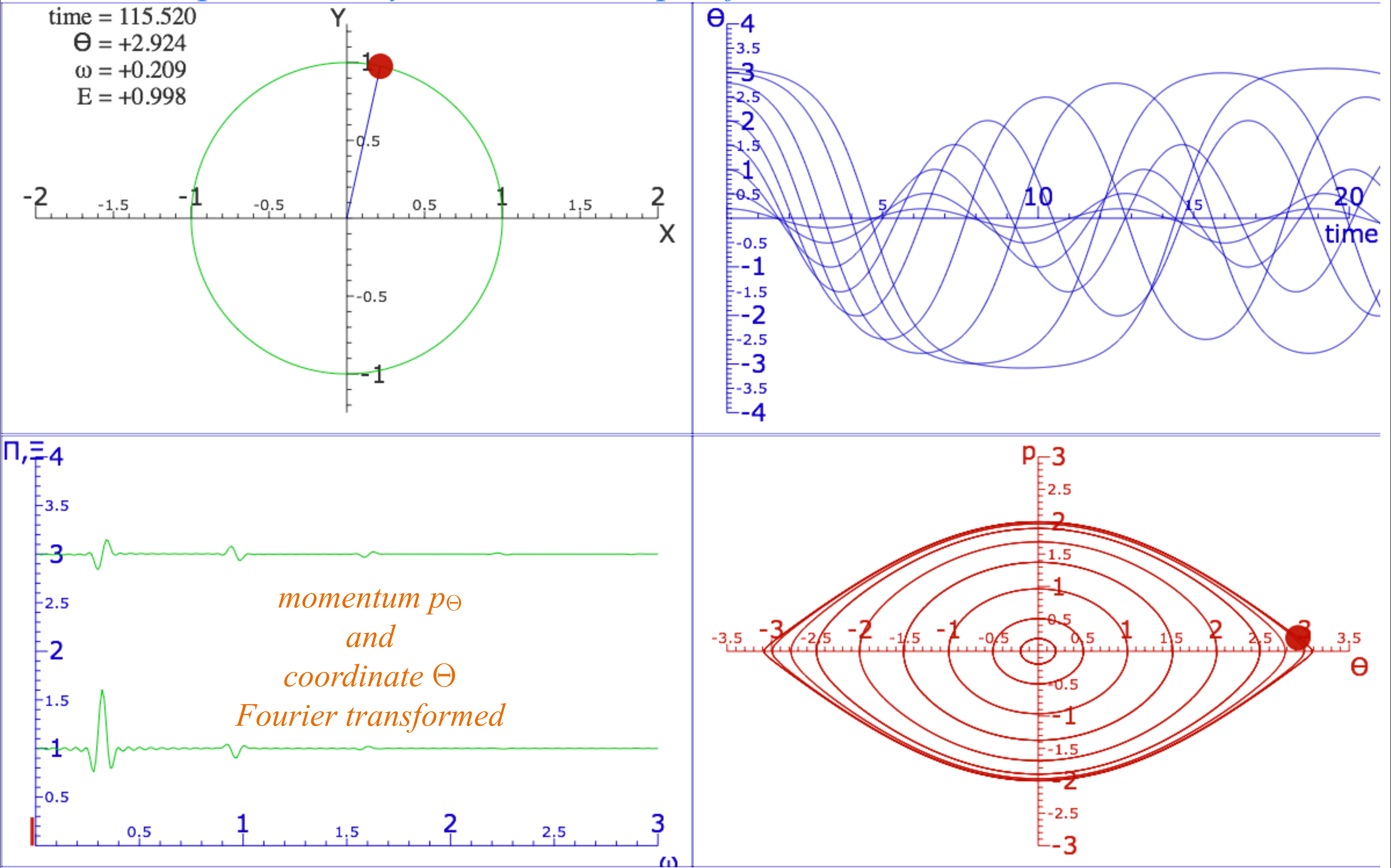




(Simulation of pendulum)

(See also: Simulation of cycloidally constrained pendulum)

Circular pendulum dynamics and elliptic functions



(Simulation of pendulum)

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Examples of Hamiltonian mechanics in phase plots

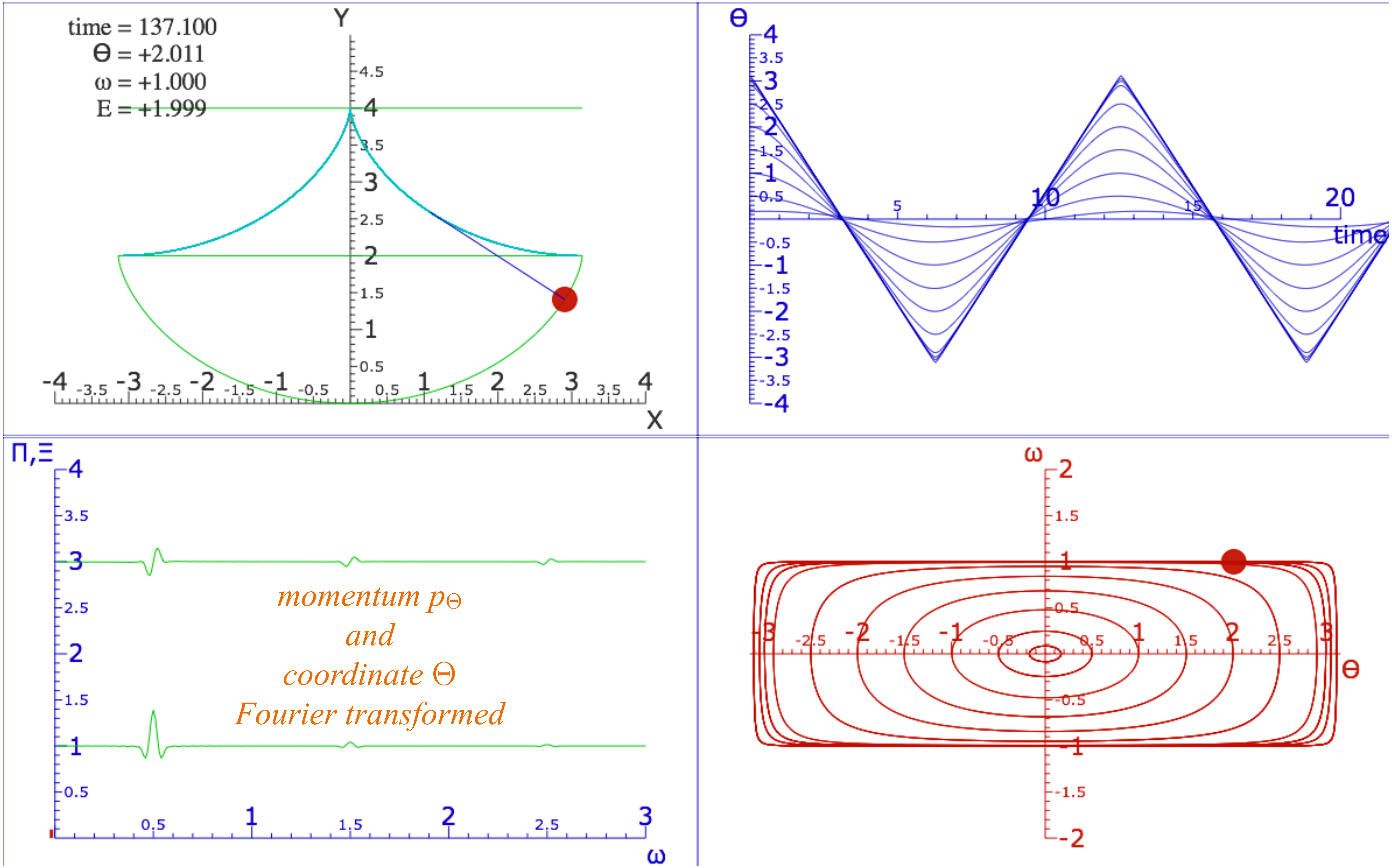
1D Pendulum and phase plot *(Web Simulations: Pendulum, Cycloidulum, JerkIt (Vert Driven Pendulum))*

Circular pendulum dynamics and elliptic functions

Cycloid pendulum dynamics and "sawtooth" functions

1D-HO phase-space control *(Web Simulation of "Catcher in the Eye")*

Cycloid pendulum dynamics and “sawtooth” functions



(Simulations of cycloidally constrained pendulum)

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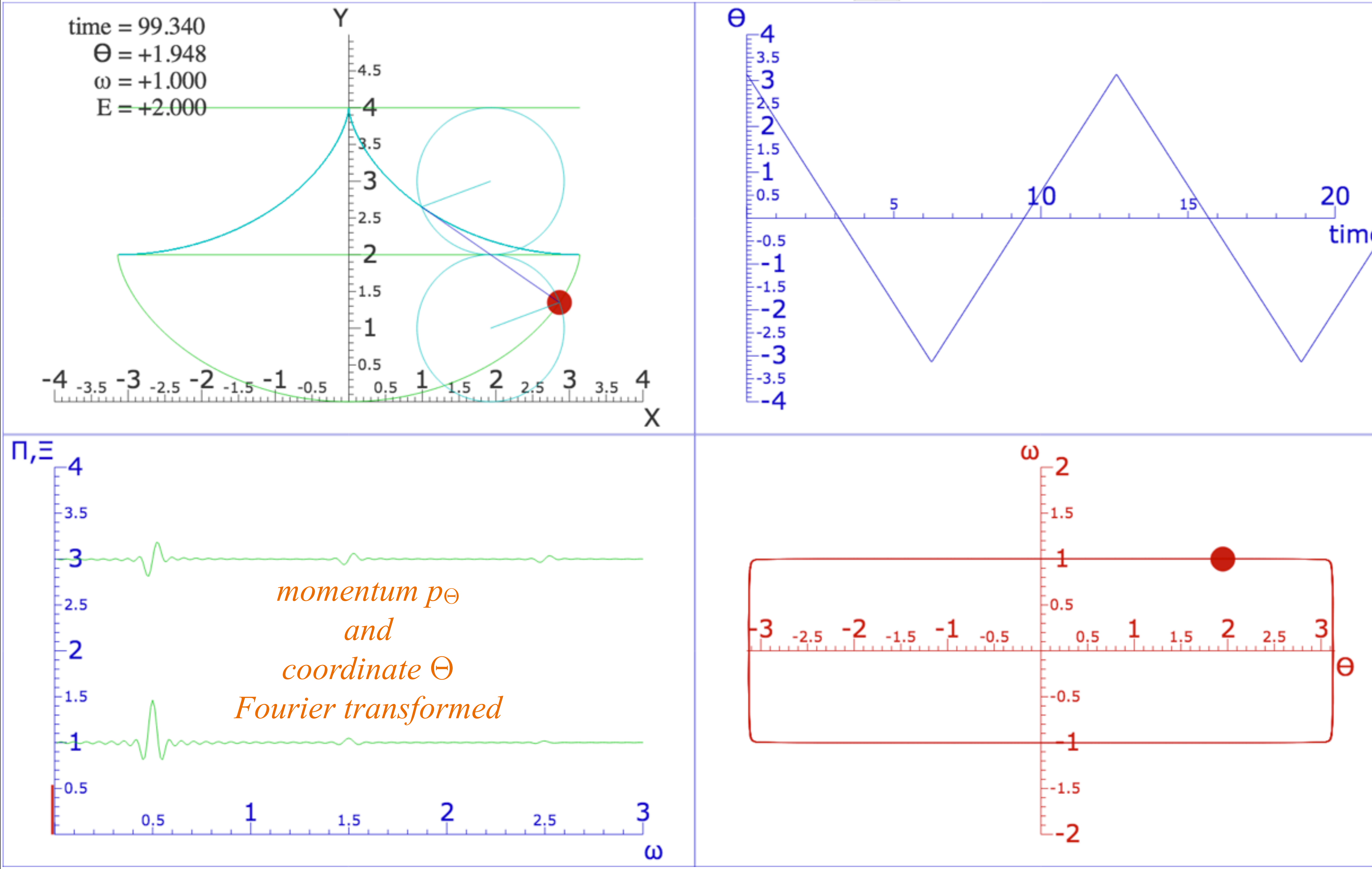

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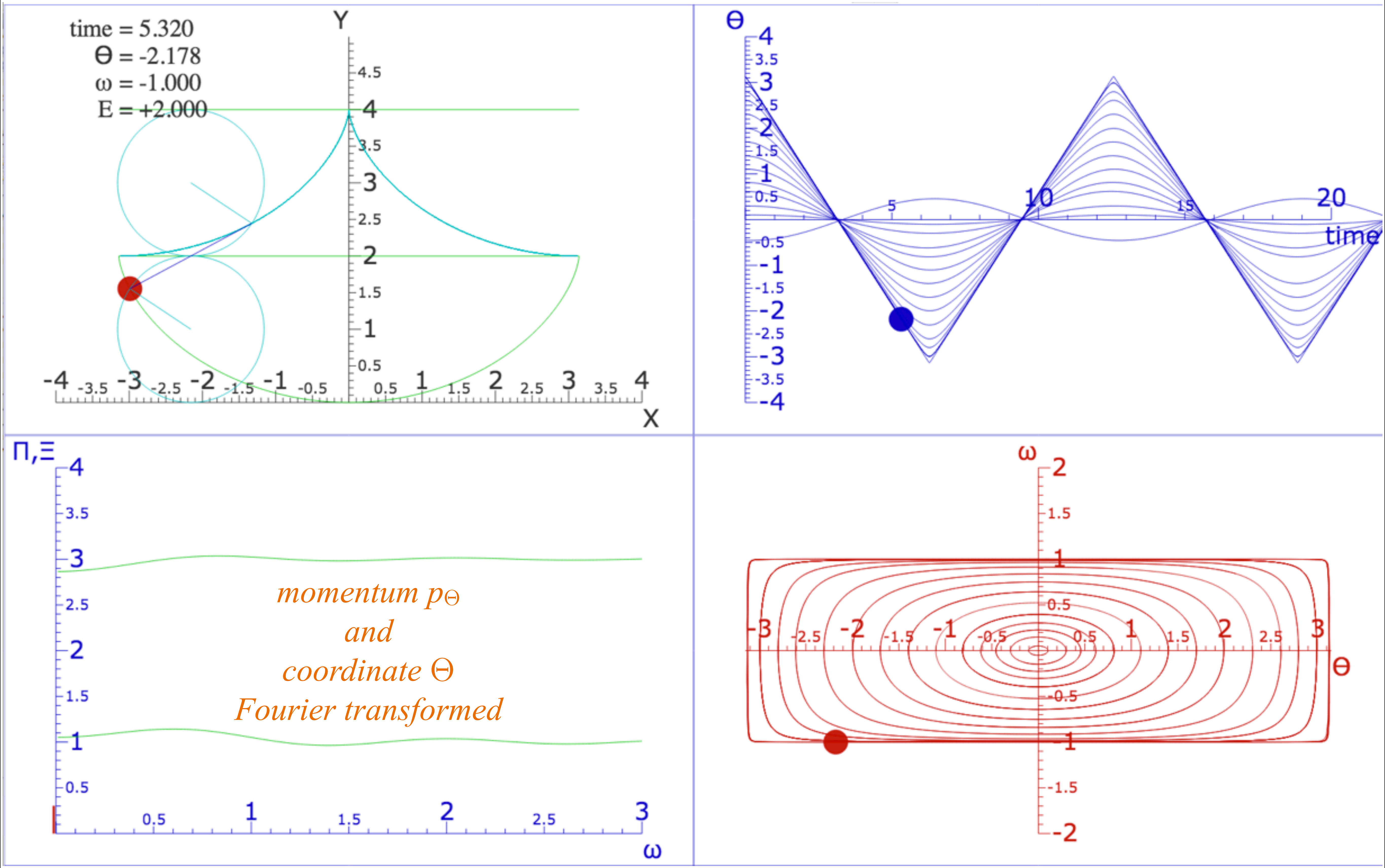

[*(Simulations of cycloidally constrained pendulum)*]

Examples of Hamiltonian mechanics in phase plots

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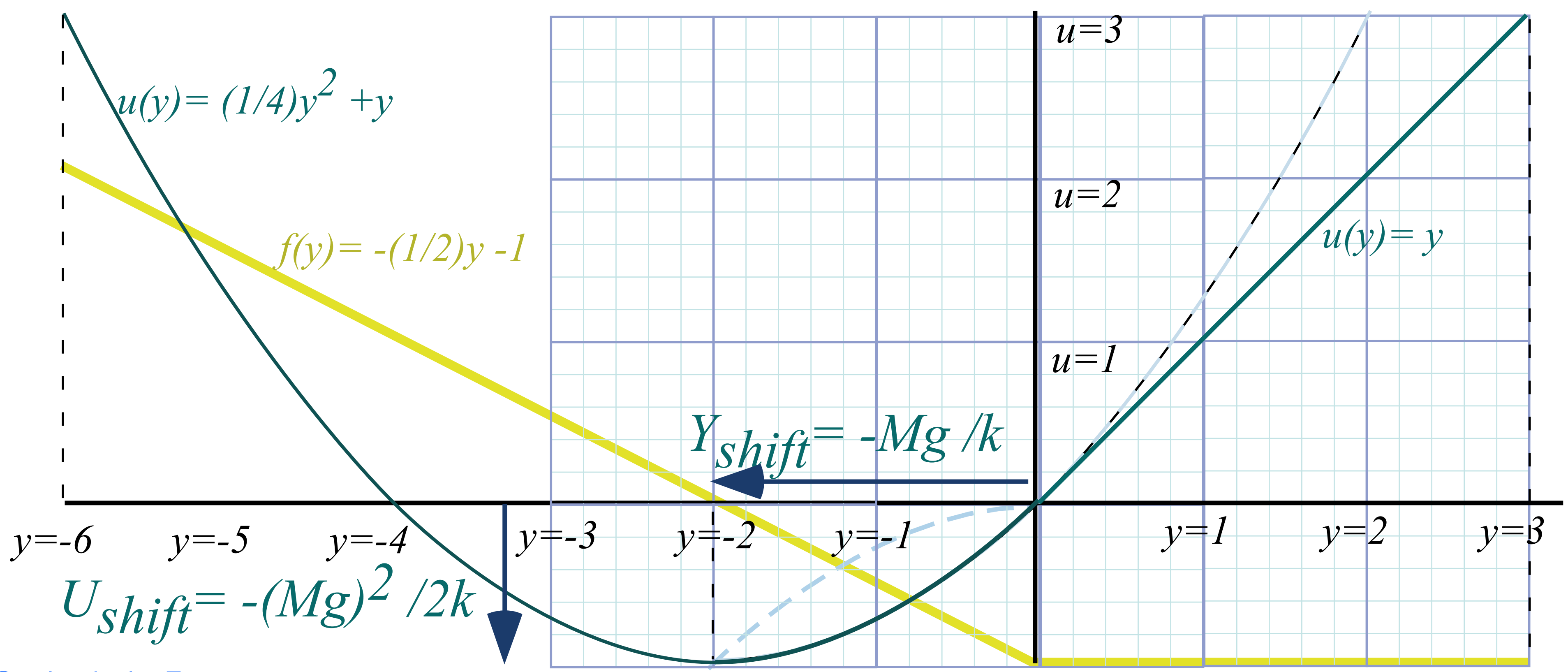
Circular pendulum dynamics and elliptic functions

Cycloid pendulum dynamics and "sawtooth" functions

1D-HO phase-space control *(Web Simulation of "Catcher in the Eye")*

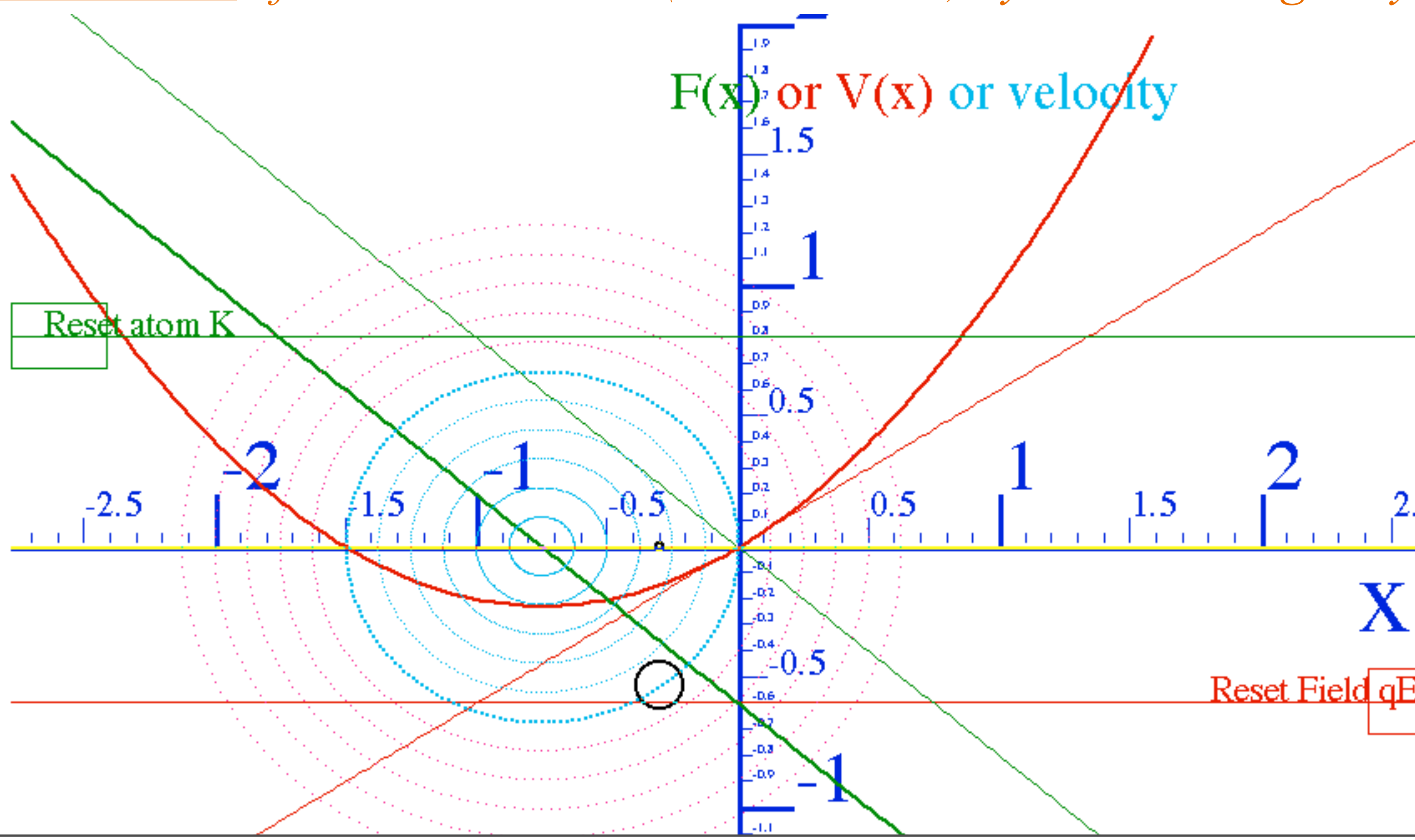
$F(Y)=-kY-Mg$ $\qquad$ $U(Y)=(1/2)kY^2+Mg\,Y$



Unit 1
Fig. 7.4

Catcher in the Eye

Web Simulation of atomic classical (or semi-classical) dynamics using varying phase control



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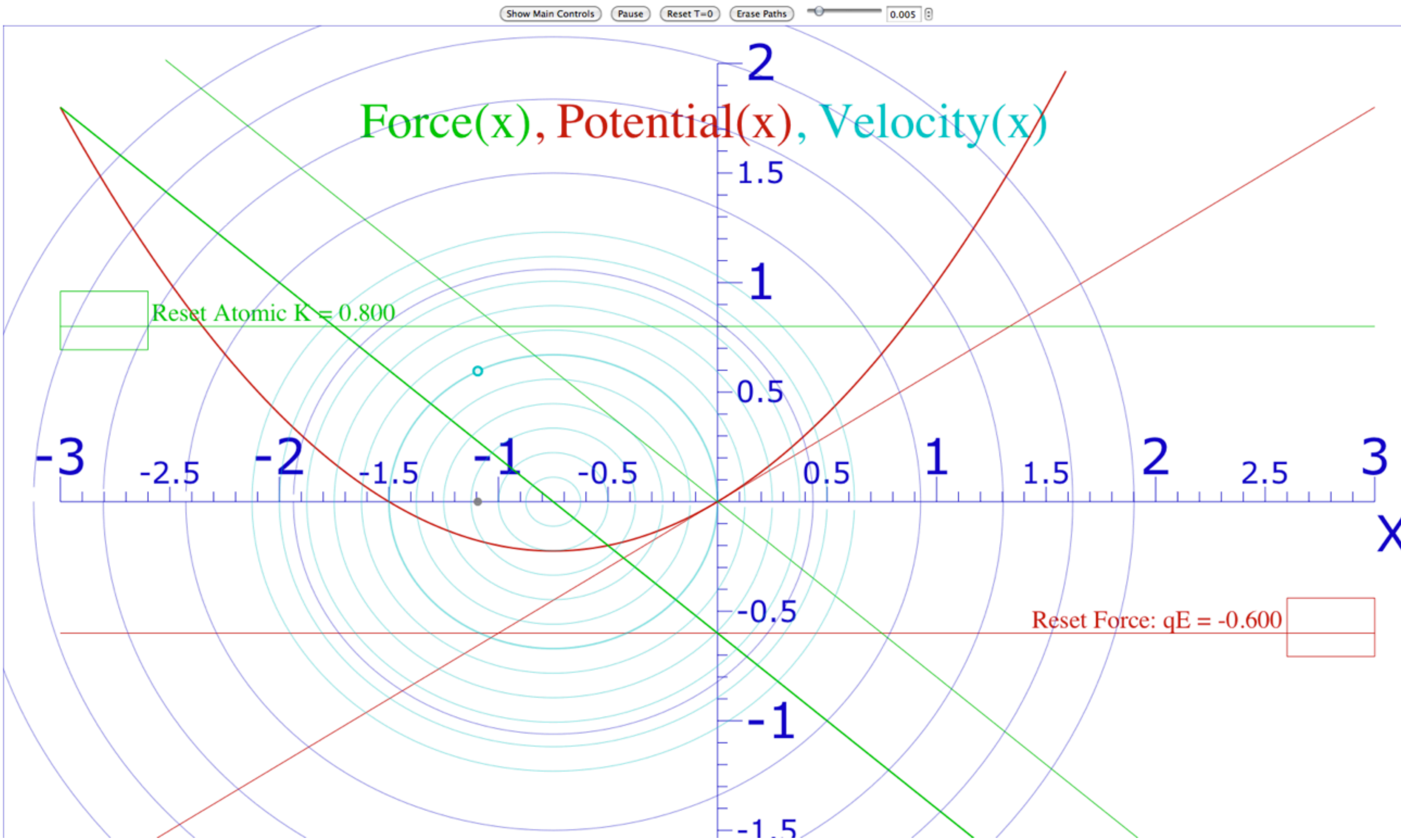


Web Simulation: JerkIt - "Catcher in the Eye"

Exploring phase space and Lagrangian mechanics more deeply

A weird "derivation" of Lagrange's equations

Poincare identity and Action, Jacobi-Hamilton *equations*

How Classicists might have "derived" quantum equations

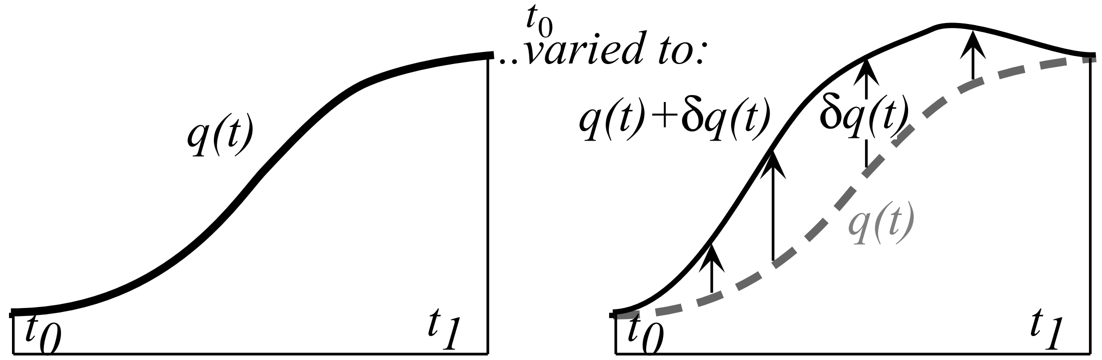
Huygen's contact transformations enforce minimum action

How to do quantum mechanics if you only know classical mechanics

A strange "derivation" of Lagrange's equations by Calculus of Variation

Variational calculus finds extreme (minimum or maximum) values to entire integrals

Minimize (or maximize): $S(q) = \int_{t_0}^{t_1} dt\ L(q(t), \dot{q}(t), t).$



An arbitrary but small variation function $\delta q(t)$ is allowed at every point t in the figure along the curve except at the end points t_0 and t_1. There we demand it not vary at all.(1)

$$\delta q(t_0) = 0 = \delta q(t_1) \qquad (1)$$

1st order L(q+δq) approximate:

$$S(q+\delta q) = \int_{t_0}^{t_1} dt \left[L(q, \dot{q}, t) + \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q} \right] \text{ where: } \delta \dot{q} = \frac{d}{dt}\delta q$$

A weird "derivation" of Lagrange's equations

Variational calculus finds extreme (minimum or maximum) values to entire integrals

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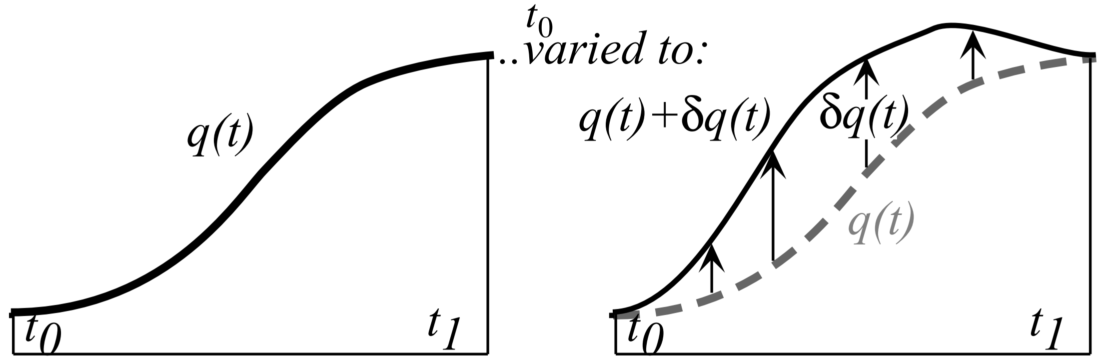

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A weird "derivation" of Lagrange's equations

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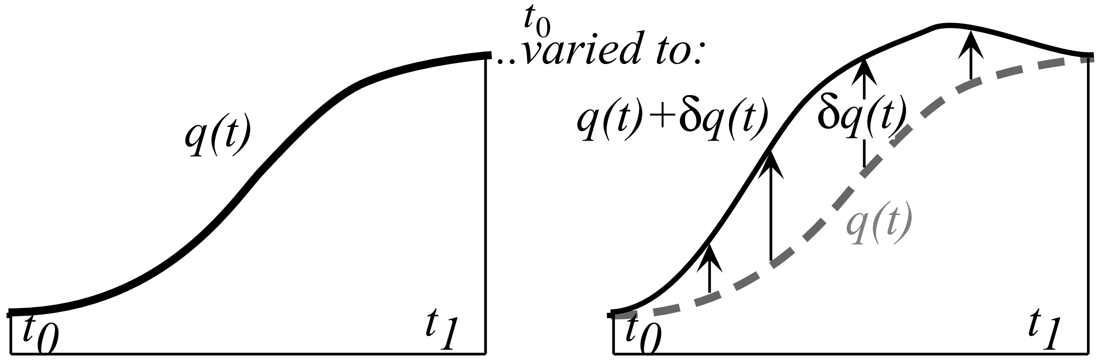

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$$S(q+\delta q)=\int_{t_0}^{t_1} dt\left[L(q,\dot{q},t)+\frac{\partial L}{\partial q}\delta q-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q\right]+\int_{t_0}^{t_1} dt\,\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\delta q\right)$$

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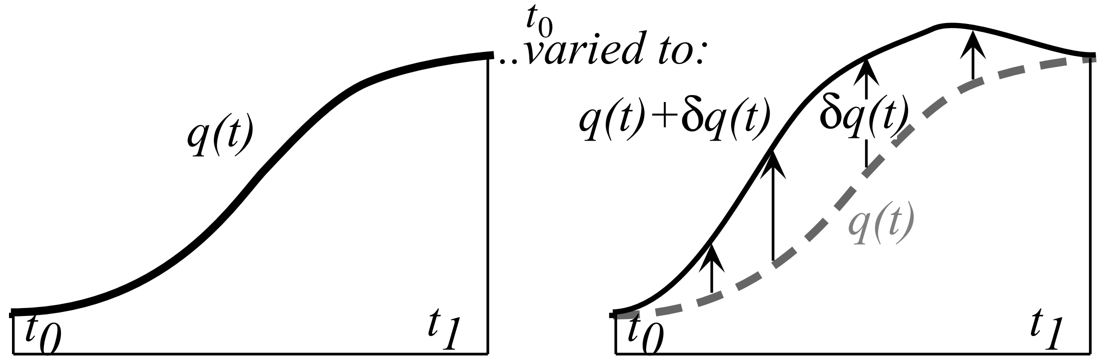

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$$=\int_{t_0}^{t_1} dt\ L(q,\dot{q},t)+\int_{t_0}^{t_1} dt\left[\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right]\delta q+\left(\frac{\partial L}{\partial \dot{q}}\delta q\right)\Bigg|_{t_0}^{t_1}$$

A weird "derivation" of Lagrange's equations

Variational calculus finds extreme (minimum or maximum) values to entire integrals

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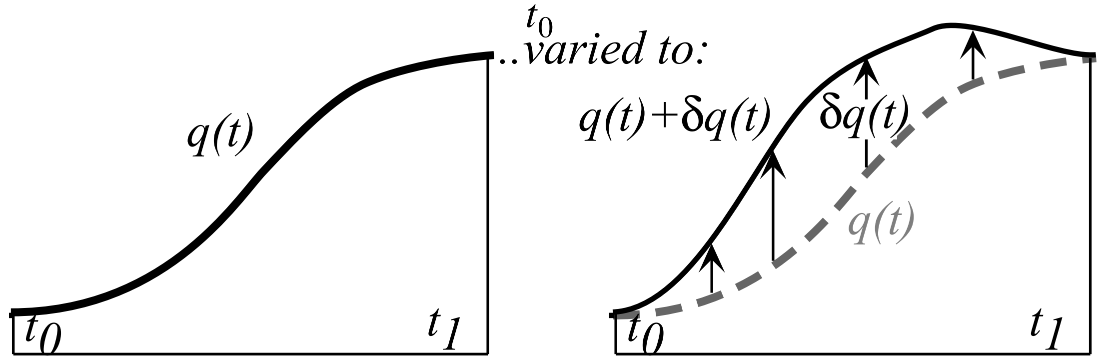

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due to requiring (1)

Third term vanishes by (1). This leaves first order variation: $\delta S = S(q+\delta q) - S(q) = \int_{t_0}^{t_1} dt \left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right]\delta q$

Extreme value (actually *minimum* value) of *S(q)* occurs *if and only if* Lagrange equation is satisfied!

$$\delta S = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

Euler-Lagrange equation(s)

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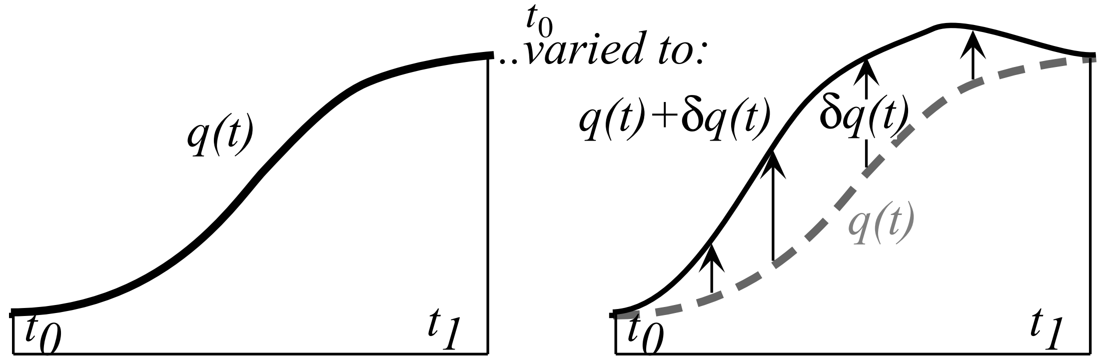

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Replace $\frac{\partial L}{\partial \dot{q}}\delta \dot{q}$ with $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\delta q\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q$

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Euler-Lagrange equation(s)

But, WHY is nature so inclined to fly JUST SO as to <u>minimize</u> the Lagrangian L = T - U???

Exploring phase space and Lagrangian mechanics more deeply

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Poincare identity and Action, Jacobi-Hamilton *equations*

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Legendre-Poincare identity and Action

Legendre transform $L(\mathbf{v}) = \mathbf{p}\bullet\mathbf{v} - H(\mathbf{p})$ becomes *Poincare's invariant differential* if dt is cleared.

$$L\cdot dt = \mathbf{p}\bullet\mathbf{v}\cdot dt - H\cdot dt = \mathbf{p}\bullet d\mathbf{r} - H\cdot dt \qquad \left(\mathbf{v}=\frac{d\mathbf{r}}{dt} \textit{ implies: } \mathbf{v}\cdot dt = d\mathbf{r}\right)$$

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This is the time differential dS of *action* $S = \int L\cdot dt$ whose time derivative is rate L of *quantum phase*.

$$dS = L\cdot dt = \mathbf{p}\bullet d\mathbf{r} - H\cdot dt \quad \text{where:} \quad L = \frac{dS}{dt}$$

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Unit 8 shows *DeBroglie law* $\mathbf{p}=\hbar\mathbf{k}$ and *Planck law* $H=\hbar\omega$ make *quantum plane wave phase* Φ:

$$\Phi = S/\hbar = \int L\cdot dt/\hbar$$

Legendre-Poincare identity and Action

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$$\Phi = S/\hbar = \int L\cdot dt/\hbar$$

$$\psi(\mathbf{r},t) = e^{iS/\hbar} = e^{i(\mathbf{p}\bullet\mathbf{r}-H\cdot t)/\hbar} = e^{i(\mathbf{k}\bullet\mathbf{r}-\omega\cdot t)}$$

Legendre-Poincare identity and Action

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This is the time differential dS of *action* $S = \int L\cdot dt$ whose time derivative is rate L of *quantum phase*.

$$dS = L\cdot dt = \mathbf{p}\bullet d\mathbf{r} - H\cdot dt \quad \text{where:} \quad L = \frac{dS}{dt}$$

Unit 8 shows $\boxed{\textit{DeBroglie law } \mathbf{p}=\hbar\mathbf{k}}$ and $\boxed{\textit{Planck law } H=\hbar\omega}$ make *quantum plane wave phase* Φ:

$$\Phi = S/\hbar = \int L\cdot dt/\hbar$$

$$\psi(\mathbf{r},t) = e^{iS/\hbar} = e^{i(\mathbf{p}\bullet\mathbf{r}-H\cdot t)/\hbar} = e^{i(\mathbf{k}\bullet\mathbf{r}-\omega\cdot t)}$$

Q:When is the *Action*-differential dS integrable?

A: A differential $dW=f_x(x,y)dx+f_y(x,y)dy$ is *integrable* to a $W(x,y)$ if: $f_x = \frac{\partial W}{\partial x}$ and: $f_y = \frac{\partial W}{\partial y}$

Legendre-Poincare identity and Action

Legendre transform $L(\mathbf{v}) = \mathbf{p}\bullet\mathbf{v} - H(\mathbf{p})$ becomes *Poincare's invariant differential* if dt is cleared.

$$L\cdot dt = \mathbf{p}\bullet\mathbf{v}\cdot dt - H\cdot dt = \mathbf{p}\bullet d\mathbf{r} - H\cdot dt \qquad \mathbf{v} = \frac{d\mathbf{r}}{dt}$$

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Similar to conditions for integrating work differential $dW=\mathbf{f}\bullet d\mathbf{r}$ *to get potential* $W(\mathbf{r})$. *That condition is* <u>no curl allowed</u>: $\nabla\times\mathbf{f}=\mathbf{0}$ *or* <u>∂-symmetry</u> *of* W:

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dS is integrable if: $\boxed{\frac{\partial S}{\partial \mathbf{r}} = \mathbf{p}}$ and: $\boxed{\frac{\partial S}{\partial t} = -H}$

These conditions are known as Jacobi-Hamilton *equations*

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How Classicists might have "derived" quantum equations

Huygen's contact transformations enforce minimum action

How to do quantum mechanics if you only know classical mechanics

How Jacobi-Hamilton *could have "derived" Schrodinger equations*

(Given "quantum wave")

$$\psi(\mathbf{r},t) = e^{iS/\hbar} = e^{i(\mathbf{p}\bullet\mathbf{r}-H\cdot t)/\hbar} = e^{i(\mathbf{k}\bullet\mathbf{r}-\omega\cdot t)}$$

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Try 1st **r**-*derivative of wave* ψ

$$\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \frac{\partial}{\partial \mathbf{r}} e^{iS/\hbar} = \frac{\partial\left(iS/\hbar\right)}{\partial \mathbf{r}} e^{iS/\hbar} = \left(i/\hbar\right)\frac{\partial S}{\partial \mathbf{r}}\psi(\mathbf{r},t)$$

$$\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \left(i/\hbar\right)\mathbf{p}\psi(\mathbf{r},t) \quad \text{or:} \quad \boxed{\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \mathbf{p}\psi(\mathbf{r},t)}$$

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$$\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = (i/\hbar)\mathbf{p}\psi(\mathbf{r},t) \quad \text{or:} \quad \boxed{\frac{\hbar}{i}\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \mathbf{p}\psi(\mathbf{r},t)}$$

Momentum Operator or **p***-op in* **r***-basis*

$$\mathbf{p} \Rightarrow \frac{\hbar}{i}\frac{\partial}{\partial \mathbf{r}}$$

How Jacobi-Hamilton *could have "derived" Schrodinger equations*

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Momentum Operator
or **p**-*op in* **r**-*basis*

$$\mathbf{p} \Rightarrow \frac{\hbar}{i}\frac{\partial}{\partial \mathbf{r}}$$

Try 1st t-derivative of wave ψ

$$\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \frac{\partial}{\partial t} e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial t} e^{iS/\hbar} = (i/\hbar)\frac{\partial S}{\partial t}\psi(\mathbf{r},t)$$

$$= (i/\hbar)(-H)\psi(\mathbf{r},t) \quad \text{or:} \quad \boxed{i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = H\psi(\mathbf{r},t)}$$

How Jacobi-Hamilton *could have "derived" Schrodinger equations*

(Given "quantum wave")

$$\psi(\mathbf{r},t) = e^{iS/\hbar} = e^{i(\mathbf{p}\bullet\mathbf{r}-H\cdot t)/\hbar} = e^{i(\mathbf{k}\bullet\mathbf{r}-\omega\cdot t)}$$

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Try 1st t-derivative of wave ψ

$$\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \frac{\partial}{\partial t}e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial t}e^{iS/\hbar} = (i/\hbar)\frac{\partial S}{\partial t}\psi(\mathbf{r},t)$$

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Schrodinger time equation

$$i\hbar\dot{\psi}(\mathbf{r},t) = H\psi(\mathbf{r},t)$$

Exploring phase space and Lagrangian mechanics more deeply

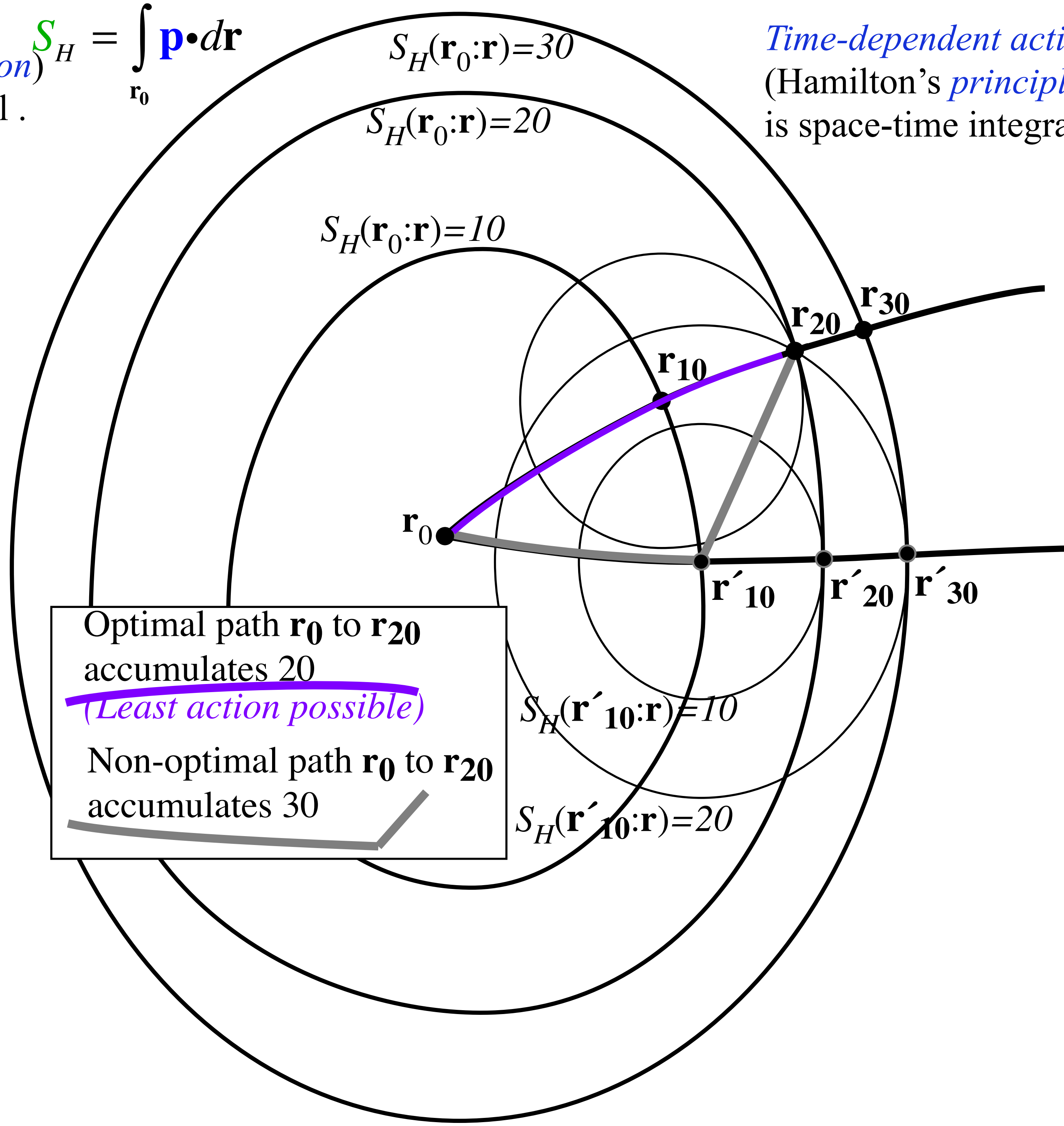
- *A weird "derivation" of Lagrange's equations*
- *Poincare identity and Action,* Jacobi-Hamilton *equations*
- *How Classicists might have "derived" quantum equations*
- ***Huygen's contact transformations enforce minimum action***
- *How to do quantum mechanics if you only know classical mechanics*

Huygen's contact transformations enforce minimum action

Each point $\mathbf{r}_k$ *on a wavefront "broadcasts" in all directions.*
Only ***minimum action*** *path interferes constructively*

Time-independent action $S_H = \int_{\mathbf{r}_0}^{\mathbf{r}_1} \mathbf{p} \bullet d\mathbf{r}$
(Hamilton's *reduced action*)
is a purely spatial integral .

Time-dependent action $S_p = \int_{\mathbf{r}_0 t_0}^{\mathbf{r}_1 t_1} (\mathbf{p} \bullet d\mathbf{r} - H \cdot dt)$
(Hamilton's *principle action*)
is space-time integral .



Unit 1
Fig. 12.12

Huygen's contact transformations enforce minimum action

Each point $\mathbf{r}_k$ *on a wavefront "broadcasts" in all directions.*
*Only **minimum action** path interferes constructively*

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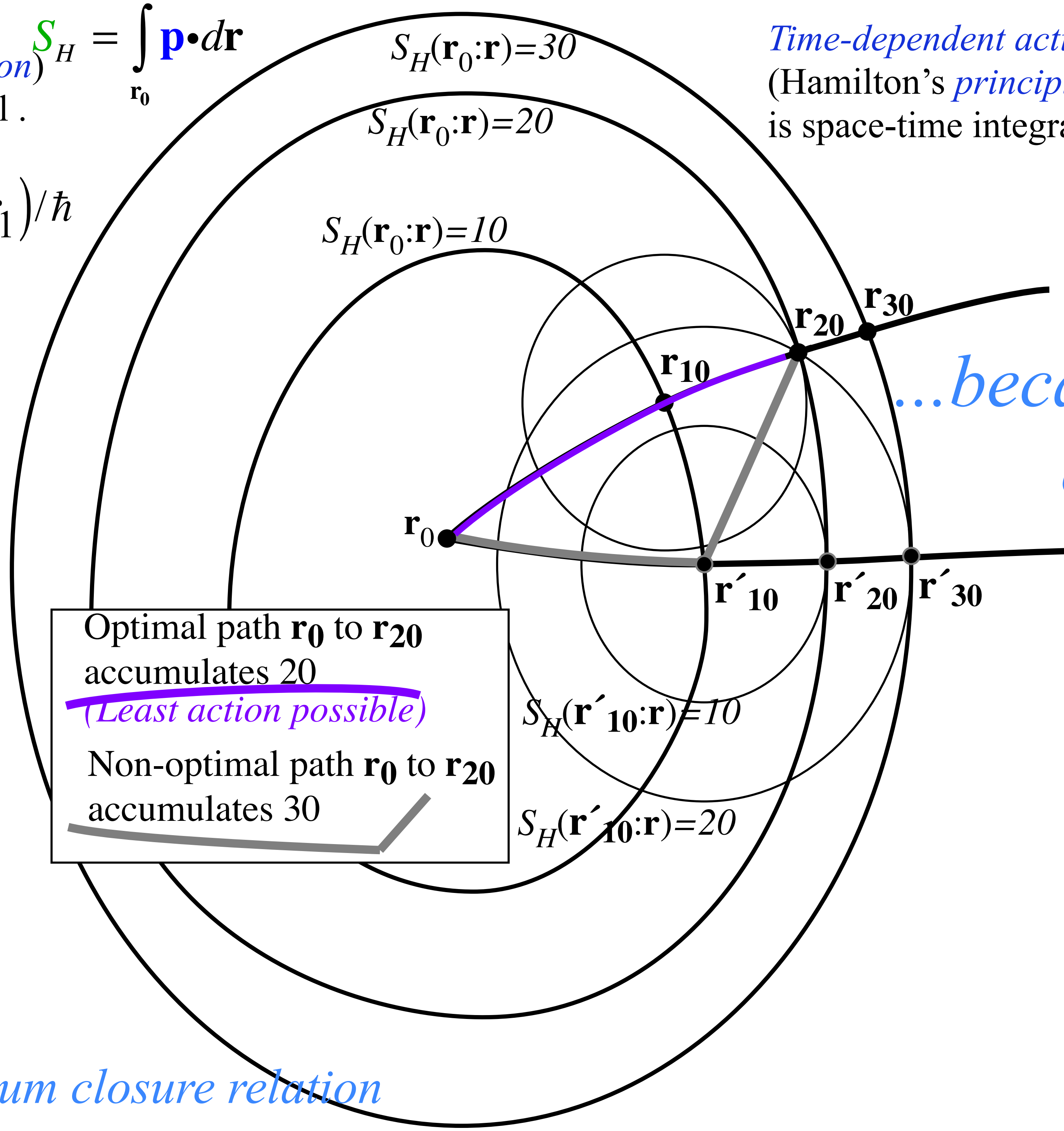
is space-time integral .

$$\langle \mathbf{r}_1 | \mathbf{r}_0 \rangle = e^{i S_H(\mathbf{r}_0 : \mathbf{r}_1)/\hbar}$$

$$\langle \mathbf{r}_1, t_1 | \mathbf{r}_0, t_0 \rangle = e^{i S(\mathbf{r}_0, t_0 : \mathbf{r}_1, t_1)/\hbar}$$



...because action is quantum wave phase

Unit 1
Fig. 12.12

Feynman's path-sum closure relation

$$\sum_{\mathbf{r}'} \langle \mathbf{r}_1 | \mathbf{r}' \rangle \langle \mathbf{r}' | \mathbf{r}_0 \rangle \cong \sum_{\mathbf{r}'} e^{i\left(S_H(\mathbf{r}_0:\mathbf{r}') + S_H(\mathbf{r}':\mathbf{r}_1)\right)/\hbar} = e^{i S_H(\mathbf{r}_0:\mathbf{r}_1)/\hbar} = \langle \mathbf{r}_1 | \mathbf{r}_0 \rangle$$

Exploring phase space and Lagrangian mechanics more deeply

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How Classicists might have "derived" quantum equations

Huygen's contact transformations enforce minimum action

How to do quantum mechanics if you only know classical mechanics

Davis-Heller "Color-Quantization" or "Classical Chromodynamics"

How to do quantum mechanics if you only know classical mechanics

Bohr quantization requires quantum phase $S_H/\hbar$ in amplitude to be an integral multiple n of 2π after a closed loop integral $S_H(\mathbf{r}_0:\mathbf{r}_0)=\int_{r_0}^{r_0}\mathbf{p}\bullet d\mathbf{r}$. The integer n ($n = 0, 1, 2,...$) is a *quantum number*.

$$1=\langle\mathbf{r}_0|\mathbf{r}_0\rangle=e^{iS_H(\mathbf{r}_0:\mathbf{r}_0)/\hbar}=e^{i\Sigma_H/\hbar}=1 \quad \text{for: } \Sigma_H=2\pi\hbar n=hn$$

Numerically integrate Hamilton's equations and Lagrangian L. Color the trajectory according to the current accumulated value of action $S_H(\mathbf{0}:\mathbf{r})/\hbar$. Adjust energy to quantized pattern (if closed system*)

$$S_H(\mathbf{0}:\mathbf{r})=S_p(\mathbf{0},0:\mathbf{r},t)+Ht=\int_0^t L\,dt+Ht\,.$$

How to do quantum mechanics if you only know classical mechanics

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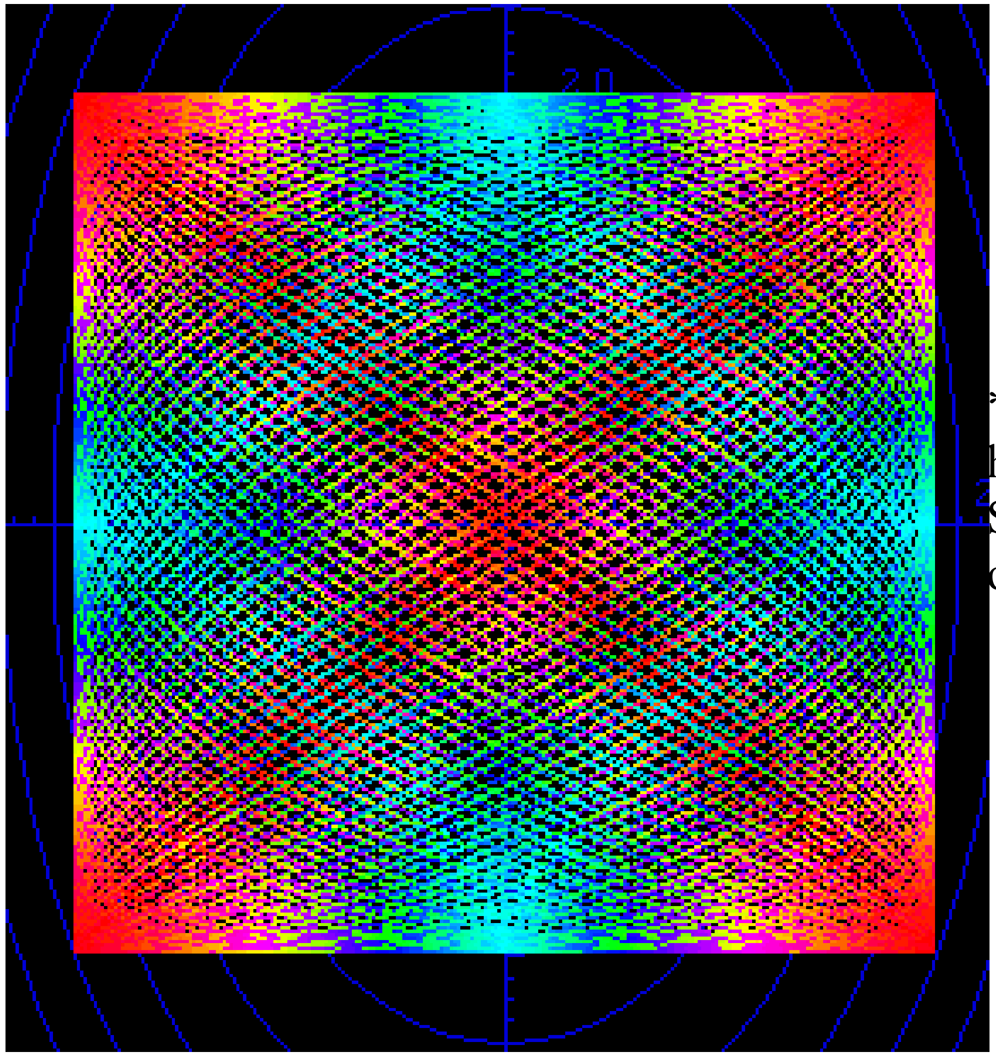
$$S_H(\mathbf{0} : \mathbf{r}) = S_p(\mathbf{0}, 0 : \mathbf{r}, t) + Ht = \int_0^t L\, dt + Ht \,.$$

The hue should represent the phase angle $S_H(\mathbf{0} : \mathbf{r})/\hbar$ *modulo* 2π as, for example,
0=red, *π/4=orange*, *π/2=yellow*, *3π/4=green*, *π=cyan* (opposite of red), *5π/4=indigo*, *3π/2=blue*, *7π/4=purple*, *and 2π=red* (full color circle).
Interpolating action on a palette of *32* colors is enough precision for low quanta.



simulation by "Color U(2)"

Unit 1 Fig. 12.13

*closed system has quantized E. Standing wave has only two phases(±) *cyan* and *red*

Wavepacket and Color-quantization:
M. J. Davis and E. J. Heller, J. Chem. Phys. 75, 246 (1981)

How to do quantum mechanics if you only know classical mechanics

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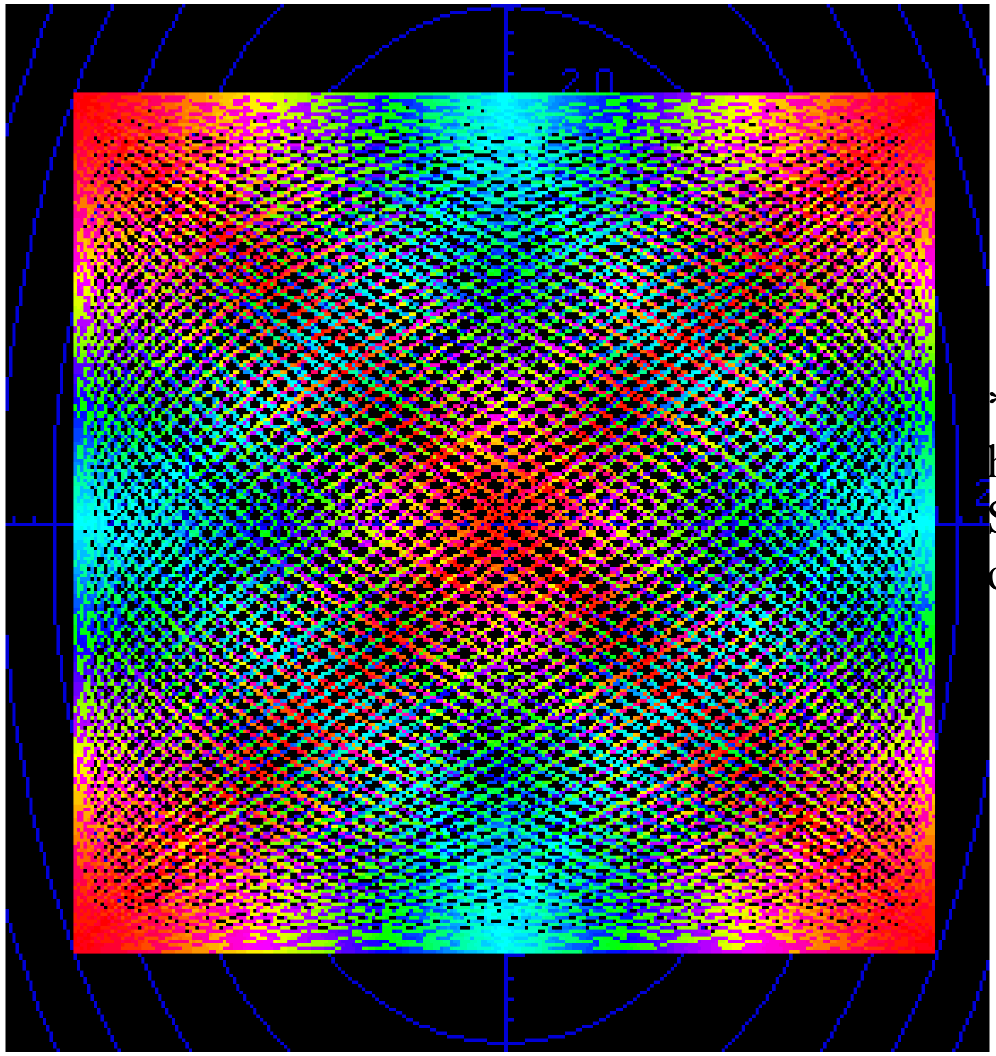
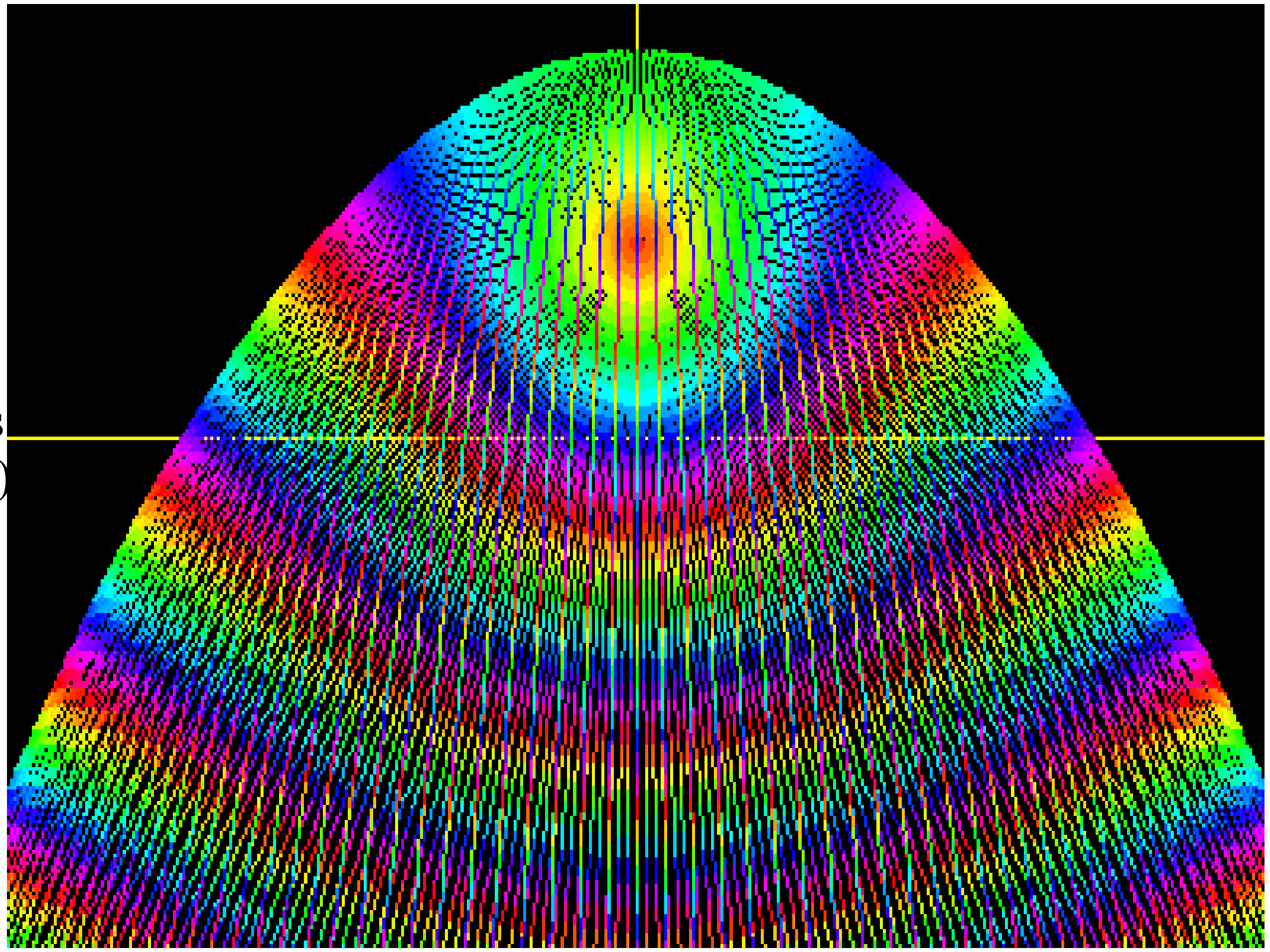


simulation by "Color U(2)"

Unit 1 Fig. 12.13

*closed system has quantized E
Standing wave has only two phases(±) *cyan* and *red*

*open system has continuous energy



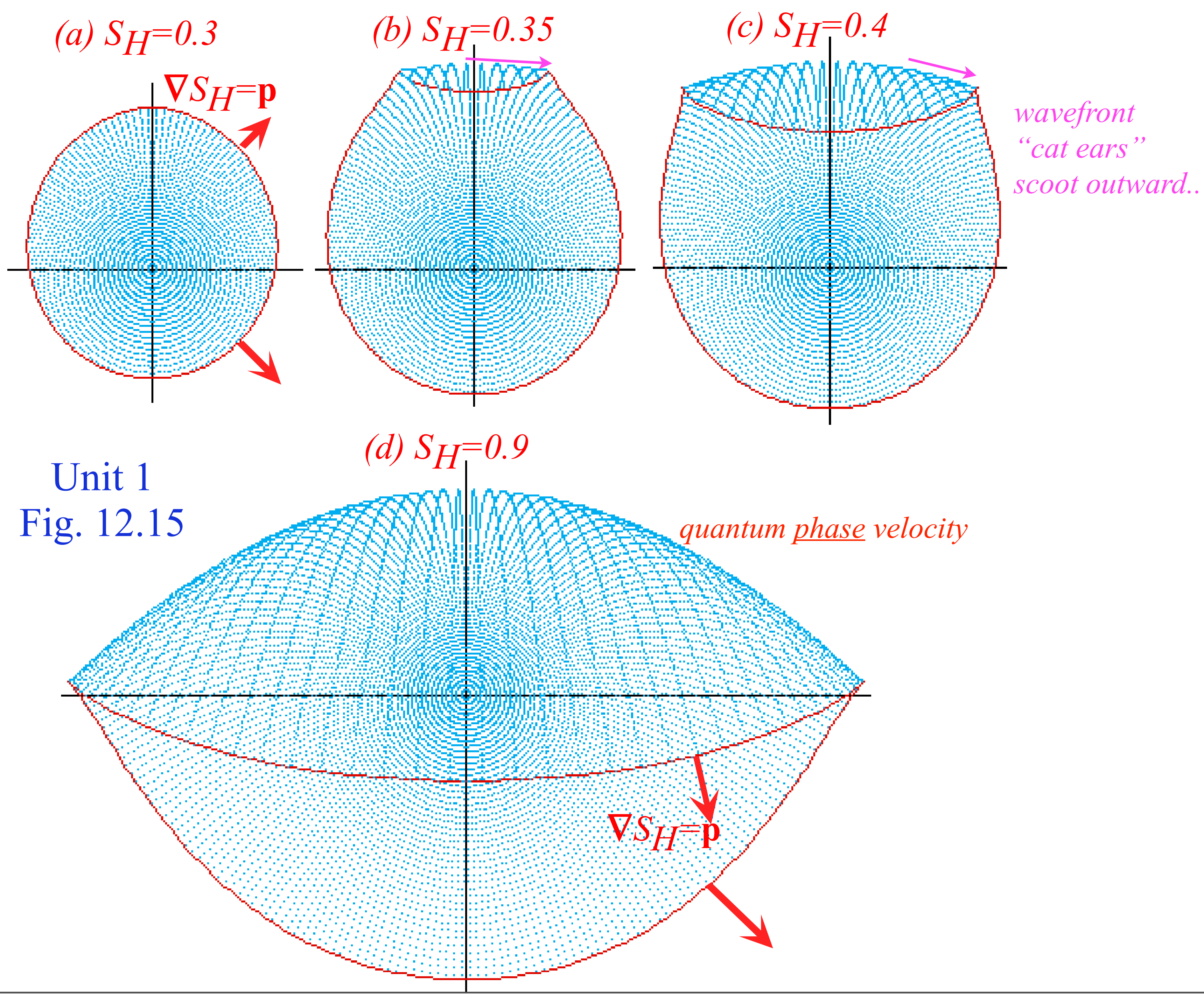
Unit 1 Fig. 12.14

[*Web Simulation by "CoulIt"*]

A moving wave has a *quantum phase velocity* found by setting $S=const.$ or $dS(0,0:r,t)=0=\mathbf{p}\bullet d\mathbf{r}-H dt$.

$$\mathbf{V}_{phase} = \frac{d\mathbf{r}}{dt} = \frac{H}{\mathbf{p}} = \frac{\omega}{\mathbf{k}}$$

Quantum "phase wavefronts"



Unit 1
Fig. 12.15

A moving wave has a *quantum phase velocity* found by setting $S=const.$ or $dS(0,0{:}\mathbf{r},t)=0=\mathbf{p}\bullet d\mathbf{r}-H dt$.

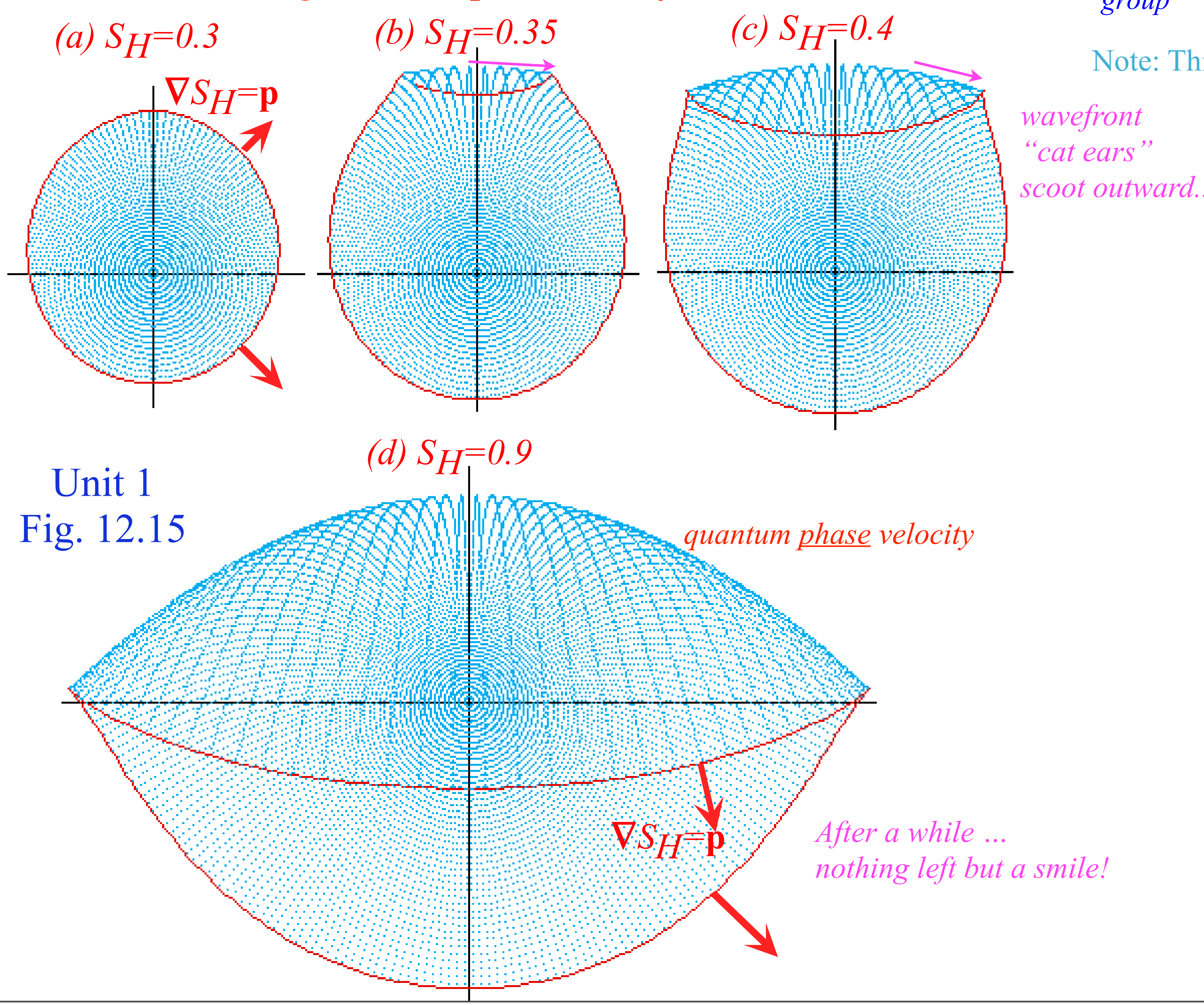
$$\mathbf{V}_{phase}=\frac{d\mathbf{r}}{dt}=\frac{H}{\mathbf{p}}=\frac{\omega}{\mathbf{k}}$$

This is quite the opposite of classical particle velocity which is *quantum group velocity*.

$$\mathbf{V}_{group}=\frac{d\mathbf{r}}{dt}=\dot{\mathbf{r}}=\frac{\partial H}{\partial \mathbf{p}}=\frac{\partial \omega}{\partial \mathbf{k}}$$

Note: This is Hamilton's 1st Equation

Quantum "phase wavefronts"



Unit 1
Fig. 12.15

A moving wave has a *quantum phase velocity* found by setting $S=const.$ or $dS(0,0{:}r,t)=0=\mathbf{p}\bullet d\mathbf{r}-Hdt$.

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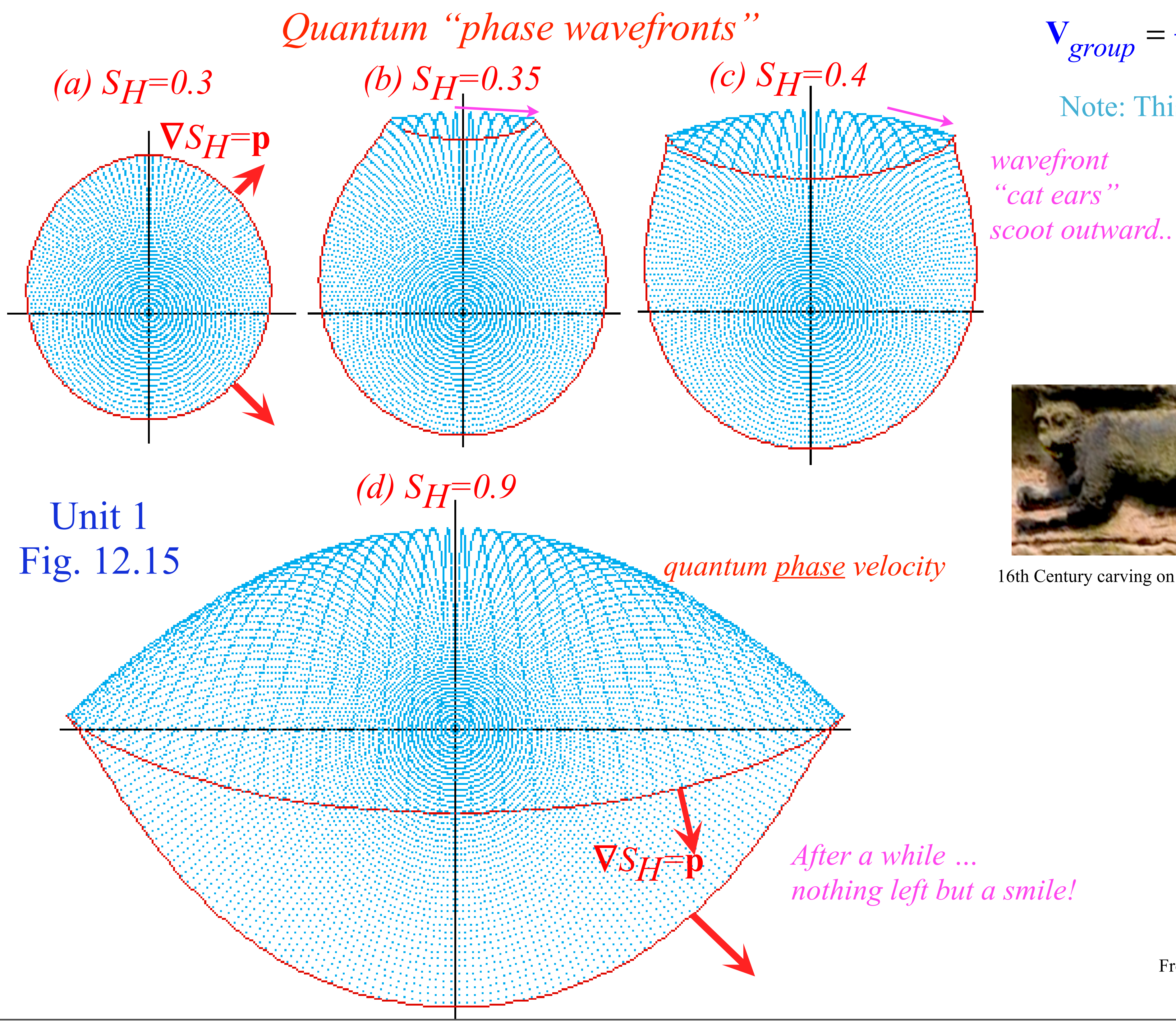

Unit 1
Fig. 12.15

16th Century carving on St. Wifred's in Grappenhall

...on St. Nicolas

From *Alice's Adventures in Wonderland* by Lewis Carrol (1865)

A moving wave has a *quantum phase velocity* found by setting $S=const.$ or $dS(0,0:r,t)=0=\mathbf{p}\bullet d\mathbf{r}-H dt$.

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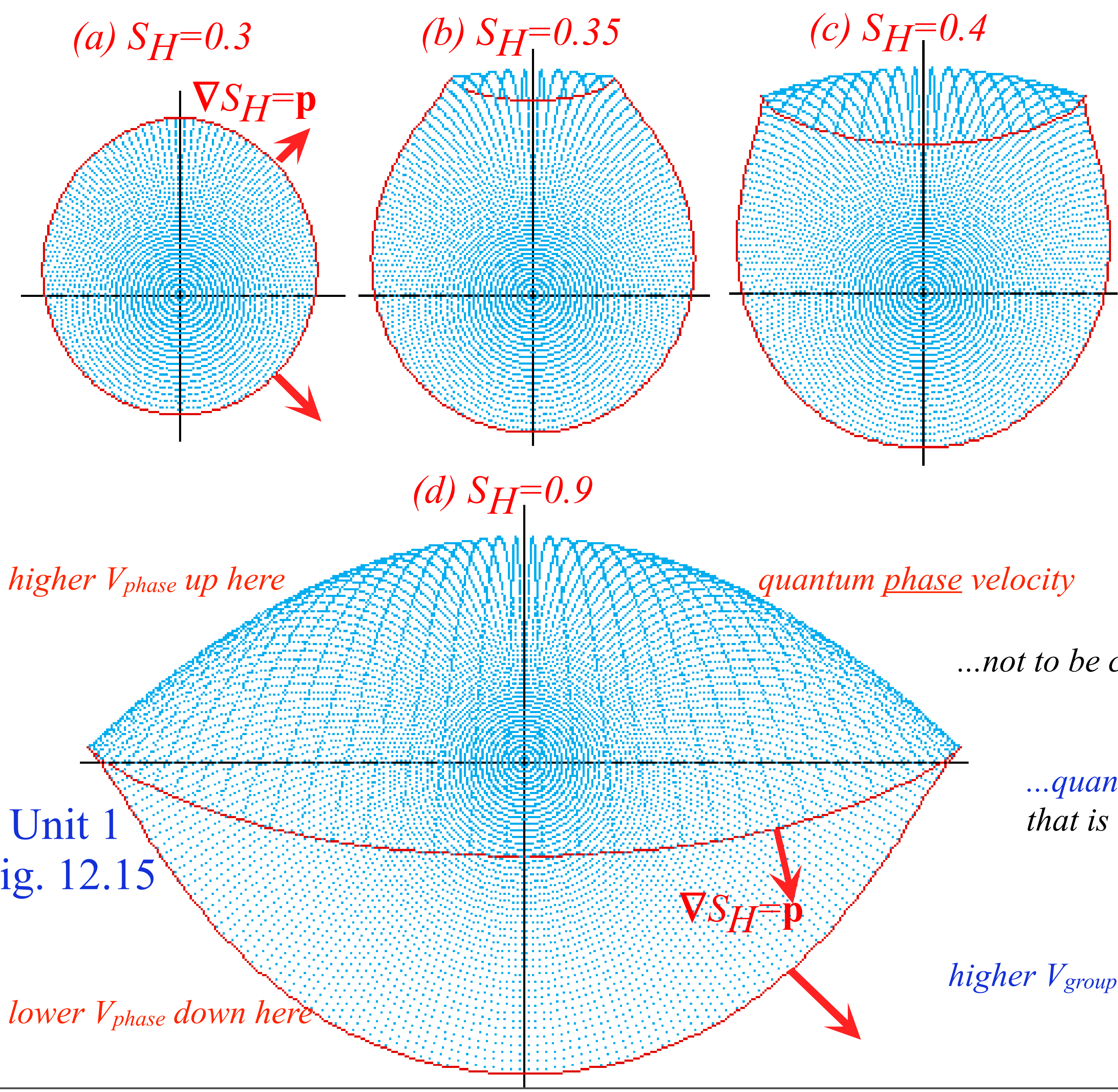
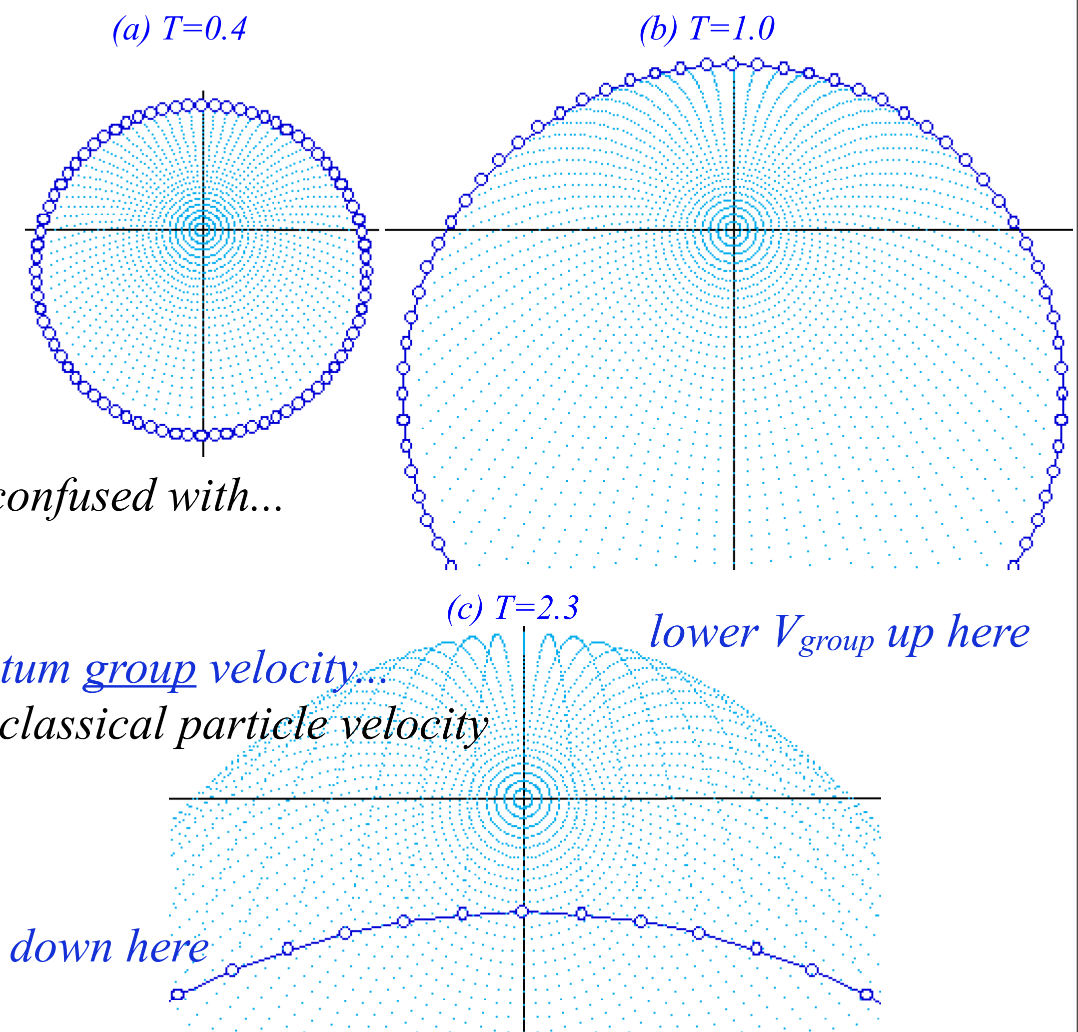
Quantum "phase wavefronts"



Unit 1
Fig. 12.15

Classical "blast wavefronts"

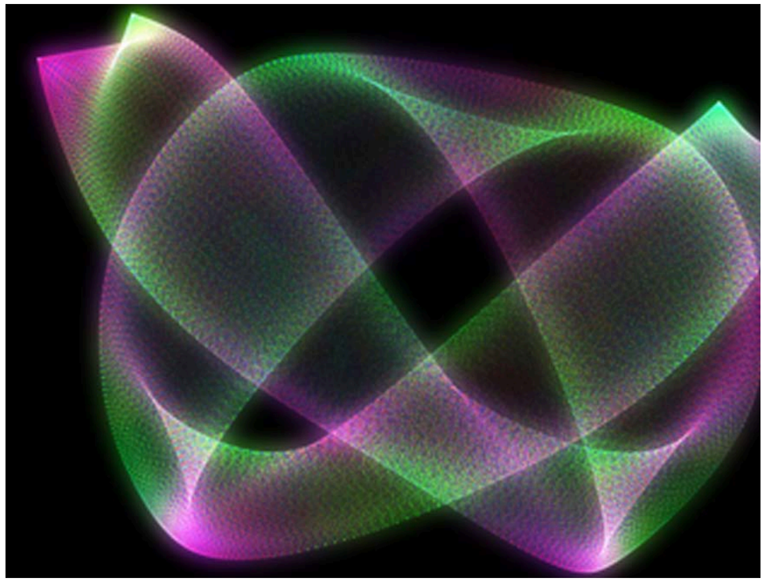
...not to be confused with...



...quantum group velocity...
that is classical particle velocity

Web Simulation

Check out the Heller Galleries http://www.ericjhellergallery.com/index.pl?page=image;iid=76



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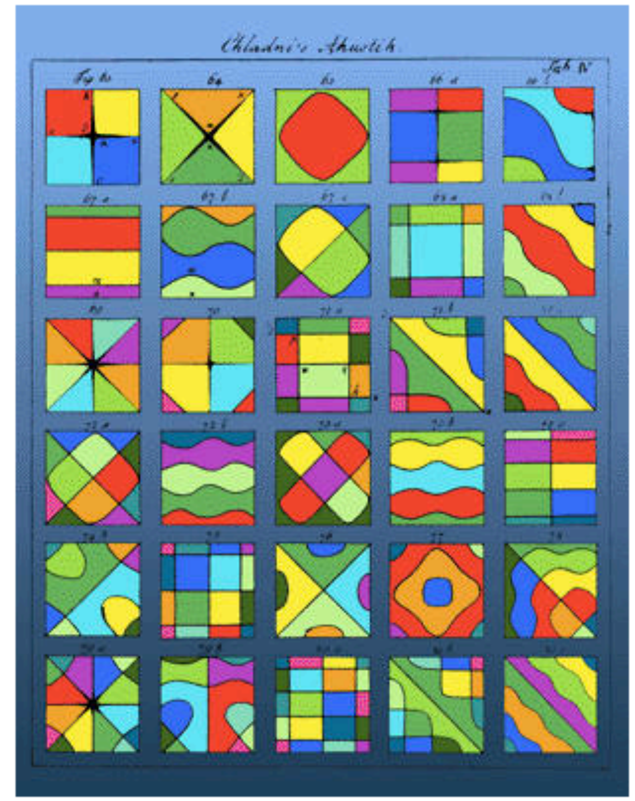
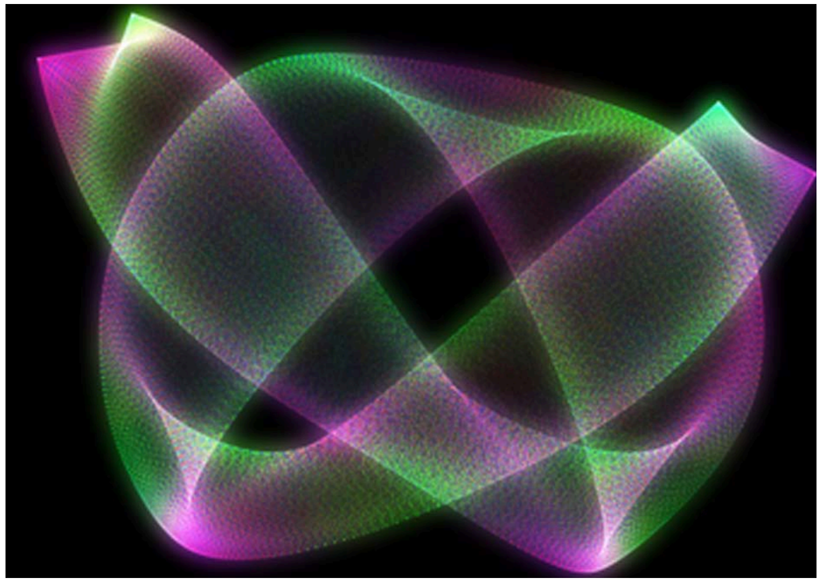
Chladni



The diagrams of Ernst Chladni (1756-1827) are the scientific, artistic, and even the sociological birthplace of the modern field of wave physics and quantum chaos. Educated in Law at the University of Leipzig, and an amateur musician, Chladni soon followed his love of science and wrote one of the first treatises on acoustics, "Discovery of the Theory of Pitch". Chladni had an inspired idea: to make waves in a solid material visible. This he did by getting metal plates to vibrate, stroking them with a violin bow. Sand or a similar substance spread on the surface of the plate naturally settles to the places where the metal vibrates the least, making such places visible. These places are the so-called nodes, which are wavy lines on the surface. The plates vibrate at pure, audible pitches, and each pitch has a unique nodal pattern. Chladni took the trouble to carefully diagram the patterns, which helped to popularize his work. Then he hit the lecture circuit, fascinating audiences in Europe with live demonstrations. This culminated with a command performance for Napoleon, who was so impressed that he offered a prize to anyone who could explain the patterns. More than that, according to Chladni himself, Napoleon remarked that irregularly shaped plate would be much harder to understand! While this was surely also known to Chladni, it is remarkable that Napoleon had this insight. Chladni received a sum of 6000 francs from Napoleon, who also offered 3000 francs to anyone who could explain the patterns. The mathematician Sophie Germain took he prize in 1816, although her solutions were not completed until the work of Kirchoff thirty years later. Even so, the patterns for irregular shapes remained (and to some extent remains) unexplained. Government funding of waves research goes back a long way! (Chladni was also the first to maintain that meteorites were extraterrestrial; before that, the popular theory was that they were of volcanic origin.) One of his diagrams is the basis for image, which is a playfully colored version of Chaldni's original line drawing. Chladni's original work on waves confined to a region was followed by equally remarkable progress a few years later.

Check out the Heller Galleries

http://www.ericjhellergallery.com/index.pl?page=image;iid=76



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University Museum, University of Arkansas, Fayetteville, AK

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"Approaching Chaos: Visions from the Quantum Frontier"

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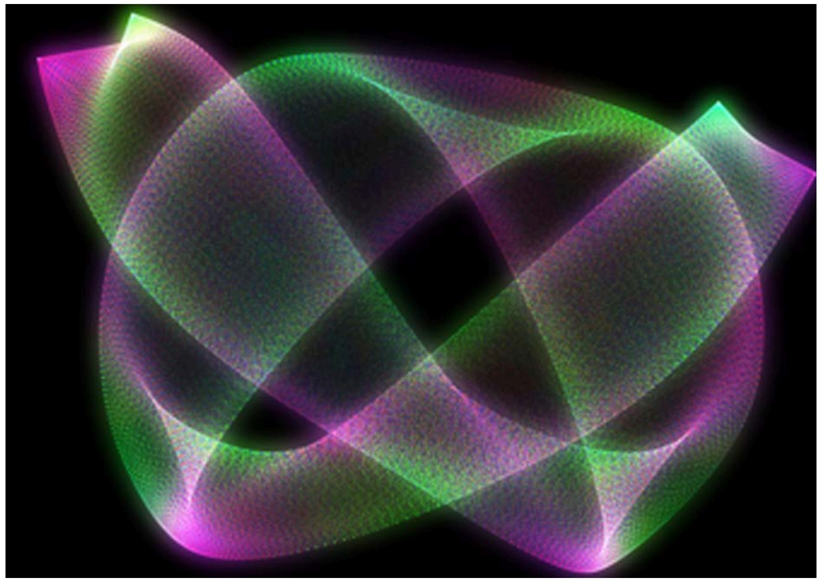

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*UAF *Museum closed after this exhibit*

Lecture 12 ends here
Thur 10.8.2015