Due Thursday Oct. 20: Assignment 7 Based on Unit 1 Chapter 10 and 12 and Lectures 13-14 (2015).

"Professional" Parabolic and Hyperbolic Coordinates (Relates to Fig. 1.10.7) *50points*

1.12.1 Consider GCC definition: $q^1=\Phi= x^2 - y^2$, $q^2 =A= 2xy$. Both $(x^1=x,x^2=y)$ and $(q^1=u=\Phi,q^2=v=A)$ are Orthogonal Curvilinear Coordinates (OCC) related by an analytic function $w=z^2$ or $(u+iv)=(x+iy)^2$. You can treat *either one* as Cartesian. (This is based on the analytic function $f(z)=2z$ whose complex potential is $\phi=$__________)

(a) Plot $(q^1=u,q^2=v)$ coordinate curves in a Cartesian $(x^1=x,x^2=y)$ graph. Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_k$ and $\mathbf{E}^k$ and metric tensors g_{mn} and g^{mn} for this GCC.

(b) Plot $(x^1=x,x^2=y)$ coordinate curves in a Cartesian $(q^1=u,q^2=v)$ graph. Derive the Jacobian, Kajobian, unitary vectors and metric tensors for this GCC.

Galaxy Grids *40points*

1.12.2 Consider the monopole field function $f(z)=e^{i\alpha}/z$ with complex source $e^{i\alpha}$ discussed in Lectures 13-14.

(a) Derive its $(q^1=\Phi,q^2=A)$ scalar and vector potential coordinate functions.

(b) Plot examples for angle $\alpha=30°$ and $\alpha=45°$.

Fun with Exponents & more of the Story of e *30points*

1.12.3 Consider a sequence of functions , $f_1(z)=z^z, f_2(z)=z^{f_1(z)}=z^{z^z}, f_3(z)=z^{f_2(z)}=z^{z^{z^z}}$,.... The function $f_N(z)$ has a finite limit $f_\infty(z)$ for N approaching infinity if argument z is small enough . ($z=1$ works! But, so does $z=\sqrt{2}$.)

(a) Find $f_\infty(\sqrt{2})=$____?

(b) Find an analytic expression for the limiting real z_{max} that involves the Euler constant. $e=2.718281828...$

Vector analysis problems involving Levi-Civita relations $\varepsilon_{ijk}\varepsilon_{mnk}=\delta_{im}\delta_{jn}-\delta_{in}\delta_{jm}$. *30points*

1.12.4 Reduce double-crosses to ordinary • or · products with Levi-Civita relations (and without peeking at answers).

(a) Reduce $\mathbf{B}\times(\mathbf{C}\times\mathbf{A})$. Also reduce $(\mathbf{A}\times\mathbf{B})\bullet(\mathbf{C}\times\mathbf{D})$.

(b) Reduce $\nabla\times(\nabla\times\mathbf{A})$ and $\nabla\times(\mathbf{A}\times\mathbf{B})$.

(c) Reduce $\mathbf{v}\times(\nabla\times\mathbf{A})$. Then show how it simplifies assuming classical mechanical constraint $(\nabla\mathbf{v})=\mathbf{0}$.

1.12.5 Derive surface shapes of rotating fluid whose curl $\nabla\times\mathbf{v}$ of velocity fields is given: *30points*

(a) $|\nabla\times\mathbf{v}|=0=\nabla\bullet\mathbf{v}$ (Whirlpool or Vortex: First describe complex velocity field $f(z^*)=v_x(x,y)+i\,v_y(x,y)=i/z^*$.)

(b) $\nabla\times\mathbf{v}=const.$ (Rigid rotation: First describe complex velocity field $f(z)=v_x(x,y)+i\,v_y(x,y)=i\omega z$.)