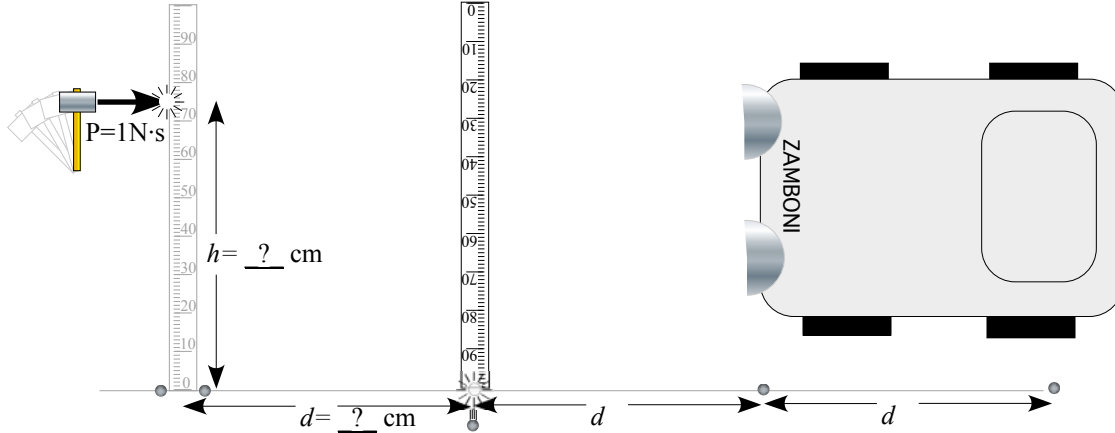


Assignment 10 - Classical Mechanics 5103 12/08/15 Due at Final Exam Tue Dec. 15

Main Reading: In new text ( *Classical Mechanics with a BANG!* ) Unit 2 thru 2.9 and Unit 3 thru 3.8.

Due Tue. Dec. 15 An icy cycloid problem

2.A.1 (a) A meter stick lies on a smooth icy hockey rink surface with two marbles sitting at the lower end on either side of the 0.0cm mark. (See figure) A hammer give impulse  $\mathbf{P}=(1\text{N}\cdot\text{s})\mathbf{e}_x$  to the stick at the  $h$ -cm. mark. What height  $h$  is *least* likely to disturb the marbles.

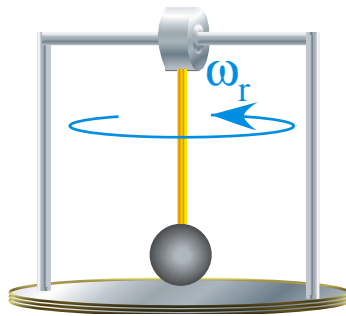


(b) Now assume  $h$ -value from (a) and friction-free “icy” surface. At what distances  $d, 2d, 3d, \dots$  along  $x$ -axis should the 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup>, ... marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time  $\Delta t$  interval snapshots of the stick as it flips by 180° and then to 360°. What is  $\Delta t$  for a 1kg stick?

Due Tue. Dec. 15 Electromagnetic cycloids

2.8.1 Suppose a vertical frictionless surface subject to Earth gravity (Say  $g=10\text{m/s}^2$ ) with a unit mass  $m=1 \text{ kg}$  and charge  $Q=1 \text{ Coul.}$  (Dangerous!) that is dropped from  $(x=0=y)$  in a strong magnetic  $\mathbf{B}$ -field.

- How many Tesla of magnetic field  $\mathbf{B}$  and in what direction would cause the mass to move to the right on a normal cycloid made by circle of one meter diameter? Where would it hit the horizontal  $x$ -axis?
- What initial speed and direction of throw would cause the mass to fly straight along the  $x$ -axis?
- Describe and plot the resulting trajectory if the mass is thrown down with a speed of  $2\text{m/s}$ .



Pendulum on turntable

Due Tue. Dec. 15

3.8.5 Suppose a pendulum supported by a circular ball bearing may swing without friction in the vertical plane of the bearing. The bearing plane is secured to a turntable that rotates at a constant angular frequency  $\omega_r$ . The pendulum consists of a mass  $m$  at the end of a rod of length  $l=1\text{m}$  and negligible mass with natural frequency of small  $\theta$ -angle motion at zero- $\omega_r$  in gravity acceleration (Say  $g=10\text{m/s}^2$ ) given by  $\omega_\theta(\omega_r=0)=$ \_\_\_\_\_.

- Derive the Lagrangian and Hamiltonian using spherical coordinates in the rotating frame.
- Derive the  $\theta$ -equilibrium points and small-oscillation frequency as a function of the frequency  $\omega_r$  and  $\omega_\theta$ . Overlay plots of effective  $\theta$ -potential for several key values of  $\omega_r$ . What  $\omega_r$  value makes  $\theta=0$  angle unstable?