Lecture 27.

Relativity of transverse waves and 4-vectors

(Ch. 2-5 of Unit 2   4.05.12)

Introducing per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Reviewing the stellar aberration angle σ vs. rapidity ρ in Epstein's cosmic speedometer†

Reviewing "Sin-Tan Rosetta" geometry

Reviewing relativistic quantum Lagrangian-Hamiltonian contact relations

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)†

Time contraction-dilation revisited

Length contraction-dilation revisited

Twin-paradox revisited

Velocity addition revisited

Lorentz symmetry effects

How it makes momentum and energy be conserved

†Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press,*
Insight Press, San Francisco, CA 94107

Reviewing the stellar aberration angle σ vs. rapidity ρ

Together, rapidity $\rho=\ln b$ and stellar aberration angle σ are parameters of relative velocity

The rapidity $\rho=\ln b$ is based on longitudinal wave Doppler shift $b=e^{\rho}$ defined by $u/c=\tanh(\rho)$. At low speed: $u/c\sim\rho$.

The stellar aberration angle σ is based on the transverse wave rotation $R=e^{i\sigma}$ defined by $u/c=\sin(\sigma)$. At low speed: $u/c\sim\sigma$.

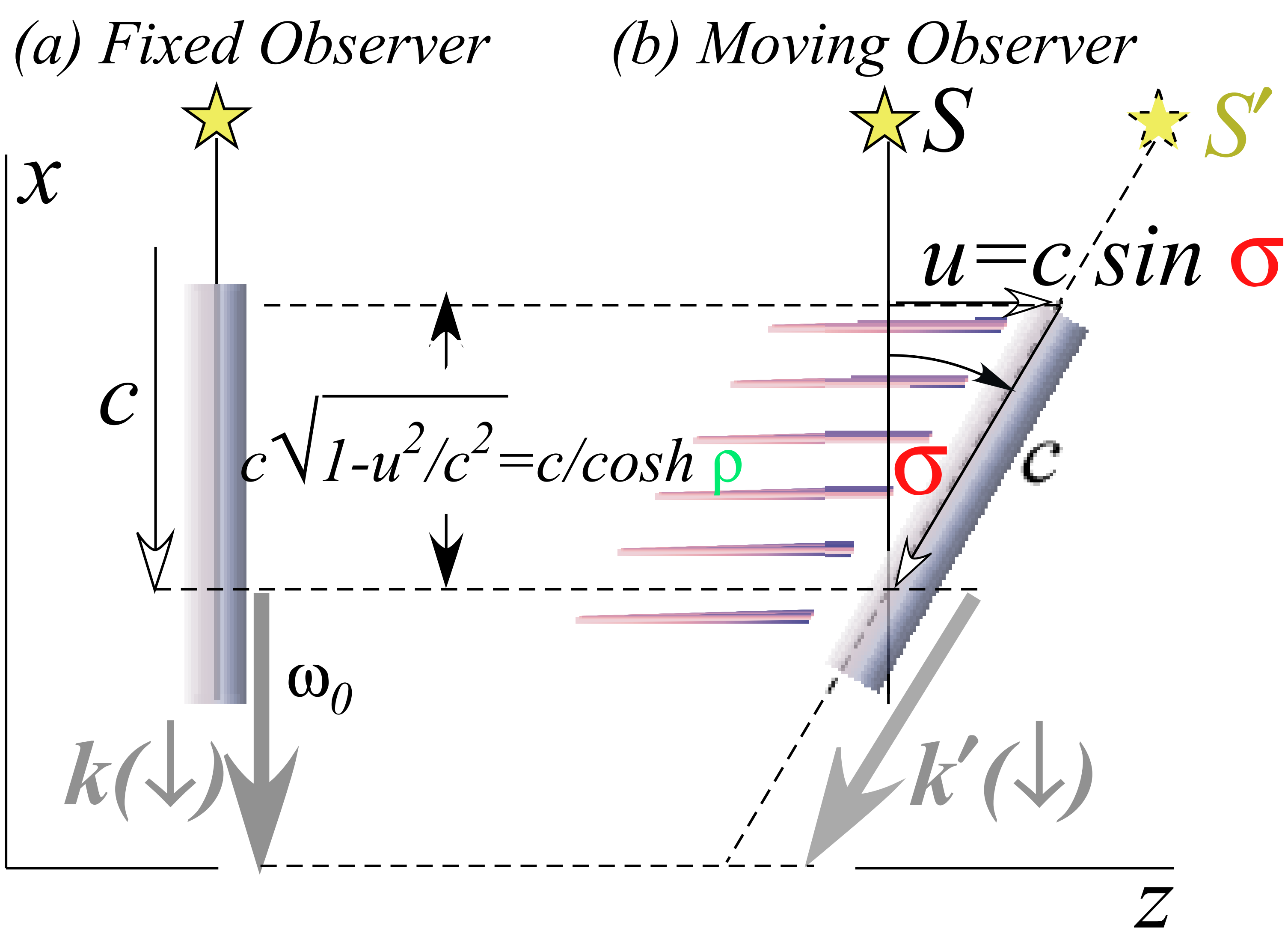


Fig. 5.6 Epstein's cosmic speedometer with aberration angle σ and transverse Doppler shift $\cosh\upsilon_Z$.

NOTE: Angle ϕ is now called stellar aberration angle σ

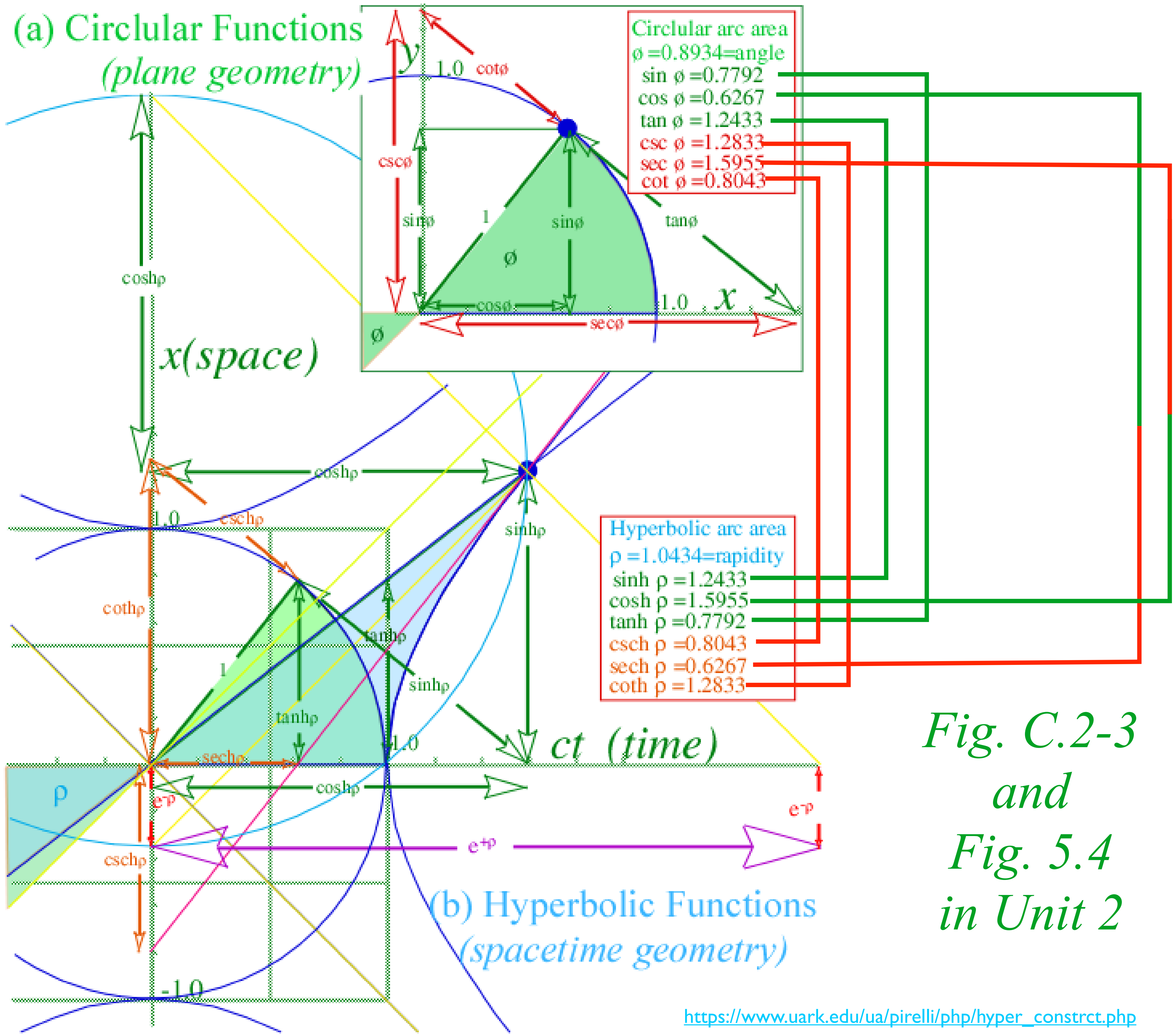
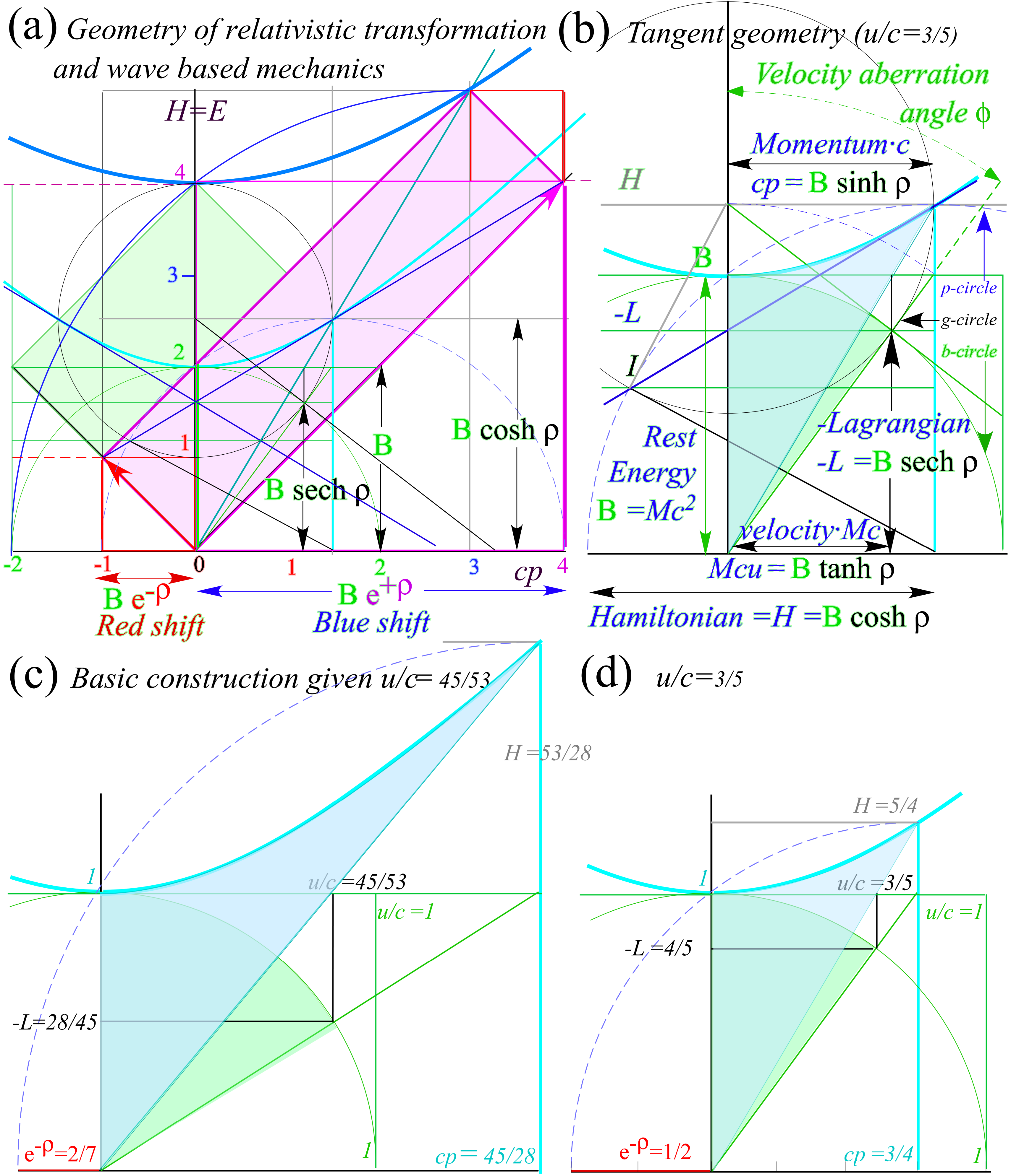


Fig. C.2-3
and
Fig. 5.4
in Unit 2

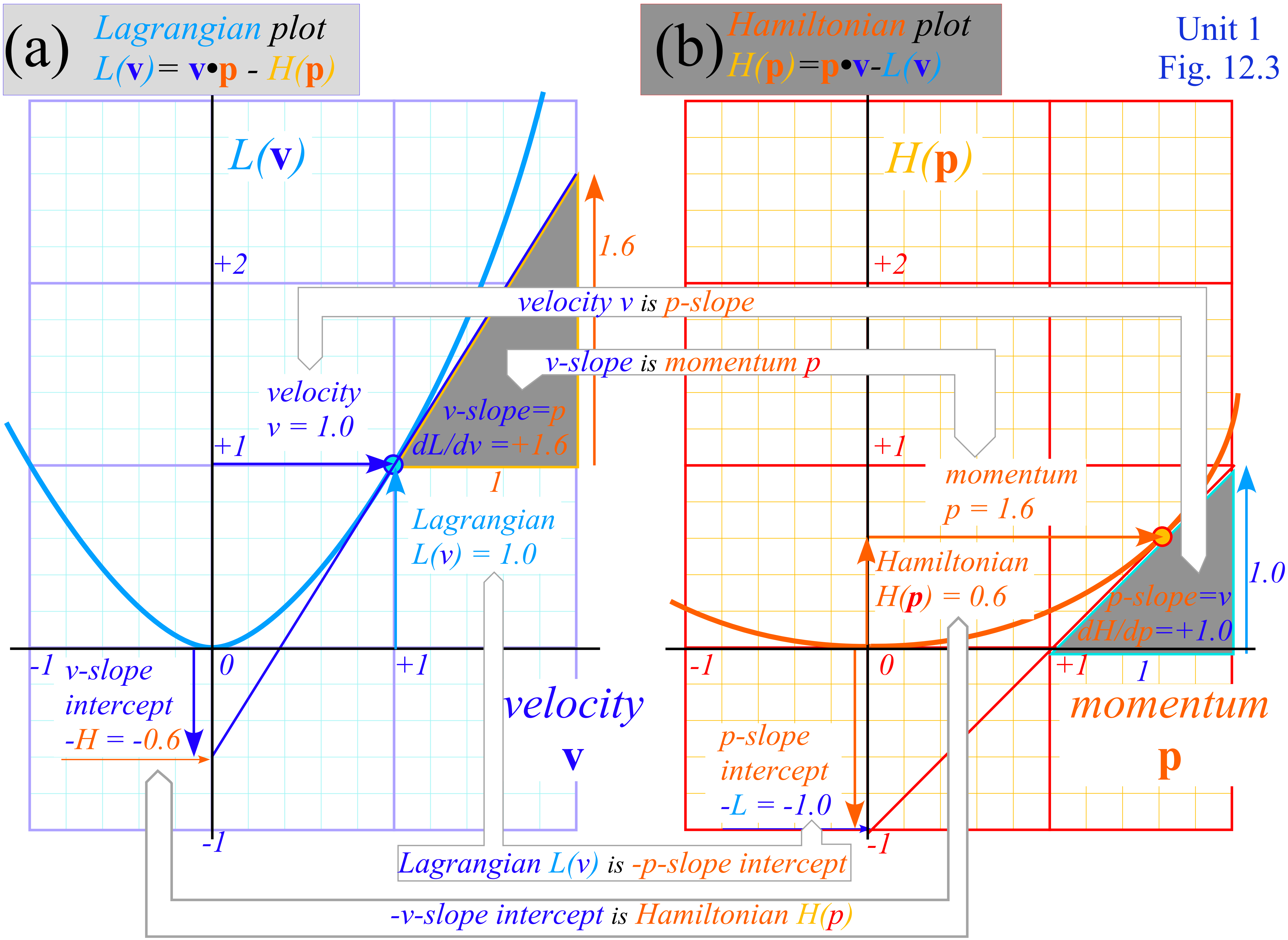
https://www.uark.edu/ua/pirelli/php/hyper_constrct.php



(a) Geometry of relativistic transformation and wave based mechanics
H=E
cp
B cosh ρ
B sech ρ
B
B e^-ρ
Red shift
B e^+ρ
Blue shift
(b) Tangent geometry (u/c=3/5)
Velocity aberration angle ϕ
Momentum·c
cp = B sinh ρ
H
B
-L
I
p-circle
g-circle
b-circle
Rest Energy
B =Mc²
-Lagrangian
-L =B sech ρ
velocity·Mc
Mcu = B tanh ρ
Hamiltonian =H =B cosh ρ
(c) Basic construction given u/c= 45/53
H =53/28
u/c =45/53
u/c =1
-L=28/45
e^-ρ=2/7
cp= 45/28
(d) u/c=3/5
H =5/4
u/c =3/5
u/c =1
-L =4/5
e^-ρ=1/2
cp =3/4

Reviewing classical quantum Lagrangian-Hamiltonian relations

Unit 1
Fig. 12.3



Reviewing relativistic quantum Lagrangian-Hamiltonian relations

Start with phase Φ *and set* $k=0$ *to get product of proper frequency* $\mu = Mc^2/\hbar$ *and proper time* τ

$$d\Phi = k dx - \omega dt = -\mu\ d\tau = -(Mc^2/\hbar)\ d\tau. \qquad d\tau = dt\sqrt{(1-u^2/c^2)} = dt\ \mathrm{sech}\ \rho$$

Differential action: $dS = L dt = p\cdot dx - H\cdot dt = \hbar k\cdot dx - \hbar\omega\cdot dt = \hbar d\Phi$
is Planck scale $\hbar$ times differential phase: $dS = \hbar d\Phi$

For constant u the Lagrangian is: $L = -\hbar\mu\tau = -Mc^2\sqrt{(1-u^2/c^2)} = -Mc^2\mathrm{sech}\ \rho = -Mc^2\cos\sigma$

...with Poincare invariant: $L = p\cdot\dot{x} - H = p\cdot u - H$

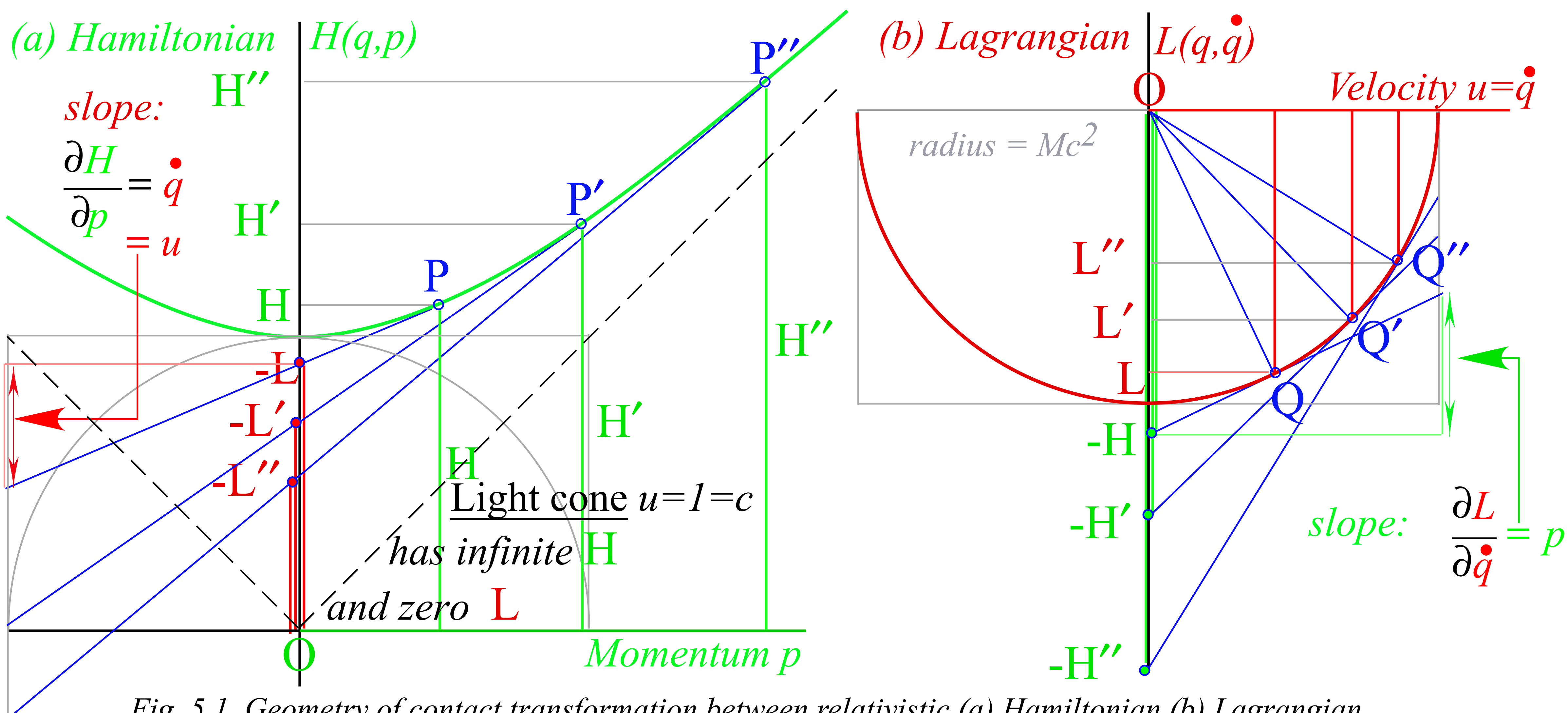
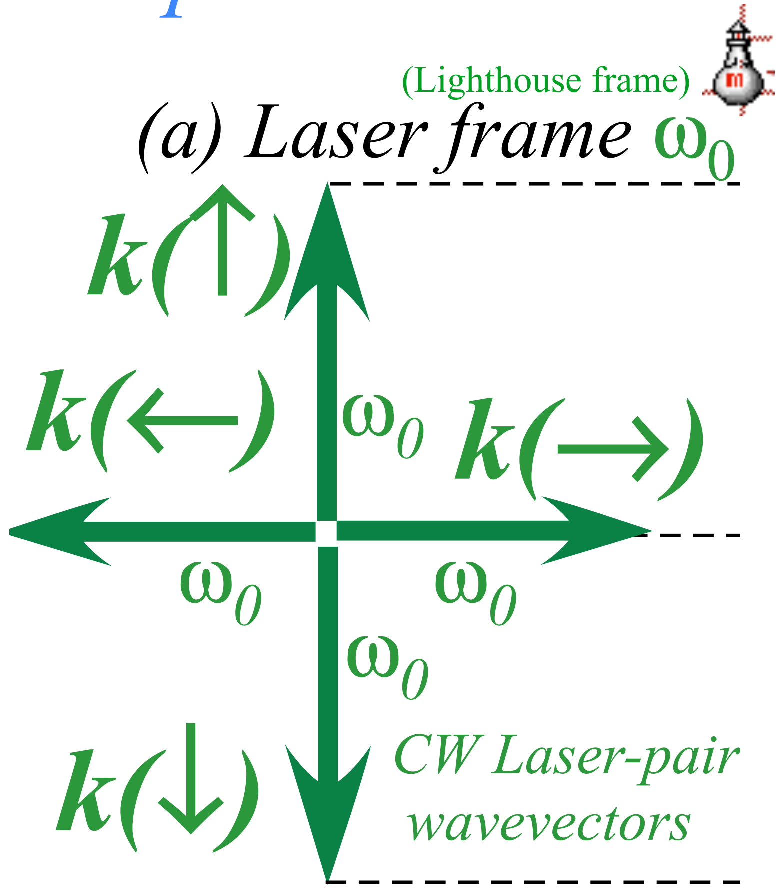
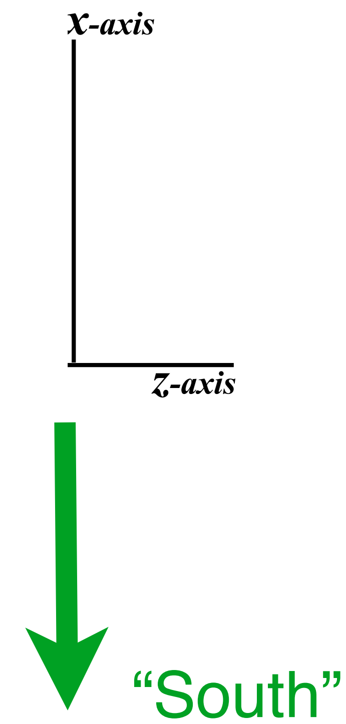


Fig. 5.1. Geometry of contact transformation between relativistic (a) Hamiltonian (b) Lagrangian

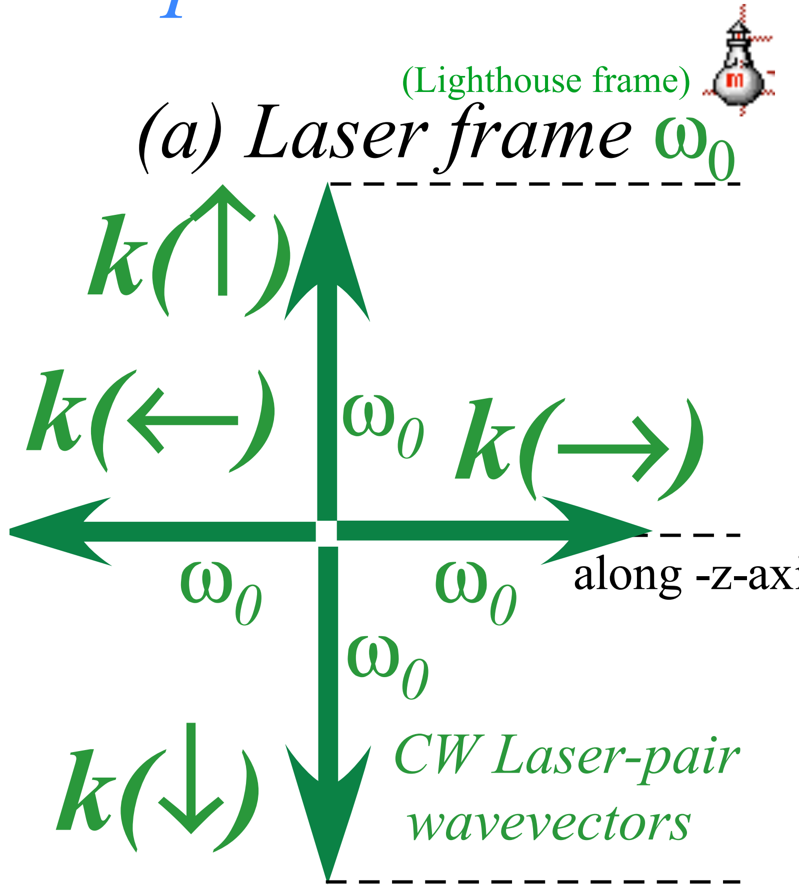
Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation



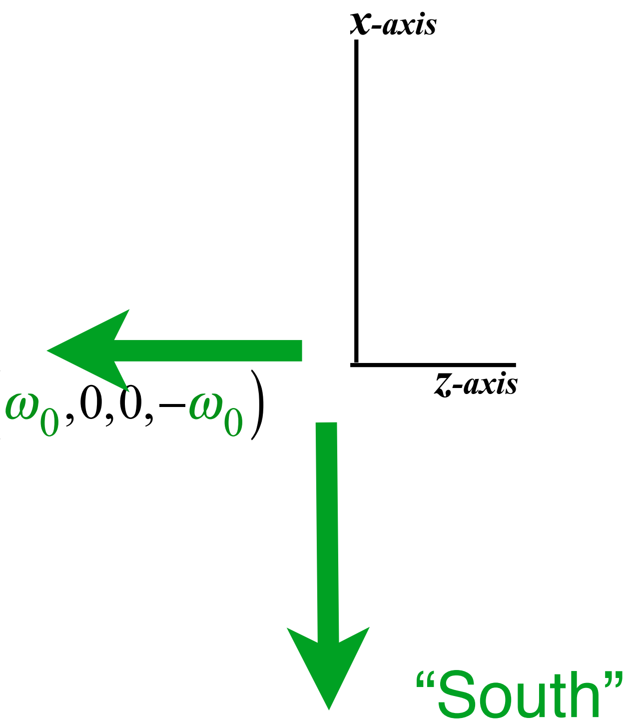


Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{y\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ *transformation*



along -z-axis : $\left(\omega_{\leftarrow},ck_{x\leftarrow},ck_{y\leftarrow},ck_{z\leftarrow}\right)=\left(\omega_0,0,0,-\omega_0\right)$



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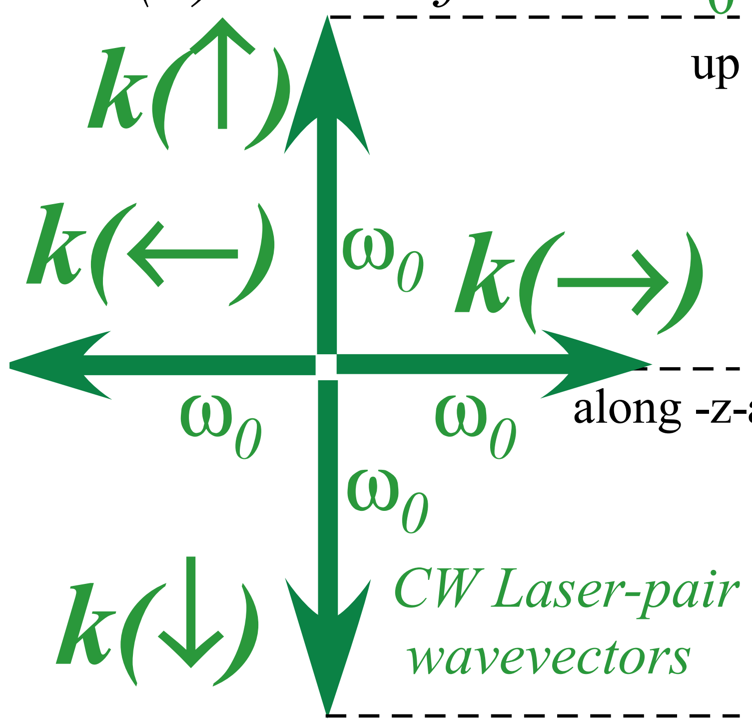
Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

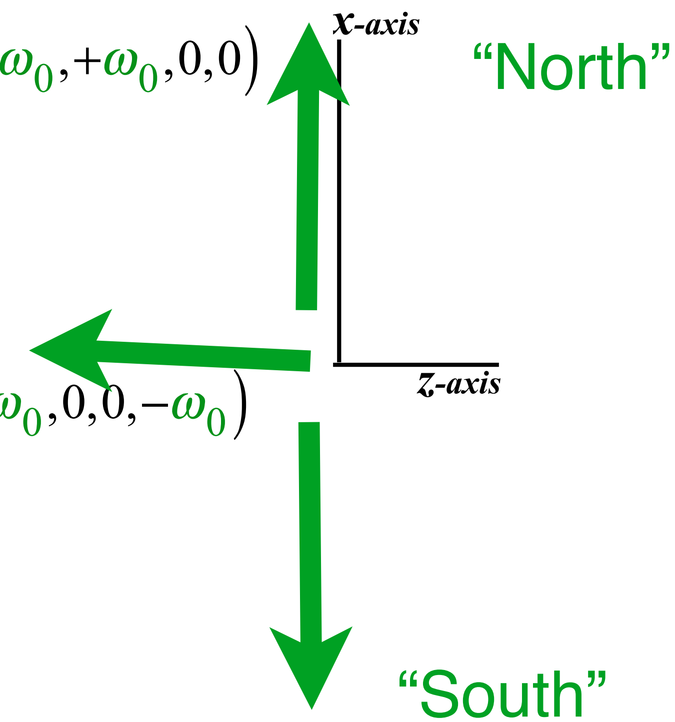
(Lighthouse frame)

(a) Laser frame ω_0



up +x-axis : $\left(\omega_{\uparrow}, ck_{x\uparrow}, ck_{y\uparrow}, ck_{z\uparrow}\right) = \left(\omega_0, +\omega_0, 0, 0\right)$

along -z-axis : $\left(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\right) = \left(\omega_0, 0, 0, -\omega_0\right)$

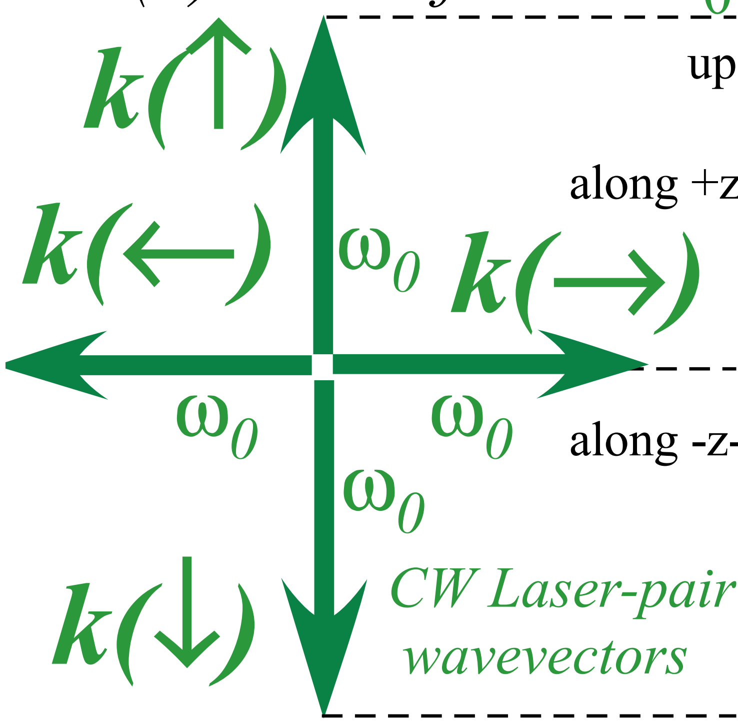


Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

(Lighthouse frame)

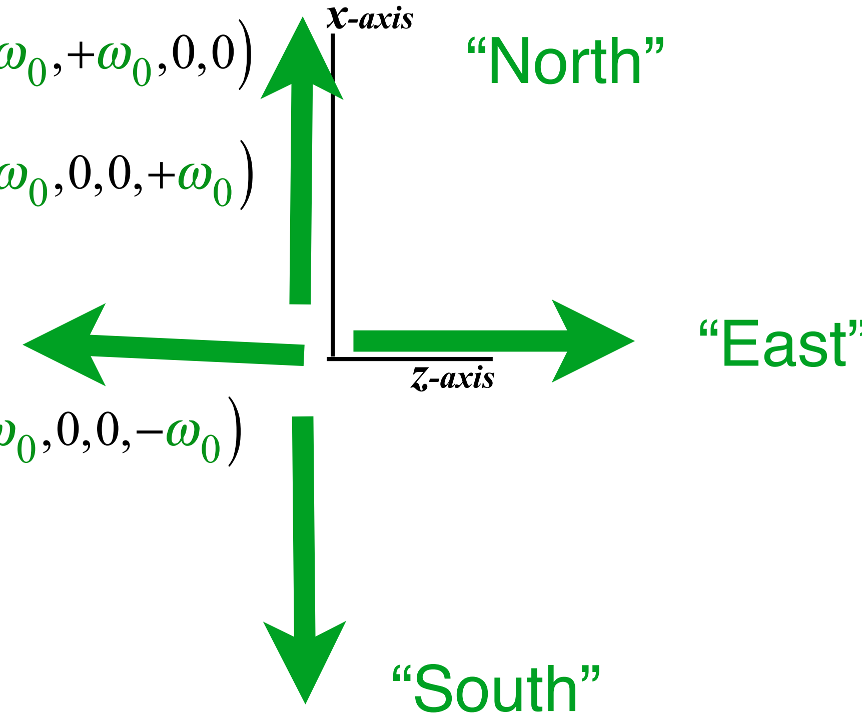
(a) Laser frame ω_0



up +x-axis : $\left(\omega_{\uparrow}, ck_{x\uparrow}, ck_{y\uparrow}, ck_{z\uparrow}\right) = \left(\omega_0, +\omega_0, 0, 0\right)$

along +z-axis : $\left(\omega_{\rightarrow}, ck_{x\rightarrow}, ck_{y\rightarrow}, ck_{z\rightarrow}\right) = \left(\omega_0, 0, 0, +\omega_0\right)$

along -z-axis : $\left(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\right) = \left(\omega_0, 0, 0, -\omega_0\right)$



Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

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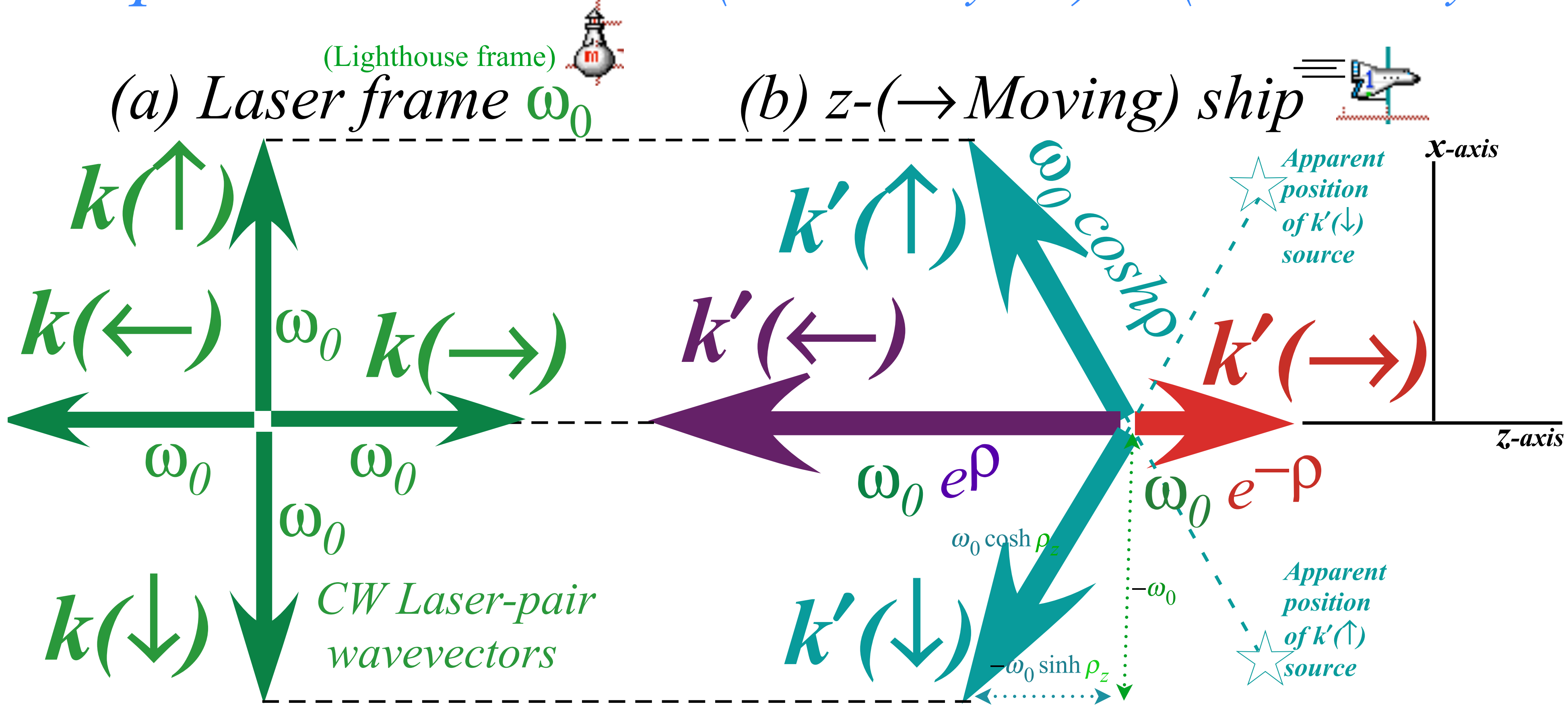

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$+\rho_z$-rapidity ship frame sees starlight Lorentz transformed to : $\left(\omega'_{\downarrow}, ck'_{x\downarrow}, ck'_{y\downarrow}, ck'_{z\downarrow}\right) = \left(\omega_0 \cosh\rho_z, -\omega_0, 0, -\omega_0 \sinh\rho_z\right)$

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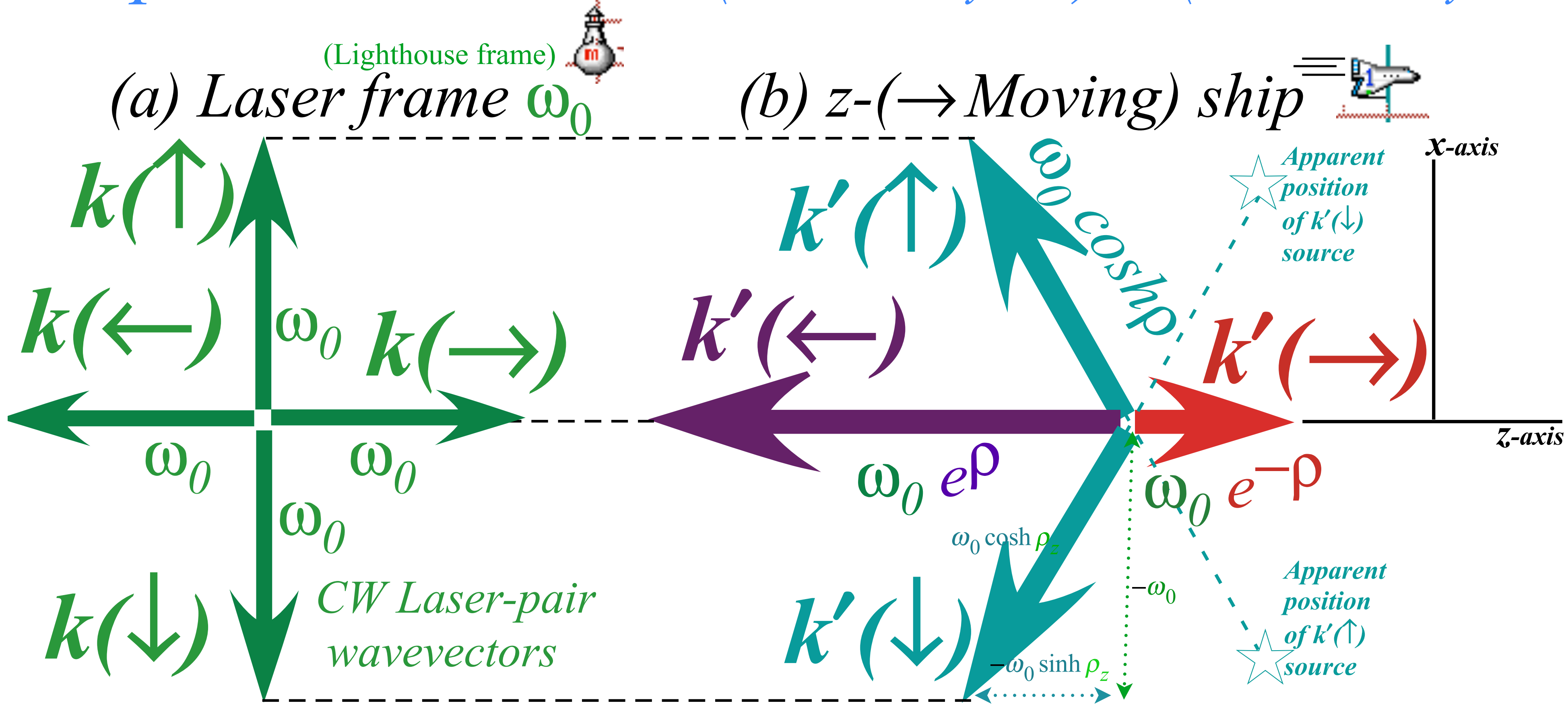

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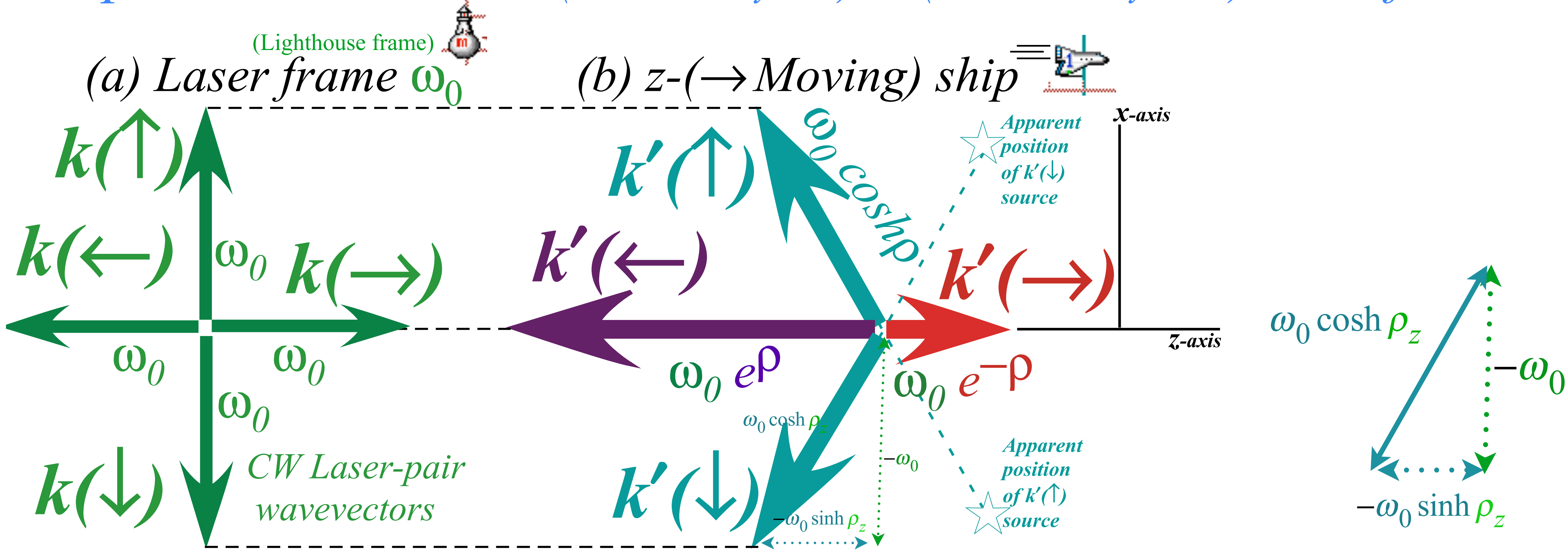

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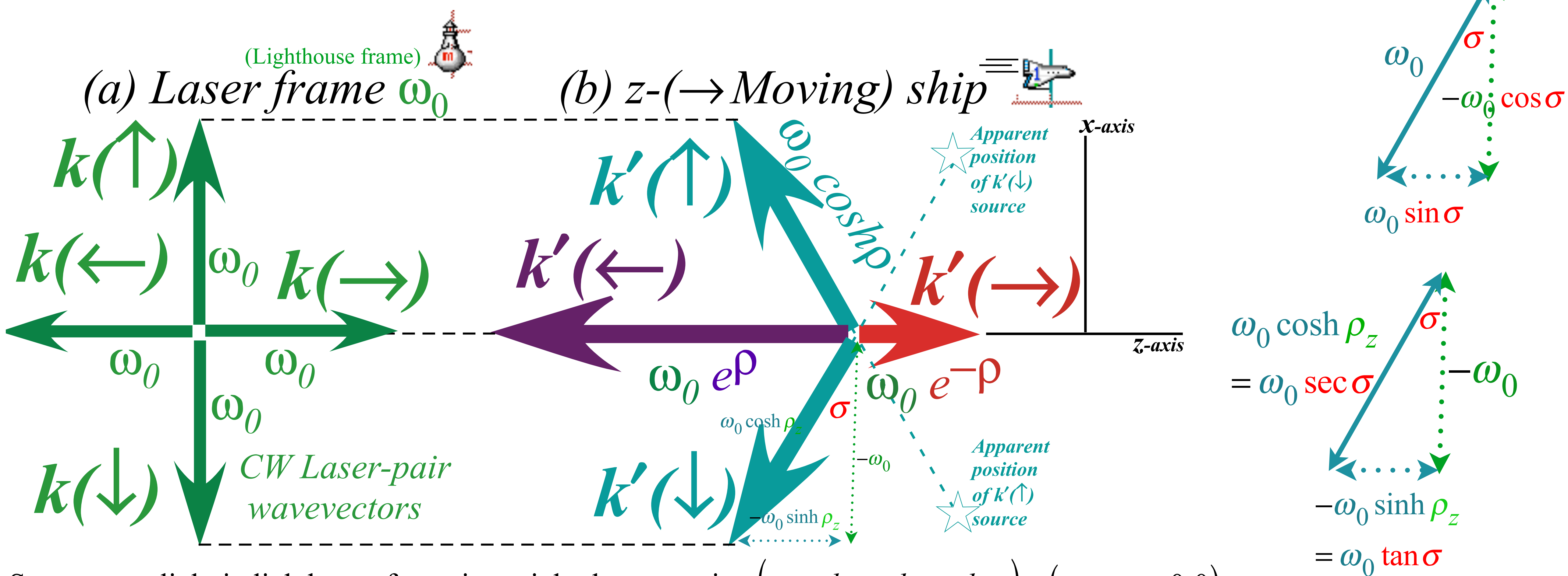
After the 4-vector transformation, $\omega_0 = \omega_\downarrow$ is *transverse Doppler shifted* to $\omega_0 \cosh\rho_z$, while $ck_z = 0$ becomes $ck_z' = -\omega_0 \sinh\rho_z$.
(The x-component is unchanged: $ck_x' = -\omega_0 = ck_x$ and so is y-component: $ck_y' = -\omega_0 = ck_y$.)

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation



Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

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After the 4-vector transformation, $\omega_0=\omega_\downarrow$ is *transverse Doppler shifted* to $\omega_0 \cosh\rho_z$, while $ck_z=0$ becomes $ck_z' = -\omega_0 \sinh\rho_z$.
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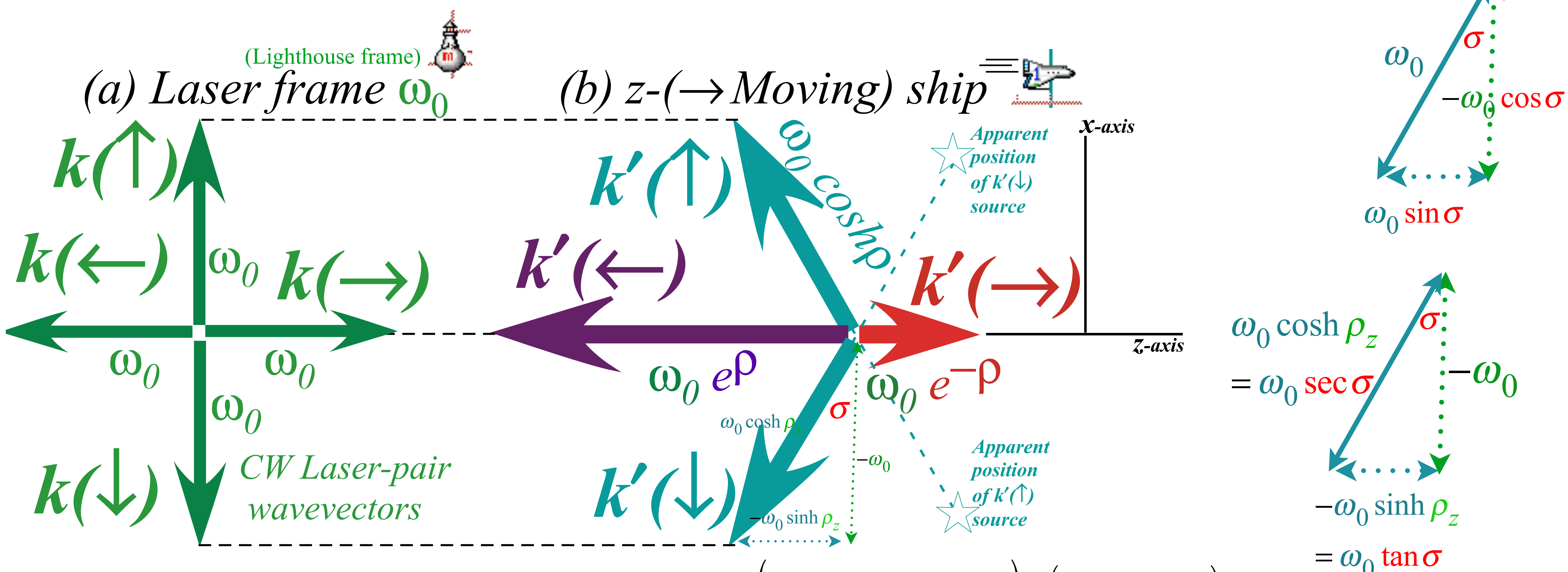
Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation



Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_\downarrow, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

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$$\begin{pmatrix} \omega'_\downarrow \\ ck'_{x\downarrow} \\ ck'_{y\downarrow} \\ ck'_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_\downarrow \\ ck_{x\downarrow} \\ ck_{y\downarrow} \\ ck_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ -\omega_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_0 \cosh\rho_z \\ -\omega_0 \\ 0 \\ -\omega_0 \sinh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 \sec\sigma \\ -\omega_0 \\ 0 \\ -\omega_0 \tan\sigma \end{pmatrix}$$

After the 4-vector transformation, $\omega_0=\omega_\downarrow$ is *transverse Doppler shifted* to $\omega_0 \cosh\rho_z$, while $ck_z=0$ becomes $ck_z' = -\omega_0 \sinh\rho_z$.
(The x-component is unchanged: $ck_x' = -\omega_0 = ck_x$ and so is y-component: $ck_y' = -\omega_0 = ck_y$.)

Recall hyperbolic invariant to Lorentz transform: $\omega^2 - c^2k^2 = \omega'^2 - c^2k'^2$ (=0 for 1-CW light)

The 4-vector form of this is: $\omega^2 - c^2\mathbf{k}\bullet\mathbf{k} = \omega'^2 - c^2\mathbf{k}'\bullet\mathbf{k}'$ (=0 " ")

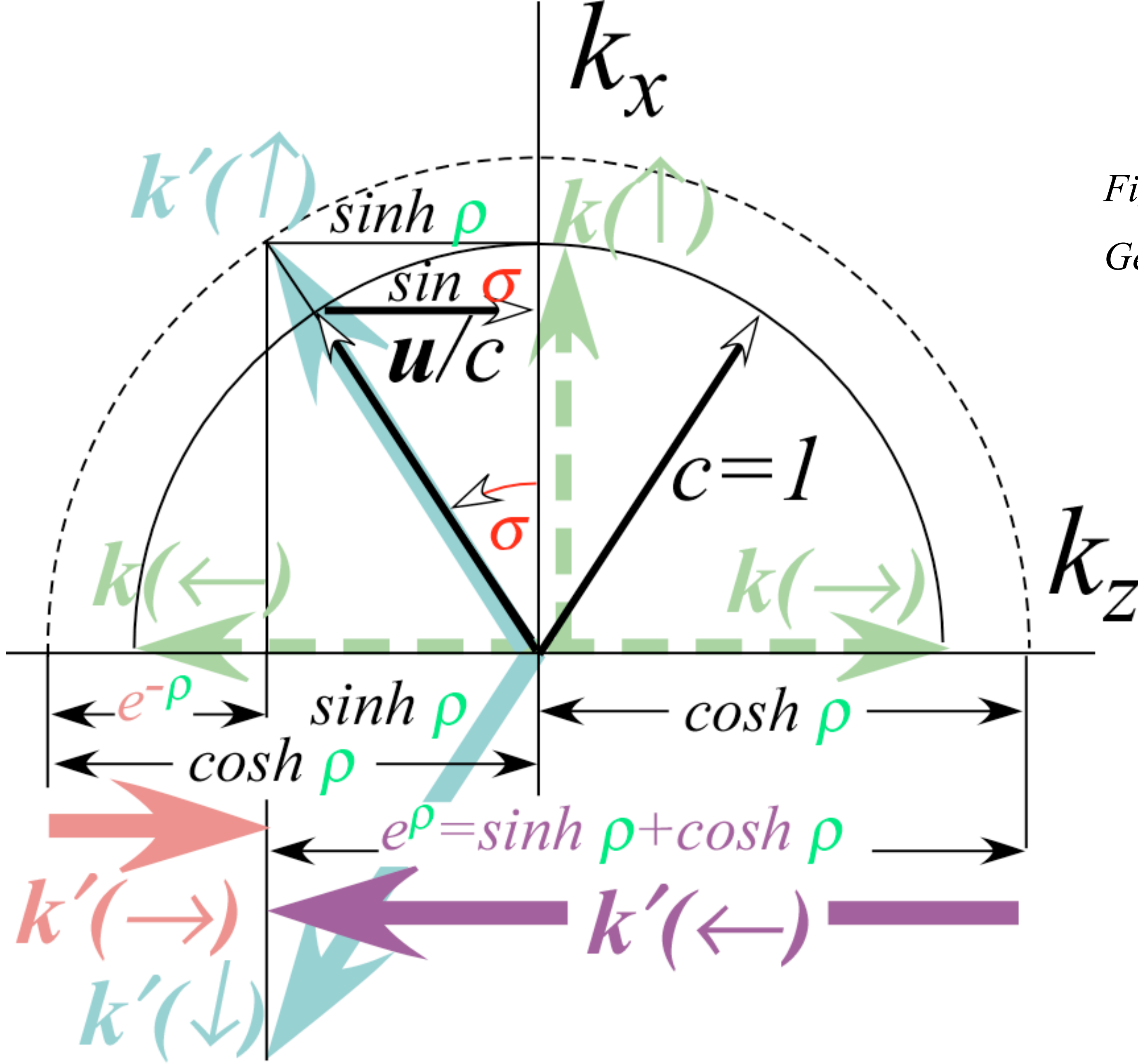
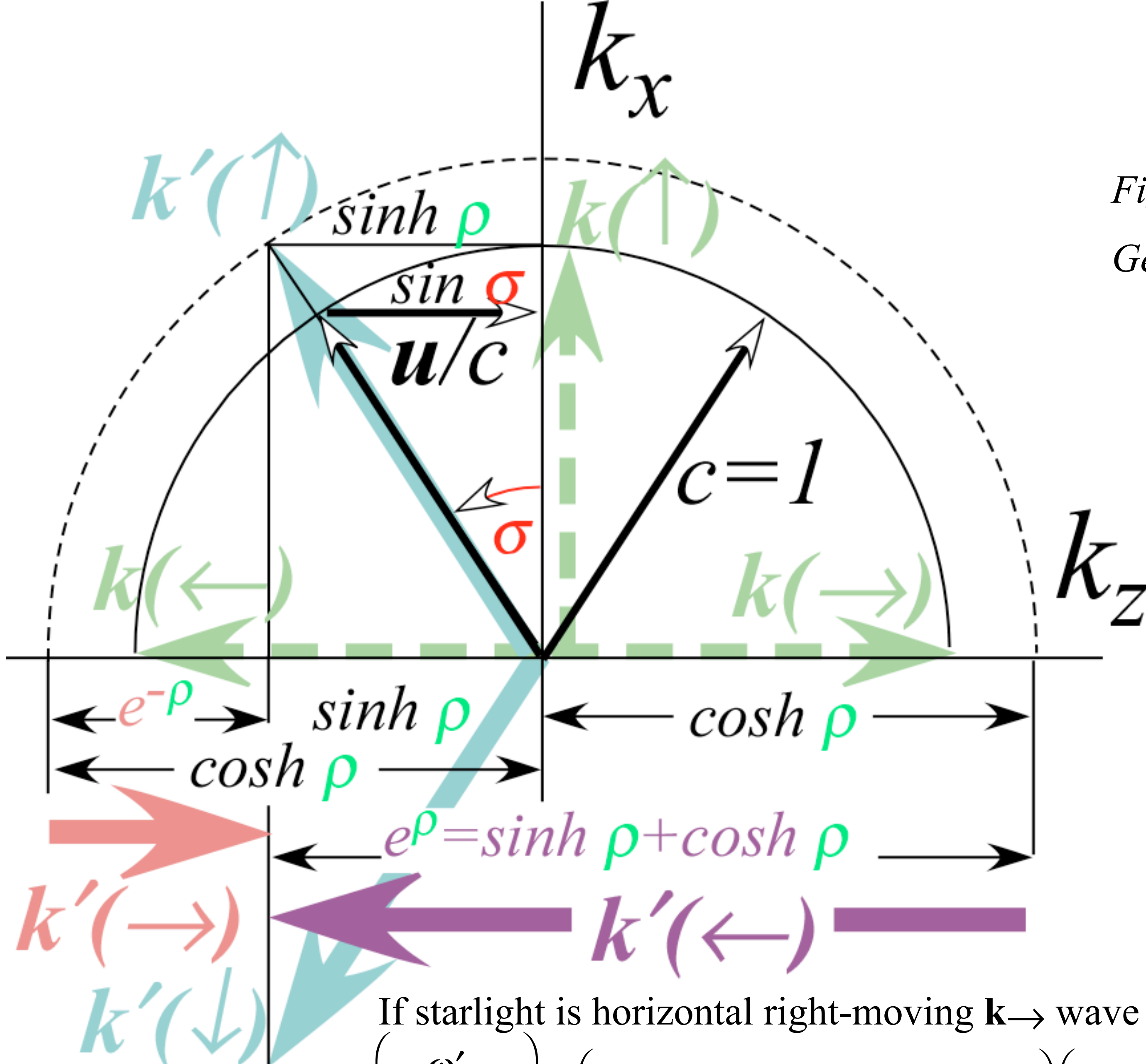


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.



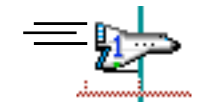


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.

If starlight is horizontal right-moving $\mathbf{k}_{\rightarrow}$ wave then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\rightarrow} \\ ck'_{x\rightarrow} \\ ck'_{y\rightarrow} \\ ck'_{z\rightarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ 0 \\ 0 \\ +\omega_0 \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z - \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z + \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{-\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{-\rho_z} \end{pmatrix}$$

If starlight is horizontal left-moving $\mathbf{k}_{\leftarrow}$ wave then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\leftarrow} \\ ck'_{x\leftarrow} \\ ck'_{y\leftarrow} \\ ck'_{z\leftarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ 0 \\ 0 \\ -\omega_0 \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z - \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{+\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{+\rho_z} \end{pmatrix}$$

The usual longitudinal Doppler blue shifts $e^{+\rho_z}$ or Doppler red shifts $e^{-\rho_z}$ appear on both k-vector and frequency ω_0.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Time contraction-dilation revisited

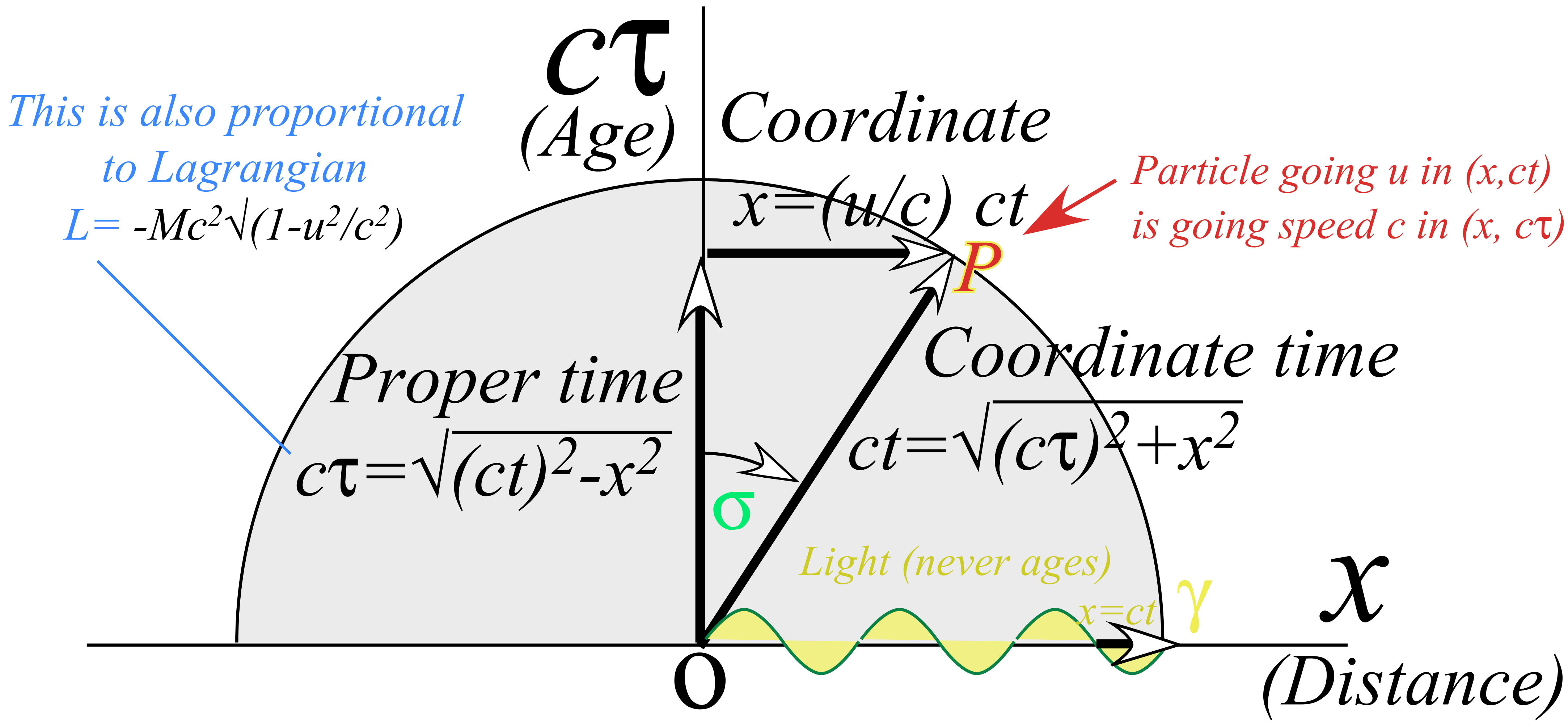
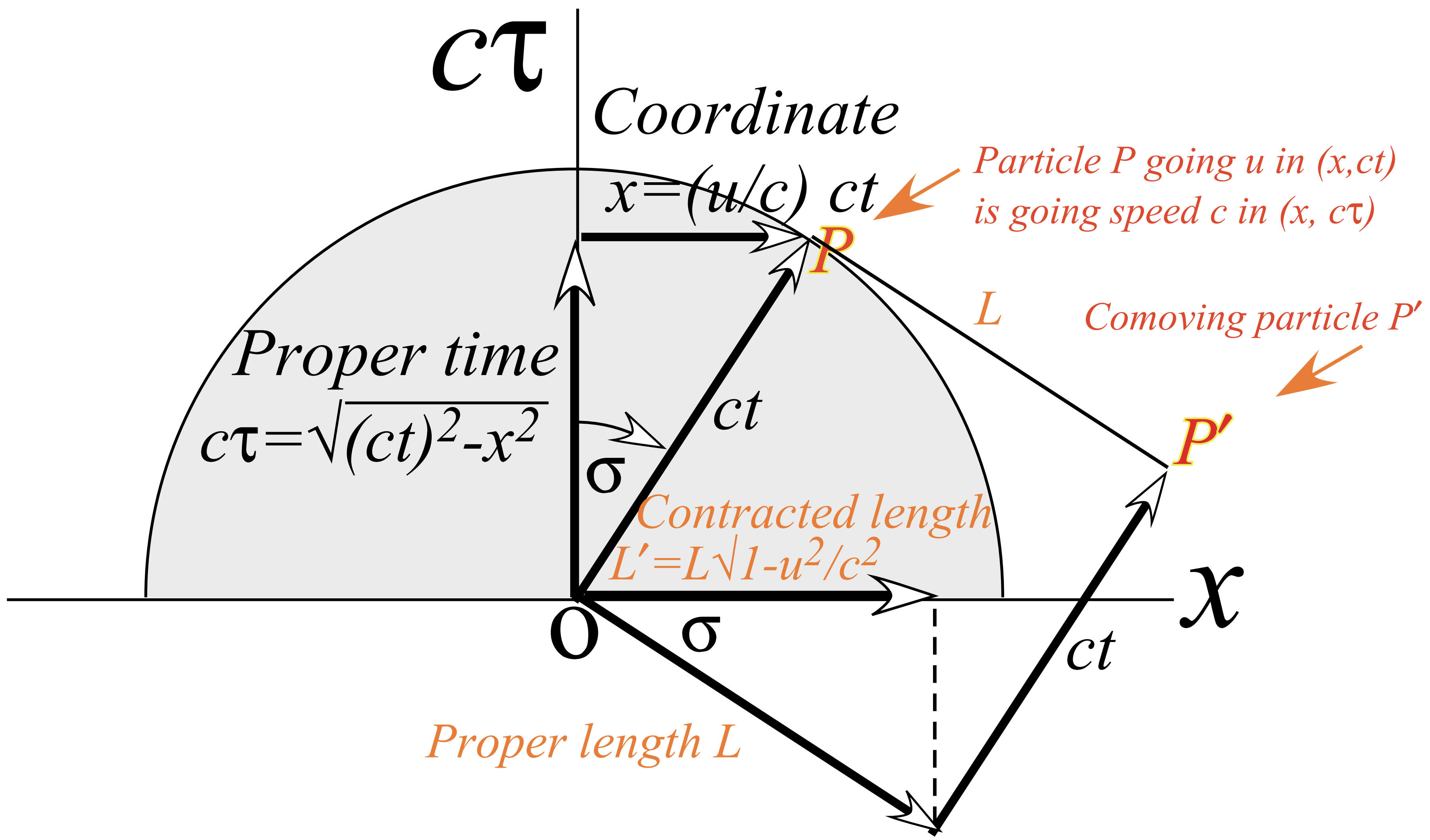


Fig. 5.8 Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

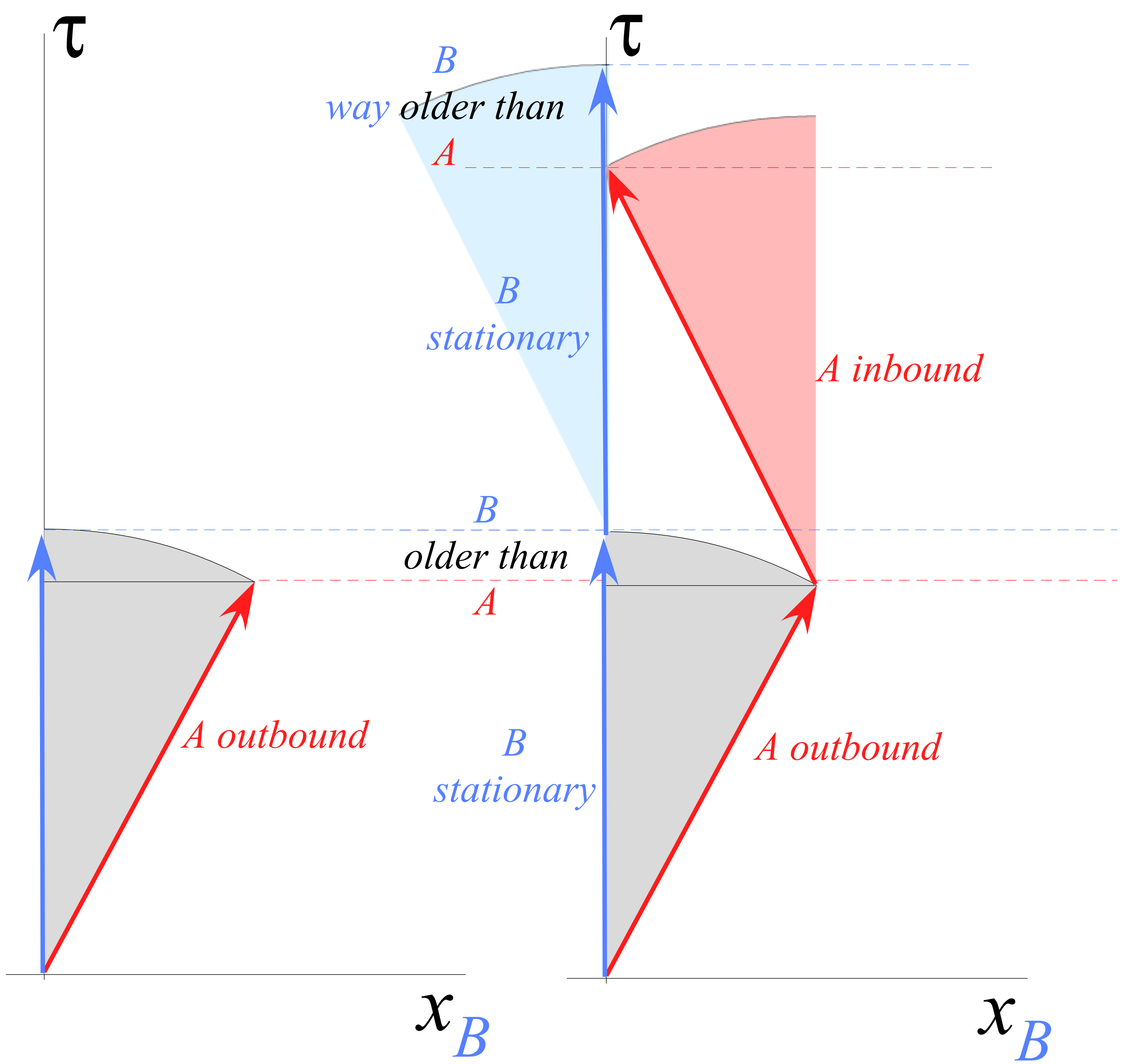
Length contraction-dilation revisited

A cute Epstein feature is that Lorentz-Fitzgerald contraction of a proper length L to $L'=L\sqrt{1-u^2/c^2}$ is simply rotational projection onto the x-axis of a length L rotated by σ.



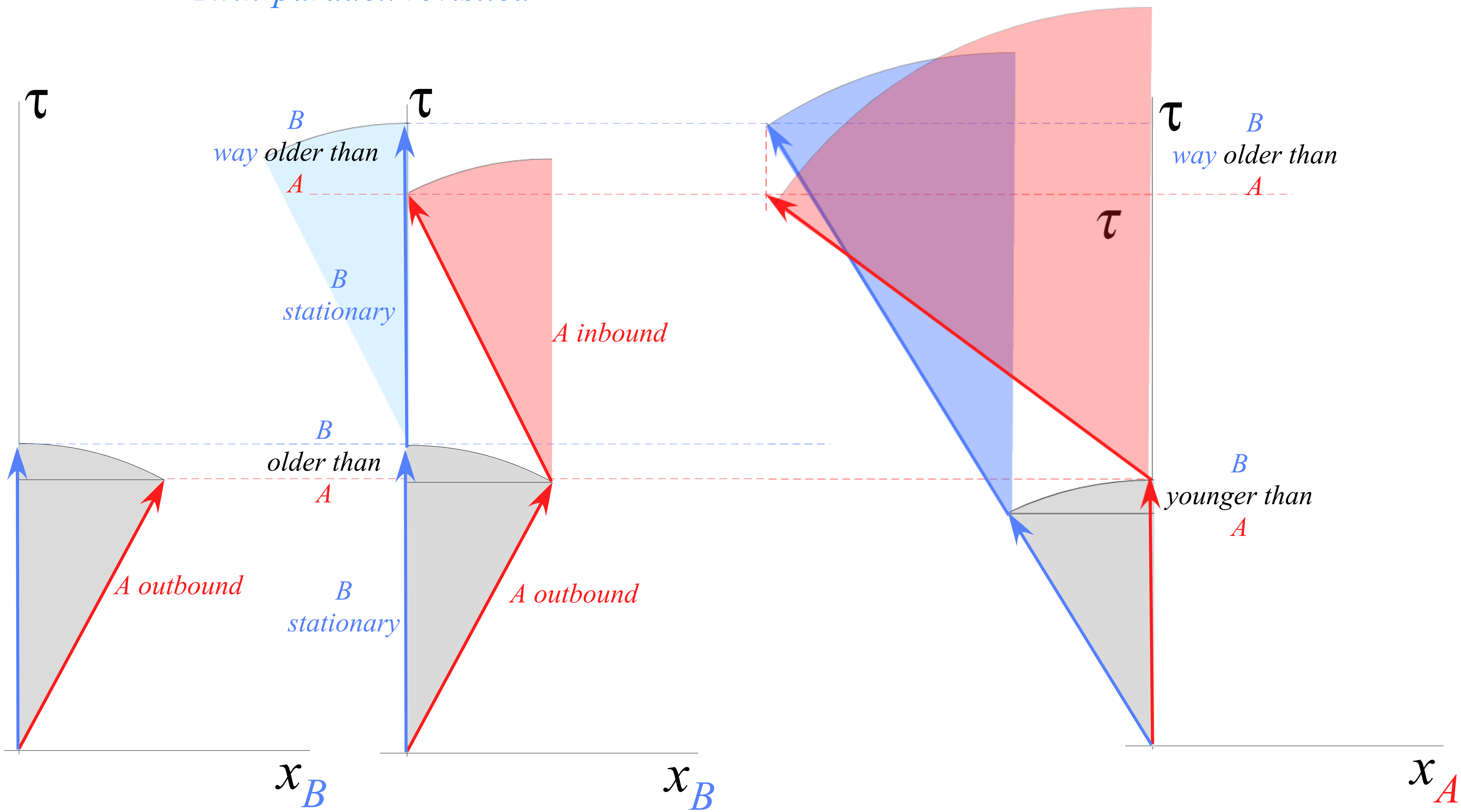
Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Twin-paradox revisited



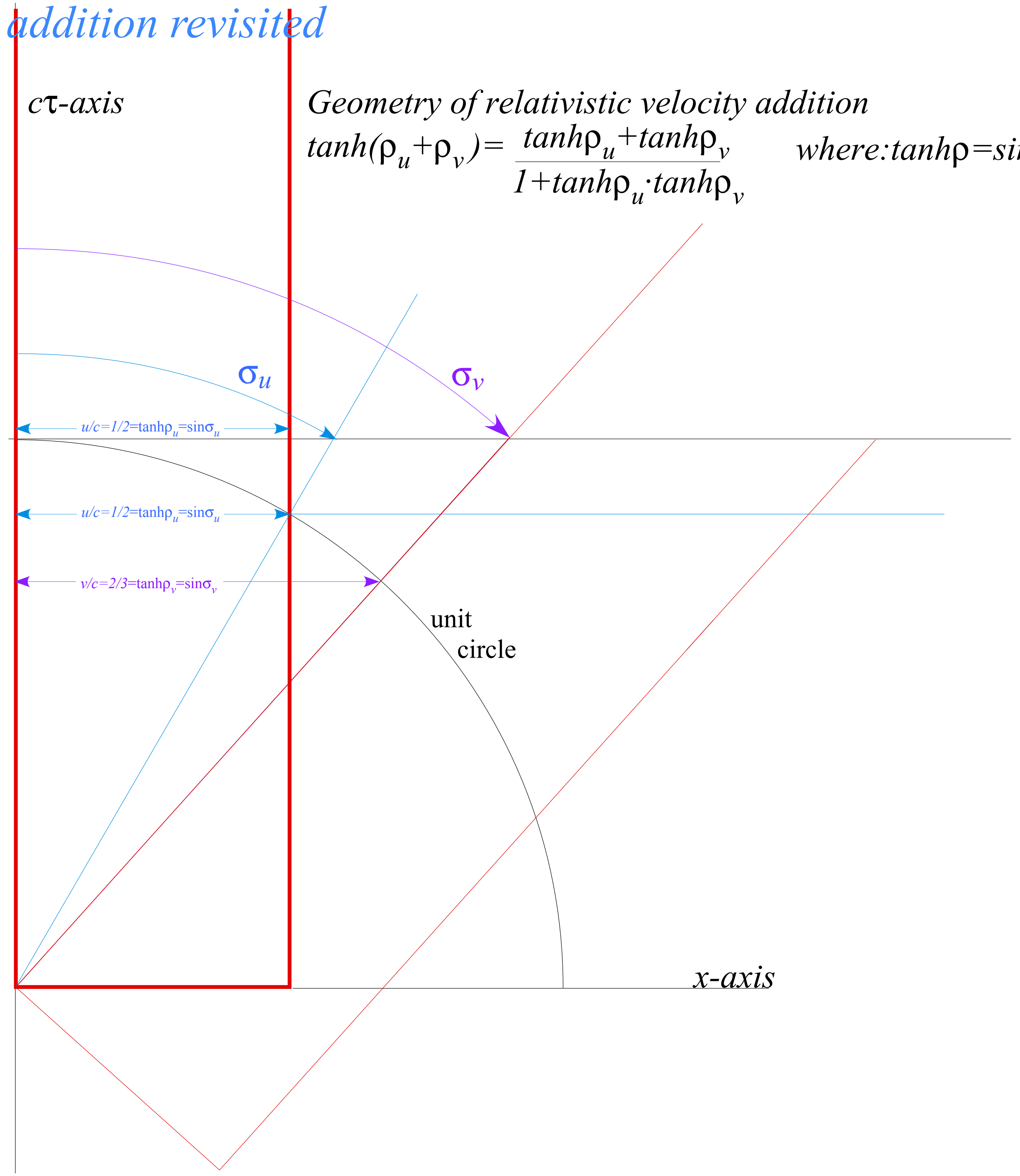
Epstein's space-proper-time $(x, c\tau)$ plots ("c-tau" plots)

Twin-paradox revisited



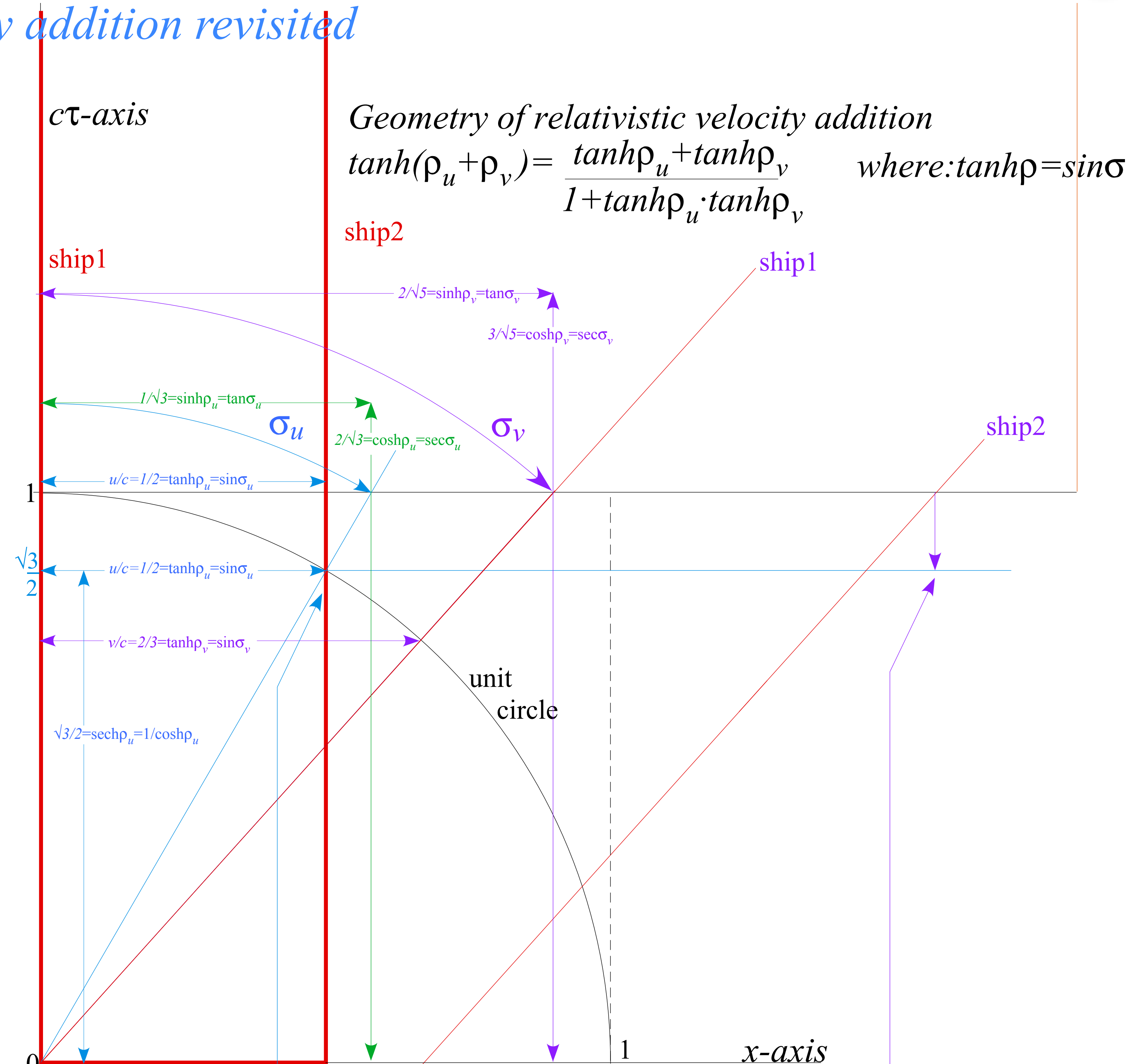
Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited



Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited

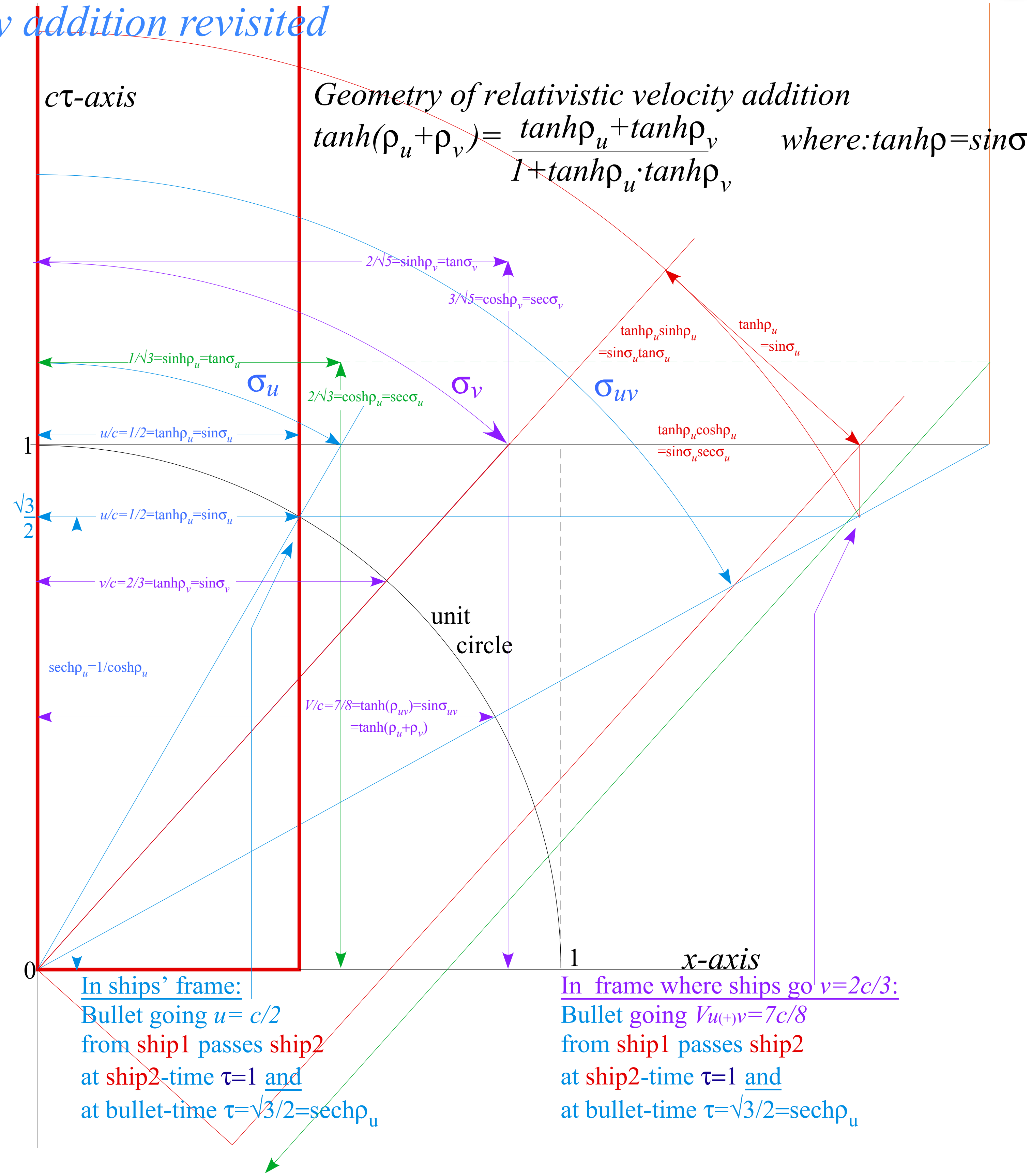


In ships' frame:
Bullet going $u= c/2$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=sech\rho_u$

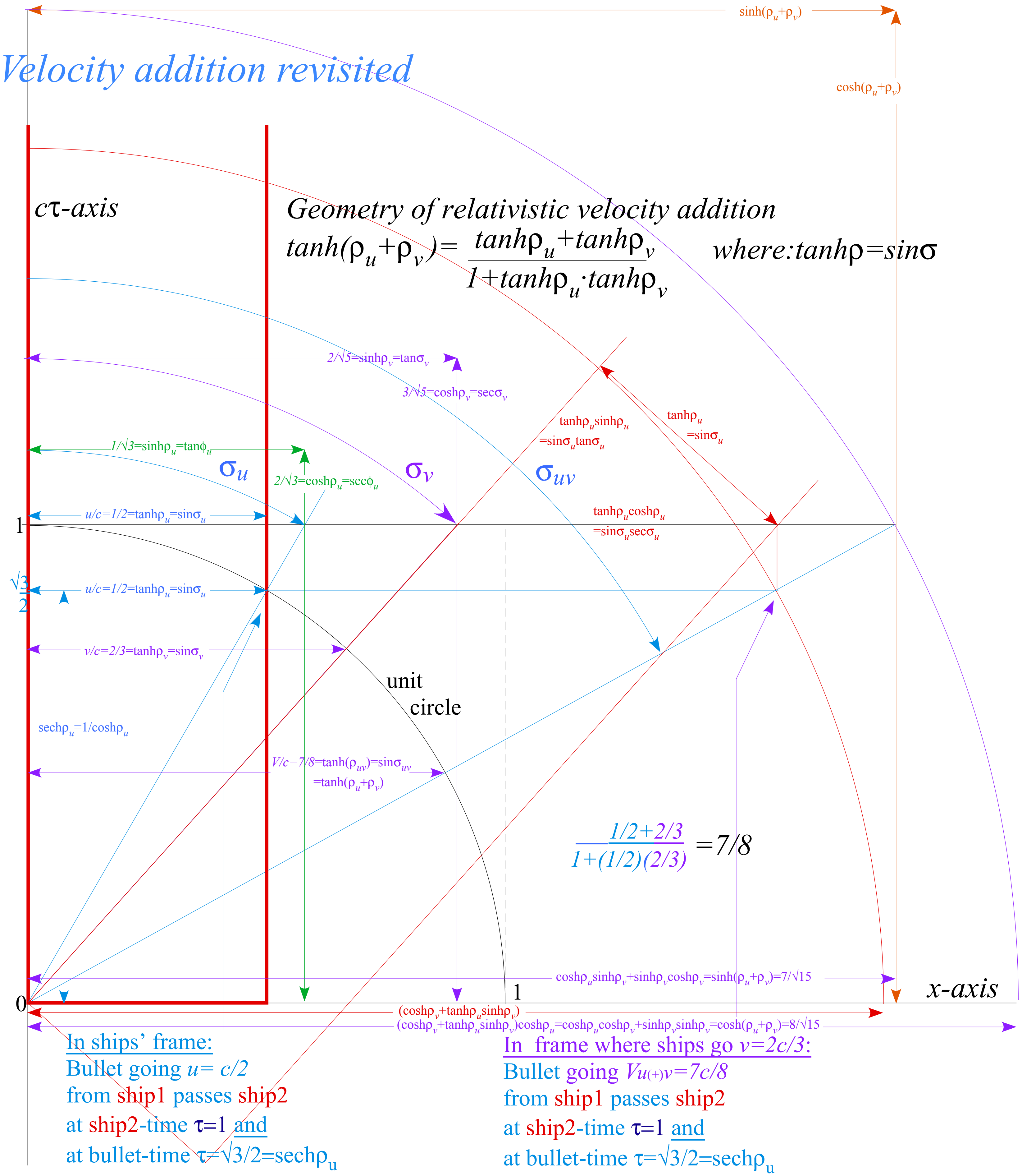
In frame where ships go $v=2c/3$:
Bullet going $V_{u(+)v}= ??$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=sech\rho_u$

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited

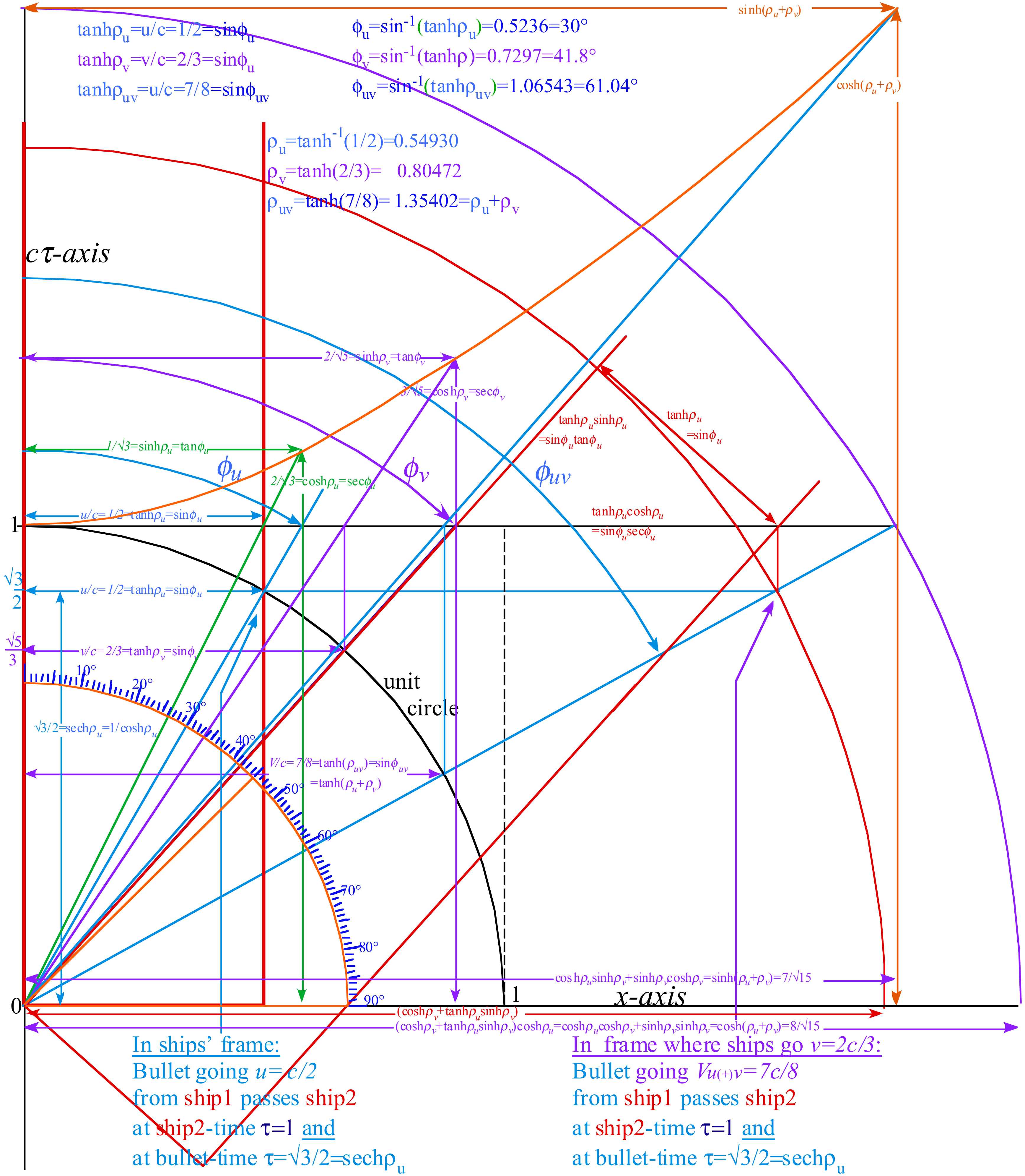


Velocity addition revisited



<u>In ships' frame:</u>
Bullet going $u = c/2$
from ship1 passes ship2
at ship2-time $\tau=1$ <u>and</u>
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$

<u>In frame where ships go $v=2c/3$:</u>
Bullet going $V_{u(+)v}=7c/8$
from ship1 passes ship2
at ship2-time $\tau=1$ <u>and</u>
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$


tanhρu=u/c=1/2=sinφu
tanhρv=v/c=2/3=sinφu
tanhρuv=u/c=7/8=sinφuv
φu=sin-1(tanhρu)=0.5236=30°
φv=sin-1(tanhρ)=0.7297=41.8°
φuv=sin-1(tanhρuv)=1.06543=61.04°
ρu=tanh-1(1/2)=0.54930
ρv=tanh(2/3)= 0.80472
ρuv=tanh(7/8)= 1.35402=ρu+ρv
sinh(ρu+ρv)
cosh(ρu+ρv)
cτ-axis
2/√5=sinhρv=tanφv
3/√5=coshρv=secφv
1/√3=sinhρu=tanφu
2/√3=coshρu=secφu
φu
φv
φuv
tanhρu sinhρu
=sinφu tanφu
tanhρu
=sinφu
tanhρu coshρu
=sinφu secφu
u/c=1/2=tanhρu=sinφu
u/c=1/2=tanhρu=sinφu
v/c=2/3=tanhρv=sinφv
1
√3/2
√5/3
10°
20°
30°
40°
50°
60°
70°
80°
90°
√3/2=sechρu=1/coshρu
V/c=7/8=tanh(ρuv)=sinφuv
=tanh(ρu+ρv)
unit
circle
coshρu sinhρv+sinhρv coshρv=sinh(ρu+ρv)=7/√15
0
1
x-axis
(coshρv+tanhρu sinhρv)
(coshρv+tanhρu sinhρv)coshρu=coshρu coshρv+sinhρv sinhρv=cosh(ρu+ρv)=8/√15
In ships' frame:
Bullet going u=c/2
from ship1 passes ship2
at ship2-time τ=1 and
at bullet-time τ=√3/2=sechρu
In frame where ships go v=2c/3:
Bullet going Vu(+)v=7c/8
from ship1 passes ship2
at ship2-time τ=1 and
at bullet-time τ=√3/2=sechρu

Lorentz symmetry effects

How it makes momentum and energy be conserved

A strength (and also, weakness) of CW axioms (1.1-2) is that they are *symmetry* principles

due to the Lorentz-Poincare isotropy of space-time (invariance to space-time translation $\mathbf{T}(\delta,\tau)$ in the vacuum).

Operator $\mathbf{T}$ has plane wave eigenfunctions $\psi_{k,\omega} = Ae^{i(kx-\omega t)}$ with roots-of-unity eigenvalues $e^{i(k\cdot\delta-\omega\cdot\tau)}$.

$$\left\langle \psi_{k,\omega} \right| \mathbf{T}^\dagger = \left\langle \psi_{k,\omega} \right| e^{-i(k\cdot\delta-\omega\cdot\tau)} \qquad (5.18a) \qquad\qquad \mathbf{T}\left| \psi_{k,\omega} \right\rangle = e^{i(k\cdot\delta-\omega\cdot\tau)} \left| \psi_{k,\omega} \right\rangle \qquad (5.18b)$$

This also applies to 2-part or "2-particle" product states $\Psi_{K,\Omega} = \psi_{k_1,\omega_1}\psi_{k_2,\omega_2}$ where exponents add *(k,*ω*)*-values of

each constituent to $K=k_1+k_2$ and $\Omega=\omega_1+\omega_2$, and $\mathbf{T}(\delta,\tau)$-eigenvalues also have that form $e^{i(K\cdot\delta-\Omega\cdot\tau)}$.

Matrix $\left\langle \Psi'_{K',\Omega'} \right| \mathbf{U} \left| \Psi_{K,\Omega} \right\rangle$ of **T**-symmetric evolution **U** is zero unless $K' = k'_1 + k'_2 = K$ and $\Omega' = \omega'_1 + \omega'_2 = \Omega$.

$$\left\langle \Psi'_{K',\Omega'} \right| \mathbf{U} \left| \Psi_{K,\Omega} \right\rangle = \left\langle \Psi'_{K',\Omega'} \right| \mathbf{T}^\dagger(\delta,\tau)\mathbf{U}\mathbf{T}(\delta,\tau) \left| \Psi_{K,\Omega} \right\rangle \qquad (\text{if } \mathbf{UT}=\mathbf{TU} \text{ for all } \delta \text{ and } \tau)$$

$$= e^{-i(K'\cdot\delta-\Omega'\cdot\tau)} e^{i(K\cdot\delta-\Omega\cdot\tau)} \left\langle \Psi'_{K',\Omega'} \right| \mathbf{U} \left| \Psi_{K,\Omega} \right\rangle = 0 \;\; \text{unless: } K' = K \text{ and: } \Omega' = \Omega$$

That's momentum (P=hK) and energy (E=hW) conservation!