Lecture 24.

Relativity of lightwaves and Lorentz-Minkowski coordinates III.

(Ch. 0-3 of Unit 2   3.29.12)

4. Einstein-Lorentz symmetry *(Includes Lecture 23 review)*

What happened to Galilean symmetry? (It moved to "gauge" space!)

Thale's construction and Euclid's means

Lecture 23 ended (about) here ←

Time reversal symmetry gives hyperbolic invariants

per-space-time hyperbola

space-time hyperbola

Phase invariance

5. That "old-time" relativity (Circa 600BCE- 1905CE)

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

Lecture 24 ended (about) here ←

A politically incorrect analogy of rotational transformation and Lorentz transformation

The straight scoop on "angle" and "rapidity" (They're area!)

Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)

How Minkowski's space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts

4. Einstein-Lorentz symmetry

What happened to Galilean symmetry? (It moved to "gauge" space!)

Thale's construction and Euclid's means

Time reversal symmetry gives hyperbolic invariants

per-space-time hyperbola

space-time hyperbola

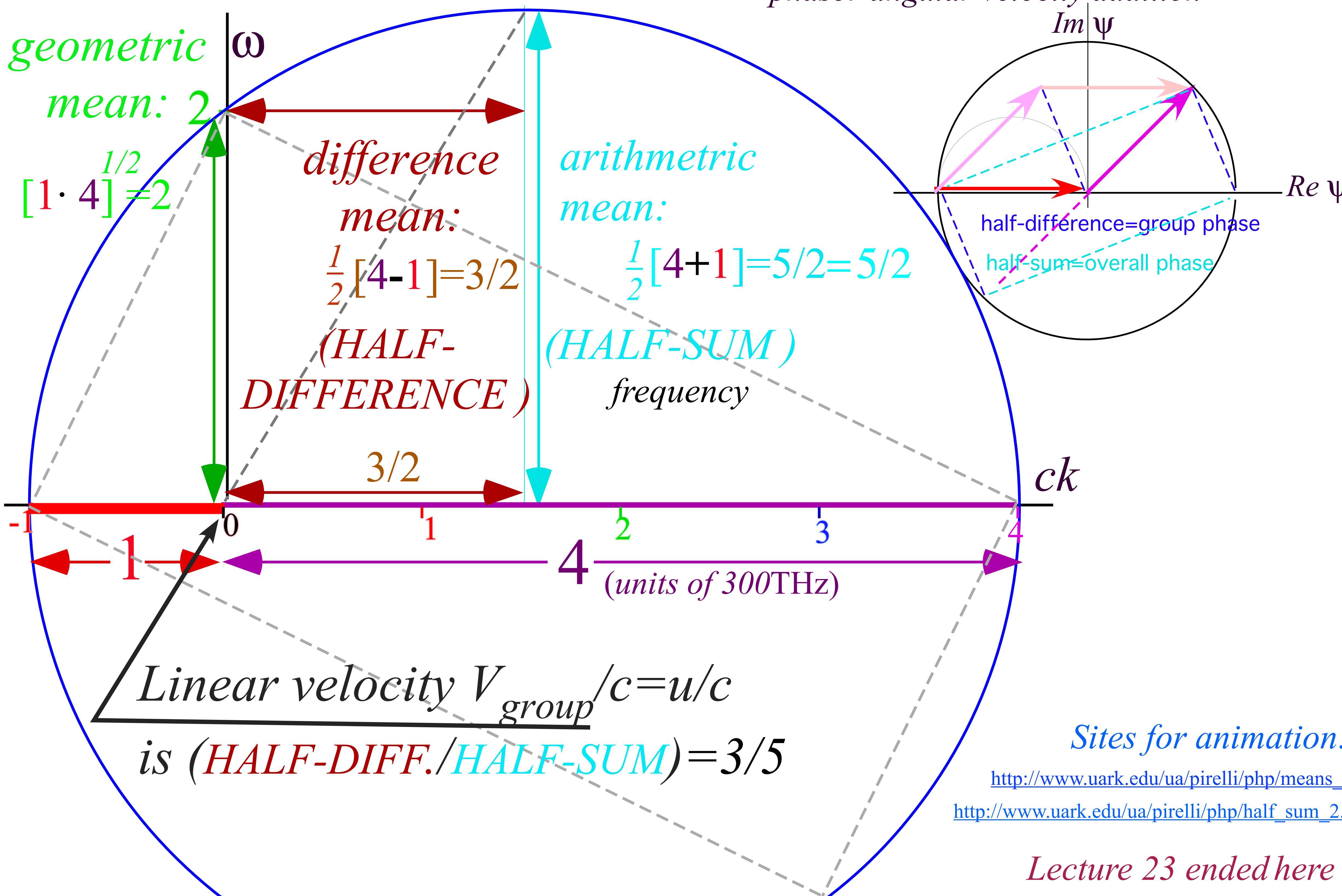
Phase invariance

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition



Sites for animation:

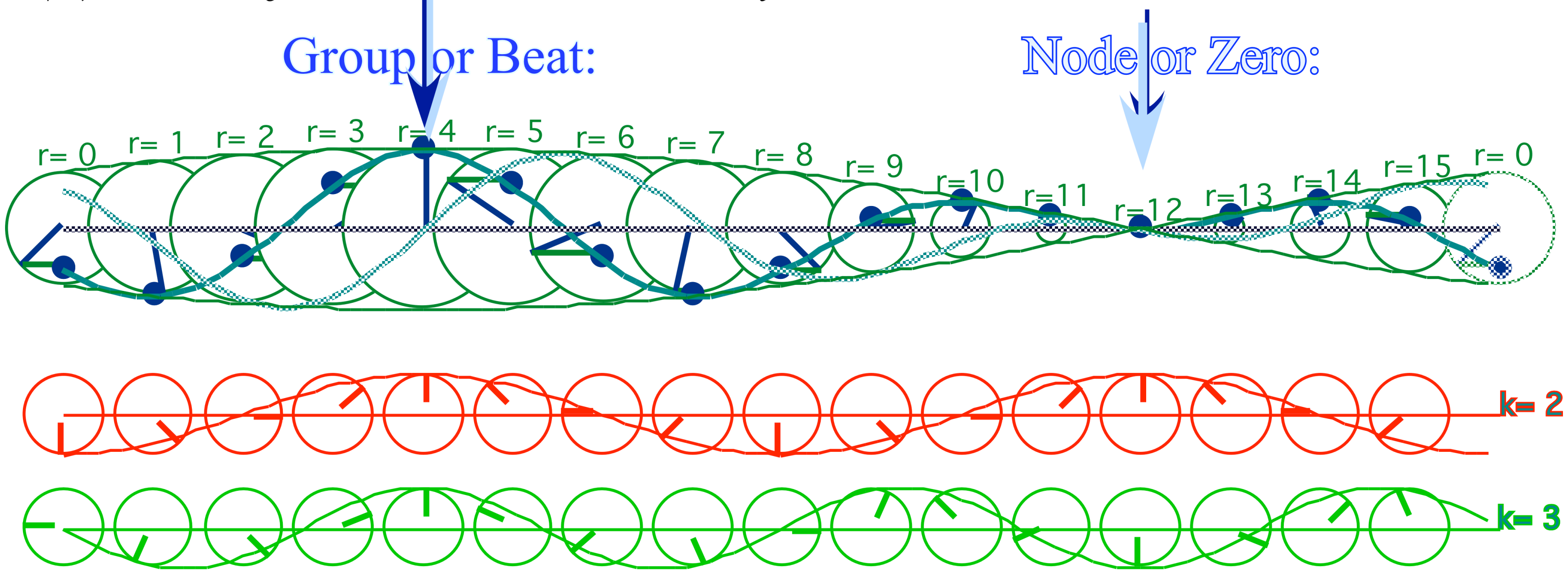
http://www.uark.edu/ua/pirelli/php/means_1.php

http://www.uark.edu/ua/pirelli/php/half_sum_2.php

Lecture 23 ended here

Fig. 3.3a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

(a) Sum of Wave Phasor Array



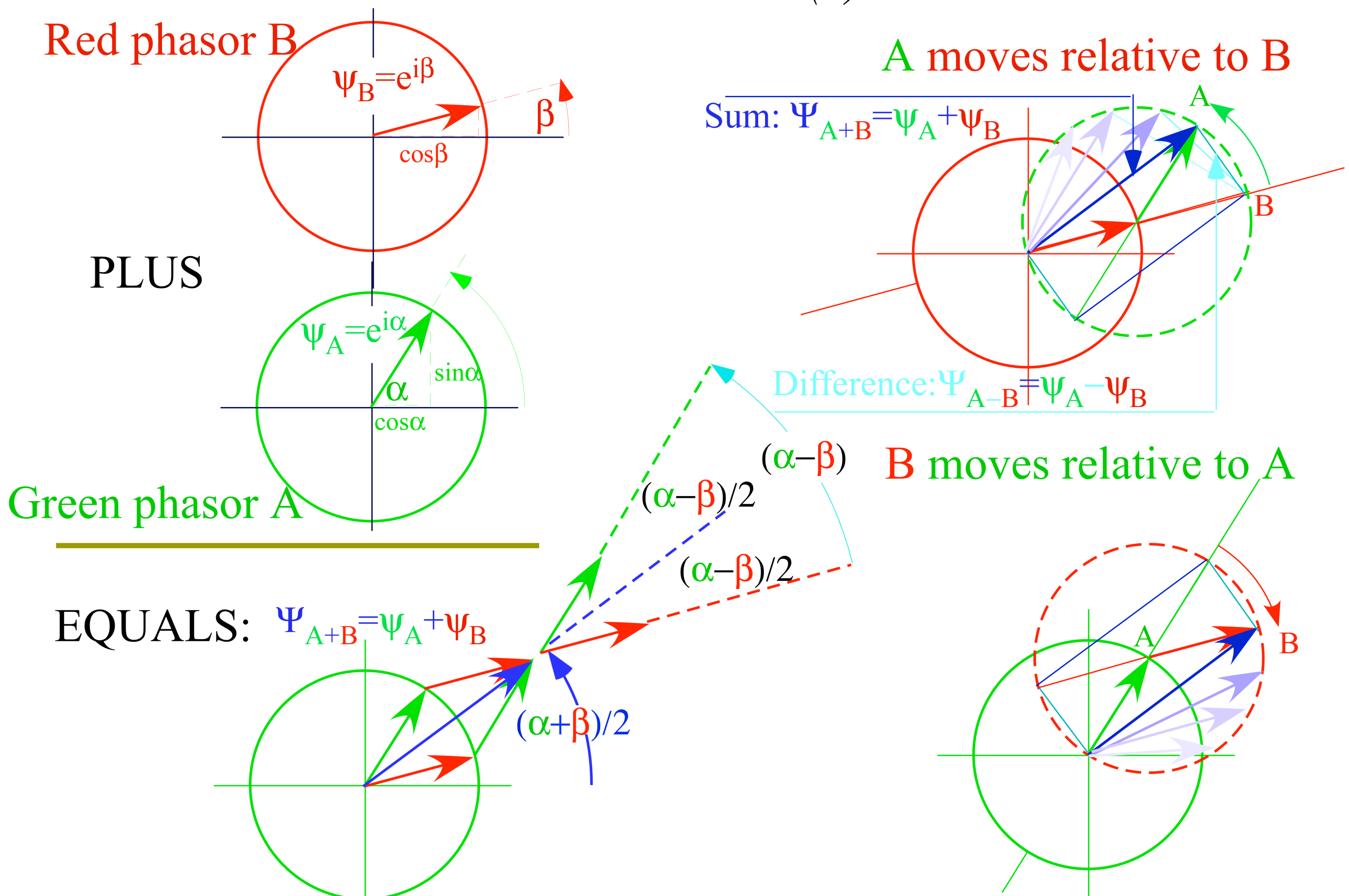
Sites for animations:

http://www.uark.edu/ua/pirelli/php/means_1.php

http://www.uark.edu/ua/pirelli/php/half_sum_5.php

(b) Typical Phasor Sum:

(c) Phasor-relative views



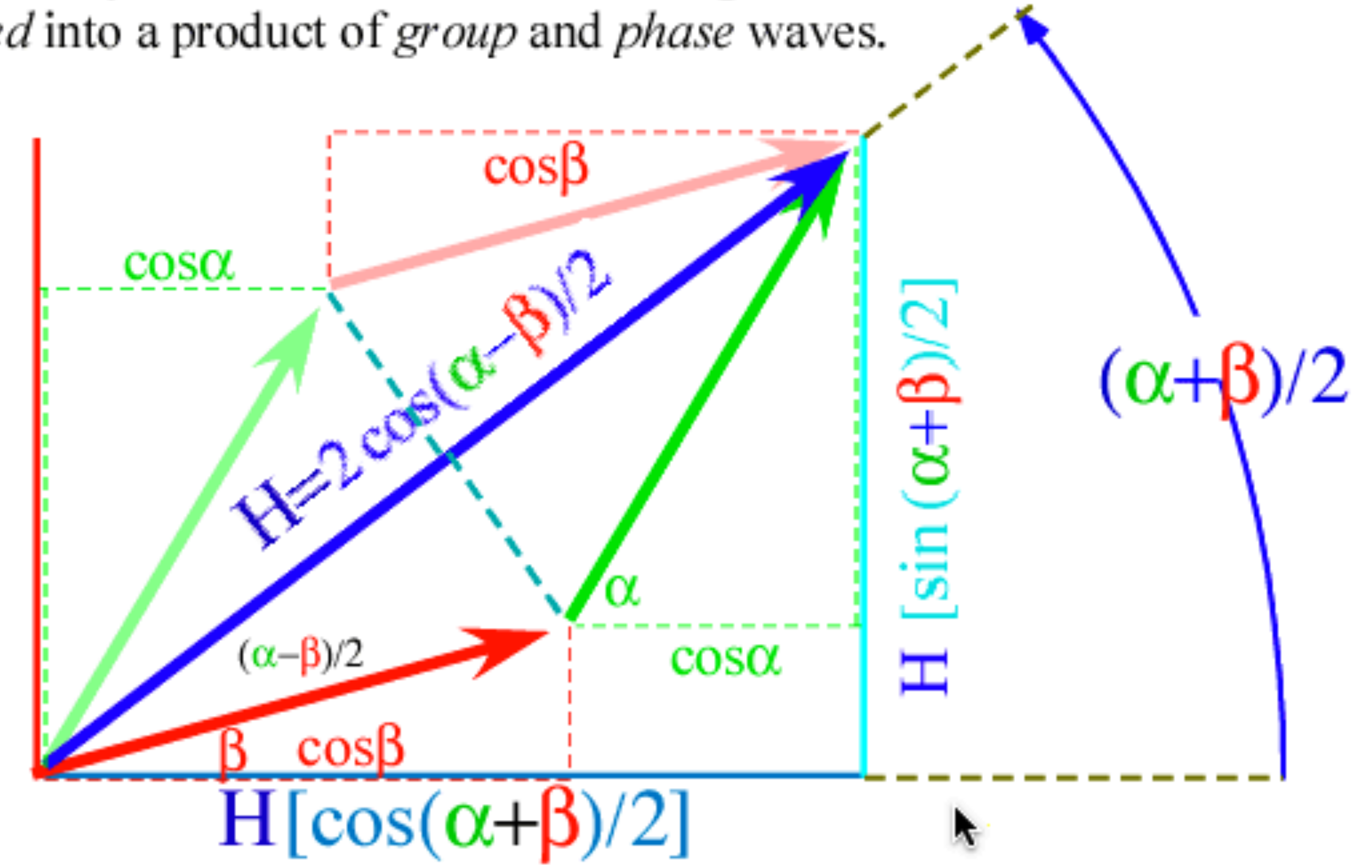
Galileo's revenge!

Now we use Galilean relativity to add angular velocity, that is frequency ω_a and ω_b, in phase or "gauge" space. No "c-limit" evident. (So far at 18-fig. precision.)

Fig. 3.1 Wave phasor addition. (a) Each phasor in a wave array is a sum (b) of two component phasors.

Half-Sum & Difference Rules of Phase Relativity (contd.)

The detailed trigonometry of half-sum & difference angles is shown below.
The wave is *factored* into a product of *group* and *phase* waves.



Main Result: Factoring algebraic sums helps to locate *wave zeros.*

$$\cos\alpha + \cos\beta = 2\cos(\alpha-\beta)/2 \bullet [\cos(\alpha+\beta)/2]$$

$$\sin\alpha + \sin\beta = 2\cos(\alpha-\beta)/2 \bullet [\sin(\alpha+\beta)/2]$$

Sum is zeroed by *either* factor. Each factor's zero line is a *spacetime coordinate line.*

<Home> <Back> <Next>

www.uark.edu/ua/pirelli/php/half_sum_5.php

4. Einstein-Lorentz symmetry

What happened to Galilean symmetry? (It moved to "gauge" space!)

Thale's construction and Euclid's means

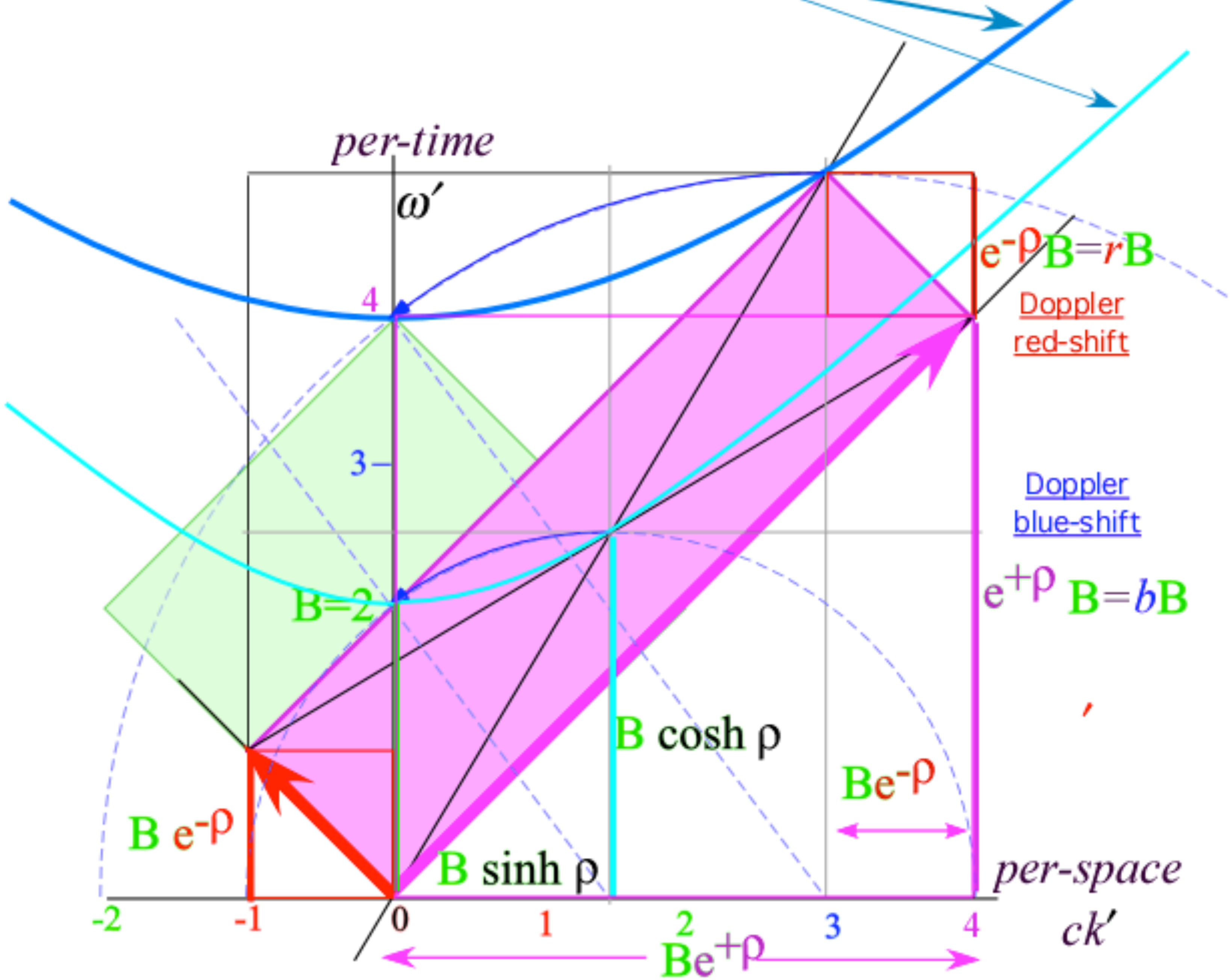
Time reversal symmetry gives hyperbolic invariants

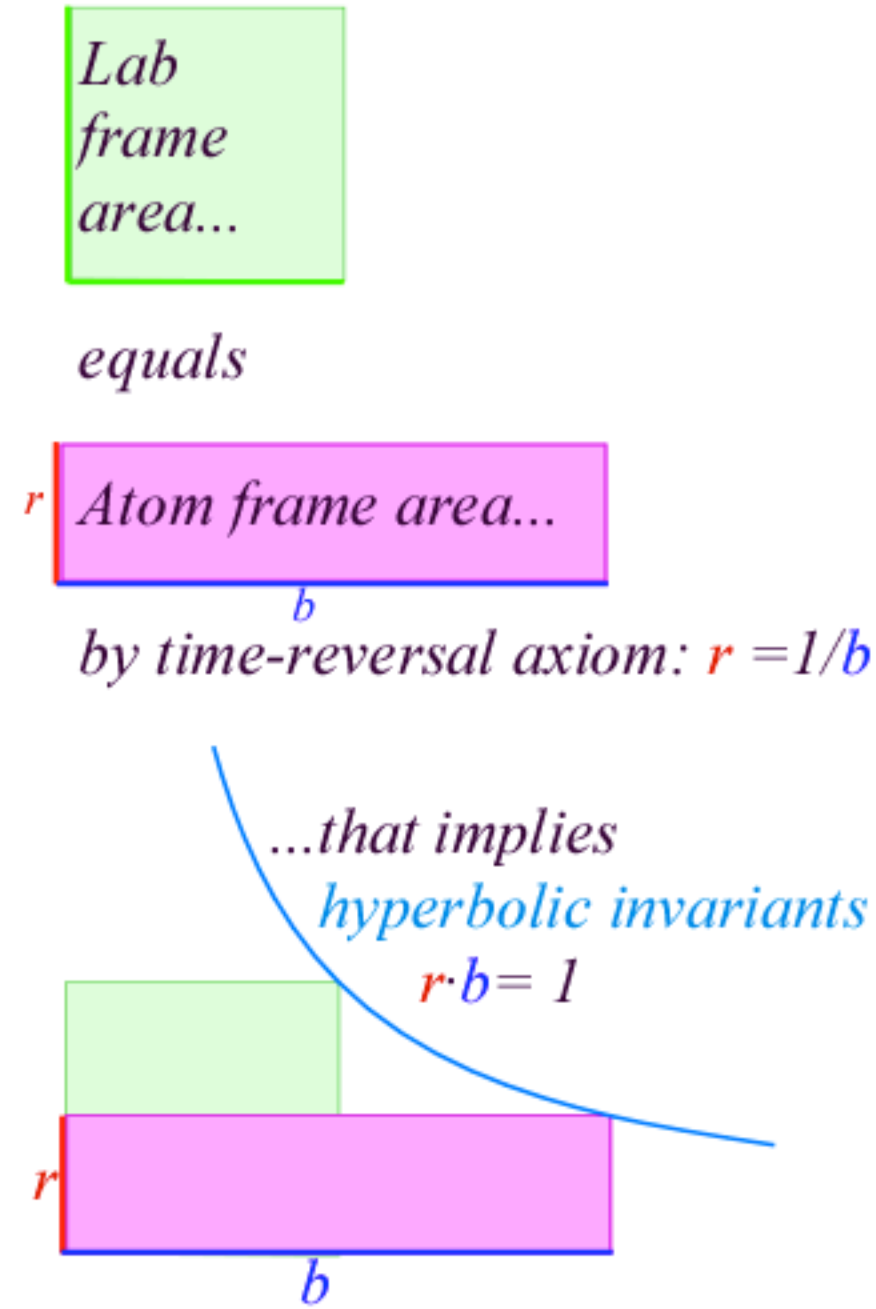
per-space-time hyperbola

space-time hyperbola

Phase invariance

Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = nB\cosh\rho$





$B\sinh\rho = (Be^{+\rho} - Be^{-\rho})/2$ $B\cosh\rho = (Be^{+\rho} + Be^{-\rho})/2$

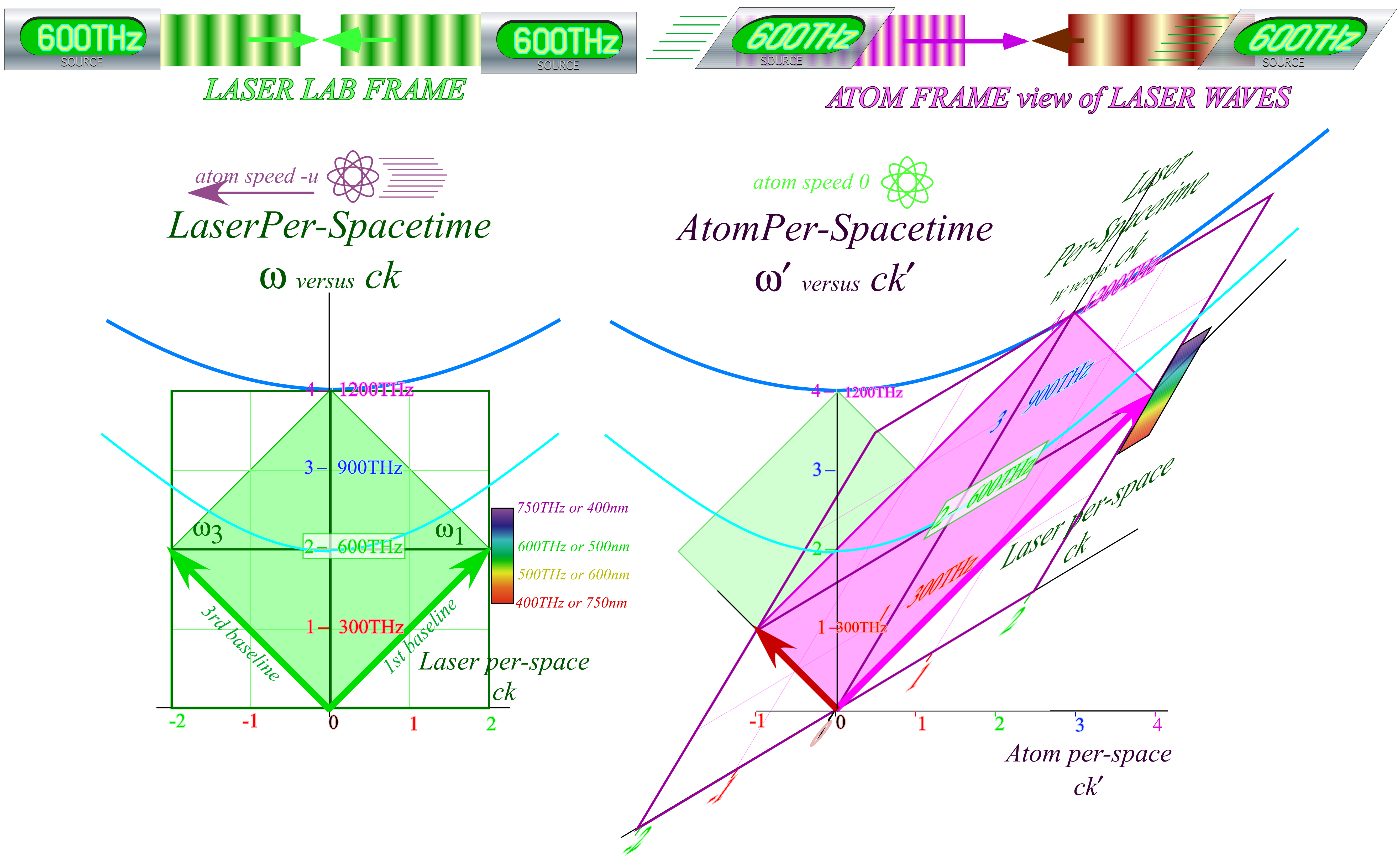


Fig. 3.5 Dispersion hyperbolas for 2-CW interference (a) Laser lab view. (b)Atom frame view.

4. Einstein-Lorentz symmetry

What happened to Galilean symmetry? (It moved to "gauge" space!)

Thale's construction and Euclid's means

Time reversal symmetry gives hyperbolic invariants

per-space-time hyperbola

space-time hyperbola

Phase invariance

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 *is called "proper frequency" or rate of "aging"*

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 *is called "proper time" or "age":*

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix}$$

$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

Proof: ?

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}} = \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}} = t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

Proof:

$$ck' \cdot x' \quad - \quad \omega' \cdot ct'$$

	$x\cdot\cosh$	$ct\cdot\sinh$		$x\cdot\sinh$	$ct\cdot\cosh$
$ck\cdot\cosh$	$ck\cdot x\cdot\cosh^2$	$ck\cdot ct\cdot\cosh\cdot\sinh$	$ck\cdot\sinh$	$ck\cdot x\cdot\sinh^2$	$ck\cdot ct\cdot\sinh\cdot\cosh$
$\omega\cdot\sinh$	$\omega\cdot x\cdot\sinh\cdot\cosh$	$\omega\cdot ct\cdot\sinh^2$	$\omega\cdot\cosh$	$\omega\cdot x\cdot\cosh\cdot\sinh$	$\omega\cdot ct\cdot\cosh^2$

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

Proof:

$$ck'\cdot x' \;-\; \omega'\cdot ct'$$

ck'	$x\cdot\cosh$	$ct\cdot\sinh$	ω'	$x\cdot\sinh$	$ct\cdot\cosh$
$ck\cdot\cosh$	$ck\cdot x\cdot\cosh^2$	~~$ck\cdot ct\cdot\cosh\cdot\sinh$~~	$ck\cdot\sinh$	$ck\cdot x\cdot\sinh^2$	~~$ck\cdot ct\cdot\sinh\cdot\cosh$~~
$\omega\cdot\sinh$	~~$\omega\cdot x\cdot\sinh\cdot\cosh$~~	$\omega\cdot ct\cdot\sinh^2$	$\omega\cdot\cosh$	~~$\omega\cdot x\cdot\cosh\cdot\sinh$~~	$\omega\cdot ct\cdot\cosh^2$

$$ck\cdot x\cdot\cosh^2 - ck\cdot x\cdot\sinh^2 = ck\cdot x$$

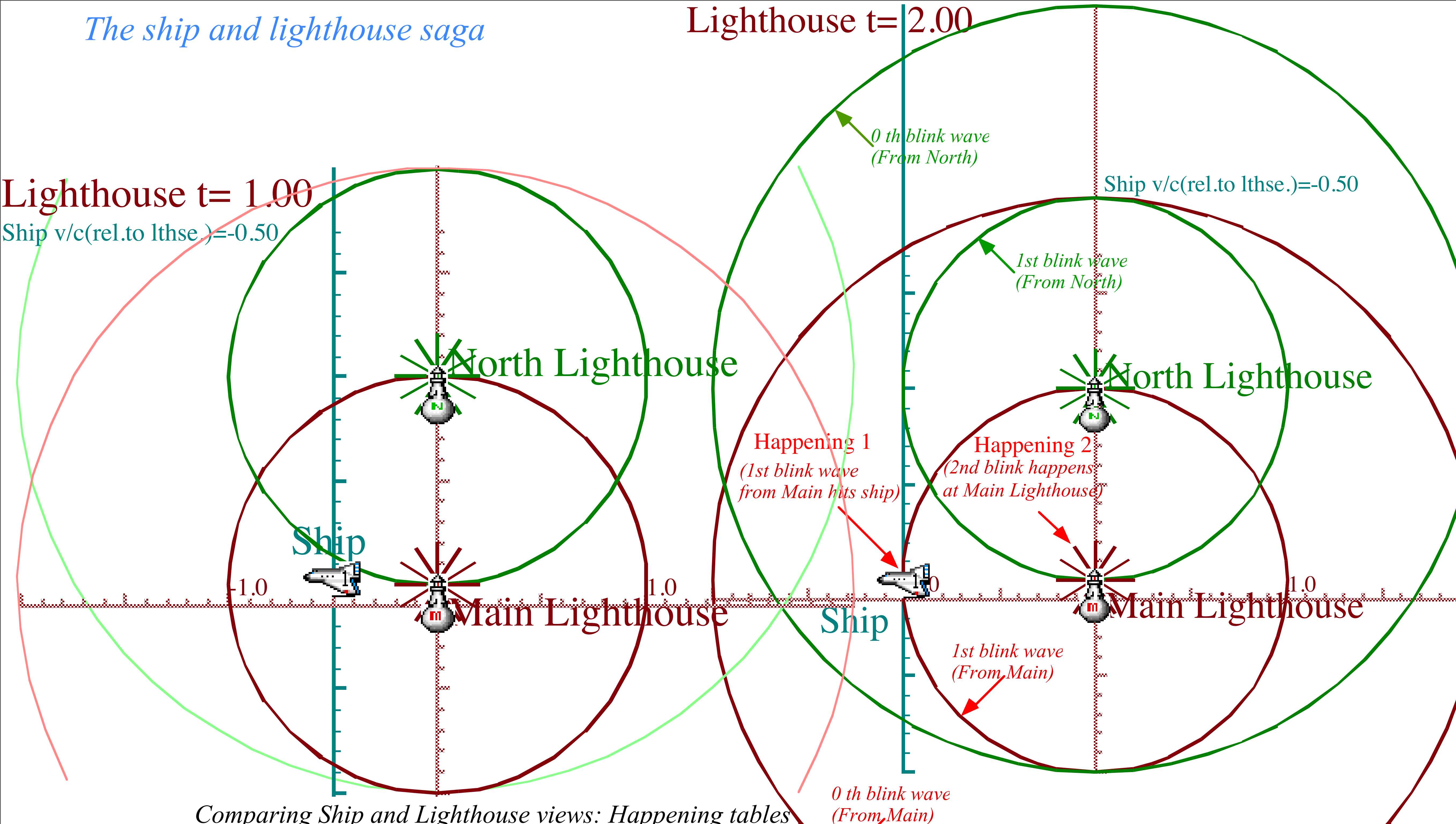
$$\omega\cdot ct\cdot\sinh^2 - \omega\cdot ct\cdot\cosh^2 = -\omega\cdot ct$$

5. That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

- *The Ship and Lighthouse saga*
- *A politically incorrect analogy of rotational transformation and Lorentz transformation*
    - *The straight scoop on "angle" and "rapidity" (They're area!)*
    - *Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)*
- *How Minkowski's space-time graphs help visualize relativity*
- *Group vs. phase velocity and tangent contacts*

The ship and lighthouse saga



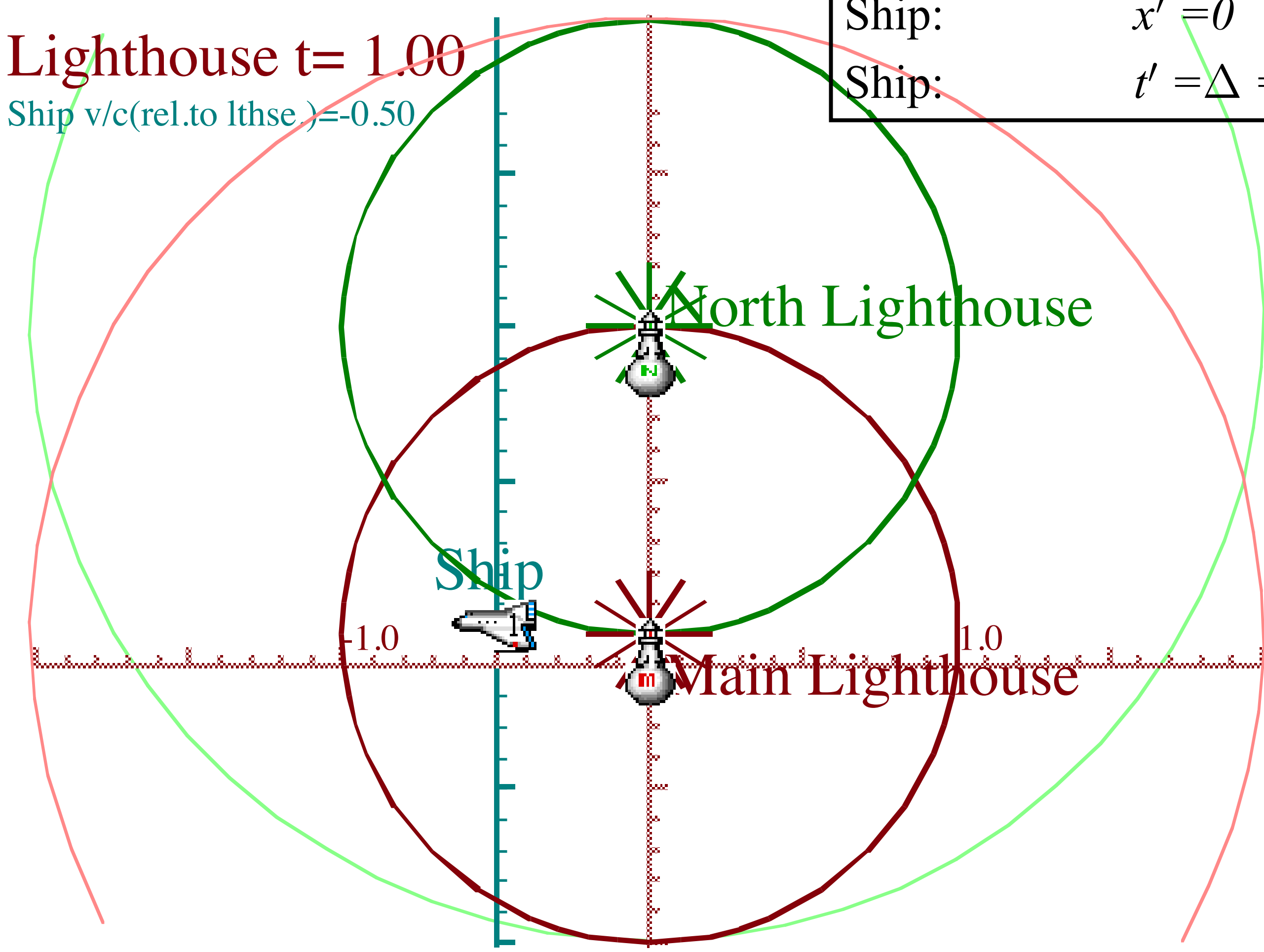
Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\ c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

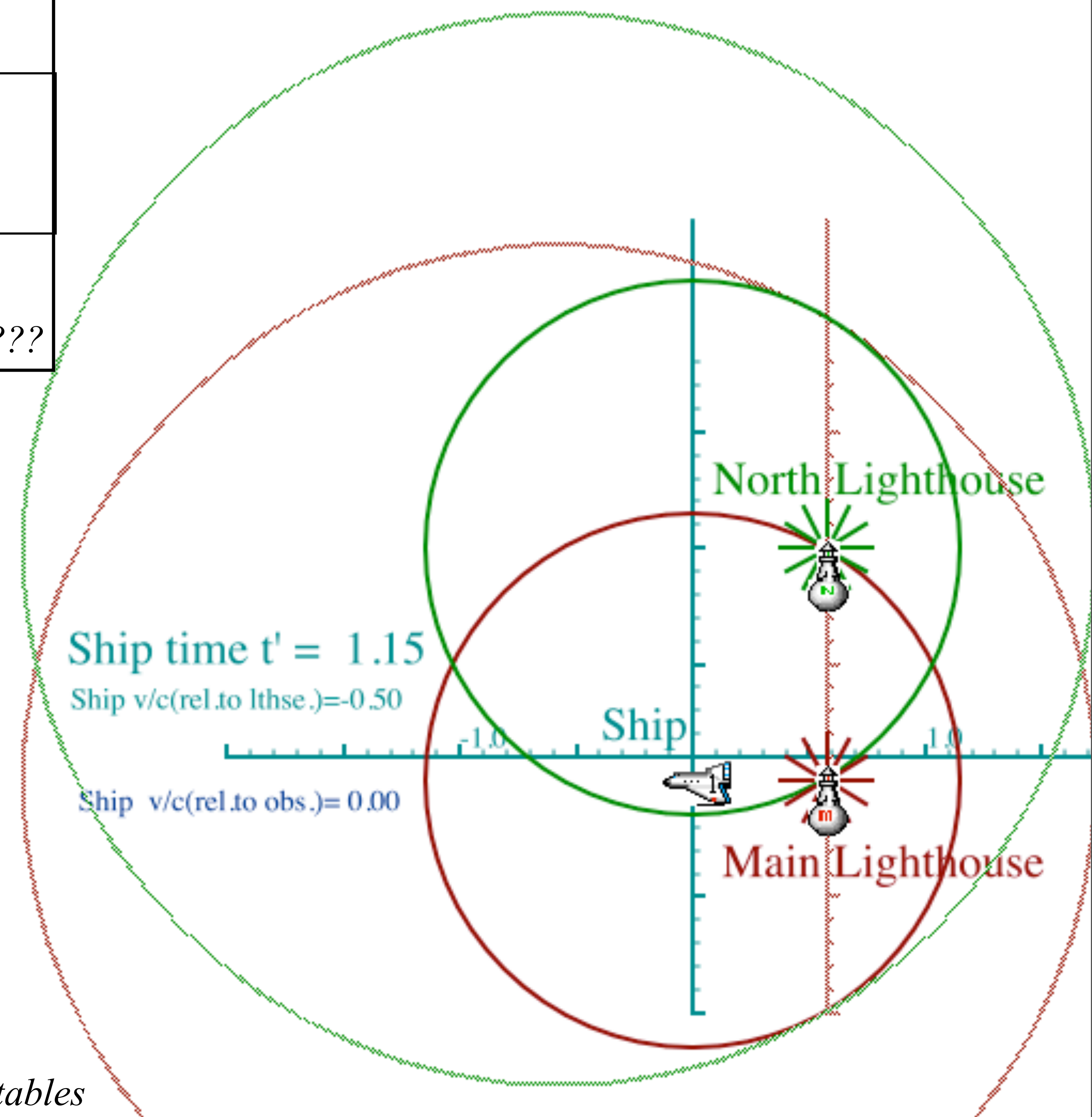
Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$



Ship Time $t'=\Delta=$??



Comparing Ship and Lighthouse views: Happening tables

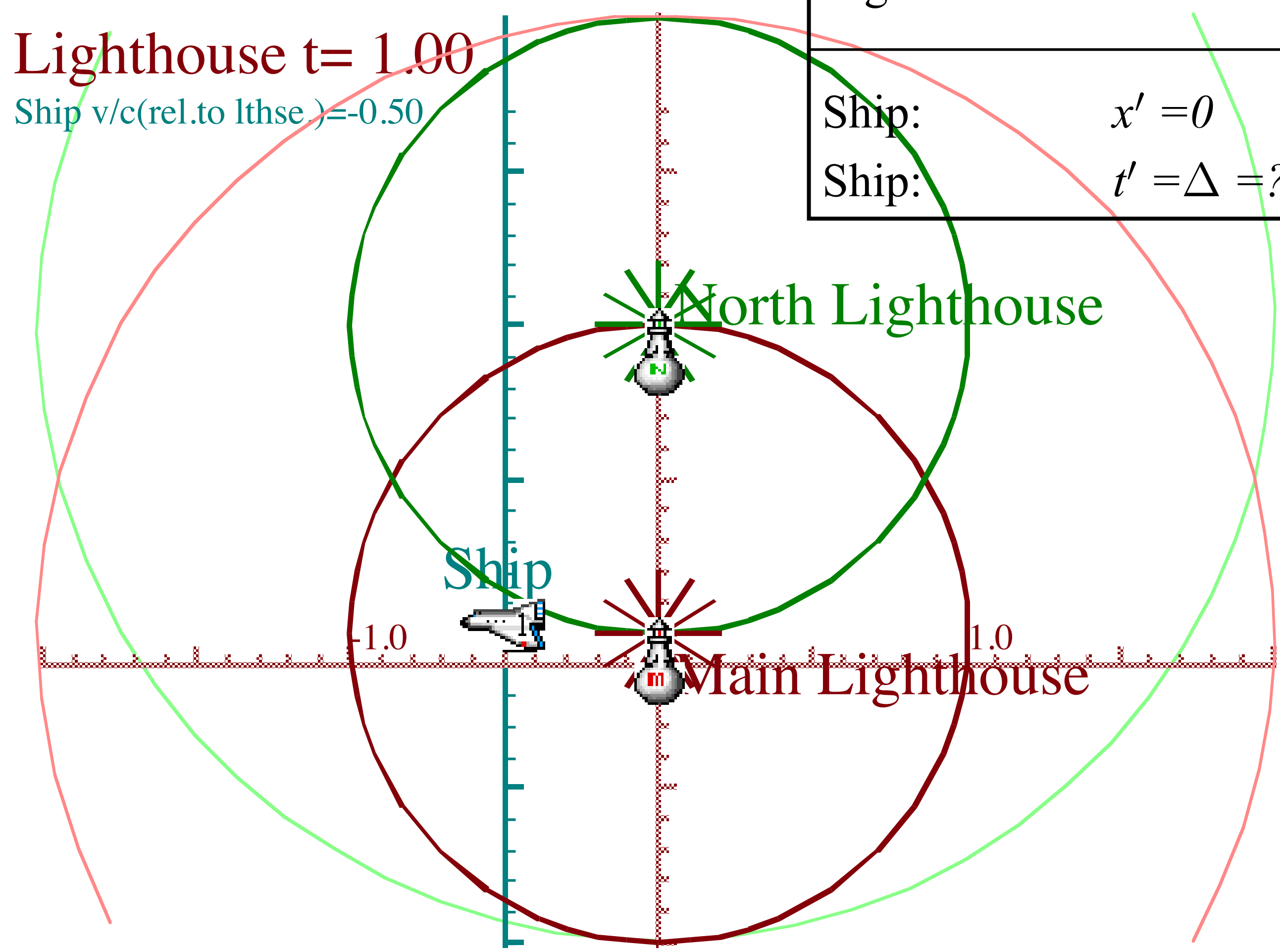
Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\ c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\ \Delta$ $t' = 2\Delta = 2.30$

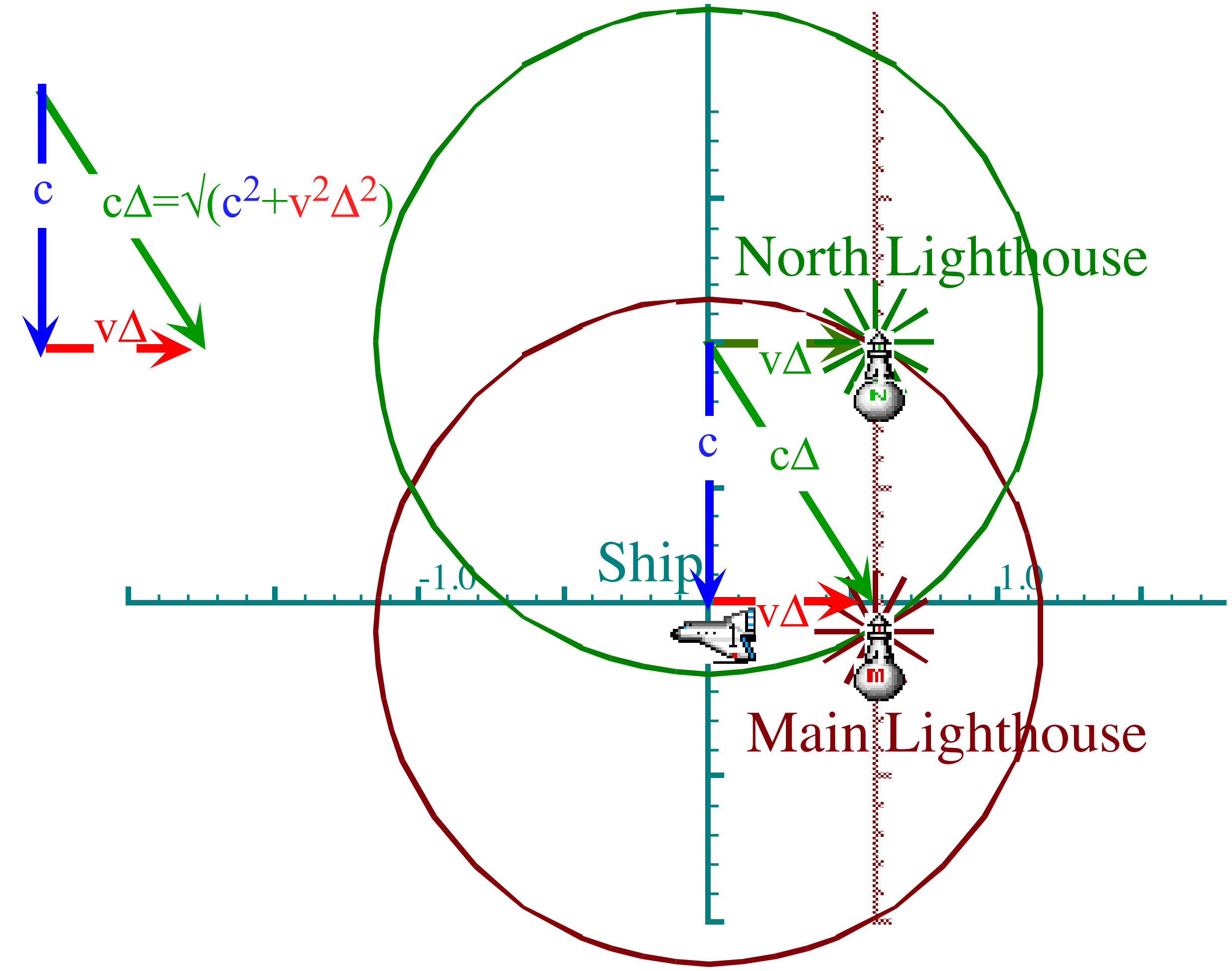
Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

Ship Time $t' = \Delta = ???$





Comparing Ship and Lighthouse views: Happening tables

	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

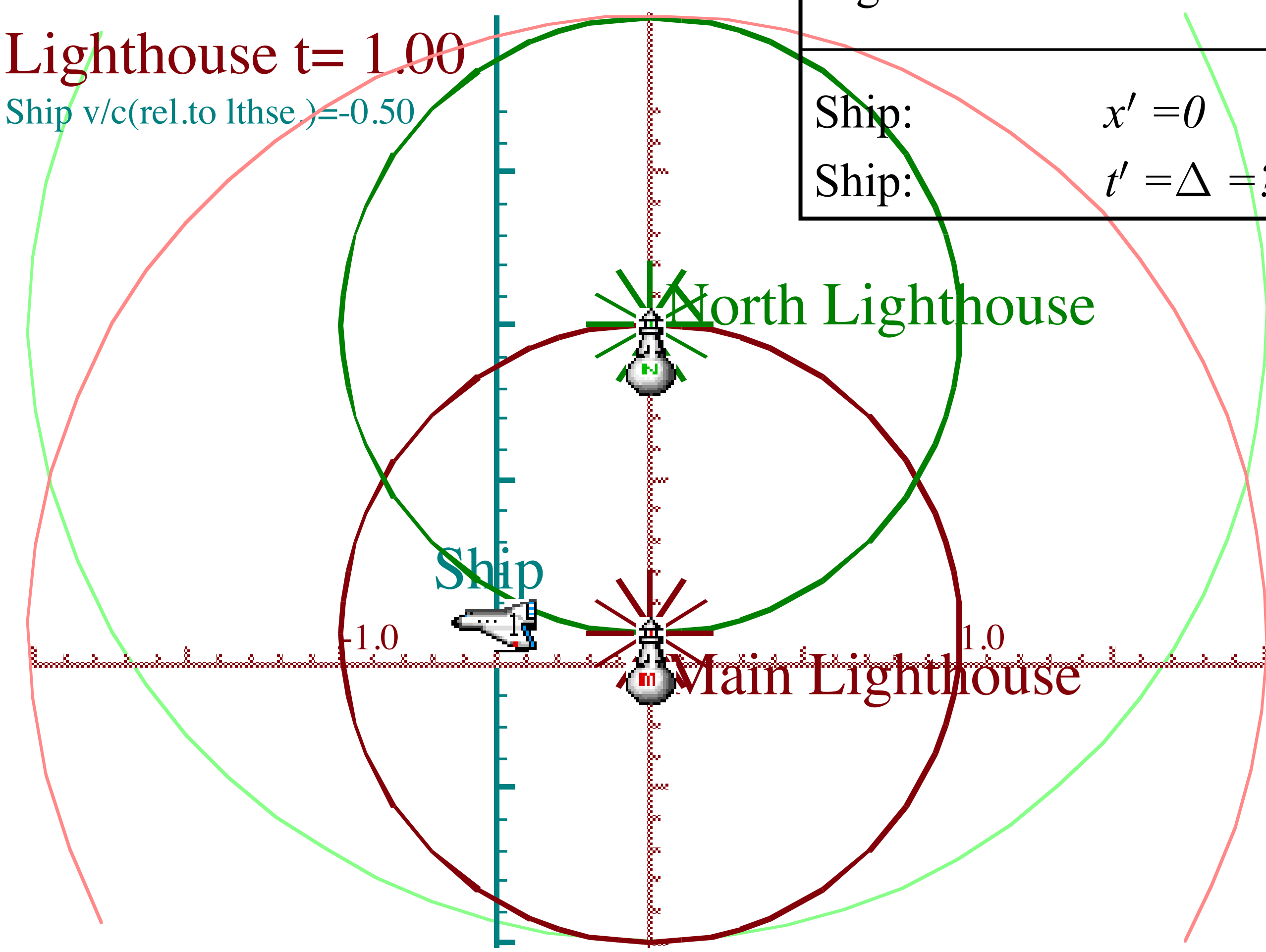
Happening 0.5:
Main Lite
blinks first time.

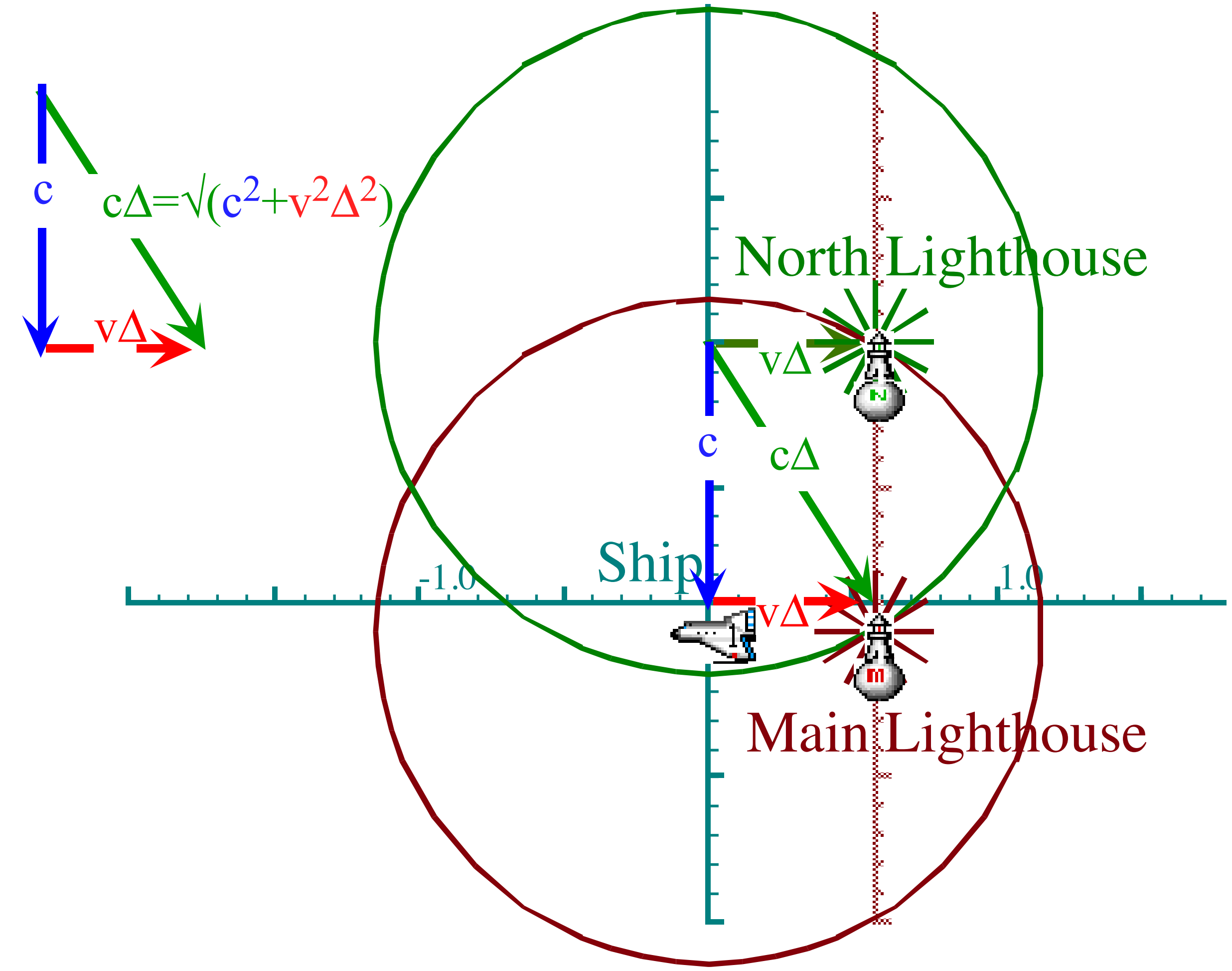
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

Ship Time $t' = \Delta = ???$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$





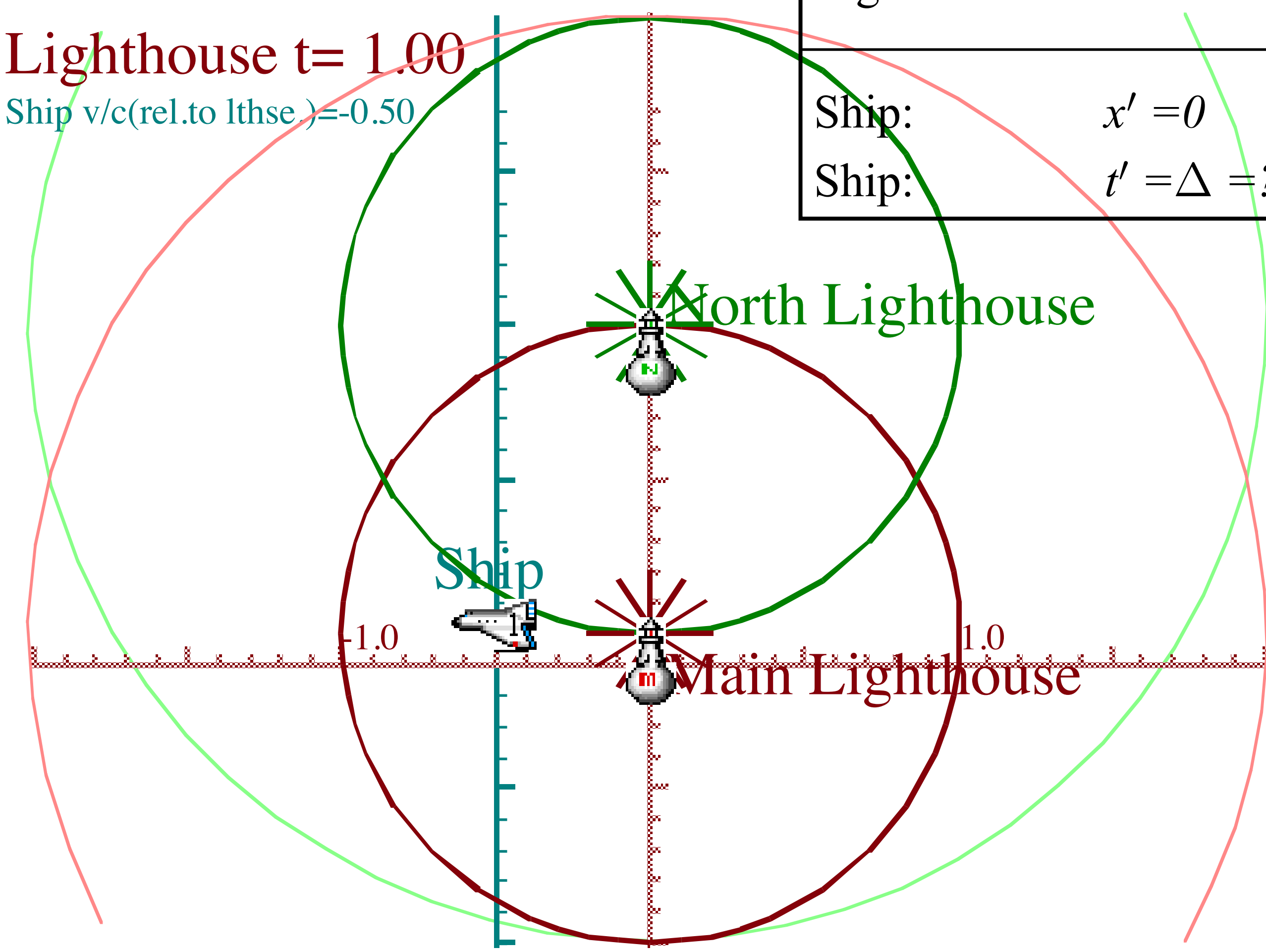
Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\ c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

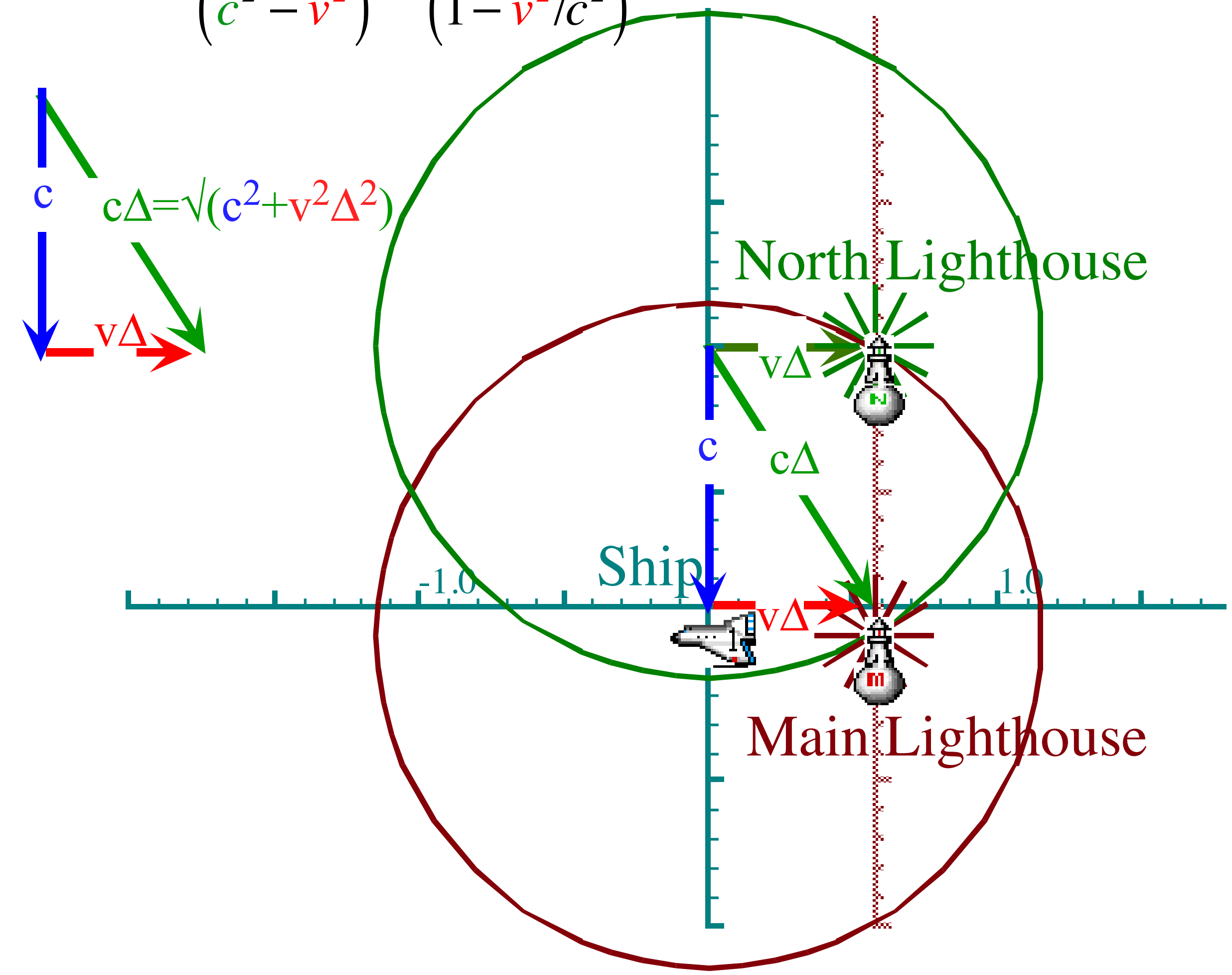


Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{\left(c^2 - v^2\right)} = \frac{1}{\left(1 - v^2/c^2\right)}$$



Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

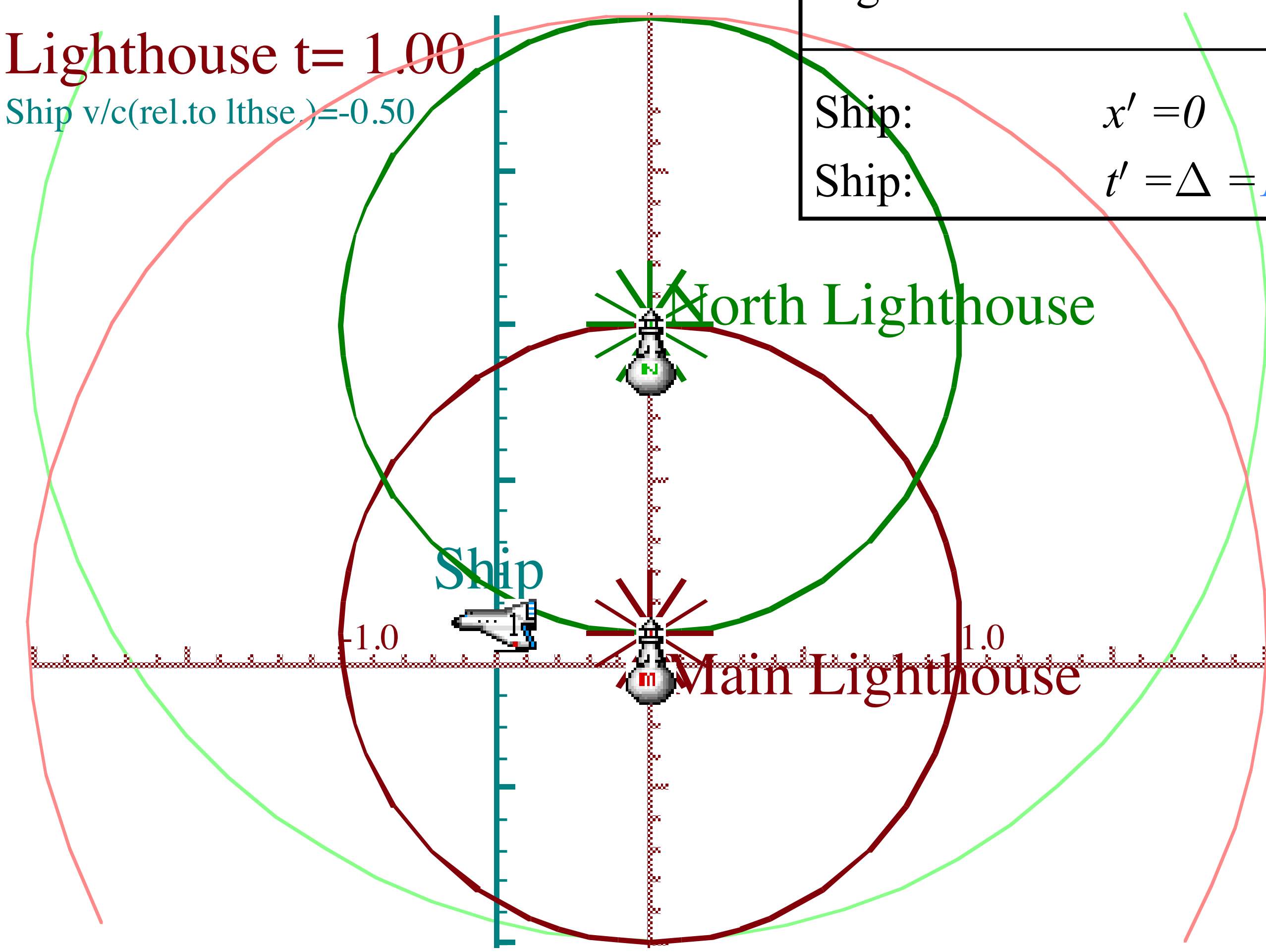
Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = 1.15$

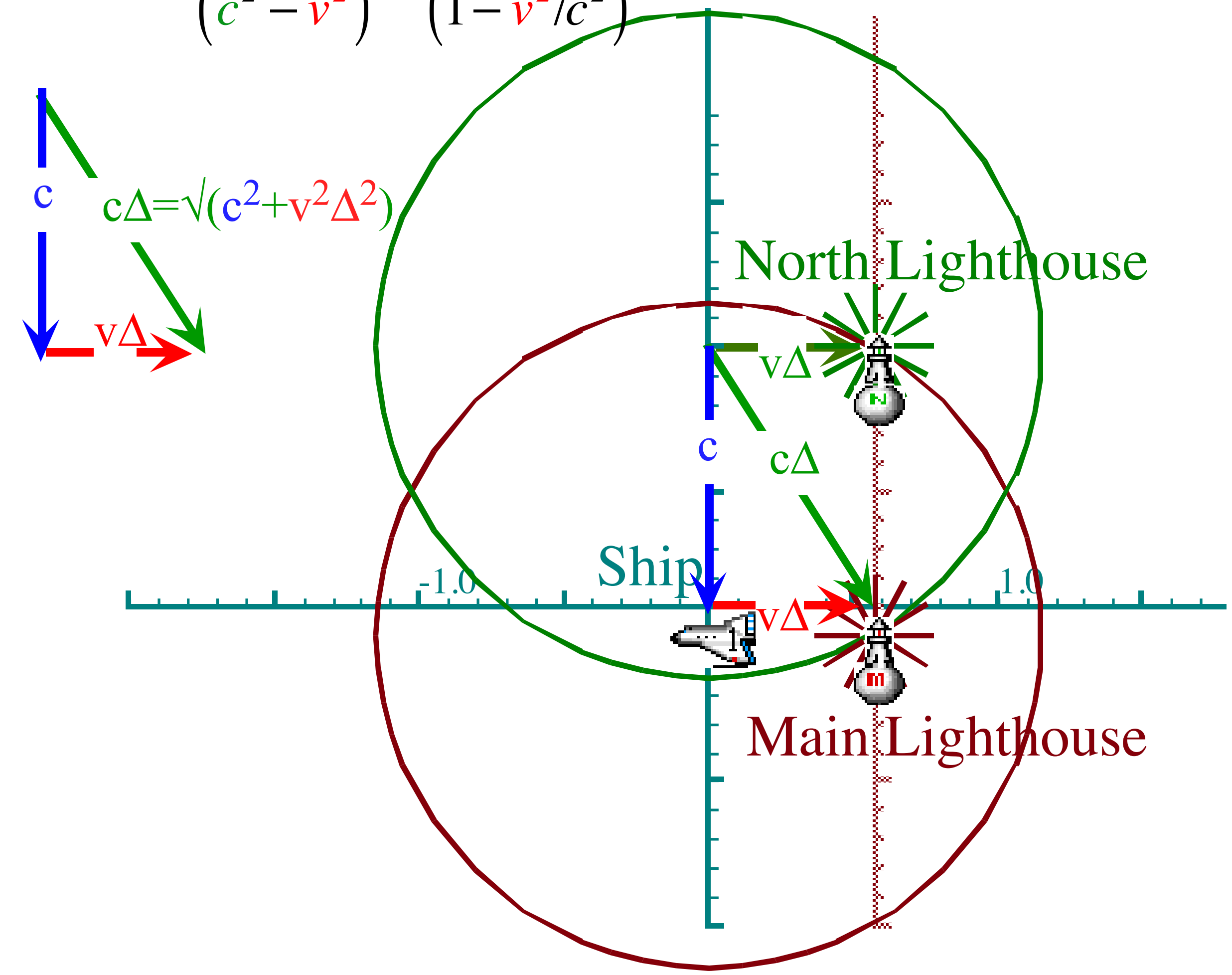
Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$(c^2 - v^2)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{(c^2 - v^2)} = \frac{1}{(1 - v^2/c^2)}$$





For u/c=1/2

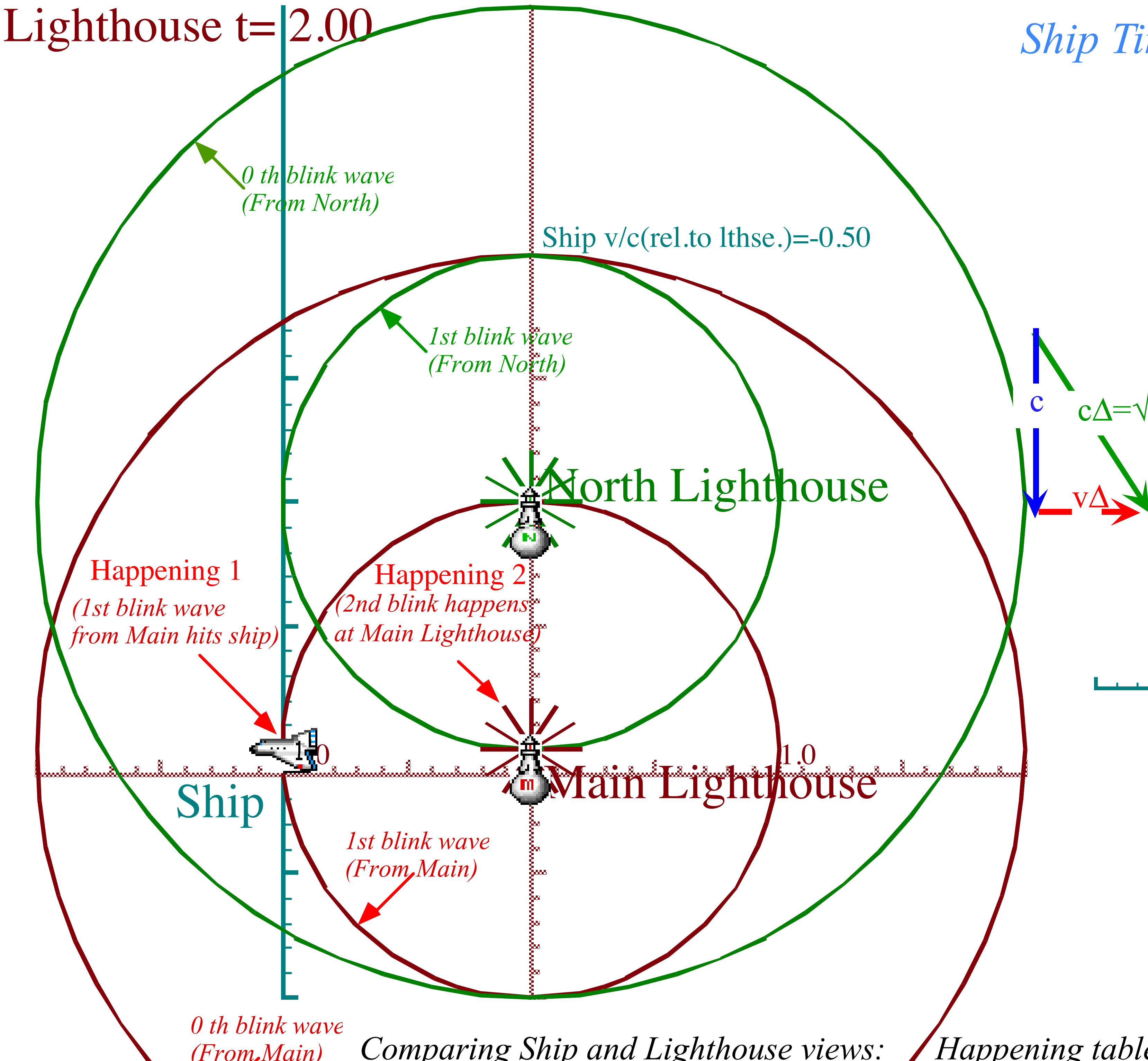
$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

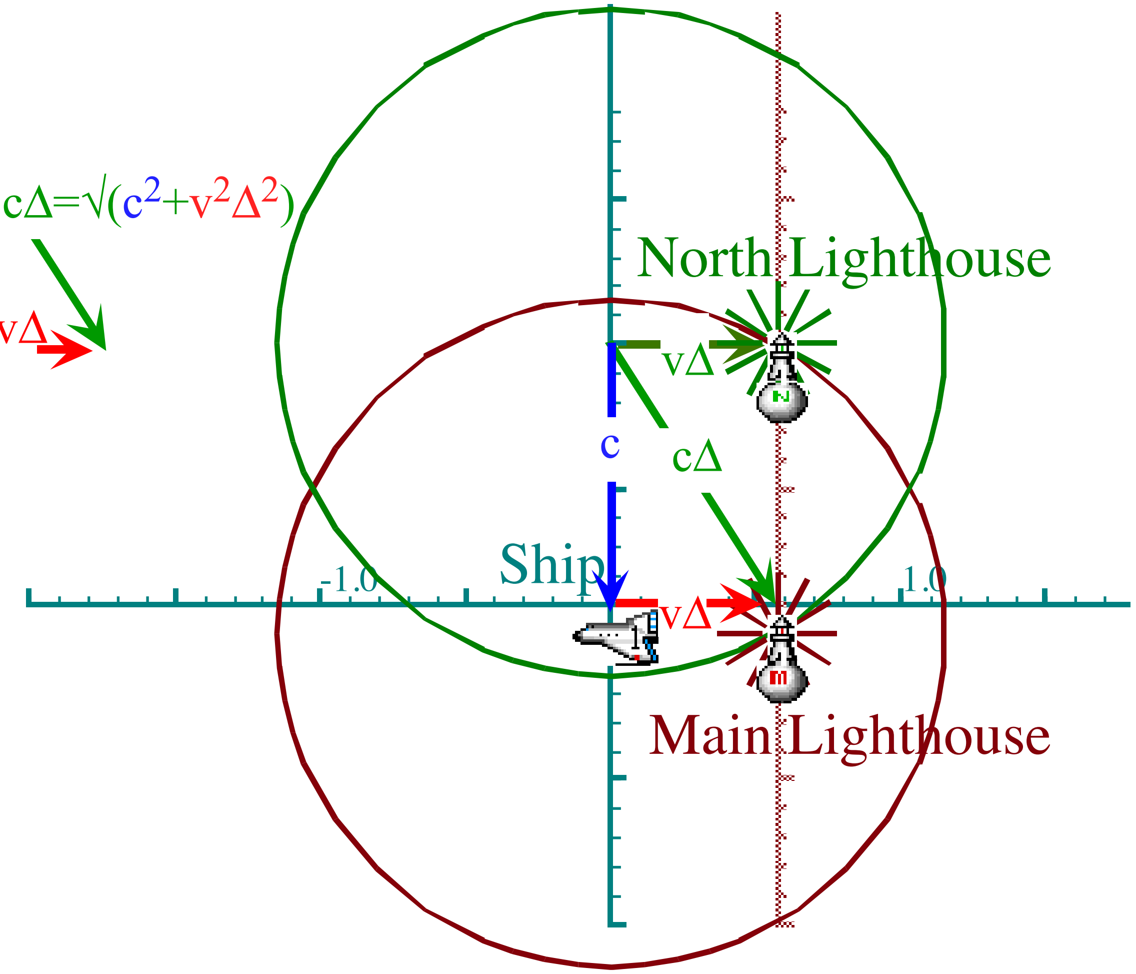
Comparing Ship and Lighthouse views: Happening tables

	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$





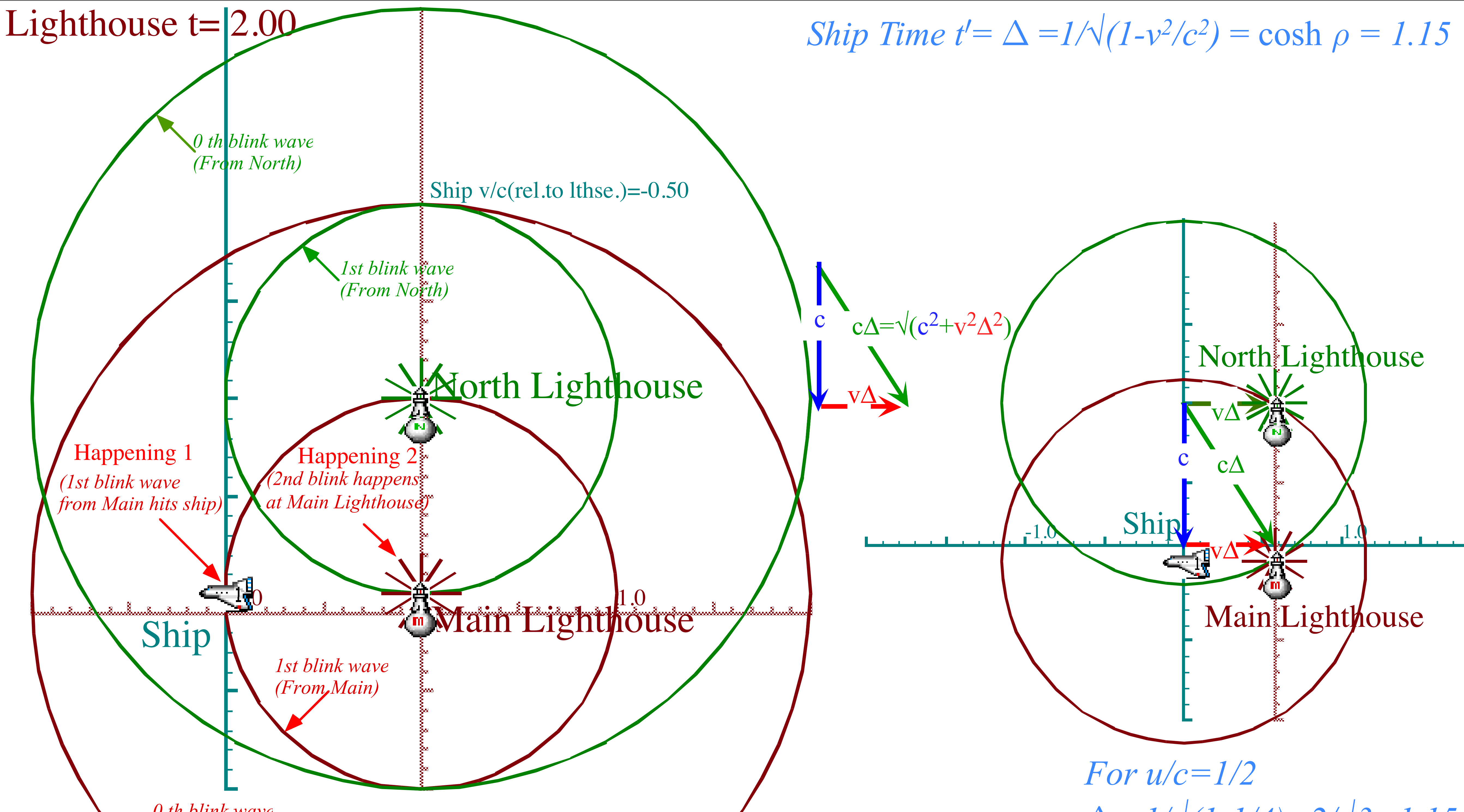
For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

Comparing Ship and Lighthouse views: *Happening tables*

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.



Comparing Ship and Lighthouse views: *Happening tables*

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -vc/(c-v)$ $t = c/(c-v)$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = (v+c)\Delta/c$	$x' = 2v\Delta$ $t' = 2\Delta$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lecture 24 ended here

5. That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

A politically incorrect analogy of rotational transformation and Lorentz transformation

The straight scoop on "angle" and "rapidity" (They're area!)

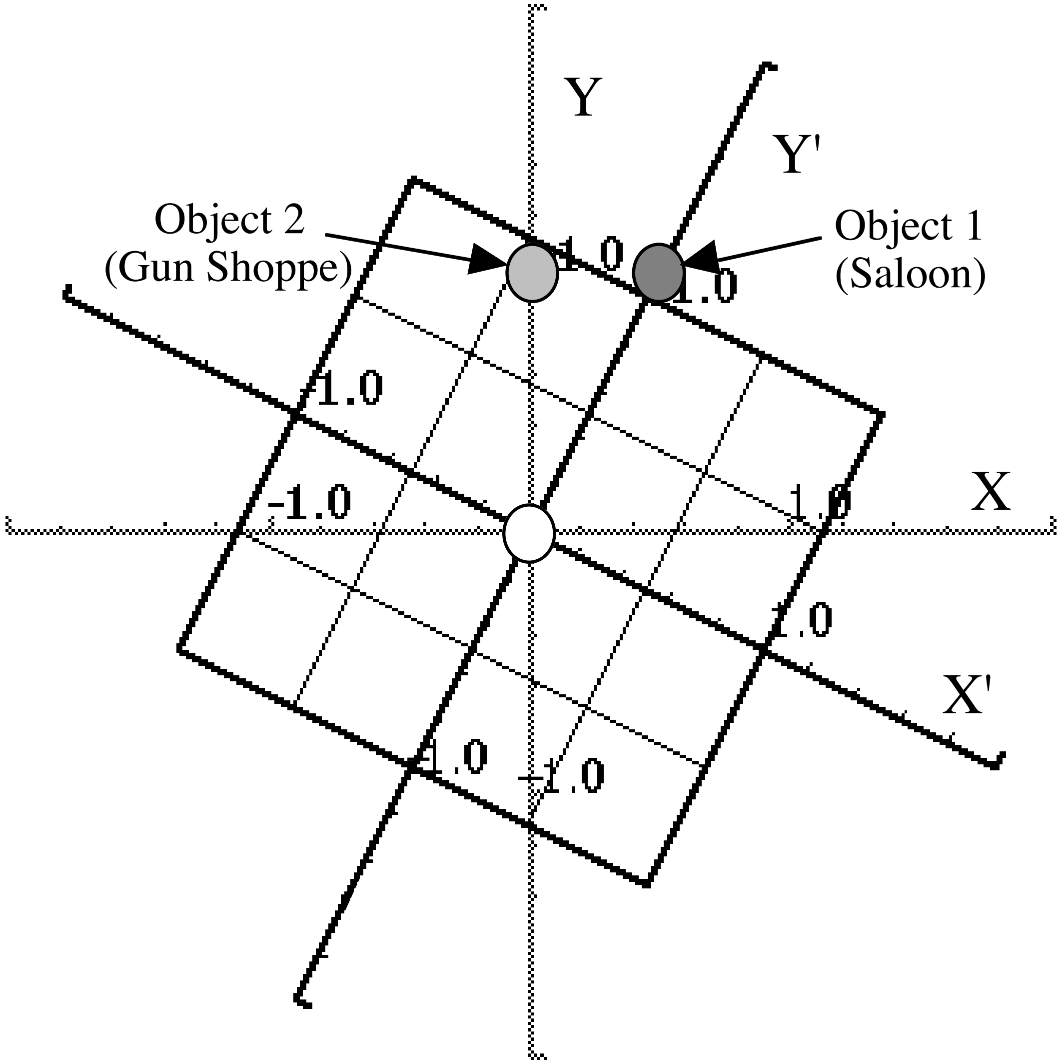
Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)

How Minkowski's space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts

A politically incorrect analogy of rotational transformation and Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.



Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$ $y = 0$	$x = 0.5$ $y = 1.0$	$x = 0$ $y = 1.0$
(French surveyor) $x' = 0$ $y' = 0$	$x' = 0$ $y' = 1.1$	$x' = -0.45$ $y' = 0.89$

A politically incorrect analogy of rotational transformation and Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

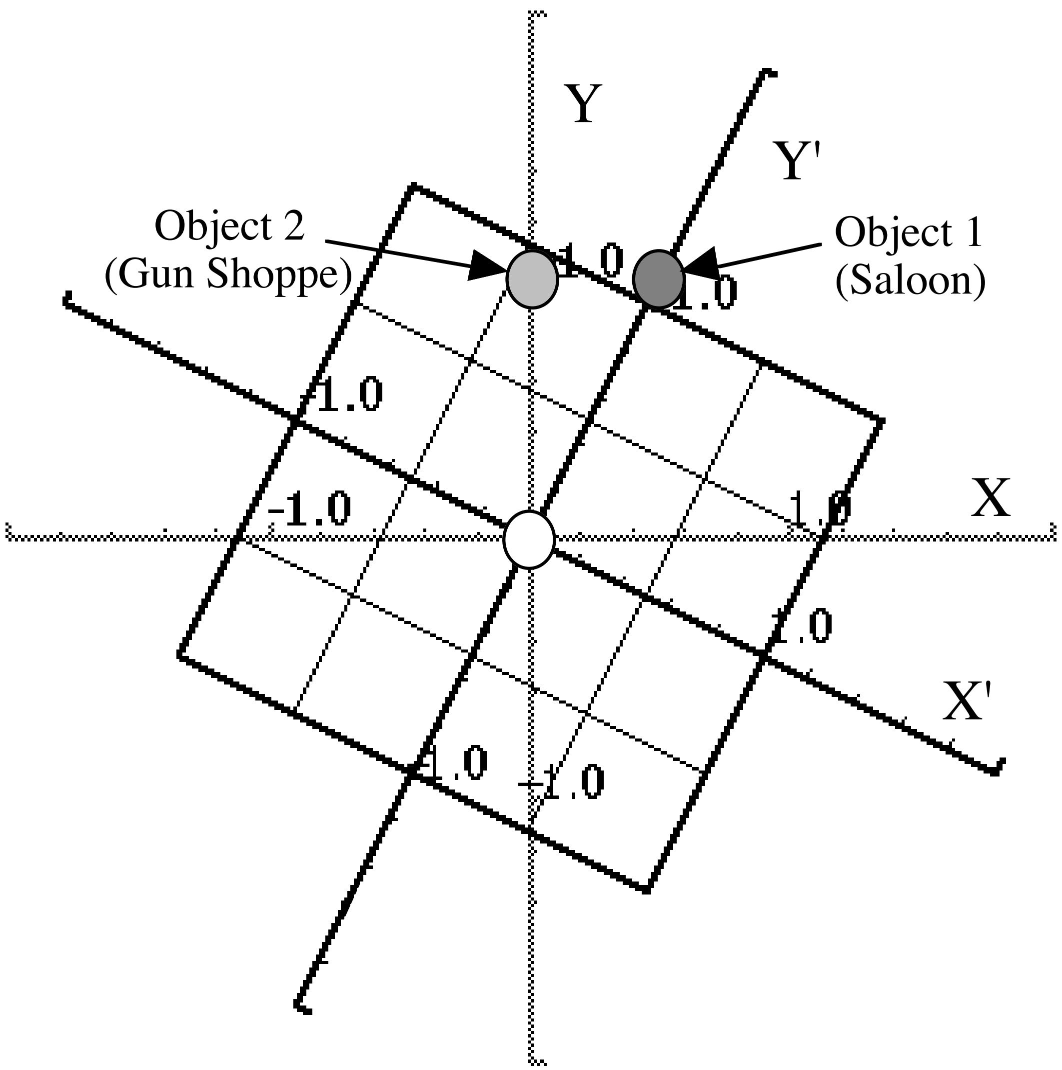
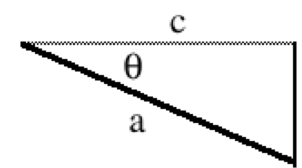
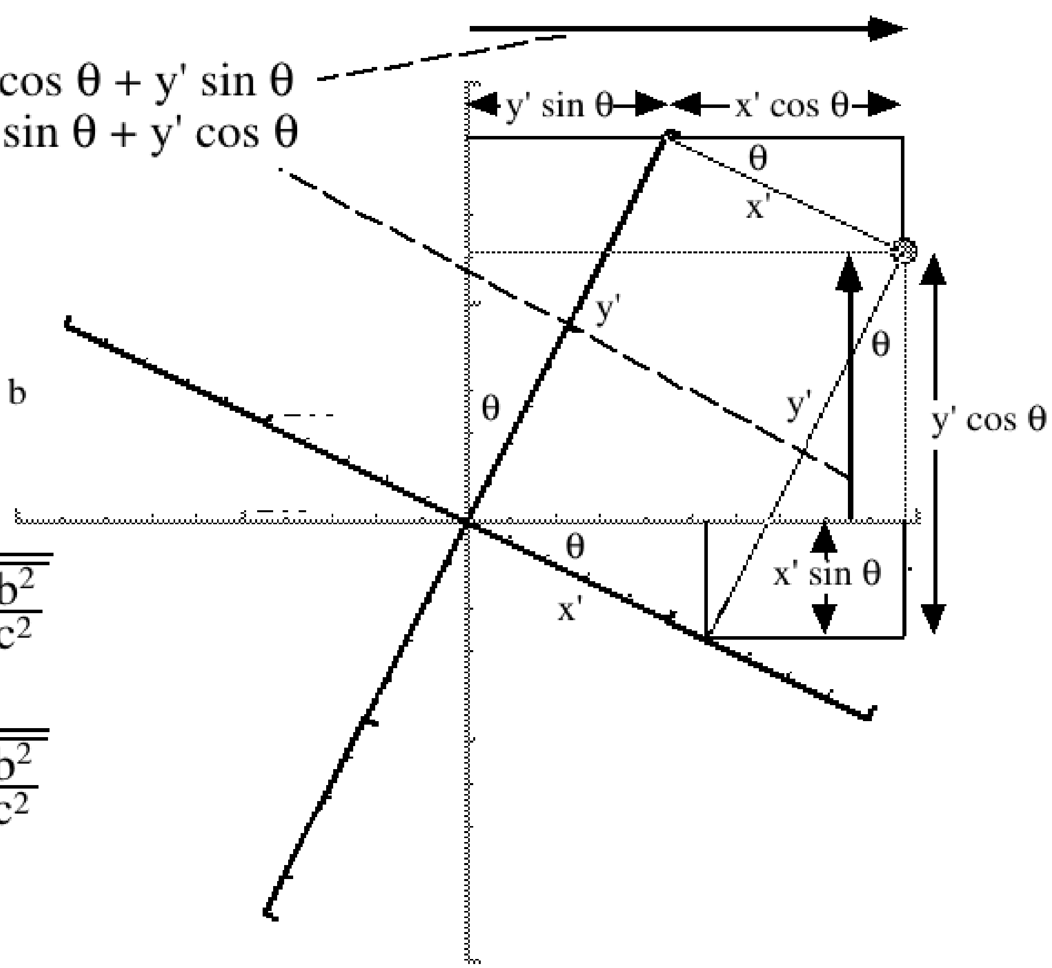


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.





$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

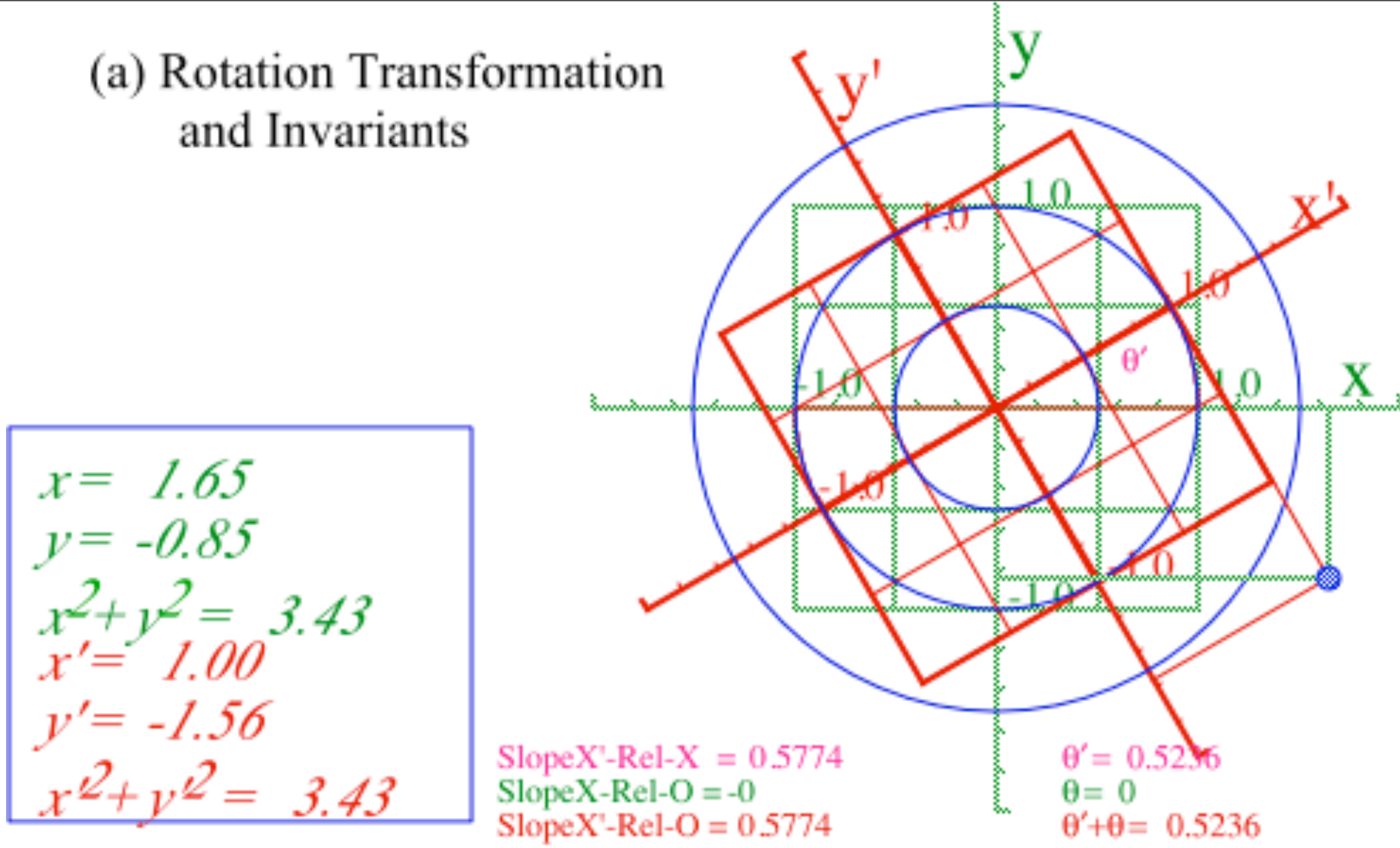
$$\sin\theta = \frac{b/c}{\sqrt{1+\frac{b^2}{c^2}}}$$

Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

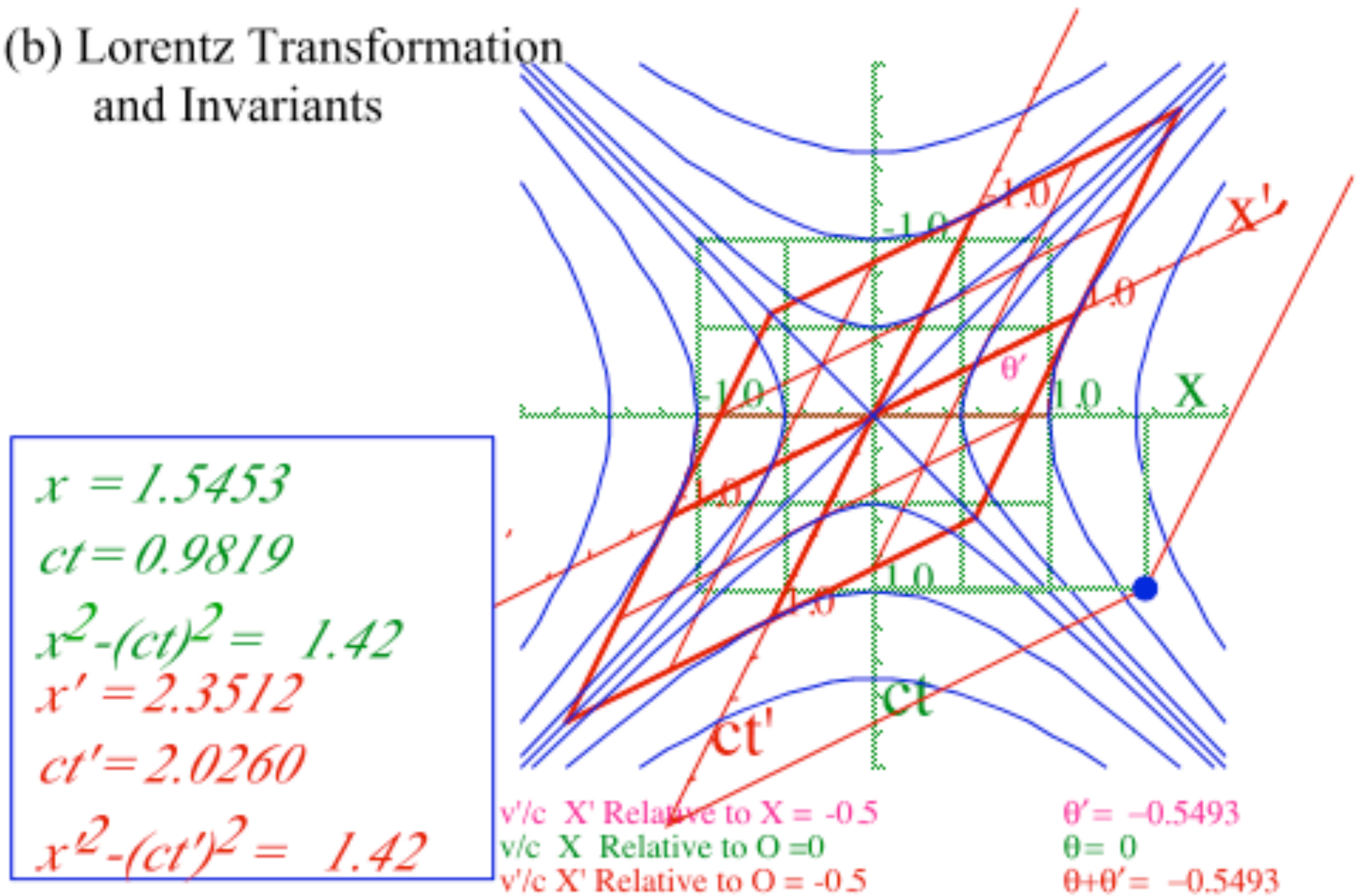
(a) Rotation Transformation and Invariants



$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

(b) Lorentz Transformation and Invariants



$$x' = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\frac{v}{c}ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\cosh\rho + y\sinh\rho$$

$$ct' = \frac{\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\sinh\rho + y\cosh\rho$$

5. That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

A politically incorrect analogy of rotational transformation and Lorentz transformation

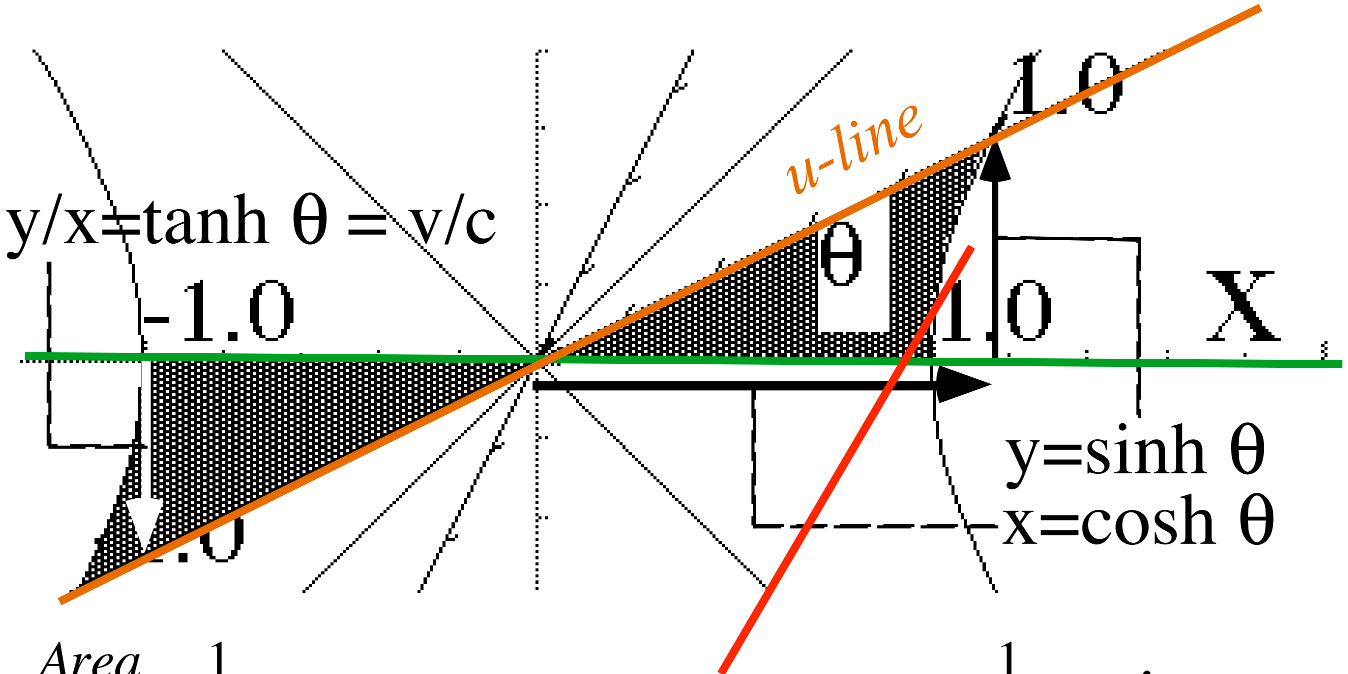
The straight scoop on "angle" and "rapidity" (They're area!)

Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)

How Minkowski's space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts

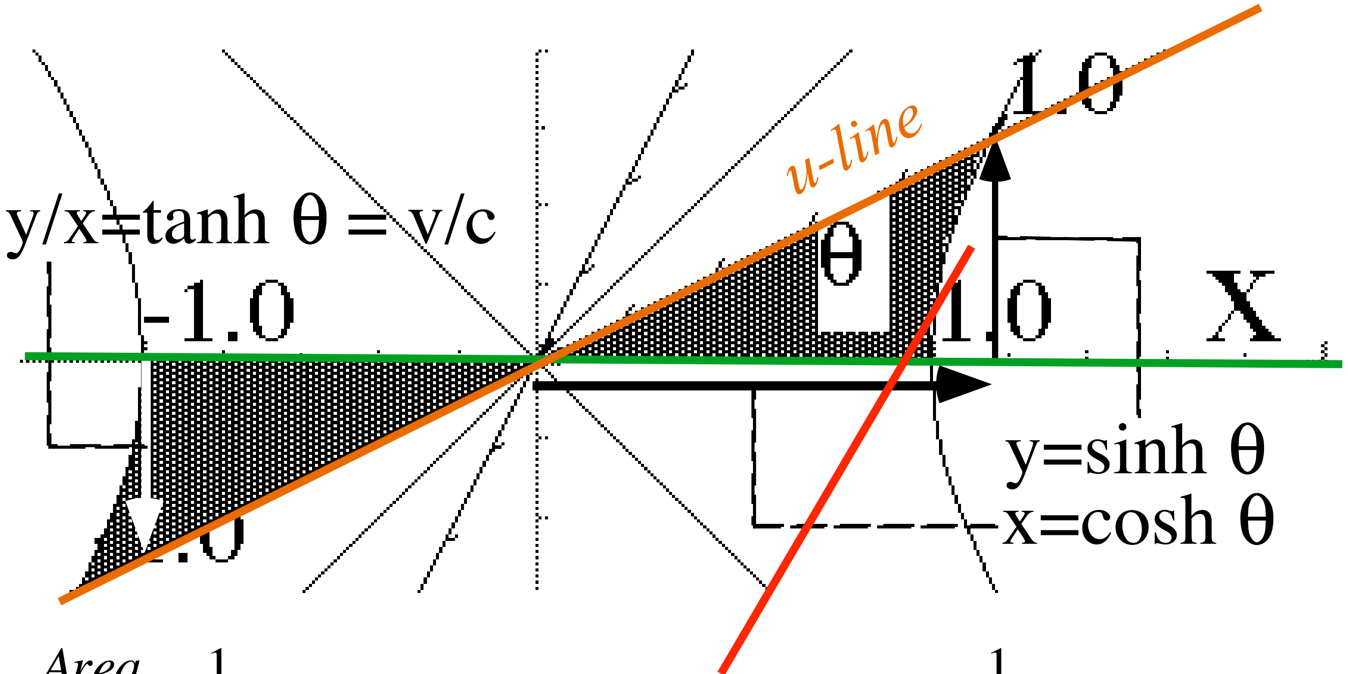
The straight scoop on "angle" and "rapidity" (They're area!)



$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2} xy - \int y\, dx$$

The "Area" being calculated is the *<u>total</u>* **gray** *area between hyperbola pairs, X axis, and sloping u-line*

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the <u>total</u> **gray** *area between hyperbola pairs, X axis, and sloping u-line*

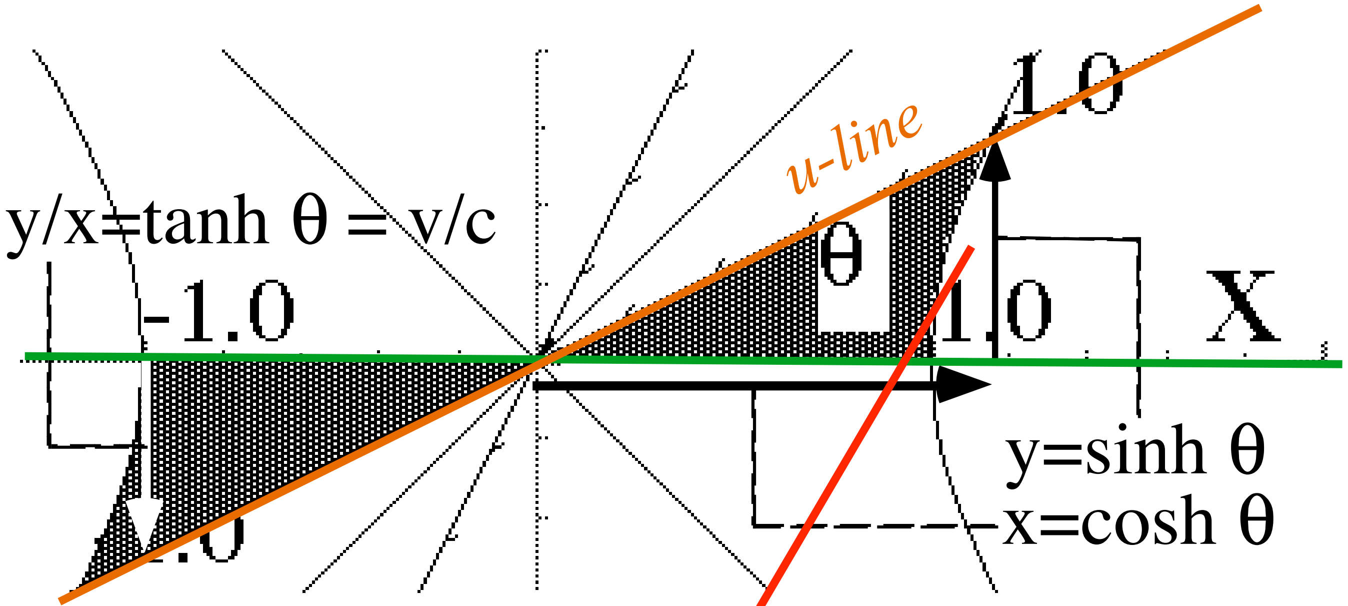
$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2} xy - \int y\,dx$$

$$\frac{Area}{2} = \frac{1}{2} \sinh\theta \cosh\theta - \int \sinh\theta \, d\left(\cosh\theta\right)$$

$$\sinh^2\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)^2 = \frac{1}{4}\left(e^{2\theta} + e^{-2\theta} - 2\right) = \frac{\cosh 2\theta - 1}{2}$$

$$\sinh\theta \cosh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)\left(\frac{e^{\theta} + e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta} - e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the total **gray** *area between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2} xy - \int y\, dx$$

$$\frac{Area}{2} = \frac{1}{2} \sinh\theta \cosh\theta - \int \sinh\theta\, d(\cosh\theta)$$

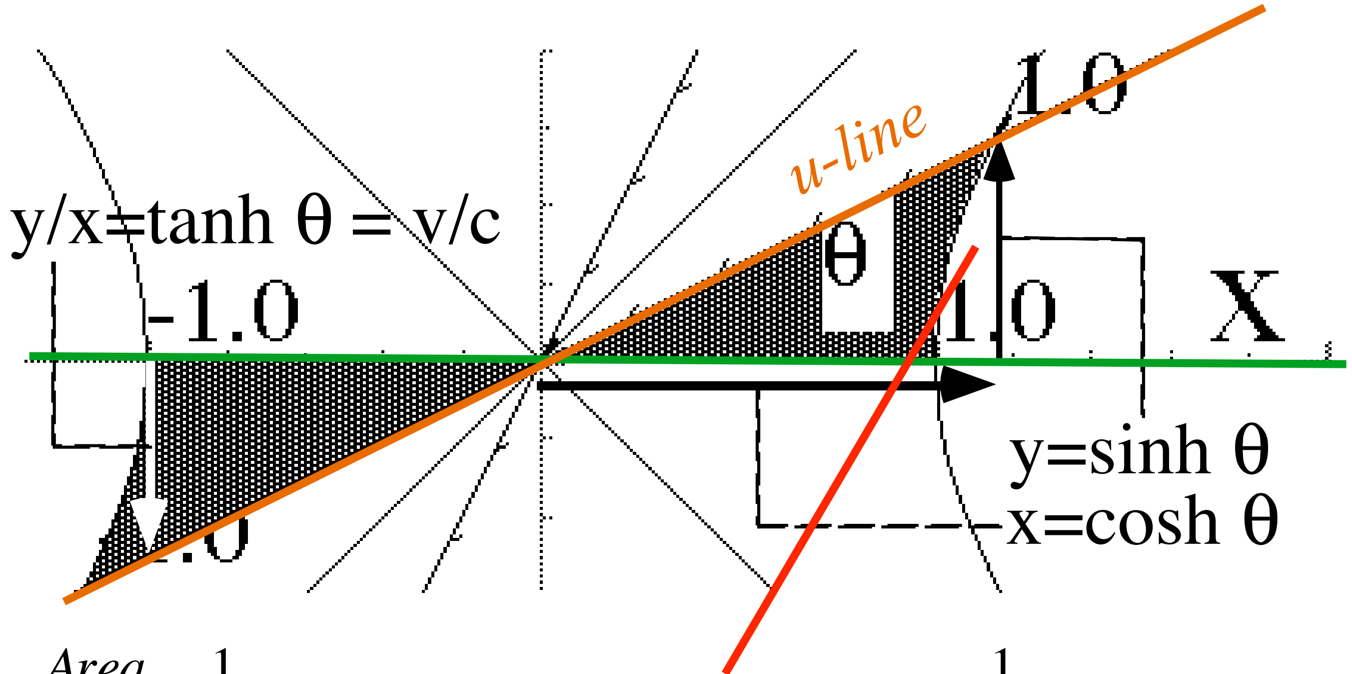
$$\sinh^2\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)^2 = \frac{1}{4}\left(e^{2\theta} + e^{-2\theta} - 2\right) = \frac{\cosh 2\theta - 1}{2}$$

$$\sinh\theta \cosh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)\left(\frac{e^{\theta} + e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta} - e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\frac{Area}{2} = \frac{1}{2} \sinh\theta \cosh\theta - \int \sinh^2\theta\, d\theta = \frac{1}{4}\sinh 2\theta - \int \frac{\cosh 2\theta - 1}{2}\, d\theta$$

$$\int \cosh a\theta\, d\theta = \frac{1}{a}\sinh a\theta$$

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the total **gray** *area between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2}\,base \cdot altitude - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\theta\cosh\theta - \int \sinh\theta\, d(\cosh\theta)$$

$$\sinh^2\theta = \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)^2 = \frac{1}{4}\left(e^{2\theta}+e^{-2\theta}-2\right) = \frac{\cosh 2\theta - 1}{2}$$

$$\sinh\theta\cosh\theta = \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)\left(\frac{e^{\theta}+e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta}-e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\theta\cosh\theta - \int \sinh^2\theta\, d\theta = \frac{1}{4}\sinh 2\theta - \int \frac{\cosh 2\theta - 1}{2}\, d\theta$$

$$\int \cosh a\theta\, d\theta = \frac{1}{a}\sinh a\theta$$

Amazing result: ***Area*** $= \theta = \rho$ *is rapidity*

5. That "old-time" relativity *(Circa 600BCE- 1905CE)*

("Bouncing-photons" in smoke & mirrors and Thales, again)

The Ship and Lighthouse saga

A politically incorrect analogy of rotational transformation and Lorentz transformation

The straight scoop on "angle" and "rapidity" (They're area!)

Introducing the "Sin-Tan Rosetta Stone" (Thanks, Thales!)

How Minkowski's space-time graphs help visualize relativity

Group vs. phase velocity and tangent contacts

Introducing the "Sin-Tan Rosetta Stone"

(a) Circlular Functions
(plane geometry)

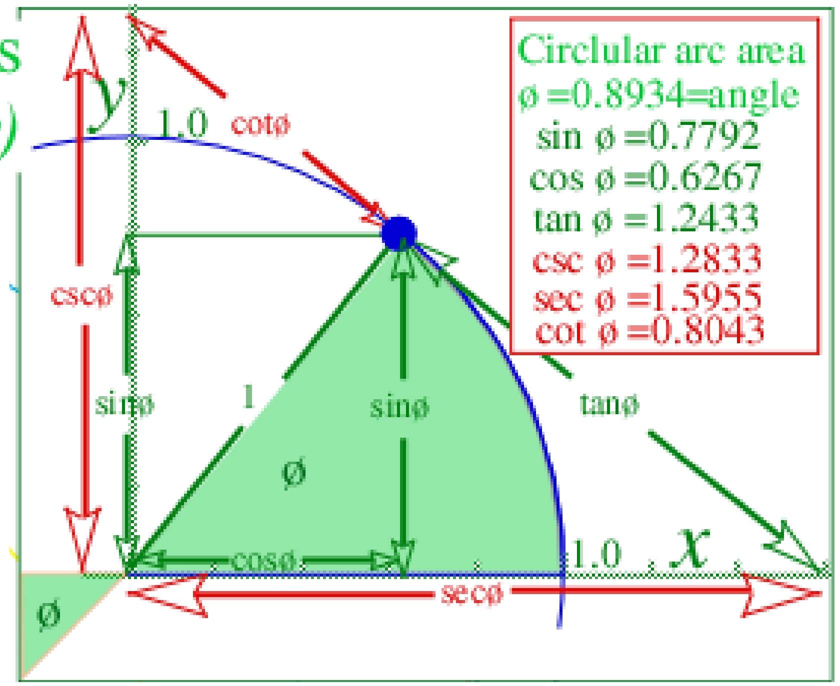


Fig. C.2-3 and Fig. 5.4 in Unit 2

Introducing the "Sin-Tan Rosetta Stone"

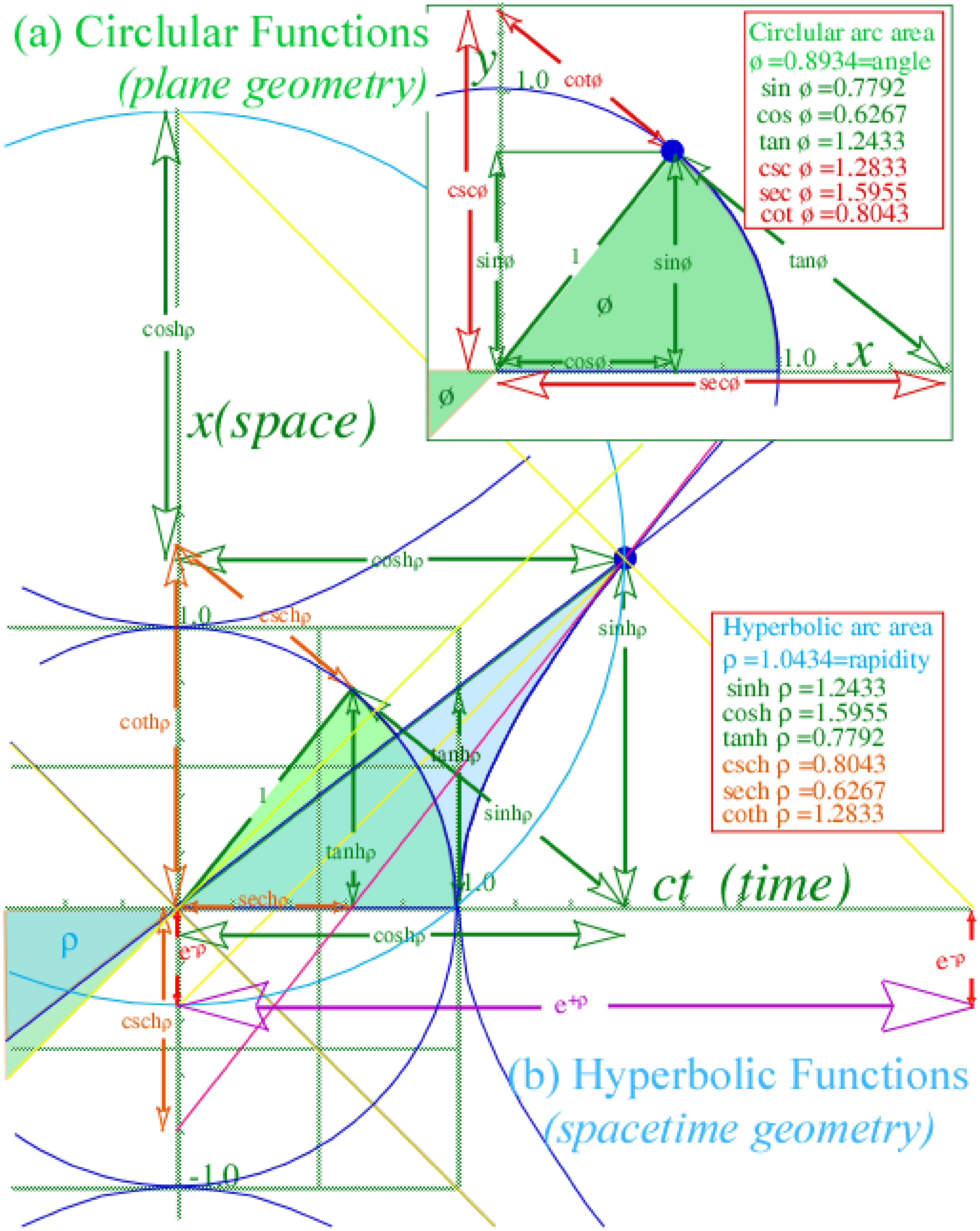


Fig. C.2-3
and
Fig. 5.4
in Unit 2