Lecture 4

Revised 12.21.12 from 8.30.2012

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations (From Lect. 3)

Special mass ratio $M_1/m_2 = 3$

High mass ratio $M_1/m_2 = 49$

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)$=const. ln(y)

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

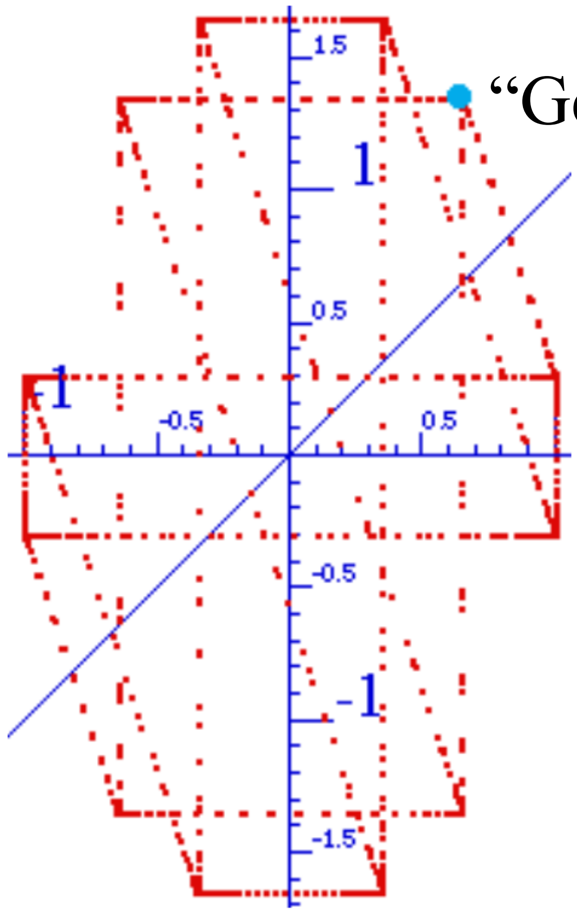
An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)],[Harter, Li IMSS (2012)]

A lesson in geometry of fractions: Ford Circles and Farey Sums

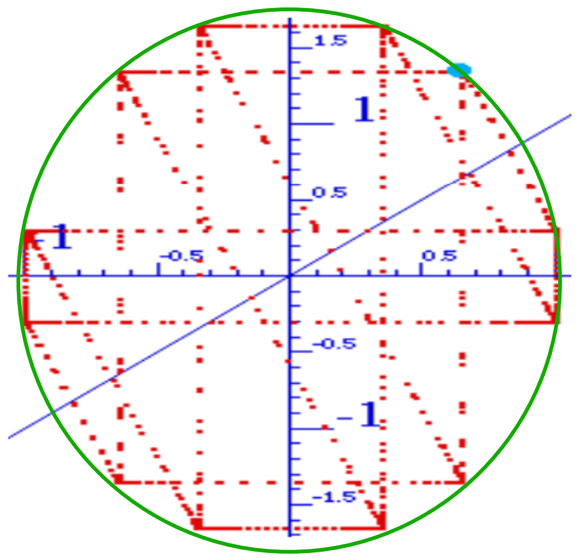
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)] [John Farey, Phil. Mag.(1816)]

Review of $(V_1,V_2) \to (y_1,y_2)$ relations

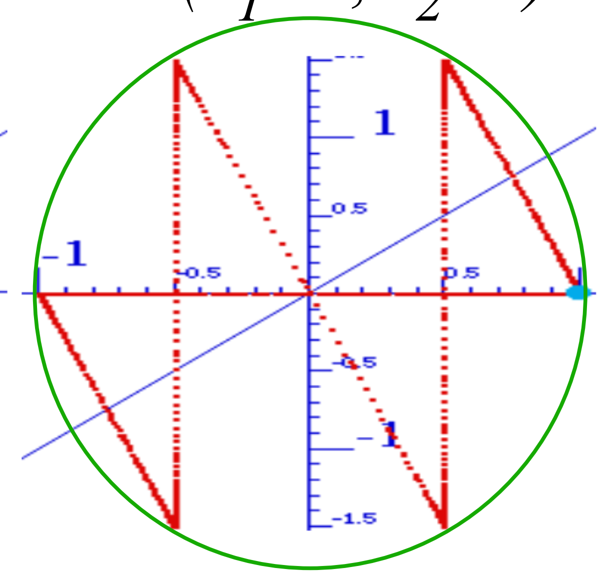
→ *Special mass ratio $M_1/m_2 = 3$*
High mass ratio $M_1/m_2 = 49$

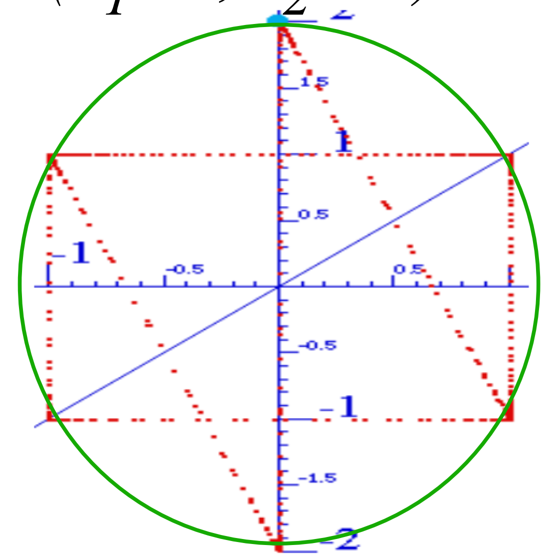


“Generic” initial velocity
$(v_1=1.0, v_2=0.1)$

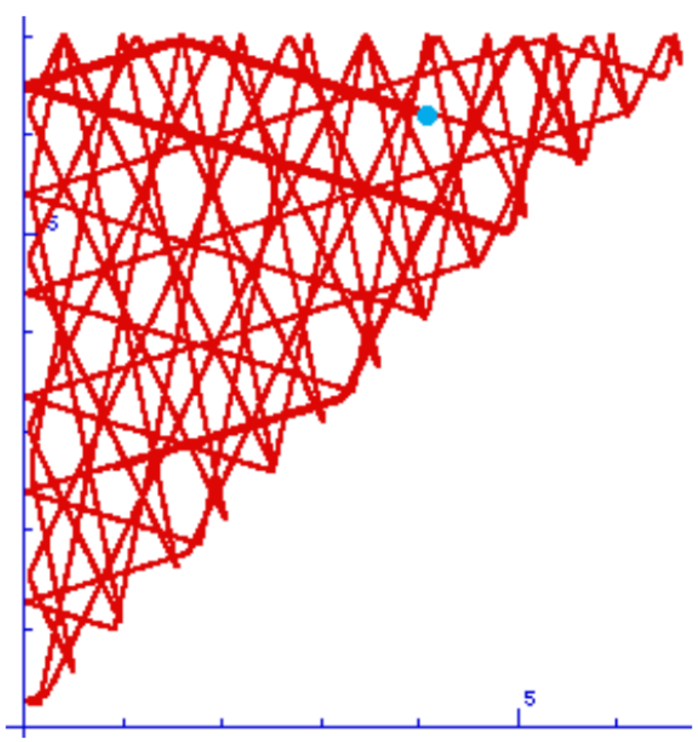


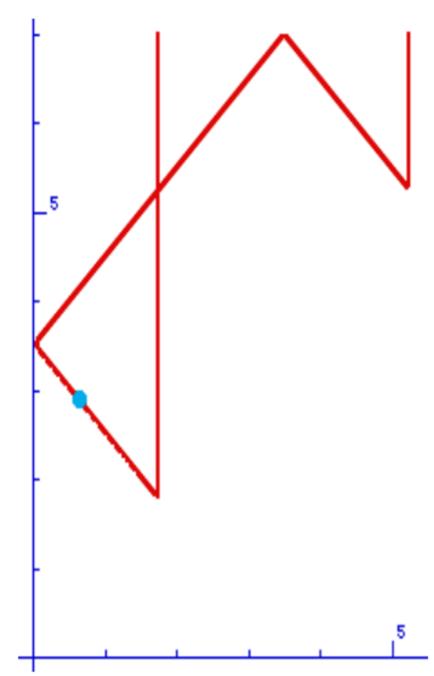
“Symmetric” initial velocity
$(v_1=1, v_2=0)$ or $(v_1=1, v_2=-1)$

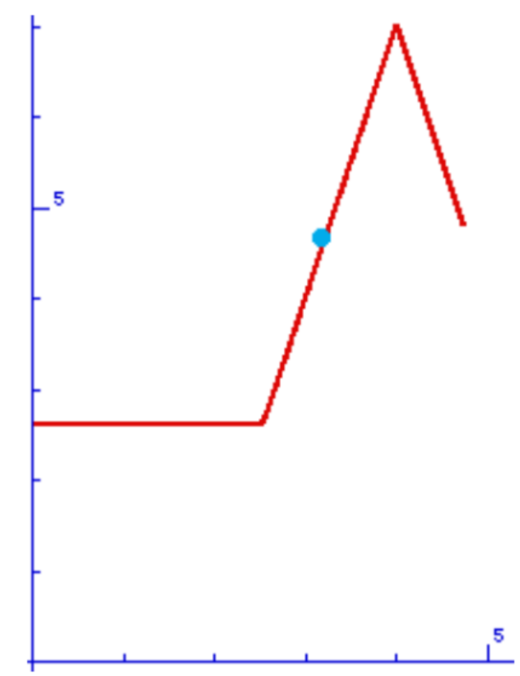


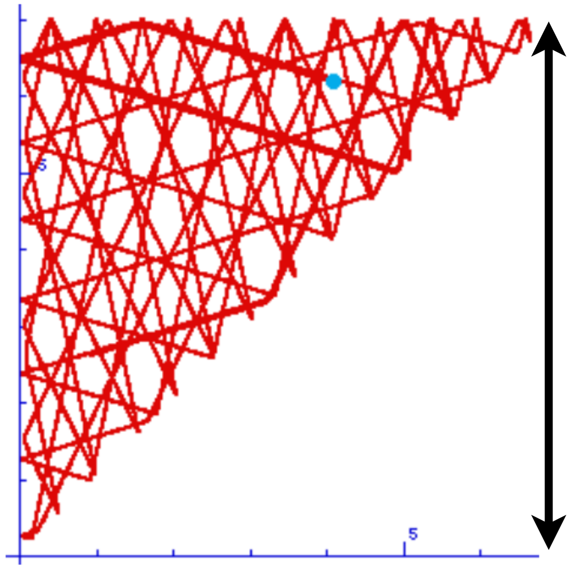


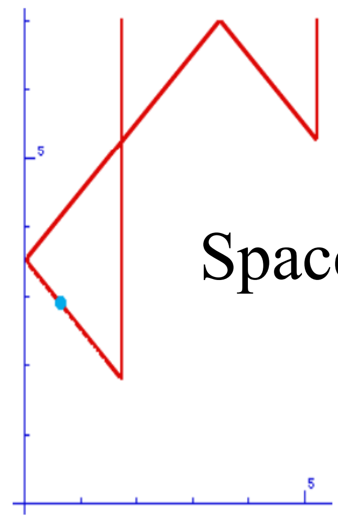
Corresponding space-space (y_1, y_2) paths



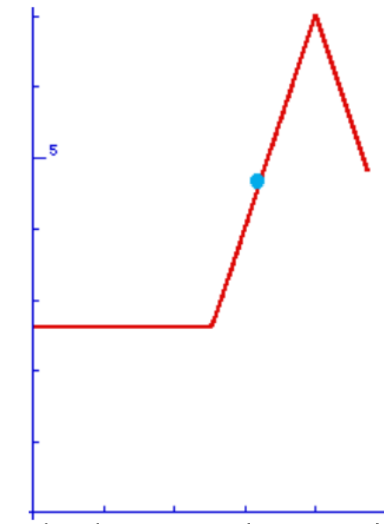






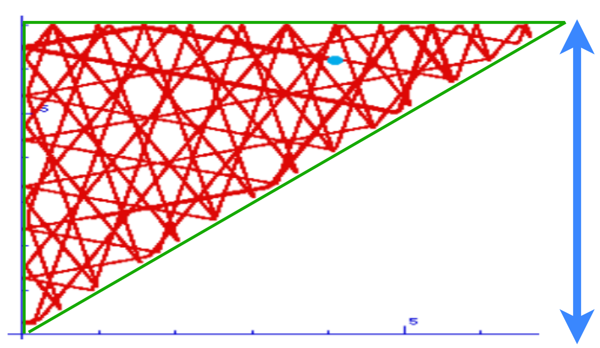


Space-space (y_1, y_2) paths

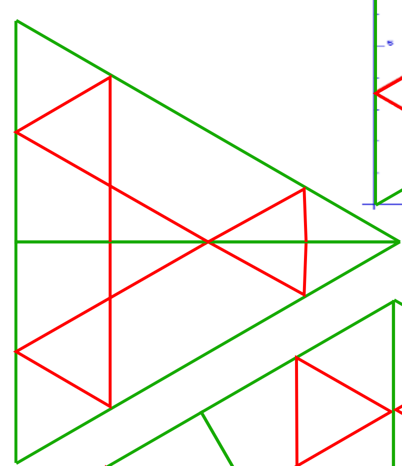


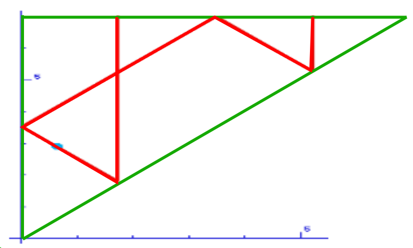
Space-space (y_1, y_2) paths scaled down by $1/\sqrt{3}$...

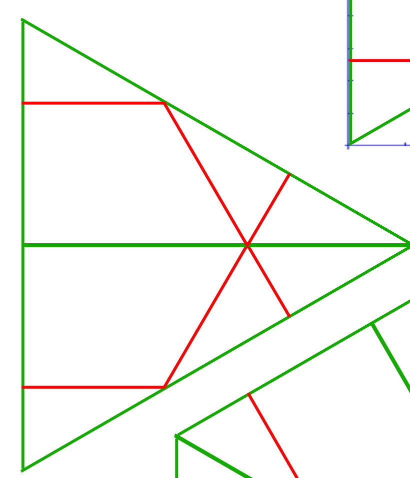
Scaled y down by $1/\sqrt{3}=0.577$

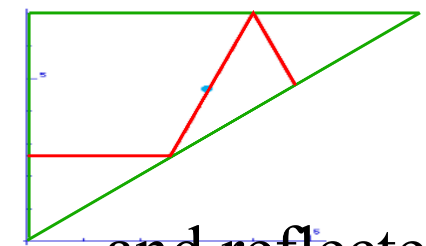


..or could have scaled x up by $\sqrt{3}=1.732$

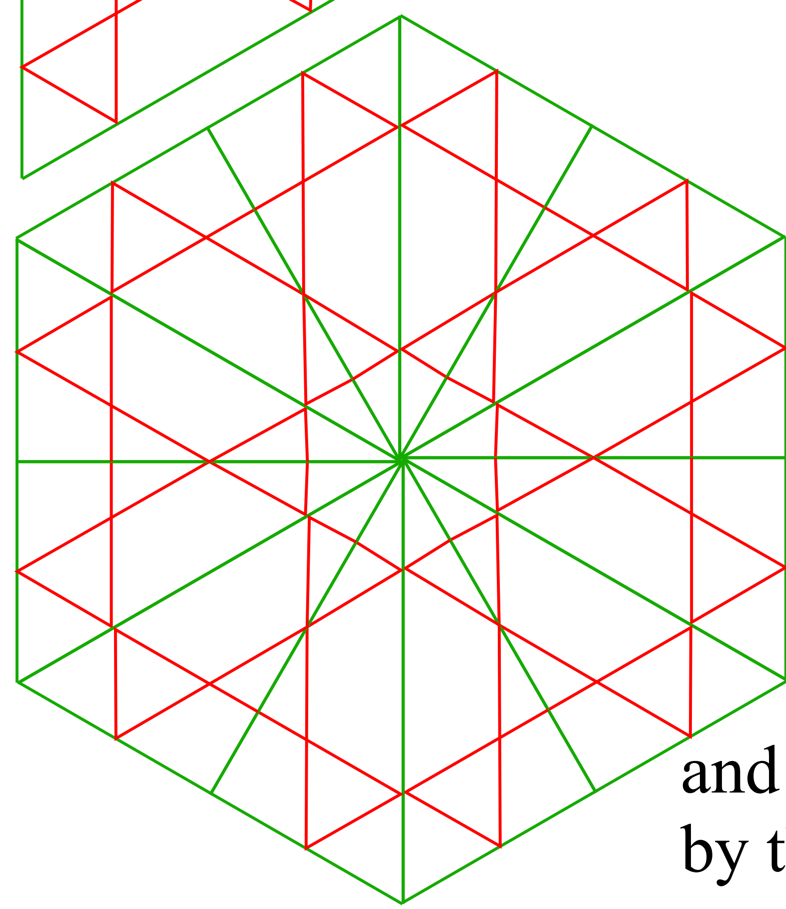




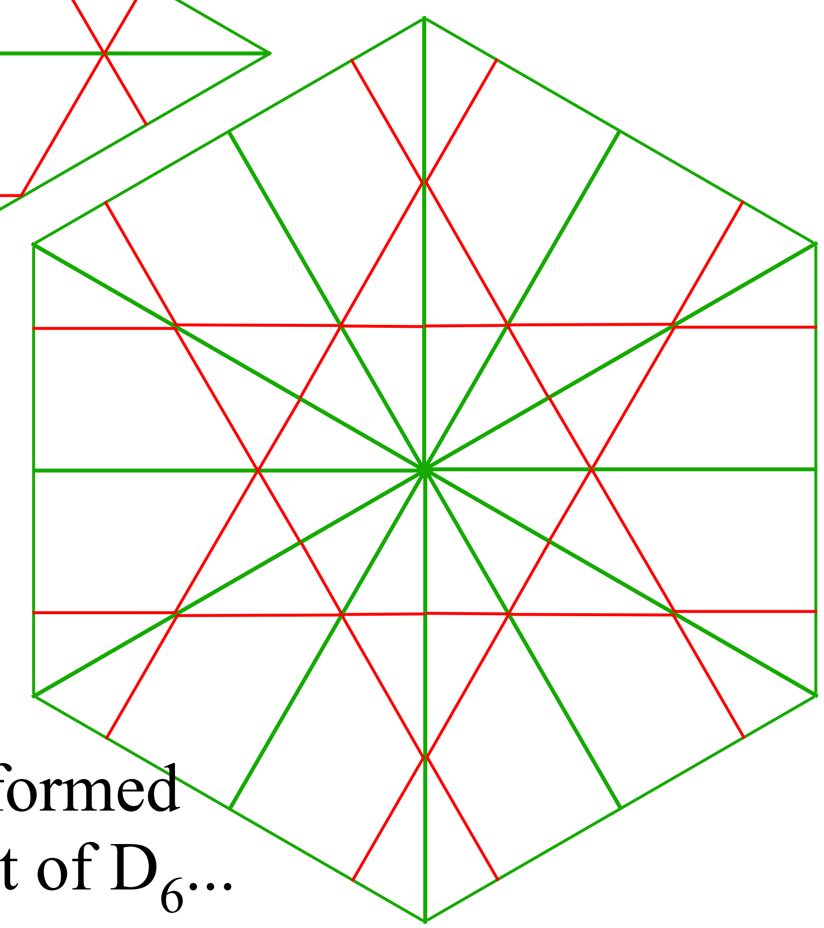


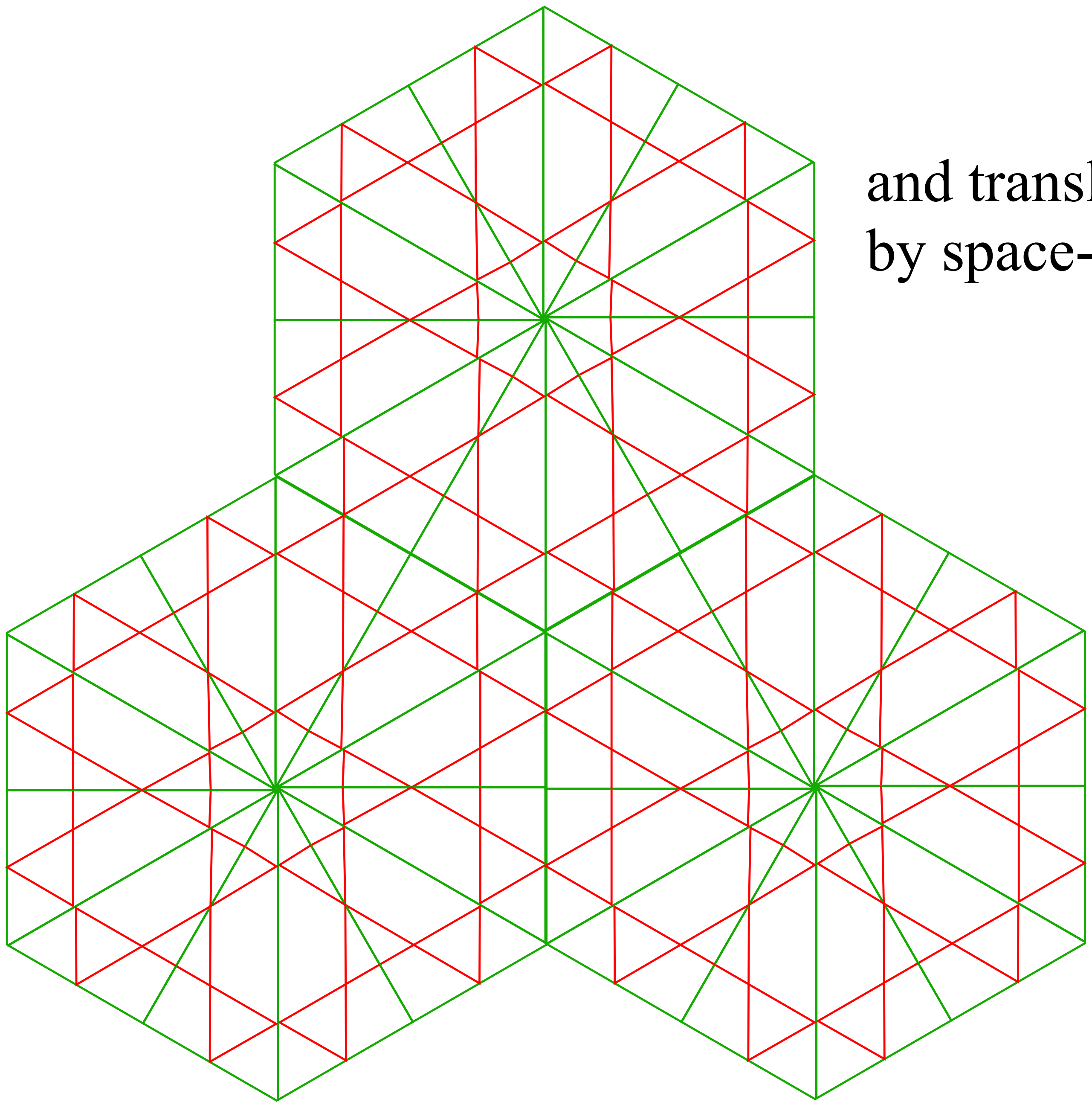


and reflected...

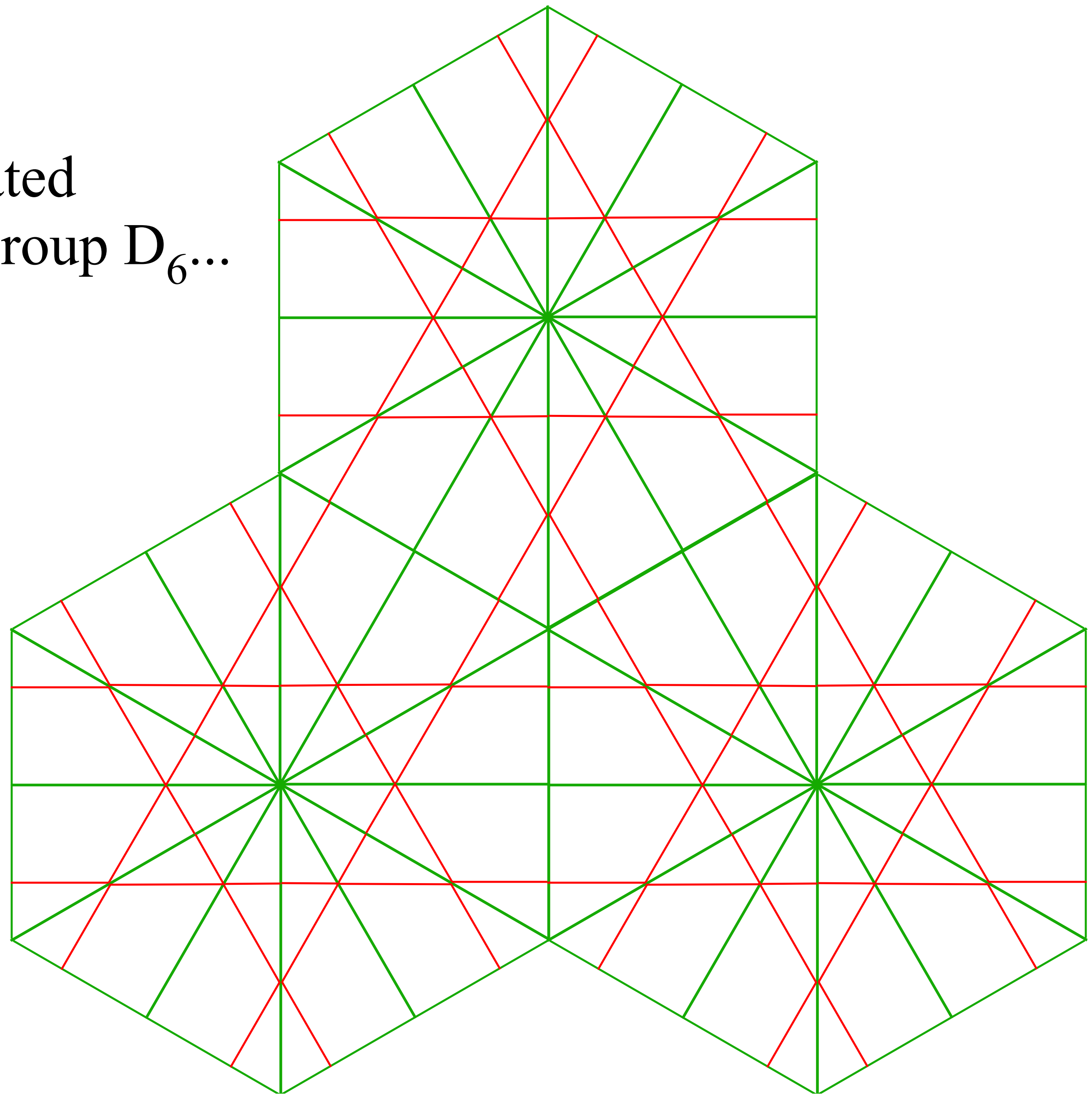


and transformed by the rest of D_6...





and translated
by space-group D_6...



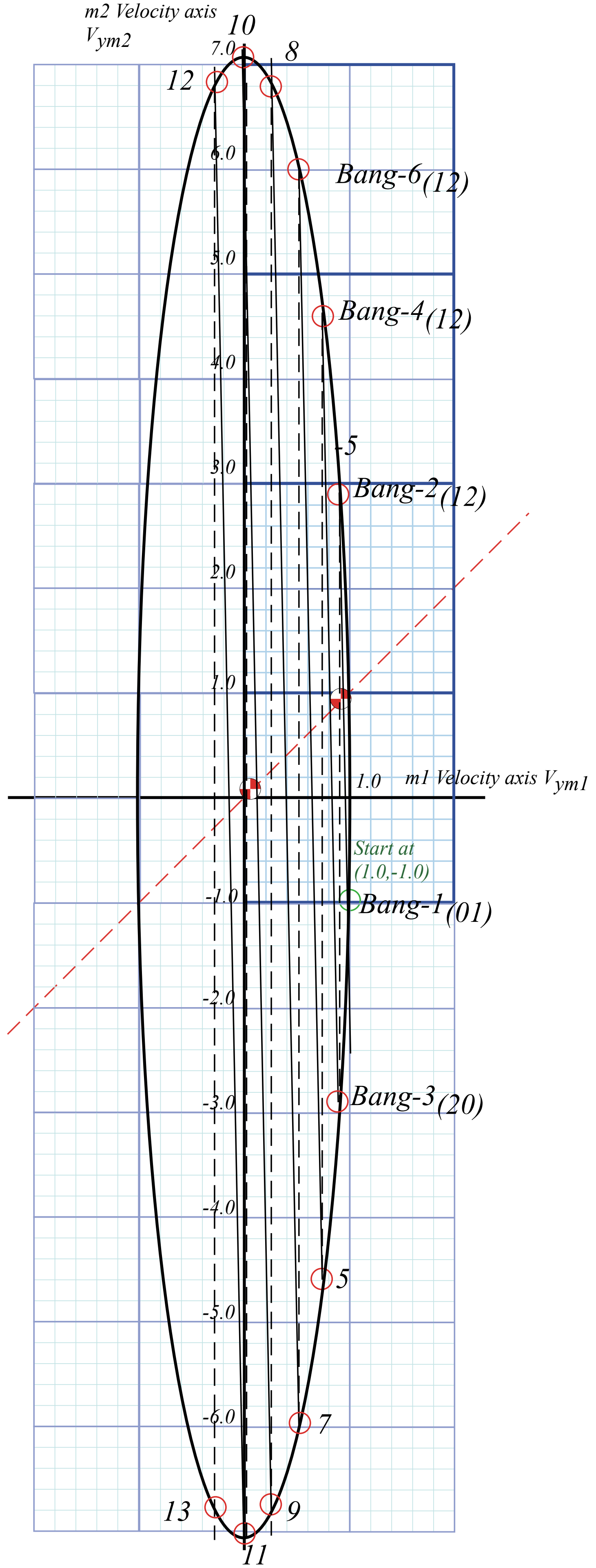
...they're just straight lines going forever.

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

Special mass ratio $M_1/m_2 = 3$

→ *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

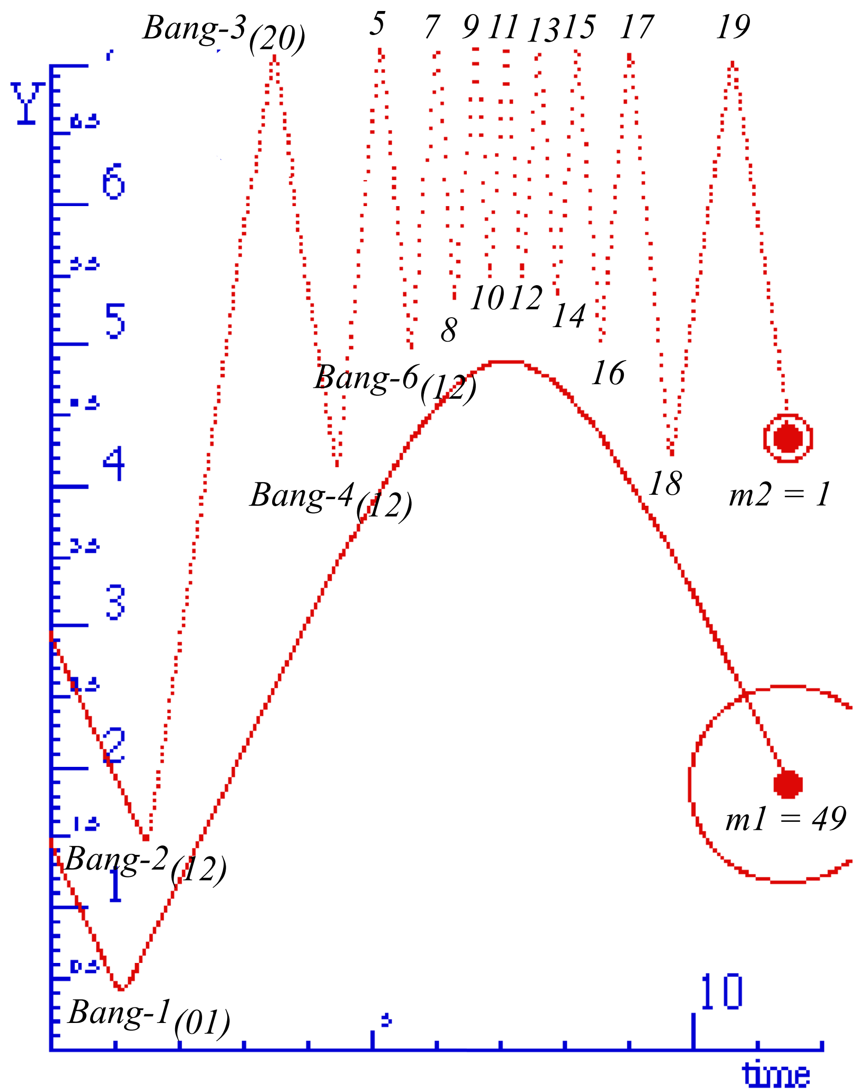


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

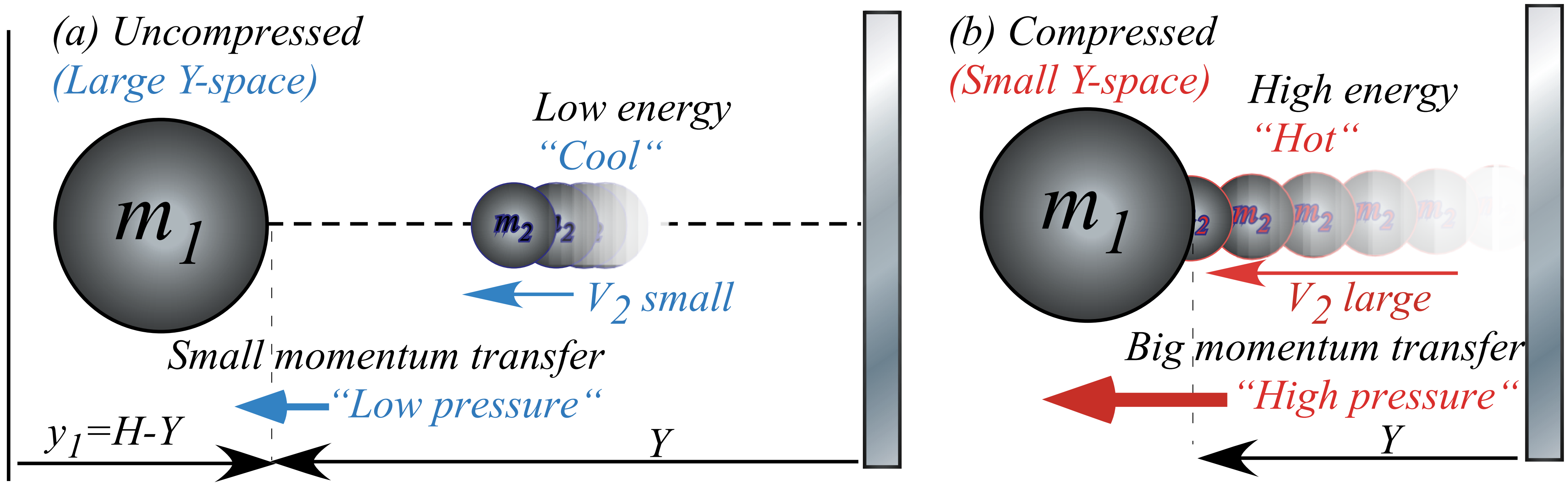
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and *the 1D-Adiabatic force field* $F(y)=const./y^2$

Big mass-m_1 ball feeling "force-field" or "pressure" of small (m_2 << m_1) rapidly (v_2 >>v_1) bouncing ball

Unit 1
Fig. 6.1



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Unit 1
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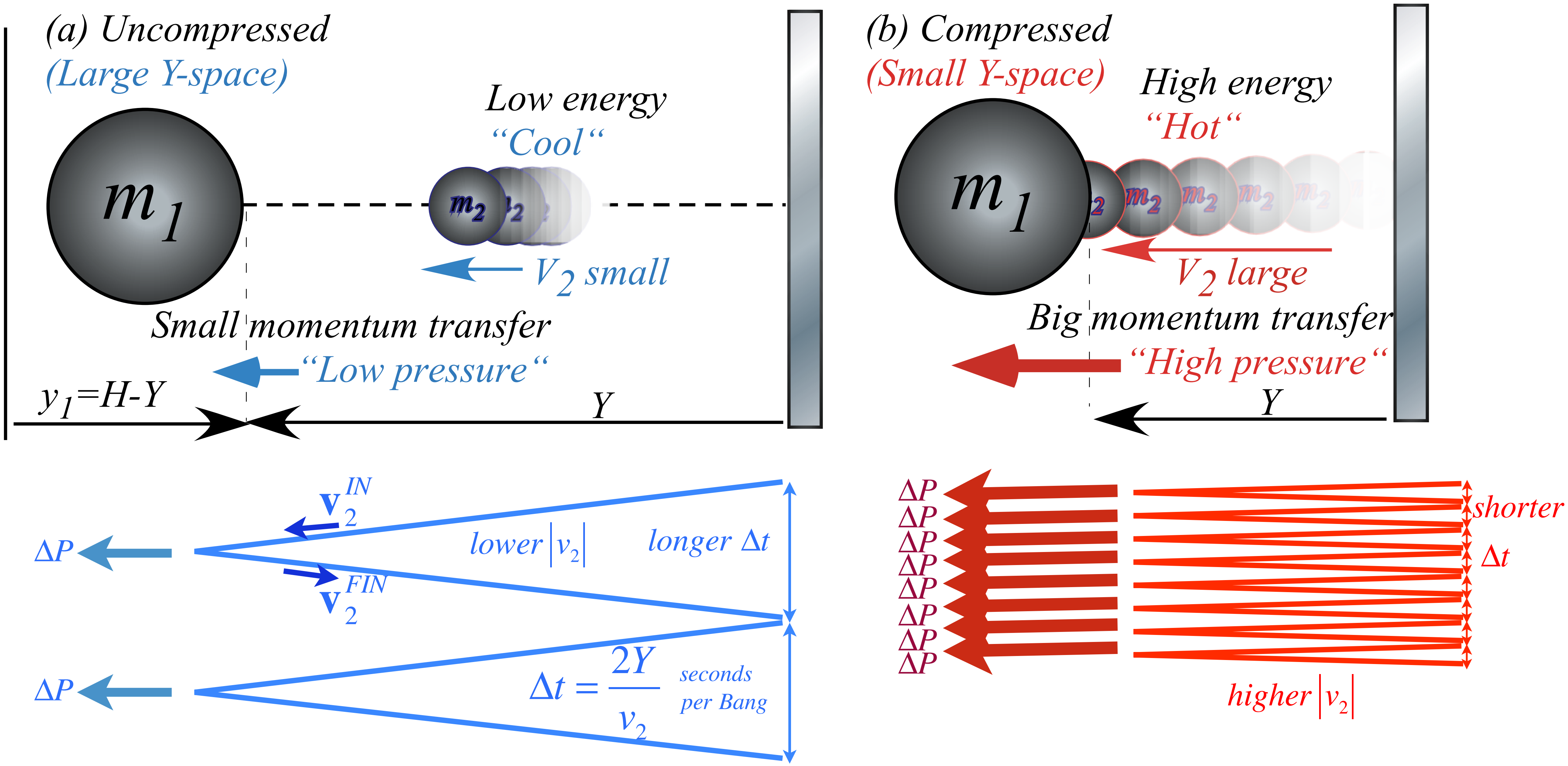

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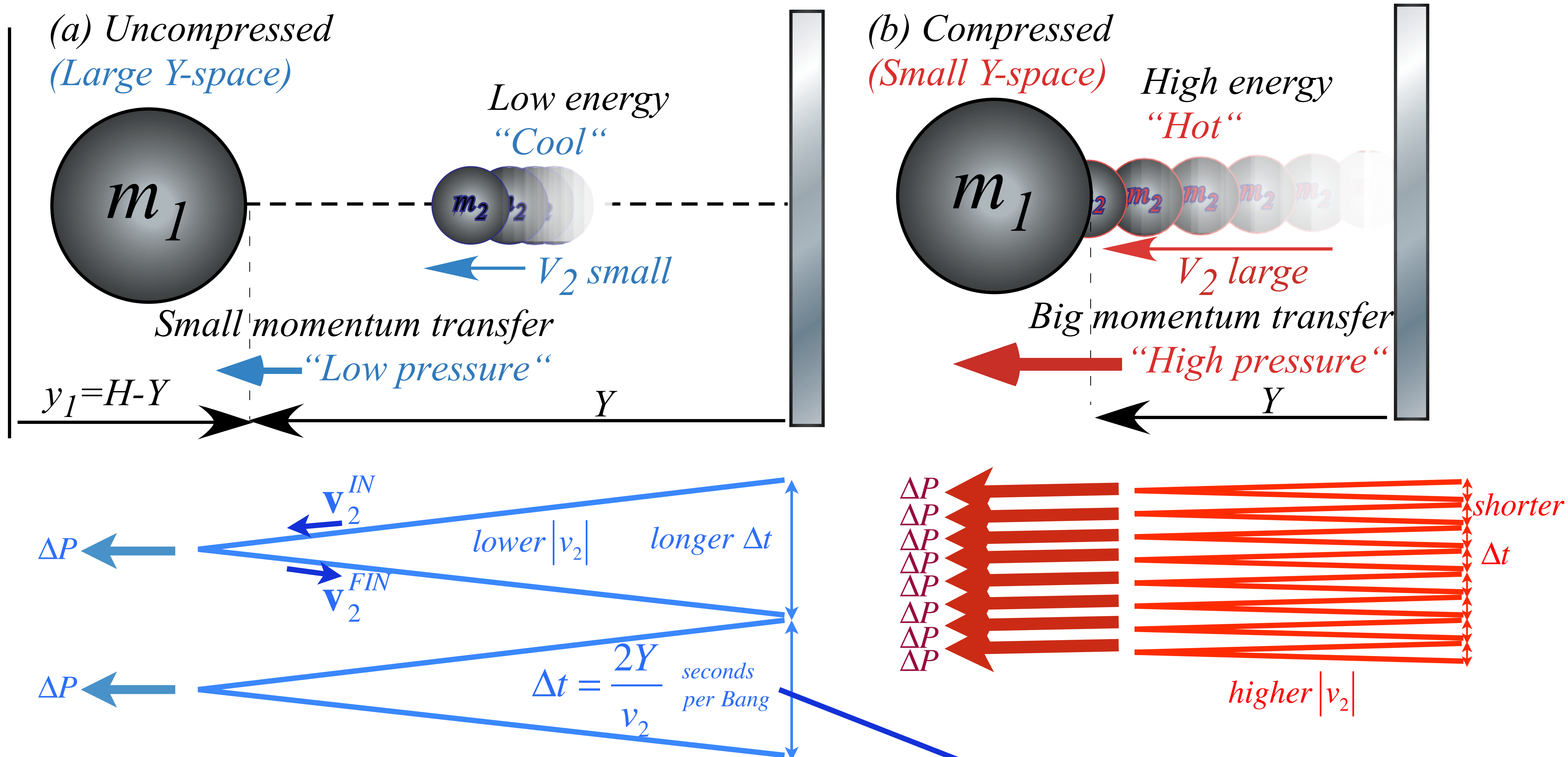
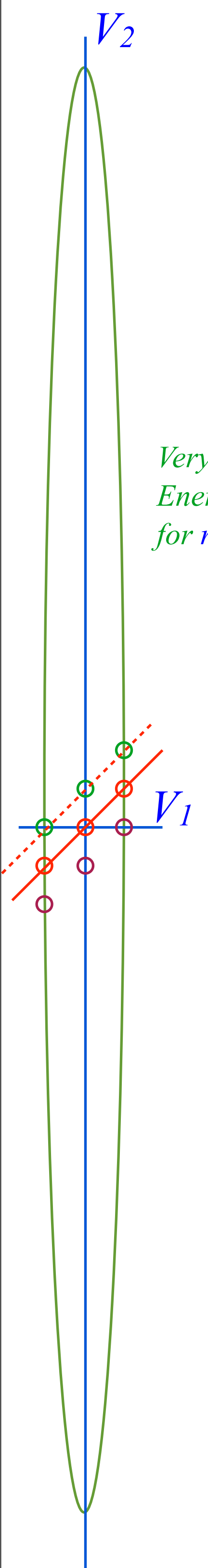
Unit 1
Fig. 6.1

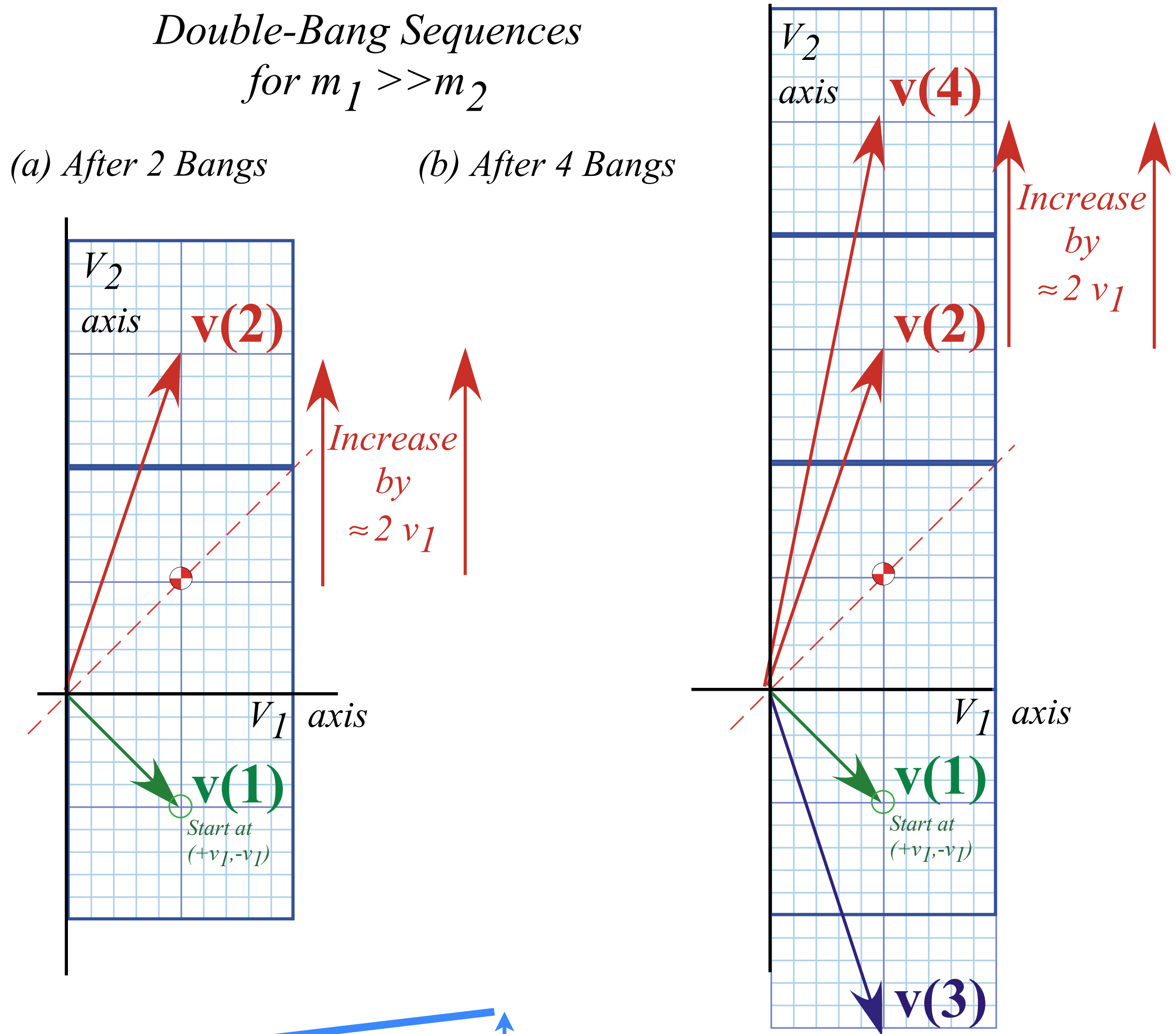


$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

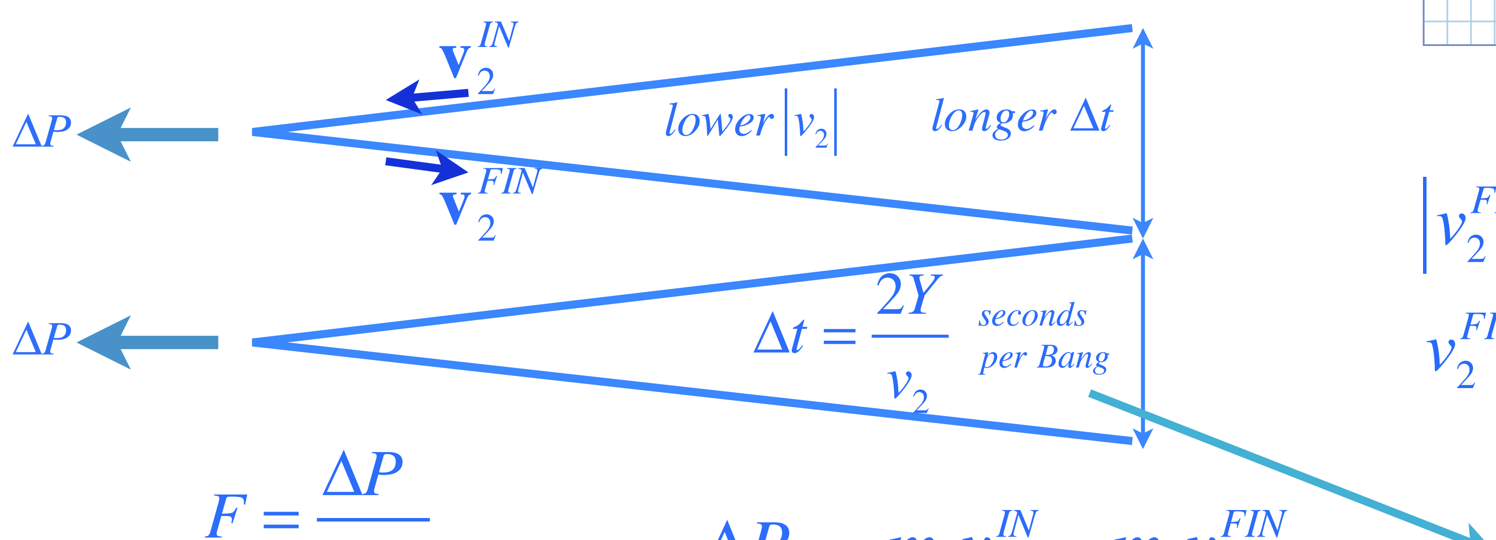
Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)



Double-Bang Sequences for $m_1 >> m_2$



Unit 1
Fig. 6.2



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

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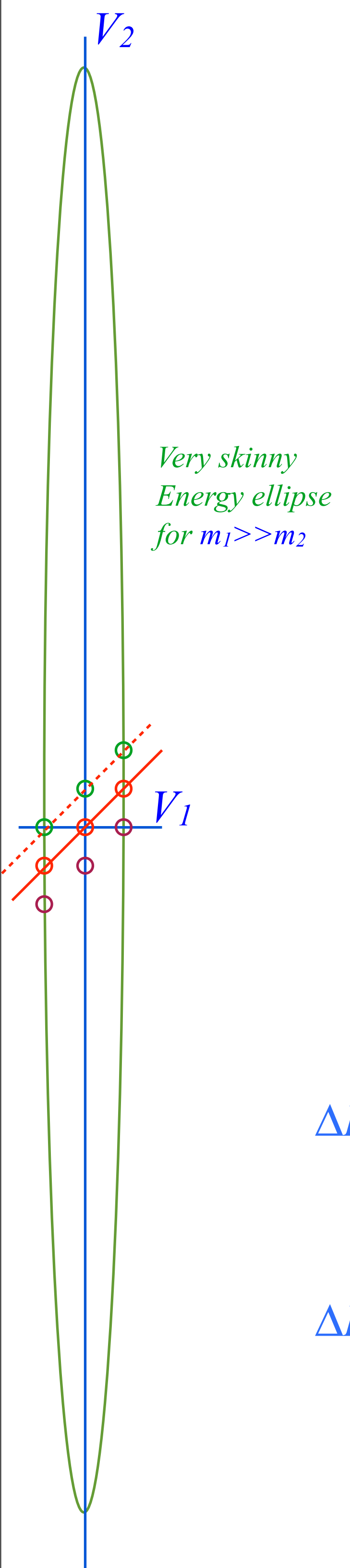
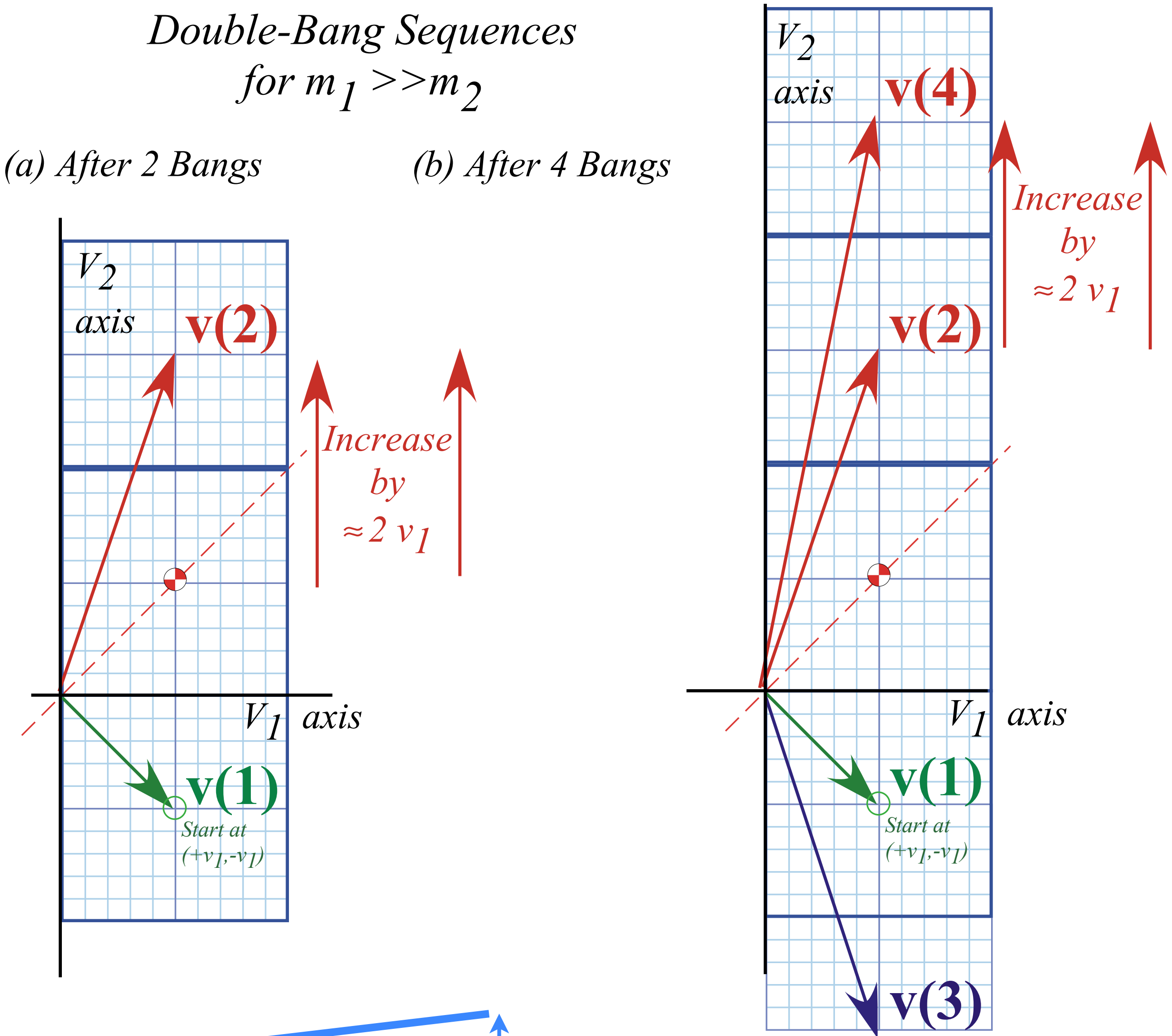
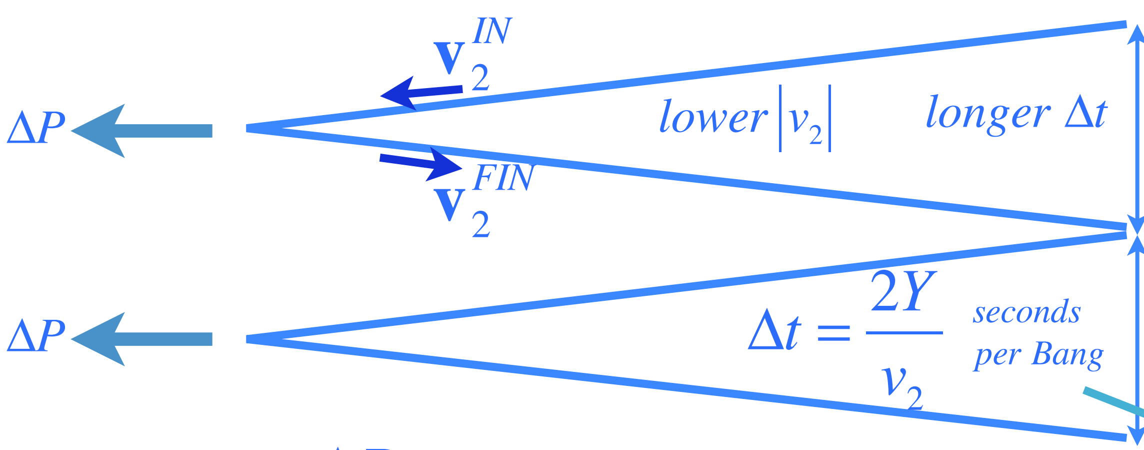




Unit 1
Fig. 6.2



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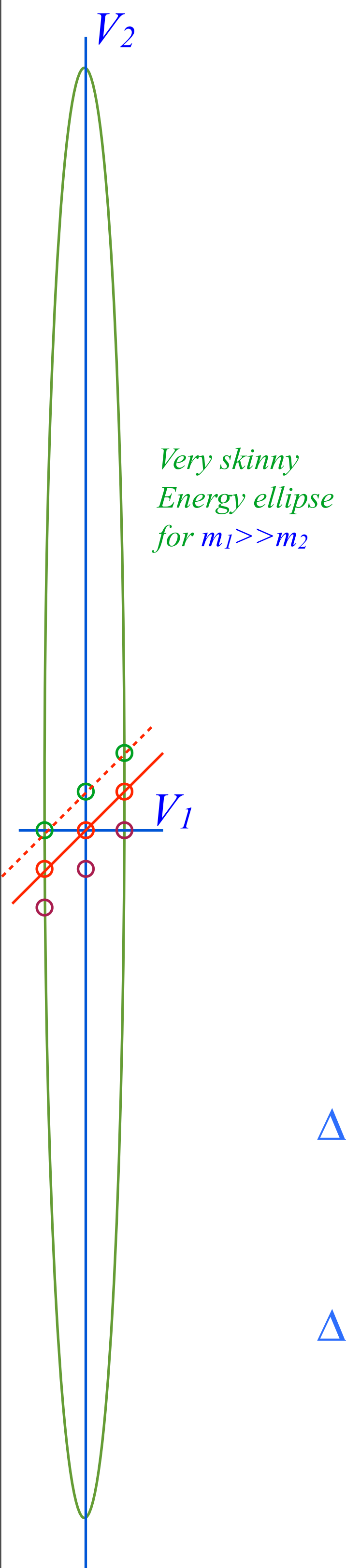
$$\frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1$$



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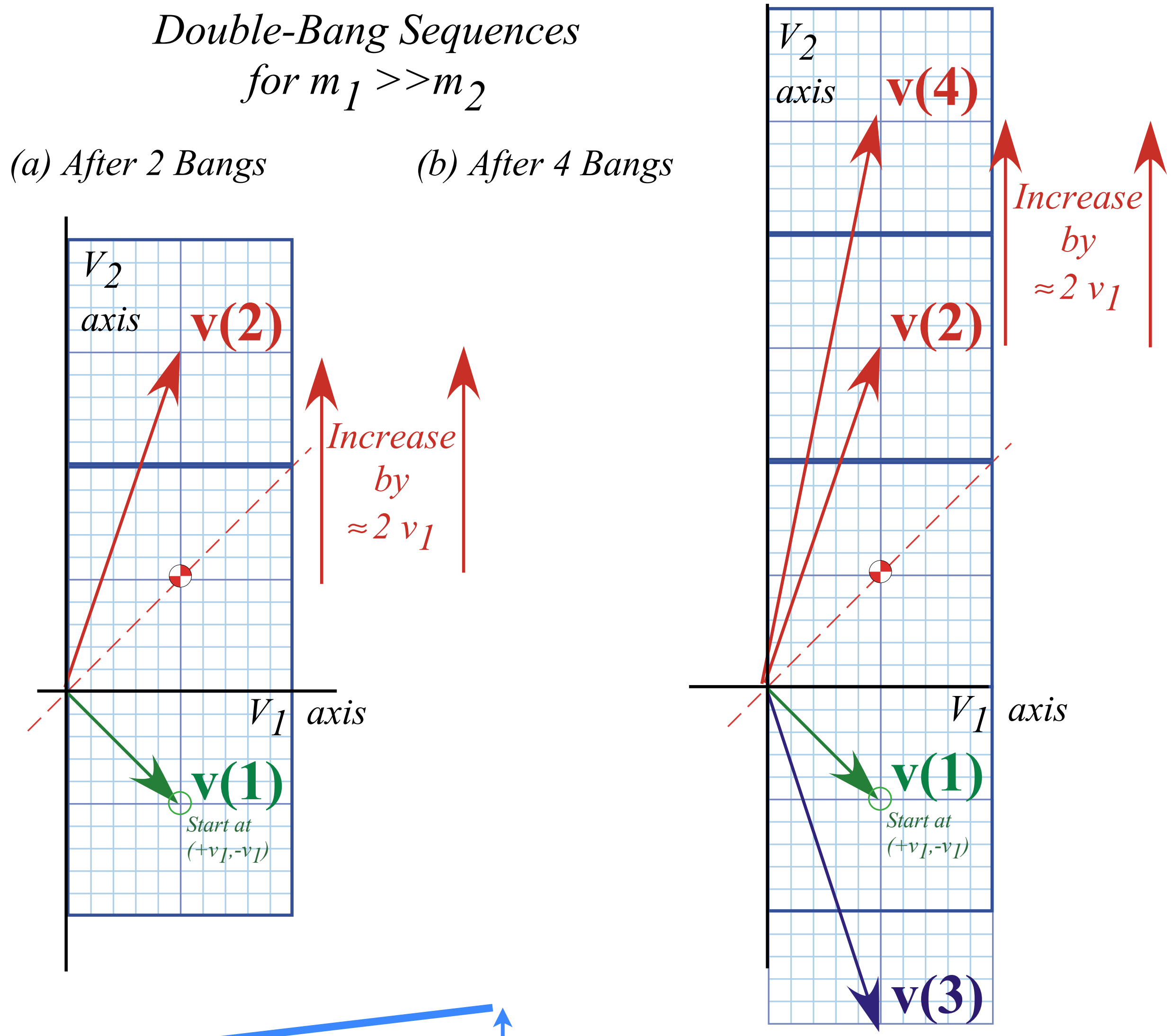
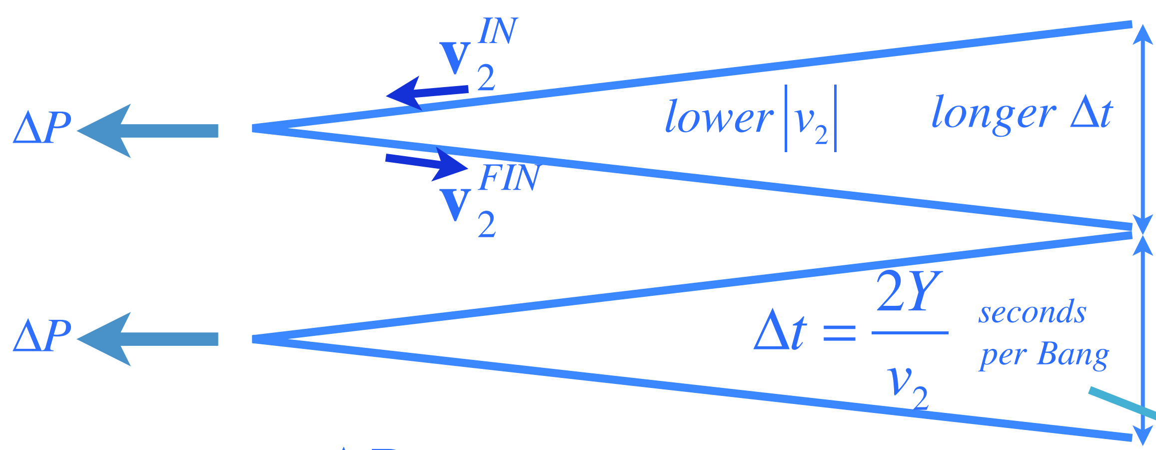


Unit 1
Fig. 6.2



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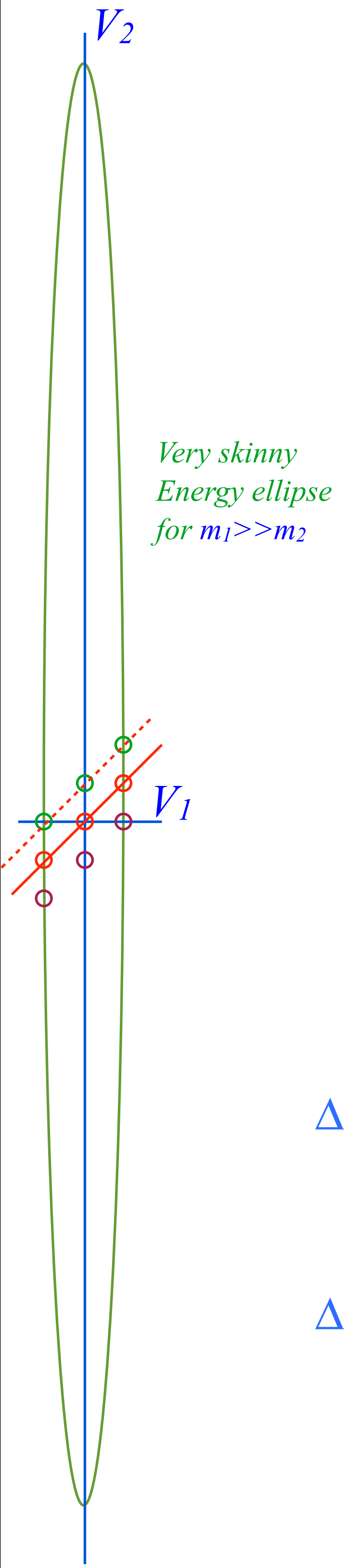
$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

Assuming slow m_1 : $v_1 \ll v_2$

Unit 1
Fig. 6.2



Double-Bang Sequences
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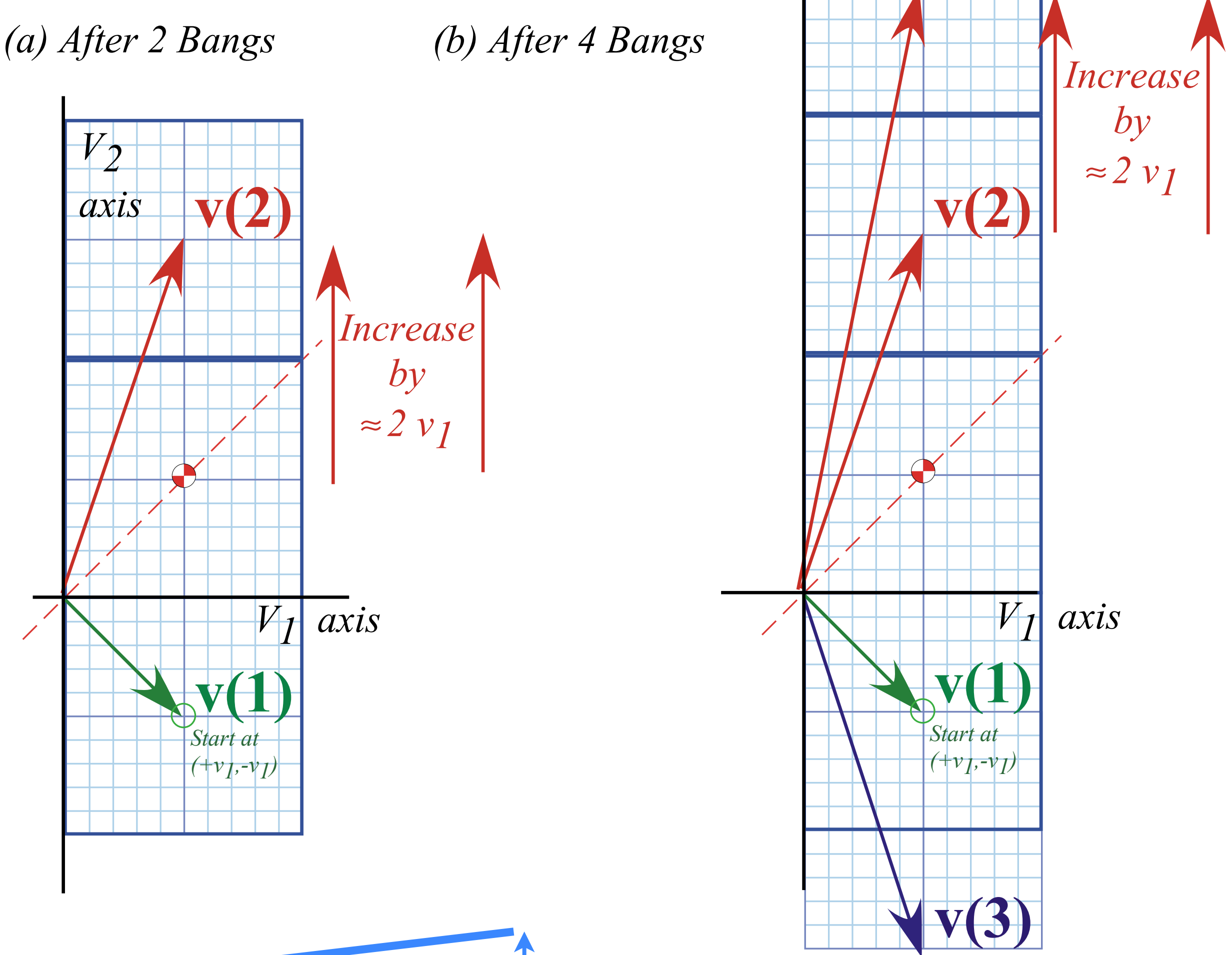

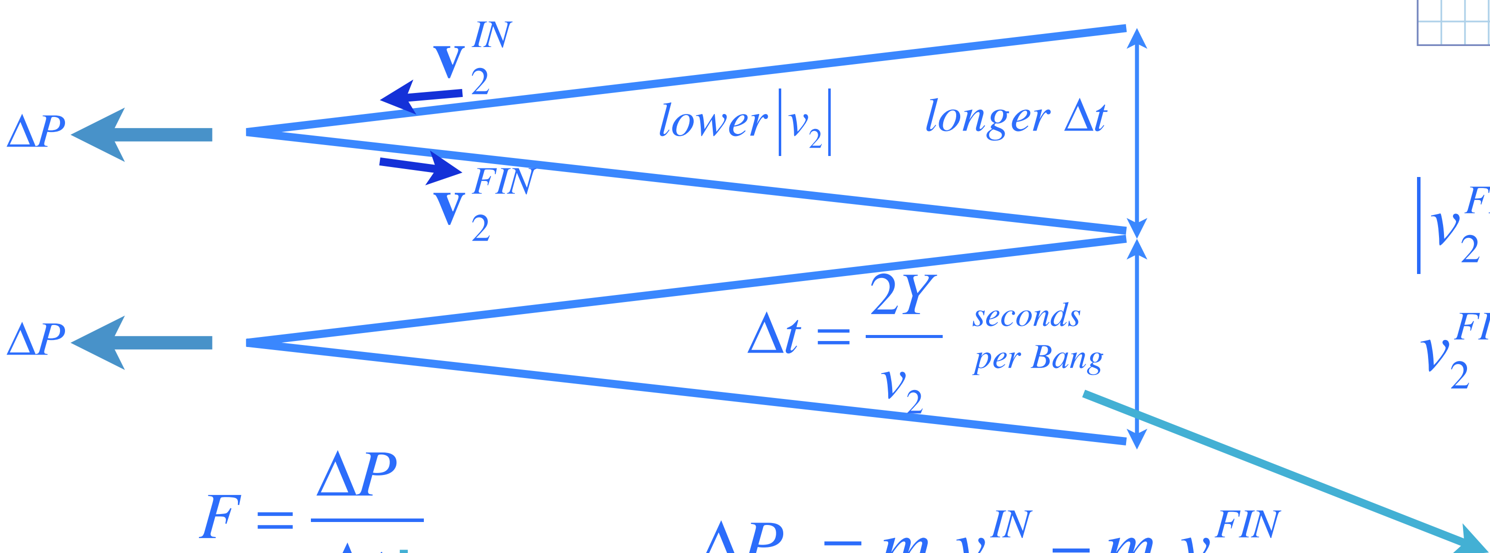

$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

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Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

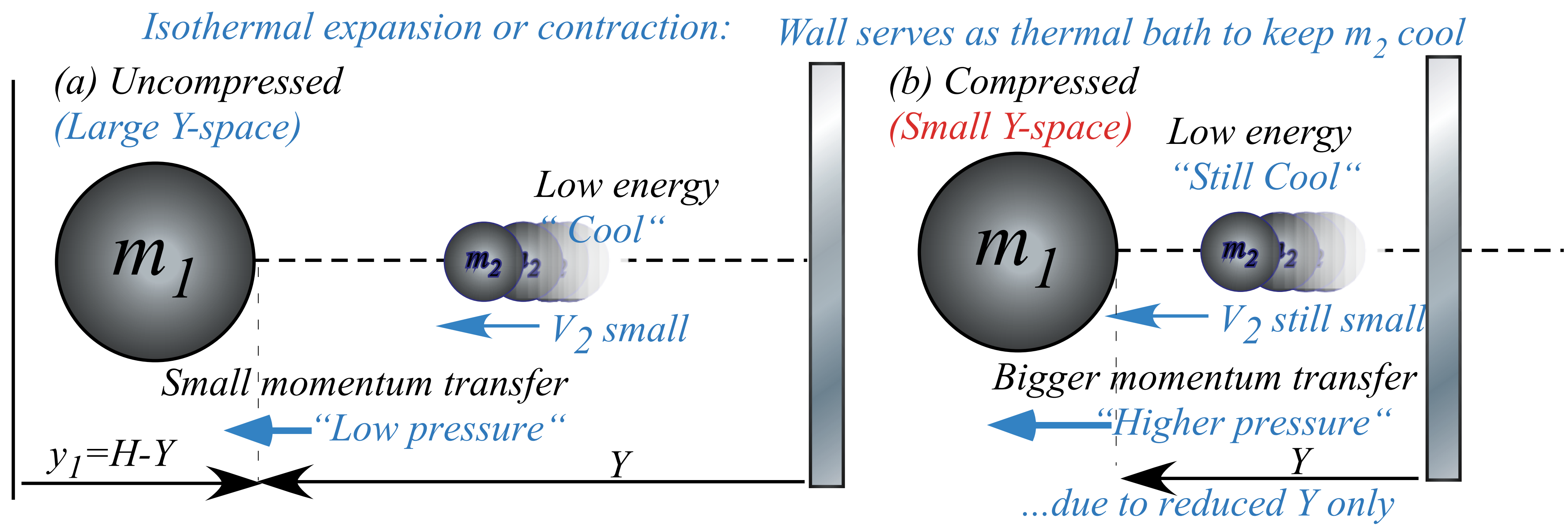
$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Assuming slow m_1 : $v_1 \ll v_2$

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$



Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field F(y)=const./y and *the 1D-Adiabatic force field F(y)=const./y^3*

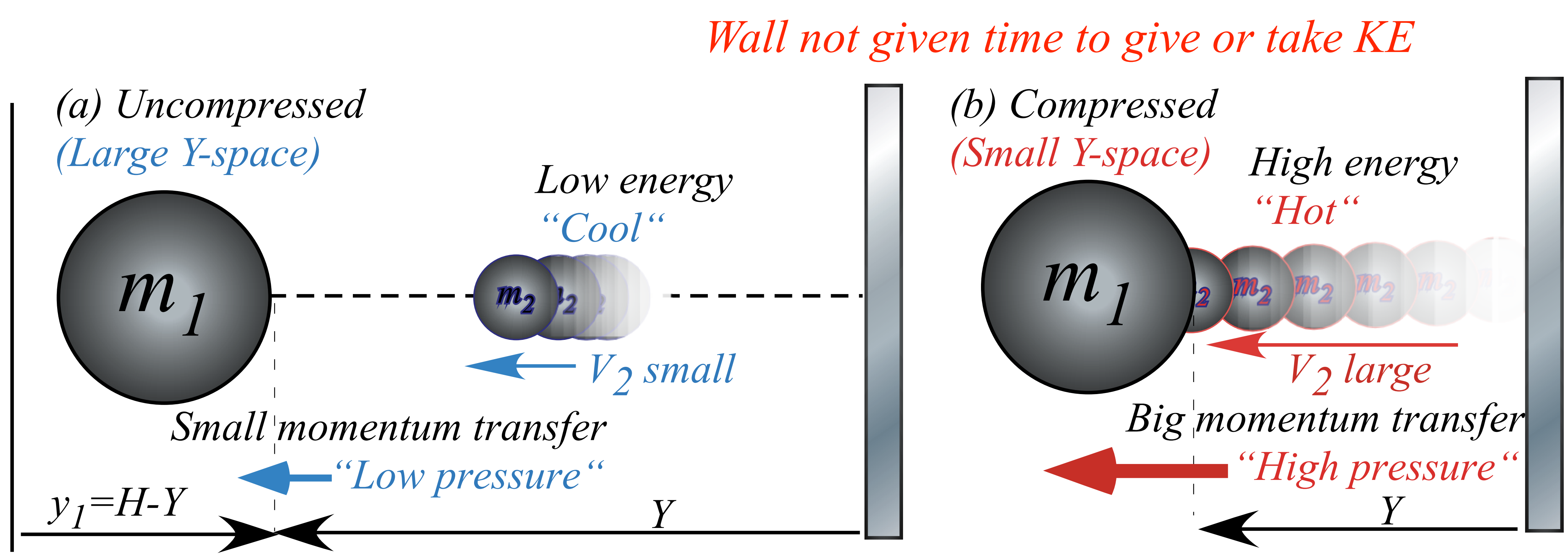
$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y}$$



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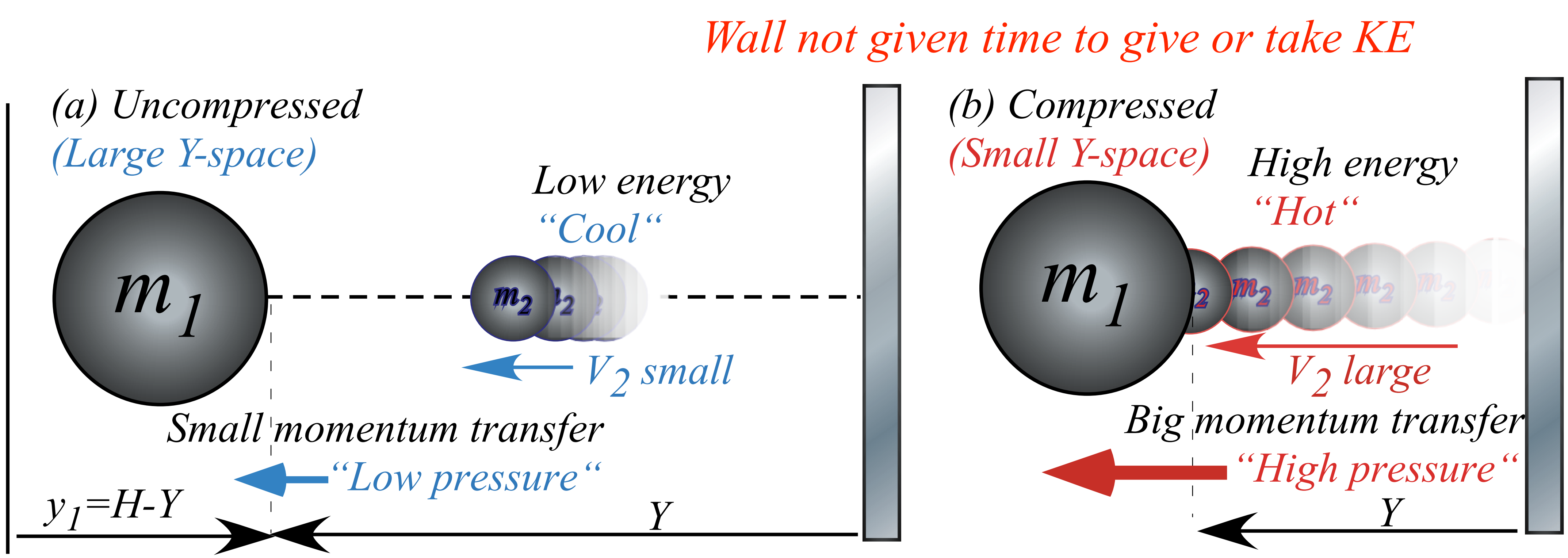
$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y}$$

Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$\frac{dv_2}{v_2} = -\frac{dY}{Y}$ integrates to: $\ln v_2 = -\ln Y + C$ or: $\ln v_2 = \ln\frac{const.}{Y}$ or: $v_2 = \frac{const.}{Y}$



$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

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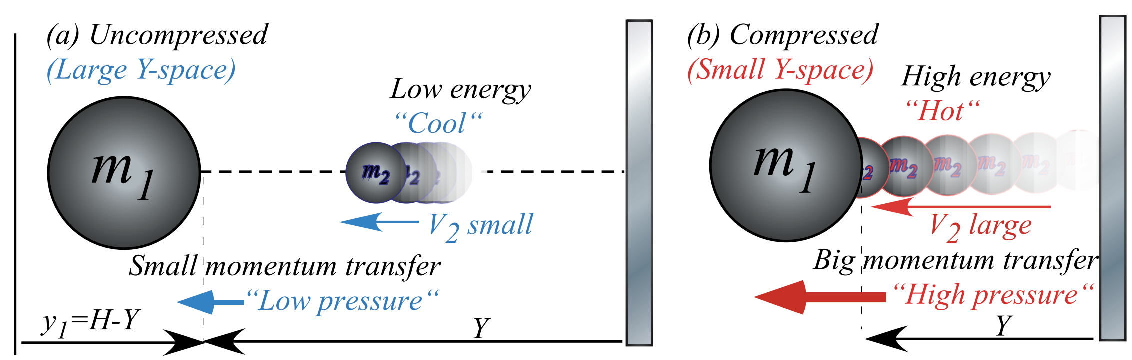
$\frac{dv_2}{v_2} = -\frac{dY}{Y}$ integrates to: $\ln v_2 = -\ln Y + C$ or: $\ln v_2 = \ln\frac{const.}{Y}$ or: $v_2 = \frac{const.}{Y}$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$*):* $F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}$



Potential field due to many small bounces

→ *Example of 1D-Adiabatic potential $U(y)=const./y^2$*

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const\ ln(y)$

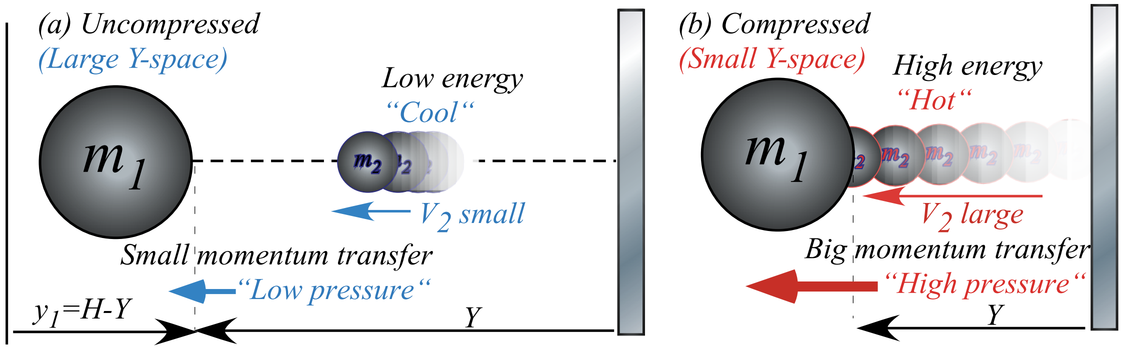
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small (m_2 << m_1) rapidly (v_2 >>v_1) bouncing ball

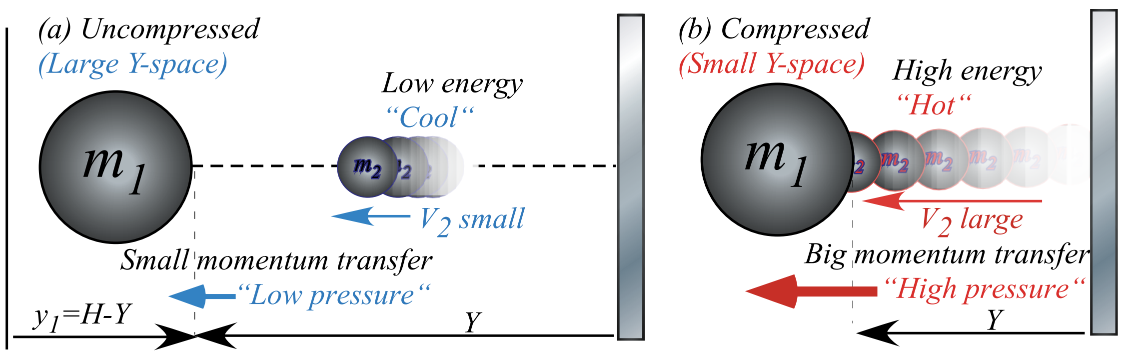
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.



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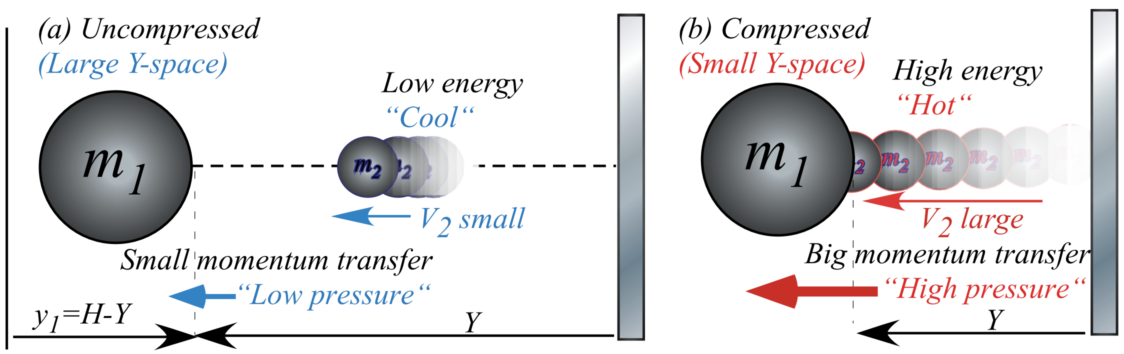
Q*?Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$



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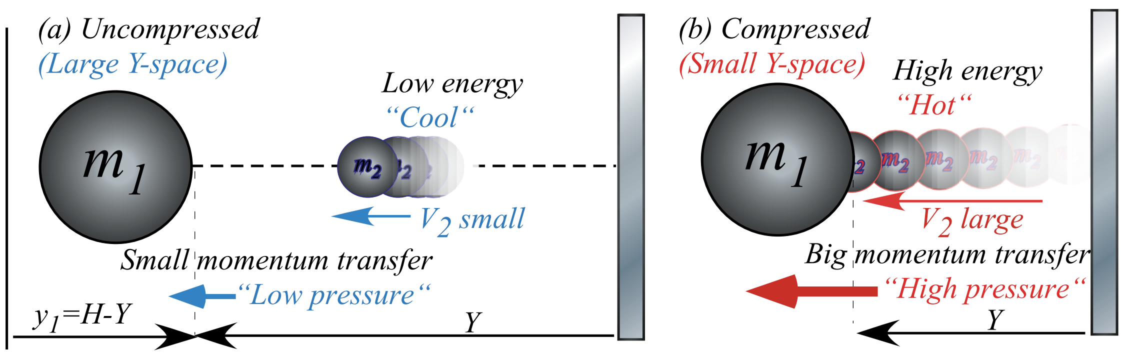
or else : $F\cdot\frac{dY}{dt} = \frac{dp}{dt}\cdot V = \frac{d(mV)}{dt}\cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$



Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

→ *Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$*

Example of 1D-Isothermal potential $U(y)$=const ln(y)

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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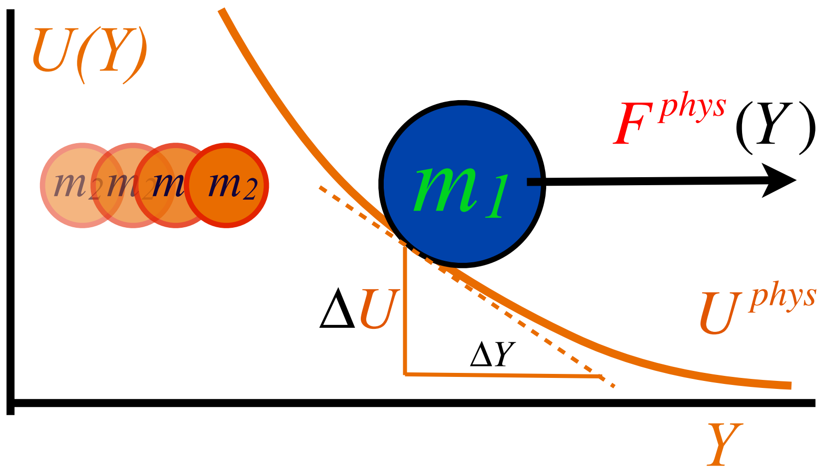
Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)$=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F\cdot dY = \pm dU$

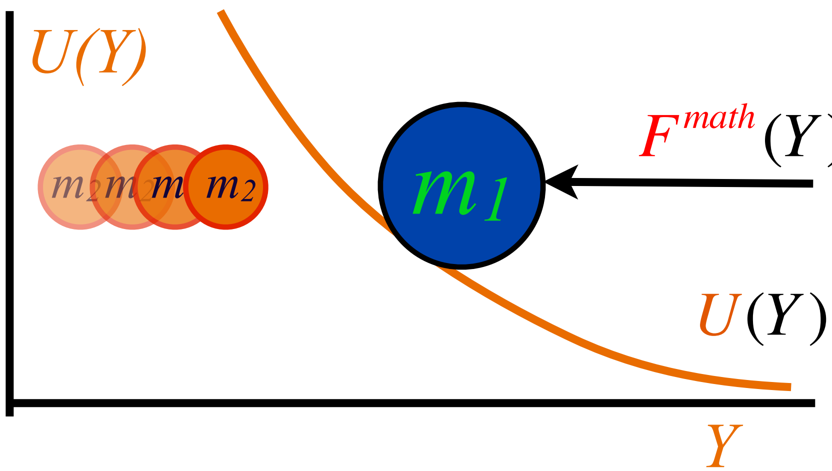
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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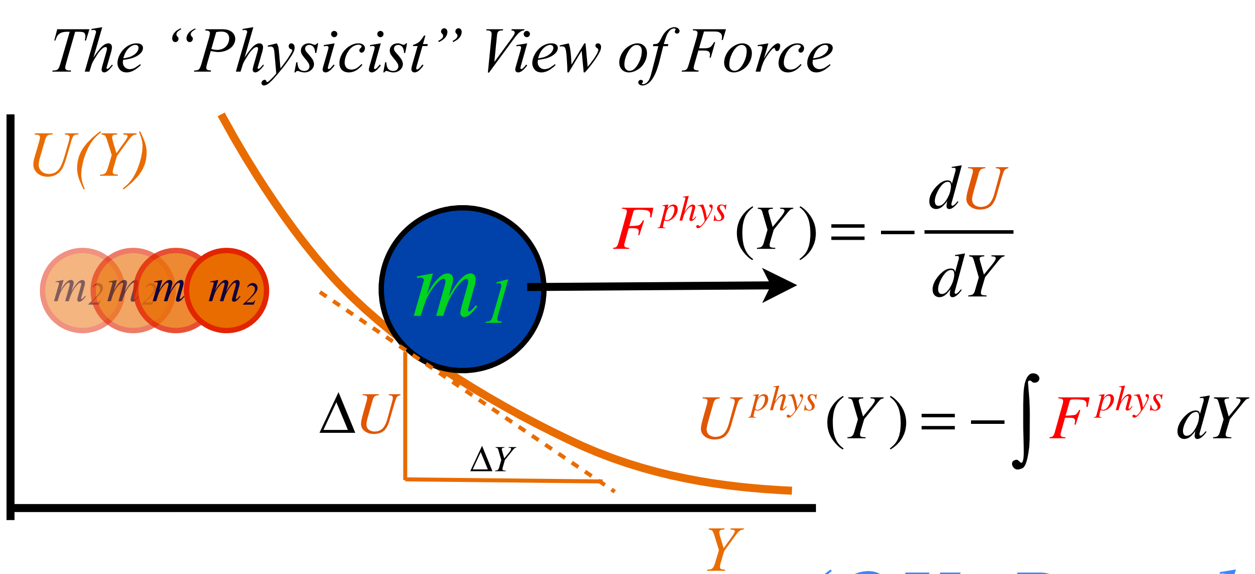
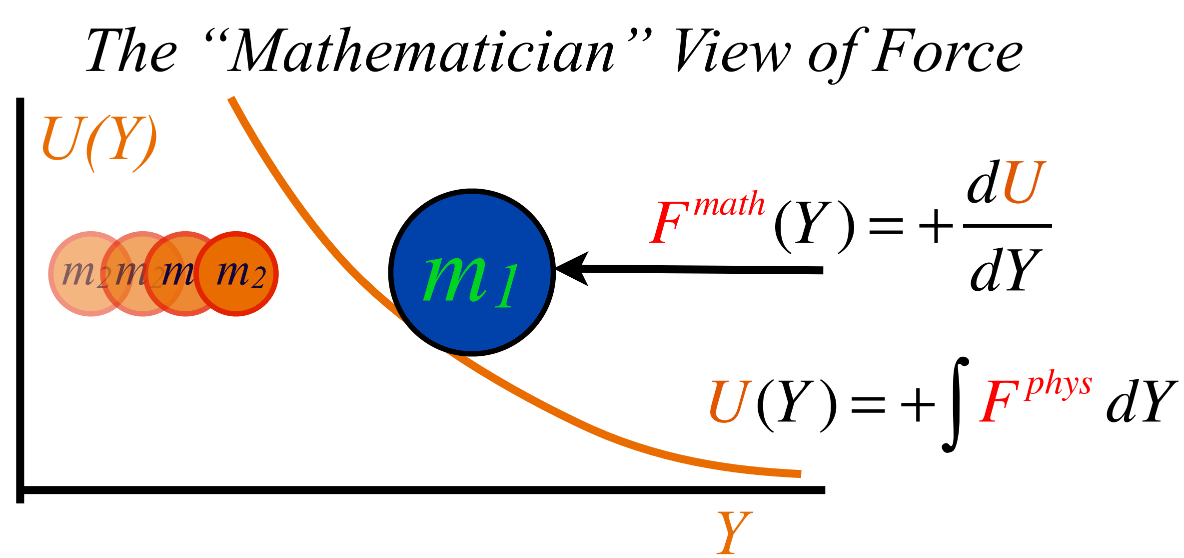
Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$





(OK, But, does it work?)

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

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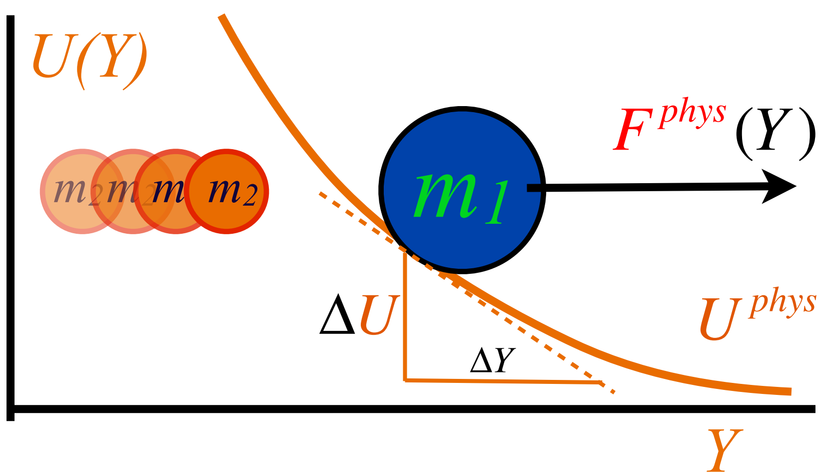
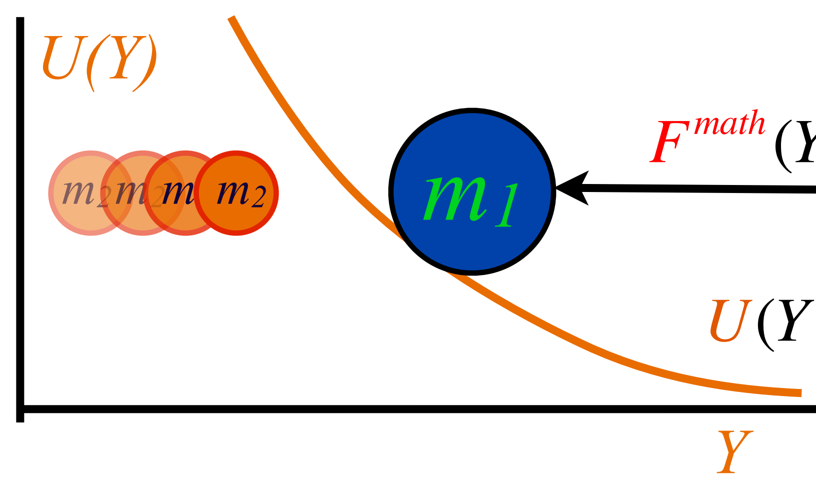
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, does it work?)

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with:

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

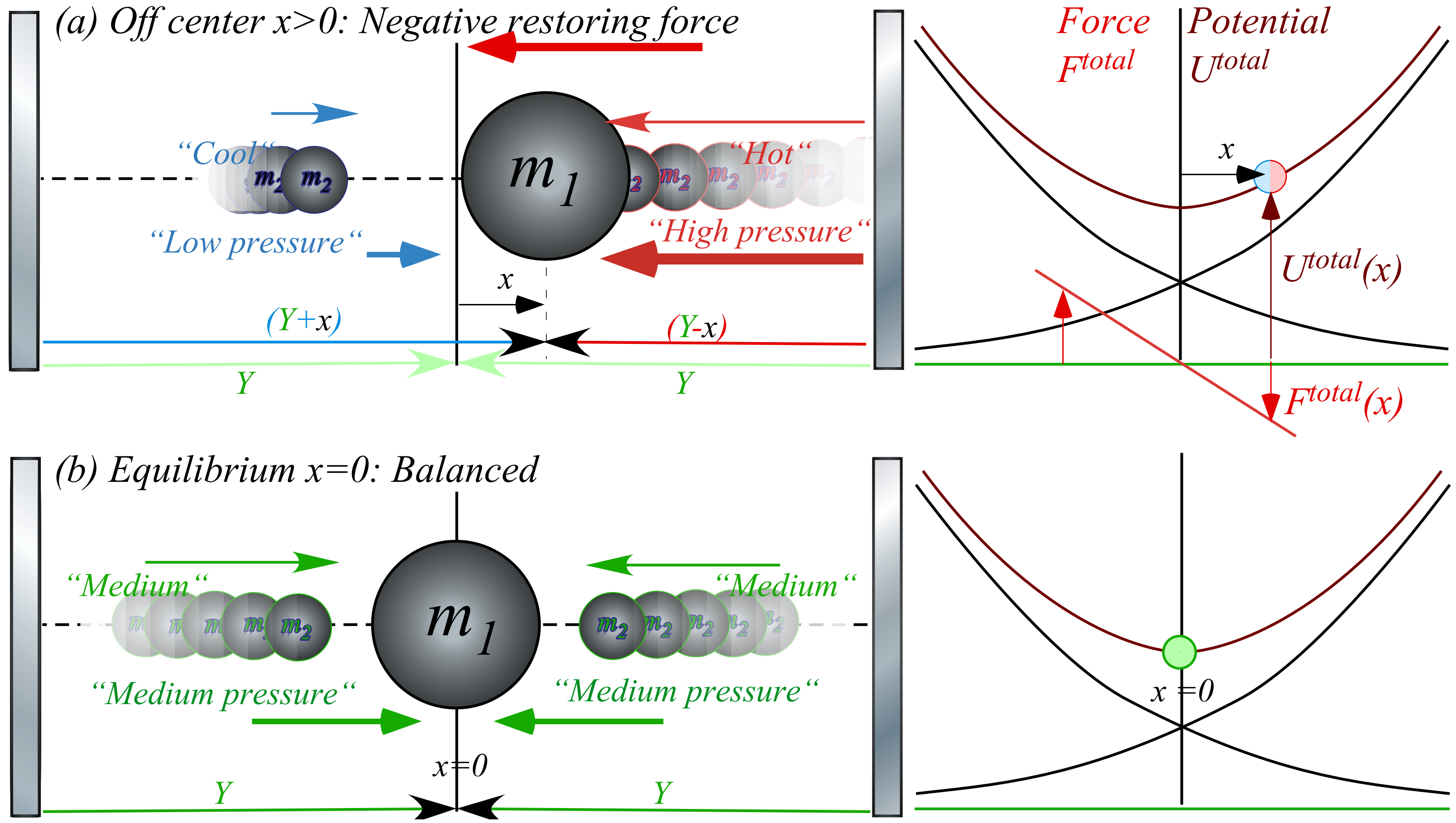
Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential $U(y)=const\ ln(y)$*

1D-Isothermal Force Law (*assume* v_2 *is constant for all Y*): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

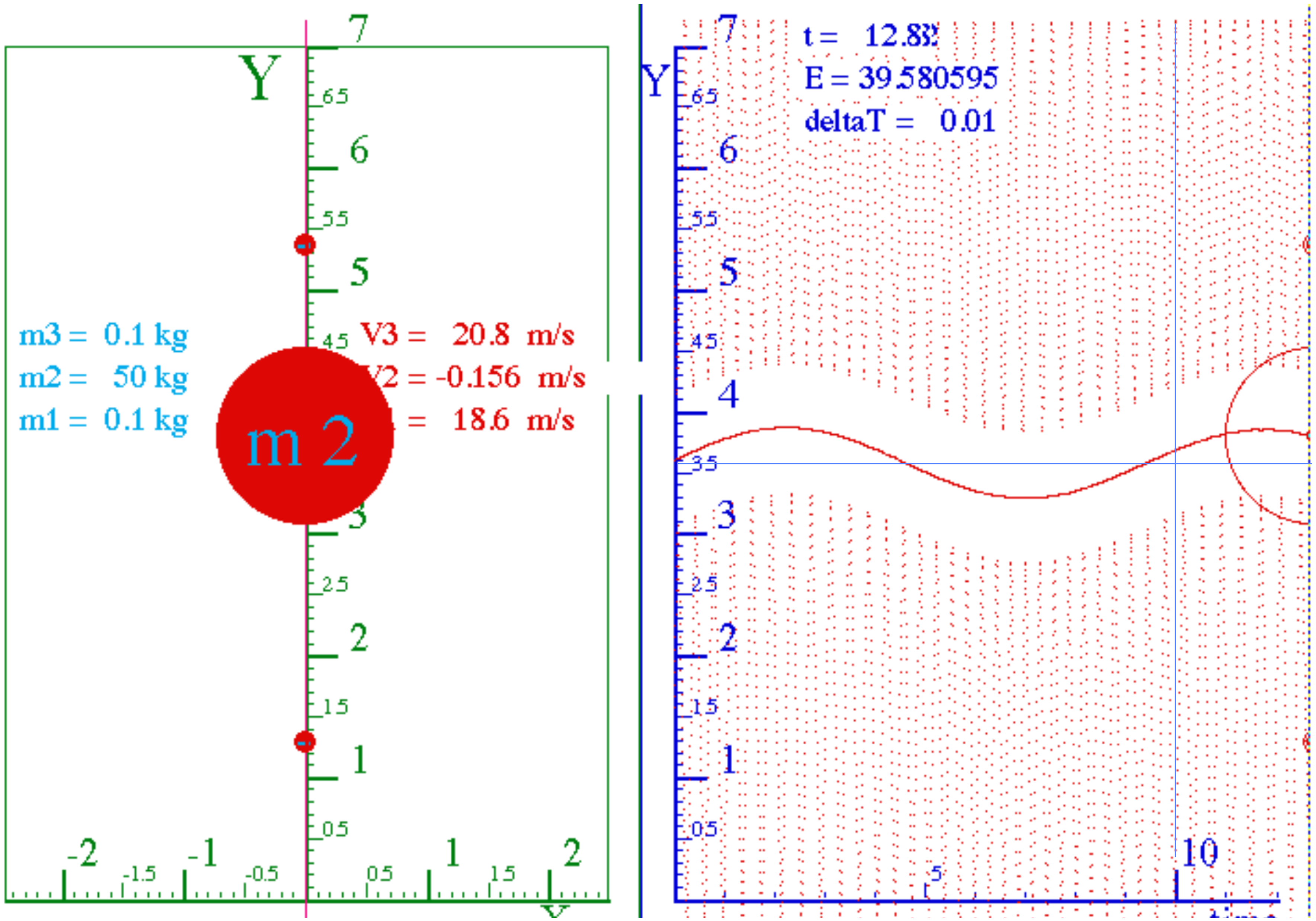
Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f\left[1 - x + x^2 - x^3 \ldots\right] - f\left[1 + x + x^2 + x^3 \ldots\right] = -2f\cdot x - 2f\cdot x^3 - \ldots$$

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x} \qquad U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx \qquad \text{HO ∡frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\upsilon$$



Unit 1
Fig. 6.3

Simulation of

the adiabatic case

See Homework problem 1.6.1: *Compute frequency and/or period for both isoT and adiabatic cases*

“Monster Mash” classical segue to Heisenberg action relations

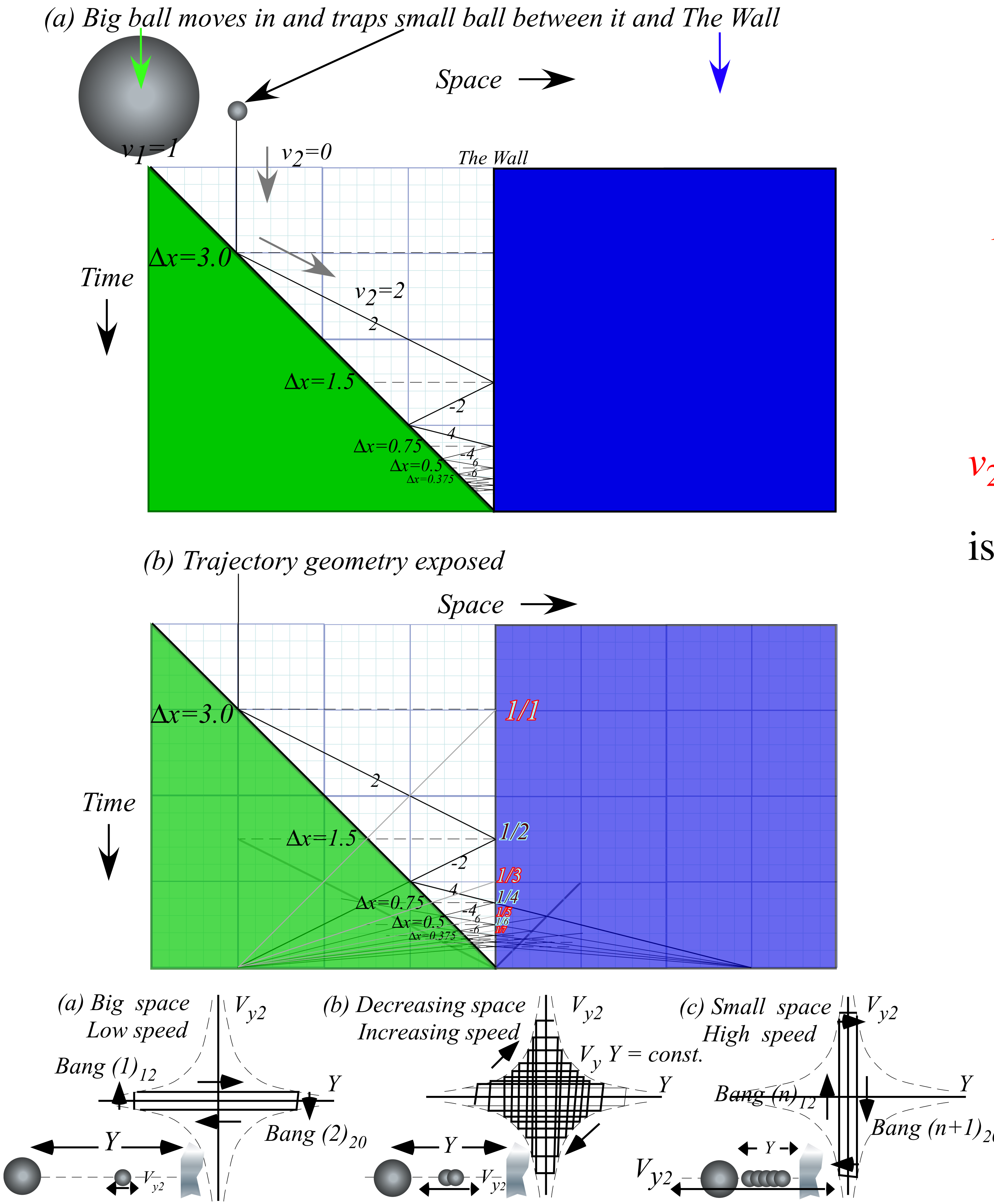
Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The “Tiny-Big-Bang” [Harter, J. Mol. Spec. 210, 166-182 (2001)],[Harter, Li IMSS (2012)]

A lesson in geometry of fractions: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)] [John Farey, Phil. Mag.(1816)]



The Classical "Monster Mash"

Classical introduction to

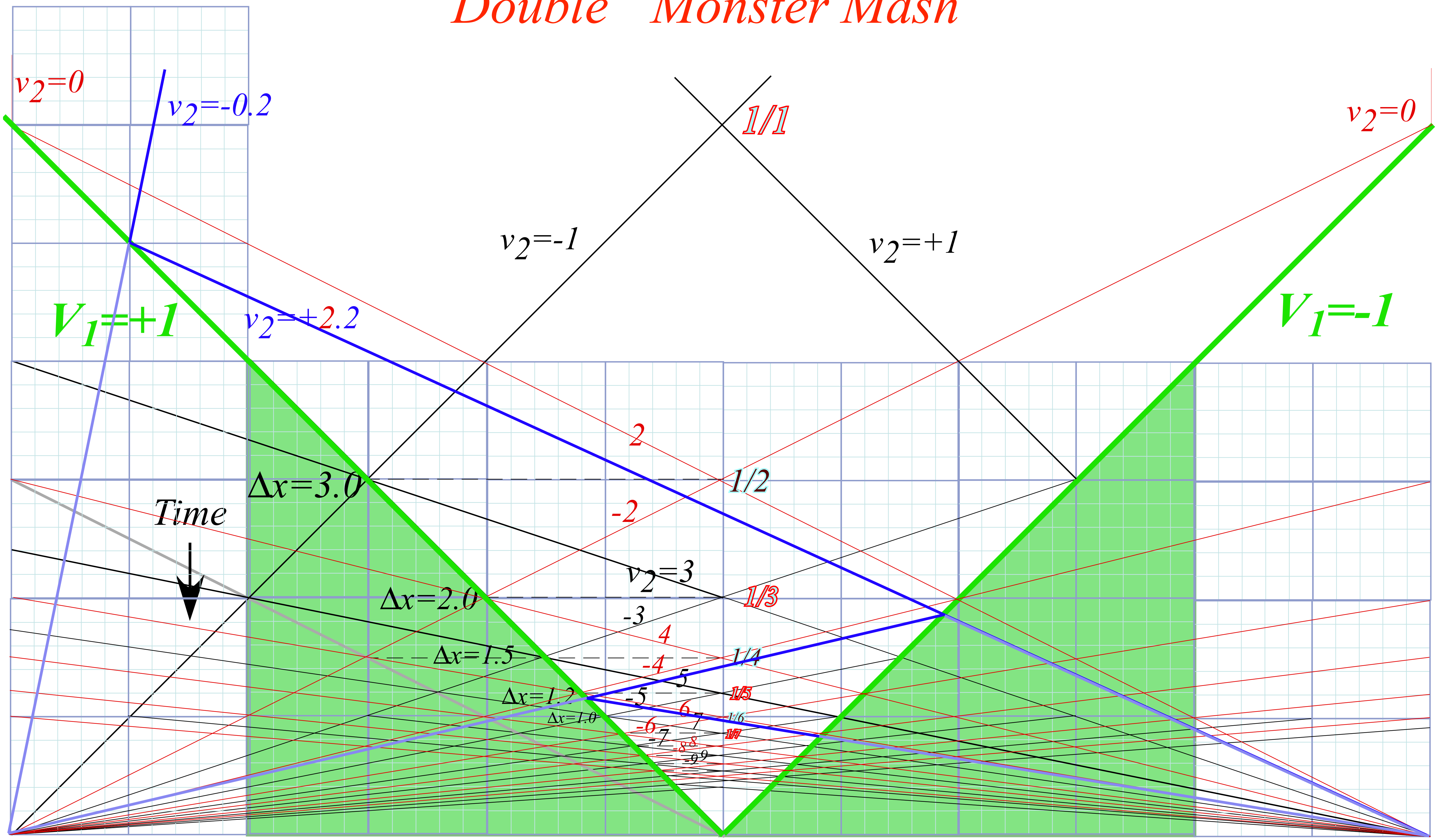
Heisenberg "Uncertainty" Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

Unit 1
Fig. 6.4

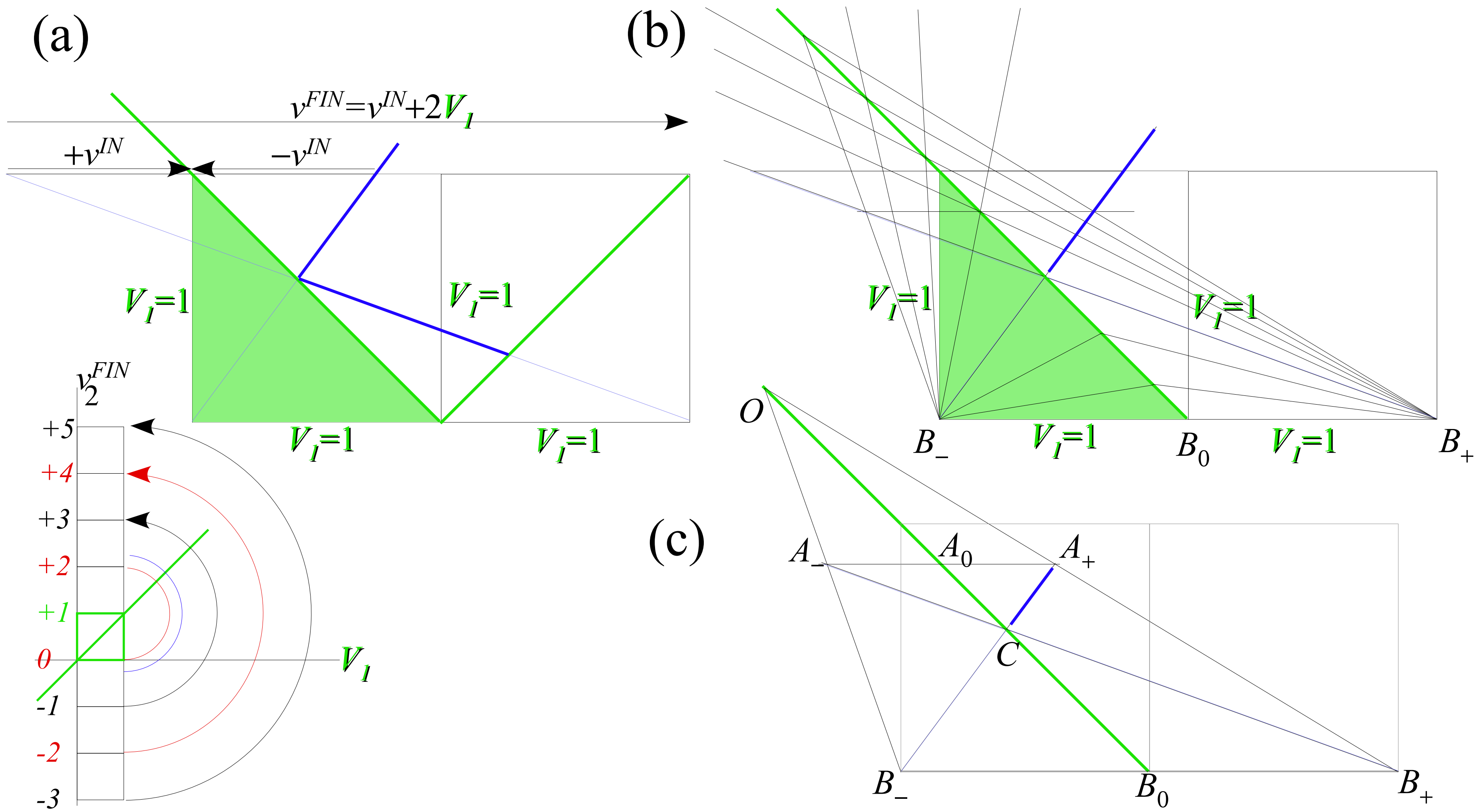
Double "Monster Mash"

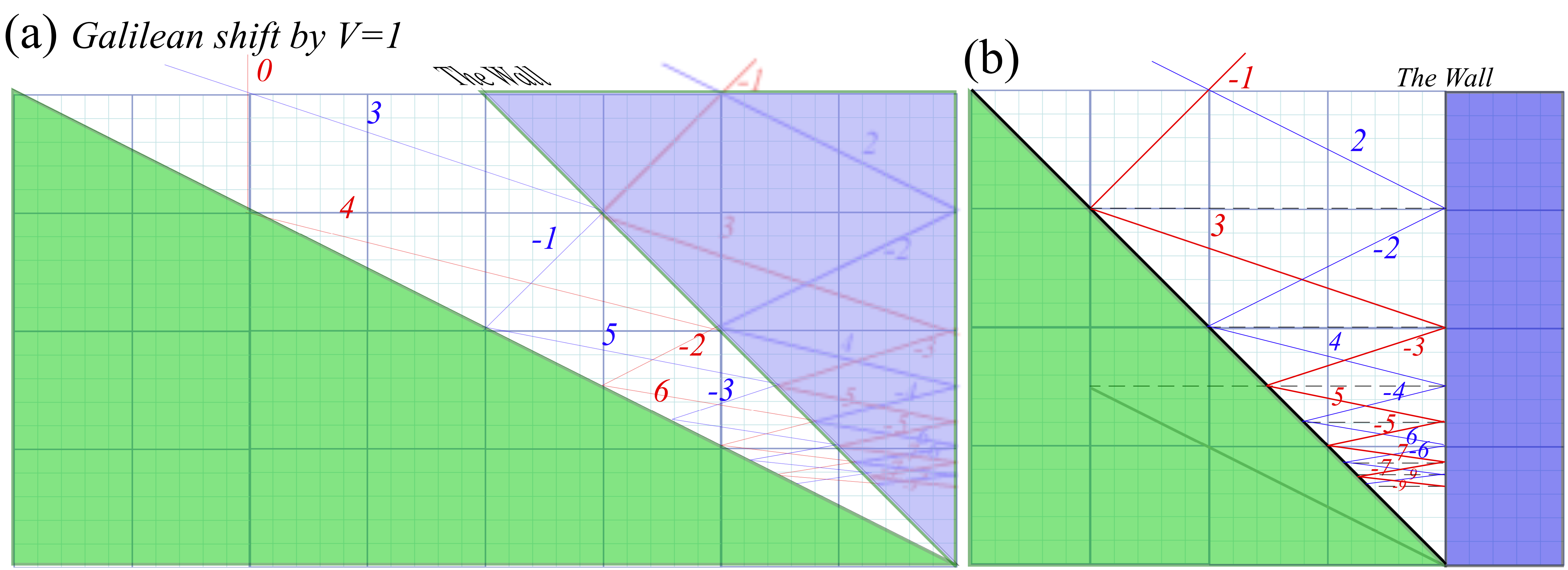


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





"Monster Mash" classical segue to Heisenberg action relations

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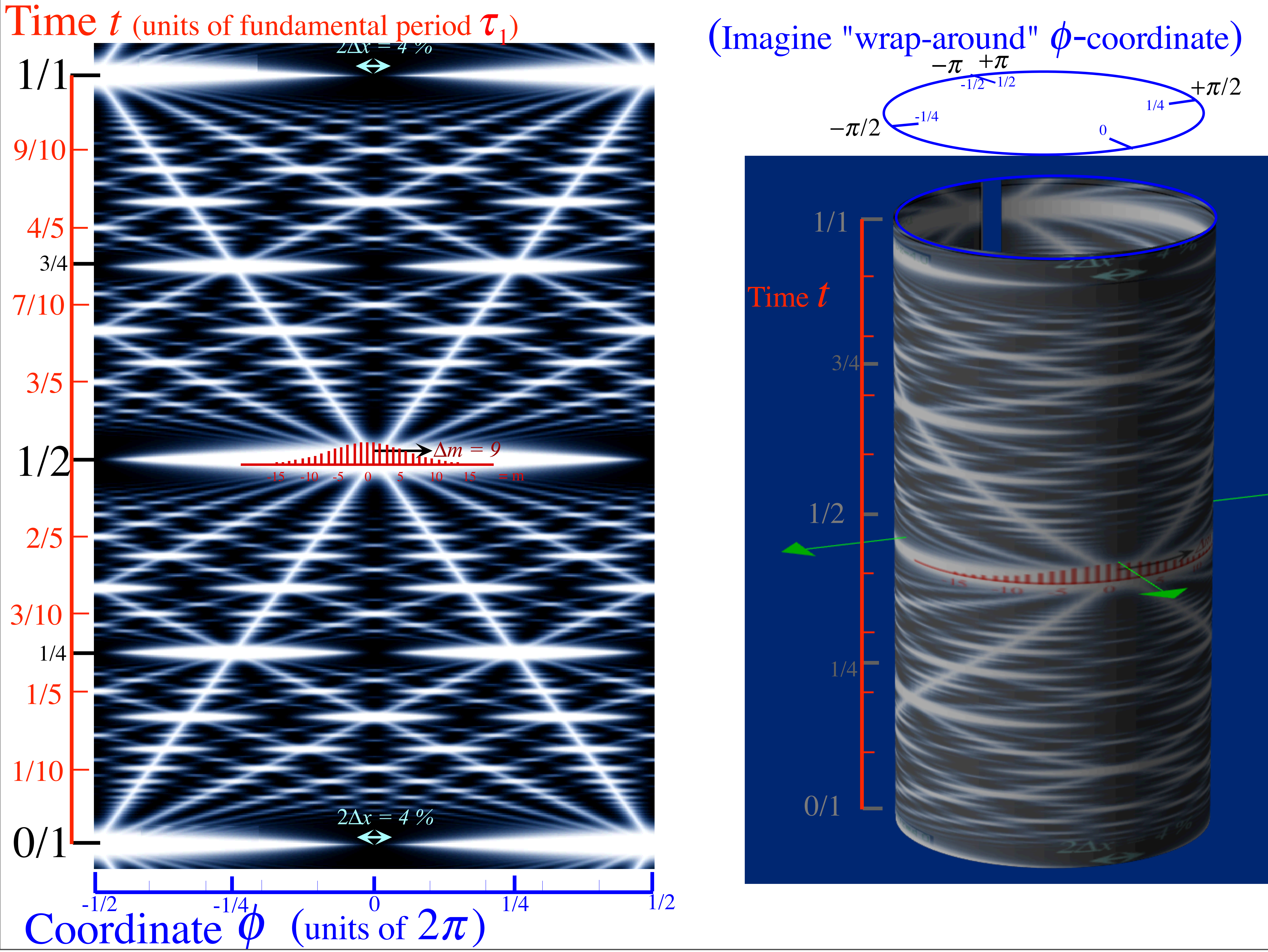
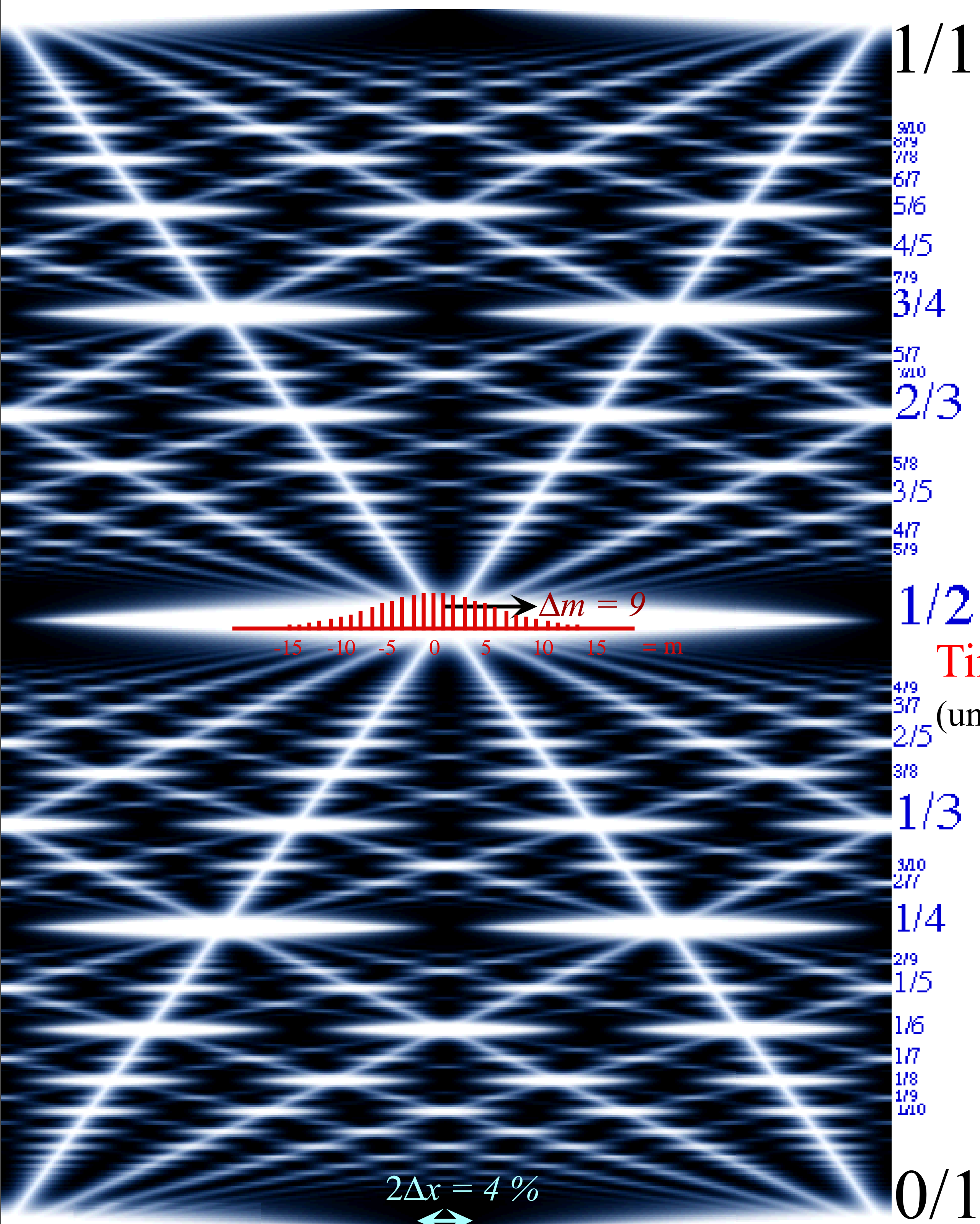

Time t (units of fundamental period τ_1)
$2\Delta x = 4\ \%$
1/1
9/10
4/5
3/4
7/10
3/5
1/2
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4\ \%$
-1/2 -1/4 0 1/4 1/2
Coordinate ϕ (units of 2π)
(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
$-\pi/2$
-1/4
0
1/1
Time t
3/4
1/2
1/4
0/1

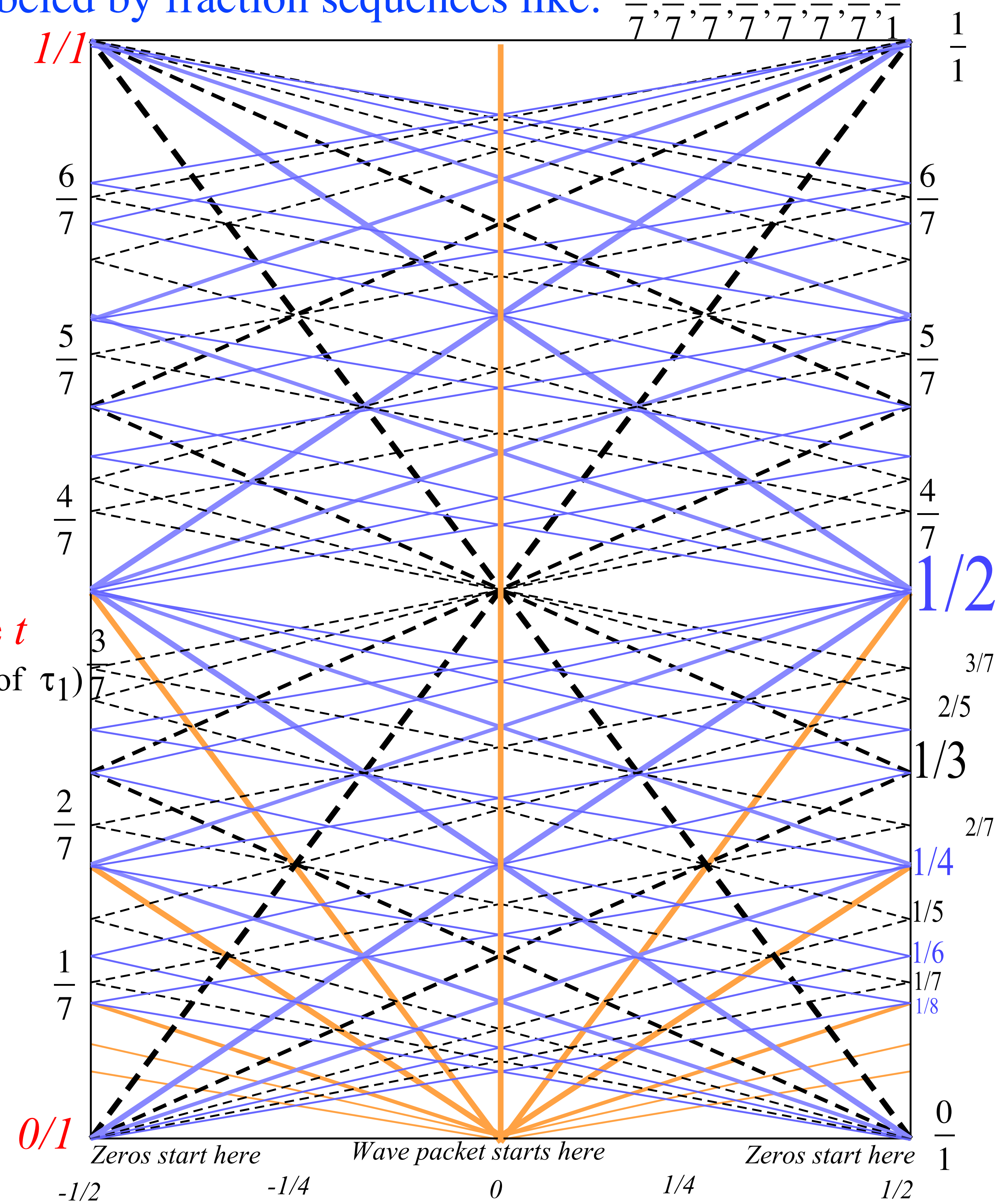
N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7},\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7},\frac{1}{1}$



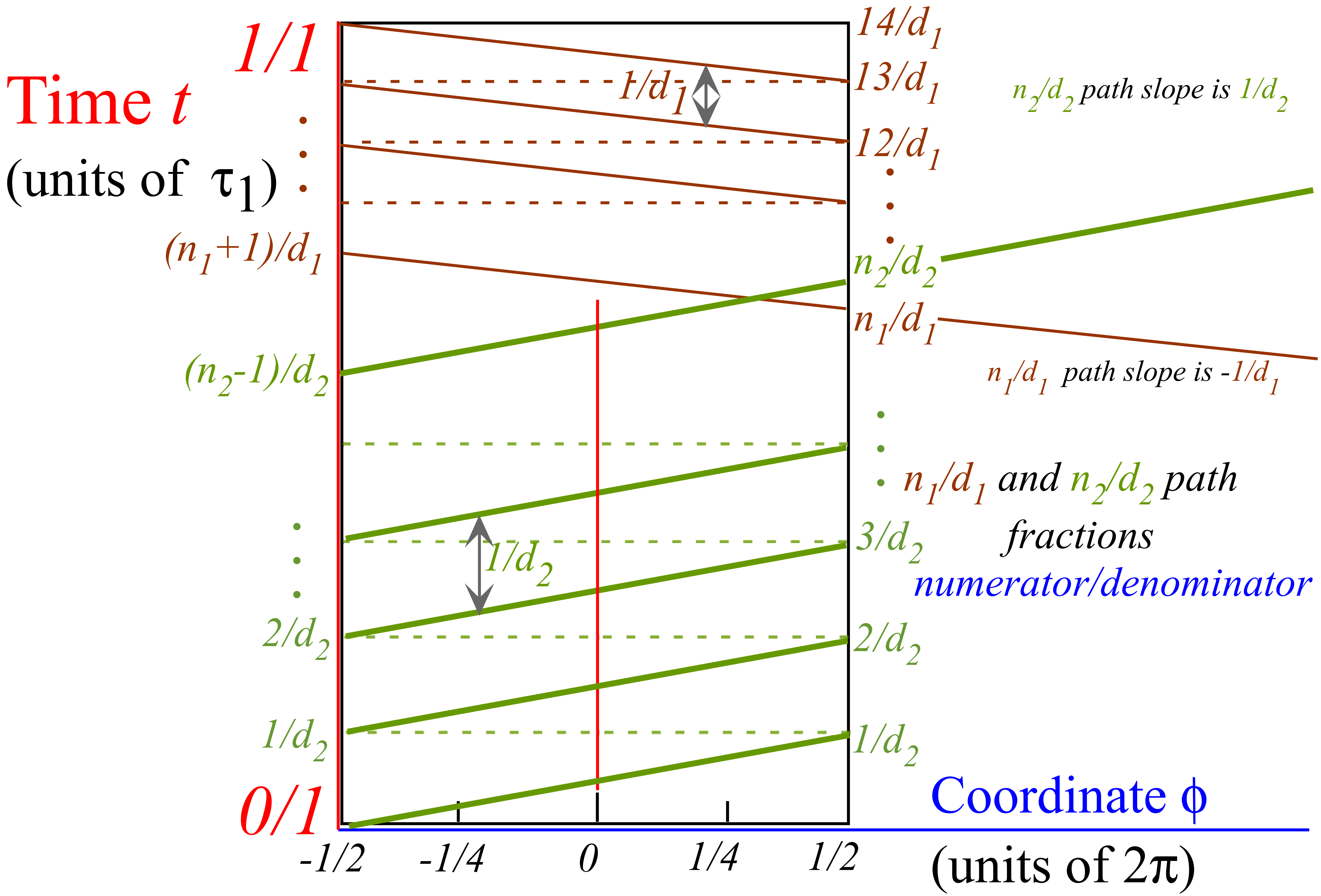
Time *t*
(units of τ_1)



Coordinate ϕ
(units of 2π)

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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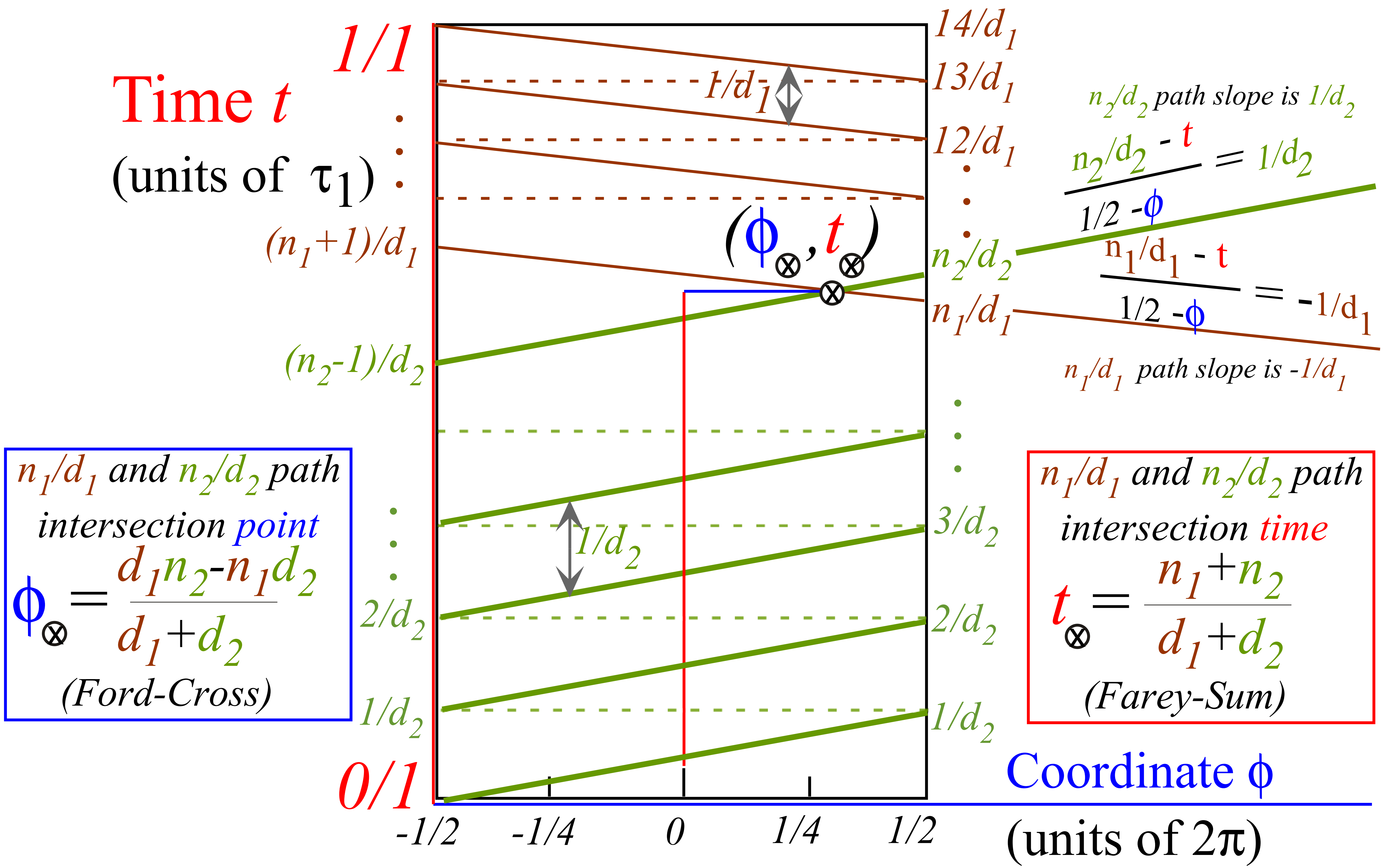

"Monster Mash" classical segue to Heisenberg action relations

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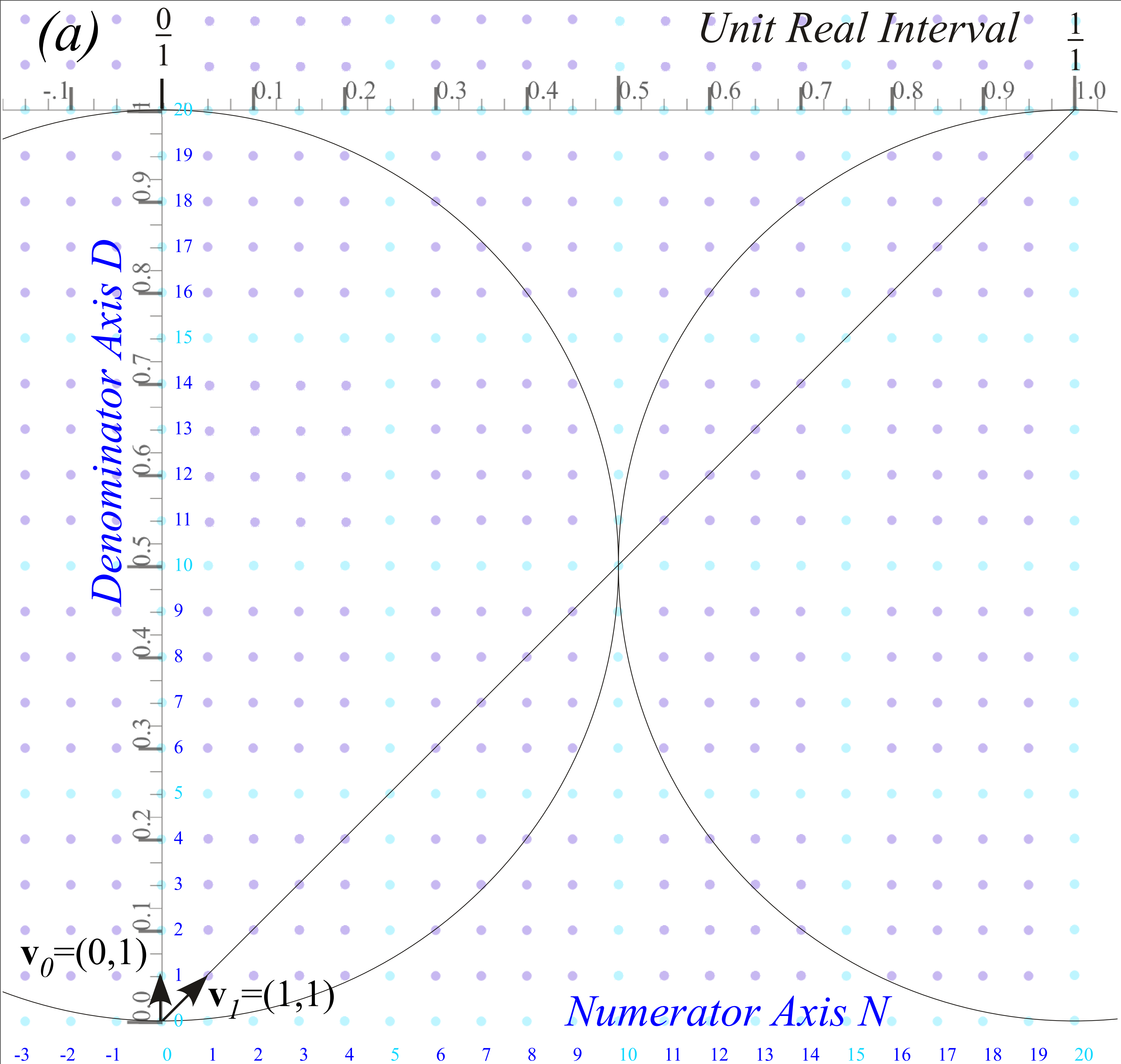
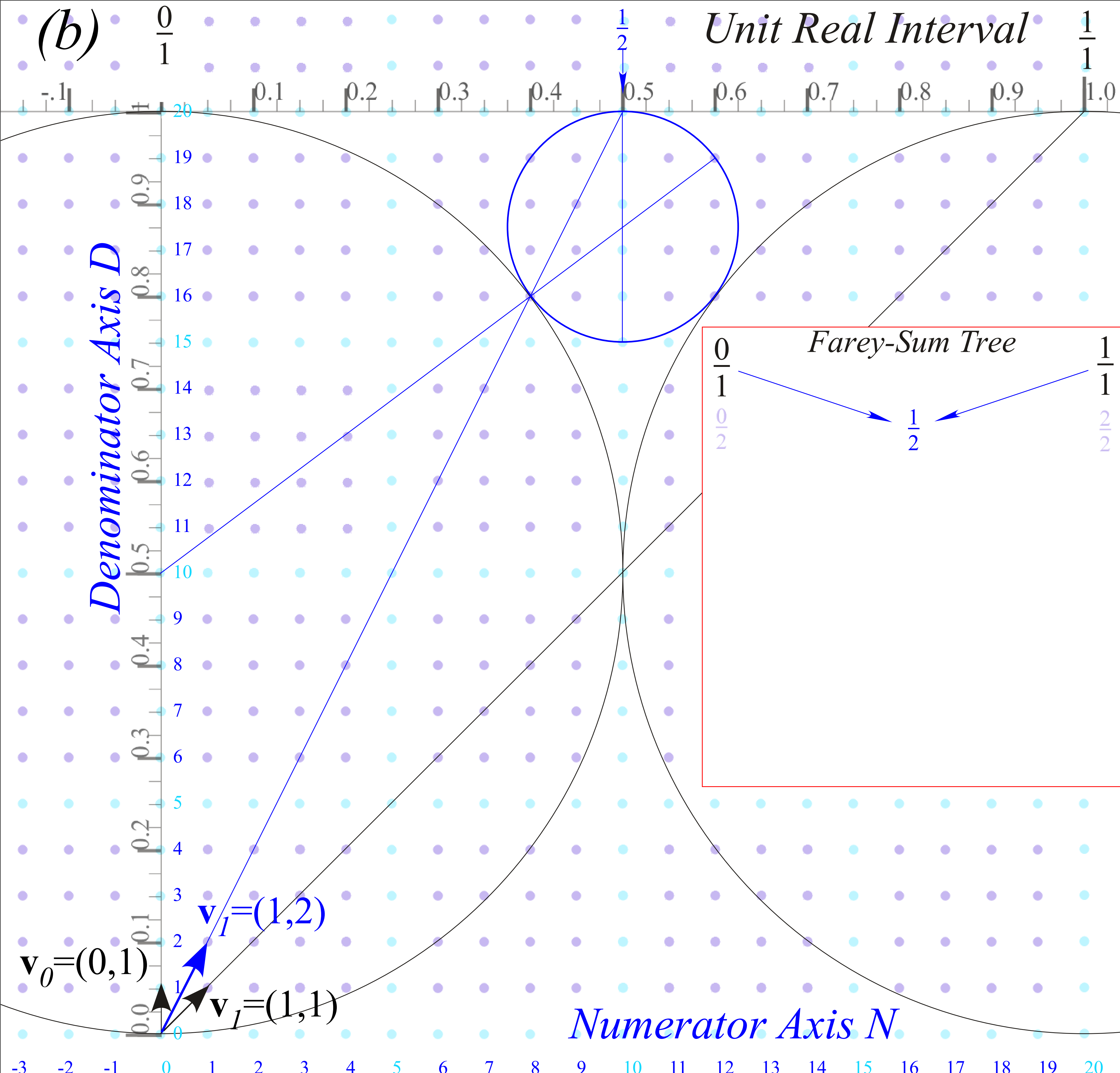


Farey Sum related to vector sum and *Ford Circles*

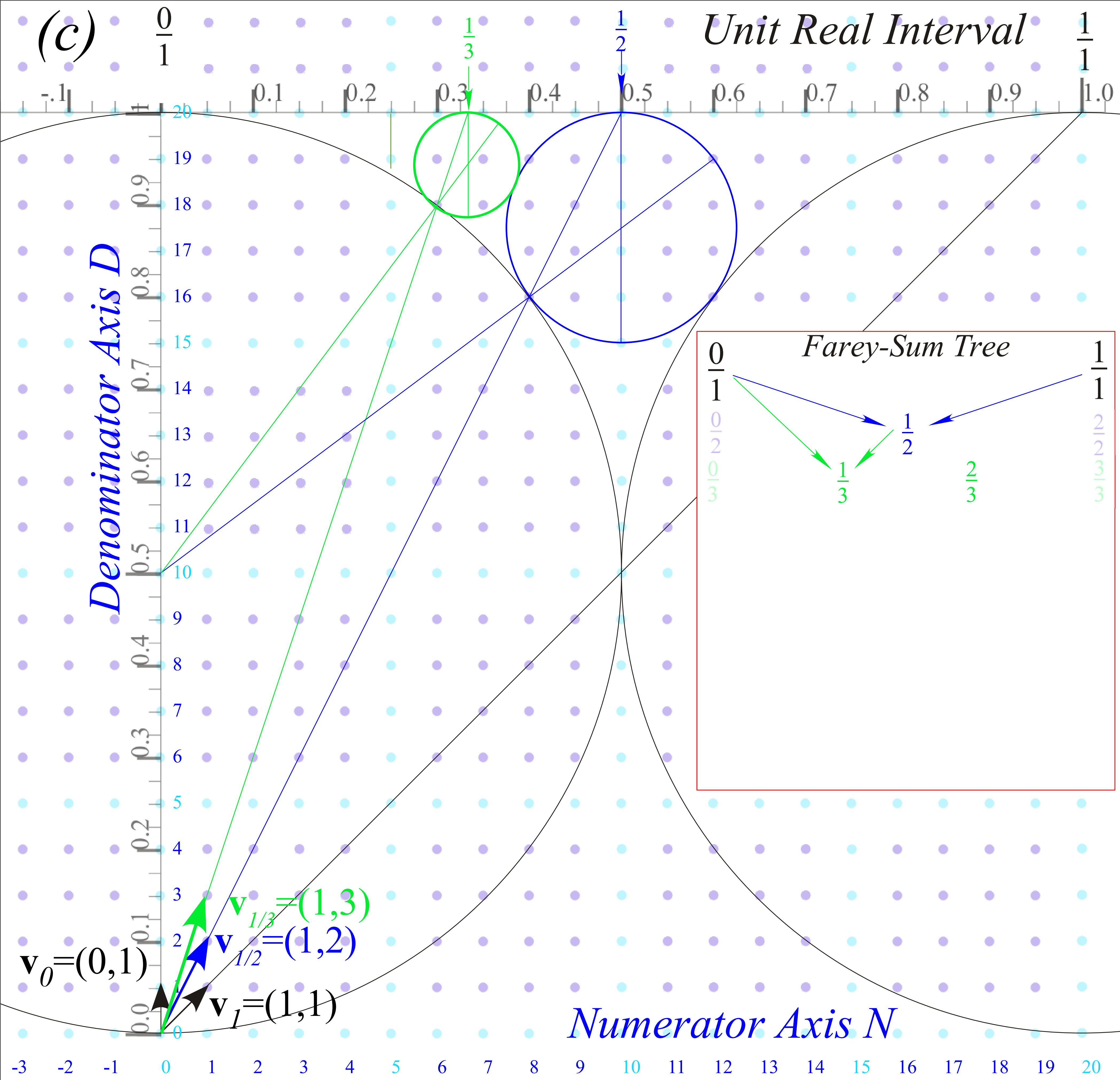
1/1-circle has diameter *1*



Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

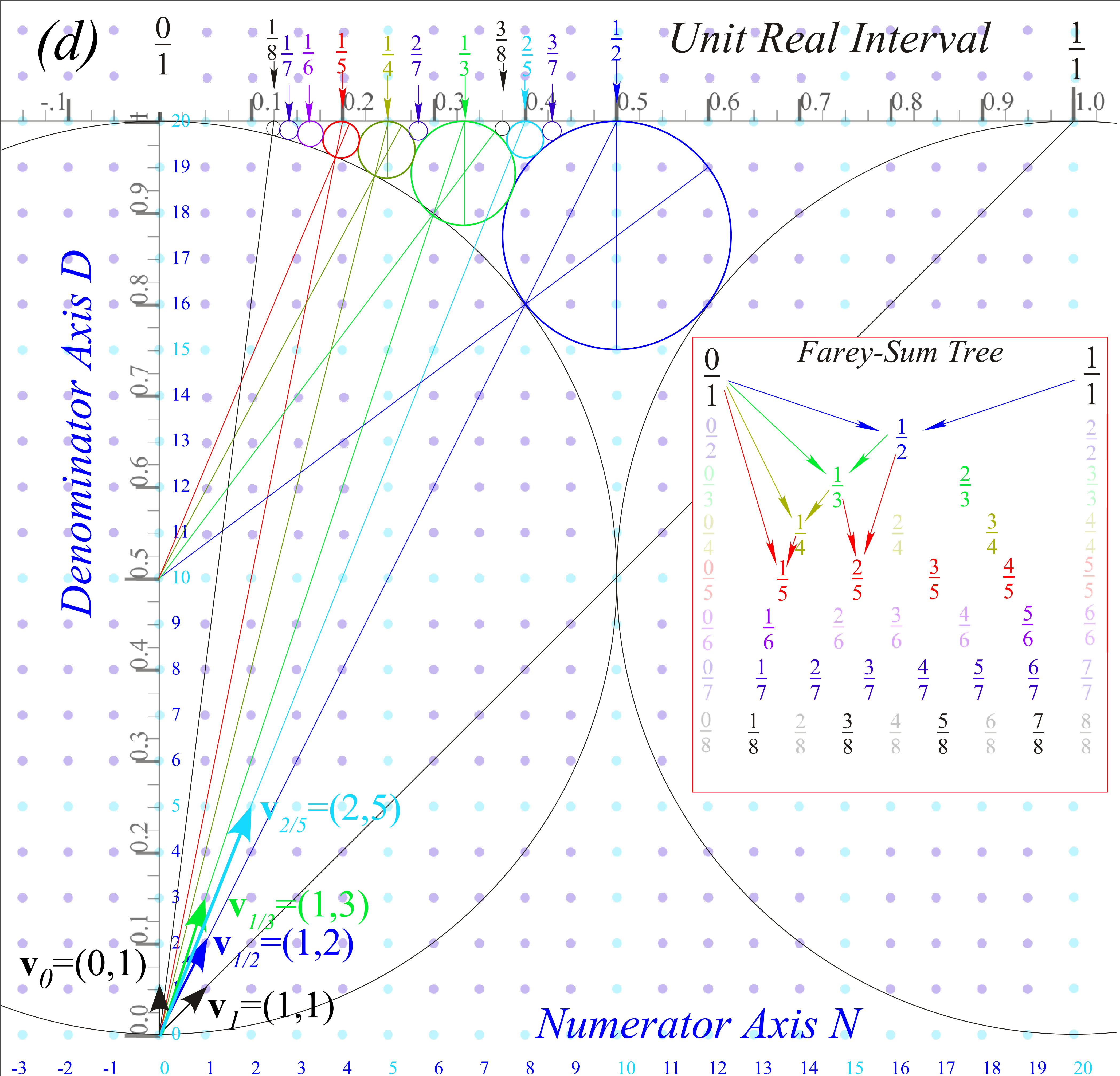
1/2-circle has diameter *1/2*^{2}*=1/4*



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

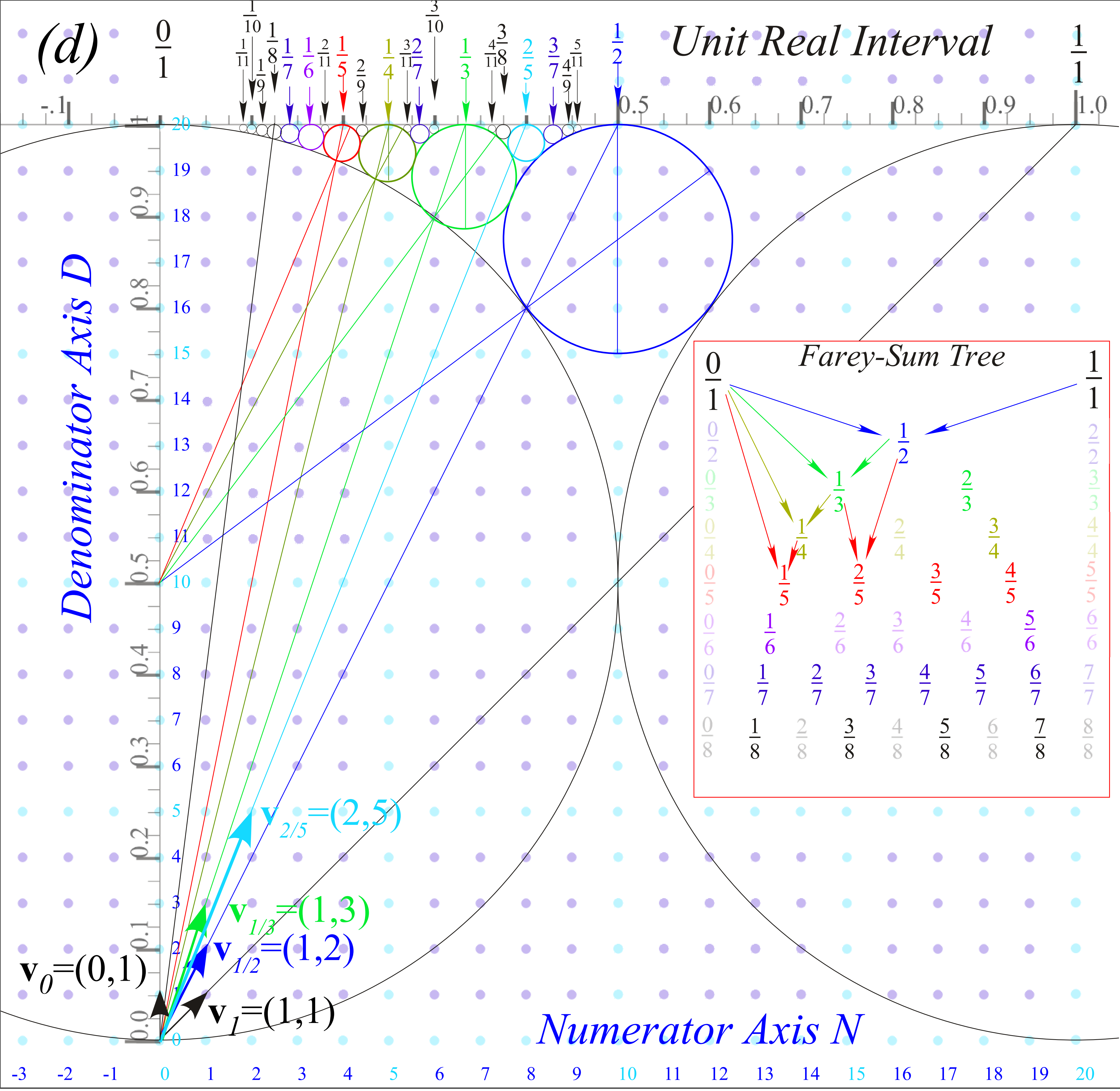


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

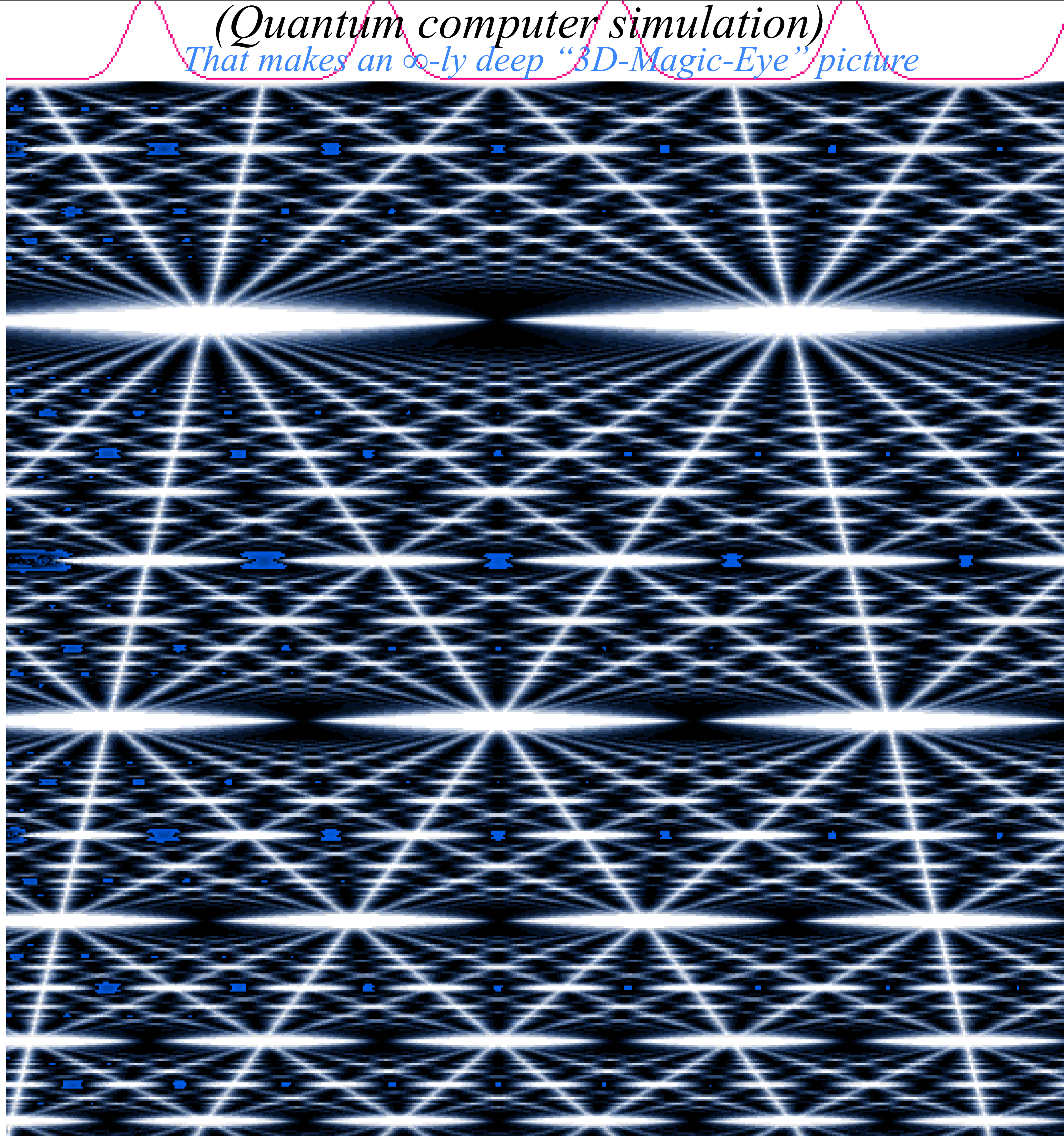
1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

See Homework problem 1.6.3: *Construct related spacetime case*

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



Geometric "Integration" (Converting Velocity data to Spacetime)

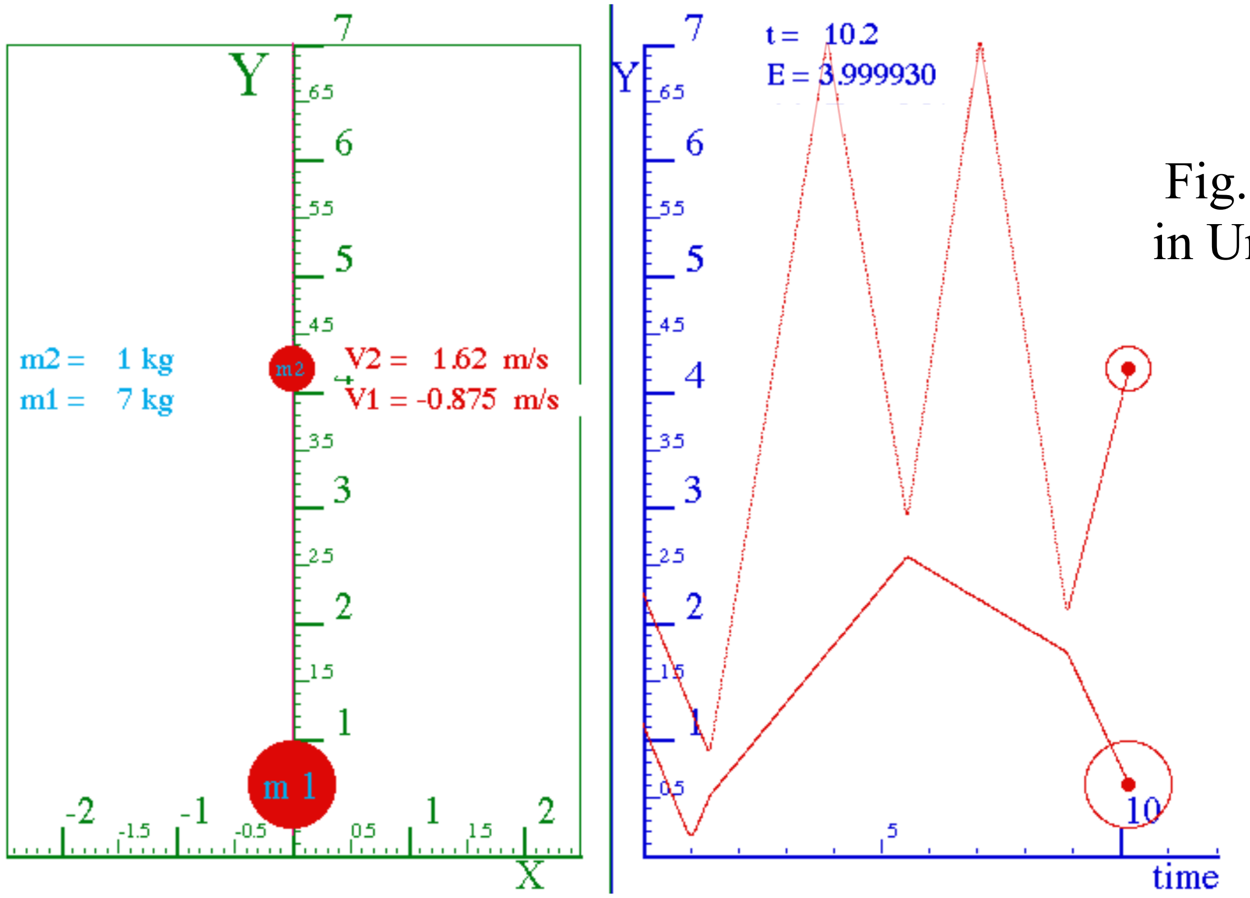


Fig. 4.8
in Unit 1

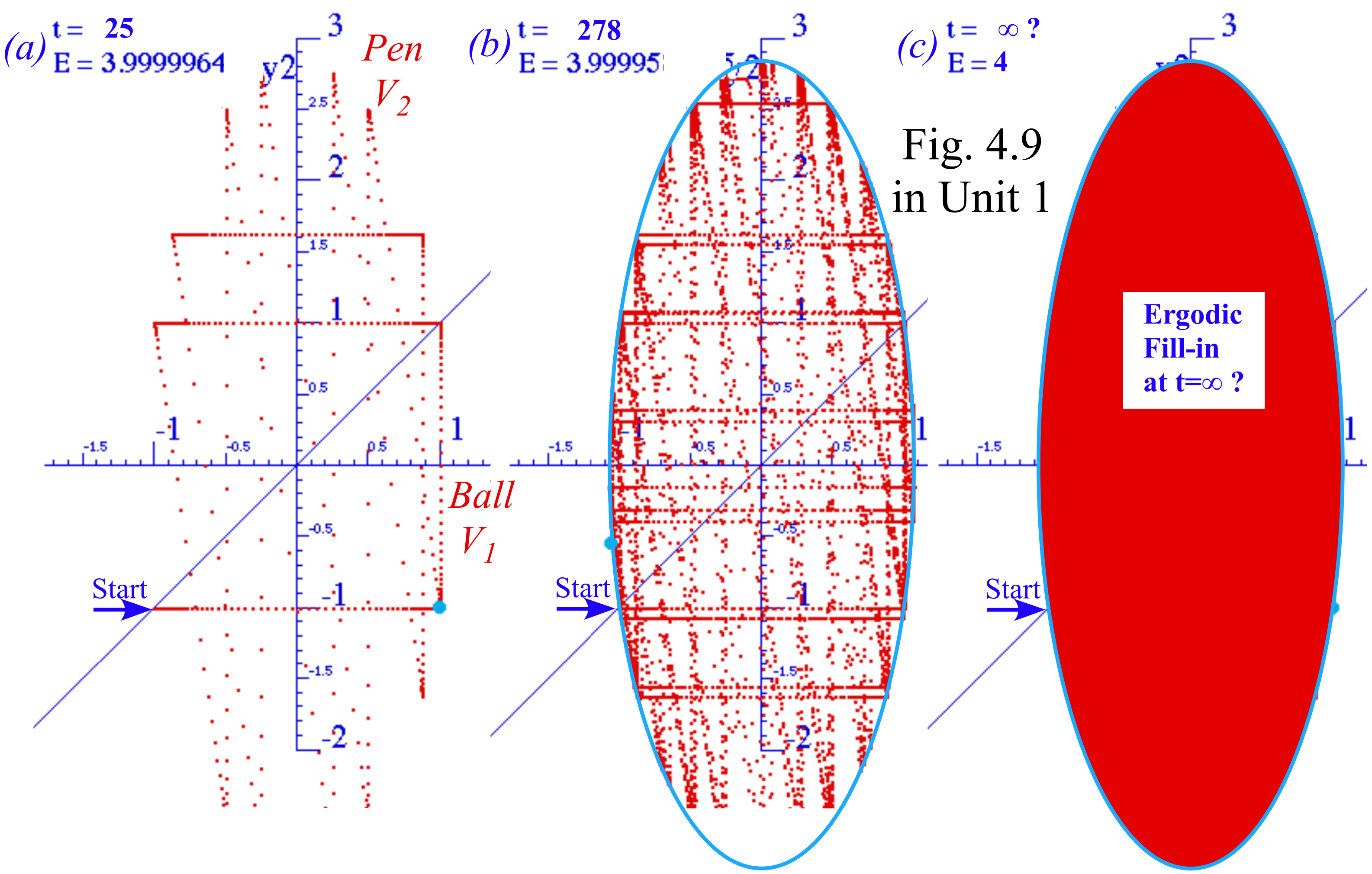
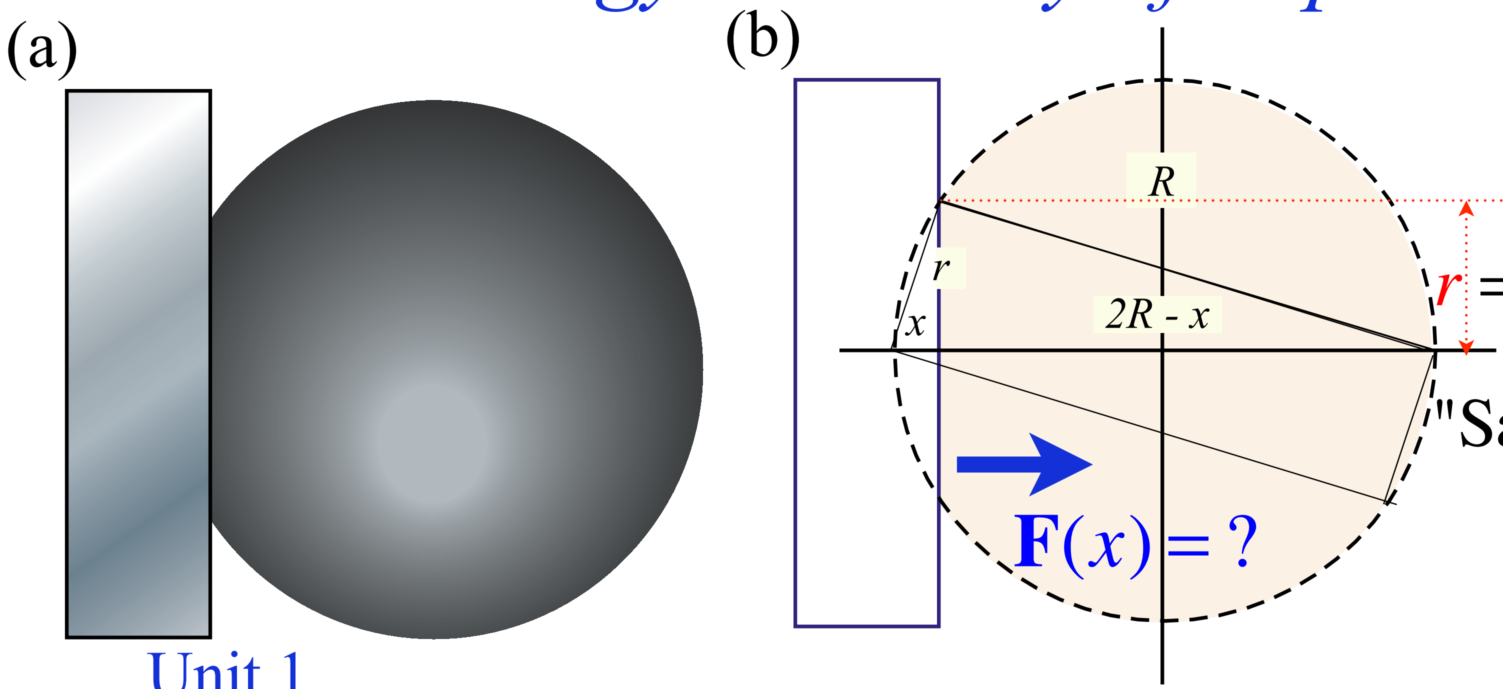


Fig. 4.9
in Unit 1

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)

$r = \sqrt{x(2R-x)}$ $\left(\approx \sqrt{2Rx} \quad for: \; x << R\right)$

"Sagittal†" Theorem due to Thales' geometry

† "bow"

If superball was a balloon its bounce force law would be linear *F=-k·x* *(Hooke Law)*

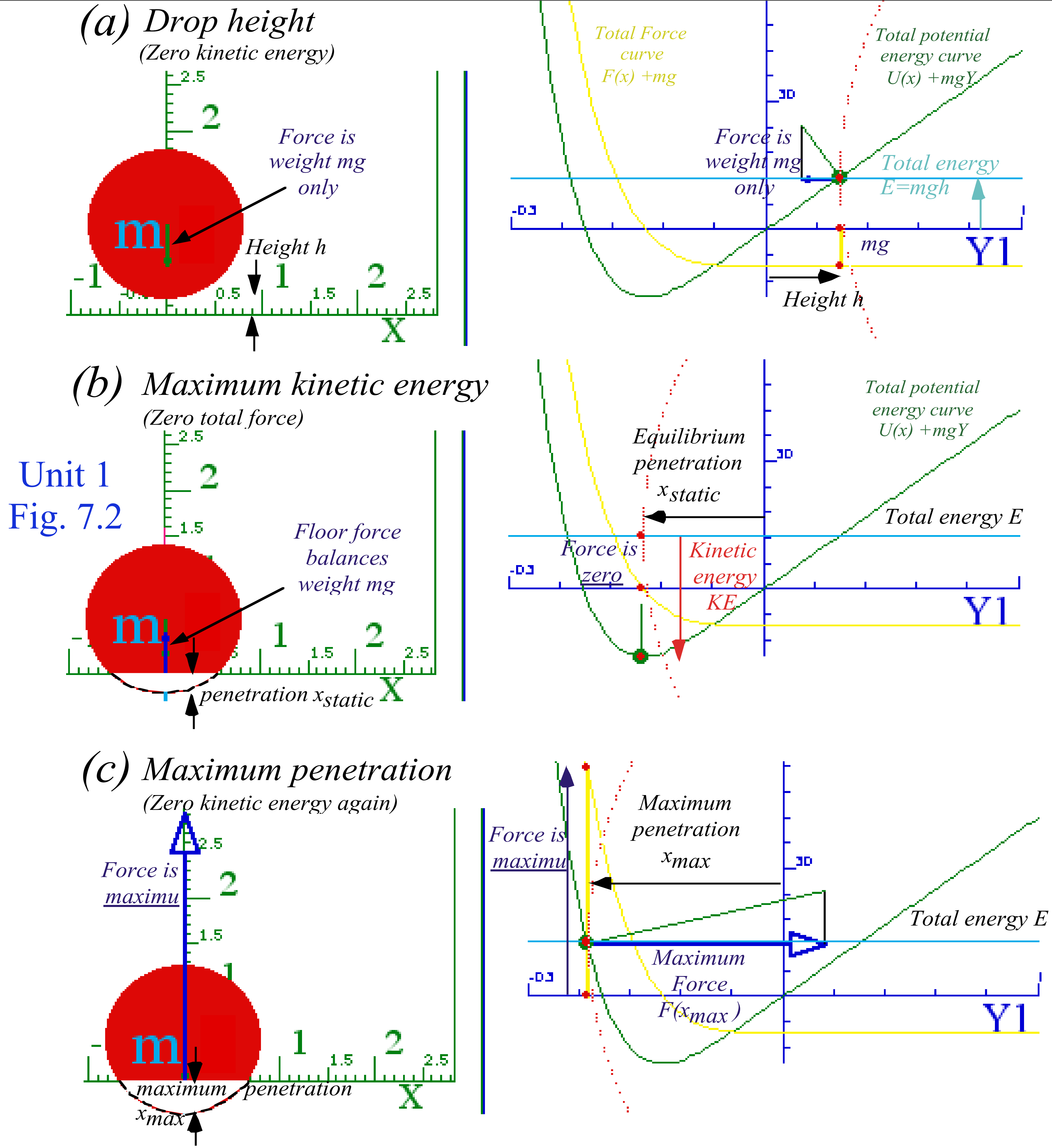
$$F_{balloon}(x) = \overset{(Pressure)}{P} \cdot A = P \cdot \pi r^2 \approx 2\pi PRx$$

Instead superball force law depends on bulk modulus and is non-linear *F~ x^p? +?* *(Power Law?)*

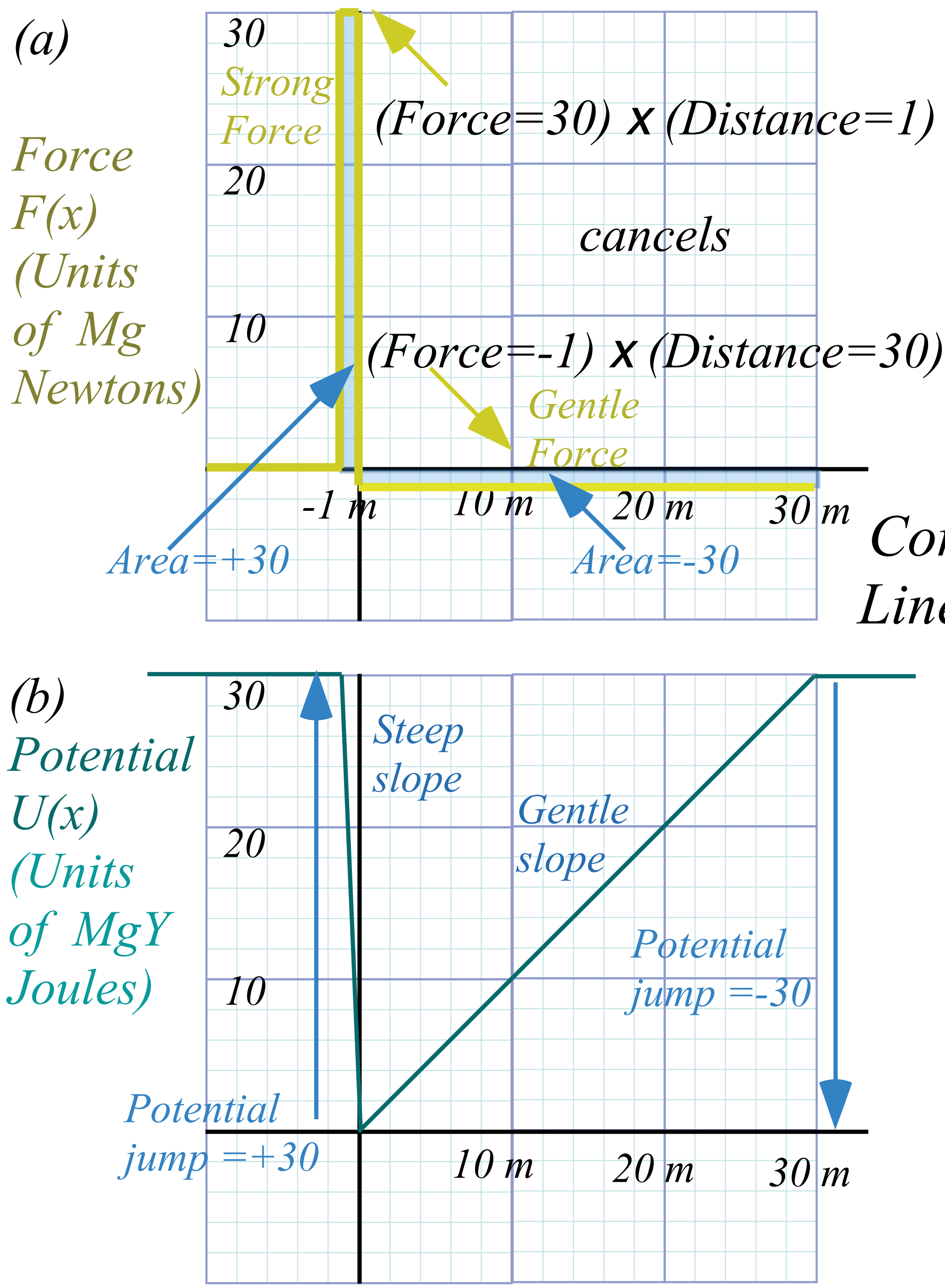
$$Volume(X) = \int_0^X \pi r^2\, dx = \int_0^X \pi x(2R-x)\, dx = \int_0^X 2R\pi x\, dx - \int_0^X \pi x^2\, dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (for: X << R) \\ \frac{4}{3}\pi R^3 & (for: X = 2R) \end{cases}$$

It also depends on velocity $\dot{x}=\frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by *"Project-Ball*"*

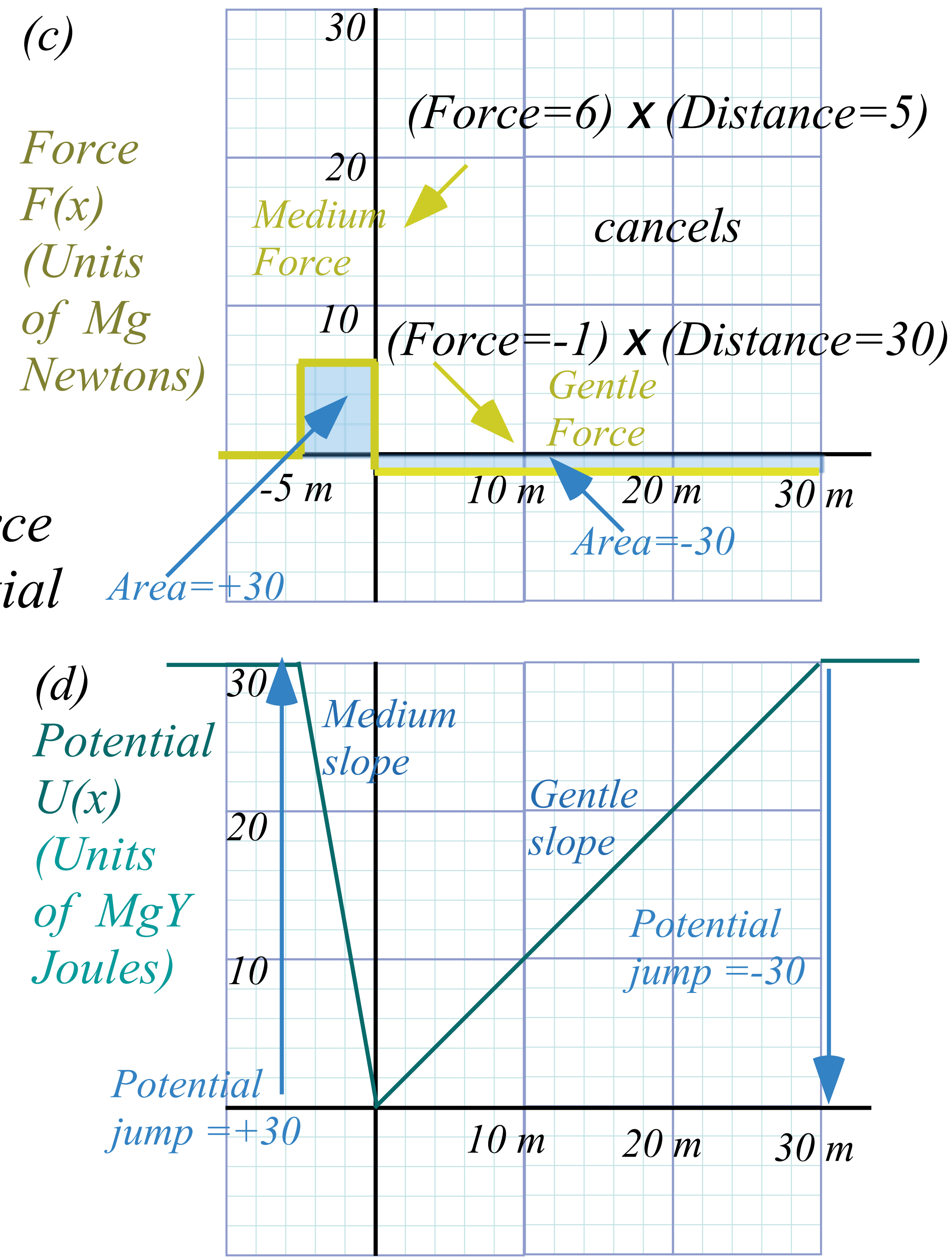
* *Am. J. Phys.* **39**, 656 (1971)



(a) Drop height
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Force is weight mg only
Total energy E=mgh
mg
Height h
Y1
(b) Maximum kinetic energy
(Zero total force)
Unit 1
Fig. 7.2
Floor force balances weight mg
penetration x_{static}
Equilibrium penetration x_{static}
Total potential energy curve U(x) +mgY
Total energy E
Force is zero
Kinetic energy KE
Y1
(c) Maximum penetration
(Zero kinetic energy again)
Force is maximu
maximum penetration x_{max}
Force is maximu
Maximum penetration x_{max}
Total energy E
Maximum Force $F(x_{max})$
Y1



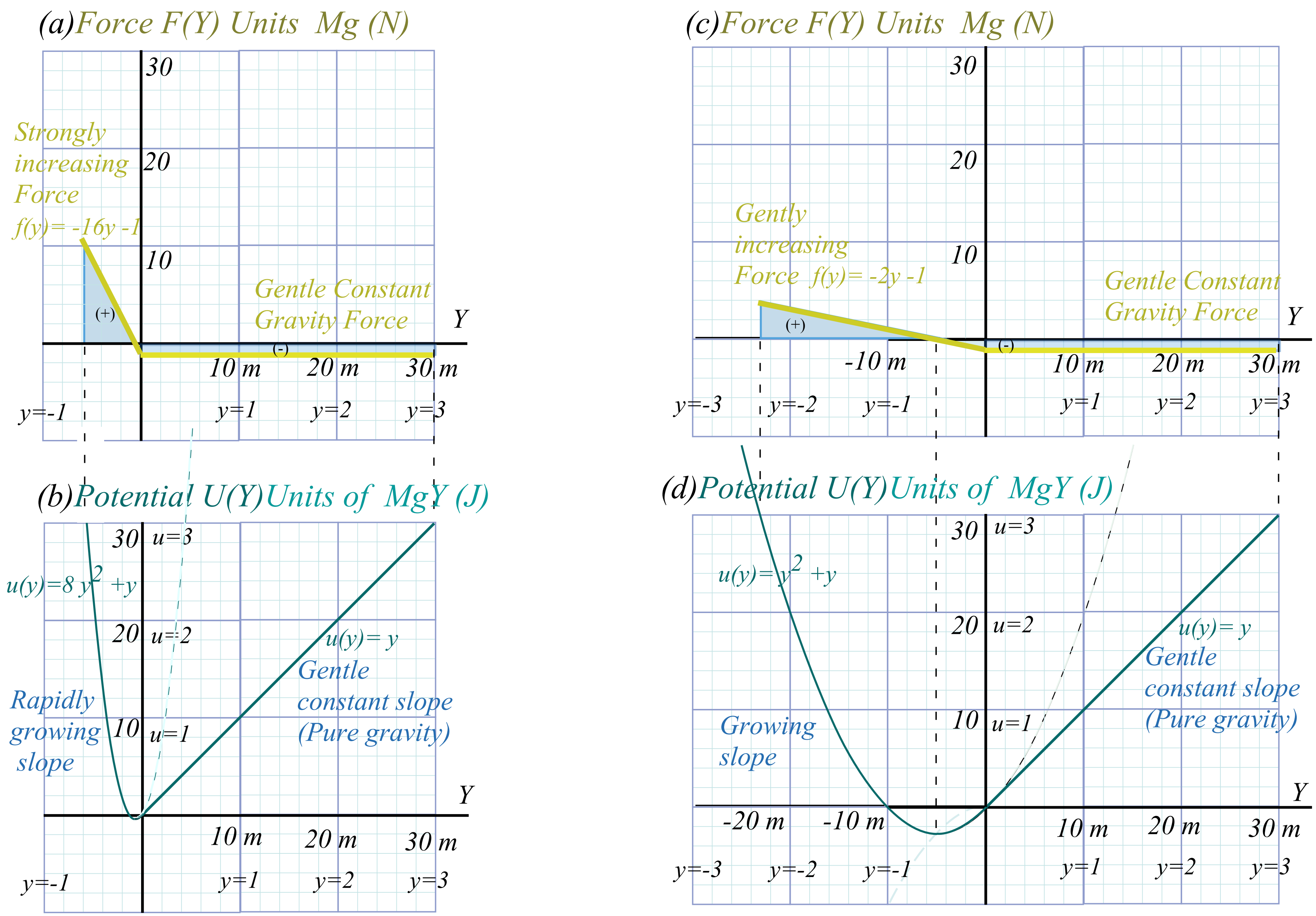
Constant Force Linear Potential Models:
$F(x)=k$, $U(x)=-kx$



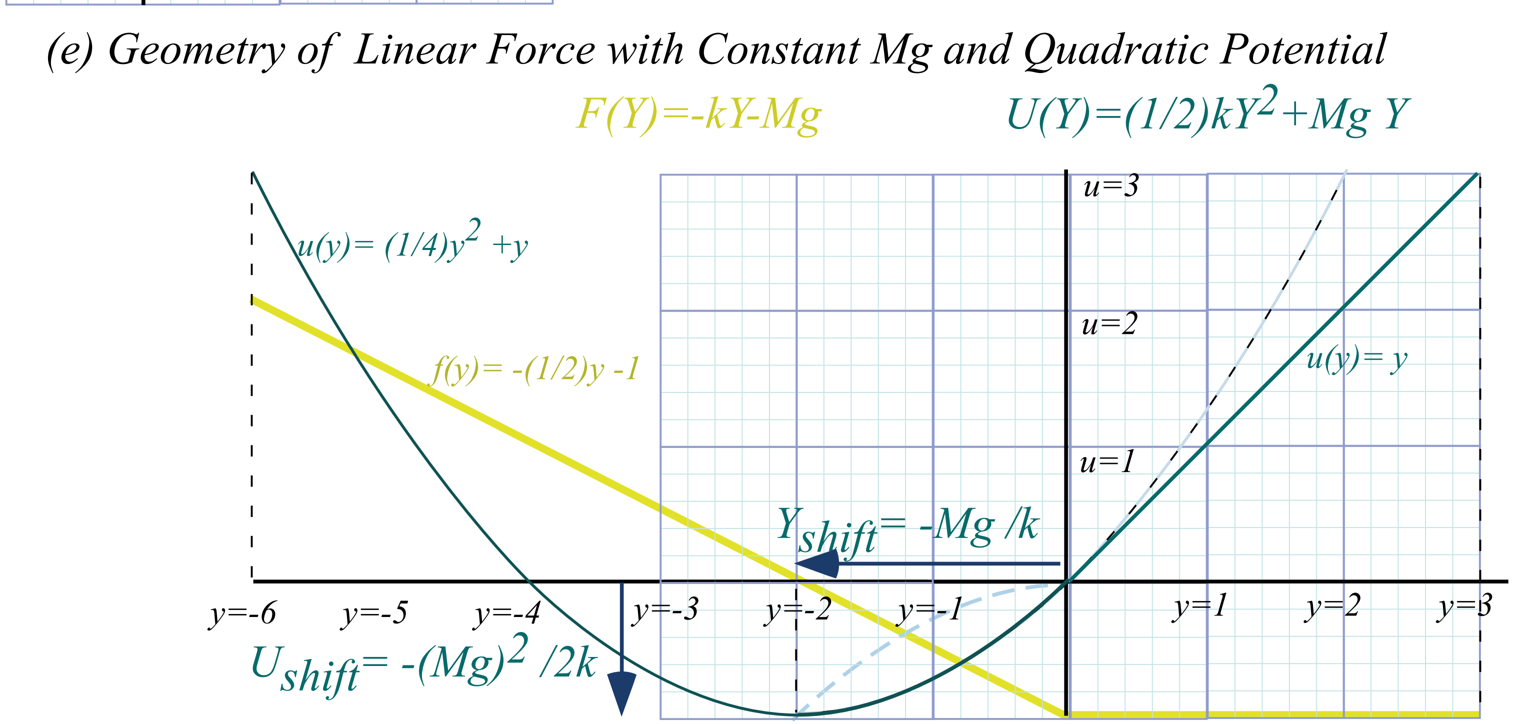
Unit 1
Fig. 7.3

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

$$Impulse = P = \int F(t)\,dt = Momentum\ acquired = Area\ of\ F(t) = P(t) \qquad F(t) = \frac{dP(t)}{dt}$$



Unit 1
Fig. 7.4

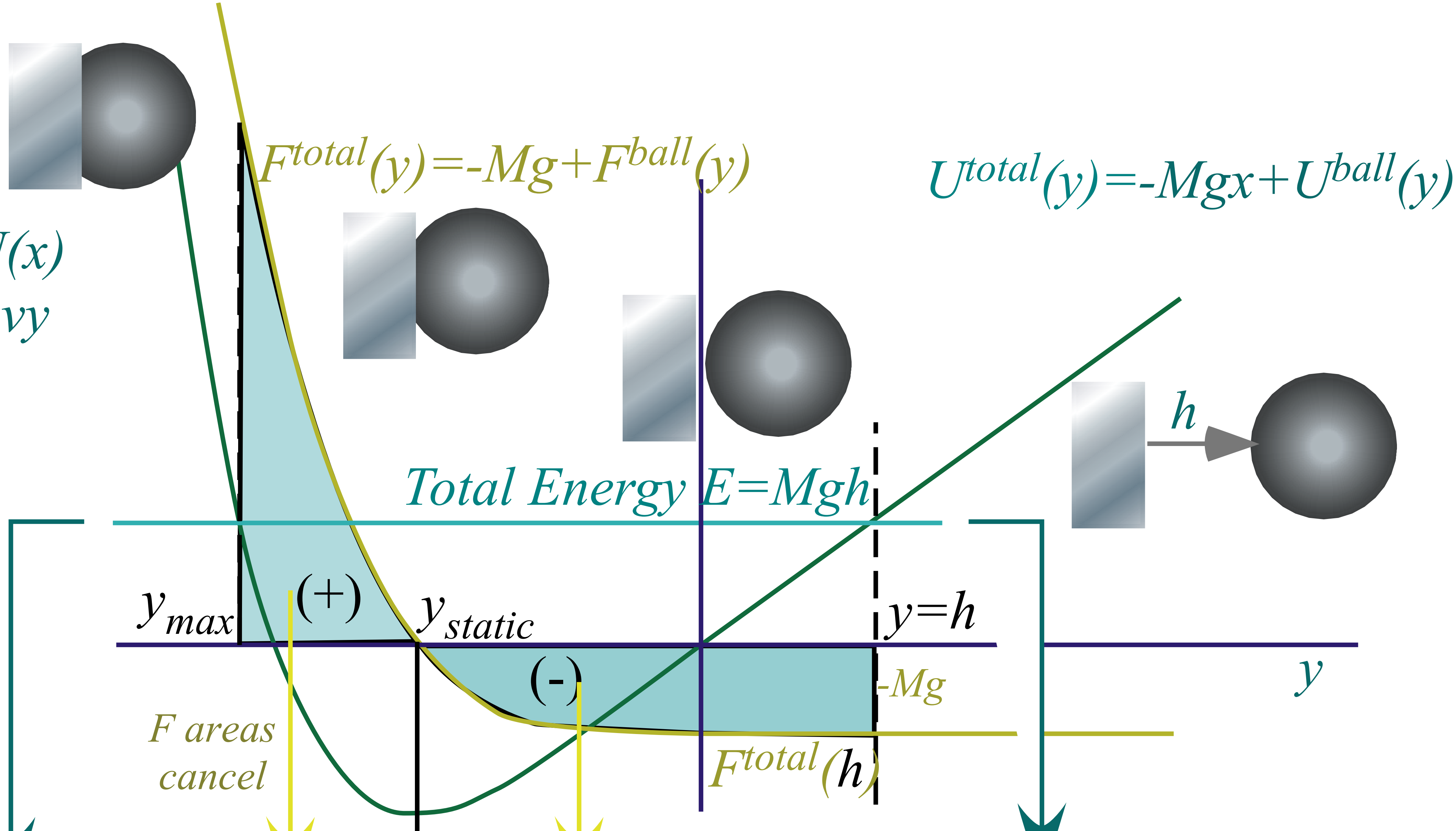


$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

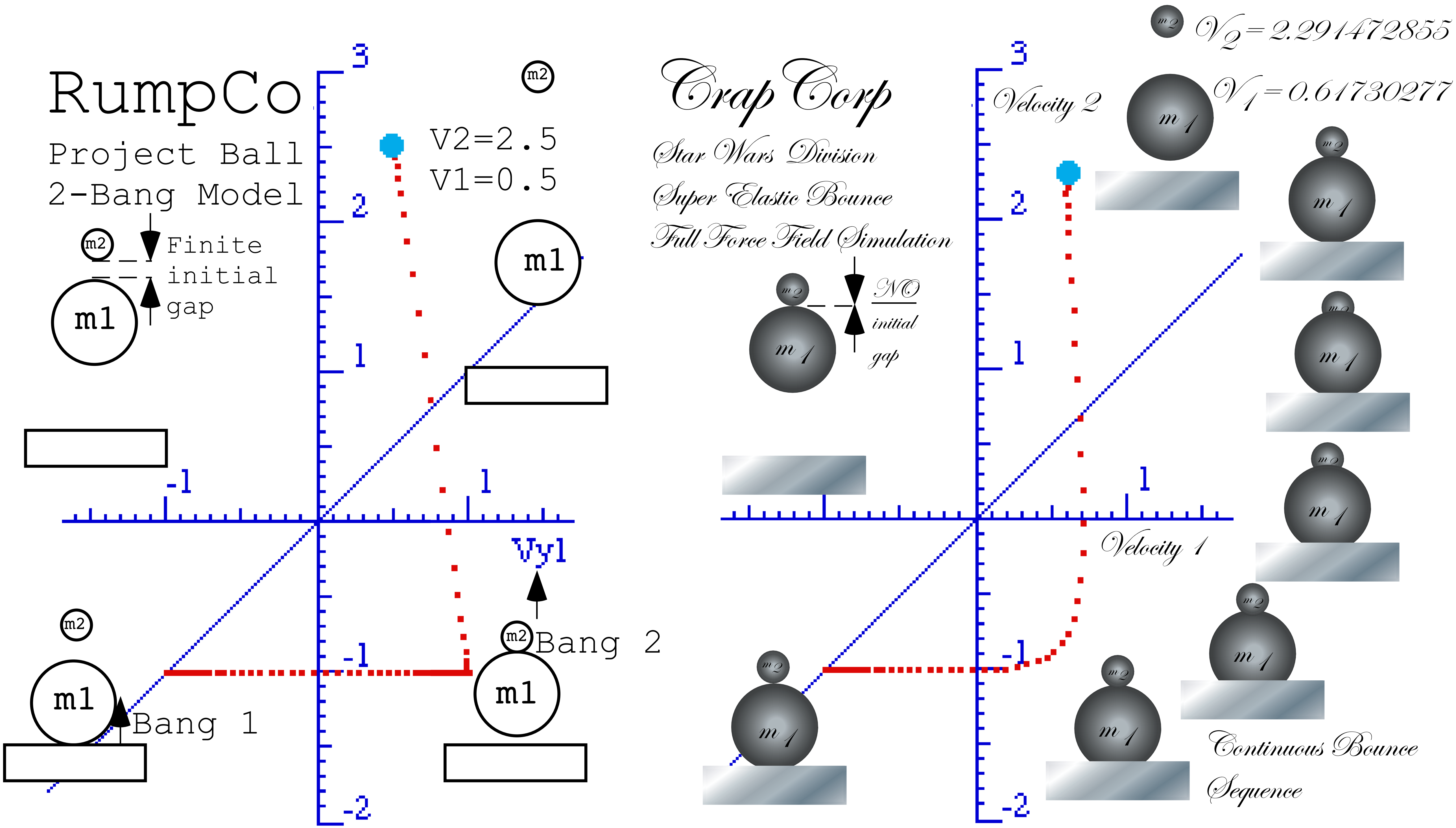
$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg\ y & (y \geq 0) \\ Mg\ y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

Force F(x) and Potential U(x) for soft heavy non-linear superball

Unit 1
Fig. 7.5

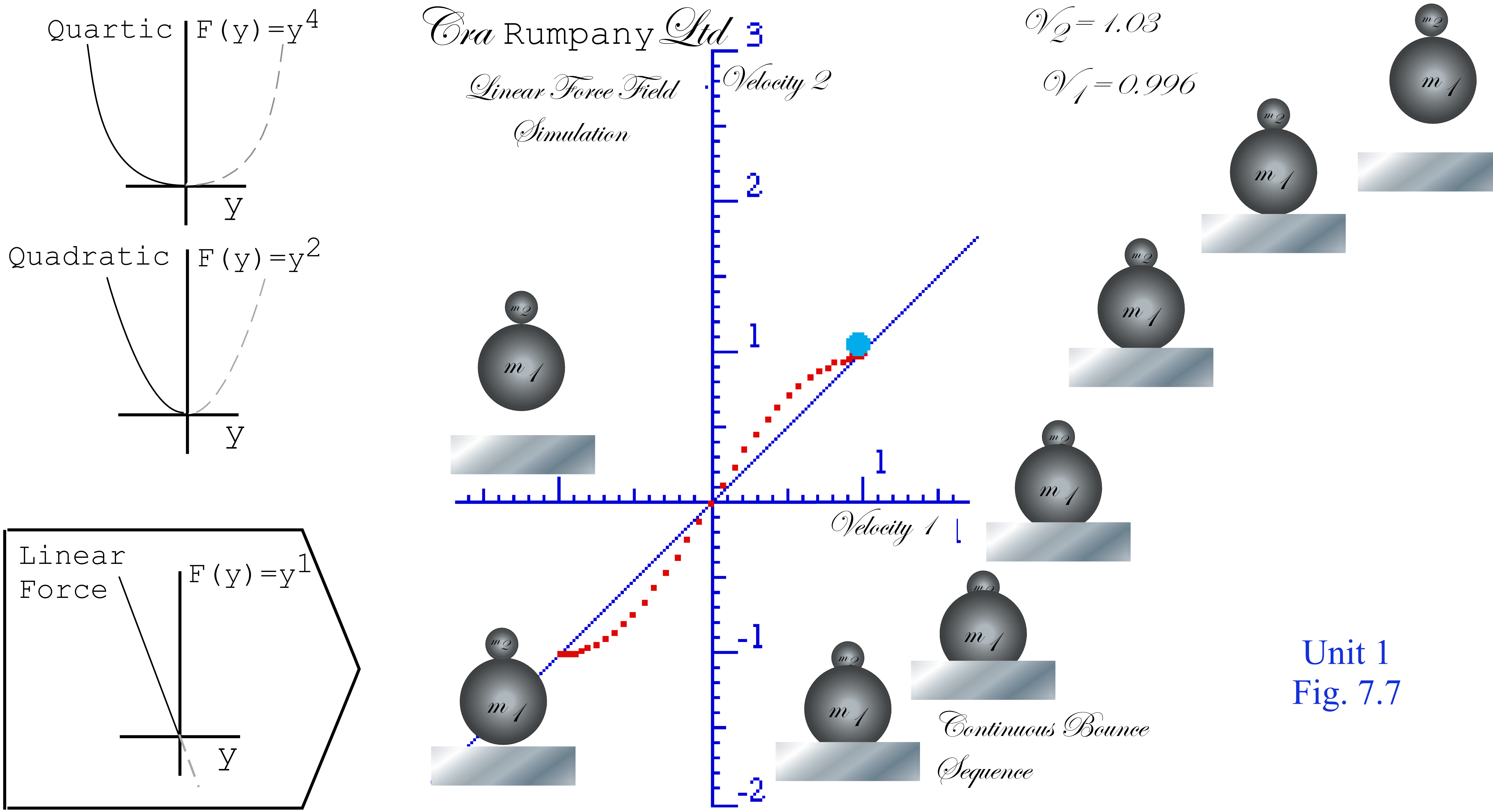


$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) = U(h) = E$$

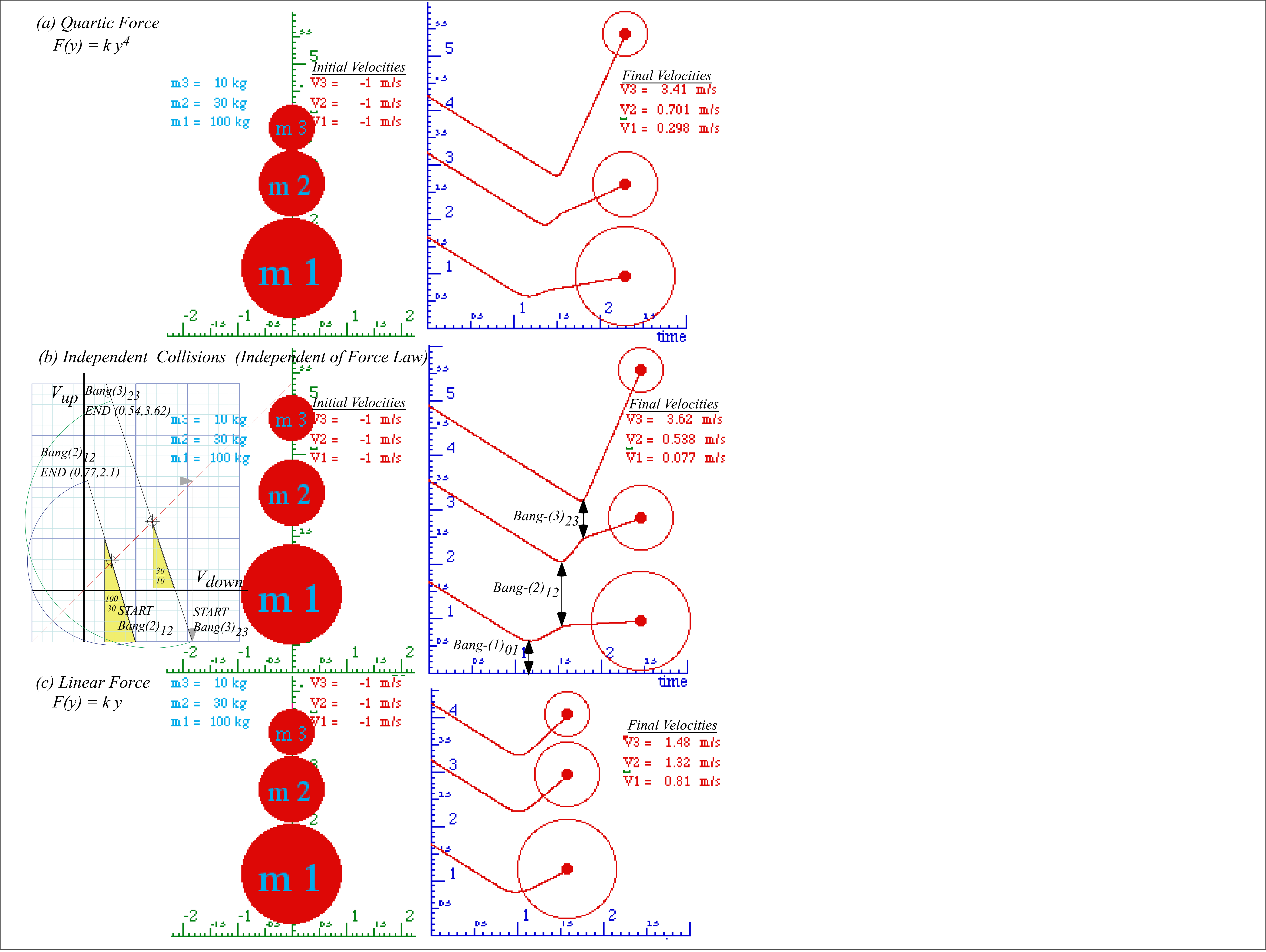


RumpCo
Project Ball
2-Bang Model
Finite
initial
gap
m1
m2
V2=2.5
V1=0.5
Vy1
Bang 1
Bang 2
Crap Corp
Star Wars Division
Super Elastic Bounce
Full Force Field Simulation
NO
initial
gap
Velocity 2
Velocity 1
V2=2.291472855
V1=0.61730277
Continuous Bounce
Sequence

Unit 1
Fig. 7.6

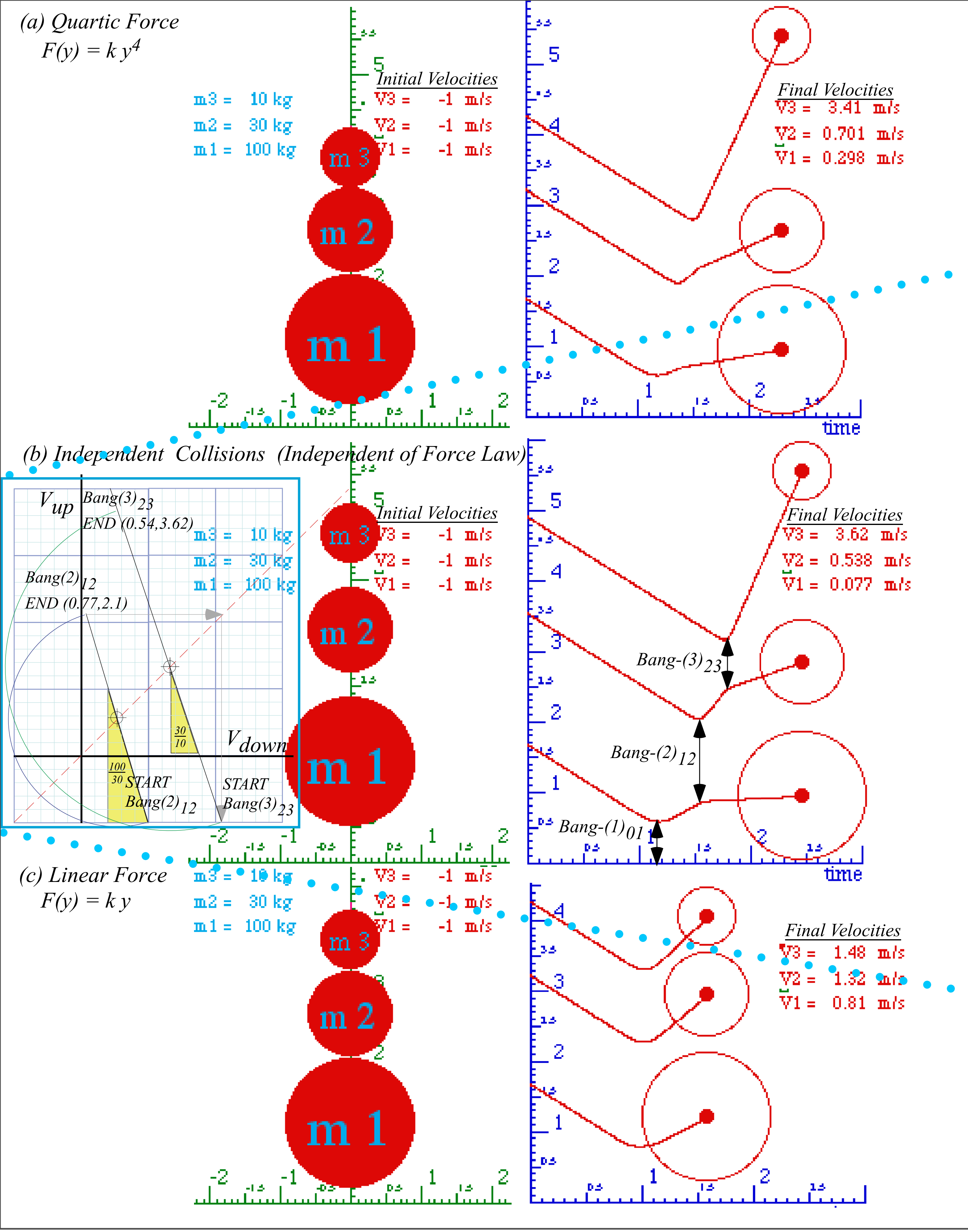


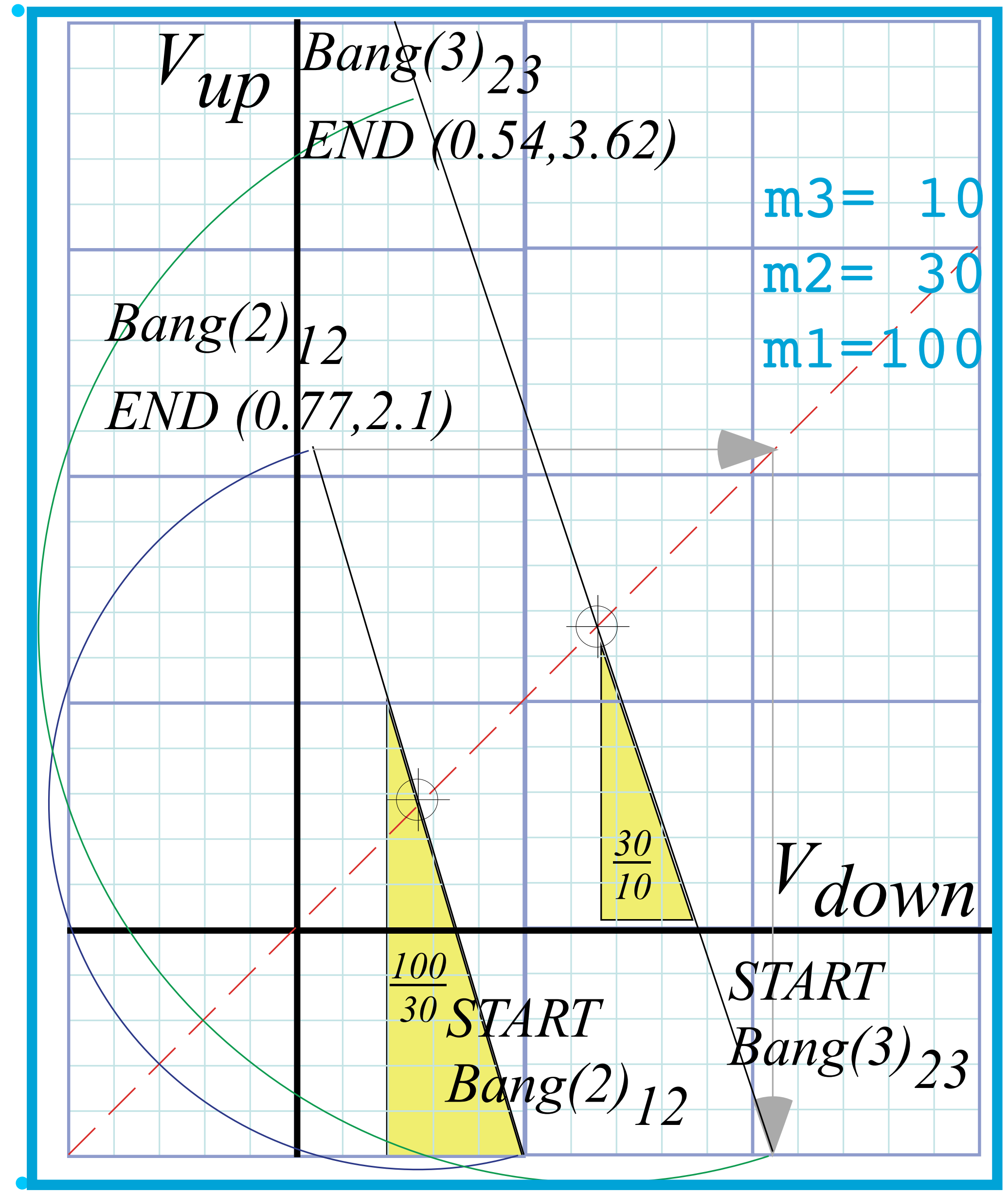
Quartic
F(y)=y4
y
Quadratic
F(y)=y2
y
Linear
Force
F(y)=y1
y
Cra Rumpany Ltd
Linear Force Field
Simulation
Velocity 2
Velocity 1
V2= 1.03
V1= 0.996
Continuous Bounce
Sequence
Unit 1
Fig. 7.7

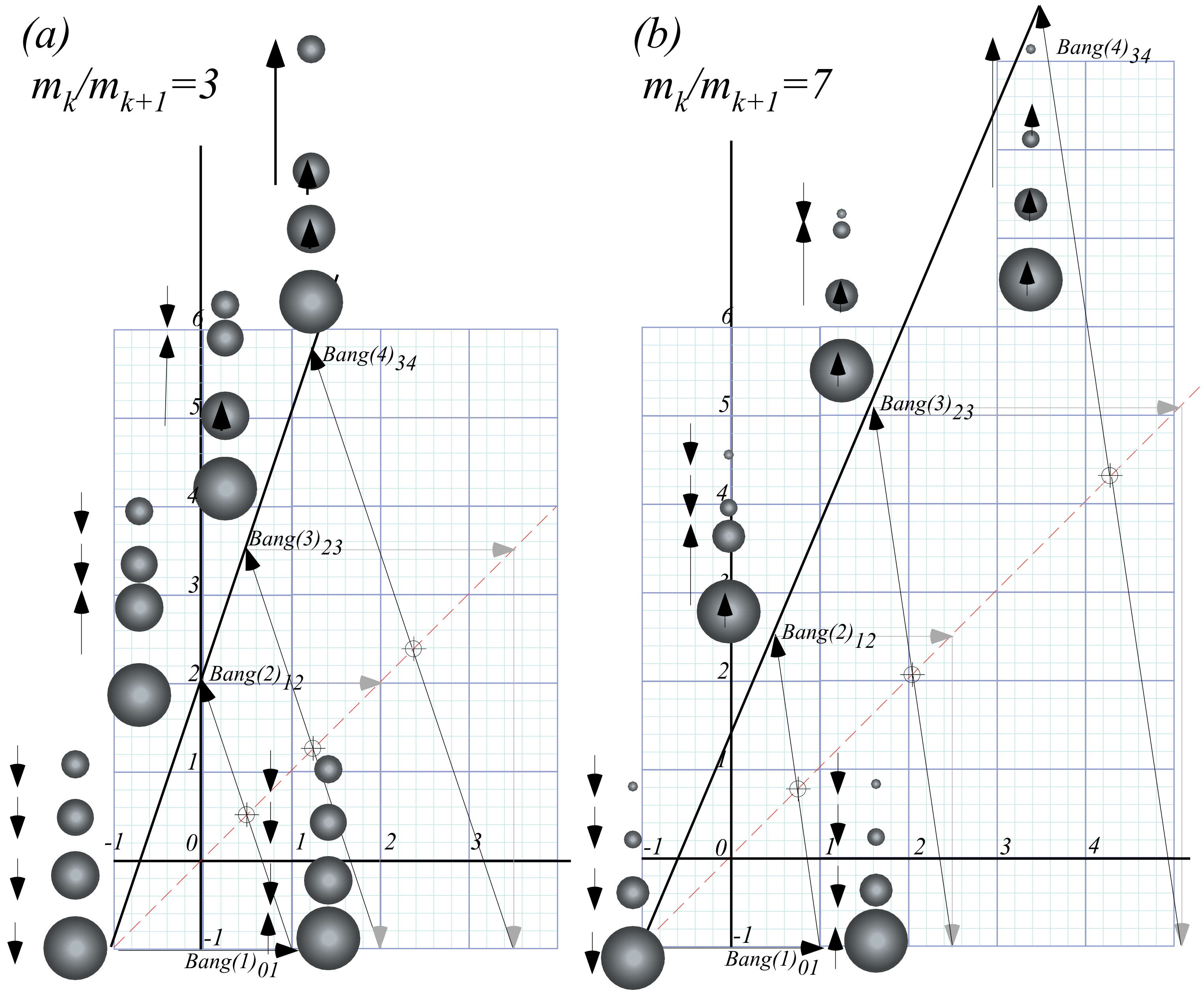


(a) Quartic Force
F(y) = k y^4
m3 = 10 kg
m2 = 30 kg
m1 = 100 kg
Initial Velocities
V3 = -1 m/s
V2 = -1 m/s
V1 = -1 m/s
m 3
m 2
m 1
Final Velocities
V3 = 3.41 m/s
V2 = 0.701 m/s
V1 = 0.298 m/s
time
(b) Independent Collisions (Independent of Force Law)
Vup
Vdown
Bang(3)23
END (0.54,3.62)
Bang(2)12
END (0.77,2.1)
100/30
30/10
START
Bang(2)12
START
Bang(3)23
m3 = 10 kg
m2 = 30 kg
m1 = 100 kg
Initial Velocities
V3 = -1 m/s
V2 = -1 m/s
V1 = -1 m/s
Final Velocities
V3 = 3.62 m/s
V2 = 0.538 m/s
V1 = 0.077 m/s
Bang-(3)23
Bang-(2)12
Bang-(1)01
time
(c) Linear Force
F(y) = k y
m3 = 10 kg
m2 = 30 kg
m1 = 100 kg
V3 = -1 m/s
V2 = -1 m/s
V1 = -1 m/s
Final Velocities
V3 = 1.48 m/s
V2 = 1.32 m/s
V1 = 0.81 m/s

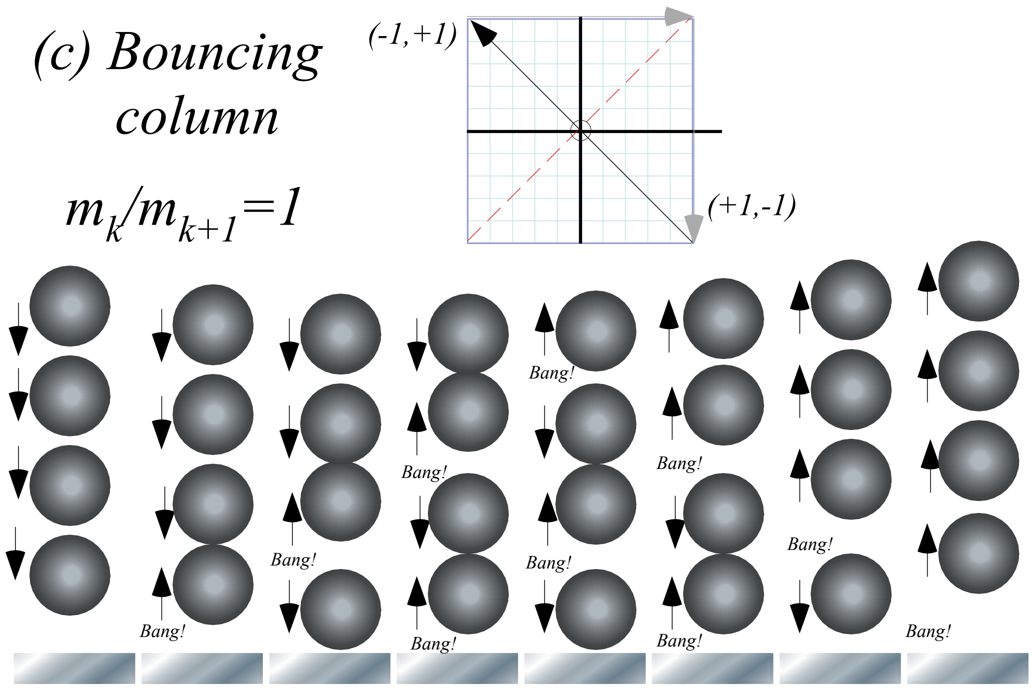
Unit 1
Fig. 8.1b
Independent Bang Model (IBM)
3-Body Geometry

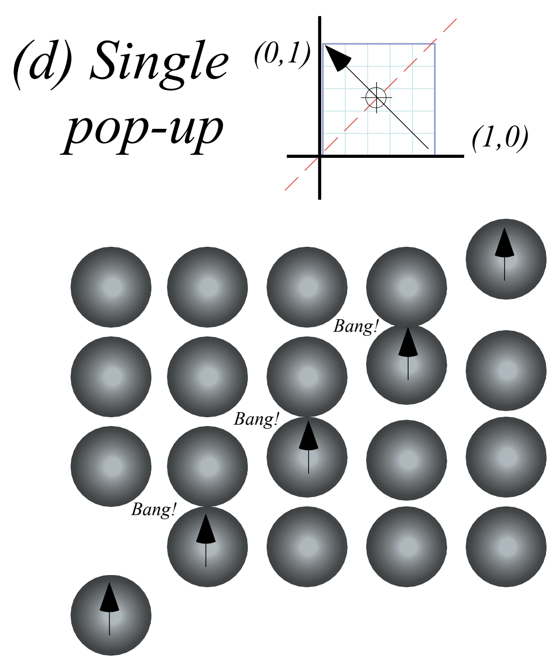






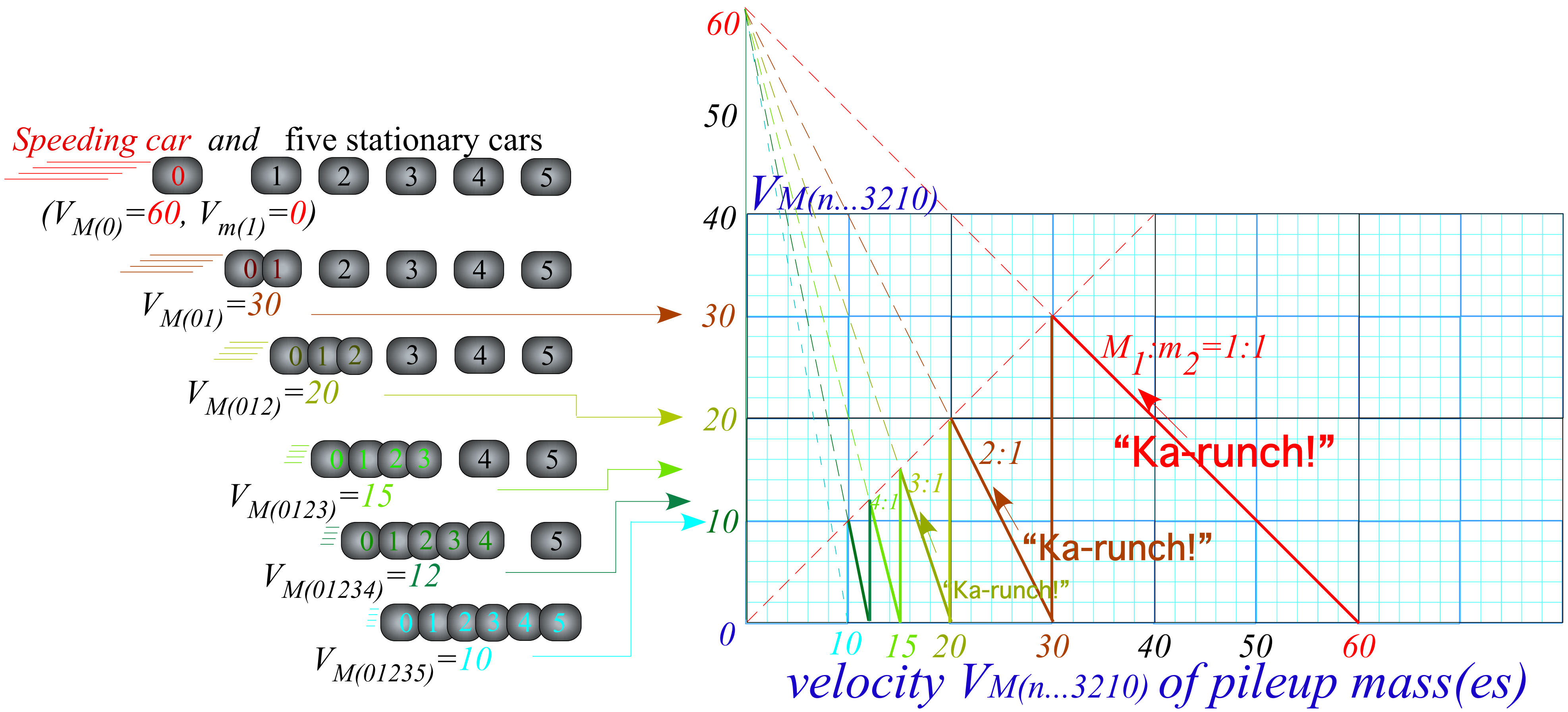
Unit 1
Fig. 8.1a-b
4-Body IBM Geometry
Fig. 8.1c-d
4-Equal-Body Geometry



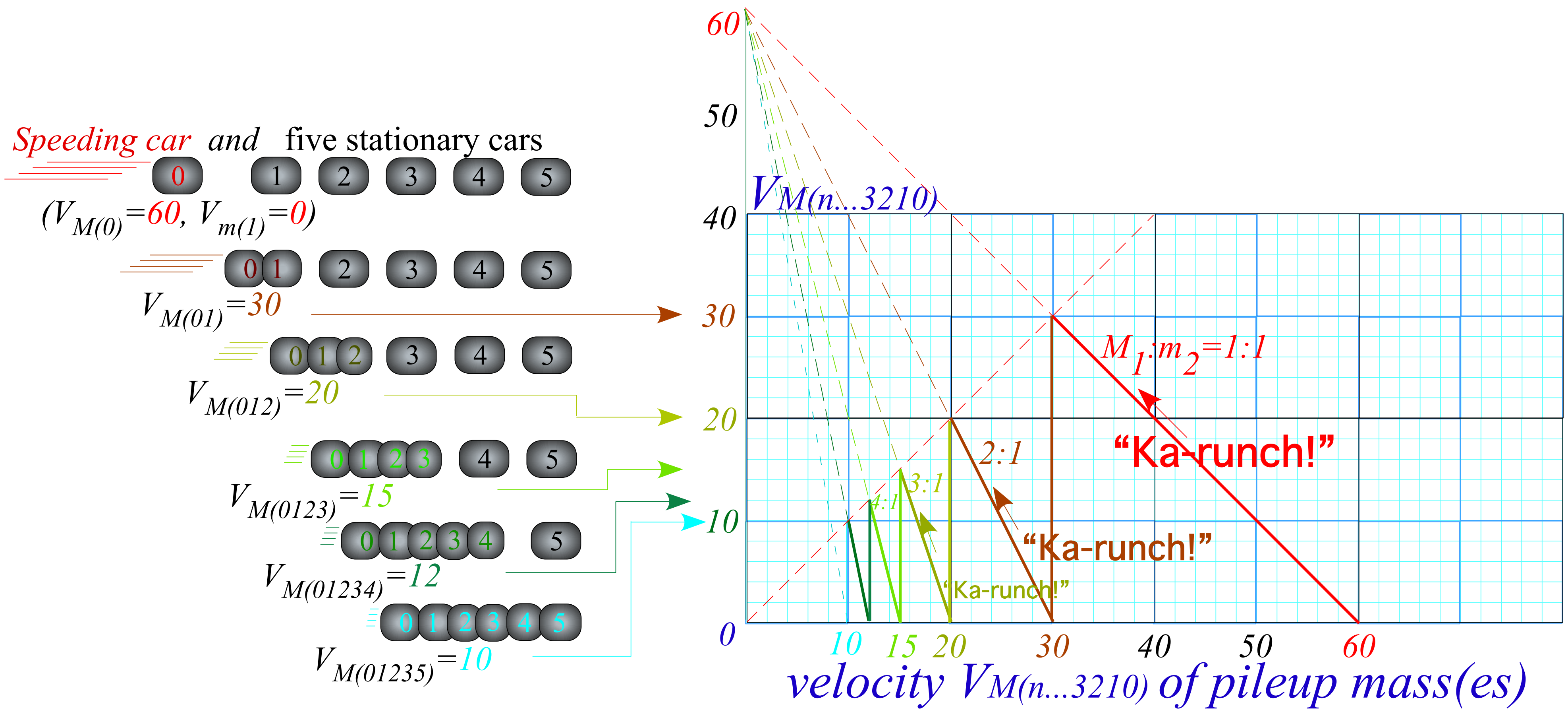


4-Equal-Body “Shockwave” or pulse wave Dynamics

Opposite of continuous wave dynamics introduced in Unit 2



Unit 1
Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars



Unit 1

Fig. 8.5

Pile-up:
One *60mph* car
hits
five standing cars

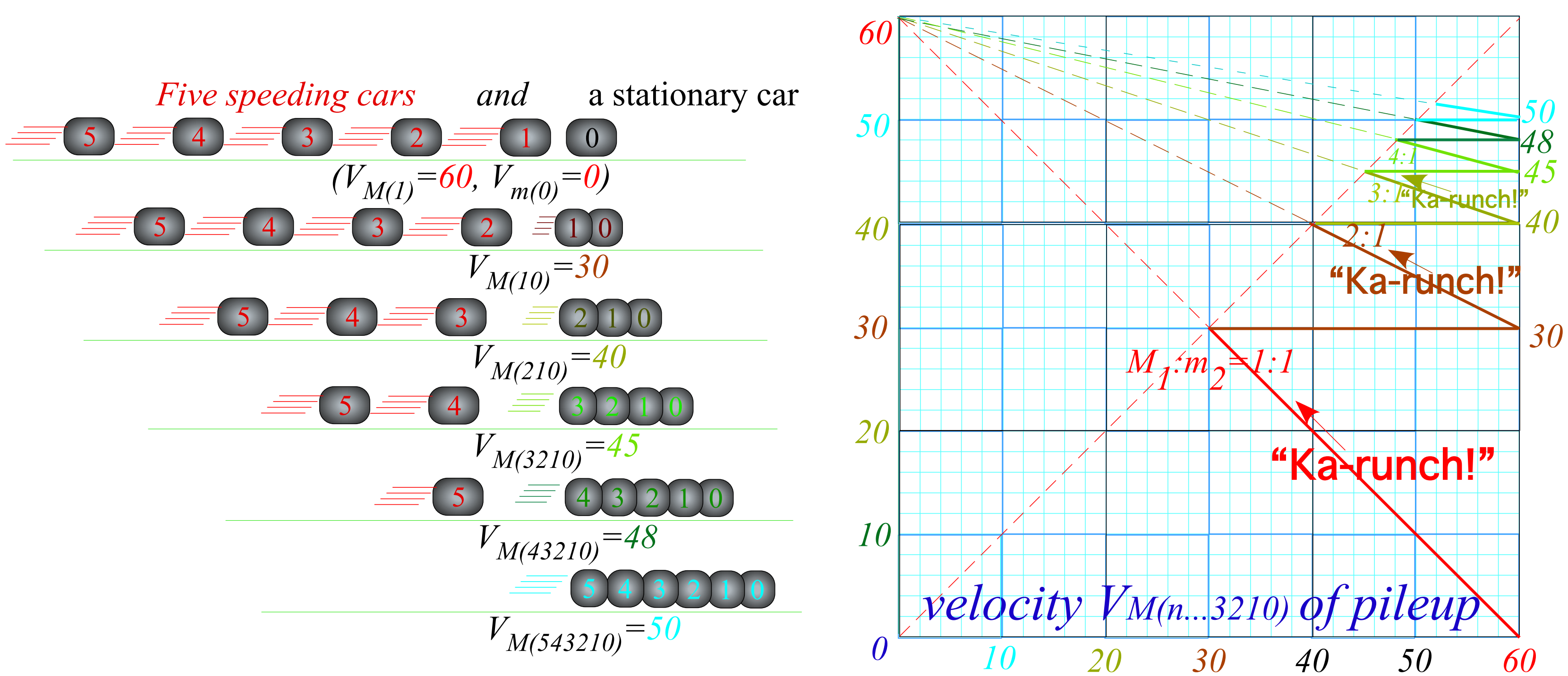


Fig. 8.6

Pile-up:
Five *60mph* cars
hit
one standing cars

Unit 1

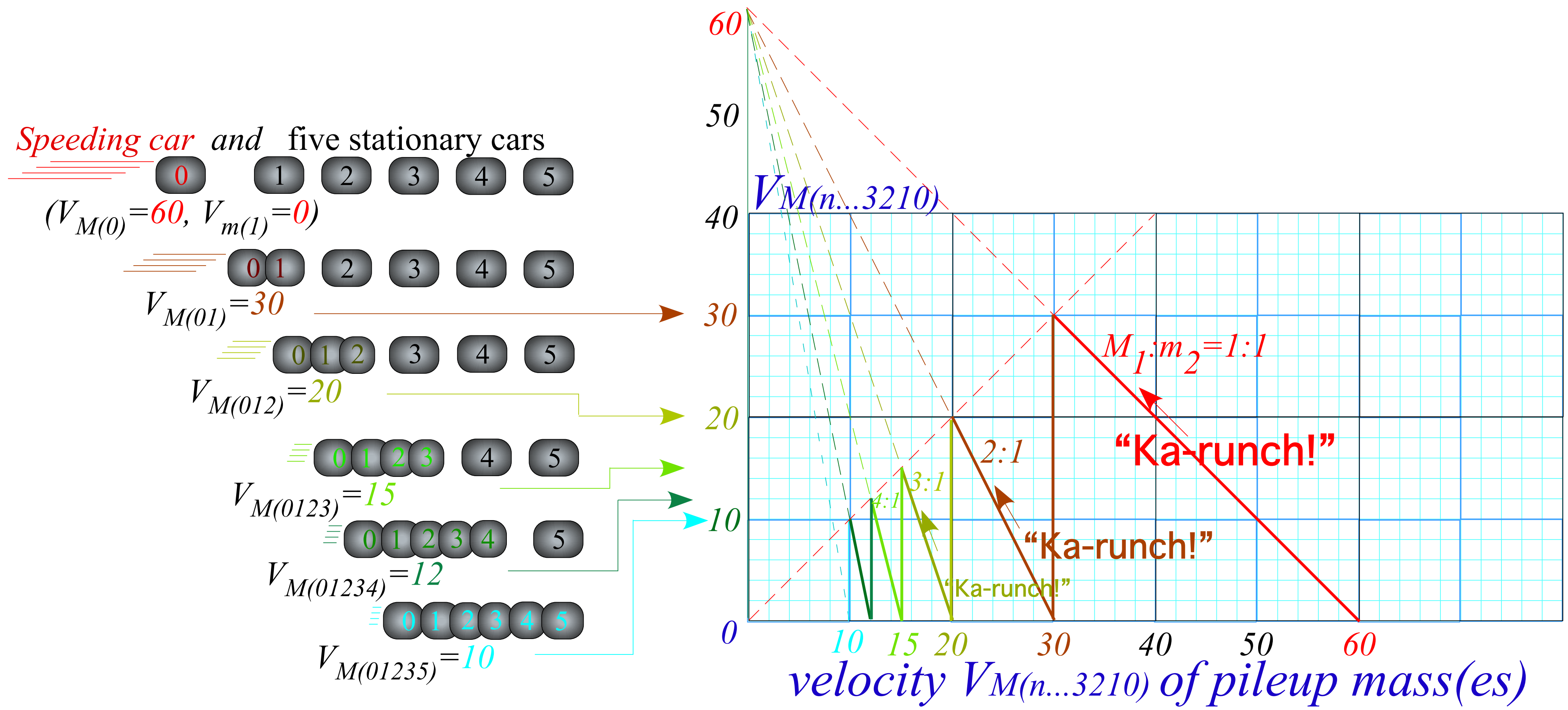


Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars

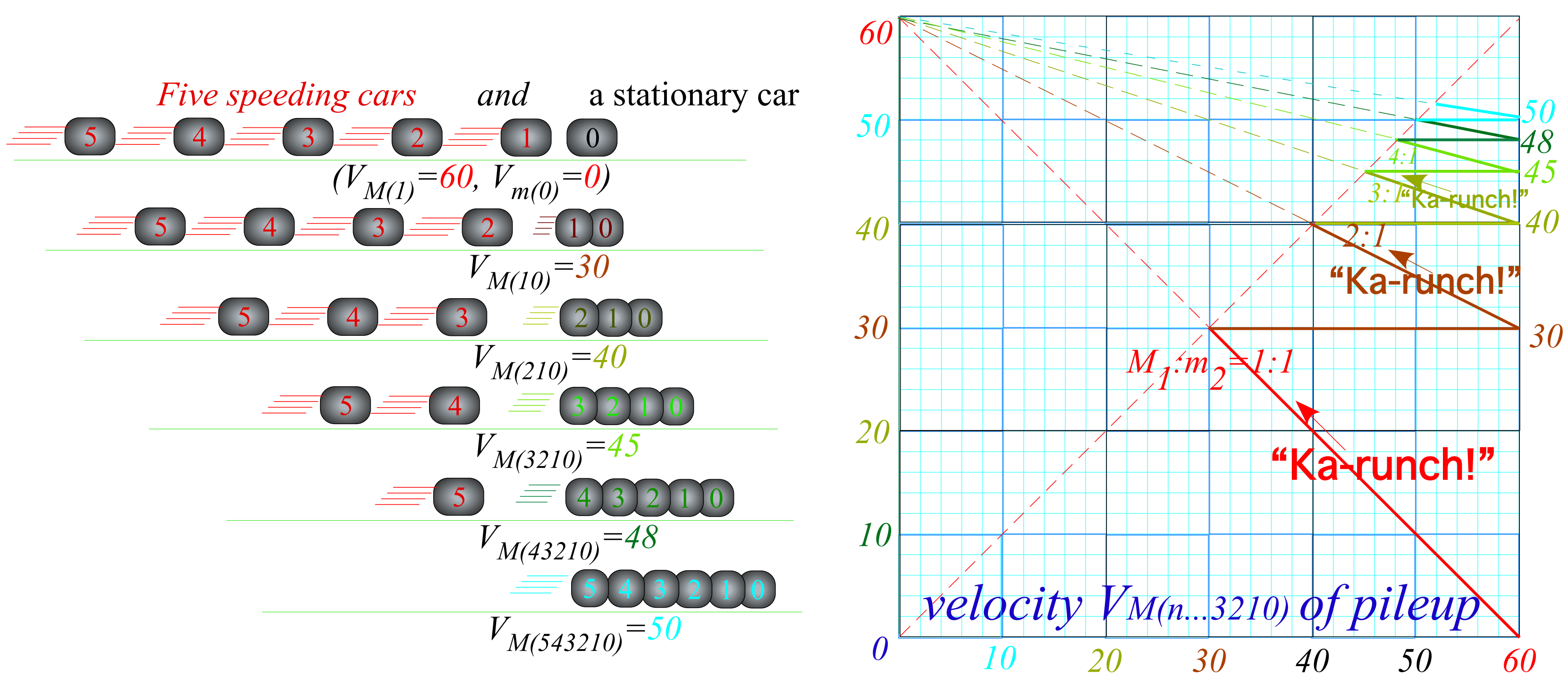


Fig. 8.6
Pile-up:
Five *60mph* cars
hit
one standing cars

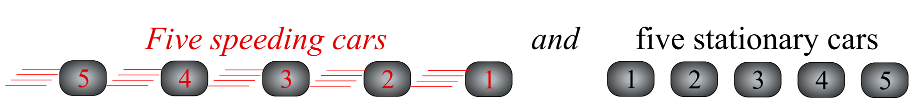
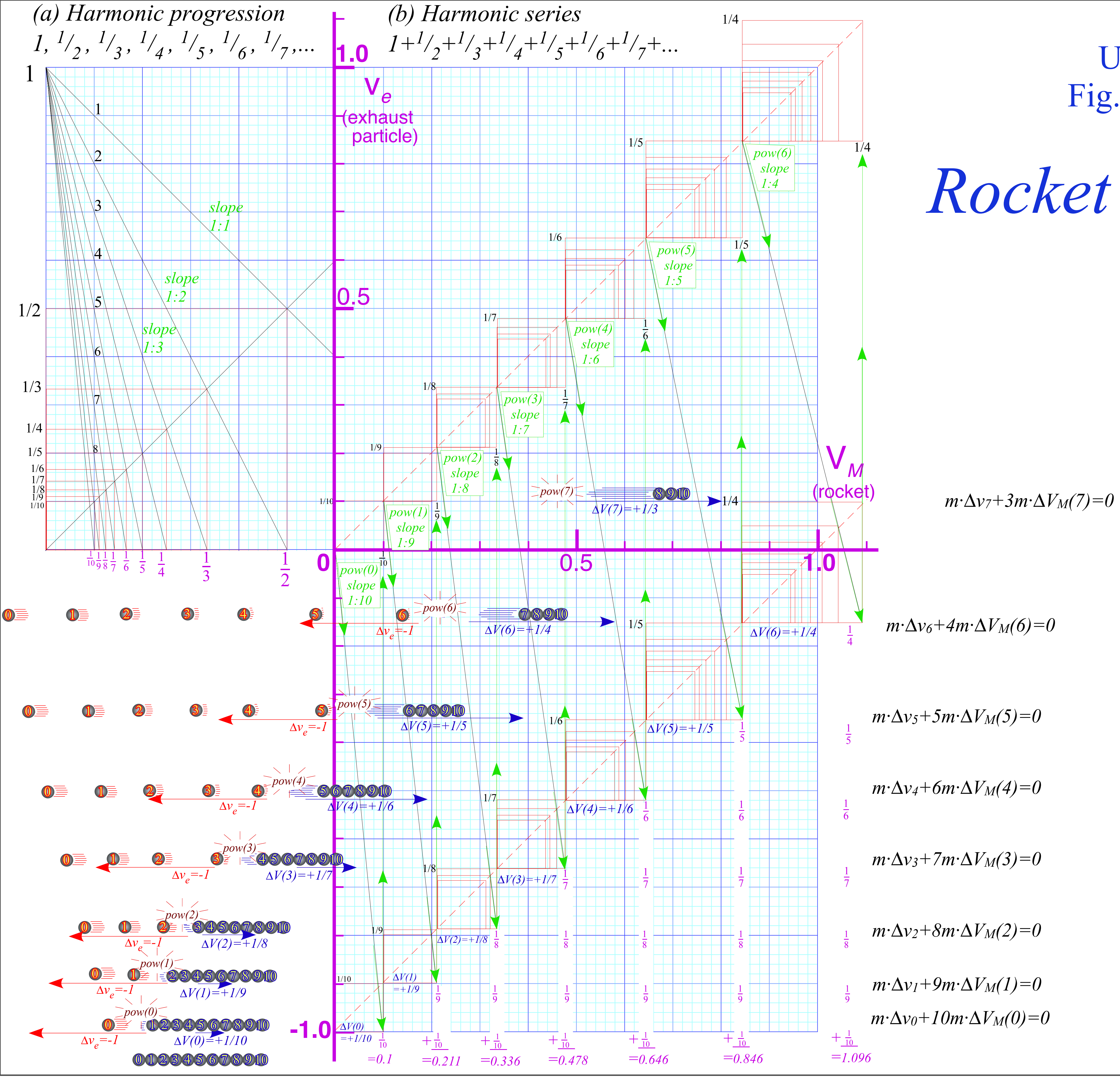


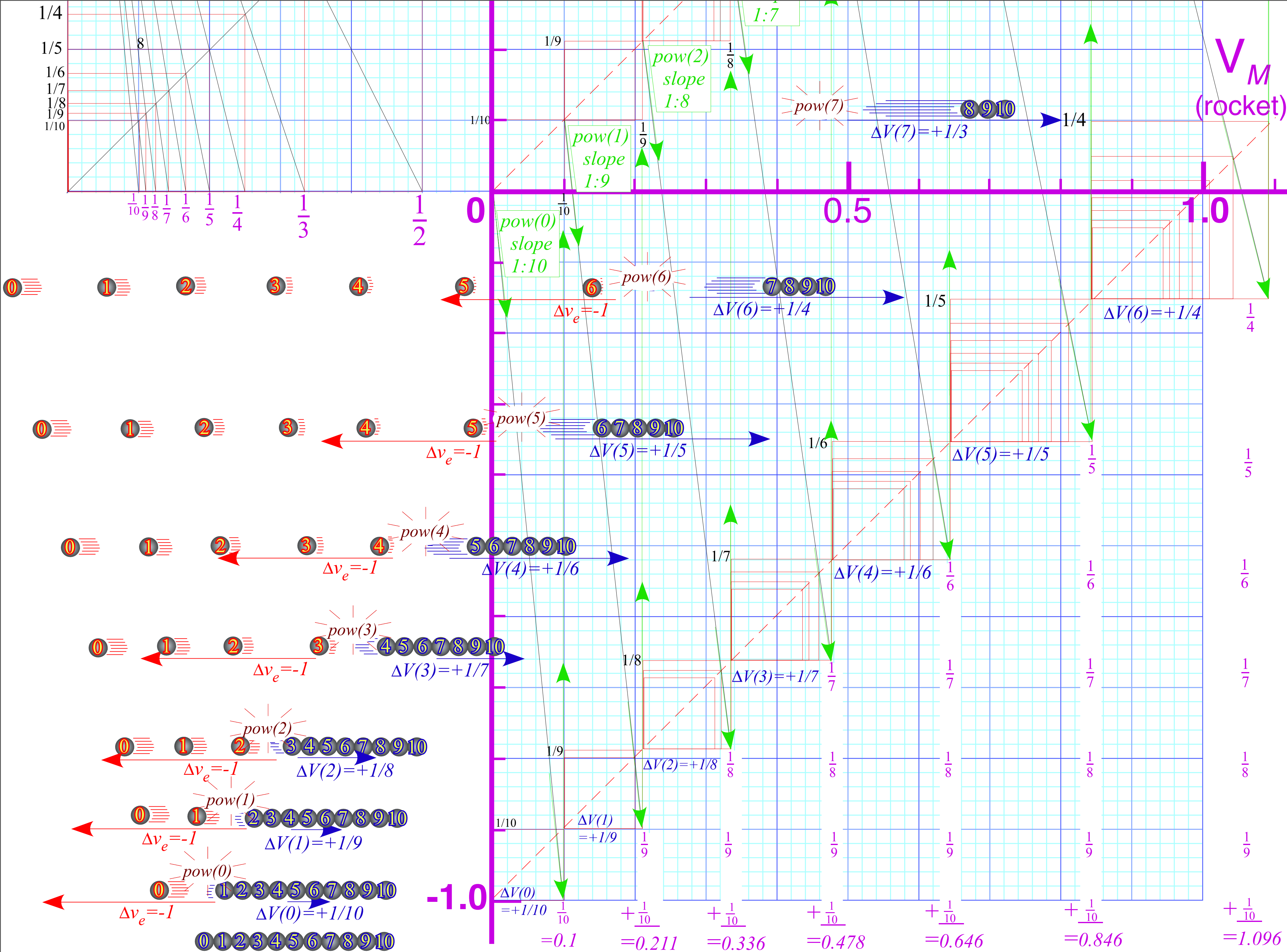
Fig. 8.7
Pile-up:
Five *60mph* cars
hit
five standing cars

(Fug-gedda-aboud-dit!!)

Unit 1
Fig. 8.8a-b

Rocket Science!





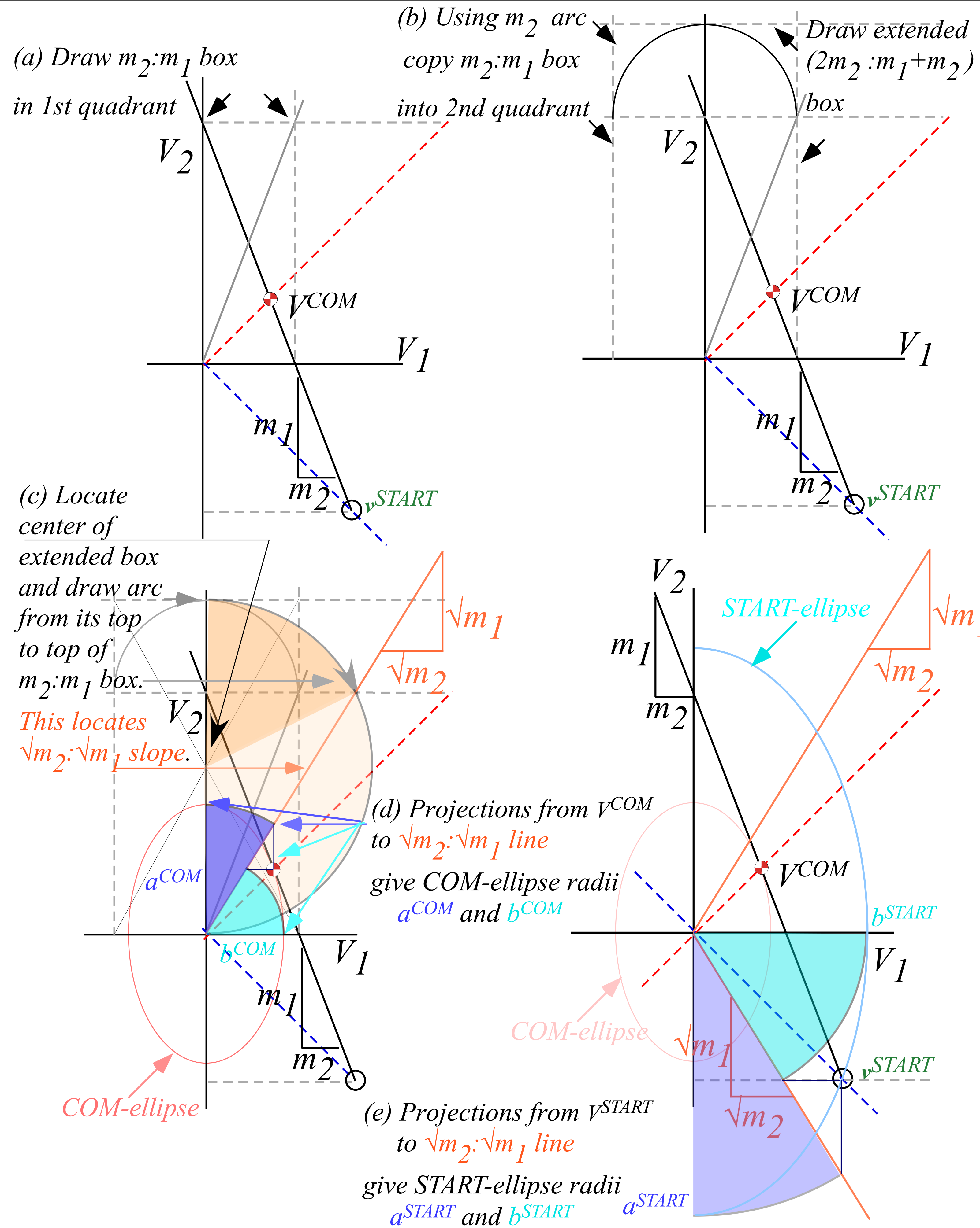
0^{th}: $V(0)=1/10=0.1$ 1^{st}: $V(1)=1/10+1/9=0.211$ 2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

3^{rd}: $V(3)=V(2)+1/7=0.478$ 4^{th}: $V(4)=V(3)+1/6=0.646$ 5^{th}: $V(5)=V(4)+1/5=0.846$

6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$


(a) Draw m_2:m_1 box
in 1st quadrant
V_2
V^{COM}
V_1
m_1
m_2
v^{START}
(b) Using m_2 arc
copy m_2:m_1 box
into 2nd quadrant
Draw extended
($2m_2$:m_1+m_2)
box
V_2
V^{COM}
V_1
m_1
m_2
v^{START}
(c) Locate
center of
extended box
and draw arc
from its top
to top of
m_2:m_1 box.
This locates
$\sqrt{m_2}$:$\sqrt{m_1}$ slope.
$\sqrt{m_1}$
$\sqrt{m_2}$
V_2
a^{COM}
b^{COM}
V_1
m_1
m_2
COM-ellipse
(d) Projections from V^{COM}
to $\sqrt{m_2}$:$\sqrt{m_1}$ line
give COM-ellipse radii
a^{COM} and b^{COM}
V_2
m_1
m_2
START-ellipse
$\sqrt{m_1}$
$\sqrt{m_2}$
V^{COM}
b^{START}
V_1
COM-ellipse
$\sqrt{m_1}$
$\sqrt{m_2}$
v^{START}
(e) Projections from V^{START}
to $\sqrt{m_2}$:$\sqrt{m_1}$ line
give START-ellipse radii
a^{START} and b^{START}
a^{START}