Lecture 2

Revised 12.21.12 from 8.23.2012

Analysis of 1D 2-Body Collisions

(Ch. 3 and Ch. 4 of Unit 1)

Review of elastic Kinetic Energy ellipse geometry

The X2 Superball pen launcher

Perfectly elastic "ka-bong" velocity amplification effects (Faux-Flubber)

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

Integration of (V_1,V_2) data to space-space plots (y_1, y_2)

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

Review of elastic Kinetic Energy ellipse geometry

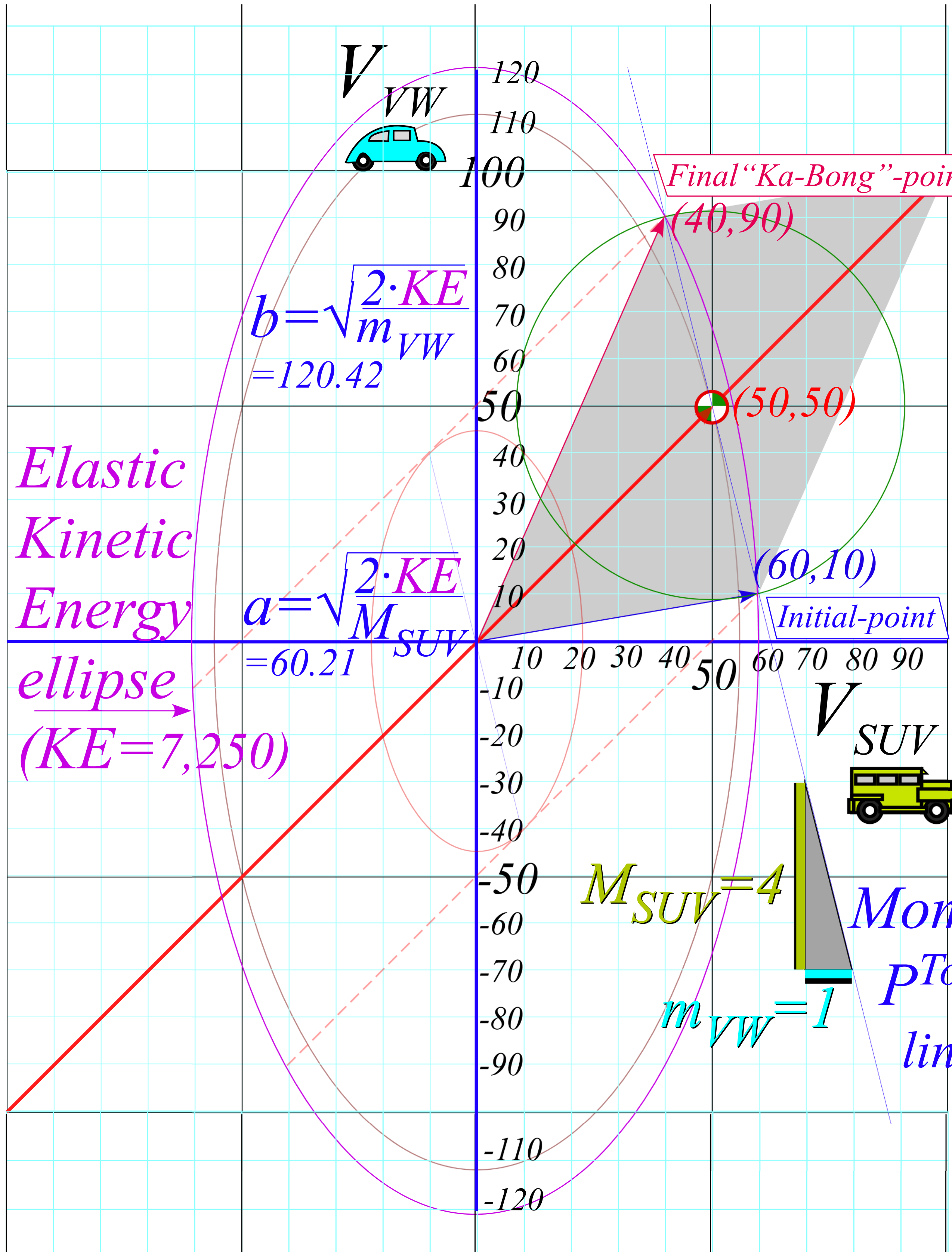


Fig. 3.1 a
in Unit 1

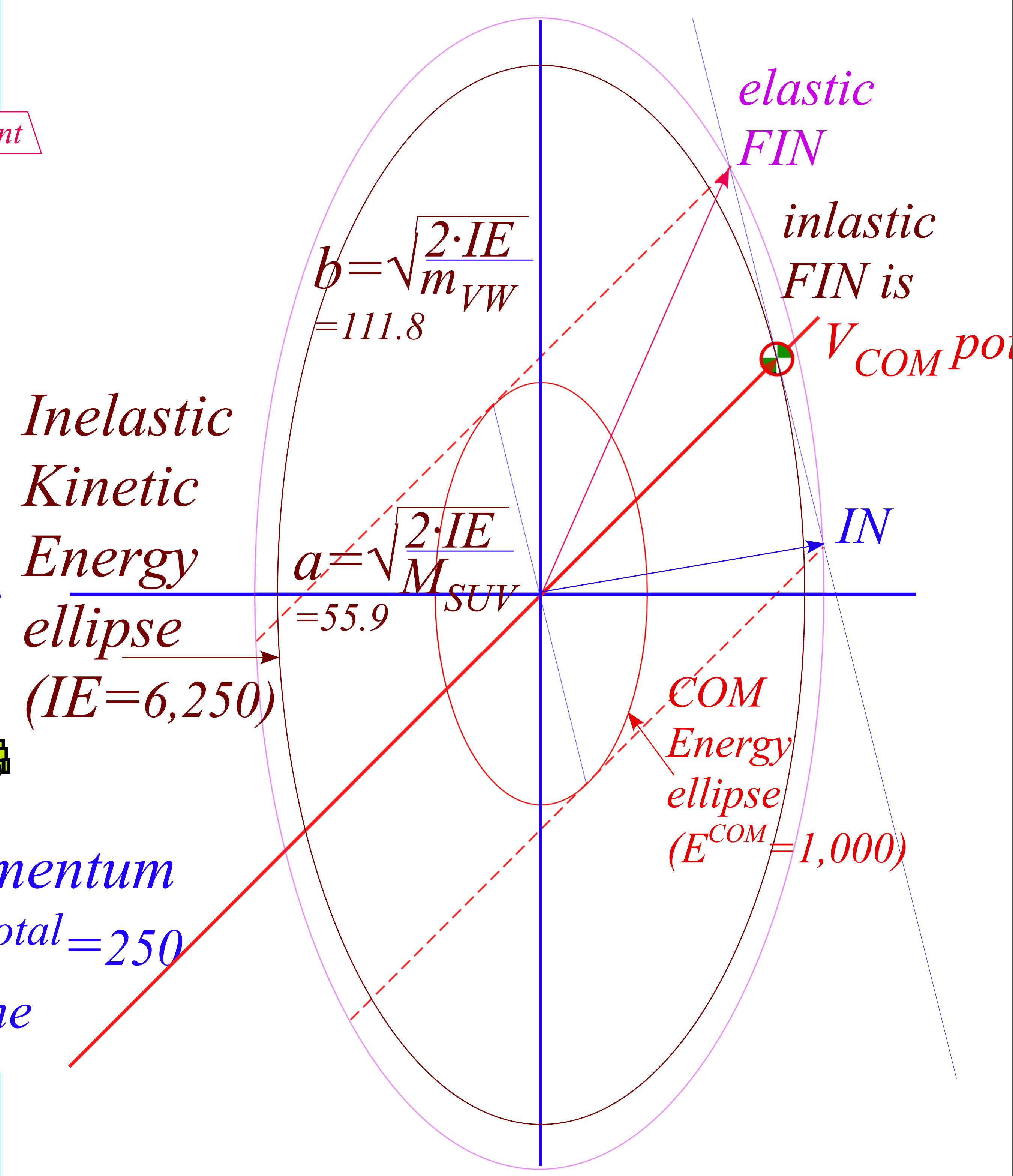
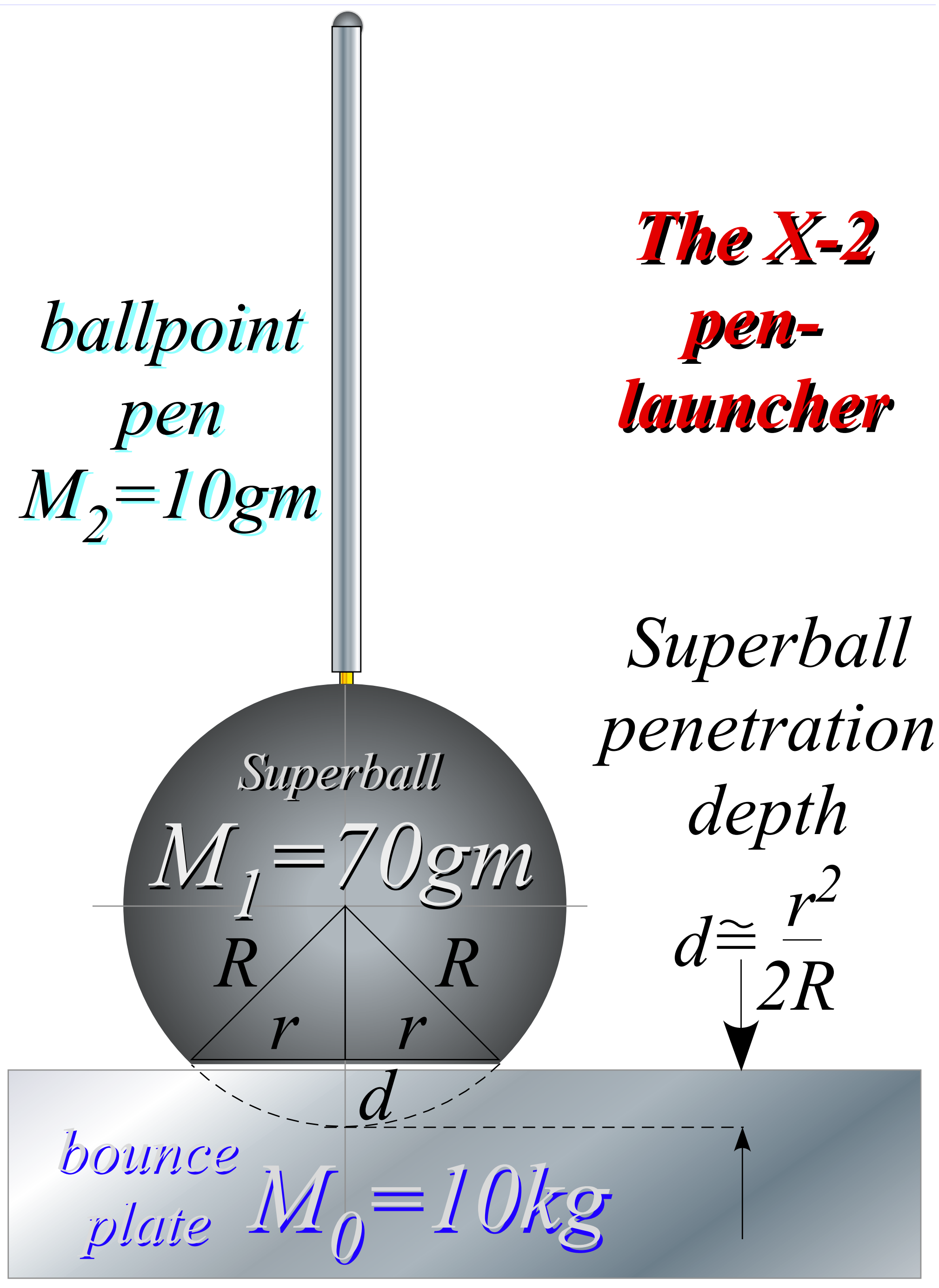
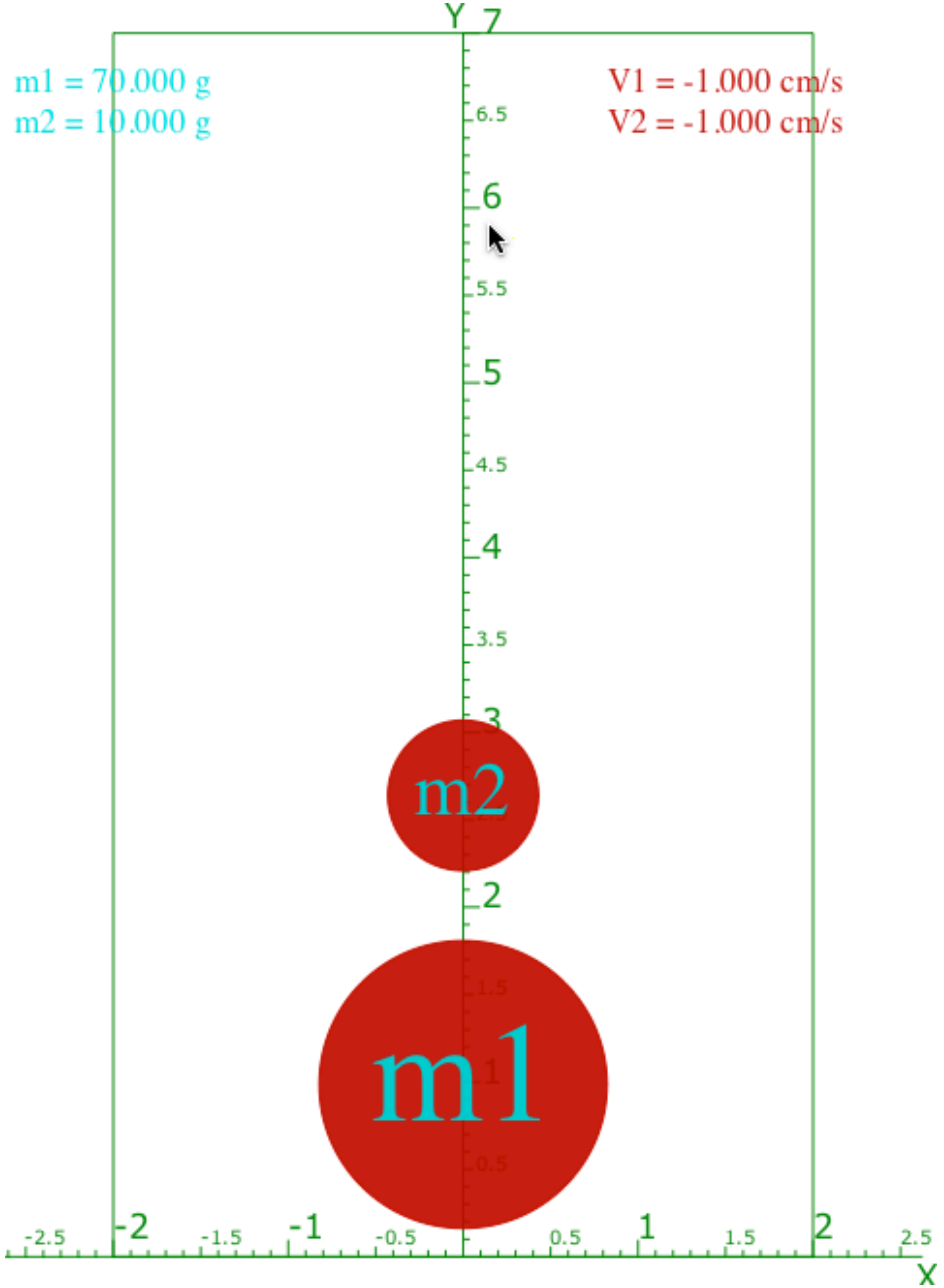


Fig. 3.1 b
in Unit 1

The X-2 Pen launcher and Superball Collision Simulator*





*Simulator Website: http://www.uark.edu/rso/modphys/animations/BounceItWeb.html

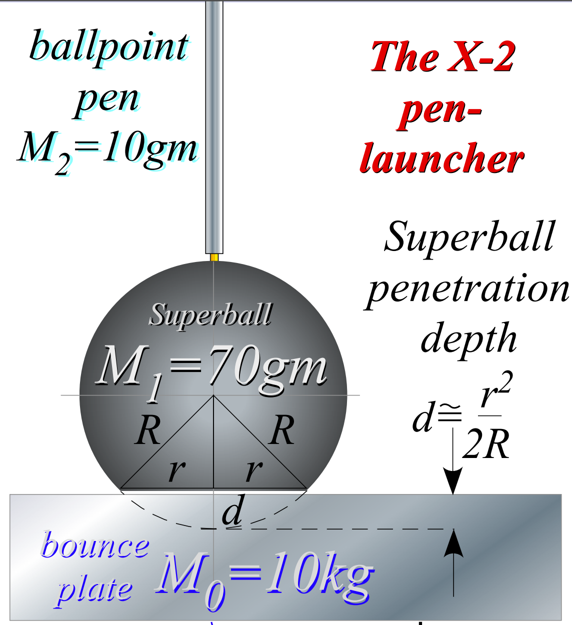
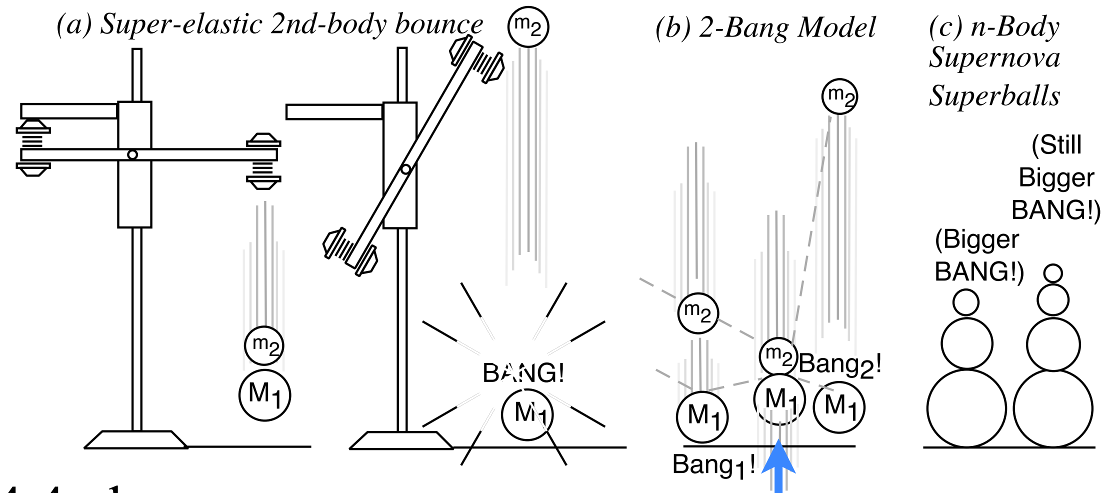


Fig. 4.1 and Fig. 4.3
in Unit 1

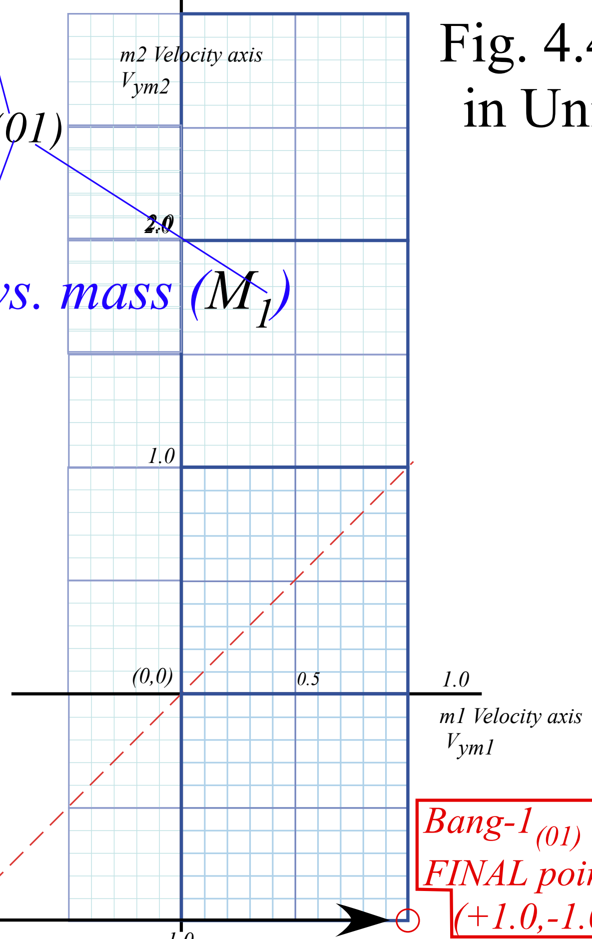


(a)

Bang-1$_{(01)}$

1^{st} bang:
mass (M_0)vs. mass (M_1)

Fig. 4.4a-b
in Unit 1

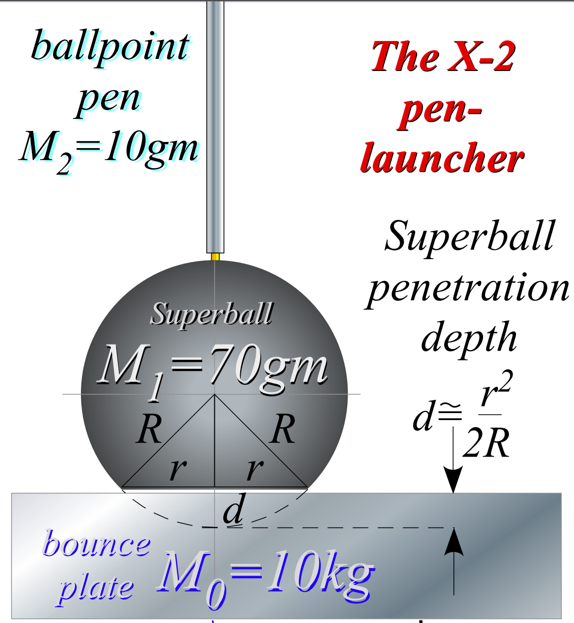


Bang-1$_{(01)}$
INIT point at
(-1.0,-1.0)

Bang-1$_{(01)}$
FINAL point
(+1.0,-1.0)

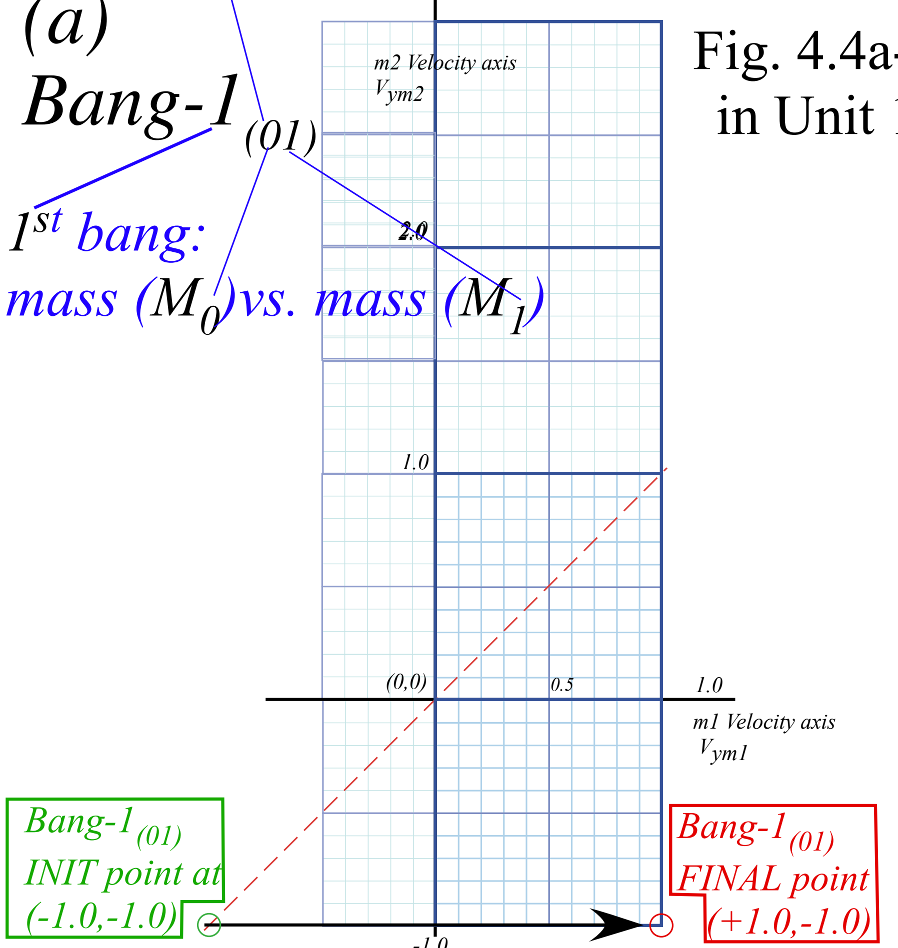
This 1st bang is a floor-bounce of
M_1 off very massive plate/Earth M_0

Next: 2^{nd} or 3^{rd} bangs:
mass (M_1) vs. mass (M_2)



(a) Bang-$1_{(01)}$

1^{st} bang: mass (M_0) vs. mass (M_1)



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 4.1 and Fig. 4.3 in Unit 1

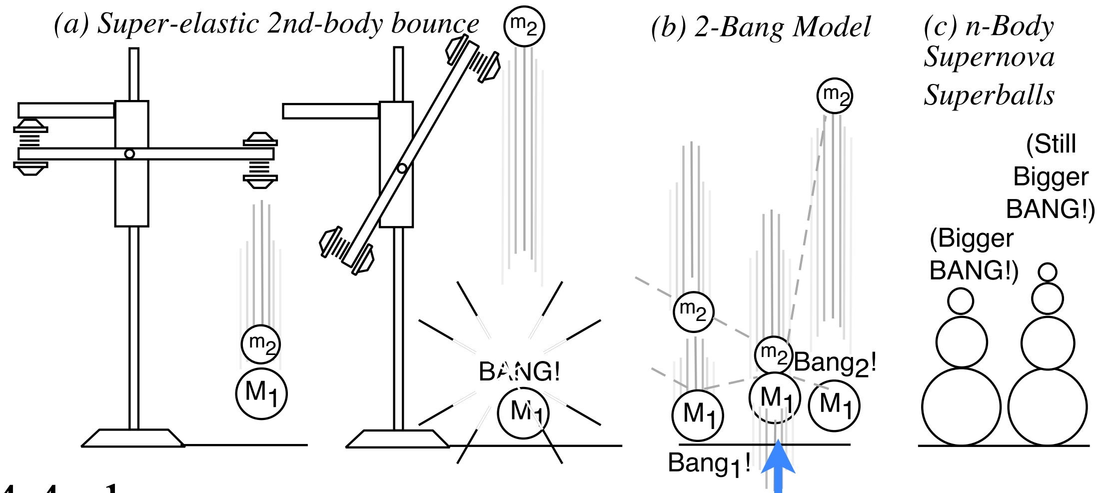
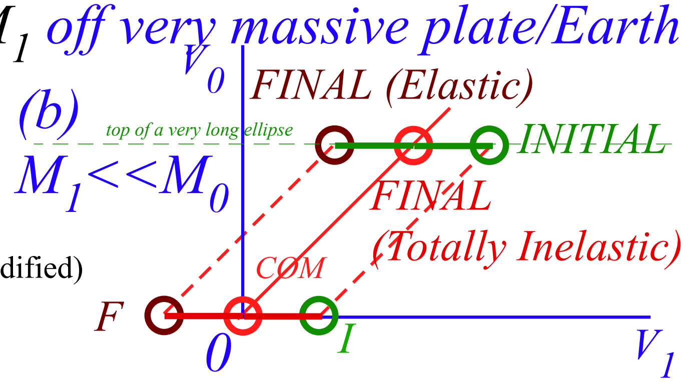
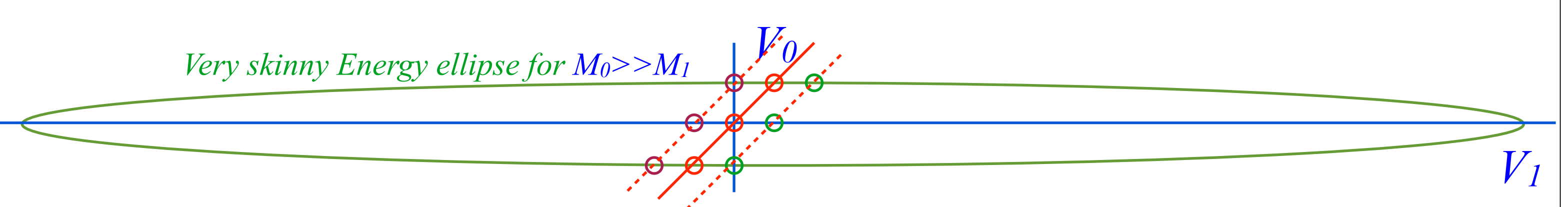


Fig. 4.4a-b in Unit 1

Next: 2^{nd} or 3^{rd} bangs: mass (M_1) vs. mass (M_2)

Fig. 4.2b in Unit 1 (slightly modified)





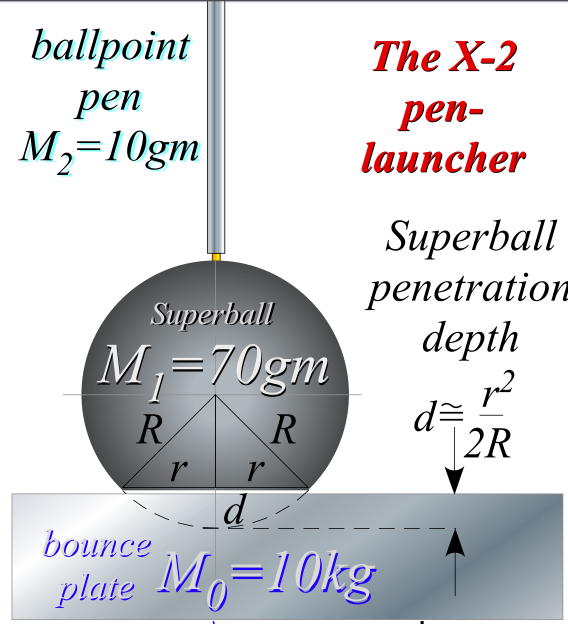
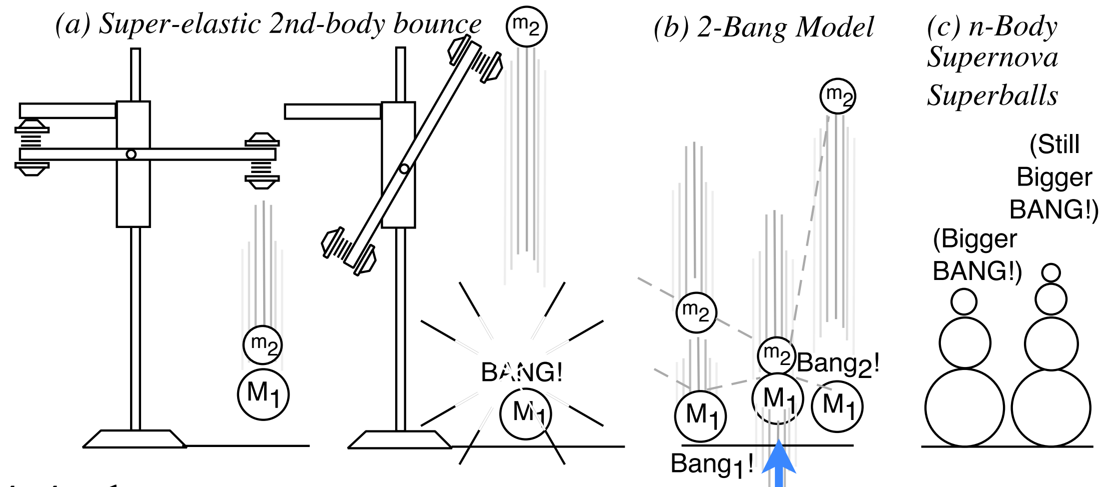


Fig. 4.1 and Fig. 4.3 in Unit 1



(a)

Bang-1$_{(01)}$

1st bang: mass (M_0) vs. mass (M_1)

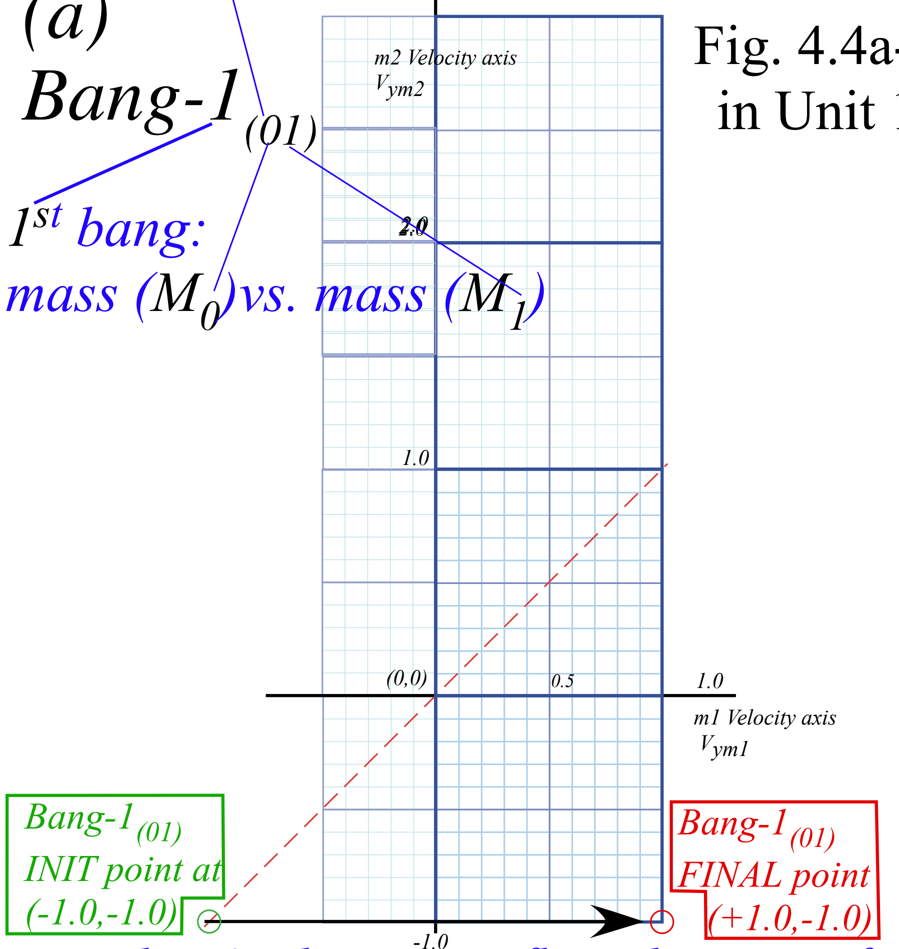


Fig. 4.4a-b in Unit 1

This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Next: 2nd or 3rd bangs: mass (M_1) vs. mass (M_2)

Later: (a) $M_1 >> M_2$

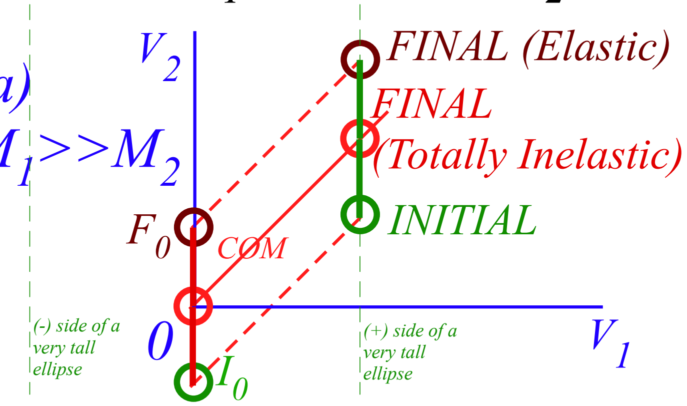


Fig. 4.2a in Unit 1 (slightly modified)

Later is a ceiling-bounce of M_2 off ceiling/Earth M_1

(b) $M_1 << M_0$

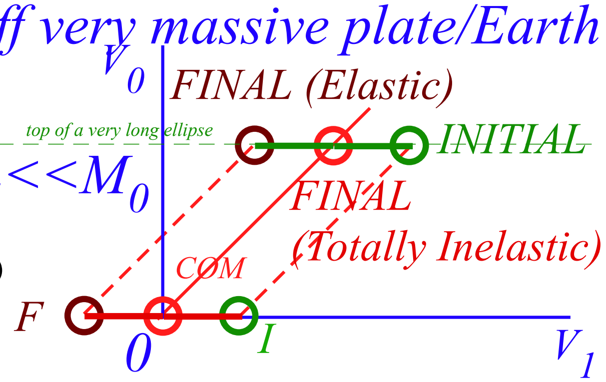
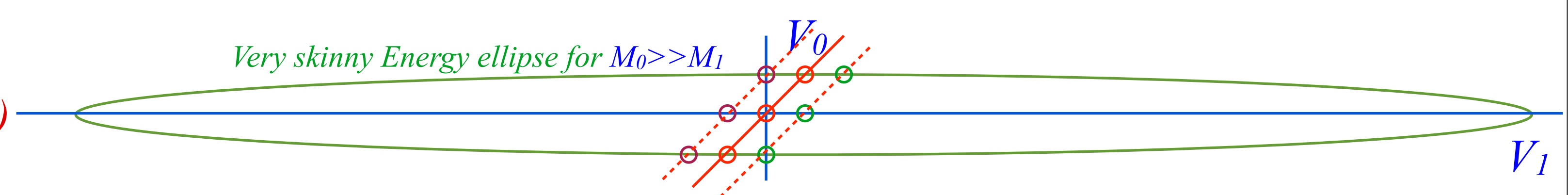


Fig. 4.2b in Unit 1 (slightly modified)



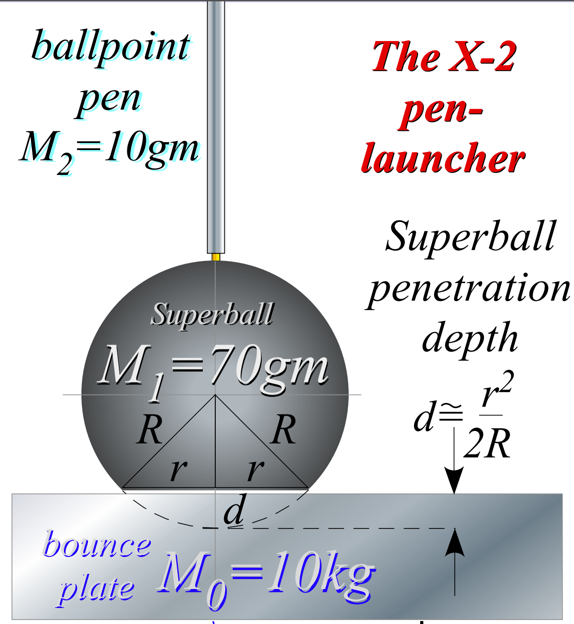
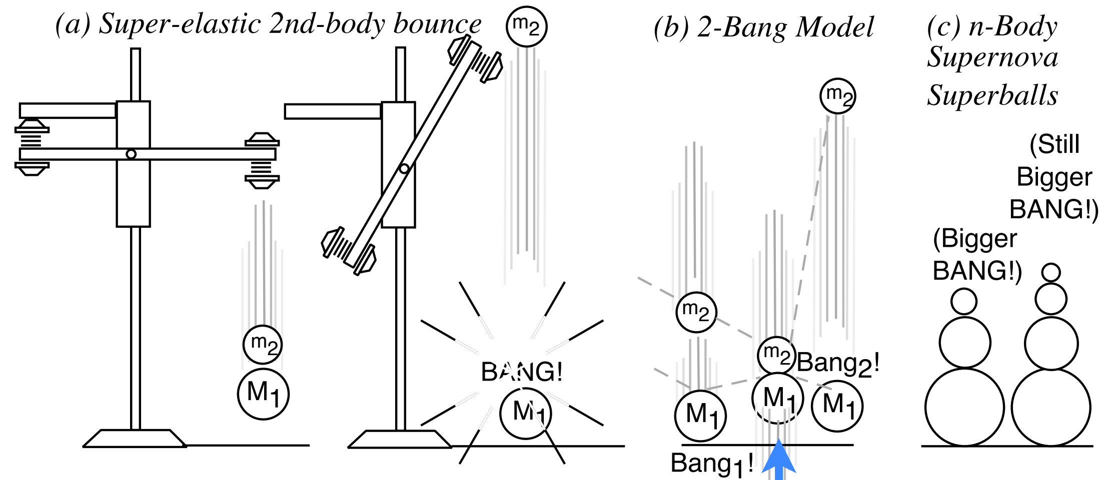


Fig. 4.1 and Fig. 4.3 in Unit 1



(a) Bang-1 (01)

1st bang: mass (M_0) vs. mass (M_1)

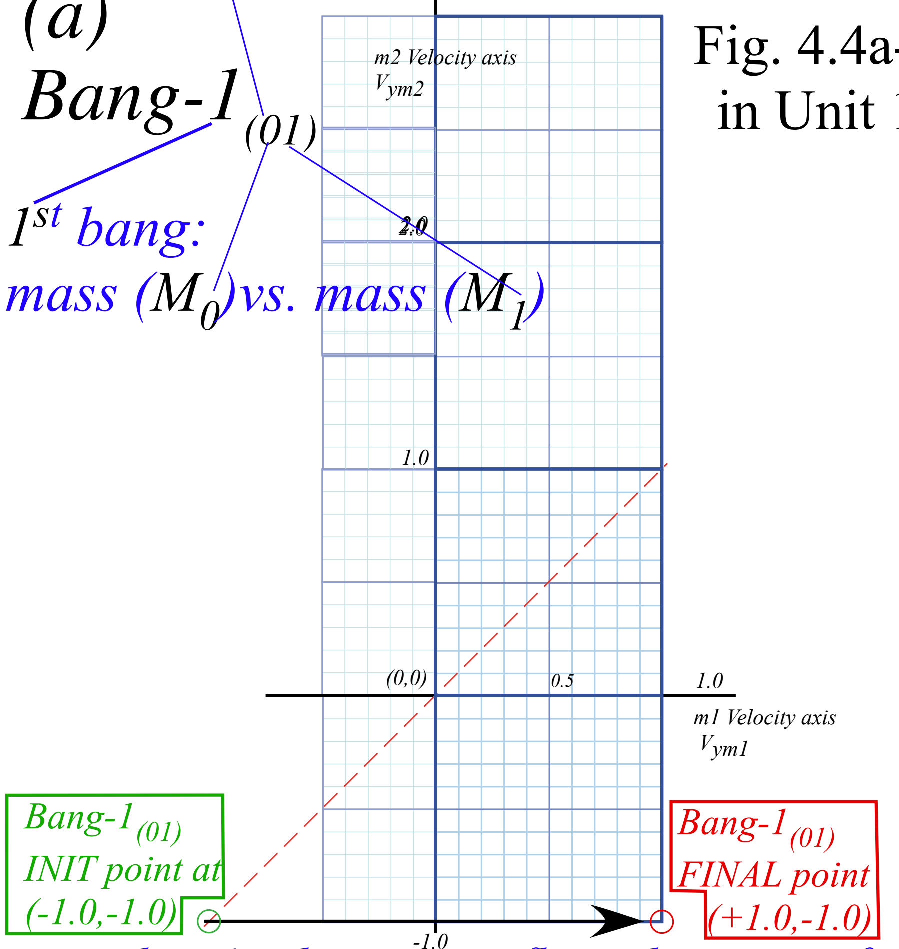


Fig. 4.4a-b in Unit 1

This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

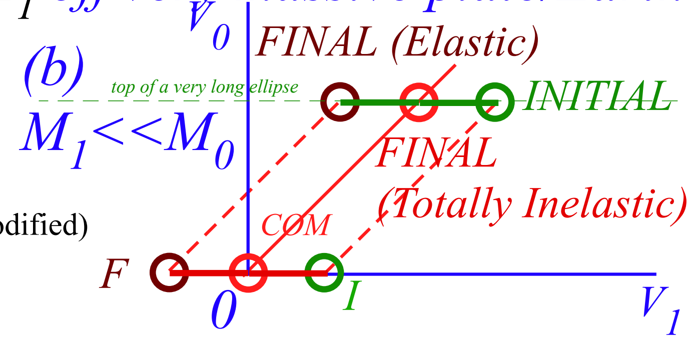


Fig. 4.2b in Unit 1 (slightly modified)

Next: 2^{nd} or 3^{rd} bangs: mass (M_1) vs. mass (M_2)

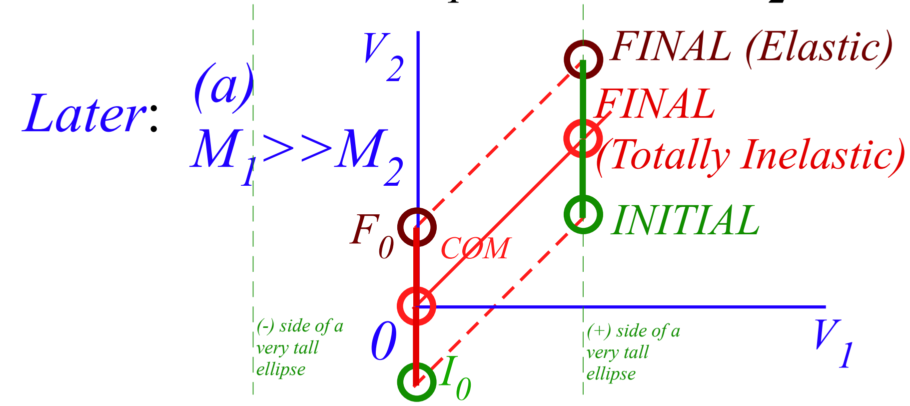
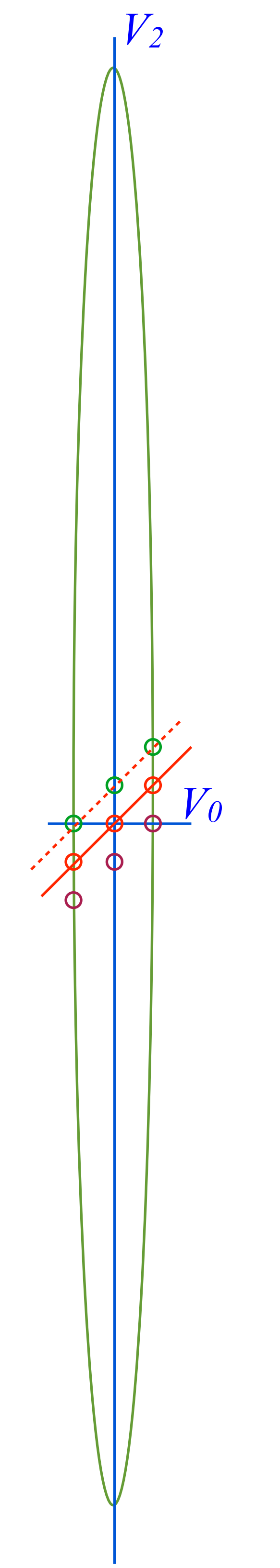
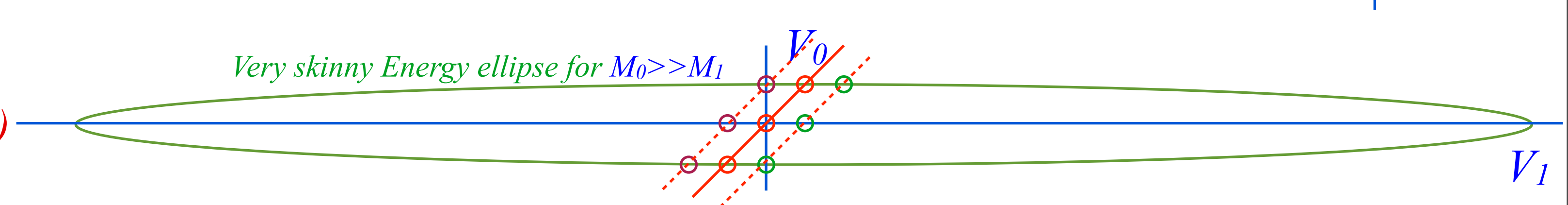


Fig. 4.2a in Unit 1 (slightly modified)

Later is a ceiling-bounce of M_2 off ceiling/Earth M_1





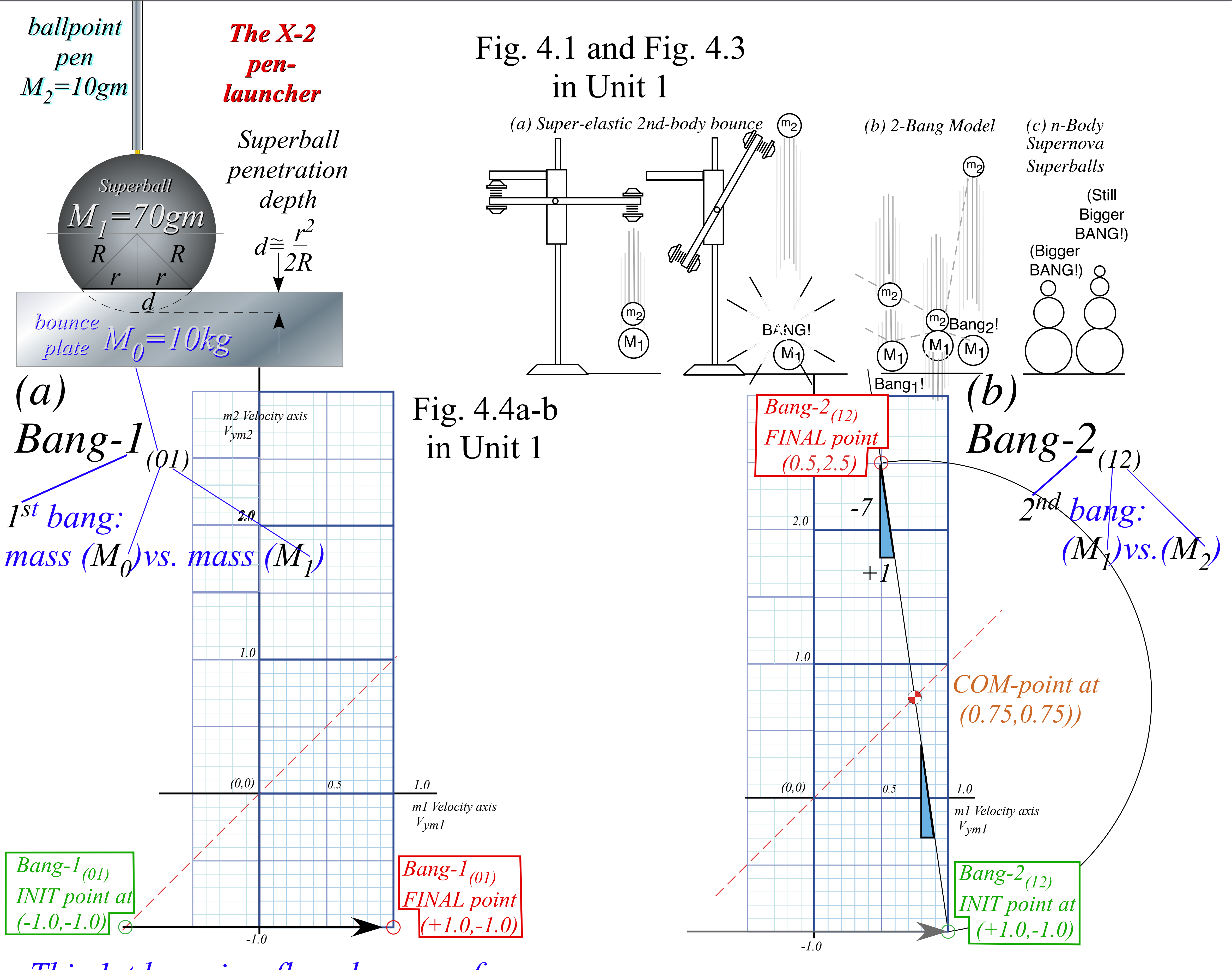
Geometry of X2 launcher bouncing in box

→ *Independent Bounce Model (IBM)*

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots ($y_1(t)$,t) and ($y_2(t)$,t) plots

Integration of (V_1,V_2) data to space-space plots (y_1, y_2)



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

→ *Geometric optimization and range-of-motion calculation(s)*

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

Integration of (V_1,V_2) data to space-space plots (y_1, y_2)

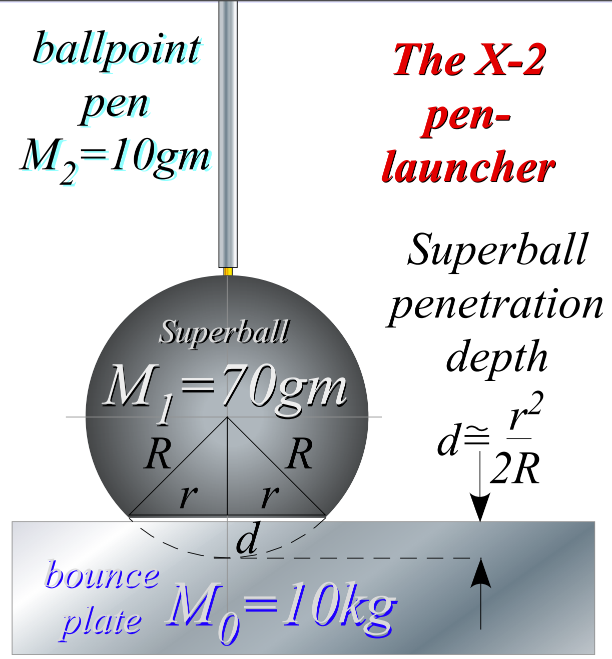
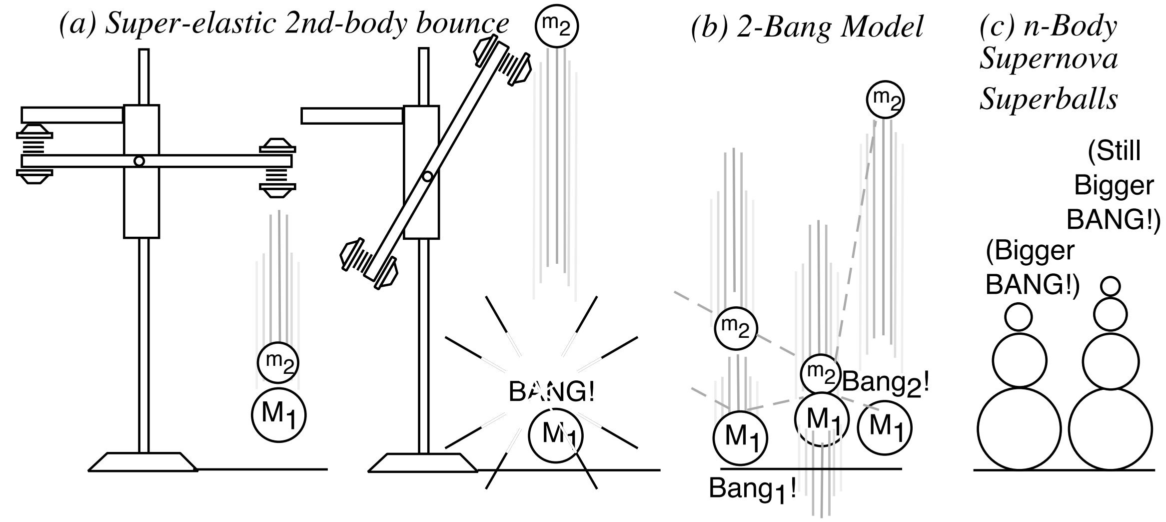


Fig. 4.1 and Fig. 4.3 in Unit 1



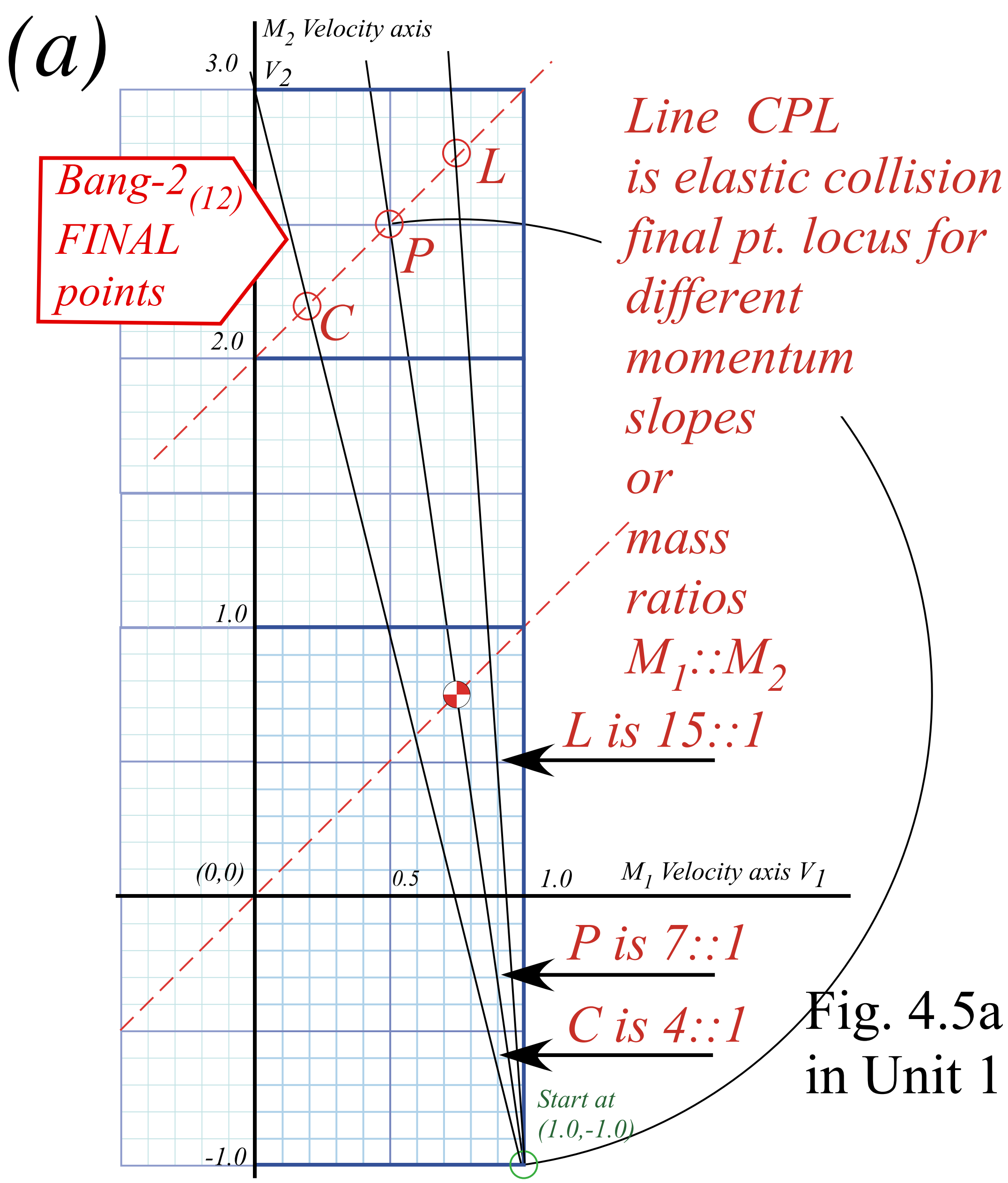


Fig. 4.5a in Unit 1

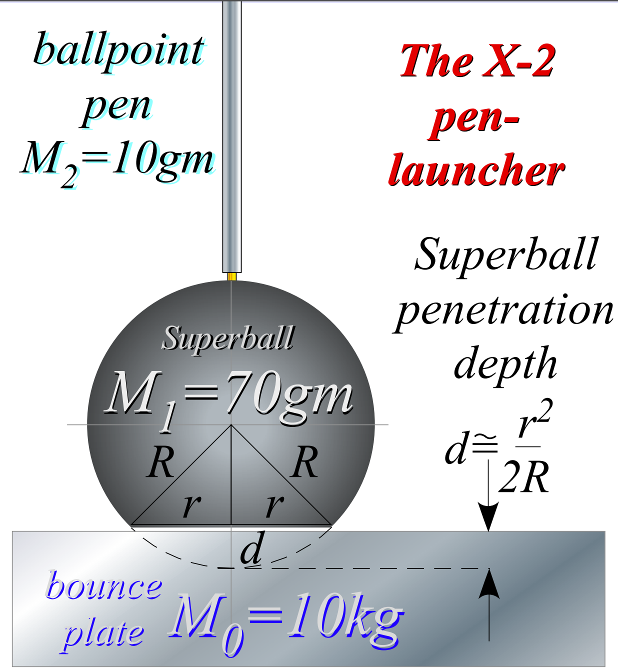
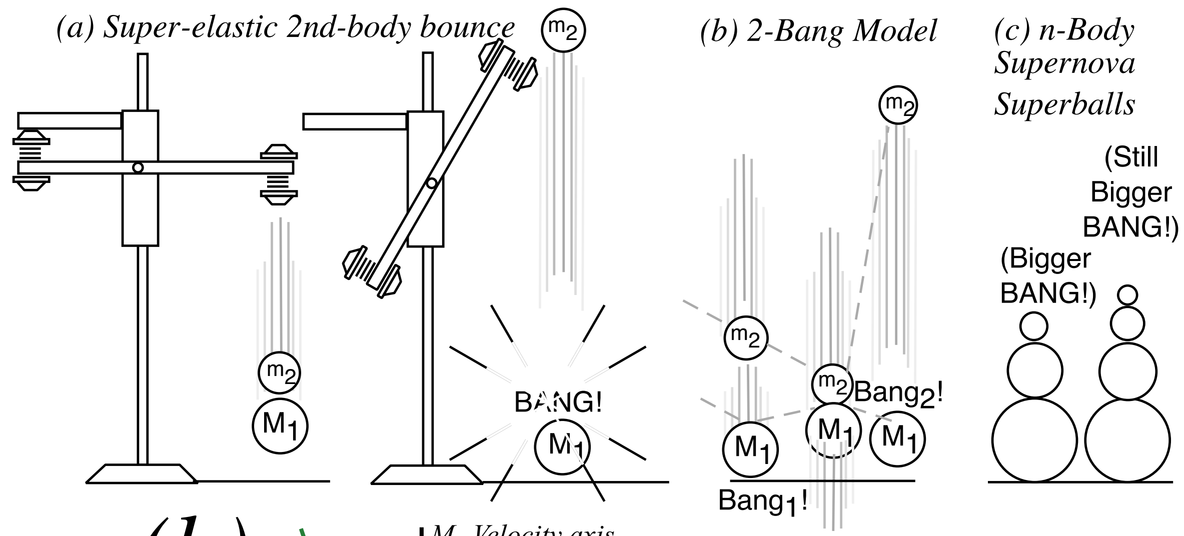


Fig. 4.1 and Fig. 4.3 in Unit 1



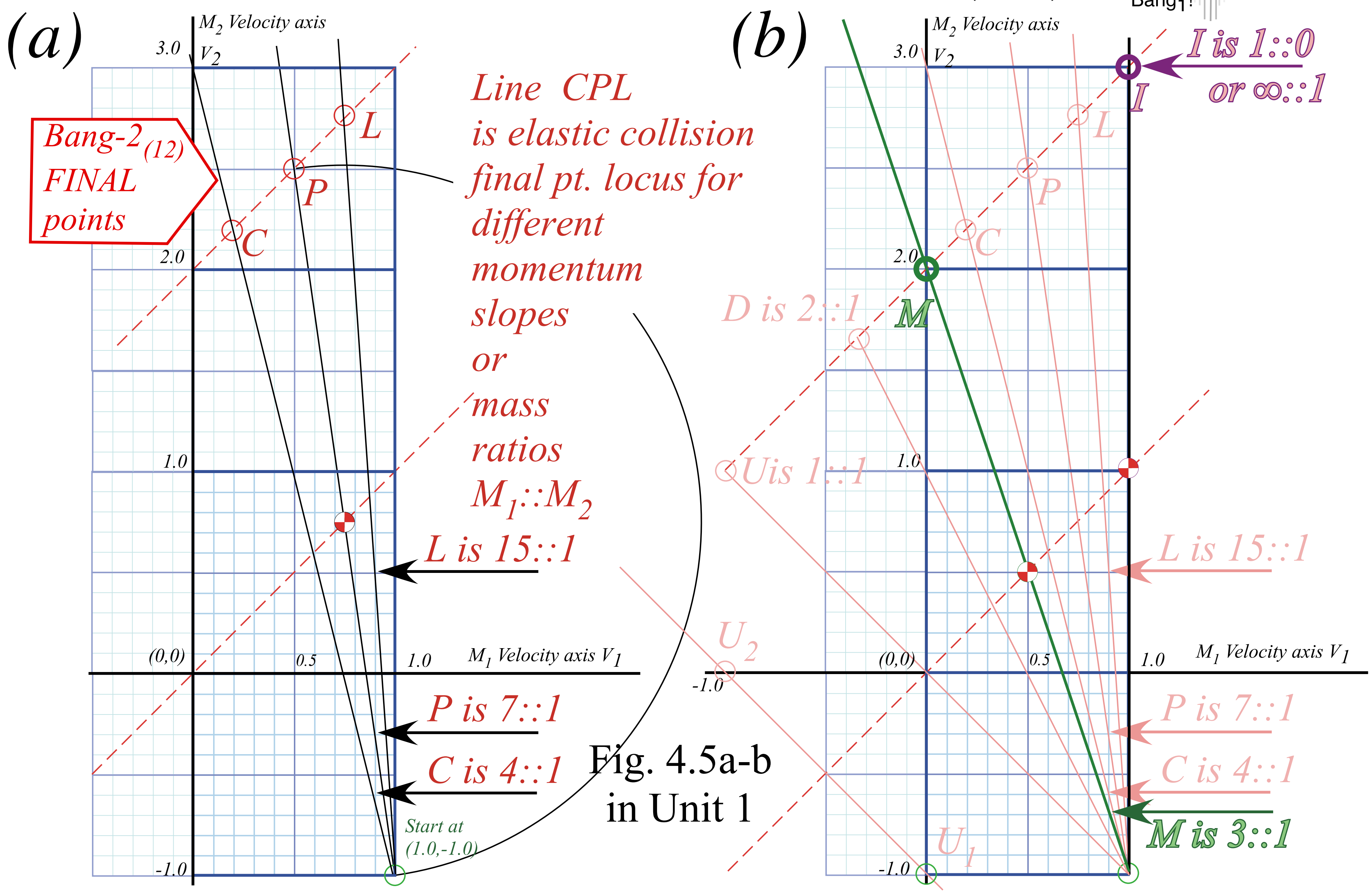


Fig. 4.5a-b in Unit 1

Geometry of X2 launcher bouncing in box

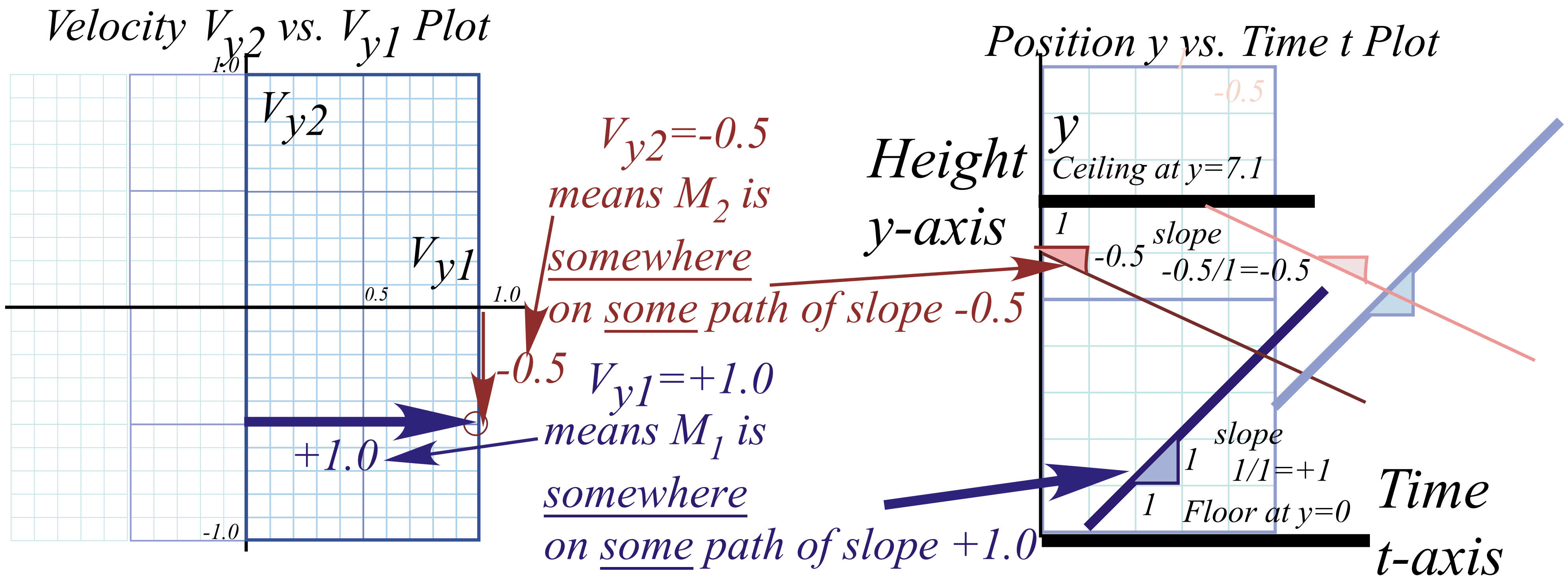
Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

→ *Integration of* (V_1,V_2) *data to space-time plots* $(y_1(t),t)$ *and* $(y_2(t),t)$ *plots*

Integration of (V_1,V_2) *data to space-space plots* (y_1, y_2)

Geometric "Integration" (Converting Velocity data to Spacetime)



Geometric "Integration" (Converting Velocity data to Spacetime)

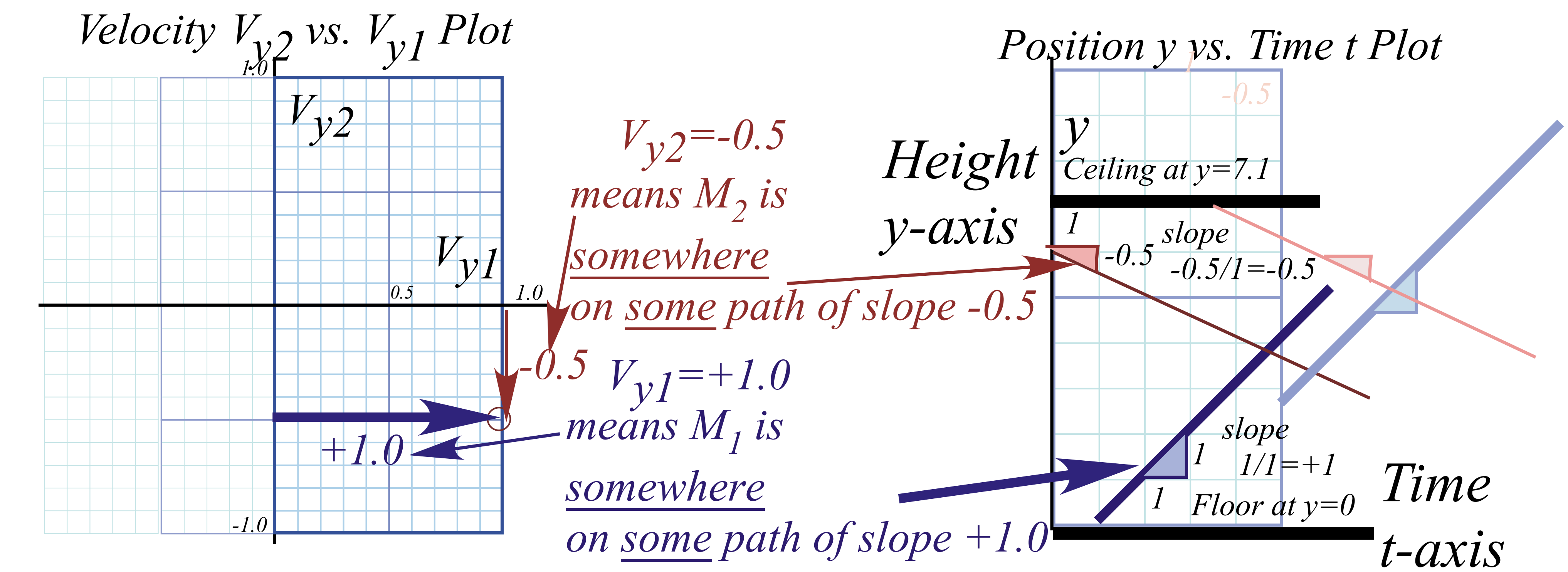
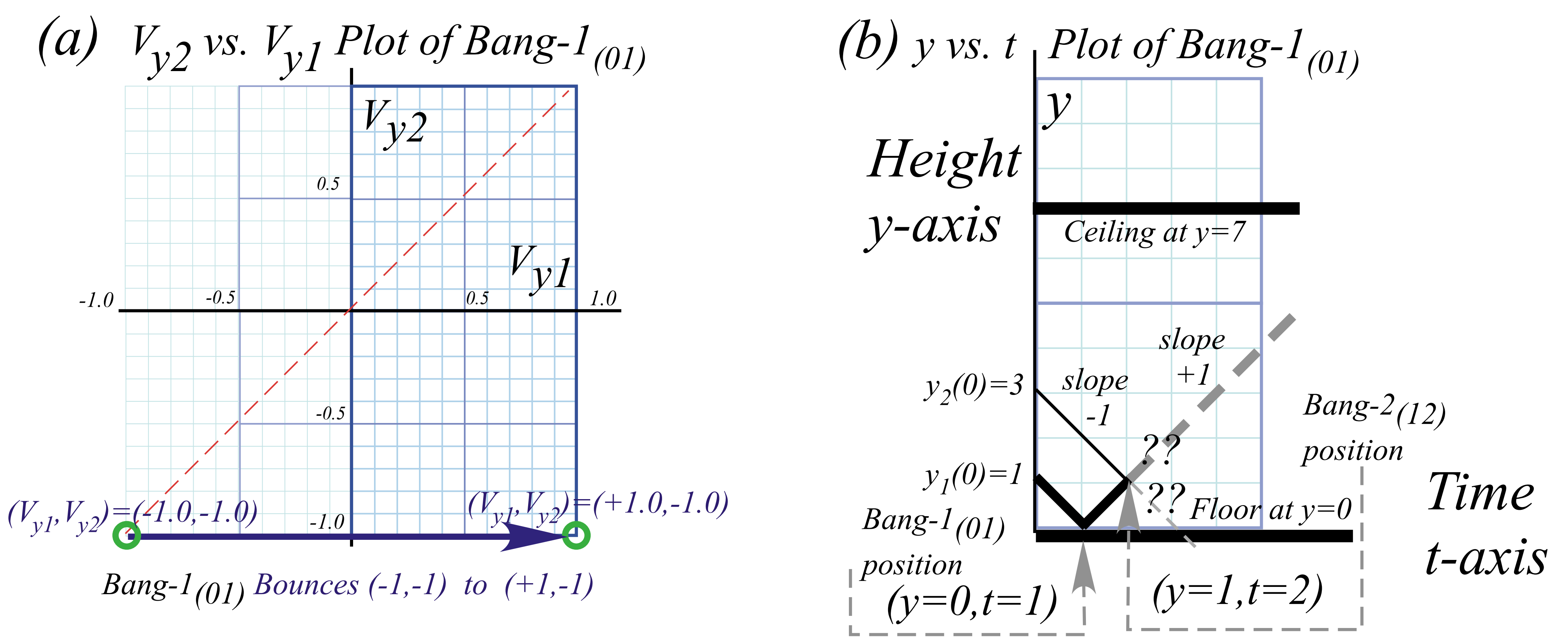
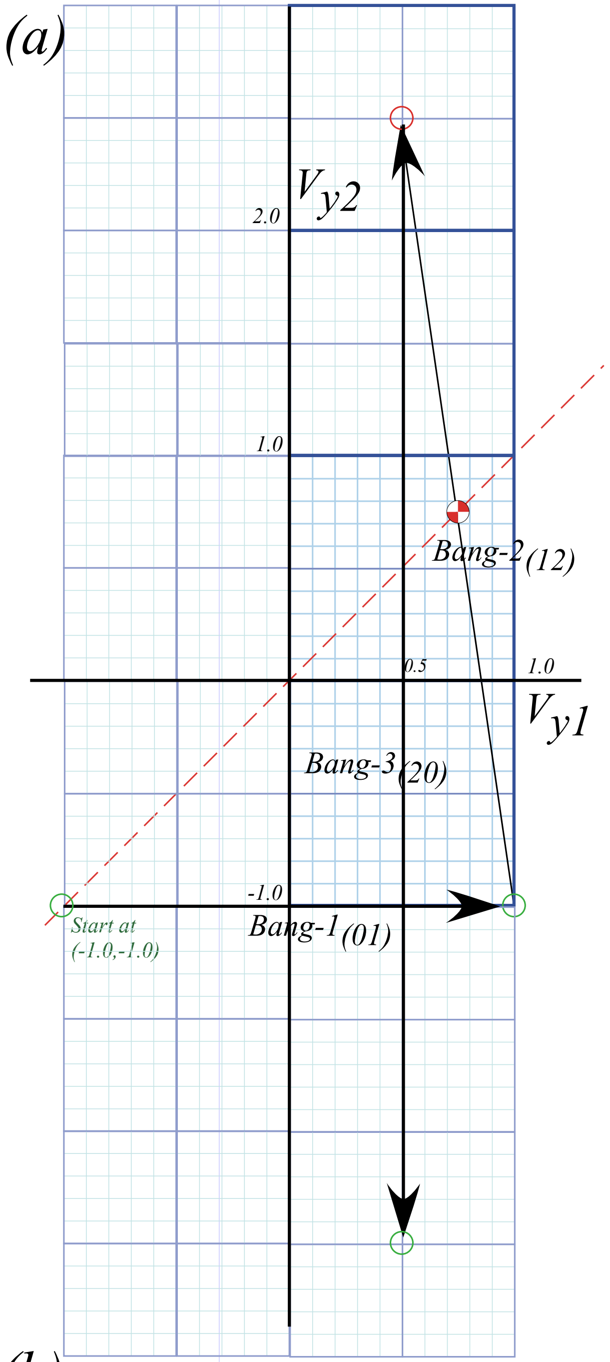
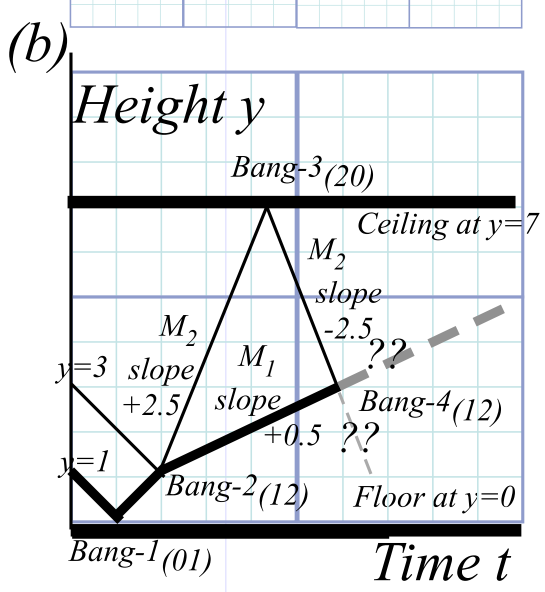


Fig. 4.6a-b
in Unit 1



Geometric "Integration" (Converting Velocity data to Spacetime)





Geometric "Integration" (Converting Velocity data to Spacetime)

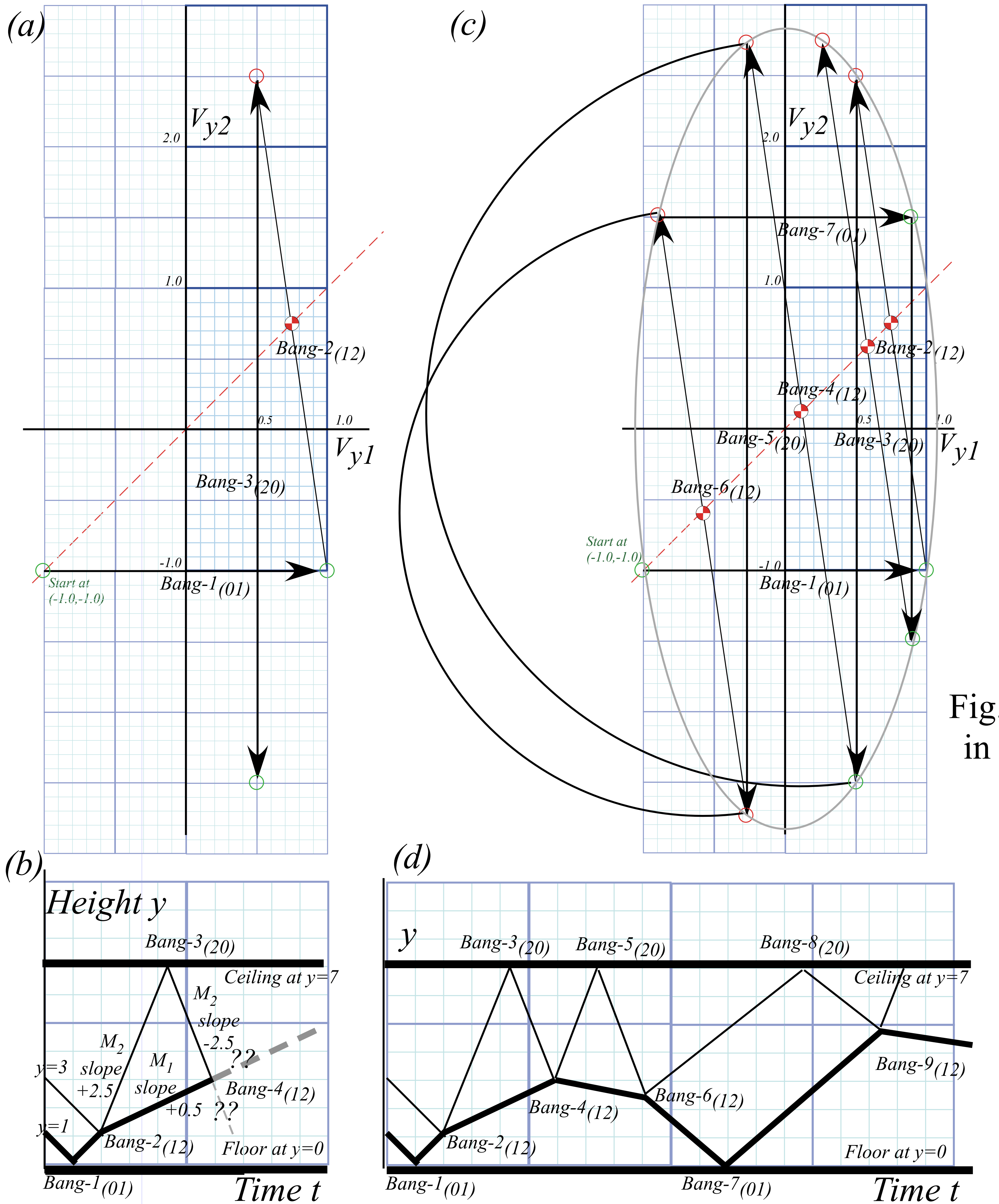


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Geometric "Integration" (Converting Velocity data to Spacetime)

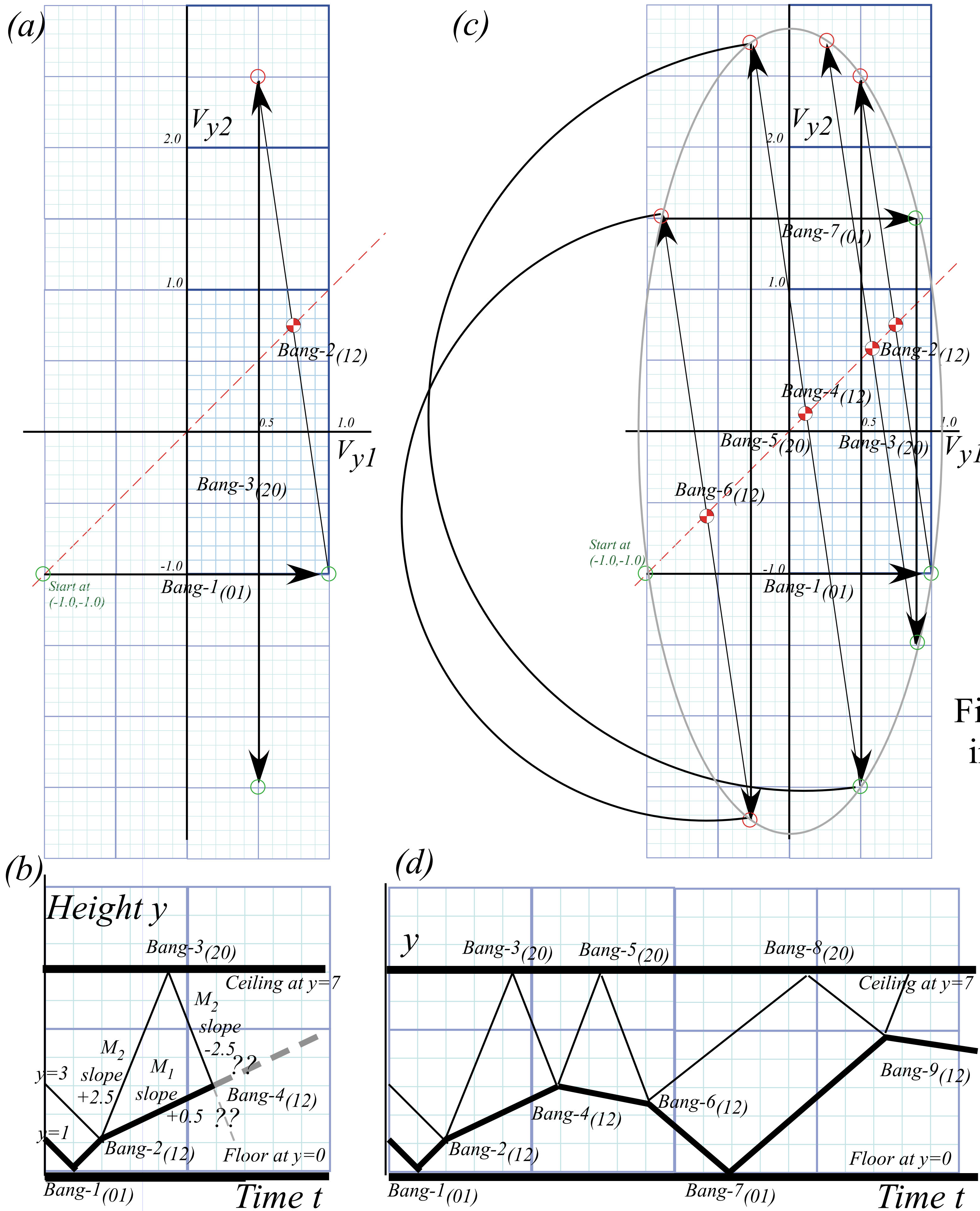


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

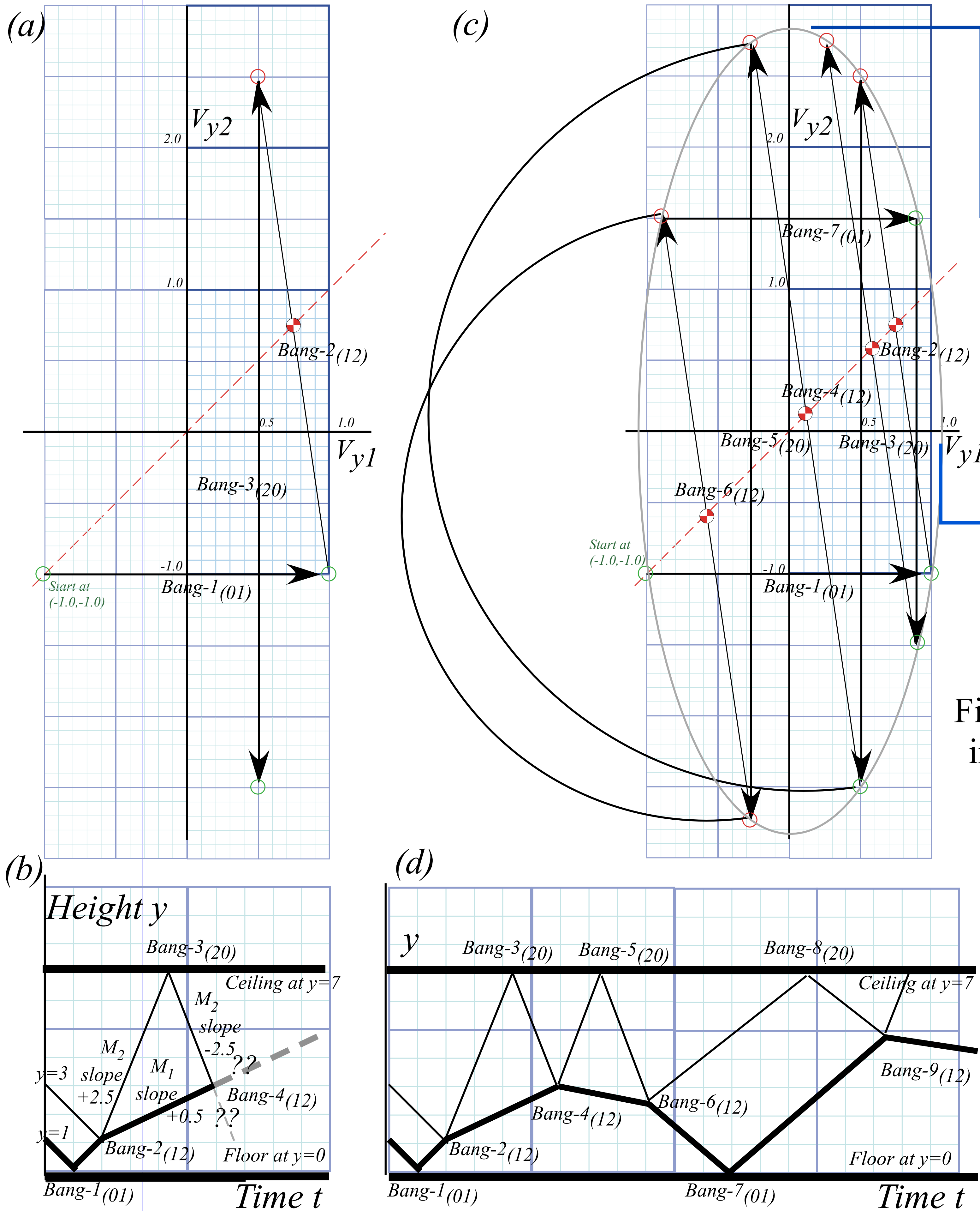
Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1}$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1}$$

Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Fig. 4.7a-d in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)

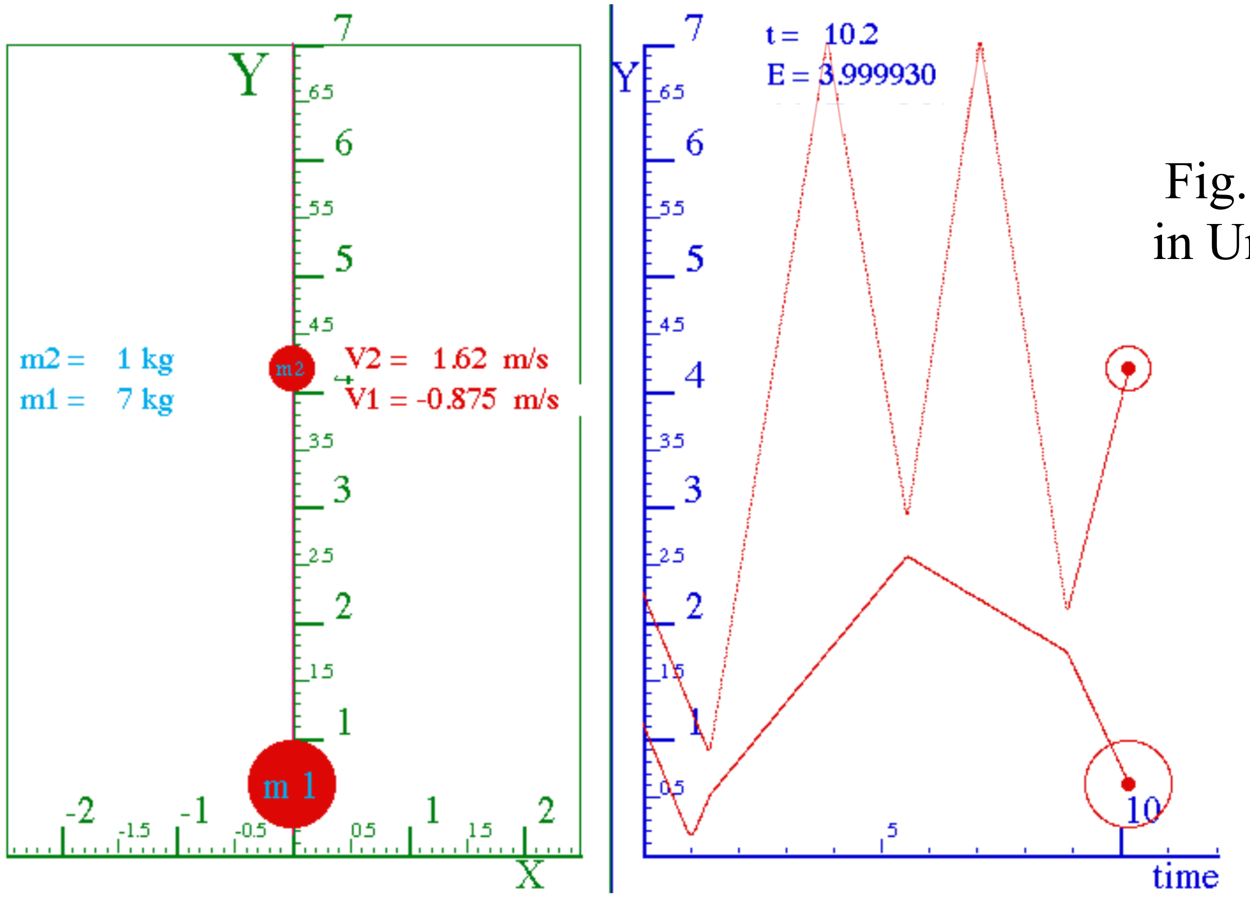


Fig. 4.8
in Unit 1

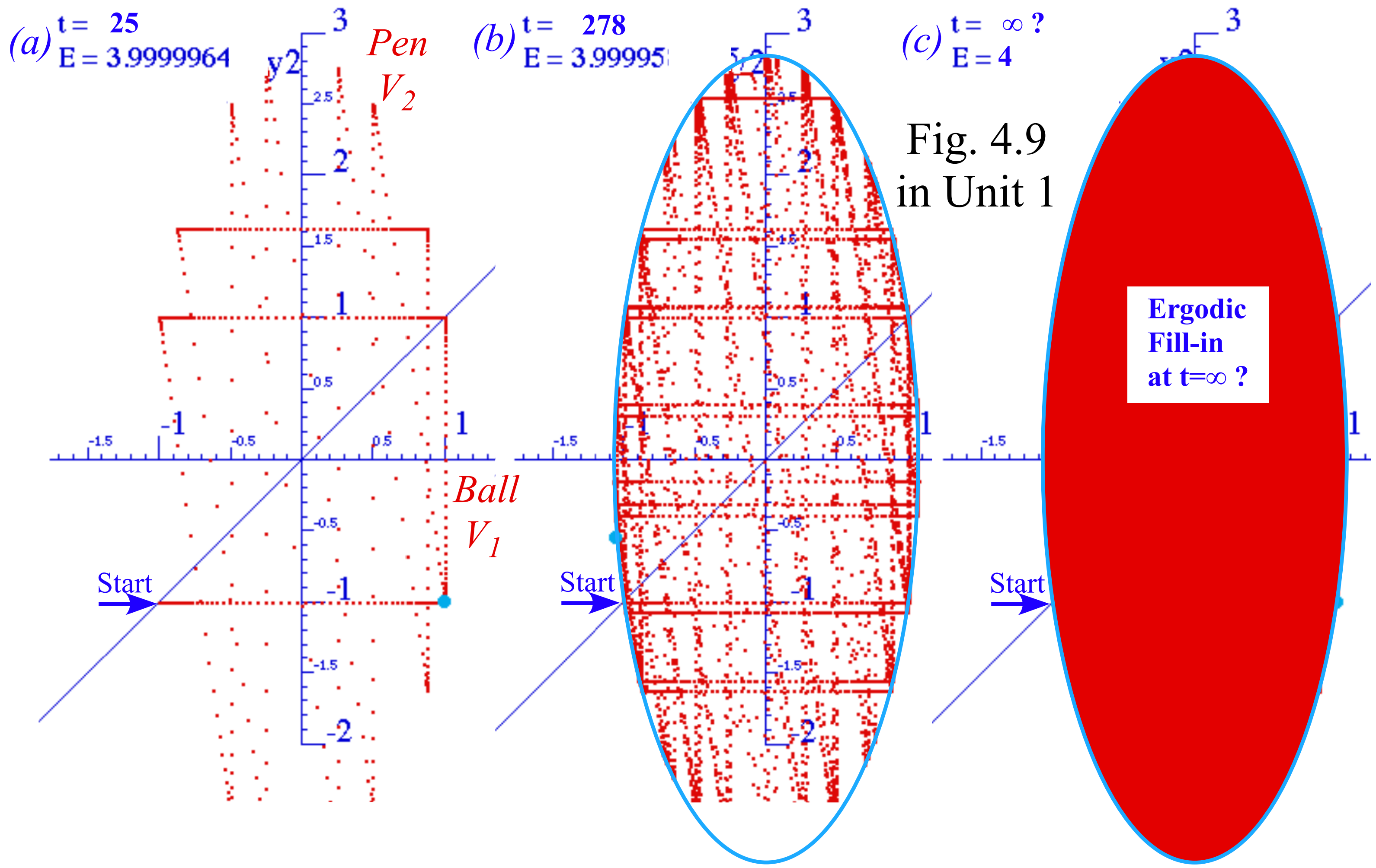


Fig. 4.9
in Unit 1

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

→ *Integration of (V_1,V_2) data to space-space plots (y_1, y_2)*

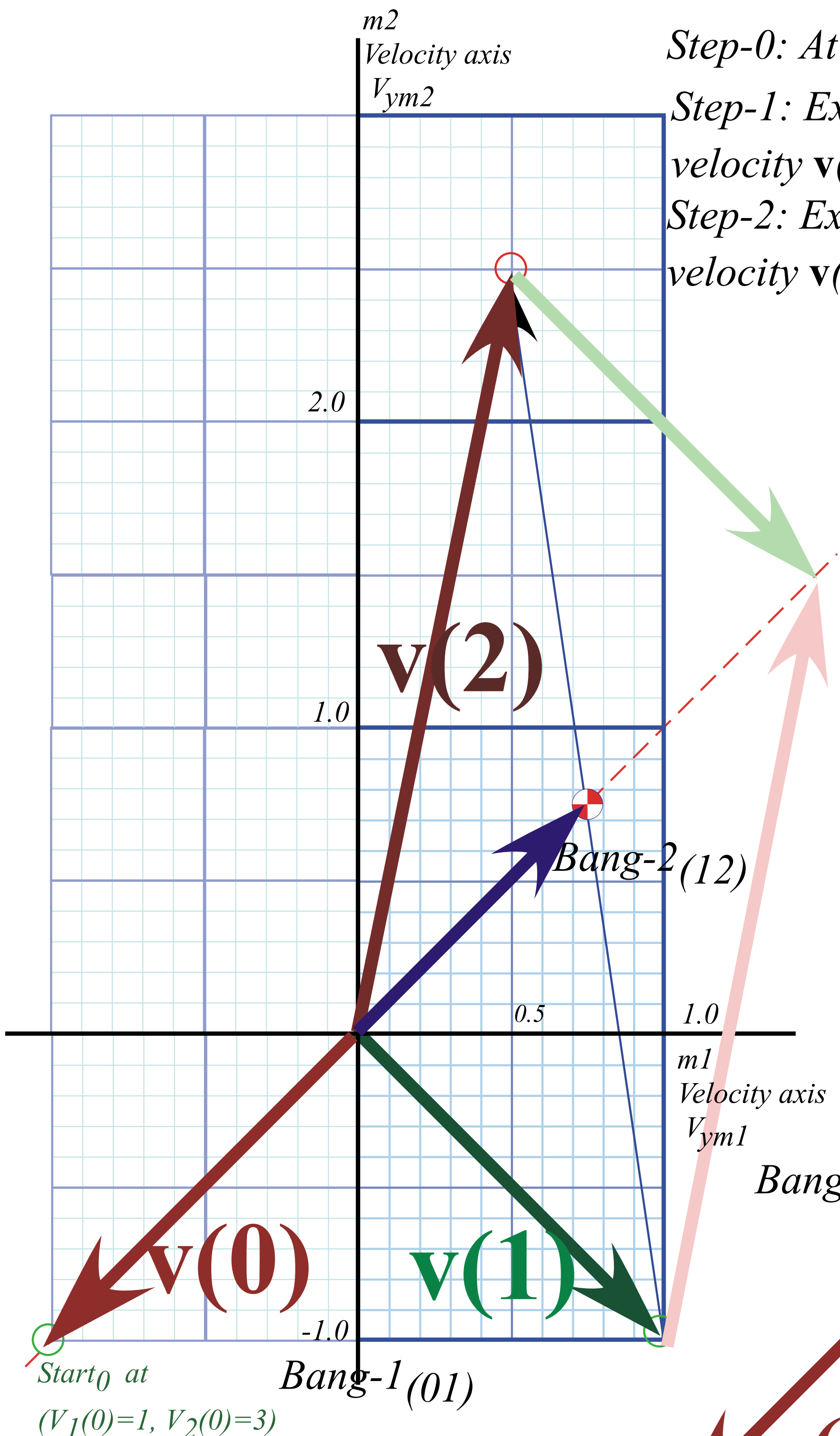
Geometric "Integration" (Converting Velocity data to Space-space trajectory)

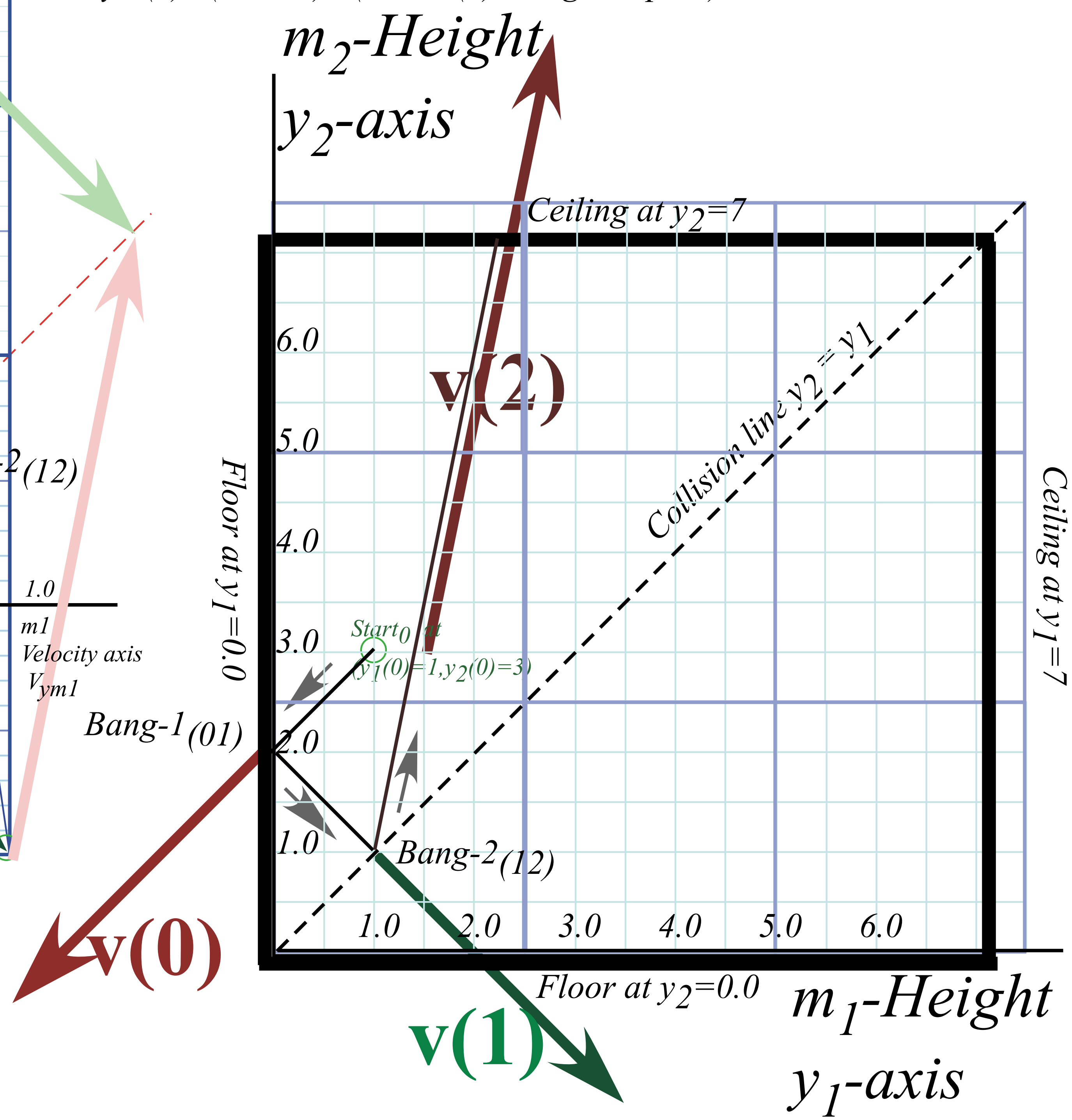
Fig. 4.11
in Unit 1

Step-0: At starting position **y***(0)=(1,3) draw initial velocity* **v***(0)=(-1,-1) line.*

Step-1: Extend **v***(0) line to floor point* **y***(0)=(0,?) and draw Bang-1$_{(01)}$ velocity* **v***(1)=(1,-1) line. (Find* **v***(1) using V-V plot.)*

Step-2: Extend **v***(1) line to collision point* **y***(0)=(?,?) and draw Bang-2$_{(12)}$ velocity* **v***(2)=(0.5,2.5). (Find* **v***(2) using V-V plot.)*





Geometric "Integration" (Converting Velocity data to Space-space trajectory)

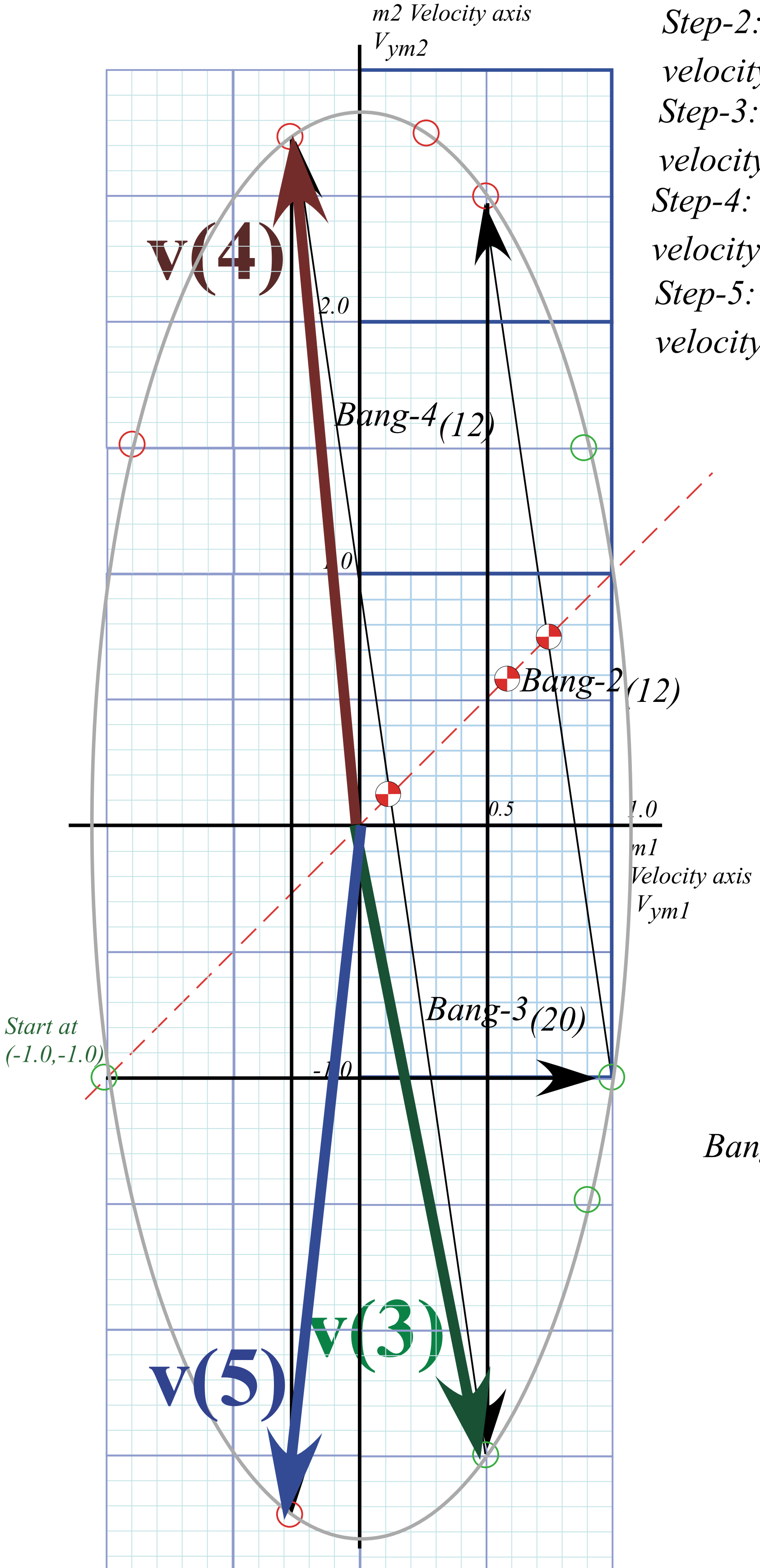
Fig. 4.11
in Unit 1

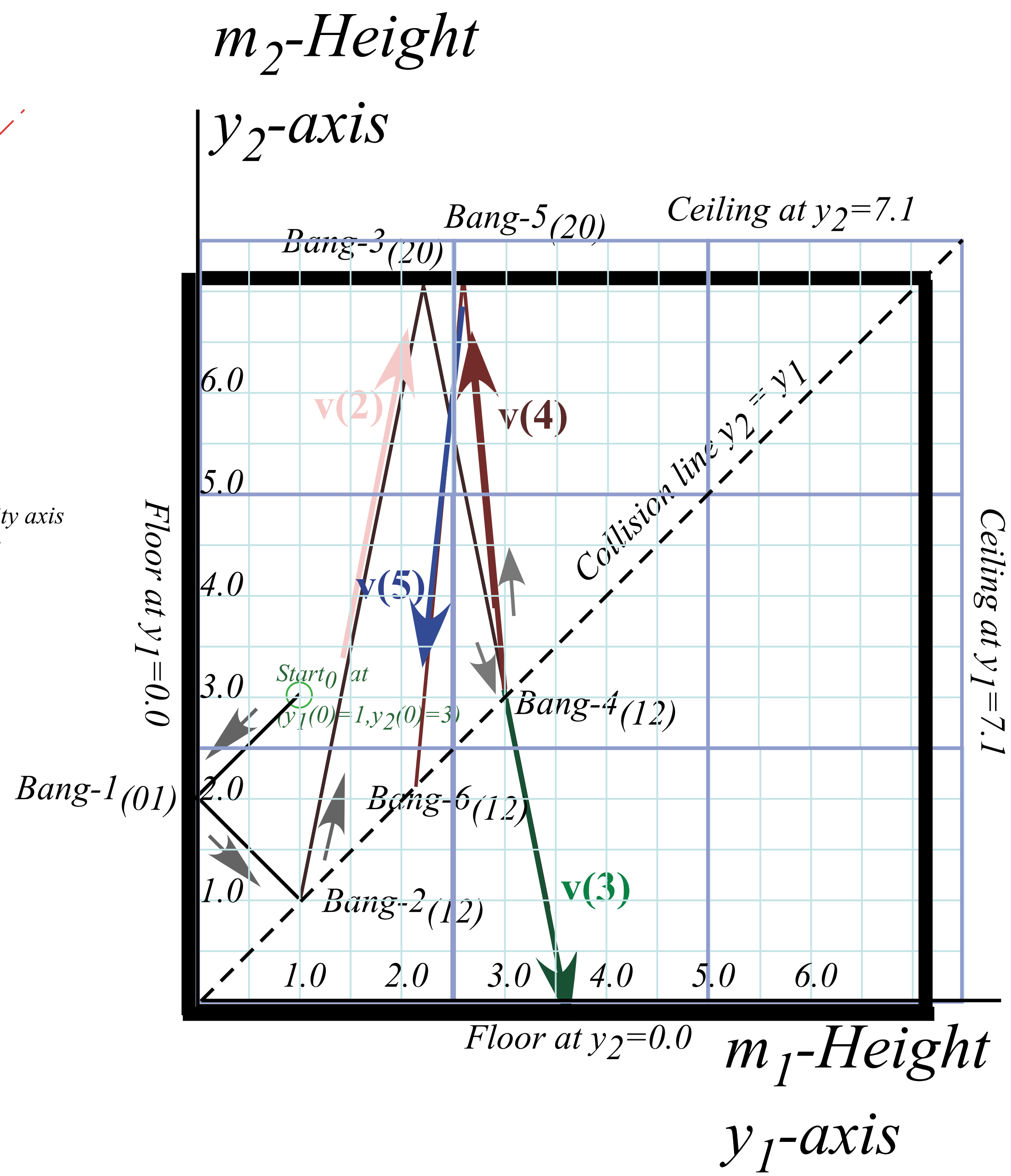
Step-2: Extend **v**(2) *line to ceiling point* **y**(3)=(?,7.1) *and draw Bang-3*$_{(20)}$ *velocity* **v**(3)=(1,-1) *line. (Find* **v**(3) *using V-V plot.)*

Step-3: Extend **v**(3) *line to collision point* **y**(4)=(?,?) *and draw Bang-4*$_{(12)}$ *velocity* **v**(4)=(0.5,2.5). *(Find* **v**(4) *using V-V plot.)*

Step-4: Extend **v**(4) *line to ceiling point* **y**(4)=(?,7.1) *and draw Bang-5*$_{(20)}$ *velocity* **v**(5)=(1,-1) *line. (Find* **v**(5) *using V-V plot.)*

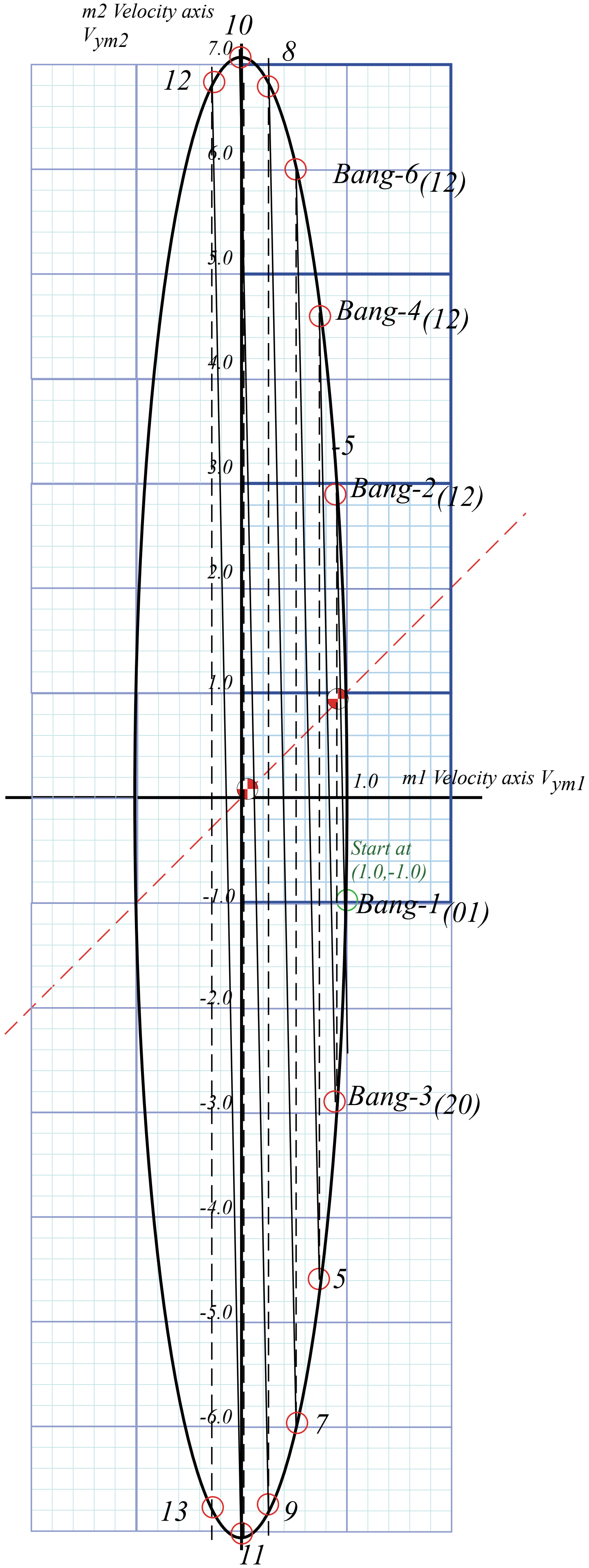
Step-5: Extend **v**(5) *line to collision point* **y**(6)=(?,?) *and draw Bang-6*$_{(12)}$ *velocity* **v**(6)=(0.5,2.5). *(Find* **v**(6) *using V-V plot.)*





Geometric "Integration" (Converting Velocity data to Space-time trajectory)

Example with masses: $m_1=49$ and $m_2=1$



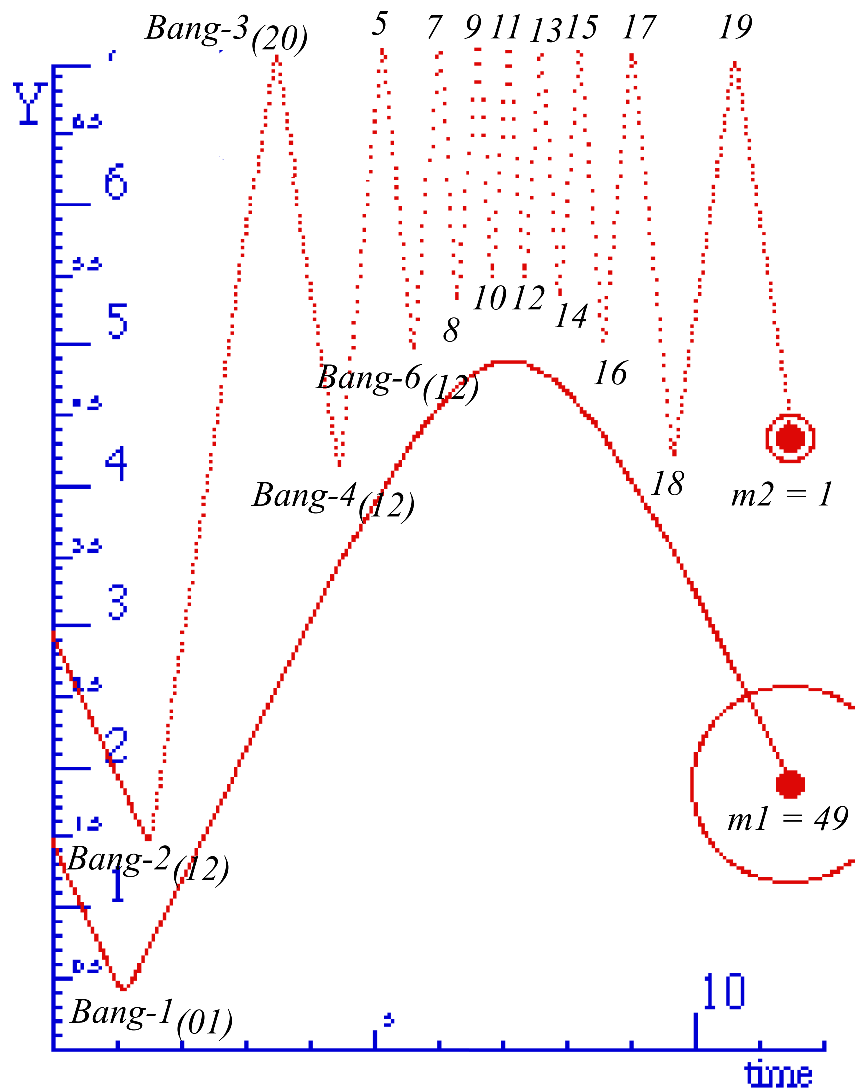
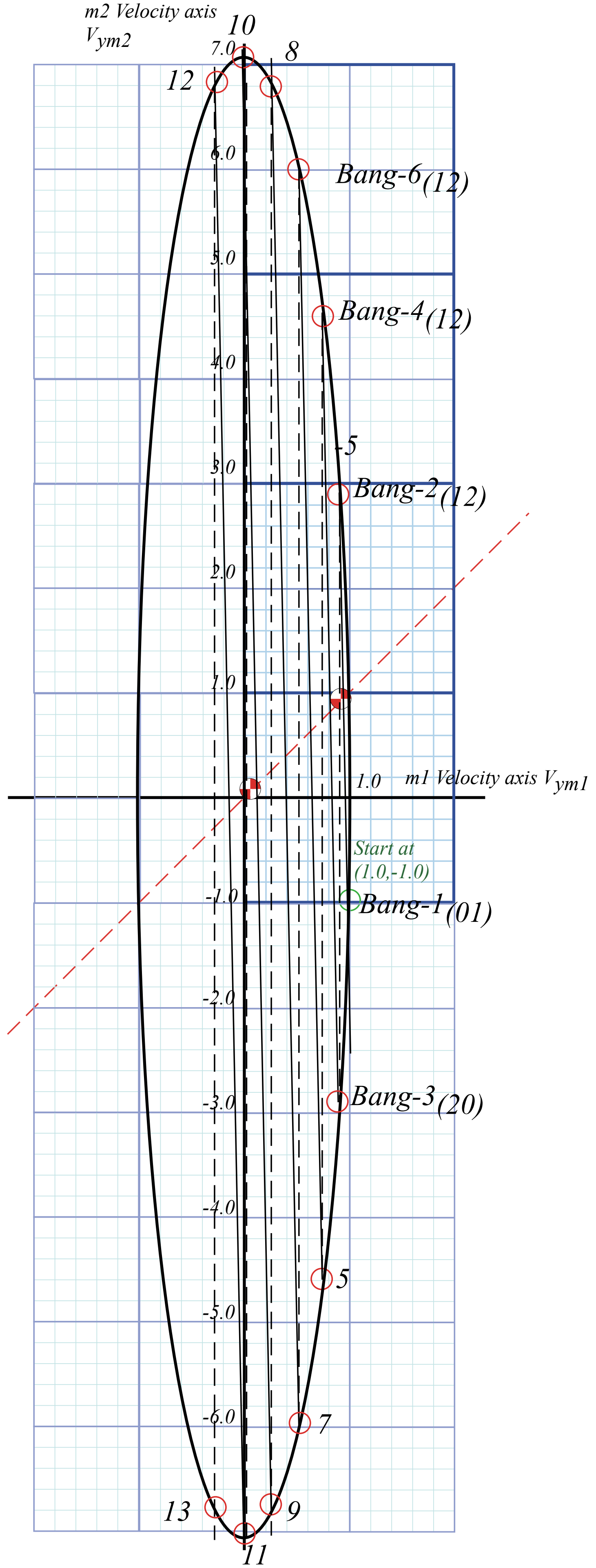


Fig. 5.1
in Unit 1

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

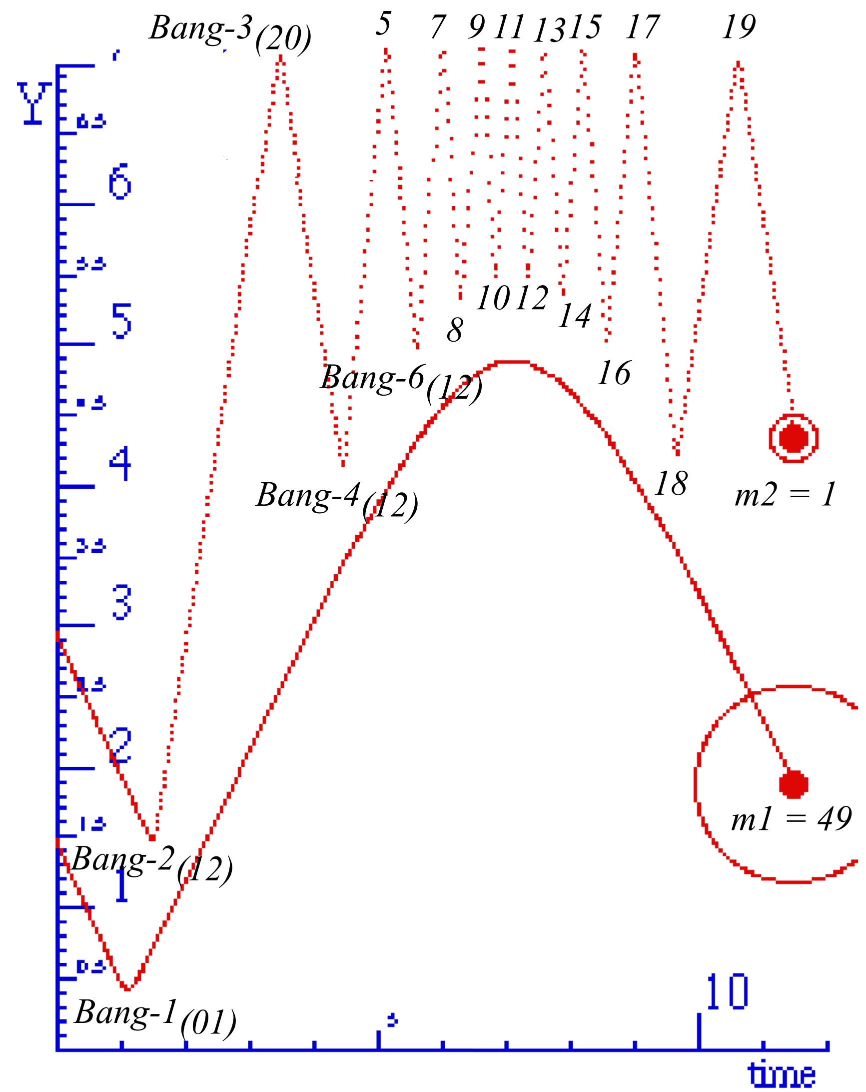


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor bounce **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass collision **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling bounce **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1 = 49$ *and* $m_2 = 1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define a "rotation" **R** as *group product*: $\mathbf{R} = \mathbf{C} \bullet \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

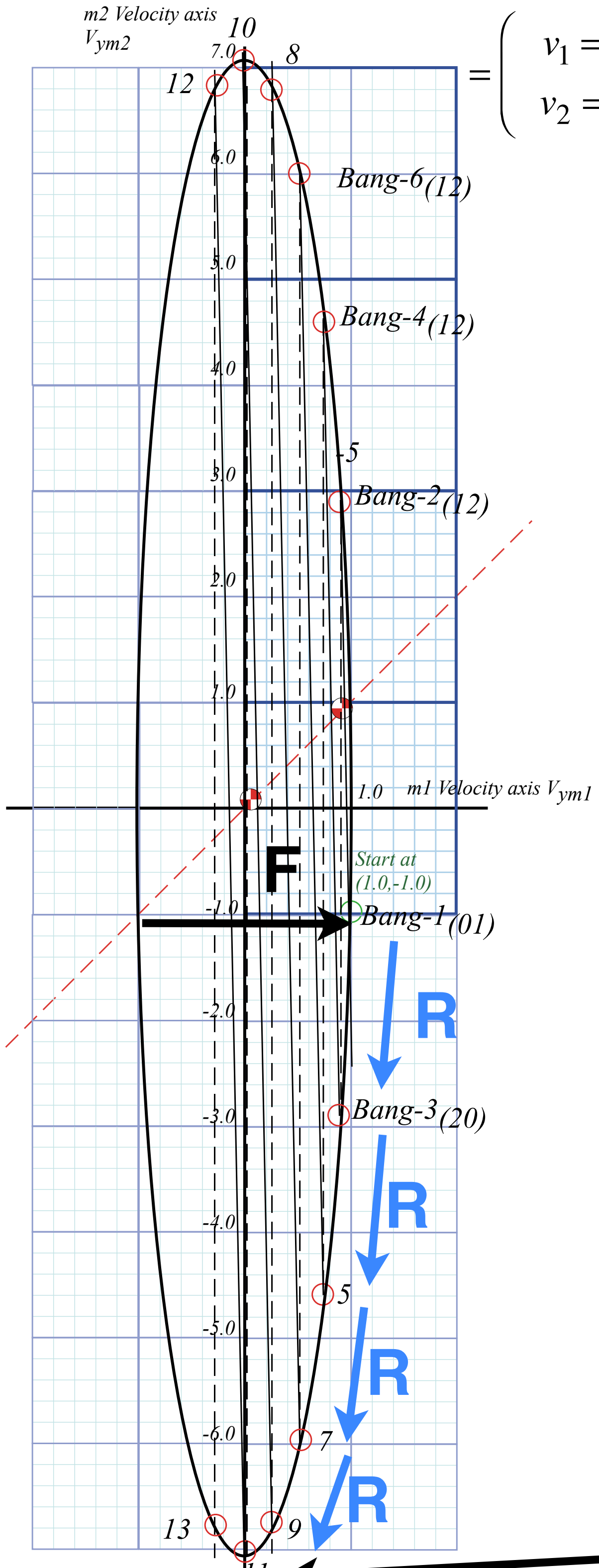
$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

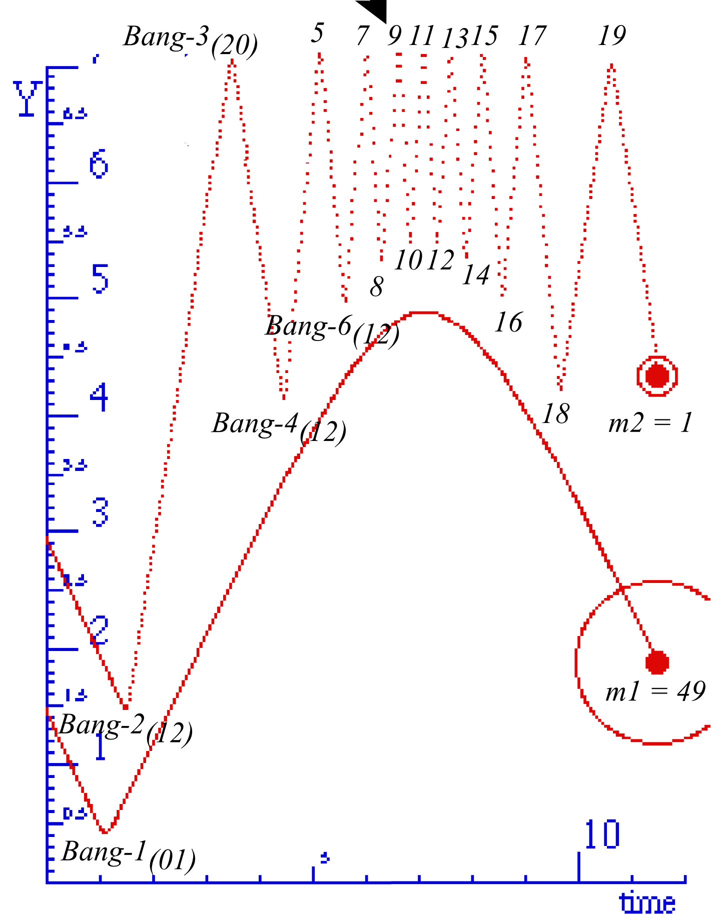
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1=1 \\ v_2=-1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{\text{(after Bang-11)}}$$