Lecture 30 Relawavity-Applications 4
Thursday 4.28.2016

Relawavity: Relativistic quantum mechanics I Basic theory

(Unit 3 p.45-61 - 4.26.16)

➔ Using (some) wave parameters to develop relativistic quantum theory
- Low rapidity approximations to υ_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum
  - How Mc^2 pops right up
- Exact υ_{phase} gives exact Planck-Einstein energy formulas (1900-1905)
- Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)
  - Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

"*What's the Matter with Mass?*" Definition(s) of relativistic and quantum mechanical mass
- (1) Einsteinian rest mass (2) Galilean momentum mass (3)Newtonian effective mass
  - Three Faces of Eve: A photon's split-mass personality

Relativistic action S and Lagrangian-Hamiltonian relations: How invariant phase works
- The Legendre transformation relations
- Deriving Lagrangian and Hamiltonian functions
  - Geometry of 1st Lagrangian and 1st Hamiltonian equations
- Poincare invariant action differential
  - Hamilton-Jacobi equations
    - How Hamilton-Jacobi derives Schrodinger-op equations
      - How Huygens contact transformations determine motion

Using (some) wave parameters to develop relativistic quantum theory

➔ Low rapidity approximations to υ_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

Exact υ_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the* Mc^2 *?!!*)

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Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

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At low speeds:

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2$

$\sinh\rho \approx \rho$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

Using (some) wave parameters to develop relativistic quantum theory

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation

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Lucky coincidences?? *Cheap trick??*

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

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(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to υ_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

➔ Exact υ_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the* Mc^2 *?!!*)

"*What's the Matter with Mass?*" Definition(s) of relativistic and quantum mechanical mass

(1) Einsteinian rest mass (2) Galilean momentum mass (3)Newtonian effective mass

Three Faces of Eve: A photon's split personality

Relativistic action *S* and Lagrangian-Hamiltonian relations: How invariant phase works

The Legendre transformation relations

Deriving Lagrangian and Hamiltonian functions

Geometry of 1st Lagrangian and 1st Hamiltonian equations

Poincare invariant action differential

Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion

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At low speeds:

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So attach scale factor h to match units.

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Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation

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$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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(old-fashioned notation)

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So attach scale factor h to match units.

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Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

(old-fashioned notation)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Max Planck 1858-1947

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

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$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h *(or hN)* to match units.

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$

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Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

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$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

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This motivates the "particle" normalization

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For more visit the Pirelli Challenge Site
Quantized amplitude

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Resolution and dirty secret: E, N, and υ_{phase} are all frequencies!

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Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to υ_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

How Mc^2 pops right up

Exact υ_{phase} gives exact Planck-Einstein energy formulas (1900-1905)

➔ Exact $c\kappa_{phase}$ gives exact Bohr momentum and dispersion formulas (1921-1927)

Bohr-Schrodinger approximation to dispersion (*Who threw away the Mc^2 ?!!*)

"*What's the Matter with Mass?*" Definition(s) of relativistic and quantum mechanical mass

(1) Einsteinian rest mass (2) Galilean momentum mass (3)Newtonian effective mass

Three Faces of Eve: A photon's split personality

Relativistic action *S* and Lagrangian-Hamiltonian relations: How invariant phase works

The Legendre transformation relations

Deriving Lagrangian and Hamiltonian functions

Geometry of 1st Lagrangian and 1st Hamiltonian equations

Poincare invariant action differential

Hamilton-Jacobi equations

How Hamilton-Jacobi derives Schrodinger-op equations

How Huygens contact transformations determine motion

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$


Max Planck 1858-1947 | Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

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~~Lucky coincidences??~~ *Natural wave conspiracy* ~~Cheap~~ *Expensive trick??*

...Try exact υ_{phase} and κ_{phase}...

Need to replace ~~h~~ with *hN to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

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Need to replace ~~h~~ with hN *to match e.m. energy density* $\varepsilon_0 \boldsymbol{E}^{*}\boldsymbol{\cdot}\boldsymbol{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{(old-fashioned notation)}$$

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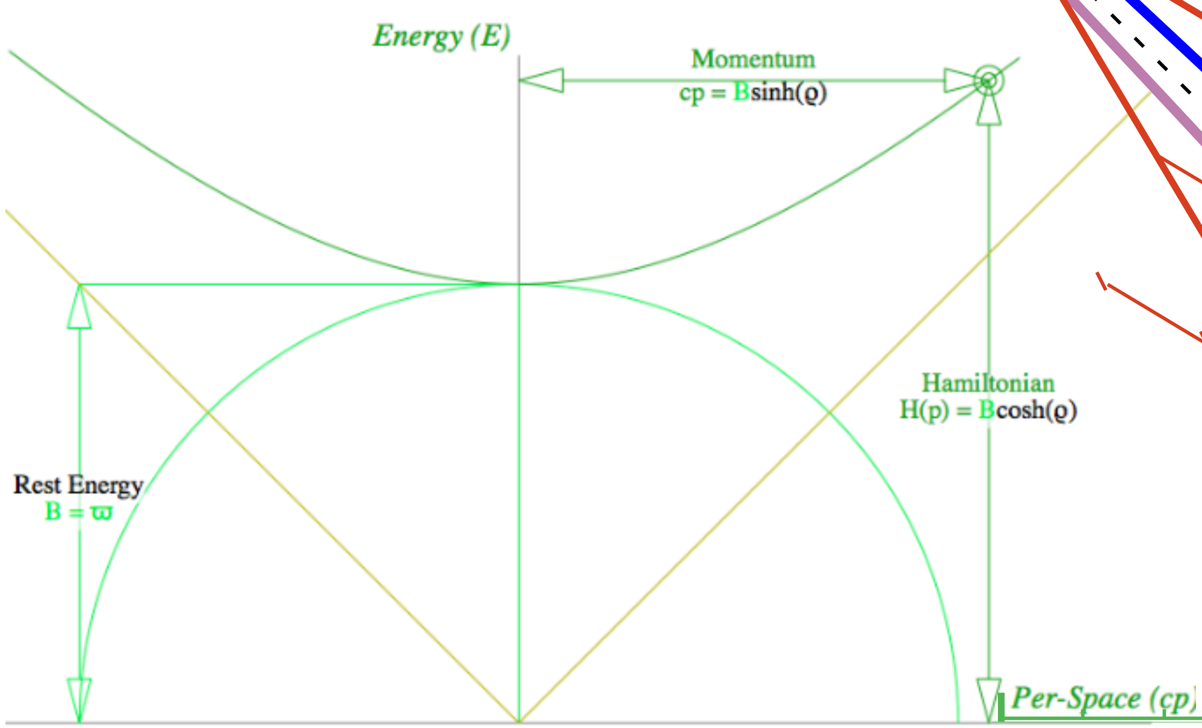
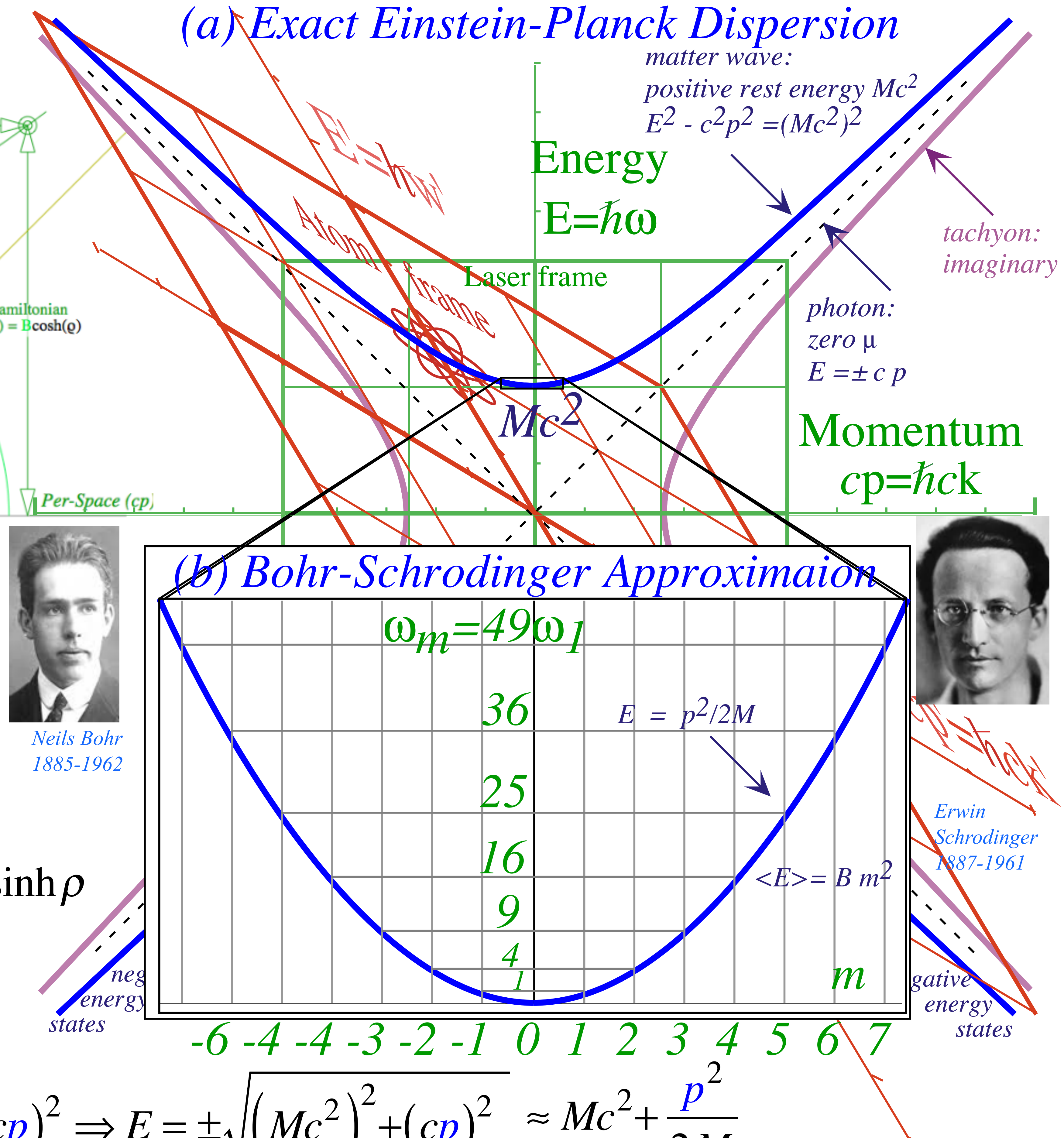
RelaWavity Web Simulation
Relativistic *Terms - Einstein-Plank Dispersion*





Neils Bohr
1885-1962

Erwin Schrodinger
1887-1961

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

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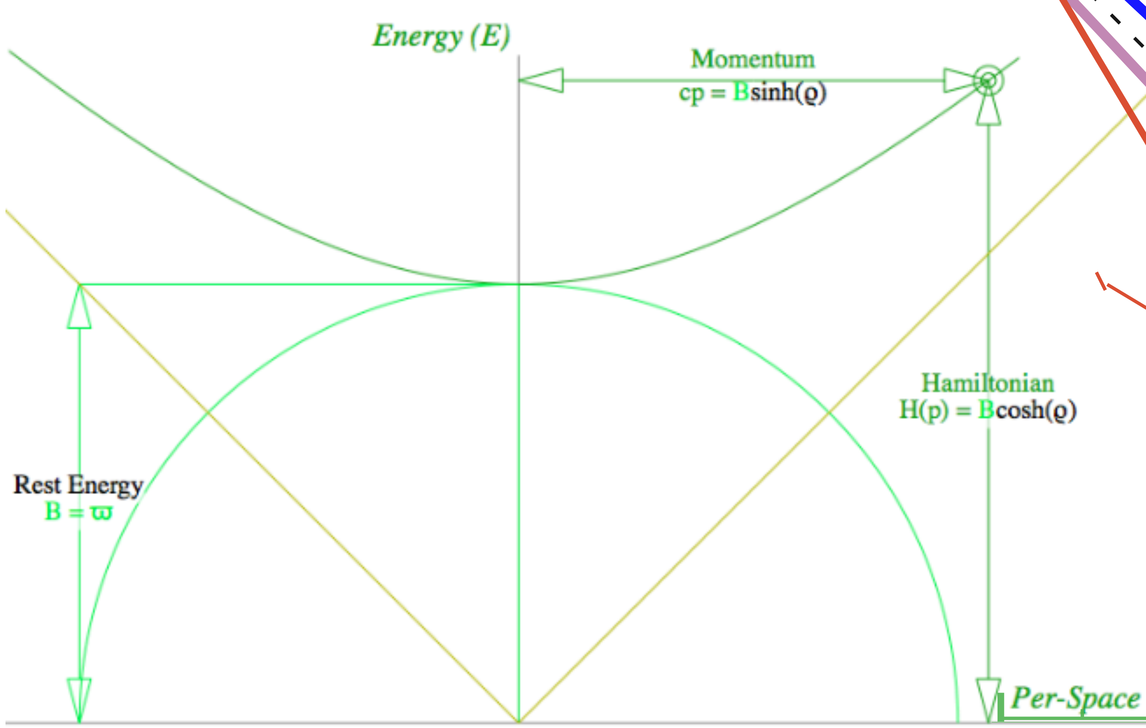

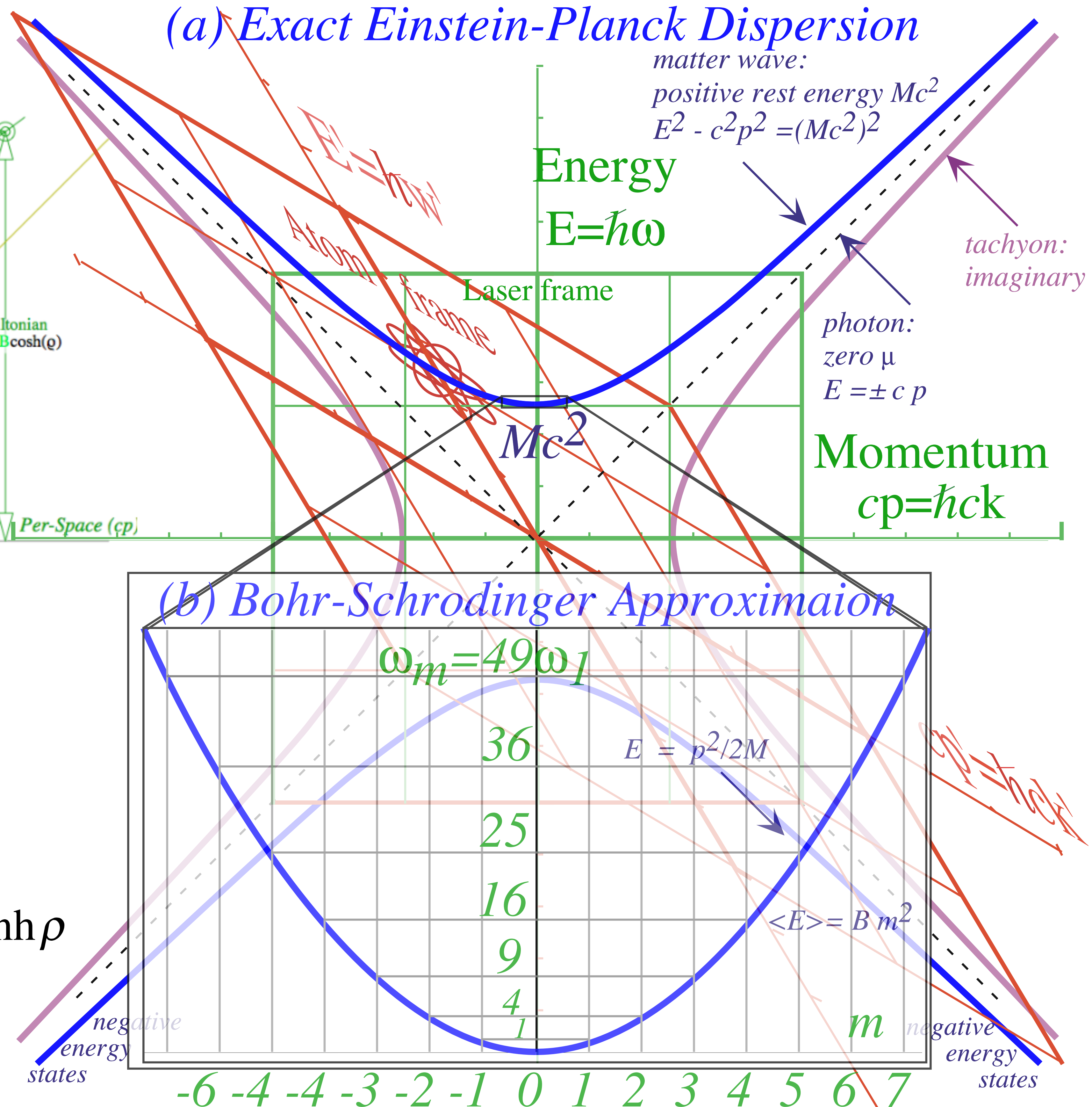

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<u>*Rest Mass*</u> *M_{rest} (Einstein's mass)*

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Defines invariant hyperbola(s)

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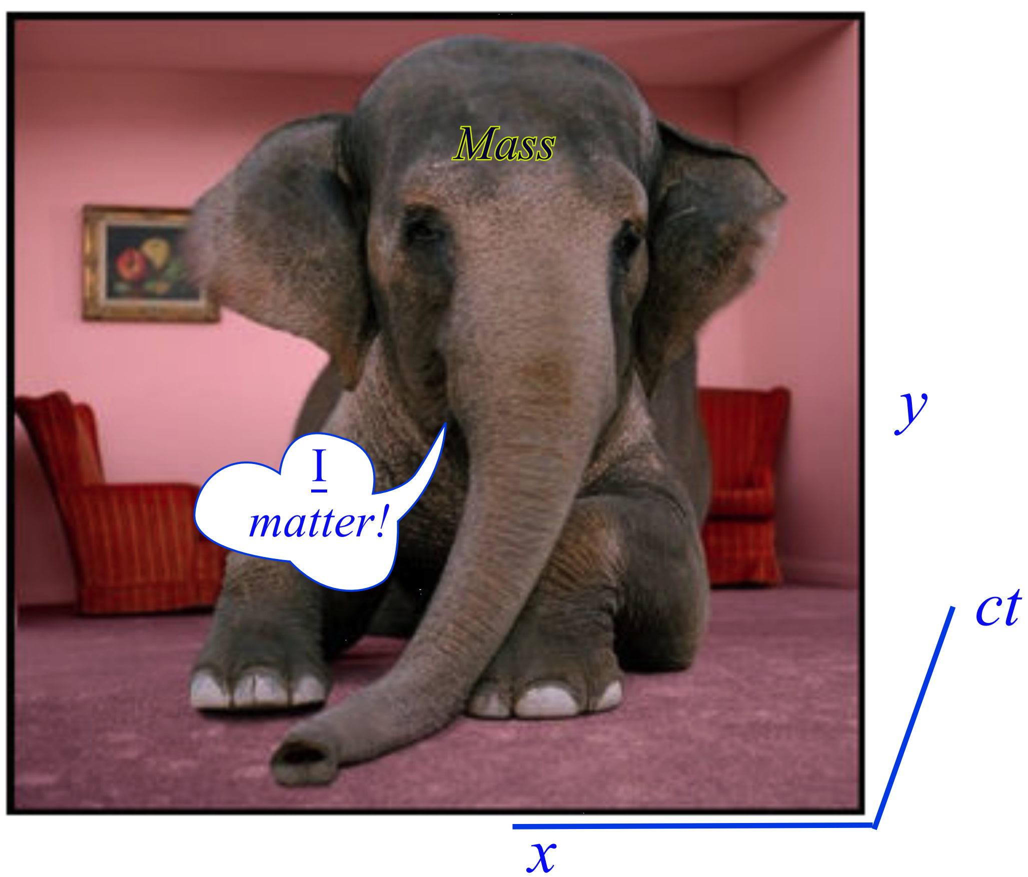
- ***What's the matter with Mass?***



Shining some light on the elephant in the spacetime room

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$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}$$ Momentum Mass

Limiting cases: $M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$

$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho$$ Effective Mass

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Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

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$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

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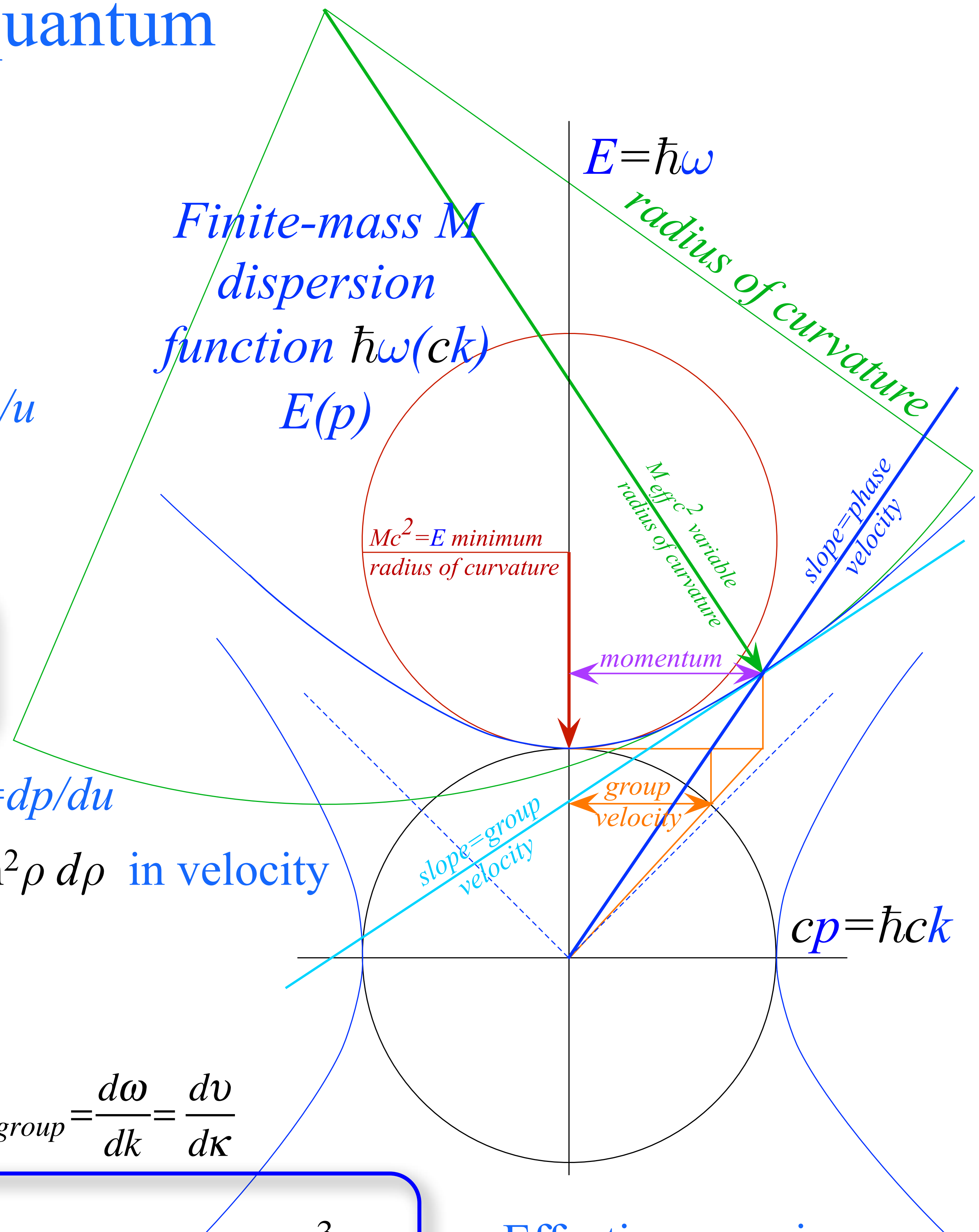

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion.*

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Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass $(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0$,

Momentum Mass $(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \dfrac{p}{c} = \dfrac{h\kappa}{c} = \dfrac{h\upsilon}{c^2}$,

Effective Mass $(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty$.

Definition(s) of mass for relativity/quantum

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s$$

$$= 4.5 \cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

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Newton complained about *his* “corpuscles” of light having “fits” (going crazy).

(For Newton these 3 mass values would be evidence of triple Schizophrenia.)

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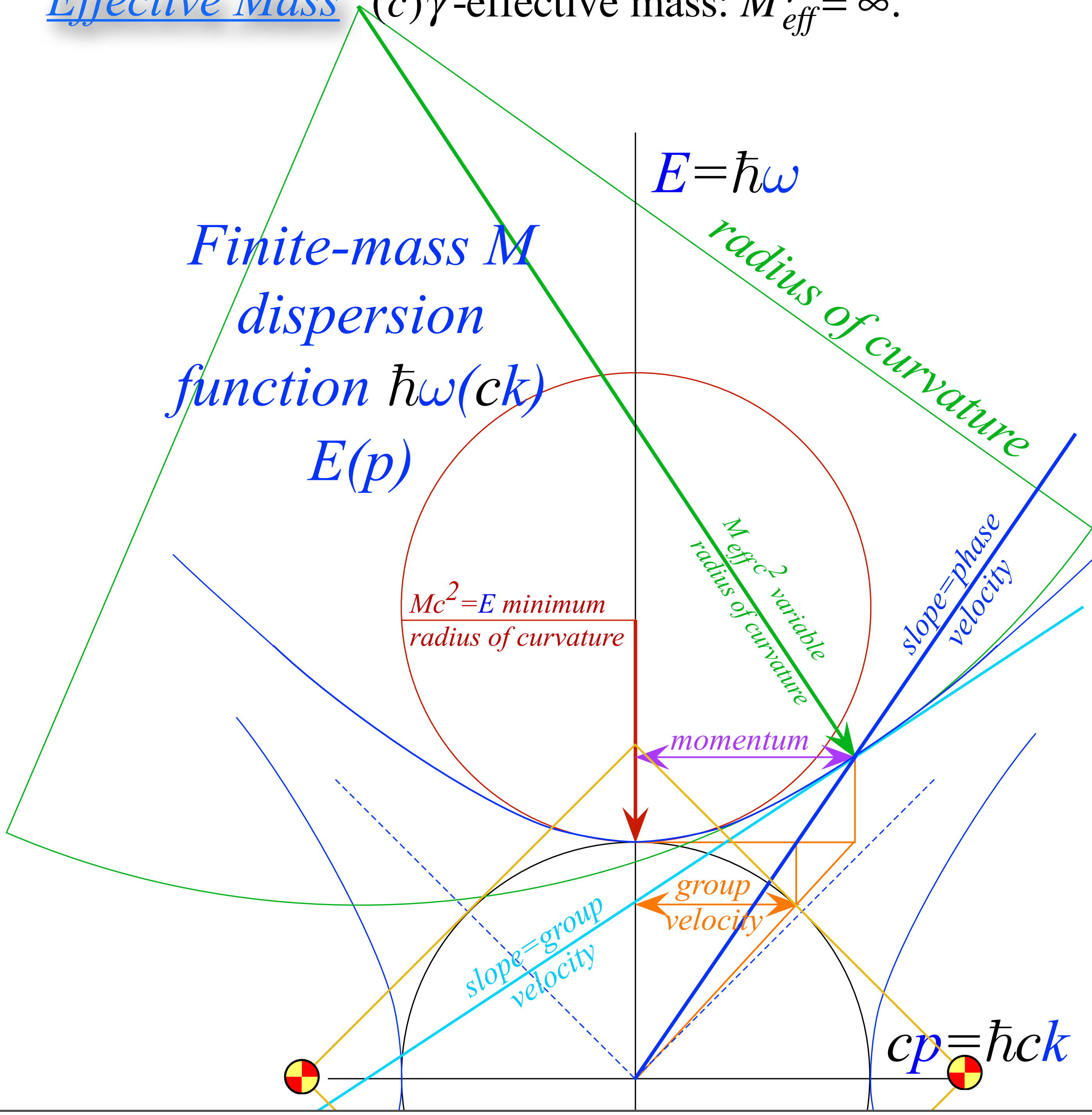
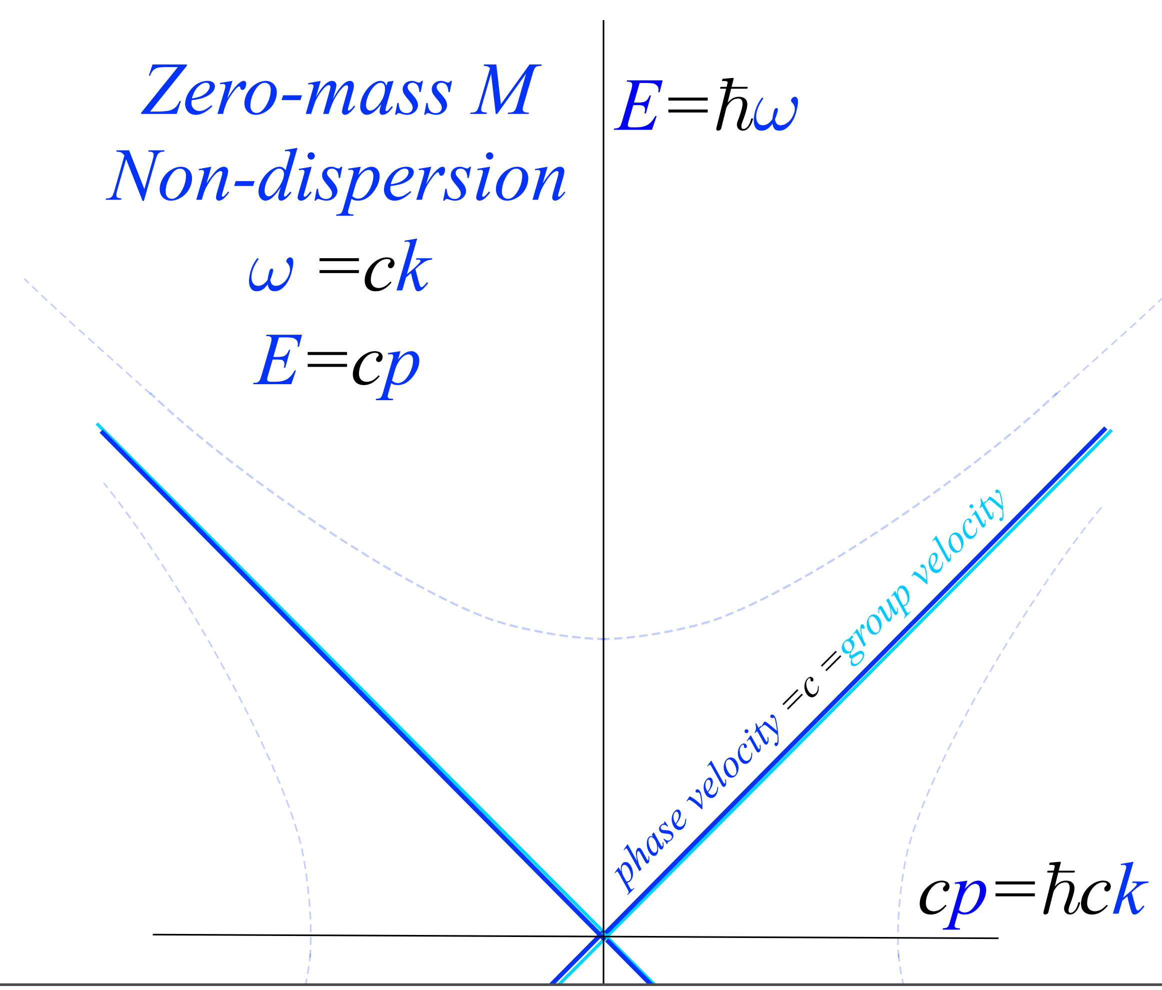


γ-(non)-dispersion has INFINITE radius of curvature



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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

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←linear Hz format

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Legendre transformation

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Use *Group velocity*: $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

Also: $cp = Mc^2 \sinh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho$$

$$= Mc^2\sqrt{1 + \sinh^2\rho} = Mc^2\sqrt{1 + (cp)^2}$$

$\hbar = h/2\pi$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

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←linear Hz format

angular phasor→ format

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Including stellar angle σ

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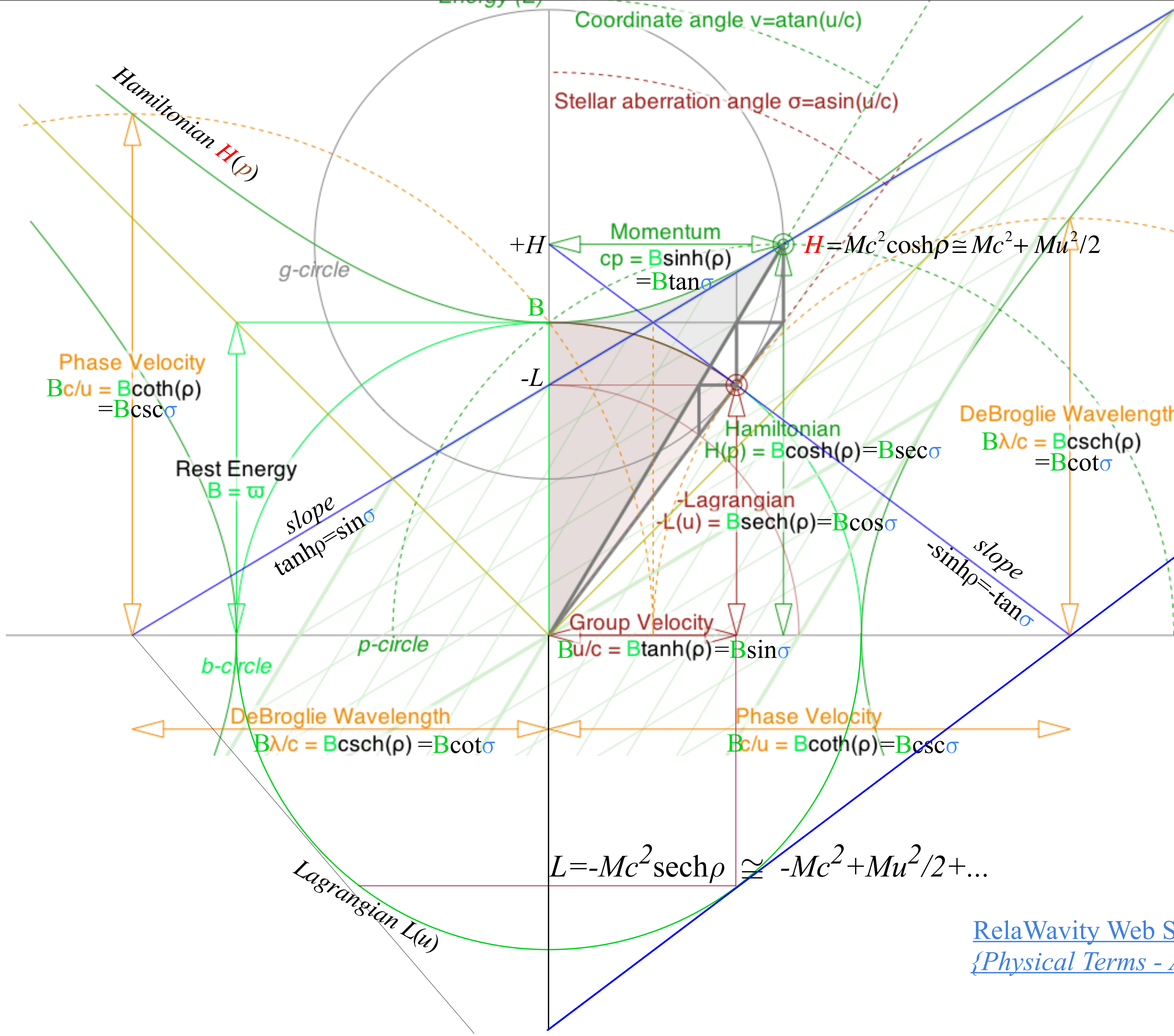
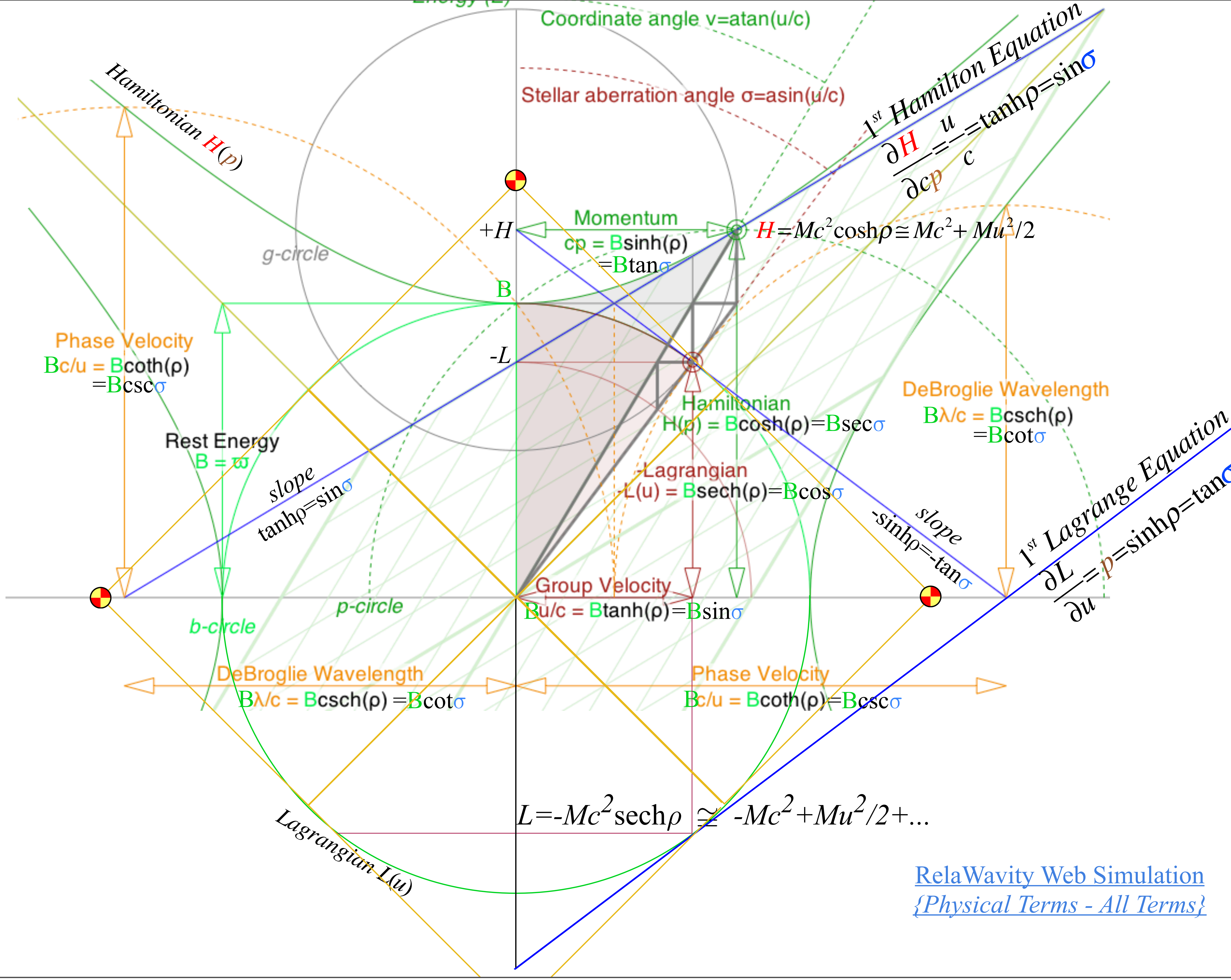


RelaWavity Web Simulation
{Physical Terms - All Terms}


Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(p)
1st Hamilton Equation
∂H/∂cp = u/c = tanhρ=sinσ
g-circle
+H
Momentum
cp = Bsinh(ρ)
=Btanσ
H=Mc²coshρ≅Mc²+ Mu²/2
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
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DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
tanhρ=sinσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
∂L/∂u = p=sinhρ=tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≅ -Mc²+Mu²/2+...
Lagrangian L(u)
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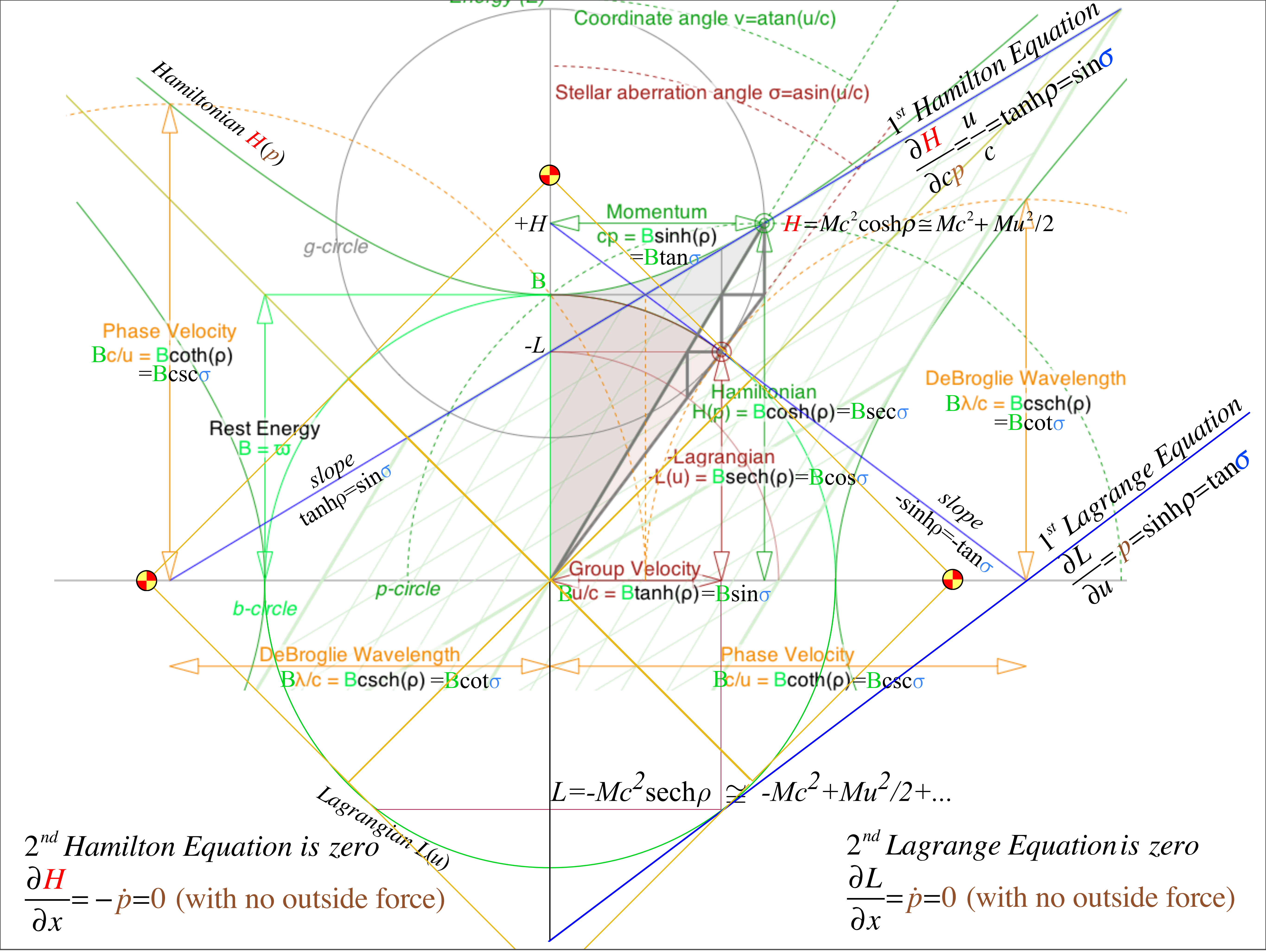
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Momentum
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-L
Phase Velocity
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B = ϖ
slope
tanhρ=sinσ
Hamiltonian
H(p) = Bcosh(ρ)=Bsecσ
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Lagrangian L(u)
2nd Hamilton Equation is zero
∂H/∂x = −ṗ=0 (with no outside force)
2nd Lagrange Equation is zero
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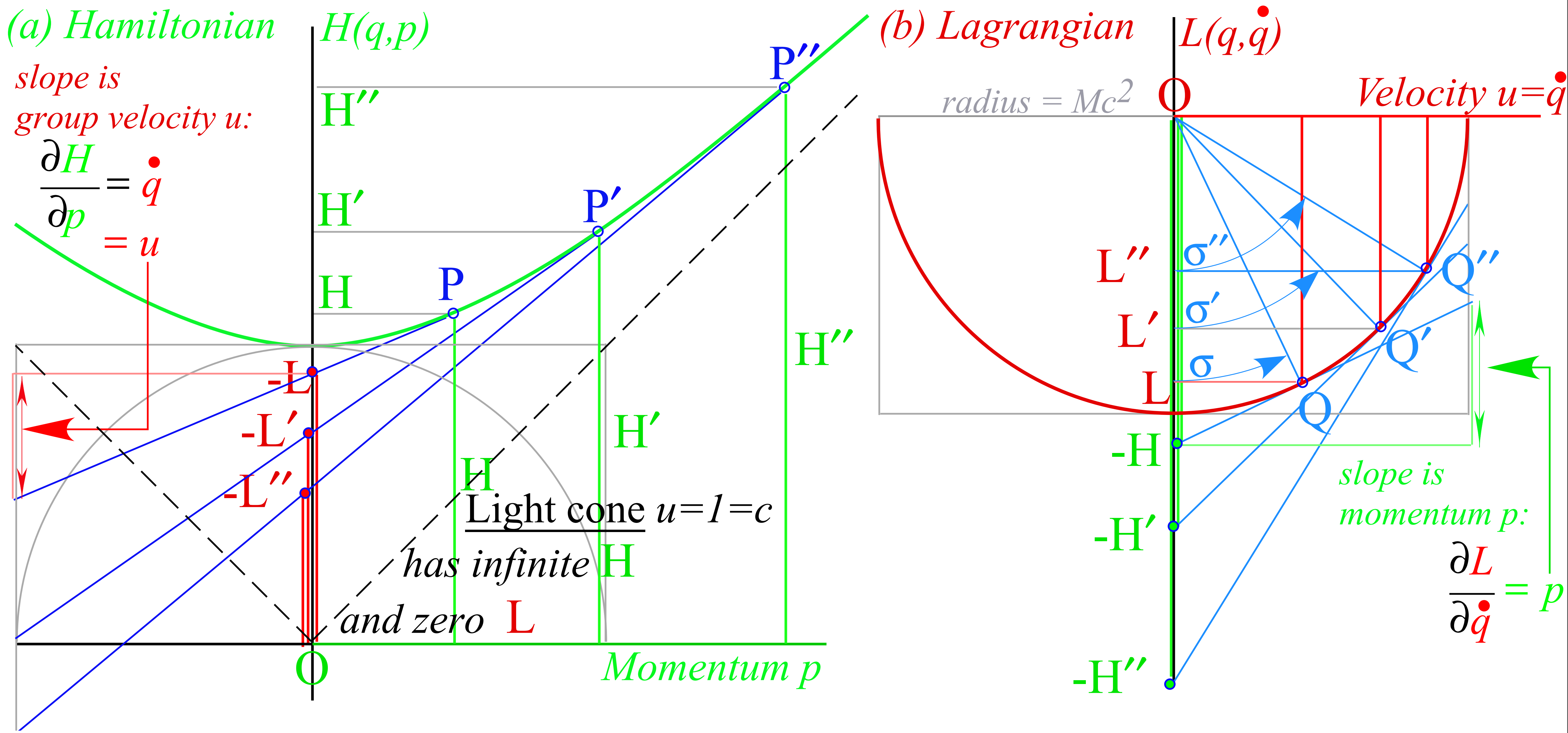
Poincare Invariant Action $dS = L\,dt = p\,dq - H\,dt = \hbar d\Phi$ *(phase)*

Hamiltonian $H(p,q) = p\dot{q} - L$ vs. *Lagrangian* $L(\dot{q}q) = p\dot{q} - H$

Contact transformation: *(slope,-intercept)* of *H* (or *L*) *tangent* determines the *(X,Y coordinates)* of *L* (or *H*).

(Also, called a *Legendre contact transformation* which is a special case of a *Huygens transformation* that uses contacting tangent *curves* instead of *lines*.)



Here *slope* is group velocity $u=\dot{q}$

Y-coordinate is *energy* $H=\hbar\omega$

Here *slope* is momentum *p*

Y-coordinate is *phase rate* $L=\hbar\Phi$

SRQTbyR&C Unit 3
Fig. 26

Using (some) wave parameters to develop relativistic quantum theory

Low rapidity approximations to υ_{phase} and $c\kappa_{phase}$ match to Newtonian KE and momentum

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Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2\cos\sigma$$

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Define *Action* $S = \hbar\Phi$

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Legendre transformation

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\,dx - H\,dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

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$$dS \equiv L dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2 \cos\sigma$$

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How Jacobi-Hamilton *derives Schrodinger equations*

(Given "quantum wave")

$$\psi(\mathbf{r},t) = e^{iS/\hbar} = e^{i(\mathbf{p}\bullet\mathbf{r}-H\cdot t)/\hbar} = e^{i(\mathbf{k}\bullet\mathbf{r}-\omega\cdot t)}$$

dS is integrable if: $\frac{\partial S}{\partial \mathbf{r}} = \mathbf{p}$ and: $\frac{\partial S}{\partial t} = -H$

These conditions are known as Jacobi-Hamilton *equations*

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Q: When is the *Action*-differential dS integrable?

A: Differential $dW=f_x(x,y)dx+f_y(x,y)dy$ is *integrable* to a $W(x,y)$ if: $f_x = \frac{\partial W}{\partial x}$ and: $f_y = \frac{\partial W}{\partial y}$

Similar to conditions for integrating work differential $dW=\mathbf{f}\bullet d\mathbf{r}$ *to get potential* $W(\mathbf{r})$*. That condition is* no curl allowed*:* $\nabla\times\mathbf{f}=\mathbf{0}$ *or ∂-symmetry of W:*

$$\frac{\partial f_x}{\partial y} = \frac{\partial^2 W}{\partial y \partial x} = \frac{\partial^2 W}{\partial x \partial y} = \frac{\partial f_y}{\partial x}$$

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Try 1st **r**-*derivative of wave* ψ

$$\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \frac{\partial}{\partial \mathbf{r}} e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial \mathbf{r}} e^{iS/\hbar} = (i/\hbar)\frac{\partial S}{\partial \mathbf{r}}\psi(\mathbf{r},t)$$

$$\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = (i/\hbar)\mathbf{p}\psi(\mathbf{r},t) \quad \text{or:} \quad \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial \mathbf{r}}\psi(\mathbf{r},t) = \mathbf{p}\psi(\mathbf{r},t)$$

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Momentum Operator or **p***-op in* **r***-basis*

$$\mathbf{p} \Rightarrow \frac{\hbar}{i}\frac{\partial}{\partial \mathbf{r}}$$

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Try 1st t-derivative of wave ψ

$$\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \frac{\partial}{\partial t}e^{iS/\hbar} = \frac{\partial(iS/\hbar)}{\partial t}e^{iS/\hbar} = (i/\hbar)\frac{\partial S}{\partial t}\psi(\mathbf{r},t)$$

$$= (i/\hbar)(-H)\psi(\mathbf{r},t) \quad \text{or:} \quad i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = H\psi(\mathbf{r},t)$$

How Jacobi-Hamilton *derives Schrodinger equations*

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Try 1st *t-derivative of wave* ψ

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Schrodinger time equation

$$i\hbar\dot{\psi}(\mathbf{r},t) = H\psi(\mathbf{r},t)$$

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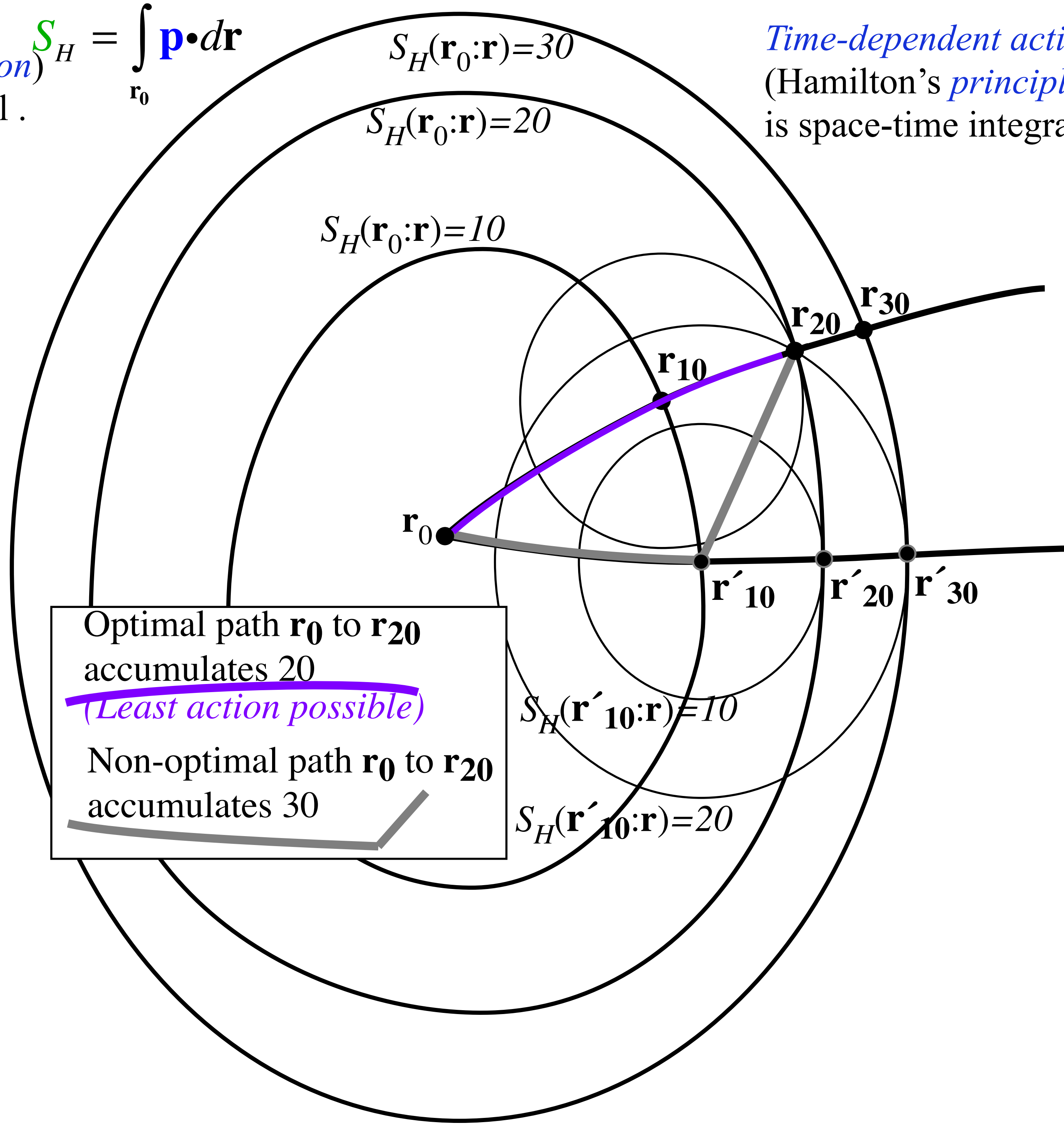
Huygen's contact transformations enforce minimum action

Each point $\mathbf{r}_k$ *on a wavefront "broadcasts" in all directions.*
Only ***minimum action*** *path interferes constructively*

Time-independent action $S_H = \int_{\mathbf{r}_0}^{\mathbf{r}_1} \mathbf{p} \bullet d\mathbf{r}$
(Hamilton's *reduced action*)
is a purely spatial integral .

Time-dependent action $S_p = \int_{\mathbf{r}_0 t_0}^{\mathbf{r}_1 t_1} \left(\mathbf{p} \bullet d\mathbf{r} - H \cdot dt\right)$
(Hamilton's *principle action*)
is space-time integral .



CMwBang! Unit 1
Fig. 12.12

Huygen's contact transformations enforce minimum action

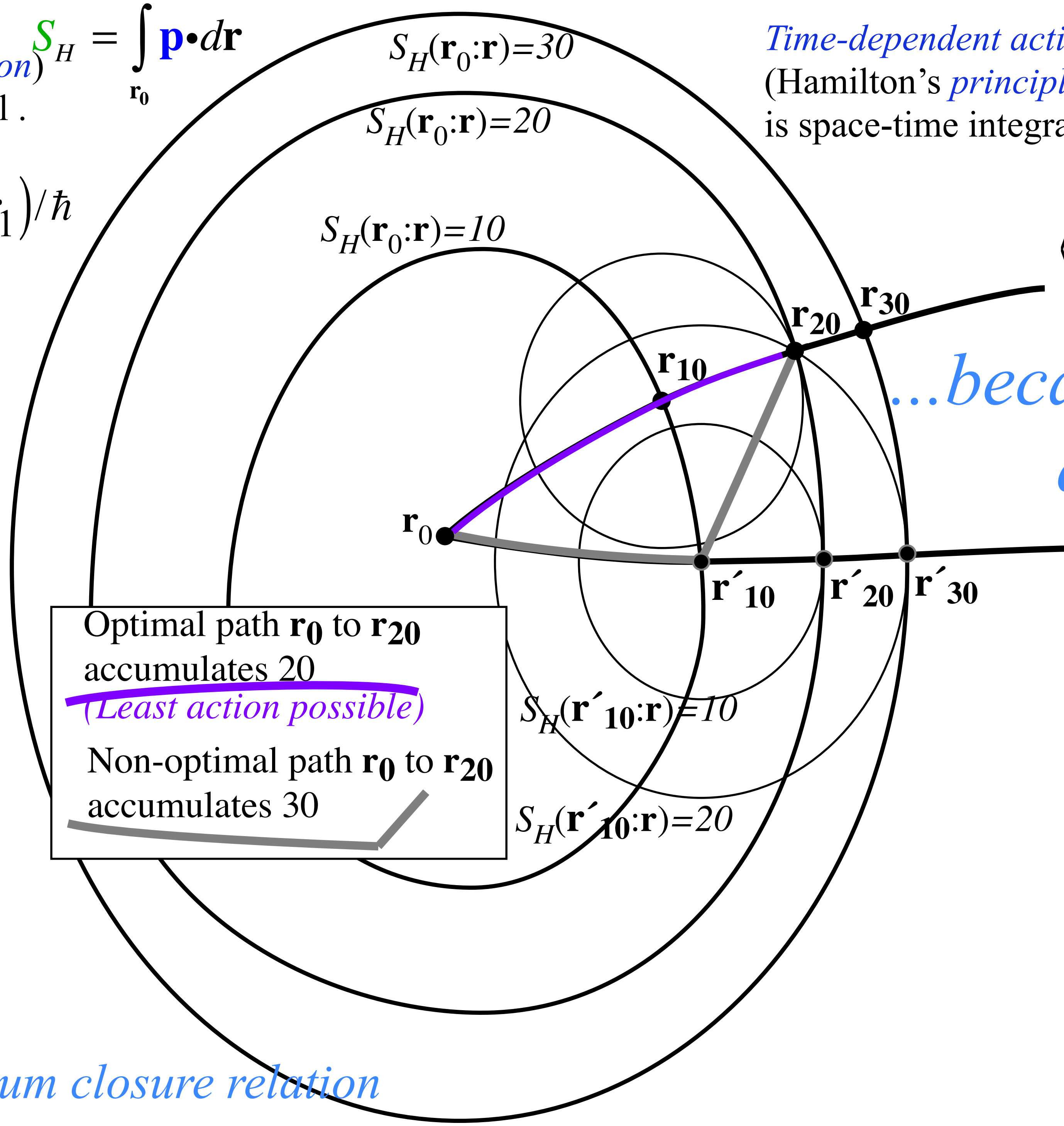
Each point $\mathbf{r}_k$ on a wavefront "broadcasts" in all directions.
*Only **minimum action** path interferes constructively*

Time-independent action $S_H = \int_{\mathbf{r}_0}^{\mathbf{r}_1} \mathbf{p} \bullet d\mathbf{r}$
(Hamilton's *reduced action*)
is a purely spatial integral .

$$\langle \mathbf{r}_1 | \mathbf{r}_0 \rangle = e^{i S_H (\mathbf{r}_0 : \mathbf{r}_1)/\hbar}$$

Time-dependent action $S_p = \int_{\mathbf{r}_0 t_0}^{\mathbf{r}_1 t_1} (\mathbf{p} \bullet d\mathbf{r} - H \cdot dt)$
(Hamilton's *principle action*)
is space-time integral .

$$\langle \mathbf{r}_1, t_1 | \mathbf{r}_0, t_0 \rangle = e^{i S(\mathbf{r}_0, t_0 : \mathbf{r}_1, t_1)/\hbar}$$



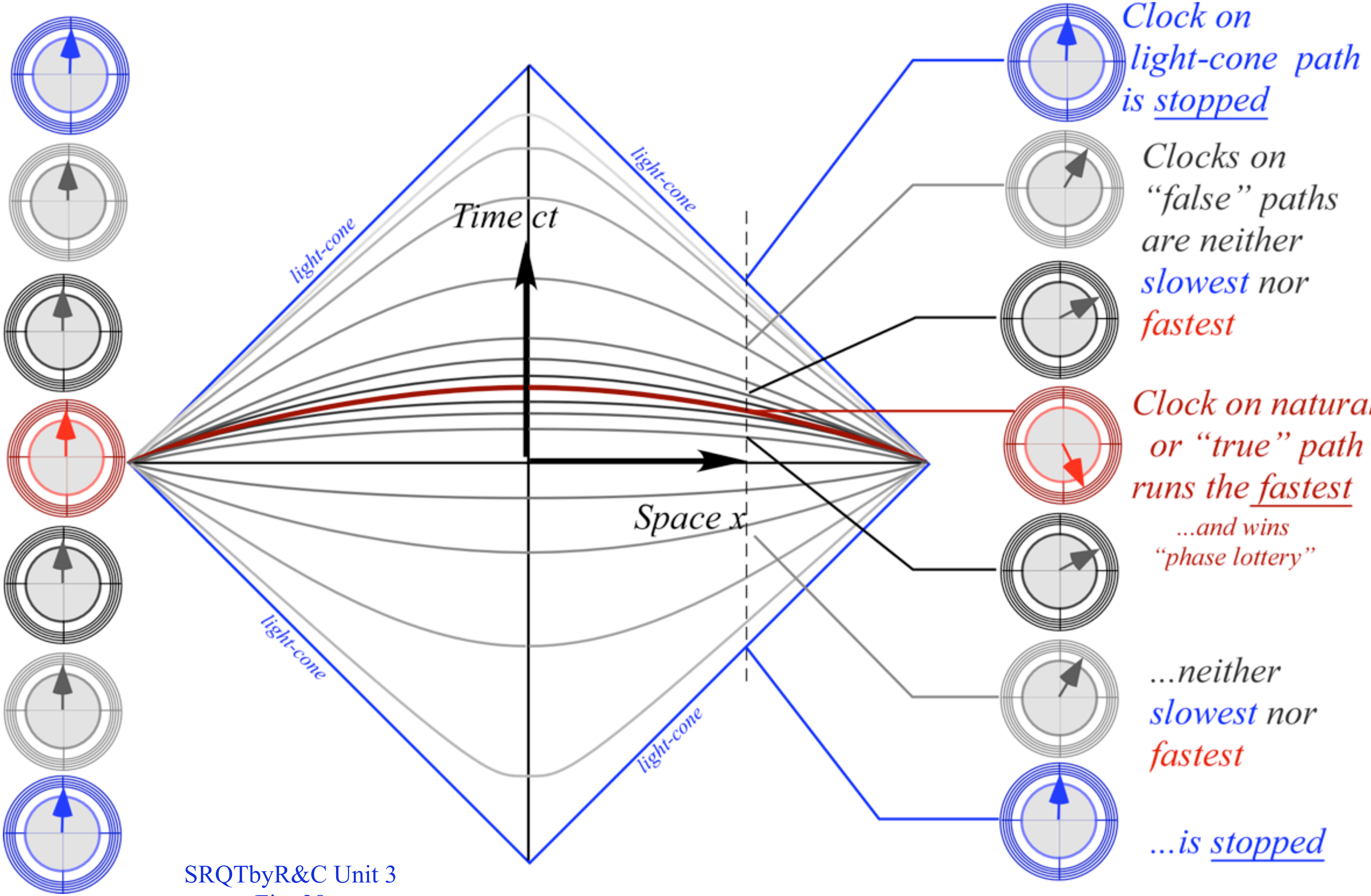
...because action is quantum wave phase

CMwBang! Unit 1
Fig. 12.12

Feynman's path-sum closure relation

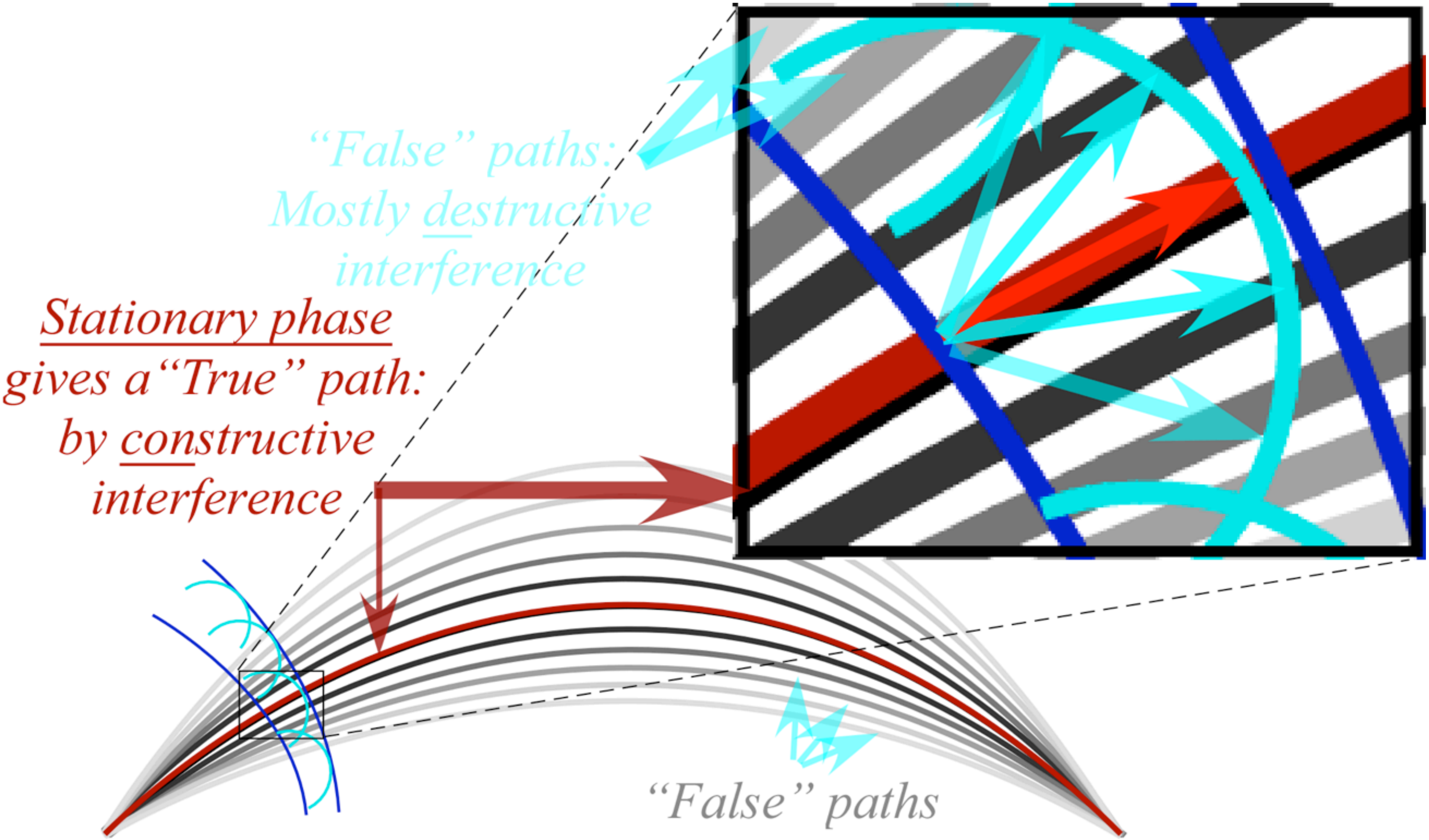
$$\sum_{\mathbf{r}'} \langle \mathbf{r}_1 | \mathbf{r}' \rangle \langle \mathbf{r}' | \mathbf{r}_0 \rangle \cong \sum_{\mathbf{r}'} e^{i(S_H(\mathbf{r}_0 : \mathbf{r}') + S_H(\mathbf{r}' : \mathbf{r}_1))/\hbar} = e^{i S_H (\mathbf{r}_0 : \mathbf{r}_1)/\hbar} = \langle \mathbf{r}_1 | \mathbf{r}_0 \rangle$$

Huygen's contact transformations enforce minimum action

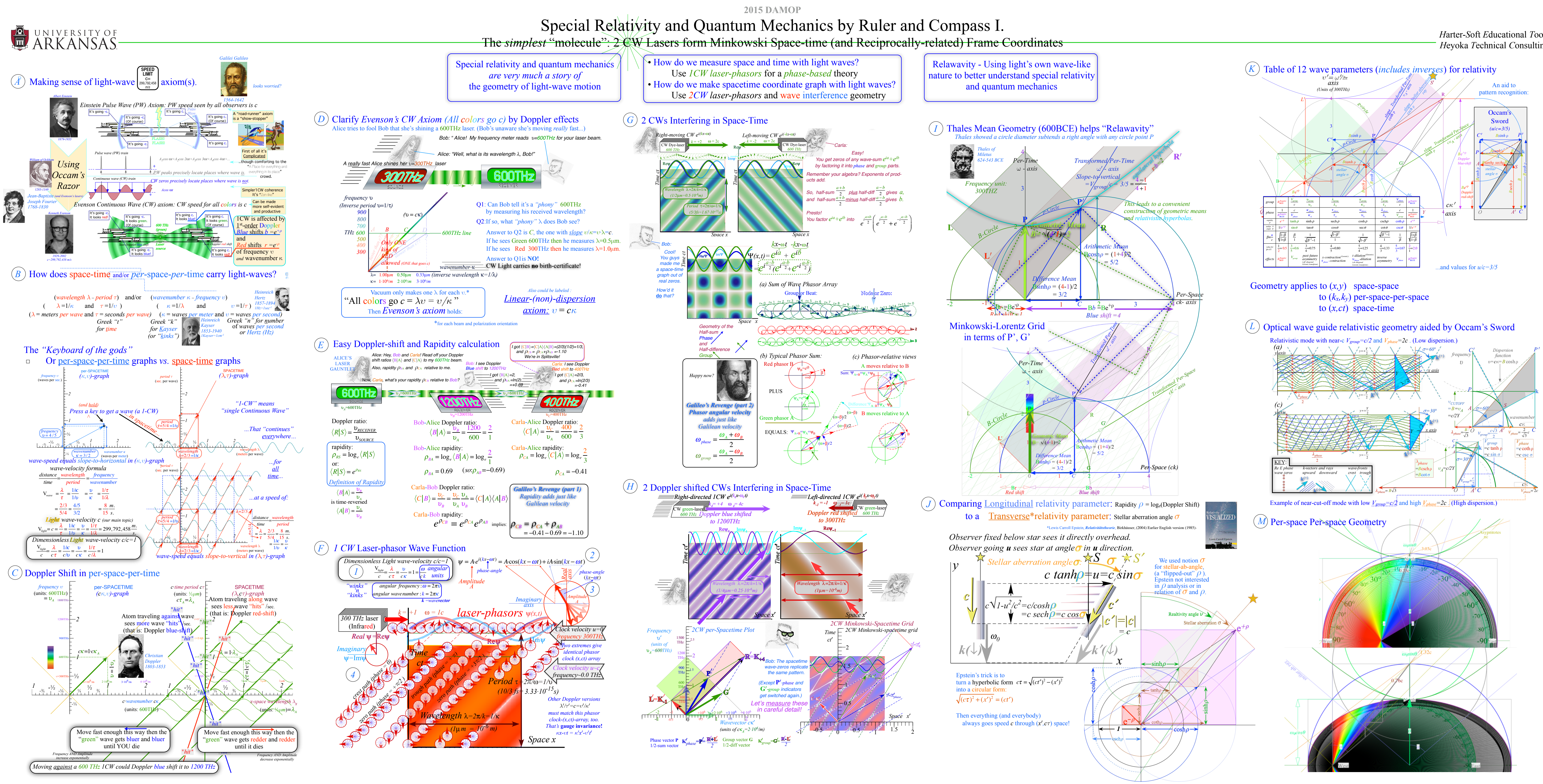


SRQTbyR&C Unit 3
Fig. 28

Huygen's contact transformations enforce minimum action

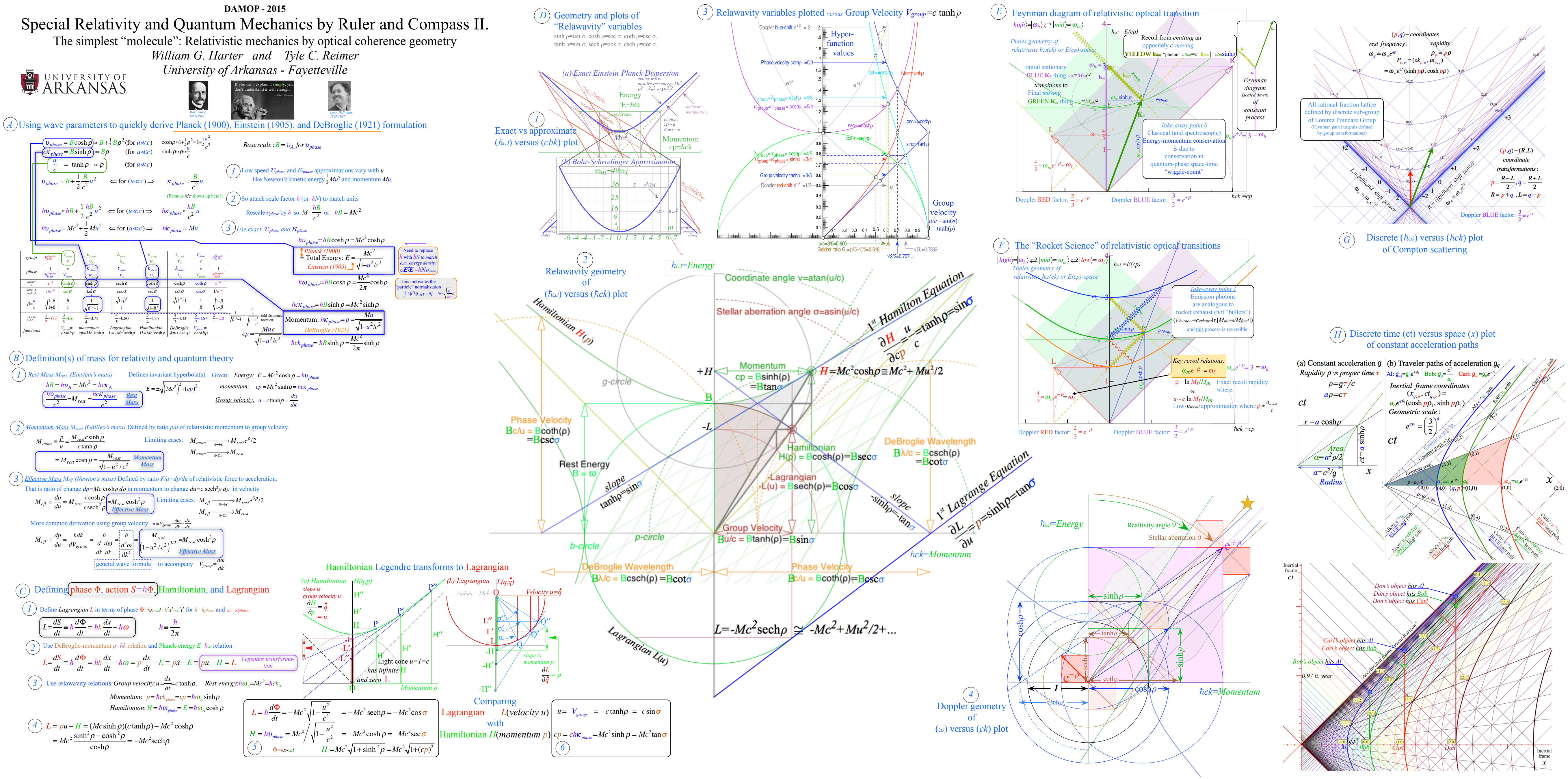


SRQTbyR&C Unit 3
Fig. 29



[Link to pdf version of *Part I* online]

Note: When printed at their optimal resolution, each poster is *7 feet across!*



[Link to pdf version of *Part II* online]

Note: When printed at their optimal resolution, each poster is *7 feet across!*