Read Unit 2 Chapter 4 (all) and Chapter 5 thru part (9).

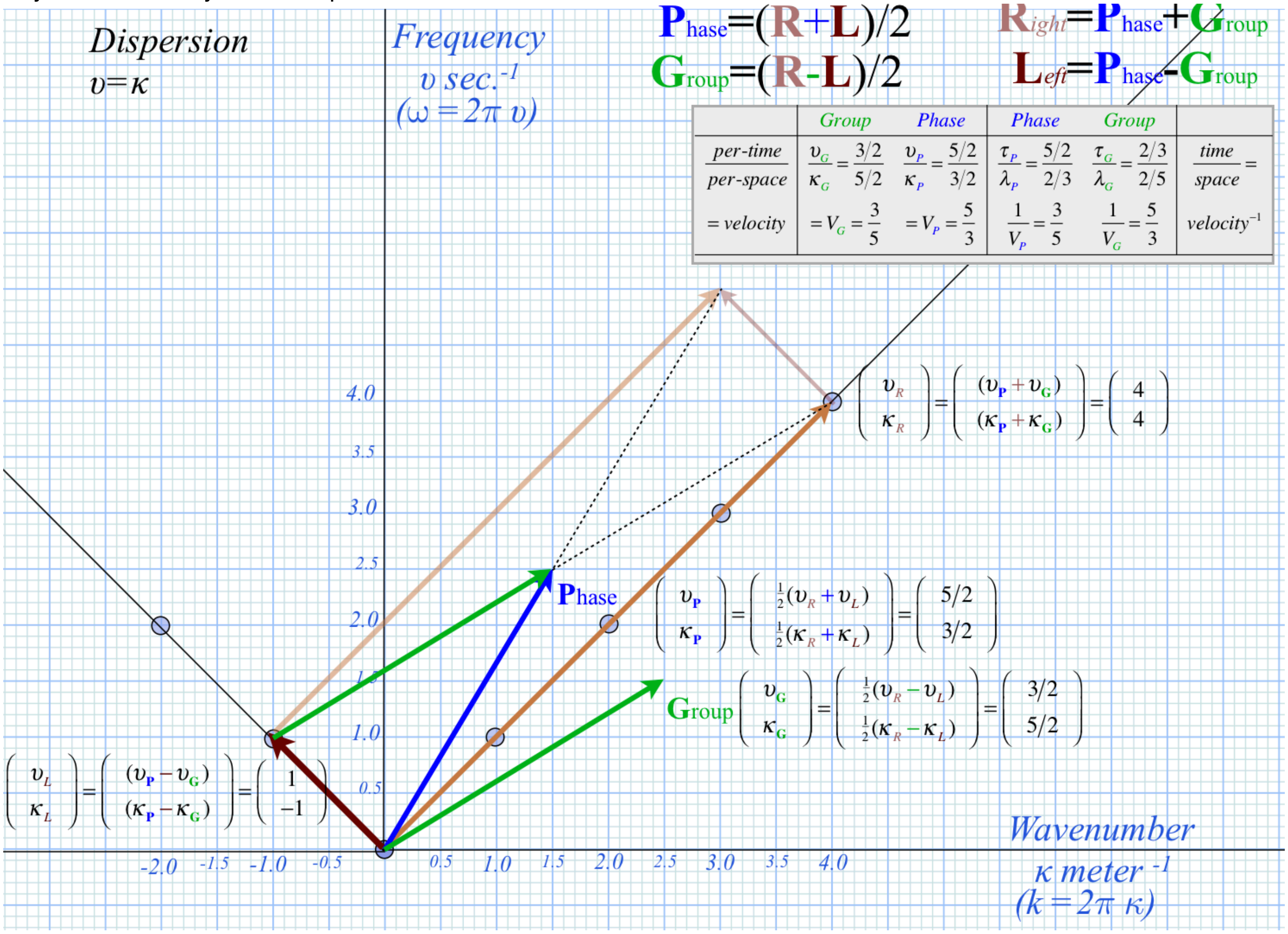
1.In class† we constructed a quadratic dispersion (υ,κ)-plot (frequency υ *vs* wavenumber κ) for the case of quadratic dispersion $\upsilon=\kappa^2$. The case involved a 1-CW (single coherent wave) with wavenumber $(\upsilon=-1)$ colliding with another 1-CW of wavenumber $(\upsilon=+2)$ and required you to derive and plot 2-CW (pair of interfering coherent waves) parameters *frequency* $\upsilon^{2\text{-}CW}_{Phase}$ *and* $\upsilon^{2\text{-}CW}_{Group}$ with *wavenumbers* κ^{2-CW}_{Phase}, κ^{2-CW}_{Group}.
With these we found wave velocities $V^{2\text{-}CW}_{Phase}$ *and* $V^{2\text{-}CW}_{Group}$.

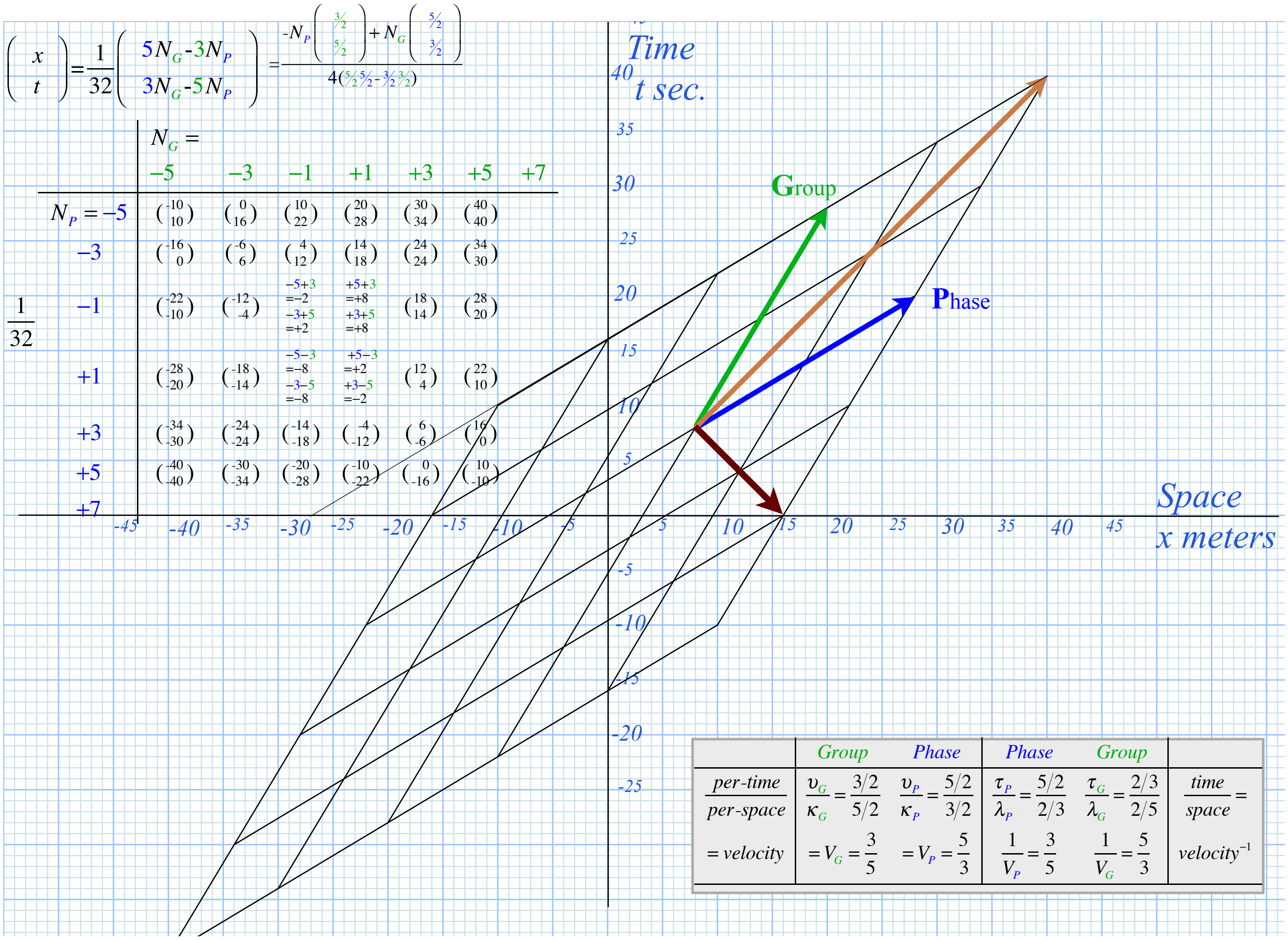
Now do this for the case of linear dispersion $\upsilon=\kappa^1$ involving a 1-CW (single coherent wave) with wavenumber $(\upsilon=-1)$ colliding with another 1-CW of wavenumber $(\upsilon=+4)$. Use per-spacetime graph paper provided in class† to find 2-CW parameters $\upsilon^{2\text{-}CW}_{Phase}$, $\upsilon^{2\text{-}CW}_{Group}$, κ^{2-CW}_{Phase}, κ^{2-CW}_{Group}, and velocities $V^{2\text{-}CW}_{Phase}$ *and* $V^{2\text{-}CW}_{Group}$.
Make a table of the wave per-space-time parameters and (reciprocal) space-time ones as done in class.

2.The second part of the class† construction involved using the space-time 2-CW parameters that are reciprocals of $\upsilon^{2\text{-}CW}_{Phase}$, $\upsilon^{2\text{-}CW}_{Group}$, κ^{2-CW}_{Phase}, κ^{2-CW}_{Group}, namely periods $\tau^{2\text{-}CW}_{Phase}$, $\tau^{2\text{-}CW}_{Group}$ and wavelengths λ^{2-CW}_{Phase}, λ^{2-CW}_{Group}.

Now do this for the case of linear dispersion $\upsilon=\kappa^1$ in part 1. and use the provided spacetime graph paper to plot and label a lattice for ideal 2-CW real-zeros in space and time. Label the line segments that correspond to periods and wavelengths as was done in class. Choose points so you make a symmetric array around origin (0,0) having at least 16 cells.

† Class step-by-step constructions are in Lecture 22 ranging from p. 40 to around p.70.
BohrIt animations in lecture show space-time lattices. First example is around p. 40.


Dispersion
υ=κ
Frequency
υ sec.^-1
(ω = 2π υ)
Phase=(R+L)/2
Group=(R-L)/2
Right=Phase+Group
Left=Phase-Group
Group Phase Phase Group
per-time / per-space = velocity
υG/κG = (3/2)/(5/2) = VG = 3/5
υP/κP = (5/2)/(3/2) = VP = 5/3
τP/λP = (5/2)/(2/3); 1/VP = 3/5
τG/λG = (2/3)/(2/5); 1/VG = 5/3
time / space = velocity^-1
(υR, κR) = ((υP + υG), (κP + κG)) = (4, 4)
Phase
(υP, κP) = (½(υR + υL), ½(κR + κL)) = (5/2, 3/2)
Group
(υG, κG) = (½(υR − υL), ½(κR − κL)) = (3/2, 5/2)
(υL, κL) = ((υP − υG), (κP − κG)) = (1, −1)
4.0
3.5
3.0
2.5
2.0
1.0
0.5
-2.0
-1.5
-1.0
-0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Wavenumber
κ meter^-1
(k = 2π κ)



$$\begin{pmatrix} x \\ t \end{pmatrix} = \frac{1}{32}\begin{pmatrix} 5N_G - 3N_P \\ 3N_G - 5N_P \end{pmatrix} = \frac{-N_P\begin{pmatrix} 3/2 \\ 5/2 \end{pmatrix} + N_G\begin{pmatrix} 5/2 \\ 3/2 \end{pmatrix}}{4\left(\frac{5}{2}\frac{5}{2} - \frac{3}{2}\frac{3}{2}\right)}$$

$\frac{1}{32}$

	$N_G = -5$	-3	-1	$+1$	$+3$	$+5$	$+7$
$N_P = -5$	$\binom{-10}{10}$	$\binom{0}{16}$	$\binom{10}{22}$	$\binom{20}{28}$	$\binom{30}{34}$	$\binom{40}{40}$	
-3	$\binom{-16}{0}$	$\binom{-6}{6}$	$\binom{4}{12}$	$\binom{14}{18}$	$\binom{24}{24}$	$\binom{34}{30}$	
-1	$\binom{-22}{-10}$	$\binom{-12}{-4}$	$-5+3$ $=-2$ $-3+5$ $=+2$	$+5+3$ $=+8$ $+3+5$ $=+8$	$\binom{18}{14}$	$\binom{28}{20}$	
$+1$	$\binom{-28}{-20}$	$\binom{-18}{-14}$	$-5-3$ $=-8$ $-3-5$ $=-8$	$+5-3$ $=+2$ $+3-5$ $=-2$	$\binom{12}{4}$	$\binom{22}{10}$	
$+3$	$\binom{-34}{-30}$	$\binom{-24}{-24}$	$\binom{-14}{-18}$	$\binom{-4}{-12}$	$\binom{6}{-6}$	$\binom{16}{0}$	
$+5$	$\binom{-40}{-40}$	$\binom{-30}{-34}$	$\binom{-20}{-28}$	$\binom{-10}{-22}$	$\binom{0}{-16}$	$\binom{10}{-10}$	
$+7$							

	Group	Phase	Phase	Group	
$\frac{per\text{-}time}{per\text{-}space}$	$\frac{\upsilon_G}{\kappa_G} = \frac{3/2}{5/2}$	$\frac{\upsilon_P}{\kappa_P} = \frac{5/2}{3/2}$	$\frac{\tau_P}{\lambda_P} = \frac{5/2}{2/3}$	$\frac{\tau_G}{\lambda_G} = \frac{2/3}{2/5}$	$\frac{time}{space} =$
$= velocity$	$= V_G = \frac{3}{5}$	$= V_P = \frac{5}{3}$	$\frac{1}{V_P} = \frac{3}{5}$	$\frac{1}{V_G} = \frac{5}{3}$	$velocity^{-1}$