Physics 3922H Physics Colloquium Thur. 3.17.2016   Exercise Set 8   Due Thur 3.31

Read Unit 2 Chapter 3 (all) and Chapter 4 thru part (b).

The following deals with spinors, matrix eigensolutions, and applications of them 2D-HO

2.4.1 Derive multiplication table for spinor matrix operators:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_C = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

	σ_0	σ_A	σ_B	σ_C
σ_0				
σ_A				
σ_B				
σ_C				

(a)Two of the operators are real mirror-plane reflections (Recall superball Theory in Ch. 4-5)
Describe what reflection each of these did.
Consider a normal combination $\sigma(\theta) = \sigma_A \cos\theta + \sigma_B \sin\theta$. Does it square like a reflection $\sigma(\theta)^2 = ?$

(b)What is its effect on vectors $|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$? Display on graph for θ=30° and θ=45°.

(c) Use the functional spectral decomposition (Lect. 18 around p.61-63) to derive the following matrix functions of the spinor matrices: (Show that more than one answer exists for each.)

$\sqrt{\sigma_A} =$ ________, $\sqrt{\sigma_B} =$ ________, $\sqrt{\sigma_C} =$ ________, $\sqrt{\sigma(\theta)} =$ ________,

$e^{-i\phi\sigma_A} =$ ________, $e^{-i\phi\sigma_B} =$ ________, $e^{-i\phi\sigma_C} =$ ________, $e^{-i\phi\sigma(\theta)} =$ ________,

Compare last four results with what you get from the "Crazy-Thing-Theorem" (Lect. 18 p.31).

(d) 2D-HO phasor space and quantum spin ½ is described by a state vector with 4-parameters

$$|\psi\rangle = \begin{pmatrix} x_1 + ip_1 \\ x_2 + ip_2 \end{pmatrix} = r \begin{pmatrix} e^{-i\frac{\alpha}{2}} \cos\frac{\beta}{2} \\ e^{+i\frac{\alpha}{2}} \sin\frac{\beta}{2} \end{pmatrix} e^{-i\frac{\gamma}{2}}$$

. Express the 4 phase phase variables (x_1, p_1, x_2, p_2) in terms of the radius r, three Euler-polar angles (azimuth α, polar angle β, and phase γ) of spin (Stokes) vector **S**.
Also express (x_1, p_1, x_2, p_2) in terms of (A,B,C,D) following projector method in Lect. 18 p.74-75.