Physics 3922H Physics Colloquium Thur.3.03.2016 Exercise Set 7 Due Thur 3.10

The following is to acquaint you with some more exponential properties and phasor views of 2D-HO

Fun with Exponents & more of the Story of e

1.10.1 Consider a sequence of functions , $f_1(z)=z^z, f_2(z)=z^{f_1(z)}=z^{z^z}, f_3(z)=z^{f_2(z)}=z^{z^{z^z}}$,.... The function $f_N(z)$ has a finite limit $f_\infty(z)$ (as *N* approaches infinity) if number *z* is small enough .
(Hints: *z=1* works! But, so does *z=√2*. Try solving for *z* and looking for a max value.)

(a) Find $f_\infty(\sqrt{2})=$____?

(b) Find an analytic expression for the limiting real z_{max} that involves the Euler constant. *e=2.718281828*... and check its numerical value.

This is to introduce phasor views of 2D-HO Geometry of *Phasor-to-Cartesian and vice-versa relations*

1.10.2 *A day in the life of a neutron starlet*

Suppose neutron starlet orbits inside the "Sophomore-Physics-Earth" (SPE) of radius $R_{SPE}=10$ units. (One unit is 637*km*.) Let it start at position **r***(0)=(8.66,0)* with velocity **v***(0)=(5.0,-7.07)*. (Let time unit be determined by setting SPE angular frequency to one ($\omega_{SPE}=1=2\pi\, \upsilon_{SPE}$).

(a) Use position-velocity-phasor graph paper to set times (phases) and amplitudes of x and y phasors.

(b) Use the phasors to locate the starlet position vector **r***(t)* and its velocity **v***(t)* for a complete orbit.

(c) Label times of each 12 orbit points at equal time intervals defined as follows by period $\tau_{SPE}=1/\upsilon_{SPE}$: $t=0$ is 12:00PM, $t=\tau_{SPE}/12$ is 1:00PM, $t=2\tau_{SPE}/12$ is 2:00PM, $t=3\tau_{SPE}/12$ is 3:00PM, and so forth. Label velocity vector points, too.

(d) Does the starlet ever penetrate the Earth surface? If it does, how might that affect its orbit?

1.10.3 *12 days for a neutron starlet with an increasingly retarded x-phasor*

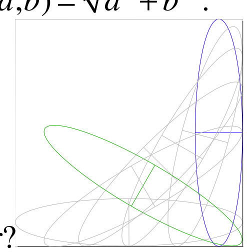
This project involves the "pincushion" graph paper for which the *x* and *y* phasors have the same unit-amplitude but their relative phase shifts so the x-phasor is retarded by ½ hour after each 24 hour orbit.

(a) Start by plotting an orbit starting with both phasors at 3:00 o'clock. (Should be a straight 45° line.) Then plot an orbit with the x-phasor starting at 2:30PM or 15° behind the y-phasor and maintaining that phase lag through the orbit. (Should be very narrow or eccentric ellipse.) Then plot an orbit with the x-phasor starting at 2:00PM or 30° behind the y-phasor and maintaining that phase lag through the orbit. (Should be a more rounded or less eccentric ellipse.) Continue plotting ellipses with phase lag 45°, 60°, 75°, 90°,…, 165°, and finally 180°, that last one being called "PI out of phase." Note what is special about the 90° case.

(b) Each of the ellipses drawn in part (a) has its own major radius *a* and minor radius *b* and may be circumscribed in its own *2a*-by-*2b* rectangle. With a ruler carefully sketch each of these rectangles.

(c) The hypotenuse of rectangular radii $r^{hypot}(a,b)=\sqrt{a^2+b^2}$ should be related. How?

(d) Derive the total energy of a 2D-IHO orbit and show it is proportional to $r^{hypot}(a,b)=\sqrt{a^2+b^2}$.



(e) Elliptical *5-by-1* surfboard rotates tangent to wall and floor. Where is its center?

Solution to 1.10.1

Consider a sequence of functions , $f_1(z) = z^z, f_2(z) = z^{f_1(z)} = z^{z^z}, f_3(z) = z^{f_2(z)} = z^{z^{z^z}}, \ldots$. The function $f_N(z)$ has a limit for N approaching infinity if argument z is small enough . ($z=1$ works! But, so does $z=\sqrt{2}$.) Find an analytic expression for the limiting real z that involves Euler constant. $e=2.718281828\ldots$

Solution: Assume limiting case can be reached. Then: $f(z) = z^{f(z)}$ or: $z = f^{\frac{1}{f}}$. Let's plot this function: $y(x) = x^{\frac{1}{x}}$

$1^{\frac{1}{1}} = 1.00, \quad 1.5^{\frac{1}{1.5}} = 1.30, \quad 2^{\frac{1}{2}} = 1.414, \quad 3^{\frac{1}{3}} = 1.442, \quad 4^{\frac{1}{4}} = 1.414,$ ...Has max when:

$\frac{dy}{dx} = 0 \; with: \ln y = \ln x^{\frac{1}{x}}$

$\frac{dy}{dx} = 0 \; with: \ln y = \ln x^{\frac{1}{x}}$ Take implicit derivative:

$$\frac{d}{dx} x \ln y = \frac{d}{dx} \ln x \quad or: \quad \ln y + \frac{x}{y}\frac{dy}{dx} = \frac{1}{x} \quad or: y = e^{\frac{1}{x}} = x^{\frac{1}{x}} = e^{\frac{1}{e}} = 1.44466786$$

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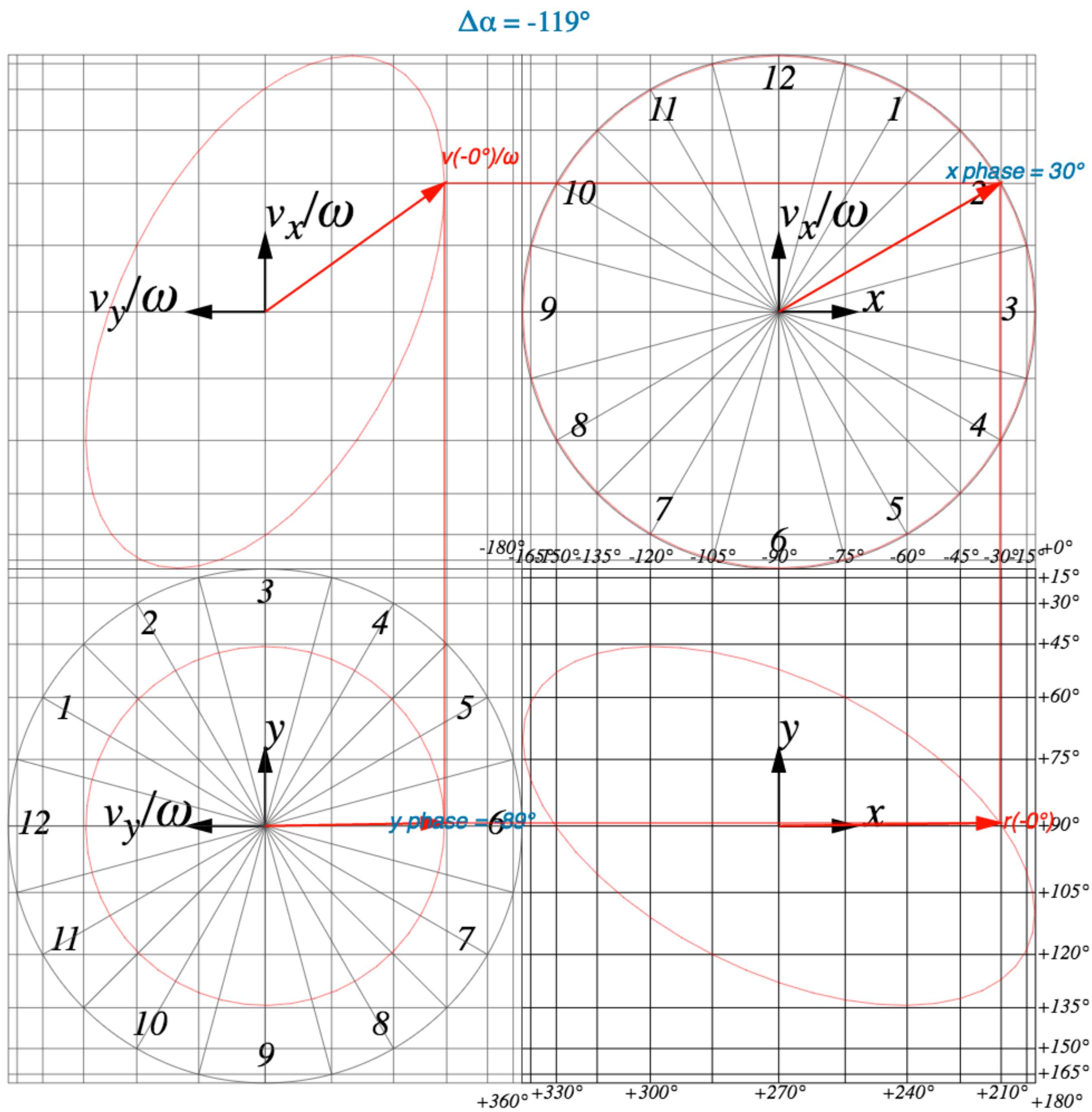

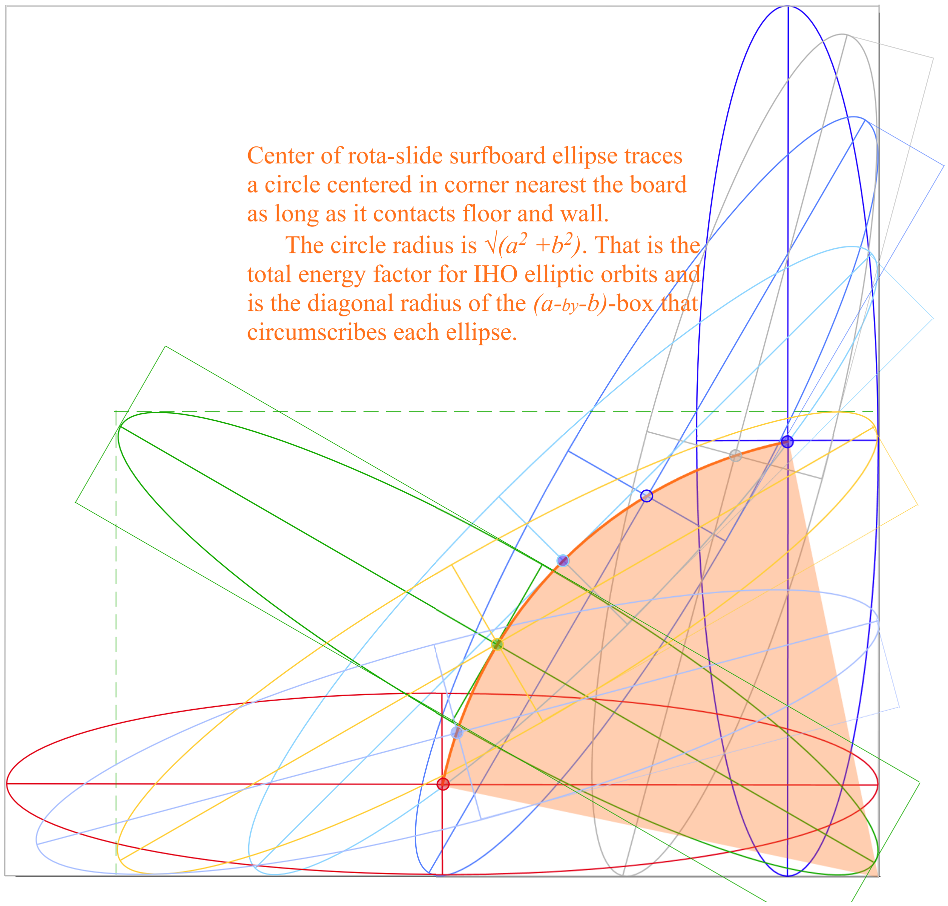

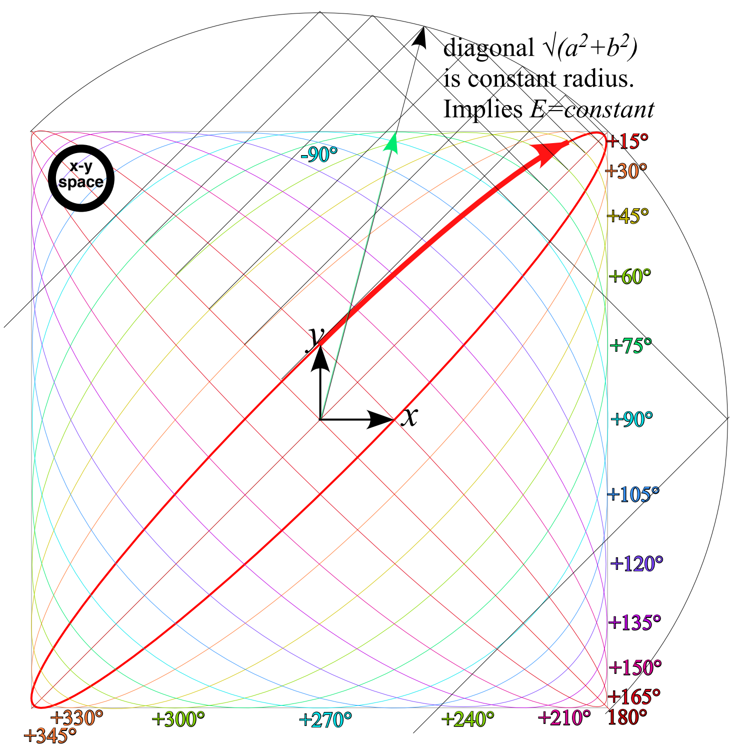

This shows total energy is not function of phase.