Group Theory in Quantum Mechanics

Lecture 12.5 *(2.28.17)*

Dynamics of Group vs. Phase Waves and space-time lattices

(Special Relativity and Quantum Theory by Ruler and Compass)
(Quantum Theory for the Computer Age - Unit 2)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 3-7 of Ch. 2)

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$*: Newton's "corpuscle" tracks vs.wave-zero paths*

Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$*: Half-sum factor* $S=e^{i(\mathbf{L}+\mathbf{R})/2}$

Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$*: Half-difference factor* $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L*-laser 600THz and* **R***-laser 600THZ (Laser lab frame)*

Phase **P***-vector and group* **G***-vector span Cartesian spacetime coordinates*

$\mathbf{L}'$*-laser 300THz and* $\mathbf{R}'$*-laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted $\mathbf{L}'$*-vector and* $\mathbf{R}'$*-vector in (***L***, ***R***)-per-spacetime*

Vectors of phase $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ *and group* $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$

Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

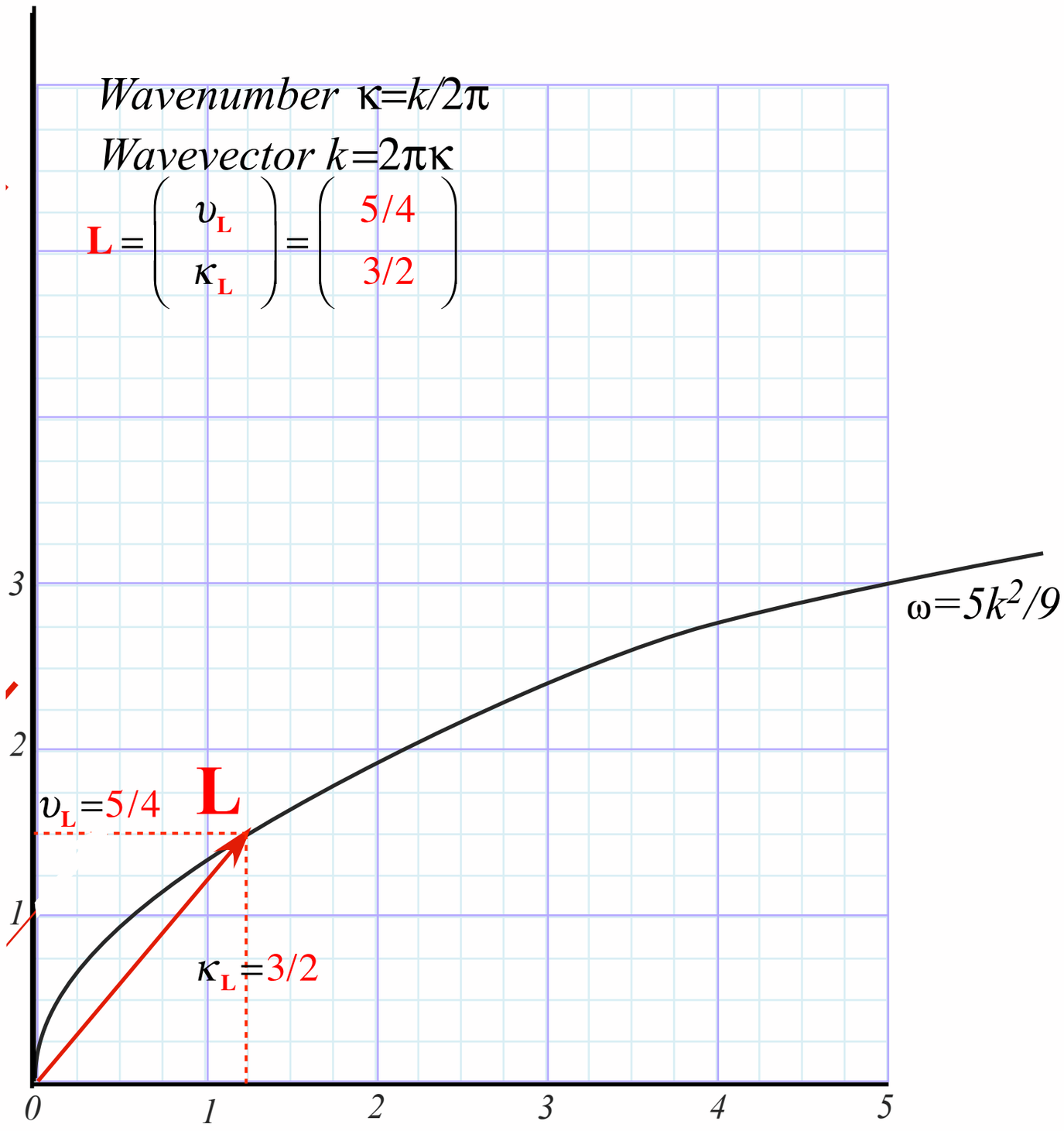
Brief tour of and relativistic mechanics by geometry

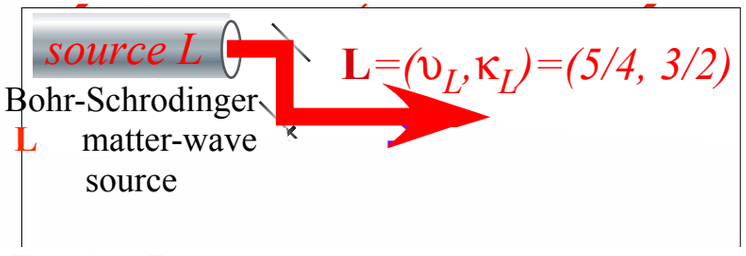
Summary of optical wave parameters for relativity and QM

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Spacetime (x,t)

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$





Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

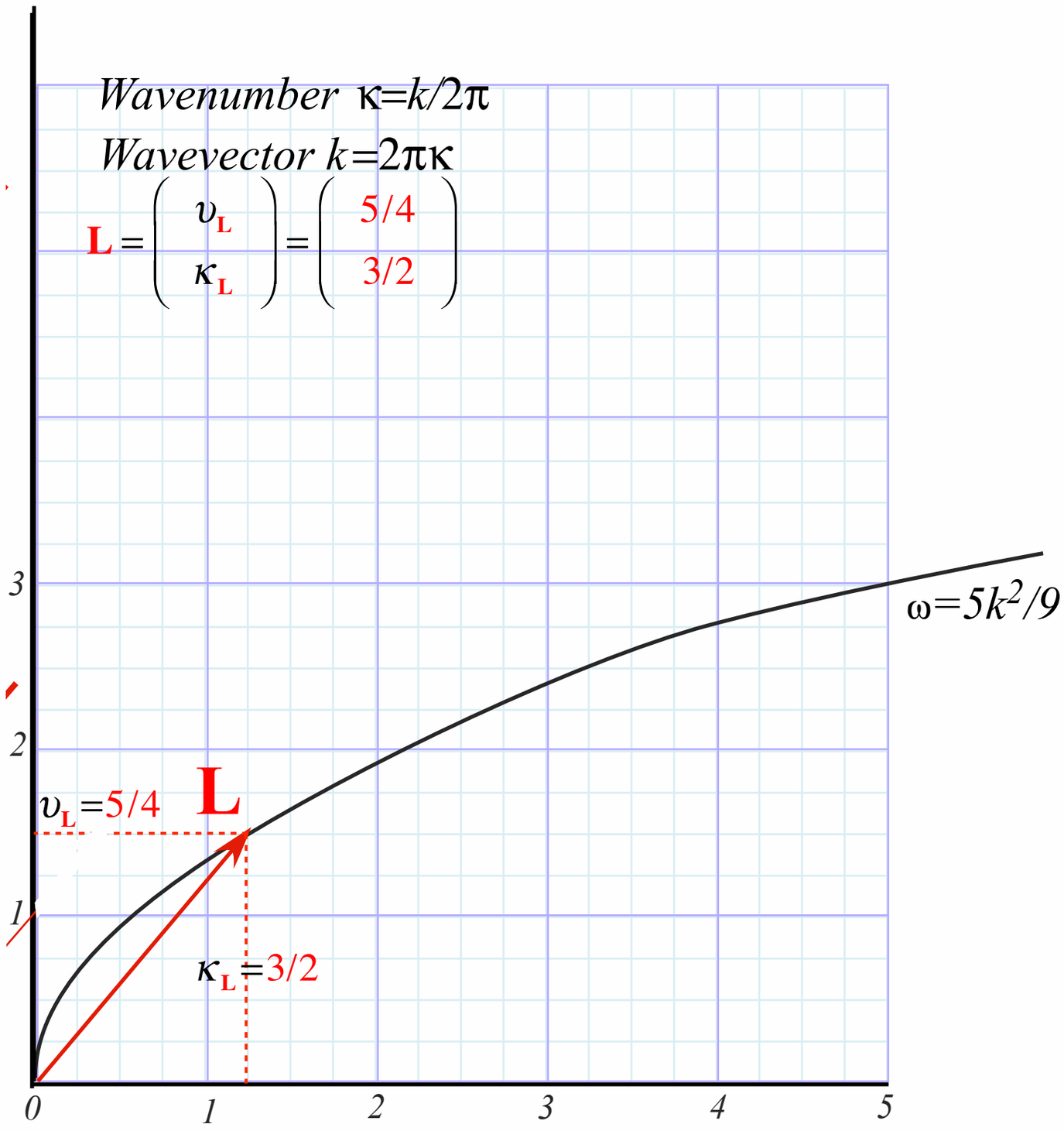
Spacetime (x,t)

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Phase velocity for simple wave $e^{i(kx-\omega t)}$
is $V=\omega/k=\upsilon/\kappa$ *where:*
υ *=waves per second*
and
κ *=waves per meter*

or:

$\omega = 2\pi\upsilon$ *=radians per second*
and
$k = 2\pi\kappa$ *=radians per meter*



Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

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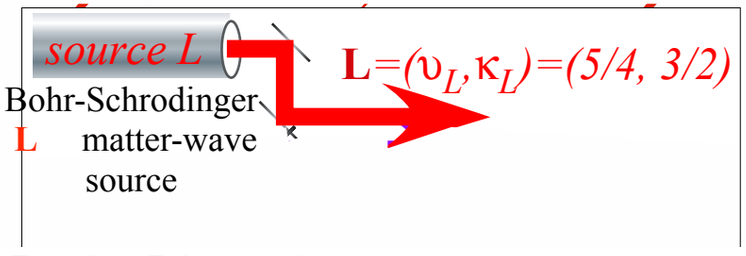
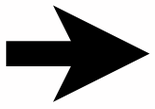


Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$: Newton's "corpuscle" tracks vs.wave-zero paths



Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

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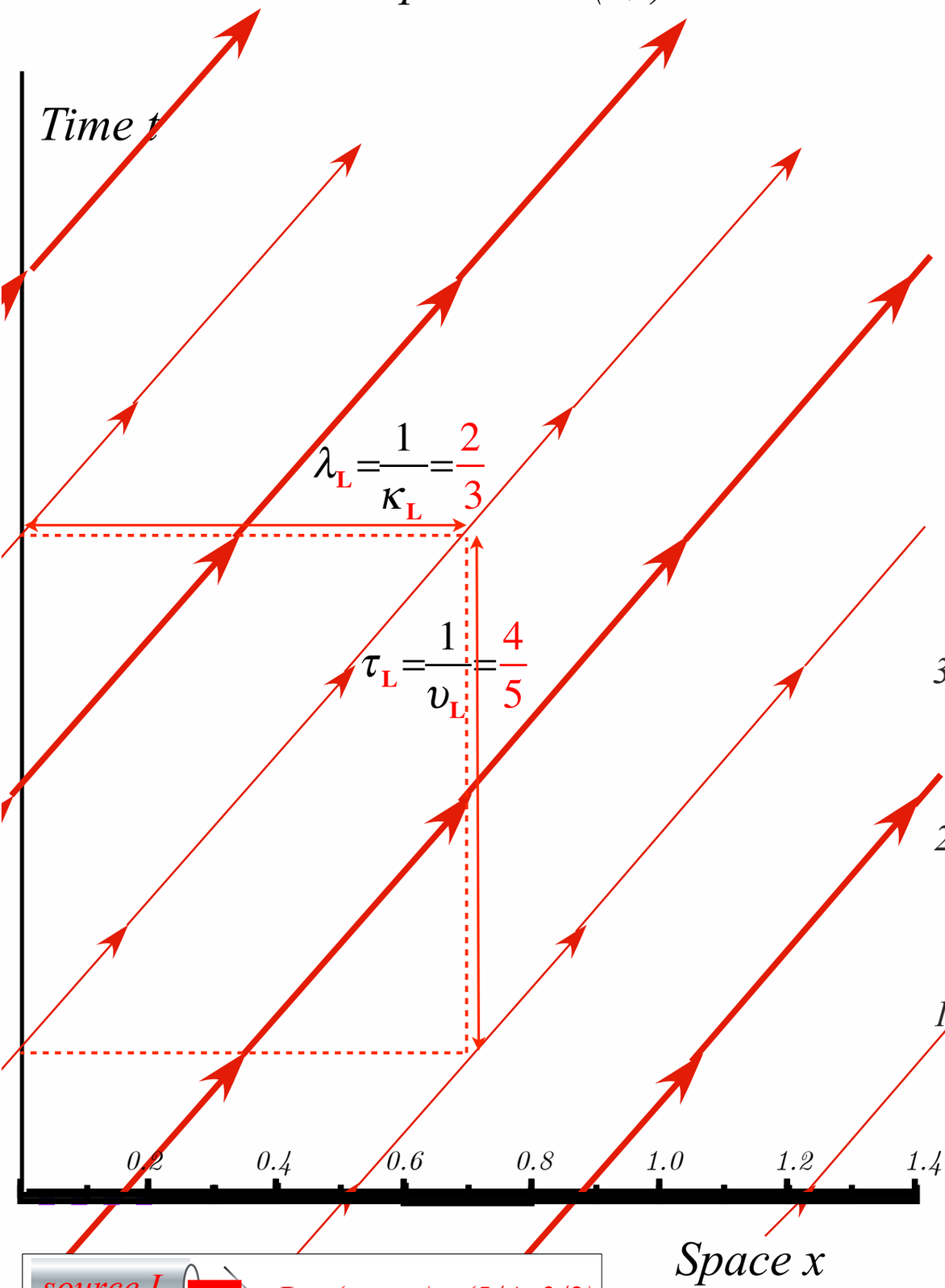
Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Spacetime (x,t)

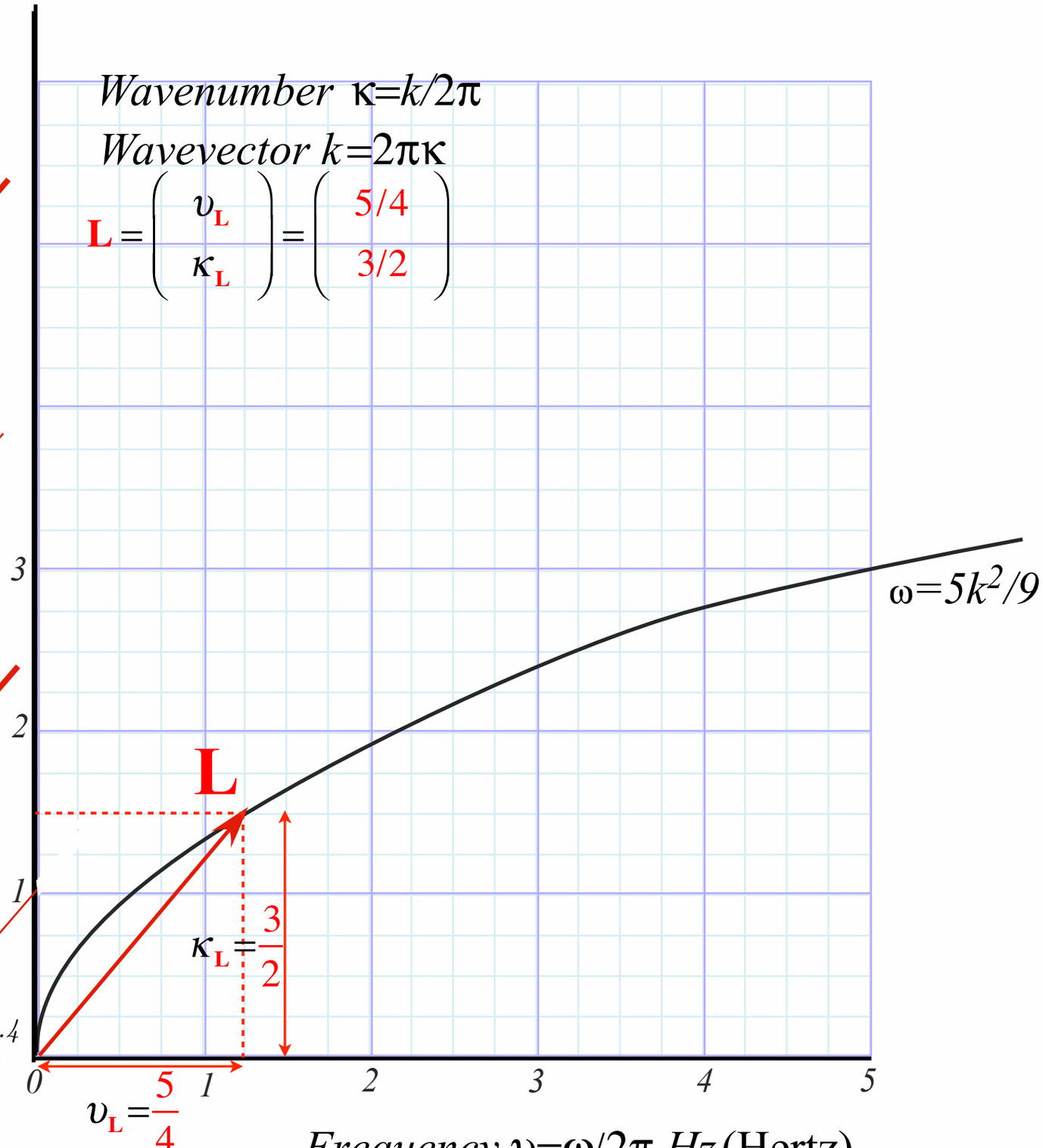


Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Wavenumber $\kappa=k/2\pi$

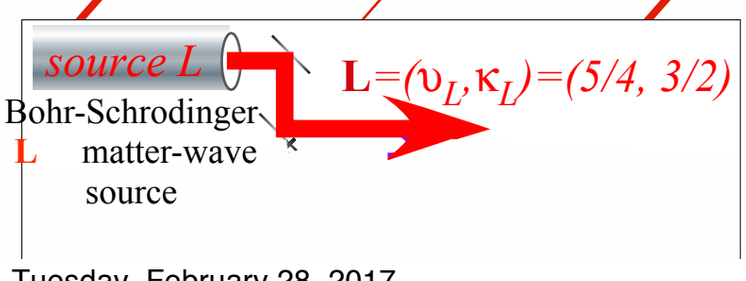
Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}$$



Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Spacetime (x,t)

$$\frac{\lambda_L}{\tau_L}=\frac{\upsilon_L}{\kappa_L}=\frac{2}{3}\Big/\frac{4}{5}=\frac{2}{3}\cdot\frac{5}{4}$$

Time t

$$V_{phase}=\frac{5}{6}$$

Velocity v of waves (or "corpuscles") in Minkowski $(t(x))$ is slope-to-vertical

$$\frac{\Delta x}{\Delta t}=\frac{\lambda_L}{\tau_L}=\frac{\upsilon_L}{\kappa_L}$$

$$\lambda_L=\frac{1}{\kappa_L}=\frac{2}{3}$$

$$\tau_L=\frac{1}{\upsilon_L}=\frac{4}{5}$$

0.2 0.4 0.6 0.8 1.0 1.2 1.4

Space x

source L

Bohr-Schrodinger matter-wave source

$\mathbf{L}=(\upsilon_L,\kappa_L)=(5/4,\ 3/2)$

[Web Simulation](#)

[1 (BW) Bohr Wave *ct vs x* Plot](#)

[*(ck = 1.0 (App)= 1.5)*](#)

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}$$

Velocity v of waves (or "corpuscles") in Minkowski $(\kappa(\upsilon))$ is slope-to-vertical

$$\frac{\Delta x}{\Delta t}=\frac{\upsilon_L}{\kappa_L}=\frac{\lambda_L}{\tau_L}$$

$\omega=5k^2/9$

$$\frac{\Delta y}{\Delta x}=\frac{\kappa_L}{\upsilon_L}=\frac{\tau_L}{\lambda_L}$$

L

$$\kappa_L=\frac{3}{2}$$

0 1 2 3 4 5

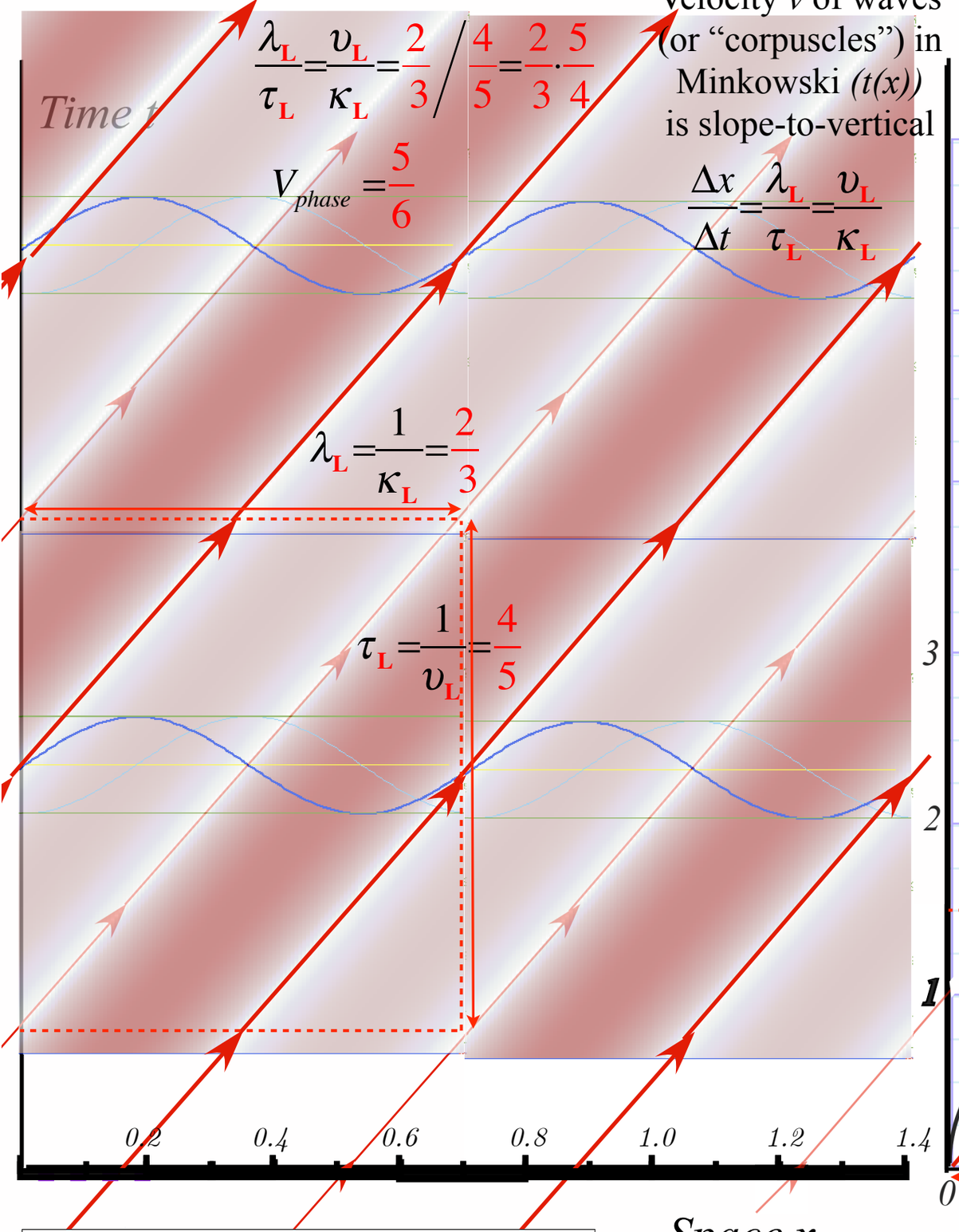
$$\upsilon_L=\frac{5}{4}$$

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

Slow L-wave $e^{iL}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Spacetime (x,t)



Velocity v of waves (or "corpuscles") in Minkowski $(t(x))$ is slope-to-vertical

$$\frac{\Delta x}{\Delta t}=\frac{\lambda_L}{\tau_L}=\frac{\upsilon_L}{\kappa_L}$$

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

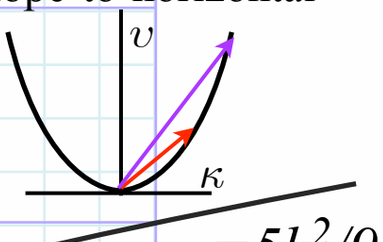
BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}$$

NOTE:
Conventional dispersion space is $(\upsilon(\kappa))$ or $(\omega(k))$ where velocity v of waves will be slope-to-horizontal



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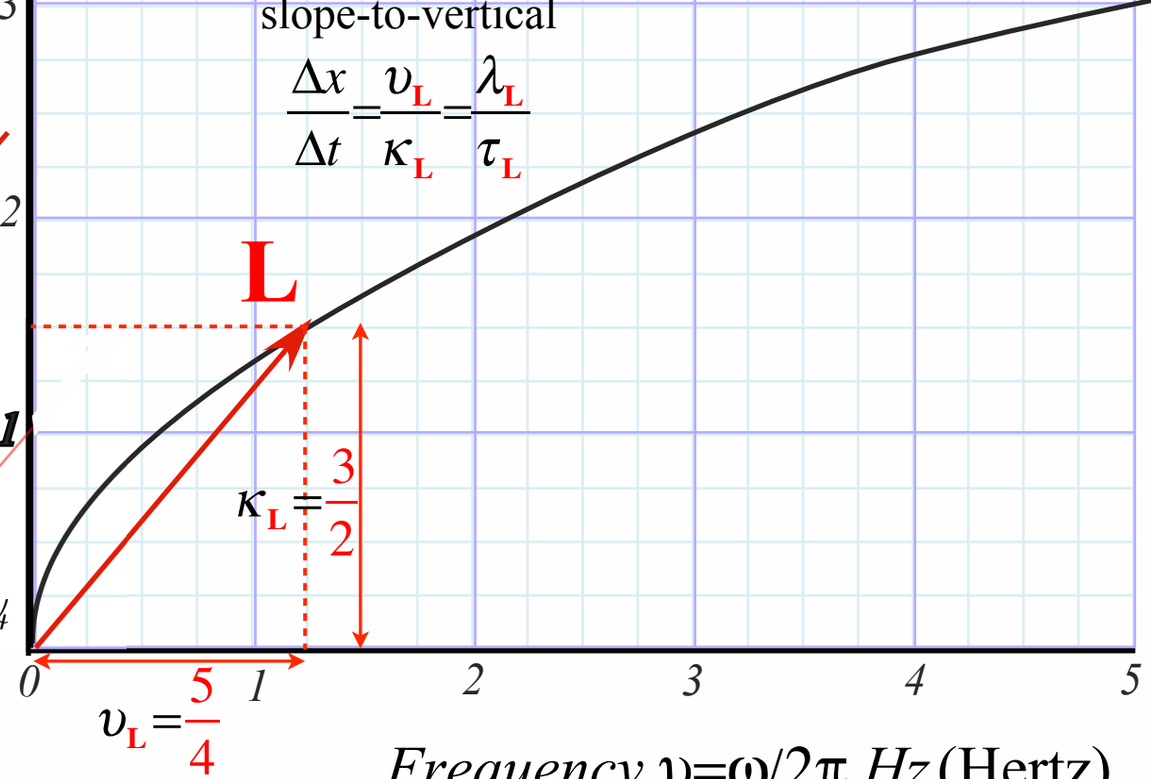
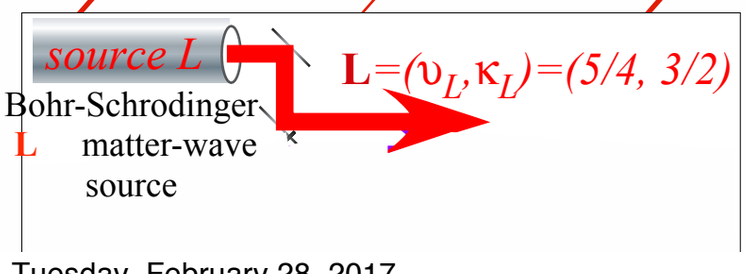
$$\frac{\Delta x}{\Delta t}=\frac{\upsilon_L}{\kappa_L}=\frac{\lambda_L}{\tau_L}$$



Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Web Simulation
1 (BW) Bohr Wave *ct vs x* Plot
(ck = 1.0 (App)= 1.5)

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$: Newton's "corpuscle" tracks vs.wave-zero paths

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Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

Spacetime (x,t)

Time t

$$\frac{\lambda_R}{\tau_R}=\frac{\upsilon_R}{\kappa_R}=\frac{1}{3}\Big/\frac{1}{5}$$

$$V_{phase}=\frac{5}{3}$$

$$\lambda_R=\frac{1}{\kappa_R}=\frac{1}{3}$$

$$\tau_R=\frac{1}{\upsilon_R}=\frac{1}{5}$$

0.2 0.4 0.6 0.8 1.0 1.2 1.4

Space x

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}$$

$$\mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

υ

$\upsilon_R=5$

κ

$\omega=5k^2/9$

R

$\kappa_R=3$

L

0 1 2 3 4 5

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

Bohr-Schrodinger
R matter-wave
source

source R

$\mathbf{R}=(\upsilon_R,\kappa_R)=(5.0, 3.0)$

BohrIt Web Simulation

1 (BW) ck = 2.0 (App) = 3.0)

Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

BohrItWeb (2,1)
Resonance

Spacetime (x,t)

$$\frac{\lambda_R}{\tau_R}=\frac{\upsilon_R}{\kappa_R}=\frac{1}{3}\Big/\frac{1}{5}$$

Time t

$$V_{phase}=\frac{5}{3}$$

$$\lambda_R=\frac{1}{\kappa_R}=\frac{1}{3}$$

$$\tau_R=\frac{1}{\upsilon_R}=\frac{1}{5}$$

Space x

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

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$\upsilon_R=5$

$\kappa_R=3$

$\omega=5k^2/9$

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

Bohr-Schrodinger
R matter-wave source
source R

$\mathbf{R}=(\upsilon_R,\kappa_R)=(5.0, 3.0)$

BohrIt Web Simulation
1 (BW) ck = 2.0 (App) = 3.0)

Newton's "corpuscle" tracks of overlapped Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$ *& Fast R-wave* $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

Spacetime (x,t)

$$\frac{\lambda_R}{\tau_R}=\frac{\upsilon_R}{\kappa_R}=\frac{1}{3}\Big/\frac{1}{5}$$

$$\frac{\lambda_L}{\tau_L}=\frac{\upsilon_L}{\kappa_L}=\frac{2}{3}\Big/\frac{4}{5}=\frac{2}{3}\cdot\frac{5}{4}$$

$$V_{phase}=\frac{5}{3}$$

Time t

$$V_{phase}=\frac{5}{6}$$

$$\lambda_L=\frac{1}{\kappa_L}=\frac{2}{3}$$

$$\tau_L=\frac{1}{\upsilon_L}=\frac{4}{5}$$

$$\lambda_R=\frac{1}{\kappa_R}=\frac{1}{3}$$

$$\tau_R=\frac{1}{\upsilon_R}=\frac{1}{5}$$

Space x

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}$$

$$\mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$\upsilon_R=5$

$\omega=5k^2/9$

$\kappa_R=3$

$\upsilon_L=5/4$

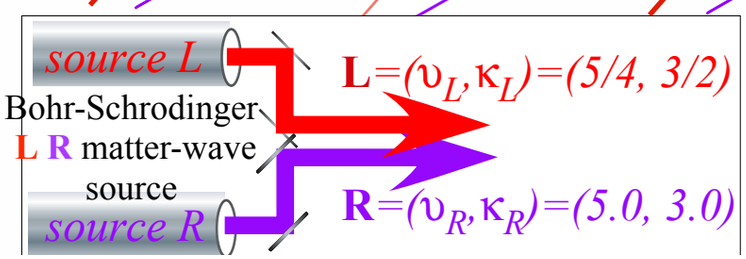
$\kappa_L=3/2$

L

R

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$: Newton's "corpuscle" tracks vs.wave-zero paths

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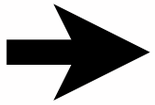
Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$



Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-sum factor $S=e^{i(\mathbf{L}+\mathbf{R})/2}$

Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-difference factor $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$

Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: *Half-sum factor* $S=e^{i(\mathbf{L}+\mathbf{R})/2}$

Spacetime (x,t) $\quad \frac{\lambda_P}{\tau_P}=\frac{\upsilon_P}{\kappa_P}=\frac{4}{9}\Big/\frac{8}{25}$

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Phase}=\frac{25}{18}$

$\tau_P=\frac{1}{\upsilon_P}=\frac{8}{25}$

$\lambda_P=\frac{1}{\kappa_P}=\frac{4}{9}$

0.2 0.4 0.6 0.8 1.0 1.2 1.4

Space x

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$$\mathbf{P}=\frac{\mathbf{R}+\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R+\upsilon_L\\ \kappa_R+\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}25/8\\ 9/4\end{pmatrix}$$

$\omega=5k^2/9$

$\upsilon_P=\frac{25}{8}$

$\mathbf{P}=(\mathbf{L}+\mathbf{R})/2$

$\kappa_L=\frac{3}{2}$

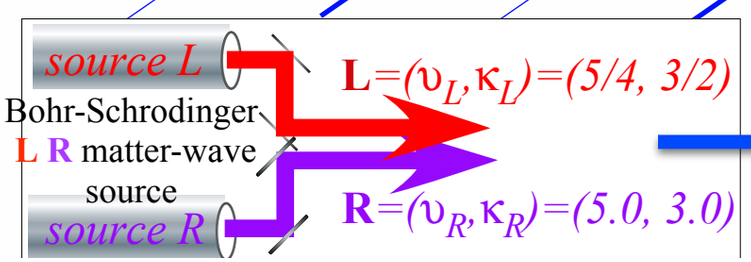
$\kappa_P=\frac{9}{4}$

$\upsilon_L=\frac{5}{4}$

0 1 2 3 4 5

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Phase factor

BohrIt Web Simulation - *2 Colinear BW with ck = 1.5, 3.0 = 1, 2 (App)*

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➔ *Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-difference factor $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$*

Group velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-difference factor $D=e^{i(L-R)/2}+e^{-i(L-R)/2}$

Spacetime (x,t) $\quad \frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Group}=\frac{5}{2}$

$\tau_G=\frac{1}{\upsilon_G}=\frac{8}{15}$

$\lambda_G=\frac{1}{\kappa_G}=\frac{4}{3}$

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$$\mathbf{P}=\frac{\mathbf{R}+\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R+\upsilon_L\\ \kappa_R+\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}25/8\\ 9/4\end{pmatrix}$$

$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

$\omega=5k^2/9$

R

$\mathbf{P}=(\mathbf{L}+\mathbf{R})/2$

$\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$

L

$\upsilon_G=\frac{15}{8}$ $\quad \kappa_G=\frac{3}{4}$

$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

$\upsilon_G=\frac{15}{8}1$

Space x

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

source L

Bohr-Schrodinger

L R matter-wave source

source R

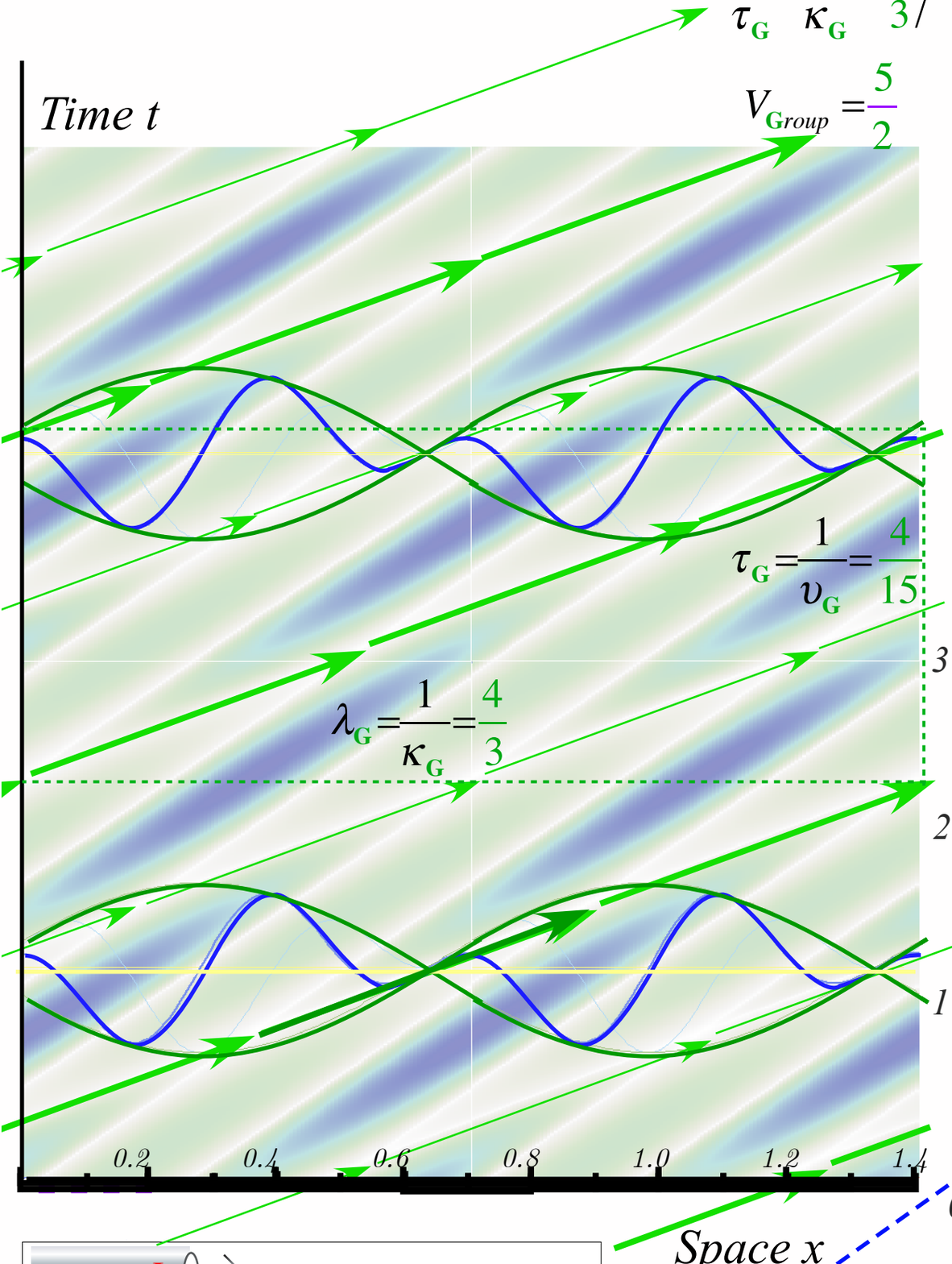
$\mathbf{L}=(\upsilon_L,\kappa_L)=(5/4,\ 3/2)$

$\mathbf{R}=(\upsilon_R,\kappa_R)=(5.0,\ 3.0)$

Phase factor and Group factor

Group velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-difference factor $D=e^{i(L-R)/2}+e^{-i(L-R)/2}$

Spacetime (x,t) $\frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$ *Per-spacetime* $(\omega,k)=2\pi(\upsilon,\kappa)$



BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

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$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

$\omega=5k^2/9$

$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

$\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$

$\upsilon_G=\frac{15}{8}$ $\kappa_G=\frac{3}{4}$

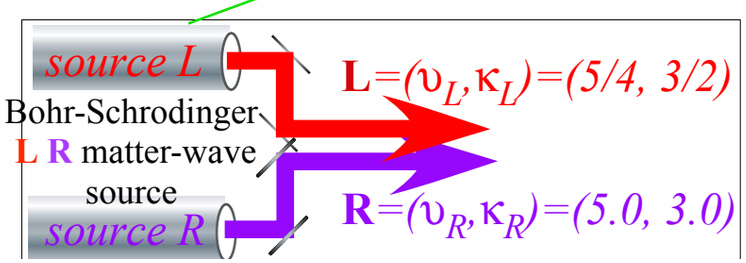
$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

0 1 2 3 4 5

$\upsilon_G=\frac{15}{8}$

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Group factor

BohrIt Web Simulation - *2 Colinear BW with ck = 1.5, 3.0 = 1, 2 (App)*

Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: *Half-difference factor* $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$

Spacetime (x,t) $\quad \frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$

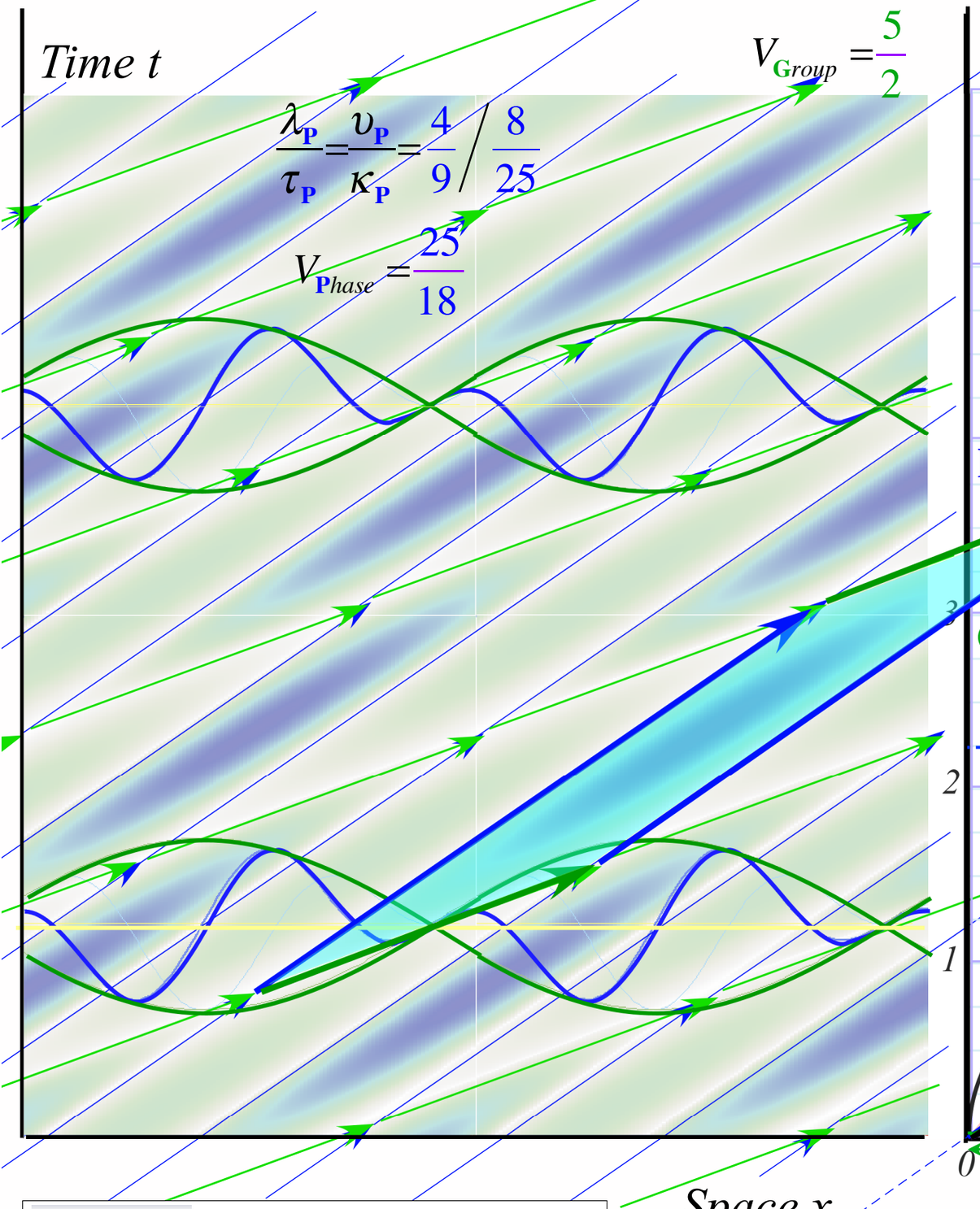
Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Group}=\frac{5}{2}$

$\frac{\lambda_P}{\tau_P}=\frac{\upsilon_P}{\kappa_P}=\frac{4}{9}\Big/\frac{8}{25}$

$V_{Phase}=\frac{25}{18}$



Space x

BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$$\mathbf{P}=\frac{\mathbf{R}+\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R+\upsilon_L\\ \kappa_R+\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}25/8\\ 9/4\end{pmatrix}$$

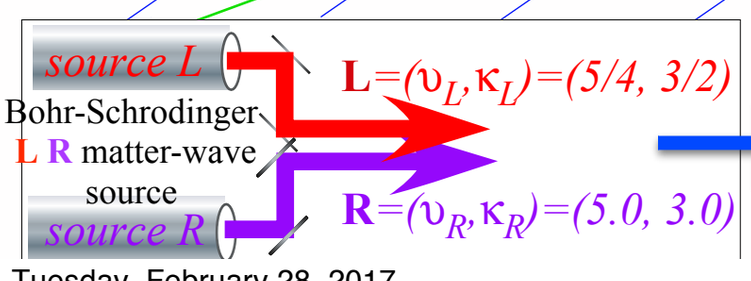
$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

$\omega=5k^2/9$

$\upsilon_P=\frac{25}{8}$ $\quad \kappa_P=\frac{9}{4}$ $\quad \kappa_G=\frac{3}{4}$ $\quad \upsilon_G=\frac{15}{8}$

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Phase factor and Group factor

Group velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-difference factor $D=e^{i(L-R)/2}+e^{-i(L-R)/2}$

Spacetime (x,t) $\quad \frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$ $\qquad$ *Per-spacetime* $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Group}=\frac{5}{2}$

BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$$\mathbf{P}=\frac{\mathbf{R}+\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R+\upsilon_L\\ \kappa_R+\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}25/8\\ 9/4\end{pmatrix}$$

$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

$\omega=5k^2/9$

G **R** **L** **P=(R+L)/2**

$\upsilon_G=\frac{15}{8}$ $\quad \kappa_G=\frac{3}{4}$

G=(R-L)/2

Space x

$\upsilon_G=\frac{15}{8}$

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

source L

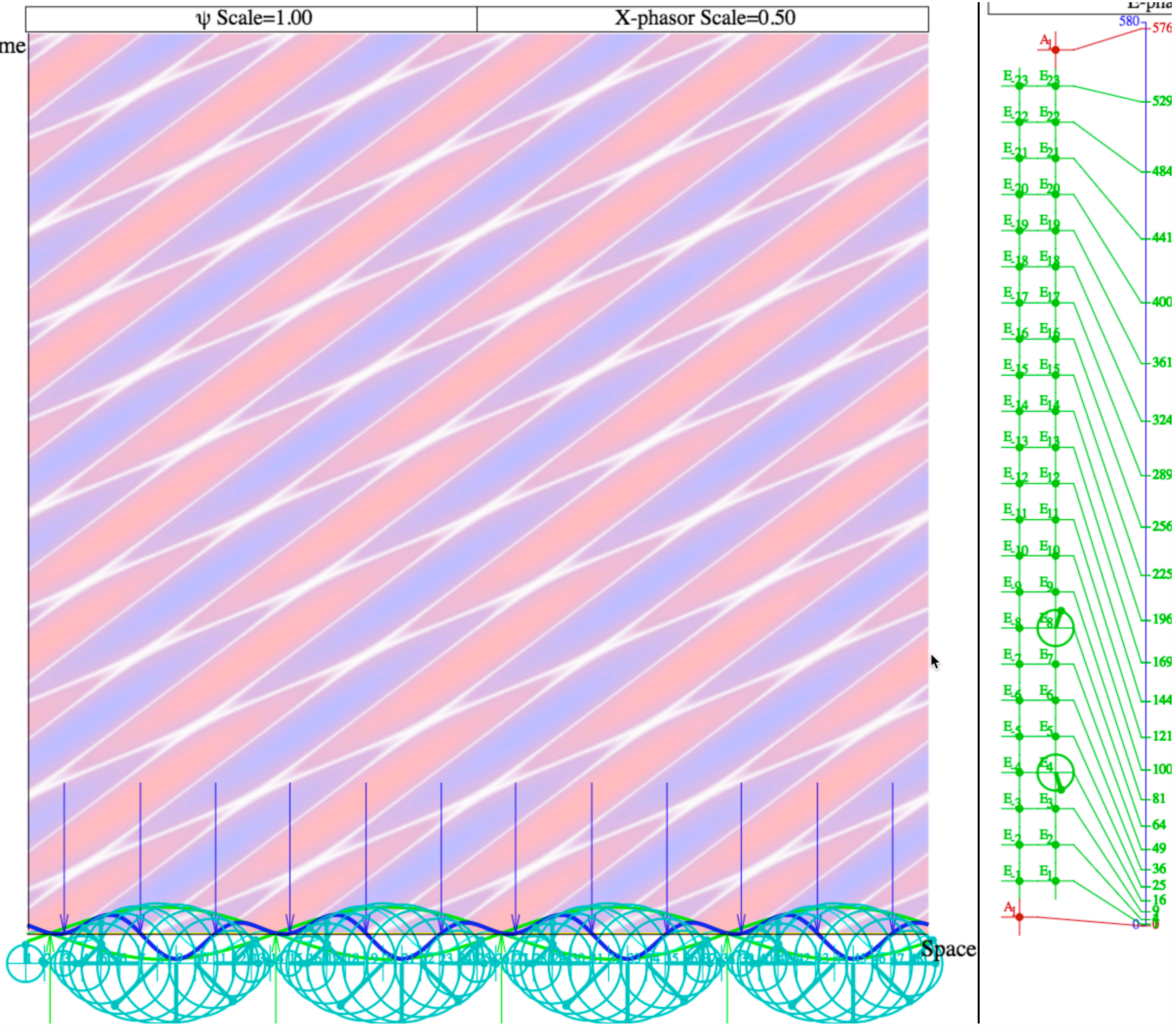
Bohr-Schrodinger
L R matter-wave
source

source R

$\mathbf{L}=(\upsilon_L,\kappa_L)=(5/4,\ 3/2)$

$\mathbf{R}=(\upsilon_R,\kappa_R)=(5.0,\ 3.0)$

Phase factor and Group factor

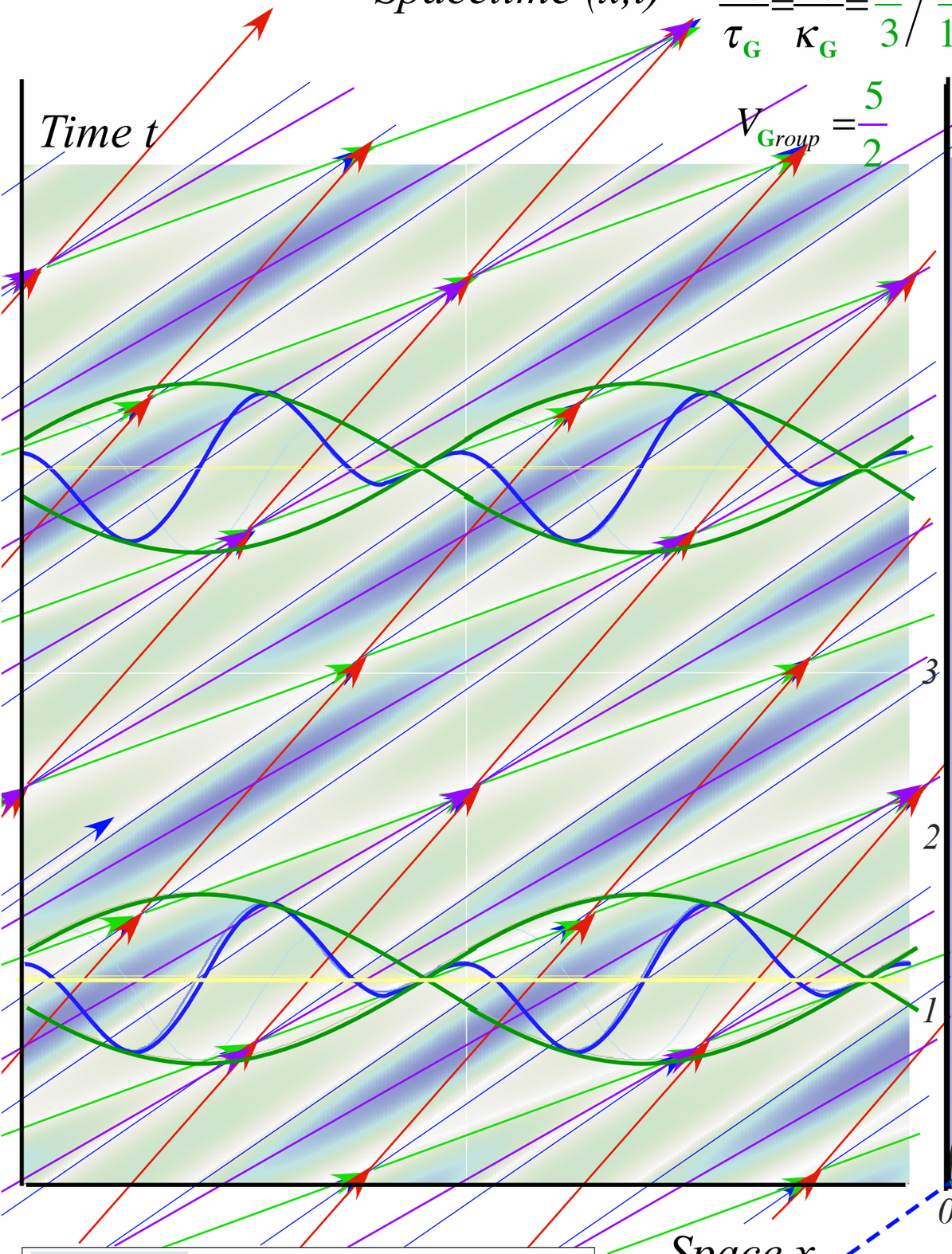

ψ Scale=1.00
X-phasor Scale=0.50
Time
Space
580
576
529
484
441
400
361
324
289
256
225
196
169
144
121
100
81
64
49
36
25
16
9
4
1
0

Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: *Half-difference factor* $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$

Spacetime (x,t) $\quad \frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$ $\quad$ *Per-spacetime* $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Group}=\frac{5}{2}$



Space x

BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

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$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

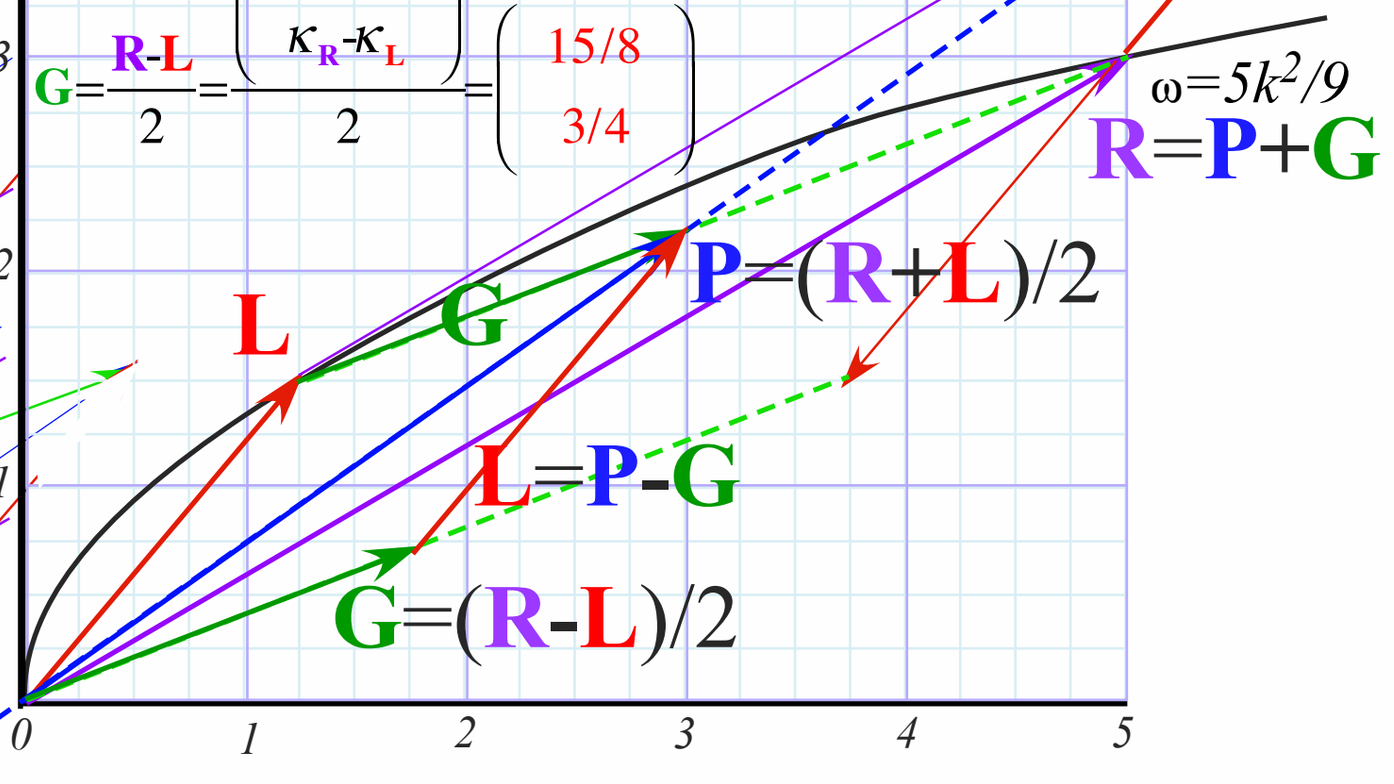
$\omega=5k^2/9$

$\mathbf{R}=\mathbf{P}+\mathbf{G}$

$\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$

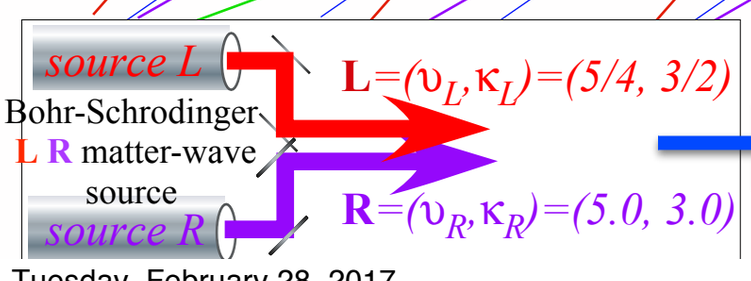
$\mathbf{L}=\mathbf{P}-\mathbf{G}$

$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$



Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Phase factor and Group factor

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Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$V_{Group}=\frac{5}{2}$

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$\omega=5k^2/9$

$\mathbf{R}=\mathbf{P}+\mathbf{G}$

$\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$

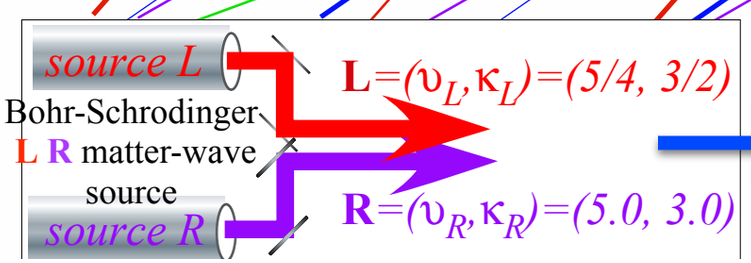
$\mathbf{L}=\mathbf{P}-\mathbf{G}$

$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

Space x

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$



Phase factor and Group factor

Group velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-difference factor $D=e^{i(L-R)/2}+e^{-i(L-R)/2}$

Spacetime (x,t)

$$\frac{\lambda_G}{\tau_G}=\frac{\upsilon_G}{\kappa_G}=\frac{4}{3}\Big/\frac{8}{15}$$

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$

Time t

$$V_{Group}=\frac{5}{2}$$

BohrItWeb (2,1)
Resonance

Wavenumber $\kappa=k/2\pi$

Wavevector $k=2\pi\kappa$

$$\mathbf{L}=\begin{pmatrix}\upsilon_L\\ \kappa_L\end{pmatrix}=\begin{pmatrix}5/4\\ 3/2\end{pmatrix}\qquad \mathbf{R}=\begin{pmatrix}\upsilon_R\\ \kappa_R\end{pmatrix}=\begin{pmatrix}5\\ 3\end{pmatrix}$$

$$\mathbf{P}=\frac{\mathbf{R}+\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R+\upsilon_L\\ \kappa_R+\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}25/8\\ 9/4\end{pmatrix}$$

$$\mathbf{G}=\frac{\mathbf{R}-\mathbf{L}}{2}=\frac{\begin{pmatrix}\upsilon_R-\upsilon_L\\ \kappa_R-\kappa_L\end{pmatrix}}{2}=\begin{pmatrix}15/8\\ 3/4\end{pmatrix}$$

$\omega=5k^2/9$

$\mathbf{R}=\mathbf{P}+\mathbf{G}$

$\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$

$\mathbf{L}=\mathbf{P}-\mathbf{G}$

$\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

L G

0 1 2 3 4 5

1 2 3

Space x

Frequency $\upsilon=\omega/2\pi$ *Hz* (Hertz)

Angular-Frequency $\omega=2\pi\upsilon$

source L

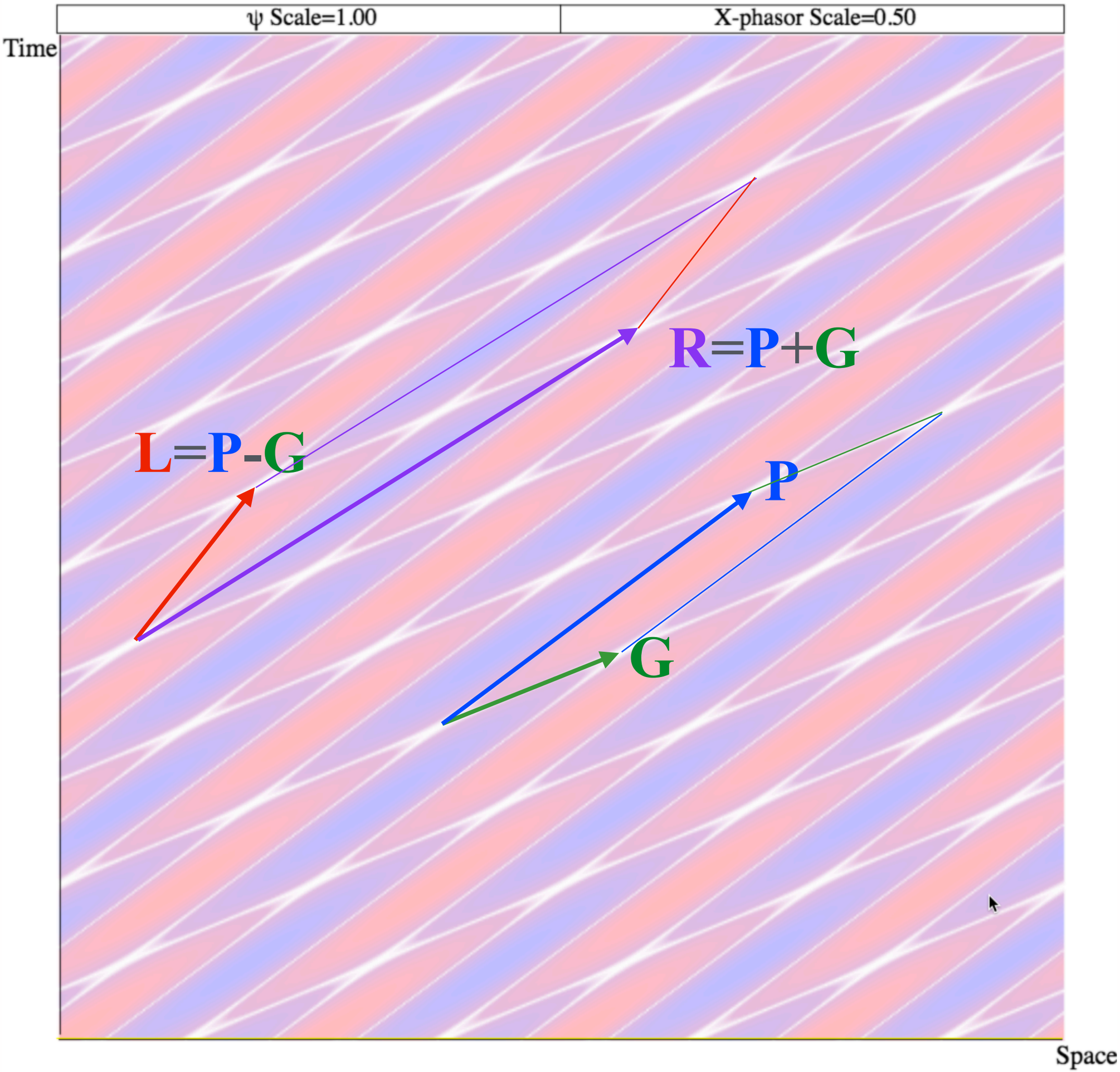
Bohr-Schrodinger
L R matter-wave
source

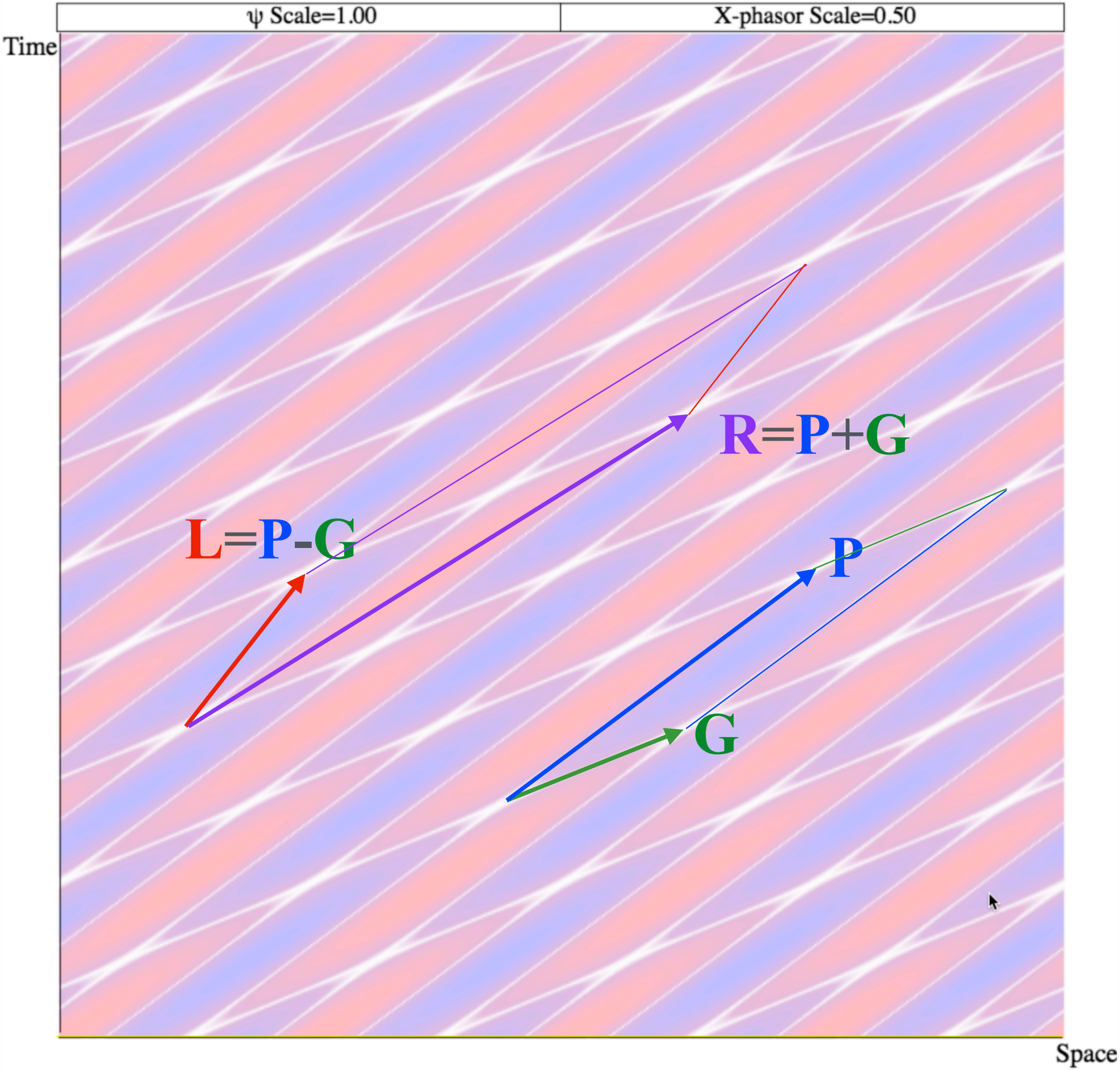
source R

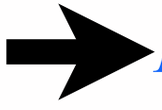
$\mathbf{L}=(\upsilon_L,\kappa_L)=(5/4,\ 3/2)$

$\mathbf{R}=(\upsilon_R,\kappa_R)=(5.0,\ 3.0)$

Phase factor and Group factor


ψ Scale=1.00
X-phasor Scale=0.50
Time
R=P+G
L=P-G
P
G
Space


ψ Scale=1.00
X-phasor Scale=0.50
Time
R=P+G
L=P-G
P
G
Space



Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (*__L__*,* __R__*)-per-spacetime*

Vectors of phase **P**′=(**R**′+**L**′)/2 *and group* **G**′=(**R**′-**L**′)/2

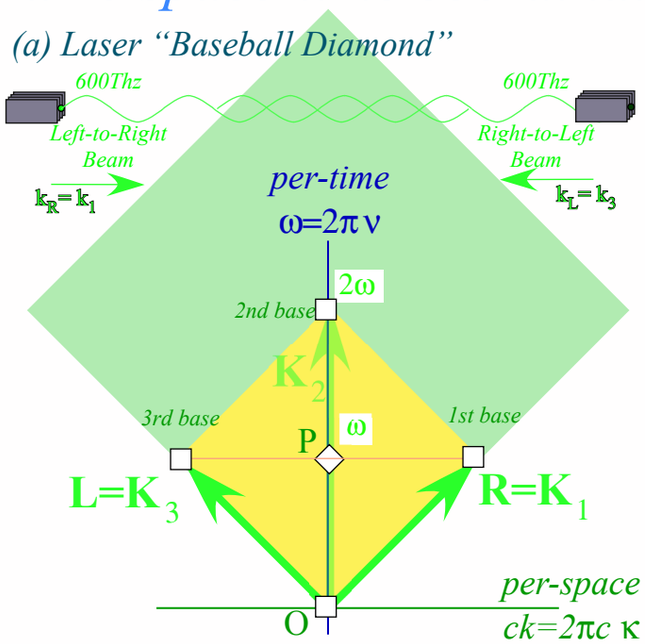
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

Brief tour of and relativistic mechanics by geometry

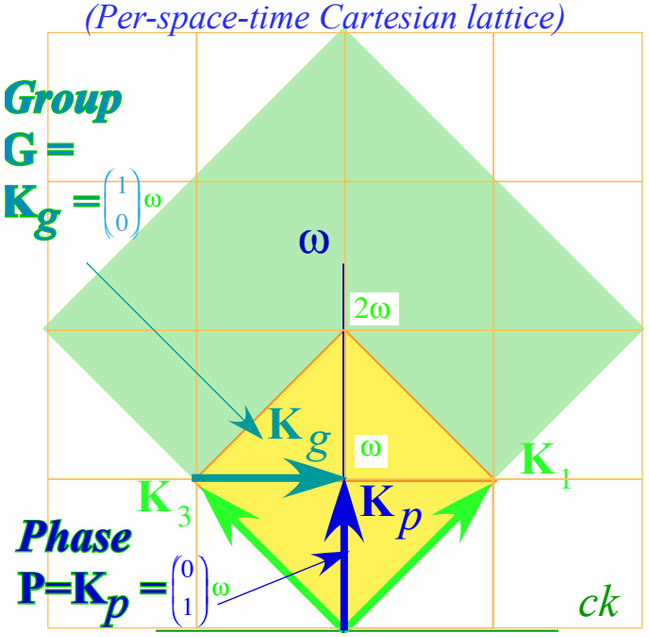
*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

u=0 space-time coordinates

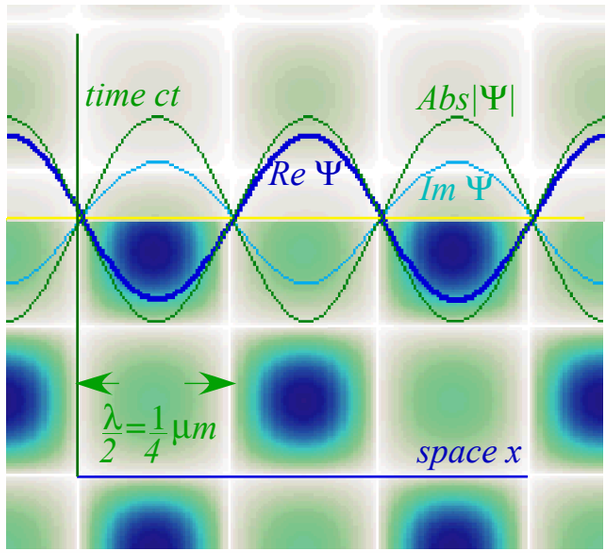
(a) Laser "Baseball Diamond"



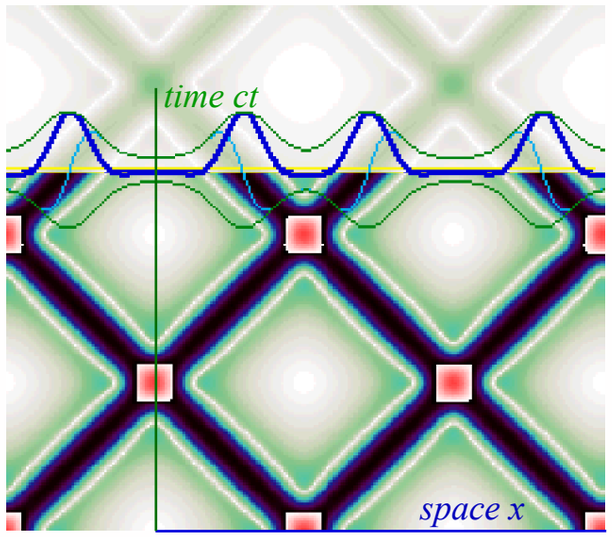
(b) Laser group and phase wavevectors
(Per-space-time Cartesian lattice)



(c) Laser Coherent Wave (CW) paths
(Space-time Cartesian grid)



(d) Laser Pulse Wave (PW) Paths
(Space-time Diamond grid)



Wave coordinates with Linear Dispersion

Continuous Wave (CW) coordinates discussed in following pages…

…starting with standing-wave case shown here

BohrIt Web Simulation
2 CW *ct vs x* Plot
(ck = ±2)

Pulse Wave (PW) coordinates ("Packet-Wave" or "particle-like") PW dynamics is discussed in the following Lecture 13

BohrIt Web Simulation
2 PW *ct vs x* Plot
(k mod 2 = 0)

u=0 space-time pulse waves

CM with a BANG! Fig. 8.2.1

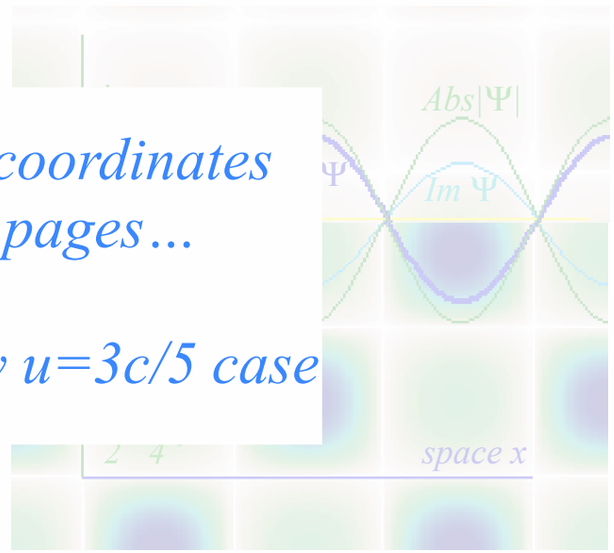
CM with a BANG! Unit 8

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

u=0 space-time coordinates

(a) Laser "Baseball Diamond"

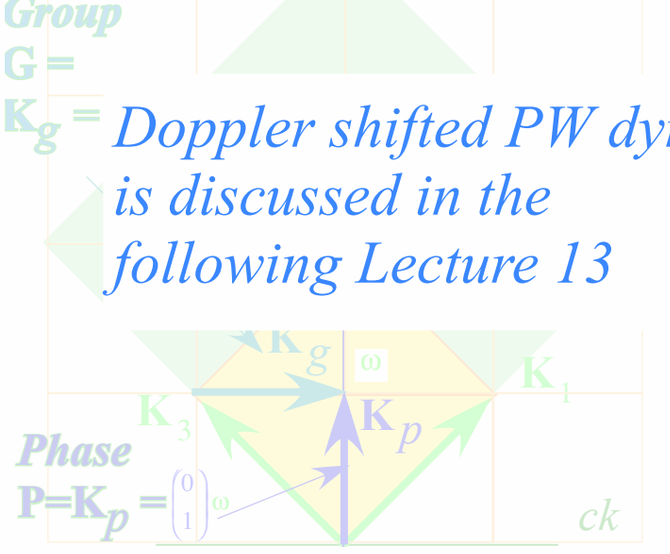
(c) Laser Coherent Wave (CW) paths (Space-time Cartesian grid)



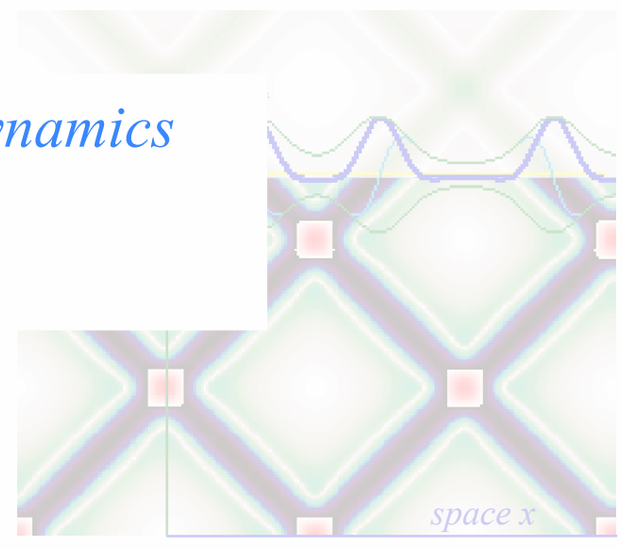
Doppler shifted (CW) coordinates discussed in following pages…

…starting with velocity u=3c/5 case shown here

(b) Laser group and phase wavevectors (Per-space-time Cartesian lattice)



(d) Laser Pulse Wave (PW) Paths (Space-time Diamond grid)

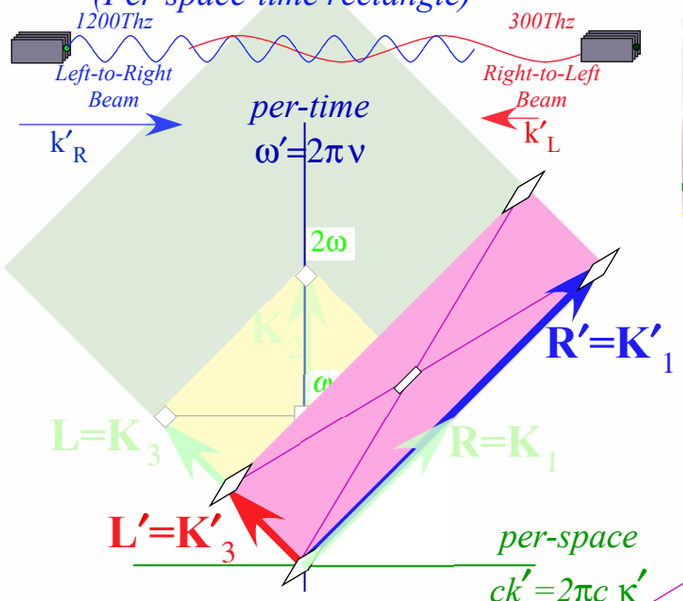


Doppler shifted PW dynamics is discussed in the following Lecture 13

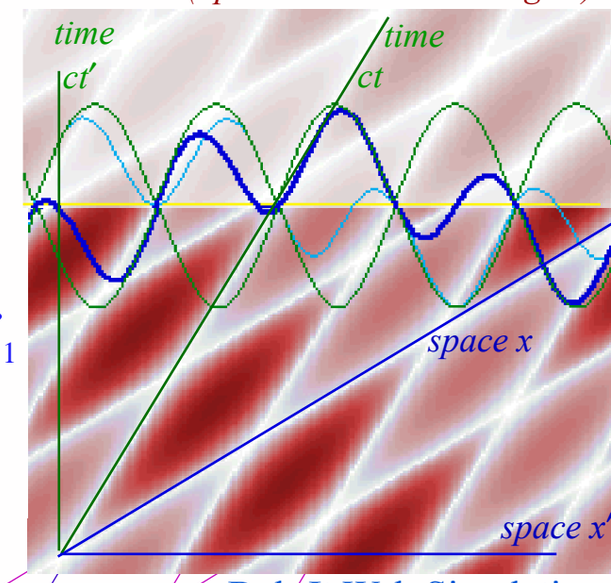
u=0 space-time pulse waves

Gives u=3c/5 Einstein-Lorentz-Minkowski coordinates

(a) Boosted Laser "Baseball Diamond" (Per-space-time rectangle)

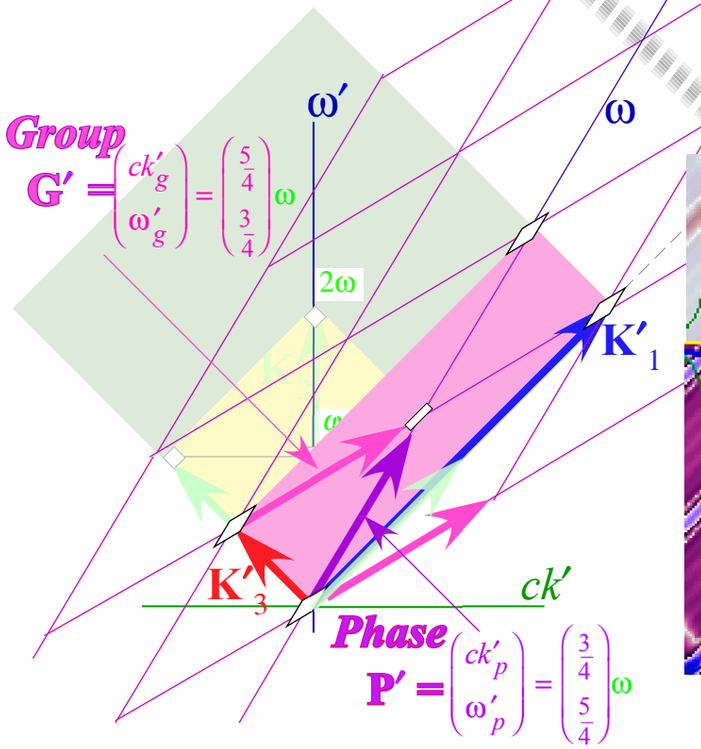


(c) (u=3c/5)-Boosted CW paths (Space-time Minkowski grid)

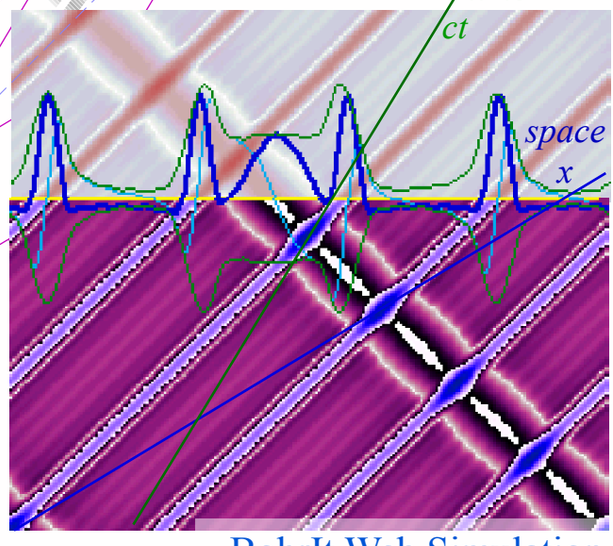


[BohrIt Web Simulation 2 CW *Minkowski* Plot (ck = -1, +4)]

(b) Boosted group and phase wavevectors (Per-space-time Minkowski lattice)



(d) Boosted PW Paths (Rectangular grid)

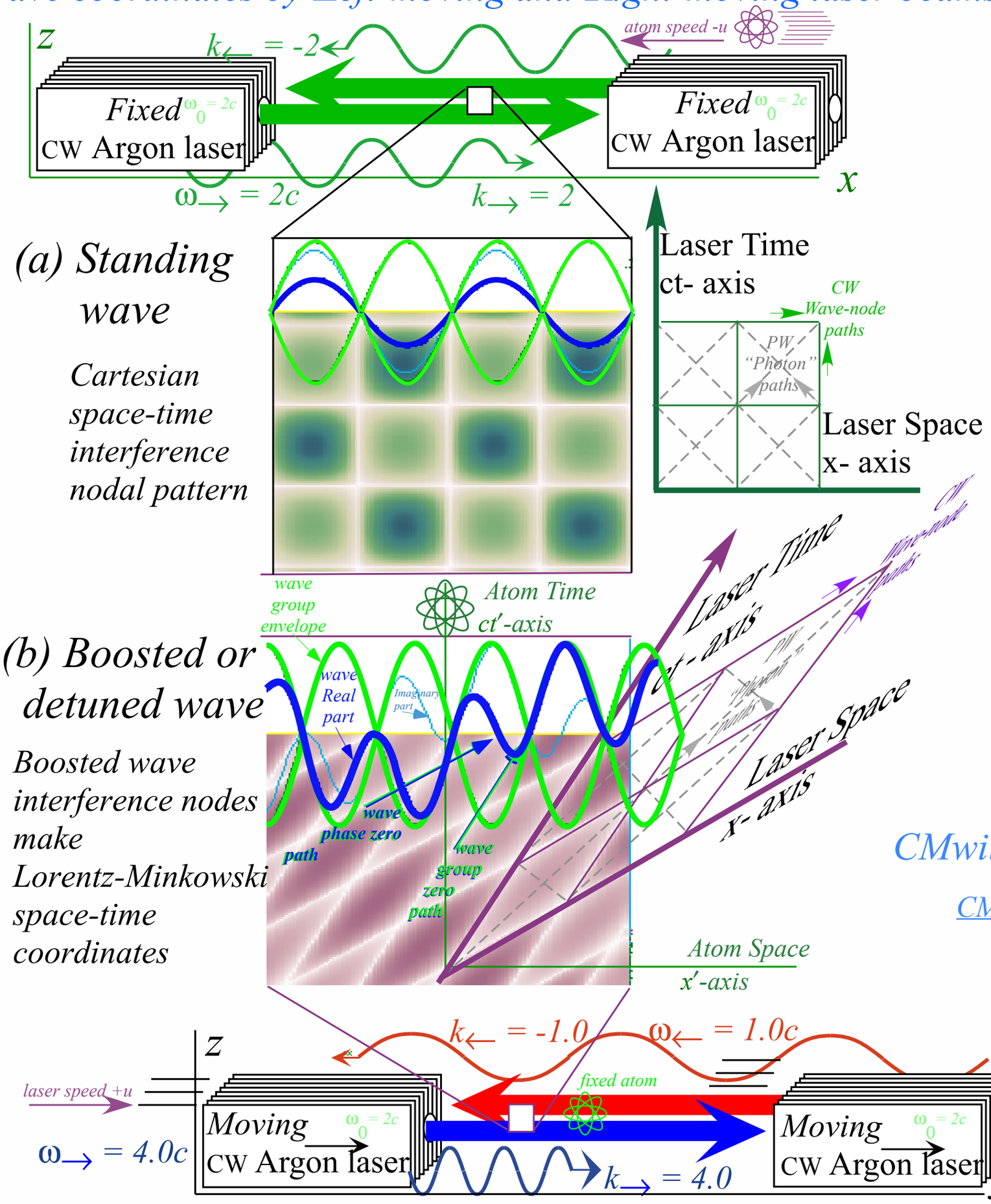


[BohrIt Web Simulation 2 PW *ct vs x* Plot (k = -n, 4n, n natural)]

u=3c/5 space-time pulse waves

CM with a BANG! Fig. 8.2.1 [CM with a BANG! Unit 8] *CM with a BANG! Fig. 8.2.2*

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*



BohrIt Web Simulation
2 CW *ct vs x* Plot
(ck = ±2)

CMwith a BANG! Fig. 8.2.3

CM with a BANG! Unit 8

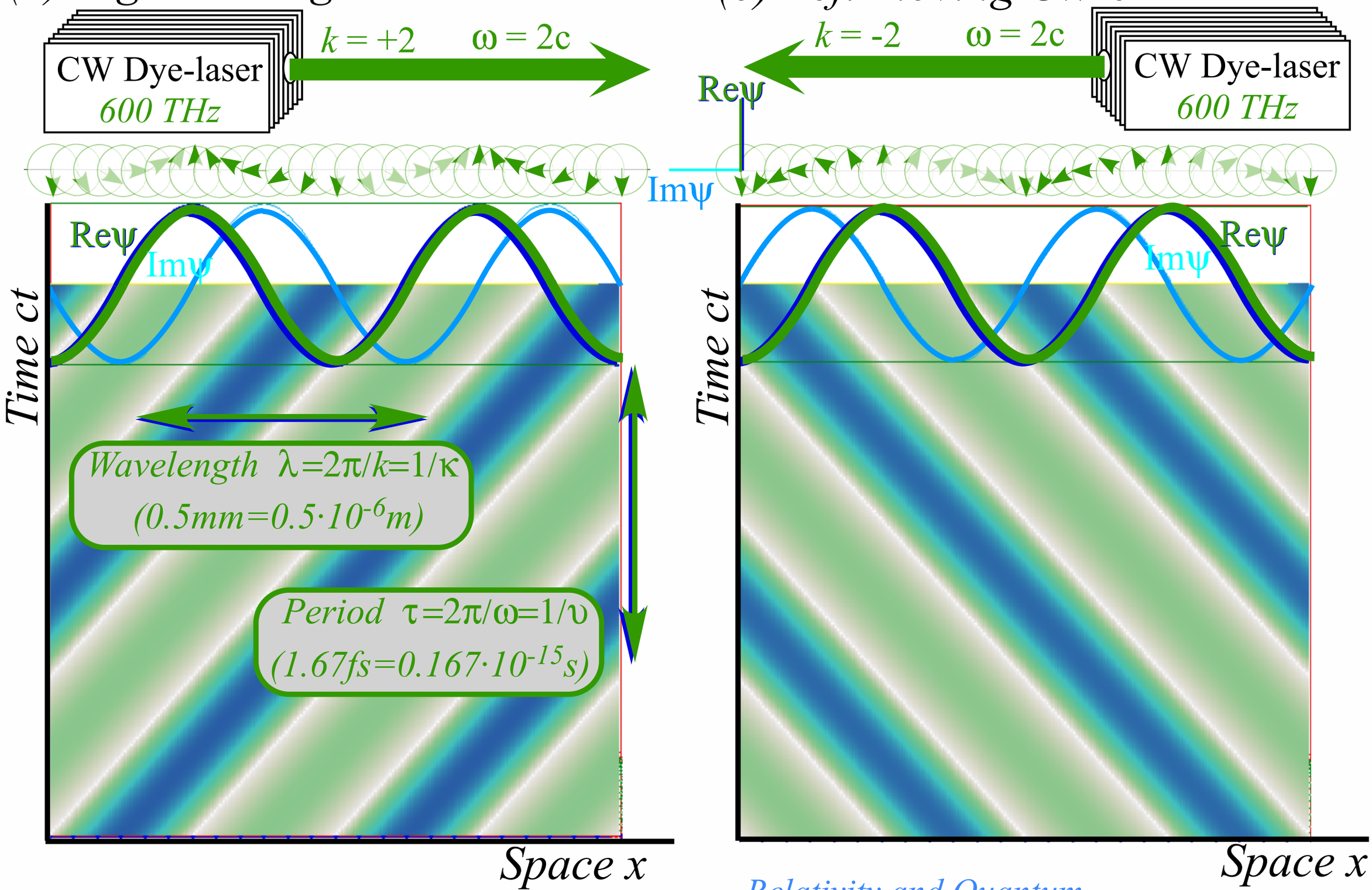
BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*
L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*
Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*
L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*
Doppler shifted **L**′-*vector and* **R**′-*vector in (***L***,* **R**)-*per-spacetime*
Vectors of phase **P**′=(**R**′+**L**′)/2 *and group* **G**′=(**R**′-**L**′)/2
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates
Brief tour of and relativistic mechanics by geometry
Summary of optical wave parameters for relativity and QM

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

(a) Right-moving CW $e^{i(kx-\omega t)}$

(b) Left-moving CW $e^{i(-kx-\omega t)}$

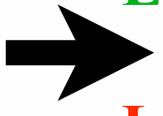


BohrIt Web Simulation
2 CW *ct vs x* Plot
(ck = ±2)

Relativity and Quantum Theory by Ruler and Compass
Fig. 5

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*



Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (*__L__*,* __R__*)-per-spacetime*

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Brief tour of and relativistic mechanics by geometry

Summary of optical wave parameters for relativity and QM

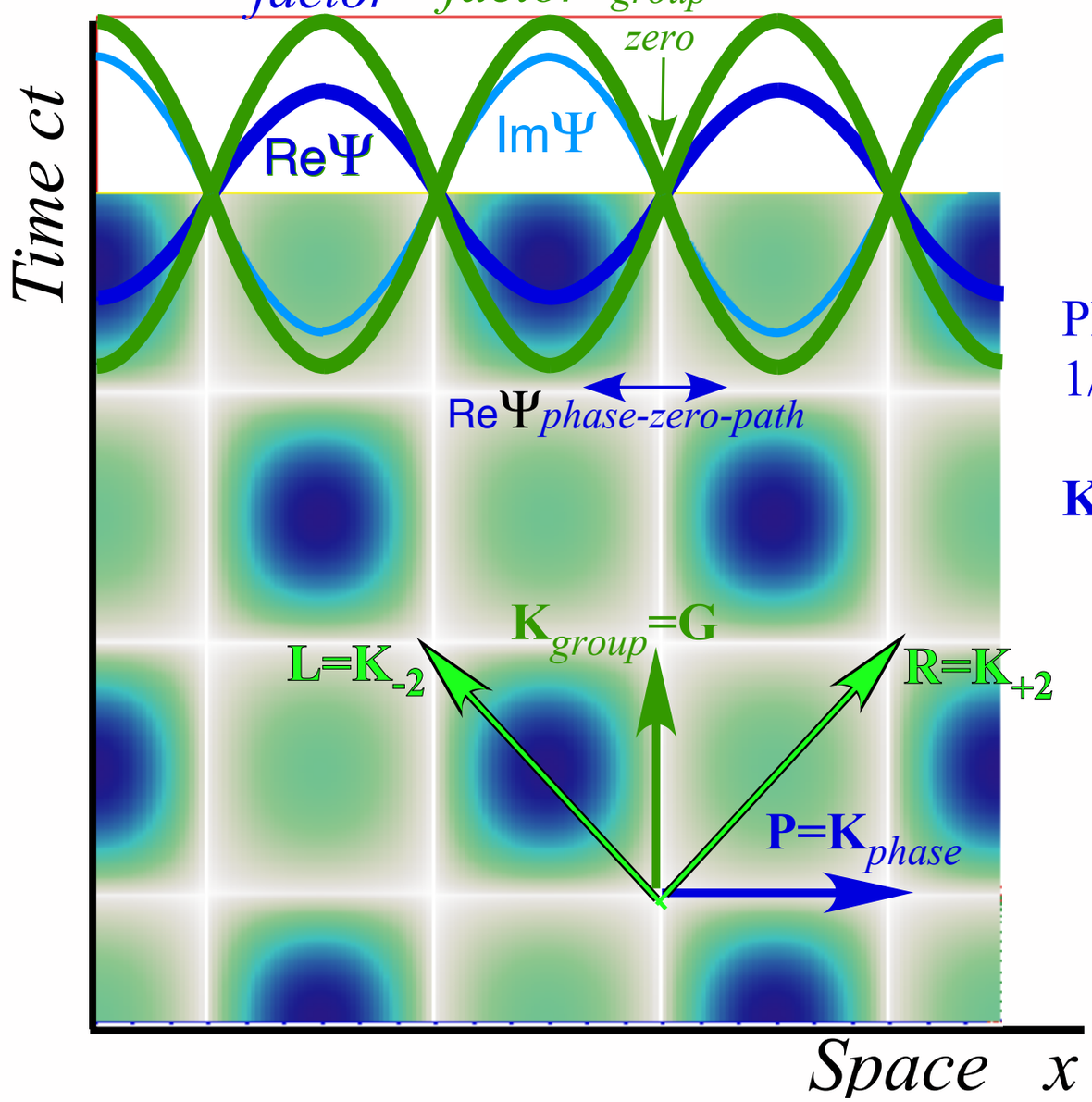
L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

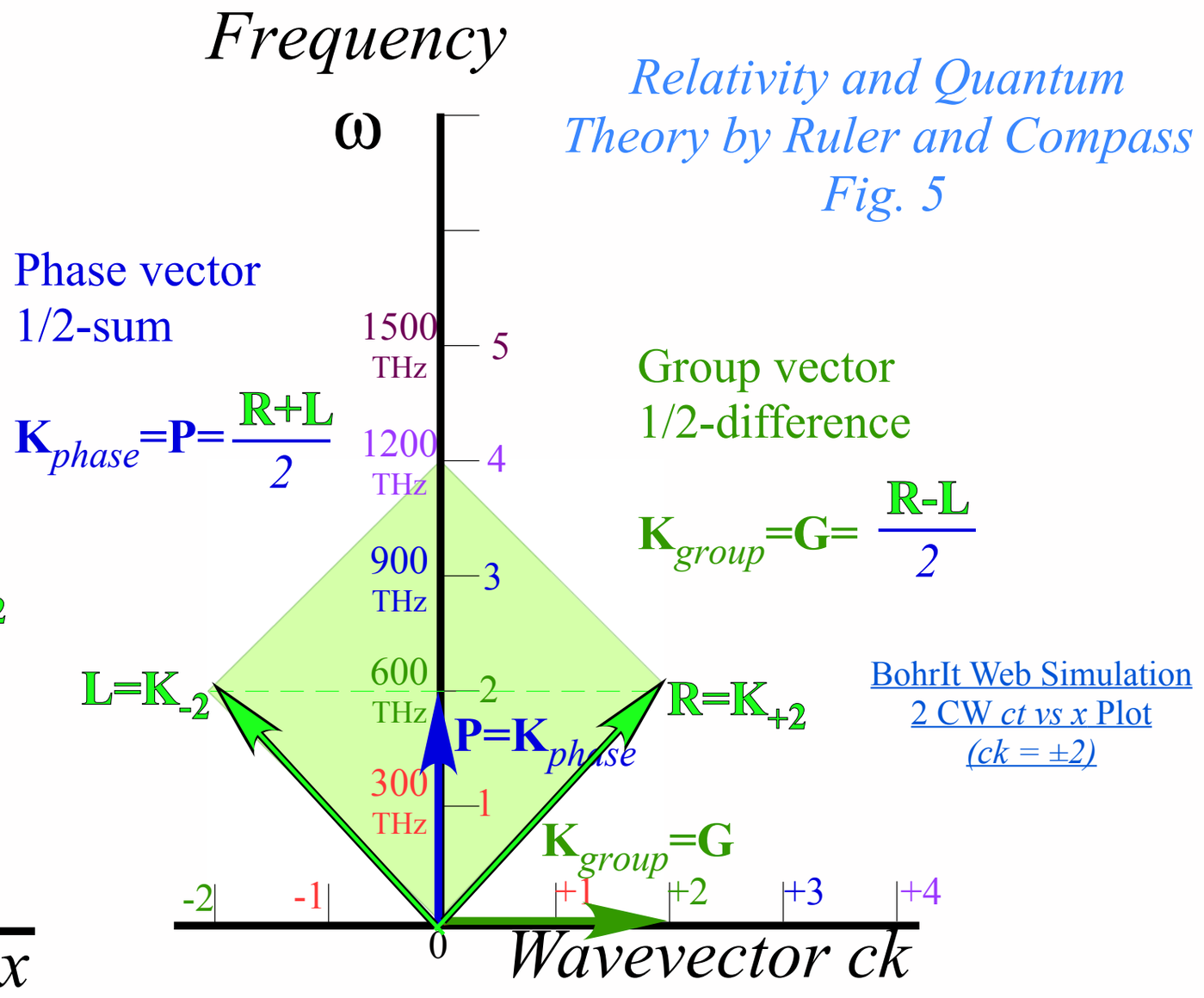
(c) Standing CW in space-time

$$\Psi(x,t) = (e^{-i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}$$

phase factor · *group factor* · $|\Psi|$ *group zero*



(d) Dispersion plot in per-space-time



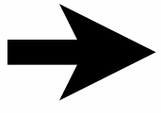
Relativity and Quantum Theory by Ruler and Compass Fig. 5

BohrIt Web Simulation
2 CW *ct vs x* Plot
(ck = ±2)

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*



L′-*laser 300THz and* **R′**-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L′**-*vector and* **R′**-*vector in (***L***,* **R***)-per-spacetime*

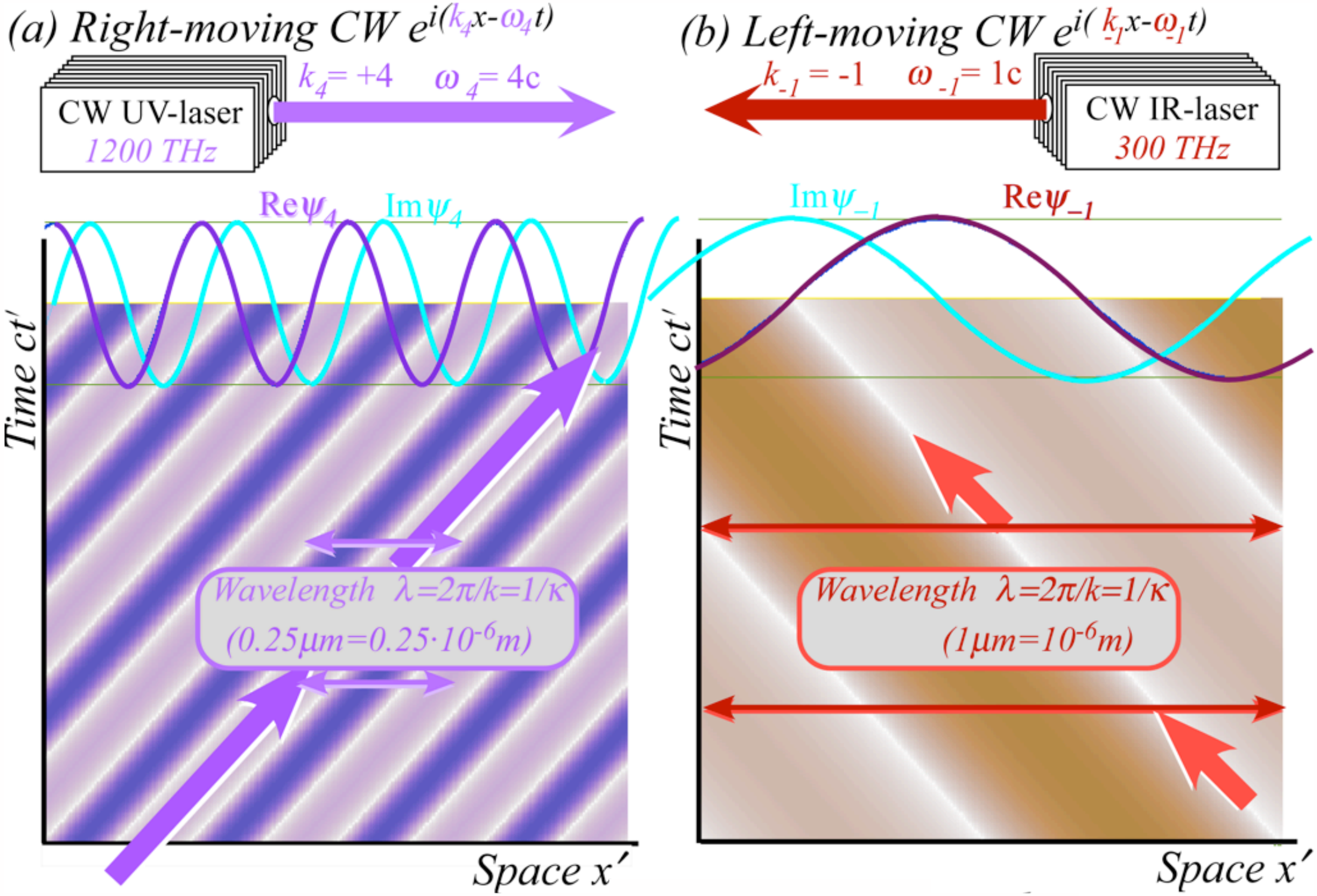
Vectors of phase **P′**=*(***R′**+**L′***)/2 and group* **G′**=*(***R′**-**L′***)/2*

Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

Brief tour of and relativistic mechanics by geometry

Summary of optical wave parameters for relativity and QM

L′-laser 300THz and R′-laser 1200THZ (Doppler shifted in moving frame)



BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)

Relativity and Quantum Theory by Ruler and Compass
Fig. 5
Relativity and Quantum Theory by Ruler and Compass

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (***L***,* **R***)-per-spacetime*

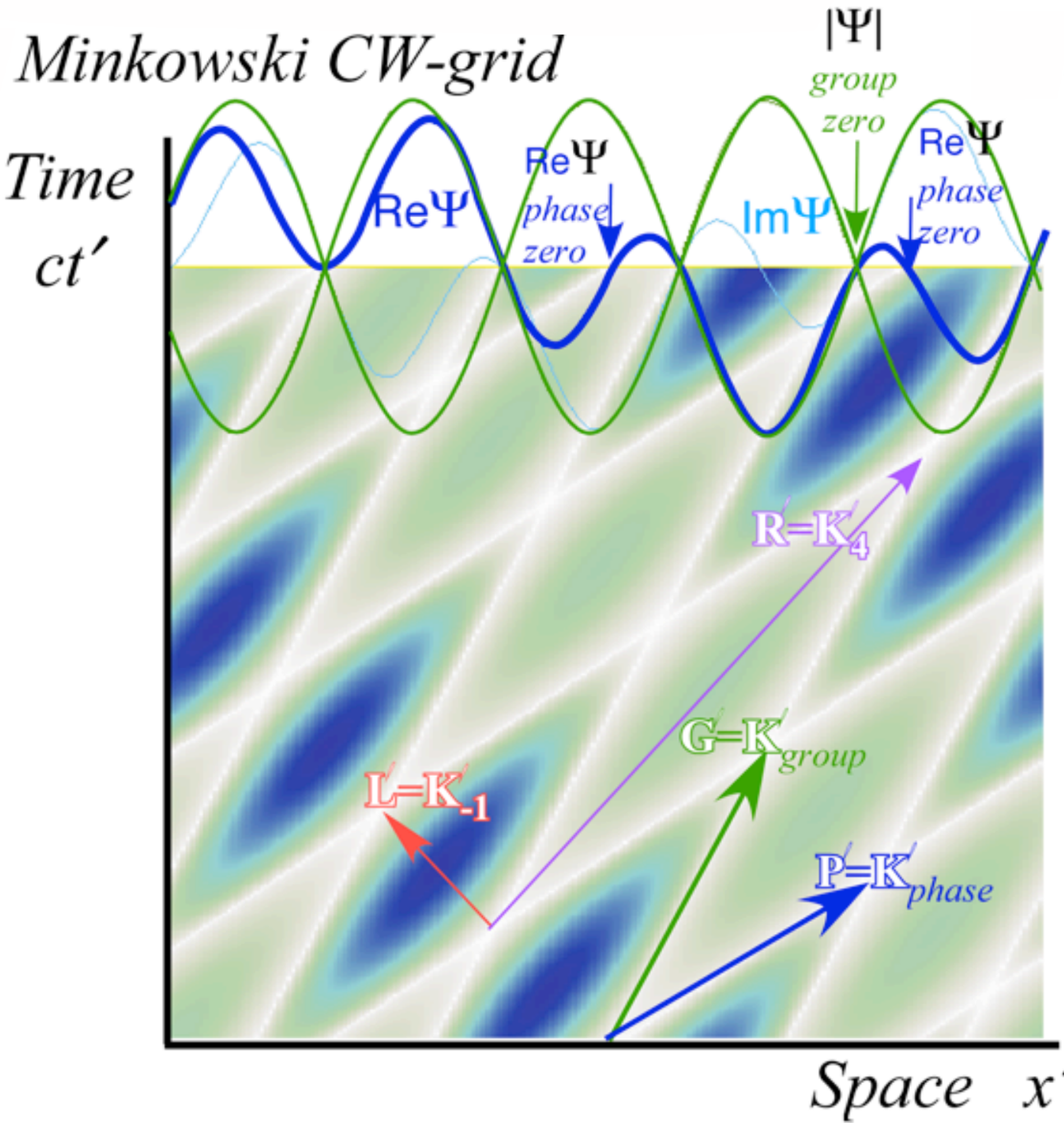
Vectors of phase **P**′=(**R**′+**L**′)/2 *and group* **G**′=(**R**′-**L**′)/2

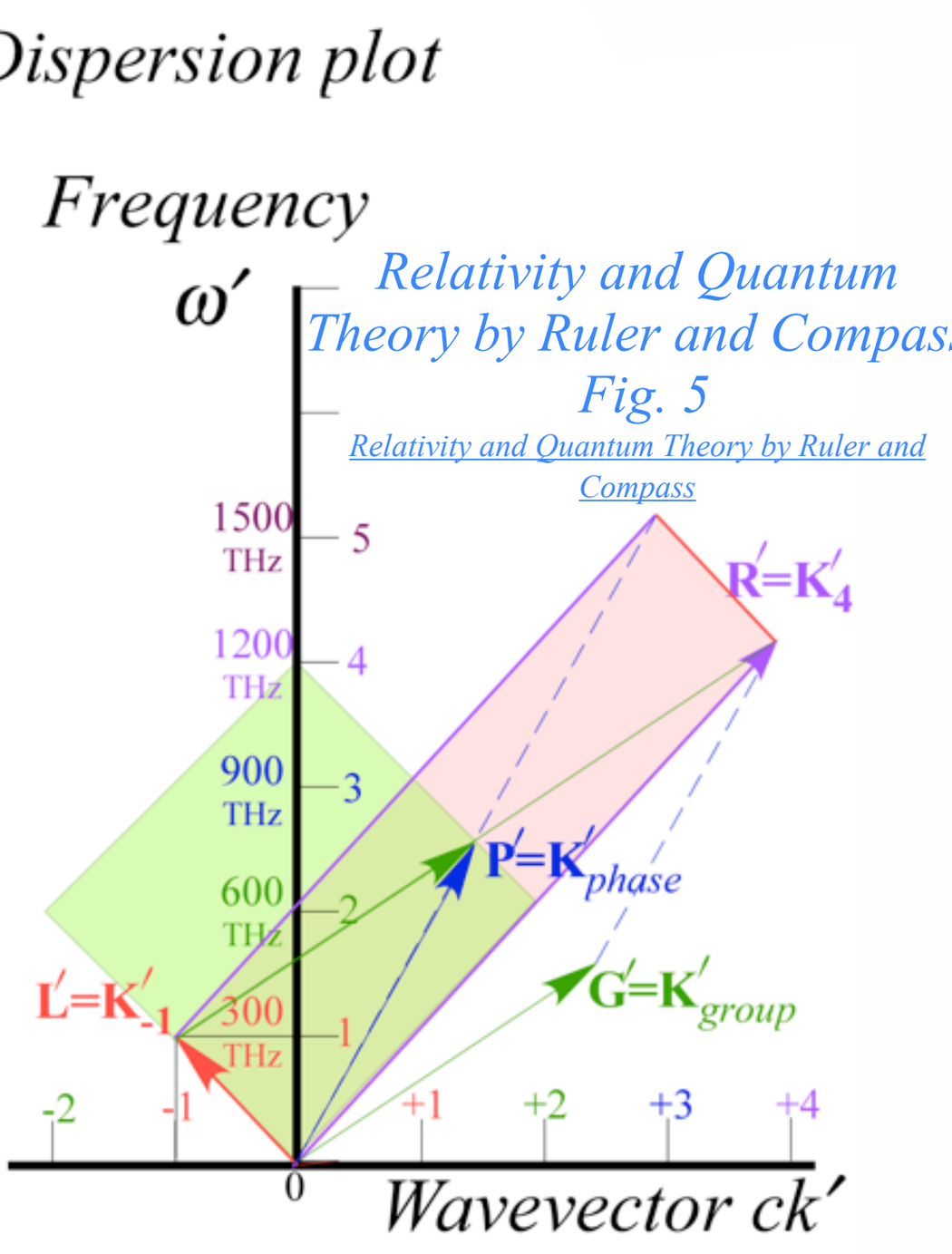
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

Brief tour of and relativistic mechanics by geometry

Summary of optical wave parameters for relativity and QM

Vectors of phase $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ *and group* $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$



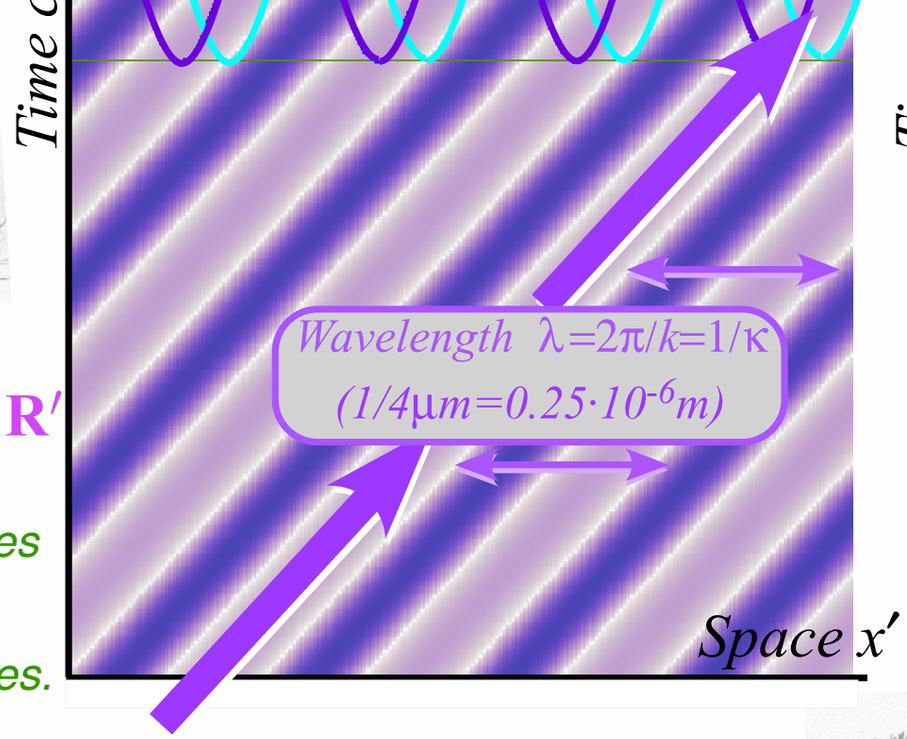


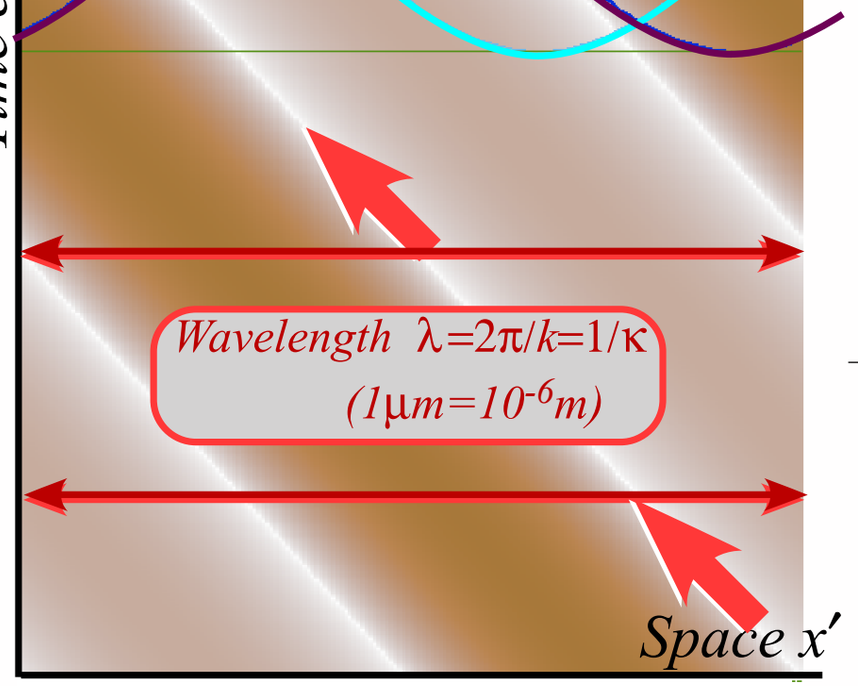
Relativity and Quantum Theory by Ruler and Compass

Fig. 5

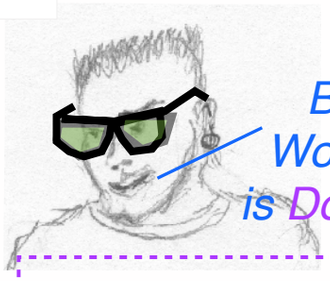
Relativity and Quantum Theory by Ruler and Compass

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



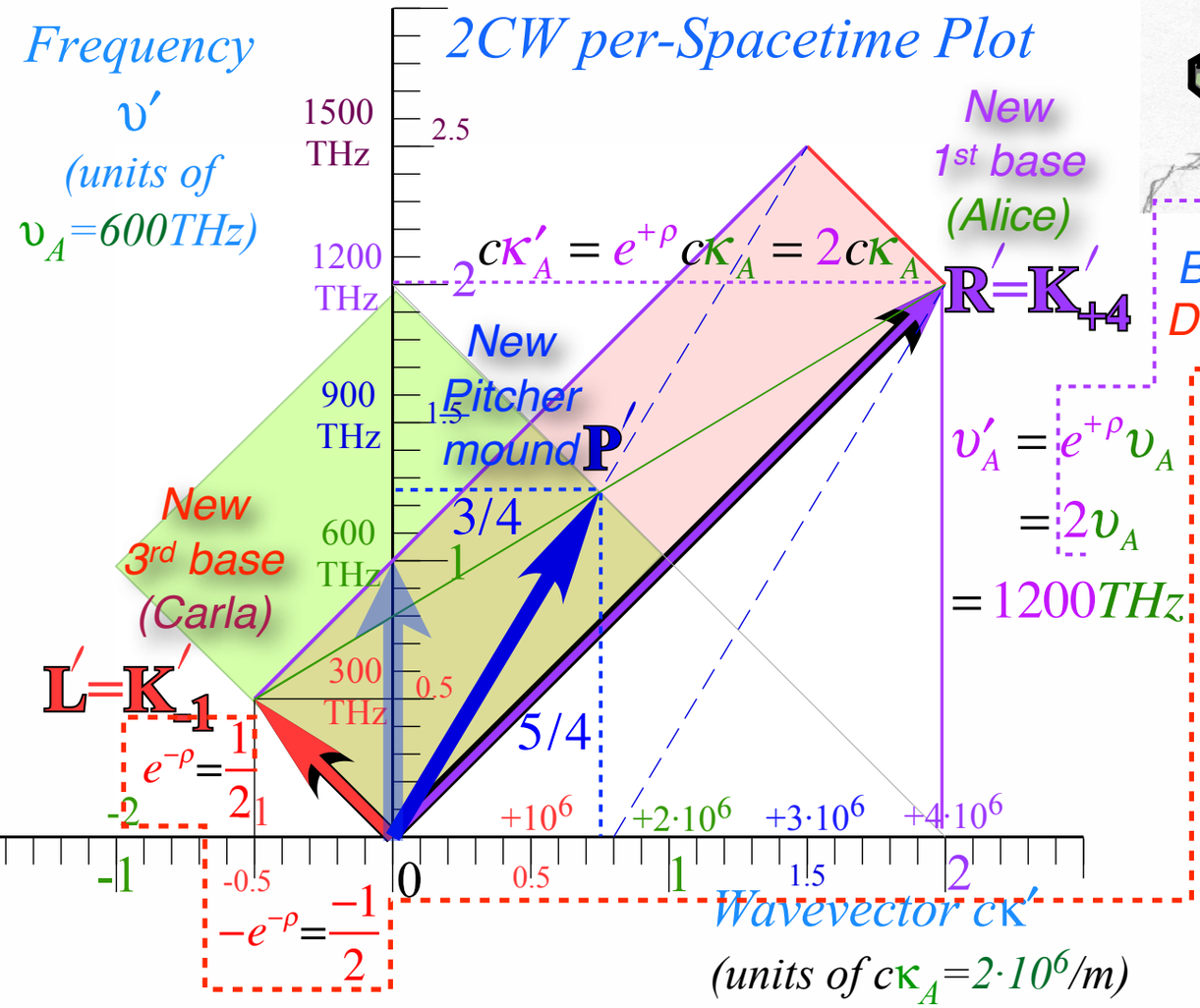


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$



But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
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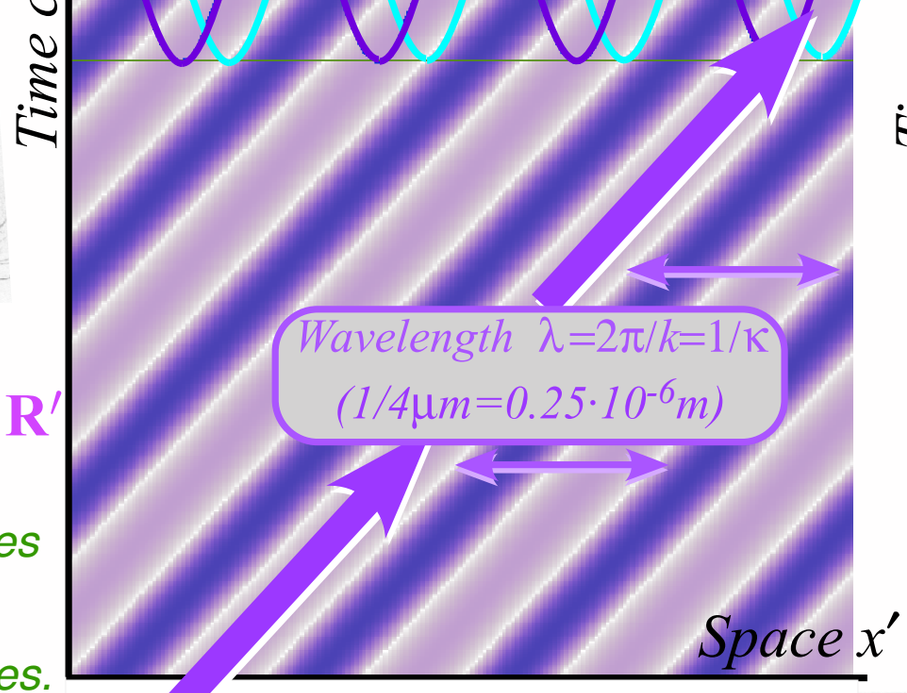

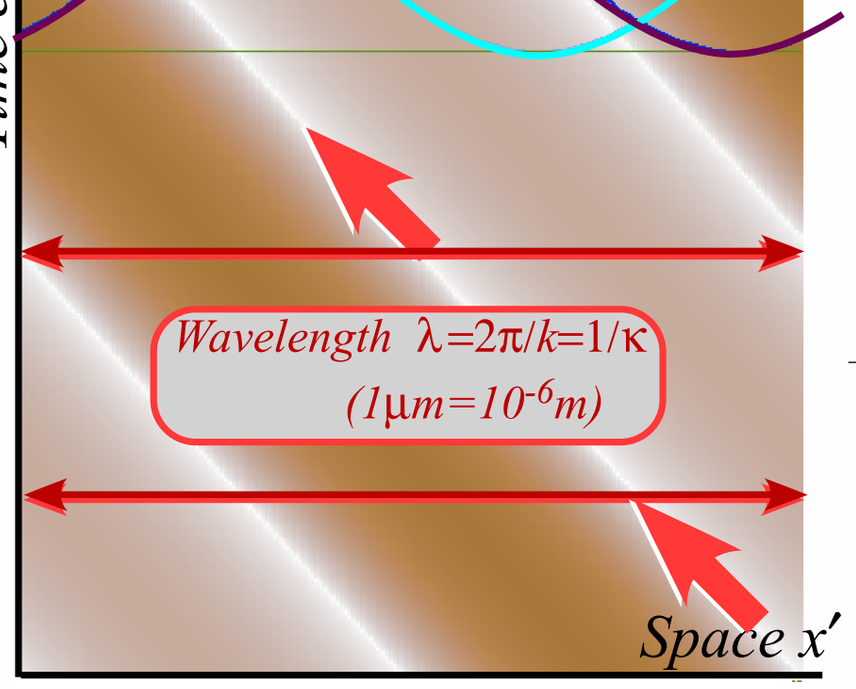

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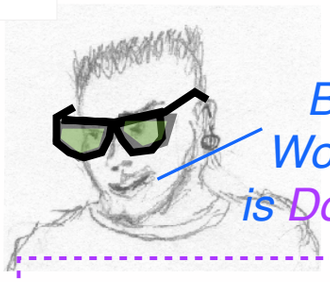

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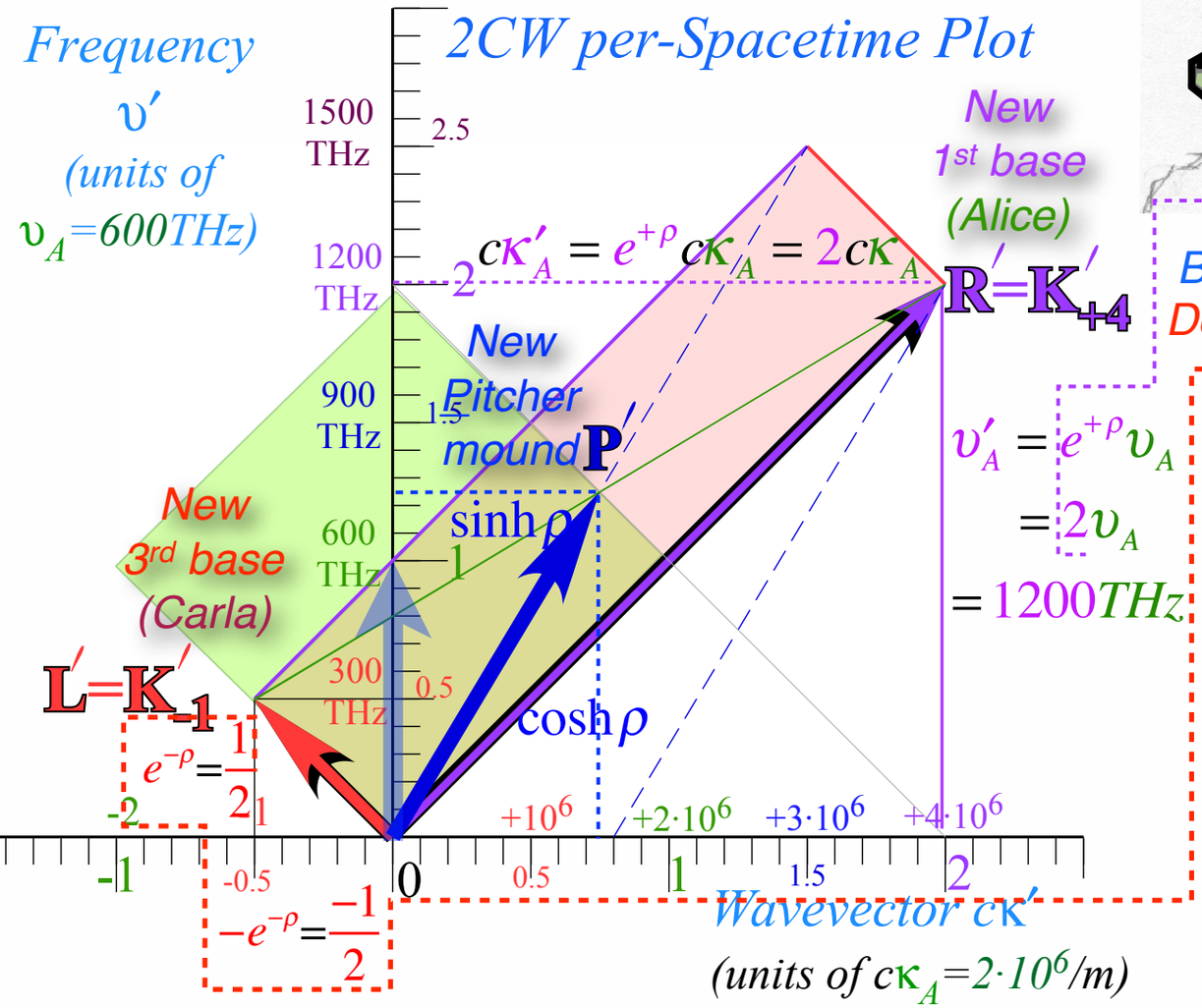

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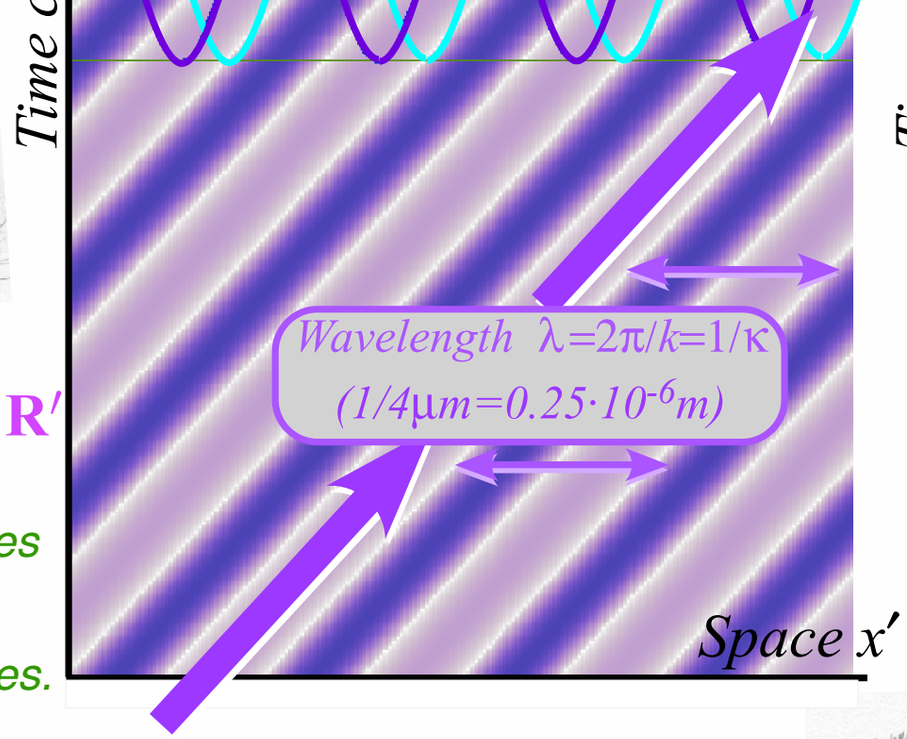
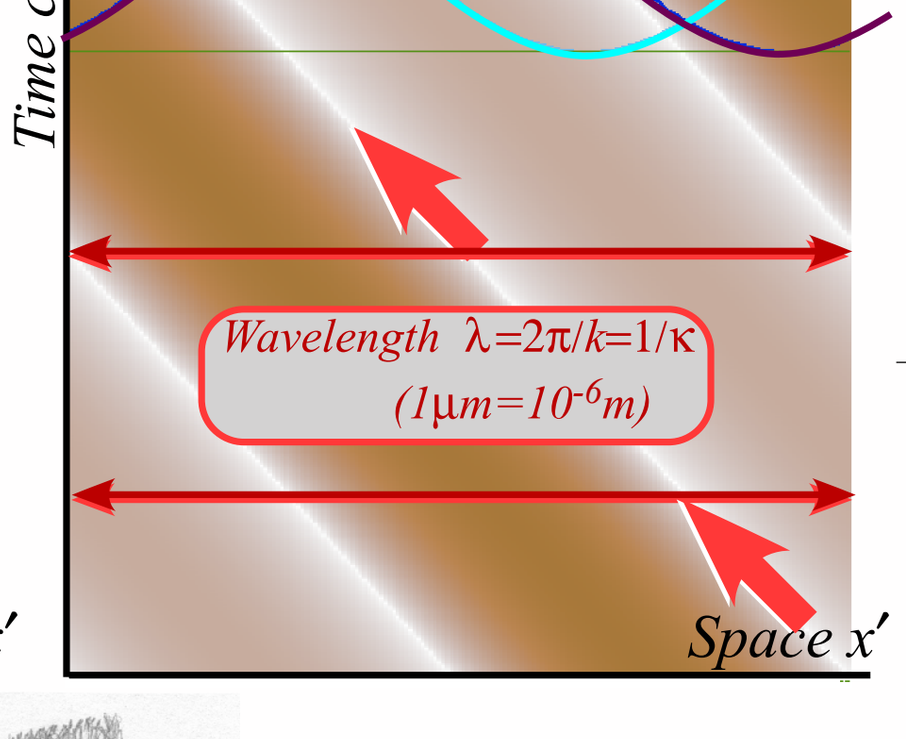
$$= \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.





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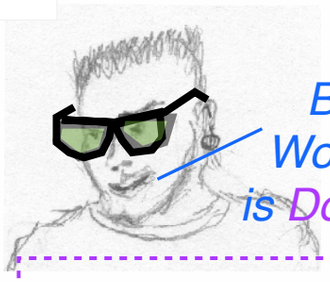
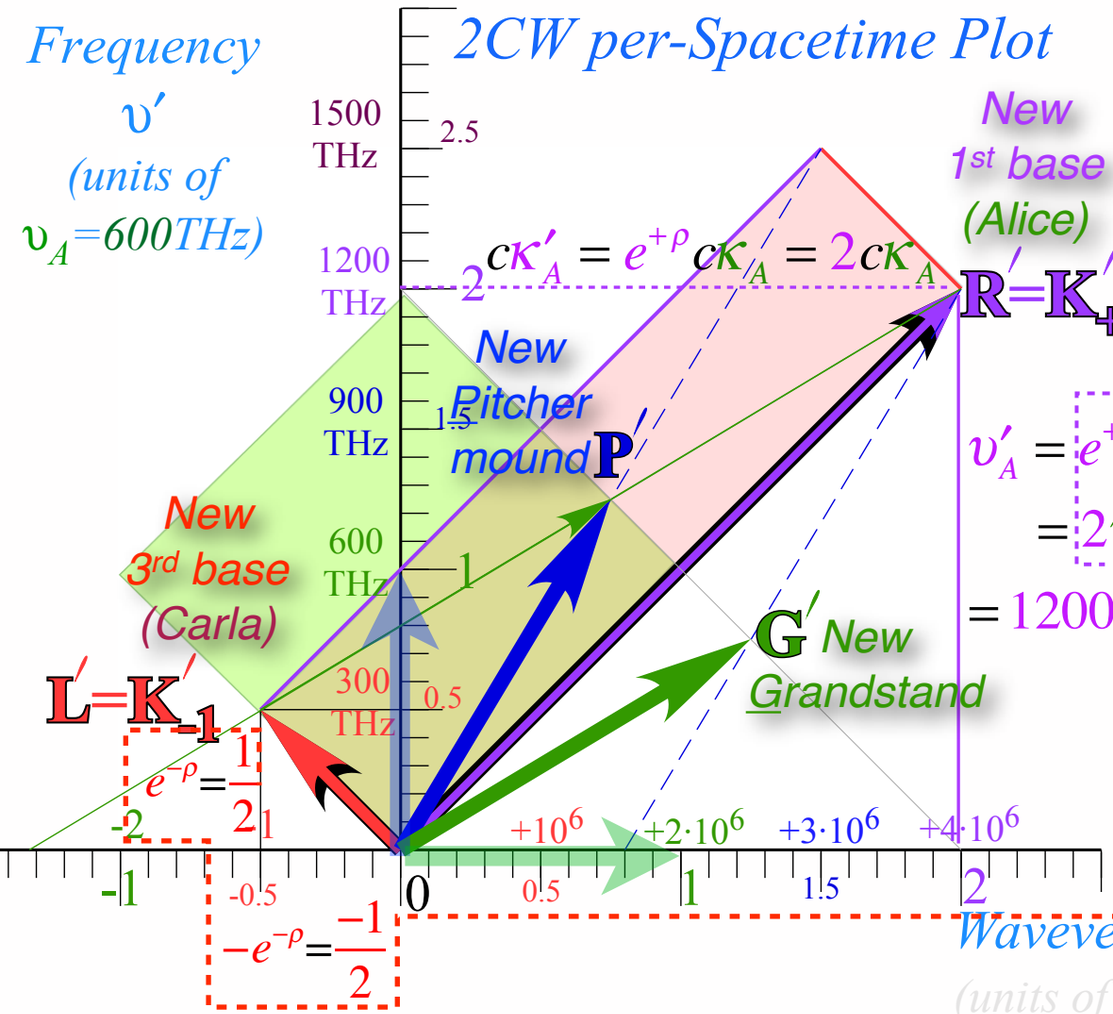


Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

2CW per-Spacetime Plot



$$\upsilon'_A = e^{+\rho}\upsilon_A = 2\upsilon_A = 1200THz$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

Group vector $\mathbf{G}$ 1/2-diff vector

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

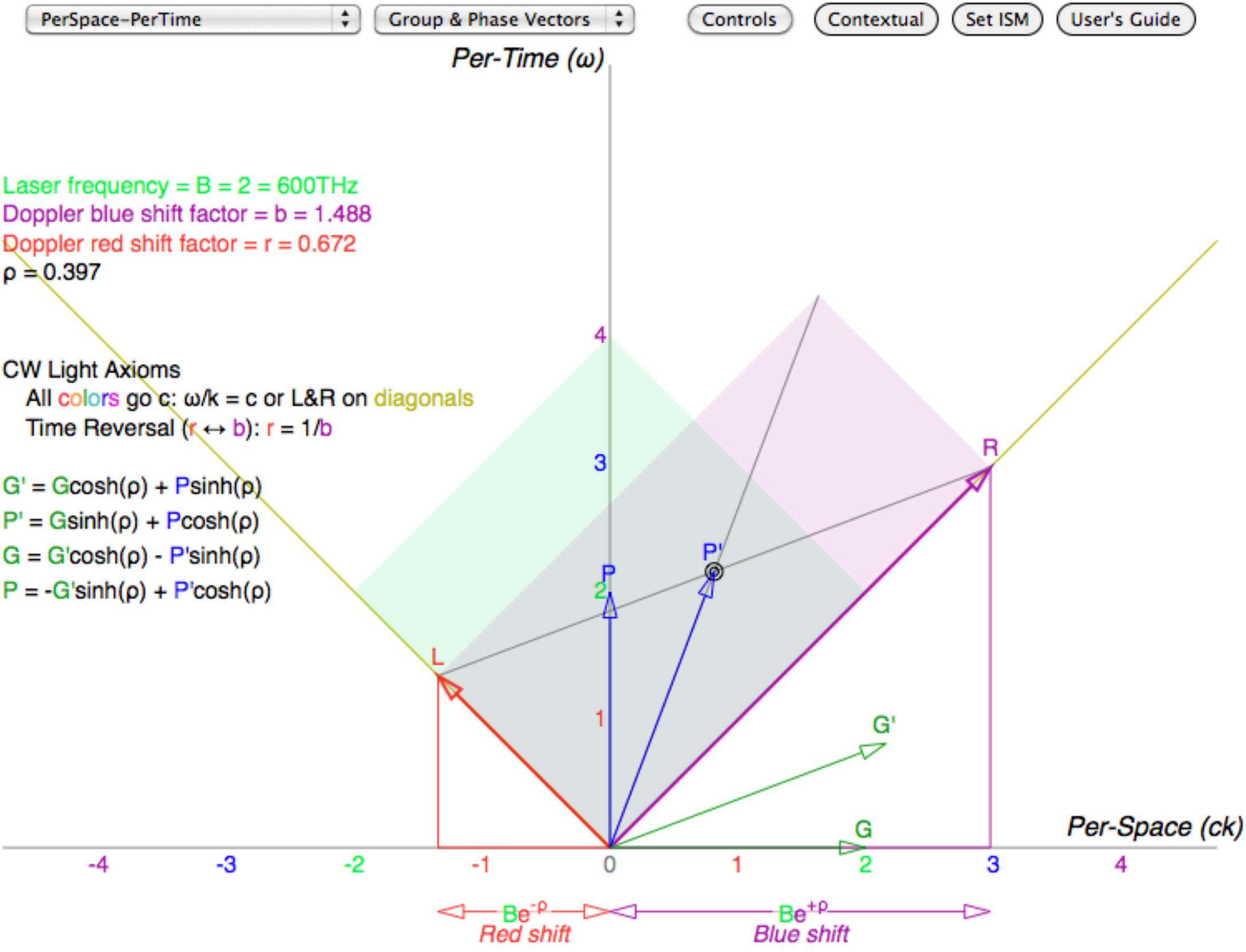
$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$$

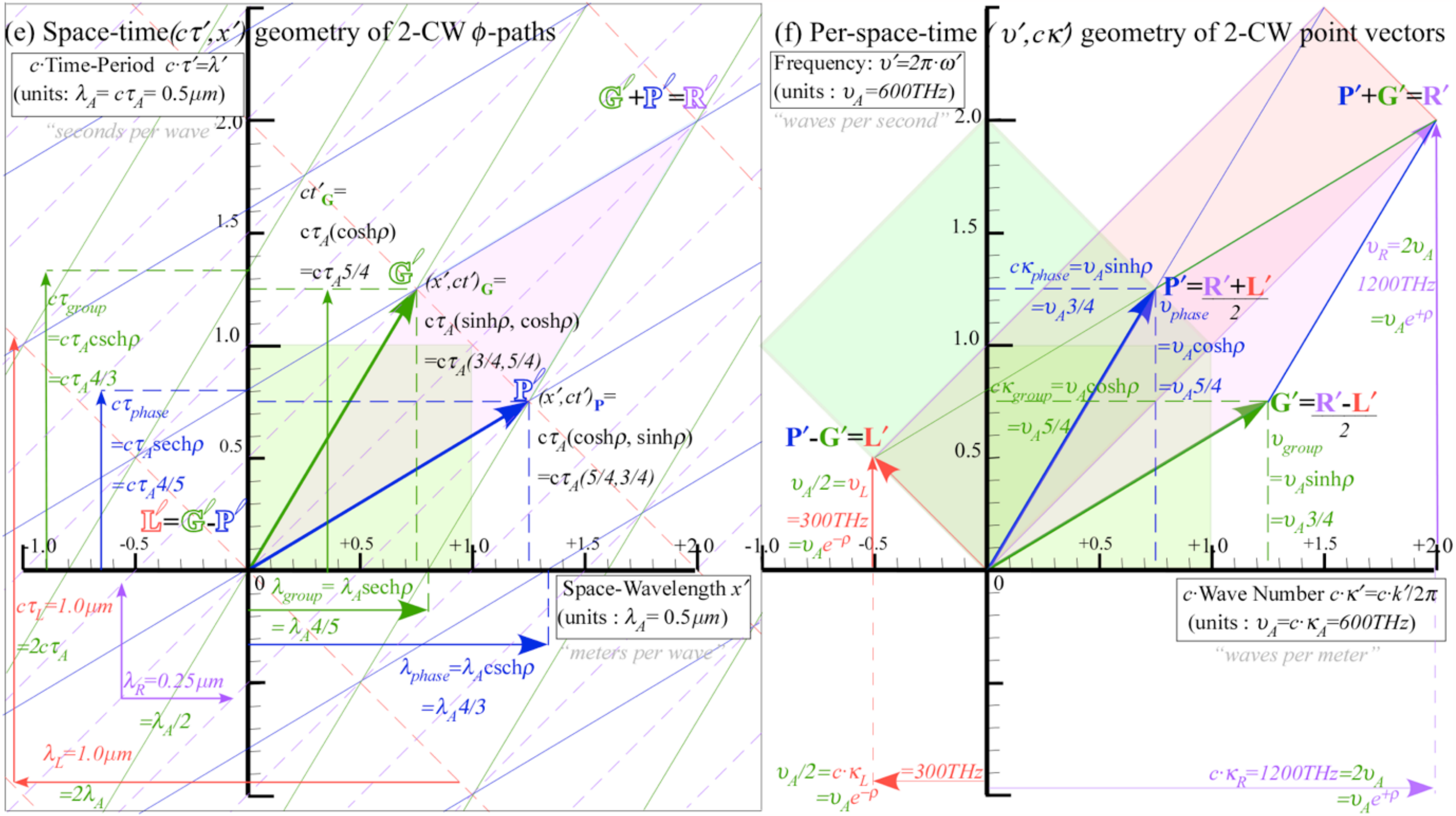
New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix}$$

$$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

RelaWavity WebApp




(e) Space-time$(c\tau',x')$ geometry of 2-CW ϕ-paths
c·Time-Period $c\cdot\tau'=\lambda'$
(units: $\lambda_A=c\tau_A=0.5\mu m$)
"seconds per wave"
$\mathbb{G}'+\mathbb{P}'=\mathbb{R}'$
$ct'_{\mathbf{G}}=$
$c\tau_A(\cosh\rho)$
$=c\tau_A 5/4$
$(x',ct')_{\mathbf{G}}=$
$c\tau_A(\sinh\rho,\cosh\rho)$
$=c\tau_A(3/4,5/4)$
$(x',ct')_{\mathbf{P}}=$
$c\tau_A(\cosh\rho,\sinh\rho)$
$=c\tau_A(5/4,3/4)$
$c\tau_{group}$
$=c\tau_A\operatorname{csch}\rho$
$=c\tau_A 4/3$
$c\tau_{phase}$
$=c\tau_A\operatorname{sech}\rho$
$=c\tau_A 4/5$
$\mathbb{L}'=\mathbb{G}'-\mathbb{P}'$
$c\tau_L=1.0\mu m$
$=2c\tau_A$
$\lambda_R=0.25\mu m$
$=\lambda_A/2$
$\lambda_L=1.0\mu m$
$=2\lambda_A$
$\lambda_{group}=\lambda_A\operatorname{sech}\rho$
$=\lambda_A 4/5$
$\lambda_{phase}=\lambda_A\operatorname{csch}\rho$
$=\lambda_A 4/3$
Space-Wavelength x'
(units : $\lambda_A=0.5\mu m$)
"meters per wave"
(f) Per-space-time $(\upsilon',c\kappa')$ geometry of 2-CW point vectors
Frequency: $\upsilon'=2\pi\cdot\omega'$
(units : $\upsilon_A=600THz$)
"waves per second"
$\mathbf{P}'+\mathbf{G}'=\mathbf{R}'$
$\upsilon_R=2\upsilon_A$
$1200THz$
$=\upsilon_A e^{+\rho}$
$c\kappa_{phase}=\upsilon_A\sinh\rho$
$=\upsilon_A 3/4$
$\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$
υ_{phase}
$=\upsilon_A\cosh\rho$
$=\upsilon_A 5/4$
$c\kappa_{group}=\upsilon_A\cosh\rho$
$=\upsilon_A 5/4$
$\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}$
υ_{group}
$=\upsilon_A\sinh\rho$
$=\upsilon_A 3/4$
$\mathbf{P}'-\mathbf{G}'=\mathbf{L}'$
$\upsilon_A/2=\upsilon_L$
$=300THz$
$=\upsilon_A e^{-\rho}$
$\upsilon_A/2=c\cdot\kappa_L$
$=\upsilon_A e^{-\rho}$
$=300THz$
$c\cdot\kappa_R=1200THz=2\upsilon_A$
$=\upsilon_A e^{+\rho}$
c·Wave Number $c\cdot\kappa'=c\cdot k'/2\pi$
(units : $\upsilon_A=c\cdot\kappa_A=600THz$)
"waves per meter"

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

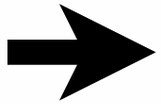
L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R′**-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L′**-*vector and* **R′**-*vector in (*__L__*,* __R__*)-per-spacetime*

Vectors of phase **P′**=(**R′**+**L′**)/2 *and group* **G′**=(**R′**-**L′**)/2



Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

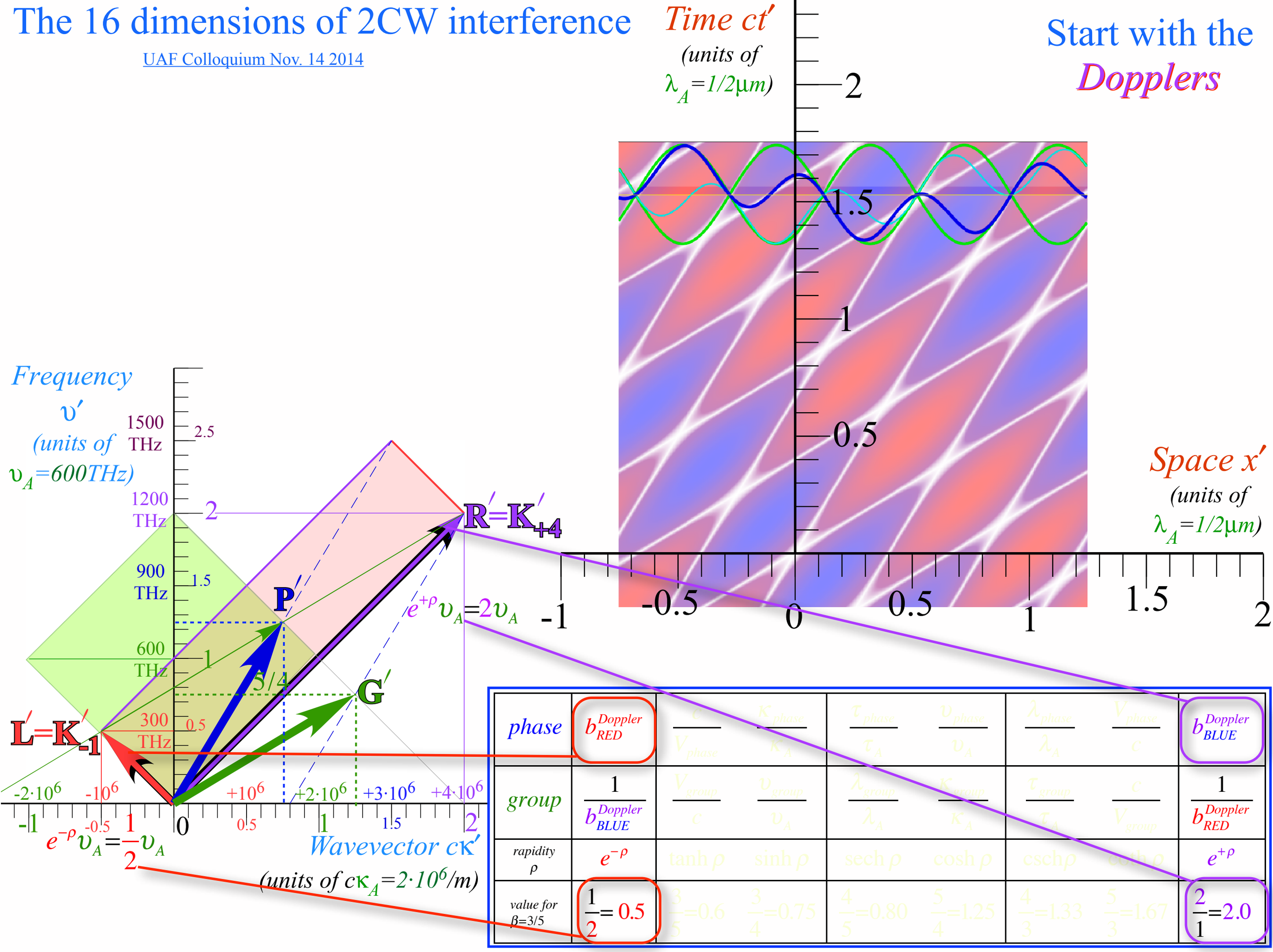
Brief tour of and relativistic mechanics by geometry

Summary of optical wave parameters for relativity and QM

The 16 dimensions of 2CW interference

UAF Colloquium Nov. 14 2014

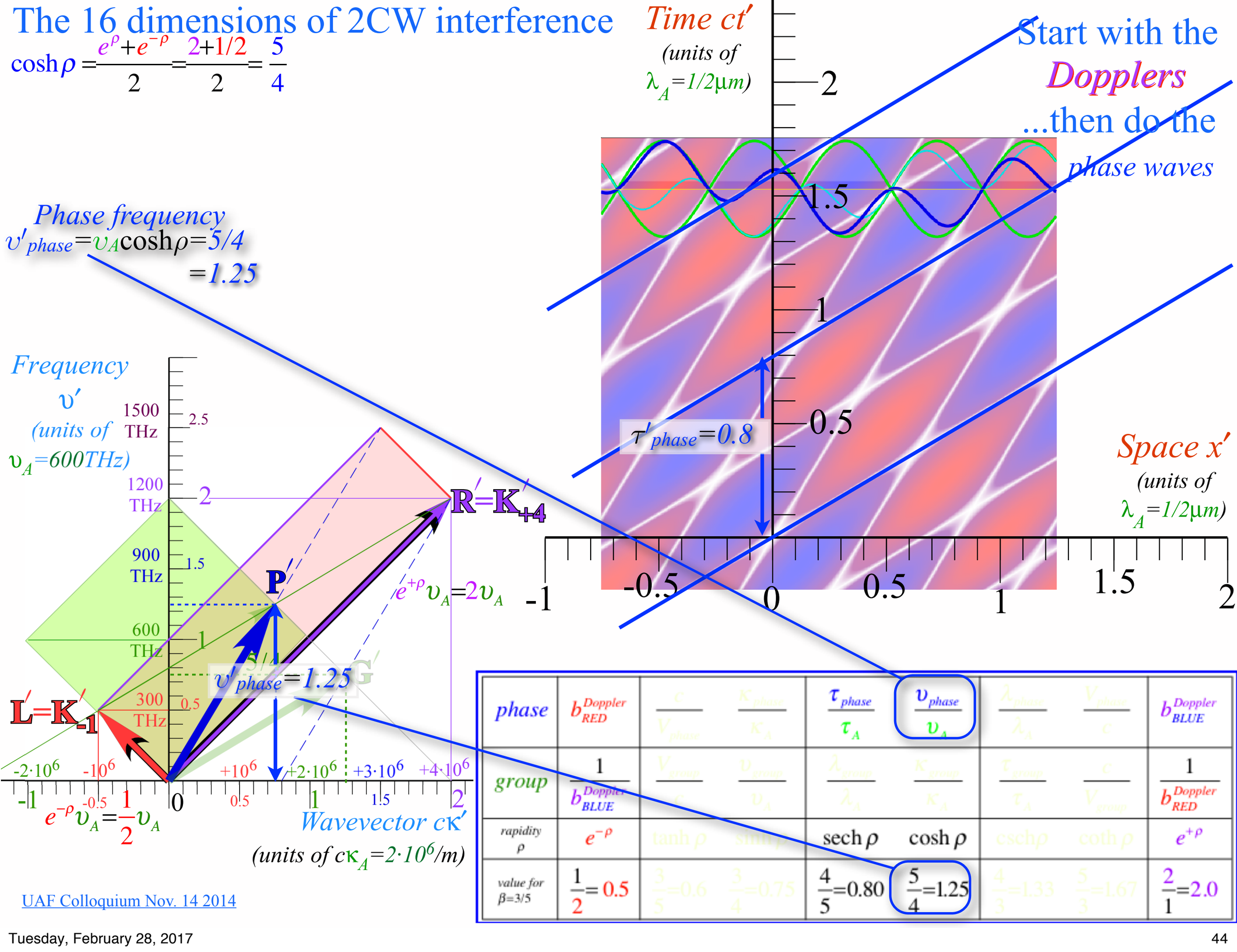
Start with the *Dopplers*



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

$$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4}$$



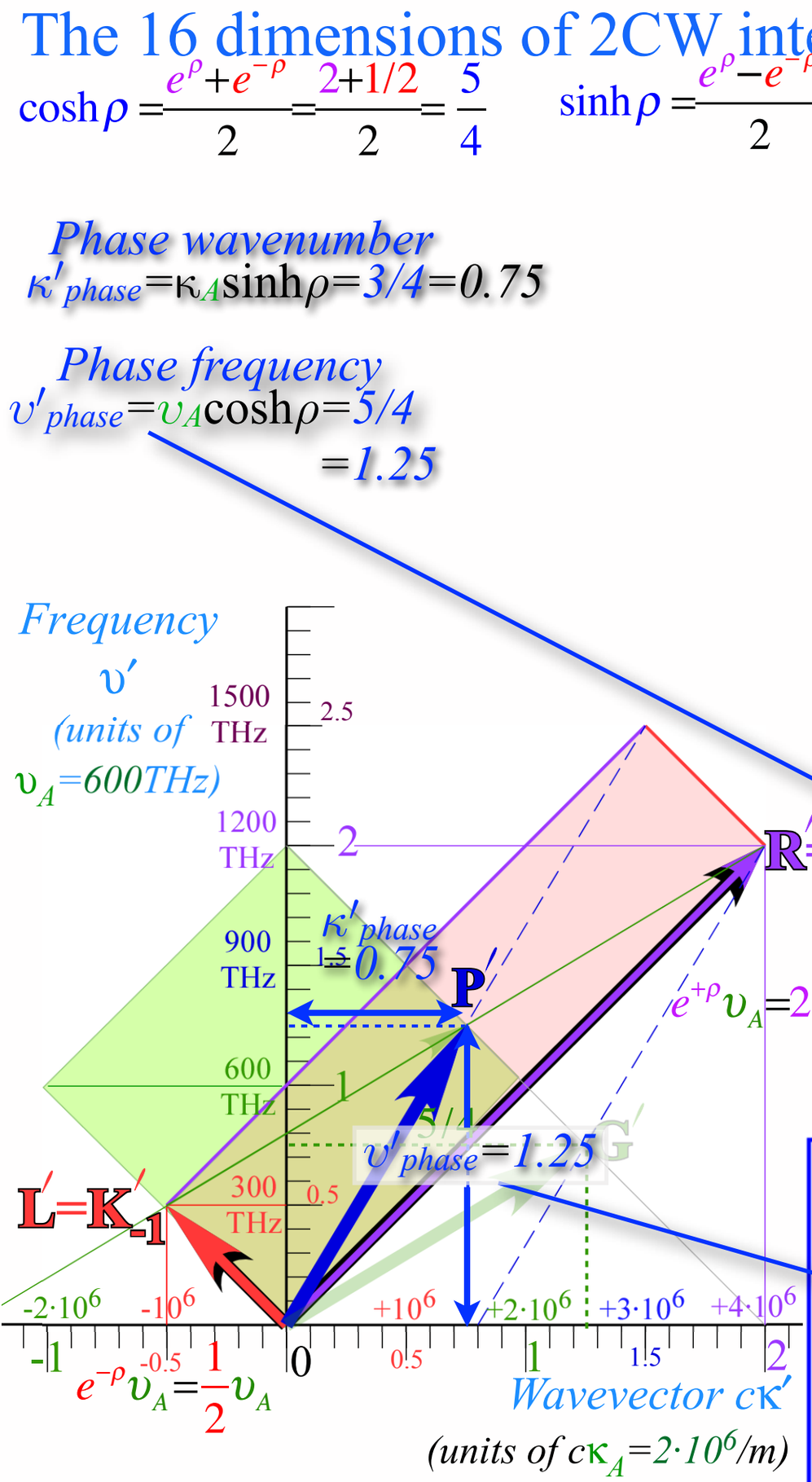
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

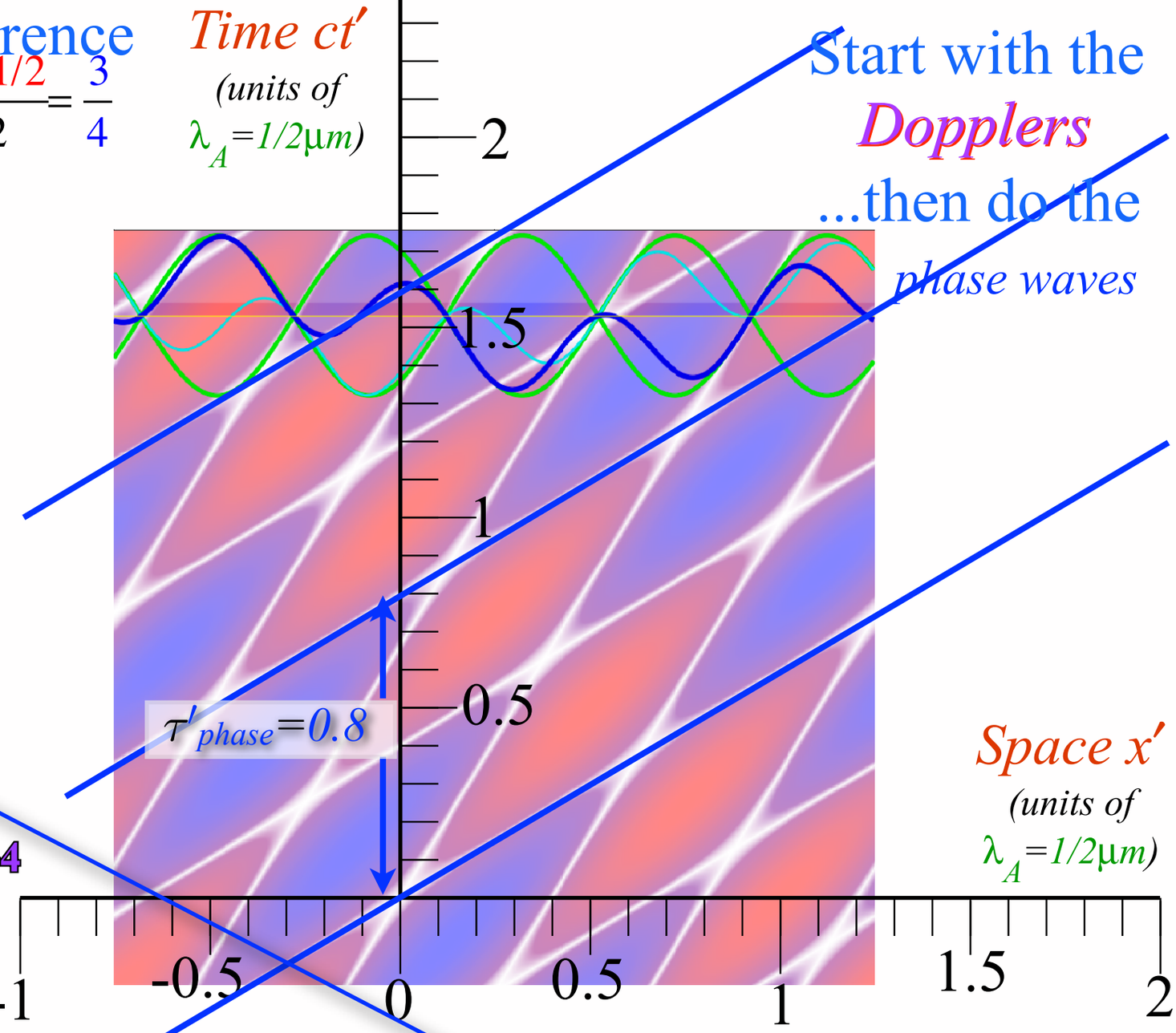
The 16 dimensions of 2CW interference

$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4}$ $\sinh\rho = \frac{e^{\rho}-e^{-\rho}}{2} = \frac{2-1/2}{2} = \frac{3}{4}$

Phase wavenumber
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4 = 0.75$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$





Start with the *Dopplers*
...then do the *phase waves*

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

$$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4} \qquad \sinh\rho = \frac{e^{\rho}-e^{-\rho}}{2} = \frac{2-1/2}{2} = \frac{3}{4}$$

$$\mathbf{P'} = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$ flips to *Phase period* $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*

Time ct′ (units of $\lambda_A = 1/2\mu m$)

Space x′ (units of $\lambda_A = 1/2\mu m$)

$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$\tau'_{phase} = 0.8$

Frequency υ' (units of $\upsilon_A = 600THz$)

1500 THz, 1200 THz, 900 THz, 600 THz, 300 THz

$\mathbf{R'} = \mathbf{K'_{+4}}$

$\mathbf{P'}$

$e^{+\rho}\upsilon_A = 2\upsilon_A$

$\upsilon'_{phase} = 1.25$

$\mathbf{L'} = \mathbf{K'_{-1}}$

$e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A$

Wavevector $c\kappa'$ (units of $c\kappa_A = 2 \cdot 10^6/m$)

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber $\kappa'_{phase}=\kappa_A\sinh\rho=3/4$ flips to *Phase wavelength* $\lambda'_{phase}=\lambda_A\operatorname{csch}\rho=4/3$ $\lambda=1/\kappa$

$$\mathbf{P'}=\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix}=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase}=\upsilon_A\cosh\rho=5/4$ flips to *Phase period* $\tau'_{phase}=\tau_A\operatorname{sech}\rho=4/5$ $\tau=1/\upsilon$

P-slope$=V_{phase}/c$

$$=\frac{\upsilon'_{phase}}{\kappa'_{phase}}=\frac{\cosh\rho}{\sinh\rho}=\frac{5/4}{3/4}=\frac{5}{3}$$

Time ct′ (units of $\lambda_A=1/2\mu m$)

$\lambda'_{phase}=\lambda_A\operatorname{csch}\rho=4/3$

$\tau'_{phase}=0.8=4/5$

$\tau'_{phase}=\tau_A\operatorname{sech}\rho=4/5$

$\tau'_{phase}=0.8$

$\lambda'_{phase}=1.33$

slope$=\frac{V_{phase}}{c}=\frac{\lambda'_{phase}}{\tau'_{phase}}=\frac{4/3}{4/5}=\frac{5}{3}=\coth\rho$

faster than light!

Space x′ (units of $\lambda_A=1/2\mu m$)

Frequency υ' (units of $\upsilon_A=600THz$)

$\kappa'_{phase}=0.75$

$\upsilon'_{phase}=1.25$

$\mathbf{R}=\mathbf{K}'_{+4}$ $\mathbf{L}=\mathbf{K}'_{-1}$ $\mathbf{P}'$ $\mathbf{G}'$

Wavevector $c\kappa'$ (units of $c\kappa_A=2\cdot10^6/m$)

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Group wavenumber $\kappa'_{group}=\kappa_A\cosh\rho=5/4=1.25$

Group wavelength $\lambda=1/\kappa$ $\lambda'_{group}=\lambda_A\operatorname{sech}\rho=4/5=0.8$

$$\mathbf{G'}=\left(\begin{array}{c} c\kappa'_{group} \\ \upsilon'_{group} \end{array}\right)=\upsilon_A\left(\begin{array}{c} \cosh\rho \\ \sinh\rho \end{array}\right)=\upsilon_A\left(\begin{array}{c} 5/4 \\ 3/4 \end{array}\right)$$

Group frequency $\upsilon'_{group}=\upsilon_A\sinh\rho=3/4=0.75$

flips to

Group period $\tau=1/\upsilon$ $\tau'_{group}=\tau_A\operatorname{csch}\rho=4/3=1.33$

G-slope=V_{group}/c

$$=\frac{\upsilon'_{group}}{\kappa'_{group}}=\frac{\sinh\rho}{\cosh\rho}=\frac{3/4}{5/4}=\frac{3}{5}$$

$\tau'_{group}=\tau_A\operatorname{csch}\rho=4/3$

Frequency υ' *(units of* $\upsilon_A=600THz$*)*

1500 THz, 1200 THz, 900 THz, 600 THz, 300 THz

$\mathbf{R'=K'_{+4}}$ $\mathbf{P'}$ $\mathbf{G'}$ $\mathbf{L'=K'_{-1}}$

$\kappa'_{group}=1.25$

$\upsilon'_{group}=0.75$

$-2\cdot10^6$ -10^6 $+10^6$ $+2\cdot10^6$ $+3\cdot10^6$ $+4\cdot10^6$

Wavevector $c\kappa'$

(units of $c\kappa_A=2\cdot10^6/m$*)*

Time ct' *(units of* $\lambda_A=1/2\mu m$*)*

$\lambda'_{group}=\lambda_A\operatorname{sech}\rho=4/5$

$\tau'_{group}=1.33$

$\lambda'_{group}=0.8$

2, 1.5, 1, 0.5

-1, -0.5, 0, 0.5, 1, 1.5, 2

Space x' *(units of* $\lambda_A=1/2\mu m$*)*

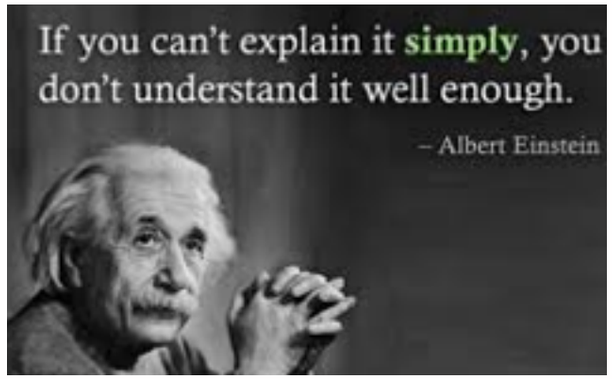
slope=V_{group}/c=$\frac{\lambda'_{group}}{\tau'_{group}}=\frac{4/5}{4/3}=\frac{3}{5}$=tanh ρ

slower than light!

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

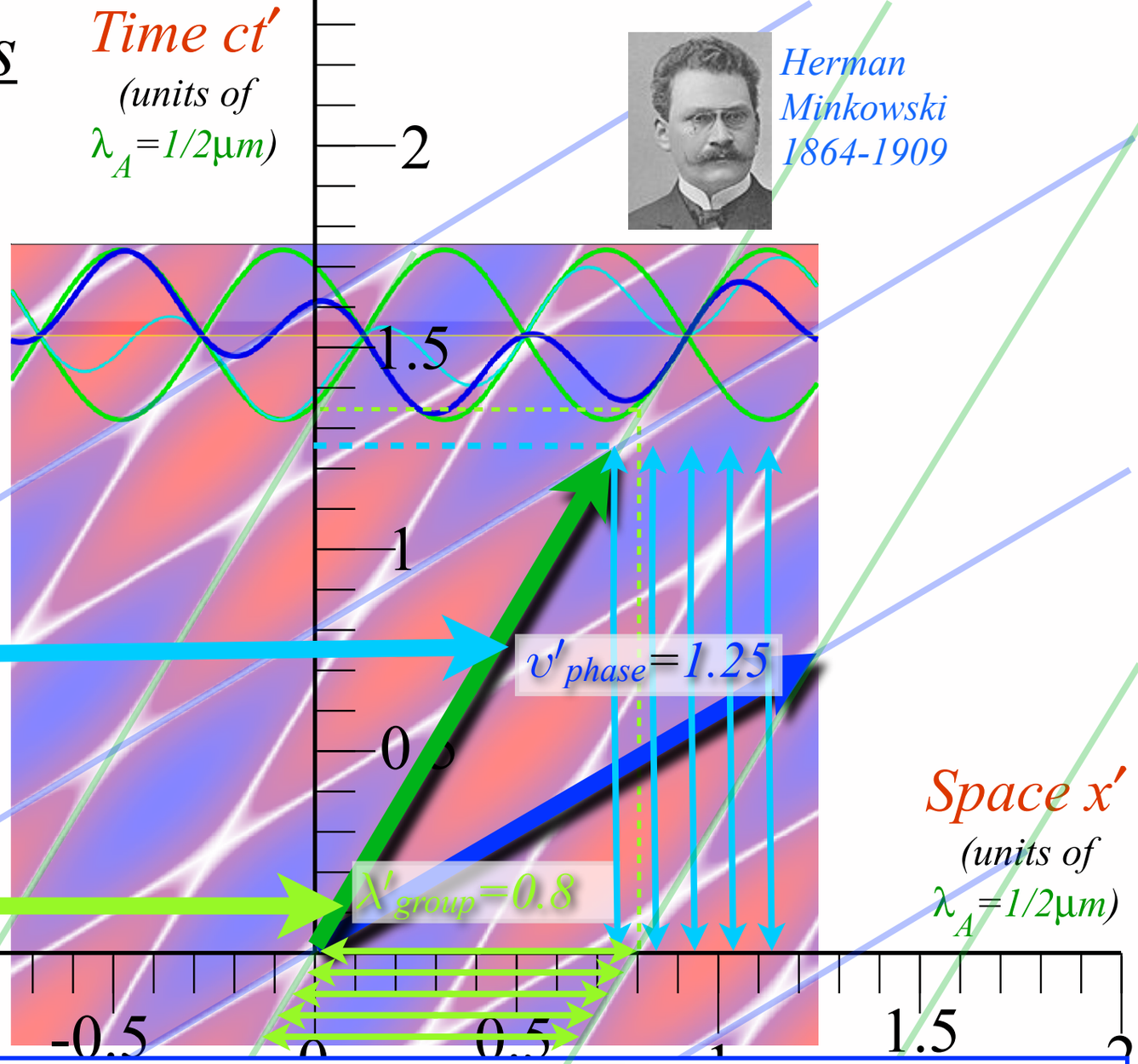
Two Famous-Name Coefficients

UAF Colloquium Nov. 14 2014



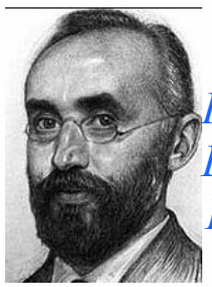
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*



Hendrik A. Lorentz 1853-1928

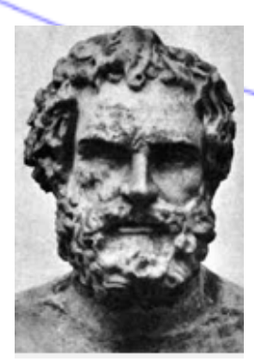
Old-Fashioned Notation

RelaWavity Web Simulation - Relativistic *Terms* *(Expanded Table)*

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Thales Mean Geometry (600BCE)

helps "Relawavity"



Thales of Miletus 624-543 BCE

Per-Time
ω - axis

Transformed Per-Time
ω' - axis

equilateral hyperbola
r·b=2
due to Doppler T-symmetry

Transformed Per-Space
ck' - axis

p-Circle

B-Circle

Geometric Mean
B= √(4·1)=2

Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2

Difference Mean
Bsinhρ = (4-1)/2
= 3/2

Per-Space (ck)

Br
Red shift

Bb
Blue shift

[RelaWavity Web Simulation
Detailed *Thales Geometry*](#)

[UAF Colloquium Nov. 14 2014](#)

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

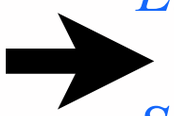
Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (*__L__*,* __R__*)-per-spacetime*

Vectors of phase **P**′=(**R**′+**L**′)/2 *and group* **G**′=(**R**′-**L**′)/2

Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates



Brief tour of and relativistic mechanics by geometry

Summary of optical wave parameters for relativity and QM

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

At low speeds:

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2$

$\sinh\rho \approx \rho$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$v_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

At low speeds:

$v_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2$ $\Leftarrow$ for $(u \ll c)$

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = v_A$

$B = v_A = c\kappa_A$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$$

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2$ ⇐ for $(u \ll c)$ ⇒ $\kappa_{phase} \approx \frac{B}{c^2}u$

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

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Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
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Max Planck
1858-1947

For more visit the Pirelli Challenge Site
Quantized amplitude

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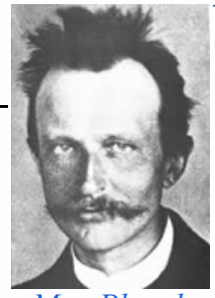

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Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy $= \frac{1}{2}A^2\upsilon^2$

Resolution and dirty secret: E, N, and υ_{phase} are all frequencies!

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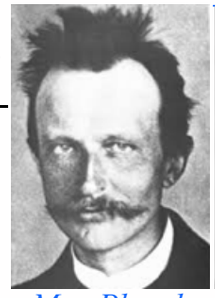

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Louis DeBroglie 1892-1987

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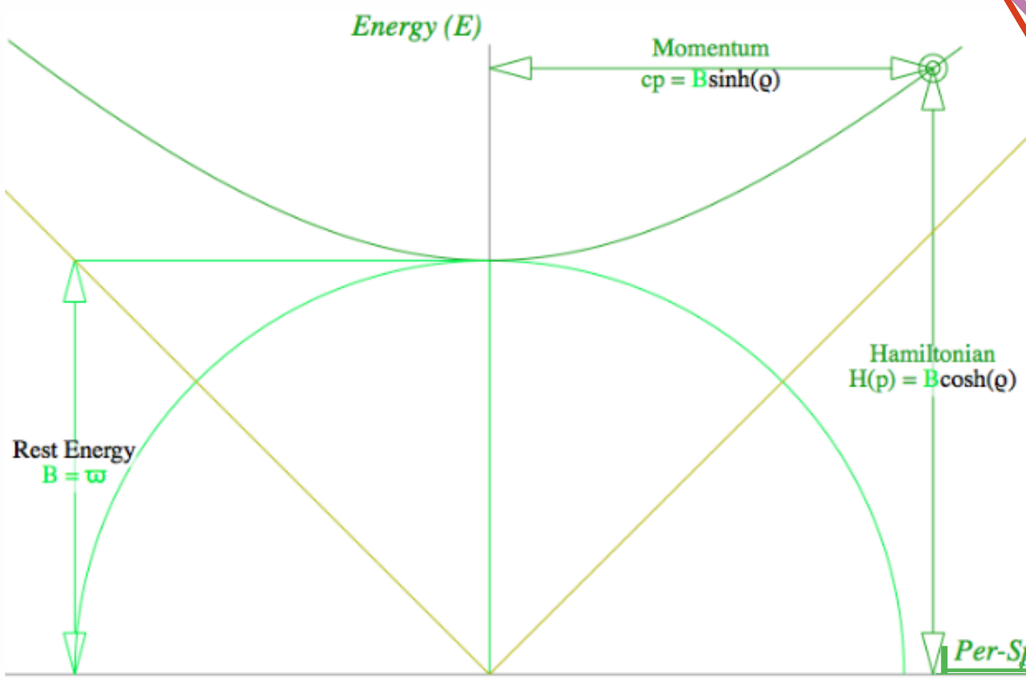
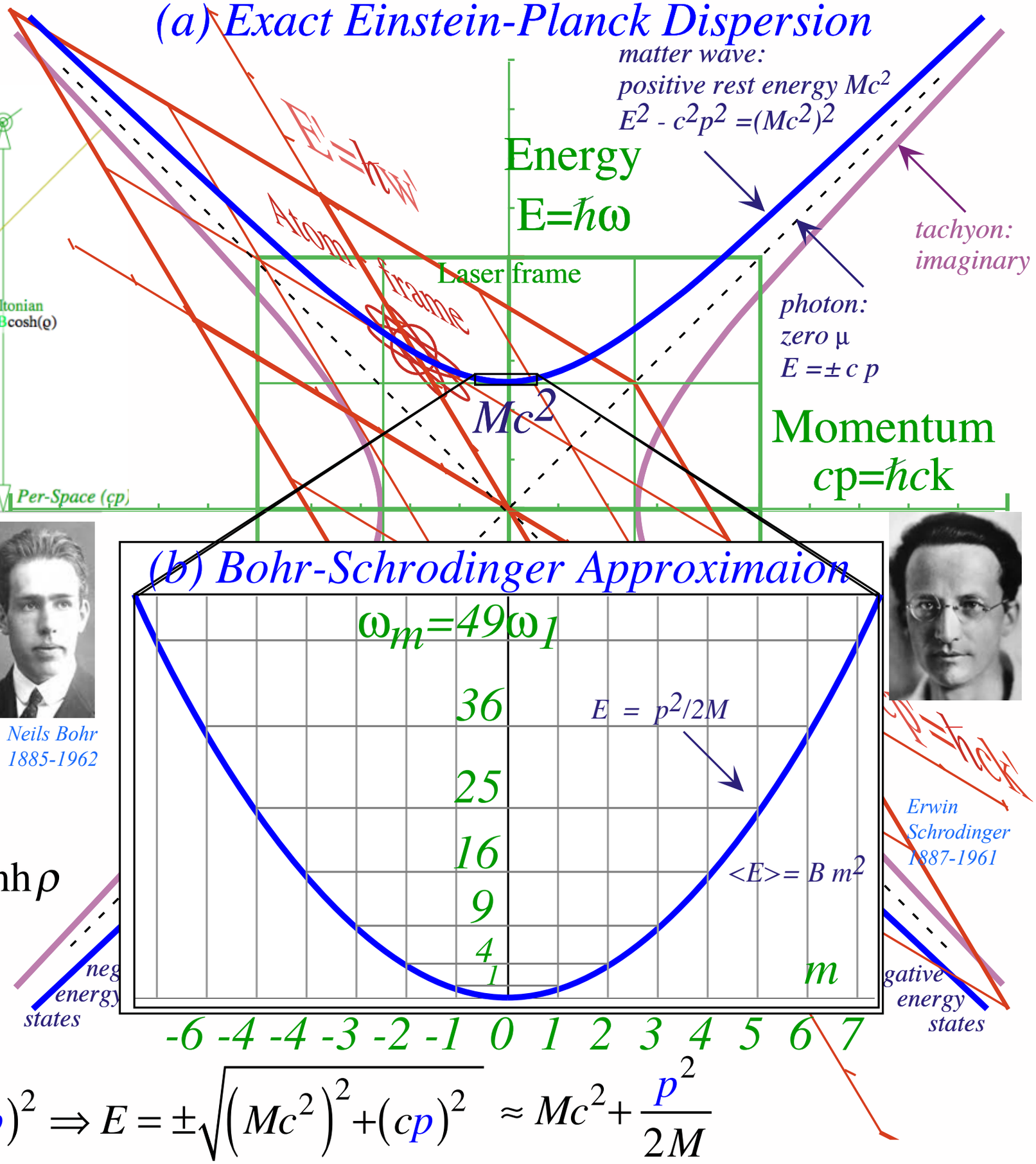
Using (some) wave coordinates for relativistic quantum theory

UAF Colloquium Nov. 14 2014





Neils Bohr
1885-1962

Erwin
Schrodinger
1887-1961

Mass (resting)

$$h\mathrm{B} = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

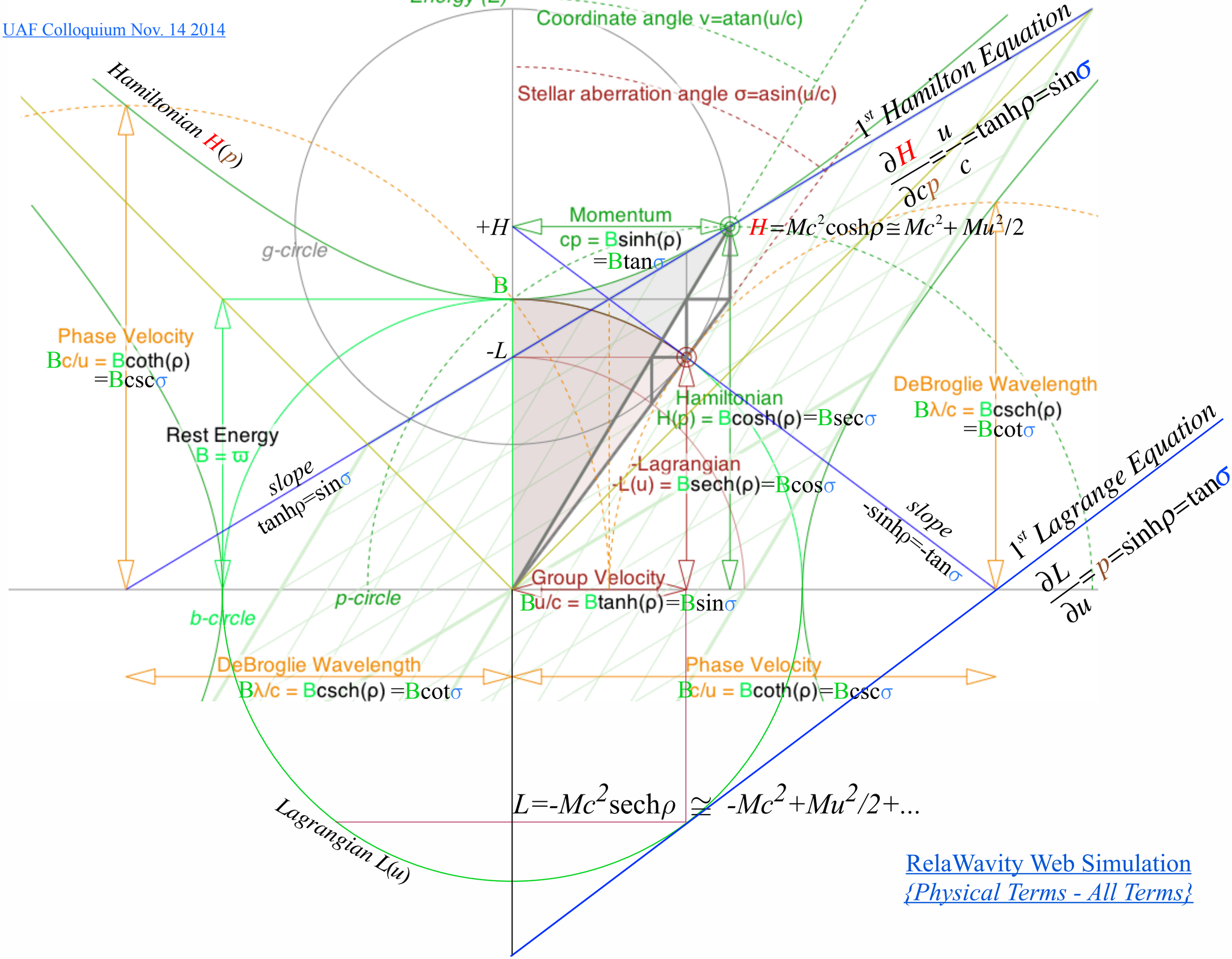
Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$



RelaWavity Web Simulation
{Physical Terms - All Terms}

Relawavity WebApp

Physical Terms | All | Controls | Contextual | Set ISM | User's Guide

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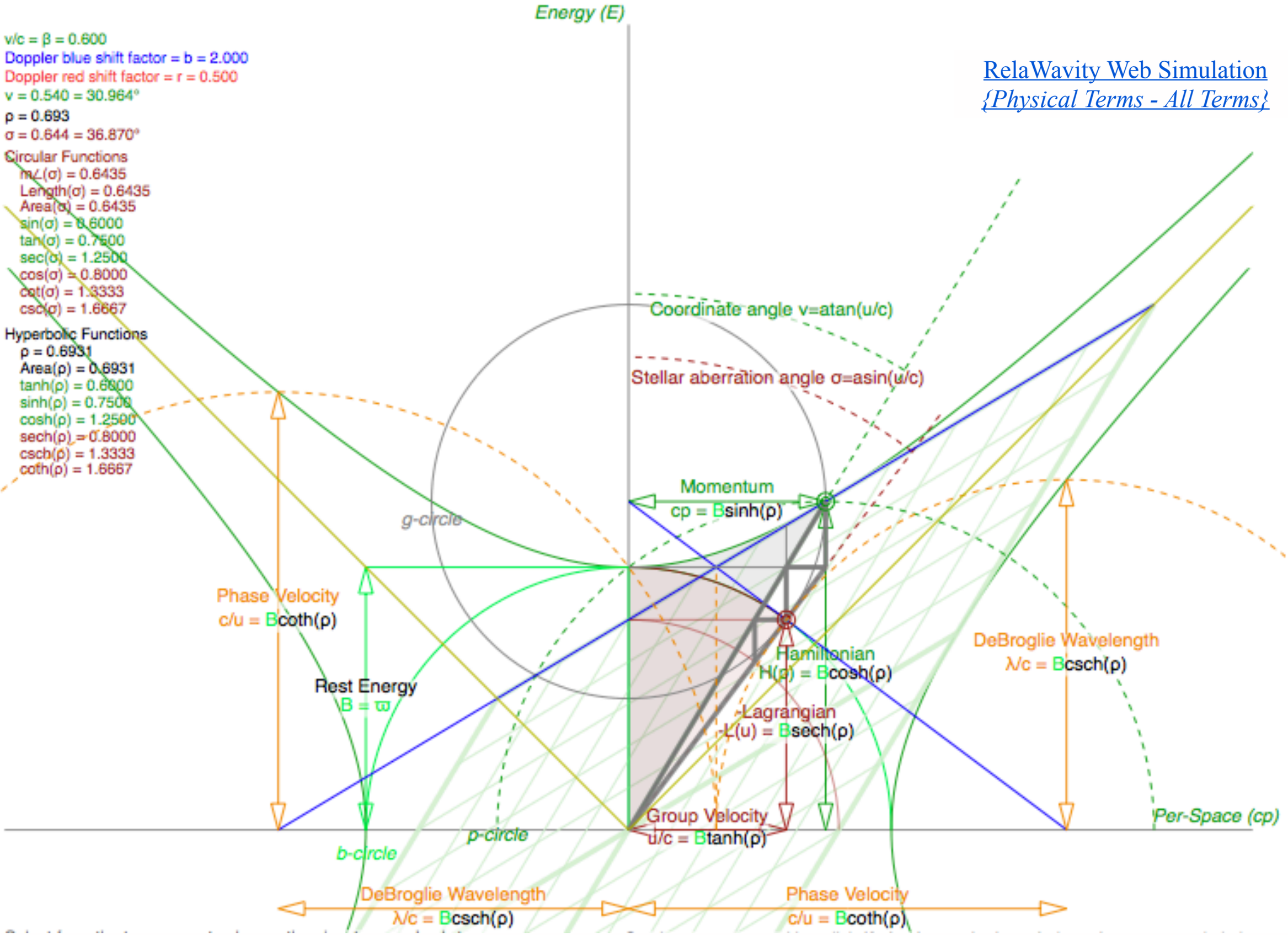


Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (*__L__*,* __R__*)-per-spacetime*

Vectors of phase **P**′=(**R**′+**L**′)/2 *and group* **G**′=(**R**′-**L**′)/2

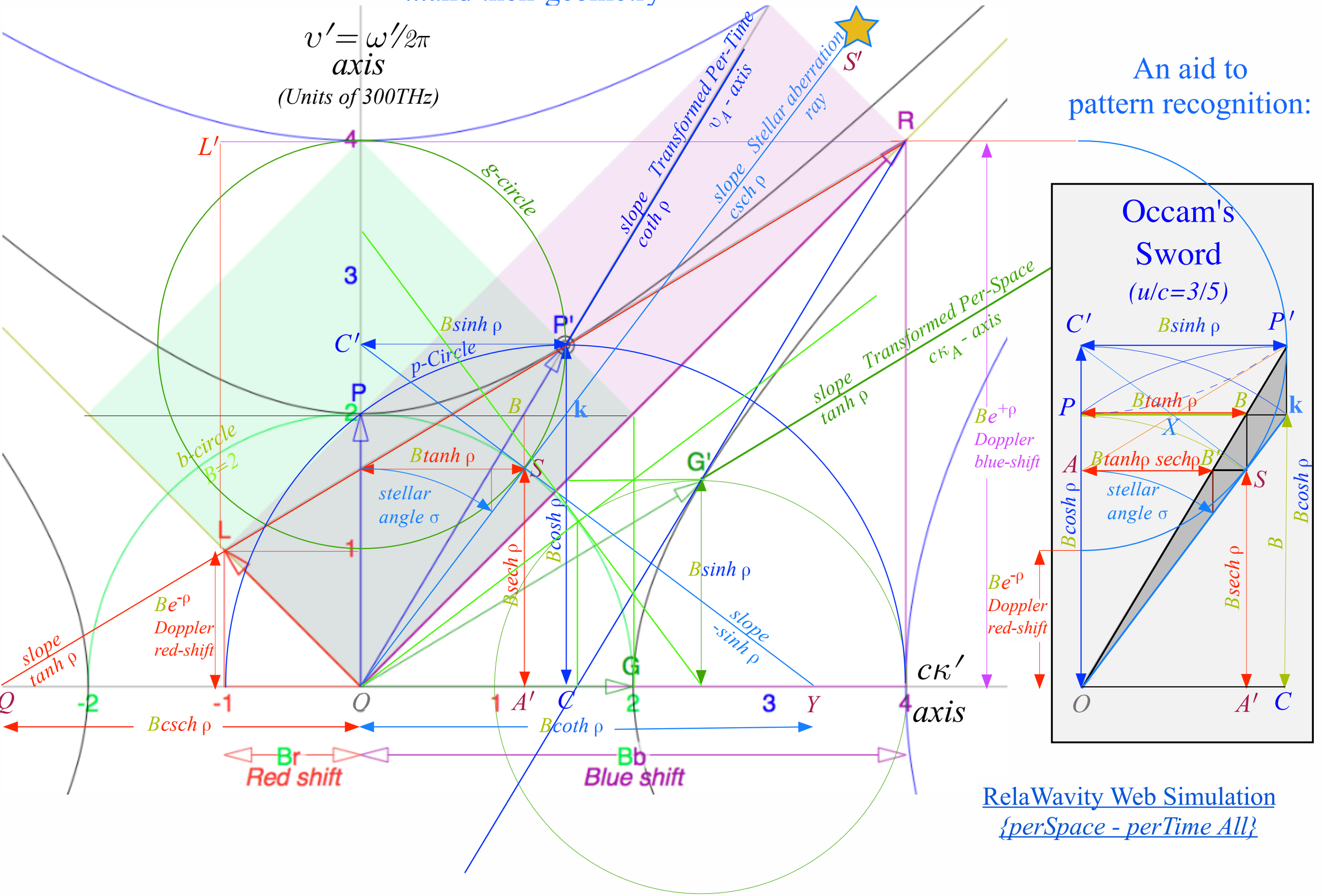
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

Brief tour of and relativistic mechanics by geometry

→ *Summary of optical wave parameters for relativity and QM*

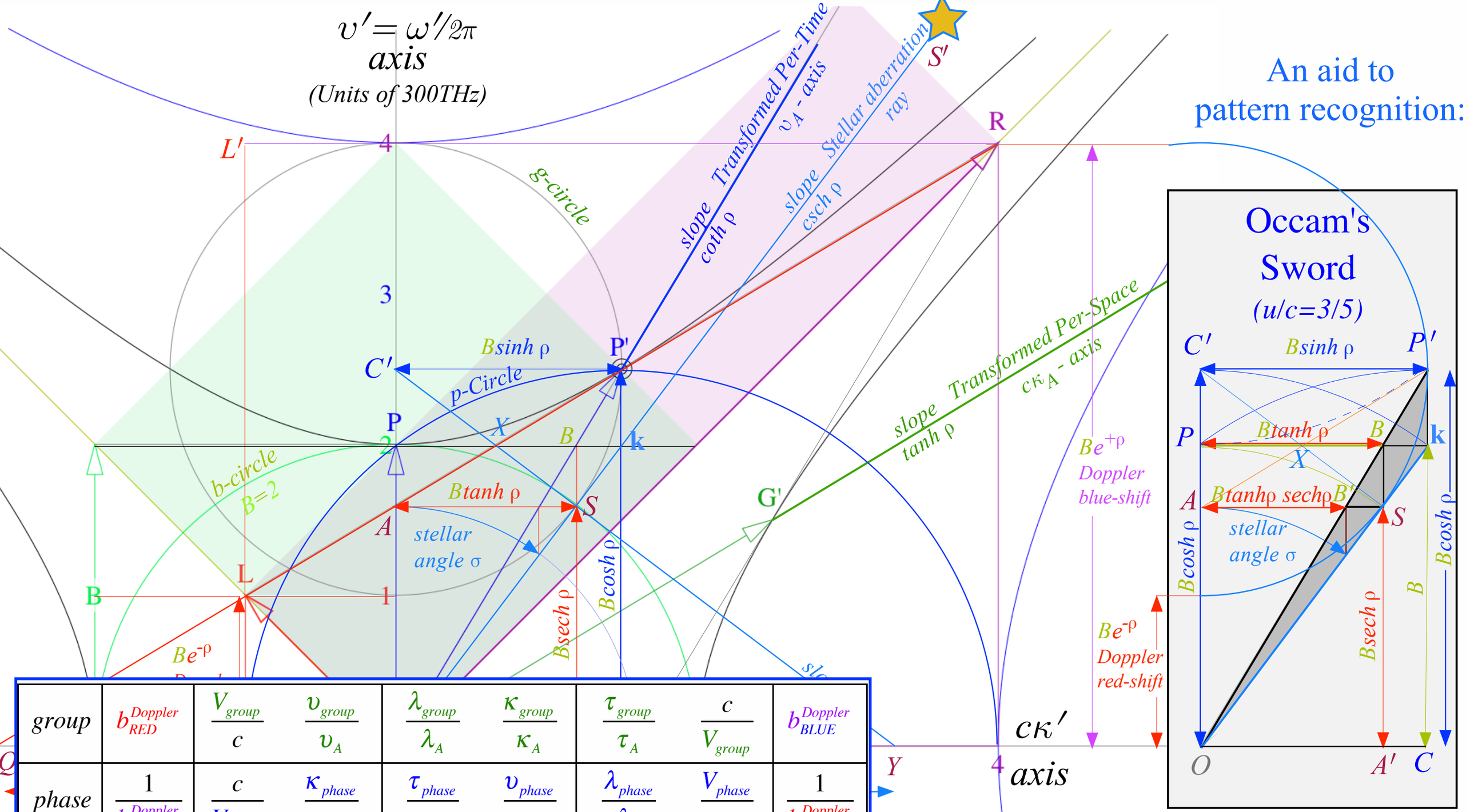
Summary of optical wave parameters for relativity and QM

...and their geometry



An aid to pattern recognition:

RelaWavity Web Simulation
{perSpace - perTime All}



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

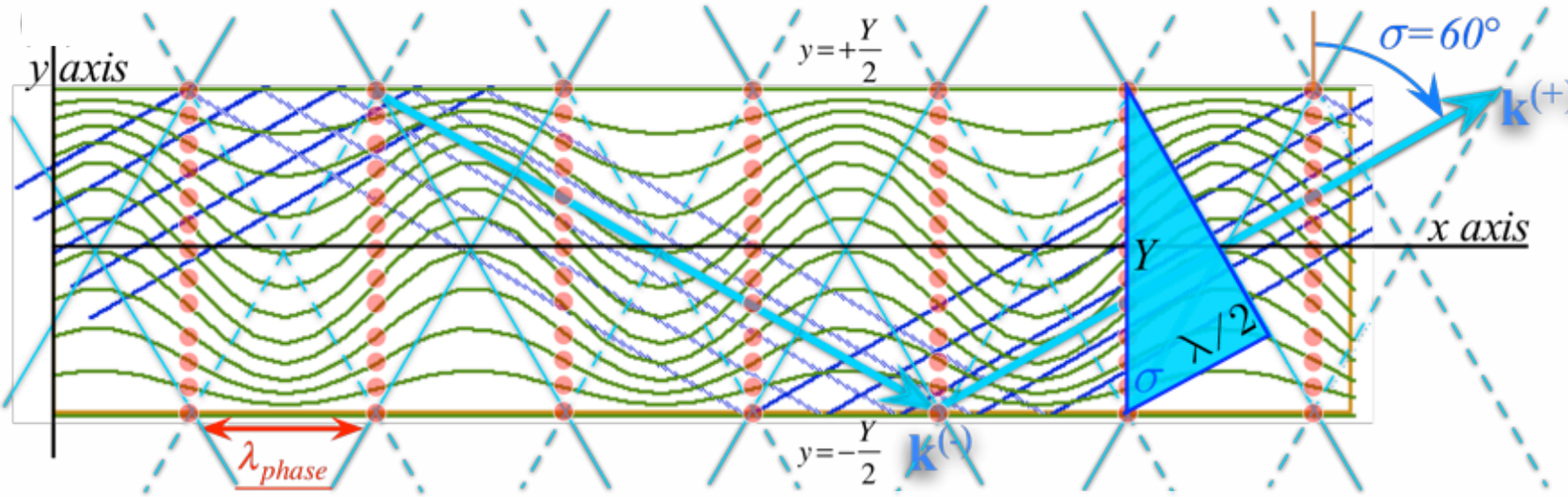
Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

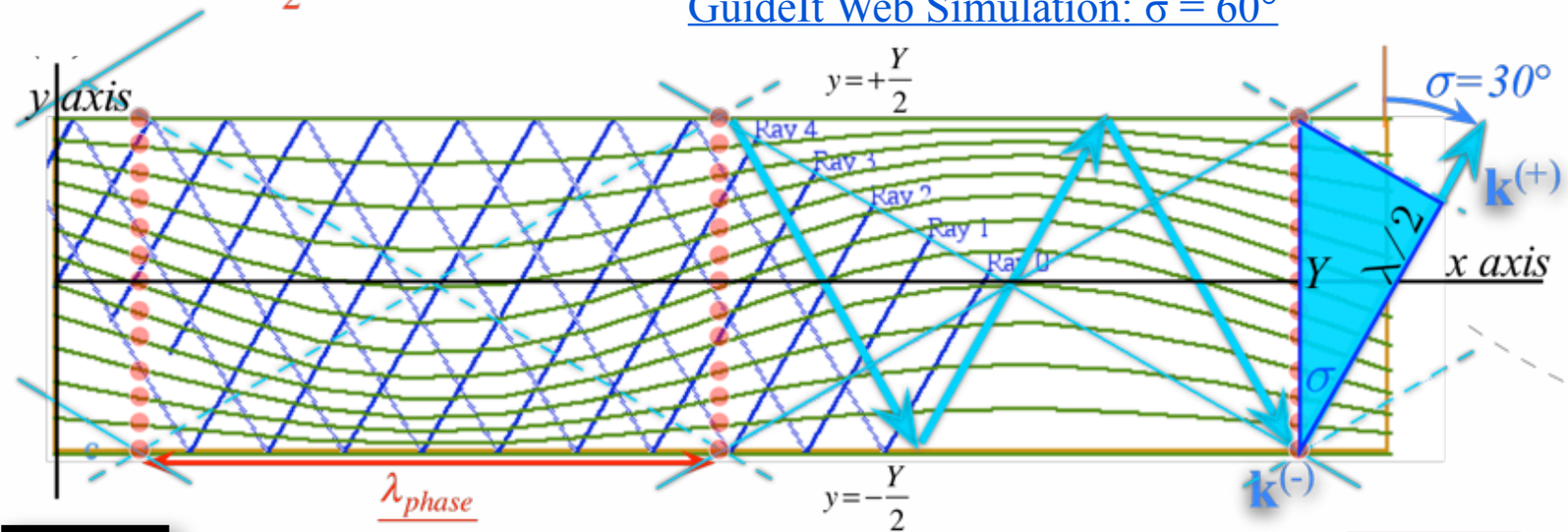
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

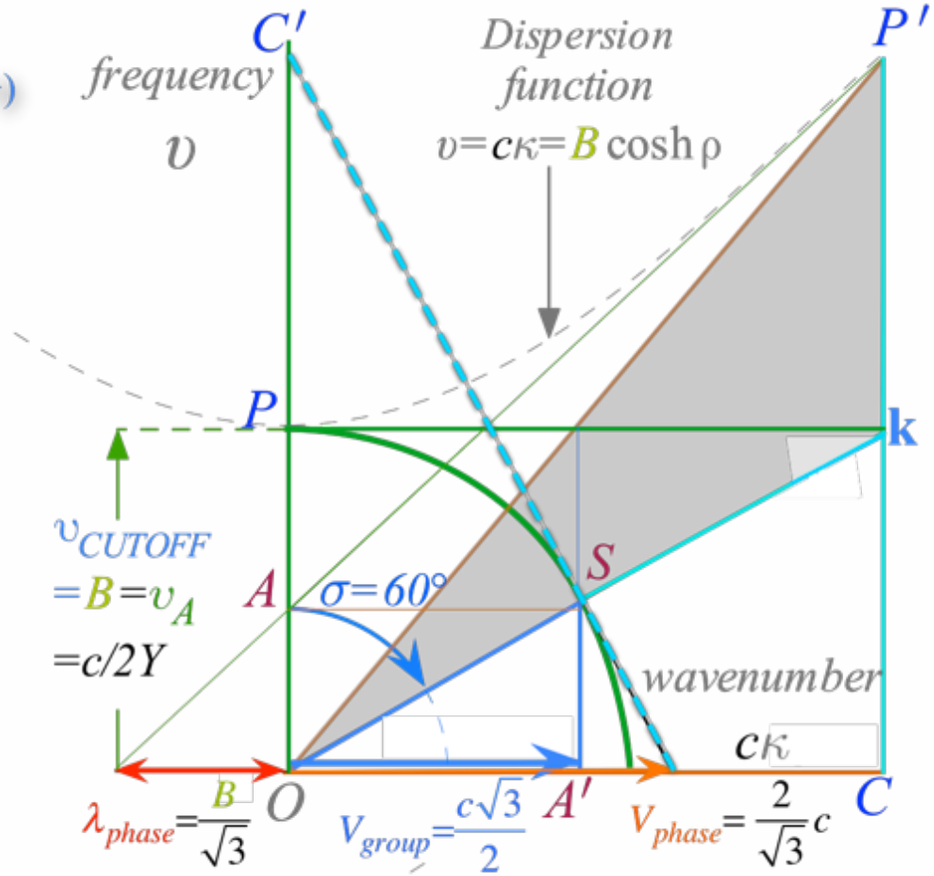
Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)

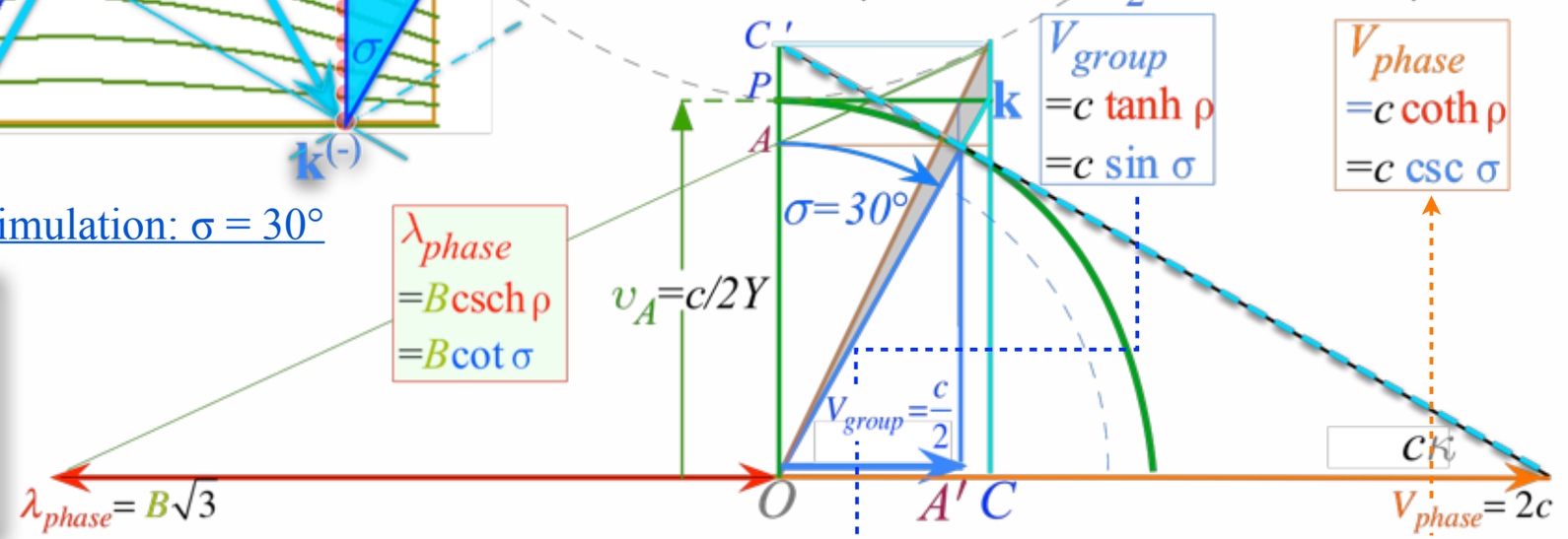


GuideIt Web Simulation: σ = 60°



GuideIt Web Simulation: σ = 30°





Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)