Group Theory in Quantum Mechanics

Lecture 2 *(1.19.2017)*

Quantum amplitudes, analyzers, and axioms

(Quantum Theory for Computer Age - Ch. 1 of Unit 1)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 1-2 of Ch. 1)

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)
Did Max Planck Goof? What's 1-photon worth?
Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?
Sample calculations
Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters
The "Do-Nothing" analyzer
Feynman amplitude axiom 4
Some "Do-Something" analyzers
Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Quantum Theory for Computer Age
http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_1_Ch._1_2013.pdf
http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

See also the *2005 Pirelli Challenge Relativity Site:*
http://www.uark.edu/ua/pirelli/html/light_energy_flux_1.html
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"Abstraction" of bra and ket vectors from a Transformation Matrix

Ket or column vectors *Bra or row vectors*

Given Transformation Matrix $T_{m,n'}$:
$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

"Abstraction" of bra and ket vectors from a Transformation Matrix

Ket or column vectors

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$$\Downarrow \qquad \Downarrow$$

$$|x'\rangle = \begin{pmatrix} \langle x|x'\rangle \\ \langle y|x'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Abstracting ket $|\, n'\rangle$ *state vectors*
from
Transformation Matrix
$T_{m,n'} = \langle m|\, n'\rangle$

"Abstraction" of bra and ket vectors from a Transformation Matrix

Ket or column vectors

Given Transformation Matrix $T_{m,n'}$:

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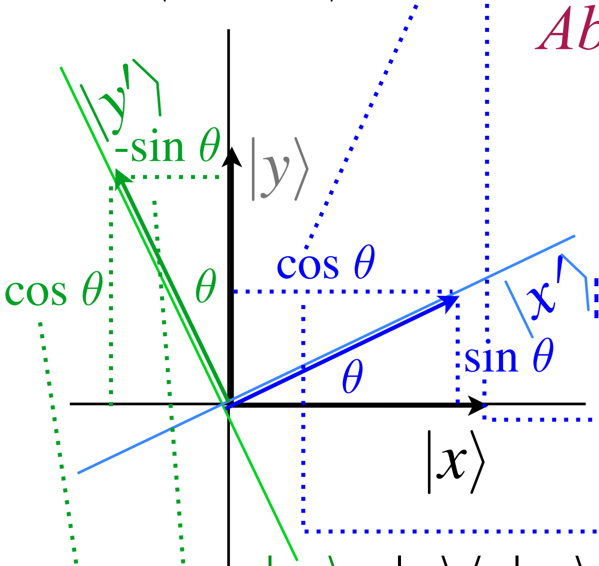
$$|x'\rangle = \begin{pmatrix} \langle x|x'\rangle \\ \langle y|x'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad |y'\rangle = \begin{pmatrix} \langle x|y'\rangle \\ \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

Abstracting ket $|n'\rangle$ *state vectors from Transformation Matrix* $T_{m,n'} = \langle m|n'\rangle$



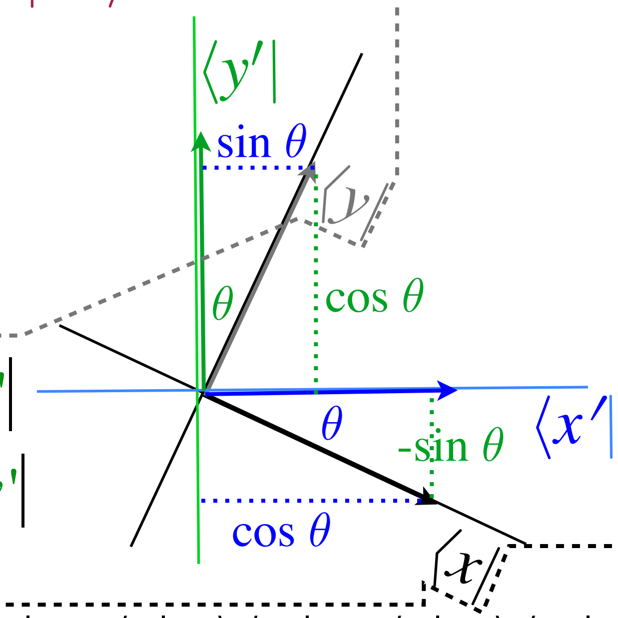
$$|x'\rangle = |x\rangle\langle x|x'\rangle + |y\rangle\langle y|x'\rangle$$
$$= |x\rangle(\cos\theta) + |y\rangle(\sin\theta)$$

$$|y'\rangle = |x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle$$
$$= |x\rangle(-\sin\theta) + |y\rangle(\cos\theta)$$

Bra or row vectors

$$\langle x| = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \end{pmatrix}$$
$$\langle y| = \begin{pmatrix} \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta \end{pmatrix}$$

Abstracting bra $\langle m|$ *state vectors from Transformation Matrix* $T_{m,n'} = \langle m|n'\rangle$



$$\langle y| = \langle y|x'\rangle\langle x'| + \langle y|y'\rangle\langle y'|$$
$$= (\sin\theta)\langle x'| + (\cos\theta)\langle y'|$$

$$\langle x| = \langle x|x'\rangle\langle x'| + \langle x|y'\rangle\langle y'|$$
$$= (\cos\theta)\langle x'| + (-\sin\theta)\langle y'|$$

($\theta=+30°$)-Rotated kets $\{|\mathbf{x}'\rangle, |\mathbf{y}'\rangle\}$ or $\{\mathbf{x}', \mathbf{y}'\}$ represented in page-aligned $\{|\mathbf{x}\rangle, |\mathbf{y}\rangle\}$ basis.

($\theta=-30°$)-Rotated bras $\{\langle\mathbf{x}|, \langle\mathbf{y}|\}$ or $\{\mathbf{x}, \mathbf{y}\}$ represented in page-aligned $\{|\mathbf{x}'\rangle, |\mathbf{y}'\rangle\}$ basis.

Ket vector algebra has the order of $T_{m,n'}$ *<u>transposed</u>*

$$|x'\rangle = |x\rangle\langle x|x'\rangle + |y\rangle\langle y|x'\rangle = |x\rangle(\cos\theta) + |y\rangle(\sin\theta)$$
$$|y'\rangle = |x\rangle\langle x|y'\rangle + |y\rangle\langle y|y'\rangle = |x\rangle(-\sin\theta) + |y\rangle(\cos\theta)$$

Bra vector algebra has the <u>same</u> order as $T_{m,n'}$

$$\langle x| = \langle x|x'\rangle\langle x'| + \langle x|y'\rangle\langle y'| = (\cos\theta)\langle x'| + (-\sin\theta)\langle y'|$$
$$\langle y| = \langle y|x'\rangle\langle x'| + \langle y|y'\rangle\langle y'| = (\sin\theta)\langle x'| + (\cos\theta)\langle y'|$$

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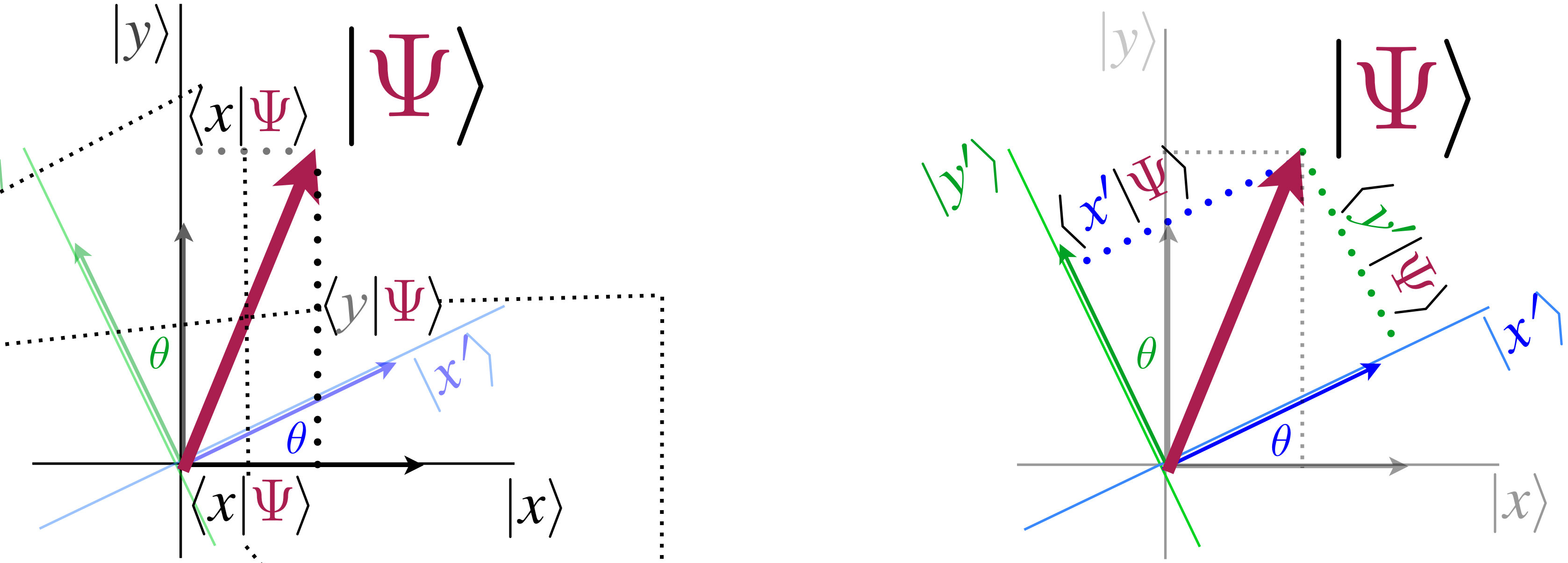
Transformation matrix $T_{m,n'} = \langle m | n' \rangle$ is array of dot or *scalar products* (dot products) of unit vectors or *direction cosines*.

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} (\mathbf{x}\bullet\mathbf{x}') & (\mathbf{x}\bullet\mathbf{y}') \\ (\mathbf{y}\bullet\mathbf{x}') & (\mathbf{y}\bullet\mathbf{y}') \end{pmatrix}$$

$\{\langle x|,\langle y|\}$ components of $|\Psi\rangle$:

$\langle x|\Psi\rangle = \Psi_x$

$\langle y|\Psi\rangle = \Psi_y$



$\{\langle x'|,\langle y'|\}$ components of $|\Psi\rangle$:

$\langle x'|\Psi\rangle = \Psi_{x'}$

$\langle y'|\Psi\rangle = \Psi_{y'}$

Any state $|\Psi\rangle$ can be expanded in any basis $\{\langle x|,\langle y|\}$, or $\{\langle x'|,\langle y'|\}$, ...*etc*.

$$|\Psi\rangle = |x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle = |x'\rangle\langle x'|\Psi\rangle + |y'\rangle\langle y'|\Psi\rangle$$

Transformation matrix $T_{m,n'}$ relates $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$ amplitudes to $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix}$$

Hybrid Gibbs-Dirac notation *(Ug-ly!)*

Proof: $\langle x| = \langle x|x'\rangle\langle x'| + \langle x|y'\rangle\langle y'|$ implies: $\langle x|\Psi\rangle = \langle x|x'\rangle\langle x'|\Psi\rangle + \langle x|y'\rangle\langle y'|\Psi\rangle$

$\langle y| = \langle y|x'\rangle\langle x'| + \langle y|y'\rangle\langle y'|$ implies: $\langle y|\Psi\rangle = \langle y|x'\rangle\langle x'|\Psi\rangle + \langle y|y'\rangle\langle y'|\Psi\rangle$

Transformation matrix $T_{m,n'}=\langle m|\,n'\rangle$ is array of dot or *scalar products* (dot products) of unit vectors or *direction cosines*.

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} (\mathbf{x}\bullet\mathbf{x}') & (\mathbf{x}\bullet\mathbf{y}') \\ (\mathbf{y}\bullet\mathbf{x}') & (\mathbf{y}\bullet\mathbf{y}') \end{pmatrix}$$

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$\langle x|\Psi\rangle=\Psi_x$

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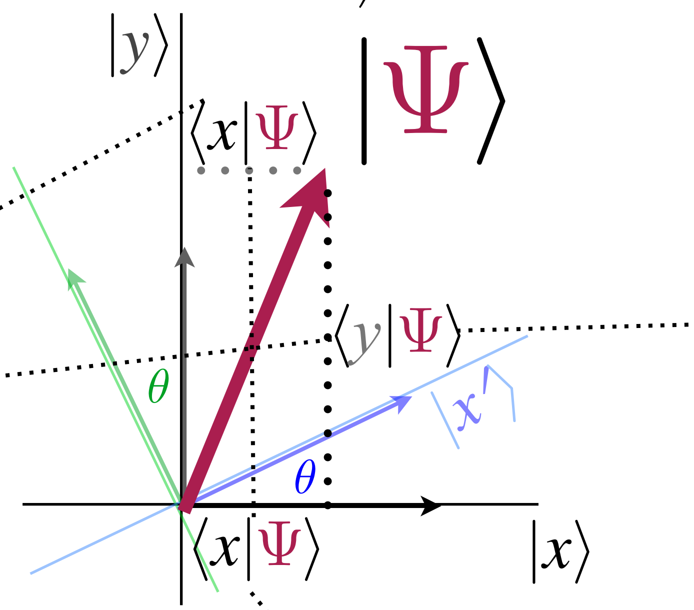

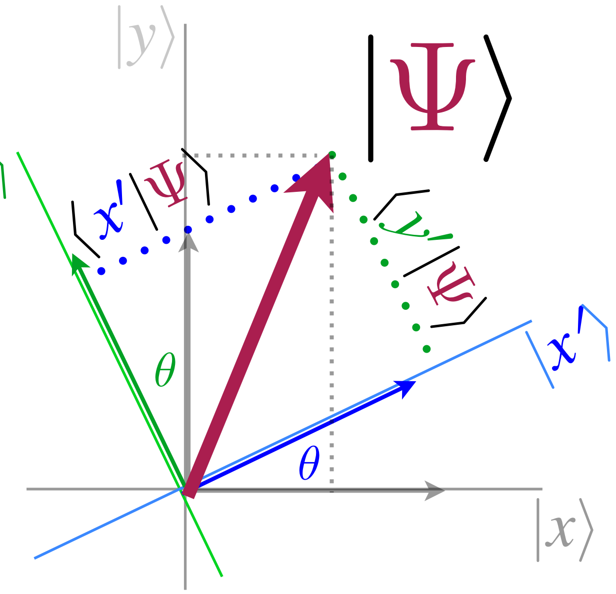

$\{\langle x'|,\langle y'|\}$ components of $|\Psi\rangle$:

$\langle x'|\Psi\rangle=\Psi_{x'}$

$\langle y'|\Psi\rangle=\Psi_{y'}$

Any state $|\Psi\rangle$ can be expanded in any basis $\{\langle x|,\langle y|\}$, or $\{\langle x'|,\langle y'|\}$, ...*etc*.

$$|\Psi\rangle = |x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle = |x'\rangle\langle x'|\Psi\rangle + |y'\rangle\langle y'|\Psi\rangle$$

Transformation matrix $T_{m,n'}$ relates $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$ amplitudes to $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix}$$

Hybrid Gibbs-Dirac notation *(Ug-ly!)*

Inverse ($\dagger$=$\top$*=-1) matrix $T_{n',m}$ relates $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$ amplitudes to $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

Hybrid Gibbs-Dirac notation *(Still Ug-ly!)*

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Planck's energy and N-quanta *(Cavity/Beam of volume V with wave mode of frequency* $\omega=2\pi\nu$*)*

Planck axiom: $\boldsymbol{E}$-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

$h=2\pi\hbar=6.6310^{-34}Js$ *Planck constant*

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_1_Ch._1_2013.pdf
http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

Planck's energy and *N*-quanta *(Cavity/Beam of volume V with wave mode of frequency* $\omega=2\pi\nu$*)*

Planck axiom: ***E***-field energy density *U* in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

$h=2\pi\hbar=6.63\cdot10^{-34}Js$ *Planck constant*

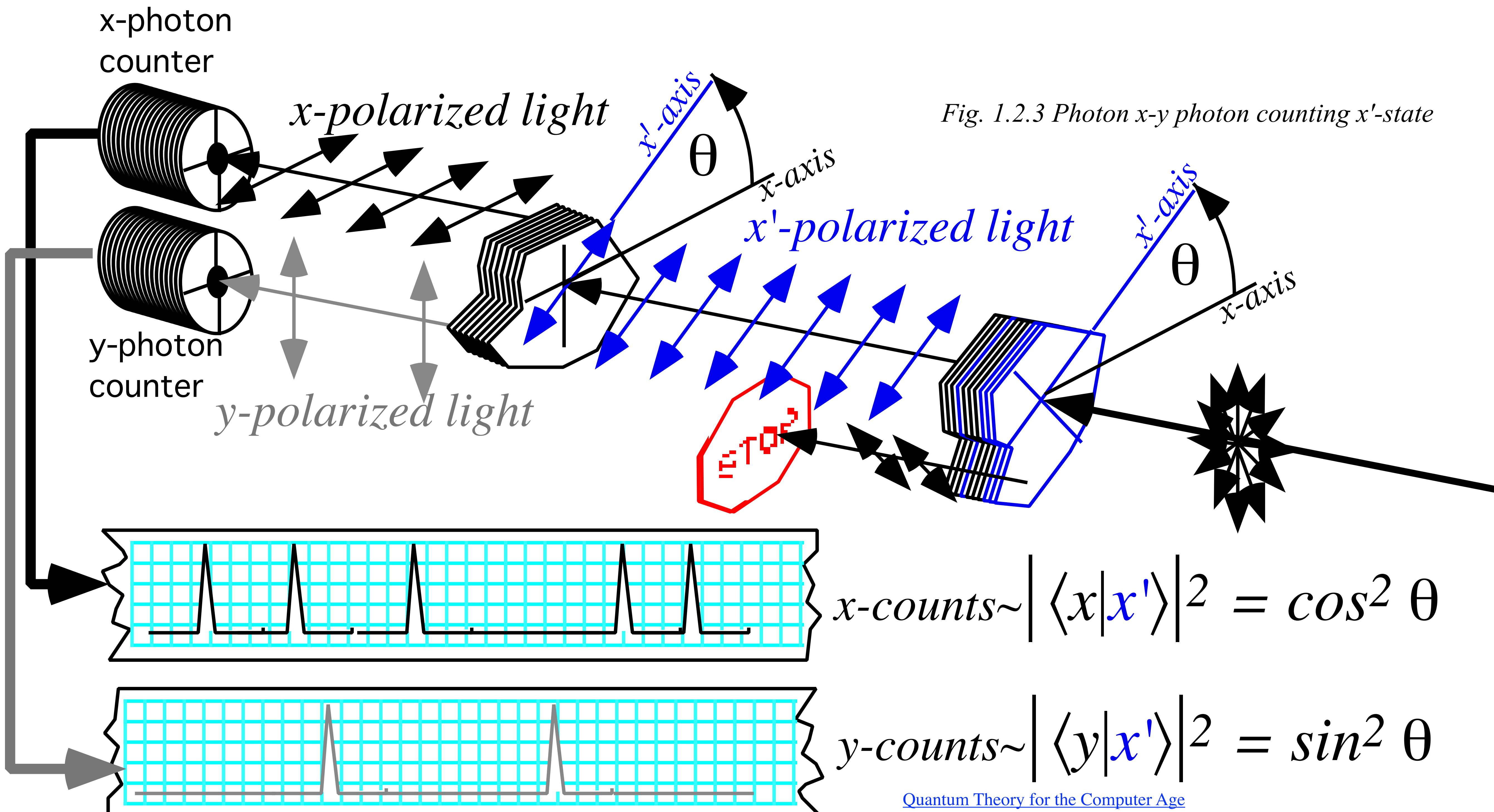


Fig. 1.2.3 Photon x-y photon counting x'-state

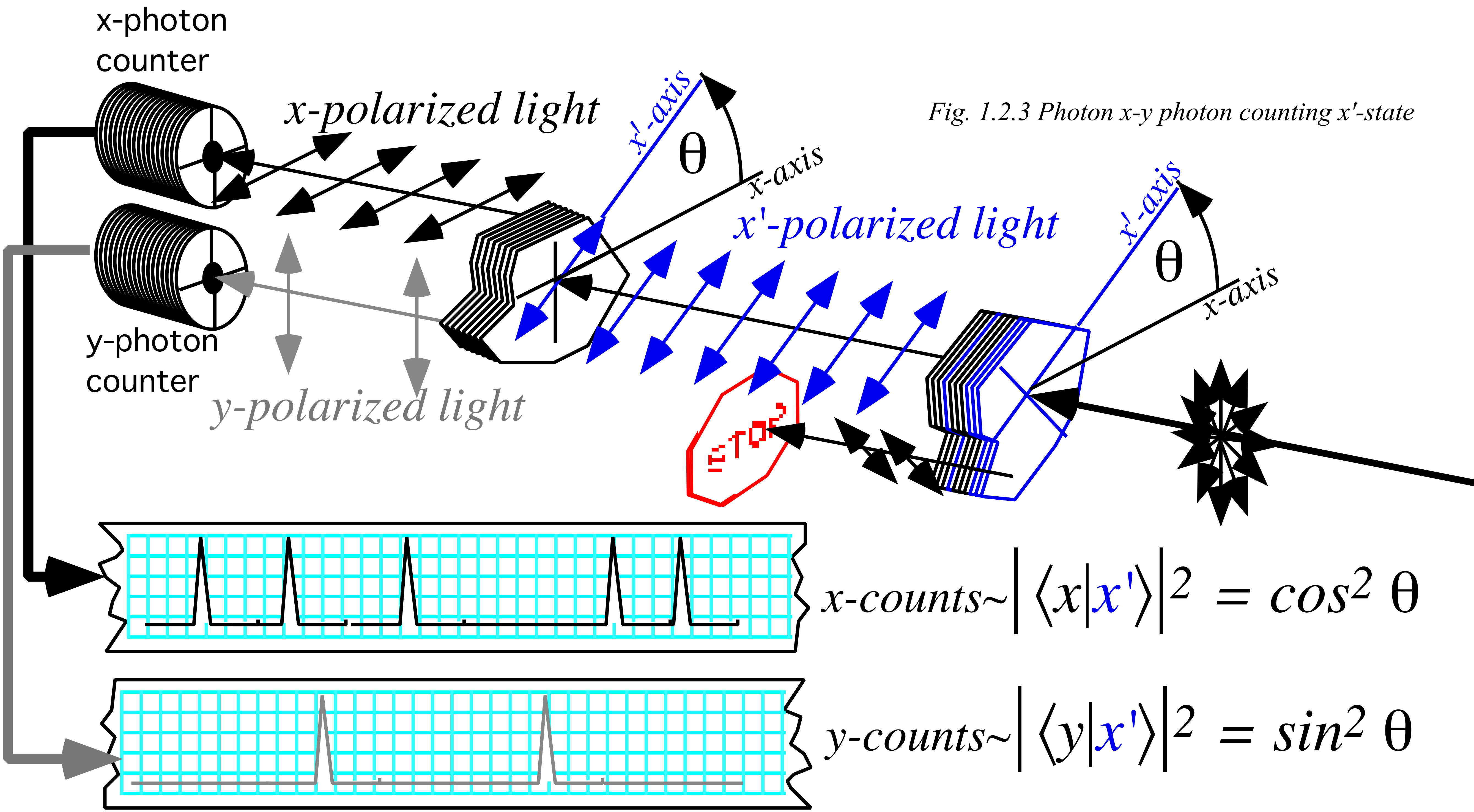
Quantum Theory for the Computer Age
URL is "http://www.uark.edu/ua/modphys/markup/QTCAWeb.html"

Planck's energy and N-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

Planck axiom: **E**-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

E-field vector $(E_x, E_y) = f\cdot(\Psi_x, \Psi_y)$ where *quantum field proportionality constant is f*.

$h=2\pi\hbar=6.63\cdot10^{-34}Js$	*Planck constant*



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$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_x(0)e^{ikz-i\omega t} \\ E_y(0)e^{ikz-i\omega t} \end{pmatrix} \cong \begin{pmatrix} E_x(0)e^{-i\omega t} \\ E_y(0)e^{-i\omega t} \end{pmatrix} = f\begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

$h=2\pi\hbar=6.63\cdot10^{-34}Js$ *Planck constant*

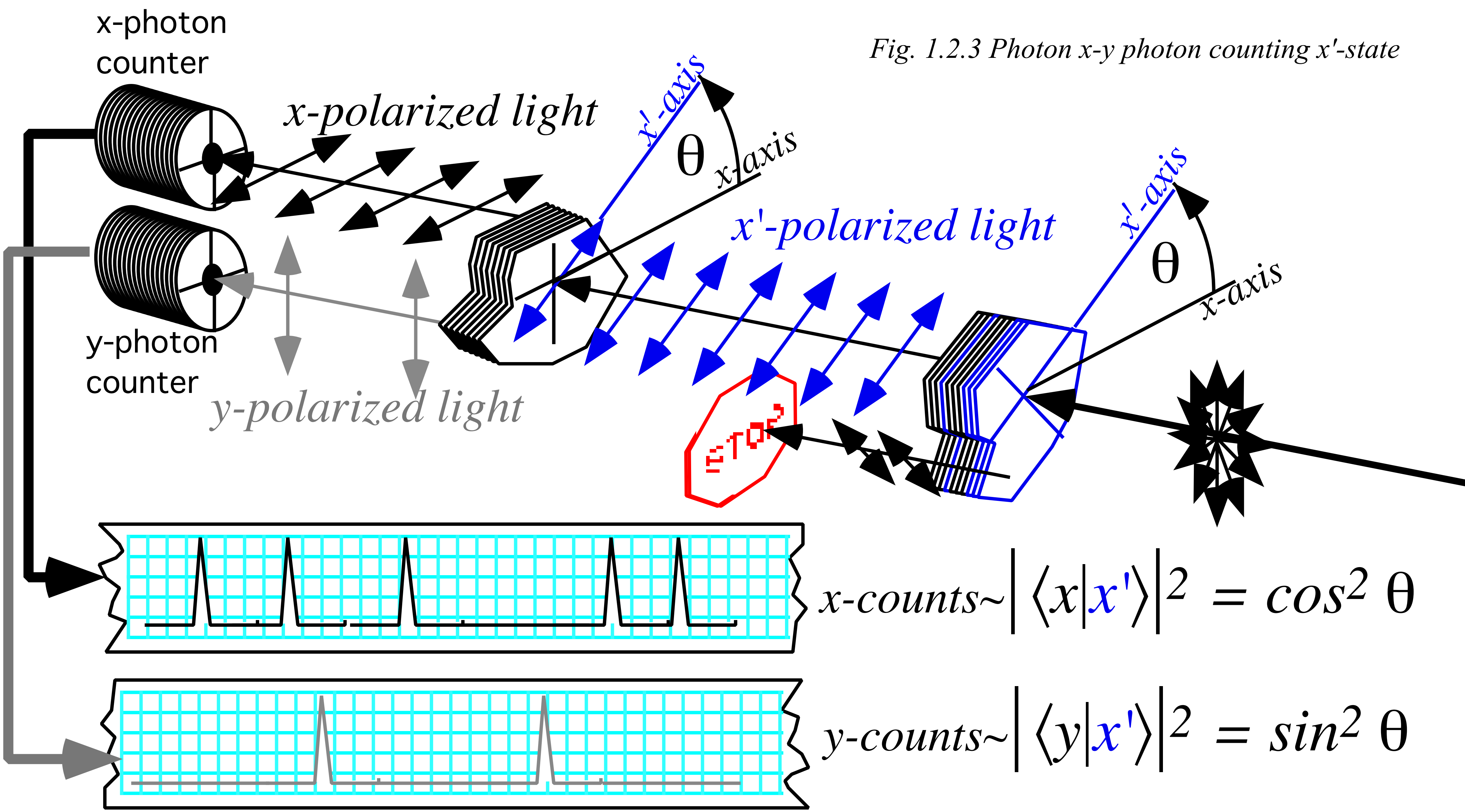

Fig. 1.2.3 Photon x-y photon counting x'-state

$$x\text{-}counts \sim \left|\langle x|x'\rangle\right|^2 = \cos^2\theta$$

$$y\text{-}counts \sim \left|\langle y|x'\rangle\right|^2 = \sin^2\theta$$

Planck's energy and N-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

Planck axiom: $\boldsymbol{E}$-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

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$h=2\pi\hbar=6.626\,075\cdot10^{-34}Js$	Planck constant
$c=2.997\,924\,58\cdot10^{8}\,ms^{-1}$	Light speed
$\varepsilon_0=8.854\cdot10^{-12}C^2N^{-1}m^{-2}$	Electrostatic constant

Coulomb constant: $k=1/4\pi\varepsilon_0$

$= 9\cdot10^9\ J/C$

Ψ-amplitude-squares sum to *exciton-number N*. (...or *photon-number N*)

$$N=\left|\Psi_x\right|^2+\left|\Psi_y\right|^2=\langle x|\Psi\rangle^*\langle x|\Psi\rangle+\langle y|\Psi\rangle^*\langle y|\Psi\rangle$$

Quantum em wave theory

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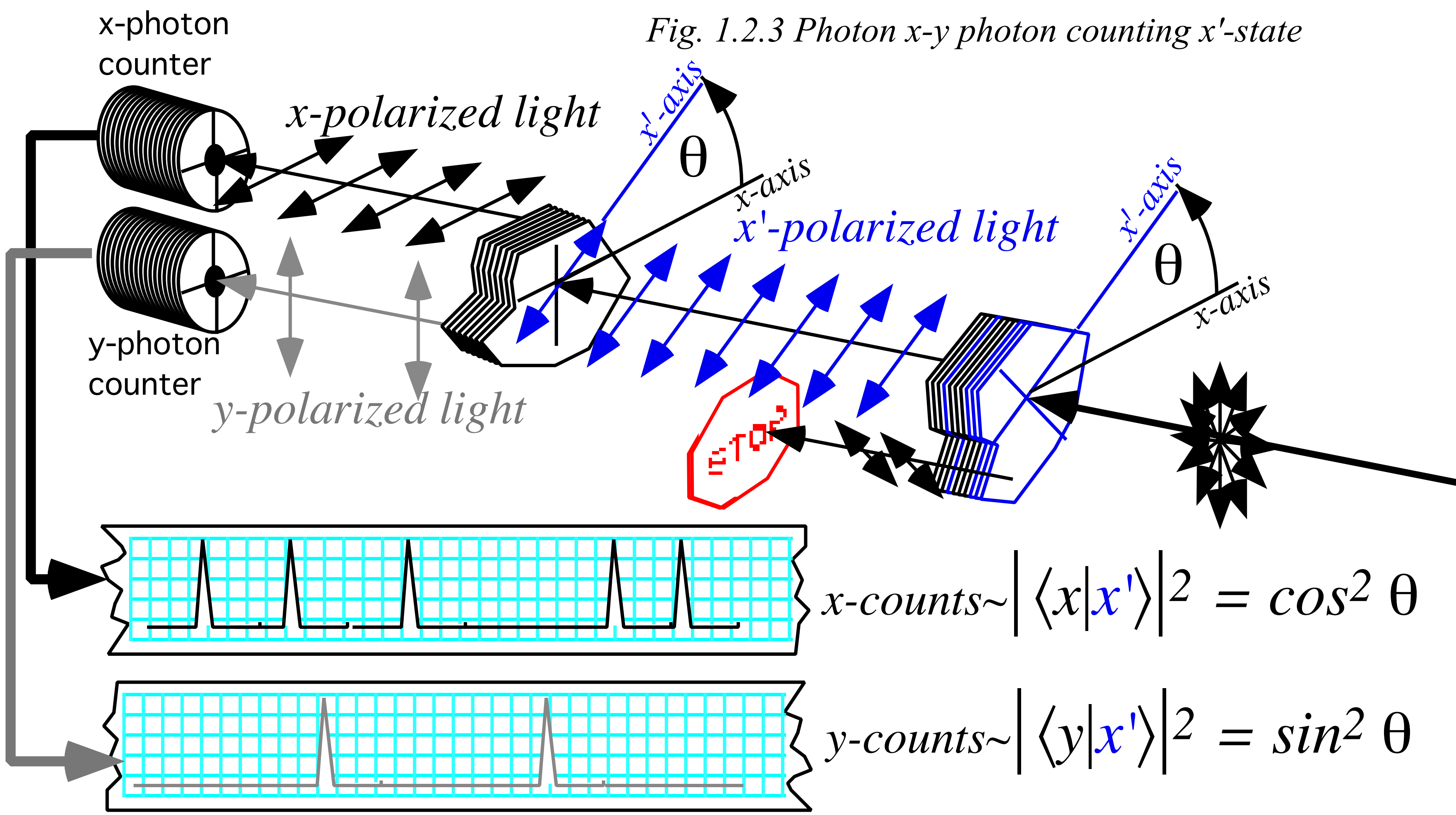

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$$N = |\Psi_x|^2 + |\Psi_y|^2 = \langle x|\Psi\rangle^*\langle x|\Psi\rangle + \langle y|\Psi\rangle^*\langle y|\Psi\rangle$$

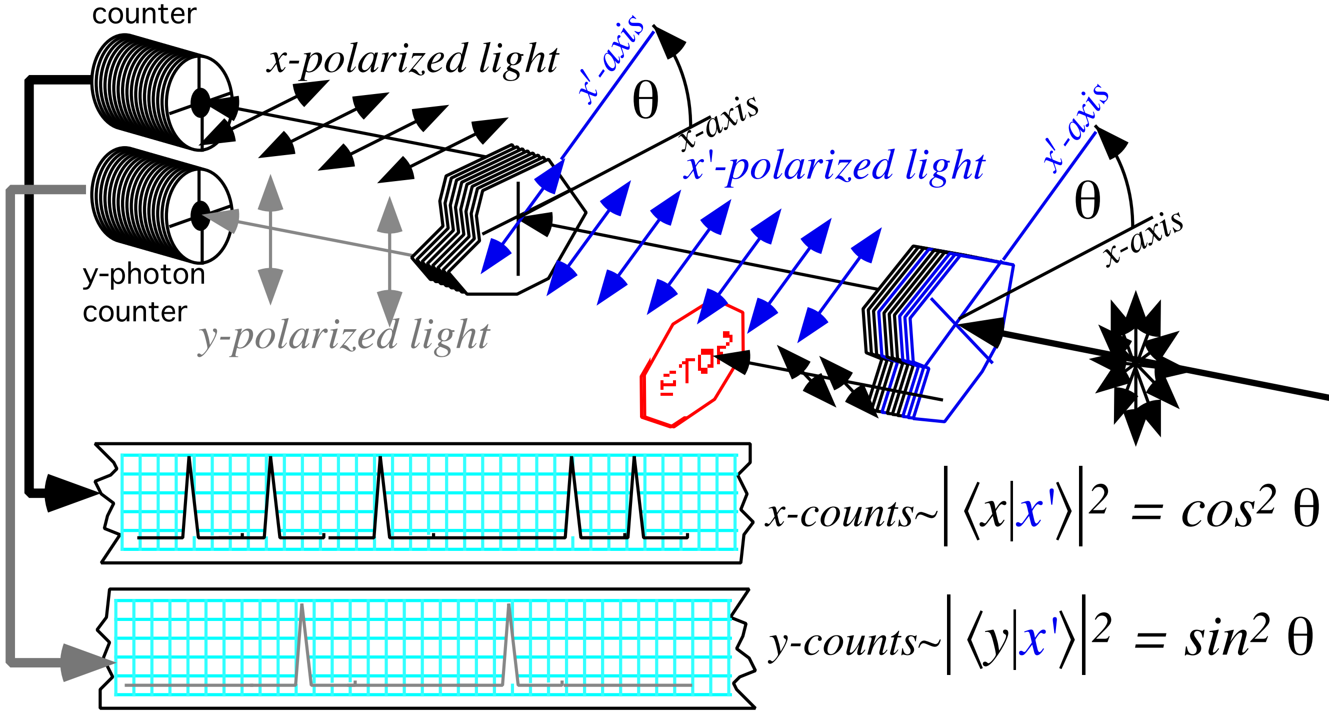
Quantum em wave theory

Poynting energy flux $S(J/m^2s)$ or *energy density $U(J/m^3)$* of light beam.

$$S = cU, \quad \text{where: } U = \varepsilon_0\left(|E_x|^2 + |E_y|^2\right) = \varepsilon_0\left(E_x^*E_x + E_y^*E_y\right) = \varepsilon_0\left(E_x(0)^2 + E_y(0)^2\right)$$

Classical em wave theory

Fig. 1.2.3 Photon x-y photon counting x'-state



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Classical em wave theory

Equate U to Planck's N-*"photon"* quantum energy density $N\hbar\omega/V$

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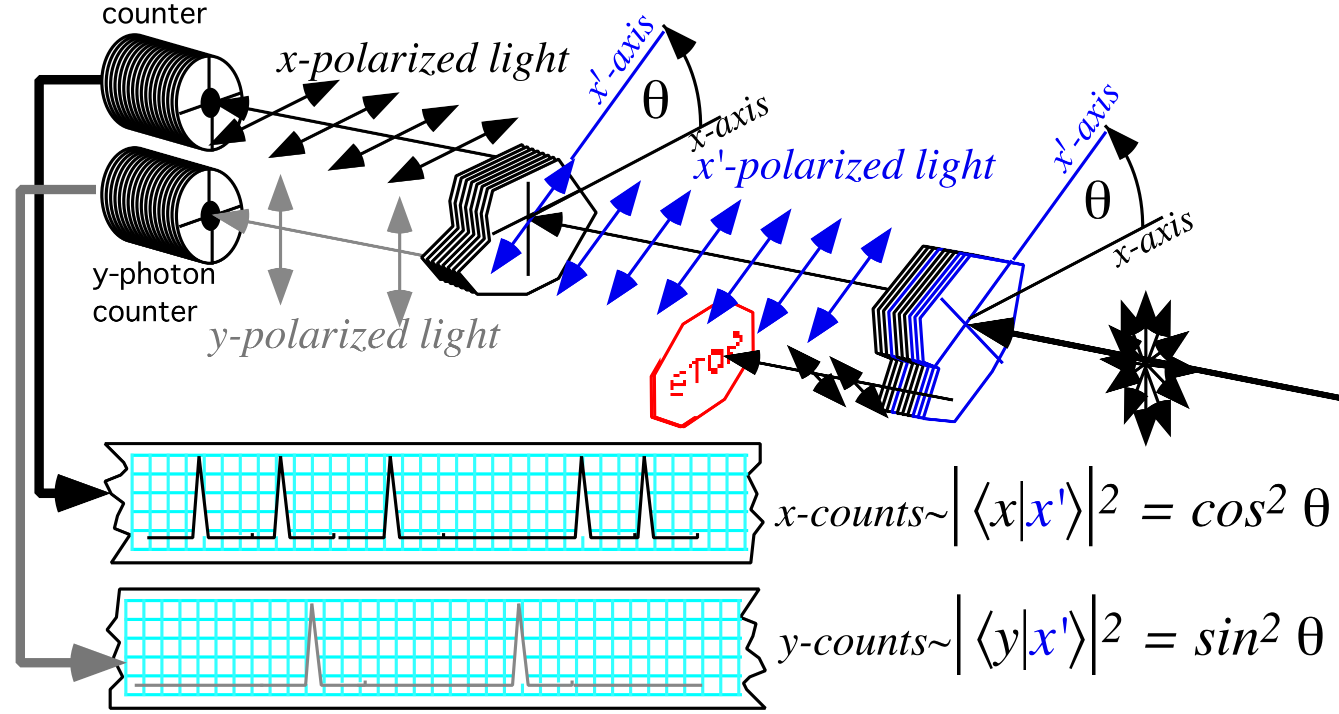

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$= 9\cdot10^{9}\,J/C$

Ψ-amplitude-squares sum to *exciton-number N*. (...or *photon-number N*)

$$N=\left|\Psi_x\right|^2+\left|\Psi_y\right|^2=\langle x|\Psi\rangle^*\langle x|\Psi\rangle+\langle y|\Psi\rangle^*\langle y|\Psi\rangle$$

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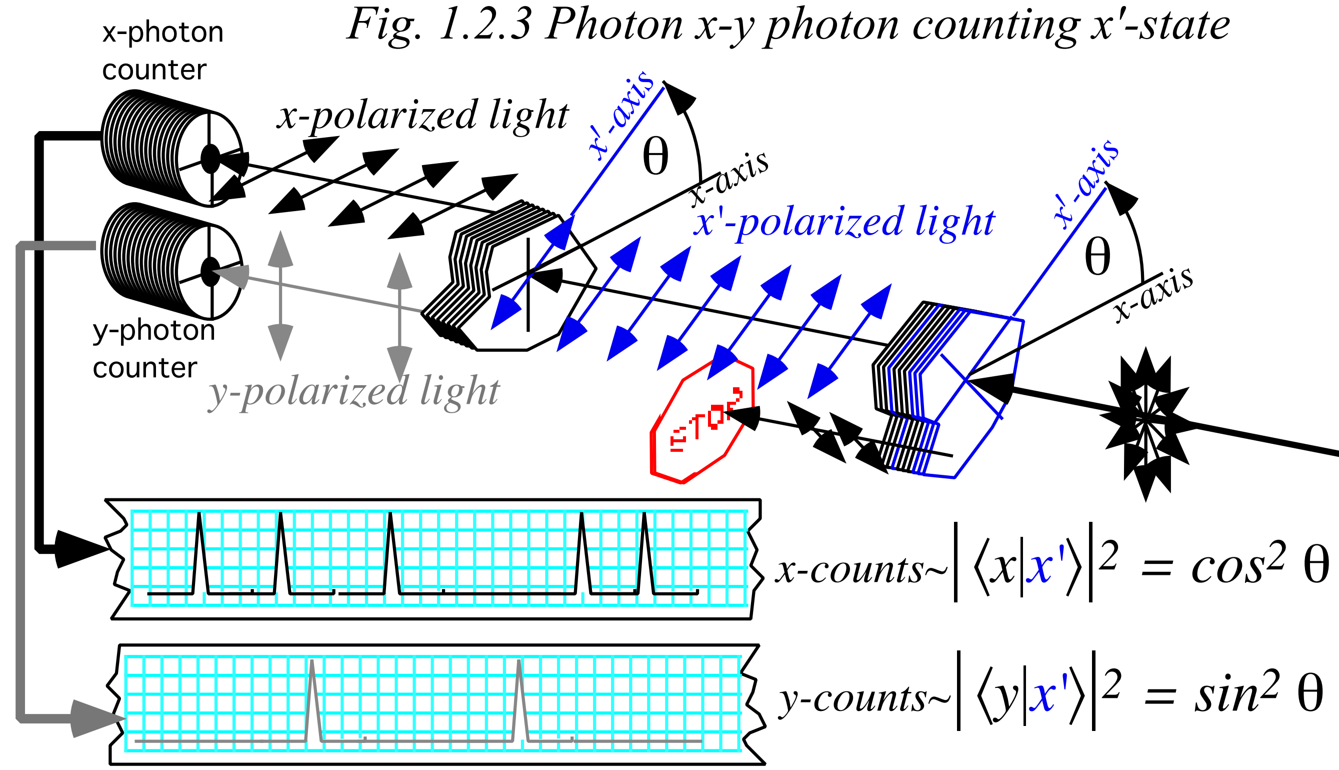
Fig. 1.2.3 Photon x-y photon counting x'-state



x-counts~$\left|\langle x|x'\rangle\right|^2=\cos^2\theta$

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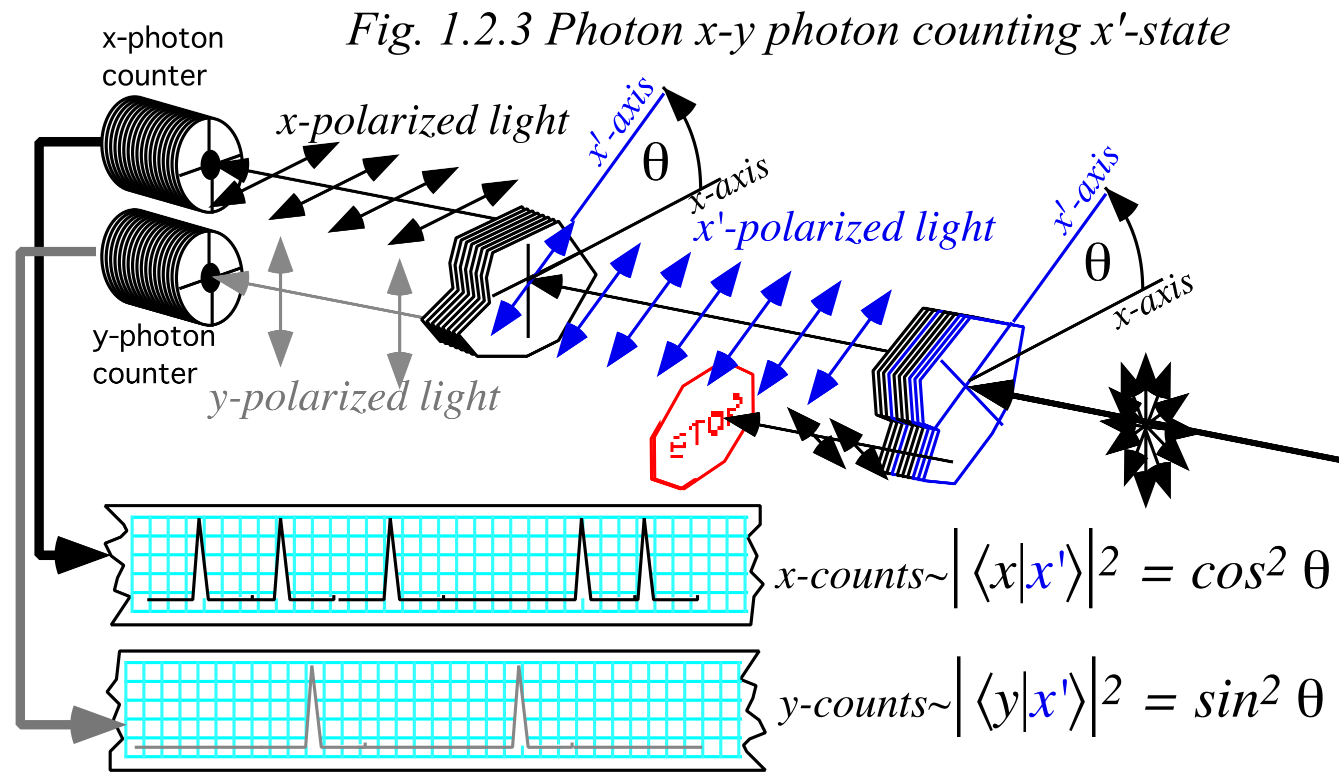

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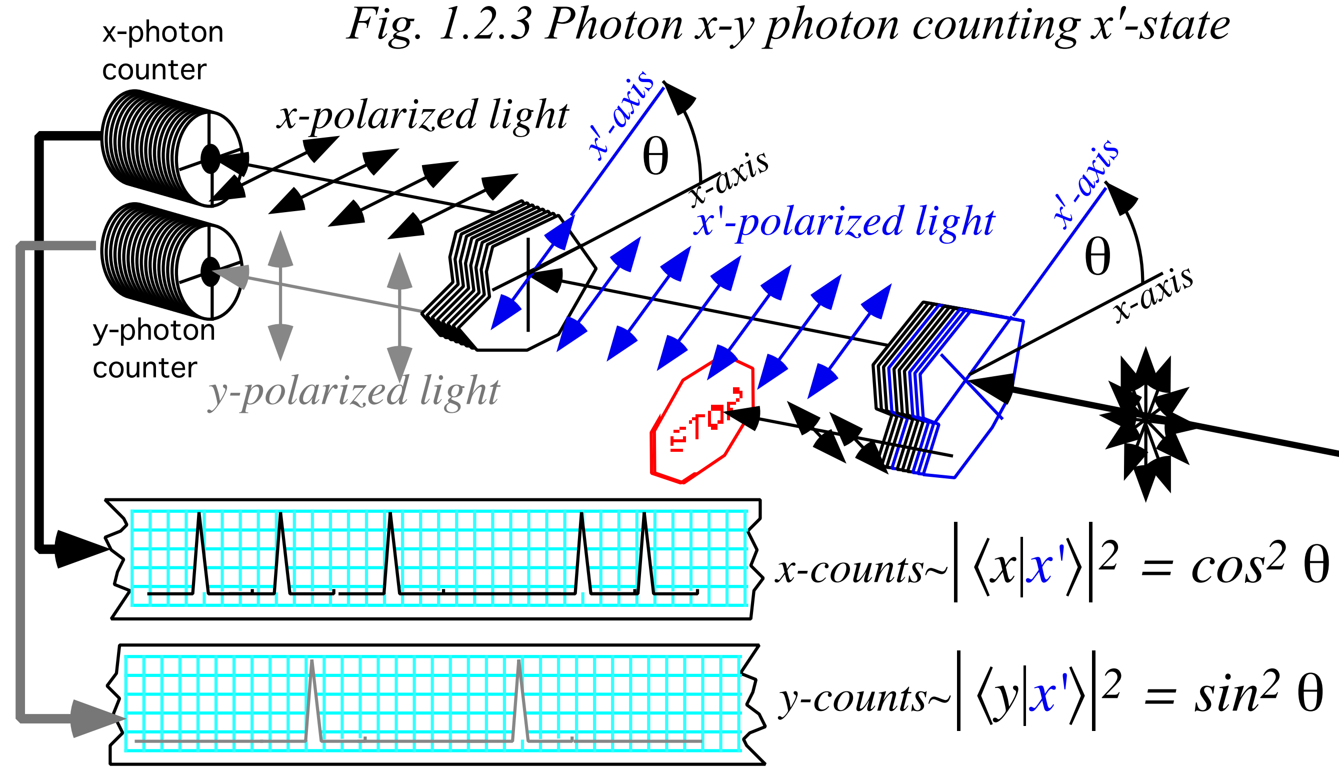

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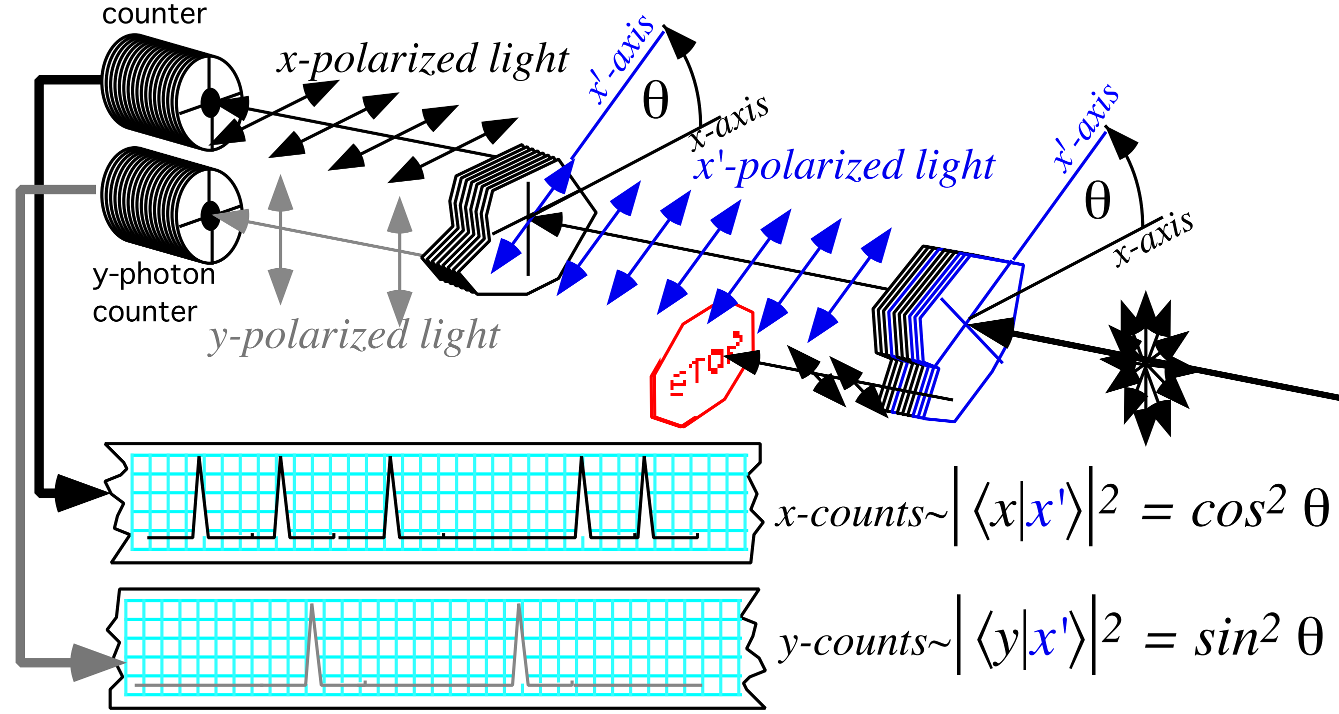

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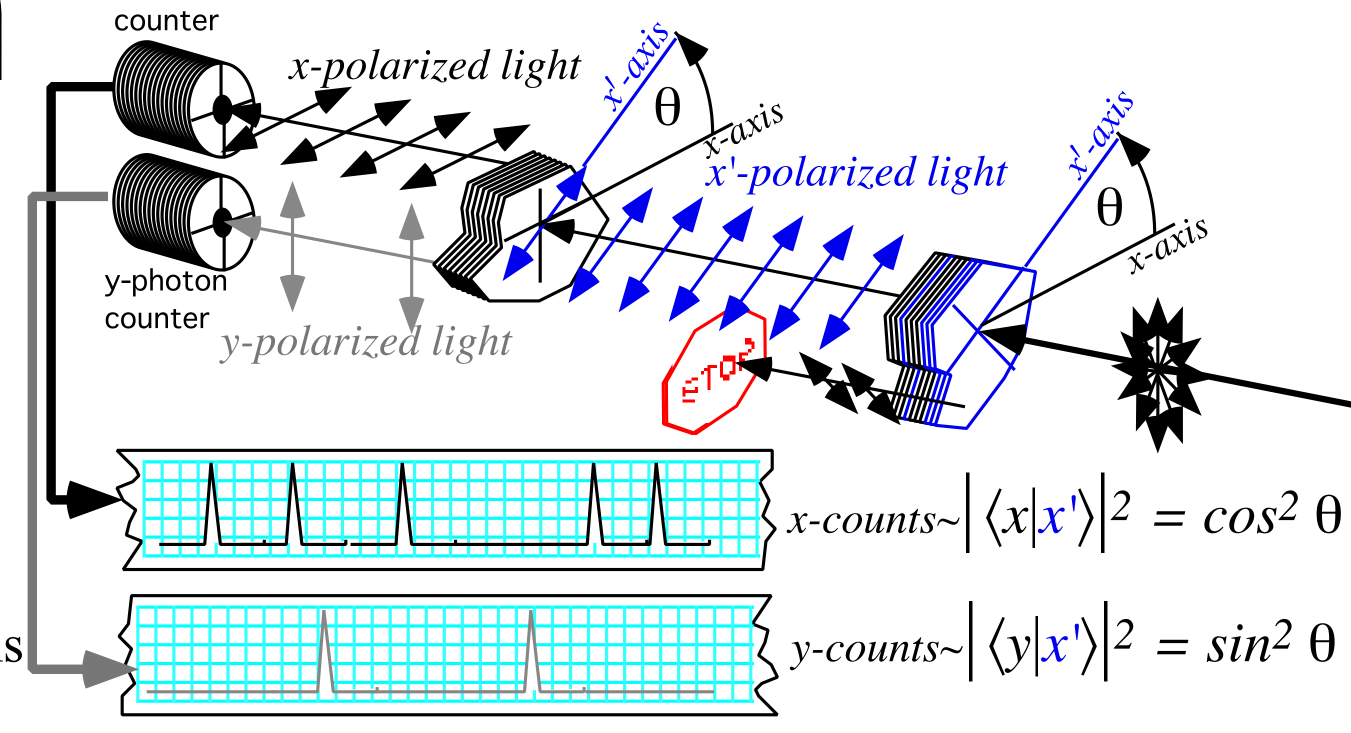

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http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_7_Ch._22_2005.pdf

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What's a photon worth? *What field is there for N=1 in a cavity of energy ($U=\hbar\omega N$) ?*

or n=1 in a beam of Poynting flux ($S=\hbar\omega n/\sigma$)?

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quantum field constant f

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E-field for $N=1$ photon $= |\mathbf{E}| = \sqrt{(h\upsilon/V\varepsilon_0)} = 1.7\cdot 10^{6}$Volts/m

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Energy per $N=10^6$ photons $= 10^6\ h\upsilon = 10^6\ 6.63\cdot 10^{-34} Js\cdot 600\cdot 10^{12}/s = 4.0\cdot 10^{-13} J = 2.5 MeV$

Was Planck's linear-in-frequency-υ energy axiom a Goof ?

Not at all. It's really not linear-in-frequency after all.

The $\varepsilon_0 \, |\mathbf{E}|^2 = h \, N \cdot \upsilon$ *axiom IS in fact a product of TWO frequencies!*

The 2nd "frequency" is COUNT RATE N.

As shown later: Both frequencies transform by Relativistic Doppler factor $e^{\pm\rho}$:

Coherent frequency is light quality: $\upsilon' = e^{\pm\rho} \, \upsilon$. *Incoherent frequency is light quantity:* $N' = e^{\pm\rho} N$.

This would imply that the $|\mathbf{E}|$*-field also transforms like a frequency:* $|\mathbf{E}|' = = e^{\pm\rho} \, |\mathbf{E}|$.

Indeed, E-field amplitude is a frequency, too!

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(1) The probability axiom

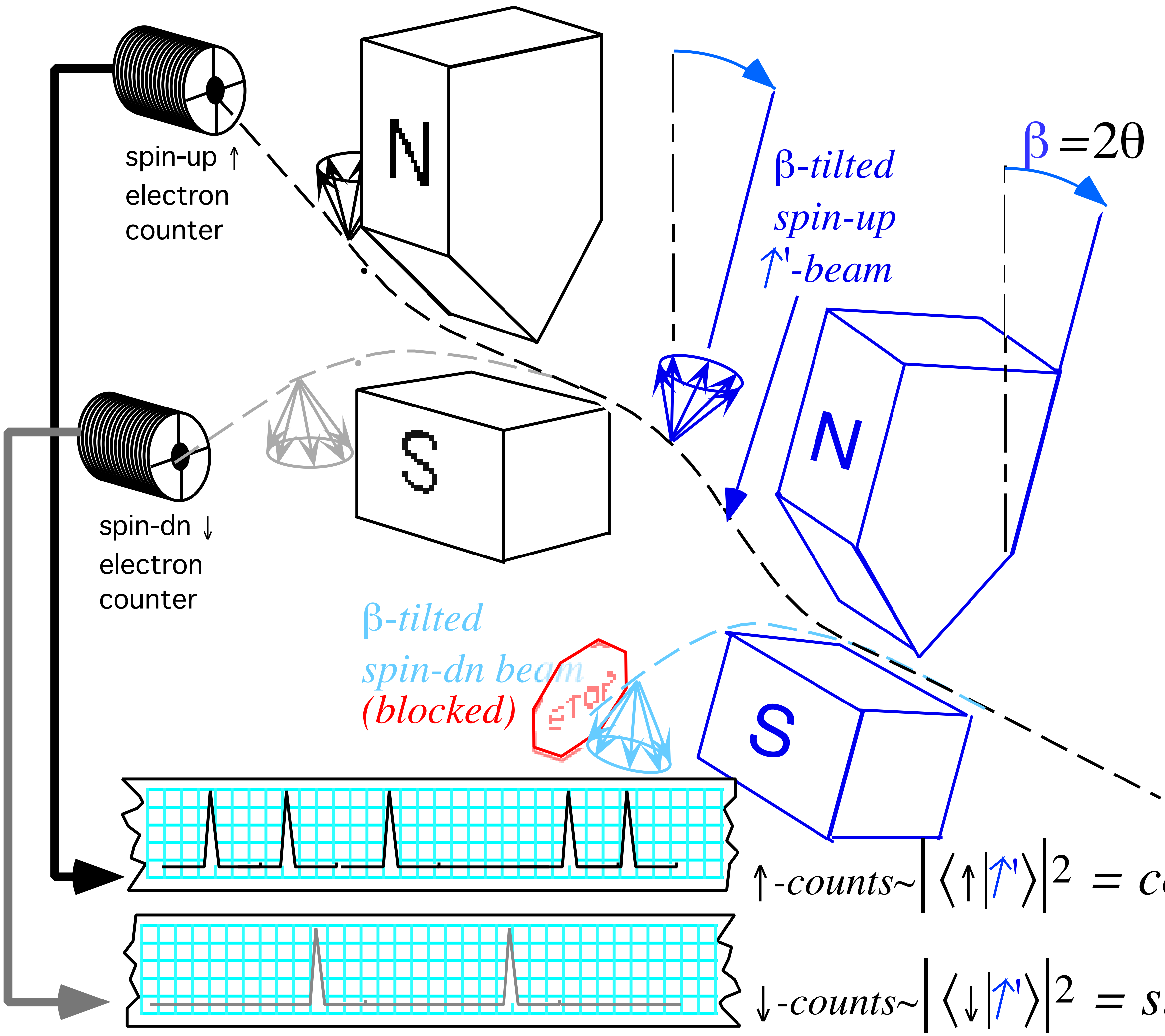
The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in state-j of a system that started in state-k′=1′,2′,..,or n′ from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

Amplitude axioms apply to all intensity-conserving systems

probability

This includes, first of all, spin-1/2 electron, proton,…,^{13}C,… particles (Fermions)



$$\uparrow\text{-counts} \sim \left|\langle\uparrow|\uparrow'\rangle\right|^2 = \cos^2\beta/2 = \cos^2\theta$$

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Fig. 1.2.4 Electron up-dn-spin counting of a tilted spin-up (↑′)-state

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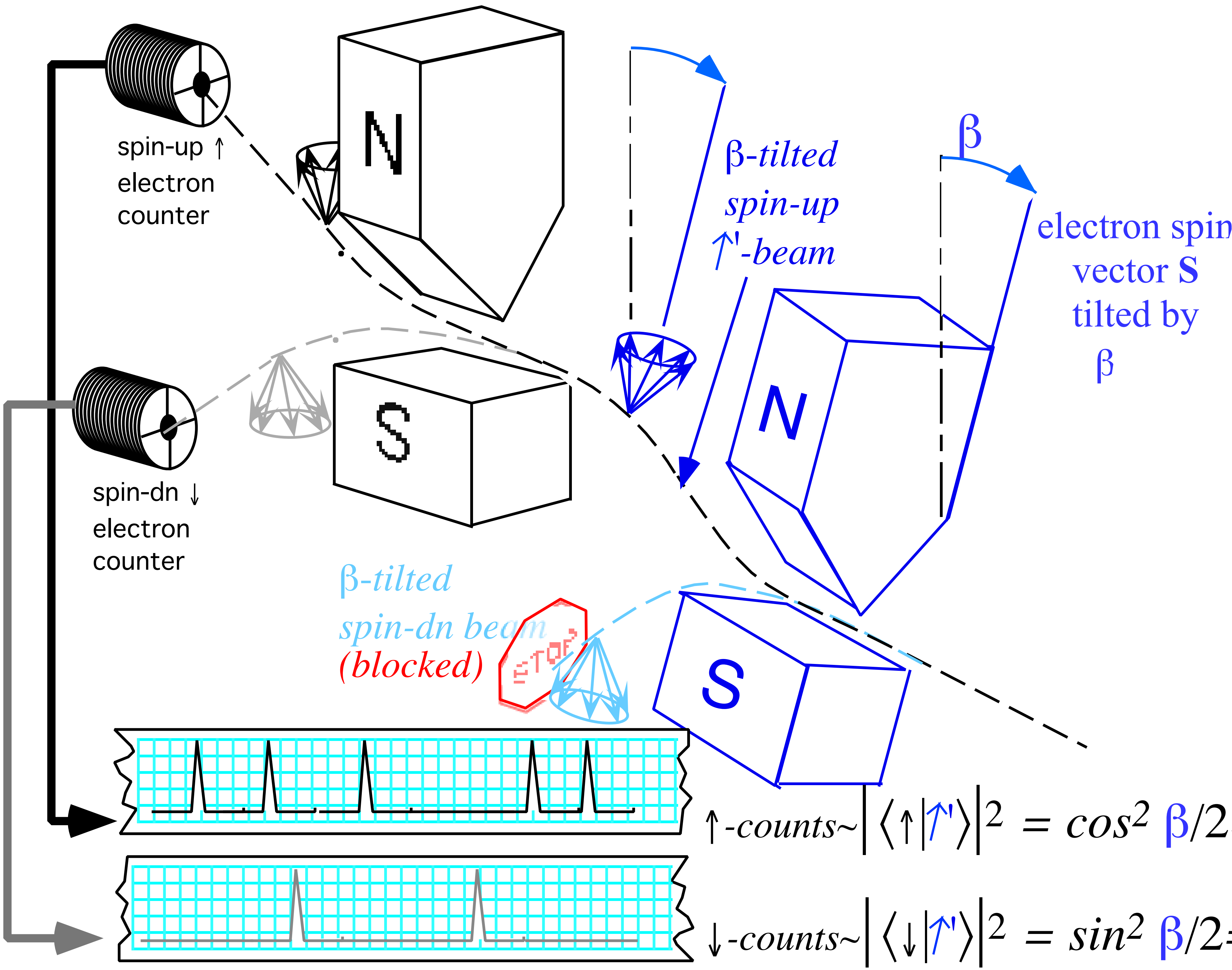

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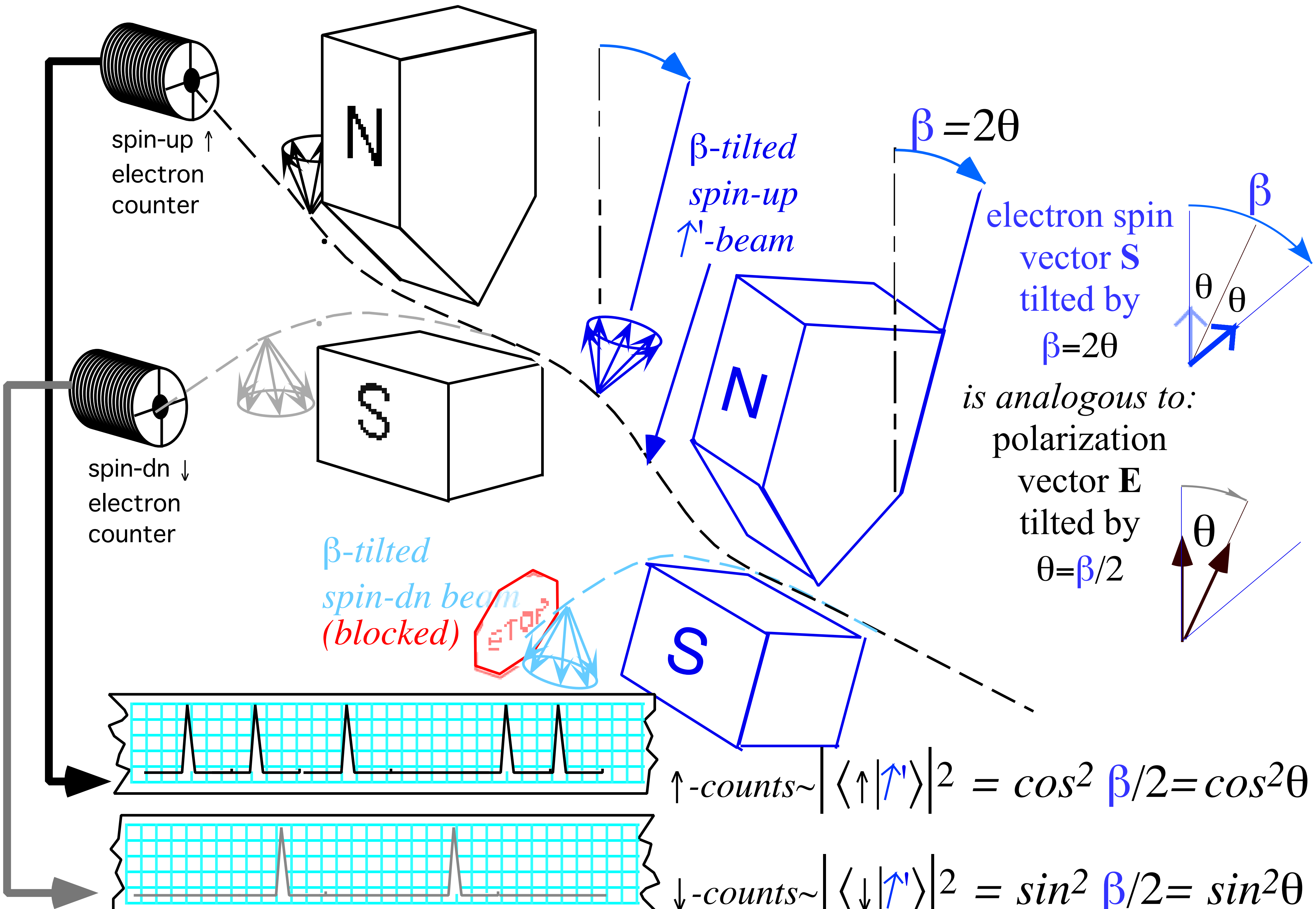

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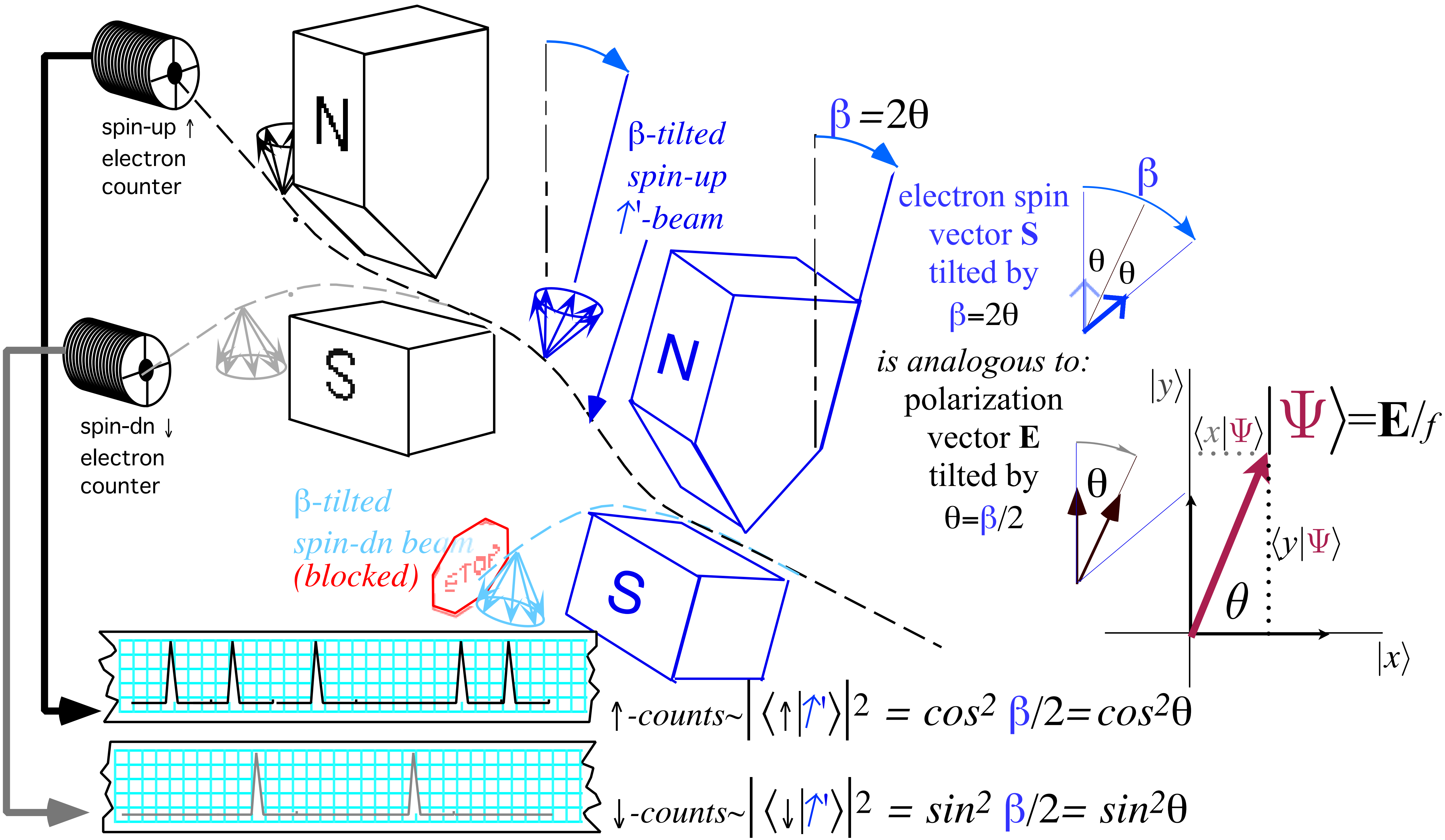

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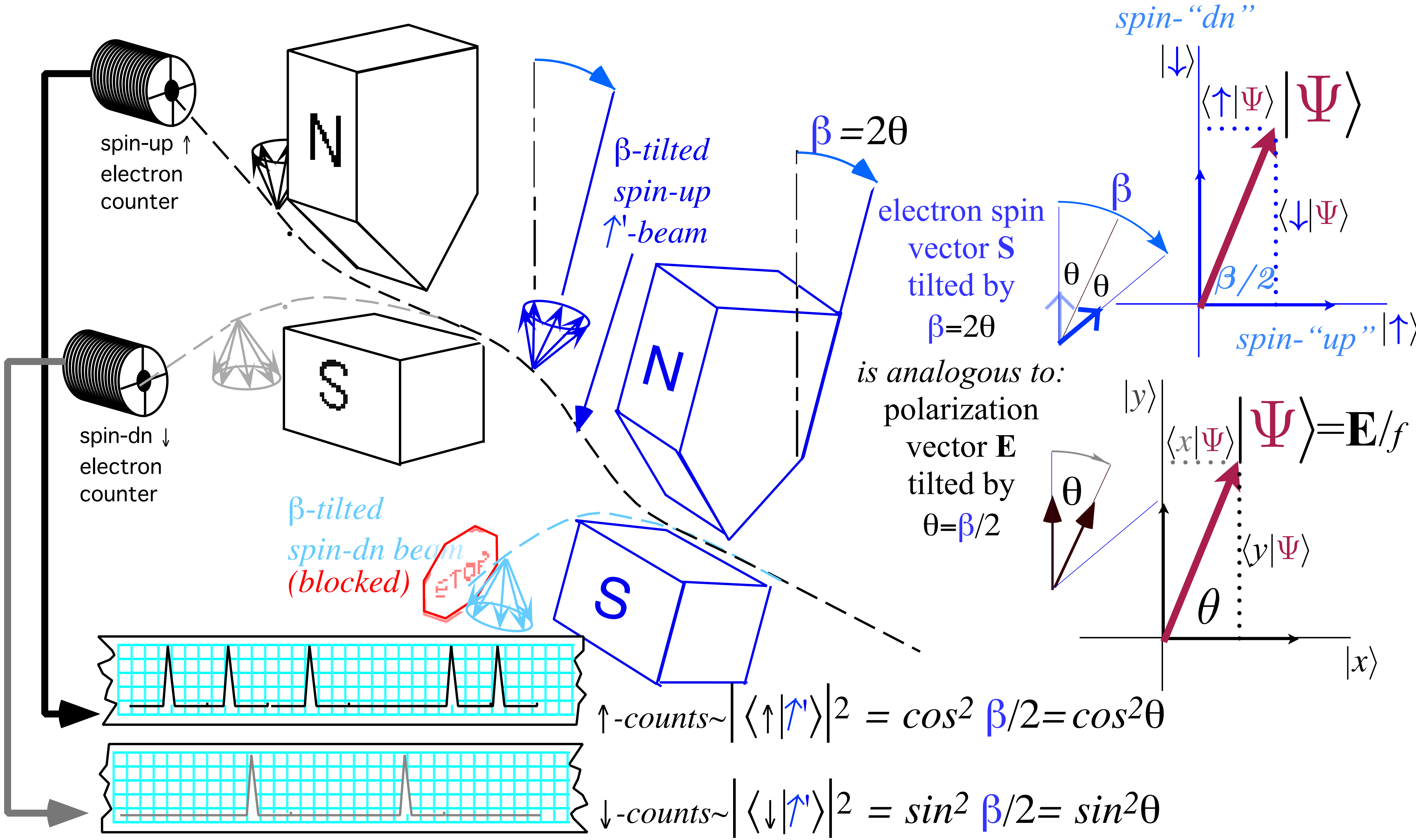

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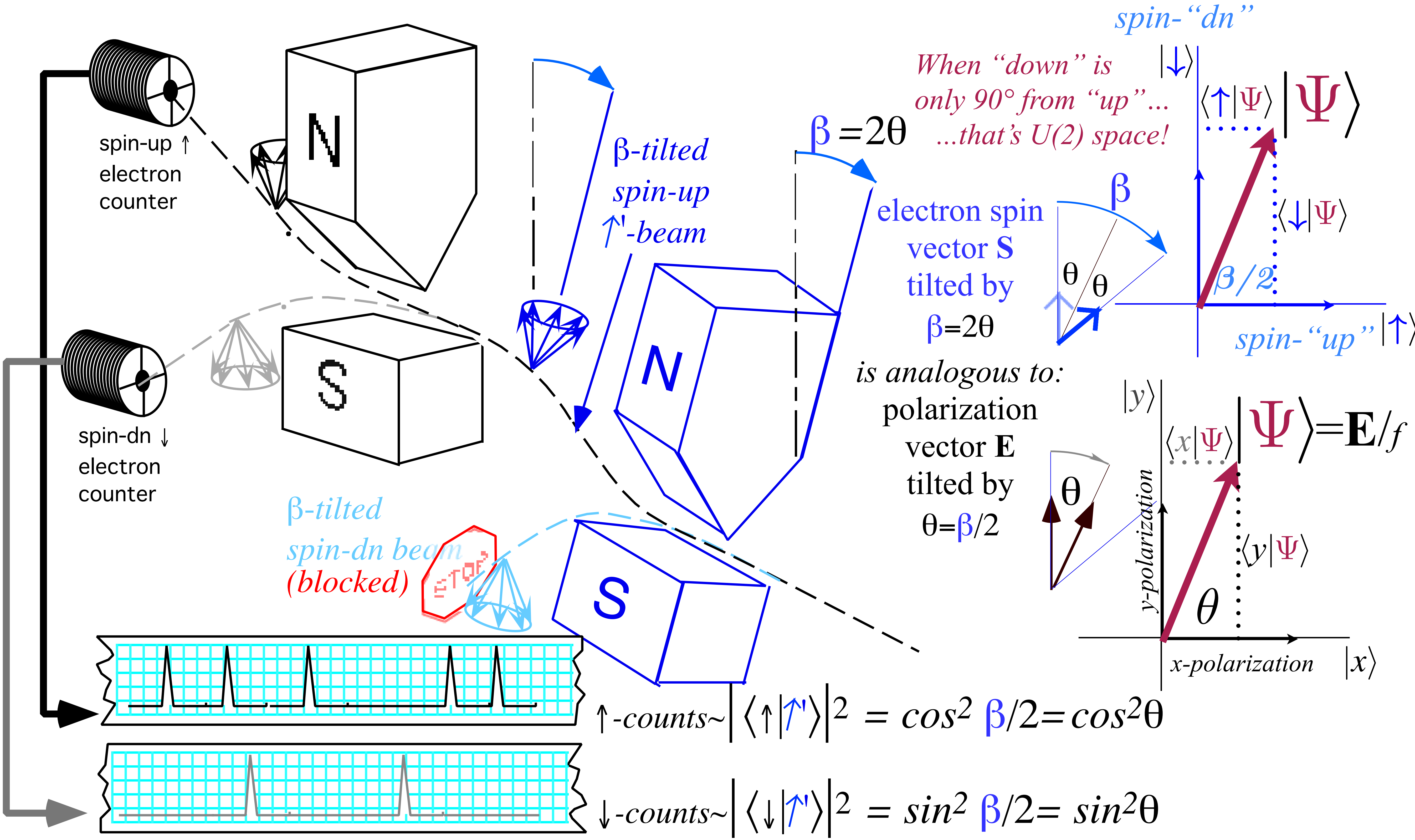

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A: Want to determine or calculate:

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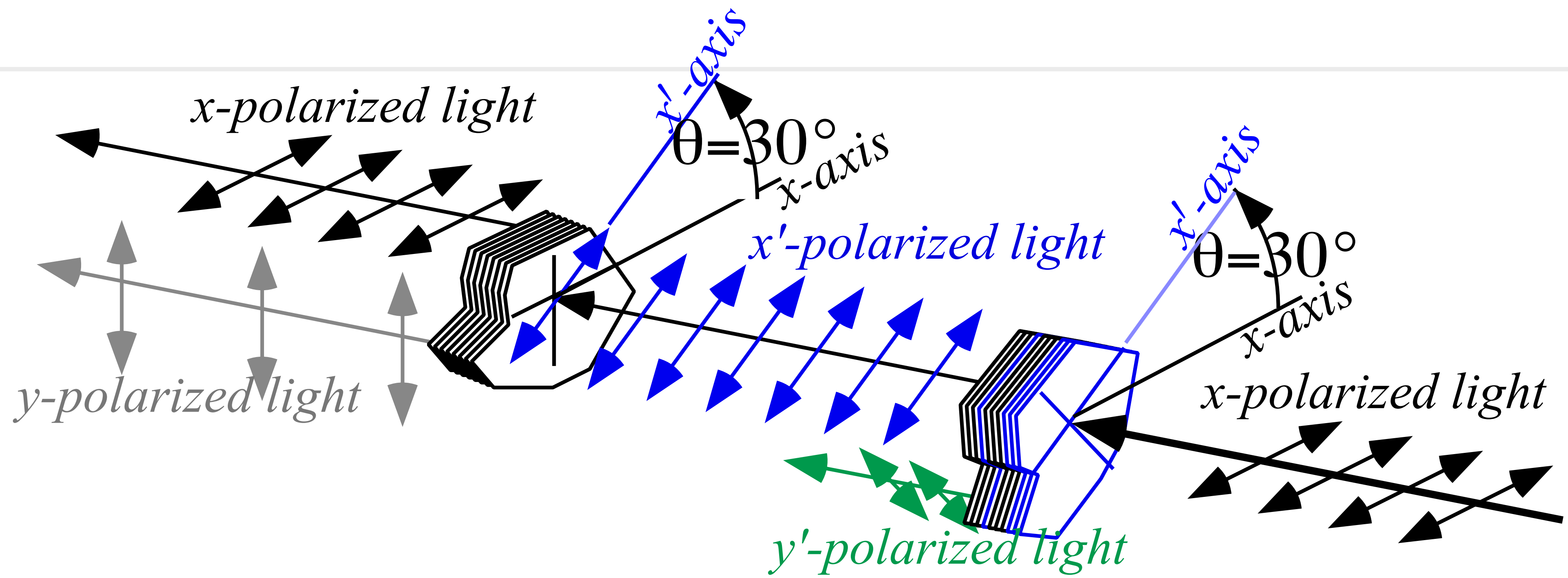

http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_Text_2013/QTCA_Unit_1_Ch._1_2013.pdf

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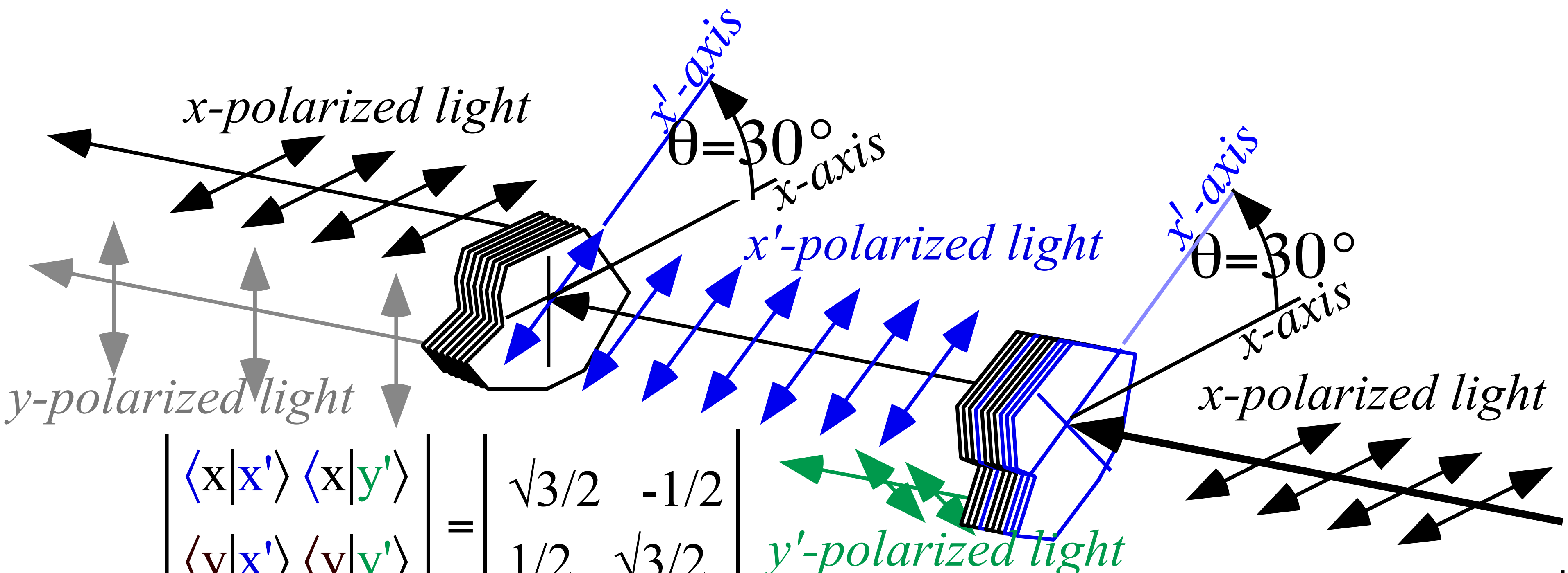

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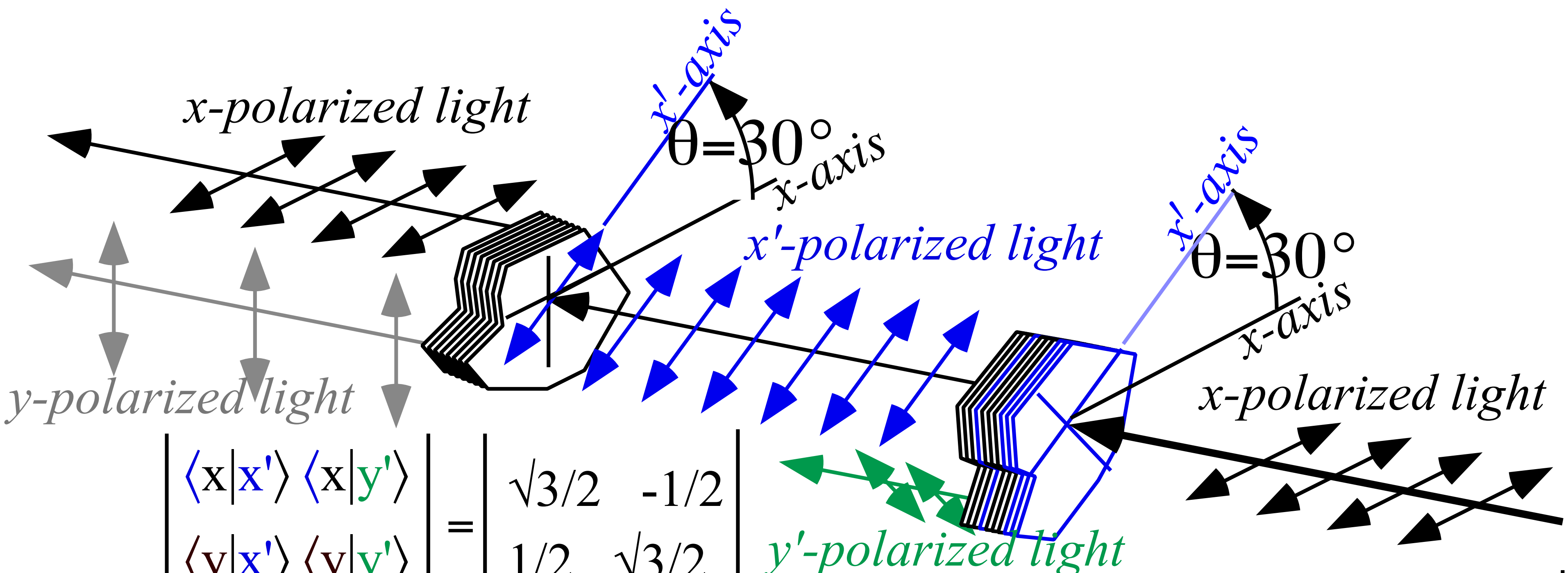

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Base State
$|x\rangle$

Amplitude=
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Probability=
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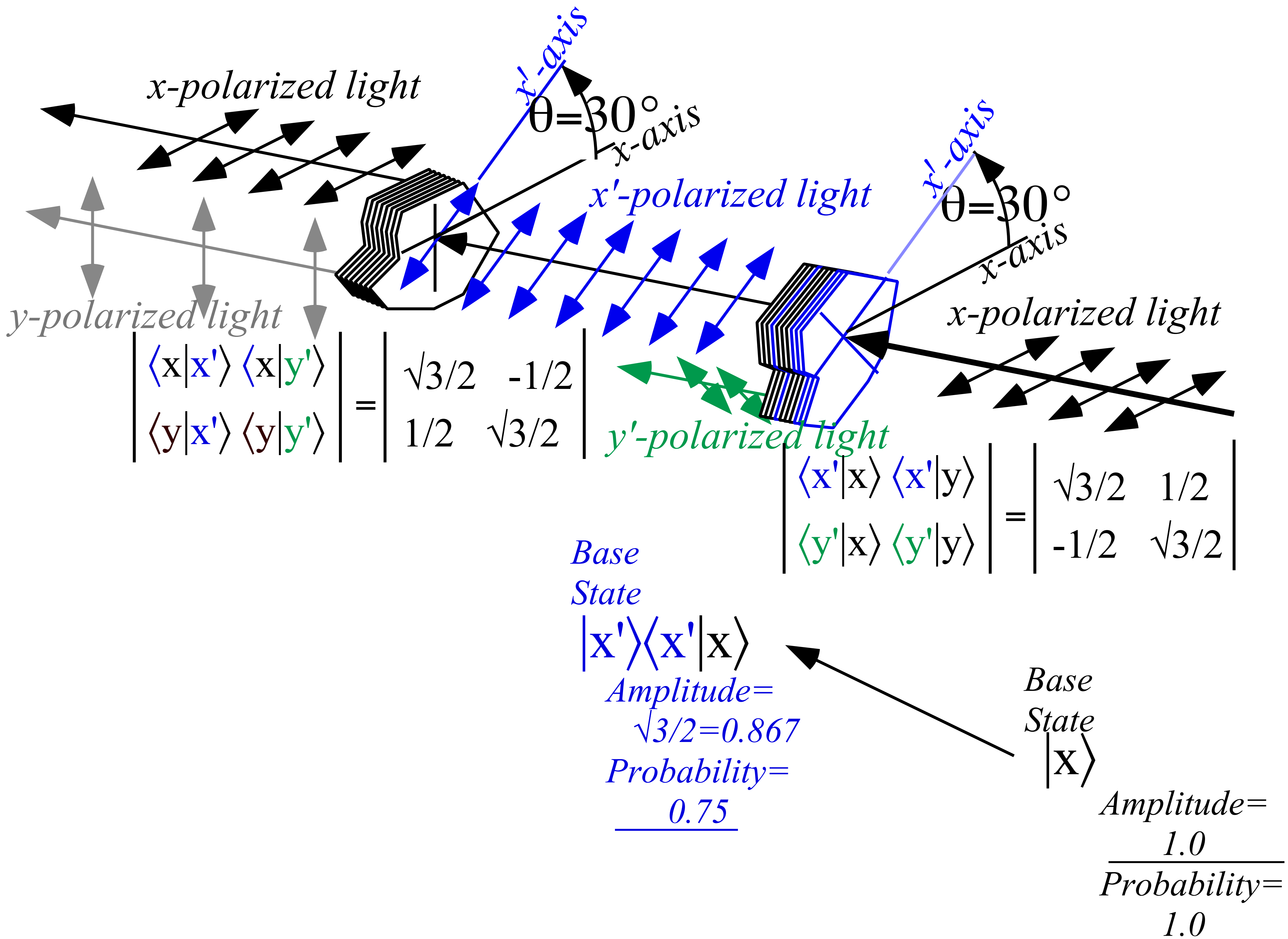

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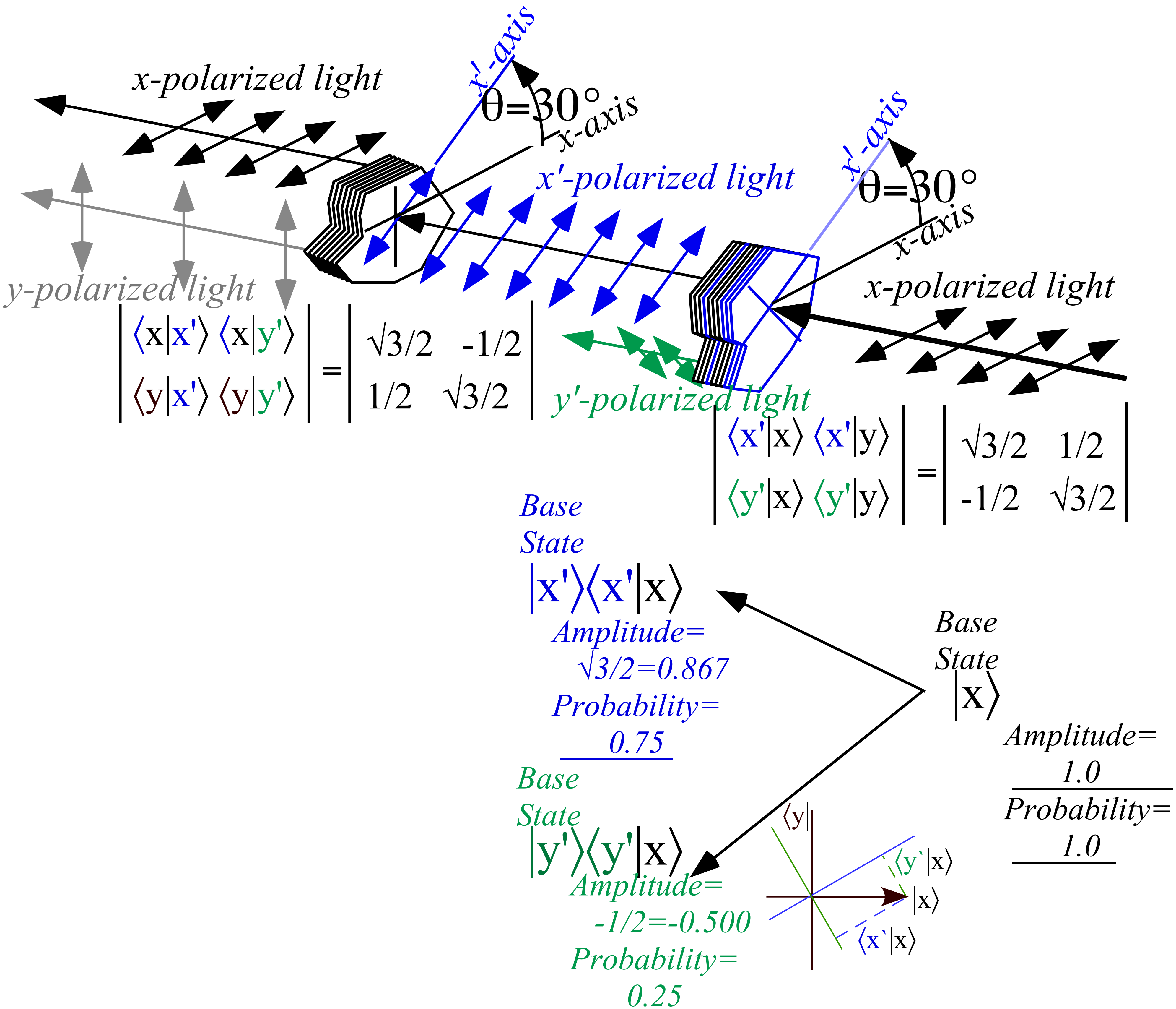

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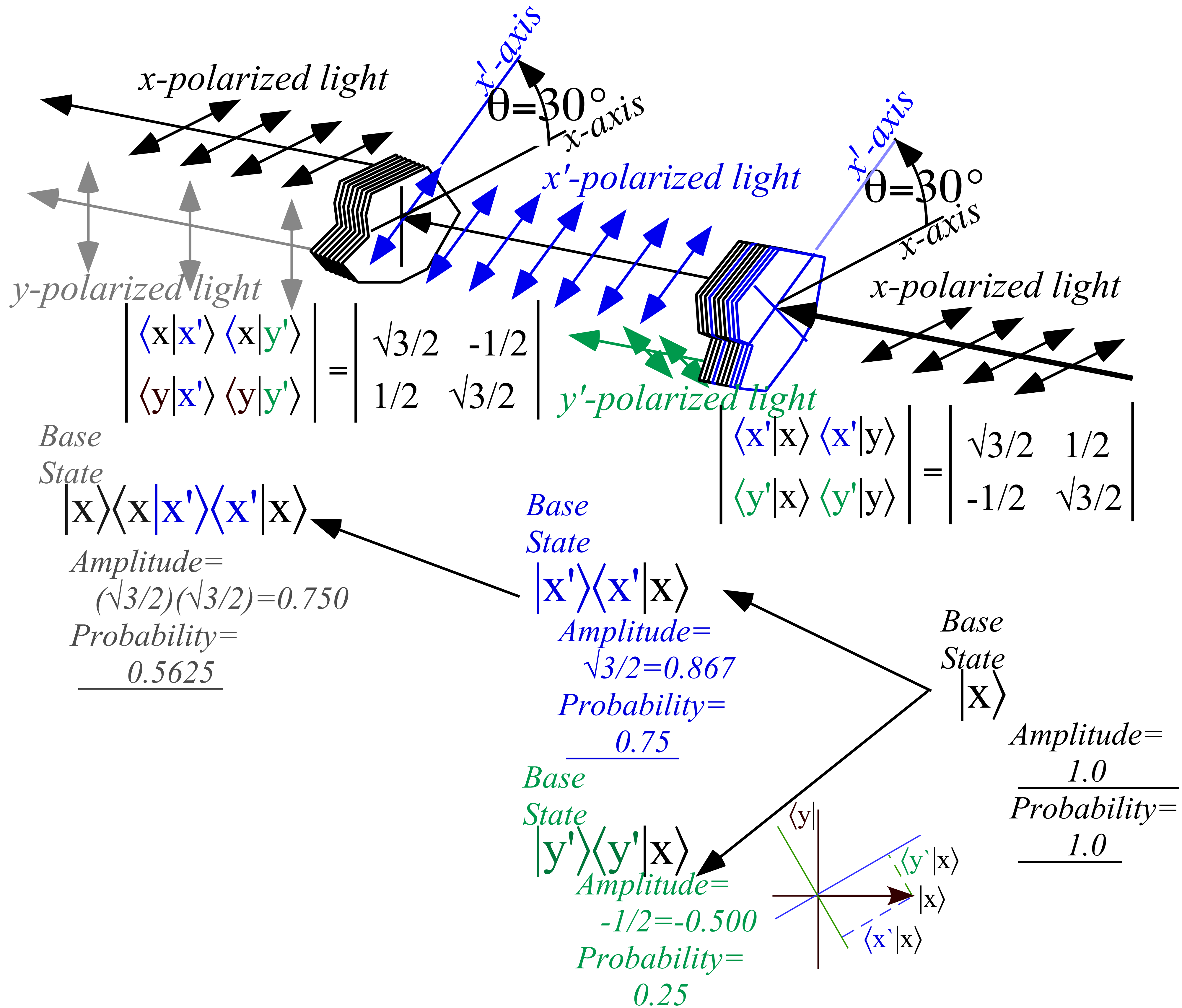

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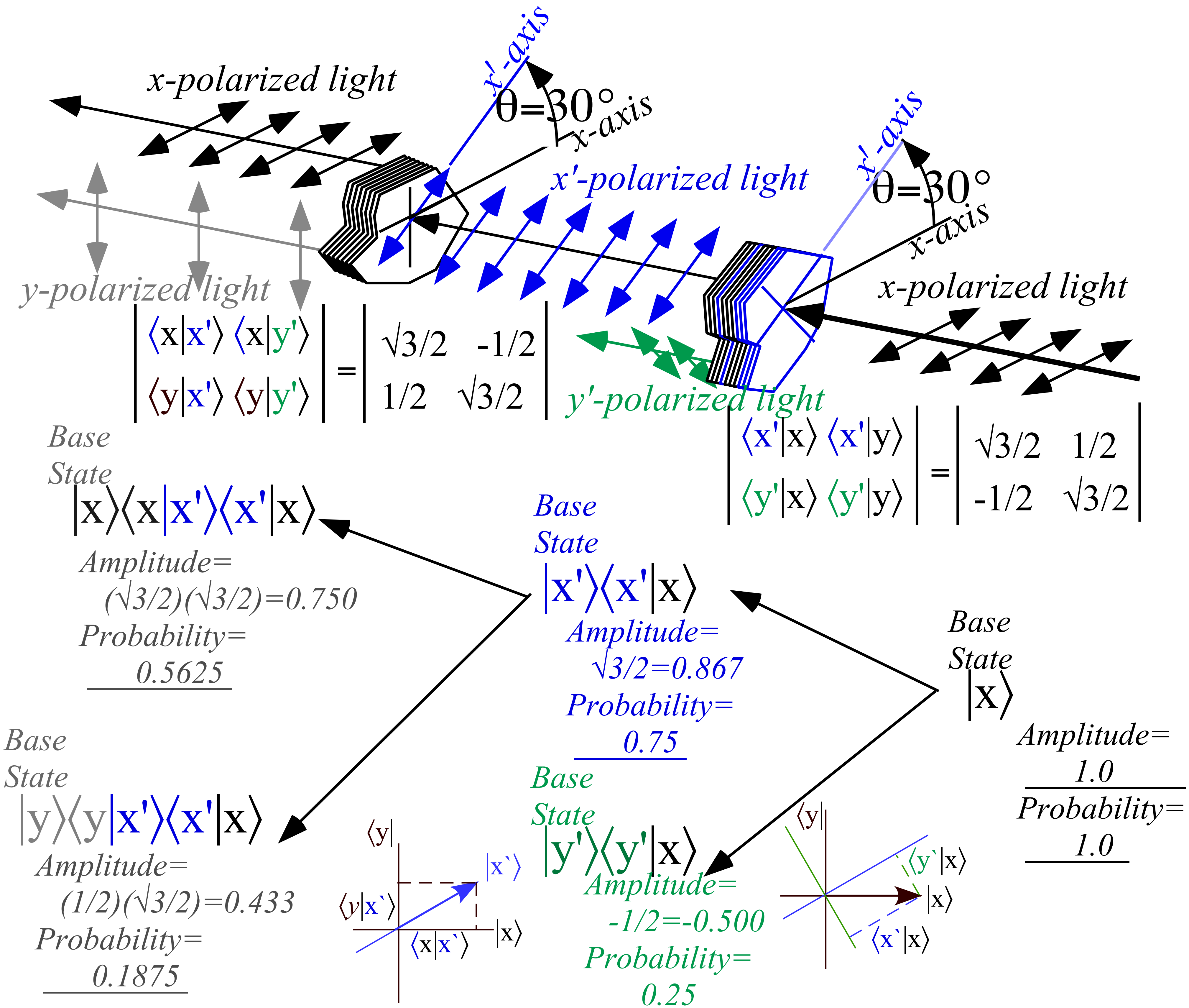

$$\left\langle e_p \middle| D_o C_n B_m \middle| \Psi \right\rangle = \left\langle e_p \middle| d_o \right\rangle \left\langle d_o \middle| c_n \right\rangle \left\langle c_n \middle| b_m \right\rangle \left\langle b_m \middle| \Psi \right\rangle$$

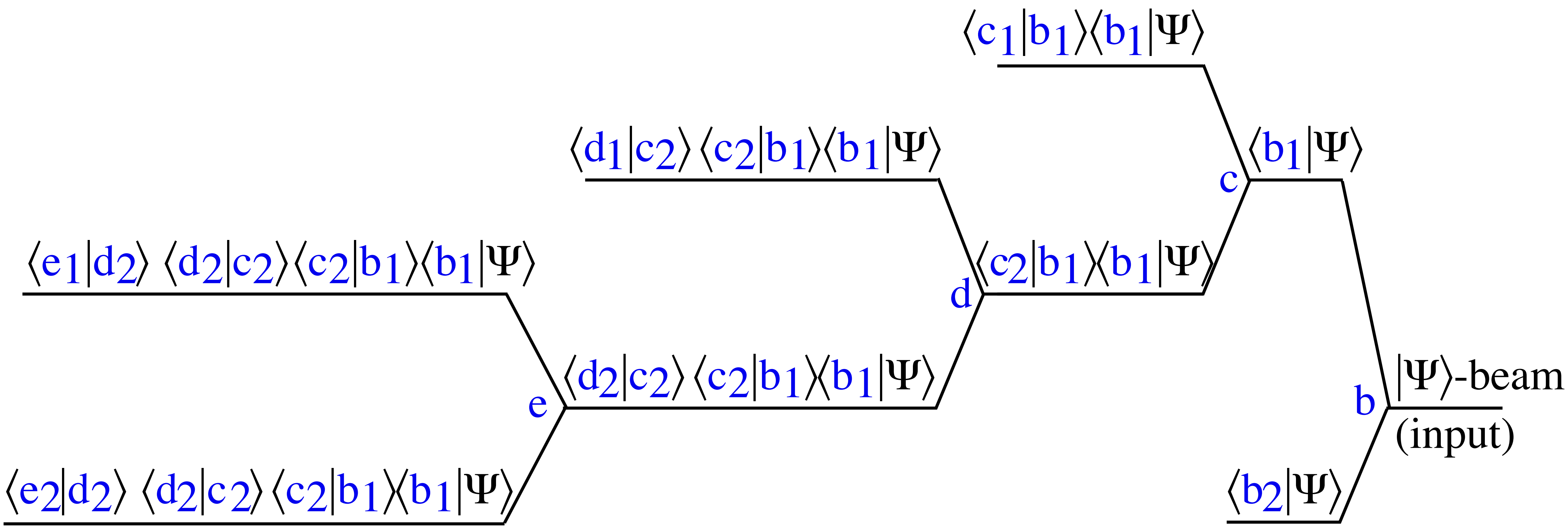

Fig. 1.3.10 Beams-amplitude products for successive beam sorting

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Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in state-j of a system that started in state-k′=1′,2′,..,or n′ from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

Feynman amplitude axioms 1-2

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

*=Amplitude of **state-j** after state-k′ is forced to choose from available **m-type** states*

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(2) The conjugation or inversion axiom *(time reversal symmetry)*

The second axiom concerns going backwards through a sorter or the reversal of amplitudes.

Axiom 2: The complex conjugate $\langle j | k' \rangle^*$ *of an amplitude* $\langle j | k' \rangle$ *equals its reverse:* $\langle j | k' \rangle^* = \langle k' | j \rangle$

Feynman amplitude axioms 1-3

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=Amplitude of state-j after state-k′ is forced to choose from available m-type states

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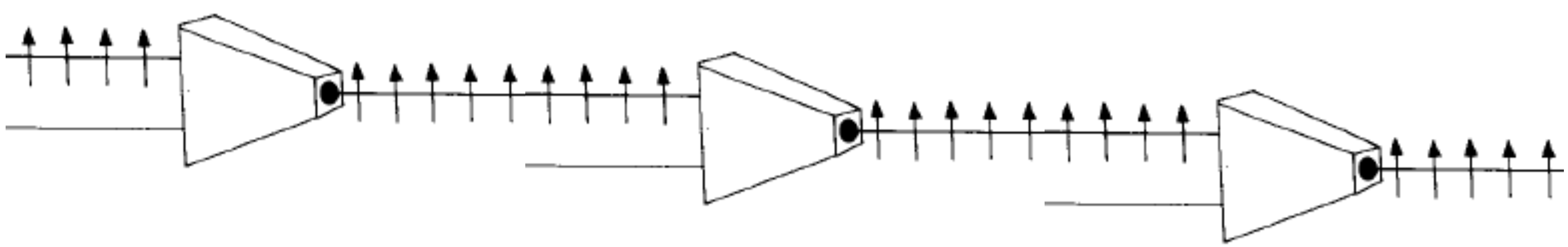


Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*
Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)
Did Max Planck Goof? What's 1-photon worth?
Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?
Sample calculations
Feynman amplitude axioms 2-3

→ *Beam analyzers: Sorter-unsorters*
The "Do-Nothing" analyzer
Feynman amplitude axiom 4
Some "Do-Something" analyzers
Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

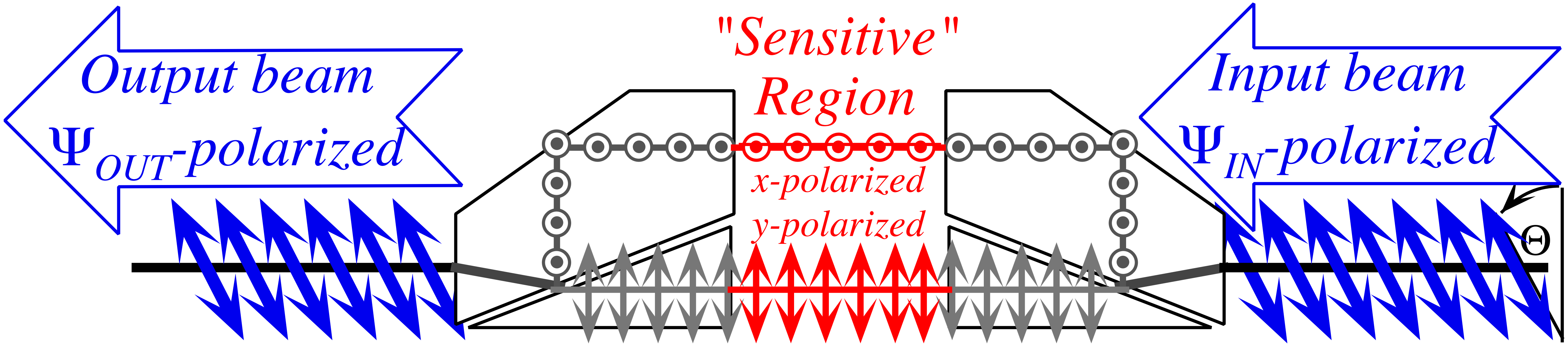
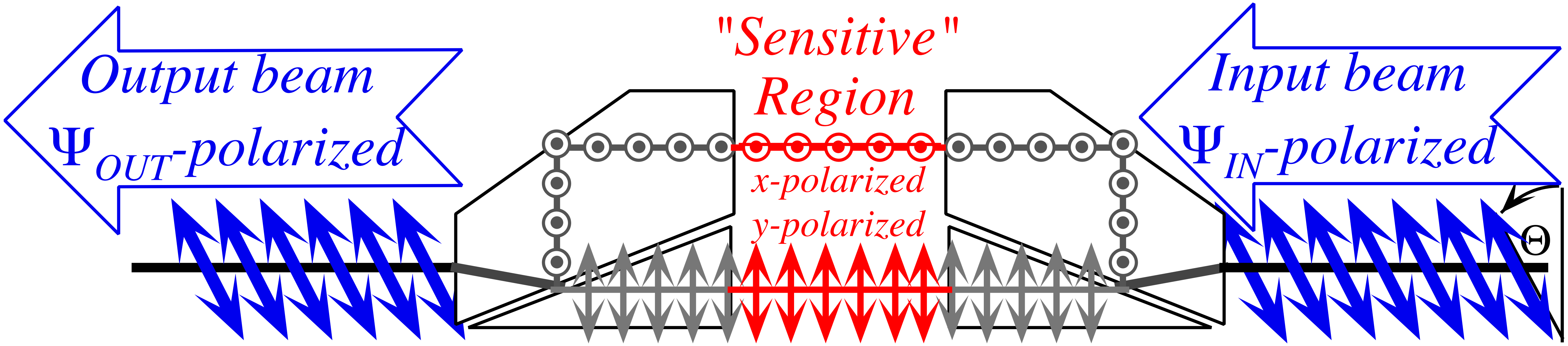


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)



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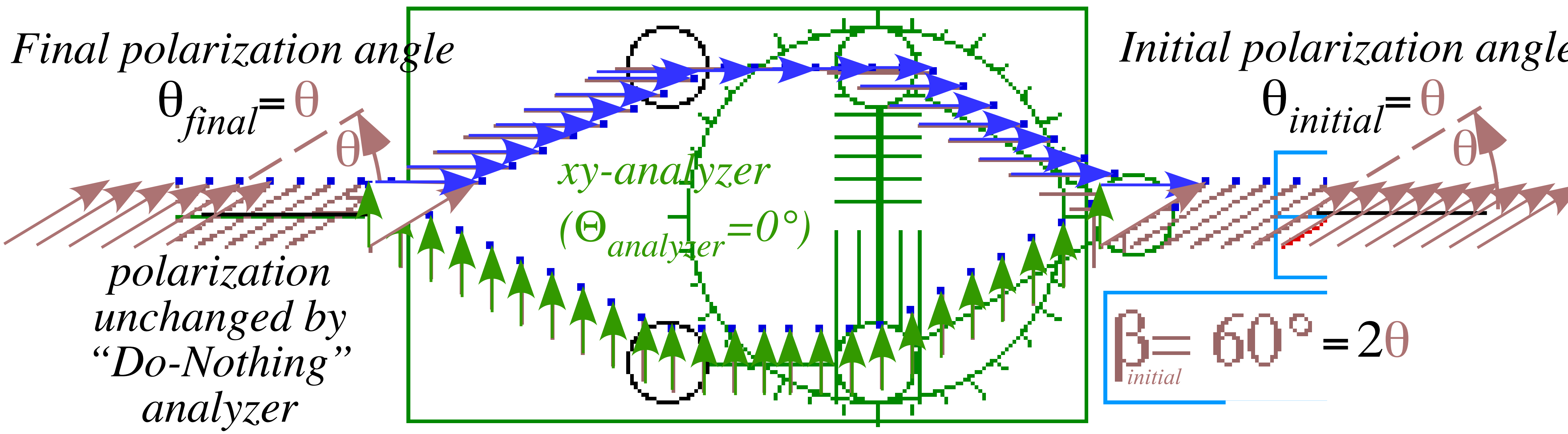

Fig. 1.3.2 Computer sketch of simulated polarization analyzer in "do-nothing" mode

Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*

Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)

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Some "Do-Something" analyzers

Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

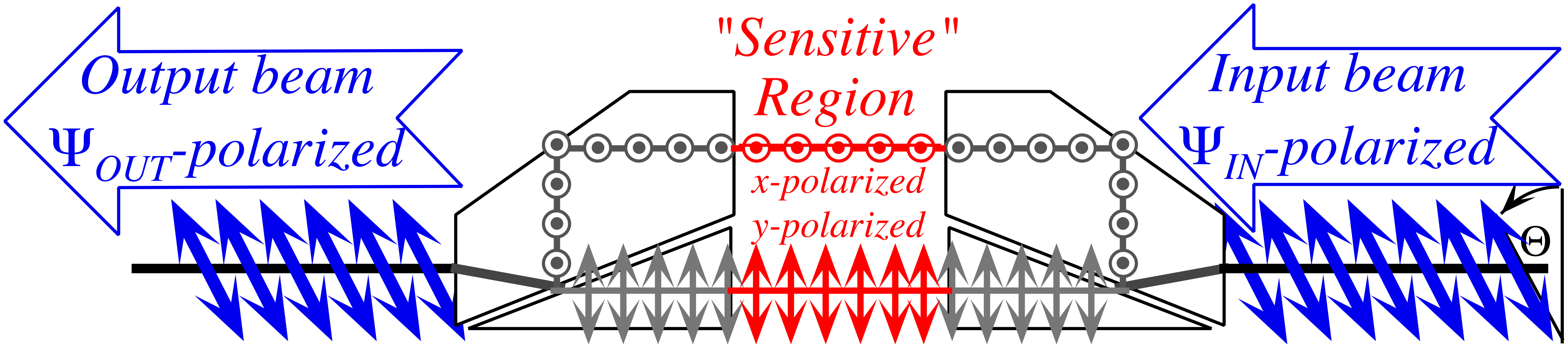


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

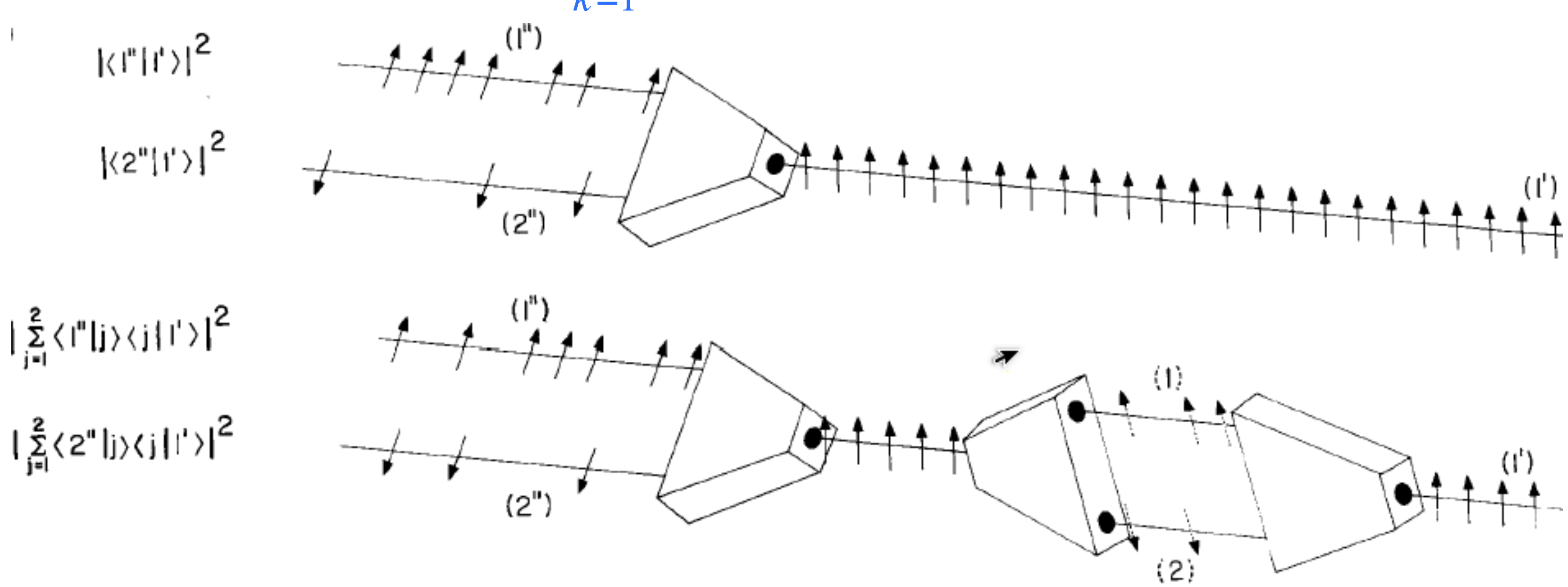
Feynman amplitude axiom 4

(4) The completeness or closure axiom

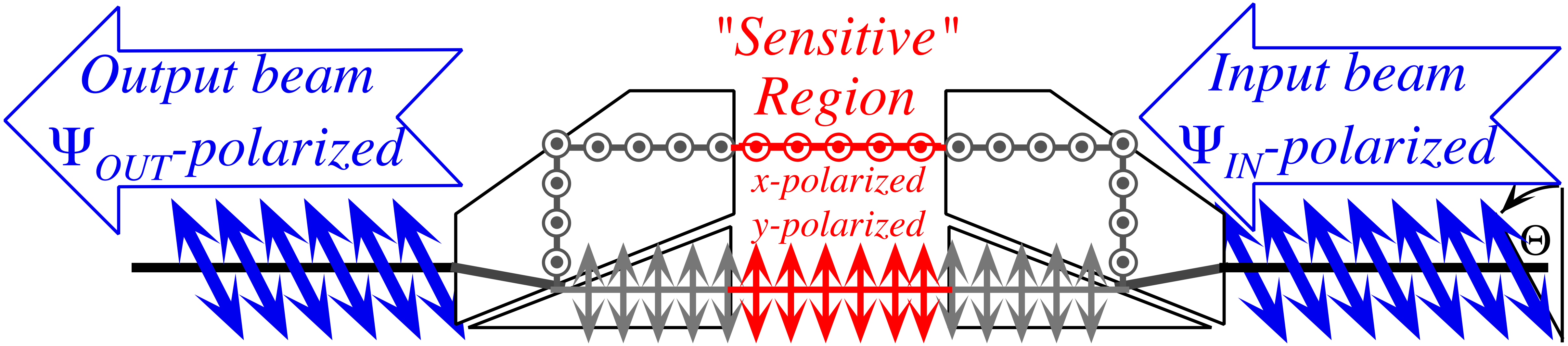
The fourth axiom concerns the "*Do-nothing*" property of an ideal analyzer, that is, a sorter followed by an "unsorter" or "put-back-togetherer" as sketched above.

Axiom 4. Ideal sorting followed by ideal recombination of amplitudes has no effect:

$$\langle j'' | m' \rangle = \sum_{k=1}^{n} \langle j'' | k \rangle \langle k | m' \rangle$$



Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)



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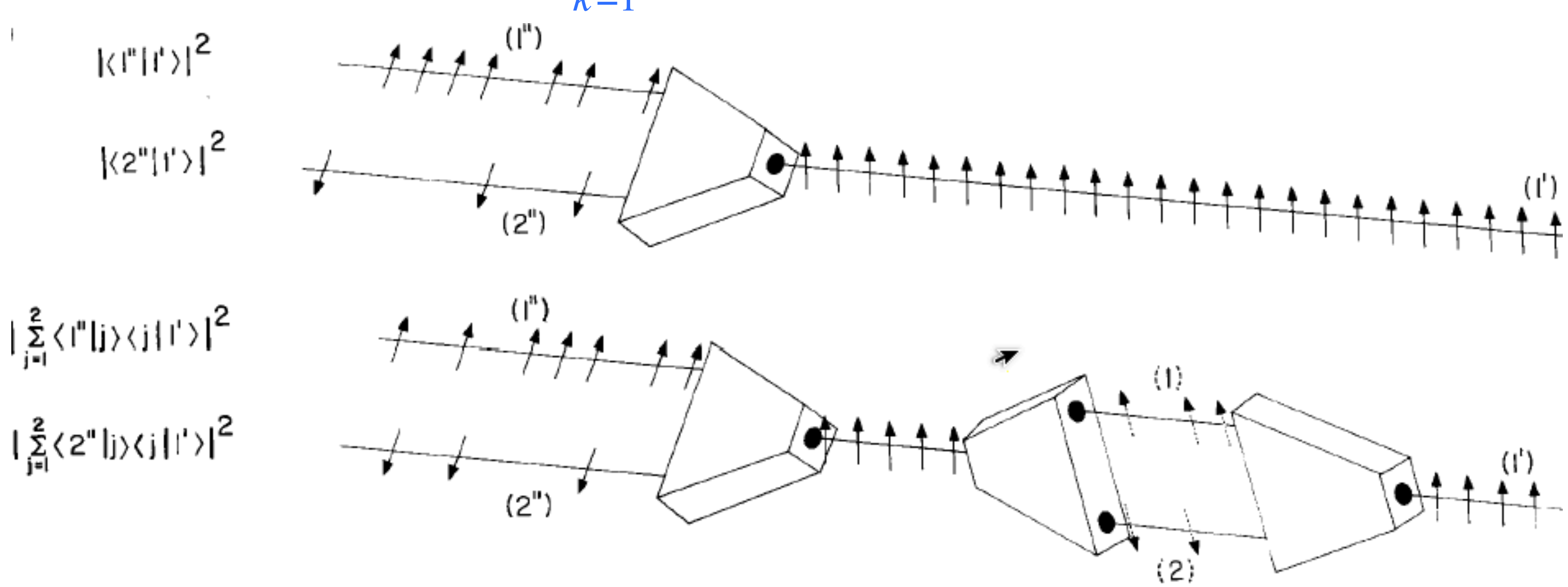

May use axioms 1-3 to prove special case:

$$1 = \langle m'|m'\rangle \quad \text{by 3}$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} \left|\langle k|m'\rangle\right|^2$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} \langle k|m'\rangle^* \langle k|m'\rangle$$

$$\Updownarrow_{\text{by 2}}$$

$$\sum_{k=1}^{n} \langle m'|k\rangle\langle k|m'\rangle$$

$$= \langle m'|m'\rangle$$

Feynman amplitude axioms 1-4

(Go to Lect. 3 p.3-5)

Feynman-Dirac Interpretation of

$\langle j | k' \rangle$

=Amplitude of state-j after state-k′ is forced to choose from available m-type states

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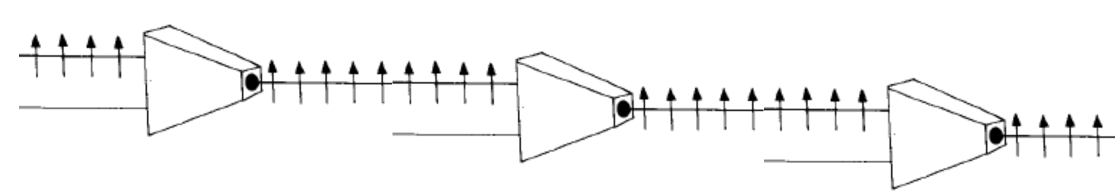

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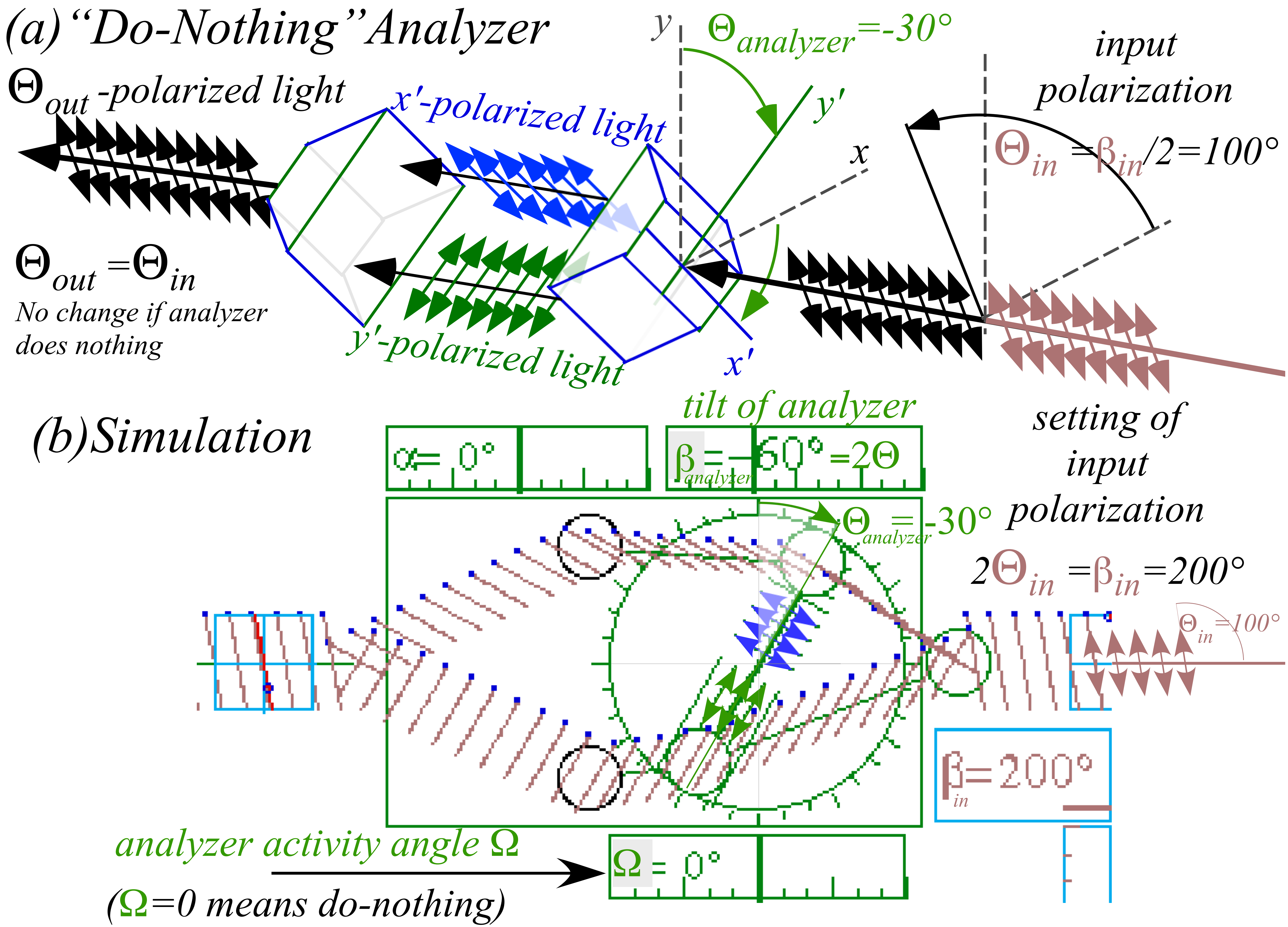
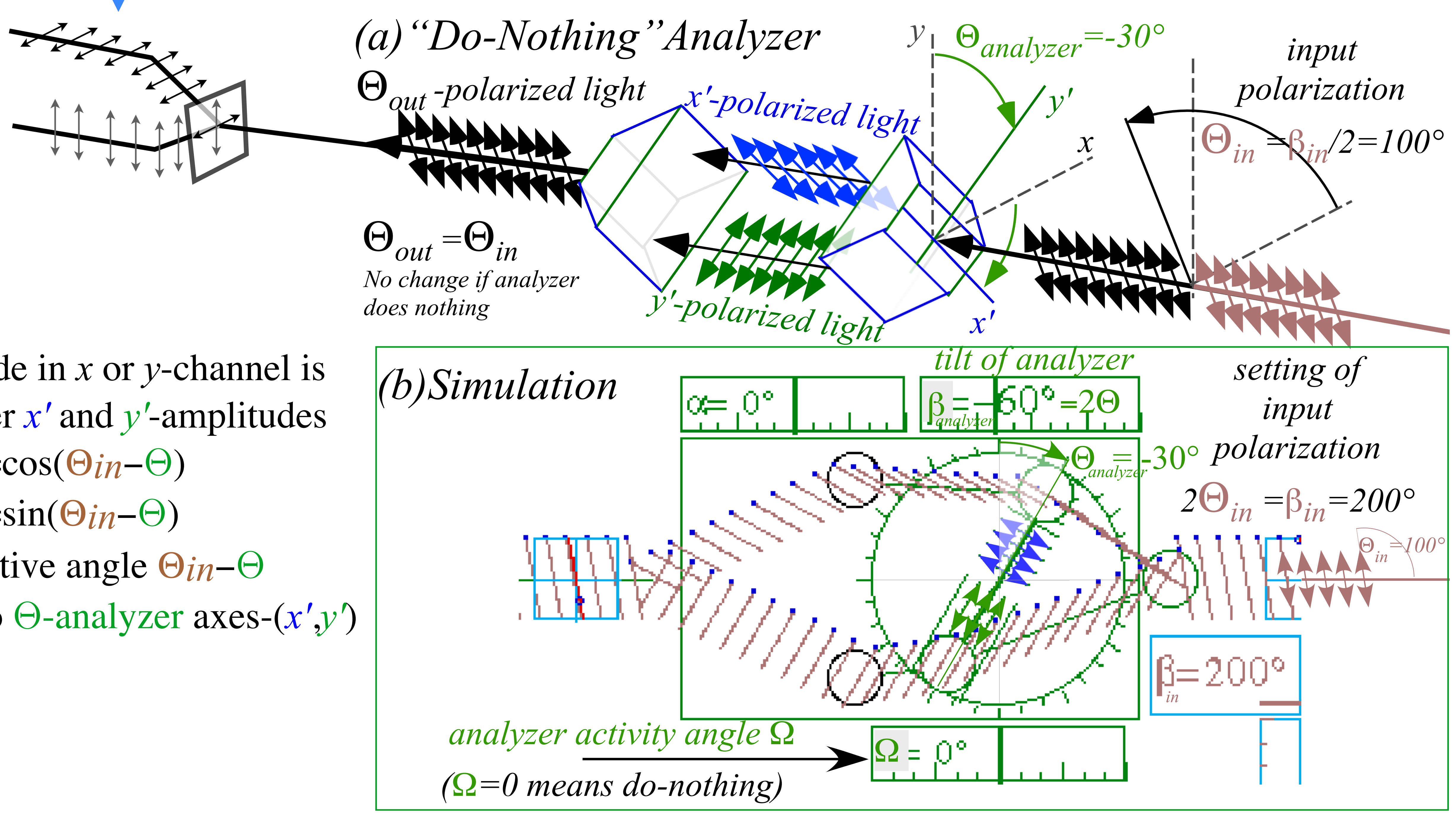


(a) "Do-Nothing" Analyzer
Θ_out -polarized light
x'-polarized light
y
Θ_analyzer=-30°
y'
x
input polarization
Θ_in =β_in/2=100°
Θ_out =Θ_in
No change if analyzer does nothing
y'-polarized light
x'
(b) Simulation
tilt of analyzer
α= 0°
β_analyzer=-60°=2Θ
Θ_analyzer=-30°
setting of input polarization
2Θ_in =β_in=200°
Θ_in=100°
β_in=200°
analyzer activity angle Ω
Ω= 0°
(Ω=0 means do-nothing)

Imagine final xy-sorter analyzes output beam into x and y-components.



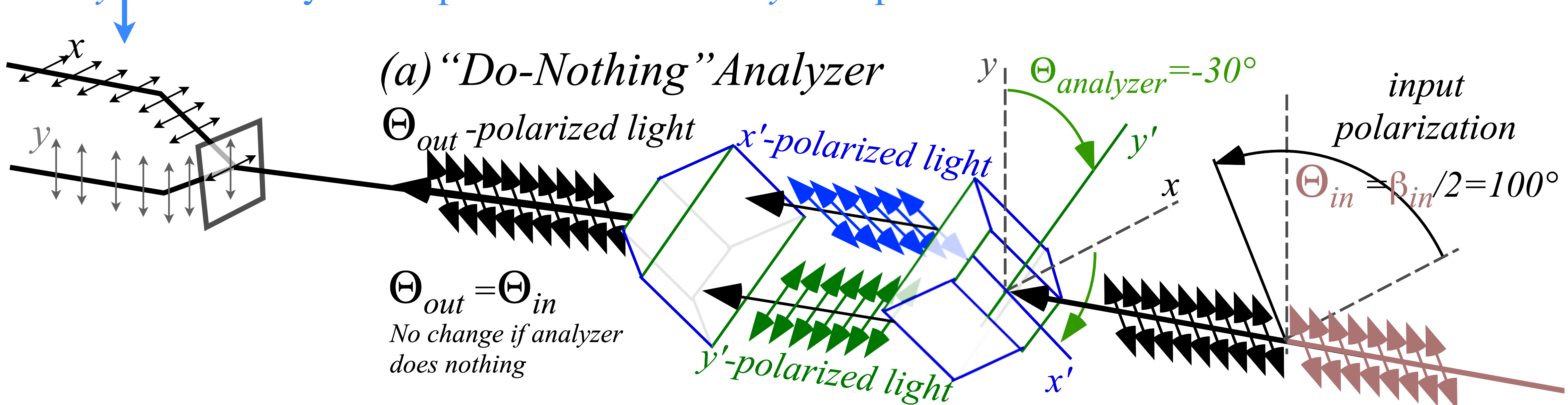
Amplitude in x or y-channel is sum over x' and y'-amplitudes

$\langle x'|\Theta in\rangle = \cos(\Theta in - \Theta)$

$\langle y'|\Theta in\rangle = \sin(\Theta in - \Theta)$

with relative angle $\Theta in - \Theta$ of Θin to Θ-analyzer axes-(x',y')

Imagine final xy-sorter analyzes output beam into x and y-components.



Amplitude in x or y-channel is sum over x' and y'-amplitudes

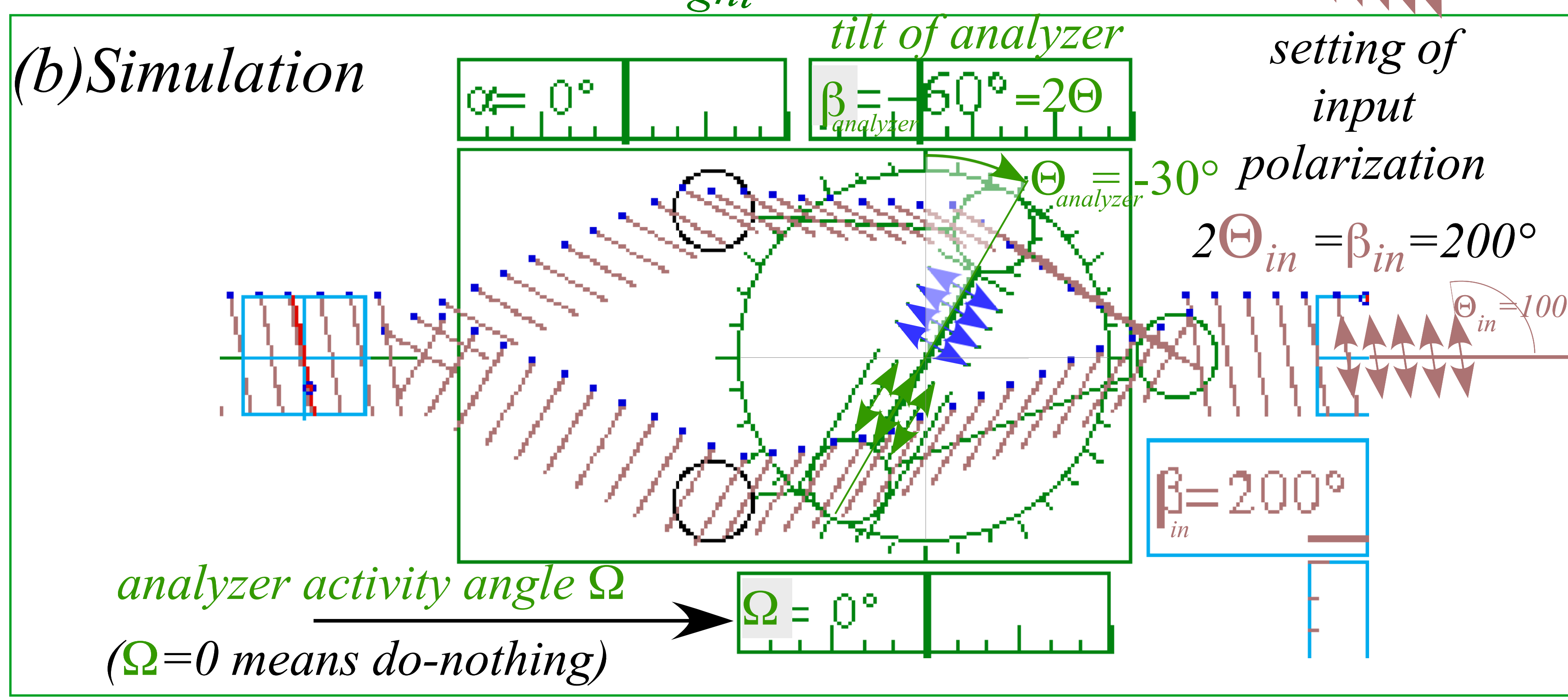
$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$

$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$

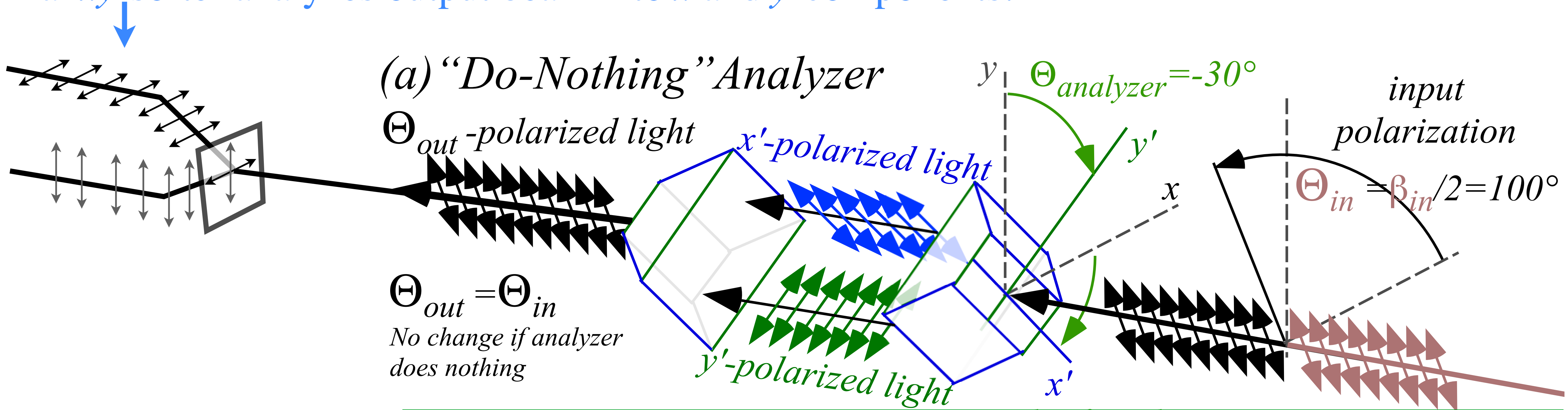
with relative angle $\Theta_{in}-\Theta$ of Θ_{in} to Θ-analyzer axes-(x',y') in products with final xy-sorter:

lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$

y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.



Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



Amplitude in *x* or *y*-channel is sum over *x′* and *y′*-amplitudes

$\langle x'|\Theta_{in}\rangle = \cos(\Theta_{in} - \Theta)$

$\langle y'|\Theta_{in}\rangle = \sin(\Theta_{in} - \Theta)$

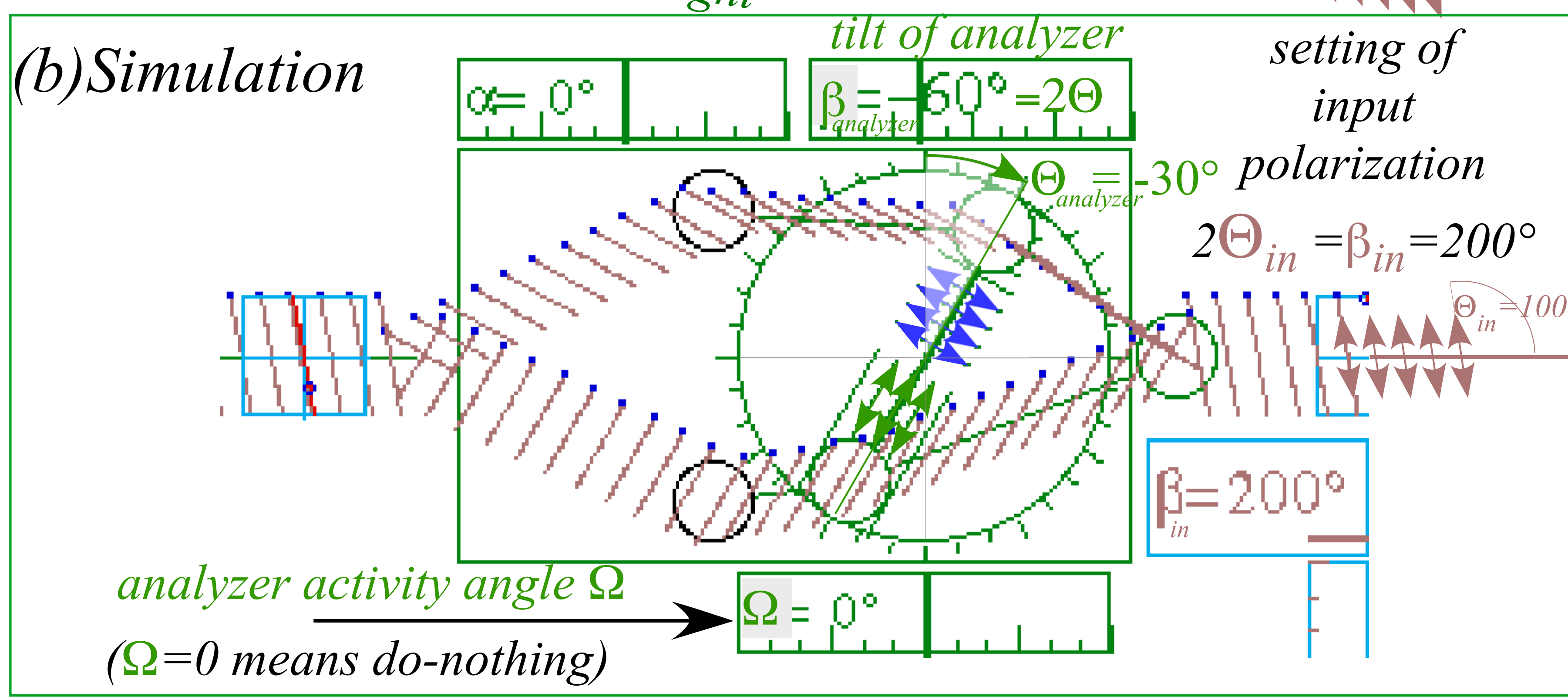
with relative angle $\Theta_{in} - \Theta$

of Θ_{in} to Θ-analyzer axes-(x', y')

in products with final *xy*-sorter:

lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$

y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.

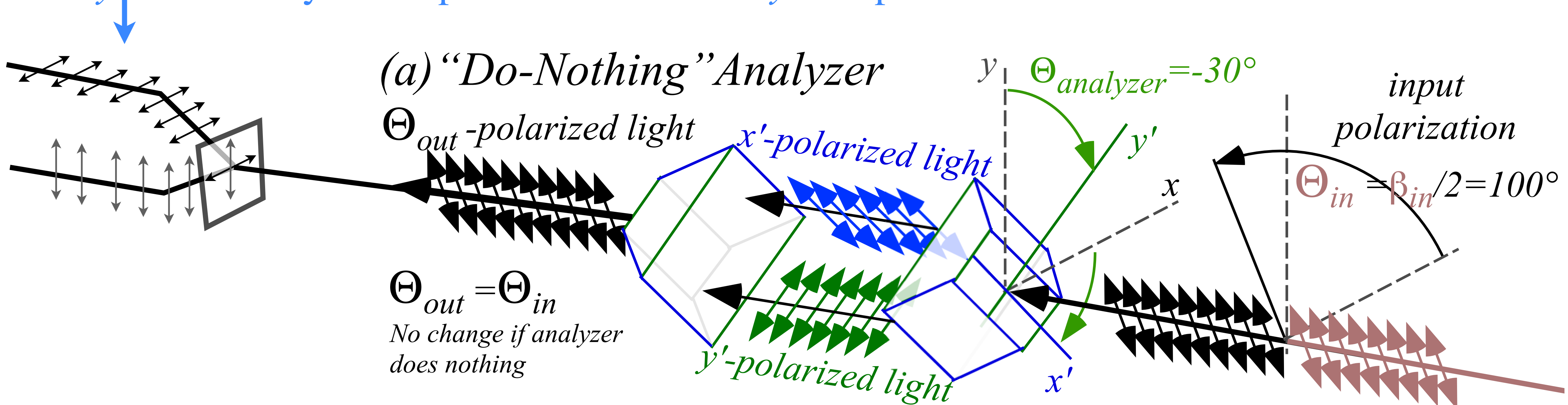


x-Output is: $\langle x|\Theta_{out}\rangle = \langle x|x'\rangle\langle x'|\Theta_{in}\rangle + \langle x|y'\rangle\langle y'|\Theta_{in}\rangle = \cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta) = \cos\Theta_{in}$

y-Output is: $\langle y|\Theta_{out}\rangle = \langle y|x'\rangle\langle x'|\Theta_{in}\rangle + \langle y|y'\rangle\langle y'|\Theta_{in}\rangle = \sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta) = \sin\Theta_{in}$.

(Recall $\cos(a+b) = \cos a \cos b - \sin a \sin b$ and $\sin(a+b) = \sin a \cos b + \cos a \sin b$)

Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



Amplitude in *x* or *y*-channel is sum over x' and y'-amplitudes

$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$

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with relative angle $\Theta_{in}-\Theta$ of Θ_{in} to Θ-analyzer axes-(x',y') in products with final *xy*-sorter:

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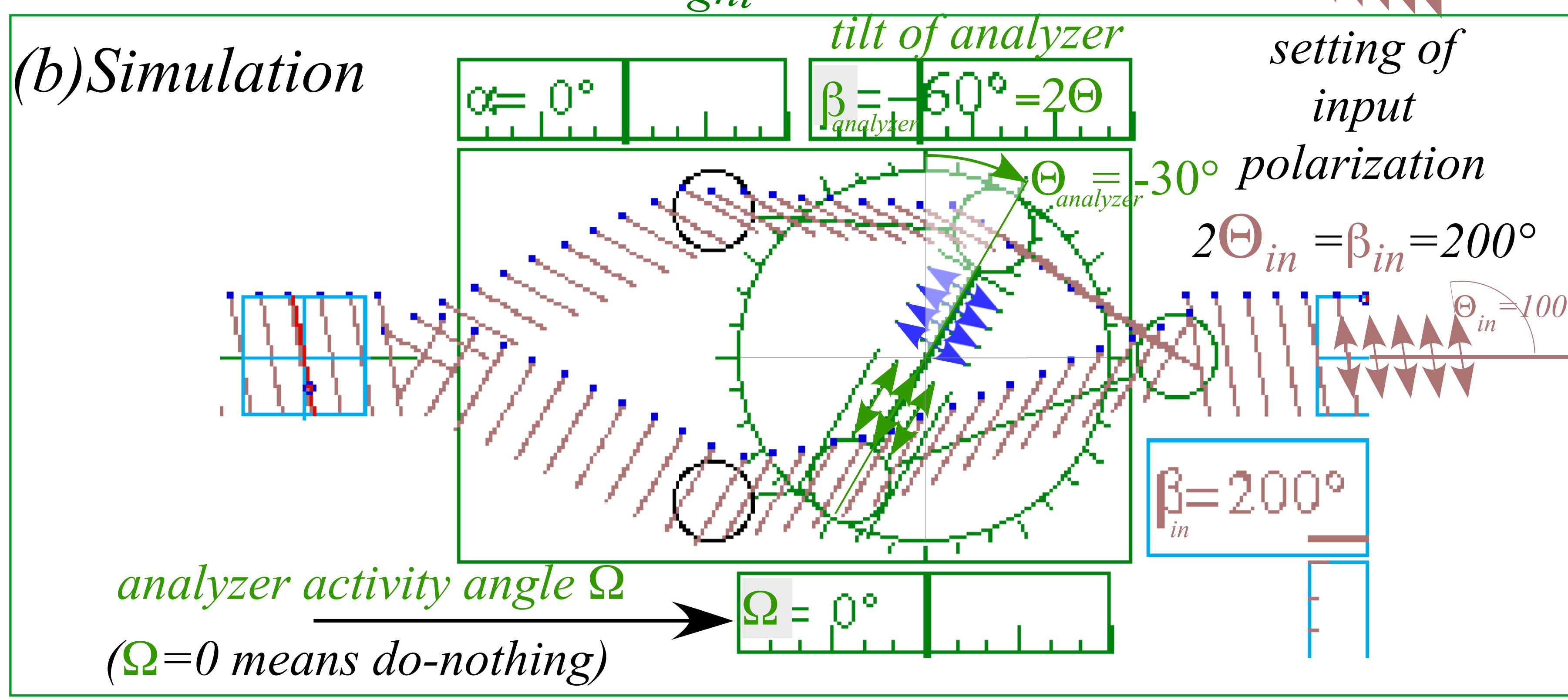

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y-Output is: $\langle y|\Theta_{out}\rangle= \langle y|x'\rangle\langle x'|\Theta_{in}\rangle+\langle y|y'\rangle\langle y'|\Theta_{in}\rangle=\sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta)=\sin\Theta_{in}$.

(Recall $\cos(a+b)=\cos a\cos b-\sin a\sin b$ and $\sin(a+b)=\sin a\cos b+\cos a\sin b$)

Conclusion:

$\langle x|\Theta_{out}\rangle = \cos\Theta_{out} = \cos\Theta_{in}$ or: $\Theta_{out}= \Theta_{in}$ so "Do-Nothing" Analyzer in fact does nothing.

Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*

Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)

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Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters

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Feynman amplitude axiom 4

Some "Do-Something" analyzers

Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

(1) Optical analyzer in sorter-counter configuration

Analyzer reduced to a simple sorter-counter by blocking output of x-high-road and y-low-road with counters

(Go to Lect. 3 p.62-68)

$x\text{-}counts \sim |\langle x|x'\rangle|^2$

$= \cos^2\theta = 0.75$

$y\text{-}counts \sim |\langle y|x'\rangle|^2$

$= \sin^2\theta = 0.25$

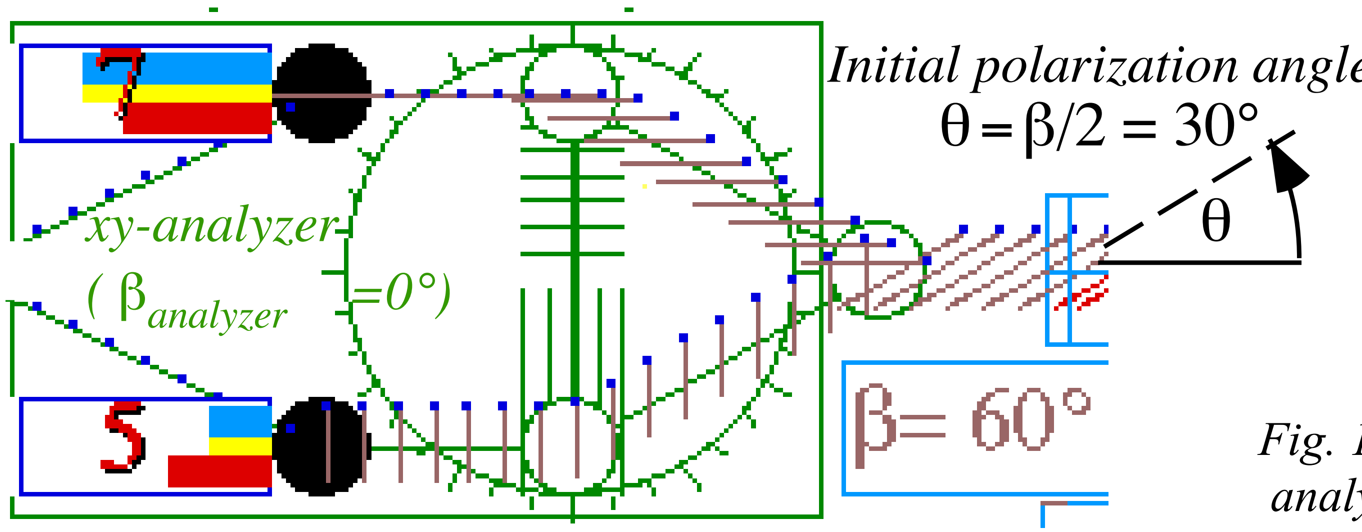

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(1) Optical analyzer in sorter-counter configuration

Analyzer reduced to a simple sorter-counter by blocking output of x-high-road and y-low-road with counters

$$x\text{-}counts \sim \left|\langle x|x'\rangle\right|^2$$

$$= \cos^2\theta = 0.75$$

$$y\text{-}counts \sim \left|\langle y|x'\rangle\right|^2$$

$$= \sin^2\theta = 0.25$$

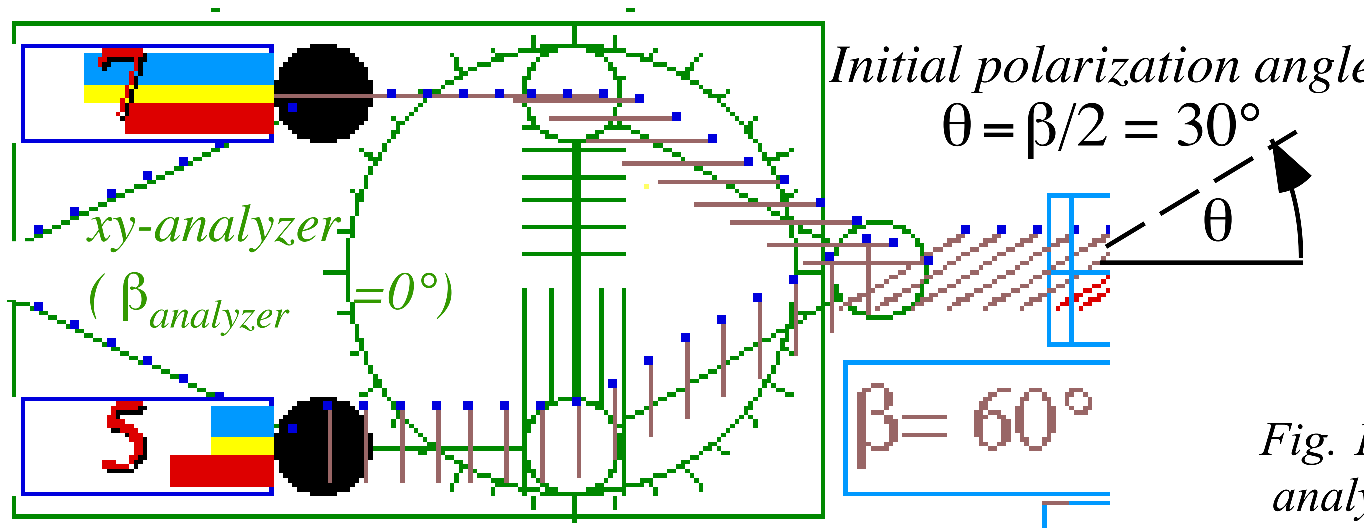

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(2) Optical analyzer in a filter configuration (Polaroid© sunglasses)

Analyzer blocks one path which may have photon counter without affecting function.

$$x\text{-}counts \sim \left|\langle y|x'\rangle\right|^2 = 0.75$$

(Blocked and filtered out)

$$y\text{-}output \sim \left|\langle y|x'\rangle\right|^2$$

$$= \sin^2\theta = 0.25$$

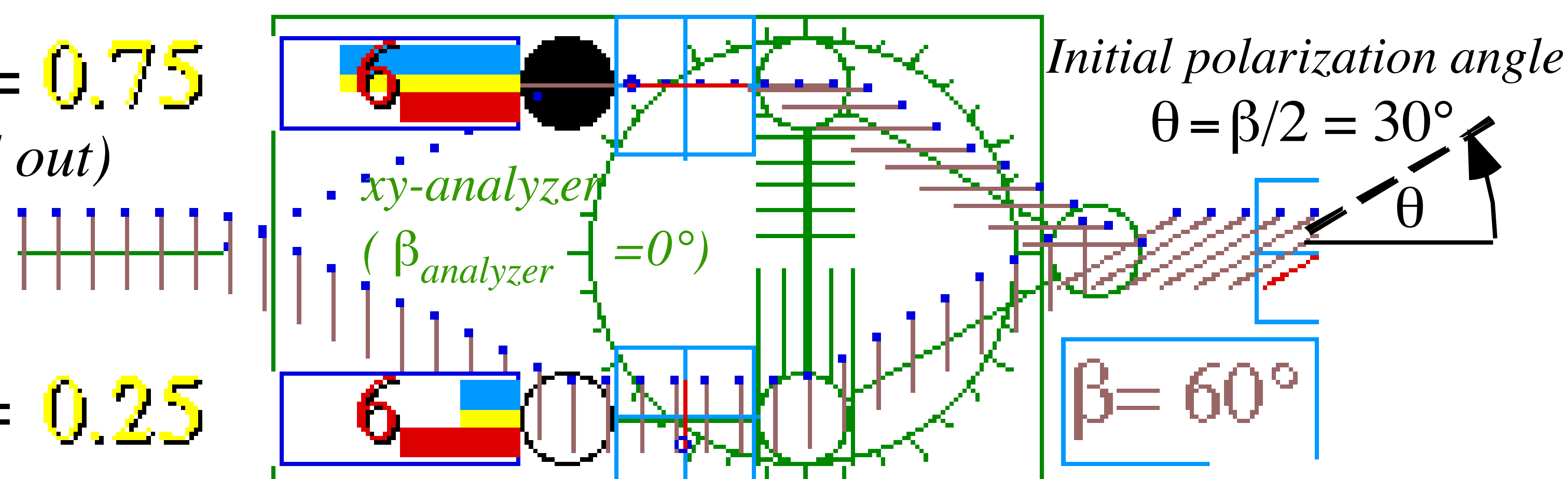

Fig. 1.3.4 Simulated polarization analyzer set up to filter out the x-polarized photons

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix

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Some "Do-Something" analyzers

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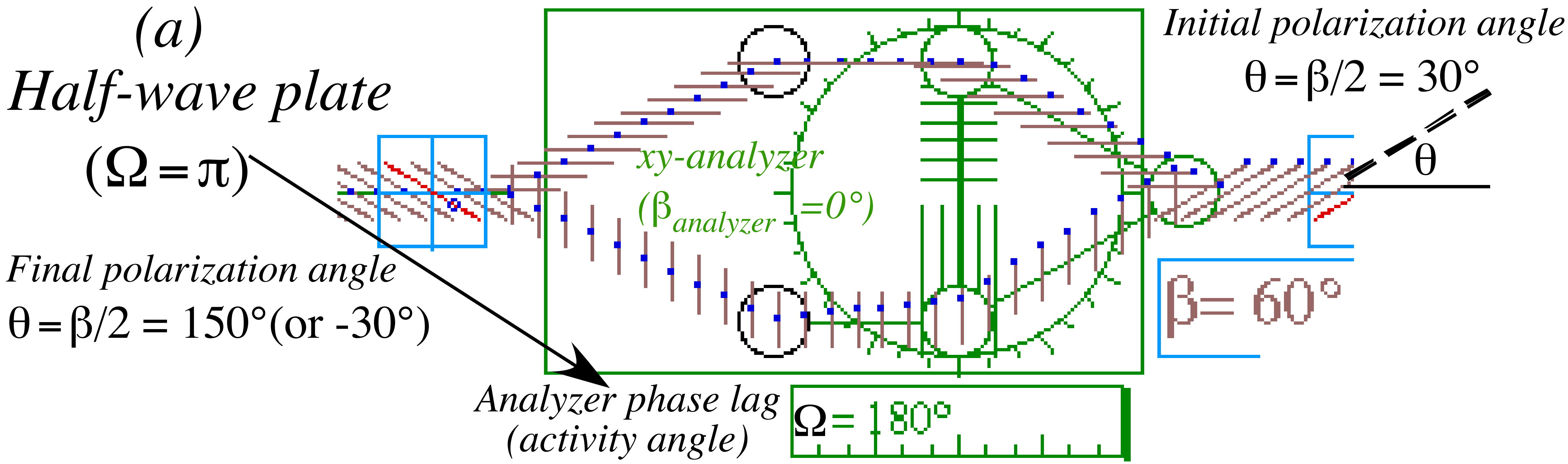
(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates



(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

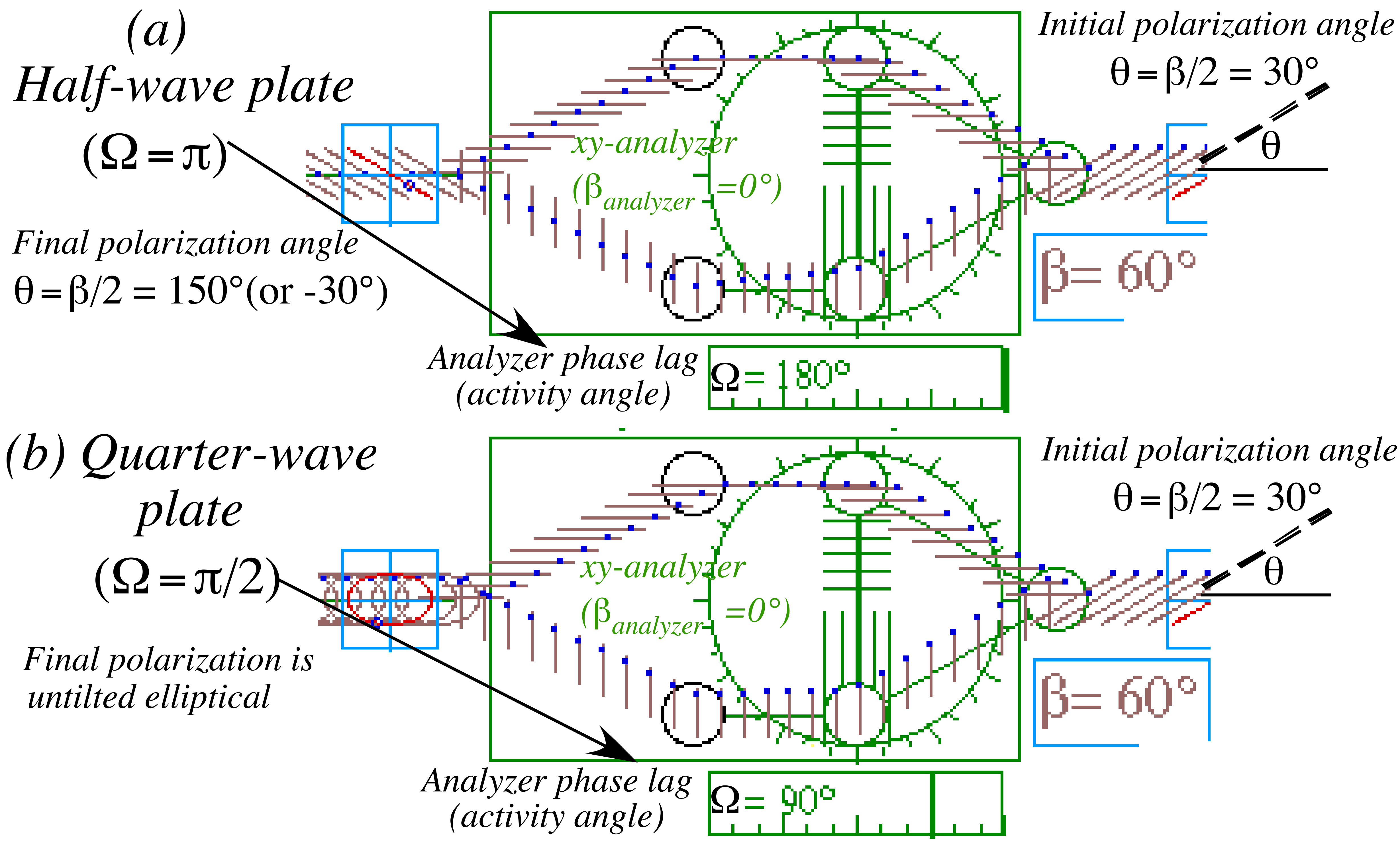
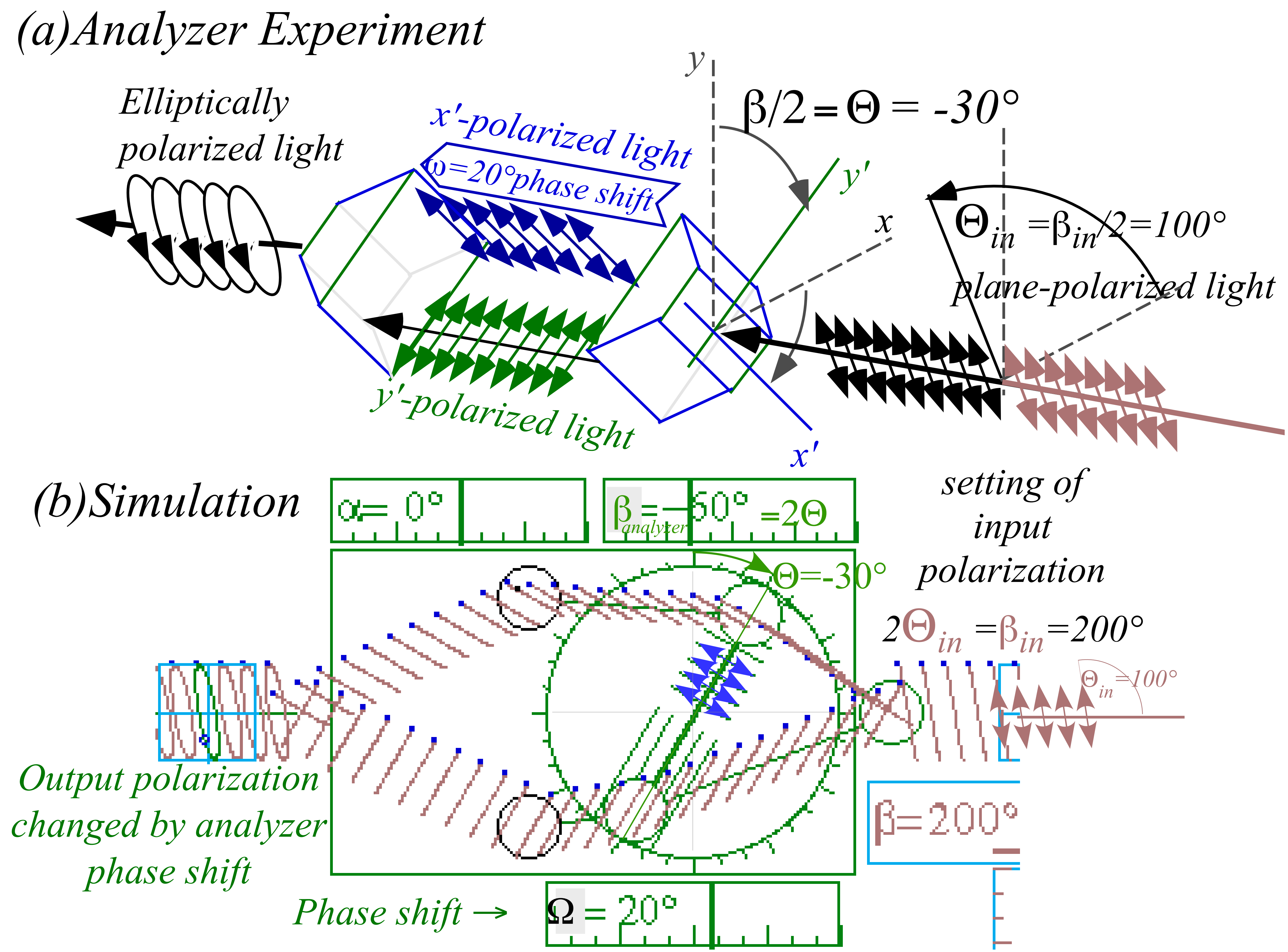


Fig. 1.3.5 Polarization control set to shift phase by (a) Half-wave ($\Omega = \pi$) , (b) Quarter wave ($\Omega = \pi/2$)



Similar to "do-nothing" analyzer but has extra phase factor $e^{-i\Omega} = 0.94-i\ 0.34$ on the x'-path .

x-output:
$$\langle x|\Psi_{out}\rangle = \langle x|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle x|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta)$$

y-output:
$$\langle y|\Psi_{out}\rangle = \langle y|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle y|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\sin\Theta\cos(\Theta_{in}-\Theta) + \cos\Theta\sin(\Theta_{in}-\Theta)$$

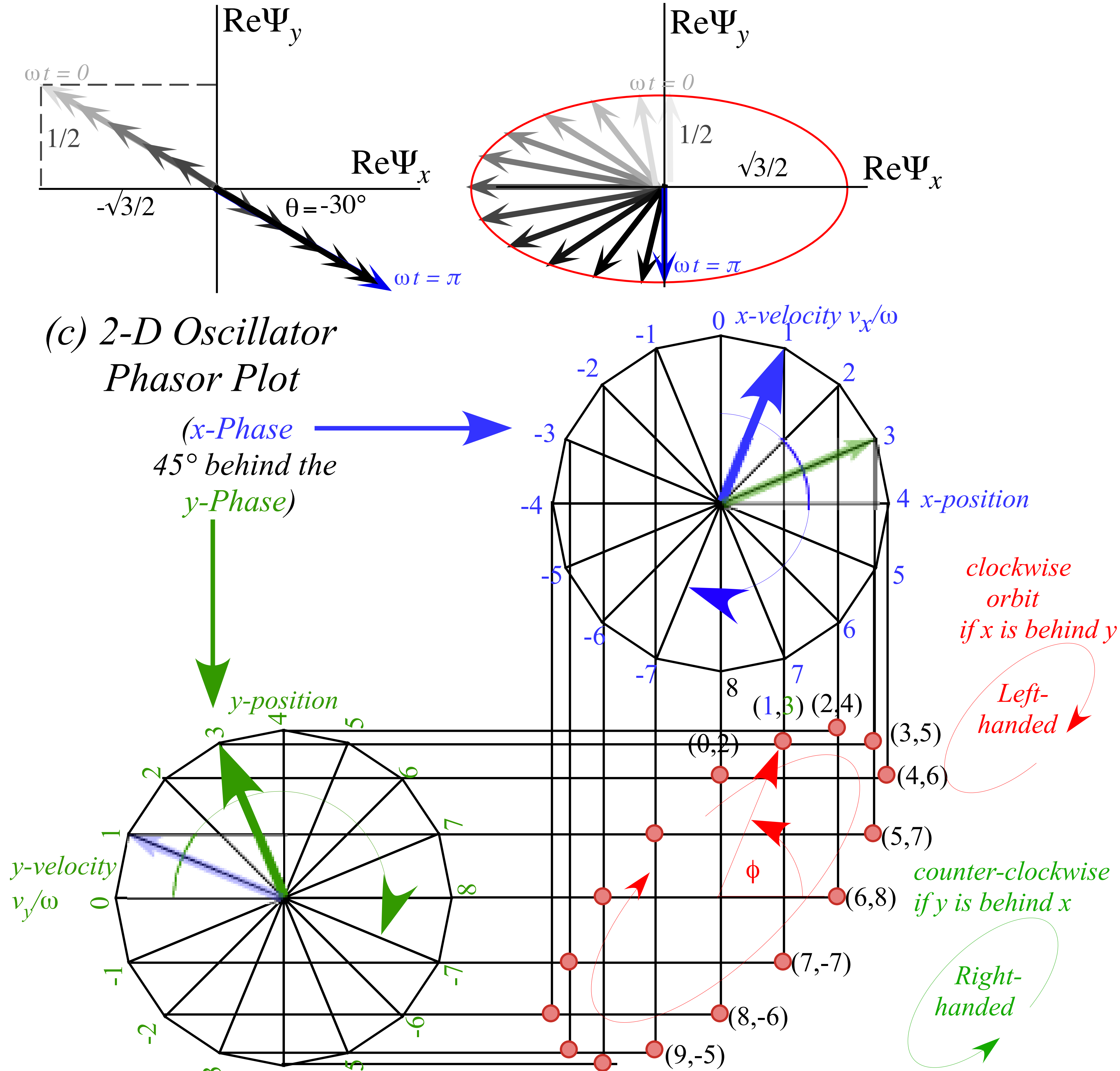


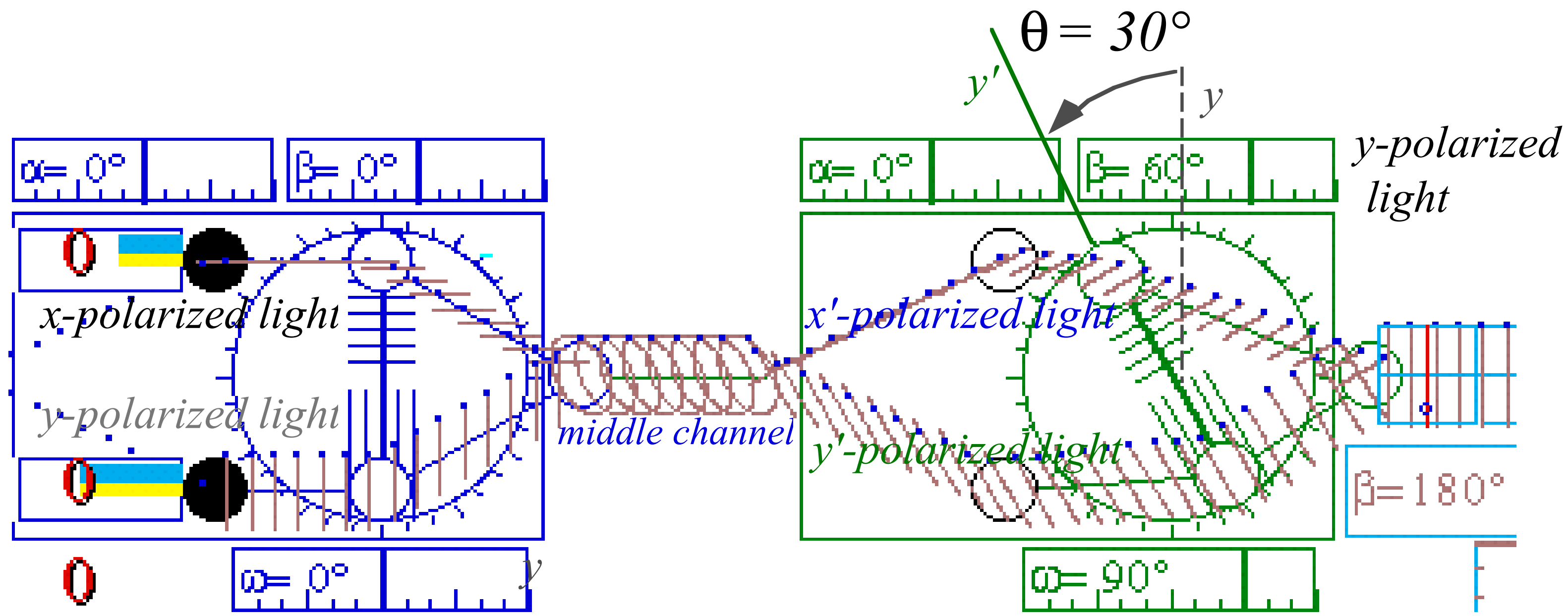
Fig. 1.3.6 Polarization states for (a) Half-wave (Ω = π) , (b) Quarter wave (Ω= π/2) (c) (Ω=−π/4)

[*BoxIt* web simulation - *With y-Phasor is on other side of xy plot*] [*RelaWavity* web simulation - *Contact ellipsometry*]

Ellipsis in Middle

1.3.1 A y-polarized light beam of unit amplitude (1 photon/sec.) enters an active analyzer that is tilted by 30° as shown below. The active analyzer puts a ω = 90° phase factor e-iω in the x' beam.

Fill in the blanks with numbers or symbols that tell as much as possible about what is present at each channel or branch.



$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{vmatrix} \qquad \begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{vmatrix}$$

State of output x channel

$|\_\_\rangle$ __________

Amplitude=

____

Probability=

____

State of x' channel

$|\_\_\rangle$ __________

Amplitude=

____

Probability=

____

State of input channel

$|y\rangle$ 1.0

Amplitude=

1.0

Probability=

1.0

State of middle channel

____

Probability=

____

State of output y channel

$|\_\_\rangle$ __________

Amplitude=

____

Probability=

____

State of y' channel

$|\_\_\rangle$ __________

Amplitude=

____

Probability=

____

Note: The x' and y' quantities will not be graded. Use for work.