Group Theory in Quantum Mechanics

Lecture 20 (4.07.15)

Octahedral-tetrahedral $O \sim T_d$ representations and spectra

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , QTCA Unit 5 Ch. 15)
(PSDS - Ch. 4)

Review Octahedral $O_h \supset O$ group operator structure
Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ subgroup chain correlations

Comparison of $O \supset D_4 \supset C_4$ and $O \supset D_4 \supset D_2$ correlations and level/projector splitting
- $O \supset D_4 \supset C_4$ subgroup chain splitting
- $O \supset D_4 \supset D_2$ subgroup chain splitting (nOrmal D_2 vs. unOrmal D_2)
- $O_h \supset O \supset D_4 \supset C_{4v}$ and $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ subgroup splitting

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O \supset C_4$
- Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ and representations $D^{\mu}_{m_4 m_4}$
- Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ induced representation $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples
- Elementary induced representation $0_4(C_4)\uparrow O$
- Projection reduction of induced representation $0_4(C_4)\uparrow O$
- Introduction to ortho-complete eigenvalue-parameter relations
  - Examples in SF_6 spectroscopy

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Elementary induced representation $0_4(C_4)\uparrow O$

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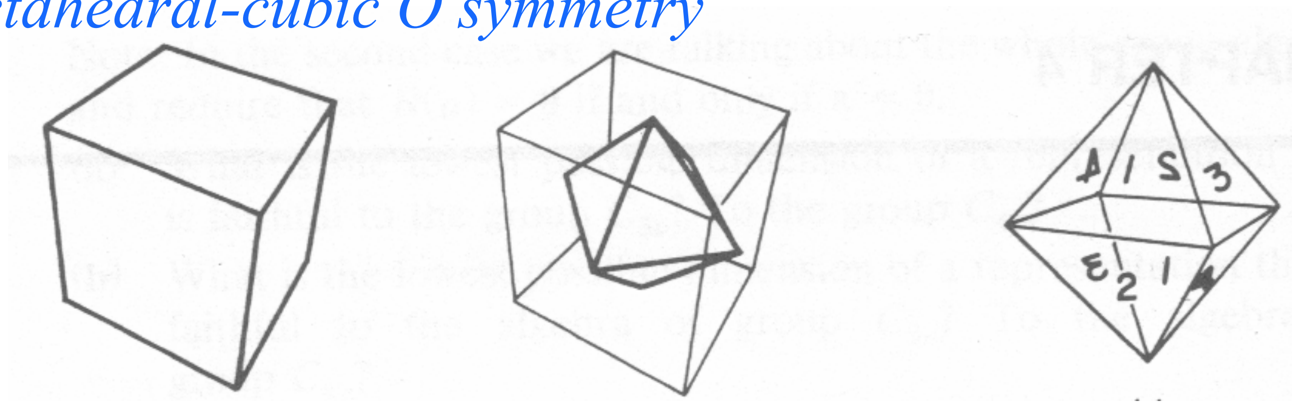
Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 spectroscopy

Introduction to octahedral/ tetrahedral symmetry $O_h \supset O \sim T_d \supset T$

Octahedral-cubic O symmetry

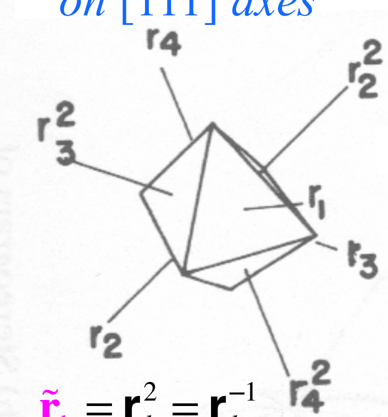


Order °O=6 hexahedron squares · 4 pts =24
=8 octahedron triangles · 3 pts =24
=12 lines · 2 pts =24 positions

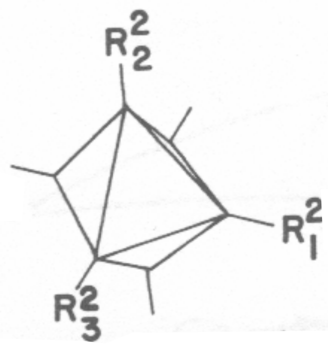
Octahedral group O operations

Class of 1: **1**
$\mathbf{r}_k = \mathbf{r}_k$

Class of 8:
±120° rotations
on [111] axes



$\tilde{\mathbf{r}}_k = \mathbf{r}_k^2 = \mathbf{r}_k^{-1}$

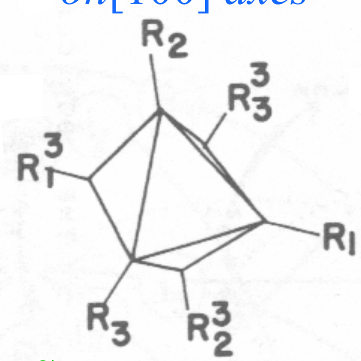


Class of 3:
180° rotations
on [100] axes

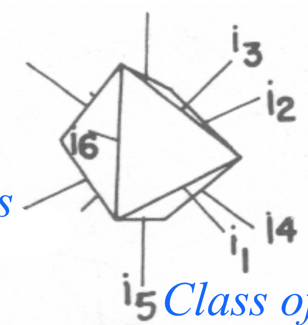
$\rho_{x,y,z} = \mathbf{R}_{1,2,3}^2$

$\mathbf{R}_{x,y,z} = \mathbf{R}_{1,2,3}$

Class of 6:
±90° rotations
on[100] axes

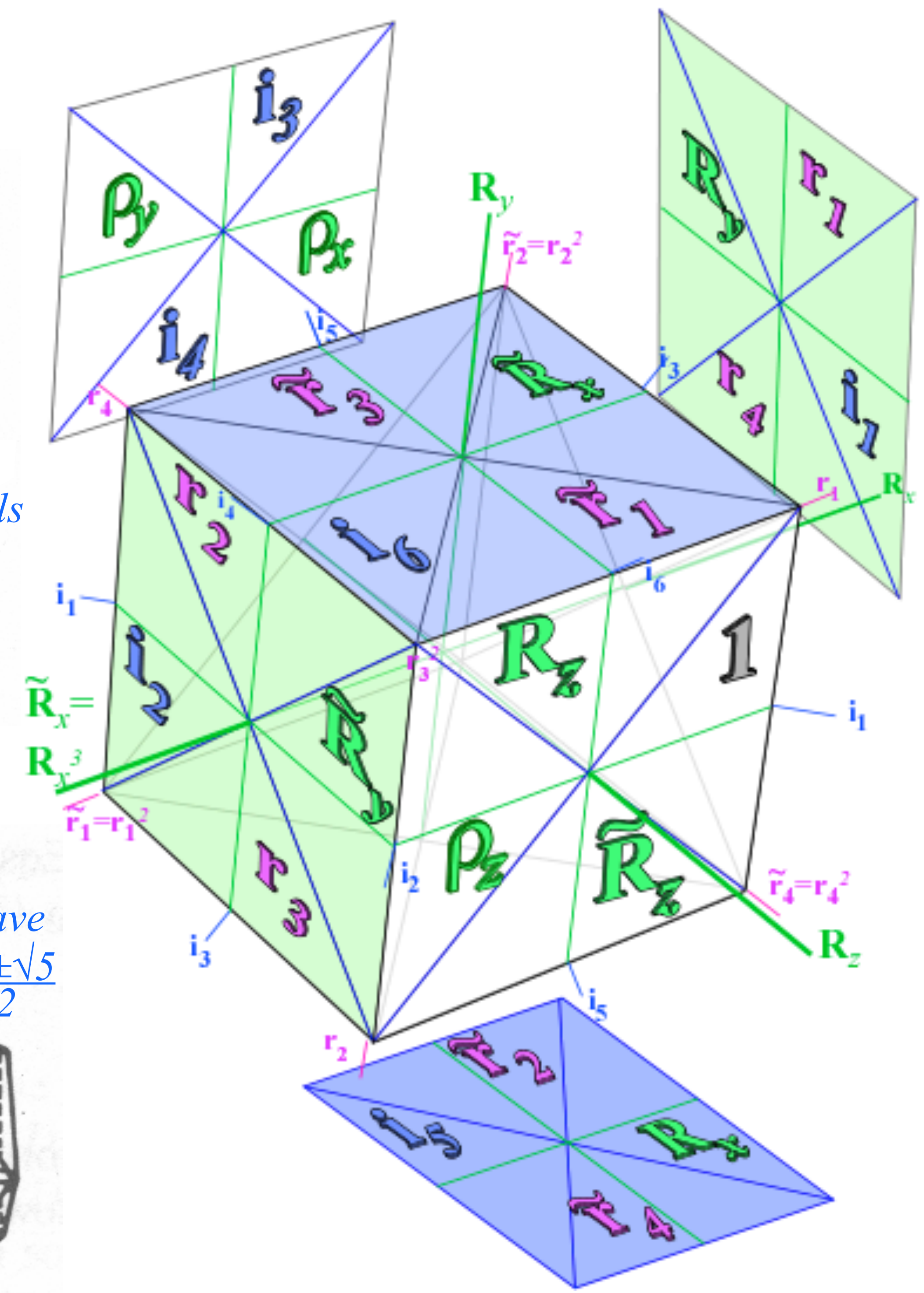


$\tilde{\mathbf{R}}_{x,y,z} = \mathbf{R}_{1,2,3}^3 = \mathbf{R}_{1,2,3}^{-1}$

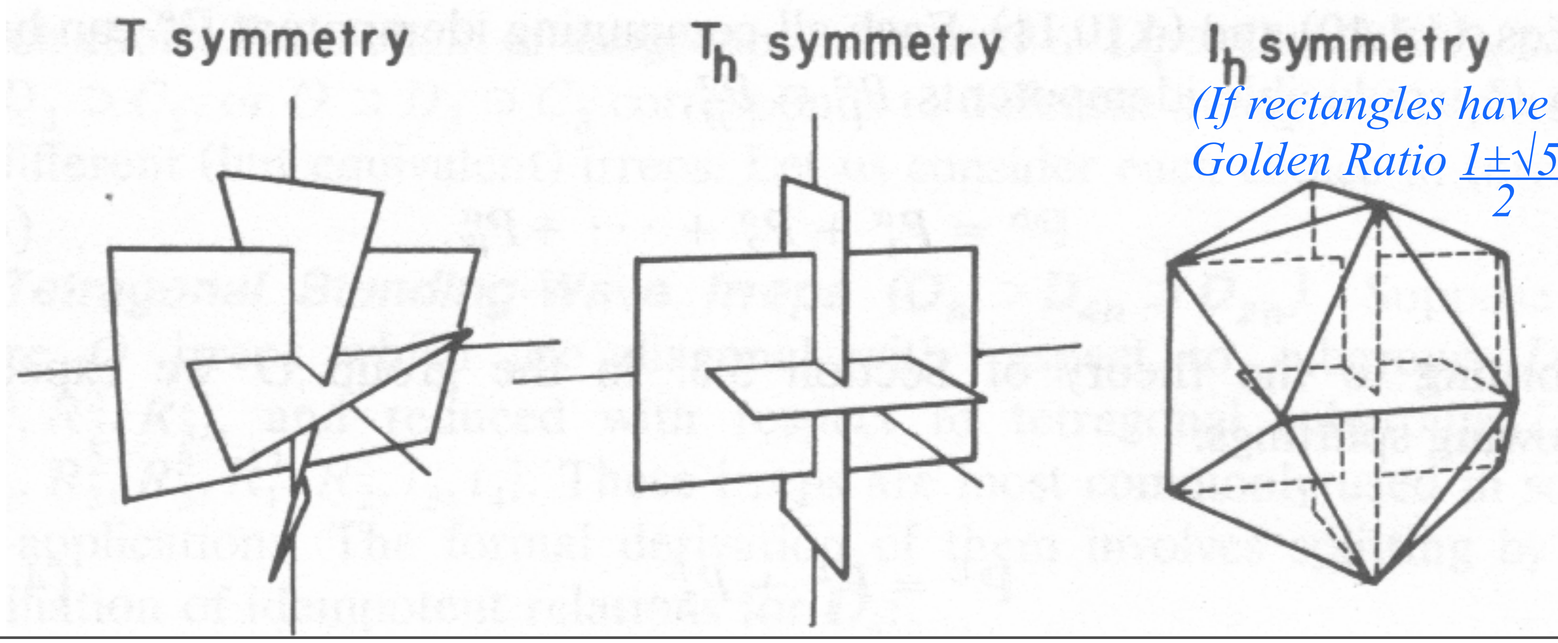


Class of 6:
180° rotations
on [110] diagonals

$\mathbf{i}_k = \mathbf{i}_k$



Tetrahedral symmetry becomes Icosahedral



(If rectangles have Golden Ratio $\frac{1 \pm \sqrt{5}}{2}$

Introduction to octahedral tetrahedral symmetry $O_h \supset O \sim T_d \supset T$

Octahedral groups $O_h \supset O \sim T_d$ and $O_h \supset T_h \supset T$

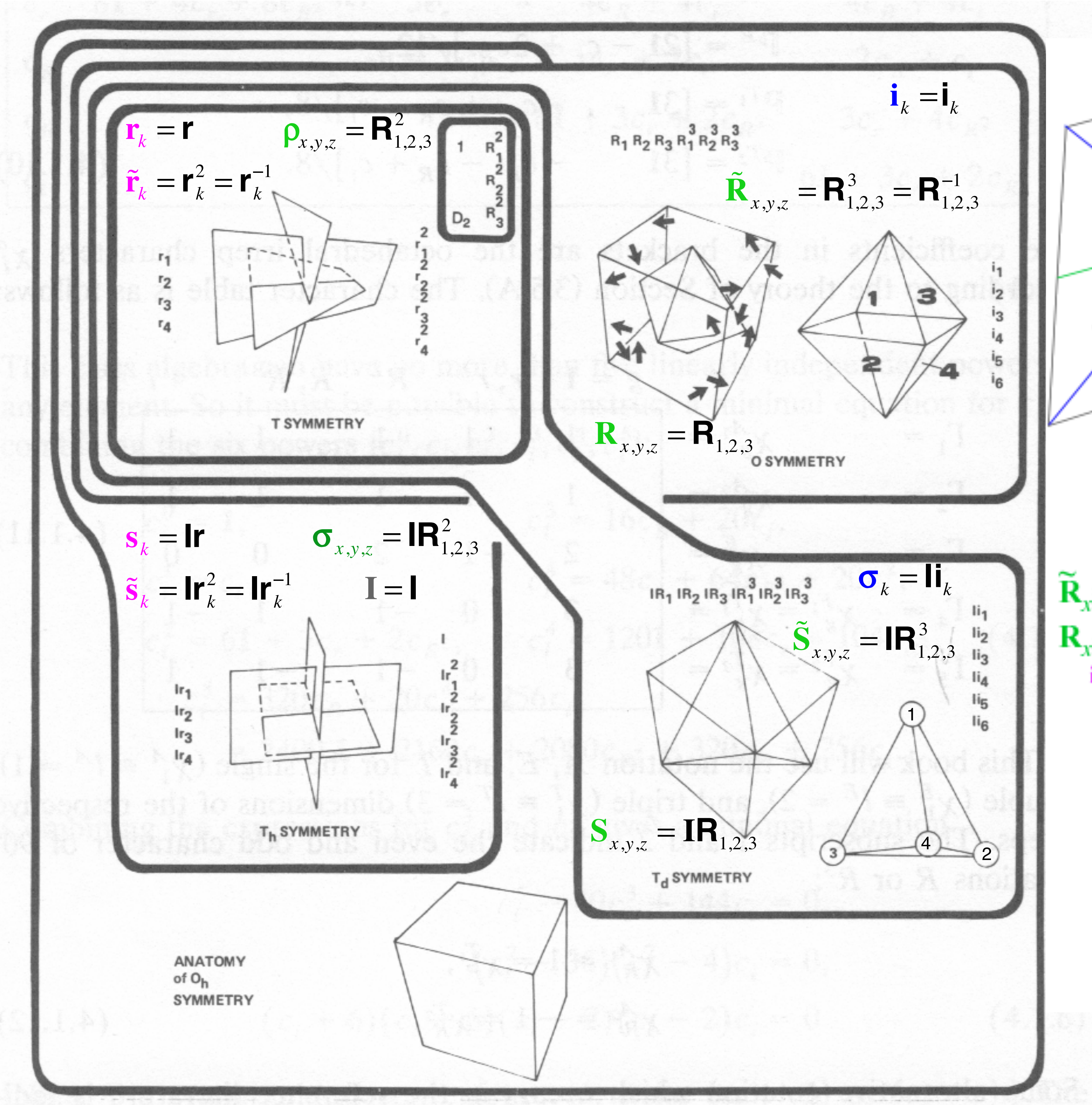
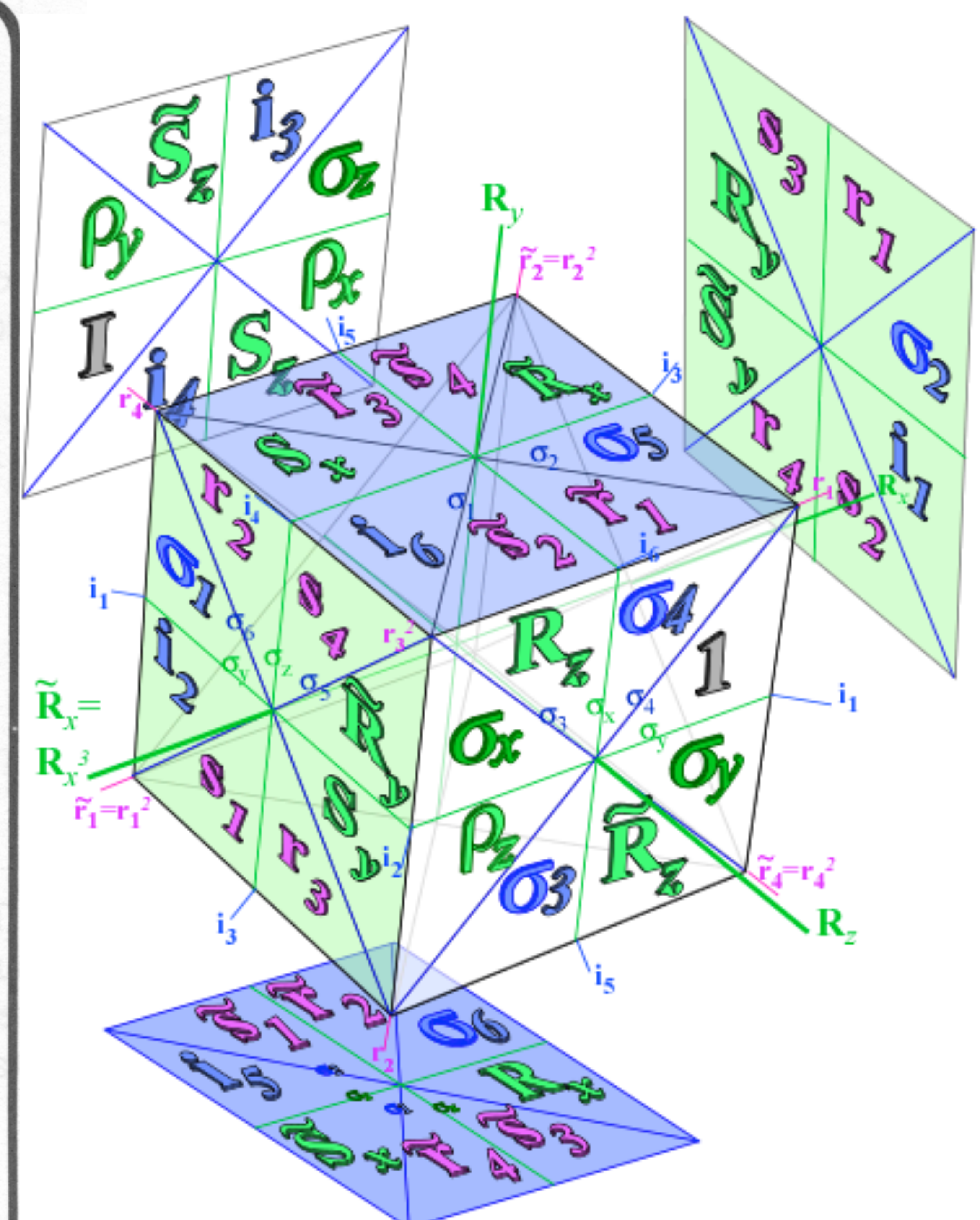


Figure 4.1.5 The full octahedral group (O_h) and four non-Abelian subgroups T, T_h, T_d, and O. The Abelian D_2 subgroup of T is indicated also.

Fig. 4.1.5 from Principles of Symmetry, Dynamics and Spectroscopy



Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

$O \supset D_4 \supset C_4$ subgroup chain splitting

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Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 spectroscopy

Octahedral $O_h \supset O \supset D_4 \supset C_4$ subgroup correlations

$\chi^{\mu}_g(O)$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1...4}$	180° $\boldsymbol{\rho}_{xyz}$	90° $\mathbf{R}_{xyz}$	180° $\mathbf{i}_{1...6}$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
T_1	3	0	−1	1	−1
T_2	3	0	−1	−1	1

$O \downarrow D_4$ subduction

D_4: $\mathbf{1}, \boldsymbol{\rho}_{z180°}, \mathbf{R}_{z\pm 90°}, \boldsymbol{\rho}_{x,y180°}, \mathbf{i}_{3,4}$

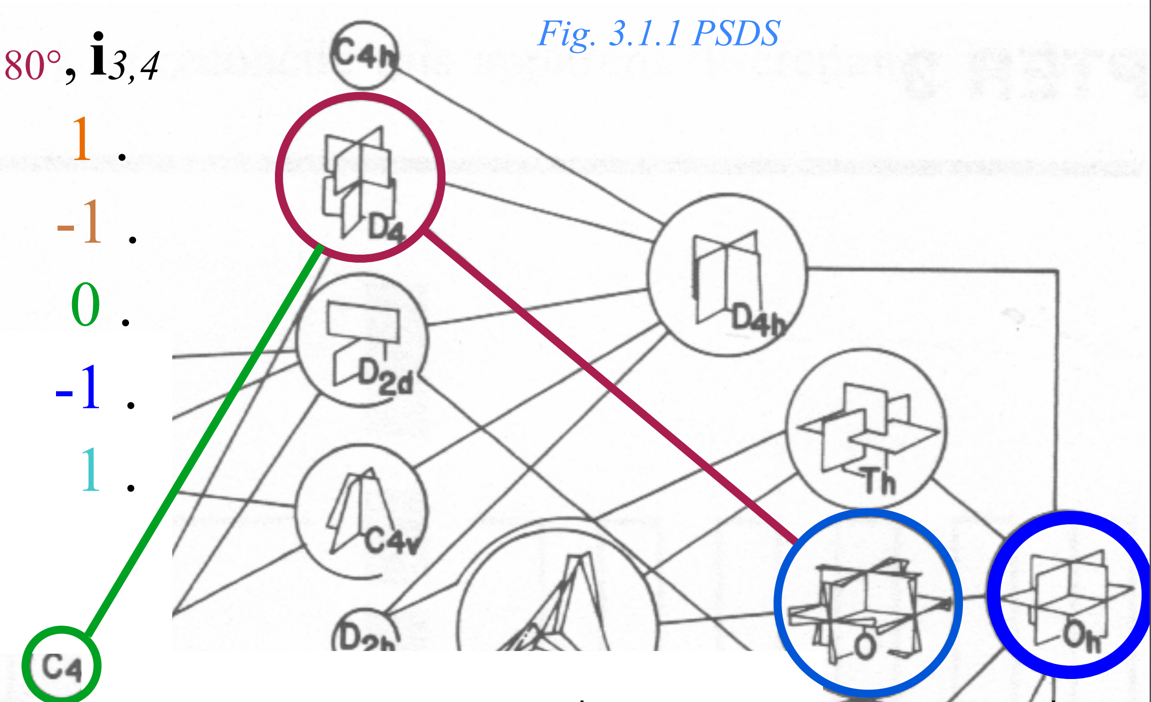
$A_1(O)\downarrow D_4 = 1, \; 1, \; 1, \; 1, \; 1.$

$A_2(O)\downarrow D_4 = 1, \; 1, \; -1, \; 1, \; -1.$

$E(O)\downarrow D_4 = 2, \; 2, \; 0, \; 2, \; 0.$

$T_2(O)\downarrow D_4 = 3, \; -1, \; 1, \; -1, \; -1.$

$T_2(O)\downarrow D_4 = 3, \; -1, \; -1, \; -1, \; 1.$



$\chi^{\mu}_g(D_4)$	$\mathbf{g}=\mathbf{1}$	$\boldsymbol{\rho}_{z180°}$	$\mathbf{R}_{z\pm 90°}$	$\boldsymbol{\rho}_{x,y180°}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	−1	1	−1
A_2	1	1	1	−1	−1
B_2	1	1	−1	−1	1
E	2	−2	0	0	0

$D_4 \downarrow C_4$ subduction

C_4: $\mathbf{1}, \mathbf{R}_{z+90°}, \boldsymbol{\rho}_{z180°}, \mathbf{R}_{z-90°}$

$A_1(D_4)\downarrow C_4 = 1, \; 1, \; 1, \; 1. \quad =(0)_4$

$B_1(D_4)\downarrow C_4 = 1, \; -1, \; 1, \; -1. \quad =(2)_4$

$A_2(D_4)\downarrow C_4 = 1, \; 1, \; 1, \; 1. \quad =(0)_4$

$B_2(D_4)\downarrow C_4 = 1, \; -1, \; 1, \; -1. \quad =(2)_4$

$E(D_4)\downarrow C_4 = 2, \; 0, \; -2, \; 0. \quad =(1)_4 \oplus (3)_4$

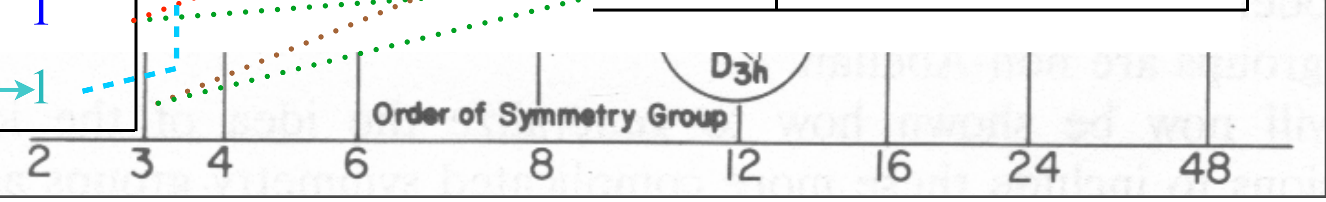
$O\downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$\chi^{\mu}_g(C_4)$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}_{z+90°}$	$\mathbf{R}_{z+180°}$	$\mathbf{R}_{z-90°}$
$(0)_4$	1	1	1	1
$(1)_4$	1	i	−1	$-i$
$(2)_4$	1	−1	1	−1
$(3)_4$	1	$-i$	−1	i

$O \downarrow C_4$ subduction

$O\downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	→1	→1	→1

$D_4\downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	→1	·
E	·	→1	·	→1



Octahedral $O_h \supset O \supset D_4 \supset C_4$ subgroup correlations

$O \downarrow D_4$ subduction

D_4: $\mathbf{1}, \rho_{z180°}, \mathbf{R}_{z\pm 90°}, \rho_{x,y180°}, \mathbf{i}_{3,4}$

$A_1(O)\downarrow D_4 = 1,\ 1,\ 1,\ 1,\ 1\ .$

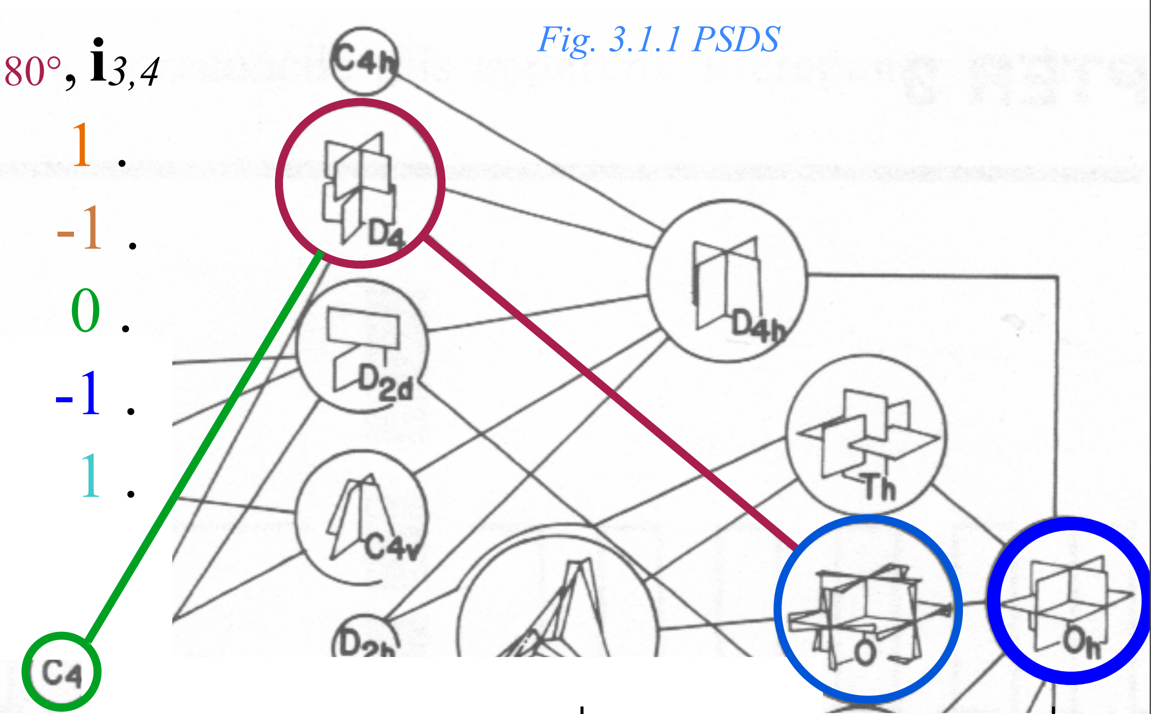
$A_2(O)\downarrow D_4 = 1,\ 1,\ -1,\ 1,\ -1\ .$

$E(O)\downarrow D_4 = 2,\ 2,\ 0,\ 2,\ 0\ .$

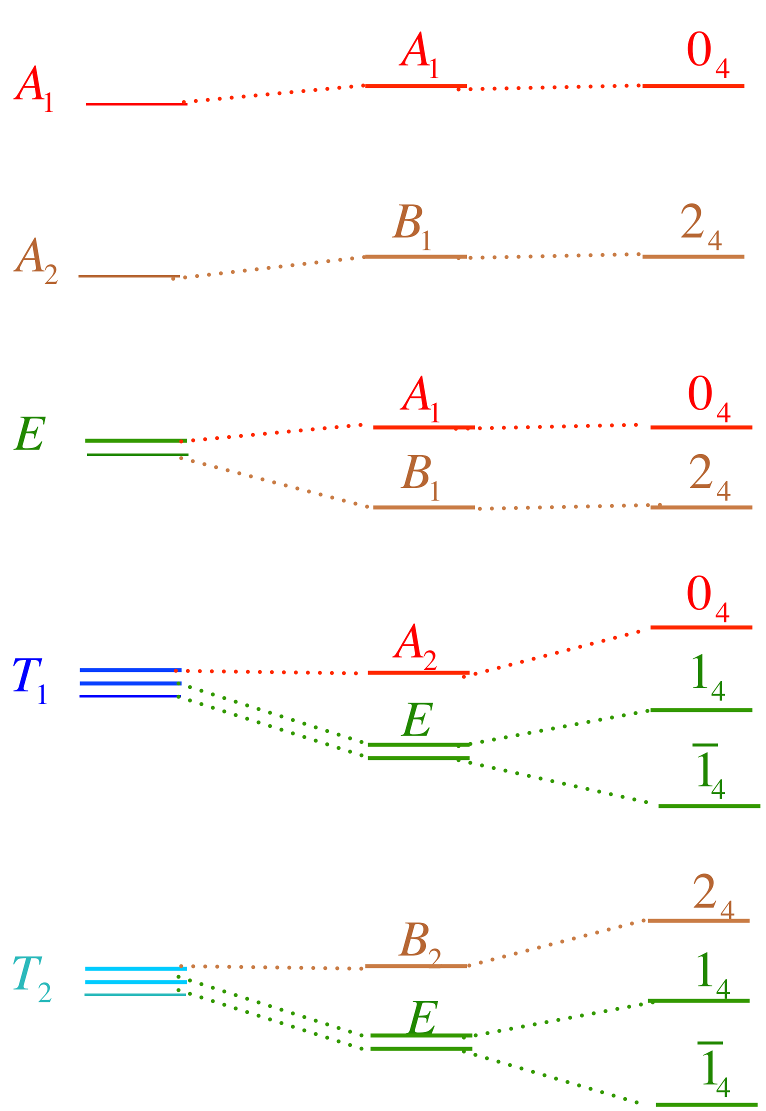
$T_2(O)\downarrow D_4 = 3,\ -1,\ 1,\ -1,\ -1\ .$

$T_2(O)\downarrow D_4 = 3,\ -1,\ -1,\ -1,\ 1\ .$

Fig. 3.1.1 PSDS



$O \supset D_4 \supset C_4$ subgroup and level-splitting/relabeling correlations



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$O\downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$O \downarrow C_4$ subduction

$O\downarrow C_4$	0_4	1_4	2_4	$3_4=\bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	→1	→1	→1

$D_4\downarrow C_4$	0_4	1_4	2_4	$3_4=\bar{1}_4$
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	→1	·
E	·	→1	·	→1

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$O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*

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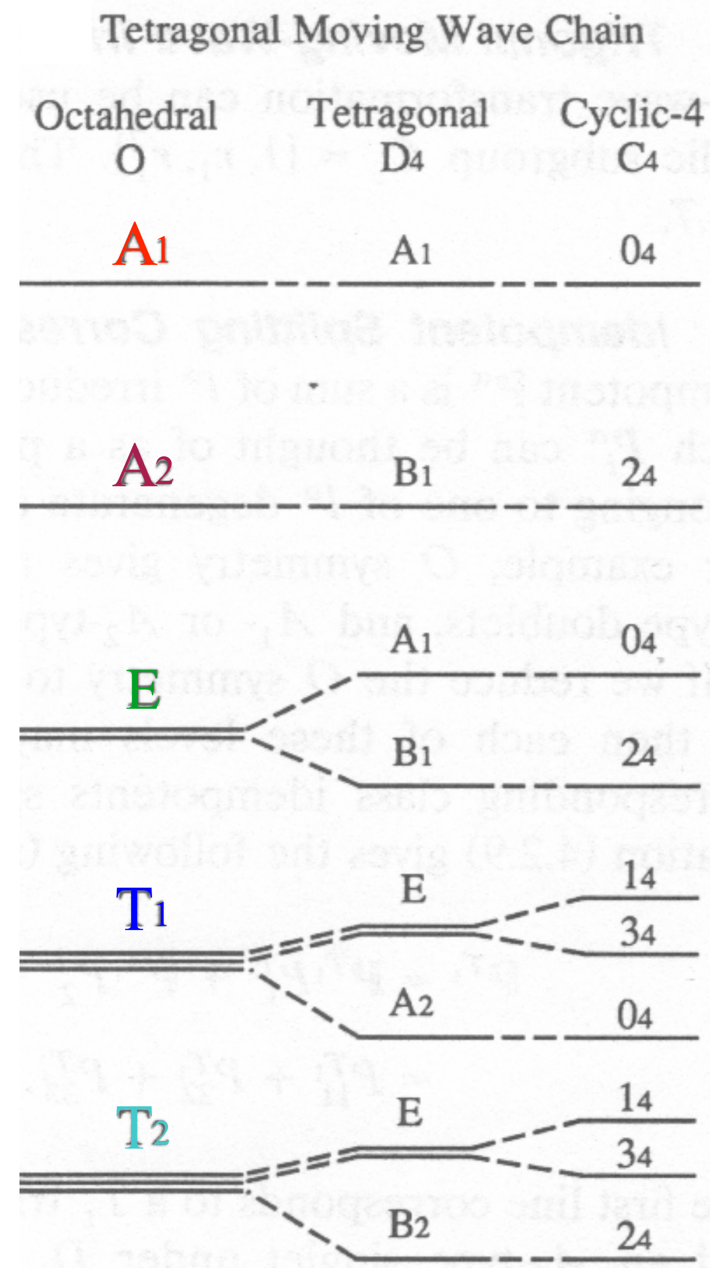
Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 *spectroscopy*

D_4	$\mathbf{1}$	ρ_z	$\mathbf{R}_z$	$\rho_{x,y}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	-1	1	-1
A_2	1	1	1	-1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

O⊃D_4⊃C_4 level splitting



C_4 { $\mathbf{1}, \mathbf{R}_z^1, \mathbf{R}_z^2, \mathbf{R}_z^3$ }
{ $\mathbf{1}, \mathbf{R}_3^2, \mathbf{R}_3^2, \mathbf{R}_3^3$ }

0_4	1	1	1	1
1_4	1	i	-1	$-i$
2_4	1	-1	1	-1
3_4	1	$-i$	-1	i

$-1_4 =$

$D_4 \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	.	.	.
B_1	.	.	1	.
A_2	1	.	.	.
B_2	.	.	1	.
E	.	1	.	1

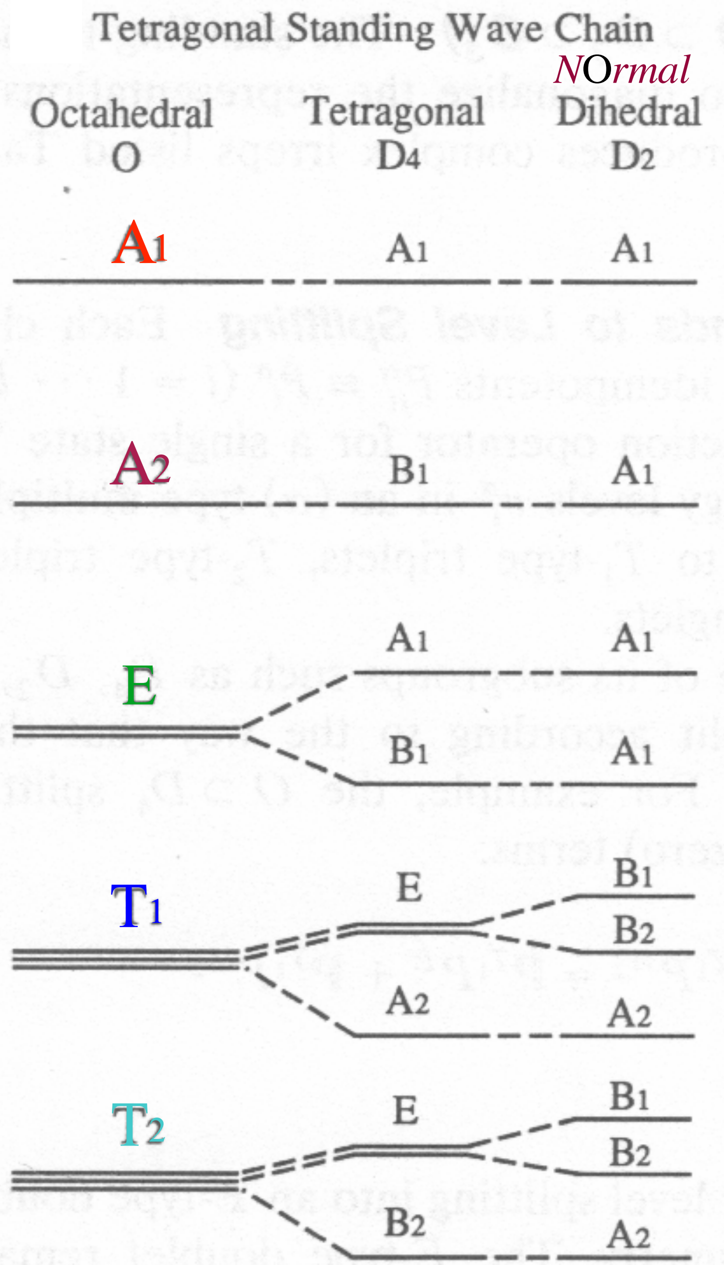
$\mathbf{r}, \tilde{\mathbf{r}}_i$ ρ_{xyz} $\mathbf{R}, \tilde{\mathbf{R}}_{xyz}$

O	$\mathbf{1}$	$\mathbf{r}$	$\mathbf{R}^2$	$\mathbf{R}^3$	$\mathbf{i}_k$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$O \downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	.	.	.	.
A_2	.	1	.	.	.
E	1	1	.	.	.
T_1	.	.	1	.	1
T_2	.	.	.	1	1

$O \downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	.	.	.
A_2	.	.	1	.
E	1	.	1	.
T_1	1	1	.	1
T_2	.	1	1	1

O⊃D_4⊃D_2 level splitting

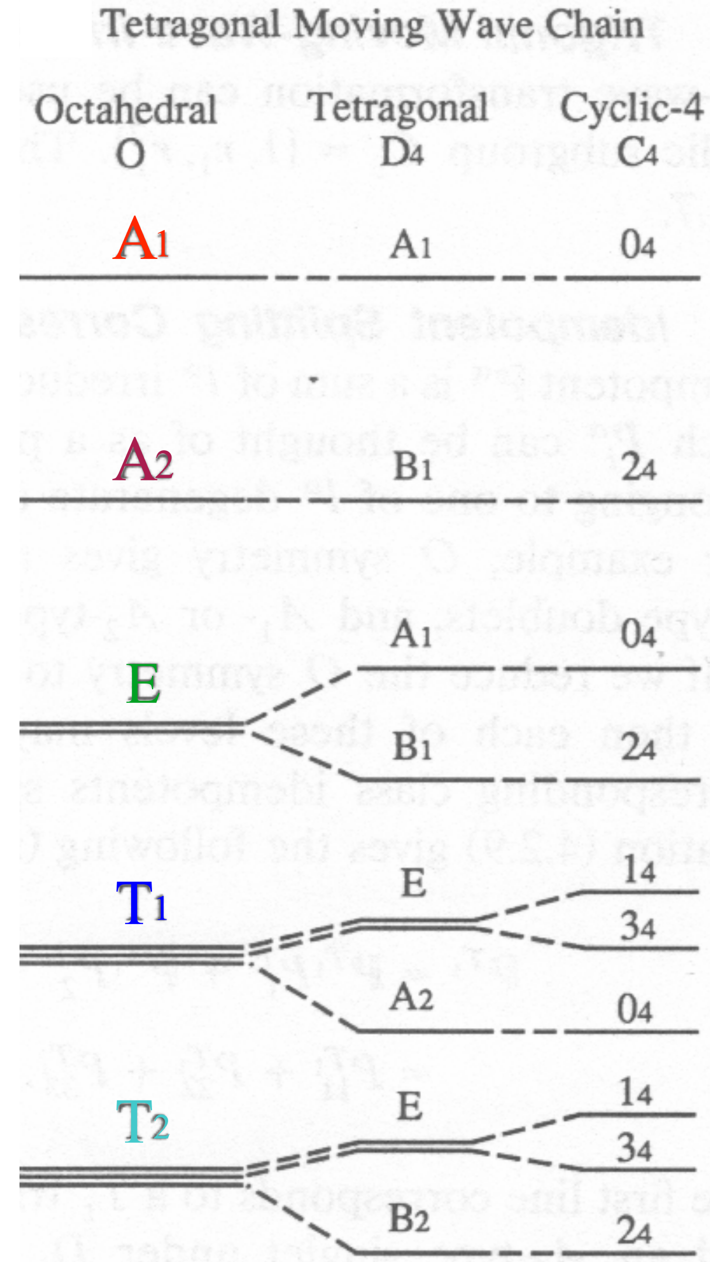


D_4	**1**	ρ_z	$\mathbf{R}_z$	$\rho_{x,y}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	-1	1	-1
A_2	1	1	1	-1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	1	·	·	·
A_2	·	·	1	·
B_2	·	·	1	·
E	·	1	·	1

O⊃D_4⊃C_4 level splitting



D_2^{Nm}	**1**	$\mathbf{R}_z^2$	$\mathbf{R}_x^2$	$\mathbf{R}_y^2$
A_1	1	1	1	1
B_1	1	-1	1	-1
A_2	1	1	-1	-1
B_2	1	-1	-1	1

$-1_4 =$

$D_4 \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	1	·
E	·	1	·	1

	$\mathbf{r}, \tilde{\mathbf{r}}_i$	ρ_{xyz}	$\mathbf{R}, \tilde{\mathbf{R}}_{xyz}$		
O	**1**	**r**	$\mathbf{R}^2$	$\mathbf{R}^3$	$\mathbf{i}_k$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	1	·	·	·
E	2	·	·	·
T_1	·	1	1	1
T_2	·	1	1	1

$O \downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$O \downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	1	1	1

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Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

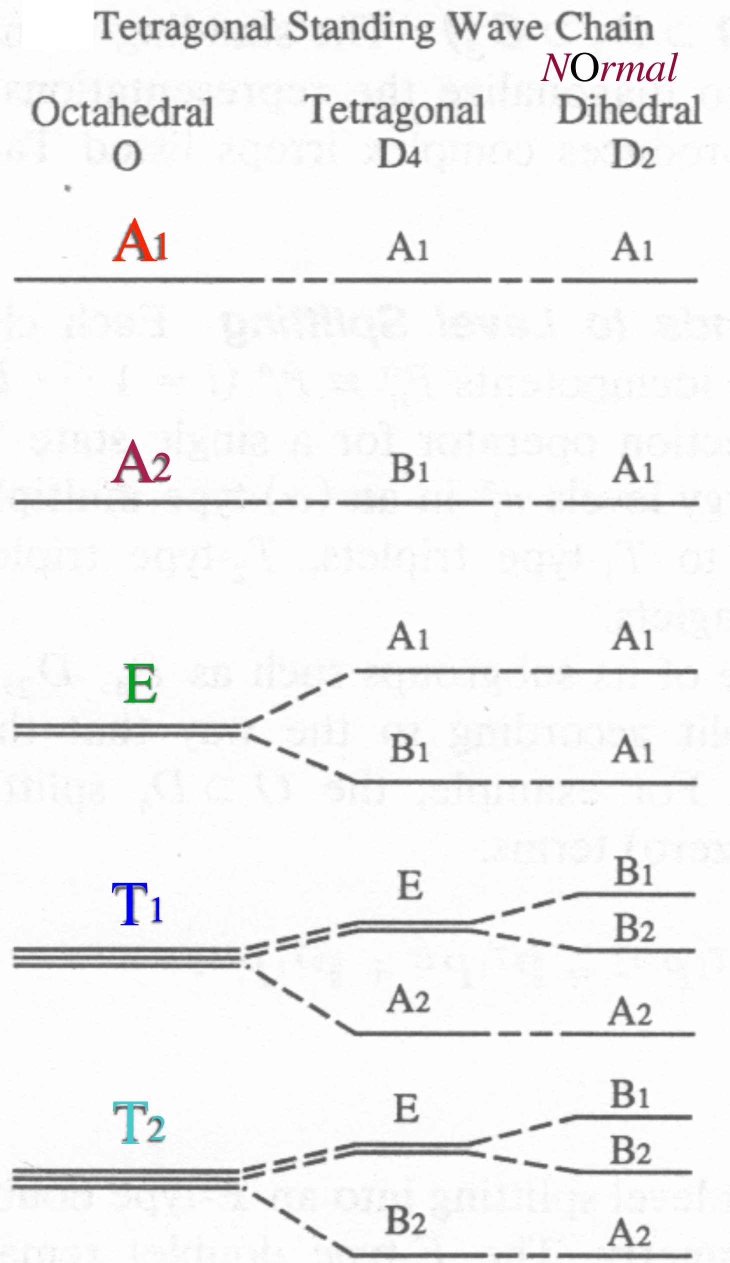
Elementary induced representation $0_4(C_4)\uparrow O$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 *spectroscopy*

$O_h \supset O \supset D_4 \supset D_2$ subgroup splitting



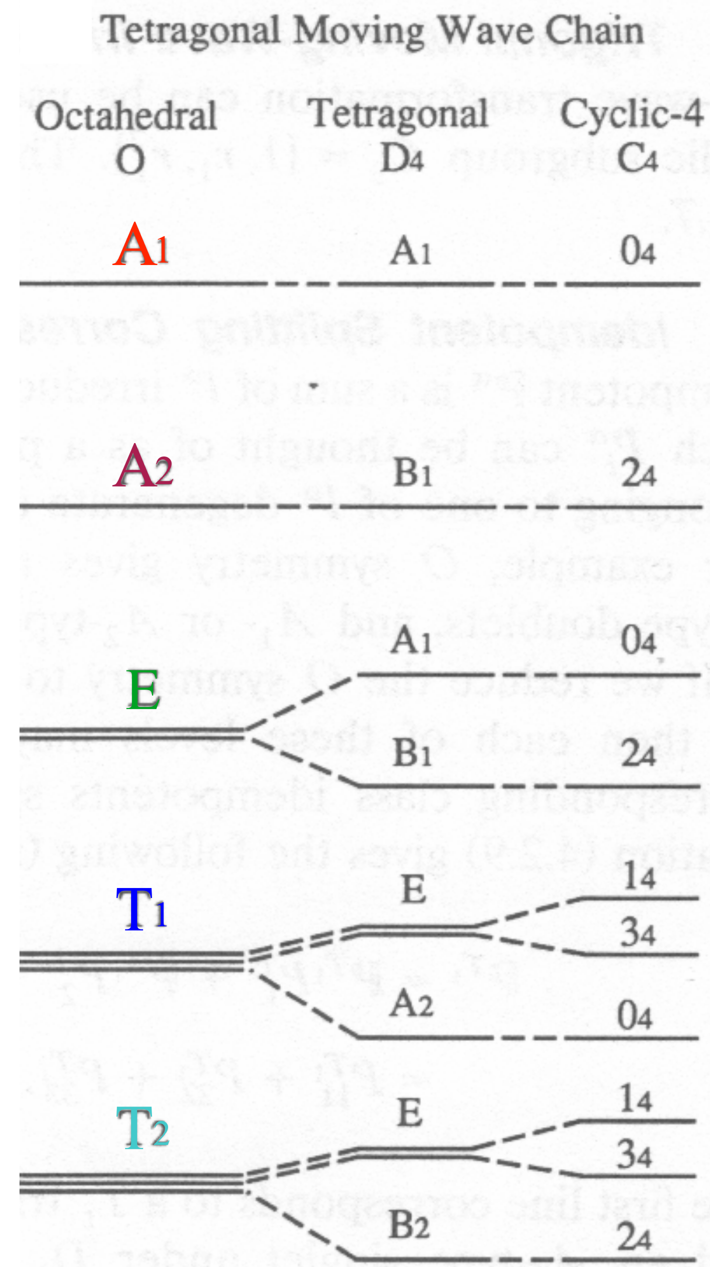
D_4	$\mathbf{1}$	ρ_z	$\mathbf{R}_z$	$\rho_{x,y}$	$\mathbf{i}_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	-1	1	-1
A_2	1	1	1	-1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	1	·	·	·
A_2	·	·	1	·
B_2	·	·	1	·
E	·	1	·	1

UnOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{i}_3, \mathbf{i}_4\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	·	·	1	·
A_2	·	·	1	·
B_2	1	·	·	·
E	·	1	·	1



$D_2^{Nm}\{\ \mathbf{1},\ \mathbf{R}_z^2,\ \mathbf{R}_x^2,\ \mathbf{R}_y^2\}$

$D_2^{Un}\{\ \mathbf{1},\ \mathbf{R}_z^2,\ \mathbf{i}_3,\ \mathbf{i}_4\}$

A_1	1	1	1	1
B_1	1	-1	1	-1
A_2	1	1	-1	-1
B_2	1	-1	-1	1

$-1_4 =$

$D_4 \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	1	·
E	·	1	·	1

O	$\mathbf{1}$	$\mathbf{r}$ ($\mathbf{r},\tilde{\mathbf{r}}_i$)	$\mathbf{R}^2$ (ρ_{xyz})	$\mathbf{R}^3$ ($\mathbf{R},\tilde{\mathbf{R}}_{xyz}$)	$\mathbf{i}_k$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

two kinds of D_2 subgroup

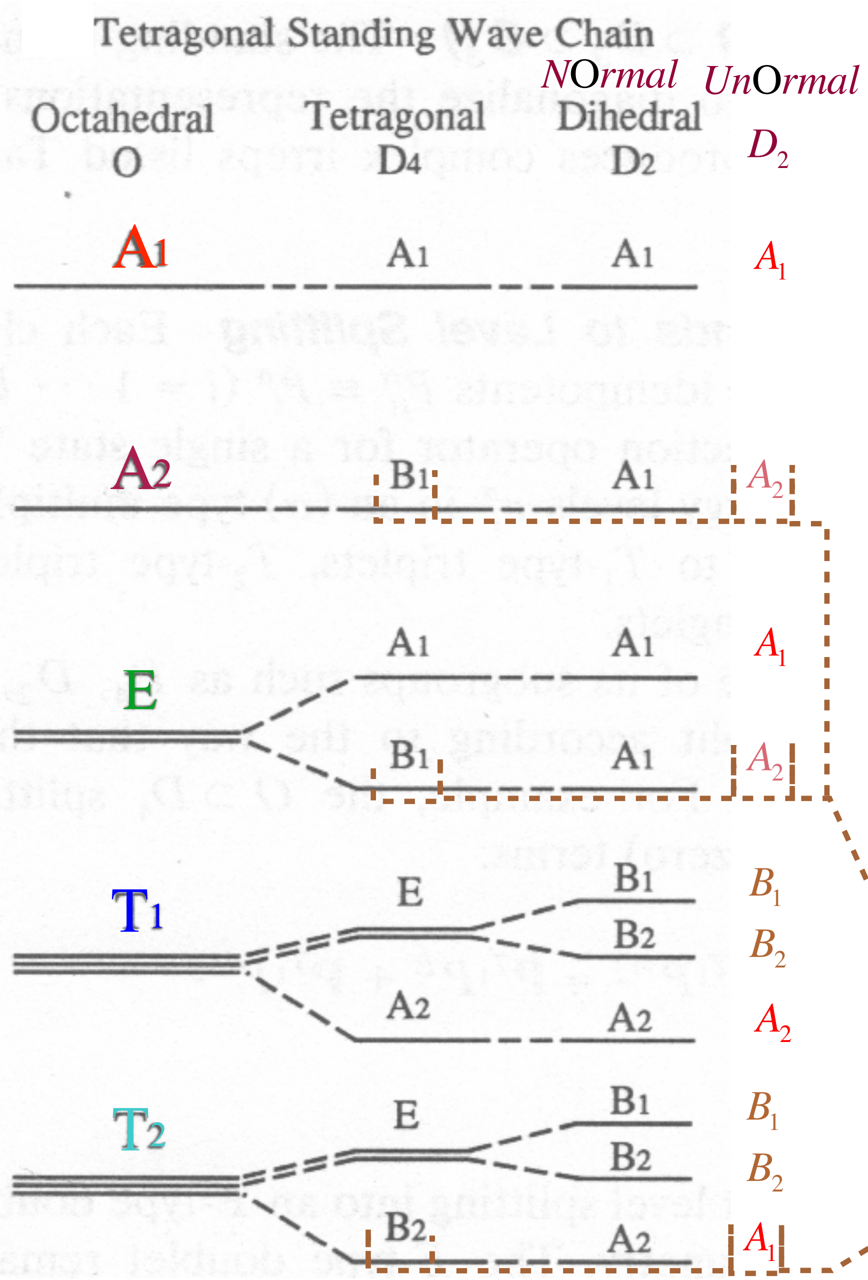
NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	1	·	·	·
E	2	·	·	·
T_1	·	1	1	1
T_2	·	1	1	1

$O \downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$O \downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	1	1	1

$O_h \supset O \supset D_4 \supset D_2$ subgroup splitting



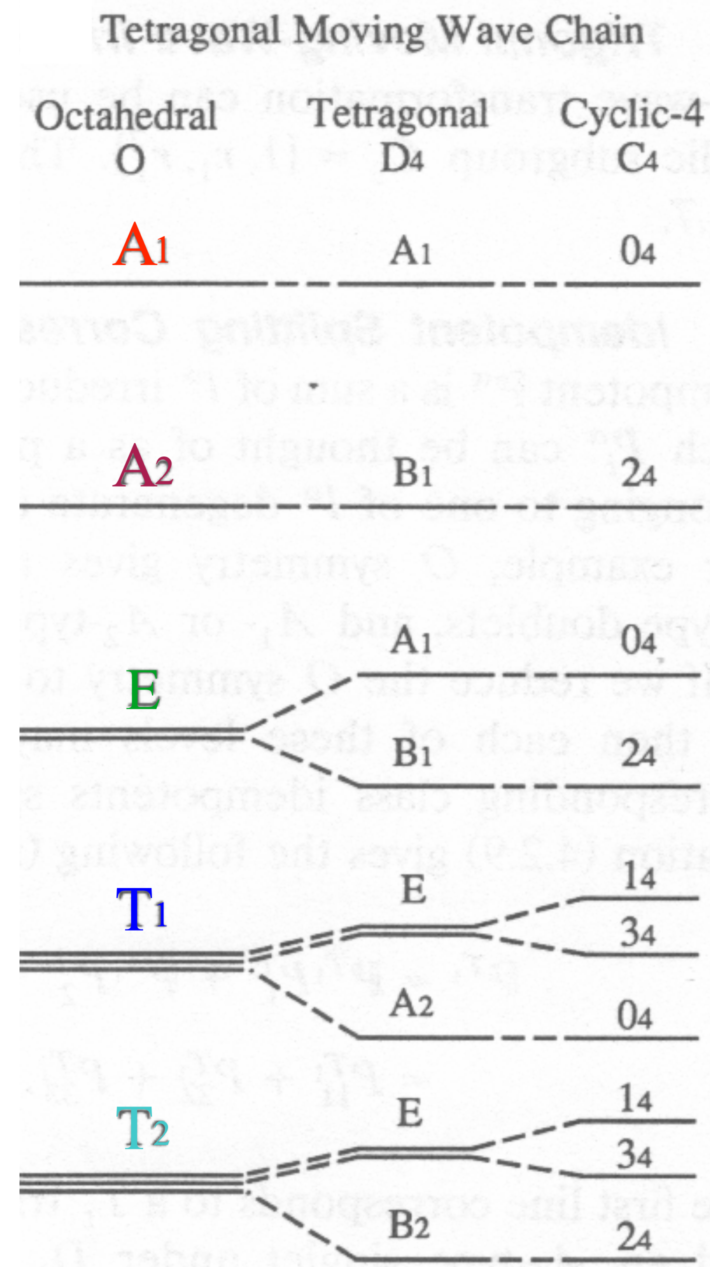
D_4	1	ρ_z	R_z	$\rho_{x,y}$	$i_{3,4}$
A_1	1	1	1	1	1
B_1	1	1	-1	1	-1
A_2	1	1	1	-1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	1	·	·	·
A_2	·	·	1	·
B_2	·	·	1	·
E	·	1	·	1

UnOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{i}_3, \mathbf{i}_4\}$

$D_4 \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
B_1	·	·	1	·
A_2	·	·	1	·
B_2	1	·	·	·
E	·	1	·	1



$D_2^{Nm}\{\mathbf{1}, \mathbf{R}_z^2, \mathbf{R}_x^2, \mathbf{R}_y^2\}$

$D_2^{Un}\{\mathbf{1}, \mathbf{R}_z^2, \mathbf{i}_3, \mathbf{i}_4\}$

A_1	1	1	1	1
B_1	1	-1	1	-1
A_2	1	1	-1	-1
B_2	1	-1	-1	1

$-1_4 =$

$D_4 \downarrow C_4$	0_4	1_4	2_4	3_4
A_1	1	·	·	·
B_1	·	·	1	·
A_2	1	·	·	·
B_2	·	·	1	·
E	·	1	·	1

O	1	$\mathbf{r}, \tilde{\mathbf{r}}_i$	ρ_{xyz} $\mathbf{R}^2$	$\mathbf{R}, \tilde{\mathbf{R}}_{xyz}$ $\mathbf{R}^3$	$\mathbf{i}_k$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

two kinds of D_2 subgroup splitting

NOrmal $D_2 = \{1, \mathbf{R}_3^2, \mathbf{R}_1^2, \mathbf{R}_2^2\}$ *UnOrmal* $D_2 = \{1, \mathbf{R}_3^2, \mathbf{i}_3, \mathbf{i}_4\}$

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	1	·	·	·
E	2	·	·	·
T_1	·	1	1	1
T_2	·	1	1	1

degeneracy ambiguity

$O \downarrow D_2$	A_1	B_1	A_2	B_2
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	·	1	1	1
T_2	1	1	·	1

$O \downarrow D_4$	A_1	B_1	A_2	B_2	E
A_1	1	·	·	·	·
A_2	·	1	·	·	·
E	1	1	·	·	·
T_1	·	·	1	·	1
T_2	·	·	·	1	1

$O \downarrow C_4$	0_4	1_4	2_4	$3_4 = \bar{1}_4$
A_1	1	·	·	·
A_2	·	·	1	·
E	1	·	1	·
T_1	1	1	·	1
T_2	·	1	1	1

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

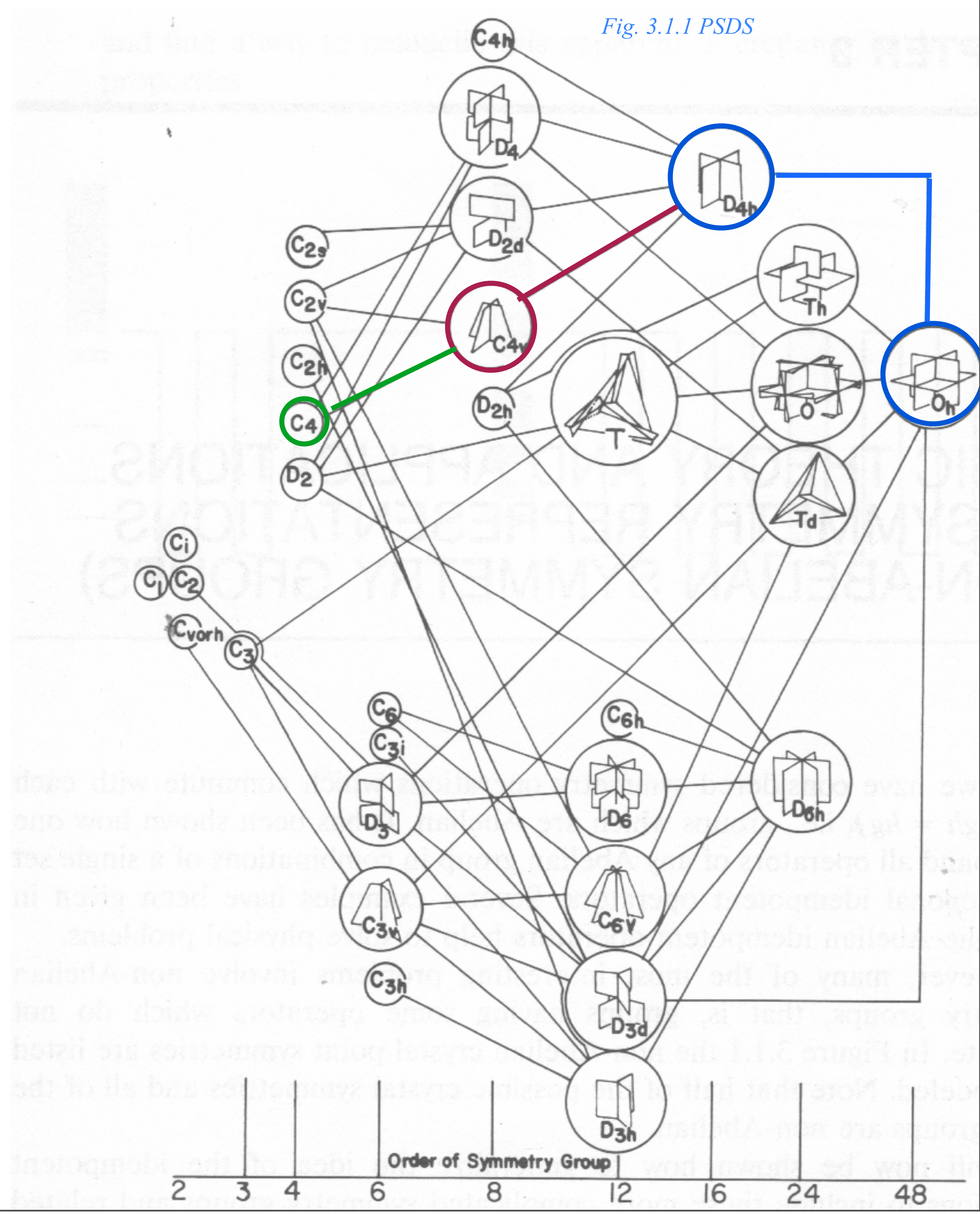
$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*

$O_h \supset D_{4h} \supset C_{4v} \supset C_4$ subgroup splitting

$O_h \supset C_{4v}$	A'	B'	A''	B''	E
$A_{1g} \downarrow C_{4v}$	1	·	·	·	·
$A_{2g} \downarrow C_{4v}$	·	1	·	·	·
$E_g \downarrow C_{4v}$	1	1	·	·	·
$T_{1g} \downarrow C_{4v}$	·	·	1	·	1
$T_{2g} \downarrow C_{4v}$	·	·	·	1	1
$A_{1g} \downarrow C_{4v}$	·	·	1	·	·
$A_{2u} \downarrow C_{4v}$	·	·	·	1	·
$E_u \downarrow C_{4v}$	·	·	1	1	·
$T_{1u} \downarrow C_{4v}$	1	·	·	·	1
$T_{2u} \downarrow C_{4v}$	·	1	·	·	1

Fig. 3.1.1 PSDS



$O_h \supset D_{4h} \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

$O_h \supset C_{4v}$	A'	B'	A''	B''	E
$A_{1g} \downarrow C_{4v}$	1	·	·	·	·
$A_{2g} \downarrow C_{4v}$	·	1	·	·	·
$E_g \downarrow C_{4v}$	1	1	·	·	·
$T_{1g} \downarrow C_{4v}$	·	·	1	·	1
$T_{2g} \downarrow C_{4v}$	·	·	·	1	1
$A_{1g} \downarrow C_{4v}$	·	·	1	·	·
$A_{2u} \downarrow C_{4v}$	·	·	·	1	·
$E_u \downarrow C_{4v}$	·	·	1	1	·
$T_{1u} \downarrow C_{4v}$	1	·	·	·	1
$T_{2u} \downarrow C_{4v}$	·	1	·	·	1

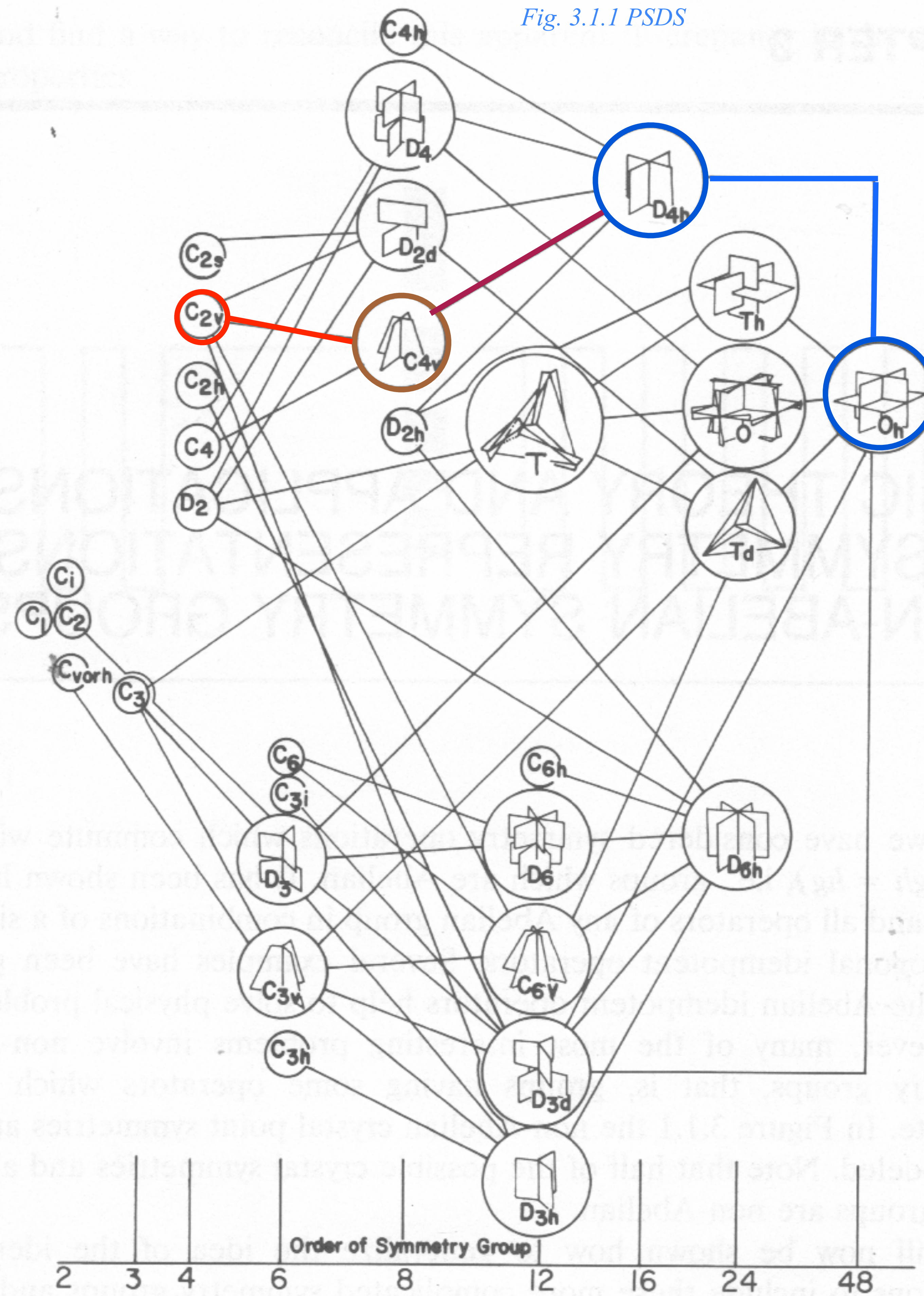
has degeneracy ambiguity

$O_h \supset C_{2v}^z$	A'	B'	A''	B''
$A_{1g} \downarrow C_{2v}^z$	1	·	·	·
$A_{2g} \downarrow C_{2v}^z$	1	·	·	·
$E_g \downarrow C_{2v}^z$	2	·	·	·
$T_{1g} \downarrow C_{2v}^z$	·	1	1	1
$T_{2g} \downarrow C_{2v}^z$	·	1	1	1
$A_{1g} \downarrow C_{2v}^z$	·	·	1	·
$A_{2u} \downarrow C_{2v}^z$	·	·	1	·
$E_u \downarrow C_{2v}^z$	·	·	2	·
$T_{1u} \downarrow C_{2v}^z$	1	1	·	1
$T_{2u} \downarrow C_{2v}^z$	1	1	·	1

has no degeneracy ambiguity

$O_h \supset C_{2v}^i$	A'	B'	A''	B''
$A_{1g} \downarrow C_{2v}^i$	1	·	·	·
$A_{2g} \downarrow C_{2v}^i$	·	1	·	·
$E_g \downarrow C_{2v}^i$	1	1	·	·
$T_{1g} \downarrow C_{2v}^i$	·	1	1	1
$T_{2g} \downarrow C_{2v}^i$	1	·	1	1
$A_{1g} \downarrow C_{2v}^i$	·	·	1	·
$A_{2u} \downarrow C_{2v}^i$	·	·	·	1
$E_u \downarrow C_{2v}^i$	·	·	1	1
$T_{1u} \downarrow C_{2v}^i$	1	1	·	1
$T_{2u} \downarrow C_{2v}^i$	1	1	1	·

Fig. 3.1.1 PSDS



Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

➡ *Splitting* O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$ ⬅

- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

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  - *Examples in* SF_6 *spectroscopy*

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O \supset C_4$

$O \supset C_4$ Correlation table shows which $\mathbf{P}^{\mu}$ splittings are allowed:

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2} = \mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

O characters

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4: d^{m_4}_{\mathbf{R}^p}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}^1_z$	$\rho_z = \mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z = \mathbf{R}^3_z$
$m_4 = 0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. *and Newer* Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O \supset C_4$

$O \supset C_4$ Correlation table shows which $\mathbf{P}^{\mu}$ splittings are allowed:

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2} = \mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

$1 \cdot \mathbf{P}^{\mu} =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4}) \cdot \mathbf{P}^{\mu}$
$1 \cdot \mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$1 \cdot \mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$1 \cdot \mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$1 \cdot \mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$1 \cdot \mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

O characters

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4: d^{m_4}_{\mathbf{R}^p}$	$\mathbf{g}=1$	$\mathbf{R}^1_z$	$\rho_z = \mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z = \mathbf{R}^3_z$
$m_4 = 0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

O operators (Two notations: Older *Princ.of Symm.Dynamics and Spectra.* and Newer *Int.J.Mol.Sci*)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

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$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	.	.	.
$A_2 \downarrow C_4$	.	.	1	.
$E \downarrow C_4$	1	.	1	.
$T_1 \downarrow C_4$	1	1	.	1
$T_2 \downarrow C_4$	.	1	1	1

$\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2} = \mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

$\mathbf{1}\cdot\mathbf{P}^{\mu} =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4: d^{m_4}_{\mathbf{R}^p}$	$\mathbf{g}=1$	$\mathbf{R}^1_z$	$\rho_z = \mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z = \mathbf{R}^3_z$
$m_4 = 0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$O \supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{P}^{E} = \frac{2}{8}\mathbf{1} - \frac{1}{8}\mathbf{c}_r + \frac{2}{8}\mathbf{c}_\rho + \frac{0}{8}\mathbf{c}_R - \frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1} = \frac{3}{8}\mathbf{1} + \frac{0}{8}\mathbf{c}_r - \frac{1}{8}\mathbf{c}_\rho + \frac{1}{8}\mathbf{c}_R - \frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2} = \frac{3}{8}\mathbf{1} + \frac{0}{8}\mathbf{c}_r - \frac{1}{8}\mathbf{c}_\rho - \frac{1}{8}\mathbf{c}_R + \frac{1}{8}\mathbf{c}_i$$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. and Newer Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O\supset C_4$

$O\supset C_4$ Correlation table shows which $\mathbf{P}^{\mu}$ splittings are allowed:

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1}=\mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2}=\mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

$\mathbf{1}\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2}=$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1}=$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4:d^{m_4}_{\mathbf{R}^p}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{R}^1_z$	$\rho_z=\mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z=\mathbf{R}^3_z$
$m_4=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$O\supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}\underbrace{e^{\frac{2\pi i m_4\cdot p}{4}}}_{C_4\ characters}\mathbf{R}^p_z=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$d^{m_4}_{R^p}=e^{\frac{2\pi i m_4\cdot p}{4}}$$

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. *and Newer* Int.J.Mol.Sci*)*

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Splitting $\mathbf{O}$ class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}{}_{m_4 m_4}$ for $\mathbf{O}\supset C_4$

$\mathbf{O}\supset C_4$ Correlation table shows which $\mathbf{P}^{\mu}$ splittings are allowed:

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1}=\mathbf{P}^{A_1}_{0_40_4}$ and $\mathbf{P}^{A_2}=\mathbf{P}^{A_2}_{2_42_4}$ *cannot* split

$\mathbf{1}\cdot\mathbf{P}^{\mu}=$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1}=$	$\mathbf{P}^{A_1}_{0_40_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2}=$	0	$+0$	$+\mathbf{P}^{A_2}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E}=$	$\mathbf{P}^{E}_{0_40_4}$	$+0$	$+\mathbf{P}^{E}_{2_42_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1}=$	$\mathbf{P}^{T_1}_{0_40_4}$	$+\mathbf{P}^{T_1}_{1_41_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_43_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2}=$	0	$+\mathbf{P}^{T_2}_{1_41_4}$	$+\mathbf{P}^{T_2}_{2_42_4}$	$+\mathbf{P}^{T_2}_{3_43_4}$

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=1$	$\mathbf{r}_{1-4}$, $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$, $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4:d^{m_4}_{\mathbf{R}^p}$	$\mathbf{g}=1$	$\mathbf{R}^1_z$	$\rho_z=\mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z=\mathbf{R}^3_z$
$m_4=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

$\mathbf{O}\supset C_4$ splitting done by C_4 projectors applied to O class projectors

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}\underbrace{e^{\frac{2\pi i m_4\cdot p}{4}}}_{C_4\ \text{characters}}\mathbf{R}^p_z=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$d^{m_4}_{R^p}=e^{\frac{2\pi i m_4\cdot p}{4}}$$

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

Following development of irreducible projectors:

$$\mathbf{P}^{\mu}_{m_4m_4}\equiv\mathbf{p}_{m_4}\mathbf{P}^{\mu}=\mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. and Newer Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for O⊃C_4

O⊃C_4 Correlation table shows which $\mathbf{P}^{\mu}$ splittings are allowed:

O⊃C_4	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1}=\mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2}=\mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

$\mathbf{1}\cdot\mathbf{P}^{\mu} =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\mu}$
$\mathbf{1}\cdot\mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$\mathbf{1}\cdot\mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$\mathbf{1}\cdot\mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ / $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ / $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4:d^{m_4}_{\mathbf{R}^p}$	g=1	$\mathbf{R}^1_z$	$\rho_z=\mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z=\mathbf{R}^3_z$
$m_4=0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

O⊃C_4 splitting done by C_4 projectors applied to O class projectors

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}\underbrace{e^{\frac{2\pi i m_4\cdot p}{4}}}_{C_4\ \text{characters}}\mathbf{R}_z^p=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$d^{m_4}_{R^p}=e^{\frac{2\pi i m_4\cdot p}{4}}$$

$$\mathbf{P}^{E}=\frac{2}{8}\mathbf{1}-\frac{1}{8}\mathbf{c}_r+\frac{2}{8}\mathbf{c}_\rho+\frac{0}{8}\mathbf{c}_R-\frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho+\frac{1}{8}\mathbf{c}_R-\frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2}=\frac{3}{8}\mathbf{1}+\frac{0}{8}\mathbf{c}_r-\frac{1}{8}\mathbf{c}_\rho-\frac{1}{8}\mathbf{c}_R+\frac{1}{8}\mathbf{c}_i$$

Following development of irreducible projectors:

$$\mathbf{P}^{\mu}_{m_4 m_4}\equiv\mathbf{p}_{m_4}\mathbf{P}^{\mu}=\mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

...with examples:

$$\mathbf{P}^{T_1}_{0_4 0_4}\equiv\mathbf{p}_{0_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{0_4}$$
$$\mathbf{P}^{T_1}_{1_4 1_4}\equiv\mathbf{p}_{1_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{1_4}$$
etc.

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. *and Newer* Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* O⊃C_4

O⊃C_4 *Correlation table shows which* $\mathbf{P}^{\mu}$ *splittings are allowed:*

O⊃C_4	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_4 0_4}$ and $\mathbf{P}^{A_2} = \mathbf{P}^{A_2}_{2_4 2_4}$ *cannot* split

$1 \cdot \mathbf{P}^{\mu} =$	$(\mathbf{p}_{0_4}$	$+\mathbf{p}_{1_4}$	$+\mathbf{p}_{2_4}$	$+\mathbf{p}_{3_4}) \cdot \mathbf{P}^{\mu}$
$1 \cdot \mathbf{P}^{A_1} =$	$\mathbf{P}^{A_1}_{0_4 0_4}$	$+0$	$+0$	$+0$
$1 \cdot \mathbf{P}^{A_2} =$	0	$+0$	$+\mathbf{P}^{A_2}_{2_4 2_4}$	$+0$
$1 \cdot \mathbf{P}^{E} =$	$\mathbf{P}^{E}_{0_4 0_4}$	$+0$	$+\mathbf{P}^{E}_{2_4 2_4}$	$+0$
$1 \cdot \mathbf{P}^{T_1} =$	$\mathbf{P}^{T_1}_{0_4 0_4}$	$+\mathbf{P}^{T_1}_{1_4 1_4}$	$+0$	$+\mathbf{P}^{T_1}_{3_4 3_4}$
$1 \cdot \mathbf{P}^{T_2} =$	0	$+\mathbf{P}^{T_2}_{1_4 1_4}$	$+\mathbf{P}^{T_2}_{2_4 2_4}$	$+\mathbf{P}^{T_2}_{3_4 3_4}$

O characters

$O: \chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$, $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$, $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$C_4 : d^{m_4}_{\mathbf{R}^p}$	g=1	$\mathbf{R}^1_z$	$\boldsymbol{\rho}_z = \mathbf{R}^2_z$	$\tilde{\mathbf{R}}_z = \mathbf{R}^3_z$
$m_4 = 0_4$	1	1	1	1
1_4	1	$-i$	-1	i
2_4	1	-1	1	-1
3_4	1	$-i$	-1	$-i$

O⊃C_4 *splitting done by* C_4 *projectors applied to O class projectors*

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} \underbrace{e^{\frac{2\pi i m_4 \cdot p}{4}}}_{C_4 \text{ characters}} \mathbf{R}^p_z = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{P}^{E} = \frac{2}{8}\mathbf{1} - \frac{1}{8}\mathbf{c}_r + \frac{2}{8}\mathbf{c}_\rho + \frac{0}{8}\mathbf{c}_R - \frac{0}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_1} = \frac{3}{8}\mathbf{1} + \frac{0}{8}\mathbf{c}_r - \frac{1}{8}\mathbf{c}_\rho + \frac{1}{8}\mathbf{c}_R - \frac{1}{8}\mathbf{c}_i$$

$$\mathbf{P}^{T_2} = \frac{3}{8}\mathbf{1} + \frac{0}{8}\mathbf{c}_r - \frac{1}{8}\mathbf{c}_\rho - \frac{1}{8}\mathbf{c}_R + \frac{1}{8}\mathbf{c}_i$$

Following development of irreducible projectors:

$$\mathbf{P}^{\mu}_{m_4 m_4} \equiv \mathbf{p}_{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

...with examples:

$$\mathbf{P}^{T_1}_{0_4 0_4} \equiv \mathbf{p}_{0_4}\mathbf{P}^{T_1} = \mathbf{P}^{T_1}\mathbf{p}_{0_4}$$

$$\mathbf{P}^{T_1}_{1_4 1_4} \equiv \mathbf{p}_{1_4}\mathbf{P}^{T_1} = \mathbf{P}^{T_1}\mathbf{p}_{1_4}$$

etc.

...uses left-coset combinations... *...and projector "factoring"...*

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\},\ \rho_x C_4 = \{\rho_x, \rho_y, \mathbf{i}_4, \mathbf{i}_3\},\ \mathbf{r}_1 C_4 = \{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\},\ \mathbf{r}_2 C_4 = \{\mathbf{r}_2, \mathbf{r}_3, \mathbf{i}_2, \tilde{\mathbf{R}}_y\},\ \tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_3, \tilde{\mathbf{R}}_x, \mathbf{i}_6\},\ \tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2, \tilde{\mathbf{r}}_4, \mathbf{R}_x, \mathbf{i}_5\}.$$

O operators (Two notations: Older *Princ.of Symm.Dynamics and Spectra.* *and Newer* *Int.J.Mol.Sci)*

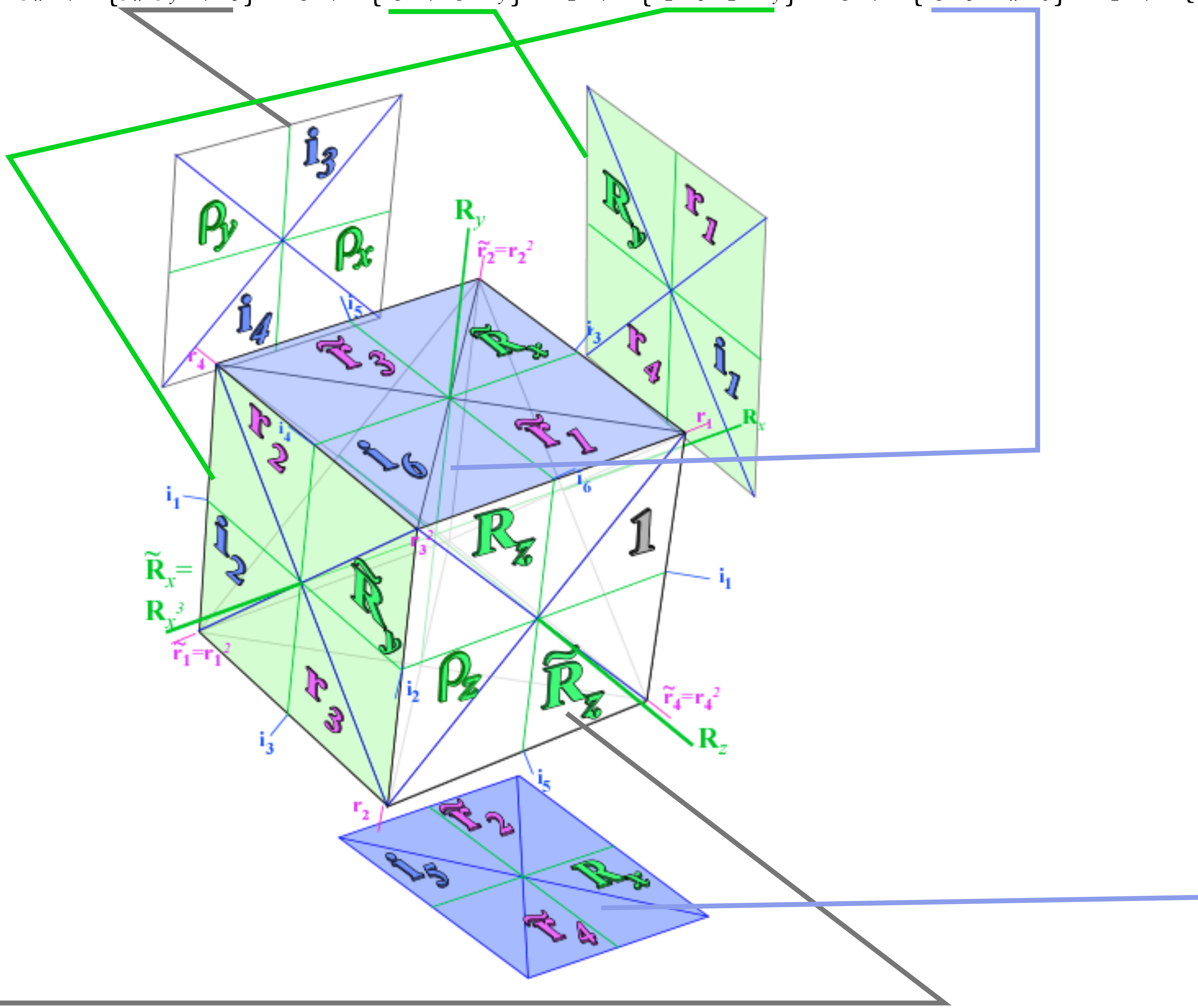
PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}^2_1$	$\mathbf{r}^2_2$	$\mathbf{r}^2_3$	$\mathbf{r}^2_4$	$\mathbf{R}^2_1$	$\mathbf{R}^2_2$	$\mathbf{R}^2_3$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}^3_1$	$\mathbf{R}^3_2$	$\mathbf{R}^3_3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Following development of irreducible projectors:

$$\mathbf{P}^{\mu}_{m_4 m_4} \equiv \mathbf{p}_{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}_{m_4}$$

...uses left-coset combinations... *...and projector "factoring"...*

$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}$, $\boldsymbol{\rho}_x C_4 = \{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\}$, $\mathbf{r}_1 C_4 = \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}$, $\mathbf{r}_2 C_4 = \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}$, $\tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}$, $\tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$



Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*

General development of O⊃C_4 *irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

Deriving diagonal irreducible *O*-representation components $D^{\mu *}_{m_4 m_4}(g)$

$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} (\chi^{\mu *}_{g}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$

$= \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\mathbf{1}}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$

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$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$

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O characters

$O:\chi^{\mu}_{\mathbf{g}}$	**g=1**	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$

C_4 characters

$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$

$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ *for subgroup chain* $O \supset D_4 \supset C_4$

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} (\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$

$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$

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$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$

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O characters

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$

C_4 characters

$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$

$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4m_4} = \sum_g^{\circ O} \frac{\ell^\mu}{\circ O} D^{\mu *}_{m_4m_4}(g)\,\mathbf{g}$ *for subgroup chain* O⊃D_4⊃C_4

$\mathbf{P}^{\mu}_{m_4m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4m_4}(g)$)

$= \sum_g^{\circ O} \frac{\ell^\mu}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$

$= \left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$

$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$

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$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\}$ *Coset*

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O characters

$O:\chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$

C_4 characters

$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$

$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\}$ *Coset*

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ *for subgroup chain* $O \supset D_4 \supset C_4$

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} \quad = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1 \cdot \mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1 \cdot \boldsymbol{\rho}_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right)$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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O characters

$O: \chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{°O} \frac{\ell^{\mu}}{°O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ *for subgroup chain* $O \supset D_4 \supset C_4$

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$= \sum_g^{°O} \frac{\ell^{\mu}}{°O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z + d^{m4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

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$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\}\,Coset$$

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O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 *characters*

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\}\,Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ for subgroup chain O⊃D_4⊃C_4

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *}) \cdot \mathbf{R}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x, \boldsymbol{\rho}_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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O characters

$O: \chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^\mu}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ for subgroup chain $O \supset D_4 \supset C_4$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^\mu}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m_4}_{R_z}\boldsymbol{\rho}_z + d^{m_4}_{\tilde{R}_z}\mathbf{1}\right) =$$

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$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} \textit{Coset}$$

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O characters

$O: \chi^\mu_{\mathbf{g}}$	$\mathbf{g=1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
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$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \textit{Coset}$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^{\mu}}{{}^{\circ}O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ for subgroup chain $O \supset D_4 \supset C_4$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{{}^{\circ}O} (\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *}) \cdot \rho_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *}) \cdot \mathbf{R}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m_4}_{R_z}\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\tilde{R}_z}\mathbf{1} + d^{m_4}_{R_z}\rho_z + 1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

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$$\rho_x C_4 = \rho_x\left\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\rho_x, \rho_y, \mathbf{i}_4, \mathbf{i}_3\right\} Coset$$

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O characters

$O: \chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
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$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{°O} \frac{\ell^\mu}{°O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ for subgroup chain $O \supset D_4 \supset C_4$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{°O} \frac{\ell^\mu}{°O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m_4}_{R_z}\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^\mu \chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\tilde{R}_z}\mathbf{1} + d^{m_4}_{R_z}\rho_z + 1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m_4}_{\rho_z}\mathbf{R}_z + d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_z\right) = \left(\frac{\ell^\mu \chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_z + d^{m_4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} \textit{Coset}$$

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O characters

$O:\chi^\mu_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \textit{Coset}$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{°O} \frac{\ell^{\mu}}{°O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ *for subgroup chain* O⊃D_4⊃C_4

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$)

$$= \sum_{g}^{°O} \frac{\ell^{\mu}}{°O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d_{\rho_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\tilde{\mathbf{R}}_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d_{\rho_z}^{m4}\mathbf{1} + 1\cdot\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z + d_{R_z}^{m4}\rho_z + d_{\tilde{R}_z}^{m4}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d_{\tilde{R}_z}^{m4}\mathbf{1} + d_{R_z}^{m4}\rho_z + 1\cdot\mathbf{R}_z + d_{\rho_z}^{m4}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d_{\rho_z}^{m4}\mathbf{R}_z + d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d_{R_z}^{m4}\mathbf{1} + d_{\tilde{R}_z}^{m4}\rho_z + d_{\rho_z}^{m4}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \rho_x\left(\mathbf{1} + d_{\rho_z}^{m4}\rho_z + d_{R_z}^{m4}\mathbf{R}_z + d_{\tilde{R}_z}^{m4}\tilde{\mathbf{R}}_z\right)\frac{1}{4} =$$

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O characters

$O:\chi_{\mathbf{g}}^{\mu}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
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T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d_{R^p}^{m_4} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ for subgroup chain $O \supset D_4 \supset C_4$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$)

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z + d^{m4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m4}_{R_z}\rho_z + d^{m4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m4}_{\tilde{R}_z}\mathbf{1} + d^{m4}_{R_z}\rho_z + 1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m4}_{\rho_z}\mathbf{R}_z + d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\rho_z + d^{m4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \rho_x\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d^{m4}_{\rho_z}\rho_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right) =$$

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O characters

$O:\chi^{\mu}_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ *for subgroup chain O⊃D₄⊃C₄*

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z + d^{m4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m4}_{R_z}\rho_z + d^{m4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m4}_{\tilde{R}_z}\mathbf{1} + d^{m4}_{R_z}\rho_z + 1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m4}_{\rho_z}\mathbf{R}_z + d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\rho_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\rho_z + d^{m4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$\rho_x C_4 = \rho_x\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\} = \{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}$ *Coset*

$$+\left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \rho_x\left(\mathbf{1} + d^{m4}_{\rho_z}\rho_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d^{m4}_{\rho_z}\rho_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x + d^{m4}_{\rho_z}\rho_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

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O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
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$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C₄ characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$$

$\mathbf{r}_1 C_4 = \mathbf{r}_1\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\} = \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}$ *Coset*

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^\mu}{\circ O} D^{\mu *}_{m_4 m_4}(g)\, \mathbf{g}$ for subgroup chain O⊃D_4⊃C_4

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$)

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^\mu}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^\mu \chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \rho_z\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\rho_z+d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^\mu \chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1}+1\cdot\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z+d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m_4}_{R_z}\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^\mu \chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\tilde{R}_z}\mathbf{1}+d^{m_4}_{R_z}\rho_z + 1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z+d^{m_4}_{\rho_z}\mathbf{R}_z + d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_z\right) = \left(\frac{\ell^\mu \chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{R_z}\mathbf{1}+d^{m_4}_{\tilde{R}_z}\rho_z + d^{m_4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} \textit{Coset}$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \rho_x\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^\mu \chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x+d^{m_4}_{\rho_z}\rho_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^\mu \chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\rho_x+d^{m_4}_{\rho_z}\rho_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

$$+\left(\frac{\ell^\mu}{24}\right)(\chi_{\rho_y}^{\mu *})\cdot \rho_y\left(\mathbf{1}+d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4}$$

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O characters

$O:\chi^\mu_{\mathbf{g}}$	g=1	$\mathbf{r}_{1-4}$ $\tilde{\mathbf{r}}_{1-4}$	ρ_{xyz}	$\mathbf{R}_{xyz}$ $\tilde{\mathbf{R}}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
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$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} \textit{Coset}$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ for subgroup chain $O \supset D_4 \supset C_4$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{1}}^{\mu *}\right)\cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_z}^{\mu *}\right)\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\mathbf{R}_z}^{\mu *}\right)\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m_4}_{R_z}\boldsymbol{\rho}_z + d^{m_4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{\tilde{R}_z}\mathbf{1} + d^{m_4}_{R_z}\boldsymbol{\rho}_z + 1\cdot\mathbf{R}_z + d^{m_4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\tilde{\mathbf{R}}_z}^{\mu *}\right)\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m_4}_{\rho_z}\mathbf{R}_z + d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\boldsymbol{\rho}_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\boldsymbol{\rho}_z + d^{m_4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\boldsymbol{\rho}_x C_4 = \boldsymbol{\rho}_x\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\} Coset$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_x}^{\mu *}\right)\cdot \boldsymbol{\rho}_x\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m_4}_{\rho_z}\boldsymbol{\rho}_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m_4}_{\rho_z}\boldsymbol{\rho}_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)\left(\chi_{\rho_y}^{\mu *}\right)\cdot \boldsymbol{\rho}_y\left(\mathbf{1} + d^{m_4}_{\rho_z}\boldsymbol{\rho}_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_y + d^{m_4}_{\rho_z}\boldsymbol{\rho}_x + d^{m_4}_{R_z}\mathbf{i}_3 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_4\right) =$$

$+$

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$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

General development of irep projectors $\mathbf{P}^{\mu}_{m_4 m_4} = \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} D^{\mu *}_{m_4 m_4}(g)\,\mathbf{g}$ *for subgroup chain* $O \supset D_4 \supset C_4$

$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}^{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}^{m_4}$

(Deriving diagonal irreducible *O*-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O}(\chi_g^{\mu *})\cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z + d^{m4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m4}_{R_z}\boldsymbol{\rho}_z + d^{m4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m4}_{\tilde{R}_z}\mathbf{1} + d^{m4}_{R_z}\boldsymbol{\rho}_z + 1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m4}_{\rho_z}\mathbf{R}_z + d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_z + d^{m4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \boldsymbol{\rho}_x\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m4}_{\rho_z}\boldsymbol{\rho}_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m4}_{\rho_z}\boldsymbol{\rho}_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

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+

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$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\} Coset$$

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$$= \sum_{g}^{\circ O} \frac{\ell^{\mu}}{\circ O} (\chi_g^{\mu *}) \cdot \mathbf{g}\left(\mathbf{p}^{m_4}\right)$$

$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{1}}^{\mu *})\cdot \mathbf{1}\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{1}}^{\mu *}}{96}\right)\left(1\cdot\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_z}^{\mu *})\cdot \boldsymbol{\rho}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_z + d^{m4}_{\rho_z}\mathbf{1} + d^{m4}_{R_z}\tilde{\mathbf{R}}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi_{\rho_z}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\mathbf{1} + 1\cdot\boldsymbol{\rho}_z + d^{m4}_{\tilde{R}_z}\mathbf{R}_z + d^{m4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{R}_z}^{\mu *})\cdot \mathbf{R}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z + d^{m4}_{R_z}\boldsymbol{\rho}_z + d^{m4}_{\tilde{R}_z}\mathbf{1}\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{R}_z}^{\mu *}}{96}\right)\left(d^{m4}_{\tilde{R}_z}\mathbf{1} + d^{m4}_{R_z}\boldsymbol{\rho}_z + 1\cdot\mathbf{R}_z + d^{m4}_{\rho_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\tilde{\mathbf{R}}_z}^{\mu *})\cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m4}_{\rho_z}\mathbf{R}_z + d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_z\right) = \left(\frac{\ell^{\mu}\chi_{\tilde{\mathbf{R}}_z}^{\mu *}}{96}\right)\left(d^{m4}_{R_z}\mathbf{1} + d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_z + d^{m4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_x}^{\mu *})\cdot \boldsymbol{\rho}_x\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m4}_{\rho_z}\boldsymbol{\rho}_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi_{\rho_x}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_x + d^{m4}_{\rho_z}\boldsymbol{\rho}_y + d^{m4}_{R_z}\mathbf{i}_4 + d^{m4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\rho_y}^{\mu *})\cdot \boldsymbol{\rho}_y\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(1\cdot\boldsymbol{\rho}_y + d^{m4}_{\rho_z}\boldsymbol{\rho}_x + d^{m4}_{R_z}\mathbf{i}_3 + d^{m4}_{\tilde{R}_z}\mathbf{i}_4\right) = \left(\frac{\ell^{\mu}\chi_{\rho_y}^{\mu *}}{96}\right)\left(d^{m4}_{\rho_z}\boldsymbol{\rho}_x + 1\cdot\boldsymbol{\rho}_y + d^{m4}_{\tilde{R}_z}\mathbf{i}_4 + d^{m4}_{R_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi_{\mathbf{i}_4}^{\mu *})\cdot \mathbf{i}_4\left(\mathbf{1} + d^{m4}_{\rho_z}\boldsymbol{\rho}_z + d^{m4}_{R_z}\mathbf{R}_z + d^{m4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(1\cdot\mathbf{i}_4 + d^{m4}_{\rho_z}\mathbf{i}_3 + d^{m4}_{R_z}\boldsymbol{\rho}_y + d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_x\right) = \left(\frac{\ell^{\mu}\chi_{\mathbf{i}_4}^{\mu *}}{96}\right)\left(d^{m4}_{\tilde{R}_z}\boldsymbol{\rho}_x + d^{m4}_{R_z}\boldsymbol{\rho}_y + 1\cdot\mathbf{i}_4 + d^{m4}_{\rho_z}\mathbf{i}_3\right)$$

$+$

$$\mathbf{r}_1 C_4 = \mathbf{r}_1\left\{\mathbf{1}, \boldsymbol{\rho}_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\right\} = \left\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\right\} \textit{Coset}$$

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$$= \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\mathbf{1}}) \cdot \mathbf{1}\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{1}}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{1}}}{96}\right)\left(1\cdot\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\rho_z}) \cdot \rho_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_z}}{96}\right)\left(1\cdot\rho_z + d^{m_4}_{\rho_z}\mathbf{1} + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_z}}{96}\right)\left(d^{m_4}_{\rho_z}\mathbf{1} + 1\cdot\rho_z + d^{m_4}_{\tilde{R}_z}\mathbf{R}_z + d^{m_4}_{R_z}\tilde{\mathbf{R}}_z\right)$$

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$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\tilde{\mathbf{R}}_z}) \cdot \tilde{\mathbf{R}}_z\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\tilde{\mathbf{R}}_z}}{96}\right)\left(1\cdot\tilde{\mathbf{R}}_z + d^{m_4}_{\rho_z}\mathbf{R}_z + d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_z\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\tilde{\mathbf{R}}_z}}{96}\right)\left(d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_z + d^{m_4}_{\rho_z}\mathbf{R}_z + 1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\rho_x C_4 = \rho_x\left\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\} = \left\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\right\} \textit{Coset}$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\rho_x}) \cdot \rho_x\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_x}}{96}\right)\left(1\cdot\rho_x + d^{m_4}_{\rho_z}\rho_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_x}}{96}\right)\left(1\cdot\rho_x + d^{m_4}_{\rho_z}\rho_y + d^{m_4}_{R_z}\mathbf{i}_4 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\rho_y}) \cdot \rho_y\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_y}}{96}\right)\left(1\cdot\rho_y + d^{m_4}_{\rho_z}\rho_x + d^{m_4}_{R_z}\mathbf{i}_3 + d^{m_4}_{\tilde{R}_z}\mathbf{i}_4\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\rho_y}}{96}\right)\left(d^{m_4}_{\rho_z}\rho_x + 1\cdot\rho_y + d^{m_4}_{\tilde{R}_z}\mathbf{i}_4 + d^{m_4}_{R_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\mathbf{i}_4}) \cdot \mathbf{i}_4\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{i}_4}}{96}\right)\left(1\cdot\mathbf{i}_4 + d^{m_4}_{\rho_z}\mathbf{i}_3 + d^{m_4}_{R_z}\rho_y + d^{m_4}_{\tilde{R}_z}\rho_x\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{i}_4}}{96}\right)\left(d^{m_4}_{\tilde{R}_z}\rho_x + d^{m_4}_{R_z}\rho_y + 1\cdot\mathbf{i}_4 + d^{m_4}_{\rho_z}\mathbf{i}_3\right)$$

$$+ \left(\frac{\ell^{\mu}}{24}\right)(\chi^{\mu *}_{\mathbf{i}_3}) \cdot \mathbf{i}_3\left(\mathbf{1} + d^{m_4}_{\rho_z}\rho_z + d^{m_4}_{R_z}\mathbf{R}_z + d^{m_4}_{\tilde{R}_z}\tilde{\mathbf{R}}_z\right)\frac{1}{4} = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{i}_3}}{96}\right)\left(1\cdot\mathbf{i}_3 + d^{m_4}_{\rho_z}\mathbf{i}_4 + d^{m_4}_{R_z}\mathbf{1} + d^{m_4}_{\tilde{R}_z}\rho_y\right) = \left(\frac{\ell^{\mu}\chi^{\mu *}_{\mathbf{i}_3}}{96}\right)\left(d^{m_4}_{R_z}\rho_x + d^{m_4}_{\tilde{R}_z}\rho_y + d^{m_4}_{\rho_z}\mathbf{i}_4 + 1\cdot\mathbf{i}_3\right)$$

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etc. etc.

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$\mathbf{P}^{\mu}_{m_4 m_4} = \mathbf{p}_{m_4}\mathbf{P}^{\mu} = \mathbf{P}^{\mu}\mathbf{p}_{m_4}$

(Deriving diagonal irreducible O-representation ("irep") components $D^{\mu *}_{m_4 m_4}(g)$

$$= \sum_{g}^{°O} \frac{\ell^{\mu}}{°O}(\chi_g^{\mu *})\cdot \mathbf{g}\cdot\left(\mathbf{p}_{m_4}\right) = \sum_{g}^{°O} \frac{\ell^{\mu}}{4°O}(\chi_g^{\mu *})\cdot \mathbf{g}\cdot\left(d_1^{m_4}\mathbf{1} + d_{\rho_z}^{m_4}\boldsymbol{\rho}_z + d_{\mathbf{R}_z}^{m_4}\mathbf{R}_z + d_{\tilde{\mathbf{R}}_z}^{m_4}\tilde{\mathbf{R}}_z\right)$$

O characters $\chi^{\mu}_{\mathbf{g}}$

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}^{p}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}^{p}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}_z^{p} = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}$ $\boldsymbol{\rho}_x C_4 = \{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\}$ $\mathbf{r}_1 C_4 = \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}$ $\mathbf{r}_2 C_4 = \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}$ $\tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}$ $\tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$

$$\begin{aligned}
&= \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{1}}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\boldsymbol{\rho}_x}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{r}_1}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{r}_2}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{r}}_1}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{r}}_2}(1, d^{m4}_{\rho_z}, d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}) \\
&+ \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\rho_z}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\boldsymbol{\rho}_y}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{r}_4}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{r}_3}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{r}}_3}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{r}}_4}(d^{m4}_{\rho_z}, 1, d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}) \\
&+ \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{R}_z}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_4}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_1}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_2}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{R}}_x}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{R}_x}(d^{m4}_{\tilde{R}_z}, d^{m4}_{R_z}, 1, d^{m4}_{\rho_z}) \\
&+ \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{R}}_z}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_3}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{R}_y}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\tilde{\mathbf{R}}_y}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_6}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1) + \tfrac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{i}_5}(d^{m4}_{R_z}, d^{m4}_{\tilde{R}_z}, d^{m4}_{\rho_z}, 1)
\end{aligned}$$

Each of 24 columns is a sum of 4 products $\frac{\ell^\mu}{96}\chi^{\mu*}_{\mathbf{g}} d^{m4}_{\rho_p}$ that gives coefficient $? = \frac{\ell^{\mu}}{°O} D^{\mu *}_{m_4 m_4}(g)$ of $\mathbf{P}^{\mu}_{m_4 m_4}$

$$\tfrac{1}{96}(\underline{?}\mathbf{1} + \underline{?}\boldsymbol{\rho}_z + \underline{?}\mathbf{R}_z + \underline{?}\tilde{\mathbf{R}}_z + \underline{?}\boldsymbol{\rho}_x + \underline{?}\boldsymbol{\rho}_y + \underline{?}\mathbf{i}_4 + \underline{?}\mathbf{i}_3 + \underline{?}\mathbf{r}_1 + \underline{?}\mathbf{r}_4 + \underline{?}\mathbf{i}_1 + \underline{?}\mathbf{R}_y + \underline{?}\mathbf{r}_2 + \underline{?}\mathbf{r}_3 + \underline{?}\mathbf{i}_2 + \underline{?}\tilde{\mathbf{R}}_y + \underline{?}\tilde{\mathbf{r}}_1 + \underline{?}\tilde{\mathbf{r}}_3 + \underline{?}\tilde{\mathbf{R}}_x + \underline{?}\mathbf{i}_6 + \underline{?}\tilde{\mathbf{r}}_2 + \underline{?}\tilde{\mathbf{r}}_4 + \underline{?}\mathbf{R}_x + \underline{?}\mathbf{i}_5)$$

This $\mathbf{P}^{\mu}_{m_4 m_4}$-sum is in order of left cosets $\mathbf{g}\cdot C_4$ of C_4 in O. (Examples follow.)

$\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}$ $\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\}$ $\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}$ $\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}$ $\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}$ $\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal D_2 vs. unOrmal D_2)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g=1}$	$\mathbf{r}^{p}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}^{p}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu = A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}^{p}_{z} = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4} = \mathbf{p}_{0_4}\mathbf{P}^{E} = \mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^E}{\circ O}(\chi^{E}_{g}) \cdot \mathbf{g} \cdot \left(\mathbf{p}_{0_4}\right) = \sum_{g}^{\circ O} \frac{2}{96}(\chi^{E}_{g}) \cdot \mathbf{g} \cdot \left(1 \cdot \mathbf{1} + 1 \cdot \boldsymbol{\rho}_z + 1 \cdot \mathbf{R}_z + 1 \cdot \tilde{\mathbf{R}}_z \right)$$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}^{p}_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}^{p}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p}=e^{\frac{2\pi i\, m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i\, m_4\cdot p}{4}}\mathbf{R}^{p}_{z}=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^E}{\circ O}(\chi^E_g)\cdot\mathbf{g}\cdot\left(\mathbf{p}_{0_4}\right)=\sum_{g}^{\circ O}\frac{2}{96}(\chi^E_g)\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}+1\cdot\boldsymbol{\rho}_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4=\mathbf{1}\left\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\right\}\quad \boldsymbol{\rho}_x C_4=\left\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\right\}\quad \mathbf{r}_1C_4=\left\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\right\}\quad \mathbf{r}_2C_4=\left\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\right\}\quad \tilde{\mathbf{r}}_1C_4=\left\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\right\}\quad \tilde{\mathbf{r}}_2C_4=\left\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\right\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\boldsymbol{\rho}_x}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})+\tfrac{1}{48}\chi^E_{\boldsymbol{\rho}_y}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)$$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}^{p}_{1-4}$	ρ_{xyz}	$\mathbf{R}^{p}_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4 \cdot p}{4}}$$

C4 characters

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4 \cdot p}{4}} \mathbf{R}^{p}_{z} = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4} = \mathbf{p}_{0_4}\mathbf{P}^{E} = \mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$= \sum_{g}^{\circ O} \frac{\ell^{E}}{\circ O}(\chi^{E}_{g})\cdot\mathbf{g}\cdot(\mathbf{p}_{0_4}) = \sum_{g}^{\circ O} \frac{2}{96}(\chi^{E}_{g})\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^{E}_{\mathbf{1}}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^{E}_{\rho_x}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{r}_1}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{r}_2}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{r}}_1}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{r}}_2}(1,d^{0_4}_{\rho_z},d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^{E}_{\rho_z}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z}) +\tfrac{1}{48}\chi^{E}_{\rho_y}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{r}_4}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{r}_3}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z}) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{r}}_3}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z}) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{r}}_4}(d^{0_4}_{\rho_z},1,d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^{E}_{\mathbf{R}_z}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_4}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_1}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_2}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z}) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{R}}_x}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z}) +\tfrac{1}{48}\chi^{E}_{\mathbf{R}_x}(d^{0_4}_{\tilde{R}_z},d^{0_4}_{R_z},1,d^{0_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{R}}_z}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_3}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1) +\tfrac{1}{48}\chi^{E}_{\mathbf{R}_y}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1) +\tfrac{1}{48}\chi^{E}_{\tilde{\mathbf{R}}_y}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_6}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1) +\tfrac{1}{48}\chi^{E}_{\mathbf{i}_5}(d^{0_4}_{R_z},d^{0_4}_{\tilde{R}_z},d^{0_4}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,+1,+1) +\tfrac{1}{48}(+2)(1,+1,+1,+1) +\tfrac{1}{48}(-1)(1,+1,+1,+1) +\tfrac{1}{48}(-1)(1,+1,+1,+1) +\tfrac{1}{48}(-1)(1,+1,+1,+1) +\tfrac{1}{48}(-1)(1,+1,+1,+1)$$

$$+\tfrac{1}{48}(+2)(+1,1,+1,+1) +\tfrac{1}{48}(+2)(+1,1,+1,+1) +\tfrac{1}{48}(-1)(+1,1,+1,+1) +\tfrac{1}{48}(-1)(+1,1,+1,+1) +\tfrac{1}{48}(-1)(+1,1,+1,+1) +\tfrac{1}{48}(-1)(+1,1,+1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,1,+1) +\tfrac{1}{48}(0)(+1,+1,1,+1) +\tfrac{1}{48}(0)(+1,+1,1,+1) +\tfrac{1}{48}(0)(+1,+1,1,+1) +\tfrac{1}{48}(0)(+1,+1,1,+1) +\tfrac{1}{48}(0)(+1,+1,1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,+1,1) +\tfrac{1}{48}(0)(+1,+1,+1,1) +\tfrac{1}{48}(0)(+1,+1,+1,1) +\tfrac{1}{48}(0)(+1,+1,+1,1) +\tfrac{1}{48}(0)(+1,+1,+1,1) +\tfrac{1}{48}(0)(+1,+1,+1,1)$$

4, 4, 4, 4, 4, 4, 4, 4, -2,-2,-2,-2, -2,-2,-2,-2, -2,-2,-2,-2, -2,-2,-2,-2,

Calculating $\mathbf{P}^{E}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}^p_{1-4}$	ρ_{xyz}	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p}=e^{\frac{2\pi i m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}^p_z=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{E}_{0_4 0_4}=\mathbf{p}_{0_4}\mathbf{P}^{E}=\mathbf{P}^{E}\mathbf{p}_{0_4}$$

$$=\sum_{g}^{°O}\frac{\ell^E}{°O}(\chi^E_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{0_4})=\sum_{g}^{°O}\frac{2}{96}(\chi^E_g)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\rho_x}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})+\tfrac{1}{48}\chi^E_{\rho_y}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})+\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})+\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)+\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(+2)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)+\tfrac{1}{48}(-1)(1,+1,+1,+1)$$

$$+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(+2)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)+\tfrac{1}{48}(-1)(+1,1,+1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)+\tfrac{1}{48}(0)(+1,+1,1,+1)$$

$$+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)+\tfrac{1}{48}(0)(+1,+1,+1,1)$$

4, 4, 4, 4,   4, 4, 4, 4,   -2,-2,-2,-2,   -2,-2,-2,-2,   -2,-2,-2,-2,   -2,-2,-2,-2,

$$\mathbf{P}^{E}_{0_4 0_4}=\tfrac{1}{12}(1\mathbf{1}+1\rho_z+1\mathbf{R}_z+1\tilde{\mathbf{R}}_z+1\rho_x+1\rho_y+1\mathbf{i}_4+1\mathbf{i}_3\ -\tfrac{1}{2}\mathbf{r}_1-\tfrac{1}{2}\mathbf{r}_4-\tfrac{1}{2}\mathbf{i}_1-\tfrac{1}{2}\mathbf{R}_y\ -\tfrac{1}{2}\mathbf{r}_2-\tfrac{1}{2}\mathbf{r}_3-\tfrac{1}{2}\mathbf{i}_2-\tfrac{1}{2}\tilde{\mathbf{R}}_y\ -\tfrac{1}{2}\tilde{\mathbf{r}}_1-\tfrac{1}{2}\tilde{\mathbf{r}}_3-\tfrac{1}{2}\tilde{\mathbf{R}}_x-\tfrac{1}{2}\mathbf{i}_6\ -\tfrac{1}{2}\tilde{\mathbf{r}}_2-\tfrac{1}{2}\tilde{\mathbf{r}}_4-\tfrac{1}{2}\mathbf{R}_x-\tfrac{1}{2}\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^{E}_{0_4 0_4}=\tfrac{1}{12}[(1)\cdot\mathbf{1}\mathbf{p}_{0_4}+(1)\cdot\rho_x\mathbf{p}_{0_4}+(-\tfrac{1}{2})\cdot\mathbf{r}_1\mathbf{p}_{0_4}+(-\tfrac{1}{2})\cdot\mathbf{r}_2\mathbf{p}_{0_4}+(-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{0_4}+(-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{0_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^{E}_{0_4 0_4}=\tfrac{1}{12}(1\cdot\mathbf{1}\ -\tfrac{1}{2}\mathbf{r}_1-\tfrac{1}{2}\mathbf{r}_2\ -\tfrac{1}{2}\mathbf{r}_3-\tfrac{1}{2}\mathbf{r}_4\ -\tfrac{1}{2}\tilde{\mathbf{r}}_1-\tfrac{1}{2}\tilde{\mathbf{r}}_2\ -\tfrac{1}{2}\tilde{\mathbf{r}}_3-\tfrac{1}{2}\tilde{\mathbf{r}}_4\ +1\rho_x+1\rho_y+1\rho_z\ -\tfrac{1}{2}\mathbf{R}_x-\tfrac{1}{2}\mathbf{R}_y+1\mathbf{R}_z\ -\tfrac{1}{2}\tilde{\mathbf{R}}_x-\tfrac{1}{2}\tilde{\mathbf{R}}_y+1\tilde{\mathbf{R}}_z\ -\tfrac{1}{2}\mathbf{i}_1-\tfrac{1}{2}\mathbf{i}_2\ +1\mathbf{i}_3+1\mathbf{i}_4\ -\tfrac{1}{2}\mathbf{i}_5-\tfrac{1}{2}\mathbf{i}_6)$$

Review Octahedral $O_h \supset O$ group operator structure

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ subgroup chain correlations

Comparison of $O \supset D_4 \supset C_4$ and $O \supset D_4 \supset D_2$ correlations and level/projector splitting

$O \supset D_4 \supset C_4$ subgroup chain splitting

$O \supset D_4 \supset D_2$ subgroup chain splitting (nOrmal D_2 vs. unOrmal D_2)

$O_h \supset O \supset D_4 \supset C_{4v}$ and $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ subgroup splitting

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O \supset C_4$

Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ and representations $D^{\mu}_{m_4 m_4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ induced representation $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples

Elementary induced representation $0_4(C_4)\uparrow O$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 spectroscopy

Calculating $\mathbf{P}^{E}_{2_4 2_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	(1)	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}^p_{1-4}$	$\boldsymbol{\rho}_{xyz}$	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

C_4 characters

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$$

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^E_{2_4 2_4} = \mathbf{p}_{2_4}\mathbf{P}^E = \mathbf{P}^E\mathbf{p}_{2_4}$$

$$= \sum_g^{\circ O} \frac{\ell^E}{\circ O}(\chi_g^E)\cdot\mathbf{g}\cdot(\mathbf{p}_{2_4}) = \sum_g^{\circ O}\frac{2}{96}(\chi_g^E)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\boldsymbol{\rho}_z-1\cdot\mathbf{R}_z-1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \boldsymbol{\rho}_x C_4=\{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{48}\chi^E_{\mathbf{1}}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^E_{\rho_x}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^E_{\mathbf{r}_1}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^E_{\mathbf{r}_2}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_1}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z}) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_2}(1,d^{2_4}_{\rho_z},d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{48}\chi^E_{\rho_z}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z}) +\tfrac{1}{48}\chi^E_{\rho_y}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z}) +\tfrac{1}{48}\chi^E_{\mathbf{r}_4}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z}) +\tfrac{1}{48}\chi^E_{\mathbf{r}_3}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z}) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_3}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z}) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{r}}_4}(d^{2_4}_{\rho_z},1,d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z})$$

$$+\tfrac{1}{48}\chi^E_{\mathbf{R}_z}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z}) +\tfrac{1}{48}\chi^E_{\mathbf{i}_4}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z}) +\tfrac{1}{48}\chi^E_{\mathbf{i}_1}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z}) +\tfrac{1}{48}\chi^E_{\mathbf{i}_2}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z}) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_x}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z}) +\tfrac{1}{48}\chi^E_{\mathbf{R}_x}(d^{2_4}_{\tilde{R}_z},d^{2_4}_{R_z},1,d^{2_4}_{\rho_z})$$

$$+\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_z}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1) +\tfrac{1}{48}\chi^E_{\mathbf{i}_3}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1) +\tfrac{1}{48}\chi^E_{\mathbf{R}_y}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1) +\tfrac{1}{48}\chi^E_{\tilde{\mathbf{R}}_y}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1) +\tfrac{1}{48}\chi^E_{\mathbf{i}_6}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1) +\tfrac{1}{48}\chi^E_{\mathbf{i}_5}(d^{2_4}_{R_z},d^{2_4}_{\tilde{R}_z},d^{2_4}_{\rho_z},1)$$

$$=\tfrac{1}{48}(+2)(1,+1,-1,-1)\ =\tfrac{1}{48}(+2)(1,+1,-1,-1)\ +\tfrac{1}{48}(-1)(1,+1,-1,-1)\ +\tfrac{1}{48}(-1)(1,+1,-1,-1)\ +\tfrac{1}{48}(-1)(1,+1,-1,-1)\ +\tfrac{1}{48}(-1)(1,+1,-1,-1)$$

$$+\tfrac{1}{48}(+2)(+1,1,-1,-1)\ +\tfrac{1}{48}(+2)(+1,1,-1,-1)\ +\tfrac{1}{48}(-1)(+1,1,-1,-1)\ +\tfrac{1}{48}(-1)(+1,1,-1,-1)\ +\tfrac{1}{48}(-1)(+1,1,-1,-1)\ +\tfrac{1}{48}(-1)(+1,1,-1,-1)$$

$$+\tfrac{1}{48}(0)(-1,-1,1,+1)\ +\tfrac{1}{48}(0)(-1,-1,1,+1)\ +\tfrac{1}{48}(0)(-1,-1,1,+1)\ +\tfrac{1}{48}(0)(-1,-1,1,+1)\ +\tfrac{1}{48}(0)(-1,-1,1,+1)\ +\tfrac{1}{48}(0)(-1,-1,1,+1)$$

$$+\tfrac{1}{48}(0)(-1,-1,+1,1)\ +\tfrac{1}{48}(0)(-1,-1,+1,1)\ +\tfrac{1}{48}(0)(-1,-1,+1,1)\ +\tfrac{1}{48}(0)(-1,-1,+1,1)\ +\tfrac{1}{48}(0)(-1,-1,+1,1)\ +\tfrac{1}{48}(0)(-1,-1,+1,1)$$

4, 4,-4,-4, 4, 4,-4,-4, -2,-2, 2, 2, -2,-2, 2, 2, -2,-2, 2, 2, -2,-2, 2, 2,

$$\tfrac{1}{12}(1\mathbf{1}+1\boldsymbol{\rho}_z-1\mathbf{R}_z-1\tilde{\mathbf{R}}_z+1\boldsymbol{\rho}_x+1\boldsymbol{\rho}_y-1\mathbf{i}_4-1\mathbf{i}_3\quad -\tfrac{1}{2}\mathbf{r}_1-\tfrac{1}{2}\mathbf{r}_4+\tfrac{1}{2}\mathbf{i}_1+\tfrac{1}{2}\mathbf{R}_y\quad -\tfrac{1}{2}\mathbf{r}_2-\tfrac{1}{2}\mathbf{r}_3+\tfrac{1}{2}\mathbf{i}_2+\tfrac{1}{2}\tilde{\mathbf{R}}_y\quad -\tfrac{1}{2}\tilde{\mathbf{r}}_1-\tfrac{1}{2}\tilde{\mathbf{r}}_3+\tfrac{1}{2}\tilde{\mathbf{R}}_x+\tfrac{1}{2}\mathbf{i}_6\quad -\tfrac{1}{2}\tilde{\mathbf{r}}_2-\tfrac{1}{2}\tilde{\mathbf{r}}_4+\tfrac{1}{2}\mathbf{R}_x+\tfrac{1}{2}\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^E_{2_4 2_4} = \tfrac{1}{12}[(1)\cdot\mathbf{1}\mathbf{p}_{2_4} + (1)\cdot\boldsymbol{\rho}_x\mathbf{p}_{2_4} + (-\tfrac{1}{2})\cdot\mathbf{r}_1\mathbf{p}_{2_4} + (-\tfrac{1}{2})\cdot\mathbf{r}_2\mathbf{p}_{2_4} + (-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{2_4} + (-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{2_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^E_{2_4 2_4} = \tfrac{1}{12}(1\mathbf{1}\ -\tfrac{1}{2}\mathbf{r}_1-\tfrac{1}{2}\mathbf{r}_2\ -\tfrac{1}{2}\mathbf{r}_3-\tfrac{1}{2}\mathbf{r}_4\ -\tfrac{1}{2}\tilde{\mathbf{r}}_1-\tfrac{1}{2}\tilde{\mathbf{r}}_2\ -\tfrac{1}{2}\tilde{\mathbf{r}}_3-\tfrac{1}{2}\tilde{\mathbf{r}}_4\ +1\boldsymbol{\rho}_x+1\boldsymbol{\rho}_y+1\boldsymbol{\rho}_z\ +\tfrac{1}{2}\mathbf{R}_x+\tfrac{1}{2}\mathbf{R}_y-1\mathbf{R}_z\ +\tfrac{1}{2}\tilde{\mathbf{R}}_x+\tfrac{1}{2}\tilde{\mathbf{R}}_y-1\tilde{\mathbf{R}}_z\ +\tfrac{1}{2}\mathbf{i}_1+\tfrac{1}{2}\mathbf{i}_2\ -1\mathbf{i}_3-1\mathbf{i}_4\ +\tfrac{1}{2}\mathbf{i}_5+\tfrac{1}{2}\mathbf{i}_6)$$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

$O \supset D_4 \supset C_4$ *subgroup chain splitting*

$O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*

$O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

Elementary induced representation $0_4(C_4)\uparrow O$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 *spectroscopy*

Calculating $\mathbf{P}^{T_1}{}_{0_4 0_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}=\mathbf{1}$	$\mathbf{r}^p_{1-4}$	ρ_{xyz}	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}^p_z = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{T_1}_{0_4 0_4} = \mathbf{p}_{0_4}\mathbf{P}^{T_1} = \mathbf{P}^{T_1}\mathbf{p}_{0_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{T_1}}{\circ O}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{0_4}) = \sum_g^{\circ O}\frac{3}{96}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\rho_z+1\cdot\mathbf{R}_z+1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_1}_{\mathbf{1}}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_1}_{\rho_x}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_1}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_2}(1,d^{04}_{\rho_z},d^{04}_{R_z},d^{04}_{\tilde{R}_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\rho_z}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z}) +\tfrac{1}{32}\chi^{T_1}_{\rho_y}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z}) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_3}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z}) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_4}(d^{04}_{\rho_z},1,d^{04}_{\tilde{R}_z},d^{04}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_z}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_4}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_1}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_2}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z}) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_x}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z}) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_x}(d^{04}_{\tilde{R}_z},d^{04}_{R_z},1,d^{04}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_z}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_3}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_y}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_y}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_6}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_5}(d^{04}_{R_z},d^{04}_{\tilde{R}_z},d^{04}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,+1,+1,+1) +\tfrac{1}{32}(-1)(1,+1,+1,+1) +\tfrac{1}{32}(0)(1,+1,+1,+1) +\tfrac{1}{32}(0)(1,+1,+1,+1) +\tfrac{1}{32}(0)(1,+1,+1,+1) +\tfrac{1}{32}(0)(1,+1,+1,+1)$$

$$+\tfrac{1}{32}(-1)(+1,1,+1,+1) +\tfrac{1}{32}(-1)(+1,1,+1,+1) +\tfrac{1}{32}(0)(+1,1,+1,+1) +\tfrac{1}{32}(0)(+1,1,+1,+1) +\tfrac{1}{32}(0)(+1,1,+1,+1) +\tfrac{1}{32}(0)(+1,1,+1,+1)$$

$$+\tfrac{1}{32}(+1)(+1,+1,1,+1) +\tfrac{1}{32}(-1)(+1,+1,1,+1) +\tfrac{1}{32}(-1)(+1,+1,1,+1) +\tfrac{1}{32}(-1)(+1,+1,1,+1) +\tfrac{1}{32}(+1)(+1,+1,1,+1) +\tfrac{1}{32}(+1)(+1,+1,1,+1)$$

$$+\tfrac{1}{32}(+1)(+1,+1,+1,1) +\tfrac{1}{32}(-1)(+1,+1,+1,1) +\tfrac{1}{32}(+1)(+1,+1,+1,1) +\tfrac{1}{32}(+1)(+1,+1,+1,1) +\tfrac{1}{32}(-1)(+1,+1,+1,1) +\tfrac{1}{32}(-1)(+1,+1,+1,1)$$

4, 4, 0, 0, -4,-4,-4,-4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

$$\tfrac{1}{8}(\underline{1}\mathbf{1}+\underline{1}\rho_z+\underline{1}\mathbf{R}_z+\underline{1}\tilde{\mathbf{R}}_z\ \underline{-1}\rho_x\ \underline{-1}\rho_y\ \underline{-1}\mathbf{i}_4\ \underline{-1}\mathbf{i}_3 +\underline{0}\mathbf{r}_1+\underline{0}\mathbf{r}_4+\underline{0}\mathbf{i}_1+\underline{0}\mathbf{R}_y +\underline{0}\mathbf{r}_2+\underline{0}\mathbf{r}_3+\underline{0}\mathbf{i}_2+\underline{0}\tilde{\mathbf{R}}_y +\underline{0}\tilde{\mathbf{r}}_1+\underline{0}\tilde{\mathbf{r}}_3+\underline{0}\tilde{\mathbf{R}}_x+\underline{0}\mathbf{i}_6 +\underline{0}\tilde{\mathbf{r}}_2+\underline{0}\tilde{\mathbf{r}}_4+\underline{0}\mathbf{R}_x+\underline{0}\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^{T_1}_{0_4 0_4} = \tfrac{1}{8}[(1)\cdot\mathbf{1}\mathbf{p}_{0_4} + (-1)\cdot\rho_x\mathbf{p}_{0_4} + (0)\cdot\mathbf{r}_1\mathbf{p}_{0_4} + (0)\cdot\mathbf{r}_2\mathbf{p}_{0_4} + (0)\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{0_4} + (0)\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{0_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^{T_1}_{0_4 0_4} = \tfrac{1}{8}(1\cdot\mathbf{1} +0+0+0+0+0+0+0+0 +1\rho_z -1\rho_x -1\rho_y +0+0+1\mathbf{R}_z +0+0+1\tilde{\mathbf{R}}_z +0+0+0+0 -1\mathbf{i}_4 -1\mathbf{i}_3)$$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*

Calculating $\mathbf{P}^{T_1}{}_{1_4 1_4}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}^p_{1-4}$	ρ_{xyz}	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p}=e^{\frac{2\pi i m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}e^{\frac{2\pi i m_4\cdot p}{4}}\mathbf{R}^p_z=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

$$\mathbf{P}^{T_1}_{1_4 1_4}=\mathbf{p}_{1_4}\mathbf{P}^{T_1}=\mathbf{P}^{T_1}\mathbf{p}_{1_4}$$

$$=\sum_{g}^{\circ O}\frac{\ell^{T_1}}{\circ O}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{1_4})=\sum_{g}^{\circ O}\frac{3}{96}(\chi^{T_1}_g)\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}-1\cdot\rho_z+i\cdot\mathbf{R}_z-i\cdot\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_1}_{\mathbf{1}}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_x}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\rho_z}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\rho_y}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{r}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{r}}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde R_z},d^{1_4}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_z}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_4}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_1}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_2}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_x}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_x}(d^{1_4}_{\tilde R_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_z}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_3}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{R}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\tilde{\mathbf{R}}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_6}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)+\tfrac{1}{32}\chi^{T_1}_{\mathbf{i}_5}(d^{1_4}_{R_z},d^{1_4}_{\tilde R_z},d^{1_4}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,-1,+i,-i)+\tfrac{1}{32}(-1)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)+\tfrac{1}{32}(0)(1,-1,+i,-i)$$

$$+\tfrac{1}{32}(-1)(-1,1,-i,+i)+\tfrac{1}{32}(-1)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)+\tfrac{1}{32}(0)(-1,1,-i,+i)$$

$$+\tfrac{1}{32}(+1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(-1)(-i,+i,1,-1)+\tfrac{1}{32}(+1)(-i,+i,1,-1)+\tfrac{1}{32}(+1)(-i,+i,1,-1)$$

$$+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(+1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)+\tfrac{1}{32}(-1)(+i,-i,-1,1)$$

+4,-4, 4i,-4i, 0, 0, 0, 0, +2i, -2i,-2, +2, +2i, -2i,-2, +2, -2i,+2i,+2, -2 -2i,+2i,+2, -2.

$$\tfrac{1}{8}(1\mathbf{1}-1\rho_z+i\mathbf{R}_z-i\tilde{\mathbf{R}}_z+0\rho_x+0\rho_y+0\mathbf{i}_4+0\mathbf{i}_3+\tfrac{i}{2}\mathbf{r}_1-\tfrac{i}{2}\mathbf{r}_4-\tfrac{1}{2}\mathbf{i}_1+\tfrac{1}{2}\mathbf{R}_y+\tfrac{i}{2}\mathbf{r}_2-\tfrac{i}{2}\mathbf{r}_3-\tfrac{1}{2}\mathbf{i}_2+\tfrac{1}{2}\tilde{\mathbf{R}}_y-\tfrac{i}{2}\tilde{\mathbf{r}}_1+\tfrac{i}{2}\tilde{\mathbf{r}}_3+\tfrac{1}{2}\tilde{\mathbf{R}}_x-\tfrac{1}{2}\mathbf{i}_6-\tfrac{i}{2}\tilde{\mathbf{r}}_2+\tfrac{i}{2}\tilde{\mathbf{r}}_4+\tfrac{1}{2}\mathbf{R}_x-\tfrac{1}{2}\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^{T_1}_{1_4 1_4}=\tfrac{1}{8}[(1)\cdot\mathbf{1}\mathbf{p}_{1_4}+(0)\cdot\rho_x\mathbf{p}_{1_4}+(\tfrac{i}{2})\cdot\mathbf{r}_1\mathbf{p}_{1_4}+(\tfrac{i}{2})\cdot\mathbf{r}_2\mathbf{p}_{1_4}+(-\tfrac{i}{2})\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{1_4}+(-\tfrac{i}{2})\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{1_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^{T_1}_{1_4 1_4}=\tfrac{1}{8}(1\cdot\mathbf{1}+\tfrac{i}{2}\mathbf{r}_1+\tfrac{i}{2}\mathbf{r}_2-\tfrac{i}{2}\mathbf{r}_3-\tfrac{i}{2}\mathbf{r}_4-\tfrac{i}{2}\tilde{\mathbf{r}}_1-\tfrac{i}{2}\tilde{\mathbf{r}}_2+\tfrac{i}{2}\tilde{\mathbf{r}}_3+\tfrac{i}{2}\tilde{\mathbf{r}}_4+0\rho_x+0\rho_y-1\rho_z+\tfrac{1}{2}\mathbf{R}_x+\tfrac{1}{2}\mathbf{R}_y+i\mathbf{R}_z+\tfrac{1}{2}\tilde{\mathbf{R}}_x+\tfrac{1}{2}\tilde{\mathbf{R}}_y-i\tilde{\mathbf{R}}_z-\tfrac{i}{2}\mathbf{i}_1-\tfrac{i}{2}\mathbf{i}_2+0\mathbf{i}_3+0\mathbf{i}_4-\tfrac{i}{2}\mathbf{i}_5-\tfrac{i}{2}\mathbf{i}_6)$$

Review Octahedral $O_h \supset O$ group operator structure

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ subgroup chain correlations

Comparison of $O \supset D_4 \supset C_4$ and $O \supset D_4 \supset D_2$ correlations and level/projector splitting

$O \supset D_4 \supset C_4$ subgroup chain splitting

$O \supset D_4 \supset D_2$ subgroup chain splitting (nOrmal D_2 vs. unOrmal D_2)

$O_h \supset O \supset D_4 \supset C_{4v}$ and $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ subgroup splitting

Splitting O class projectors $\mathbf{P}^{\mu}$ into irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ for $O \supset C_4$

Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ and representations $D^{\mu}_{m_4 m_4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ induced representation $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples

Elementary induced representation $0_4(C_4)\uparrow O$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 spectroscopy

Calculating $\mathbf{P}^{T_2}_{2_4 2_4}$

O⊃C_4	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	.	.	.
$A_2\downarrow C_4$	.	.	1	.
$E\downarrow C_4$	1	.	1	.
$T_1\downarrow C_4$	1	1	.	1
$T_2\downarrow C_4$	.	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	**g**=1	$\mathbf{r}^p_{1-4}$	ρ_{xyz}	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p} = e^{\frac{2\pi i m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i m_4\cdot p}{4}} \mathbf{R}_z^p = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{T_2}_{2_4 2_4} = \mathbf{p}_{2_4}\mathbf{P}^{T_2} = \mathbf{P}^{T_2}\mathbf{p}_{2_4}$$

$$= \sum_g^{°O} \frac{\ell^{T_2}}{°O}(\chi_g^{T_2})\cdot \mathbf{g}\cdot(\mathbf{p}_{2_4}) = \sum_g^{°O} \frac{3}{96}(\chi_g^{T_2})\cdot \mathbf{g}\cdot(1\cdot\mathbf{1}+1\cdot\rho_z-1\cdot\mathbf{R}_z-1\cdot\tilde{\mathbf{R}}_z)$$

$$\mathbf{1}C_4 = \mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\} \quad \rho_x C_4 = \{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\} \quad \mathbf{r}_1 C_4 = \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\} \quad \mathbf{r}_2 C_4 = \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\} \quad \tilde{\mathbf{r}}_1 C_4 = \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\} \quad \tilde{\mathbf{r}}_2 C_4 = \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_2}_{\mathbf{1}}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_x}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_1}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_2}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_1}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_2}(1,d^{24}_{\rho_z},d^{24}_{R_z},d^{24}_{\tilde{R}_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\rho_z}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\rho_y}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_4}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_3}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_3}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_4}(d^{24}_{\rho_z},1,d^{24}_{\tilde{R}_z},d^{24}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_z}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_4}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_1}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_2}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_x}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z}) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_x}(d^{24}_{\tilde{R}_z},d^{24}_{R_z},1,d^{24}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_z}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_3}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_y}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_y}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_6}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1)) + \tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_5}(d^{24}_{R_z},d^{24}_{\tilde{R}_z},d^{24}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,+1,-1,-1) + \tfrac{1}{32}(-1)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1) + \tfrac{1}{32}(0)(1,+1,-1,-1)$$

$$+\tfrac{1}{32}(-1)(+1,1,-1,-1) + \tfrac{1}{32}(-1)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1) + \tfrac{1}{32}(0)(+1,1,-1,-1)$$

$$+\tfrac{1}{32}(-1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(+1)(-1,-1,1,+1) + \tfrac{1}{32}(-1)(-1,-1,1,+1) + \tfrac{1}{32}(-1)(-1,-1,1,+1)$$

$$+\tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1) + \tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(-1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1) + \tfrac{1}{32}(+1)(-1,-1,+1,1)$$

4, 4,-4,-4, *-4,-4, 4, 4,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,* *0, 0, 0, 0,*

$$\tfrac{1}{8}(1\mathbf{1}+1\rho_z-1\mathbf{R}_z-1\tilde{\mathbf{R}}_z \; -1\rho_x-1\rho_y+1\mathbf{i}_4+1\mathbf{i}_3 \; +0\mathbf{r}_1+0\mathbf{r}_4+0\mathbf{i}_1+0\mathbf{R}_y \; +0\mathbf{r}_2+0\mathbf{r}_3+0\mathbf{i}_2+0\tilde{\mathbf{R}}_y \; +0\tilde{\mathbf{r}}_1+0\tilde{\mathbf{r}}_3+0\tilde{\mathbf{R}}_x+0\mathbf{i}_6 \; +0\tilde{\mathbf{r}}_2+0\tilde{\mathbf{r}}_4+0\mathbf{R}_x+0\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^{T_2}_{2_4 2_4} = \tfrac{1}{8}[(1)\cdot\mathbf{1}\mathbf{p}_{2_4} + (1)\cdot\rho_x\mathbf{p}_{2_4} + (0)\cdot\mathbf{r}_1\mathbf{p}_{2_4} + (0)\cdot\mathbf{r}_2\mathbf{p}_{2_4} + (0)\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{2_4} + (0)\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{2_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^{T_2}_{2_4 2_4} = \tfrac{1}{8}(1\cdot\mathbf{1} \; +0+0+0+0+0+0+0+0 \; -1\rho_x-1\rho_y+1\rho_z \; +0+0-1\mathbf{R}_z+0+0-1\tilde{\mathbf{R}}_z \; +0+0+0+0+1\mathbf{i}_4+1\mathbf{i}_3)$$

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$O \supset C_4$ induced representation $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples

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Calculating $\mathbf{P}^{T_2}_{1_4 1_4}$

O⊃C_4	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

O characters

$O:\chi^{\mu}_{\mathbf{g}}$	$\mathbf{g}$=1	$\mathbf{r}^p_{1-4}$	ρ_{xyz}	$\mathbf{R}^p_{xyz}$	$\mathbf{i}_{1-6}$
$\mu=A_1$	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	1	-1
T_2	3	0	-1	-1	1

$$d^{m_4}_{R^p} = e^{\frac{2\pi i\, m_4\cdot p}{4}}$$

C_4 characters

$$\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{\frac{2\pi i\, m_4\cdot p}{4}}\mathbf{R}^p_z = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$$\mathbf{P}^{T_2}_{1_4 1_4} = \mathbf{p}_{1_4}\mathbf{P}^{T_2} = \mathbf{P}^{T_2}\mathbf{p}_{1_4}$$

$$= \sum_g^{\circ O} \frac{\ell^{T_2}}{\circ O}(\chi^{T_2}_g)\cdot\mathbf{g}\cdot(\mathbf{p}_{1_4}) = \sum_g^{\circ O}\frac{3}{96}(\chi^{T_2}_g)\cdot\mathbf{g}\cdot\left(1\cdot\mathbf{1}-1\cdot\rho_z+i\cdot\mathbf{R}_z-i\cdot\tilde{\mathbf{R}}_z\right)$$

$$\mathbf{1}C_4=\mathbf{1}\{\mathbf{1},\rho_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\quad \rho_x C_4=\{\rho_x,\rho_y,\mathbf{i}_4,\mathbf{i}_3\}\quad \mathbf{r}_1C_4=\{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\quad \mathbf{r}_2C_4=\{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\quad \tilde{\mathbf{r}}_1C_4=\{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\quad \tilde{\mathbf{r}}_2C_4=\{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$=\tfrac{1}{32}\chi^{T_2}_{\mathbf{1}}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_2}_{\rho_x}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_1}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z}) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_2}(1,d^{1_4}_{\rho_z},d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\rho_z}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z}) +\tfrac{1}{32}\chi^{T_2}_{\rho_y}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{r}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z}) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_3}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z}) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{r}}_4}(d^{1_4}_{\rho_z},1,d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_z}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_4}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_1}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_2}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z}) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z}) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_x}(d^{1_4}_{\tilde{R}_z},d^{1_4}_{R_z},1,d^{1_4}_{\rho_z})$$

$$+\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_z}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_3}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{R}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_2}_{\tilde{\mathbf{R}}_y}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_6}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1) +\tfrac{1}{32}\chi^{T_2}_{\mathbf{i}_5}(d^{1_4}_{R_z},d^{1_4}_{\tilde{R}_z},d^{1_4}_{\rho_z},1)$$

$$=\tfrac{1}{32}(+3)(1,-1,+i,-i) +\tfrac{1}{32}(-1)(1,-1,+i,-i) +\tfrac{1}{32}(0)(1,-1,+i,-i) +\tfrac{1}{32}(0)(1,-1,+i,-i) +\tfrac{1}{32}(0)(1,-1,+i,-i) +\tfrac{1}{32}(0)(1,-1,+i,-i)$$

$$+\tfrac{1}{32}(-1)(-1,1,-i,+i) +\tfrac{1}{32}(-1)(-1,1,-i,+i) +\tfrac{1}{32}(0)(-1,1,-i,+i) +\tfrac{1}{32}(0)(-1,1,-i,+i) +\tfrac{1}{32}(0)(-1,1,-i,+i) +\tfrac{1}{32}(0)(-1,1,-i,+i)$$

$$+\tfrac{1}{32}(-1)(-i,+i,1,-1) +\tfrac{1}{32}(+1)(-i,+i,1,-1) +\tfrac{1}{32}(+1)(-i,+i,1,-1) +\tfrac{1}{32}(+1)(-i,+i,1,-1) +\tfrac{1}{32}(-1)(-i,+i,1,-1) +\tfrac{1}{32}(-1)(-i,+i,1,-1)$$

$$+\tfrac{1}{32}(-1)(+i,-i,-1,1) +\tfrac{1}{32}(+1)(+i,-i,-1,1) +\tfrac{1}{32}(-1)(+i,-i,-1,1) +\tfrac{1}{32}(-1)(+i,-i,-1,1) +\tfrac{1}{32}(+1)(+i,-i,-1,1) +\tfrac{1}{32}(+1)(+i,-i,-1,1)$$

+4,-4, 4i,-4i,   0,0,0,0,   -2i,2i,2,-2,   -2i,2i,2,-2,   2i,-2i,-2,2,   2i,-2i,-2,2.

$$\tfrac{1}{8}(1\mathbf{1}-1\rho_z+i\mathbf{R}_z-i\tilde{\mathbf{R}}_z \quad +0\rho_x+0\rho_y+0\mathbf{i}_4+0\mathbf{i}_3 \quad -\tfrac{i}{2}\mathbf{r}_1+\tfrac{i}{2}\mathbf{r}_4+\tfrac{1}{2}\mathbf{i}_1-\tfrac{1}{2}\mathbf{R}_y \quad -\tfrac{i}{2}\mathbf{r}_2+\tfrac{i}{2}\mathbf{r}_3+\tfrac{1}{2}\mathbf{i}_2-\tfrac{1}{2}\tilde{\mathbf{R}}_y \quad +\tfrac{i}{2}\tilde{\mathbf{r}}_1-\tfrac{i}{2}\tilde{\mathbf{r}}_3-\tfrac{1}{2}\tilde{\mathbf{R}}_x+\tfrac{1}{2}\mathbf{i}_6 \quad +\tfrac{i}{2}\tilde{\mathbf{r}}_2-\tfrac{i}{2}\tilde{\mathbf{r}}_4-\tfrac{1}{2}\mathbf{R}_x+\tfrac{1}{2}\mathbf{i}_5)$$

Coset-factored sum:

$$\mathbf{P}^{T_2}_{1_4 1_4} = \tfrac{1}{8}[(1)\cdot\mathbf{1}\mathbf{p}_{1_4} \quad +(0)\cdot\rho_x\mathbf{p}_{1_4} \quad +(-\tfrac{i}{2})\cdot\mathbf{r}_1\mathbf{p}_{1_4} \quad +(-\tfrac{i}{2})\cdot\mathbf{r}_2\mathbf{p}_{1_4} \quad +(-\tfrac{i}{2})\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{1_4} \quad +(-\tfrac{i}{2})\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{1_4}]$$

Broken-class-ordered sum:

$$\mathbf{P}^{T_2}_{1_4 1_4} = \tfrac{1}{8}(1\cdot\mathbf{1} \quad -\tfrac{i}{2}\mathbf{r}_1-\tfrac{i}{2}\mathbf{r}_2 \quad +\tfrac{i}{2}\mathbf{r}_3+\tfrac{i}{2}\mathbf{r}_4 \quad +\tfrac{i}{2}\tilde{\mathbf{r}}_1+\tfrac{i}{2}\tilde{\mathbf{r}}_2 \quad -\tfrac{i}{2}\tilde{\mathbf{r}}_3-\tfrac{i}{2}\tilde{\mathbf{r}}_4 \quad +0\rho_x+0\rho_y-1\rho_z \quad -\tfrac{1}{2}\mathbf{R}_x-\tfrac{1}{2}\mathbf{R}_y+i\mathbf{R}_z \quad -\tfrac{1}{2}\tilde{\mathbf{R}}_x-\tfrac{1}{2}\tilde{\mathbf{R}}_y-i\tilde{\mathbf{R}}_z \quad +\tfrac{1}{2}\mathbf{i}_1+\tfrac{1}{2}\mathbf{i}_2 \quad +0\mathbf{i}_3+0\mathbf{i}_4 \quad +\tfrac{1}{2}\mathbf{i}_5+\tfrac{1}{2}\mathbf{i}_6)$$

Review Octahedral $O_h \supset O$ *group operator structure*
Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*
- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$
- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

➡ $O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples* ⬅
- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*

O⊃C_4 induced representation 0_4↑O~A_1⊕T_1⊕E and spectral analysis examples

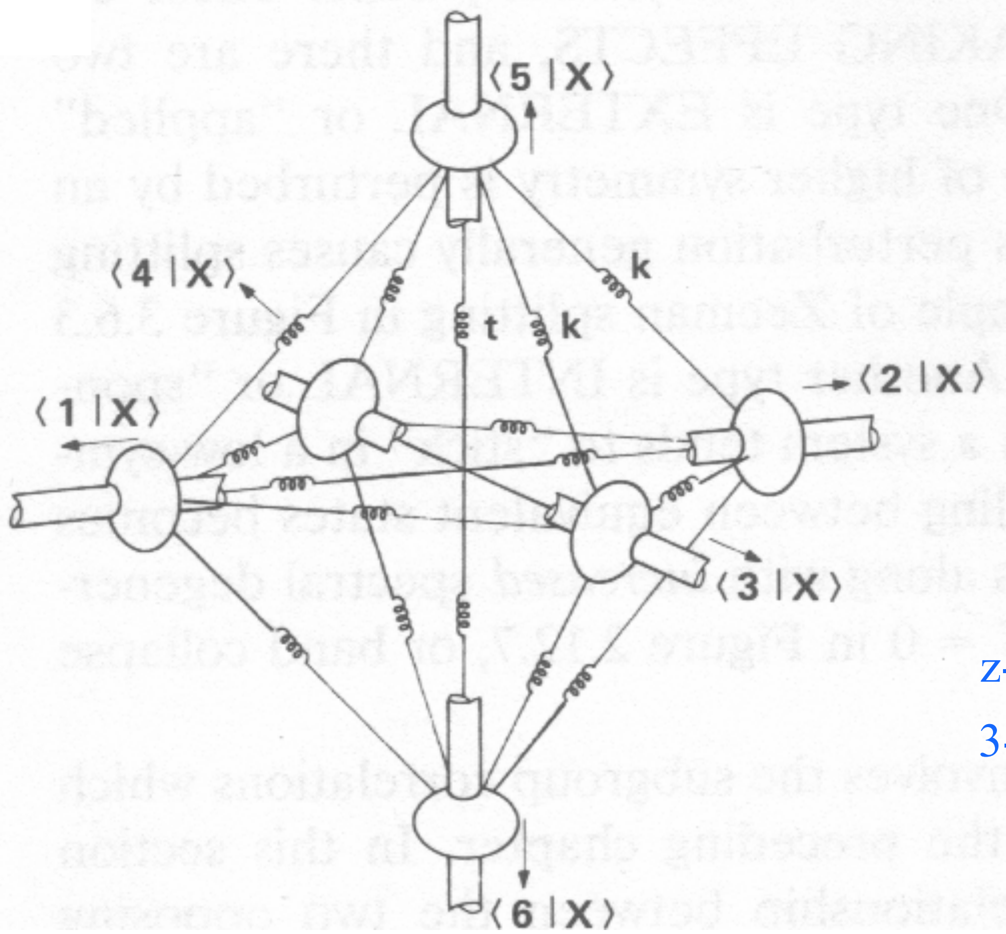
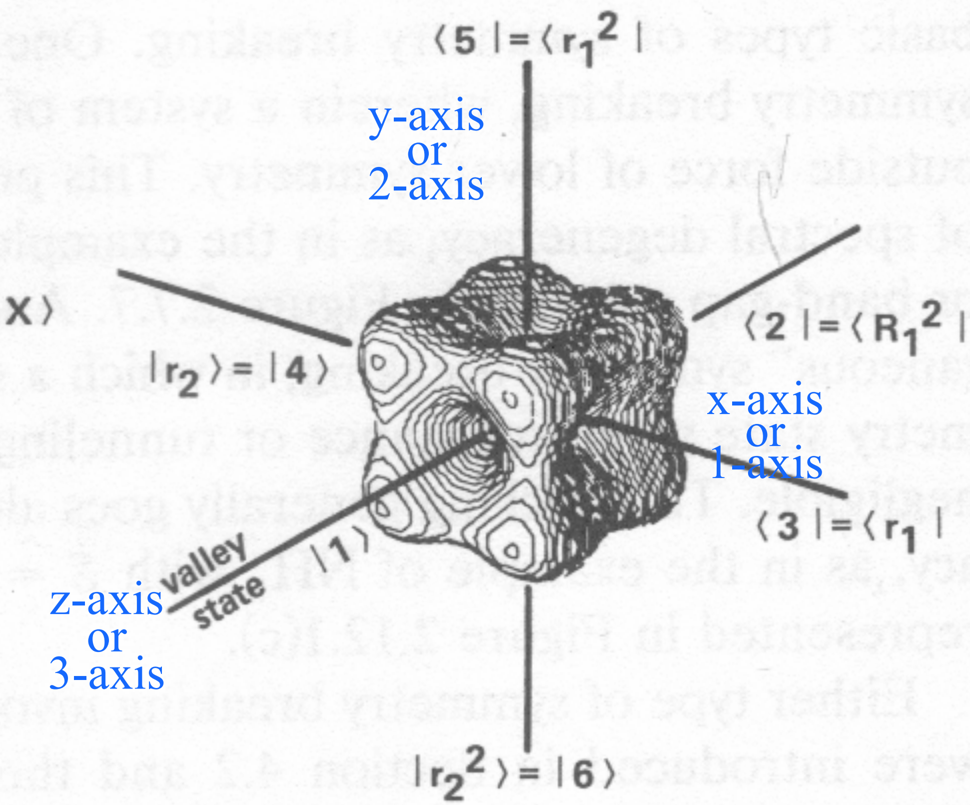


Fig. 4.3.1 PSDS



Solve XY_6 radial vibration **K**=**a**-matrix

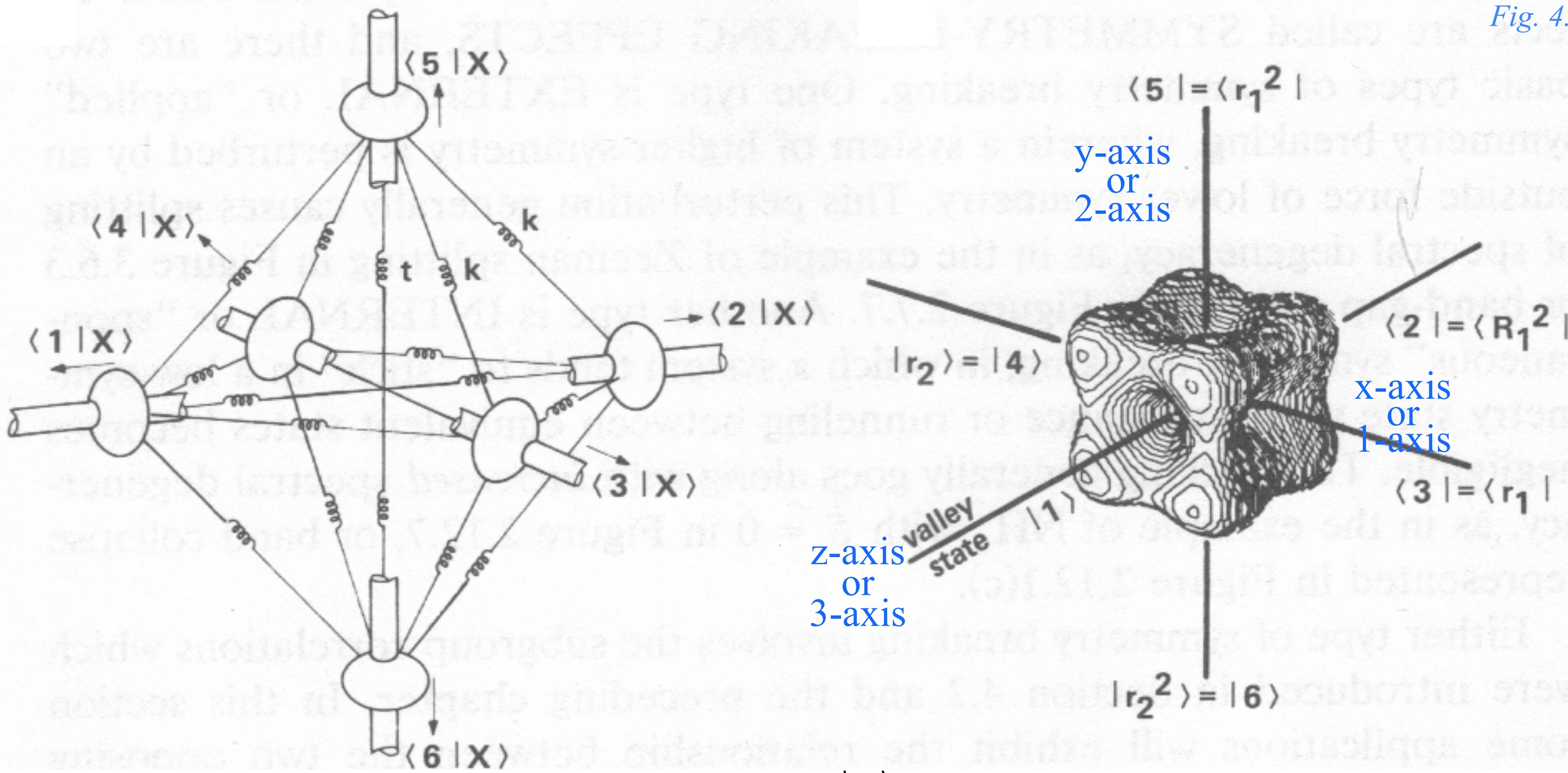
$$\begin{pmatrix} \langle 1|\mathbf{a}|1\rangle & \langle 1|\mathbf{a}|2\rangle & \cdots & \langle 1|\mathbf{a}|6\rangle \\ \langle 2|\mathbf{a}|1\rangle & \langle 2|\mathbf{a}|2\rangle & \cdots & \langle 2|\mathbf{a}|6\rangle \\ \cdot & & & \\ \cdot & h=2k+t, & & \\ \cdot & s=k/2 & & \\ \langle 6|\mathbf{a}|1\rangle & \langle 6|\mathbf{a}|2\rangle & \cdots & \langle y|\mathbf{a}|6\rangle \end{pmatrix} = \begin{pmatrix} h & t & s & s & s & s \\ t & h & s & s & s & s \\ s & s & h & t & s & s \\ s & s & t & h & s & s \\ s & s & s & s & h & t \\ s & s & s & s & t & h \end{pmatrix},$$

Solve SF_6 J-tunneling Hamiltonian **H**

$$\begin{pmatrix} \langle 1|\mathbf{H}|1\rangle & \langle 1|\mathbf{H}|2\rangle & \cdots & \langle 1|\mathbf{H}|6\rangle \\ \langle 2|\mathbf{H}|1\rangle & \langle 2|\mathbf{H}|2\rangle & \cdots & \langle 2|\mathbf{H}|6\rangle \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \langle 6|\mathbf{H}|1\rangle & \langle 6|\mathbf{H}|2\rangle & \cdots & \langle 6|\mathbf{H}|6\rangle \end{pmatrix} = \begin{pmatrix} H & T & S & S & S & S \\ T & H & S & S & S & S \\ S & S & H & T & S & S \\ S & S & T & H & S & S \\ S & S & S & S & H & T \\ S & S & S & S & T & H \end{pmatrix}.$$

O⊃C_4 *induced representation* $0_4\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

Fig. 4.3.1 PSDS



Assuming *C_4-local symmetry conditions* for $|\mathbf{1}\rangle$ state

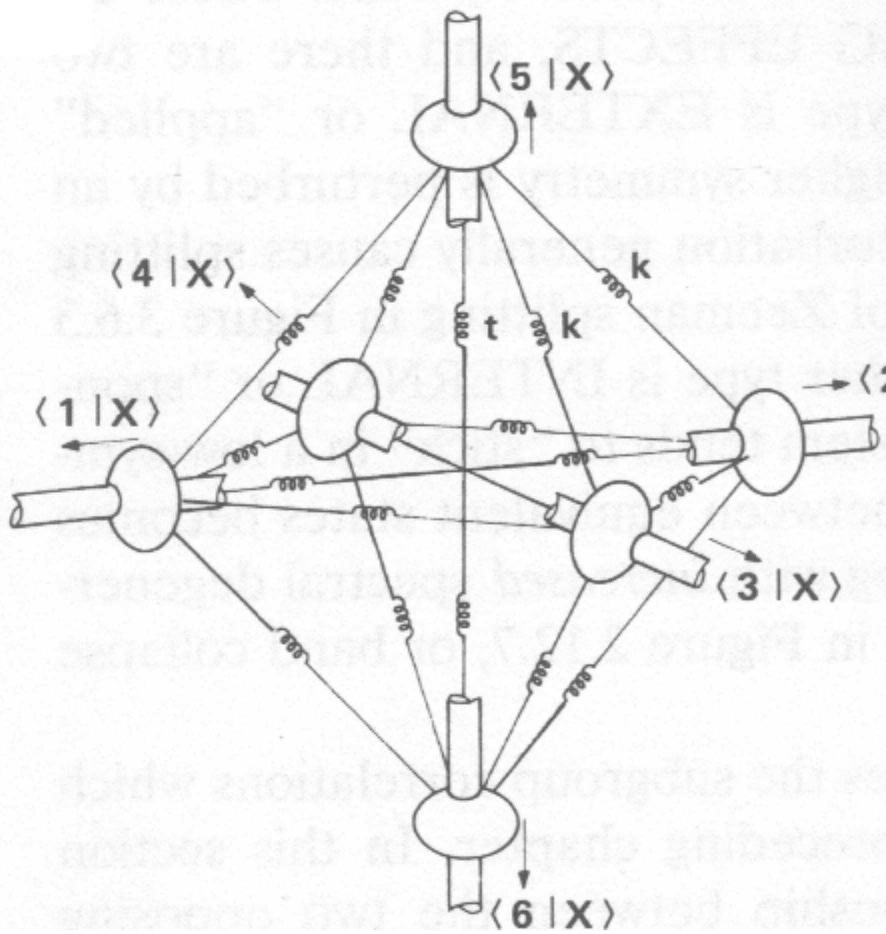
$$|1\rangle = \mathbf{1}|1\rangle = R_3|1\rangle = R_3^2|1\rangle = R_3^3|1\rangle$$

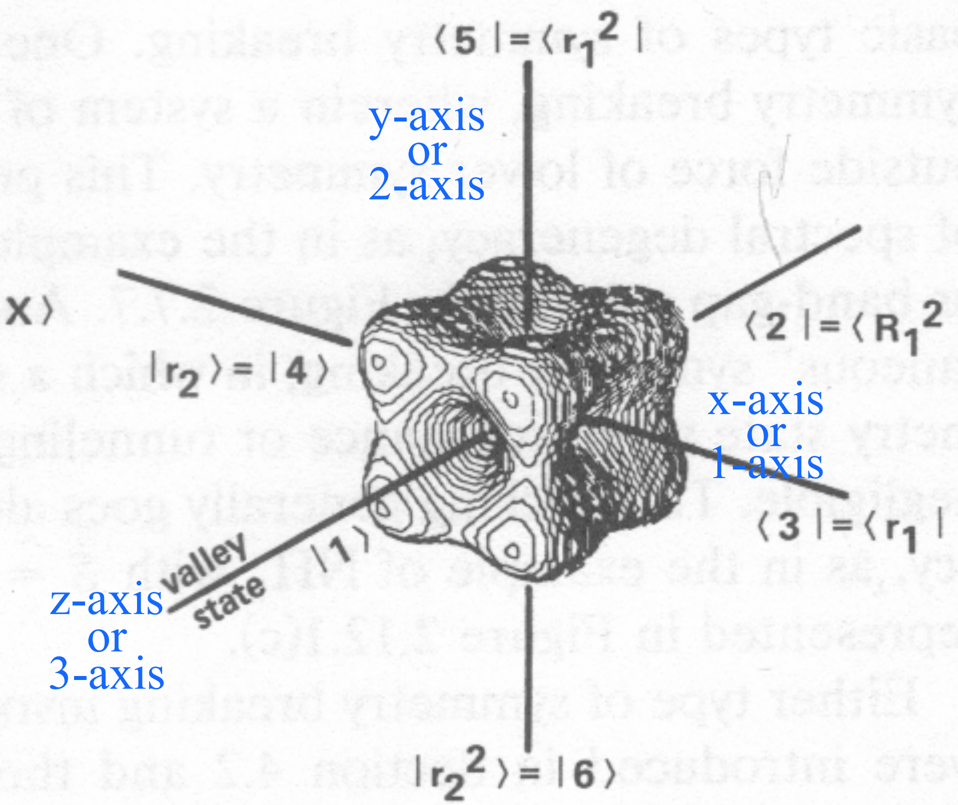
O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. and Newer Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1^2$	$\mathbf{r}_2^2$	$\mathbf{r}_3^2$	$\mathbf{r}_4^2$	$\mathbf{R}_1^2$	$\mathbf{R}_2^2$	$\mathbf{R}_3^2$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_1^3$	$\mathbf{R}_2^3$	$\mathbf{R}_3^3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

O⊃C_4 *induced representation* $0_4\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

Fig. 4.3.1 PSDS





Assuming *C_4-local symmetry conditions* for $|1\rangle$ state

$$|1\rangle = \mathbf{1}|1\rangle = R_3|1\rangle = R_3^2|1\rangle = R_3^3|1\rangle$$

Using *C_4-local symmetry projector equations* $P^A \equiv P^{0_4} = (\mathbf{1} + R_3 + R_3^2 + R_3^3)/4.$

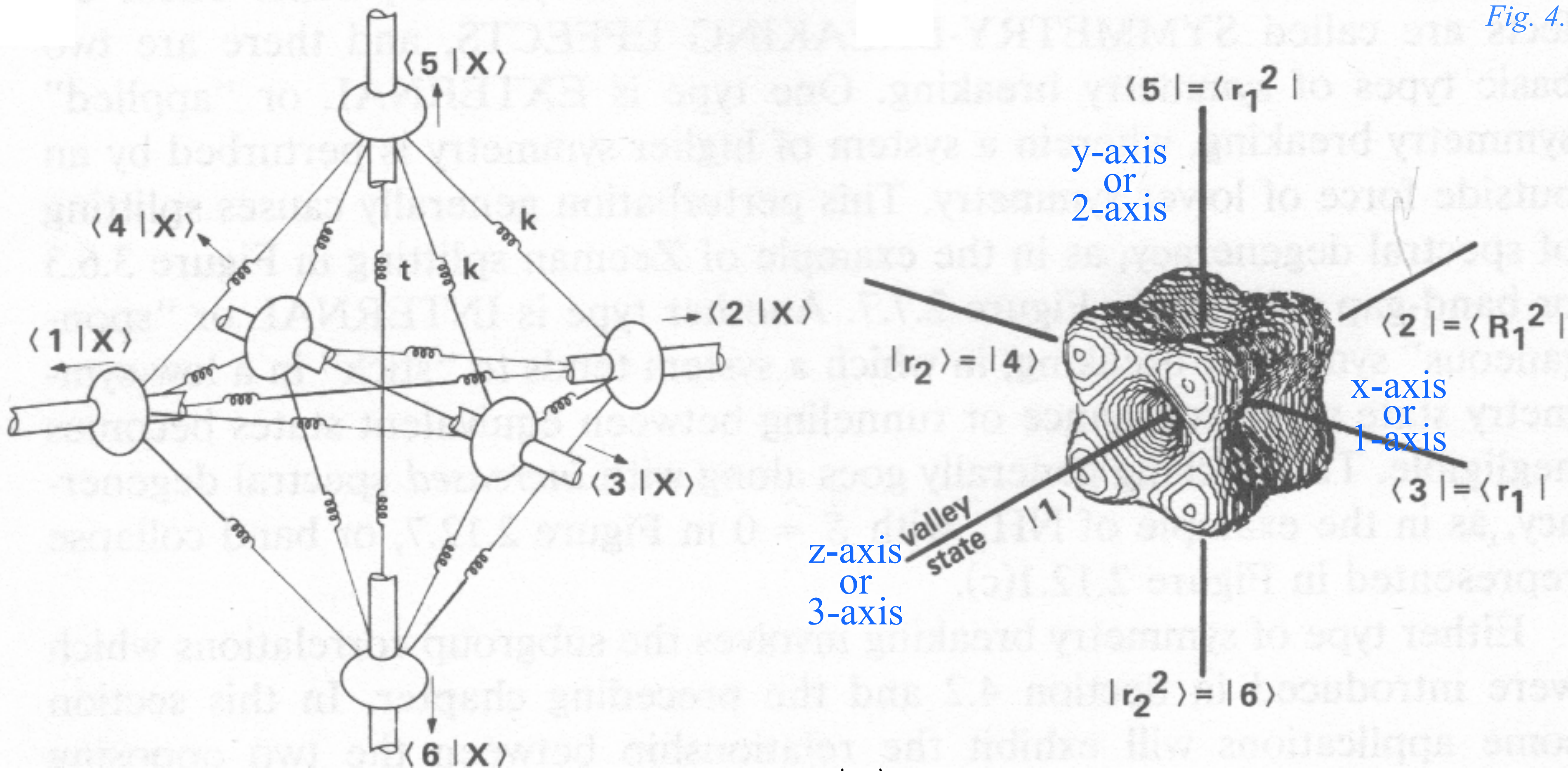
$$|1\rangle = P^{0_4}|1\rangle = (\mathbf{1} + R_3 + R_3^2 + R_3^3)|1\rangle/4.$$

O operators (Two notations: Older *Princ.of Symm.Dynamics and Spectra.* *and Newer* *Int.J.Mol.Sci)*

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1^2$	$\mathbf{r}_2^2$	$\mathbf{r}_3^2$	$\mathbf{r}_4^2$	$\mathbf{R}_1^2$	$\mathbf{R}_2^2$	$\mathbf{R}_3^2$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_1^3$	$\mathbf{R}_2^3$	$\mathbf{R}_3^3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

O⊃C_4 *induced representation* $0_4\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

Fig. 4.3.1 PSDS



Assuming *C_4-local symmetry conditions* for $|\mathbf{1}\rangle$ state

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$$|g\rangle = g|1\rangle = gR_3|1\rangle = gR_3^2|1\rangle = gR_3^3|1\rangle$$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. *and Newer* Int.J.Mol.Sci*)*

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1^2$	$\mathbf{r}_2^2$	$\mathbf{r}_3^2$	$\mathbf{r}_4^2$	$\mathbf{R}_1^2$	$\mathbf{R}_2^2$	$\mathbf{R}_3^2$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_1^3$	$\mathbf{R}_2^3$	$\mathbf{R}_3^3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

$O \supset C_4$ induced representation $0_4 \uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples

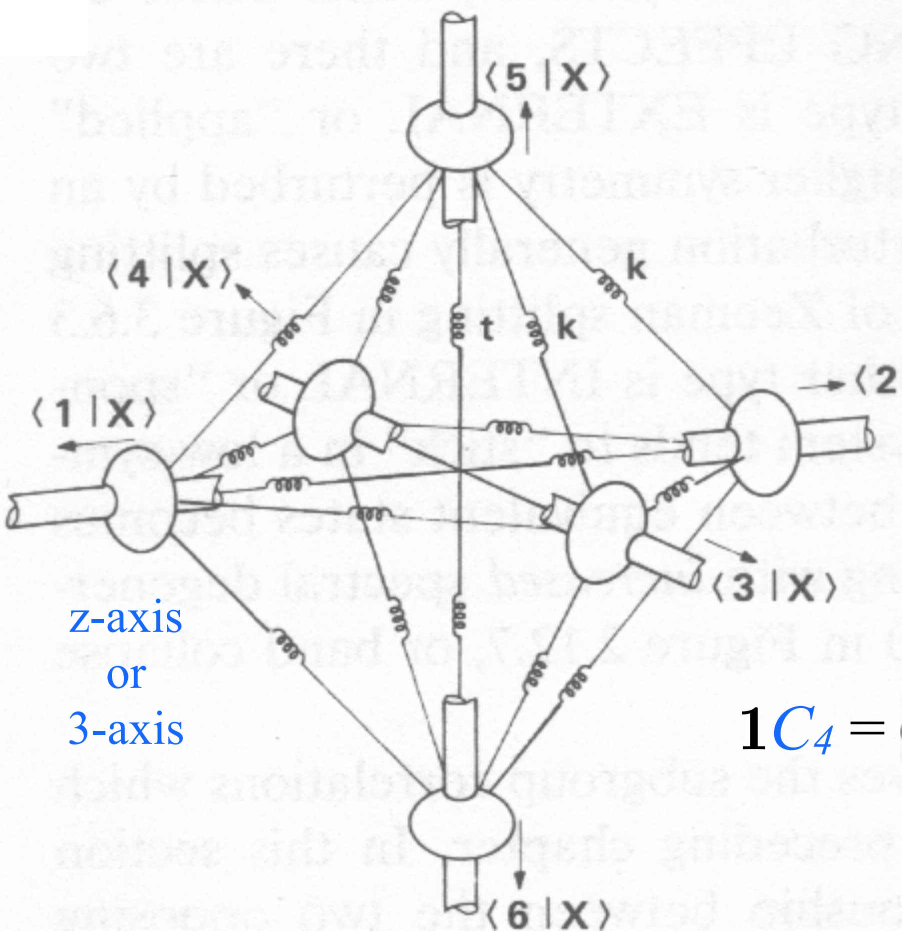
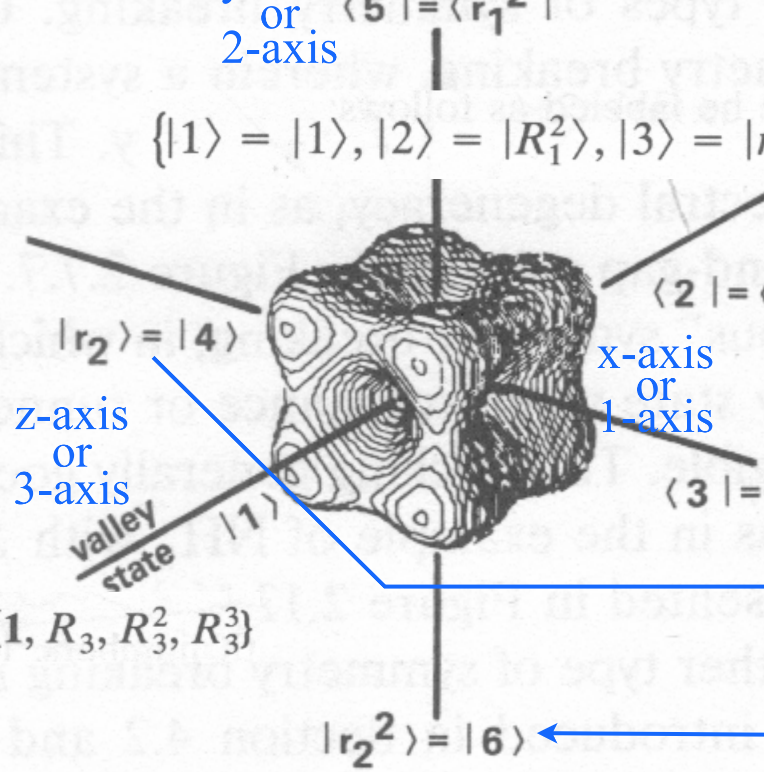


Fig. 4.3.1 PSDS



Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

$$\{|1\rangle = |1\rangle, |2\rangle = |R_1^2\rangle, |3\rangle = |r_1\rangle, |4\rangle = |r_2\rangle, |5\rangle = |r_1^2\rangle, |6\rangle = |r_2^2\rangle$$

$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$

$$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$$

$$R_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (R_1^2, i_4, R_2^2, i_3),$$
$$r_1(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1, i_1, r_4, R_2),$$
$$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$$
$$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$$
$$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$$

Assuming *C_4-local symmetry conditions* for $|\mathbf{1}\rangle$ state

$$|1\rangle = \mathbf{1}|1\rangle = R_3|1\rangle = R_3^2|1\rangle = R_3^3|1\rangle$$

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These apply to all six $|\mathbf{g}\rangle = \mathbf{g}|\mathbf{1}\rangle$-base states. $\quad |g\rangle = |gR_3\rangle = |gR_3^2\rangle = |gR_3^3\rangle$

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O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. and Newer Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1^2$	$\mathbf{r}_2^2$	$\mathbf{r}_3^2$	$\mathbf{r}_4^2$	$\mathbf{R}_1^2$	$\mathbf{R}_2^2$	$\mathbf{R}_3^2$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_1^3$	$\mathbf{R}_2^3$	$\mathbf{R}_3^3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

$O \supset C_4$ induced representation $0_4 \uparrow O \sim A_1 \oplus T_1 \oplus E$ and spectral analysis examples

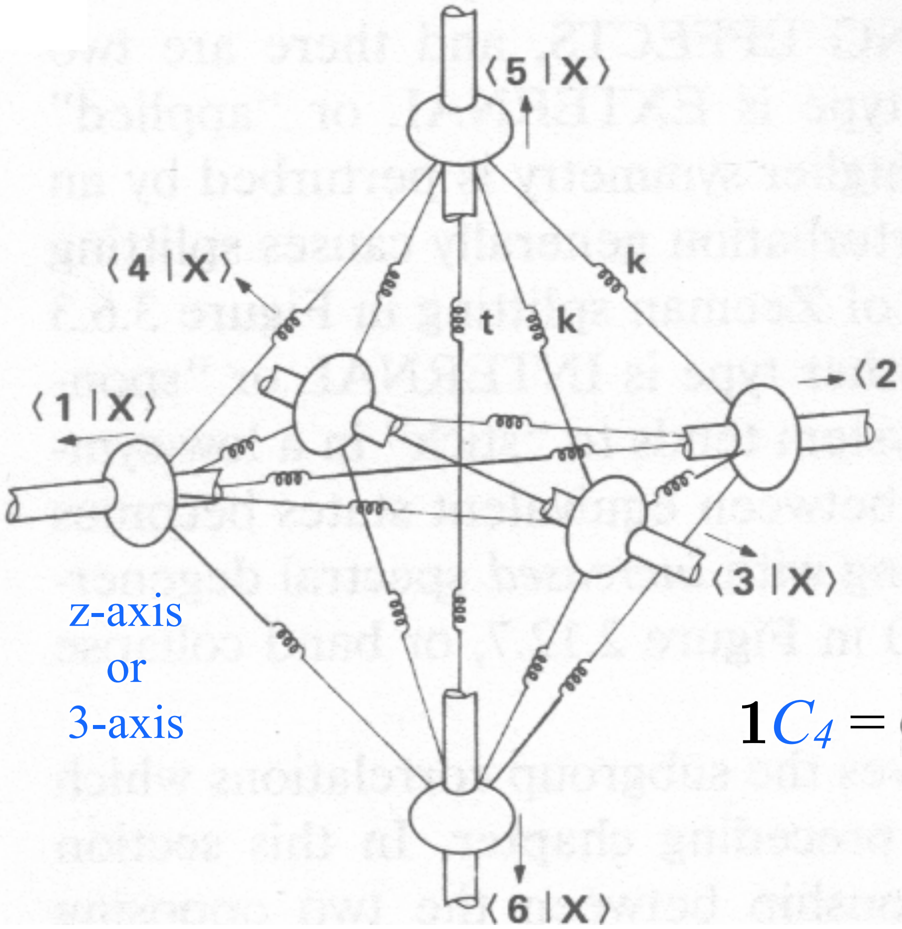
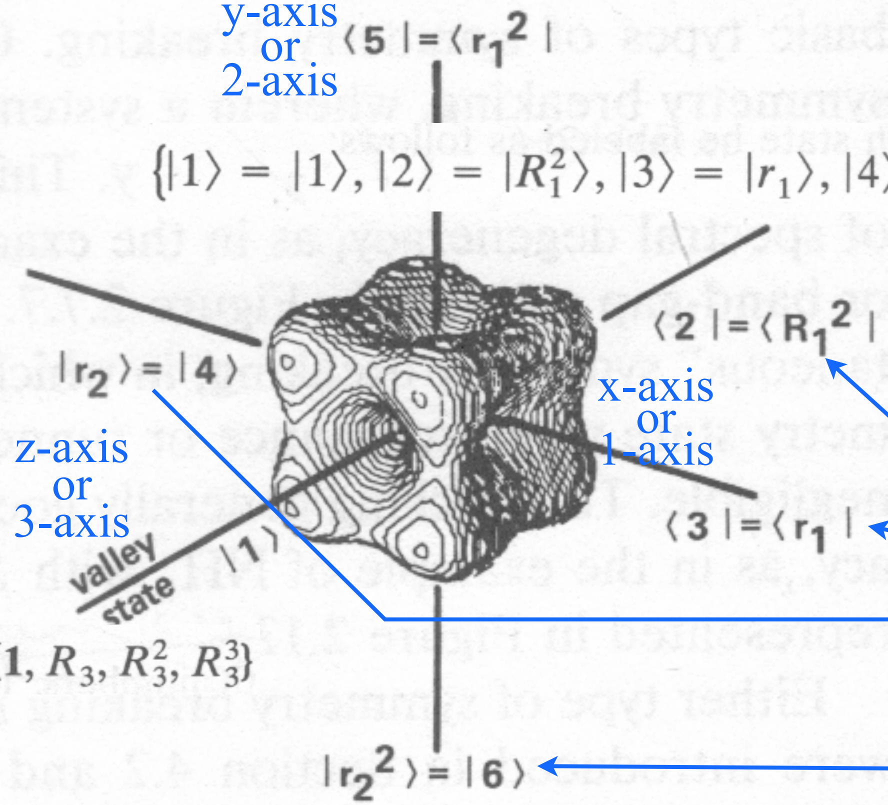


Fig. 4.3.1 PSDS



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$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$

$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$

$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$

Compare to IJMS cosets on pages 25 -60:

Assuming *C_4-local symmetry conditions* for $|\mathbf{1}\rangle$ state

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These apply to all six $|\mathbf{g}\rangle = \mathbf{g}|\mathbf{1}\rangle$-base states. $|g\rangle = |gR_3\rangle = |gR_3^2\rangle = |gR_3^3\rangle$

$$|g\rangle = g|1\rangle = gR_3|1\rangle = gR_3^2|1\rangle = gR_3^3|1\rangle$$

Switch columns 1 with 2

$\{\mathbf{1}, \rho_z, \mathbf{R}_z, \tilde{\mathbf{R}}_z\}$

$\{\rho_x, \rho_y, \mathbf{i}_4, \mathbf{i}_3\}$

$\{\mathbf{r}_1, \mathbf{r}_4, \mathbf{i}_1, \mathbf{R}_y\}$

$\{\mathbf{r}_2, \mathbf{r}_3, \mathbf{i}_2, \tilde{\mathbf{R}}_y\}$

$\{\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_3, \tilde{\mathbf{R}}_x, \mathbf{i}_6\}$

$\{\tilde{\mathbf{r}}_2, \tilde{\mathbf{r}}_4, \mathbf{R}_x, \mathbf{i}_5\}$

O operators (Two notations: Older Princ.of Symm.Dynamics and Spectra. and Newer Int.J.Mol.Sci)

PSDS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\mathbf{r}_1^2$	$\mathbf{r}_2^2$	$\mathbf{r}_3^2$	$\mathbf{r}_4^2$	$\mathbf{R}_1^2$	$\mathbf{R}_2^2$	$\mathbf{R}_3^2$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_1^3$	$\mathbf{R}_2^3$	$\mathbf{R}_3^3$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$
IJMS:	$\mathbf{1}$	$\mathbf{r}_1$	$\mathbf{r}_2$	$\mathbf{r}_3$	$\mathbf{r}_4$	$\tilde{\mathbf{r}}_1$	$\tilde{\mathbf{r}}_2$	$\tilde{\mathbf{r}}_3$	$\tilde{\mathbf{r}}_4$	ρ_x	ρ_y	ρ_z	$\mathbf{R}_x$	$\mathbf{R}_y$	$\mathbf{R}_z$	$\tilde{\mathbf{R}}_x$	$\tilde{\mathbf{R}}_y$	$\tilde{\mathbf{R}}_z$	$\mathbf{i}_1$	$\mathbf{i}_2$	$\mathbf{i}_3$	$\mathbf{i}_4$	$\mathbf{i}_5$	$\mathbf{i}_6$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

$O \supset D_4 \supset C_4$ *subgroup chain splitting*

$O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*

$O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$

Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

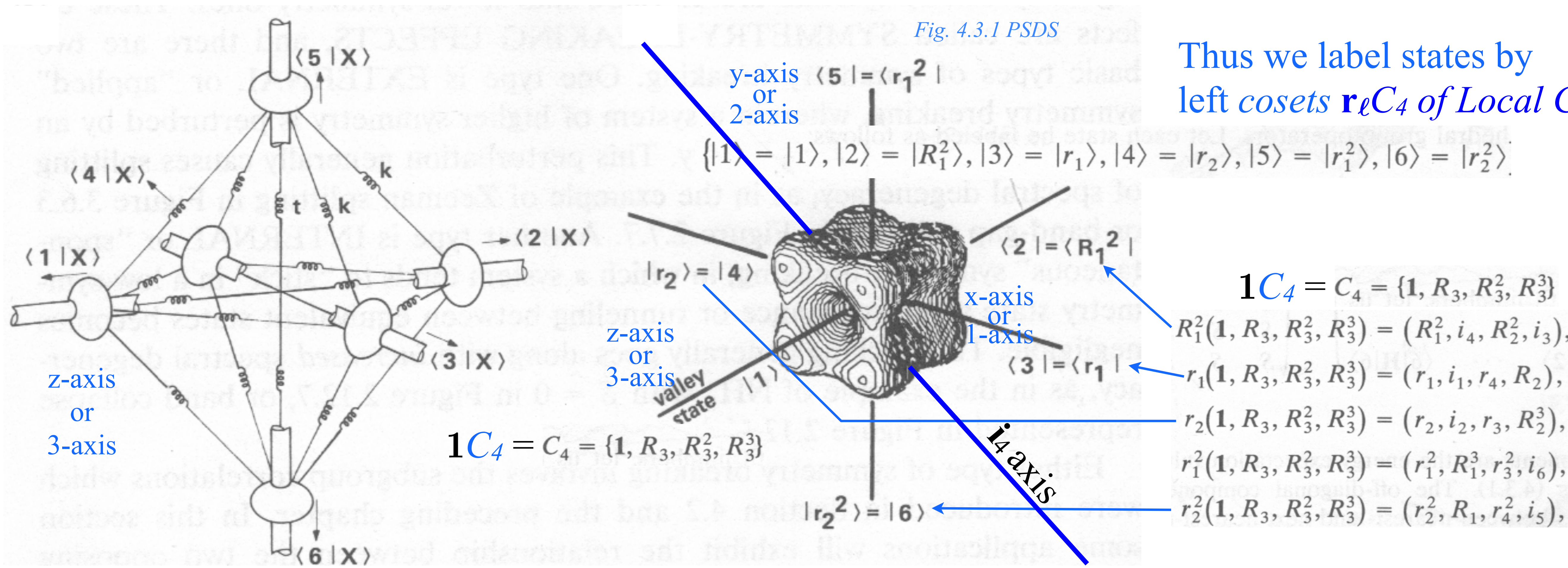
➡ *Elementary induced representation* $0_4(C_4)\uparrow O$ ⬅

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Introduction to ortho-complete eigenvalue-parameter relations

Examples in SF_6 *spectroscopy*

Elementary induced representation $0_4(C_4)\uparrow O$



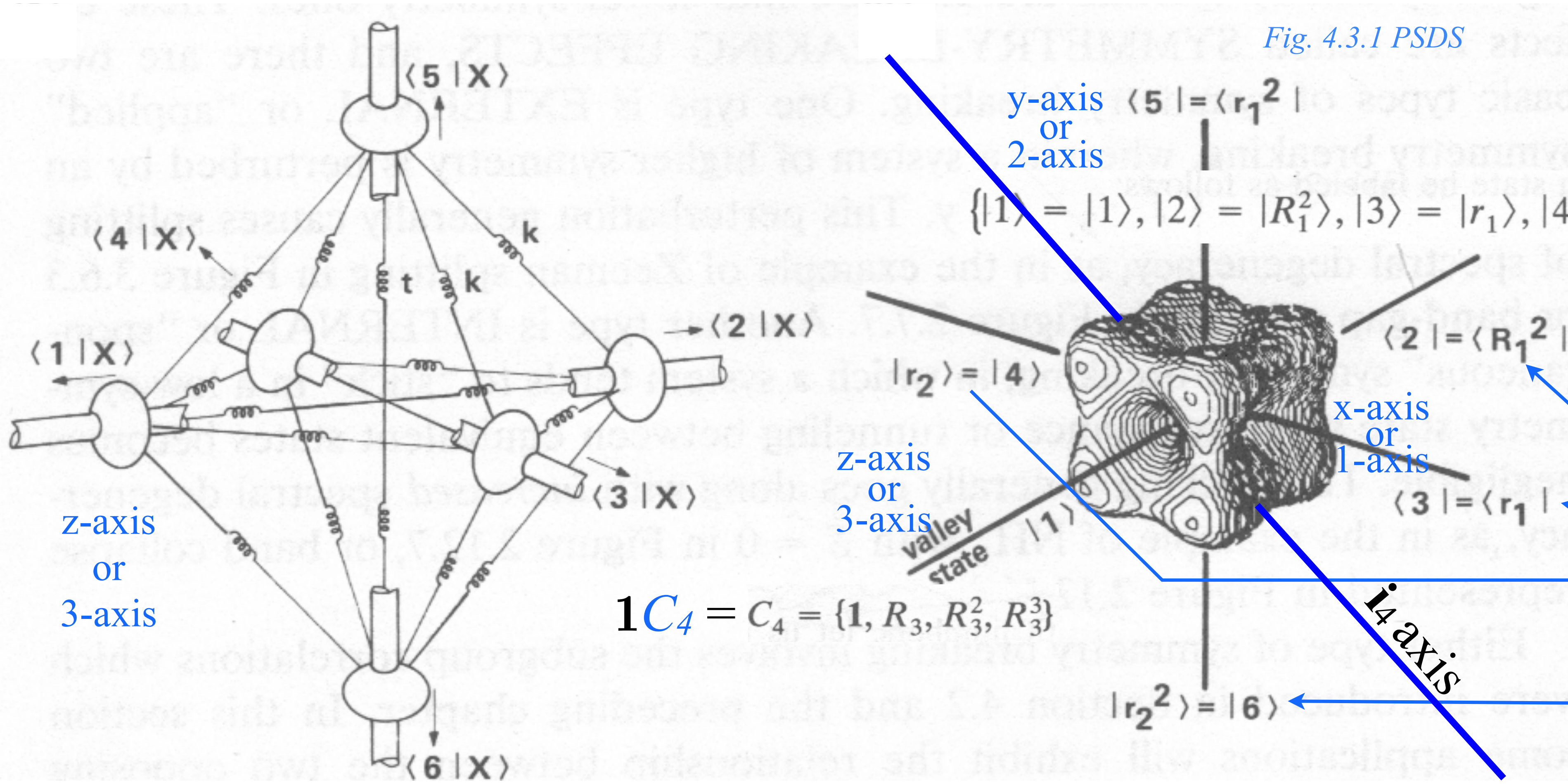
This "coset-basis" spans a *scalar* $0_4(C_4)$ *induced representation* $0_4(C_4)\uparrow O$

$$\mathbf{i}_4|1\rangle=\mathbf{i}_4|1\rangle,\quad \mathbf{i}_4|2\rangle=\mathbf{i}_4\mathbf{R}_1^2|1\rangle,\quad \mathbf{i}_4|3\rangle=\mathbf{i}_4\mathbf{r}_1|1\rangle,\quad \mathbf{i}_4|4\rangle=\mathbf{i}_4\mathbf{r}_2|1\rangle,\quad \mathbf{i}_4|5\rangle=\mathbf{i}_4\mathbf{r}_1^2|1\rangle,\quad \mathbf{i}_4|6\rangle=\mathbf{i}_4\mathbf{r}_2^2|1\rangle.$$

$$=\mathbf{R}_1^2|1\rangle,\quad =\mathbf{R}_3^3|1\rangle,\quad =\mathbf{i}_5|1\rangle,\quad =\mathbf{i}_6|1\rangle,\quad =\mathbf{i}_2|1\rangle,\quad =\mathbf{i}_1|1\rangle.$$

$$=|2\rangle,\quad =|1\rangle,\quad =|6\rangle,\quad =|5\rangle,\quad =|4\rangle,\quad =|3\rangle.$$

Elementary induced representation $0_4(C_4)\uparrow O$



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$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$

Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

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$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$

$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$

$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$

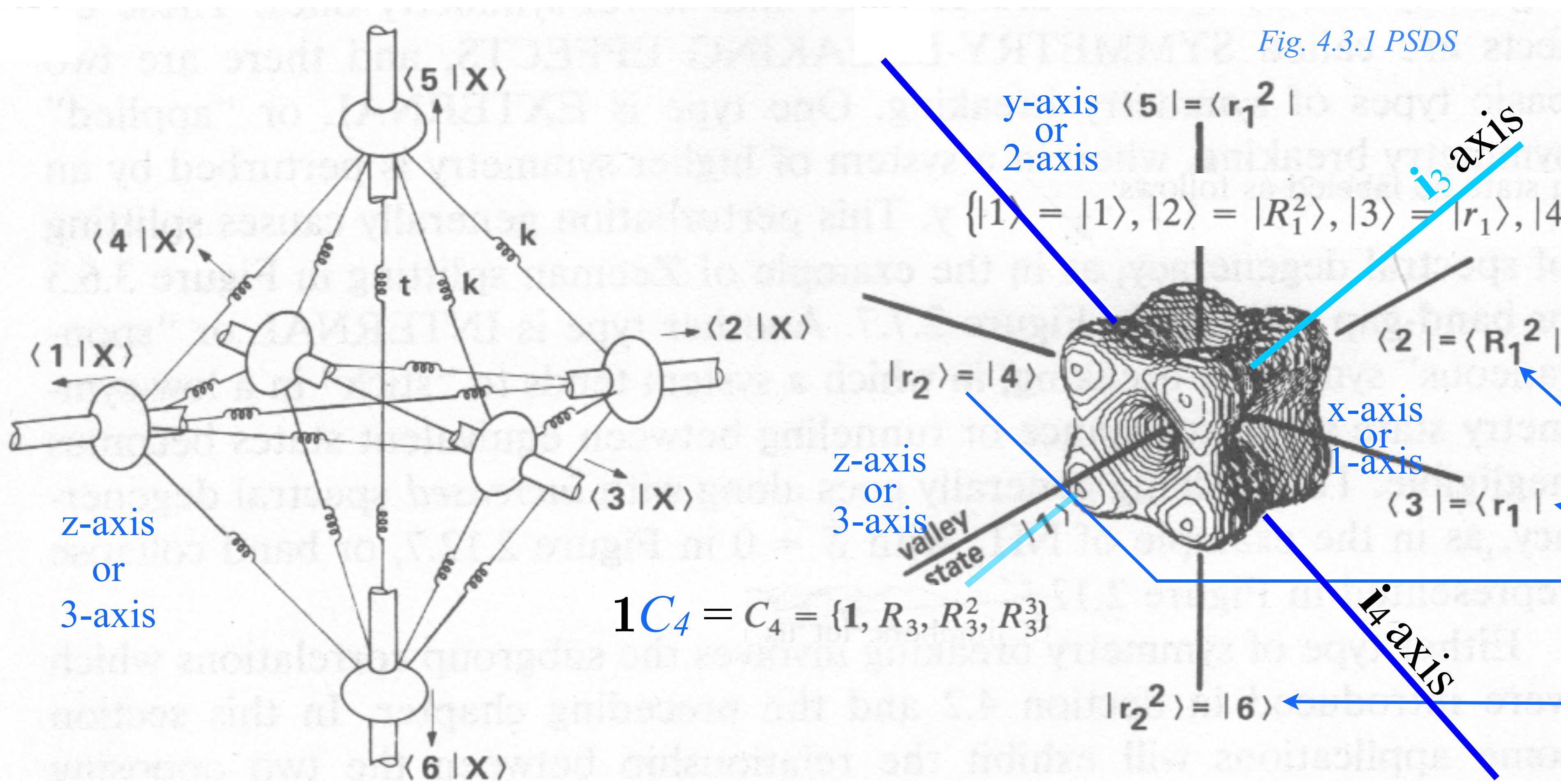
This "coset-basis" spans a *scalar* $0_4(C_4)$ *induced representation* $0_4(C_4)\uparrow O$

$$\begin{array}{llllll}
\mathbf{i}_4|1\rangle=\mathbf{i}_4|1\rangle, & \mathbf{i}_4|2\rangle=\mathbf{i}_4\mathbf{R}_1^2|1\rangle, & \mathbf{i}_4|3\rangle=\mathbf{i}_4\mathbf{r}_1|1\rangle, & \mathbf{i}_4|4\rangle=\mathbf{i}_4\mathbf{r}_2|1\rangle, & \mathbf{i}_4|5\rangle=\mathbf{i}_4\mathbf{r}_1^2|1\rangle, & \mathbf{i}_4|6\rangle=\mathbf{i}_4\mathbf{r}_2^2|1\rangle. \\
=\mathbf{R}_1^2|1\rangle, & =\mathbf{R}_3^3|1\rangle, & =\mathbf{i}_5|1\rangle, & =\mathbf{i}_6|1\rangle, & =\mathbf{i}_2|1\rangle, & =\mathbf{i}_1|1\rangle. \\
=|2\rangle, & =|1\rangle, & =|6\rangle, & =|5\rangle, & =|4\rangle, & =|3\rangle.
\end{array}$$

For example here is $0_4(C_4)$ *induced representation* $0_4(C_4)\uparrow O(\mathbf{i}_4)$

$$\mathscr{I}^{0_4\uparrow O}(\mathbf{i}_4)=\begin{pmatrix}
\langle 1|\mathbf{i}_4|1\rangle & \langle 1|\mathbf{i}_4|2\rangle & \cdots & \langle 1|\mathbf{i}_4|6\rangle \\
\langle 2|\mathbf{i}_4|1\rangle & \langle 2|\mathbf{i}_4|2\rangle & & \vdots \\
\vdots & & & \vdots \\
\langle 6|\mathbf{i}_4|1\rangle & \langle 6|\mathbf{i}_4|2\rangle & & \langle 1|\mathbf{i}_4|6\rangle
\end{pmatrix}=
\begin{array}{c|cccccc}
 & |1\rangle & |2\rangle & |3\rangle & |4\rangle & |5\rangle & |6\rangle \\
\hline
\langle 1| & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\langle 2| & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\langle 3| & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\langle 4| & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\
\langle 5| & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\langle 6| & \cdot & \cdot & 1 & \cdot & \cdot & \cdot
\end{array}$$

Elementary induced representation $0_4(C_4)\uparrow O$



Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

$$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$$
$$R_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (R_1^2, i_4, R_2^2, i_3),$$
$$r_1(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1, i_1, r_4, R_2),$$
$$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$$
$$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$$
$$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$$

This "coset-basis" spans a *scalar* $0_4(C_4)$ *induced representation* $0_4(C_4)\uparrow O$

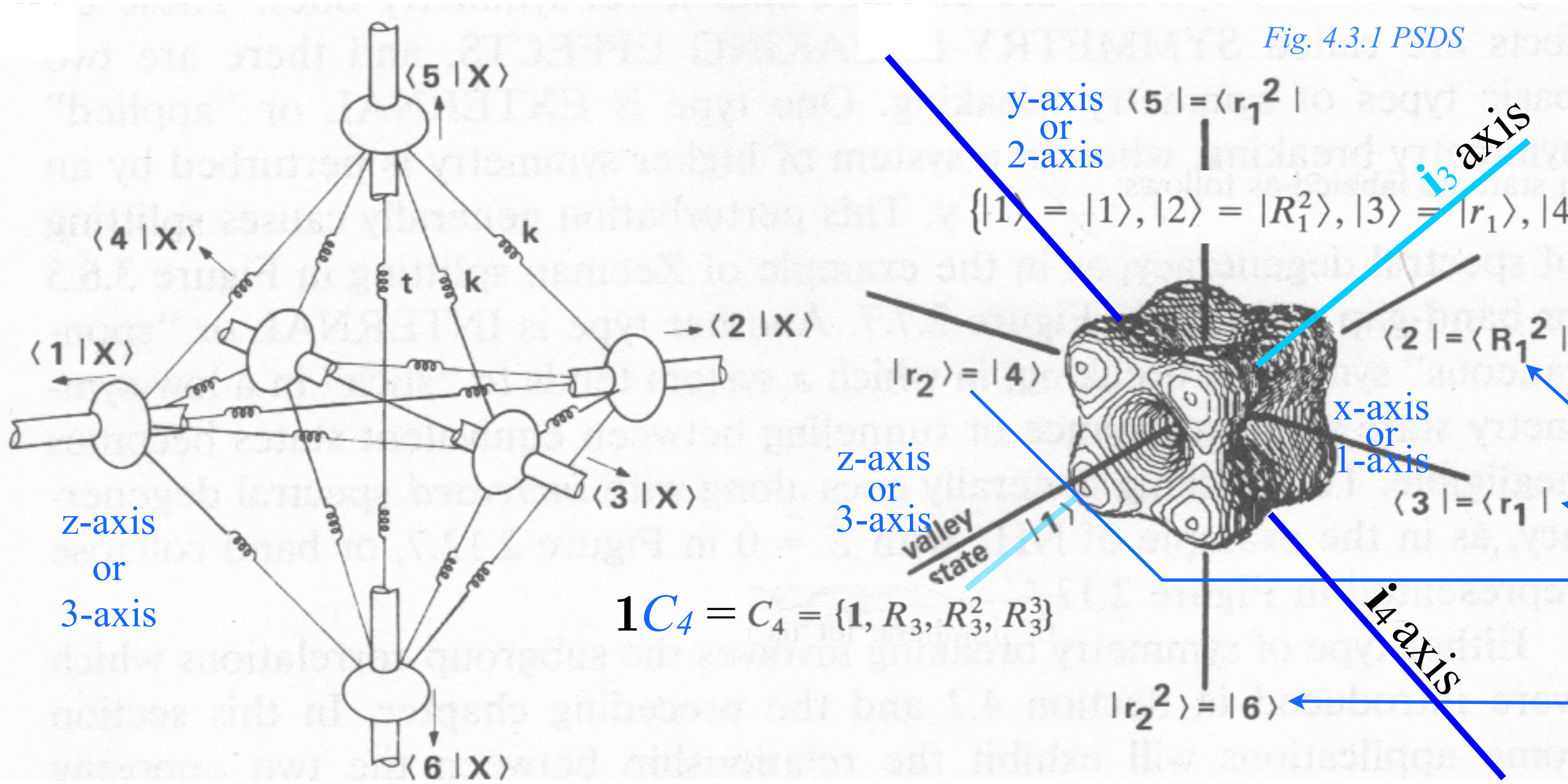
$$\begin{array}{llllll}
\mathbf{i}_4|1\rangle=\mathbf{i}_4|1\rangle, & \mathbf{i}_4|2\rangle=\mathbf{i}_4\mathbf{R}_1^2|1\rangle, & \mathbf{i}_4|3\rangle=\mathbf{i}_4\mathbf{r}_1|1\rangle, & \mathbf{i}_4|4\rangle=\mathbf{i}_4\mathbf{r}_2|1\rangle, & \mathbf{i}_4|5\rangle=\mathbf{i}_4\mathbf{r}_1^2|1\rangle, & \mathbf{i}_4|6\rangle=\mathbf{i}_4\mathbf{r}_2^2|1\rangle. \\
=\mathbf{R}_1^2|1\rangle, & =\mathbf{R}_3^3|1\rangle, & =\mathbf{i}_5|1\rangle, & =\mathbf{i}_6|1\rangle, & =\mathbf{i}_2|1\rangle, & =\mathbf{i}_1|1\rangle. \\
=|2\rangle, & =|1\rangle, & =|6\rangle, & =|5\rangle, & =|4\rangle, & =|3\rangle.
\end{array}$$

For example here is $0_4(C_4)$ *induced representation* $0_4(C_4)\uparrow O(\mathbf{i}_4)$ *and* $0_4(C_4)\uparrow O(\mathbf{i}_3)$

$$\mathscr{I}^{0_4\uparrow O}(\mathbf{i}_4)=\begin{pmatrix} \langle 1|\mathbf{i}_4|1\rangle & \langle 1|\mathbf{i}_4|2\rangle & \cdots & \langle 1|\mathbf{i}_4|6\rangle \\ \langle 2|\mathbf{i}_4|1\rangle & \langle 2|\mathbf{i}_4|2\rangle & & \vdots \\ \vdots & & & \vdots \\ \langle 6|\mathbf{i}_4|1\rangle & \langle 6|\mathbf{i}_4|2\rangle & & \langle 1|\mathbf{i}_4|6\rangle \end{pmatrix} = \begin{array}{c|cccccc} & |1\rangle & |2\rangle & |3\rangle & |4\rangle & |5\rangle & |6\rangle \\ \hline \langle 1| & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \langle 2| & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \langle 3| & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \langle 4| & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \langle 5| & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \langle 6| & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \end{array}$$

$$\mathscr{I}^{0_4\uparrow O}(\mathbf{i}_3)=\begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \end{pmatrix}$$

Elementary induced representation $0_4(C_4)\uparrow O$



Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$

$R_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (R_1^2, i_4, R_2^2, i_3),$

$r_1(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1, i_1, r_4, R_2),$

$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$

$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$

$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$

Here is $0_4(C_4)$ *induced representation* $\mathscr{I}^{0_4\uparrow O}(\mathbf{I_i})$ *of a linear combination of* **i**-*class rotations*

$$\mathbf{I_i} = i_1\mathbf{i_1} + i_2\mathbf{i_2} + i_3\mathbf{i_3} + i_4\mathbf{i_4} + i_5\mathbf{i_5} + i_6\mathbf{i_6}$$

$\mathscr{I}^{0_4\uparrow O}(\mathbf{I_i}) =$

	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$	$\vert 5\rangle$	$\vert 6\rangle$
$\langle 1\vert$	1	i_3+i_4	i_1	i_2	i_6	i_5
$\langle 2\vert$	i_3+i_4	1	i_2	i_1	i_5	i_6
$\langle 3\vert$	i_1	i_2	1	i_5+i_6	i_3	i_4
$\langle 4\vert$	i_2	i_1	i_5+i_6	1	i_4	i_3
$\langle 5\vert$	i_6	i_5	i_3	i_4	1	i_1+i_2
$\langle 6\vert$	i_5	i_6	i_4	i_3	i_1+i_2	1

$$\mathscr{I}^{0_4\uparrow O}(\mathbf{i}_3) = \begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \end{pmatrix}$$

Review Octahedral $O_h \supset O$ *group operator structure*
Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*
$O \supset D_4 \supset C_4$ subgroup chain splitting
$O \supset D_4 \supset D_2$ subgroup chain splitting (nOrmal D_2 vs. unOrmal D_2)
$O_h \supset O \supset D_4 \supset C_{4v}$ and $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ subgroup splitting

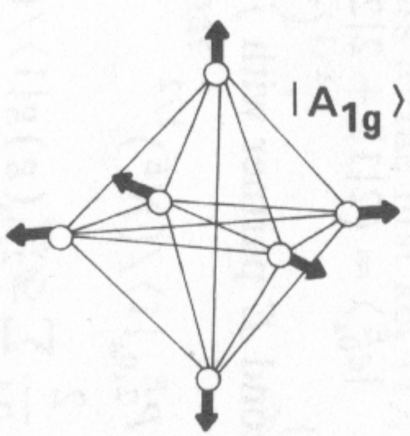
Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$
Development of irreducible projectors $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
Calculating $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

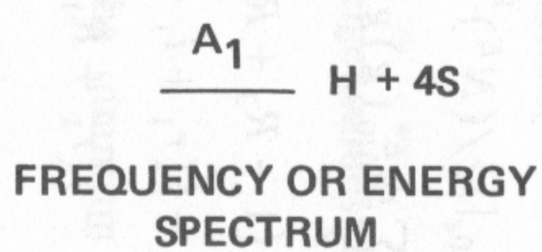
$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*
Elementary induced representation $0_4(C_4)\uparrow O$
➡ *Projection reduction of induced representation* $0_4(C_4)\uparrow O$ ⬅
Introduction to ortho-complete eigenvalue-parameter relations
Examples in SF_6 spectroscopy

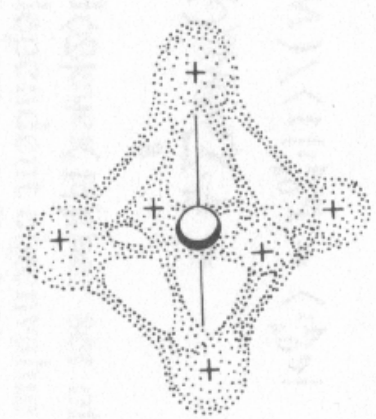
Projection reduction of induced representation $0_4(C_4)\uparrow O$
Scalar A_1 eigenket

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \mathbf{P}^{A_1}_{0_4 0_4}\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{A_1 *}_{0_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \left(\left|1\right\rangle + \left|2\right\rangle + \left|3\right\rangle + \left|4\right\rangle + \left|5\right\rangle + \left|6\right\rangle\right)/\sqrt{6}$$

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix}\frac{1}{\sqrt{6}}$$







$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix}\frac{1}{\sqrt{6}}$$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Scalar A_1 eigenket $0_4 0_4$

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \mathbf{P}^{A_1}_{0_4 0_4}\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{A_1 *}_{0_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \left(\left|1\right\rangle + \left|2\right\rangle + \left|3\right\rangle + \left|4\right\rangle + \left|5\right\rangle + \left|6\right\rangle\right)/\sqrt{6}$$

Tensor E-eigenket $0_4 0_4$

Diagonal (idempotent) Projector $\mathbf{P}^{\mu}_{jj}$

$$\left|e^{E}_{0_4 0_4}\right\rangle = \mathbf{P}^{E}_{0_4 0_4}\left|1\right\rangle / \sqrt{N^{E}}$$
$$= \frac{2}{24}\sum_{p=1}^{24} D^{E *}_{0_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{E}}$$

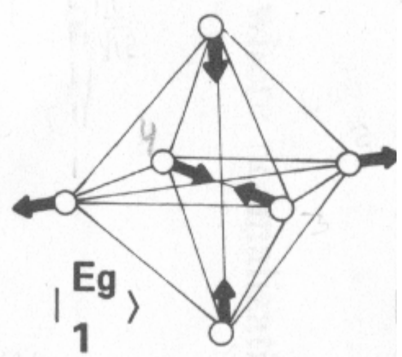
From p.49-50:

$$= \left(\left|1\right\rangle + \left|2\right\rangle - \tfrac{1}{2}\left|3\right\rangle - \tfrac{1}{2}\left|4\right\rangle - \tfrac{1}{2}\left|5\right\rangle - \tfrac{1}{2}\left|6\right\rangle\right)/\sqrt{3}$$

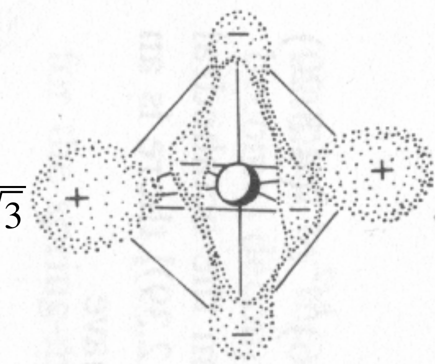
$$\mathbf{P}^{E}_{0_4 0_4} = \tfrac{1}{12}\left[(1)\cdot\mathbf{1p}_{0_4} + (1)\cdot\boldsymbol{\rho}_x\mathbf{p}_{0_4} + (-\tfrac{1}{2})\cdot\mathbf{r}_1\mathbf{p}_{0_4} + (-\tfrac{1}{2})\cdot\mathbf{r}_2\mathbf{p}_{0_4} + (-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{0_4} + (-\tfrac{1}{2})\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{0_4}\right]$$

$$\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\ \{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_3,\mathbf{i}_4\}\ \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\ \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\ \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\ \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

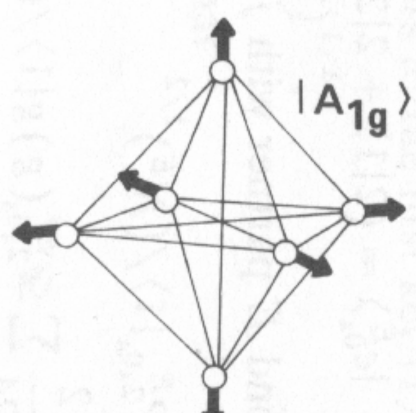
$$\left|e^{E}_{0_4 0_4}\right\rangle = \begin{pmatrix} 2\\ 2\\ -1\\ -1\\ -1\\ -1 \end{pmatrix}\frac{1}{2\sqrt{3}}$$



$$\left|e^{E}_{0_4 0_4}\right\rangle = \begin{pmatrix} 2\\ 2\\ -1\\ -1\\ -1\\ -1 \end{pmatrix}\frac{1}{2\sqrt{3}}$$



$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1\\ 1\\ 1\\ 1\\ 1\\ 1 \end{pmatrix}\frac{1}{\sqrt{6}}$$



A1 ——— H + 4S

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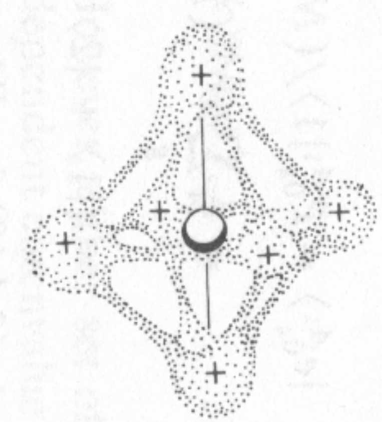


$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1\\ 1\\ 1\\ 1\\ 1\\ 1 \end{pmatrix}\frac{1}{\sqrt{6}}$$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Scalar A_1 eigenket $0_4 0_4$

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \mathbf{P}^{A_1}_{0_4 0_4}\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{A_1 *}_{0_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \left(\left|1\right\rangle + \left|2\right\rangle + \left|3\right\rangle + \left|4\right\rangle + \left|5\right\rangle + \left|6\right\rangle\right)/\sqrt{6}$$

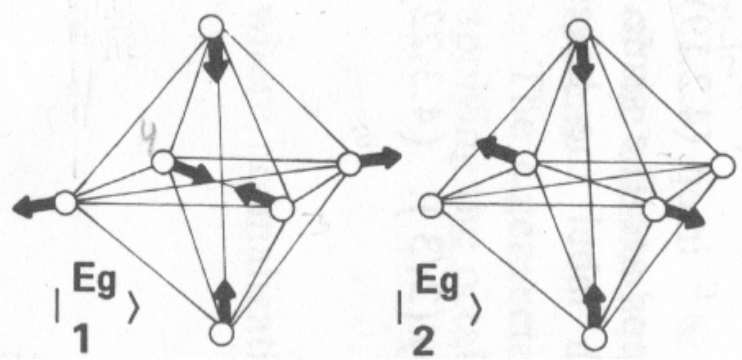
Off-Diagonal (nilpotent) Projector $\mathbf{P}^{\mu}_{jk}$

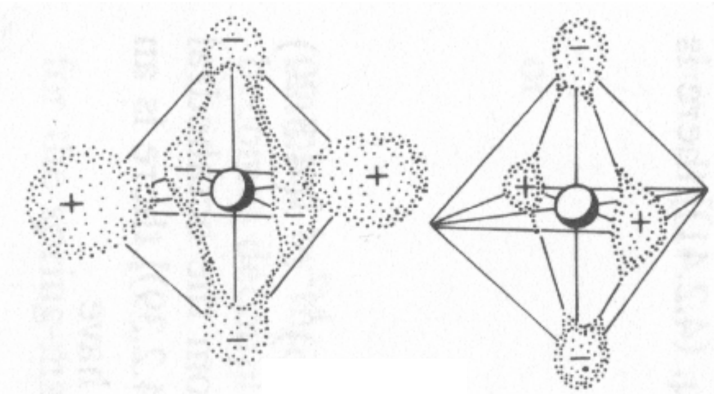
Derived next lectures

Tensor E-eigenket $2_4 0_4$

$$\left|e^{E}_{2_4 0_4}\right\rangle = \mathbf{P}^{E}_{2_4 0_4}\left|1\right\rangle / \sqrt{N^{E}}$$
$$= \frac{2}{24}\sum_{p=1}^{24} D^{E *}_{2_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{E}}$$
$$= \left(\left|3\right\rangle + \left|4\right\rangle - \left|5\right\rangle - \left|6\right\rangle\right)/2$$

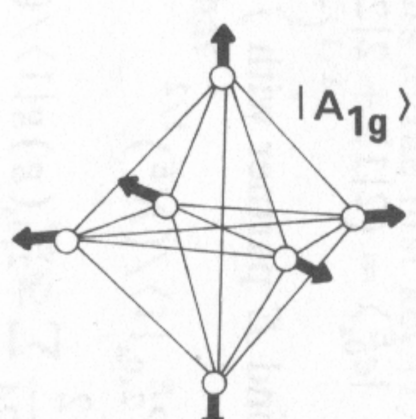
$$\left|e^{E}_{0_4 0_4}\right\rangle = \begin{pmatrix} 2 \\ 2 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \frac{1}{2\sqrt{3}}$$

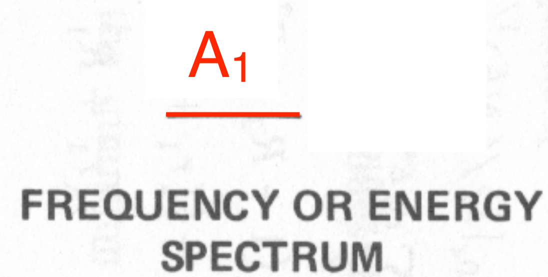


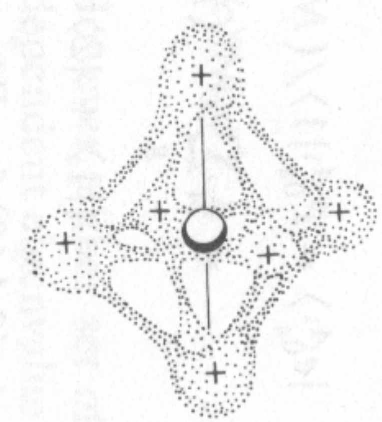


$$\left|e^{E}_{2_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \frac{1}{2}$$

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{6}}$$







$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{6}}$$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Scalar A_1 eigenket 0_40_4

$$\left|e^{A_1}_{0_40_4}\right\rangle = \mathbf{P}^{A_1}_{0_40_4}\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{A_1 *}_{0_40_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \left(\left|1\right\rangle + \left|2\right\rangle + \left|3\right\rangle + \left|4\right\rangle + \left|5\right\rangle + \left|6\right\rangle\right)/\sqrt{6}$$

Diagonal (idempotent) Projector $\mathbf{P}^{\mu}_{jj}$

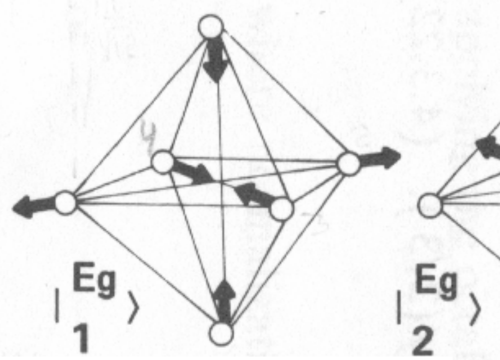
From p.53:

Vector T_1-eigenket 0_40_4

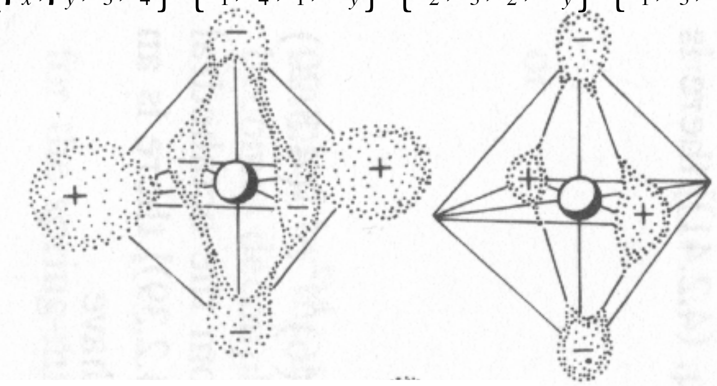
$$\left|e^{T_1}_{0_40_4}\right\rangle = \mathbf{P}^{T_1}_{0_40_4}\left|1\right\rangle / \sqrt{N^{T_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{T_1 *}_{0_40_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{T_1}}$$
$$= \left(\left|1\right\rangle - \left|2\right\rangle + 0 + 0 + 0 + 0\right)/\sqrt{2}$$

$$\mathbf{P}^{T_1}_{0_40_4} = \tfrac{1}{8}\left[(1)\cdot\mathbf{1}\mathbf{p}_{0_4} + (-1)\cdot\boldsymbol{\rho}_x\mathbf{p}_{0_4} + (0)\cdot\mathbf{r}_1\mathbf{p}_{0_4} + (0)\cdot\mathbf{r}_2\mathbf{p}_{0_4} + (0)\cdot\tilde{\mathbf{r}}_1\mathbf{p}_{0_4} + (0)\cdot\tilde{\mathbf{r}}_2\mathbf{p}_{0_4}\right]$$
$$\{\mathbf{1},\boldsymbol{\rho}_z,\mathbf{R}_z,\tilde{\mathbf{R}}_z\}\ \{\boldsymbol{\rho}_x,\boldsymbol{\rho}_y,\mathbf{i}_3,\mathbf{i}_4\}\ \{\mathbf{r}_1,\mathbf{r}_4,\mathbf{i}_1,\mathbf{R}_y\}\ \{\mathbf{r}_2,\mathbf{r}_3,\mathbf{i}_2,\tilde{\mathbf{R}}_y\}\ \{\tilde{\mathbf{r}}_1,\tilde{\mathbf{r}}_3,\tilde{\mathbf{R}}_x,\mathbf{i}_6\}\ \{\tilde{\mathbf{r}}_2,\tilde{\mathbf{r}}_4,\mathbf{R}_x,\mathbf{i}_5\}$$

$$\left|e^{E}_{0_40_4}\right\rangle = \begin{pmatrix} 2\\ 2\\ -1\\ -1\\ -1\\ -1 \end{pmatrix}\frac{1}{2\sqrt{3}}$$

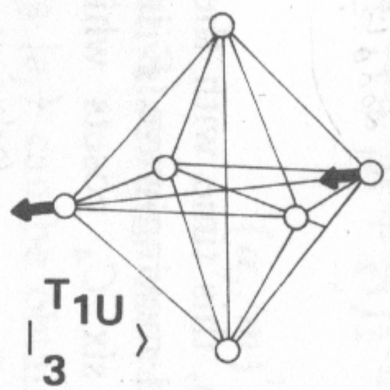


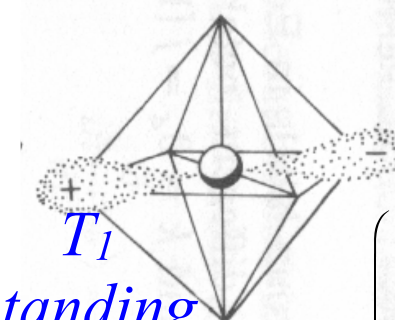
E



$$\left|e^{E}_{2_40_4}\right\rangle = \begin{pmatrix} 0\\ 0\\ 1\\ 1\\ -1\\ -1 \end{pmatrix}\frac{1}{2}$$

T1

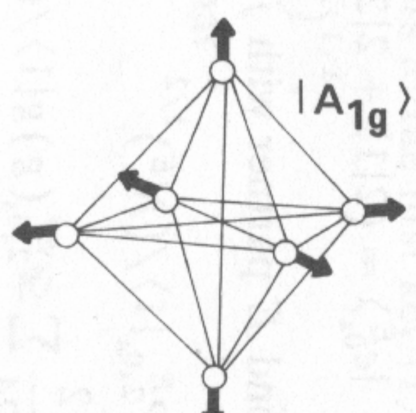




T_1 standing wave $0_4 = z$

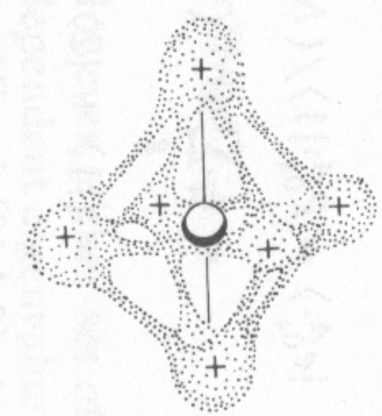
$$\left|e^{T_1}_{0_40_4}\right\rangle = \begin{pmatrix} 1\\ -1\\ 0\\ 0\\ 0\\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$$

$$\left|e^{A_1}_{0_40_4}\right\rangle = \begin{pmatrix} 1\\ 1\\ 1\\ 1\\ 1\\ 1 \end{pmatrix}\frac{1}{\sqrt{6}}$$



A1

FREQUENCY OR ENERGY SPECTRUM



$$\left|e^{A_1}_{0_40_4}\right\rangle = \begin{pmatrix} 1\\ 1\\ 1\\ 1\\ 1\\ 1 \end{pmatrix}\frac{1}{\sqrt{6}}$$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

Scalar A_1 eigenket $0_4 0_4$

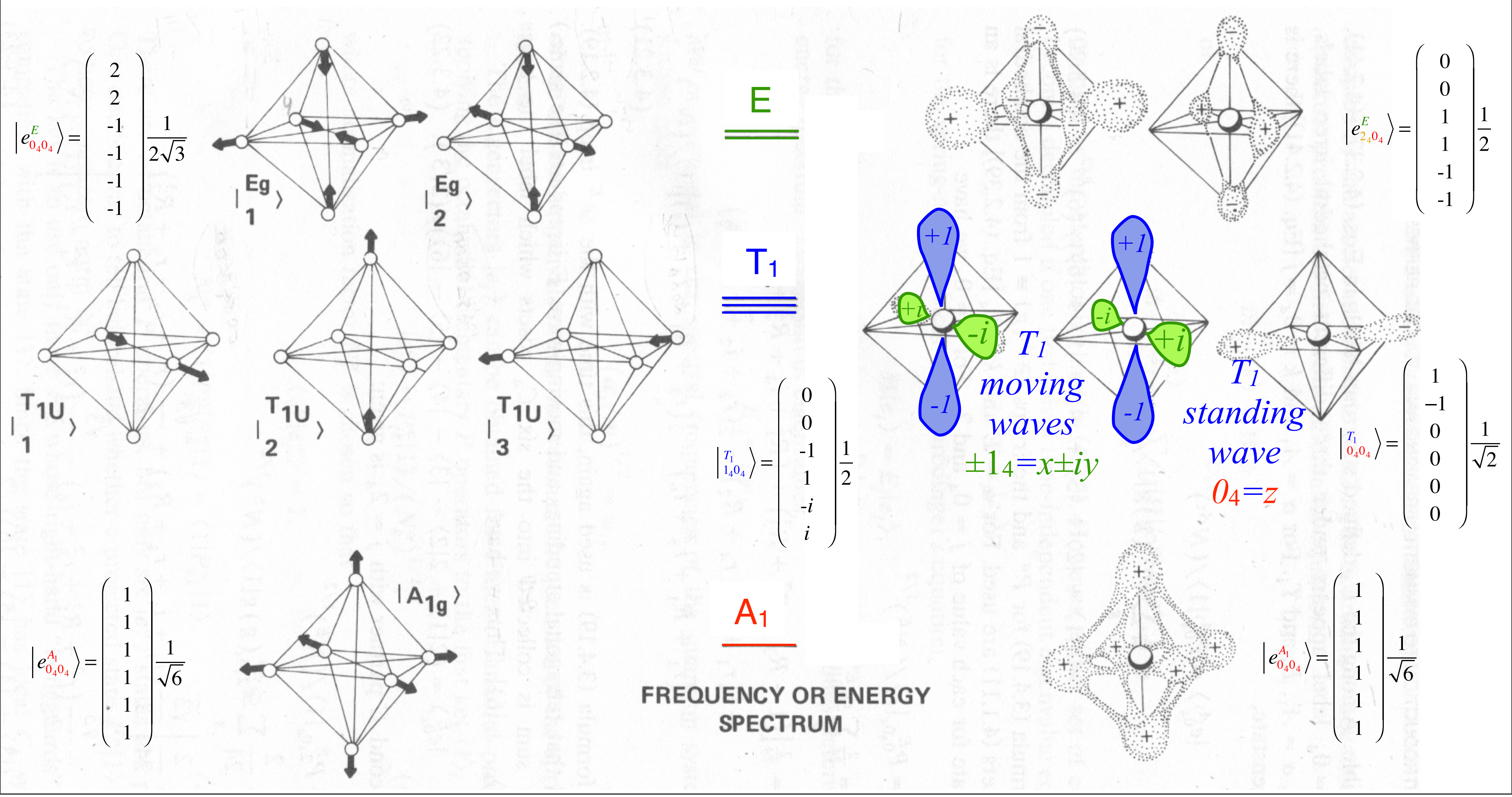
$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \mathbf{P}^{A_1}_{0_4 0_4}\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{A_1 *}_{0_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{A_1}}$$
$$= \left(\left|1\right\rangle + \left|2\right\rangle + \left|3\right\rangle + \left|4\right\rangle + \left|5\right\rangle + \left|6\right\rangle\right)/\sqrt{6}$$

Off-Diagonal (nilpotent) Projector $\mathbf{P}^{\mu}_{jk}$

Derived next lectures

Vector T_1-eigenket $\pm 1_4 0_4$ and $0_4 0_4$

$$\left|e^{T_1}_{\pm 1_4 0_4}\right\rangle = \mathbf{P}^{T_1}_{1_4 0_4}\left|1\right\rangle / \sqrt{N^{T_1}}$$
$$= \frac{1}{24}\sum_{p=1}^{24} D^{T_1 *}_{\pm 1_4 0_4}(g_p)\,\mathbf{g}_p\left|1\right\rangle / \sqrt{N^{T_1}}$$
$$= \left(0 + 0 + \left|3\right\rangle + \left|4\right\rangle \pm i\left|5\right\rangle \pm i\left|6\right\rangle\right)/2$$



Projection reduction of induced representation $0_4(C_4)\uparrow O$

$$E^{A_1} = H + T + 4S$$
$$E^{T_1} = H - T$$
$$E^{E} = H + T - 2S$$

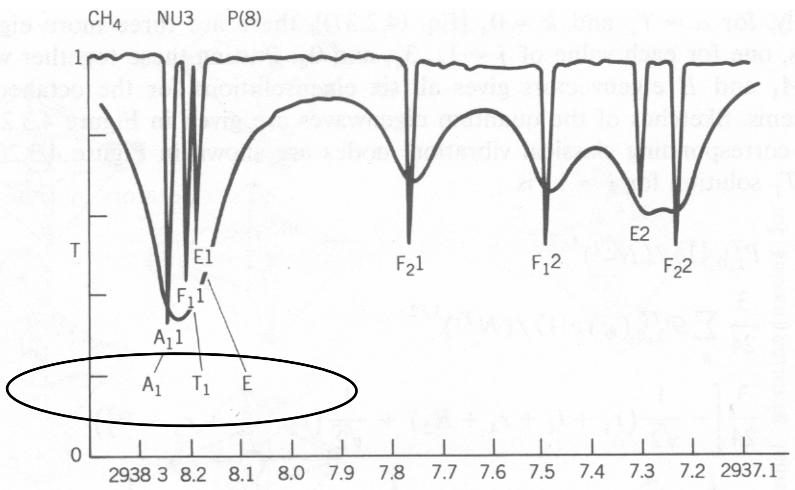
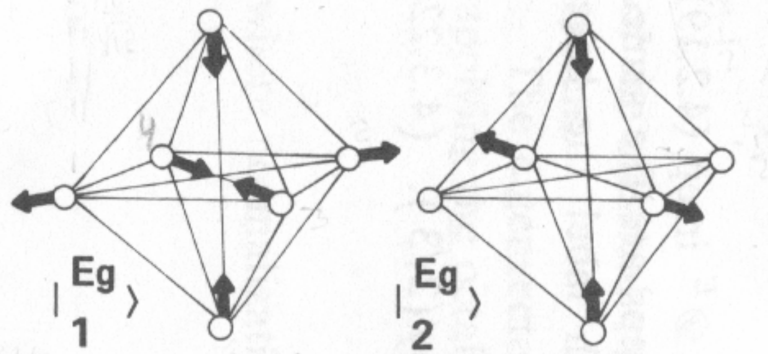
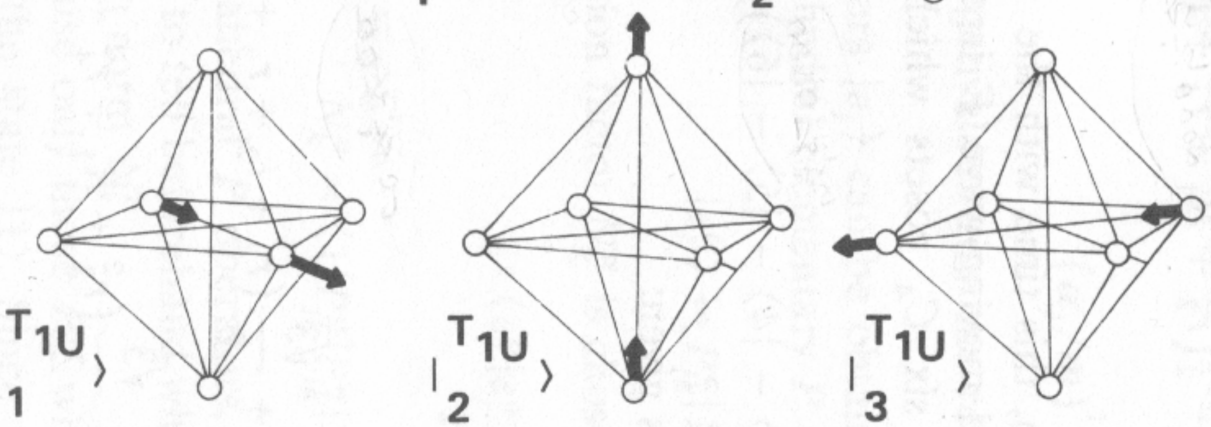


Figure 4.3.3 Evidence of an (A_1T_1E) spectral cluster in methane laser spectra. (Courtesy of Dr. Allan Pine, MIT Lincoln Laboratories, from *Journal of Optical Society of America* **66**, 97 (1976)). The ordering and approximate spacing of the A_1T_1 and E lines is consistent with that of Figure 4.3.2.

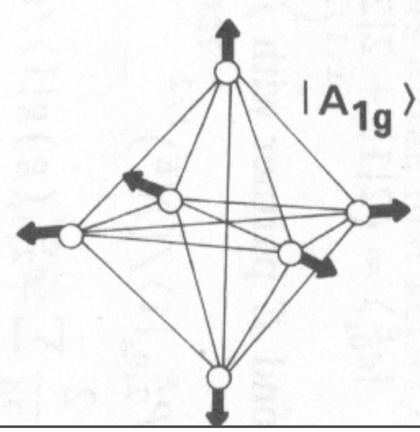
$$\begin{pmatrix} \langle 1|\mathbf{H}|1\rangle & \langle 1|\mathbf{H}|2\rangle & \cdots & \langle 1|\mathbf{H}|6\rangle \\ \langle 2|\mathbf{H}|1\rangle & \langle 2|\mathbf{H}|2\rangle & \cdots & \langle 2|\mathbf{H}|6\rangle \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \langle 6|\mathbf{H}|1\rangle & \langle 6|\mathbf{H}|2\rangle & \cdots & \langle 6|\mathbf{H}|6\rangle \end{pmatrix} = \begin{pmatrix} H & T & S & S & S & S \\ T & H & S & S & S & S \\ S & S & H & T & S & S \\ S & S & T & H & S & S \\ S & S & S & S & H & T \\ S & S & S & S & T & H \end{pmatrix}.$$

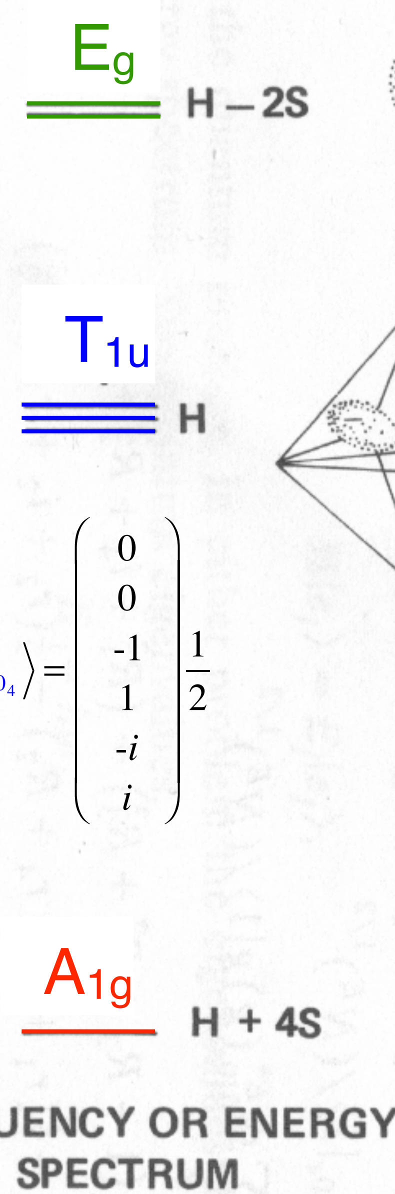
$$\left|{}^{E}_{0_4 0_4}\right\rangle = \begin{pmatrix} 2 \\ 2 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \frac{1}{2\sqrt{3}}$$

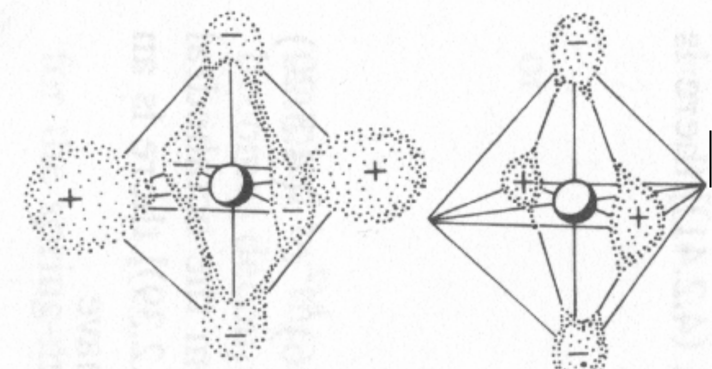




$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{6}}$$

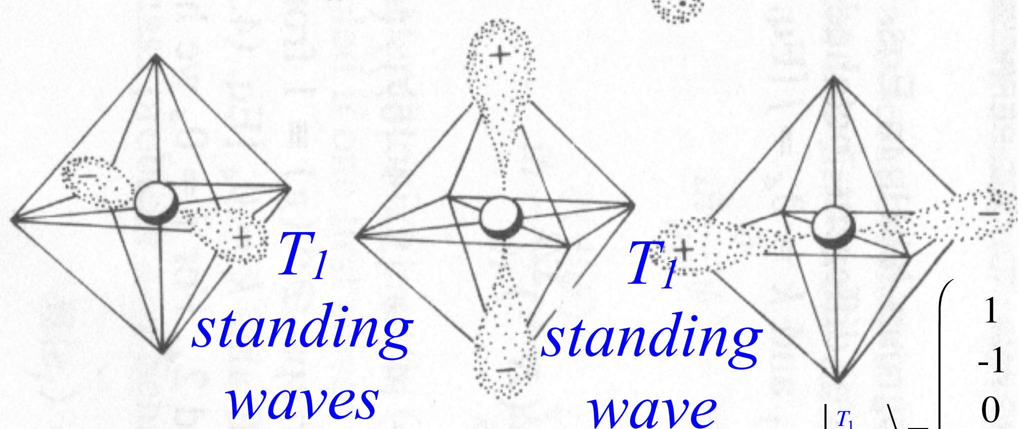






$$\left|{}^{E}_{2_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \frac{1}{2}$$

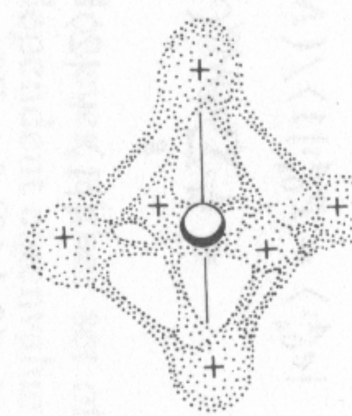
$$\left|{}^{T_1}_{1_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ -i \\ i \end{pmatrix} \frac{1}{2}$$



T_1 standing waves x and y

T_1 standing wave 0_4=z

$$\left|{}^{T_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}}$$



$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{6}}$$

Projection reduction of induced representation $0_4(C_4)\uparrow O$

$$E^{A_1} = H + T + 4S,$$
$$E^{T_1} = H - T,$$
$$E^{E} = H + T - 2S.$$

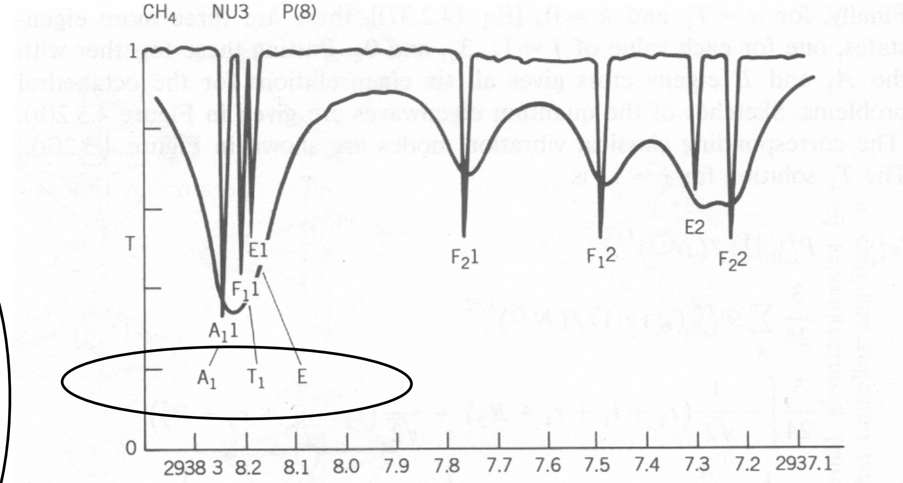


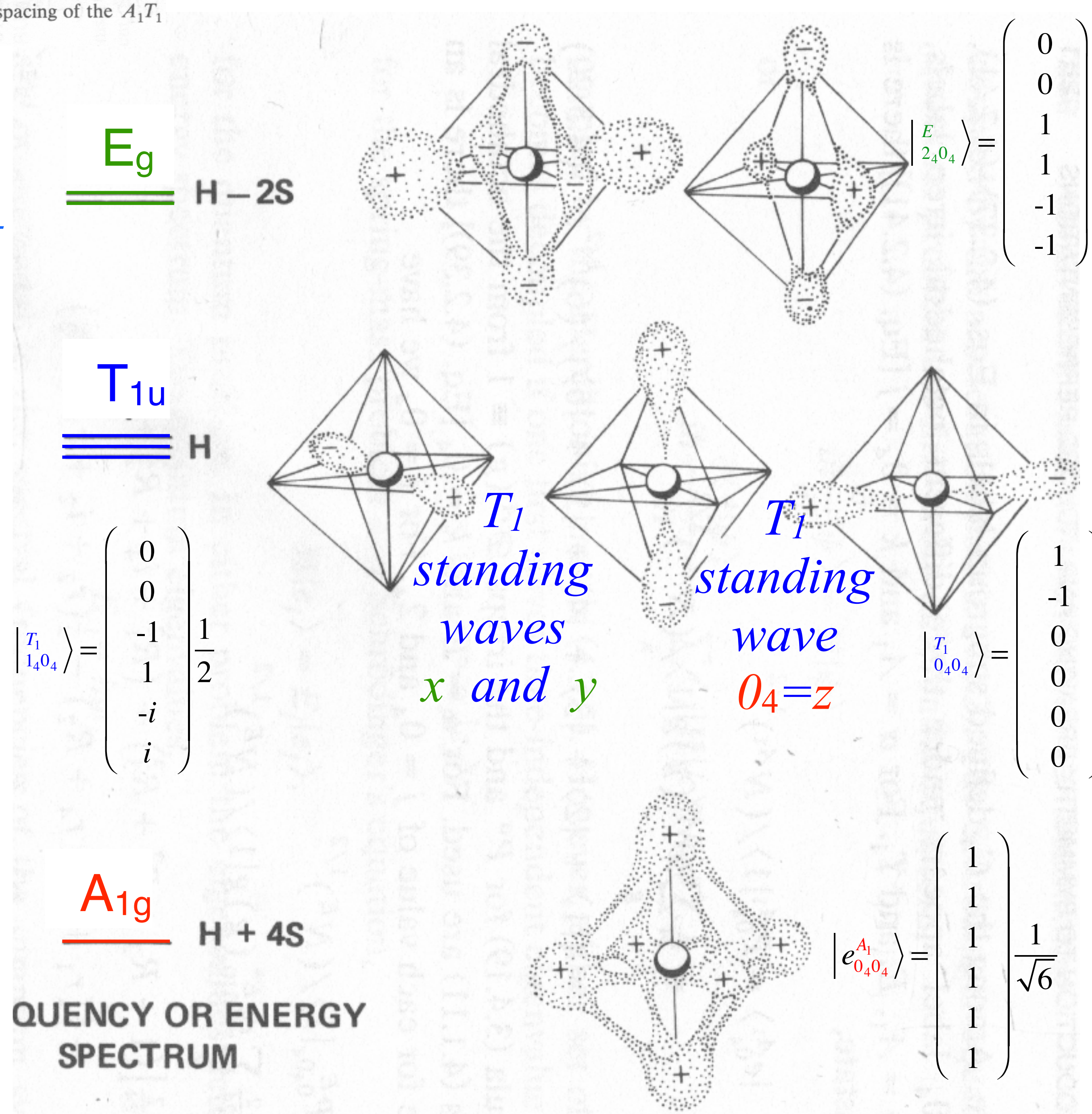
Figure 4.3.3 Evidence of an (A_1T_1E) spectral cluster in methane laser spectra. (Courtesy of Dr. Allan Pine, MIT Lincoln Laboratories, from *Journal of Optical Society of America* **66**, 97 (1976)). The ordering and approximate spacing of the A_1T_1 and E lines is consistent with that of Figure 4.3.2.

$$\begin{pmatrix} \langle 1|\mathbf{H}|1\rangle & \langle 1|\mathbf{H}|2\rangle & \cdots & \langle 1|\mathbf{H}|6\rangle \\ \langle 2|\mathbf{H}|1\rangle & \langle 2|\mathbf{H}|2\rangle & \cdots & \langle 2|\mathbf{H}|6\rangle \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \langle 6|\mathbf{H}|1\rangle & \langle 6|\mathbf{H}|2\rangle & \cdots & \langle 6|\mathbf{H}|6\rangle \end{pmatrix} = \begin{pmatrix} H & T & S & S & S & S \\ T & H & S & S & S & S \\ S & S & H & T & S & S \\ S & S & T & H & S & S \\ S & S & S & S & H & T \\ S & S & S & S & T & H \end{pmatrix}.$$

$O_h \supset D_{4h} \supset C_{4v} \supset C_{2v}$ subgroup splitting

$O_h \supset C_{4v}$	A'	B'	A''	B''	E
$A_{1g} \downarrow C_{4v}$	1	·	·	·	·
$A_{2g} \downarrow C_{4v}$	·	1	·	·	·
$E_{g} \downarrow C_{4v}$	1	1	·	·	·
$T_{1g} \downarrow C_{4v}$	·	·	1	·	1
$T_{2g} \downarrow C_{4v}$	·	·	·	1	1
$A_{1g} \downarrow C_{4v}$	·	·	1	·	·
$A_{2u} \downarrow C_{4v}$	·	·	·	1	·
$E_{u} \downarrow C_{4v}$	·	·	1	1	·
$T_{1u} \downarrow C_{4v}$	1	·	·	·	1
$T_{2u} \downarrow C_{4v}$	·	1	·	·	1

Labels correct u or g parity!



$$\left|{}^{E}_{2_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \frac{1}{2}$$

$$\left|{}^{T_1}_{1_4 0_4}\right\rangle = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ -i \\ i \end{pmatrix} \frac{1}{2}$$

$$\left|{}^{T_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\left|e^{A_1}_{0_4 0_4}\right\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{6}}$$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

* $O \supset D_4 \supset C_4$ *subgroup chain splitting*
* $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
* $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

*   *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
*   *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

*   *Elementary induced representation* $0_4(C_4)\uparrow O$
*   *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
*   ➡ *Introduction to ortho-complete eigenvalue-parameter relations* ⬅
    *   *Examples in* SF_6 *spectroscopy*

$\ell^{A_1}=1$

$\ell^{A_2}=1$

$\ell^{E}=2$

$\ell^{T_1}=3$

$\ell^{T_2}=3$

Example: $G=O$

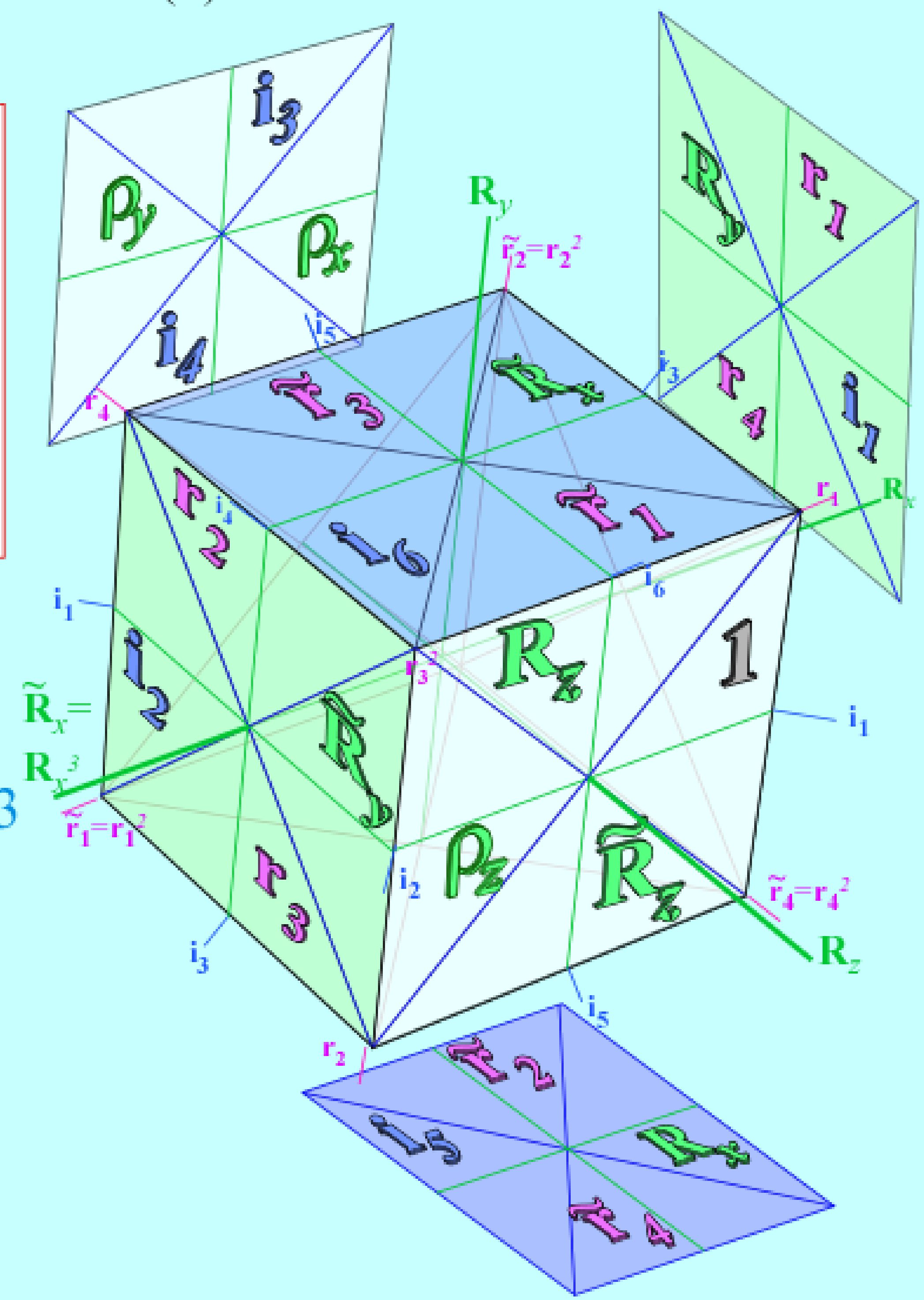
Cubic-Octahedral Group O

Centrum: $\kappa(O)=\Sigma_{(\alpha)}\,(\ell^{\alpha})^{0}=1^{0}+1^{0}+2^{0}+3^{0}+3^{0}=5$

Rank: $\rho(O)=\Sigma_{(\alpha)}\,(\ell^{\alpha})^{1}=1^{1}+1^{1}+2^{1}+3^{1}+3^{1}=10$

Order: $^{o}(O)=\Sigma_{(\alpha)}\,(\ell^{\alpha})^{0}=1^{2}+1^{2}+2^{2}+3^{2}+3^{2}=24$

	O group $\chi^{\alpha}_{\kappa_g}$	$g=1$	r_{1-4} $\tilde{r}_{1-4}$	ρ_{xyz}	R_{xyz} $\tilde{R}_{xyz}$	i_{1-6}
s-orbital r^2	$\alpha=A_1$	1	1	1	1	1
d-orbitals	A_2	1	1	1	−1	−1
$\{x^2+y^2-2z^2, x^2-y^2\}$	E	2	−1	2	0	0
p-orbitals $\{x, y, z\}$	T_1	3	0	−1	1	−1
$\{xz, yz, xy\}$ d-orbitals	T_2	3	0	−1	−1	1



$O\supset C_4$

	$(0)_4$	$(1)_4$	$(2)_4$	$(3)_4=(-1)_4$
A_1	1	•	•	•
A_2	•	•	1	•
E	1	•	1	•
T_1	1	1	•	1
T_2	•	1	1	1

$O\supset C_3$

	$(0)_3$	$(1)_3$	$(2)_3=(-1)_3$
A_1	1	•	•
A_2	1	•	•
E	•	1	1
T_1	1	1	1
T_2	1	1	1

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$$\mathbf{1}\cdot\mathbf{P}^{\alpha} = (\,\mathbf{p}_{0_4} + \mathbf{p}_{1_4} + \mathbf{p}_{2_4} + \mathbf{p}_{3_4}\,)\cdot\mathbf{P}^{\alpha}$$

$$\mathbf{1}\cdot\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_4 0_4} + 0 + 0 + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{A_2} = 0 + 0 + \mathbf{P}^{A_2}_{2_4 2_4} + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{E} = \mathbf{P}^{E}_{0_4 0_4} + 0 + \mathbf{P}^{E}_{2_4 2_4} + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{T_1} = \mathbf{P}^{T_1}_{0_4 0_4} + \mathbf{P}^{T_1}_{1_4 1_4} + 0 + \mathbf{P}^{T_1}_{3_4 3_4}$$

$$\mathbf{1}\cdot\mathbf{P}^{T_2} = 0 + \mathbf{P}^{T_2}_{1_4 1_4} + \mathbf{P}^{T_2}_{2_4 2_4} + \mathbf{P}^{T_2}_{3_4 3_4}$$

where: $\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{im\cdot p/4}\mathbf{R}_z^p$

$$\mathbf{p}_{m_4} = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

$\mathbf{P}^{(\alpha)}_{n_4 n_4}(O\supset C_4)$	$\mathbf{1}$	$r_1r_2\tilde{r}_3\tilde{r}_4$	$\tilde{r}_1\tilde{r}_2r_3r_4$	$\rho_x\rho_y$	ρ_z	$R_x\tilde{R}_xR_y\tilde{R}_y$	R_z	$\tilde{R}_z$	$i_1i_2i_5i_6$	i_3i_4
$24\cdot\mathbf{P}^{A_1}_{0_4 0_4}$	1	1	1	1	1	1	1	1	1	1
$24\cdot\mathbf{P}^{A_2}_{2_4 2_4}$	1	1	1	1	1	-1	-1	-1	-1	-1
$12\cdot\mathbf{P}^{E}_{0_4 0_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1
$12\cdot\mathbf{P}^{E}_{2_4 2_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$+\frac{1}{2}$	-1	-1	$+\frac{1}{2}$	-1
$8\cdot\mathbf{P}^{T_1}_{1_4 1_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	-1	$+\frac{1}{2}$	$-i$	$+i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{3_4 3_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	-1	$+\frac{1}{2}$	$+i$	$-i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{0_4 0_4}$	1	0	0	-1	1	0	1	1	0	-1
$8\cdot\mathbf{P}^{T_2}_{1_4 1_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	-1	$-\frac{1}{2}$	$-i$	$+i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{3_4 3_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	-1	$-\frac{1}{2}$	$+i$	$-i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{2_4 2_4}$	1	0	0	-1	1	0	-1	-1	0	1

Summary of $O\supset C_4$ diagonal (idempotent) projectors $\mathbf{P}^{\mu}_{jj}$

$O\supset C_4$	0_4	1_4	2_4	3_4
$A_1\downarrow C_4$	1	·	·	·
$A_2\downarrow C_4$	·	·	1	·
$E\downarrow C_4$	1	·	1	·
$T_1\downarrow C_4$	1	1	·	1
$T_2\downarrow C_4$	·	1	1	1

$$\mathbf{1}\cdot\mathbf{P}^{\alpha}=(\mathbf{p}_{0_4}+\mathbf{p}_{1_4}+\mathbf{p}_{2_4}+\mathbf{p}_{3_4})\cdot\mathbf{P}^{\alpha}$$

$$\mathbf{1}\cdot\mathbf{P}^{A_1}=\mathbf{P}^{A_1}_{0_40_4}+0+0+0$$

$$\mathbf{1}\cdot\mathbf{P}^{A_2}=0+0+\mathbf{P}^{A_2}_{2_42_4}+0$$

$$\mathbf{1}\cdot\mathbf{P}^{E}=\mathbf{P}^{E}_{0_40_4}+0+\mathbf{P}^{E}_{2_42_4}+0$$

$$\mathbf{1}\cdot\mathbf{P}^{T_1}=\mathbf{P}^{T_1}_{0_40_4}+\mathbf{P}^{T_1}_{1_41_4}+0+\mathbf{P}^{T_1}_{3_43_4}$$

$$\mathbf{1}\cdot\mathbf{P}^{T_2}=0+\mathbf{P}^{T_2}_{1_41_4}+\mathbf{P}^{T_2}_{2_42_4}+\mathbf{P}^{T_2}_{3_43_4}$$

where: $\mathbf{p}_{m_4}=\frac{1}{4}\sum_{p=0}^{3}e^{im\cdot p/4}\mathbf{R}_z^p$

$$\mathbf{p}_{m_4}=\begin{cases}\mathbf{p}_{0_4}=(\mathbf{1}+\mathbf{R}_z+\rho_z+\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{1_4}=(\mathbf{1}+i\mathbf{R}_z-\rho_z-i\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{2_4}=(\mathbf{1}-\mathbf{R}_z+\rho_z-\tilde{\mathbf{R}}_z)/4\\ \mathbf{p}_{3_4}=(\mathbf{1}-i\mathbf{R}_z-\rho_z+i\tilde{\mathbf{R}}_z)/4\end{cases}$$

Summary of $O\supset C_4$ diagonal (idempotent) projectors $\mathbf{P}^{\mu}_{jj}$

$\mathbf{P}^{(\alpha)}_{n_4n_4}(O\supset C_4)$	$\mathbf{1}$	$r_1r_2\tilde{r}_3\tilde{r}_4$	$\tilde{r}_1\tilde{r}_2r_3r_4$	$\rho_x\rho_y$	ρ_z	$R_x\tilde{R}_xR_y\tilde{R}_y$	R_z	$\tilde{R}_z$	$i_1i_2i_5i_6$	i_3i_4
$24\cdot\mathbf{P}^{A_1}_{0_40_4}$	1	1	1	1	1	1	1	1	(+1)	1
$24\cdot\mathbf{P}^{A_2}_{2_42_4}$	1	1	1	1	1	−1	−1	−1	−1	−1
$12\cdot\mathbf{P}^{E}_{0_40_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1	1	($-\frac{1}{2}$)	1
$12\cdot\mathbf{P}^{E}_{2_42_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$+\frac{1}{2}$	−1	−1	$+\frac{1}{2}$	−1
$8\cdot\mathbf{P}^{T_1}_{1_41_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	−1	$+\frac{1}{2}$	$-i$	$+i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{3_43_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	−1	$+\frac{1}{2}$	$+i$	$-i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{0_40_4}$	1	0	0	−1	1	0	1	1	(0)	−1
$8\cdot\mathbf{P}^{T_2}_{1_41_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	−1	$-\frac{1}{2}$	$-i$	$+i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{3_43_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	−1	$-\frac{1}{2}$	$+i$	$-i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{2_42_4}$	1	0	0	−1	1	0	−1	−1	0	1

The $0_4\uparrow$ cluster

i_{16} split *i_{34} split*

$\mathbf{P}^{A_1}_{0_40_4}$ +1

$\mathbf{P}^{T_1}_{0_40_4}$ 0

$\mathbf{P}^{E}_{0_40_4}$ −1/2

$O \supset C_4$	0_4	1_4	2_4	3_4
$A_1 \downarrow C_4$	1	·	·	·
$A_2 \downarrow C_4$	·	·	1	·
$E \downarrow C_4$	1	·	1	·
$T_1 \downarrow C_4$	1	1	·	1
$T_2 \downarrow C_4$	·	1	1	1

$$\mathbf{1}\cdot\mathbf{P}^{\alpha} = (\,\mathbf{p}_{0_4} + \mathbf{p}_{1_4} + \mathbf{p}_{2_4} + \mathbf{p}_{3_4}\,)\cdot\mathbf{P}^{\alpha}$$

$$\mathbf{1}\cdot\mathbf{P}^{A_1} = \mathbf{P}^{A_1}_{0_40_4} + 0 + 0 + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{A_2} = 0 + 0 + \mathbf{P}^{A_2}_{2_42_4} + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{E} = \mathbf{P}^{E}_{0_40_4} + 0 + \mathbf{P}^{E}_{2_42_4} + 0$$

$$\mathbf{1}\cdot\mathbf{P}^{T_1} = \mathbf{P}^{T_1}_{0_40_4} + \mathbf{P}^{T_1}_{1_41_4} + 0 + \mathbf{P}^{T_1}_{3_43_4}$$

$$\mathbf{1}\cdot\mathbf{P}^{T_2} = 0 + \mathbf{P}^{T_2}_{1_41_4} + \mathbf{P}^{T_2}_{2_42_4} + \mathbf{P}^{T_2}_{3_43_4}$$

where: $\mathbf{p}_{m_4} = \frac{1}{4}\sum_{p=0}^{3} e^{im\cdot p/4}\mathbf{R}_z^p$

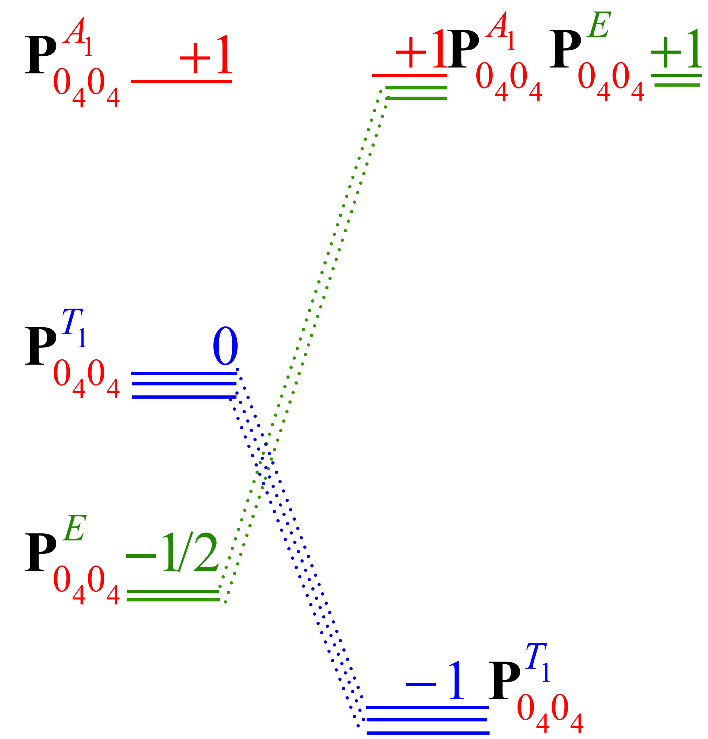
$$\mathbf{p}_{m_4} = \begin{cases} \mathbf{p}_{0_4} = (\mathbf{1} + \mathbf{R}_z + \rho_z + \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{1_4} = (\mathbf{1} + i\mathbf{R}_z - \rho_z - i\tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{2_4} = (\mathbf{1} - \mathbf{R}_z + \rho_z - \tilde{\mathbf{R}}_z)/4 \\ \mathbf{p}_{3_4} = (\mathbf{1} - i\mathbf{R}_z - \rho_z + i\tilde{\mathbf{R}}_z)/4 \end{cases}$$

Summary of $O \supset C_4$ diagonal (idempotent) projectors $\mathbf{P}^{\mu}_{jj}$

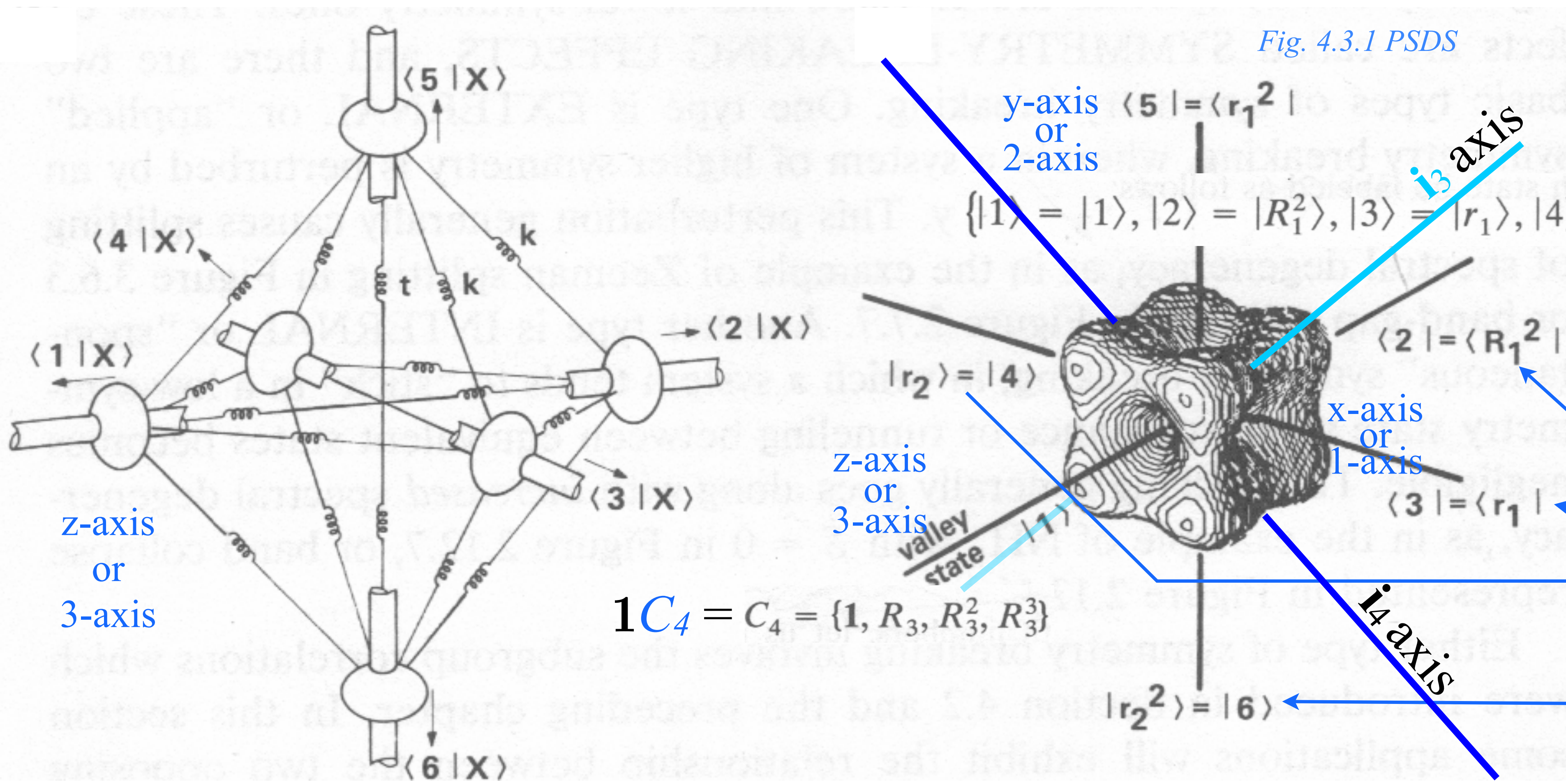
$\mathbf{P}^{(\alpha)}_{n_4n_4}(O \supset C_4)$	**1**	$r_1r_2\tilde{r}_3\tilde{r}_4$	$\tilde{r}_1\tilde{r}_2r_3r_4$	$\rho_x\rho_y$	ρ_z	$R_x\tilde{R}_xR_y\tilde{R}_y$	R_z	$\tilde{R}_z$	$i_1i_2i_5i_6$	i_3i_4
$24\cdot\mathbf{P}^{A_1}_{0_40_4}$	1	1	1	1	1	1	1	1	+1	+1
$24\cdot\mathbf{P}^{A_2}_{2_42_4}$	1	1	1	1	1	−1	−1	−1	−1	−1
$12\cdot\mathbf{P}^{E}_{0_40_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	+1
$12\cdot\mathbf{P}^{E}_{2_42_4}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	$+\frac{1}{2}$	−1	−1	$+\frac{1}{2}$	−1
$8\cdot\mathbf{P}^{T_1}_{1_41_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	−1	$+\frac{1}{2}$	$-i$	$+i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{3_43_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	−1	$+\frac{1}{2}$	$+i$	$-i$	$-\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_1}_{0_40_4}$	1	0	0	−1	1	0	1	1	0	−1
$8\cdot\mathbf{P}^{T_2}_{1_41_4}$	1	$+\frac{i}{2}$	$-\frac{i}{2}$	0	−1	$-\frac{1}{2}$	$-i$	$+i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{3_43_4}$	1	$-\frac{i}{2}$	$+\frac{i}{2}$	0	−1	$-\frac{1}{2}$	$+i$	$-i$	$+\frac{1}{2}$	0
$8\cdot\mathbf{P}^{T_2}_{2_42_4}$	1	0	0	−1	1	0	−1	−1	0	1

The $0_4\uparrow$ cluster

i_{16} split *i_{34} split*



Elementary induced representation $0_4(C_4)\uparrow O$



Thus we label states by left *cosets* $\mathbf{r}_\ell C_4$ *of Local* C_4

$\mathbf{1}C_4 = C_4 = \{\mathbf{1}, R_3, R_3^2, R_3^3\}$

$R_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (R_1^2, i_4, R_2^2, i_3),$

$r_1(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1, i_1, r_4, R_2),$

$r_2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2, i_2, r_3, R_2^3),$

$r_1^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_1^2, R_1^3, r_3^2, i_6),$

$r_2^2(\mathbf{1}, R_3, R_3^2, R_3^3) = (r_2^2, R_1, r_4^2, i_5),$

Here is $0_4(C_4)$ *induced representation* $\mathscr{I}^{0_4\uparrow O}(\mathbf{I_i})$ *of a linear combination of* **i**-*class rotations*

$$\mathbf{I_i} = i_1\mathbf{i_1} + i_2\mathbf{i_2} + i_3\mathbf{i_3} + i_4\mathbf{i_4} + i_5\mathbf{i_5} + i_6\mathbf{i_6} \longrightarrow \mathbf{I_i} = i_{34}(\mathbf{i_3} + \mathbf{i_4}) + i_{16}(\mathbf{i_1} + \mathbf{i_2} + \mathbf{i_5} + \mathbf{i_6})$$

$\mathscr{I}^{0_4\uparrow O}(\mathbf{I_i}) =$

	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$
$\langle 1\rvert$	1	i_3+i_4	i_1	i_2	i_6	i_5
$\langle 2\rvert$	i_3+i_4	1	i_2	i_1	i_5	i_6
$\langle 3\rvert$	i_1	i_2	1	i_5+i_6	i_3	i_4
$\langle 4\rvert$	i_2	i_1	i_5+i_6	1	i_4	i_3
$\langle 5\rvert$	i_6	i_5	i_3	i_4	1	i_1+i_2
$\langle 6\rvert$	i_5	i_6	i_4	i_3	i_1+i_2	1

Let:
$i_3 = i_{34} = i_4$

	$\lvert 1\rangle$	$\lvert 2\rangle$	$\lvert 3\rangle$	$\lvert 4\rangle$	$\lvert 5\rangle$	$\lvert 6\rangle$
$\langle 1\rvert$	1	$2i_{34}$	i_{16}	i_{16}	i_{16}	i_{16}
$\langle 2\rvert$	$2i_{34}$	1	i_{16}	i_{16}	i_{16}	i_{16}
$\langle 3\rvert$	i_{16}	i_{16}	1	$2i_{16}$	i_{34}	i_{34}
$\langle 4\rvert$	i_{16}	i_{16}	$2i_{16}$	1	i_{34}	i_{34}
$\langle 5\rvert$	i_{16}	i_{16}	i_{34}	i_{34}	1	$2i_{16}$
$\langle 6\rvert$	i_{16}	i_{16}	i_{34}	i_{34}	$2i_{16}$	1

and/or:
$i_{16} = i_1 = i_2 = i_5 = i_6$

Review Octahedral $O_h \supset O$ *group operator structure*

Review Octahedral $O_h \supset O \supset D_4 \supset C_4$ *subgroup chain correlations*

Comparison of $O \supset D_4 \supset C_4$ *and* $O \supset D_4 \supset D_2$ *correlations and level/projector splitting*

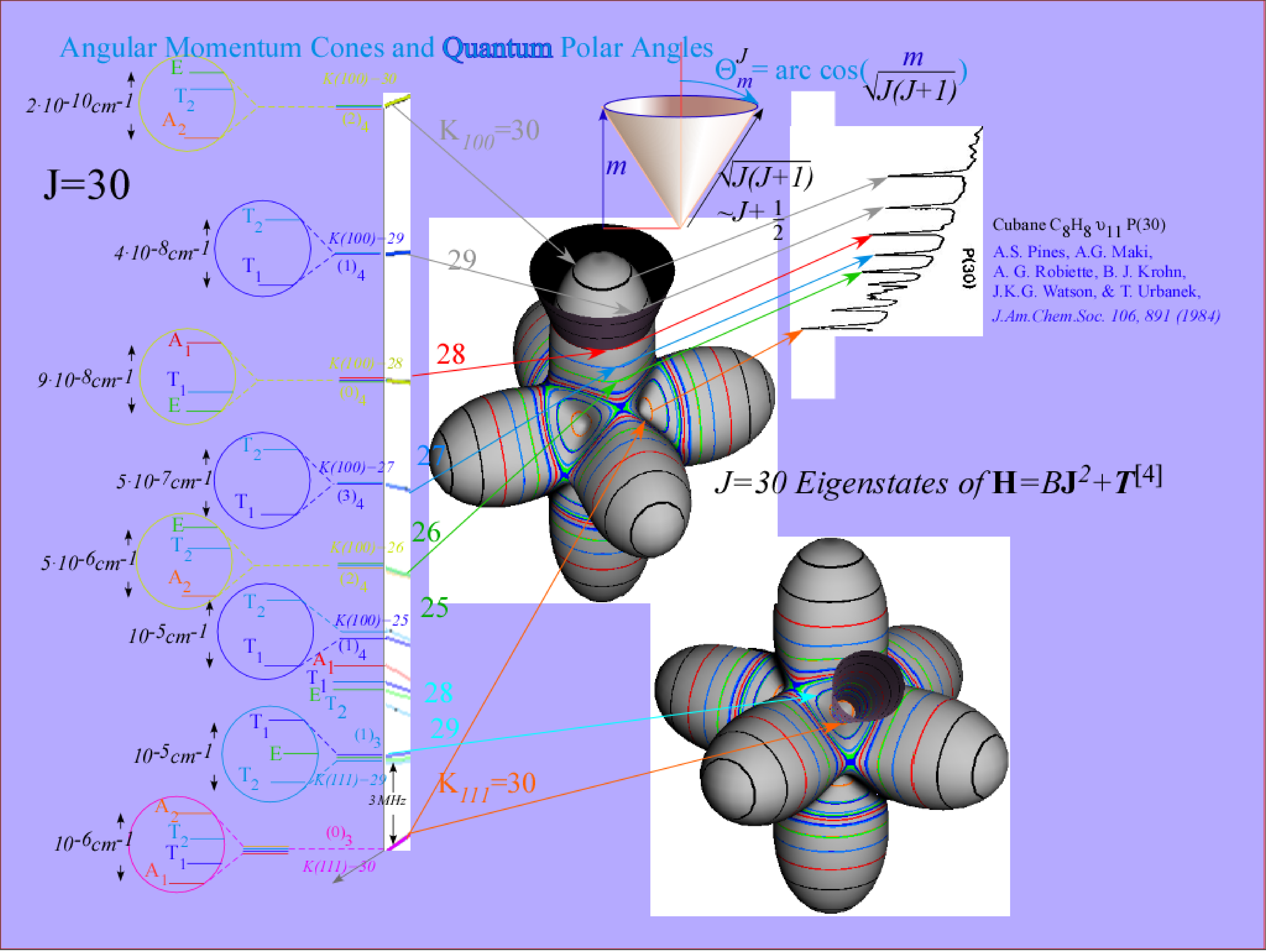
- $O \supset D_4 \supset C_4$ *subgroup chain splitting*
- $O \supset D_4 \supset D_2$ *subgroup chain splitting (nOrmal* D_2 *vs. unOrmal* D_2*)*
- $O_h \supset O \supset D_4 \supset C_{4v}$ *and* $O_h \supset O \supset D_4 \supset C_{4v} \supset C_{2v}$ *subgroup splitting*

Splitting O *class projectors* $\mathbf{P}^{\mu}$ *into irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *for* $O \supset C_4$

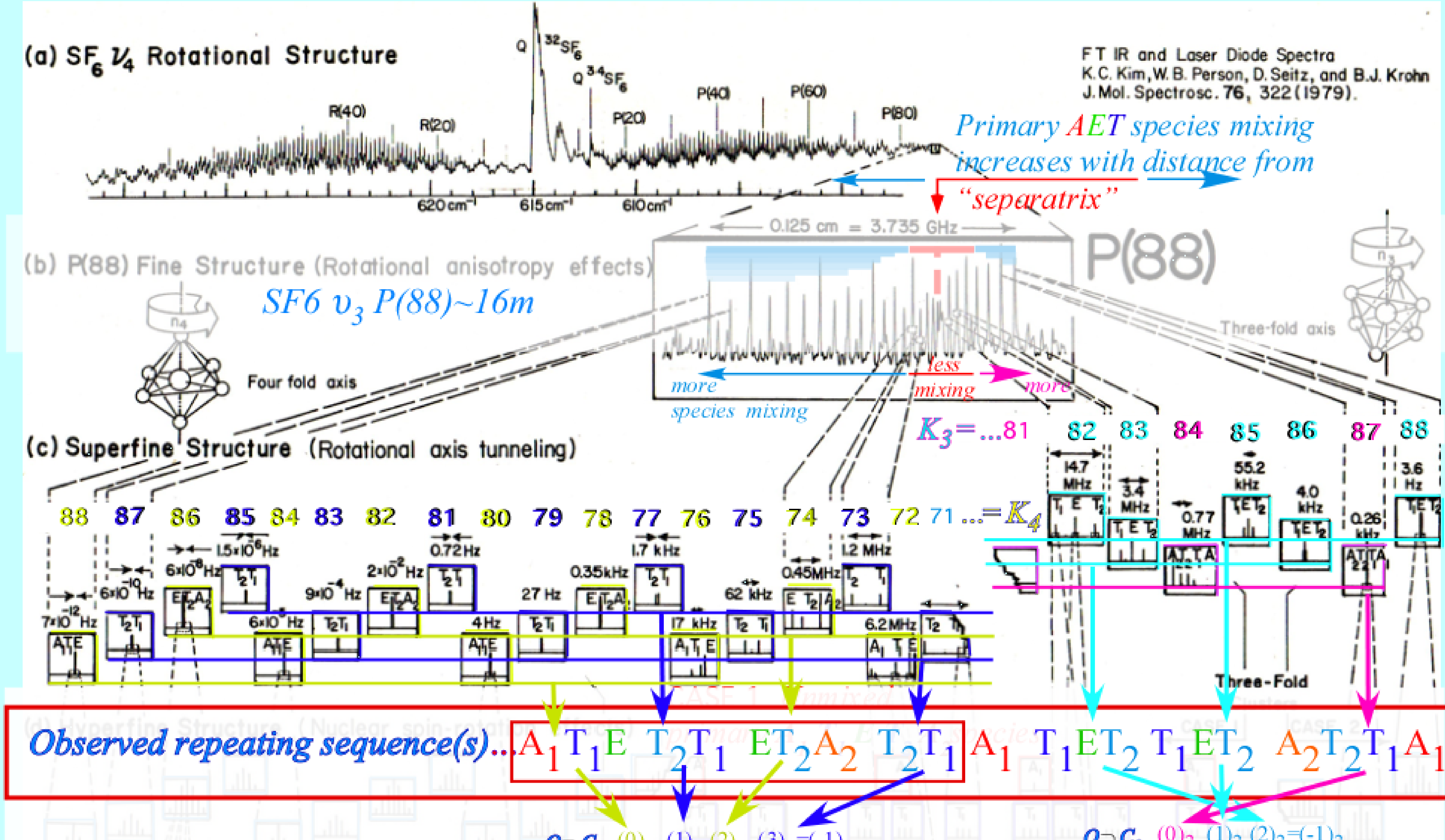
- *Development of irreducible projectors* $\mathbf{P}^{\mu}_{m_4 m_4}$ *and representations* $D^{\mu}_{m_4 m_4}$
- *Calculating* $\mathbf{P}^{E}_{0_4 0_4}$, $\mathbf{P}^{E}_{2_4 2_4}$, $\mathbf{P}^{T_1}_{0_4 0_4}$, $\mathbf{P}^{T_1}_{1_4 1_4}$, $\mathbf{P}^{T_2}_{2_4 2_4}$, $\mathbf{P}^{T_2}_{1_4 1_4}$,

$O \supset C_4$ *induced representation* $0_4(C_4)\uparrow O \sim A_1 \oplus T_1 \oplus E$ *and spectral analysis examples*

- *Elementary induced representation* $0_4(C_4)\uparrow O$
- *Projection reduction of induced representation* $0_4(C_4)\uparrow O$
- *Introduction to ortho-complete eigenvalue-parameter relations*
  - *Examples in* SF_6 *spectroscopy*



Angular Momentum Cones and Quantum Polar Angles
$\Theta_m^J = \arccos\left(\frac{m}{\sqrt{J(J+1)}}\right)$
J=30
m
$\sqrt{J(J+1)}$
$\sim J+\frac{1}{2}$
K100=30
29
28
27
26
25
28
29
K111=30
2·10-10cm-1
4·10-8cm-1
9·10-8cm-1
5·10-7cm-1
5·10-6cm-1
10-5cm-1
10-5cm-1
10-6cm-1
3 MHz
Cubane C8H8 υ11 P(30)
A.S. Pines, A.G. Maki,
A. G. Robiette, B. J. Krohn,
J.K.G. Watson, & T. Urbanek,
J.Am.Chem.Soc. 106, 891 (1984)
P(30)
J=30 Eigenstates of H=BJ2+T[4]



	$(0)_4$	$(1)_4$	$(2)_4$	$(3)_4$
A_1	1	•	•	•
A_2	•	•	1	•
E	1	•	1	•
T_1	1	1	•	1
T_2	•	1	1	1

	$(0)_3$	$(1)_3$	$(2)_3$
A_1	1	•	•
A_2	1	•	•
E	•	1	1
T_1	1	1	1
T_2	1	1	1

Eigenvalues of $\mathbf{H}=B\mathbf{J}^2+\cos\phi\,\mathbf{T}^{[4]}+\sin\phi\,\mathbf{T}^{[6]}$ *vs. mix angle* ϕ*:* $0<\phi<\pi$