Assignment 5 - **Group Theory and Quantum Physics 5093** Due Fri. 3/17/17

Main Reading: In text *QTCA* Unit 3-5 and *U(2) Character Geometry p.5-18*. Lectures 9-12.

Galloping Gertie

1. Galloping phase velocity of general 2-frequency plane $A_1 e^{i(kx-\omega t)} + A_2 e^{i(kx-\omega t)}$ helps us understand relativistic QM. Derive this velocity $V_{gallop}(t)$ as a function of *Standing Wave Ratio* $SWR=(A_1-A_2)/(A_1+A_2)$, time t, and $V_{phase}=c=\omega/k$. Then give a formula for maximum $V_{gallop\text{-}max}$ and minimum $V_{gallop\text{-}min}$ phase velocities achieved in terms of SWR.

$H_{2x2}=\begin{pmatrix} A & B \\ B & A \end{pmatrix}$ *and high school algebra revisited*

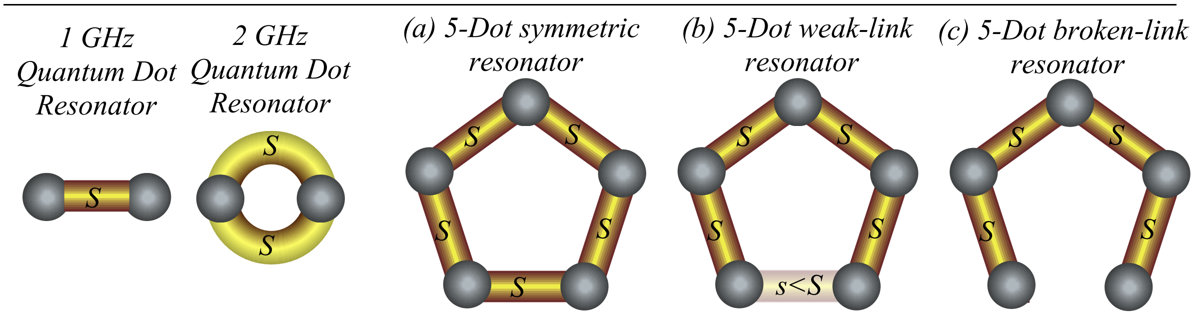
2. Being able to solve eigenvalue problems using symmetry projection implies that such projection can also solve certain types of algebraic polynomial equations, that is, secular equations. Indeed, the $C_2(B)$ solution amounts to derivation of the quadratic formula for solutions to $ax^2+bx^2+c=0$. Show eigenvalues of H_{2x2}*(above)* given by $C_2(B)$ algebra also re-derive quadratic equation. (Relate A and B to b/a and c/a. Next we begin to generalize these results.)

$H_{3x3}=\begin{pmatrix} 0 & A & B \\ B & 0 & A \\ A & B & 0 \end{pmatrix}$ *and college algebra revisited*

3. A C_3-symmetric *3x3* Hamiltonian matrix H_{3x3}*(above)* has a cubic secular equation. Use C_3-solutions to derive a cubic formula for roots of $ax^3+bx^2+cx+d=0$. Hint: Simplify by dealing with secular equation $x^3+0+cx+d=0$ of traceless H_{3x3}. Show that missing a and b parts are easy to tack on afterwards.

$H_{4x4}=\left(?\right)$ *and beyond (This is an extra credit problem)*

4. A C_4-symmetric *4x4* Hamiltonian matrix H_{4x4}*(derive this)* has a quartic secular equation. Use C_4-solutions to derive its eigensolutions. Discuss whether this give roots for a *general* quartic $ax^4+bx^3+cx^2+dx+e=0$ or just for a special case.



Holey Pentagram Batman!

5. Five identical quantum dots are originally connected by five identical evanescent tunneling "weak-links" as shown in Figure (a) above. Let a quantum dot pair linked by a *single* path of tunneling amplitude S (1st sketch) have a *1.0 GHz* resonance as its lowest frequency and a *2.0 GHz* resonance if linked by a *two* S-paths.) A link may have slightly weaker tunneling amplitude $s<S$, as in Figure (b) or be removed entirely, as sketched in Figure (c).

(a) Derive and solve a model Hamiltonian matrix for the Figure (a) device. This should include a matrix representation and its eigenvectors and eigenvalues. These may be given in terms of the tunneling amplitude S, but a numerical value of S in Hz should be used, too. Sketch ω_m levels and ω_m-projecting polygon. Discuss its beats. Can it do perfect revivals?

(b) Approximate the Hamiltonian and energy levels perturbed by reducing lower link to $s=0.9S$ in Figure (b). (Perturbation theory in Sec. 3.2(b) of *QTforCA* may be helpful.)

(c) Find Hamiltonian and its eigensolutions missing a link as in Figure (c). Give eigenvectors and values in n-dot linear chain models given in class. (See also p. 13-16 of *U(2) Character Geometry or* Sec. 14.1 of *QTforCA* Unit 5.)

6. List all *distinct* commutative (Abelian) groups in terms of *distinct* $C_p \times C_q \times$... cross products.
(a) for order $N=8$. **(b)** $N=9$. **(c)** $N=10$. **(d)** $N=11$. **(e)** $N=12$. **(f)** $N=16$.

7. Consider character tables for Abelian groups using $(m)_n \times (p)_q \times$ …notation where $(m)_n$ means *m-waves*-modulo-*n*.

(a) List character tables of all *distinct* Abelian groups of order 8.
(b) List character tables of $C_3 \times C_2$ using $(m)_3 \times (p)_2$ notation and C_6 using $(m)_6$ notation.
(c) Show these two "six-groups" are not *distinct* Abelian groups, and relate $(m)_3 \times (p)_2$ notation to $(m)_6$ notation.
(d) Using notation $(p)_2 \times (m)_3$ for $C_2 \times C_3$, show how that alters character table ordering.