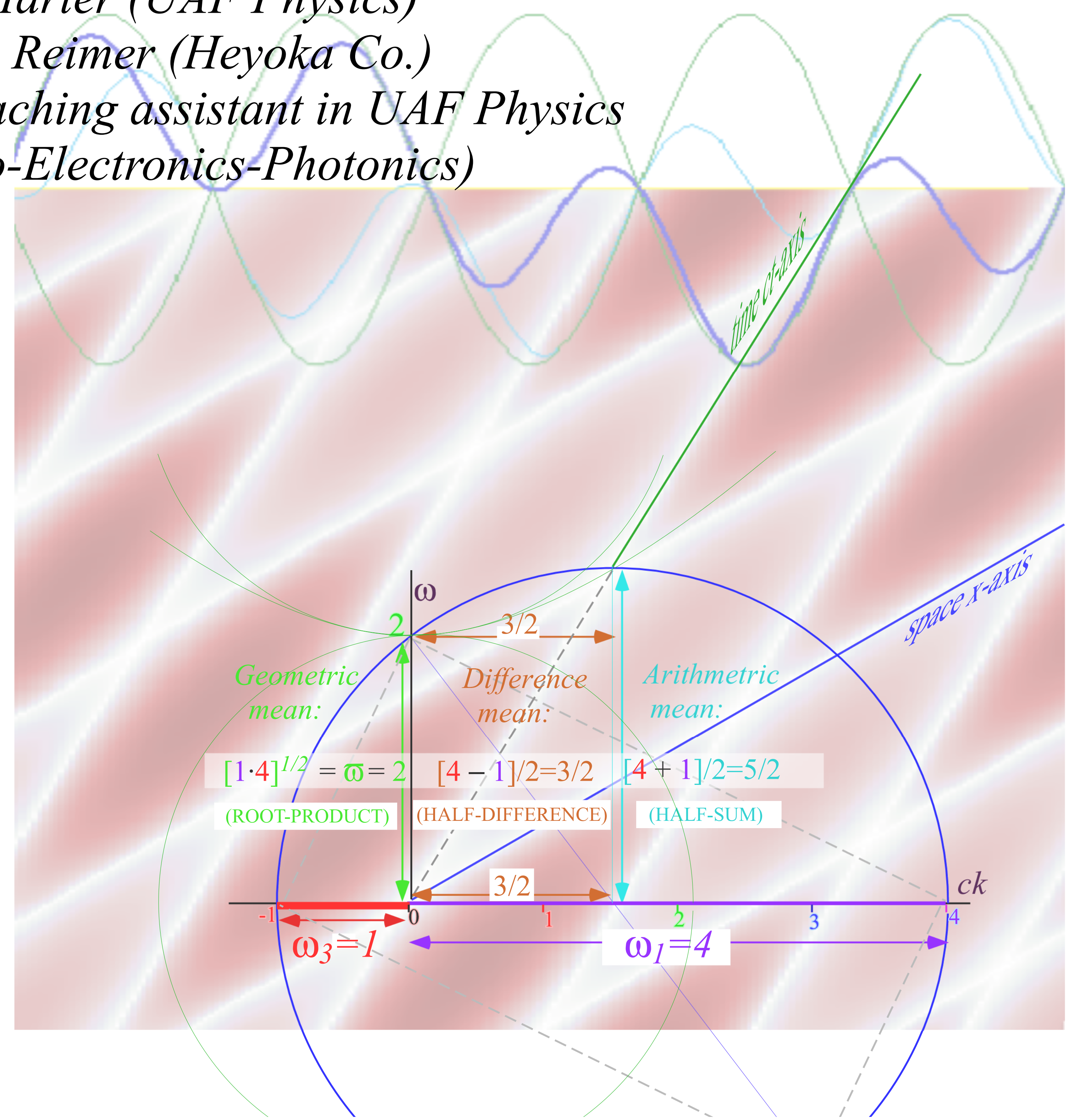
EVENSON'S LASERS AND OCCAM'S RAZORS: Some dirty little geometric secrets of relativity and quantum theory

Presentation and production by
W. G. Harter (UAF Physics)
Dr. T.C. Reimer (Heyoka Co.)
Al Calabrese (Teaching assistant in UAF Physics and Micro-Electronics-Photonics)



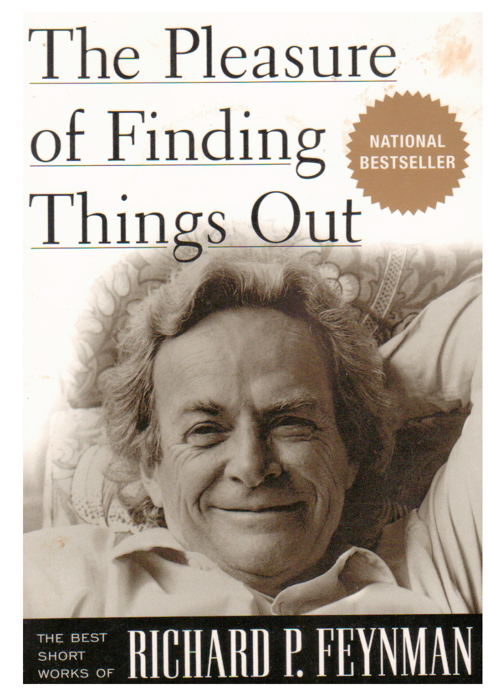
Shout-out to
Graphene pseudo-Relativity
UAF theory team:
Salvador Barraza-Lopez
Physics Department
Edmund Harriss
Mathematics Department

Bob: Don’t worry Alice, I don’t understand this relativity *or* the quantum theory, but I bet the professor doesn’t either.

I like relativity and
quantum theories
Because I don’t under-
stand them
and they make me feel
as if space shifted
about like a swan
that can’t settle,
refusing to sit still and
be measured:
and as if the atom were
an impulsive thing
always changing its
mind.

—D. H. Lawrence

From Jargodzki and Potter *“Mad About Physics”*



“If you think you understand quantum mechanics, you don’t...”
Quote from R. P. Feynman in “Character of Physical Law” University Lecture

The quote, exact words, "If you think you understand quantum mechanics, you don't…” in Google hits about 16,500 pages. But I can't find anywhere that actually gives a written source! What to do? Possibly, originated with Niels Bohr: "Anyone who is not shocked by quantum theory has not understood it." Similar problems with checking a *much* older quote “Only 12 people understand relativity…”

My personal opinion about my first graduate advisor: I doubt he meant to attach a Catch-22 to understanding physics.

Current understanding of relativity and QM at UAF

NWAT photo by David Gottschalk

Current understanding of relativity and QM at UAF(and the World)

NWAT photo by David Gottschalk

[1] D. F. Styer, M. S. Balkin, K. M. Becker, M. R. Burns, C. E. Dudley, S. T. Forth, J. S. Gaumer, M. A. Kramer, D. C. Oertel, L. H. Park, M. T. Rinkoski, C. T. Smith, and T. D. Wotherspoon, "Nine Formulations of Quantum Mechanics", *Am. J. Phys.* **70**, 288 (2002).

Current understanding of relativity and QM at UAF (and the World)

Can we <u>clarify</u>? ...and <u>simplify</u>?

Current understanding of relativity and QM at UAF (and the World)

Can we clarify *by* simplifying?

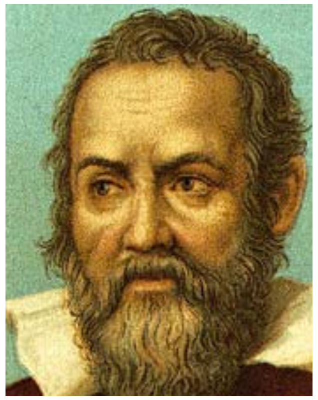
Level 1 Secrets *(which really shouldn't be secrets at all!)*

Some have forgotten... Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

looks worried?



Galilei Galileo
1564-1642

Need to review...

- Where Galilean relativity fails for light waves,

...and where it doesn't.

and then see...

- How to make sense of light-wave **SPEED LIMIT c= 299,792,458 *m/s*** axiom(s)

Good approximation:
c ≅ 300 million m/s
300 Megameter/s

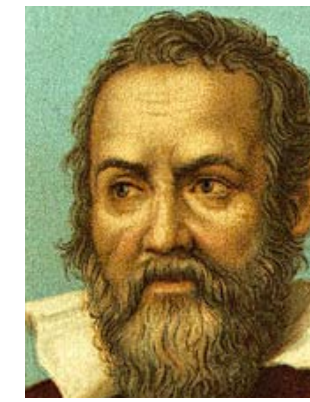
(We'll use frequencies divisible by 3)

Some have forgotten... Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

looks worried?



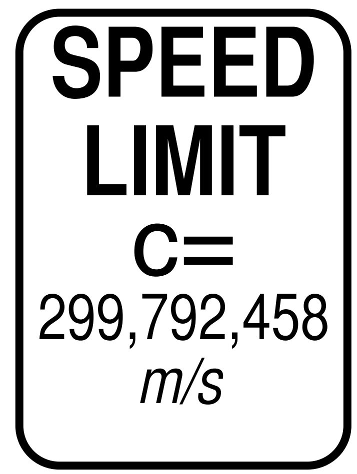
Galilei Galileo
1564-1642

Need to review...

- Where Galilean relativity fails for light waves,
...and where it doesn't.

and then see...

- How to make sense of light-wave



axiom(s)

by comparing *Einstein Pulse Wave* (PW) axiom

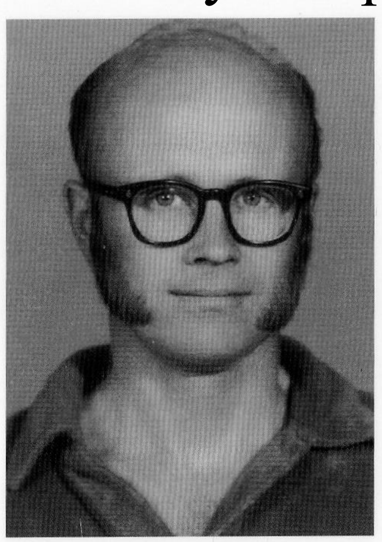
with

Evenson Continuous Wave (CW) axiom

in *space-time* and *inverse space-time*

Good approximation:
c ≅ 300 million m/s
300 Megameter/s

(We'll use frequencies divisible by 3)



PAULINIA, BRASIL 1976

THE SPEED OF LIGHT IS
299,792,458 METERS PER SECOND!

Kenneth M. Evenson
1932-2002

K. M. Evenson, J.S. Wells, F.R. Peterson, B.L. Danielson, G.W. Day, R.L. Barger and J.L. Hall, Phys. Rev. Letters 29, 1346(1972).

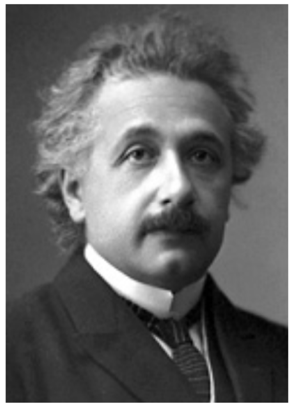
In 2005 the Nobel Prize in physics was awarded to Glauber, Hall, and Hensch†† for laser optics and metrology.

†† *The Nobel Prize in Physics, 2005*. http://nobelprize.org/

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

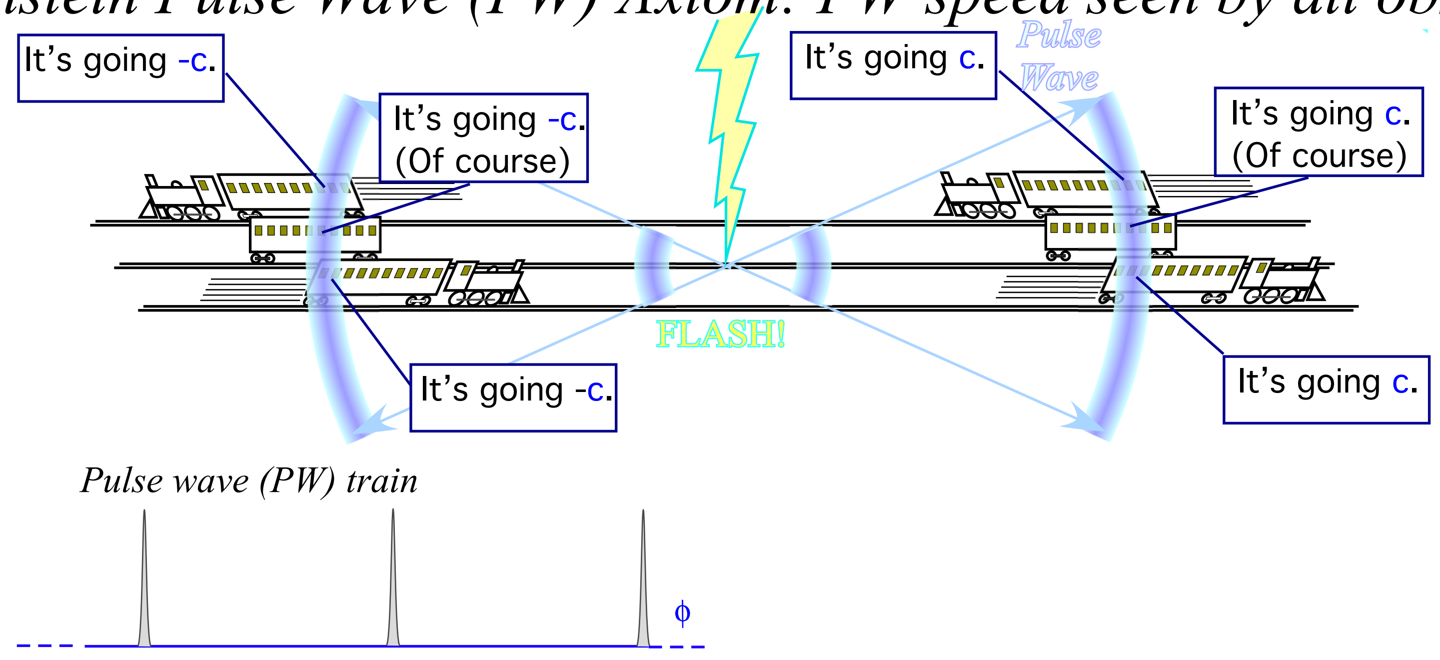
And, HE-eee-rRE'S Albert!

Albert Einstein

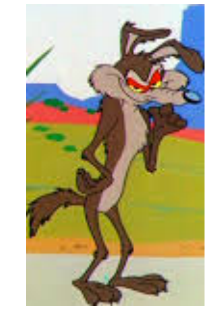


1879-1955

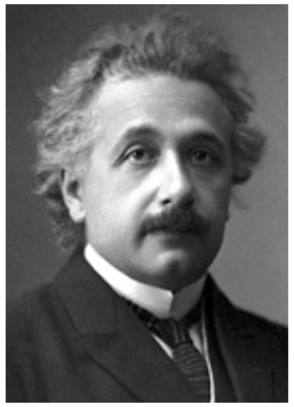
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

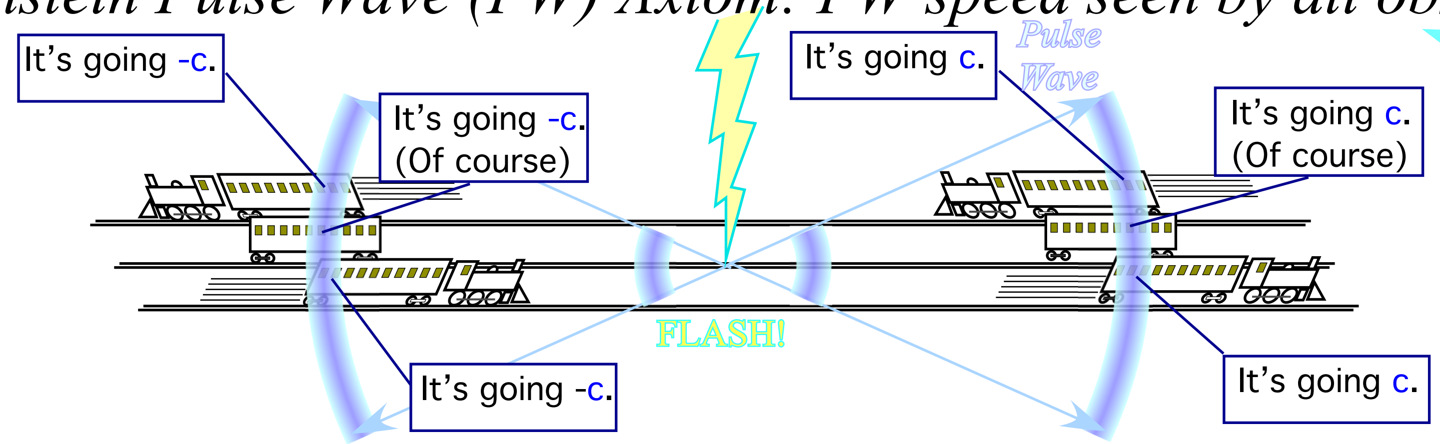


Albert Einstein

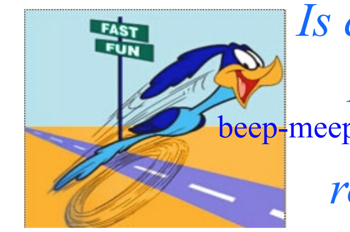


1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

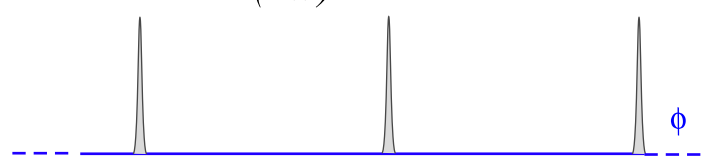


A "road-runner" axiom is a "show-stopper"

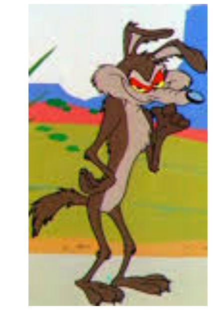


Is cartoon physics a reality?!

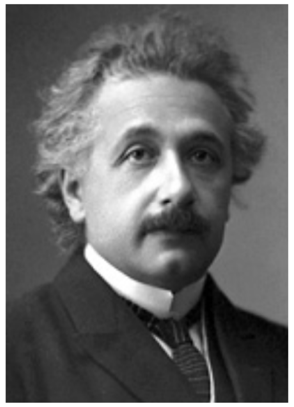
Pulse wave (PW) train



• How do you make sense of light-wave [SPEED LIMIT c= 299,792,458 m/s] axiom(s)?

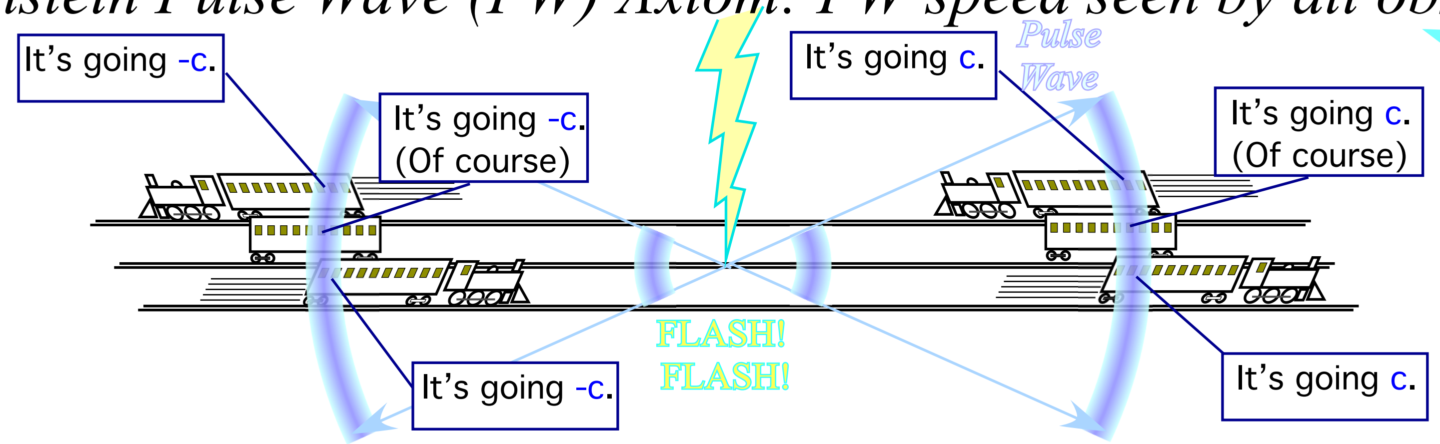


Albert Einstein



1879-1955

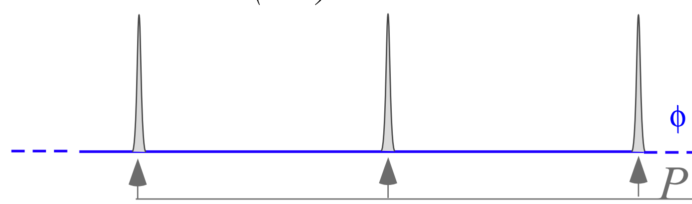
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"

Is cartoon physics a reality?!

Pulse wave (PW) train



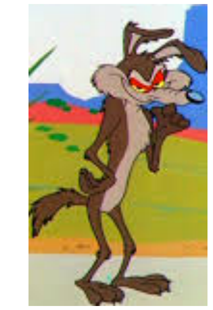
$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + \ldots$

PW Axiom is <u>complicated</u>

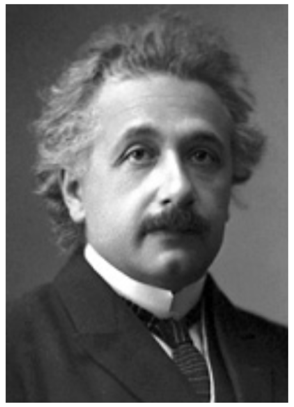
..though it has a Newtonian "Place for everything & everything in place" feel.

PW peaks precisely locate places where wave <u>is</u>.

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

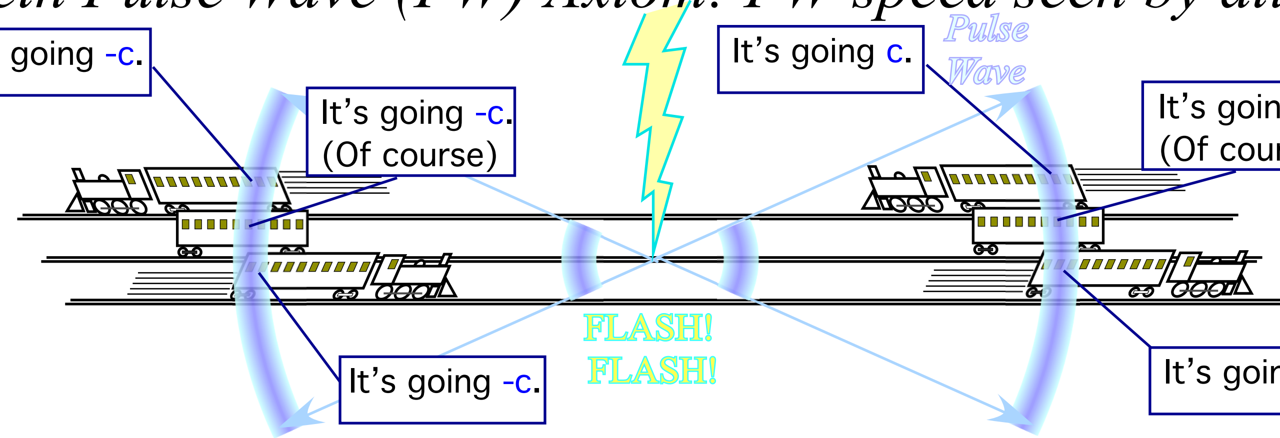


Albert Einstein



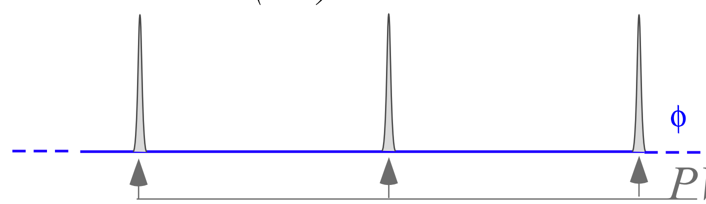
1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"

Pulse wave (PW) train



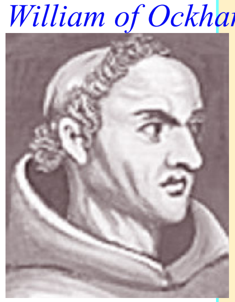
$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots$

PW Axiom is <u>complicated</u>

..though it has a Newtonian "Place for everything & everything in place" feel.

PW peaks precisely locate places where wave <u>is</u>.

William of Ockham

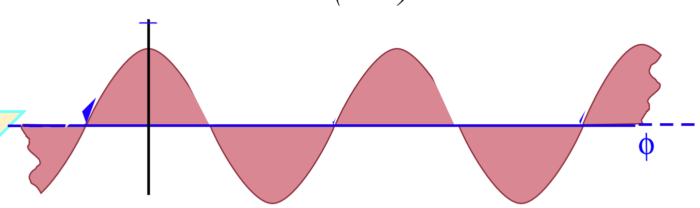


1285-1349

Using Occam's Razor

(and Evenson's lasers)

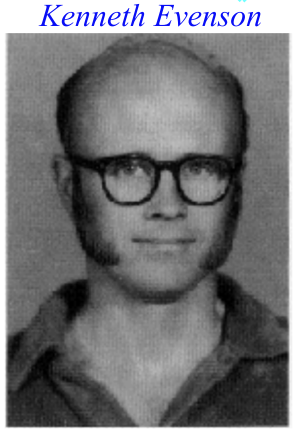
Continuous wave (CW) train



$A \cos \omega t$

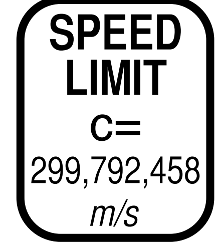
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

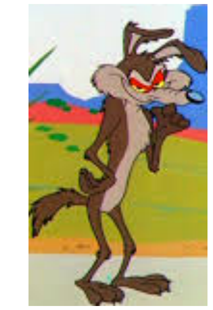
Kenneth Evenson



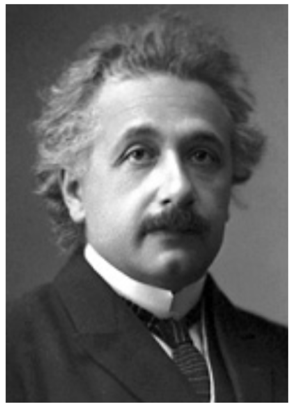
1932-2002

Cut a *PW* to just *one* Continuous Wave

• How do you make sense of light-wave  axiom(s)?

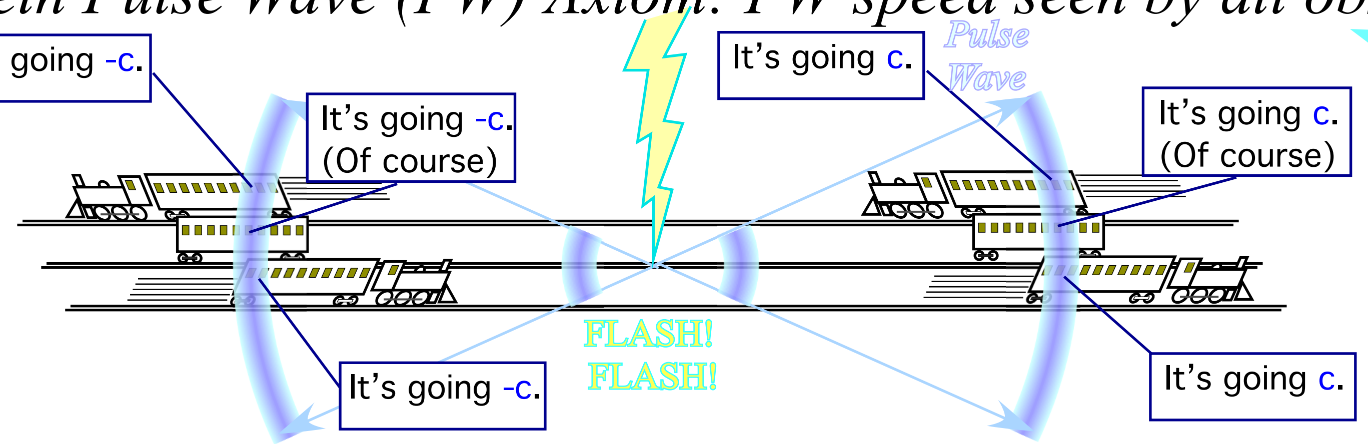


Albert Einstein



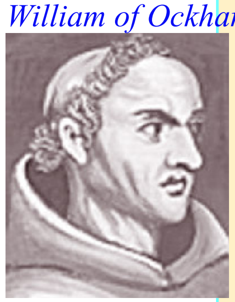
1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"

William of Ockham

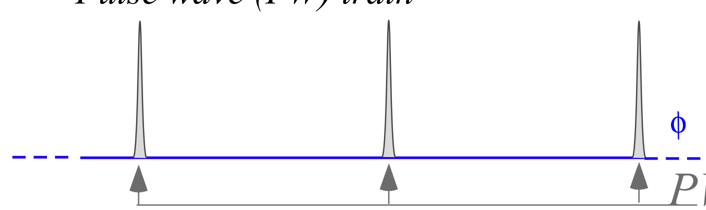


1285-1349

Using Occam's Razor

(and Evenson's lasers)

Pulse wave (PW) train



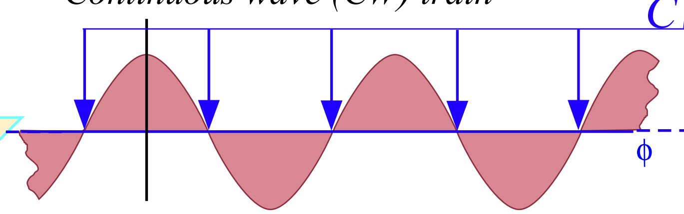
~~$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots$~~

PW Axiom is complicated

..though it has a Newtonian "Place for everything & everything in place" feel.

PW peaks precisely locate places where wave is.

Continuous wave (CW) train



CW zeros precisely locate places where wave is not.

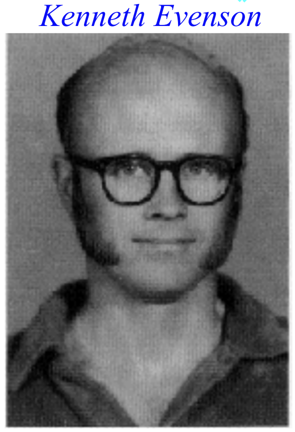
$A \cos \omega t$

Simpler 1CW coherence is more "Zen-like"

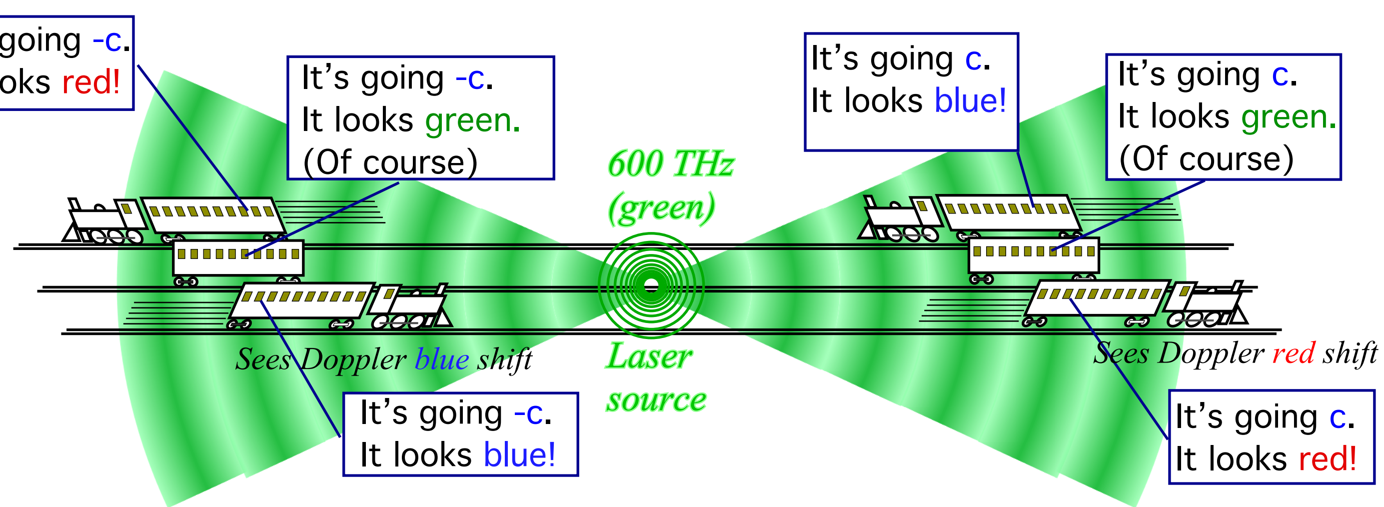
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Can be made more self-evident and productive

Kenneth Evenson

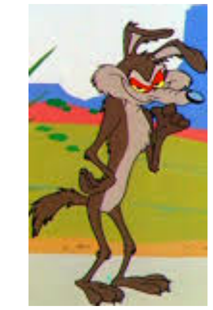


1932-2002

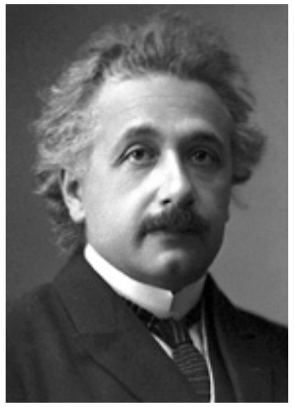


Cut a *PW* to just *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

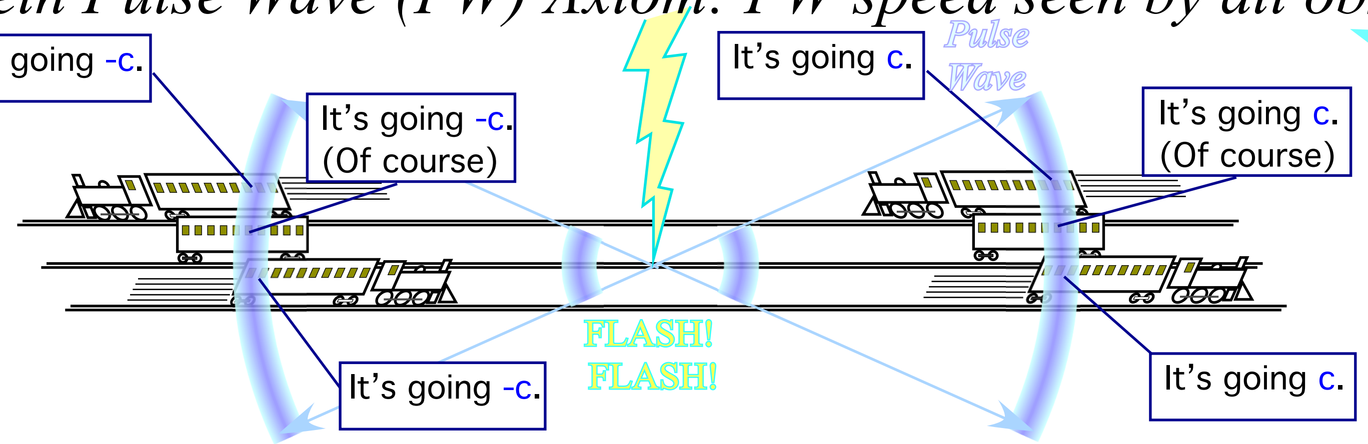


Albert Einstein



1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

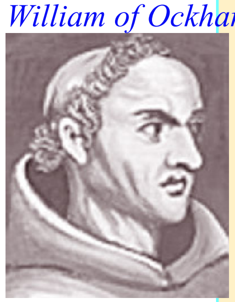


A "road-runner" axiom is a "show-stopper"

First of all it's Complicated

..though comforting to the "A Place for everything and everything in its place" crowd.

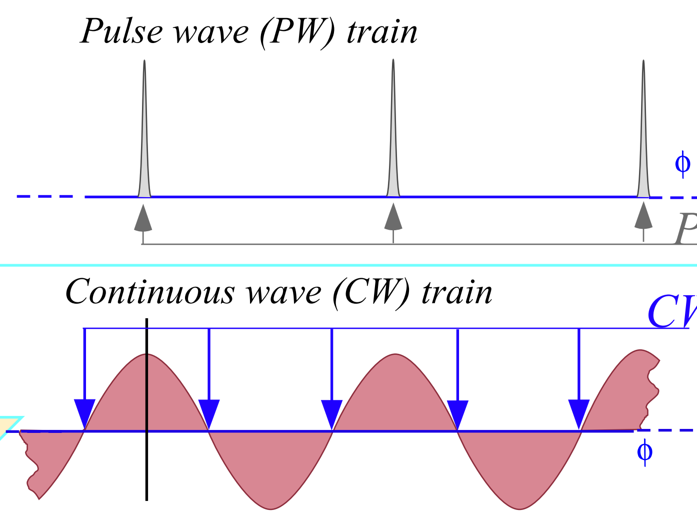
William of Ockham



1285-1349

Using Occam's Razor

(and Evenson's lasers)



$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + ...$

PW peaks precisely locate places where wave is.

CW zeros precisely locate places where wave is not.

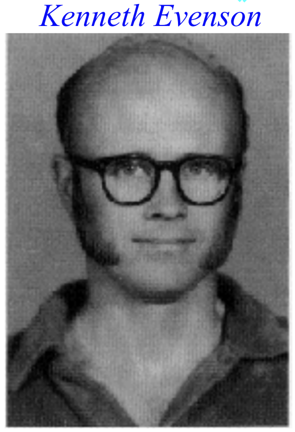
$A\cos\omega t$

Simpler 1CW coherence It's "Zen-like"

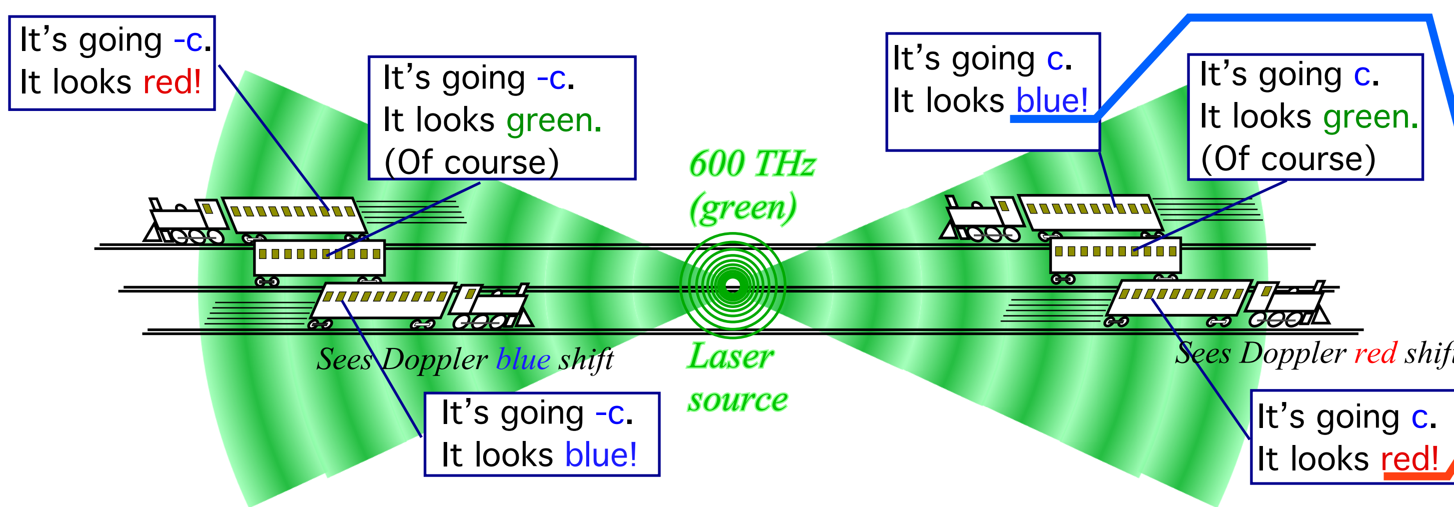
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Can be made more self-evident and productive

Kenneth Evenson



1932-2002



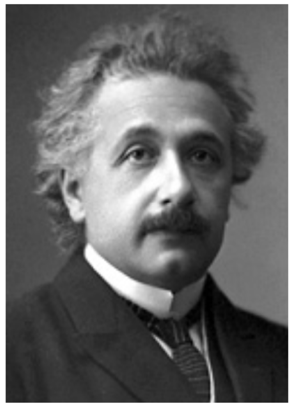
1CW is affected by 1st-order Doppler *Blue* shifts $b = e^{+\rho}$ and *Red* shifts $r = e^{-\rho}$ of frequency υ *and* wavenumber κ

Cut a *PW* to just *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

CW also stands for "Cosine Wave" or "Coherent Wave" or "Colored Wave" (all helpful things!)

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

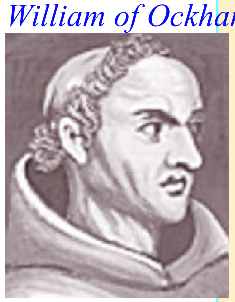
Albert Einstein



1879-1955

A major objection to relativity/QM theory:
It's the only major theoretical development that starts with $\underline{2}^{nd}$-order (and quite mysterious!) (and very very very tiny!) effects.

William of Ockham

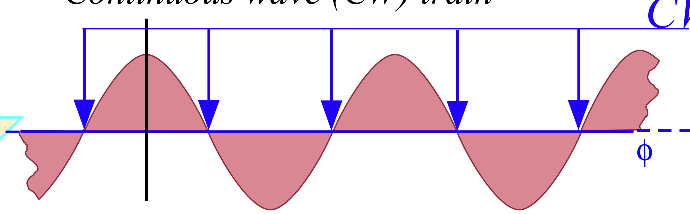


1285-1349

Using Occam's Razor

(and Evenson's lasers)

Continuous wave (CW) train



CW zeros precisely locate places where wave is not.

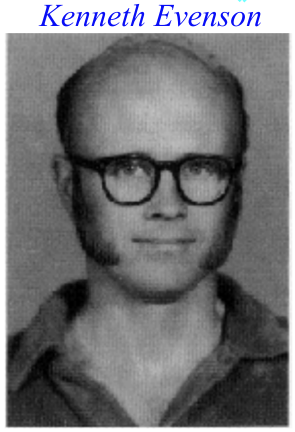
A cos ωt

Simpler 1CW coherence
It's "Zen-like"

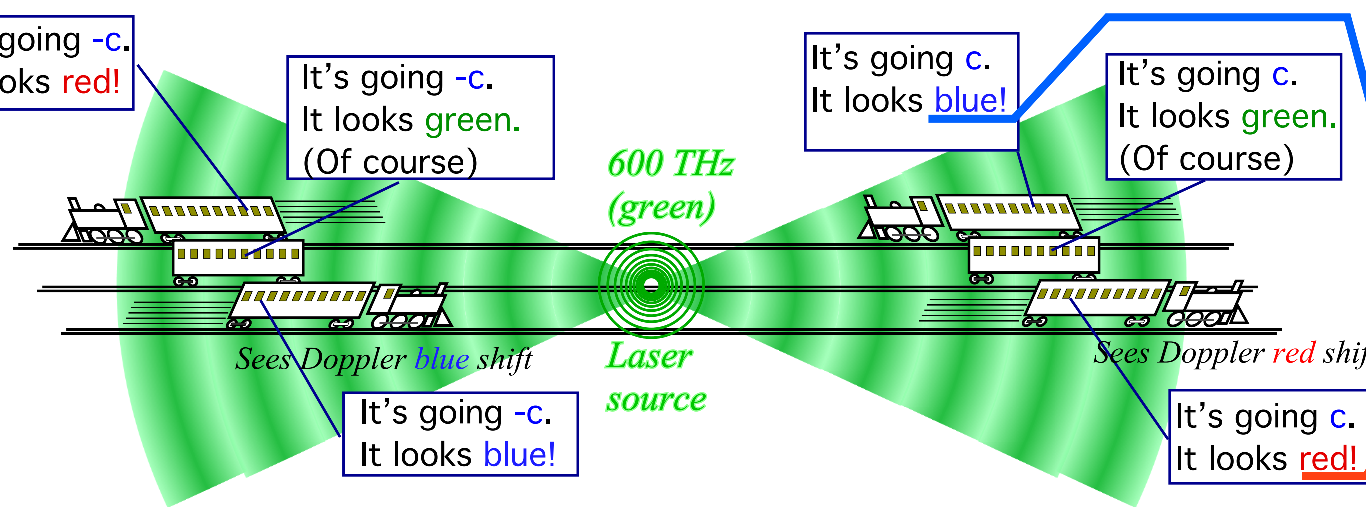
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Can be made more self-evident and productive

Kenneth Evenson



1932-2002



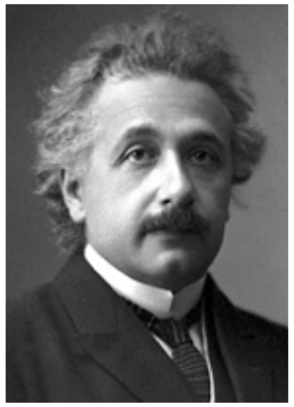
1CW is affected by 1^{st}-order Doppler
Blue shifts $b = e^{+\rho}$
and
Red shifts $r = e^{-\rho}$
of frequency υ
and wavenumber κ

Cut a *PW* to just *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

CW also stands for "Cosine Wave" or "Coherent Wave" or "Colored Wave" (all helpful things!)

- How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

Albert Einstein



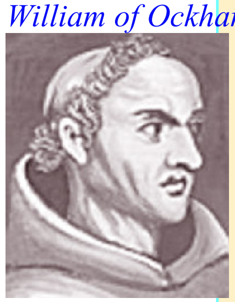
1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A major objection to relativity/QM theory:
It's the only major theoretical development that starts with *2nd*-order (and quite mysterious!) (and very very very tiny!) effects.

So lets try doing first-things first!

William of Ockham



1285-1349

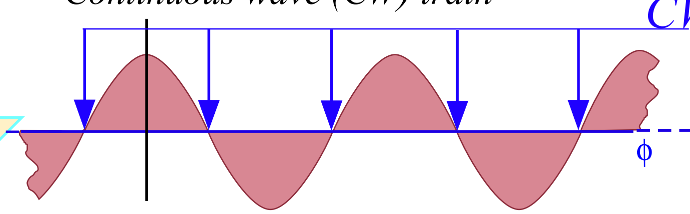
Using Occam's Razor

(and Evenson's lasers)

PW peaks precisely locate places where wave *is*.

..though comforting to the "A Place for everything and everything in its place" crowd.

Continuous wave (CW) train



CW zeros precisely locate places where wave is not.

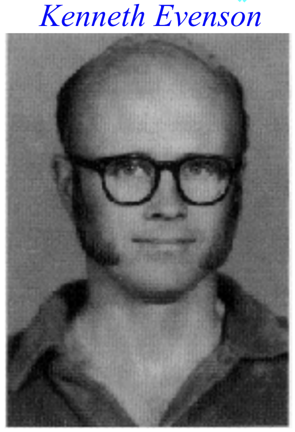
A cos ωt

Simpler 1 CW coherence
It's "Zen-like"

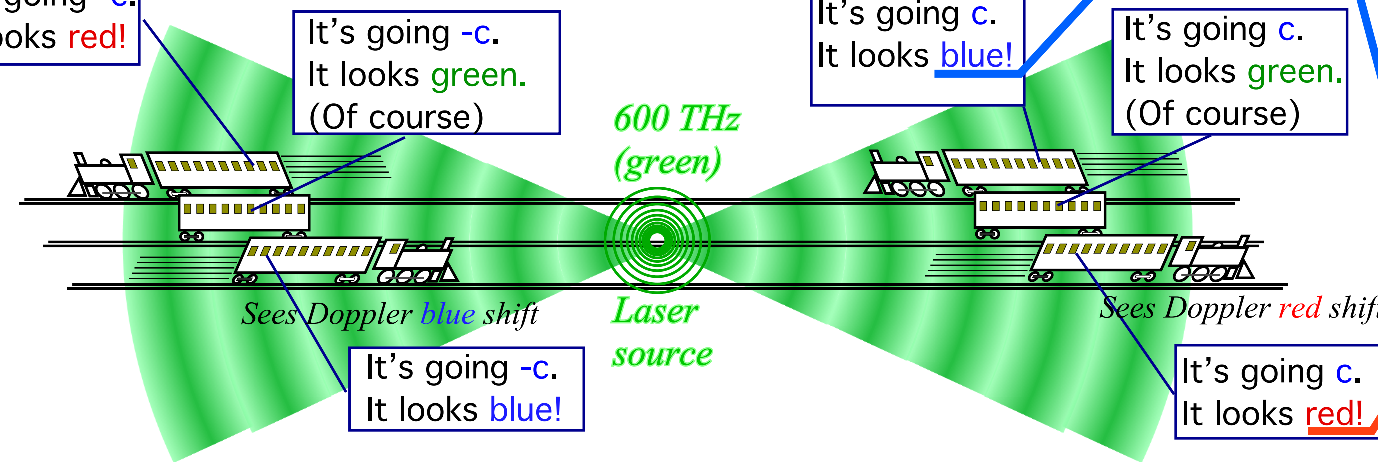
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Can be made more self-evident and productive

Kenneth Evenson



1932-2002



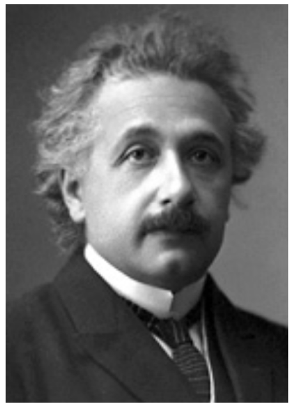
1CW is affected by 1^{st}-order Doppler
Blue shifts $b = e^{+\rho}$
and
Red shifts $r = e^{-\rho}$
of frequency υ *and* wavenumber κ

Cut a *PW* to just *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

CW also stands for "Cosine Wave" or "Coherent Wave" or "Colored Wave" (all helpful things!)

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

Albert Einstein



1879-1955

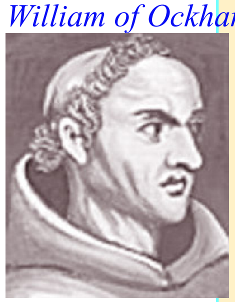
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A major objection to relativity/QM theory:
It's the only major theoretical development that starts with *2nd*-order (and quite mysterious!) (and very very very tiny!) effects.

So lets try doing first-things first!
...and start off by dealing with this sucker...

PW peaks precisely locate places where wave is *is.*

William of Ockham

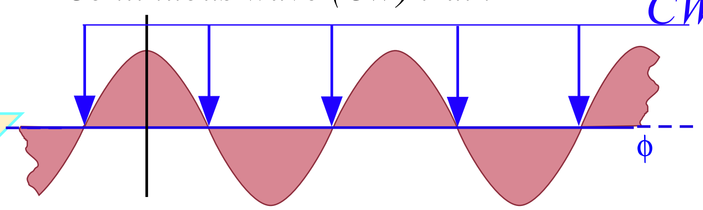


1285-1349

Using Occam's Razor

(and Evenson's lasers)

Continuous wave (CW) train



CW zeros precisely locate places where wave is not.

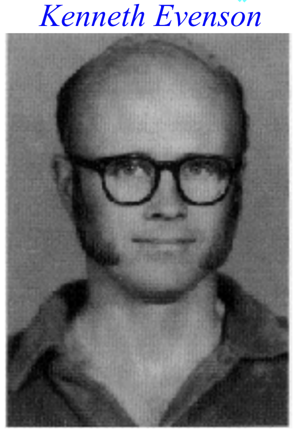
Acos ωt

Simpler 1CW coherence
It's "Zen-like"

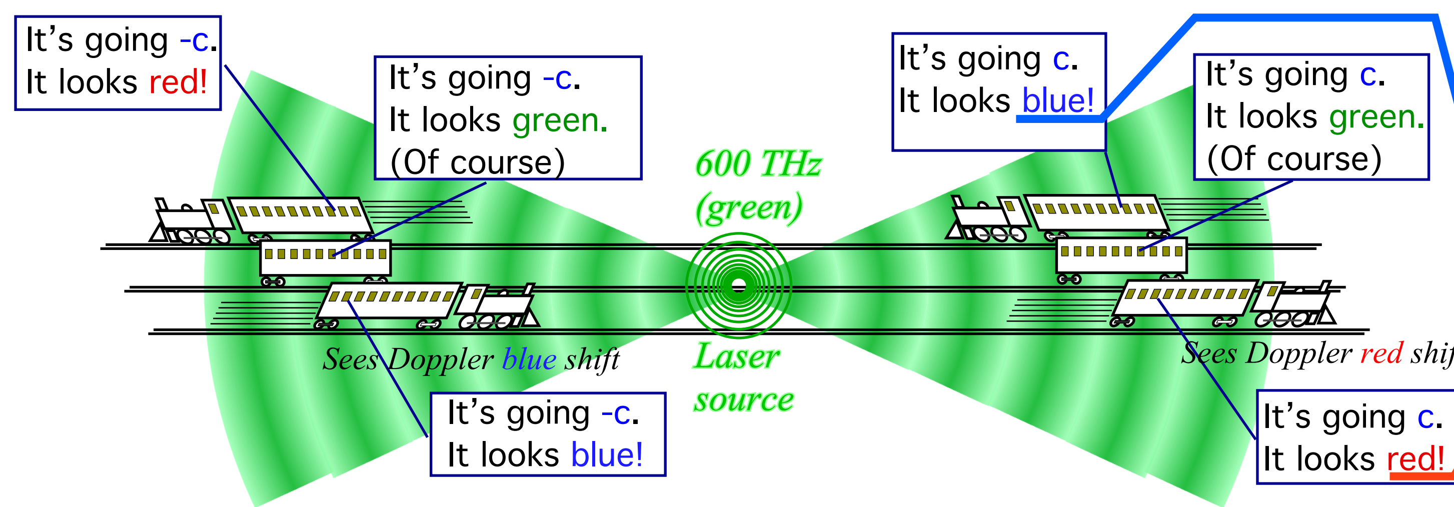
Can be made more self-evident and productive

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Kenneth Evenson

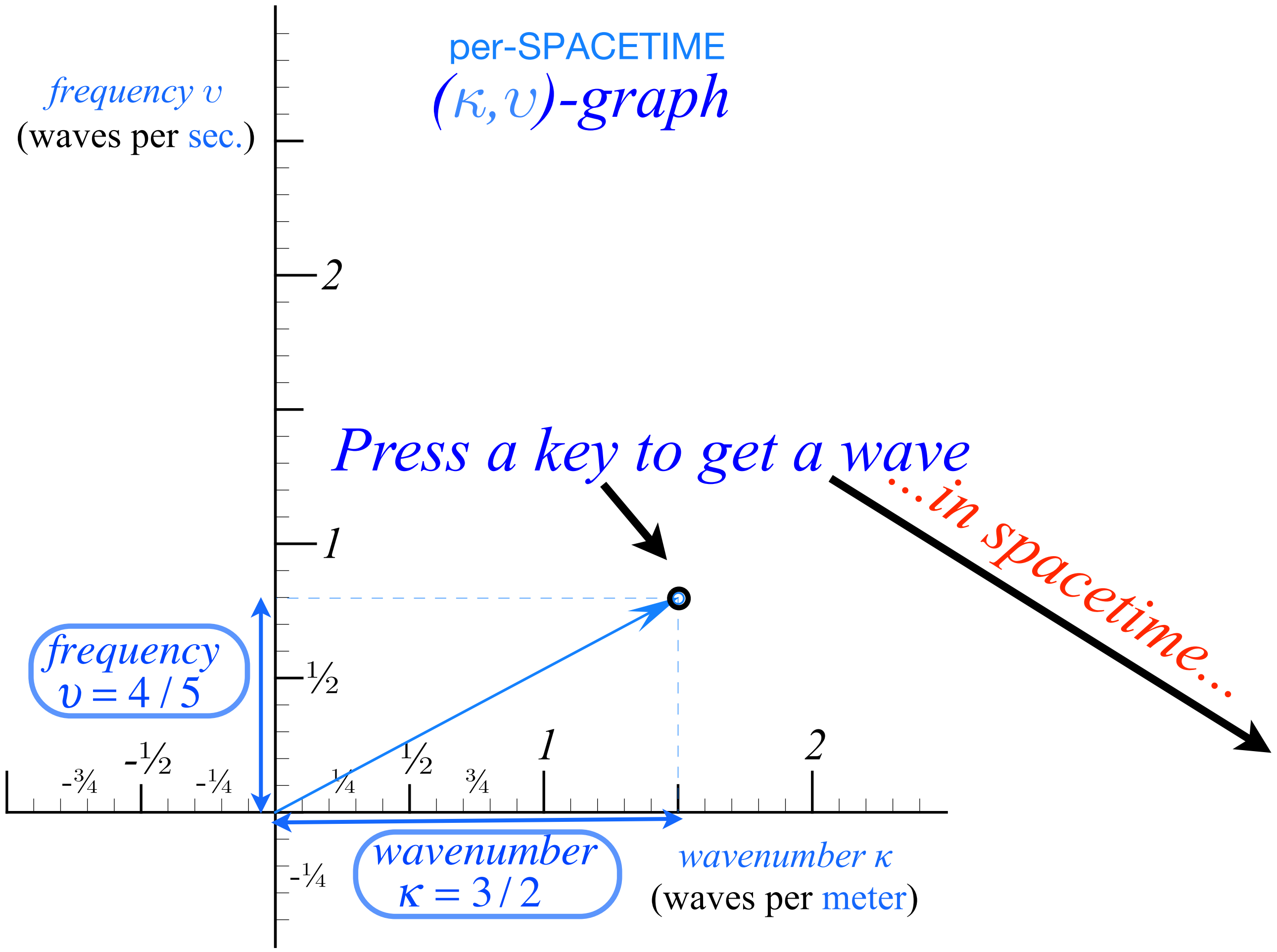


1932-2002

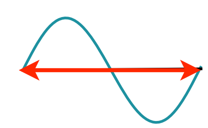


1CW is affected by 1st-order Doppler
Blue shifts $b = e^{+\rho}$
and
Red shifts $r = e^{-\rho}$
of frequency υ
and wavenumber κ

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

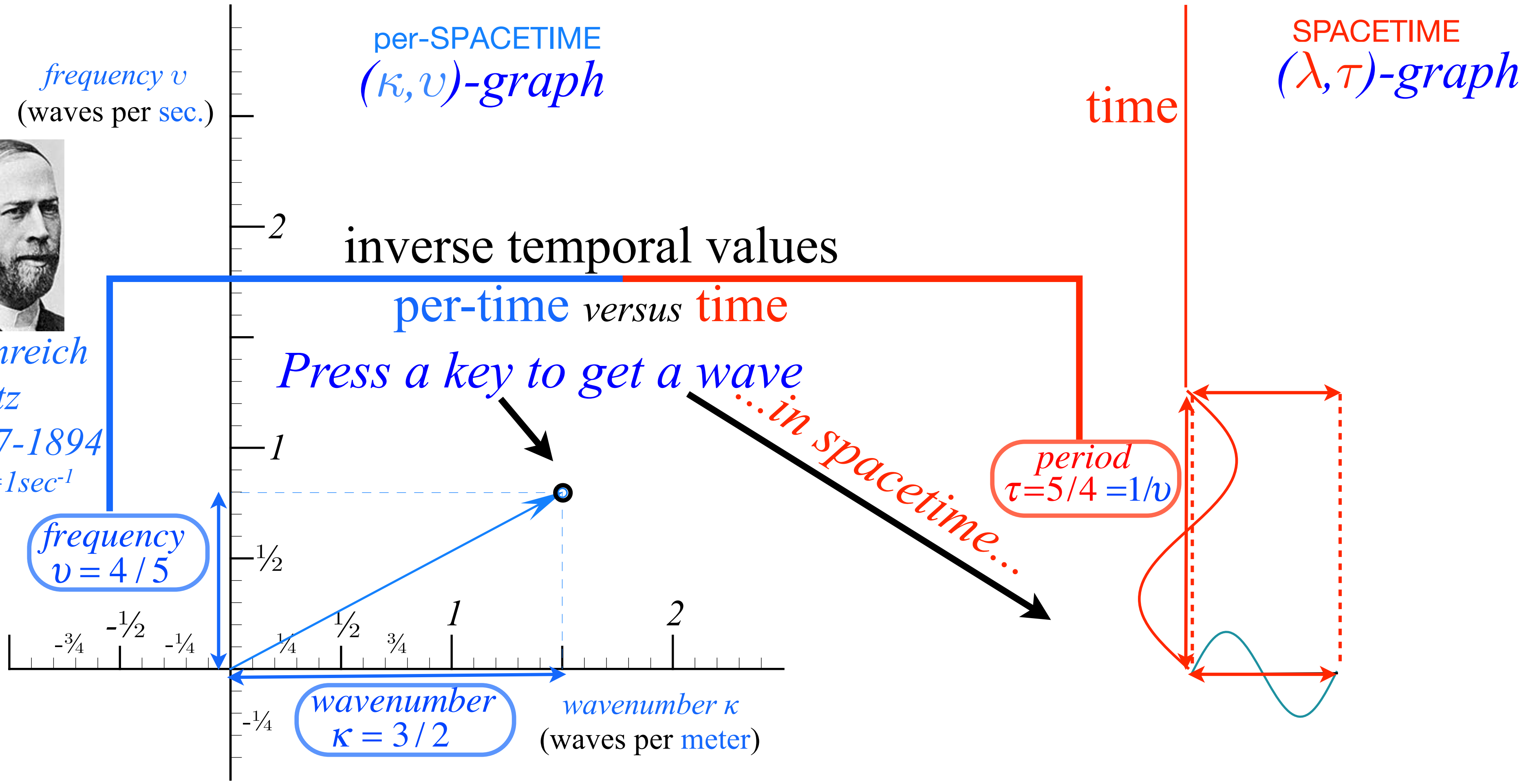
Jean-Baptiste
Joseph Fourier
1768-1830

•*How to understand waves*
and
wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

Heinreich Hertz 1857-1894

1Hz=1sec⁻¹

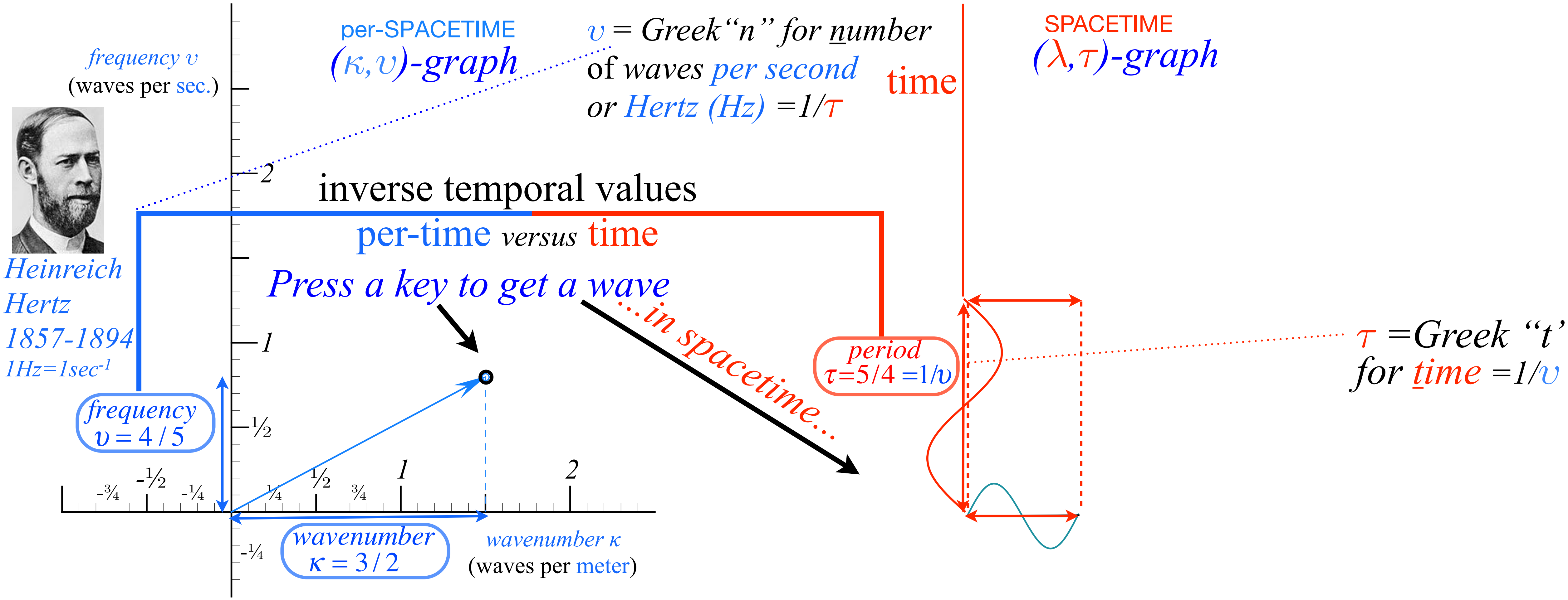


"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

> •*How to understand waves and wave velocity* V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

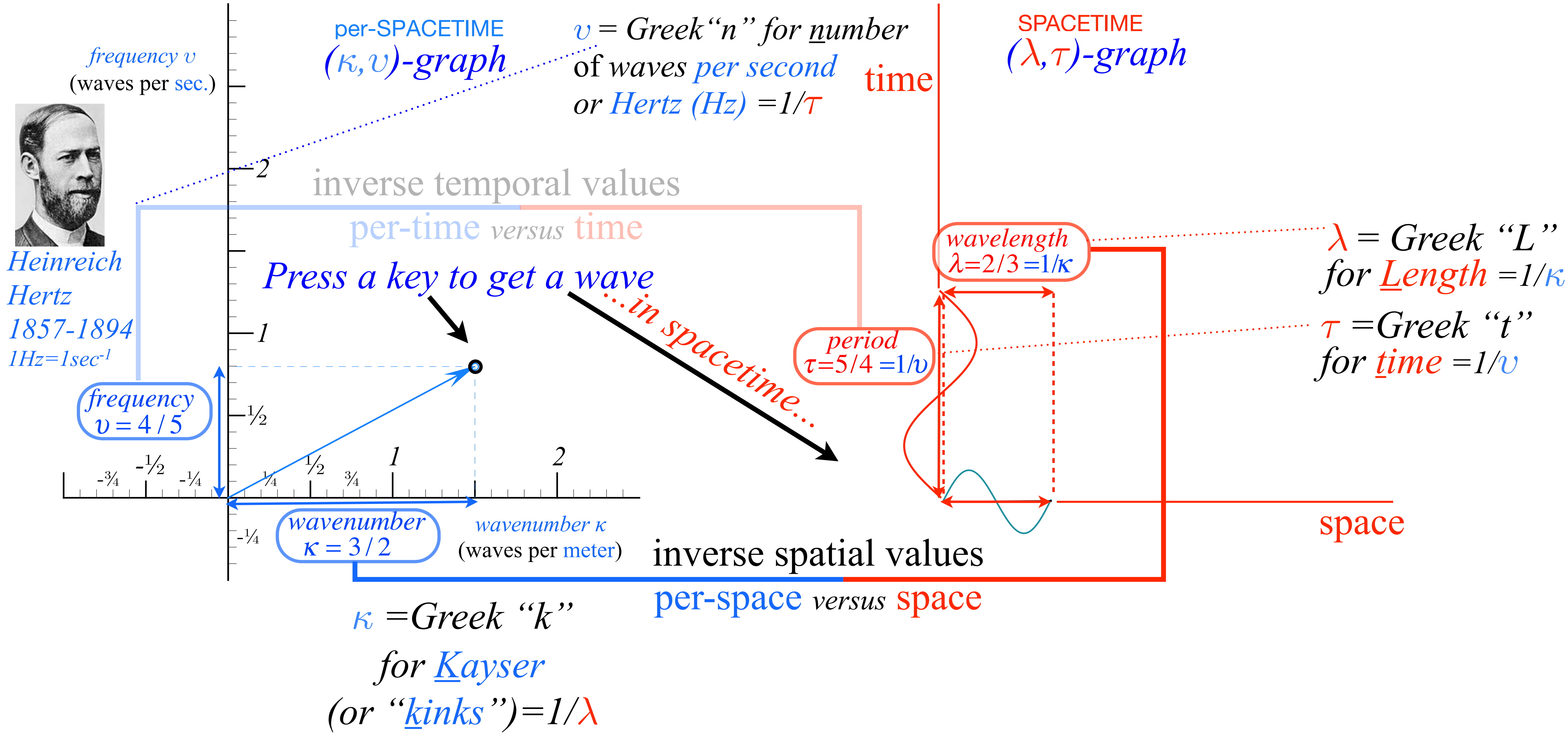


"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
and
wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

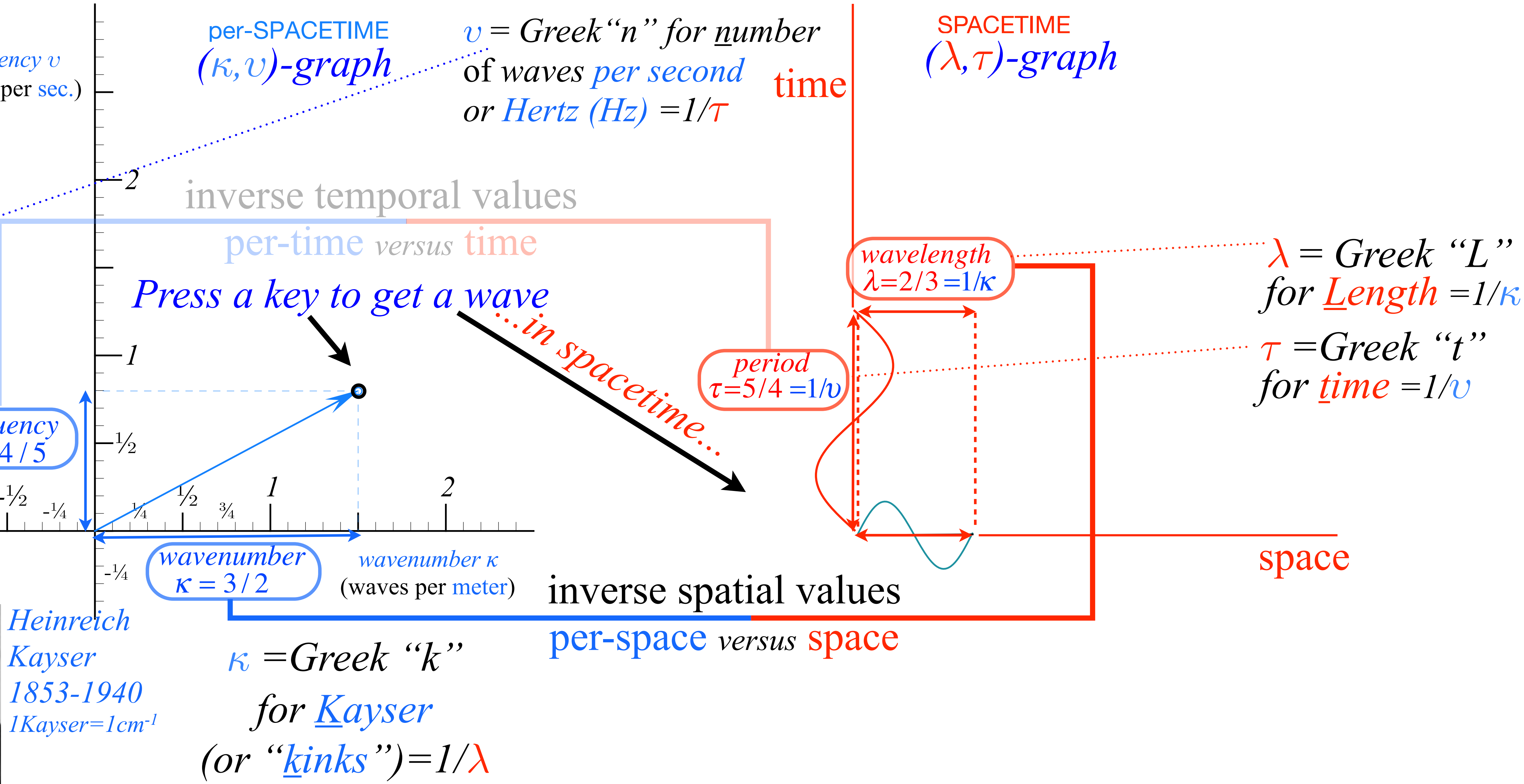


"Keyboard of the gods" is known as "Fourier-space"

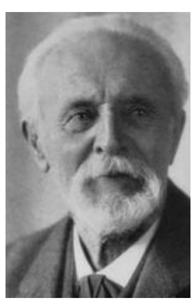
Jean-Baptiste
Joseph Fourier
1768-1830

> •*How to understand waves*
> *and*
> *wave velocity V_{wave}*

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



Heinreich Hertz 1857-1894
1Hz=1sec^{-1}



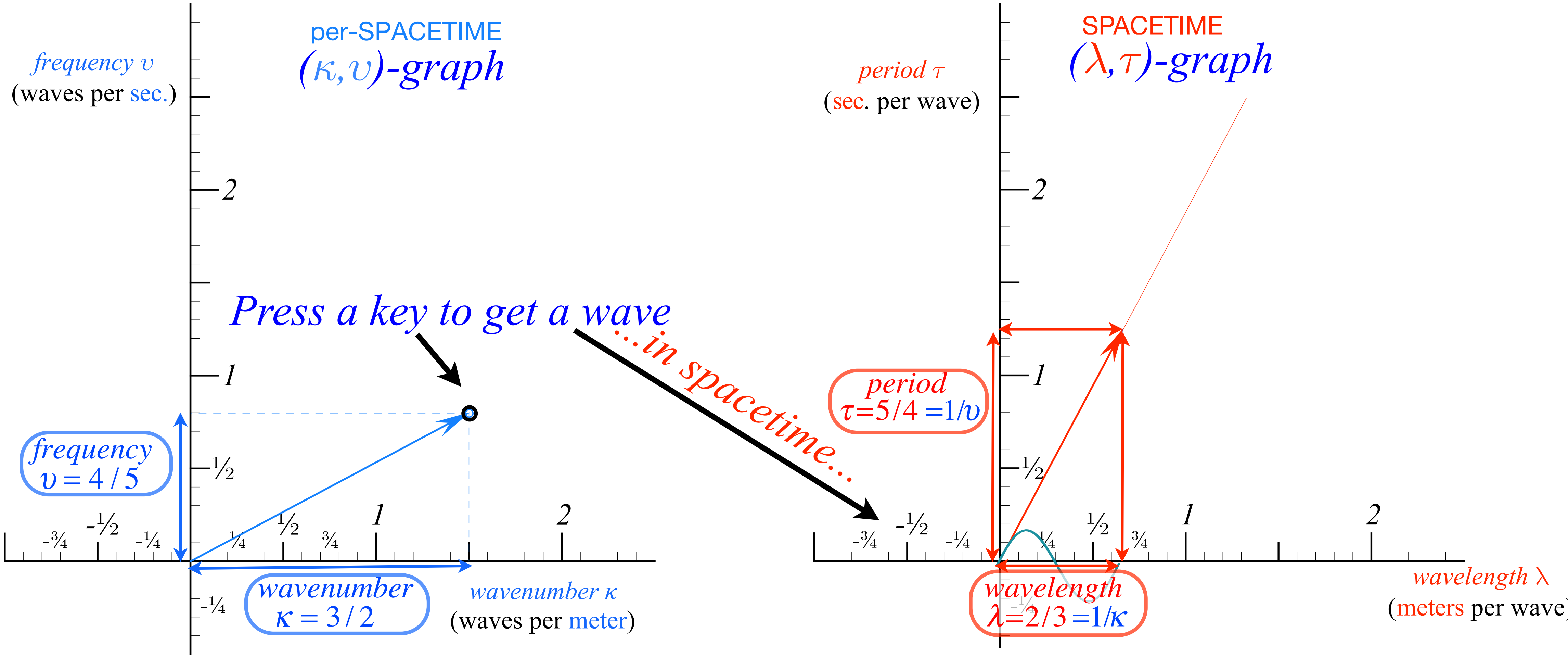
Heinreich Kayser 1853-1940
1Kayser=1cm^{-1}

"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

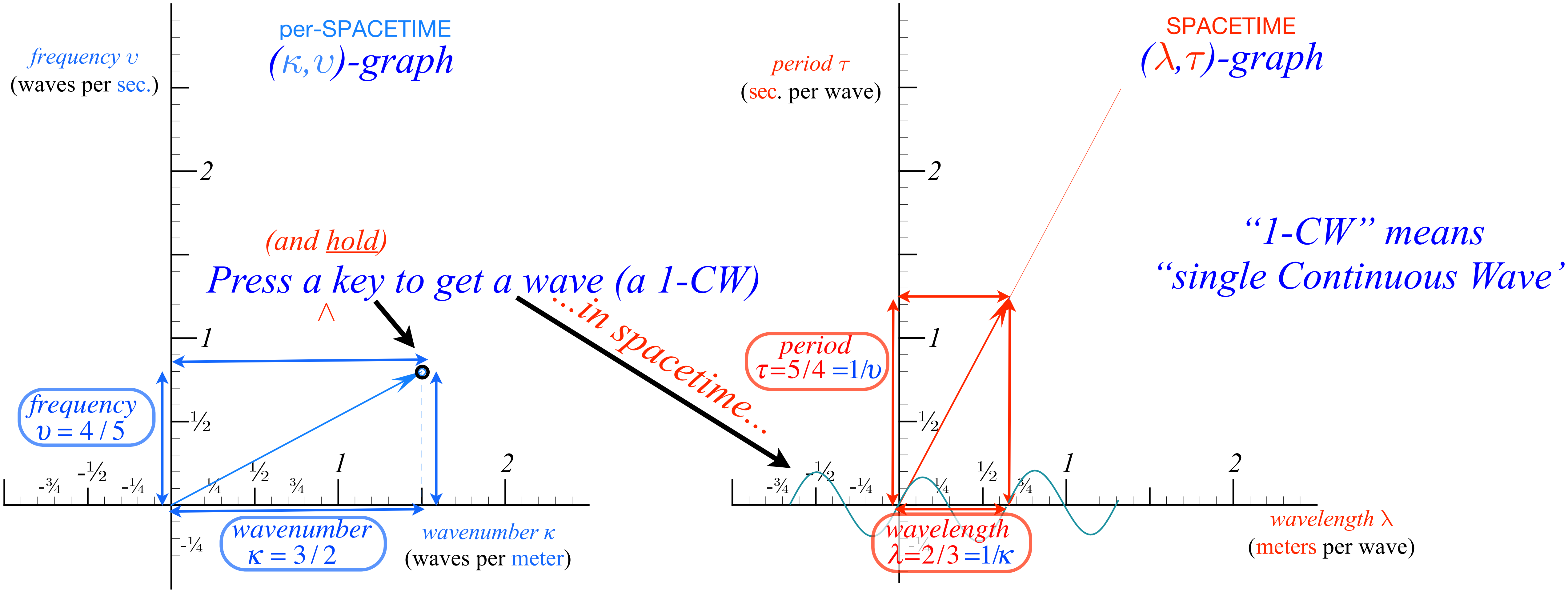


"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

•*How to understand waves*
and
wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

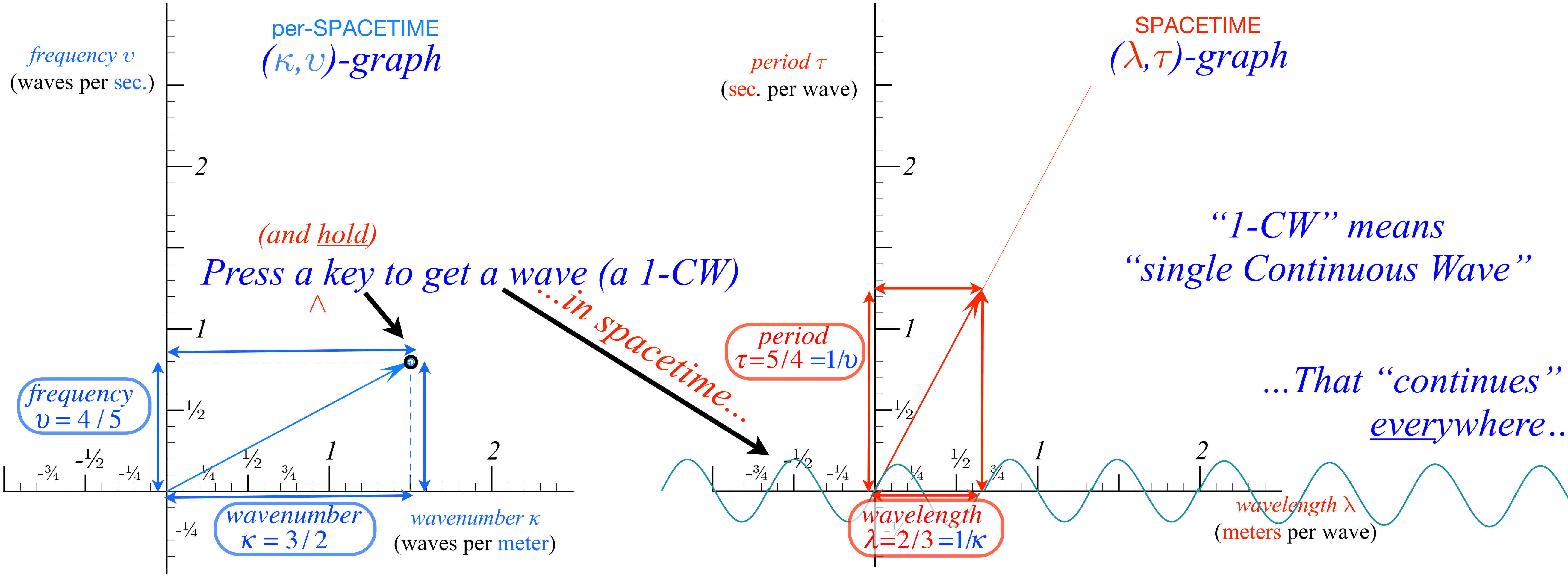


"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
and
wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

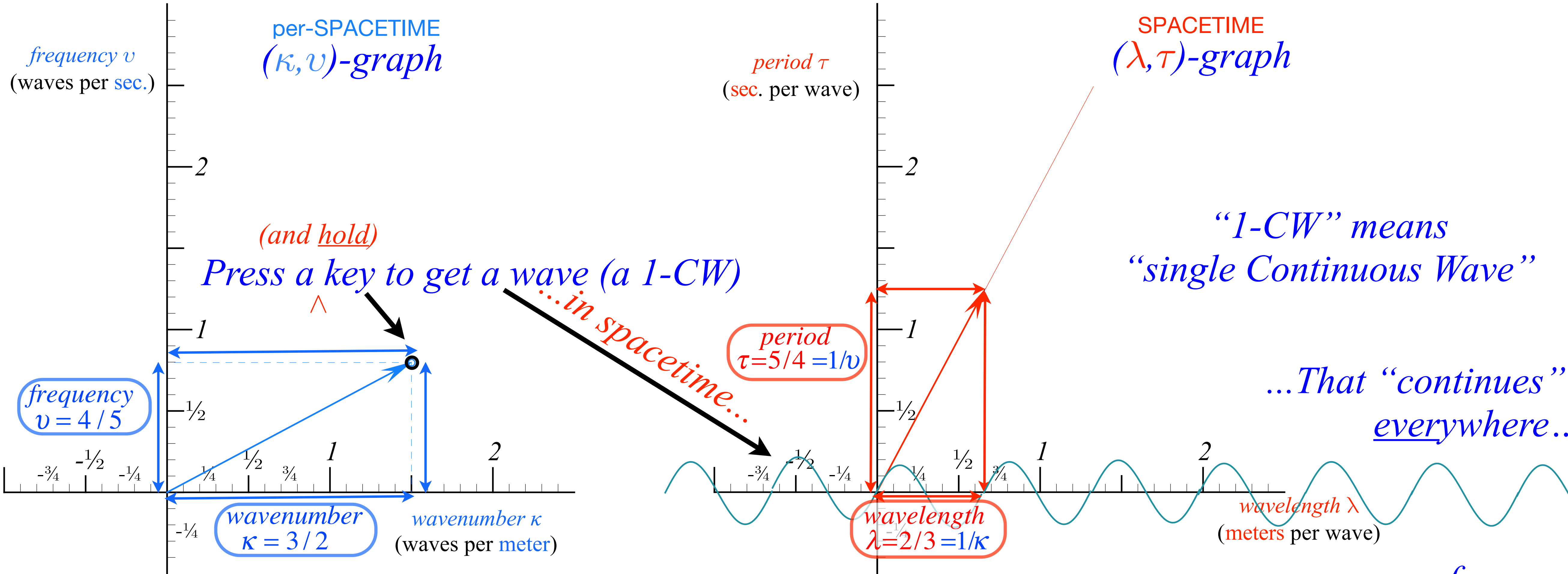


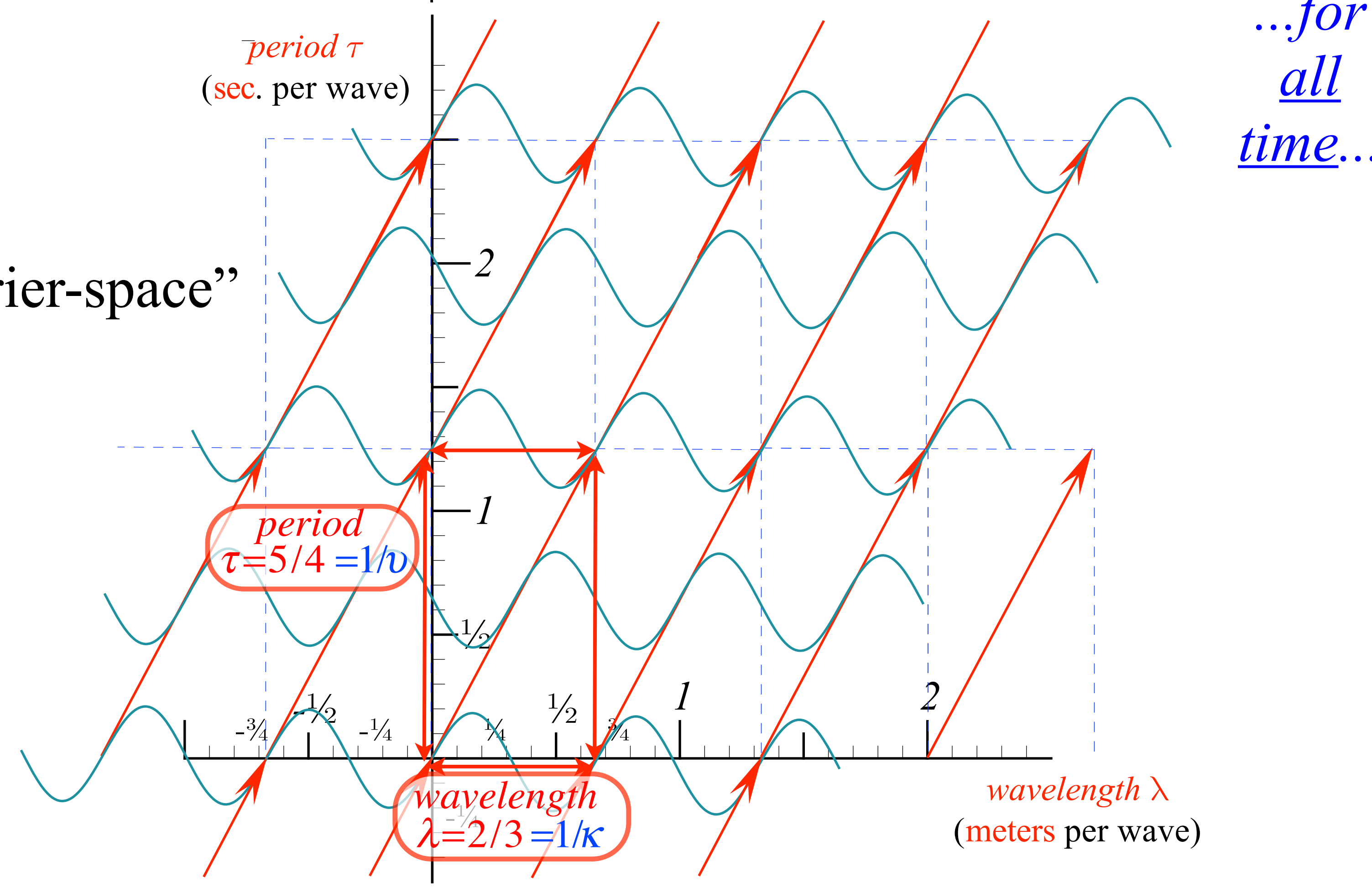
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

•*How to understand waves*
and
wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



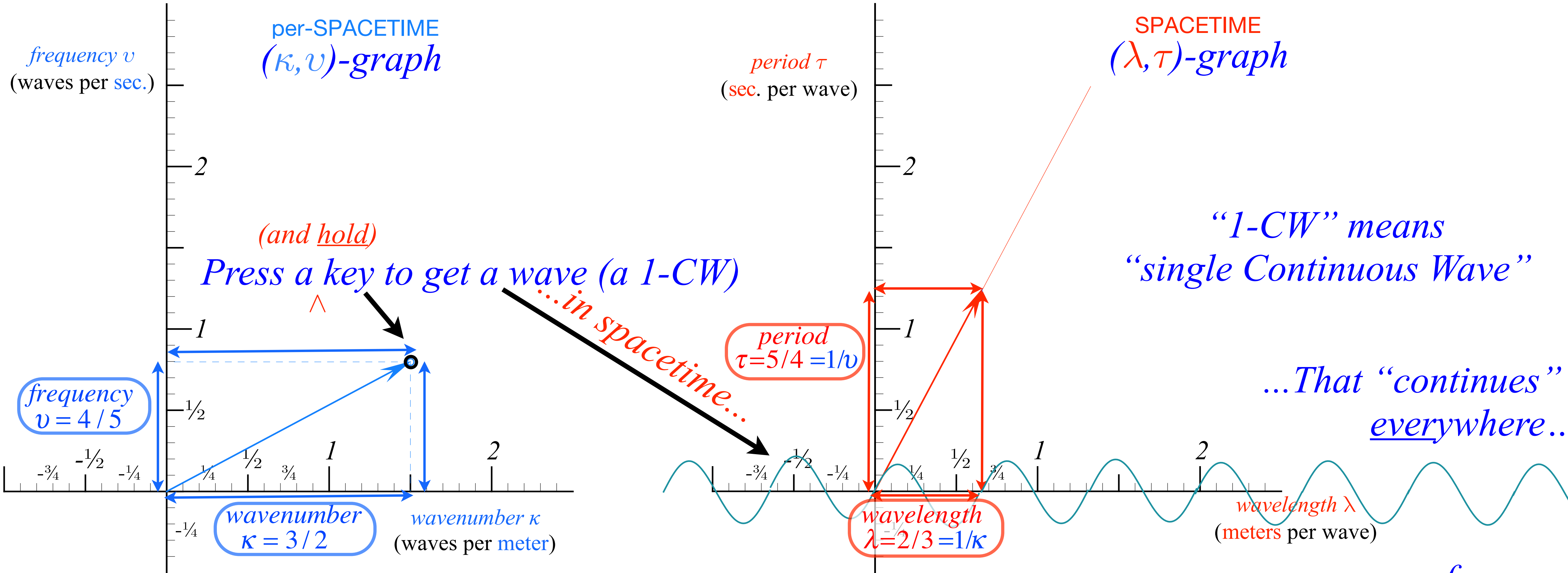


"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

- *How to understand waves*
  and
  wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs

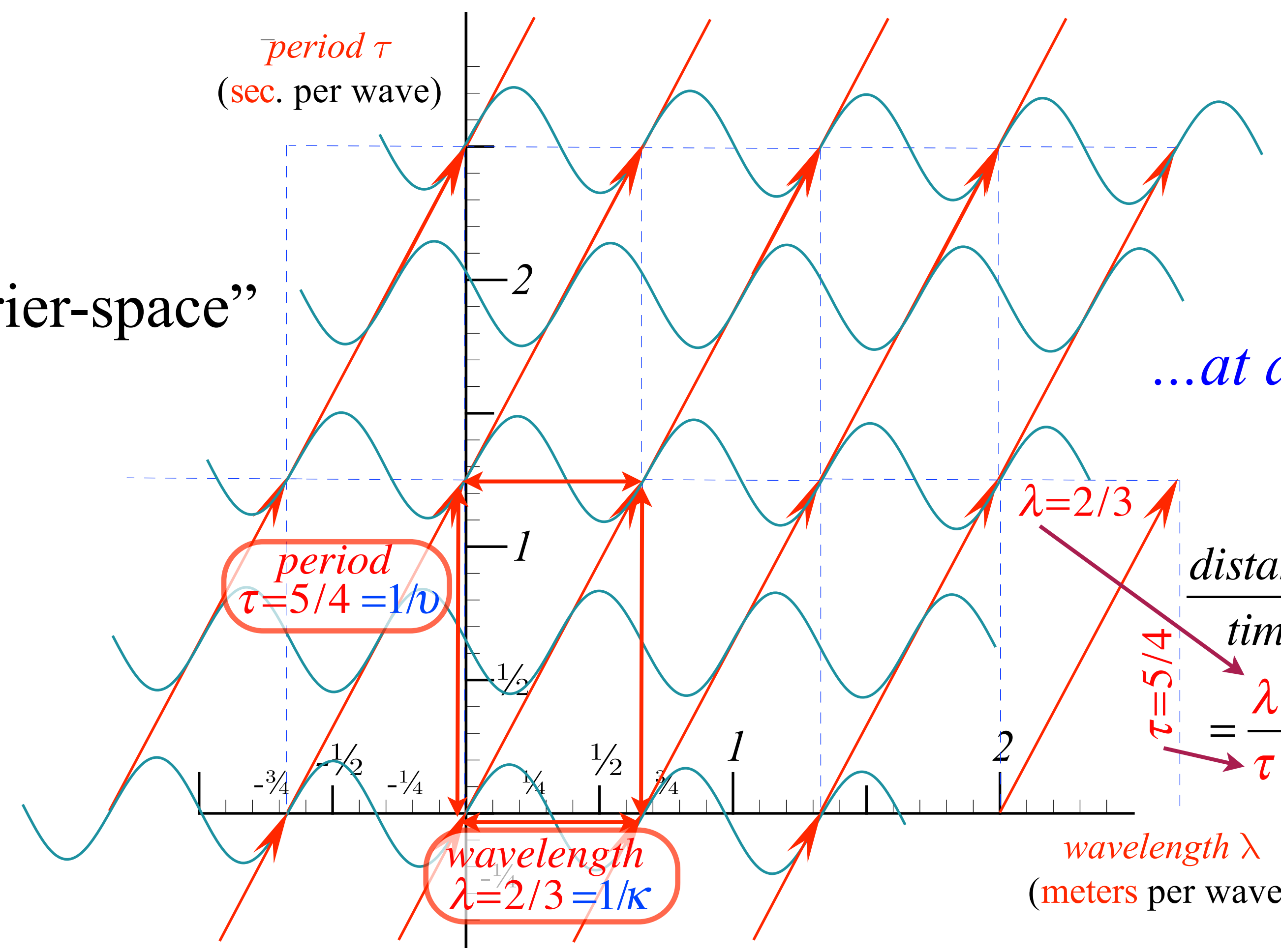


"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

...for all time...

...at a speed of:



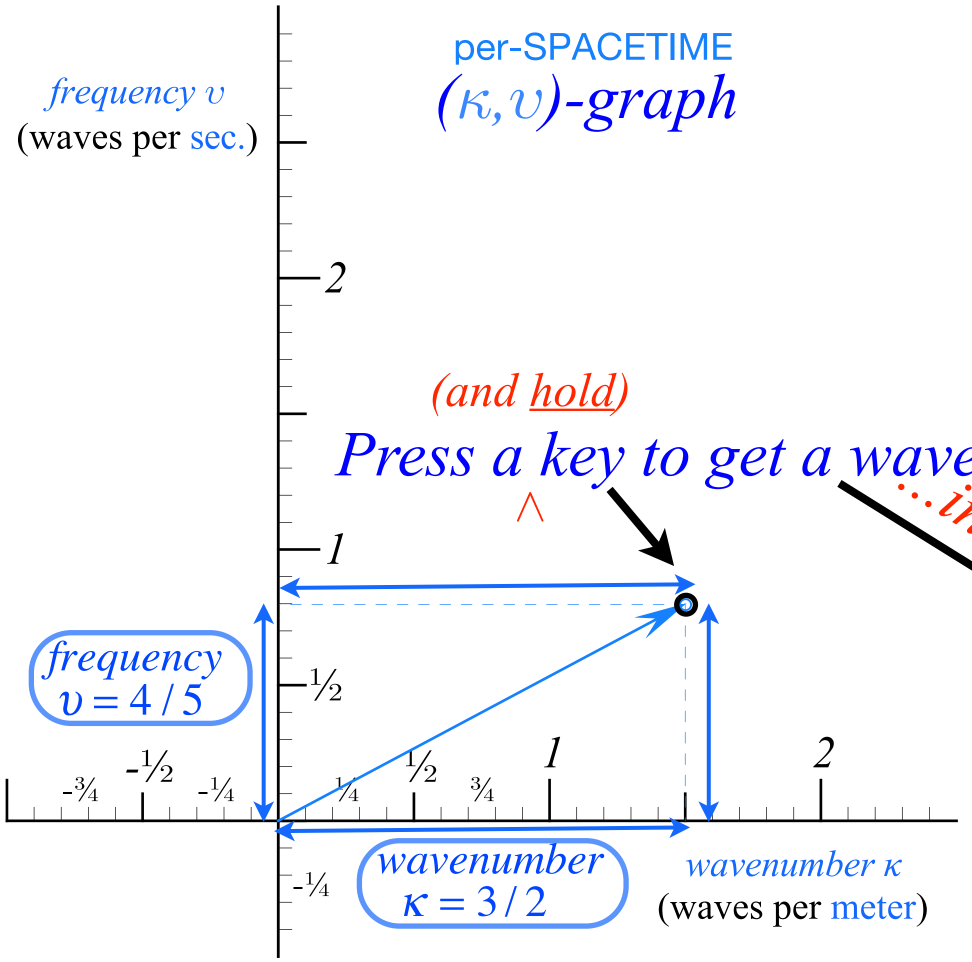
$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

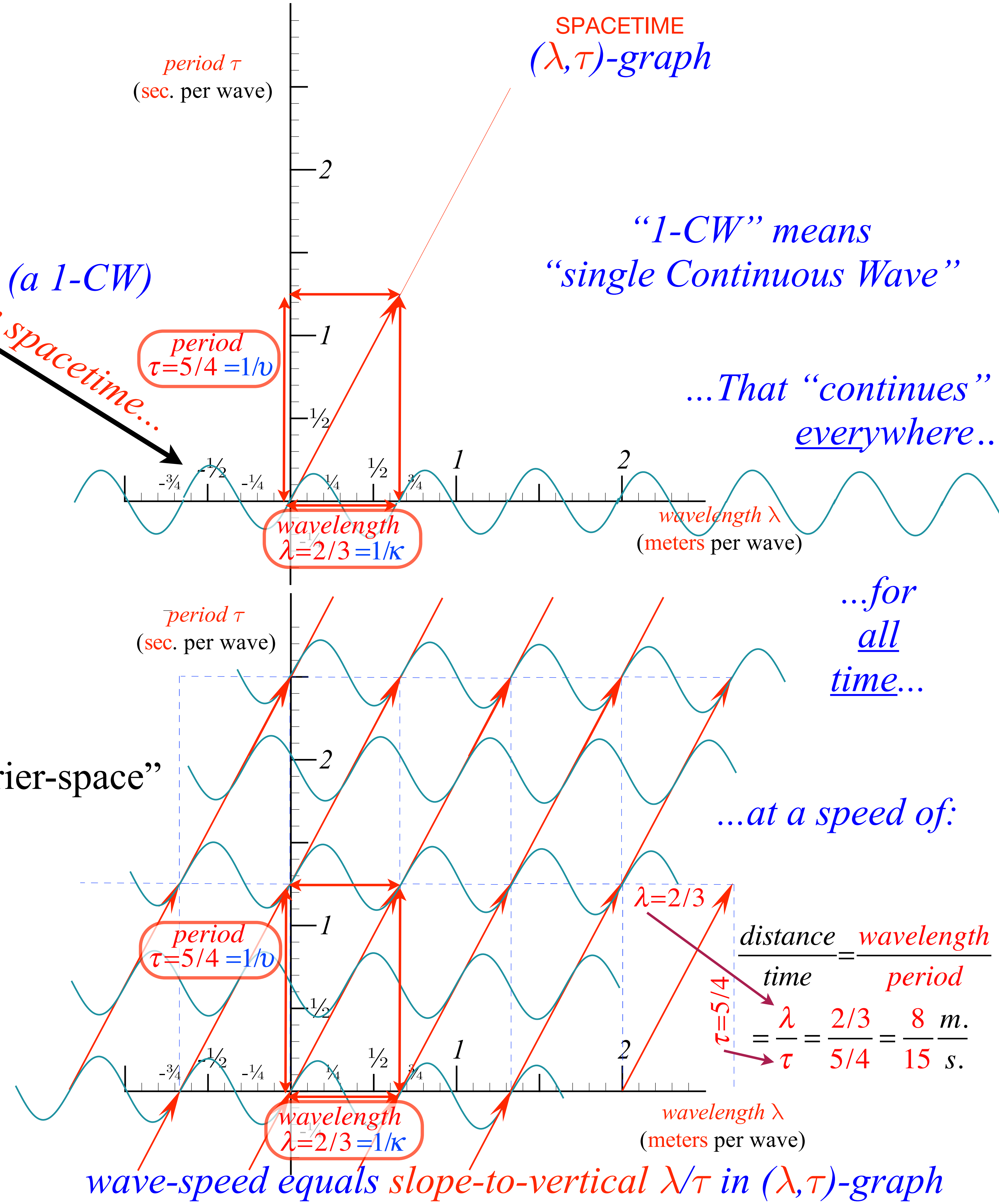
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



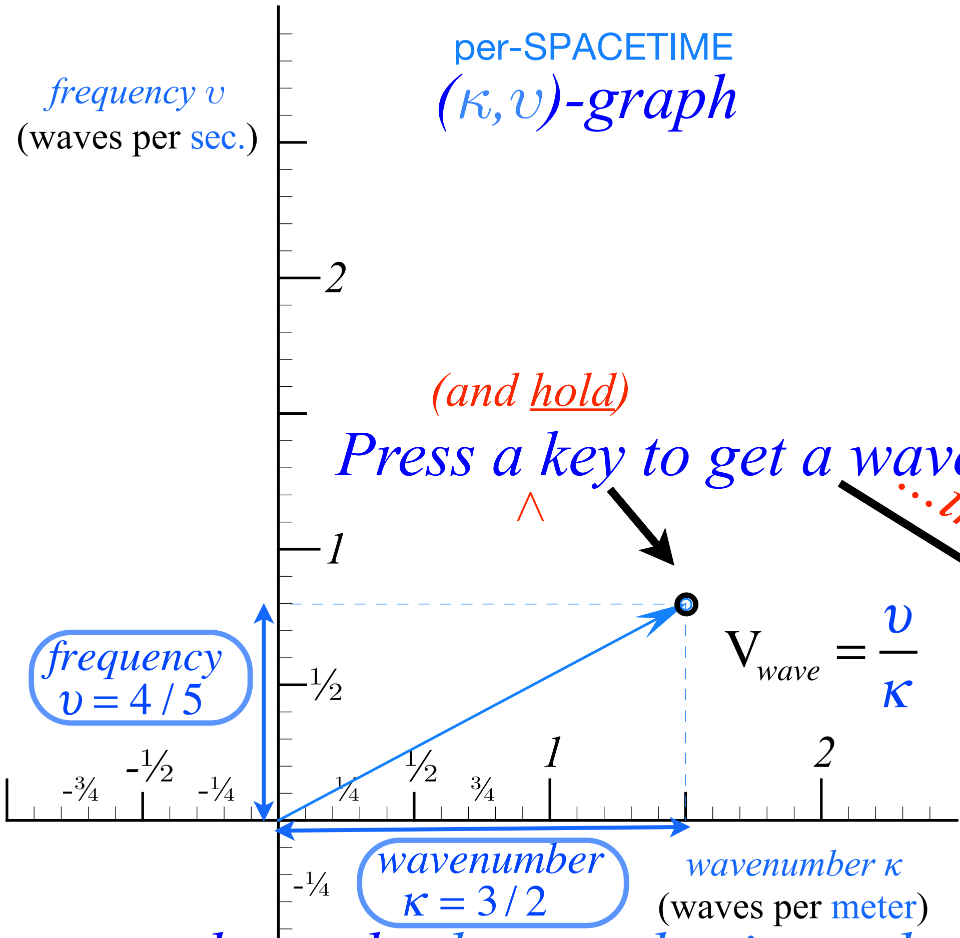


"Keyboard of the gods" is known as "Fourier-space"

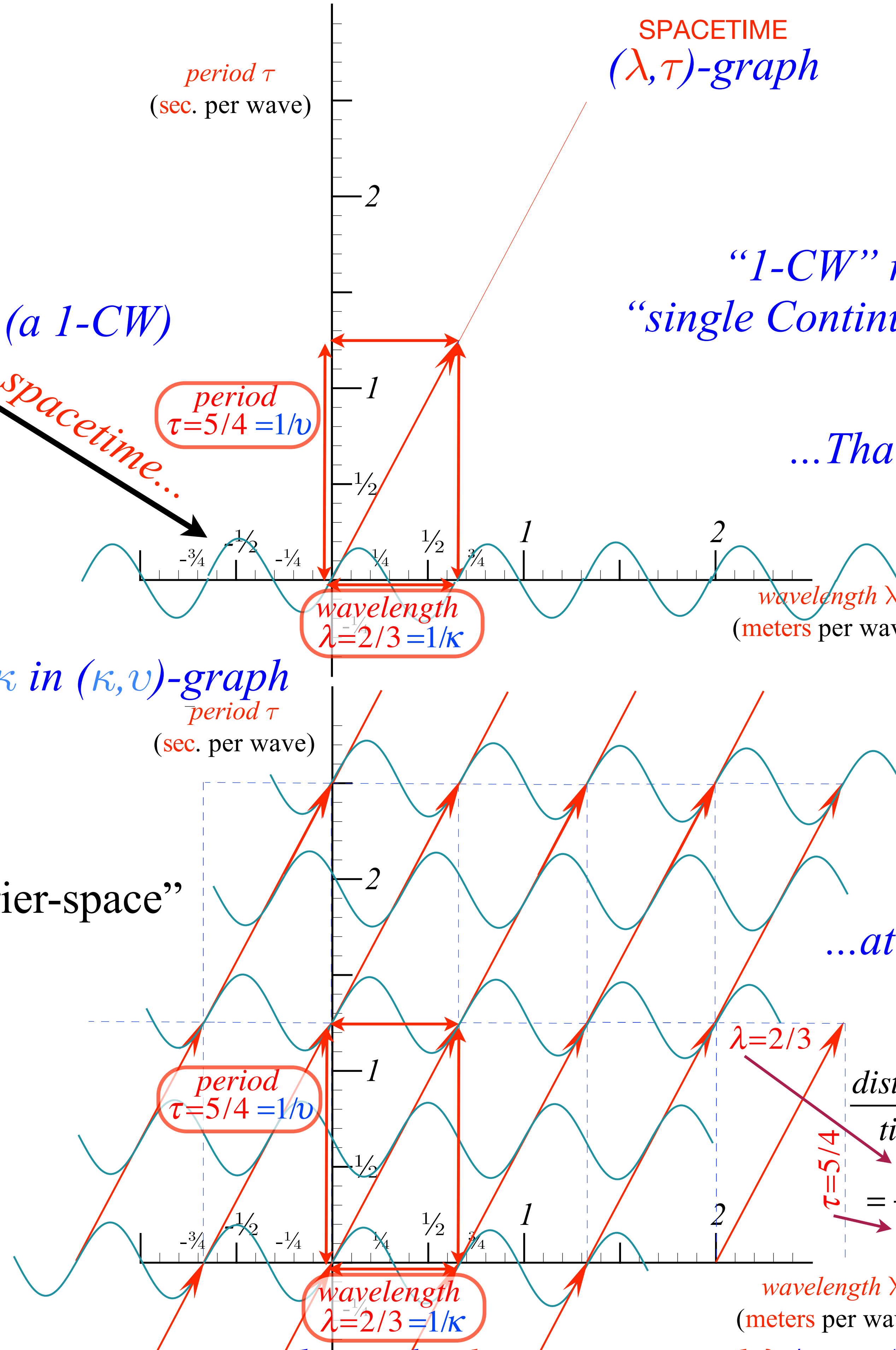
Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
and
wave velocity V_{wave}

The “Keyboard of the gods” or per-space-per-time graphs *versus* space-time graphs



wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph



“1-CW” means “single Continuous Wave”

...That “continues” everywhere...

...for all time...

...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

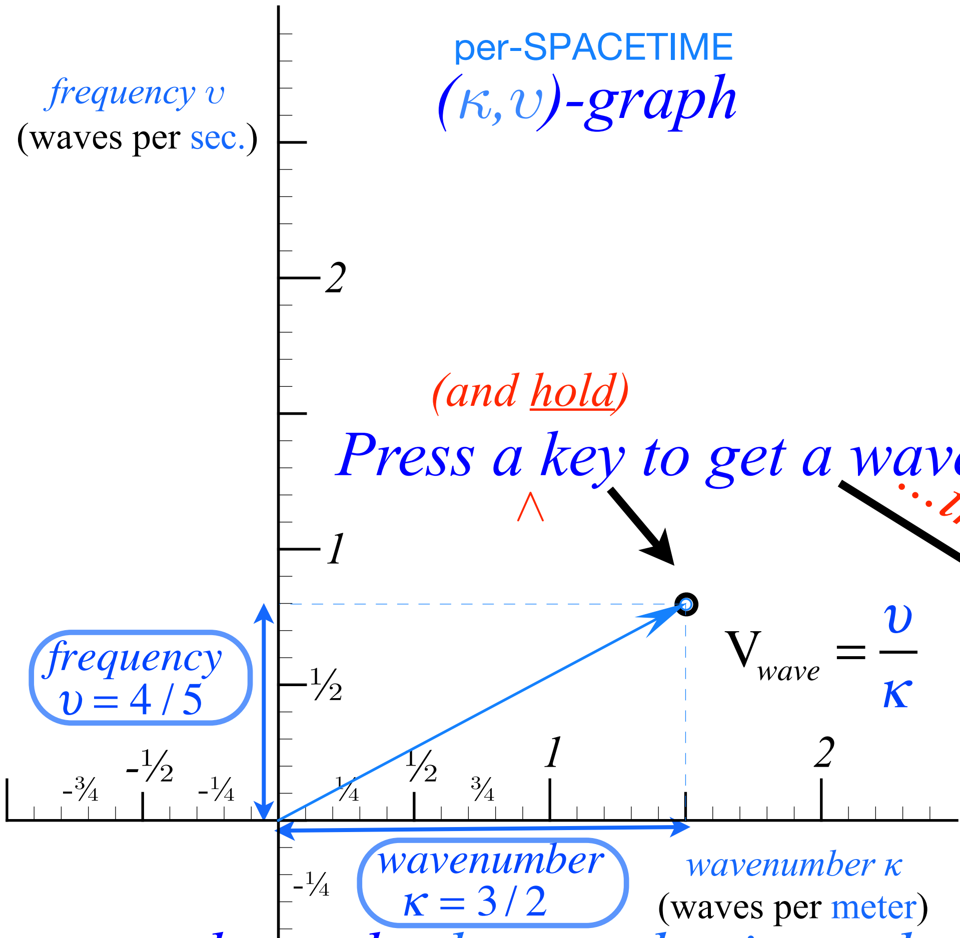
“Keyboard of the gods” is known as “Fourier-space”

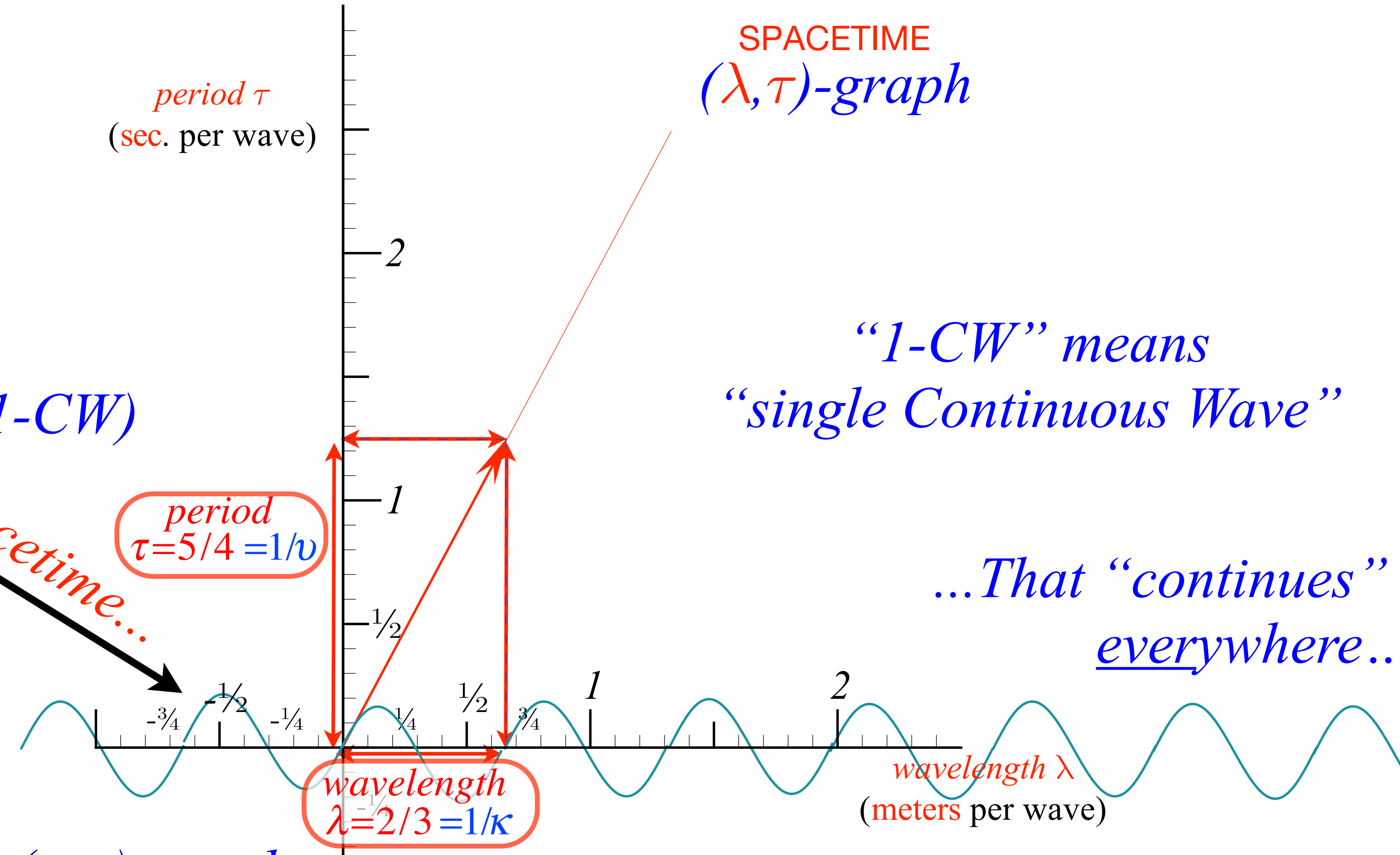
Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs





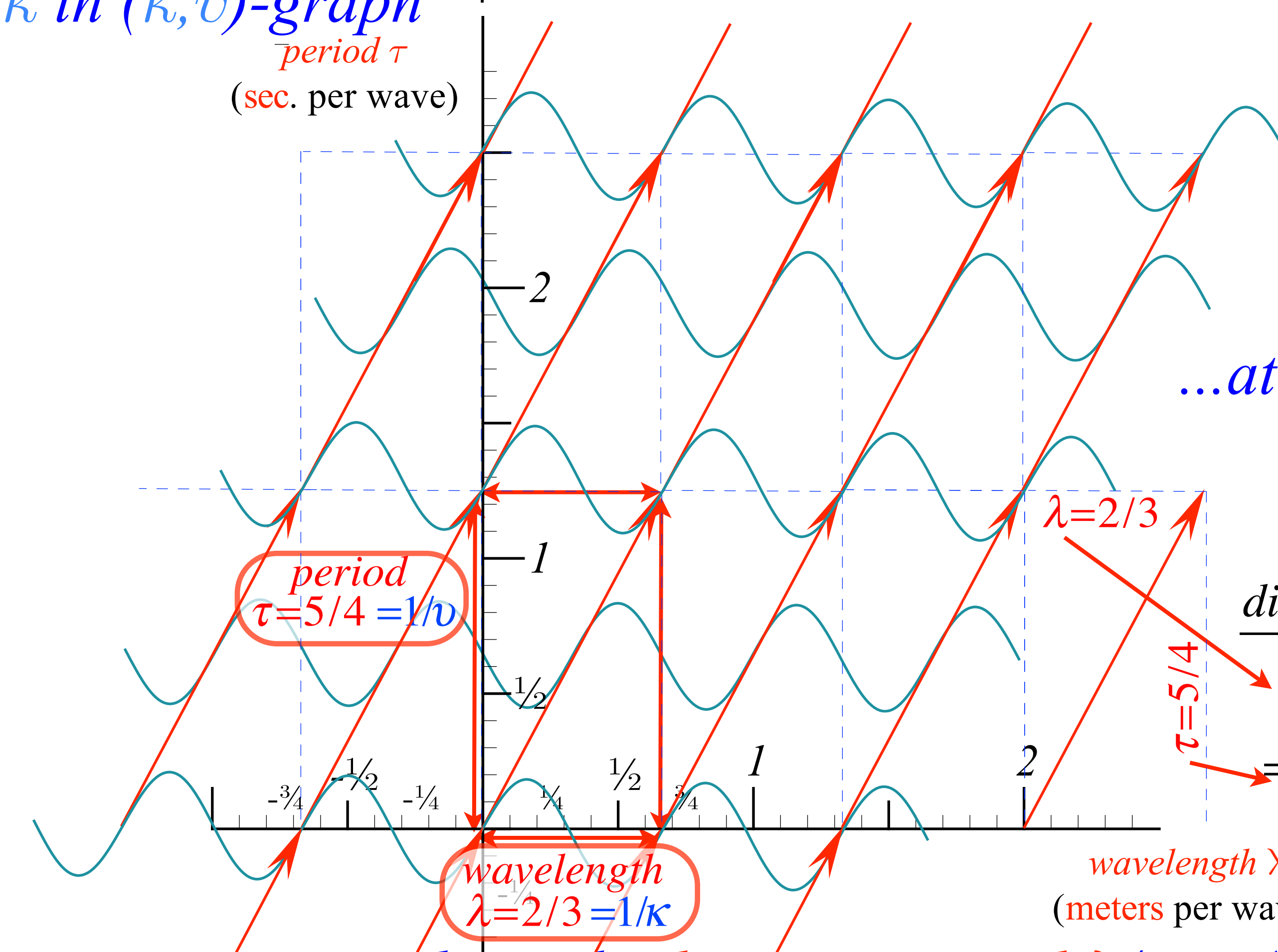
"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph

...for all time...

...at a speed of:



$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

wave-velocity formulas

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

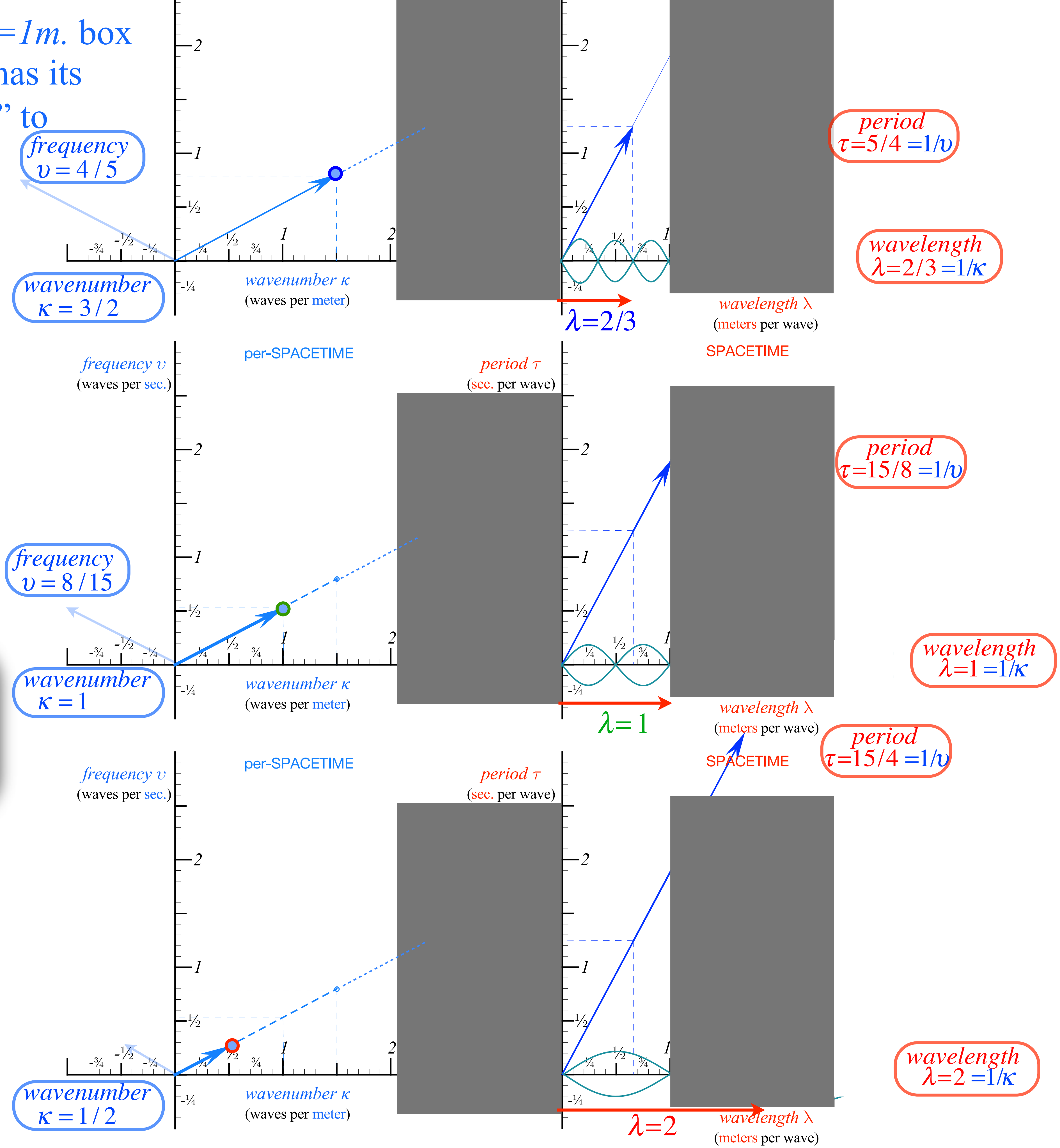
$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

- *How to understand waves and "1st quantization"*

If a wave is confined to an *L=1m.* box *the "Keyboard of the gods"* has its wavenumber κ is "quantized" to multiples of *1/2L=1/2*.

$$\kappa = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \ldots$$



- *How to understand waves and "1st quantization" or κ-quantization*

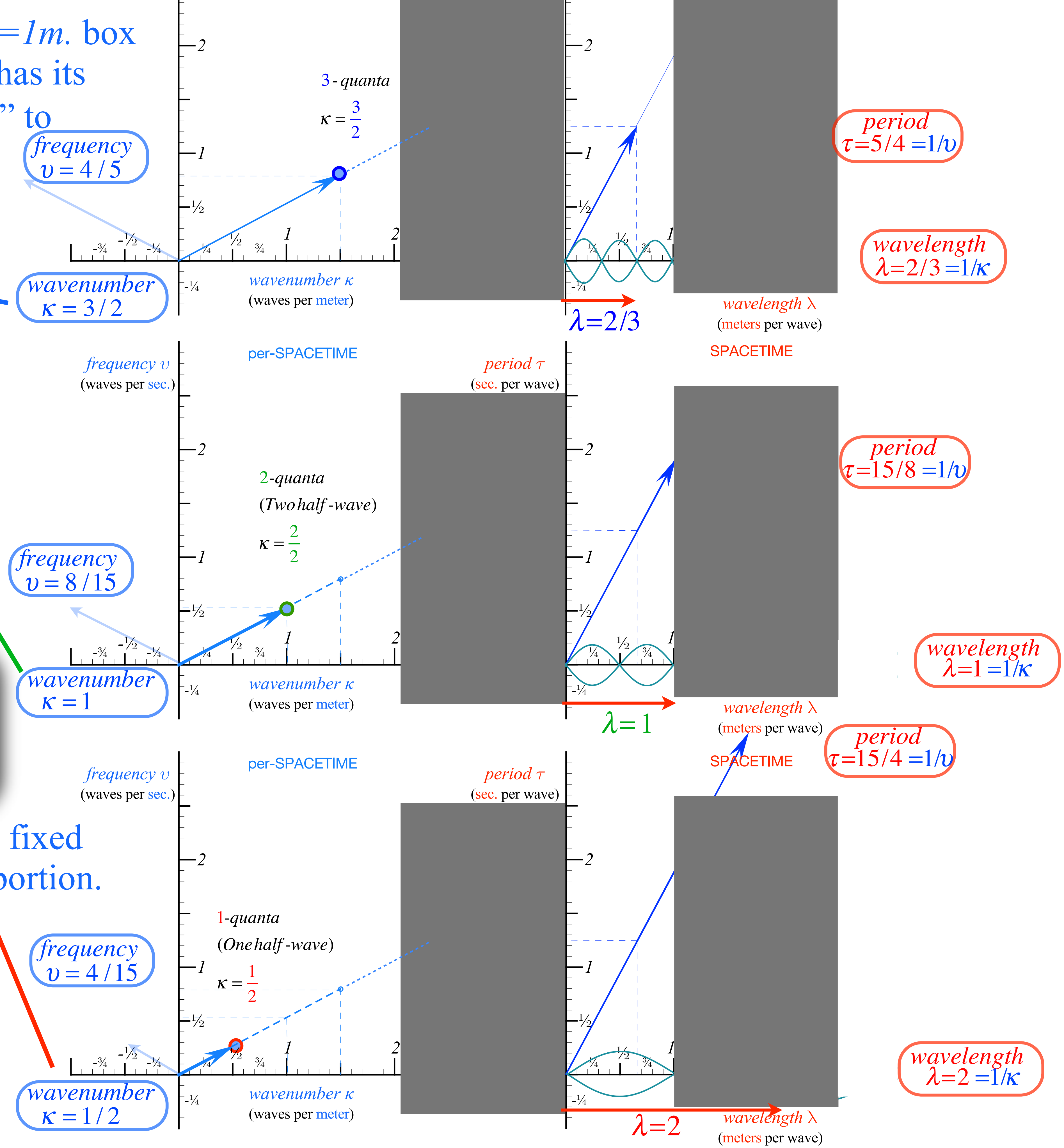
If a wave is confined to an *L=1m.* box *the "Keyboard of the gods"* has its wavenumber κ is "quantized" to multiples of *1/2L=1/2*.

$$\kappa = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \ldots$$

κ = *1/2, 1, 3/2, 2,...*

•How to understand waves
and
"1st quantization"
or κ-quantization

If wave velocity V_{wave}=8/15 is fixed *frequency* is quantized in proportion.



If a wave is confined to an *L=1m.* box *the "Keyboard of the gods"* has its wavenumber κ is "quantized" to multiples of *1/2L=1/2*.

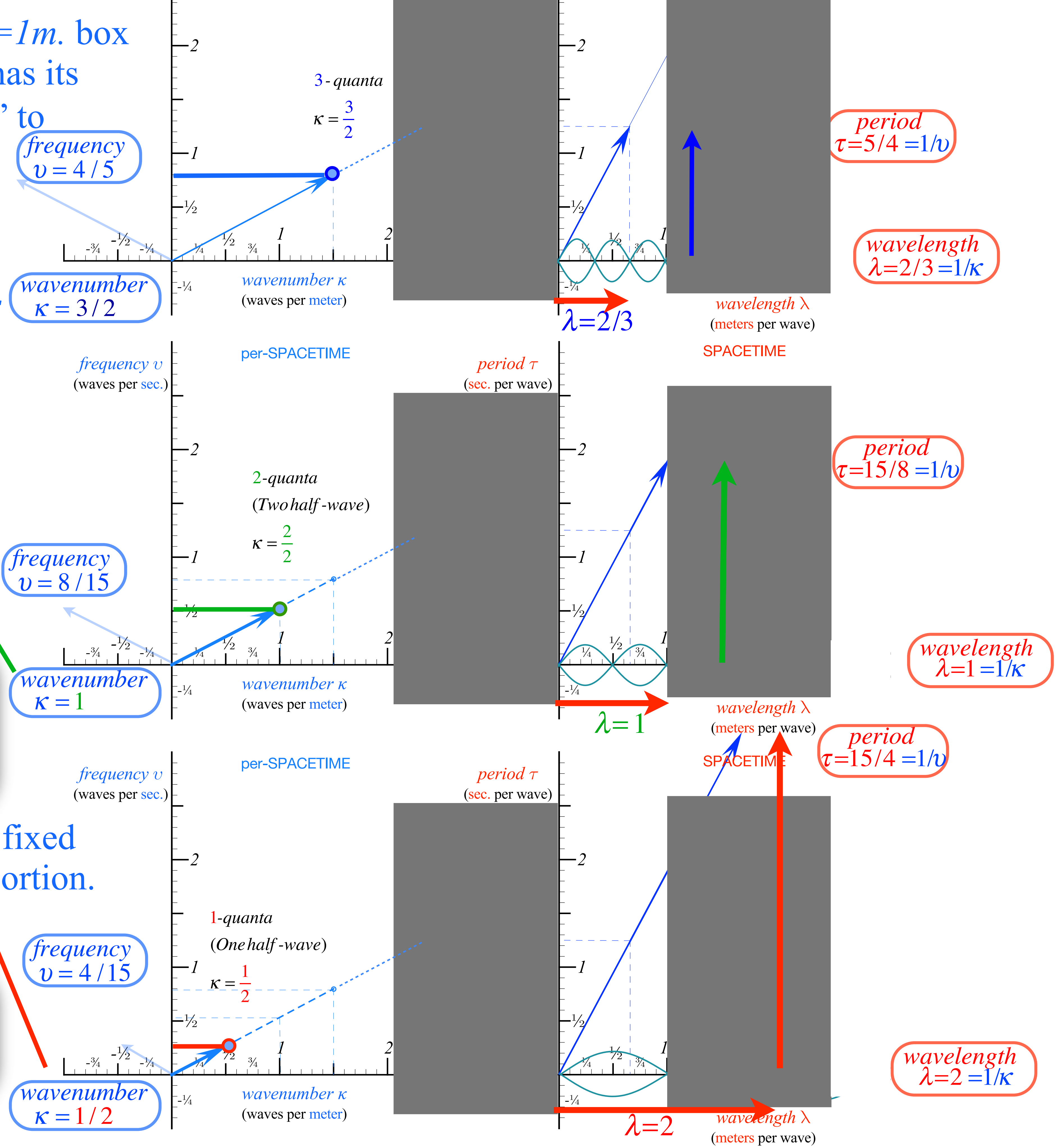
$$\kappa = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \ldots$$

κ = *1/2*, *1*, *3/2*, *2*,...

•*How to understand waves and "1st quantization" or κ-quantization*

If wave velocity V_{wave}=8/15 is fixed *frequency* is quantized in proportion.

•*Amplitude A-quantization is called "2nd quantization"*



If a wave is confined to an *L=1m.* box *the "Keyboard of the gods"* has its wavenumber κ is "quantized" to multiples of *1/2L=1/2*.

$$\kappa = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \ldots$$

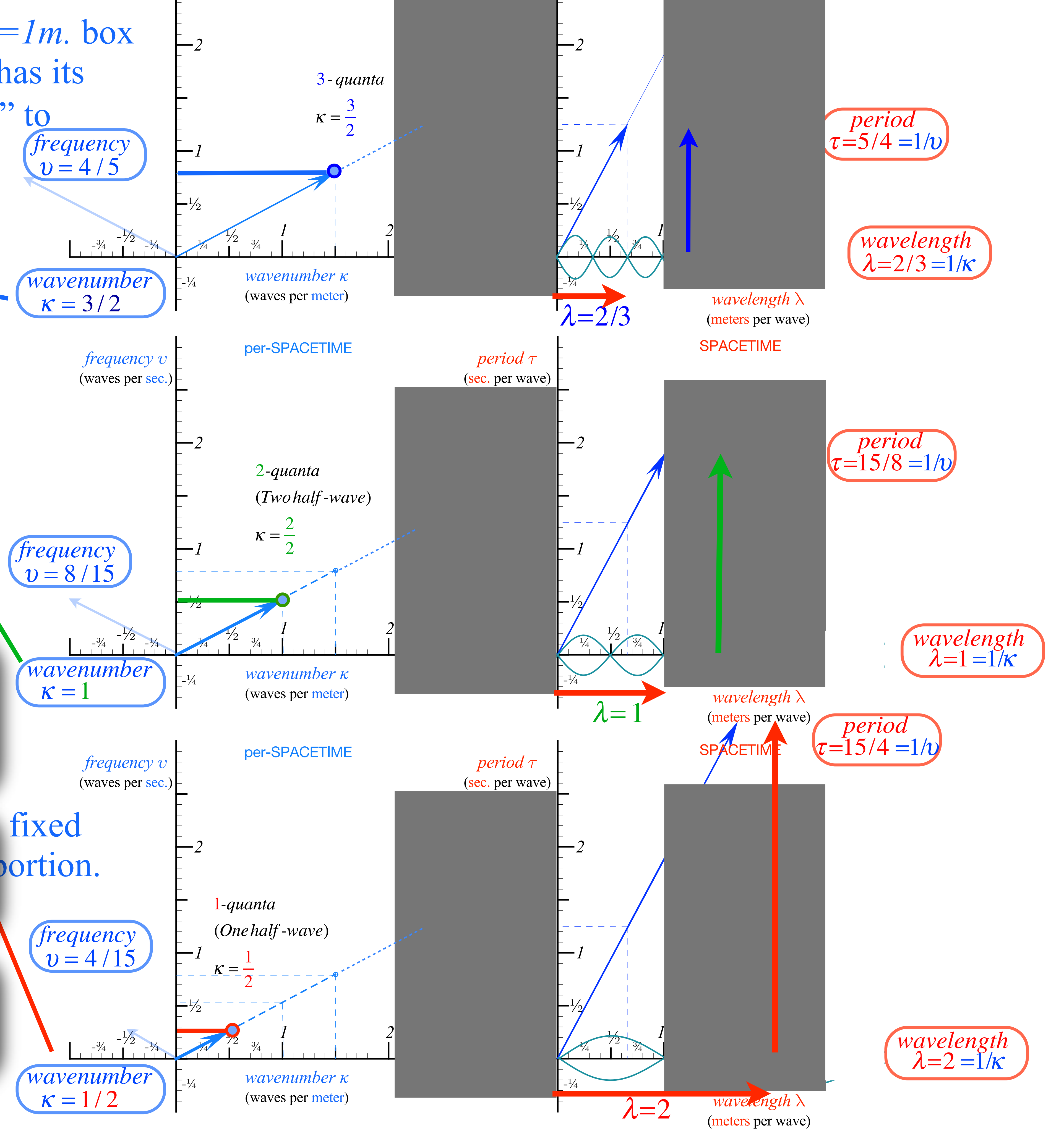
κ = *1/2*, *1*, *3/2*, *2*,...

- *How to understand waves and "1st quantization" or* κ*-quantization*

...as QUALITY (color) versus QUANTITY (Number of "photons")

s fixed
portion.

- *Amplitude A-quantization is called "2nd quantization"*



As will be shown:

Light wave-velocity c is VERY fixed

$$V_{light} = c = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda} = 299{,}792{,}458 \frac{m.}{s.}$$

As will be shown:

Light wave-velocity c is VERY fixed

$$V_{light} = c = \frac{\upsilon}{\kappa} = \frac{1/\kappa}{1/\upsilon} = \frac{\lambda}{\tau} = \frac{1/\tau}{1/\lambda} = 299{,}792{,}458\,\frac{m.}{s.}$$

Then it's convenient to use:

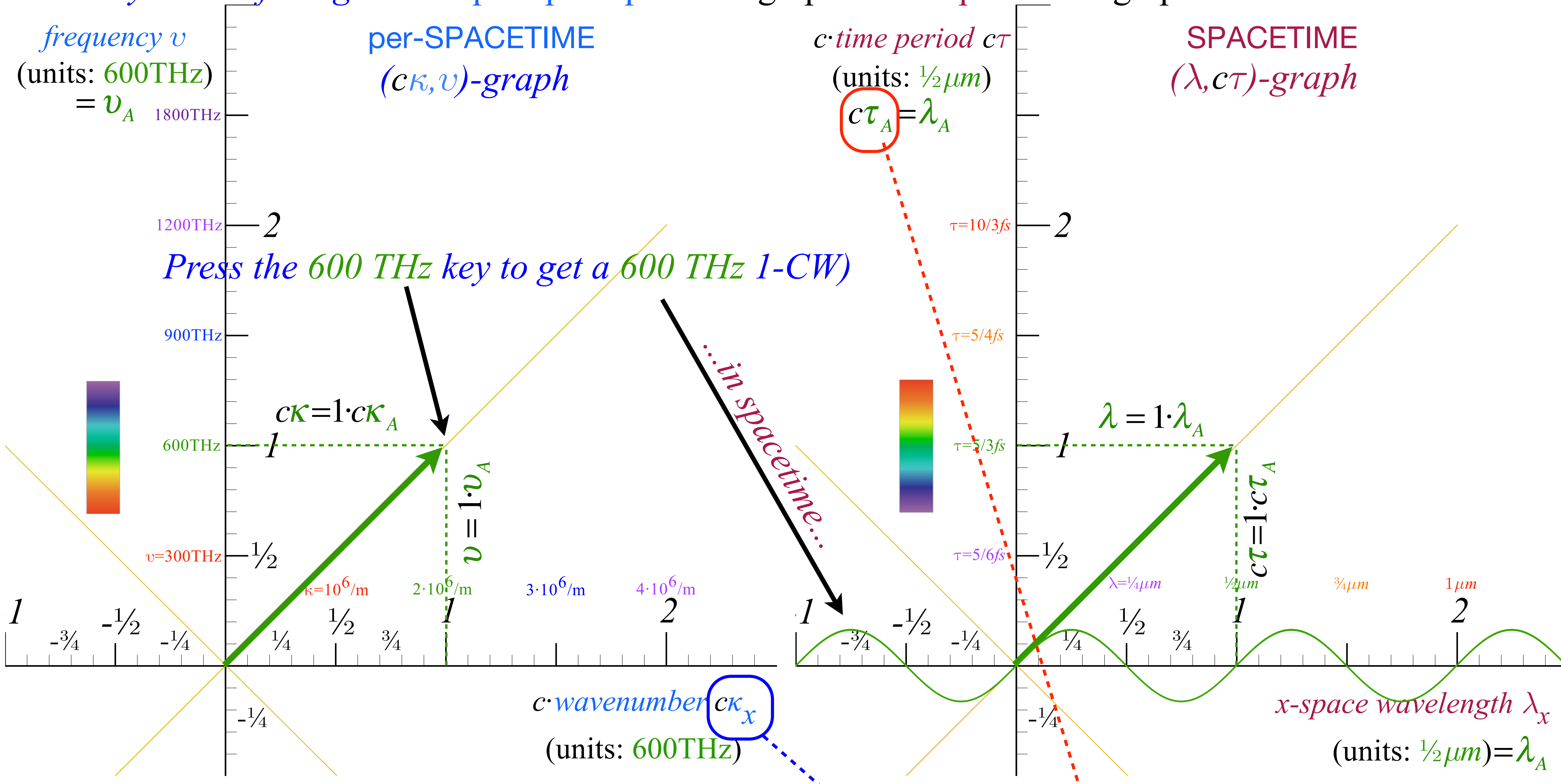
Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\upsilon}{c\kappa} = \frac{\lambda}{c\tau} = 1 \quad \textit{instead of:} \quad \frac{\upsilon}{\kappa} = \frac{\lambda}{\tau} = c$$

Such graphs use c-units of per-time $\upsilon = c\kappa$ and length $\lambda = c\tau$.

$$\frac{V_{light}}{c} = \frac{\upsilon}{c\kappa} = \frac{1/\kappa}{c/\upsilon} = \frac{\lambda}{c\tau} = \frac{1/\tau}{c/\lambda} = 1$$

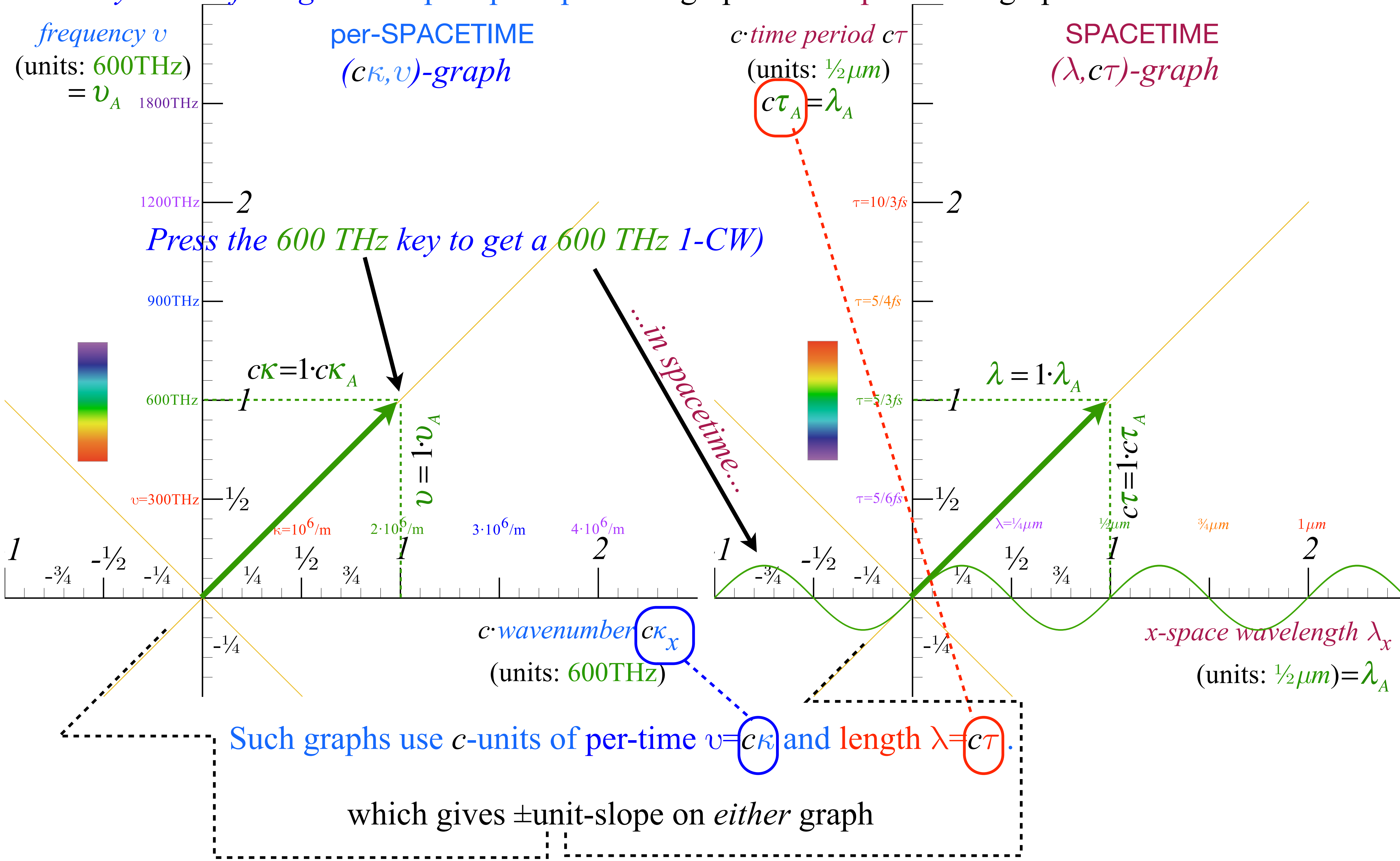
The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



Such graphs use c-units of per-time $\upsilon=c\kappa$ and length $\lambda=c\tau$.

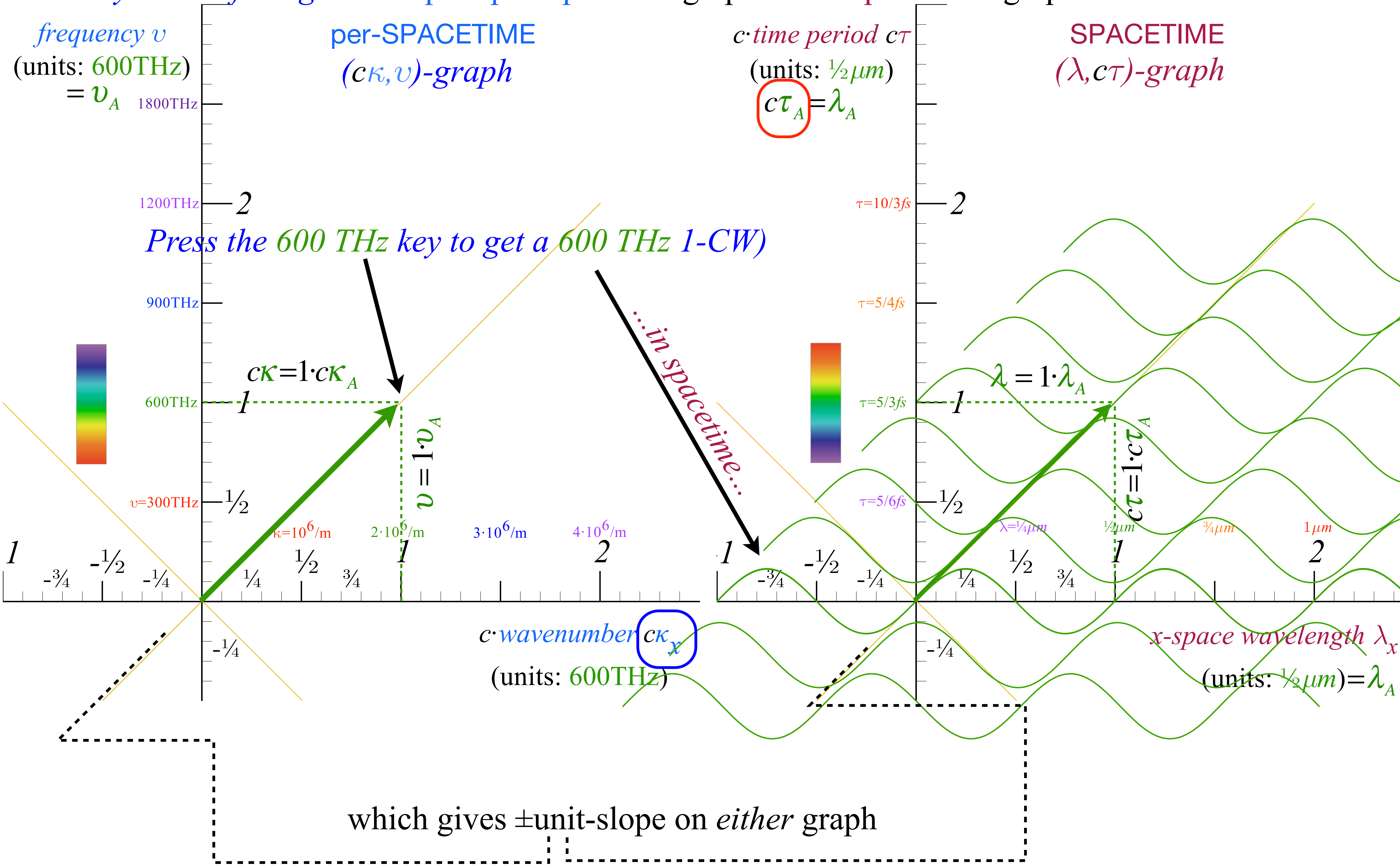
*Ways to quantify **light** waves (600 THz example)*

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



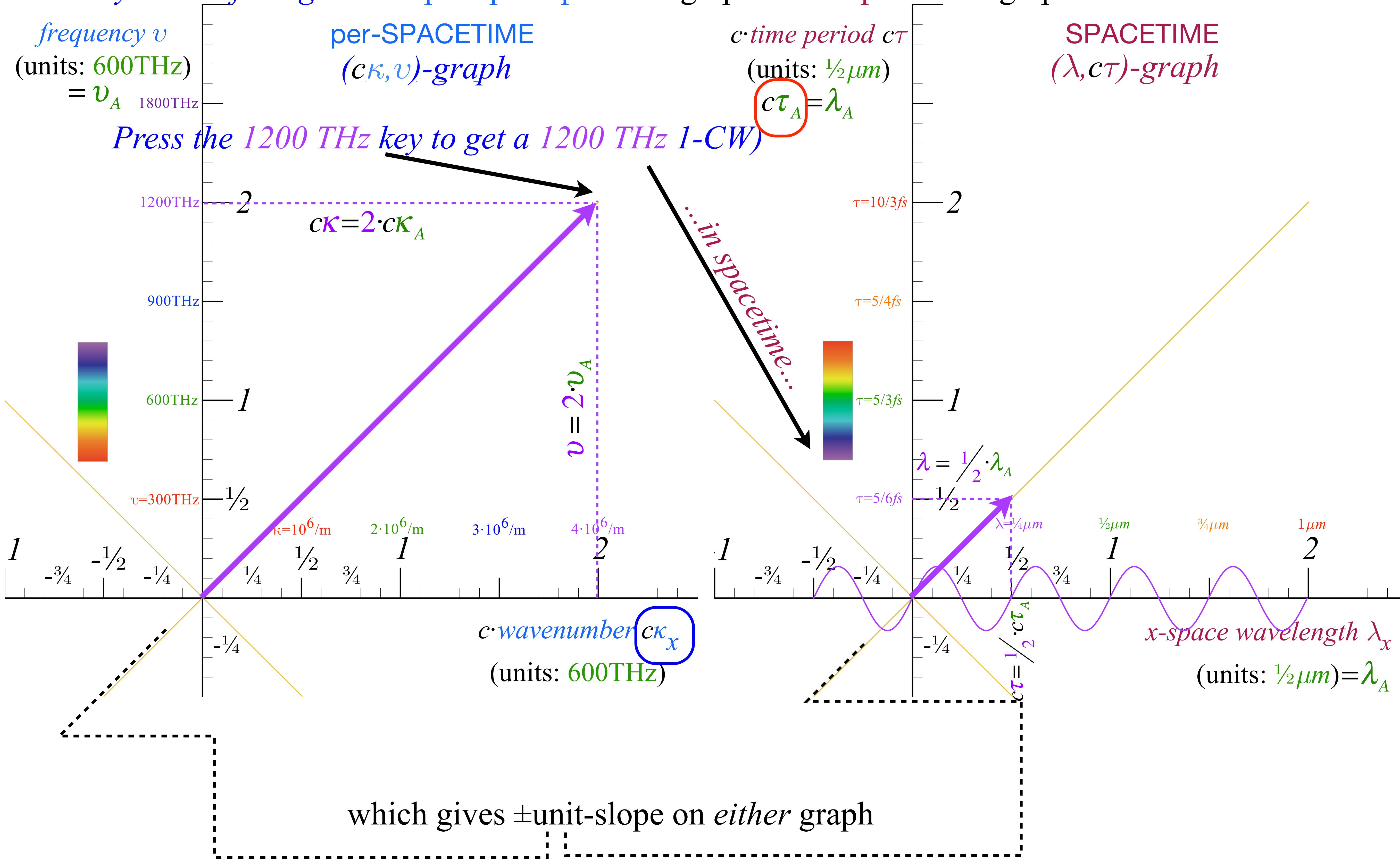
Ways to quantify light waves (600 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



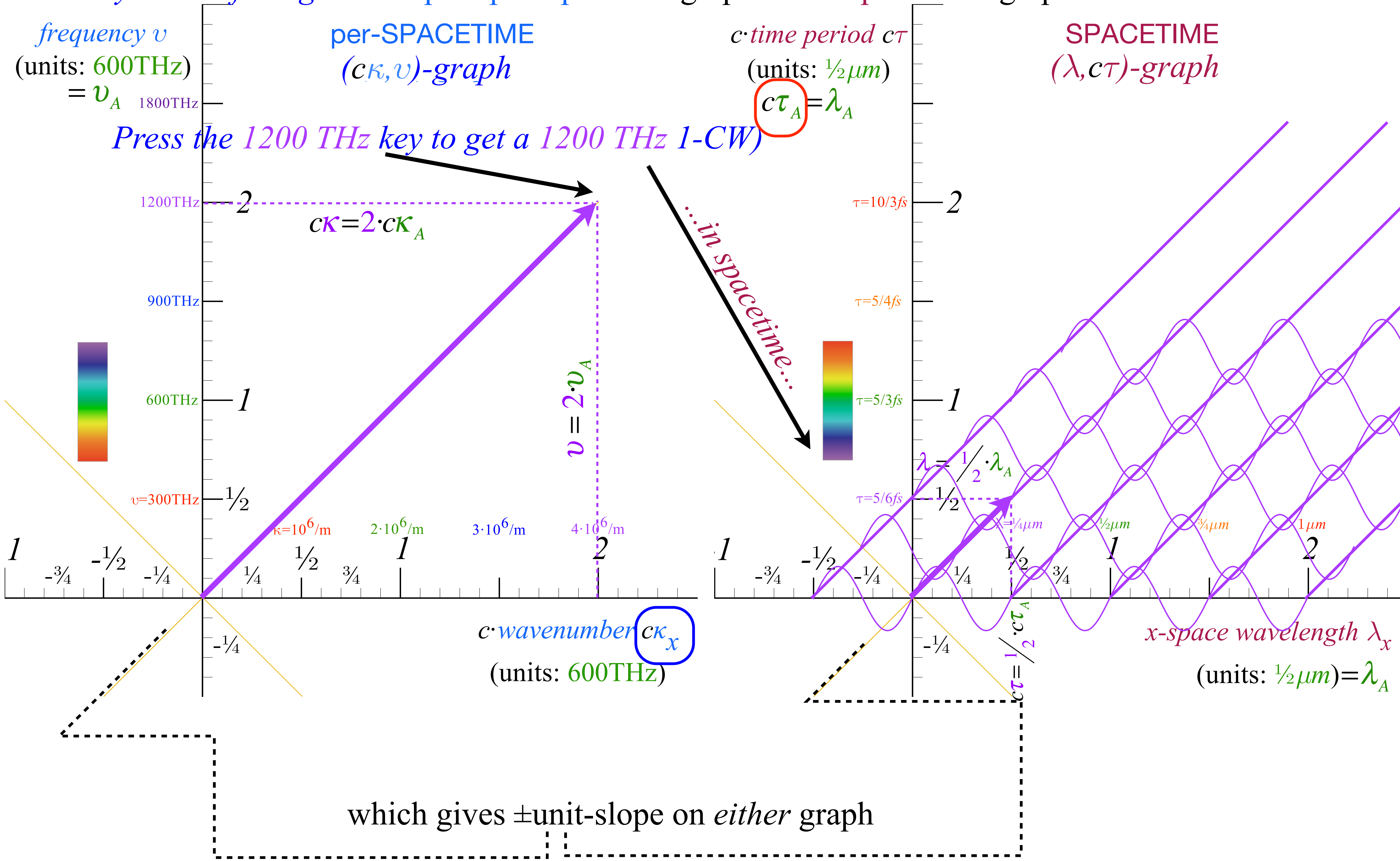
*Ways to quantify **light** waves (600 THz example)*

The “Keyboard of the gods” or per-space-per-time graphs *versus* space-time graphs



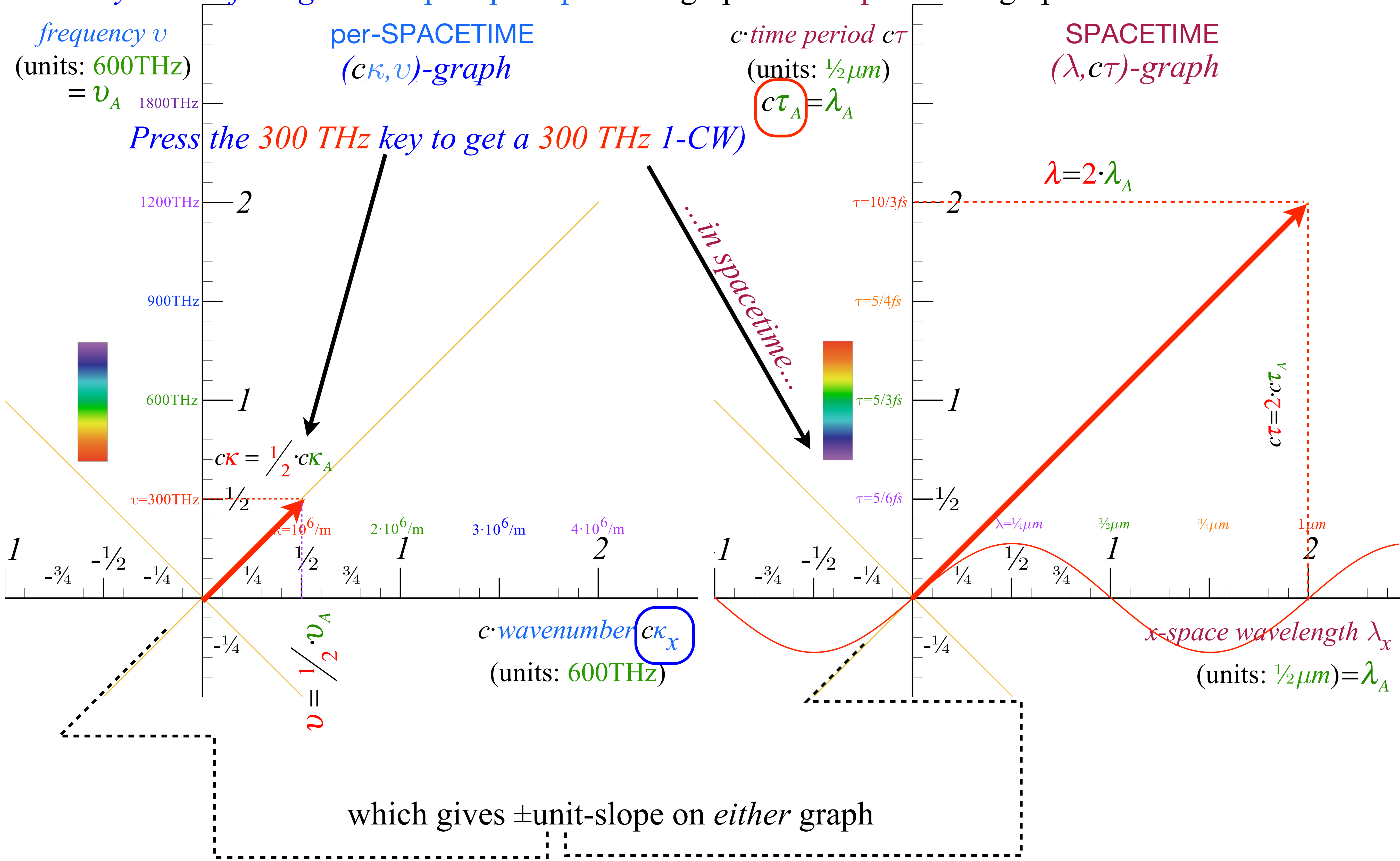
Ways to quantify light waves (1200 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



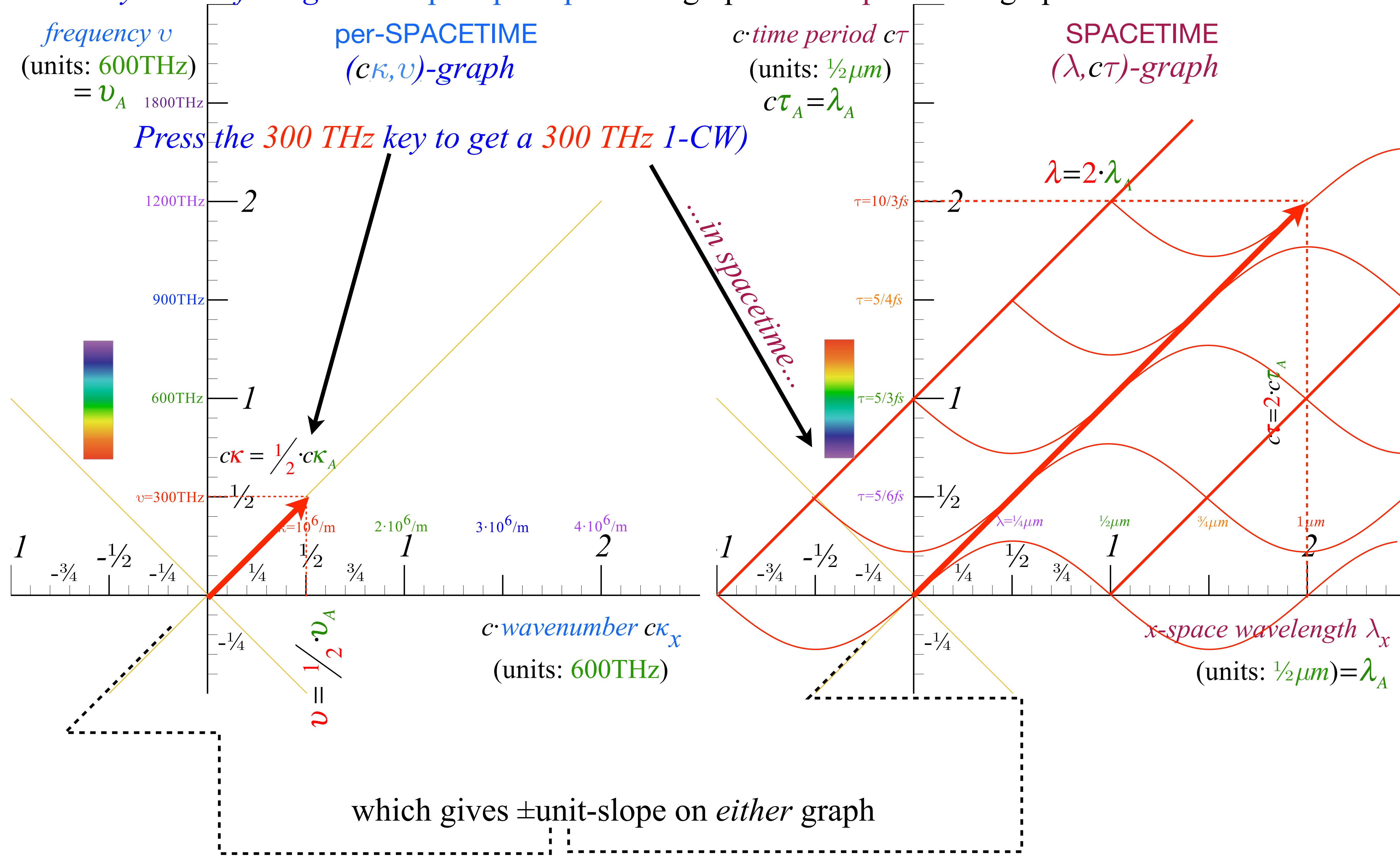
Ways to quantify light waves (1200 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



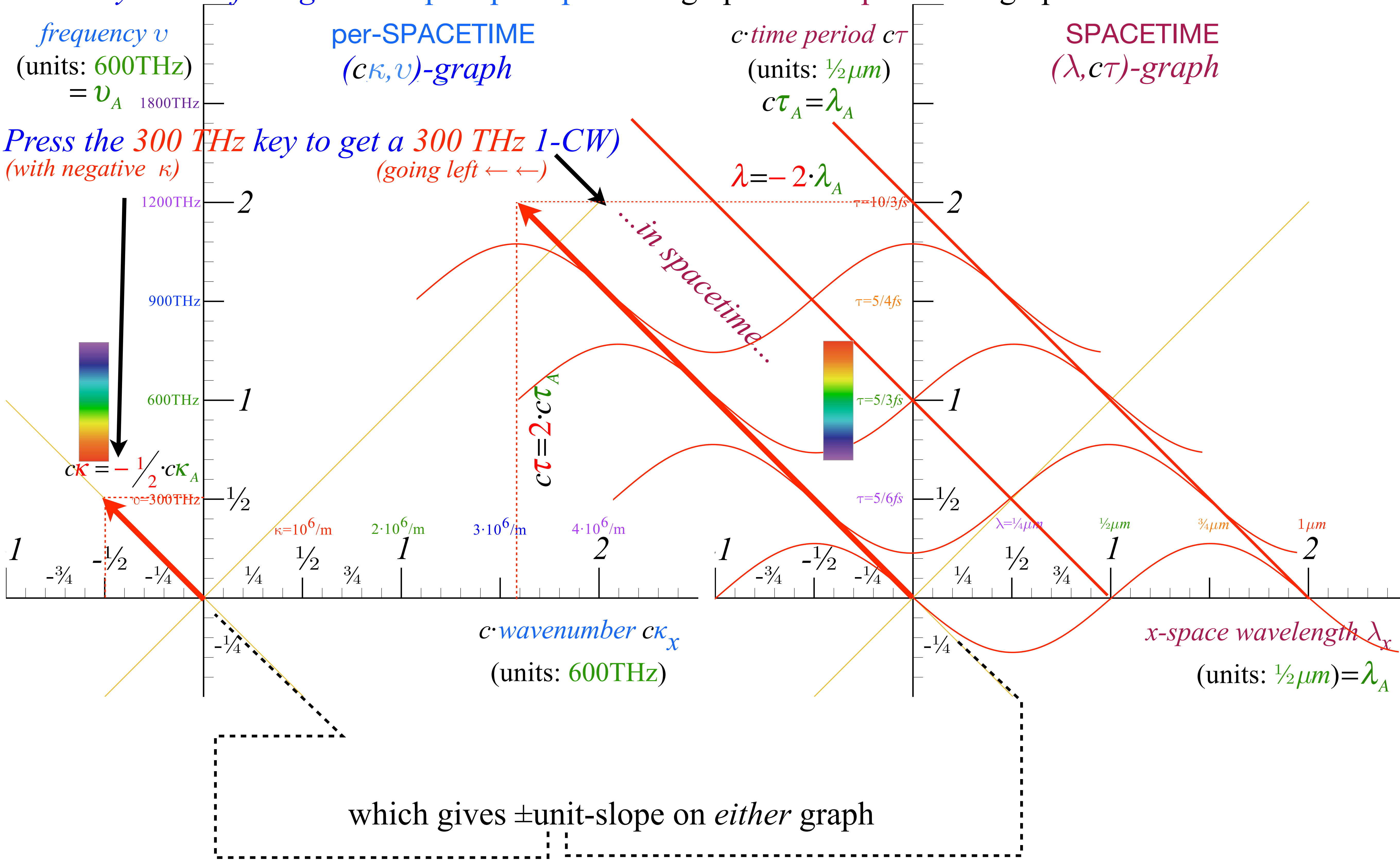
Ways to quantify light waves (300 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



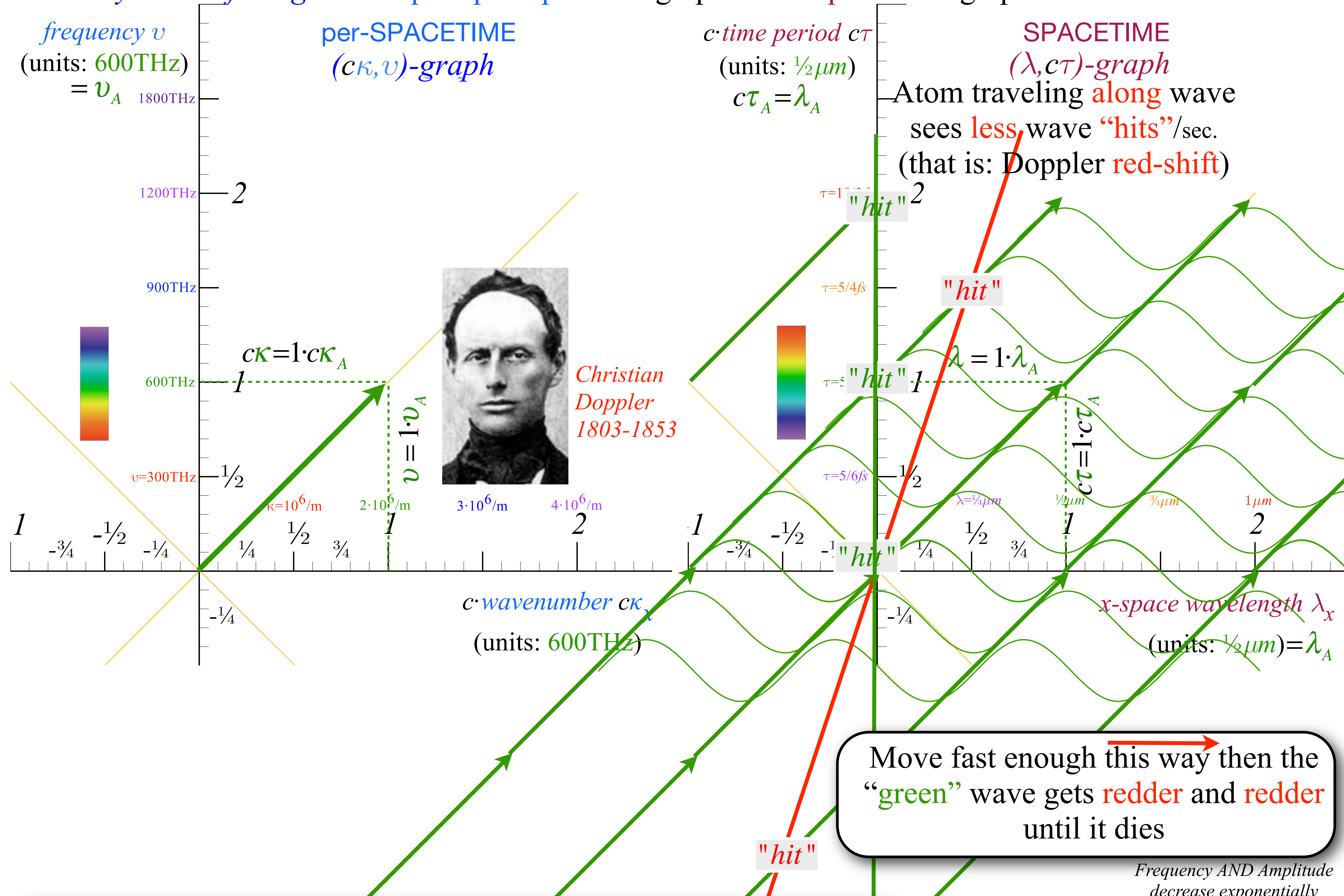
Ways to quantify light waves (300 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



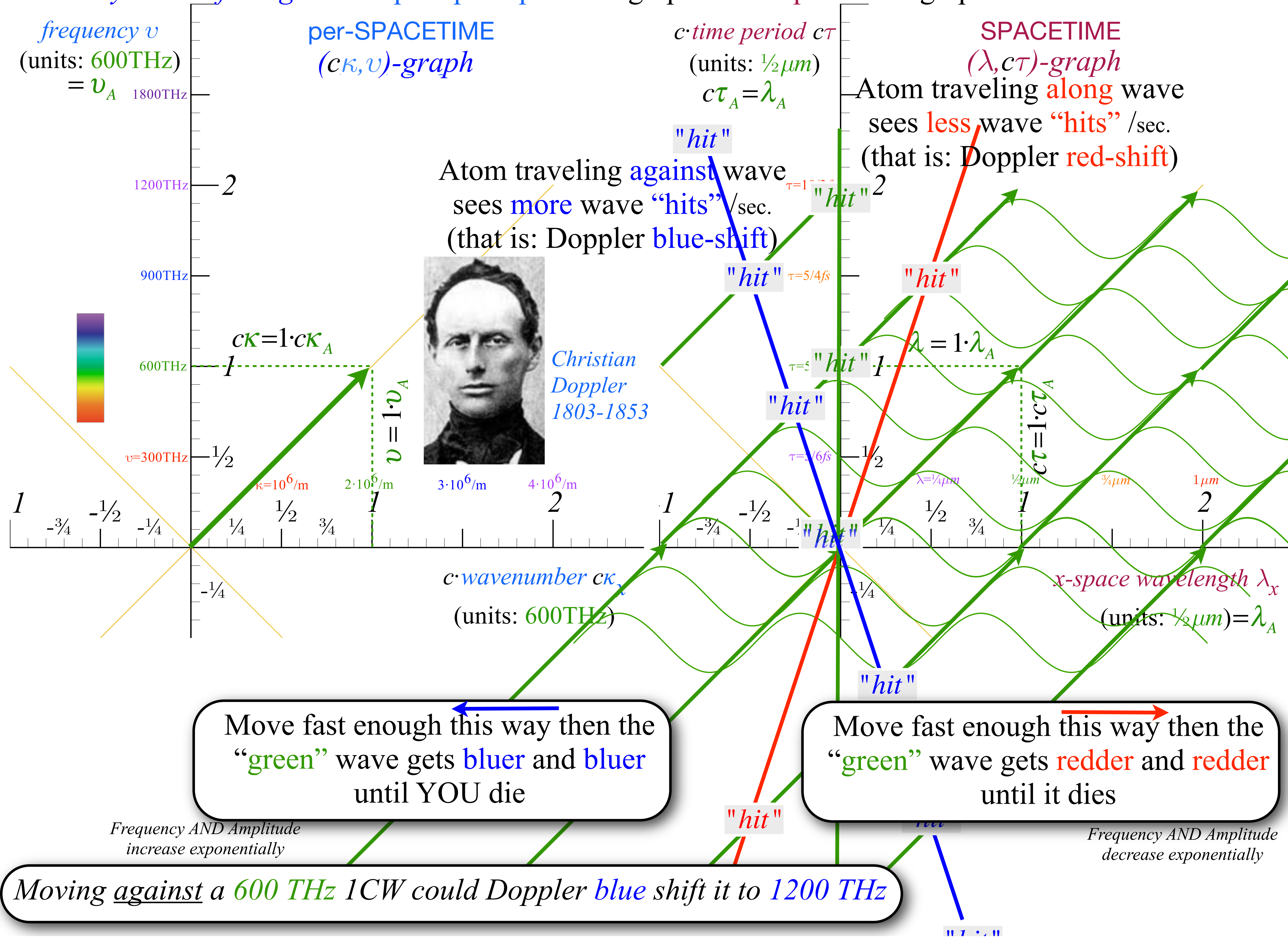
Ways to quantify light waves (300 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



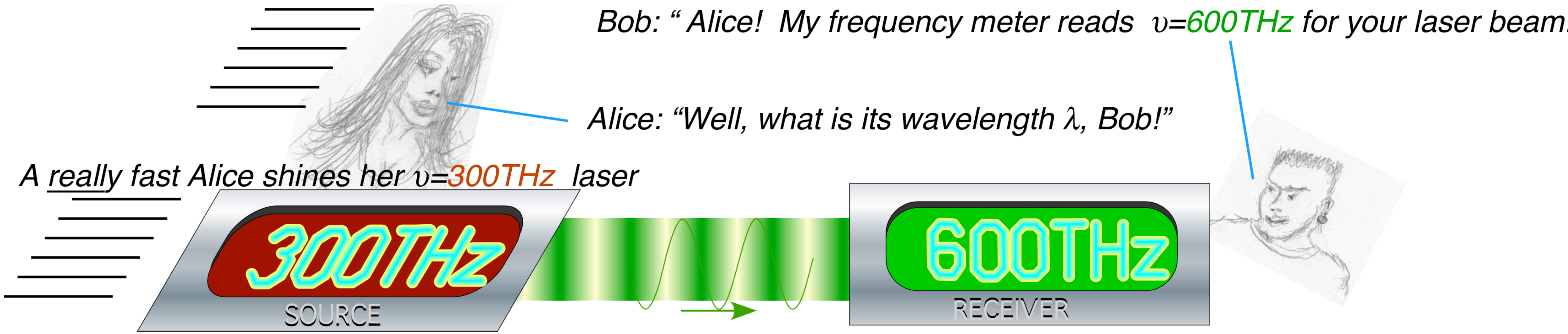
Moving along a 600 THz 1CW could Doppler red shift it to 300 THz

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



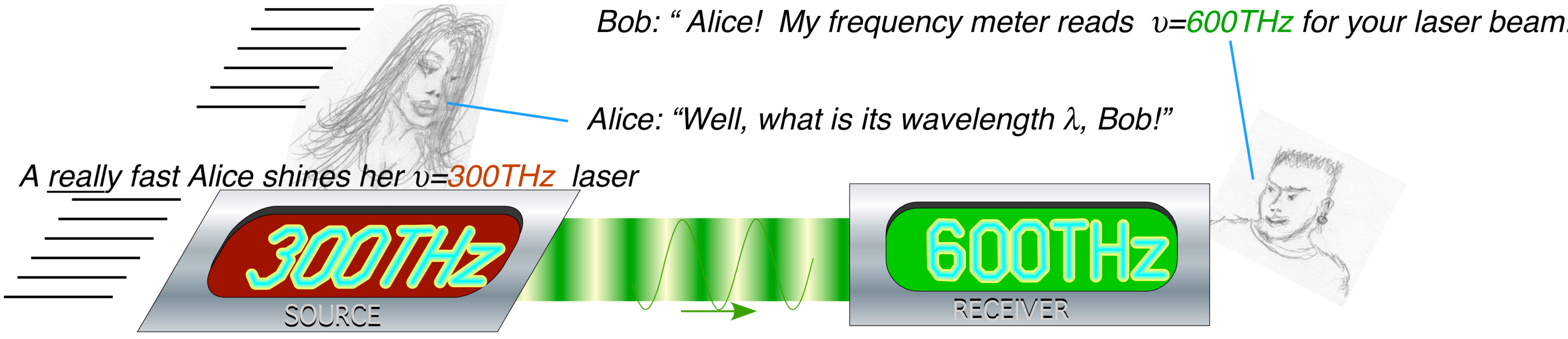
Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)



Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

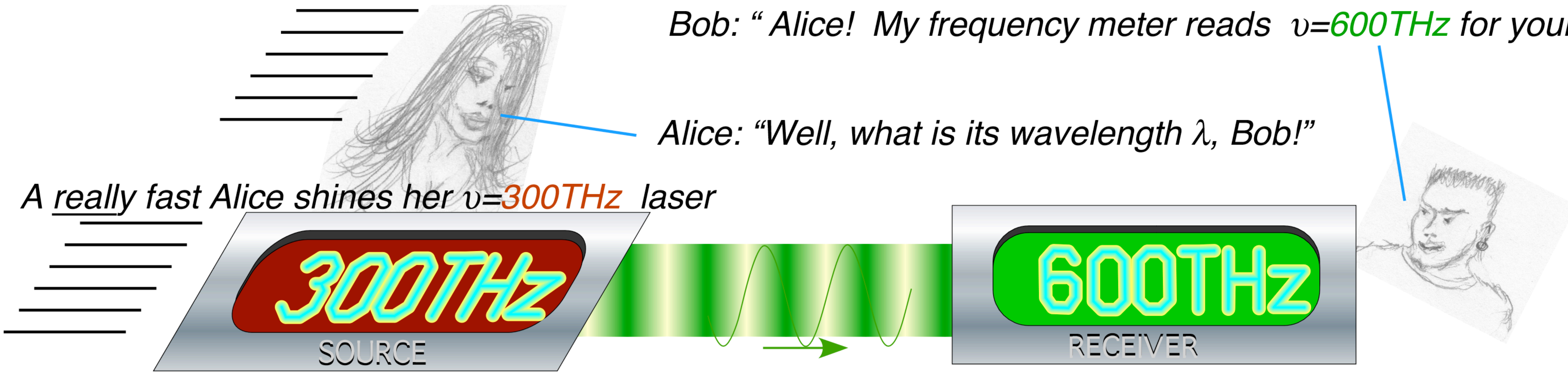
Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)



Q1: Can Bob tell it's a *"phony"* 600THz by measuring his received wavelength?

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)

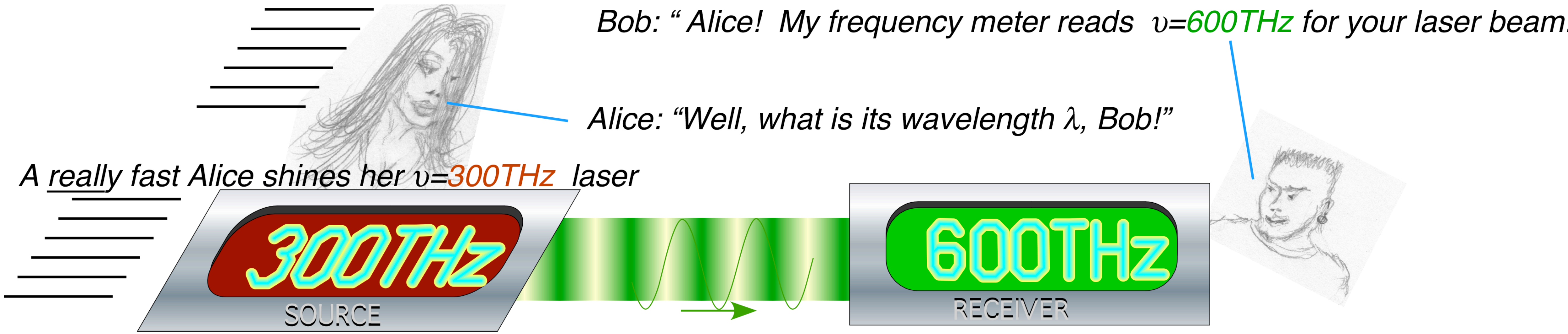


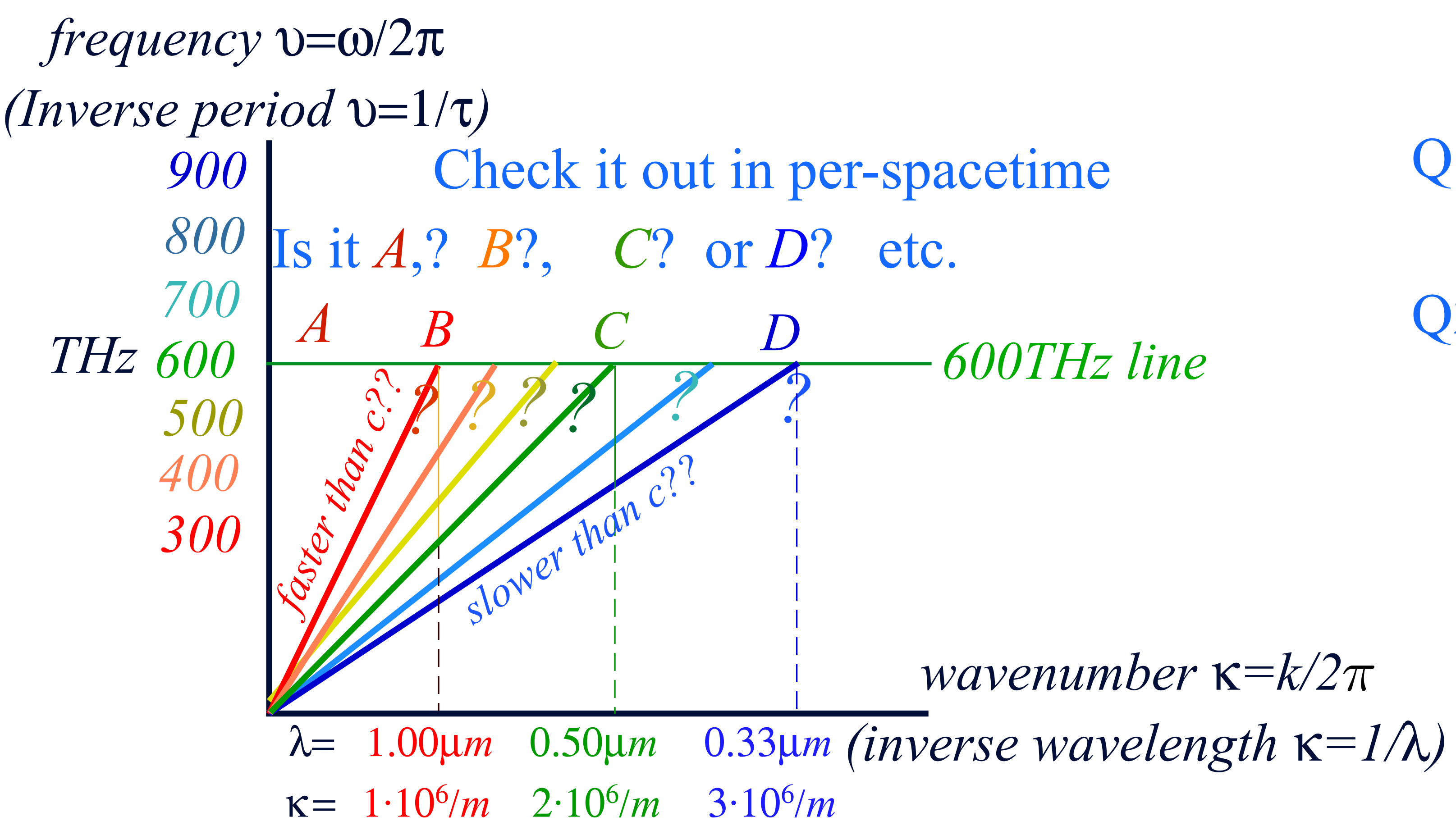
Q1: Can Bob tell it's a *"phony"* 600THz by measuring his received wavelength?

Q2:If so, what *"phony"* λ does Bob see?

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)



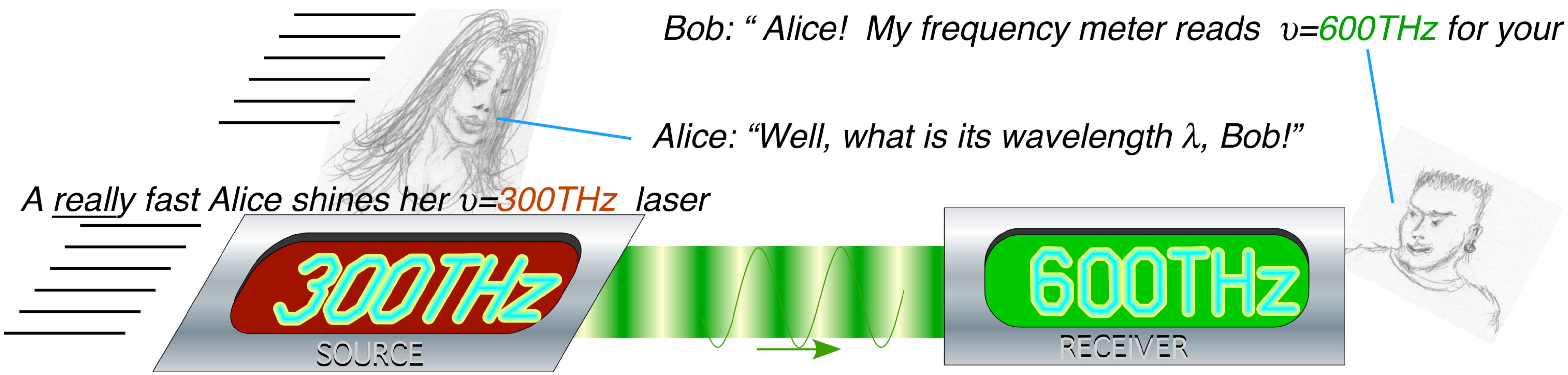


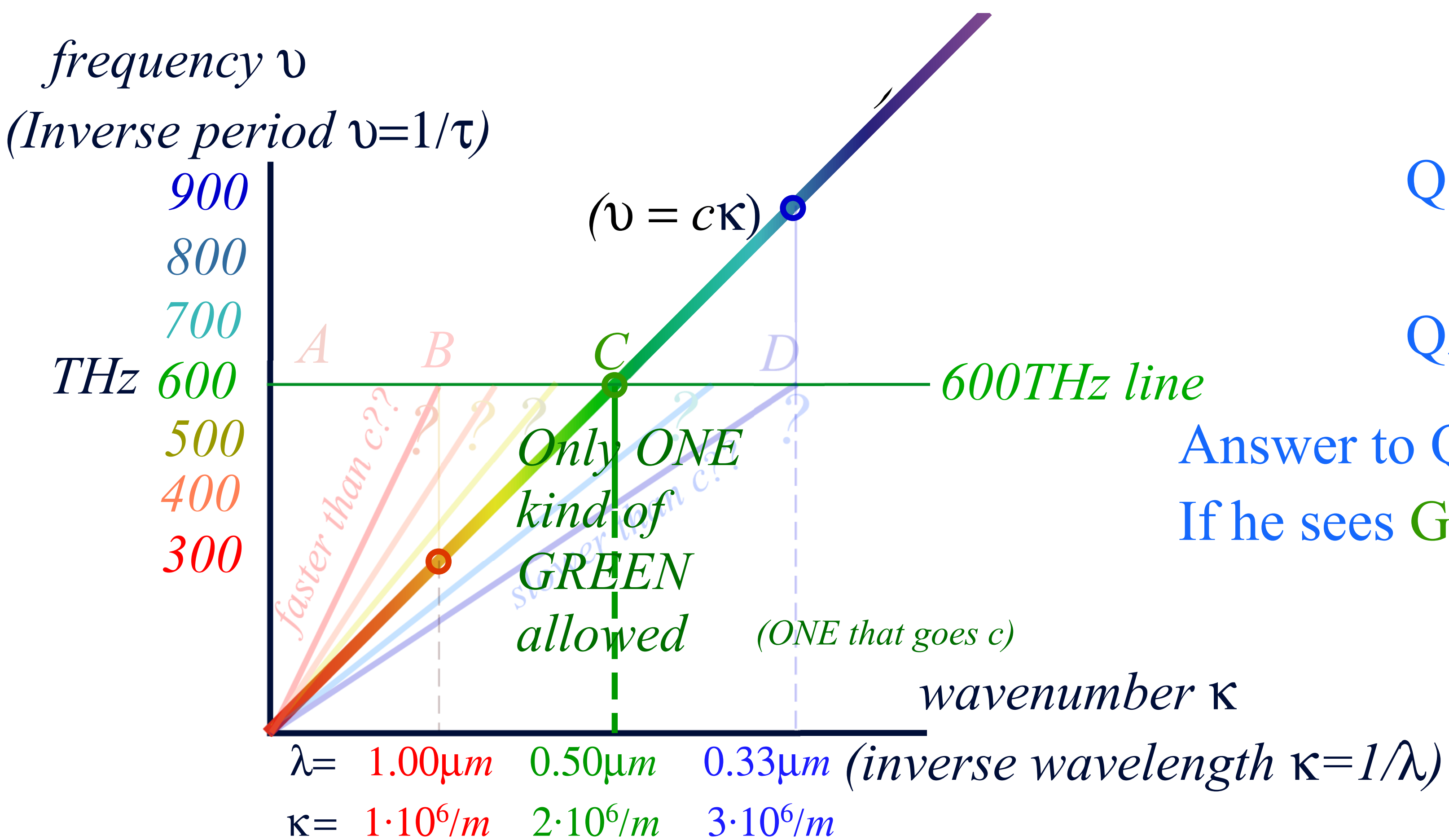
Q1: Can Bob tell it's a *"phony"* 600THz by measuring his received wavelength?

Q2:If so, what *"phony"* λ does Bob see?

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)





Q1: Can Bob tell it's a *"phony"* 600THz by measuring his received wavelength?

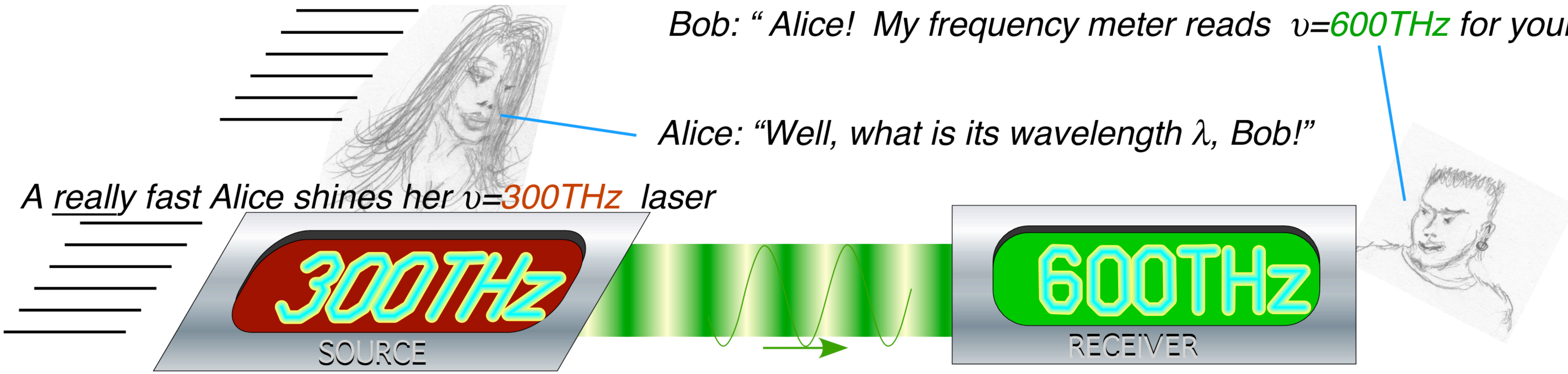
Q2: If so, what *"phony"* λ does Bob see?

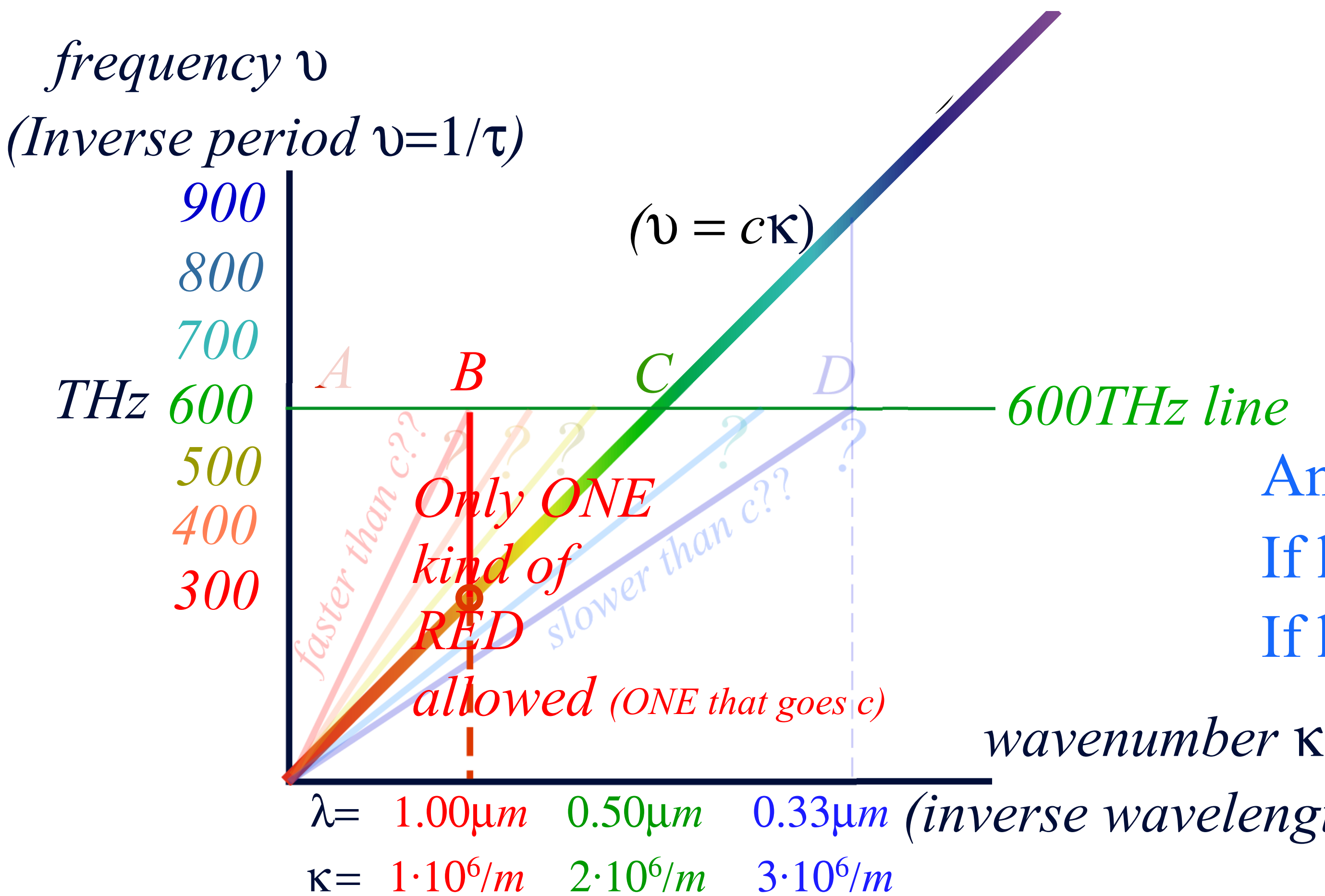
Answer to Q2 is *C*, the one with <u>*slope*</u> $\upsilon/\kappa=\upsilon\cdot\lambda=c$.

If he sees Green 600THz then he measures λ=0.5μm.

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)





Q1: Can Bob tell it's a *"phony"* 600THz
by measuring his received wavelength?

Q2:If so, what *"phony"* λ does Bob see?

Answer to Q2 is *C*, the one with slope $\upsilon/\kappa=\upsilon\cdot\lambda=c$.

If he sees Green 600THz then he measures λ=0.5μm.

If he sees Red 300THz then he measures λ=1.0μm.

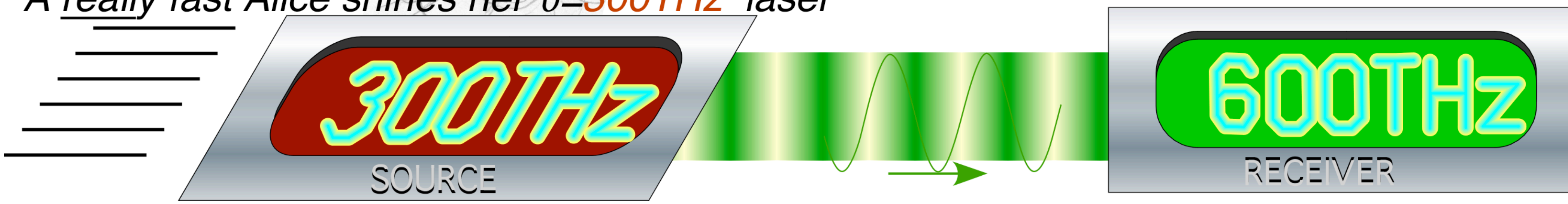
Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

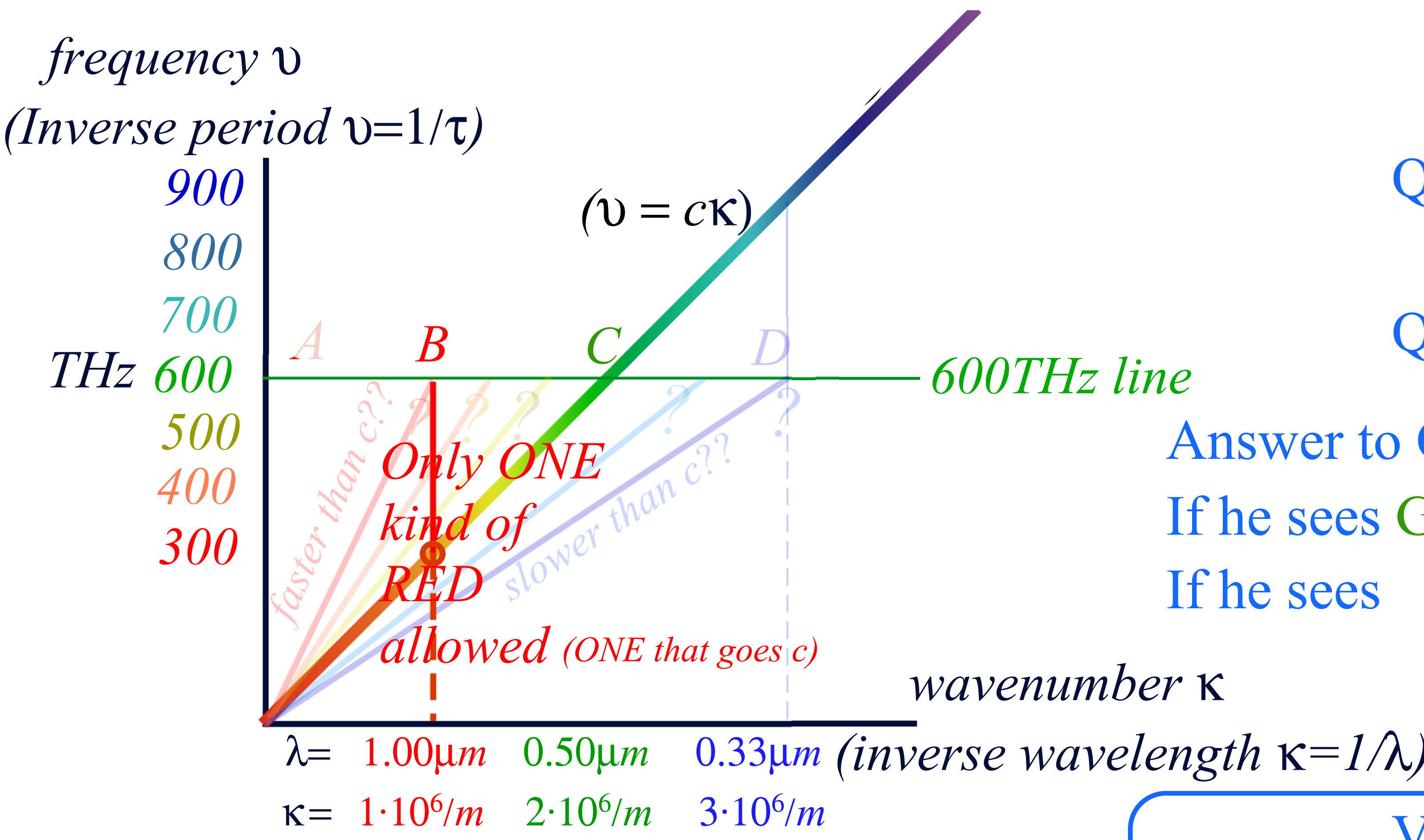
Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)

Bob: " Alice! My frequency meter reads υ=600THz for your laser beam.

Alice: "Well, what is its wavelength λ, Bob!"

A really fast Alice shines her υ=300THz laser





Q1: Can Bob tell it's a *"phony"* 600THz
by measuring his received wavelength?

Q2: If so, what *"phony"* λ does Bob see?

Answer to Q2 is *C*, the one with *slope* $\upsilon/\kappa=\upsilon\cdot\lambda=c$.

If he sees Green 600THz then he measures λ=0.5μm.

If he sees Red 300THz then he measures λ=1.0μm.

Answer to Q1 is **NO!**

CW Light carries **no** birth-certificate!

Vacuum only makes one λ for each υ.*

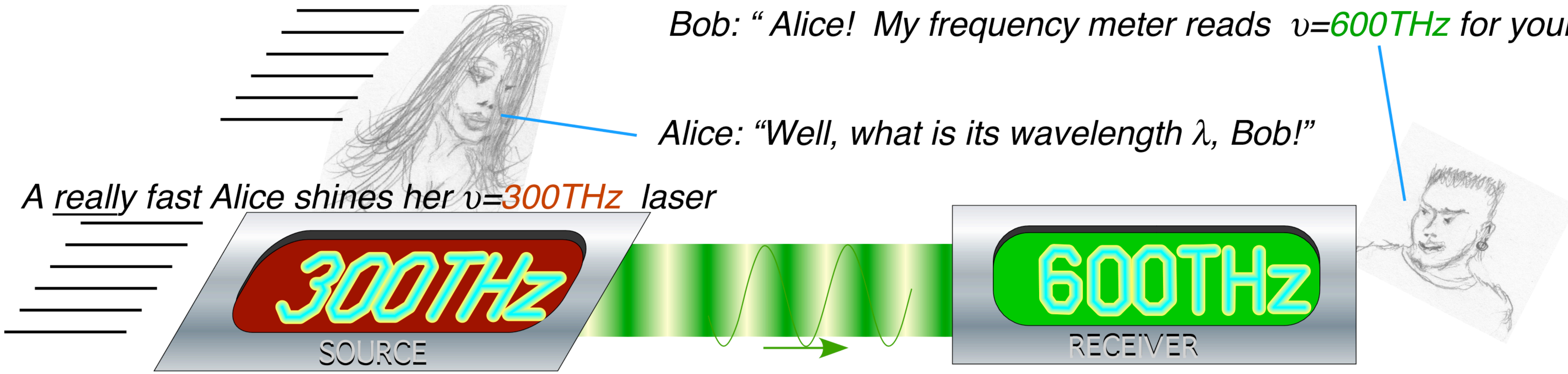
"All colors go $c = \lambda\upsilon = \upsilon/\kappa$ "

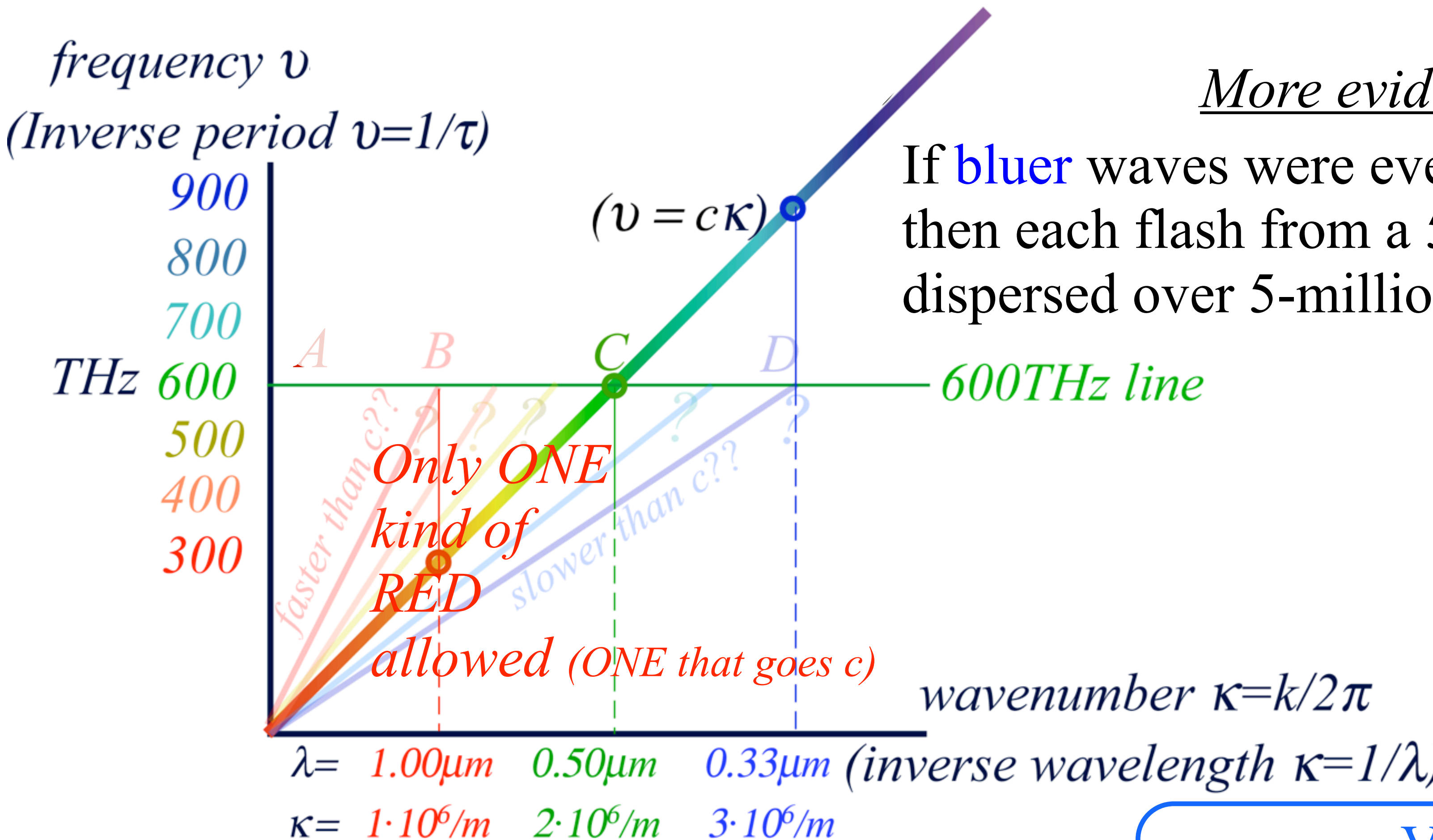
Then *Evenson's axiom* holds:

*for each beam and polarization orientation

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)





More evidence supporting Evenson's axiom

If bluer waves were even 0.1% faster (or slower) than redder ones then each flash from a 5-billion light-year distant galaxy shows up dispersed over 5-million years. (*Goodbye galactic astronomy!*)

Also could be labeled :

Linear-(non)-dispersion axiom: $\upsilon = c\kappa$

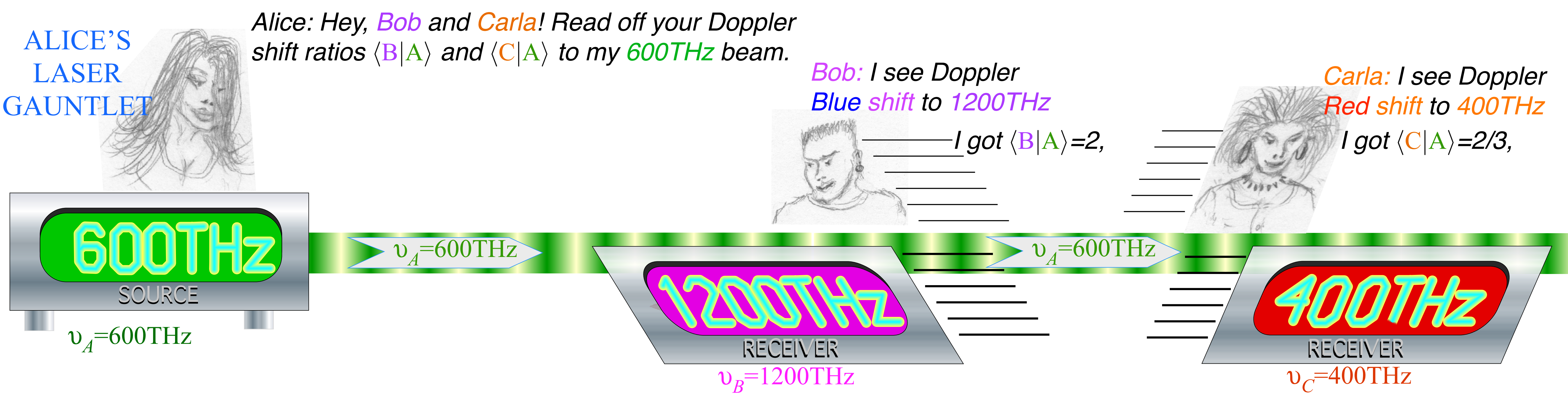
Vacuum only makes one λ for each υ.*

"All colors go $c = \lambda\upsilon = \upsilon/\kappa$ "

Then *Evenson's axiom* holds:

*for each beam and polarization orientation

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

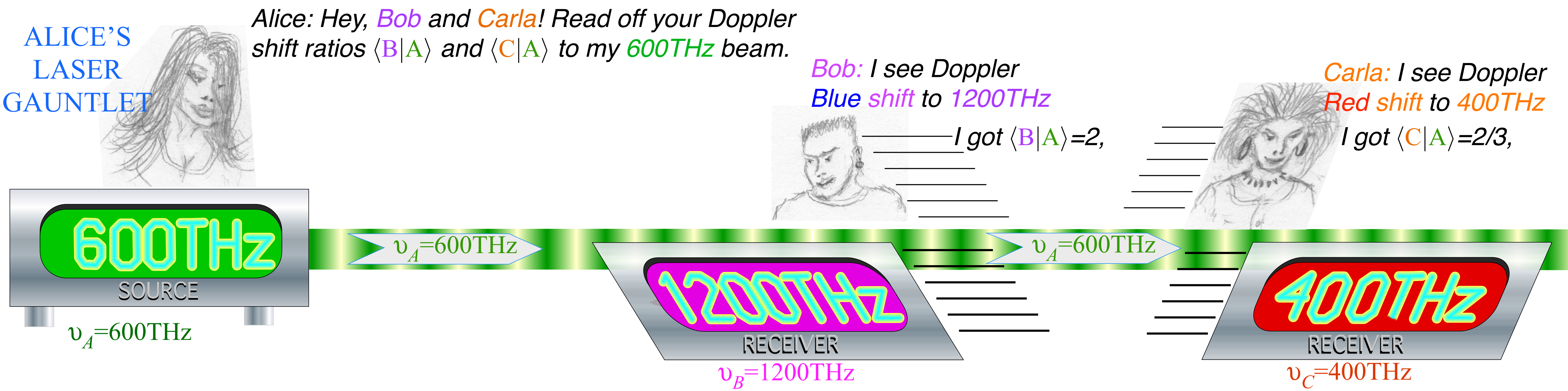
Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

Bob-Alice Doppler ratio:

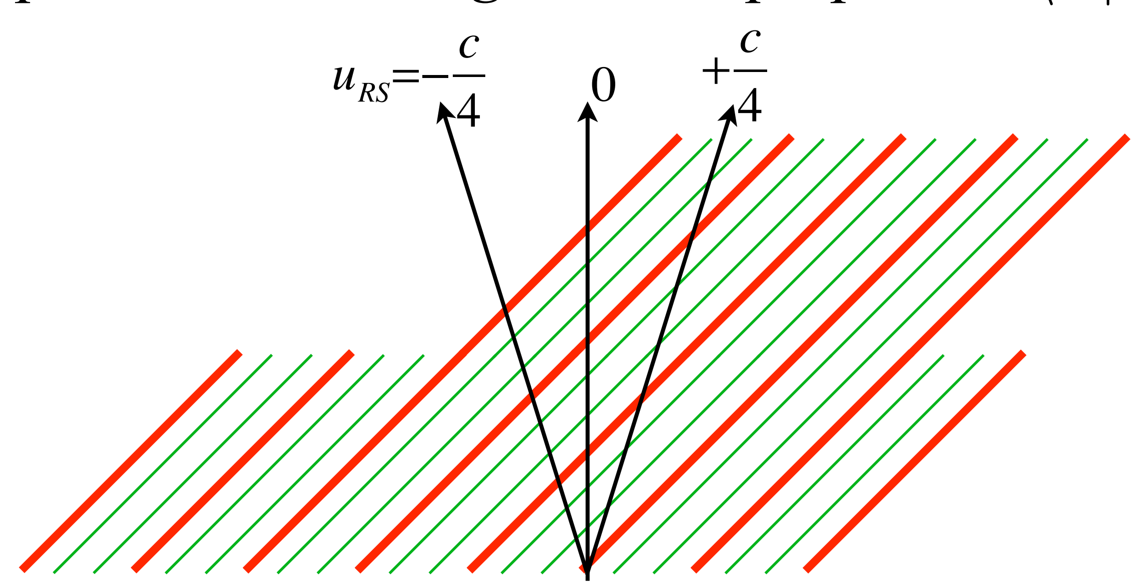
$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

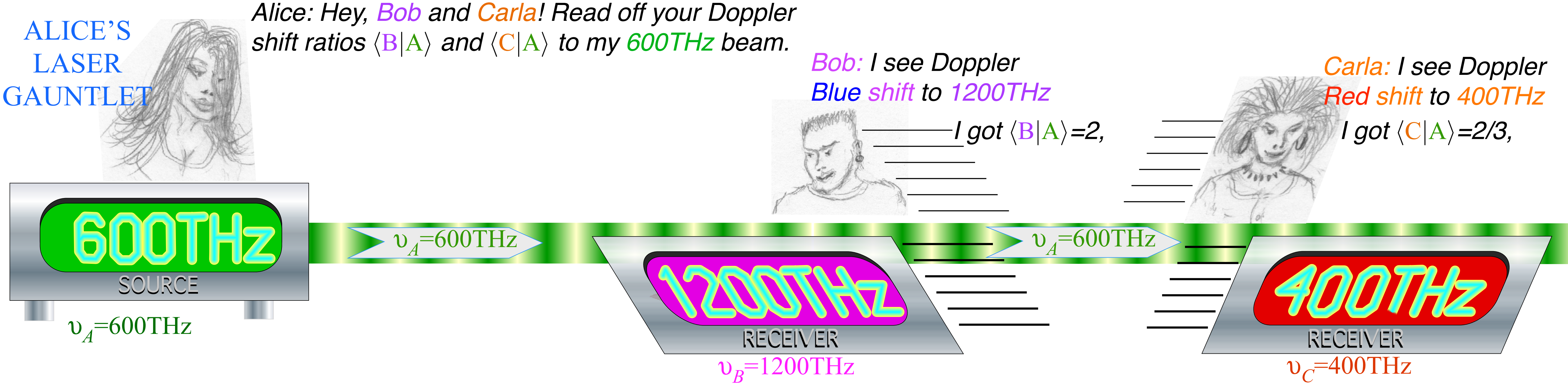
$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

IMPORTANT POINT:

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion ⟨R|S⟩.



Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

IMPORTANT POINT:

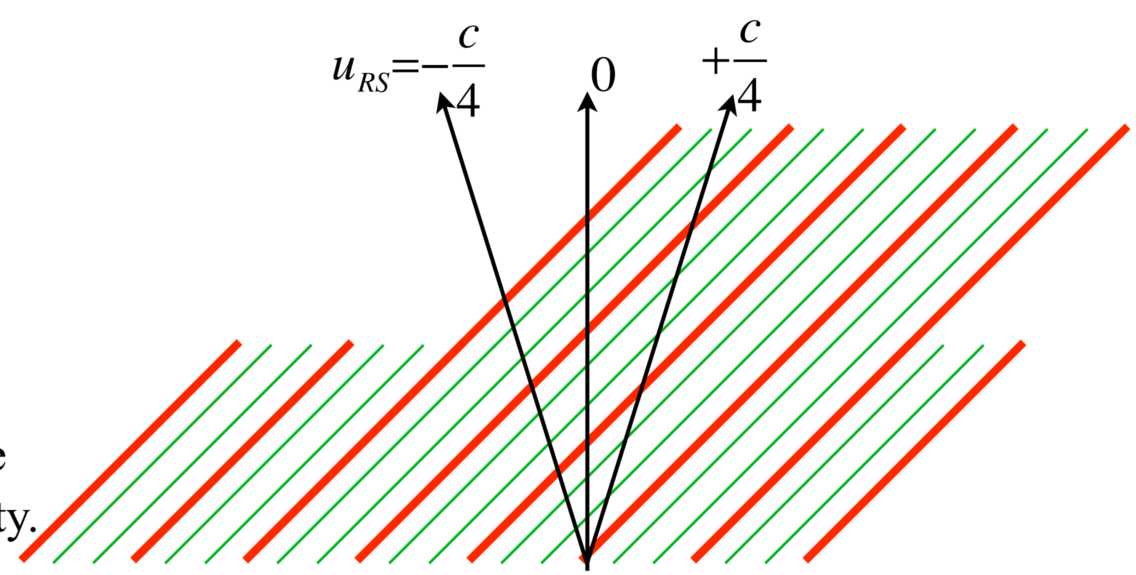
Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion ⟨R|S⟩.

If Alice sends υ_A=600THz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=1200THz

If Alice sends υ_A=60 THz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=120THz

If Alice sends υ_A=6 Hz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=12 Hz

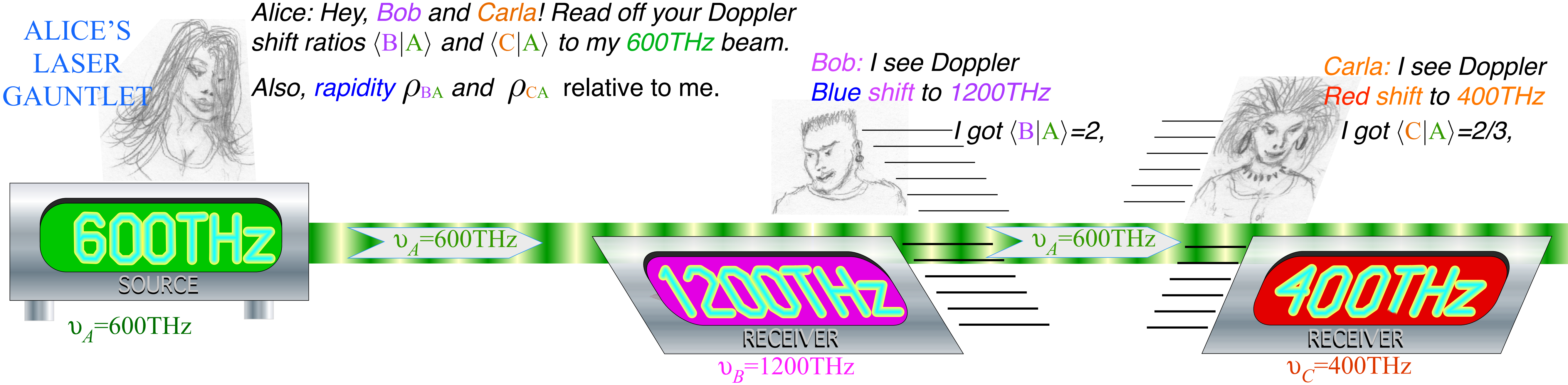
$\langle B|A\rangle$=2 for any frequency Alice and Bob use while they maintain their relative velocity.



Easy Doppler-shift and Rapidity calculation

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ to my 600THz beam.

Also, rapidity ρ_{BA} *and* ρ_{CA} relative to me.



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

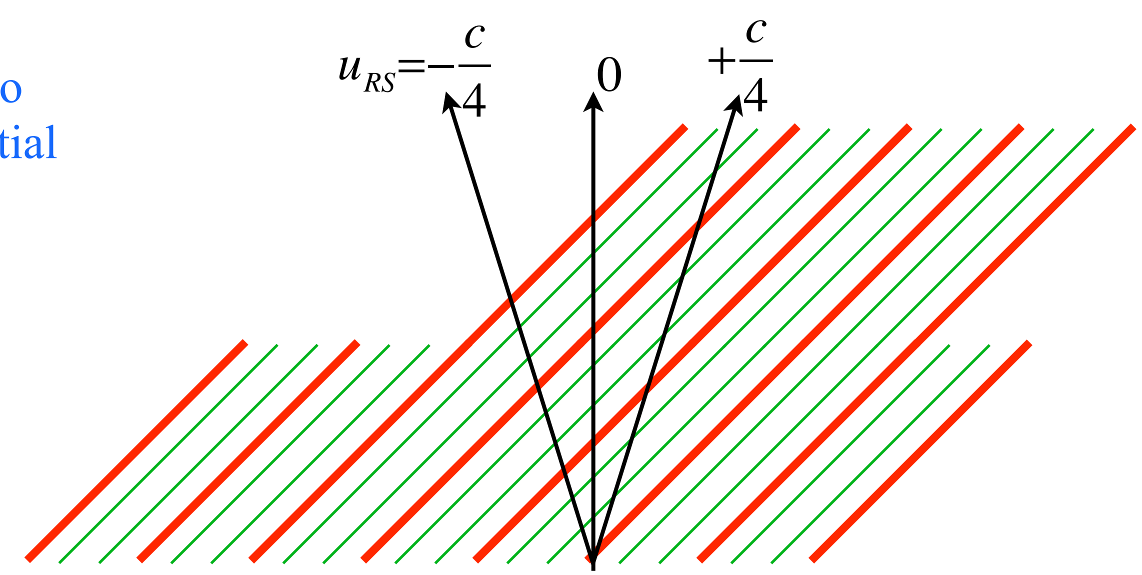
Rapidity is most convenient!

1TeV proton has
u=0.999995598·c (Pain in the A)
or: ⟨R|S⟩=2131.6 (Better)
or: ρ_{RS}*=7.6646 (Best)*

IMPORTANT POINTS:

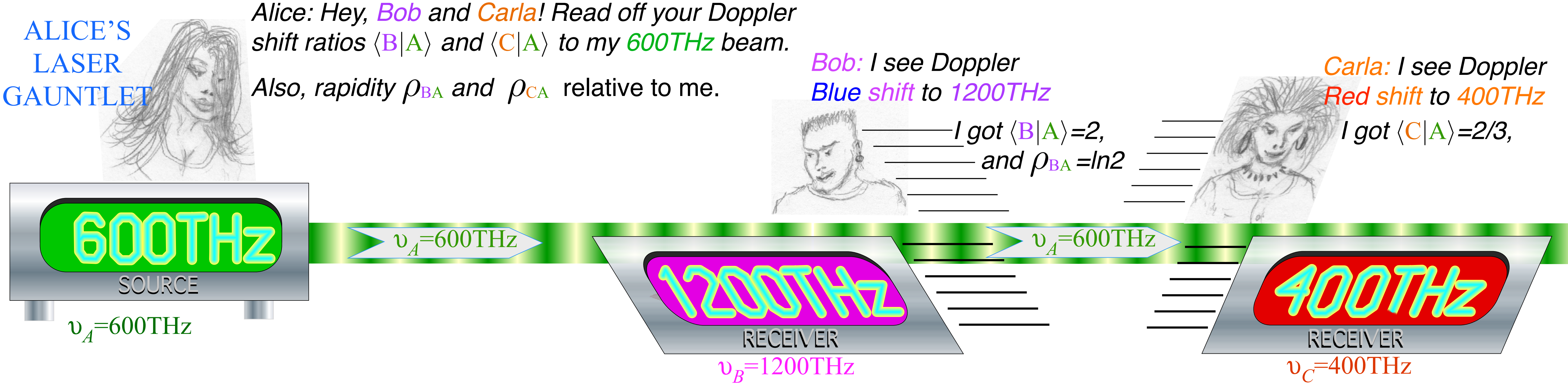
Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion ⟨R|S⟩.

Geometric phenomena tend to involve logarithmic/exponential functionality!



For low velocity u<<c rapidity ρ_{RS} *approaches u/c*

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

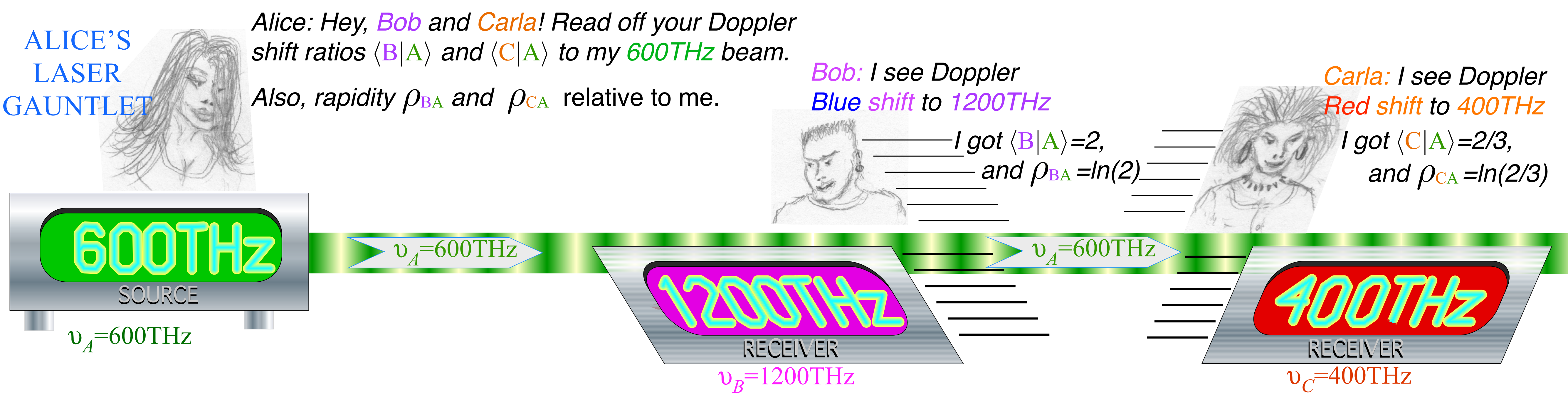
Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

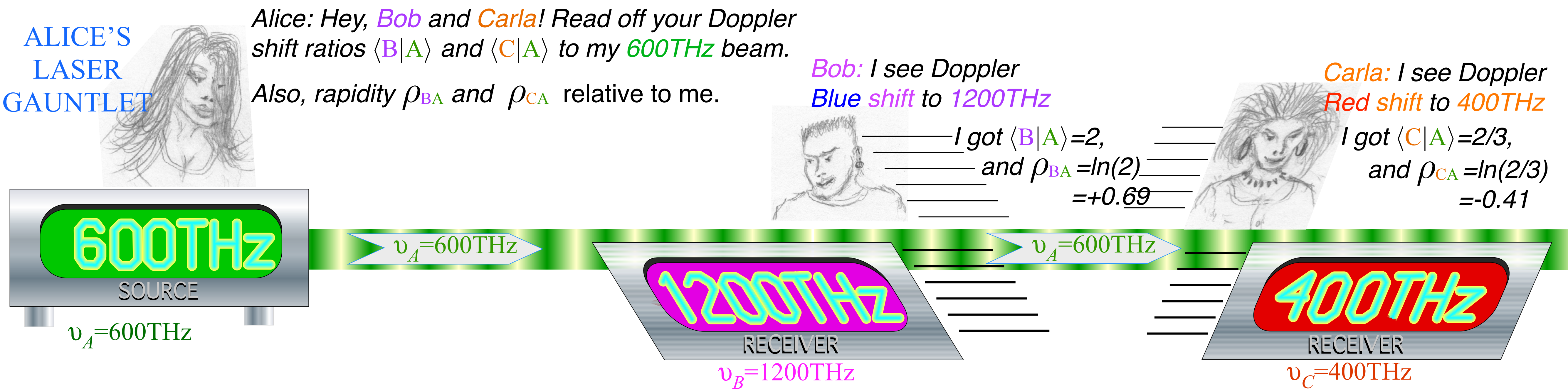
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{2}{1}$$

is time-reversal of:

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B} = \frac{1}{2}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \qquad \text{(time-reversed)} \; (so{:}\rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

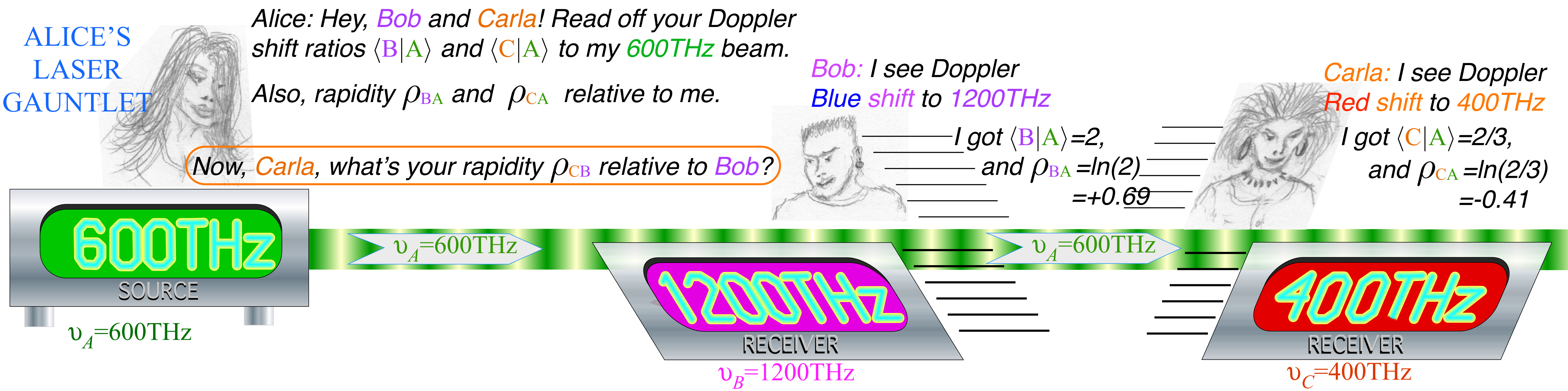
$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

Mnemonic: You can think of rapidity ρ_{BA} as "R" for "Romance"… (+) positive on approach, (-) negative on reproach

Do the stars hate us?

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{2}{1}$$

is time-reversal of:

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B} = \frac{1}{2}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

(time-reversed)

$$\rho_{BA} = 0.69 \qquad (so{:}\ \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

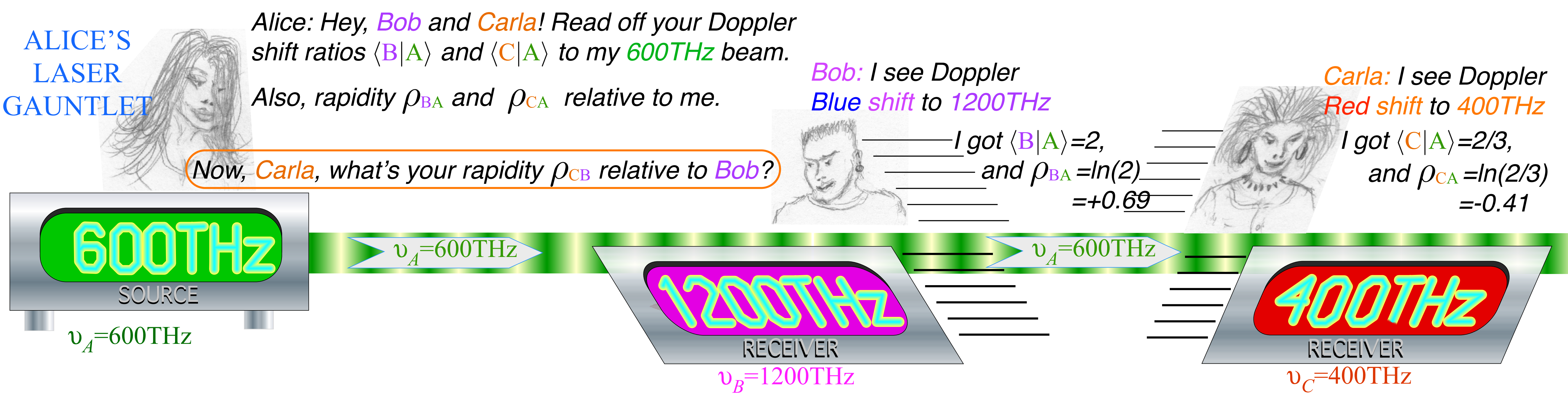
$$\rho_{CA} = -0.41$$

Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Mnemonic: You can think of rapidity ρ_{BA} as "R" for "Romance"… (+) positive on approach, (-) negative on reproach

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

so:

$$\langle R|S\rangle = e^{\rho_{RS}}$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \qquad (so{:}\rho_{AB}=-0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

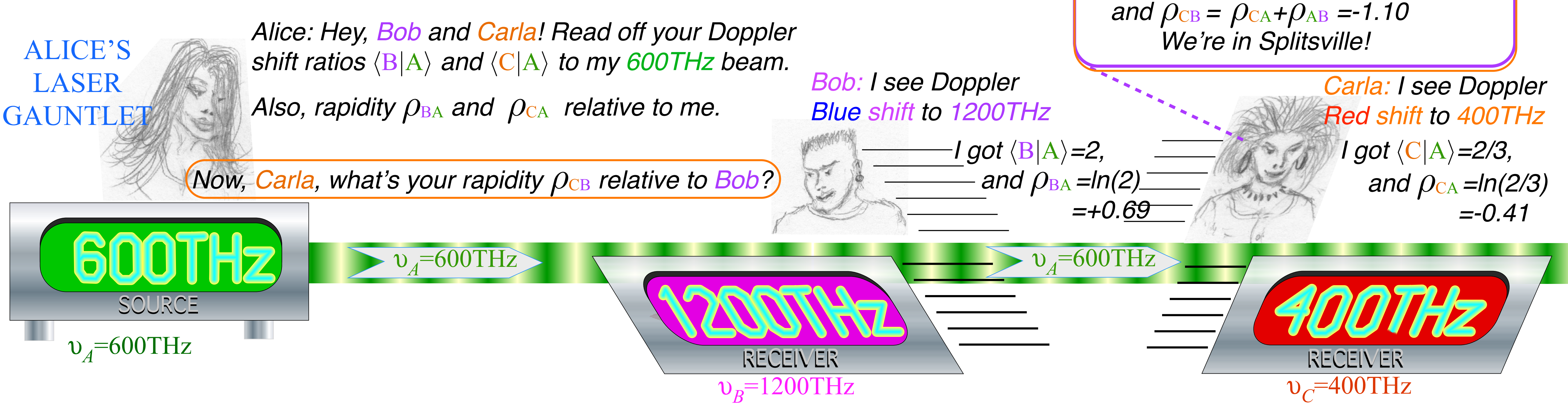
Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \qquad (so: \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

implies: $\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10$

Easy Doppler-shift and Rapidity calculation

ALICE'S LASER GAUNTLET

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ to my 600THz beam.

Also, rapidity ρ_{BA} and ρ_{CA} relative to me.

Now, Carla, what's your rapidity ρ_{CB} relative to Bob?

Bob: I see Doppler Blue shift to 1200THz

I got ⟨B|A⟩=2, and ρ_{BA} =ln(2) =+0.69

Carla: I see Doppler Red shift to 400THz

I got ⟨C|A⟩=2/3, and ρ_{CA} =ln(2/3) =-0.41

I got ⟨C|B⟩=⟨C|A⟩⟨A|B⟩=(2/3)(1/2)=1/3, and $\rho_{CB} = \rho_{CA}+\rho_{AB}$ =-1.10 We're in Splitsville!

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}}$$

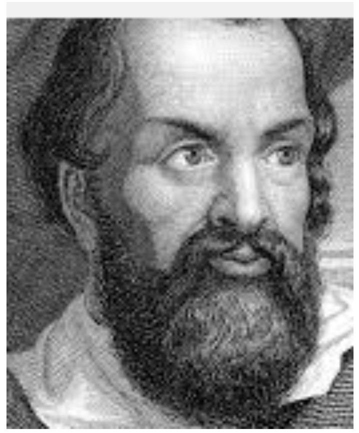
Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Happy now, Galileo?



Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \qquad (so: \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}$$ implies:

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10$$

SPEED
LIMIT
c=
299,792,458
m/s

Level 2 Secrets *(which also shouldn't be secrets!)*

Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

- How do we measure space and time with light waves?

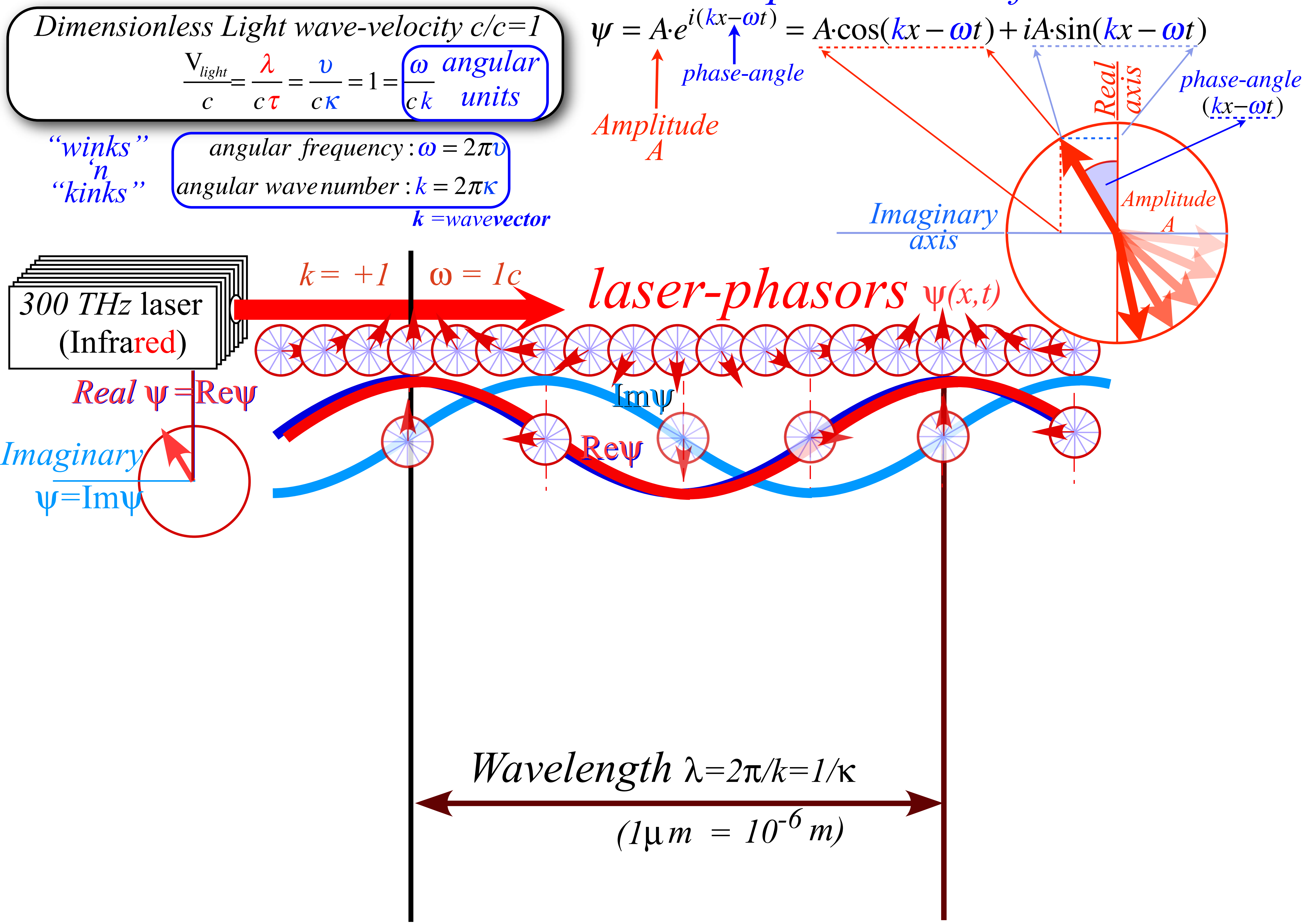
    Use *1CW laser-phasors* for a *phase-based* theory

- How do we make spacetime coordinate graph with light waves?

    Use *2CW laser-phasors* and wave interference geometry

    Get Einstein-Lorentz-Minkowski graphs for free!

1CW Laser-phasor wave function



1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck} \quad \textit{angular units}$$

"winks" 'n "kinks"

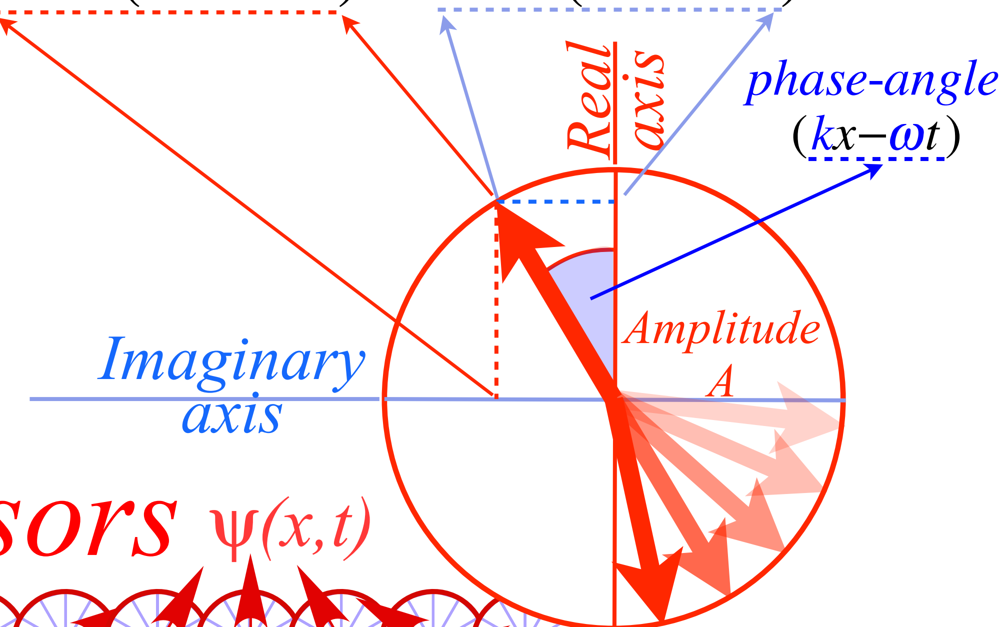
angular frequency: $\omega = 2\pi\upsilon$

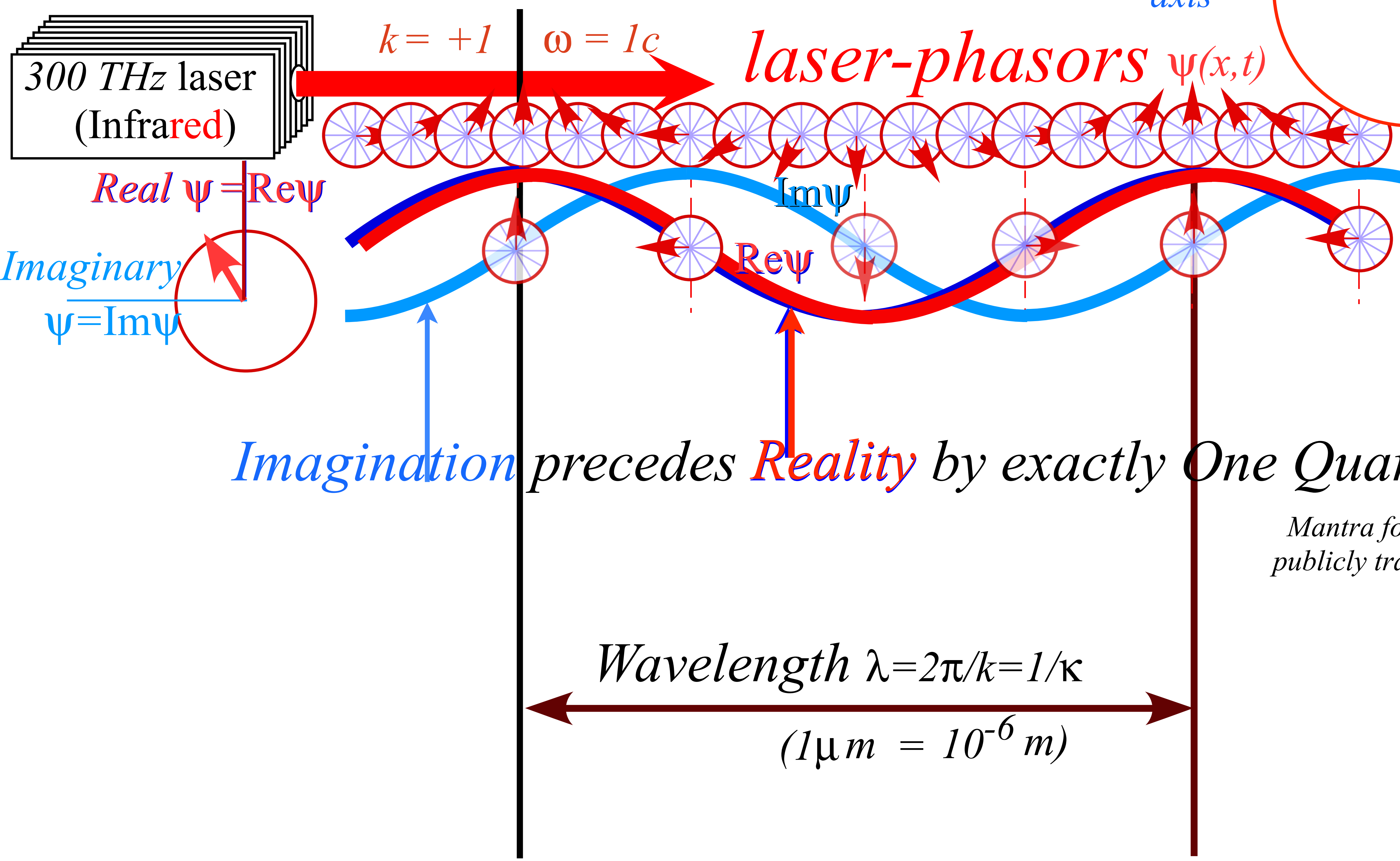
angular wavenumber: $k = 2\pi\kappa$

k = wave**vector**

$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Amplitude A — phase-angle





Mantra for most of the US publicly traded corporations

1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cos(kx-\omega t) + iA\sin(kx-\omega t)$$

Amplitude A — phase-angle

Dimensionless Light wave-velocity c/c=1

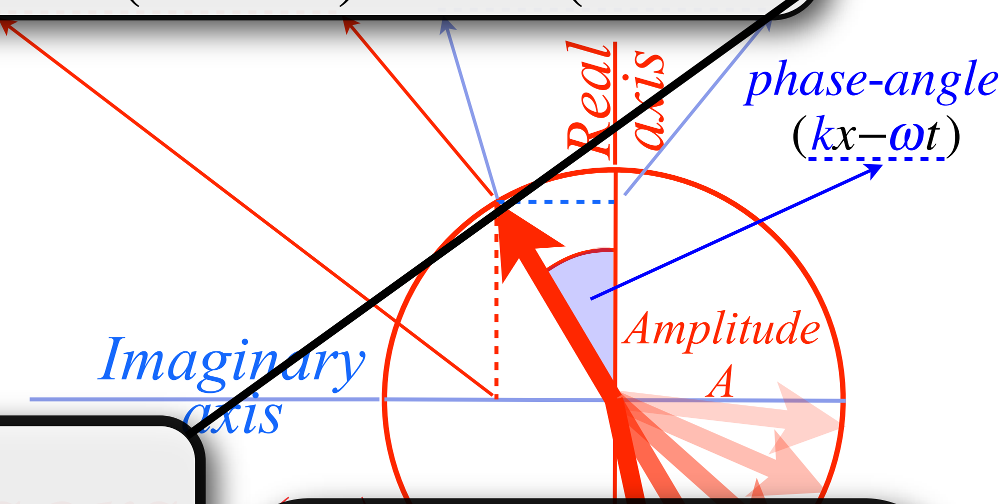
$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ angular units

"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

k = wave**vector**

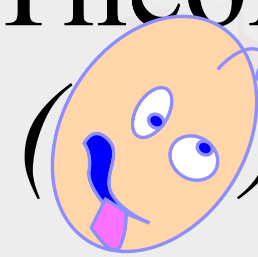


300 THz laser (Infrared)

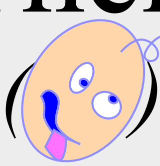
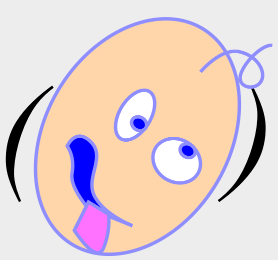
Real ψ = Reψ

Imaginary ψ=Imψ

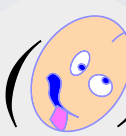
The Crazy-Thing Theorem:

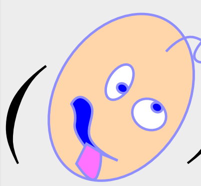
If ()^2 = **-1**

Then:

e^( a) = **1**cos*a* + ()sin*a*

Examples of Crazy Things

() $= i = \sqrt{-1}$

() $= \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$

(one of four Hamilton quaternions)

1CW Laser-phasor wave function

$\psi = A \cdot e^{i(kx-\omega t)} = A\cos(kx-\omega t) + iA\sin(kx-\omega t)$

phase-angle

Amplitude A

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$

angular units

"winks" n "kinks"

angular frequency: $\omega = 2\pi\upsilon$

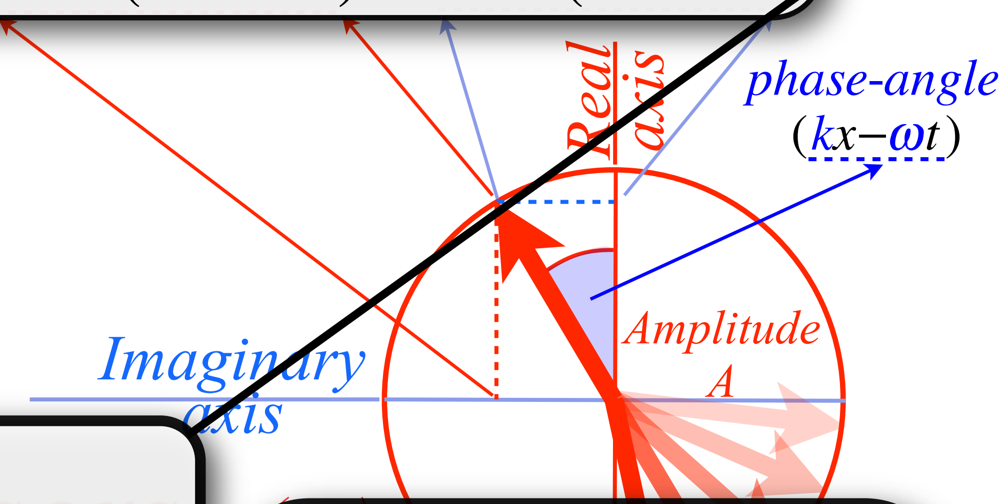
angular wavenumber: $k = 2\pi\kappa$

k = *wavevector*

300 THz laser (Infrared)

Real ψ = Reψ

Imaginary ψ=Imψ



The Crazy-Thing Theorem:

If $(\;)^2 = -1$

Then:

$e^{(\;)a} = \mathbf{1}\cos a + (\;)\sin a$

and even crazier thing:

$e^{(i\;)a} = \mathbf{1}\cosh a + (i\;)\sinh a$

Examples of Crazy Things

$(\;) = i = \sqrt{-1}$

$(\;) = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$

(one of four Hamilton quaternions)

even crazier thing

$(i\;) = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}$

is *Pauli matrix* $\boldsymbol{\sigma}_y$

(one of three)

1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

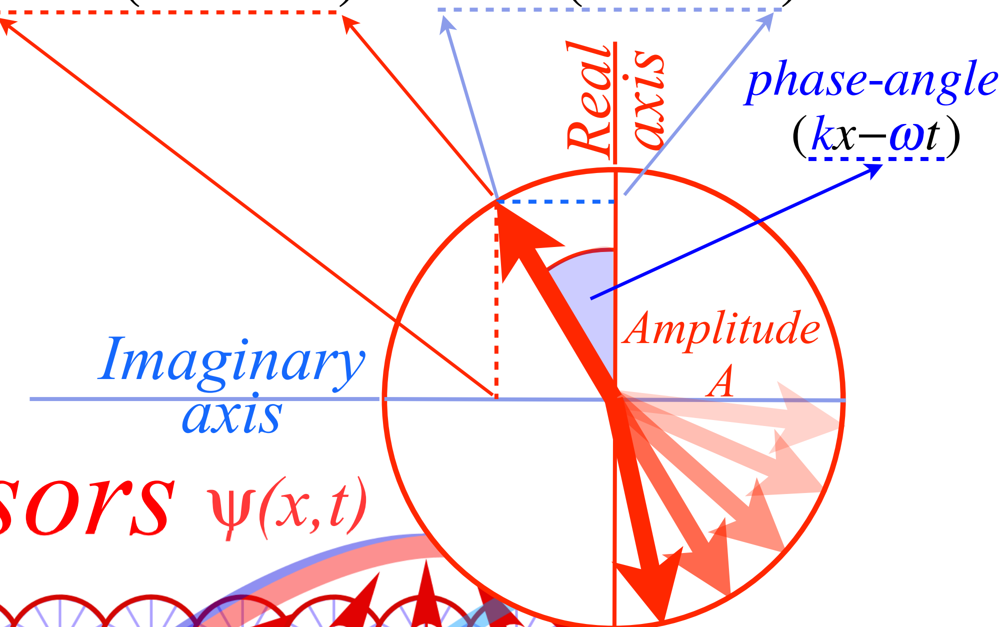
Amplitude A — phase-angle

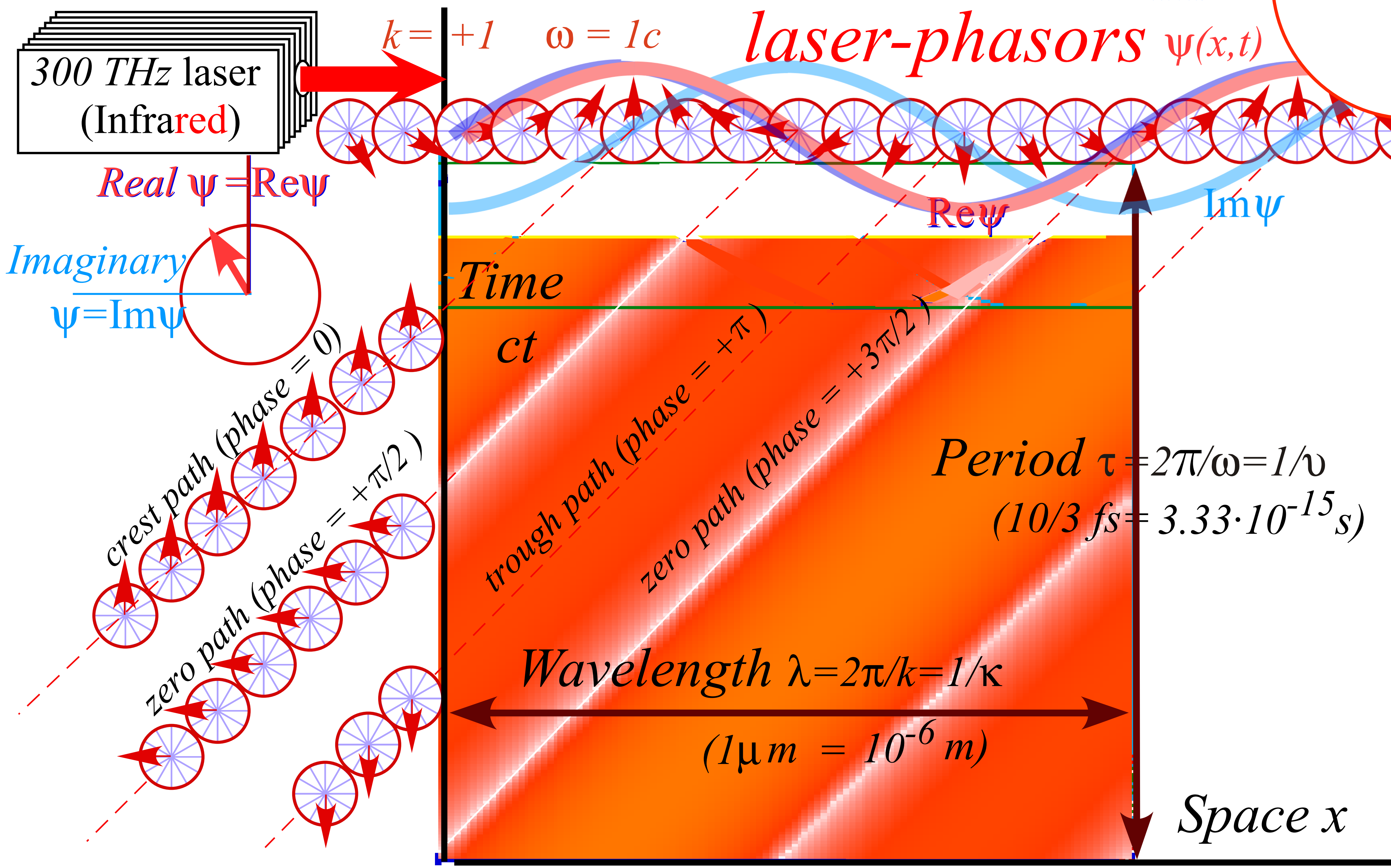
Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck} \text{ angular units}$$

Q: Where is phase=($kx-\omega t$)=0?

A: It is wherever this is: $\frac{x}{t} = \frac{\omega}{k}$





1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Dimensionless Light wave-velocity c/c=1

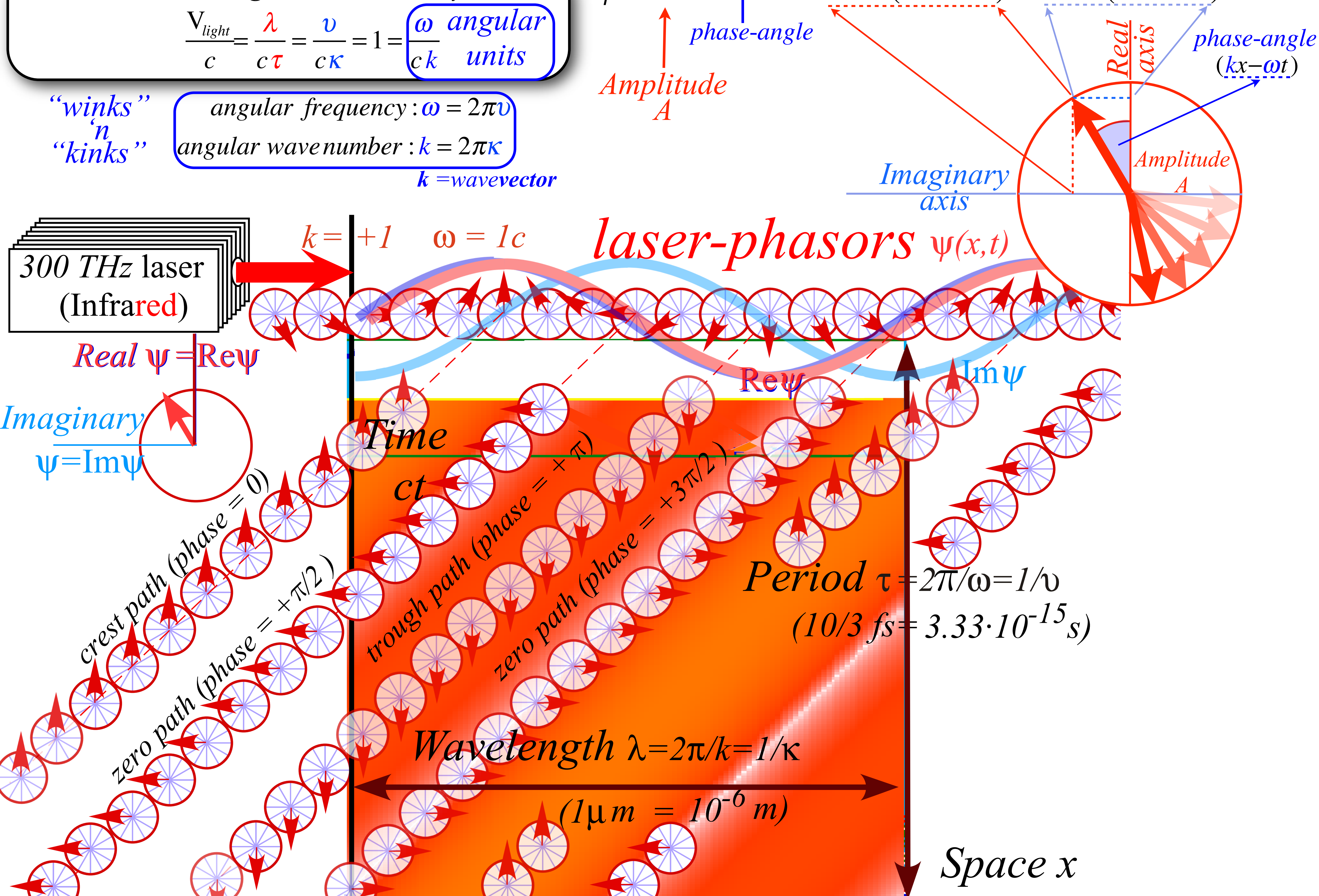
$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ angular units

"winks" 'n "kinks"

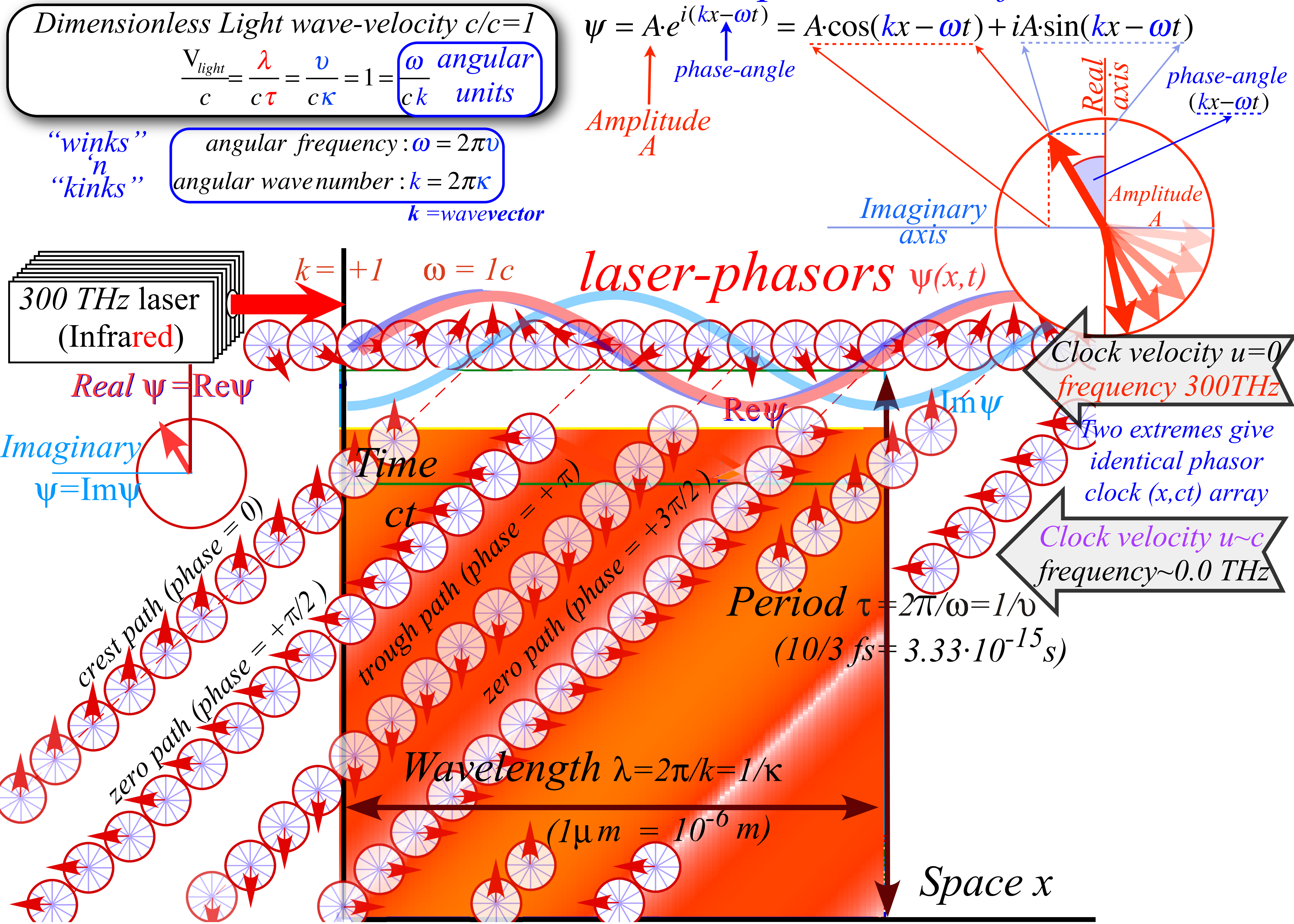
angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

k =wave**vector**



1CW Laser-phasor wave function



1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck} \quad \textit{angular units}$$

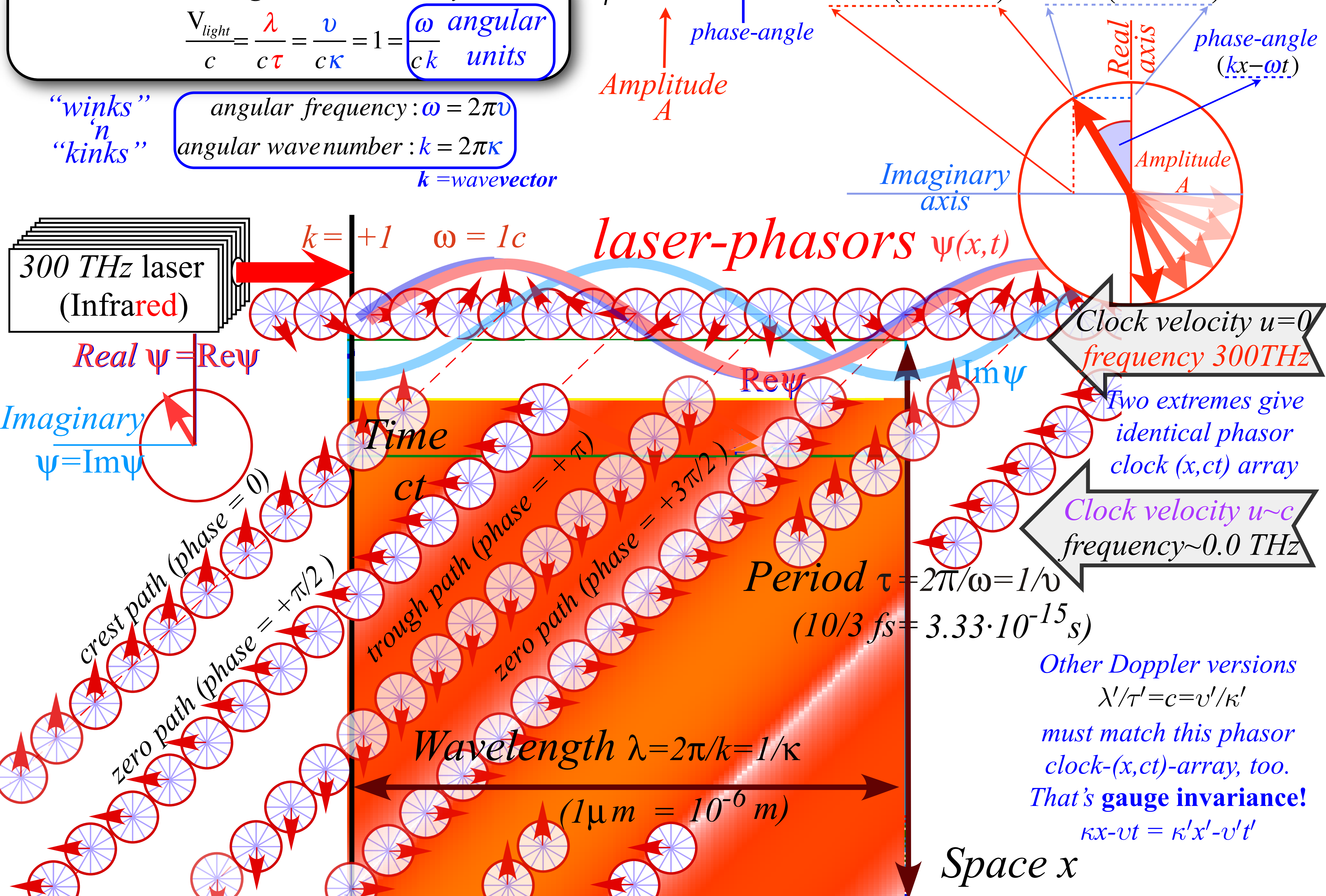
"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

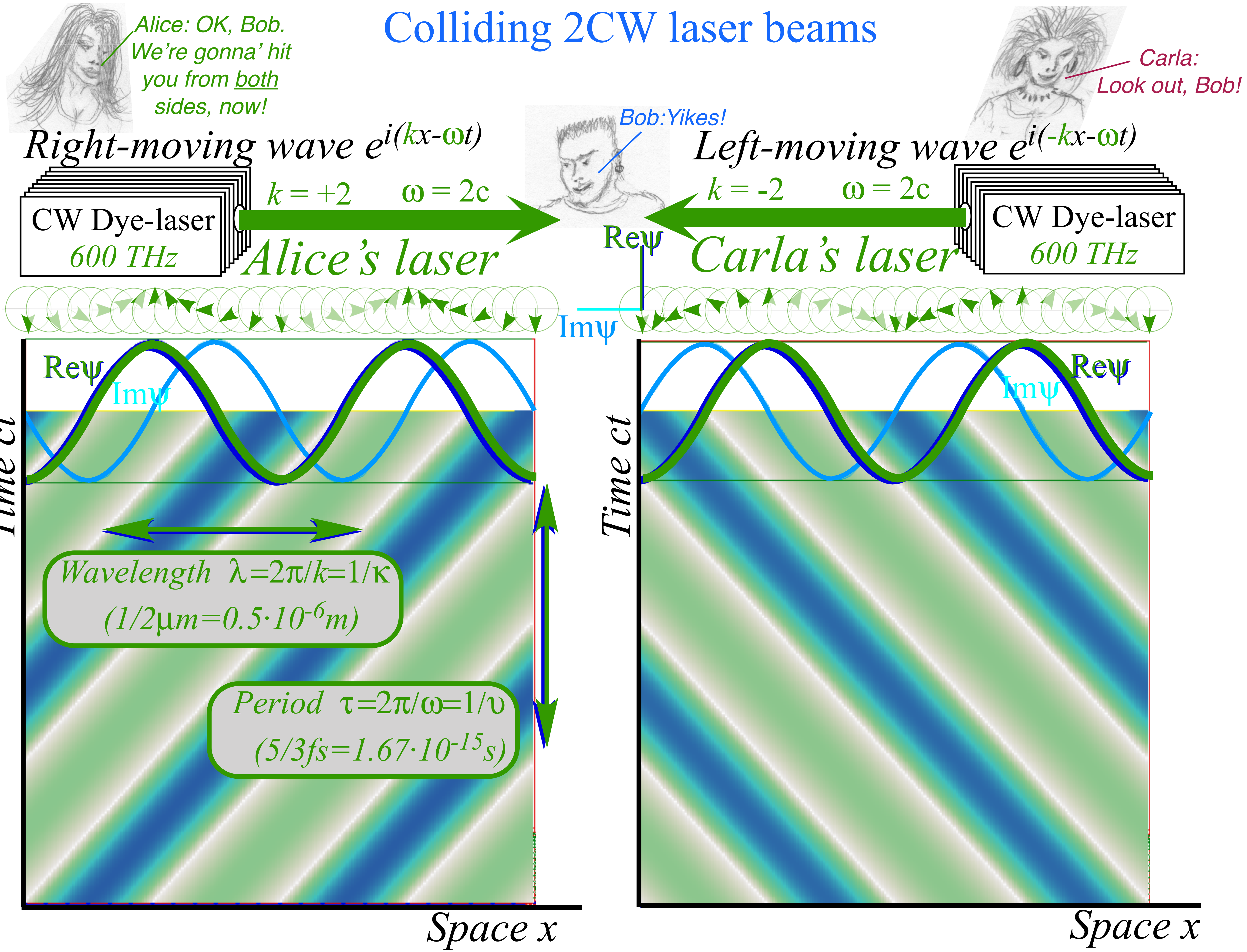
$\mathbf{k}$ = wave**vector**

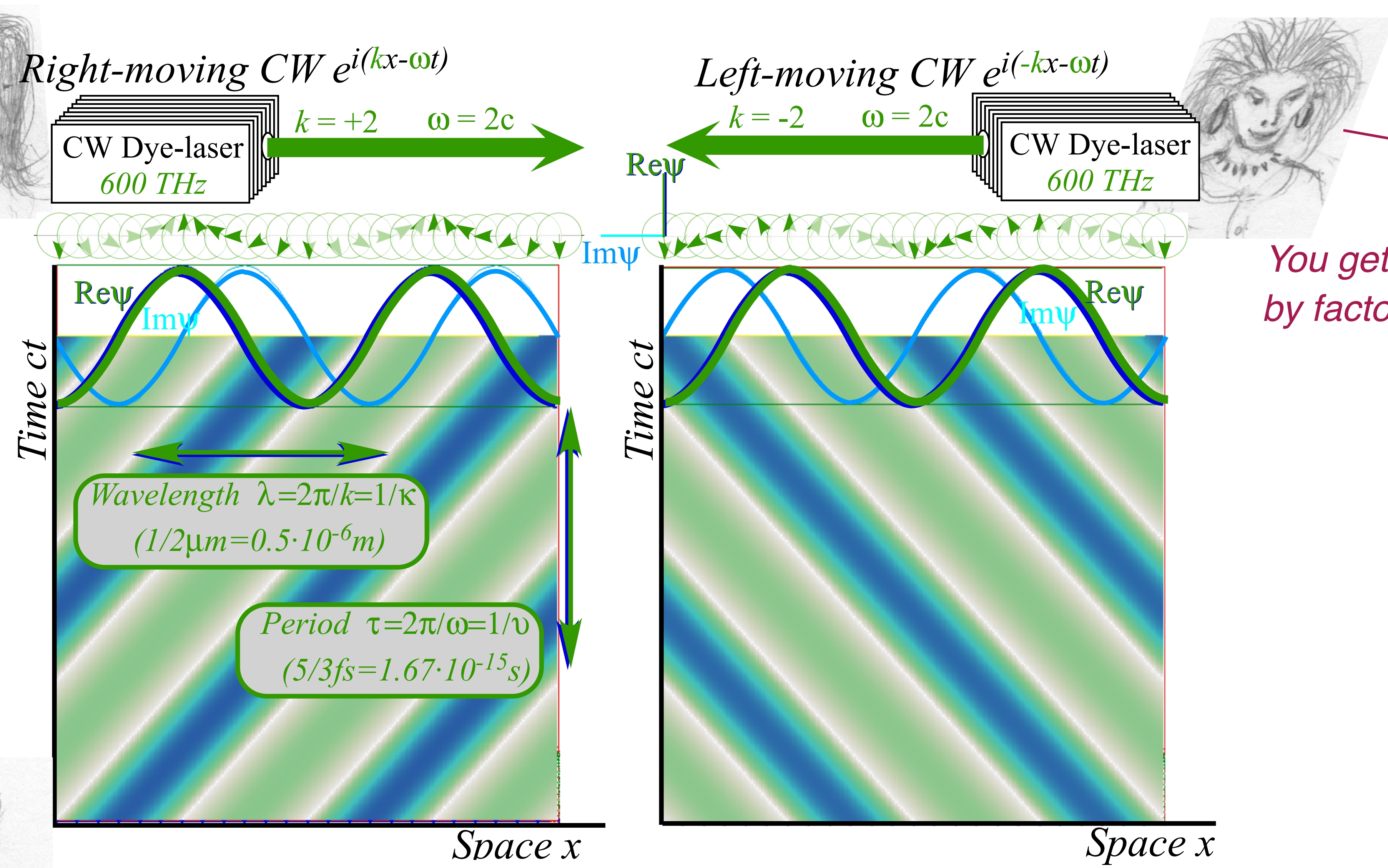
$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$



Other Doppler versions $\lambda'/\tau' = c = \upsilon'/\kappa'$ must match this phasor clock-(x,ct)-array, too. That's **gauge invariance!** $\kappa x - \upsilon t = \kappa' x' - \upsilon' t'$

Colliding 2CW laser beams





Carla:

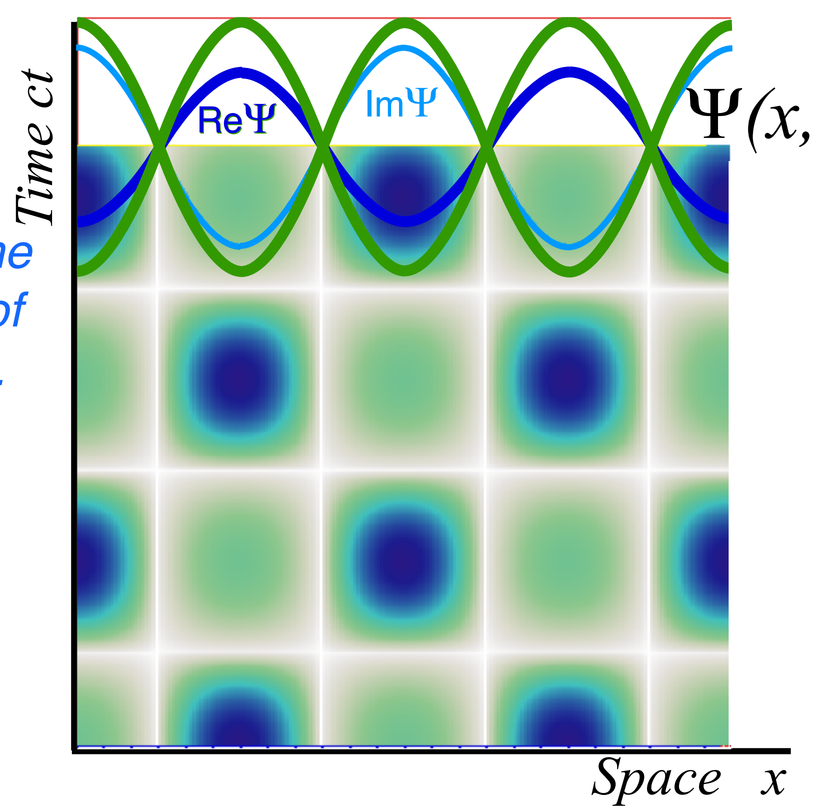
Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

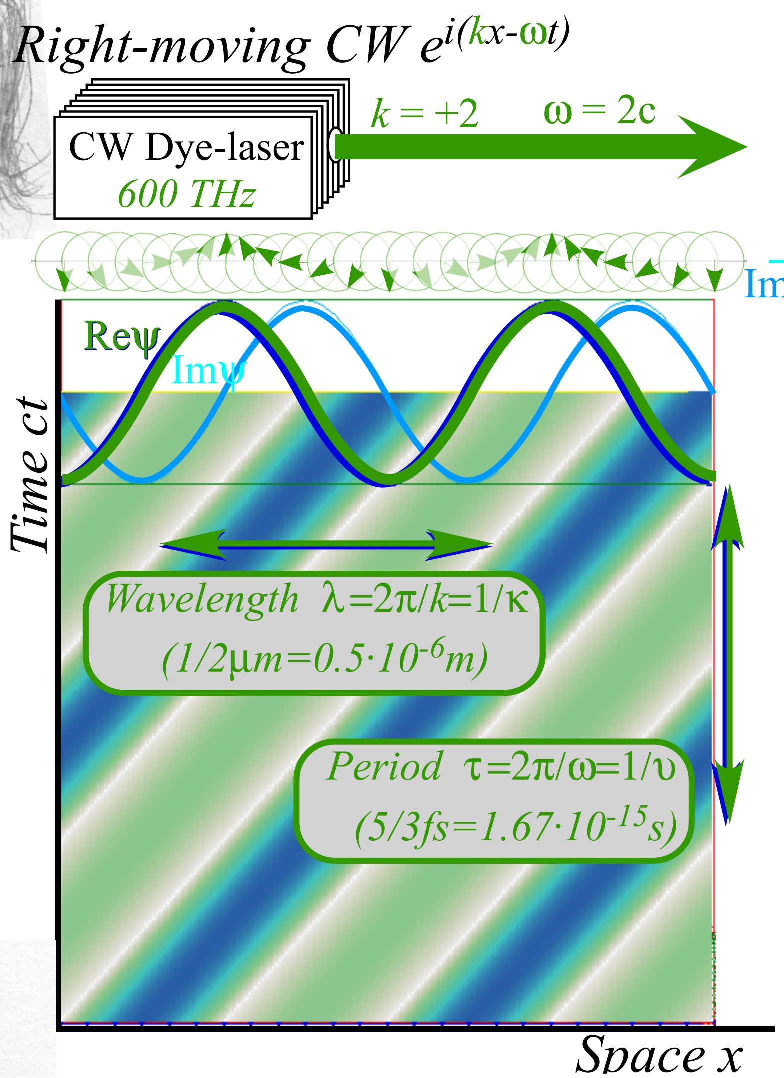
Bob:

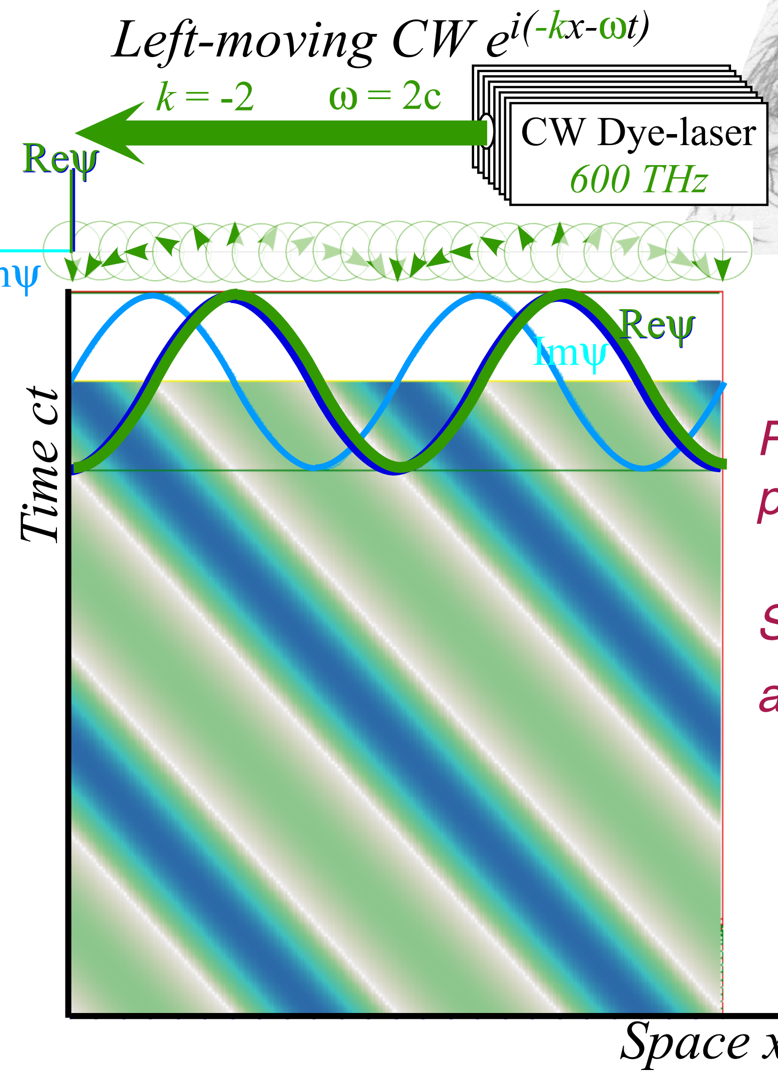
Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



$$\Psi(x,t)=e^{i\overbrace{a}^{kx-\omega t}}+e^{i\overbrace{b}^{-kx-\omega t}}$$





Carla:

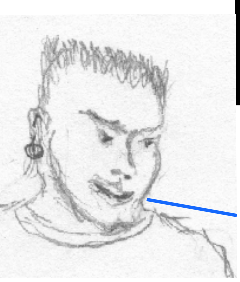
Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,

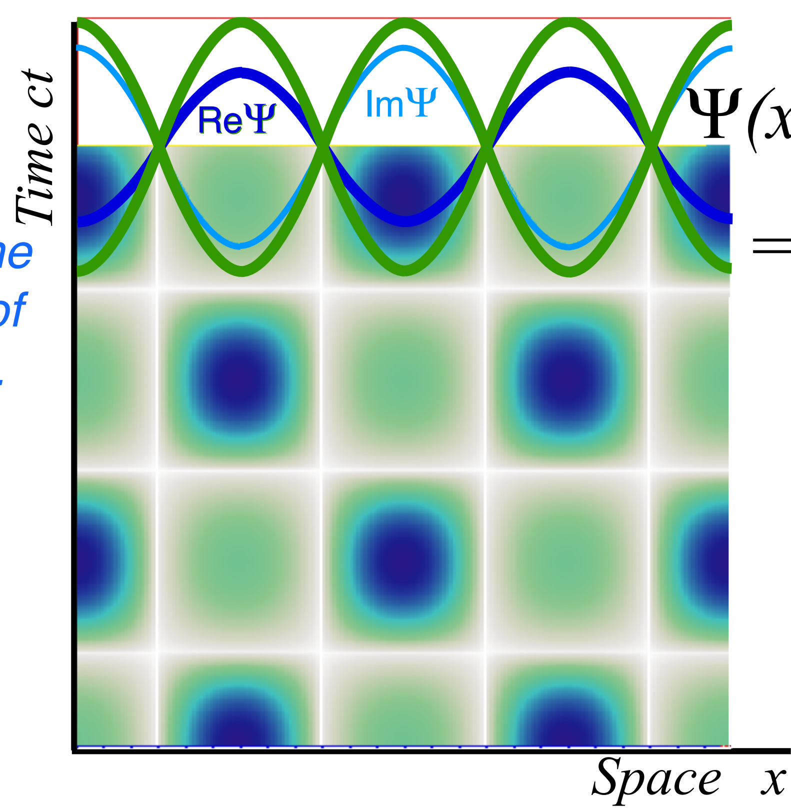
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.



Bob:

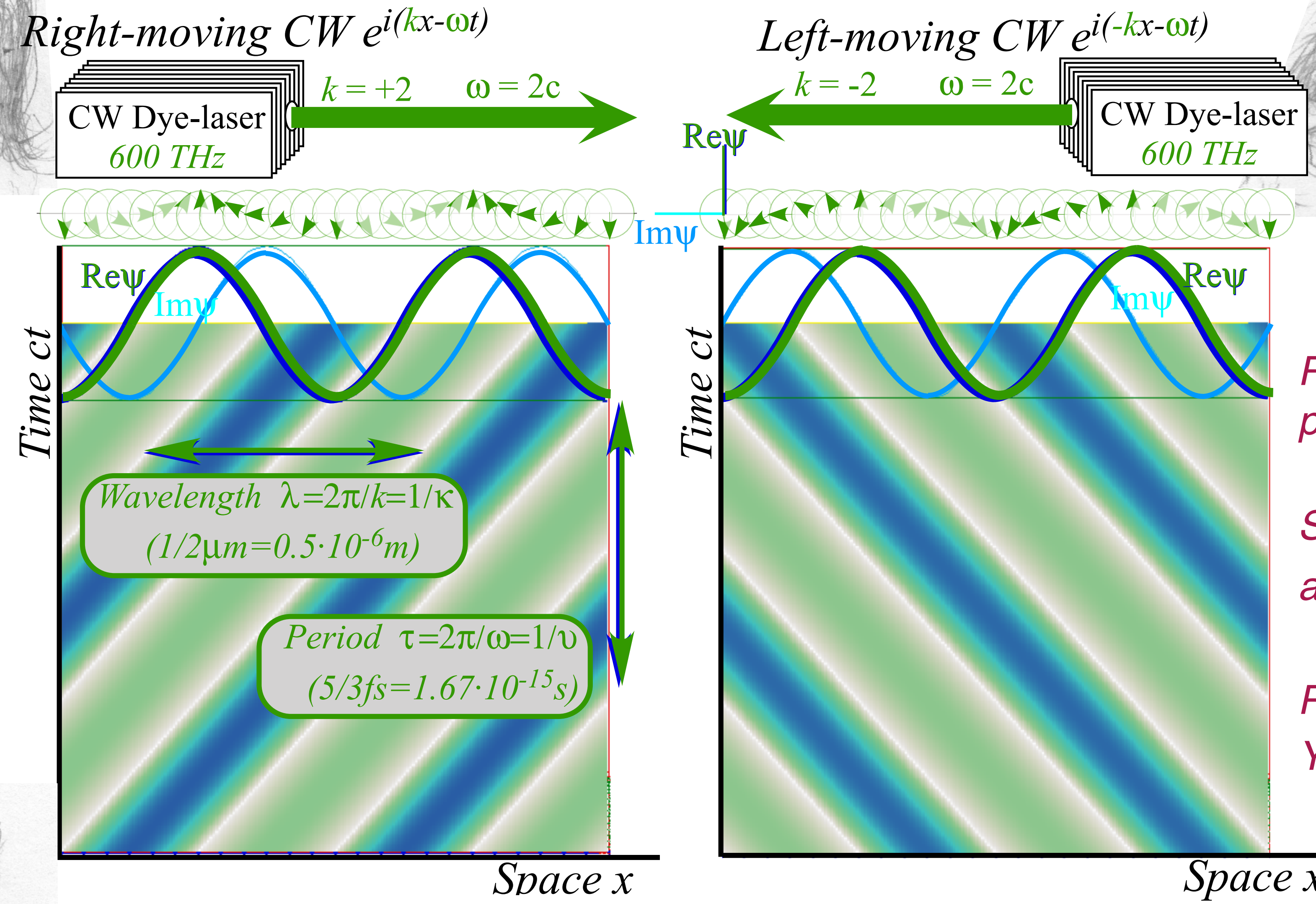
Cool! You guys made me a space-time graph out of real zeros.

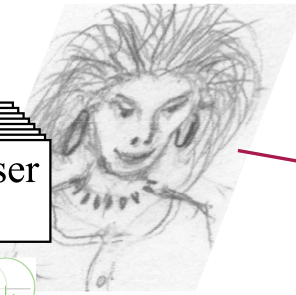
How'd it **do** that?



$$\Psi(x,t)=e^{ia}+e^{ib}\quad (a=kx-\omega t,\; b=-kx-\omega t)$$

$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$





Carla:

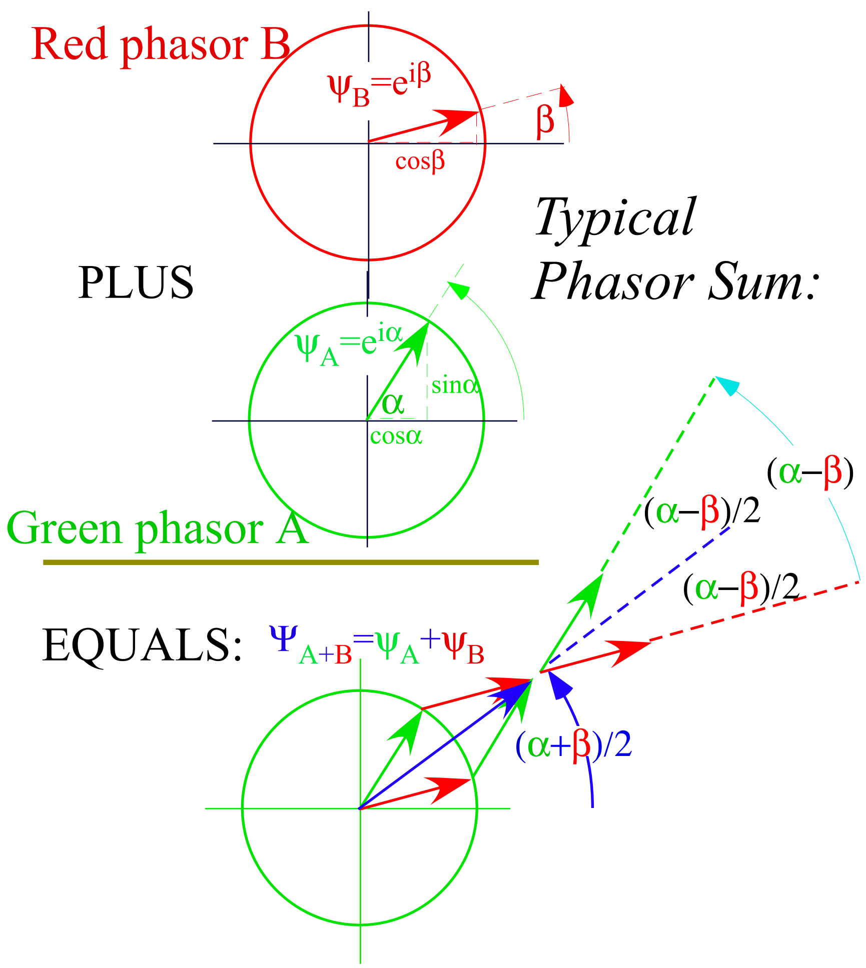
Easy!

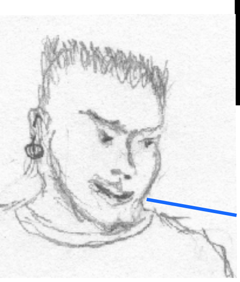
You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

Presto!
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

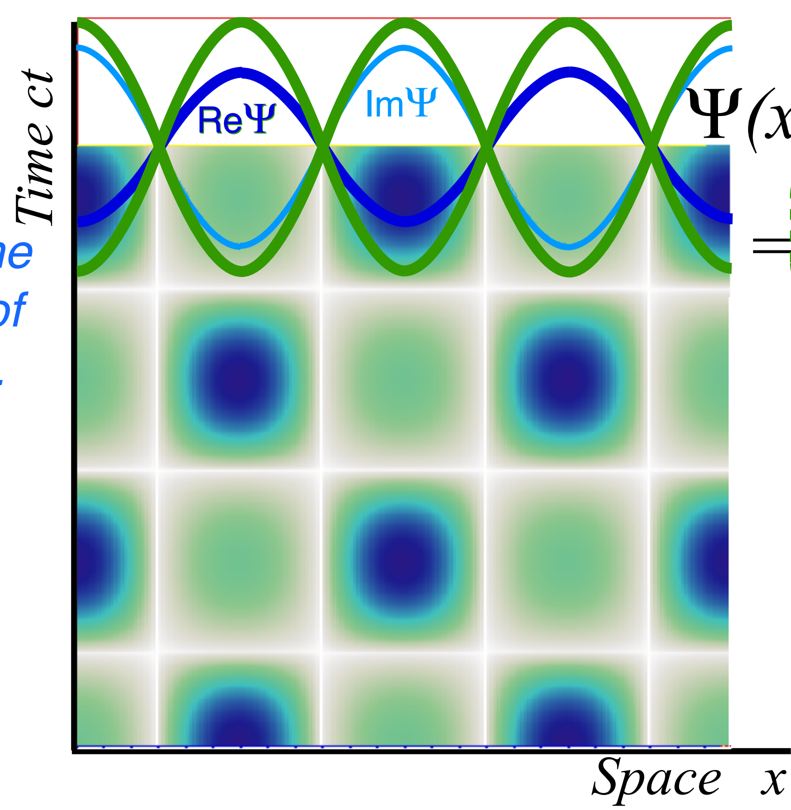




Bob:

Cool!
You guys made me a space-time graph out of real zeros.

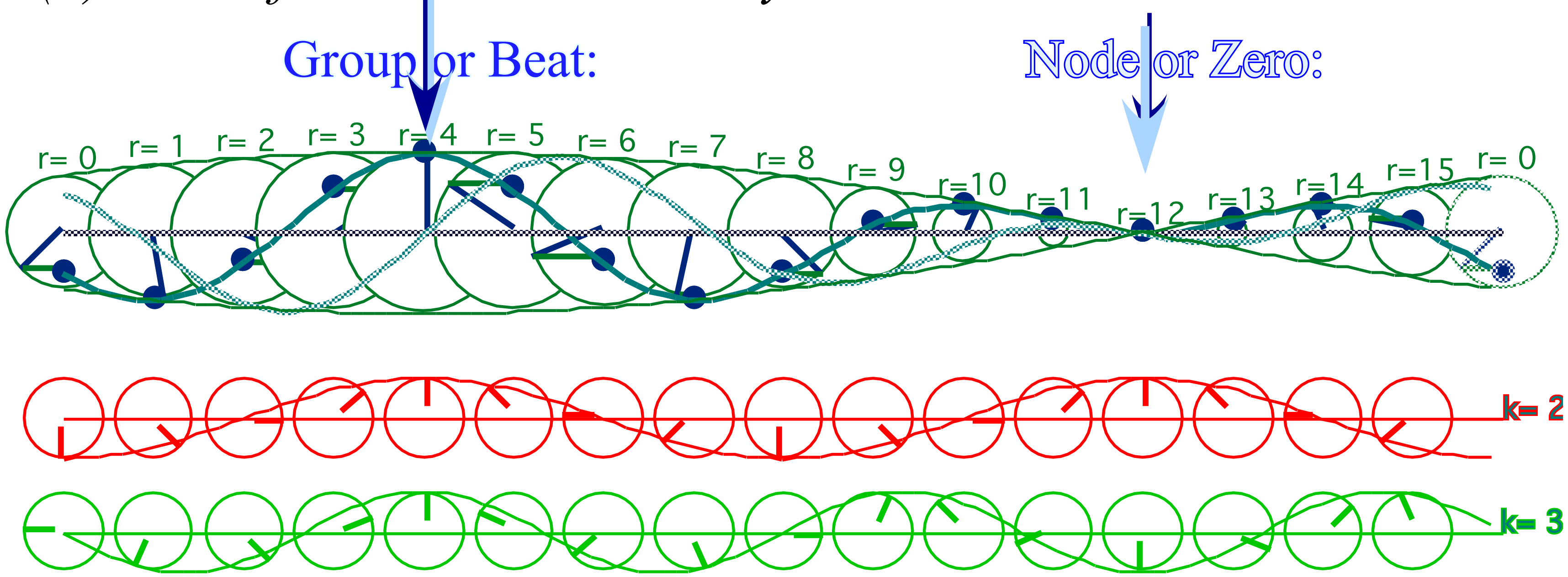
How'd it **do** that?



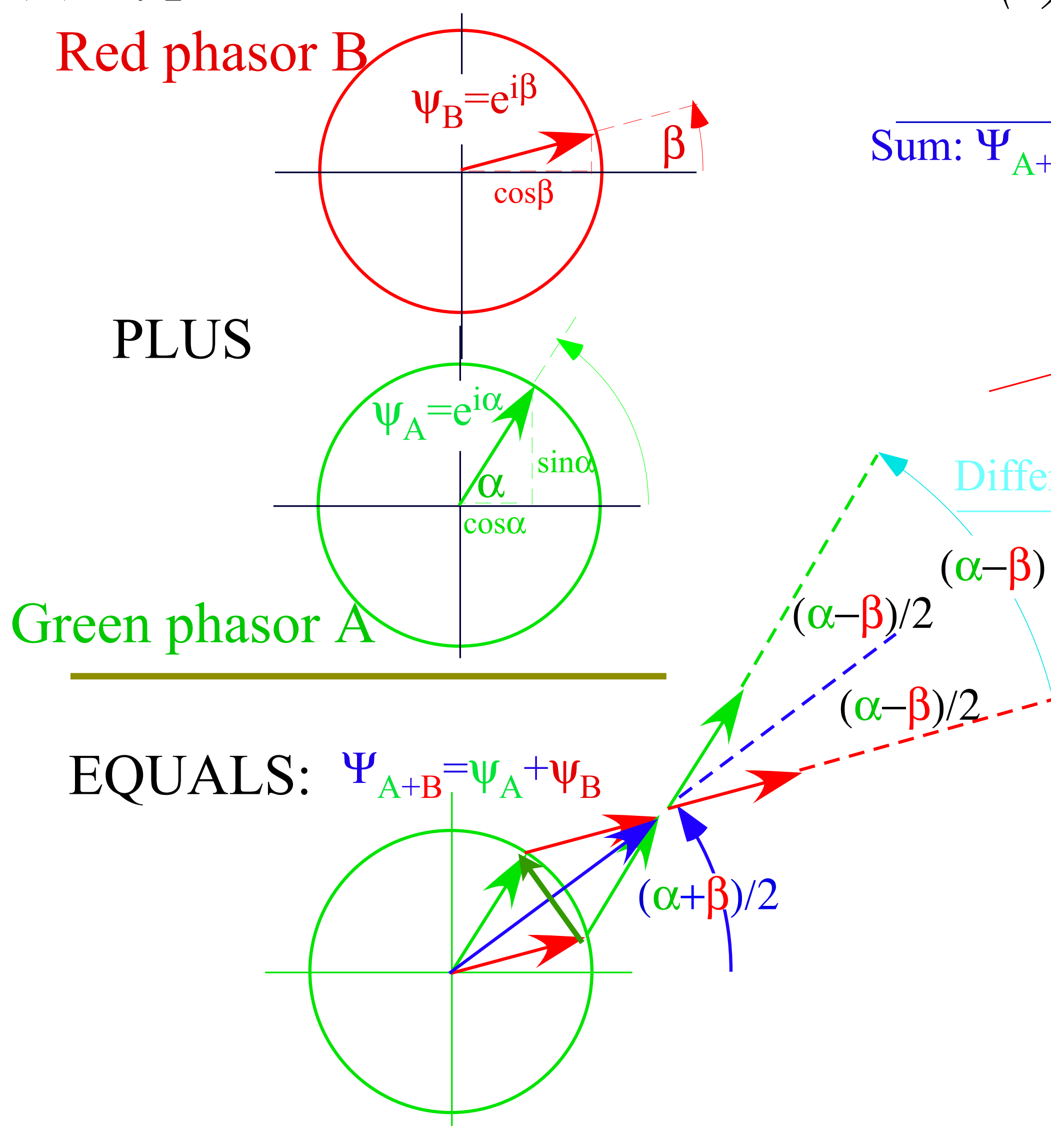
$$\Psi(x,t)=e^{ia}+e^{ib}=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

where $a = kx-\omega t$ and $b = -kx-\omega t$.

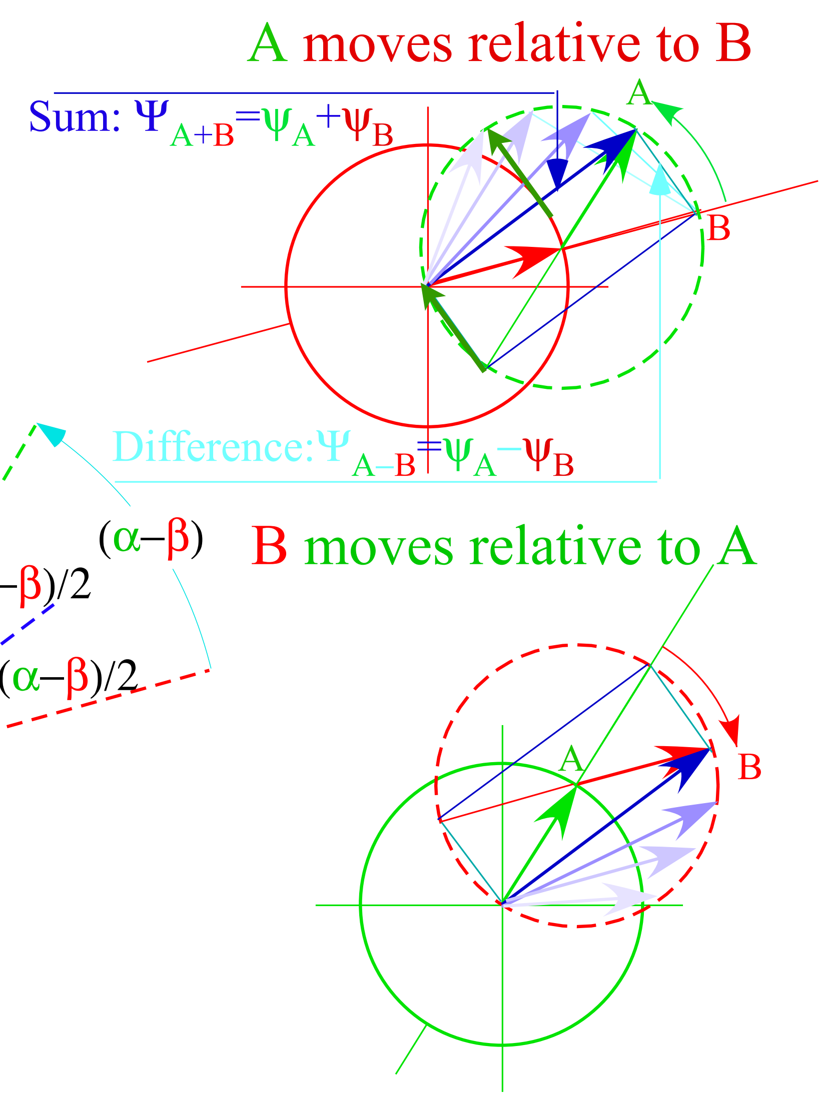
(a) Sum of Wave Phasor Array



(b) Typical Phasor Sum:

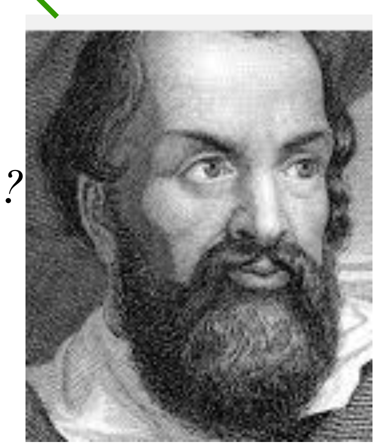


(c) Phasor-relative views

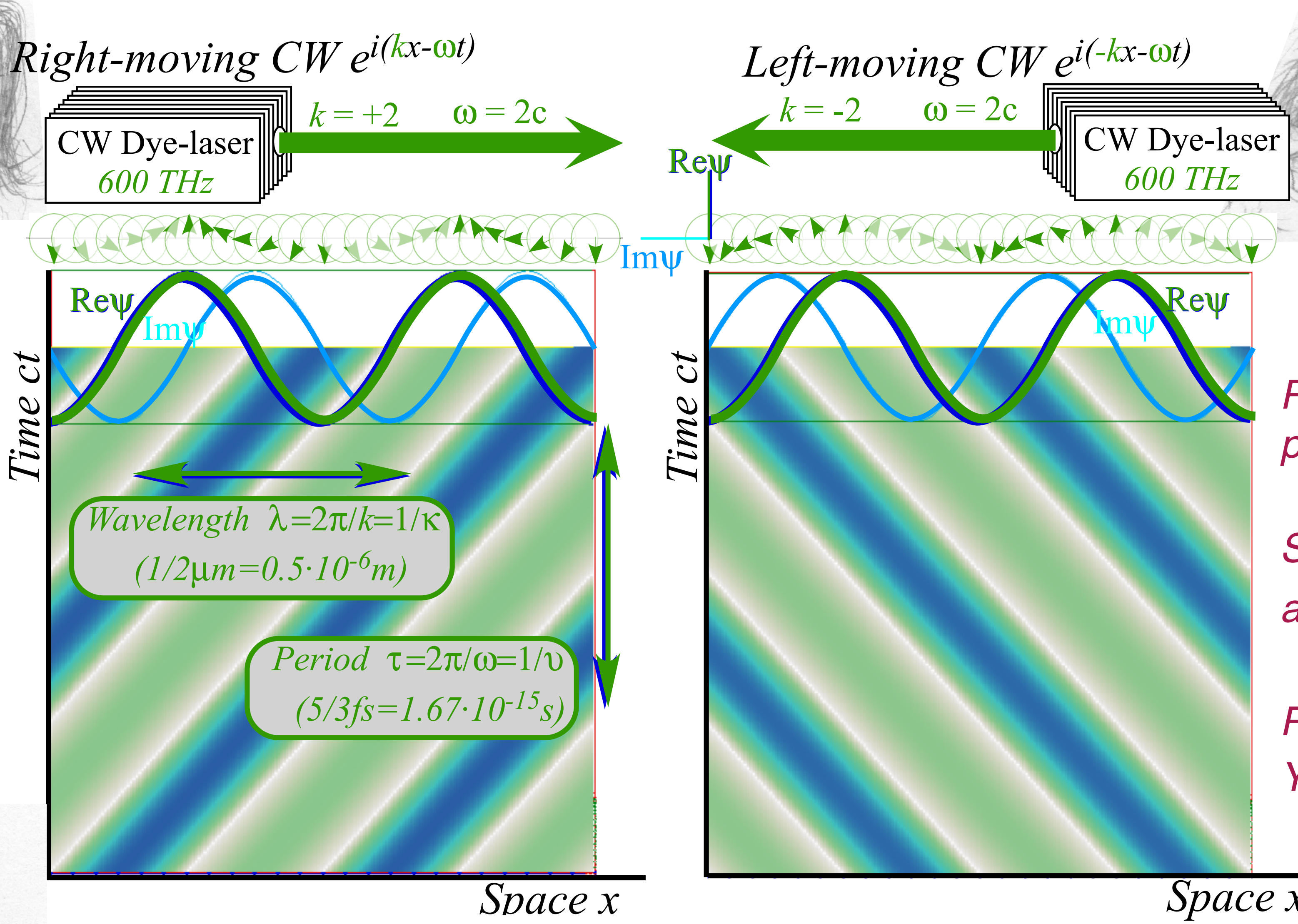


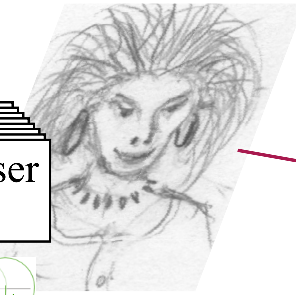
Geometry of the
Half-sum
Phase
and
Half-difference
Group

Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity





Carla:

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

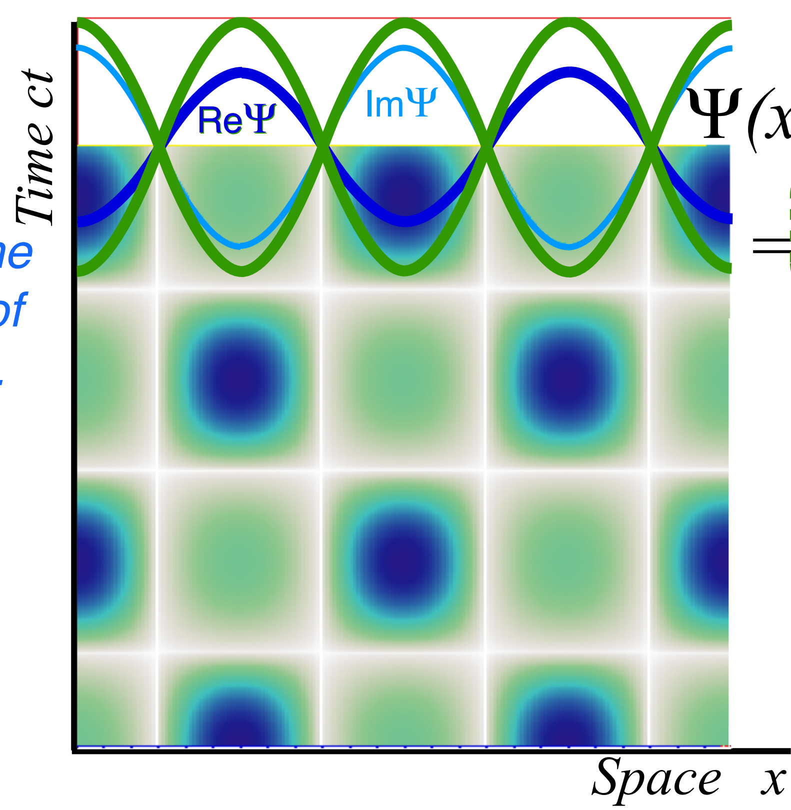
Presto!
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

Alice 1CW phase: $a = kx - \omega t$
Carla 1CW phase: $b = -kx - \omega t$

Bob:

Cool!
You guys made me a space-time graph out of real zeros.

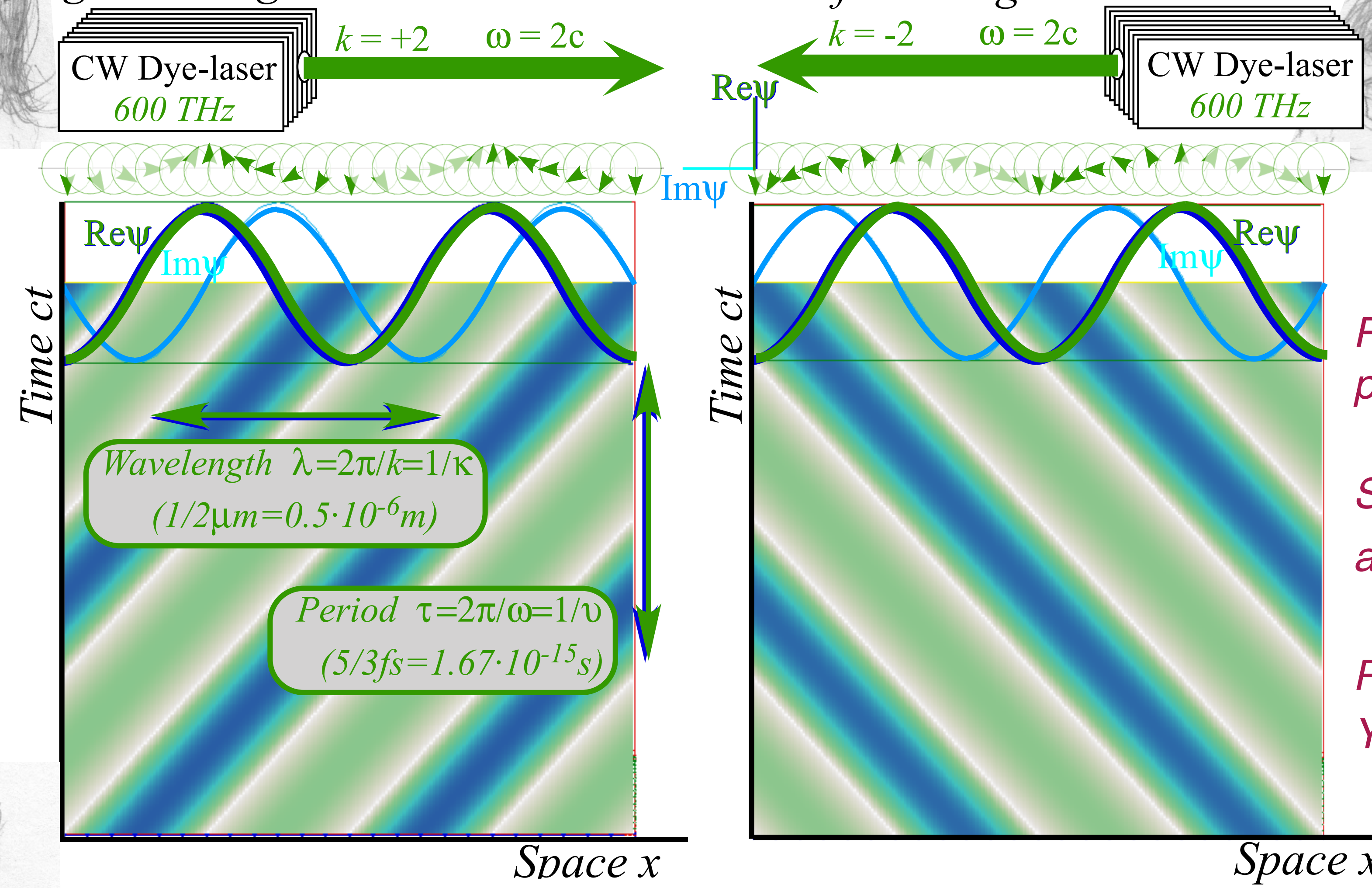
How'd it **do** that?



$$\Psi(x,t) = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

with $a = kx-\omega t$, $b = -kx-\omega t$

Right-moving CW $e^{i(kx-\omega t)}$ *Left-moving CW* $e^{i(-kx-\omega t)}$



Carla:

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,

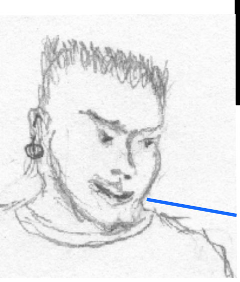
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

Presto!

You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

Alice 1CW phase: $a = kx - \omega t$

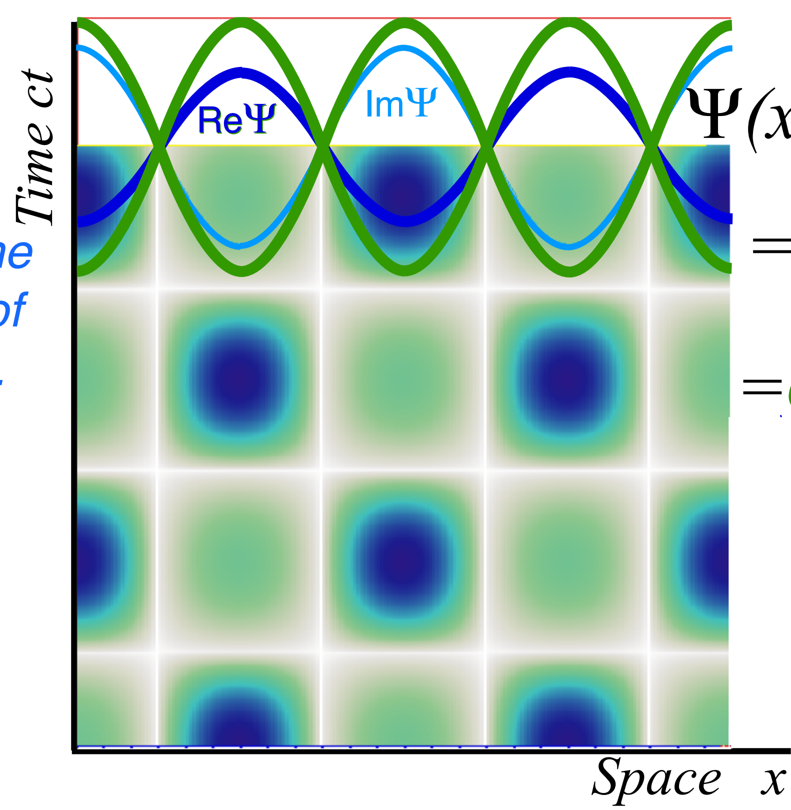
Carla 1CW phase: $b = -kx - \omega t$



Bob:

Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



$$\Psi(x,t) = \underbrace{e^{ia}}_{kx-\omega t} + \underbrace{e^{ib}}_{-kx-\omega t}$$

$$= e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

$$= e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

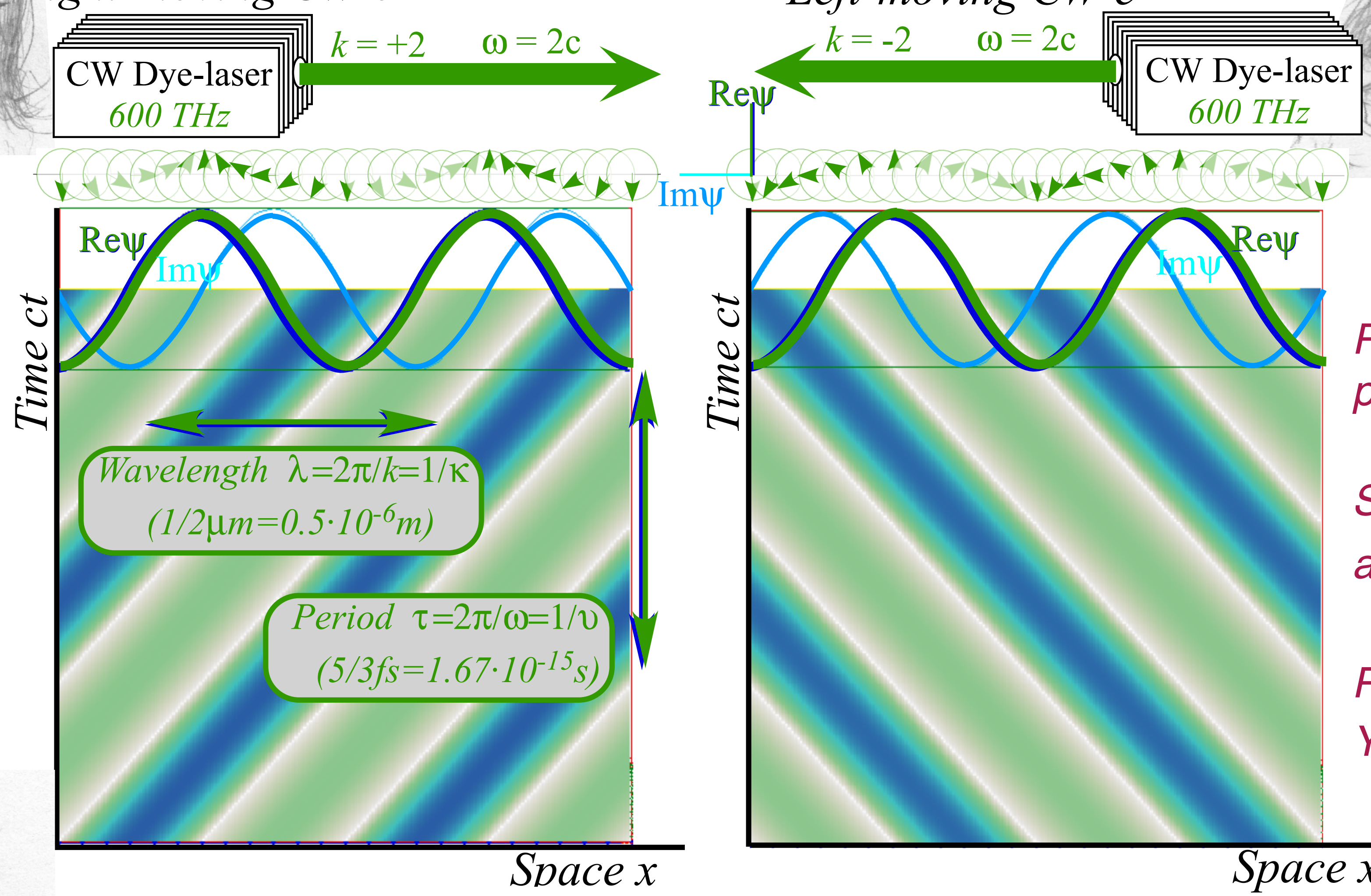
Bob's 2CW **Group**-phase: $+k = \frac{a-b}{2}$
Wave

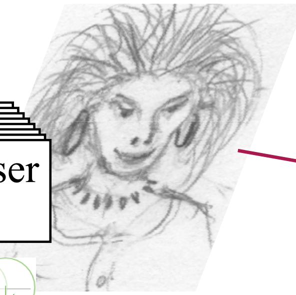
Group wave: $e^{-ikx} + e^{-ikx} = 2\cos kx$

is *standing* wave (does not vary with time t)

Right-moving CW $e^{i(kx-\omega t)}$

Left-moving CW $e^{i(-kx-\omega t)}$





Carla:

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ *by factoring it into phase and group parts.*

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ *plus half-diff* $\frac{a-b}{2}$ *gives* a,
and half-sum $\frac{a+b}{2}$ *minus half-diff* $\frac{a-b}{2}$ *gives* b.

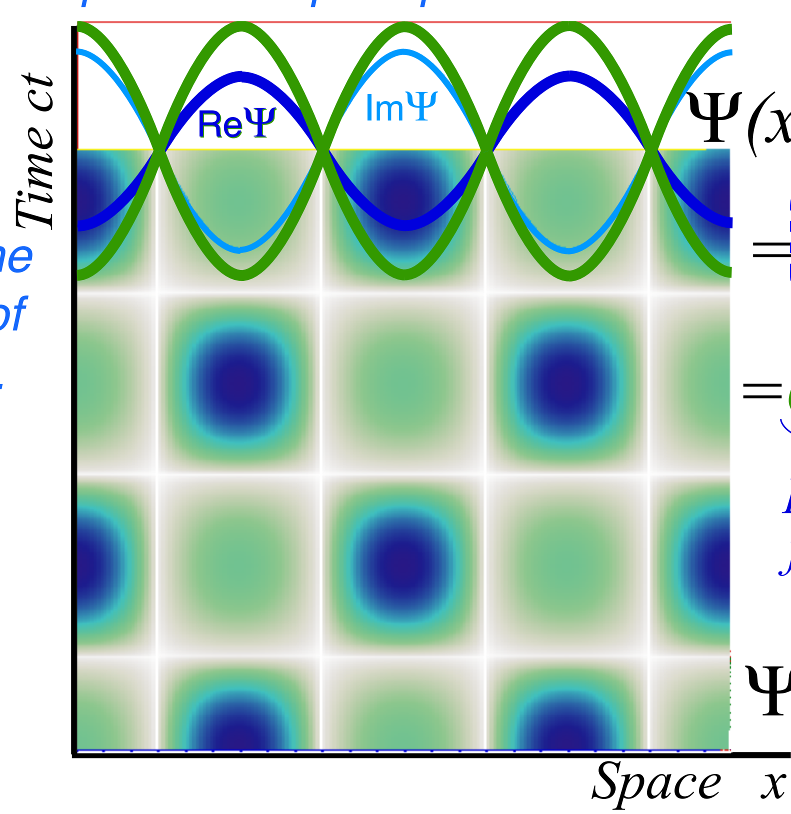
Presto!
You factor $e^{ia}+e^{ib}$ *into* $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

Alice 1CW phase: $a = kx-\omega t$
Carla 1CW phase: $b = -kx-\omega t$

Bob: Let's plot this in per-spacetime?!

Cool! You guys made me a space-time graph out of real zeros.

How'd it do that?



$kx-\omega t \quad -kx-\omega t$

$$\Psi(x,t)=e^{ia}+e^{ib}$$

$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

$$=e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

phase factor   *group factor*

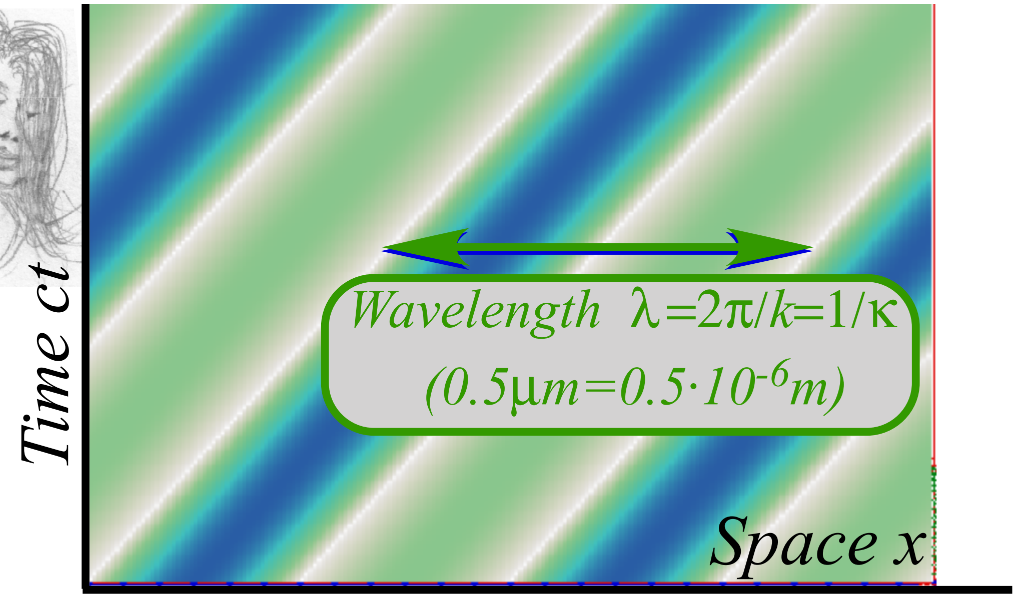
$$\Psi(x,t)=e^{-i\omega t}2\cos kx$$

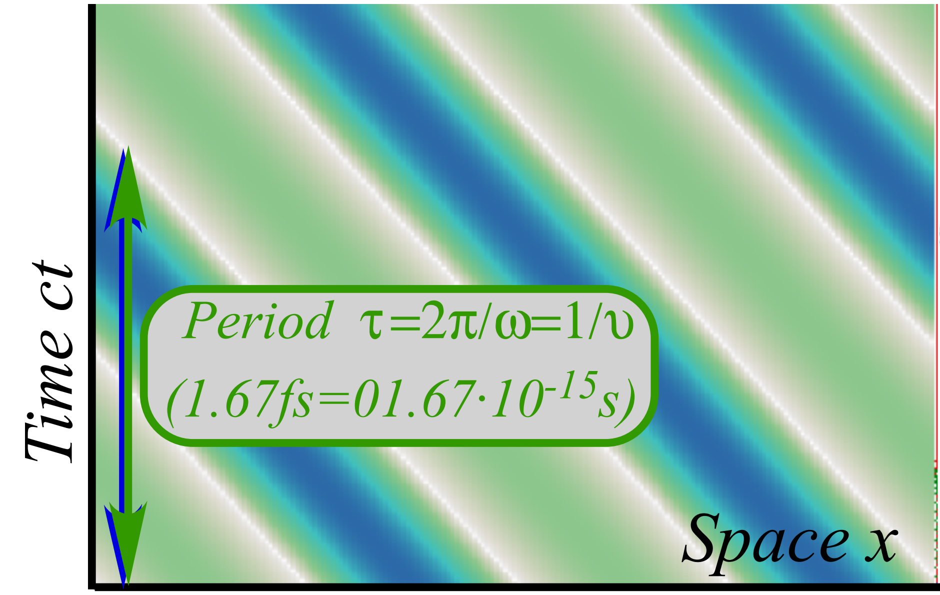
Bob's 2CW **Group**-*phase:* **Wave** $+k=\frac{a-b}{2}$

Group wave: $e^{-ikx}+e^{-ikx}=2\cos kx$
is *standing* wave (does not vary with time t)

Bob's 2CW **Phase**-*phase:* **Wave** $-\omega=\frac{a+b}{2}$

Phase wave real part: $\mathrm{Re}(e^{-i\omega t})=\cos(\omega t)$
is *"instanton"* wave (does not vary in space x)



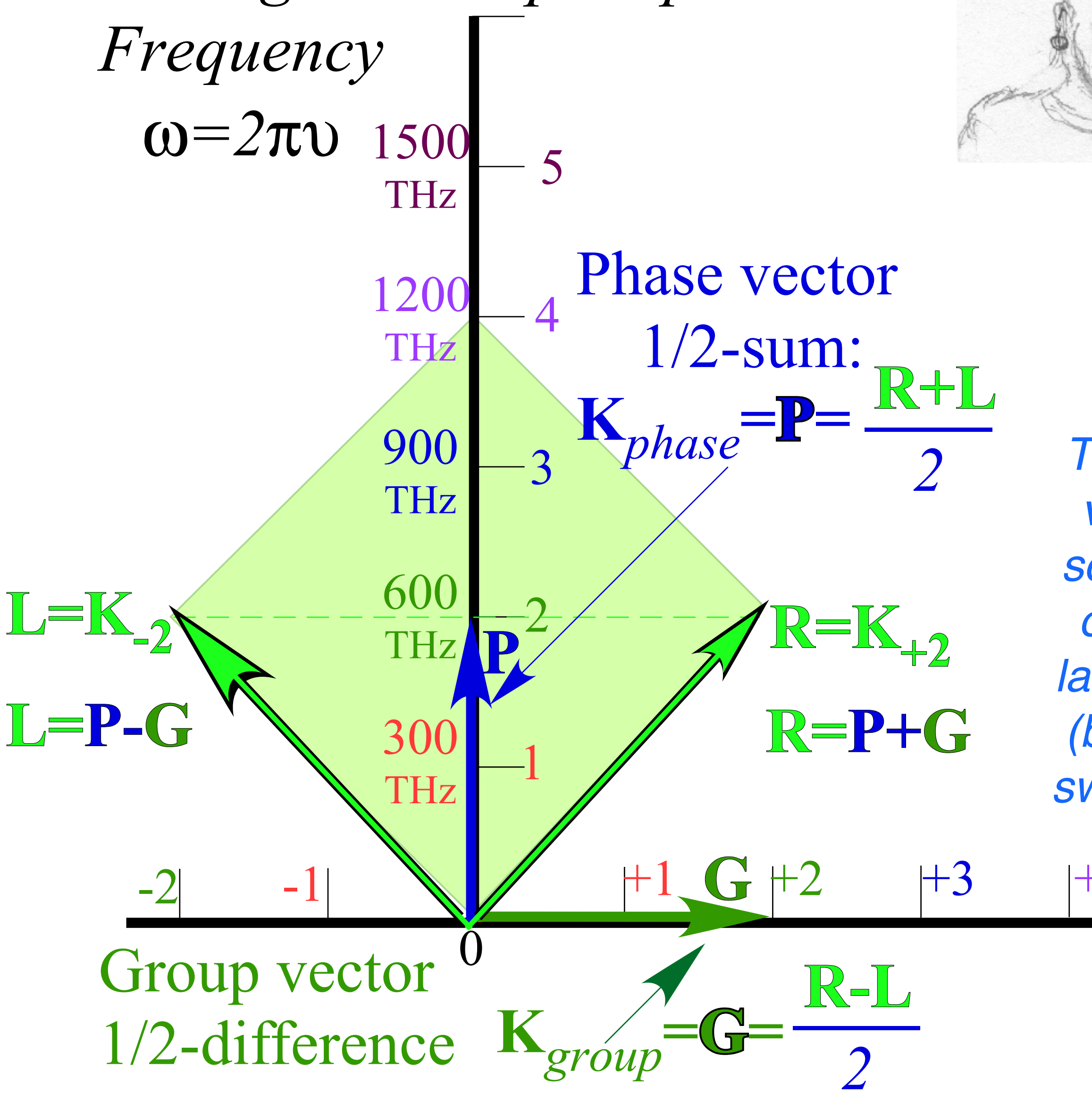


Carla:
OK, Bob!
It looks like a baseball diamond with
P at Pitcher's mound
and
G at the Grandstand*.
I'm on 1st base! (**R**)

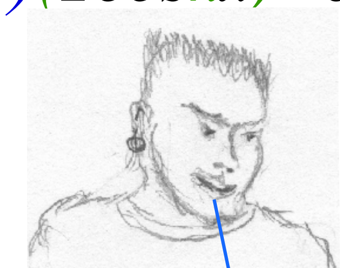
*Thanks, Woody!

$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$

Standing 2CW in per-space-time

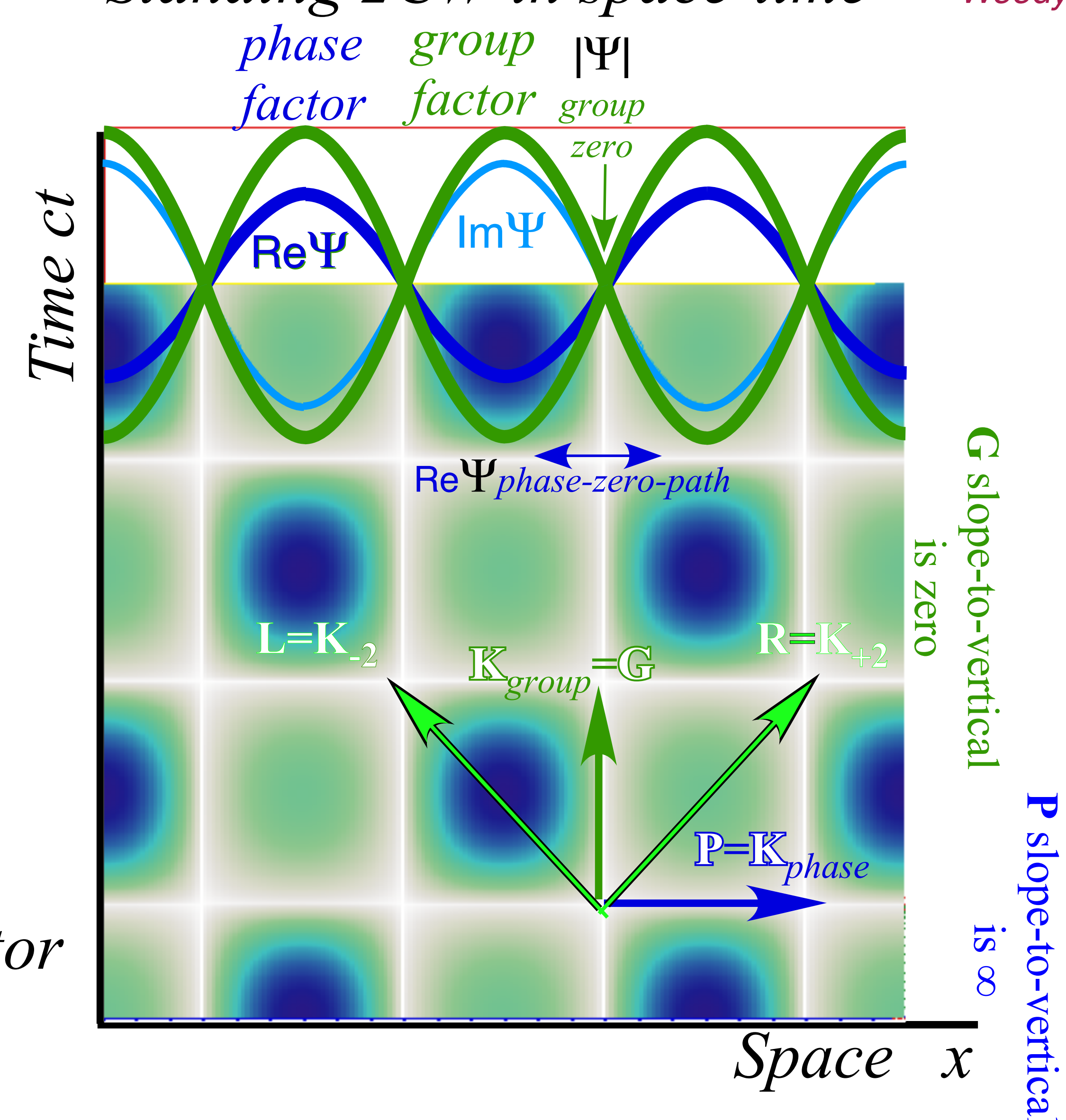


Wavevector
$ck=2\pi\kappa c$

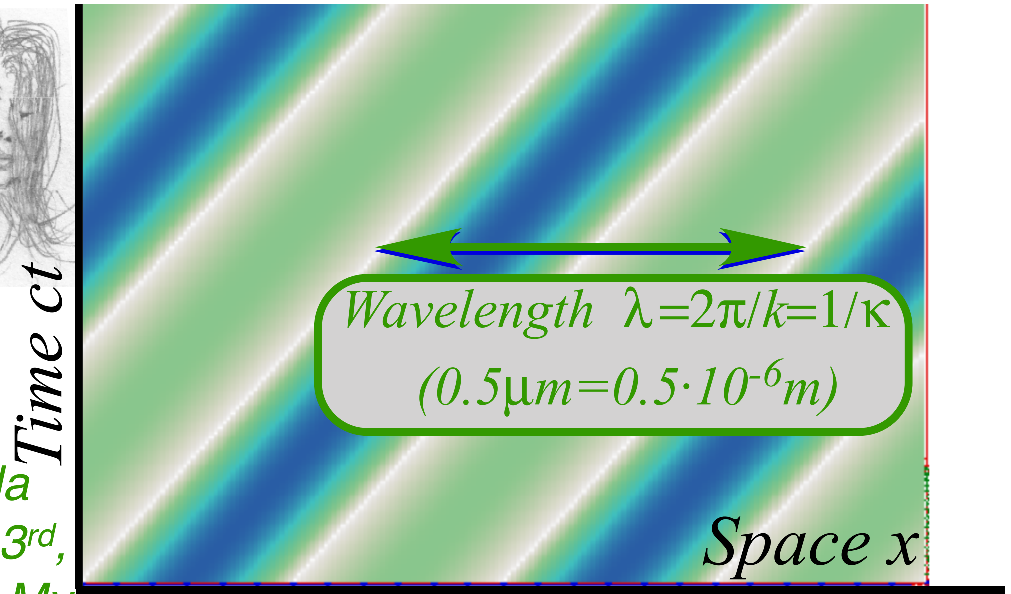


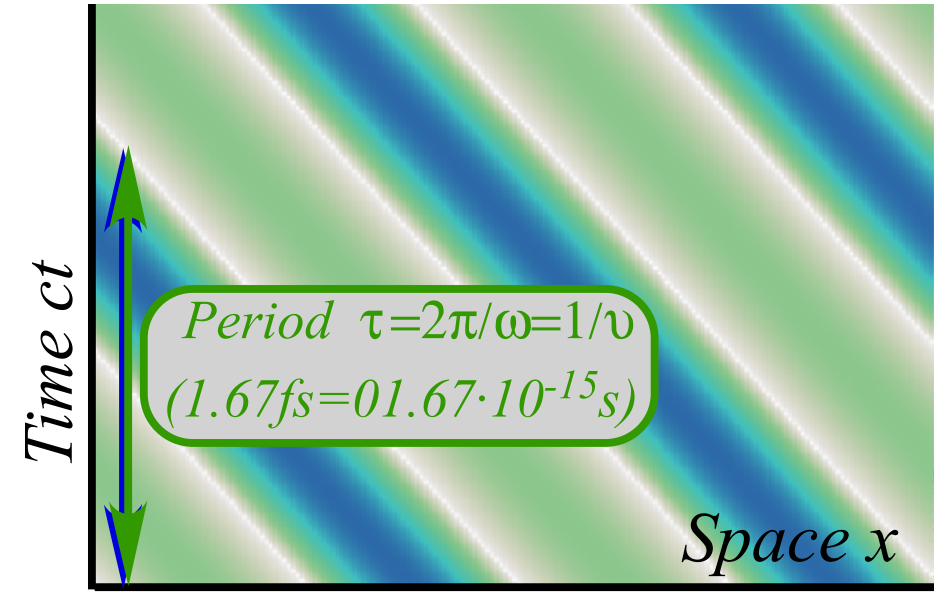
Bob:
The **P** and **G** vectors are scale models of zero-grid lattice vectors (but **P** and **G** switch places)

Standing 2CW in space-time



Alice:
No, Carla you're on 3rd, I'm on 1st. My laser points **R**ight
Yours points **L**eft!

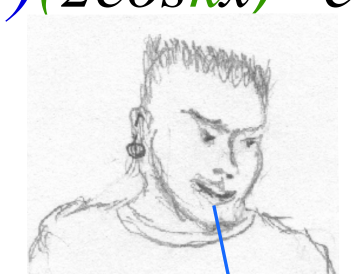




Carla:
OK, Bob!
It looks like a baseball diamond with
P at Pitcher's mound
and
G at the Grandstand*.
Ok, I'm on 3rd base **L**.

*Thanks, Woody!

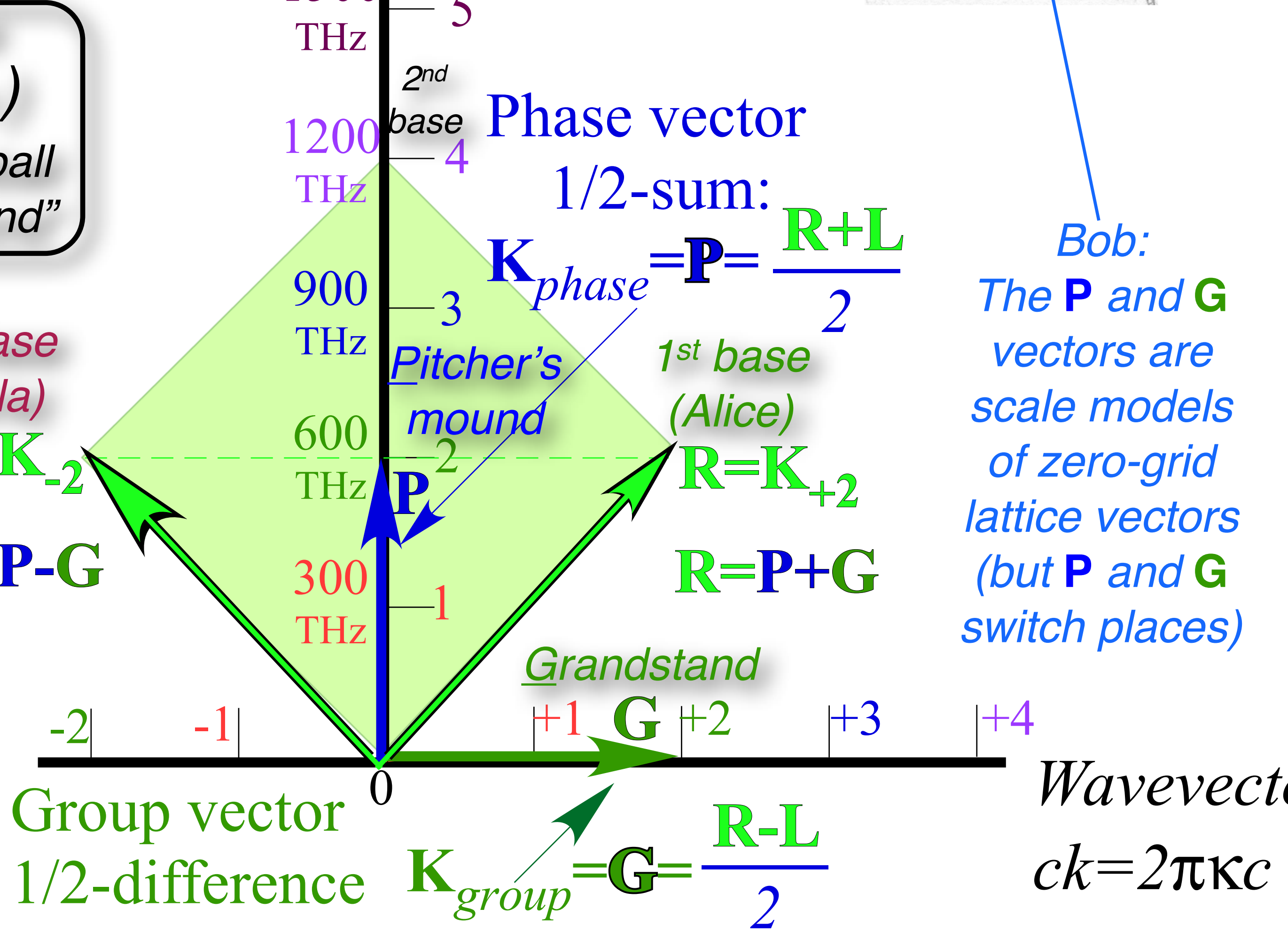
$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$



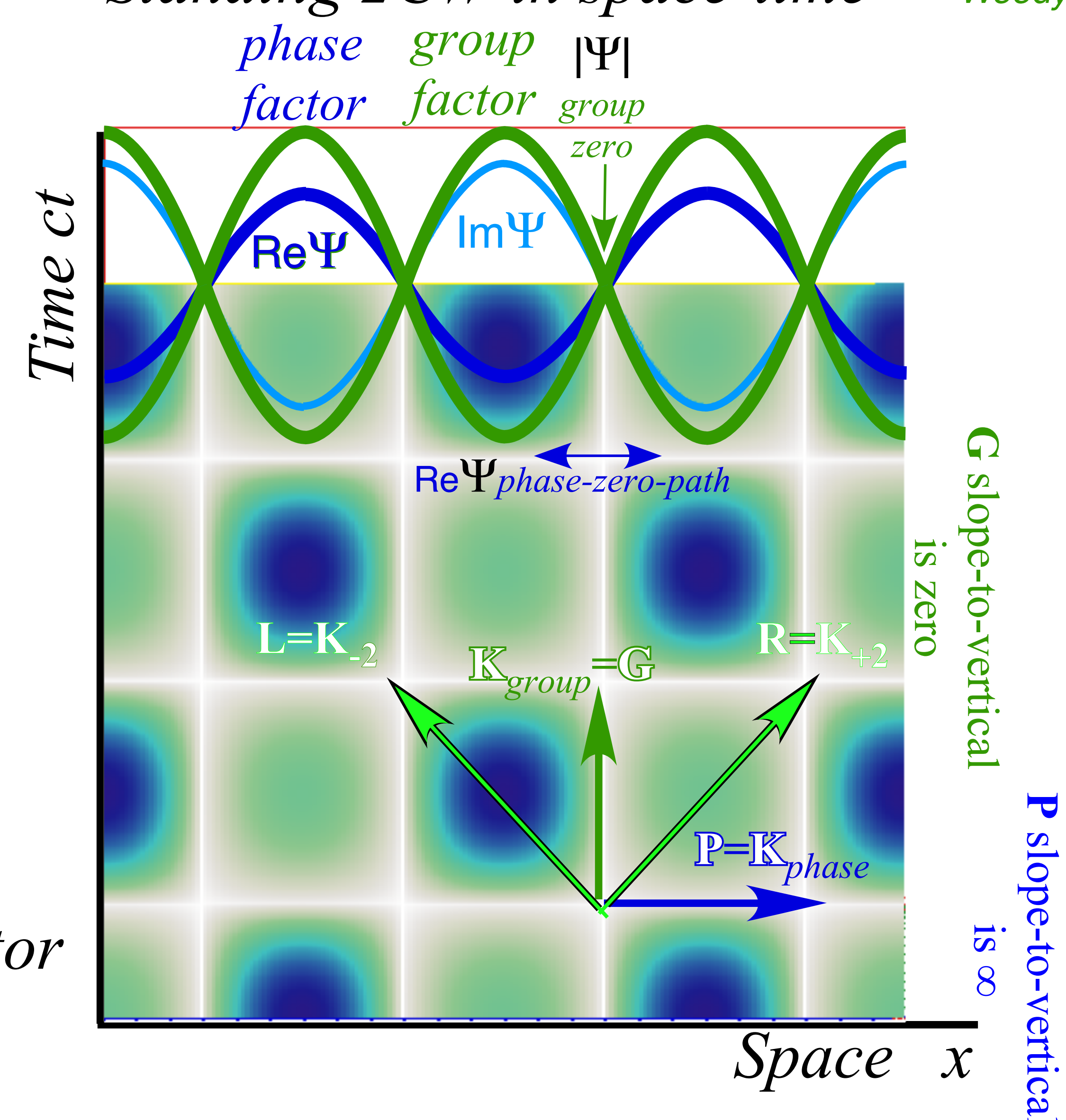
Bob:
The **P** and **G** vectors are scale models of zero-grid lattice vectors (but **P** and **G** switch places)

Standing 2CW in per-space-time

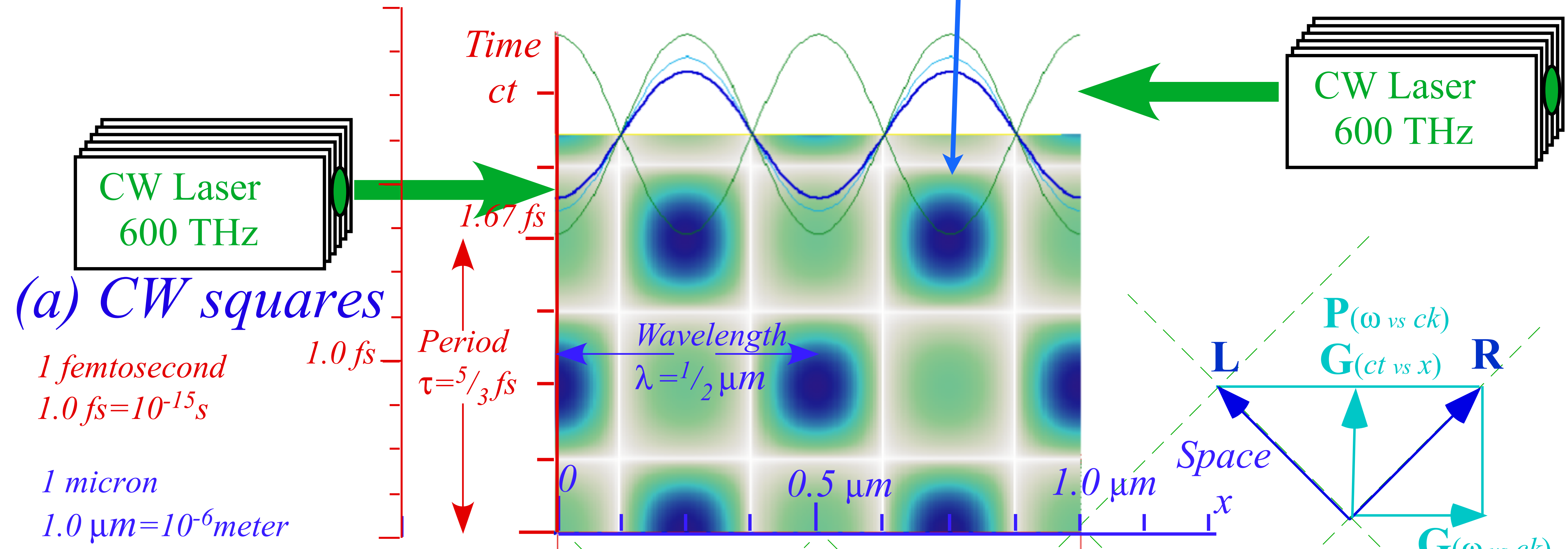
The (υ,κ) "Baseball Diamond"



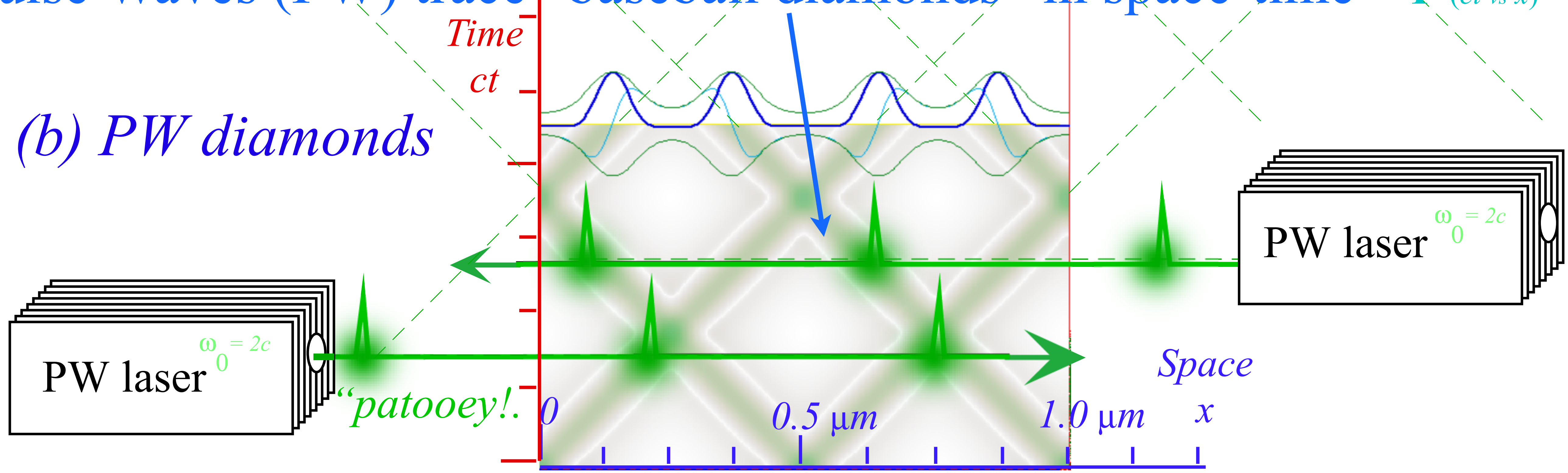
Standing 2CW in space-time

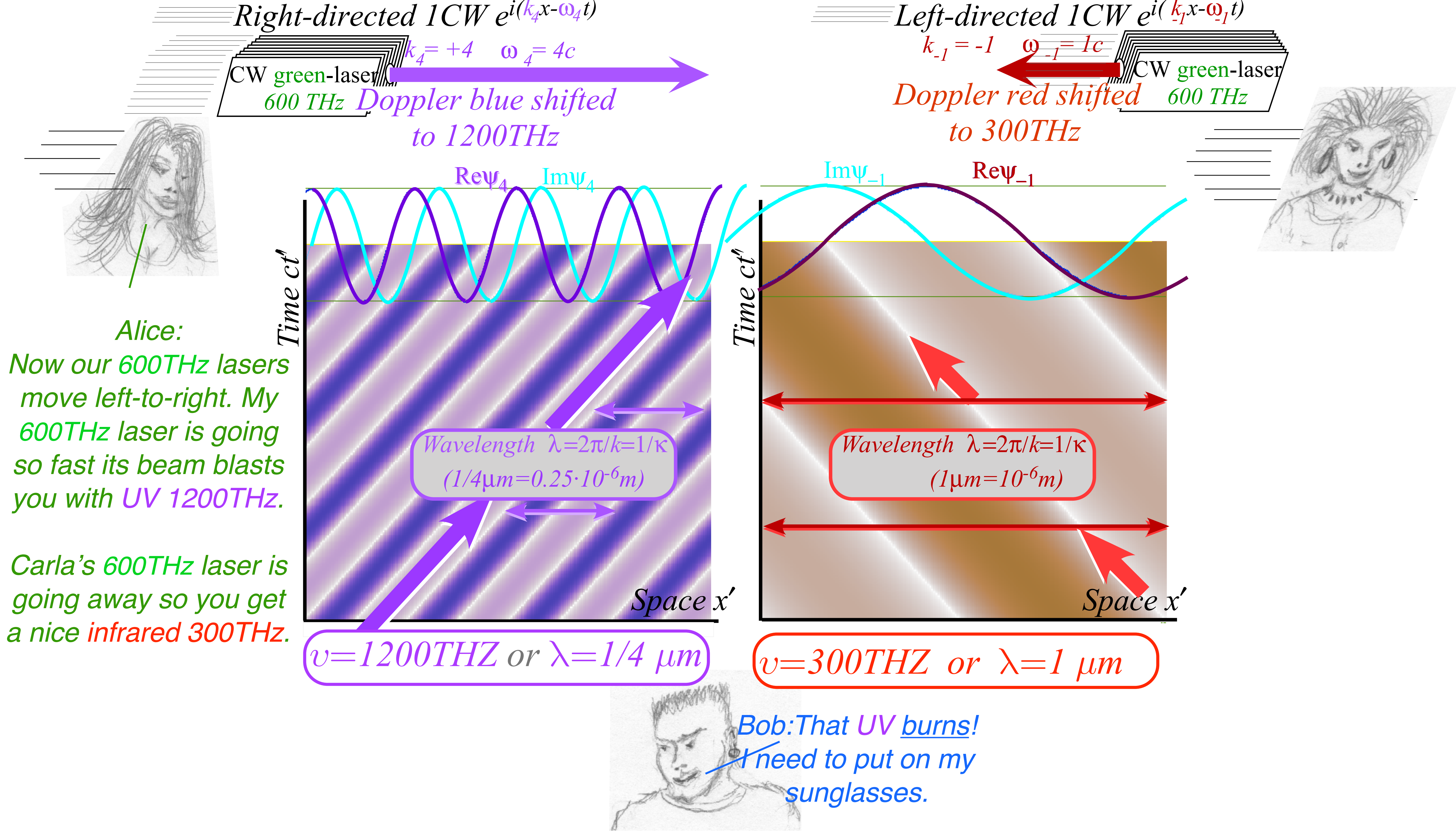


Continuous Waves (CW) trace "Cartesian squares" in space-time

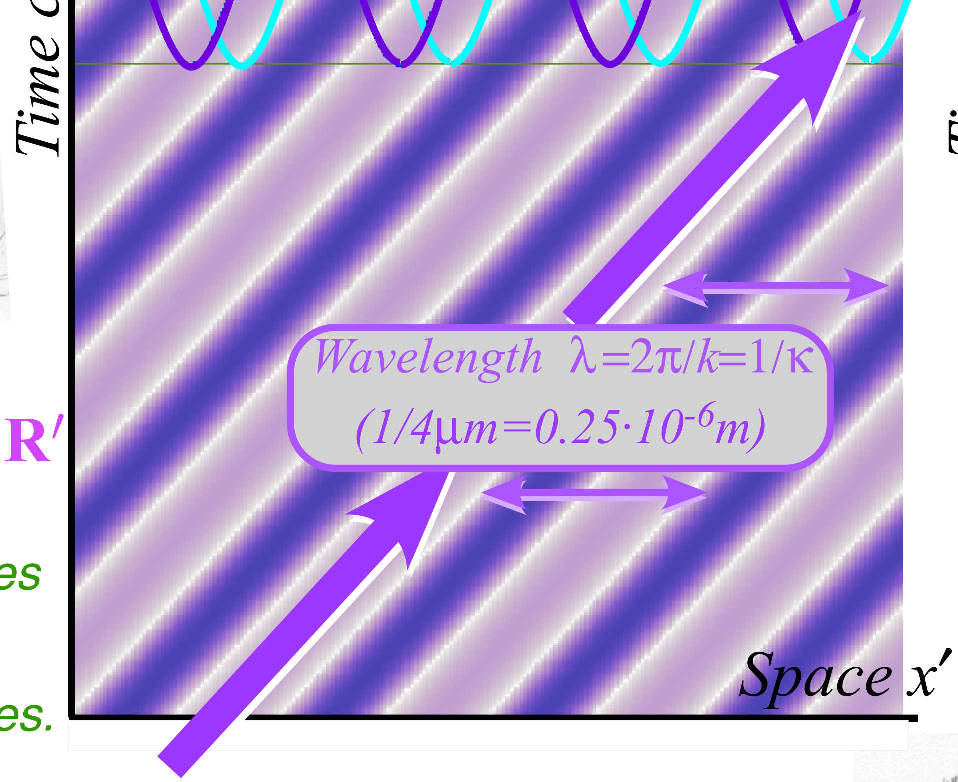


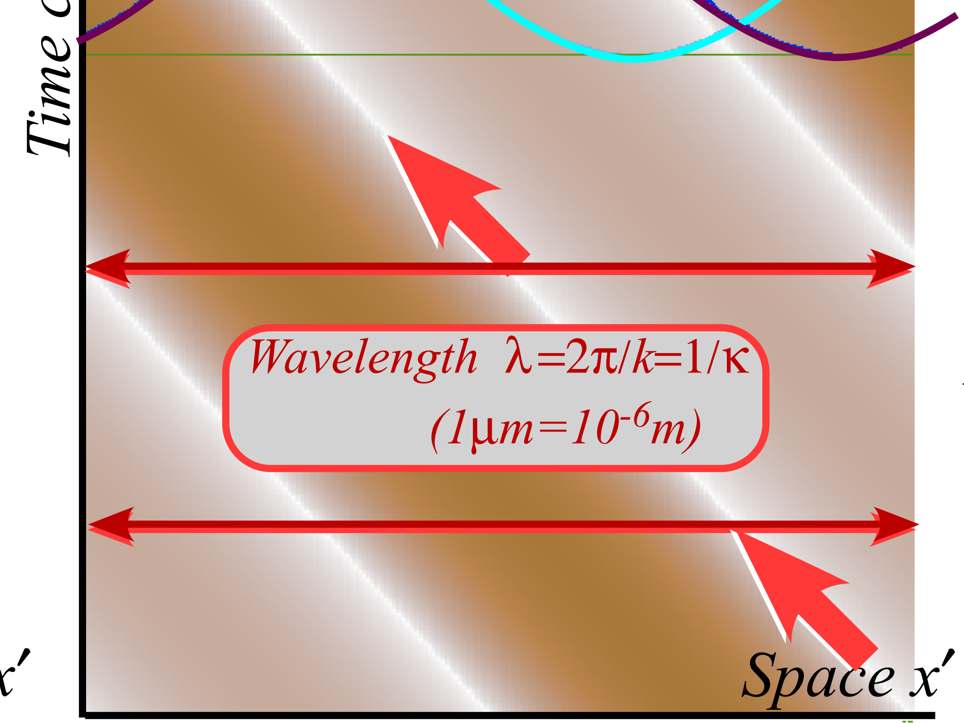
Pulse Waves (PW) trace "baseball diamonds" in space-time

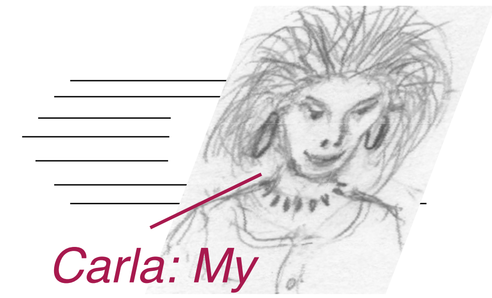



Right-directed 1CW $e^{i(k_4 x-\omega_4 t)}$
$k_4 = +4$ $\omega_4 = 4c$
CW green-laser
600 THz
Doppler blue shifted
to 1200THz
Left-directed 1CW $e^{i(k_{-1} x-\omega_{-1} t)}$
$k_{-1} = -1$ $\omega_{-1} = 1c$
CW green-laser
600 THz
Doppler red shifted
to 300THz
Reψ4 Imψ4
Imψ−1 Reψ−1
Time ct′
Space x′
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10⁻⁶m)
Wavelength λ=2π/k=1/κ
(1μm=10⁻⁶m)
υ=1200THZ or λ=1/4 μm
υ=300THZ or λ=1 μm
Alice:
Now our 600THz lasers move left-to-right. My 600THz laser is going so fast its beam blasts you with UV 1200THz.
Carla's 600THz laser is going away so you get a nice infrared 300THz.
Bob:That UV burns!
I need to put on my sunglasses.

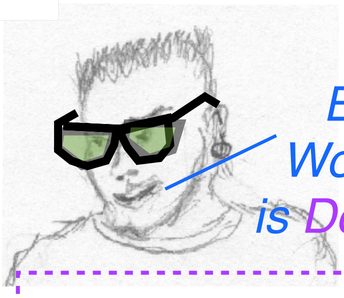
Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



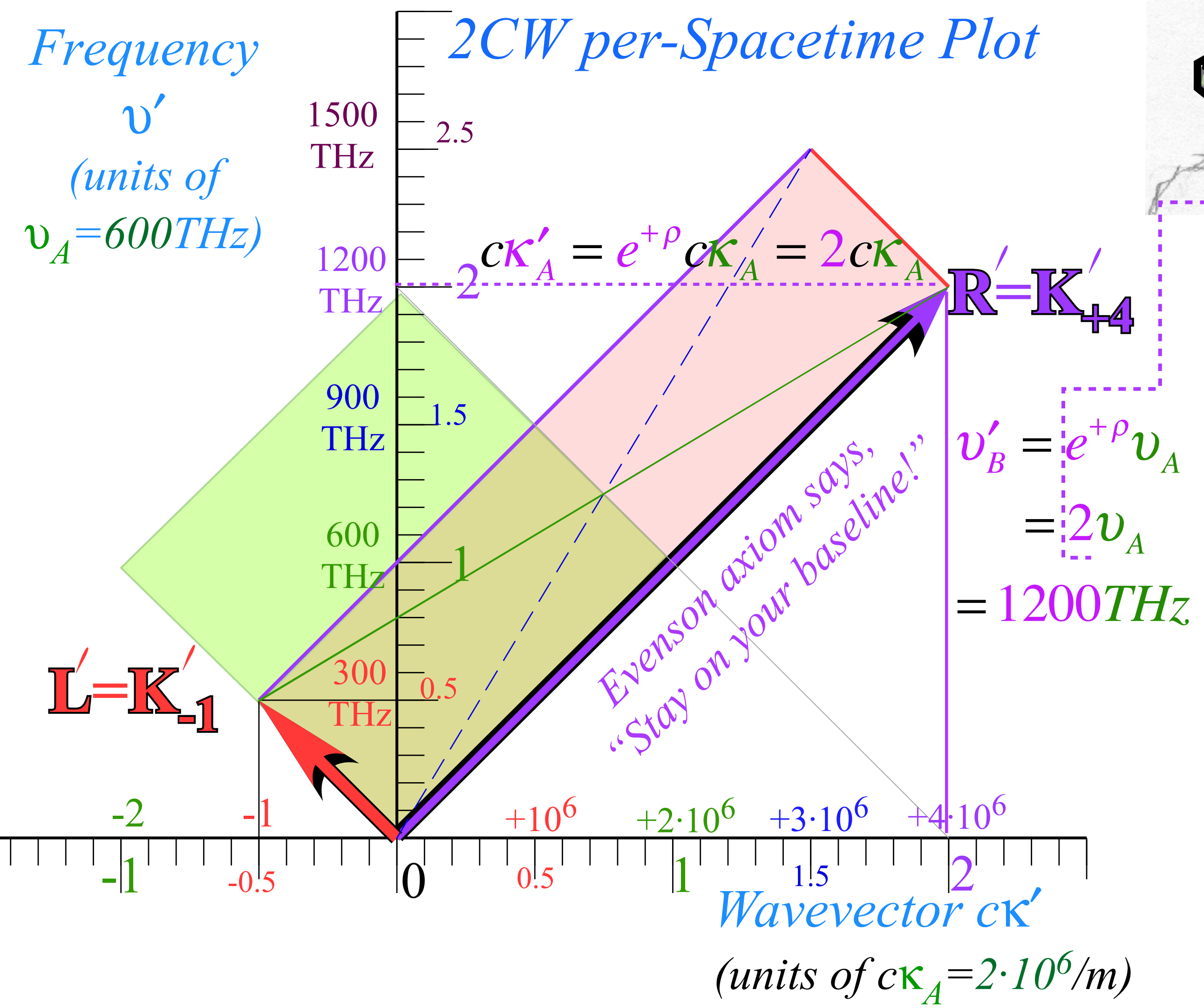


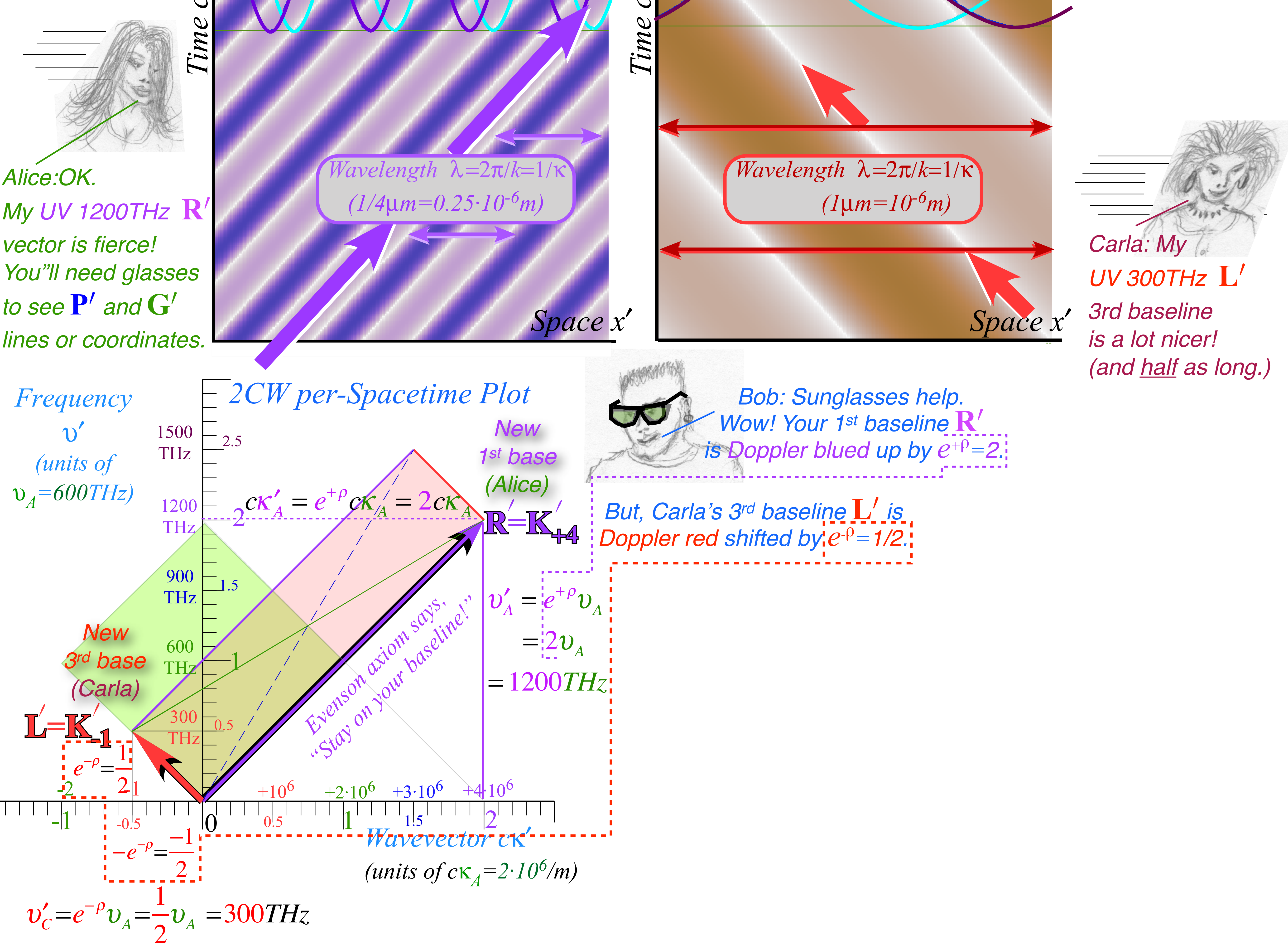


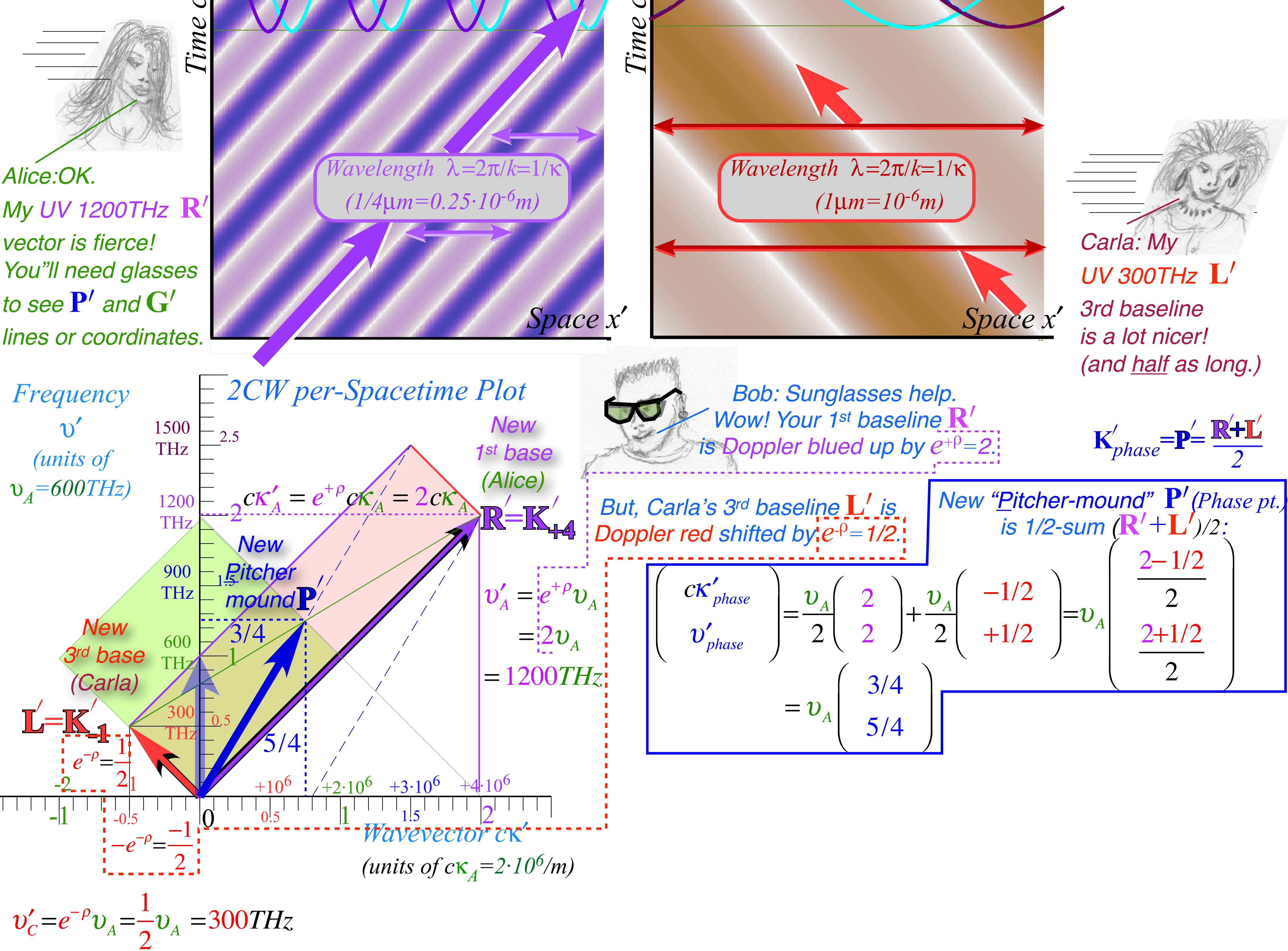
Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

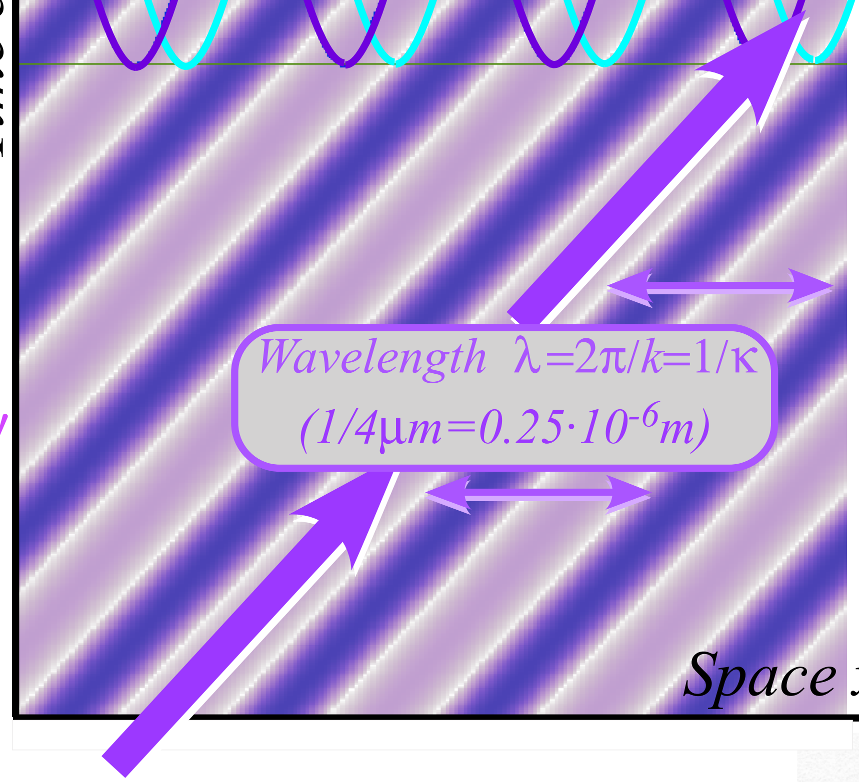


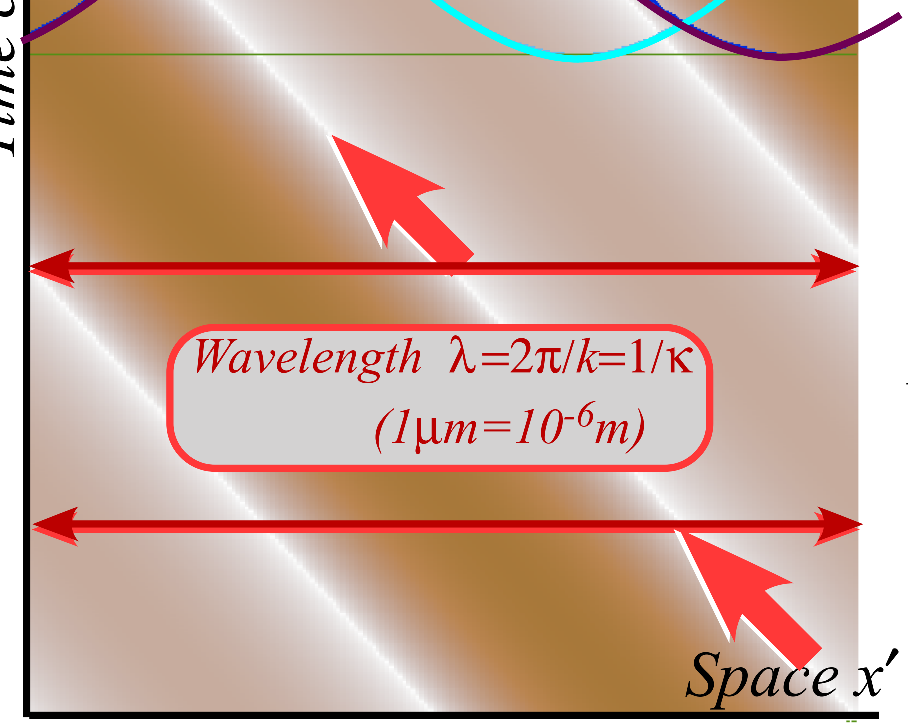

Alice:OK.
My UV 1200THz R′
vector is fierce!
You"ll need glasses
to see P′ and G′
lines or coordinates.
Time
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6 m)
Space x′
Time
Wavelength λ=2π/k=1/κ
(1μm=10^-6 m)
Space x′
Carla: My
UV 300THz L′
3rd baseline
is a lot nicer!
(and half as long.)
Frequency
υ′
(units of
υ_A=600THz)
2CW per-Spacetime Plot
1500 THz
1200 THz
900 THz
600 THz
300 THz
2.5
2
1.5
1
0.5
cκ′_A = e^{+ρ} cκ_A = 2cκ_A
New
1st base
(Alice)
R′=K′_{+4}
Bob: Sunglasses help.
Wow! Your 1st baseline R′
is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is
Doppler red shifted by e^{-ρ}=1/2.
υ′_A = e^{+ρ} υ_A
= 2υ_A
= 1200THz
New
3rd base
(Carla)
L′=K′_{-1}
e^{-ρ} = 1/2
Evenson axiom says,
"Stay on your baseline!"
+10^6
+2·10^6
+3·10^6
+4·10^6
-2
-1
-0.5
0
0.5
1
1.5
2
−e^{-ρ} = −1/2
Wavevector cκ′
(units of cκ_A=2·10^6/m)
υ′_C = e^{-ρ} υ_A = (1/2) υ_A = 300THz



Alice:OK.
My UV 1200THz R′ vector is fierce!
You"ll need glasses to see P′ and G′ lines or coordinates.
Time
Space x′
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6m)
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Carla: My UV 300THz L′
3rd baseline is a lot nicer!
(and half as long.)
Frequency υ′ (units of υ_A=600THz)
2CW per-Spacetime Plot
New 1st base (Alice)
New Pitcher mound P′
New 3rd base (Carla)
1500 THz
1200 THz
900 THz
600 THz
300 THz
cκ′_A = e^{+ρ}cκ_A = 2cκ_A
R′=K′_{+4}
L′=K′_{-1}
3/4
5/4
e^{-ρ}=1/2
−e^{-ρ}=−1/2
υ′_A = e^{+ρ}υ_A = 2υ_A = 1200THz
+10^6 +2·10^6 +3·10^6 +4·10^6
Wavevector cκ′
(units of cκ_A=2·10^6/m)
υ′_C=e^{-ρ}υ_A=1/2 υ_A =300THz
Bob: Sunglasses help.
Wow! Your 1st baseline R′ is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is Doppler red shifted by e^{-ρ}=1/2.
K′_phase=P′=(R′+L′)/2
New "Pitcher-mound" P′ (Phase pt.) is 1/2-sum (R′+L′)/2:
(cκ′_phase, υ′_phase) = υ_A/2 (2, 2) + υ_A/2 (−1/2, +1/2) = υ_A ((2−1/2)/2, (2+1/2)/2) = υ_A (3/4, 5/4)

Alice: OK. My UV 1200THz $\mathbf{R}'$ vector is fierce! You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.





Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)

Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

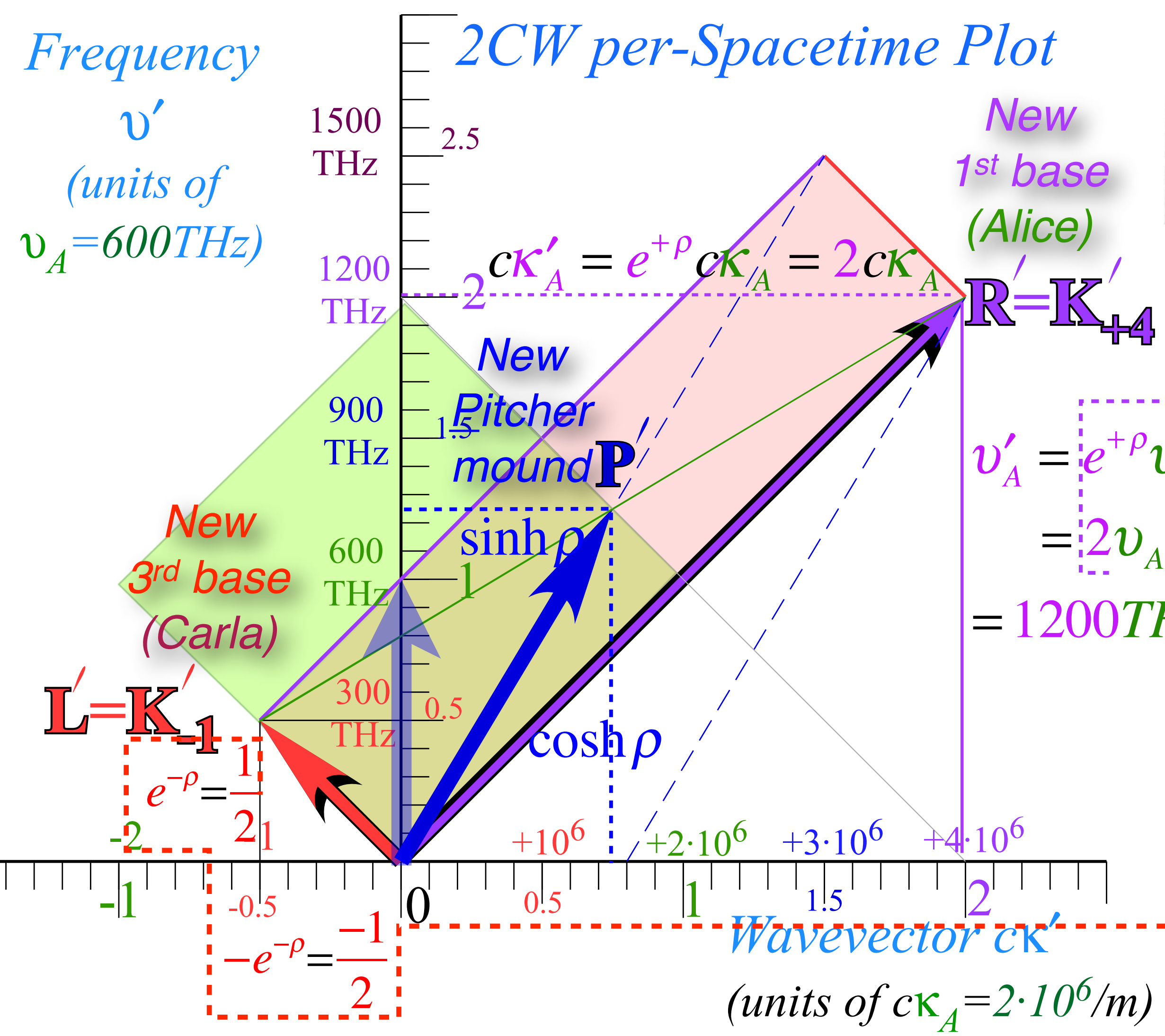
$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase}\end{pmatrix}=\frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho}\end{pmatrix}+\frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho}\end{pmatrix}=\upsilon_A\begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}$$

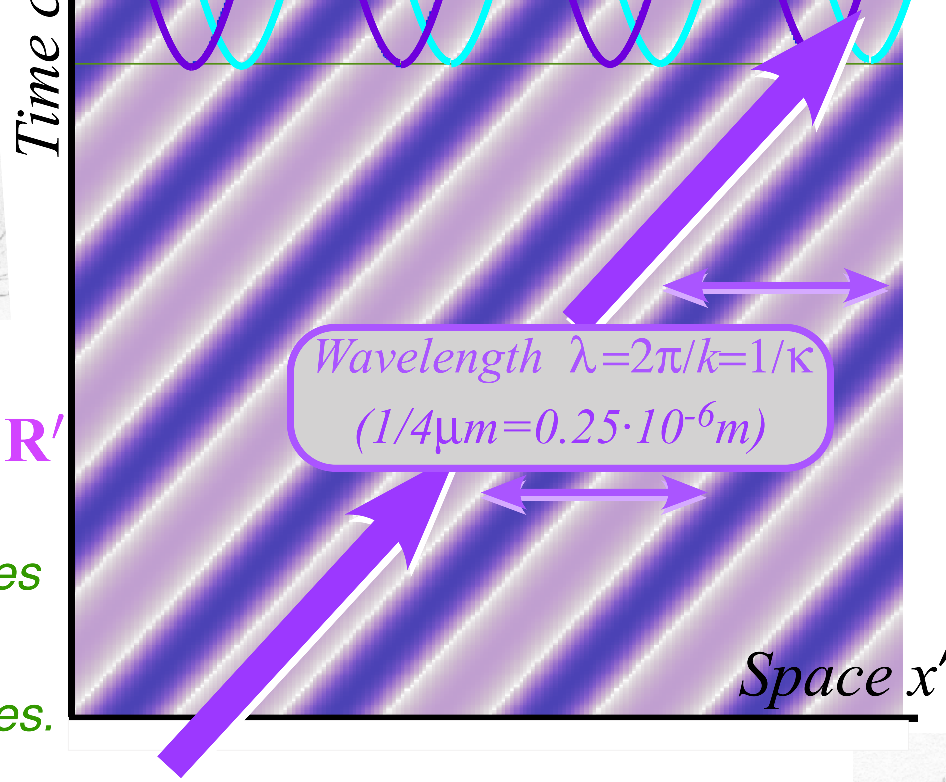
$$=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix} 3/4 \\ 5/4\end{pmatrix}$$

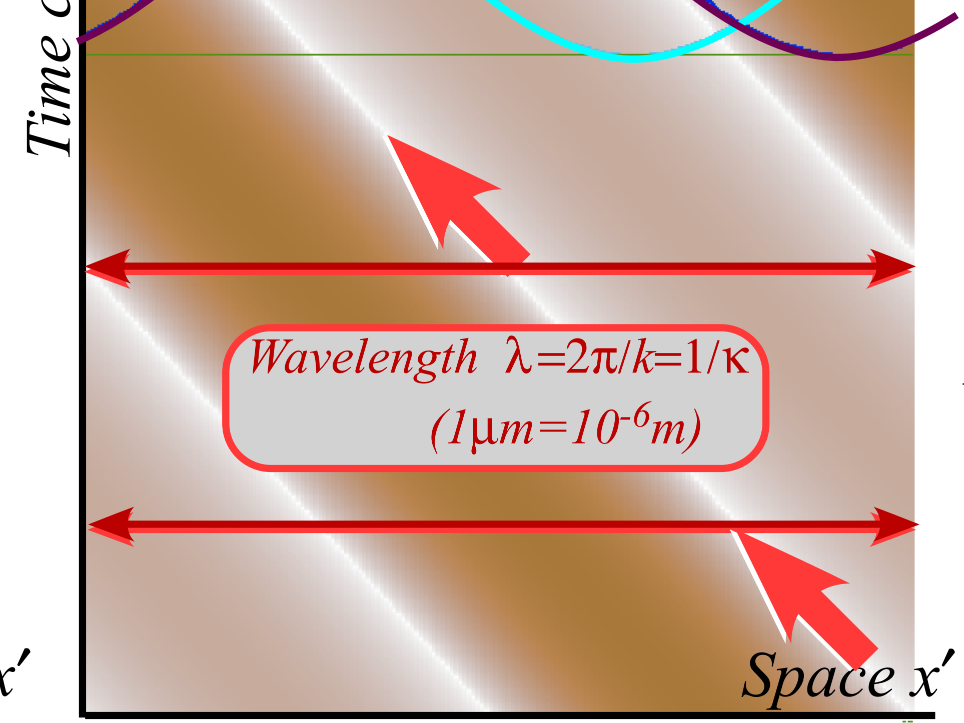


$$\upsilon'_A=e^{+\rho}\upsilon_A=2\upsilon_A=1200THz$$

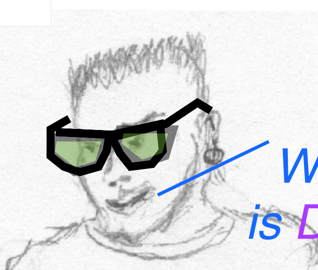
$$\upsilon'_C=e^{-\rho}\upsilon_A=\frac{1}{2}\upsilon_A=300THz$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$
vector is fierce!
You"ll need glasses
to see $\mathbf{P}'$ and $\mathbf{G}'$
lines or coordinates.





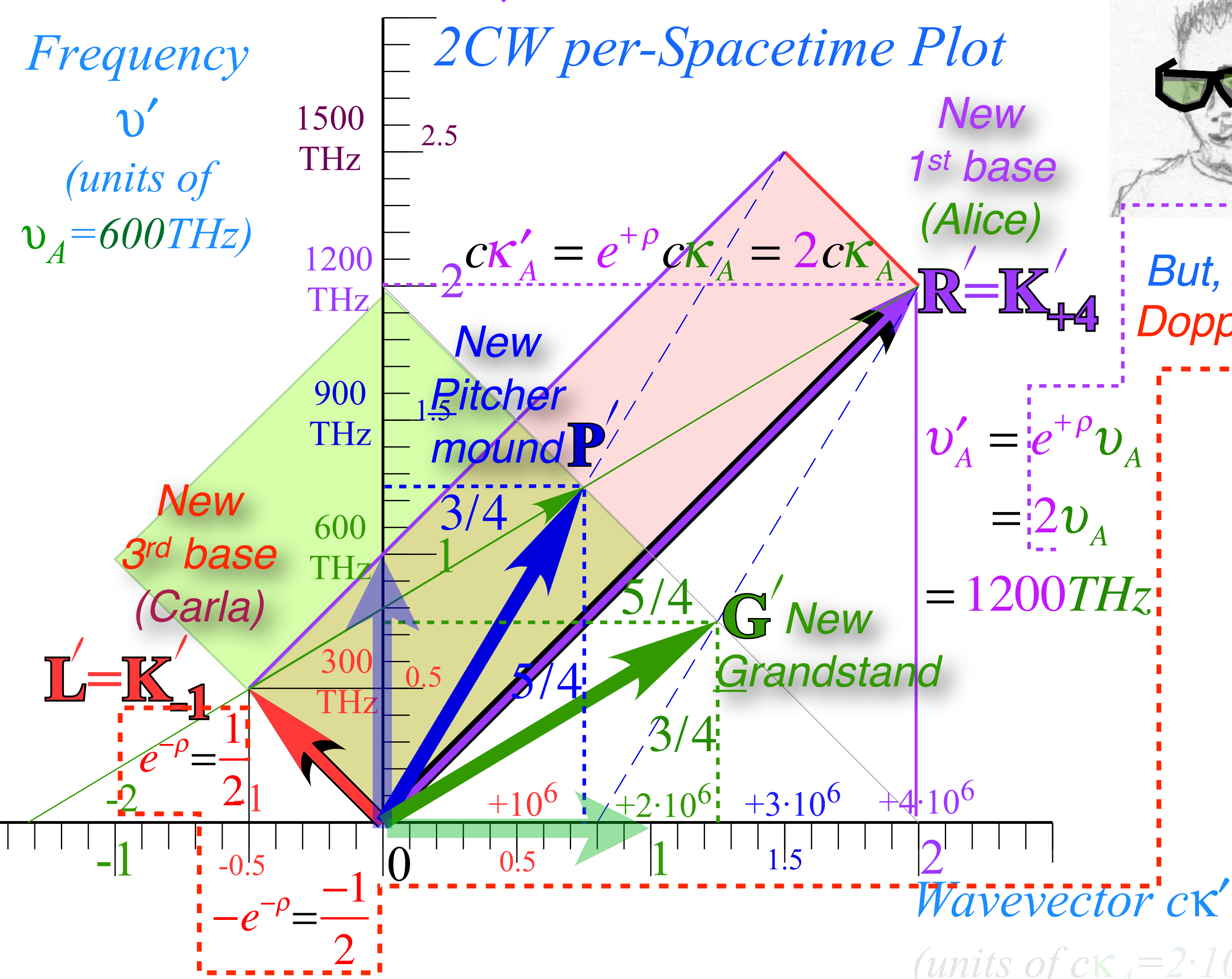
Carla: My
UV 300THz $\mathbf{L}'$
3rd baseline
is a lot nicer!
(and half as long.)



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$
is Doppler blued up by $e^{+\rho}=2$.

$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

But, Carla's 3rd baseline $\mathbf{L}'$ is
Doppler red shifted by $e^{-\rho}=1/2$.



$$\upsilon'_C=e^{-\rho}\upsilon_A=\frac{1}{2}\upsilon_A=300THz$$

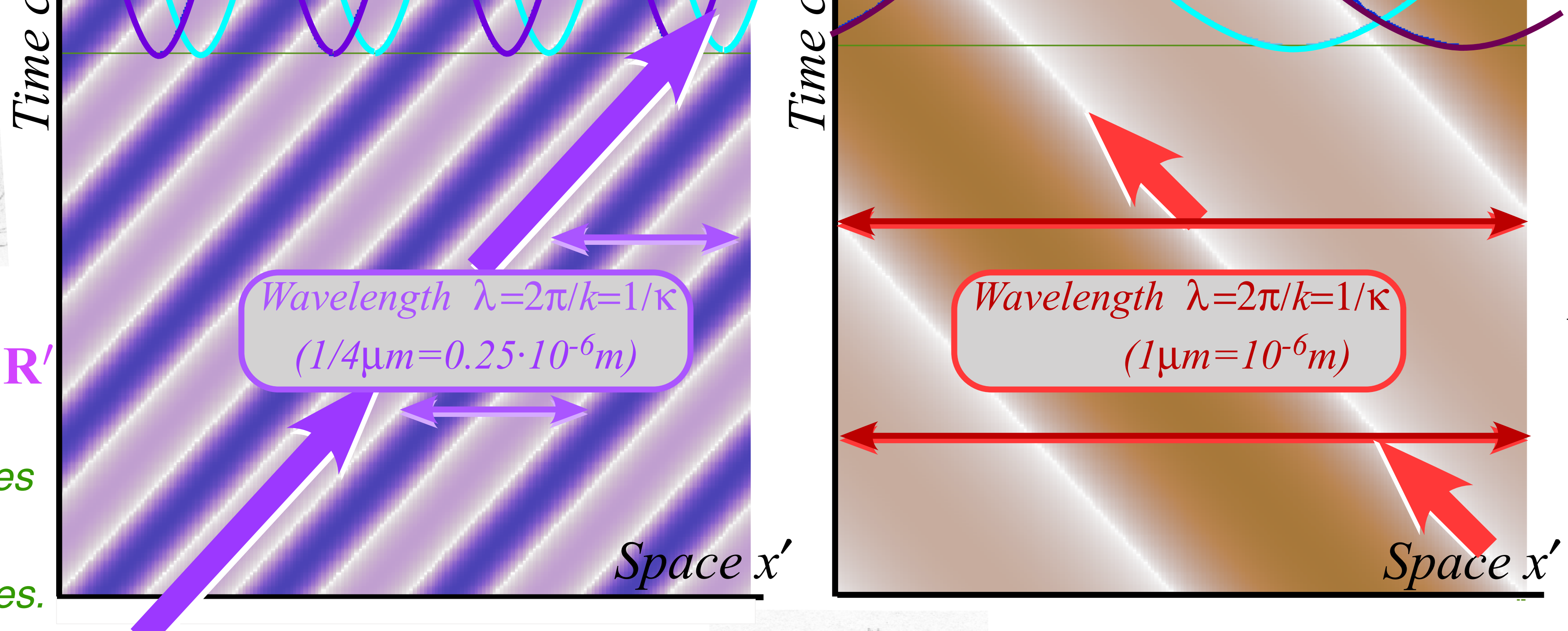
New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.)
is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

New "Grandstand" $\mathbf{G}'$ (Group pt.)
is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2+1/2}{2} \\ \frac{2-1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

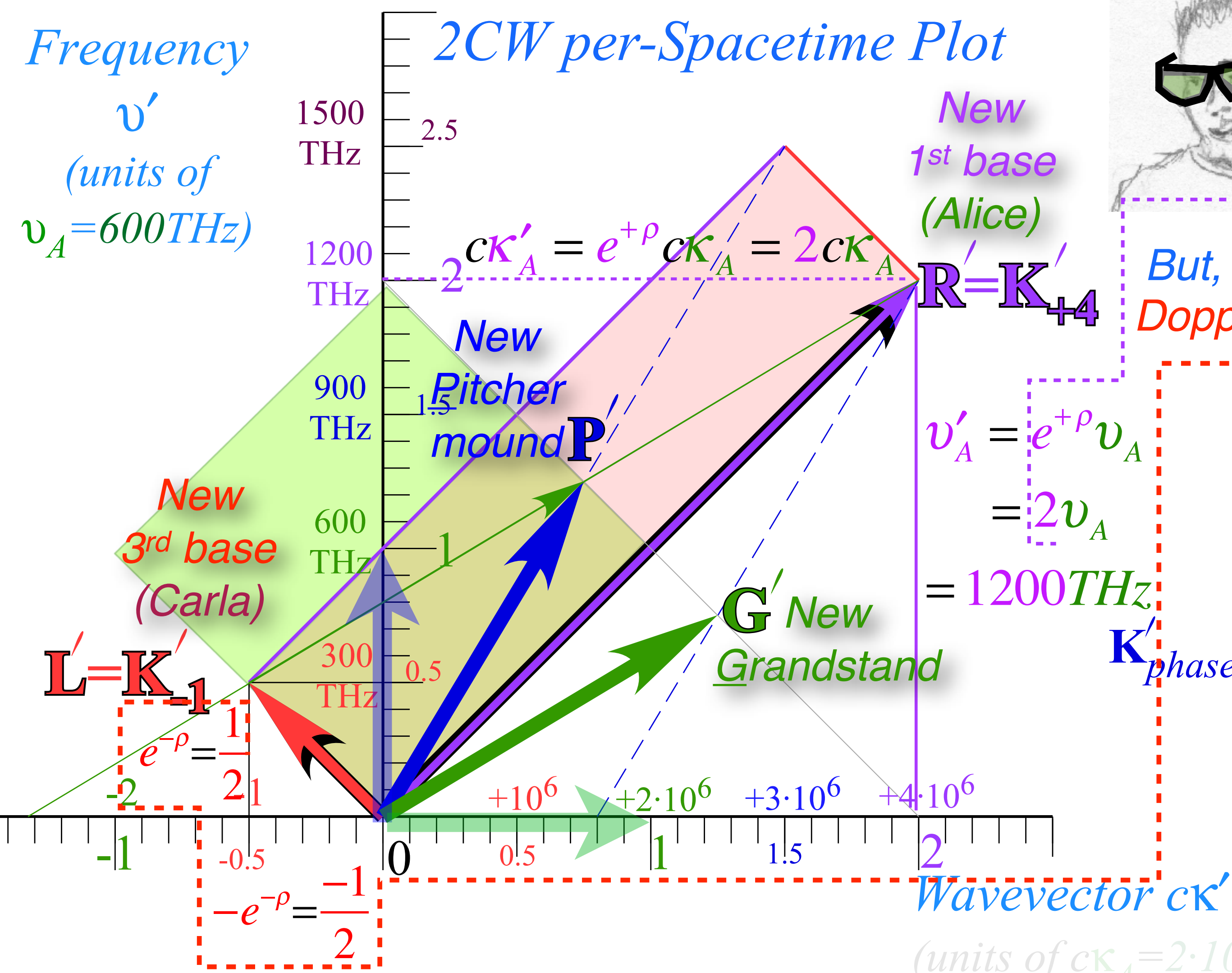
Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)

Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.



$$\upsilon'_A = e^{+\rho}\upsilon_A = 2\upsilon_A = 1200THz$$

$$-e^{-\rho}=\frac{1}{2} \qquad -e^{-\rho}=\frac{-1}{2}$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

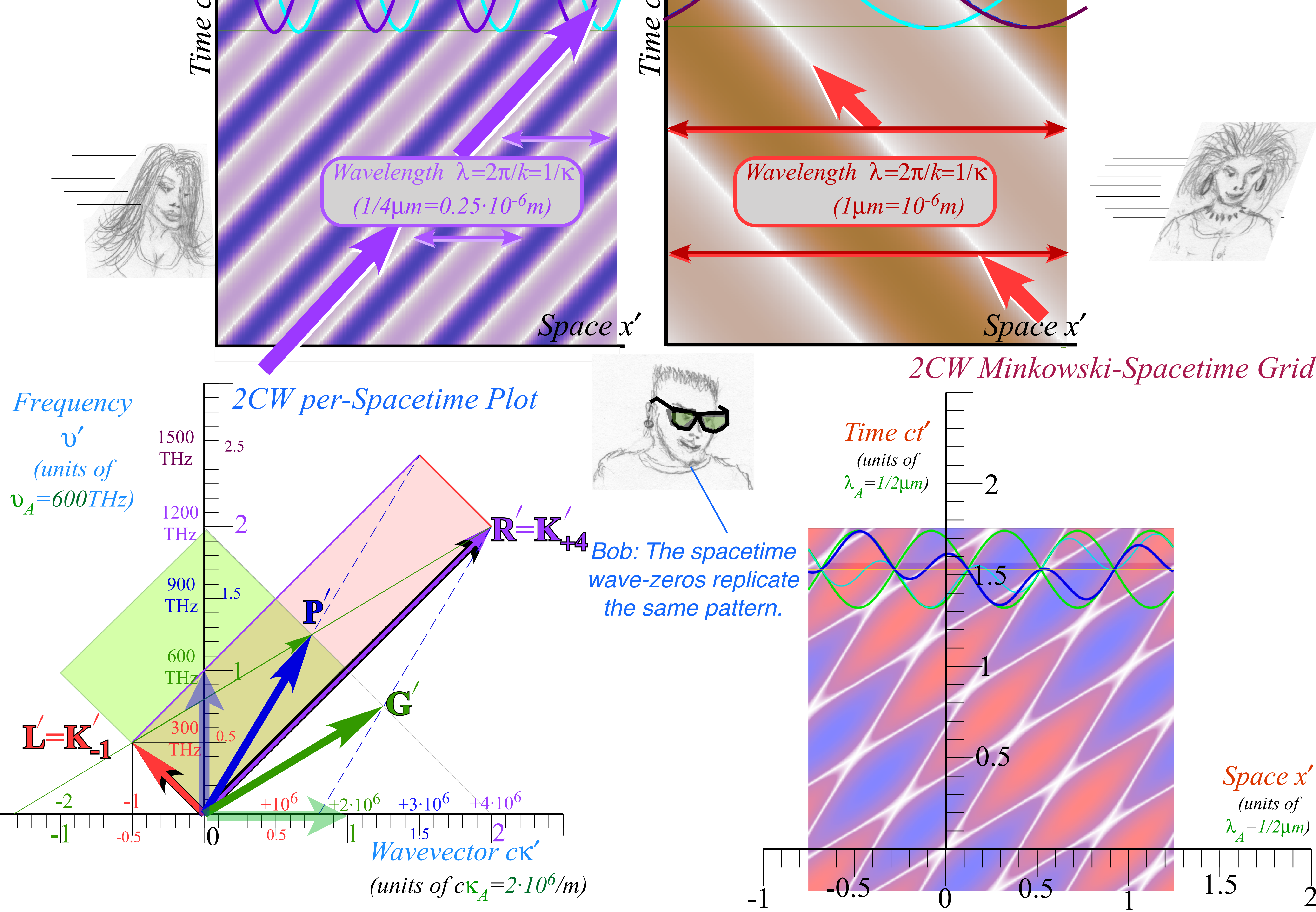
$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}+\mathbf{L}}{2} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$$

New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix}$$

Group vector $\mathbf{G}$
1/2-diff vector

$$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}-\mathbf{L}}{2} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

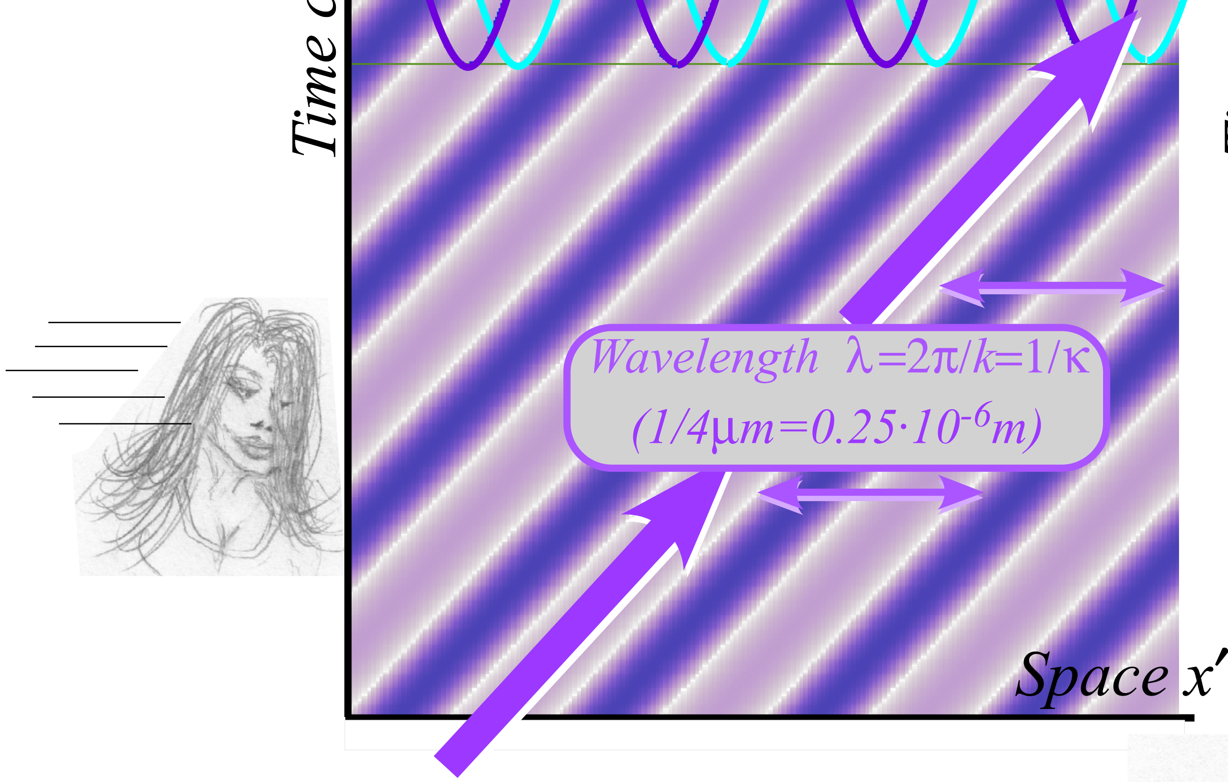


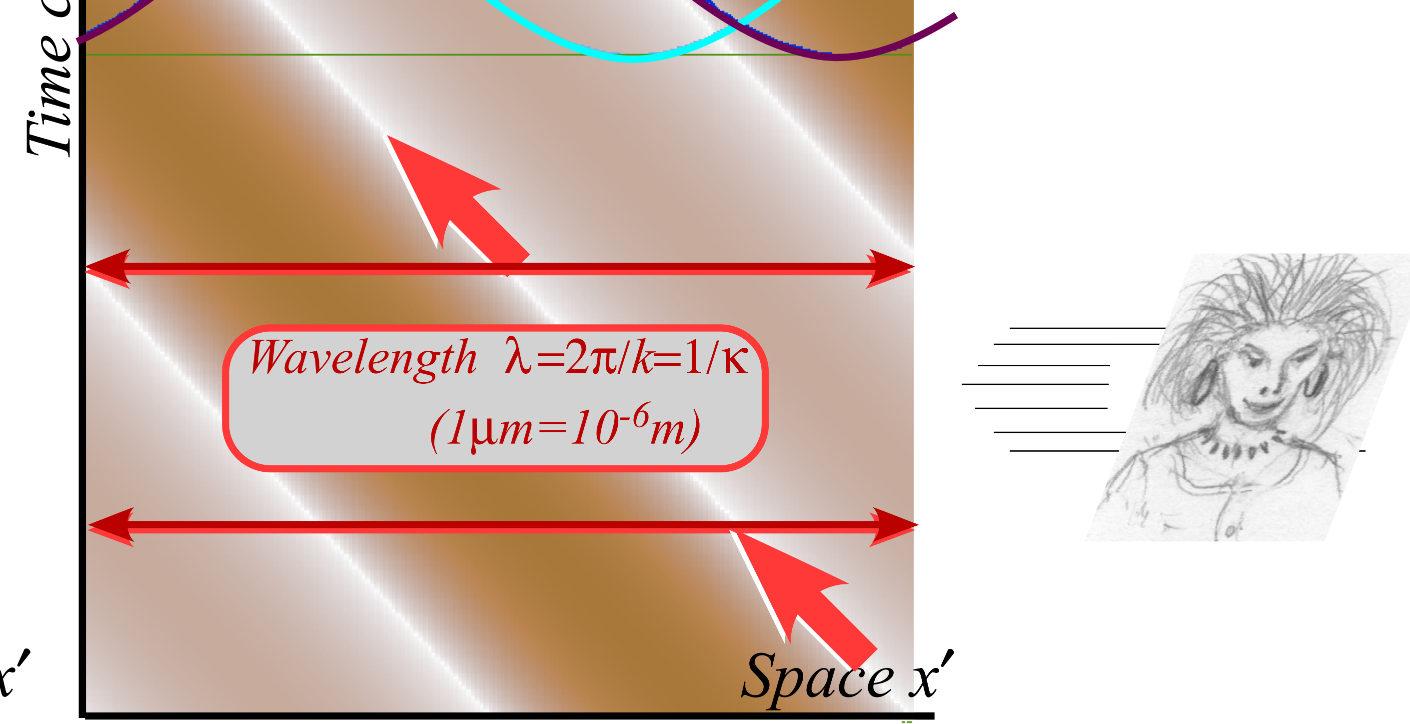
Phase vector **P**
1/2-sum vector

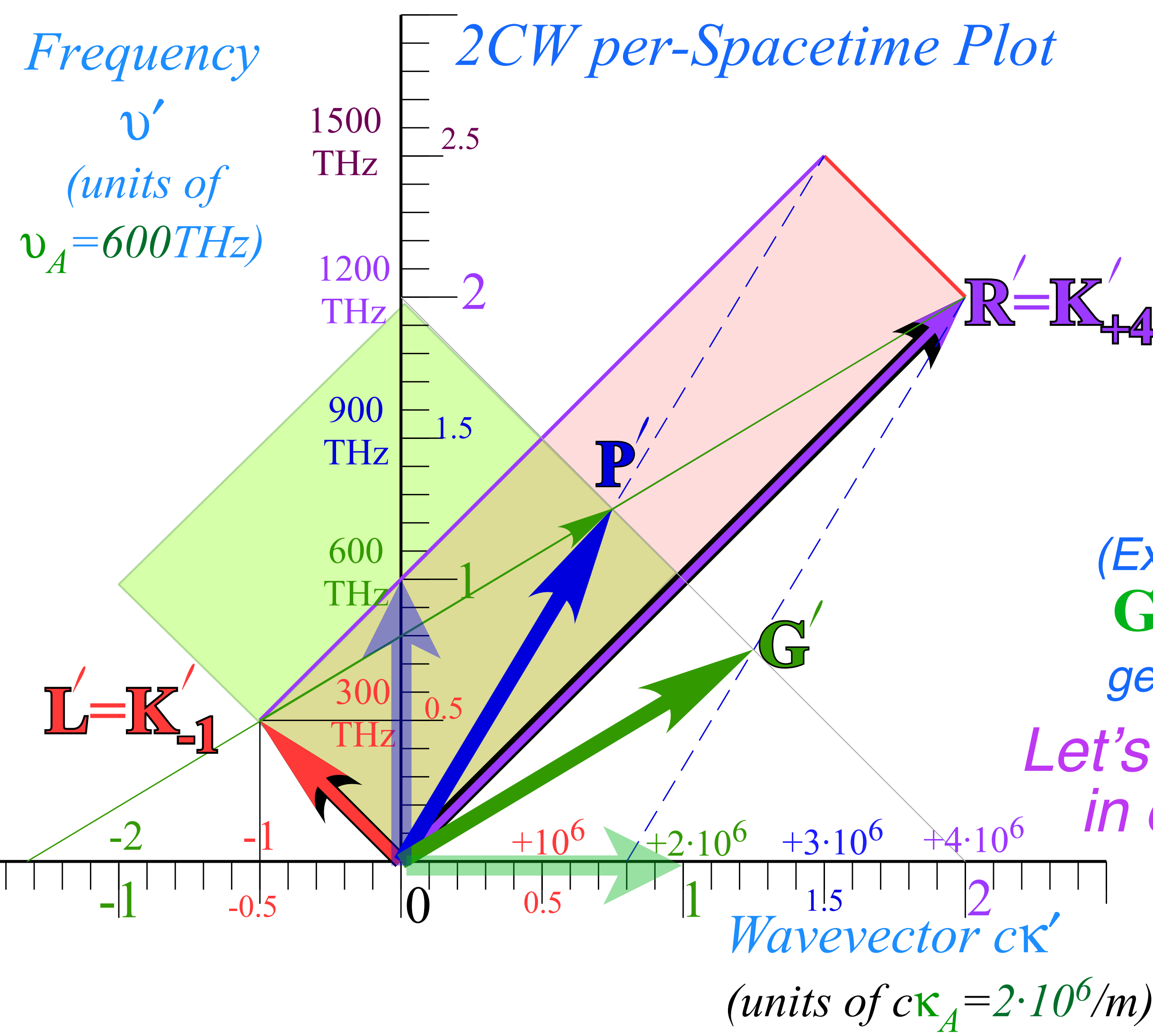
$K'_{phase} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2}$

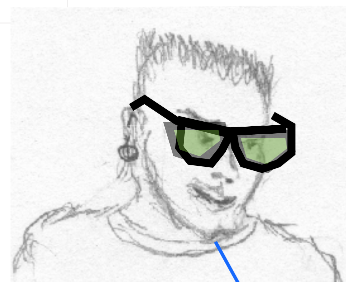
Group vector **G**
1/2-diff vector

$K'_{group} = \mathbf{G}' = \frac{\mathbf{R}' - \mathbf{L}'}{2}$





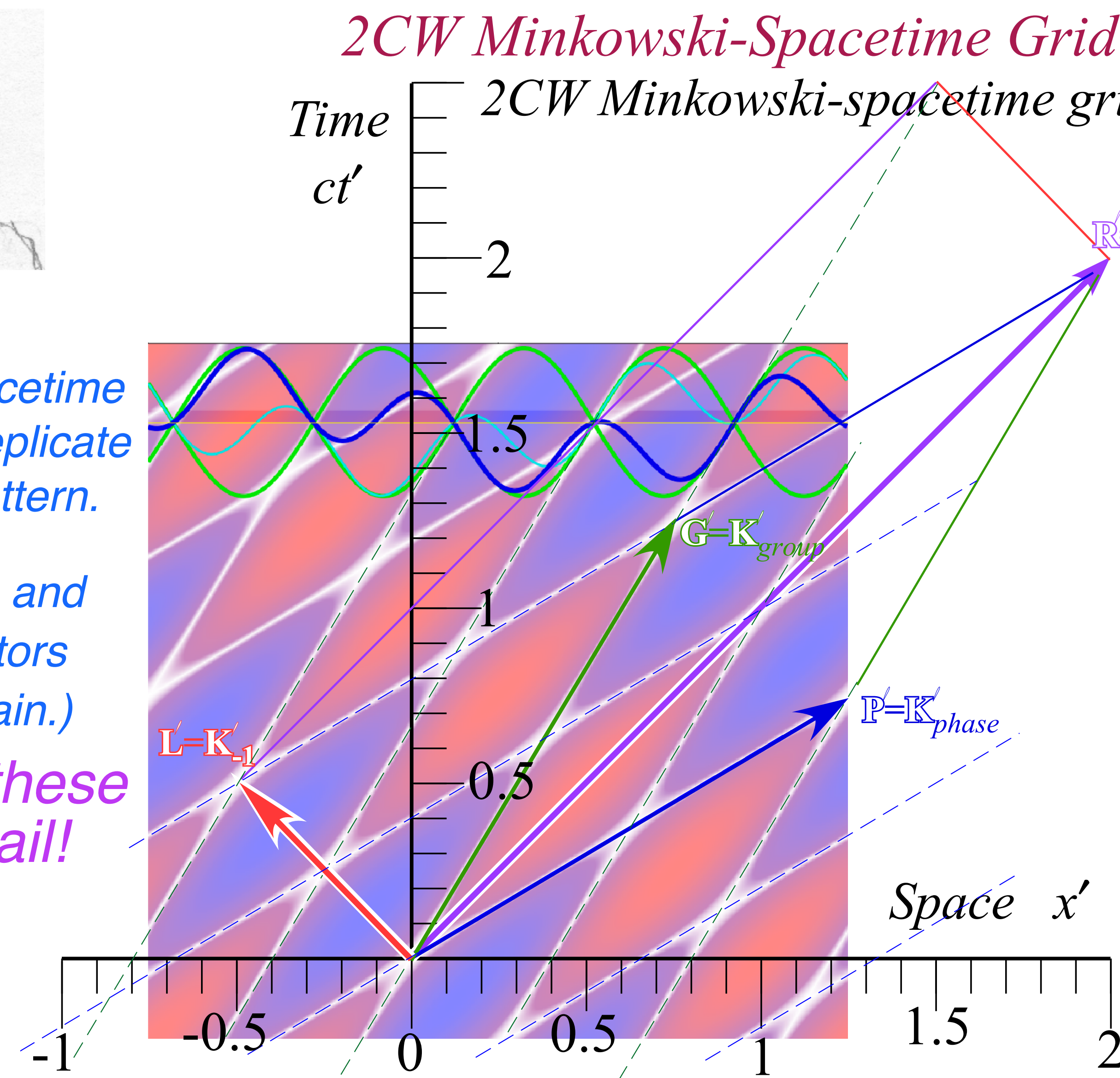




Bob: The spacetime wave-zeros replicate the same pattern.

(Except **P′**-*phase* and **G′**-*group* indicators get switched again.)

Let's <u>measure</u> these in careful detail!

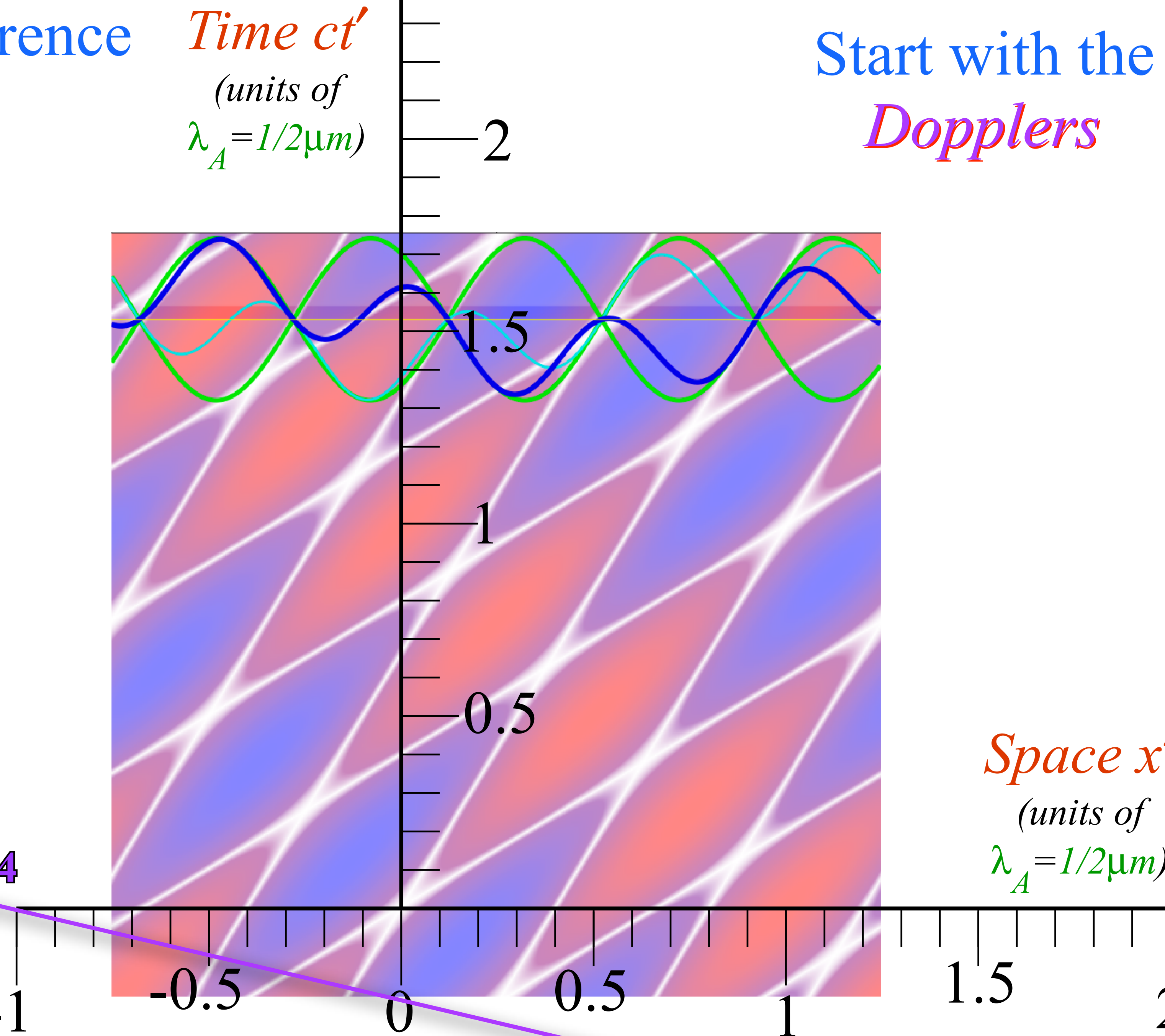


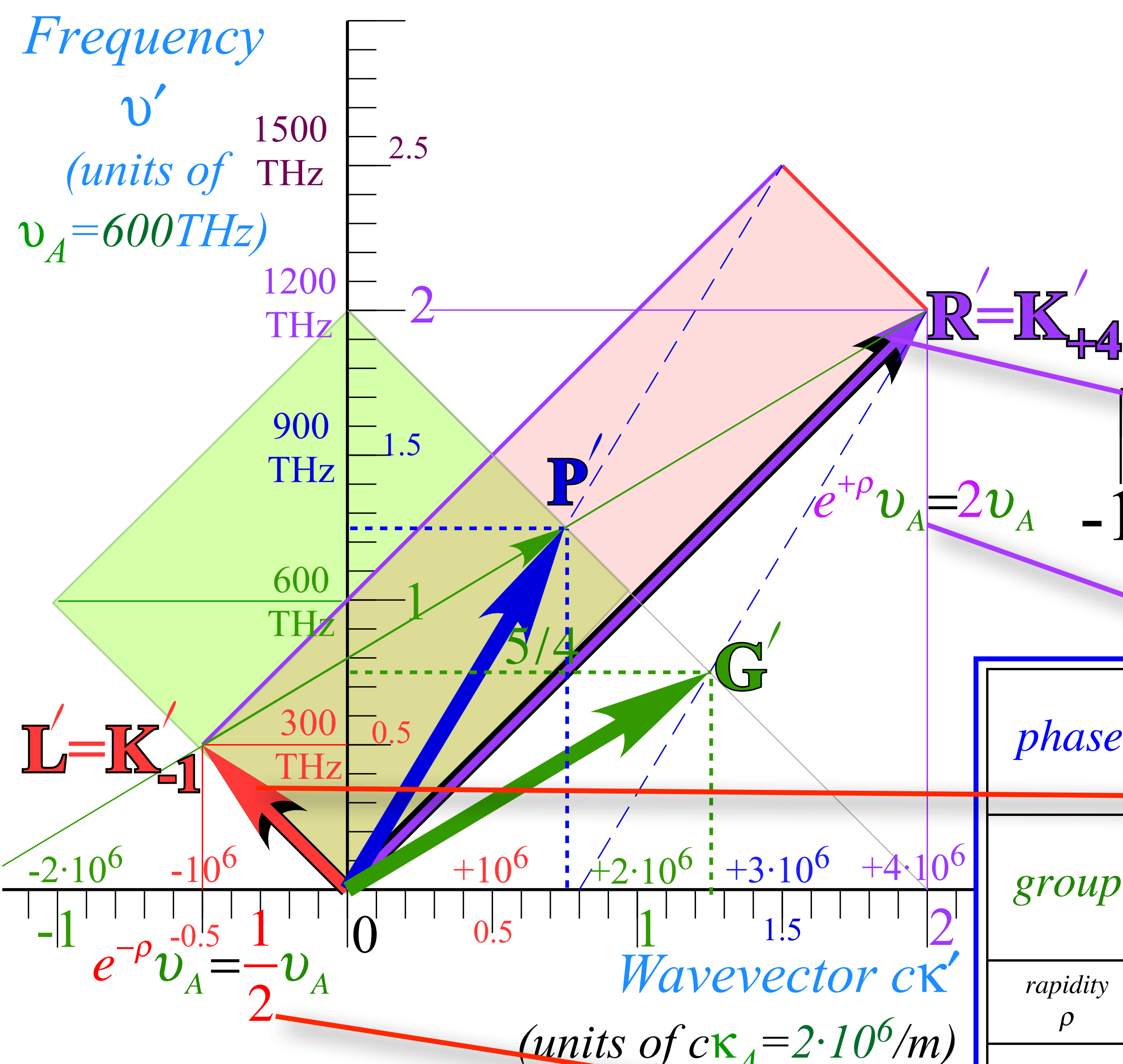
Phase vector **P** 1/2-sum vector $K'_{phase}=P'=\frac{R'+L'}{2}$

Group vector **G** 1/2-diff vector $K'_{group}=G'=\frac{R'-L'}{2}$

The 16 dimensions of 2CW interference

Start with the *Dopplers*





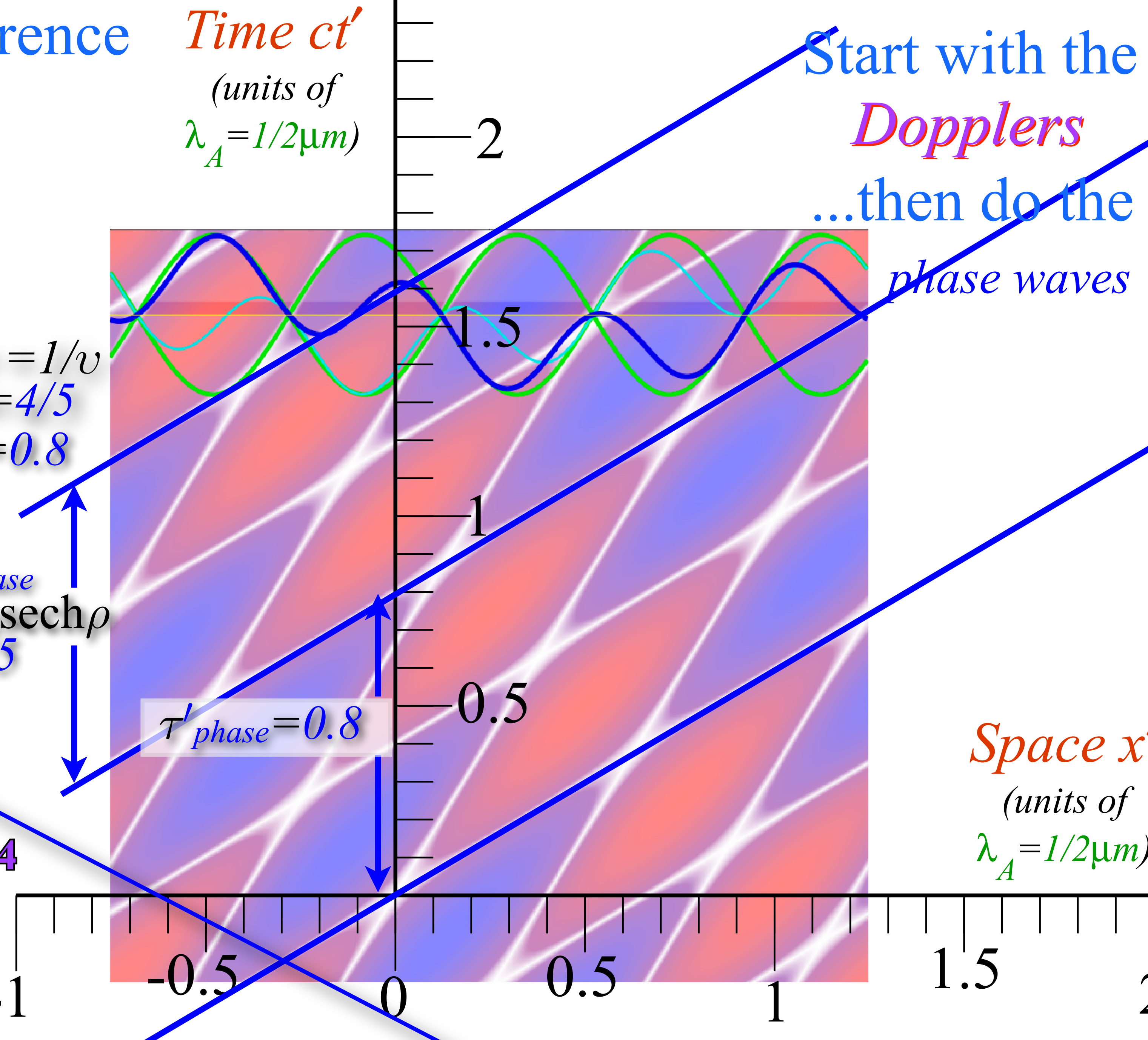
phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

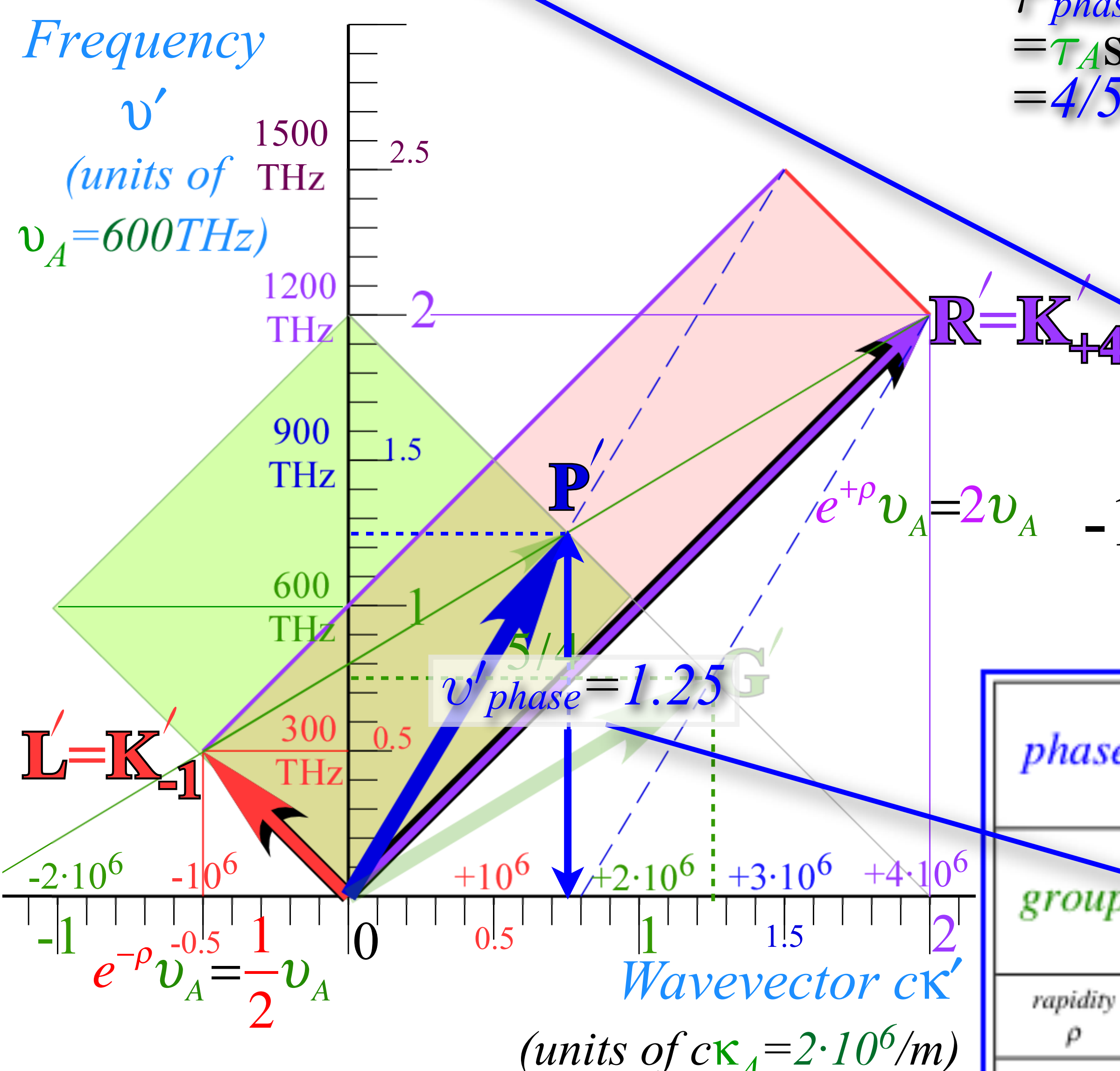
The 16 dimensions of 2CW interference

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$ flips to *Phase period* $\tau = 1/\upsilon$: $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

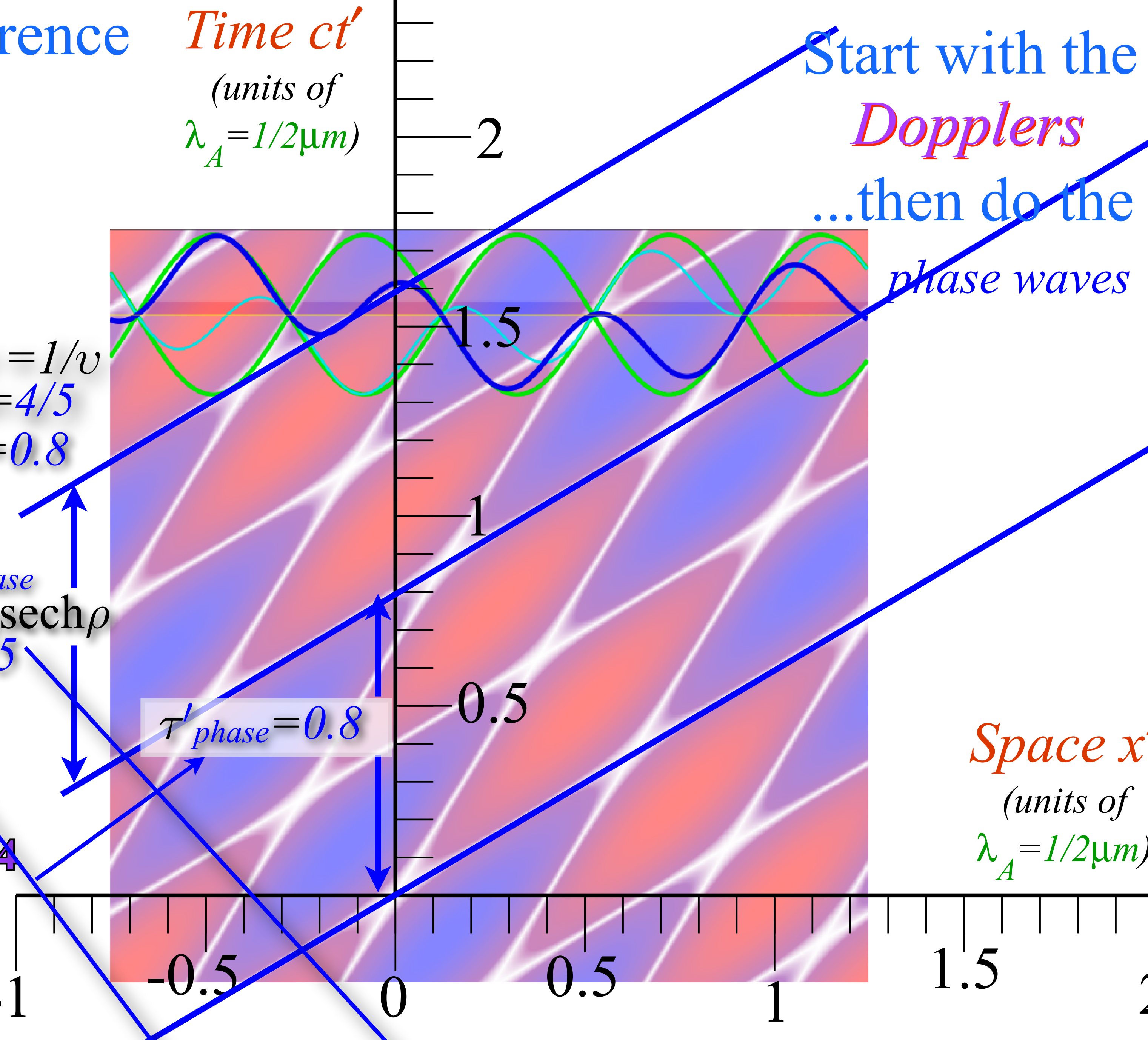
The 16 dimensions of 2CW interference

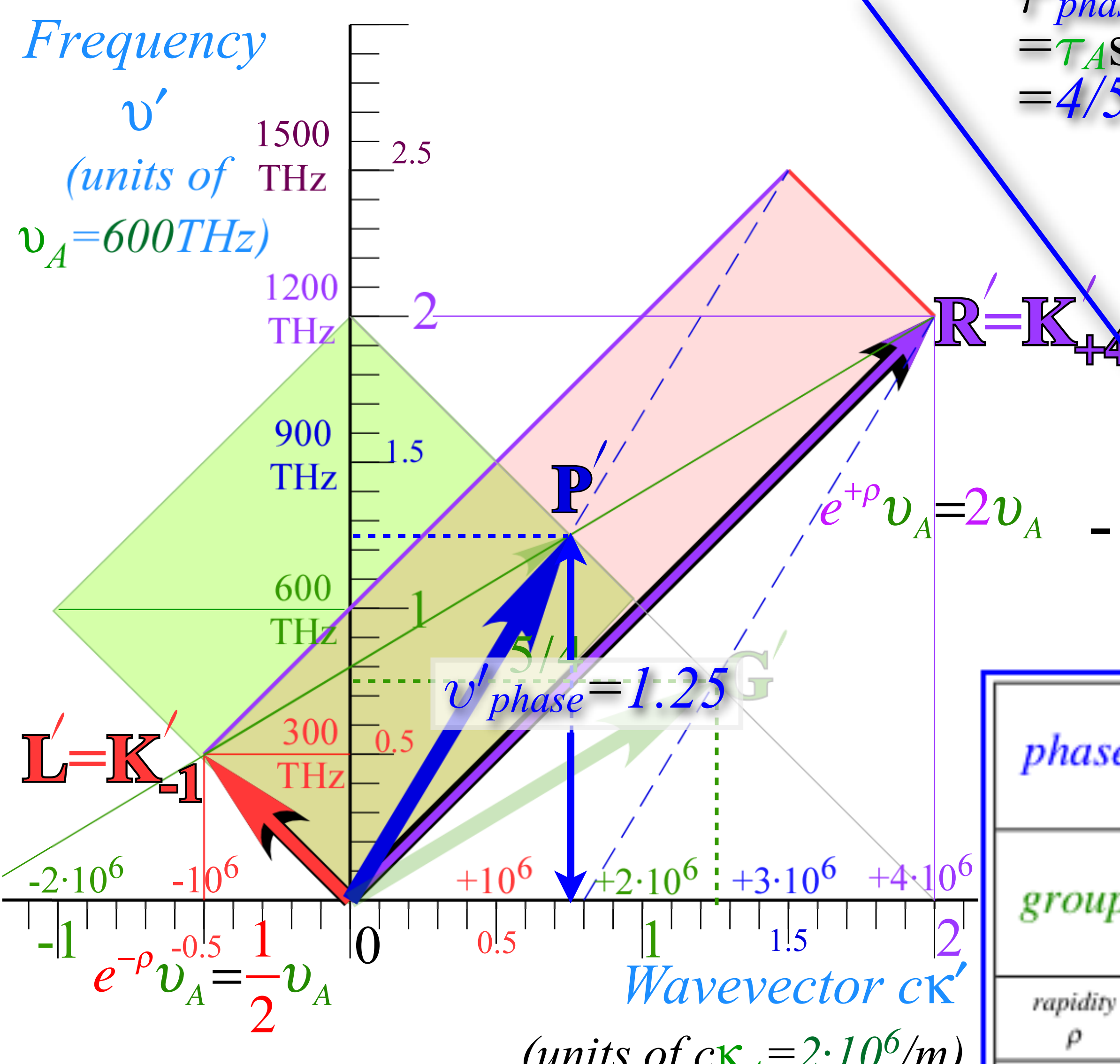
$$\mathbf{P'} = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

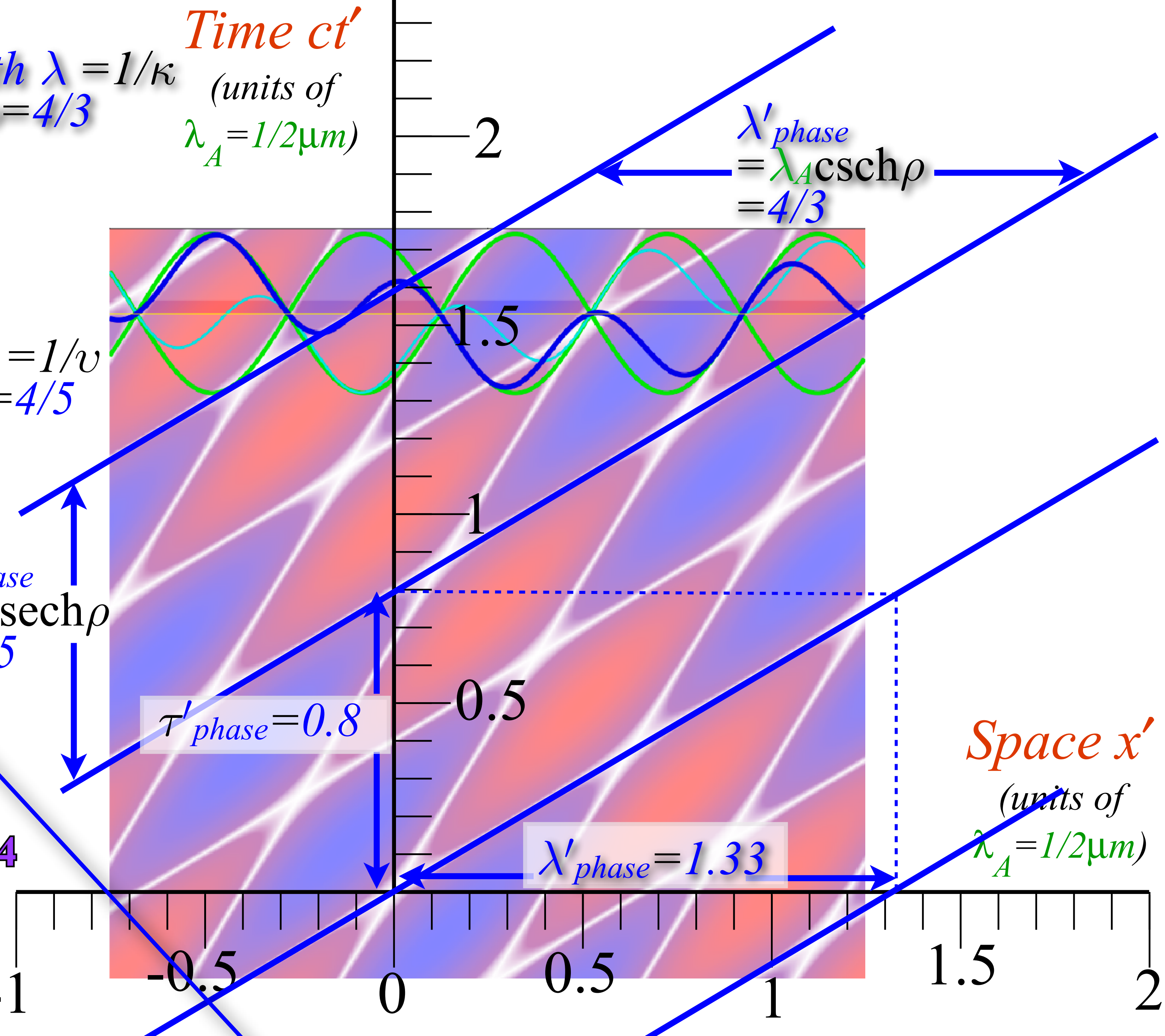
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

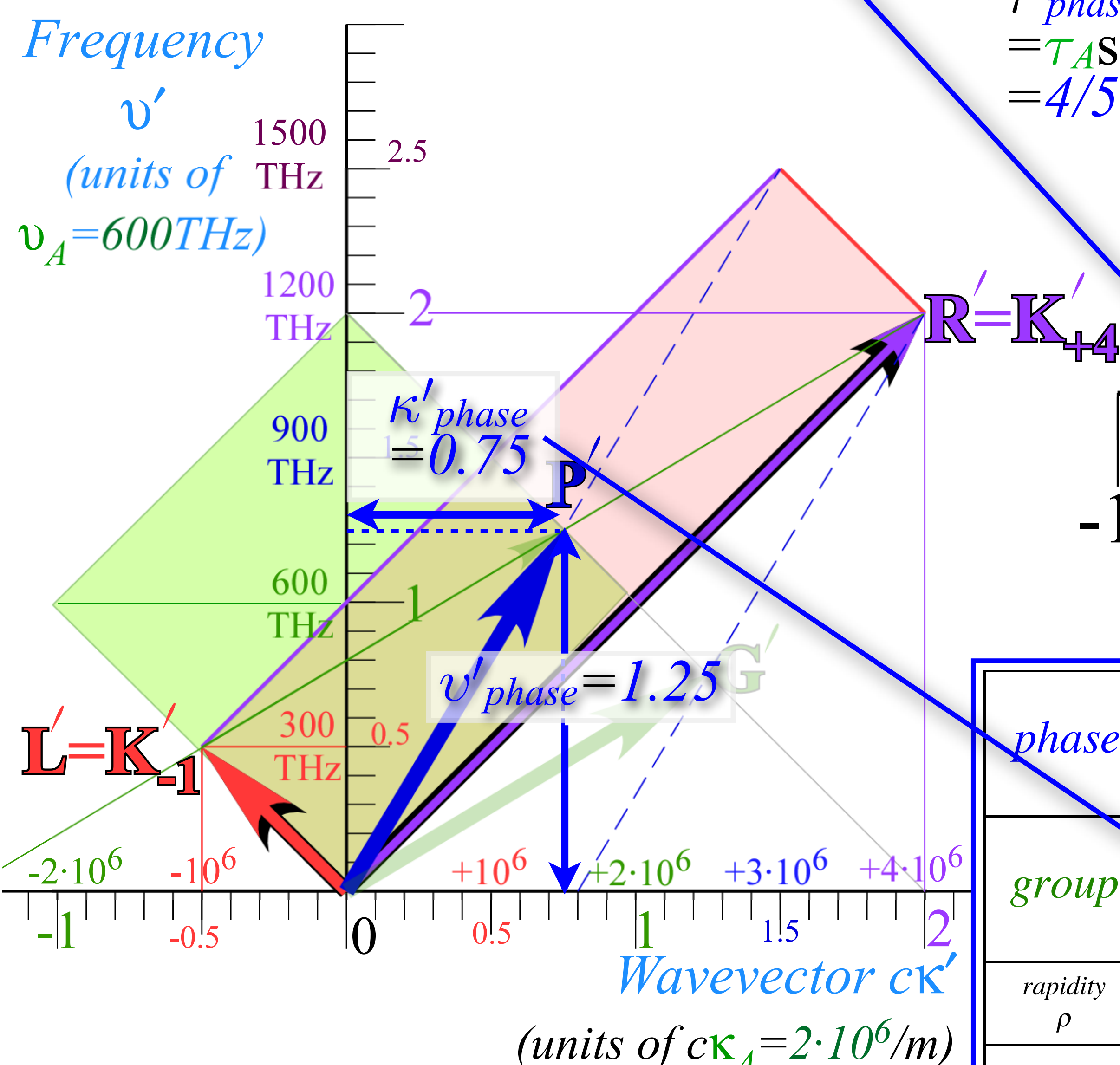
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

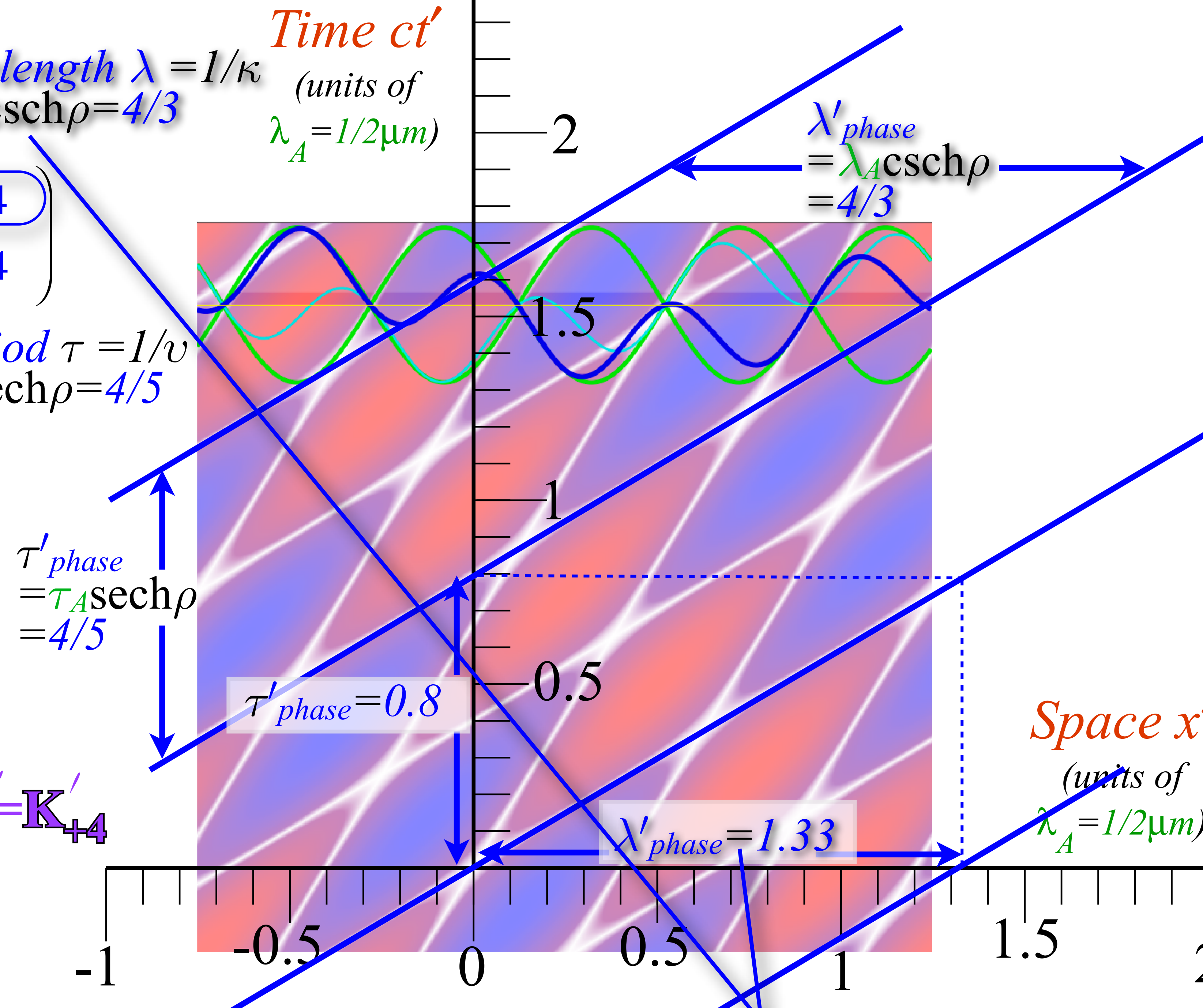
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

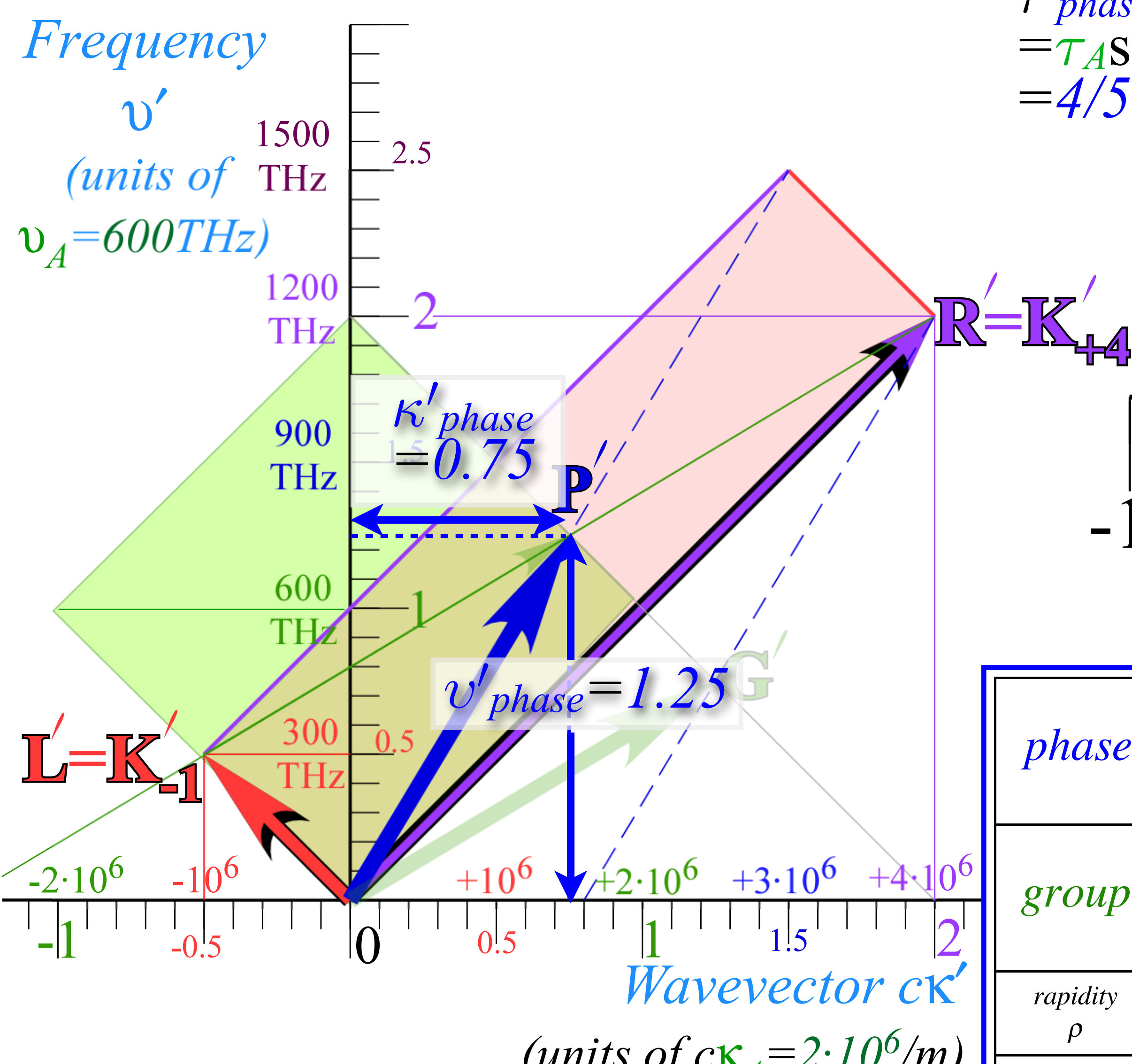
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

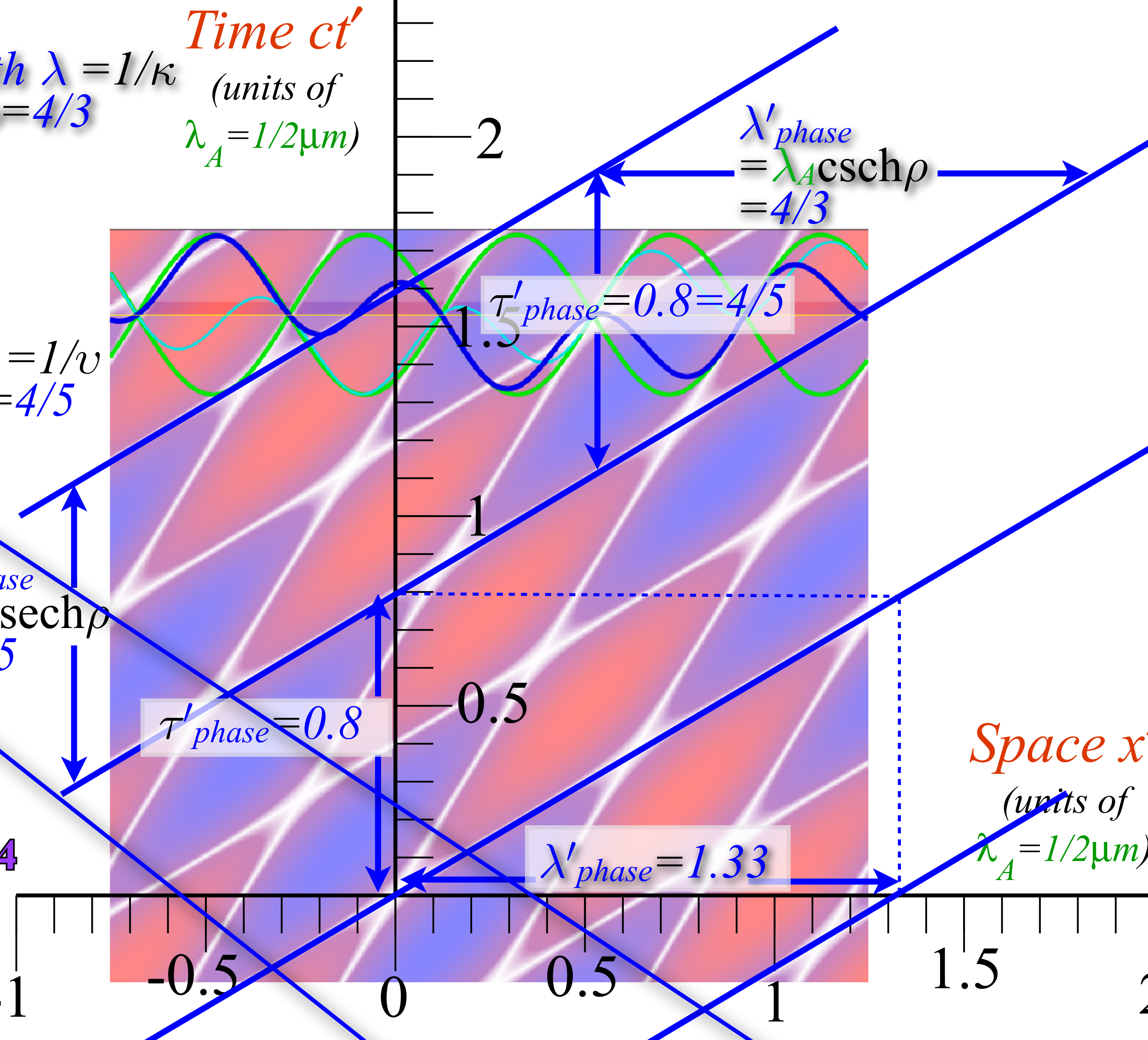
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

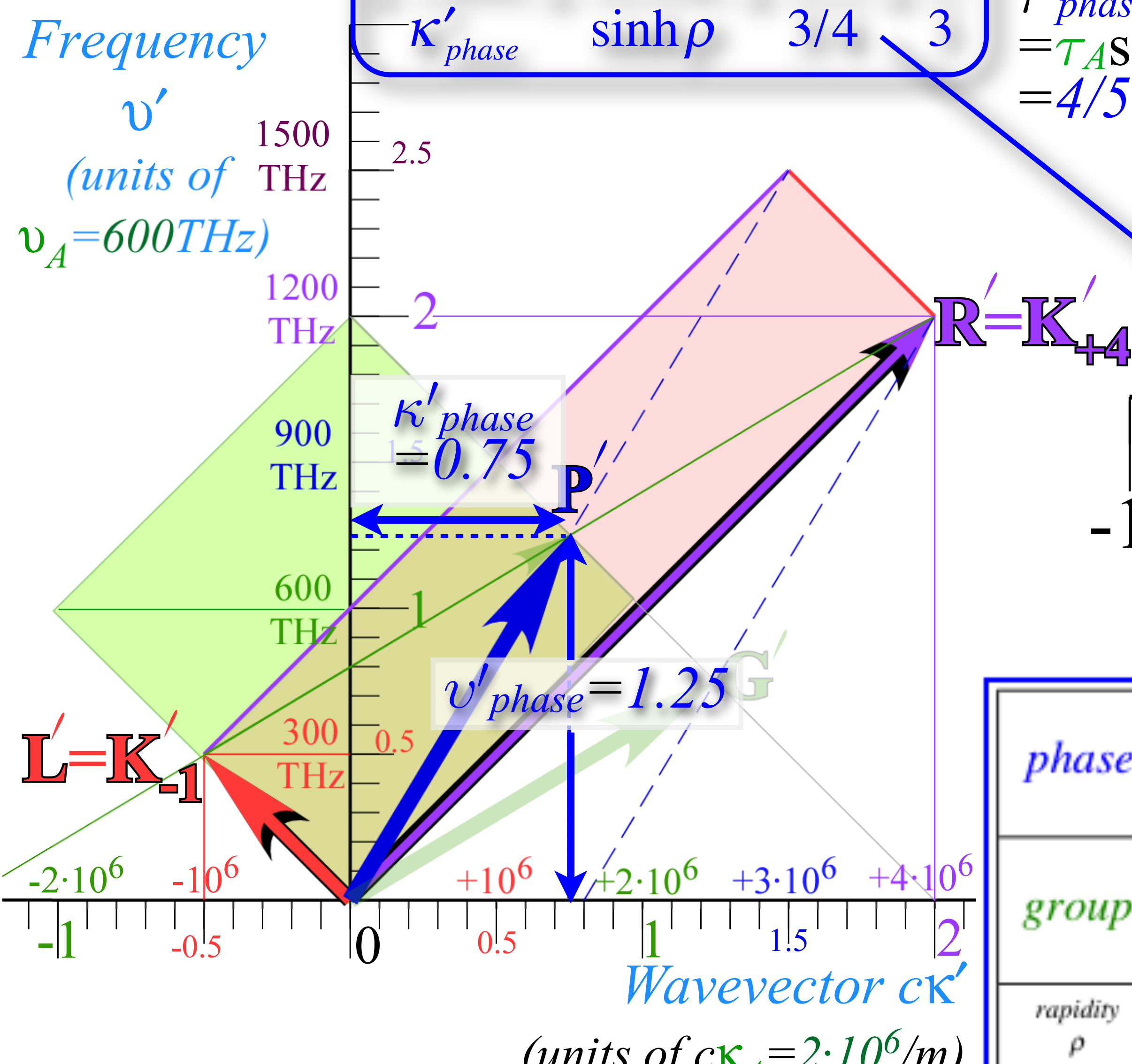
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

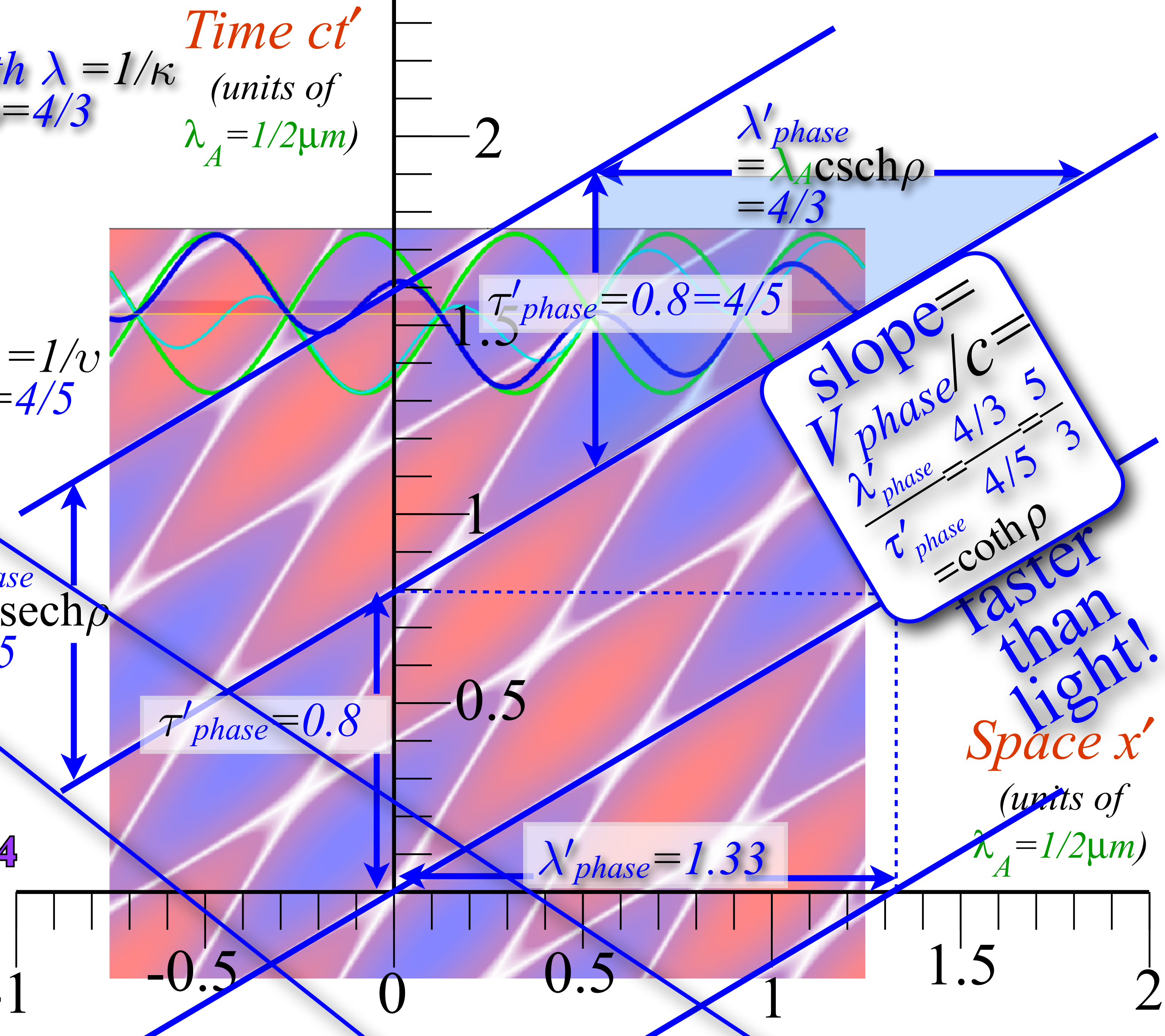
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

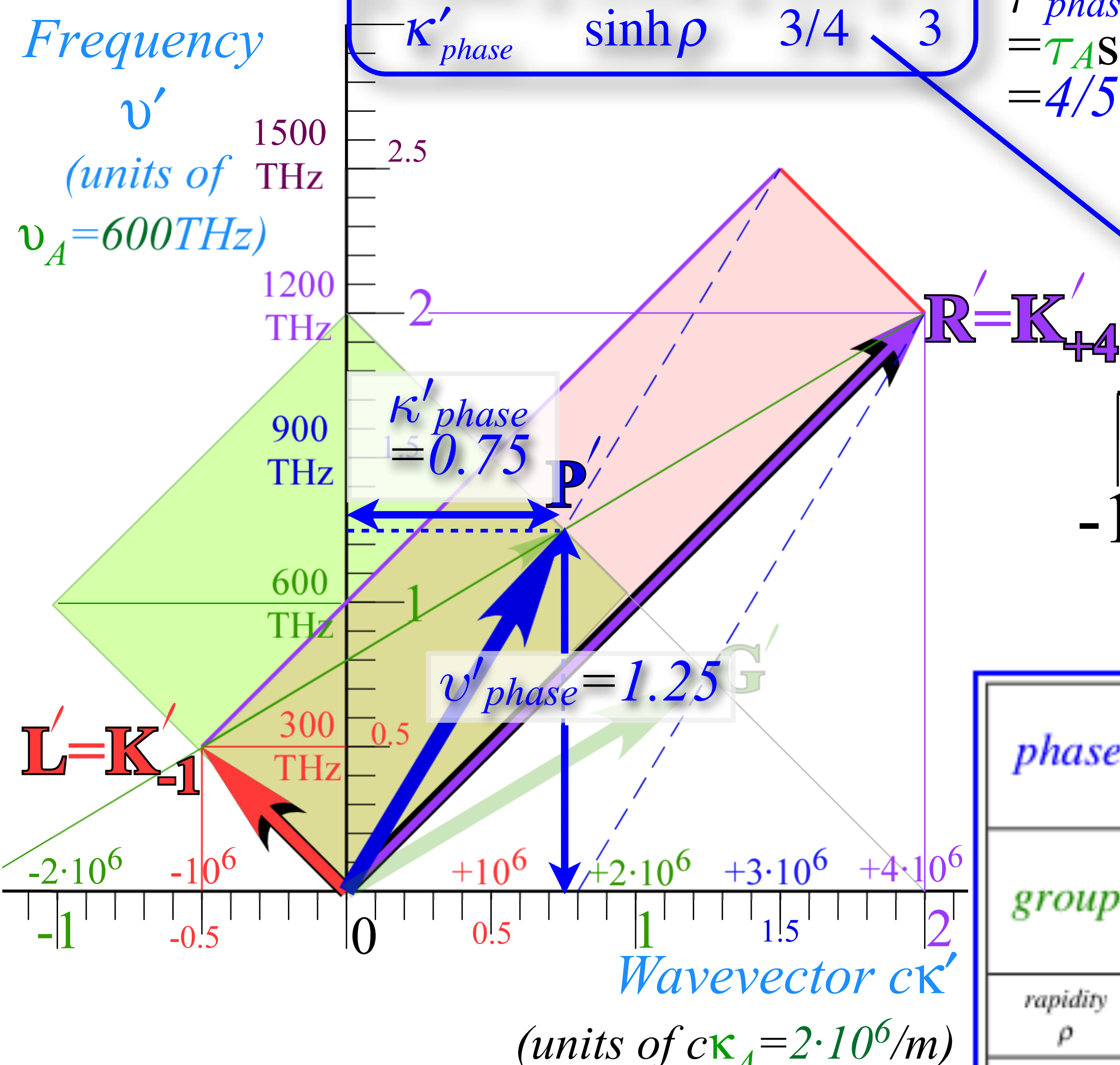
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c$

$$= \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$



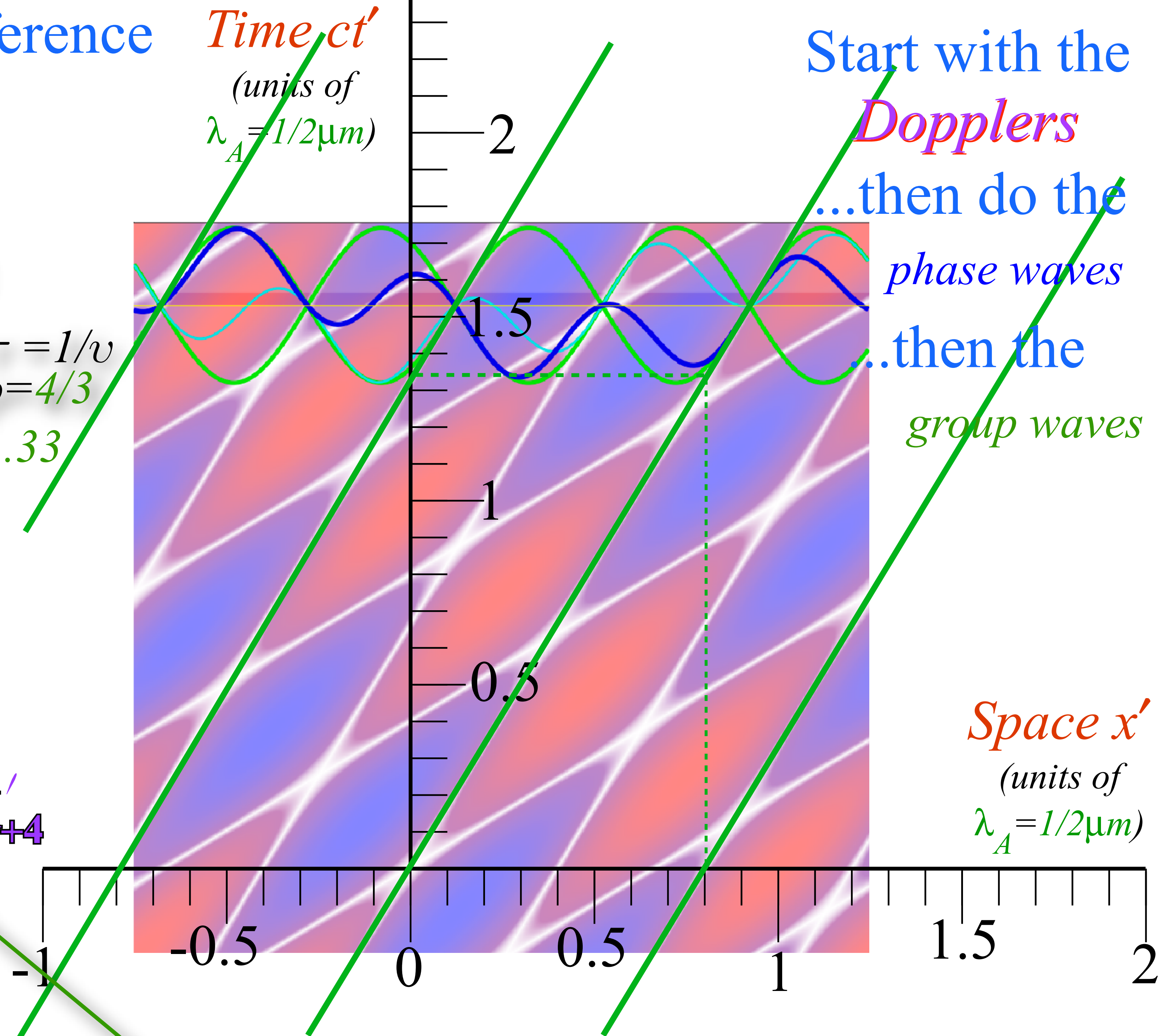


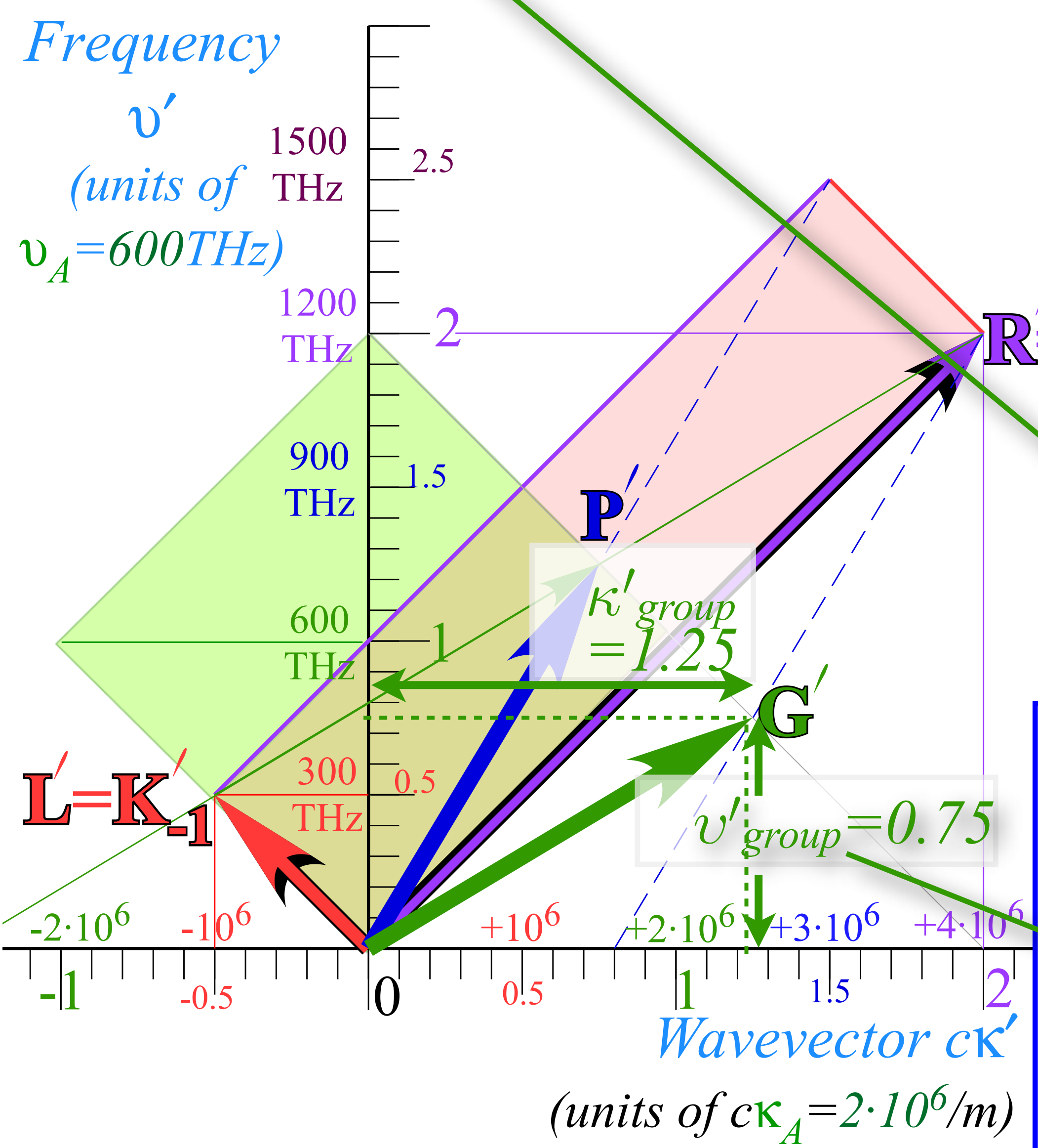
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$ flips to Group period $\tau = 1/\upsilon$ $\tau'_{group} = \tau_A \text{csch}\rho = 4/3 = 1.33$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

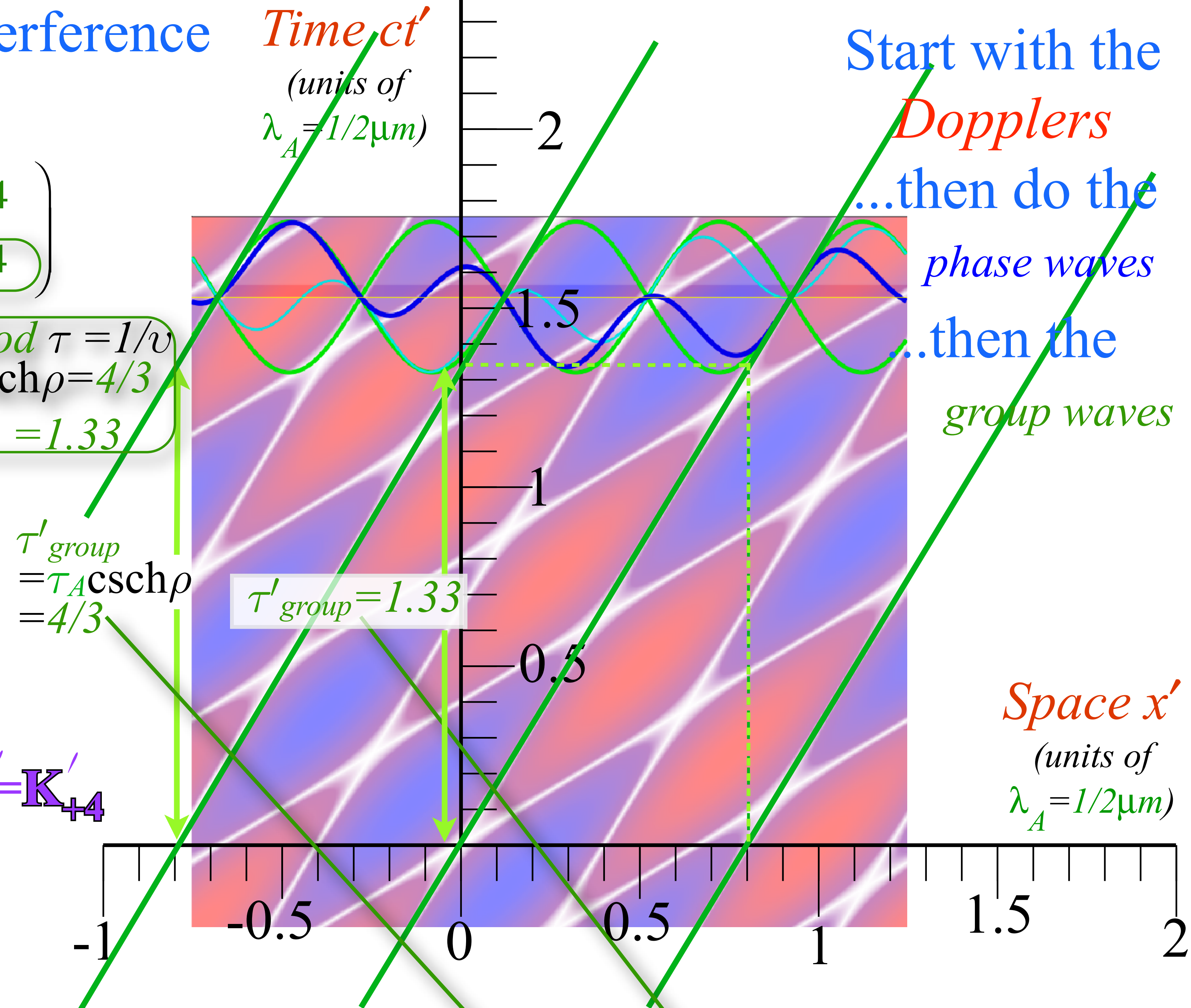
The 16 dimensions of 2CW interference

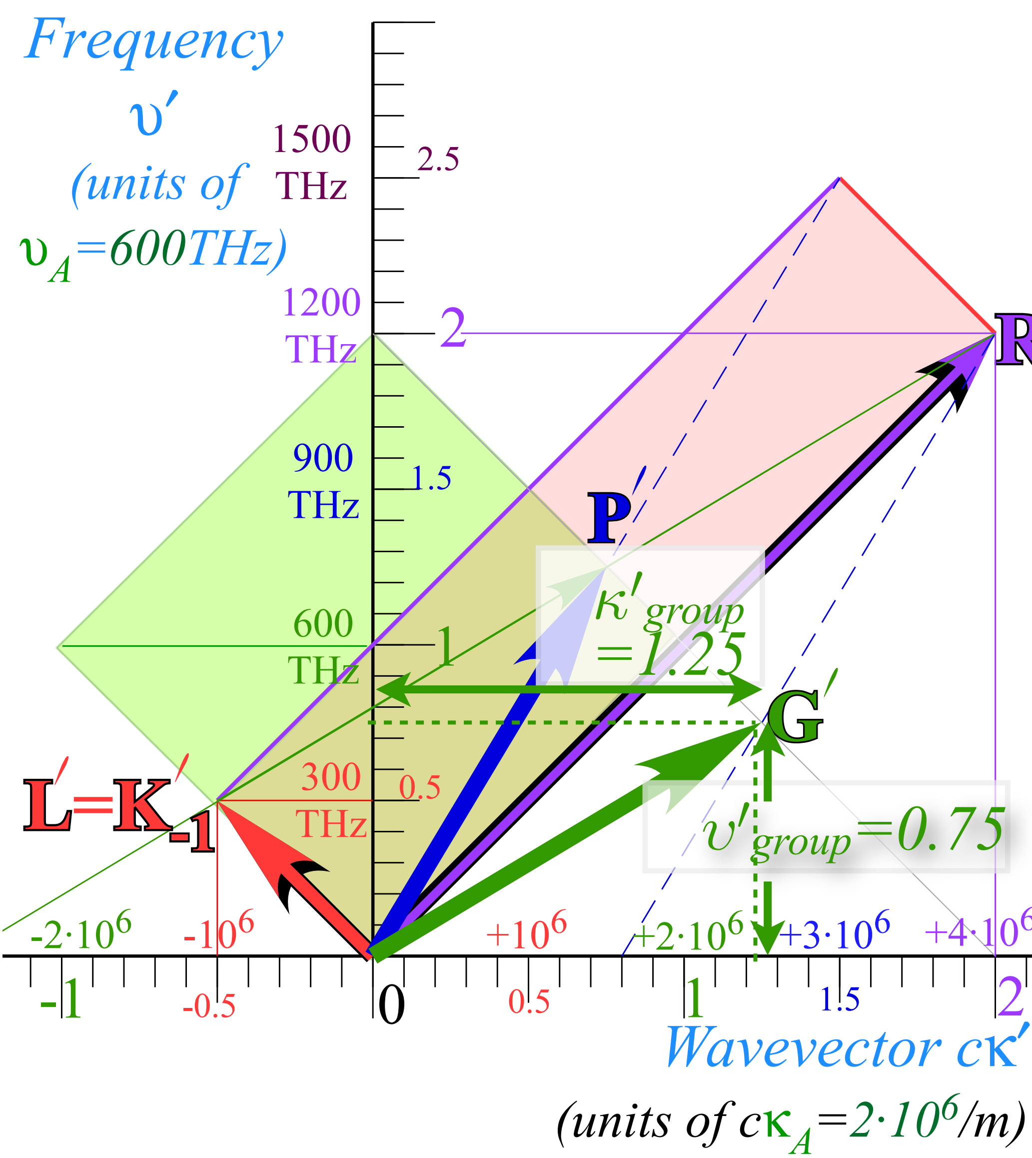
$$\mathbf{G'} = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \mathrm{csch}\rho = 4/3 = 1.33$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Group wavenumber $\kappa'_{group}=\kappa_A\cosh\rho=5/4=1.25$

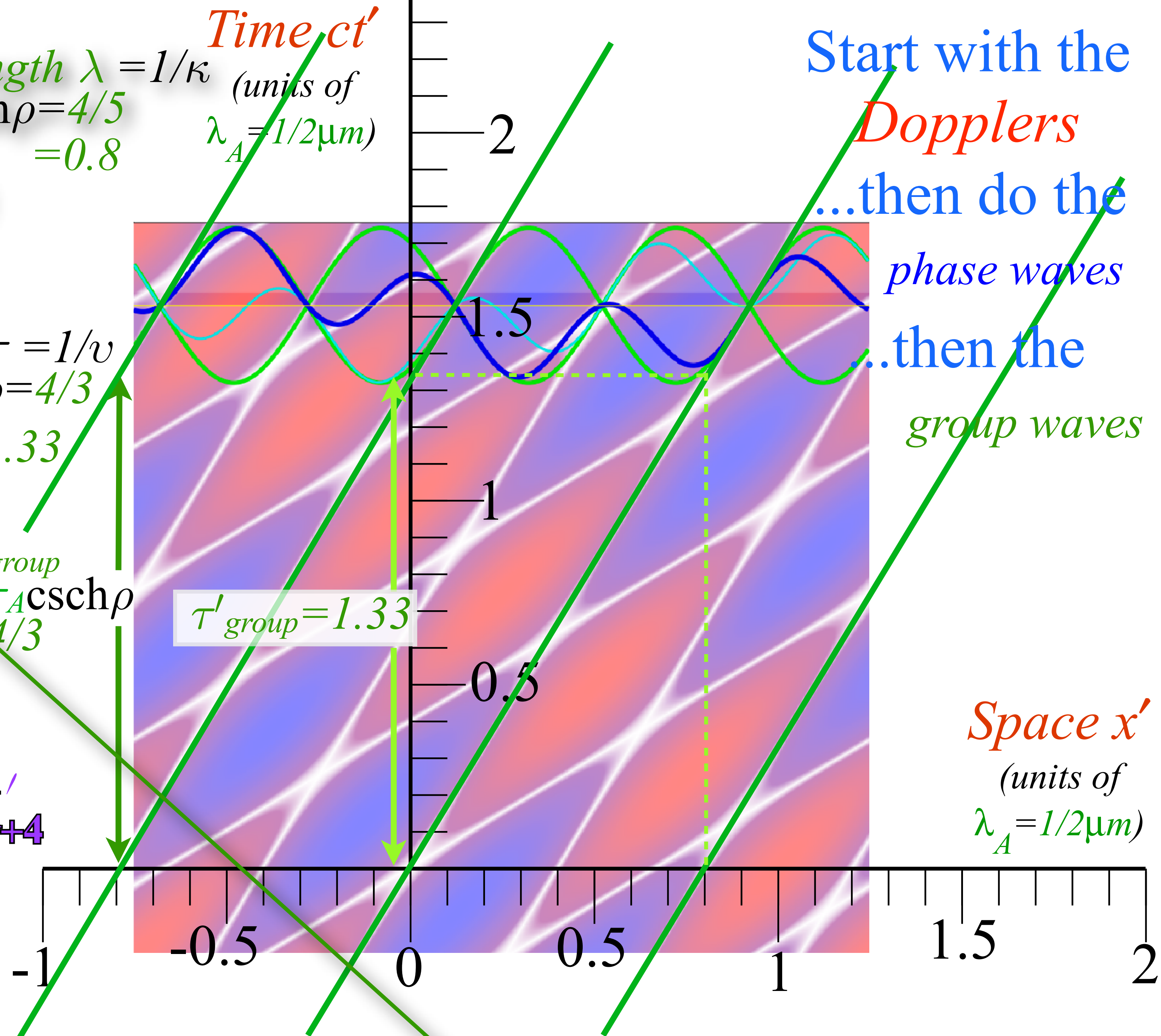
Group wavelength $\lambda=1/\kappa$ $\quad \lambda'_{group}=\lambda_A\mathrm{sech}\rho=4/5=0.8$

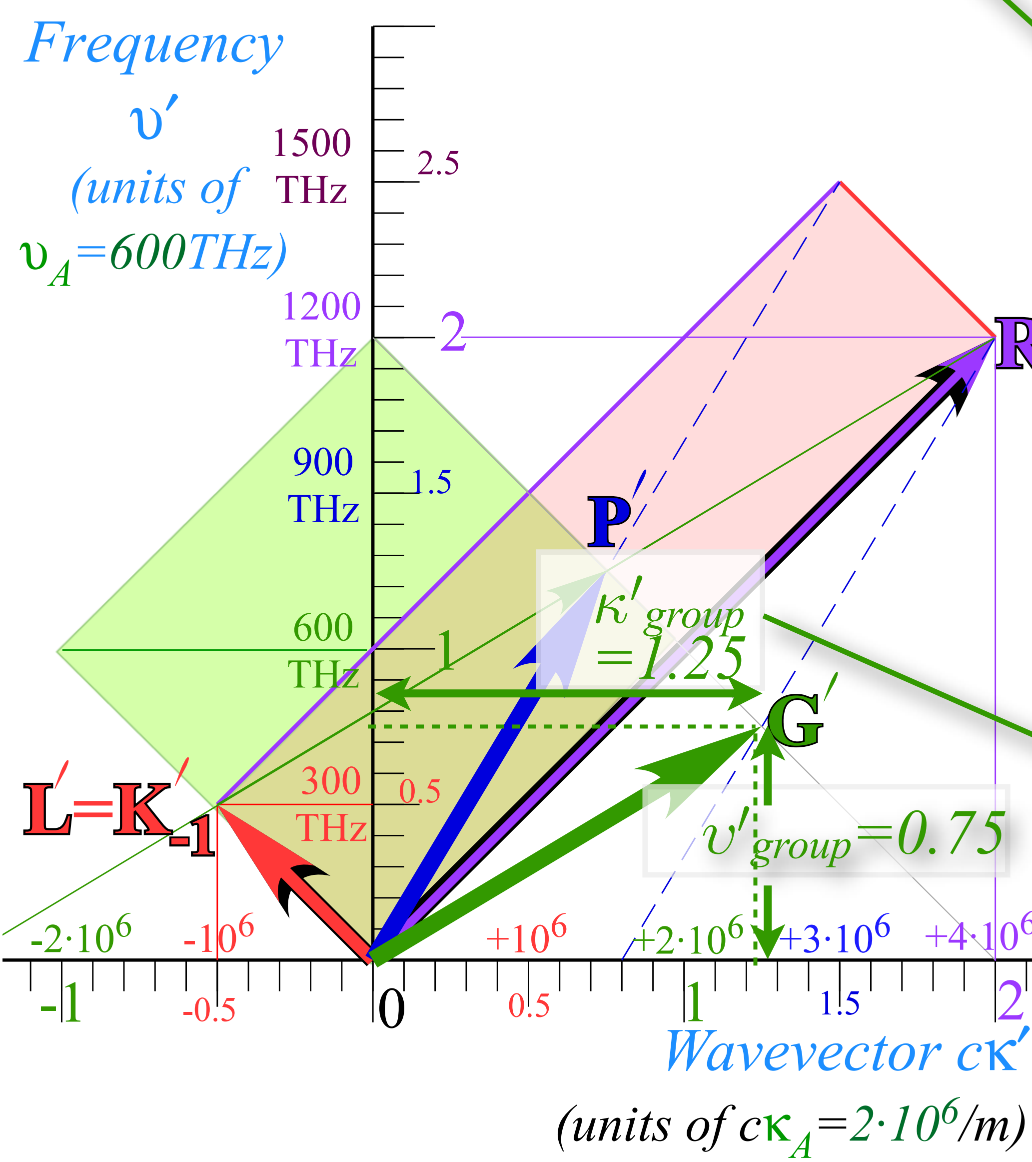
$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho \\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4 \\ 3/4\end{pmatrix}$$

Group frequency $\upsilon'_{group}=\upsilon_A\sinh\rho=3/4=0.75$

flips to

Group period $\tau=1/\upsilon$ $\quad \tau'_{group}=\tau_A\mathrm{csch}\rho=4/3=1.33$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Group wavenumber
$\kappa'_{group}=\kappa_A\cosh\rho=5/4$
$=1.25$

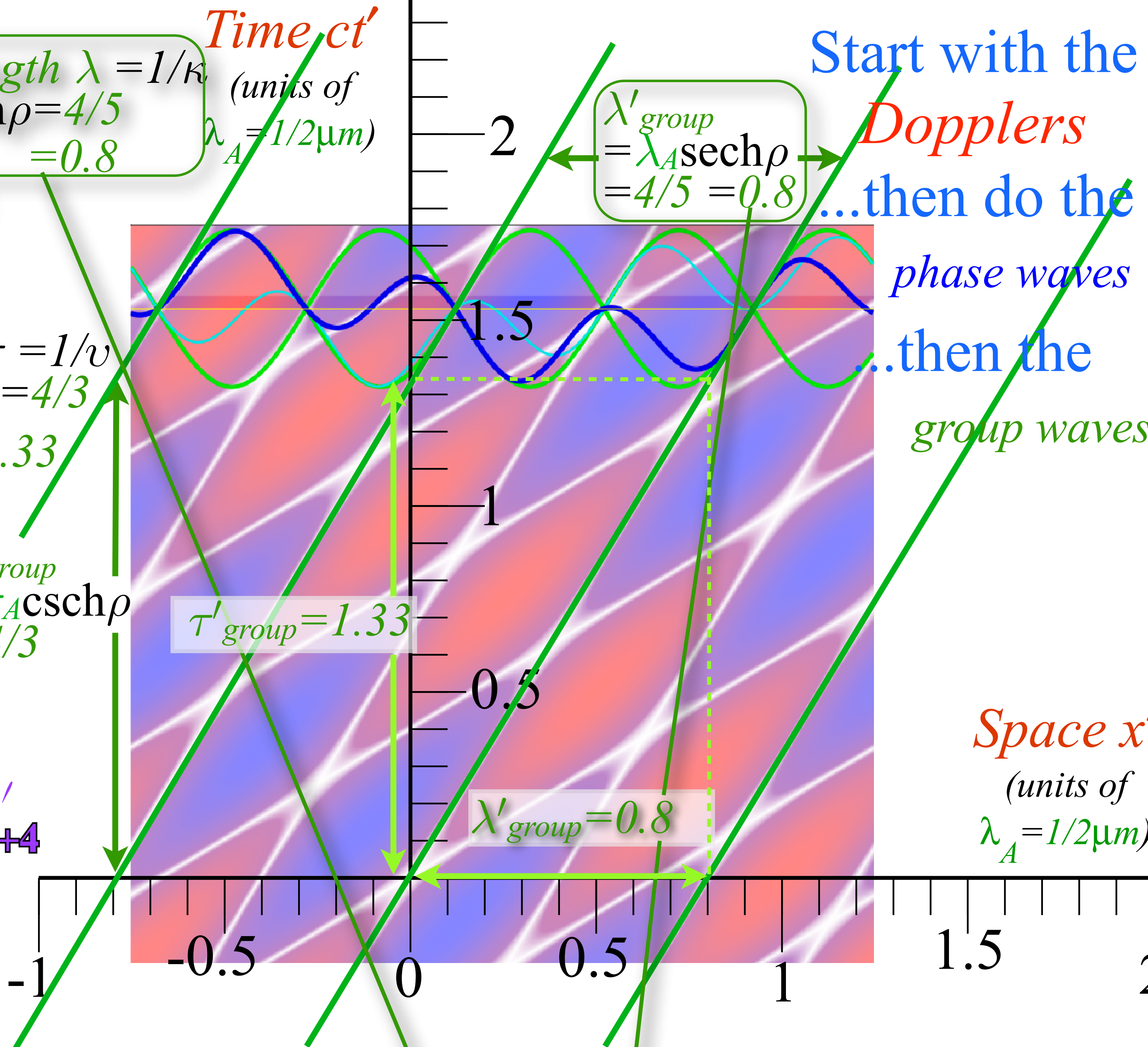
Group wavelength $\lambda=1/\kappa$
$\lambda'_{group}=\lambda_A\text{sech}\rho=4/5$
$=0.8$

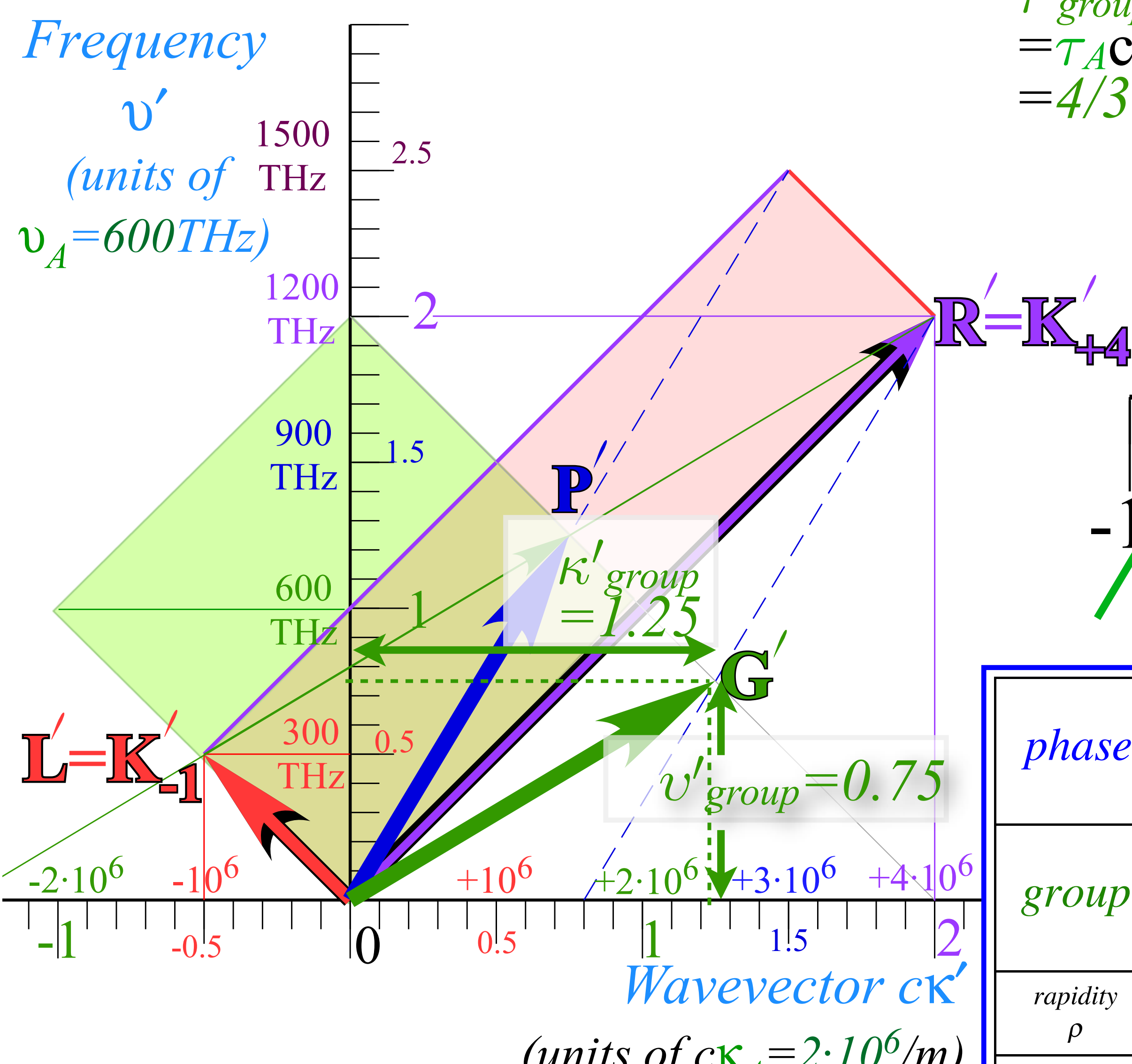
$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho \\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4 \\ 3/4\end{pmatrix}$$

Group frequency
$\upsilon'_{group}=\upsilon_A\sinh\rho=3/4$
$=0.75$

flips to

Group period $\tau=1/\upsilon$
$\tau'_{group}=\tau_A\text{csch}\rho=4/3$
$=1.33$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4$
$= 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \mathrm{sech}\rho = 4/5$
$= 0.8$

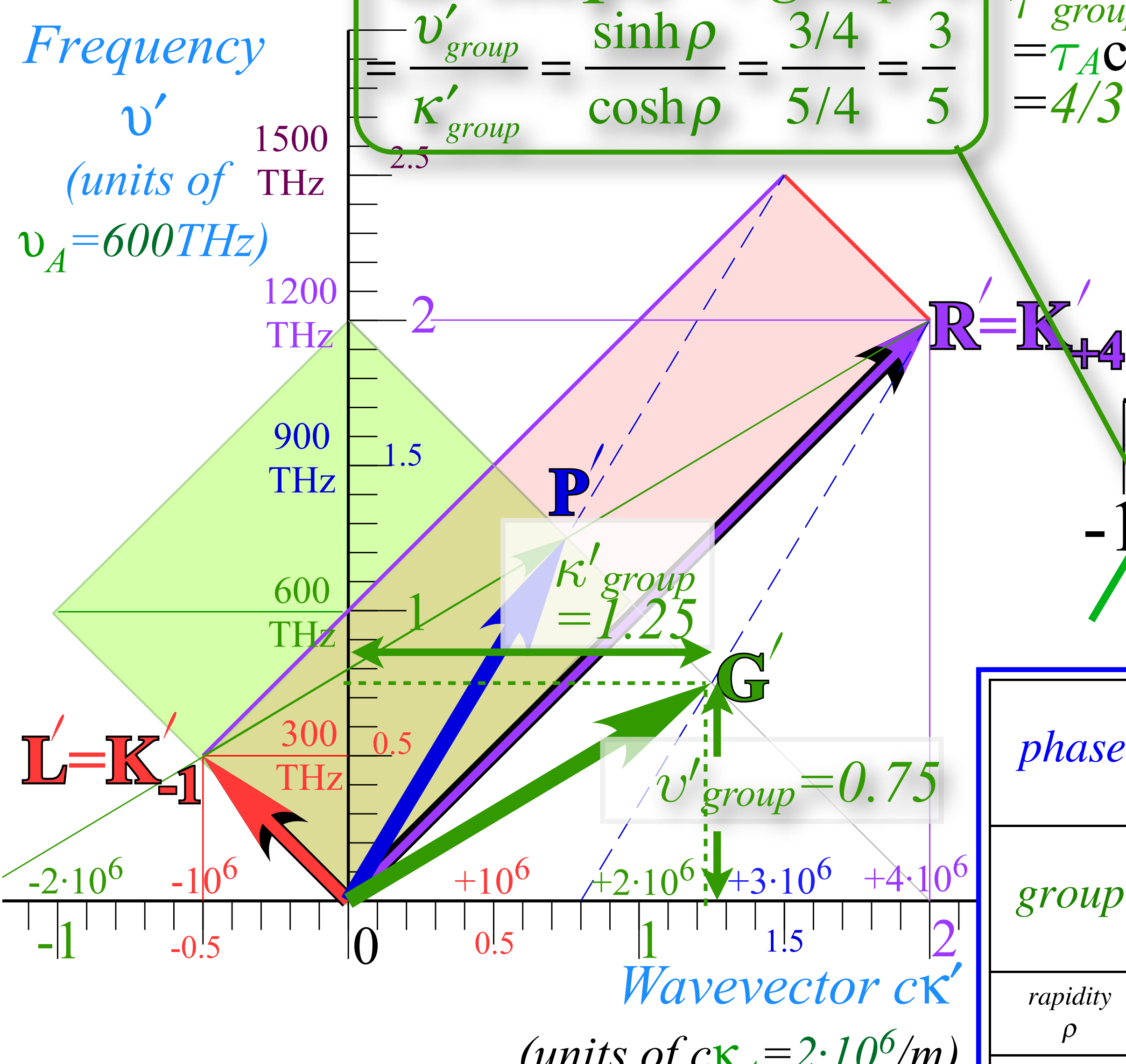
$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

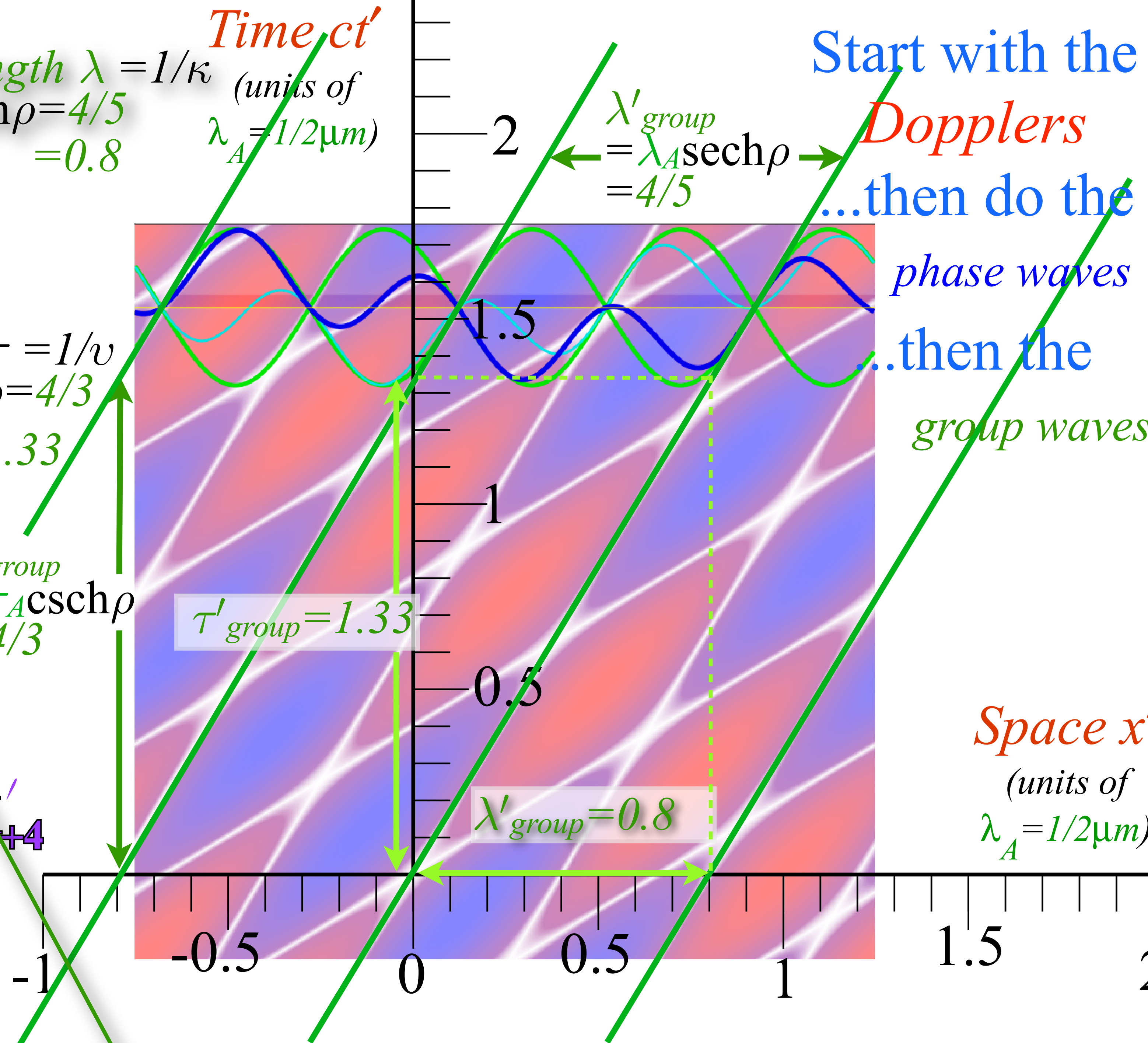
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4$
$= 0.75$

flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \mathrm{csch}\rho = 4/3$
$= 1.33$

G-slope $= V_{group}/c$
$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$





Start with the
Dopplers
...then do the
phase waves
...then the
group waves

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

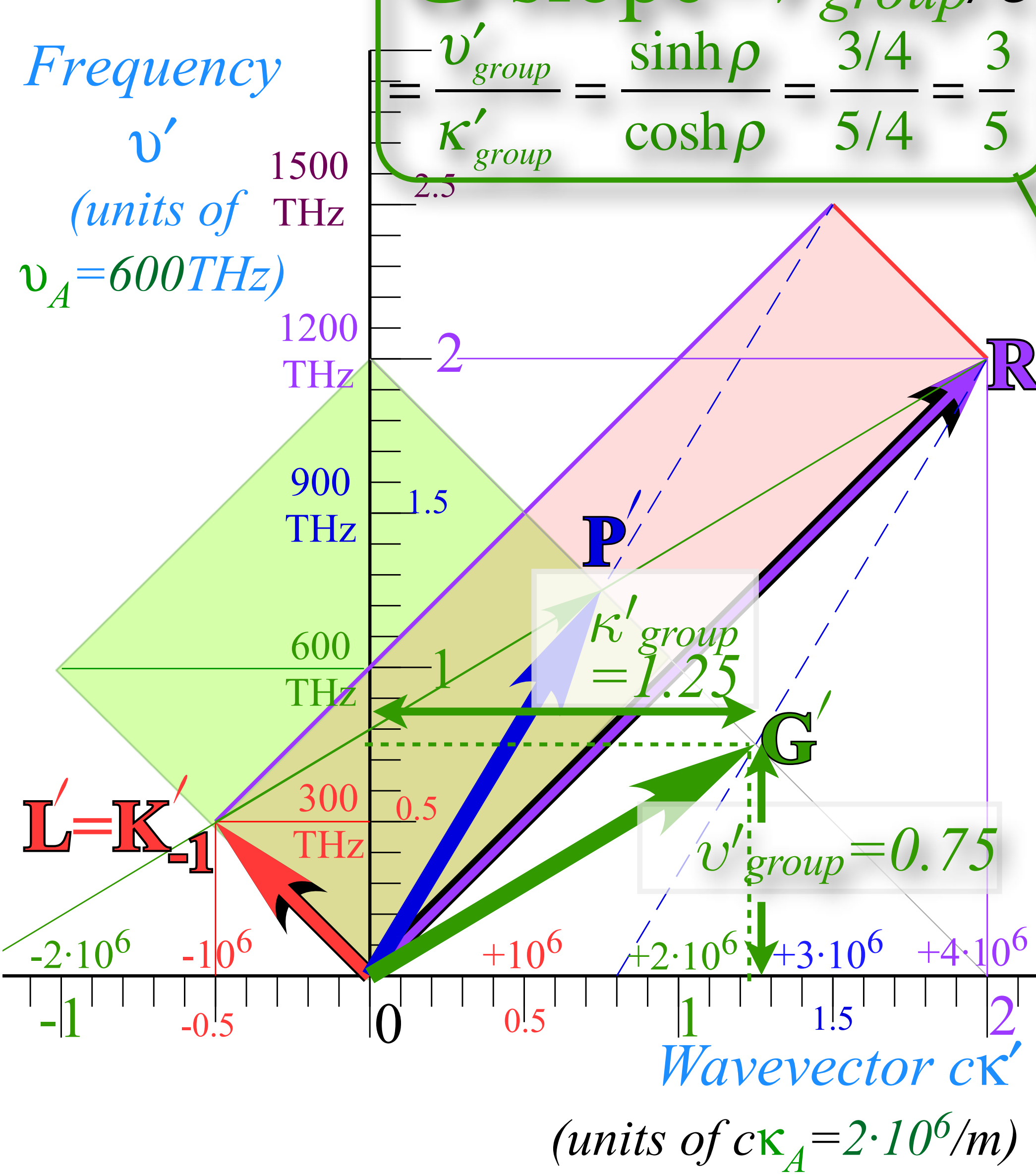
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

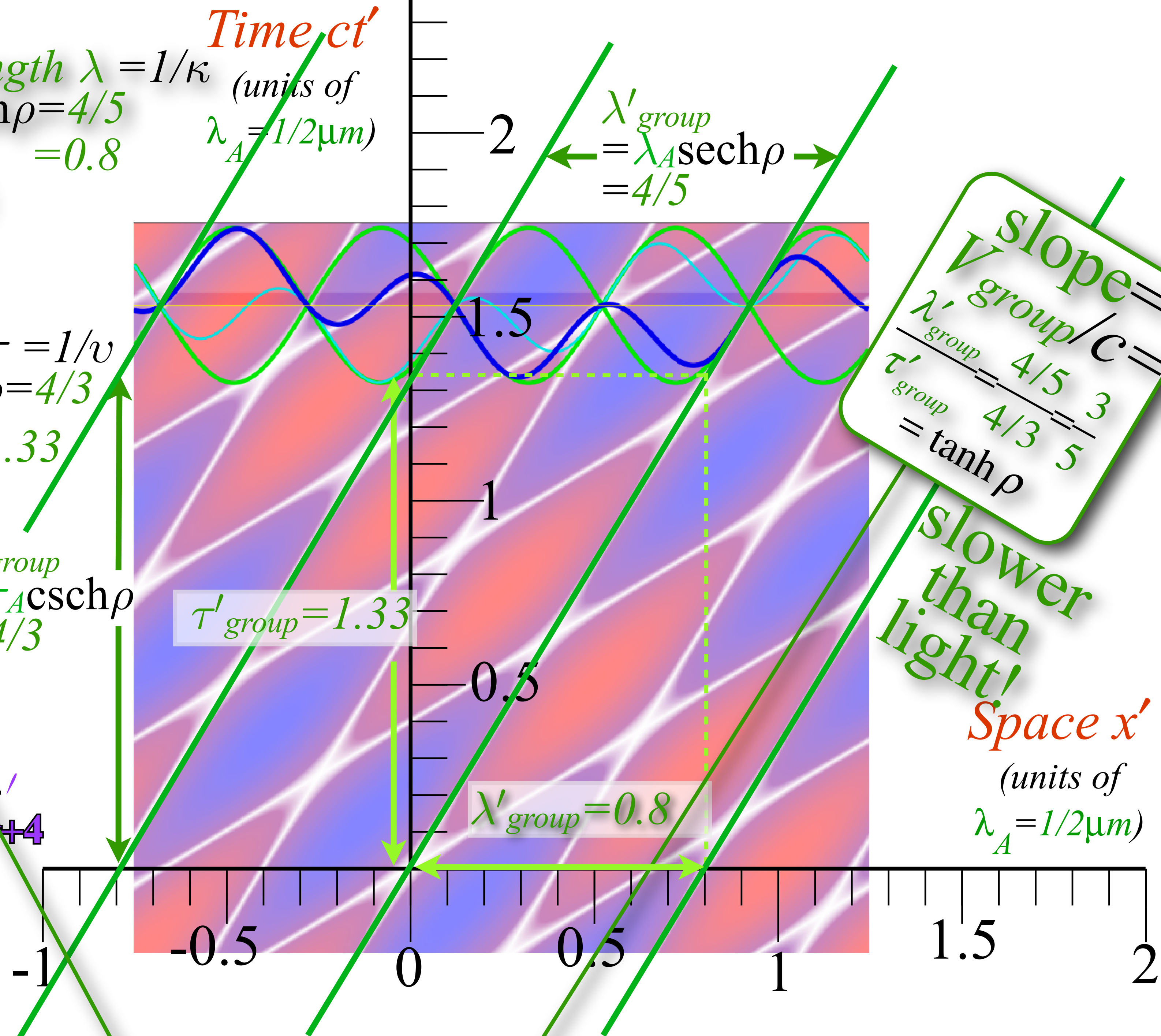
flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c = \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Lorentz transformations...

write **G′** *and* **P′** *in terms of* **G** *and* **P** *using* coshρ *and* sinhρ

$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho \\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4 \\ 3/4\end{pmatrix}$$

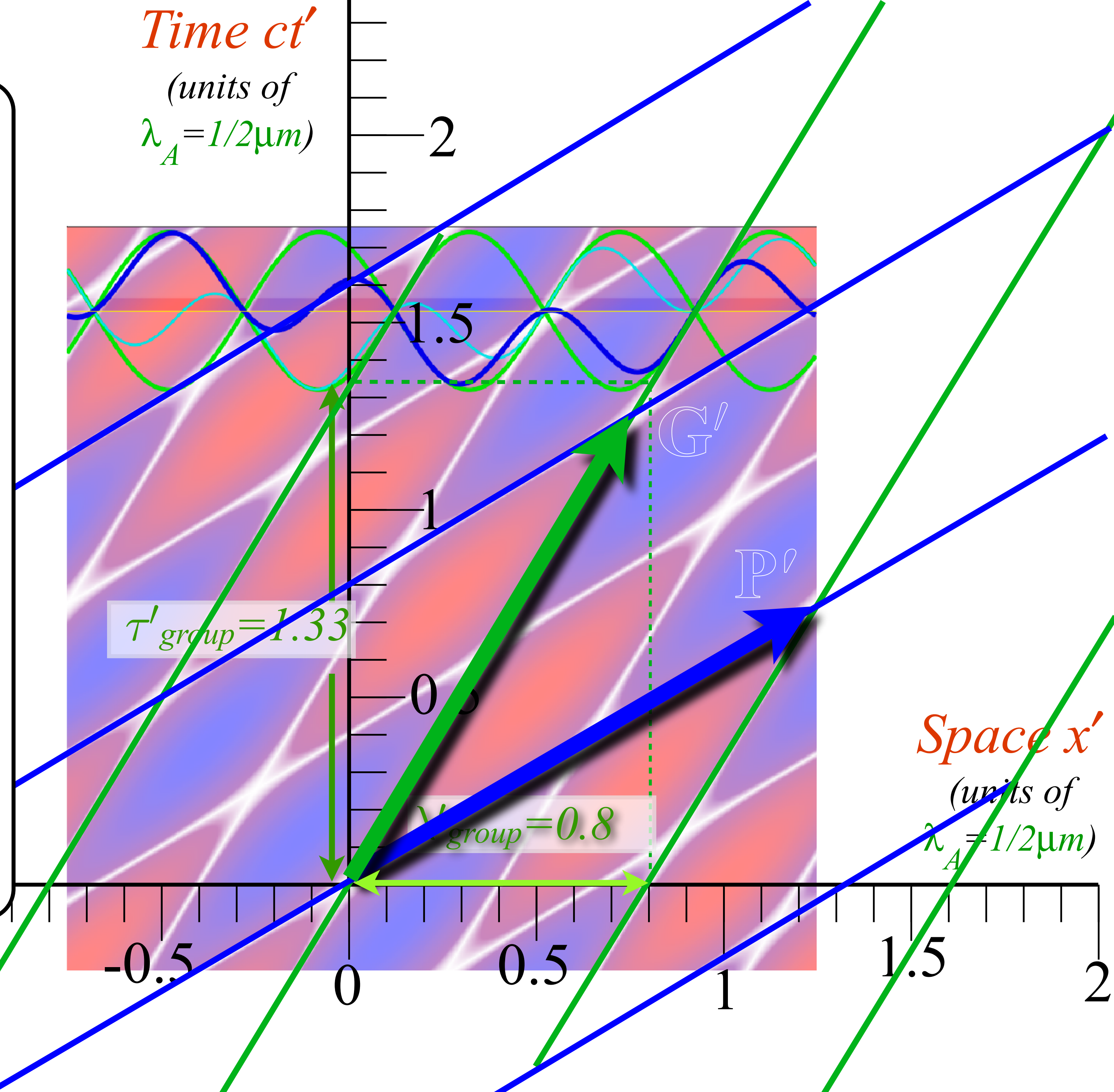
$$=\upsilon_A\begin{pmatrix}1 \\ 0\end{pmatrix}\cosh\rho+\upsilon_A\begin{pmatrix}0 \\ 1\end{pmatrix}\sinh\rho$$

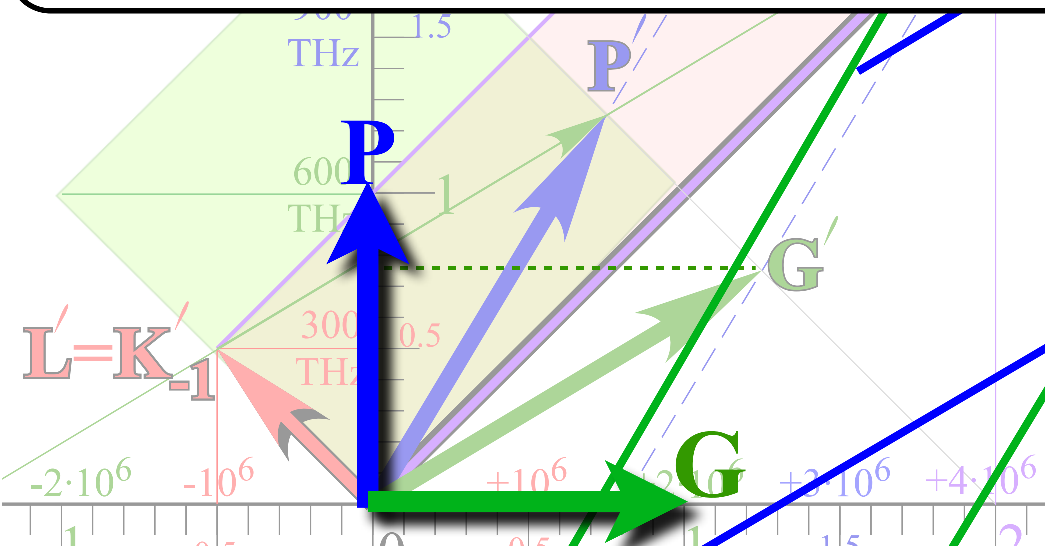
$$\mathbf{G}'=\quad\mathbf{G}\quad\cosh\rho+\quad\mathbf{P}\quad\sinh\rho$$

$$\mathbf{P}'=\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho \\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}3/4 \\ 5/4\end{pmatrix}$$

$$=\upsilon_A\begin{pmatrix}1 \\ 0\end{pmatrix}\sinh\rho+\upsilon_A\begin{pmatrix}0 \\ 1\end{pmatrix}\cosh\rho$$

$$\mathbf{P}'=\quad\mathbf{G}\quad\sinh\rho+\quad\mathbf{P}\quad\cosh\rho$$

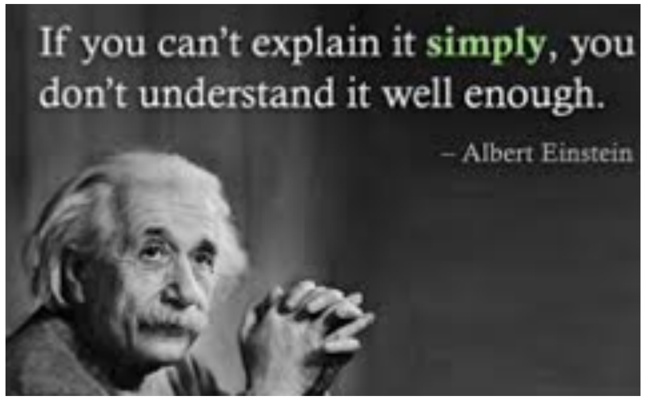




$$\begin{pmatrix}\cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho\end{pmatrix}$$ Lorentz transform matrix

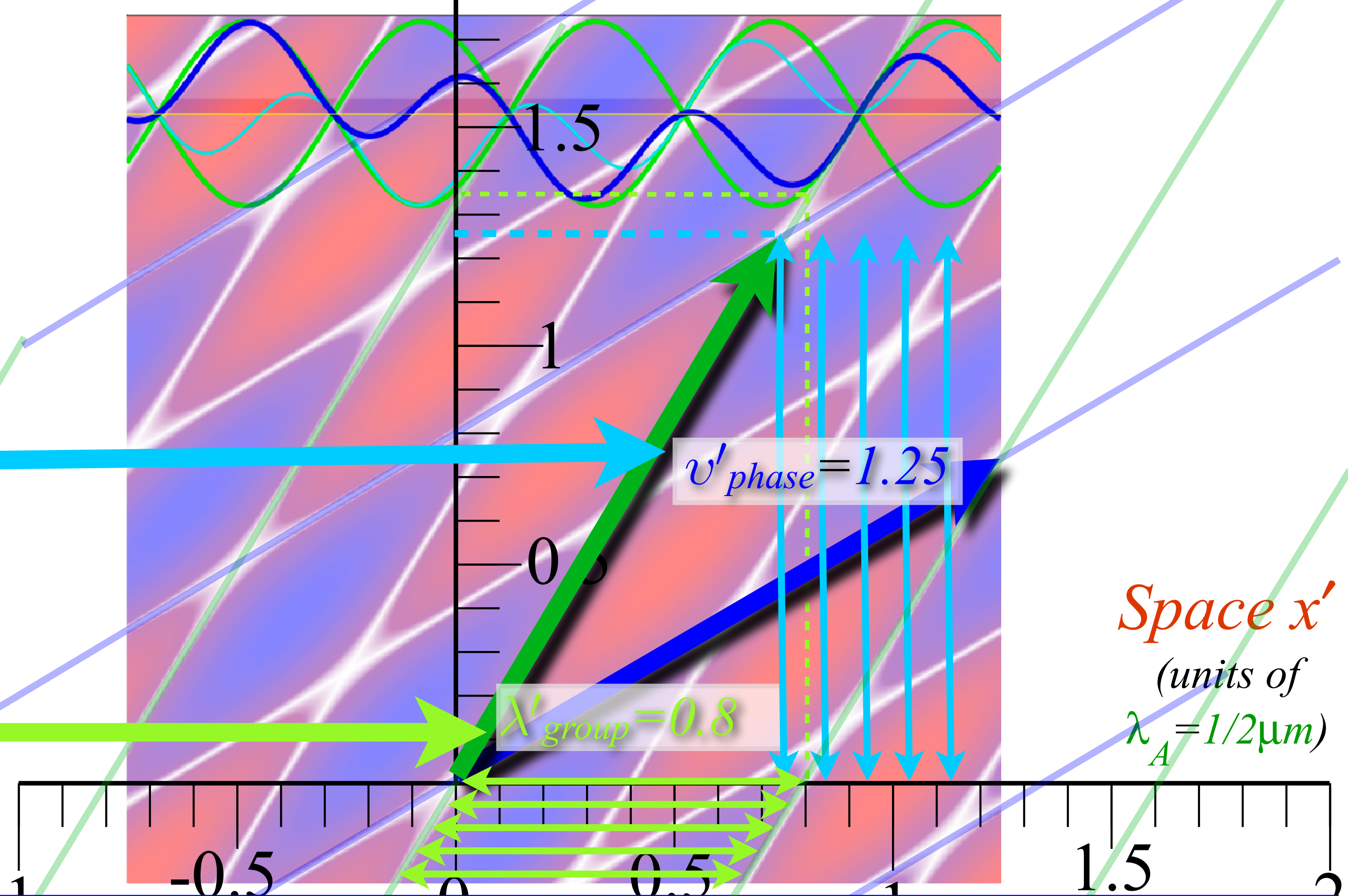
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Two Famous-Name Coefficients



Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)

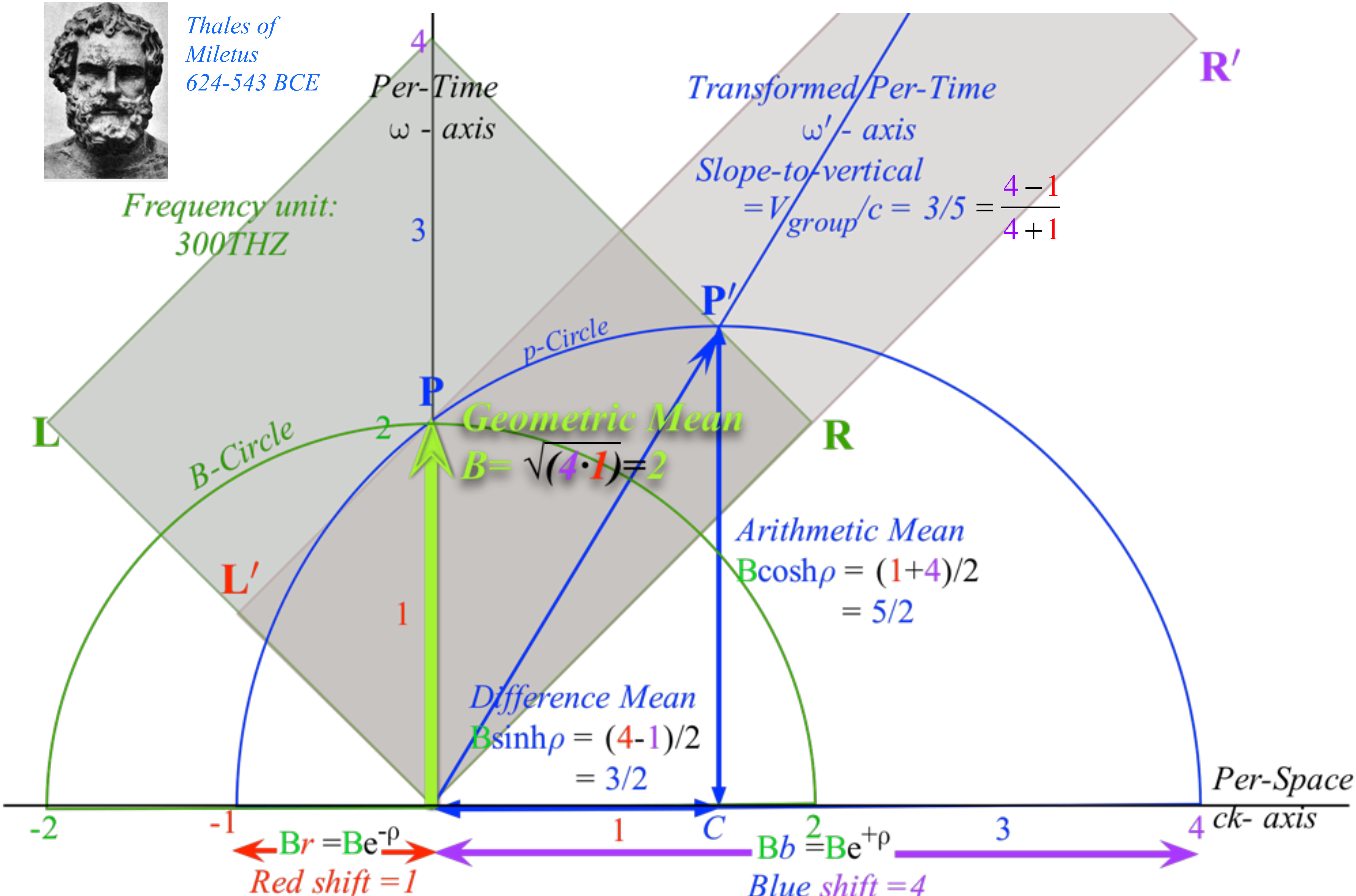
Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

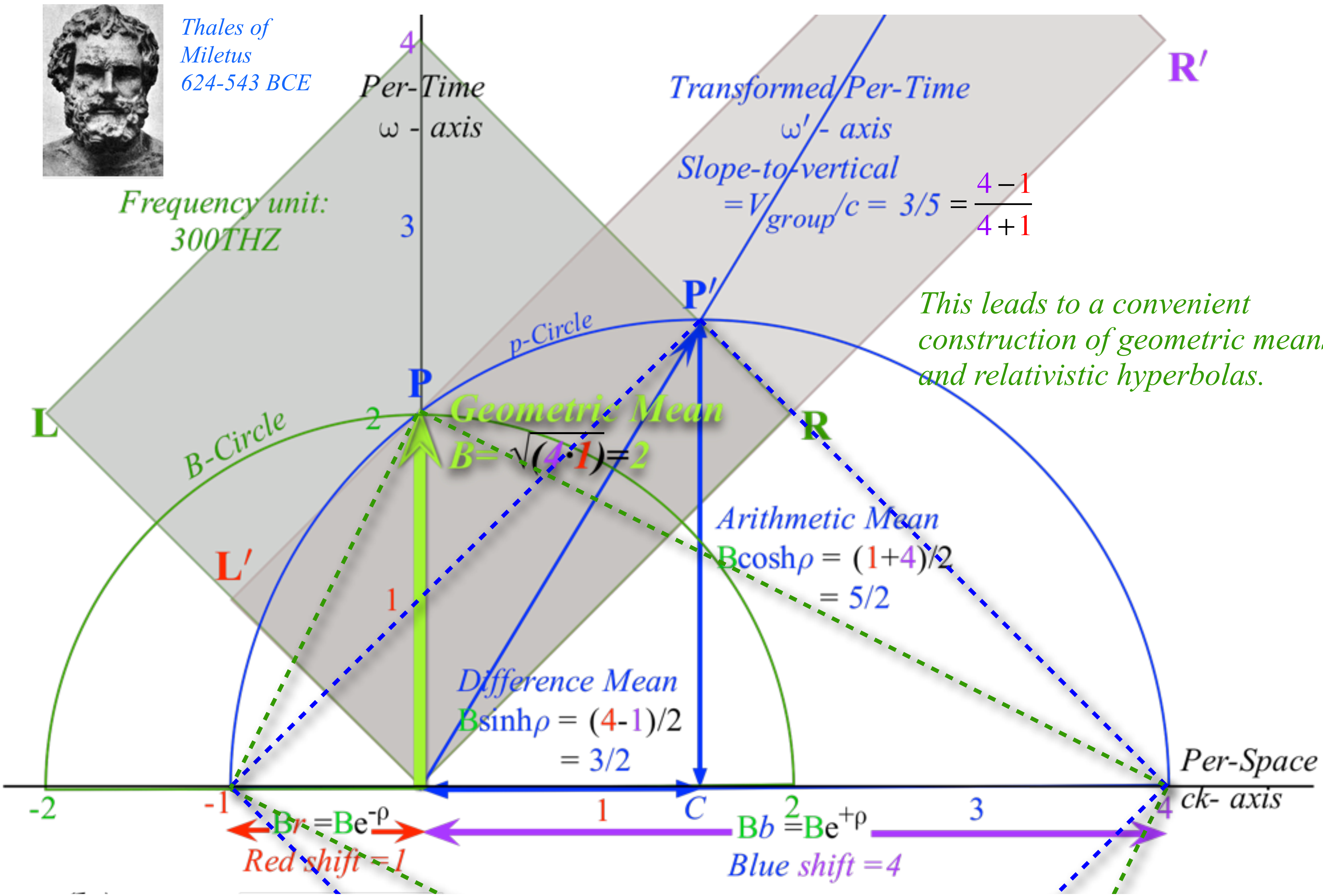
Thales Mean Geometry (600BCE)

helps “Relawavity”



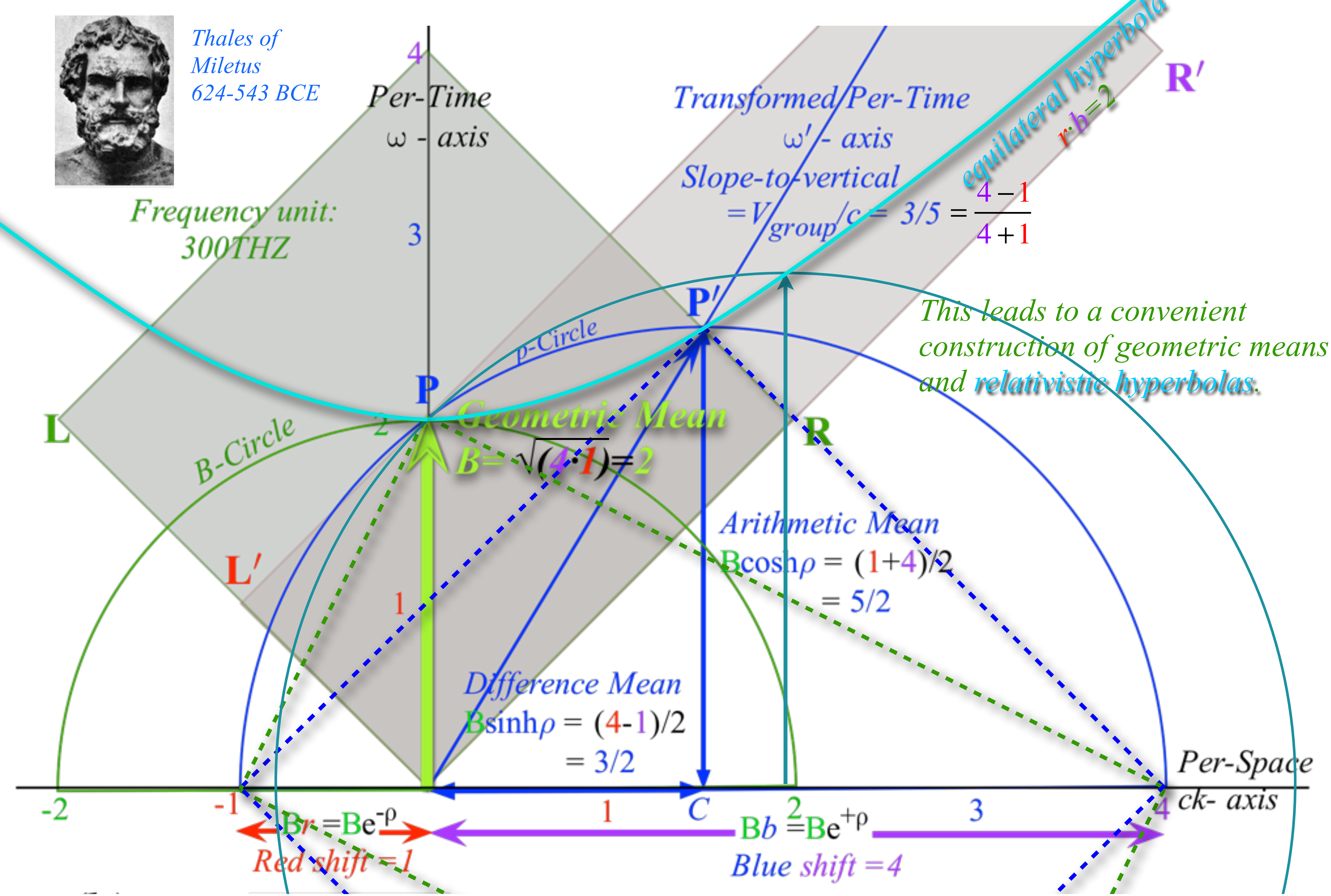
Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



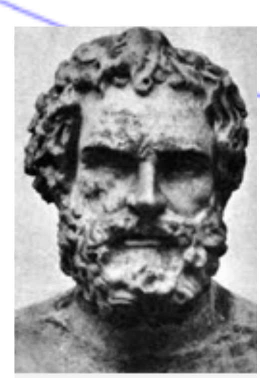
Thales Mean Geometry (600BCE)

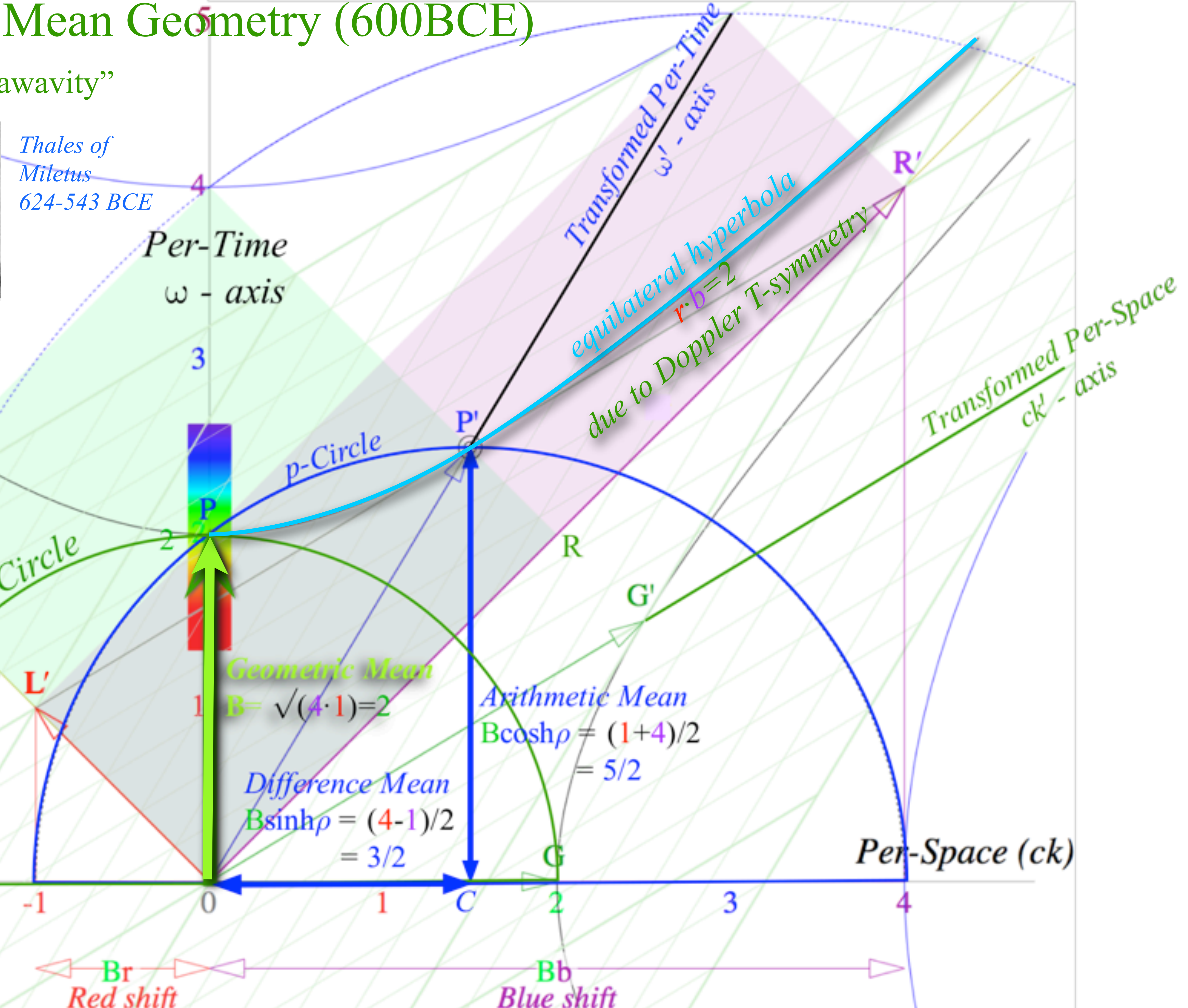
helps “Relawavity” *Thales showed a circle diameter subtends a right angle with any circle point P*



Thales Mean Geometry (600BCE)

helps "Relawavity"

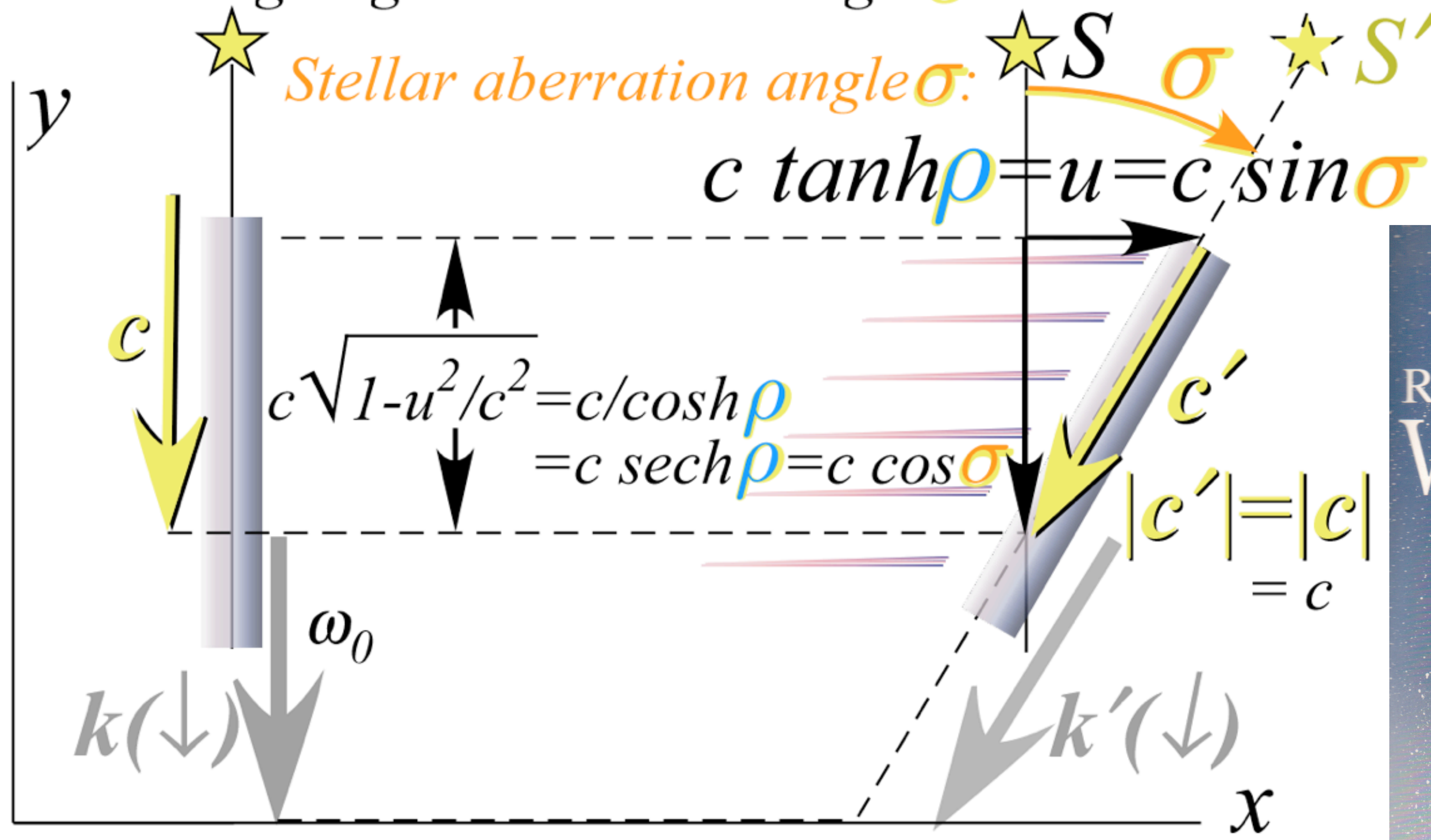




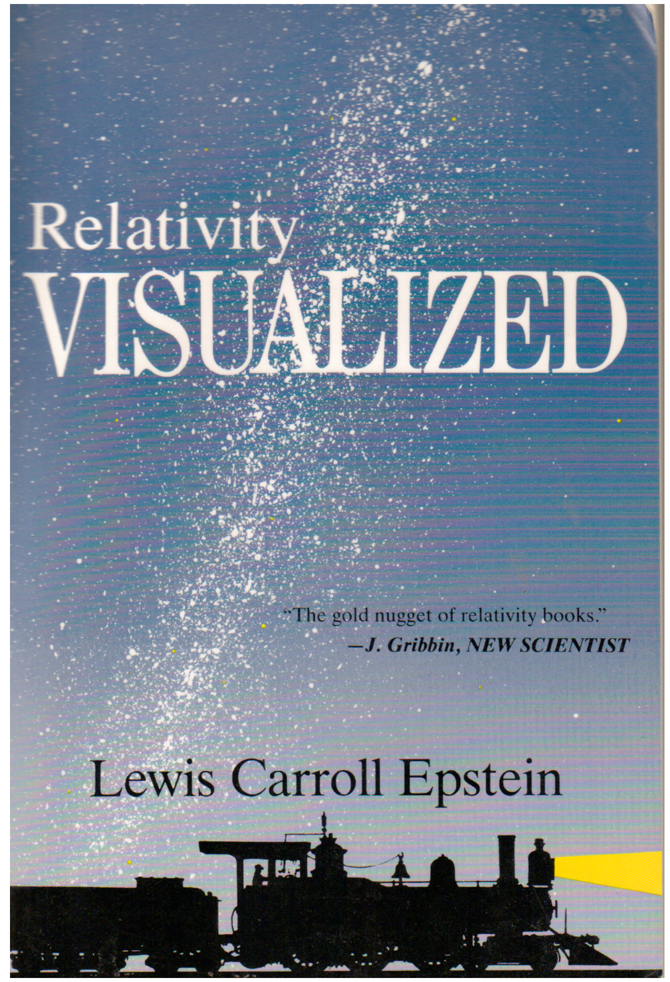
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.
*Observer going **u** sees star at angle σ in **u** direction.*



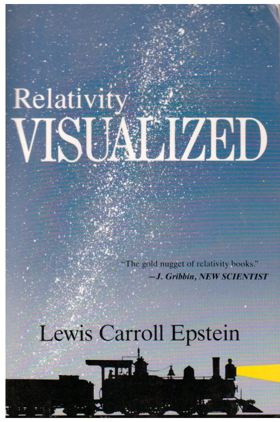
We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

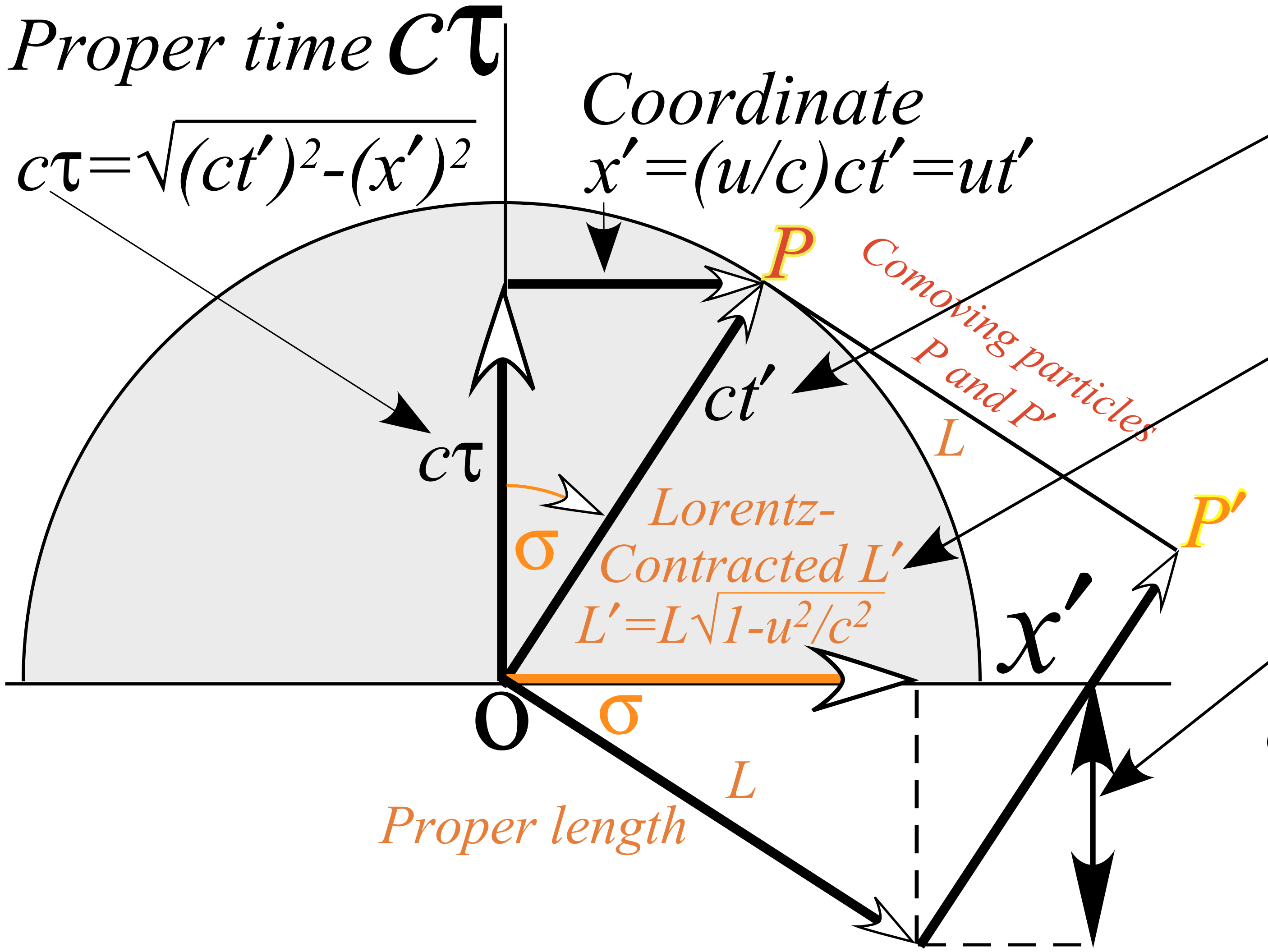
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\text{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

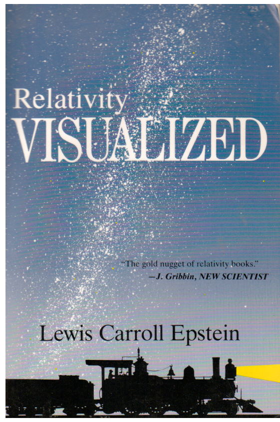
$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

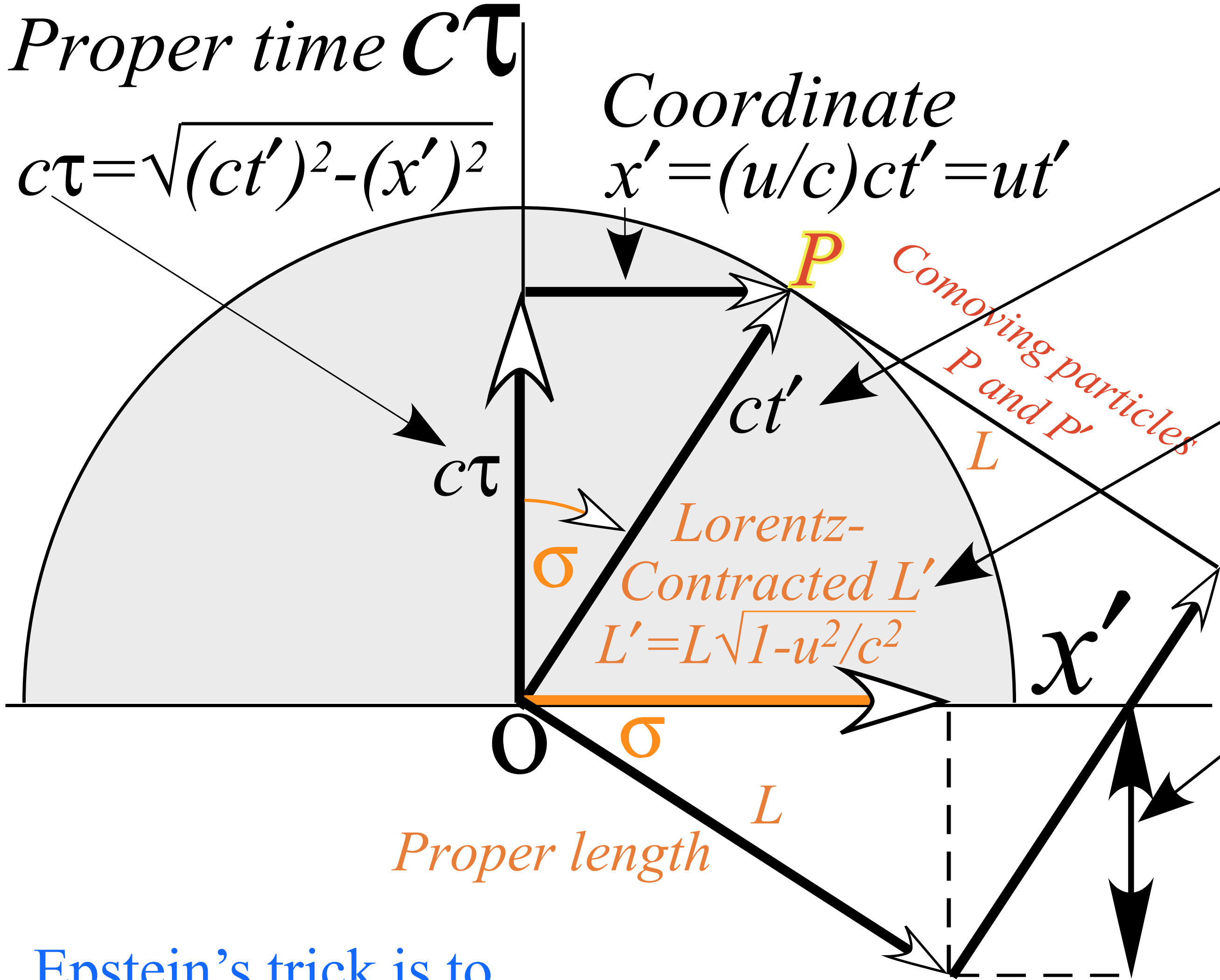
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P' have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct'=c\tau\sec\sigma=c\tau\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'= L\,\text{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

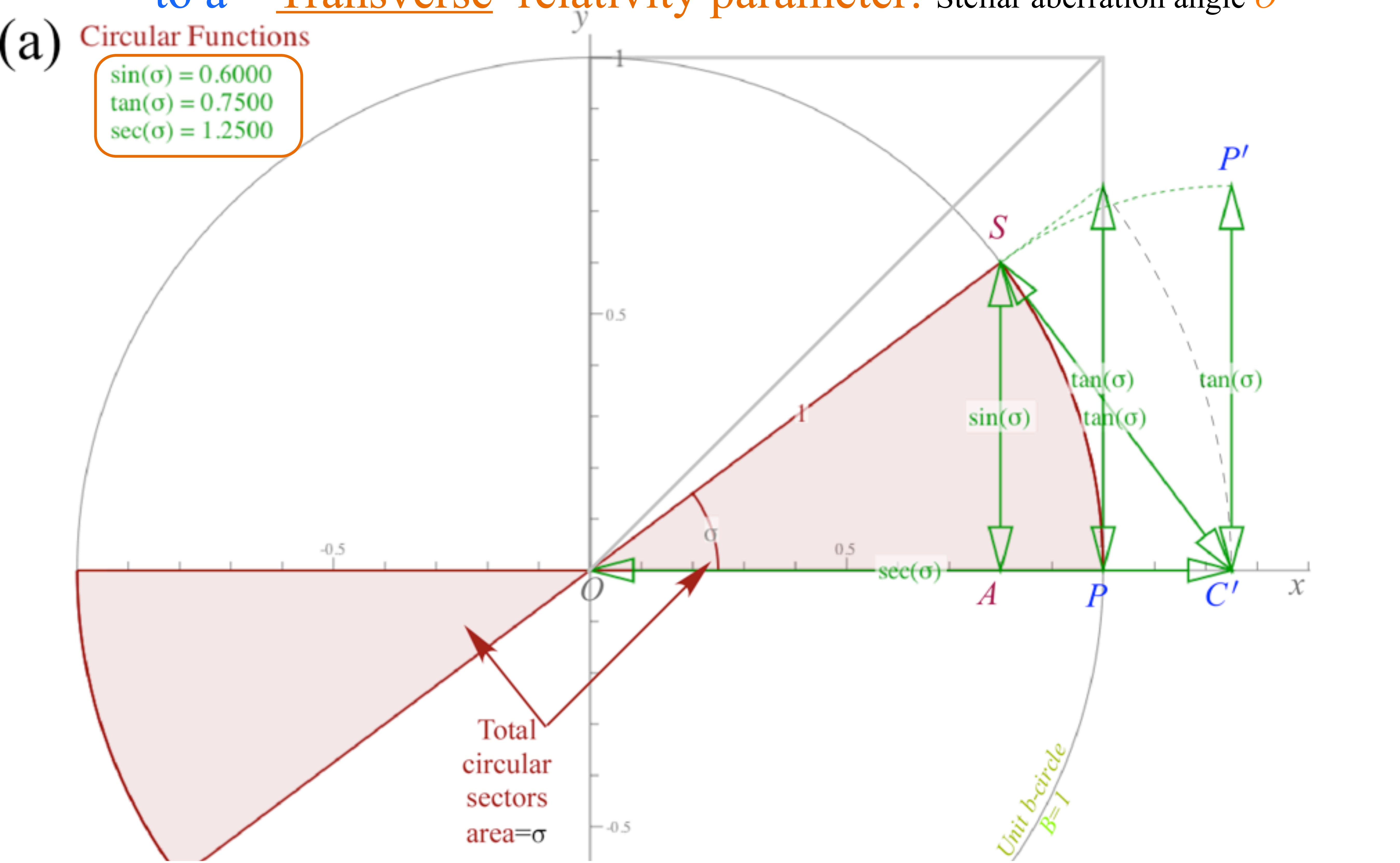
Epstein's trick is to turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2+(x')^2}=(ct')$$

Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!

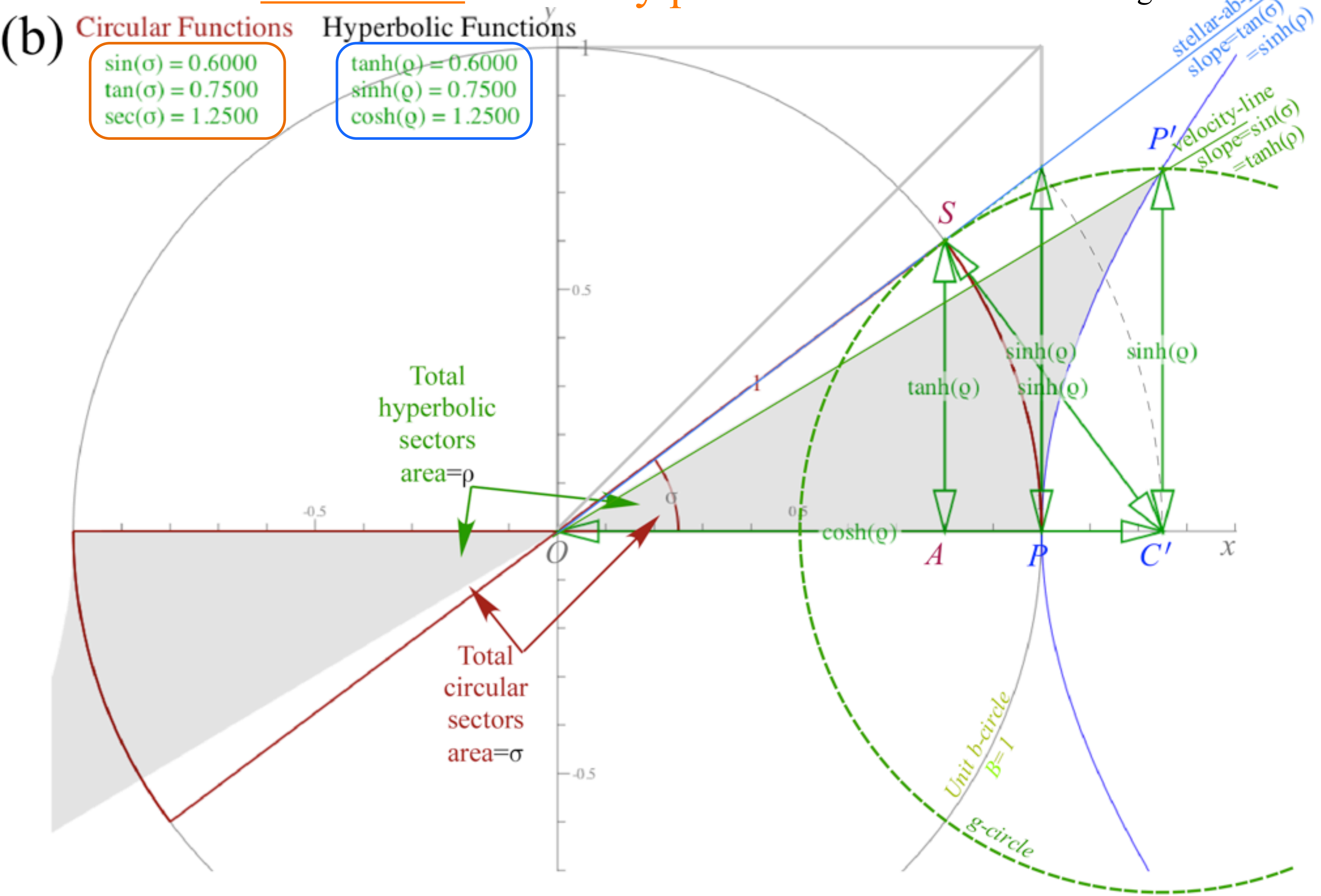
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

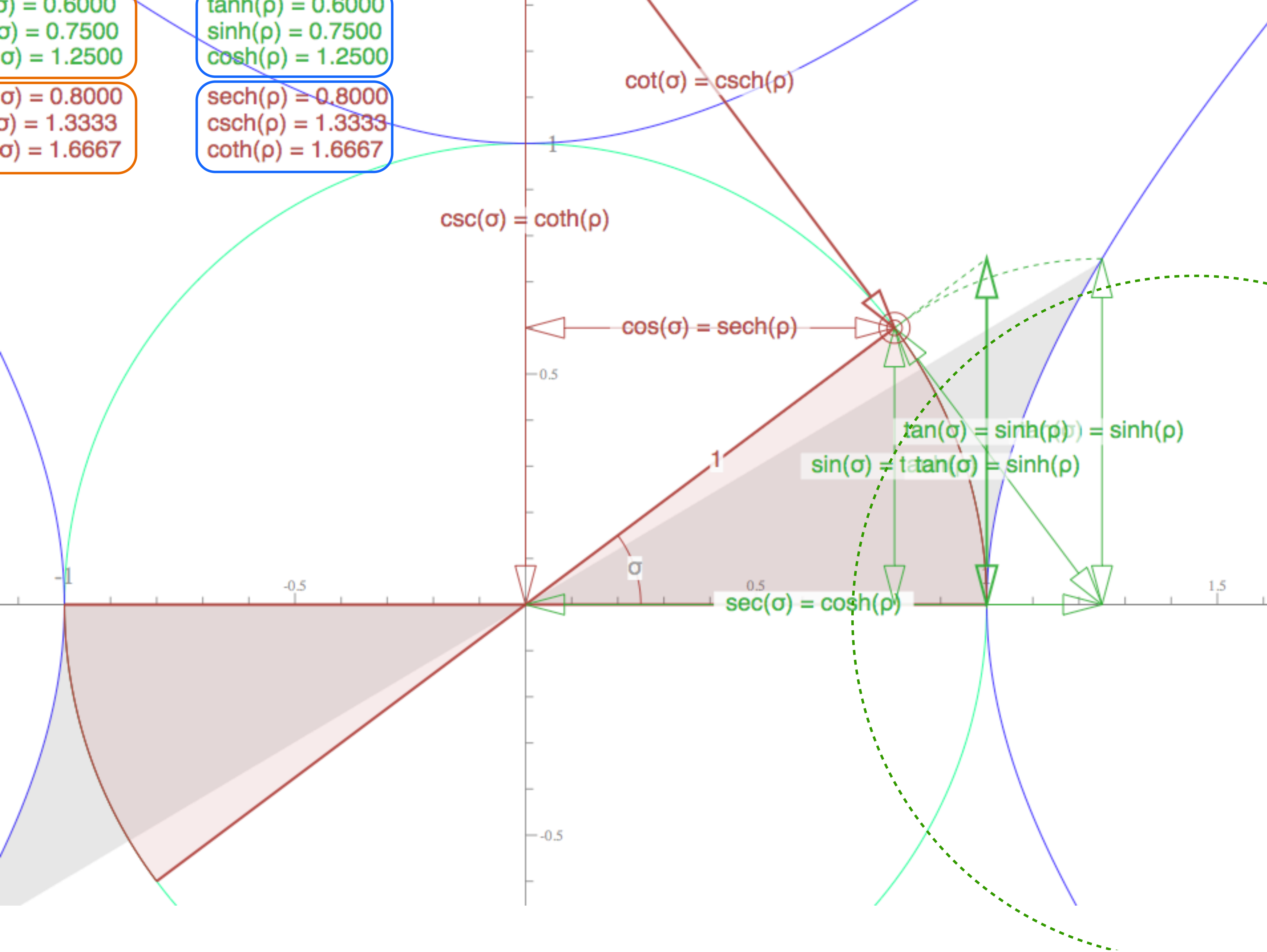
to a Transverse relativity parameter: Stellar aberration angle σ



Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse relativity parameter: Stellar aberration angle σ

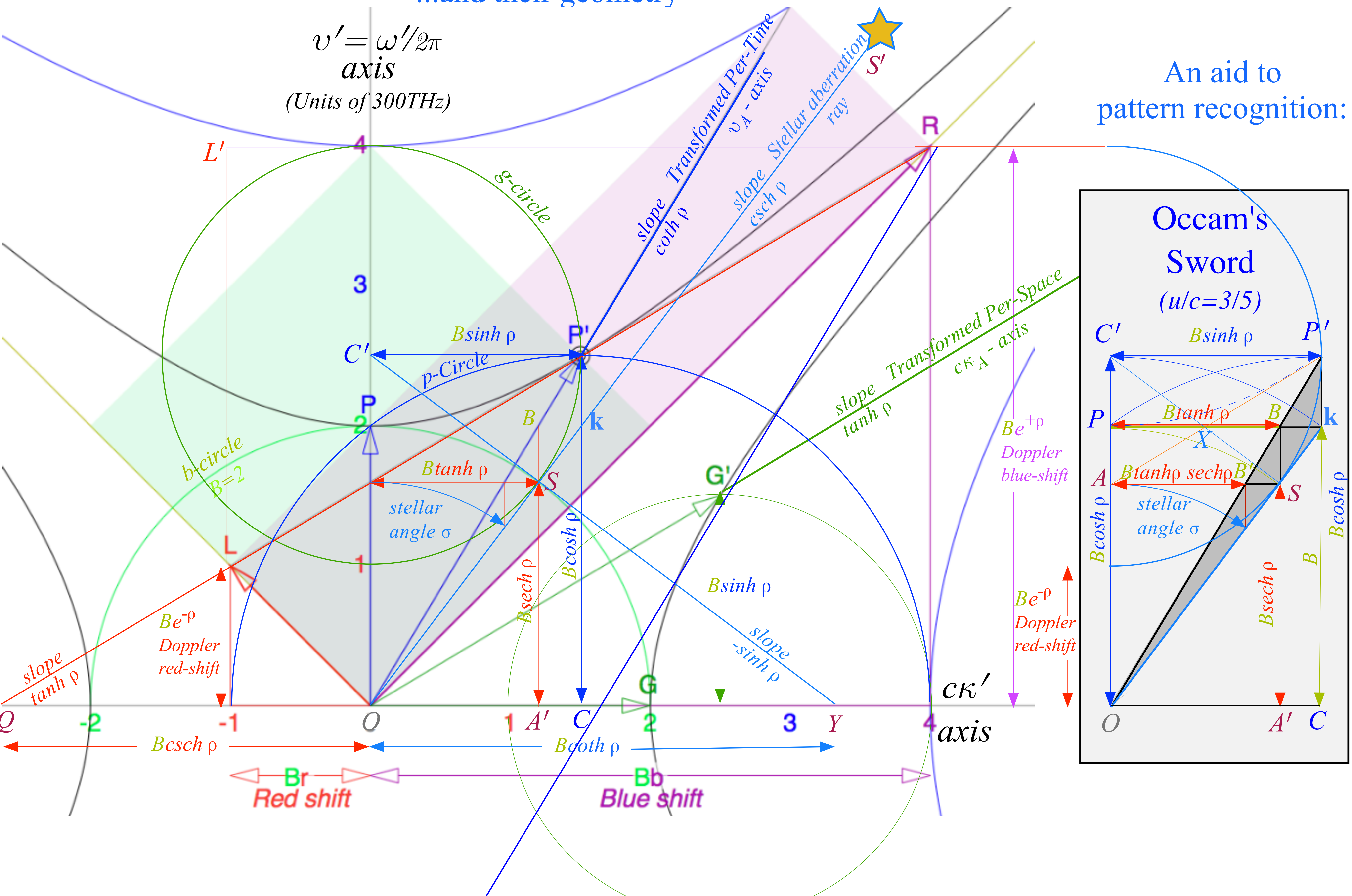




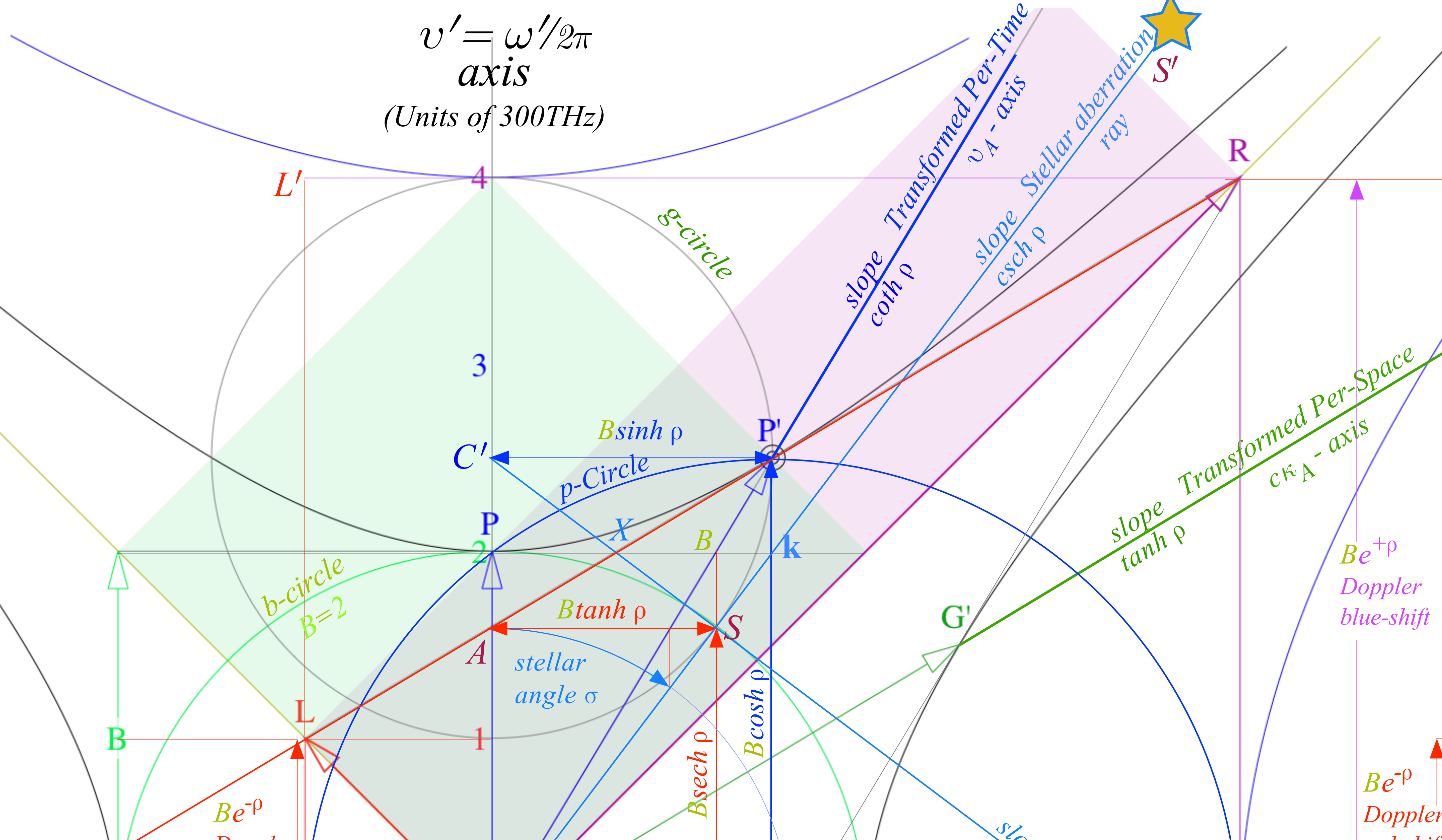
σ) = 0.6000
σ) = 0.7500
σ) = 1.2500
tanh(ρ) = 0.6000
sinh(ρ) = 0.7500
cosh(ρ) = 1.2500
σ) = 0.8000
σ) = 1.3333
σ) = 1.6667
sech(ρ) = 0.8000
csch(ρ) = 1.3333
coth(ρ) = 1.6667
cot(σ) = csch(ρ)
csc(σ) = coth(ρ)
cos(σ) = sech(ρ)
tan(σ) = sinh(ρ)
sin(σ) = tan(σ) = sinh(ρ)
sec(σ) = cosh(ρ)
σ
1
0.5
-0.5
1.5
-1

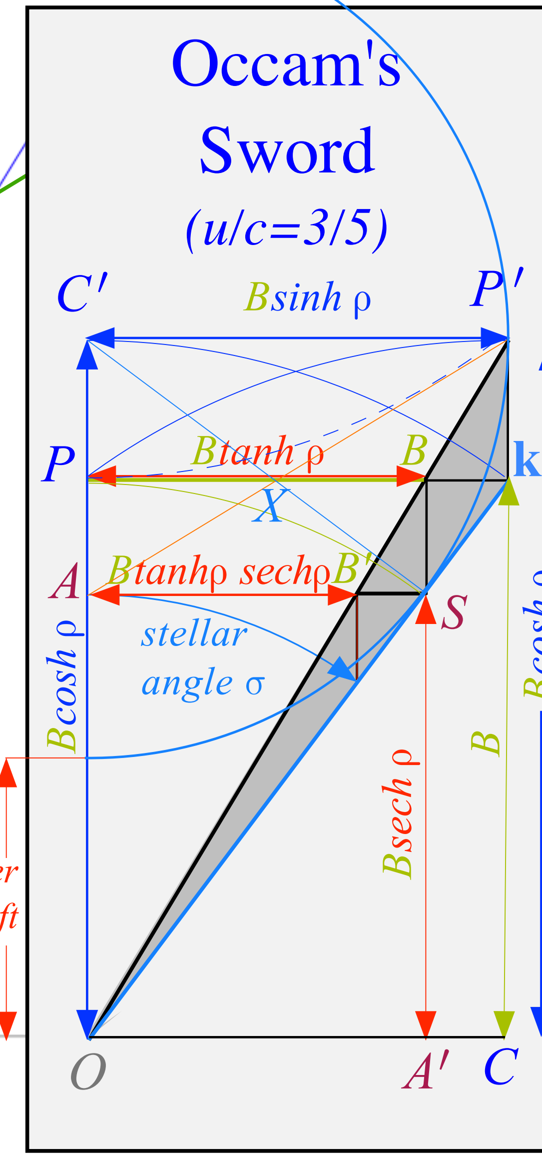
Summary of optical wave parameters for relativity and QM

...and their geometry



An aid to pattern recognition:





group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

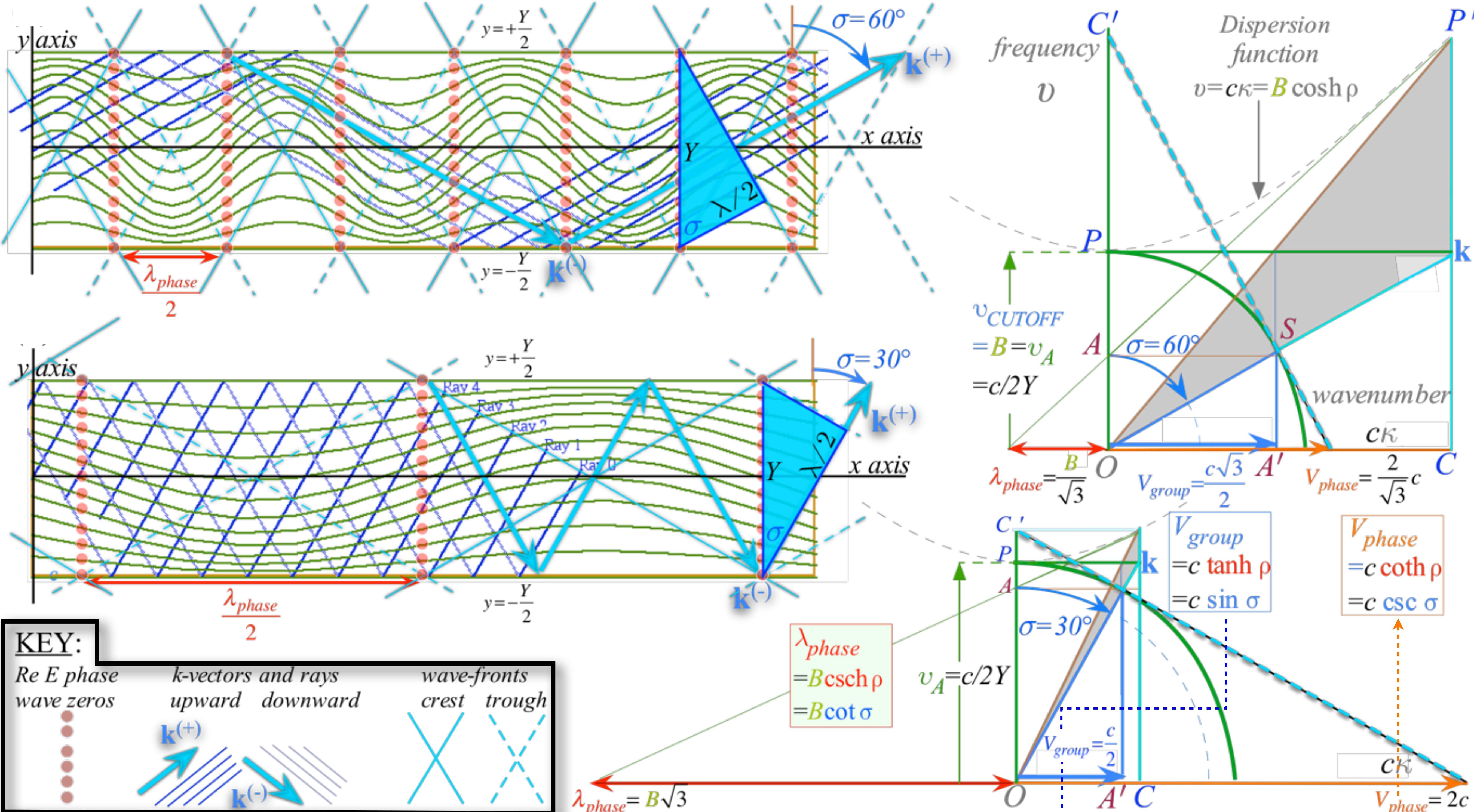
Table of 12 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)

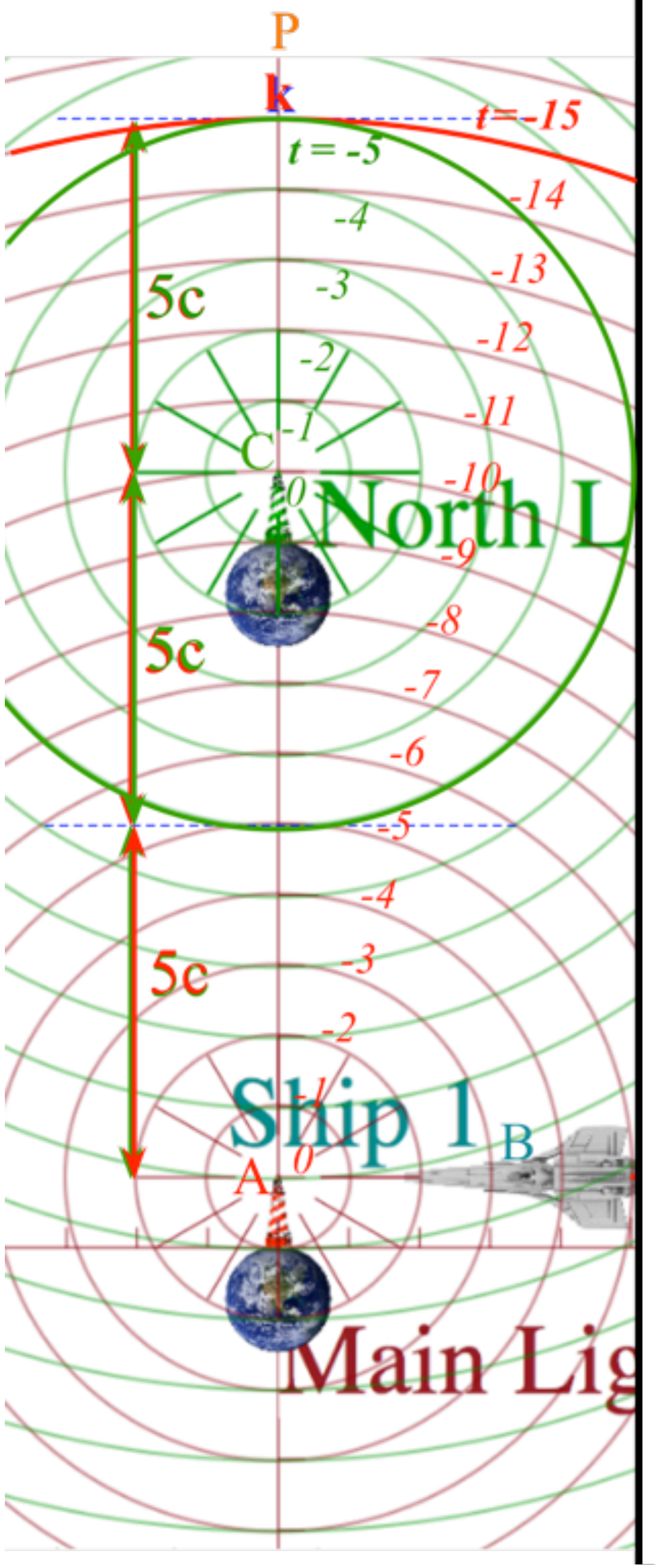


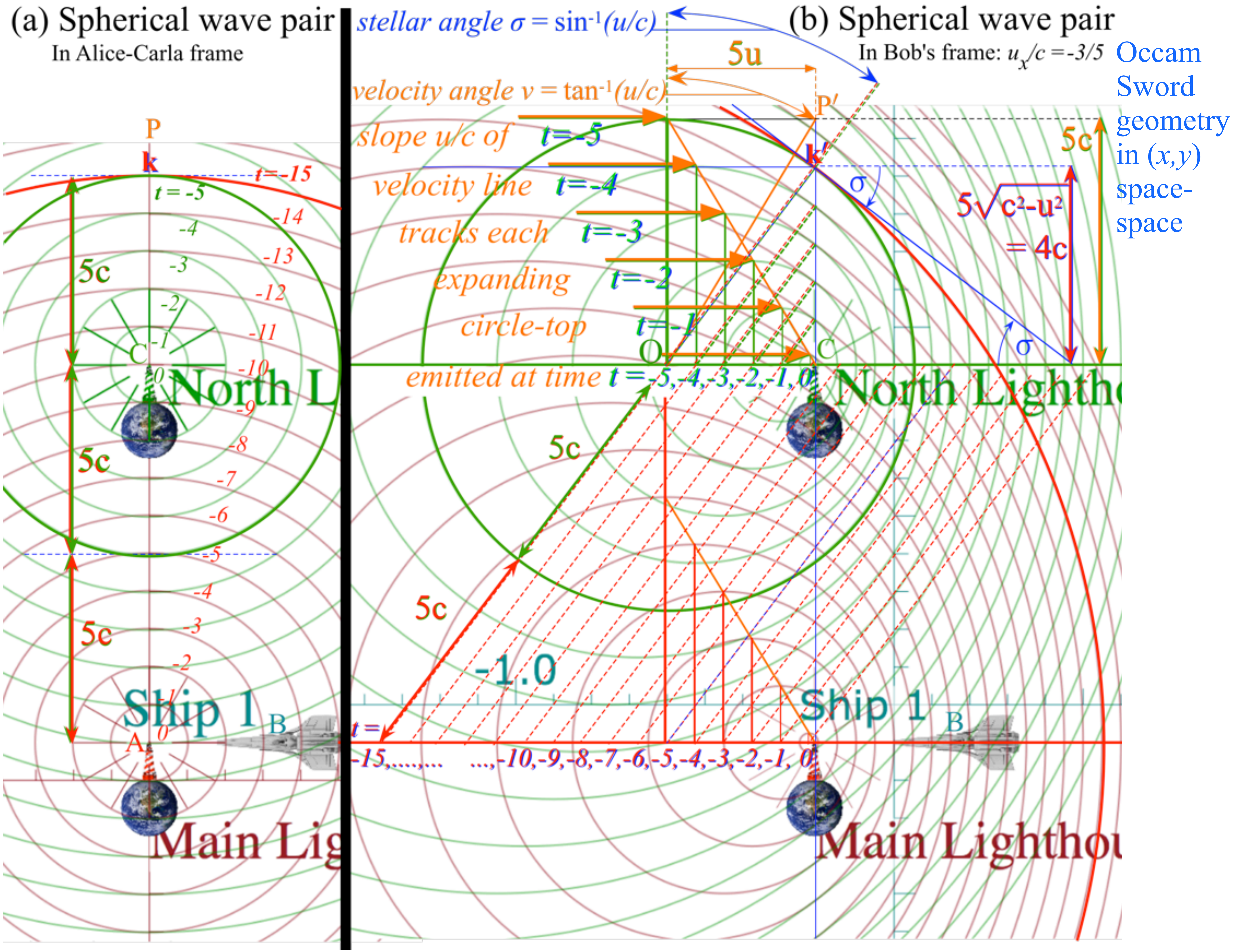
Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Spherical wave relativistic geometry

Also, aided by Occam's Sword

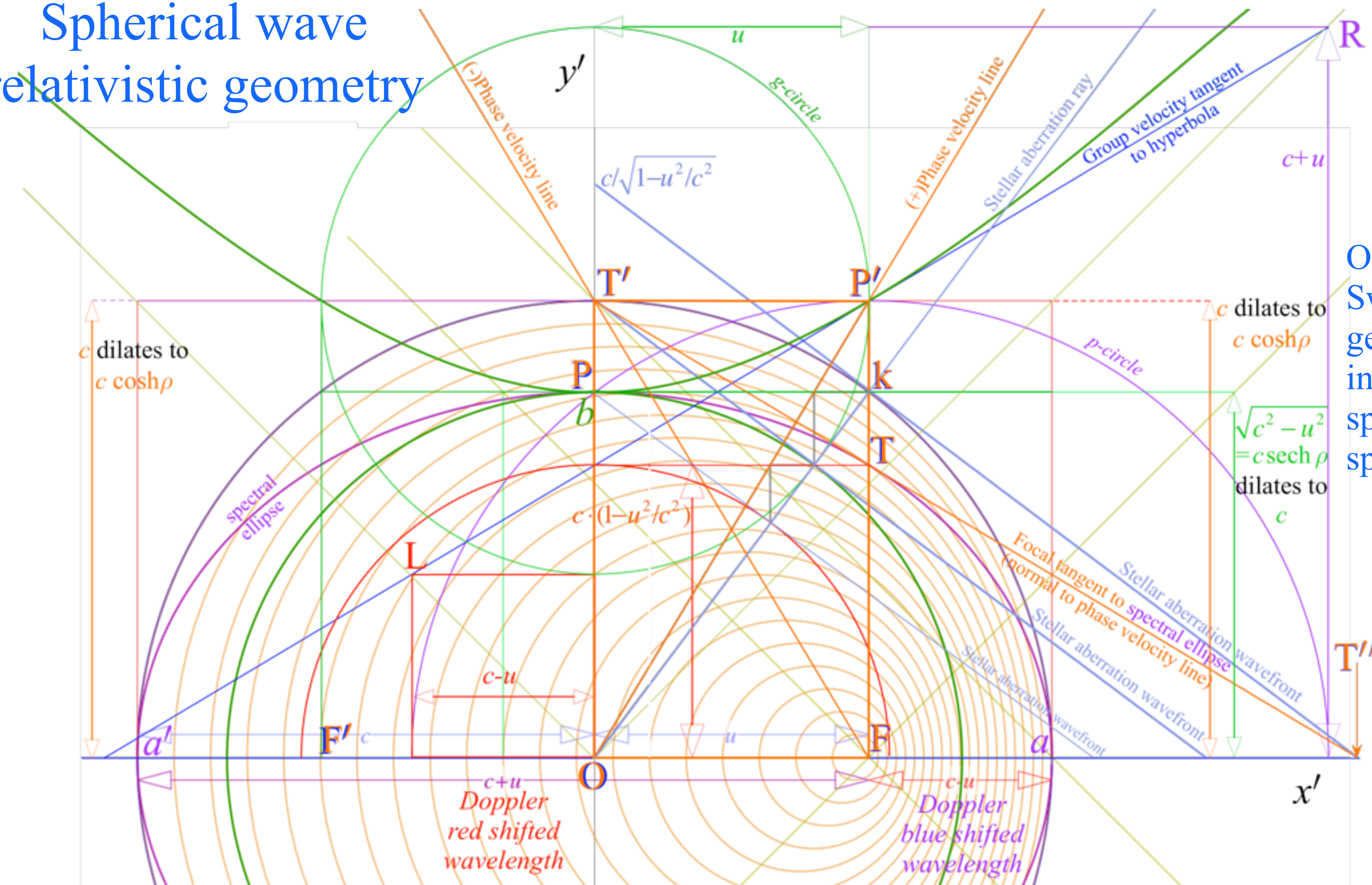
(a) Spherical wave pair
In Alice-Carla frame





(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of velocity line tracks each expanding circle-top emitted at time t = -5,-4,-3,-2,-1,0
5u
5c
$5\sqrt{c^2-u^2} = 4c$
North Lighthouse
Main Lighthouse
Ship 1
-1.0
t = -15,......, ...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0
Occam Sword geometry in (x,y) space-space

Spherical wave relativistic geometry



Occam Sword geometry in (x,y) space-space

Doppler Red λ=c+u

dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse focal length **FO**= $u = c\tanh\rho$

dilates to: $u\cosh\rho \;= c\sinh\rho$

Doppler Blue λ=$c-u$

dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

ellipse major radius a=**OF**a=c

dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:

$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse latus radius **FT**=$c(1-u^2/c^2)$

dilates to: $c(1-u^2/c^2)\cosh\rho$

$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

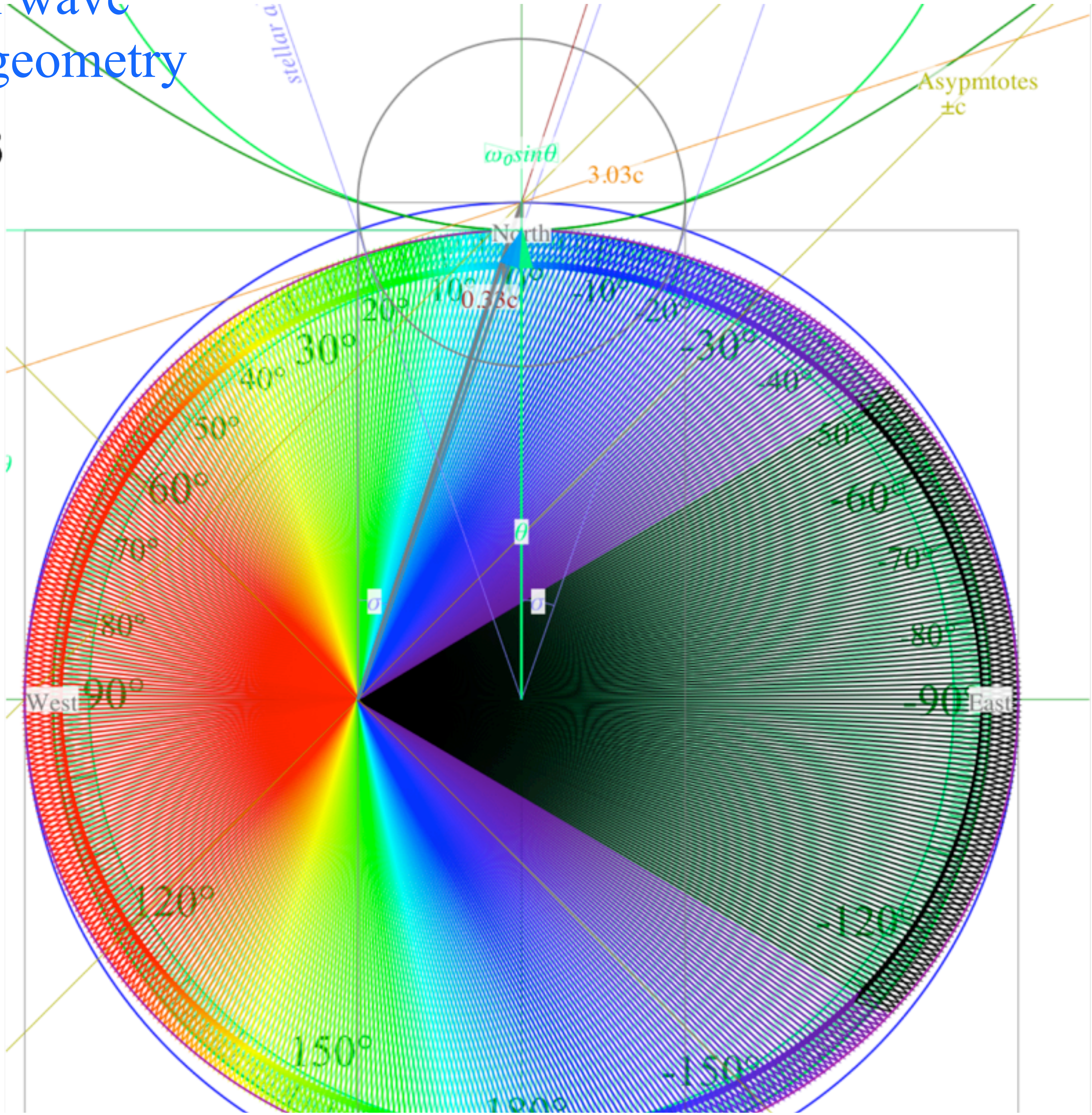
Base height **FTk**=$\sqrt{c^2-u^2}$

dilates to: $\sqrt{c^2-u^2}\ \cosh\rho = c$

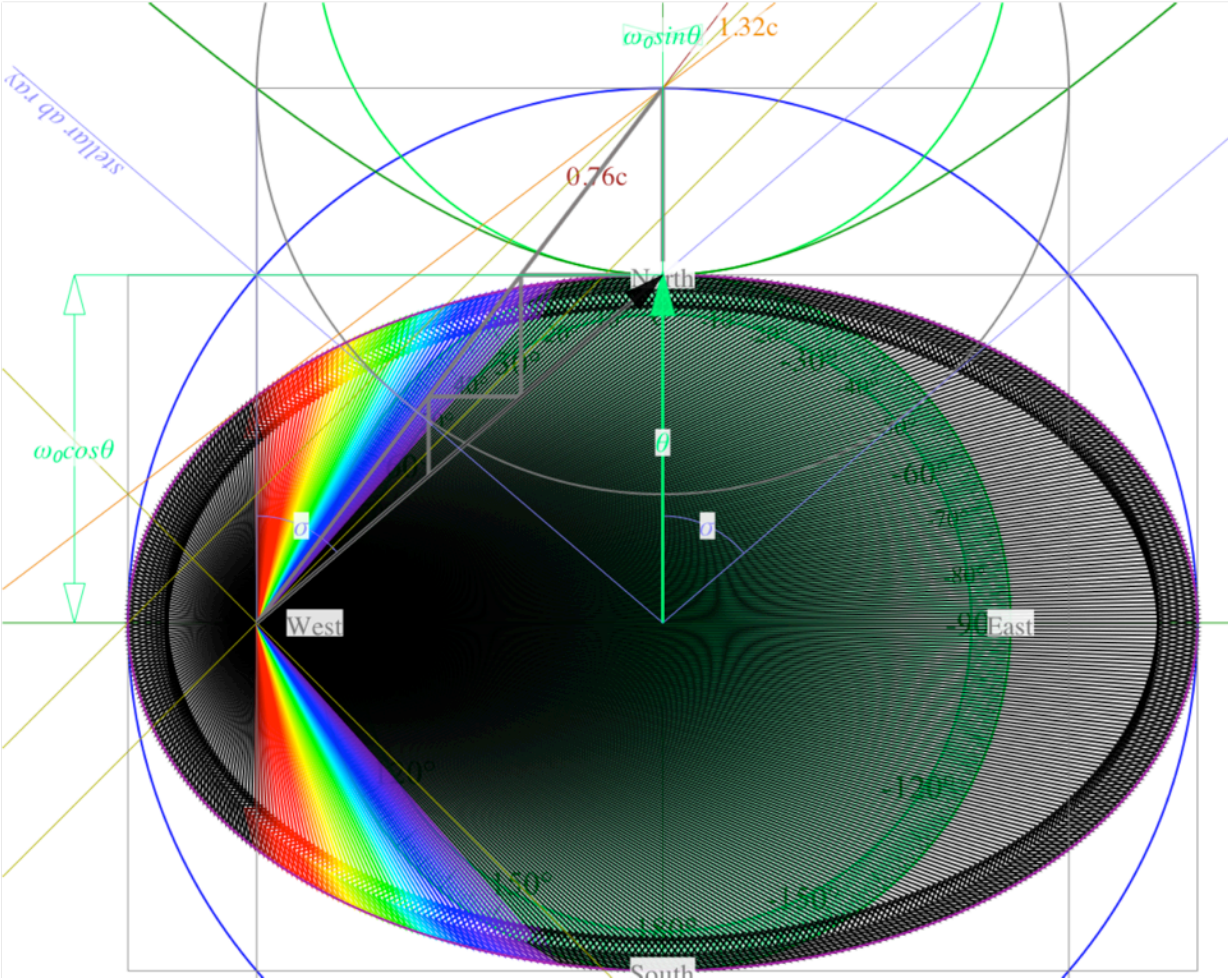
(equal to ellipse minor radius b)

Spherical wave relativistic geometry

(a) u/c=1/3



(b) u/c=3/4



$\omega_0 sin\theta$
1.32c
stellar ab ray
0.76c
North
$\omega_0 cos\theta$
θ
σ
σ
West
East
-30°
-60°
-90
-120°
-150°
120°
150°
South

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \text{ (for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$$

$$\sinh\rho \approx \rho$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \text{ (for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \text{ (for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c)$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$v_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1+\frac{1}{2}\rho^2 \approx 1+\frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = v_A$

$B = v_A = c\kappa_A$

At low speeds:

$v_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$

v_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale v_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$hv_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$

$hv_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

Lucky coincidences?? Cheap trick??

...Try exact v_{phase} ...

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1+\frac{1}{2}\rho^2 \approx 1+\frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

(old-fashioned notation)

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \Leftarrow \text{for } (u \ll c) \Rightarrow h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow \text{for } (u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

(old-fashioned notation)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Max Planck 1858-1947

Using (some) wave parameters to develop relativistic quantum theory

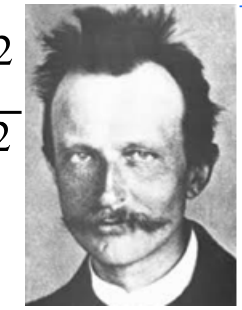
$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$



Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$

So attach scale factor h (or hN) to match units.

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

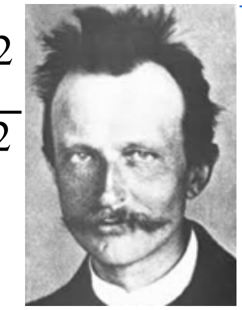
$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$



Max Planck 1858-1947

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? Cheap trick??

...Try exact υ_{phase} ...

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^*\!\cdot\!\mathbf{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$			
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

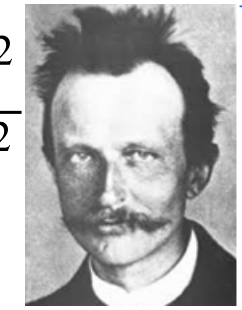
Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2} \qquad \sinh\rho \approx \rho \approx \frac{u}{c}$$



Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \boldsymbol{E}^*\!\cdot\!\boldsymbol{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	[illegible]
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	[illegible]	[illegible]	[illegible]
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	[illegible]	[illegible]	[illegible]
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy $= \frac{1}{2}A^2\upsilon^2$

Using (some) wave parameters to develop relativistic quantum theory

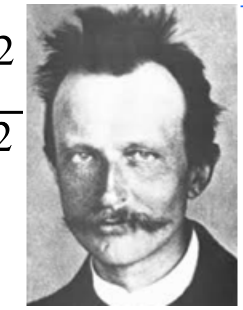
$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$



Max Planck 1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} ...

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N \quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy$= \frac{1}{2}A^2\upsilon^2$

Resolution and dirty secret: E, N, and υ_{phase} are all frequencies!

Using (some) wave parameters to develop relativistic quantum theory

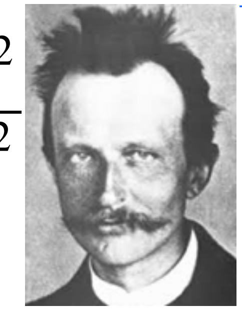
$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$



Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *and* κ_{phase}*...*

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 E^* \cdot E = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

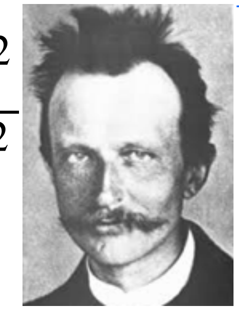
$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$



Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2$ ⇐ for $(u \ll c)$ ⇒ $\kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2$ ⇐ for $(u \ll c)$ ⇒ $h\kappa_{phase} \approx \frac{hB}{c^2}u$

So attach scale factor h *(or hN)* to match units.

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2$ ⇐ for $(u \ll c)$ ⇒ $h\kappa_{phase} \approx Mu$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with *hN to match e.m. energy density* $\varepsilon_0 E^* \cdot E = hN\upsilon_{phase}$

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

$cp = \frac{Mcu}{\sqrt{1-u^2/c^2}}$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phas...}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

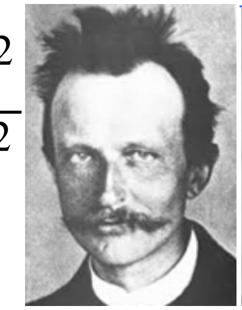
$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$


Max Planck 1858-1947


Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$

So attach scale factor h (or hN) to match units.

~~Lucky coincidences??~~ Natural wave conspiracy

~~Cheap~~ Expensive trick??

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow$ for $(u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

Momentum: $h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$

DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	[illegible]
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck 1858-1947

Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

~~Lucky coincidences~~?? Natural wave conspiracy

~~Cheap~~ Expensive trick??

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N \qquad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}} \text{ (old-fashioned notation)}$$

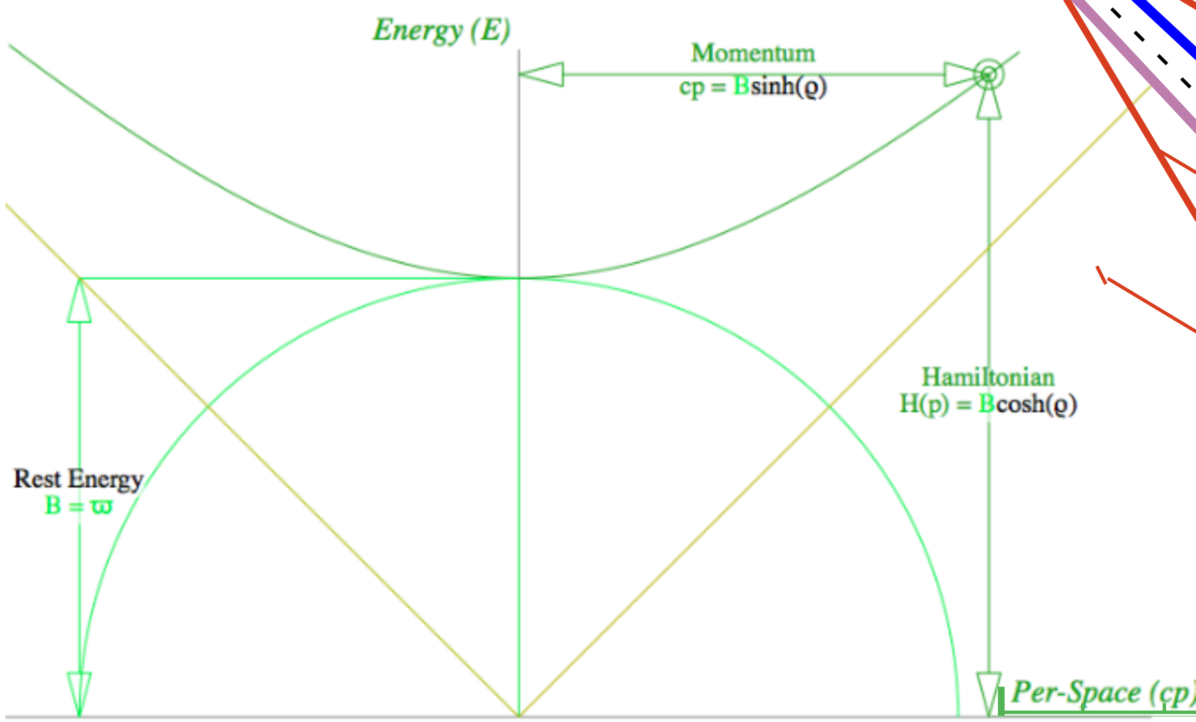
$$cp = \frac{Muc}{\sqrt{1 - u^2/c^2}}$$

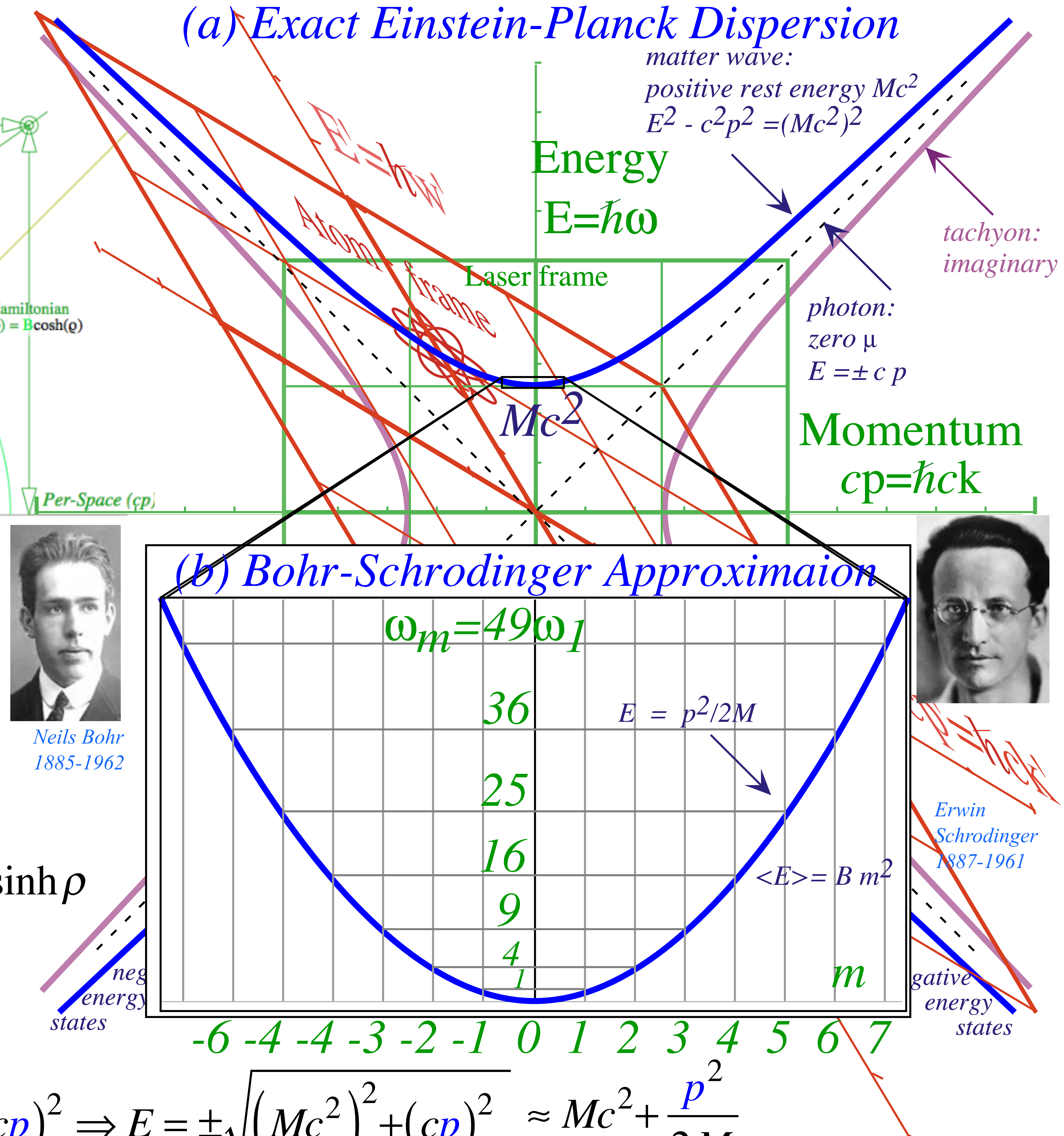
Momentum: $h\kappa_{phase} = p = \dfrac{Mu}{\sqrt{1 - u^2/c^2}}$

DeBroglie (1921)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$			
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave coordinates for relativistic quantum theory





Neils Bohr
1885-1962

Erwin
Schrodinger
1887-1961

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

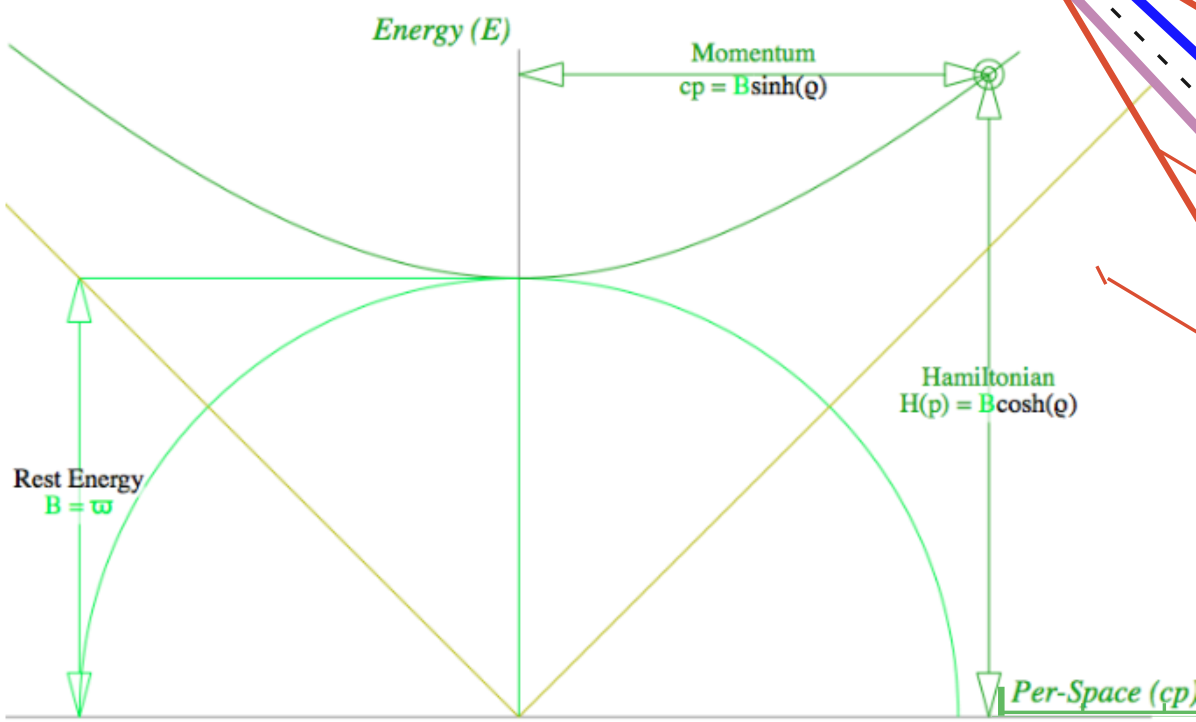
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

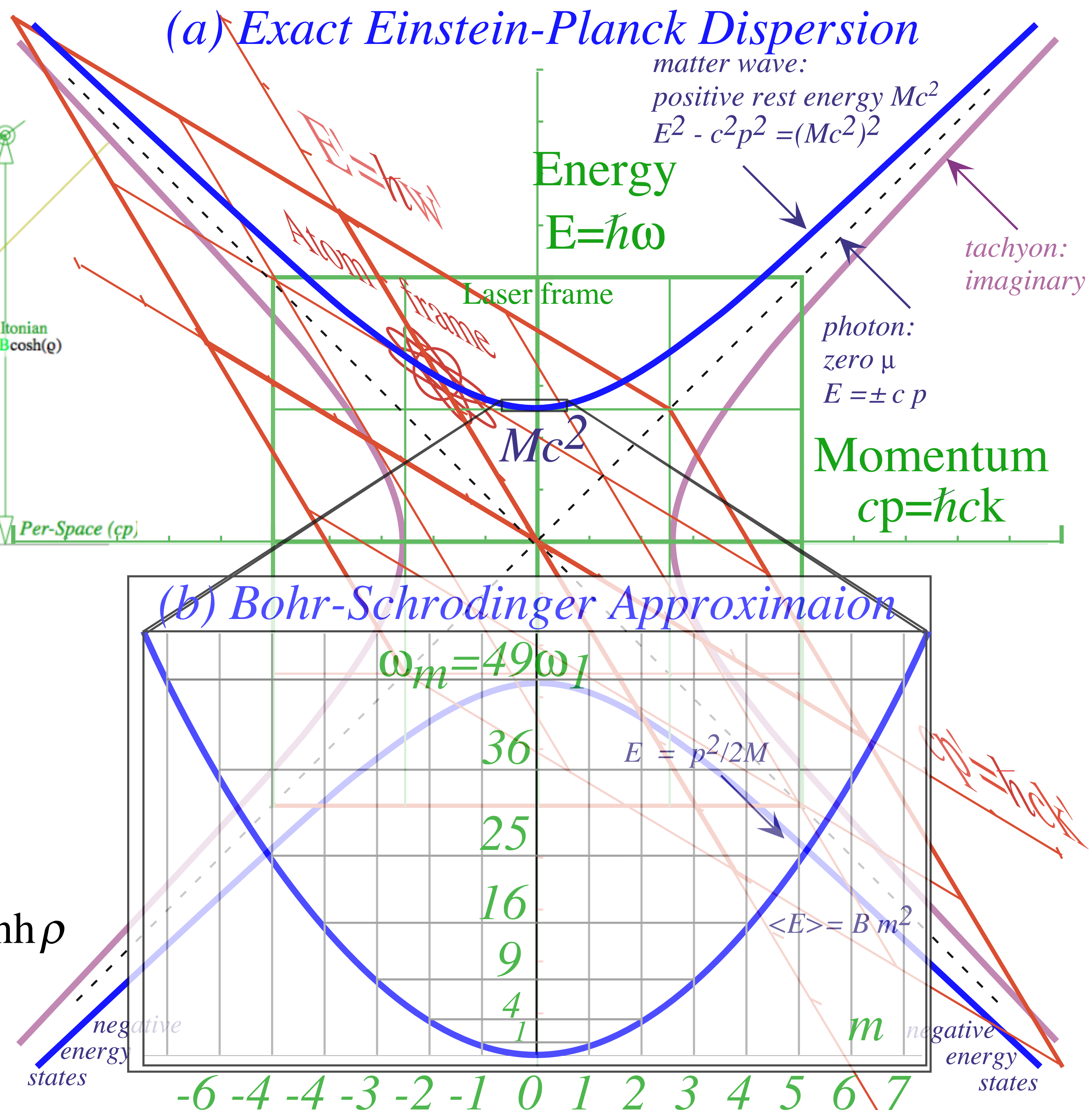
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2+(cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory





Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2+(cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

low speed approximation

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

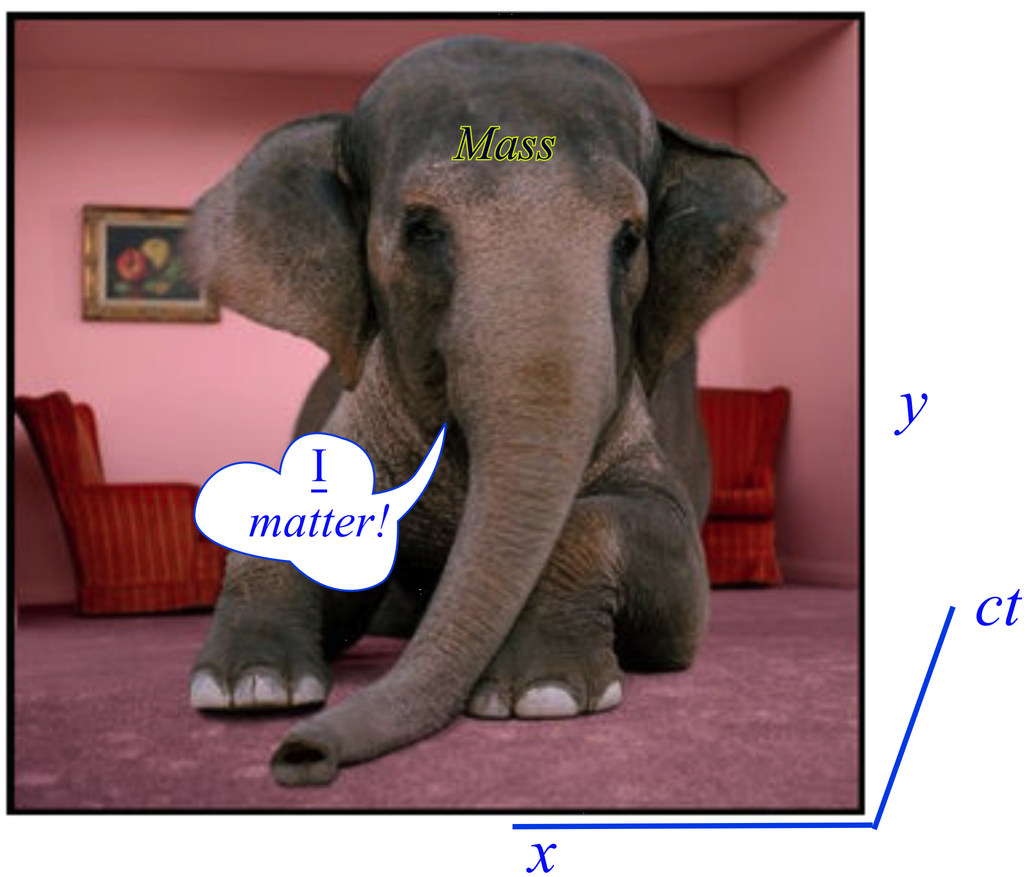
Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

Definition(s) of mass for relativity/quantum

Given: <u>*Energy:*</u> $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

<u>*momentum:*</u> $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

<u>*velocity:*</u> $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

<u>*Rest Mass*</u> *M_{rest} (Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ <u>*Rest Mass*</u>

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + \left(cp\right)^2}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho = h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho = hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ *Rest Mass*

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ *Rest Mass*

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}$$ *Momentum Mass*

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho = h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho = hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ *Rest Mass*

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1 - u^2/c^2}}$$ *Momentum Mass*

Limiting cases:

$$M_{mom} \xrightarrow[u \to c]{} M_{rest} e^{\rho}/2$$

$$M_{mom} \xrightarrow[u \ll c]{} M_{rest}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ *Rest Mass*

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}$$ *Momentum Mass*

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ Rest Mass

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh\rho}{c \tanh\rho}$$

$$= M_{rest} \cosh\rho = \frac{M_{rest}}{\sqrt{1 - u^2 / c^2}}$$ Momentum Mass

Limiting cases:

$$M_{mom} \xrightarrow[u \to c]{} M_{rest} e^{\rho}/2$$

$$M_{mom} \xrightarrow[u \ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh\rho \, d\rho$ in momentum to change $du = c \operatorname{sech}^2\rho \, d\rho$ in velocity

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \text{Rest Mass}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest} e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.

That is ratio of change $dp=Mc\cosh\rho\, d\rho$ in momentum to change $du=c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho = h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho = hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \text{Rest Mass}$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{(Mc^2)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}$$

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \text{Rest Mass}$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Limiting cases: $M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$

$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\text{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\text{sech}^2\rho} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}$$

Limiting cases: $M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$

$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
$= hc\kappa_{phase}$

Group velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \text{Rest Mass}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.

That is ratio of change $dp=Mc\cosh\rho\, d\rho$ in momentum to change $du=c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}$$

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}$$

general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass $(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0$,

Momentum Mass $(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2}$,

Effective Mass $(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty$.

Newton complained about *his* "corpuscles" of light having "fits" (going crazy).

(This would be evidence of triple Schizophrenia.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor format→

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian* L using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \qquad \hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho \qquad\qquad E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor format→

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar ck_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian* L using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar c k_A$

$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$

$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is}: Mc^2\frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar ck_A$

$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$

$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc \sinh\rho)(c \tanh\rho) - Mc^2 \cosh\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar ck_A$

$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$

$\hbar ck_{phase} = cp = \hbar\omega_A \sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Note: $Mcu=Mc^2\tanh\rho$

$$= Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

with *Hamiltonian* $H=E$

$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho$$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$

$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv pu-H=L$$

Legendre transformation

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$p=\hbar k=Mc\sinh\rho \qquad E=\hbar\omega=Mc^2\cosh\rho=H$$

$$L=pu-H=(Mc\sinh\rho)(c\tanh\rho)-Mc^2\cosh\rho$$

$$=Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho}=-Mc^2\operatorname{sech}\rho$$

Note: $Mcu=Mc^2\tanh\rho$

Also: $cp=Mc^2\sinh\rho$

Compare *Lagrangian L*

$$L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Prior wave relations

$\hbar=h/2\pi$

$$h\upsilon_A=Mc^2=hc\kappa_A$$
$$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A=Mc^2=\hbar ck_A$$
$$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho = c\sin\sigma$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Note: $Mcu = Mc^2\tanh\rho = Mc^2\sin\sigma$

Also: $cp = Mc^2\sinh\rho = \hbar ck = Mc^2\tan\sigma$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Including stellar angle σ

Prior wave relations

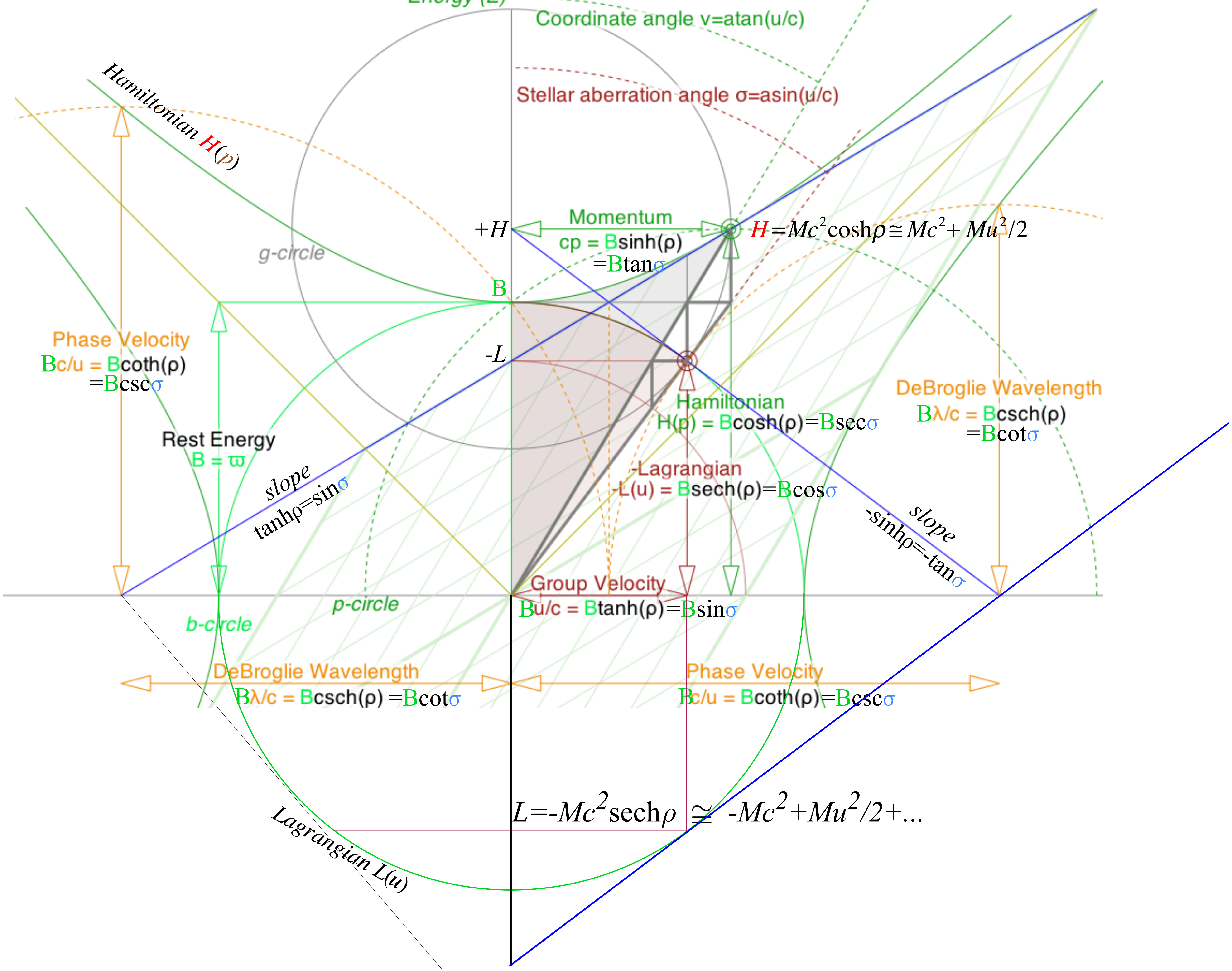
$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$$

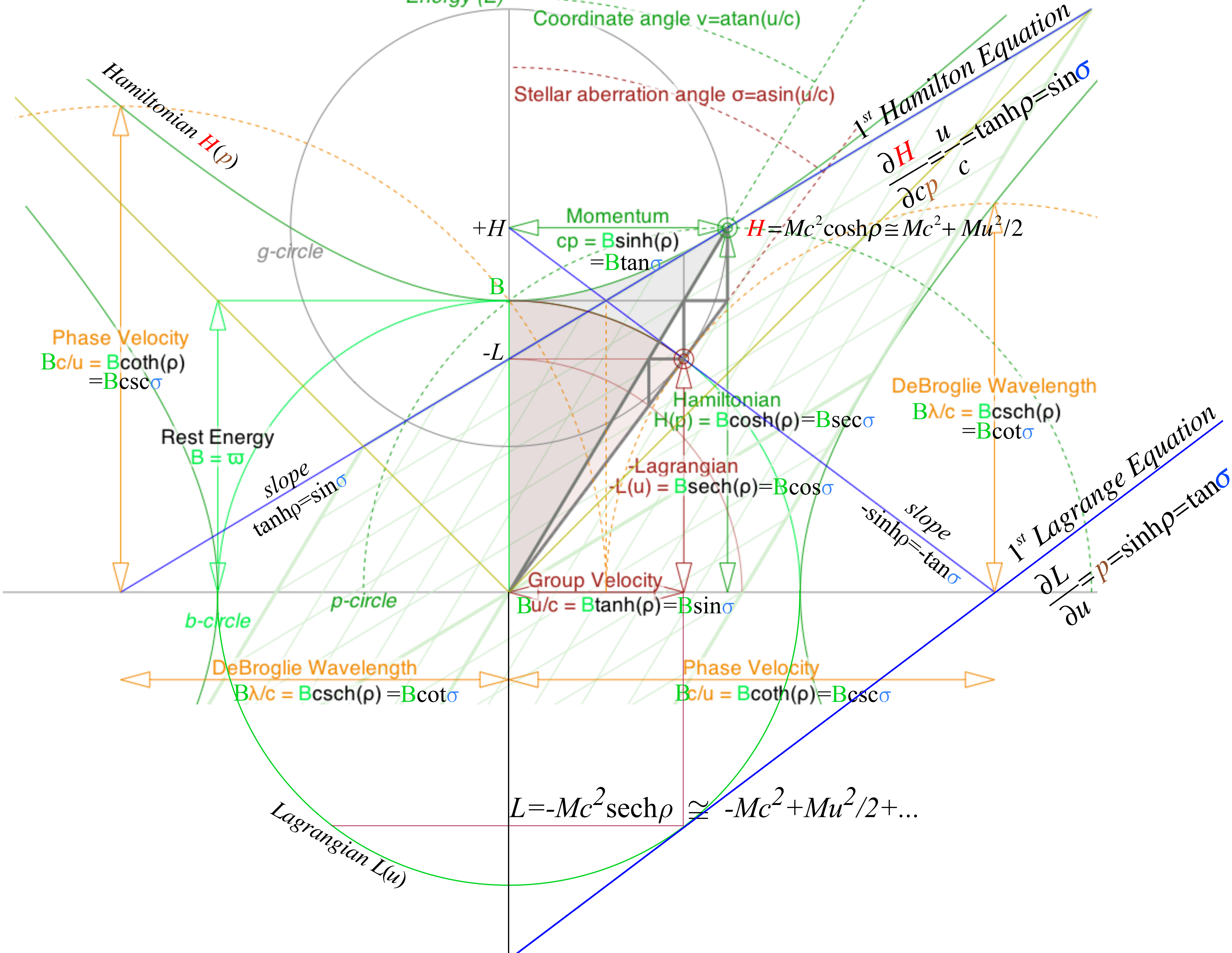
←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$$

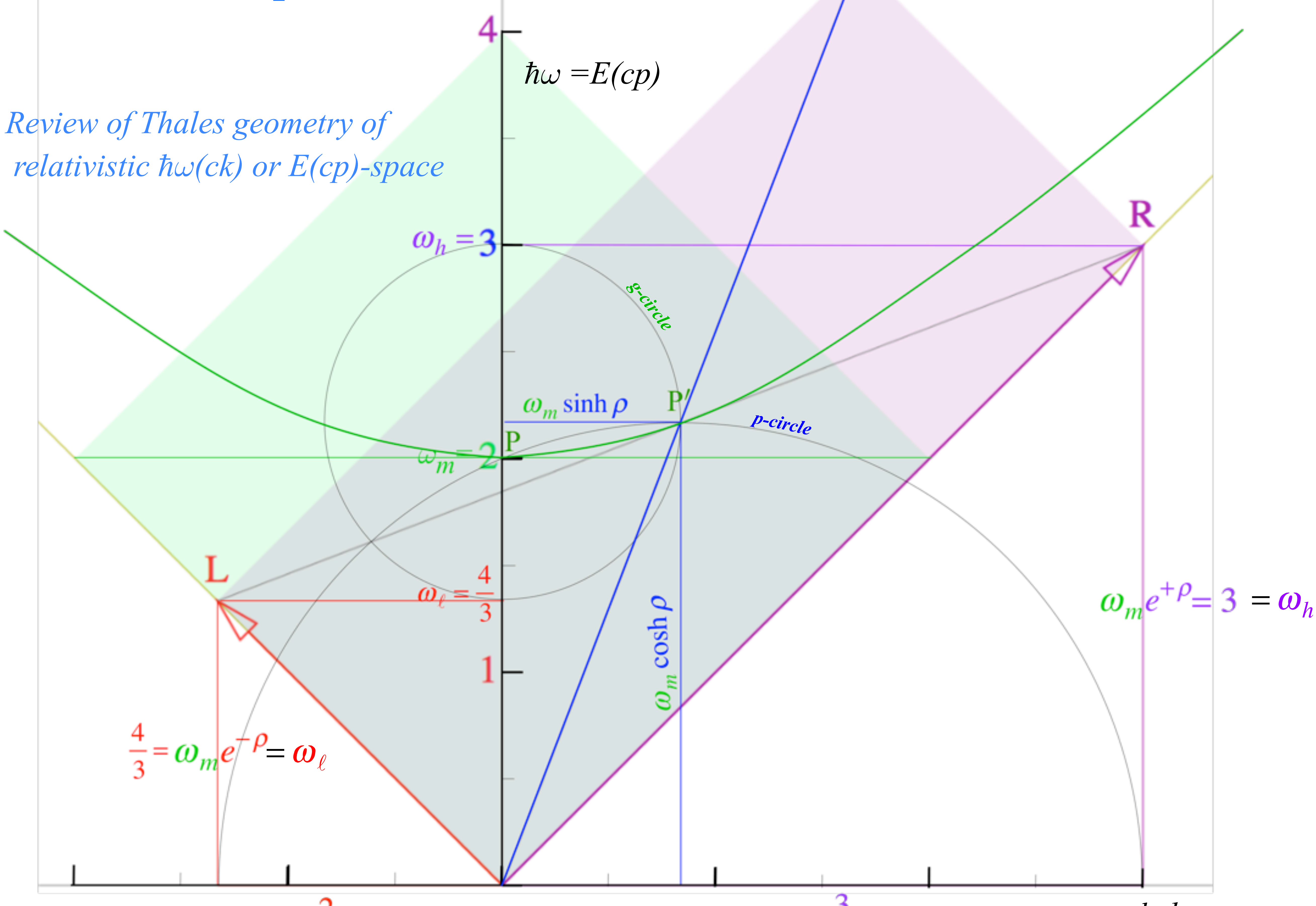
$$\hbar \equiv \frac{h}{2\pi}$$


Energy (E)
Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(ρ)
+H
Momentum
cp = Bsinh(ρ)
=Btanσ
H=Mc²coshρ≅Mc²+ Mu²/2
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
tanhρ=sinσ
slope
-sinhρ=-tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≅ -Mc²+Mu²/2+...
Lagrangian L(u)


Energy (E)
Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
1st Hamilton Equation
∂H/∂cp = u/c = tanhρ = sinσ
Hamiltonian H(p)
+H
Momentum
cp = Bsinh(ρ)
=Btanσ
H=Mc²coshρ≅Mc²+ Mu²/2
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
Hamiltonian
H(p) = Bcosh(ρ)=Bsecσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Rest Energy
B = ϖ
slope
tanhρ=sinσ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
∂L/∂u = p = sinhρ = tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≃ -Mc²+Mu²/2+...
Lagrangian L(u)

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

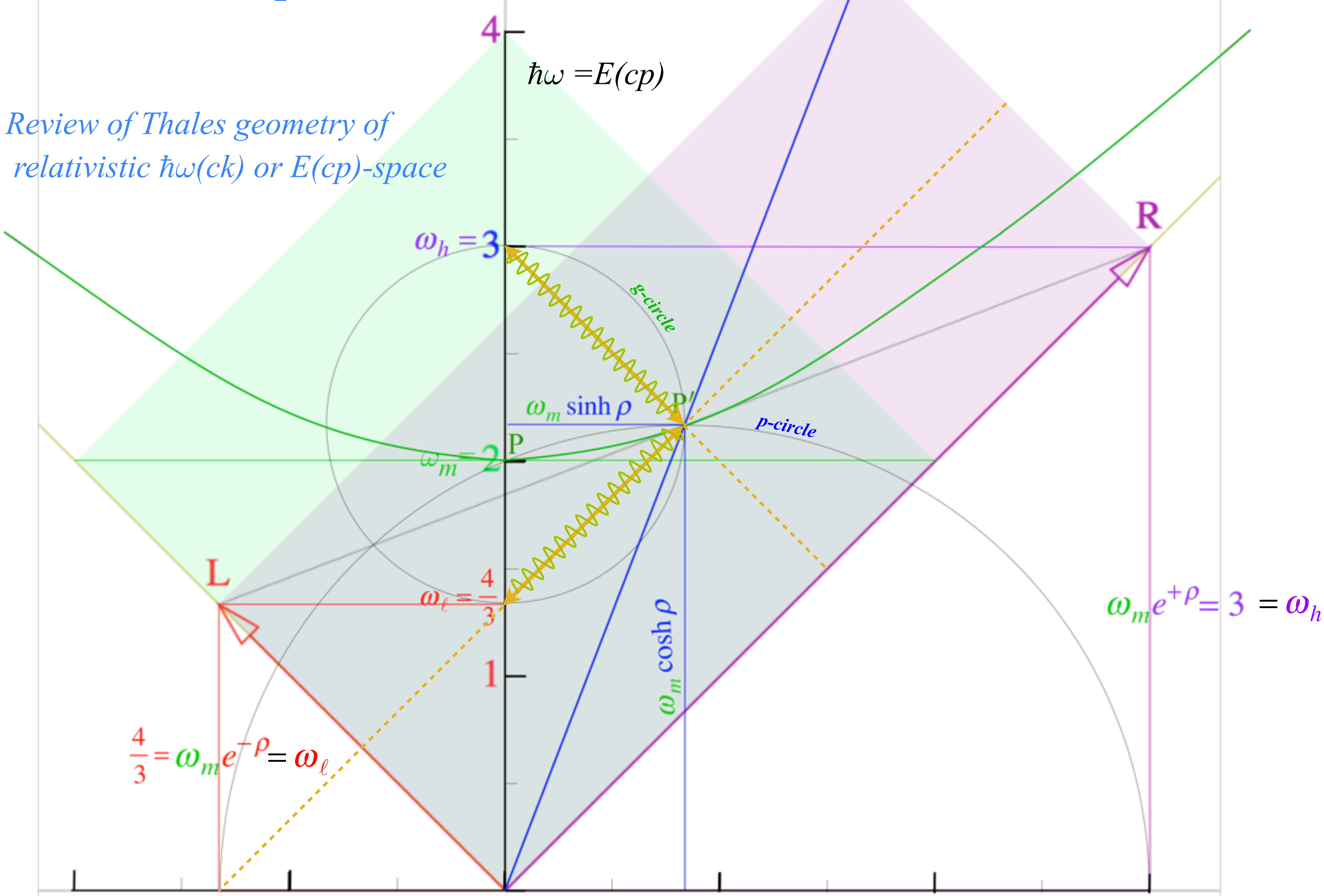
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

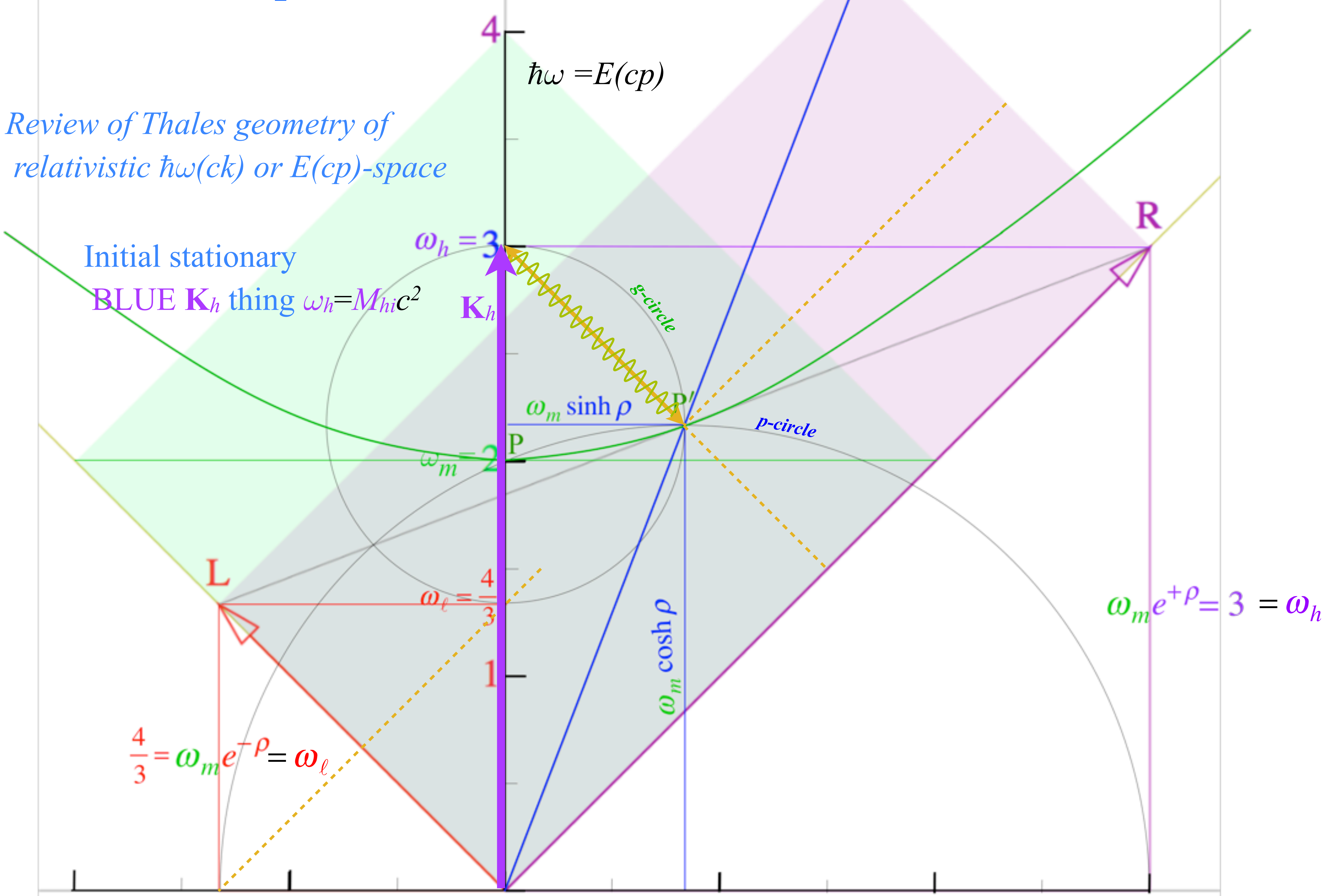
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



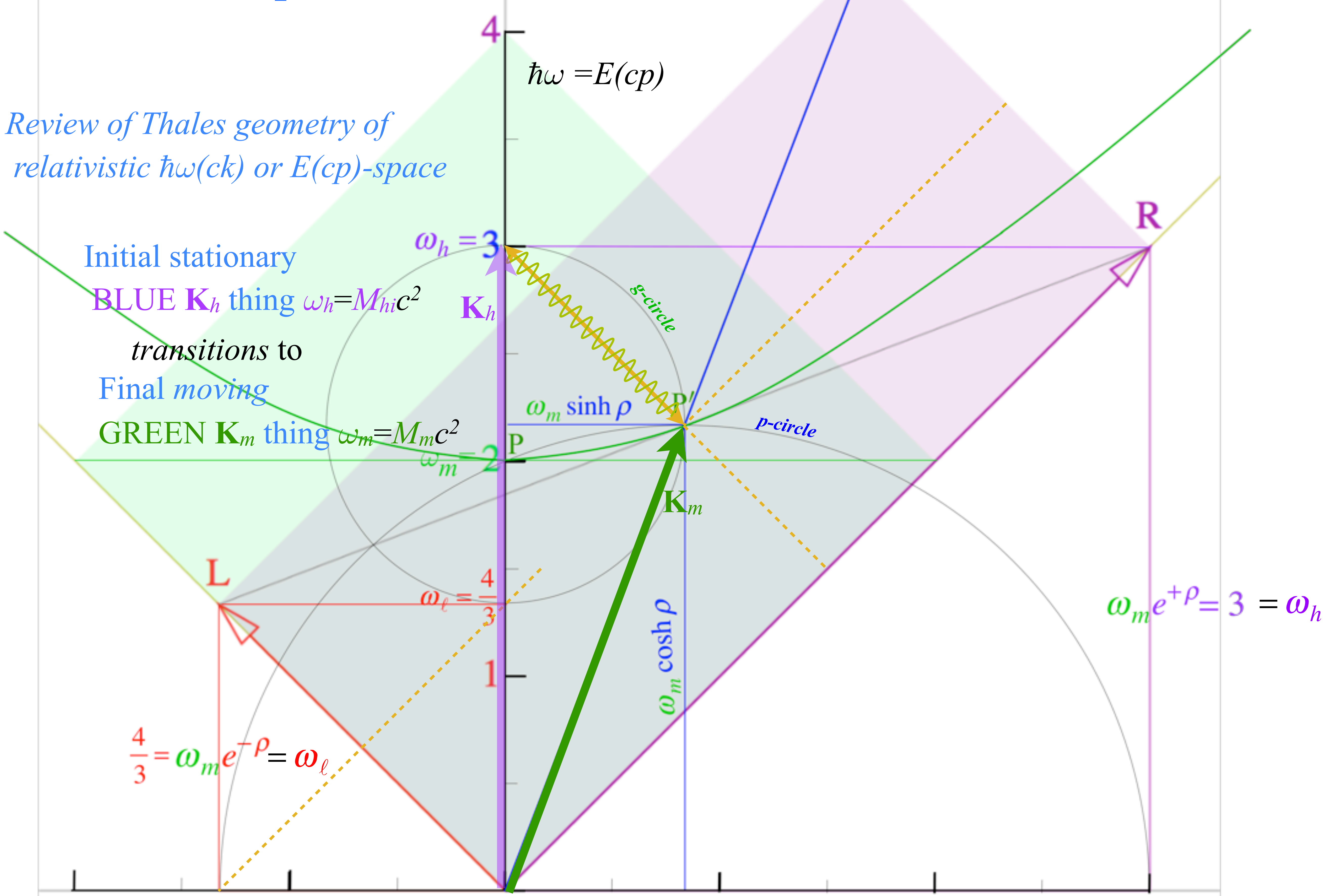
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



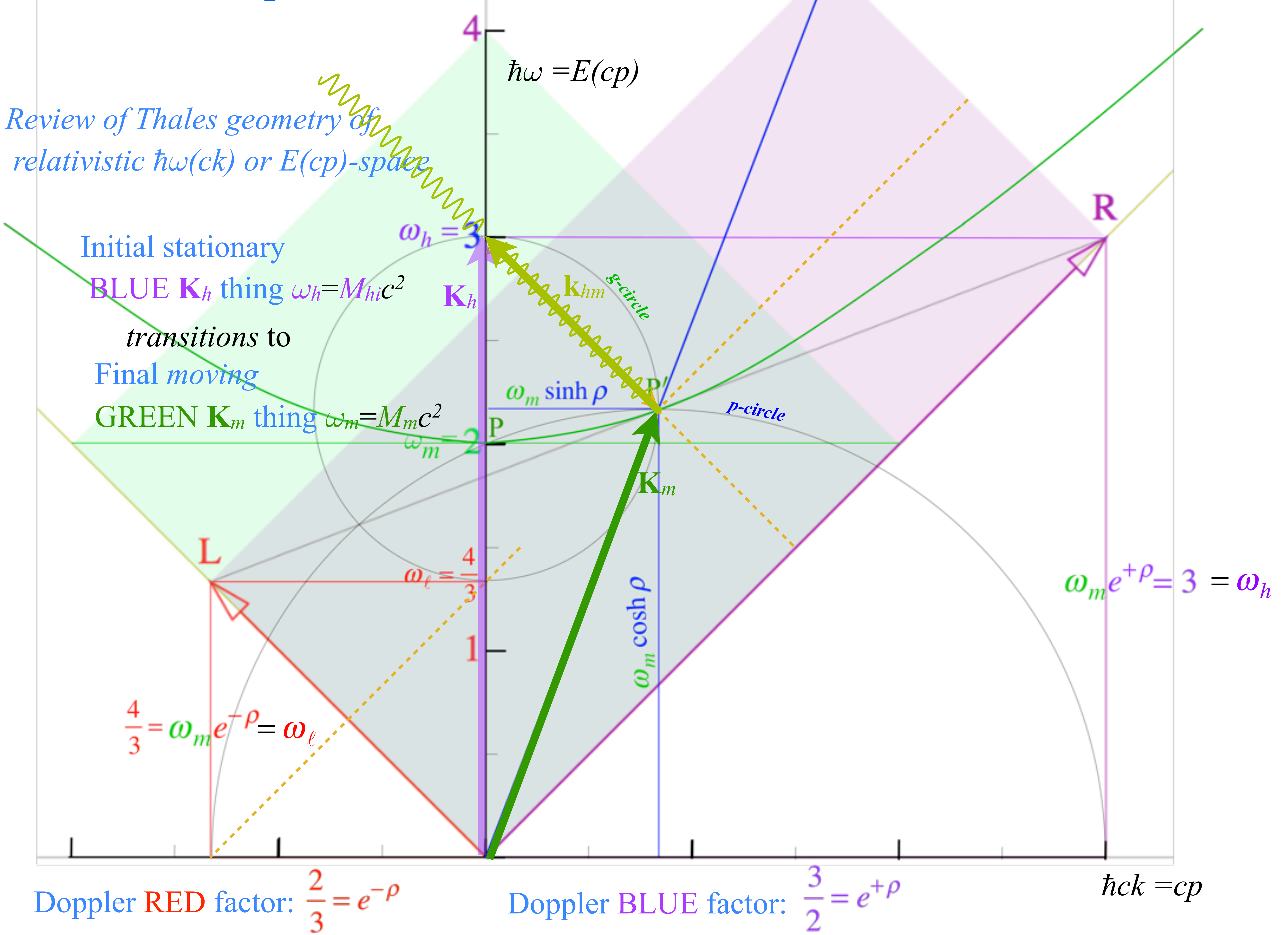
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

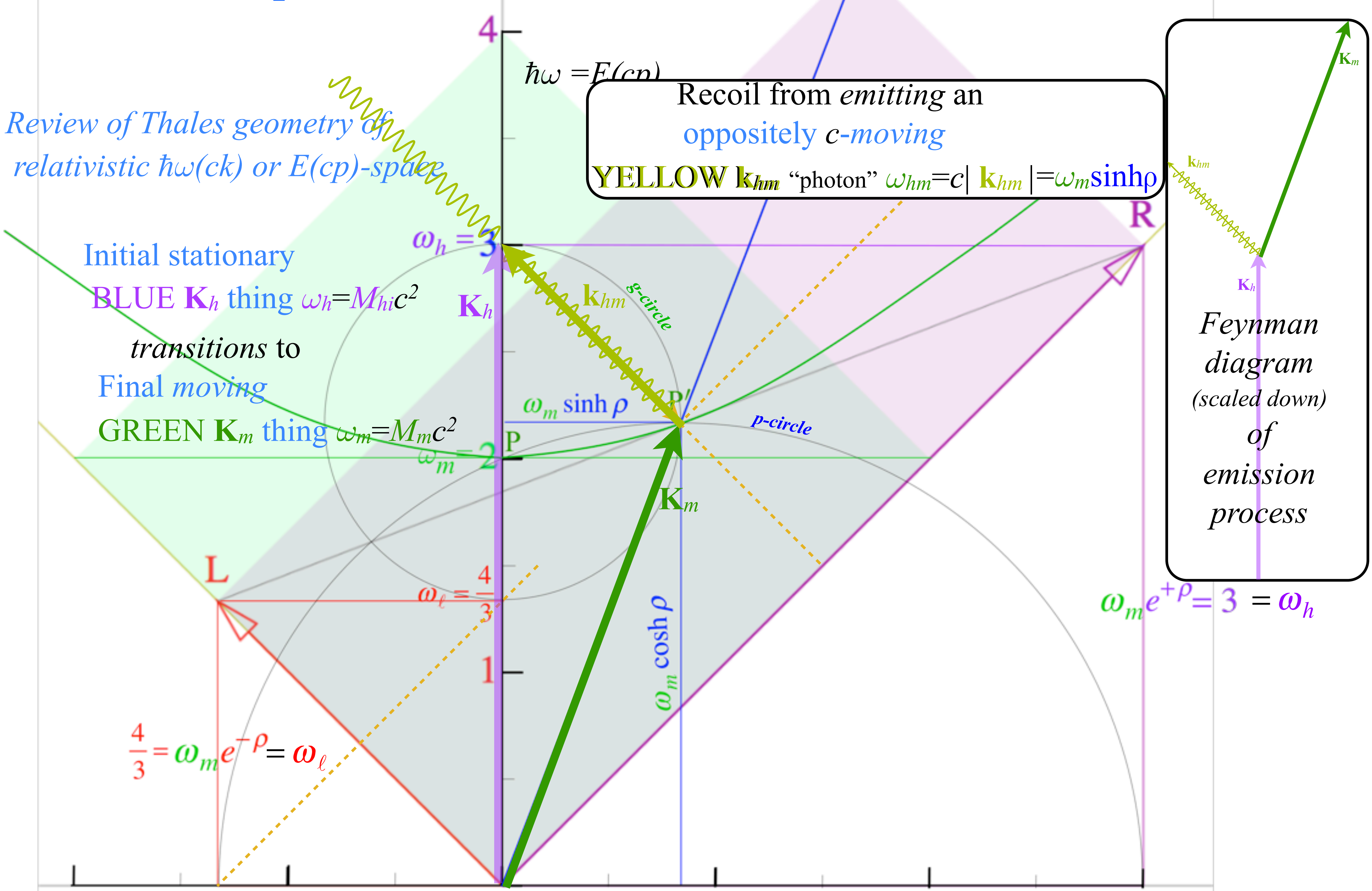


Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

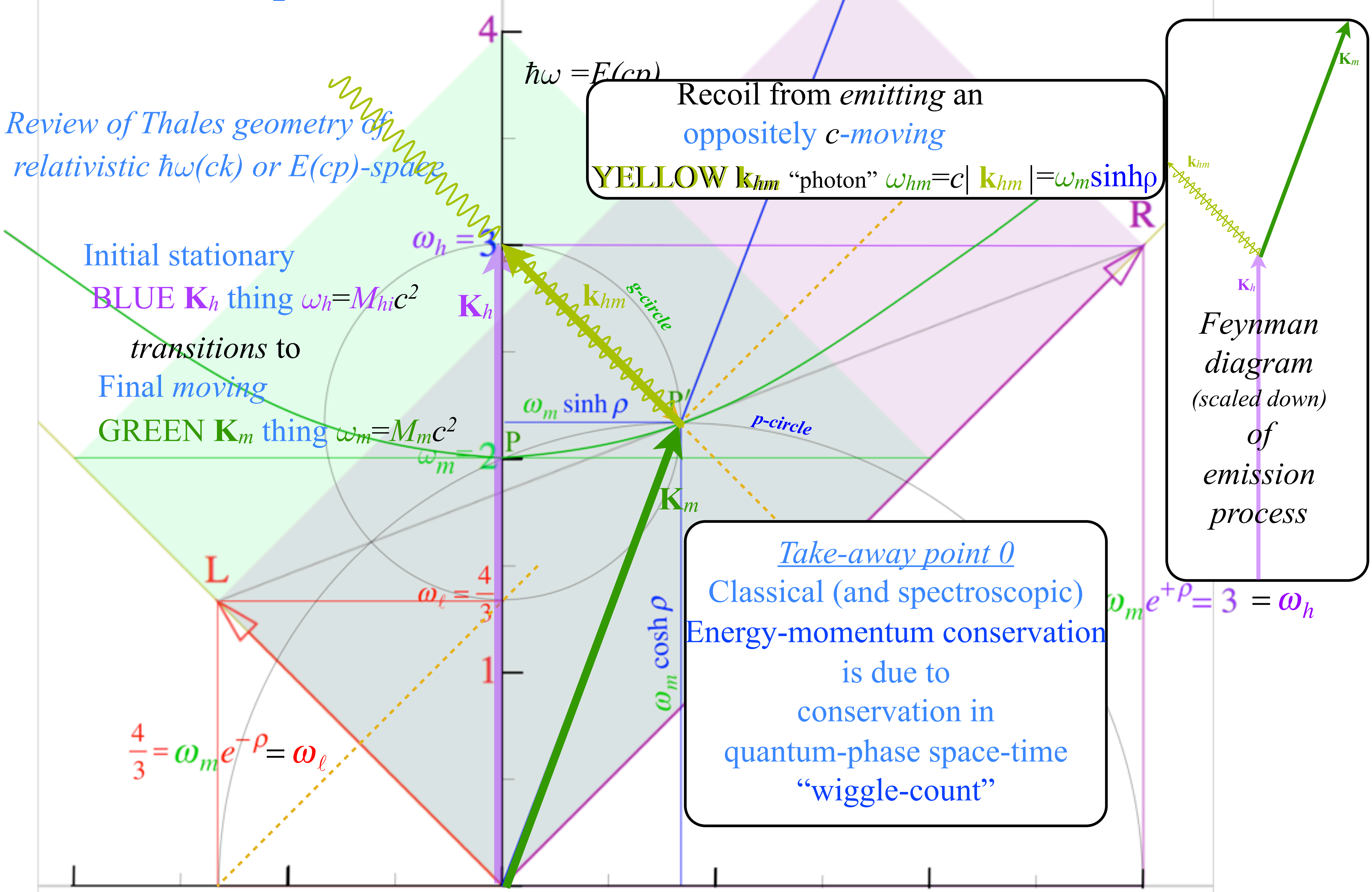


Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3}=e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2}=e^{+\rho}$

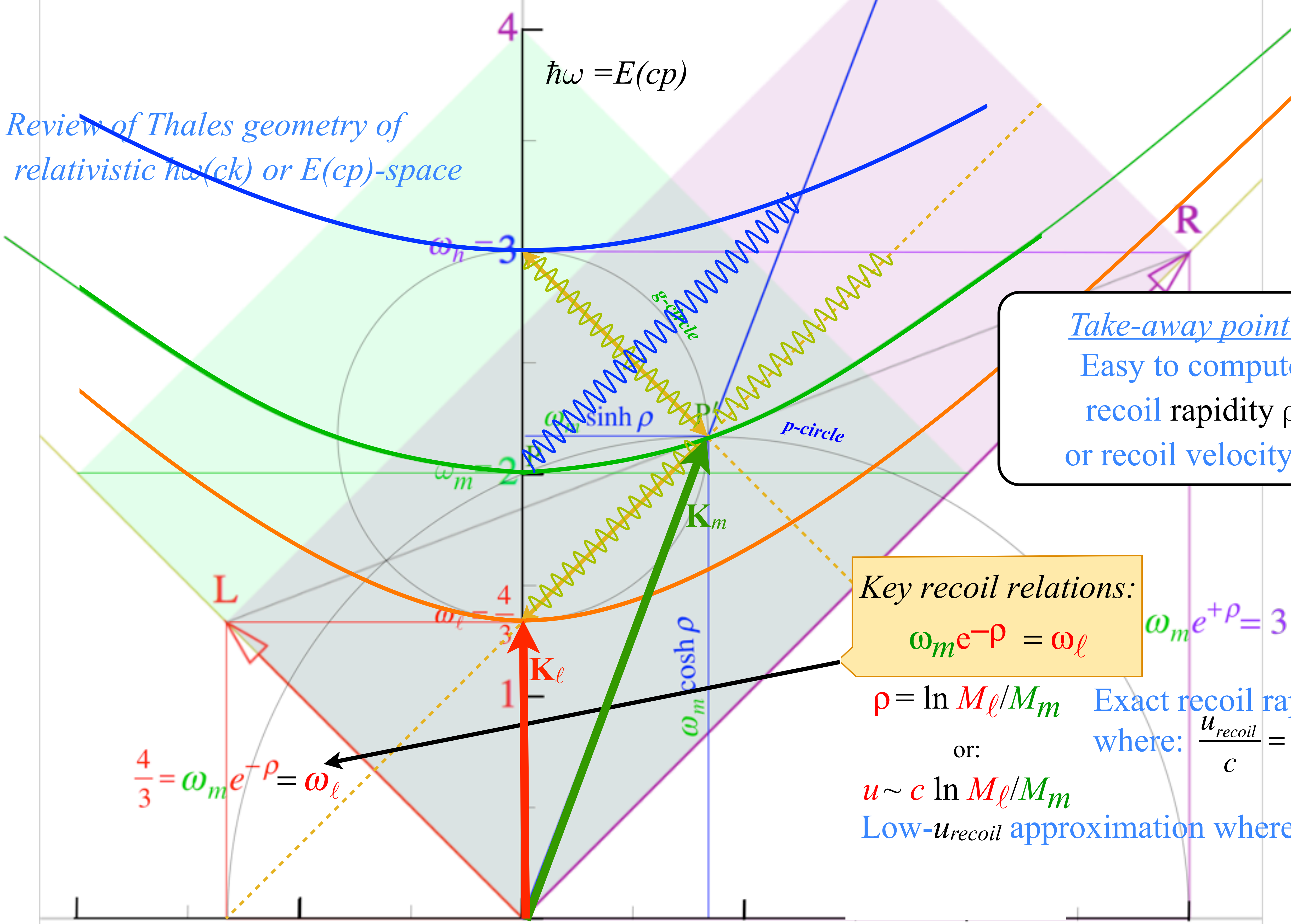
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space



<u>*Take-away point 2*</u>
Easy to compute
recoil rapidity ρ
or recoil velocity u

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

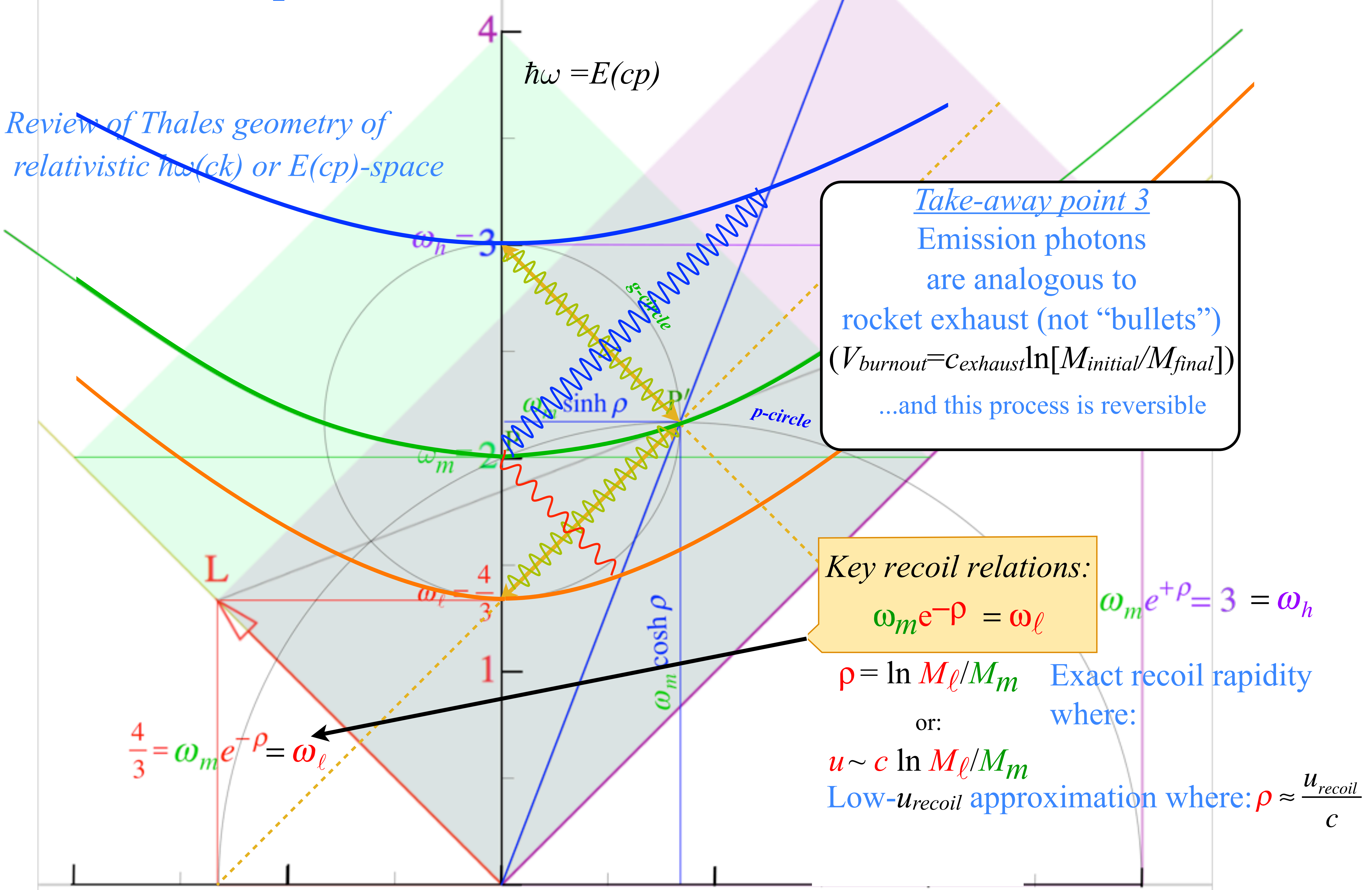
Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh \rho$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

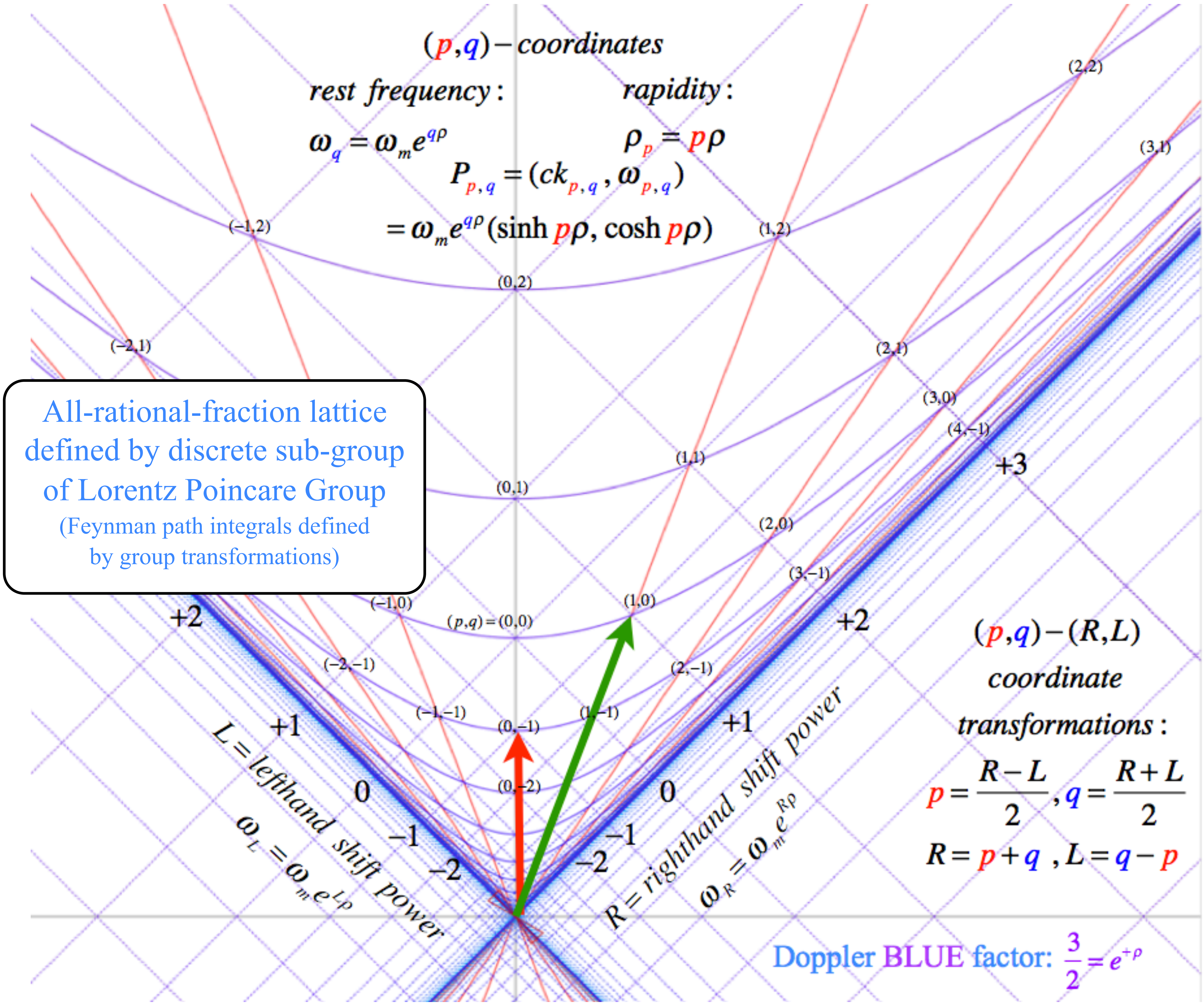
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



(p,q)−coordinates
rest frequency:
$\omega_q = \omega_m e^{q\rho}$
rapidity:
$\rho_p = p\rho$
$P_{p,q} = (ck_{p,q}, \omega_{p,q})$
$= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)$
All-rational-fraction lattice defined by discrete sub-group of Lorentz Poincare Group
(Feynman path integrals defined by group transformations)
(p,q)−(R,L) coordinate transformations:
$p = \frac{R-L}{2}, q = \frac{R+L}{2}$
$R = p+q, L = q-p$
L = lefthand shift power
$\omega_L = \omega_m e^{L\rho}$
R = righthand shift power
$\omega_R = \omega_m e^{R\rho}$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
(2,2)
(3,1)
(−1,2)
(1,2)
(0,2)
(−2,1)
(2,1)
(3,0)
(4,−1)
(1,1)
(0,1)
(2,0)
(3,−1)
(−1,0)
(p,q)=(0,0)
(1,0)
(−2,−1)
(2,−1)
(−1,−1)
(0,−1)
(1,−1)
(0,−2)
+3
+2
+1
0
−1
−2

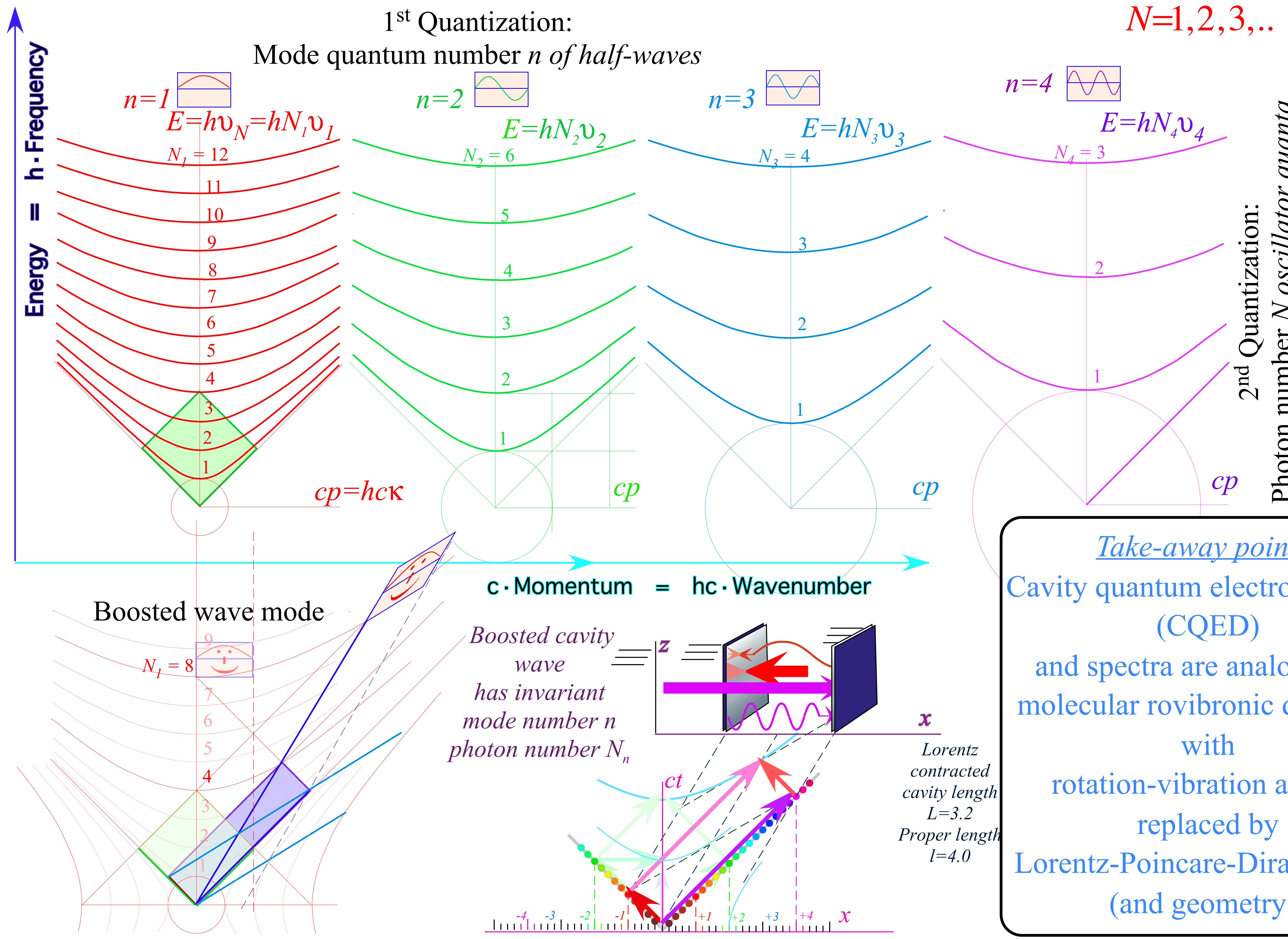
That's All Folks!

A "road-runner" axiom
is a "show-stopper"

2^{nd} Quantization: NEWS FLASH!!! $h\upsilon$ *is actually* $hN\upsilon$

($h\upsilon_{phase}=E=h\upsilon_A \cosh\rho$) *is actually* ($hN\upsilon_{phase}=E_N=hN\upsilon_A \cosh\rho$ with *quantum* numbers)

$N=1,2,3,..$

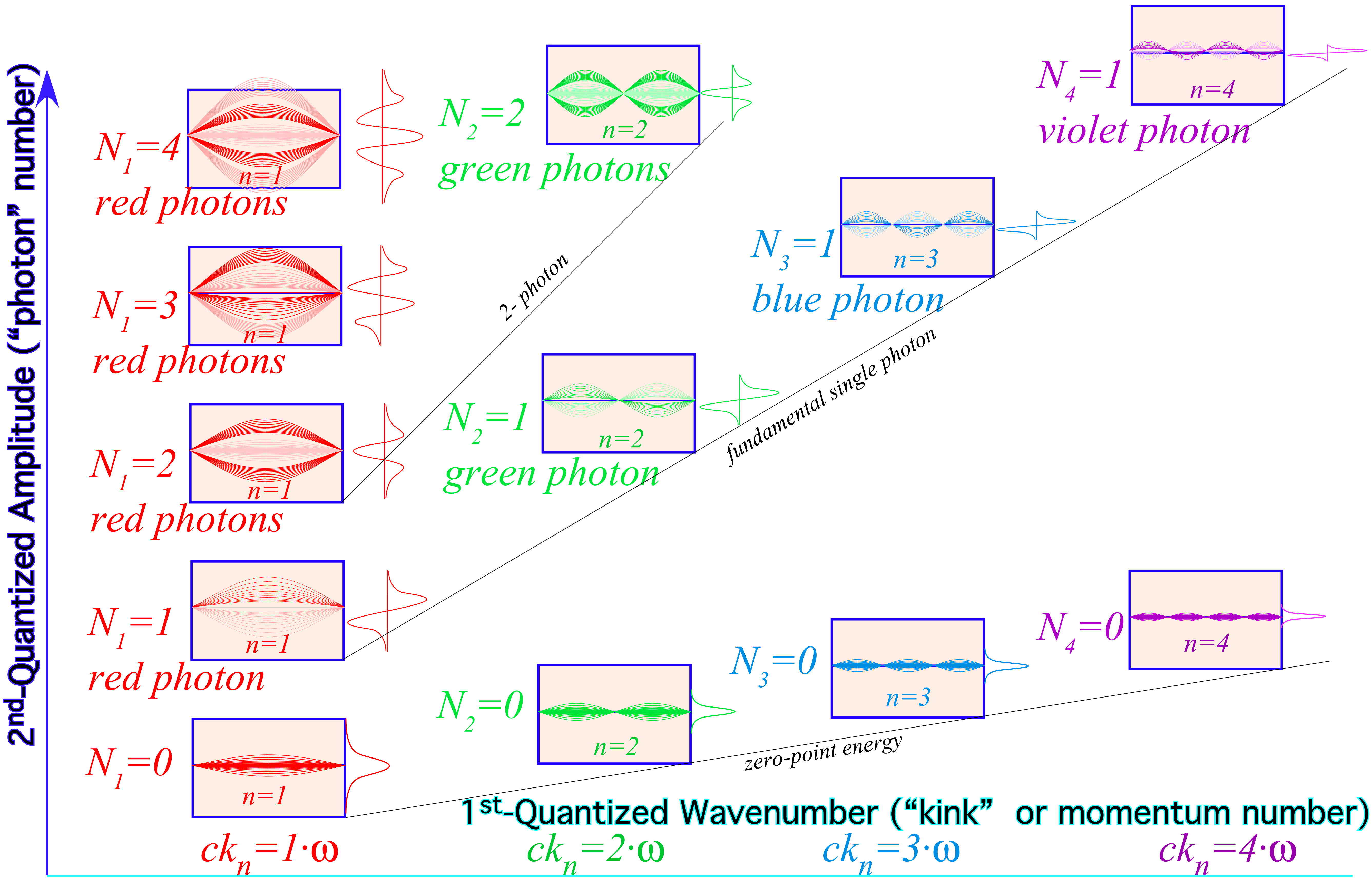


Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2^{nd} Quantization: NEWS FLASH!!! $h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(\ hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..)\ )$



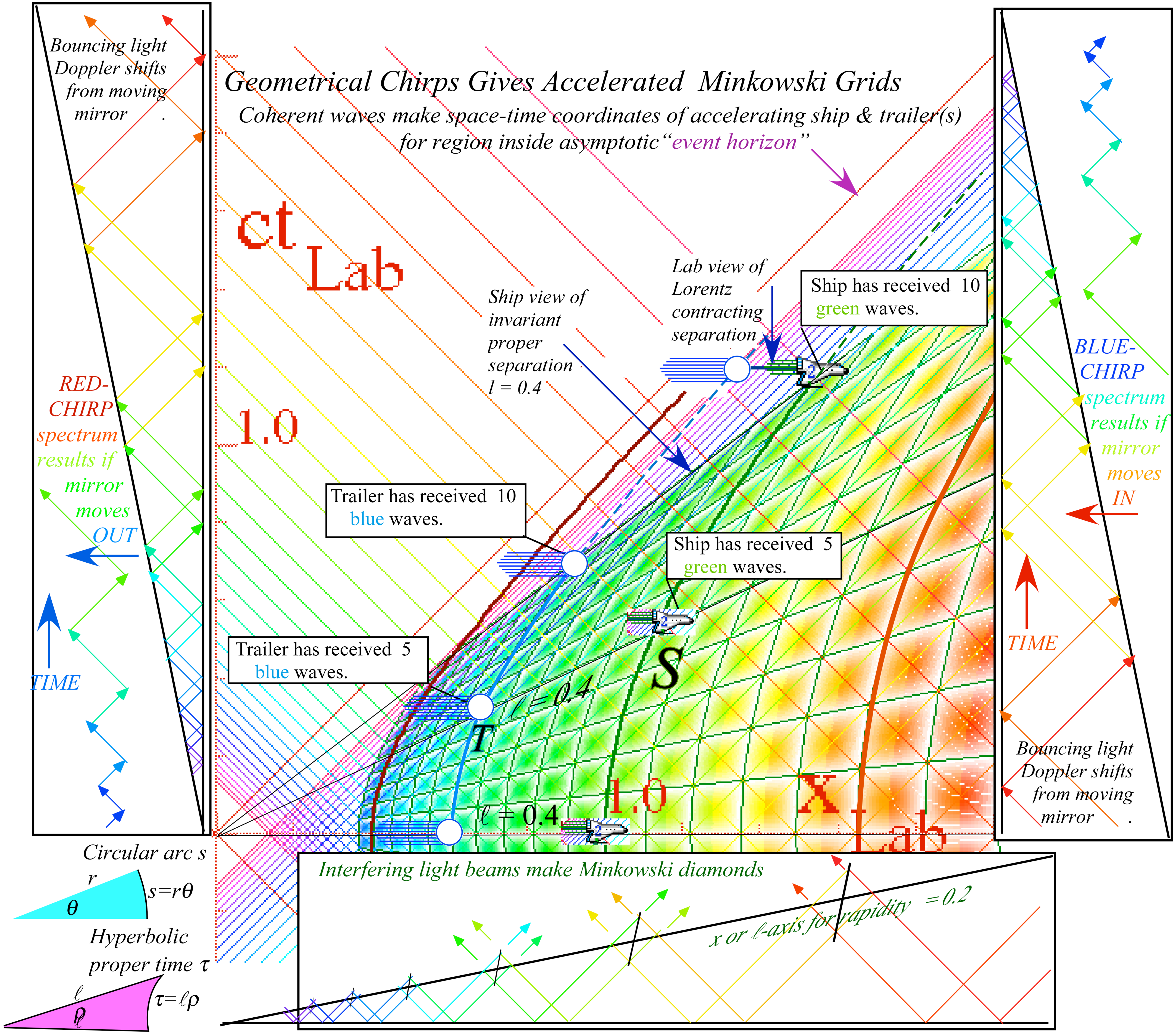


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S = \hbar\Phi$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A = Mc^2 = \hbar ck_A$

$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$

$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$\frac{dS}{dt}=L\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv pu-H=L$$

Legendre transformation

$$dS\equiv Ldt\equiv\hbar\, d\Phi=\hbar k\,dx-\hbar\omega\,dt=p\,dx-H\,dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S}=L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$

$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$\frac{dS}{dt} = L \equiv \hbar\frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

$$dS \equiv L dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S = \hbar\Phi$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar ck_A$

$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$

$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$

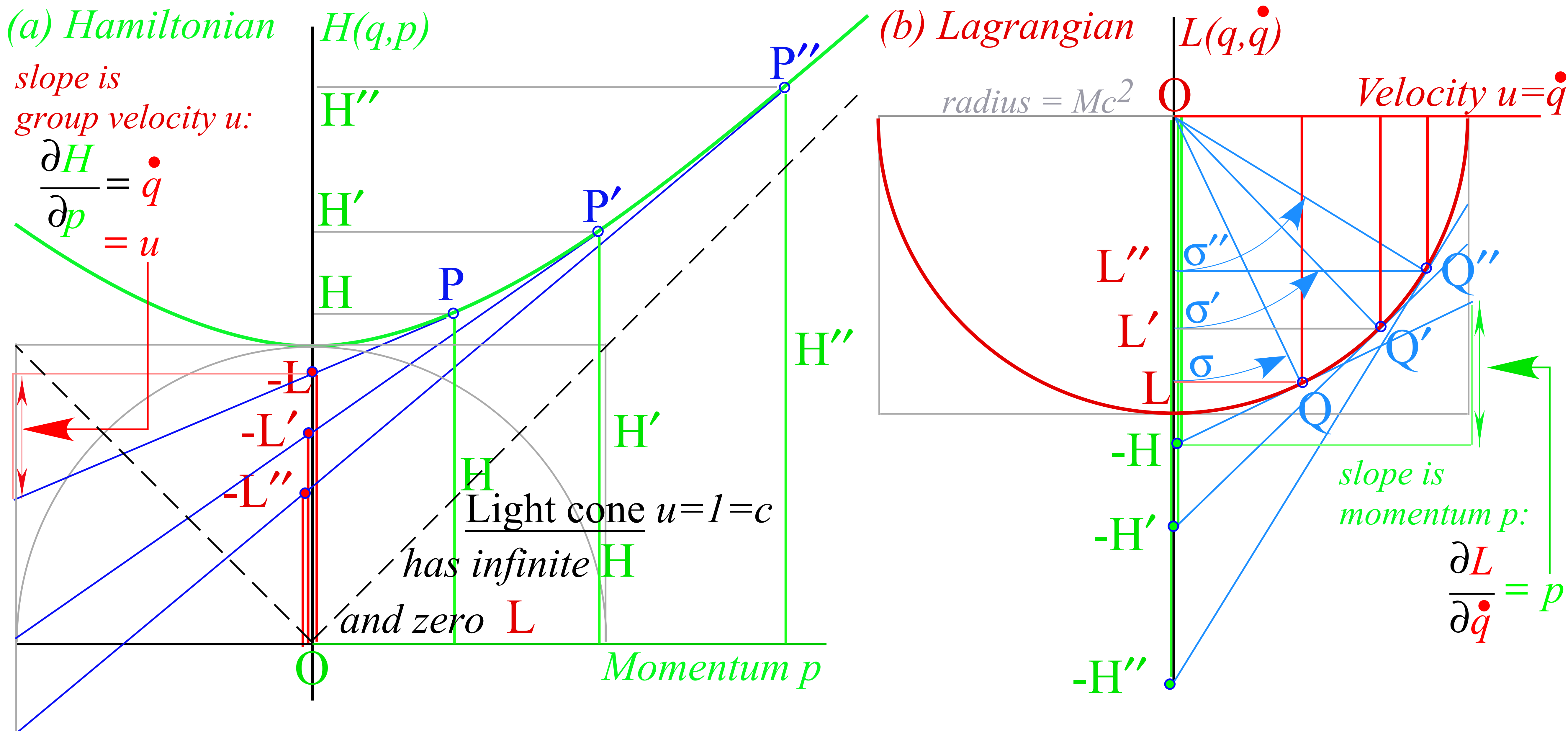
$$\hbar \equiv \frac{h}{2\pi}$$

Poincare Invariant Action $dS = L dt = p\,dq - H\,dt = \hbar d\Phi$ *(phase)*

Hamiltonian $H(p,q) = p\dot{q} - L$ vs. *Lagrangian* $L(\dot{q}q) = p\dot{q} - H$

Contact transformation: *(slope,-intercept)* of *H* (or *L*) *tangent* determines the *(X,Y coordinates)* of *L* (or *H*).

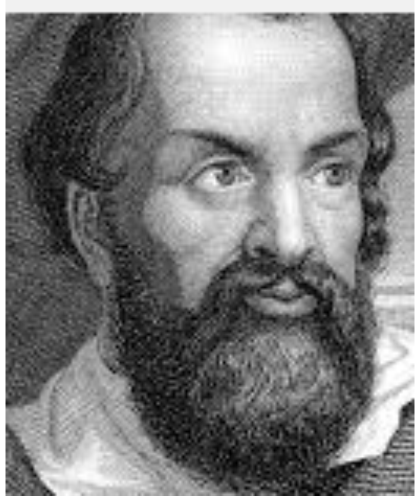
(Also, called a *Legendre contact transformation* which is a special case of a *Huygens transformation* that uses contacting tangent *curves* instead of *lines*.)



Here *slope* is group velocity $u=\dot{q}$
Y-coordinate is *energy* $H=\hbar\omega$

Here *slope* is momentum p
Y-coordinate is *phase rate* $L=\hbar\Phi$

Happy now?

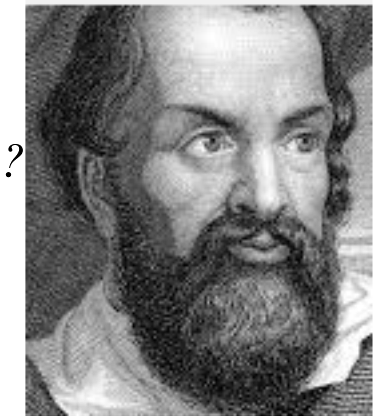


Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

$$\omega_{phase} = \frac{\omega_A + \omega_B}{2}$$

$$\omega_{group} = \frac{\omega_A - \omega_B}{2}$$

Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

$$\omega_{phase} = \frac{\omega_A + \omega_B}{2}$$

$$\omega_{group} = \frac{\omega_A - \omega_B}{2}$$

I got $\langle C|B\rangle = \langle C|A\rangle\langle A|B\rangle$ *=(2/3)(1/2)=1/3,*
and $\rho_{CB} = \rho_{CA} + \rho_{AB}$ *=-1.10*
We're in Splitsville!

Carla-Bob Doppler ratio:

Carla-Bob rapidity: