Rapidity \( \rho \) related to stellar aberration angle \( \sigma \) and L. C. Epstein’s approach to relativity
Longitudinal hyperbolic \( \rho \)-geometry connects to transverse circular \( \sigma \)-geometry
“Occams Sword” and summary of 16 parameter functions of \( \rho \) and \( \sigma \)
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What’s the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare’ and Hamilton-Jacobi equations

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Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2\(^{nd}\)-quantization “photon” number \( N \) and 1\(^{st}\)-quantization wavenumber \( \kappa \)

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes \( g \)-acceleration grid
Analysis of constant-\( g \) grid compared to zero-\( g \) Minkowski grid
Animation of mechanics and metrology of constant-\( g \) grid
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   Two famous ones: Extremes and plot vs. $\rho$
   Doppler jeopardy
   Geometric mean and Relativistic hyperbolas
   Animation of $e^\rho=2$ spacetime and per-spacetime plots

Rapidity $\rho$ related to stellar aberration angle $\sigma$ and L. C. Epstein’s approach to relativity
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      Analysis of constant-$g$ grid compared to zero-$g$ Minkowski grid
      Animation of mechanics and metrology of constant-$g$ grid
Introducing optical space-time grids and per-space-time “baseball-diamonds”
Introducing optical space-time grids and per-space-time “baseball-diamonds”

\[ e^{iR} + e^{iL} = e^{i \frac{R+L}{2}} \left( e^{i \frac{R-L}{2}} + e^{-i \frac{R-L}{2}} \right) \]

\[ = 2e^{i \frac{R+L}{2} \cos \frac{R-L}{2}} \]

\[ = 2e^{-i\omega t} \cos kx \]

\[ R = kx - \omega t \text{ and: } L = -kx - \omega t \]
Three scenarios that look the same to Bob

Alice’s laser moving right at $u = \frac{3c}{5}$ tuned to $v_A = 600\text{THz}$

Bob stationary

Carla’s laser moving right at $u = \frac{3c}{5}$ tuned to $v_A = 600\text{THz}$

Alice’s laser stationary tuned to $v_A = 600\text{THz}$

Bob moving left at $u = -\frac{3c}{5}$

Carla’s laser stationary tuned to $v_A = 600\text{THz}$

Alice’s laser stationary tuned up to $v_A = 1200\text{THz}$

Bob stationary

Carla’s laser stationary tuned down to $v_A = 300\text{THz}$

Much cheaper to do this one!$!
\[ \psi = Ae^{i(k_R x - \omega_R t)} + Ae^{i(-k_L x - \omega_L t)} \]

\[ P' = \frac{1}{2}(R' + L') \]

\[ G' = \frac{1}{2}(R' - L') \]

\[ G = \frac{1}{2}(R - L) \]

\[ P = \frac{1}{2}(R + L) \]

\[ e^{iR} + e^{iL'} = e^{i\frac{R' + L'}{2}} \left( e^{i\frac{R' - L'}{2}} + e^{-i\frac{R' - L'}{2}} \right) \]

\[ = e^{i\frac{R' + L'}{2}} \cdot 2 \cos \frac{R' - L'}{2} \]

\[ = \psi'_{\text{phase}} \psi'_{\text{group}} \]

\[ R' = k_R x - \omega_R t \quad \text{and} \quad L' = -k_L x - \omega_L t \]

**Fig. 10 in text**

*Relativawity...*
The 16 dimensions of 2CW interference

Frequency

\( v' \)  
(units of \( v_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)  
(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

Phase frequency \( v'_\text{phase} \)  
\( v'_\text{phase} = v_A \cosh \rho = \frac{5}{4} = 1.25 \)

Phase period \( \tau' = 1/v' \)  
\( \tau'_\text{phase} = \tau_A \text{sech} \rho = \frac{4}{5} \)

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( v' )</th>
<th>( v_A' )</th>
<th>( \kappa' )</th>
<th>( \kappa_A )</th>
<th>( \tau' )</th>
<th>( \tau_A )</th>
<th>( v'_{\text{phase}} )</th>
<th>( v_A'_{\text{phase}} )</th>
<th>( v'_{\text{group}} )</th>
<th>( v_A'_{\text{group}} )</th>
<th>( b_{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>( 1 / b_{\text{Doppler BLUE}} )</td>
<td>( V'_{\text{group}} )</td>
<td>( V_A'_{\text{group}} )</td>
<td>( \lambda'_{\text{group}} )</td>
<td>( \lambda_A'_{\text{group}} )</td>
<td>( \tau'_{\text{group}} )</td>
<td>( \tau_A'_{\text{group}} )</td>
<td>( c' )</td>
<td>( c_A' )</td>
<td>( 1 / b_{\text{Doppler RED}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rapidity ( \rho )</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \text{sech} \rho )</td>
<td>( \cosh \rho )</td>
<td>( \text{csch} \rho )</td>
<td>( \coth \rho )</td>
<td>( e^\rho )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.8 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The 16 dimensions of 2CW interference

\[
P' = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \nu_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( \nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4 \Rightarrow 1.25 \)

Phase period \( \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \Rightarrow 0.8 \)

Frequency \( \nu' \)
(units of \( \nu_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \times 10^6 / \text{m} \))

Space \( x' \)
(units of \( \lambda_A = 1/2 \mu \text{m} \))

The 16 dimensions of 2CW interference...then do the Dopplers...
Phase wavenumber $\kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4$

Phase wavelength $\lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3$

Frequency $\nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4$

Phase frequency flips to $\nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4$

Phase period $\tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5$

Phase wavelength $\lambda'_{\text{phase}} = \lambda_A \text{csch} \rho = 4/3$

Phase period $\tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5$

Phase wavenumber $\kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4$

Phase wavelength $\lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3$

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Phase wavelength $\lambda'_{\text{phase}} = \lambda_A \cosh \rho = 4/3$

Phase period $\tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5$
Lorentz transformations...

Write $G'$ and $P'$ in terms of $G$ and $P$ using $\cosh \rho$ and $\sinh \rho$

$$G' = \begin{pmatrix} c \kappa'_{\text{group}} \\ \nu'_{\text{group}} \end{pmatrix} = \nu_A \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = \nu_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$P' = G' \sinh \rho + P' \cosh \rho$$

$$G' = G \cosh \rho + P \sinh \rho$$

$$P' = G \sinh \rho + P \cosh \rho$$

Lorentz transform matrix

\[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix}
\]
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Two Famous-Name Coefficients

Review of Lect. 30 p.106

If you can't explain it simply, you don't understand it well enough.  
— Albert Einstein

Albert Einstein 1859-1955

This number is called a: Einstein time-dilation
(dilated by 25% here)

This number is called a: Lorentz length-contraction
(contracted by 20% here)

Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

RelaWavity Web Simulation - Relativistic Terms
(Expanded Table)

Time $ct'$
(units of $\lambda_A = 1/2\mu m$)

Space $x'$
(units of $\lambda_A = 1/2\mu m$)

This number $\psi'_{\text{phase}} = 1.25$

$\lambda'_{\text{group}} = 0.8$

$|\rho_s| = 0.5$

$1.5$

$1$

$0$

$-0.5$

$-1$

$-1.5$

$1.5$

$1$

$0$

$-0.5$

$2$

$|\rho_t| = 0.75$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_r| = 0.6$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_n| = 0.8$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_s| = 0.5$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_t| = 1.25$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_r| = 1.33$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_n| = 1.67$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$|\rho_s| = 2.0$

$1.5$

$1$

$0.5$

$-0.5$

$-1$

$1.5$

$2$

$1.25$

$1$

$0.5$

$-0.5$

$-1$

$1.25$

$1$

$0.5$

$-0.5$

$-1$

$1.25$

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$-0.5$

$-1$

$1.25$

$1$

$0.5$

$-0.5$

$-1$

$1.25$

$1$

$0.5$
Fig. 11 in text  Relawavity...

(a) Per-space-time ($\nu', c\kappa'$) geometry of 2-CW vectors

- Frequency: $\nu' = 2\pi \cdot \omega$
  (units: $\omega = 600 THz$)
  "waves per second"

- $P' + G' = R'$
- $P' - G' = L'$

(b) Space-time ($c\tau', x'$) geometry of 2-CW paths

- $c$-Time-Period $\tau' = \lambda'$
  (units: $\lambda' = c\tau_A = 2$ micron)

- $P' = c\tau_A \sinh \rho$
- $G' = c\tau_A \cosh \rho$

- $\nu' = \nu_A \sinh \rho$
- $\nu_{\text{group}} = \nu_A \cosh \rho$

- $\tau' = \tau_A$
- $\nu_{\text{ephase}} = \nu_A \cosh \rho$
- $\nu_{\text{group}} = \nu_A \cosh \rho$

- $\lambda_{\text{group}} = c\tau_A \cosh \rho$
- $\lambda' = c\tau_A \cosh \rho$

- $\kappa' = \nu_A \sinh \rho$
- $\kappa_{\text{group}} = \nu_A \cosh \rho$

- $b_{\text{Dopper RED}} = \frac{c}{V_{\text{phase}}}$
- $b_{\text{Dopper BLUE}} = \frac{c}{V_{\text{phase}}}$

<table>
<thead>
<tr>
<th>phase</th>
<th>$b'_{\text{Dopper RED}}$</th>
<th>$c$</th>
<th>$\kappa_{\text{phase}}$</th>
<th>$\kappa_A$</th>
<th>$\tau_{\text{phase}}$</th>
<th>$\tau_A$</th>
<th>$\nu_{\text{phase}}$</th>
<th>$\nu_A$</th>
<th>$\lambda_{\text{phase}}$</th>
<th>$\lambda_A$</th>
<th>$V_{\text{phase}}$</th>
<th>$V_A$</th>
<th>$b'_{\text{Dopper BLUE}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>1</td>
<td>$c$</td>
<td>$\nu_{\text{group}}$</td>
<td>$\nu_A$</td>
<td>$\kappa_{\text{group}}$</td>
<td>$\kappa_A$</td>
<td>$\tau_{\text{group}}$</td>
<td>$\tau_A$</td>
<td>$\nu_{\text{group}}$</td>
<td>$\nu_A$</td>
<td>$\lambda_{\text{group}}$</td>
<td>$\lambda_A$</td>
<td>$V_{\text{group}}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\frac{1-\beta}{1+\beta}$</td>
<td>$\beta$</td>
<td>1</td>
<td>$\frac{1}{\sqrt{\beta^2-1}}$</td>
<td>$\frac{1}{1-\sqrt{1-\beta^2}}$</td>
<td>$\frac{1}{1+\beta}$</td>
<td>$\frac{1}{\sqrt{1-\beta^2}}$</td>
<td>$\frac{1}{\beta}$</td>
<td>$\sqrt{1-\beta^2}$</td>
<td>$\frac{1}{\beta}$</td>
<td>$\frac{1}{\sqrt{1-\beta^2}}$</td>
<td>$\frac{1}{\beta}$</td>
<td>$\frac{1}{\sqrt{1-\beta^2}}$</td>
</tr>
</tbody>
</table>

- $\rho = e^{-\rho}$
- $\tanh \rho = \sinh \rho$
- $\sech \rho = \cosh \rho$
- $\cosh \rho = \cosh \rho$

- $\beta = \frac{u}{c}$

- $\frac{1}{2} = 0.5$
- $\frac{3}{5} = 0.6$
- $\frac{4}{3} = 0.75$
- $\frac{5}{4} = 0.80$
- $\frac{5}{4} = 1.25$
- $\frac{4}{3} = 1.33$
- $\frac{5}{3} = 1.67$
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two parameters become exactly equal:

\[
\frac{c}{c_{\tau_A}} = \frac{\lambda_g}{\lambda_A} = \frac{\tau_{\text{phase}}}{\tau_A} = \frac{\sinh \rho}{\cosh \rho} = 0.786... = \sqrt{G_-} = 0.786...
\]

and

\[
\frac{x_p'}{\lambda_A} = \frac{\cosh \rho}{\lambda_A} = \frac{\tau_{\text{phase}}}{\tau_A} = \frac{\cosh \rho}{\tau_A} = 1.272... = \frac{1}{\sqrt{G_-}} = 1.272..
\]

If \( \frac{u}{c} = \tanh \rho = 0.618... \) (Golden-Mean \( G_- \))

Solve:

\[
\sinh \rho = \sinh \rho
\]

or:

\[
\sinh \rho \cosh \rho = 1
\]

or:

\[
\sinh 2\rho = 2
\]

\[
\rho = \frac{1}{2} \sinh^{-1} 2 = 0.7218...
\]

\[
\tanh \rho = 0.618... = \frac{\sqrt{5} - 1}{2}
\]
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  Animation of mechanics and metrology of constant-$g$ grid
(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\nu_E$?
(2.) What is that frequency $\nu_E$?
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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{\text{group}} = \frac{u_{\text{group}}}{K_{\text{group}}} = \frac{\nu_R - \nu_L}{K_R - K_L} = c \frac{\nu_R - \nu_L}{\nu_R + \nu_L}$$
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Query (2.) similarly: What $\nu_E$ is blue-shift $b\nu_L$ of $\nu_L$ and red-shift $\nu_R/b$ of $\nu_R$?

$$\nu_E = b\nu_L = \nu_R/b \quad \Rightarrow \quad b = \sqrt{\nu_R/\nu_L} \quad \Rightarrow \quad \nu_E = \sqrt{\nu_R \cdot \nu_L}$$
Doppler Jeopardy

\( \omega_R = 2\pi \nu_R \quad \Rightarrow \quad \nu_R = 600\text{THz} \)

\( \omega_L = 2\pi \nu_L \quad \Rightarrow \quad \nu_L = 300\text{THz} \)

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\[ \nu_E = b \nu_L = \nu_R / b \quad \Rightarrow \quad b = \sqrt{\nu_R / \nu_L} \quad \Rightarrow \quad \nu_E = \sqrt{\nu_R \cdot \nu_L} \]

Geometric mean
(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $v_E$?

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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

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$$V_{\text{group}} = c \frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c \frac{600 - 300}{600 + 300} = \frac{1}{3} c$$

Query (2.) similarly: What $v_E$ is blue-shift $b v_L$ of $v_L$ and red-shift $v_R/b$ of $v_R$?

$$v_E = b v_L = v_R/b \quad \Rightarrow \quad b = \sqrt{v_R/v_L} \quad \Rightarrow \quad v_E = \sqrt{v_R \cdot v_L}$$

*Geometric mean*
(1.) To what velocity $u_E$ must Bob accelerate so he sees beams with equal frequency $\nu_E$?
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$$\nu_E = \sqrt{\nu_R \cdot \nu_L} = \sqrt{180000} = 424$$

Geometric mean
Doppler Jeopardy

(1.) To what velocity \( u_E \) must Bob accelerate so he sees beams with equal frequency \( \nu_E \)?

(2.) What is that frequency \( \nu_E \)?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

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\nu_E = V_{\text{group}} = \frac{\nu_R - \nu_L}{\nu_R + \nu_L} = \frac{c}{\nu_R + \nu_L}
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\]

\[ V_{\text{group}}/c \] is ratio of difference mean \( \nu_{\text{group}} = \frac{\nu_R - \nu_L}{2} \) to arithmetic mean \( \nu_{\text{phase}} = \frac{\nu_R + \nu_L}{2} \). Frequency \( \nu_E = B \) is the geometric mean \( \sqrt{\nu_R \cdot \nu_L} \) of left and right-moving frequencies defining the geometry.
Review: Relawavity $\rho$ functions Two famous ones Extremes and plot vs. $\rho$
Doppler jeopardy ➤Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity $\rho$ related to *stellar aberration angle* $\sigma$ and L. C. Epstein’s approach to relativity
Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
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*Learning about* $\sin$ and $\cos$ *and...*

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Thales Mean Geometry (600BCE)

helps “Relativity”

Frequency unit: 300THZ

Thales of Miletus 624-543 BCE

\[ \frac{4 - 1}{4 + 1} \]

**Geometric Mean**

\[ B = \sqrt{(4 \cdot 1)} = 2 \]

**Arithmetic Mean**

\[ Bcosh\rho = \frac{(1+4)}{2} = \frac{5}{2} \]

**Difference Mean**

\[ Bsinh\rho = \frac{(4-1)}{2} = \frac{3}{2} \]

**Per-Time**

\[ \omega - axis \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

**Per-Space**

\[ ck- axis \]

\[ -1 \]

\[ 1 \]

\[ 3 \]

\[ 4 \]
Thales Mean Geometry (600BCE)

helps “Relawavity”  

Thales showed a circle diameter subtends a right angle with any circle point $P$

This leads to a convenient construction of geometric means and relativistic hyperbolas.

Thales of Miletus  
624-543 BCE

RelaWavity Web Simulation  
Detailed Thales Geometry
Thales Mean Geometry (600BCE)

helps “Relawavity”

Thales of Miletus
624-543 BCE

Thales Mean Geometry

\[ r \cdot b = 2 \]
due to Doppler T-symmetry

Relawavity Web Simulation
Detailed Thales Geometry

Geometric Mean
\[ B = \sqrt{4 \cdot 1} = 2 \]

Arithmetic Mean
\[ B \cosh \rho = \frac{(1+4)}{2} = \frac{5}{2} \]

Difference Mean
\[ B \sinh \rho = \frac{(4-1)}{2} = \frac{3}{2} \]
(a) Laser lab view

(b) Atom view
(a) Laser lab view

(b) Atom view

Time Dilation
\[ \frac{\Delta t'}{t'} = \frac{1}{\sqrt{1-v^2/c^2}} \]

Length Contraction
\[ \frac{\Delta L'}{L'} = \sqrt{1-v^2/c^2} \]
OK! But...
What about “Time Contraction”?
or
“Length dilation”?

(a) Laser lab view

(b) Atom view

Atom Time
ct' - axis

Time Dilation
\[
\frac{\Delta t'}{t'} = \frac{1}{\sqrt{1-v^2/c^2}}
\]

Length Contraction
\[
\frac{\Delta L'}{L'} = \sqrt{1-v^2/c^2}
\]

Atom Space
x' - axis
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Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


*Observer fixed below star sees it directly overhead.*

*Observer going $\mathbf{u}$ sees star at angle $\sigma$ in $\mathbf{u}$ direction.*

Stellar aberration angle $\sigma$:

$$c \tanh \rho = u = c \sin \sigma$$

- We used notion $\sigma$ for stellar-ab-angle, (a “flipped-out” $\rho$).
- Epstein not interested in $\rho$ analysis or in relation of $\sigma$ and $\rho$. 

Purchase at: [Amazon](https://www.amazon.com/Relativit%C3%A4tstheorie-Lewis-Carroll-Epstein/dp/3764368688)
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**Proper time** $c\tau$ vs. **coordinate space** $x$ - (L. C. Epstein’s “Cosmic Speedometer”)

Particles $P$ and $P'$ have speed $u$ in $(x',ct')$ and speed $c$ in $(x, c\tau)$

**Proper time** $c\tau$

$\tau = \sqrt{(ct')^2 - (x')^2}$

**Coordinate** $x' = (u/c)ct' = ut'$

Einstein time dilation:

$ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau/\sqrt{1-u^2/c^2}$

Lorentz length contraction:

$L' = L \sech \rho = L \cos \sigma = L \cdot \sqrt{1-u^2/c^2}$

Proper Time asimultaneity:

$c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho$

$= L \cos \sigma \tan \sigma$

$= L \sin \sigma = L / \sqrt{c^2/u^2 - 1} \sim L u/c$

Epstein’s trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$
Comparing **Longitudinal** relativity parameter: \( \text{Rapidity } \rho = \log_e(\text{Doppler Shift}) \)

to a **Transverse** \*relativity parameter: Stellar aberration angle \( \sigma \)


**Proper time** \( c\tau \) vs. **coordinate space** \( x \) - (L. C. Epstein’s “Cosmic Speedometer”)

Particles \( P \) and \( P' \) have speed \( u \) in \((x', c\tau')\) and speed \( c \) in \((x, c\tau)\)

**Proper time** \( c\tau \)

\[
c\tau = \sqrt{(ct')^2 - (x')^2}
\]

**Coordinate** \( x' = (u/c)ct' = ut' \)

**Einstein time dilation:**

\[
ct' = c\tau \sec \sigma = c\tau \cosh \rho = c\tau / \sqrt{1 - u^2 / c^2}
\]

**Lorentz length contraction:**

\[
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\]

**Proper Time simultaneity:**

\[
c \Delta \tau = L' \sinh \rho = L \cos \sigma \sinh \rho
\]

\[
= L \cos \sigma \tan \sigma
\]

\[
= L \sin \sigma = L / \sqrt{c^2 / u^2 - 1} \sim L u / c
\]

Epstein’s trick is to turn a hyperbolic form \( c\tau = \sqrt{(ct')^2 - (x')^2} \) into a circular form: \( \sqrt{(c\tau)^2 + (x')^2} = (ct') \)

Then everything (and everybody) always goes speed \( c \) through \((x', c\tau)\) space!
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This map has circle sector arc-area $\sigma = 0.6435$ set to angle $\angle \sigma = 36.87^\circ = 0.6435 \text{radian}$

\[
\begin{align*}
\sin(\sigma) &= 0.6000 = \tanh(\rho) = \frac{3}{5} \\
\tan(\sigma) &= 0.7500 = \sinh(\rho) = \frac{3}{4} \\
\sec(\sigma) &= 1.2500 = \cosh(\rho) = \frac{5}{4} \\
\cos(\sigma) &= 0.8000 = \sech(\rho) = \frac{4}{5} \\
\cot(\sigma) &= 1.3333 = \csc(\rho) = \frac{4}{3} \\
\csc(\sigma) &= 1.6667 = \coth(\rho) = \frac{5}{3}
\end{align*}
\]

\[
\begin{align*}
\cosh(\rho) + \sinh(\rho) &= \frac{5}{4} + \frac{3}{4} = 2.0 = e^\rho \\
\cosh(\rho) - \sinh(\rho) &= \frac{5}{4} - \frac{3}{4} = 1/2 = e^{-\rho}
\end{align*}
\]

\[
\begin{align*}
\cosh(\rho) &= \frac{e^\rho + e^{-\rho}}{2} \quad \text{Half-Sum} \\
\sinh(\rho) &= \frac{e^\rho - e^{-\rho}}{2} \quad \text{Half-Difference}
\end{align*}
\]

Half-Sum-Half-Difference Trig-Formulae for exponentials $e^{\pm \rho}$

Also it is set to hyperbola sector arc-area $\rho = 0.6931$ angle $\angle \rho = \nu = 30.96^\circ$

\[
\begin{align*}
B\cosh(\rho) + B\sinh(\rho) &= Be^{\rho} \\
B\cosh(\rho) - B\sinh(\rho) &= Be^{-\rho}
\end{align*}
\]
Summary of optical wave parameters for relativity and QM
...and their geometry

\[ \nu' = \omega' / 2\pi \]
(Units of 300THz)

An aid to pattern recognition:

RelaWavity Web Simulation
\{perSpace - perTime All\}
\[ v' = \omega'/2\pi \]

(Units of 300THz)

<table>
<thead>
<tr>
<th>group</th>
<th>phase</th>
<th>rapidity ( \rho )</th>
<th>stellar angle ( \sigma )</th>
<th>( \beta \equiv u/c )</th>
<th>value for ( \beta = 3/5 )</th>
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<td>( \frac{1 - \beta}{1 + \beta} )</td>
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Optical wave guide relativistic geometry aided by Occam’s Sword geometry applies to \((x,y)\) space-space to \((k_x,k_y)\)per-space-per-space to \((x,ct)\) space-time

Relativistic mode with near-\(c\) \(V_{\text{group}}=c/2\) and \(V_{\text{phase}}=2c\). (Low dispersion.)

Example of near-cut-off mode with low \(V_{\text{group}}=c/2\) and high \(V_{\text{phase}}=2c\). (Low dispersion.)

**KEY:**
- \(Re\ E\ phase\) k-vectors and rays
- \(k\)-vectors and rays
- wave-fronts upward downward crest trough

\[\sigma=60^\circ\]

\[\nu_{\text{CUTOFF}}=c/2\]

\[\nu_{\text{CUTOFF}}=B=v_A\]

\[\nu_{\text{CUTOFF}}=c/2\]

\[\lambda_{\text{phase}}=\sqrt{3}\]

\[\lambda_{\text{phase}}=\frac{B}{\sqrt{3}}\]

\[V_{\text{group}}=\frac{c\sqrt{3}}{2}\]

\[V_{\text{group}}=c\ tanh \rho\]

\[V_{\text{phase}}=c\ coth \rho\]

\[V_{\text{phase}}=c\ sin \sigma\]

\[V_{\text{phase}}=c\ csc \sigma\]
Optical wave guide relativistic geometry aided by Occam’s Sword geometry applies to \((x,y)\) space-space to \((k_x,k_y)\) per-space-per-space to \((x,ct)\) space-time

Example of near-cut-off mode with low \(V_{\text{group}} = c/2\) and high \(V_{\text{phase}} = 2c\). (High dispersion.)
Optical wave guide relativistic geometry aided by Occam’s Sword geometry applies to (x,y) space-space to (k_x,k_y)per-space-per-space to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)
Spherical wave relativistic geometry
Also, aided by Occam’s Sword
(a) Spherical wave pair
In Alice-Carla frame

Stellar angle \( \sigma = \sin^{-1}(u/c) \)

Velocity angle \( \nu = \tan^{-1}(u/c) \)

Slope \( u/c \) of velocity line

Tracks each expanding circle-top

Emitted at time \( t = -5, -4, -3, -2, -1, 0 \)

Main Lighthouse’s Frame ← RelativIt Web Simulation - Space-Time with many blinks → Ship’s Frame

(b) Spherical wave pair
In Bob’s frame: \( u_x/c = -3/5 \)

Occam Sword geometry in \((x, y)\) space-space

\[ 5\sqrt{c^2-u^2} = 4c \]
Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in Space-Space

Applications of Einstein dilation factor:
- \( \gamma = \cosh \rho = \sqrt{1 - u^2/c^2} \)
- Base height \( FTk = \sqrt{c^2 - u^2} \) (equal to ellipse minor radius \( b \))

Doppler Red \( \lambda = c+u \)
- Major radius \( a = OF = c \)
- Ellipse focal length \( FO = u = ctanh \rho \)
- Dilates to: \( (c+u) \cosh \rho = c \sqrt{\frac{c+u}{c-u}} = ce^\rho \)
- Dilates to: \( u \cosh \rho = c \sinh \rho \)
- Ellipse latus radius \( FT = c(1-u^2/c^2) \)

Doppler Blue \( \lambda = c-u \)
- Major radius \( a = OF = c \)
- Ellipse focal length \( FO = u = ctanh \rho \)
- Dilates to: \( (c-u) \cosh \rho = c \sqrt{\frac{c-u}{c+u}} = ce^\rho \)
- Dilates to: \( u \cosh \rho = c \sinh \rho \)
- Ellipse latus radius \( FT = c(1-u^2/c^2) \)

Sunday, January 29, 2017
Spherical wave relativistic geometry

\[ u/c = 1/3 \]
Spherical wave relativistic geometry

\[ u/c = 3/4 \]
Review: Relawavity $\rho$ functions  Two famous ones  Extremes and plot vs. $\rho$
Doppler jeopardy  Geometric mean and Relativistic hyperbolas
  Animation of $e^\rho=2$ spacetime and per-spacetime plots

Rapidity $\rho$ related to stellar aberration angle $\sigma$ and L. C. Epstein’s approach to relativity
Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
“Occams Sword” and summary of 16 parameter functions of $\rho$ and $\sigma$
Applications to optical waveguide, spherical waves, and accelerator radiation

Learning about $\sin!$ and $\cos$ and...

Derivation of relativistic quantum mechanics
What’s the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
  Poincare’ and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
  Compton recoil related to rocket velocity formula
  Comparing 2$^{nd}$-quantization “photon” number $N$ and 1$^{st}$-quantization wavenumber $\kappa$

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes $g$-acceleration grid
  Analysis of constant-$g$ grid compared to zero-$g$ Minkowski grid
  Animation of mechanics and metrology of constant-$g$ grid
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \]

\[ c\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad (\text{for } u \ll c) \]

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \]

\[ \sinh \rho \approx \rho \]

At low speeds:

\[ B = \nu_A \]

\[ B = \nu_A = c\kappa_A \]

---

**Table: Relativistic Terms (Expanded Table)**

<table>
<thead>
<tr>
<th>( \nu_{\text{phase}} )</th>
<th>( b_{\text{Doppler}} )</th>
<th>( V_{\text{group}} )</th>
<th>( \nu_\text{group} )</th>
<th>( \nu_A )</th>
<th>( \lambda_{\text{group}} )</th>
<th>( \lambda_A )</th>
<th>( \kappa_{\text{group}} )</th>
<th>( \kappa_A )</th>
<th>( \tau_{\text{group}} )</th>
<th>( \tau_A )</th>
<th>( V_{\text{phase}} )</th>
<th>( c )</th>
<th>( b_{\text{Doppler}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta \equiv \frac{u}{c} )</td>
<td>[ \sqrt{1-\beta^2} ]</td>
<td>[ \frac{1}{\sqrt{1+\beta}} ]</td>
<td>[ \frac{1}{\sqrt{1-\beta^2}} ]</td>
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<td>value for ( \beta=3/5 )</td>
<td>[ \frac{1}{2} = 0.5 ]</td>
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<td></td>
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Using (some) wave parameters to develop relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]
\[ c\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \]
\[ \sinh \rho \approx \rho \approx \frac{u}{c} \]

\[ B = v_A \]
\[ B = v_A = c\kappa_A \]

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<tr>
<th>( b_{\text{Doppler}}^\text{RED} )</th>
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<th>( \lambda_{\text{group}} )</th>
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<tr>
<td>phase</td>
<td>( \frac{1}{b_{\text{Doppler}}^\text{BLUE}} )</td>
<td>( \frac{c}{V_{\text{phase}}} )</td>
<td>( \frac{\kappa_{\text{phase}}}{\kappa_A} )</td>
<td>( \frac{\tau_{\text{phase}}}{\tau_A} )</td>
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<td>rapidity ( \rho )</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \sech \rho )</td>
<td>( \cosh \rho )</td>
<td>( \csch \rho )</td>
<td>( \coth \rho )</td>
</tr>
<tr>
<td>stellar angle ( \sigma )</td>
<td>( \frac{1}{e^{+\rho}} )</td>
<td>( \sin \sigma )</td>
<td>( \tan \sigma )</td>
<td>( \cos \sigma )</td>
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\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]

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\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:

\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \]

\[ \cos \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \]

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At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{time} & b_D^{\text{Doppler}} & \frac{V_{\text{group}}}{c} & \frac{\nu_{\text{group}}}{\nu_A} & \frac{\tau_{\text{phase}}}{\tau_A} & \frac{\nu_{\text{phase}}}{\nu_A} & \frac{\tau_{\text{group}}}{\tau_A} & \frac{V_{\text{phase}}}{V_A} & b_D^{\text{Doppler}} \\
\hline
\text{space} & \frac{1}{b_D^{\text{BLUE}}} & \frac{1}{V_{\text{phase}}} & \frac{\kappa_{\text{phase}}}{\kappa_A} & \frac{\lambda_{\text{group}}}{\lambda_A} & \frac{\lambda_{\text{phase}}}{\lambda_A} & \frac{1}{V_{\text{group}}} & \frac{1}{b_D^{\text{RED}}} \\
\hline
\text{rapidity } \rho & e^{-\rho} & \tanh \rho & \sinh \rho & \sech \rho & \cosh \rho & \csch \rho & \coth \rho & e^{+\rho} \\
\hline
\text{stellar angle } \sigma & 1/e^{+\rho} & \sin \sigma & \tan \sigma & \cos \sigma & \sec \sigma & \cot \sigma & \csc \sigma & 1/e^{-\rho} \\
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\beta \equiv \frac{u}{c} & \frac{1}{\sqrt{1+\beta}} & \frac{\beta}{1} & \frac{1}{\sqrt{\beta^2-1}} & \frac{\sqrt{1-\beta^2}}{1} & \frac{1}{\sqrt{1-\beta^2}} & \frac{\sqrt{\beta^2-1}}{1} & \frac{1}{\beta} & \frac{\sqrt{1+\beta}}{1} \\
\hline
\text{value for } \beta = 3/5 & \frac{1}{2} = 0.5 & \frac{3}{5} = 0.6 & \frac{3}{4} = 0.75 & \frac{4}{5} = 0.80 & \frac{5}{4} = 1.25 & \frac{4}{3} = 1.33 & \frac{5}{3} = 1.67 & \frac{2}{1} = 2.0 \\
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\]
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\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c \text{)} \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{ (for } u \ll c \text{)} \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{ (for } u \ll c \text{)} \]

At low speeds:

\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{ for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \mathcal{U}_{\text{phase}} \text{ and } \kappa_{\text{phase}} \text{ resemble formulae for Newton’s kinetic energy and momentum} \]

Resembles: \( \text{const.} + \frac{1}{2} M u^2 \)  

Resembles: \( M u \)

\[ \begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{group} & b_{\text{Doppler}}^{\text{RED}} & \frac{V_{\text{group}}}{c} & v_A & \lambda_{\text{group}} & \kappa_{\text{group}} & \tau_{\text{group}} & \frac{V_{\text{phase}}}{c} & b_{\text{Doppler}}^{\text{BLUE}} \\
\hline
\text{phase} & \frac{1}{b_{\text{Doppler}}^{\text{BLUE}}} & \frac{c}{V_{\text{phase}}} & \frac{\kappa_{\text{phase}}}{\kappa_A} & \frac{\tau_{\text{phase}}}{\tau_A} & \frac{v_{\text{phase}}}{v_A} & \frac{\lambda_{\text{phase}}}{\lambda_A} & \frac{c}{V_{\text{group}}} & \frac{1}{b_{\text{Doppler}}^{\text{RED}}} \\
\hline
\text{rapidly} & \quad e^{-\rho} & \quad \tanh \rho & \quad \sinh \rho & \quad \sech \rho & \quad \cosh \rho & \quad \csch \rho & \quad \coth \rho & \quad e^{+\rho} \\
\text{stellar} & \quad 1/e^{+\rho} & \quad \sin \sigma & \quad \tan \sigma & \quad \cos \sigma & \quad \sec \sigma & \quad \cot \sigma & \quad \csc \sigma & \quad 1/e^{-\rho} \\
\text{angle} & \quad \beta \equiv \frac{u}{c} & \quad \sqrt{1-\beta^2} & \frac{1}{1+\beta} & \quad \frac{\beta}{1} & \quad \frac{1}{\sqrt{1-\beta^2}} & \quad \sqrt{1-\frac{\beta^2}{\beta}} & \quad \frac{1}{1-\beta} & \quad \frac{1+\beta}{\sqrt{1-\beta}} \\
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Using (some) wave parameters to develop relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \]
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\[ \frac{u}{c} = \tanh \rho \approx \rho \]

At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \] and \[ \kappa_{\text{phase}} \] resemble formulae for Newton’s kinetic energy and momentum.

So attach scale factor \( h \) to match units.

Resembles: \( \text{const} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]
\[ \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ \mu = \frac{hB}{c^2} \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)

So attach scale factor \( h \) to match units.

Sunday, January 29, 2017
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \] (for \( u \ll c \))

\[ c\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \] (for \( u \ll c \))

\[ \frac{u}{c} = \tanh \rho \approx \rho \] (for \( u \ll c \))

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

\[ \frac{\nu_{\text{phase}}}{B} \approx \frac{u}{c} \quad \iff \quad \frac{c}{v_{\text{phase}}} \approx \frac{B}{c^2} u \]

\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ M = \frac{hB}{c^2} \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)

So attach scale factor \( h \) to match units.

---

**Table:**

| Group | \( b_{\text{Doppler}} \) | \( V_{\text{group}} \) | \( \nu_{\text{group}} \) | \( \lambda_{\text{group}} \) | \( \kappa_{\text{group}} \) | \( \tau_{\text{group}} \) | \( V_{\text{phase}} \) | \( b_{\text{Doppler}} \)
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| phase | 1 | \( \frac{c}{\nu_{\text{phase}}} \) | \( \frac{1}{\kappa_{\text{phase}}} \) | \( \frac{\nu_{\text{phase}}}{\tau_{\text{phase}}} \) | \( \frac{\lambda_{\text{phase}}}{\kappa_{\text{phase}}} \) | \( \frac{\tau_{\text{group}}}{\nu_{\text{phase}}} \) | \( \frac{c}{V_{\text{group}}} \) | \( \frac{c}{\nu_{\text{phase}}} \)
| rapidly | \( e^{-\rho} \) | \( \tanh \rho \) | \( \sinh \rho \) | \( \cosh \rho \) | \( \csc \rho \) | \( \cot \rho \) | \( \frac{1}{\nu_{\text{phase}}} \) | \( \frac{1}{e^{-\rho}} \)
| stellar angle | \( \frac{\beta}{e^{\rho}} \) | \( \sin \sigma \) | \( \tan \sigma \) | \( \cos \sigma \) | \( \sec \sigma \) | \( \cot \sigma \) | \( \csc \sigma \) | \( \frac{1}{\nu_{\text{phase}}} \)
| \( \beta = \frac{u}{c} \) | \( \frac{1}{\sqrt{1+\beta^2}} \) | \( \frac{\beta}{1} \) | \( \frac{1}{\beta} \) | \( \frac{1}{\beta} \) | \( \frac{1}{\beta} \) | \( \frac{1}{\beta} \) | \( \frac{1}{\beta} \)
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---

\( B = \nu_A \)

\( B = \nu_A = c\kappa_A \)

\( \cos \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \)

\( \sinh \rho \approx \rho \approx \frac{u}{c} \)
Using (some) wave parameters to develop relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

Rescale \( v_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \)

\[ h v_{\text{phase}} = hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)

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<th>group</th>
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<td>( \frac{V_{\text{phase}}}{\lambda_{\text{Doppler}}} )</td>
</tr>
</tbody>
</table>

| rapidity | \( \rho \) | \( e^{-\rho} \) | \( \tanh \rho \) | \( \sinh \rho \) | \( \text{sech} \rho \) | \( \cosh \rho \) | \( \text{csch} \rho \) | \( \text{coth} \rho \) | \( e^{+\rho} \) |
| stellar angle | \( \sigma \) | \( 1/e^{+\rho} \) | \( \sin \sigma \) | \( \tan \sigma \) | \( \cos \sigma \) | \( \sec \sigma \) | \( \cot \sigma \) | \( \csc \sigma \) | \( 1/e^{-\rho} \) |

\[ \beta \equiv \frac{u}{c} = \frac{\sqrt{1-\beta^2}}{1+\beta} \]

| value for \( \beta = \frac{3}{5} \) | \[ \frac{1}{2} = 0.5 \] | \[ \frac{3}{5} = 0.6 \] | \[ \frac{3}{4} = 0.75 \] | \[ \frac{4}{5} = 0.80 \] | \[ \frac{5}{4} = 1.25 \] | \[ \frac{4}{3} = 1.33 \] | \[ \frac{5}{3} = 1.67 \] | \[ \frac{2}{1} = 2.0 \] |
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftrightarrow \text{ for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

The famous \( Mc^2 \) shows up here!

So attach scale factor \( h \) to match units.

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

resp.
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad (\text{for } u \ll c) \]

At low speeds:

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad (\text{for } u \ll c) \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

So attach scale factor \( h \) to match units.

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \]

\[ h\kappa_{\text{phase}} \approx Mu \]
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
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At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]
(The famous \( Mc^2 \) shows up here!)

Lucky coincidences??? Cheap trick??

cheap trick?

Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]
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Cheap trick??

Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
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At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]
\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]
(The famous \( Mc^2 \) shows up here!)

Lucky coincidences?? Cheap trick??

Cheap trick??
Using (some) wave parameters to develop relativistic quantum theory

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

Rescale \( \nu_{\text{phase}} \) by \( \hbar \) so: \( M = \frac{hB}{c^2} \) or \( hB = Mc^2 \) (The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \]

So attach scale factor \( \hbar \) to match units.

Lucky coincidences?? Cheap trick??

...Try exact \( U_{\text{phase}} \) ...
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

\[ \nu_{\text{phase}} \approx \frac{1}{2} Mc^2 + \frac{1}{2} Mu \]

\[ h \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu \]

\[ h \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu \]

\[ h \nu_{\text{phase}} \approx hB \cosh \rho \]

\[ h \nu_{\text{phase}} \approx hB \cosh \rho \]

\[ B = \nu_A \]

\[ B = \nu_A = c \kappa_A \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so: \[ M = \frac{hB}{c^2} \]

or: \[ hB = Mc^2 \]

(The famous \( Mc^2 \) shows up here!)

So attach scale factor \( h \) to match units.

Lucky coincidences?? Cheap trick??

...Try exact \( \nu_{\text{phase}} \) ...

\[ h \nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]
Using (some) wave parameters to develop relativistic quantum theory

\( \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \) (for \( u \ll c \))

\( c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \) (for \( u \ll c \))

\( \frac{u}{c} = \tanh \rho \approx \rho \) (for \( u \ll c \))

At low speeds:

\( \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} \rho^2 \) \( \Leftrightarrow \) for \( (u \ll c) \) \( \Rightarrow \)

\( \kappa_{\text{phase}} \approx \frac{B}{c^2} \rho \)

\( U_{\text{phase}} \) and \( \kappa_{\text{phase}} \) resemble formulae for Newton’s kinetic energy \( \frac{1}{2} M u^2 \) and momentum \( M u \).

So attach scale factor \( h \) to match units.

<table>
<thead>
<tr>
<th>group</th>
<th>( b_{\text{Doppler}} )</th>
<th>( \nu_{\text{group}} )</th>
<th>( \lambda_{\text{group}} )</th>
<th>( \kappa_{\text{group}} )</th>
<th>( \tau_{\text{group}} )</th>
<th>( \nu_{\text{phase}} )</th>
<th>( \beta_{\text{Doppler}} )</th>
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<td>( \frac{\nu_{\text{phase}}}{\nu_{A}} )</td>
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<td>( e^{-\rho} )</td>
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<td>( \sinh \rho )</td>
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<td>( \cosh \rho )</td>
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</tr>
<tr>
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<td>( \frac{1}{e^+ \rho} )</td>
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</tr>
<tr>
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<td>( \frac{1}{\sqrt{1+\beta}} )</td>
<td>( \frac{\sqrt{1-\beta^2}}{1} )</td>
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<td>( \frac{\sqrt{\beta^2-1}}{1} )</td>
<td>( \frac{1}{\sqrt{1-\beta^2}} )</td>
<td>( \frac{1+\beta}{\sqrt{1-\beta}} )</td>
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<td>value for ( \beta = 3/5 )</td>
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</tr>
</tbody>
</table>

\( h \nu_{\text{phase}} = h B \cosh \rho = M c^2 \cosh \rho \)

\( \sqrt{1-u^2/c^2} \)

Max Planck
1858-1947
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ cK_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{ (for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \text{ (for } u \ll c) \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so: \[ M = \frac{hB}{c^2} \]

or: \[ hB = Mc^2 \] (The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \text{ (for } u \ll c) \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \text{ (for } u \ll c) \]

\[ h\nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]

\[ B = \nu_A \]

\[ B = \nu_A = cK_A \]

**Max Planck**

1858-1947

**Einstein**

1879-1955

---

\[ \nu_{\text{phase}} \] and \( K_{\text{phase}} \) resemble formulae for Newton’s kinetic energy \( \frac{1}{2} Mu^2 \) and momentum \( Mu \).

So attach scale factor \( h \) (or \( hN \)) to match units.

---

Lucky coincidences?? Cheap trick??

...Try exact \( \nu_{\text{phase}} \)...

\[ \nu_{\text{phase}} = h\nu \]

\[ \epsilon_{\text{E}} \cdot E = h\nu \]

Total Energy: \[ E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ cK_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \text{or} \quad \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \)
\( \text{or} \): \( hB = Mc^2 \)

The famous \( Mc^2 \)

\[ h\nu_{\text{phase}} = hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \text{or} \quad h\nu_{\text{phase}} = hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \quad \text{or} \quad h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Lucky coincidences?? Cheap trick??

\[ h\nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]

Need to replace \( h \) with \( hN \) to match units.

\[ \text{Planck (1900)} \]
\[ E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

\[ \text{Einstein (1905)} \]

For more visit the Pirelli Challenge Site

Quantized amplitude
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ m = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

(The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx Mu \]

Hence, \( h \nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \)

**Einstein (1905)**

\[ E = \sqrt{1 - \frac{u^2}{c^2}} \]

**Planck (1900)**

\[ E = \frac{Mc^2}{h\nu} \]

This motivates the “particle” normalization

\[ \int \Psi^* \Psi \, dV = N \quad \Psi = \sqrt{\frac{E}{h\nu}} \]

<table>
<thead>
<tr>
<th>Group</th>
<th>( b_{\text{Doppler}} )</th>
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<th>( \nu_{\text{group}} / \nu_A )</th>
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\( \beta = \frac{u}{c} \)

\( \frac{1}{2} = 0.5 \)

| \( \frac{3}{5} \) | 0.6 | 0.75 | | | | |
| \( \frac{5}{4} \) | 0.80 | 1.25 | | | | |
| \( \frac{4}{3} \) | 1.33 | 1.67 | | | | |
| \( \frac{2}{1} \) | 2.0 | |

Big worry: Is not oscillator energy quadratic in frequency \( \nu \)?

HO energy = \( \frac{1}{2} A^2 \nu^2 \)
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \]  
\[ \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \] 
\[ \frac{u}{c} = \tanh \rho \approx \rho \]  

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]  
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \] 

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \]  
\[ \text{or: } hB = Mc^2 \]  
(The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} = hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]  
\[ \text{or } (u \ll c) \implies h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \] 

\[ h\nu_{\text{phase}} = Mc^2 + \frac{1}{2} Mu^2 \]  
\[ \text{or } (u \ll c) \implies h\kappa_{\text{phase}} \approx Mu \]

This motivates the "particle" normalization
\[ \int \Psi^* \Psi \, dV = N \]
\[ \Psi = \sqrt{\frac{E}{h\nu}} \]

Need to replace \( h \) with \( hN \) to match units.

\[ \text{Planck (1900)} \]
\[ = \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

\[ \text{Einstein (1905)} \]

Big worry: Is not oscillator energy quadratic in frequency \( \nu \)?

\[ \text{HO energy} = \frac{1}{2} A^2 \nu^2 \]

Resolution and dirty secret: \( E, N, \) and \( \nu_{\text{phase}} \) are all frequencies!

So \( E \cdot N = hN\nu_{\text{phase}} \) is quadratic in frequency
Using (some) wave parameters to develop relativistic quantum theory

\[ v_{phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ c \kappa_{phase} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

\[ \cos \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \]

\[ \sinh \rho \approx \rho \approx \frac{u}{c} \]

\[ B = v_A \]

\[ B = v_A = c \kappa_A \]

At low speeds:

\[ v_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

\[ \kappa_{phase} \approx \frac{B}{c^2} u \]

Rescale \( v_{phase} \) by \( h \) so: \[ M \approx \frac{hB}{c^2} \] or: \[ hB = Mc^2 \] (The famous \( Mc^2 \) shows up here!)

\[ h\nu_{phase} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h\kappa_{phase} \approx \frac{hB}{c^2} u \]

\[ h\nu_{phase} \approx Mc^2 + \frac{1}{2} Mu^2 \]

\[ h\kappa_{phase} \approx Mu \]

So attach scale factor \( h \) (or \( hN \)) to match units.

Lucky coincidences?? Cheap trick??

\[ \nu_{phase} = hB \cos \rho = Mc^2 \cos \rho \]

Planck (1900)

\[ = \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

Einstein (1905)

\[ h\kappa_{phase} = hB \sinh \rho = Mc^2 \sinh \rho \]

- Need to replace \( h \) with \( hN \) to match e.m. energy density \( \varepsilon = E \cdot E = hN \nu_{phase} \)
Using (some) wave parameters to develop relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( v_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \text{ or: } hB = Mc^2 \]
(The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} = hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]
\[ h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]
\[ h\kappa_{\text{phase}} \approx Mu \]

Need to replace \( h \) with \( hN \) to match units.

Lucky coincidences?? Cheap trick??

...Try exact \( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \)...

Planck (1900)
\[ \hbar = \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

Einstein (1905)
\[ E = \sqrt{1-u^2/c^2} \]

\[ h\kappa_{\text{phase}} = hB \sinh \rho = Mc^2 \sinh \rho \]

\[ cp = \frac{Mcu}{\sqrt{1-u^2/c^2}} \]

Max Planck
1858-1947
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

(The famous \( Mc^2 \) shows up here!)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx Mu \]

Need to replace \( h \) with \( hN \) to match units.

**Natural wave conspiracy?** **Expensive cheap trick??**

...Try exact \( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \)...

**Planck (1900)**

\[ h\nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]

**Einstein (1905)**

\[ E = \frac{1}{2} Mu^2 \]

\[ \Rightarrow \text{Total Energy: } E = \sqrt{1 - u^2/c^2} \]

\[ h\kappa_{\text{phase}} = hB \sinh \rho = Mc^2 \sinh \rho \]

**DeBroglie (1921)**

\[ cp = \frac{Mu}{\sqrt{1 - u^2/c^2}} \]

Momentum:

\[ h\kappa_{\text{phase}} = p = \frac{Mu}{\sqrt{1 - u^2/c^2}} \]

\[ 1 = \frac{u}{c} = \frac{1}{\sqrt{\beta^2 - 1}} \quad \text{(old-fashioned notation)} \]
Using (some) wave parameters to develop relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 (\text{for } u \ll c) \]

\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad (\text{for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \quad (\text{for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]

\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \nu_{\text{phase}} \approx \frac{hB}{c^2} u \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]

\[ h \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]

\[ h \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \]

\[ h \kappa_{\text{phase}} \approx \frac{hB}{c} \]

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 (\text{for } u \ll c) \]

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{u^2}{c^2} \]

\[ \sinh \rho \approx \frac{u}{c} \]

\[ B = \nu_A \]

Max Planck 1858-1947

\[ \beta = \frac{u}{c} \]

\[ \frac{1}{2} = 0.5 \]

\[ \frac{3}{4} = 0.75 \]

\[ \frac{4}{5} = 0.80 \]

\[ \frac{5}{4} = 1.25 \]

\[ \frac{4}{3} = 1.33 \]

\[ \frac{5}{3} = 1.67 \]

\[ \frac{2}{1} = 2.0 \]

\[ h \kappa_{\text{phase}} = hB \sinh \rho \approx Mc^2 \sinh \rho \]

\[ \frac{1}{\sqrt{\beta^2 - 1}} = \frac{u}{c \sqrt{1 - u^2/c^2}} \quad \text{(old-fashioned notation)} \]

\[ cp = \frac{Mc}{\sqrt{1 - u^2/c^2}} \]

\[ \beta = \frac{u}{c} \]

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\[ B = \nu_A = c \kappa_A \]

\[ B = \nu_A = c \kappa_A \]

\[ \nu_{\text{phase}} \text{ and } \kappa_{\text{phase}} \text{ resemble} \]

\[ \text{formulae for Newton’s kinetic energy } \frac{1}{2} Mu^2 \text{ and momentum } Mu. \]

\[ \text{So attach scale factor } h \text{ (or } hN) \text{ to match units.} \]

\[ \text{Natural wave conspiracy } ?? \text{ Expensive} \]

\[ \text{Cheap trick??} \]

\[ \text{...Try } \text{exact } \nu_{\text{phase}} \text{ and } \kappa_{\text{phase}}... \]

\[ h \nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]

\[ \text{This motivates the “particle” normalization } \int \Psi^* \Psi \ dV = N \quad \Psi = \sqrt{\frac{\varepsilon}{h \nu}} \]

\[ \text{Planck (1900)} \]

\[ \text{Einstein (1905)} \]

\[ \text{Momentum: } h \kappa_{\text{phase}} = p = \frac{Mu}{c} \text{(for } \beta = 3/5) \]

\[ \text{DeBroglie (1921)} \]

\[ \text{DeBroglie (1921)} \]
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

Energy
\[ E = \hbar \omega \]

Momentum wave:
\[ \text{positive rest energy } M c^2 \]
\[ E^2 - c^2 p^2 = (M c^2)^2 \]

\[ \text{tachyon: imaginary } \mu \]
\[ E = \pm c p \]

Mass (resting)
\[ \hbar B = \hbar \nu_A = M c^2 = \hbar c \kappa_A \]

Energy
\[ \hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho \]

Momentum
\[ \hbar c \kappa_{\text{phase}} = cp = \hbar c \kappa_A \sinh \rho = \hbar \nu_A \sinh \rho \]

(b) Bohr-Schrödinger Approximation

\[ \omega_m = 49 \omega_I \]

\[ E = \frac{p^2}{2M} \]

\[ \langle E \rangle = B m^2 \]

\[ M c^2 \]

\[ E = \pm \sqrt{(M c^2)^2 + (cp)^2} \approx M c^2 + \frac{p^2}{2M} \]

Energy versus Momentum
\[ E^2 = (M c^2)^2 \cosh^2 \rho \]

\[ = (M c^2)^2 (1 + \sinh^2 \rho) = (M c^2)^2 + (cp)^2 \Rightarrow E = \pm \sqrt{(M c^2)^2 + (cp)^2} \approx M c^2 + \frac{p^2}{2M} \]
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

- **Energy wave**: positive rest energy $Mc^2$
  \[ E^2 - c^2p^2 = (Mc^2)^2 \]
  \[ E = \pm cp \]

(b) Bohr-Schrodinger Approximation

- **Energy and Momentum**
  \[ h\kappa = cp = h\kappa_A \sinh \rho = h\nu_A \sinh \rho \]
  \[ E^2 = (Mc^2)^2 \cosh^2 \rho \]

**Energy**
\[ h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \]

**Momentum**
\[ h\kappa_{\text{phase}} = cp = h\kappa_A \sinh \rho = h\nu_A \sinh \rho \]

**Approximation**
\[ E \approx Mc^2 + \frac{p^2}{2M} \]

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Sunday, January 29, 2017
### Relavativity variable tables

<table>
<thead>
<tr>
<th>group</th>
<th>$b_{\text{Doppler RED}}$</th>
<th>$\frac{V_{\text{group}}}{c}$</th>
<th>$\frac{v_{\text{group}}}{v_A}$</th>
<th>$\frac{\lambda_{\text{group}}}{\lambda_A}$</th>
<th>$\frac{\kappa_{\text{group}}}{\kappa_A}$</th>
<th>$\frac{\tau_{\text{group}}}{\tau_A}$</th>
<th>$\frac{V_{\text{phase}}}{c}$</th>
<th>$b_{\text{Doppler BLUE}}$</th>
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<tbody>
<tr>
<td>phase</td>
<td>$\frac{1}{b_{\text{Doppler BLUE}}}$</td>
<td>$\frac{c}{V_{\text{phase}}}$</td>
<td>$\frac{\kappa_{\text{phase}}}{\kappa_A}$</td>
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<td>$e^{-\rho}$</td>
<td>tanh $\rho$</td>
<td>sinh $\rho$</td>
<td>sech $\rho$</td>
<td>cosh $\rho$</td>
<td>csch$\rho$</td>
<td>coth $\rho$</td>
<td>$e^{+\rho}$</td>
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<tr>
<td>stellar angle $\sigma$</td>
<td>$1/e^{+\rho}$</td>
<td>$\sin\sigma$</td>
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<td>$\cot\sigma$</td>
<td>$\csc\sigma$</td>
<td>$1/e^{-\rho}$</td>
</tr>
<tr>
<td>$\beta = \frac{u}{c}$</td>
<td>$\sqrt{\frac{1-\beta}{1+\beta}}$</td>
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<td>$\frac{1}{\sqrt{1-\beta^2}}$</td>
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<tr>
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<td>effects $b_{\text{Doppler RED}}$</td>
<td>$V_{\text{group}}$</td>
<td>$\text{past-future asymmetry}$</td>
<td>$\tau_{\text{phase}}$</td>
<td>$x$-contraction $^{(\text{Lorentz})}$</td>
<td>$t$-dilation $^{(\text{Einstein})}$</td>
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### Relativistic quantum mechanics variable tables

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<td>functions</td>
<td>$\frac{V_{\text{group}}}{c} \tan \rho$</td>
<td>momentum $= Mc^2 \sinh \rho$</td>
<td>$L = -Mc^2 \sech \rho$</td>
<td>$H = Mc^2 \cosh \rho$</td>
<td>$\lambda = \alpha \csc \rho$</td>
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Review: Relawavity $\rho$ functions  Two famous ones  Extremes and plot vs. $\rho$
Doppler jeopardy  Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity $\rho$ related to stellar aberration angle $\sigma$ and L. C. Epstein’s approach to relativity
Longitudinal hyperbolic $\rho$-geometry connects to transverse circular $\sigma$-geometry
“Occams Sword” and summary of 16 parameter functions of $\rho$ and $\sigma$
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What’s the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare’ and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2$^{\text{nd}}$-quantization “photon” number $N$ and 1$^{\text{st}}$-quantization wavenumber $\kappa$

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes $g$-acceleration grid
Analysis of constant-$g$ grid compared to zero-$g$ Minkowski grid
Animation of mechanics and metrology of constant-$g$ grid
Definition(s) of mass for relativity/quantum

Rest Mass $M_{\text{rest}}$ (Einstein’s mass)

$\ hB = h\nu_A = Mc^2 = h\kappa_A$

Defines invariant hyperbola(s)

$E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$

Given:

Energy: $E = Mc^2 \cosh \rho = h\nu_{\text{phase}}$

Momentum: $cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}}$

Velocity: $u = c \tanh \rho = \frac{du}{d\kappa}$

• What’s the matter with Mass?

Shining some light on the elephant in the spacetime room
Definition(s) of mass for relativity/quantum

Rest Mass \( M_{\text{rest}} \) (Einstein’s mass)

\[
hB = h\nu_A = Mc^2 = h\kappa A
\]

\[
\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}}
\]

Defines invariant hyperbola(s)

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E = \pm \sqrt{(Mc^2)^2 + (cp)^2}
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Given:

Energy: \( E = Mc^2 \cosh \rho \)

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Definition(s) of mass for relativity/quantum

Rest Mass $M_{\text{rest}}$ (Einstein’s mass) Defines invariant hyperbola(s)

\[ hB = h\nu_A = Mc^2 = hc\kappa_A \]

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Given: \[ E = Mc^2 \cosh \rho \]
\[ = h\nu_{\text{phase}} \]
\[ \text{momentum: } cp = Mc^2 \sinh \rho \]
\[ = hc\kappa_{\text{phase}} \]
\[ \text{velocity: } u = c \tanh \rho = \frac{dv}{d\kappa} \]

Momentum Mass $M_{\text{mom}}$ (Galileo’s mass) Defined by ratio $p/u$ of relativistic momentum to group velocity.

\[ M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c \sinh \rho}{c \tanh \rho} \]
Definition(s) of mass for relativity/quantum

**Rest Mass**\( M_{\text{rest}} \) (Einstein's mass)

\[
M_{\text{rest}} \equiv M = \frac{h\nu}{c^2} = h\kappa_A
\]

Defines invariant hyperbola(s)

\[
E = \pm \sqrt{(Mc^2)^2 + (cp)^2}
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Given:

Energy: \( E = Mc^2 \cosh \rho = h\nu_{\text{phase}} \)

Momentum: \( cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}} \)

Velocity: \( u = c \tanh \rho = \frac{dv}{dk} \)

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Defined by ratio \( p/u \) of relativistic momentum to group velocity.

\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} = M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}
\]
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass)

\[ hB = h\nu_A = Mc^2 = h\kappa \]

Defines invariant hyperbola(s)

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**Given:**

- **Energy:** $E = Mc^2 \cosh \rho = h\nu_{\text{phase}}$
- **Momentum:** $cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}}$
- **Velocity:** $u = c \tanh \rho = \frac{d\nu}{d\kappa}$

Limiting cases:

\[ M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^\rho / 2 \]

\[ M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}} \]
Definition(s) of mass for relativity/quantum

**Rest Mass** \( M_{\text{rest}} \) (Einstein’s mass)

\[
hB = h\nu_A = Mc^2 = h\kappa_A
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Defines invariant hyperbola(s)

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E = \pm \sqrt{(Mc^2)^2 + (cp)^2}
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M_{\text{mom}} = \frac{p}{u} = \frac{M_{\text{rest}}c\sinh\rho}{c\tanh\rho}
\]

\[
= M_{\text{rest}} \cosh\rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}
\]

**Effective Mass** \( M_{\text{eff}} \) (Newton’s mass)

Defined by ratio \( F/a = dp/du \) of relativistic force to acceleration.

Given:

- **Energy:** \( E = Mc^2 \cosh\rho = h\nu_{\text{phase}} \)
- **Momentum:** \( cp = Mc^2 \sinh\rho = h\kappa_{\text{phase}} \)
- **Velocity:** \( u = c\tanh\rho = \frac{d\nu}{dk} \)

Limiting cases:

\[
M_{\text{mom}} \quad \xrightarrow{u \to c} \quad M_{\text{rest}} \cosh\rho / 2
\]

\[
M_{\text{mom}} \quad \xrightarrow{u \ll c} \quad M_{\text{rest}}
\]
Definition(s) of mass for relativity/quantum

Rest Mass $M_{\text{rest}}$ (Einstein’s mass)

$\frac{h\nu}{c^2} = M_{\text{rest}}$

Defines invariant hyperbola(s)

$E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$

Momentum Mass $M_{\text{mom}}$ (Galileo’s mass)

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$M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho}$

Limiting cases:

$M_{\text{mom}} \xrightarrow{u \rightarrow c} M_{\text{rest}} e^{\rho}/2$

$M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}}$

Effective Mass $M_{\text{eff}}$ (Newton’s mass)

Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh \rho \ d\rho$ in momentum to change $du = c \sech^2 \rho \ d\rho$ in velocity.
**Definition(s) of mass for relativity/quantum**

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass) Defined by invariant hyperbola(s)

\[ E = \pm \sqrt{\left(Mc^2\right)^2 + (cp)^2} \]

- **Energy:** $E = Mc^2 \cosh \rho = h\nu_{\text{phase}}$
- **Momentum:** $cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}}$
- **Velocity:** $u = c \tanh \rho = \frac{du}{d\kappa}$

**Momentum Mass** $M_{\text{mom}}$ (Galileo’s mass) Defined by ratio $p/u$ of relativistic momentum to group velocity.

\[ M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} \]

\[ = M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2/c^2}} \]

Limiting cases: \[ M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^{\rho}/2 \]
\[ M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}} \]

**Effective Mass** $M_{\text{eff}}$ (Newton’s mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh \rho \, d\rho$ in momentum to change $du = c \, \sech^2 \rho \, d\rho$ in velocity

\[ M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho \]

Sunday, January 29, 2017
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass)

Defines invariant hyperbola(s)

$$E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$$

**Momentum Mass** $M_{\text{mom}}$ (Galileo’s mass)

Defined by ratio $p/u$ of relativistic momentum to group velocity.

$$M_{\text{mom}} = \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} = M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}$$

**Effective Mass** $M_{\text{eff}}$ (Newton’s mass)

Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.

That is ratio of change $dp=Mc \cosh \rho \, d\rho$ in momentum to change $du=c \sech^2 \rho \, d\rho$ in velocity.

$$M_{\text{eff}} = \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho$$

Given:

- **Energy:** $E = Mc^2 \cosh \rho = \hbar \nu_{\text{phase}}$
- **Momentum:** $cp = Mc^2 \sinh \rho = \hbar c \kappa_{\text{phase}}$
- **Velocity:** $u = c \tanh \rho = \frac{du}{d\kappa}$

Limiting cases:

- $M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^\rho / 2$
- $M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}}$
- $M_{\text{eff}} \xrightarrow{u \to c} M_{\text{rest}} e^{3\rho / 2}$
- $M_{\text{eff}} \xrightarrow{u \ll c} M_{\text{rest}}$
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass)

Defines invariant hyperbola(s)

\[ E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \]

\[ \frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} \]

**Momentum Mass** $M_{\text{mom}}$ (Galileo’s mass)

Defined by ratio $p/u$ of relativistic momentum to group velocity.

\[ M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} \sinh \rho}{c \tanh \rho} \]

\[ = M_{\text{rest}} \cosh \rho = \sqrt{1 - u^2 / c^2} \]

**Effective Mass** $M_{\text{eff}}$ (Newton’s mass)

Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.

That is ratio of change $dp=Mc\cosh \rho \, d\rho$ in momentum to change $du=c\text{sech}^2 \rho \, d\rho$ in velocity

\[ M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{\cosh \rho}{\text{sech}^2 \rho} = M_{\text{rest}} \cosh^3 \rho \]

**More common derivation using group velocity:**

\[ u \equiv V_{\text{group}} = \frac{d\omega}{dk} = \frac{d\nu}{dk} \]

\[ M_{\text{eff}} \equiv \frac{dp}{du} = \frac{\hbar \, dk}{dV_{\text{group}}} = \frac{\hbar}{d\omega/dk} = \frac{\hbar}{d^2\omega/dk^2} = \left(1 - u^2 / c^2\right)^{3/2} \]

Limiting cases:

\[ M_{\text{mom}} \underbrace{\text{u \to c}}_{\text{u \to c}} \rightarrow M_{\text{rest}} e^{\rho / 2} \]

\[ M_{\text{mom}} \underbrace{\text{u \ll c}}_{\text{u \ll c}} \rightarrow M_{\text{rest}} \]

\[ M_{\text{eff}} \underbrace{\text{u \to c}}_{\text{u \to c}} \rightarrow M_{\text{rest}} e^{3\rho / 2} \]

\[ M_{\text{eff}} \underbrace{\text{u \ll c}}_{\text{u \ll c}} \rightarrow M_{\text{rest}} \]
Definition(s) of mass for relativity/quantum

**Rest Mass** \( M_{\text{rest}} \) (Einstein’s mass)

\[
hB = h\nu_A = Mc^2 = h\kappa_A
\]

\[
\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} \quad \text{(Rest Mass)}
\]

**Momentum Mass** \( M_{\text{mom}} \) (Galileo’s mass)

Defined by ratio \( p/u \) of relativistic momentum to group velocity.

\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} = M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2/c^2}} \quad \text{(Momentum Mass)}
\]

**Effective Mass** \( M_{\text{eff}} \) (Newton’s mass)

Defined by ratio \( F/a = dp/du \) of relativistic force to acceleration.

That is ratio of change \( dp = Mc \cosh \rho \, d\rho \) in momentum to change \( du = c \text{ sech}^2 \rho \, d\rho \) in velocity

\[
M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \text{ sech}^2 \rho} = M_{\text{rest}} \cosh^3 \rho \quad \text{(Effective Mass)}
\]

Limiting cases:

\[
M_{\text{mom}} \quad u \to c \rightarrow M_{\text{rest}} e^{\rho/2}
\]

\[
M_{\text{mom}} \quad u \ll c \rightarrow M_{\text{rest}}
\]

More common derivation using group velocity:

\[
M_{\text{eff}} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{\text{group}}} = \frac{\hbar}{d} \frac{d\omega}{dk} = \frac{\hbar}{d^2 \omega} \frac{dk}{dk^2} = \frac{M_{\text{rest}}}{\left(1 - u^2/c^2\right)^{3/2}} = M_{\text{rest}} \cosh^3 \rho \quad \text{(Effective Mass)}
\]

General wave formula to accompany \( V_{\text{group}} = \frac{d\omega}{dk} \)
Definition(s) of mass for relativity/quantum

Rest Mass \( M_{\text{rest}} \) (Einstein’s mass)
\[
hB = h\nu_A = Mc^2 = hc\kappa_A
\]
\[
\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} = \frac{hc\kappa_{\text{phase}}}{c^2}
\]

Momentum Mass \( M_{\text{mom}} \) (Galileo’s mass) Defined by \( p/u \)
\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c\sinh \rho}{c \tanh \rho}
\]
\[
= M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}
\]

Effective Mass \( M_{\text{eff}} \) (Newton’s mass) Defined by \( F/a = dp/du \)
That is ratio of \( dp = Mc \cosh \rho \ d\rho \) to change \( du = c \sech^2 \rho \ d\rho \) in velocity
\[
M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho
\]

More common derivation using group velocity: \( u \equiv V_{\text{group}} = \frac{d\omega}{dk} = \frac{d\nu}{dk} \)
\[
M_{\text{eff}} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{\text{group}}} = \frac{\hbar}{d \frac{d\omega}{dk} \frac{dk}{dk}} = \frac{\hbar}{d \frac{d\omega}{dk} \frac{dk}{dk}} = \frac{M_{\text{rest}}}{(1 - u^2 / c^2)^{3/2}} = M_{\text{rest}} \cosh^3 \rho
\]

Effective mass is proportional to the radius of curvature of \( \omega(k) \) dispersion.
Definition(s) of mass for relativity/quantum

How much mass does a $\gamma$-photon have?

- **Rest Mass** (a) $\gamma$-rest mass: $M_{\text{rest}}^\gamma = 0$.

- **Momentum Mass** (b) $\gamma$-momentum mass: $M_{\text{mom}}^\gamma = \frac{p}{c} = \frac{h\kappa}{c} = \frac{hv}{c^2}$.

- **Effective Mass** (c) $\gamma$-effective mass: $M_{\text{eff}}^\gamma = \infty$.

$$M_{\text{mom}}^\gamma = \frac{hv}{c^2} = \nu(1.2 \cdot 10^{-51}) \text{ kg} \cdot s = 4.5 \cdot 10^{-36} \text{ kg} \quad \text{(for: } \nu = \text{600THz})$$

Newton complained about *his* “corpuscles” of light having “fits” (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia.*)

Sunday, January 29, 2017
Review: Relawavity \( \rho \) functions
  Two famous ones Extremes and plot vs. \( \rho \)
Doppler jeopardy Geometric mean and Relativistic hyperbolas
  Animation of \( e^{\rho}=2 \) spacetime and per-spacetime plots

Rapidity \( \rho \) related to stellar aberration angle \( \sigma \) and L. C. Epstein’s approach to relativity
Longitudinal hyperbolic \( \rho \)-geometry connects to transverse circular \( \sigma \)-geometry
“Occams Sword” and summary of 16 parameter functions of \( \rho \) and \( \sigma \)
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
  What’s the matter with mass? Shining some light on the Elephant in the room
  ➤ Relativistic action and Lagrangian-Hamiltonian relations
    Poincare’ and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
  Feynman diagram geometry
    Compton recoil related to rocket velocity formula
    Comparing 2nd-quantization “photon” number \( N \) and 1st-quantization wavenumber \( \kappa \)

Relawavity in accelerated frames
  Laser up-tuning by Alice and down-tuning by Carla makes \( g \)-acceleration grid
  Analysis of constant-\( g \) grid compared to zero-\( g \) Minkowski grid
  Animation of mechanics and metrology of constant-\( g \) grid
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

- $h\nu_A = Mc^2 = h\kappa_A$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\nu_A \sinh \rho$

- $\hbar \omega_A = Mc^2 = h\kappa_A$
- $\hbar \omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\omega_A \sinh \rho$

$h \equiv \frac{h}{2\pi}$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = k x - \omega t = k' x' - \omega' t'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega
\]

$p = \hbar k = Mc \sinh \rho$ \hspace{2cm} $E = \hbar \omega = Mc^2 \cosh \rho$

Prior wave relations

$h \nu_A = Mc^2 = h c \kappa_A$

$h \nu_{\text{phase}} = E = h \nu_A \cosh \rho$

$h c \kappa_{\text{phase}} = c p = h \nu_A \sinh \rho$

$\hbar \omega_A = Mc^2 = \hbar c k_A$

$\hbar \omega_{\text{phase}} = E = h \omega_A \cosh \rho$

$\hbar c k_{\text{phase}} = c p = h \omega_A \sinh \rho$

$\hbar \equiv \frac{h}{2\pi}$
Relativistic action \( S \) and Lagrangian-Hamiltonian relations

Define Lagrangian \( L \) using invariant wave phase \( \Phi = kx - \omega t = k'x' - \omega' t' \) for wave of \( k = k_{\text{phase}} \) and \( \omega = \omega_{\text{phase}} \).

Use DeBroglie-momentum \( p = \hbar k \) relation and Planck-energy \( E = \hbar \omega \) relation

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E
\]

\[\begin{align*}
p &= \hbar k = Mc \sinh \rho \\
E &= \hbar \omega = Mc^2 \cosh \rho
\end{align*}\]

\[
h\nu_A = Mc^2 = hck_A \\
h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \\
hck_{\text{phase}} = cp = h\omega_A \sinh \rho
\]

Prior wave relations

linear Hz format

angular phasor format

\[\begin{align*}
h\omega_A &= Mc^2 = hck_A \\
h\omega_{\text{phase}} &= E = h\omega_A \cosh \rho \\
hck_{\text{phase}} &= cp = h\omega_A \sinh \rho
\end{align*}\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi= kx-\omega t= k'|x' - \omega'|t'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

$p=\hbar k=Mc \sinh \rho$

$E=\hbar \omega=Mc^2 \cosh \rho = H$

Prior wave relations

$$h\nu_A=Mc^2=hc\kappa_A$$
$$h\nu_{\text{phase}}=E=h\nu_A \cosh \rho$$
$$hck_{\text{phase}}=cp=h\omega_A \sinh \rho$$

Legendre transformation
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = k x - \omega t = k' x' - \omega' t'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p \dot{x} - E \equiv pu - H = L
\]

Use Group velocity $u = \frac{dx}{dt} = c \tanh \rho$

\[
p = \hbar k = Mc \sinh \rho \quad \quad E = \hbar \omega = Mc^2 \cosh \rho = H
\]

Prior wave relations

- Linear Hz format: $h \nu_A = Mc^2 = h c \kappa_A$
- Angular phasor format: $h \nu_{\text{phase}} = E = h \nu_A \cosh \rho$ and $h c \kappa_{\text{phase}} = cp = h \nu_A \sinh \rho$

- Legendre transformation $u = \frac{dx}{dt} = c \tanh \rho$

- $\hbar \equiv \frac{\hbar}{2\pi}$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

\[
p = \hbar k = Mc \sinh \rho
\]

\[
E = \hbar \omega = Mc^2 \cosh \rho = H
\]

\[
L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho
\]

Prior wave relations

- Linear Hz format: $h\nu_A = Mc^2 = \hbar c \kappa_A$
- Angular phasor format: $h\omega_{\text{phase}} = E = h\nu_A \cosh \rho$
- $h\nu_{\text{phase}} = cp = h\nu_A \sinh \rho$
- $h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $h\omega_A = Mc^2 = \hbar c k_A$
- $hck_{\text{phase}} = cp = h\omega_A \sinh \rho$

\[
\hbar \equiv \frac{h}{2\pi}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

$$
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
$$

Legendre transformation

Use Group velocity $u = \frac{dx}{dt} = c \tanh \rho$

$p = \hbar k = M c \sinh \rho$

$E = \hbar \omega = M c^2 \cosh \rho = H$

$L = pu - H = (M c \sinh \rho)(c \tanh \rho) - M c^2 \cosh \rho$

$$
= M c^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = -M c^2 \text{sech} \rho
$$

$L$ is:

$$
M c^2 \frac{-1}{\cosh \rho} = -M c^2 \text{sech} \rho
$$

Prior wave relations

- $h \nu_A = M c^2 = h c \kappa_A$
- $h \nu_{\text{phase}} = E = h \nu_A \cosh \rho$
- $h c \kappa_{\text{phase}} = cp = h \nu_A \sinh \rho$

- $h \omega_A = M c^2 = h c k_A$
- $h \omega_{\text{phase}} = E = h \omega_A \cosh \rho$
- $h c k_{\text{phase}} = cp = h \omega_A \sinh \rho$

$\hbar \equiv \frac{h}{2\pi}$

Sunday, January 29, 2017
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$.

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p \dot{x} - E \equiv pu - H = L
\]

Use Group velocity: $u=\frac{dx}{dt}=c \tanh \rho$

\[
p = \hbar k = Mc \sinh \rho
\]

\[
E = \hbar \omega = Mc^2 \cosh \rho = H
\]

\[
L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho
\]

\[
= Mc^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = -Mc^2 \text{sech} \rho
\]

Compare Lagrangian $L$

\[
L = \hbar \dot{\Phi} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \text{sech} \rho
\]

\[
\hbar \nu_A = Mc^2 = \hbar c k_A
\]

\[
\hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho
\]

\[
\hbar c k_{\text{phase}} = cp = \hbar \nu_A \sinh \rho
\]

Prior wave relations

Linear Hz format

\[
\hbar \omega_A = Mc^2 = \hbar c k_A
\]

Angular phasor format

\[
\hbar \omega_{\text{phase}} = E = \hbar \omega_A \cosh \rho
\]

\[
\hbar c k_{\text{phase}} = cp = \hbar \omega_A \sinh \rho
\]

\[
\hbar \equiv \frac{h}{2\pi}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

$$\begin{align*}
p &= \hbar k = Mc \sinh \rho \\
E &= \hbar \omega = Mc^2 \cosh \rho = H
\end{align*}$$

Note: $Mcu = Mc^2 \tanh \rho$

Compare Lagrangian $L$

$$L = \hbar \dot{\Phi} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \text{sech} \rho$$

with Hamiltonian $H = E$

$$H = \hbar \omega = Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho$$

Prior wave relations

$$\begin{align*}
\hbar \nu_A &= Mc^2 = \hbar c k_A \\
\hbar \nu_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
\hbar c k_{\text{phase}} &= cp = \hbar \nu_A \sinh \rho
\end{align*}$$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p \dot{x} - E \equiv p u - H = L
\]

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

\[
p = \hbar k = Mc \sinh \rho
\]

\[
E = \hbar \omega = M c^2 \cosh \rho = H
\]

\[
L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - M c^2 \cosh \rho
\]

\[
= M c^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = -M c^2 \text{sech} \rho
\]

Compare Lagrangian $L$

\[
L = \hbar \dot{\Phi} = -M c^2 \sqrt{1 - \frac{u^2}{c^2}} = -M c^2 \text{sech} \rho
\]

with Hamiltonian $H = E$

\[
H = \hbar \omega = M c^2 \sqrt{1 - \frac{u^2}{c^2}} = M c^2 \cosh \rho
\]

\[
= M c^2 \sqrt{1 + \sinh^2 \rho} = M c^2 \sqrt{1 + (cp)^2}
\]

\[
h \nu_A = M c^2 = h \kappa_A
\]

\[
h \nu_{\text{phase}} = E = h \nu_A \cosh \rho
\]

\[
h \kappa_{\text{phase}} = cp = h \nu_A \sinh \rho
\]

Prior wave relations

\[
\hbar = h/2\pi
\]

\[
h \omega_A = M c^2 = h \kappa_A
\]

\[
h \omega_{\text{phase}} = E = h \omega_A \cosh \rho
\]

\[
h \kappa_{\text{phase}} = cp = h \omega_A \sinh \rho
\]

\[
\hbar \equiv \frac{h}{2\pi}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

\[
\frac{dS}{dt} = L = \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

Note: $Mcu = Mc^2 \tanh \rho$

Also: $cp = Mc^2 \sin \sigma$

Note: $Mck = Mc^2 \tan \sigma$

Including stellar angle $\sigma$

\[
H = \hbar \omega = Mc^2 \cosh \rho = H
\]

Compare Lagrangian $L$

\[
\dot{S} = L = \hbar \Phi = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \sech \rho = -Mc^2 \cos \sigma
\]

with Hamiltonian $H = E$

\[
H = \hbar \omega = Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma
\]

\[
= Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (cp)^2}
\]

Define Action $S = \hbar \Phi$

Prior wave relations

- Linear Hz format
- Angular phasor format

\[
\begin{align*}
\nu_A &= Mc^2 = c \kappa_A \\
\nu_{\text{phase}} &= E = h \nu_A \cosh \rho \\
h \kappa_{\text{phase}} &= cp = h \nu_A \sinh \rho \\
\hbar = \frac{\hbar}{2\pi}
\end{align*}
\]

\[
\begin{align*}
\hbar \omega_A &= Mc^2 = \hbar c \kappa_A \\
\hbar \omega_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} &= cp = \hbar \nu_A \sinh \rho
\end{align*}
\]

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Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

$$\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Compare Lagrangian $L$

$$\dot{S} = L = \hbar \dot{\Phi} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}}$$

with Hamiltonian $H = E$

$$H = \hbar \omega = Mc^2 \sqrt{1 - \frac{u^2}{c^2}}$$

Define Action $S = \hbar \Phi$

Prior wave relations

$$\nu_A = Mc^2 = \hbar c \kappa_A$$
$$\nu_{\text{phase}} = E = \nu_A \cosh \rho$$
$$h c \kappa_{\text{phase}} = c p = \nu_A \sinh \rho$$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

Define Action $S = \hbar \Phi$

Prior wave relations

- Linear Hz format: $\hbar \nu_A = M c^2 = \hbar c \kappa_A$
- Angular phasor format: $\hbar \omega_A = M c^2 = \hbar c k_A$

Legendre transformation

Poincare Invariant action differential

Compare Lagrangian $L$

$$\dot{S} = \dot{L} = \hbar \dot{\Phi} = - M c^2 \sqrt{1 - \frac{u^2}{c^2}} = - M c^2 \operatorname{sech} \rho = - M c^2 \cos \sigma$$

with Hamiltonian $H = E$

$$H = \hbar \omega = M c^2 \sqrt{1 - \frac{u^2}{c^2}} = M c^2 \cosh \rho = M c^2 \sec \sigma$$

$$= M c^2 \sqrt{1 + \sinh^2 \rho} = M c^2 \sqrt{1 + (cp)^2}$$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p \dot{x} - E \equiv pu - H = L
\]

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

Poincare Invariant action differential

Hamilton-Jacobi equations

\[
\frac{\partial S}{\partial x} = p \quad \frac{\partial S}{\partial t} = -H
\]

Compare Lagrangian $L$ with Hamiltonian $H = E$

\[
\hat{S} = L = \hbar \Phi = -Mc^2 \left( \frac{u^2}{c^2} \right) = -Mc^2 \text{sech} \rho = -Mc^2 \cos \sigma
\]

with Hamiltonian $H = E$

\[
H = \hbar \omega = \frac{Mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma
\]

Define Action $S = \hbar \Phi$

\[
\nu_A = Mc^2 = \hbar c \kappa_A
\]

Prior wave relations

\[
\nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho
\]

\[
h \kappa_{\text{phase}} = cp = \hbar \nu_A \sinh \rho
\]

\[
\nu_A = Mc^2 = \hbar c_k A
\]

\[
h \omega_{\text{phase}} = E = \hbar \omega_A \cosh \rho
\]

\[
h \omega_A = \hbar c_k A
\]

\[
h \kappa_{\text{phase}} = cp = \hbar \omega_A \sinh \rho
\]

\[
h \equiv \frac{\hbar}{2\pi}
\]

Sunday, January 29, 2017
Review: Relawavity $\rho$ functions Two famous ones Extremes and plot vs. $\rho$
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^\rho=2$ spacetime and per-spacetime plots

Rapidity $\rho$ related to stellar aberration angle $\sigma$ and L. C. Epstein’s approach to relativity
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“Occams Sword” and summary of 16 parameter functions of $\rho$ and $\sigma$
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Derivation of relativistic quantum mechanics
What’s the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare’ and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2\textsuperscript{nd}-quantization “photon” number $N$ and 1\textsuperscript{st}-quantization wavenumber $\kappa$

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes g-acceleration grid
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Animation of mechanics and metrology of constant-g grid
Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space

Relativistic optical transitions

$|\text{high}\rangle = |\omega_h\rangle \iff |\text{mid}\rangle = |\omega_m\rangle \iff |\text{low}\rangle = |\omega_\ell\rangle$

$\hbar\omega = E(cp)$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

$\omega_h = 3$

$\omega_m, \sinh \rho$

$\omega_m e^{+\rho} = 3 = \omega_h$

$\omega = \frac{4}{3}$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

$\omega_m \cosh \rho$

$\hbar ck = cp$
Relativistic optical transitions: $|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar \omega(ck)$ or $E(cp)$-space

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Sunday, January 29, 2017
Relativistic optical transitions $|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space

Initial stationary BLUE $K_h$ thing $\omega_h = M_h \hbar c^2$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
Relativistic optical transitions $|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space

Initial stationary BLUE $K_h$ thing $\omega_h = M_{hi}c^2$

transitions to

Final moving GREEN $K_m$ thing $\omega_m = M_{mi}c^2$

$\omega_h = \frac{4}{3}$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
Relativistic optical transitions \( |high\rangle = |\omega_h\rangle \iff |mid\rangle = |\omega_m\rangle \iff |low\rangle = |\omega_\ell\rangle \)

Review of Thales geometry of relativistic \( \hbar \omega(ck) \) or \( E(cp) \)-space

Initial stationary
BLUE \( K_h \) thing \( \omega_h = M_{hi} c^2 \)

transitions to
Final moving
GREEN \( K_m \) thing \( \omega_m = M_{m} c^2 \)

\( \frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell \)

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)

\( \hbar c k = cp \)
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Rapidity $\rho$ related to stellar aberration angle $\sigma$ and L. C. Epstein’s approach to relativity
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Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \quad \Leftrightarrow \quad |\text{mid}\rangle = |\omega_m\rangle \quad \Leftrightarrow \quad |\text{low}\rangle = |\omega_\ell\rangle \]

\begin{align*}
\hbar \omega &= E(cp) \\
\hbar \omega &= E(cp)
\end{align*}

Recoil from emitting an oppositely \(c\)-moving "photon" \(\omega_{hm} = c \mid k_{hm} \mid = \omega_m \sinh \rho \)

Feynman diagram (scaled down) of emission process

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_l\rangle \]

**Review of Thales geometry of relativistic** \( h\omega(ck) \) **or** \( E(cp) \)-space

**Initial stationary**

BLUE \( K_h \) **thing** \( \omega_h = M_{hi} c^2 \)

**transitions to**

Final **moving**

GREEN \( K_m \) **thing** \( \omega_m = M_{mi} c^2 \)

Recoil from emitting an oppositely \( c \)-moving

**YELLOW** \( k_{hm} \) **“photon”** \( \omega_{hm} = c \) | \( k_{hm} | = \omega_m \text{ sinh } \rho \)

**Feynman diagram** (scaled down)

of emission process

**Take-away point 0**

Classical (and spectroscopic)

Energy-momentum conservation is due to conservation in quantum-phase space-time “wiggle-count”

Doppler RED factor: \[ \frac{2}{3} = e^{-\rho} \]

Doppler BLUE factor: \[ \frac{3}{2} = e^{+\rho} \]
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Review of Thales geometry of relativistic \( \omega(ck) \) or \( E(cp) \)-space

\[ \hbar \omega = E(cp) \]

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)

\[ \hbar c k = cp \]

**Take-away point 2**
Easy to compute recoil rapidity \( \rho \) or recoil velocity \( u \)

**Key recoil relations:**
\[ \omega_m e^{-\rho} = \omega_\ell \]

\[ \rho = \ln \frac{M_\ell}{M_m} \]

Exact recoil rapidity where:
\[ \frac{u_{\text{recoil}}}{c} = \tanh \rho \]

or:
\[ u \sim c \ln \frac{M_\ell}{M_m} \]

Low-\( u_{\text{recoil}} \) approximation where:
\[ \rho \approx \frac{u_{\text{recoil}}}{c} \]
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle \]

Review of Thales geometry of relativistic \( \hbar \omega (ck) \) or \( E(cp) \)-space

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)

\( \hbar ck = cp \)

Take-away point 3

Emission photons are analogous to rocket exhaust (not “bullets”)

\((V_{\text{burnout}} = c_{\text{exhaust}} \ln\left[ M_{\text{initial}}/M_{\text{final}} \right])\)

...and this process is reversible

Key recoil relations:

\[ \omega_m e^{-\rho} = \omega_\ell \]

\[ \rho = \ln \frac{M_\ell}{M_m} \]

Exact recoil rapidity where:

\[ u \sim c \ln \frac{M_\ell}{M_m} \]

Low-\( u_{\text{recoil}} \) approximation where:

\[ \rho \approx \frac{u_{\text{recoil}}}{c} \]

\( \omega_m e^{+\rho} = 3 = \omega_h \)
All-rational-fraction lattice defined by discrete sub-group of Lorentz Poincare Group (Feynman path integrals defined by group transformations)

\[(p,q)\text{-coordinates}

Rest frequency: \[\omega_q = \omega_m e^{q\rho}\]

Rapidity: \[\rho_p = p\rho\]

\[P_{p,q} = (ck_{p,q}, \omega_{p,q})\]

\[= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)\]

Relativity Web Simulation

\{(Compton Scattering)\}

Doppler BLUE factor: \[\frac{3}{2} = e^{\rho}\]
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2nd Quantization: \( h\nu \) is actually \( hN\nu \)
\(( h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \) is actually \(( hN\nu_{\text{phase}} = E_N = hN\nu_A \cosh \rho \) with quantum numbers\)

\[ N = 1, 2, 3, \ldots \]

1st Quantization:
Mode quantum number \( n \) of half-waves

\[ E = hN\nu \]

\[ N_1 = 12 \]
\[ N_2 = 6 \]
\[ N_3 = 4 \]
\[ N_4 = 3 \]

Boosted wave mode

\[ E = \hbar c \kappa \]

\[ c \cdot \text{Momentum} = \hbar c \cdot \text{Wavenumber} \]

Take-away point 4
Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

Sunday, January 29, 2017
Quantization: \( h\nu \) is actually \( hN\nu \)

\( (h\nu_{\text{phase}}=E=h\nu A\cosh \rho) \) is actually \( (hN\nu_{\text{phase}}=E_N=hN\nu A\cosh \rho \) \( (N=1,2,\ldots) \)
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*Learning about* $\sin!$ and $\cos$ and...

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**Acceleration by chirping laser pairs**

*Varying acceleration (General case)*

Varying local acceleration \( \rho = \rho(\tau) \)

\[
\begin{align*}
u &= \frac{dx}{dt} = c \tanh(\tau) \\
dt/d\tau &= \cosh \rho(\tau) \\
c(t) &= c \int \cosh \rho(\tau) \, d\tau \\
x &= c \int \sinh \rho(\tau) \, d\tau
\end{align*}
\]

Constant local acceleration \( \rho = \frac{g \tau}{c} \) "Einstein Elevator"

\[
\begin{align*}
ct &= c \int \cosh \frac{g \tau}{c} \, d\tau \\
x &= c \int \sinh \frac{g \tau}{c} \, d\tau \\
&= \frac{c^2}{g} \sinh \frac{g \tau}{c} \\
&= \frac{c^2}{g} \cosh \frac{g \tau}{c}
\end{align*}
\]

Previous examples involved constant velocity

Constant velocity \( \rho = \rho_0 = \text{const.} \) "Lorentz transformation"

\[
\begin{align*}
ct &= c \int \cosh \rho_0 \, d\tau \\
x &= c \int \sinh \rho_0 \, d\tau \\
&= \tau \cosh \rho_0 \\
&= \tau \sinh \rho_0
\end{align*}
\]

From Lect. 35

*ModPhys (2012)*

Only green-light is seen by observers on the green accelerated trajectory

At \( x = x - ct \) frequency is \( e^{-\rho} \)

At \( x = x + ct \) frequency is \( e^{+\rho} \)

Only green-light is seen by observers on the green constant-\( g \) hyperbola

At \( x \to x - ct \) frequency is \( \omega \to \omega_0 e^{-\rho} \)

At \( x \to x + ct \) frequency is \( \omega \to \omega_0 e^{+\rho} \)

Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant \( g \)
(a) Constant acceleration $g$
Rapidity $\rho$ vs proper time $\tau$

\[ \rho = g\tau/c \]
\[ a\rho = c\tau \]

\[ x = a \cosh \rho \]
\[ ct = a \sinh \rho \]

Radius \[ a = c^2/g \]

(b) Traveler paths of acceleration $g_q$

Inertial frame coordinates
\[ (x_{q,p}, ct_{q,p}) = a_0 e^{q\rho_1} (\cosh p\rho_1, \sinh p\rho_1) \]

Geometric scale:
\[ e^{q\rho_1} = \left( \frac{3}{2} \right)^q \]
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RelativIt Web Simulation

{Accelerated proper-time frame}
RelativIt Web Simulation
{Accelerated proper-time frame}

- Don's object hits Al
- Don's object hits Bob
- Don's object hits Carl

- Carl's object hits Al
- Carl's object hits Bob

- Bob's object hits Al

0.97 lt. year
Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light.
\[ c \cdot \text{time} = \frac{4}{3} \]

\[ \frac{1}{2} \mu m = c \cdot \frac{5}{3} \text{fs} \]

\[ 0.8 = c \tau_{\text{phase}} = \frac{4}{5} \]

\[ 1.33 = c \tau_{\text{group}} = \frac{4}{3} \]

\[ \lambda_{\text{group}} = \frac{4}{5} = 0.8 \]

\[ \lambda_{\text{phase}} = \frac{4}{3} = 1.33 \]

\[ x\text{-space (wavelength } \lambda \text{)} \]

\[ (\text{units: } \frac{1}{2} \mu m) \]
| time | $b_{\text{Doppler RED}}$ | $V_{\text{group}}/c$ | $v_{\text{group}}/v_A$ | $\tau_{\text{phase}}/\tau_A$ | $v_{\text{phase}}/v_A$ | $\tau_{\text{group}}/\tau_A$ | $V_{\text{phase}}/V_A$ | $b_{\text{Doppler BLUE}}$ | 
|------|----------------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|------|
| space | $1/b_{\text{Doppler BLUE}}$ | $c/V_{\text{phase}}$ | $\kappa_{\text{phase}}/\kappa_A$ | $\lambda_{\text{group}}/\lambda_A$ | $\kappa_{\text{group}}/\kappa_A$ | $\lambda_{\text{phase}}/\lambda_A$ | $V_{\text{phase}}/V_A$ | $1/b_{\text{Doppler RED}}$ | 
| rapidity $\rho$ | $e^{-\rho}$ | $\tanh \rho$ | $\sinh \rho$ | $\text{sech} \rho$ | $\cosh \rho$ | $\text{csch} \rho$ | $\coth \rho$ | $e^{+\rho}$ | 
| value for $\beta=0.8$ | 0.33 | 0.80 | 1.34 | 0.60 | 1.67 | 0.75 | 1.25 | 3.01 |
Bob's coordinates for Alice's G-point
\[ x'_G = \lambda_A \sinh \rho = c \tau_A \sinh \rho \]
\[ c t'_G = \lambda_A \cosh \rho = c \tau_A \cosh \rho \]

Bob's coordinates for Alice's P-point
\[ x'_P = \lambda_A \sinh \rho = c \tau_A \sinh \rho \]
\[ c t'_P = \lambda_A \cosh \rho = c \tau_A \cosh \rho \]

RelaWavity Web Simulation
Comprehensive dual plots
with parameter table
RelaWavity Web Simulation
\((ct' \text{ vs } x')\) with parameter table

Space-time parameters
\[ \lambda_{\text{phase}} = \lambda_A \cosh \rho \]
\[ \lambda_{\text{group}} = \lambda_A \sinh \rho \]
\[ c \tau_{\text{phase}} = c \tau_A \cosh \rho \]
\[ c \tau_{\text{group}} = c \tau_A \sinh \rho \]

Per-space-time parameters
\[ c \kappa_{\text{phase}} = c \kappa_A \cosh \rho \]
\[ c \kappa_{\text{group}} = c \kappa_A \sinh \rho \]
\[ v_{\text{phase}} = v_A \cosh \rho \]
\[ v_{\text{group}} = v_A \sinh \rho \]

<table>
<thead>
<tr>
<th>Group</th>
<th>(b_{\text{Doppler RED}})</th>
<th>(\frac{v_{\text{group}}}{c})</th>
<th>(\frac{v_{\text{phase}}}{v_A})</th>
<th>(\frac{\lambda_{\text{group}}}{\lambda_A})</th>
<th>(\frac{\kappa_{\text{group}}}{\kappa_A})</th>
<th>(\frac{\tau_{\text{group}}}{\tau_A})</th>
<th>(\frac{V_{\text{phase}}}{c})</th>
<th>(b_{\text{Doppler BLUE}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase</td>
<td>(\frac{1}{b_{\text{Doppler BLUE}}\cosh \rho})</td>
<td>(\frac{\sinh \rho}{\sinh \rho})</td>
<td>(\frac{\cosh \rho}{\cosh \rho})</td>
<td>(\frac{\sinh \rho}{\cosh \rho})</td>
<td>(\frac{\cosh \rho}{\cosh \rho})</td>
<td>(\frac{1}{\cosh \rho})</td>
<td>(\frac{1}{\cosh \rho})</td>
<td>(\frac{1}{b_{\text{Doppler RED}}\cosh \rho})</td>
</tr>
<tr>
<td>Rapidity</td>
<td>(e^{-\rho})</td>
<td>(\tanh \rho)</td>
<td>(\sinh \rho)</td>
<td>(\cosh \rho)</td>
<td>(\sinh \rho)</td>
<td>(\cosh \rho)</td>
<td>(\cosh \rho)</td>
<td>(e^{-\rho})</td>
</tr>
<tr>
<td>Stellar angle</td>
<td>(\frac{1}{e^{+\rho}})</td>
<td>(\sin \sigma)</td>
<td>(\tan \sigma)</td>
<td>(\cos \sigma)</td>
<td>(\sec \sigma)</td>
<td>(\cot \sigma)</td>
<td>(\csc \sigma)</td>
<td>(\frac{1}{e^{+\rho}})</td>
</tr>
</tbody>
</table>

\[\beta = \frac{u}{c}\]
\[= \frac{1 - \beta}{1 + \beta}\]
\[= \frac{1}{1 + \beta}\]
\[= \frac{1}{\sqrt{1 - \beta^2}}\]

Value for \(\beta = 0.5\)
- 0.5
- 0.6
- 0.75
- 0.80
- 1.25
- 1.33
- 1.67
- 2.0

Effects
- \(b_{\text{Doppler RED}}\)
- \(\frac{v_{\text{group}}}{c}\)
- \(\tau_{\text{phase}}\)
- \(\frac{v_{\text{phase}}}{v_A}\)

- Past-future asymmetry (off-diagonal Lorentz transform)
- \(x\)-contraction (Lorentz)
- \(t\)-dilation (Einstein)
- Inverse asymmetry

- Value for \(\beta = 0.5\)
- \(\frac{3}{5}\)
- \(\frac{3}{4}\)
- \(\frac{4}{5}\)
- \(\frac{5}{4}\)
- \(\frac{5}{3}\)
- \(\frac{5}{3}\)
- \(\frac{2}{1}\)
A more compact circle-based geometry

<table>
<thead>
<tr>
<th>group</th>
<th>$b_{\text{Dee}}^{\text{RED}}$</th>
<th>$\frac{V_{\text{group}}}{c}$</th>
<th>$\frac{V_{\text{group}}}{\lambda_A}$</th>
<th>$\frac{\lambda_{\text{group}}}{\lambda_A}$</th>
<th>$\frac{\kappa_{\text{group}}}{\kappa_A}$</th>
<th>$\frac{\tau_{\text{group}}}{\tau_A}$</th>
<th>$\frac{c}{V_{\text{group}}}$</th>
<th>$b_{\text{Dee}}^{\text{BLUE}}$</th>
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<tbody>
<tr>
<td>phase</td>
<td>$\frac{1}{b_{\text{Dee}}^{\text{BLUE}}}$</td>
<td>$\frac{c}{V_{\text{phase}}}$</td>
<td>$\frac{\kappa_{\text{phase}}}{\kappa_A}$</td>
<td>$\frac{\tau_{\text{phase}}}{\tau_A}$</td>
<td>$\frac{V_{\text{phase}}}{\lambda_A}$</td>
<td>$\frac{\lambda_{\text{phase}}}{c}$</td>
<td>$\frac{1}{b_{\text{Dee}}^{\text{RED}}}$</td>
<td>$e^{\rho}$</td>
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<td>rapidity</td>
<td>$e^{-\rho}$</td>
<td>tanh $\rho$</td>
<td>sinh $\rho$</td>
<td>sech $\rho$</td>
<td>cosh $\rho$</td>
<td>csch $\rho$</td>
<td>coth $\rho$</td>
<td>$e^{\rho}$</td>
</tr>
<tr>
<td>stellar $\text{v}$ angle $\beta$</td>
<td>$\frac{1}{e^{\rho}}$</td>
<td>$\sin \sigma$</td>
<td>$\tan \sigma$</td>
<td>$\cos \sigma$</td>
<td>$\sec \sigma$</td>
<td>$\cot \sigma$</td>
<td>$\csc \sigma$</td>
<td>$\frac{1}{e^{\rho}}$</td>
</tr>
<tr>
<td>$\beta=\frac{u}{c}$</td>
<td>$\sqrt{\frac{1-\beta}{1+\beta}}$</td>
<td>$\frac{\beta}{1}$</td>
<td>$\frac{1}{\sqrt{\beta^2-1}}$</td>
<td>$\frac{\sqrt{1-\beta^2}}{1}$</td>
<td>$\frac{1}{\sqrt{1-\beta^2}}$</td>
<td>$\frac{1}{\beta}$</td>
<td>$\frac{\sqrt{1+\beta}}{1-\beta}$</td>
<td>$\frac{1}{e^{\rho}}$</td>
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<tr>
<td>values for $\beta=3/5$</td>
<td>$\frac{1}{2} = 0.5$</td>
<td>$\frac{3}{5} = 0.6$</td>
<td>$\frac{3}{4} = 0.75$</td>
<td>$\frac{4}{5} = 0.8$</td>
<td>$\frac{5}{4} = 1.25$</td>
<td>$\frac{4}{3} = 1.33$</td>
<td>$\frac{5}{3} = 1.67$</td>
<td>$\frac{2}{1} = 2.0$</td>
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