Current understanding of relativity and QM at UAF

NWAT photo by David Gottschalk
Current understanding of relativity and QM at UAF

Is a clearer understanding possible…?
Level 1 Secrets (which really shouldn’t be secrets at all!)
Special relativity and quantum mechanics are very much a story of the geometry of light-wave motion

• How badly does Galilean relativity fail for light waves?

• How do you make sense of light-wave axiom(s)?
  The *Einstein Pulse Wave (PW)* axiom versus
  The *Evenson Continuous Wave (CW)* axiom

SPEED LIMIT
\[ c = 299,792,458 \text{ m/s} \]
Good approximation: \( c = 300 \text{ million m/s} \)
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Special relativity and quantum mechanics
are very much a story of
the geometry of light-wave motion

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• How does space-time and/or per-space-per-time carry light-waves?

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Special relativity and quantum mechanics *are very much a story of* the geometry of light-wave motion

- How badly does Galilean relativity fail for light waves?
- How do you make sense of light-wave axiom(s)?
  - The *Einstein Pulse Wave* (PW) axiom versus
  - The *Evenson Continuous Wave* (CW) axiom
- How does space-time and/or *per-space-per-time* carry light-waves?
  - *(wavelength $\lambda$ – period $\tau$) and/or* *(wavenumber $\kappa$ – frequency $\nu$)*
  - $\lambda = 1/\kappa$ and $\tau = 1/\nu$ *and* $\kappa = 1/\lambda$ and $\nu = 1/\tau$
Level 1 Secrets (which really shouldn’t be secrets at all!)

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• How does space-time and/or per-space-per-time carry light-waves?

(wavelength \( \lambda \) – period \( \tau \)) and/or (wavenumber \( \kappa \) – frequency \( \upsilon \))

\[
\begin{align*}
\lambda &= 1/\kappa \\
\tau &= 1/\upsilon
\end{align*}
\]

\[
\begin{align*}
\lambda &= \text{meters per wave} \\
\tau &= \text{seconds per wave}
\end{align*}
\]

SPEED LIMIT

\[
c = 299,792,458 \text{ m/s}
\]

Good approximation: \( c = 300 \text{ million m/s} \)

Heinrich Hertz

1857-1894

1 Hz = 1 sec\(^{-1}\)
Level 1 Secrets (which really shouldn’t be secrets at all!)
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  The Einstein Pulse Wave (PW) axiom
  versus
  The Evenson Continuous Wave (CW) axiom
  developed in

• How does space-time and/or per-space-per-time carry light-waves?

Greek “L” for Length
Greek “t” for time
Greek “k” for Kayser (or “kinks”)
Greek “n” for number of waves per second or Hertz (Hz)

SPECIAL
LIMIT
C =
299,792,458
m/s
Good approximation: c = 300 million m/s

(wavelength $\lambda$ - period $\tau$) and/or
(wavenumber $\kappa$ - frequency $\nu$)
($\lambda = 1/\kappa$ and $\tau = 1/\nu$)
($\kappa = 1/\lambda$ and $\nu = 1/\tau$)

(\lambda = meters per wave and \tau = seconds per wave) (\kappa = waves per meter and \nu = waves per second)

Heinrich Hertz 1857-1894
1Hz=1sec^{-1}

Heinrich Kayser 1853-1940
1Kayser=1cm^{-1}

Sunday, November 2, 2014
How fast is light? Light goes one foot in a nano-second. This may seem quite fast to us. But, on a cosmic scale lightspeed is positively sub-glacial.

In your lifetime light cannot move across one pixel (\(\cdot\)) of a Hubble deep-sky photo.
• How badly does Galilean relativity fail for light waves?

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

It's going -c. (Of course)
It's going -c.
It's going -c.

FLASH! FLASH!

It’s going c. (Of course)
It’s going c.
It’s going c.

Complicated

Pulse wave (PW) train

\[ A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots \]

PW peaks precisely locate places where wave is.

A “road-runner” axiom is a “show-stopper”

"beep-meeep!"
• How badly does Galilean relativity fail for light waves?

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

PW forms are also called Wave Packets (WP) since they are interfering sums of many CW terms (10-Cosine Waves make up this pulse)

CW terms are also called Color Waves or Fourier Spectral Components

Complicated

PW peaks precisely locate places where wave is.

A “road-runner” axiom is a “show-stopper”
• How badly does Galilean relativity fail for light waves?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is c

PW forms are also called Wave Packets (WP) since they are interfering sums of many CW terms. PW widths reduce proportionally with more CW terms (greater Spectral width)

- **Space-time width** (pulse width)
  - $\Delta t = \tau$

- **Spectral width** (harmonic frequency range)
  - 1 CW term
    - $\Delta \nu = 1/\tau$
  - 2 CW terms
    - $\Delta \nu = 2\nu$
  - 5 CW terms
    - $\Delta \nu = 5\nu$
  - 10 CW terms
    - $\Delta \nu = 10\nu$
  - 50 CW terms
    - $\Delta \nu = 50\nu$

**Fourier-Heisenberg product:** $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

A “road-runner” axiom is a “show-stopper”
How do you make sense of light-wave axiom(s)?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is c

- It's going -c. (Of course)
- It's going c.

**Continuous Wave (CW) train**

- CW zeros precisely locate places where wave is not

**Pulse wave (PW) train**

- PW peaks precisely locate places where wave is

\[ A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + \ldots \]

...many waves and Amplitude parameters

\[ A \cos(\omega t) \]

...just one wave (a 1CW)

Using Occam’s Razor

- A “road-runner” axiom is a “show-stopper”
- beep-meep!

Cut a *PW* to just one Continuous Wave (1CW)

Sunday, November 2, 2014
• How do you make sense of light-wave axiom(s)?

**Einstein Pulse Wave (PW) Axiom:** PW speed seen by all observers is \( c \)

\[
A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots
\]

**Continuous wave (CW) train:** CW speed for all colors is \( c \)

\[
A \cos \omega t
\]

**Cut a PW to one Continuous Wave (1CW) that changes Color if you accelerate!**

**Simplicity vs. Complicated:**
- **Simpler:** CW affects by 1st order Doppler, \( b = e^{i\phi} \) and \( r = e^{i\phi} \) of frequency \( \nu \) and wavenumber \( \kappa \)
- **Complicated:** many waves and Amplitude parameters
• How do you make sense of light-wave axiom(s)?

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

Using Occam’s Razor
(and Evenson’s lasers)

Continuous wave (CW) train

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Cut a PW to one Continuous Wave (1CW) that changes Color if you accelerate!

CW also stands for “Cosine Wave” or “Coherent Wave” or “Colored Wave” (all helpful things)!
The “Keyboard of the gods” or \textit{per-space-per-time} graphs \textit{versus} \textit{space-time} graphs

\begin{align*}
    \text{frequency } \nu &= \text{waves per sec.} \\
    \text{wavenumber } \kappa &= \text{waves per meter} \\
    \text{wavelength } \lambda &= \frac{2}{3} = \frac{1}{\kappa} \\
    \text{period } \tau &= \frac{5}{4} = \frac{1}{\nu}
\end{align*}

\textit{Press a key to get a wave} \rightarrow \textit{...in spacetime...}

\textit{“Keyboard of the gods” is known as “Fourier-space”}

Jean-Baptiste Joseph Fourier
1768-1830

\textit{Ways to quantify general waves}
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

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**Press a key to get a wave (a 1-CW)**

...in spacetime...

“1-CW” means “single Continuous Wave”

**Ways to quantify general waves**
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"1-CW" means "single Continuous Wave"

...for all time...

...That "continues" everywhere...

\[ \lambda = \frac{2}{3} = \frac{1}{\kappa} \]

\[ \tau = \frac{5}{4} = \frac{1}{\nu} \]

\[ \kappa = \frac{3}{2} \]

\[ \nu = \frac{4}{5} \]

Sunday, November 2, 2014
The "Keyboard of the gods" or per-space-per-time graphs versus space-time graphs

Ways to quantify general waves

Jean-Baptiste Joseph Fourier 1768-1830

"Keyboard of the gods" is known as "Fourier-space"

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...at a speed of:
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Press a key to get a wave (a 1-CW)

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The "Keyboard of the gods" or per-space-per-time graphs versus space-time graphs.

Press a key to get a wave (a 1-CW)

...in spacetime...

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...for all time...

...at a speed of:

V_{wave} = \frac{\lambda}{\tau} = \frac{1}{\kappa} = \frac{\nu}{1/\kappa} = \frac{1}{\tau} = \frac{2/3}{5/4} = \frac{4}{5} = \frac{8}{15} = 8 \text{ m. per s.}

Ways to quantify general waves

wave-speed equals slope-to-horizontal in \((\kappa, \nu)\)-graph

wave-speed equals slope-to-vertical in \((\lambda, \tau)\)-graph

The wave-velocity formula:

\[
\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}}
\]

\[
V_{wave} = \frac{\lambda}{\tau} = \frac{1}{\kappa} = \frac{\nu}{1/\kappa} = \frac{1}{\tau} = \frac{2/3}{5/4} = \frac{4}{5} = \frac{8}{15} = 8 \text{ m. per s.}
\]
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Ways to quantify light waves

Light wave-velocity c (our main topic)

\[ V_{\text{light}} = c = \frac{\lambda}{\tau} = \frac{1}{\kappa} = \frac{\nu}{\kappa} = \frac{1}{\nu} = \frac{1}{\tau} = \frac{1}{\lambda} = 299,792,458 \text{ m/s} \]

Wave-speed equals slope-to-vertical in \((\lambda, \tau)-\text{graph}\)

Wave-speed equals slope-to-horizontal in \((\kappa, \nu)-\text{graph}\)

Press a key to get a wave (a 1-CW)

...in spacetime...

...That “continues” everywhere...

...for all time...

...at a speed of:

\[ \lambda = 2/3 = 1/\kappa \]

\[ \tau = 5/4 = 1/\nu \]

Distance

Time

Period

Wavelength

Wave-speed equals slope-to-vertical in \((\lambda, \tau)-\text{graph}\)

Wave-speed equals slope-to-horizontal in \((\kappa, \nu)-\text{graph}\)

Frequency

Period

Wavelength

Wave-number

Wave-number \(\kappa\) (waves per meter)

Wave-number \(\nu\) (waves per second)

Wave-speed \(V_{\text{wave}}\)

Distance

Time

Period

Wavelength

Wave-speed equals slope-to-vertical in \((\lambda, \tau)-\text{graph}\)

Wave-speed equals slope-to-horizontal in \((\kappa, \nu)-\text{graph}\)

Sunday, November 2, 2014
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Per-space-per-time graphs

Press a key to get a wave (a 1-CW)

...in spacetime...

...That “continues” everywhere...

...for all time...

...at a speed of:

Light wave-velocity formula

\[
\text{wave-speed equals slope-to-horizontal in } (\kappa, \nu)\text{-graph}
\]

\[
\text{wave-velocity} = \frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}}
\]

\[
V_{\text{wave}} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\nu} = \frac{\nu}{\kappa} = \frac{1/\tau}{1/\lambda}
\]

\[
= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15} \text{ m/s.}
\]

Light wave-velocity

\[
V_{\text{light}} = c = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\nu} = \frac{\nu}{\kappa} = \frac{1/\tau}{1/\lambda}
\]

\[
= \frac{2/3}{5/4} = \frac{4/5}{3/2} = 299,792,458 \text{ m/s.}
\]

Dimensionless Light wave-velocity

\[
\frac{V_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{1/\kappa}{c/\nu} = \frac{\nu}{c\kappa} = \frac{1}{c/\lambda} = 1
\]
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

**Ways to quantify light waves (600 THz example)**
The "Keyboard of the gods" or per-space-per-time graphs versus space-time graphs

Press the 600 THz key to get a 600 THz 1-CW)

Ways to quantify light waves (600 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Frequency \( \nu \) (units: 600 THz) = \( \nu_A \)

Per-SPACETIME \((c\kappa, \nu)\)-graph

\( c \cdot \text{time period} \ c \tau \) (units: \( \frac{1}{2} \mu \text{m} \))

\( c \tau_A = \lambda_A \)

Atom traveling along wave sees less wave “hits”/sec. (that is: Doppler red-shift)

Ways to quantify light waves (600 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

<table>
<thead>
<tr>
<th>frequency $\nu$ (units: 600THz)</th>
<th>per-SPACETIME $(c\kappa, \nu)$-graph</th>
<th>c-time period $c\tau$ (units: $\frac{1}{2} \mu$m)</th>
<th>SPACETIME $(\lambda, c\tau)$-graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_A$ 1800THz</td>
<td></td>
<td>$c\tau_A = \lambda_A$</td>
<td>Atom traveling along wave sees less wave “hits” /sec. (that is: Doppler red-shift)</td>
</tr>
<tr>
<td>$\nu = 300$THz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 600$THz</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\nu = 900$THz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 1200$THz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 1800$THz</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Atom traveling against wave sees more wave “hits” /sec. (that is: Doppler blue-shift)

Ways to quantify light waves (600 THz example)
The "Keyboard of the gods" or per-space-per-time graphs versus space-time graphs

Ways to quantify light waves (1200 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Ways to quantify light waves (1200 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Press the 300 THz key to get a 300 THz 1-CW)

Ways to quantify light waves (300 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Ways to quantify light waves (300 THz example)
The “Keyboard of the gods” or per-space-per-time graphs versus space-time graphs

Press the 300 THz key to get a 300 THz 1-CW) (with negative $\kappa$)

Ways to quantify light waves (300 THz example)
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads $\nu = 600\text{THz}$ for your laser beam.

Alice: “Well, what is its wavelength $\lambda$, Bob!”

A really fast Alice shines her $\nu = 300\text{THz}$ laser
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

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A really fast Alice shines her \( v=300\text{THz} \) laser

Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?
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A *really fast* Alice shines her \( \nu = 300THz \) laser

---

**Q1:** Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

**Q2:** If so, what “phony” \( \lambda \) does Bob see?

---

**Check it out in per-spacetime**

**Frequency** \( \nu = \omega/2\pi \)

**Wavenumber** \( \kappa = k/2\pi \)

<table>
<thead>
<tr>
<th>THz</th>
<th>900</th>
<th>800</th>
<th>700</th>
<th>600</th>
<th>500</th>
<th>400</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>1.00 ( \mu m )</td>
<td>0.50 ( \mu m )</td>
<td>0.33 ( \mu m )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>( 1 \cdot 10^6/m )</td>
<td>( 2 \cdot 10^6/m )</td>
<td>( 3 \cdot 10^6/m )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects**

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A *really* fast Alice shines her $\nu=300\text{THz}$ laser

---

**frequency $\nu=\omega/2\pi$**

(Inverse period $\nu=1/\tau$)

<table>
<thead>
<tr>
<th>THz</th>
<th>600</th>
<th>500</th>
<th>400</th>
<th>300</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</thead>
<tbody>
<tr>
<td>600</td>
<td></td>
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<td>500</td>
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<td>400</td>
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<tr>
<td>300</td>
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</tr>
</tbody>
</table>

Check it out in per-spacetime

Is it $A$, $B$, $C$ or $D$? etc.

**wavenumber $\kappa=k/2\pi$**

$\lambda=1.00\mu m$ $0.50\mu m$ $0.33\mu m$ (inverse wavelength $\kappa=1/\lambda$)

$\kappa=1\cdot10^6/m$ $2\cdot10^6/m$ $3\cdot10^6/m$

---

Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

Q2: If so, what “phony” $\lambda$ does Bob see?
Clarify Evenson’s CW Axiom (*All colors go c*) by Doppler effects

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Bob: “Alice! My frequency meter reads \( \nu = 600 \text{THz} \) for your laser beam.

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A *really fast* Alice shines her \( \nu = 300 \text{THz} \) laser

**Q1:** Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

**Q2:** If so, what “phony” \( \lambda \) does Bob see?

Answer to Q2 is C, the one with *slope* \( \nu / \kappa = \nu \cdot \lambda = c \).

If he sees Green 600THz then he measures \( \lambda = 0.5 \mu m \).

---

*Only ONE kind of GREEN allowed (ONE that goes c)*

- \( \lambda = 1.00 \mu m \)
- \( \kappa = 1 \times 10^6 / m \)
- \( \lambda = 0.50 \mu m \)
- \( \kappa = 2 \times 10^6 / m \)
- \( \lambda = 0.33 \mu m \)
- \( \kappa = 3 \times 10^6 / m \)
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A really fast Alice shines her \( \nu = 300 \text{THz} \) laser

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**Q1:** Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

**Q2:** If so, what “phony” \( \lambda \) does Bob see?

Answer to Q2 is C, the one with slope \( \nu / \kappa = \nu \cdot \lambda = c \).

If he sees Green 600THz then he measures \( \lambda = 0.5 \mu \text{m} \).
If he sees Red 300THz then he measures \( \lambda = 1.0 \mu \text{m} \).

---

**Evenson's CW Axiom (All colors go c) by Doppler effects**

\[
\nu = \frac{c}{\kappa}
\]

**wavenumber \( \kappa \)**

\[
\kappa = \frac{1}{\lambda}
\]

\( \lambda = 1.00 \mu \text{m} \)  \( 0.50 \mu \text{m} \)  \( 0.33 \mu \text{m} \)

\( \kappa = 1 \cdot 10^6 / \text{m} \)  \( 2 \cdot 10^6 / \text{m} \)  \( 3 \cdot 10^6 / \text{m} \)
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads $\nu = 600$THz for your laser beam.

Alice: “Well, what is its wavelength $\lambda$, Bob!”

A really fast Alice shines her $\nu = 300$THz laser

---

Q1: Can Bob tell it’s a “phony” 600THz by measuring his received wavelength?

Q2: If so, what “phony” $\lambda$ does Bob see?

Answer to Q2 is C, the one with slope $\nu/\kappa = \nu \cdot \lambda = c$.

If he sees Green 600THz then he measures $\lambda = 0.5\mu m$.

If he sees Red 300THz then he measures $\lambda = 1.0\mu m$.

Answer to Q1 is NO!

Light carries no birth-certificate!
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

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Alice: “Well, what is its wavelength \( \lambda \), Bob!”

A really fast Alice shines her \( \nu = 300 \text{THz} \) laser

\[ \nu = c \kappa \]

\[ \nu = \frac{c}{\lambda} \]

\[ \nu = \frac{1}{\tau} \]

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If he sees Green 600THz then he measures \( \lambda = 0.5 \mu m \).

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Answer to Q1 is NO!

Light carries no birth-certificate!

**Vacuum only makes one \( \lambda \) for each \( \nu \).**

“All colors go \( c = \lambda \nu = \nu / \kappa \)”

Then Evenson’s axiom holds:

*for each beam and polarization orientation*
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

Bob: “Alice! My frequency meter reads \( v = 600\text{THz} \) for your laser beam.

Alice: “Well, what is its wavelength \( \lambda \), Bob!”

A really fast Alice shines her \( v = 300\text{THz} \) laser

\[
(\nu = c\kappa)
\]

Vacuum only makes one \( \lambda \) for each \( v \).

Also could be labeled:

**Linear-(non)-dispersion axiom**: \( v = c \kappa \)

“**All colors go** \( c = \lambda v = v / \kappa \) ”

Then **Evenson’s axiom** holds:

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php
http://www.uark.edu/ua/pirelli/php/waveit_1way_disp2_phasor.php

Sunday, November 2, 2014
Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

Alice tries to fool Bob that she’s shining a 600THz laser. (Bob’s unaware she’s moving really fast...)

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A really fast Alice shines her \( \nu=300\text{THz} \) laser

More evidence supporting Evenson’s axiom

If bluer waves were even 0.1% faster (or slower) than redder ones then each flash from a 5-billion light-year distant galaxy shows up dispersed over 5-million years. (Goodbye galactic astronomy!)

Also could be labeled:

Linear-(non)-dispersion axiom: \( \nu = c \kappa \)

Vacuum only makes one \( \lambda \) for each \( \nu \).

“All colors go \( c = \lambda \nu = \nu/\kappa \)”

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Clarify Evenson’s CW Axiom (All colors go c) by Doppler effects

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A really fast Alice shines her $\nu=300$THz laser

Define angular frequency $\omega=2\pi\nu$ or $\omega=ck$

Angular frequency $\omega=2\pi\nu$

Angular wavenumber $k=2\pi\kappa$ is called wavevector $k$

Angular frequency $\omega=2\pi\nu$

Coming Soon:
Introduction of Laser-Phasor clock
Parameters $\omega$ and $k$

Linear-(non)-dispersion axiom: $\nu=ck$ or $\omega=ck$

Vacuum only makes one $\lambda$ for each $\nu$.

“All colors go $c = \lambda\nu = \nu/\kappa = \omega/k$”

Then Evenson’s axiom holds:

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php
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Sunday, November 2, 2014
Evenson's axiom is: "All frequencies march in lock-step." Hence, Doppler shift ratio $\langle R|S\rangle = \frac{v_R}{v_S}$ depends on relative velocity $u_{RS}$ of RECEIVER $R$ vs. SOURCE $S$ but not on source frequency $v_s$:

$$v_{RECEIVER} = \langle R|S\rangle v_{SOURCE}$$

Light is GEOMETRIC

(If light were ARITHMETRIC then $v_{RECEIVER} = v_{SOURCE} \pm \Delta_{RS}$ might be convenient.)
Evenson's axiom is: "All frequencies march in lock-step." Hence, Doppler shift ratio \( \frac{\nu_R}{\nu_S} \) depends on relative velocity \( u_{RS} \) of RECEIVER \( R \) vs. SOURCE \( S \) but not on source frequency \( \nu_S \):

\[
\nu_{\text{RECEIVER}} = \langle R \mid S \rangle \nu_{\text{SOURCE}}
\]

If Source-Receiver distance is **contracting**: 
\[
\frac{\nu_{\text{RECEIVER}}}{\nu_{\text{SOURCE}}} = \text{Blue shift} = \langle R \mid S \rangle > 1
\]

If Source-Receiver distance is **expanding**: 
\[
\frac{\nu_{\text{RECEIVER}}}{\nu_{\text{SOURCE}}} = \text{Red shift} = \langle R \mid S \rangle < 1
\]
Evenson's axiom is: "All frequencies march in lock-step." Hence, the **Doppler shift ratio** $\langle R|S\rangle = \frac{v_R}{v_S}$ depends on relative velocity $u_{RS}$ of RECEIVER $R$ vs. SOURCE $S$ but not on source frequency $v_S$:

\[ v_{RECEIVER} = \langle R|S\rangle v_{SOURCE} \]

If Source-Receiver distance is **contracting**:

\[ \frac{v_{RECEIVER}}{v_{SOURCE}} = \text{Blue shift} = \langle R|S\rangle > 1 \]

If Source-Receiver distance is **expanding**:

\[ \frac{v_{RECEIVER}}{v_{SOURCE}} = \text{Red shift} = \langle R|S\rangle < 1 \]

Logarithm of $\langle R|S\rangle$ known as **Rapidity**:

\[ \rho_{RS} = \log_e \langle R|S\rangle \quad \text{or} \quad \langle R|S\rangle = e^{\rho_{RS}} \]
Doppler shift-ratios and rapidity

Evenson's axiom is: "All frequencies march in lock-step." Hence, Doppler shift ratio \( \frac{v_R}{v_s} \) depends on relative velocity \( u_{RS} \) of RECEIVER \( R \) vs. SOURCE \( S \) but not on source frequency \( v_s \):

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Logarithm of \( \langle R|S \rangle \) known as **Rapidity**:

\[
\rho_{RS} = \log_e \langle R|S \rangle \quad \text{or} \quad \langle R|S \rangle = e^{\rho_{RS}}
\]

\( \langle R|S \rangle = e^{\rho_{RS}} \) with: \( \rho_{RS} > 0 \) for **contraction**, \( \rho_{RS} < 0 \) for **expansion**.
Doppler time-reversal symmetry

Alice: “Checkout my 600THz and 350THz beams!”

Bob: “Coming toward you and WOW! I’m seeing top 600THz doubled to 1200THz uv and 350THz doubled to 700THz!”

Note: time-reversal switches with reverses motion of Bob’s lasers and laser beams. (But, digital frequency readouts remain unchanged.)

Alice: “Well, I’m disappointed, Bob. Your so called 1200THz is a lousy 600THz, and I don’t need any more Blue! (Fortunately, 700THz turned up as a warm 350THz.)”

Bob: “I’m leaving now! But, I’ll send you a nice 1200THz uv beam and a Blue 700THz beam.”

Alice-Bob Doppler ratio: \( \langle A|B \rangle = \frac{\nu_A}{\nu_B} = \frac{600}{1200} = \frac{1}{2} \)

Bob-Alice Doppler ratio: \( \langle B|A \rangle = \frac{\nu_B}{\nu_A} = \frac{1200}{600} = \frac{2}{1} \)
**Easy Doppler-shift and Rapidity calculation**

**ALICE’S LASER GAUNTLET**

Alice: “Hey Bob and Carla! Read your Doppler shifts of my 600THz beam. What rapidity $\rho_{BA}$ or $\rho_{BC}$ do you all have relative to me and each other?”

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

Doppler ratio:

$$\langle R|S \rangle = \frac{v_{\text{RECEIVER}}}{v_{\text{SOURCE}}}$$

Bob-Alice Doppler ratio:

$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation
**Easy Doppler-shift and Rapidity calculation**

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---

**Definition of Rapidity**

Doppler ratio:

$$\langle R | S \rangle = \frac{\nu_{\text{RECEIVER}}}{\nu_{\text{SOURCE}}}$$

Rapidity:

$$\rho_{RS} = \log_e \langle R | S \rangle$$

Bob-Alice Doppler ratio:

$$\langle B | A \rangle = \frac{\nu_B}{\nu_A} = \frac{1200}{600} = \frac{2}{1}$$

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Doppler ratio:

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rapidity:

$$\rho_{RS} = \log_e \langle R|S \rangle$$

**Definition of Rapidity**

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$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1}$$

Carla-Alice Doppler ratio:

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Bob-Alice rapidity:

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Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$
Easy Doppler-shift and Rapidity calculation

**ALICE’S LASER GAUNTLET**

Alice: “Hey Bob and Carla! Read your Doppler shifts of my 600THz beam. What rapidity $\rho_{BA}$ or $\rho_{BC}$ do you all have relative to me and each other?”

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$$\langle R|S \rangle = \frac{v_{RECEIVER}}{v_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S \rangle$$

**Definition of Rapidity**

Bob-Alice Doppler ratio:

$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1} = 0.69$$

(\textit{so} $\rho_{AB} = -0.69$)

Carla-Alice Doppler ratio:

$$\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3} = -0.41$$

Blue shift to 1200THz

Red shift to 400THz
**Easy Doppler-shift and Rapidity calculation**

**ALICE’S LASER GAUNTLET**

Alice: “Hey Bob and Carla! Read your Doppler shifts of my 600THz beam. What *rapidity* $\rho_{BA}$ or $\rho_{BC}$ do you all have relative to me and each other?”

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Doppler ratio:

$$\langle R|S \rangle = \frac{v_{RECEIVER}}{v_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S \rangle$$

Definition of Rapidity

**Bob-Alice Doppler ratio:**

$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

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$$\rho_{BA} = 0.69 \quad (so: \rho_{AB} = -0.69)$$

**Carla-Alice Doppler ratio:**

$$\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

**Carla-Alice rapidity:**

$$\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

**Carla-Bob Doppler ratio:**

$$\langle C|B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \frac{v_A}{v_B} = \langle C|A \rangle \langle A|B \rangle$$
Definition of Rapidity

\[ \rho_{CA} = \frac{d_{CA}}{d_{AB}} \]

\[ \rho_{BC} = \frac{d_{BC}}{d_{AB}} \]

\[ \langle A | \rho | C \rangle = \frac{d_{CA}}{d_{AB}} = \frac{d_{BC}}{d_{AB}} = \langle B | \rho | C \rangle \]

Carla-Bob Rapidity:

\[ \langle A | \rho | C \rangle = \frac{d_{CA}}{d_{AB}} \]

\[ \langle B | \rho | C \rangle = \frac{d_{BC}}{d_{AB}} \]

Easy Doppler-shift and Rapidity calculation

Source

\[ \frac{\nu_s}{\nu_r} = \langle R | \rho | S \rangle \]

Doppler Rate:

\[ \frac{\nu_s}{\nu_r} = \langle R | \rho | S \rangle \]

Doppler shift:

\[ \text{Red shift to 400THz} \]

\[ \text{Blue shift to 1200THz} \]

Carla: I see Doppler

Bob: I see Doppler
Alice: “Hey Bob and Carla! Read your Doppler shifts of my 600THz beam. What rapidity $\rho_{BA}$ or $\rho_{BC}$ do you all have relative to me and each other?”

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Bob-Alice Doppler ratio:
$$\langle B|A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:
$$\rho_{BA} = \log_e \langle B|A \rangle = \log_e \frac{2}{1}$$
$$\rho_{BA} = 0.69 \quad (so: \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:
$$\langle C|A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$$

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$$\rho_{CA} = \log_e \langle C|A \rangle = \log_e \frac{2}{3}$$
$$\rho_{CA} = -0.41$$

Carla-Bob Doppler ratio:
$$\langle C|B \rangle = \frac{v_C}{v_B} = \frac{v_C}{v_A} \frac{v_A}{v_B} = \langle C|A \rangle \langle A|B \rangle$$

Carla-Bob rapidity:
$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies: } \rho_{CB} = \rho_{CA} + \rho_{AB}$$
$$= -0.41 - 0.69 = -1.10$$

Definition of Rapidity

$$\rho_{RS} = \log_e \langle R|S \rangle$$

or:
$$\langle R|S \rangle = e^{\rho_{RS}}$$

Easy Doppler-shift and Rapidity calculation
**Easy Doppler-shift and Rapidity calculation**

**ALICE’S LASER GAUNTLET**

Alice: “Hey Bob and Carla! Read your Doppler shifts of my 600THz beam. What rapidity \( \rho_{BA} \) or \( \rho_{BC} \) do you all have relative to me and each other?”

Bob: I see Doppler Blue shift to 1200THz

Carla: I see Doppler Red shift to 400THz

---

**Galileo’s Revenge (part 1)**

Rapidity adds just like Galilean velocity

\[
\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10
\]

---

**Definition of Rapidity**

\[
\rho_{RS} = \log_e \langle R | S \rangle = e^{\rho_{RS}}
\]

**Bob-Alice Rapidity**

\[
\rho_{BA} = \log_e \langle B | A \rangle = \log_e \frac{2}{1} = 0.69 \quad (so: \rho_{AB} = -0.69)
\]

**Carla-Alice Rapidity**

\[
\rho_{CA} = \log_e \langle C | A \rangle = \log_e \frac{2}{3} = -0.41
\]

**Bob-Alice Doppler ratio**

\[
\langle B | A \rangle = \frac{v_B}{v_A} = \frac{1200}{600} = \frac{2}{1}
\]

**Carla-Alice Doppler ratio**

\[
\langle C | A \rangle = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}
\]

---

**Doppler ratio**

\[
\langle R | S \rangle = \frac{v_{RECEIVER}}{v_{SOURCE}}
\]
Level 2 Secrets (which also shouldn’t be secrets!)
Special relativity and quantum mechanics are very much a story of
the geometry of light-wave motion

• How do we measure space and time with light waves?
  Use 1CW laser-phasors for a phase-based theory
• How do we make spacetime coordinate graph with light waves?
  Use 2CW laser-phasors and wave interference geometry
  Get Einstein-Lorentz-Minkowski graphs for free!
Dimensionless Light wave-velocity $c/c=1$

\[
\frac{v_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{1}{\kappa} = \frac{\nu}{c/k} = \frac{1/\tau}{c/\lambda} = 1 = \frac{\omega}{c \kappa \text{ units}}
\]

$V_{\text{light}} = \frac{\lambda}{c} = \frac{1}{\kappa} = \frac{\nu}{c/k} = \frac{1/\tau}{c/\lambda} = 1 = \frac{\omega}{c \kappa \text{ units}}$

$300 \text{ THz laser (Infrared)}$

$\text{Real } \psi = \text{Re} \psi$

$\psi(x,t) = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t)$

$k = +1 \quad \omega = 1c$

Wavelength $\lambda = 2\pi/k = 1/\kappa$

$(1 \mu m = 10^{-6} \text{ m})$
Dimensionless Light wave-velocity \( c/c = 1 \)

\[
\frac{v_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{1}{\kappa} = \frac{v}{c} = \frac{1}{\tau} = 1 = \frac{\omega}{c k} \text{ angular units}
\]

Q: Where is phase \((kx - \omega t) = 0\)?
A: It is wherever this is:

\[
\frac{x}{t} = \frac{\omega}{k}
\]

300 THz laser (Infrared)

Real \( \psi = \text{Re} \psi \)

Imaginary \( \psi = \text{Im} \psi \)

crest path (phase = 0)
zero path (phase = +\( \pi/2 \))

equation: \( \psi(x,t) = A \cdot e^{i(kx - \omega t)} = A \cdot \cos(kx - \omega t) + iA \cdot \sin(kx - \omega t) \)

1CW Laser-phasor wave function

Amplitude

Real axis

Phase-angle

Imaginary axis

Amplitude

\( A \)

Time

Space \( x \)

Period \( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \)

Wavelength \( \lambda = \frac{2\pi}{k} = \frac{1}{\kappa} \)

(10/3 fs = 3.33 \( \cdot 10^{-15} \) s)

(1 \( \mu \) m = 10\(^{-6} \) m)
**ICW Laser-phasor wave function**

\[ \psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t) \]

**Dimensionless Light wave-velocity**

\[ \frac{V_{\text{light}}}{c} = \frac{\lambda}{\lambda} = \frac{1}{c} = \frac{\nu}{c} = \frac{1}{\tau} = \frac{1}{\omega} = 1 = \frac{\omega}{c \text{ angular units}} \]

**300 THz laser**  
(Infrared)

\( k = +1 \quad \omega = 1c \)

**Real**  
\( \psi = \text{Re} \psi \)

**Imaginary**  
\( \psi = \text{Im} \psi \)

**Crest path**  
(phase = 0)

**Zero path**  
(phase = +\( \pi/2 \))

**Trough path**  
(phase = +\( \pi \))

**Period**  
\( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \)

\( (10/3 \, fs = 3.33 \cdot 10^{-15} \, s) \)

**Wavelength**  
\( \lambda = \frac{2\pi}{k} = \frac{1}{\kappa} \)

\( (1 \, \mu m = 10^{-6} \, m) \)

**Space x**

**Time**  
\( ct \)
1CW Laser-phasor wave function

\[ \psi = A e^{i(kx - \omega t)} = A \cos(kx - \omega t) + iA \sin(kx - \omega t) \]

**Dimensionless Light wave-velocity**

\[
\frac{V_{\text{light}}}{c} = \frac{\lambda}{c} = \frac{1}{\kappa} = \frac{v}{c} = \frac{1}{\tau} = 1 = \frac{\omega}{c \kappa} = \frac{c}{l} = \text{angular units}
\]

**300 THz laser** (Infrared)

**Real \(\psi\)**

\[ k = +1 \quad \omega = 1c \]

**Imaginary \(\psi\)**

\[ \psi = \text{Im} \psi \]

**Amplitude**

\[ A \]

**Phase-angle**

\[ \frac{\text{phase}}{\text{angle}} \]

**Real axis**

**Imaginary axis**

**Time**

\[ ct \]

**Space**

\[ x \]

**Wavelength**

\[ \lambda = 2\pi/k = 1/\kappa \]

\[ (1\mu m = 10^{-6} m) \]

**Period**

\[ \tau = 2\pi/\omega = 1/\nu \]

\[ (10/3 \text{ fs} = 3.33 \cdot 10^{-15} \text{ s}) \]

**3.3 CW Laser-phasor wave function**

\[ \psi(x, t) \]

**Amplitude**

\[ A \]

**Phase-angle**

\[ \frac{\text{phase}}{\text{angle}} \]

**Real axis**

**Imaginary axis**

**Time**

\[ ct \]

**Space**

\[ x \]
Colliding 2CW laser beams

**Right-moving wave** \( e^{i(kx - \omega t)} \)
- \( k = +2 \)
- \( \omega = 2c \)

**Left-moving wave** \( e^{i(-kx - \omega t)} \)
- \( k = -2 \)
- \( \omega = 2c \)

**CW Dye-laser**
- 600 THz

**Alice's laser**

**Carla's laser**

**Wavelength** \( \lambda = \frac{2\pi}{k} = \frac{1}{\kappa} \)
- \( 1/2\mu m = 0.5 \cdot 10^{-6} m \)

**Period** \( \tau = \frac{2\pi}{\omega} = \frac{1}{\nu} \)
- \( 5/3fs = 1.67 \cdot 10^{-15} s \)

Alice: OK, Bob. We're gonna' hit you from both sides, now!

Bob: Yikes!

Carla: Look out, Bob!
Colliding 2CW laser beams makes space-time coordinate frame

Right-moving wave $e^{i(kx-\omega t)}$

$\text{CW Dye-laser}$

600 THz

$k = +2$ $\omega = 2c$

$\text{Alice's laser}$

Bob: Yikes!

Left-moving wave $e^{i(-kx-\omega t)}$

$\text{CW Dye-laser}$

600 THz

$k = -2$ $\omega = 2c$

$\text{Carla's laser}$

The result is the “simplest molecule” (a $2-\gamma$ “thing”)...

..with a space-time frame that eventually reveals relativistic/quantum matter-wave effects!

$\omega = 2c$ $\omega = 2c$

$\text{Period } \tau = 2\pi / \omega = 1 / \nu$

$(5/3fs = 1.67 \cdot 10^{-15}s)$

$Wavelength \ \lambda = 2\pi / k = 1 / \kappa$

$(1/2\mu m = 0.5 \cdot 10^{-6}m)$

Alice: OK, Bob. We’re gonna’ hit you from both sides, now!

Carla: Look out, Bob!

Alice: OK, Bob. We’re gonna’ hit you from both sides, now!

Carla: Look out, Bob!
Bob: Cool! You guys made me a space-time graph out of real zeros. How'd it do that?

Carla: Easy! You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.
Bob: Cool! You guys made me a space-time graph out of real zeros. How’d it do that?

Carla: Easy! You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Right-moving CW $e^{i(kx-\omega t)}$

Left-moving CW $e^{i(-kx-\omega t)}$

Wavelength $\lambda = 2\pi/k = 1/\kappa$

$(1/2\mu m = 0.5 \times 10^{-6} m)$

Period $\tau = 2\pi/\omega = 1/\nu$

$(5/3 fs = 1.67 \times 10^{-15} s)$

$\Psi(x,t) = e^{ia} + e^{ib}$

$= e^{i\frac{a+b}{2}} (e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}})$
Cool!
You guys made me a space-time graph out of real zeros.

How'd it do that?

Carla:
Easy!
You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!
You factor $e^{ia} + e^{ib}$ into $e^{\frac{i}{2} (e^{\frac{i}{2} a-b} + e^{-\frac{i}{2} a-b})}$.
Bob: Cool! You guys made me a space-time graph out of real zeros.

How’d it do that?

Carla: Easy! You get zeros of any wave-sum \(e^{ia} + e^{ib}\) by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum \(\frac{a+b}{2}\) plus half-diff \(\frac{a-b}{2}\) gives \(a\), and half-sum \(\frac{a+b}{2}\) minus half-diff \(\frac{a-b}{2}\) gives \(b\).

Presto! You factor \(e^{ia} + e^{ib}\) into

\[
e^{\frac{a+b}{2}} \left( e^{\frac{a-b}{2}} + e^{-\frac{a-b}{2}} \right)
\]

Alice 1CW phase: \(a = kx - \omega t\)

Carla 1CW phase: \(b = -kx - \omega t\)
You factor $e^{ia} + e^{ib}$ into $e^{i(a+b)/2} \left( e^{i(a-b)/2} + e^{-i(a-b)/2} \right)$

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!

You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

Group wave: $e^{-ikx} + e^{-ikx} = 2\cos kx$

is standing wave (does not vary with time $t$)
Sunday, November 2, 2014

Bob: Let’s plot this in per-spacetime?!

Cool! You guys made me a space-time graph out of real zeros.

How’d it do that?

Carla: Easy!

You get zeros of any wave-sum $e^{ia} + e^{ib}$ by factoring it into phase and group parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives $a$, and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives $b$.

Presto!

You factor $e^{ia} + e^{ib}$ into $e^{ia} + e^{ib} = e^{\frac{i(a+b)}{2}} (e^{\frac{i(a-b)}{2}} + e^{-\frac{i(a-b)}{2}})$

Alice 1CW phase: $a = kx - \omega_0 t$

Carla 1CW phase: $b = -kx - \omega_0 t$

Bob’s 2CW Group-phase: $+k = \frac{a-b}{2}$

Group wave: $e^{-ikx} + e^{ikx} = 2\cos kx$

is standing wave (does not vary with time $t$)

Bob’s 2CW Phase-phase: $-\omega = \frac{a+b}{2}$

Phase wave real part: $\text{Re}(e^{-i\omega t}) = \cos(\omega t)$

is “instanton” wave (does not vary in space $x$)
Carla: OK, Bob!
It looks like a baseball diamond with
P at Pitcher’s mound and
G at the Grandstand*. I’m on 1st base! (R)

*Thanks, Woody!

Bob: The P and G vectors are scale models of zero-grid lattice vectors (but P and G switch places)
Standing 2CW in per-space-time

Frequency

\[ \omega = 2\pi \nu \]

Period \( \tau = 2\pi/\omega = 1/\nu \)

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)

(0.5 \mu m = 0.5 \cdot 10^{-6} m)

(1.67 fs = 0.167 \cdot 10^{-15} s)

\[ \Psi(x,t) = (e^{i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)} \]

Phase vector

1/2-sum:

\[ \mathbf{K}_{\text{phase}} = \mathbf{P} = \frac{\mathbf{R} + \mathbf{L}}{2} \]

Group vector

1/2-difference

\[ \mathbf{K}_{\text{group}} = \mathbf{G} = \frac{\mathbf{R} - \mathbf{L}}{2} \]

Carla:

OK, Bob!

It looks like a baseball diamond with

\( \mathbf{P} \) at Pitcher’s mound

and \( \mathbf{G} \) at the Grandstand*. Ok, I’m on 3rd base L.

*Thanks, Woody!

Bob:

The \( \mathbf{P} \) and \( \mathbf{G} \) vectors are scale models of zero-grid lattice vectors (but \( \mathbf{P} \) and \( \mathbf{G} \) switch places)

Wavevector

\[ \mathbf{c} k = 2\pi \kappa c \]
Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares

1 femtosecond
1.0 fs = 10^{-15}s

1 micron
1.0 µm = 10^{-6} meter

Time ct

Period \tau = \frac{5}{3} fs

Wavelength \lambda = \frac{1}{2} µm

CW Laser 600 THz

CW Laser 600 THz
Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares

1 femtosecond
1.0 fs = 10\(^{-15}\)s

1 micron
1.0 \(\mu\)m = 10\(^{-6}\) meter

Pulse Waves (PW) trace “baseball diamonds” in space-time

(b) PW diamonds

\[ \omega_0 = \frac{2c}{L} \]

“patooey!”

CW Laser 600 THz

PW laser \( \omega = 2c \)

\( \omega \) = 2c

Pulse Waves (PW) trace “baseball diamonds” in space-time

Continuous Waves (CW) trace “Cartesian squares” in space-time

\( \omega_0 = \frac{2c}{L} \)

“patooey!”

PW laser \( \omega = 2c \)

\( \omega \) = 2c

Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares

1 femtosecond
1.0 fs = 10\(^{-15}\)s

1 micron
1.0 \(\mu\)m = 10\(^{-6}\) meter

Pulse Waves (PW) trace “baseball diamonds” in space-time

(b) PW diamonds

\[ \omega_0 = \frac{2c}{L} \]

“patooey!”

PW laser \( \omega = 2c \)

\( \omega \) = 2c

Continuous Waves (CW) trace “Cartesian squares” in space-time

(a) CW squares

1 femtosecond
1.0 fs = 10\(^{-15}\)s

1 micron
1.0 \(\mu\)m = 10\(^{-6}\) meter

Pulse Waves (PW) trace “baseball diamonds” in space-time

(b) PW diamonds

\[ \omega_0 = \frac{2c}{L} \]

“patooey!”

PW laser \( \omega = 2c \)

\( \omega \) = 2c
Alice: Now our 600THz lasers move left-to-right. My 600THz laser is blasting you with UV 1200THz. Carla’s 600THz gives you a nice infrared 300THz.

Bob: That UV burns! I need to put on my sunglasses.
**Alice:** OK. My UV 1200THz vector is fierce! You’ll need glasses to see $P'$ and $G'$ lines or coordinates.

**Bob:** Sunglasses help. Wow! Your 1st baseline $R'$ is Doppler blued up by $e^{+\rho}=2$.

**Carla:** My UV 300THz baseline is a lot nicer!
Alice: OK. My UV 1200THz vector is fierce! You’ll need glasses to see $P'$ and $G'$ lines or coordinates.

Bob: Sunglasses help.

Carla: My UV 300THz $L'$ 3rd baseline is a lot nicer! (and half as long.)
Alice: OK. My UV 1200THz vector is fierce! You'll need glasses to see $P'$ and $G'$ lines or coordinates.

Bob: Sunglasses help. Wow! Your 1st baseline is Doppler blued up by $e^{+\psi}=2$.

But, Carla's 3rd baseline $L'$ is Doppler red shifted by $e^{-\psi}=1/2$.

New "Pitcher-mound" $P'$ (Phase pt.) is 1/2-sum $(R'+L')/2$:

$$
K'_{\text{phase}} = \mathbf{p}' = \frac{R'+L'}{2}
$$

$$
\begin{align*}
\nu'_{\text{phase}} &= \nu_A \left( \begin{array}{c} 2 \\ 2 \\ \frac{-1}{2} \\ \frac{1}{2} \end{array} \right) + \nu_A \left( \begin{array}{c} \frac{3}{4} \\ 5/4 \\ \frac{2}{2} \\ \frac{2+1/2}{2} \end{array} \right) \\
&= \nu_A \left( \begin{array}{c} 2-1/2 \\ 2 \\ \frac{2}{2} \\ \frac{2+1/2}{2} \end{array} \right)
\end{align*}
$$

New Wavelength $\lambda=2\pi/k=1/\kappa$ (1/4$\mu$m=0.25×10^{-6}$m)

3rd baseline is a lot nicer! (and half as long.)
Alice: OK. My UV 1200THz \( \mathbf{R}' \) vector is fierce! You’ll need glasses to see \( \mathbf{P}' \) and \( \mathbf{G}' \) lines or coordinates.

Bob: Sunglasses help. Wow! Your 1st baseline \( \mathbf{R}' \) is Doppler blued up by \( e^\rho = 2 \); but, Carla’s 3rd baseline \( \mathbf{L}' \) is Doppler red shifted by \( e^\rho = 1/2 \).

But, Carla’s 3rd baseline \( \mathbf{L}' \) is a lot nicer! (and half as long.)
**Alice:** OK.

My UV 1200THz R' vector is fierce! You'll need glasses to see P' and G' lines or coordinates.

**Carla:** My UV 300THz L' vector is a lot nicer! (and half as long.)

**Bob:** Sunglasses help. Wow! Your 1st baseline is Doppler blued up by $e^{+v}=2$.

But, Carla’s 3rd baseline L' is Doppler red shifted by $e^{-v}=1/2$.

New “Pitcher-mound” P' (Phase pt.) is 1/2-sum $(R'+L')/2$:

$$cK'_{\text{phase}} = \frac{v_A}{2} \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{v_A}{2} \begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \begin{pmatrix} 2-1/2 \\ 2+1/2 \end{pmatrix}$$

New “Grandstand” G' (Group pt.) is 1/2-difference $(R'-L')/2$:

$$cK'_{\text{phase}} = \frac{v_A}{2} \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \frac{v_A}{2} \begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \begin{pmatrix} 2+1/2 \\ 2-1/2 \end{pmatrix}$$

**Sunday, November 2, 2014**
My UV 1200THz vector is fierce! You'll need glasses to see $P'$ and $G'$ lines or coordinates.

New 1st base (Alice)

New 3rd base (Carla)

New Grandstand

Wavelength $\lambda = 2\pi/k = 1/\kappa$

$(1/4\mu m = 0.25 \cdot 10^{-6} m)$

$\nu' = e^{+\rho} \nu_A = 1200 \text{ THz}$

$\nu_c' = e^{-\rho} \nu_A = 1/2 \nu_A = 300 \text{ THz}$

Bob: Sunglasses help. Wow! Your 1st baseline is Doppler blued by $e^{+\rho} = 2$.

But, Carla's 3rd baseline $L'$ is Doppler red shifted by $e^{-\rho} = 1/2$.

New "Pitcher-mound" $P'$ (Phase pt.) is 1/2-sum $(R' + L')/2$:

New "Grandstand" $G'$ (Group pt.) is 1/2-difference $(R' - L')/2$:
Frequency \( \nu' \) (units of \( \nu_A = 600 \text{THz} \))

Wavevector \( c \kappa' \) (units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

Phase vector \( \mathbf{P} \)
1/2-sum vector \( \mathbf{K}_{\text{phase}}' = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} \)

Group vector \( \mathbf{G} \)
1/2-diff vector \( \mathbf{K}_{\text{group}}' = \mathbf{G}' = \frac{\mathbf{R}' - \mathbf{L}'}{2} \)

Wavelength \( \lambda = 2\pi / k = 1 / \kappa \) 
(1/4\( \mu m = 0.25 \cdot 10^{-6} \text{m} \))

Wavelength \( \lambda = 2\pi / k = 1 / \kappa \) 
(1\( \mu m = 10^{-6} \text{m} \))

Bob: The spacetime wave-zeros replicate the same pattern.
Frequency
\( \nu' \)
(units of
\( \nu_A = 600 \text{THz} \))

2CW per-Spacetime Plot

Wavelength \( \lambda = 2\pi/k = 1/\kappa \)
(1/4\( \mu \)m = 0.25 \( \times \) 10^{-6} m)

Space \( x' \)

Wavevector \( ck' \)
(units of \( c\kappa_A = 2 \times 10^6 / \text{m} \))

Phase vector \( P' \)
1/2-sum vector
\( \mathbf{K}_{\text{phase}}' = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} \)

Group vector \( G' \)
1/2-diff vector
\( \mathbf{K}_{\text{group}}' = \mathbf{G}' = \frac{\mathbf{R}' - \mathbf{L}'}{2} \)

2CW Minkowski-Spacetime Grid

Time \( ct' \)

Space \( x' \)

Bob: The spacetime wave-zeros replicate the same pattern.

(Except \( \mathbf{P}' \)-phase and \( \mathbf{G}' \)-group indicators get switched again.)
Phase frequency $\nu'_{\text{phase}} = \nu_A \cosh \rho = \frac{5}{4}$

flips to $\tau'_{\text{phase}} = \tau_A \sech \rho = \frac{4}{5}$

Phase period $\tau = \frac{1}{\nu}$

$\nu'_{\text{phase}}$ = 1.25

$\tau'_{\text{phase}}$ = 0.8

$\nu'_{\text{phase}}$ = $\nu_A \cosh \rho$ = $\frac{5}{4}$

$\tau'_{\text{phase}}$ = $\tau_A \sech \rho$ = $\frac{4}{5}$

Frequency $\nu'$
(units of $\nu_A = 600$ THz)

Wavevector $c\kappa_A$
(units of $c\kappa_A = 2 \cdot 10^6 / \text{m}$)

Space $x'$
(units of $\lambda_A = 1/2 \mu m$)

Time $ct'$
(units of $\lambda_A = 1/2 \mu m$)

<table>
<thead>
<tr>
<th>phase $b_{\text{Doppler RED}}$</th>
<th>$\nu_A$</th>
<th>$\kappa_A$</th>
<th>$V_{\text{phase}}$</th>
<th>$\tau_{\text{phase}}$</th>
<th>$\nu'_{\text{phase}}$</th>
<th>$\tau'_{\text{phase}}$</th>
<th>$\lambda_A$</th>
<th>$V_{\text{group}}$</th>
<th>$b_{\text{Doppler BLUE}}$</th>
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</thead>
<tbody>
<tr>
<td>group</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$\frac{5}{4}$</td>
<td>$\frac{5}{3}$</td>
<td>$\frac{1}{2}$</td>
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<tr>
<td>rapidity $\rho$</td>
<td>$e^{-\rho}$</td>
<td>$\tanh \rho$</td>
<td>$\sinh \rho$</td>
<td>$\sech \rho$</td>
<td>$\cosh \rho$</td>
<td>$\csch \rho$</td>
<td>$\coth \rho$</td>
<td>$e^{\rho}$</td>
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<tr>
<td>value for $\beta = 3/5$</td>
<td>$\frac{1}{2} = 0.5$</td>
<td>$\frac{5}{6}$</td>
<td>$0.6$</td>
<td>$\frac{3}{4} = 0.75$</td>
<td>$\frac{4}{5} = 0.80$</td>
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<td>$\frac{5}{3} = 1.67$</td>
<td>$\frac{2}{1} = 2.0$</td>
</tr>
</tbody>
</table>
Frequency

\( \nu' \)

(units of
\( \nu_A=600 \text{THz} \))

Wavevector \( cK' \)

(units of \( c\kappa_A=2\cdot10^6/\text{m} \))

Phase frequency

\( \nu'_{\text{phase}}=\nu_A \cosh \rho=\frac{5}{4} \)

\( =1.25 \)

Phase period

\( \tau'_{\text{phase}}=\tau_A \text{sech} \rho=\frac{4}{5} \)

\( =0.8 \)

Time \( c t' \)

(units of \( \lambda_A=1/2 \mu m \))

Space \( x' \)

(units of \( \lambda_A=1/2 \mu m \))


<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}} )</th>
<th>( \kappa'_{\text{Doppler RED}} )</th>
<th>( \tau_{\text{phase}} )</th>
<th>( \nu_{\text{phase}} )</th>
<th>( \lambda_A )</th>
<th>( V_{\text{group}} )</th>
<th>( b_{\text{Doppler BLUE}} )</th>
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<tr>
<td>group</td>
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<td>( \text{sech} \rho )</td>
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<td>( \text{csch} \rho )</td>
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<td>( \frac{2}{1} = 2.0 )</td>
<td></td>
</tr>
</tbody>
</table>
Phase wavenumber $\kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4$ flips to $\chi'_{\text{phase}} = \chi_A \cosh \rho = 4/3$

Phase frequency $v'_{\text{phase}} = v_A \cosh \rho = 5/4$ flips to $T'_{\text{phase}} = T_A \sinh \rho = 4/5$

$P' = \left( \begin{array}{c} c \kappa'_{\text{phase}} \\ v'_{\text{phase}} \end{array} \right) = V_A \left( \begin{array}{c} \sinh \rho \\ \cosh \rho \end{array} \right) = V_A \left( \begin{array}{c} 3/4 \\ 5/4 \end{array} \right)$

Frequency $v' = (\text{units of } v_A = 600 \text{THz})$

Wavevector $c \kappa' = (\text{units of } c \kappa_A = 2 \cdot 10^6 \text{m})$

Time $c't' = (\text{units of } \lambda_A = 1/2 \mu m)$

Space $x' = (\text{units of } \lambda_A = 1/2 \mu m)$

Phase wavelength $\lambda = 1/\kappa$

$\kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4$

$\chi'_{\text{phase}} = \chi_A \cosh \rho = 4/3$

$\kappa_{\text{phase}} = \kappa_A$

$\chi_{\text{phase}} = \chi_A$

$\tau = 1/v$

$T = 1/\kappa$

$V = c \kappa$

$V_A = c \kappa_A$

$V_{\text{group}} = c \kappa_{\text{group}}$

$V_{\text{group}} = c \kappa_{\text{group}}$

$\beta = 3/5$

$\rho = \tanh \rho$

$\sinh \rho$

$\cosh \rho$

$\cosh \rho$

$\cosh \rho$

$e^{-\rho}$

$e^{\rho}$

$\tanh \rho$

$\sinh \rho$

$\cosh \rho$

$\cosh \rho$

$\cosh \rho$

$e^{-\rho}$

$e^{\rho}$
Phase wavenumber \( \kappa_{phase} = \kappa_A \sinh \rho = 3/4 \) flips to \( \lambda'_{phase} = \lambda_A \text{csch} \rho = 4/3 \)

\[
P' = \begin{pmatrix} c \kappa_{phase} \\ \nu_{phase}' \end{pmatrix} = v_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = v_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( \nu'_{phase} = \nu_A \cosh \rho = 5/4 \) flips to \( \tau'_{phase} = \tau_A \text{sech} \rho = 4/5 \)

**Time** \( ct' \) (units of \( \lambda_A = 1/2 \mu m \))

**Space** \( x' \) (units of \( \lambda_A = 1/2 \mu m \))

**Phase wavenumber** \( \kappa_{phase} = \kappa_A \sinh \rho = 3/4 \)

**Phase wavelength** \( \lambda'_{phase} = \lambda_A \text{csch} \rho = 4/3 \)

**Phase frequency** \( \nu'_{phase} = \nu_A \cosh \rho = 5/4 \)

**Phase period** \( \tau'_{phase} = \tau_A \text{sech} \rho = 4/5 \)

**P-slope** \( = \frac{V_{phase}}{c} \)

**Frequency** \( v' \) (units of \( \nu_A = 600 \text{THz} \))

\[\begin{array}{c|c|c|c}
\text{Frequency} & \nu' & \nu_A & \nu_A \\hline
\text{THz} & \text{THz} & \text{THz} \\hline
1500 & 2.5 & 3.0 \\hline
1200 & 2.0 & 3.0 \\hline
900 & 1.5 & 3.0 \\hline
600 & 1.0 & 3.0 \\hline
300 & 0.5 & 3.0 \\hline
\end{array}\]

**Wavevector** \( c \kappa' \) (units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

\[\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
\text{Group} & b_{Doppler RED} & V_{wave} & \kappa_{phase} & \kappa_A & c'_{phase} & \tau_A & v_{phase} & \lambda_{phase} & \lambda_A & V_{phase} & b_{Doppler BLUE} \\hline
\text{rapidity} & \rho & e^{\rho} & \tanh \rho & \sinh \rho & \cosh \rho & \text{sech} \rho & \text{csch} \rho & \text{coth} \rho & e^{\rho} & \text{value for} & \beta=3/5 \\hline
\text{\( \frac{1}{2} = 0.5 \)} & \frac{3}{5} = 0.6 & \frac{3}{4} = 0.75 & \frac{4}{5} = 0.80 & \frac{5}{4} = 1.25 & \frac{4}{3} = 1.33 & \frac{5}{3} = 1.67 & \frac{2}{1} = 2.0 \\hline
\end{array}\]
Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \) flips to Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \text{csch} \rho = 4/3 \)

\[
P' = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}
\]

Phase frequency \( \nu'_{\text{phase}} = \nu_A \cosh \rho = 5/4 \) flips to Phase period \( \tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5 \)

Time \( ct' \) (units of \( \lambda_A = 1/2 \mu m \))

\[
\lambda'_{\text{phase}} = \lambda_A \text{csch} \rho = 4/3
\]

\[
\tau'_{\text{phase}} = \tau_A \text{sech} \rho = 4/5
\]

Phase wavenumber \( \kappa'_{\text{phase}} = \kappa_A \sinh \rho = 3/4 \) flips to Phase wavelength \( \lambda'_{\text{phase}} = \lambda_A \text{csch} \rho = 4/3 \)

\[
P - \text{slope} = \frac{\nu'_{\text{phase}}}{c} = \frac{\cosh \rho}{3/4} = \frac{5}{3} = \coth \rho
\]

\[
V_{\text{phase}} \bigg/ c = \frac{\nu'_{\text{phase}}}{c} = \frac{\cosh \rho}{3/4} = \frac{5}{3} = \coth \rho
\]

Space \( x' \) (units of \( \lambda_A = 1/2 \mu m \))

<table>
<thead>
<tr>
<th>phase</th>
<th>( b^{\text{Doppler RED}} )</th>
<th>( V_{\text{phase}} )</th>
<th>( \kappa_{\text{phase}} )</th>
<th>( \tau_{\text{phase}} )</th>
<th>( \nu_{\text{phase}} )</th>
<th>( \lambda_{\text{phase}} )</th>
<th>( V_{\text{phase}} )</th>
<th>( b^{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>1</td>
<td>( V_{\text{phase}} )</td>
<td>( \kappa_A )</td>
<td>( \tau_A )</td>
<td>( \nu_A )</td>
<td>( \lambda_A )</td>
<td>( V_{\text{phase}} )</td>
<td>1</td>
</tr>
<tr>
<td>rapidity</td>
<td>( e^{-\rho} )</td>
<td>( \tanh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \text{sech} \rho )</td>
<td>( \cosh \rho )</td>
<td>( \text{csch} \rho )</td>
<td>( \text{coth} \rho )</td>
<td>( e^{\rho} )</td>
</tr>
<tr>
<td>value for ( \beta = 3/5 )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
</tr>
</tbody>
</table>
The image contains a diagram and text related to physics, specifically discussing frequency and period in a coordinate system. The text and diagram are as follows:

**Frequency**
- \( v' \) (units of 600 THz)
- \( v'_{\text{group}} = v_A \sinh \rho = \frac{3}{4} \)
- \( v'_{\text{group}} = 0.75 \)

**Group frequency**
- \( v'_{\text{group}} = v_A \sinh \rho = \frac{3}{4} \)
- \( v'_{\text{group}} = 0.75 \)

**Group period**
- \( \tau'_{\text{group}} = \tau_A \csc h \rho = \frac{4}{3} \)
- \( \tau'_{\text{group}} = 1.33 \)

**Wavevector**
- \( c' k'_{\text{group}} = v_A \cosh \rho = \frac{5}{4} \)
- \( c' k'_{\text{group}} = 0.625 \)

**Phase**
- \( b_{\text{Doppler RED}}^{\text{group}} \)
- \( b_{\text{Doppler BLUE}}^{\text{group}} \)

**Group**
- \( v_{\text{group}} = \frac{1}{3} \)
- \( \tau_{\text{group}} = 3 \)
- \( v_{\text{group}} = 0.33 \)

**Space**
- \( x' \) (units of \( \lambda_A = 1/2 \mu m \))

**Time**
- \( c t' \) (units of \( \lambda_A = 1/2 \mu m \))
- \( t' = \frac{1}{v} \)
- \( t' = 2 \)

The diagram illustrates the transformation of frequency and group frequency, showing the relationship between \( v'_{\text{group}} \), \( \tau'_{\text{group}} \), and \( c' k'_{\text{group}} \) in a coordinate system.

The table below contains the phase, group, rapidity, and value for \( \beta = 3/5 \):

<table>
<thead>
<tr>
<th>phase</th>
<th>( b_{\text{Doppler RED}}^{\text{group}} )</th>
<th>( c_{\text{group}} )</th>
<th>( \kappa_{\text{phase}} )</th>
<th>( \tau_{\text{phase}} )</th>
<th>( v_{\text{phase}} )</th>
<th>( \lambda_{\text{phase}} )</th>
<th>( V_{\text{phase}} )</th>
<th>( b_{\text{Doppler BLUE}}^{\text{group}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>rapidity</td>
<td>( e^{-\rho} )</td>
<td>( \sinh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \sinh \rho )</td>
<td>( \sinh \rho )</td>
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<tr>
<td>value for ( \beta = 3/5 )</td>
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<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
</tr>
</tbody>
</table>
Group frequency

\[ \nu'_\text{group} = \nu_A \sinh \rho = \frac{3}{4} \]

Group period

\[ \tau'_\text{group} = \tau_A \text{csch} \rho = \frac{4}{3} \]

\[ \nu' = \nu_A \left( \frac{c}{\kappa_{\text{group}}} \right) = \nu_A \left( \frac{\cosh \rho}{\sinh \rho} \right) \]

\[ \tau = \frac{1}{\nu'} \]

\[ \nu'_\text{group} = 0.75 \]

\[ \tau'_\text{group} = 1.33 \]

Frequency

\[ \nu' \]

(units of \( \nu_A = 600 \text{THz} \))

Wavevector \( c \kappa' \)

(units of \( c \kappa_A = 2 \cdot 10^6 / \text{m} \))

Time \( c t' \)

(units of \( \lambda_A = 1/2 \mu \text{m} \))

Space \( x' \)

(units of \( \lambda_A = 1/2 \mu \text{m} \))
The image contains a diagram illustrating the relationship between group wavenumber and group wavelength. It also includes expressions for the group wavenumber and group wavelength in terms of frequency and group velocity. The diagram shows how frequency and wavevector change with group velocity and includes tables for the Doppler effect and phase shifts. The text explains that the group wavenumber and wavelength are related to the frequency through specific equations, and the diagram visually represents these relationships with arrows and labels.
Group wavenumber
\( \kappa'_{\text{group}} = \kappa_A \cosh \rho = \frac{5}{4} \)
\( = 1.25 \)

\[ G' = \begin{pmatrix} c \kappa'_{\text{group}} \\ \nu'_{\text{group}} \end{pmatrix} = V_A \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = V_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix} \]

Group frequency
\( \nu'_{\text{group}} = \nu_A \sinh \rho = \frac{3}{4} \)
\( = 0.75 \)

Group period
\( \tau'_{\text{group}} = \tau_A \cosh \rho = \frac{4}{3} \)
\( = 1.33 \)

Frequency
\( \nu' \)
(units of
\( \nu_A = 600 \text{THz} )

Wavevector \( c \kappa' \)
(units of \( c \kappa_A = 2 \cdot 10^6 \text{m} \))

Table:

<table>
<thead>
<tr>
<th>phase</th>
<th>( b'_{\text{Doppler RED}} )</th>
<th>( \frac{c}{V_{\text{phase}}} )</th>
<th>( \frac{\kappa_{\text{phase}}}{\kappa_A} )</th>
<th>( \frac{\tau_{\text{phase}}}{\tau_A} )</th>
<th>( \frac{\nu_{\text{phase}}}{\nu_A} )</th>
<th>( \frac{\lambda_{\text{phase}}}{\lambda_A} )</th>
<th>( \frac{V_{\text{phase}}}{c} )</th>
<th>( b'_{\text{Doppler BLUE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>( \frac{1}{c} )</td>
<td>( \frac{\nu_A}{c} )</td>
<td>( \frac{\nu_{\text{group}}}{\nu_A} )</td>
<td>( \frac{\kappa_{\text{group}}}{\kappa_A} )</td>
<td>( \frac{\tau_{\text{group}}}{\tau_A} )</td>
<td>( \frac{\lambda_{\text{group}}}{\lambda_A} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| rapidity \( \rho \) | \( e^{-\rho} \) | \( \tanh \rho \) | \( \sinh \rho \) | \( \sech \rho \) | \( \cosh \rho \) | \( \csch \rho \) | \( \coth \rho \) | \( e^{\rho} \)
| value for \( \beta = 3/5 \) | \( \frac{1}{2} = 0.5 \) | \( \frac{3}{5} = 0.6 \) | \( \frac{3}{4} = 0.75 \) | \( \frac{4}{5} = 0.80 \) | \( \frac{5}{4} = 1.25 \) | \( \frac{4}{3} = 1.33 \) | \( \frac{5}{3} = 1.67 \) | \( \frac{2}{1} = 2.0 \)
Lorentz transformations...

Write $G'$ and $P'$ in terms of $G$ and $P$ using $\cosh \rho$ and $\sinh \rho$.

$$G' = \begin{pmatrix} c \kappa'_{\text{group}} \\ \nu'_{\text{group}} \end{pmatrix} = \nu_{A} \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = \nu_{A} \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$P' = G \cosh \rho + P \sinh \rho$$

$$P' = \begin{pmatrix} c \kappa'_{\text{phase}} \\ \nu'_{\text{phase}} \end{pmatrix} = \nu_{A} \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \nu_{A} \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$P' = G \sinh \rho + P \cosh \rho$$

Lorentz transform matrix

\[
\begin{pmatrix}
\cosh \rho & \sinh \rho \\
\sinh \rho & \cosh \rho
\end{pmatrix}
\]
**Two Famous-Name Coefficients**

This number is called an: **Einstein time-dilation**
(dilated by 25% here)

This number is called a: **Lorentz length-contraction**
(contracted by 20% here)

### Old-Fashioned Notation

<table>
<thead>
<tr>
<th>Group</th>
<th>( \beta = \frac{u}{c} )</th>
<th>( 1 - \beta )</th>
<th>( \beta )</th>
<th>( 1 - \beta^2 )</th>
<th>( \sqrt{1 - \beta^2} )</th>
<th>( \beta^2 - 1 )</th>
<th>( \sqrt{\beta^2 - 1} )</th>
<th>( 1 + \beta )</th>
<th>( \sqrt{1 + \beta^2} )</th>
<th>( \beta \sqrt{1 + \beta^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value for ( \beta = \frac{3}{5} )</td>
<td>( \frac{1}{2} = 0.5 )</td>
<td>( \frac{3}{5} = 0.6 )</td>
<td>( \frac{3}{4} = 0.75 )</td>
<td>( \frac{4}{5} = 0.80 )</td>
<td>( \frac{5}{4} = 1.25 )</td>
<td>( \frac{4}{3} = 1.33 )</td>
<td>( \frac{5}{3} = 1.67 )</td>
<td>( \frac{2}{1} = 2.0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Two Famous-Name Coefficients

This number is called an: **Einstein time-dilation** (dilated by 67% here)

This number is called a: **Lorentz length-contraction** (contracted by 40% here)

**Old-Fashioned Notation**

<table>
<thead>
<tr>
<th>phase</th>
<th>$b_{\text{Doppler RED}}$</th>
<th>$c$</th>
<th>$\kappa_{\text{phase}}$</th>
<th>$\tau_{\text{phase}}$</th>
<th>$v_{\text{phase}}$</th>
<th>$\lambda_{\text{phase}}$</th>
<th>$V_{\text{phase}}$</th>
<th>$b_{\text{Doppler BLUE}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>$\frac{1}{b_{\text{Doppler BLUE}}}$</td>
<td>$V_{\text{group}}/c$</td>
<td>$v_{\text{group}}/\lambda_A$</td>
<td>$\lambda_{\text{group}}/\lambda_A$</td>
<td>$\tau_{\text{group}}/\tau_A$</td>
<td>$c/V_{\text{group}}$</td>
<td>$1/b_{\text{Doppler RED}}$</td>
<td></td>
</tr>
<tr>
<td>rapidity $\rho$</td>
<td>$e^{-\rho}$</td>
<td>$\tanh \rho$</td>
<td>$\sinh \rho$</td>
<td>$\sech \rho$</td>
<td>$\cosh \rho$</td>
<td>$\csch \rho$</td>
<td>$\coth \rho$</td>
<td>$e^{+\rho}$</td>
</tr>
</tbody>
</table>

$\beta \equiv \frac{u}{c} \equiv \sqrt{\frac{1-\beta}{1+\beta}}$  
$\frac{1}{3} = 0.33$  
$\frac{4}{5} = 0.8$  
$\frac{4}{3} = 1.33$  
$\frac{3}{5} = 0.60$  
$\frac{5}{3} = 1.67$  
$\frac{3}{4} = 0.75$  
$\frac{5}{4} = 1.25$  
$\frac{3}{1} = 3.0$  

$\lambda_{\text{group}} = 0.6$  
$\nu_{\text{phase}} = 1.67$  
$\lambda_{A} = \frac{1}{2} \mu m$  
$\lambda'_{\text{group}} = 0.6$
A “Lover’s Quarrel” about a 20% Lorentz contraction

\[ \lambda'_{\text{group}} = 0.8 \lambda_A \]

(You’re short! No, YOU’RE short!!, etc.)

…(The worst kind of quarrel is when both are right and wrong)
Another “Lover’s Quarrel”
...easily resolved as simple 1CW-Doppler...)

(b) Paradox-2

Alice: “No Bob, you’re the one with short laser-λ!"

Carla: “I’m outa here. They have really lost it!”

Bob: “Alice! Your laser-λ is 50% short!”

(Doppler blue-shifts 0.5μm to 0.25μm for Alice)
A “Lover’s Quarrel” about a 20% Lorentz contraction

\( \lambda'_{\text{group}} = 0.8 \lambda_A \)

(You’re short! No, **YOU’RE** short!!, etc.)

...(The worst kind of quarrel is when **both** are right and wrong)

So we learn to accept that a group-wave shortens by 20% at this enormous speed of \( \frac{3}{5} c \).

Q: But, does the steel laser cavity holding the wave also shorten by 20%??

A: ...
A “Lover’s Quarrel” about a 20% Lorentz contraction $\lambda'_{\text{group}} = 0.8 \lambda_A$

(You’re short! No, YOU’RE short!!, etc.)

…(The worst kind of quarrel is when both are right and wrong)

So we learn to accept that a group-wave shortens by 20% at this enormous speed of $\frac{3}{5}c$.

Q: But, does the steel laser cavity holding the wave also shorten by 20%??

A: Yes, or else laser does not resonate! Steel is made of waves, too. Contraction is what waves do.
Let’s do the Alice, Bob, Carla problem backwards...

Suppose Bob sees beam of frequency \( \nu_L \) coming from the \( \text{LEFT} \) and opposing beam of frequency \( \nu_R \) coming from the \( \text{RIGHT} \).

**Question 1:** To what velocity \( \nu_E \) must Bob accelerate to see beams of \( \text{EQUAL} \) frequency \( \nu_E \)?

**Question 2:** What is frequency \( \nu_E \)?

---

Alice: Hey Bob, speed up and join us!

Bob: I'm seeing \( \nu_L = 1200 \) from you and \( \nu_R = 300 \) from Carla!
Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!
Let’s do the Alice Bob Carla problem backwards...

Suppose Bob sees beam of frequency $U_L$ coming from the LEFT and opposing beam of frequency $U_R$ coming from the RIGHT.

**Question 1:** To what velocity $U_E$ must Bob accelerate to see beams of EQUAL frequency $U_E$?

**Question 2:** What is frequency $U_E$?

Question 1. has a Jeopardy-style answer-by-question: *What is beam group velocity?*

$$u_E = V_{\text{group}} = \frac{v_{\text{group}}}{k_{\text{group}}} = \frac{(v_L - v_R)/2}{(\kappa_L - \kappa_R)/2}$$

Alice: Hey Bob, speed up and join us!

Bob: I'm seeing $v_L=1200$ from you and $v_R=300$ from Carla!

Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!
Let’s do the Alice Bob Carla problem backwards...

Suppose Bob sees beam of frequency \( \nu_L \) coming from the left and opposing beam of frequency \( \nu_R \) coming from the right.

**Question 1:** To what velocity \( \nu_E \) must Bob accelerate to see beams of **EQUAL** frequency \( \nu_E \)?

**Question 2:** What is frequency \( \nu_E \)?

Question 1. has a Jeopardy-style answer-by-question: *What is beam group velocity?*

\[
\nu_E = V_{group} = \frac{\nu_{group}}{k_{group}} = \frac{\left(\nu_L - \nu_R\right)/2}{(\kappa_L - \kappa_R)/2} = c \frac{\left(\nu_L - \nu_R\right)/2}{\left(\nu_L + \nu_R\right)/2}
\]

where:

\[
\nu_L = + c \kappa_L \quad \text{and} \quad \nu_R = - c \kappa_R
\]

Alice: Hey Bob, speed up and join us!

Bob: I’m seeing \( \nu_L = 1200 \) from you and \( \nu_R = 300 \) from Carla! Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!
Let’s do the Alice, Bob, Carla problem backwards...

Suppose Bob sees beam of frequency \( u_L \) coming from the \textit{LEFT} and opposing beam of frequency \( u_R \) coming from the \textit{RIGHT}.

**Question 1:** To what velocity \( u_E \) must Bob accelerate to see beams of \textit{EQUAL} frequency \( u_E \)?

**Question 2:** What is frequency \( u_E \)?

Question 1 has a Jeopardy-style answer-by-question: \textit{What is beam group velocity?}

\[
\begin{align*}
    u_E &= V_{\text{group}} = \frac{v_{\text{group}}}{\kappa_{\text{group}}} = \frac{(\nu_L - \nu_R)/2}{(\kappa_L - \kappa_R)/2} = \frac{c(\nu_L - \nu_R)/2}{(\nu_L + \nu_R)/2} \\
    \text{where: } \quad \nu_L &= +c\kappa_L \\
    \nu_R &= -c\kappa_R \\
\end{align*}
\]

\[
\begin{align*}
    \frac{1200 - 300}{1200 + 300}c &= \frac{900}{1500}c = \frac{3}{5}c
\end{align*}
\]

Alice: Hey Bob, speed up and join us!

Bob: I’m seeing \( \nu_L=1200 \) from you and \( \nu_R=300 \) from Carla! Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!

Bob: OK. Now I know they’re both going \( \frac{3}{5}c \) relative to me.
Let’s do the Alice, Bob, Carla problem backwards...

Suppose Bob sees beam of frequency $\nu_L$ coming from the LEFT and opposing beam of frequency $\nu_R$ coming from the RIGHT.

**Question 1:** To what velocity $u_E$ must Bob accelerate to see beams of EQUAL frequency $\nu_E$?

**Question 2:** What is frequency $\nu_E$?

Question 1. has a Jeopardy-style answer-by-question: *What is beam group velocity?*

$$u_E = v_{\text{group}} = \frac{\nu_{\text{group}}}{\kappa_{\text{group}}} = \frac{(\nu_L - \nu_R)/2}{(\kappa_L - \kappa_R)/2} = c \frac{(\nu_L - \nu_R)/2}{(\nu_L + \nu_R)/2}$$

where: $\nu_L = +c\kappa_L$ and $\nu_R = -c\kappa_R$

Question 2. ...similarly: *What $\nu_E$ is blue-shift $b\nu_R$ of $\nu_R$ AND red-shift $r\nu_L = \nu_L/b$ of $\nu_L$?*

Question 1: $u_E = 900$ $c = \frac{3}{5}c$

Question 2: $\nu_E = 300$ $\nu_L = 1200$ $\nu_R = 300$ $c = \frac{3}{5}c$

Alice: Hey Bob, speed up and join us!

Bob: I’m seeing $\nu_L = 1200$ from you and $\nu_R = 300$ from Carla! Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!

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**Question 2:** What is frequency \( u_E \)?

**Question 1.** has a Jeopardy-style answer-by-question: *What is beam group velocity?*

\[
u_E = V_{\text{group}} = \frac{\nu_{\text{group}}}{\kappa_{\text{group}}} = \frac{(\nu_L - \nu_R)/2}{(\kappa_L - \kappa_R)/2} = c \frac{(\nu_L - \nu_R)/2}{(\nu_L + \nu_R)/2}
\]

where:

\[
u_L = +c\kappa_L \quad \text{and} \quad \nu_R = -c\kappa_R
\]

**Question 2.** ...similarly: *What \( u_E \) is blue-shift \( b\nu_R \) of \( \nu_R \) \( \text{AND} \) red-shift \( r\nu_L = \nu_L/b \) of \( \nu_L \)?

\[
u_E = b\nu_R = r\nu_L = \frac{\nu_L}{b} \quad \text{implies:} \quad b^2 = \frac{\nu_L}{\nu_R}
\]

or: \( b = \sqrt{\frac{\nu_L}{\nu_R}} \)

Alice: Hey Bob, speed up and join us!

Bob: I’m seeing \( \nu_L = 1200 \) from you and \( \nu_R = 300 \) from Carla!

Give me frequencies and speeds for you and for Carla.

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Bob: OK. Now I know they’re both going \( \frac{3}{5}c \) relative to me.
Let's do the Alice-Bob-Carla problem backwards...

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**Question 2:** What is frequency $\nu_E$?

Question 1. has a Jeopardy-style answer-by-question: \textit{What is beam group velocity?}

$$u_E = \nu_{\text{group}} = \frac{\nu_{\text{group}}}{\kappa_{\text{group}}} = \frac{(\nu_L - \nu_R)/2}{(\kappa_L - \kappa_R)/2} = c \frac{(\nu_L - \nu_R)/2}{(\nu_L + \nu_R)/2} \quad \text{where:} \quad \nu_L = +c\kappa_L \quad \text{and} \quad \nu_R = -c\kappa_R$$

Question 2. ...similarly: \textit{What $\nu_E$ is blue-shift $b\nu_R$ of $\nu_R$ AND red-shift $r\nu_L$?}

$$\nu_E = b\nu_R = r\nu_L / b \quad \text{implies:} \quad b^2 = \frac{\nu_L}{\nu_R} \quad \text{or:} \quad b = \sqrt{\frac{\nu_L}{\nu_R}} \quad \text{so:} \quad \nu_E = b\nu_R = \sqrt{\frac{\nu_L}{\nu_R}} \nu_R = \sqrt{\nu_L} \nu_R$$

Alice: Hey Bob, speed up and join us!

Bob: I'm seeing $\nu_L = 1200$ from you and $\nu_R = 300$ from Carla! Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!

Bob: OK. Now I know they're both going $\frac{3}{5}c$ relative to me.
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Suppose Bob sees beam of frequency $\nu_L$ coming from the LEFT and opposing beam of frequency $\nu_R$ coming from the RIGHT.

**Question 1:** To what velocity $\nu_E$ must Bob accelerate to see beams of EQUAL frequency $\nu_E$?

**Question 2:** What is frequency $\nu_E$?

**Question 1.** has a Jeopardy-style answer-by-question: What is beam group velocity?

\[
\nu_E = V_{\text{group}} = \frac{\nu_{\text{group}}}{\kappa_{\text{group}}} = \frac{(\nu_L - \nu_R)/2}{(\kappa_L - k_R)/2} = c \frac{(\nu_L - \nu_R)/2}{(\nu_L + \nu_R)/2}
\]

\[
\nu_L = +c\kappa_L \quad \text{and} \quad \nu_R = -c\kappa_R
\]

**Question 2.** ...similarly: What $\nu_E$ is blue-shift $b\nu_R$ of $\nu_R$ AND red-shift $r\nu_L = \nu_L/b$ of $\nu_L$?

\[
\nu_E = b\nu_R = r\nu_L = \frac{\nu_L}{b} \quad \text{implies} \quad b^2 = \frac{\nu_L}{\nu_R} \quad \text{or} \quad b = \sqrt{\frac{\nu_L}{\nu_R}} \quad \text{so} \quad \nu_E = b\nu_R = \sqrt{\frac{\nu_L}{\nu_R}}\nu_R = \sqrt{\nu_L\nu_R}
\]

Alice, Bob, and Carla: Al-RIGHT! Bob’s home. Let’s Boogie!

**Sunday, November 2, 2014**
Thales Mean Geometry (600BCE)

helps “Relawavity”

**Frequency unit:**
300THZ

**Geometric Mean**
\[ B = \sqrt{(4 \cdot 1)} = 2 \]

**Arithmetic Mean**
\[ B_{\cosh \rho} = \frac{(1+4)}{2} = \frac{5}{2} \]

**Difference Mean**
\[ B_{\sinh \rho} = \frac{(4-1)}{2} = \frac{3}{2} \]

\[ Br = e^{-\rho} \quad \text{Red shift} = 1 \]

\[ Bb = e^{+\rho} \quad \text{Blue shift} = 4 \]
Thales Mean Geometry (600BCE) helps “Relativity.” Thales showed a circle diameter subtends a right angle with any circle point $P$. This leads to a convenient construction of geometric means and relativistic hyperbolas.
Thales Mean Geometry (600BCE) helps “Relawavity” Thales showed a circle diameter subtends a right angle with any circle point $P$. This leads to a convenient construction of geometric means and relativistic hyperbolas.

Frequency unit: 300THZ

\[ r \cdot b = 2 \]

This leads to a convenient construction of geometric means and relativistic hyperbolas.
Thales Mean Geometry (600BCE) helps “Relawavity”
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


**Observer fixed below star sees it directly overhead.**

**Observer going $u$ sees star at angle $\sigma$ in $u$ direction.**

\[
c \tanh \rho = u = c \sin \sigma
\]

\[
c \sqrt{1 - u^2/c^2} = c / \cosh \rho = c \text{ sech} \rho = c \cos \sigma
\]

\[
|c'| = |c| = c
\]
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(Doppler Shift)$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


**Proper time** $c\tau$ **vs. coordinate space** $x$ - (L. C. Epstein’s “Cosmic Speedometer”)

Particles $P$ and $P'$ have speed $u$ in $(x',ct')$ and speed $c$ in $(x, c\tau)$

**Proper time** $c\tau = \sqrt{(ct')^2 - (x')^2}$

**Coordinate**

$x' = (u/c)ct' = ut'$

**Einstein time dilation:**

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

**Lorentz length contraction:**

$$L' = L \sech\rho = L\cos\sigma = L \cdot \sqrt{1-u^2/c^2}$$

**Proper Time asimultaneity:**

$$c \Delta\tau = L' \sinh\rho = L \cos\sigma \sinh\rho$$

$$= L \cos\sigma \tan\sigma$$

$$= L \sin\sigma = L /\sqrt{c^2/u^2-1} \sim L u/c$$
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

... to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$


---

**Proper time $c\tau$ vs. coordinate space $x$ -** (L. C. Epstein’s “Cosmic Speedometer”)

**Particles $P$ and $P'$ have speed $u$ in $(x', ct')$ and speed $c$ in $(x, c\tau)$**

**Proper time** $c\tau$

\[ c\tau = \sqrt{(ct')^2 - (x')^2} \]

**Coordinate**

\[ x' = (u/c)ct' = u' \]

**Einstein time dilation:**

\[ ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau / \sqrt{1 - u^2/c^2} \]

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\[ = L \cos\sigma \tan\sigma \]

\[ = L \sin\sigma = L / \sqrt{c^2/u^2 - 1} \sim L u/c \]

---

Epstein’s trick is to turn a hyperbolic form into a circular form:

\[ \sqrt{(c\tau)^2 + (x')^2} = (ct') \]
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$

(a) Circular Functions

$$\sin(\sigma) = 0.6000$$
$$\tan(\sigma) = 0.7500$$
$$\sec(\sigma) = 1.2500$$
Comparing **Longitudinal** relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$

to a **Transverse** relativity parameter: Stellar aberration angle $\sigma$

(b) Circular Functions
- $\sin(\sigma) = 0.6000$
- $\tan(\sigma) = 0.7500$
- $\sec(\sigma) = 1.2500$

Hyperbolic Functions
- $\tanh(\rho) = 0.6000$
- $\sinh(\rho) = 0.7500$
- $\cosh(\rho) = 1.2500$
Each of 6 trig (or trigh) functions serves at least once as a hyperbolic $x, y,$ and $z$ coordinate, $x, y,$ and $z$ tangent intercept, and tangent slope, and a circular $x, y,$ and $z$ coordinate, $x, y,$ and $z$ tangent intercept, and tangent slope.

A bewildering array of relativity parameters…

(Need ways to sort them out!)
Summary of optical wave parameters for relativity and QM

...and their geometry

\[ v' = \frac{\omega'}{2\pi} \]

(Units of 300THz)

**An aid to pattern recognition:**

**Occam's Sword**

(u/c = 3/5)
\[ v' = \omega/2\pi \]

axis

(Units of 300THz)

\[ p \cdot \text{circle} \]

\[ b \cdot \text{circle} \]

\[ cs \]

\[ \text{sech} \]

\[ \cosh \]

\[ \text{csch} \]

\[ \text{tanh} \]

\[ \text{coth} \]

\[ \text{slope} \]

\[ e^\rho \]

\[ \text{tanh} \rho \]

\[ \sinh \rho \]

\[ \text{sech} \rho \]

\[ \cosh \rho \]

\[ \text{csch} \rho \]

\[ \text{coth} \rho \]

\[ e^{-\rho} \]

\[ 1/e^\rho \]

\[ \sin \sigma \]

\[ \tan \sigma \]

\[ \cos \sigma \]

\[ \sec \sigma \]

\[ \cot \sigma \]

\[ \csc \sigma \]

\[ 1/e^{-\rho} \]

\[ \beta = u/c \]

\[ \frac{\beta}{1 + \beta} \]

\[ \frac{\sqrt{1 - \beta^2}}{1} \]

\[ \frac{1}{\sqrt{1 - \beta^2}} \]

\[ \frac{1}{\beta} \]

\[ \frac{1}{1 - \beta} \]

\[ 1 + \beta \]

\[ 1 - \beta \]

\[ \text{value for } \beta = 3/5 \]

\[ \frac{1}{2} = 0.5 \]

\[ \frac{3}{5} = 0.6 \]

\[ \frac{3}{4} = 0.75 \]

\[ \frac{4}{5} = 0.80 \]

\[ \frac{5}{4} = 1.25 \]

\[ \frac{4}{3} = 1.33 \]

\[ \frac{5}{3} = 1.67 \]

\[ \frac{2}{1} = 2.0 \]

Table of 12 wave parameters (includes inverses) for relativity

...and values for \( u/c = 3/5 \)

An aid to pattern recognition:

Occam's Sword

(\( u/c = 3/5 \))

\[\]
Using (some) wave parameters for relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \]
\[ \sinh \rho \approx \rho \]

At low speeds:

\[ B = v_A \]
\[ B = v_A = c \kappa_A \]
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \]

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\[ \frac{u}{c} = \tanh \rho \approx \rho \quad (\text{for } u \ll c) \]

At low speeds:

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2} \]

\[ \sinh \rho \approx \rho \approx \frac{u}{c} \]

\[ B = \nu_A \]

\[ B = \nu_A = c \kappa_A \]
Using (some) wave parameters for relativistic quantum theory

\[
\nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c)
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c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c)\]

\[
\frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c)\]

\[
\nu_{\text{phase}} \approx B + \frac{1}{2} B \frac{u^2}{c^2} \quad \text{for } (u \ll c)
\]

At low speeds:

\[
c \approx B \cosh \rho \approx B \left(1 + \frac{1}{2} \rho^2 \right) \\
\sinh \rho \approx \rho \quad \text{for } (u \ll c)
\]

Using (some) wave parameters for relativistic quantum theory

\[
B = \nu_A
\]

\[
B = \nu_A = c \kappa_A
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Using (some) wave parameters for relativistic quantum theory

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B = \nu_A
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\nu_{\text{phase}} \approx B + \frac{1}{2} B \frac{u^2}{c^2} \quad \text{for } (u \ll c)
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c \approx B \cosh \rho \approx B \left(1 + \frac{1}{2} \rho^2 \right) \\
\sinh \rho \approx \rho \quad \text{for } (u \ll c)
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Using (some) wave parameters for relativistic quantum theory

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Using (some) wave parameters for relativistic quantum theory

\[
\nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \quad \text{cosh}\rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{1}{2} \frac{u^2}{c^2}
\]

\[
\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \quad \sinh \rho \approx \rho \approx \frac{u}{c}
\]

\[
\frac{u}{c} = \tanh \rho \approx \rho \quad \text{At low speeds:}
\]

\[
\nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u
\]

<table>
<thead>
<tr>
<th>time</th>
<th>(b_{\text{Doppler}}^{\text{RED}})</th>
<th>(V_{\text{group}}/c)</th>
<th>(V_{\text{group}}/\nu_A)</th>
<th>(\tau_{\text{phase}}/\tau_A)</th>
<th>(V_{\text{phase}}/\nu_A)</th>
<th>(\tau_{\text{group}}/\tau_A)</th>
<th>(V_{\text{phase}}/c)</th>
<th>(b_{\text{Doppler}}^{\text{BLUE}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>space</td>
<td>(1)</td>
<td>(c)</td>
<td>(\kappa_{\text{phase}}/\kappa_A)</td>
<td>(\lambda_{\text{group}}/\lambda_A)</td>
<td>(\lambda_{\text{phase}}/\lambda_A)</td>
<td>(1)</td>
<td>(\lambda_{\text{group}}/V_{\text{phase}})</td>
<td>(1)</td>
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<tr>
<td>rapidity (\rho)</td>
<td>(e^{-\rho})</td>
<td>(\tanh \rho)</td>
<td>(\sinh \rho)</td>
<td>(\text{sech } \rho)</td>
<td>(\cosh \rho)</td>
<td>(\text{csch } \rho)</td>
<td>(\text{coth } \rho)</td>
<td>(e^{+\rho})</td>
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<tr>
<td>stellar angle (\sigma)</td>
<td>(1/e^{+\rho})</td>
<td>(\sin \sigma)</td>
<td>(\tan \sigma)</td>
<td>(\cos \sigma)</td>
<td>(\sec \sigma)</td>
<td>(\cot \sigma)</td>
<td>(\csc \sigma)</td>
<td>(1/e^{-\rho})</td>
</tr>
<tr>
<td>(\beta \equiv \frac{u}{c})</td>
<td>(\begin{array}{c} \sqrt{1-\beta^2} \ \sqrt{1+\beta^2} \end{array})</td>
<td>(\frac{\beta}{1})</td>
<td>(\frac{1}{\sqrt{1-\beta^2}})</td>
<td>(\frac{1}{\sqrt{1+\beta^2}})</td>
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Using (some) wave parameters for relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ \rho \equiv \frac{u}{c} \]

\[ \beta \equiv \frac{u}{c} \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)
Using (some) wave parameters for relativistic quantum theory:

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
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\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( v_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \)

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \) \( \kappa_{\text{phase}} \)

\[ B = \nu_A \]
\[ B = \nu_A = c \kappa_A \]

Resembles: \( Mu \)
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]

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At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ M = \frac{hB}{c^2} \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \) \quad \text{Resembles: } Mu \]

\( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \) resemble formulæ for Newton's kinetic energy \( \frac{1}{2} Mu^2 \) and momentum \( Mu \).

So attach scale factor \( h \) (or \( hN \)) to match units.
Using (some) wave parameters for relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
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At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} B \frac{u^2}{c^2} \]
\[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( v_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

Resembles: \( Mu \)

Respect to scale factor \( h \) (or \( hN \)) to match units.

\[ \nu_{\text{phase}} \approx h B + \frac{1}{2} \frac{hB}{c^2} u^2 \]
\[ h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

So attach scale factor \( h \) (or \( hN \)) to match units.
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c) \]

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\[ \frac{u}{c} = \tanh \rho \approx \rho \quad (\text{for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]

\[ c\kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \]

\[ h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

Resembles: \( \text{const.} + \frac{1}{2} Mu^2 \)

\[ \text{Resembles: } M \]

\[ \text{Rescale } \nu_{\text{phase}} \text{ by } h \text{ so: } M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

(\text{The famous } \text{Mc}^2 \text{ shows up!})

\[ \text{So attach scale factor } h \text{ (or } hN) \text{ to match units.} \]
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

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\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \) \quad or: \( hB = Mc^2 \) \quad (The famous \( Mc^2 \) shows up!)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \iff \text{for } (u \ll c) \Rightarrow h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} Mu^2 \quad \iff \text{for } (u \ll c) \Rightarrow h\kappa_{\text{phase}} \approx Mu \]

---

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Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c\text{)} \]

\[ c\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c\text{)} \]

\[
\frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c\text{)}
\]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} M u^2 \quad \Leftrightarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx Mu \]

Lucky coincidences??

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<th>( \beta )</th>
<th>( \nu_{\text{Doppler \ RED}} )</th>
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<th>( \lambda_{\text{group}} )</th>
<th>( \kappa_{\text{group}} )</th>
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Sunday, November 2, 2014
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \] (for \( u \ll c \))
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \] (for \( u \ll c \))
\[ \frac{u}{c} = \tanh \rho \approx \rho \] (for \( u \ll c \))

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \] \leftarrow \text{for} \( (u \ll c) \) \Rightarrow \[ \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{h B}{c^2} \] or: \( h B = M c^2 \)

\[ h \nu_{\text{phase}} \approx h B + \frac{1}{2} \frac{h B}{c^2} u^2 \] \leftarrow \text{for} \( (u \ll c) \) \Rightarrow \[ h \kappa_{\text{phase}} \approx \frac{h B}{c^2} u \]

\[ h \nu_{\text{phase}} \approx M c^2 + \frac{1}{2} M u^2 \] \leftarrow \text{for} \( (u \ll c) \) \Rightarrow \[ h \kappa_{\text{phase}} \approx M u \]

**Lucky coincidences??**

\[ \text{...Try exact } \nu_{\text{phase}} \ldots \]

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<tr>
<th>group</th>
<th>( b_{\text{Doppler BLUE}} )</th>
<th>( V_{\text{group}} )</th>
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<th>( \lambda_{\text{group}} )</th>
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Using (some) wave parameters for relativistic quantum theory

\[ v_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ c \kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad \text{(for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ v_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( v_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]

\[ h \nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h \kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h \nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} M u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h \kappa_{\text{phase}} \approx Mu \]

So attach scale factor \( h \) (or \( hN \)) to match units.

---

Lucky coincidences??

...Try exact \( U_{\text{phase}} \) ...

\[ h \nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho \]
Using (some) wave parameters for relativistic quantum theory

\[
\nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \quad (\text{for } u \ll c)
\]

\[
\kappa_{\text{phase}} = B \sinh \rho \approx B \rho \quad (\text{for } u \ll c)
\]

\[
\frac{u}{c} = \tanh \rho \approx \rho
\quad (\text{for } u \ll c)
\]

At low speeds:

\[
\nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2
\]

\[
\kappa_{\text{phase}} \approx \frac{B}{c^2} u
\]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:

\[
M = \frac{hB}{c^2}
\]

or:

\[
hB = Mc^2
\]

\[
h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2
\]

\[
h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u
\]

So attach scale factor \( h \) (or \( hN \)) to match units.

\[
\nu_{\text{phase}} = hB \cosh \rho = Mc^2 \cosh \rho
\]

= Total Energy: \( E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \)

Einstein (1905)
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ cK_{\text{phase}} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \leftrightarrow \text{for } (u \ll c) \Rightarrow \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so: \( M = \frac{hB}{c^2} \) or: \( hB = M c^2 \)

\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \leftrightarrow \text{for } (u \ll c) \Rightarrow h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]

\[ h\nu_{\text{phase}} \approx M c^2 + \frac{1}{2} M u^2 \leftrightarrow \text{for } (u \ll c) \Rightarrow h\kappa_{\text{phase}} \approx M u \]

...Try exact \( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \)...

\[ h\nu_{\text{phase}} = hB \cosh \rho = M c^2 \cosh \rho \]

\[ = \text{Total Energy: } E = \frac{M c^2}{\sqrt{1-\frac{u^2}{c^2}}} \]

Einstein (1905)

\[ h\kappa_{\text{phase}} = hB \sinh \rho = M c^2 \sinh \rho \]

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</table>
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| stellar angle \( \sigma \) | \( \frac{1}{e^{+\rho}} \) | \( \sin \sigma \) | \( \tan \sigma \) | \( \cos \sigma \) | \( \sec \sigma \) | \( \cot \sigma \) | \( \csc \sigma \) | \( \frac{1}{e^{-\rho}} \)
| \( \beta = \frac{u}{c} \) | \( \frac{1}{\sqrt{1+\beta}} \) | \( \frac{\beta}{1} \) | \( \frac{1}{\sqrt{1-\beta^2}} \) | \( \frac{\sqrt{1-\beta^2}}{1} \) | \( \frac{1}{\beta} \) | \( \frac{1}{\sqrt{1-\beta}} \) | \( \frac{\sqrt{1+\beta}}{1} \) |
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Using (some) wave parameters for relativistic quantum theory

\[ \nu_{\text{phase}} = B \cosh \rho = B + \frac{1}{2} B \rho^2 \quad \text{(for } u \ll c) \]
\[ c\kappa_{\text{phase}} = B \sinh \rho = B \rho \quad \text{(for } u \ll c) \]
\[ \frac{u}{c} = \tanh \rho \approx \rho \quad \text{(for } u \ll c) \]

At low speeds:
\[ \nu_{\text{phase}} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \quad \leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{\text{phase}} \approx \frac{B}{c^2} u \]

Rescale \( \nu_{\text{phase}} \) by \( h \) so:
\[ M = \frac{hB}{c^2} \quad \text{or: } hB = Mc^2 \]
\[ h\nu_{\text{phase}} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \quad \leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx \frac{hB}{c^2} u \]
\[ h\nu_{\text{phase}} \approx Mc^2 + \frac{1}{2} \frac{Mc^2}{u^2} \quad \leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{\text{phase}} \approx Mu \]

...Try exact \( \nu_{\text{phase}} \) and \( \kappa_{\text{phase}} \)...
Using (some) wave parameters for relativistic quantum theory

\[ \nu_{phase} = B \cosh \rho \approx B + \frac{1}{2} B \rho^2 \text{ (for } u \ll c) \]

\[ cK_{phase} = B \sinh \rho \approx B \rho \text{ (for } u \ll c) \]

\[ \frac{u}{c} = \tanh \rho \approx \rho \text{ (for } u \ll c) \]

At low speeds:

\[ \nu_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \]

\[ \nu_{phase} \approx B + \frac{1}{2} \frac{B}{c^2} u^2 \text{ (for } u \ll c) \Rightarrow \]

\[ \kappa_{phase} \approx \frac{B}{c^2} u \]

\[ h\nu_{phase} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \]

\[ h\nu_{phase} \approx hB + \frac{1}{2} \frac{hB}{c^2} u^2 \text{ (for } u \ll c) \Rightarrow \]

\[ h\kappa_{phase} \approx \frac{hB}{c^2} u \]

Rescale \( \nu_{phase} \) by \( h \) so: \( M = \frac{hB}{c^2} \) or: \( hB = Mc^2 \)

So attach scale factor \( h \) (or \( hN \)) to match units.

Lucky coincidences??

\[ h\nu_{phase} = hB \cosh \rho = Mc^2 \cosh \rho \]

\[ h\kappa_{phase} = hB \sinh \rho = Mc^2 \sinh \rho \]

At low speeds:

\[ \cosh \rho \approx 1 + \frac{1}{2} \rho^2 \approx 1 + \frac{u^2}{c^2} \]

\[ \sinh \rho \approx \rho \approx \frac{u}{c} \]

\[ \nu_{phase} \text{ and } \kappa_{phase} \text{ resemble formulae for Newton's kinetic energy } \frac{1}{2} Mu^2 \text{ and momentum } M u. \]

\[ E = \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \]

\[ cP = \frac{Mu}{\sqrt{1-u^2/c^2}} \]

Momentum: \[ h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}} \]

\[ \begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{group} & b_{Doppler}^{\text{RED}} & \nu_{group} & \lambda_{group} & \kappa_{group} & \tau_{group} & \nu_{phase} & b_{Doppler}^{\text{BLUE}} \\
\hline
\text{phase} & \nu_{A} & \lambda_{A} & \kappa_{A} & \tau_{A} & \nu_{A} & \lambda_{A} \bigg/ \kappa_{A} \\
\hline
\text{rapidity } \rho & e^{-\rho} & \tanh \rho & \sinh \rho & \sech \rho & \cosh \rho & \csc \rho \bigg/ \cot \rho & e^{+\rho} \\
\hline
\text{stellar angle } \sigma & 1/e^{\rho} & \sin \sigma & \tan \sigma & \cos \sigma & \sec \sigma & \cot \sigma & 1/e^{-\rho} \\
\hline
\beta & \frac{u}{c} & \sqrt{\frac{1-\beta^2}{1+\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} & \frac{1}{1-\beta} & \frac{1}{\sqrt{1-\beta^2}} & \frac{1}{1-\beta} & \frac{1}{\sqrt{1-\beta^2}} \\
\hline
\text{value for } \beta=3/5 & 0.5 & 0.6 & 0.75 & 0.8 & 0.8 & 0.8 & 0.8 \\
\hline
\end{array} \]
Using (some) wave coordinates for relativistic quantum theory

\[ \text{Energy (E)} \]
\[ \text{Momentum} \]
\[ \text{cp} = \text{E}\sinh(\rho) \]

\[ \text{Rest Energy} \]
\[ B = \omega \]

\[ \text{Hamiltonian} \]
\[ H(p) = B\cosh(\rho) \]

\[ \text{Per-Space (cp)} \]

\[ \text{Mass (resting)} \]
\[ hB = h\nu_A = Mc^2 = hc\kappa_A \]

\[ \text{Energy} \]
\[ h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \]

\[ \text{(a) Exact Einstein-Planck Dispersion} \]

\[ \text{Energy} \]
\[ E = \hbar\omega \]

\[ \text{Momentum} \]
\[ \text{cp} = \hbar c k \]

\[ \text{Laser frame} \]

\[ \text{positive rest energy} \]
\[ Mc^2 \]

\[ \text{E}^2 - c^2 p^2 = (Mc^2)^2 \]

\[ \text{tachyon: imaginary} \]

\[ \text{photon: zero } \mu \]
\[ E = \pm c p \]

\[ \text{massive} \]
\[ E = \pm \sqrt{h\nu_{\text{phase}}} \]

\[ \text{zero energy states} \]

\[ \text{negative energy states} \]
Using (some) wave coordinates for relativistic quantum theory

\[ E = mc^2 \]

\[ E = h\omega \]

\[ E = \pm c \rho \]

\[ h\kappa_{\text{phase}} = cp = h\kappa_A \sinh \rho = h\nu_A \sinh \rho \]

\[ hB = h\nu_A = Mc^2 = hc\kappa_A \]

\[ h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \]

\[ \text{Energy} \]

\[ \text{Mass (resting)} \]

\[ \text{Momentum} \]

\[ \text{Hamiltonian} \]

\[ H(p) = B \cosh(\rho) \]

\[ \text{Per-Space } (cp) \]

\[ \text{Laser frame} \]

\[ \text{Momentum } \]

\[ cp = h\kappa c \]

\[ \text{Negative energy states} \]

\[ \text{Photon: imaginary} \]

\[ \text{Tachyon:} \]
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

Energy
\[ E = \hbar \omega \]

Momentum
\[ cp = \hbar c k \]

Mass (resting)
\[ hB = h\nu_A = Mc^2 = h\kappa_A \]

Energy
\[ h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \]

Momentum
\[ h\kappa_{\text{phase}} = cp = h\kappa_A \sinh \rho = h\nu_A \sinh \rho \]

Energy versus Momentum
\[ E^2 = \left( Mc^2 \right)^2 \cosh^2 \rho \]
\[ = \left( Mc^2 \right)^2 (1 + \sinh^2 \rho) = \left( Mc^2 \right)^2 + (cp)^2 \]
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

Energy $E = \hbar \omega$

Momentum $cp = \hbar \kappa$

Mass (resting) $hB = h\nu_A = Mc^2 = \hbar \kappa_A$

Energy $h\nu_{phase} = E = h\nu_A \cosh \rho$

Momentum $h\kappa_{phase} = cp = \hbar \kappa_A \sinh \rho = h\nu_A \sinh \rho$

Energy versus Momentum $E^2 = (Mc^2)^2 \cosh^2 \rho$

$= (Mc^2)^2 (1 + \sinh^2 \rho) = (Mc^2)^2 + (cp)^2 \Rightarrow E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$

The need for Negative Frequency arises!

Phase conjugate light!

Counter-clockwise phasors!

Mass (resting)

Energy

Momentum

Energy versus Momentum

The need for Negative Frequency arises!

Phase conjugate light!

Counter-clockwise phasors!
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

Energy

\[ E = h\omega \]

Momentum

\[ cp = \hbar c k \]

Mass (resting)

\[ hB = h\nu_A = Mc^2 = h\kappa_A \]

Energy

\[ h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \]

Momentum

\[ h\kappa_{\text{phase}} = cp = h\kappa_A \sinh \rho = h\nu_A \sinh \rho \]

Energy versus Momentum

\[ E^2 = (Mc^2)^2 \cosh^2 \rho \]

\[ = (Mc^2)^2 (1 + \sinh^2 \rho) = (Mc^2)^2 + (cp)^2 \quad \Rightarrow \quad E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M} \]
Using (some) wave coordinates for relativistic quantum theory

(a) Exact Einstein-Planck Dispersion

\[ E = \hbar \omega \]

positive rest energy \( Mc^2 \)
\[ E^2 - c^2 p^2 = (Mc^2)^2 \]

matter wave:

tachyon: imaginary \( \kappa \) 
\[ E = \pm c p \]

photon: zero \( \mu \) 
\[ B \]

Mass (resting)
\[ \hbar B = \hbar \omega_A = Mc^2 = \hbar c \kappa_A \]

Energy
\[ \hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho \]

Momentum
\[ \hbar c \kappa_{\text{phase}} = cp = \hbar c \kappa_A \sinh \rho = \hbar \nu_A \sinh \rho \]

Energy versus Momentum
\[ E^2 = \left( Mc^2 \right)^2 \cosh^2 \rho \]

\[ = \left( Mc^2 \right)^2 (1 + \sinh^2 \rho) = \left( Mc^2 \right)^2 + (cp)^2 \Rightarrow E = \pm \sqrt{\left( Mc^2 \right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M} \]
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ (Einstein's mass)

\[ hB = h\nu_A = Mc^2 = hck_A \]

Defines invariant hyperbola(s)

\[ E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \]

Given:

**Energy:** \[ E = Mc^2 \cosh \rho \]
\[ = h\nu_{\text{phase}} \]

**Momentum:** \[ cp = Mc^2 \sinh \rho \]
\[ = hck_{\text{phase}} \]

**Velocity:** \[ u = c \tanh \rho = \frac{dv}{dk} \]
Definition(s) of mass for relativity/quantum

Rest Mass \( M_{\text{rest}} \) (Einstein's mass)

\[
hB = h\nu_A = Mc^2 = h\kappa_A
\]

\[
\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} = \frac{h\kappa_{\text{phase}}}{c^2}
\]

Defines invariant hyperbola(s)

\[
E = \pm \sqrt{(Mc^2)^2 + (cp)^2}
\]

Given:

**Energy:** \( E = Mc^2 \cosh \rho \)

\[= h\nu_{\text{phase}}\]

**Momentum:** \( cp = Mc^2 \sinh \rho \)

\[= h\kappa_{\text{phase}}\]

**Velocity:** \( u = c \tanh \rho = \frac{dv}{d\kappa} \)
Definition(s) of mass for relativity/quantum

**Rest Mass** \( M_{\text{rest}} \) (Einstein’s mass)
- Defines invariant hyperbola(s)
- \( E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \)

\[
\begin{align*}
\frac{h\nu_{\text{phase}}}{c^2} &= M_{\text{rest}} = \frac{h c \kappa_{\text{phase}}}{c^2} \\
\text{Rest} &\quad \text{Mass}
\end{align*}
\]

**Momentum Mass** \( M_{\text{mom}} \) (Galileo’s mass)
- Defined by ratio \( p/u \) of relativistic momentum to group velocity.
- \( M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} \)

\[
\begin{align*}
\text{Given:} \quad &\text{Energy: } E = Mc^2 \cosh \rho \\
&= h\nu_{\text{phase}} \\
&= h c \kappa_{\text{phase}} \\
\text{momentum: } &cp = Mc^2 \sinh \rho \\
&= h c \kappa_{\text{phase}} \\
\text{velocity: } &u = c \tanh \rho = \frac{d\nu}{dk}
\end{align*}
\]
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass)

- Defines invariant hyperbola(s)
- $E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$

- $h\nu_{\text{phase}} = \frac{h c \kappa}{c^2} = M_{\text{rest}}$ (Rest Mass)

**Momentum Mass** $M_{\text{mom}}$ (Galileo’s mass)

- Defined by ratio $p/u$ of relativistic momentum to group velocity.

- $M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho}$

- $= M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}$ (Momentum Mass)

**Given:**

- **Energy:** $E = Mc^2 \cosh \rho$
  
  - $= h\nu_{\text{phase}}$

- **Momentum:** $cp = Mc^2 \sinh \rho$
  
  - $= h c \kappa$

- **Velocity:** $u = c \tanh \rho = \frac{d\nu}{dk}$
Definition(s) of mass for relativity/quantum mechanics

**Rest Mass** $M_{\text{rest}}$ (Einstein’s mass)

$$hB = h\nu_A = Mc^2 = h\kappa_A$$

$$\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} = \frac{h\kappa_{\text{phase}}}{c^2}$$

**Momentum Mass** $M_{\text{mom}}$ (Galileo’s mass)

Defined by ratio $p/u$ of relativistic momentum to group velocity.

$$M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c \sinh \rho}{c \tanh \rho}$$

Limited cases:

$$M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^{\rho/2}$$

$$M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}}$$

**Energy:**

$$E = Mc^2 \cosh \rho = h\nu_{\text{phase}}$$

**Momentum:**

$$cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}}$$

**Velocity:**

$$u = c \tanh \rho = \frac{d\nu}{d\kappa}$$
Definition(s) of mass for relativity/quantum

Rest Mass \( M_{\text{rest}} \) (Einstein's mass)

\[
hB = h\nu_A = Mc^2 = h\kappa_A
\]

Defines invariant hyperbola(s)

\[
E = \pm \sqrt{\left(Mc^2\right)^2 + (cp)^2}
\]

Momentum Mass \( M_{\text{mom}} \) (Galileo's mass)

Defined by ratio \( p/u \) of relativistic momentum to group velocity.

\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c \sinh \rho}{c \tanh \rho}
\]

\[
= M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}
\]

Effective Mass \( M_{\text{eff}} \) (Newton's mass)

Defined by ratio \( F/a = dp/du \) of relativistic force to acceleration.

Given:

\[
\text{Energy:} \quad E = Mc^2 \cosh \rho = h\nu_{\text{phase}}
\]

\[
\text{momentum:} \quad cp = Mc^2 \sinh \rho = h\kappa_{\text{phase}}
\]

\[
\text{velocity:} \quad u = c \tanh \rho = \frac{d\nu}{dk}
\]

Limiting cases:

\[
M_{\text{mom}} \underset{u \to c}{\longrightarrow} M_{\text{rest}}e^{\rho/2}
\]

\[
M_{\text{mom}} \underset{u \ll c}{\longrightarrow} M_{\text{rest}}
\]
Definition(s) of mass for relativity/quantum

Rest Mass $M_{\text{rest}}$ (Einstein’s mass)

\[ hB = h\nu_A = Mc^2 = h\kappa_A \]

\[ \frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} = \frac{h\kappa_{\text{phase}}}{c^2} \]

Momentum Mass $M_{\text{mom}}$ (Galileo’s mass)

Defined by ratio $p/u$ of relativistic momentum to group velocity.

\[ M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho} \]

\[ = M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}} \]

Energy: \[ E = \pm \sqrt{\left( Mc^2 \right)^2 + (cp)^2} \]

\[ = h\nu_{\text{phase}} \]

\[ \text{momentum:} \quad cp = Mc^2 \sinh \rho \]

\[ = h\kappa_{\text{phase}} \]

\[ \text{velocity:} \quad u = c \tanh \rho = \frac{du}{d\kappa} \]

Limiting cases:

\[ M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^\rho / 2 \]

\[ M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}} \]

Effective Mass $M_{\text{eff}}$ (Newton’s mass)

Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh \rho \, d\rho$ in momentum to change $du = c \, \text{sech}^2 \rho \, d\rho$ in velocity.
Definition(s) of mass for relativity/quantum

**Rest Mass** $M_{\text{rest}}$ *(Einstein's mass)*

- Defines invariant hyperbola(s)
  - $E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$

**Momentum Mass** $M_{\text{mom}}$ *(Galileo's mass)*

- Defined by ratio $p/u$ of relativistic momentum to group velocity.
  - $M_{\text{mom}} = \frac{p}{u} = \frac{M_{\text{rest}}}{c \tanh \rho} c \sinh \rho$
  - Limiting cases:
    - $M_{\text{mom}} \rightarrow c \rightarrow M_{\text{rest}} = \frac{e^\rho}{2}$
    - $M_{\text{mom}} \rightarrow u \ll c \rightarrow M_{\text{rest}}$

**Effective Mass** $M_{\text{eff}}$ *(Newton's mass)*

- Defined by ratio $F/a=dp/du$ of relativistic force to acceleration.
  - That is ratio of change $dp = Mc \cosh \rho \rho \rho$ in momentum to change $du = c \sech^2 \rho \rho \rho$ in velocity
  - $M_{\text{eff}} = \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho }{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho$
Definition(s) of mass for relativity/quantum

**Rest Mass** \( M_{\text{rest}} \) (Einstein's mass)

\[
hB = h\nu_A = Mc^2 = hck_A \]

Defines invariant hyperbola(s)

\[
E = \pm \sqrt{(Mc^2)^2 + (cp)^2} \]

**Momentum Mass** \( M_{\text{mom}} \) (Galileo's mass)

Defined by ratio \( \frac{p}{u} \) of relativistic momentum to group velocity.

\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c\sinh\rho}{c\tanh\rho} = M_{\text{rest}} \cosh\rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2/c^2}} \]

**Effective Mass** \( M_{\text{eff}} \) (Newton's mass)

Defined by ratio \( \frac{F}{a} = \frac{dp}{du} \) of relativistic force to acceleration.

That is ratio of change \( dp = Mc\cosh\rho \, d\rho \) in momentum to change \( du = c\sech^2\rho \, d\rho \) in velocity

\[
M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c\cosh\rho}{c\sech^2\rho} = M_{\text{rest}} \cosh^3\rho \]

**Given:**
- **Energy:** \( E = Mc^2 \cosh\rho = h\nu_{\text{phase}} \)
- **Momentum:** \( cp = Mc^2 \sinh\rho = hck_{\text{phase}} \)
- **Velocity:** \( u = c\tanh\rho = \frac{du}{dk} \)

**Limiting cases:**
- \( M_{\text{mom}} \to c \quad M_{\text{rest}} e^{\rho/2} \)
- \( M_{\text{mom}} \to 0 \quad M_{\text{rest}} \)
- \( M_{\text{eff}} \to c \quad M_{\text{rest}} e^{3\rho/2} \)
- \( M_{\text{eff}} \to 0 \quad M_{\text{rest}} \)
Definition(s) of mass for relativity/quantum

Rest Mass $M_{\text{rest}}$ (Einstein’s mass)

$$hB = h\nu_A = Mc^2 = \h_bar \kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm \sqrt{(Mc^2)^2 + (cp)^2}$$

Given:

Energy: $E = Mc^2 \cosh \rho = h\nu_{\text{phase}}$

momentum: $cp = Mc^2 \sinh \rho = \h_bar \kappa_{\text{phase}}$

velocity: $u = c \tanh \rho = \frac{dv}{dk}$

Momentum Mass $M_{\text{mom}}$ (Galileo’s mass)

Defined by ratio $p/u$ of relativistic momentum to group velocity.

$$M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}} c \sinh \rho}{c \tanh \rho}$$

$$= M_{\text{rest}} \cosh \rho = M_{\text{rest}} \sqrt{1 - u^2 / c^2}$$

Limiting cases:

$$M_{\text{mom}} \xrightarrow{u \to c} M_{\text{rest}} e^{\rho/2}$$

$$M_{\text{mom}} \xrightarrow{u \ll c} M_{\text{rest}}$$

Effective Mass $M_{\text{eff}}$ (Newton’s mass)

Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc \cosh \rho \, d\rho$ in momentum to change $du = c \sech^2 \rho \, d\rho$ in velocity

$$M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho$$

Limiting cases:

$$M_{\text{eff}} \xrightarrow{u \to c} M_{\text{rest}} e^{3\rho/2}$$

$$M_{\text{eff}} \xrightarrow{u \ll c} M_{\text{rest}}$$

More common derivation using group velocity:

$$u = \nu_{\text{group}} = \frac{d\omega}{dk} = \frac{dv}{dk}$$

$$M_{\text{eff}} \equiv \frac{dp}{du} = \h_bar \frac{dk}{dV_{\text{group}}} = \h_bar \frac{d}{dk} \frac{d\omega}{dk} = \h_bar \frac{d^2\omega}{dk^2} = \left(1 - u^2 / c^2\right)^{3/2}$$

$$M_{\text{rest}}$$
Definition(s) of mass for relativity/quantum

**Rest Mass** \( M_{\text{rest}} \) (Einstein’s mass)

\[
hB = h\nu_A = Mc^2 = h\kappa_A
\]

\[
\frac{h\nu_{\text{phase}}}{c^2} = M_{\text{rest}} = \frac{h\kappa_{\text{phase}}}{c^2}
\]

**Momentum Mass** \( M_{\text{mom}} \) (Galileo’s mass) Defined by ratio \( p/u \) of relativistic momentum to group velocity.

\[
M_{\text{mom}} \equiv \frac{p}{u} = \frac{M_{\text{rest}}c \sinh \rho}{c \tanh \rho}
\]

\[
= M_{\text{rest}} \cosh \rho = \frac{M_{\text{rest}}}{\sqrt{1 - u^2 / c^2}}
\]

**Effective Mass** \( M_{\text{eff}} \) (Newton’s mass) Defined by ratio \( F/a = dp/du \) of relativistic force to acceleration.

That is ratio of change \( dp = Mc \cosh \rho \, d\rho \) in momentum to change \( du = c \sech^2 \rho \, d\rho \) in velocity

\[
M_{\text{eff}} \equiv \frac{dp}{du} = M_{\text{rest}} \frac{c \cosh \rho}{c \sech^2 \rho} = M_{\text{rest}} \cosh^3 \rho
\]

\[
\text{Limiting cases: } M_{\text{eff}} \xrightarrow{u \to c} M_{\text{rest}} e^{3\rho/2}
\]

More common derivation using group velocity: \( u \equiv V_{\text{group}} = \frac{d\omega}{dk} = \frac{d\nu}{d\kappa} \) to accompany

\[
M_{\text{eff}} \equiv \frac{dp}{du} = \frac{\hbar \kappa}{dV_{\text{group}}} = \frac{\hbar}{d^2 \omega/dk^2} = \frac{\hbar}{d\omega/dk} \left( \frac{1}{1 - u^2 / c^2} \right)^{3/2} = M_{\text{rest}} \cosh^3 \rho
\]

\[
\text{Effective Mass}
\]

\[
\text{given: } E = Mc^2 \cosh \rho = h\nu_{\text{phase}}
\]

\[
momentum: \ c p = Mc^2 \sinh \rho = h\kappa_{\text{phase}}
\]

\[
\text{Group velocity: } u = c \tanh \rho = \frac{d\nu}{d\kappa}
\]
Definition(s) of mass for relativity/quantum

How much mass does a $\gamma$-photon have?

(a) $\gamma$-rest mass: $M^\gamma_{\text{rest}} = 0$,

(b) $\gamma$-momentum mass: $M^\gamma_{\text{mom}} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\nu}{c^2}$,

(c) $\gamma$-effective mass: $M^\gamma_{\text{eff}} = \infty$.

Newton complained about his “corpuscles” of light having “fits” (going crazy).

$$M^\gamma_{\text{mom}} = \frac{h\nu}{c^2} = \nu(1.2 \cdot 10^{-51}) kg \cdot s = 4.5 \cdot 10^{-36} kg \quad \text{(for: } \nu=600 \text{THz})$$
Definition(s) of mass for relativity/quantum

How much does a $\gamma$-photon weigh?

(a) $\gamma$-rest mass: $M^\gamma_{rest} = 0$,

(b) $\gamma$-momentum mass: $M^\gamma_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\nu}{c^2}$,

(c) $\gamma$-effective mass: $M^\gamma_{eff} = \infty$.

Newton complained about *his* “corpuscles” of light having “fits” (going crazy).
For him this would be evidence of optical-triple-schizophrenia!

$$M^\gamma_{mom} = \frac{h\nu}{c^2} = \nu(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36} kg \quad \text{(for: } \nu=600\text{THz})$$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$

Prior wave relations

- Linear Hz format
  - $\hbar \nu_A = Mc^2 = \hbar c \kappa_A$
  - $\hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho$
  - $\hbar c \kappa_{\text{phase}} = cp = \hbar \nu_A \sinh \rho$

- Angular phasor format
  - $\hbar \omega_A = Mc^2 = \hbar c k_A$
  - $\hbar \omega_{\text{phase}} = E = \hbar \omega_A \cosh \rho$
  - $\hbar c k_{\text{phase}} = cp = \hbar \omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$p = \hbar k = Mc \sinh \rho$

$E = \hbar \omega = Mc^2 \cosh \rho$

Prior wave relations

$$h \nu_A = Mc^2 = h c \kappa_A$$

$$h \nu_{\text{phase}} = E = h \nu_A \cosh \rho$$

$$h c \kappa_{\text{phase}} = c p = h \nu_A \sinh \rho$$

$$h \omega_A = Mc^2 = h c k_A$$

$$h \omega_{\text{phase}} = E = h \omega_A \cosh \rho$$

$$h c k_{\text{phase}} = c p = h \omega_A \sinh \rho$$

Sunday, November 2, 2014
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$. Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E$$

$p = \hbar k = Mc \sinh \rho$

$E = \hbar \omega = Mc^2 \cosh \rho$

Prior wave relations

- Linear Hz format: $h\nu_A = Mc^2 = hck_A$
- Angular phasor format: $h\omega_A = E = h\nu_A \cosh \rho$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $hc\kappa_{\text{phase}} = cp = h\nu_A \sinh \rho$
- $h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $hc\kappa_{\text{phase}} = cp = h\omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

Prior wave relations:

- $h\nu_A = Mc^2 = h\omega_A$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $hck_{\text{phase}} = cp = h\nu_A \sinh \rho$

Legendre transformation

$\frac{d\Phi}{dt} = \hbar \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

$p = \hbar k = Mc \sinh \rho$

$E = \hbar \omega = Mc^2 \cosh \rho = H$

Prior wave relations

\[
\begin{align*}
\hbar \nu_A &= Mc^2 = \hbar c \kappa_A \\
\hbar \nu_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} &= cp = \hbar \nu_A \sinh \rho
\end{align*}
\]

\[
\begin{align*}
\hbar \omega A &= Mc^2 = \hbar c \kappa_A \\
\hbar \omega_{\text{phase}} &= E = \hbar \omega_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} &= cp = \hbar \omega_A \sinh \rho
\end{align*}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$

$$L \equiv \hbar \frac{d \Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p \dot{x} - E \equiv pu - H = L$$

Use Group velocity: $u=\frac{dx}{dt}=c \tanh \rho$

$p = \hbar k = Mc \sinh \rho$

$E = \hbar \omega = Mc^2 \cosh \rho = H$

$L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho$

Prior wave relations

$\hbar \nu_A = Mc^2 = \hbar c \kappa_A$

$\hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho$

$\hbar c \kappa_{\text{phase}} = cp = \hbar \nu_A \sinh \rho$

$\hbar \omega_A = Mc^2 = \hbar ck_A$

$\hbar \omega_{\text{phase}} = E = \hbar \omega_A \cosh \rho$

$\hbar ck_{\text{phase}} = cp = \hbar \omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$.

\[ L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L \]

Legendre transformation

Use Group velocity $u = \frac{dx}{dt} = c \tanh \rho$

\[ p = \hbar k = M c \sinh \rho \]

\[ E = \hbar \omega = M c^2 \cosh \rho = H \]

\[ L = pu - H = (M c \sinh \rho)(c \tanh \rho) - M c^2 \cosh \rho \]

\[ = M c^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = - M c^2 \text{sech} \rho \]

$L$ is: $M c^2 \frac{-1}{\cosh \rho} = - M c^2 \text{sech} \rho$

Prior wave relations

- $h\nu_A = M c^2 = hc \kappa_A$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $h c \kappa_{\text{phase}} = c p = h\nu_A \sinh \rho$

Legendre transformation

- $h\omega_A = M c^2 = hc k_A$
- $h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $h c k_{\text{phase}} = c p = h\omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

\[
p = \hbar k = Mc \sinh \rho
\]

\[
E = \hbar \omega = Mc^2 \cosh \rho = H
\]

\[
L = pu - H = (Mc \sinh \rho) (c \tanh \rho) - Mc^2 \cosh \rho
\]

\[
= Mc^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = - Mc^2 \text{sech} \rho
\]

Compare Lagrangian $L$

\[
L = \hbar \dot{\Phi} = - Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = - Mc^2 \text{sech} \rho
\]

Prior wave relations

- Linear Hz format: $\hbar \nu_A = Mc^2 = h c \kappa_A$
- Angular phasor format: $\hbar \omega_A = Mc^2 = h c k_A$
- $\hbar \omega_{phase} = E = \hbar \nu_A \cosh \rho$
- $\hbar \kappa_{phase} = cp = \hbar \nu_A \sinh \rho$

Legendre transformation

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi= kx-\omega t= k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

\[p = \hbar k=Mc \sinh \rho\]

\[E = \hbar \omega = Mc^2 \cosh \rho = H\]

\[
L = pu - H = (Mc \sinh \rho)(c \tanh \rho) - Mc^2 \cosh \rho
\]

\[= Mc^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = - Mc^2 \text{sech} \rho
\]

Compare Lagrangian $L$

\[L = \hbar \dot{\Phi} = - Mc^2 \sqrt{1- \frac{u^2}{c^2}} = - Mc^2 \text{sech} \rho\]

with Hamiltonian $H=E$

\[H = \hbar \omega = - Mc^2 \sqrt{1- \frac{u^2}{c^2}} = Mc^2 \cosh \rho\]

Prior wave relations

- $h\nu_A = Mc^2 = h\kappa_A$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\nu_A \sinh \rho$

- $h\omega_A = Mc^2 = h\kappa_A$
- $h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

\[ L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv p u - H = L \]

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

\[ p = \hbar k = Mc \sinh \rho \]
\[ E = \hbar \omega = Mc^2 \cosh \rho = H \]

Note: $Mcu = Mc^2 \tanh \rho$

Also: $c p = Mc^2 \sinh \rho$

Compare Lagrangian $L$

\[ L = \hbar \dot{\Phi} = - Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = - Mc^2 \sech \rho \]

with Hamiltonian $H = E$

\[ H = \hbar \omega = - Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho \]
\[ = Mc^2 \sqrt{1 + \sinh^2 \rho} = Mc^2 \sqrt{1 + (c p)^2} \]

$h\nu_A = Mc^2 = h c \kappa_A$

$h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$

$h c \kappa_{\text{phase}} = c p = h\nu_A \sinh \rho$

Prior wave relations

$\longrightarrow$ linear Hz format

angular phasor format

$h\omega_A = Mc^2 = h c k_A$

$h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$

$h c k_{\text{phase}} = c p = h\omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Use Group velocity $u = \frac{dx}{dt} = c \tanh \rho$

\[
p = \hbar k = Mc \sinh \rho
\]

\[
E = \hbar \omega = Mc^2 \cosh \rho = H
\]

Note: $Mcu = Mc^2 \tanh \rho = Mc^2 \sin \sigma$

Also: $c p = Mc^2 \sinh \rho = \hbar c k = Mc^2 \tan \sigma$

Including stellar angle $\sigma$

Prior wave relations

\[
\begin{align*}
\hbar \nu_A &= Mc^2 = \hbar c \kappa_A \\
\hbar \nu_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} &= cp = \hbar \nu_A \sinh \rho
\end{align*}
\]

\[
\begin{align*}
\hbar \omega_A &= Mc^2 = \hbar c k_A \\
\hbar \omega_{\text{phase}} &= E = \hbar \omega_A \cosh \rho \\
\hbar c k_{\text{phase}} &= cp = \hbar \omega_A \sinh \rho
\end{align*}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{\text{phase}}$ and $\omega=\omega_{\text{phase}}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar \omega$ relation to define Hamiltonian $H=E$

\[
\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Legendre transformation

Compare Lagrangian $L$

\[
\dot{S} = L = \hbar \Phi = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} \]

with Hamiltonian $H=E$

\[
H = \hbar \omega = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma
\]

Define Action $S=\hbar \Phi$

\[
\begin{align*}
\nu_A &= Mc^2 = \hbar c \kappa_A \\
\nu_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
hc \kappa_{\text{phase}} &= cp = \hbar \nu_A \sinh \rho
\end{align*}
\]

Prior wave relations

linear Hz format \hspace{1cm} \text{angular phasor format}

\[
\begin{align*}
\nu_A &= Mc^2 = \hbar c \kappa_A \\
\nu_{\text{phase}} &= E = \hbar \nu_A \cosh \rho \\
hc \kappa_{\text{phase}} &= cp = \hbar \omega_A \sinh \rho
\end{align*}
\]
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$

\[
\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Prior wave relations

- $h\nu_A = Mc^2 = \hbar c \kappa_A$
- $h\nu_{\text{phase}} = E = h\nu_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\nu_A \sinh \rho$

- $h\omega_A = Mc^2 = \hbar c \kappa_A$
- $h\omega_{\text{phase}} = E = h\omega_A \cosh \rho$
- $h\kappa_{\text{phase}} = cp = h\omega_A \sinh \rho$
Relativistic action $S$ and Lagrangian-Hamiltonian relations

Define Lagrangian $L$ using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega 't'$ for wave of $k = k_{\text{phase}}$ and $\omega = \omega_{\text{phase}}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation to define Hamiltonian $H = E$.

\[
\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = p \frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L
\]

Use Group velocity: $u = \frac{dx}{dt} = c \tanh \rho$

\[
dS \equiv Ldt \equiv \hbar d\Phi = \hbar kdx - \hbar \omega dt = pdx - H dt
\]

Poincare Invariant action differential

Hamilton-Jacobi equations

\[
\frac{\partial S}{\partial x} = p \quad \frac{\partial S}{\partial t} = -H
\]

Compare Lagrangian $L$

\[
\dot{S} = L = \hbar \dot{\Phi} = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \text{sech} \rho = -Mc^2 \cos \sigma
\]

with Hamiltonian $H = E$

\[
H = \hbar \omega = -Mc^2 \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh \rho = Mc^2 \sec \sigma
\]

Define Action $S = \hbar \Phi$

\[
\hbar \nu_A = Mc^2 = \hbar c \kappa_A \\
\hbar \nu_{\text{phase}} = E = \hbar \nu_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} = cp = \hbar \nu_A \sinh \rho
\]

Prior wave relations

$\longleftrightarrow$ linear Hz format

$\longleftrightarrow$ angular phasor format

\[
\hbar \omega_A = Mc^2 = \hbar c \kappa_A \\
\hbar \omega_{\text{phase}} = E = \hbar \omega_A \cosh \rho \\
\hbar c \kappa_{\text{phase}} = cp = \hbar \omega_A \sinh \rho
\]
Poincare Invariant Action \( dS=\mathcal{L}dt=p\,dq-H\,dt=\hbar d\Phi \) (phase)

Hamiltonian \( H(p,q) = pq - L \) vs. Lagrangian \( \mathcal{L}(\dot{q},q) = p\dot{q} - H \)

Contact transformation: (slope, intercept) of \( H \) (or \( \mathcal{L} \)) tangent determines the \((X,Y \text{ coordinates})\) of \( \mathcal{L} \) (or \( H \)).

(Also, called a Legendre contact transformation which is a special case of a Huygens transformation that uses contacting tangent curves instead of lines.)

Here slope is group velocity \( u = \dot{q} \)

Y-coordinate is energy \( H = \hbar \omega \)

Here slope is momentum \( p \)

Y-coordinate is phase rate \( L = \hbar \Phi \)
Relativistic optical transitions  \(|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle\)

Review of Thales geometry of relativistic \(\hbar\omega(ck)\) or \(E(cp)\)-space

\(\hbar\omega = E(cp)\)

Doppler RED factor: \(\frac{2}{3} = e^{-\rho}\)

Doppler BLUE factor: \(\frac{3}{2} = e^+\rho\)

\(\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell\)
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h \rangle \iff |\text{mid}\rangle = |\omega_m \rangle \iff |\text{low}\rangle = |\omega_\ell \rangle \]

Review of Thales geometry of relativistic \( \hbar \omega(ck) \) or \( E(cp) \)-space

\[ \omega_h = 3 \]

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)

\( \hbar ck = cp \)

\( \omega_m e^{+\rho} = 3 = \omega_h \)
Relativistic optical transitions \( |\text{high}\rangle = |\omega_h\rangle \iff |\text{mid}\rangle = |\omega_m\rangle \iff |\text{low}\rangle = |\omega_{\ell}\rangle \)

Review of Thales geometry of relativistic \(\hbar \omega(ck)\) or \(E(cp)\)-space

Initial stationary
BLUE \(K_h\) thing \(\omega_h = M_{hi}c^2\)

\[ \frac{4}{3} = \omega_m e^{-\rho} = \omega_{\ell} \]

\[ \frac{2}{3} = e^{-\rho} \quad \text{Doppler RED factor} \]
\[ \frac{3}{2} = e^{+\rho} \quad \text{Doppler BLUE factor} \]

\[ \hbar ck = cp \]
Relativistic optical transitions $|\text{high}\rangle = |\omega_{h}\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_{m}\rangle \Leftrightarrow |\text{low}\rangle = |\omega_{\ell}\rangle$

Review of Thales geometry of relativistic $h\omega(ck)$ or $E(cp)$-space

Initial stationary
BLUE $K_{h}$ thing $\omega_{h} = Mc_{2}^{2}$
transitions to
Final moving
GREEN $K_{m}$ thing $\omega_{m} = Mc_{2}^{2}$

$\frac{4}{3} = \omega_{m} e^{-\rho} = \omega_{\ell}$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle \]

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space.

Initial stationary BLUE $K_h$ thing $\omega_h = M_{hi}c^2$

transitions to

Final moving GREEN $K_m$ thing $\omega_m = M_{mi}c^2$

$\omega_h = \frac{4}{3}

\omega_m e^{+\rho} = 3 = \omega_h$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \iff |\text{mid}\rangle = |\omega_m\rangle \iff |\text{low}\rangle = |\omega_\ell\rangle \]

Initial stationary BLUE \( K_h \) thing \( \omega_h = M_{hi}c^2 \)

transitions to

Final moving GREEN \( K_m \) thing \( \omega_m = M_{mi}c^2 \)

\[ \frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell \]

Doppler RED factor: \( \frac{2}{3} = e^{-\rho} \)

Doppler BLUE factor: \( \frac{3}{2} = e^{+\rho} \)

\[ \hbar c k = \text{cp} \]

by emitting an oppositely \( c \)-moving YELLOW \( k_{hm} \) “photon” \( \omega_{hm} = c \mid k_{hm} \mid = \omega_m \sinh \rho \)

Feynman diagram (scaled down) of emission process

\[ \omega_m e^{+\rho} = 3 = \omega_h \]
Relativistic optical transitions

\[ |\text{high}\rangle = |\omega_h\rangle \ \Leftrightarrow \ |\text{mid}\rangle = |\omega_m\rangle \ \Leftrightarrow \ |\text{low}\rangle = |\omega_\ell\rangle \]

Doppler RED factor: \[ \frac{2}{3} = e^{-\rho} \]

Doppler BLUE factor: \[ \frac{3}{2} = e^{+\rho} \]

Review of Thales geometry of relativistic \( \hbar \omega(ck) \) or \( E(cp) \)-space

Initial stationary BLUE \( K_h \) thing \( \omega_h = M_{hi} c^2 \)

transitions to

Final moving GREEN \( K_m \) thing \( \omega_m = M_{mi} c^2 \)

by emitting an oppositely \( c \)-moving YELLOW \( k_{hm} \) “photon” \( \omega_{hm} = c \mid k_{hm} \mid = \omega_m \sinh \rho \)

Feynman diagram (scaled down) of emission process

Take-away point 1

Classical (and spectroscopic)
Energy-momentum conservation is due to conservation quantum-phase space-time “wiggle-count”
Relativistic optical transitions \[ |\text{high}\rangle = |\omega_h\rangle \iff |\text{mid}\rangle = |\omega_m\rangle \iff |\text{low}\rangle = |\omega_\ell\rangle \]

Review of Thales geometry of relativistic \(\hbar\omega(ck)\) or \(E(cp)\)-space

\[ \hbar\omega = E(cp) \]

Take-away point 1.5
It's *very* easy to compute recoil rapidity \(\rho\) or recoil velocity \(u\)

Key recoil relations:
\[ \omega_m e^{-\rho} = \omega_\ell \]
\[ \rho = \ln \frac{M_\ell}{M_m} \]

or:
\[ u \sim c \ln \frac{M_\ell}{M_m} \]

Photons are more like "rockets" than "bullets"
Relativistic optical transitions: $|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\omega(ck)$ or $E(cp)$-space

Take-away point 2
Emission photons are more like rocket exhaust than bullets ($V_{\text{burnout}} = c \ln[M_{\text{initial}}/M_{\text{final}}]$)

Key recoil relations:

\[ \omega_m e^{-\rho} = \omega_\ell \]
\[ \rho = \ln \frac{M_\ell}{M_m} \]

or:

\[ u \sim c \ln \frac{M_\ell}{M_m} \]
Photons are more like “rockets” than “bullets”

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

$\hbar c k = cp$
Relativistic optical transitions $|\text{high}\rangle = |\omega_h\rangle \Leftrightarrow |\text{mid}\rangle = |\omega_m\rangle \Leftrightarrow |\text{low}\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $h\omega(ck)$ or $E(cp)$-space

Take-away point 2.5
Emission photons are more like 
*rocket-exhaust* than bullets
($V_{\text{burnout}} = c \text{exhaust} \ln[M_{\text{initial}}/M_{\text{final}}]$)

Same for absorption processes except those rockets suck.
(Raman-Compton processes are not rocket science.)

Key recoil relations:

$$\omega_m e^{-\rho} = \omega_\ell$$

$$\rho = \ln \frac{M_\ell}{M_m}$$

or:

$$u \sim c \ln \frac{M_\ell}{M_m}$$

Photons are more like "rockets" than "bullets"

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$
2nd Quantization: NEWS FLASH!!!  \( h\nu \) is actually \( hN\nu \)

( \( h\nu_{\text{phase}} = E = h\nu_A \cosh \rho \) is actually \( hN\nu_{\text{phase}} = E_N = hN\nu_A \cosh \rho \) with quantum numbers)

\( N=1,2,3,\ldots \)

1st Quantization:
Mode quantum number \( n \) of half-waves

<table>
<thead>
<tr>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n=1 )</td>
</tr>
<tr>
<td>( n=2 )</td>
</tr>
<tr>
<td>( n=3 )</td>
</tr>
<tr>
<td>( n=4 )</td>
</tr>
</tbody>
</table>

\( E = h\nu_N = hN_1 \nu_1 \)  
\( E = hN_2 \nu_2 \)  
\( E = hN_3 \nu_3 \)  
\( E = hN_4 \nu_4 \)

1st Quantization:  
Mode quantum number \( n \) of half-waves

2nd Quantization:  
Photon number \( N \) oscillator quanta

Take-away point 3  
Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

Boosted cavity wave has invariant mode number \( n \) photon number \( N_n \)

Boosted wave mode

Boosted cavity wave has invariant mode number \( n \) photon number \( N_n \)
Quantization: NEWS FLASH!!! \( h \nu \) is actually \( hN \nu \)

\[ (h \nu_{\text{phase}} = E = h \nu_A \cosh \rho) \text{ is actually } (hN \nu_{\text{phase}} = E_N = hN \nu_A \cosh \rho \quad (N=1,2,\ldots)) \]

\( N_1 = 4 \) red photons
\( N_1 = 3 \) red photons
\( N_1 = 2 \) red photons
\( N_1 = 1 \) red photon
\( N_1 = 0 \)

\( N_2 = 2 \) green photons
\( N_2 = 1 \) green photon
\( N_2 = 0 \)

\( N_3 = 1 \) blue photon
\( N_3 = 0 \)

\( N_4 = 1 \) violet photon
\( N_4 = 0 \)

1st-Quantized Wavenumber ("kink" or momentum number)
\[ c k_n = 1 \cdot \omega \]
\[ c k_n = 2 \cdot \omega \]
\[ c k_n = 3 \cdot \omega \]
\[ c k_n = 4 \cdot \omega \]
Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light