Lecture 9
Wed. 9.25.2019

Equations of Lagrange and Hamilton mechanics in GeneralizedCurvilinear Coordinates (GCC)

(Ch. 12 of Unit 1 and Ch. 1-5 of Unit 2 and Ch. 1-5 of Unit 3)

Quick Review of Lagrange Relations in Lectures 7-8

Using differential chain-rules for coordinate transformations
- *Polar coordinate example of Generalized Curvilinear Coordinates (GCC)*
- *Getting the GCC ready for mechanics: Generalized velocity and Jacobian Lemma 1*
- *Getting the GCC ready for mechanics: Generalized acceleration and Lemma 2*

How to say Newton's "F=ma" in Generalized Curvilinear Coords.
- *Use Cartesian KE quadratic form* $KE=T=1/2\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *and* $\mathbf{F}=\mathbf{M}\bullet\mathbf{a}$ *to get GCC force*
- *Lagrange GCC trickery gives Lagrange force equations*
- *Lagrange GCC trickery gives Lagrange potential equations (Lagrange 1 and 2)*

GCC Cells, base vectors, and metric tensors
- *Polar coordinate examples: Covariant* $\mathbf{E}_m$ *vs. Contravariant* $\mathbf{E}^m$
- *Covariant* g_{mn} *vs. Invariant* δ_m^n *vs. Contravariant* g^{mn}

Lagrange prefers Covariant g_{mn} *with Contravariant velocity*
- *GCC Lagrangian definition*
- *GCC "canonical" momentum* p_m *definition*
- *GCC "canonical" force* F_m *definition*
  - *Coriolis "fictitious" forces (... and weather effects)*

This Lecture's Reference Link Listing

Lecture #9

CMwithBang Lecture 8, page=20
WWW.sciencenewsforstudents.org: Cassini - Saturnian polar vortex

Select, exciting, and related Research & Articles of Interest:

These *are* hot off the presses. Out in MISC for quick reference.

Burning a hole in reality—design for a new laser may be powerful enough to pierce space-time - Sumner-Daily KOS-2019
Trampoline mirror may push laser pulse through fabric of the Universe - Lee-ArsTechnica-2019
Achieving_Extreme_Light_Intensities_using_Optically_Curved_Relativistic_Plasma_Mirrors_-_Vincenti-prl-2019

A_Soft_Matter_Computer_for_Soft_Robots_-_Garrad-sr-2019
Correlated_Insulator_Behaviour_at_Half-Filling_in_Magic-Angle_Graphene_Superlattices_-_cao-n-2018

Sorting_ultracold_atoms_in_a_three-dimensional_optical_lattice_in_a_realization_of_Maxwell's_Demon - Kumar-n-2018
Synthetic_three-dimensional_atomic_structures_assembled_atom_by_atom - Barredo-n-2018

Older ones:
Wave-particle_duality_of_C60_molecules - Arndt-ltn-1999
Optical_Vortex_Knots_-_One_Photon__At_A_Time - Tempone-Wiltshire-Sr-2018
Baryon_Deceleration_by_Strong_Chromofields_in_Ultrarelativistic_Nuclear_Collisions - Mishustin-PhysRevC-2007, *APS Link & Abstract*
Hadronic Molecules - Guo-x-2017
Hidden-charm_pentaquark_and_tetraquark_states - Chen-pr-2016

Running Reference Link Listing

Lectures #8 through #7

In reverse order

"RelaWavity" Web Simulations:
2-CW laser wave, Lagrangian vs Hamiltonian,
Physical Terms Lagrangian L(u) vs Hamiltonian H(p)
CoulIt Web Simulation of the Volcanoes of Io
BohrIt Multi-Panel Plot:
Relativistically shifted Time-Space plots of 2 CW light waves

NASA Astronomy Picture of the Day -
Io: The Prometheus Plume *(Just Image)*
NASA Galileo - *Io's Alien Volcanoes*
New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*
NASA Galileo - A Hawaiian-Style Volcano on Io

AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Aux. slides-2018

BoxIt Web Simulations:
Generic/Default
Most Basic A-Type
Basic A-Type w/reference lines
Basic A-Type A-Type with Potential energy
A-Type with Potential energy and Stokes Plot
A-Type w/3 time rates of change
A-Type w/3 time rates of change with Stokes Plot
B-Type (A=1.0, B=-0.05, C=0.0, D=1.0)

RelaWavity Web Elliptical Motion Simulations:
Orbits with b/a=0.125
Orbits with b/a=0.5
Orbits with b/a=0.7
Exegesis with b/a=0.125
Exegesis with b/a=0.5
Exegesis with b/a=0.7
Contact Ellipsometry

Pirelli Site: Phasors animimation
CMwithBang Lecture #6, page=70 (9.10.18)

Running Reference Link Listing

Lectures #6 through #1

In reverse order

RelaWavity Web Simulation: Contact Ellipsometry
BoxIt Web Simulation: Elliptical Motion (A-Type)
CMwBang Course: Site Title Page
Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors
UAF Physics UTube channel

Velocity Amplification in Collision Experiments Involving Superballs - Harter, 1971
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

BounceIt Web Animation - Scenarios:
Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

BounceItIt Web Animation - Scenarios:
49:1 y vs t, 49:1 V2 vs V1, 1:500:1 - 1D Gas Model w/ faux restorative force (Cool),
1:500:1 - 1D Gas (Warm), 1:500:1 - 1D Gas Model (Cool, Zoomed in),
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, Quantum_Carpet_wMBars,
Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Dual plots
$m_1:m_2$ = 3:1
v_2 vs v_1 and V_2 vs V_1, $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$
y_2 vs y_1 plots: $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$, $(v_1, v_2)=(1, -1)$
Estrangian plot *V_2 vs V_1: $(v_1, v_2)=(0, 1)$, $(v_1, v_2)=(1, -1)$*
$m_1:m_2$ = 4:1
v2 vs v1, y2 vs y1
$m_1:m_2$ = 100:1, *$(v_1, v_2)=(1, 0)$: V2 vs V1 Estrangian plot, y2 vs y1 plot*

With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>

More Advanced QM and classical references at the end of this Lecture

Quick Review of Lagrange Relations in Lectures 7-8

→ *0th and 1st equations of Lagrange and Hamilton*

Quick Review of Lagrange Relations in Lectures 7-8

0th and 1st equations of Lagrange and Hamilton

Starts out with simple demands for explicit-dependence, "loyalty" or "fealty to the colors"

Lagrangian and **Estrangian** have no explicit dependence on **momentum p**

$$\frac{\partial L}{\partial p_k} \equiv 0 \equiv \frac{\partial E}{\partial p_k}$$

Hamiltonian and **Estrangian** have no explicit dependence on **velocity v**

$$\frac{\partial H}{\partial v_k} \equiv 0 \equiv \frac{\partial E}{\partial v_k}$$

Lagrangian and **Hamiltonian** have no explicit dependence on **speedinum V**

$$\frac{\partial L}{\partial V_k} \equiv 0 \equiv \frac{\partial H}{\partial V_k}$$

Such non-dependencies hold in spite of "under-the-table" matrix and partial-differential connections

$$\nabla_v L = \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}}{2} = \mathbf{M}\bullet\mathbf{v} = \mathbf{p}$$

$$\begin{pmatrix} \frac{\partial L}{\partial v_1} \\ \frac{\partial L}{\partial v_2} \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

Lagrange's 1st equation(s)

$$\frac{\partial L}{\partial v_k} = p_k \quad \text{or:} \quad \frac{\partial L}{\partial \mathbf{v}} = \mathbf{p}$$

$$\nabla_p H = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}}{2} = \mathbf{M}^{-1}\bullet\mathbf{p} = \mathbf{v}$$

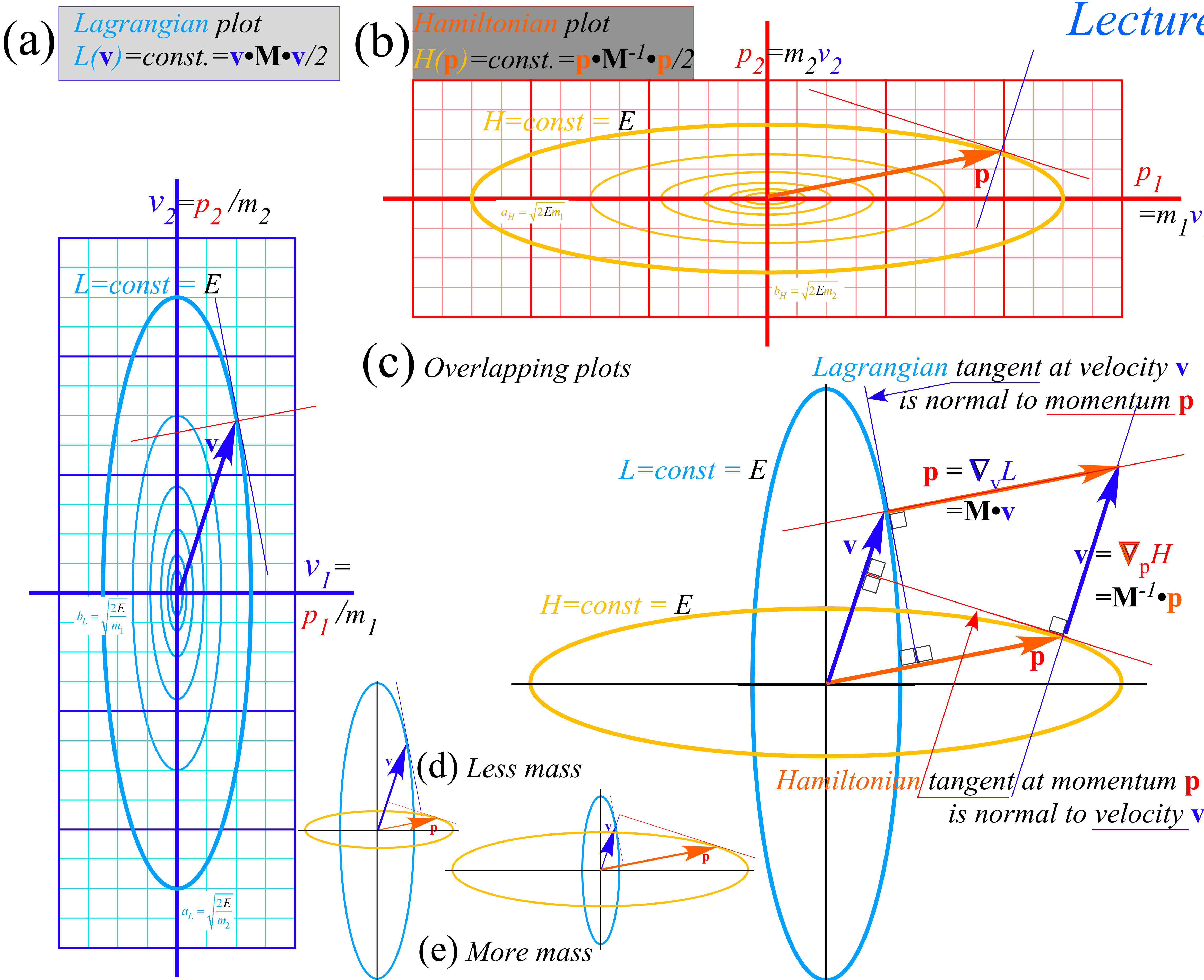
$$\begin{pmatrix} \frac{\partial H}{\partial p_1} \\ \frac{\partial H}{\partial p_2} \end{pmatrix} = \begin{pmatrix} m_1^{-1} & 0 \\ 0 & m_2^{-1} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

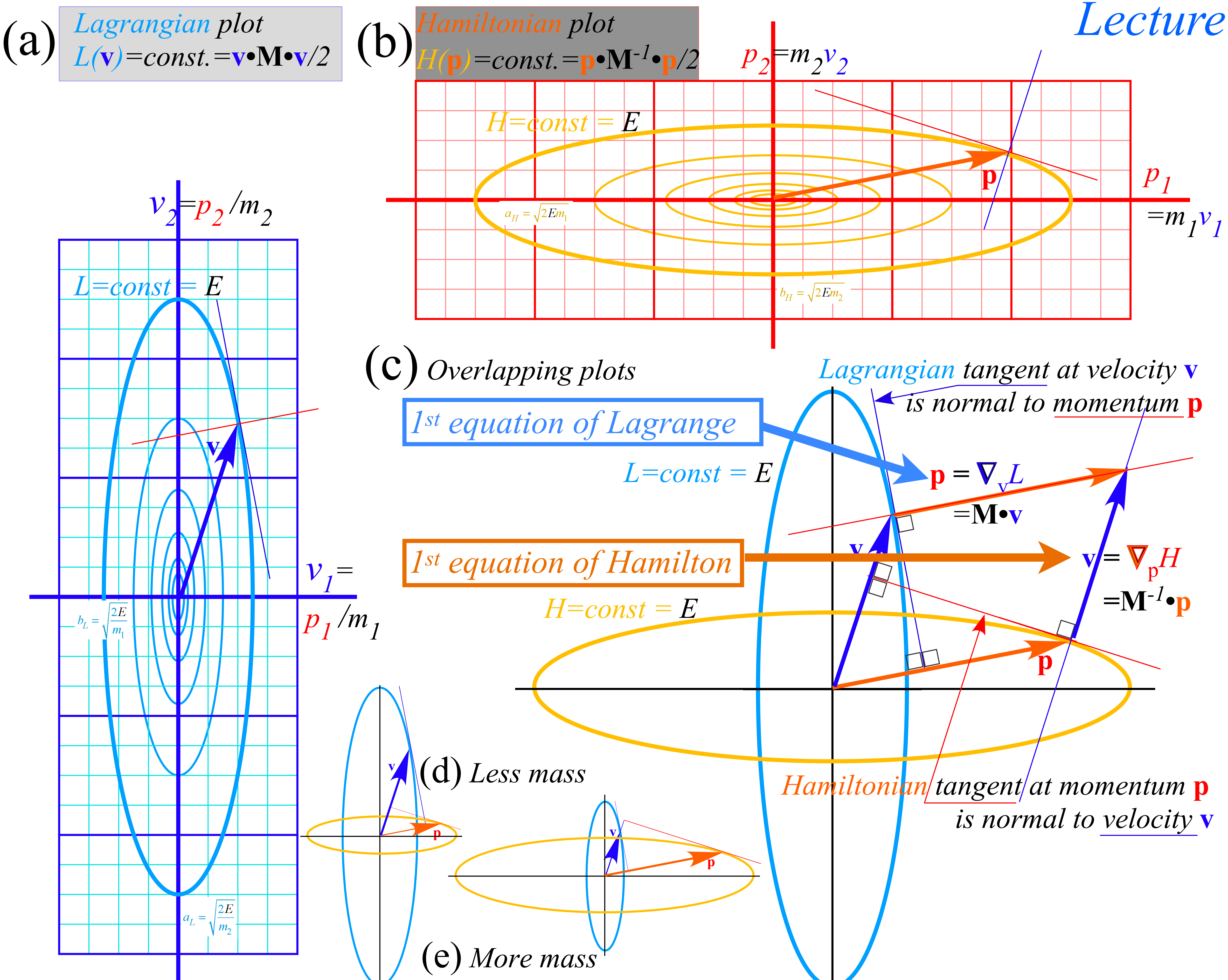
Hamilton's 1st equation(s)

$$\frac{\partial H}{\partial p_k} = v_k \quad \text{or:} \quad \frac{\partial H}{\partial \mathbf{p}} = \mathbf{v}$$

(Forget Estrangian for now)

p. 28 of Lecture 8


(a) Lagrangian plot
L(v)=const.=v•M•v/2
(b) Hamiltonian plot
H(p)=const.=p•M-1•p/2
p2=m2v2
H=const = E
p
p1
=m1v1
v2=p2/m2
L=const = E
v
v1=
p1/m1
(c) Overlapping plots
Lagrangian tangent at velocity v
is normal to momentum p
L=const = E
p = ∇vL
=M•v
v
v = ∇pH
=M-1•p
H=const = E
p
Hamiltonian tangent at momentum p
is normal to velocity v
(d) Less mass
(e) More mass


(a) Lagrangian plot
L(v)=const.=v•M•v/2
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H=const = E
p2=m2v2
p1
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(c) Overlapping plots
Lagrangian tangent at velocity v
is normal to momentum p
1st equation of Lagrange
L=const = E
p = ∇vL
=M•v
1st equation of Hamilton
v = ∇pH
=M-1•p
H=const = E
Hamiltonian tangent at momentum p
is normal to velocity v
(d) Less mass
(e) More mass

Using differential chain-rules for coordinate transformations

- → *Polar coordinate example of Generalized Curvilinear Coordinates (GCC)*
- *Getting the GCC ready for mechanics: Generalized velocity and Jacobian Lemma 1*
- *Getting the GCC ready for mechanics: Generalized acceleration and Lemma 2*

Using differential chain-rules† for coordinate transformations

A pair of 2-variable functions f(x,y) and g(x,y) can define a coordinate system on (x,y)-space

$$df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$dg(x,y) = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy$$

for example: polar coordinates

$r^2(x,y)= x^2+y^2$ and $\theta(x,y)=atan2(y,x)$

(Not in text. Recall Lecture 8 p. 6-22)†

$$dr(x,y) = \frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy$$

$$d\theta(x,y) = \frac{\partial \theta}{\partial x}dx + \frac{\partial \theta}{\partial y}dy$$

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$$d\theta(x,y) = \frac{\partial \theta}{\partial x}dx + \frac{\partial \theta}{\partial y}dy$$

Easy to invert differential chain relations (even if functions are not easily inverted)

$$dx = \frac{\partial x}{\partial f}df + \frac{\partial y}{\partial g}dg$$

$$dy = \frac{\partial y}{\partial f}df + \frac{\partial y}{\partial g}dg$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$dx = \frac{\partial x}{\partial r}dr + \frac{\partial x}{\partial \theta}d\theta$$

$$dy = \frac{\partial y}{\partial r}dr + \frac{\partial y}{\partial \theta}d\theta$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} \begin{pmatrix} dr \\ d\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \begin{pmatrix} dr \\ d\theta \end{pmatrix}$$

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$r^2(x,y)=x^2+y^2$ and $\theta(x,y)=atan2(y,x)$

$$dr(x,y)=\frac{\partial r}{\partial x}dx+\frac{\partial r}{\partial y}dy$$

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Notation for differential GCC (Generalized Curvilinear Coordinates $\{q^1, q^2, q^3,\ldots\}$)

$$dx^j=\frac{\partial x^j}{\partial q^m}dq^m\left(\equiv\sum_{m=1}^{N}\frac{\partial x^j}{\partial q^m}dq^m \quad \left\{\begin{matrix}\text{Defining a shorthand}\\ \text{dummy-index } m\text{-sum}\end{matrix}\right\}\right)$$

What does "*q*" stand for?
One guess: "*Queer*"
And they do get pretty queer!

These x^j are plain old CC (Cartesian Coordinates $\{dx^1=dx, dx^2=dy, dx^3=dx, dx^4=dt\}$)

Using differential chain-rules† for coordinate transformations

A pair of 2-variable functions $f(x,y)$ and $g(x,y)$ can define a coordinate system on (x,y)-space

$$df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$dg(x,y) = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy$$

for example: polar coordinates

$r^2(x,y) = x^2 + y^2$ and $\theta(x,y) = atan2(y,x)$

(Not in text. Recall Lecture 8 p. 6-22)†

$$dr(x,y) = \frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy$$

$$d\theta(x,y) = \frac{\partial \theta}{\partial x}dx + \frac{\partial \theta}{\partial y}dy$$

Easy to invert differential chain relations (even if functions are not easily inverted)

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$$x = r\cos\theta$$

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$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} \begin{pmatrix} dr \\ d\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \begin{pmatrix} dr \\ d\theta \end{pmatrix}$$

Index m REPEATED on SAME side of $=$ is SUMMED

Notation for differential GCC (Generalized Curvilinear Coordinates $\{q^1, q^2, q^3, \ldots\}$)

$$dx^j = \frac{\partial x^j}{\partial q^m} dq^m \left(\equiv \sum_{m=1}^{N} \frac{\partial x^j}{\partial q^m} dq^m \quad \left\{ \begin{array}{l} \text{Defining a shorthand} \\ \text{dummy-index } m\text{-sum} \end{array} \right\} \right)$$

What does "q" stand for?
One guess: "*Queer*"
And they do get pretty queer!

Connection lines may help to indicate summation (OK on scratch paper...Difficult in text)

These x^j are plain old CC (Cartesian Coordinates $\{dx^1 = dx, dx^2 = dy, dx^3 = dx, dx^4 = dt\}$)

Using differential chain-rules for coordinate transformations

Polar coordinate example of Generalized Curvilinear Coordinates (GCC)

→ *Getting the GCC ready for mechanics: Generalized velocity and Jacobian Lemma 1*

Getting the GCC ready for mechanics: Generalized acceleration and Lemma 2

Getting the GCC ready for mechanics:

Generalized velocity relation follows from GCC chain rule

$$dx^j = \frac{\partial x^j}{\partial q^m} dq^m$$

Same kind of linear relation exists between CC velocity $v^j \equiv \dot{x}^j \equiv \frac{dx^j}{dt}$ and GCC velocity $\upsilon^m \equiv \dot{q}^m \equiv \frac{dq^m}{dt}$

$$\dot{x}^j = \frac{\partial x^j}{\partial q^m} \dot{q}^m$$

Getting the GCC ready for mechanics:

Generalized velocity relation follows from GCC chain rule

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$$\dot{x}^j = \frac{\partial x^j}{\partial q^m} \dot{q}^m$$

This is a key "lemma-1" for setting up mechanics:

or:

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m}$$

lemma-1

Getting the GCC ready for mechanics:

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or:

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m}$$ *lemma-1*

Jacobian $J_m{}^j$ matrix gives each CCC differential dx^j or velocity $\dot{x}^j$ in terms of GCC dq^m or $\dot{q}^m$.

$$J^j_m \equiv \frac{\partial x^j}{\partial q^m} = \frac{\partial \dot{x}^j}{\partial \dot{q}^m}$$ {Defining *Jacobian* matrix component}

Recall polar coordinate transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

Getting the GCC ready for mechanics:

Generalized velocity relation follows from GCC chain rule

$$dx^j = \frac{\partial x^j}{\partial q^m} dq^m$$

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This is a key "lemma-1" for setting up mechanics:

$$\dot{x}^j = \frac{\partial x^j}{\partial q^m} \dot{q}^m$$

or:

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \quad \textit{lemma-1}$$

Jacobian J_m^j matrix gives each CCC differential dx^j or velocity $\dot{x}^j$ in terms of GCC dq^m or $\dot{q}^m$.

$$J^j_m \equiv \frac{\partial x^j}{\partial q^m} = \frac{\partial \dot{x}^j}{\partial \dot{q}^m}$$

{Defining *Jacobian* matrix component}

Recall polar coordinate transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

Inverse (so-called) *Kajobian* K_j^m matrix is flipped partial derivatives of J_m^j.

$$K^m_j \equiv \frac{\partial q^m}{\partial x^j} = \frac{\partial \dot{q}^m}{\partial \dot{x}^j}$$

{Defining *"Kajobian"* (inverse to Jacobian)}

Polar coordinate inverse transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{pmatrix} = \frac{\begin{pmatrix} r\cos\theta & r\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}{(\det J = r)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{r} & \frac{\cos\theta}{r} \end{pmatrix}$$

Defining 2x2 matrix inverse: *(always test inverse matrices!)*

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \frac{\begin{pmatrix} D & -B \\ -C & A \end{pmatrix}}{AD - BC}$$

Getting the GCC ready for mechanics:

Generalized velocity relation follows from GCC chain rule

$$dx^j = \frac{\partial x^j}{\partial q^m} dq^m$$

Same kind of linear relation exists between CC velocity $v^j \equiv \dot{x}^j \equiv \frac{dx^j}{dt}$ and GCC velocity $\upsilon^m \equiv \dot{q}^m \equiv \frac{dq^m}{dt}$

This is a key "lemma-1" for setting up mechanics:

$$\dot{x}^j = \frac{\partial x^j}{\partial q^m} \dot{q}^m$$

or:

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \quad \text{lemma-1}$$

Jacobian J_m^j matrix gives each CCC differential dx^j or velocity $\dot{x}^j$ in terms of GCC dq^m or $\dot{q}^m$.

$$J_m^j \equiv \frac{\partial x^j}{\partial q^m} = \frac{\partial \dot{x}^j}{\partial \dot{q}^m}$$

{Defining *Jacobian* matrix component}

Recall polar coordinate transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

Inverse (so-called) *Kajobian* K_j^m matrix is flipped partial derivatives of J_m^j.

$$K_j^m \equiv \frac{\partial q^m}{\partial x^j} = \frac{\partial \dot{q}^m}{\partial \dot{x}^j}$$

{Defining *"Kajobian"* (inverse to Jacobian)}

Polar coordinate inverse transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{pmatrix} = \frac{\begin{pmatrix} r\cos\theta & r\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}{(\det J = r)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{r} & \frac{\cos\theta}{r} \end{pmatrix}$$

Defining 2x2 matrix inverse: *(always test inverse matrices!)*

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \frac{\begin{pmatrix} D & -B \\ -C & A \end{pmatrix}}{AD - BC} = \begin{pmatrix} \frac{D}{AD-BC} & \frac{-B}{AD-BC} \\ \frac{-C}{AD-BC} & \frac{A}{AD-BC} \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix} = \begin{pmatrix} AD-BC & 0 \\ 0 & AD-BC \end{pmatrix}$$

Getting the GCC ready for mechanics:

Generalized velocity relation follows from GCC chain rule

$$dx^j = \frac{\partial x^j}{\partial q^m} dq^m$$

Same kind of linear relation exists between CC velocity $v^j \equiv \dot{x}^j \equiv \frac{dx^j}{dt}$ and GCC velocity $\upsilon^m \equiv \dot{q}^m \equiv \frac{dq^m}{dt}$

$$\dot{x}^j = \frac{\partial x^j}{\partial q^m} \dot{q}^m$$

This is a key "lemma-1" for setting up mechanics: *or:*

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \quad \textit{lemma-1}$$

Jacobian J_m^j matrix gives each CCC differential dx^j or velocity $\dot{x}^j$ in terms of GCC dq^m or $\dot{q}^m$.

$$J_m^j \equiv \frac{\partial x^j}{\partial q^m} = \frac{\partial \dot{x}^j}{\partial \dot{q}^m}$$
{Defining *Jacobian* matrix component}

Recall polar coordinate transformation matrix:

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

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$$K_j^m \equiv \frac{\partial q^m}{\partial x^j} = \frac{\partial \dot{q}^m}{\partial \dot{x}^j}$$
{Defining *"Kajobian"* (inverse to Jacobian)}

$$\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{pmatrix} = \frac{\begin{pmatrix} r\cos\theta & r\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}{(\det J = r)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{r} & \frac{\cos\theta}{r} \end{pmatrix}$$

Product of matrix J_m^j and K_j^m is a unit matrix by definition of partial derivatives. *(always test inverse matrices!)*

$$K_j^m \cdot J_n^j \equiv \frac{\partial q^m}{\partial x^j} \cdot \frac{\partial x^j}{\partial q^n} = \frac{\partial q^m}{\partial q^n} = \delta_n^m = \begin{cases} 1 & \textit{if } m = n \\ 0 & \textit{if } m \neq n \end{cases}$$

$$\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{r} & \frac{\cos\theta}{r} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Using differential chain-rules for coordinate transformations

Polar coordinate example of Generalized Curvilinear Coordinates (GCC)

Getting the GCC ready for mechanics: Generalized velocity and Jacobian Lemma 1

→ *Getting the GCC ready for mechanics: Generalized acceleration and Lemma 2*

Getting the GCC ready for mechanics (2nd part)

Generalized acceleration relations are a little more complicated (It's curved coords, after all!)

First apply $\frac{d}{dt}$ to velocity $\dot{x}^j$ and use product rule: $\frac{d}{dt}(u \cdot v) = \frac{du}{dt} \cdot v + u \cdot \frac{dv}{dt}$

$$\ddot{x}^j \equiv \frac{d}{dt}\dot{x}^j = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\dot{q}^m\right) = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)\dot{q}^m + \frac{\partial x^j}{\partial q^m}\ddot{q}^m$$

Getting the GCC ready for mechanics (2nd part)

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$$\ddot{x}^j \equiv \frac{d}{dt}\dot{x}^j = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m} \dot{q}^m \right) = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m} \right)\dot{q}^m + \frac{\partial x^j}{\partial q^m}\ddot{q}^m$$

Apply derivative chain sum to Jacobian.

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m} \right) = \frac{\partial}{\partial q^n}\left(\frac{\partial x^j}{\partial q^m} \right)\frac{dq^n}{dt} = \left(\frac{\partial^2 x^j}{\partial q^n \, \partial q^m} \right)\frac{dq^n}{dt}$$

Getting the GCC ready for mechanics (2nd part)

Generalized acceleration relations are a little more complicated (It's curved coords, after all!)

First apply $\frac{d}{dt}$ to velocity $\dot{x}^j$ and use product rule: $\frac{d}{dt}(u\cdot v)=\frac{du}{dt}\cdot v+u\cdot\frac{dv}{dt}$

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(Not in text. Recall Lecture 9 p. 15-19)†

Apply derivative chain sum to Jacobian. Partial derivatives are reversible. $\partial_m\partial_n=\partial_n\partial_m$

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)=\frac{\partial}{\partial q^n}\left(\frac{\partial x^j}{\partial q^m}\right)\frac{dq^n}{dt}=\left(\frac{\partial^2 x^j}{\partial q^n\,\partial q^m}\right)\frac{dq^n}{dt}=\left(\frac{\partial^2 x^j}{\partial q^m\,\partial q^n}\right)\frac{dq^n}{dt}=\frac{\partial}{\partial q^m}\left(\frac{\partial x^j}{\partial q^n}\frac{dq^n}{dt}\right)$$

Important thing about mechanics to recall: coordinates q^n *<u>independent</u> of velocities* $\frac{dq^m}{dt}=\dot{q}^m$

Getting the GCC ready for mechanics (2nd part)

Generalized acceleration relations are a little more complicated (It's curved coords, after all!)

First apply $\frac{d}{dt}$ to velocity $\dot{x}^j$ and use product rule: $\frac{d}{dt}(u \cdot v) = \frac{du}{dt} \cdot v + u \cdot \frac{dv}{dt}$

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By chain-rule def. of CC velocity:

$$= \frac{\partial}{\partial q^m}\left(\dot{x}^j\right)$$

Important thing about mechanics to recall: coordinates q^n *<u>independent</u> of velocities* $\frac{dq^m}{dt} = \dot{q}^m$

Getting the GCC ready for mechanics (2nd part)

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First apply $\frac{d}{dt}$ to velocity $\dot{x}^j$ and use product rule: $\frac{d}{dt}(u \cdot v) = \frac{du}{dt} \cdot v + u \cdot \frac{dv}{dt}$

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By chain-rule def. of CC velocity: $= \frac{\partial}{\partial q^m}\left(\dot{x}^j\right)$

Important thing about mechanics to recall: coordinates q^n *<u>independent</u> of velocities* $\frac{dq^m}{dt} = \dot{q}^m$

This is the key "lemma-2" for setting up Lagrangian mechanics .

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right) = \frac{\partial \dot{x}^j}{\partial q^m}$$

lemma 2

Getting the GCC ready for mechanics (2nd part)

Generalized acceleration relations are a little more complicated (It's curved coords, after all!)

First apply $\frac{d}{dt}$ to velocity $\dot{x}^j$ and use product rule: $\frac{d}{dt}(u \cdot v) = \frac{du}{dt} \cdot v + u \cdot \frac{dv}{dt}$

$$\ddot{x}^j \equiv \frac{d}{dt}\dot{x}^j = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\dot{q}^m\right) = \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)\dot{q}^m + \frac{\partial x^j}{\partial q^m}\ddot{q}^m$$

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$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right) = \frac{\partial}{\partial q^n}\left(\frac{\partial x^j}{\partial q^m}\right)\frac{dq^n}{dt} = \left(\frac{\partial^2 x^j}{\partial q^n \partial q^m}\right)\frac{dq^n}{dt} = \left(\frac{\partial^2 x^j}{\partial q^m \partial q^n}\right)\frac{dq^n}{dt} = \frac{\partial}{\partial q^m}\left(\frac{\partial x^j}{\partial q^n}\frac{dq^n}{dt}\right)$$

By chain-rule def. of CC velocity: $$= \frac{\partial}{\partial q^m}\left(\dot{x}^j\right)$$

The "lemma-1" was in the GCC velocity analysis just before this one for acceleration.

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \qquad \textit{lemma 1}$$

This is the key "lemma-2" for setting up Lagrangian mechanics .

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right) = \frac{\partial \dot{x}^j}{\partial q^m} \qquad \textit{lemma 2}$$

How to say Newton's "F=ma" in Generalized Curvilinear Coords.

→ *Use Cartesian KE quadratic form* $KE=T=1/2\,\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *and* $\mathbf{F}=\mathbf{M}\bullet\mathbf{a}$ *to get GCC force*

Lagrange GCC trickery gives Lagrange force equations

Lagrange GCC trickery gives Lagrange potential equations (Lagrange 1 and 2)

Deriving GCC mechanics from Cartesian Coord. (CC) Newton I-II

Start with stuff we know...(sort of)

Multidimensional CC version of kinetic energy $\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$

$$T = \frac{1}{2} M_{jk} v^j v^k = \frac{1}{2} M_{jk} \dot{x}^j \dot{x}^k$$

where: M_{jk} are CC inertia constants

Multidimensional CC version of Newt-II (**F**=**M**•**a**) using M_{jk} *constants*

$$f_j = M_{jk} a^k = M_{jk} \ddot{x}^k$$

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where: M_{jk} *are inertia* *<u>constants</u>* *that are* *<u>symmetric</u>*: $M_{jk}=M_{kj}$

Multidimensional CC version of Newt-II (**F**=**M•a**) using M_{jk} *constants*

$$f_j = M_{jk}\, a^k = M_{jk}\, \ddot{x}^k$$

Multidimensional CC version of work-energy differential (dW= **F•**d**x**). *Insert GCC differentials* dq^m

$$dW = f_j dx^j = f_j \left(\frac{\partial x^j}{\partial q^m} dq^m \right) = M_{jk}\, \ddot{x}^k \left(\frac{\partial x^j}{\partial q^m} dq^m \right)$$

(It's time to bring in the queer q^m *!)*

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Start with stuff we know...(sort of)

Multidimensional CC version of kinetic energy $\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$

$$T = \frac{1}{2} M_{jk}\, v^j v^k = \frac{1}{2} M_{jk}\, \dot{x}^j \dot{x}^k$$

where: M_{jk} are inertia <u>constants</u> that are <u>symmetric</u>: $M_{jk}=M_{kj}$

Multidimensional CC version of Newt-II (**F**=**M•a**) using M_{jk} *constants*

$$f_j = M_{jk}\, a^k = M_{jk}\, \ddot{x}^k$$

Multidimensional CC version of work-energy differential (dW= **F**•d**x**). *Insert GCC differentials dq^m*

(It's time to bring in the queer q^m !)

$$dW = f_j dx^j = f_j \left(\frac{\partial x^j}{\partial q^m} dq^m \right) = M_{jk}\, \ddot{x}^k \left(\frac{\partial x^j}{\partial q^m} dq^m \right)$$

dq^m are independent so dq^m-sum is true term-by-term.

$$dW = f_j dx^j = F_m dq^m = f_j \frac{\partial x^j}{\partial q^m} dq^m = M_{jk}\, \ddot{x}^k \frac{\partial x^j}{\partial q^m} dq^m$$

Deriving GCC mechanics from Cartesian Coord. (CC) Newton I-II

Start with stuff we know...(sort of)

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Multidimensional CC version of work-energy differential (dW= **F**•d**x**). *Insert GCC differentials dq^m*

$$dW=f_j dx^j=f_j\left(\frac{\partial x^j}{\partial q^m}dq^m\right)=M_{jk}\,\ddot{x}^k\left(\frac{\partial x^j}{\partial q^m}dq^m\right)$$

(It's time to bring in the queer q^m !)

dq^m are independent so dq^m-sum is true term-by-term. (Still holds if all dq^m are zero but one.)

$$dW=f_j dx^j=F_m dq^m=f_j\frac{\partial x^j}{\partial q^m}dq^m=M_{jk}\,\ddot{x}^k\frac{\partial x^j}{\partial q^m}dq^m \quad\Rightarrow\quad F_m=f_j\frac{\partial x^j}{\partial q^m}=M_{jk}\,\ddot{x}^k\frac{\partial x^j}{\partial q^m}$$

Deriving GCC mechanics from Cartesian Coord. (CC) Newton I-II

Start with stuff we know...(sort of)

Multidimensional CC version of kinetic energy $\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$

$$T = \frac{1}{2} M_{jk} v^j v^k = \frac{1}{2} M_{jk} \dot{x}^j \dot{x}^k$$

where: M_{jk} are inertia constants

Multidimensional CC version of Newt-II (**F**=**M•a**) using M_{jk} *constants*

$$f_j = M_{jk} a^k = M_{jk} \ddot{x}^k$$

Multidimensional CC version of work-energy differential (dW= **F**•d**x**). *Insert GCC differentials dq^m*

$$dW = f_j dx^j = f_j \left(\frac{\partial x^j}{\partial q^m} dq^m \right) = M_{jk} \ddot{x}^k \left(\frac{\partial x^j}{\partial q^m} dq^m \right)$$

(It's time to bring in the queer q^m !)

dq^m are independent so dq^m-sum is true term-by-term. (Still holds if all dq^m are zero but one.)

$$dW = f_j dx^j = F_m dq^m = f_j \frac{\partial x^j}{\partial q^m} dq^m = M_{jk} \ddot{x}^k \frac{\partial x^j}{\partial q^m} dq^m \quad \Rightarrow \quad F_m = f_j \frac{\partial x^j}{\partial q^m} = M_{jk} \ddot{x}^k \frac{\partial x^j}{\partial q^m}$$

Here *generalized GCC force component F_m* is defined:

$$where: \quad F_m = f_j \frac{\partial x^j}{\partial q^m} = M_{jk} \ddot{x}^k \frac{\partial x^j}{\partial q^m}$$

How to say Newton's "F=ma" in Generalized Curvilinear Coords.

Use Cartesian KE quadratic form $KE=T=1/2\,\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *and* $\mathbf{F}=\mathbf{M}\bullet\mathbf{a}$ *to get GCC force*

→ *Lagrange GCC trickery gives Lagrange force equations*

Lagrange GCC trickery gives Lagrange potential equations (Lagrange 1 and 2)

Now Lagrange GCC trickery begins

Obvious stuff...(sort of, if you've looked at it for a century!)

Lagrange's clever end game: First set $A = M_{jk}\ddot{x}^k$ and $B = \frac{\partial x^j}{\partial q^m}$ with calc. formula: $\left[\ddot{A}B = \frac{d}{dt}\left(\dot{A}B\right) - \dot{A}\dot{B}\right]$

$$F_m = f_j \frac{\partial x^j}{\partial q^m} = \underbrace{M_{jk}\ddot{x}^k \frac{\partial x^j}{\partial q^m}}_{\ddot{A}B} = \frac{d}{dt}\underbrace{\left(M_{jk}\dot{x}^k \frac{\partial x^j}{\partial q^m}\right)}_{(\dot{A}B)} - \underbrace{M_{jk}\dot{x}^k \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)}_{\dot{A}\dot{B}}$$

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$$F_m = f_j \frac{\partial x^j}{\partial q^m} = M_{jk}\ddot{x}^k \frac{\partial x^j}{\partial q^m} = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial x^j}{\partial q^m}\right) - M_{jk}\dot{x}^k \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)$$

$\ddot{A}B$ $\left(\dot{A}B\right)$ $\dot{A}\dot{B}$

Cartesian M_{jk} *must be constant for this to work*

(Bye, Bye relativistic mechanics or QM!)

Now Lagrange GCC trickery begins

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Lagrange's clever end game: First set $A = M_{jk}\ddot{x}^k$ and $B = \frac{\partial x^j}{\partial q^m}$ with calc. formula: $\left[\ddot{A}B = \frac{d}{dt}(\dot{A}B) - \dot{A}\dot{B}\right]$

$$F_m = f_j \frac{\partial x^j}{\partial q^m} = M_{jk}\ddot{x}^k \frac{\partial x^j}{\partial q^m} = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial x^j}{\partial q^m}\right) - M_{jk}\dot{x}^k \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)$$

Then convert ∂x^j to $\partial \dot{x}^j$ by *Lemma 1* and *Lemma 2* on 2nd term.

$$F_m = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial \dot{x}^j}{\partial \dot{q}^m}\right) - M_{jk}\dot{x}^k \left(\frac{\partial \dot{x}^j}{\partial q^m}\right)$$

Cartesian M_{jk} must be constant for this to work

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$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \qquad \textit{lemma 1}$$

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right) = \frac{\partial \dot{x}^j}{\partial q^m} \qquad \textit{lemma 2}$$

Now Lagrange GCC trickery begins

Obvious stuff...(sort of, if you've looked at it for a century!)

Lagrange's clever end game: First set $A = M_{jk}\ddot{x}^k$ and $B = \frac{\partial x^j}{\partial q^m}$ with calc. formula: $\left[\ddot{A}B = \frac{d}{dt}(\dot{A}B) - \dot{A}\dot{B}\right]$

$$F_m = f_j \frac{\partial x^j}{\partial q^m} = M_{jk}\ddot{x}^k \frac{\partial x^j}{\partial q^m} = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial x^j}{\partial q^m}\right) - M_{jk}\dot{x}^k \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)$$

($\ddot{A}B$, $(\dot{A}B)$, $\dot{A}\dot{B}$)

Cartesian M_{jk} must be constant for this to work

(Bye, Bye relativistic mechanics or QM!)

Then convert ∂x^j to $\partial \dot{x}^j$ by *Lemma 1* and *Lemma 2* on 2nd term.

$$F_m = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial \dot{x}^j}{\partial \dot{q}^m}\right) - M_{jk}\dot{x}^k \left(\frac{\partial \dot{x}^j}{\partial q^m}\right)$$

Simplify using: $\left[M_{ij}v^i \frac{\partial v^j}{\partial q} = M_{ij}\frac{\partial}{\partial q}\frac{v^i v^j}{2}\right]$ where q may be $\dot{q}^m$ or q^m

$$F_m = \frac{d}{dt}\frac{\partial}{\partial \dot{q}^m}\left(\frac{M_{jk}\dot{x}^k \dot{x}^j}{2}\right) - \frac{\partial}{\partial q^m}\left(\frac{M_{jk}\dot{x}^k \dot{x}^j}{2}\right)$$

$$\frac{\partial \dot{x}^j}{\partial \dot{q}^m} = \frac{\partial x^j}{\partial q^m} \qquad \textit{lemma 1}$$

$$\frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right) = \frac{\partial \dot{x}^j}{\partial q^m} \qquad \textit{lemma 2}$$

Now Lagrange GCC trickery begins

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Lagrange's clever end game: First set $A = M_{jk}\ddot{x}^k$ and $B = \frac{\partial x^j}{\partial q^m}$ with calc. formula: $\left[\ddot{A}B = \frac{d}{dt}(\dot{A}B) - \dot{A}\dot{B}\right]$

$$F_m = f_j \frac{\partial x^j}{\partial q^m} = \overbrace{M_{jk}\ddot{x}^k \frac{\partial x^j}{\partial q^m}}^{\ddot{A}B} = \frac{d}{dt}\overbrace{\left(M_{jk}\dot{x}^k \frac{\partial x^j}{\partial q^m}\right)}^{(\dot{A}B)} - \overbrace{M_{jk}\dot{x}^k \frac{d}{dt}\left(\frac{\partial x^j}{\partial q^m}\right)}^{\dot{A}\dot{B}}$$

Then convert ∂x^j to $\partial \dot{x}^j$ by *Lemma 1* and *Lemma 2* on 2nd term.

$$F_m = \frac{d}{dt}\left(M_{jk}\dot{x}^k \frac{\partial \dot{x}^j}{\partial \dot{q}^m}\right) - M_{jk}\dot{x}^k \left(\frac{\partial \dot{x}^j}{\partial q^m}\right)$$

Simplify using: $\left[M_{ij}v^i \frac{\partial v^j}{\partial q} = M_{ij}\frac{\partial}{\partial q}\frac{v^i v^j}{2}\right]$ where q may be $\dot{q}^m$ or q^m

$$F_m = \frac{d}{dt}\frac{\partial}{\partial \dot{q}^m}\left(\frac{M_{jk}\dot{x}^k \dot{x}^j}{2}\right) - \frac{\partial}{\partial q^m}\left(\frac{M_{jk}\dot{x}^k \dot{x}^j}{2}\right)$$

The result is *Lagrange's GCC force equation* in terms of *kinetic energy* $T = \frac{1}{2}M_{jk}\dot{x}^j \dot{x}^k$

$$\boxed{F_m = \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^m} - \frac{\partial T}{\partial q^m}}$$

or: $\mathbf{F} = \frac{d}{dt}\frac{\partial T}{\partial \mathbf{v}} - \frac{\partial T}{\partial \mathbf{r}}$

How to say Newton's "F=ma" in Generalized Curvilinear Coords.

Use Cartesian KE quadratic form $KE=T=1/2\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}$ *and* $\mathbf{F}=\mathbf{M}\bullet\mathbf{a}$ *to get GCC force*

Lagrange GCC trickery gives Lagrange force equations

→ *Lagrange GCC trickery gives Lagrange potential equations (Lagrange 1 and 2)*

But, Lagrange GCC trickery is not yet done...
(Still another trick-up-the-sleeve!)

If the force is conservative it's a gradient $\mathbf{F} = -\nabla U$ In GCC: $F_m = -\frac{\partial U}{\partial q^m}$

$$F_m = -\frac{\partial U}{\partial q^m} = \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^m} - \frac{\partial T}{\partial q^m}$$

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Becomes *Lagrange's GCC potential equation* with a new definition for the *Lagrangian: L=T-U*.

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^m} - \frac{\partial L}{\partial q^m} \qquad L(\dot{q}^m, q^m) = T(\dot{q}^m, q^m) - U(q^m)$$

This trick requires: $\frac{\partial U}{\partial \dot{q}^m} \equiv 0$

U(r) has NO explicit velocity dependence!

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This trick requires: $\frac{\partial U}{\partial \dot{q}^m} \equiv 0$ *U(r) has NO explicit velocity dependence!*

Lagrange's 1st GCC equation (Defining GCC momentum)

$$p_m = \frac{\partial L}{\partial \dot{q}^m}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^m} = \frac{\partial L}{\partial q^m}$$

Recall : $\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}}$

Lagrange's 2nd GCC equation (Change of GCC momentum)

$$\frac{dp_m}{dt} \equiv \dot{p}_m = \frac{\partial L}{\partial q^m}$$

But, Lagrange GCC trickery is not yet done...

(Still another trick-up-the-sleeve!)

If the force is conservative it's a gradient $\mathbf{F} = -\nabla U$ In GCC: $F_m = -\frac{\partial U}{\partial q^m}$

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This trick requires: $\frac{\partial U}{\partial \dot{q}^m} \equiv 0$

U(r) has NO explicit velocity dependence!

If L has no explicit q^m dependence then:
$\dot{p}_m = 0$
or :
$p_m = const.$

Lagrange's 1st GCC equation (Defining GCC momentum)

$$p_m = \frac{\partial L}{\partial \dot{q}^m}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^m} = \frac{\partial L}{\partial q^m}$$

Recall :

$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}}$

Lagrange's 2nd GCC equation (Change of GCC momentum)

$$\frac{dp_m}{dt} \equiv \dot{p}_m = \frac{\partial L}{\partial q^m}$$

GCC Cells, base vectors, and metric tensors

→ *Polar coordinate examples:* Co*variant* $\mathbf{E}_m$ *vs.* Contra*variant* $\mathbf{E}^m$

Co*variant* g_{mn} *vs.* In*variant* δ_m^n *vs.* Contra*variant* g^{mn}

A dual set of *quasi-unit vectors* show up in Jacobian J and Kajobian K.

J-Columns are *covariant vectors* $\{\mathbf{E}_1=\mathbf{E}_r \;\; \mathbf{E}_2=\mathbf{E}_\phi\}$ K-Rows are *contravariant vectors* $\{\mathbf{E}^1=\mathbf{E}^r \;\; \mathbf{E}^2=\mathbf{E}^\phi\}$

$$\langle J\rangle = \begin{pmatrix} \frac{\partial x^1}{\partial q^1} & \frac{\partial x^1}{\partial q^2} \\ \frac{\partial x^2}{\partial q^1} & \frac{\partial x^2}{\partial q^2} \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r}=\cos\phi & \frac{\partial x}{\partial \phi}=-r\sin\phi \\ \frac{\partial y}{\partial r}=\sin\phi & \frac{\partial y}{\partial \phi}=r\cos\phi \end{pmatrix}$$

$\uparrow \mathbf{E}_1 \;\; \uparrow \mathbf{E}_2 \qquad \uparrow \mathbf{E}_r \qquad \uparrow \mathbf{E}_\phi$

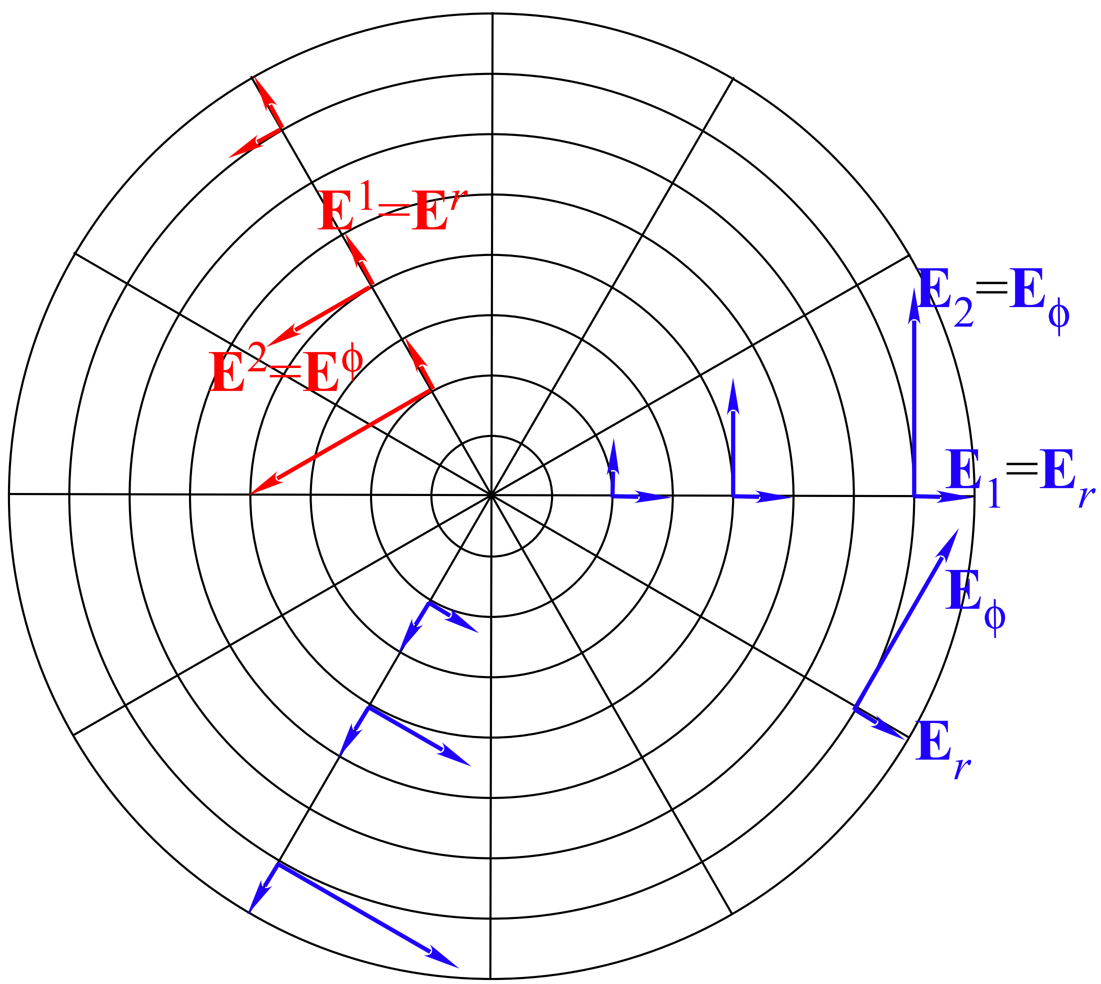
$$\langle K\rangle = \langle J^{-1}\rangle = \begin{pmatrix} \frac{\partial r}{\partial x}=\cos\phi & \frac{\partial r}{\partial y}=\sin\phi \\ \frac{\partial \phi}{\partial x}=\frac{-\sin\phi}{r} & \frac{\partial \phi}{\partial y}=\frac{\cos\phi}{r} \end{pmatrix} \begin{matrix} \leftarrow \mathbf{E}^r=\mathbf{E}^1 \\ \leftarrow \mathbf{E}^\phi=\mathbf{E}^2 \end{matrix}$$

Derived from polar definition: $x=r\cos\phi$ *and* $y=r\sin\phi$

Inverse polar definition:

$r^2=x^2+y^2$ *and* $\phi=\text{atan2}(y,x)$

(a) Polar coordinate bases



Unit 1
Fig. 12.10

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$$\uparrow \mathbf{E}_1 \quad \uparrow \mathbf{E}_2 \qquad \uparrow \mathbf{E}_r \qquad \uparrow \mathbf{E}_\phi$$

Derived from polar definition: $x=r\cos\phi$ *and* $y=r\sin\phi$

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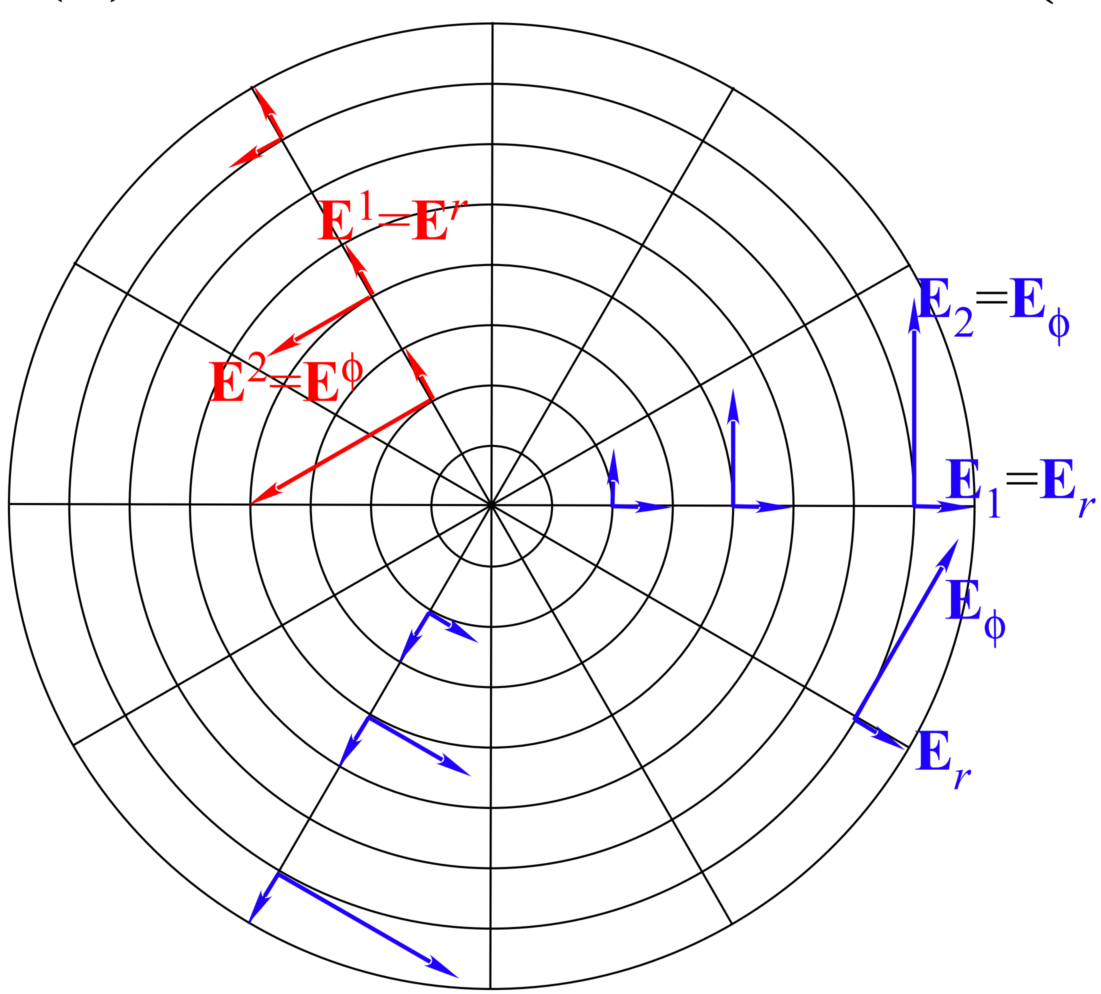
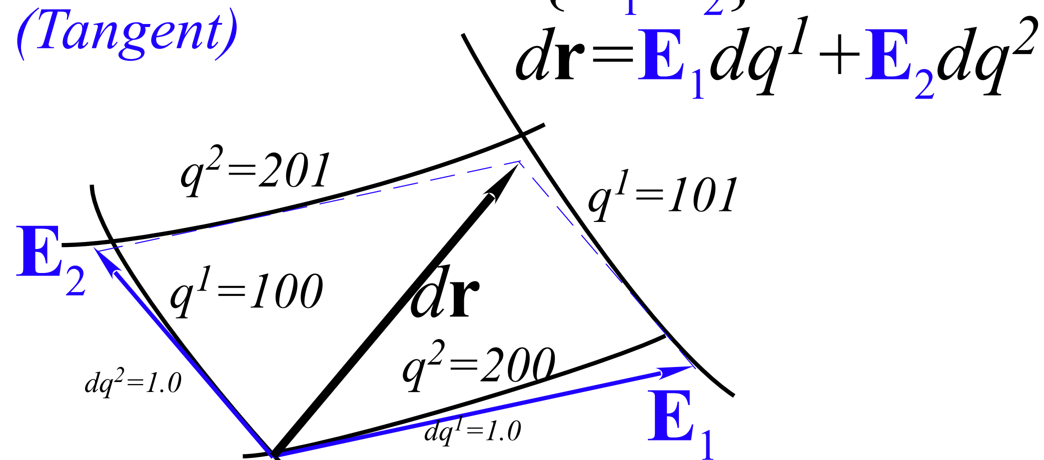
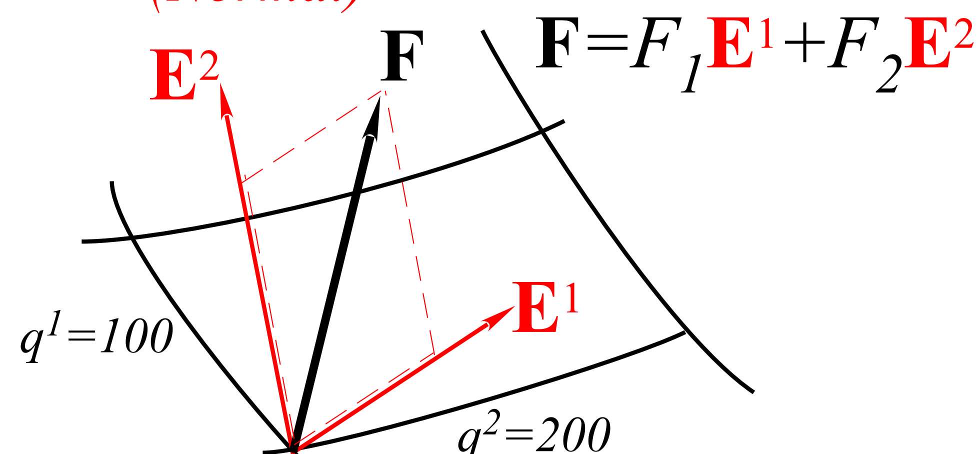
(a) Polar coordinate bases



(b) Covariant bases $\{\mathbf{E}_1\;\mathbf{E}_2\}$



(c) Contravariant bases $\{\mathbf{E}^1\;\mathbf{E}^2\}$



NOTE:These are 2D drawings! No 3D perspective

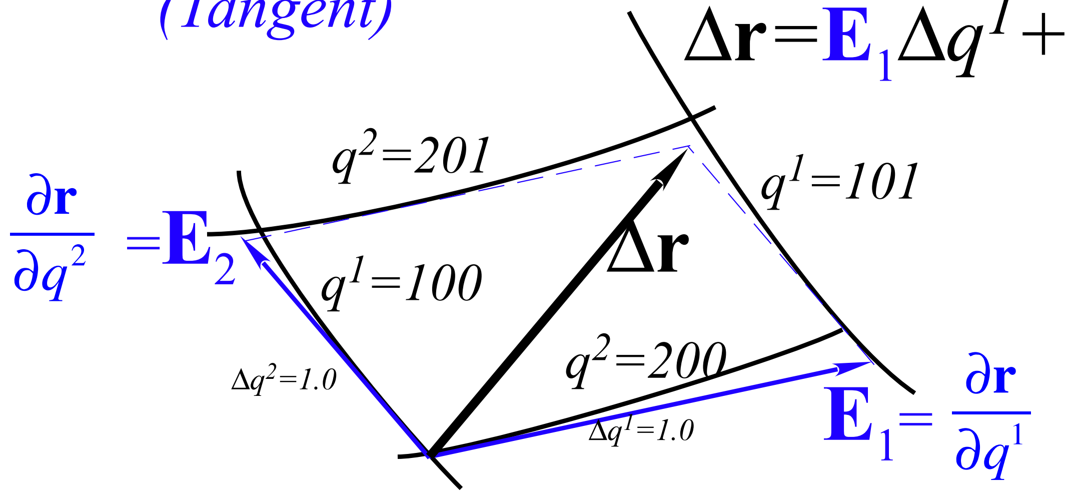
Unit 1
Fig. 12.10

Comparison: Covariant $\mathbf{E}_m = \frac{\partial \mathbf{r}}{\partial q^m}$ *vs. Contravariant* $\mathbf{E}^m = \frac{\partial q^m}{\partial \mathbf{r}} = \nabla q^m$

geometric unit

*Co*variant bases $\{\mathbf{E}_1\ \mathbf{E}_2\}$ match cell walls
(Tangent)

$\Delta \mathbf{r} = \mathbf{E}_1 \Delta q^1 + \mathbf{E}_2 \Delta q^2$ *is based on chain rule:* $d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial q^1} dq^1 + \frac{\partial \mathbf{r}}{\partial q^2} dq^2 = \mathbf{E}_1 dq^1 + \mathbf{E}_2 dq^2$



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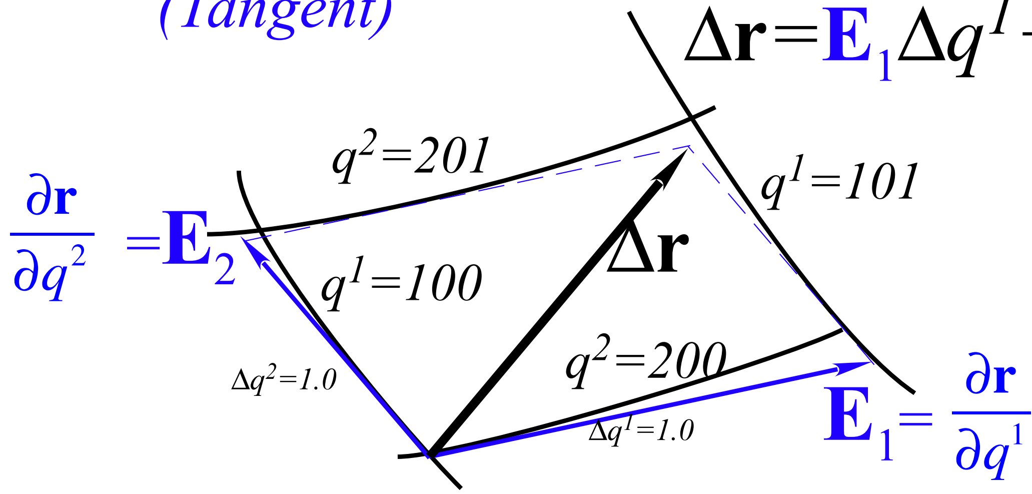
Covariant bases $\{\mathbf{E}_1\ \mathbf{E}_2\}$ match $\hat{}$ cell walls
(Tangent)

$$\Delta \mathbf{r} = \mathbf{E}_1 \Delta q^1 + \mathbf{E}_2 \Delta q^2$$

is based on chain rule: $d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial q^1} dq^1 + \frac{\partial \mathbf{r}}{\partial q^2} dq^2 = \mathbf{E}_1 dq^1 + \mathbf{E}_2 dq^2$



$\mathbf{E}_1$ *follows tangent to* q^2*=const. ...*
since only q^1 *varies in* $\frac{\partial \mathbf{r}}{\partial q^1}$
while q^2, q^3,... *remain constant*

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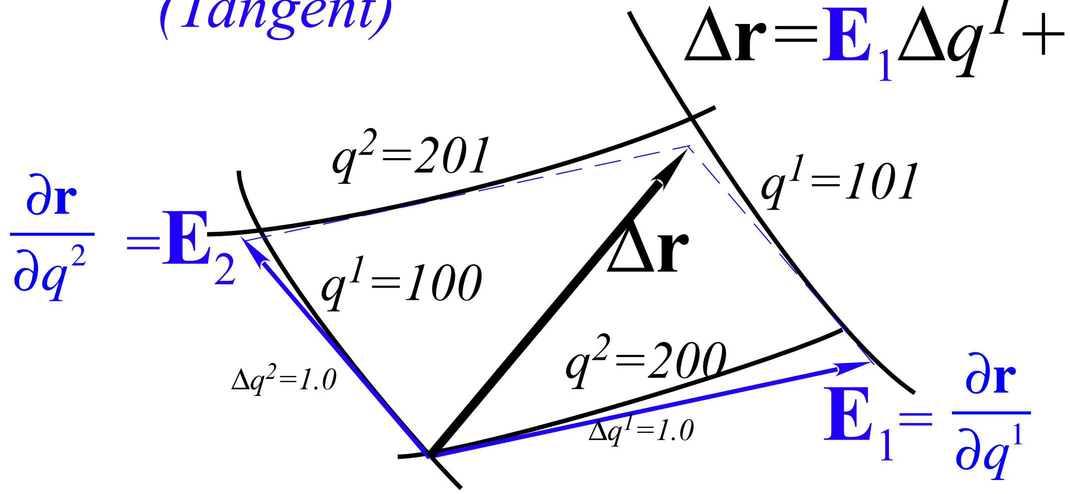

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$\mathbf{E}_m$ are convenient bases for *ex*tensive quantities like distance and velocity.

$$\mathbf{V} = V^1\mathbf{E}_1 + V^2\mathbf{E}_2 = V^1\frac{\partial \mathbf{r}}{\partial q^1} + V^2\frac{\partial \mathbf{r}}{\partial q^2}$$

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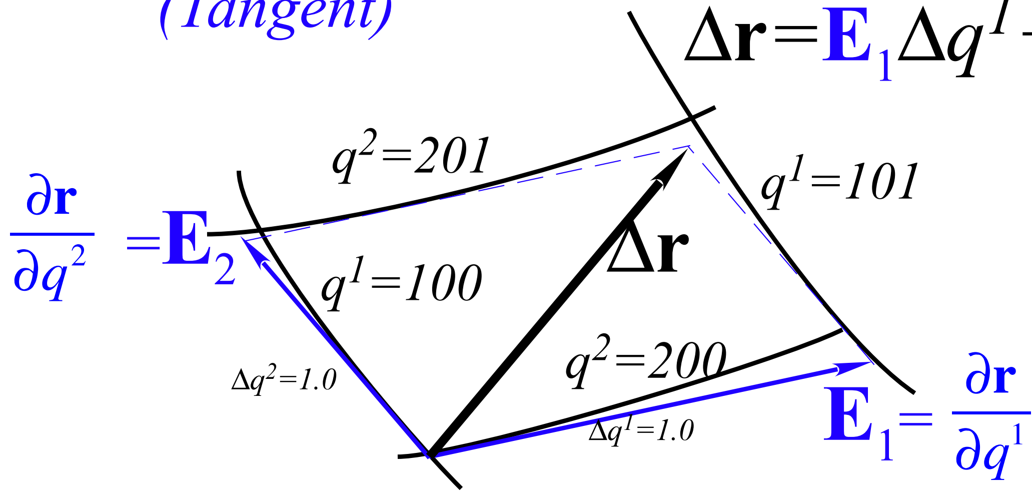

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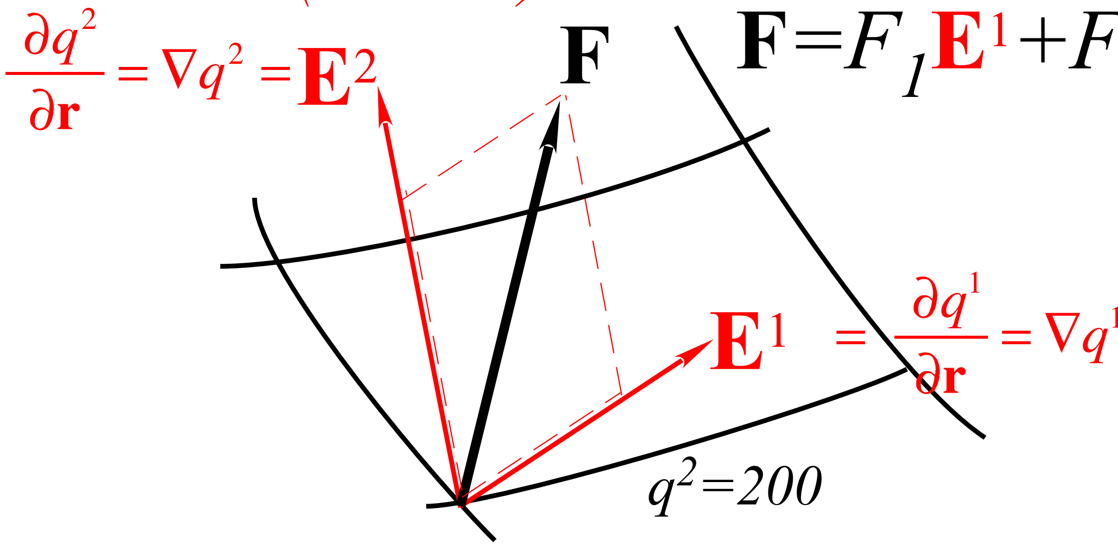
Contravariant $\{\mathbf{E}^1\ \mathbf{E}^2\}$ match reciprocal cells

(Normal)

$$\mathbf{F} = F_1 \mathbf{E}^1 + F_2 \mathbf{E}^2$$



NOTE: These are 2D drawings!
No 3D *perspective*

$\mathbf{E}^1$ *is normal to* q^1*=const. since* **gradient** *of* q^1 *is vector sum* $\nabla q^1 = \begin{pmatrix} \frac{\partial q^1}{\partial x} \\ \frac{\partial q^1}{\partial y} \end{pmatrix}$
of all its partial derivatives

Comparison: Covariant $\mathbf{E}_m = \frac{\partial \mathbf{r}}{\partial q^m}$ *vs. Contravariant* $\mathbf{E}^m = \frac{\partial q^m}{\partial \mathbf{r}} = \nabla q^m$

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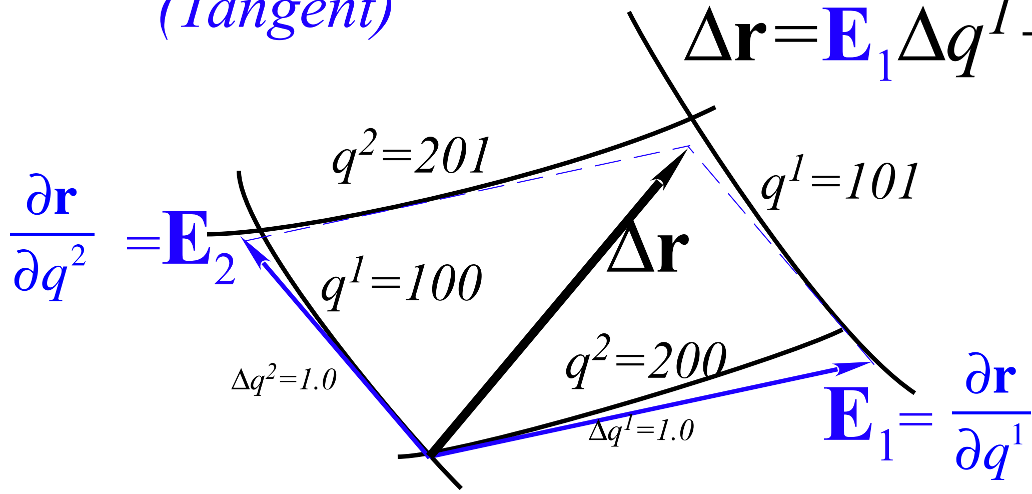
(Tangent)

$$\Delta\mathbf{r} = \mathbf{E}_1 \Delta q^1 + \mathbf{E}_2 \Delta q^2$$

is based on chain rule: $d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial q^1} dq^1 + \frac{\partial \mathbf{r}}{\partial q^2} dq^2 = \mathbf{E}_1 dq^1 + \mathbf{E}_2 dq^2$



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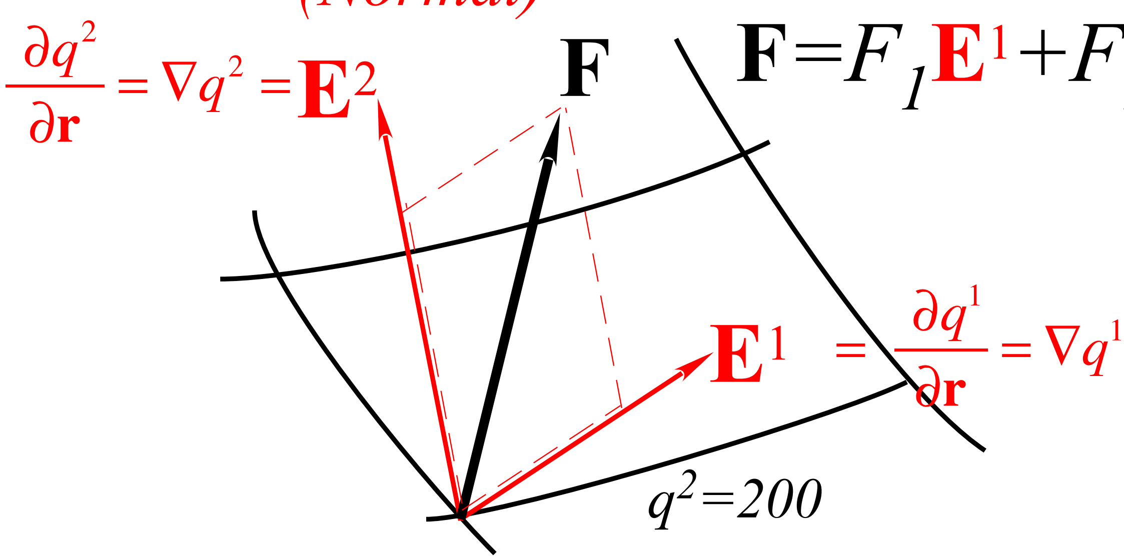
(Normal)

$$\mathbf{F} = F_1 \mathbf{E}^1 + F_2 \mathbf{E}^2$$



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of all its partial derivatives

$\mathbf{E}^m$ are convenient bases for *in*tensive quantities like force and momentum.

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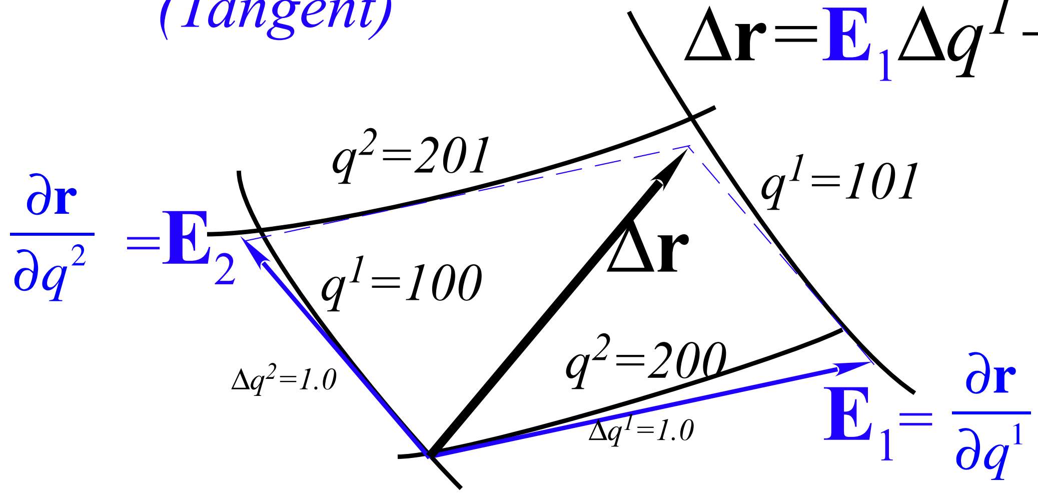
Comparison: Covariant $\mathbf{E}_m = \frac{\partial \mathbf{r}}{\partial q^m}$ vs. Contravariant $\mathbf{E}^n = \frac{\partial q^n}{\partial \mathbf{r}} = \nabla q^n$

Covariant bases $\{\mathbf{E}_1\ \mathbf{E}_2\}$ match geometric unit cell walls
(Tangent)

$$\Delta\mathbf{r} = \mathbf{E}_1 \Delta q^1 + \mathbf{E}_2 \Delta q^2$$

is based on chain rule: $d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial q^1} dq^1 + \frac{\partial \mathbf{r}}{\partial q^2} dq^2 = \mathbf{E}_1 dq^1 + \mathbf{E}_2 dq^2$



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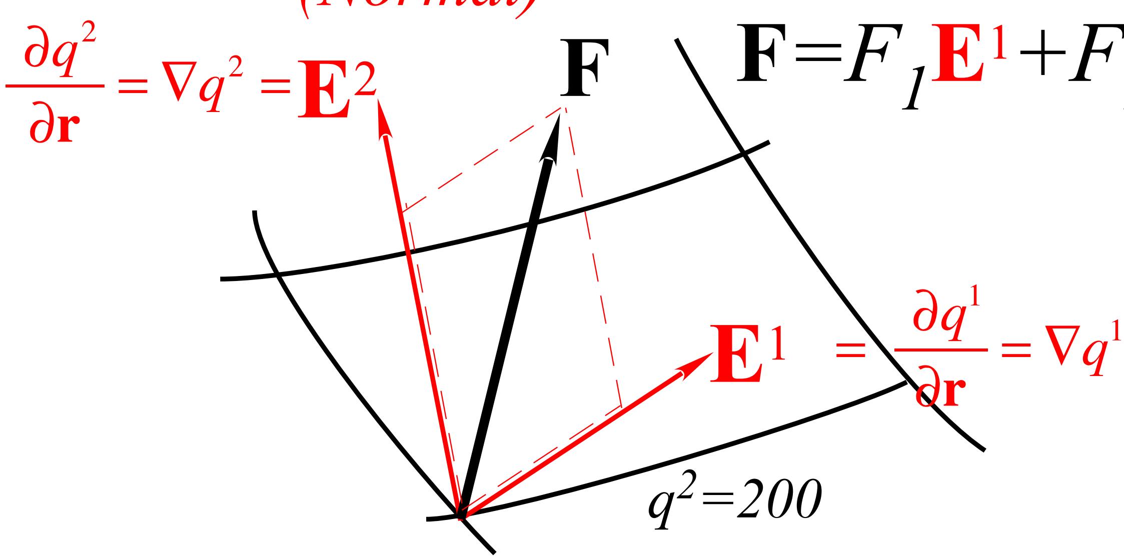
$$\mathbf{V} = V^1 \mathbf{E}_1 + V^2 \mathbf{E}_2 = V^1 \frac{\partial \mathbf{r}}{\partial q^1} + V^2 \frac{\partial \mathbf{r}}{\partial q^2}$$

Co-Contra dot products $\mathbf{E}_m \cdot \mathbf{E}^n$ are orthonormal:

$$\mathbf{E}_m \bullet \mathbf{E}^n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial q^n}{\partial \mathbf{r}} = \delta_m^n$$

Contravariant $\{\mathbf{E}^1\ \mathbf{E}^2\}$ match reciprocal cells
(Normal)

$$\mathbf{F} = F_1 \mathbf{E}^1 + F_2 \mathbf{E}^2$$



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$$\mathbf{F} = F_1 \mathbf{E}^1 + F_2 \mathbf{E}^2 = F_1 \frac{\partial q^1}{\partial \mathbf{r}} + F_2 \frac{\partial q^2}{\partial \mathbf{r}} = F_1 \nabla q^1 + F_2 \nabla q^2$$

By chain rule: $\frac{\partial q^n}{\partial q^m} = \delta_m^n$

GCC Cells, base vectors, and metric tensors

Polar coordinate examples: Co*variant* $\mathbf{E}_m$ *vs.* Contra*variant* $\mathbf{E}^m$

→ Co*variant* g_{mn} *vs.* In*variant* $\delta_m{}^n$ *vs.* Contra*variant* g^{mn}

Covariant g_{mn} vs. Invariant δ_m^n vs. Contravariant g^{mn}

$$\mathbf{E}_m \bullet \mathbf{E}_n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial \mathbf{r}}{\partial q^n} \equiv g_{mn}$$

$$\mathbf{E}_m \bullet \mathbf{E}^n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial q^n}{\partial \mathbf{r}} = \delta_m^n$$

$$\mathbf{E}^m \bullet \mathbf{E}^n = \frac{\partial q^m}{\partial \mathbf{r}} \bullet \frac{\partial q^n}{\partial \mathbf{r}} \equiv g^{mn}$$

Covariant
metric tensor
g_{mn}

Invariant
Kroneker unit tensor

$$\delta_m^n \equiv \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

Contravariant
metric tensor
g^{mn}

Covariant g_{mn} vs. Invariant $\delta_m{}^n$ vs. Contravariant g^{mn}

$$\mathbf{E}_m \bullet \mathbf{E}_n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial \mathbf{r}}{\partial q^n} \equiv g_{mn}$$

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Contravariant
metric tensor
g^{mn}

Polar coordinate examples (again):

$$\langle J \rangle = \begin{pmatrix} \frac{\partial x^1}{\partial q^1} & \frac{\partial x^1}{\partial q^2} \\ \frac{\partial x^2}{\partial q^1} & \frac{\partial x^2}{\partial q^2} \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r} = \cos\phi & \frac{\partial x}{\partial \phi} = -r\sin\phi \\ \frac{\partial y}{\partial r} = \sin\phi & \frac{\partial y}{\partial \phi} = r\cos\phi \end{pmatrix}$$

$\uparrow \mathbf{E}_1$ $\uparrow \mathbf{E}_2$ $\uparrow \mathbf{E}_r$ $\uparrow \mathbf{E}_\phi$

$$\langle K \rangle = \langle J^{-1} \rangle = \begin{pmatrix} \frac{\partial r}{\partial x} = \cos\phi & \frac{\partial r}{\partial y} = \sin\phi \\ \frac{\partial \phi}{\partial x} = \frac{-\sin\phi}{r} & \frac{\partial \phi}{\partial y} = \frac{\cos\phi}{r} \end{pmatrix} \begin{matrix} \leftarrow \mathbf{E}^r = \mathbf{E}^1 \\ \leftarrow \mathbf{E}^\phi = \mathbf{E}^2 \end{matrix}$$

Covariant g_{mn} vs. Invariant δ_m^n vs. Contravariant g^{mn}

$$\mathbf{E}_m \bullet \mathbf{E}_n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial \mathbf{r}}{\partial q^n} \equiv g_{mn}$$

$$\mathbf{E}_m \bullet \mathbf{E}^n = \frac{\partial \mathbf{r}}{\partial q^m} \bullet \frac{\partial q^n}{\partial \mathbf{r}} = \delta_m^n$$

$$\mathbf{E}^m \bullet \mathbf{E}^n = \frac{\partial q^m}{\partial \mathbf{r}} \bullet \frac{\partial q^n}{\partial \mathbf{r}} \equiv g^{mn}$$

Covariant metric tensor g_{mn}

Invariant Kroneker unit tensor

$$\delta_m^n \equiv \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

Contravariant metric tensor g^{mn}

Polar coordinate examples (again):

$$\langle J \rangle = \begin{pmatrix} \frac{\partial x^1}{\partial q^1} & \frac{\partial x^1}{\partial q^2} \\ \frac{\partial x^2}{\partial q^1} & \frac{\partial x^2}{\partial q^2} \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r} = \cos\phi & \frac{\partial x}{\partial \phi} = -r\sin\phi \\ \frac{\partial y}{\partial r} = \sin\phi & \frac{\partial y}{\partial \phi} = r\cos\phi \end{pmatrix}$$

$\uparrow \mathbf{E}_1$ $\uparrow \mathbf{E}_2$ $\uparrow \mathbf{E}_r$ $\uparrow \mathbf{E}_\phi$

$$\langle K \rangle = \langle J^{-1} \rangle = \begin{pmatrix} \frac{\partial r}{\partial x} = \cos\phi & \frac{\partial r}{\partial y} = \sin\phi \\ \frac{\partial \phi}{\partial x} = \frac{-\sin\phi}{r} & \frac{\partial \phi}{\partial y} = \frac{\cos\phi}{r} \end{pmatrix} \begin{matrix} \leftarrow \mathbf{E}^r = \mathbf{E}^1 \\ \leftarrow \mathbf{E}^\phi = \mathbf{E}^2 \end{matrix}$$

Covariant g_{mn}

$$\begin{pmatrix} g_{rr} & g_{r\phi} \\ g_{\phi r} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} \mathbf{E}_r \bullet \mathbf{E}_r & \mathbf{E}_r \bullet \mathbf{E}_\phi \\ \mathbf{E}_\phi \bullet \mathbf{E}_r & \mathbf{E}_\phi \bullet \mathbf{E}_\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

Invariant δ_m^n

$$\begin{pmatrix} \delta_r^r & \delta_r^\phi \\ \delta_\phi^r & \delta_\phi^\phi \end{pmatrix} = \begin{pmatrix} \mathbf{E}_r \bullet \mathbf{E}^r & \mathbf{E}_r \bullet \mathbf{E}^\phi \\ \mathbf{E}_\phi \bullet \mathbf{E}^r & \mathbf{E}_\phi \bullet \mathbf{E}^\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} g^{rr} & g^{r\phi} \\ g^{\phi r} & g^{\phi\phi} \end{pmatrix} = \begin{pmatrix} \mathbf{E}^r \bullet \mathbf{E}^r & \mathbf{E}^r \bullet \mathbf{E}^\phi \\ \mathbf{E}^\phi \bullet \mathbf{E}^r & \mathbf{E}^\phi \bullet \mathbf{E}^\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1/r^2 \end{pmatrix}$$

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→ *GCC Lagrangian definition*

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Coriolis "fictitious" forces (... and weather effects)

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$$= M\,r\,\dot{\phi}^2 - \frac{\partial U}{\partial r}$$

Centrifugal (center-fleeing) force equals total Centripetal (center-pulling) force

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$$L(\mathbf{v}) = \tfrac{1}{2}M\mathbf{v}\bullet\mathbf{v} - U = \tfrac{1}{2}M\dot{\mathbf{r}}\bullet\dot{\mathbf{r}} - U = \tfrac{1}{2}M(\mathbf{E}_m\dot{q}^m)\bullet(\mathbf{E}_n\dot{q}^n) - U = \tfrac{1}{2}M(g_{mn}\dot{q}^m\dot{q}^n) - U = L(\dot{q})$$

Use polar coordinate Covariant g_{mn} metric (page 53)

$$\begin{pmatrix} g_{rr} & g_{r\phi} \\ g_{\phi r} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} \mathbf{E}_r\bullet\mathbf{E}_r & \mathbf{E}_r\bullet\mathbf{E}_\phi \\ \mathbf{E}_\phi\bullet\mathbf{E}_r & \mathbf{E}_\phi\bullet\mathbf{E}_\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

This gives polar GCC form (Actually it's an OCC or Orthogonal Curvilinear Coordinate form)

$$L(\dot{r},\dot{\phi}) = \tfrac{1}{2}M(g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2) - U(r,\phi) = \tfrac{1}{2}M(1\cdot\dot{r}^2 + r^2\cdot\dot{\phi}^2) - U(r,\phi)$$

GCC Lagrange equations follow. 1st L-equation is momentum p_m definition for each coordinate q^m:

$$p_r = \frac{\partial L}{\partial \dot{r}} = M\,g_{rr}\dot{r} = M\,\dot{r}$$

Nothing too surprising; radial momentum p_r has the usual linear $M\cdot v$ form

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = M g_{\phi\phi}\dot{\phi} = Mr^2\dot{\phi}$$

Wow! $g_{\phi\phi}$ gives moment-of-inertia factor Mr^2 automatically for the angular momentum $p_\phi = Mr^2\omega$.

2nd L-equation involves total time derivative $\dot{p}_m$ for each momentum p_m:

$$\dot{p}_r = \frac{\partial L}{\partial r} = \frac{M}{2}\frac{\partial g_{\phi\phi}}{\partial r}\dot{\phi}^2 - \frac{\partial U}{\partial r} = M\,r\,\dot{\phi}^2 - \frac{\partial U}{\partial r}$$

Centrifugal force $Mr\omega^2$

$$\dot{p}_\phi = \frac{\partial L}{\partial \phi} = 0 - \frac{\partial U}{\partial \phi}$$

Angular momentum p_ϕ is conserved if potential U has no explicit ϕ-dependence

Find $\dot{p}_m$ directly from 1st L-equation: $\dot{p}_m \equiv \frac{dp_m}{dt} = \frac{d}{dt}M(g_{mn}\dot{q}^n) = M(\dot{g}_{mn}\dot{q}^n + g_{mn}\ddot{q}^n)$ Equate it to $\dot{p}_m$ in 2nd L-equation:

$$\dot{p}_r \equiv \frac{dp_r}{dt} = M\,\ddot{r}$$

$$= M\,r\,\dot{\phi}^2 - \frac{\partial U}{\partial r}$$

Centrifugal (center-fleeing) force equals total Centripetal (center-pulling) force

$$\dot{p}_\phi \equiv \frac{dp_\phi}{dt} = 2Mr\dot{r}\dot{\phi} + Mr^2\ddot{\phi}$$

Torque relates to two distinct parts: Coriolis and angular acceleration

$$= 0 - \frac{\partial U}{\partial \phi}$$

Angular momentum p_ϕ is conserved if potential U has no explicit ϕ-dependence

Rewriting GCC Lagrange equations :

$$\dot{p}_r \equiv \frac{dp_r}{dt} = M\ddot{r} = Mr\dot{\phi}^2 - \frac{\partial U}{\partial r}$$

Centrifugal (center-fleeing) force equals total Centripetal (center-pulling) force

$$\dot{p}_\phi \equiv \frac{dp_\phi}{dt} = 2Mr\dot{r}\dot{\phi} + Mr^2\ddot{\phi} = 0 - \frac{\partial U}{\partial \phi}$$

Torque relates to two distinct parts: Coriolis and angular acceleration

Angular momentum p_ϕ is conserved if potential U has no explicit ϕ-dependence

Conventional forms

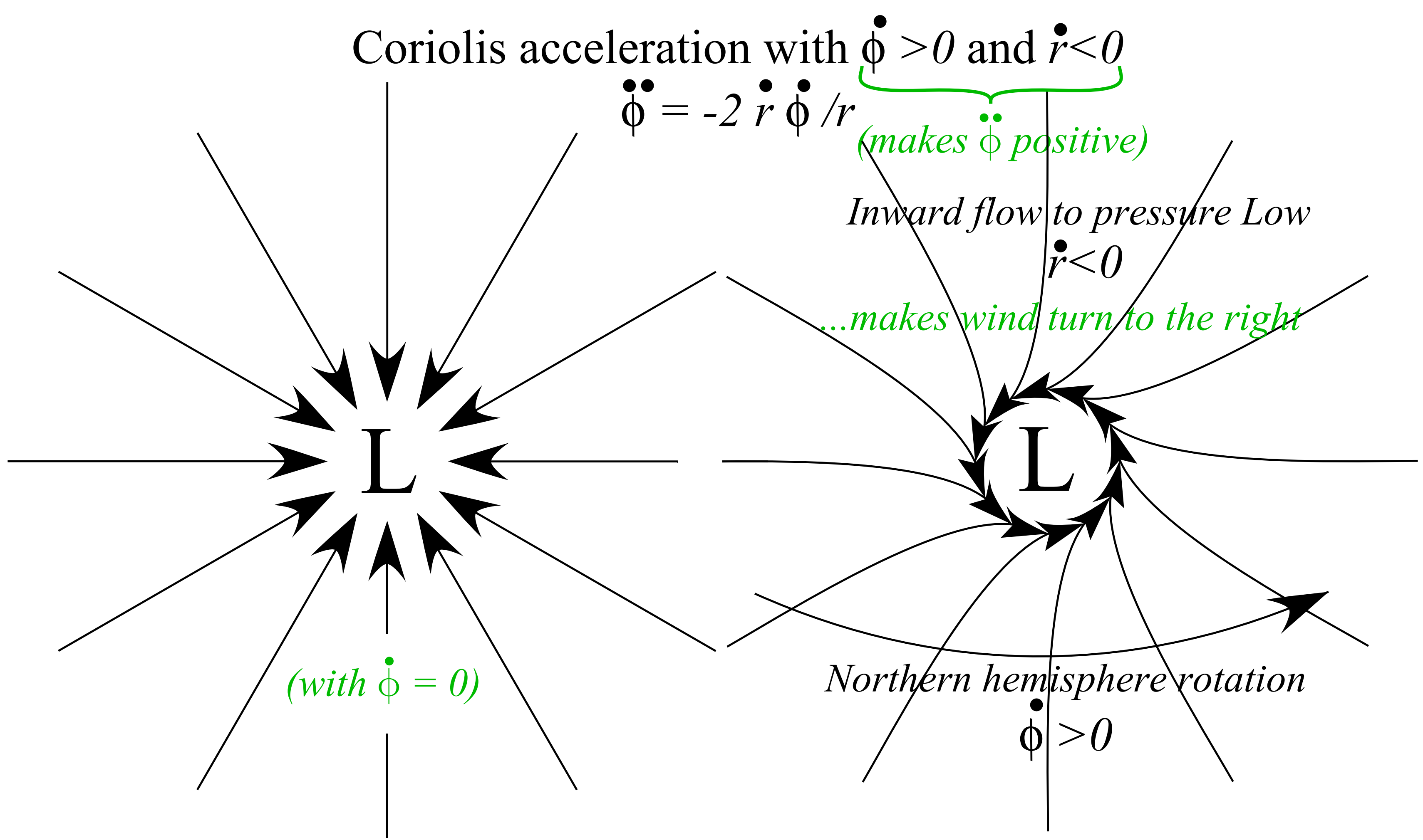
radial force: $M\ddot{r} = Mr\dot{\phi}^2 - \frac{\partial U}{\partial r}$

angular force or torque: $Mr^2\ddot{\phi} = -2Mr\dot{r}\dot{\phi} - \frac{\partial U}{\partial \phi}$

Field-free (U=0)

radial acceleration: $\ddot{r} = r\dot{\phi}^2$

angular acceleration: $\ddot{\phi} = -2\frac{\dot{r}\dot{\phi}}{r}$



Effect on <u>Northern</u> Hemisphere local weather

Cyclonic flow around lows

Rewriting GCC Lagrange equations :

$$\dot{p}_r \equiv \frac{dp_r}{dt} = M\ddot{r} = Mr\dot{\phi}^2 - \frac{\partial U}{\partial r}$$

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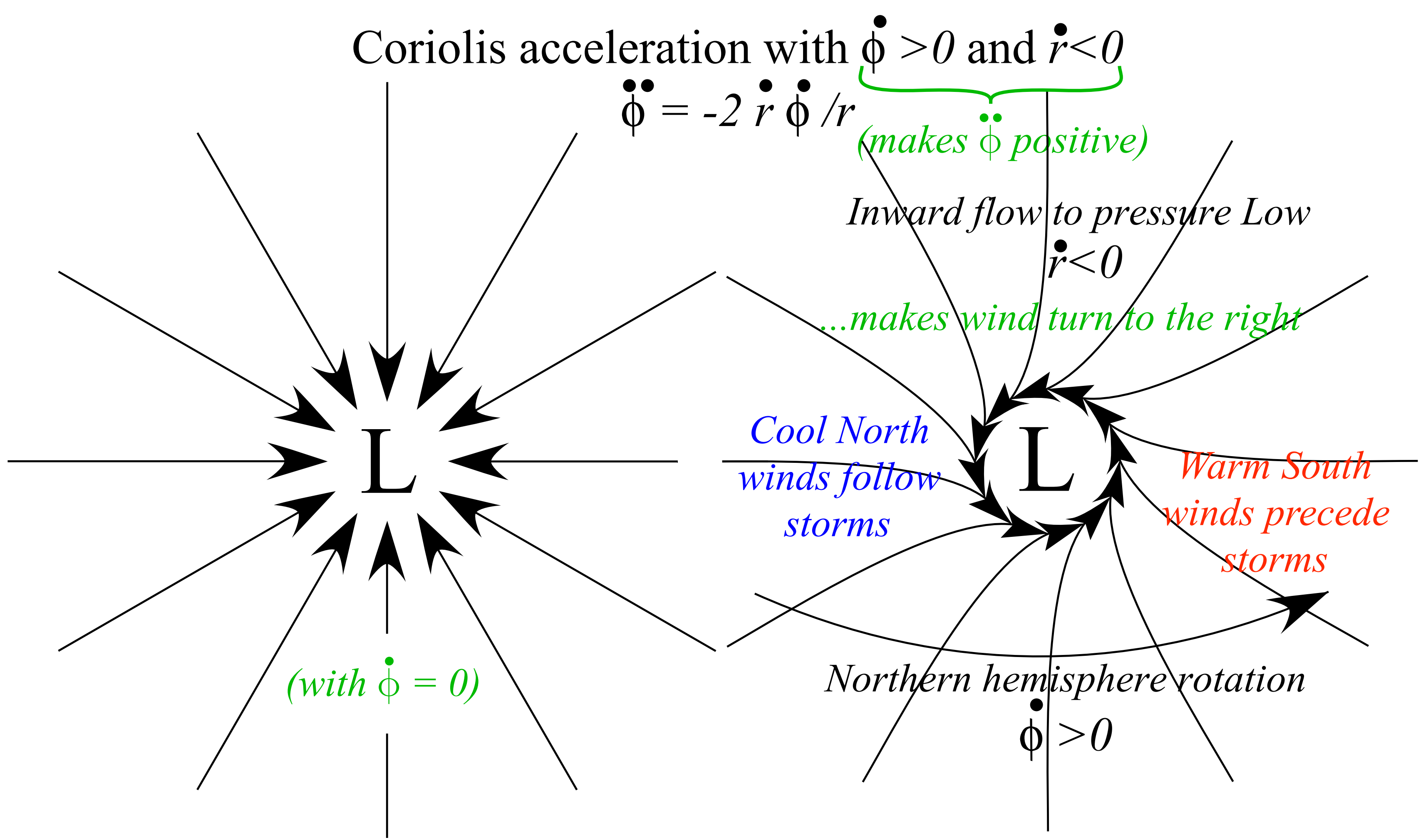

Effect on Northern Hemisphere local weather

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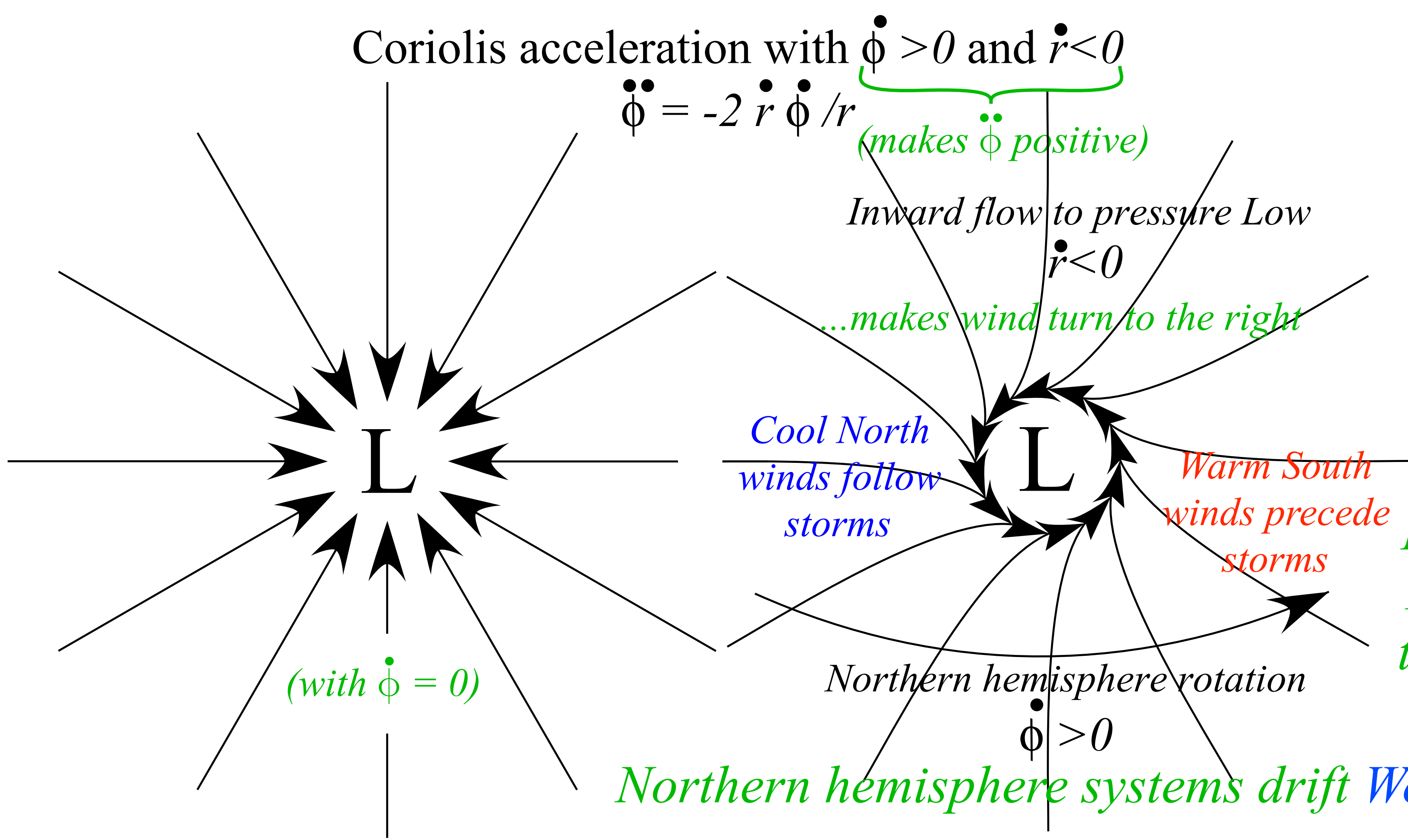
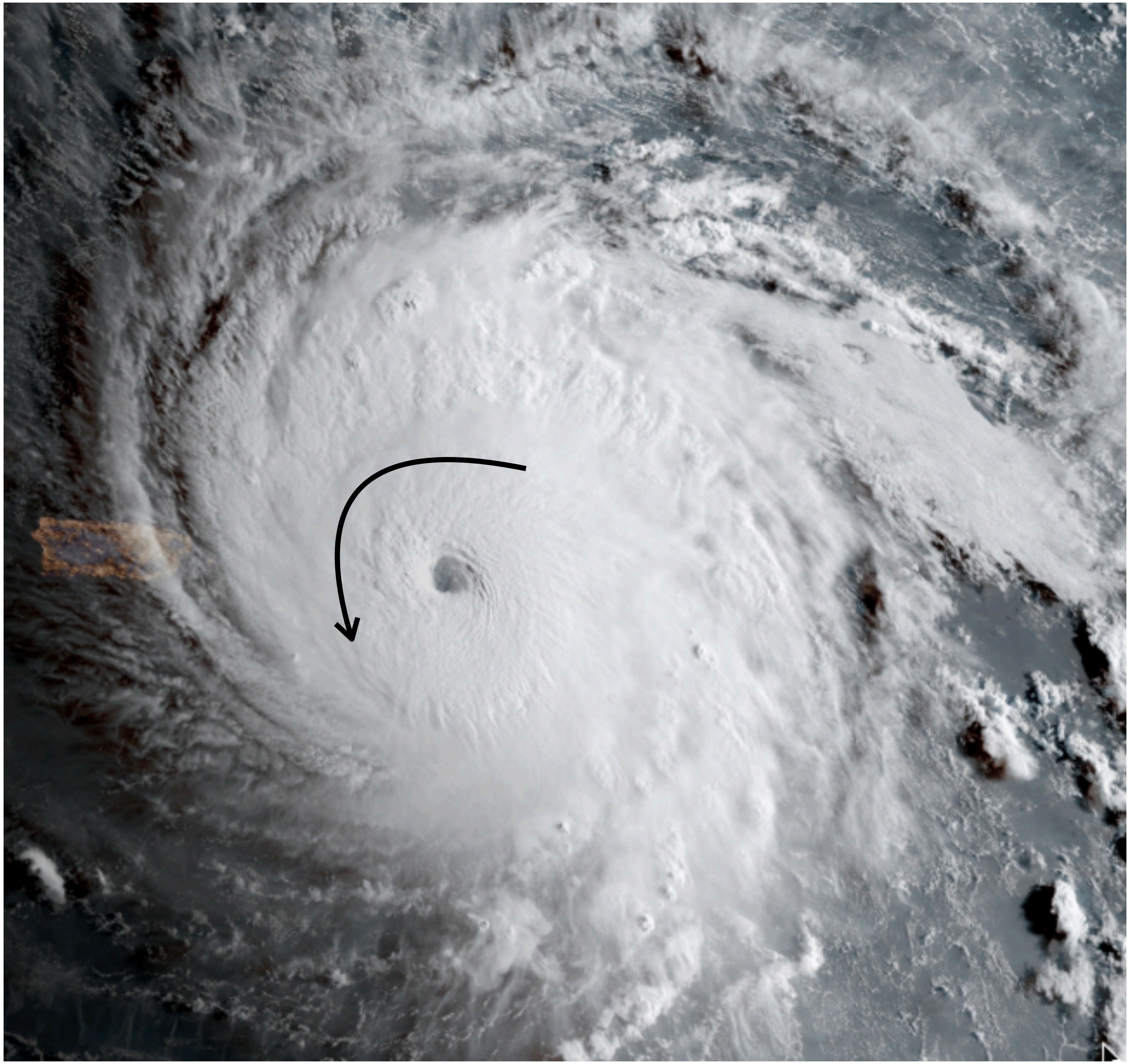


Effect on Northern Hemisphere local weather

Cyclonic flow around lows

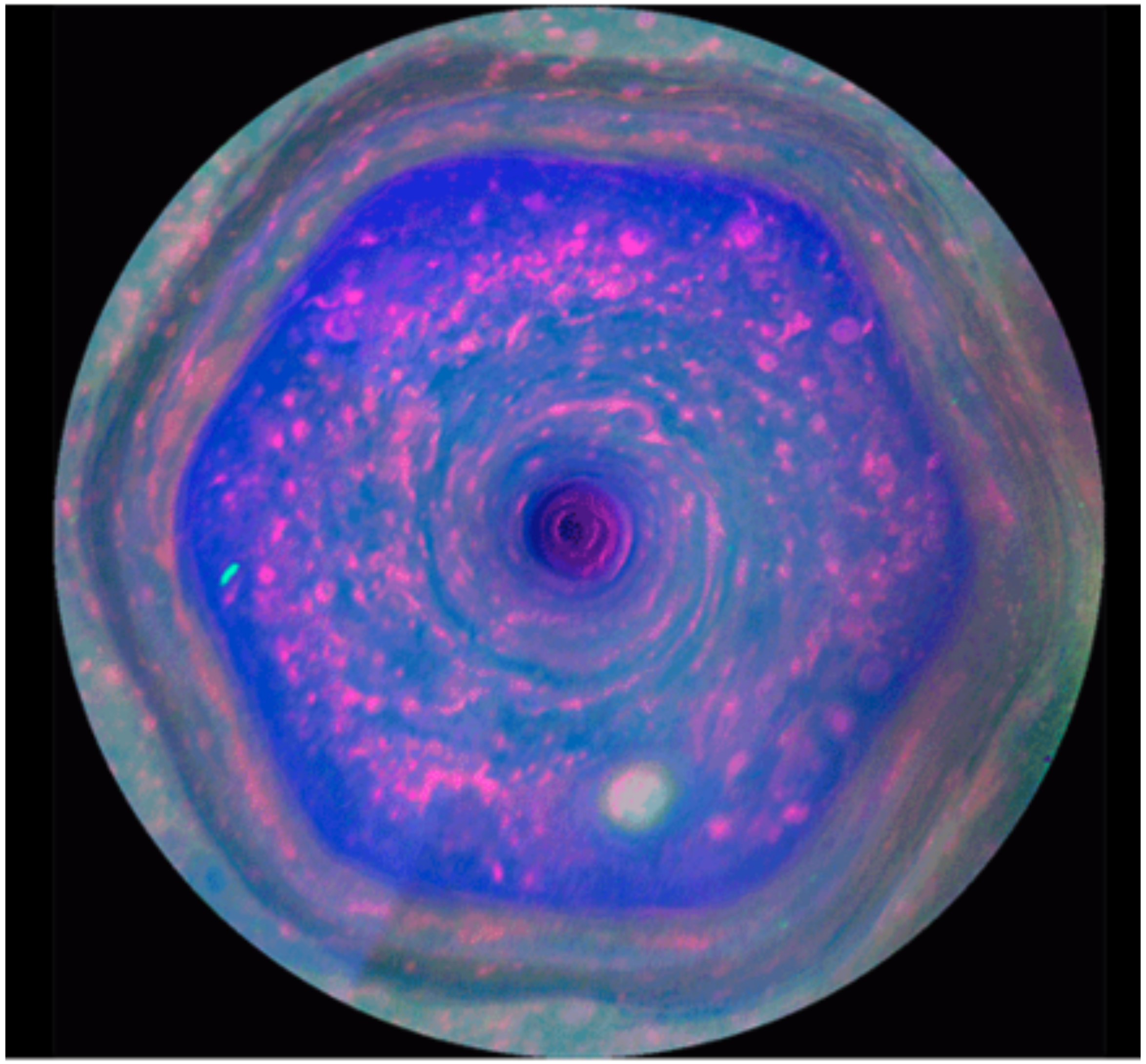
Deep quantum rule: Flow tries to mimic the external rotation (least relative v)

Northern hemisphere systems drift West to East



GOES-16 captured this geocolor image of Hurricane Irma approaching Anguilla at about 7:15 am (eastern), September 6, 2017. Irma's maximum sustained winds remain near 185 mph with higher gusts, making it a category 5 hurricane on the Saffir-Simpson Hurricane Wind Scale. According to the latest information from NOAA's National Hurricane Center (issued at 8:00 am eastern), Irma was located about 15 miles west-southwest of Anguilla and moving toward the west-northwest near 16 miles per hour.

Science News <link>



Saturn's north pole was dark when Cassini arrived in 2004. But as the seasons changed, light illuminated a bizarre six-sided swirl of gases at the pole (shown here in false color). The hexagon has been known since the 1980s. It is about 30,000 kilometers (18,600 miles) wide with a massive hurricane centered on the north pole.
JPL-CALTECH/NASA, SPACE SCIENCE INSTITUTE

Lecture 9 ends here
Wed. 9.25.2019

https://www.sciencenewsforstudents.org/article/cassini-spacecraft-takes-its-final-bow

AMOP reference links *(Updated list given on **2nd** and 3rd pages of each class presentation)*

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2014 AMOP
2017 Group Theory for QM
2018 AMOP

Representaions Of Multidimensional Symmetries In Networks - harter-jmp-1973

Alternative Basis for the Theory of Complex Spectra

- Alternative_Basis_for_the_Theory_of_Complex_Spectra_I_-_harter-pra-1973
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_II_-_harter-patterson-pra-1976
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_III_-_patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978
Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979
Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984
Galloping waves and their relativistic properties - ajp-1985-Harter
Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)
II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

Rotation–vibration spectra of icosahedral molecules.

I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)
II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)
III) Half-integral angular momentum - harter-reimer-jcp-1991

Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 (Alt scan)
Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene -Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)
Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996
Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - jcp-Reimer-Harter-1997 (HiRez)
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001
Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

Resonance and Revivals

I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank
II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)
III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)
Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013
QTCA Unit 10 Ch 30 - 2013
AMOP Ch 0 Space-Time Symmetry - 2019

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https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and 3rd pages of each class presentation)*

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , *QTCA Unit 7 Ch. 23-26),* *(PSDS - Ch. 5, 7)*

Int.J.Mol.Sci, 14, 714(2013), *QTCA Unit 8 Ch. 23-25,* *QTCA Unit 9 Ch. 26,* *PSDS Ch. 5,* *PSDS Ch. 7*

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.
Deeper theory ends p53

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande' g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Young Tableau Magic Formulae
GrpThLect29 p46-48.

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AMOP reference links *(Updated list given on 2nd and **3rd** and **4th** pages of each class presentation)*

Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

Chaos_Classical_and_Quantum_-_2018-Cvitanovic-ChaosBook
Group Theory - PUP_Lucy_Day_-_Diagrammatic_notation_-_Ch4
Simplification_Rules_for_Birdtrack_Operators_-_Alcock-Zeilinger-Weigert-zeilinger-jmp-2017
Group Theory - Birdtracks_Lies_and_Exceptional_Groups_-_Cvitanovic-2011
Simplification_rules_for_birdtrack_operators-_jmp-alcock-zeilinger-2017
Birdtracks for SU(N) - 2017-Keppeler

Frank Rioux's: UMA method of vibrational induction

Quantum_Mechanics_Group_Theory_and_C60_-_Frank_Rioux_-_Department_of_Chemistry_Saint_Johns_U
Symmetry_Analysis_for_H20-_H20GrpTheory-_Rioux
Quantum_Mechanics-Group_Theory_and_C60_-_JChemEd-Rioux-1994
Group_Theory_Problems-_Rioux-_SymmetryProblemsX
Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

Supplemental AMOP Techniques & Experiment

Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
High-resolution_spectroscopy_and_global_analysis_of_CF4_rovibrational_bands_to_model_its_atmospheric_absorption-_carlos-Boudon-jqsrt-2017
Symmetry and Chirality - Continuous_Measures_-_Avnir
*

Special Topics & Colloquial References

r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

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