2019 CMwBang! site

Class YouTube Channel

Lecture 7
Wed. 9.18.2019

Kepler Geometry of IHO (Isotropic Harmonic Oscillator) Elliptical Orbits

(Ch. 9 and Ch. 11 of Unit 1)

Constructing 2D IHO orbital phasor "clock" dynamics in uniform-body

Constructing 2D IHO orbits using Kepler anomaly plots
- *Mean-anomaly and eccentric-anomaly geometry*
- *Calculus and vector geometry of IHO orbits*
- *A confusing introduction to Coriolis-centrifugal force geometry* *(Derived better in Ch. 12)*

Some Kepler's "laws" for all central (isotropic) force F(r) fields
- *Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$* *(Derived here)*
- *Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$* *(Derived in Unit 5)*
- *Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$* *(Derived here)*
- *Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$* *(Derived in Unit 5)*

Introduction to dual matrix operator contact geometry (based on IHO orbits)
- *Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ vs.inverse form ellipse $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$*
  - *Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$)*
  - *Q-Ellipse tangents $\mathbf{r}'$ normal to dual Q^{-1}-ellipse position $\mathbf{p}$ ($\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$)*
- *Operator geometric sequences and eigenvectors*
- *Alternative scaling of matrix operator geometry*
  - *Vector calculus of tensor operation*

Q:Where is this headed? A: Lagrangian-Hamiltonian duality

Web Links

BoxIt simulation: IHO orbits w/Stokes plot
RelaWavity Simulation: IHO orbital time rates of change
RelaWavity Simulation: Exegesis Plot

This Lecture's Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2017 Group Theory for QM
2018 Adv CM
2018 AMOP
2019 Advanced Mechanics

Lecture #7

Pirelli Site: Phasors animimation
CMwithBang Lecture #6, page=70 (9.10.18)

BoxIt Web Simulations:

- *Generic/Default*
- *Most Basic A-Type*
- *Basic A-Type w/reference lines*
- *Basic A-Type A-Type with Potential energy*
- *A-Type with Potential energy and Stokes Plot*
- *A-Type w/3 time rates of change*
- *A-Type w/3 time rates of change with Stokes Plot*
- *B-Type (A=1.0, B=-0.05, C=0.0, D=1.0)*

RelaWavity Web Elliptical Motion Simulations:

- *Orbits with b/a=0.125*
- *Orbits with b/a=0.5*
- *Orbits with b/a=0.7*
- *Exegesis with b/a=0.125*
- *Exegesis with b/a=0.5*
- *Exegesis with b/a=0.7*
- *Contact Ellipsometry*

Burning a hole in reality—design for a new laser may be powerful enough to pierce space-time
Trampoline mirror may push laser pulse through fabric of the Universe
Achieving_Extreme_Light_Intensities_using_Optically_Curved_Relativistic_Plasma_Mirrors_-_Vincenti-prl-2019
A_Soft_Matter_Computer_for_Soft_Robots_-_Garrad-sr-2019
Correlated_Insulator_Behaviour_at_Half-Filling_in_Magic-Angle_Graphene_Superlattices_-_cao-n-2018

Running Reference Link Listing

RelaWavity Web Simulation: Contact Ellipsometry
BoxIt Web Simulation: Elliptical Motion (A-Type)
CMwBang Course: Site Title Page
Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors
UAF Physics UTube channel

Prior to Lecture #7

Velocity Amplification in Collision Experiments Involving Superballs -
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

These *are* hot off the presses. Out in MISC for quick reference.
Sorting_ultracold_atoms_in_a_three-dimensional_optical_lattice_in_a_realization_of_Maxwell's_Demon - Kumar-n-2018
Synthetic_three-dimensional_atomic_structures_assembled_atom_by_atom - Barredo-n-2018

Older ones:
Wave-particle_duality_of_C60_molecules - Arndt-ltn-1999
Optical_Vortex_Knots_-_One_Photon__At_A_Time - Tempone-Wiltshire-Sr-2018
Baryon_Deceleration_by_Strong_Chromofields_in_Ultrarelativistic_, Nuclear_Collisions - Mishustin-PhysRevC-2007, APS Link & Abstract
Hadronic Molecules - Guo-x-2017
Hidden-charm_pentaquark_and_tetraquark_states - Chen-pr-2016

BounceItIt Web Animation - Scenarios:
49:1 y vs t, 49:1 V2 vs V1, 1:500:1 - 1D Gas Model w/ faux restorative force (Cool),
1:500:1 - 1D Gas (Warm), 1:500:1 - 1D Gas Model (Cool, Zoomed in),
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, Quantum_Carpet_wMBars,
Quantum_Carpet_BCar, Quantum_Carpet_BCar_wMBars
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Web Animation - Scenarios:
Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

BounceIt Dual plots
m_1:m_2 = 3:1
v_2 vs v_1 and V_2 vs V_1, (v_1, v_2)=(1, 0.1), (v_1, v_2)=(1, 0)
y_2 vs y_1 plots: (v_1, v_2)=(1, 0.1), (v_1, v_2)=(1, 0), (v_1, v_2)=(1, -1)
Estrangian plot V_2 vs V_1: (v_1, v_2)=(0, 1), (v_1, v_2)=(1, -1)
m_1:m_2 = 4:1
v2 vs v1, y2 vs y1
m_1:m_2 = 100:1, (v_1, v_2)=(1, 0): V2 vs V1 Estrangian plot, y2 vs y1 plot

With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

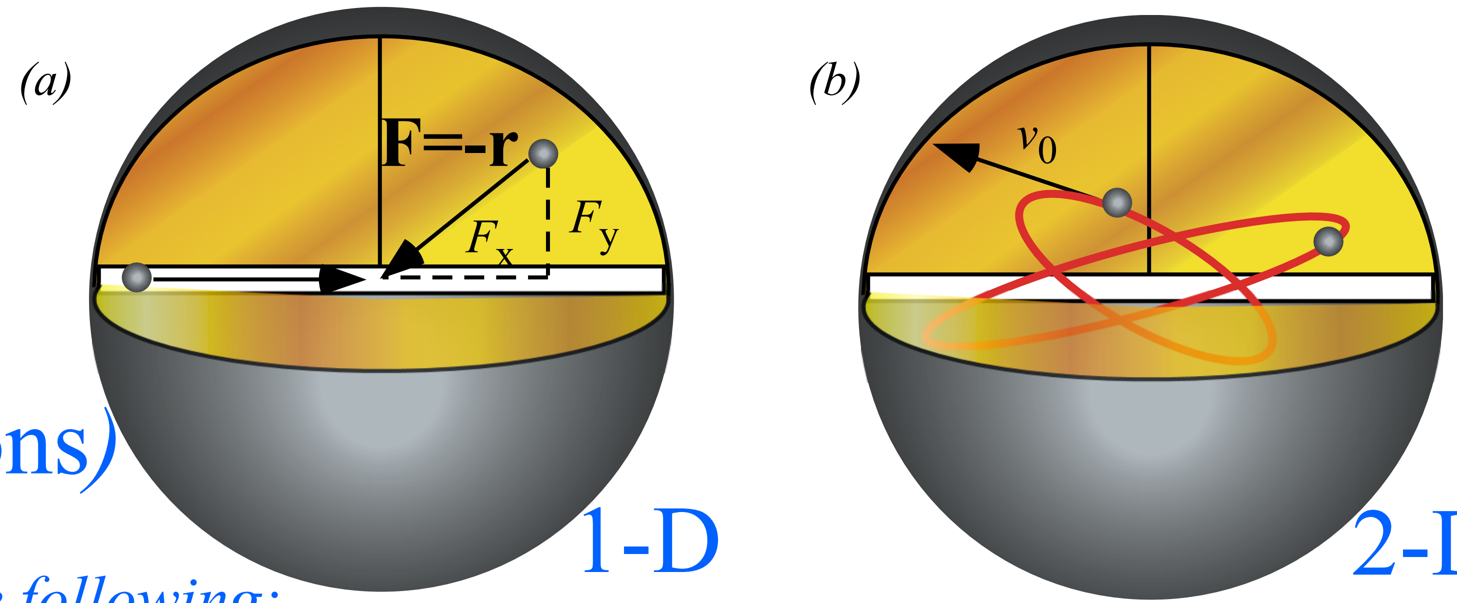
X2 paper: Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>

More Advanced QM and classical references at the end of this Lecture

→ *Introducing 2D IHO orbits and phasor geometry*

Phasor "clock" geometry

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

I.H.O. Force law

F =-x (1-Dimension)

F =-**r** (2 or 3-Dimensions)

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

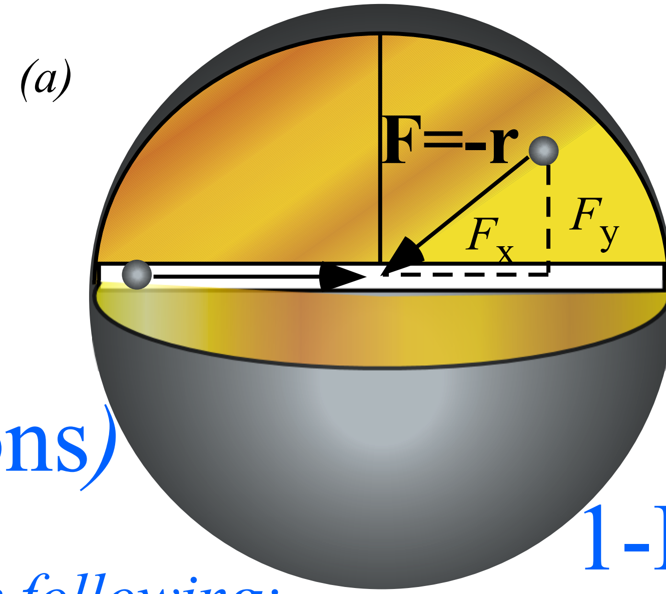
Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

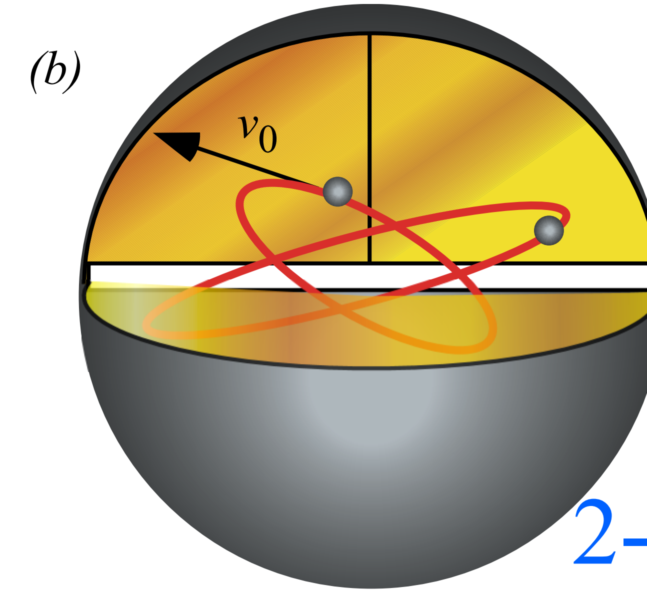
Isotropic Harmonic Oscillator phase dynamics in uniform-body

<u>*I.H.O. Force law*</u>

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)





Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal <u>isotropic</u> case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

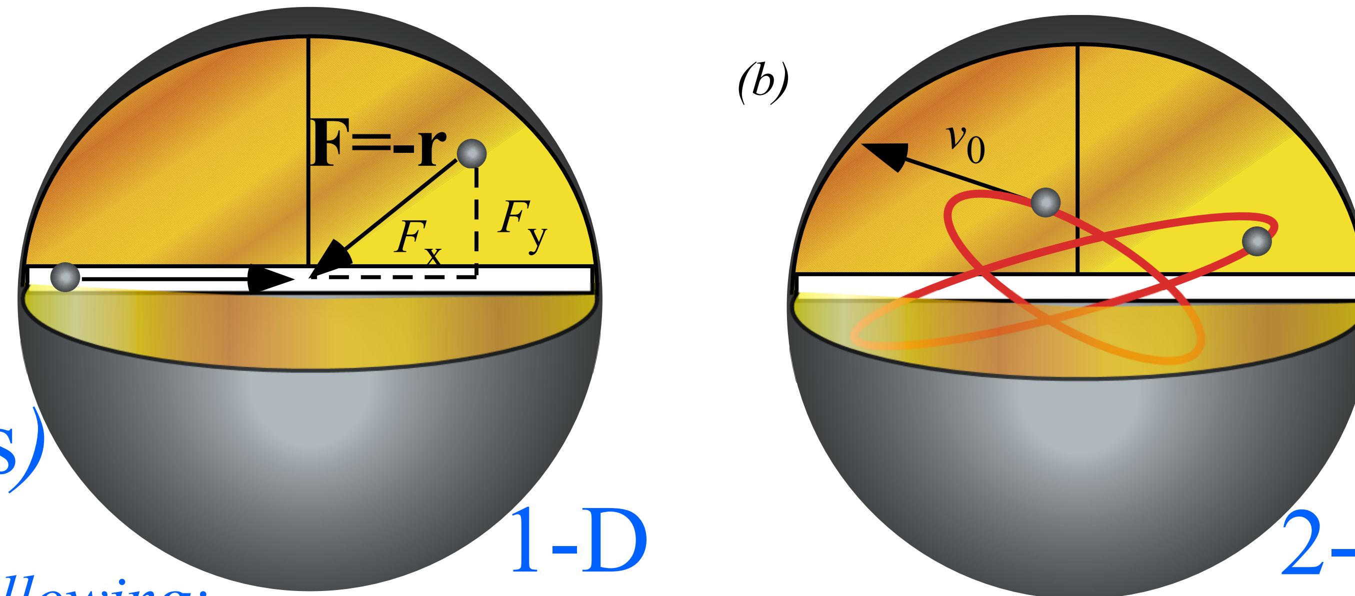
Another example of the old "scale-a-circle" trick...

velocity: *position:*

$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a) (b)



Unit 1
Fig. 9.10

I.H.O. Force law

F =-x (1-Dimension)

F =-**r** (2 or 3-Dimensions)

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

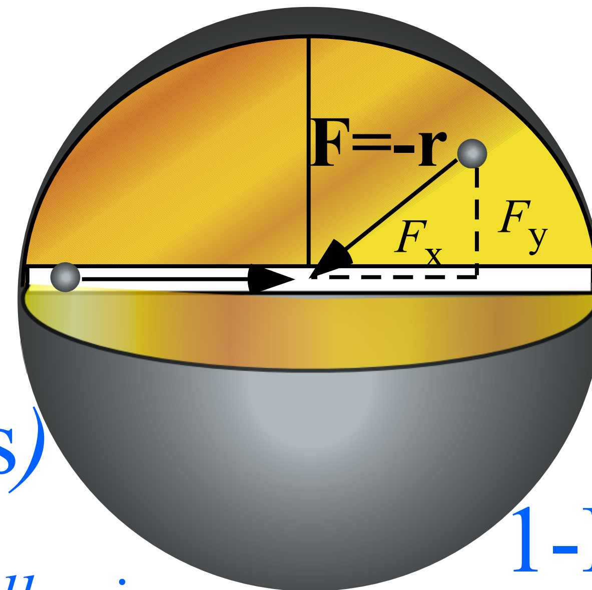
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

F =-*x* (1-Dimension)

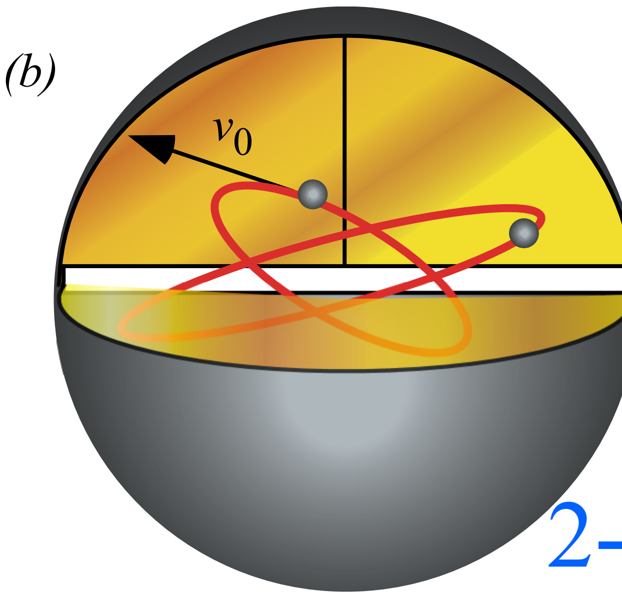
F =-**r** (2 or 3-Dimensions)

(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

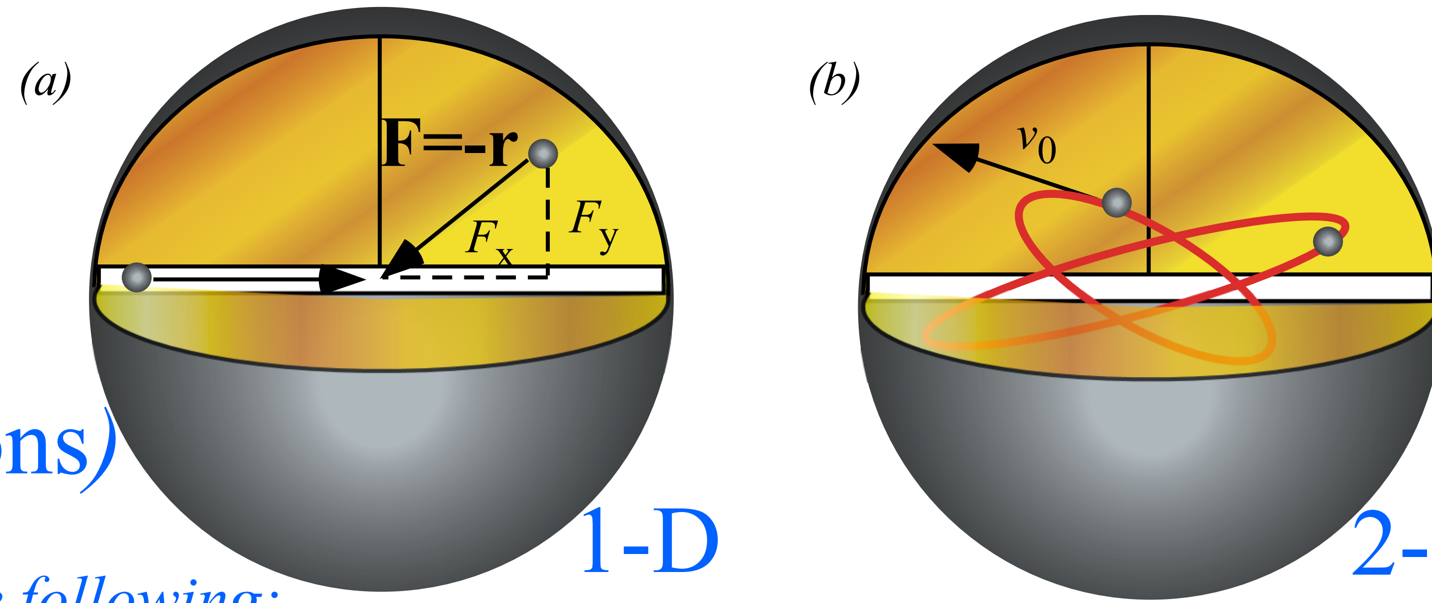
velocity: *position:* *angular velocity:*

$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta,\quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt}$$

by **(1)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 9.10

I.H.O. Force law

F =-x (1-Dimension)

F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

Let : **(1)** $v = \sqrt{2E/m}\cos\theta$, *and :* **(2)** $x = \sqrt{2E/k}\sin\theta$ *def.* **(3)** $\omega = \frac{d\theta}{dt}$

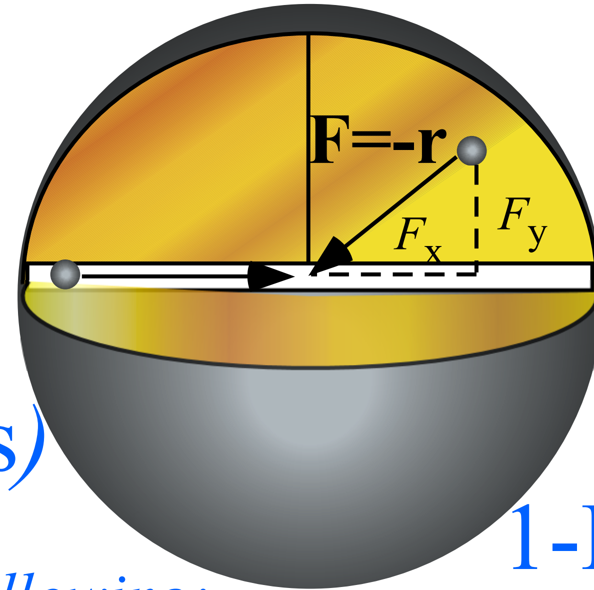
$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta}$$

by **(1)**

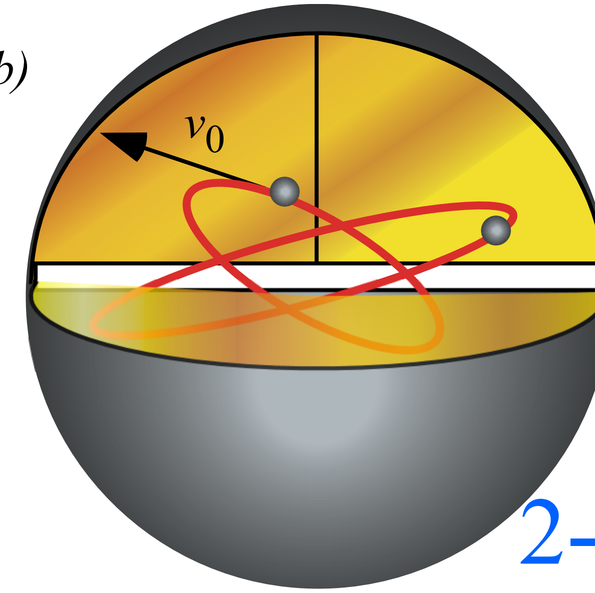
simple calculus

Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



(b)



Unit 1
Fig. 9.10

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

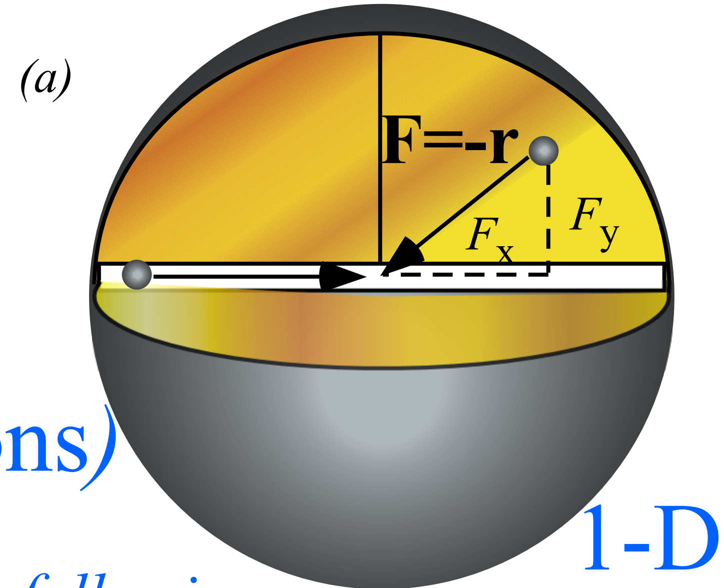
velocity: *position:* *angular velocity:*

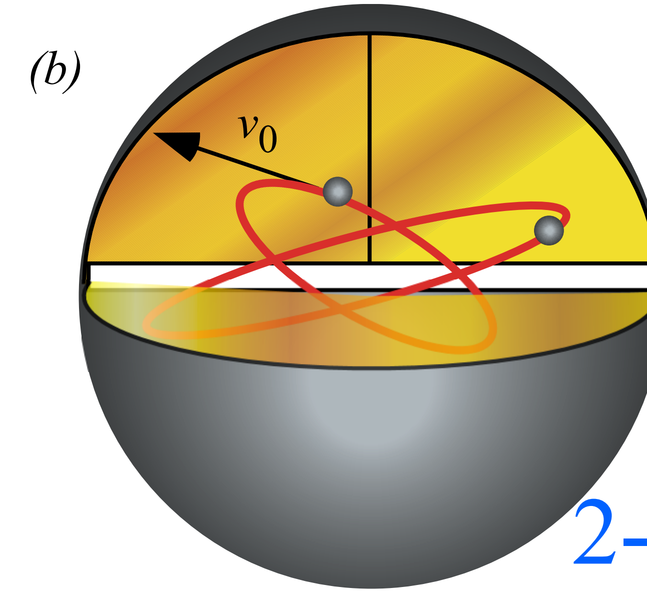
$$Let: \ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and: \ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \mathbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}$$

by **(1)** *by def.* **(3)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-*x* (1-Dimension)

F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

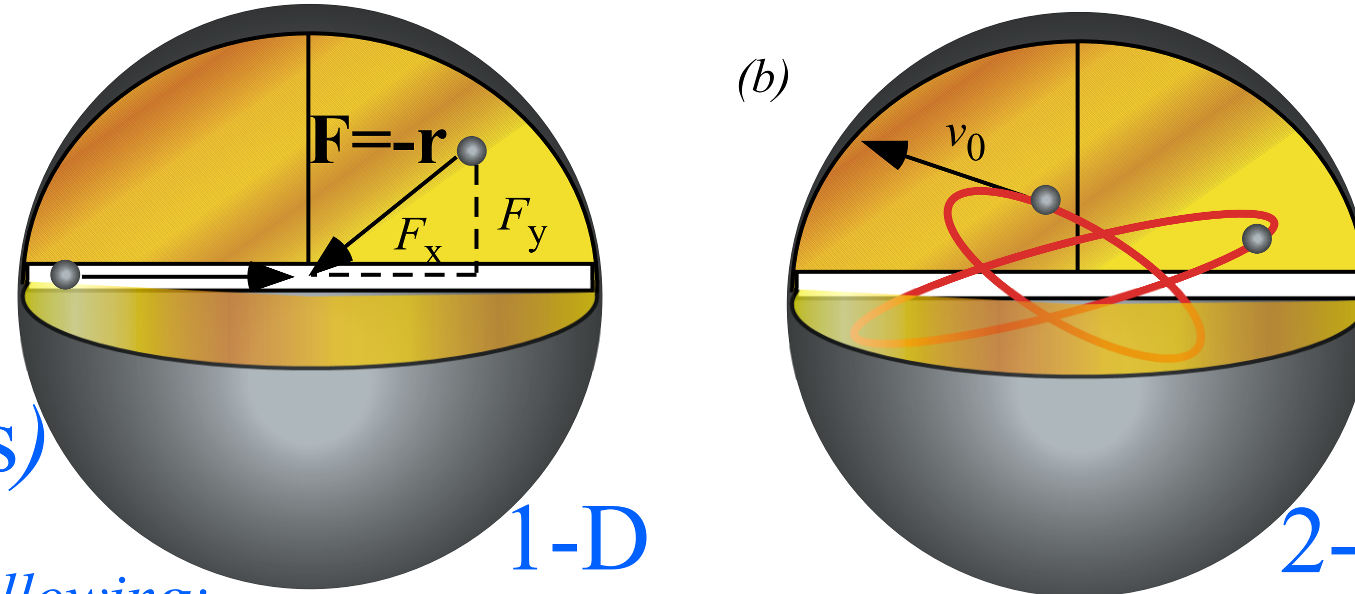
velocity: *position:* *angular velocity:*

$$Let: \ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and: \ (2)\ x = \sqrt{2E/k}\sin\theta \qquad def.\ (3)\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by (1) by def. (3) by (2)

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

def. (3) $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by (1) *by def.* (3) *by* (2)

by def. (3)

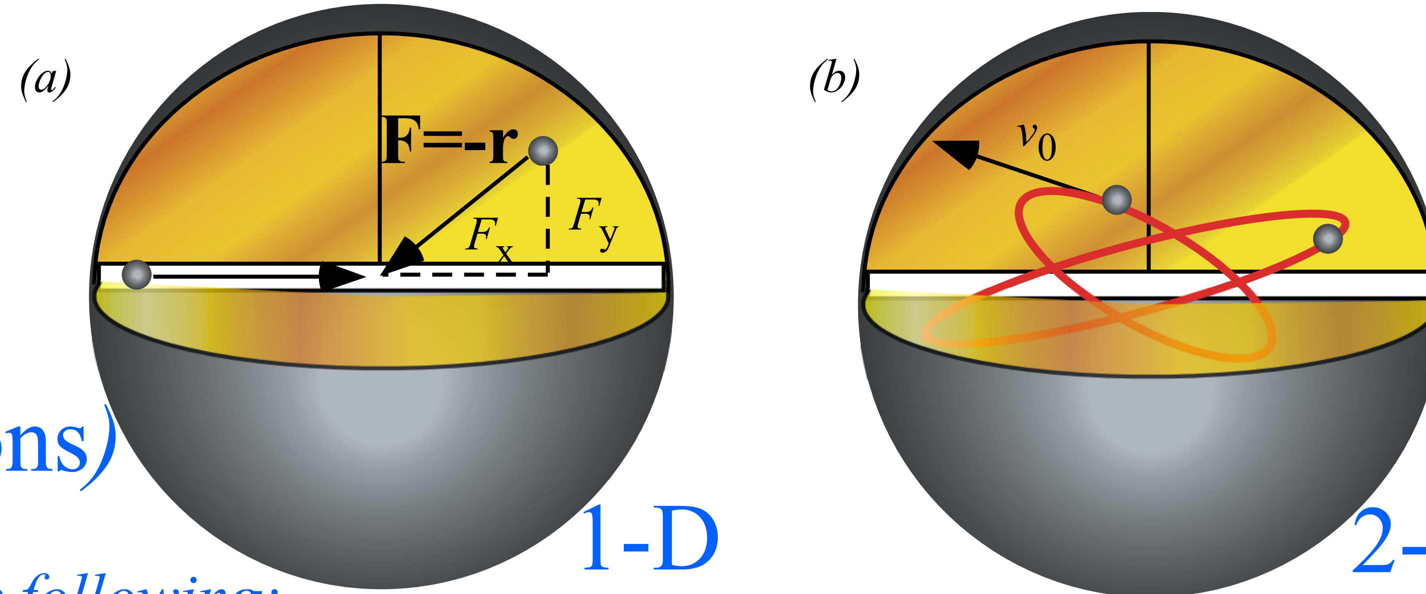
$$\omega = \frac{d\theta}{dt}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)



1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 9.10

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by **(1)** — *by def.* **(3)** — *by* **(2)**

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}}\cos\theta}{\sqrt{\frac{2E}{k}}\cos\theta}$$

divide **(1)** — *by* **(2)** *derivative*

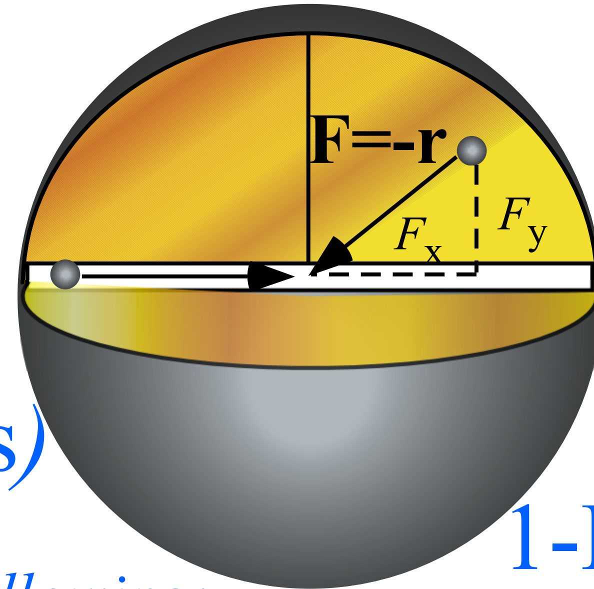
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

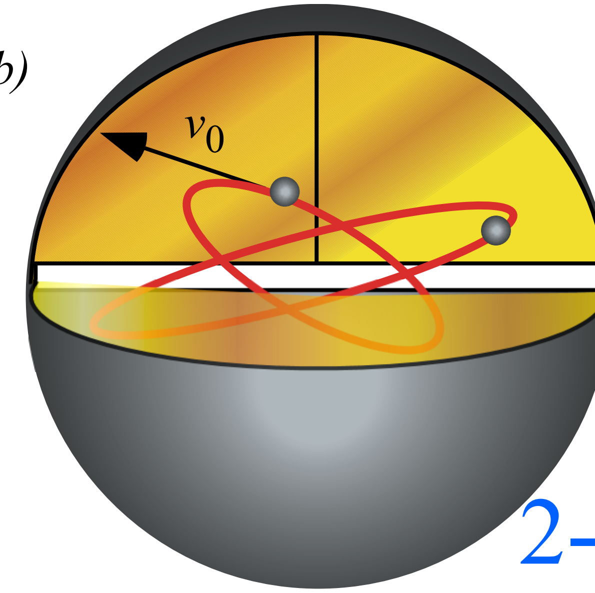
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \mathbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by (1) — by def. (3) — by (2)

by def. (3)

$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}}\cos\theta}{\sqrt{\frac{2E}{k}}\cos\theta} = \sqrt{\frac{k}{m}}$$

divide (1) by (2) derivative

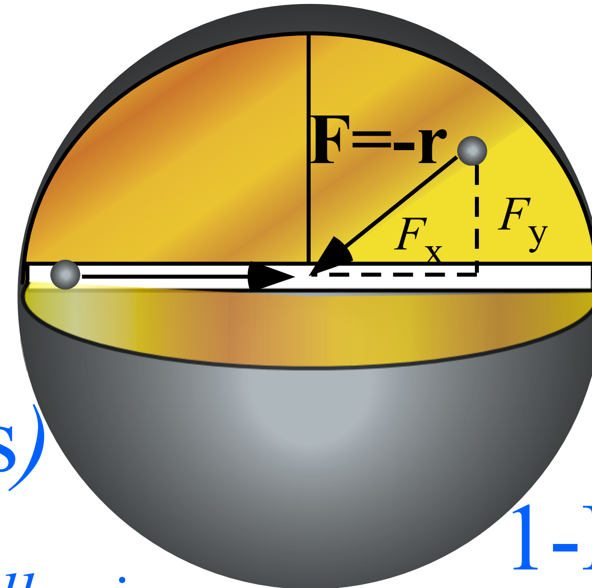
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

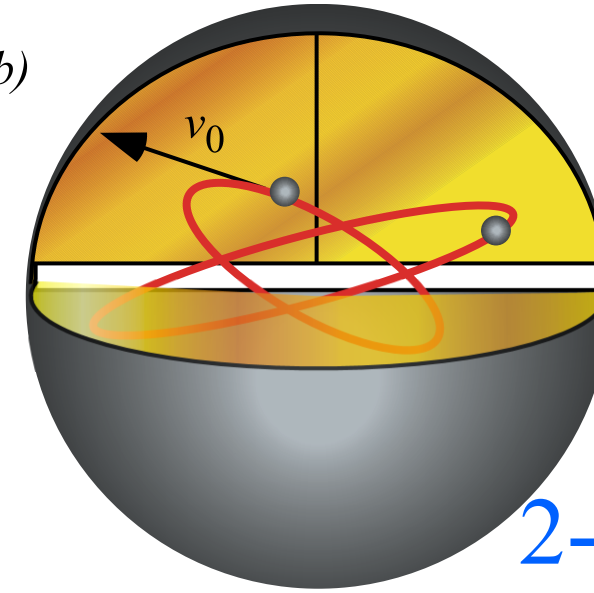
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

(a)



1-D

(b)



2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1 Fig. 9.10

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal *isotropic* *case.*

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta$$

angular velocity: *def.* **(3)** $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by **(1)** *by def.* **(3)** *by* **(2)**

by def. **(3)**

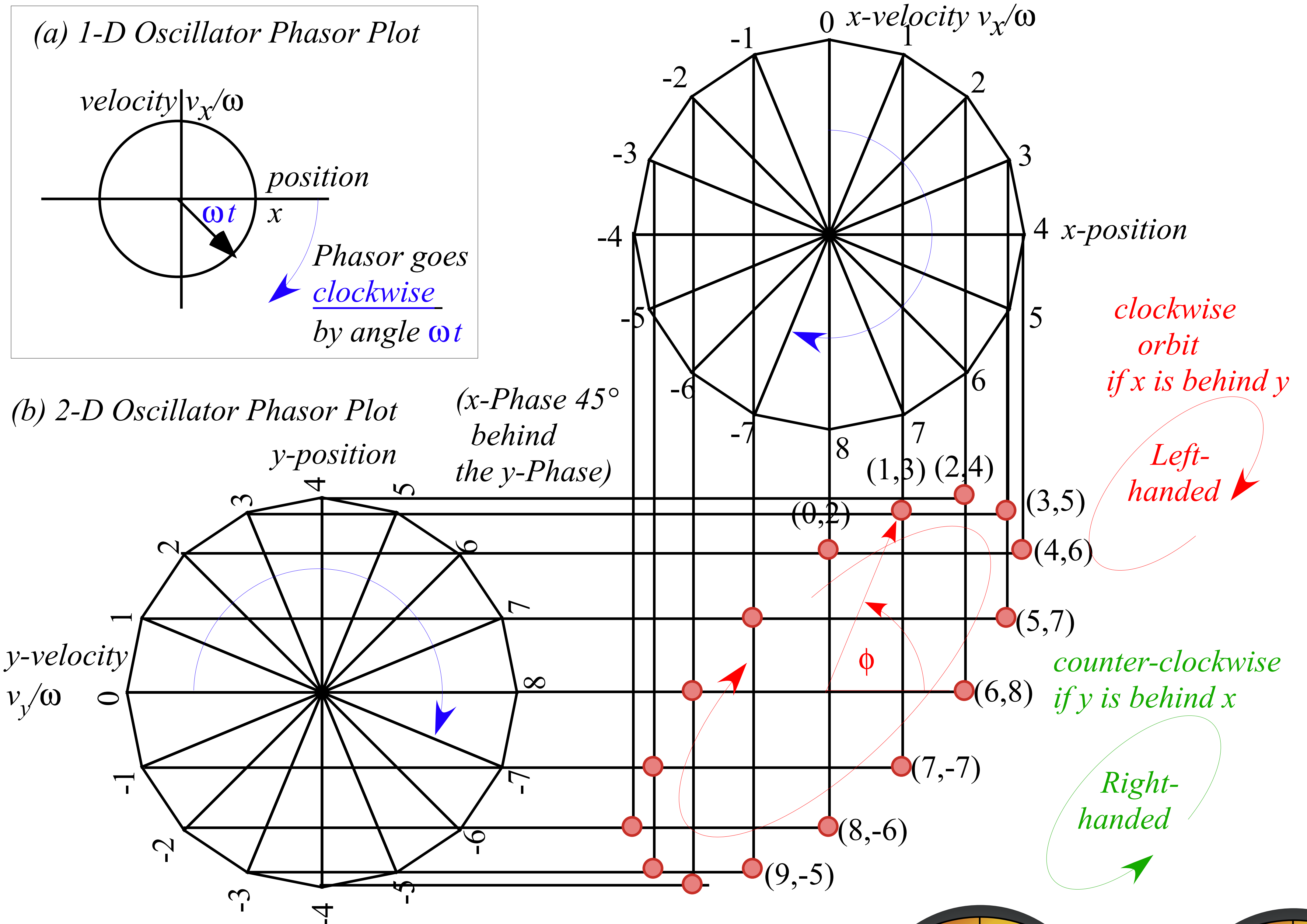
$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

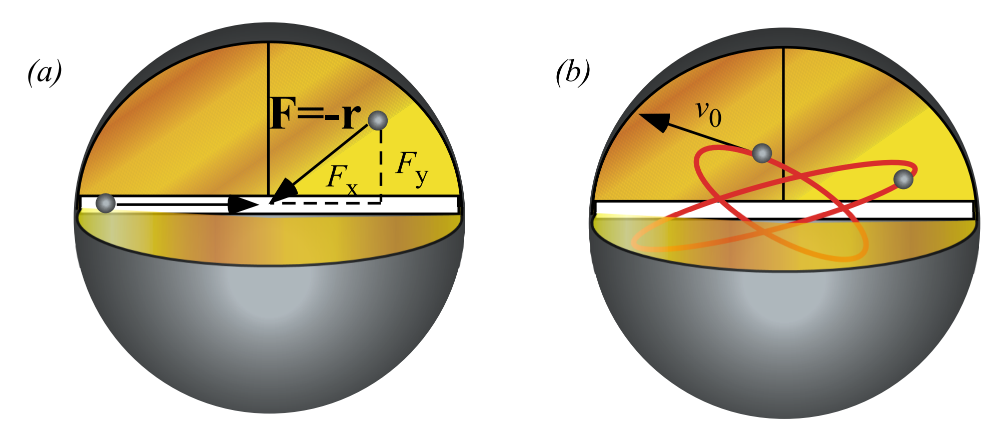
→ *Review of IHO orbital phasor "clock" dynamics in uniform-body with two "movie" examples*

Review of IHO orbital phase dynamics in uniform-body



Unit 1
Fig. 9.10

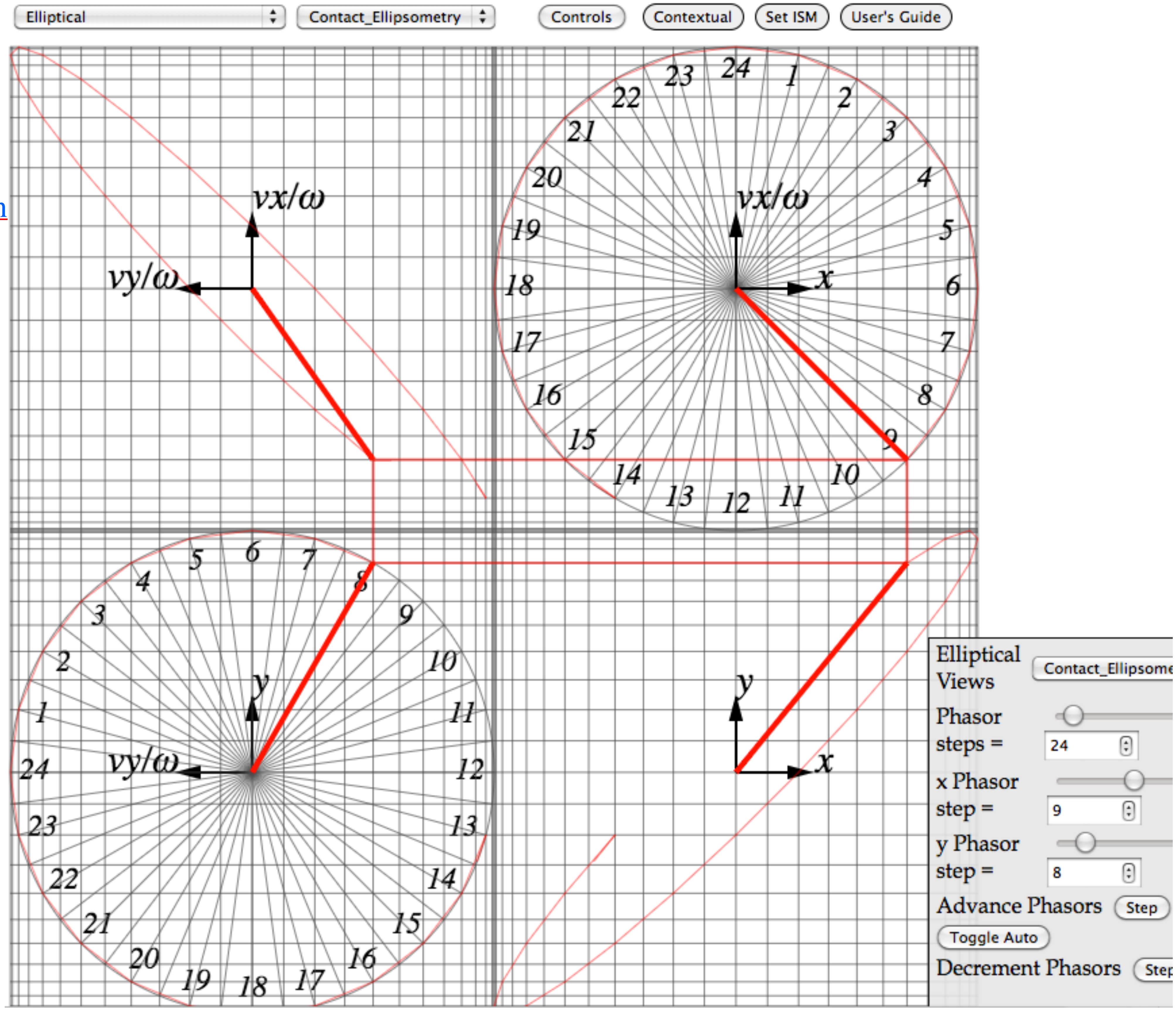
RelaWavity web simulation - Contact ellipsometry



Introduction to Phasors at our Pirelli Relativity Site

BoxIt web simulation - *With y-Phasor is on other side of xy plot*

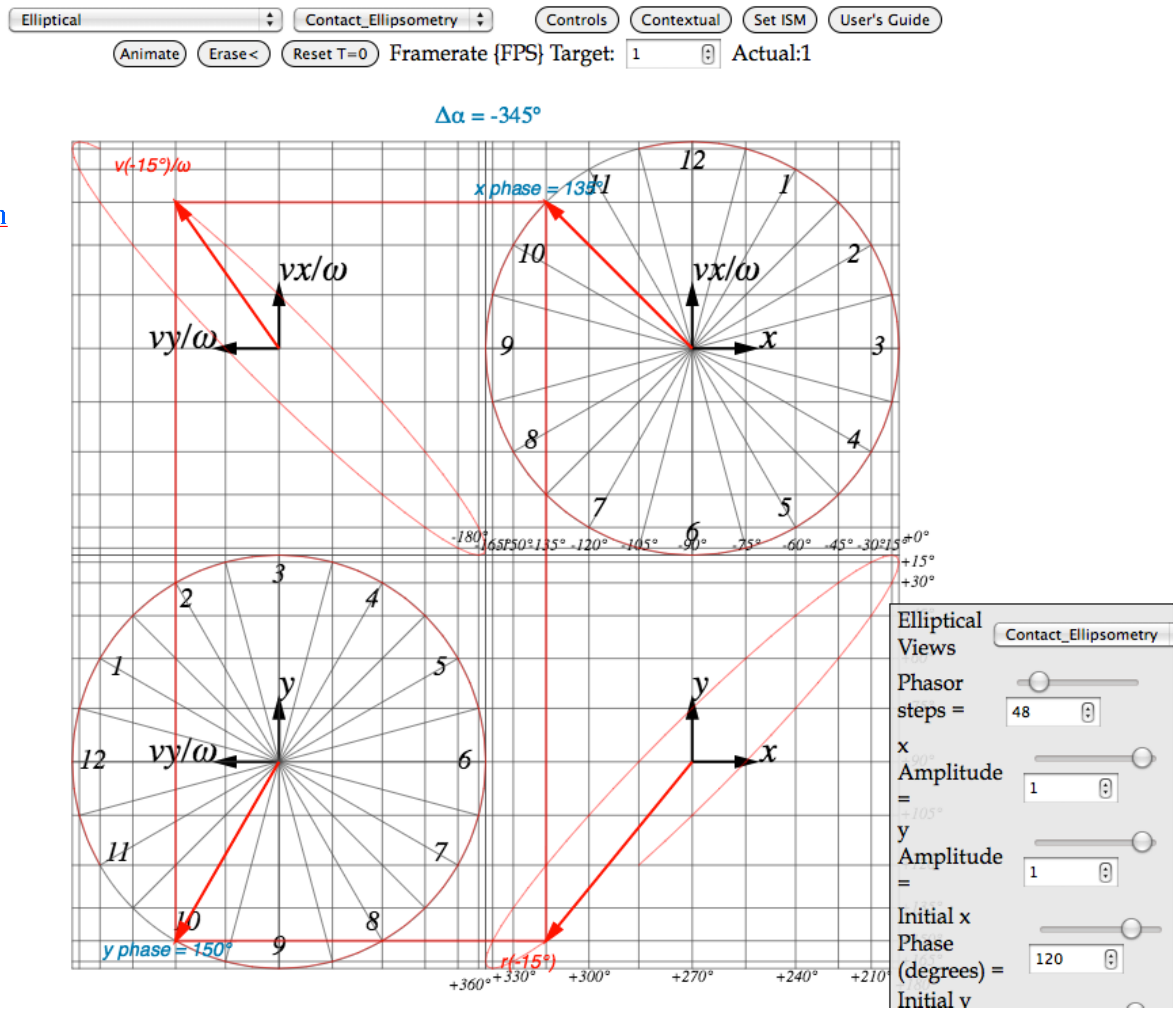
[RelaWavity Web Simulation Ellipsometry]



Geometry of Kepler anomalies for vectors [**r**(ϕ), **v**(ϕ)] *in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity described below after p.70.*

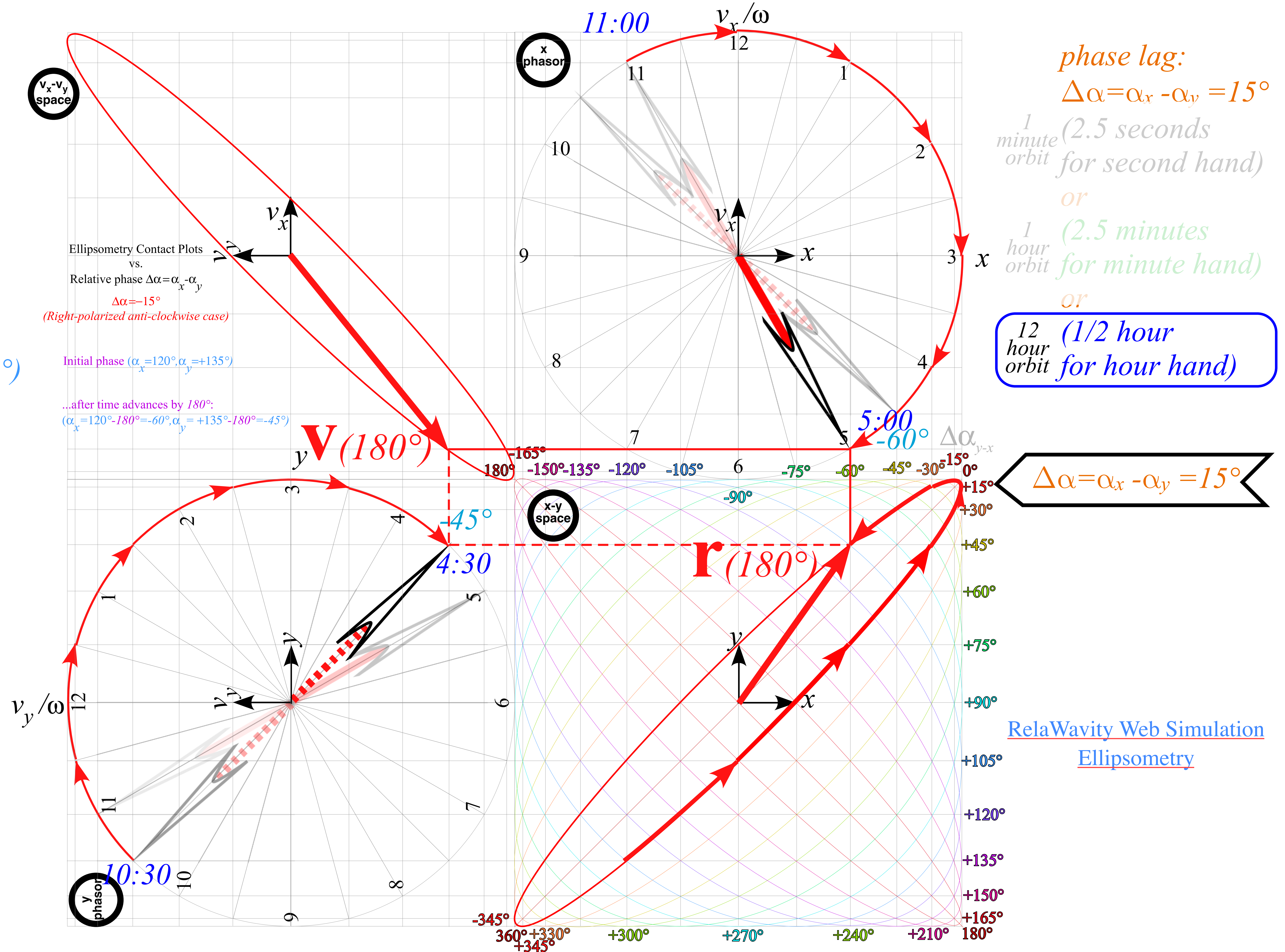
[RelaWavity web simulation - Contact ellipsometry] *(User Mouse Input allowed for setting phasor values)*

[RelaWavity Web Simulation Ellipsometry]



*Geometry of Kepler anomalies for vectors [**r**(ϕ), **v**(ϕ)] in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity described below after p.7 and p.17.*

RelaWavity web simulation - Contact ellipsometry *(User Mouse Input allowed for setting phasor values)*



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y = +135°-180°=-45°)
phase lag:
Δα=α_x -α_y =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
x phasor
v_x-v_y space
x-y space
y phasor
v_x/ω
v_y/ω
11:00
5:00
4:30
10:30
V(180°)
r(180°)
-60°
-45°
Δα_y-x
Δα=α_x -α_y =15°
RelaWavity Web Simulation
Ellipsometry

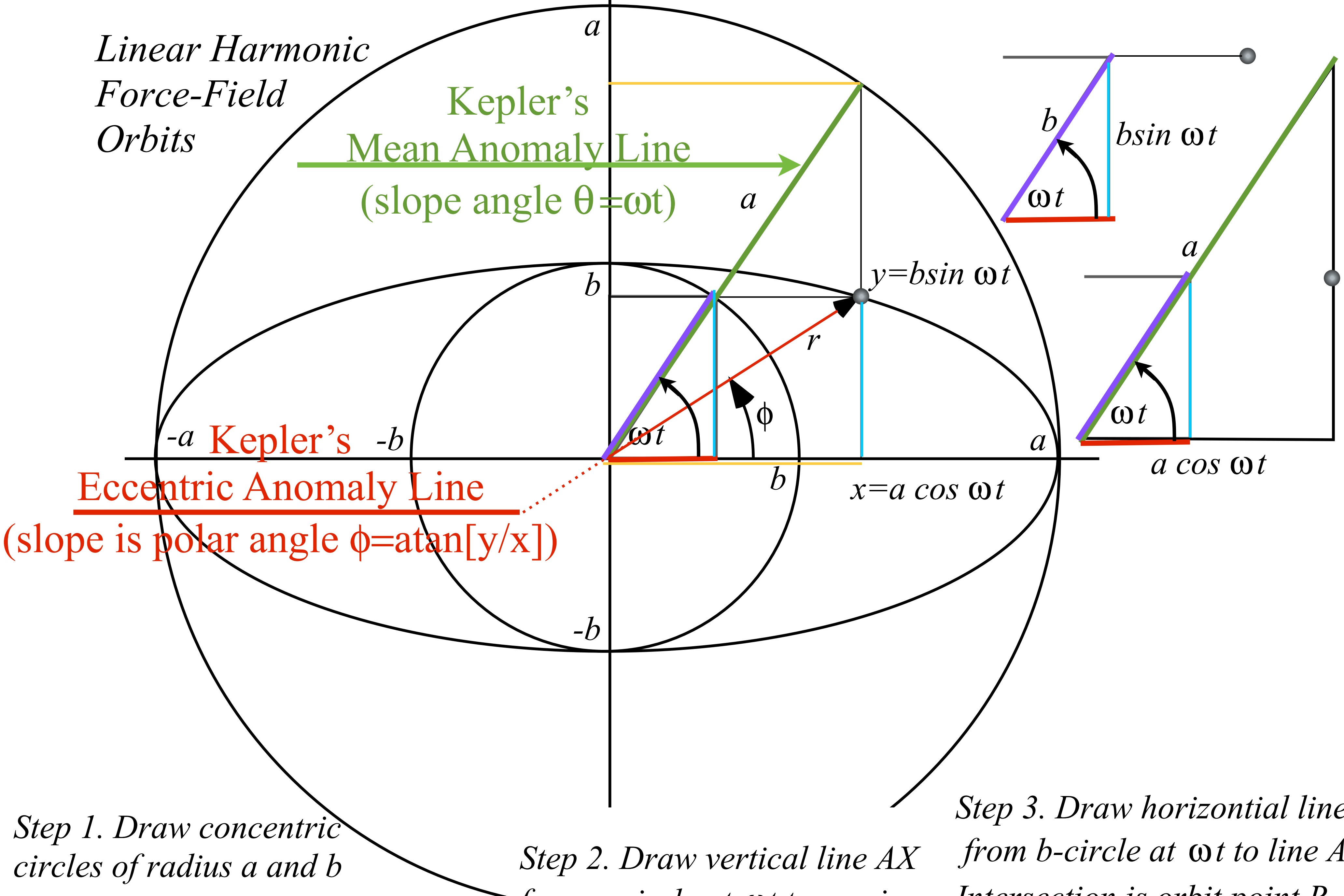
Constructing 2D IHO orbits using Kepler anomaly plots

→ *Mean-anomaly and eccentric-anomaly geometry*

Calculus and vector geometry of IHO orbits

A confusing introduction to Coriolis-centrifugal force geometry *(Derived better in Ch. 12)*

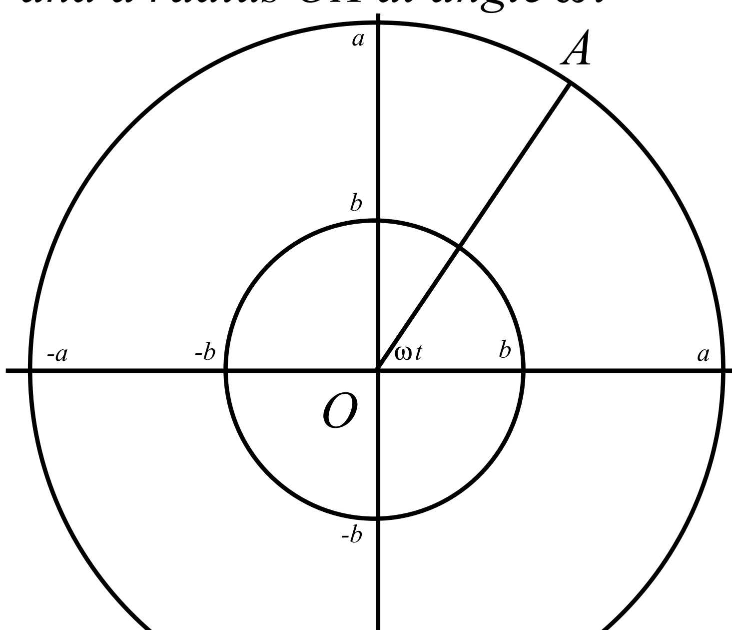
Linear Harmonic Force-Field Orbits



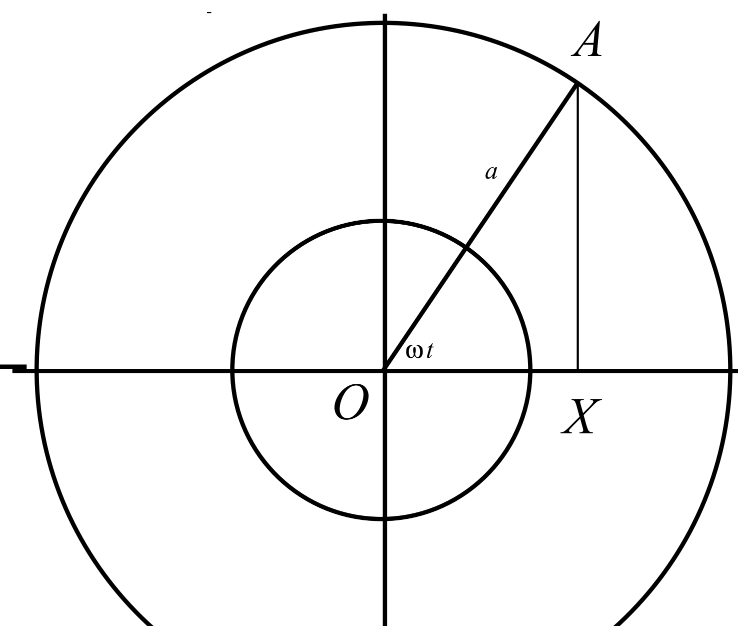
Unit 1
Fig. 11.1
(top 2/3's)

Another example of a Zig-Zag construction:

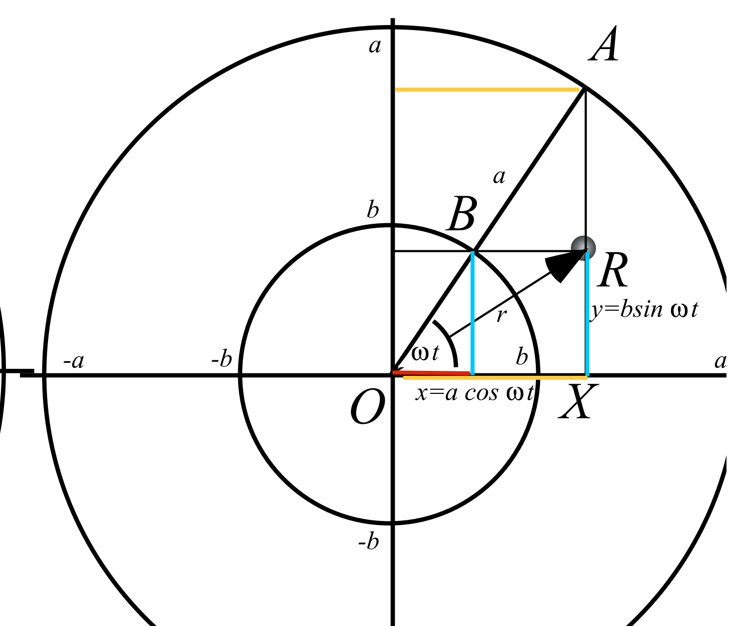
Step 1. Draw concentric circles of radius a and b and a radius OA at angle ωt

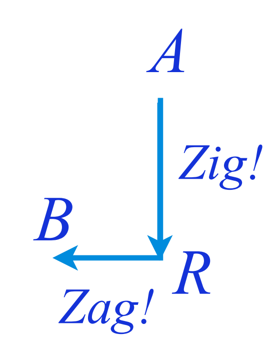


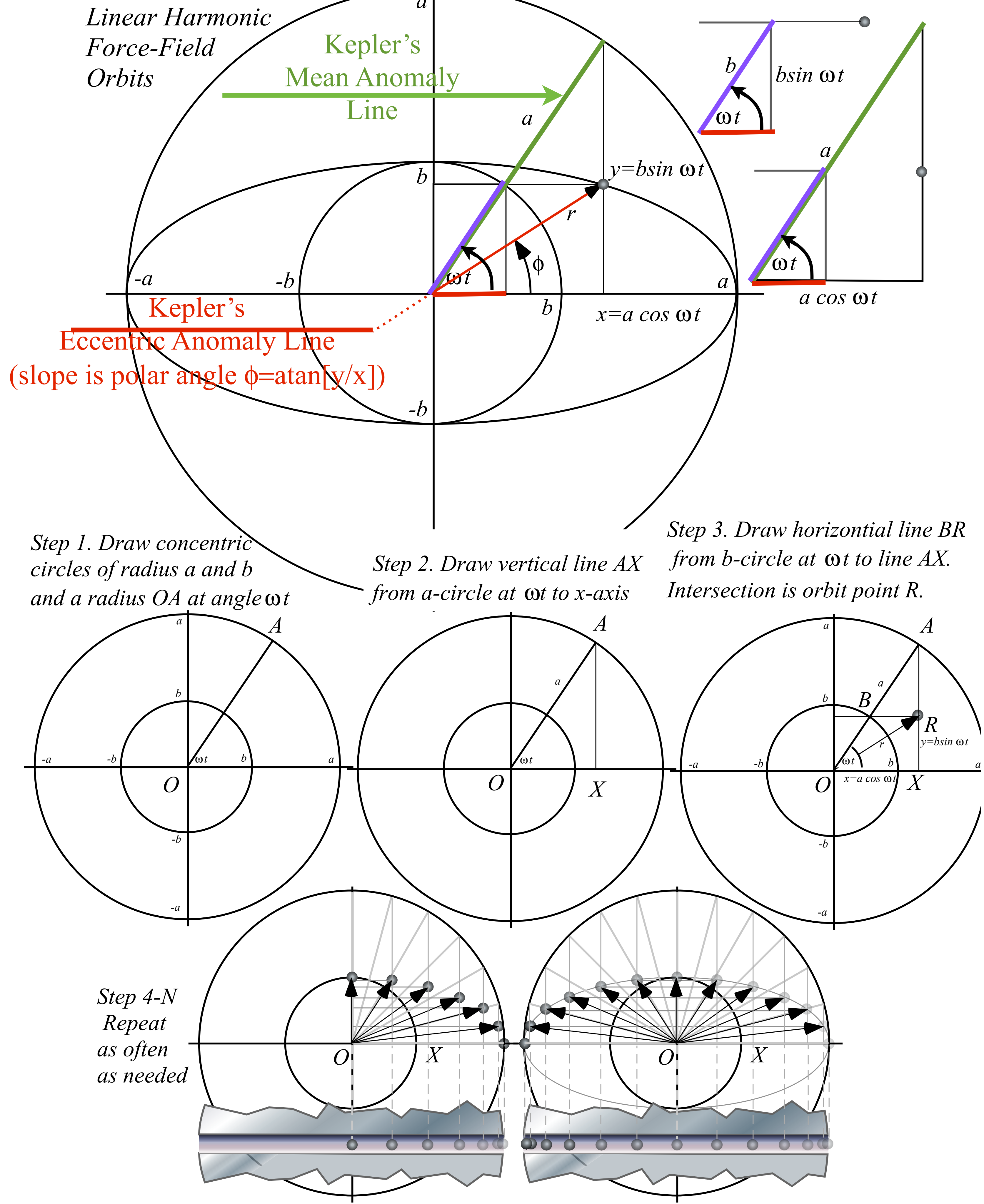
Step 2. Draw vertical line AX from a-circle at ωt to x-axis



Step 3. Draw horizontial line BR from b-circle at ωt to line AX. Intersection is orbit point R.







Unit 1
Fig. 11.1

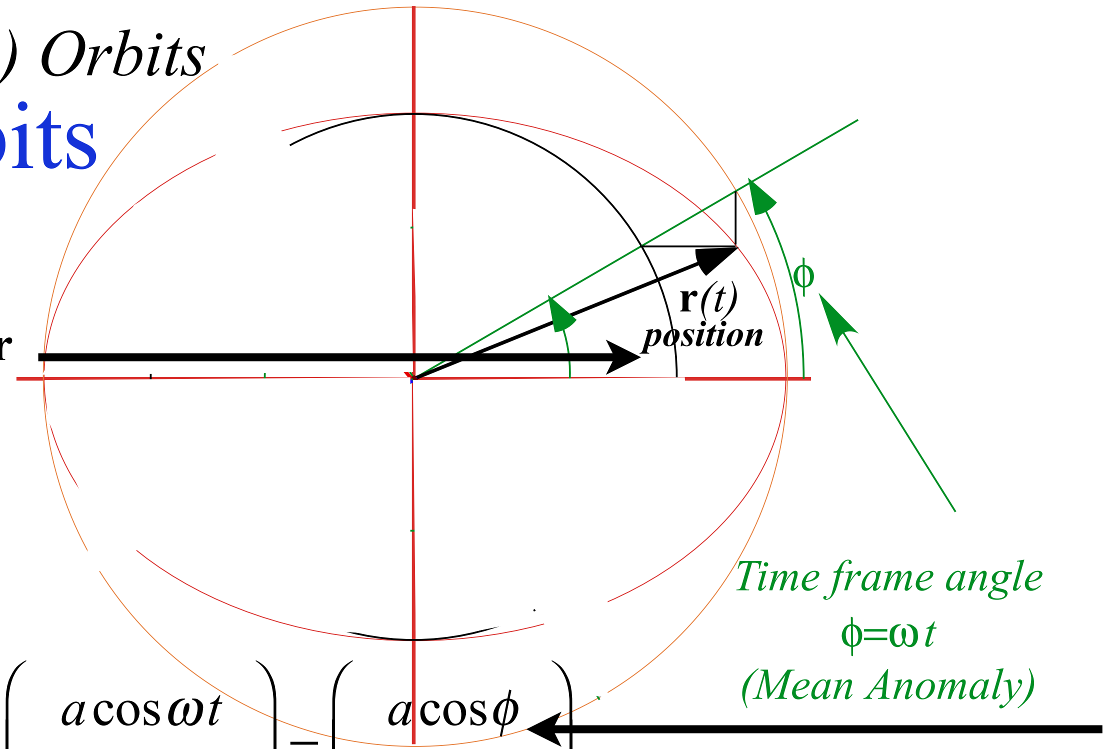
Constructing 2D IHO orbits using Kepler anomaly plots

Mean-anomaly and eccentric-anomaly geometry

→ *Calculus and vector geometry of IHO orbits*

A confusing introduction to Coriolis-centrifugal force geometry *(Derived better in Ch. 12)*

Calculus of IHO orbits

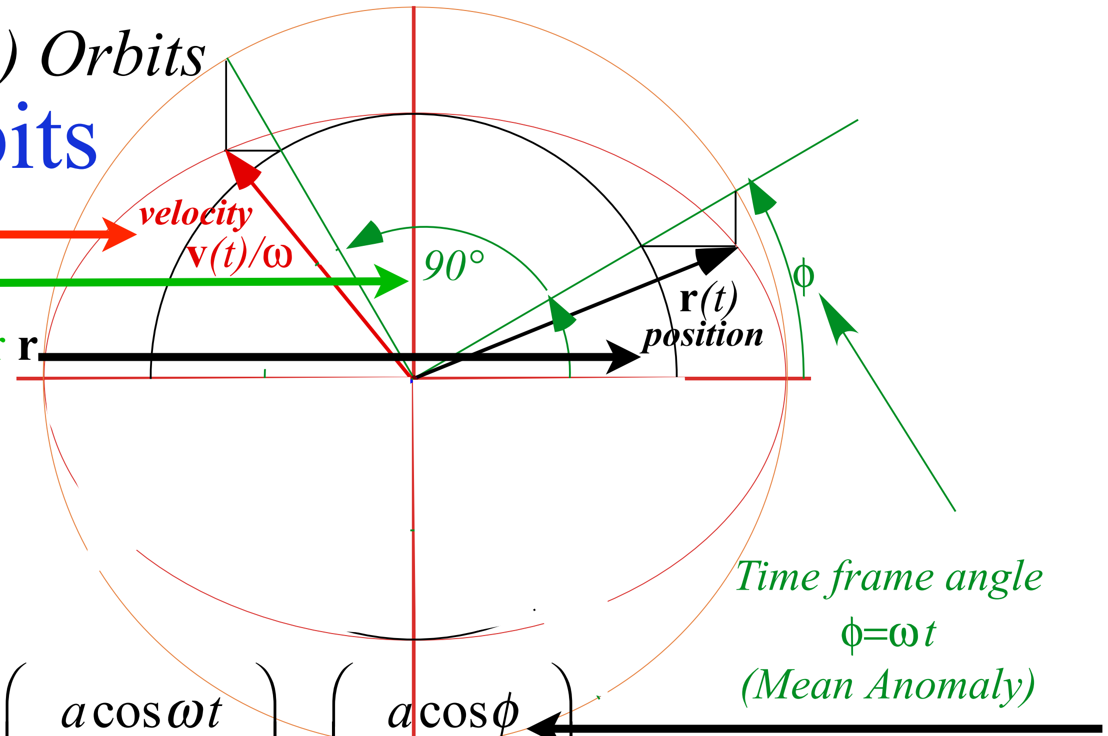


$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

Unit 1
Fig. 11.5

Calculus of IHO orbits

To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**



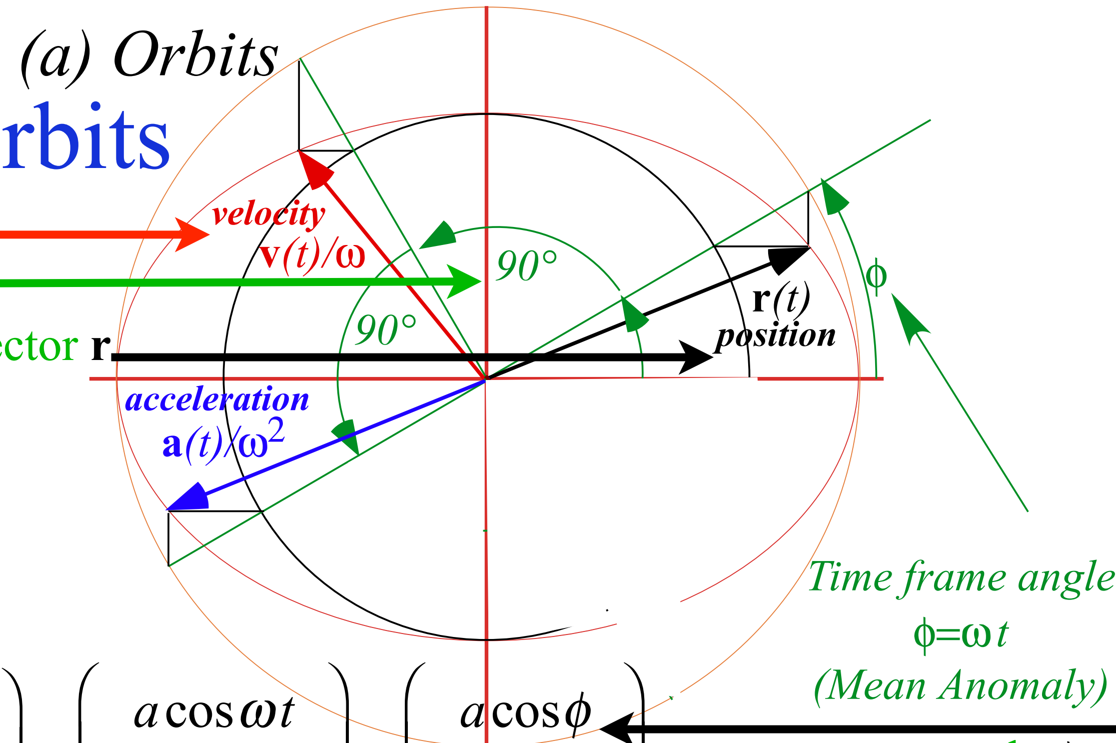
Unit 1
Fig. 11.5

$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r**
rotated by π/2 or 90° is *m.a.* of vector **v**

$$\textit{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\textit{for } \omega = 1)$$

Calculus of IHO orbits



To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**

$$\text{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r**
rotated by π/2 or 90° is *m.a.* of vector **v**

Unit 1
Fig. 11.5

$$\text{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\text{for } \omega = 1)$$

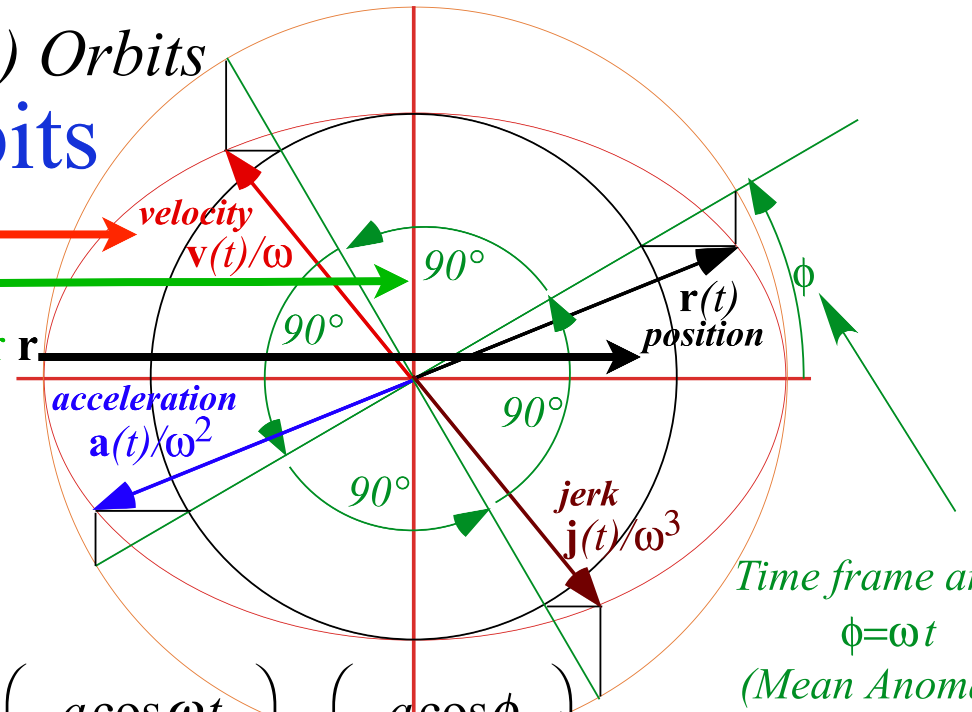
m.a. ϕ+π/2 of vector **v** rotated by
another π/2 is *m.a.* of vector **a**

$$\text{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

Calculus of IHO orbits

To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**



$$radius\ vector : \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r** rotated by π/2 or 90° is *m.a.* of vector **v**

Unit 1
Fig. 11.5

$$velocity\ vector : \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (for\ \omega = 1)$$

m.a. ϕ+π/2 of vector **v** rotated by another π/2 is *m.a.* of vector **a**

$$acceleration\ or\ force\ vector : \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

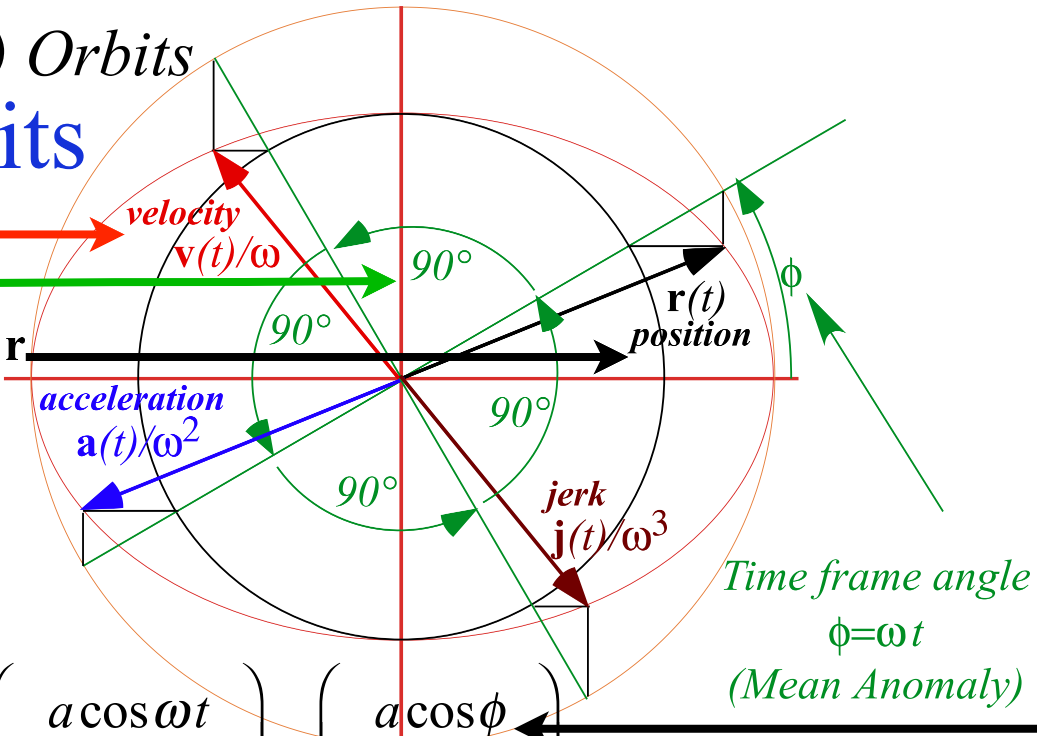
$$jerk\ or\ change\ of\ acceleration : \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi + \frac{3\pi}{2}\right) \\ b\sin\left(\phi + \frac{3\pi}{2}\right) \end{pmatrix}$$

...and so forth...

Calculus of IHO orbits

(a) Orbits

To make velocity vector **v** just rotate by π/2 or 90° the mean-anomaly ϕ of position vector **r**



Unit 1
Fig. 11.5

$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r** rotated by π/2 or 90° is *m.a.* of vector **v**

$$\textit{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\textit{for } \omega = 1)$$

m.a. ϕ+π/2 of vector **v** rotated by another π/2 is *m.a.* of vector **a**

$$\textit{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

$$\textit{jerk or change of acceleration}: \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi + \frac{3\pi}{2}\right) \\ b\sin\left(\phi + \frac{3\pi}{2}\right) \end{pmatrix}$$

...and so forth...

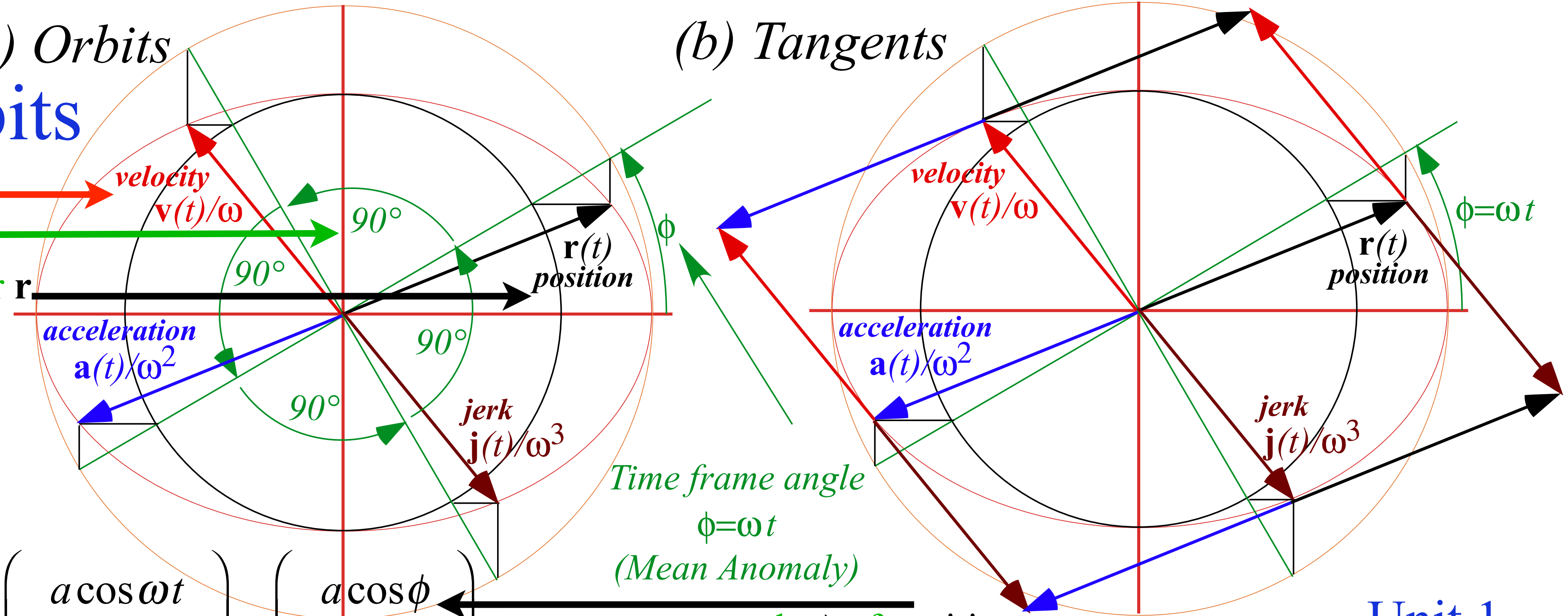
$$\textit{inauguration or change of jerk}: \mathbf{i} = \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} +a\omega^4\cos\omega t \\ +b\omega^4\sin\omega t \end{pmatrix} = \frac{d\mathbf{j}}{dt} = \dot{\mathbf{j}} = \ddot{\mathbf{a}} = \dddot{\mathbf{v}} = \ddddot{\mathbf{r}} = \frac{d^4\mathbf{r}}{dt^4} = \begin{pmatrix} a\cos\left(\phi + \frac{4\pi}{2}\right) \\ b\sin\left(\phi + \frac{4\pi}{2}\right) \end{pmatrix}$$

...and so on...
...But, now it repeats after 4 *t*-derivatives

Calculus of IHO orbits

To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**

Link ⇒ BoxIt simulation of IHO orbits



Unit 1
Fig. 11.5

Link → IHO Exegesis Plot
Link → IHO orbital time rates of change

$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r** rotated by π/2 or 90° is *m.a.* of vector **v**

$$\textit{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\textit{for } \omega = 1)$$

m.a. $\phi+\pi/2$ of vector **v** rotated by another π/2 is *m.a.* of vector **a**

$$\textit{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

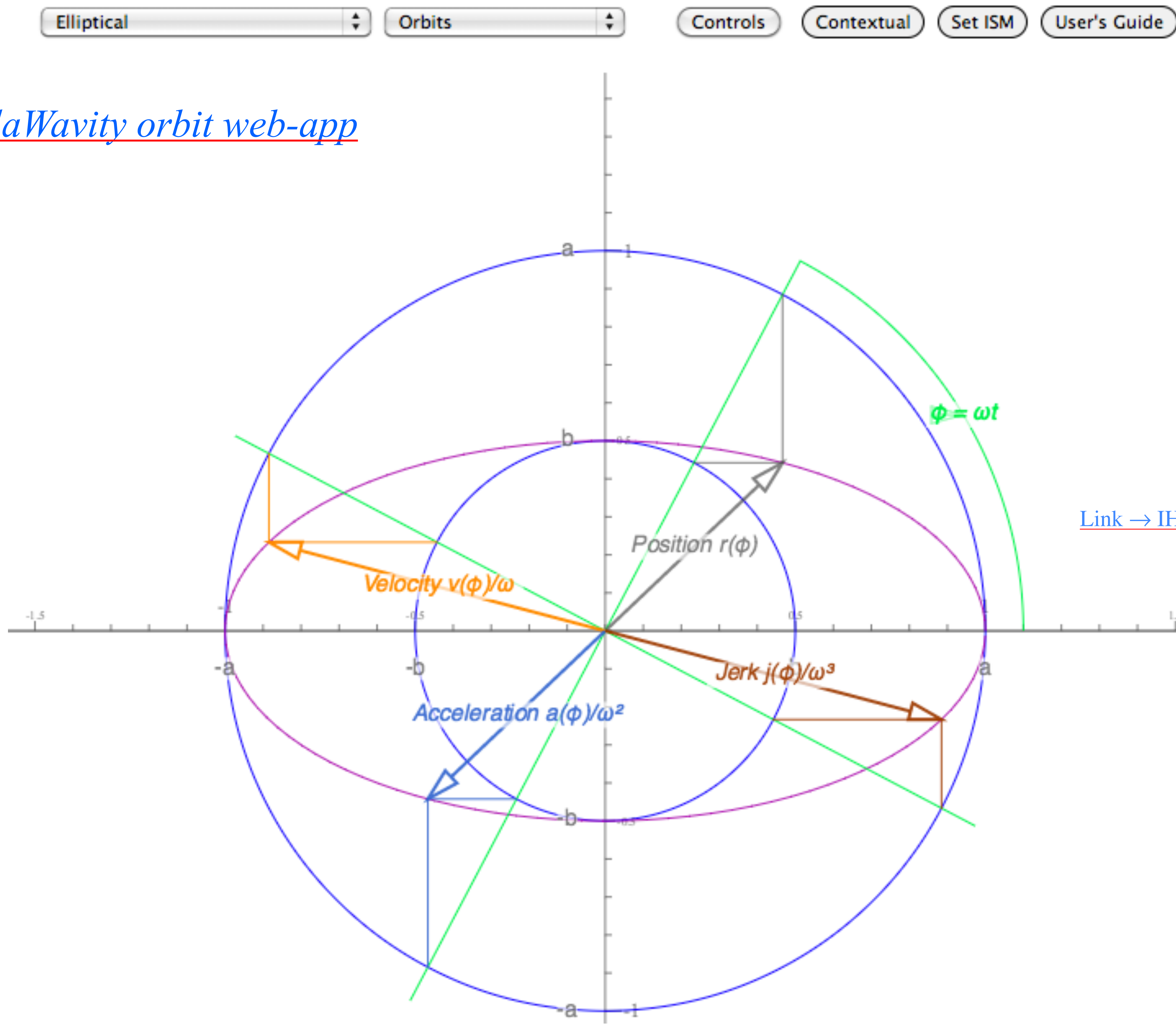
$$\textit{jerk or change of acceleration}: \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi + \frac{3\pi}{2}\right) \\ b\sin\left(\phi + \frac{3\pi}{2}\right) \end{pmatrix}$$

...and so forth...

$$\textit{inauguration or change of jerk}: \mathbf{i} = \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} +a\omega^4\cos\omega t \\ +b\omega^4\sin\omega t \end{pmatrix} = \frac{d\mathbf{j}}{dt} = \dot{\mathbf{j}} = \ddot{\mathbf{a}} = \dddot{\mathbf{v}} = \ddddot{\mathbf{r}} = \frac{d^4\mathbf{r}}{dt^4} = \begin{pmatrix} a\cos\left(\phi + \frac{4\pi}{2}\right) \\ b\sin\left(\phi + \frac{4\pi}{2}\right) \end{pmatrix}$$

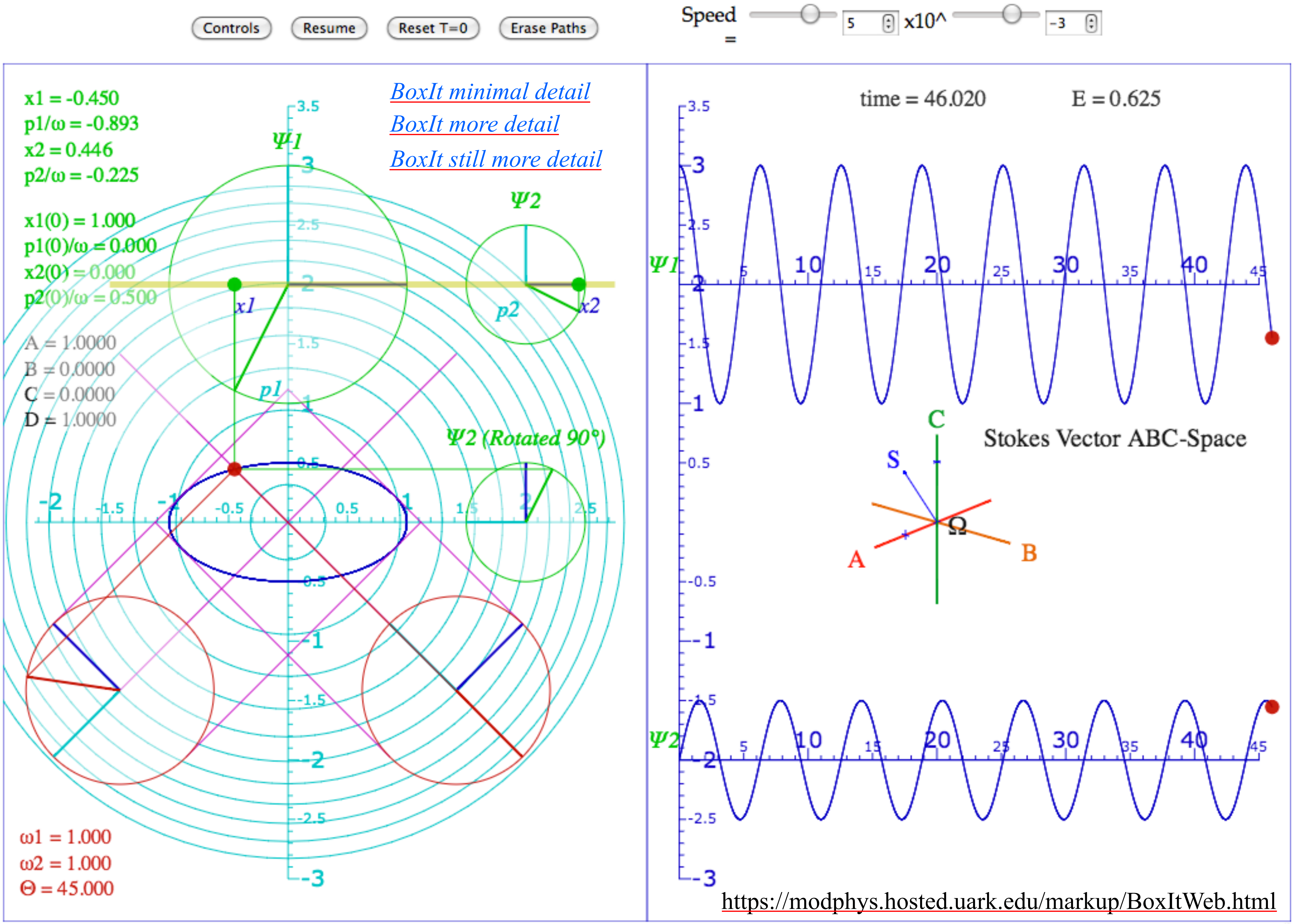
...and so on...
...But, now it repeats after 4 *t*-derivatives

RelaWavity orbit web-app



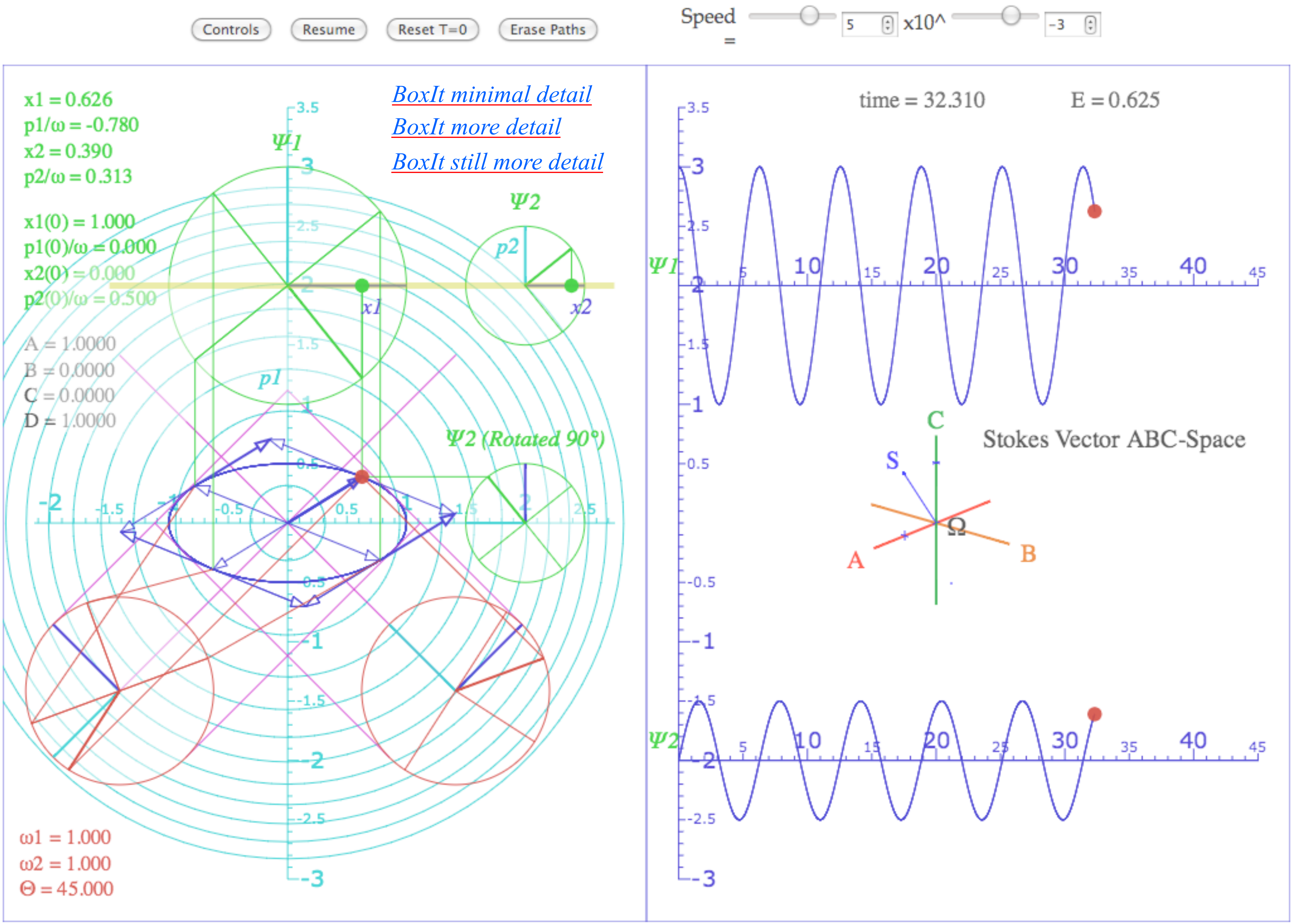
Link → IHO orbital time rates of change

Geometry of Kepler anomalies for vectors [$\mathbf{r}(\phi)$, $\mathbf{v}(\phi)$, $\mathbf{a}(\phi)$, $\mathbf{j}(\phi)$,] in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity.



Geometry of Kepler anomalies for vectors [**r**(ϕ)] *in coordinate (x,y) space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.*

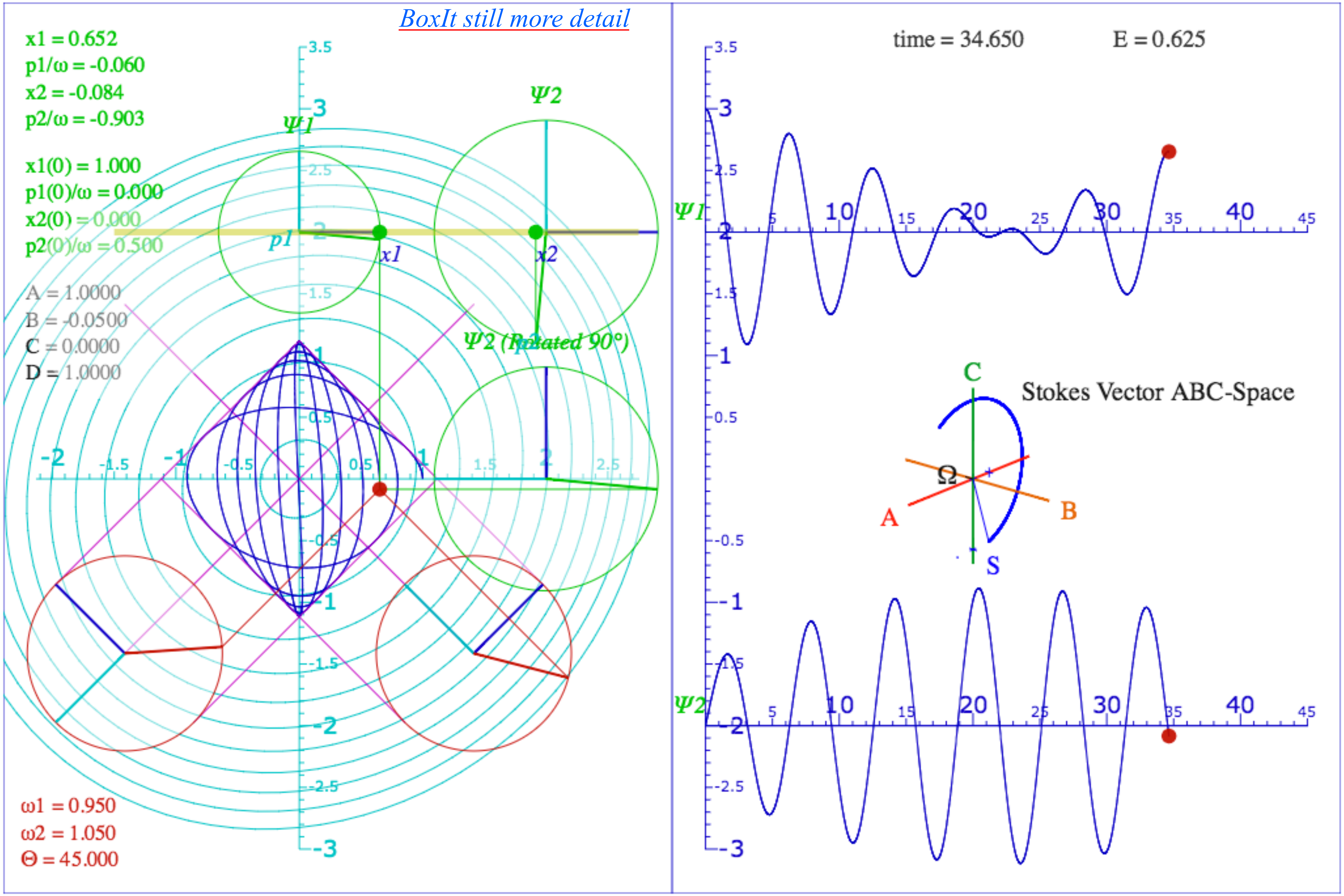
BoxIt Web Stokes Simulation



Geometry of Kepler anomalies for vectors [**r**(ϕ), **v**(ϕ), **a**(ϕ), **j**(ϕ),] *in coordinate (x,y) space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.*

BoxIt Web Simulation - *w/Derivatives*

[BoxIt minimal detail](#)
[BoxIt more detail](#)
[BoxIt still more detail](#)



Geometry of vectors [**r**(ϕ), **p**(ϕ)] *and quantum spin S-space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.*

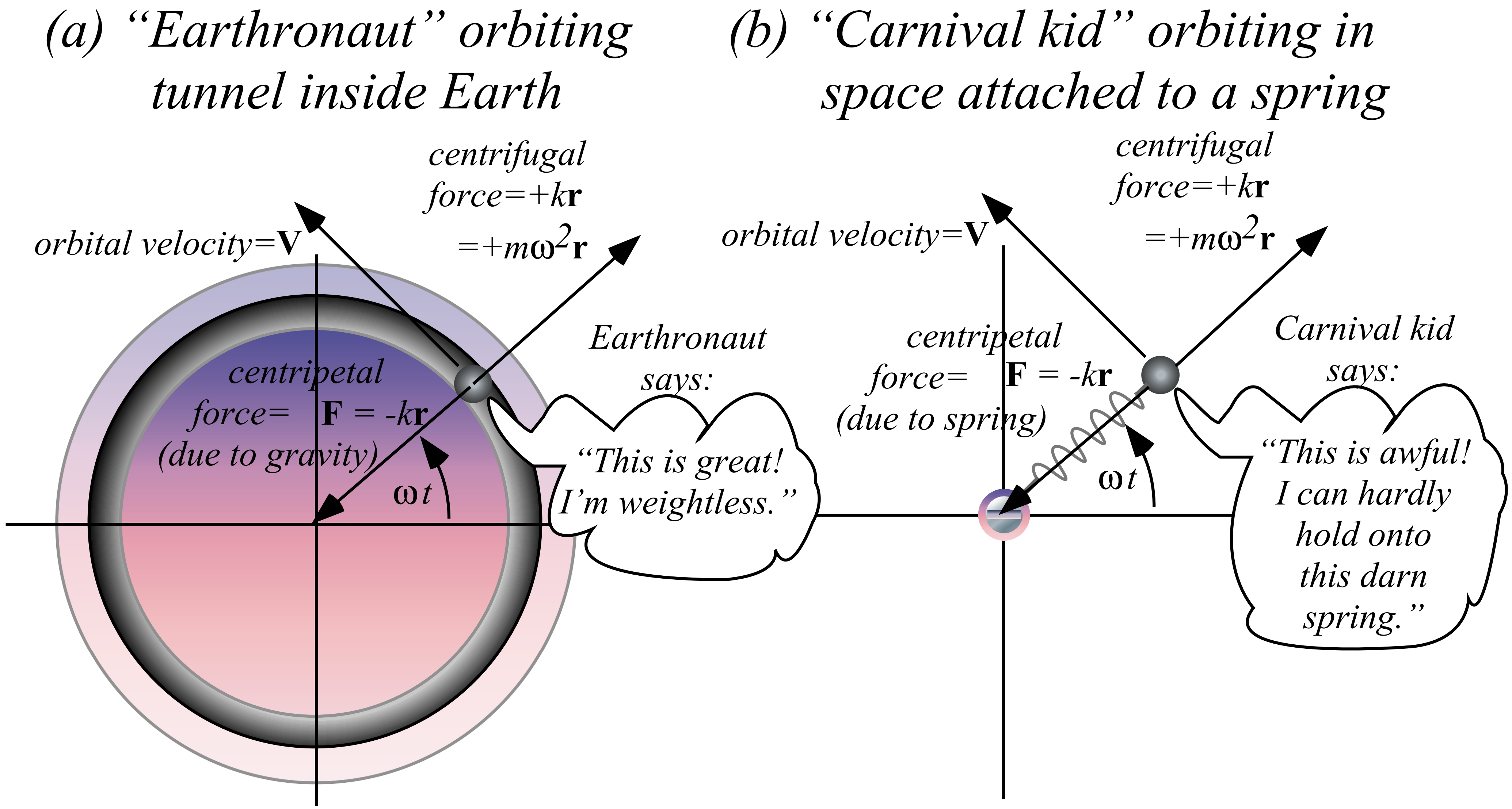
[BoxIt Web Simulation - *B-Type Motion*](#)

Constructing 2D IHO orbits using Kepler anomaly plots

Mean-anomaly and eccentric-anomaly geometry

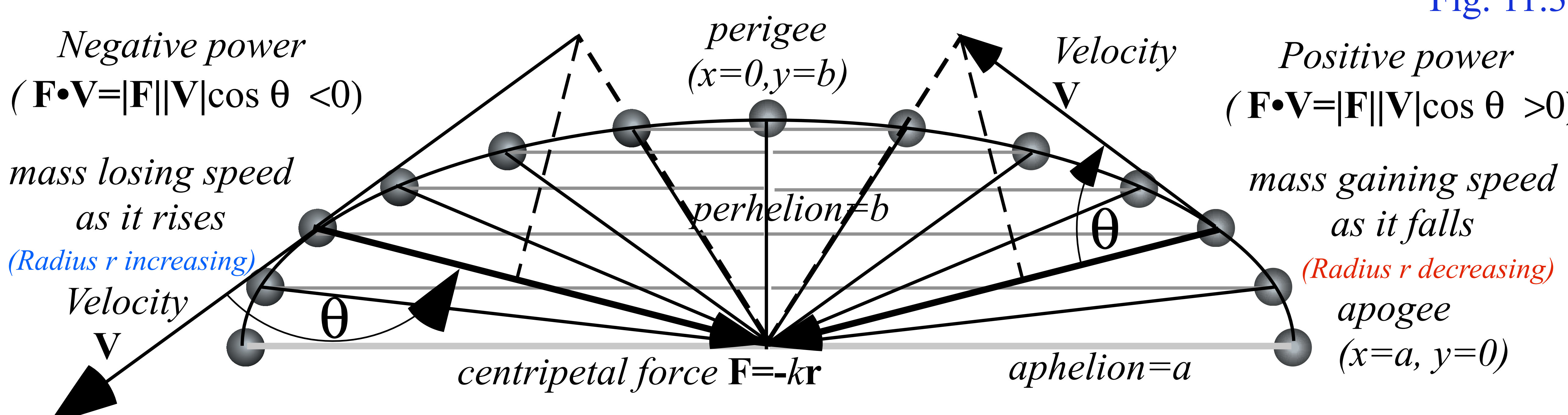
Calculus and vector geometry of IHO orbits

→ *A confusing introduction to Coriolis-centrifugal force geometry* *(Derived better in Ch. 12)*

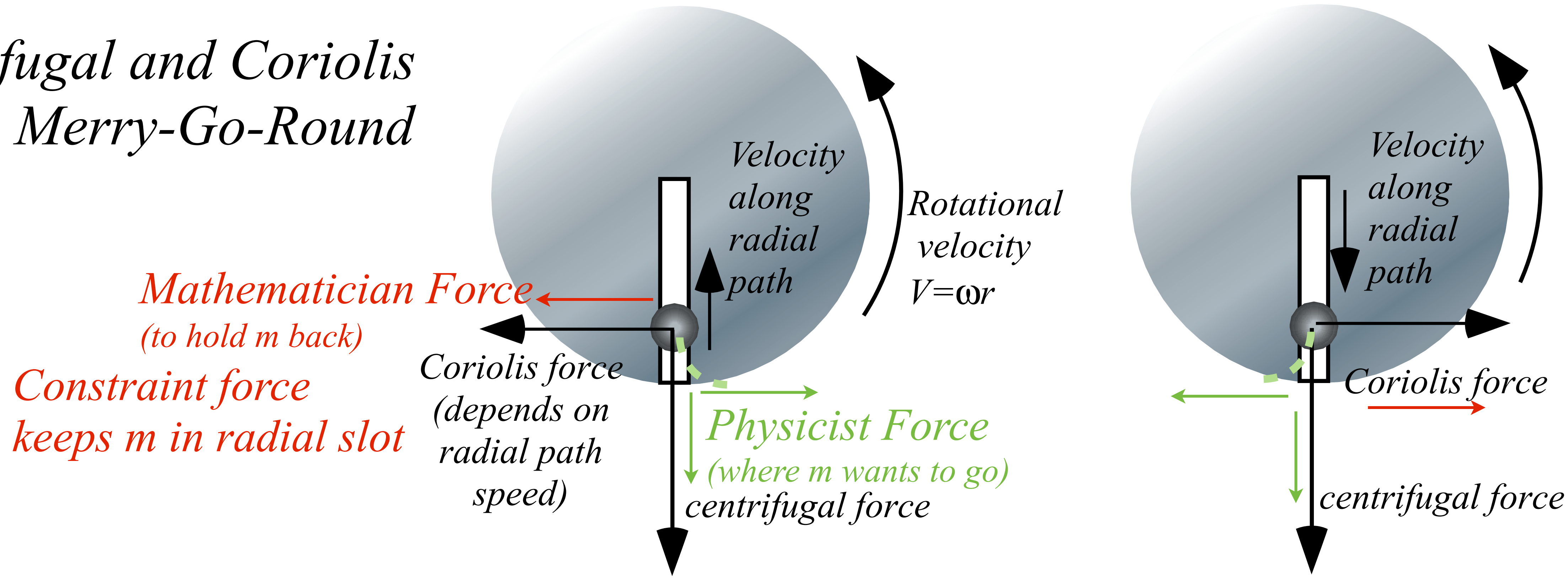


Unit 1
Fig. 11.2

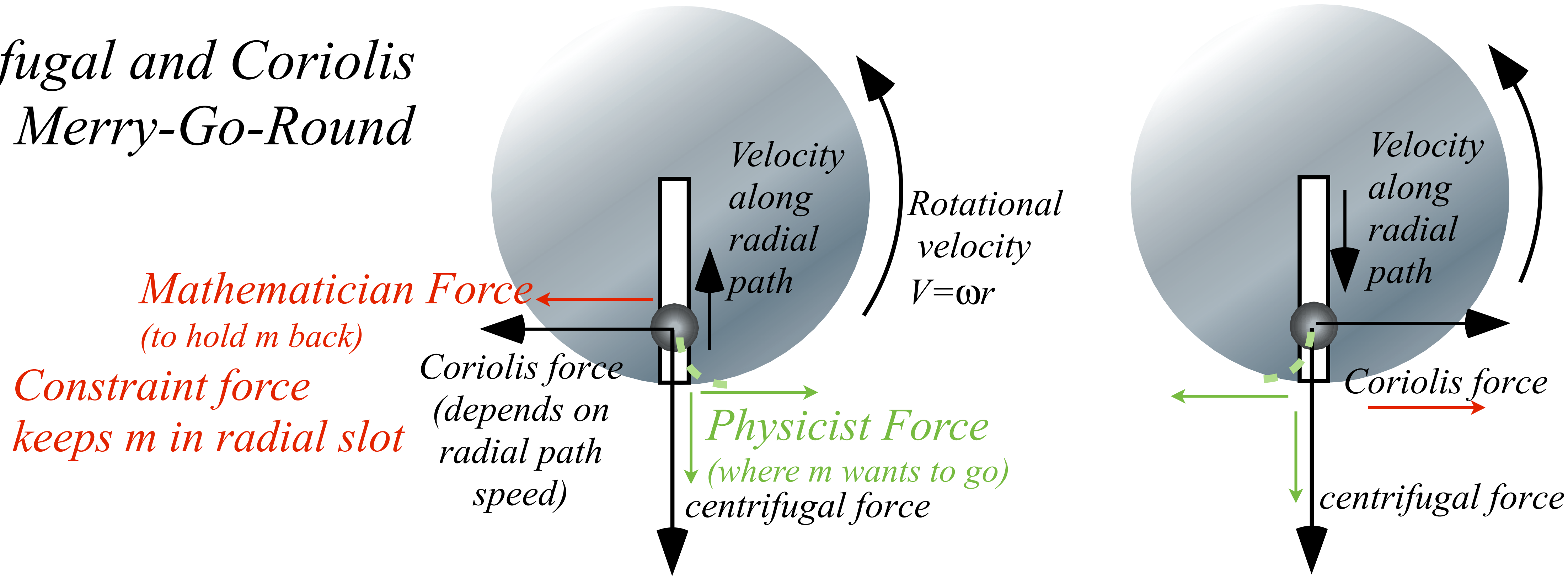
Unit 1
Fig. 11.3



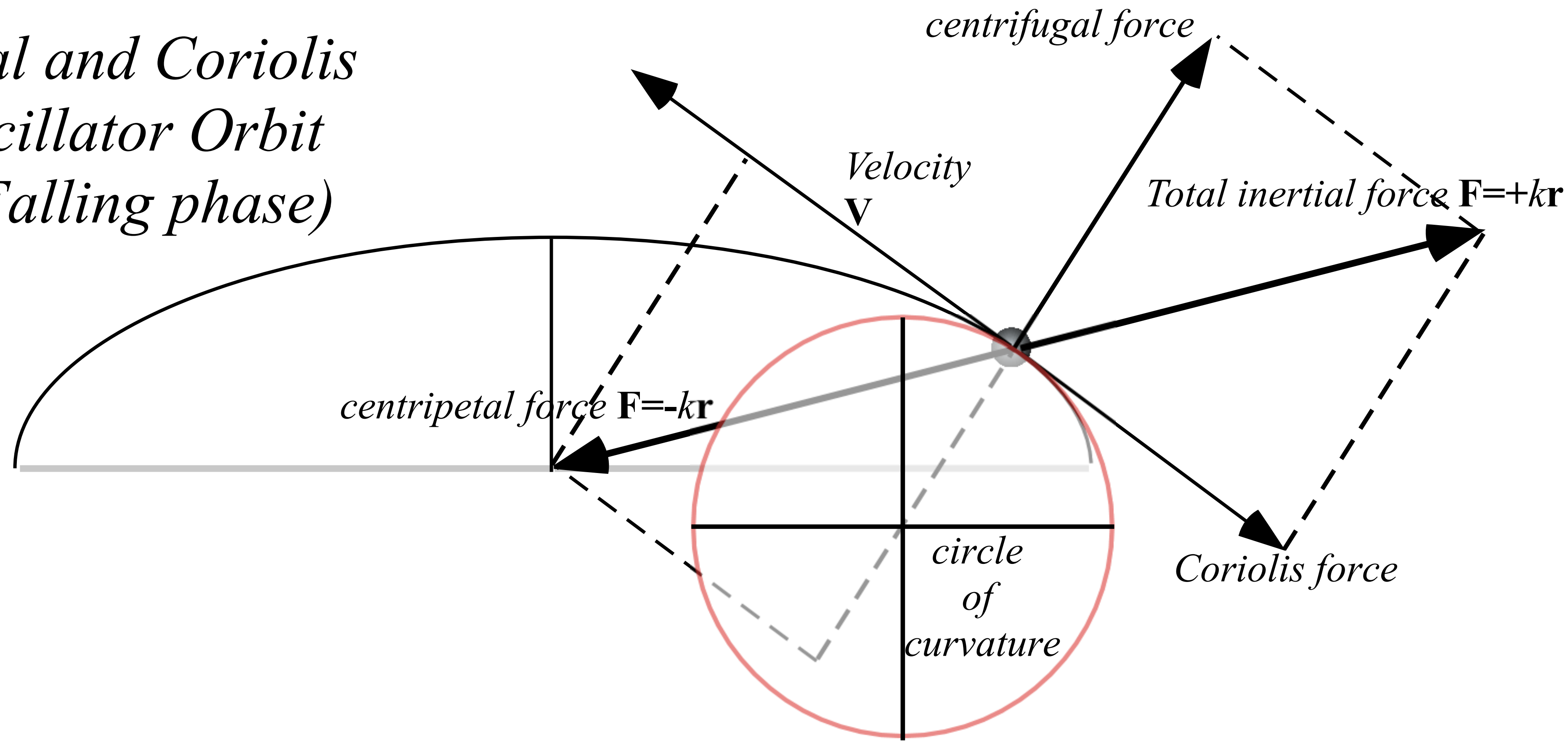
(a) Centrifugal and Coriolis Forces on Merry-Go-Round

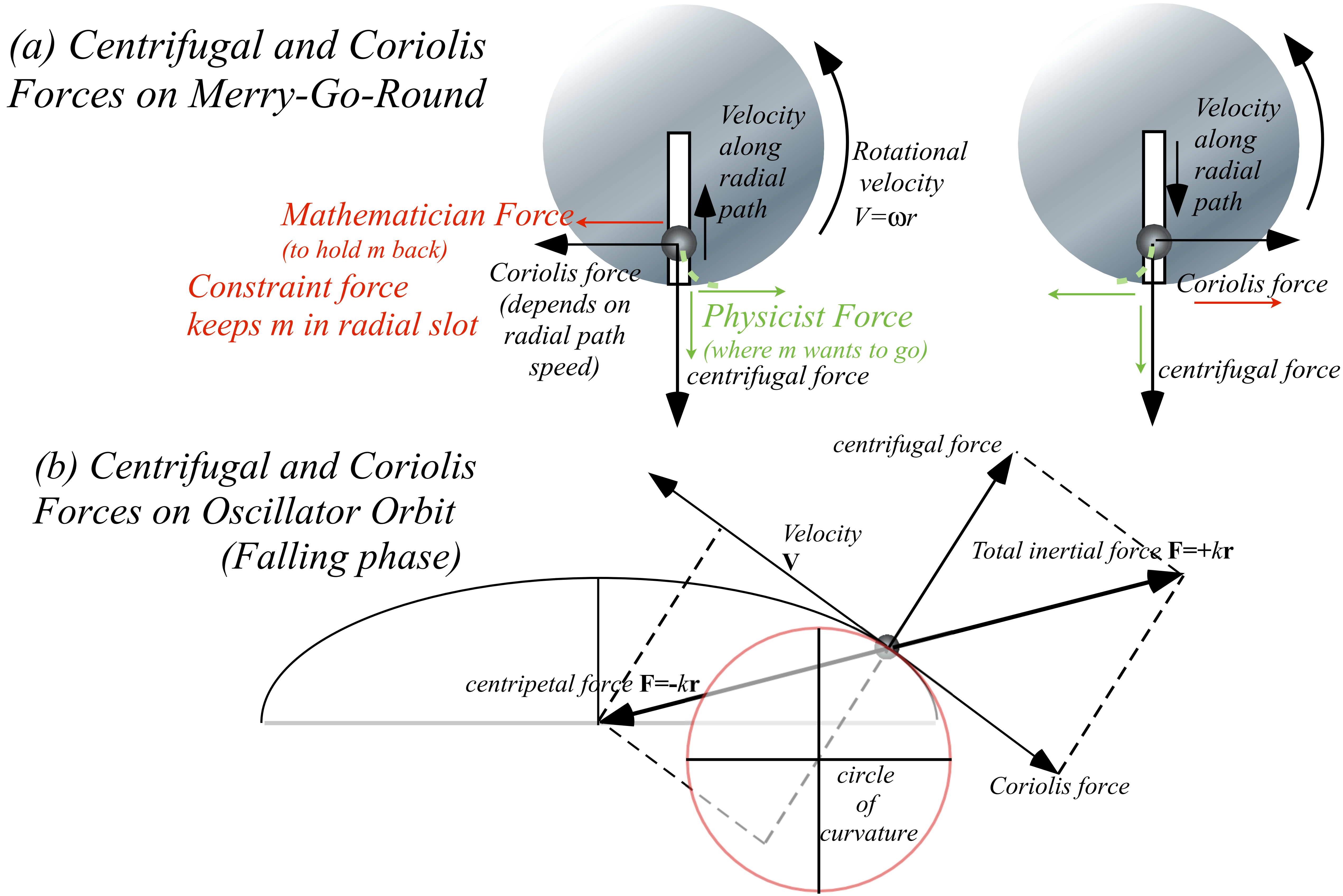


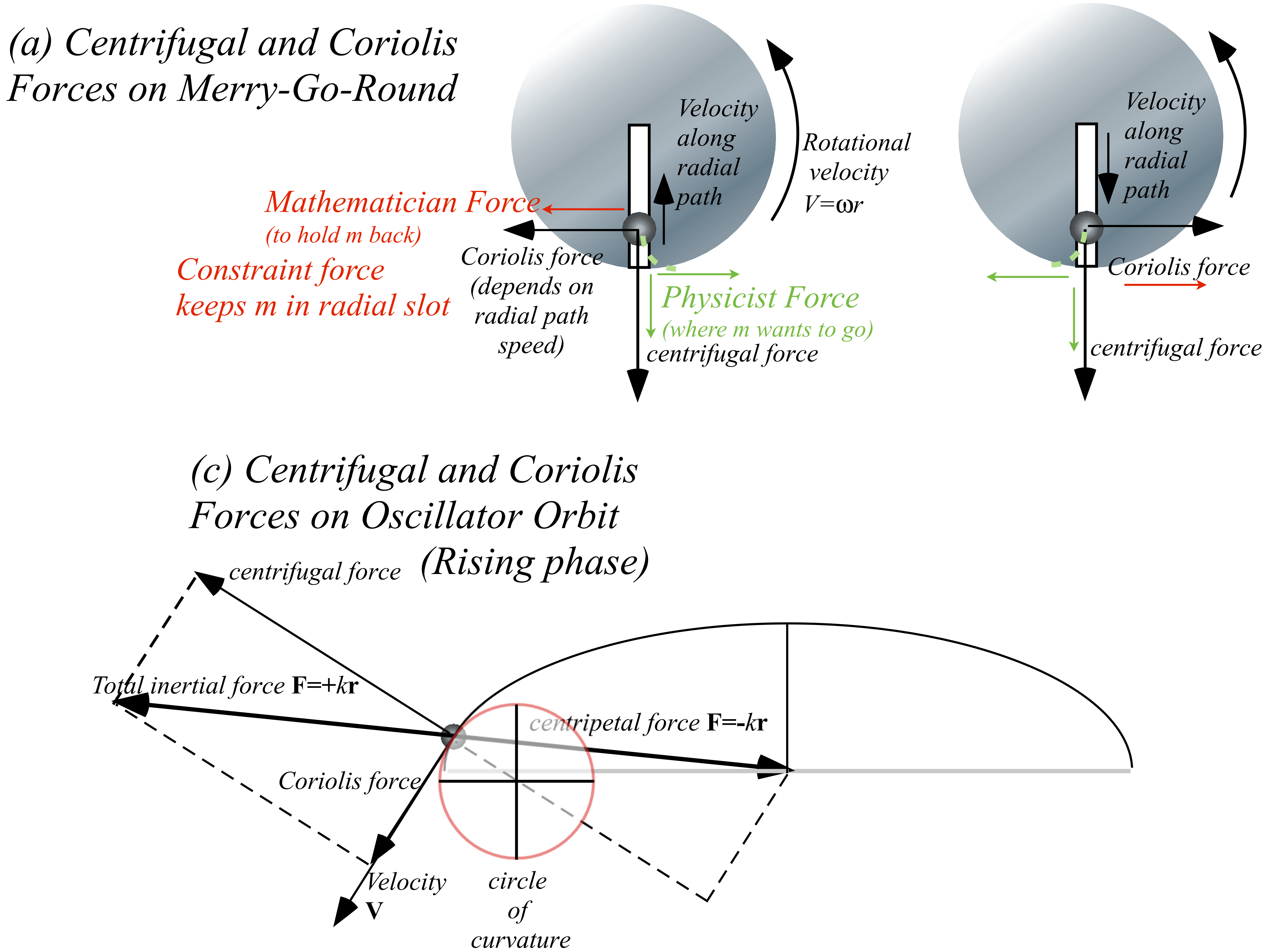
(a) Centrifugal and Coriolis Forces on Merry-Go-Round



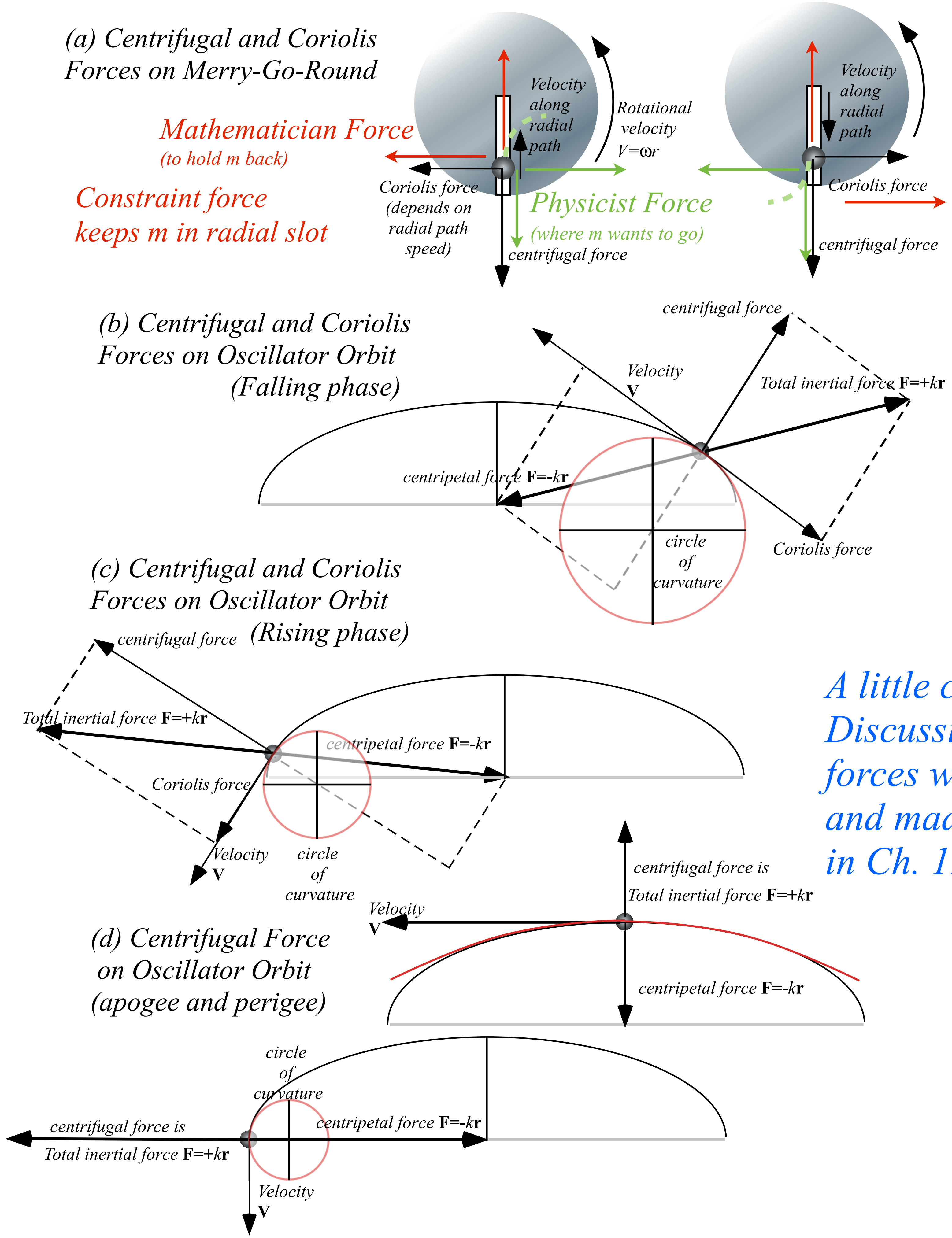
(b) Centrifugal and Coriolis Forces on Oscillator Orbit (Falling phase)




(a) Centrifugal and Coriolis Forces on Merry-Go-Round
Velocity along radial path
Rotational velocity V=ωr
Mathematician Force
(to hold m back)
Constraint force keeps m in radial slot
Coriolis force (depends on radial path speed)
Physicist Force
(where m wants to go)
centrifugal force
Velocity along radial path
Coriolis force
centrifugal force
(b) Centrifugal and Coriolis Forces on Oscillator Orbit (Falling phase)
centrifugal force
Velocity V
Total inertial force F=+kr
centripetal force F=-kr
circle of curvature
Coriolis force



(a) Centrifugal and Coriolis Forces on Merry-Go-Round
Velocity along radial path
Rotational velocity V=ωr
Mathematician Force
(to hold m back)
Constraint force keeps m in radial slot
Coriolis force (depends on radial path speed)
Physicist Force
(where m wants to go)
centrifugal force
Velocity along radial path
Coriolis force
centrifugal force
(c) Centrifugal and Coriolis Forces on Oscillator Orbit
(Rising phase)
centrifugal force
Total inertial force F=+kr
centripetal force F=-kr
Coriolis force
Velocity V
circle of curvature



Unit 1
Fig. 11.4
a-d

A little confusing?
Discussion of Coriolis forces will be done more elegantly and made more physically intuitive in Ch. 12 of Unit1 and in Unit 6.

Some Kepler's "laws" for <u>all</u> central (isotropic) force F(r) fields

→ *Angular momentum invariance of* IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ *(Derived here)*

Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*

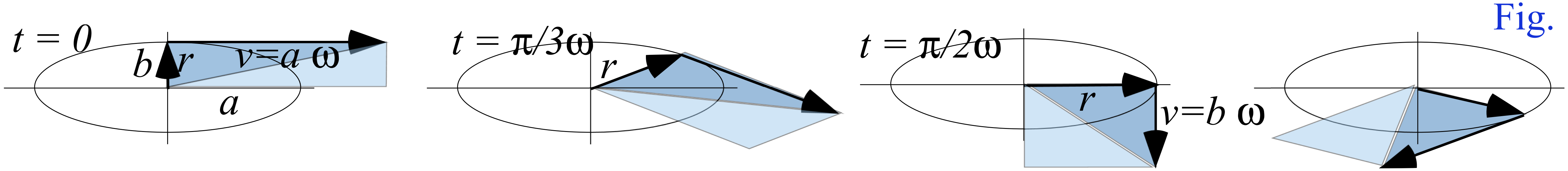
Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived in Unit 5)*

Some Kepler's "laws" for central (isotropic) force F(r)

...and certainly apply to the IHO: F(r)=-k·r with U(r)=k·r²/2

(Recall from Lecture 6: $k = Gm\frac{4\pi}{3}\rho_{\oplus}$)

Unit 1
Fig. 11.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - b\sin\omega t\cdot(-a\omega\sin\omega t) = ab\cdot\omega\left(\cos^2\omega t + \sin^2\omega t\right)$$

✓ *for IHO*

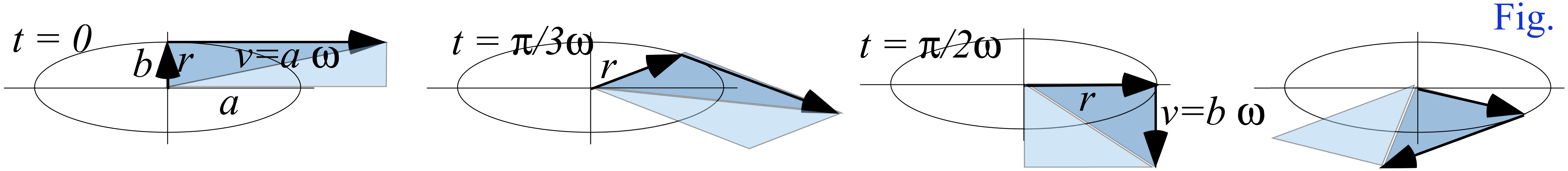
$$\begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} \qquad \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix}$$

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

(Recall from Lecture 6: $k = Gm\frac{4\pi}{3}\rho_{\oplus}$)

Unit 1
Fig. 11.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot\left(b\omega\cos\omega t\right) - a\sin\omega t\cdot\left(-b\omega\sin\omega t\right) = ab\cdot\omega$$

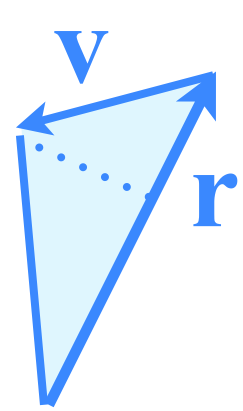
✓ *for IHO*

2. *Angular momentum* $\mathbf{L} = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,|\mathbf{r}\times\mathbf{v}| = m\left(r_x v_y - r_y v_x\right) = m\cdot ab\cdot\omega$$

✓ *for IHO*

$$|\mathbf{r}\times\mathbf{v}| = r\cdot v\cdot \sin\measuredangle_r^v$$

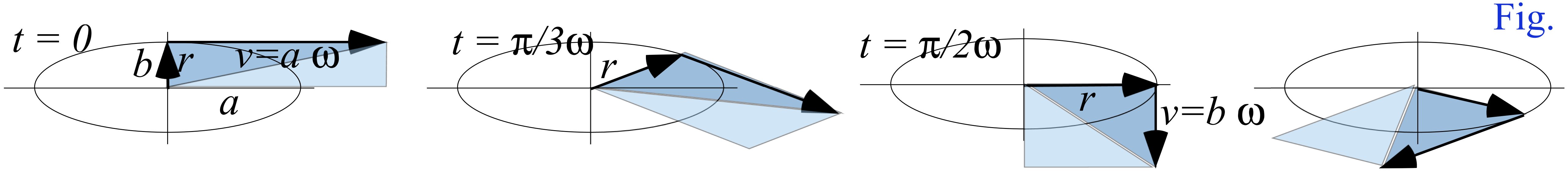


Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: F(r)=-k·r with U(r)=k·r²/2

(Recall from Lecture 6: $k = Gm\frac{4\pi}{3}\rho_{\oplus}$)

Unit 1
Fig. 11.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

✓ *for IHO*

2. *Angular momentum* $\mathbf{L} = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,|\mathbf{r}\times\mathbf{v}| = m\left(r_x v_y - r_y v_x\right) = m\cdot ab\cdot\omega$$

✓ *for IHO*

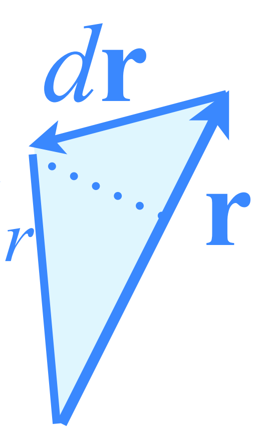
3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{2m}\int_0^T dt = \frac{L}{2m}T$$

by 2.

✓ *for IHO*

$$|\mathbf{r}\times d\mathbf{r}| = r\cdot dr\cdot \sin\measuredangle_{r}^{dr}$$

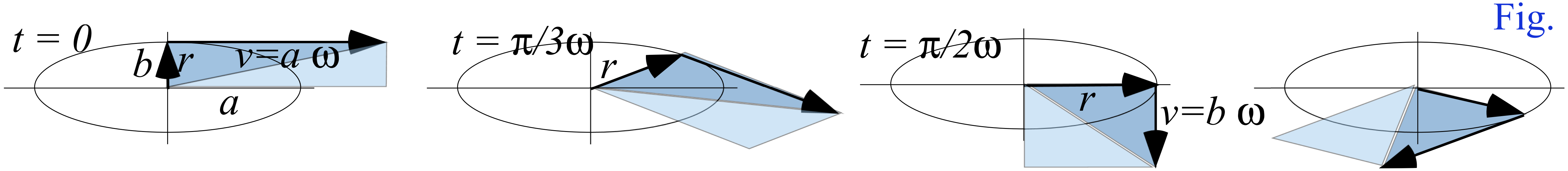


Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: F(r)=-k·r with U(r)=k·r²/2

(Recall from Lecture 6: $k = Gm\frac{4\pi}{3}\rho_\oplus$)

Unit 1
Fig. 11.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

✓ *for IHO*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = m\cdot ab\cdot\omega = m\cdot ab\cdot\frac{2\pi}{\tau}$$

✓ *for IHO*

3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{2m}\int_0^T dt = \frac{L}{2m}T$$

✓ *for IHO*

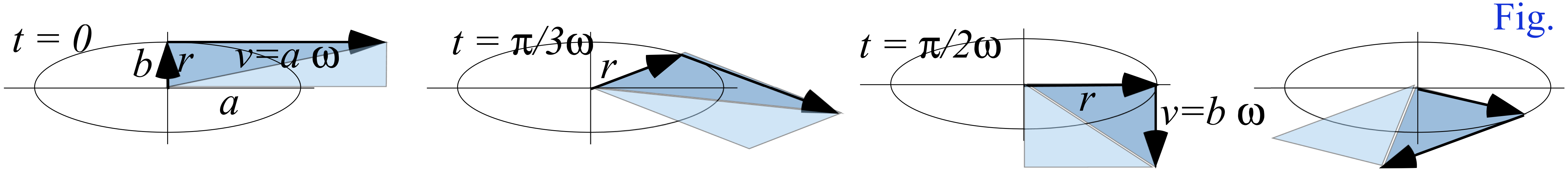
In one period: $\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L}$ the area is: $A_\tau = \frac{L\tau}{2m}$ $(= ab\cdot\pi$ for ellipse orbit$)$

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

(Recall from Lecture 6: $k = Gm\frac{4\pi}{3}\rho_\oplus$)

Unit 1
Fig. 11.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

✓ *for IHO*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m(r_x v_y - r_y v_x) = m\cdot ab\cdot\omega = m\cdot ab\cdot\frac{2\pi}{\tau}$$

✓ *for IHO*

3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{2m}\int_0^T dt = \frac{L}{2m}T$$

✓ *for IHO*

In one period: $\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L}$ the area is: $A_\tau = \frac{L\tau}{2m}$ $(= ab\cdot\pi$ for ellipse orbit$)$

(Recall from Lecture 6: $\omega = \sqrt{k/m} = \sqrt{G\rho_\oplus 4\pi/3}$)

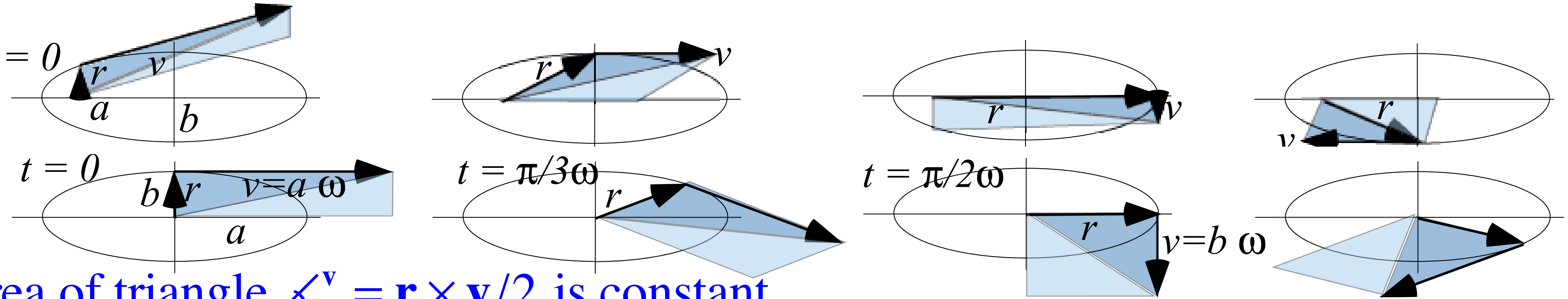
(IHO formulas from Lect. 6 p.70-79)

Some Kepler's "laws" for <u>all</u> central (isotropic) force F(r) fields

- *Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$* *(Derived here)*
- → *Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$* *(Derived in Unit 5)*
- *Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$* *(Derived here)*
- *Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$* *(Derived in Unit 5)*

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: F(r)=-k·r with U(r)=k·r²/2 and Coulomb: F(r)=-GMm/r² with U(r)=-GMm·/r



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_{\oplus}4\pi/3} & \textit{for IHO} \\ a^{-1/2}b\sqrt{GM_{\oplus}} & \textit{for Coul.} \end{cases}$$

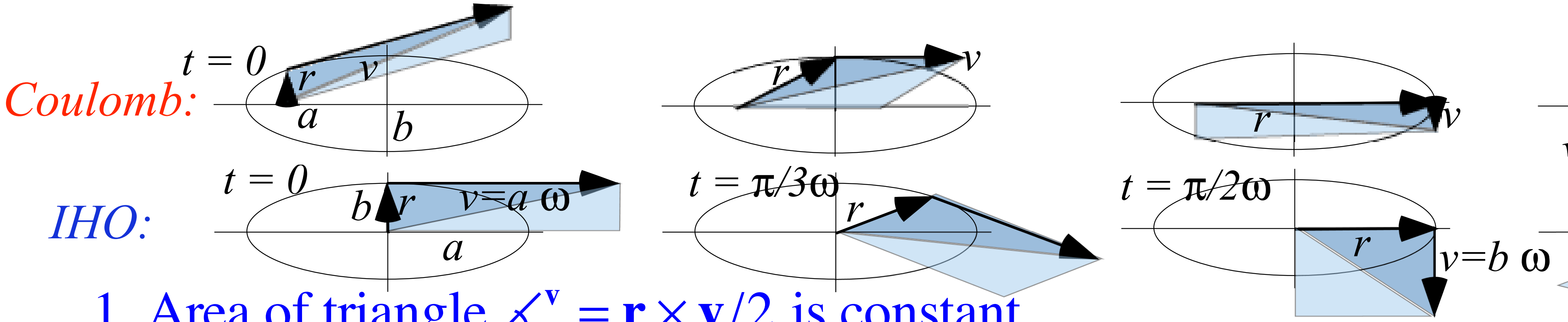
✓ *for IHO*

(Derived in Unit 5) ✓ *for Coul.*

(IHO formulas from Lect. 6 p.70-79)

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_{\oplus}4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_{\oplus}} & \text{for Coul.} \end{cases}$$

✓ *for IHO*

(Derived in Unit 5) ✓ *for Coul.*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_{\oplus}4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2}b\sqrt{GM_{\oplus}} & \text{for Coul.} \end{cases}$$

✓ *for IHO*

(... in Unit 5) ✓ *for Coul.*

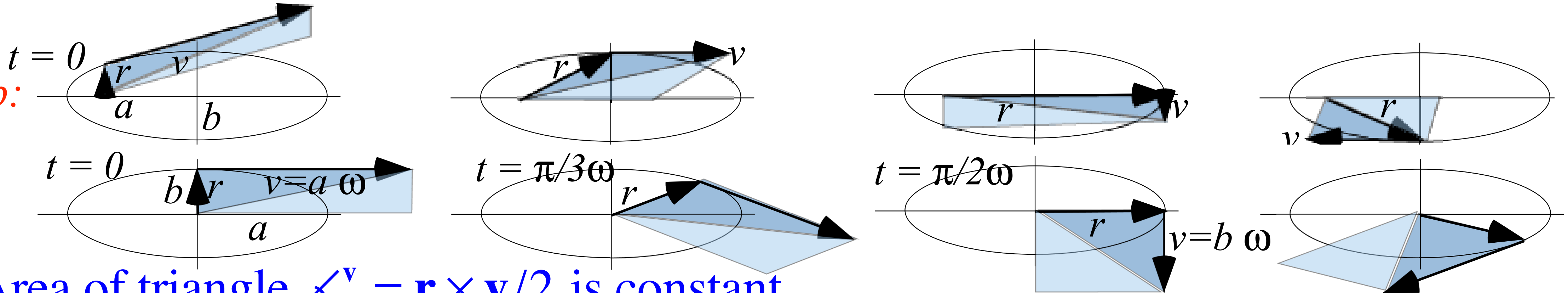
(IHO formulas from Lect. 6 p.70-79)

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$

Coulomb:

IHO:



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

✓ for IHO

(Derived in Unit 5) ✓ for Coul.

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

✓ for IHO

(... in Unit 5) ✓ for Coul.

3. Equal area is swept by radius vector in each equal time interval T

In one period:

$$\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L} = \frac{2m\cdot ab\cdot\pi}{L} = \begin{cases} \dfrac{2m\cdot ab\cdot\pi}{m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3}} \\ \dfrac{2m\cdot ab\cdot\pi}{m\cdot a^{-1/2}b\sqrt{GM_\oplus}} \end{cases}$$

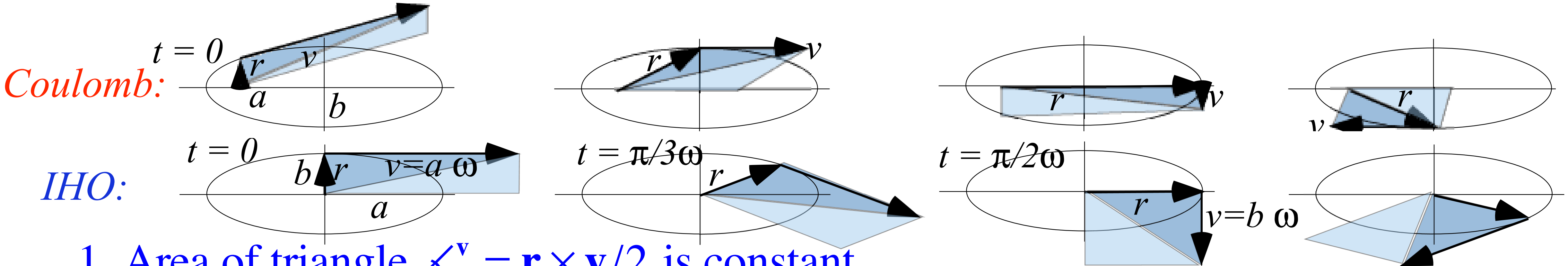
Applies to any central F(r) (for $\frac{2mA_\tau}{L}$)

Applies to IHO and Coulomb

(IHO formulas from Lect. 6 p.70-79)

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: F(r)=-k·r with U(r)=k·r²/2 and Coulomb: F(r)=-GMm/r² with U(r)=-GMm·/r



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

✓ *for IHO*

(Derived in Unit 5) ✓ *for Coul.*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

✓ *for IHO*

(... in Unit 5) ✓ *for Coul.*

3. Equal area is swept by radius vector in each equal time interval T

In one period:

$$\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L} = \frac{2m\cdot ab\cdot\pi}{L} = \begin{cases} \dfrac{2m\cdot ab\cdot\pi}{m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3}} = \dfrac{2\pi}{\sqrt{G\rho_\oplus 4\pi/3}} & \text{for IHO} \\ \dfrac{2m\cdot ab\cdot\pi}{m\cdot a^{-1/2}b\sqrt{GM_\oplus}} = \dfrac{2\pi}{a^{-3/2}\sqrt{GM_\oplus}} & \text{for Coul.} \end{cases}$$

(not a function of a or b)

that is ω_{IHO}

that is ω_{Coul}

(not a function of b)

Applies to any central F(r)

Applies to IHO and Coulomb

Some Kepler's "laws" for *<u>all</u>* *central (isotropic) force F(r) fields*

Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ *(Derived here)*

Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

→ *Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$* *(Derived here)*

Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived in Unit 5)*

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total energy $= KE + PE$ is constant

$$KE + PE = \frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v} + \frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$= \frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$= \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}kr_x^2 + \frac{1}{2}kr_y^2$$

$$= \frac{1}{2}m(-a\omega\sin\omega t)^2 + \frac{1}{2}m(b\omega\cos\omega t)^2 + \frac{1}{2}k(a\cos\omega t)^2 + \frac{1}{2}k(b\sin\omega t)^2$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} \qquad \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix}$$

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix}+\begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)$$

$Given: k=m\omega^2$

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix}+\begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)$$

$Given: k=m\omega^2$

$$E=KE+PE=\frac{1}{2}m\omega^2(a^2+b^2)=\frac{1}{2}k(a^2+b^2)$$ since: $\omega=\sqrt{\frac{k}{m}}=\sqrt{G\rho_{\oplus}4\pi/3}$ or: $m\omega^2=k$

(IHO formulas from Lect. 6 p.70-79)

Some Kepler's "laws" for all central (isotropic) force F(r) fields

- *Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$* *(Derived here)*
- *Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$* *(Derived in Unit 5)*
- *Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$* *(Derived here)*
- → *Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$* *(Derived in Unit 5)*

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE = \frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v} + \frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$= \frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$= \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}kr_x^2 + \frac{1}{2}kr_y^2$$

$$= \frac{1}{2}m(-a\omega\sin\omega t)^2 + \frac{1}{2}m(b\omega\cos\omega t)^2 + \frac{1}{2}k(a\cos\omega t)^2 + \frac{1}{2}k(b\sin\omega t)^2$$

$$= \frac{1}{2}ma^2\omega^2(\sin^2\omega t) + \frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2 + \frac{1}{2}ka^2(\cos^2\omega t) + \frac{1}{2}kb^2(\sin^2\omega t)$$

$$= \frac{1}{2}m\omega^2(a^2+b^2) \qquad Given: k = m\omega^2$$

$$E = KE+PE = \frac{1}{2}m\omega^2(a^2+b^2) = \frac{1}{2}k(a^2+b^2)$$ since: $\omega = \sqrt{\frac{k}{m}} = \sqrt{G\rho_\oplus 4\pi/3}$ or: $m\omega^2 = k$

We'll see that the Coul. orbits are simpler: (*like the period*...not a function of b)

(IHO formulas from Lect. 6 p.70-79)

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix}+\begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)$$

$Given: k=m\omega^2$

$$E=KE+PE=\frac{1}{2}m\omega^2(a^2+b^2)=\frac{1}{2}k(a^2+b^2)$$ since: $\omega=\sqrt{\frac{k}{m}}=\sqrt{G\rho_\oplus 4\pi/3}$ or: $m\omega^2=k$

We'll see that the Coul. orbits are simpler: (*like the period*...not a function of *b*)

$$E=KE+PE=\frac{1}{2}mv^2{}_x+\frac{1}{2}mv^2{}_y-\frac{k}{r}=\frac{1}{2}mv^2{}_x+\frac{1}{2}mv^2{}_y-\frac{GM_\oplus m}{r}=-\frac{GM_\oplus m}{a}$$

→ *Introduction to dual matrix operator contact geometry (based on IHO orbits)*

Quadratic form ellipse $\mathbf{r}\cdot Q\cdot\mathbf{r}=1$ *vs. inverse form ellipse* $\mathbf{p}\cdot Q^{-1}\cdot\mathbf{p}=1$

Duality norm relations ($\mathbf{r}\cdot\mathbf{p}=1$)

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}'\cdot\mathbf{p}=0=\mathbf{r}\cdot\mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation

Quadratic forms and tangent contact geometry of their ellipses

A matrix Q that generates an ellipse by **r**•*Q*•**r**=*1 is called* positive-definite (*if* **r**•*Q*•**r** *always >0*)

$$\overbrace{\begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix}}^{\mathbf{r}\bullet\mathbf{Q}\bullet\mathbf{r} \;=1} = 1 = \overbrace{\begin{pmatrix} x & y \end{pmatrix}}^{\mathbf{r}} \bullet \overbrace{\begin{pmatrix} \frac{x}{a^2} \\ \frac{y}{b^2} \end{pmatrix}}^{\mathbf{Q}\bullet\mathbf{r}} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

A inverse matrix Q^{-1} *generates an ellipse by* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$ *called* inverse *or* dual *ellipse:*

$$\overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix} \bullet \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \bullet \begin{pmatrix} p_x \\ p_y \end{pmatrix}}^{\mathbf{p}\bullet\mathbf{Q}^{-1}\bullet\mathbf{p} \;=1} = 1 = \overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix}}^{\mathbf{p}} \bullet \overbrace{\begin{pmatrix} a^2 p_x \\ b^2 p_y \end{pmatrix}}^{\mathbf{Q}^{-1}\bullet\mathbf{p}} = a^2 p_x^2 + b^2 p_y^2$$

Quadratic forms and tangent contact geometry of their ellipses

A matrix Q that generates an ellipse by **r**•*Q*•**r**=*1 is called* positive-definite (*if* **r**•*Q*•**r** *always >0*)

$$\underbrace{\begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix}}_{\mathbf{r}\bullet\mathbf{Q}\bullet\mathbf{r}\,=1} = 1 = \overbrace{\begin{pmatrix} x & y \end{pmatrix}}^{\mathbf{r}} \bullet \overbrace{\begin{pmatrix} \frac{x}{a^2} \\ \frac{y}{b^2} \end{pmatrix}}^{\mathbf{Q}\bullet\mathbf{r}=\mathbf{p}} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Defined mapping between ellipses

A inverse matrix Q^{-1} *generates an ellipse by* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$ *called* inverse *or* dual *ellipse:*

$$\underbrace{\begin{pmatrix} p_x & p_y \end{pmatrix} \bullet \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \bullet \begin{pmatrix} p_x \\ p_y \end{pmatrix}}_{\mathbf{p}\bullet\mathbf{Q}^{-1}\bullet\mathbf{p}\,=1} = 1 = \overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix}}^{\mathbf{p}} \bullet \overbrace{\begin{pmatrix} a^2 p_x \\ b^2 p_y \end{pmatrix}}^{\mathbf{Q}^{-1}\bullet\mathbf{p}=\mathbf{r}} = a^2 p_x^2 + b^2 p_y^2$$

Introduction to dual matrix operator contact geometry (based on IHO orbits)

→ *Quadratic form ellipse* $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

Duality norm relations $(\mathbf{r}\bullet\mathbf{p}=1)$

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ $(\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}')$

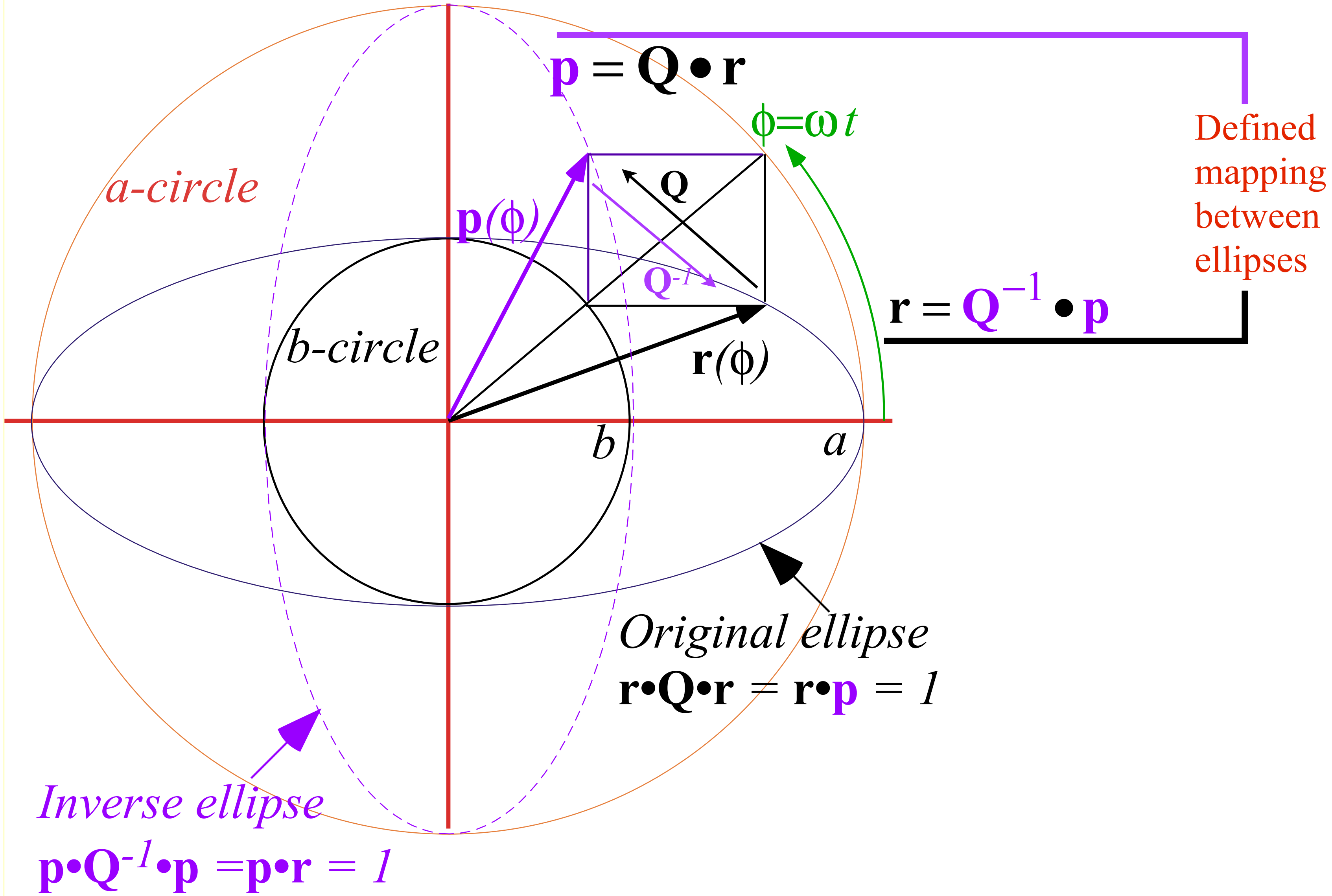
Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation

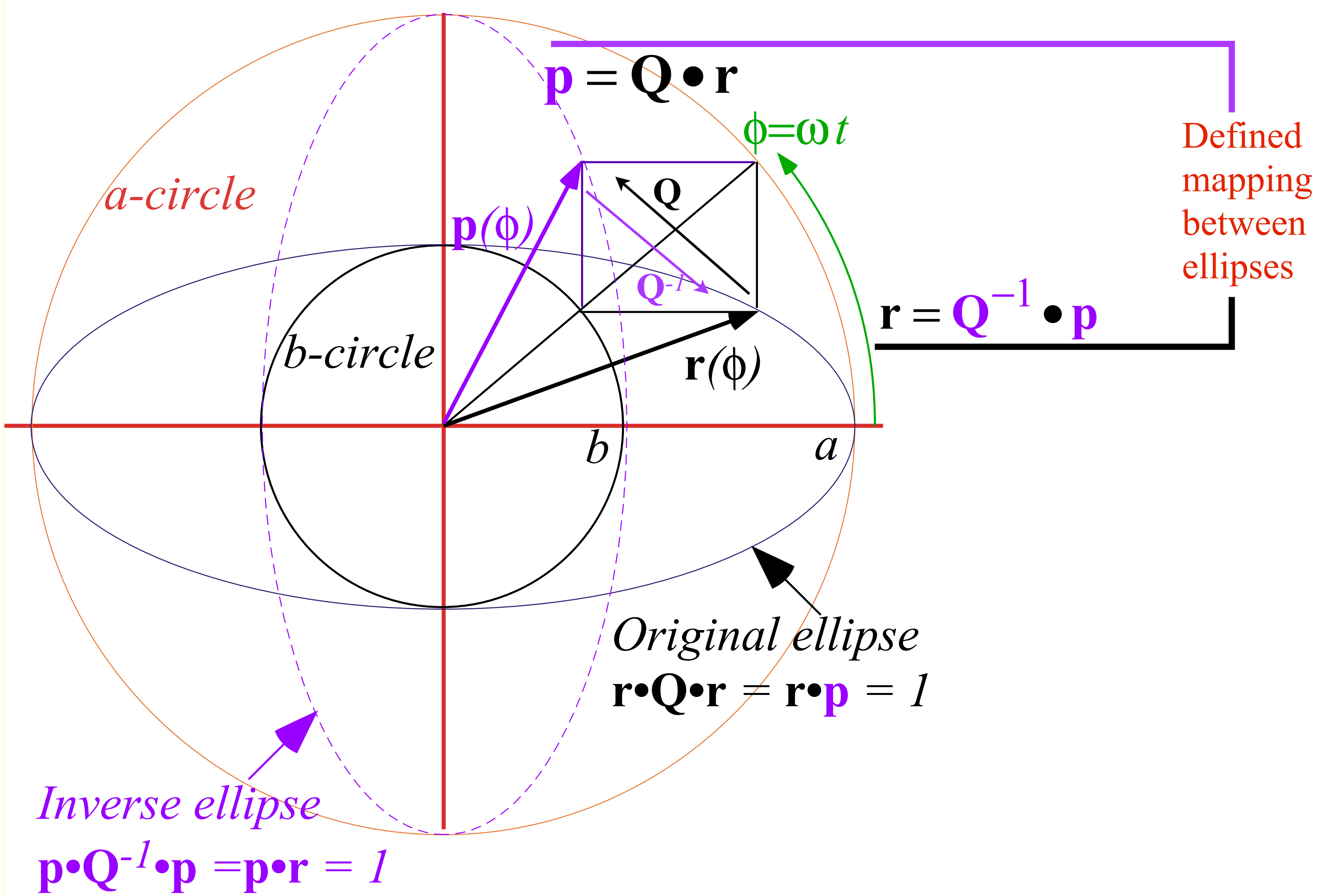
(a) Quadratic form ellipse and
Inverse quadratic form ellipse

based on
Unit 1
Fig. 11.6



(a) Quadratic form ellipse and
Inverse quadratic form ellipse

based on
Unit 1
Fig. 11.6



Here plot of **p**-ellipse is re-scaled by scalefactor $S=a \cdot b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

Introduction to dual matrix operator geometry (based on IHO orbits)

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

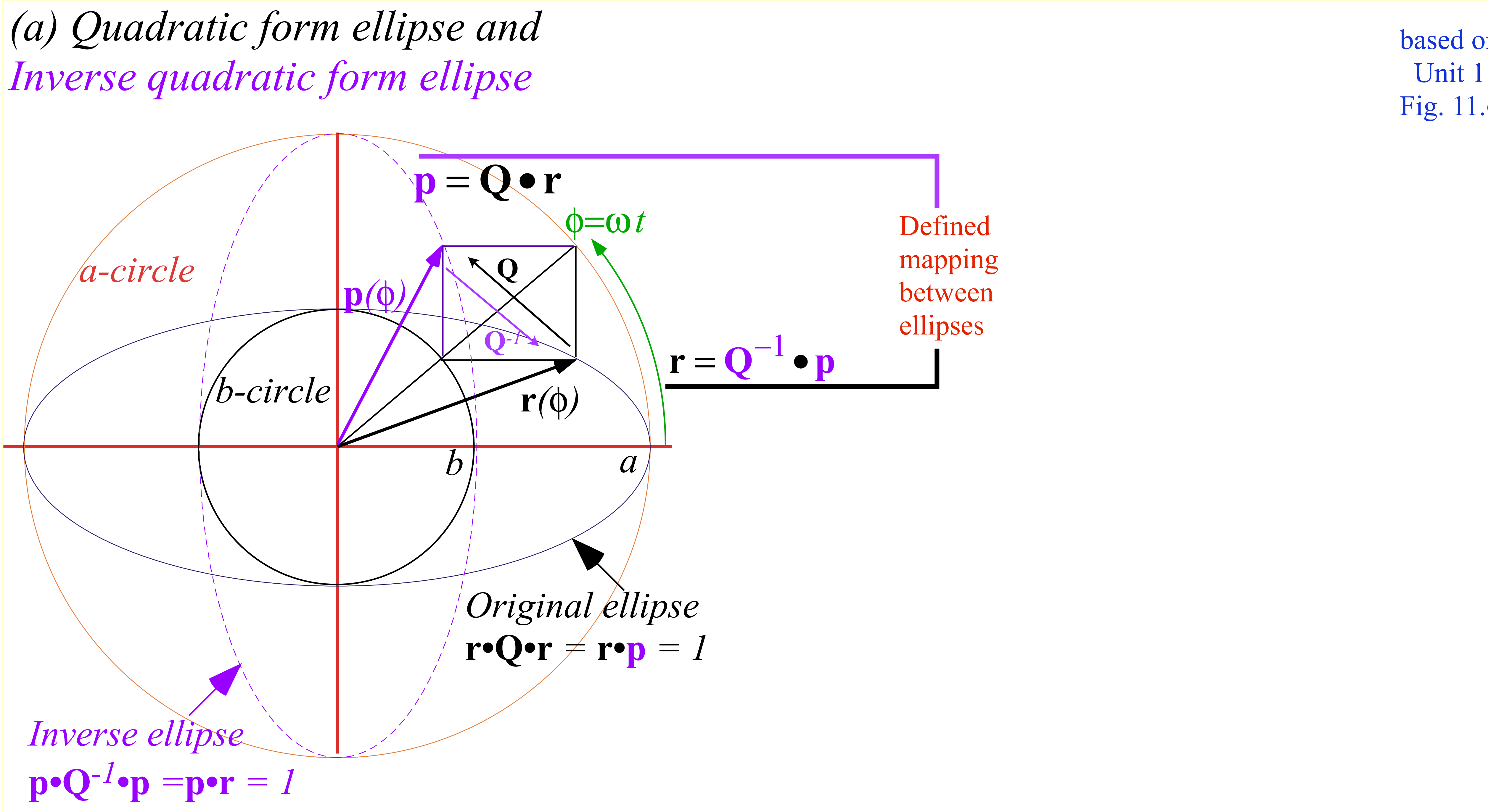
Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$)

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation



Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** *=1=* **p•r**

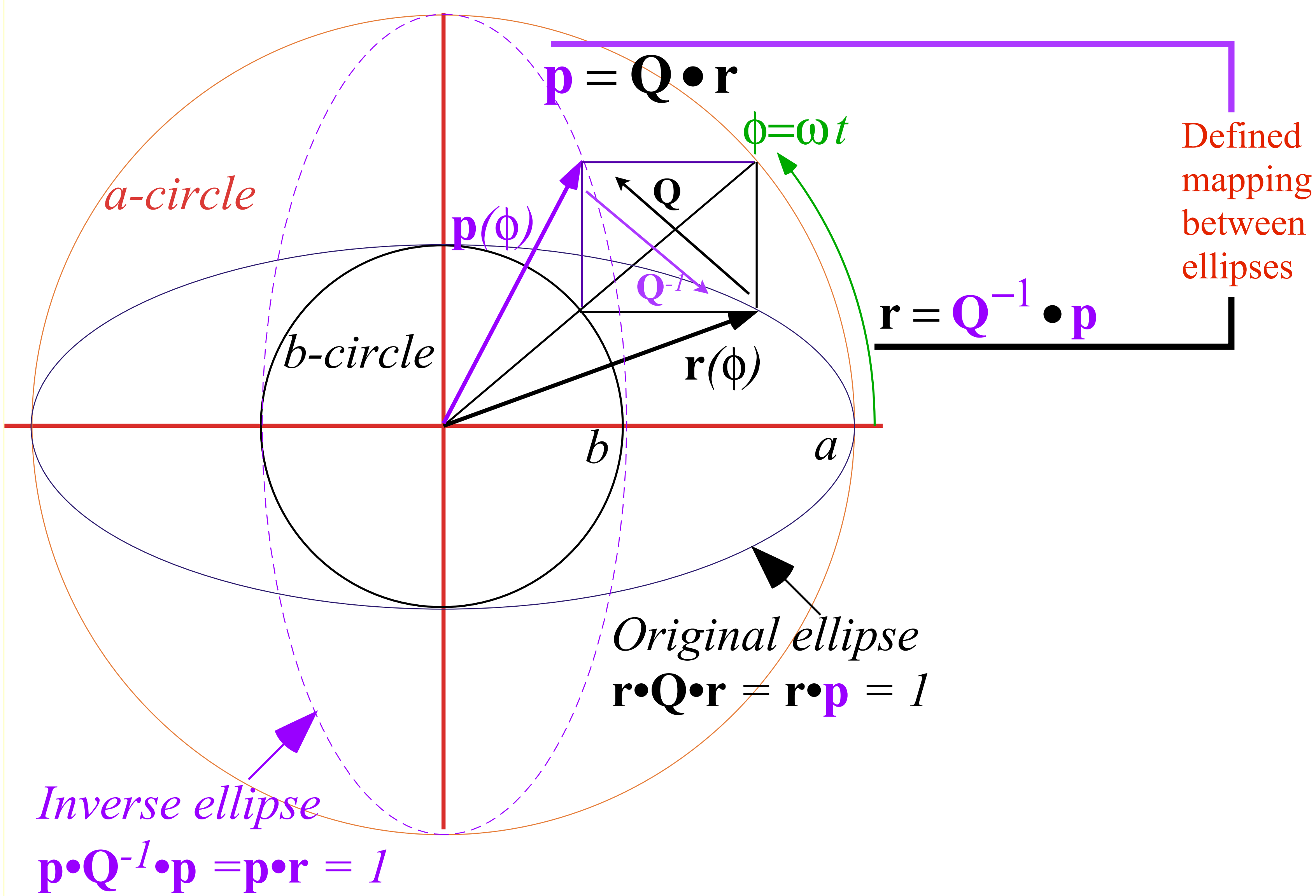
Here plot of **p**-ellipse is re-scaled by scalefactor $S=a \cdot b$

p-ellipse x-radius$=1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius$=1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

(a) Quadratic form ellipse and
Inverse quadratic form ellipse

based on
Unit 1
Fig. 11.6



Quadratic form **r•Q•r** =*1 has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** =*1*= **p•r**

$$\mathbf{p}=\mathbf{Q}\cdot\mathbf{r}=\overbrace{\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}}^{\mathbf{Q}}\bullet\overbrace{\begin{pmatrix}x\\ y\end{pmatrix}}^{\mathbf{r}}=\overbrace{\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}}^{\mathbf{p}}=\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}$$

where: $x=r_x=a\cos\phi=a\cos\omega t$, $y=r_y=b\sin\phi=b\sin\omega t$ so: $\mathbf{p}\cdot\mathbf{r}=1$

Here plot of **p**-ellipse is re-scaled by scalefactor $S=a\cdot b$

p-ellipse x-radius=1/a plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=1/b plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

Link ⇒ BoxIt simulation of IHO orbits
Link → IHO orbital time rates of change
Link → IHO Exegesis Plot

Introduction to dual matrix operator geometry (based on IHO orbits)

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

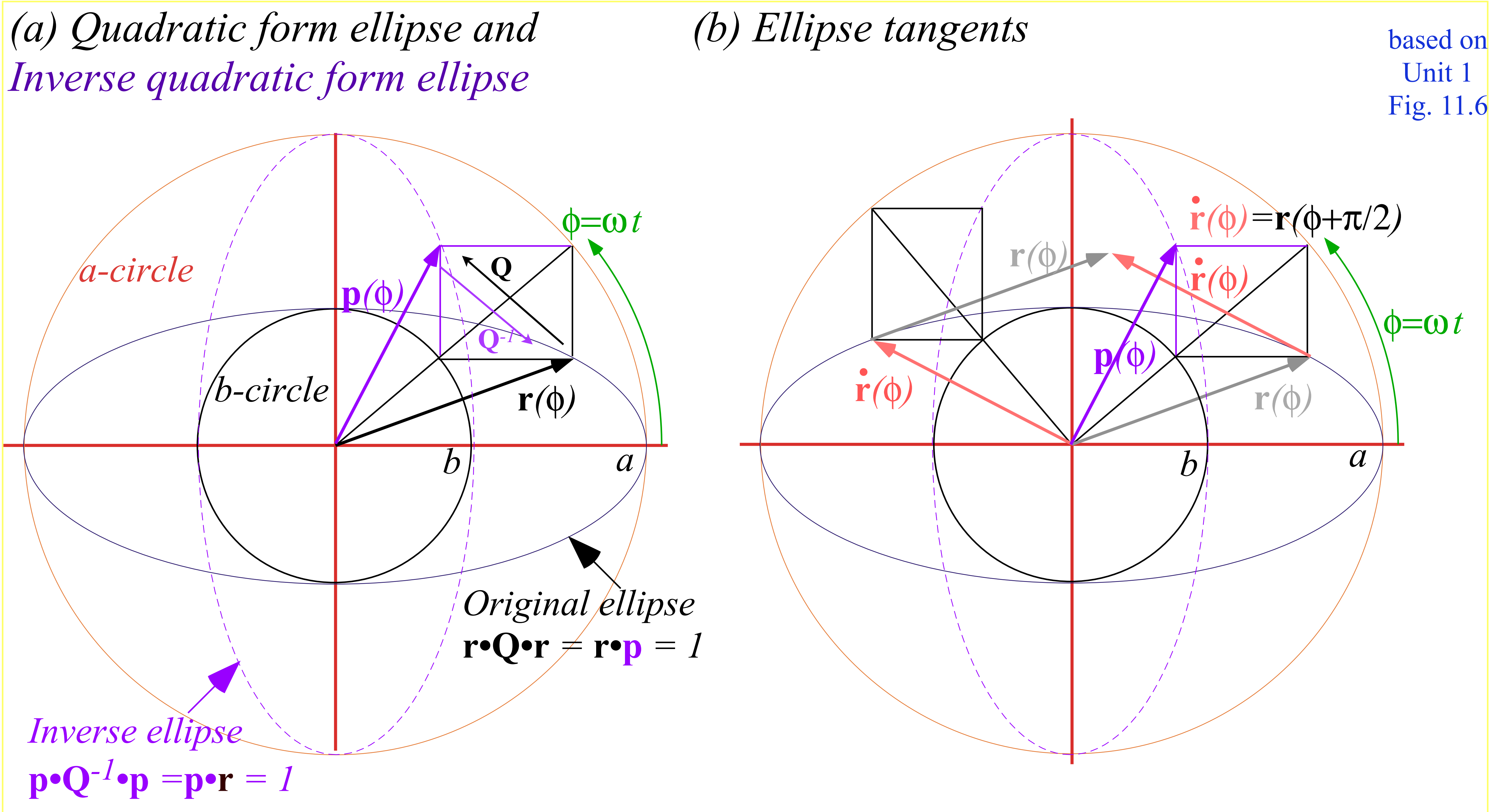
Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$)

→ *Q-Ellipse tangents* $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation



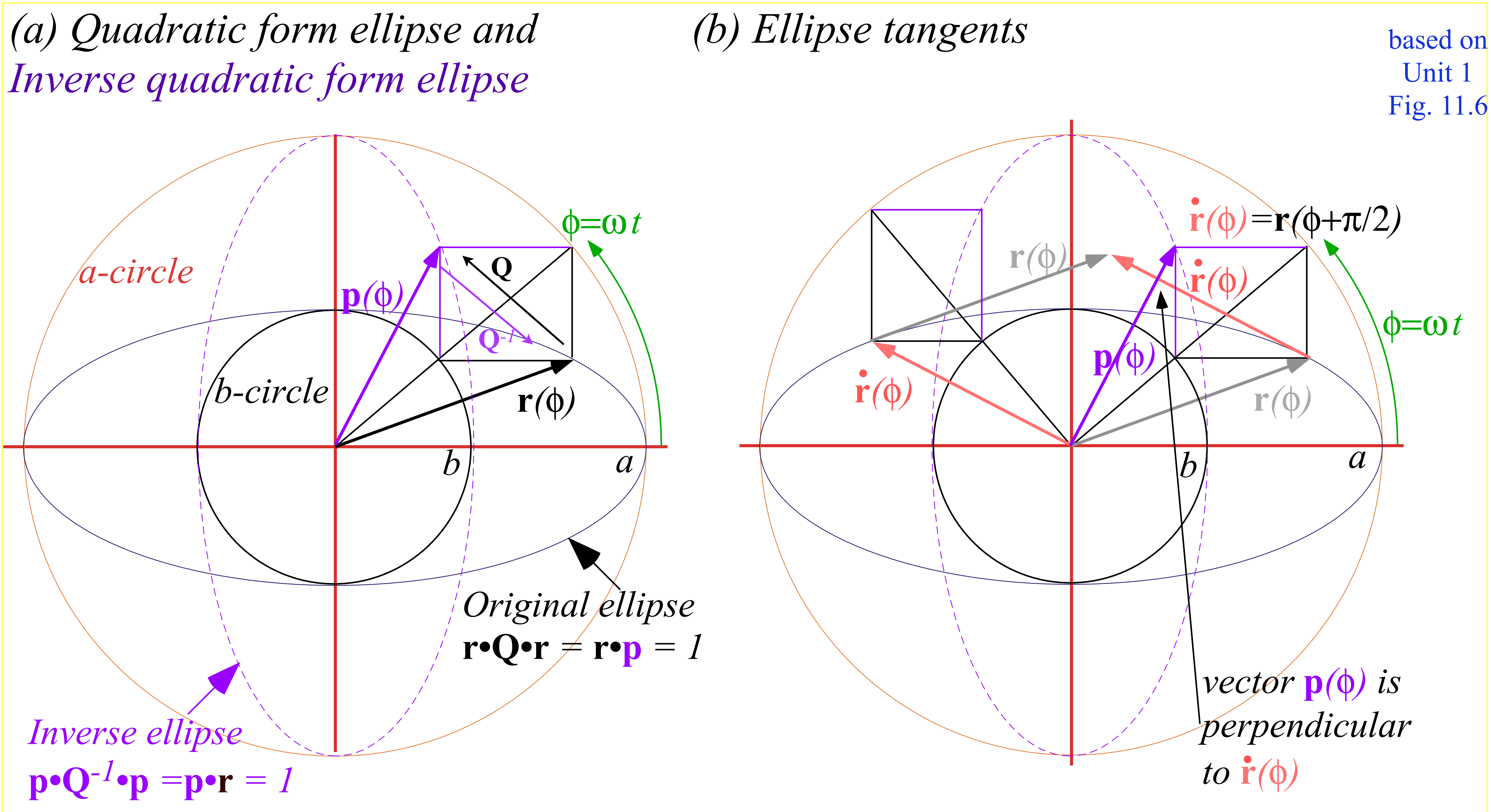
Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** *=1=* **p•r**

$$\mathbf{p} = \mathbf{Q} \cdot \mathbf{r} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix} = \begin{pmatrix} (1/a)\cos\phi \\ (1/b)\sin\phi \end{pmatrix} \quad \text{where: } \begin{matrix} x = r_x = a\cos\phi = a\cos\omega t \\ y = r_y = b\sin\phi = b\sin\omega t \end{matrix} \quad \text{so: } \boxed{\mathbf{p} \cdot \mathbf{r} = 1}$$

Here plot of **p**-ellipse is re-scaled by scalefactor $S=a \cdot b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

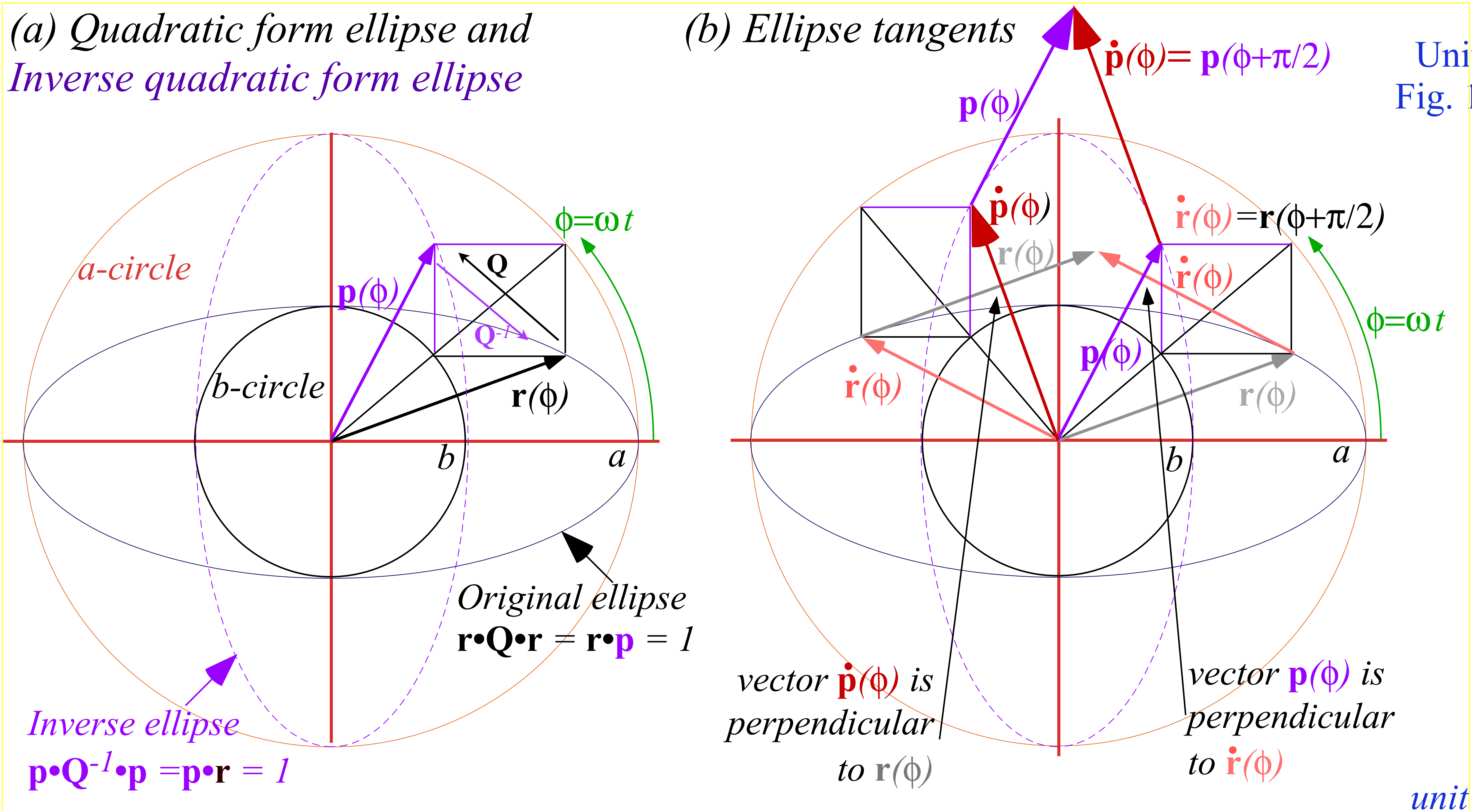


Quadratic form $\mathbf{r}\bullet\mathbf{Q}\bullet\mathbf{r} = 1$ *has mutual duality relations with inverse form* $\mathbf{p}\bullet\mathbf{Q}^{-1}\bullet\mathbf{p} = 1 = \mathbf{p}\bullet\mathbf{r}$

$$\mathbf{p} = \mathbf{Q}\bullet\mathbf{r} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix} = \begin{pmatrix} (1/a)\cos\phi \\ (1/b)\sin\phi \end{pmatrix} \quad \text{where: } \begin{matrix} x = r_x = a\cos\phi = a\cos\omega t \\ y = r_y = b\sin\phi = b\sin\omega t \end{matrix} \quad \text{so: } \boxed{\mathbf{p}\bullet\mathbf{r} = 1}$$

$\mathbf{p}$ *is perpendicular to velocity* $\mathbf{v} = \dot{\mathbf{r}}$ *, a mutual orthogonality*

$$\boxed{\dot{\mathbf{r}}\bullet\mathbf{p} = 0} = \begin{pmatrix} \dot{r}_x & \dot{r}_y \end{pmatrix} \bullet \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} -a\sin\phi & b\cos\phi \end{pmatrix} \bullet \begin{pmatrix} (1/a)\cos\phi \\ (1/b)\sin\phi \end{pmatrix} \quad \text{where: } \begin{matrix} \dot{r}_x = -a\sin\phi \\ \dot{r}_y = b\cos\phi \end{matrix} \quad \text{and: } \begin{matrix} p_x = (1/a)\cos\phi \\ p_y = (1/b)\sin\phi \end{matrix}$$



Unit 1
Fig. 11.6

unit mutual projection

Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q⁻¹•p** *=1*

$$\mathbf{p}=\mathbf{Q}\bullet\mathbf{r}=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\bullet\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}=\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}$$

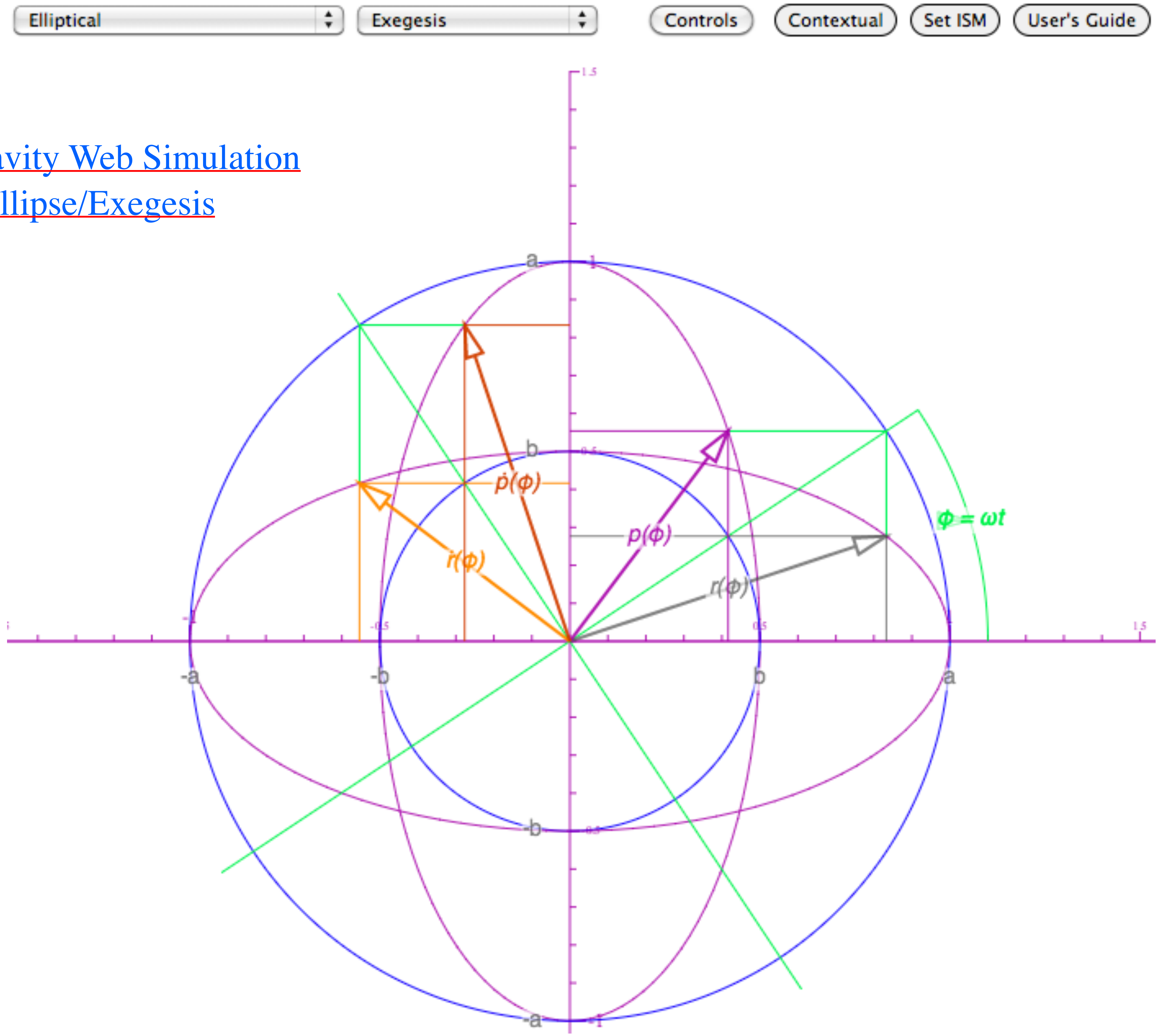
where: $x=r_x=a\cos\phi=a\cos\omega t$, $y=r_y=b\sin\phi\;=b\sin\omega t$ so: $\boxed{\mathbf{p}\bullet\mathbf{r}=1}$

p is perpendicular to velocity **v**=**ṙ** *, a mutual orthogonality. So is* **r** *perpendicular to* **ṗ**: $\boxed{\dot{\mathbf{p}}\bullet\mathbf{r}=0}$

$$\boxed{\dot{\mathbf{r}}\bullet\mathbf{p}=0}=\begin{pmatrix}\dot r_x & \dot r_y\end{pmatrix}\bullet\begin{pmatrix}p_x\\ p_y\end{pmatrix}=\begin{pmatrix}-a\sin\phi & b\cos\phi\end{pmatrix}\bullet\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}$$

where: $\dot r_x=-a\sin\phi$, $\dot r_y=b\cos\phi$ and: $p_x=(1/a)\cos\phi$, $p_y=(1/b)\sin\phi$

[RelaWavity Web Simulation](#)
[Ellipse/Exegesis](#)



Geometry of dual ellipse Kepler anomalies for vectors [$\mathbf{r}(\phi)$, $\mathbf{p}(\phi)$] and $^{d}/_{dt}$[$\mathbf{r}(\phi)$, $\mathbf{p}(\phi)$,] *in coordinate (x,y) space rendered by animation web-app in RelaWavity and described in Lect. 12-advanced.*

Introduction to dual matrix operator geometry (based on IHO orbits)

Quadratic form ellipse $\mathbf{r} \bullet Q \bullet \mathbf{r} = 1$ *vs.inverse form ellipse* $\mathbf{p} \bullet Q^{-1} \bullet \mathbf{p} = 1$

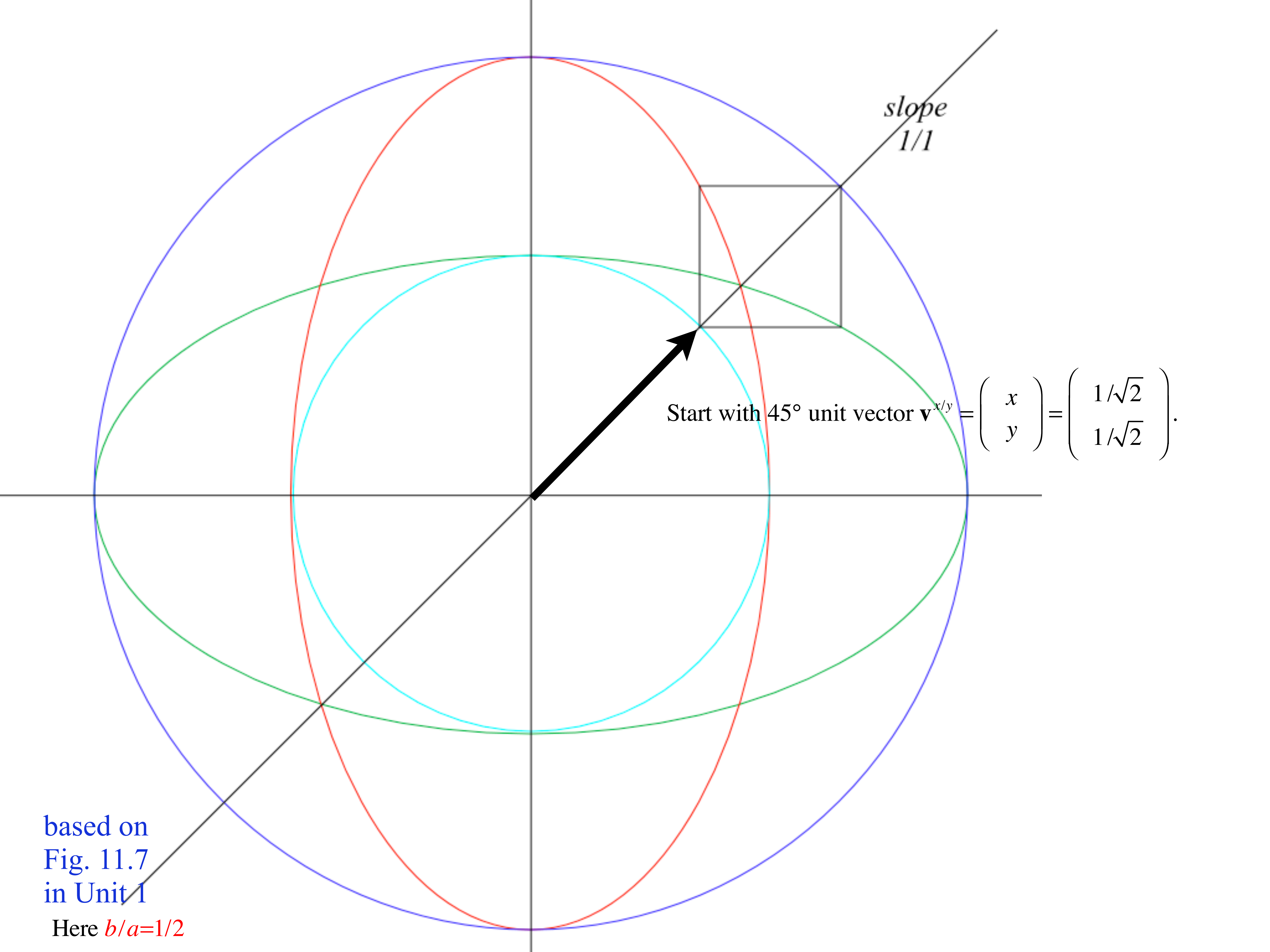
Duality norm relations ($\mathbf{r} \bullet \mathbf{p} = 1$)

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}' \bullet \mathbf{p} = 0 = \mathbf{r} \bullet \mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation



slope 1/1
Start with 45° unit vector $\mathbf{v}^{x/y} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$.
based on Fig. 11.7 in Unit 1
Here $b/a=1/2$

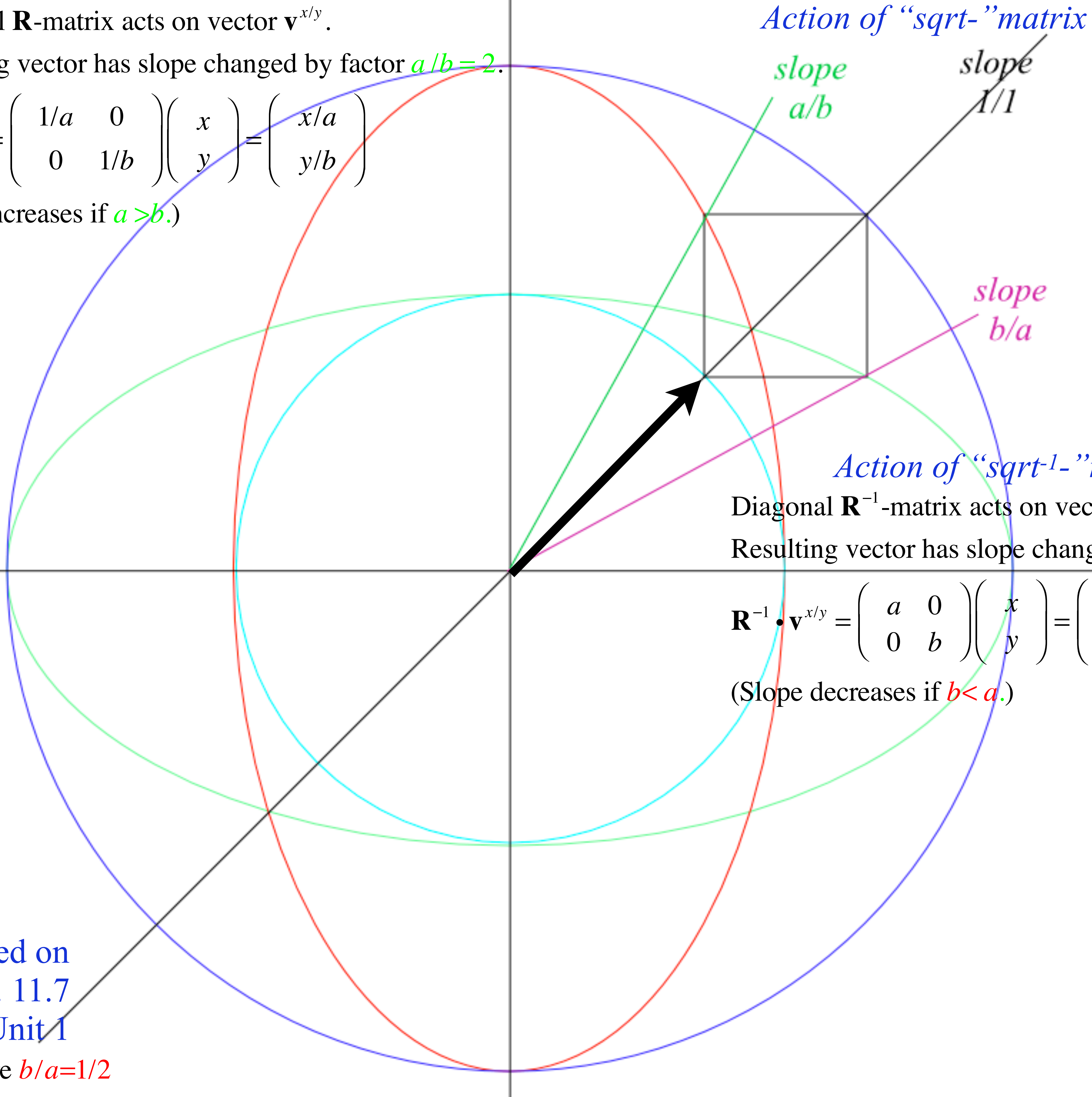
Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b=2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}1/a & 0\\ 0 & 1/b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a\\ y/b\end{pmatrix}$$

(Slope increases if $a>b$.)

Action of "sqrt-"matrix $R=\sqrt{Q}$



Action of "sqrt^{-1}-"matrix $R^{-1}=\sqrt{Q}^{-1}$

Diagonal $\mathbf{R}^{-1}$-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor b/a.

$$\mathbf{R}^{-1}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\cdot a\\ y\cdot b\end{pmatrix}$$

(Slope decreases if $b<a$.)

based on
Fig. 11.7
in Unit 1

Here $b/a=1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b=2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}1/a & 0\\ 0 & 1/b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a\\ y/b\end{pmatrix}$$

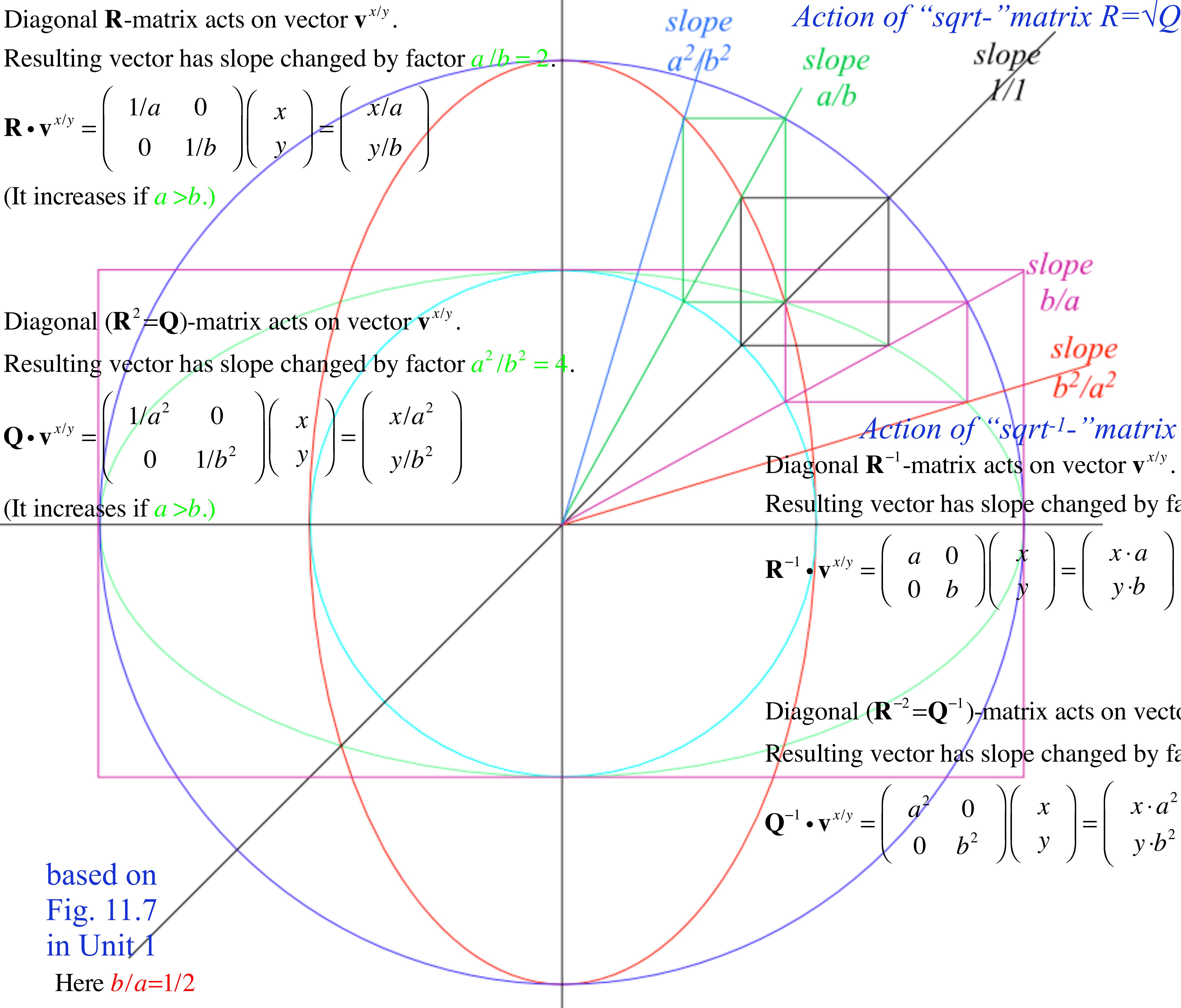
(It increases if $a>b$.)

Diagonal ($\mathbf{R}^2$=**Q**)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2=4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}$$

(It increases if $a>b$.)



Diagonal $\mathbf{R}^{-1}$-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b/a=1/2$.

$$\mathbf{R}^{-1}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\cdot a\\ y\cdot b\end{pmatrix}$$

Diagonal ($\mathbf{R}^{-2}$=$\mathbf{Q}^{-1}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^2/a^2=1/4$.

$$\mathbf{Q}^{-1}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}a^2 & 0\\ 0 & b^2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\cdot a^2\\ y\cdot b^2\end{pmatrix}$$

based on
Fig. 11.7
in Unit 1

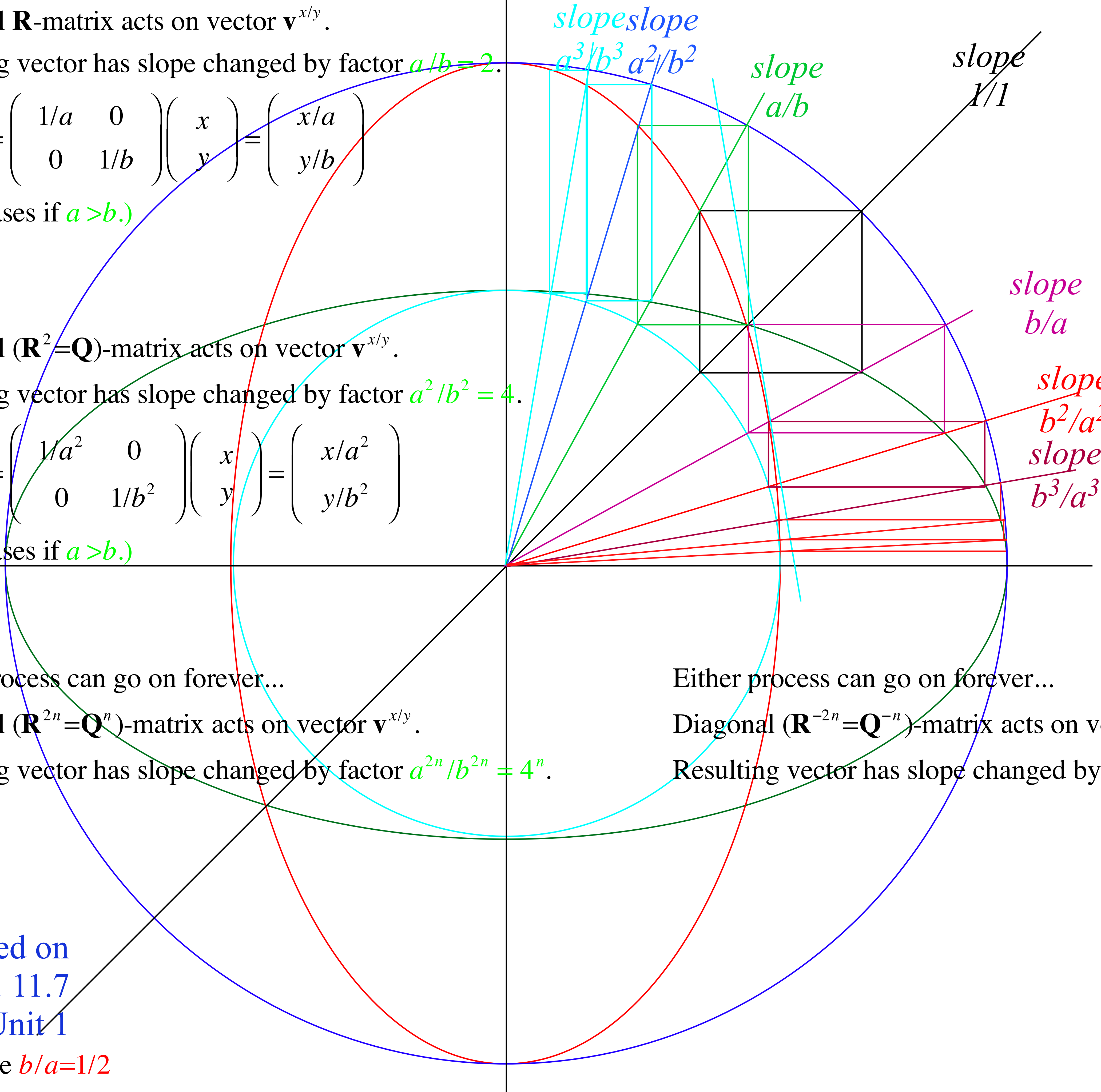
Here $b/a=1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

(It increases if $a > b$.)



Diagonal ($\mathbf{R}^2 = \mathbf{Q}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2 = 4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix}$$

(It increases if $a > b$.)

Either process can go on forever...

Diagonal ($\mathbf{R}^{2n} = \mathbf{Q}^n$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^{2n}/b^{2n} = 4^n$.

Either process can go on forever...

Diagonal ($\mathbf{R}^{-2n} = \mathbf{Q}^{-n}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^{2n}/a^{2n} = 4^{-n}$.

based on
Fig. 11.7
in Unit 1

Here $b/a = 1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

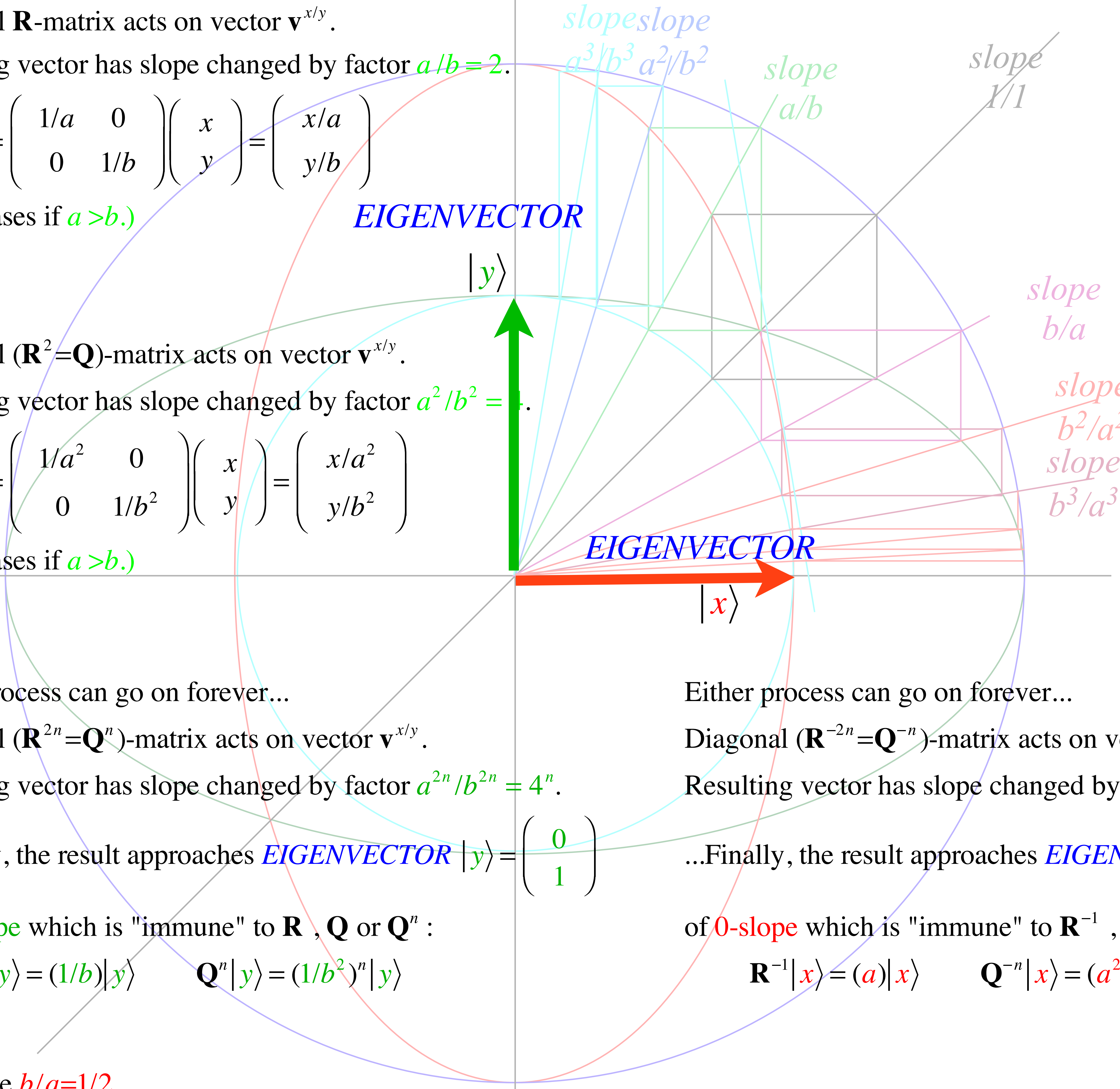
(It increases if $a > b$.)

Diagonal ($\mathbf{R}^2 = \mathbf{Q}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2 = 4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix}$$

(It increases if $a > b$.)



Either process can go on forever...

Diagonal ($\mathbf{R}^{2n} = \mathbf{Q}^n$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^{2n}/b^{2n} = 4^n$.

...Finally, the result approaches *EIGENVECTOR* $|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

of ∞-slope which is "immune" to **R**, **Q** or $\mathbf{Q}^n$:

$$\mathbf{R}|y\rangle = (1/b)|y\rangle \qquad \mathbf{Q}^n|y\rangle = (1/b^2)^n|y\rangle$$

Either process can go on forever...

Diagonal ($\mathbf{R}^{-2n} = \mathbf{Q}^{-n}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^{2n}/a^{2n} = 4^{-n}$.

...Finally, the result approaches *EIGENVECTOR* $|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

of 0-slope which is "immune" to $\mathbf{R}^{-1}$, $\mathbf{Q}^{-1}$ or $\mathbf{Q}^{-n}$:

$$\mathbf{R}^{-1}|x\rangle = (a)|x\rangle \qquad \mathbf{Q}^{-n}|x\rangle = (a^2)^n|x\rangle$$

Here $b/a = 1/2$

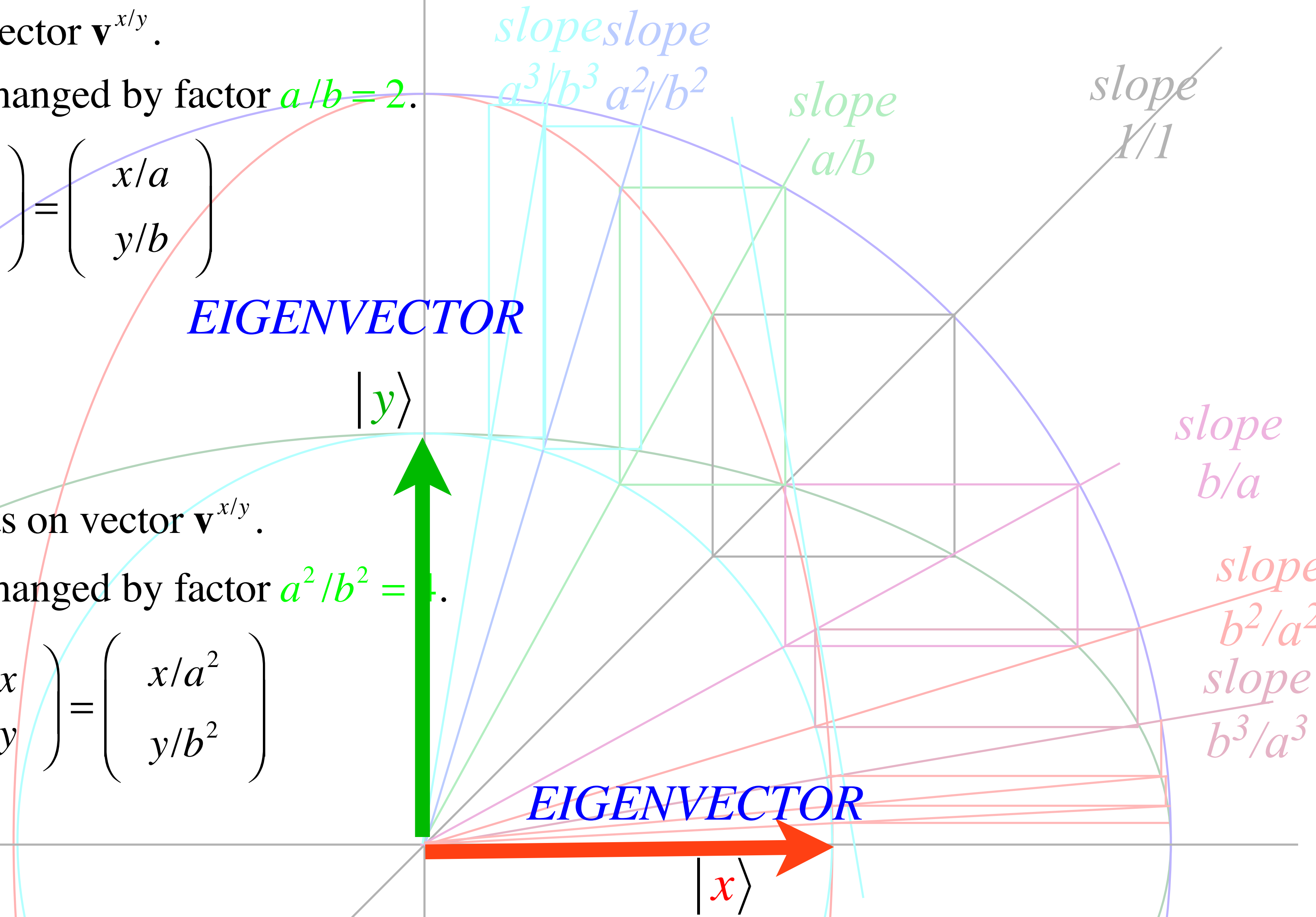
Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

(It increases if $a > b$.)

Here $b/a = 1/2$



Diagonal ($\mathbf{R}^2$=**Q**)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2 = 4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix}$$

(It increases if $a > b$.)

Either process can go on forever...

Diagonal ($\mathbf{R}^{2n}$=$\mathbf{Q}^n$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^{2n}/b^{2n} = 4^n$.

...Finally, the result approaches *EIGENVECTOR* $|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

of ∞-slope which is "immune" to **R** , **Q** or $\mathbf{Q}^n$:

Either process can go on forever...

Diagonal ($\mathbf{R}^{-2n}$=$\mathbf{Q}^{-n}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^{2n}/a^{2n} = 4^{-n}$.

...Finally, the result approaches *EIGENVECTOR* $|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

of 0-slope which is "immune" to $\mathbf{R}^{-1}$, $\mathbf{Q}^{-1}$ or $\mathbf{Q}^{-n}$:

$\mathbf{R}|y\rangle = (1/b)|y\rangle$ $\mathbf{Q}^n|y\rangle = (1/b^2)^n|y\rangle$

Eigenvalues

Eigensolution Relations

$\mathbf{R}^{-1}|x\rangle = (a)|x\rangle$ $\mathbf{Q}^{-n}|x\rangle = (a^2)^n|x\rangle$

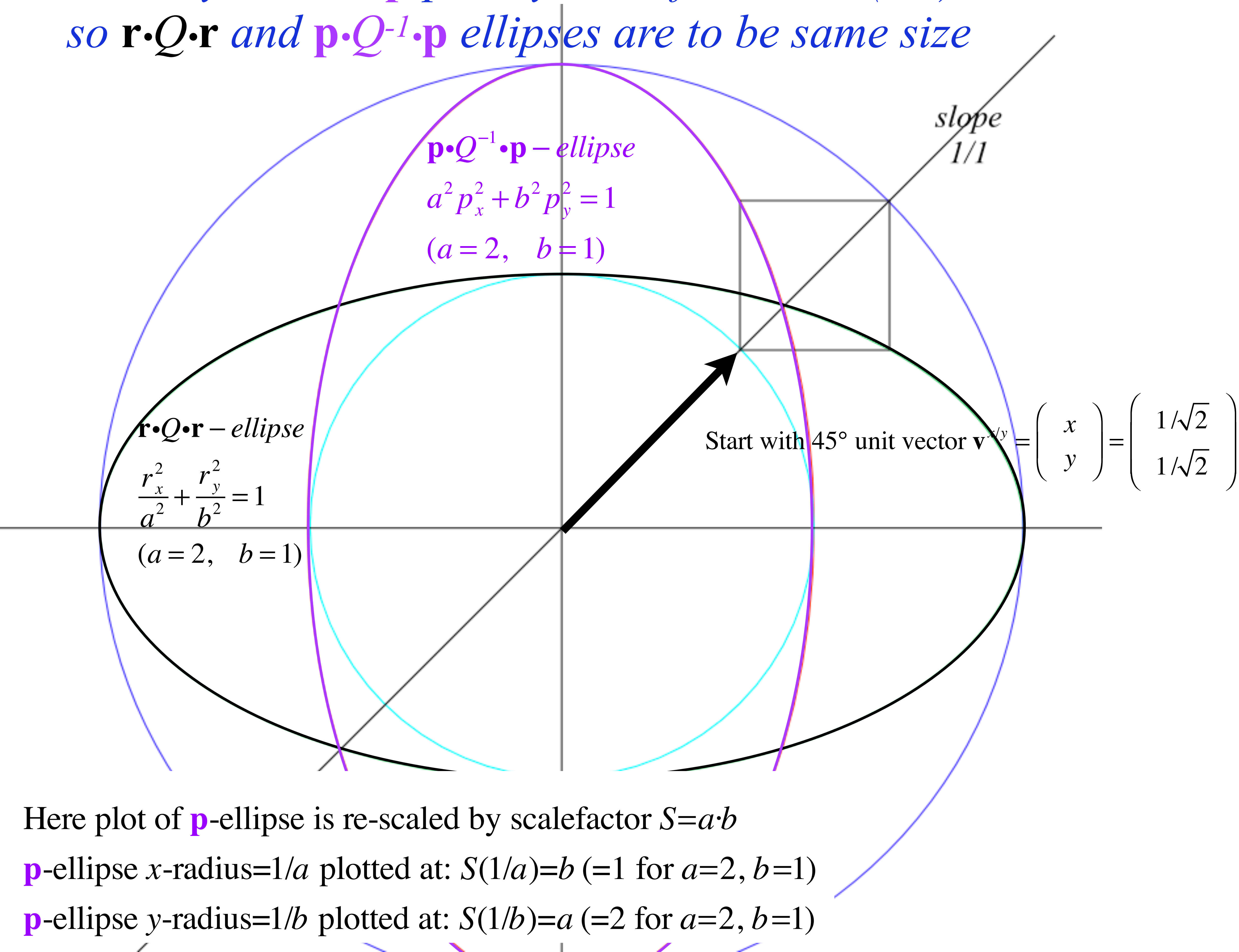
Eigenvalues

Introduction to dual matrix operator geometry (based on IHO orbits)

- *Quadratic form ellipse* $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$
  - *Duality norm relations (* $\mathbf{r}\bullet\mathbf{p}=1$*)*
  - *Q-Ellipse tangents* $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*
- *Operator geometric sequences and eigenvectors*
- → *Alternative scaling of matrix operator geometry*
  - *Vector calculus of tensor operation*

You may rescale **p***-plot by scale factor S=(a·b)*
so **r**·*Q*·**r** *and* **p**·*Q*$^{-1}$·**p** *ellipses are to be same size*

Here b/a=1/2



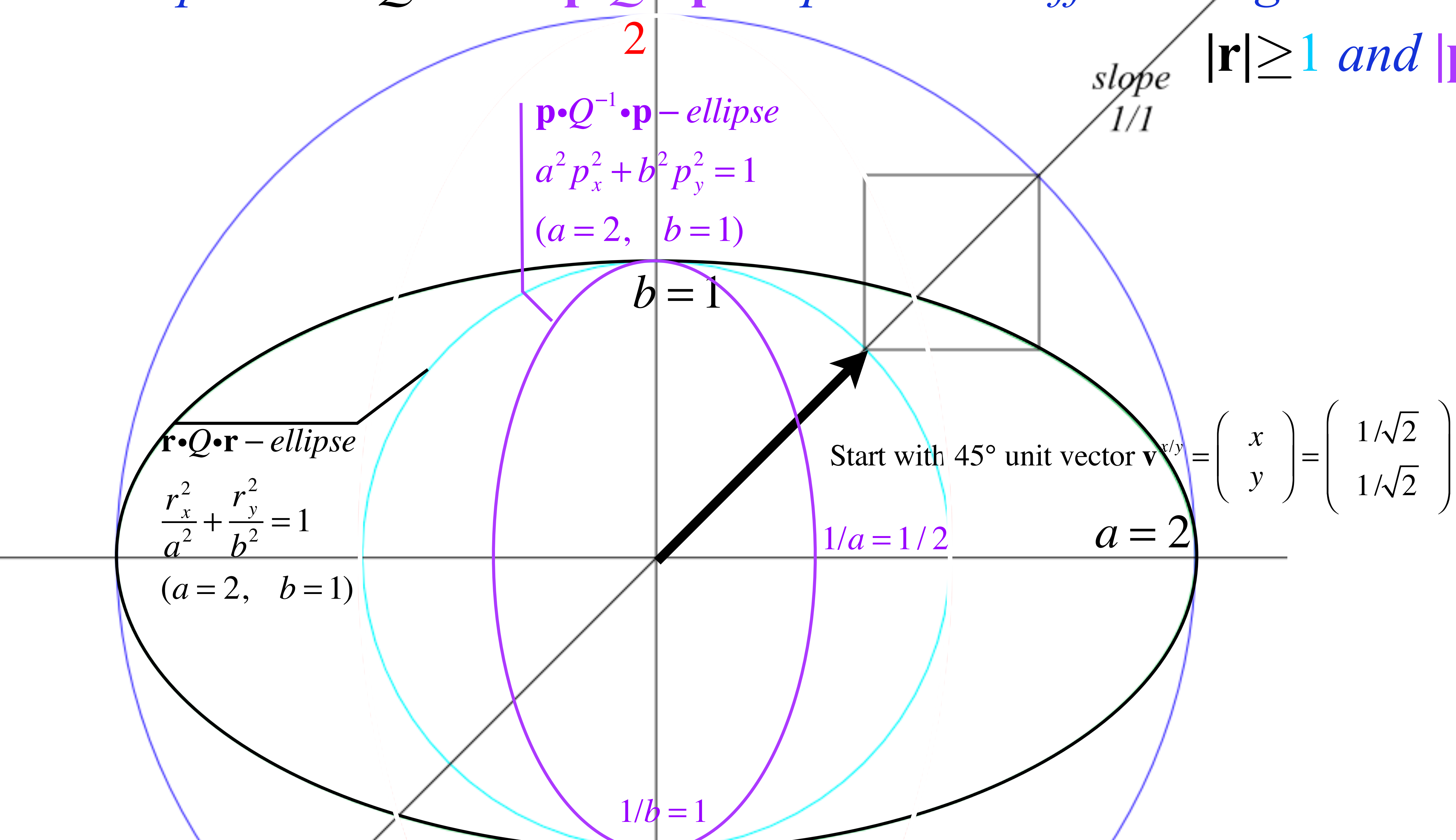
Here plot of **p**-ellipse is re-scaled by scalefactor $S=a\cdot b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

..or else rescale p-plot by scale factor $S=b$ to separate $\mathbf{r}\cdot Q\cdot\mathbf{r}$ and $\mathbf{p}\cdot Q^{-1}\cdot\mathbf{p}$ ellipses into different regions

Here $b/a=1/2$



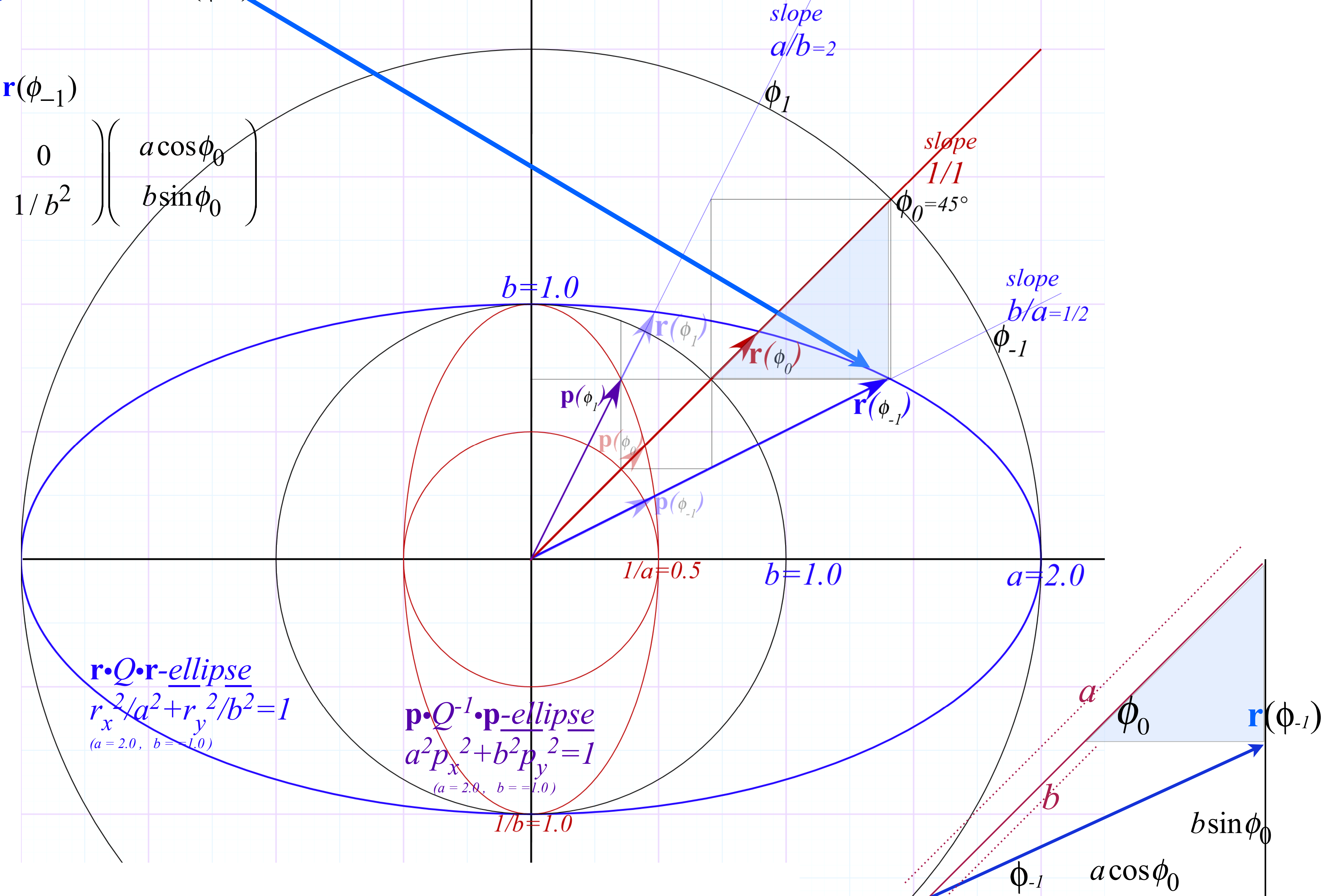
Here plot of **p**-ellipse is re-scaled by scalefactor $S=b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b/a$ (=1/2 for $a=2$, $b=1$)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=1$

*Action of matrix Q that generates an **r**-ellipse (**r**•Q• **r** =1) on a single **r**-vector **r**(ϕ-1)...*

$$\mathbf{p}(\phi_1) = \mathbf{Q} \bullet \mathbf{r}(\phi_{-1}) = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$



Variation of Fig. 11.7 in Unit 1

Here plot of **p**-ellipse is re-scaled by scalefactor $S=b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b/a$ (=1/2 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=1$

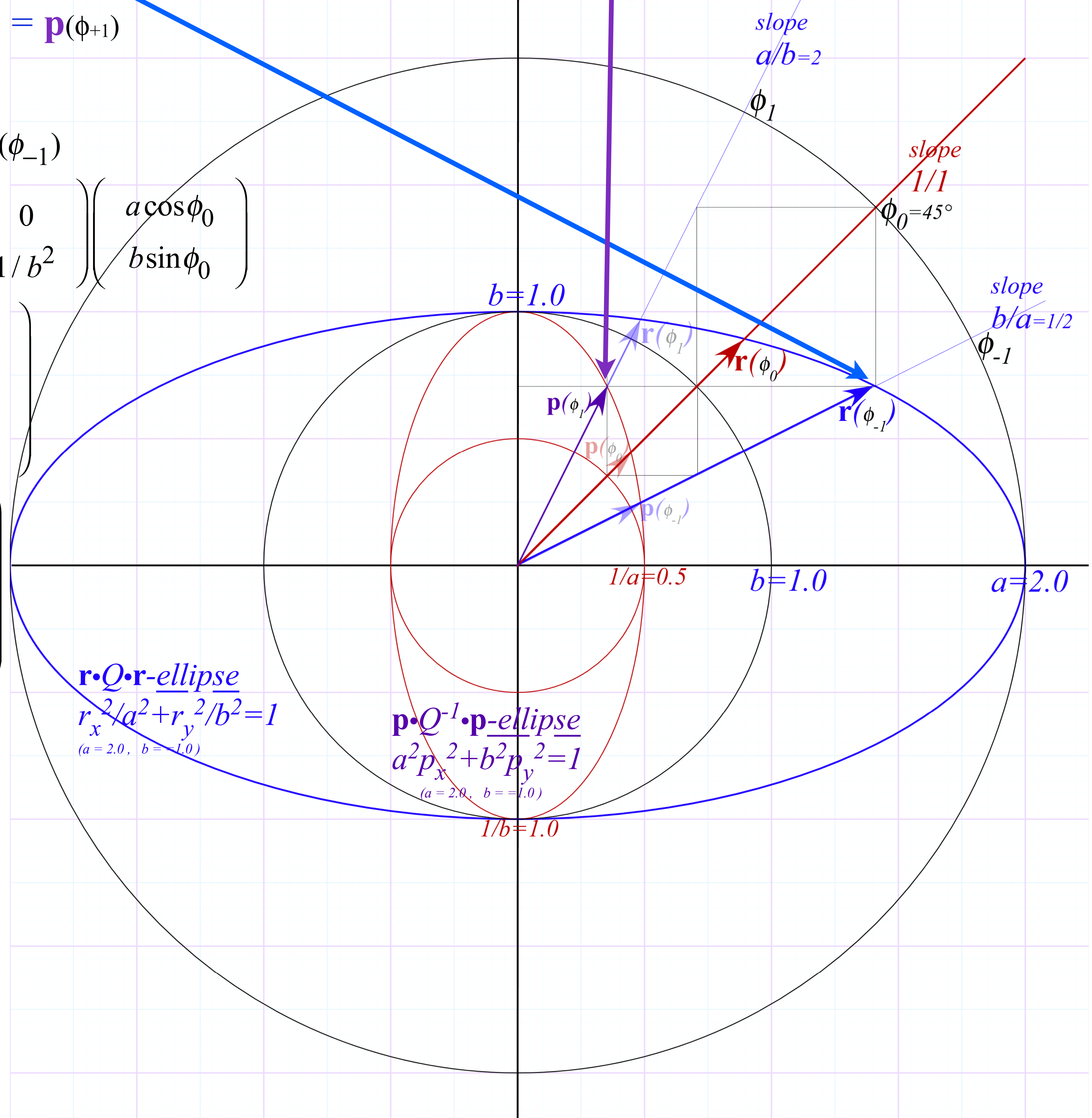
Action of matrix Q that generates an **r***-ellipse (***r•***Q***•r** *=1)*
on a single **r***-vector* **r**(ϕ_{-1})*... is to rotate it to a new vector* **p** *on the* **p***-ellipse (***p•***Q*$^{-1}$**• p** *=1),*
that is, $Q\bullet\mathbf{r}(\phi_{-1}) = \mathbf{p}(\phi_{+1})$

$$\mathbf{p}(\phi_1) = \mathbf{Q}\bullet\mathbf{r}(\phi_{-1})$$

$$= \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{a}\cos\phi_0 \\ \frac{1}{b}\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\frac{1}{\sqrt{2}} \\ \frac{1}{1}\frac{1}{\sqrt{2}} \end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1

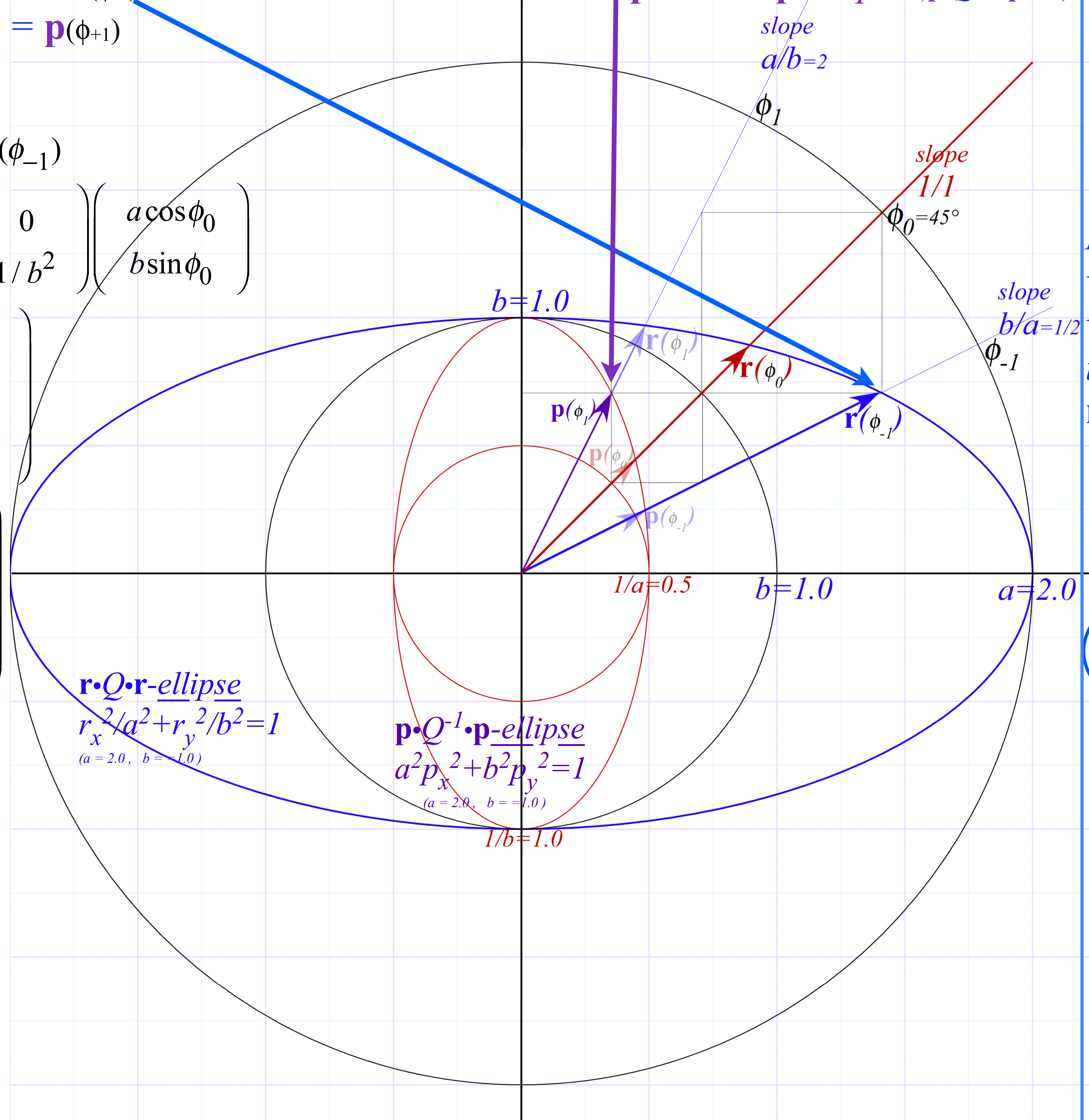
Action of matrix Q that generates an **r***-ellipse* (**r**•Q•**r** $=1$)
on a single **r***-vector* **r**(ϕ_{-1})*... is to rotate it to a new vector* **p** *on the* **p***-ellipse* (**p**•Q^{-1}• **p** $=1$),
that is, Q•**r**(ϕ_{-1}) = **p**(ϕ_{+1})

$$\mathbf{p}(\phi_1) = \mathbf{Q} \bullet \mathbf{r}(\phi_{-1})$$

$$= \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{a}\cos\phi_0 \\ \frac{1}{b}\sin\phi_0 \end{pmatrix}$$

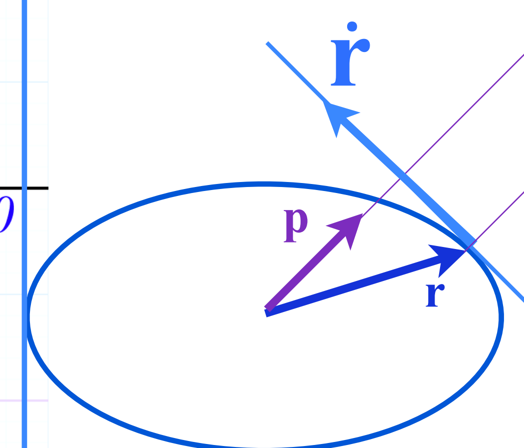
$$= \begin{pmatrix} \frac{1}{2}\frac{1}{\sqrt{2}} \\ \frac{1}{1}\frac{1}{\sqrt{2}} \end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1

Key points of matrix geometry:

Matrix Q maps any vector **r** *to a new vector* **p** *normal to the tangent* $\dot{\mathbf{r}}$ *to its* **r**•Q•**r***-ellipse.*



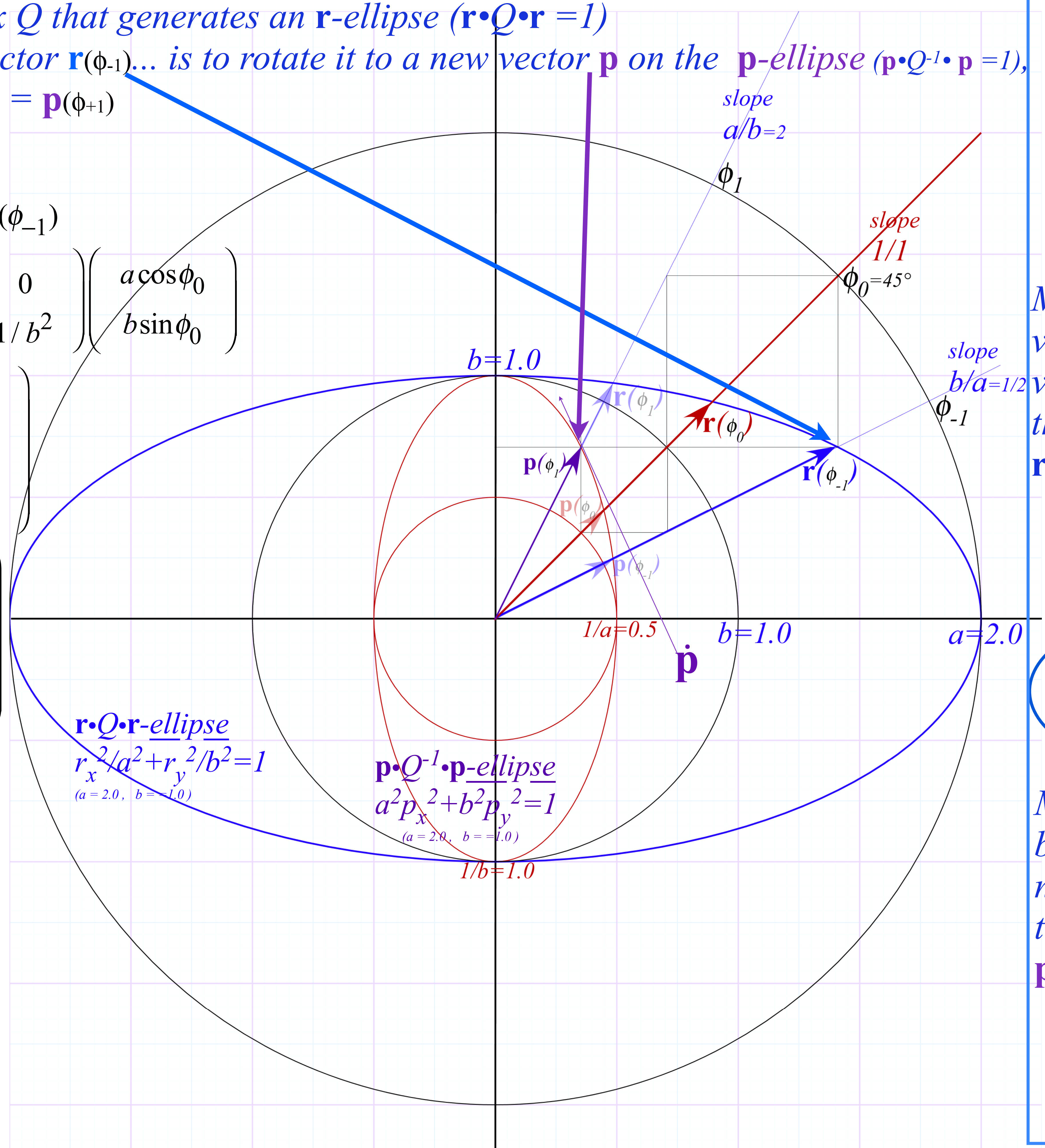
Action of matrix Q that generates an **r**-*ellipse (***r**•*Q*•**r** *=1) on a single* **r**-*vector* **r**(ϕ_{-1})... *is to rotate it to a new vector* **p** *on the* **p**-*ellipse* (**p**•Q^{-1}• **p** *=1), that is,* Q•**r**(ϕ_{-1}) = **p**(ϕ_{+1})

$$\mathbf{p}(\phi_1) = \mathbf{Q}\bullet\mathbf{r}(\phi_{-1})$$

$$= \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{a}\cos\phi_0 \\ \frac{1}{b}\sin\phi_0 \end{pmatrix}$$

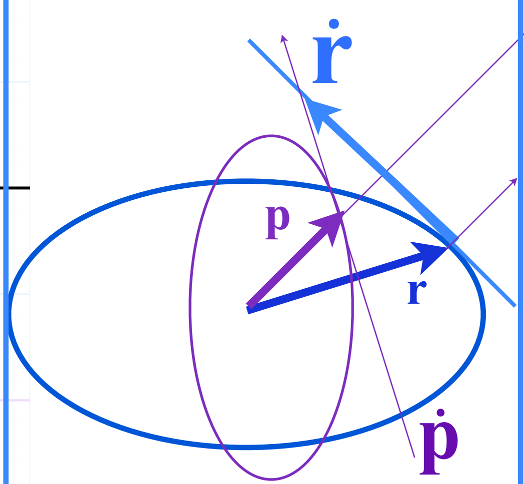
$$= \begin{pmatrix} \frac{1}{2}\frac{1}{\sqrt{2}} \\ \frac{1}{1}\frac{1}{\sqrt{2}} \end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1

Key points of matrix geometry:

Matrix Q maps any vector **r** *to a new vector* **p** *normal to the tangent* **ṙ** *to its* **r**•*Q*•**r**-*ellipse.*



Matrix Q^{-1} *maps* **p** *back to* **r** *that is normal to the tangent* **ṗ** *to its* **p**• Q^{-1}• **p**-*ellipse.*

Introduction to dual matrix operator geometry (based on IHO orbits)

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

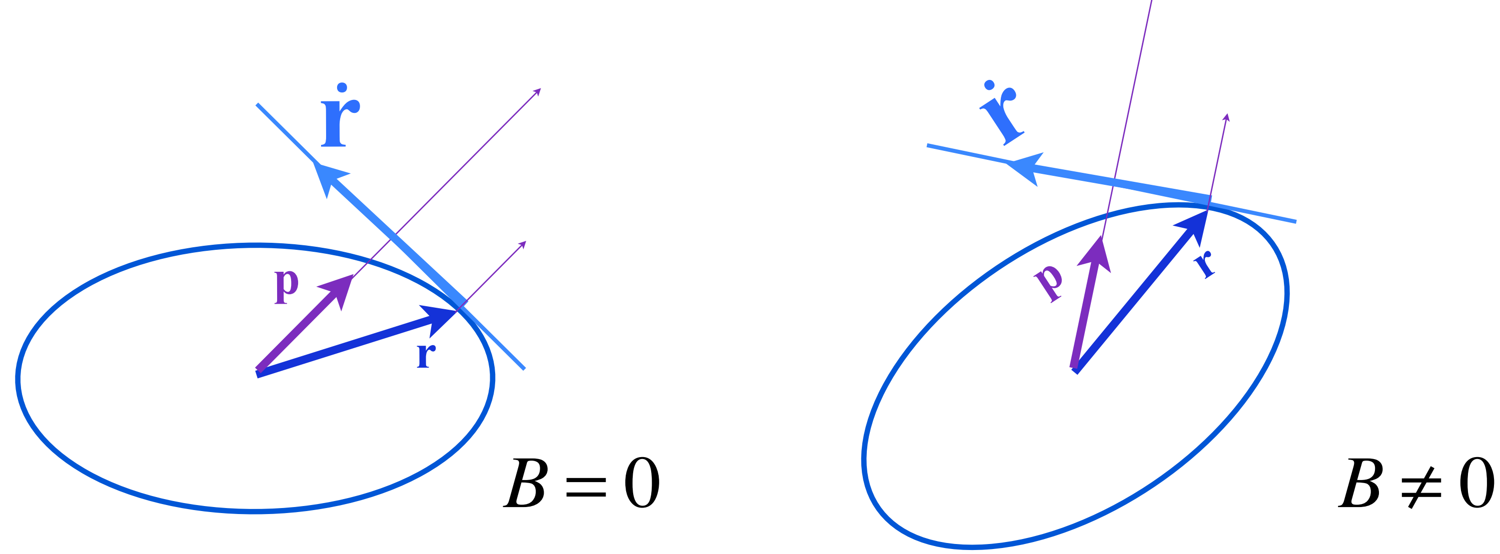
Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$*)*

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

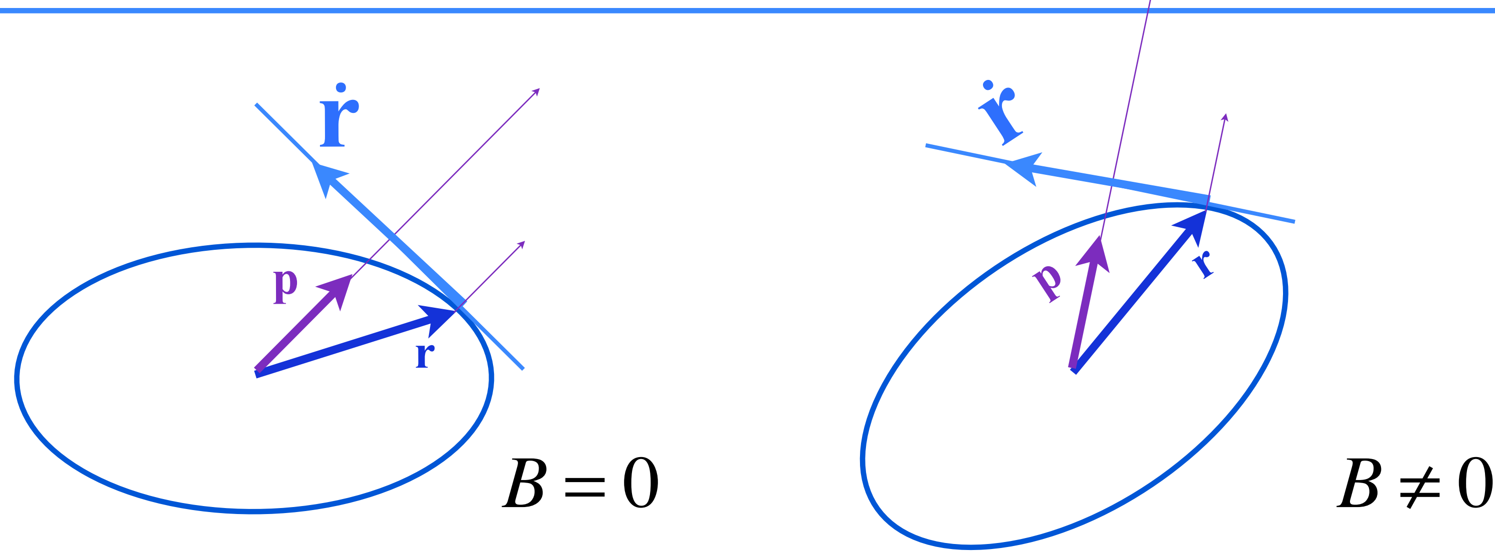
→ *Vector calculus of tensor operation*



Derive matrix “normal-to-ellipse”geometry by vector calculus:

Let matrix $Q = \begin{pmatrix} A & B \\ B & D \end{pmatrix}$

define the ellipse $1 = \mathbf{r} \bullet Q \bullet \mathbf{r} = \begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} A & B \\ B & D \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} A \cdot x + B \cdot y \\ B \cdot x + D \cdot y \end{pmatrix} = A \cdot x^2 + 2B \cdot xy + D \cdot y^2 = 1$



Derive matrix “normal-to-ellipse”geometry by vector calculus:

Let matrix $Q = \begin{pmatrix} A & B \\ B & D \end{pmatrix}$

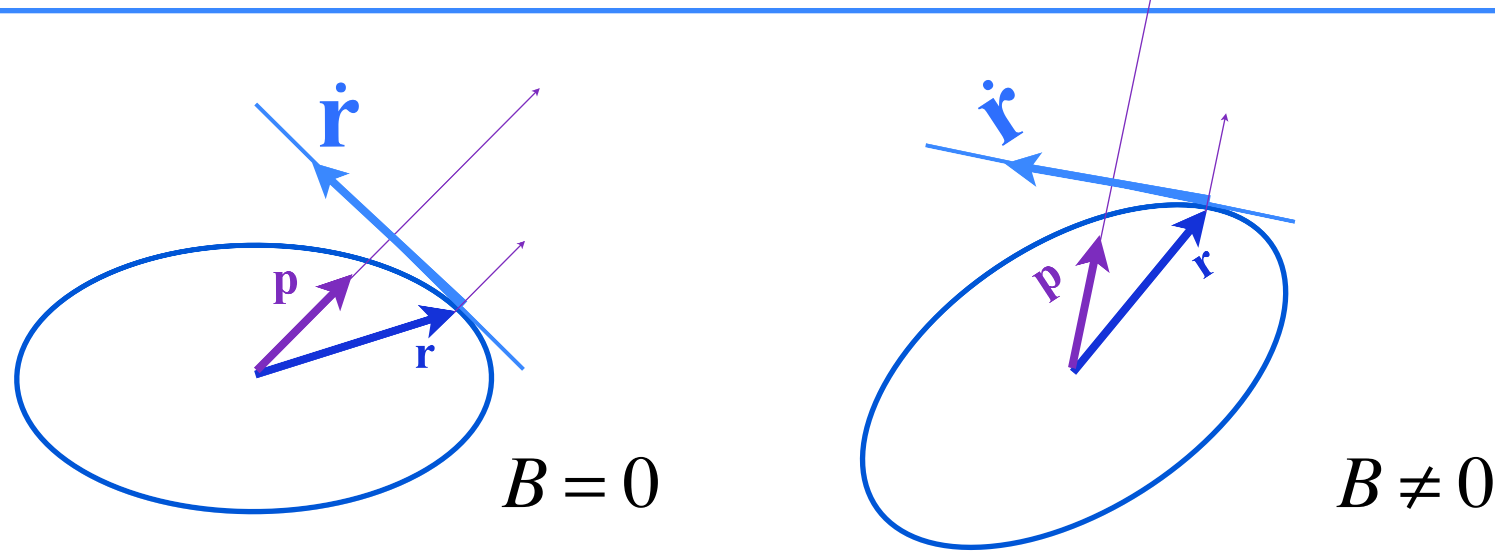
define the ellipse $1 = \mathbf{r}\cdot Q\cdot\mathbf{r} = \begin{pmatrix} x & y \end{pmatrix}\cdot\begin{pmatrix} A & B \\ B & D \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix}\cdot\begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix} = A\cdot x^2 + 2B\cdot xy + D\cdot y^2 = 1$

Compare operation by Q on vector **r** *with* *vector derivative or gradient of* $\mathbf{r}\cdot Q\cdot\mathbf{r}$

$$\frac{\partial}{\partial\mathbf{r}}(\mathbf{r}\cdot Q\cdot\mathbf{r}) = \nabla(\mathbf{r}\cdot Q\cdot\mathbf{r})$$

$$\begin{pmatrix} A & B \\ B & D \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}\left(A\cdot x^2 + 2B\cdot xy + D\cdot y^2\right) = \begin{pmatrix} 2A\cdot x + 2B\cdot y \\ 2B\cdot x + 2D\cdot y \end{pmatrix}$$



Derive matrix "normal-to-ellipse" geometry by vector calculus:

Let matrix $Q = \begin{pmatrix} A & B \\ B & D \end{pmatrix}$

define the ellipse $1 = \mathbf{r}\bullet Q\bullet\mathbf{r} = \begin{pmatrix} x & y \end{pmatrix}\bullet\begin{pmatrix} A & B \\ B & D \end{pmatrix}\bullet\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix}\bullet\begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix} = A\cdot x^2 + 2B\cdot xy + D\cdot y^2 = 1$

Compare operation by Q on vector **r** *with* *vector derivative or gradient of* $\mathbf{r}\bullet Q\bullet\mathbf{r}$

$$\begin{pmatrix} A & B \\ B & D \end{pmatrix}\bullet\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix}$$

$$\frac{\partial}{\partial\mathbf{r}}(\mathbf{r}\bullet Q\bullet\mathbf{r}) = \nabla(\mathbf{r}\bullet Q\bullet\mathbf{r})$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}\left(A\cdot x^2 + 2B\cdot xy + D\cdot y^2\right) = \begin{pmatrix} 2A\cdot x + 2B\cdot y \\ 2B\cdot x + 2D\cdot y \end{pmatrix}$$

Very simple result:

$$\frac{\partial}{\partial\mathbf{r}}\left(\frac{\mathbf{r}\bullet Q\bullet\mathbf{r}}{2}\right) = \nabla\left(\frac{\mathbf{r}\bullet Q\bullet\mathbf{r}}{2}\right) = Q\bullet\mathbf{r}$$

Introduction to dual matrix operator geometry (based on IHO orbits)

- *Quadratic form ellipse* $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$
  - *Duality norm relations (* $\mathbf{r}\bullet\mathbf{p}=1$*)*
  - *Q-Ellipse tangents* $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*
- *(Still more) Operator geometric sequences and eigenvectors*
- *Alternative scaling of matrix operator geometry*
  - *Vector calculus of tensor operation*

Action of "sqrt-"matrix $R=\sqrt{Q}$ *(R generates another ellipse* **r**•*R*•**r** *=1 not shown)*

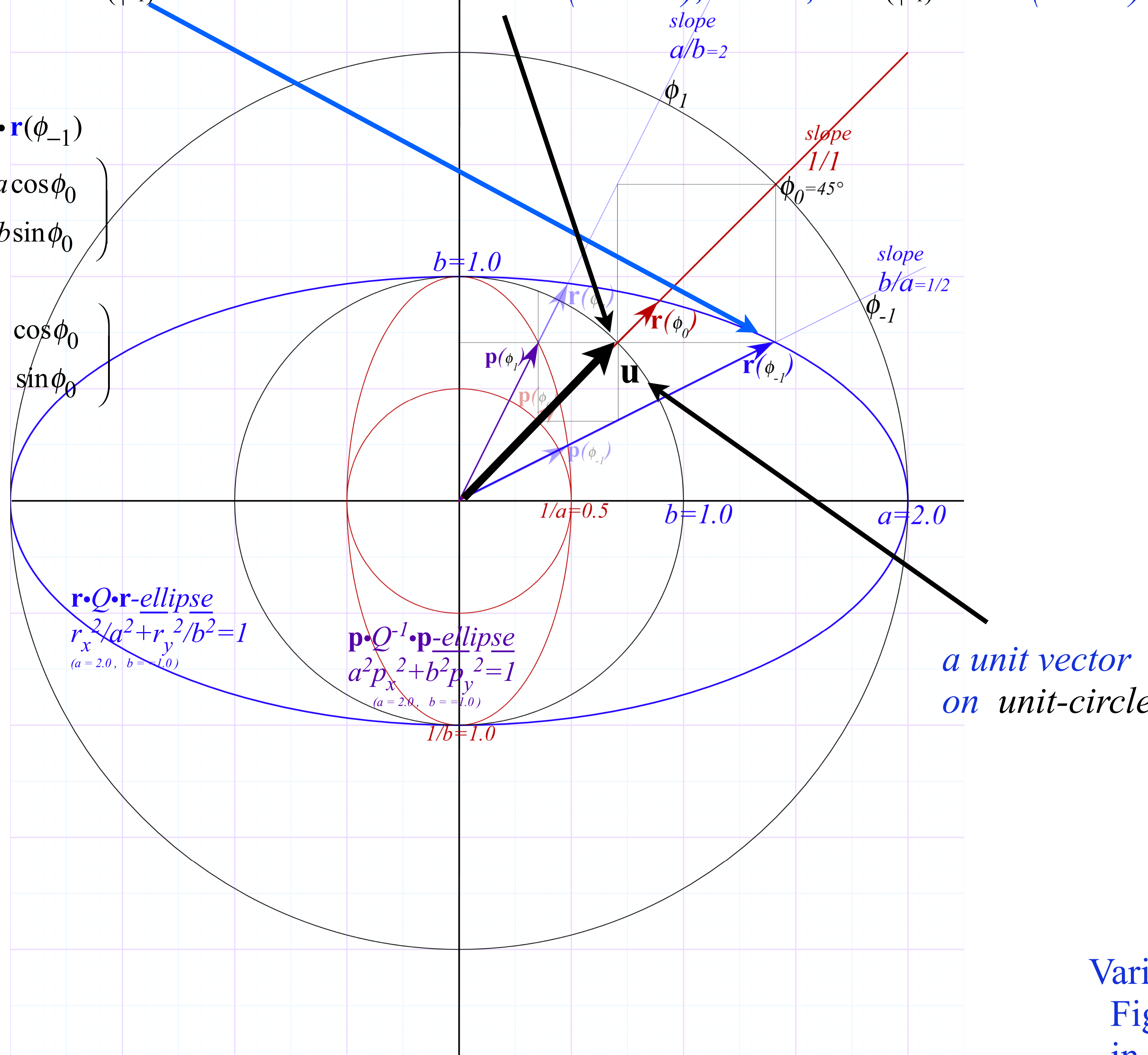
on a single **r**-*vector* **r**(ϕ_{-1})... *is to rotate it to* **u**-*circle (***u**•**u** *=1), that is,* R•**r**(ϕ_{-1}) = **u** =*(const.)***r**(ϕ_0)

$$\mathbf{u}=\sqrt{\mathbf{Q}}\bullet\mathbf{r}(\phi_{-1})=\mathbf{R}\bullet\mathbf{r}(\phi_{-1})$$

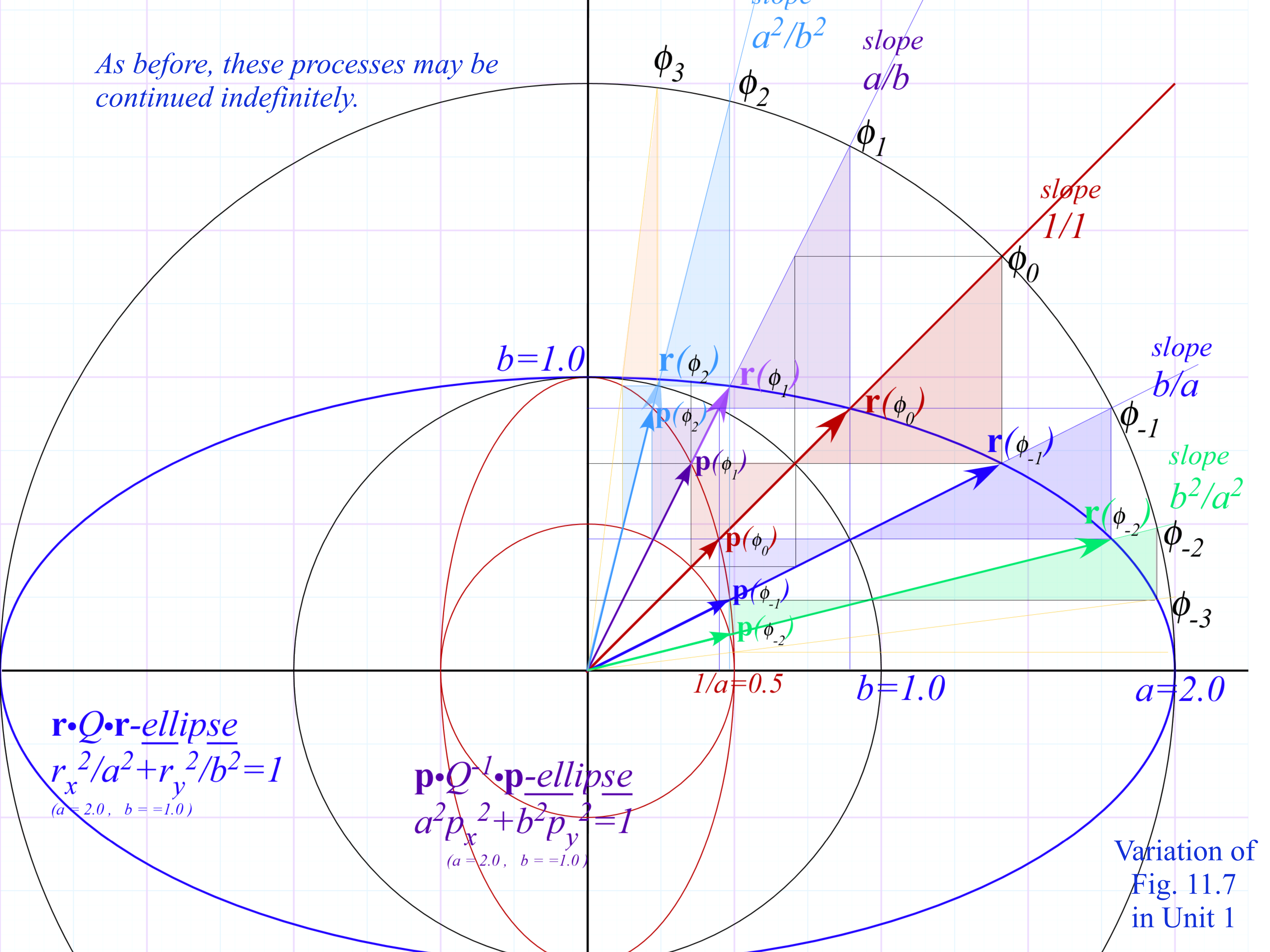
$$=\begin{pmatrix}1/a & 0\\ 0 & 1/b\end{pmatrix}\begin{pmatrix}a\cos\phi_0\\ b\sin\phi_0\end{pmatrix}$$

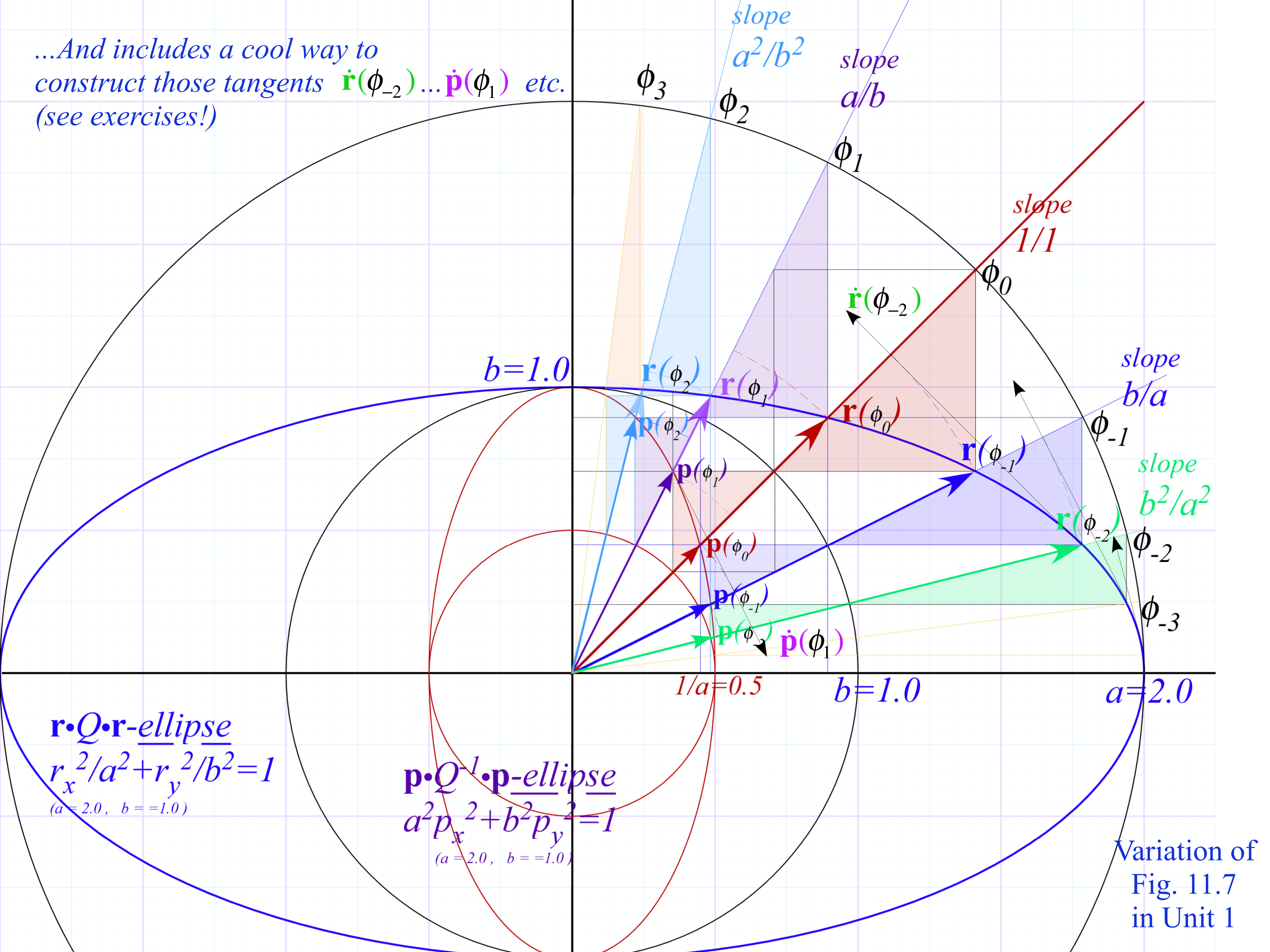
$$=\begin{pmatrix}\frac{1}{a}a\cos\phi_0\\ \frac{1}{b}b\sin\phi_0\end{pmatrix}=\begin{pmatrix}\cos\phi_0\\ \sin\phi_0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1


As before, these processes may be continued indefinitely.
slope
a²/b²
slope
a/b
slope
1/1
slope
b/a
slope
b²/a²
φ3
φ2
φ1
φ0
φ-1
φ-2
φ-3
b=1.0
r(φ2)
r(φ1)
r(φ0)
r(φ-1)
r(φ-2)
p(φ2)
p(φ1)
p(φ0)
p(φ-1)
p(φ-2)
1/a=0.5
b=1.0
a=2.0
r•Q•r-ellipse
rx²/a²+ry²/b²=1
(a = 2.0 , b = =1.0)
p•Q-1•p-ellipse
a²px²+b²py²=1
(a = 2.0 , b = =1.0)
Variation of
Fig. 11.7
in Unit 1


...And includes a cool way to
construct those tangents ṙ(φ₋₂)...ṗ(φ₁) etc.
(see exercises!)
slope
a²/b²
slope
a/b
slope
1/1
slope
b/a
slope
b²/a²
φ3
φ2
φ1
φ0
φ-1
φ-2
φ-3
ṙ(φ₋₂)
b=1.0
r(φ2)
r(φ1)
r(φ0)
r(φ-1)
r(φ-2)
p(φ2)
p(φ1)
p(φ0)
p(φ-1)
p(φ-2)
ṗ(φ1)
1/a=0.5
b=1.0
a=2.0
r•Q•r-ellipse
rx²/a²+ry²/b²=1
(a = 2.0 , b = =1.0)
p•Q⁻¹•p-ellipse
a²px²+b²py²=1
(a = 2.0 , b = =1.0)
Variation of
Fig. 11.7
in Unit 1

Q:Where is this headed? *A: Lagrangian-Hamiltonian duality*

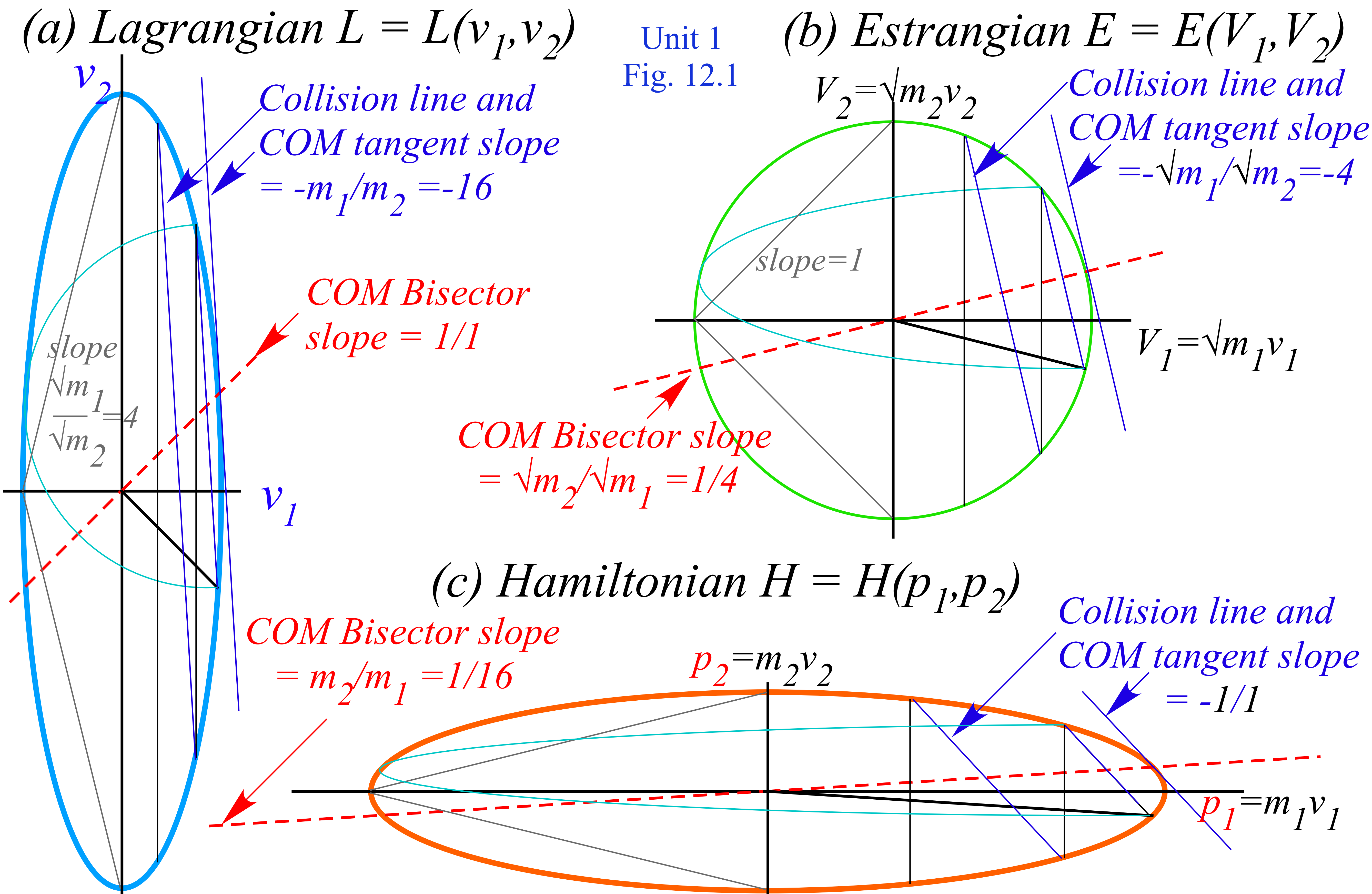
Preview of Lecture 8

The R and Q matrix transformations are like the mechanics rescaling matrices $\sqrt{\mathbf{M}}$ *and* $\mathbf{M}$:

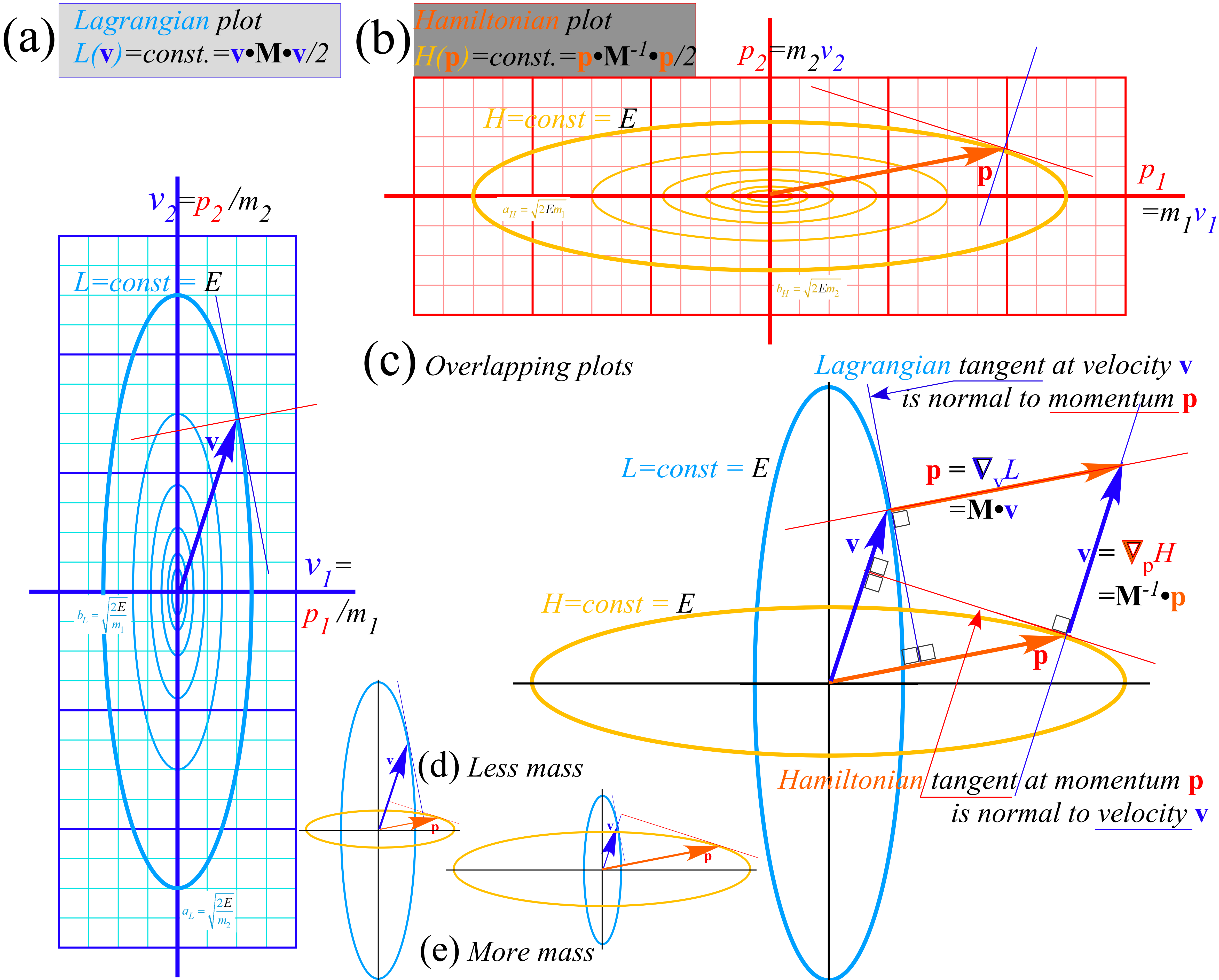
Like $Q=R^2$: $$\mathbf{M}=\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}=\mathbf{R}^2$$

Like $\sqrt{Q}=R$: $$\sqrt{\mathbf{M}}=\begin{pmatrix} \sqrt{m_1} & 0 \\ 0 & \sqrt{m_2} \end{pmatrix}=\mathbf{R}$$

Like $Q^{-1}=R^{-2}$: $$\mathbf{M}^{-1}=\begin{pmatrix} 1/m_1 & 0 \\ 0 & 1/m_2 \end{pmatrix}=\mathbf{R}^{-2}$$



Unit 1
Fig. 12.2



AMOP reference links *(Updated list given on **2nd** and 3rd pages of each class presentation)*

Web Resources - front page | Quantum Theory for the Computer Age | 2014 AMOP

UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2017 Group Theory for QM

Classical Mechanics with a Bang! | 2018 AMOP

Modern Physics and its Classical Foundations

Representaions Of Multidimensional Symmetries In Networks - harter-jmp-1973

Alternative Basis for the Theory of Complex Spectra

- Alternative_Basis_for_the_Theory_of_Complex_Spectra_I_-_harter-pra-1973
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_II_-_harter-patterson-pra-1976
- Alternative_Basis_for_the_Theory_of_Complex_Spectra_III_-_patterson-harter-pra-1977

Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978

Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979

Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984

Galloping waves and their relativistic properties - ajp-1985-Harter

Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 (Alt Scan)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

I) Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson (Alt scan)

II) Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 (Alt scan)

Rotation–vibration spectra of icosahedral molecules.

I) Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 (Alt scan)

II) Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 (Alt scan)

III) Half-integral angular momentum - harter-reimer-jcp-1991

Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 (Alt scan)

Nuclear spin weights and gas phase spectral structure of 12C60 and 13C60 buckminsterfullerene -Harter-Reimer-Cpl-1992 - (Alt1, Alt2 Erratum)

Gas Phase Level Structure of C60 Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996

Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer 12C 13C59 - jcp-Reimer-Harter-1997 (HiRez)

Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001

Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006

Resonance and Revivals

I) QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 (Talk) OSU knowledge Bank

II) Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talks)

III) Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - (2013-Li-Diss)

Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 (Talk)

Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013

Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013

QTCA Unit 10 Ch 30 - 2013

AMOP Ch 0 Space-Time Symmetry - 2019

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and 3rd pages of each class presentation)*

(Int.J.Mol.Sci, 14, 714(2013) p.755-774 , QTCA Unit 7 Ch. 23-26), (PSDS - Ch. 5, 7)

Int.J.Mol.Sci, 14, 714(2013), QTCA Unit 8 Ch. 23-25, QTCA Unit 9 Ch. 26, PSDS Ch. 5, PSDS Ch. 7

Intro spin ½ coupling
Unit 8 Ch. 24 p3

H atom hyperfine-B-level crossing
Unit 8 Ch. 24 p15

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.
Deeper theory ends p53

Intro 2p3p coupling
Unit 8 Ch. 24 p17.

Intro LS-jj coupling
Unit 8 Ch. 24 p22.

CG coupling derived (start)
Unit 8 Ch. 24 p39.

CG coupling derived (formula)
Unit 8 Ch. 24 p44.

Lande' g-factor
Unit 8 Ch. 24 p26.

Irrep Tensor building
Unit 8 Ch. 25 p5.

Irrep Tensor Tables
Unit 8 Ch. 25 p12.

Wigner-Eckart tensor Theorem.
Unit 8 Ch. 25 p17.

Tensors Applied to d,f-levels.
Unit 8 Ch. 25 p21.

Tensors Applied to high J levels.
Unit 8 Ch. 25 p63.

Intro 3-particle coupling.
Unit 8 Ch. 25 p28.

Intro 3,4-particle Young Tableaus
GrpThLect29 p42.

Young Tableau Magic Formulae
GrpThLect29 p46-48.

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*

AMOP reference links *(Updated list given on 2nd and **3rd** and **4th** pages of each class presentation)*

Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

Chaos_Classical_and_Quantum_-_2018-Cvitanovic-ChaosBook
Group Theory - PUP_Lucy_Day_-_Diagrammatic_notation_-_Ch4
Simplification_Rules_for_Birdtrack_Operators_-_Alcock-Zeilinger-Weigert-zeilinger-jmp-2017
Group Theory - Birdtracks_Lies_and_Exceptional_Groups_-_Cvitanovic-2011
Simplification_rules_for_birdtrack_operators-_jmp-alcock-zeilinger-2017
Birdtracks for SU(N) - 2017-Keppeler

Frank Rioux's: UMA method of vibrational induction

Quantum_Mechanics_Group_Theory_and_C60_-_Frank_Rioux_-_Department_of_Chemistry_Saint_Johns_U
Symmetry_Analysis_for_H20-_H20GrpTheory-_Rioux
Quantum_Mechanics-Group_Theory_and_C60_-_JChemEd-Rioux-1994
Group_Theory_Problems-_Rioux-_SymmetryProblemsX
Comment_on_the_Vibrational_Analysis_for_C60_and_Other_Fullerenes_Rioux-RSP

Supplemental AMOP Techniques & Experiment

Many Correlation Tables are Molien Sequences - Klee (Draft 2016)
High-resolution_spectroscopy_and_global_analysis_of_CF4_rovibrational_bands_to_model_its_atmospheric_absorption-_carlos-Boudon-jqsrt-2017
Symmetry and Chirality - Continuous_Measures_-_Avnir

*

Special Topics & Colloquial References

r-process_nucleosynthesis_from_matter_ejected_in_binary_neutron_star_mergers-PhysRevD-Bovard-2017

**Index/Search is disabled - a web based A.M.O.P. oriented reference page, with thumbnail/previews, greater control over the information display,*
https://modphys.hosted.uark.edu/markup/AMOP_References.html
and eventually full on Apache-SOLR *Index and search for* nuanced, whole-site content/metadata level *searching.*