Lecture 7
Thur. 9.15.2015

Geometry of common power-law potentials II.

(Ch. 9 of Unit 1)

Review of "Sophomore-physics Earth" field geometry
"Outside" Coulomb geometry of $-kr^{-1}$-potential and $-kr^{-2}$-force field
"Inside" Oscillator geometry of $kr^2/2$ potential and $-kr^1$ force field
Easy-to-remember geo-solar constants

Geometry and algebra of idealized "Sophomore-physics Earth" fields
Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*
Contact-geometry of potential curve(s) and "kite" geometry
"Ordinary-Earth" models: 3 key energy "steps" and 4 key energy "levels"
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"
Earth matter vs nuclear matter:
Introducing the "neutron starlet"
*Fantasizing a "***Black-Hole-Earth***"*

Isotropic Harmonic Oscillator phase dynamics in uniform-body orbits
Dual phasor construction of elliptic oscillator orbits
Integrating IHO equations by phasor geometry

Link ⇒ BoxIt simulation of IHO orbits
Link → IHO orbital time rates of change
Link → IHO Exegesis Plot

Review of "Sophomore-physics Earth" field geometry

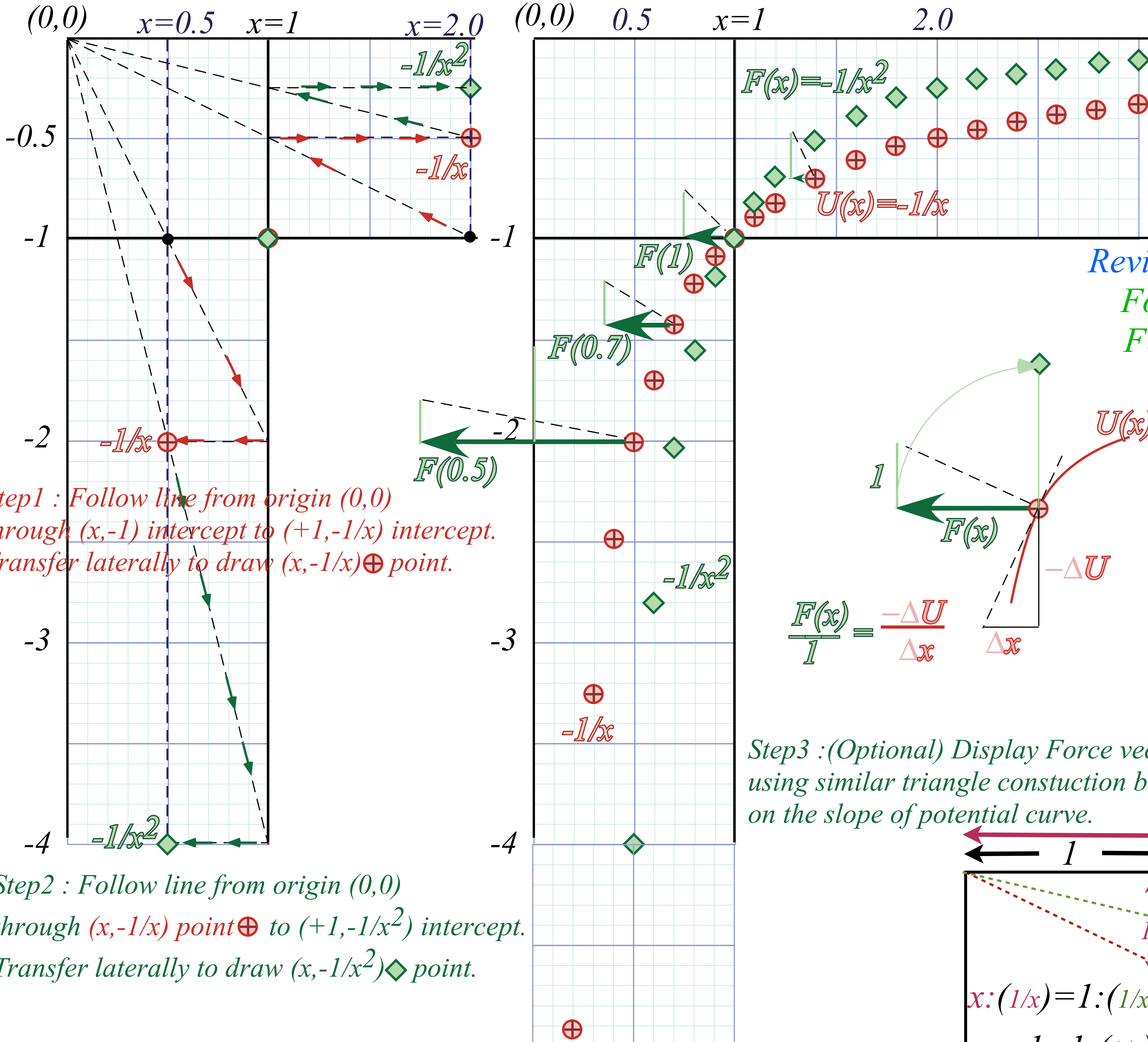
→ *"Outside" Coulomb geometry of* $-kr^{-1}$*-potential and* $-kr^{-2}$*-force field*

"Inside" Oscillator geometry of $kr^2/2$ *potential and* $-kr^1$ *force field*

Easy-to-remember geo-solar constants

Unit 1
Fig. 9.4

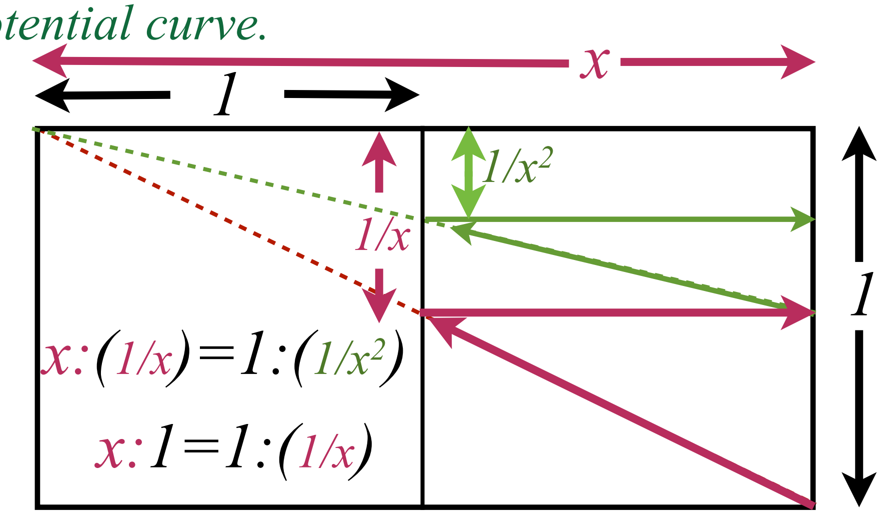
Review: Coulomb geometry
Force and Potential
$F(x)=-1/x^2$ $U(x)=-1/x$
Outside of $r=R_{\oplus}$

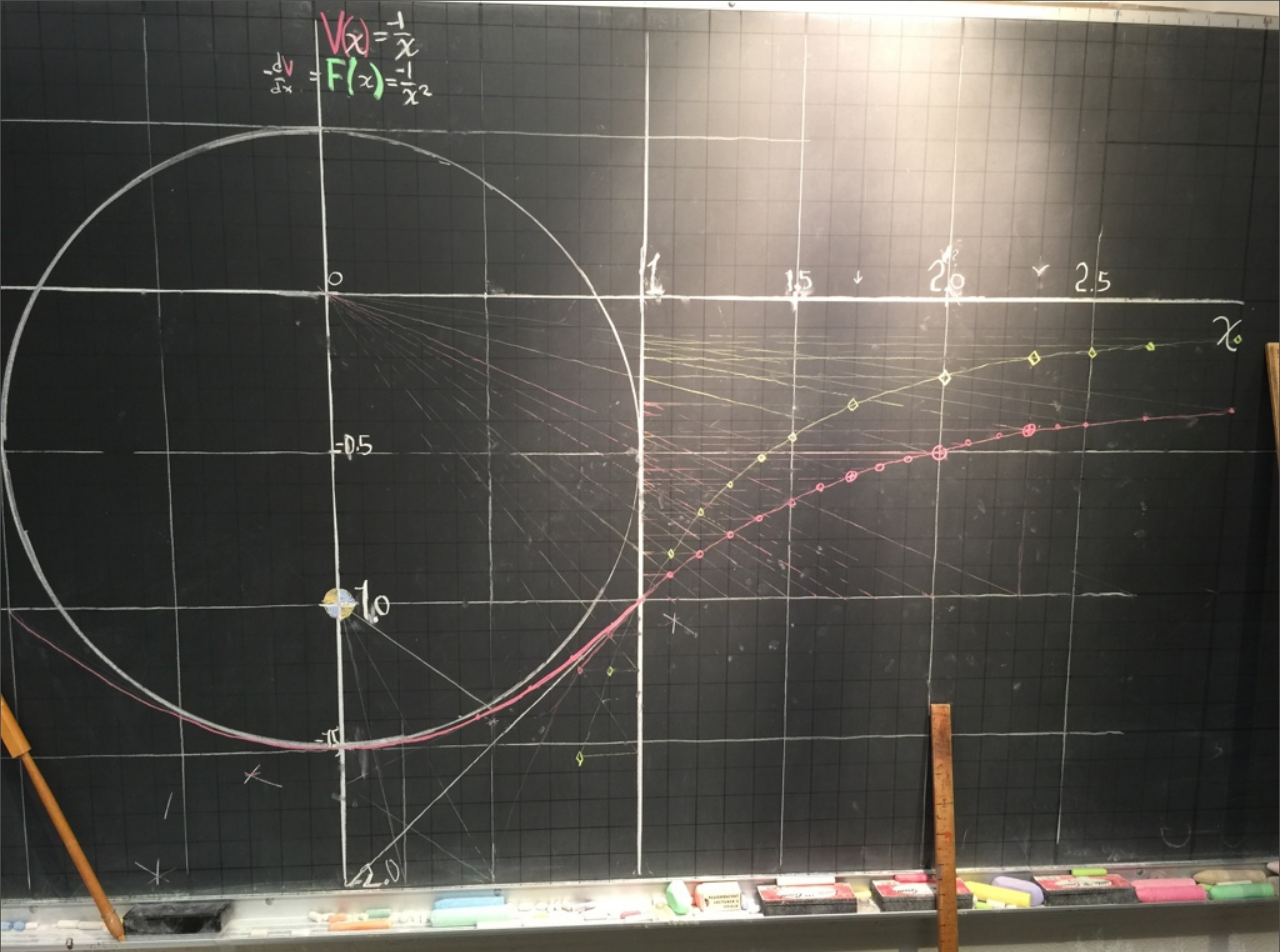


Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x^2) intercept. Transfer laterally to draw (x,-1/x^2)◇ point.

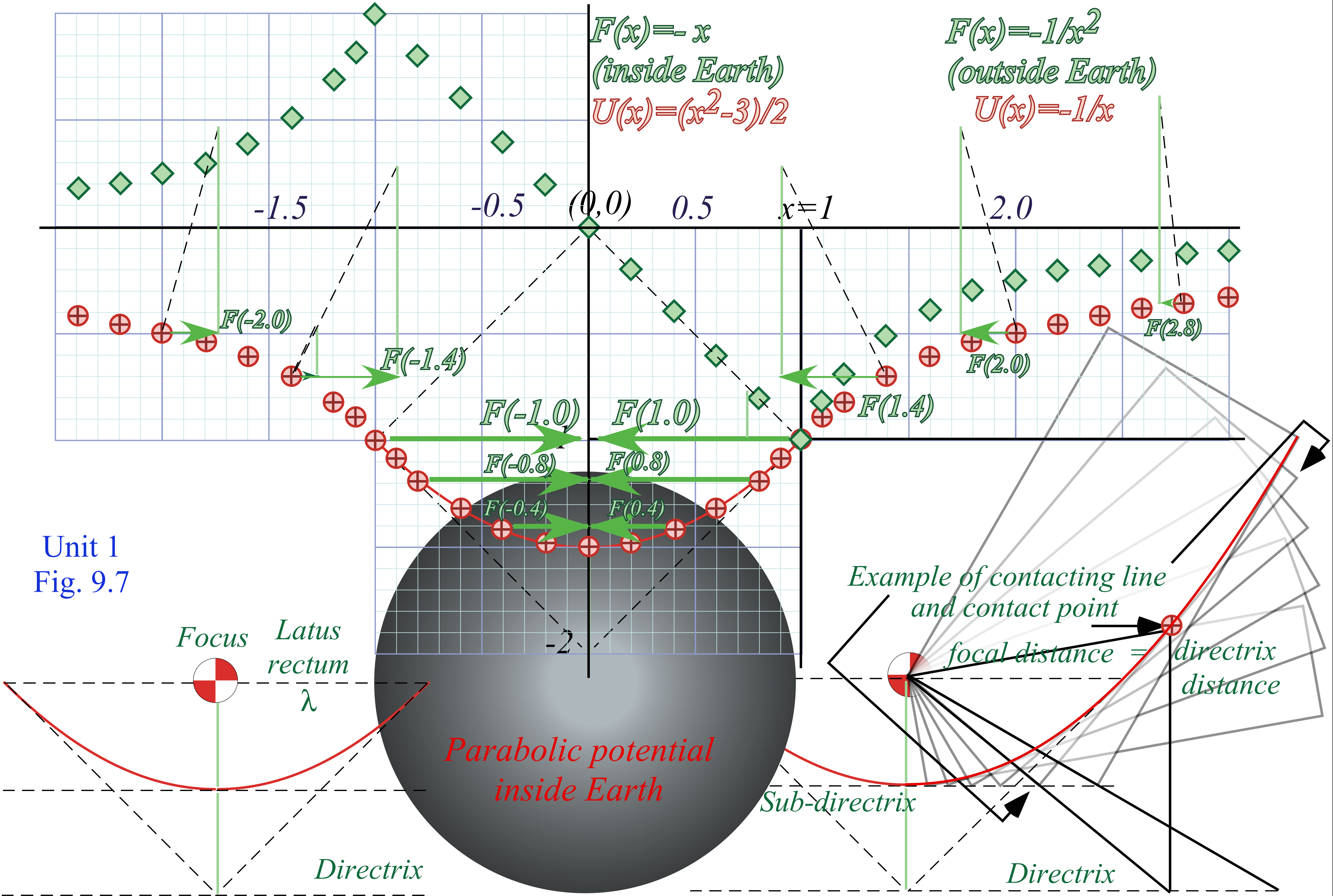
Step3 :(Optional) Display Force vector using similar triangle constuction based on the slope of potential curve.





V(x) = -1/x
-dV/dx = F(x) = -1/x²
0
1
1.5
2.0
2.5
x
-0.5
-1.0
-1.5
-2.0

The ideal "Sophomore-Physics-Earth" model of geo-gravity



Review of "Sophomore-physics Earth" field geometry

"Outside" Coulomb geometry of $-kr^{-1}$-potential and $-kr^{-2}$-force field

→ *"Inside" Oscillator geometry of $kr^2/2$ potential and $-kr^1$ force field*

Easy-to-remember geo-solar constants

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

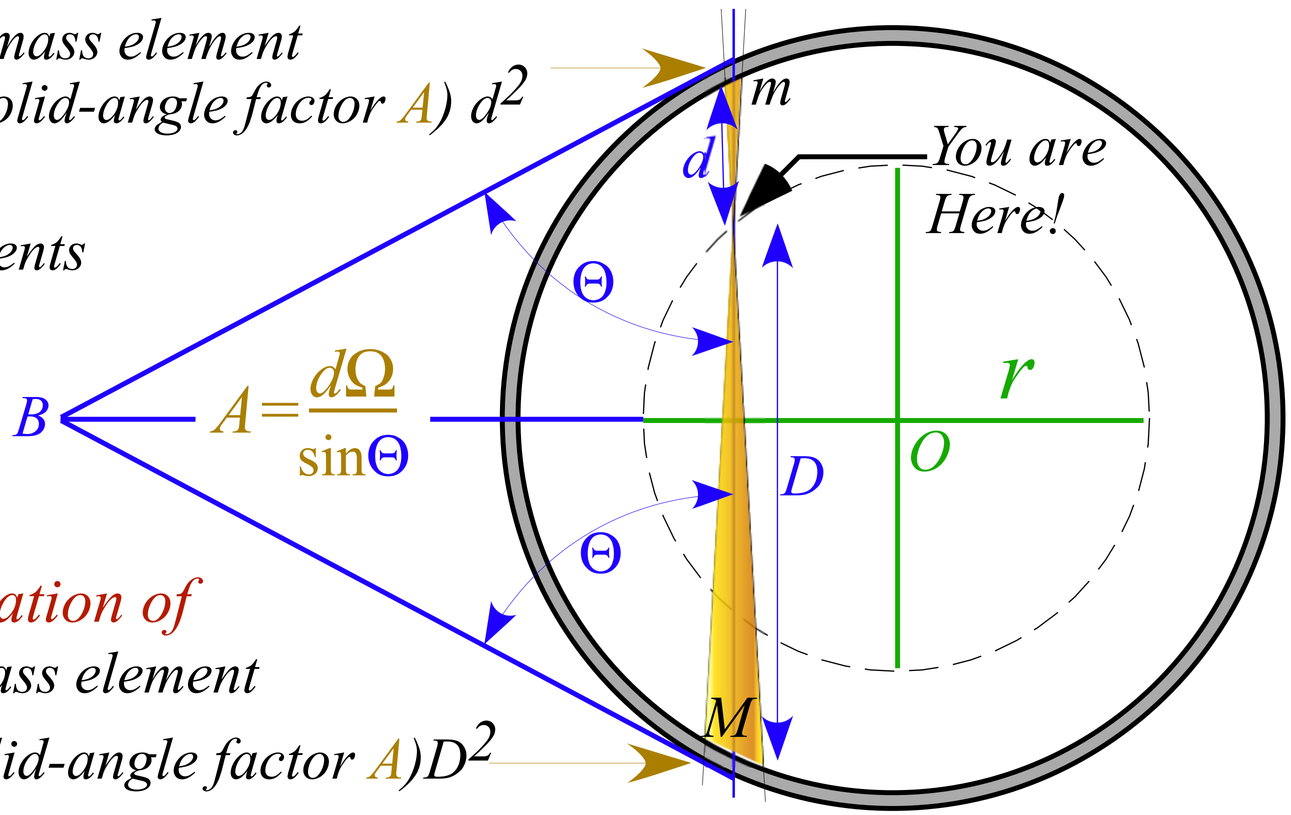
Unit 1
Fig. 9.6

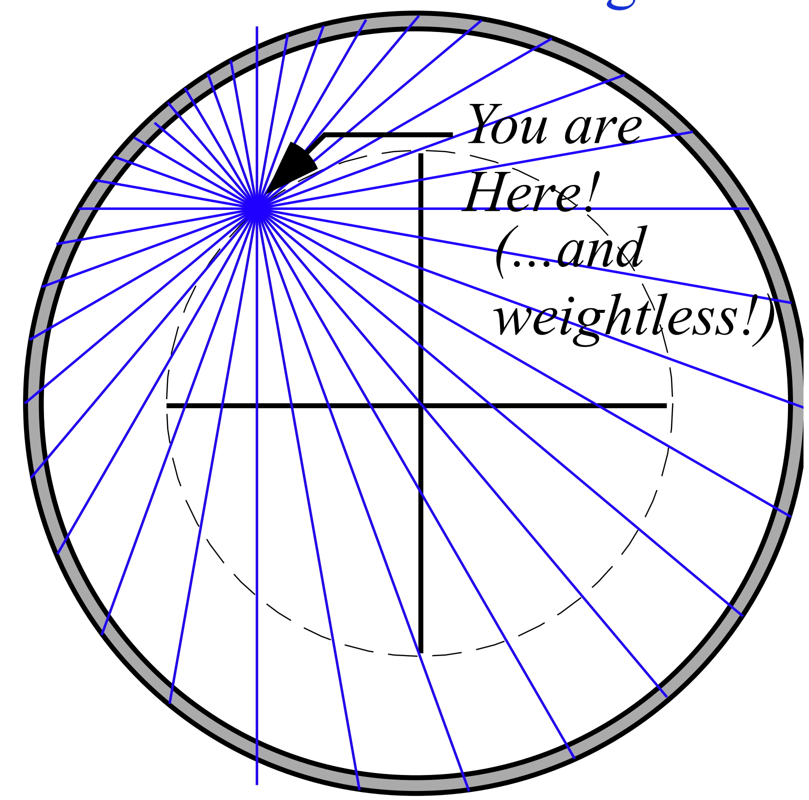
Shell mass element
m =(solid-angle factor A) d^2

Gravity at r
due to shell mass elements

$$\frac{GM}{D^2} - \frac{Gm}{d^2} =$$

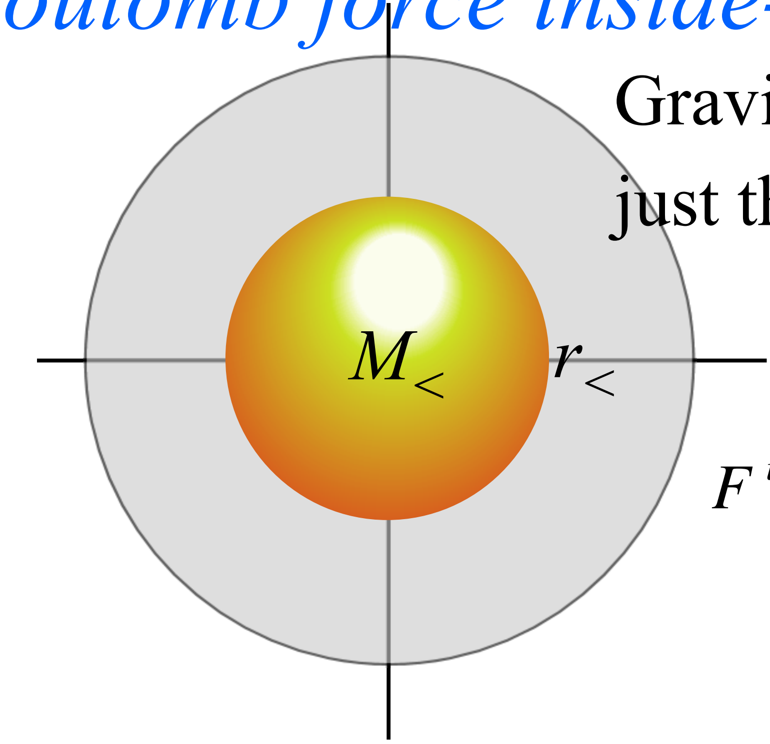
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

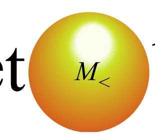




Cancellation of
Shell mass element
M =(solid-angle factor A)D^2

Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg \cdot x$$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

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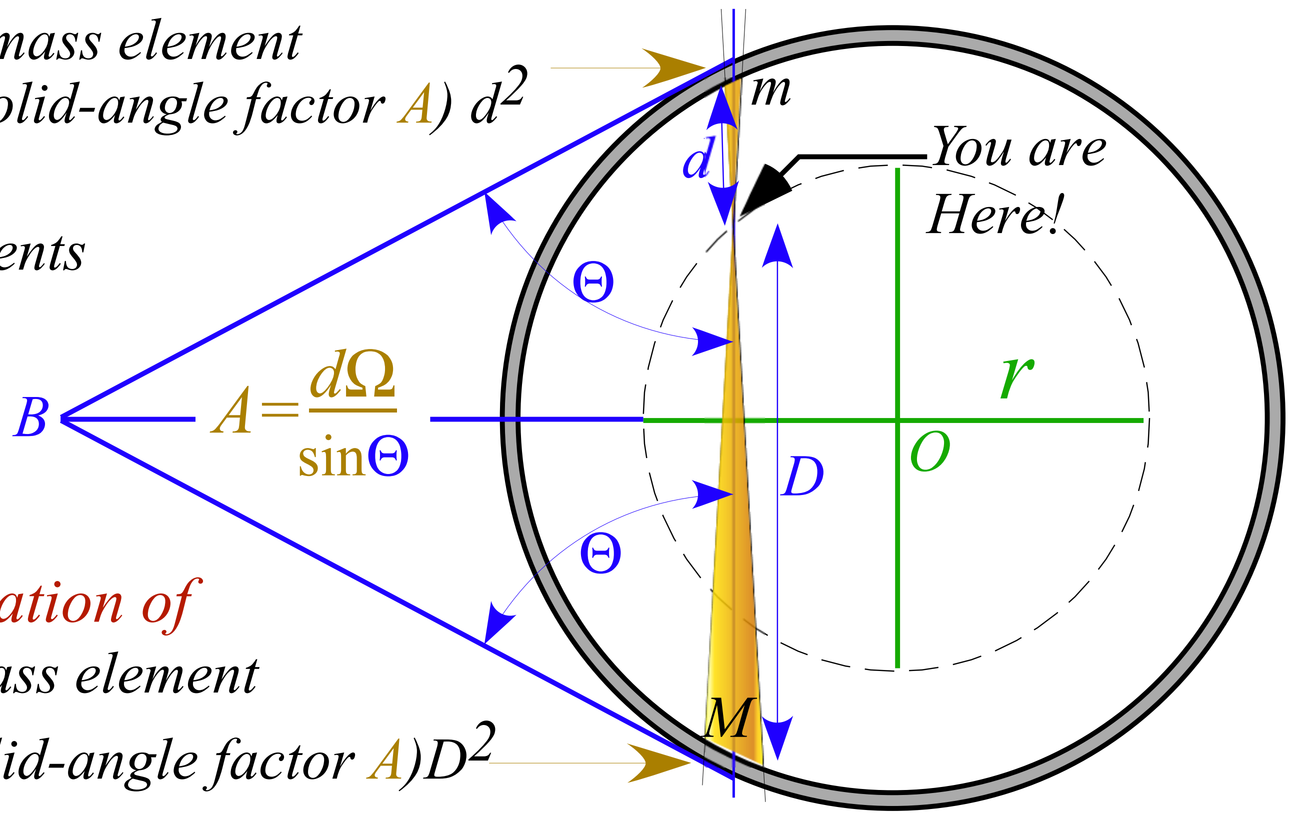

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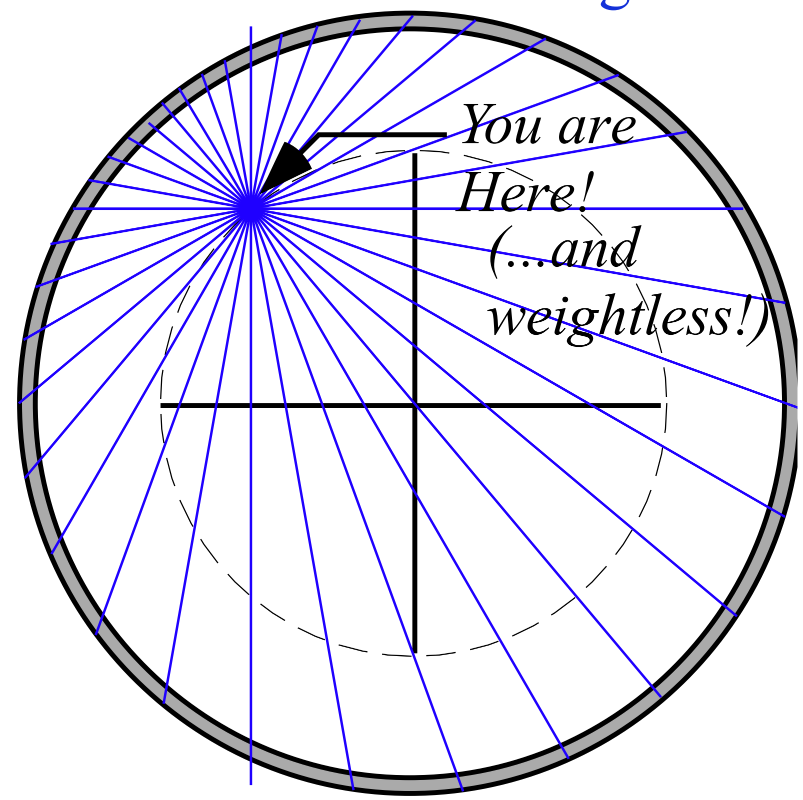
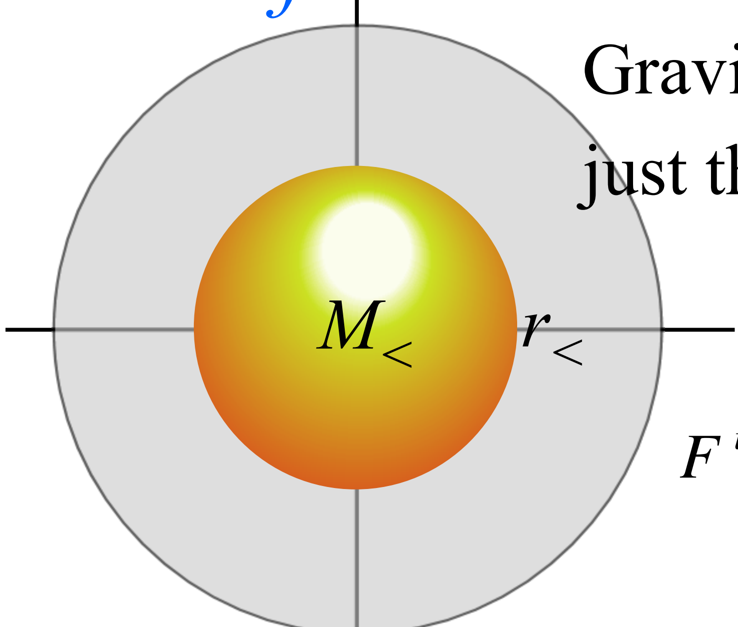


Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet $M_<$ below $r_<$

Note:
Hooke's (linear) force law for $r_<$ inside uniform body

$$F^{\,inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

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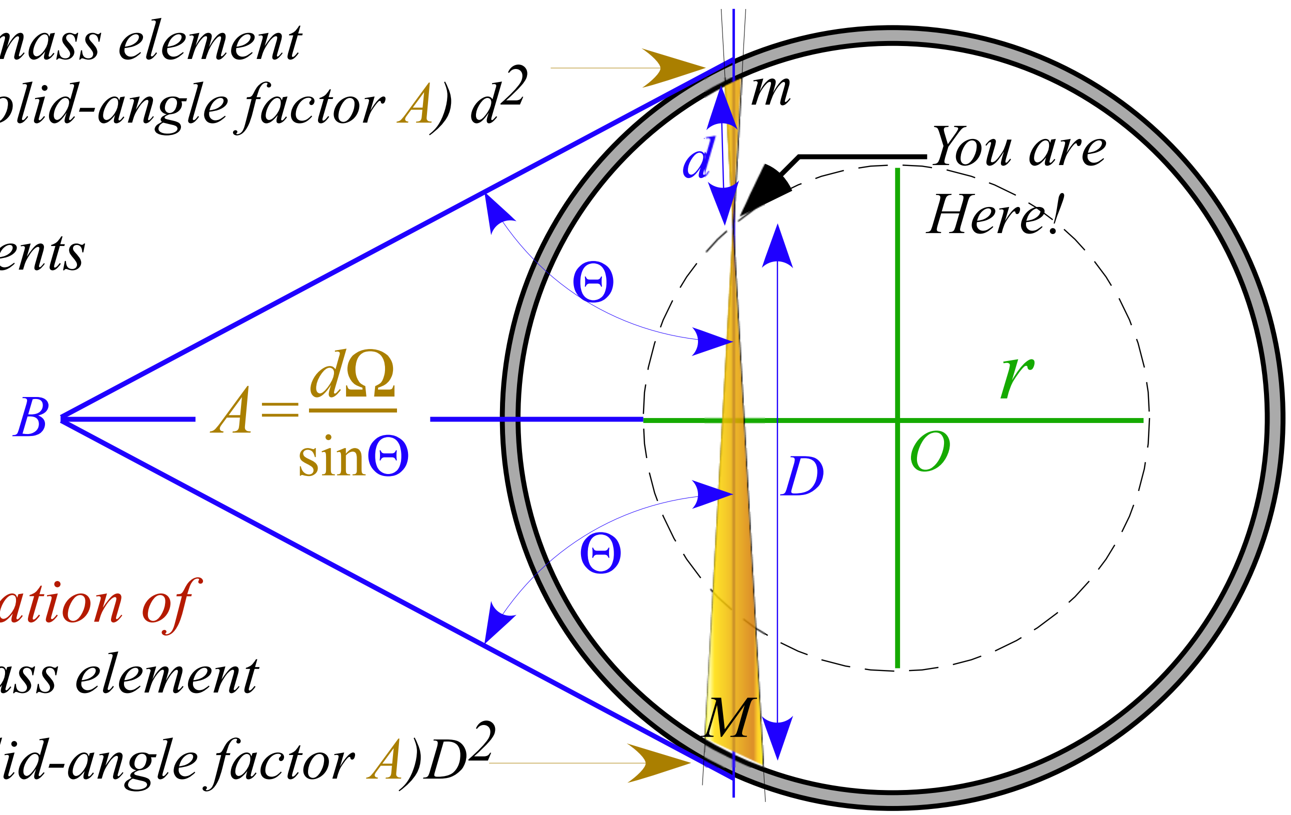
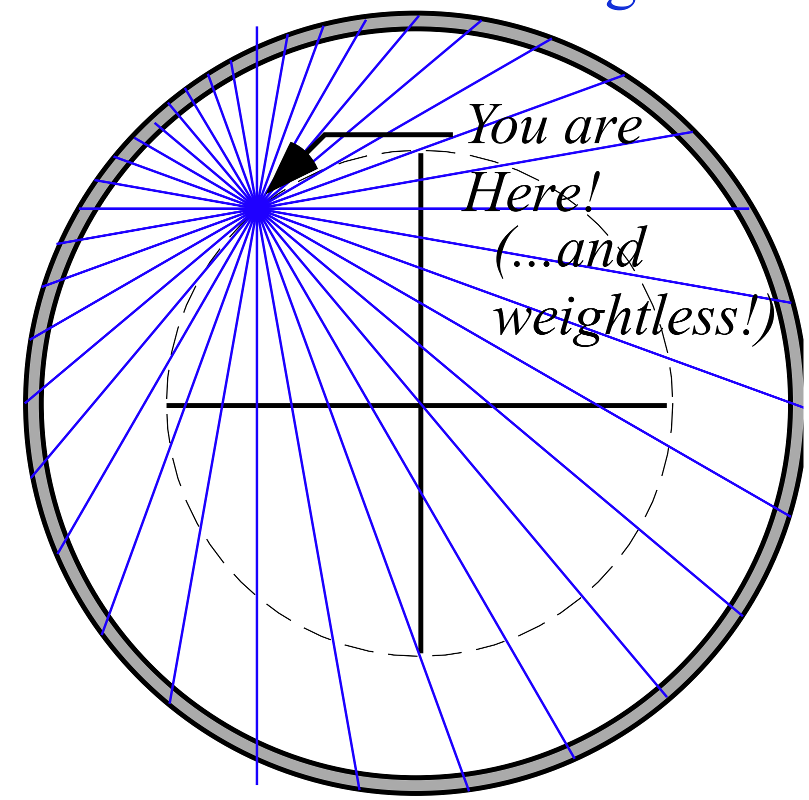
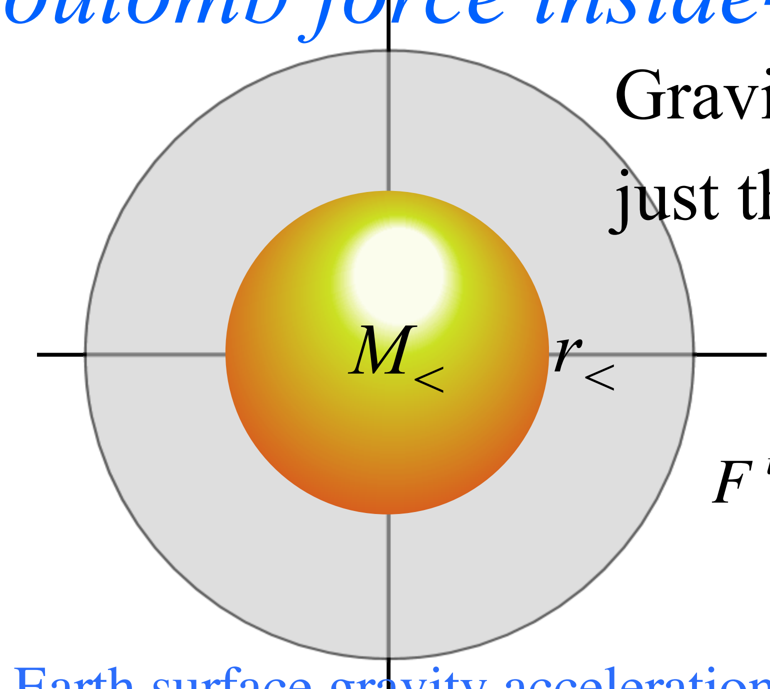




Coulomb force inside-spherical body due to stuff below you, only.



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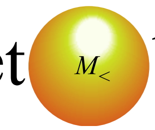
just that of planet  below $r_<$

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Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Earth radius : $R_\oplus = 6.371\cdot 10^6 m \simeq 6.4\cdot 10^6 m$

Earth mass : $M_\oplus = 5.9722 \times 10^{24} kg. \simeq 6.0\cdot 10^{24} kg.$

Solar radius : $R_\odot = 6.955 \times 10^8 m. \simeq 7.0\cdot 10^8 m.$

Solar mass : $M_\odot = 1.9889 \times 10^{30} kg. \simeq 2.0\cdot 10^{30} kg.$

Geometry and algebra of idealized "Sophomore-physics Earth" fields

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

→ *Contact-geometry of potential curve(s) and "kite" geometry*

"Ordinary-Earth" models: 3 key energy "steps" and 4 key energy "levels"

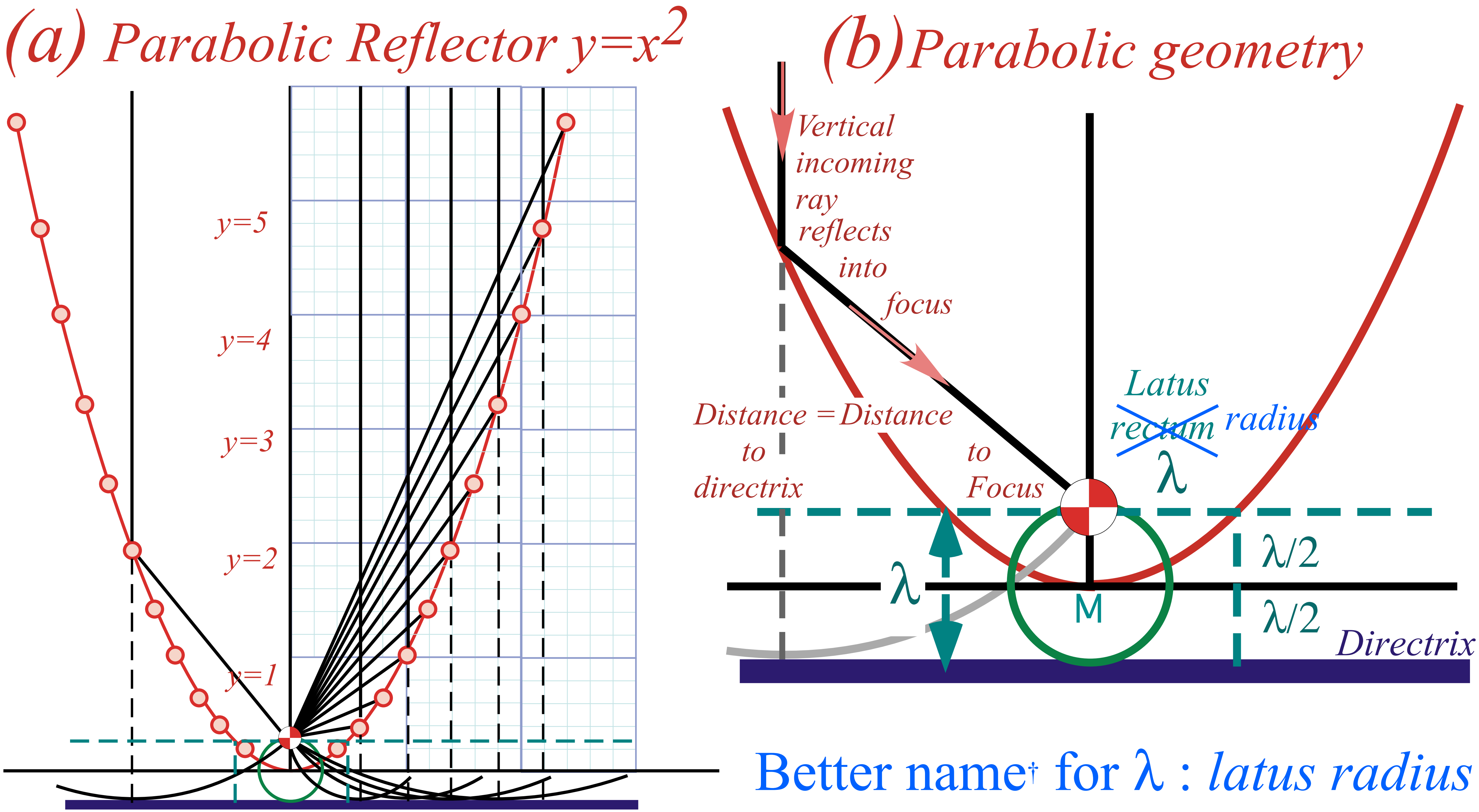
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

Introducing the "neutron starlet"

*Fantasizing a "***Black-Hole-Earth***"*

A more conventional parabolic geometry...

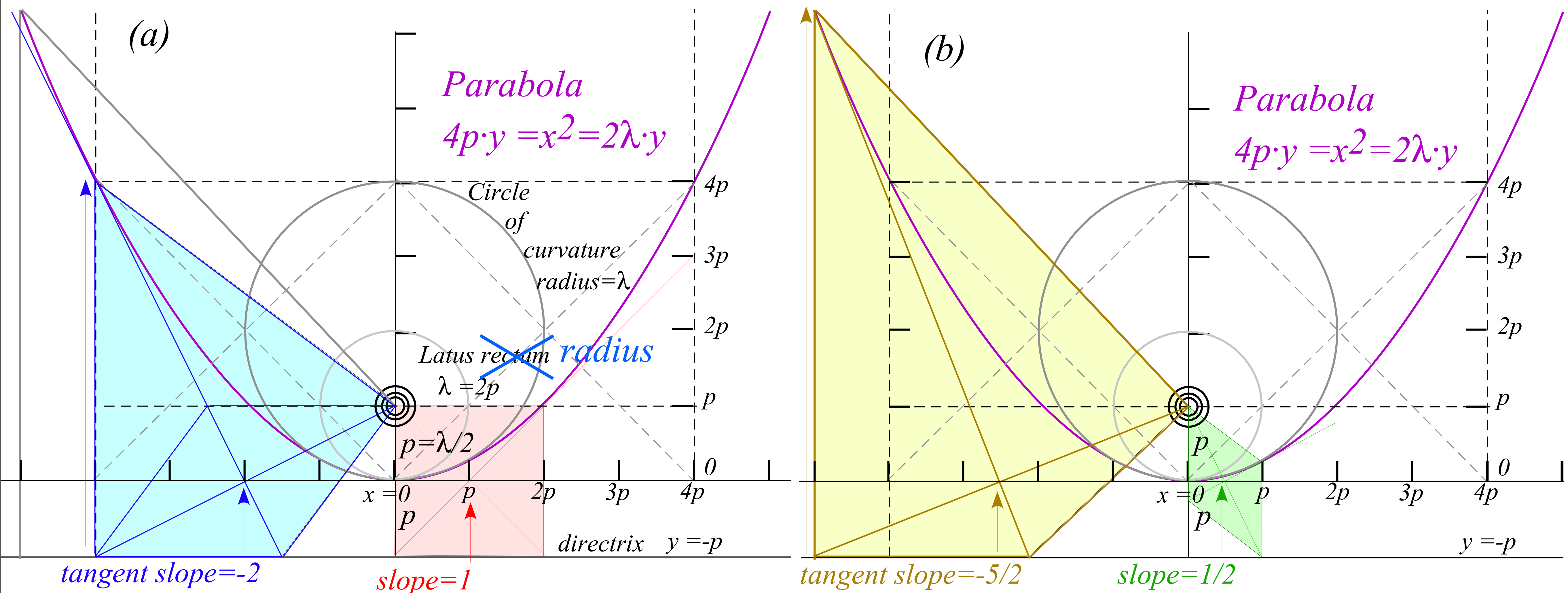


Better name[†] for λ : *latus radius*

† Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

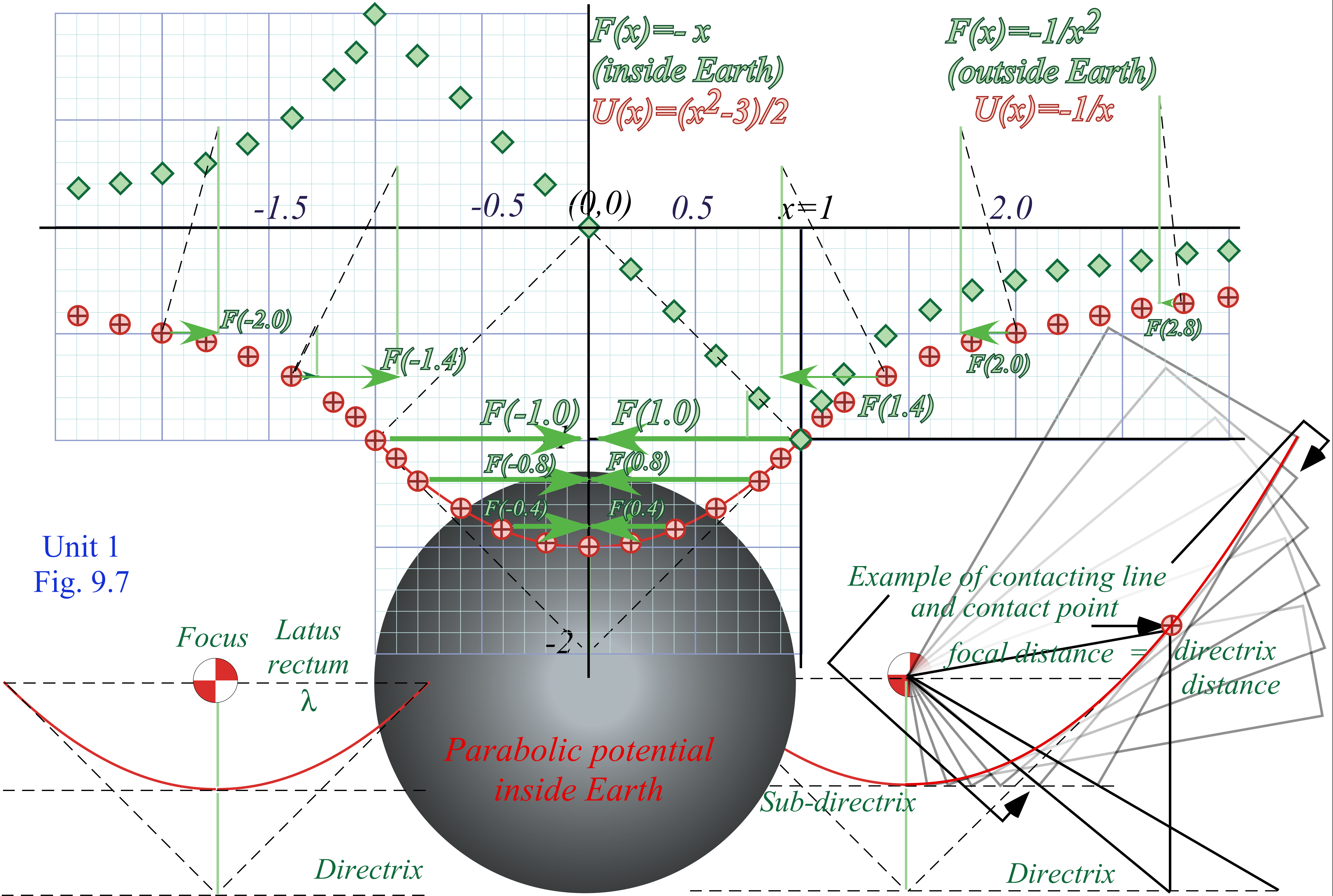
Unit 1
Fig. 9.3

Review of conventional parabolic geometry...introducing "kites"



Unit 1
Fig. 9.4

The ideal "Sophomore-Physics-Earth" model of geo-gravity



Geometry and algebra of idealized "Sophomore-physics Earth" fields

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

Contact-geometry of potential curve(s) and "kite" geometry

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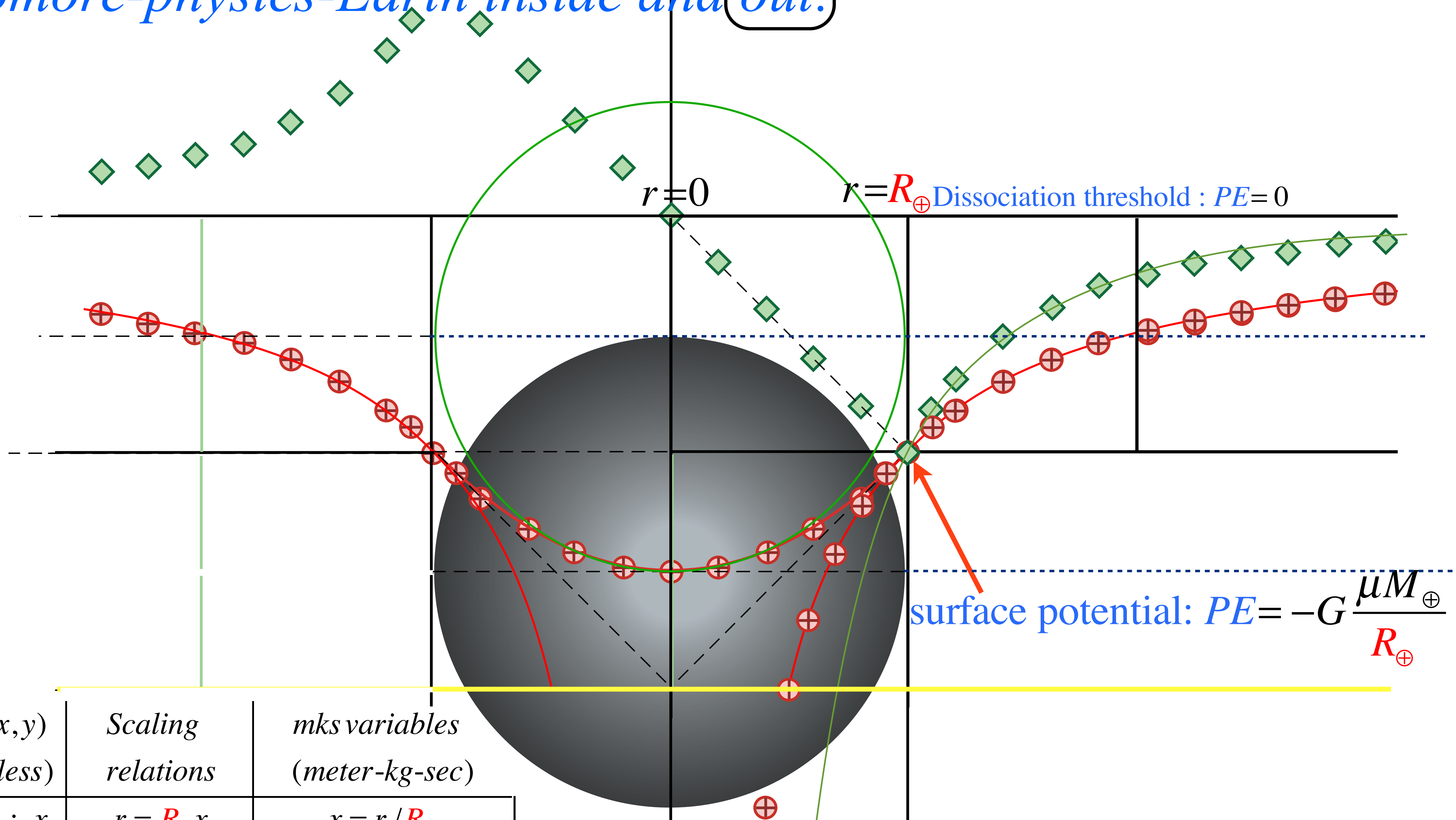
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

Introducing the "neutron starlet"

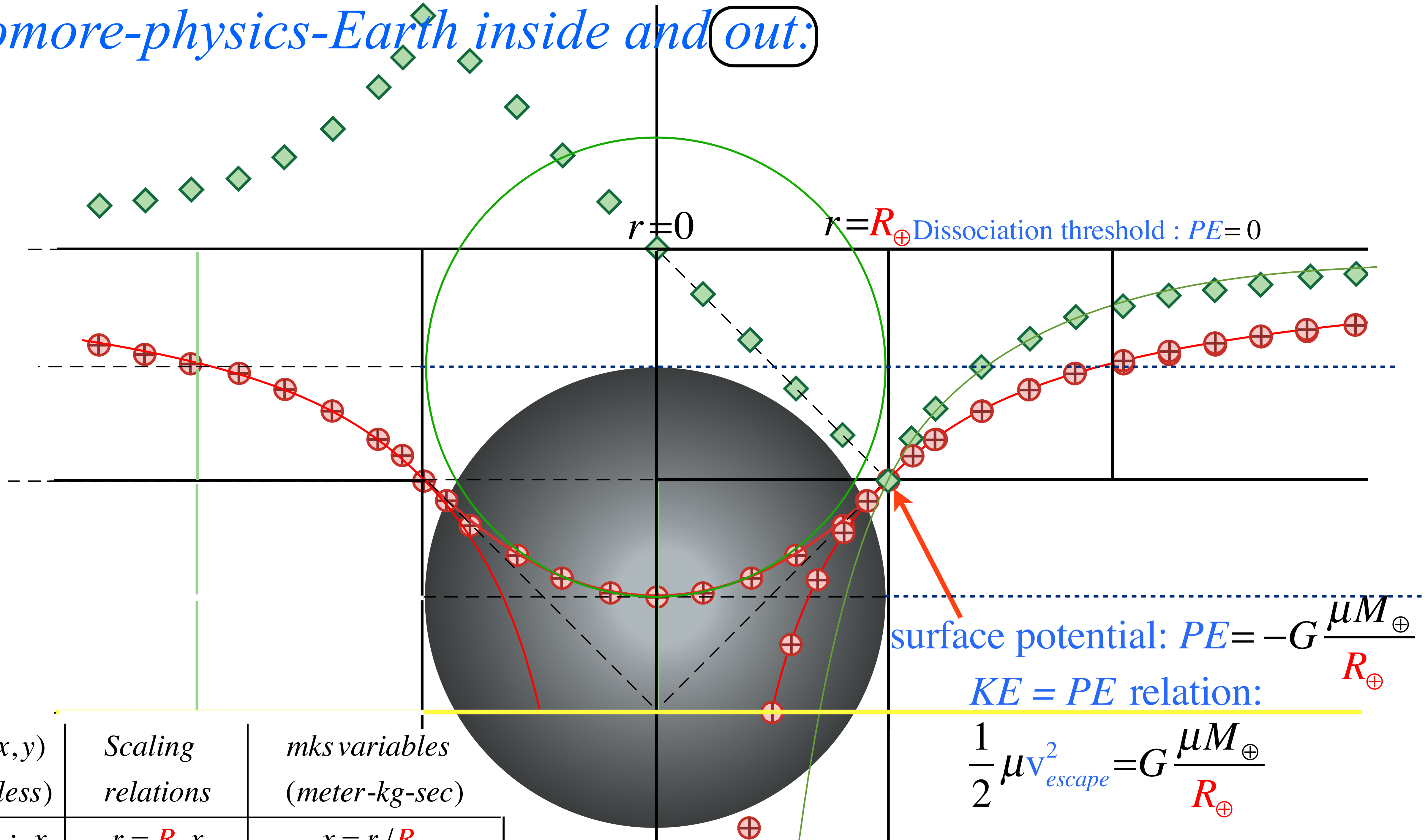
*Fantasizing a "***Black-Hole-Earth***"*

Sophomore-physics-Earth inside and out:



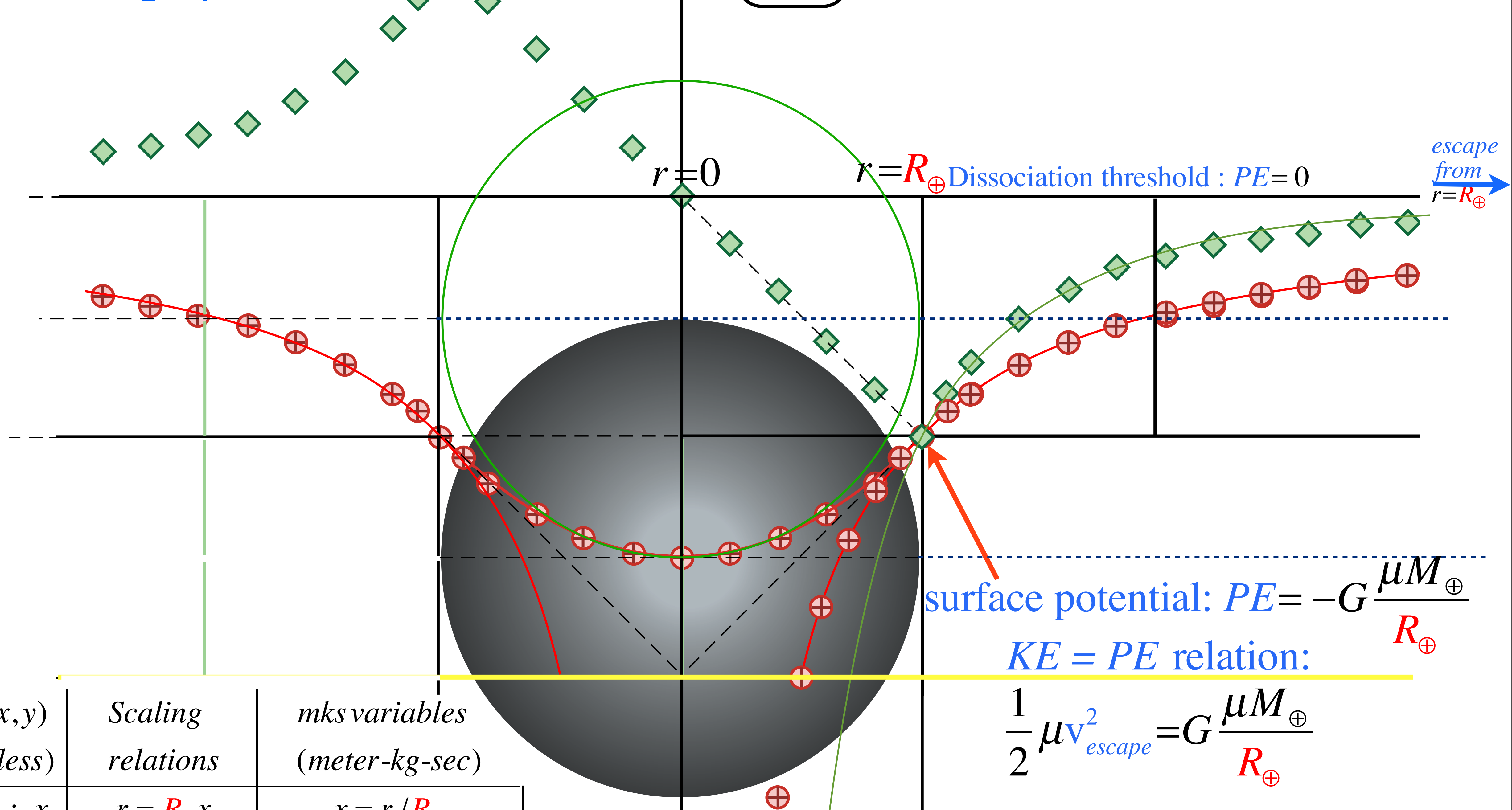
Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r=R_\oplus x$	$x=r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE}=\dfrac{-1}{x}$	$PE^{mks}(r)$ $=\dfrac{GM\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r)=-\dfrac{GM\mu}{r}$ $=-\dfrac{GM\mu}{R_\oplus}\dfrac{1}{x}$

Sophomore-physics-Earth inside and out:



Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus}x$	$x = r/R_{\oplus}$
PE for $\lvert x\rvert \geq 1$: $y^{PE}=\frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}}y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}}\frac{1}{x}$

Sophomore-physics-Earth inside and out:



surface potential: $PE = -G\dfrac{\mu M_{\oplus}}{R_{\oplus}}$

$KE = PE$ relation:

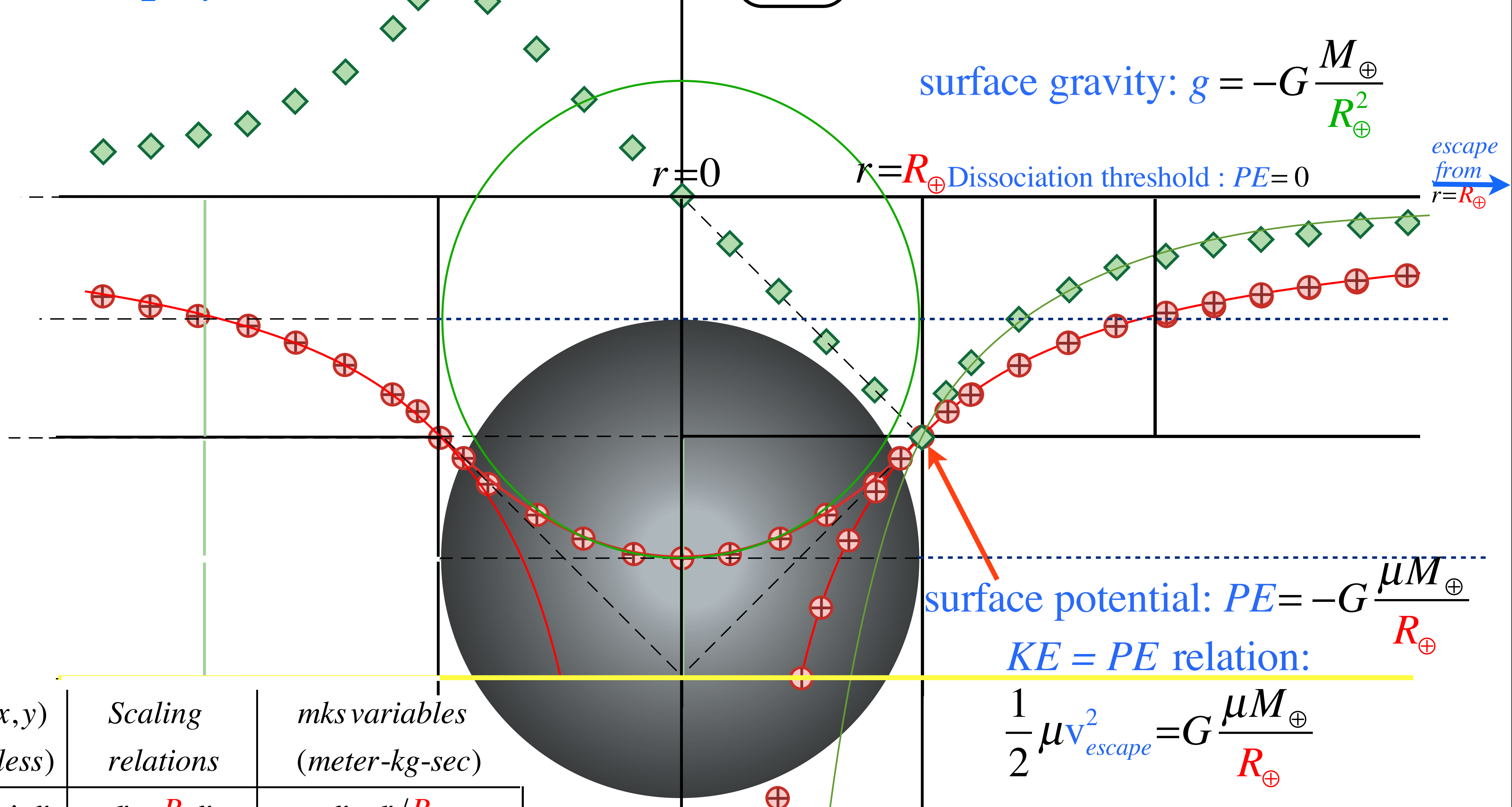
$$\frac{1}{2}\mu v_{escape}^2 = G\frac{\mu M_{\oplus}}{R_{\oplus}}$$

$R_{\oplus}$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_{\oplus}}{R_{\oplus}}}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus}x$	$x = r/R_{\oplus}$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM\mu}{r} = -\dfrac{GM\mu}{R_{\oplus}}\dfrac{1}{x}$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

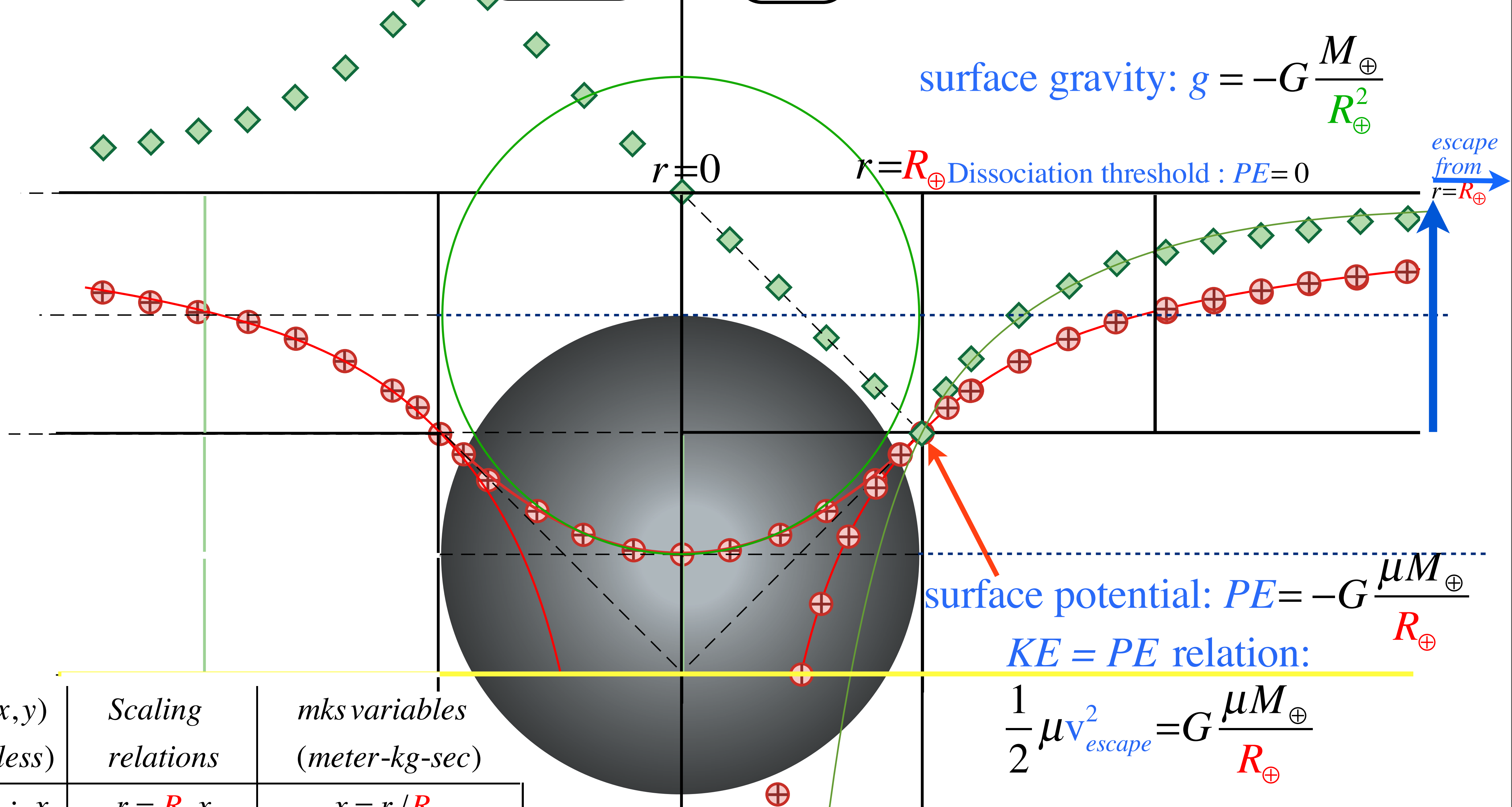
$$\frac{1}{2}\mu v_{escape}^2 = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

Geometric (*x*,*y*) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM\mu}{r} = -\dfrac{GM\mu}{R_\oplus}\dfrac{1}{x}$
Force for $\lvert x\rvert \geq 1$: $y^{Force} = \dfrac{-1}{x^2}$	$F^{mks}(r) = \dfrac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\dfrac{GM\mu}{r^2} = -\dfrac{GM\mu}{R_\oplus^2}\dfrac{1}{x^2}$

Sophomore-physics-Earth inside and out:

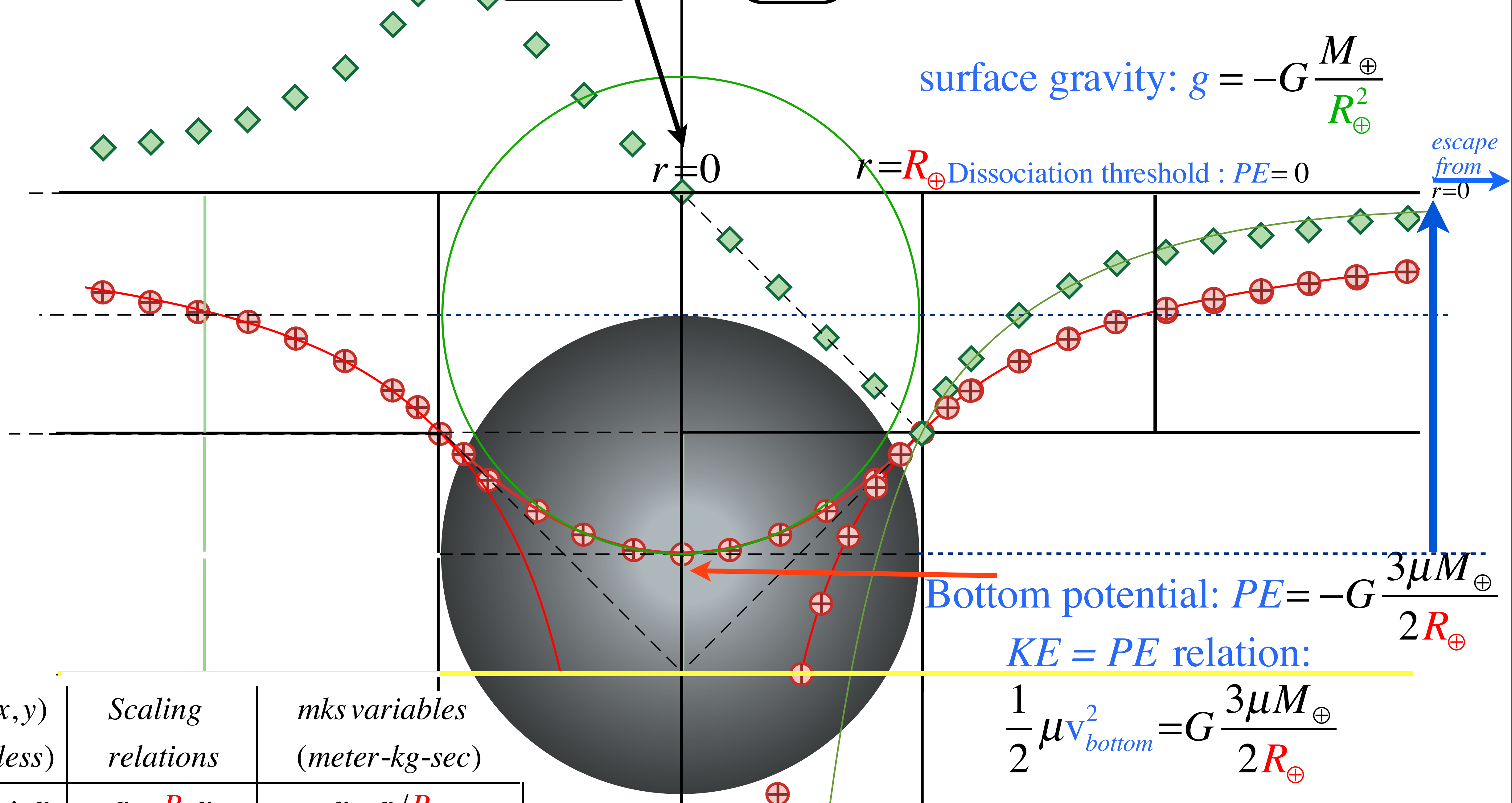


Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)		
space coord.: x	$r = R_{\oplus} x$	$x = r/R_{\oplus}$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}}\frac{1}{x}$	PE for $\lvert x\rvert<1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}}\left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2}\right)$
$Force$ for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2}\frac{1}{x^2}$	$Force$ for $\lvert x\rvert<1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

$R_{\oplus}$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_{\oplus}}{R_{\oplus}}}$$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

(r=0)-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)		
space coord.: x	$r = R_\oplus x$	$x = r / R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM\mu}{r} = -\dfrac{GM\mu}{R_\oplus}\dfrac{1}{x}$	PE for $\lvert x\rvert < 1$: $y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_\oplus}\left(\dfrac{r^2}{2R_\oplus^2} - \dfrac{3}{2}\right)$
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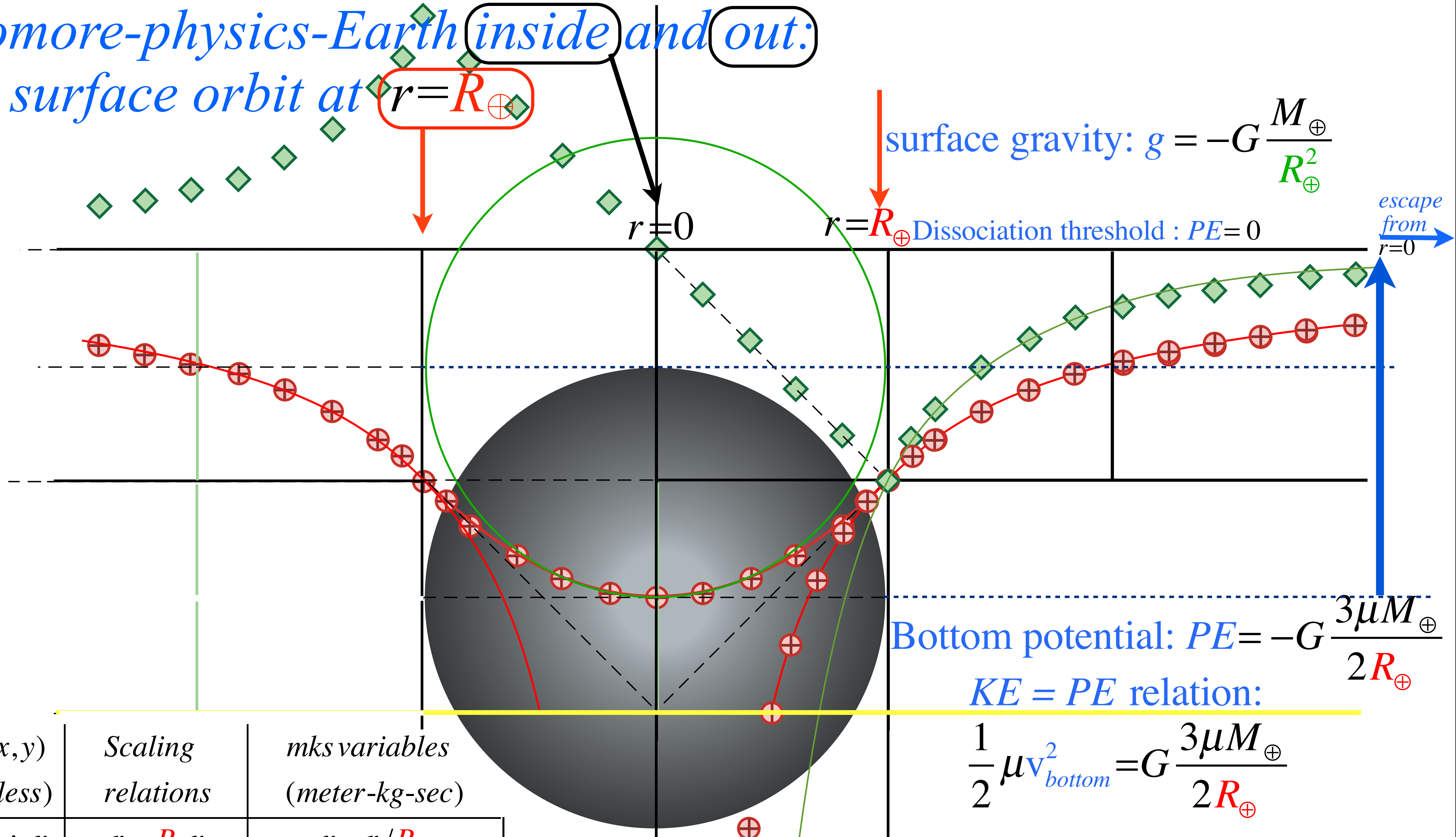
Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



Bottom potential: $PE=-G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)		
space coord.: x	$r=R_\oplus x$	$x=r/R_\oplus$		
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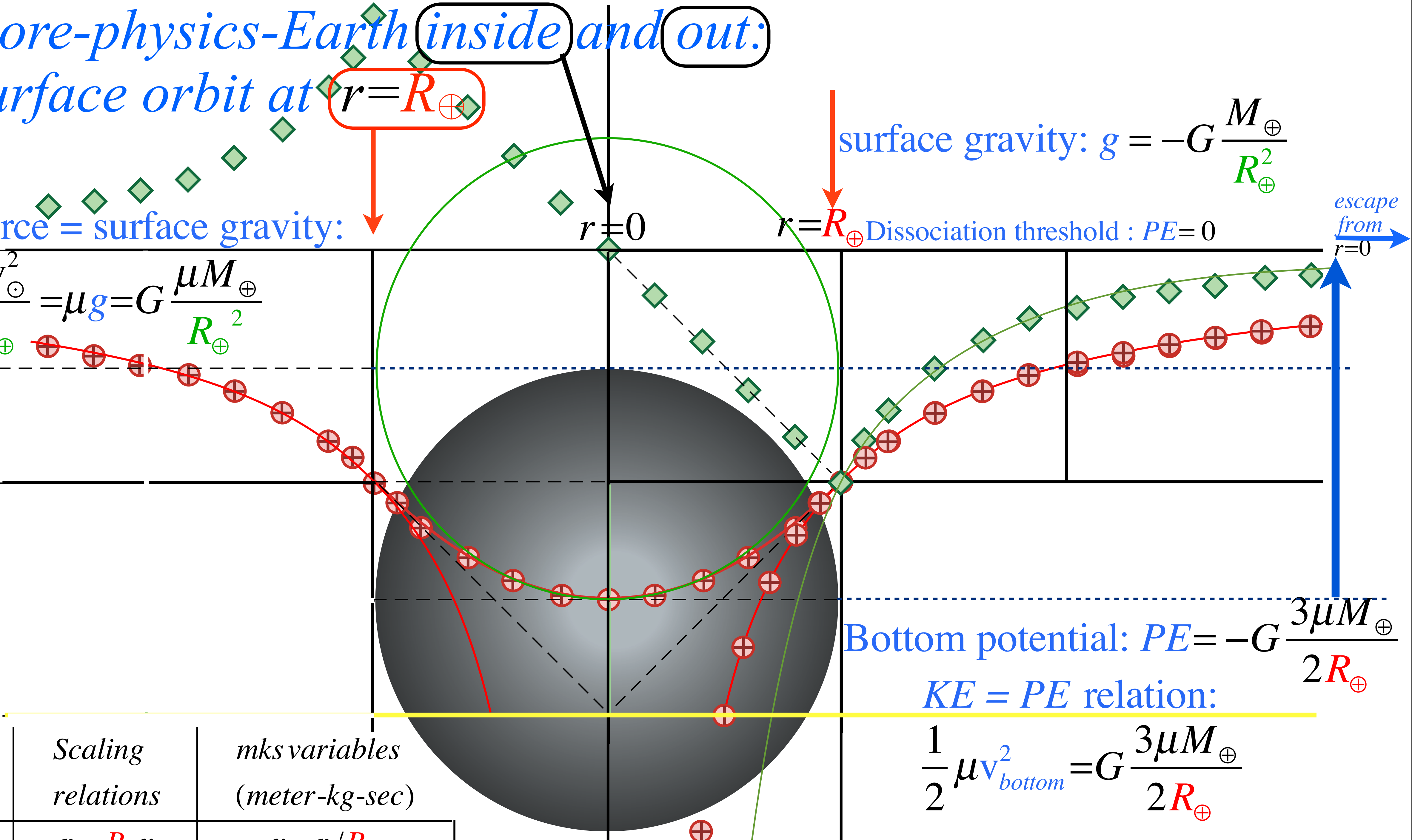
Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



surface gravity: $g=-G\frac{M_\oplus}{R_\oplus^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus{}^2}$$

Bottom potential: $PE=-G\frac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

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Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)		
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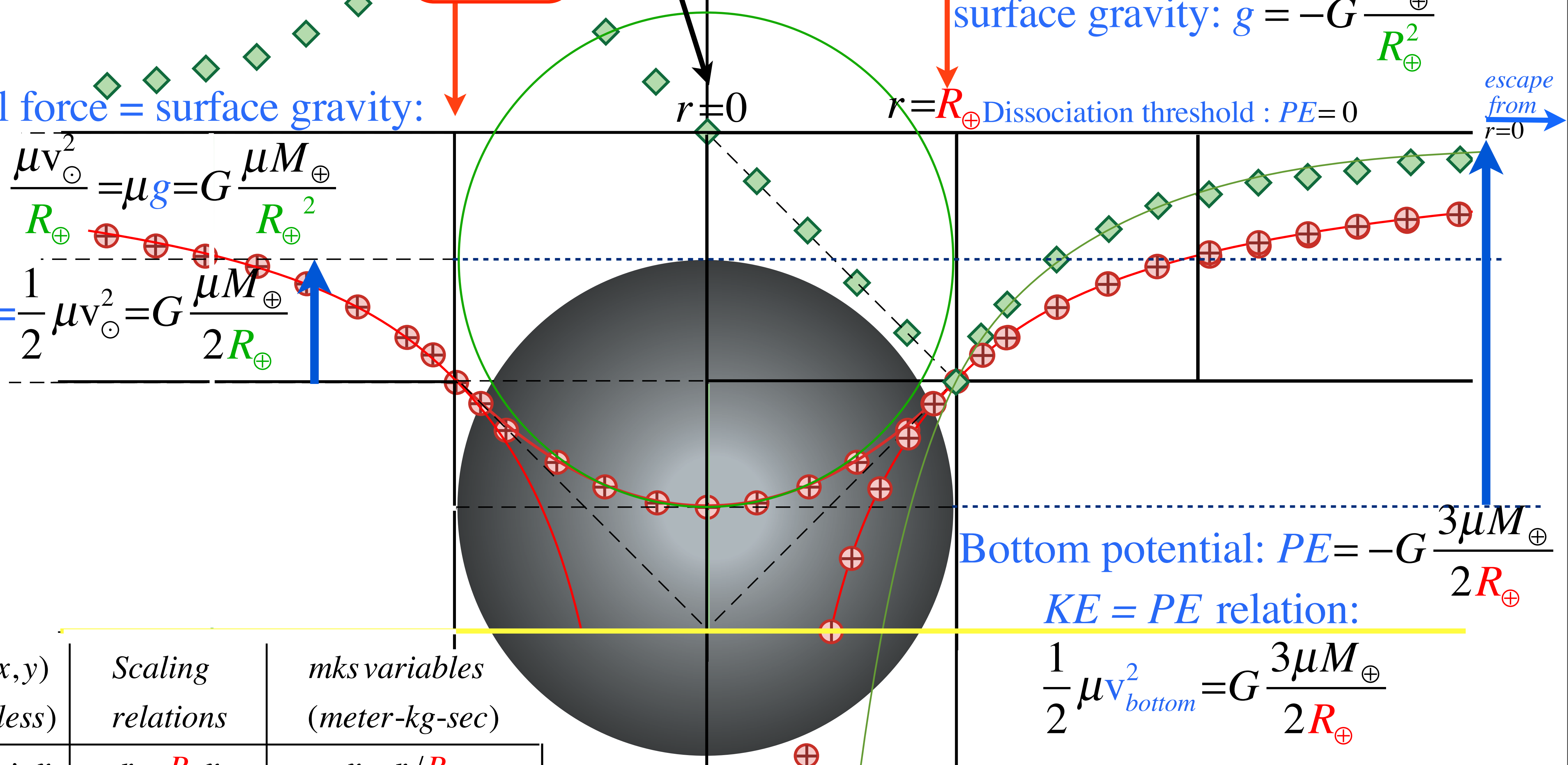
Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



Centifugal force = surface gravity: $\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^{\ 2}}$

surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2}$

Orbit KE$=\frac{1}{2}\mu v_\odot^2 = G\frac{\mu M_\oplus}{2R_\oplus}$

Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$

$(r=0)$-escape-velocity

$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$

Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)		
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$	*PE* for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$
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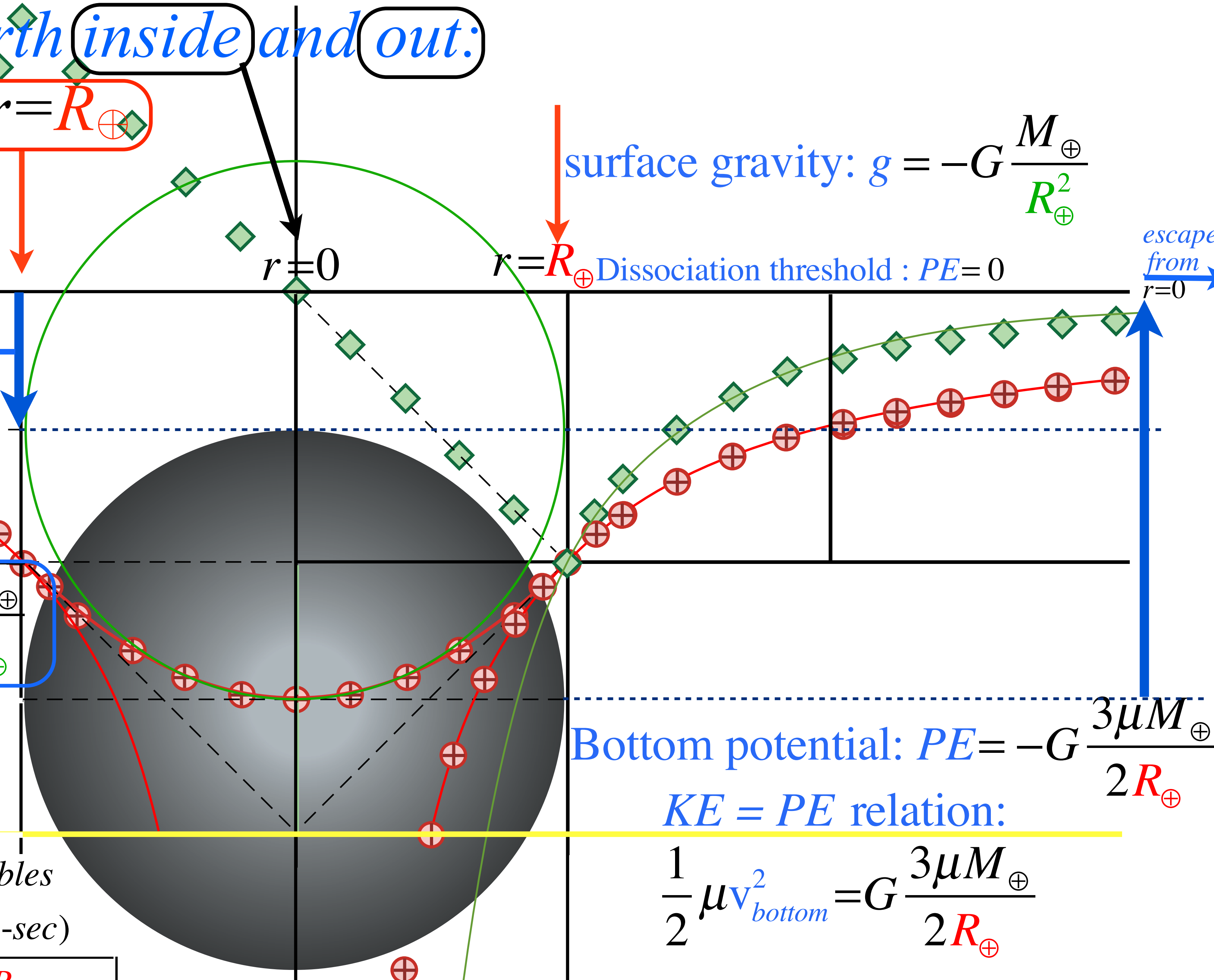

surface gravity: $g=-G\dfrac{M_\oplus}{R_\oplus^2}$

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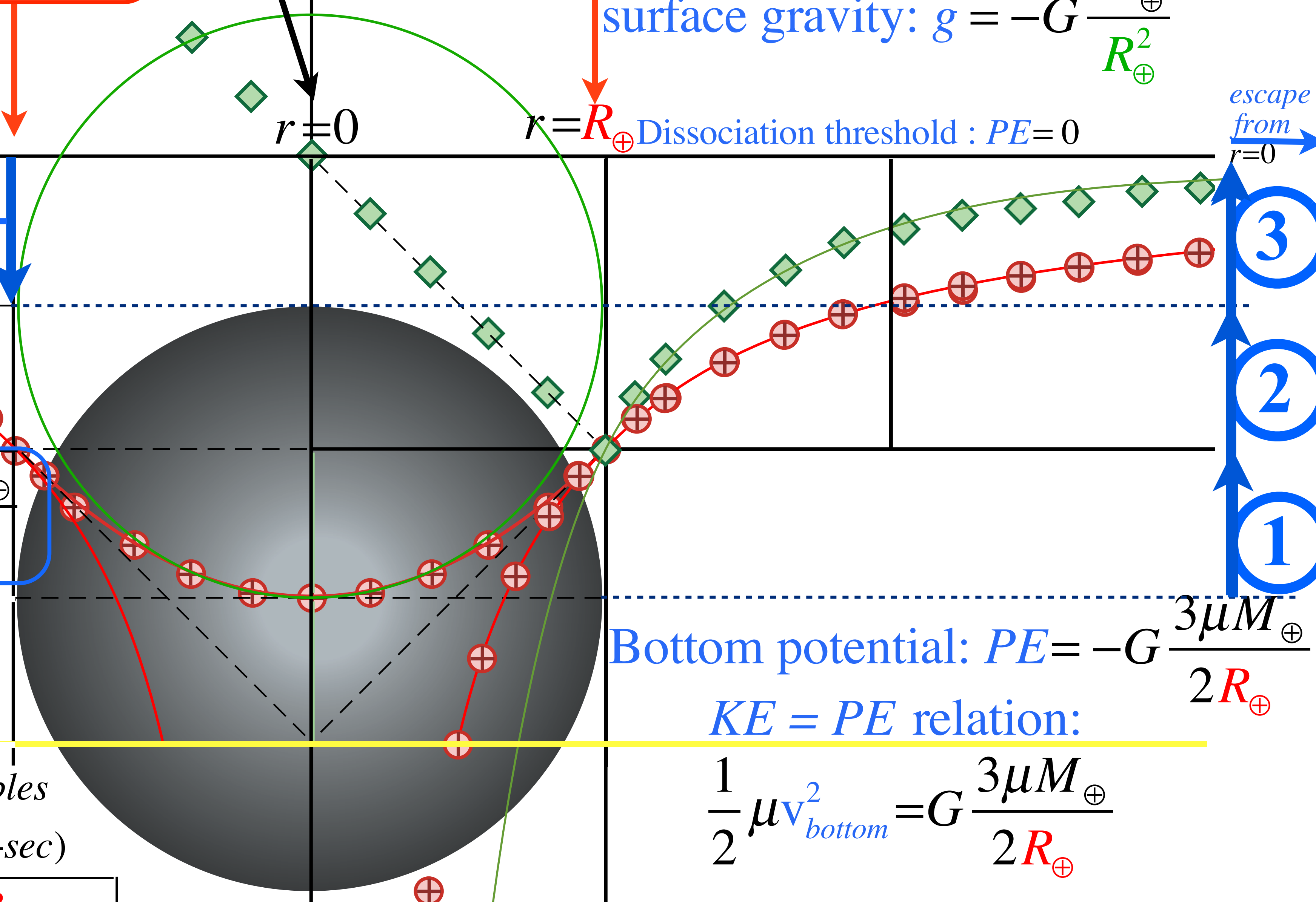
$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$



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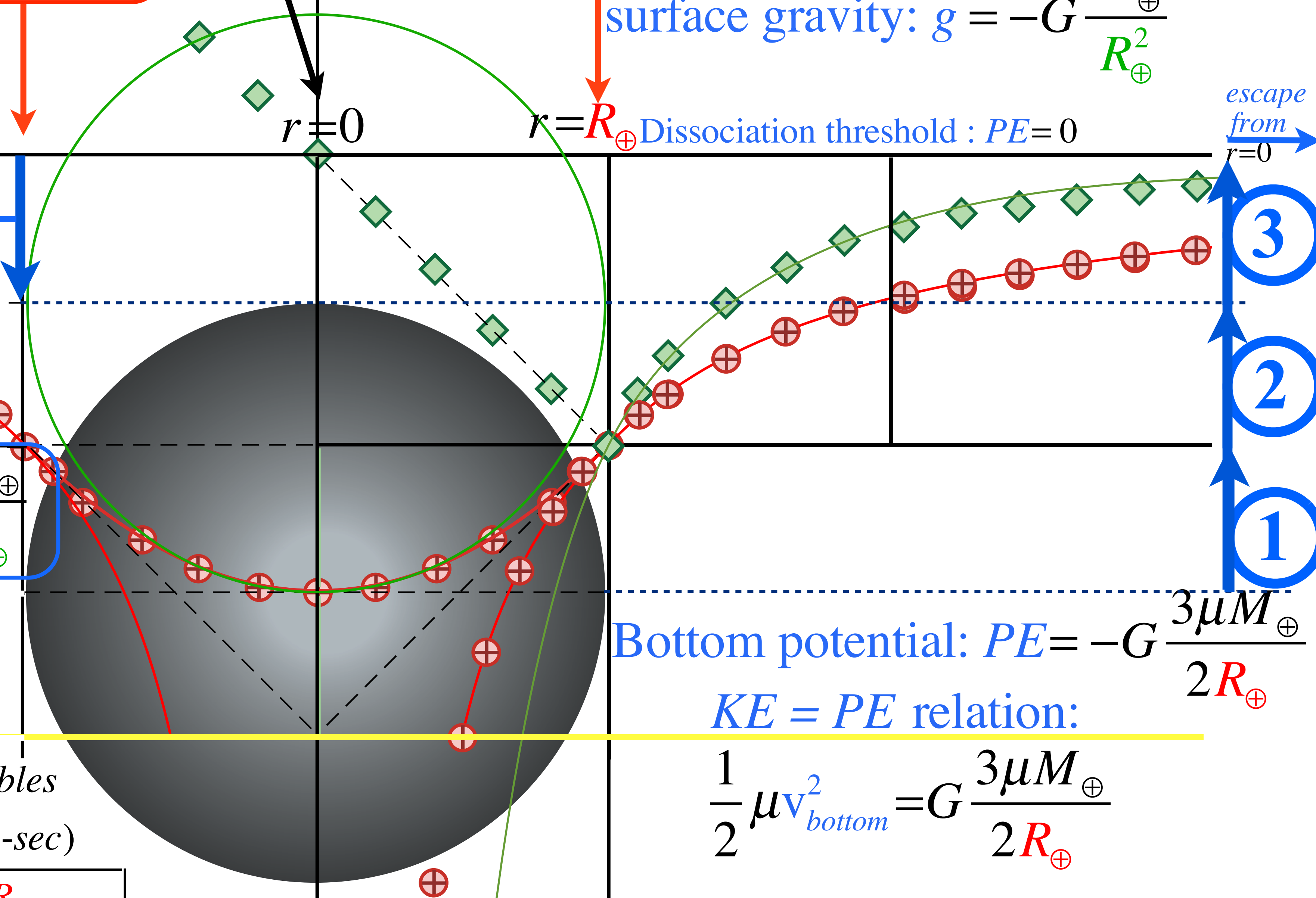

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$(r=R_\oplus)$-orbit speed:

$$v_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus}}$$

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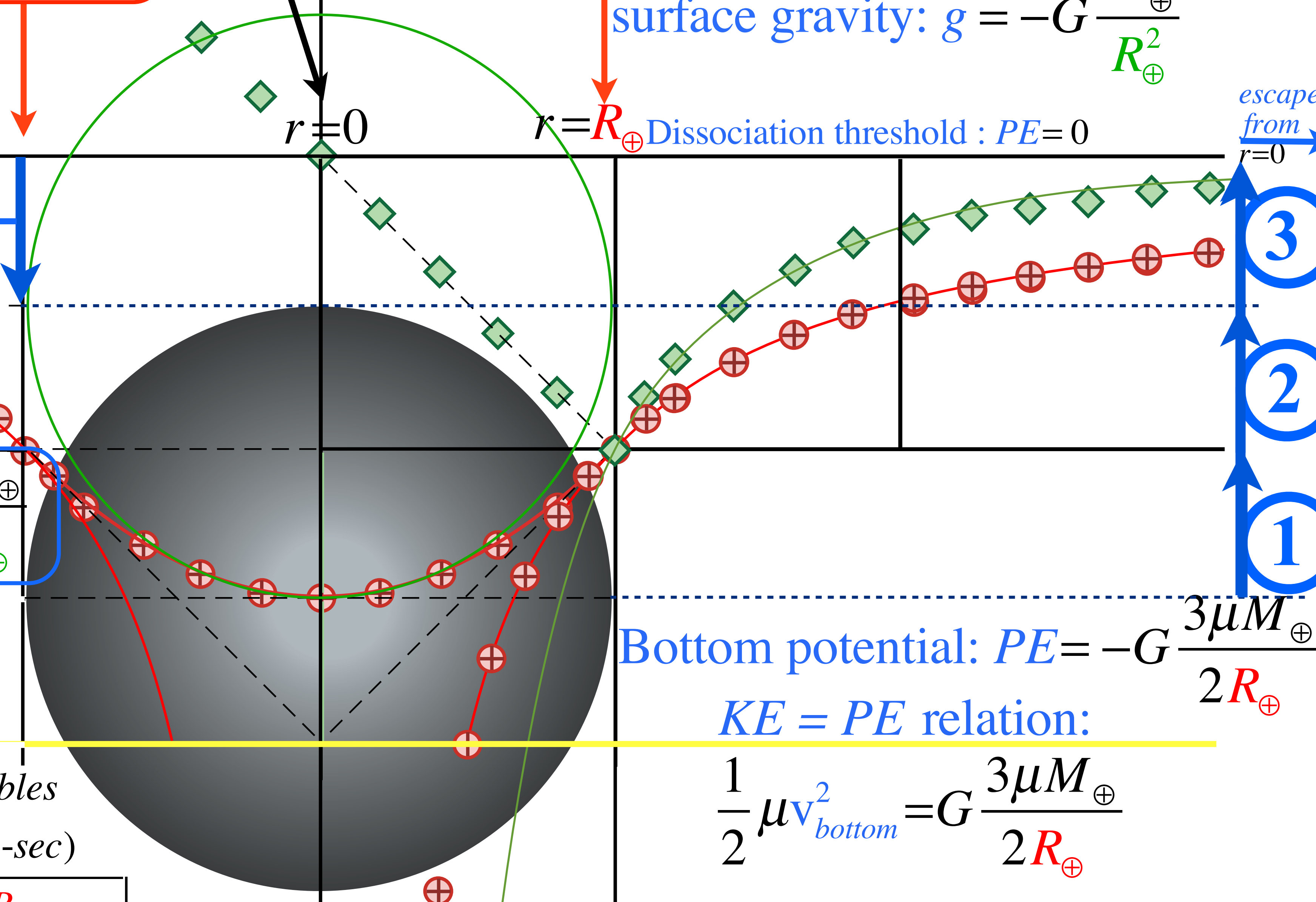

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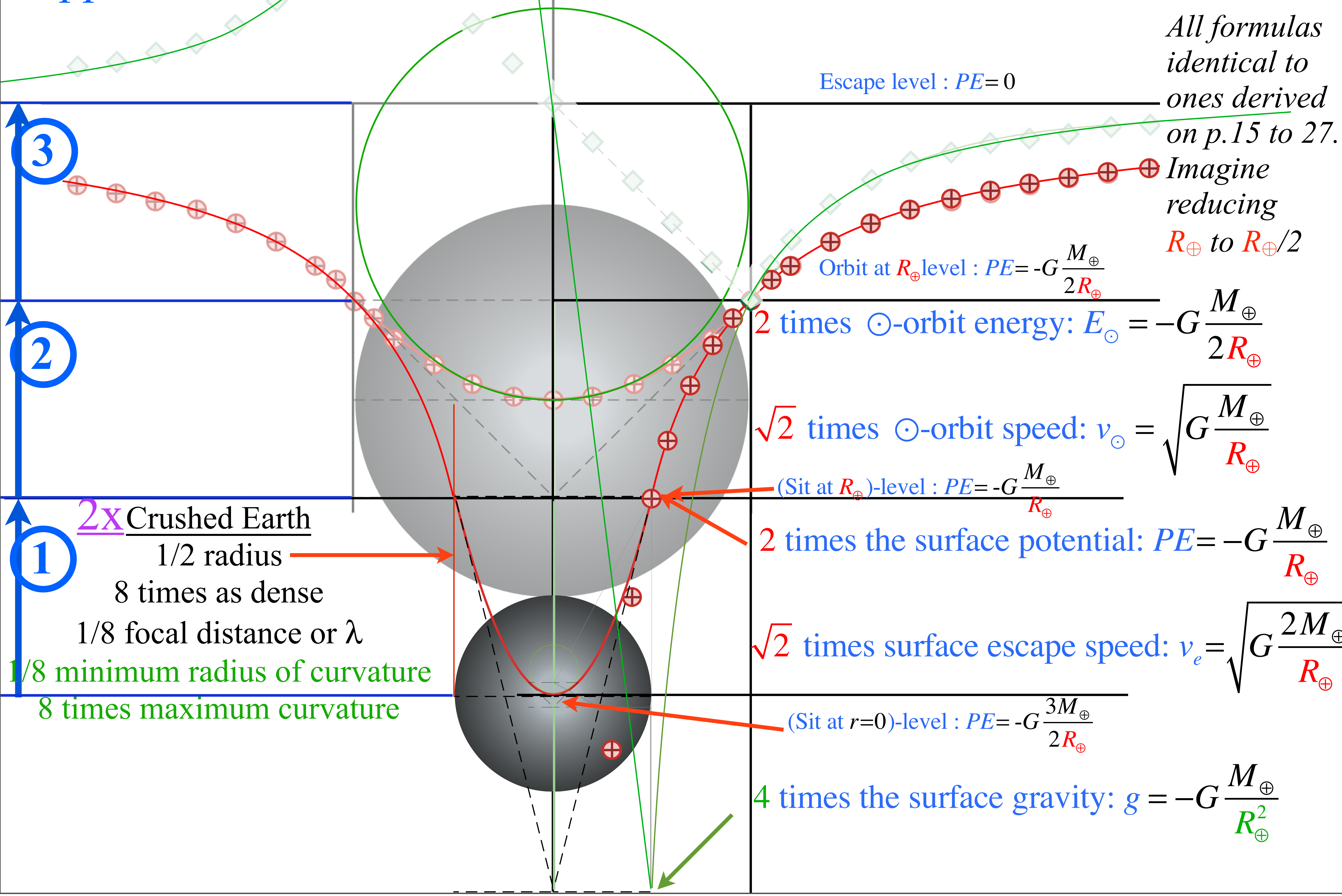
Earth matter vs nuclear matter:

Introducing the "neutron starlet"

*Fantasizing a "***Black-Hole-Earth***"*

Sophomore-physics-Earth inside and out: "3-steps to Hell"

Suppose Earth radius crushed to 1/2: ($R_\oplus$=6.4·10^{6}m crushed to $R_\oplus$/2=3.2·10^{6}m)



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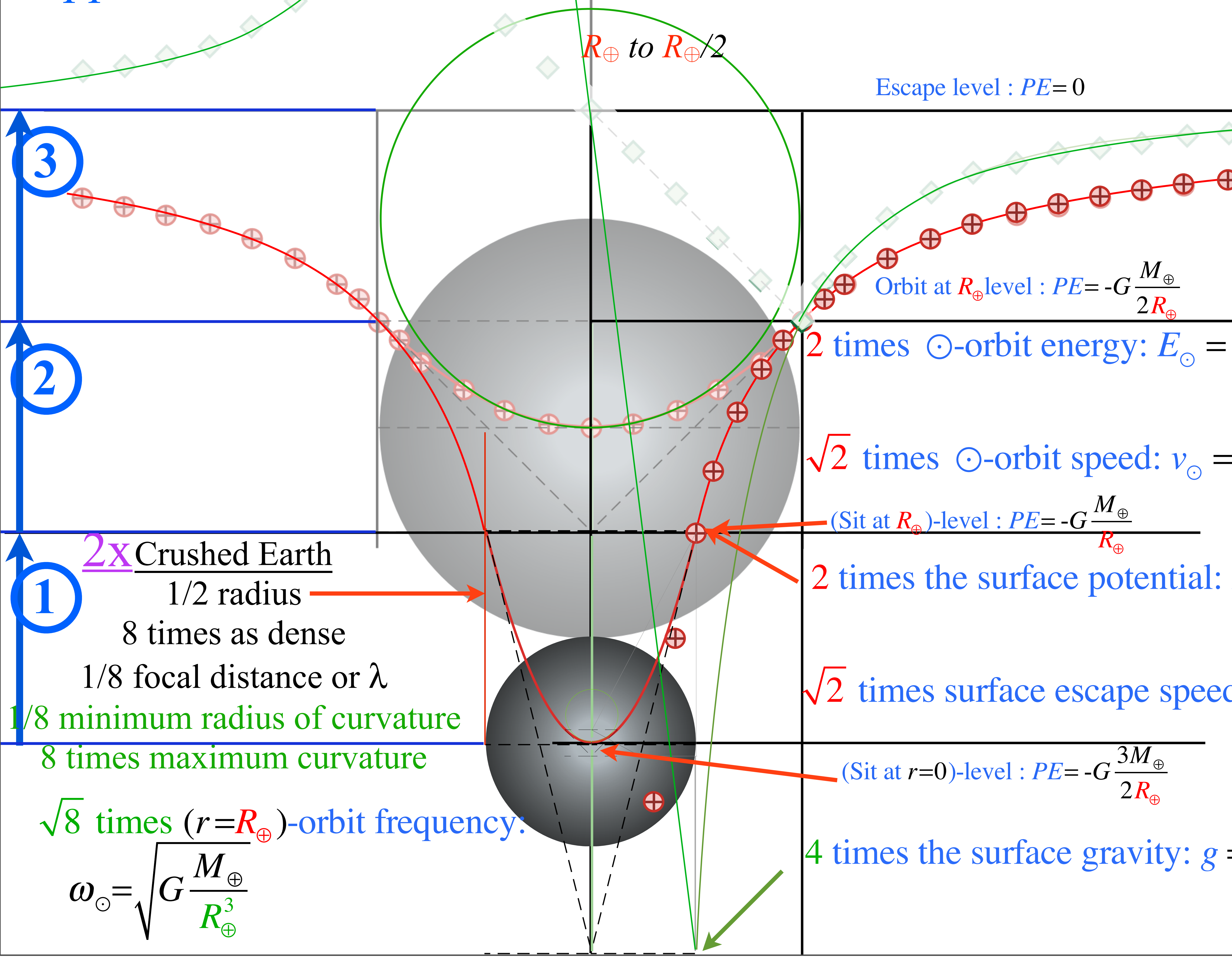

All formulas identical to ones derived on p.15 to 27. Imagine reducing $R_\oplus$ to $R_\oplus/2$

2 times $\odot$-orbit energy: $E_\odot = -G\dfrac{M_\oplus}{2R_\oplus}$

$\sqrt{2}$ times $\odot$-orbit speed: $v_\odot = \sqrt{G\dfrac{M_\oplus}{R_\oplus}}$

2 times the surface potential: $PE = -G\dfrac{M_\oplus}{R_\oplus}$

$\sqrt{2}$ times surface escape speed: $v_e = \sqrt{G\dfrac{2M_\oplus}{R_\oplus}}$

4 times the surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

2x Crushed Earth

1/2 radius

8 times as dense

1/8 focal distance or λ

1/8 minimum radius of curvature

8 times maximum curvature

$\sqrt{8}$ times ($r = R_\oplus$)-orbit frequency:

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$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}{}^{3}}$$

Earth matter

$Earth\, mass: M_{\oplus} = 5.9722 \times 10^{24}\, kg. \simeq 6.0 \cdot 10^{24}\, kg.$ Density $\rho_{\oplus} = ??$

$Earth\, radius: R_{\oplus} = 6.371 \cdot 10^{6}\, m \simeq 6.4 \cdot 10^{6}\, m \quad Earth\, volume: (4\pi/3)R_{\oplus}{}^{3} \simeq 1.083 \cdot 10^{21} \sim 10^{21}\, m^{3}$

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Density of solid Fe=$7.9\cdot 10^3$kg/m^3
Density of liquid Fe=$6.9\cdot 10^3$kg/m^3

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Nucleon mass $=1.67 \cdot 10^{-27}kg. \sim 2 \cdot 10^{-27}kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27}kg$.

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$36\pi = 113 \sim 10^2$

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip (1cc).

Earth radius crushed by a factor of $0.5\cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

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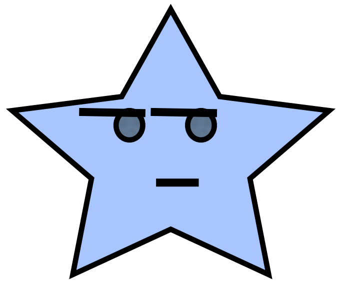
Introducing the "Neutron starlet"
1 cm^3 *of nuclear matter: mass=* 10^{12} *kg.*



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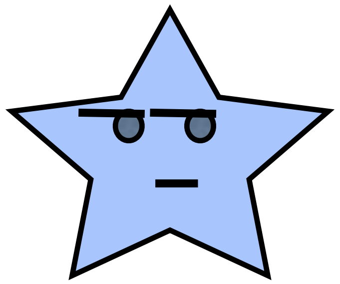

*Introducing the "***Black Hole Earth***"*

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^{8} m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$V_{escape} = \sqrt{(2GM/R_{\otimes})}$$

(from p. 15)

$G = 6.67384(80) \cdot 10^{-11} Nm^{2}/C^{2} \sim (2/3)10^{-10}$

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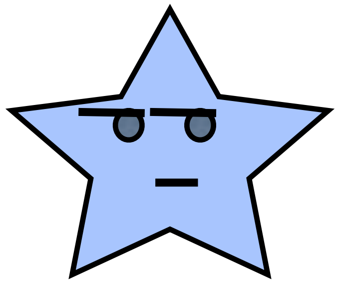
That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of ${}^{4\pi}/_{3} r^3 = {}^{4\pi}/_{3} (3 \cdot 10^{-15})^3 m^3$ or about $10^{-43} m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

Introducing the "Neutron starlet"
1 cm³ of nuclear matter: mass= 10^{12} kg.



Introducing the "**Black Hole Earth**"
Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$V_{escape} = \sqrt{(2GM/R_{\otimes})}$

(from p. 15)

$c = \sqrt{(2GM/R_{\otimes})}$

$R_{\otimes} = 2GM/c^2 = 8.9mm \sim 1cm$ *(fingertip size!)*

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

→ *Isotropic Harmonic Oscillator phase dynamics in uniform-body orbits*
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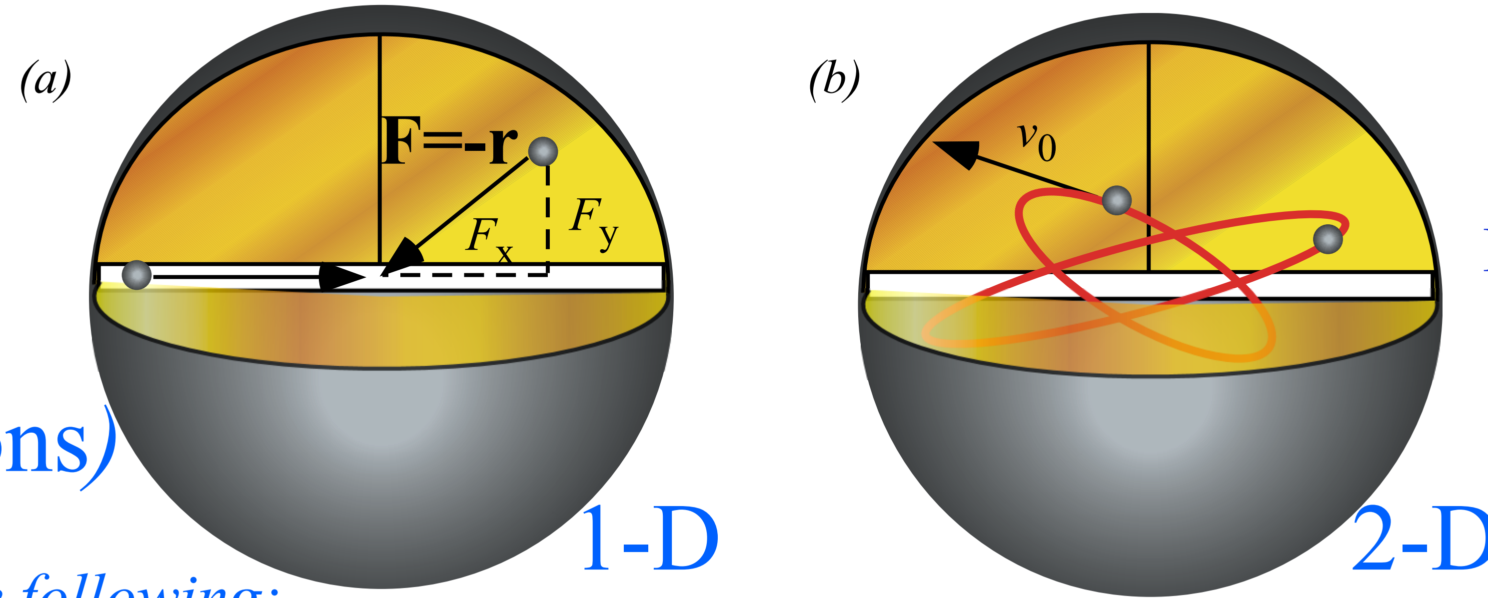
Integrating IHO equations by phasor geometry

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

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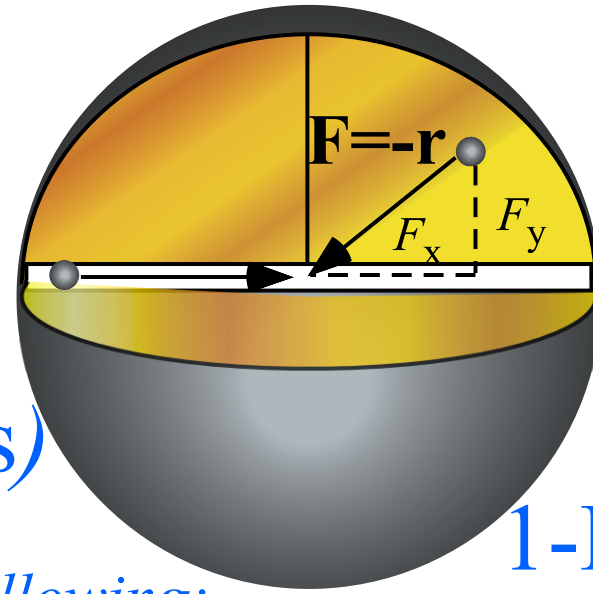
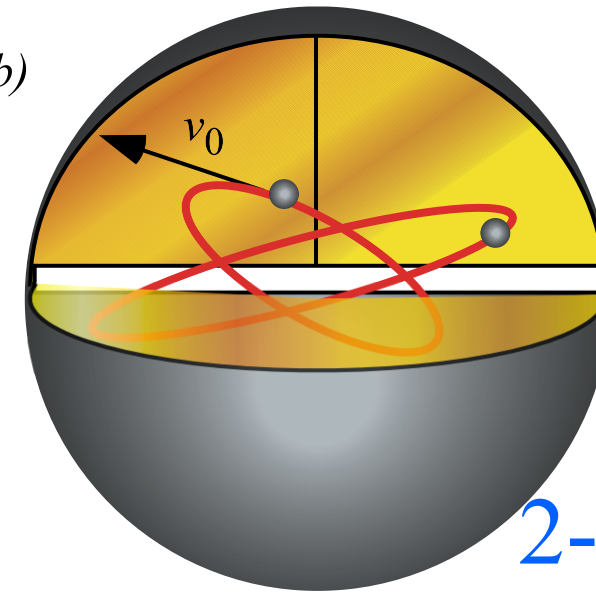
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



2-D

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Unit 1
Fig. 9.10

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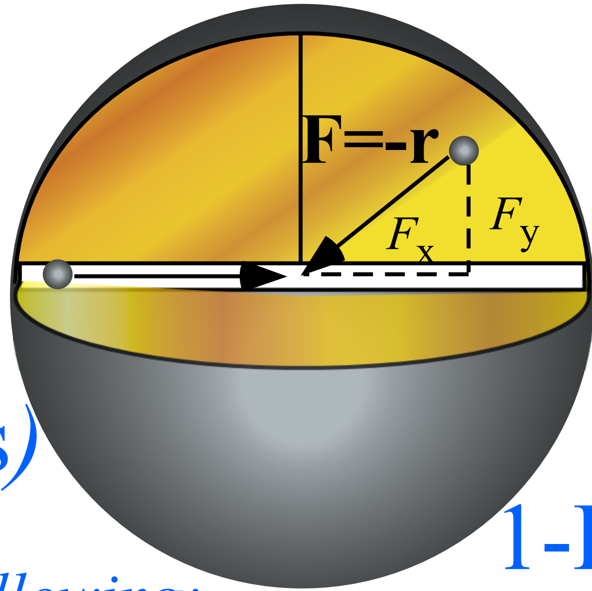
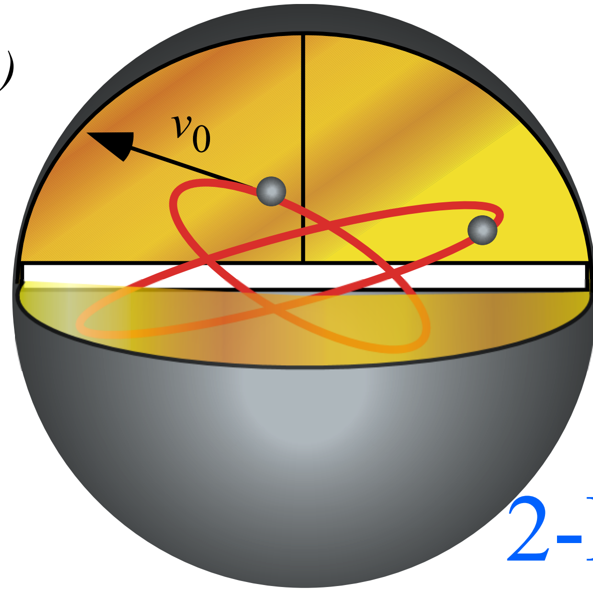
HO"Spring-constant" (from p. 26)

$$\frac{1}{2}k = \frac{GM}{2R_\oplus^3}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D



Unit 1
Fig. 9.10

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Isotropic Harmonic Oscillator phase dynamics in uniform-body

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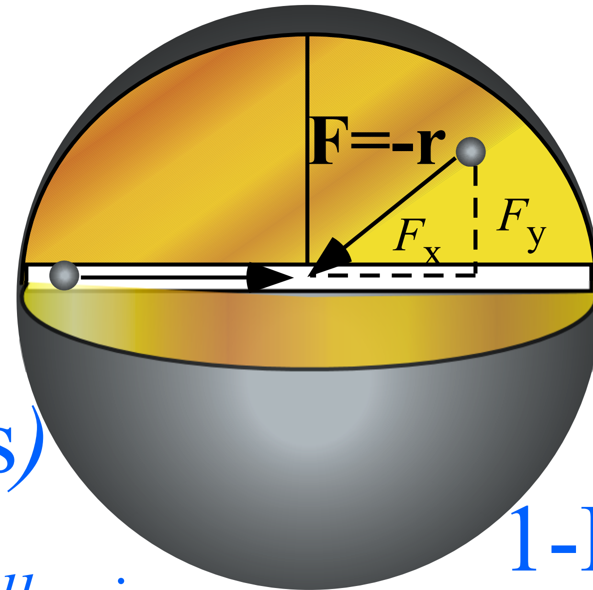
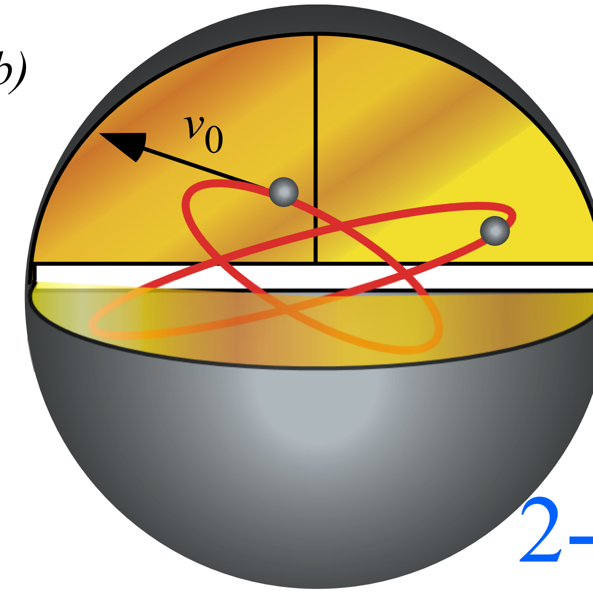


1-D

(b)



Unit 1
Fig. 9.10

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Isotropic Harmonic Oscillator phase dynamics in uniform-body

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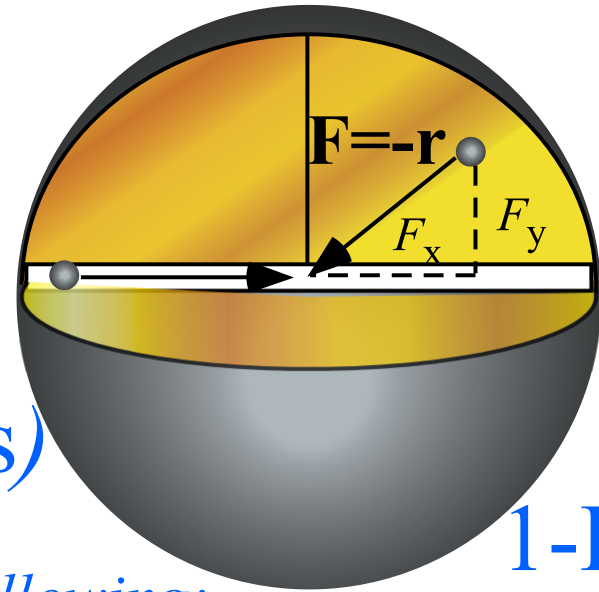
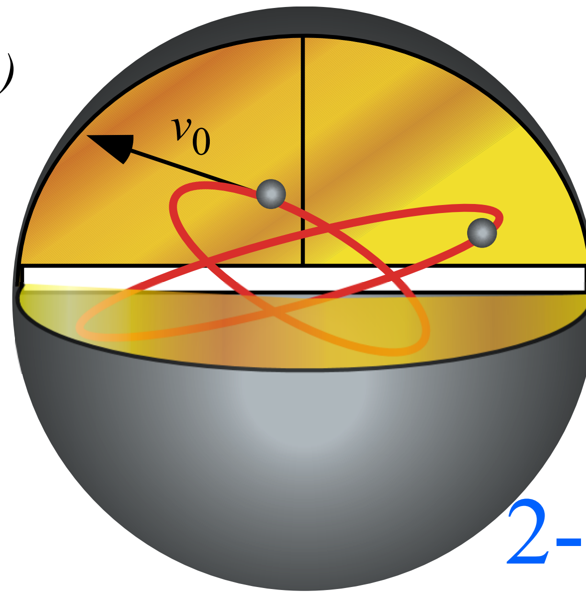


1-D

(b)



Unit 1
Fig. 9.10

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def. **(3)** $\omega = \frac{d\theta}{dt}$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega}_{by\ def.\ (3)}\frac{dx}{d\theta}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

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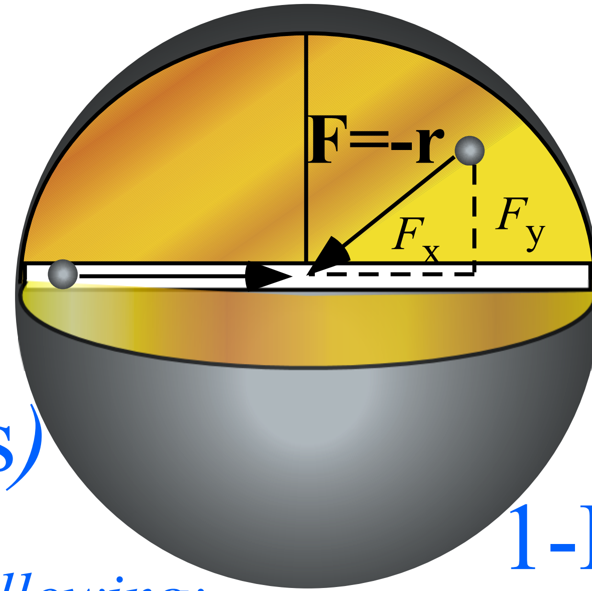
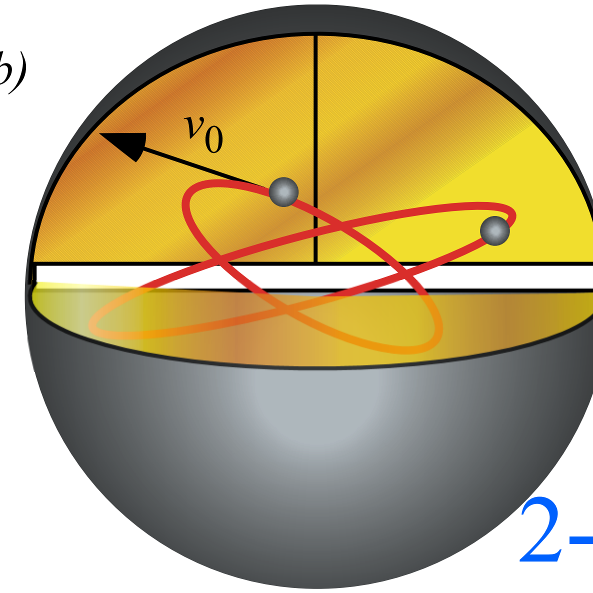


1-D

(b)



Unit 1
Fig. 9.10

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by (1) — by def. (3) — by (2)

Isotropic Harmonic Oscillator phase dynamics in uniform-body

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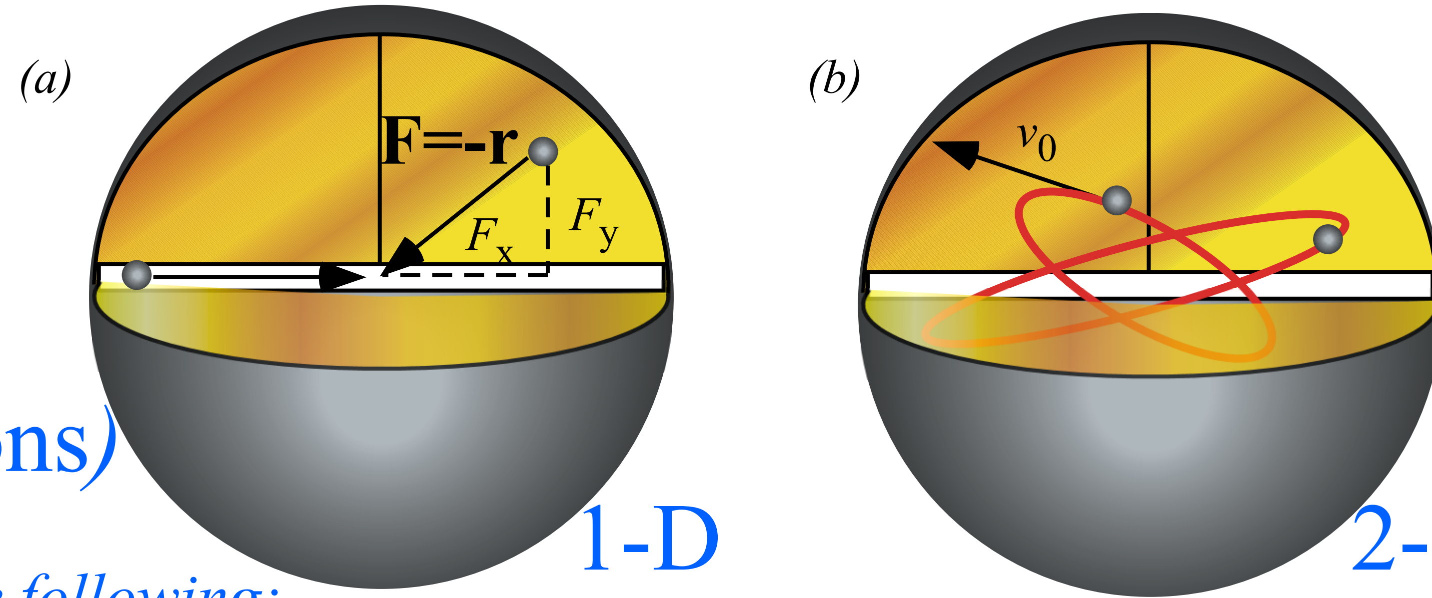

1-D

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Unit 1 Fig. 9.10

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by (1) — *by def.* (3) — *by* (2)

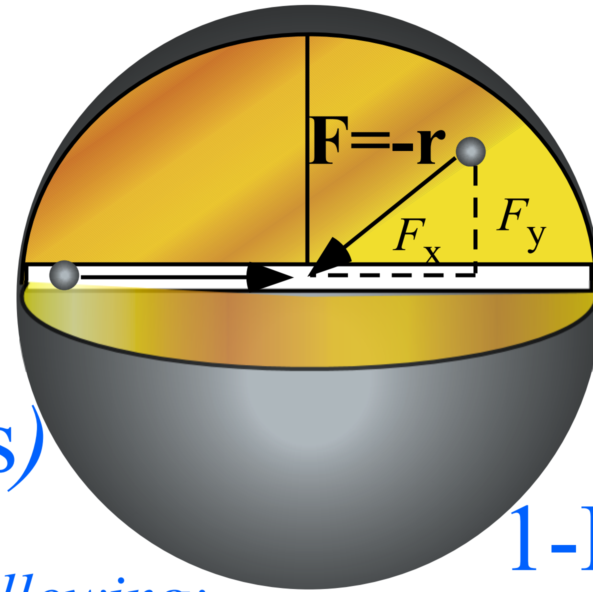
by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by (1)

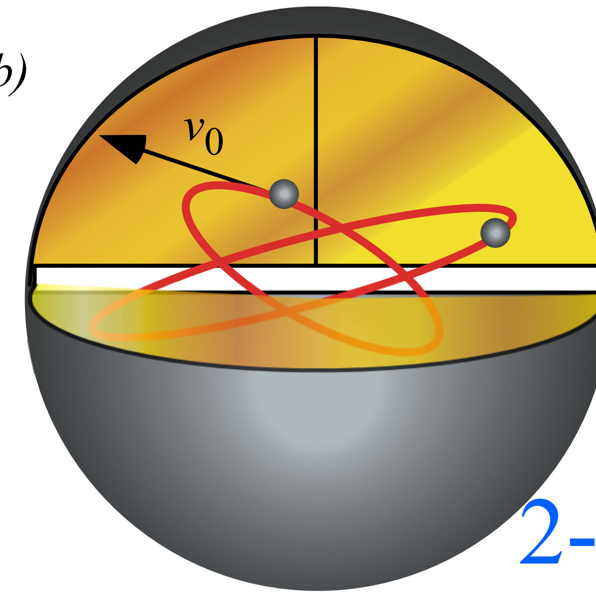
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



Unit 1
Fig. 9.10

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by **(1)** *by def.* **(3)** *by* **(2)**

by def. **(3)**

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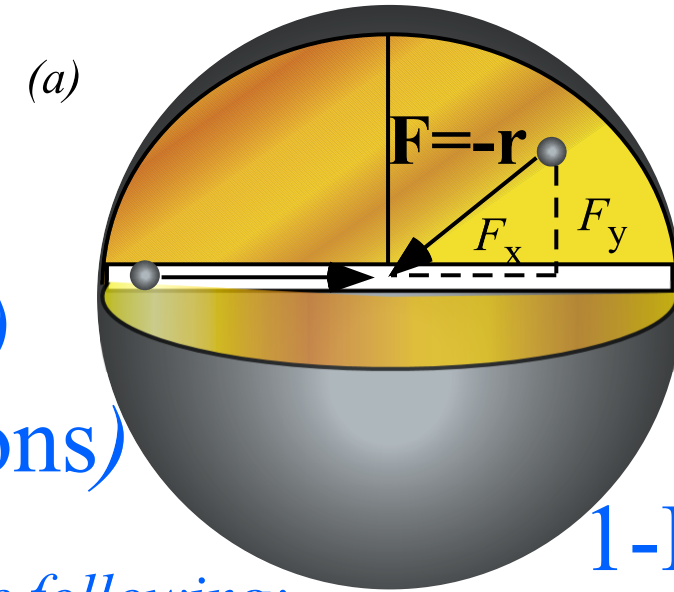
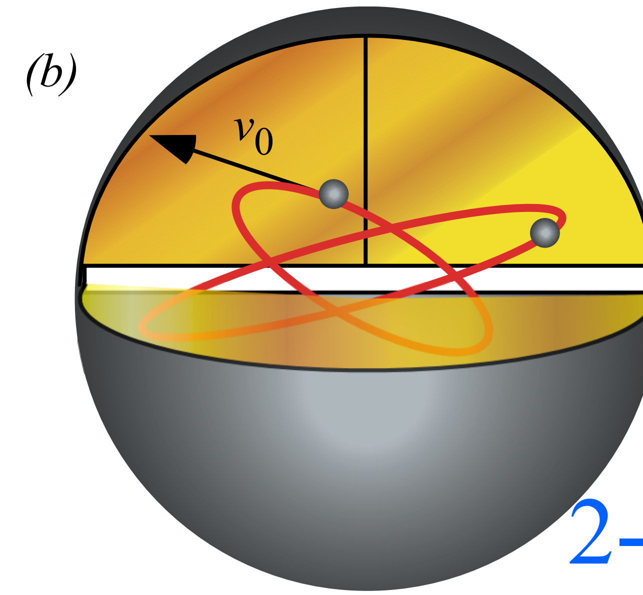
divide this by **(1)**

by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

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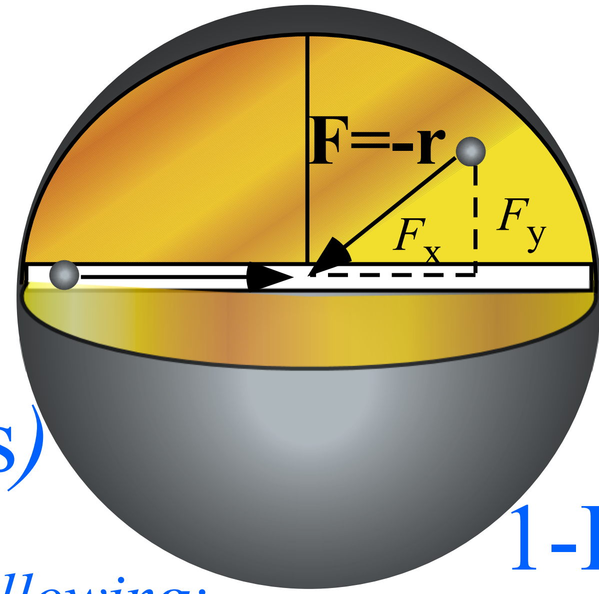
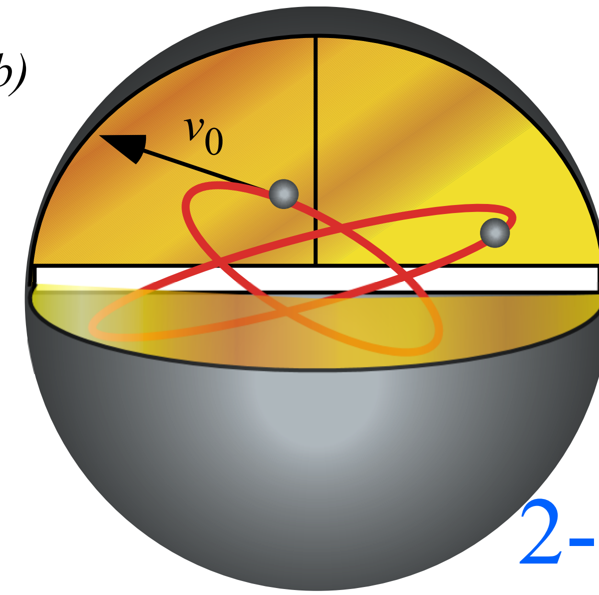


1-D

(b)



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Isotropic Harmonic Oscillator phase dynamics in uniform-body orbits
→ *Dual phasor construction of elliptic oscillator orbits*
Integrating IHO equations by phasor geometry

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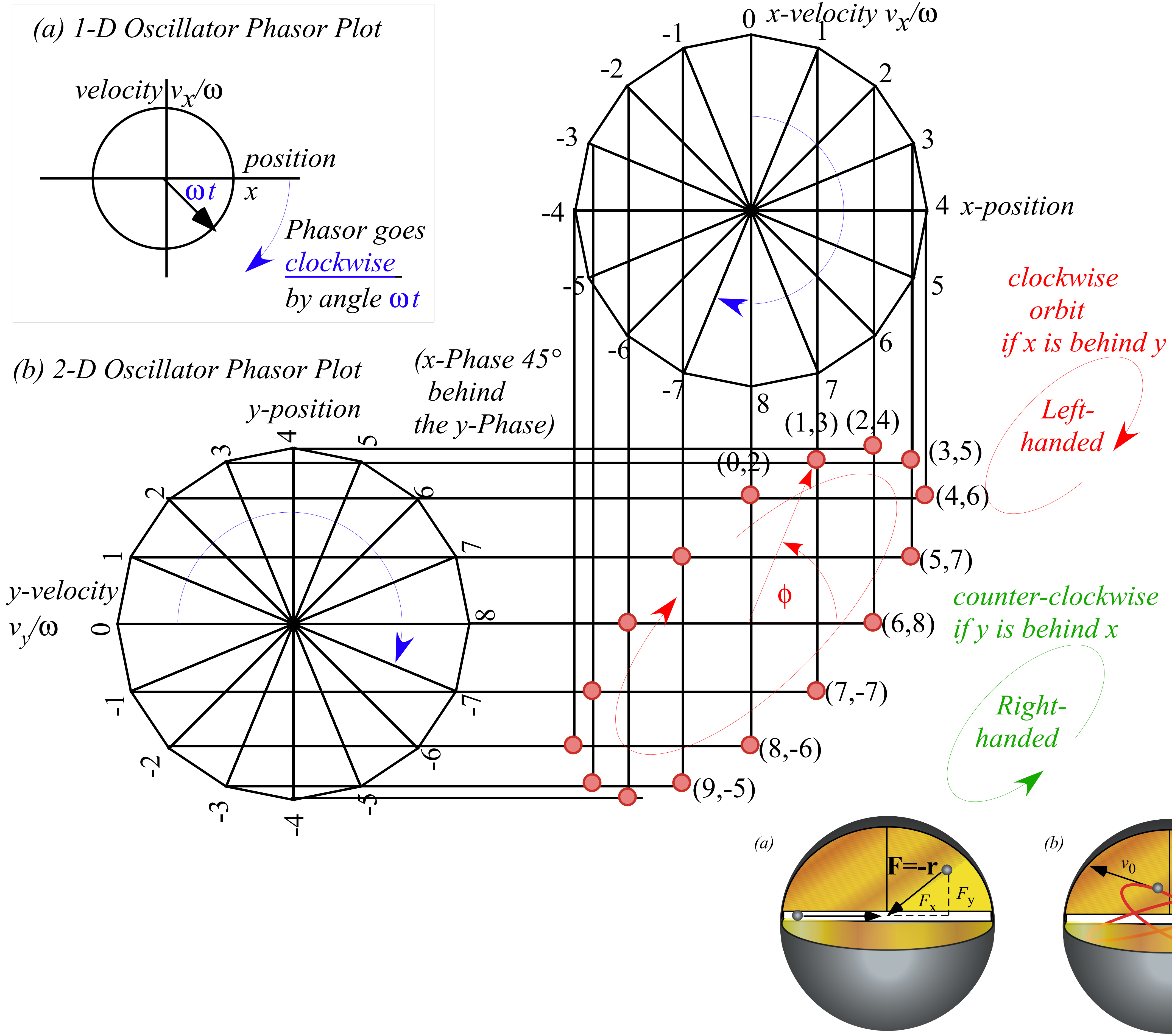
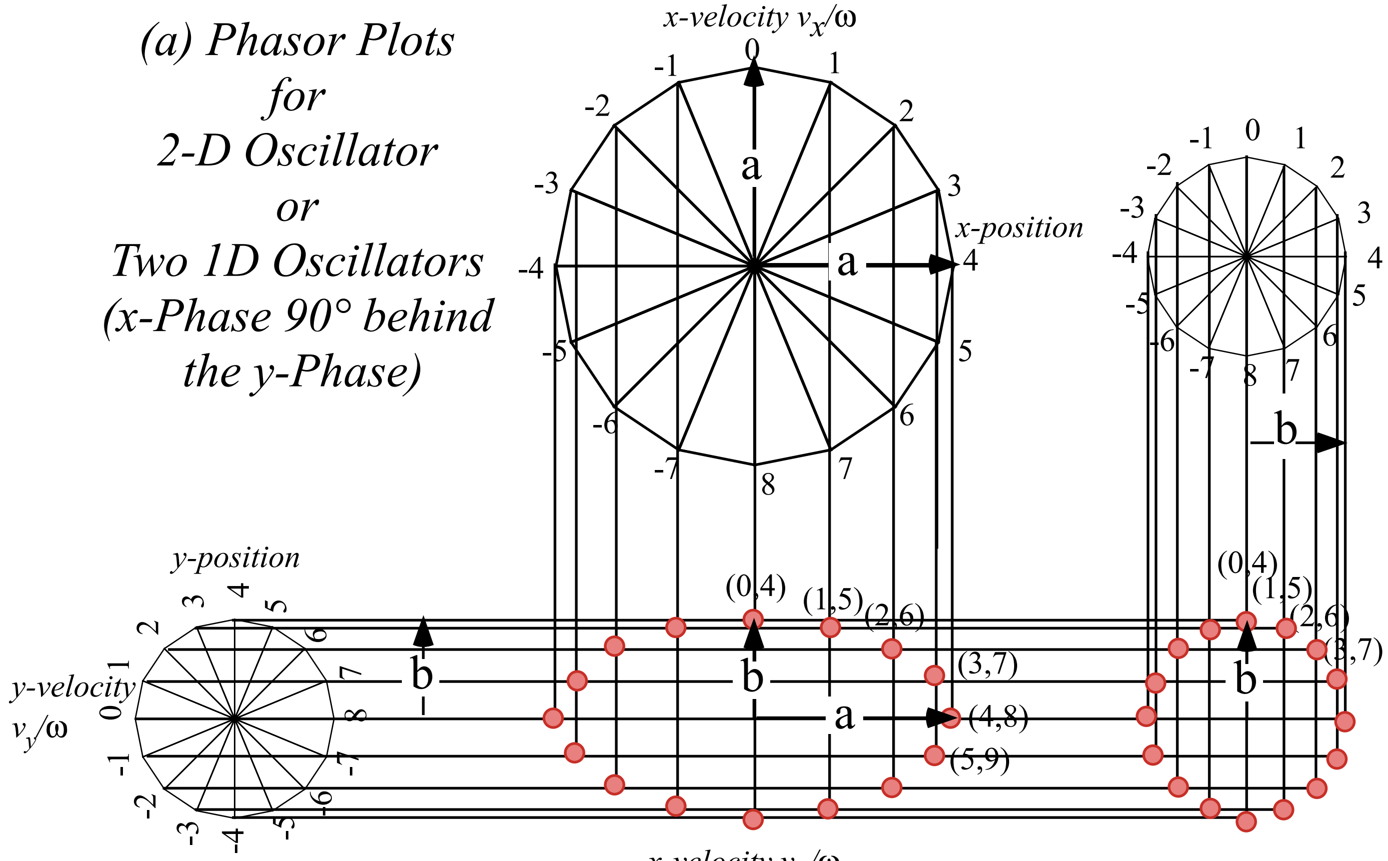


Unit 1
Fig. 9.10

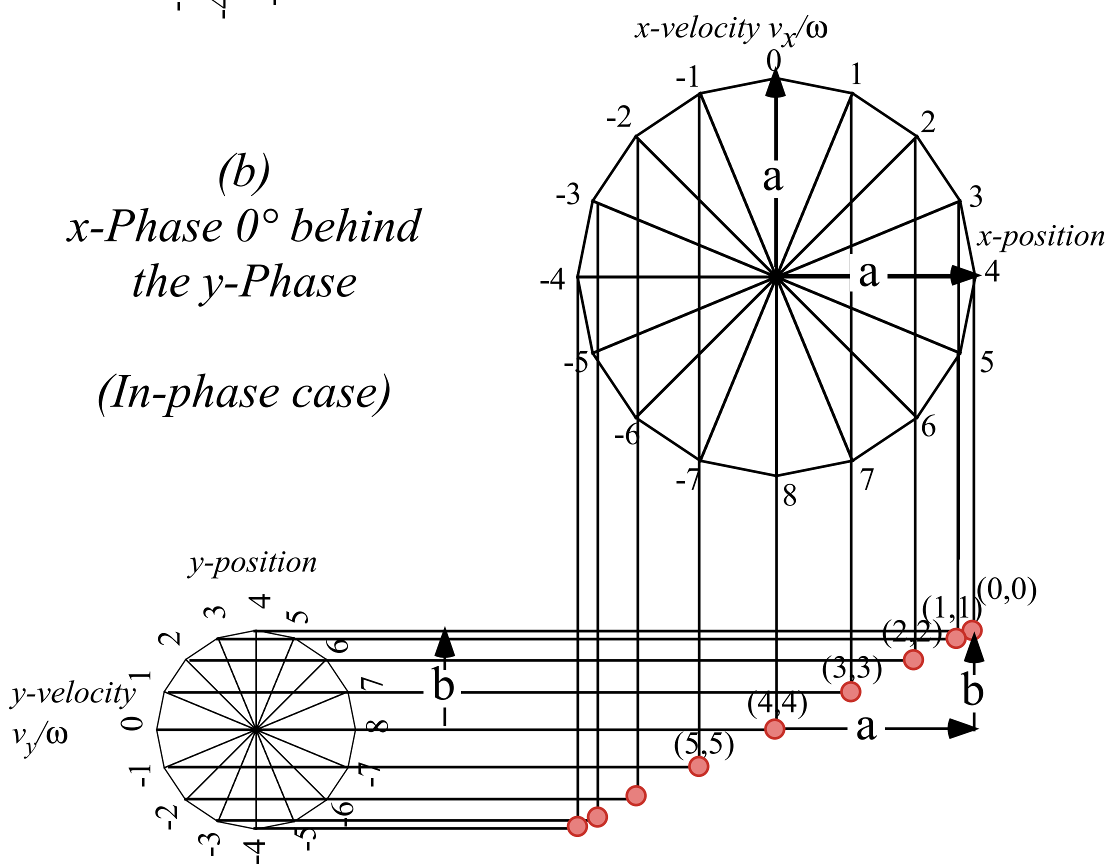
(a) Phasor Plots for 2-D Oscillator or Two 1D Oscillators (x-Phase 90° behind the y-Phase)



Unit 1
Fig. 9.12

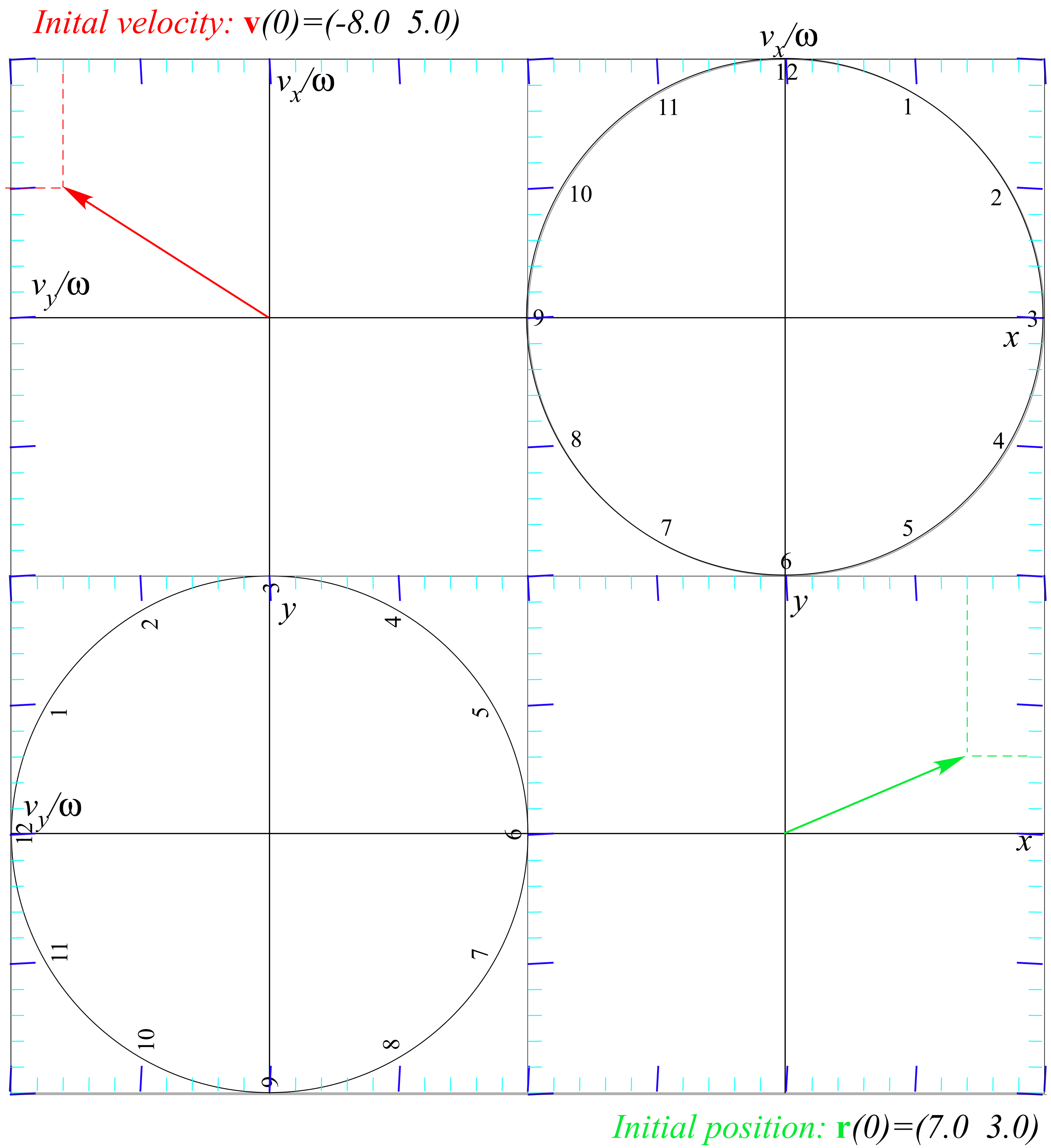
(b) x-Phase 0° behind the y-Phase

(In-phase case)



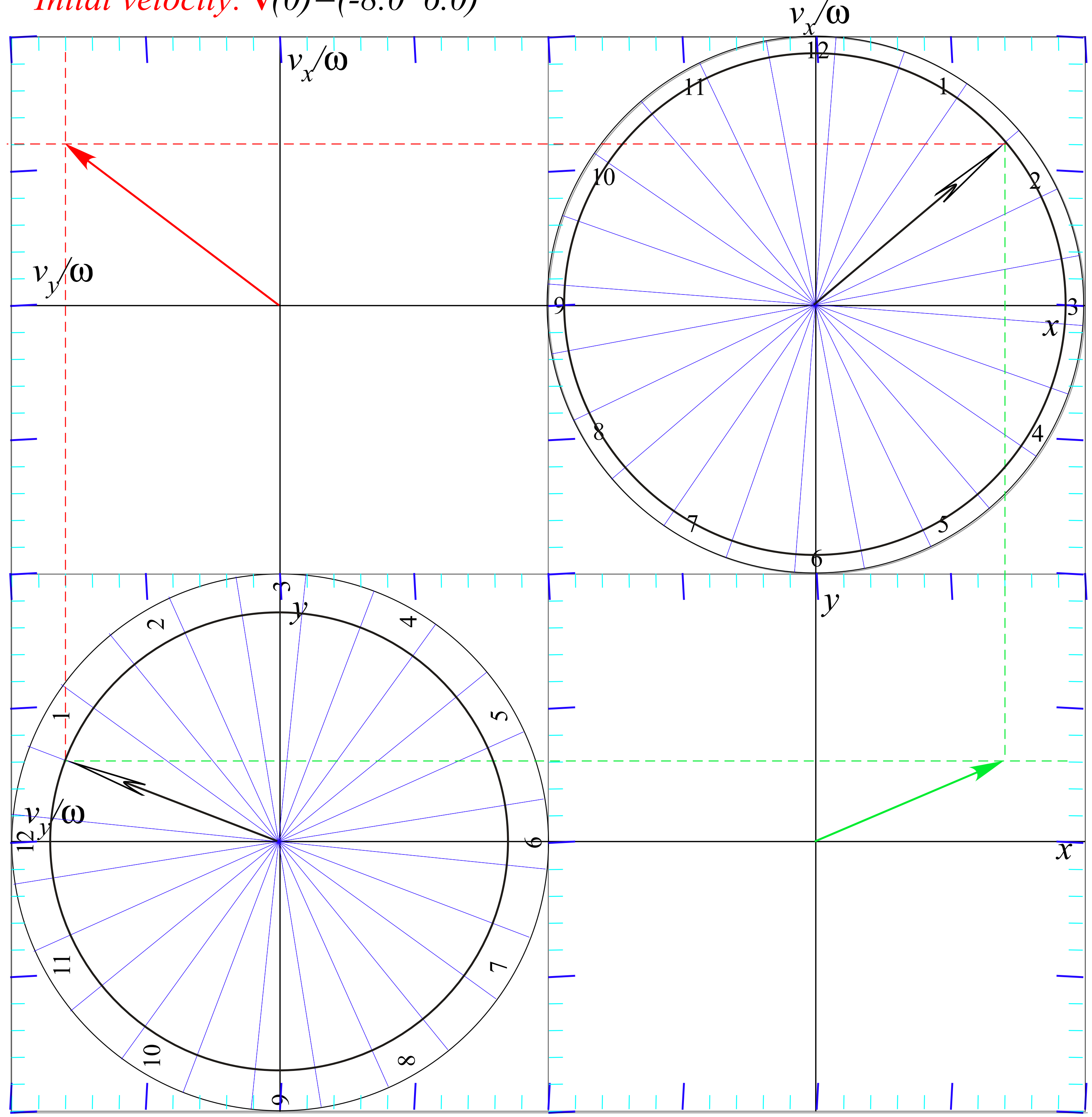
These are more generic examples with radius of x-phasor differing from that of the y-phasor.

Isotropic Harmonic Oscillator phase dynamics in uniform-body orbits
Dual phasor construction of elliptic oscillator orbits
→ *Integrating IHO equations by phasor geometry*



Link ⇒ [BoxIt simulation of IHO orbits]

Inital velocity: **v***(0)=(-8.0 6.0)*



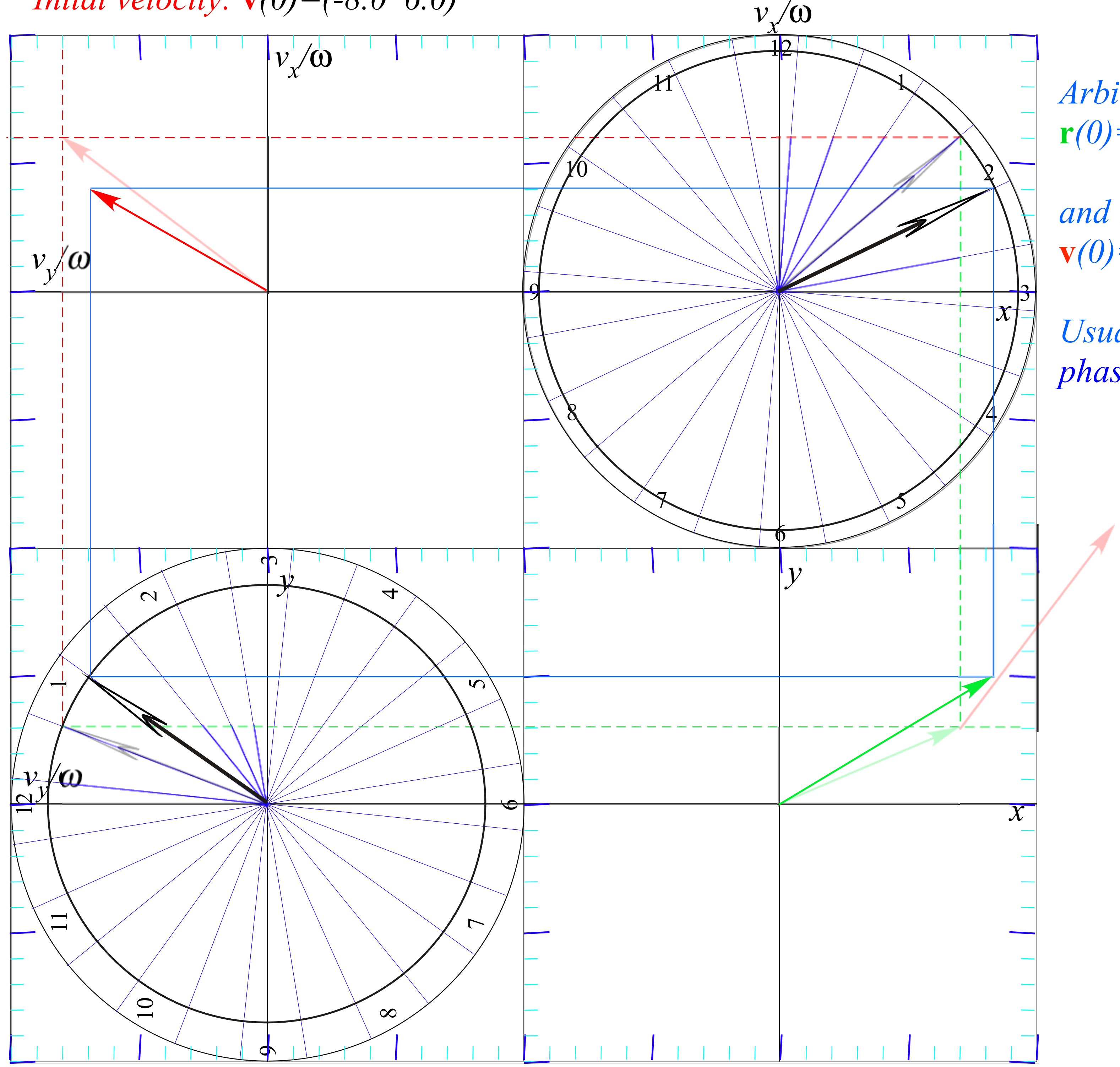
Initial position: **r***(0)=(7.0 3.0)*

Arbitrary initial position
r*(0)=(x(0),y(0))*

and initial velocity
v*(0)=(v_x(0),v_y(0))*

Usually have x and y
phasor circles of unequal size

Inital velocity: **v**(0)=(-8.0 6.0)



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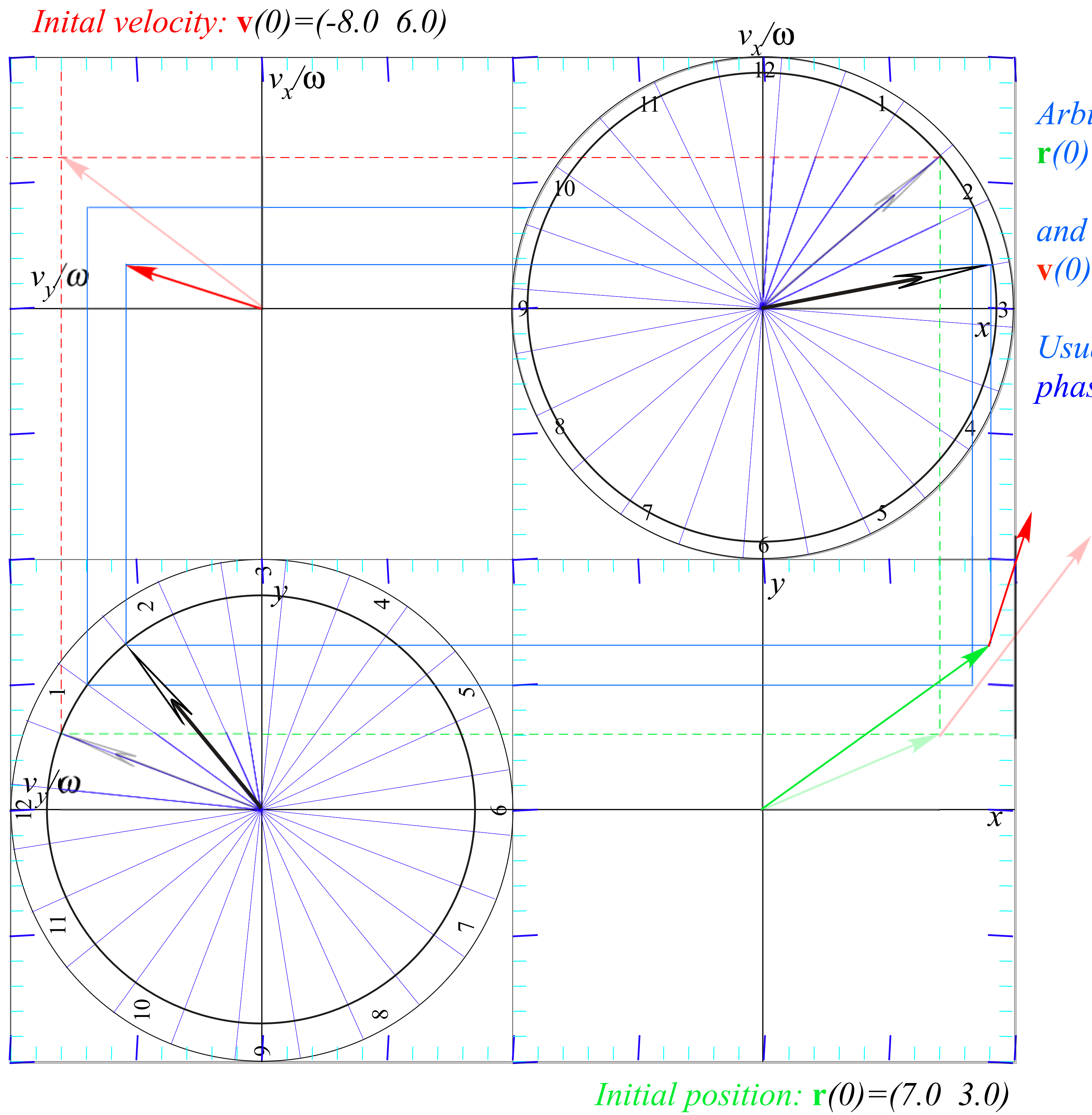
v(0)=(v_x(0),v_y(0))

Usually have x and y phasor circles of unequal size

Initial position: **r**(0)=(7.0 3.0)

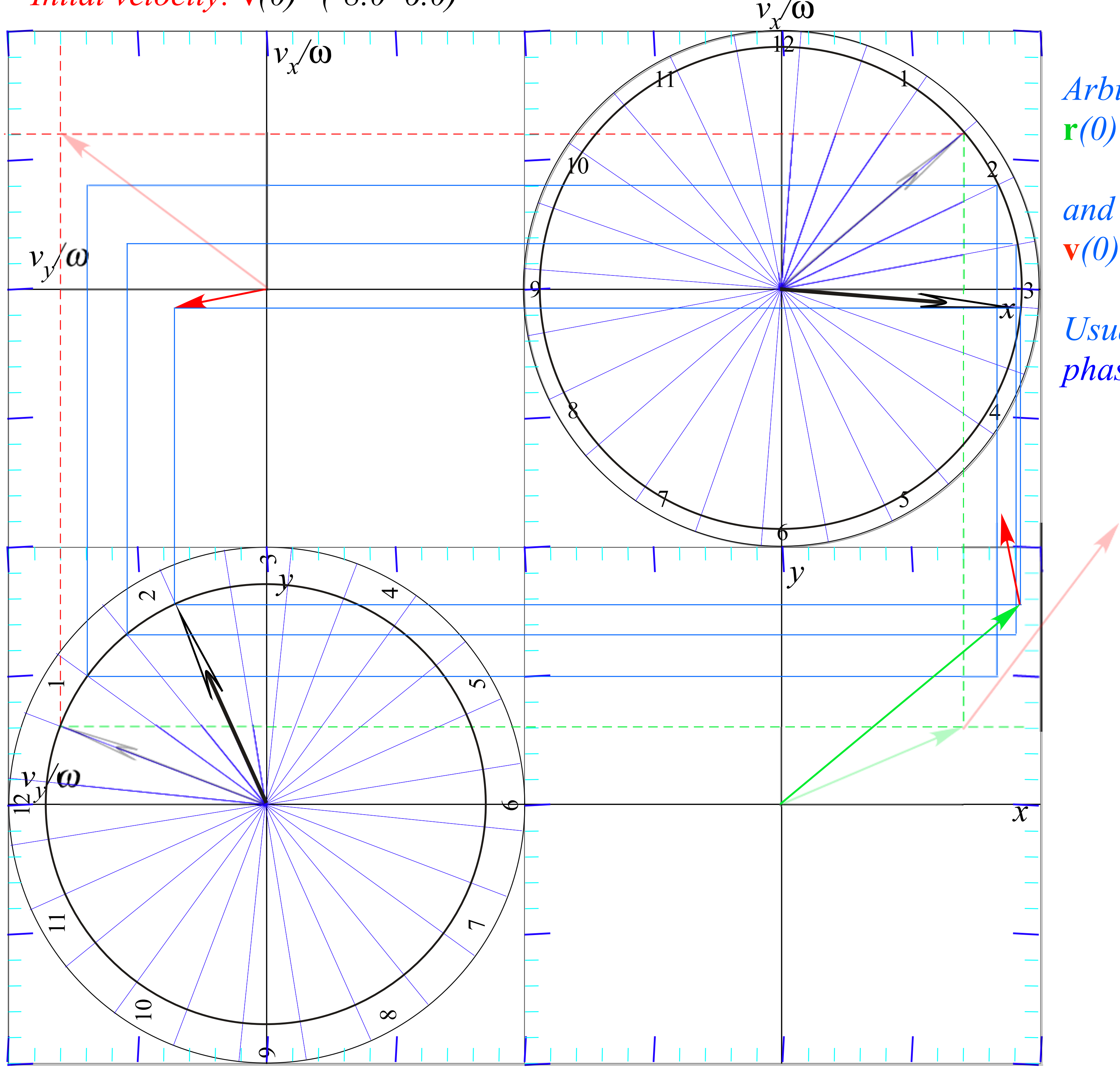


Arbitrary initial position
$\mathbf{r}(0)=(x(0),y(0))$

and initial velocity
$\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y phasor circles of unequal size

Inital velocity: **v***(0)=(-8.0 6.0)*



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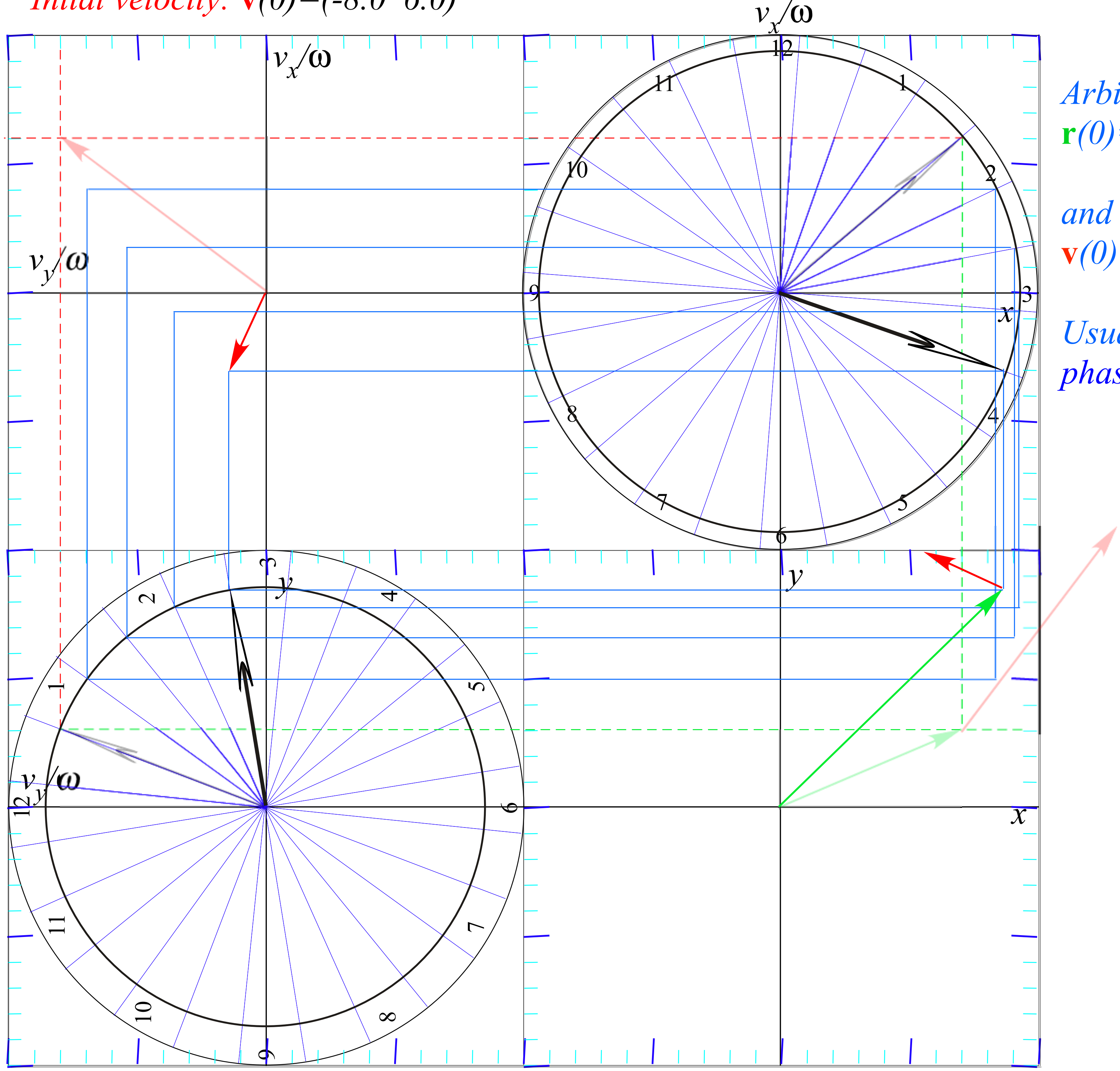

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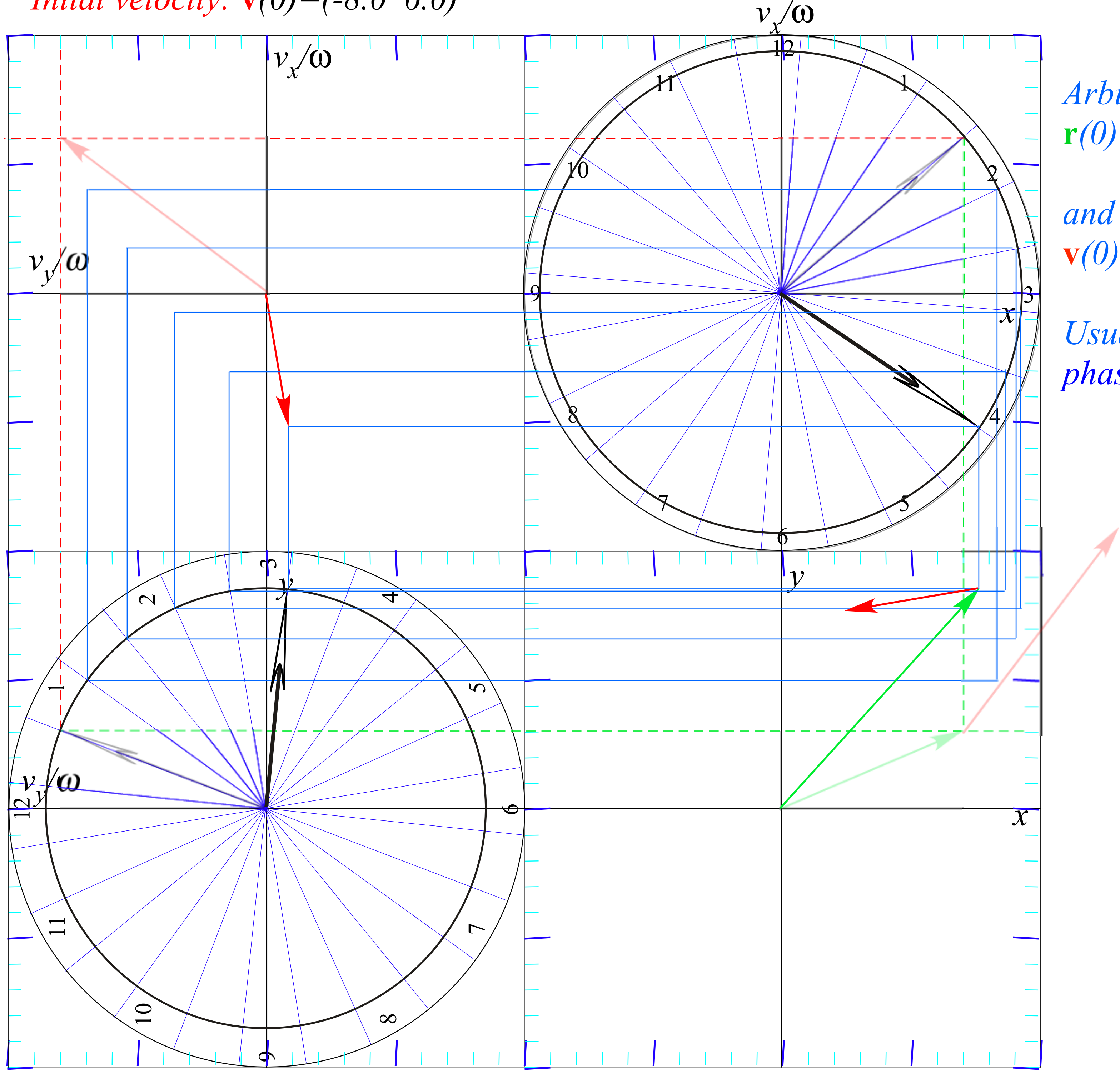

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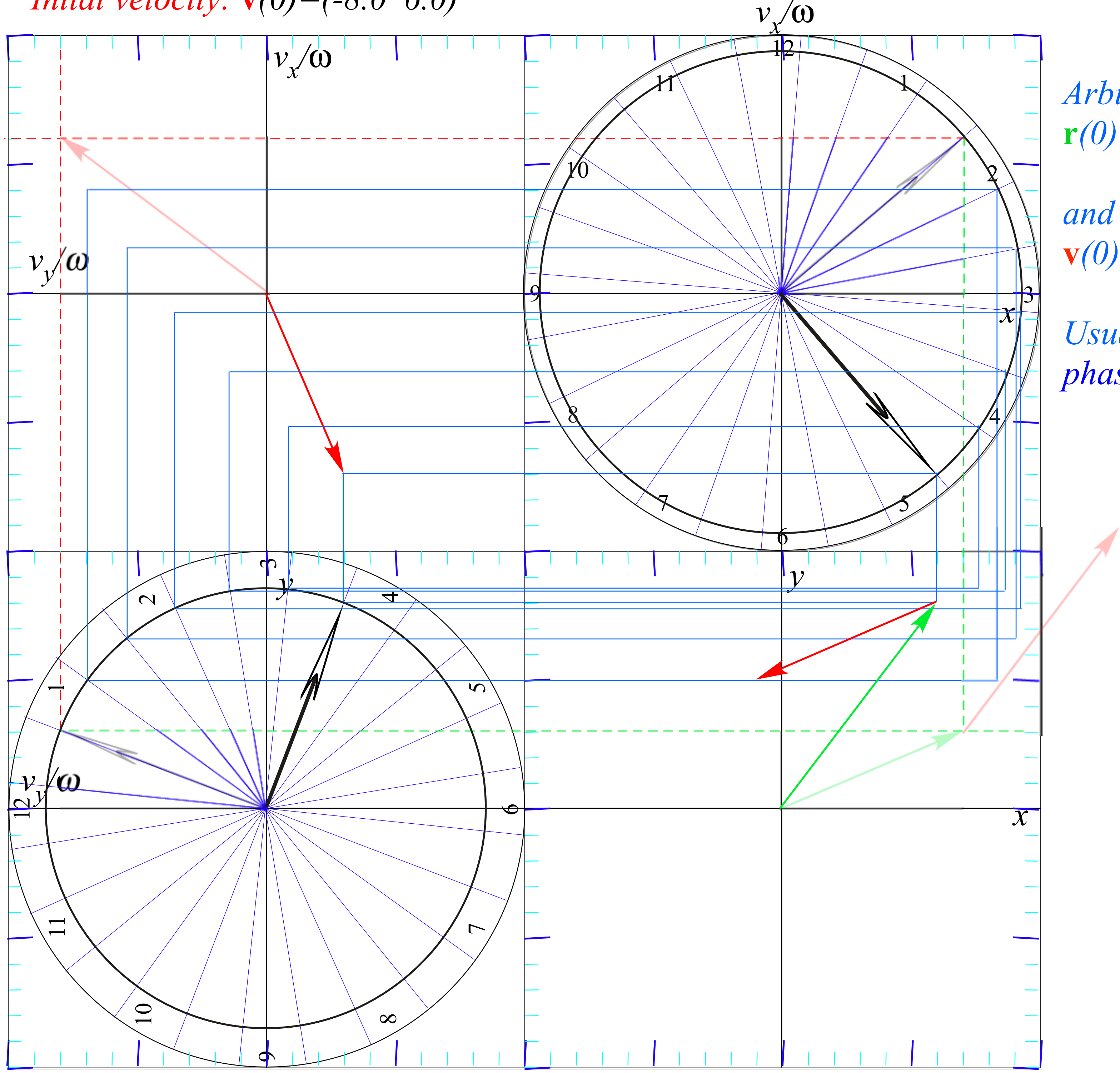

Arbitrary initial position
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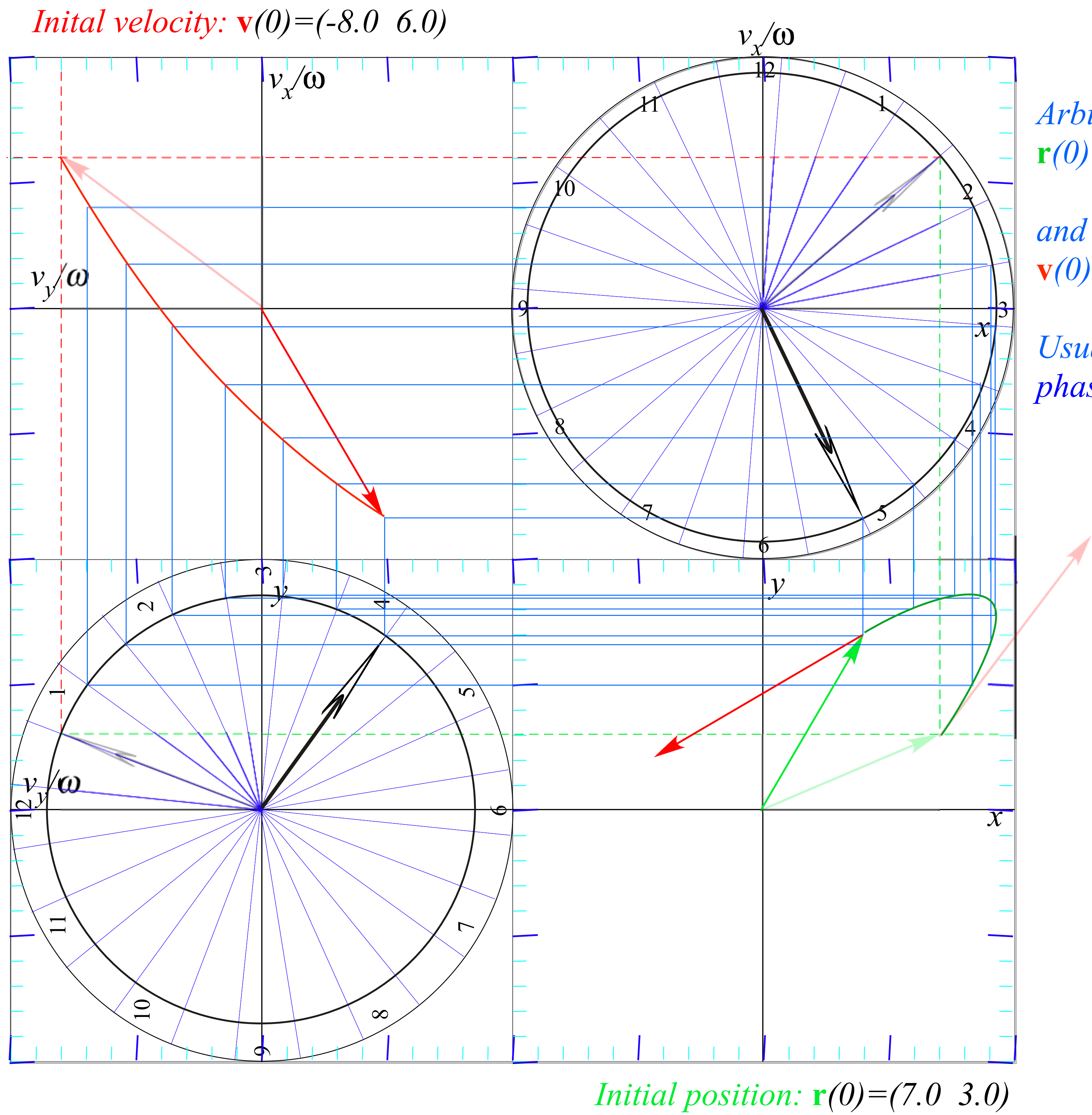

Arbitrary initial position **r**(0)=(x(0),y(0))

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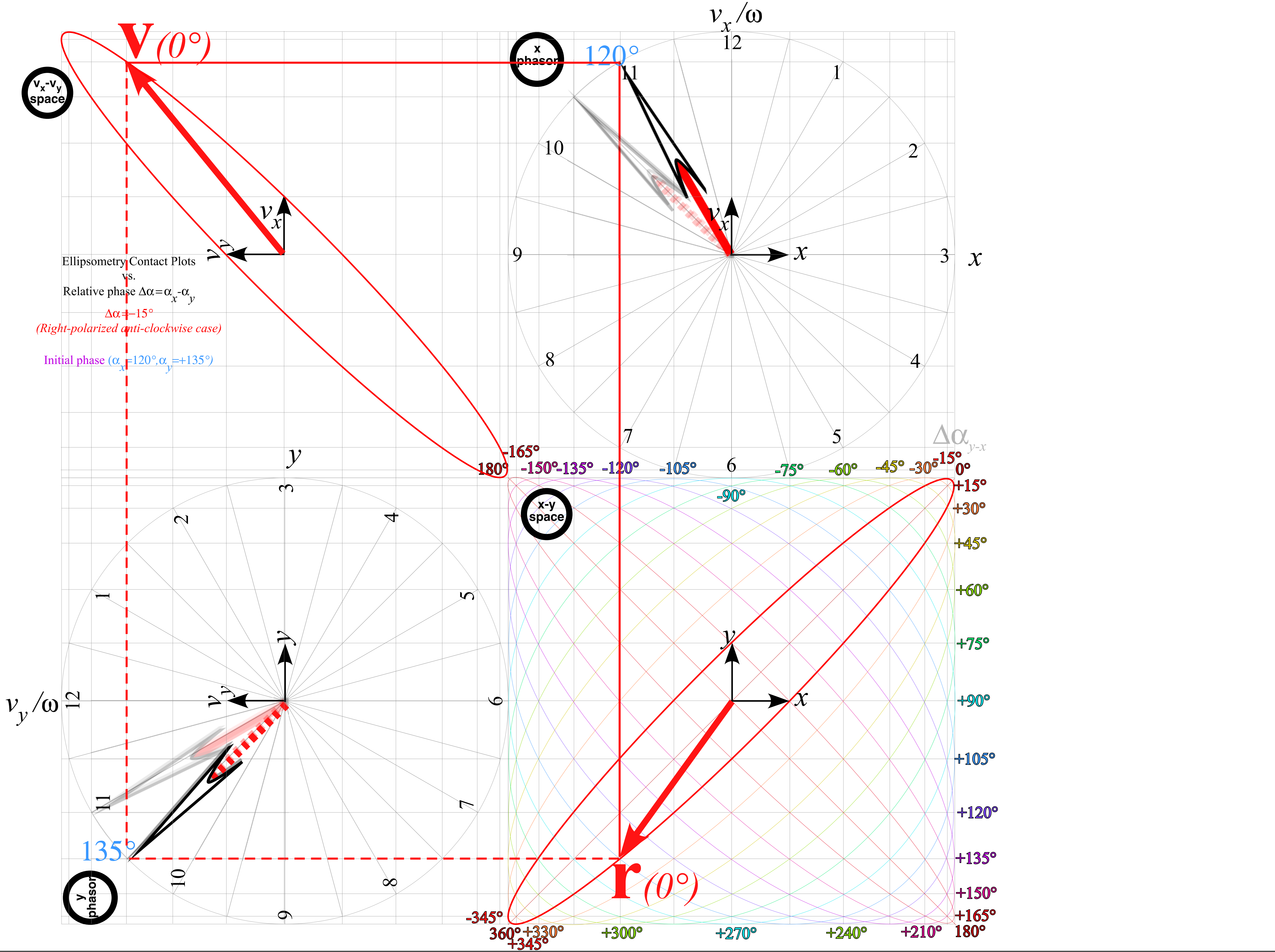
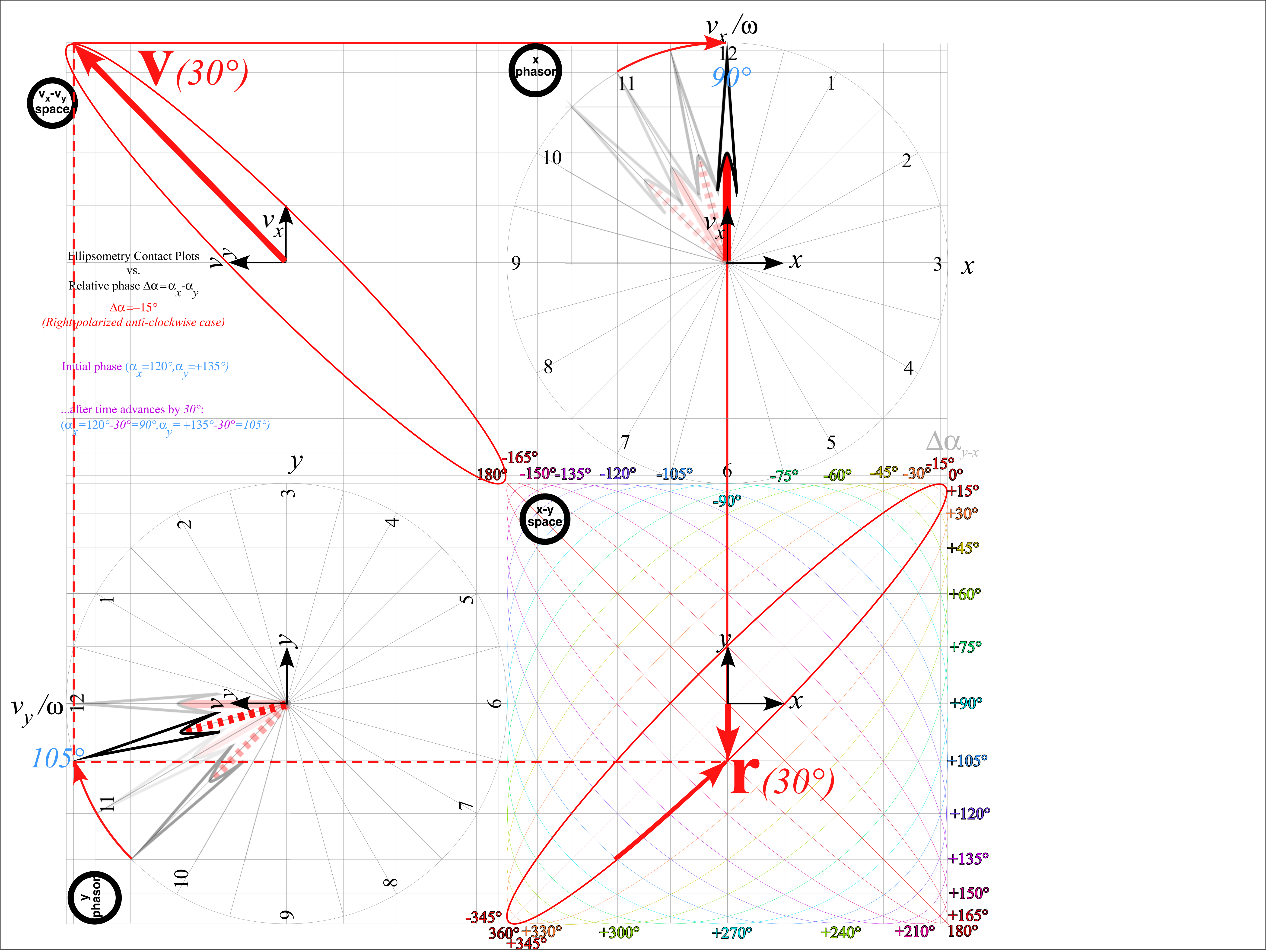
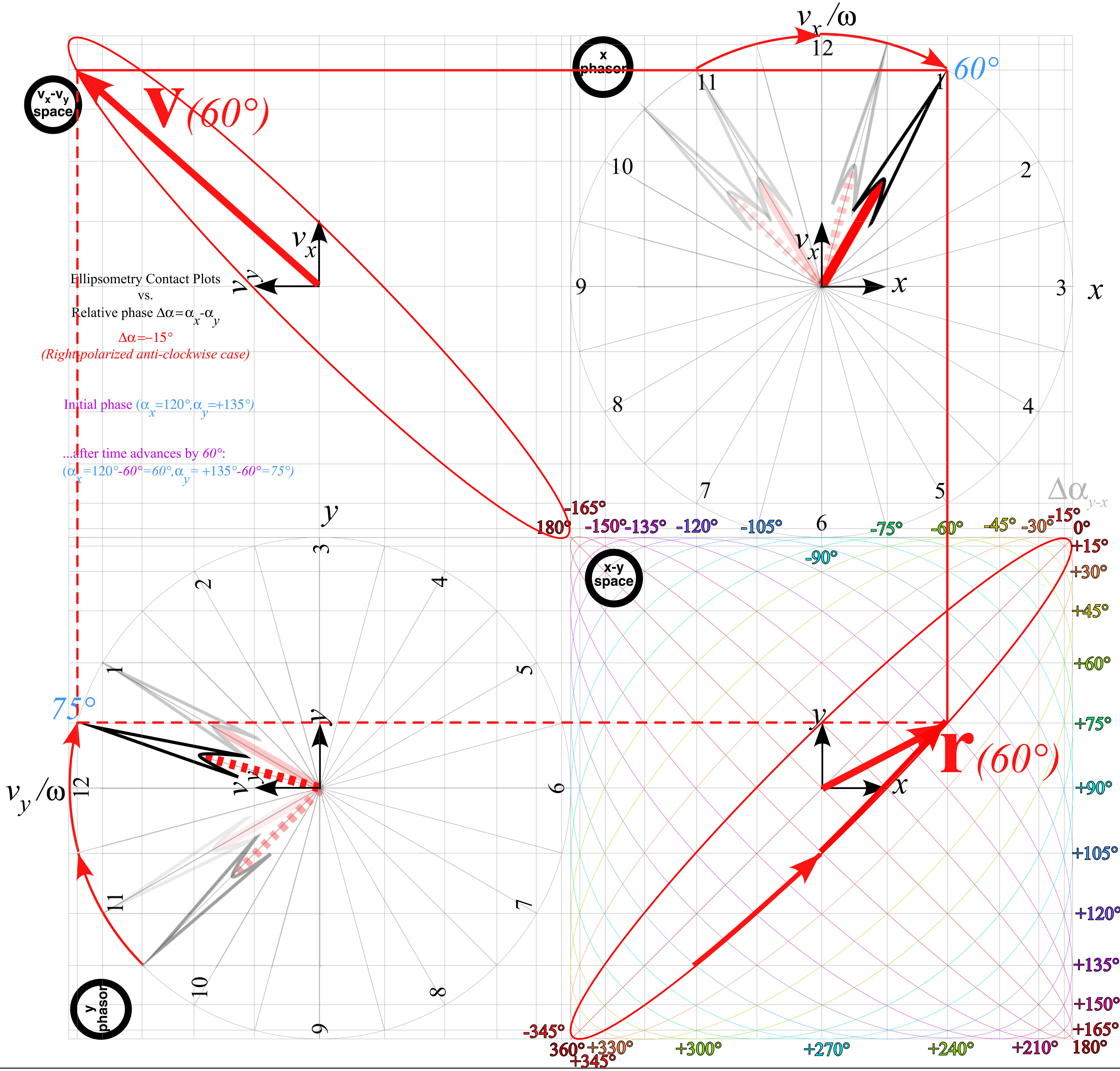
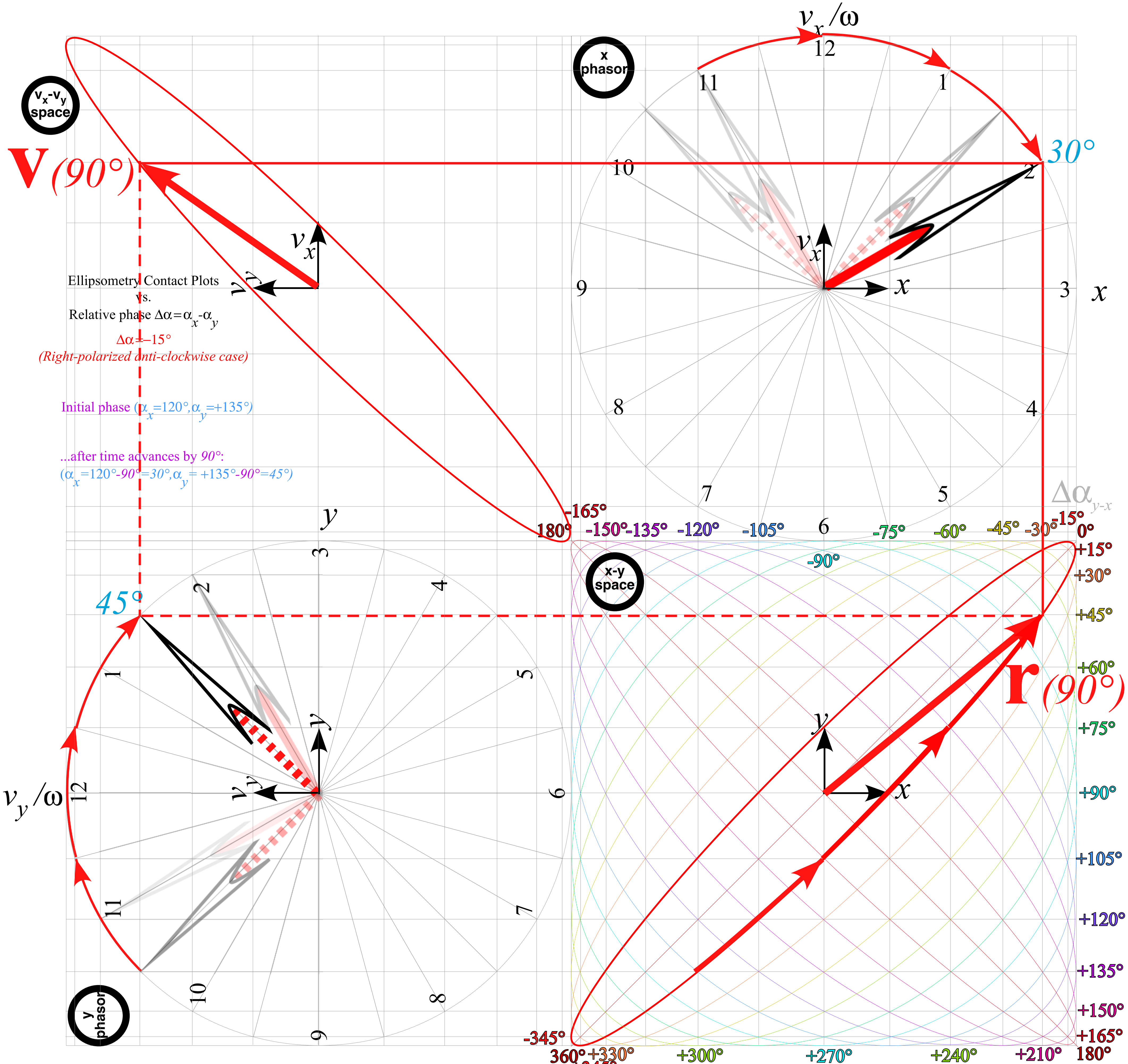


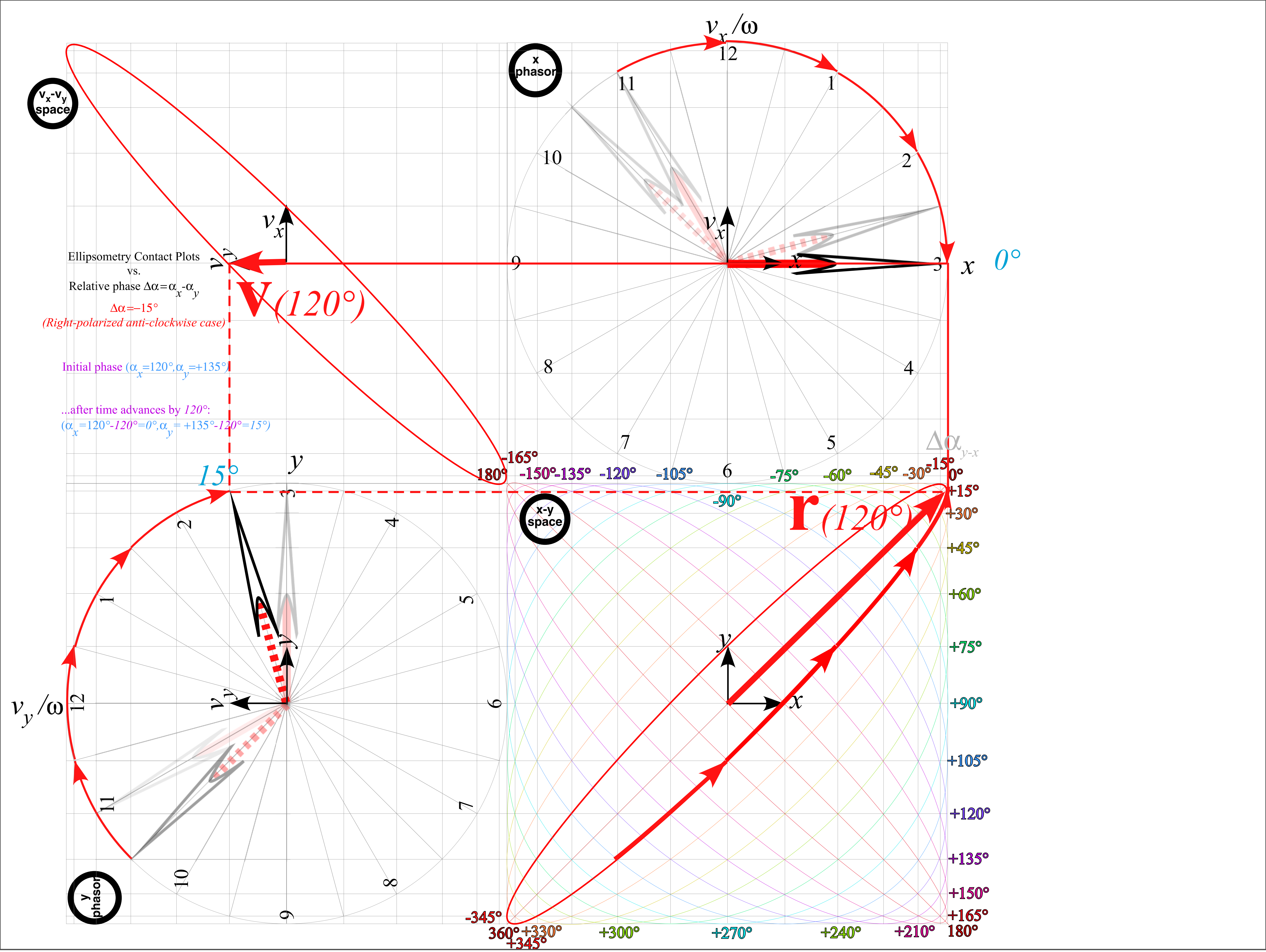
V(0°)
v_x-v_y space
x phasor
120°
v_x/ω
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°$, $\alpha_y=+135°$)
x-y space
y phasor
135°
v_y/ω
r(0°)
$\Delta\alpha_{y-x}$


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y= +135°-30°=105°)
V(30°)
r(30°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
90°
105°
Δα_y-x
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°


Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right polarized anti-clockwise case)
Initial phase ($\alpha_x=120°$, $\alpha_y=+135°$)
...after time advances by 60°:
($\alpha_x=120°-60°=60°$, $\alpha_y=+135°-60°=75°$)
v_x-v_y space
x phasor
y phasor
x-y space
V(60°)
r(60°)
v_x/ω
v_y/ω
60°
75°
$\Delta\alpha_{y-x}$
180°
-165°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
+150°
+165°
180°
-345°
360°
+345°
+330°
+300°
+270°
+240°
+210°



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 90°:
(α_x=120°-90°=30°,α_y = +135°-90°=45°)
V (90°)
r (90°)
30°
45°
v_x/ω
v_y/ω
x phasor
y phasor
v_x-v_y space
x-y space
Δα_y-x



Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 120°:
(αx=120°-120°=0°,αy= +135°-120°=15°)
vx-vy space
x phasor
y phasor
x-y space
vx/ω
vy/ω
V(120°)
r(120°)
0°
15°
Δαy-x
-165°
180°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
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180°
+210°
+240°
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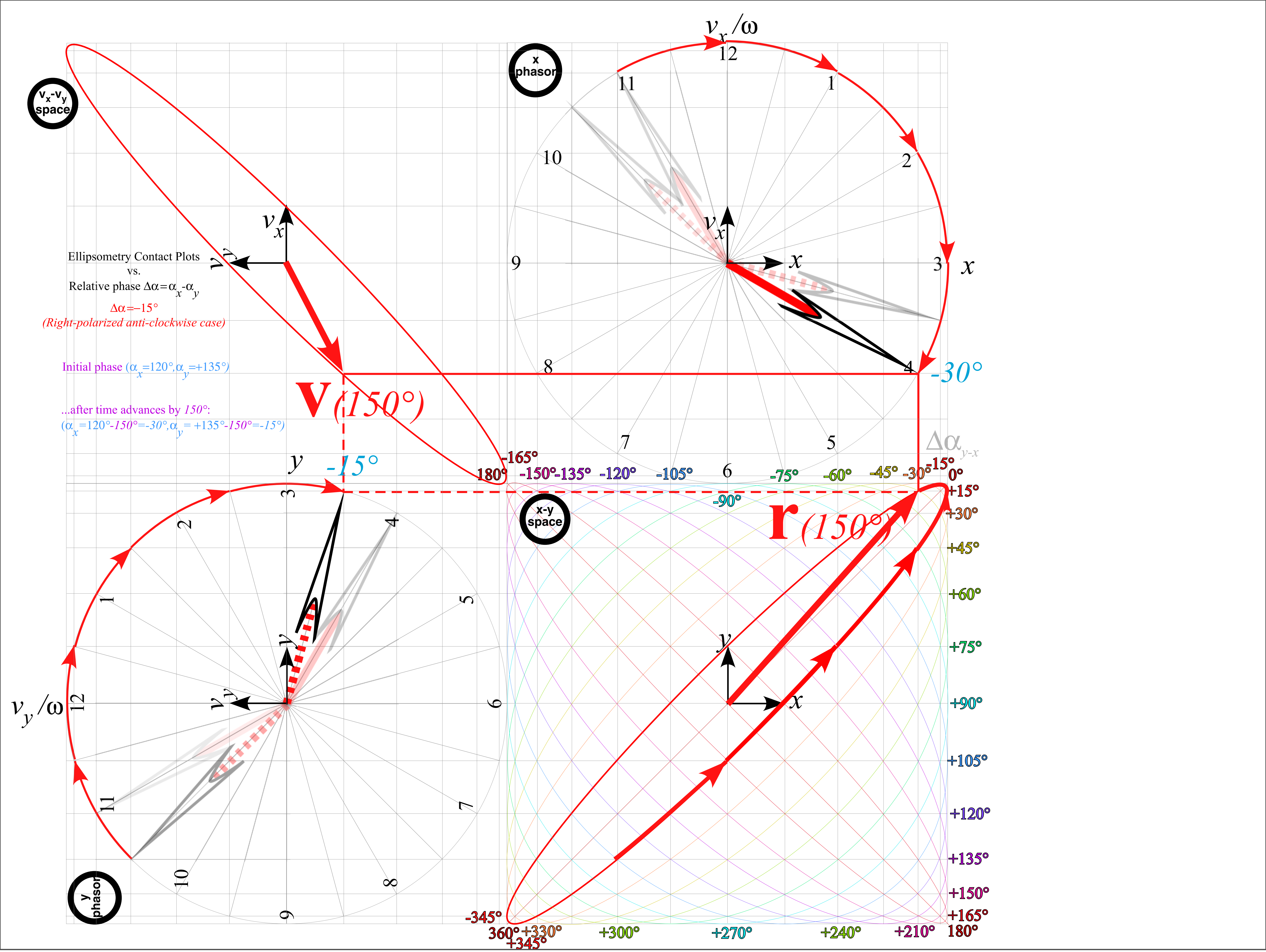
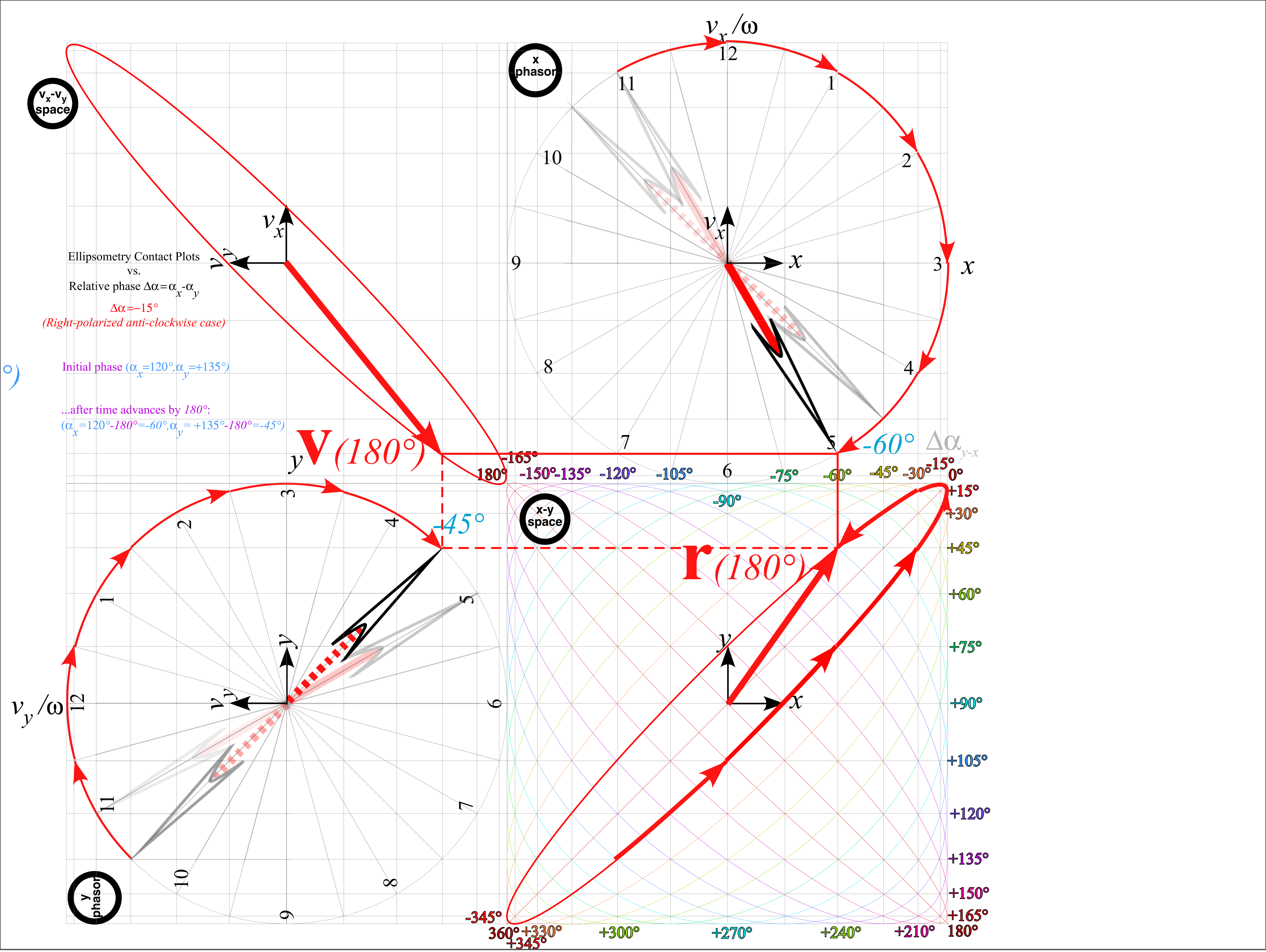
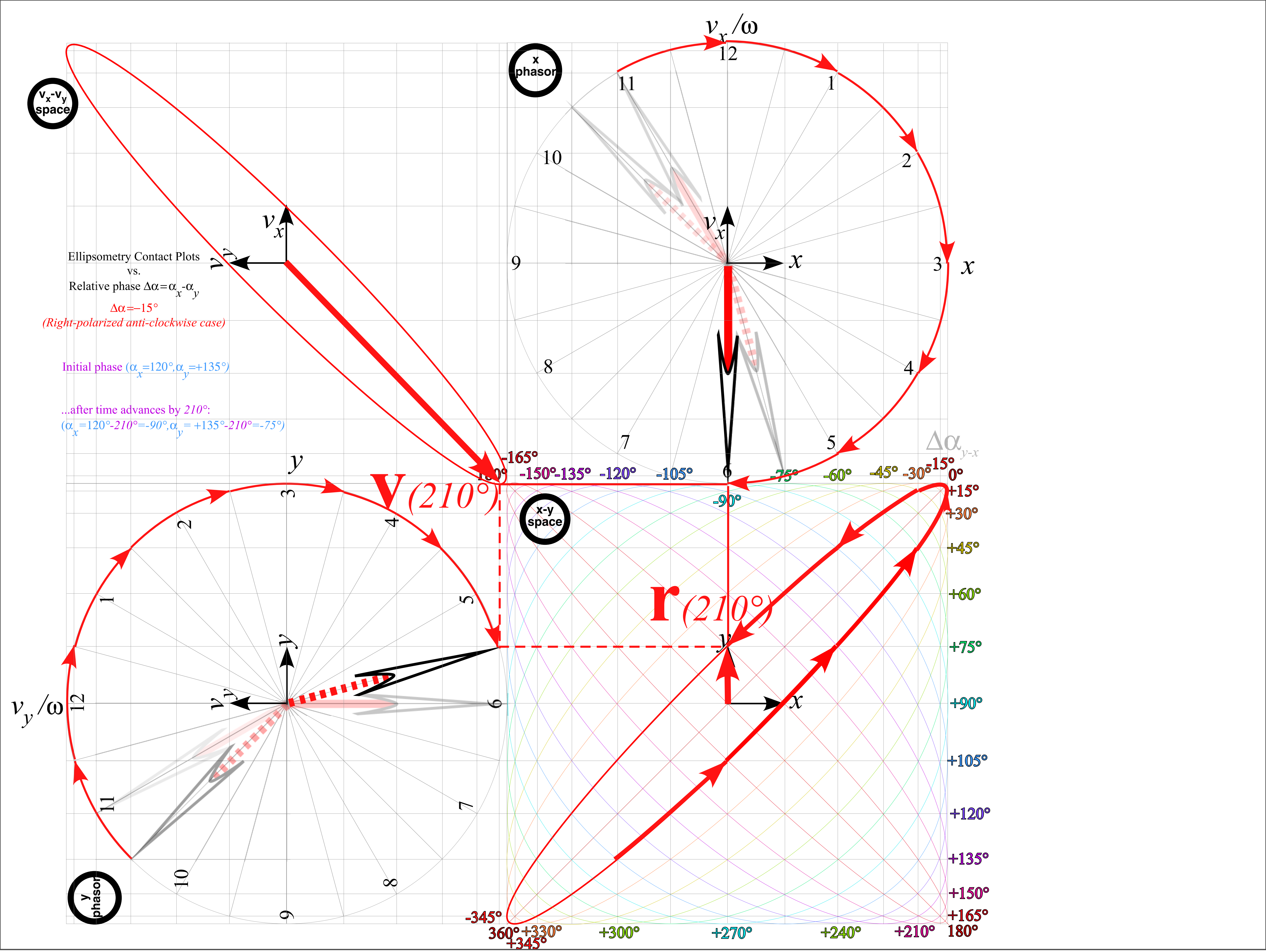
+300°
+330°
+345°
360°
-345°



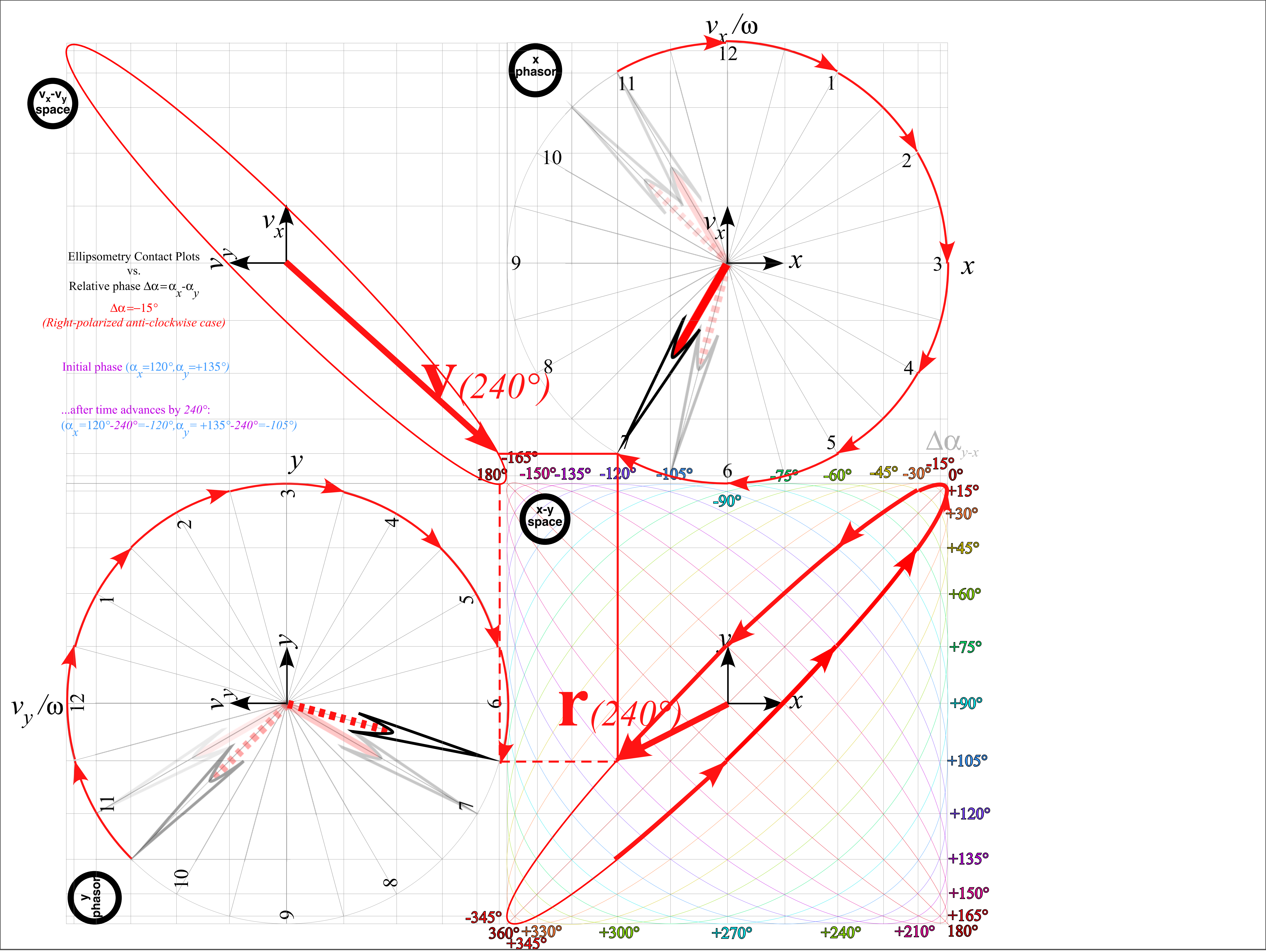
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 150°:
(α_x=120°-150°=-30°,α_y = +135°-150°=-15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(150°)
r(150°)
-30°
-15°
Δα_y-x

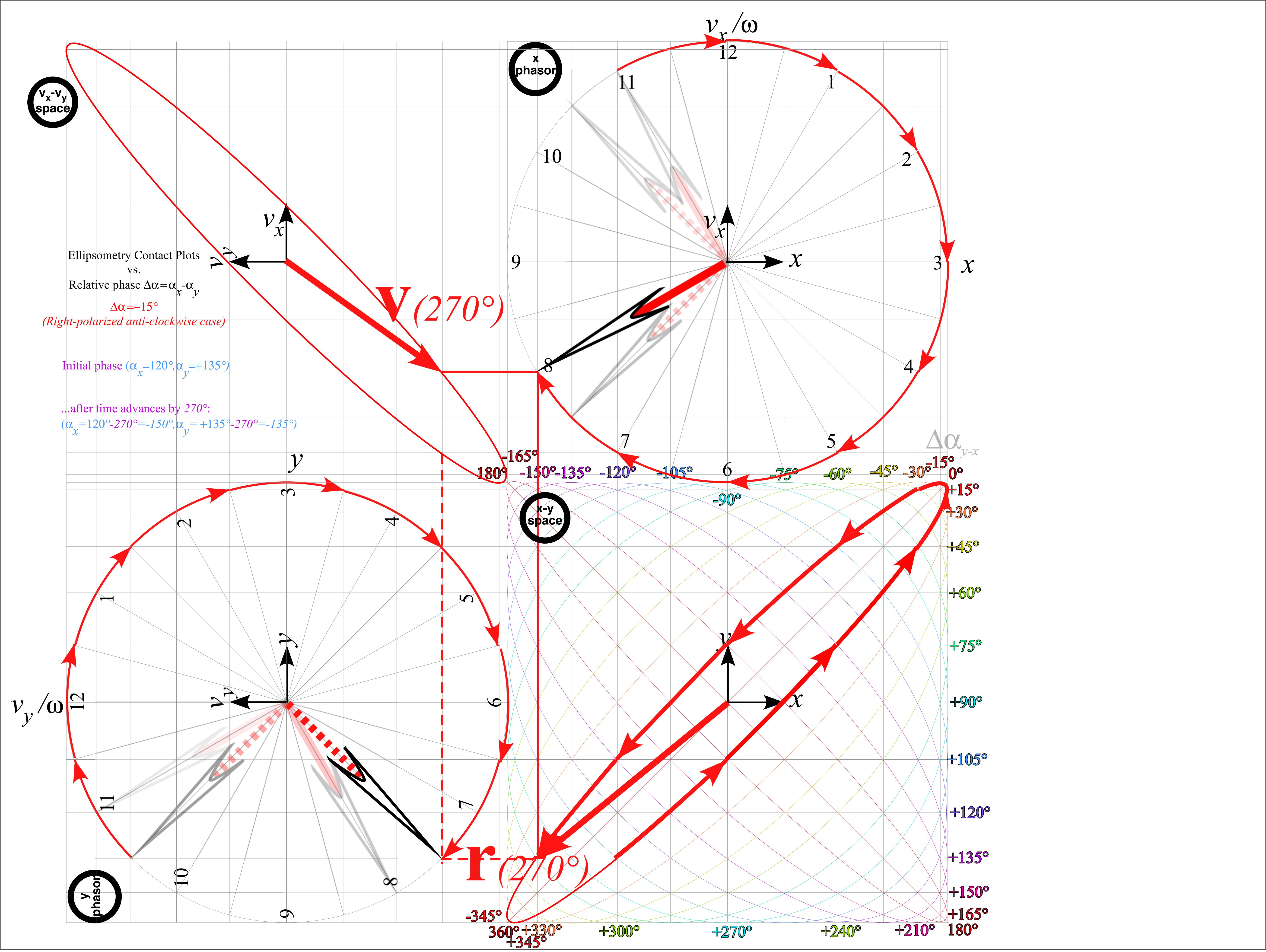


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y = +135°-180°=-45°)
v_x-v_y space
x phasor
y phasor
x-y space
V(180°)
r(180°)
v_x/ω
v_y/ω
-60°
-45°
Δα_y-x



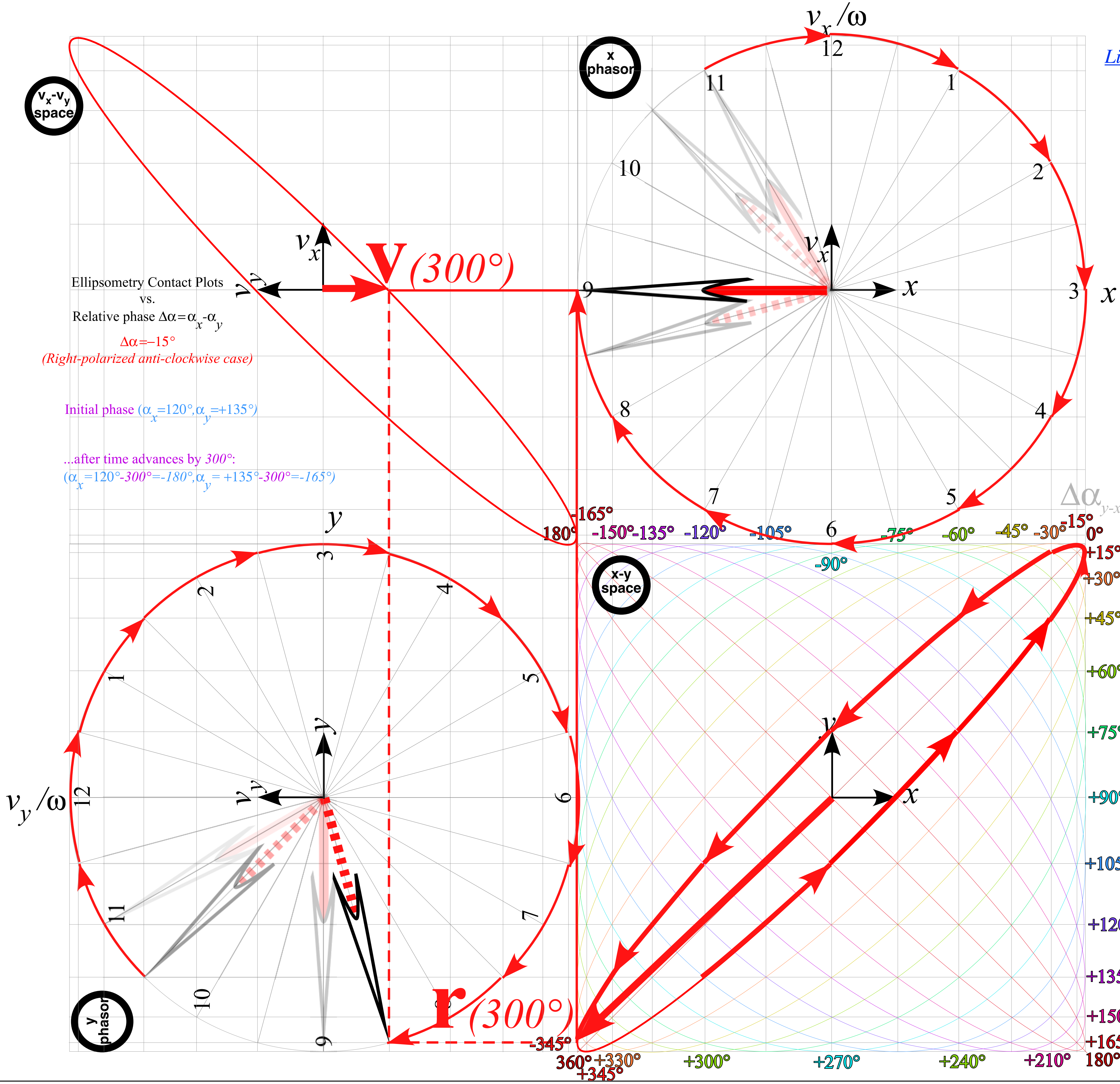
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 210°:
(α_x=120°-210°=-90°,α_y = +135°-210°=-75°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(210°)
r(210°)
Δα_y-x

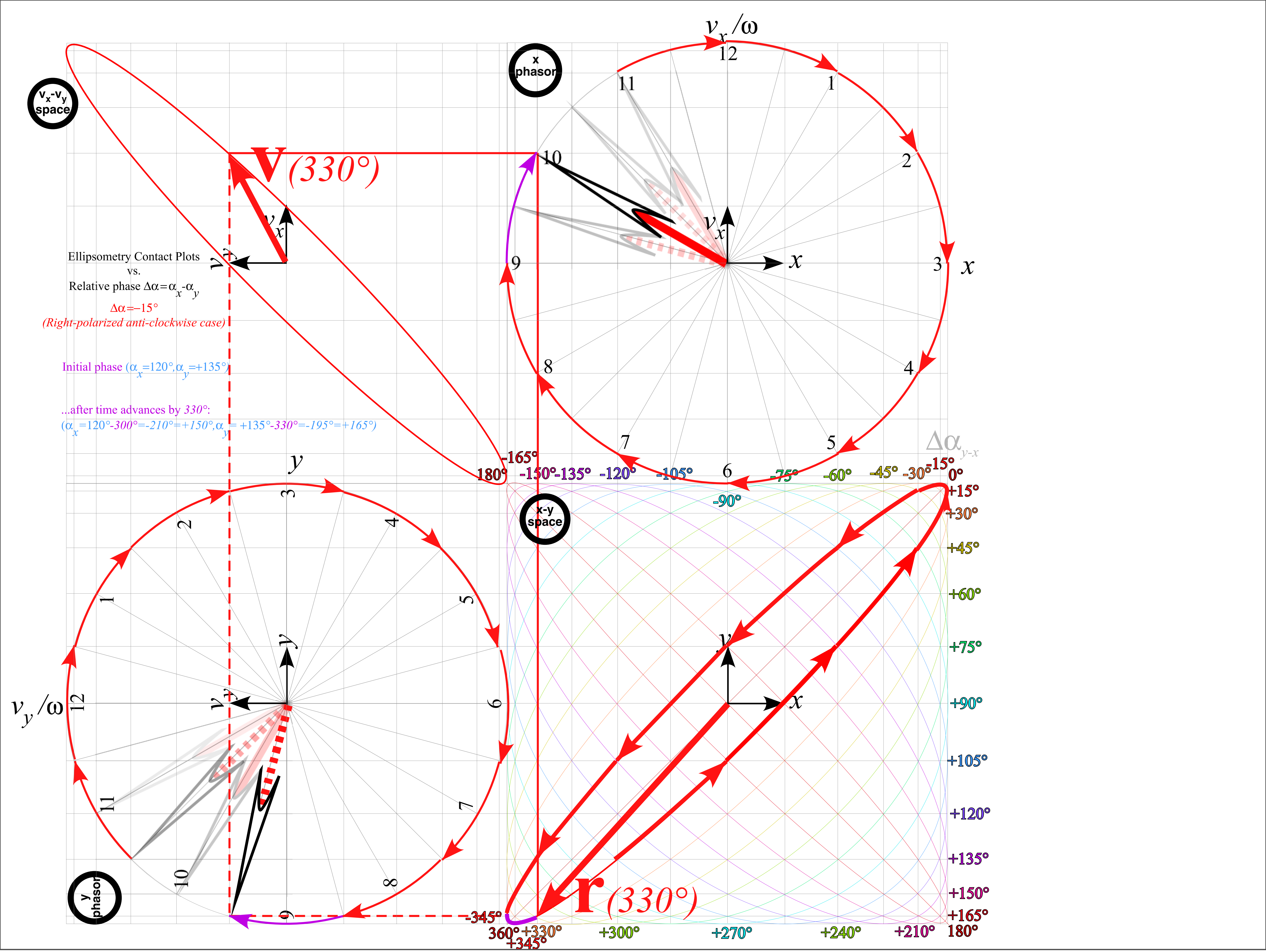

Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 240°:
(α_x=120°-240°=-120°,α_y= +135°-240°=-105°)
v_x-v_y space
x phasor
y phasor
x-y space
V(240°)
r(240°)
v_x/ω
v_y/ω
Δα_y-x


Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 270°:
(α_x=120°-270°=-150°,α_y = +135°-270°=-135°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
v(270°)
r(270°)
$\Delta\alpha_{y-x}$

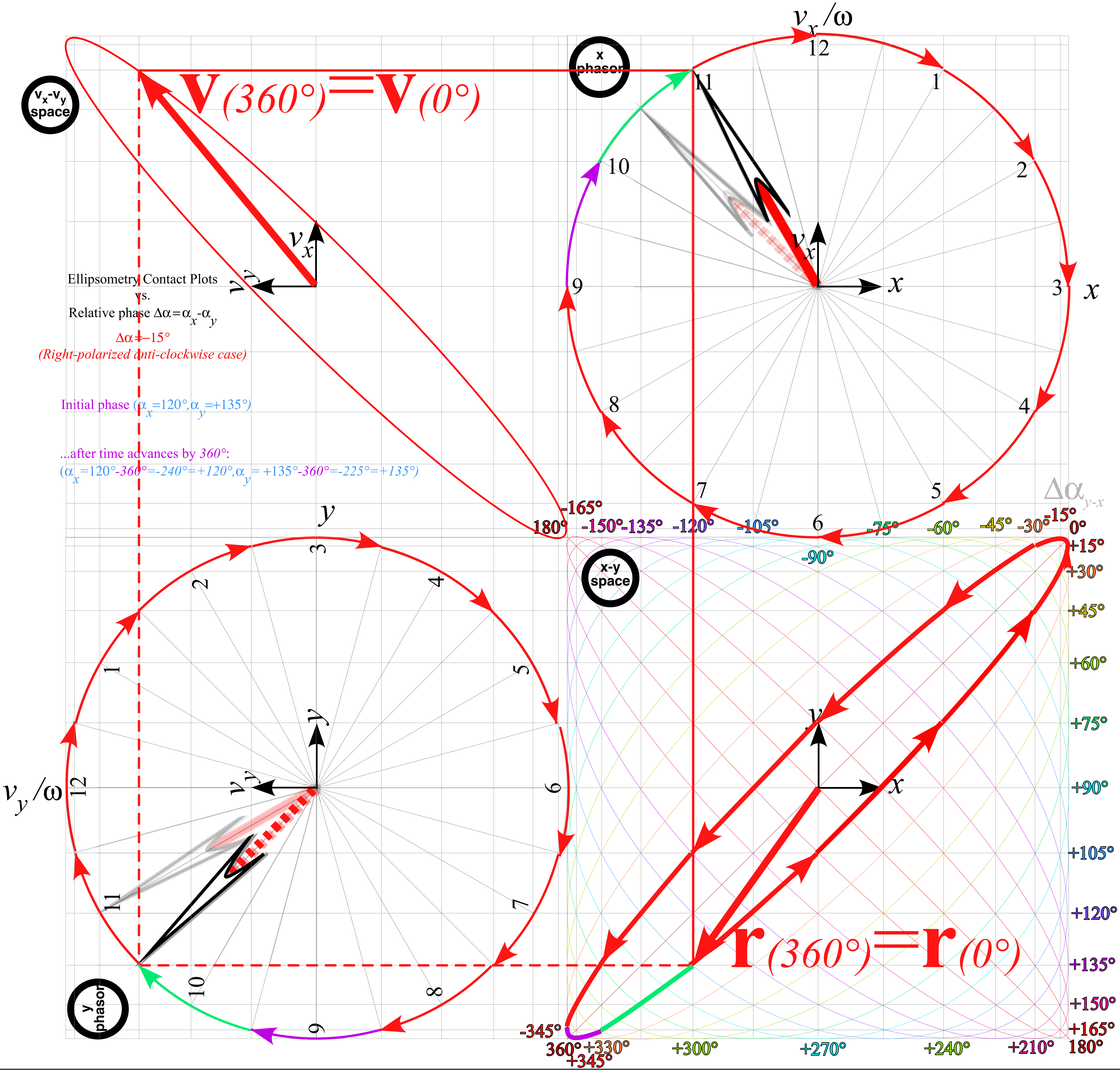
[Link → IHO orbital time rates of change]

[Link → IHO Exegesis Plot]

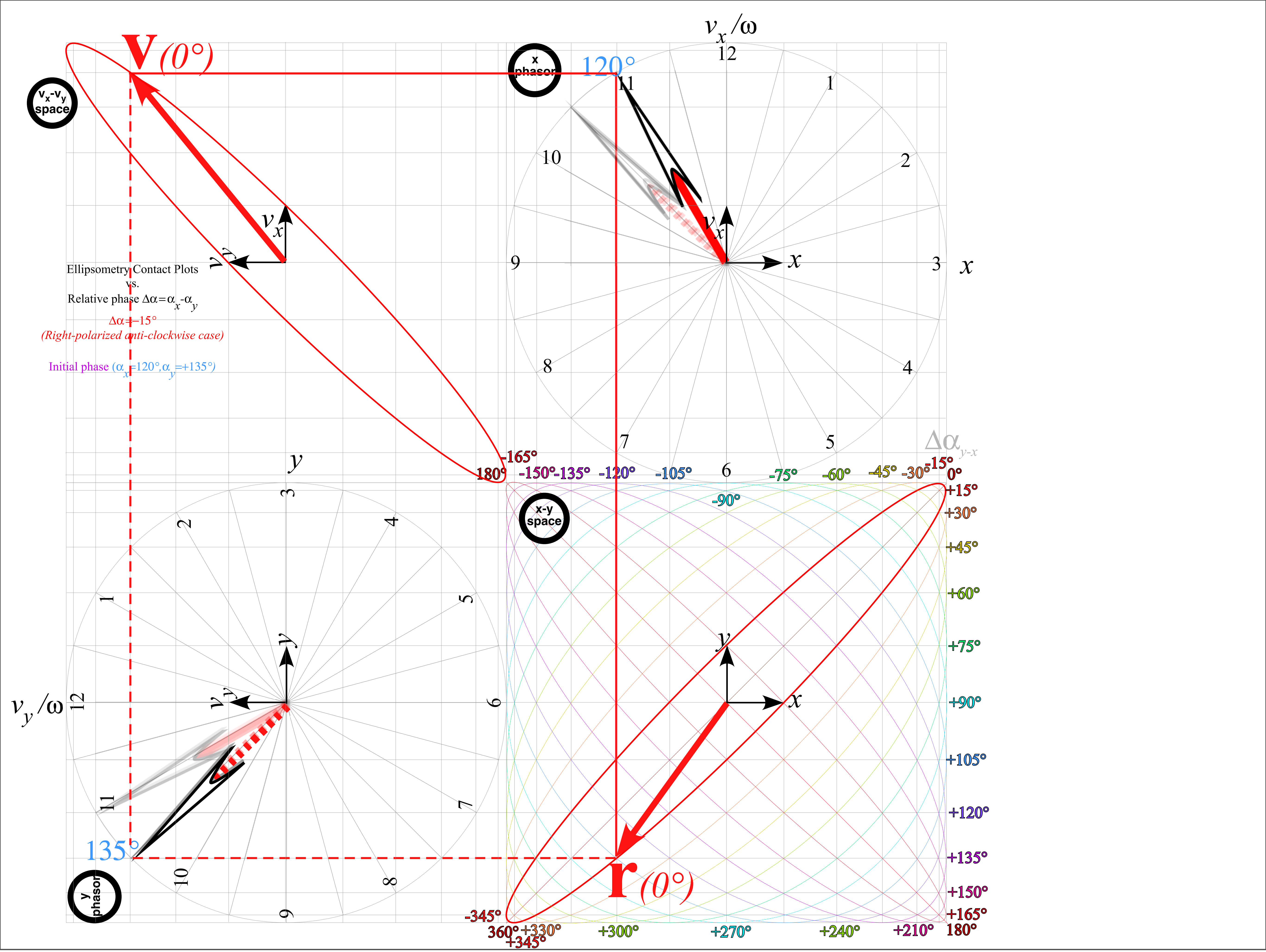




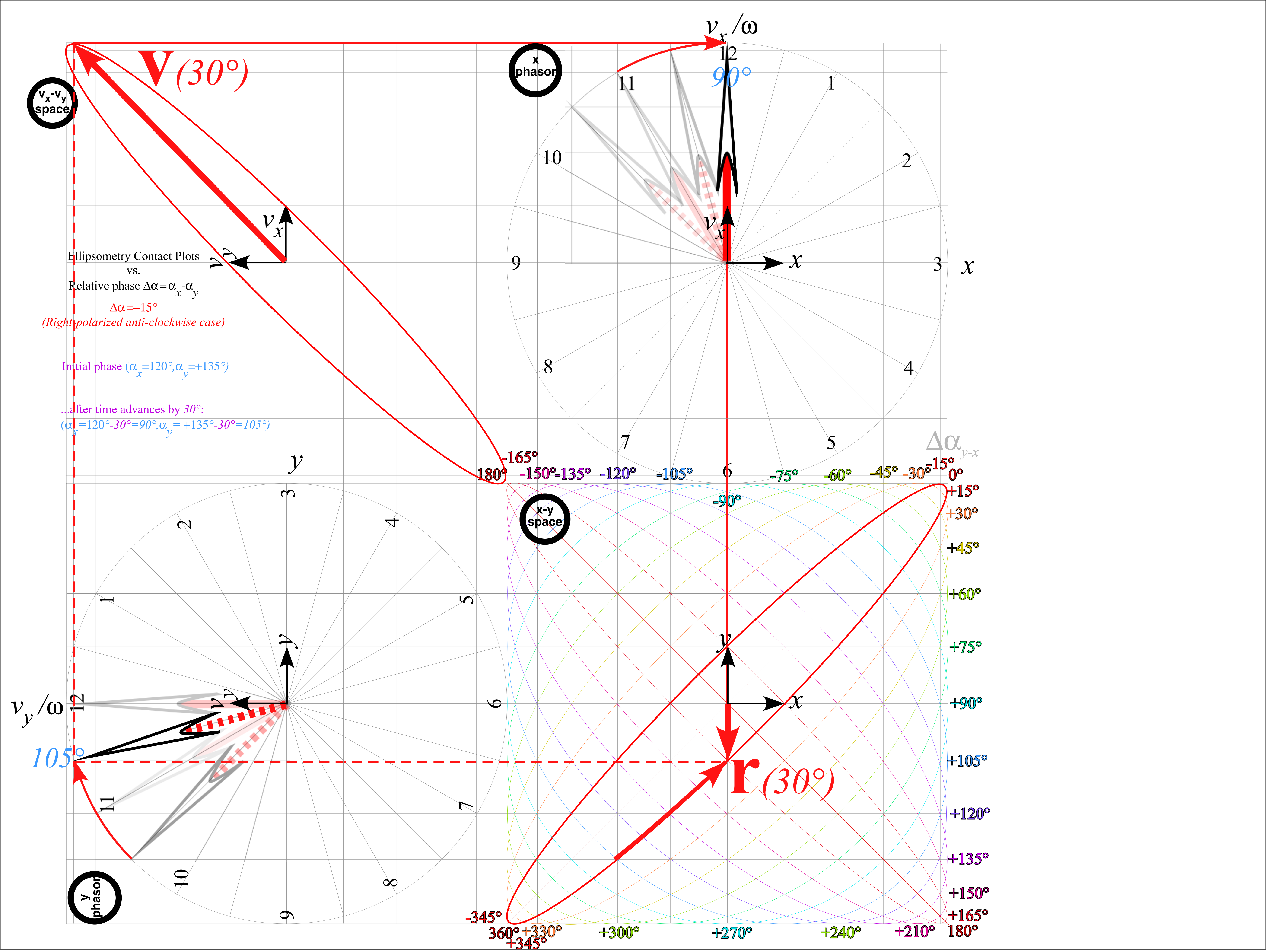
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 330°:
(αx=120°-300°=-210°=+150°,αy= +135°-330°=-195°=+165°)
vx-vy space
x phasor
y phasor
x-y space
V(330°)
r(330°)
vx/ω
vy/ω
Δαy-x
0°
-15°
-30°
-45°
-60°
-75°
-90°
-105°
-120°
-135°
-150°
-165°
180°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
+150°
+165°
180°
+210°
+240°
+270°
+300°
+330°
+345°
360°
-345°

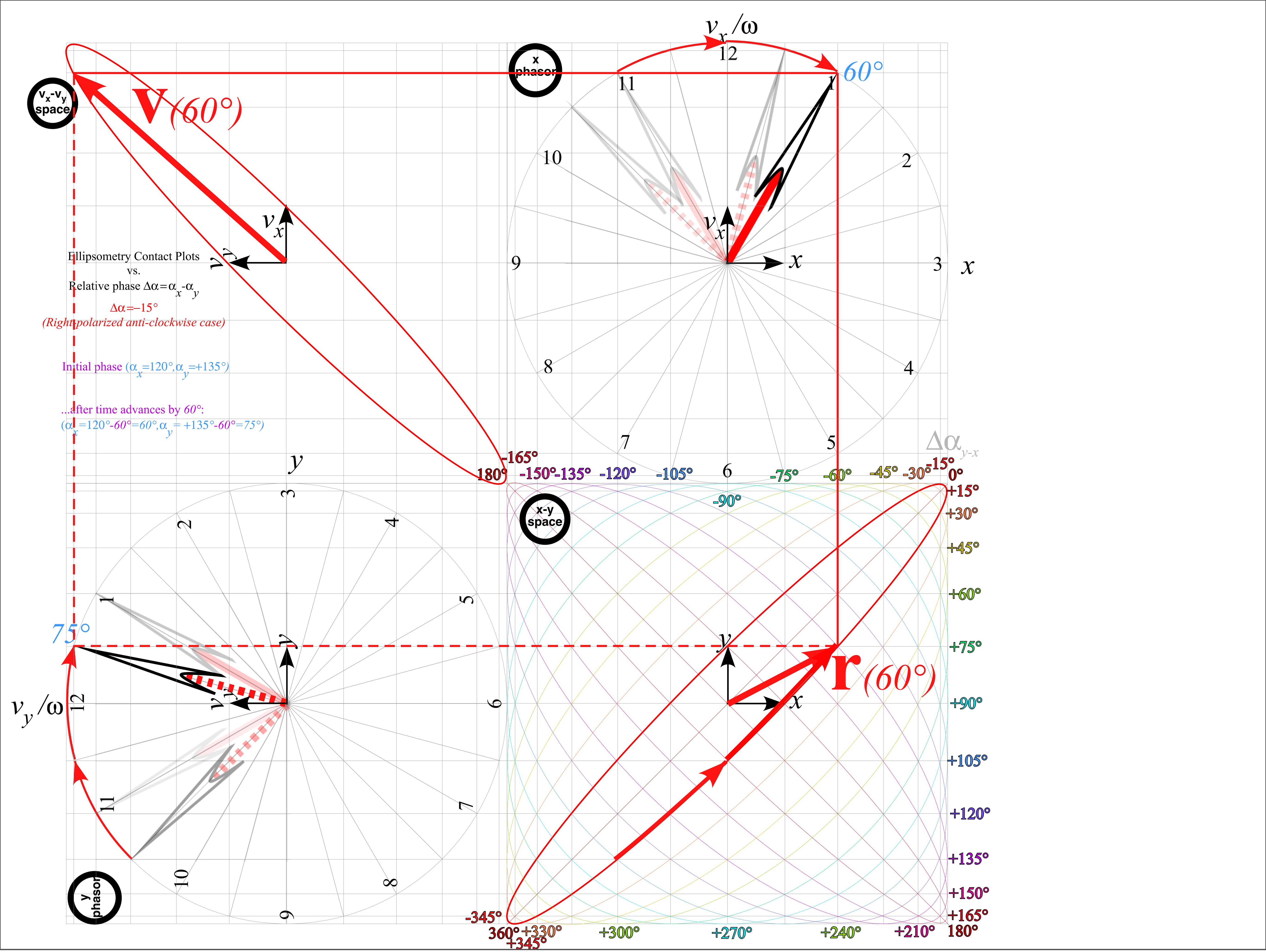


vx-vy space
V(360°)=V(0°)
x phasor
vx/ω
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 360°:
(αx=120°-360°=-240°=+120°,αy=+135°-360°=-225°=+135°)
x-y space
y phasor
vy/ω
r(360°)=r(0°)
Δαy-x

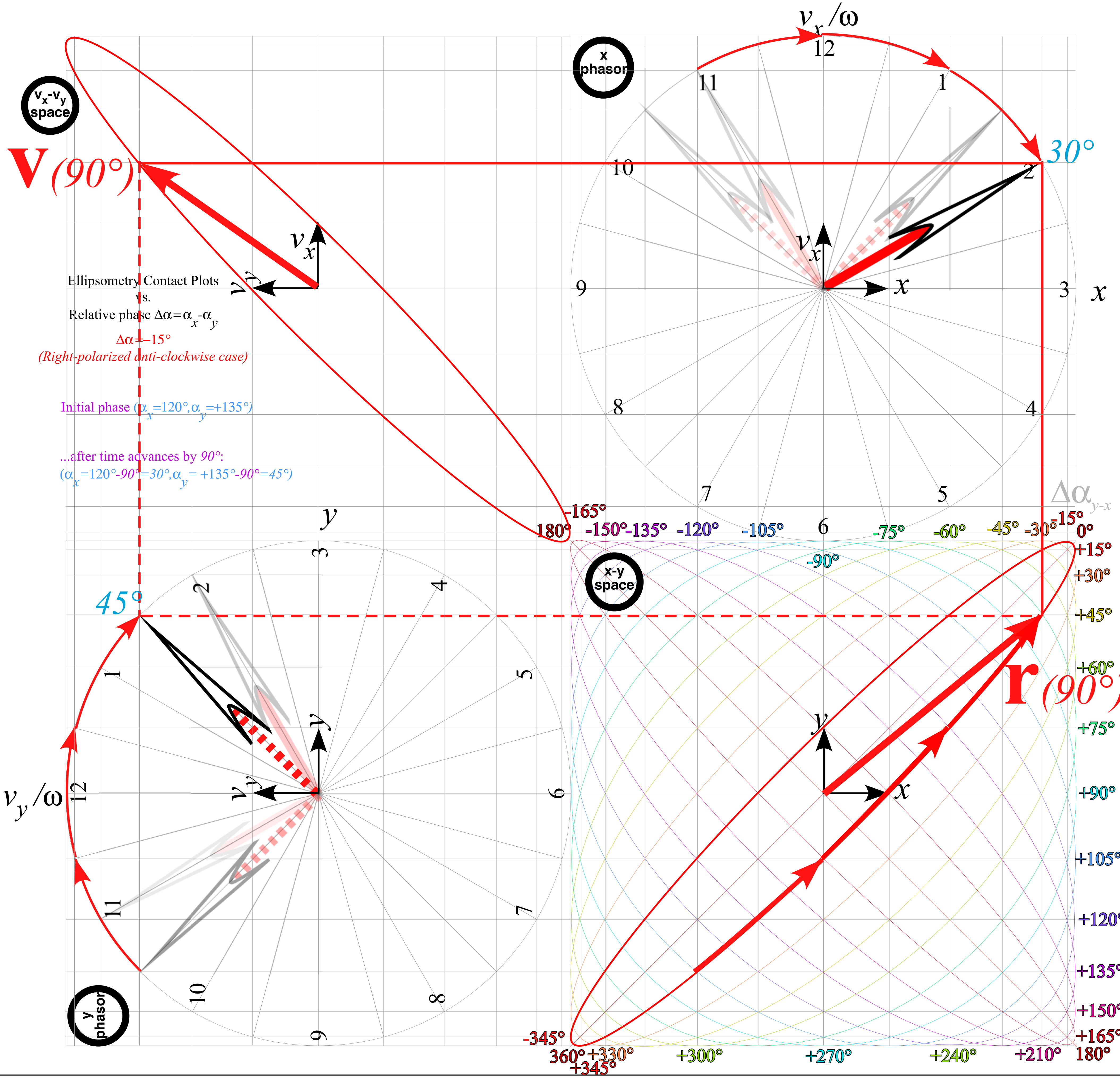


Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
V(0°)
r(0°)
vx-vy space
x phasor
y phasor
x-y space
vx/ω
vy/ω
120°
135°
Δαy-x

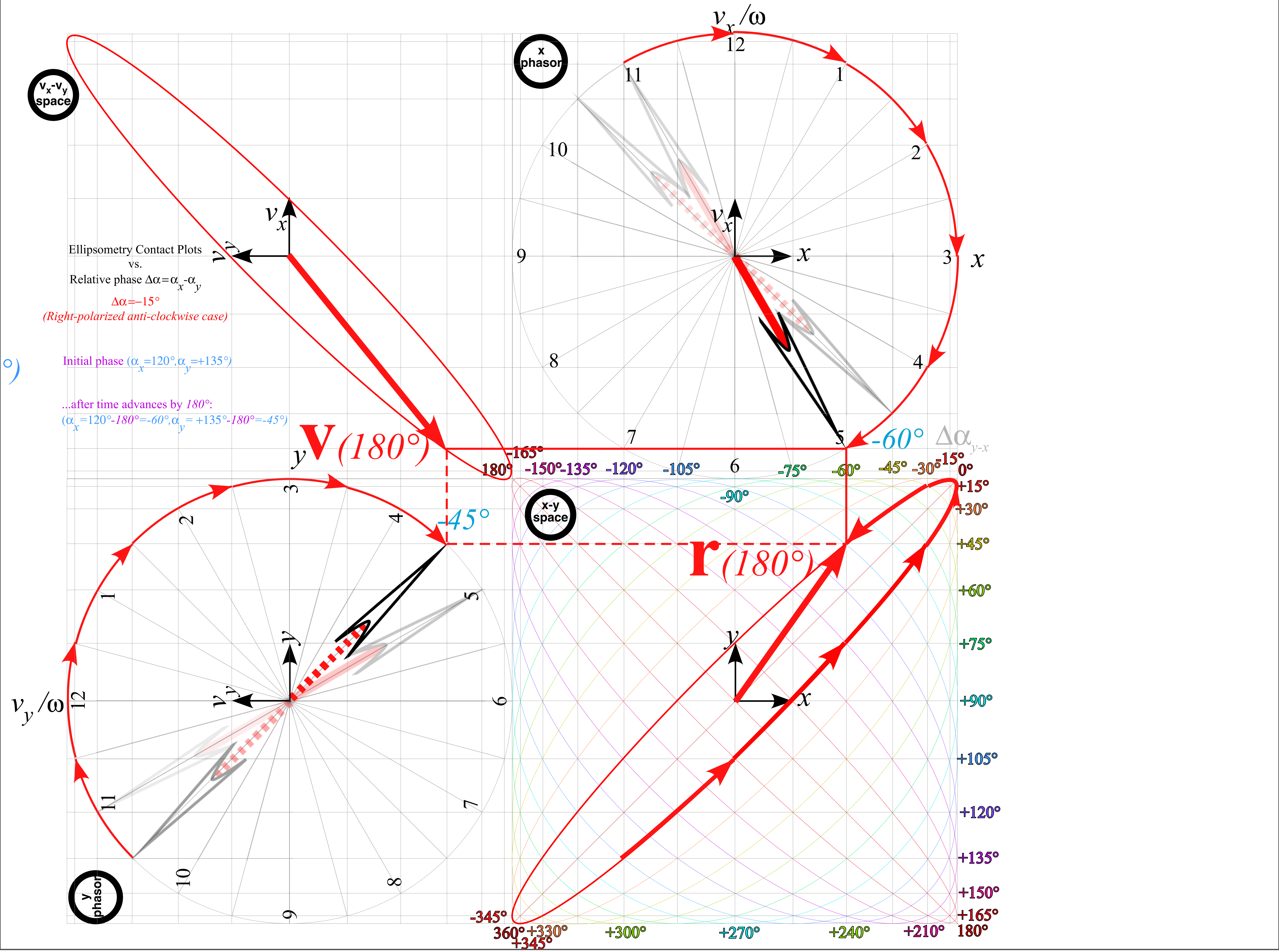

V(30°)
v_x/ω
x phasor
v_x-v_y space
90°
v_x
v_y
x
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y=+135°-30°=105°)
y
$\Delta\alpha_{y-x}$
-165°
180° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°
x-y space
y phasor
v_y/ω
105°
r(30°)



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 60°:
(α_x=120°-60°=60°,α_y=+135°-60°=75°)
v_x-v_y space
V(60°)
v_x
v_y
x phasor
v_x/ω
60°
x-y space
r(60°)
y phasor
v_y/ω
75°
Δα_y-x



v_x/ω
x phasor
v_x-v_y space
V (90°)
30°
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)
...after time advances by 90°:
($\alpha_x=120°-90°=30°, \alpha_y=+135°-90°=45°$)
$\Delta\alpha_{y-x}$
45°
x-y space
r (90°)
v_y/ω
y phasor



Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y = +135°-180°=-45°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(180°)
r(180°)
-60°
-45°
$\Delta\alpha_{y-x}$