

Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

Introducing 2D IHO orbits and phasor geometry

Phasor “clock” geometry

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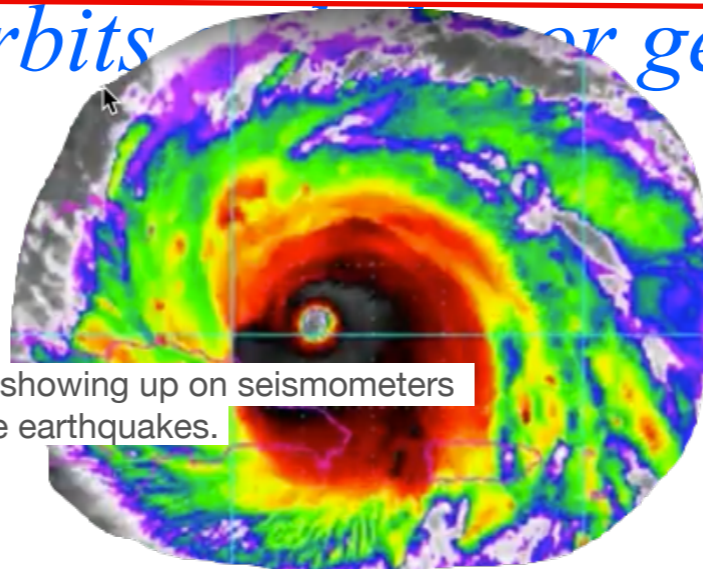
Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

MONSTERS!

Introducing 2D IHO orbits or geometry

Phasor “clock” geometry



Hurricane Dorian is so strong it's showing up on seismometers
— equipment designed to measure earthquakes.

This year it's Hurricane Dorian?

This Lecture's Reference Link Listing

[Web Resources - front page](#)

[UAF Physics UTube channel](#)

[Quantum Theory for the Computer Age](#)

[Principles of Symmetry, Dynamics, and Spectroscopy](#)

[Classical Mechanics with a Bang!](#)

[Modern Physics and its Classical Foundations](#)

[2017 Group Theory for QM](#)

[2018 Adv CM](#)

[2018 AMOP](#)

[**2019 Advanced Mechanics**](#)

Lecture #6

[RelaWavity Web Simulation: Contact Ellipsometry](#)

[BoxIt Web Simulation: Elliptical Motion \(A-Type\)](#)

[CMwBang Course: Site Title Page](#)

[Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors](#)

[UAF Physics UTube channel](#)

These *are* hot off the presses. Out in MISC for quick reference.

[Sorting ultracold atoms in a three-dimensional optical lattice in a realization of Maxwell's Demon - Kumar-n-2018](#)

[Synthetic three-dimensional atomic structures assembled atom by atom - Barredo-n-2018](#)

Older ones:

[Wave-particle duality of C60 molecules - Arndt-ltn-1999](#)

[Optical Vortex Knots - One Photon At A Time - Tempone-Wiltshire-Sr-2018](#)

References that did not make Lect 5, but that may have an affinity for the Independent Bounce Model covered there in:

[Baryon Deceleration by Strong Chromofields in Ultrarelativistic Nuclear Collisions - Mishustin-PhysRevC-2007,](#)

[APS Link & Abstract](#)

They treat collisions between ultrahigh energy nuclei as pair-wise independent collisions between “baryonic slabs”

Some Nuclei Hadronic material may behave or in times of extreme perturbation collapse or react as if it were comprised of clumps, of a size or composition more favorable, say: NP(Deuterium), He(Alpha), Carbon, ...

[Hadronic Molecules - Guo-x-2017](#)

[Hidden-charm pentaquark and tetraquark states - Chen-pr-2016](#)

One might want to check spectra or design and observe collisions to look for evidence of compositional rotation and vibration structure, that more complex phenomena that we are just now acquiring the tools and techniques needed to describe and handle, and will be even better covered in our next Atomic, Molecular, Optical (AMO) or Group Theory (GT) offering.

A lot of cool tools for handling such composite Molecular behavior, and internalize as a natural continuation of the Geometric approach that we take here in this course.

Running Reference Link Listing

Prior to Lecture #6

[Velocity Amplification in Collision Experiments Involving Superballs - Harter, 1971](#)
[MIT OpenCourseWare: High School/Physics/Impulse and Momentum](#)
[Hubble Site: Supernova - SN 1987A](#)

BounceIt Web Animation - Scenarios:

[49:1 y vs t, 49:1 V2 vs V1, 1:500:1 - 1D Gas Model w/ faux restorative force \(Cool\),](#)
[1:500:1 - 1D Gas \(Warm\), 1:500:1 - 1D Gas Model \(Cool, Zoomed in\),](#)

[Farey Sequence - Wolfram](#)
[Fractions - Ford-AMM-1938](#)

Monstermash BounceIt Animations:

[1000:1 - V2 vs V1, 1000:1 with t vs x - Minkowski Plot](#)

[Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013](#)

[Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2015](#)

[Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 \(Publ.\)](#)

[Velocity Amplification in Collision Experiments Involving Superballs-Harter-1971](#)

WaveIt Web Animation - Scenarios:

[Quantum Carpet, Quantum Carpet wMBar,](#)

[Quantum Carpet BCar, Quantum Carpet BCar wMBar](#)

[Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001](#)

[Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 \(Publ.\)](#)

[AJP article on superball dynamics](#)
[AAPT Summer Reading List](#)
[Scitation.org - AIP publications](#)
[HarterSoft Youtube Channel](#)

BounceIt Web Animation - Scenarios:

[Generic Scenario: 2-Balls dropped no Gravity \(7:1\) - V vs V Plot \(Power=4\)](#)

[1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4](#)

[7:1 - V vs V Plot: Power=1](#)

[3-Ball Stack \(10:3:1\) w/Newton plot \(y vs t\) - Power=4](#)

[3-Ball Stack \(10:3:1\) w/Newton plot \(y vs t\) - Power=1](#)

[3-Ball Stack \(10:3:1\) w/Newton plot \(y vs t\) - Power=1 w/Gaps](#)

[4-Ball Stack \(27:9:3:1\) w/Newton plot \(y vs t\) - Power=4](#)

[4-Newton's Balls \(1:1:1:1\) w/Newtonian plot \(y vs t\) - Power=4 w/Gaps](#)

[6-Ball Totally Inelastic \(1:1:1:1:1\) w/Gaps: Newtonian plot \(t vs x\), V6 vs V5 plot](#)

[5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot \(t vs x1\) w/Gaps](#)

[1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps](#)

BounceIt Dual plots

$m_1:m_2 = 3:1$

[v2 vs v1 and V2 vs V1, \(v1, v2\)=\(1, 0.1\), \(v1, v2\)=\(1, 0\)](#)

[y2 vs y1 plots: \(v1, v2\)=\(1, 0.1\), \(v1, v2\)=\(1, 0\), \(v1, v2\)=\(1, -1\)](#)

[Estrangian plot V2 vs V1: \(v1, v2\)=\(0, 1\), \(v1, v2\)=\(1, -1\)](#)

$m_1:m_2 = 4:1$

[v2 vs v1, y2 vs y1](#)

$m_1:m_2 = 100:1$, (v1, v2)=(1, 0): [V2 vs V1 Estrangian plot, y2 vs y1 plot](#)

[With g=0 and 70:10 mass ratio](#)

[With non zero g, velocity dependent damping and mass ratio of 70:35](#)

[M1=49, M2=1 with Newtonian time plot](#)

[M1=49, M2=1 with V2 vs V1 plot](#)

[Example with friction](#)

[Low force constant with drag displaying a Pass-thru, Fall-Thru, Bounce-Off](#)

[m1:m2= 3:1 and \(v1, v2\) = \(1, 0\) Comparison with Estrangian](#)

X2 paper: [Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 \(pdf\)](#)

Car Collision Web Simulator: <https://modphys.hosted.uark.edu/markup/CMMotionWeb.html>

Superball Collision Web Simulator: <https://modphys.hosted.uark.edu/markup/BounceItWeb.html>; with Scenarios: 1007

[BounceIt web simulation with g=0 and 70:10 mass ratio](#)

[With non zero g, velocity dependent damping and mass ratio of 70:35](#)

[Elastic Collision Dual Panel Space vs Space: Space vs Time \(Newton\), Time vs. Space\(Minkowski\)](#)

[Inelastic Collision Dual Panel Space vs Space: Space vs Time \(Newton\), Time vs. Space\(Minkowski\)](#)

[Matrix Collision Simulator: M1=49, M2=1 V2 vs V1 plot <<Under Construction>>](#)

[More Advanced QM and classical references at the end of this Lecture](#)

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“Zig-Zag” exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

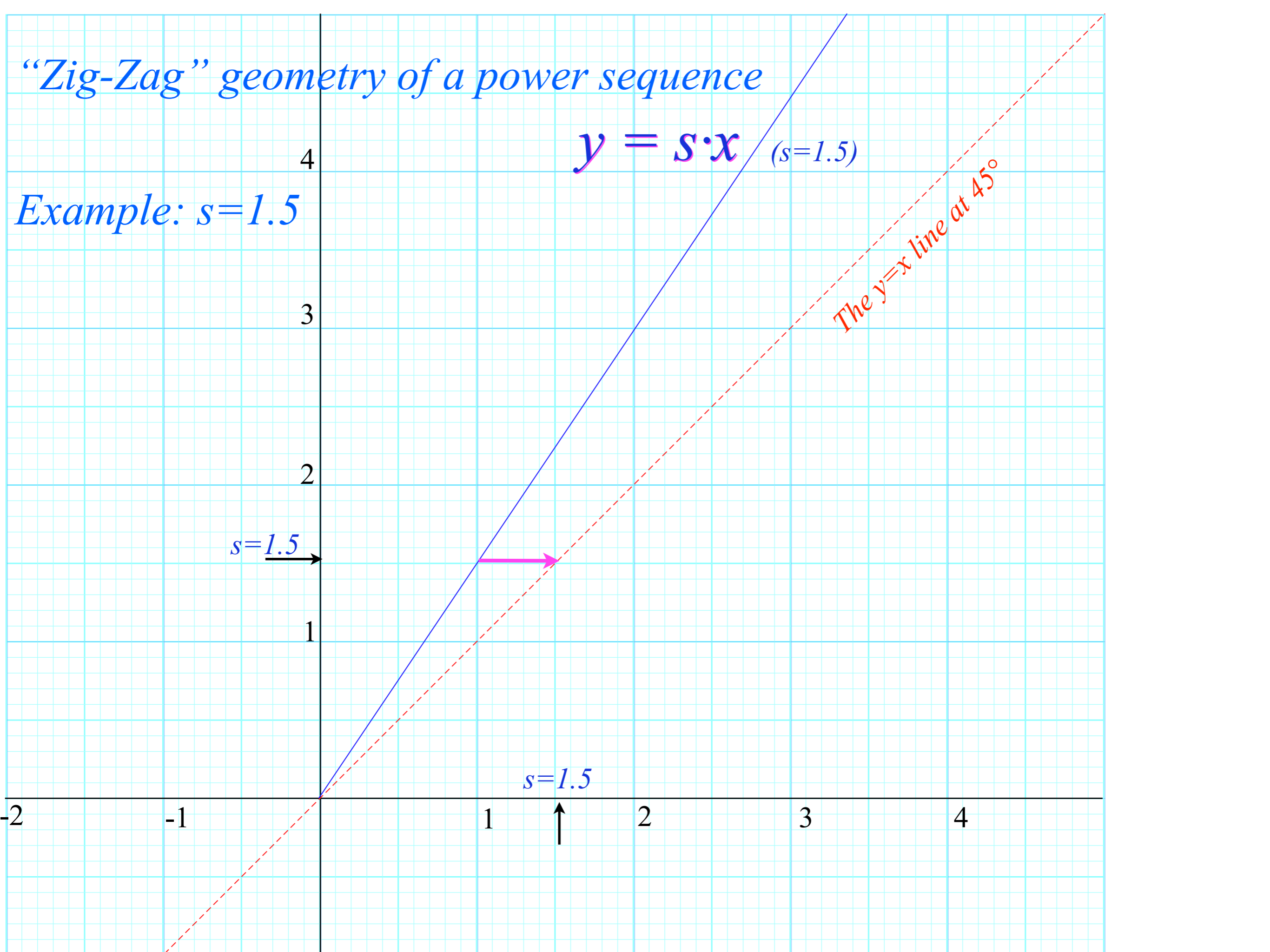
Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

“Zig-Zag” geometry of a power sequence

$$y = s \cdot x \quad (s=1.5)$$

Example: $s=1.5$

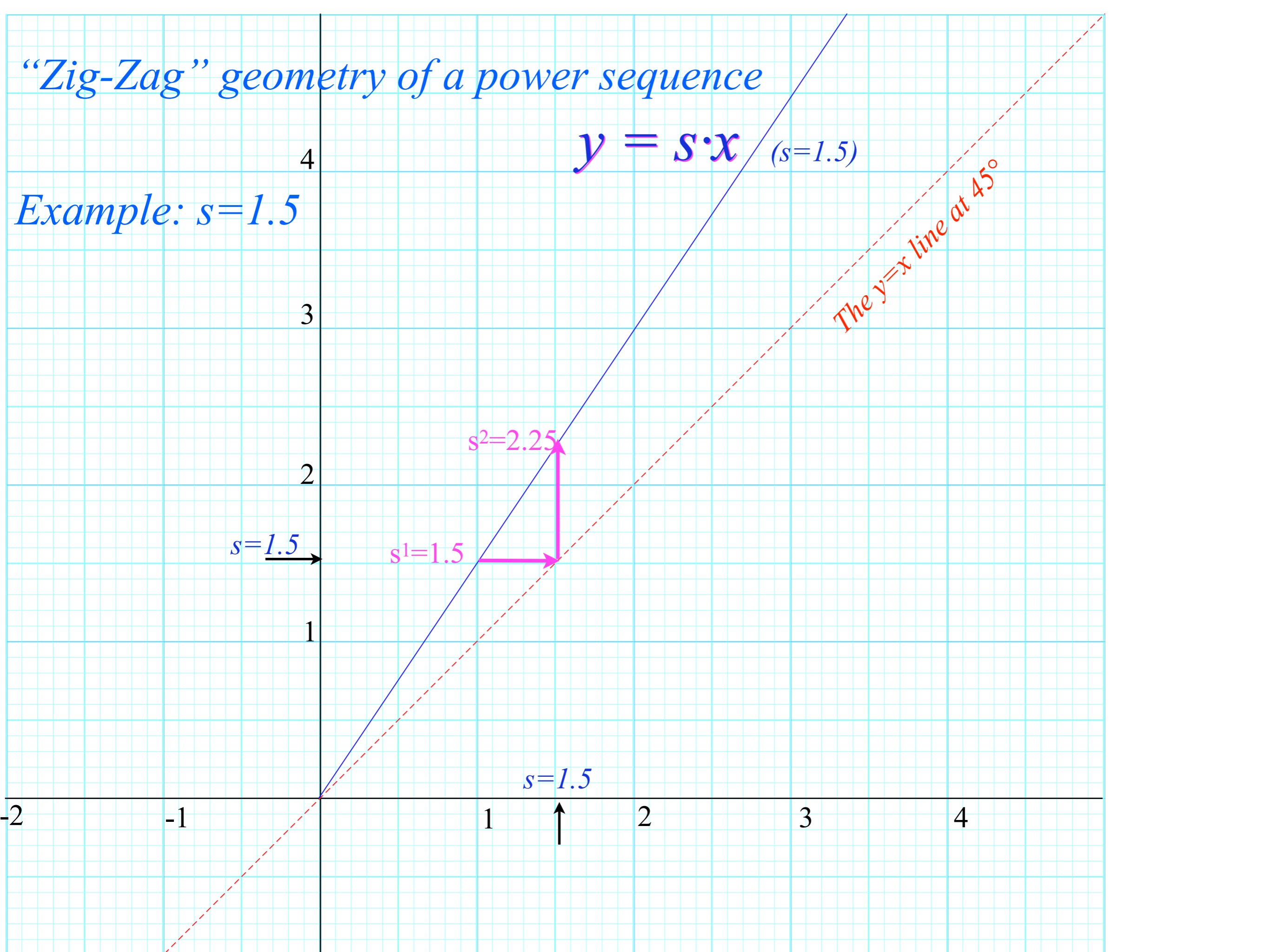


“Zig-Zag” geometry of a power sequence

$$y = s \cdot x \quad (s=1.5)$$

Example: $s=1.5$

The $y=x$ line at 45°



$s=1.5$

$s^1=1.5$

$s^2=2.25$

$s=1.5$

-2

-1

1

2

3

4

4

3

2

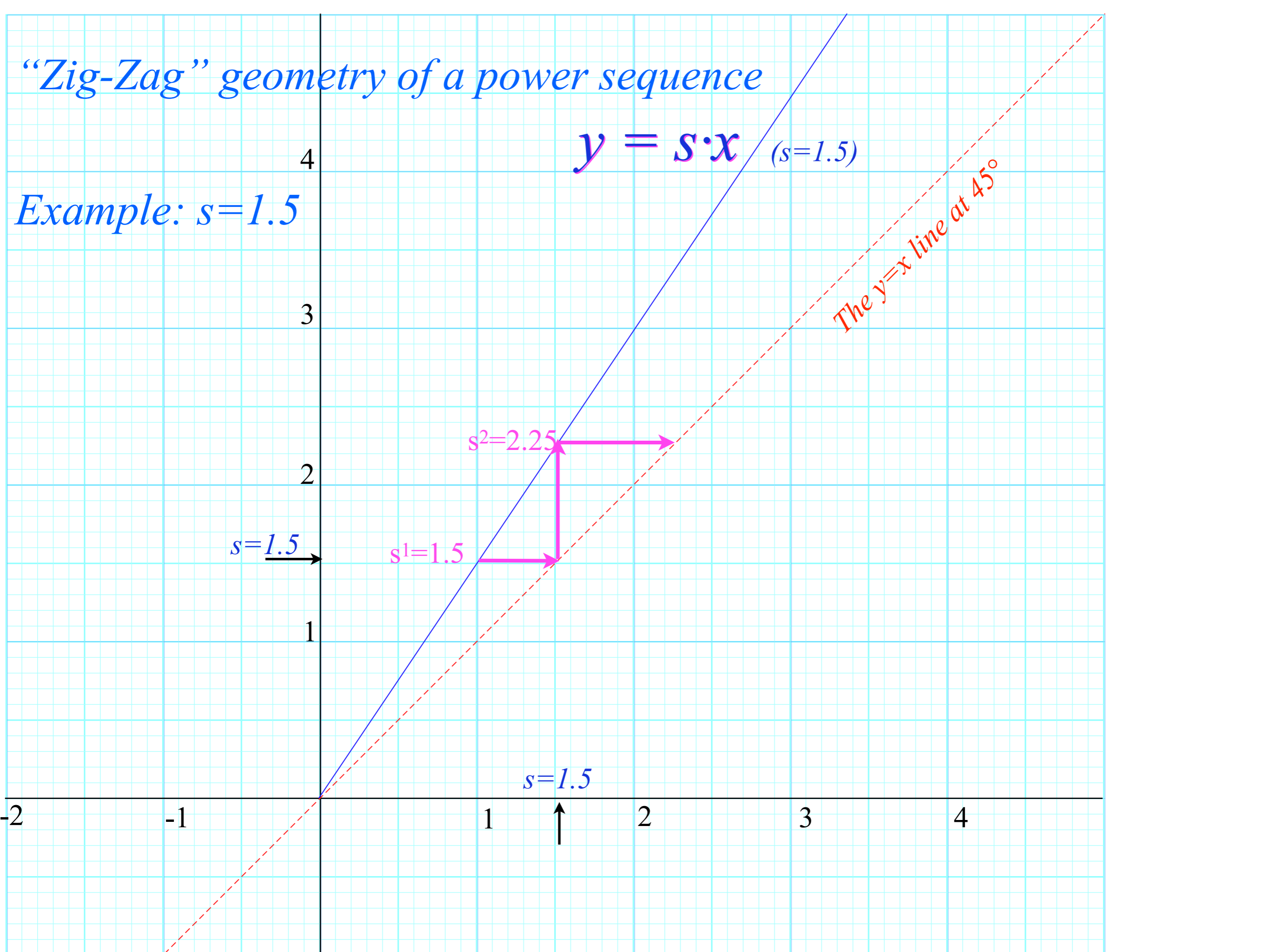
1

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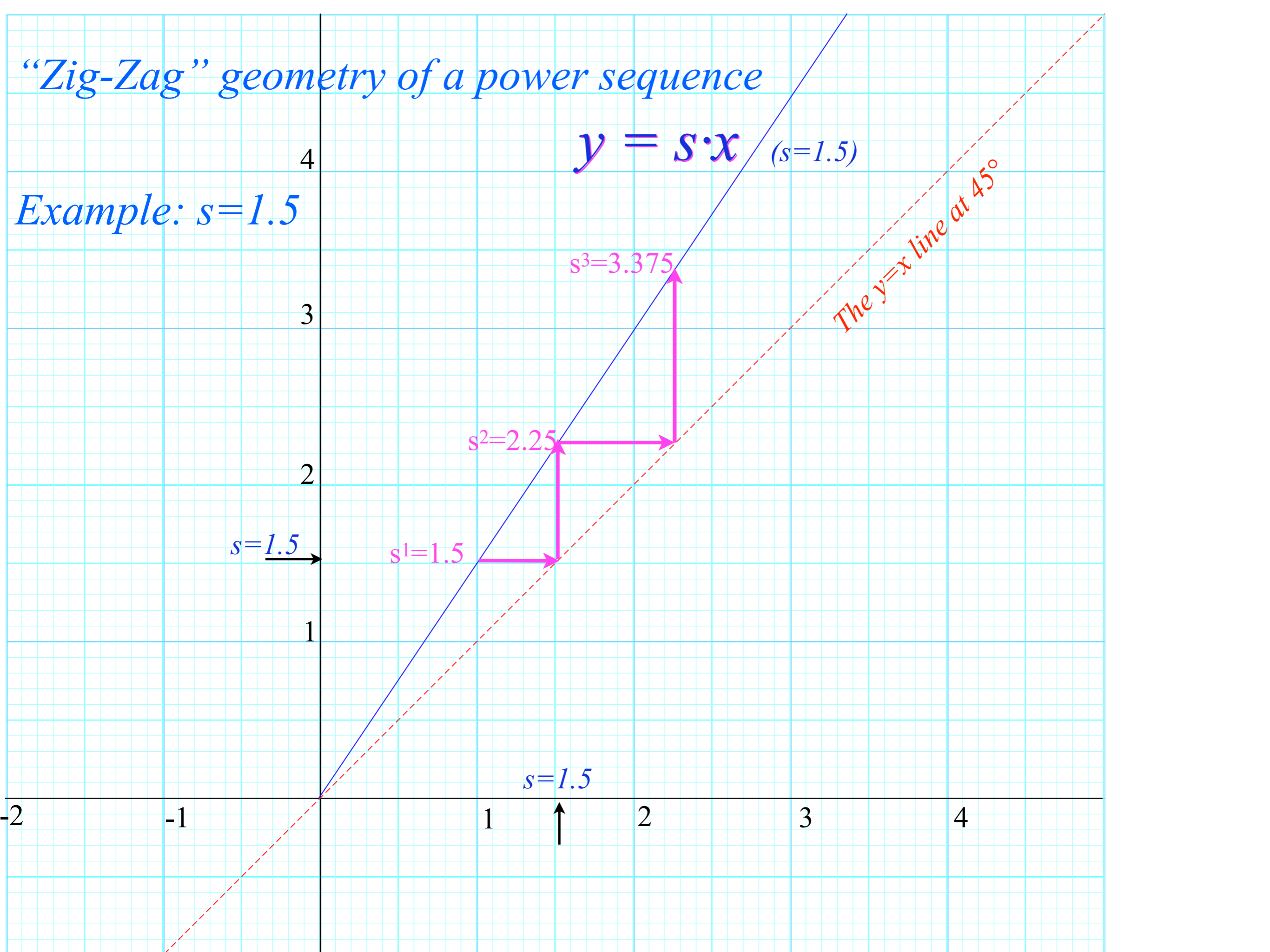
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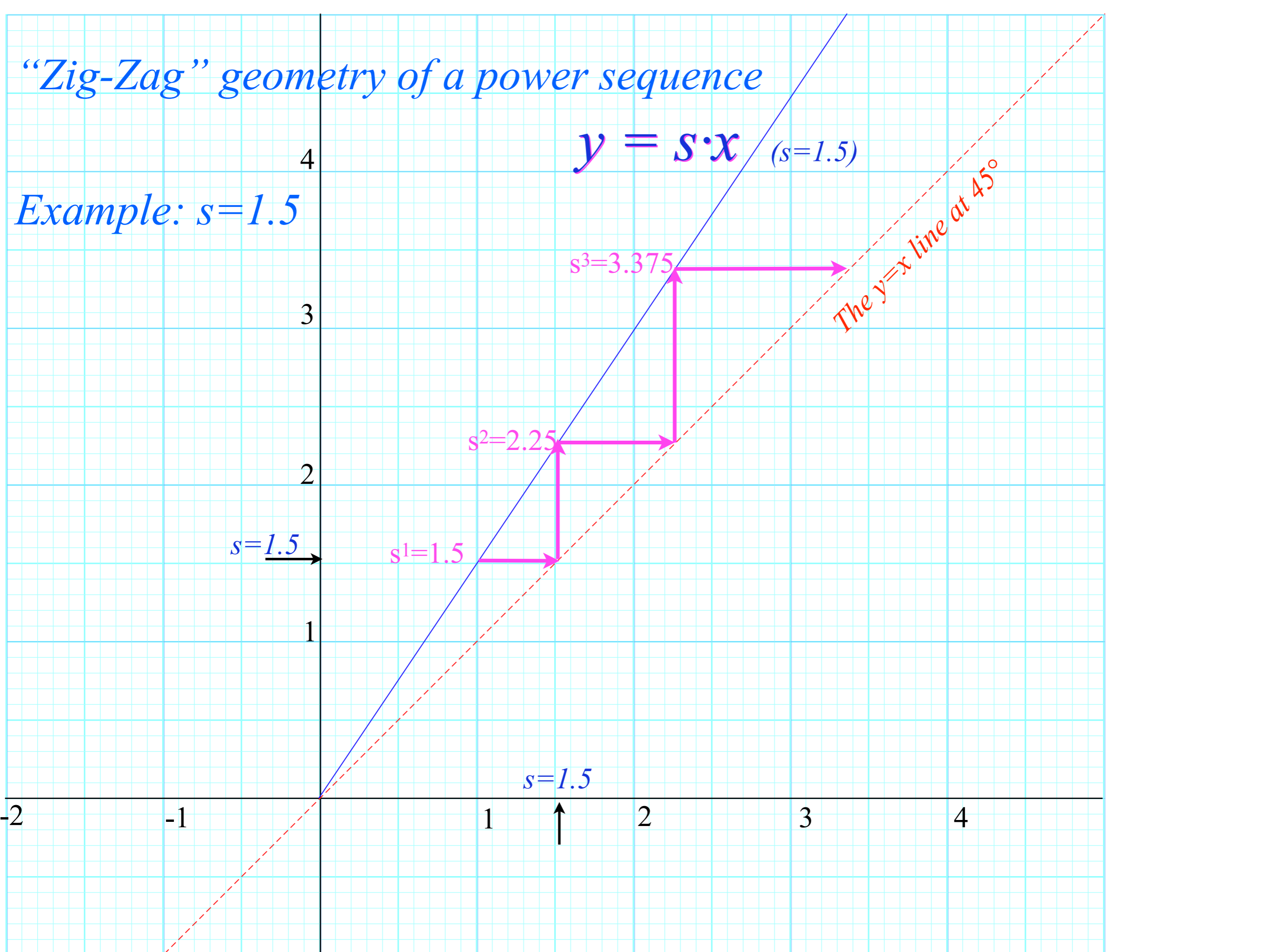


“Zig-Zag” geometry of a power sequence

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$s^1=1.5$

$s^2=2.25$

$s^3=3.375$

$s=1.5$

$s=1.5$

4

3

2

1

-2

-1

1

2

3

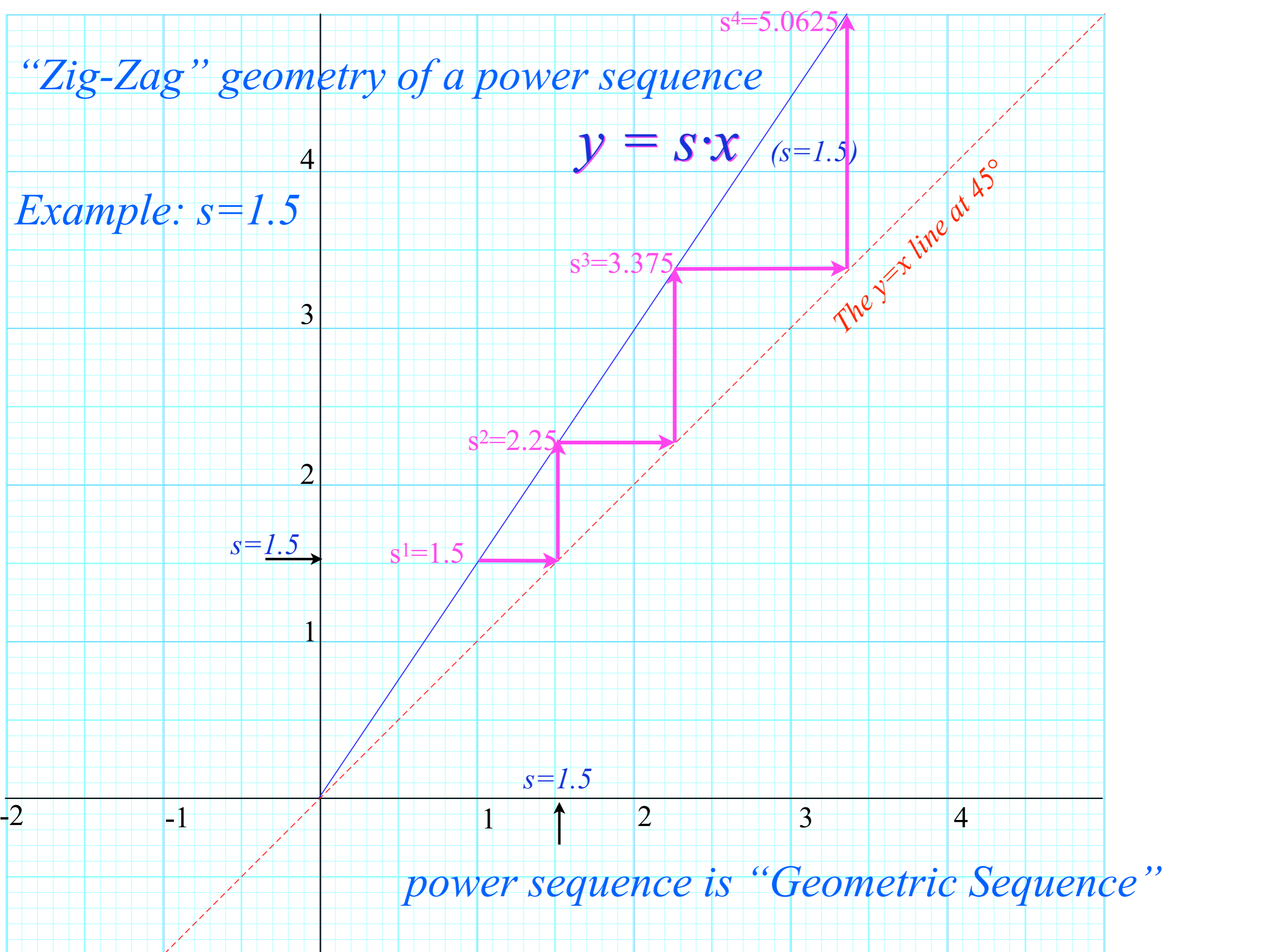
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“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

$y = s \cdot x$ ($s=1.5$)

The $y=x$ line at 45°



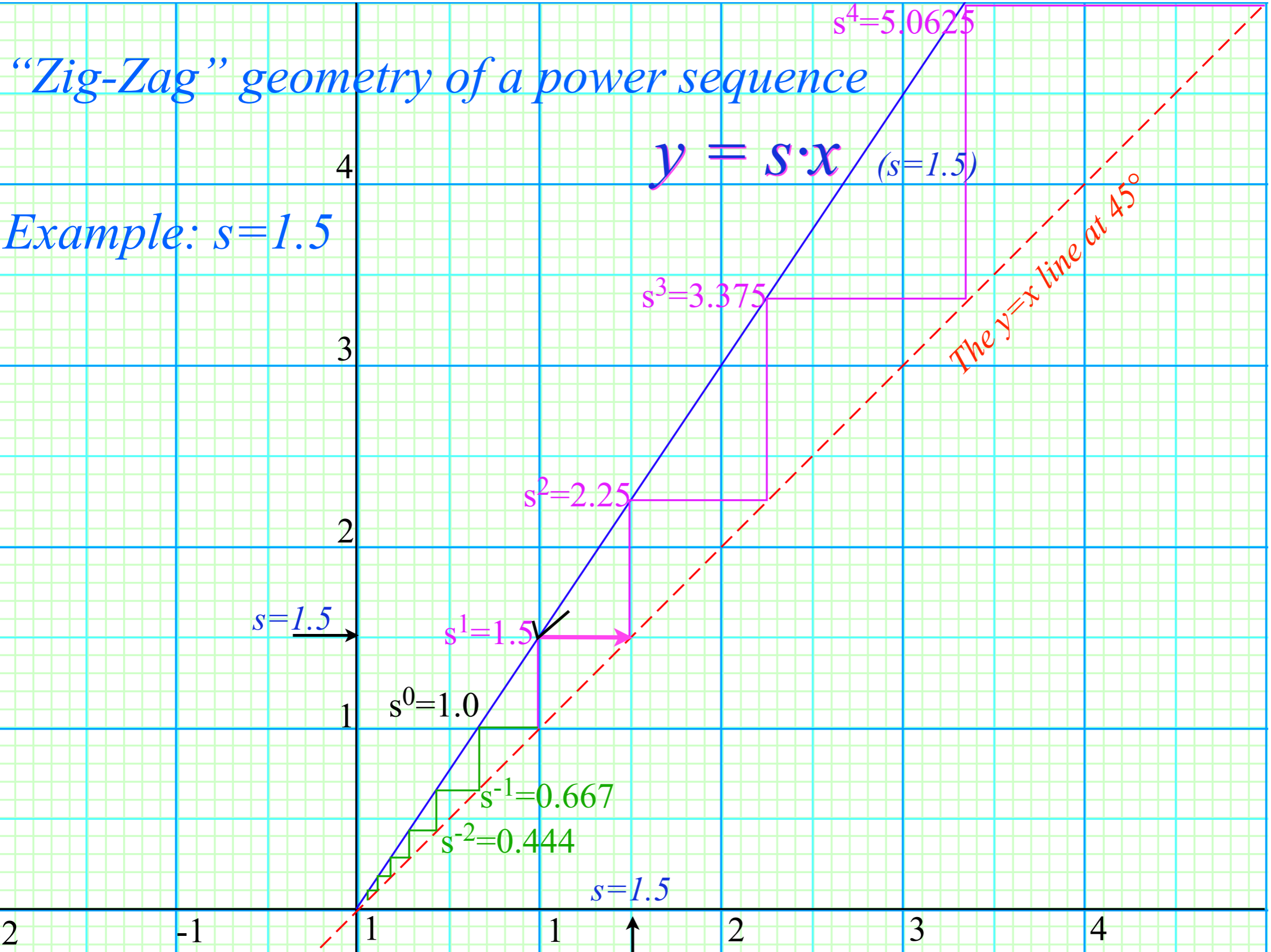
power sequence is “Geometric Sequence”

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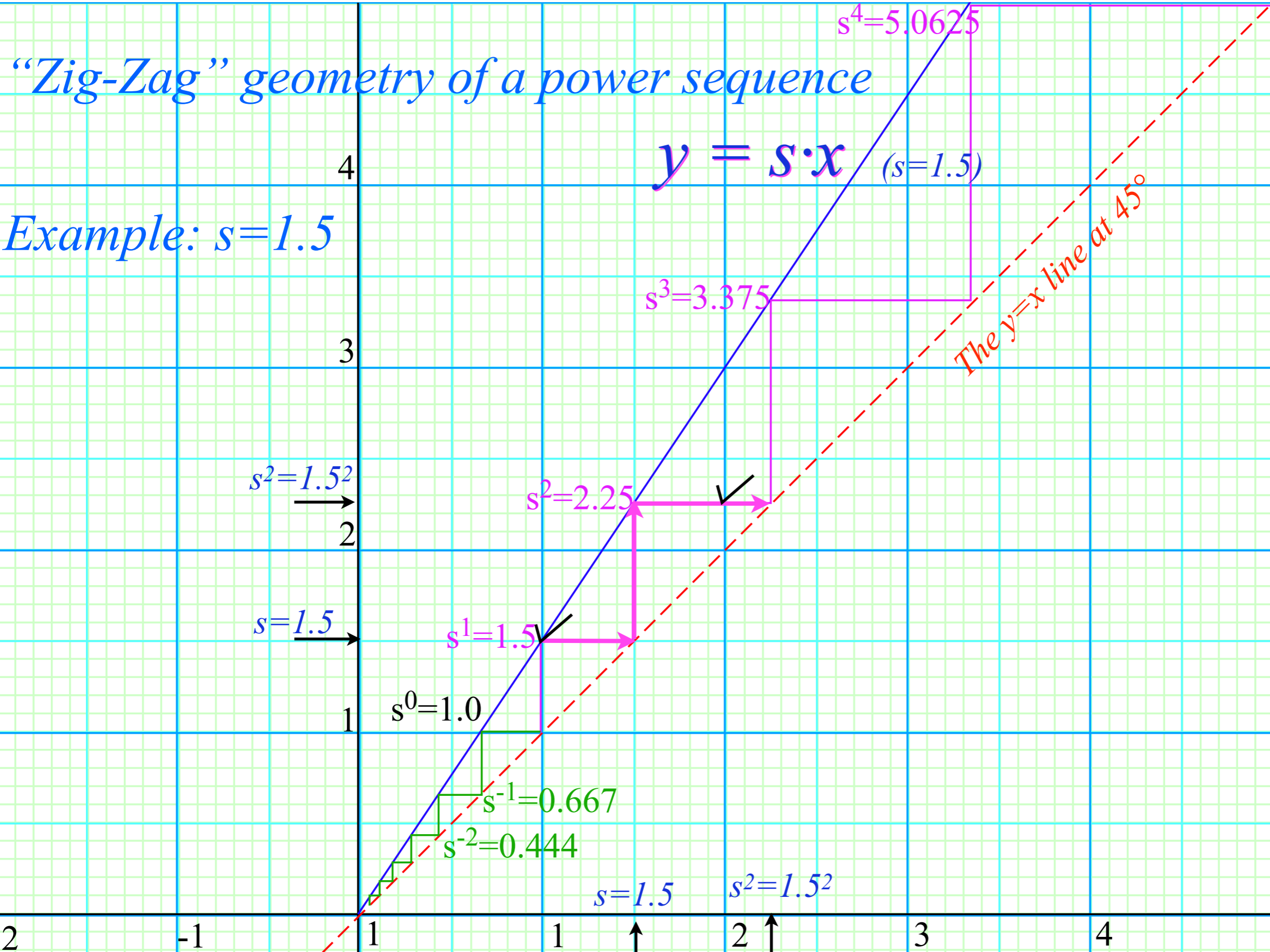
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power sequence is “Geometric Sequence”

“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

$$y = s \cdot x \quad (s=1.5)$$

The $y=x$ line at 45°

$s^3=1.5^3$

$s^2=1.5^2$

$s=1.5$

$s^0=1.0$

$s^{-1}=0.667$

$s^{-2}=0.444$

$s=1.5$

$s^2=1.5^2$

$s^4=5.0625$

$s^3=3.375$

$s^2=2.25$

$s^1=1.5$

-2

-1

1

1

2

3

4

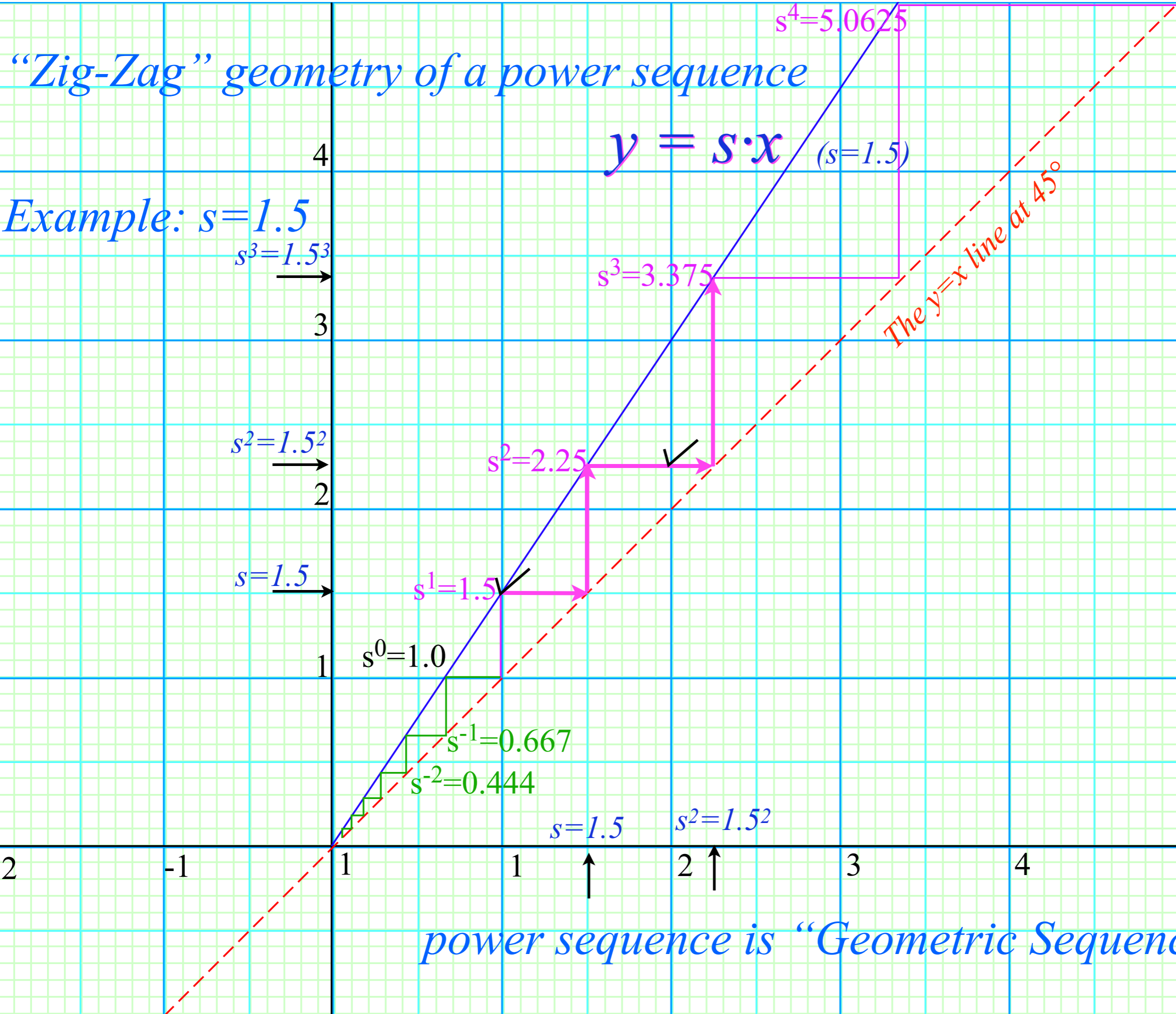
4

3

2

1

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$s^1=1.5$

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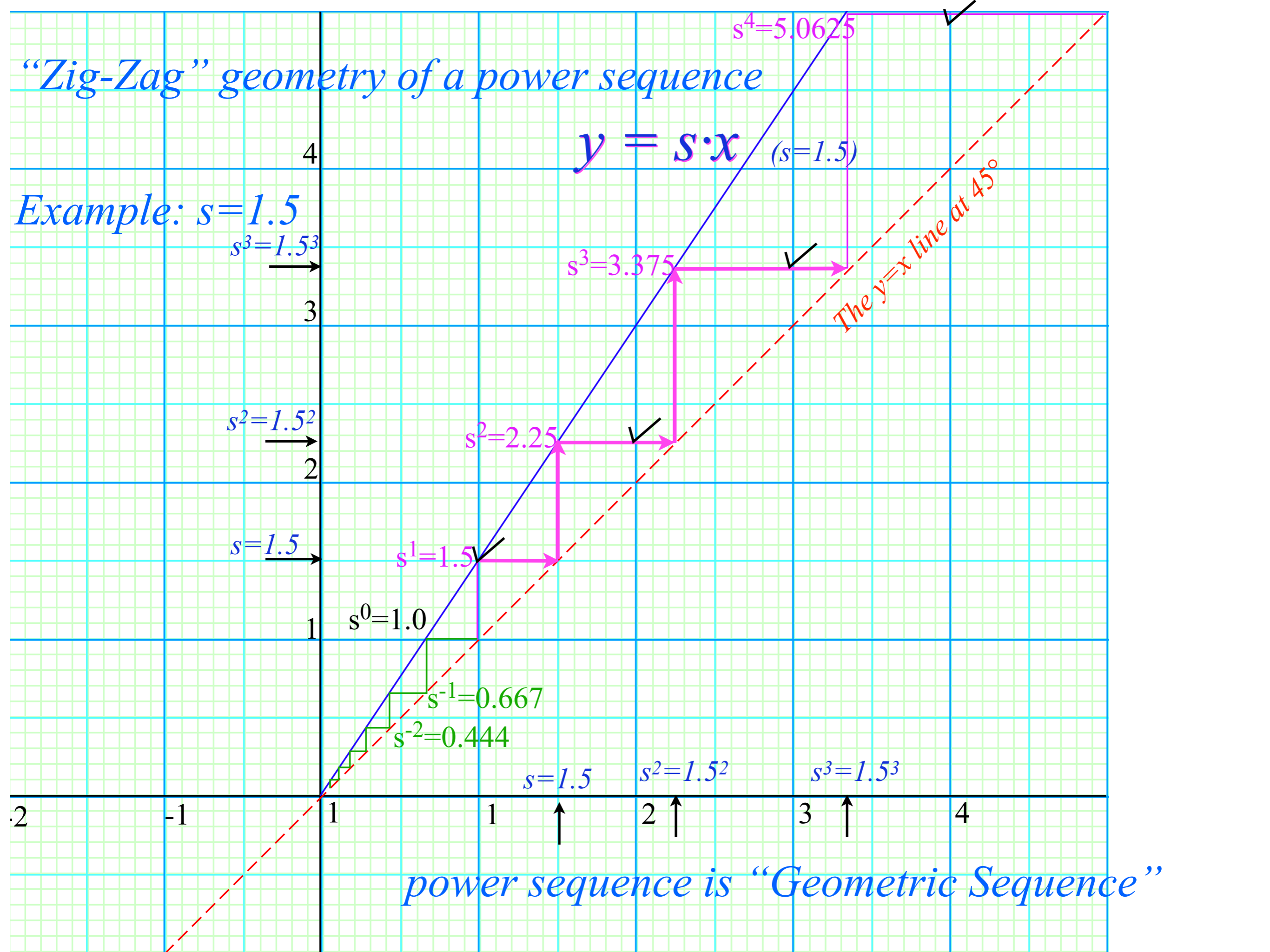
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$s=1.5$

$s^2=1.5^2$

$s^3=1.5^3$

power sequence is “Geometric Sequence”



“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

...and exponential function...

$s=1.5$

$s=1.5$

$y = s \cdot x$ ($s=1.5$)

The $y=x$ line at 45°

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$s^3=3.375$

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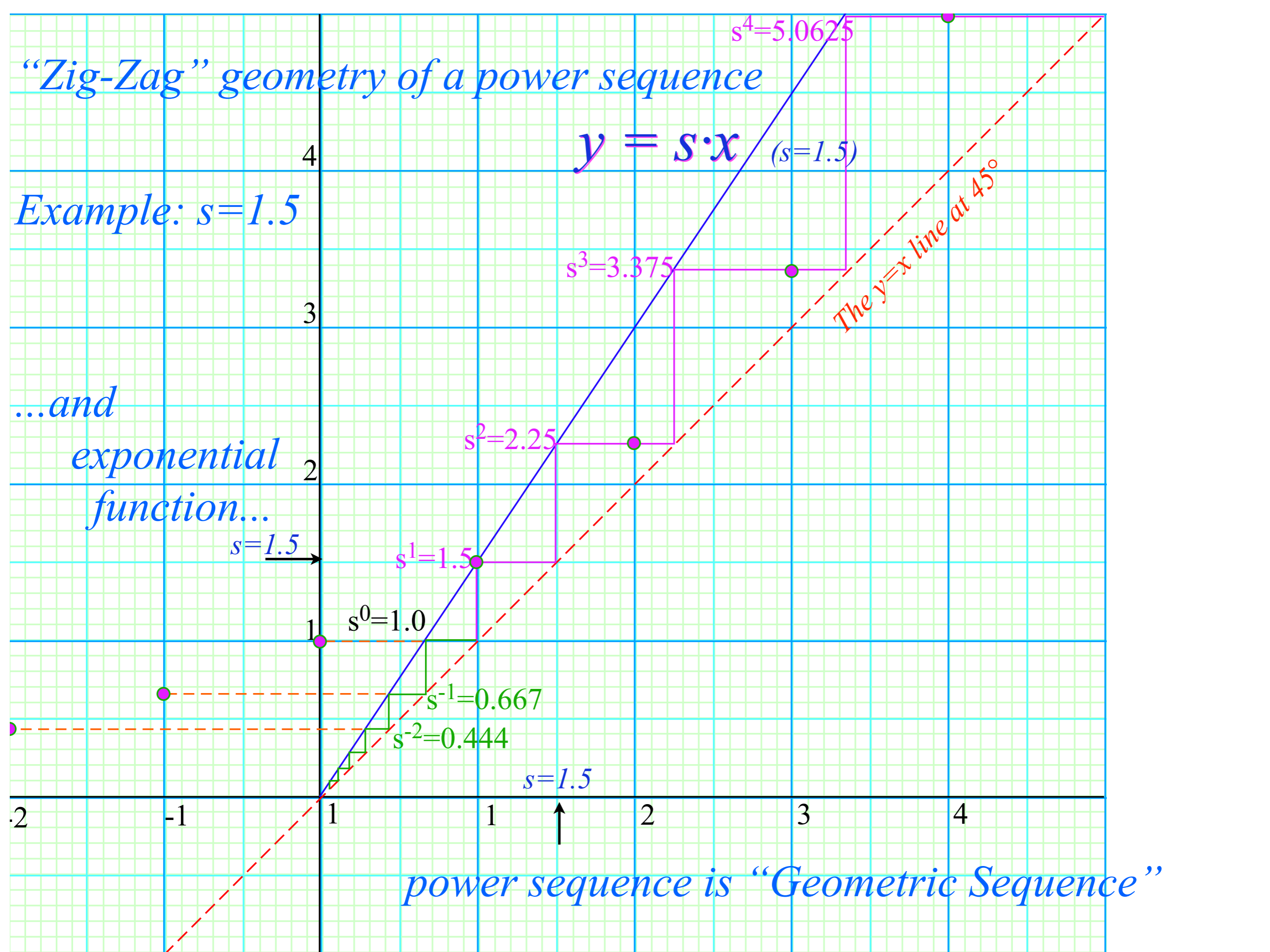
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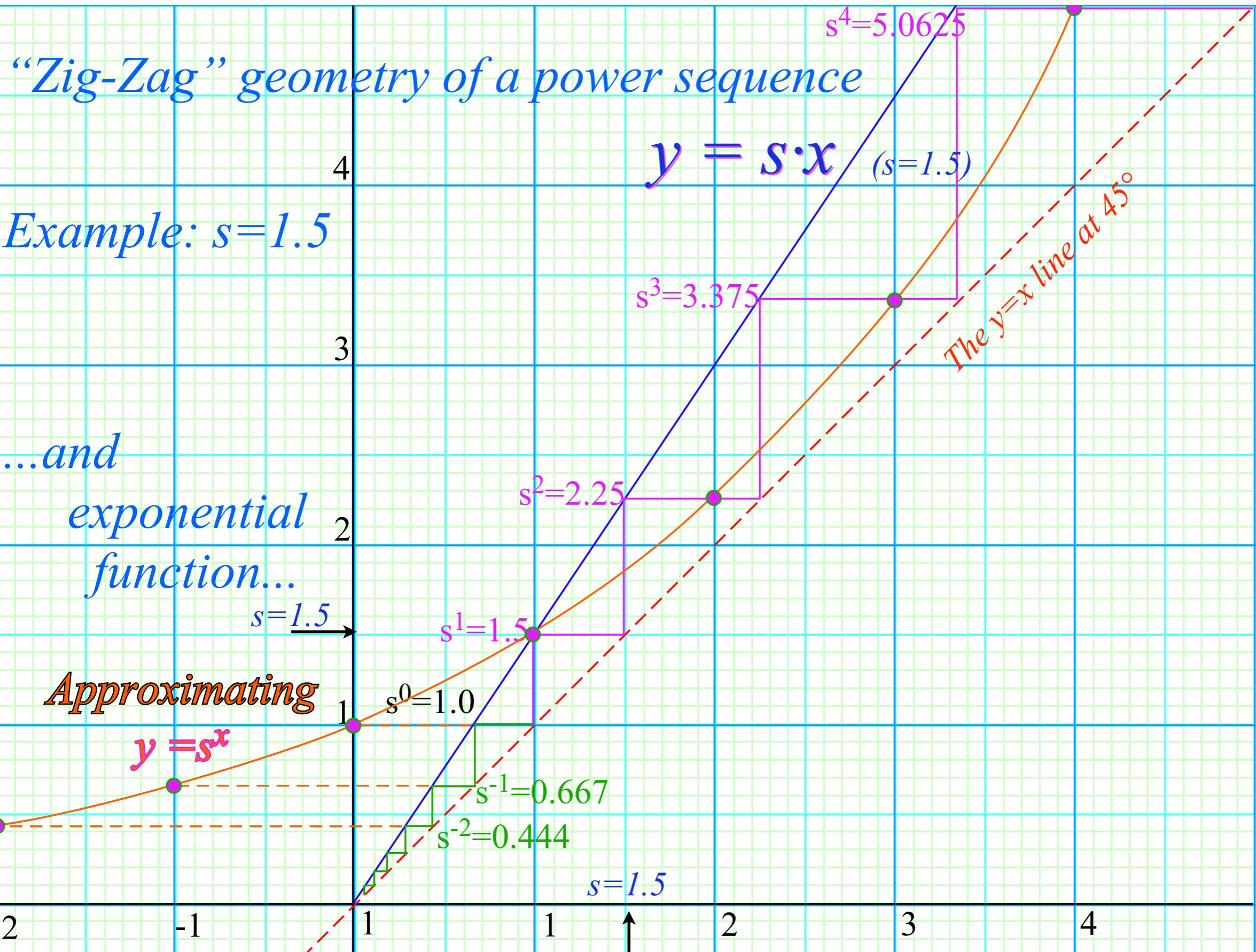


“Zig-Zag” geometry of a power sequence

Example: $s=1.5$

...and exponential function...

Approximating



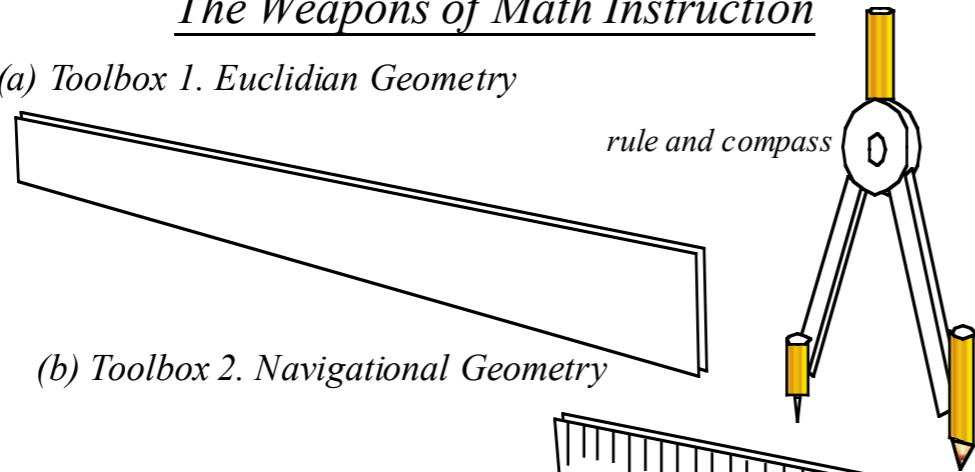
$y = s \cdot x$ ($s=1.5$)

The $y=x$ line at 45°

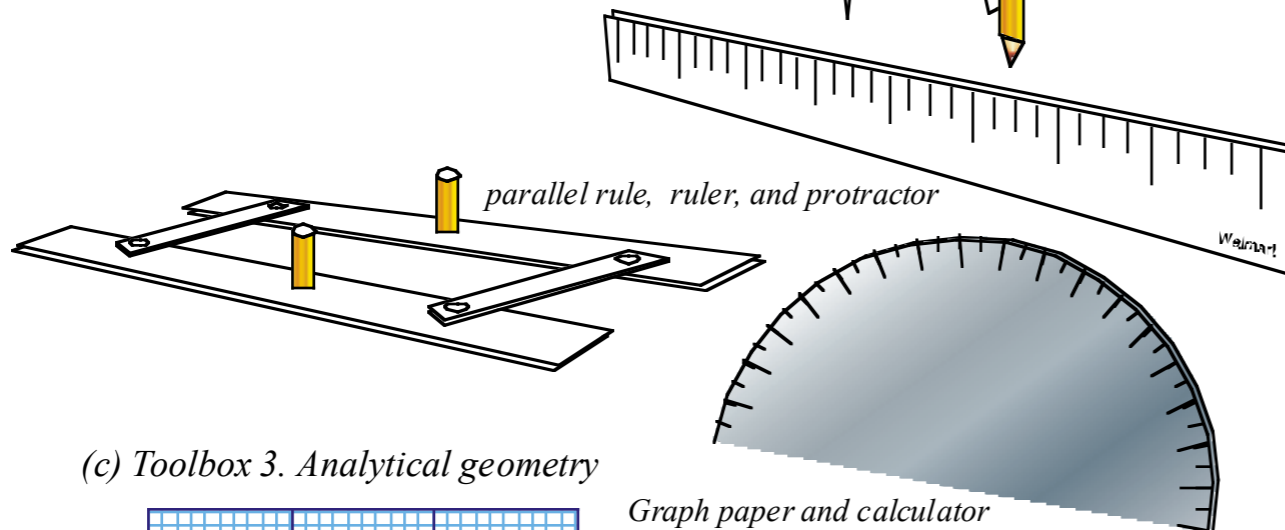
power sequence is “Geometric Sequence”

The Weapons of Math Instruction

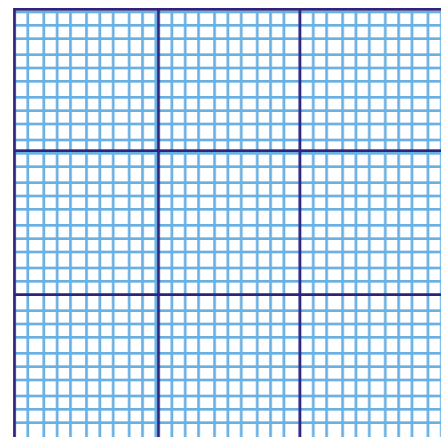
(a) Toolbox 1. Euclidian Geometry



(b) Toolbox 2. Navigational Geometry



(c) Toolbox 3. Analytical geometry

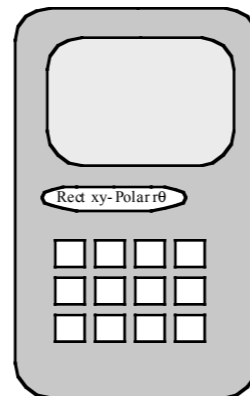


Graph paper and calculator

Complex algebra and calculus

$$1/z = r^{-1} e^{-i\theta}$$

$$\int 1/z dz = \ln z$$

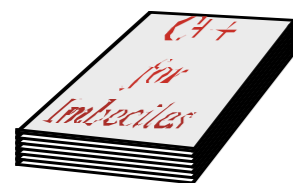


So far we mostly use
Toolbox (a-b)

What follows uses
Toolbox (c) ...

...and Toolbox (d)

(d) Toolbox 4. Computer geometry...Anything goes!



Facelt



Bandlt



Bohrlt



Bouncelt



Colorlt U2



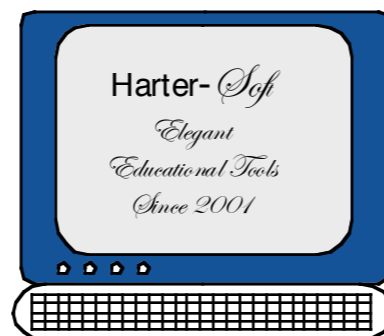
Oscillt

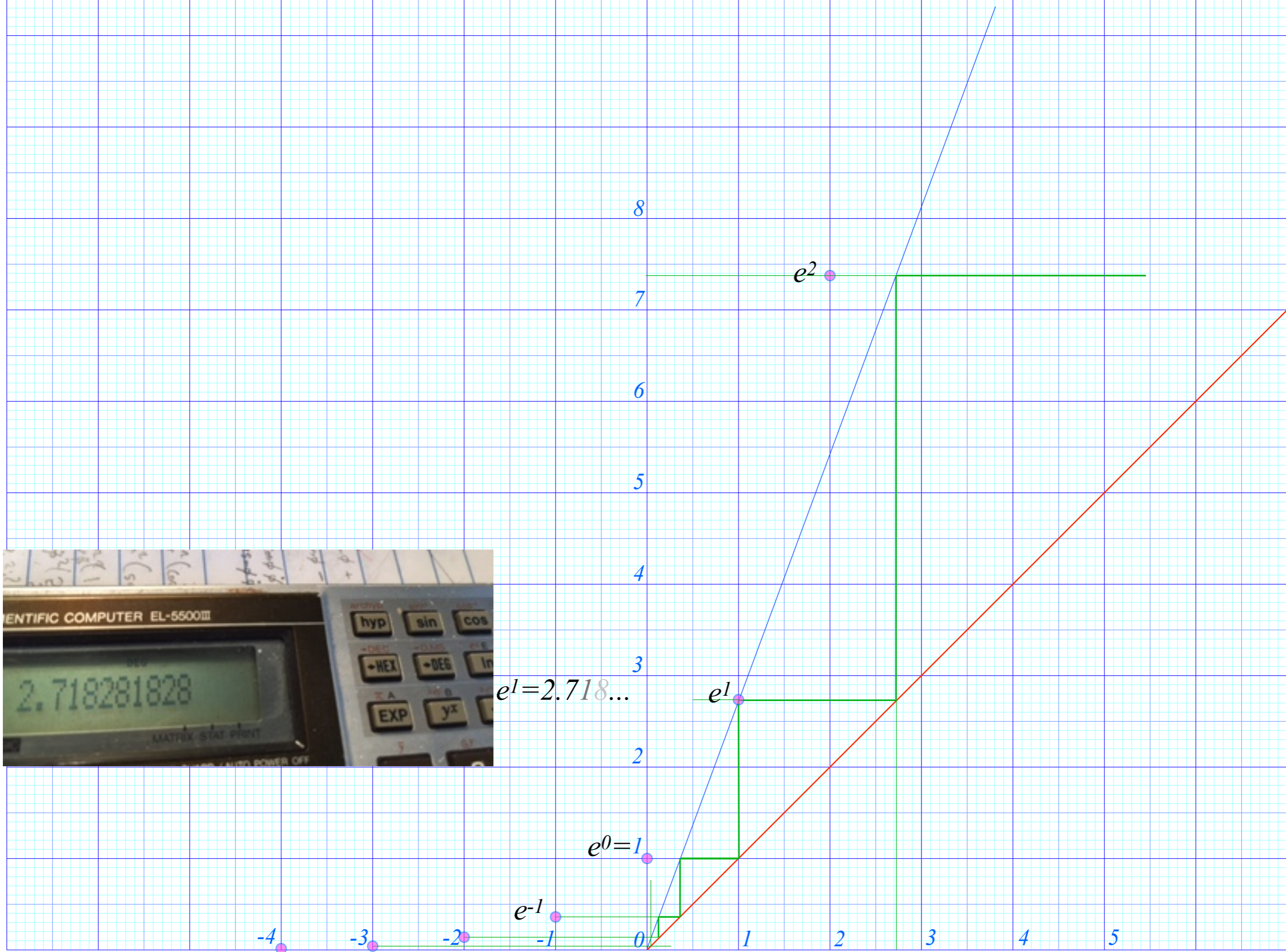
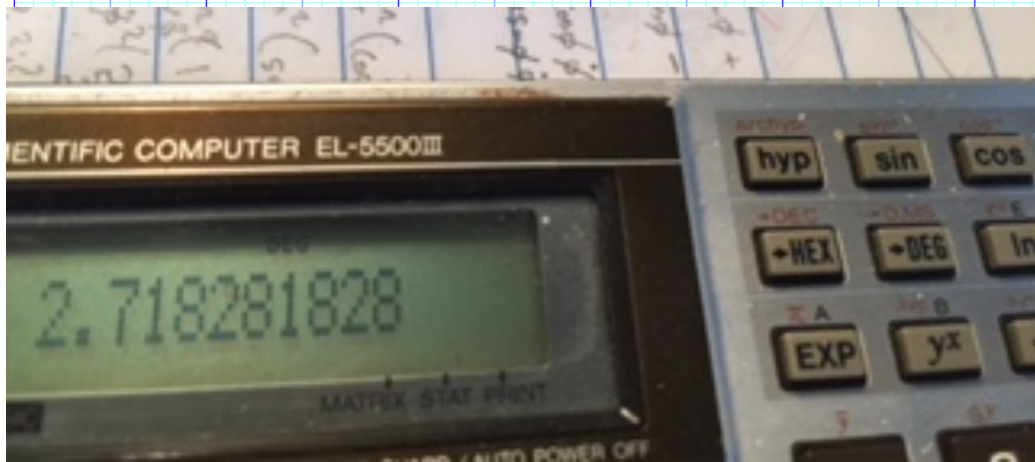


Relativt



Wavelt





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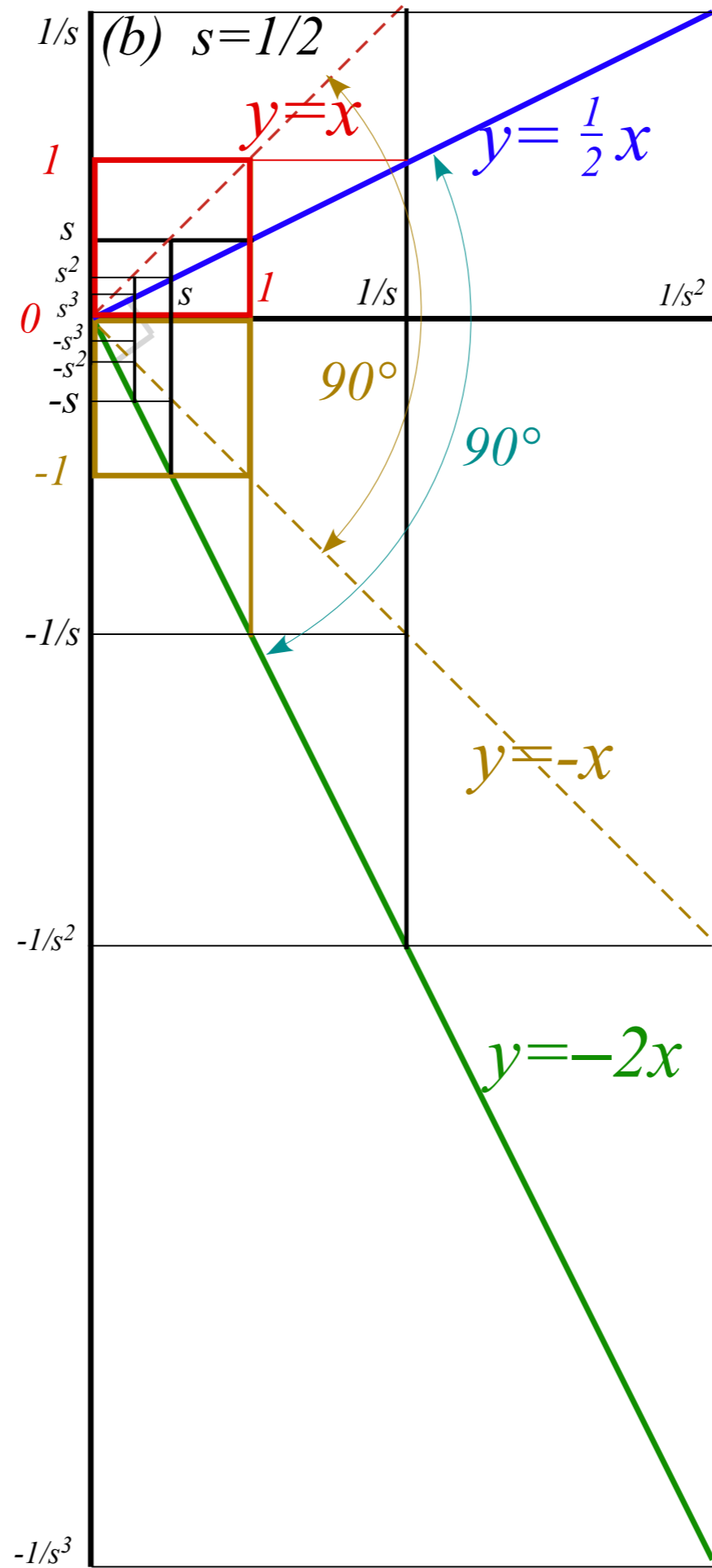
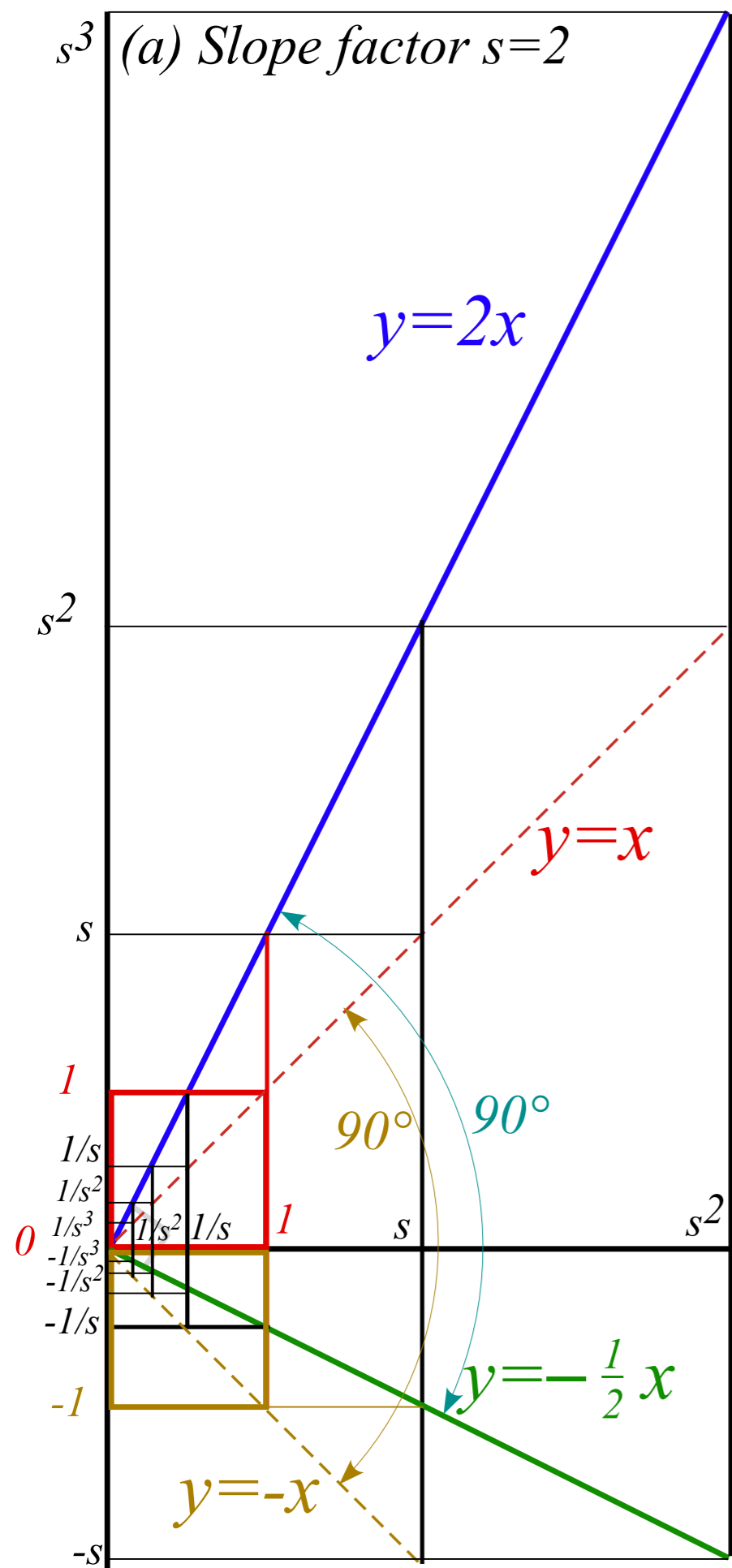


Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

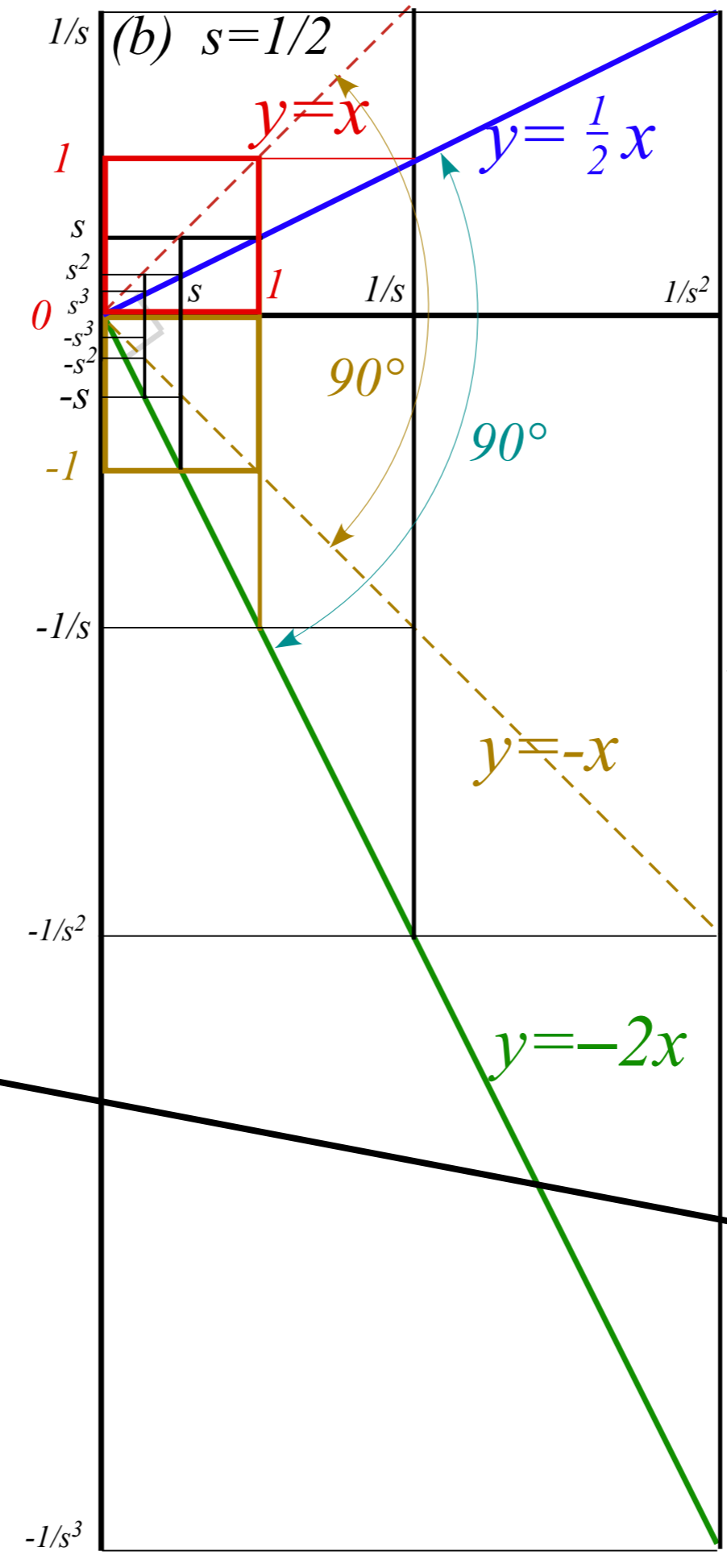
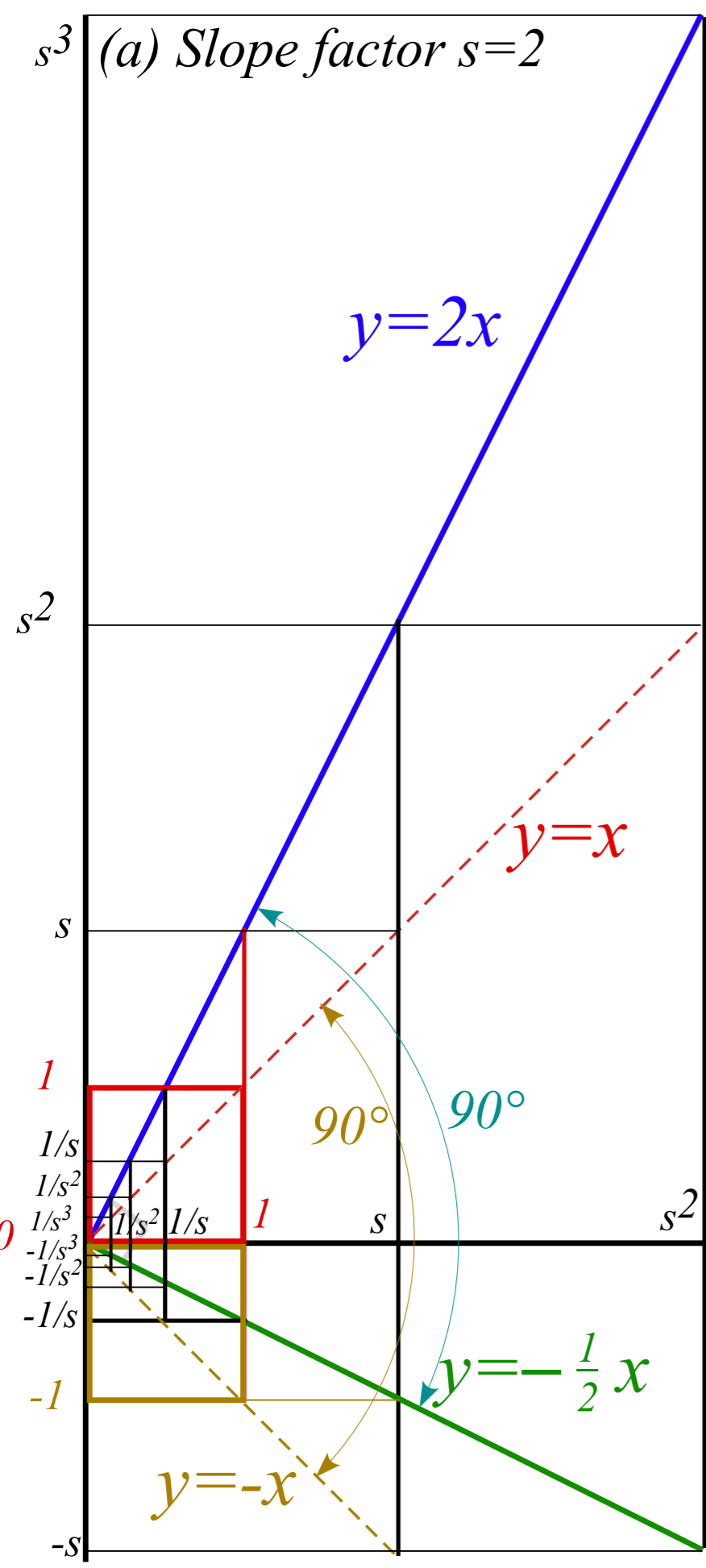
Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

Compare mks units of Coulomb Electrostatic vs. Gravity



“Zig-Zags” give perspective geometry
(1D-vanishing point)

Unit 1
Fig. 9.2



“Zig-Zags” give perspective geometry (1D-vanishing point)

Unit 1
Fig. 9.2

1st-day-of-school perspective of 12th-grader

1st-day-of-school perspective of 1st-grader

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Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

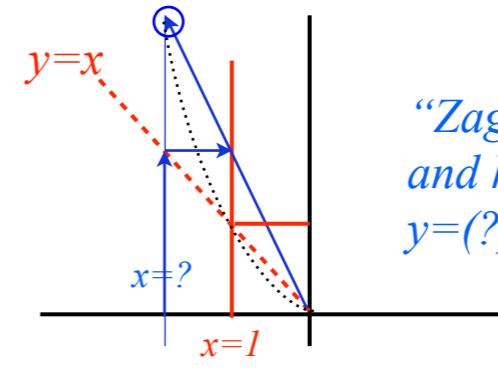
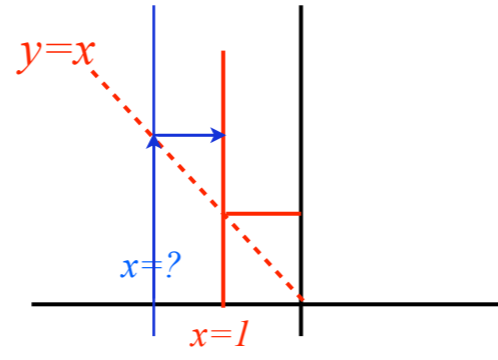
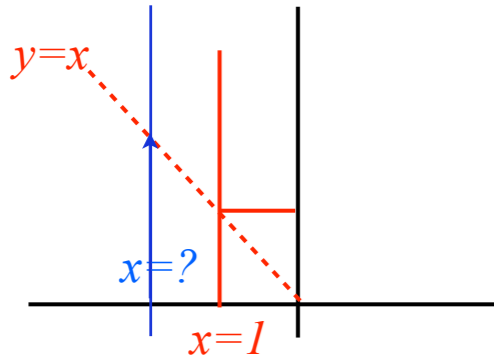
Compare mks units of Coulomb Electrostatic vs. Gravity

Each $y=x^2$ parabola point found by just one “Zig-Zag”

1. Pick an $(x=?)$ -line

2. “Zig” from its $y=x$ intersection to $x=1$ line

3. “Zag” from origin back to $(x=?)$ -line



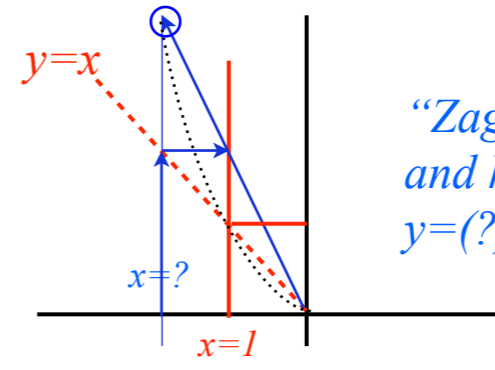
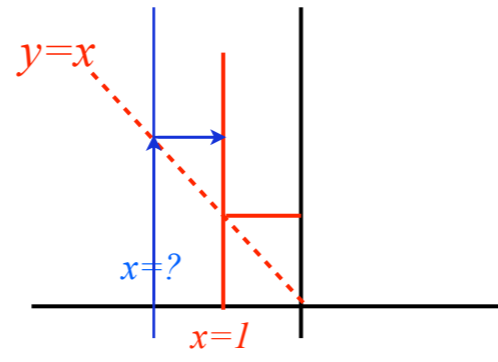
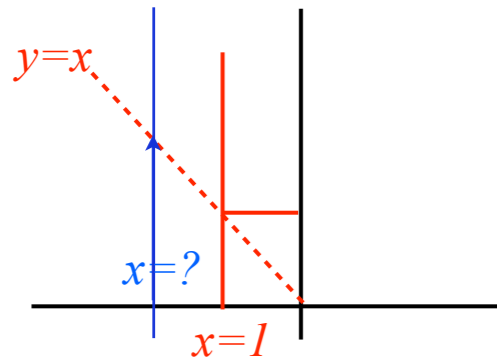
“Zag” line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$

Each $y=x^2$ parabola point found by just one “Zig-Zag”

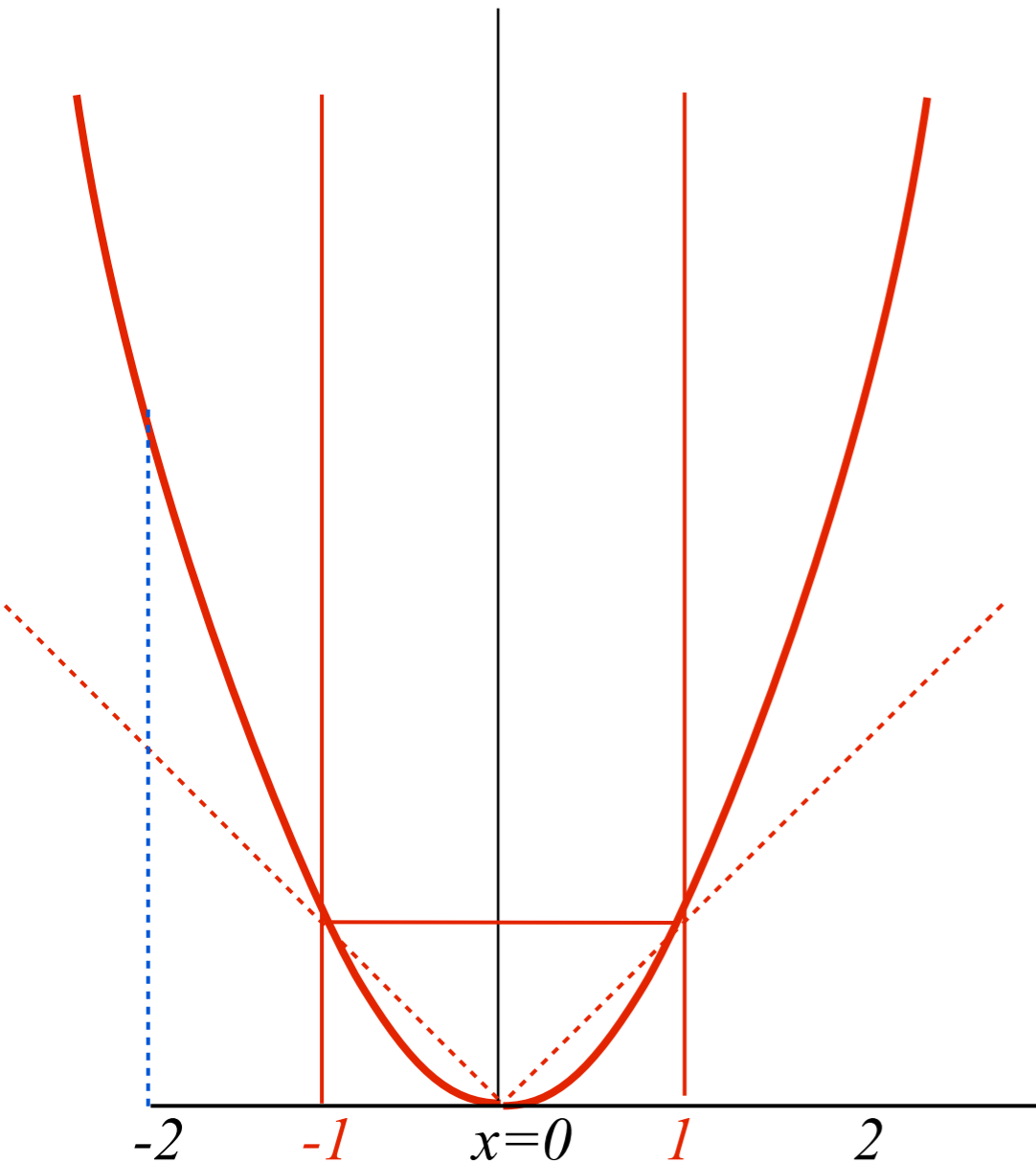
1. Pick an $(x=?)$ -line

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“Zag” line is $y=(?) \cdot x$
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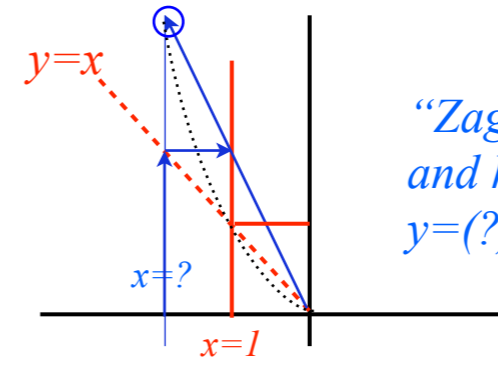
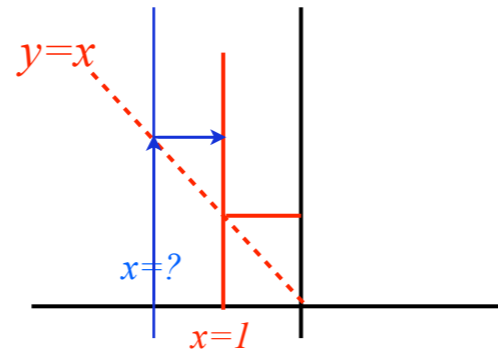
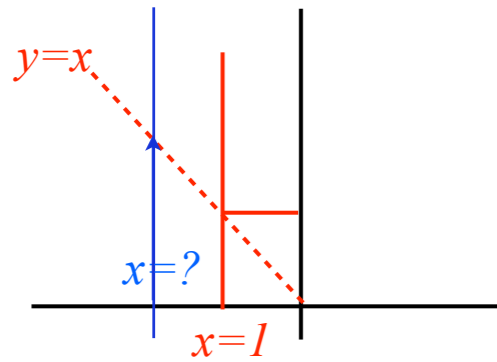
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one “Zig-Zag”

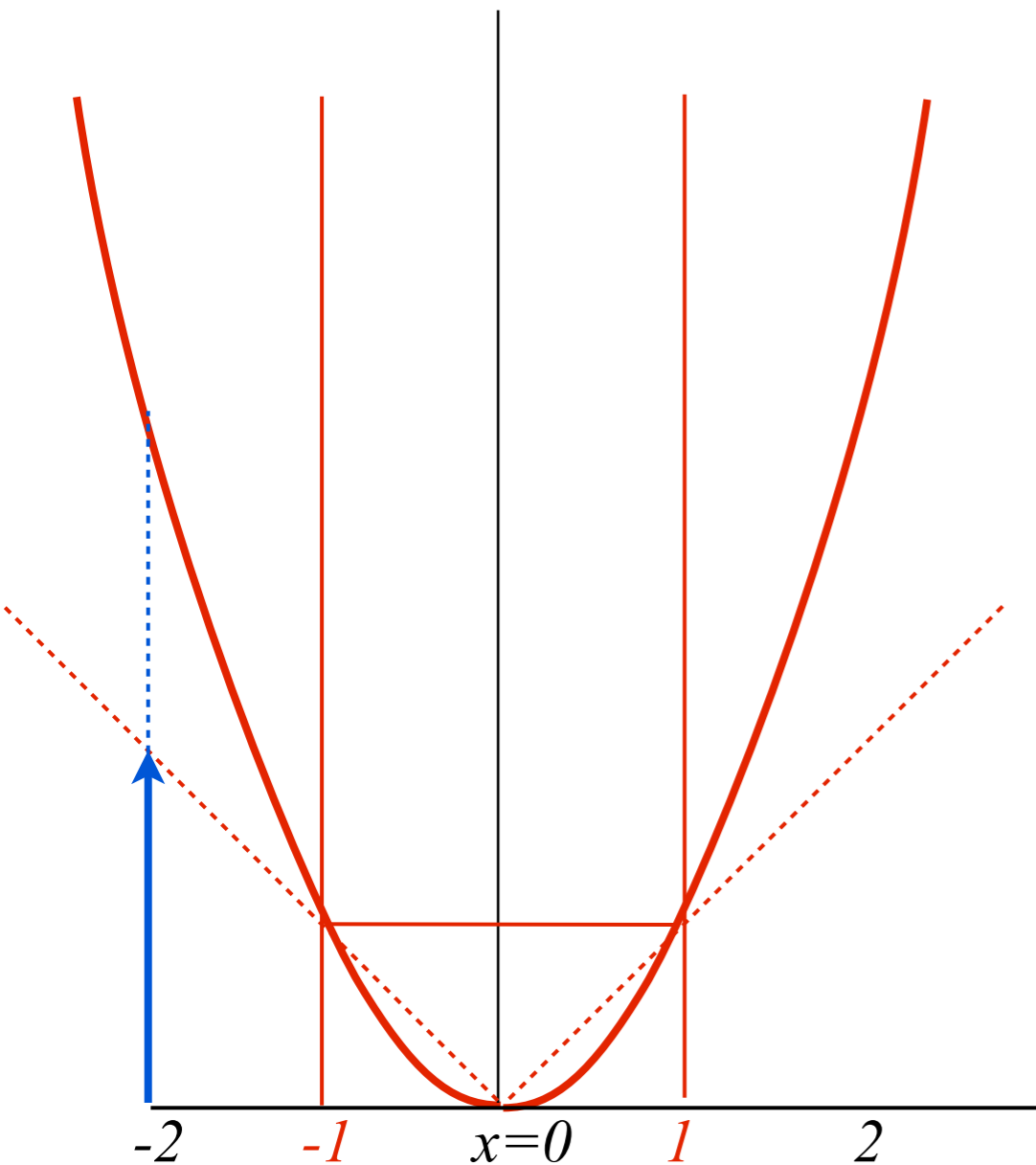
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“Zag” line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$



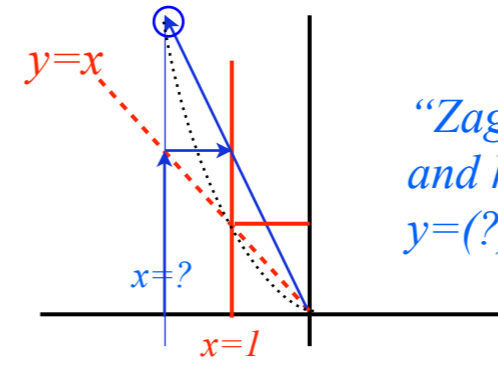
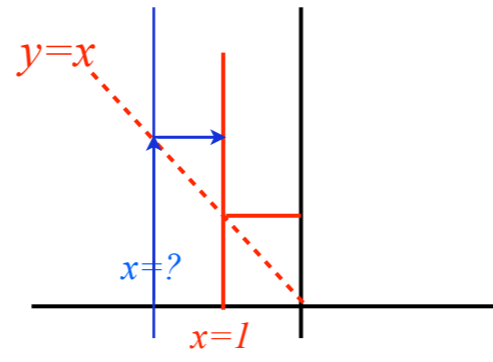
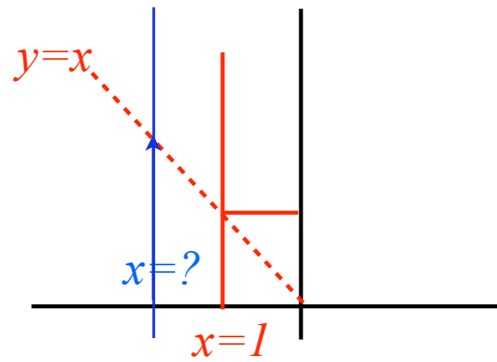
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one “Zig-Zag”

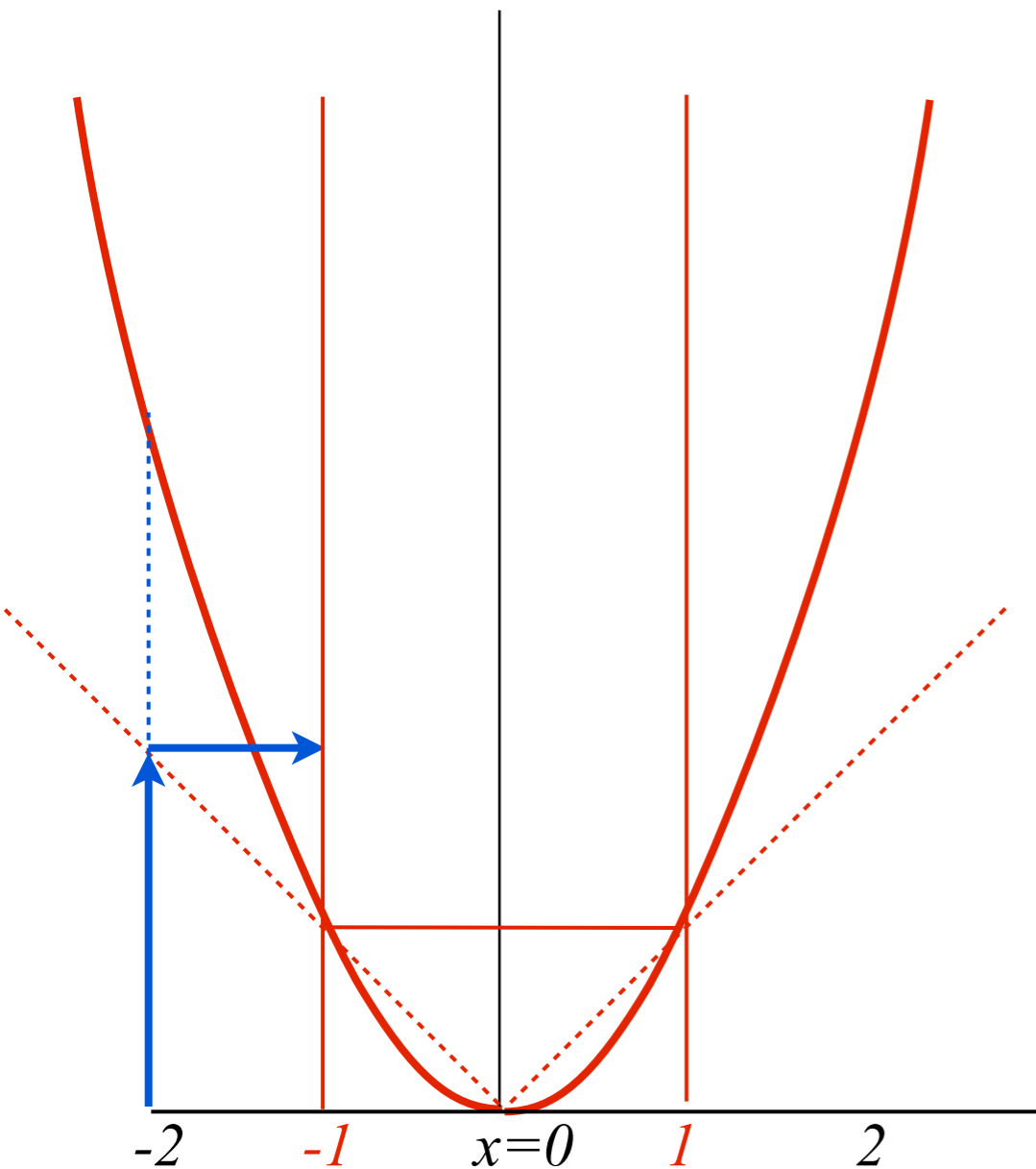
1. Pick an $(x=?)$ -line

2. “Zig” from its $y=x$ intersection to $x=1$ line

3. “Zag” from origin back to $(x=?)$ -line



“Zag” line is $y=(?) \cdot x$
and hits $(x=?)$ -line at
 $y=(?) \cdot (?) = (?)^2$



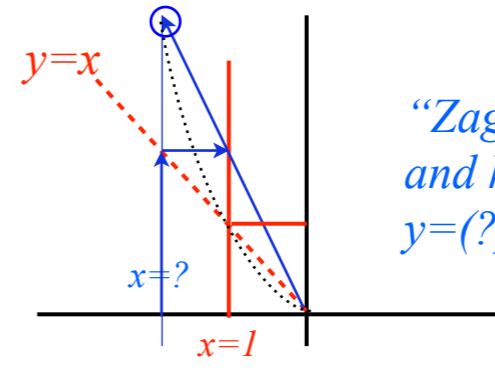
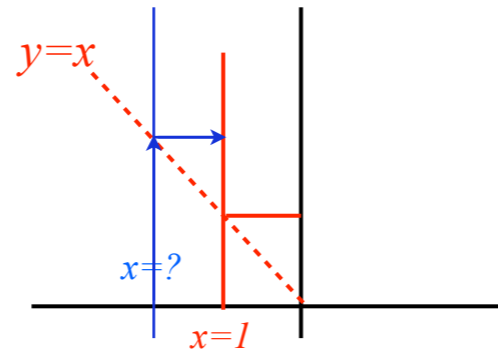
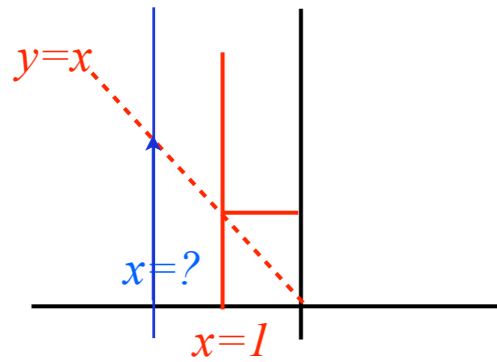
Unit 1
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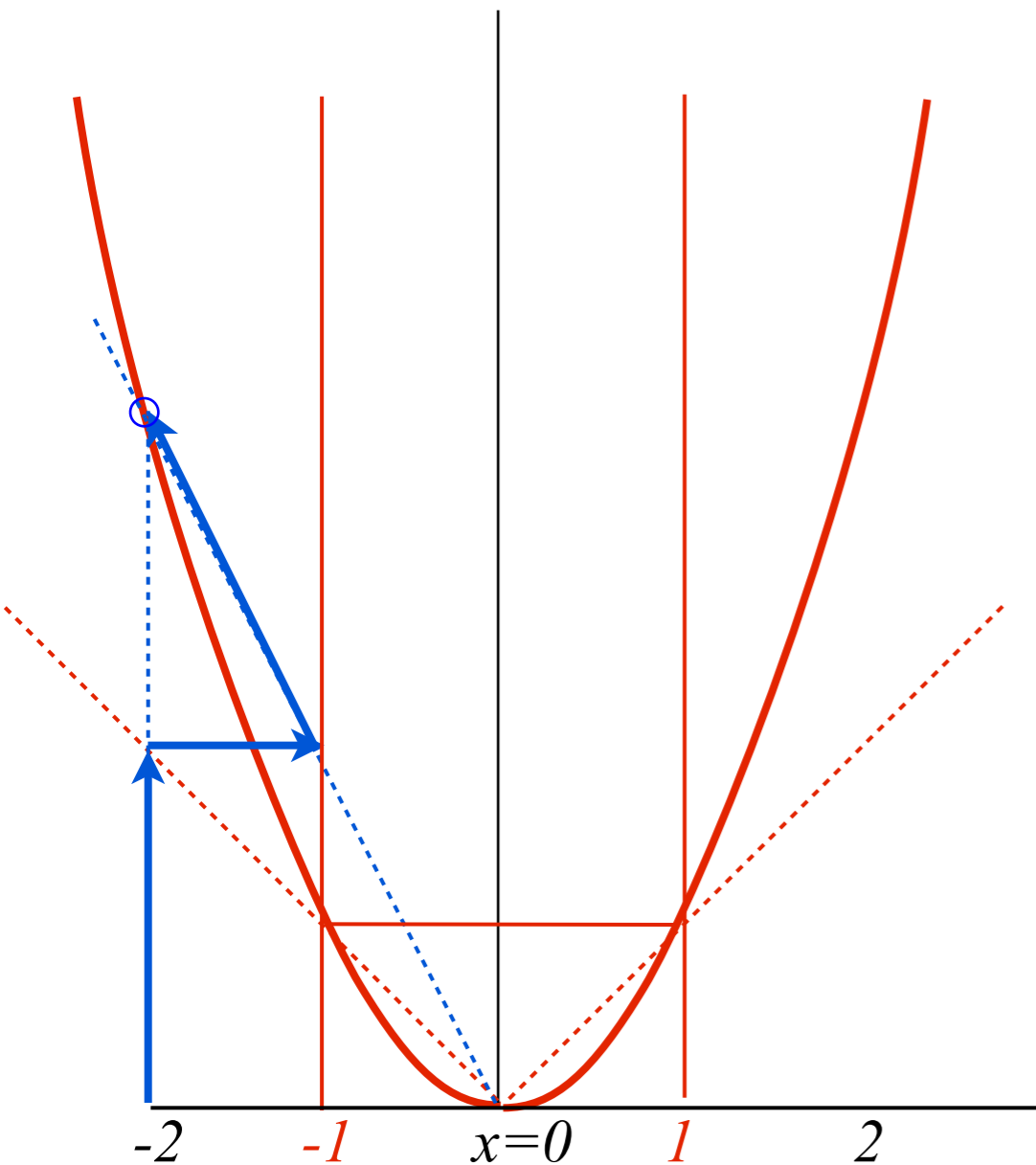
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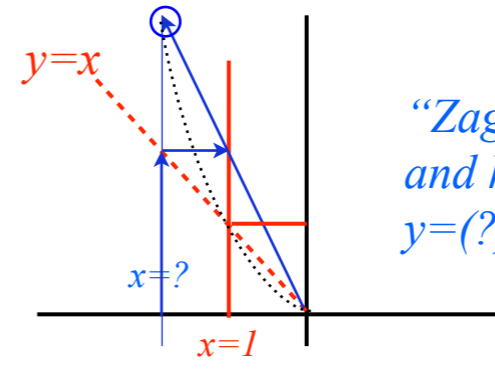
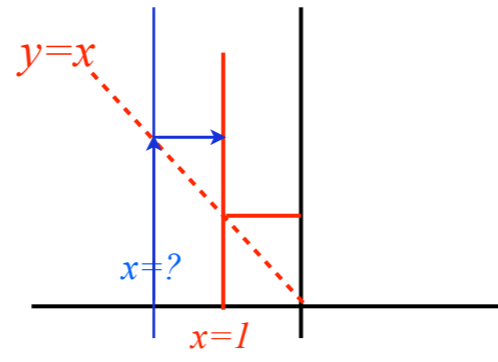
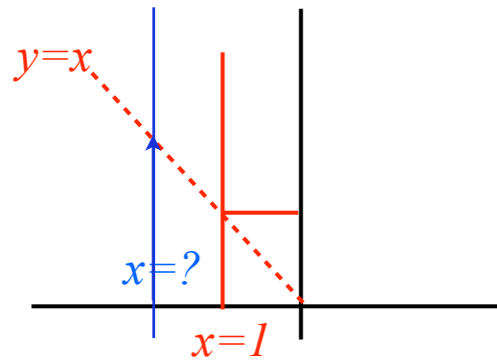
Unit 1
Fig. 9.1

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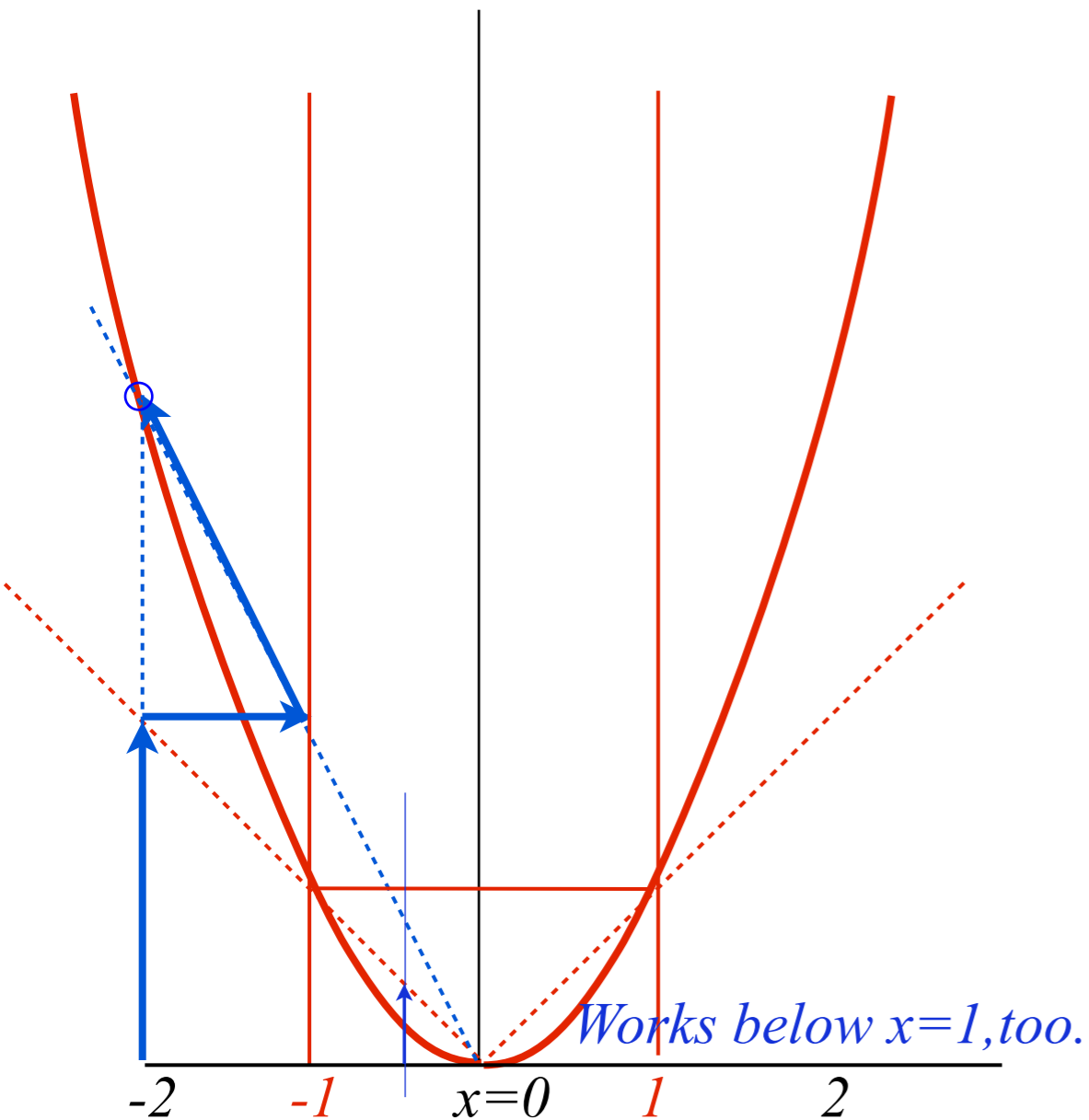
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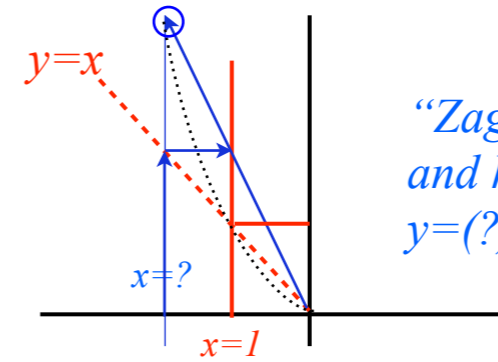
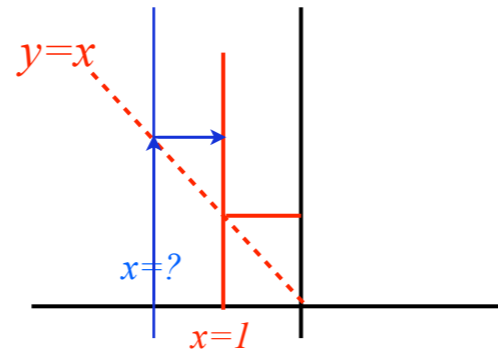
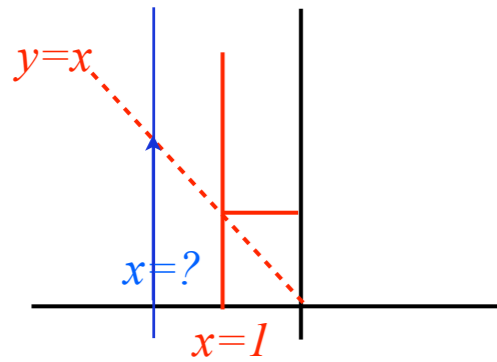


Each $y=x^2$ parabola point found by just one “Zig-Zag”

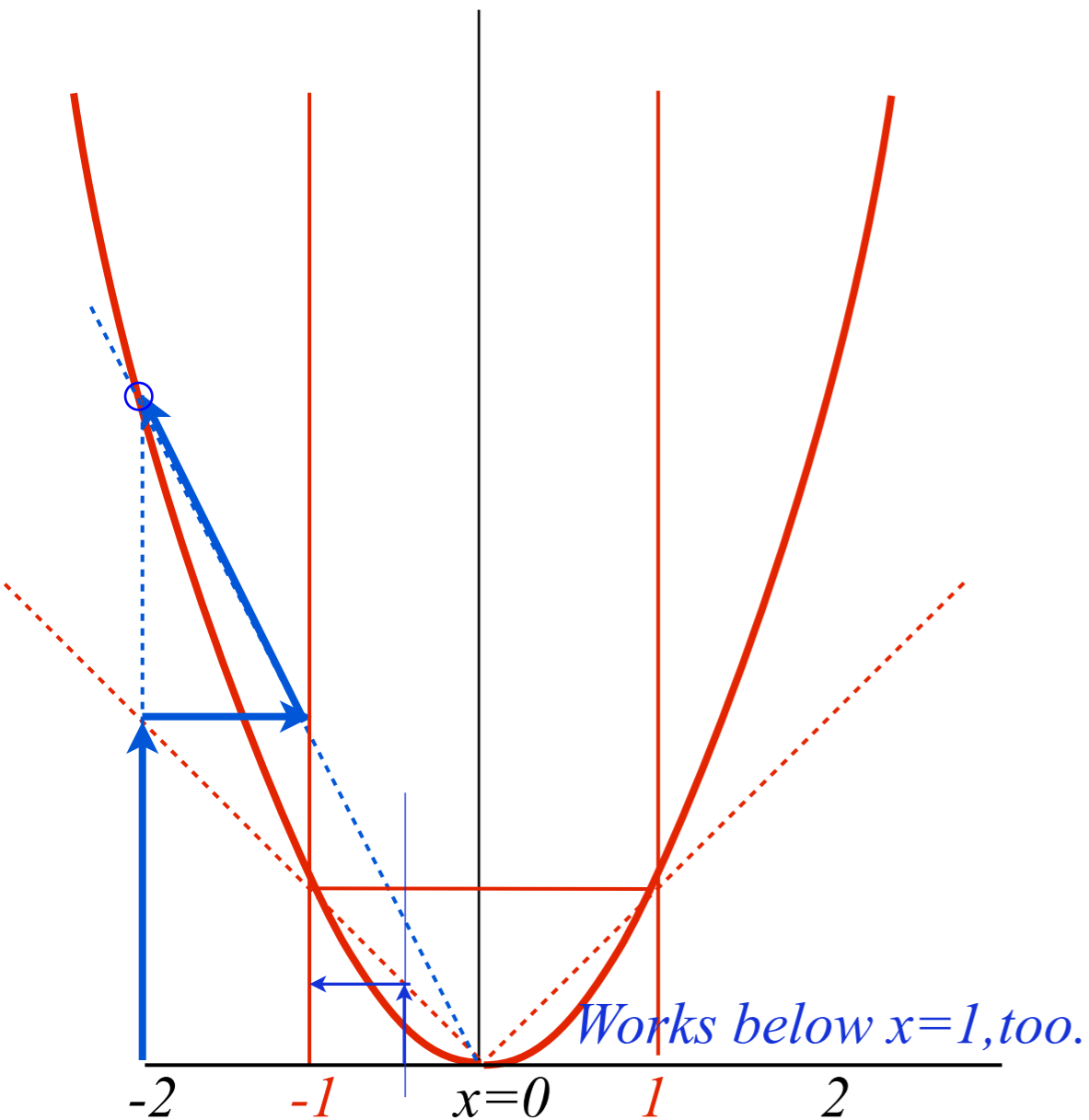
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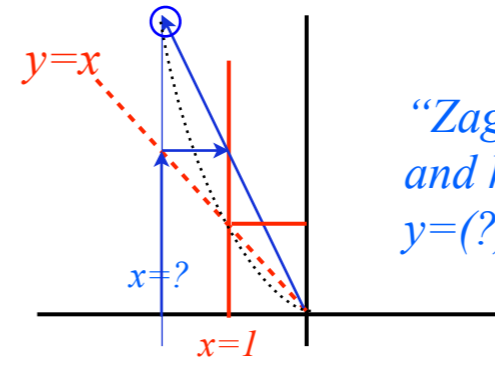
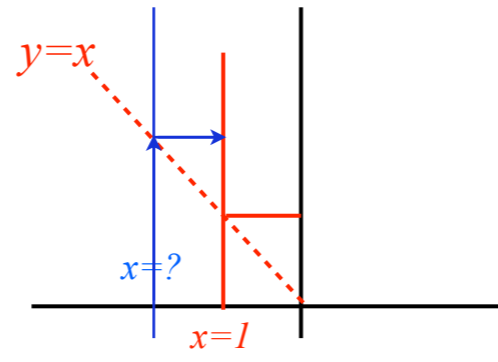
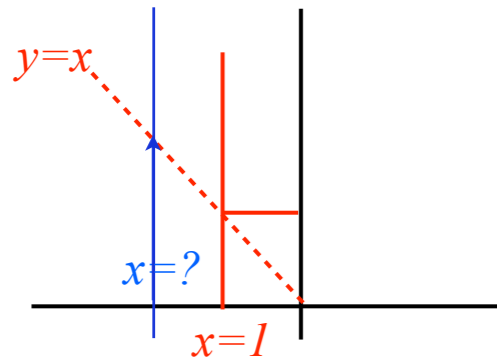


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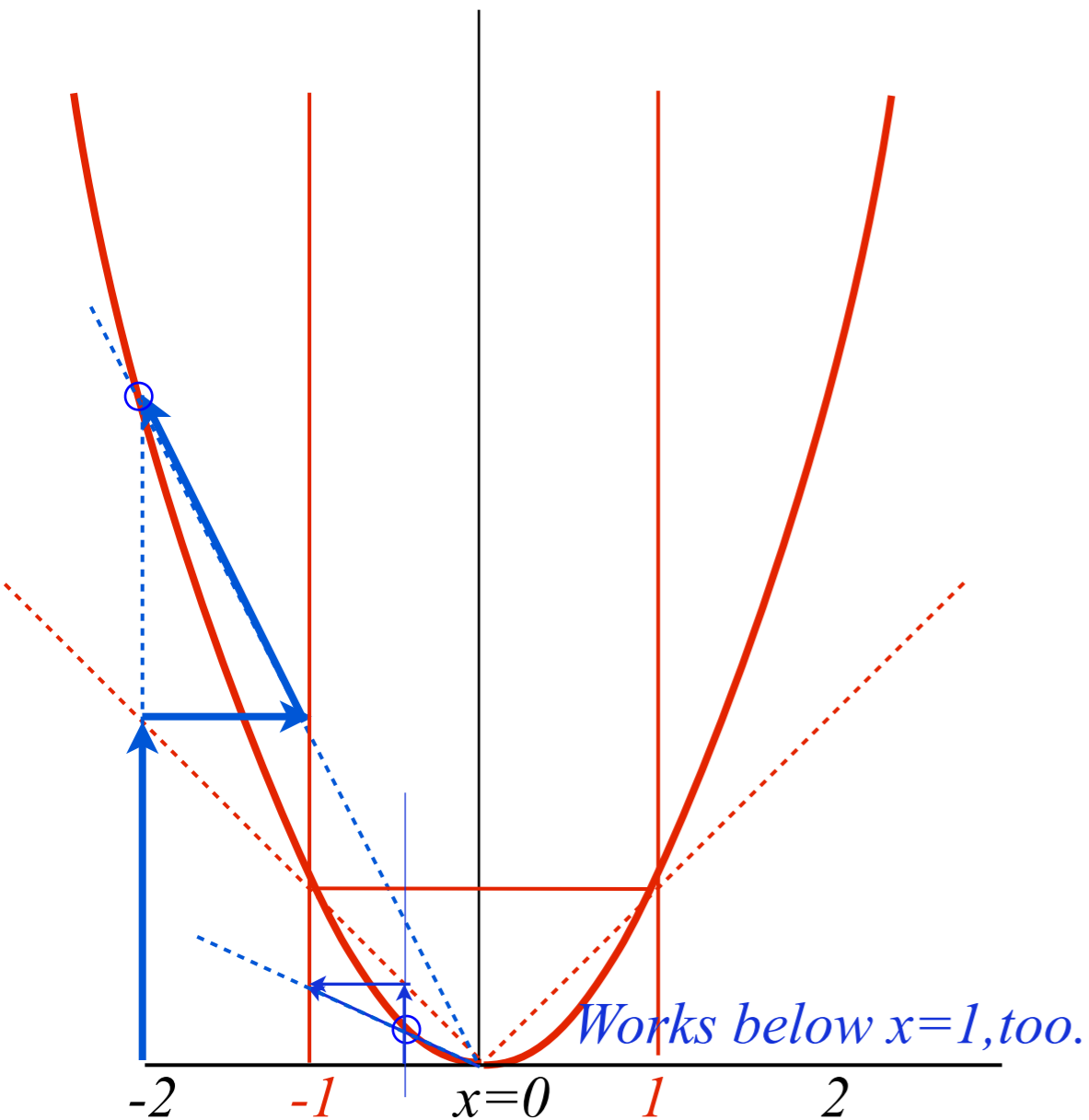
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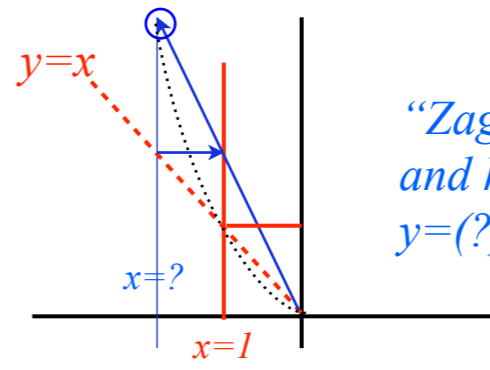
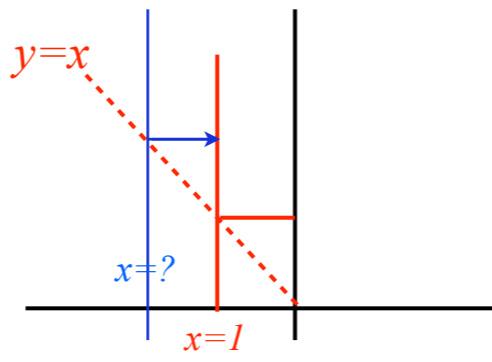
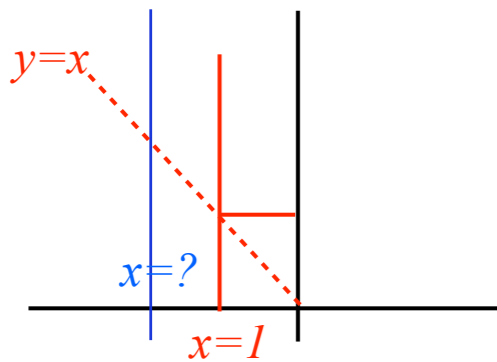
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Unit 1
Fig. 9.1

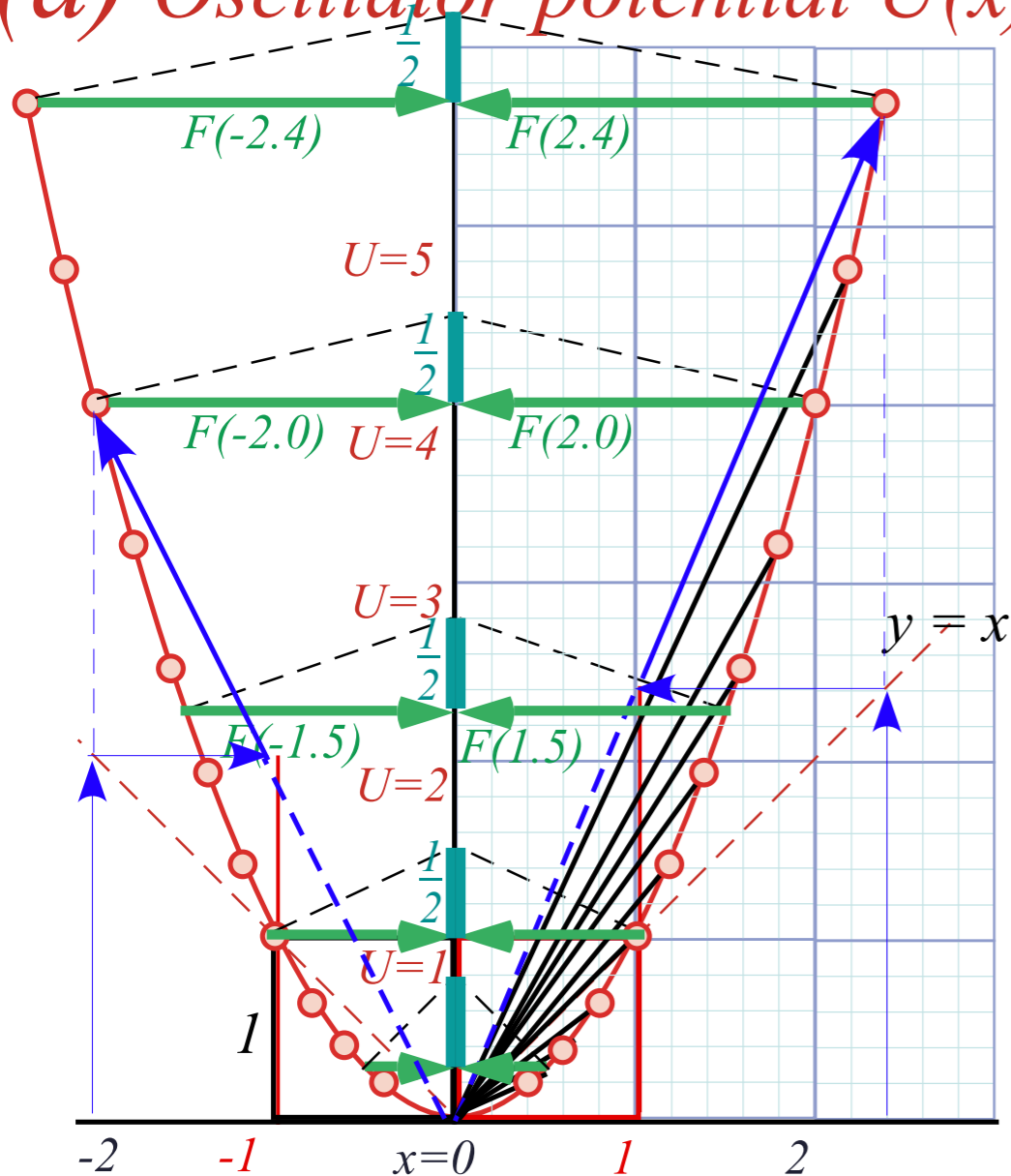
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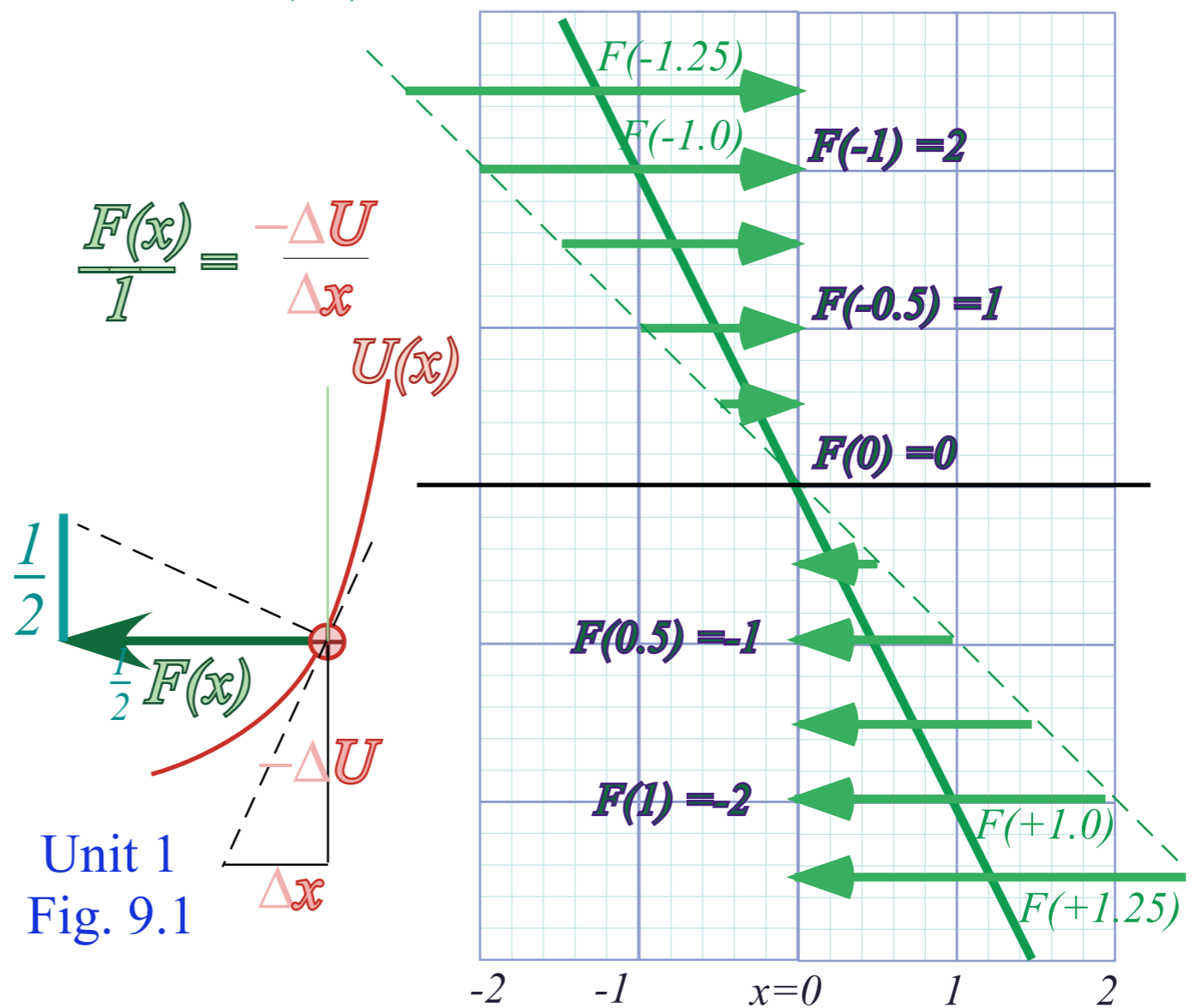


"Zag" line is $y=(?) \cdot x$ and hits $(x=?)$ -line at $y=(?) \cdot (?) = (?)^2$

(a) Oscillator potential $U(x)=x^2$



(b) Hooke-Law Force $F(x) = -2x$

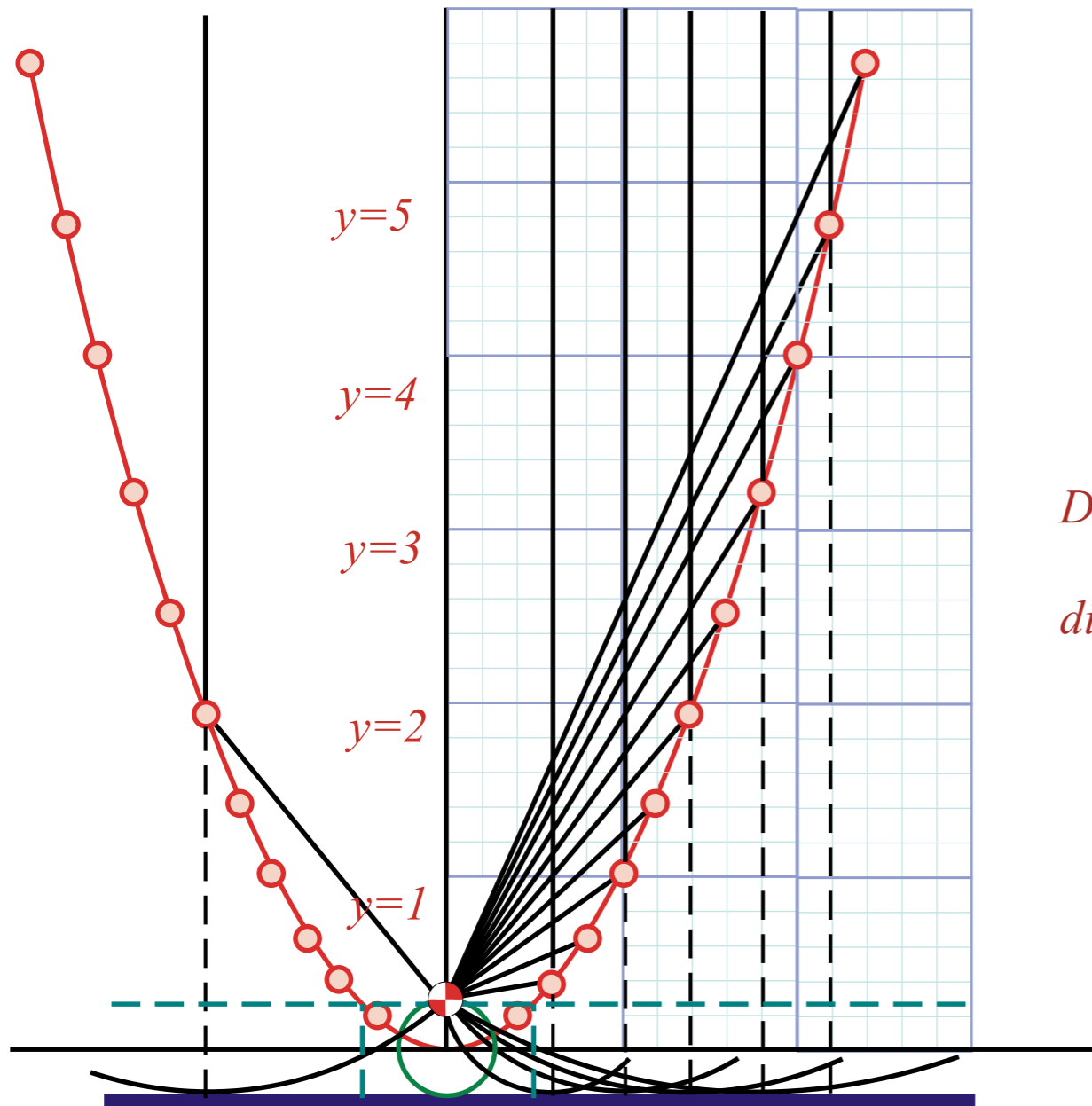


$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

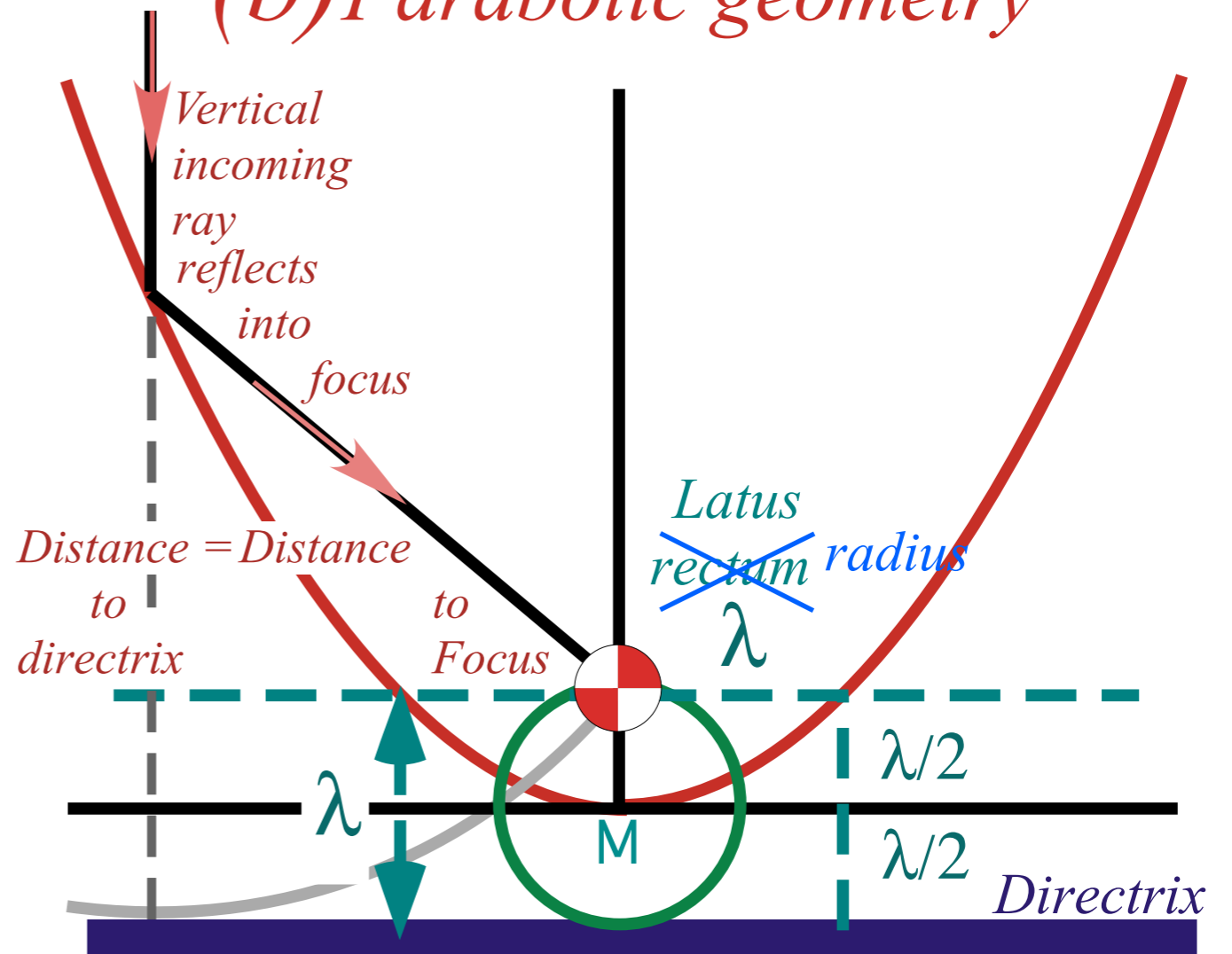
Unit 1
Fig. 9.1

A more conventional parabolic geometry...

(a) Parabolic Reflector $y=x^2$



(b) Parabolic geometry

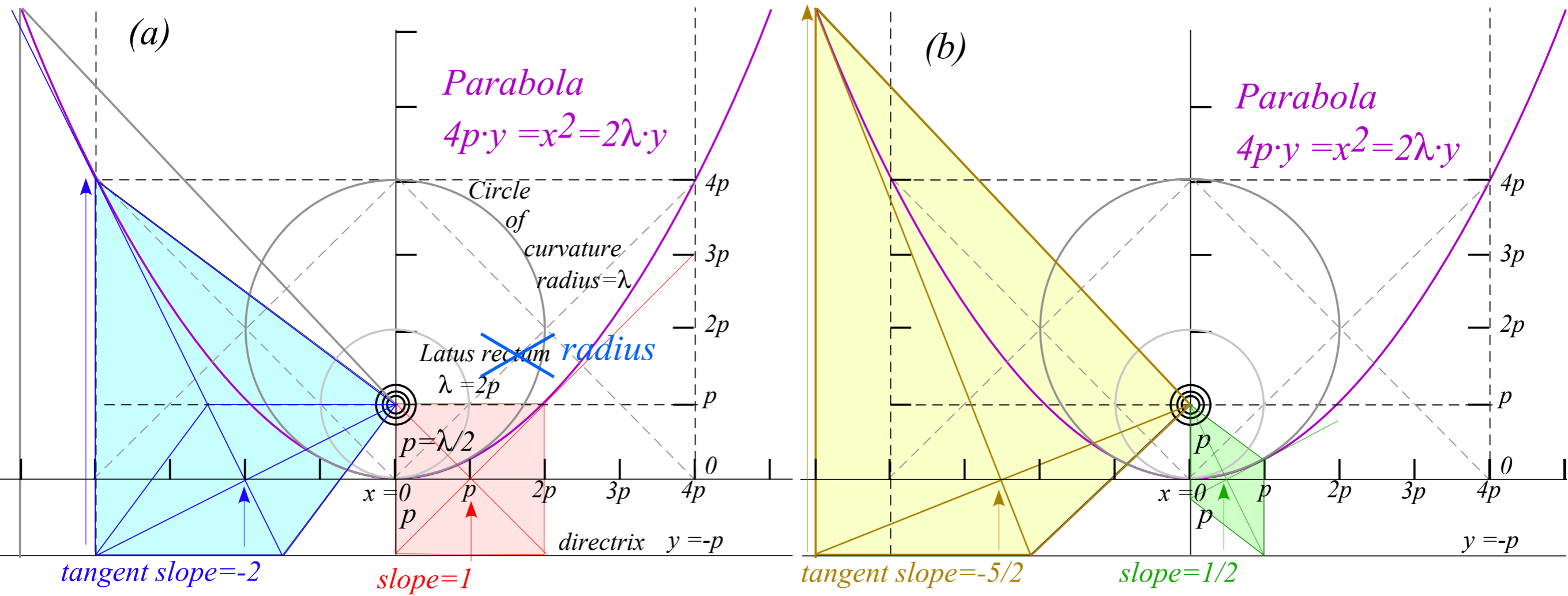


Better name† for λ : *latus radius*

Unit 1
Fig. 9.3

† Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

...conventional parabolic geometry...carried to extremes...



Unit 1
Fig. 9.4


Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

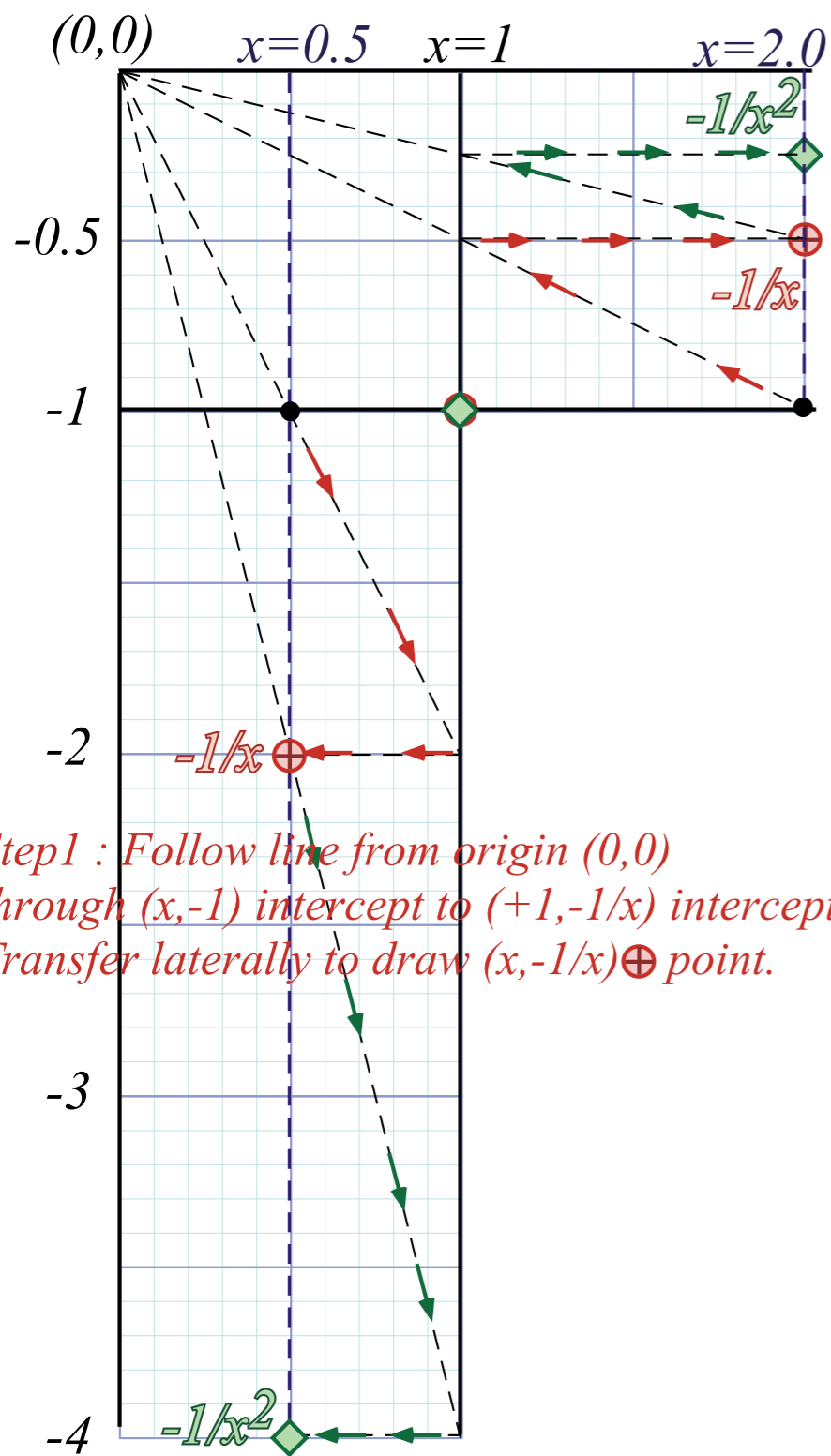
Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

 *Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields*

Compare mks units of Coulomb Electrostatic vs. Gravity

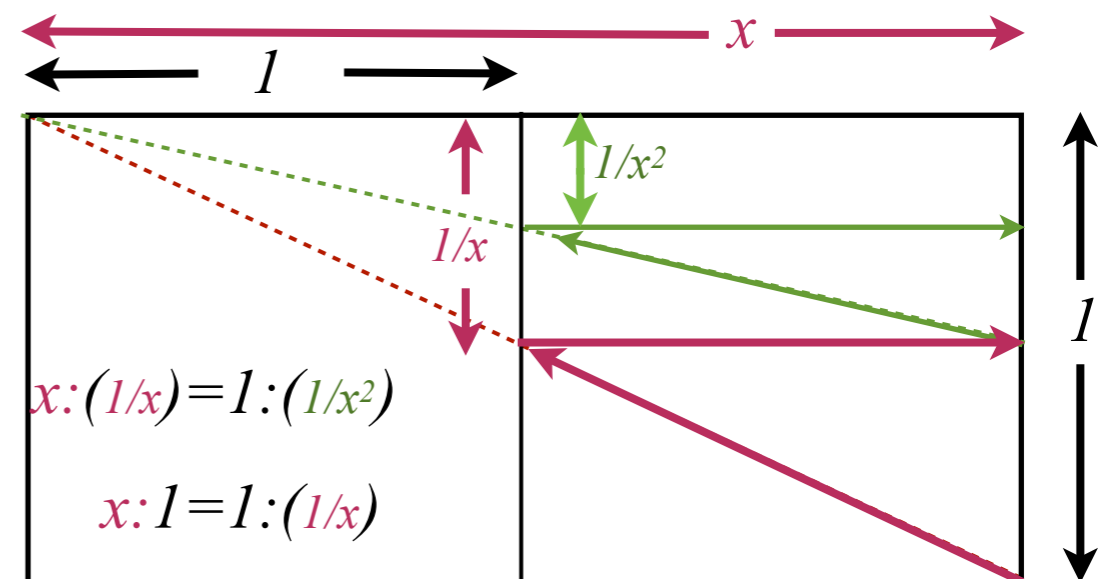
Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
 $F(x) = -1/r^2$ $U(x) = -1/r$



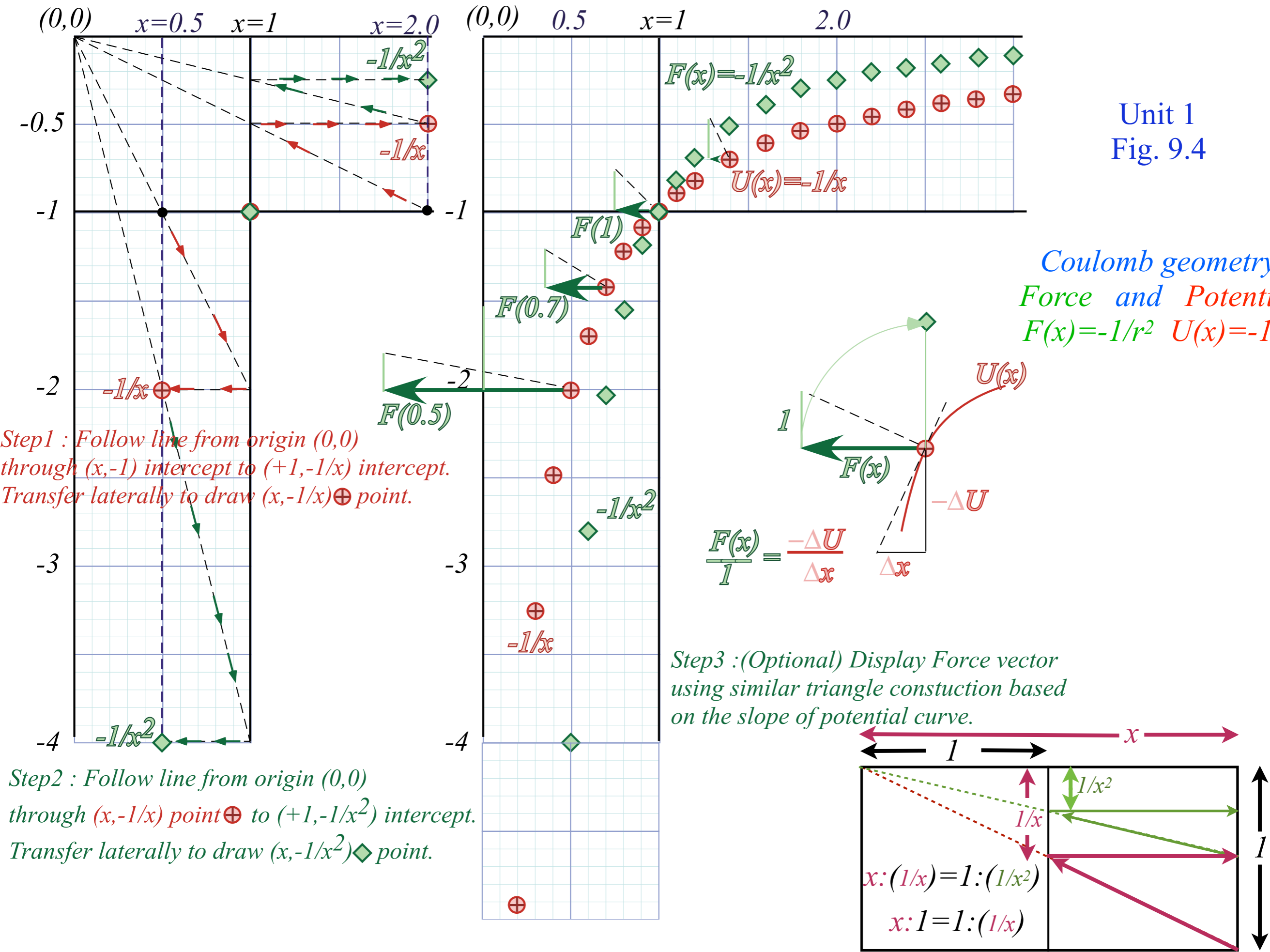
Step 1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step 2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x^2) intercept. Transfer laterally to draw (x,-1/x^2)◇ point.



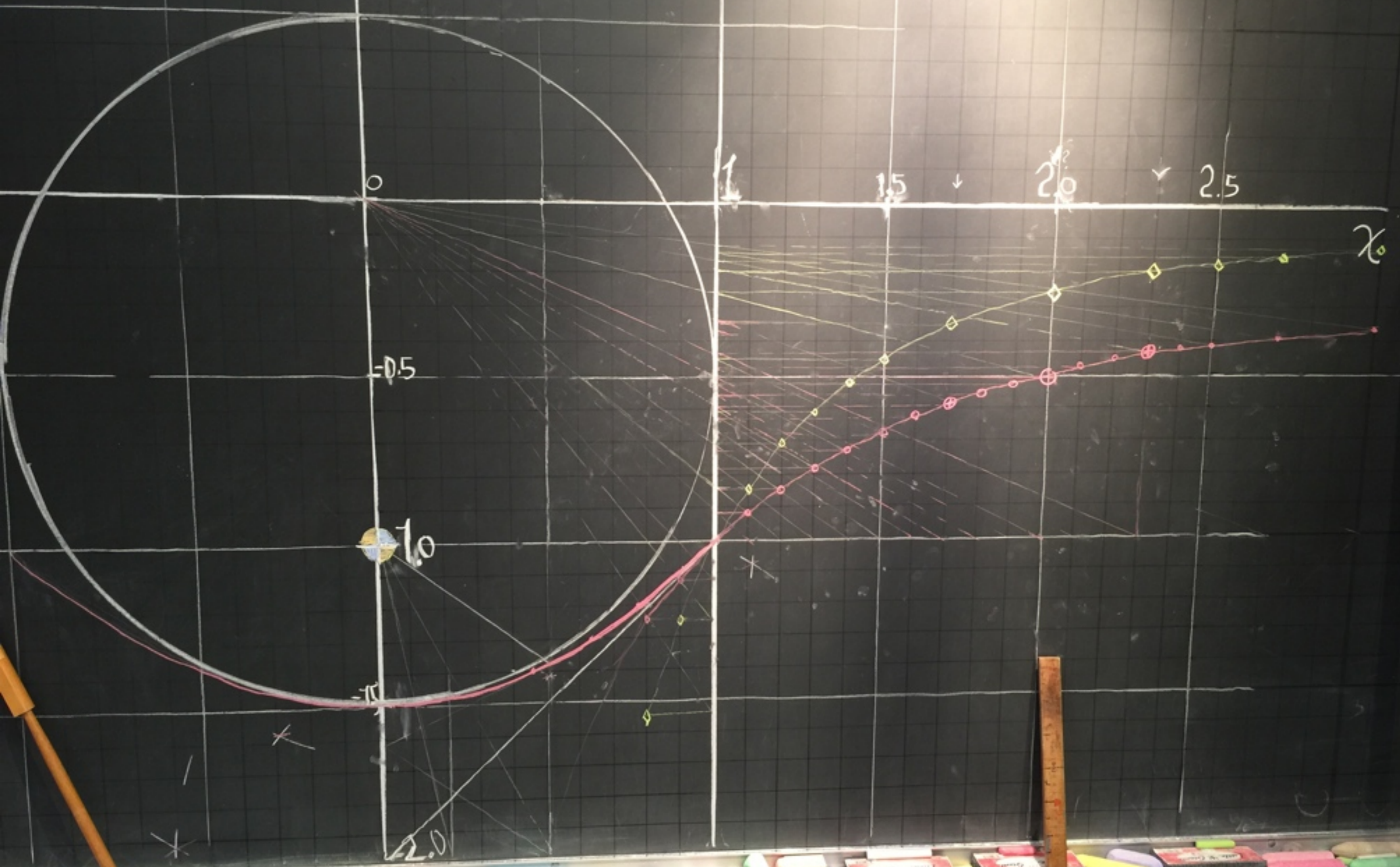
Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
 $F(x) = -1/r^2$ $U(x) = -1/r$



$$V(x) = \frac{1}{x}$$

$$F(x) = \frac{1}{x^2}$$



Geometry of common power-law potentials

Geometric (Power) series

“Zig-Zag” exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$ -potential and $-1/r^2$ -force fields

 *Compare mks units of Coulomb Electrostatic vs. Gravity*

Compare *mks* units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong \boxed{?.?.10?} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong \overset{\sim 9E9 \sim 10^{10}}{9,000,000,000} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

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More precise value for electrostatic constant : $1/4\pi\epsilon_0 = 8.987,551 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19} \text{ Coulomb}$



Repulsive (+)(+) or (-)(-)

Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

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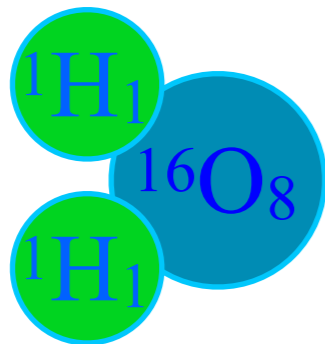
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 8 notes that 1 (cm)³ = 1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules

Avogadro's
Number

$\sim 0.3 \cdot 10^{23}$

H₂O Molecular weight ~18



Compare mks units for Coulomb fields

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$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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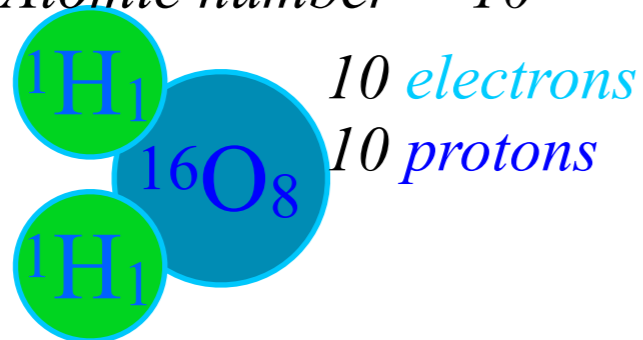
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“Fingertip Physics” of Ch. 9 notes that 1 (cm)³ = 1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or $\sim 3 \cdot 10^{23}$ electrons
 $\sim 0.3 \cdot 10^{23}$ and $\sim 3 \cdot 10^{23}$ protons.

H₂O Molecular weight ~ 18

Atomic number = 10



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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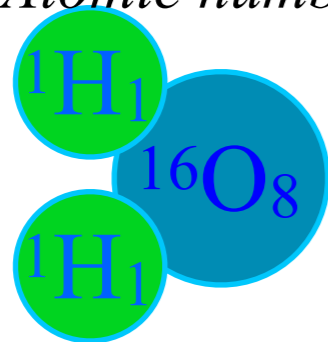
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 $\sim 0.3 \cdot 10^{23}$ and $\sim 3 \cdot 10^{23}$ protons.

H₂O Molecular weight ~ 18

Atomic number = 10



10 electrons That is $\sim -3 \cdot 10^{23} \cdot 1.6022 \cdot 10^{-19} \text{ Coulomb}$ or about $-0.5 \cdot 10^{+5} \text{ C}$ or $-50,000 \text{ Coulomb}$
 10 protons plus $\sim +3 \cdot 10^{23} \cdot 1.6022 \cdot 10^{-19} \text{ Coulomb}$ or about $+0.5 \cdot 10^{+5} \text{ C}$ or $+50,000 \text{ Coulomb}$

Equals zero total charge

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

~9E9~10¹⁰

More precise value for electrostatic constant : $1/4\pi\epsilon_0 = 8.987,551 \cdot 10^9 \text{Nm}^2/\text{C}^2 \sim 9 \cdot 10^9 \sim 10^{10}$

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Repulsive (+)(+) or (-)(-)

Attractive (+)(-) or (-)(+)

vs

Always Attractive (so far)

↑ COMPARE! ↓

BIG

vs
small



2. Gravitational force between m (kilograms) and M (kg.)

$$F^{grav.}(r) = -G \frac{mM}{r^2} \quad \text{where: } G = \boxed{?.?.? \cdot 10^?} \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.) !!!!

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More precise value for electrostatic constant : $1/4\pi\epsilon_0 = 8.987,551 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19} \text{ Coulomb}$



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vs

Always Attractive (so far)

↑ COMPARE! ↓

BIG

vs
small



2. Gravitational force between m (kilograms) and M (kg.) !!!!

$$F^{grav.}(r) = -G \frac{mM}{r^2} \quad \text{where: } G = 0.000,000,000,067 \frac{\sim 2/3 10^{-10} \sim 10^{-10} \text{ Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3) 10^{-10}$

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$

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Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}} \quad \text{!!!!}$$

Compare mks units for Coulomb fields

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Discussion of repulsive force and PE in Ch. 9...

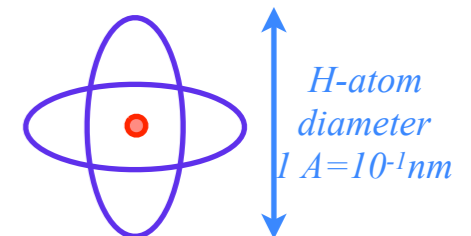
1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

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Atomic size ~ 1 Angstrom = 10^{-10} m

Nuclear size $\sim 10^{-15}$ m = 1 femtometer = 1fm



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$



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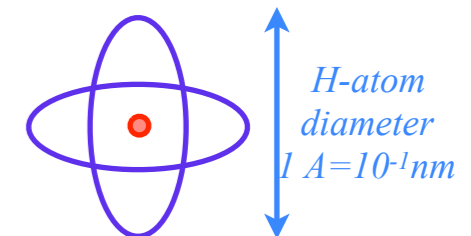
$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$



Nuclear size $\sim 10^{-15} \text{ m} = 1 \text{ femtometer} = 1 \text{ fm}$

Atomic size $\sim 1 \text{ Angstrom} = 10^{-10} \text{ m}$

Big molecule $\sim 10 \text{ Angstrom} = 10^{-9} \text{ m} = 1 \text{ nanometer} = 1 \text{ nm}$



Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

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Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$

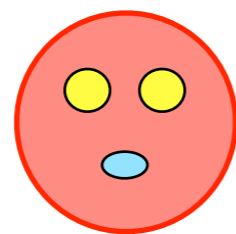


Atomic size ~ 1 Angstrom = 10^{-10} m

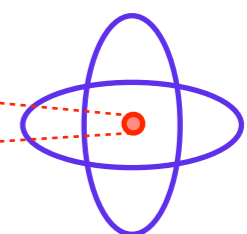
Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm

Nuclear size $\sim 10^{-15}$ m = 1 femtometer = 1fm

also: 1fm = 10^{-13} cm = 1Fermi
= 1Fm



1 Fermi



H-atom
diameter
1 A = 10^{-1} nm

Compare mks units for Coulomb fields

1. Electrostatic force between q (Coulombs) and Q (C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Newtons} \cdot \text{meter} \cdot \text{square}}{\text{per square Coulomb}}$$



Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

quantum of charge: $|e|=1.6022 \cdot 10^{-19}$ Coulomb

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q (Coulombs) and Q (C.)

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad \text{where: } \frac{1}{4\pi\epsilon_0} \cong 9,000,000,000 \frac{\text{Joule}}{\text{per square Coulomb}}$$

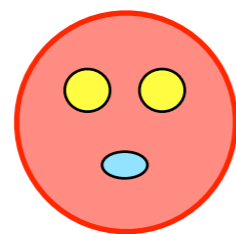


Atomic size ~ 1 Angstrom = 10^{-10} m

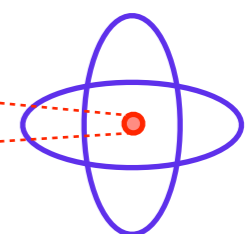
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= 1Fm



1 Fermi



H-atom
diameter
1 A = 10^{-1} nm

nuclear radii are 100,000 to 1,000,000 times smaller than atomic/chemical radii

...so nuclear qQ/r energy 100,000 to 1,000,000 times **larger** than that of atomic/chemical...

Geometry of idealized “Sophomore-physics Earth”

→ *Coulomb field outside Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

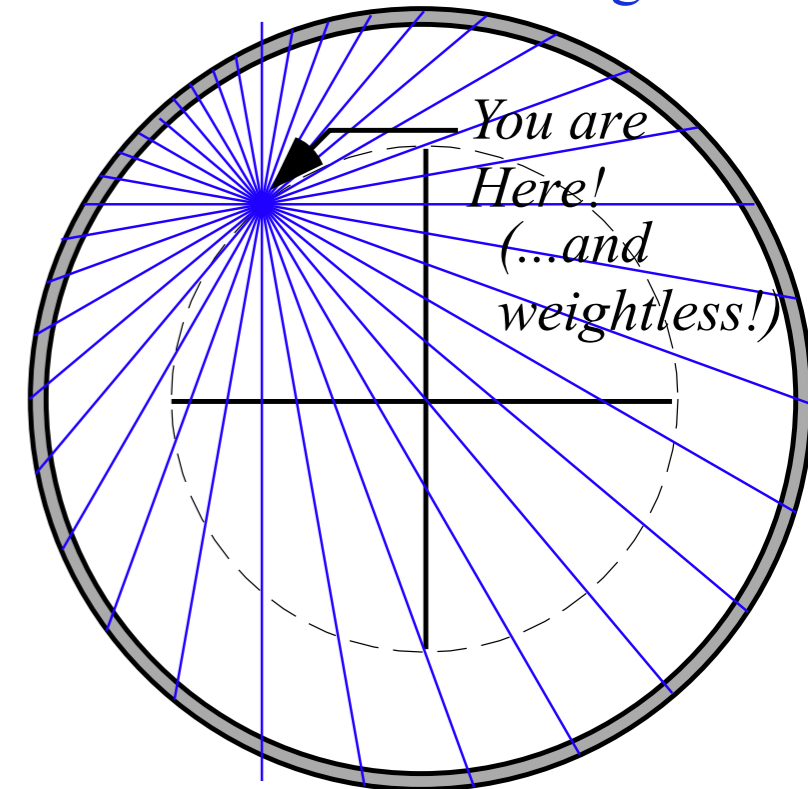
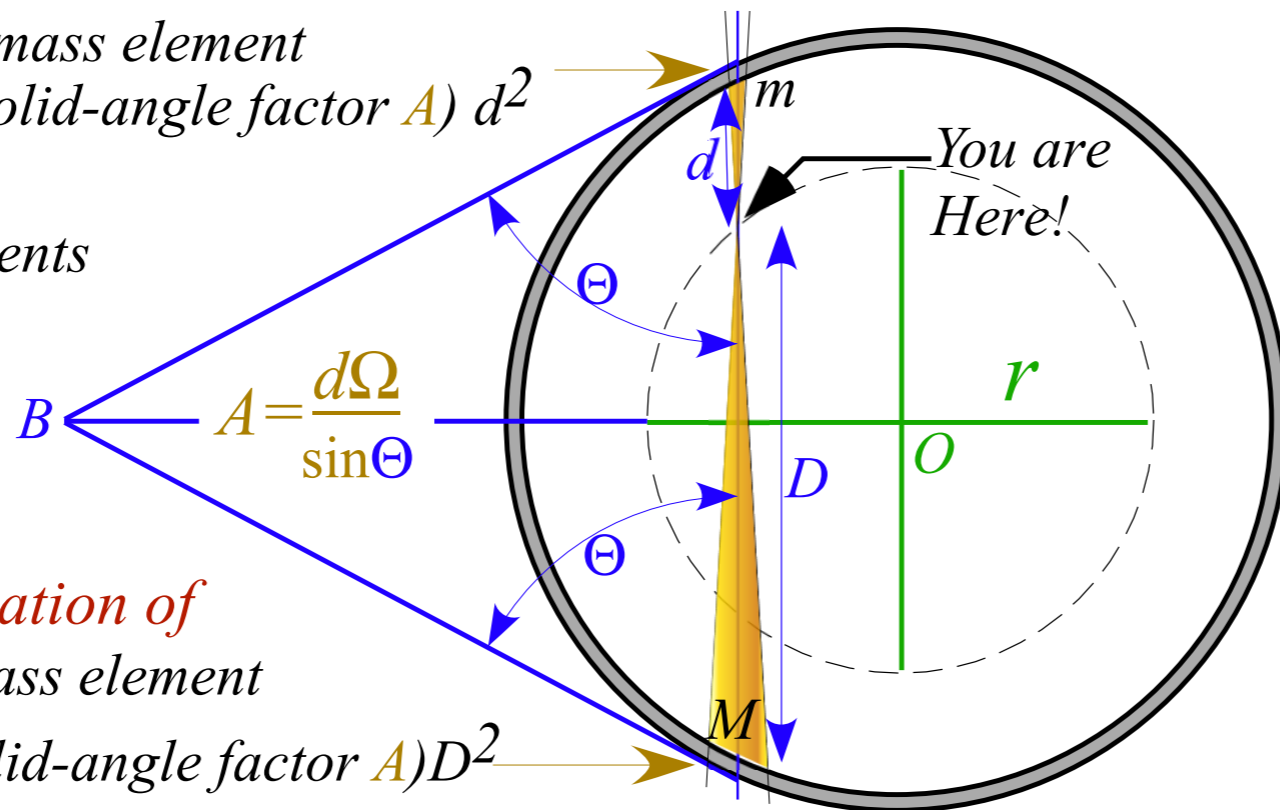
Gravity at r
due to shell mass elements

$$\frac{GM}{D^2} - \frac{Gm}{d^2} =$$

$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

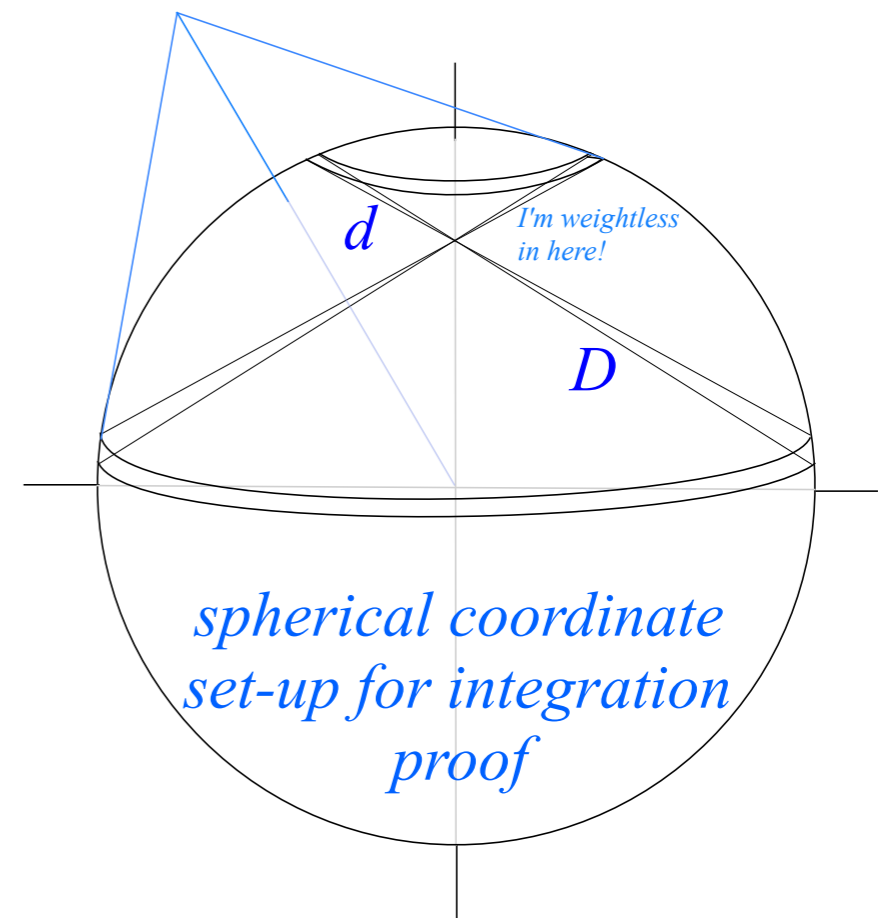
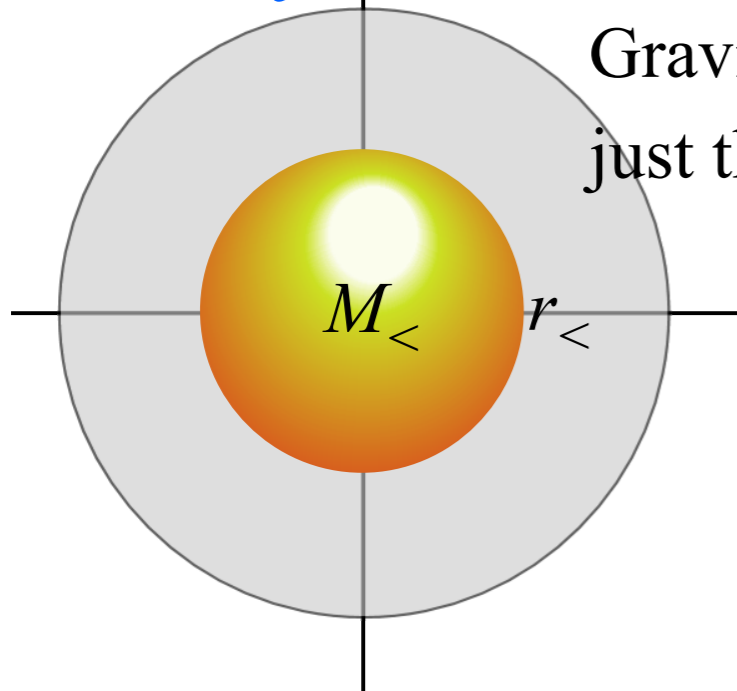
Cancellation of
Shell mass element

$M = (\text{solid-angle factor } A)D^2$



Coulomb force inside-spherical body due to stuff below you, only.

Gravitational force at $r_<$ is
just that of planet $M_<$ below $r_<$



Coulomb force vanishes inside-spherical shell (Gauss-law)

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Fig. 9.6

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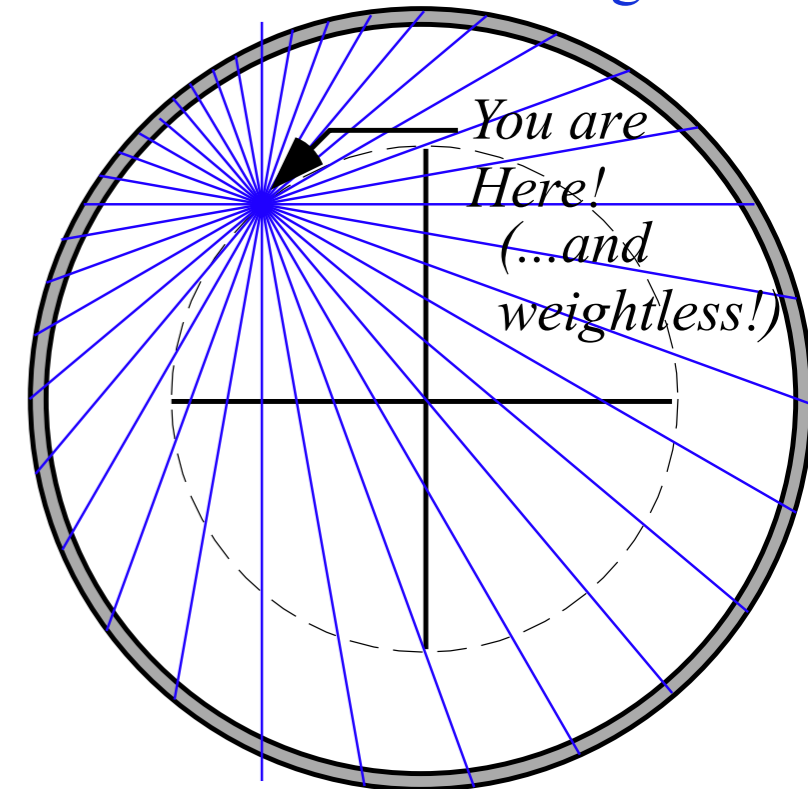
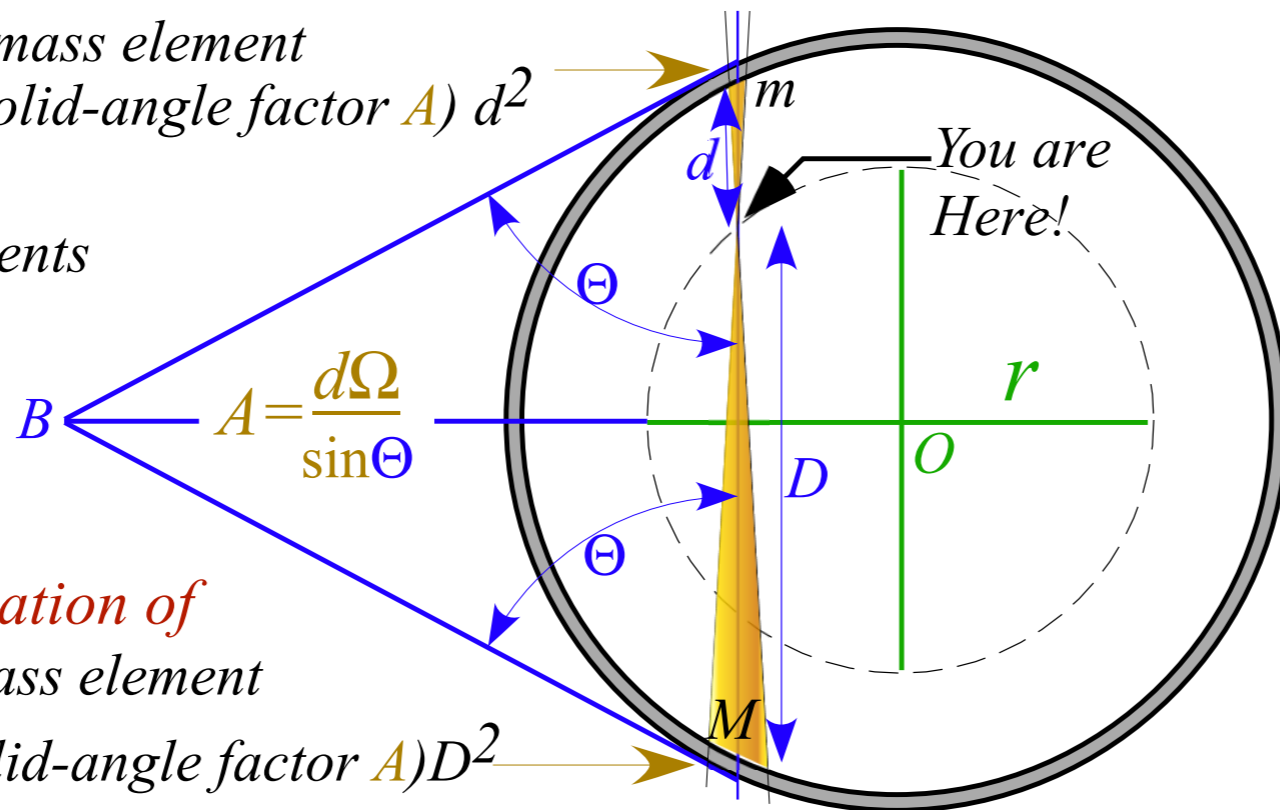
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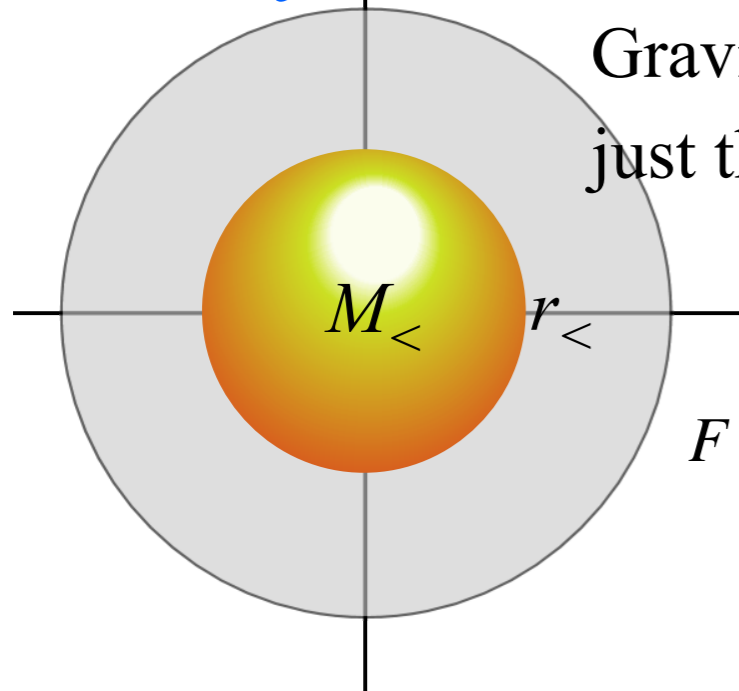
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Cancellation of
Shell mass element

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Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_{<}$ is
just that of planet $M_{<}$ below $r_{<}$

Note:
Hooke's (linear) force law
for $r_{<}$ inside uniform body

$$F^{inside}(r_{<}) = G \frac{mM_{<}}{r_{<}^2} = Gm \frac{4\pi}{3} \frac{M_{<}}{4\pi r_{<}^3} r_{<} = Gm \frac{4\pi}{3} \rho_{\oplus} r_{<} = mg \frac{r_{<}}{R_{\oplus}} \equiv mg \cdot x$$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

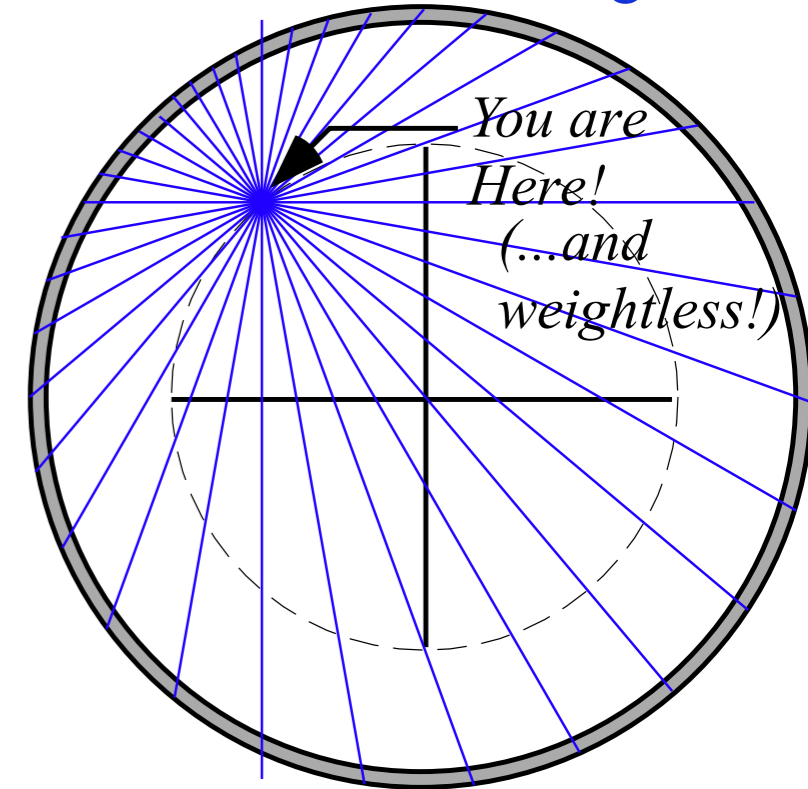
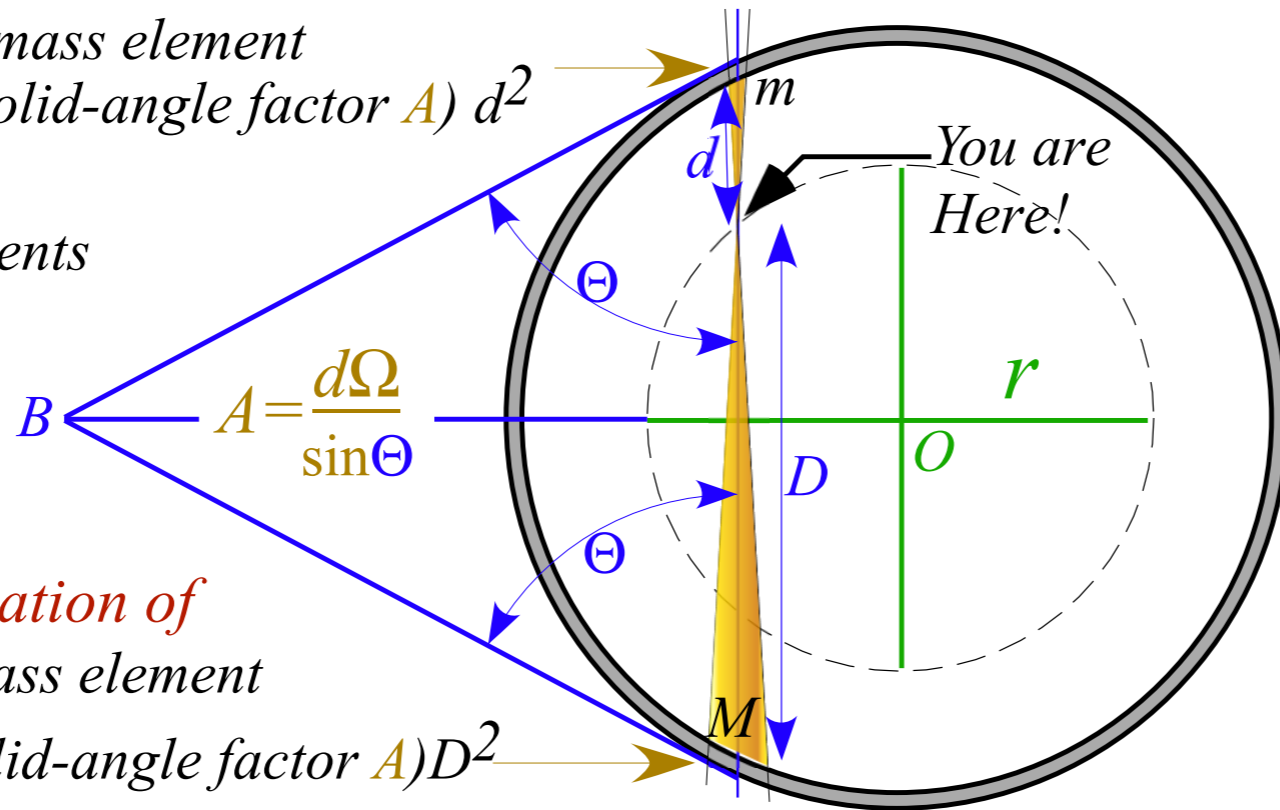
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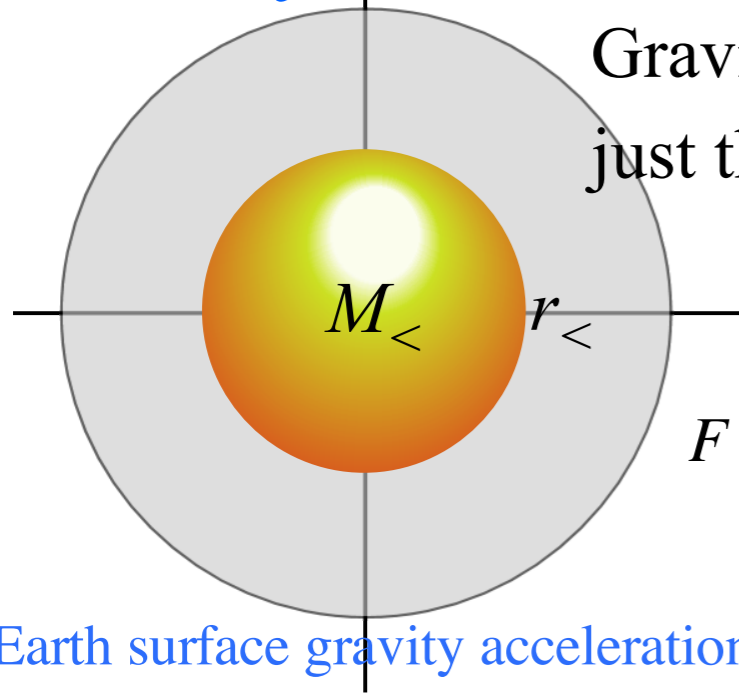
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Earth surface gravity acceleration: $g = G \frac{M_{\oplus}}{R_{\oplus}^2} = G \frac{M_{\oplus}}{R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \frac{M_{\oplus}}{4\pi R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \rho_{\oplus} R_{\oplus} = 9.8 m/s^2$

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3) 10^{-10}$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Shell mass element
 $m = (\text{solid-angle factor } A) d^2$

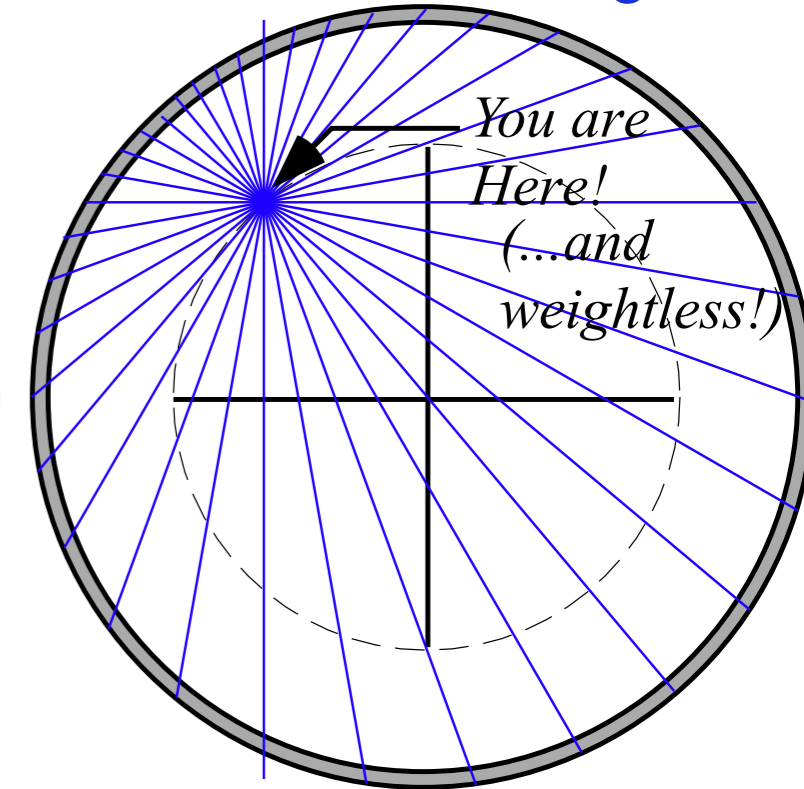
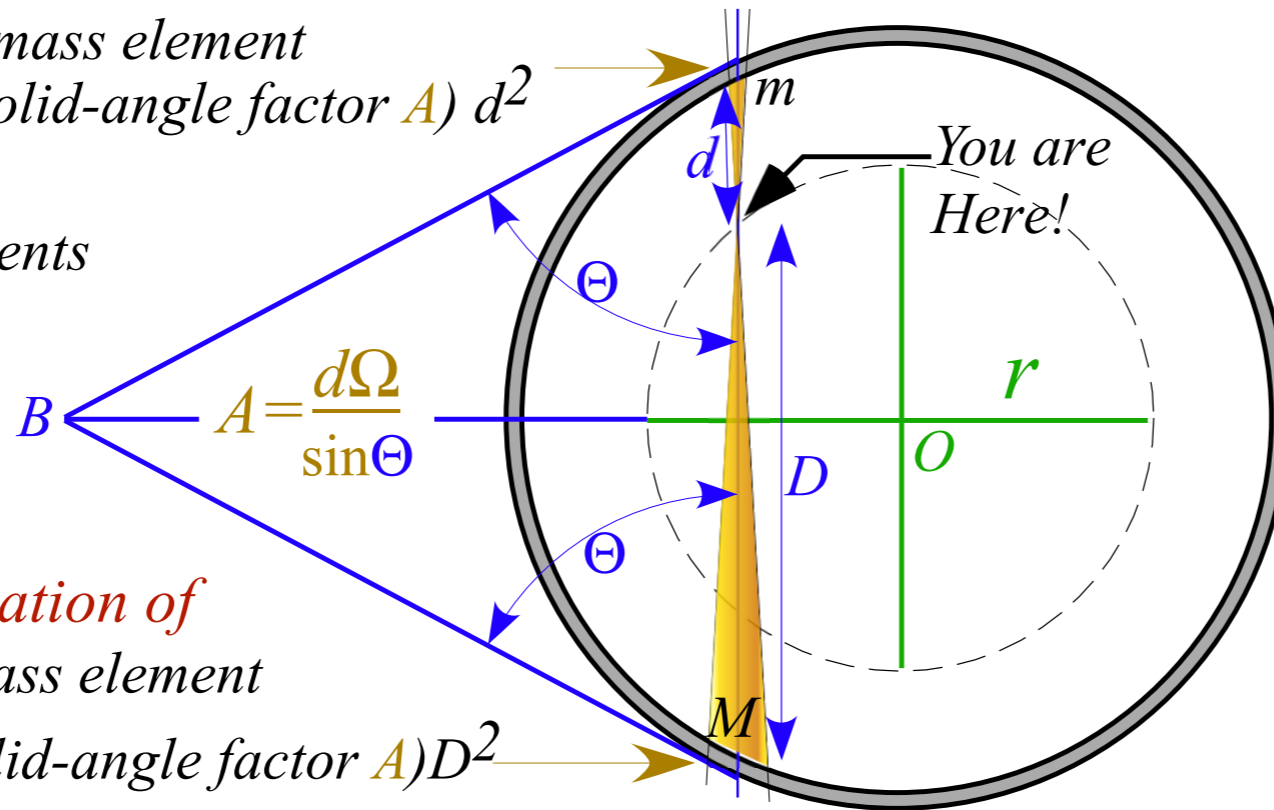
Gravity at r
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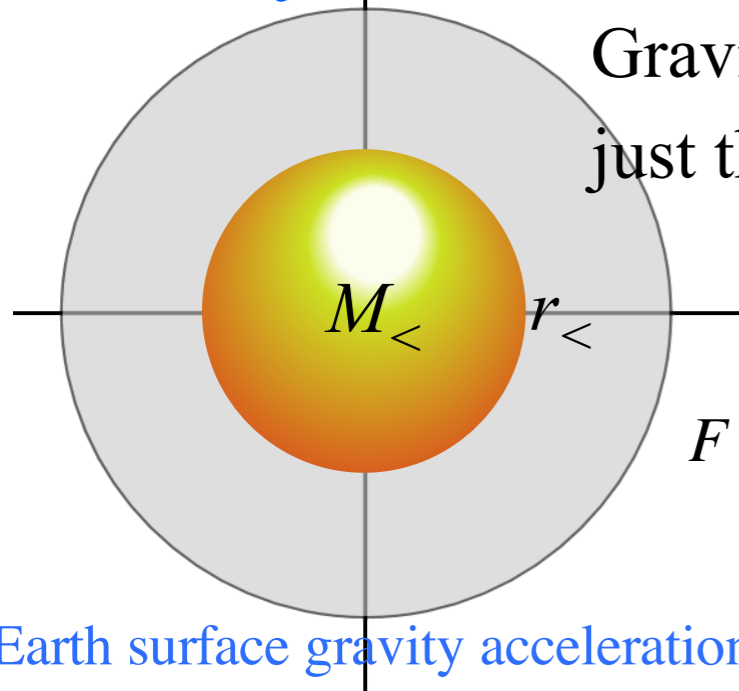
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Earth surface gravity acceleration: $g = G \frac{M_{\oplus}}{R_{\oplus}^2} = G \frac{M_{\oplus}}{R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \frac{M_{\oplus}}{4\pi R_{\oplus}^3} R_{\oplus} = G \frac{4\pi}{3} \rho_{\oplus} R_{\oplus} = 9.8 \text{ m/s}$

$G = 6.67384(80) \cdot 10^{-11} \text{ Nm}^2/\text{C}^2 \sim (2/3) 10^{-10}$

Earth radius: $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$

Earth mass: $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}$

Solar radius: $R_{\odot} = 6.955 \times 10^8 \text{ m} \approx 7.0 \cdot 10^8 \text{ m}$

Solar mass: $M_{\odot} = 1.9889 \times 10^{30} \text{ kg} \approx 2.0 \cdot 10^{30} \text{ kg}$

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

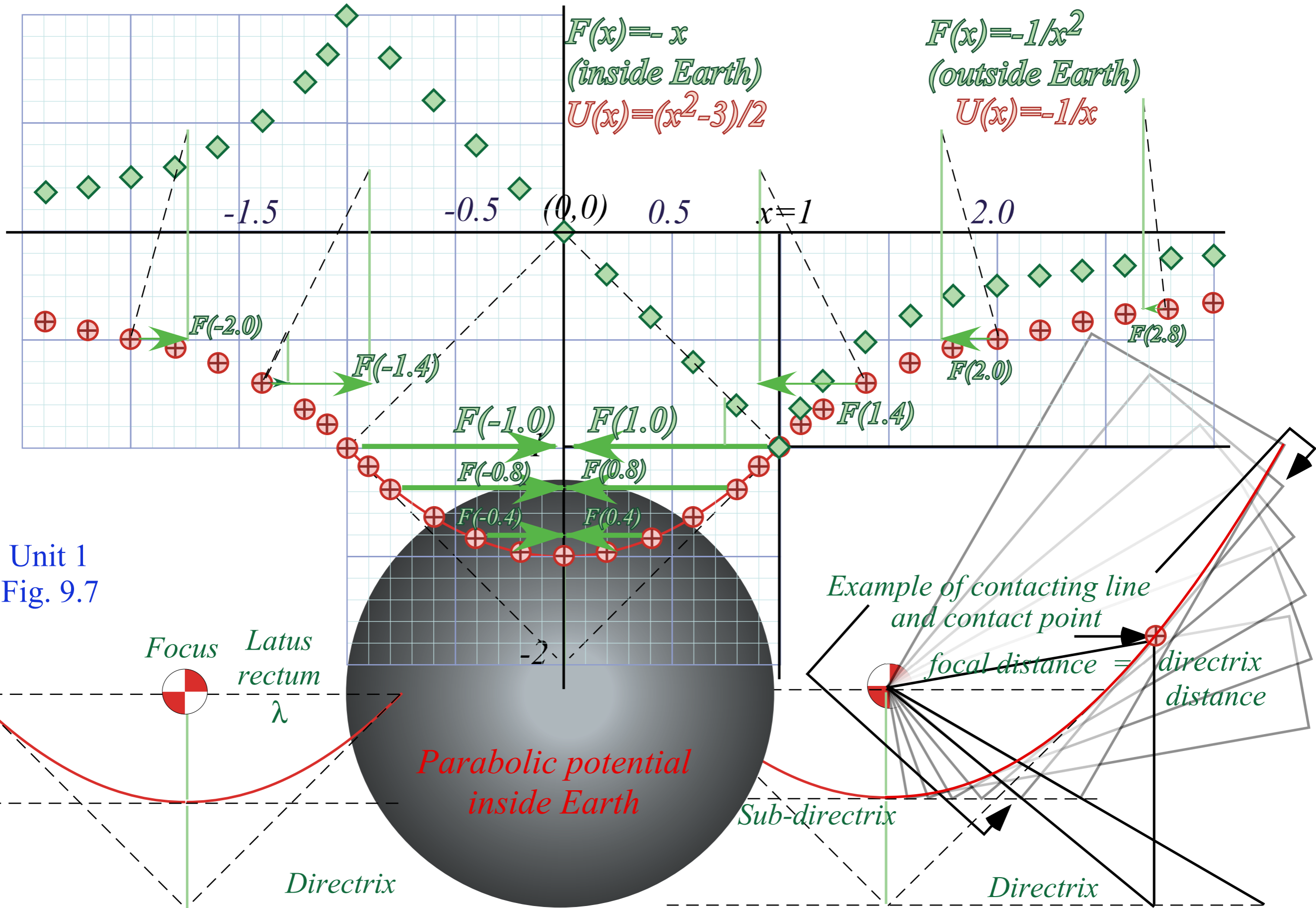
→ *Contact-geometry of potential curve(s)*

“Crushed-Earth” models: 3 key energy “steps” and 4 key energy “levels”

Earth matter vs nuclear matter:

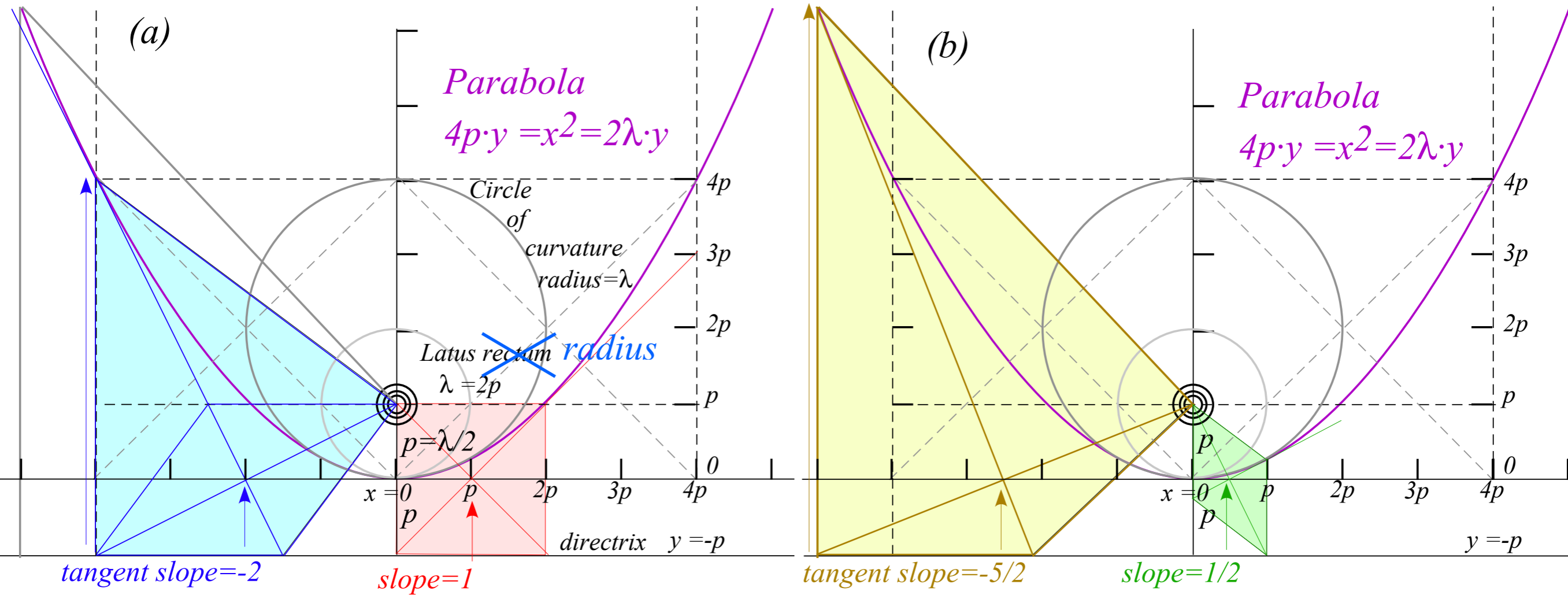
*Introducing the “neutron starlet” and **“Black-Hole-Earth”***

The ideal “Sophomore-Physics-Earth” model of geo-gravity



...conventional parabolic geometry...carried to extremes...

(From p.18)



Unit 1
Fig. 9.4

Geometry of idealized “Sophomore-physics Earth”

Coulomb field outside

Isotropic Harmonic Oscillator (IHO) field inside

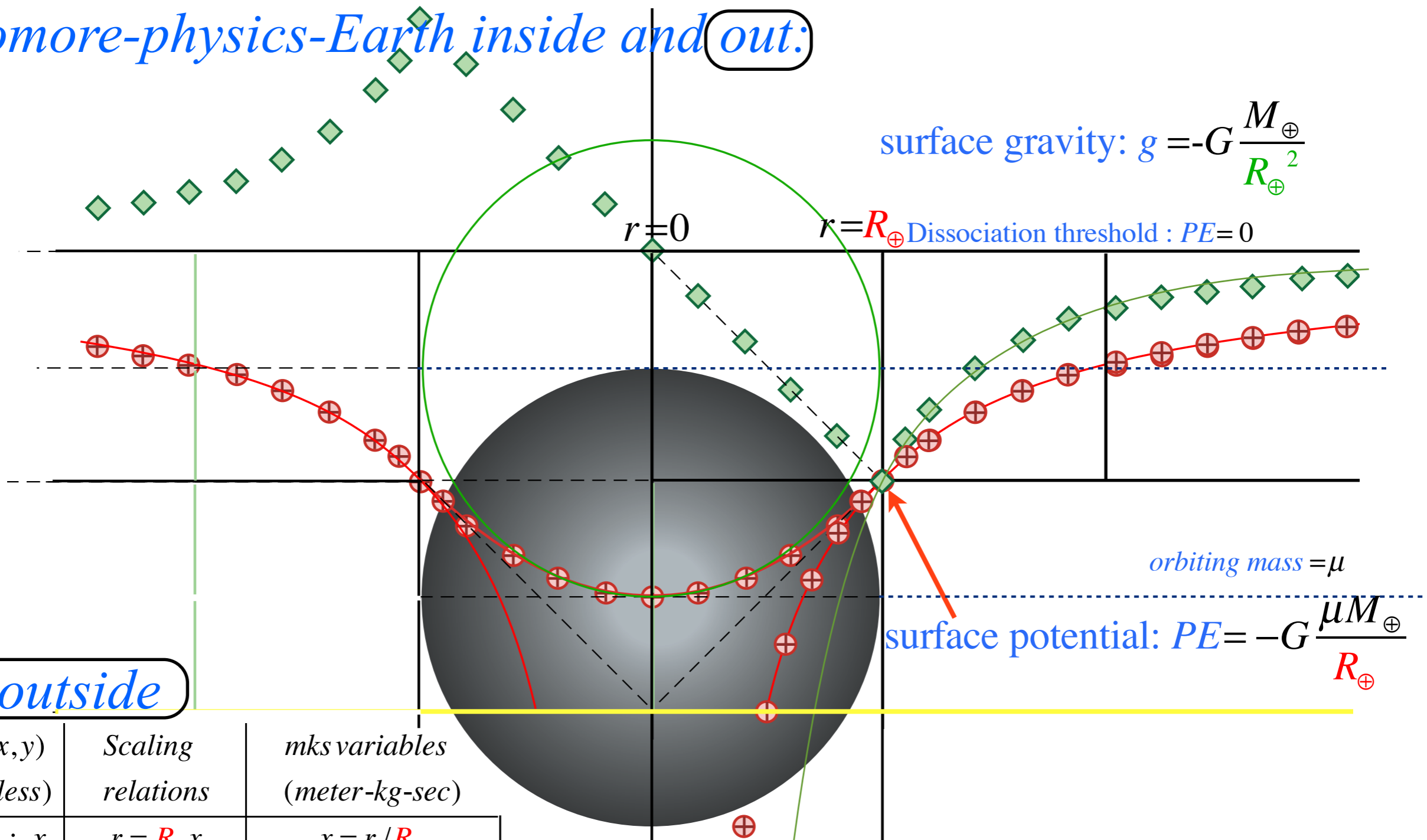
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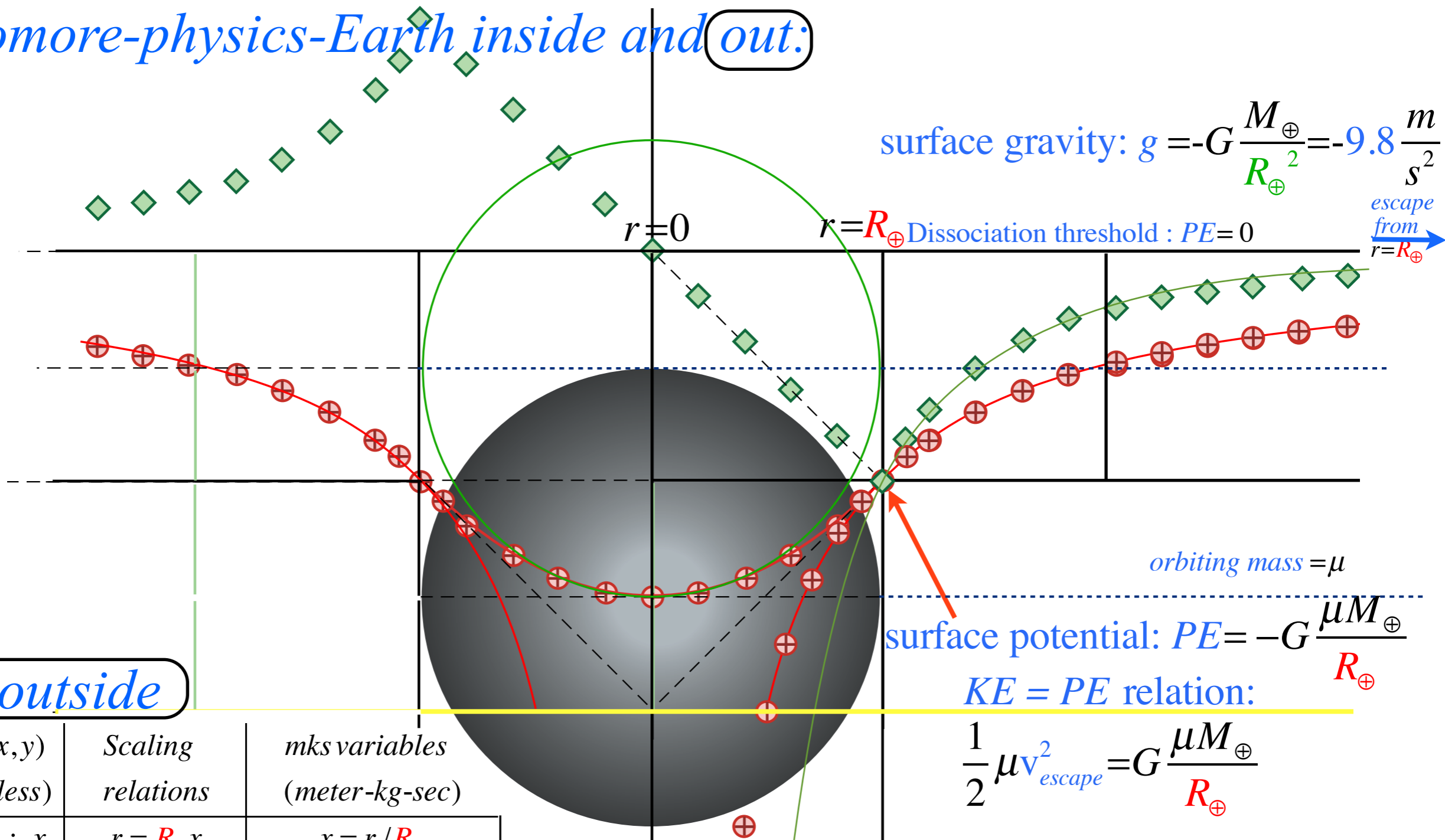
*Introducing the “neutron starlet” and **“Black-Hole-Earth”***

Sophomore-physics-Earth inside and out:



Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$

Sophomore-physics-Earth inside and out:



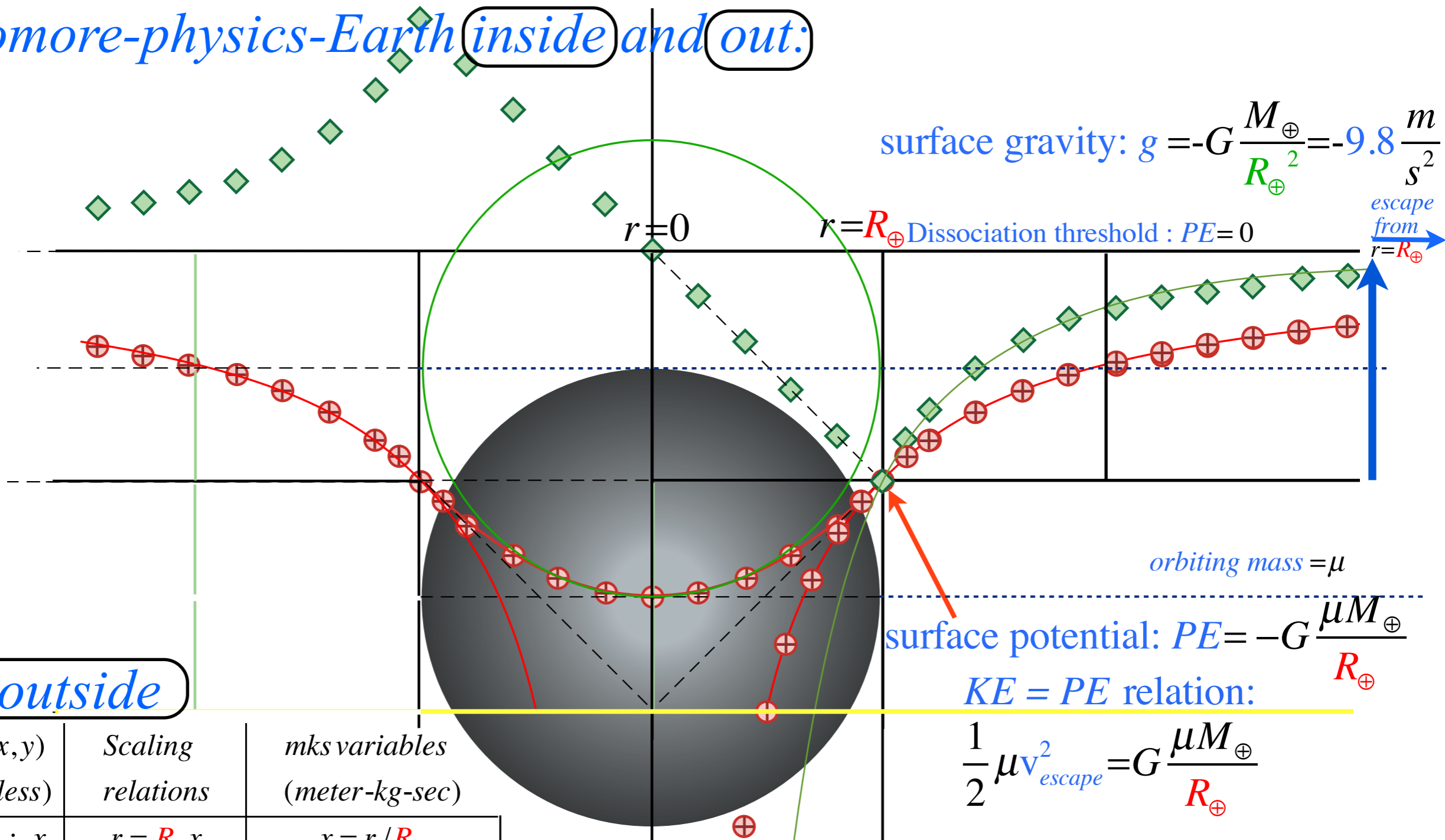
$$\frac{1}{2} \mu v_{\text{escape}}^2 = G \frac{\mu M_{\oplus}}{R_{\oplus}}$$

R_{\oplus} -escape-velocity

$$v_{\text{escape}} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$

Sophomore-physics-Earth (inside) and (out):



outside

inside

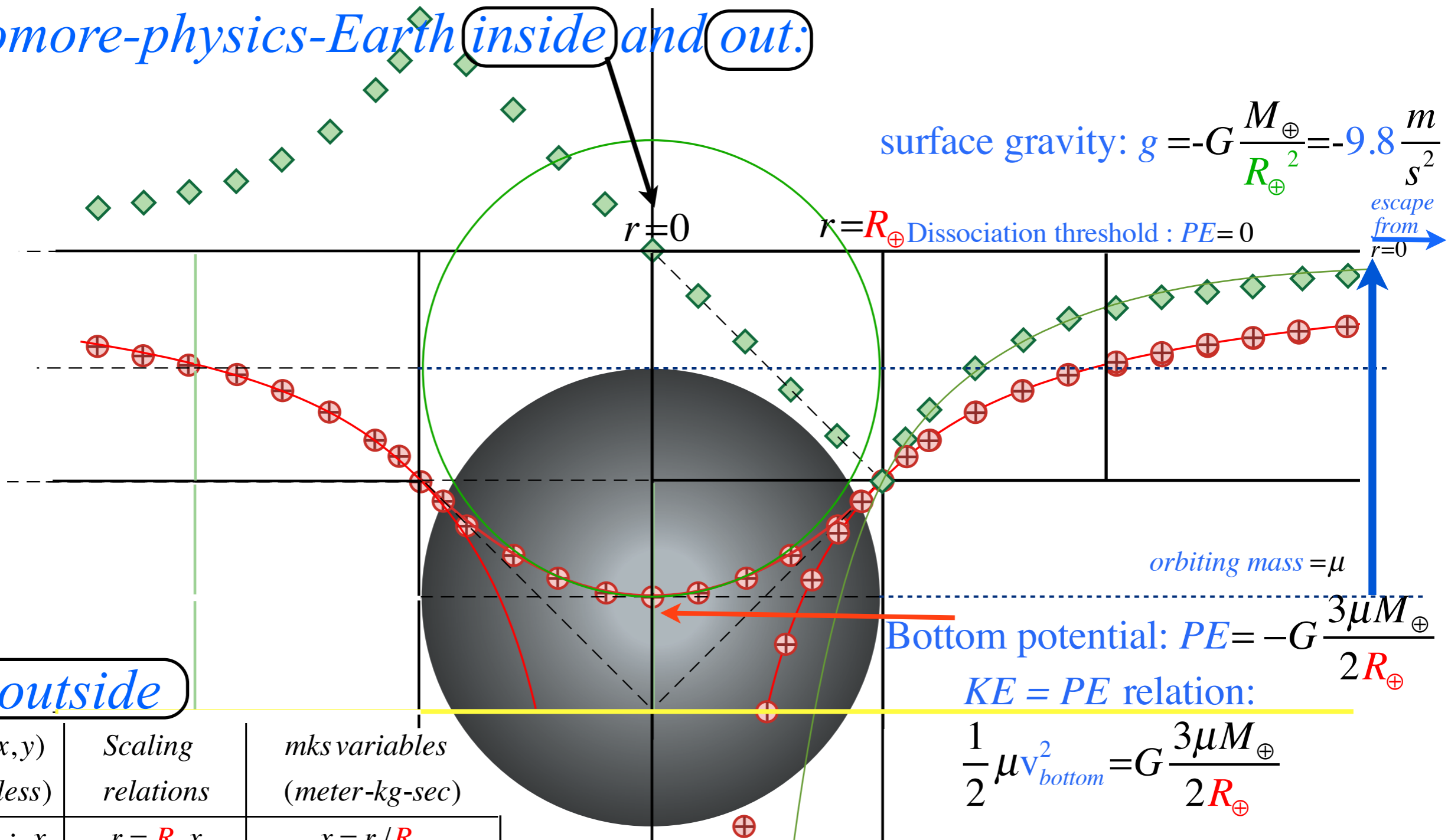
Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
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Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$
PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$		
Force for $ x < 1$: $y^{Force} = -x$		

R_{\oplus} -escape-velocity

$$v_{escape} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$$

11.1km/sec

Sophomore-physics-Earth **inside** and **out**:



outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$
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PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$		$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$
Force for $ x < 1$: $y^{Force} = -x$		$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

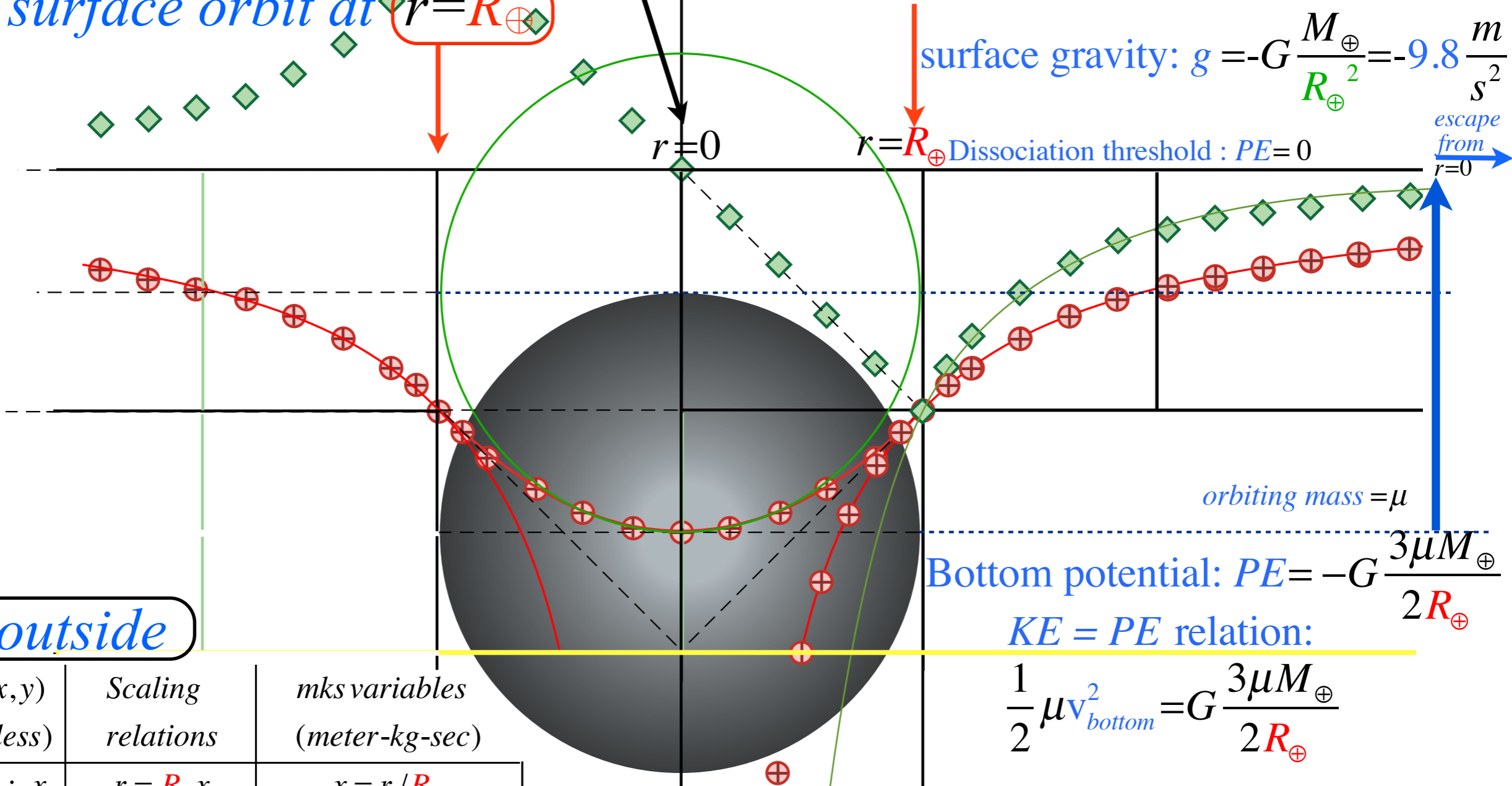
(r=0)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$



outside

inside

($r=0$)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)	
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$	
PE for $ x \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$	PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$
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			$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

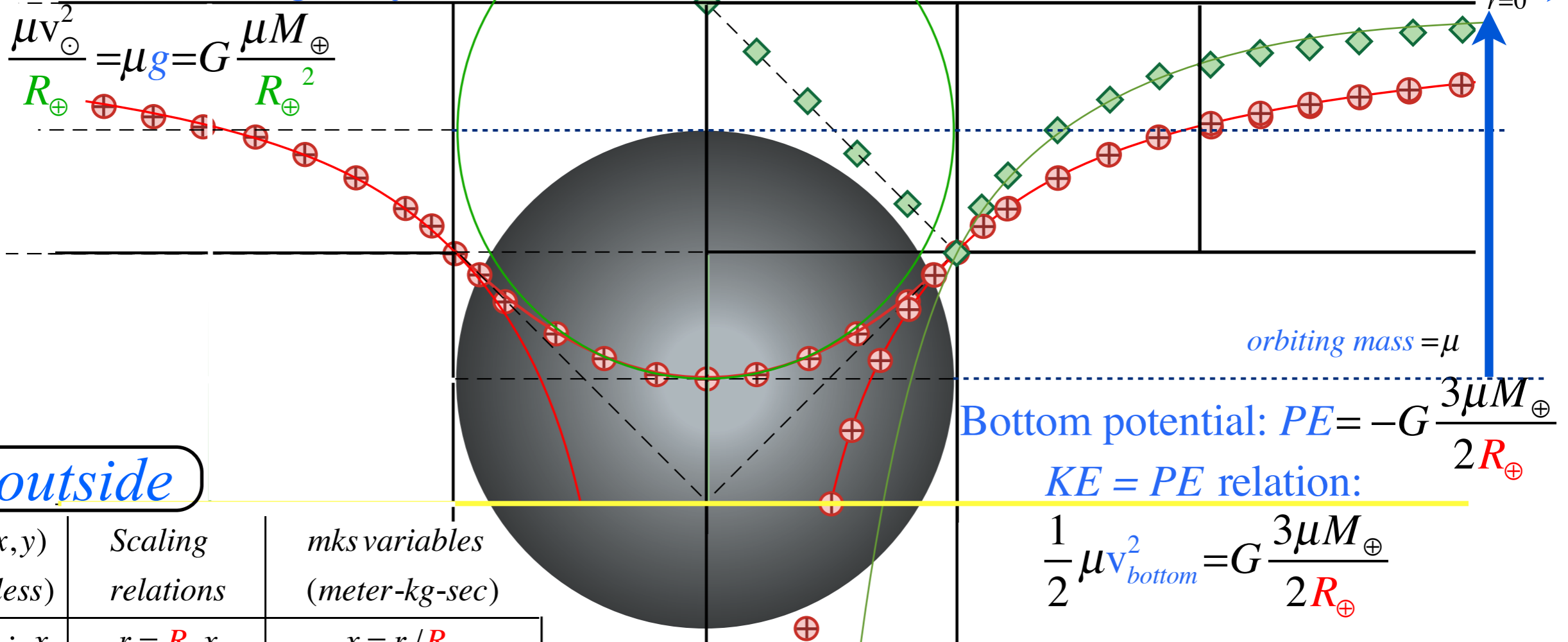
surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

Centrifugal force = surface gravity:

$$\frac{\mu v_{\oplus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Dissociation threshold : $PE=0$

escape from $r=0$



orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
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PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$		
Force for $ x < 1$: $y^{Force} = -x$		

$$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$$

$$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$$

(r=0)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$

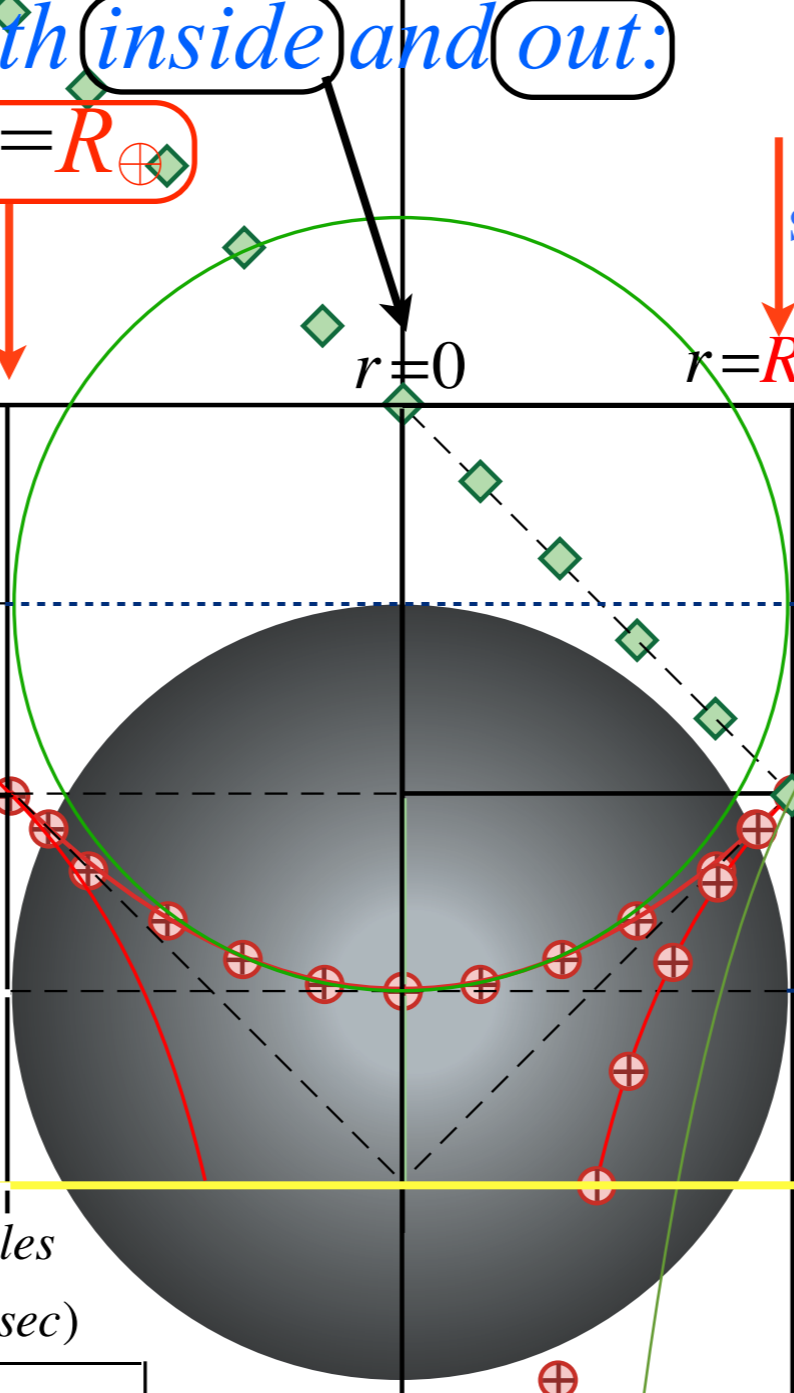
Centrifugal force = surface gravity:

$$\frac{\mu v_{\ominus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Orbit KE = $\frac{1}{2} \mu v_{\ominus}^2 = G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Dissociation threshold : $PE=0$

escape from $r=0$



orbiting mass = μ

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

outside

inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
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PE for $|x| < 1$:

$$y^{PE} = \frac{x^2}{2} - \frac{3}{2}$$

Force for $|x| < 1$:

$$y^{Force} = -x$$

$$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$$

$$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$$

($r=0$)-escape-velocity

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Sophomore-physics-Earth (inside and out):

...and surface orbit at $r=R_{\oplus}$

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Centrifugal force = surface gravity:

$$\frac{\mu v_{\oplus}^2}{R_{\oplus}} = \mu g = G \frac{\mu M_{\oplus}}{R_{\oplus}^2}$$

Orbit KE = $\frac{1}{2} \mu v_{\oplus}^2 = G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Orbit $E_{\oplus}^{Total} = \frac{1}{2} \mu v_{\oplus}^2 - G \frac{\mu M_{\oplus}}{R_{\oplus}} = -G \frac{\mu M_{\oplus}}{2R_{\oplus}}$

Dissociation threshold : $PE=0$ escape from $r=0$

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

orbiting mass = μ

outside

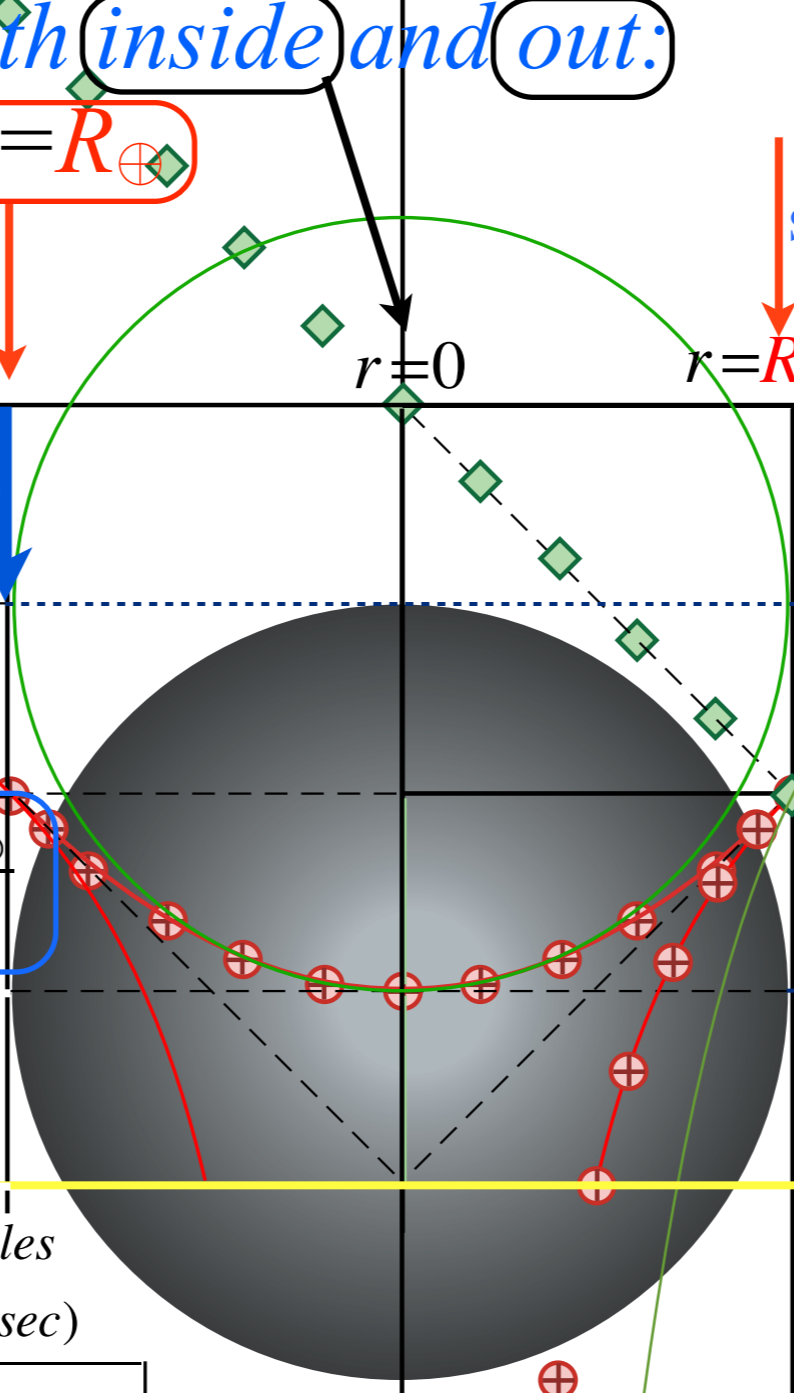
inside

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_{\oplus} x$	$x = r / R_{\oplus}$
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Force for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$
PE for $ x < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$		
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(r=0)-escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

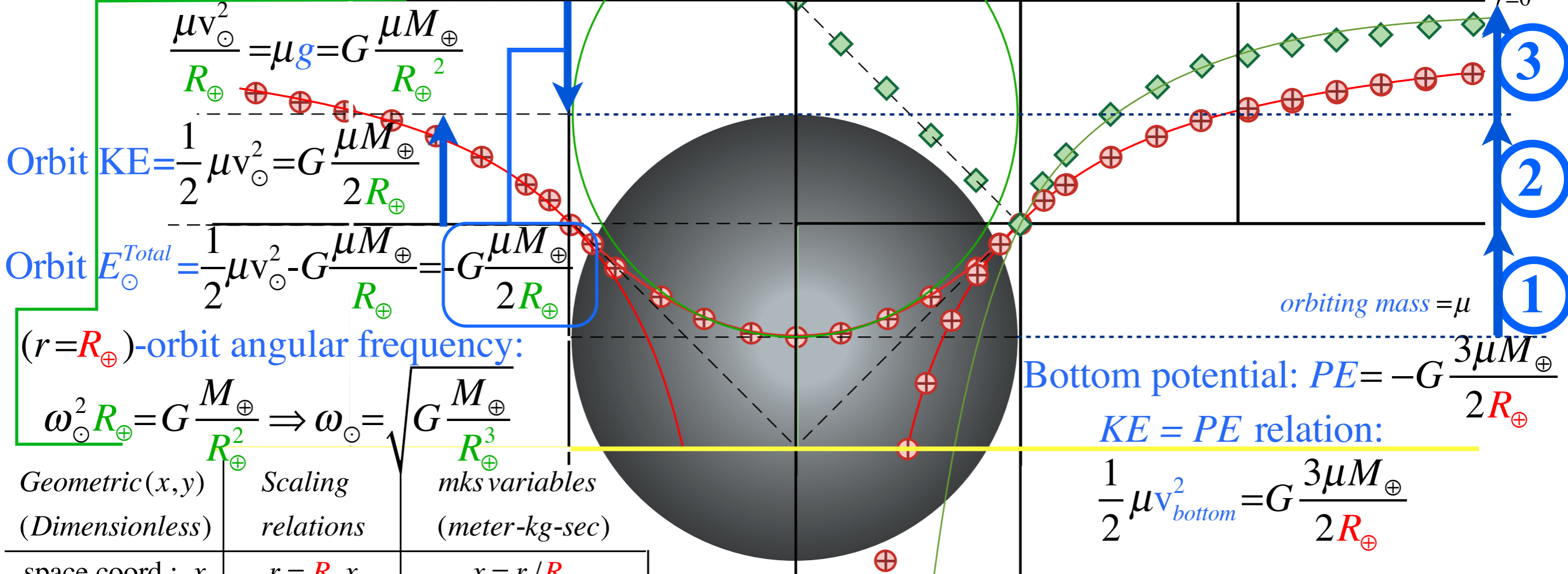


Sophomore-physics-Earth (inside and out): "3-steps out of (or into) Hell"

...and surface orbit at $r=R_{\oplus}$

Centrifugal force = surface gravity:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$



Orbit KE =

$$\frac{1}{2} \mu v_{\odot}^2 = G \frac{\mu M_{\oplus}}{2 R_{\oplus}}$$

Orbit E_{\odot}^{Total} =

$$\frac{1}{2} \mu v_{\odot}^2 - G \frac{\mu M_{\oplus}}{R_{\oplus}} = -G \frac{\mu M_{\oplus}}{2 R_{\oplus}}$$

$(r=R_{\oplus})$ -orbit angular frequency:

$$\omega_{\odot}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$$

Bottom potential: $PE = -G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$

KE = PE relation:

$$\frac{1}{2} \mu v_{bottom}^2 = G \frac{3\mu M_{\oplus}}{2R_{\oplus}}$$

Geometric (x,y)
(Dimensionless)

Scaling relations

mks variables
(meter-kg-sec)

space coord.: x

$r = R_{\oplus} x$

$x = r / R_{\oplus}$

PE for $|x| \geq 1$:

$$y^{PE} = \frac{-1}{x}$$

$PE^{mks}(r)$

$$= \frac{GM\mu}{R_{\oplus}} y^{PE}$$

$$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_{\oplus}} \frac{1}{x}$$

PE for $|x| < 1$:

$$y^{PE} = \frac{x^2}{2} - \frac{3}{2}$$

inside

$$PE^{mks}(r) = \frac{GM\mu}{R_{\oplus}} \left(\frac{r^2}{2R_{\oplus}^2} - \frac{3}{2} \right)$$

$(r=0)$ -escape-velocity

$$V_{bottom} = \sqrt{3G \frac{M_{\oplus}}{R_{\oplus}}}$$

13.7km/sec

Force for $|x| \geq 1$:

$$y^{Force} = \frac{-1}{x^2}$$

$F^{mks}(r)$

$$= \frac{GM\mu}{R_{\oplus}^2} y^{Force}$$

$$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$$

Force for $|x| < 1$:

$$y^{Force} = -x$$

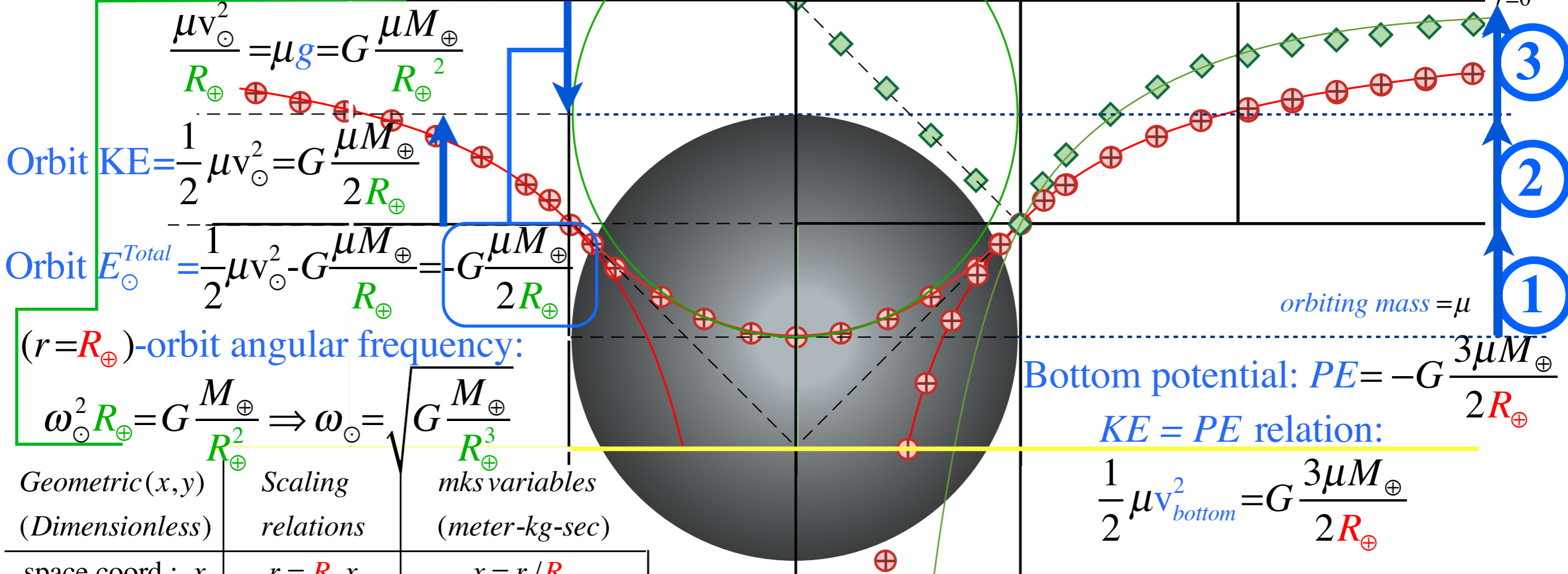
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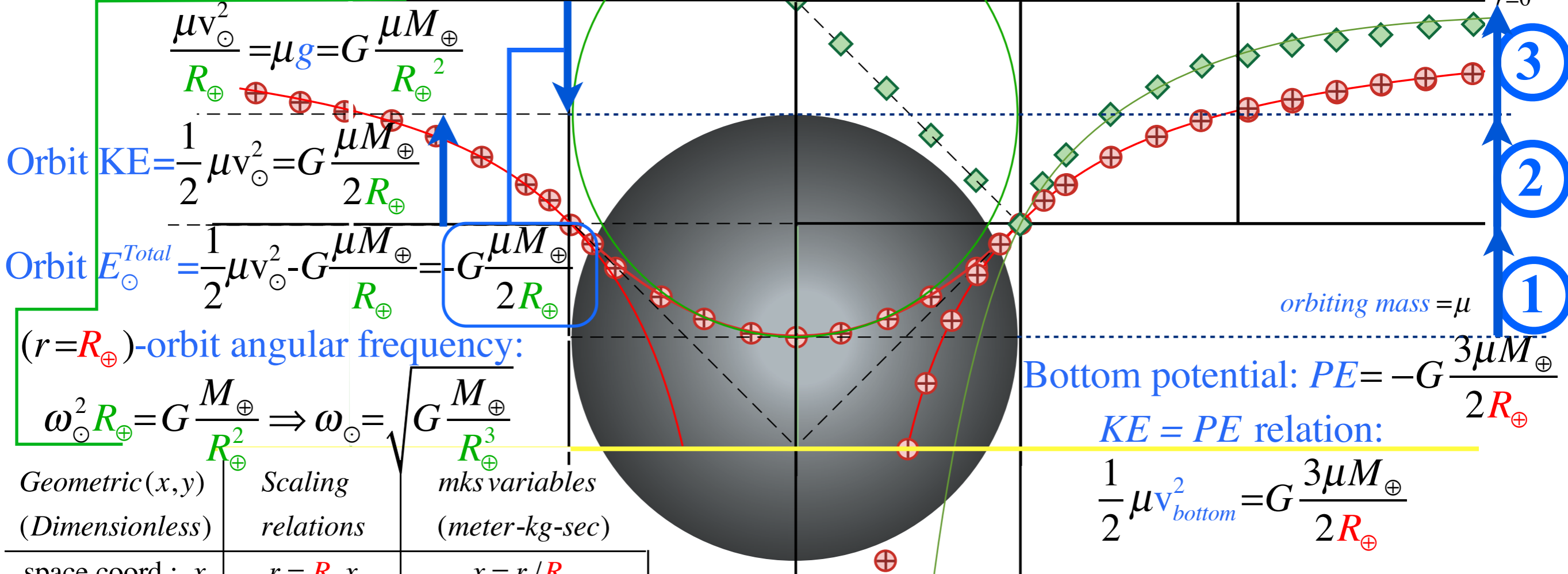
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$Force$ for $ x \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_{\oplus}^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_{\oplus}^2} \frac{1}{x^2}$	$Force$ for $ x < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_{\oplus}^3} r$	$(r=R_{\oplus})$ -orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}} = \sqrt{gR_{\oplus}}$ 7.9km/sec

Sophomore-physics-Earth (inside and out): "3-steps out of (or into) Hell"

...and surface orbit at $r=R_{\oplus}$

Centrifugal force = surface gravity:

surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2} = -9.8 \frac{m}{s^2}$



Orbit $KE = \frac{1}{2} \mu v_{\ominus}^2 = G \frac{\mu M_{\oplus}}{2 R_{\oplus}}$

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$$\omega_{\ominus}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\ominus} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$$

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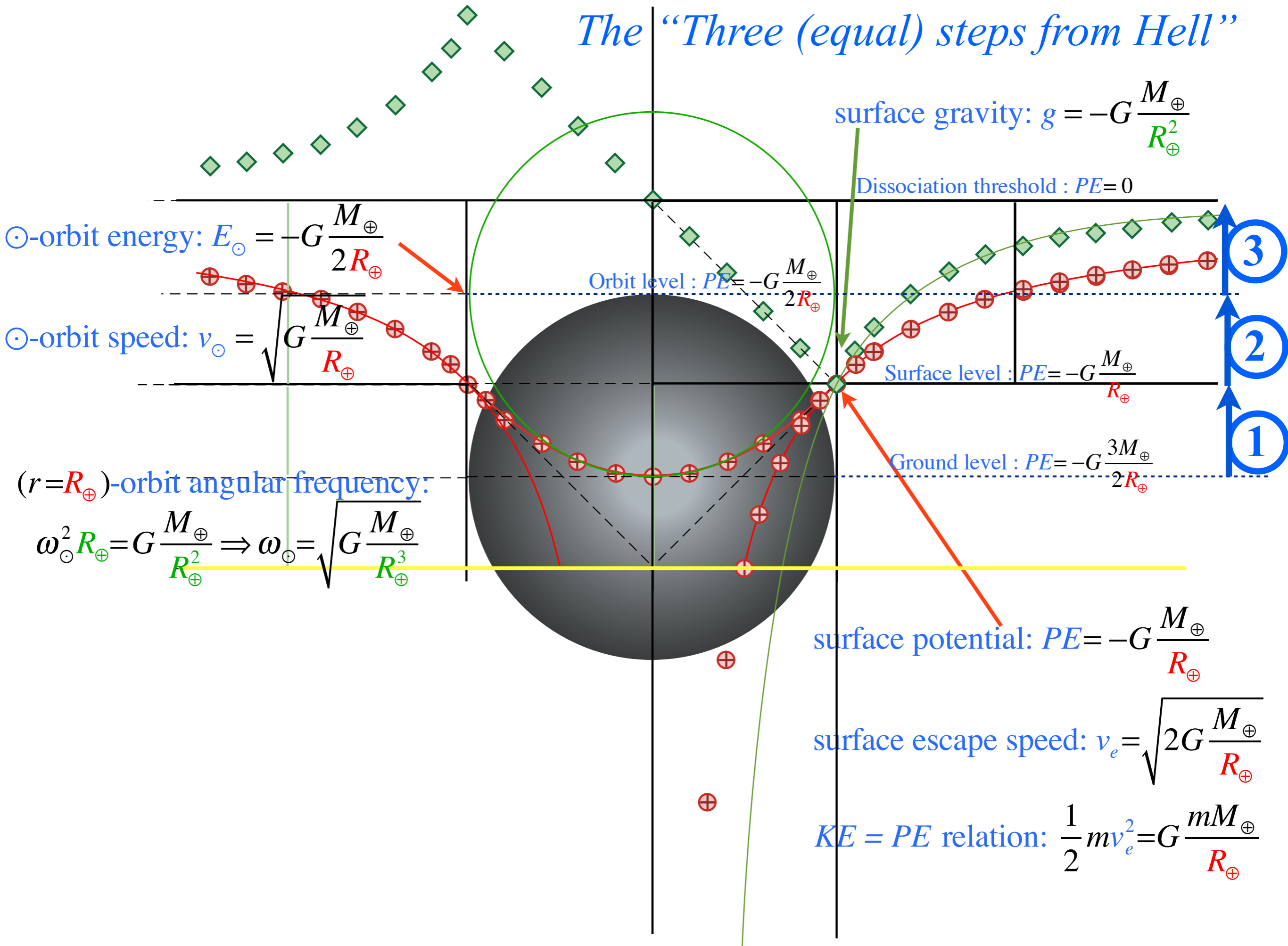
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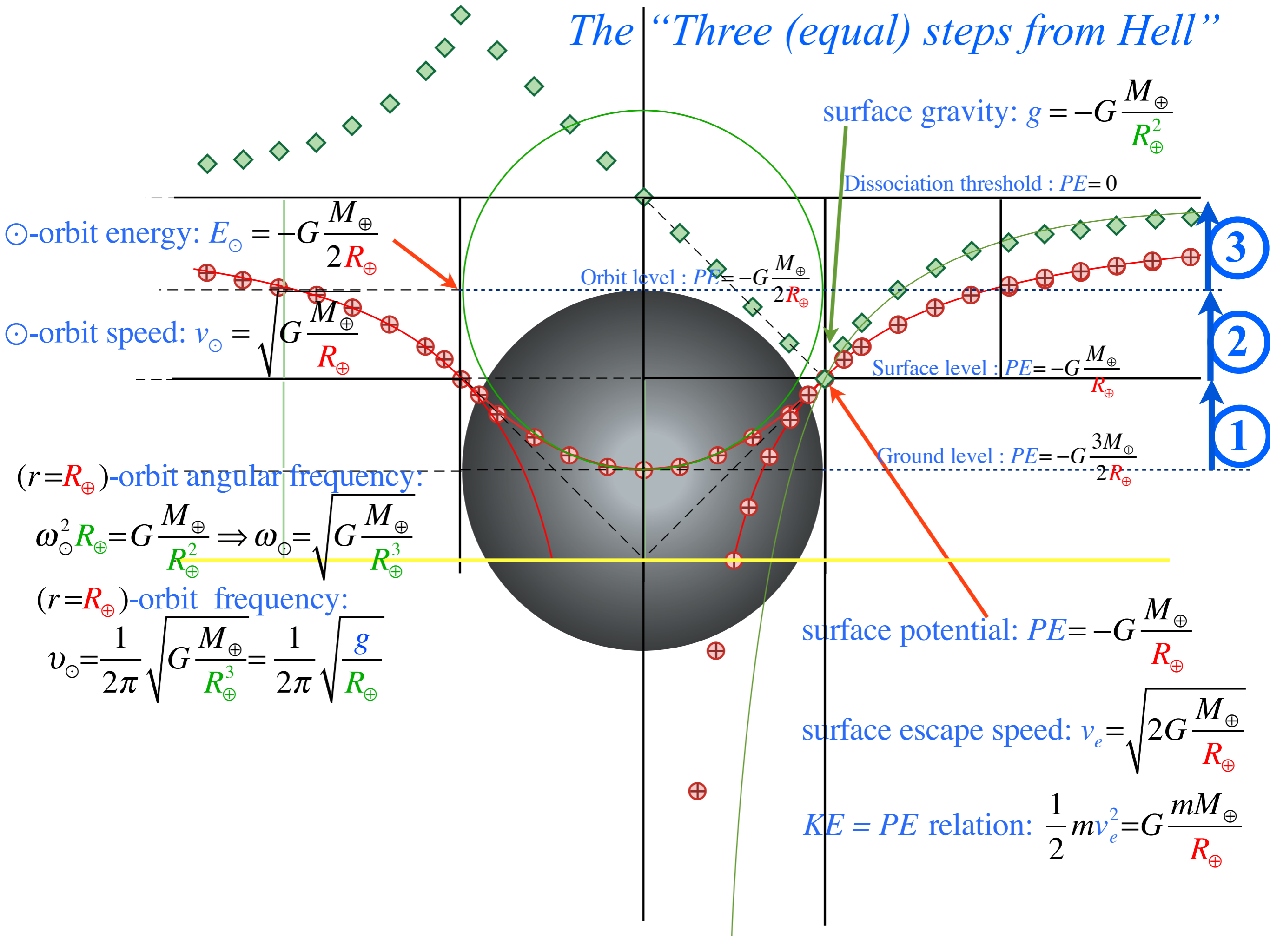
$(r=R_{\oplus})$ -escape velocity:
11.1km/sec

$$V_{escape} = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$$

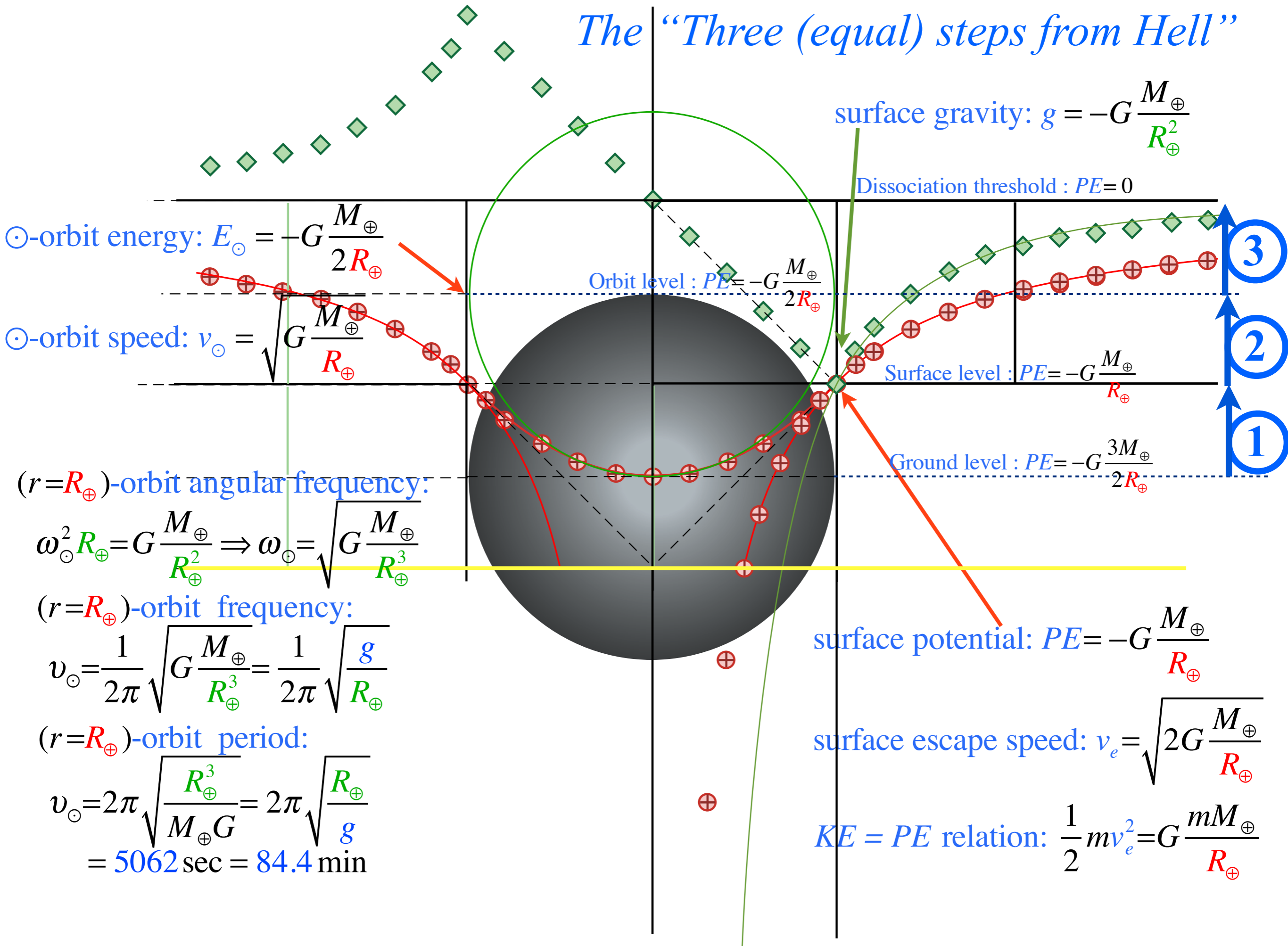
The "Three (equal) steps from Hell"



The "Three (equal) steps from Hell"



The "Three (equal) steps from Hell"



surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2}$

Dissociation threshold : $PE=0$

☉-orbit energy: $E_{\odot} = -G \frac{M_{\oplus}}{2R_{\oplus}}$

☉-orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}}$

Orbit level : $PE = -G \frac{M_{\oplus}}{2R_{\oplus}}$

Surface level : $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

Ground level : $PE = -G \frac{3M_{\oplus}}{2R_{\oplus}}$

$(r=R_{\oplus})$ -orbit angular frequency:

$$\omega_{\odot}^2 R_{\oplus} = G \frac{M_{\oplus}}{R_{\oplus}^2} \Rightarrow \omega_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}}$$

$(r=R_{\oplus})$ -orbit frequency:

$$v_{\odot} = \frac{1}{2\pi} \sqrt{G \frac{M_{\oplus}}{R_{\oplus}^3}} = \frac{1}{2\pi} \sqrt{\frac{g}{R_{\oplus}}}$$

$(r=R_{\oplus})$ -orbit period:

$$v_{\odot} = 2\pi \sqrt{\frac{R_{\oplus}^3}{M_{\oplus} G}} = 2\pi \sqrt{\frac{R_{\oplus}}{g}} = 5062 \text{ sec} = 84.4 \text{ min}$$

surface potential: $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

surface escape speed: $v_e = \sqrt{2G \frac{M_{\oplus}}{R_{\oplus}}}$

KE = PE relation: $\frac{1}{2} m v_e^2 = G \frac{m M_{\oplus}}{R_{\oplus}}$

- 3
- 2
- 1

Suppose Earth radius crushed to 1/2: ($R_{\oplus} = 6.4 \cdot 10^6 m$ crushed to $R_{\oplus}/2 = 3.2 \cdot 10^6 m$)

All formulas identical to ones derived on p.63 to 78.

Imagine reducing R_{\oplus} to $R_{\oplus}/2$

- 3
- 2
- 1

⊙ - Orbit level : $PE = -G \frac{M_{\oplus}}{2R_{\oplus}}$

2 times ⊙-orbit energy: $E_{\odot} = -G \frac{M_{\oplus}}{2R_{\oplus}}$

$\sqrt{2}$ times ⊙-orbit speed: $v_{\odot} = \sqrt{G \frac{M_{\oplus}}{R_{\oplus}}}$

2 times the surface potential: $PE = -G \frac{M_{\oplus}}{R_{\oplus}}$

$\sqrt{2}$ times surface escape speed: $v_e = \sqrt{G \frac{2M_{\oplus}}{R_{\oplus}}}$

4 times the surface gravity: $g = -G \frac{M_{\oplus}}{R_{\oplus}^2}$

2x Crushed Earth

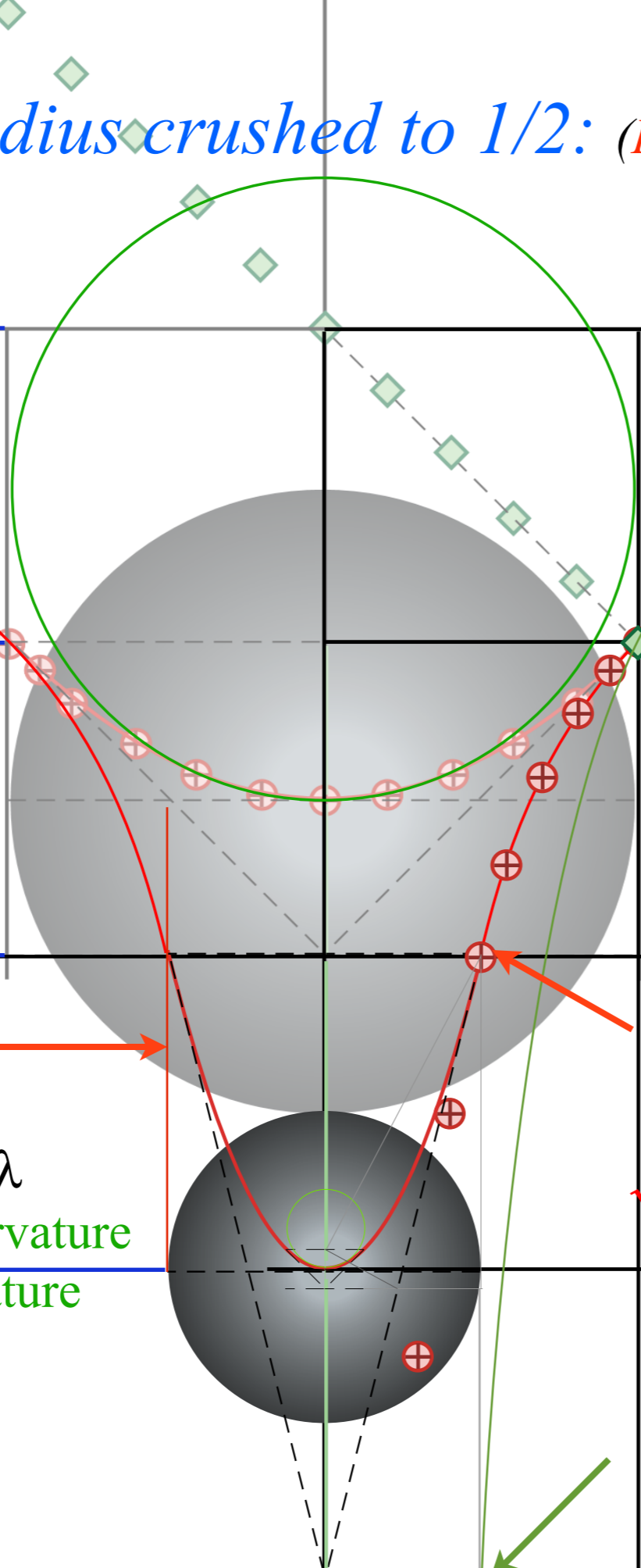
1/2 radius

8 times as dense

1/8 focal distance or λ

1/8 minimum radius of curvature

8 times maximum curvature



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Earth matter vs nuclear matter:



*Introducing the “neutron starlet” and “**Black-Hole-Earth**”*

Examples of “crushed” matter

Earth matter Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \simeq 6.0 \cdot 10^{24} \text{ kg}$. Density $\rho_{\oplus} = ??$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \simeq 6.4 \cdot 10^6 \text{ m}$ Earth volume : $(4\pi / 3)R_{\oplus}^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21} \text{ m}^3$

$(6.4)^3 \sim 262$ and $(4\pi/3)260 = 1098 \sim 10^3$

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$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}^3}$$

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Density of solid Fe = $7.9 \cdot 10^3 \text{ kg/m}^3$

Density of liquid Fe = $6.9 \cdot 10^3 \text{ kg/m}^3$

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Say a nucleus of atomic weight 50 has a radius of 3 fm, or 50 nucleons each with a mass $2 \cdot 10^{-27} \text{ kg}$.

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$$[4\pi 3^2 = 36\pi = 113 \sim 10^2] 10^{-45}$$

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$$(1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$$

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Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{\text{crush}\oplus} \approx 300 \text{ m}$ would approach neutron-star density. $(1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$



Geometry and algebra of idealized “Sophomore-physics Earth” fields

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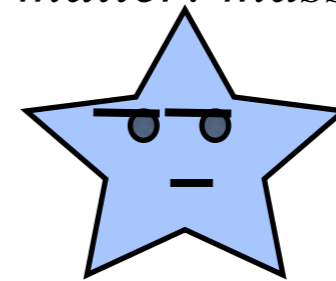
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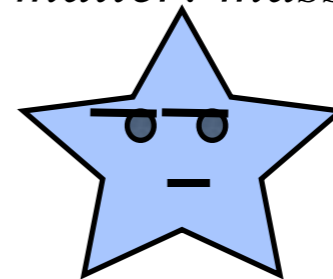
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Fantasizing the “Black Hole Earth” Suppose Earth is crushed so that its

surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 \text{ m/s}$.

$$V_{\text{escape}} = \sqrt{(2GM/R_{\oplus})}$$

(from p. 67,...,82)

$$G = 6.673 \cdot 10^{-11} \text{ Nm}^2/\text{C}^2$$

$$\sim (2/3) 10^{-10}$$

(from p. 60)

$$c \equiv 299,792,458 \text{ m/s (EXACTLY)}$$

$$\text{Uncrushed Earth radius : } R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$$

$$\text{Earth mass : } M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}.$$

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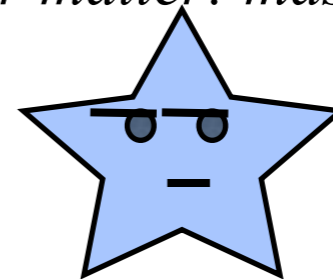
Say a nucleus of atomic weight 50 has a radius of 3 fm, or 50 nucleons each with a mass $2 \cdot 10^{-27} \text{ kg}$.

That’s $100 \cdot 10^{-27} = 10^{-25} \text{ kg}$ packed into a volume of $\frac{4\pi}{3}r^3 = \frac{4\pi}{3} (3 \cdot 10^{-15})^3 \text{ m}^3$ or about 10^{-43} m^3 .

Nuclear density is $10^{-25+43} = 10^{18} \text{ kg/m}^3$ or a trillion (10^{12}) kilograms in the size of a **fingertip**.
[$4\pi 3^2 = 36\pi = 113 \sim 10^2$] 10^{-45}

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{\text{crush}\oplus} \approx 300 \text{ m}$ would approach neutron-star density.

Introducing the “Neutron starlet” 1 cm^3 of nuclear matter: mass = 10^{12} kg .



Fantasizing the “Black Hole Earth” Suppose Earth is crushed so that its

surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 \text{ m/s}$.

$c \equiv 299,792,458 \text{ m/s}$ (EXACTLY) $\sim 3 \cdot 10^8 \text{ m/s}$

$$V_{\text{escape}} = \sqrt{(2GM/R_{\oplus})}$$

(from p. 67,...,82)

$$c = \sqrt{(2GM/R_{\oplus})}$$

Uncrushed Earth radius : $R_{\oplus} = 6.371 \cdot 10^6 \text{ m} \approx 6.4 \cdot 10^6 \text{ m}$

Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} \text{ kg} \approx 6.0 \cdot 10^{24} \text{ kg}$.

$$G = 6.673 \cdot 10^{-11} \text{ Nm}^2/\text{C}^2$$

$\sim (2/3) 10^{-10}$

(from p. 60)

$$R_{\oplus} = 2GM/c^2 = 2 \cdot 2/3 \cdot 10^{-10} \cdot 6 \cdot 10^{24} / (9 \cdot 10^{16}) = 8/9 \cdot 10^{-2} = 8.9 \text{ mm} \sim 1 \text{ cm}$$

(Then Earth would be **fingertip** size!)



→ *Introducing 2D IHO orbits and phasor geometry*
Phasor “clock” geometry

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

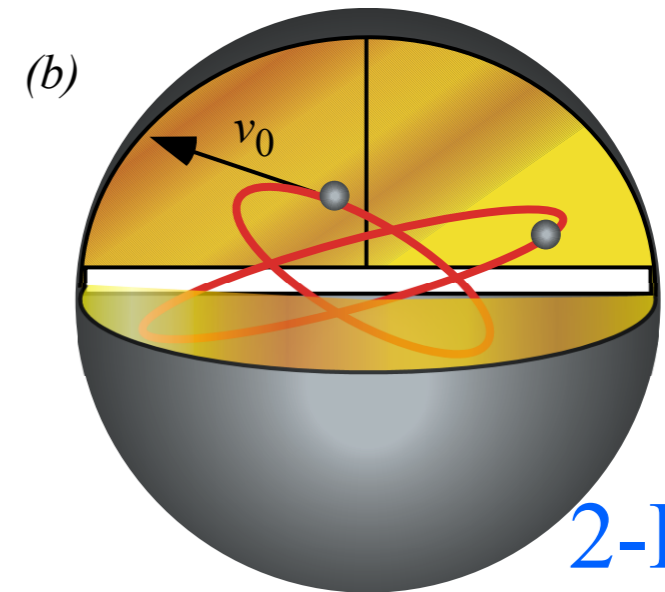
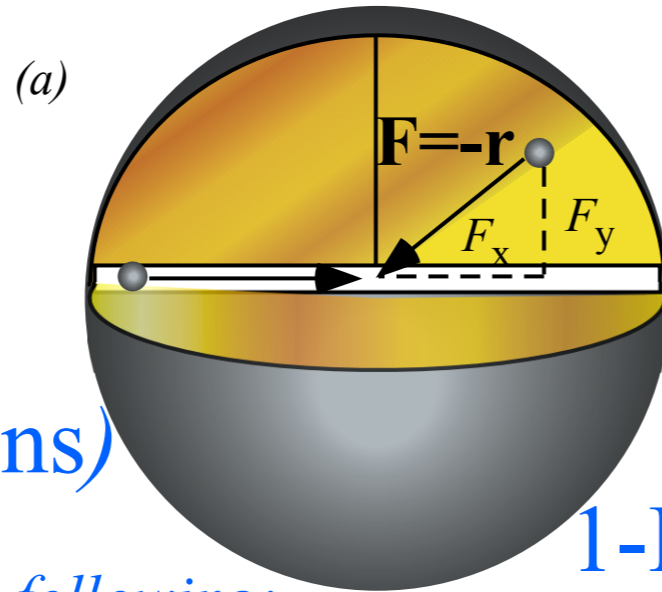
I.H.O. Force law

$$F = -x \quad (1\text{-Dimension})$$

$$\mathbf{F} = -\mathbf{r} \quad (2 \text{ or } 3\text{-Dimensions})$$

Each dimension x , y , or z obeys the following:

$$\text{Total } E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const.}$$



Unit 1
Fig. 9.10

(Paths are *always*
2-D ellipses if
viewed right!)

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

I.H.O. Force law

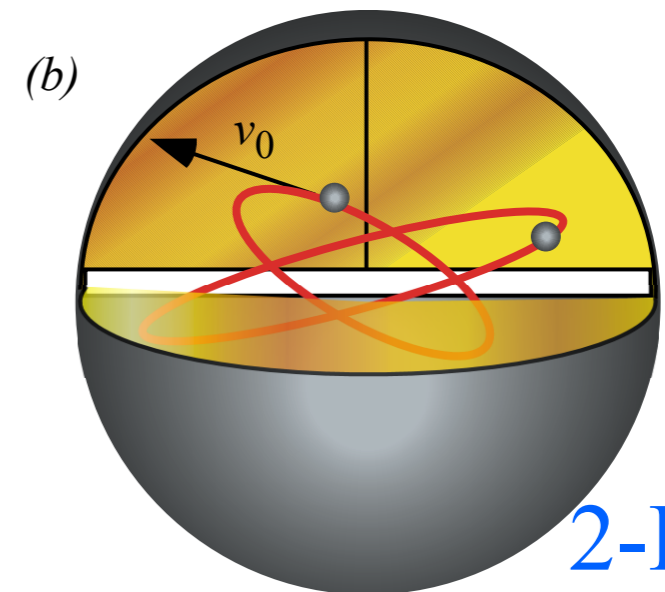
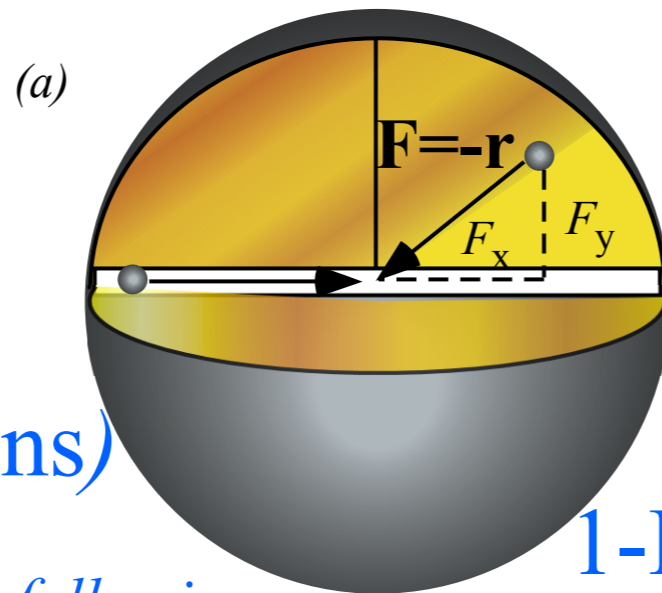
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Equations for x -motion
[$x(t)$ and $v_x=v(t)$] are
given first. They apply
as well to dimensions
[$y(t)$ and $v_y=v(t)$] and
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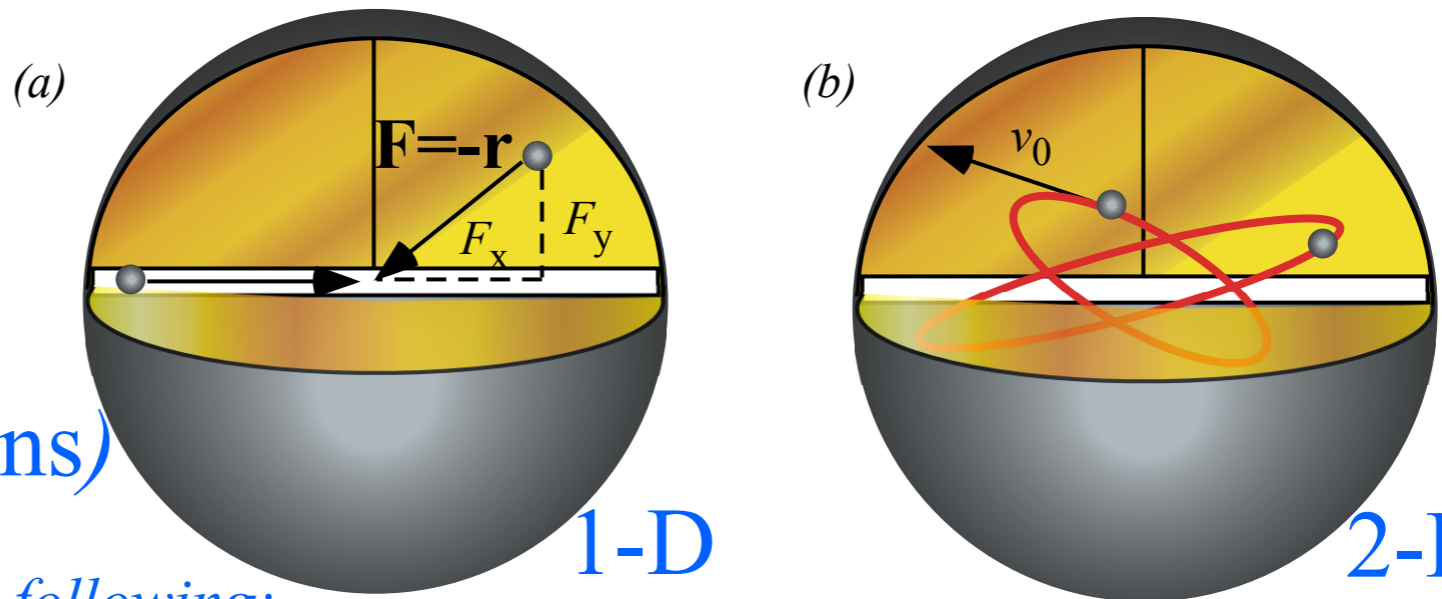


Unit 1
Fig. 9.10

(Paths are *always* 2-D
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Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

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$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old “scale-a-circle” trick...

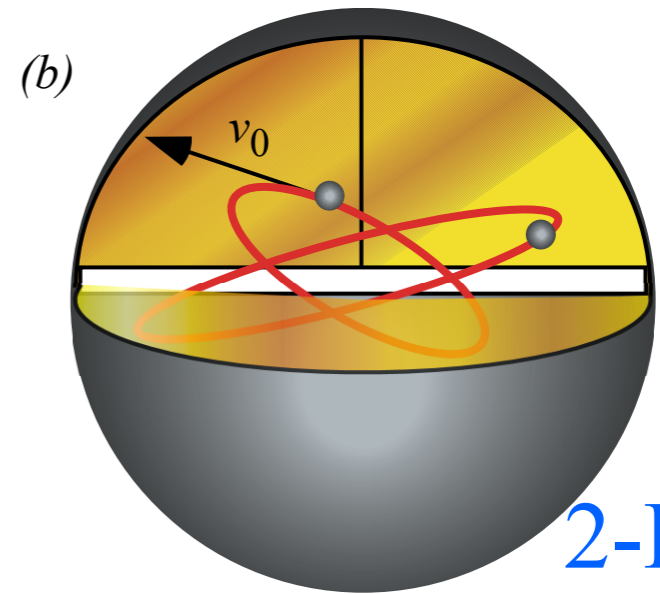
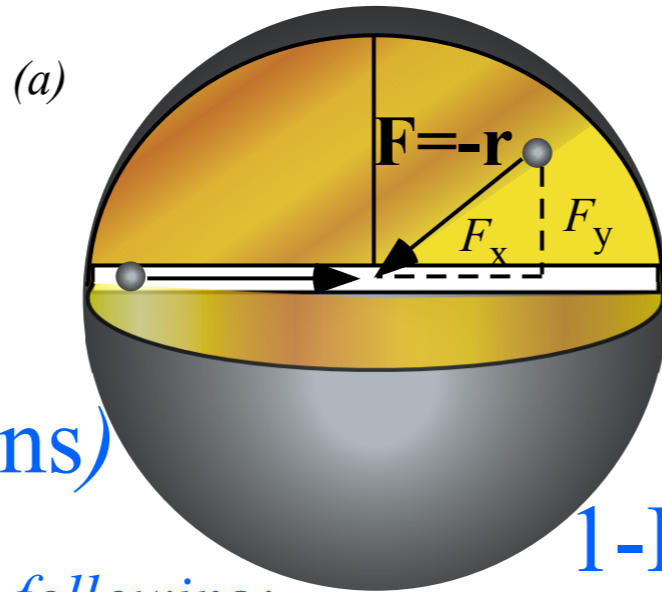
velocity:

position:

Let : **(1)** $v = \sqrt{2E/m} \cos\theta$, and : **(2)** $x = \sqrt{2E/k} \sin\theta$

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



2-D or 3-D
(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

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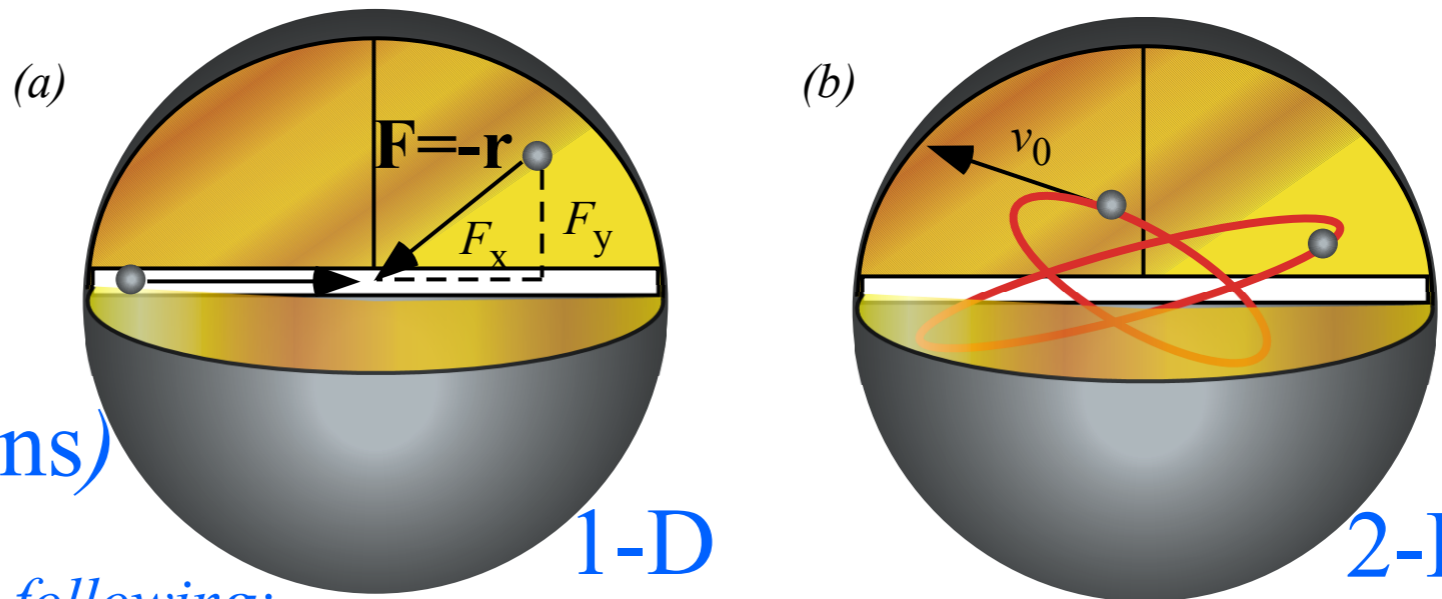
$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old “scale-a-circle” trick...

Let : **(1)** *velocity:* $v = \sqrt{2E/m} \cos\theta$, and : **(2)** *position:* $x = \sqrt{2E/k} \sin\theta$ *angular velocity:* def. **(3)** $\omega = \frac{d\theta}{dt}$

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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Let : **(1)** $v = \sqrt{2E/m} \cos\theta$, and : **(2)** $x = \sqrt{2E/k} \sin\theta$ def. **(3)** $\omega = \frac{d\theta}{dt}$

velocity:

position:

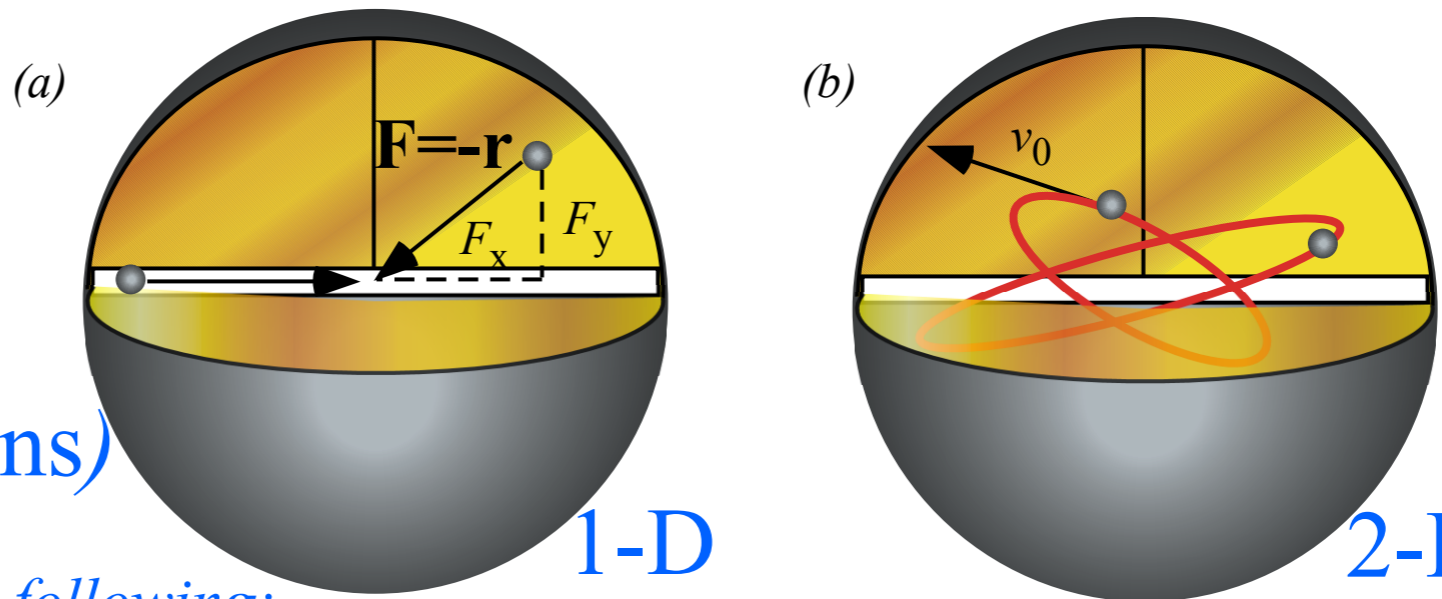
angular velocity:

$$\sqrt{\frac{2E}{m}} \cos\theta = v = \frac{dx}{dt}$$

by (1)

Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

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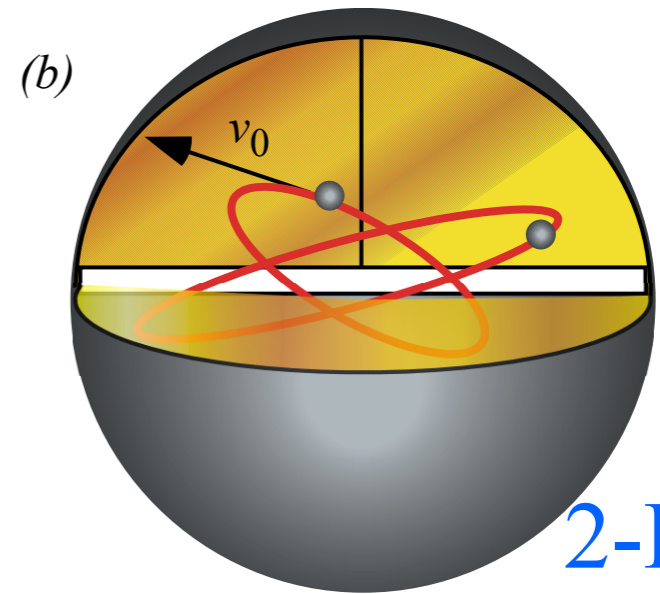
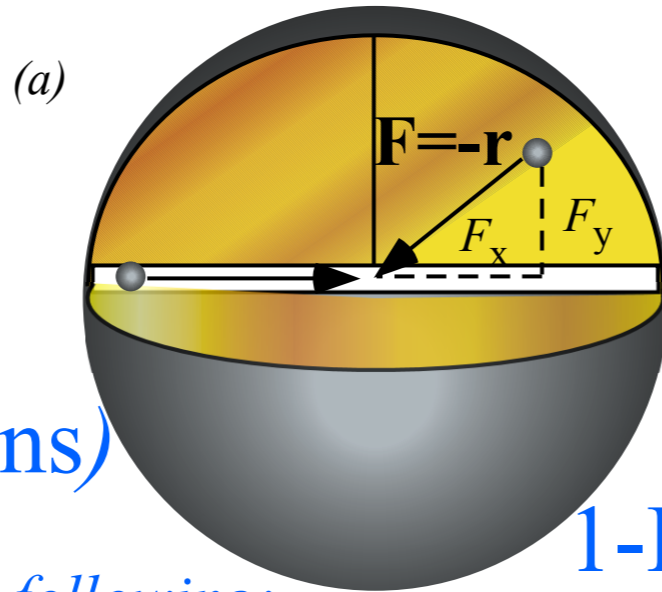
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by (1)

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



2-D or 3-D
(Paths are *always* 2-D ellipses if viewed right!)

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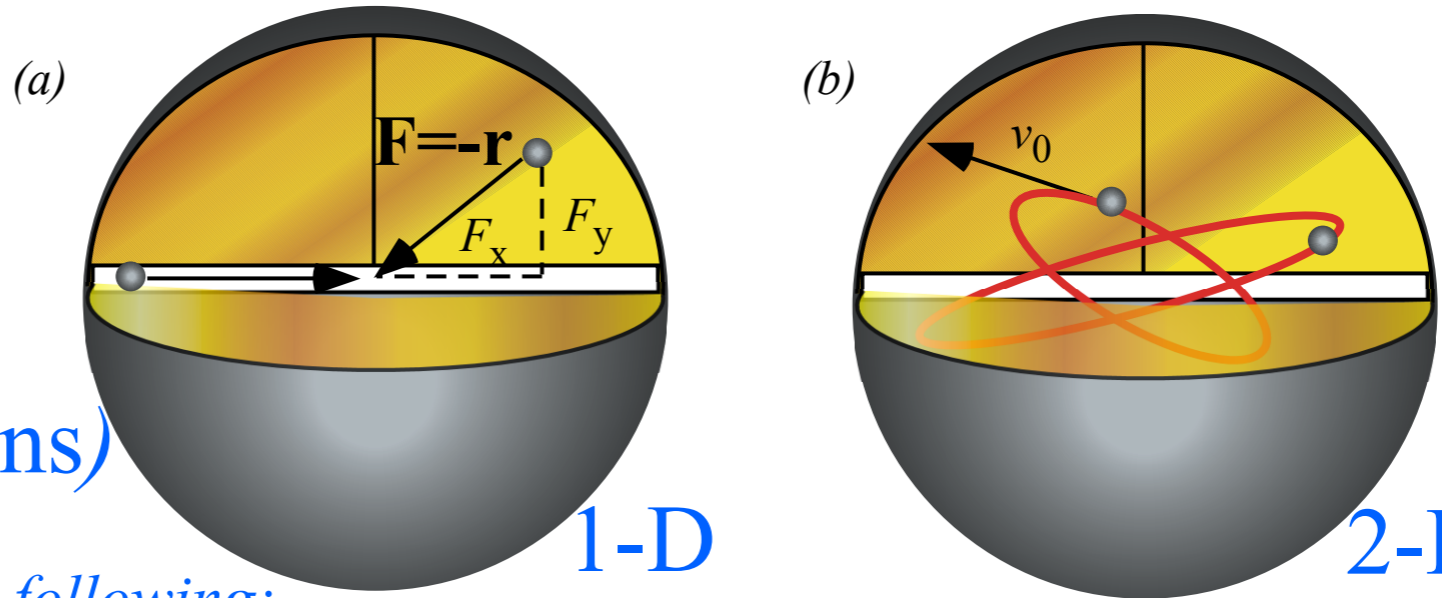
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$$\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{by (1)}}{=} v = \frac{dx}{dt} \stackrel{\text{by def. (3)}}{=} \frac{d\theta}{dt} \frac{dx}{d\theta} \stackrel{\text{by (2)}}{=} \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos\theta$$

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

Unit 1
Fig. 9.10



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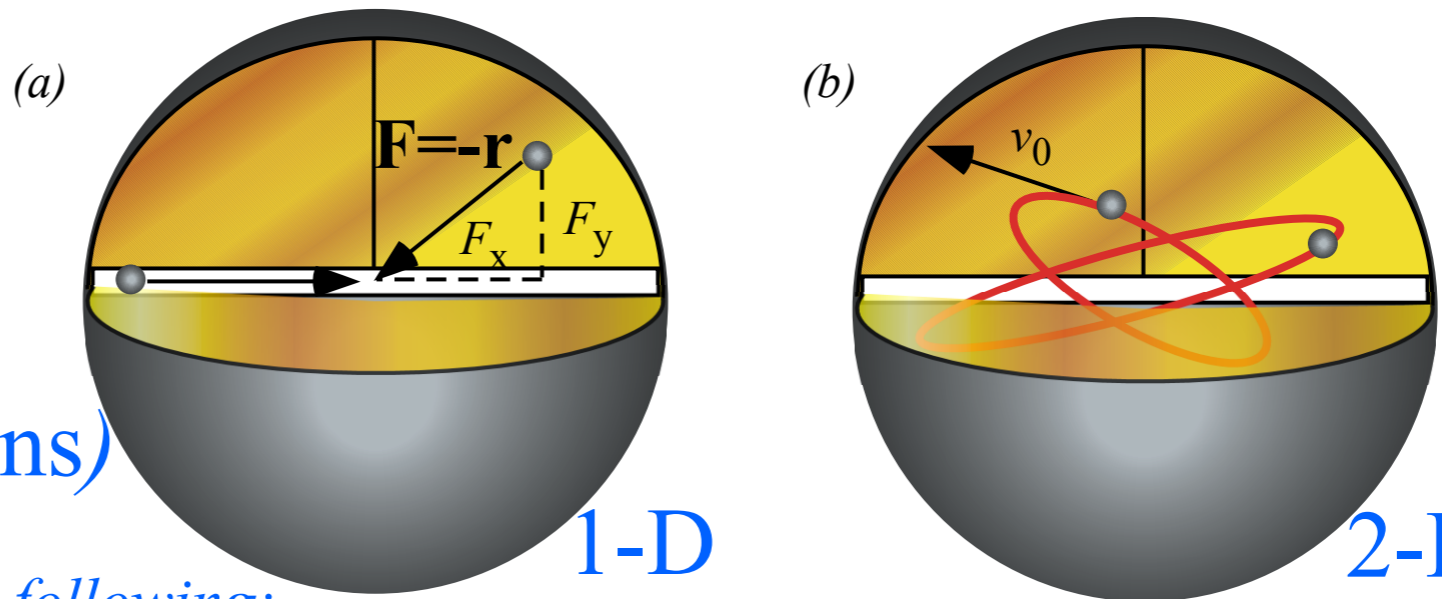
$$\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{by (1)}}{=} v = \frac{dx}{dt} \stackrel{\text{by def. (3)}}{=} \frac{d\theta}{dt} \frac{dx}{d\theta} \stackrel{\text{by (2)}}{=} \omega \frac{dx}{d\theta} = \omega \sqrt{\frac{2E}{k}} \cos\theta$$

by def. (3)

$$\omega = \frac{d\theta}{dt}$$

Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



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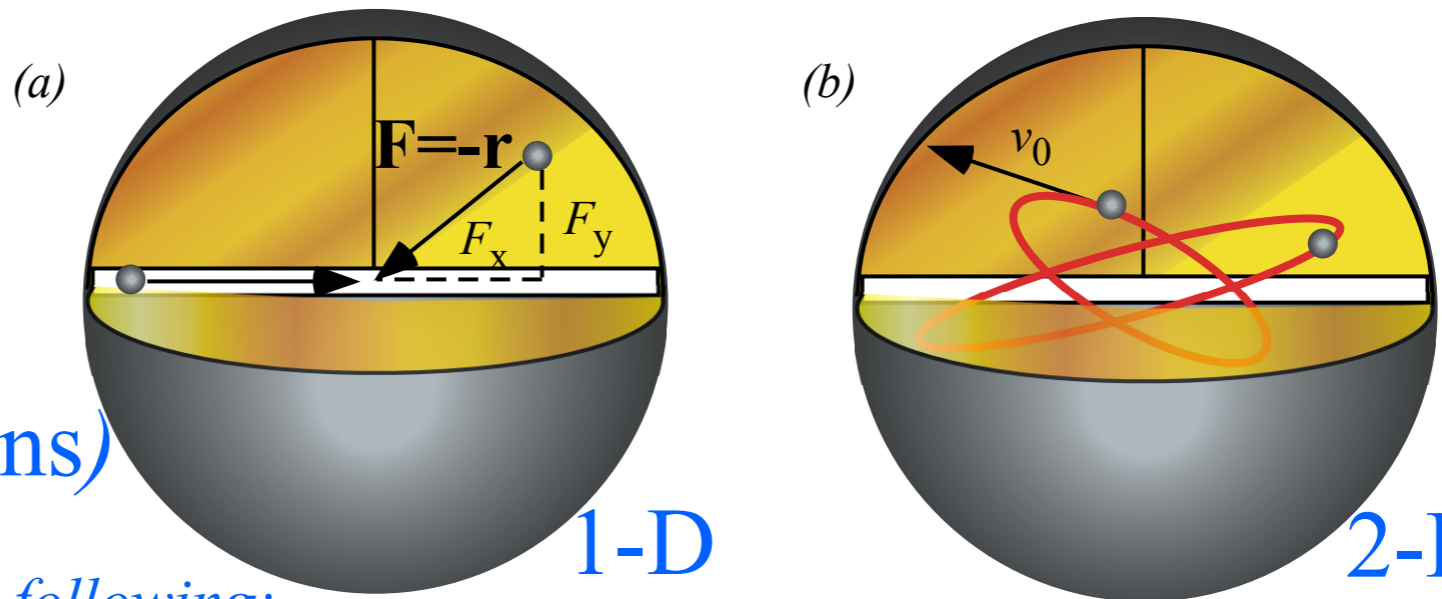
$$\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{by (1)}}{=} v = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} \stackrel{\text{by (2)}}{=} \omega \sqrt{\frac{2E}{k}} \cos\theta$$

$$\omega = \frac{d\theta}{dt} \stackrel{\text{by def. (3)}}{=} \frac{\sqrt{\frac{2E}{m}} \cos\theta}{\sqrt{\frac{2E}{k}} \cos\theta} \stackrel{\text{divide (1)}}{=} \sqrt{\frac{k}{m}}$$

by (2) derivative

Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



1-D

2-D or 3-D

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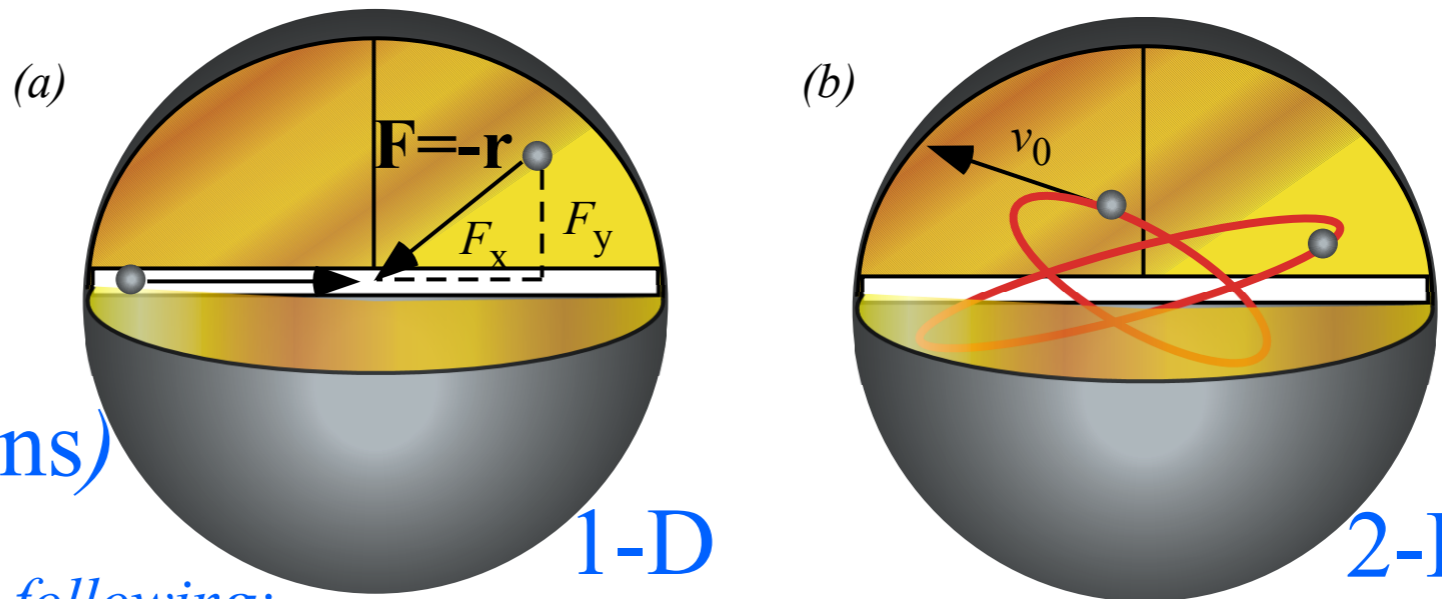
Let : (1) $v = \sqrt{2E/m} \cos\theta$, and : (2) $x = \sqrt{2E/k} \sin\theta$ def. (3) $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{by (1)}}{=} v = \frac{dx}{dt} = \frac{d\theta}{dt} \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} \stackrel{\text{by (2)}}{=} \omega \sqrt{\frac{2E}{k}} \cos\theta$$

$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}} \cos\theta \stackrel{\text{divide (1)}}{}}{\sqrt{\frac{2E}{k}} \cos\theta \stackrel{\text{by (2) derivative}}{}}} = \sqrt{\frac{k}{m}}$$

Isotropic Harmonic Oscillator *phase dynamics* in uniform-body

Unit 1
Fig. 9.10



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2-D or 3-D

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by (1) by def. (3) by (2)

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

by integration given constant ω :

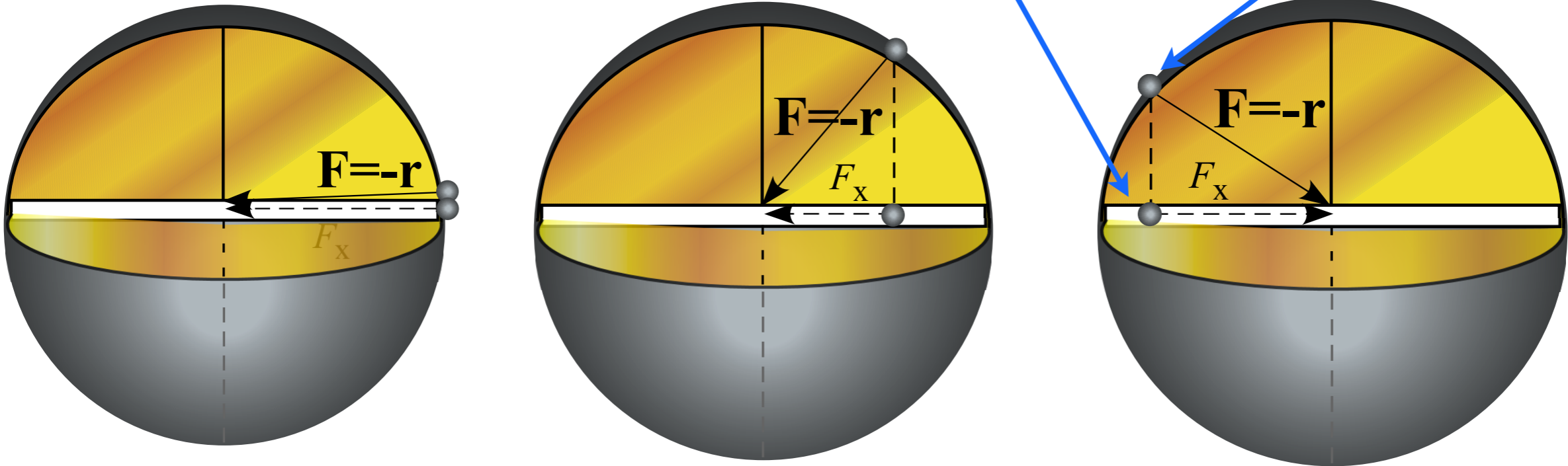
$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$



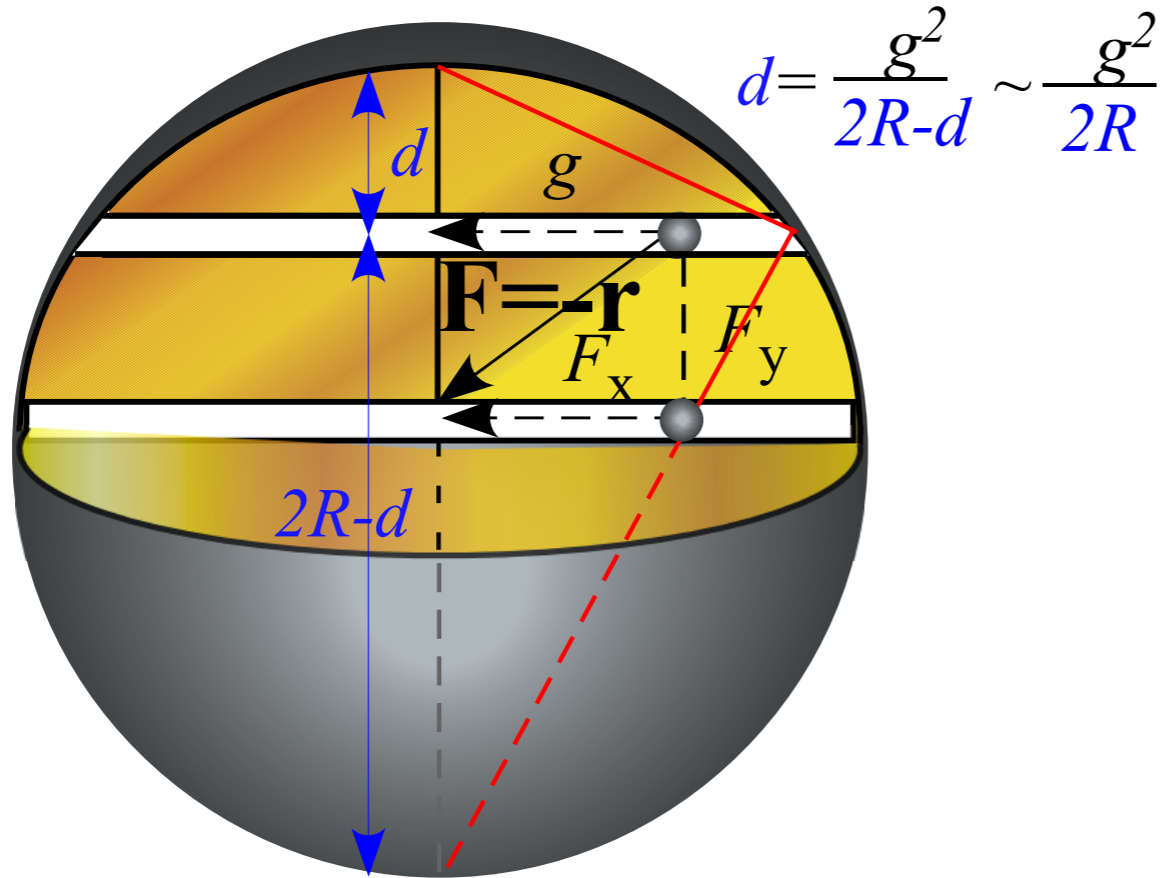
Introducing 2D IHO orbits and phasor geometry

Phasor “clock” geometry

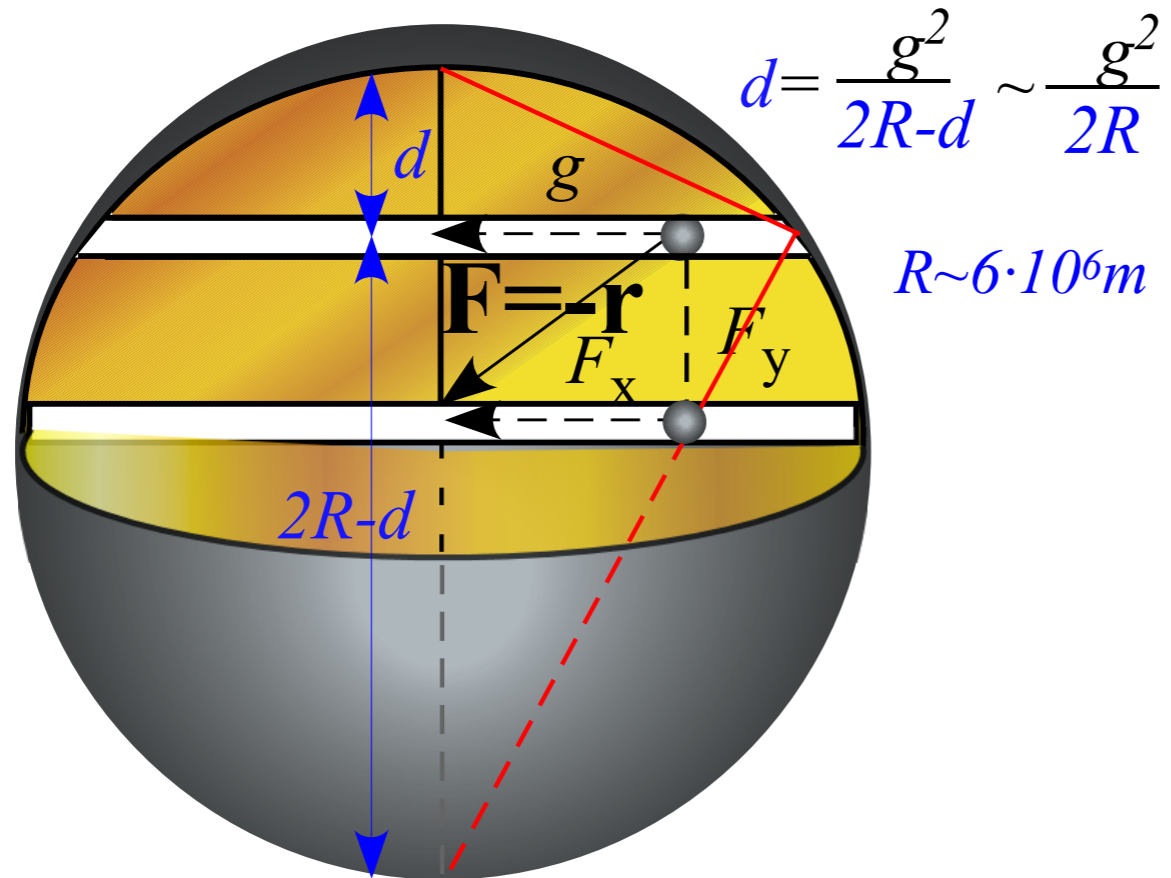
Isotropic Harmonic Oscillator makes tunneling ball track orbiting ball



Isotropic Harmonic Oscillator makes balls in parallel tunnel track each other



Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...

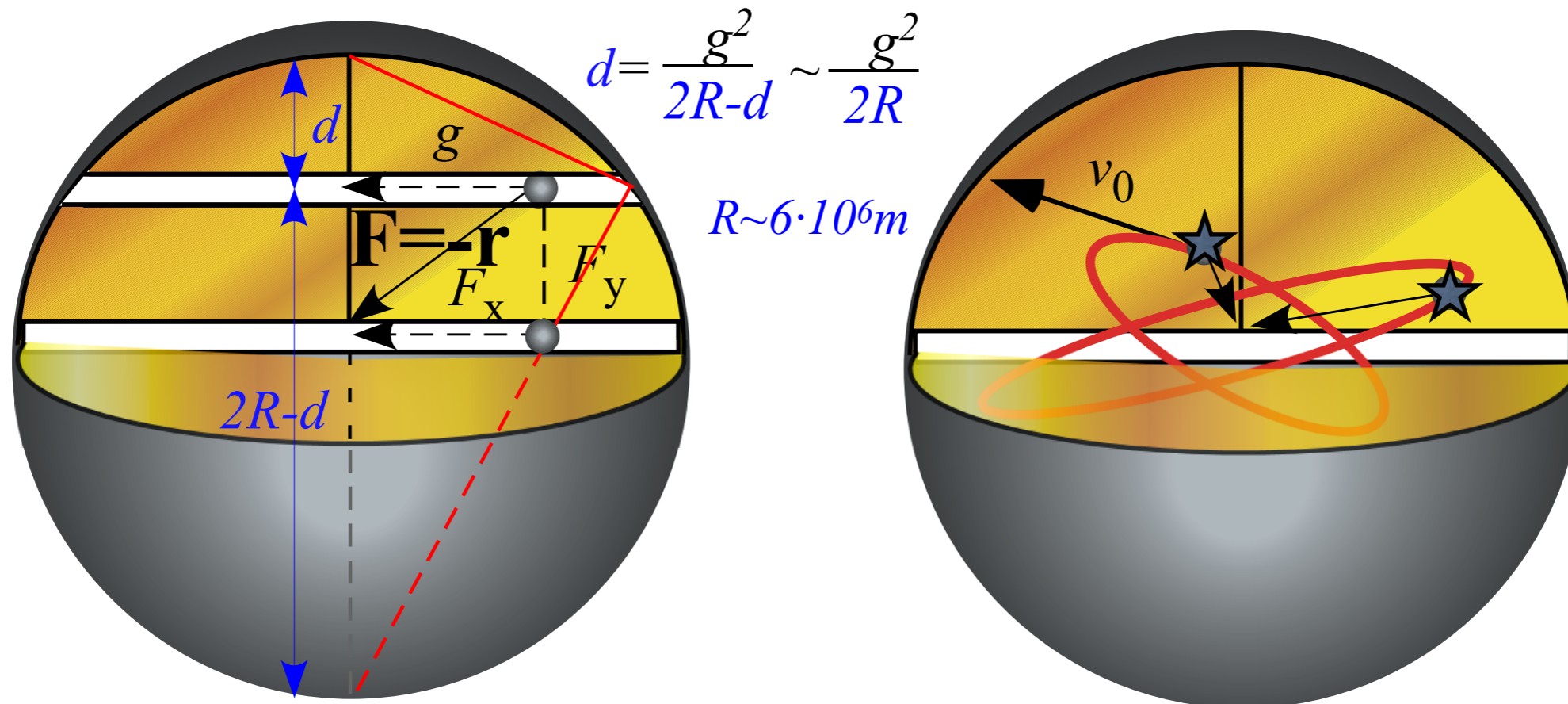


$$d \sim \frac{1}{2R}$$

...even if track length is just $g = 1\text{m}$ so $d \sim (1/12)\text{micron}$

They all take about 84 minutes to go from right to left and back, again.

Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...



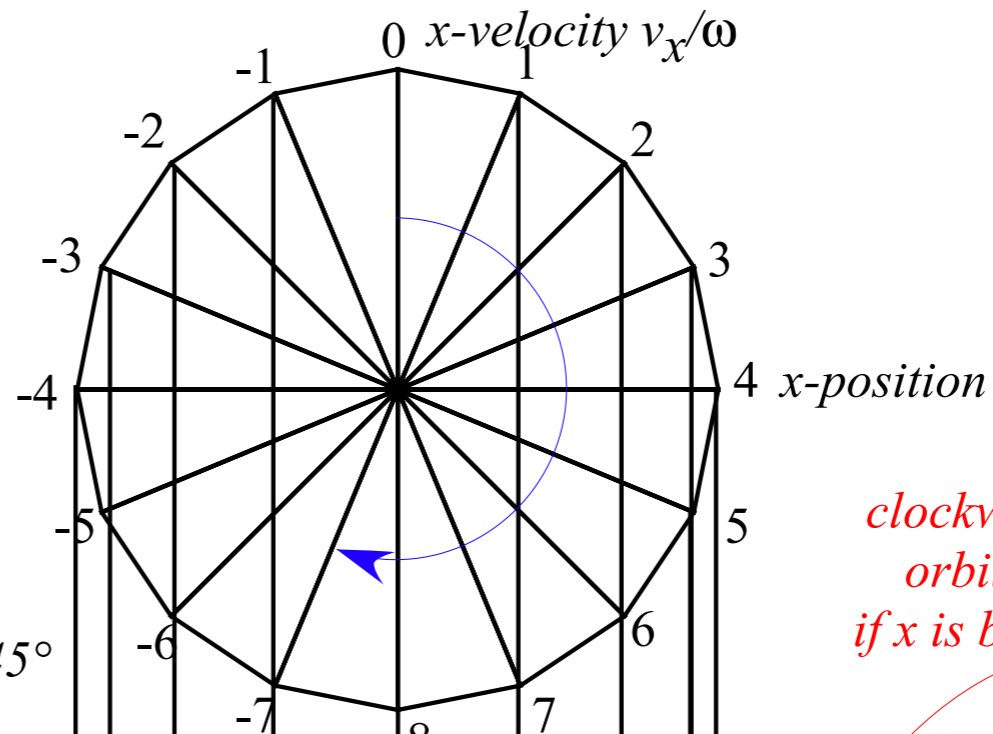
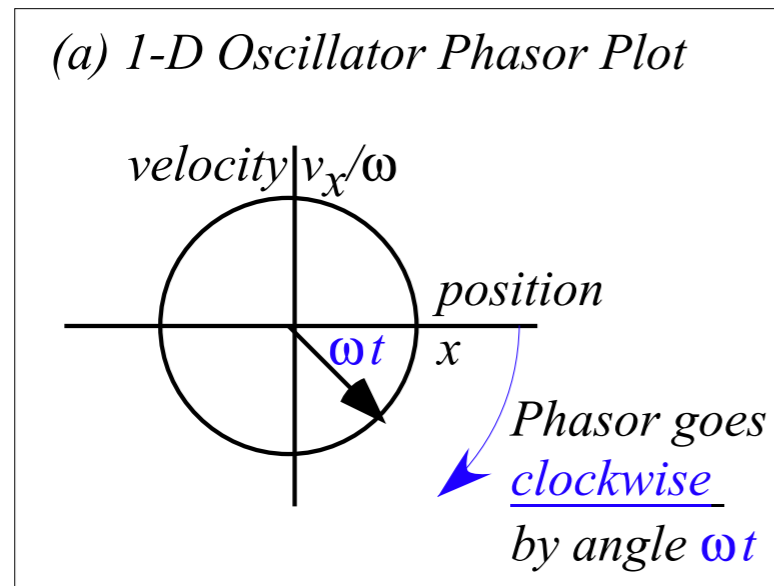
...even if track length is just $g = 1m$ so $d = (1/12)micron$

The all take about 84 minutes to go from right to left and back, again.

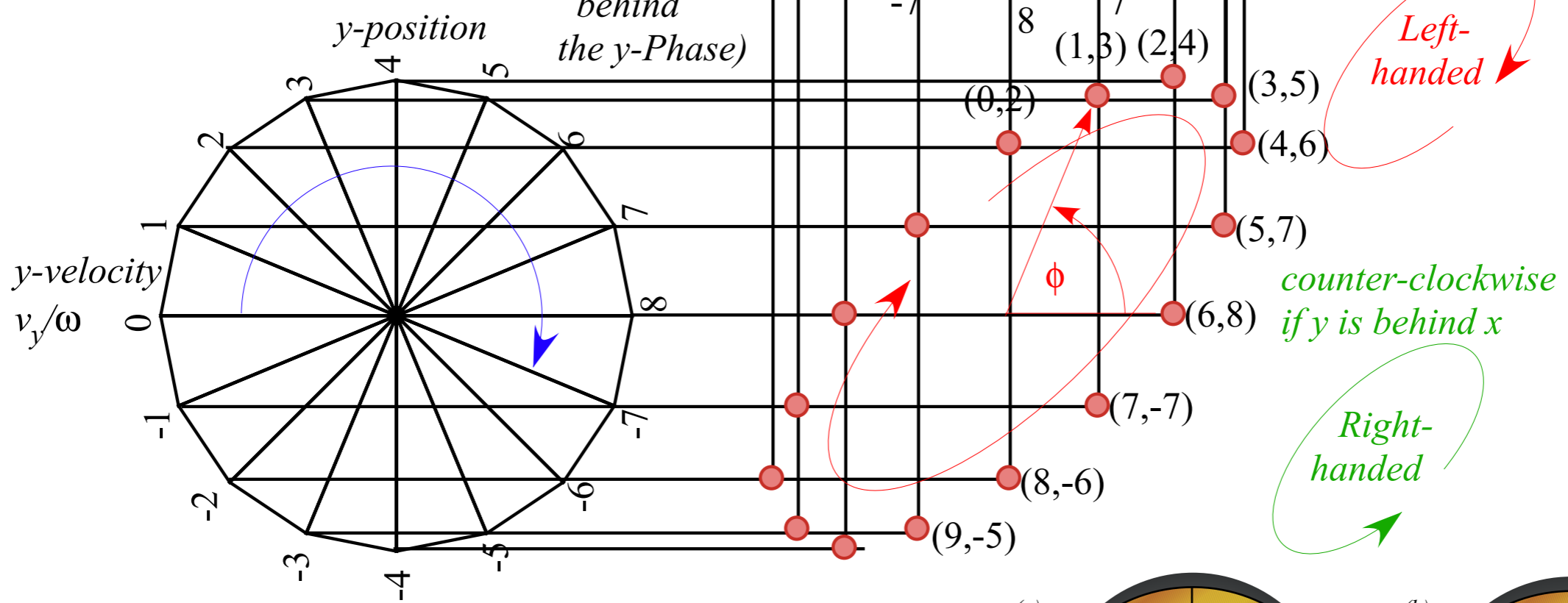
Most neutron starlet (★) orbits are centered ellipses

Isotropic Harmonic Oscillator *phase dynamics in uniform-body*

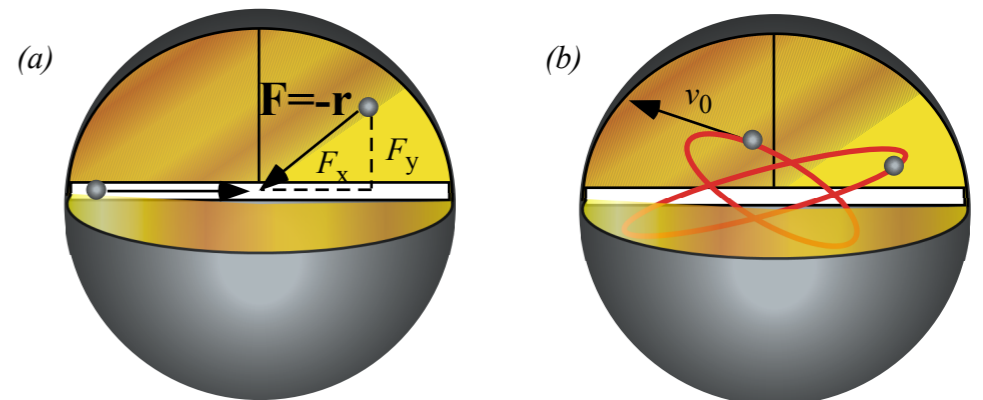
Unit 1
Fig. 9.10



(b) 2-D Oscillator Phasor Plot

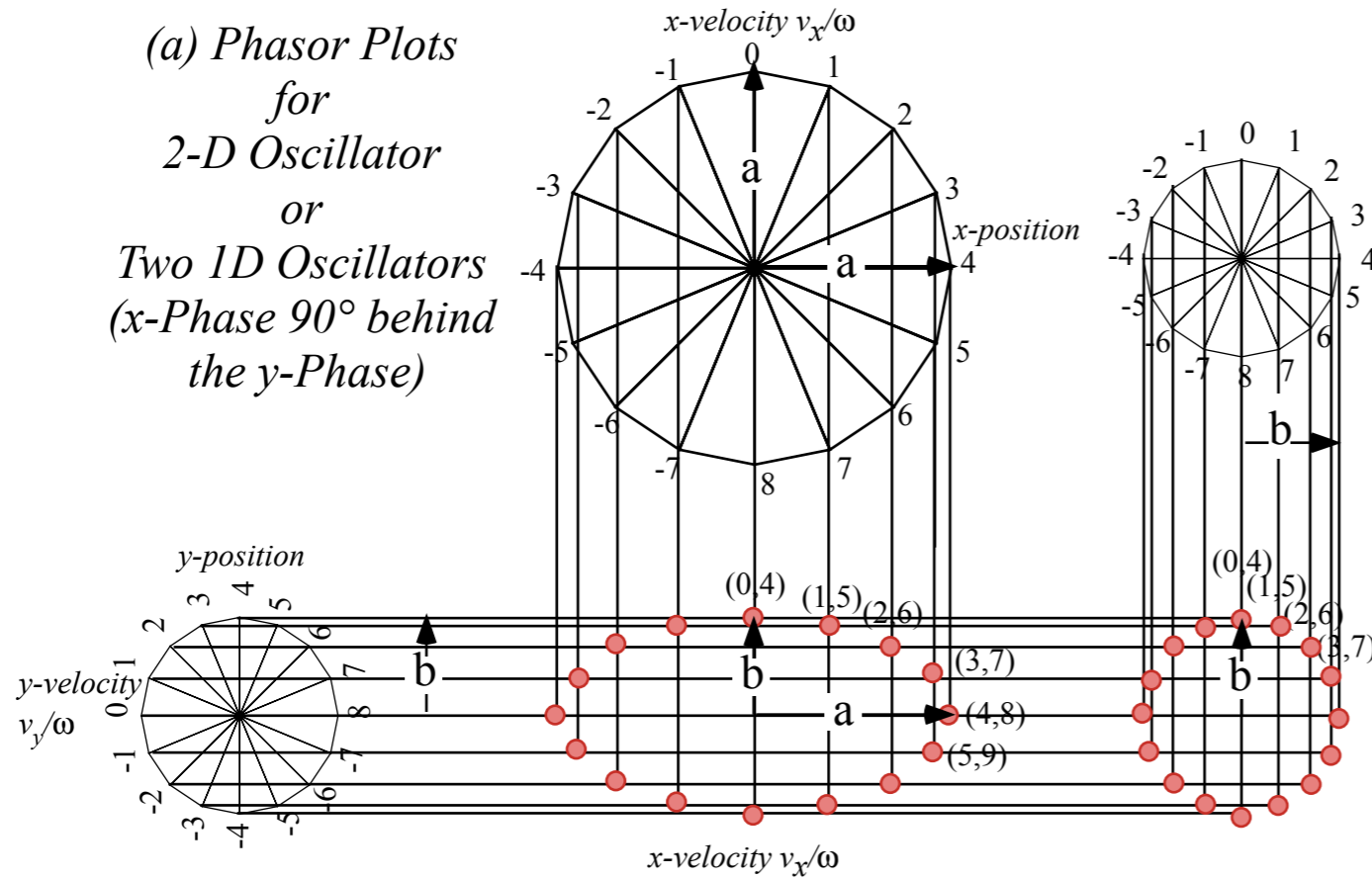


[RelaWavity web simulation - Contact ellipsometry](#)

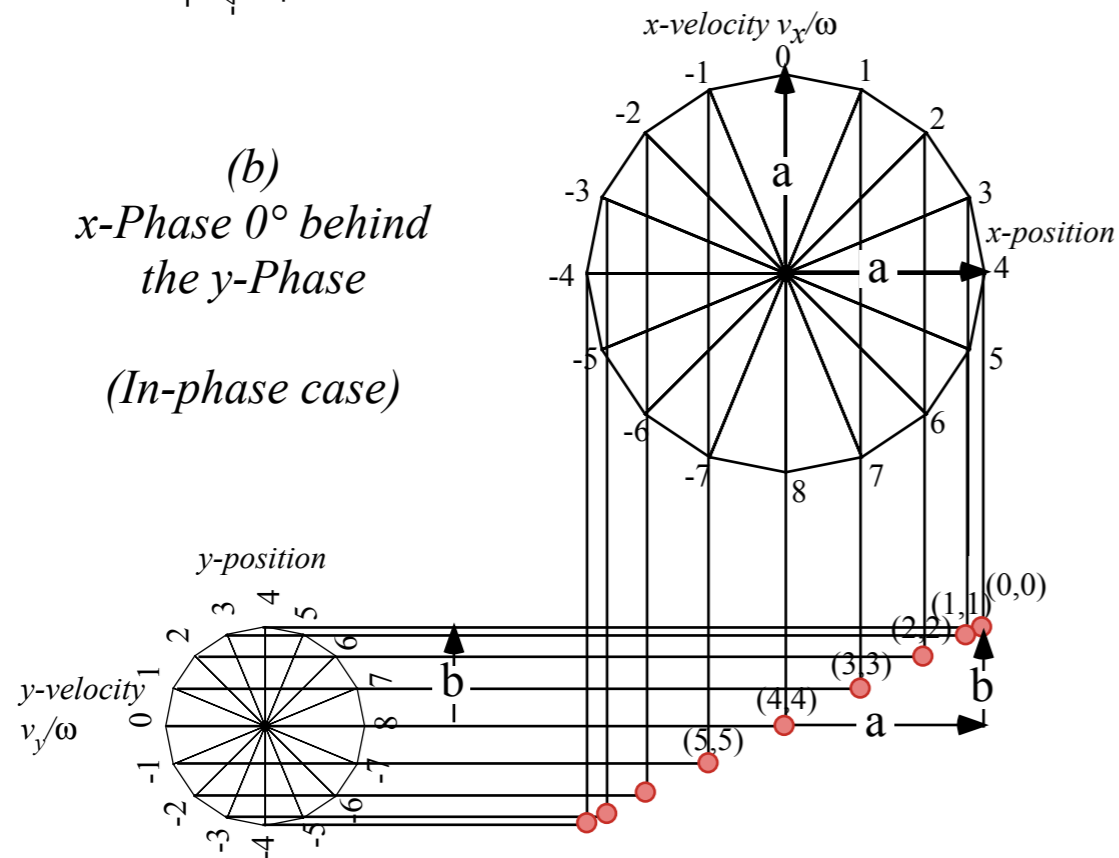


[Introduction to Phasors at our Pirelli Relativity Site](#)
[BoxIt web simulation - With \$y\$ -Phasor is on other side of \$xy\$ plot](#)

(a) Phasor Plots
for
2-D Oscillator
or
Two 1D Oscillators
(x -Phase 90° behind
the y -Phase)



(b)
 x -Phase 0° behind
the y -Phase
(In-phase case)



*These are more generic examples
with radius of x -phasor differing
from that of the y -phasor.*

AMOP reference links (Updated list given on 2nd and 3rd pages of each class presentation)

[Web Resources - front page](#)

[UAF Physics UTube channel](#)

[Quantum Theory for the Computer Age](#)

[Principles of Symmetry, Dynamics, and Spectroscopy](#)

[Classical Mechanics with a Bang!](#)

[Modern Physics and its Classical Foundations](#)

[2014 AMOP](#)

[2017 Group Theory for QM](#)

[2018 AMOP](#)

[Representations Of Multidimensional Symmetries In Networks - harter-jmp-1973](#)

Alternative Basis for the Theory of Complex Spectra

[Alternative Basis for the Theory of Complex Spectra I - harter-pra-1973](#)

[Alternative Basis for the Theory of Complex Spectra II - harter-patterson-pra-1976](#)

[Alternative Basis for the Theory of Complex Spectra III - patterson-harter-pra-1977](#)

[Frame Transformation Relations And Multipole Transitions In Symmetric Polyatomic Molecules - RMP-1978](#)

[Asymptotic eigensolutions of fourth and sixth rank octahedral tensor operators - Harter-Patterson-JMP-1979](#)

[Rotational energy surfaces and high- J eigenvalue structure of polyatomic molecules - Harter - Patterson - 1984](#)

[Galloping waves and their relativistic properties - ajp-1985-Harter](#)

[Rovibrational Spectral Fine Structure Of Icosahedral Molecules - Cpl 1986 \(Alt Scan\)](#)

Theory of hyperfine and superfine levels in symmetric polyatomic molecules.

I) [Trigonal and tetrahedral molecules: Elementary spin-1/2 cases in vibronic ground states - PRA-1979-Harter-Patterson \(Alt scan\)](#)

II) [Elementary cases in octahedral hexafluoride molecules - Harter-PRA-1981 \(Alt scan\)](#)

Rotation-vibration spectra of icosahedral molecules.

I) [Icosahedral symmetry analysis and fine structure - harter-weeks-jcp-1989 \(Alt scan\)](#)

II) [Icosahedral symmetry, vibrational eigenfrequencies, and normal modes of buckminsterfullerene - weeks-harter-jcp-1989 \(Alt scan\)](#)

III) [Half-integral angular momentum - harter-reimer-jcp-1991](#)

[Rotation-vibration scalar coupling zeta coefficients and spectroscopic band shapes of buckminsterfullerene - Weeks-Harter-CPL-1991 \(Alt scan\)](#)

[Nuclear spin weights and gas phase spectral structure of ¹²C₆₀ and ¹³C₆₀ buckminsterfullerene -Harter-Reimer-Cpl-1992 - \(Alt1, Alt2 Erratum\)](#)

[Gas Phase Level Structure of C₆₀ Buckyball and Derivatives Exhibiting Broken Icosahedral Symmetry - reimer-diss-1996](#)

[Fullerene symmetry reduction and rotational level fine structure/ the Buckyball isotopomer ¹²C ¹³C₅₉ - jcp-Reimer-Harter-1997 \(HiRez\)](#)

[Wave Node Dynamics and Revival Symmetry in Quantum Rotors - harter - jms - 2001](#)

[Molecular Symmetry and Dynamics - Ch32-Springer Handbooks of Atomic, Molecular, and Optical Physics - Harter-2006](#)

Resonance and Revivals

I) [QUANTUM ROTOR AND INFINITE-WELL DYNAMICS - ISMSLi2012 \(Talk\) OSU knowledge Bank](#)

II) [Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 \(Talks\)](#)

III) [Quantum Resonant Beats and Revivals in the Morse Oscillators and Rotors - \(2013-Li-Diss\)](#)

[Resonance and Revivals in Quantum Rotors - Comparing Half-integer Spin and Integer Spin - Alva-ISMS-Ohio2013-R777 \(Talk\)](#)

[Molecular Eigensolution Symmetry Analysis and Fine Structure - IJMS-harter-mitchell-2013](#)

[Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-cpl-2013](#)

[QTCA Unit 10 Ch 30 - 2013](#)

[AMOP Ch 0 Space-Time Symmetry - 2019](#)

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(Int.J.Mol.Sci, 14, 714(2013) p.755-774 ,

QTCA Unit 7 Ch. 23-26),

(PSDS - Ch. 5, 7)

[Int.J.Mol.Sci, 14, 714\(2013\),](#)

[QTCA Unit 8 Ch. 23-25,](#)

[QTCA Unit 9 Ch. 26,](#)

[PSDS Ch. 5,](#)

[PSDS Ch. 7](#)

Intro spin ½ coupling

[Unit 8 Ch. 24 p3](#)

H atom hyperfine-B-level crossing

[Unit 8 Ch. 24 p15](#)

Hyperf. theory Ch. 24 p48.

Hyperf. theory Ch. 24 p48.

[Deeper theory ends p53](#)

Intro 2p3p coupling

[Unit 8 Ch. 24 p17.](#)

Intro LS-jj coupling

[Unit 8 Ch. 24 p22.](#)

CG coupling derived (start)

[Unit 8 Ch. 24 p39.](#)

CG coupling derived (formula)

[Unit 8 Ch. 24 p44.](#)

Lande' g-factor

[Unit 8 Ch. 24 p26.](#)

Irrep Tensor building

[Unit 8 Ch. 25 p5.](#)

Irrep Tensor Tables

[Unit 8 Ch. 25 p12.](#)

Wigner-Eckart tensor Theorem.

[Unit 8 Ch. 25 p17.](#)

Tensors Applied to d,f-levels.

[Unit 8 Ch. 25 p21.](#)

Tensors Applied to high J levels.

[Unit 8 Ch. 25 p63.](#)

Intro 3-particle coupling.

[Unit 8 Ch. 25 p28.](#)

Intro 3,4-particle Young Tableaus

[GrpThLect29 p42.](#)

Young Tableau Magic Formulae

[GrpThLect29 p46-48.](#)

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Predrag Cvitanovic's: Birdtrack Notation, Calculations, and Simplification

[Chaos Classical and Quantum - 2018-Cvitanovic-ChaosBook](#)
[Group Theory - PUP Lucy Day - Diagrammatic notation - Ch4](#)
[Simplification Rules for Birdtrack Operators - Alcock-Zeilinger-Weigert-zeilinger-jmp-2017](#)
[Group Theory - Birdtracks Lies and Exceptional Groups - Cvitanovic-2011](#)
[Simplification rules for birdtrack operators- jmp-alcock-zeilinger-2017](#)
[Birdtracks for SU\(N\) - 2017-Keppeler](#)

Frank Rioux's: UMA method of vibrational induction

[Quantum Mechanics Group Theory and C60 - Frank Rioux - Department of Chemistry Saint Johns U](#)
[Symmetry Analysis for H2O- H2OGrpTheory- Rioux](#)
[Quantum Mechanics-Group Theory and C60 - JChemEd-Rioux-1994](#)
[Group Theory Problems- Rioux- SymmetryProblemsX](#)
[Comment on the Vibrational Analysis for C60 and Other Fullerenes Rioux-RSP](#)

Supplemental AMOP Techniques & Experiment

[Many Correlation Tables are Molien Sequences - Klee \(Draft 2016\)](#)
[High-resolution spectroscopy and global analysis of CF4 rovibrational bands to model its atmospheric absorption- carlos-Boudon-iqsrt-2017](#)
[Symmetry and Chirality - Continuous Measures - Avnir](#)

*

Special Topics & Colloquial References

[r-process nucleosynthesis from matter ejected in binary neutron star mergers-PhysRevD-Bovard-2017](#)

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