Lecture 5
Tue. 9.10.2013

Dynamics of Potentials and Force Fields

(Ch. 7 and Ch. 8 of Unit 1)

(From Lect 4.) A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)] [John Farey, Phil. Mag.(1816)]

Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"
Geometry and dynamics of single ball bounce
Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)
Some physics of dare-devil-divers
Non-linear force (like superball-floor or ball-bearing-anvil)
Geometry and dynamics of 2-ball bounce (again with feeling)
The parable of RumpCo. vs CrapCorp.
The story of USC pre-meds visiting Whammo Manufacturing Co.
Geometry and dynamics of 3-ball bounce
A story of Stirling Colgate (Palmolive) and core-collapse supernovae
The story of USC pre-meds visiting Whammo Manufacturing Co.
Other bangings-on: The western buckboard and Newton's balls

Crunch energy geometry of freeway crashes and related things

*Crunch energy played backwards: This really **is** "Rocket-Science"*
A Thales construction for momentum-energy

Potential energy geometry of Superballs and related things

→ *Thales geometry and "Sagittal approximation"*

Geometry and dynamics of single ball bounce

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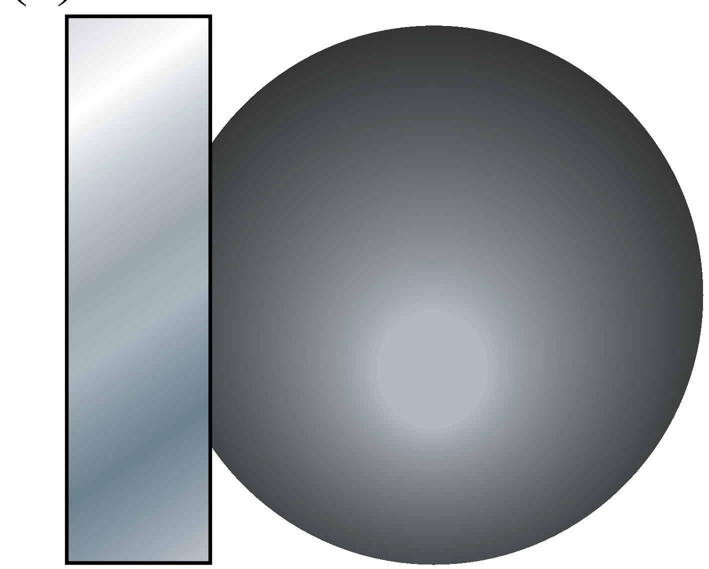
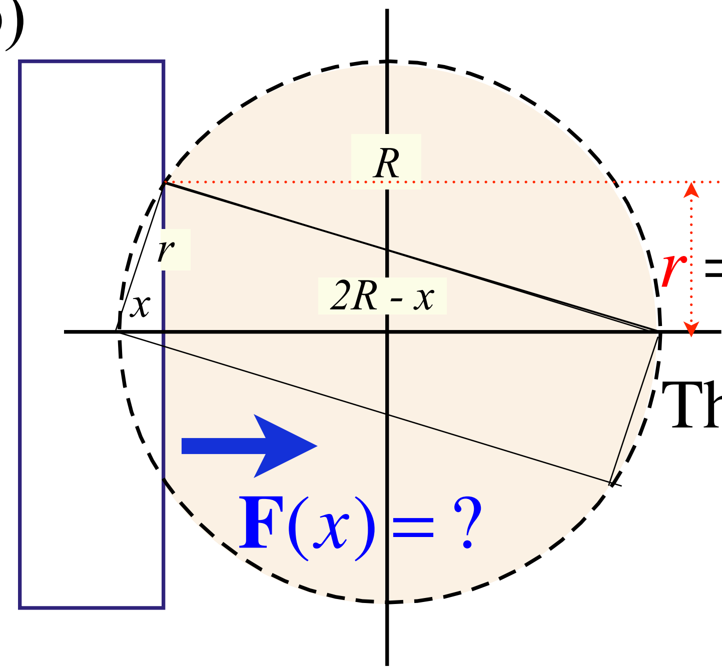
Potential Energy Geometry of Superballs and Related things

(a)



Unit 1
Fig. 7.1
(modified)

(b)



$$r = \sqrt{x(2R-x)} \quad \left(\approx \sqrt{2Rx} \quad for: \ x << R\right)$$

Thales' geometry and "Sagittal†" approx.

† "bow"

If superball was a balloon its bounce force law would be linear *F=-k·x* *(Hooke Law)*

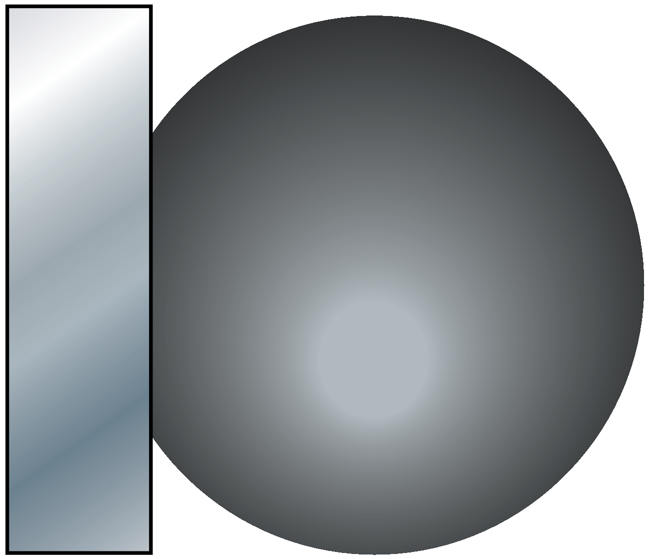
(Pressure)

$$F_{balloon}(x) = P \cdot A = P \cdot \pi r^2$$

$$\approx P \cdot \pi 2Rx$$

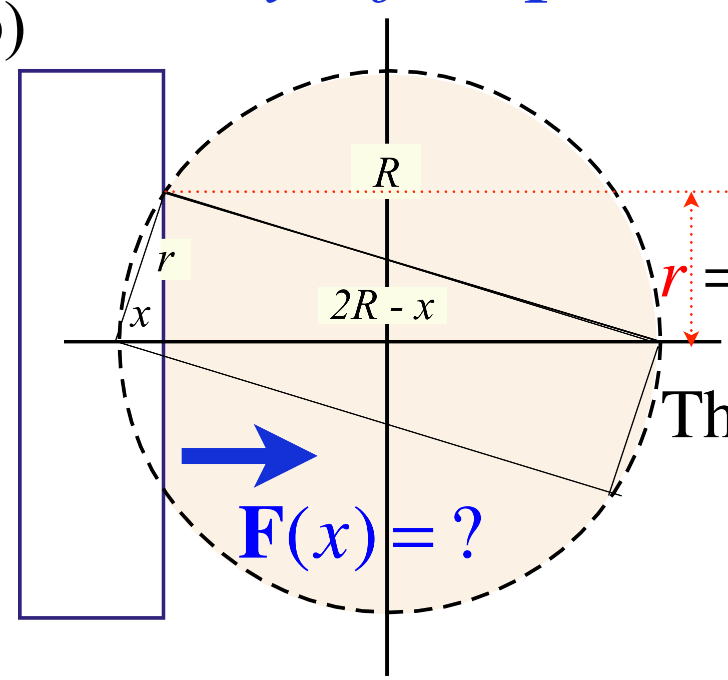
Potential Energy Geometry of Superballs and Related things

(a)



Unit 1
Fig. 7.1
(modified)

(b)



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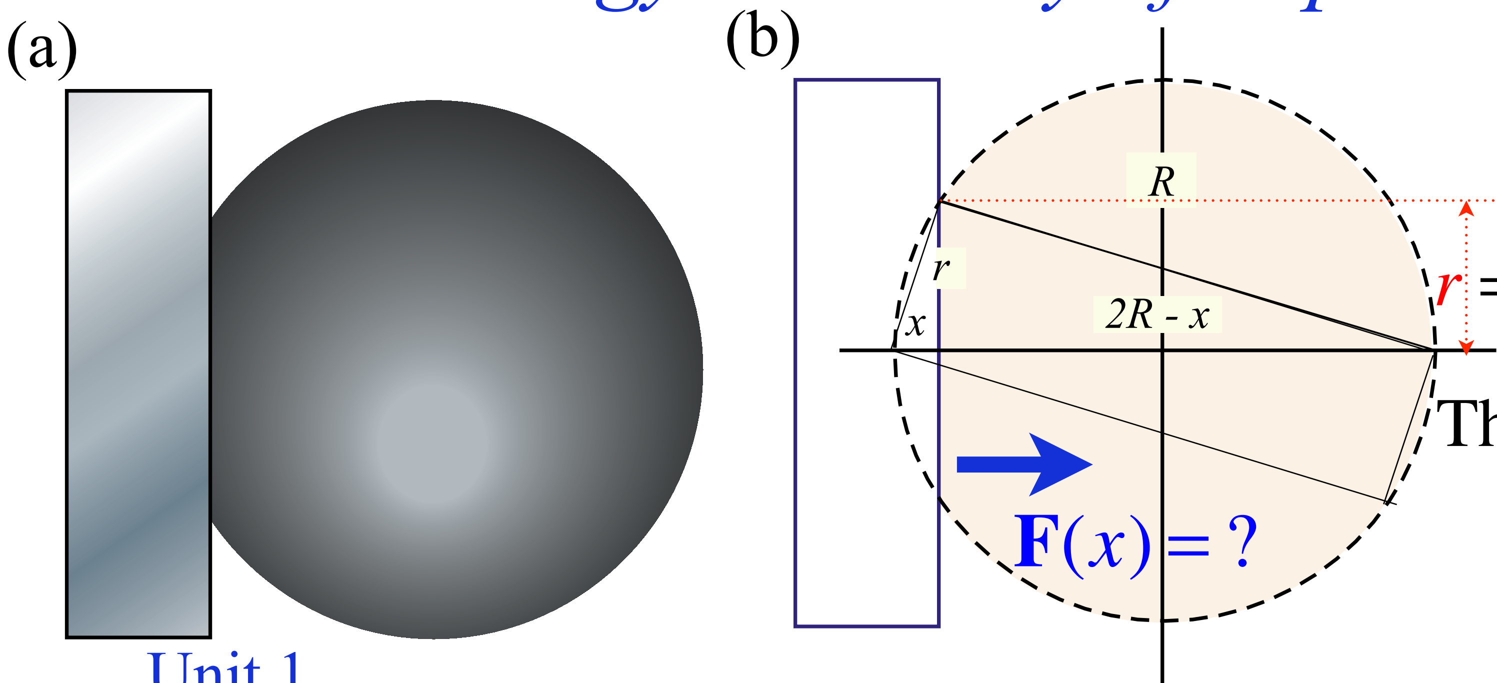
If superball was a balloon its bounce force law would be linear $F=-k\cdot x$ (Hooke Law)

$$F_{balloon}(x) = \underset{(Pressure)}{P} \cdot A = P \cdot \pi r^2$$

$$\approx P \cdot \pi 2Rx = \underbrace{P \cdot 2\pi R}_{\text{(Hooke spring constant } k\text{)}} x$$

$$= kx$$

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)

$r = \sqrt{x(2R-x)} \quad \left(\approx \sqrt{2Rx} \quad for: \ x << R\right)$

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If superball was a balloon its bounce force law would be linear $F=-k\cdot x$ *(Hooke Law)*

$$F_{balloon}(x) = \underset{(Pressure)}{P} \cdot A = P \cdot \pi r^2$$

$$\approx P \cdot \pi 2Rx = \underbrace{P \cdot 2\pi R}_{(Hooke\ spring\ constant\ k)} x$$

$$= kx$$

Instead superball force law depends on bulk *volume* modulus and is non-linear $F\sim x^{p?}+?$ *(Power Law?)*

$$Volume(X) = \int_0^X \pi r^2\, dx = \int_0^X \pi x(2R-x)\,dx = \int_0^X 2R\pi x\, dx - \int_0^X \pi x^2\, dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (for: X << R) \\ \frac{4}{3}\pi R^3 & (for: X = 2R) \end{cases}$$

It also depends on velocity $\dot{x}=\frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by *"Project-Ball*"*

* *Am. J. Phys.* **39**, 656 (1971)

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Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce *(See Simulation)*

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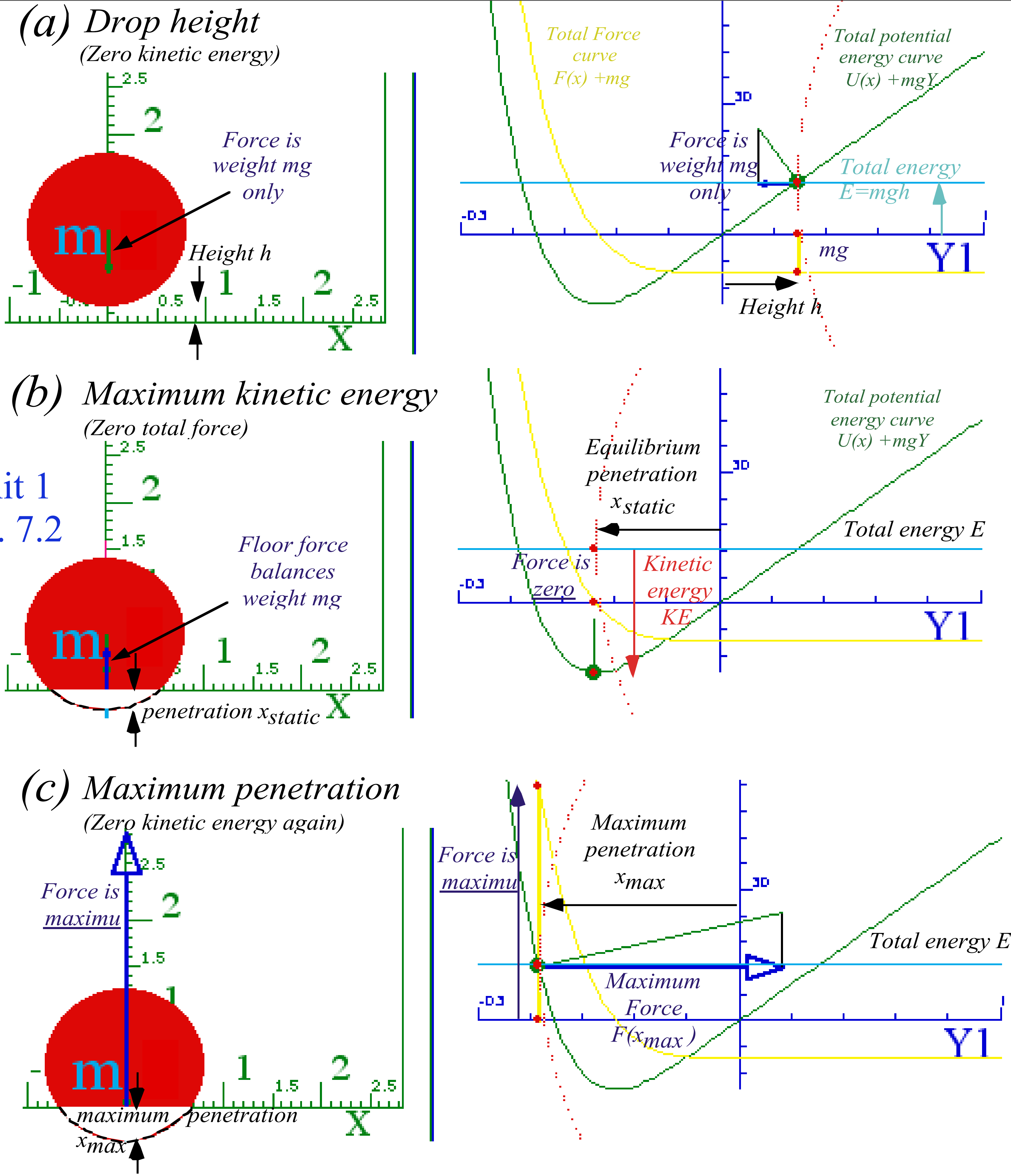

(a) Drop height
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Force is weight mg only
Total energy E=mgh
mg
Y1
Height h
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x static
Equilibrium penetration x static
Total potential energy curve U(x) +mgY
Total energy E
Force is zero
Kinetic energy KE
Y1
(c) Maximum penetration
(Zero kinetic energy again)
Force is maximu
maximum penetration
x max
Force is maximu
Maximum penetration x max
Total energy E
Maximum Force F(x max)
Y1

Unit 1
Fig. 7.2

Main Control Panel

Start | Resume

- Let mouse set: (x,y,Vx,Vy) (selected)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths (selected)
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1 (selected)
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed (selected)
- No preset initial values

Number of masses: 1 Balls

Acceleration of gravity: 0.5 100x{cm/s^2}

- [x] Draw force vectors
- [x] Pause (once) at top
- [x] Constrain motion to Y-axis
- [x] Plot v2 vs v1
- [ ] Plot p2 vs p1
- [ ] Plot V2 vs V1
- [ ] Plot Ellipses
- [ ] Plot Bisector Lines
- [ ] Old Color Scheme

Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 5.45 x10^ -1 {g}

Force power law exponent: 1

Force Constant: 500

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

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		y Max =	7
Initial x1 =	0.5	y Min =	0
Max x PE plot =	0.5	T Max =	6
F-Vector scale =	0.003	V2y Max =	3
Error step =	0.000	V2y Min =	-2

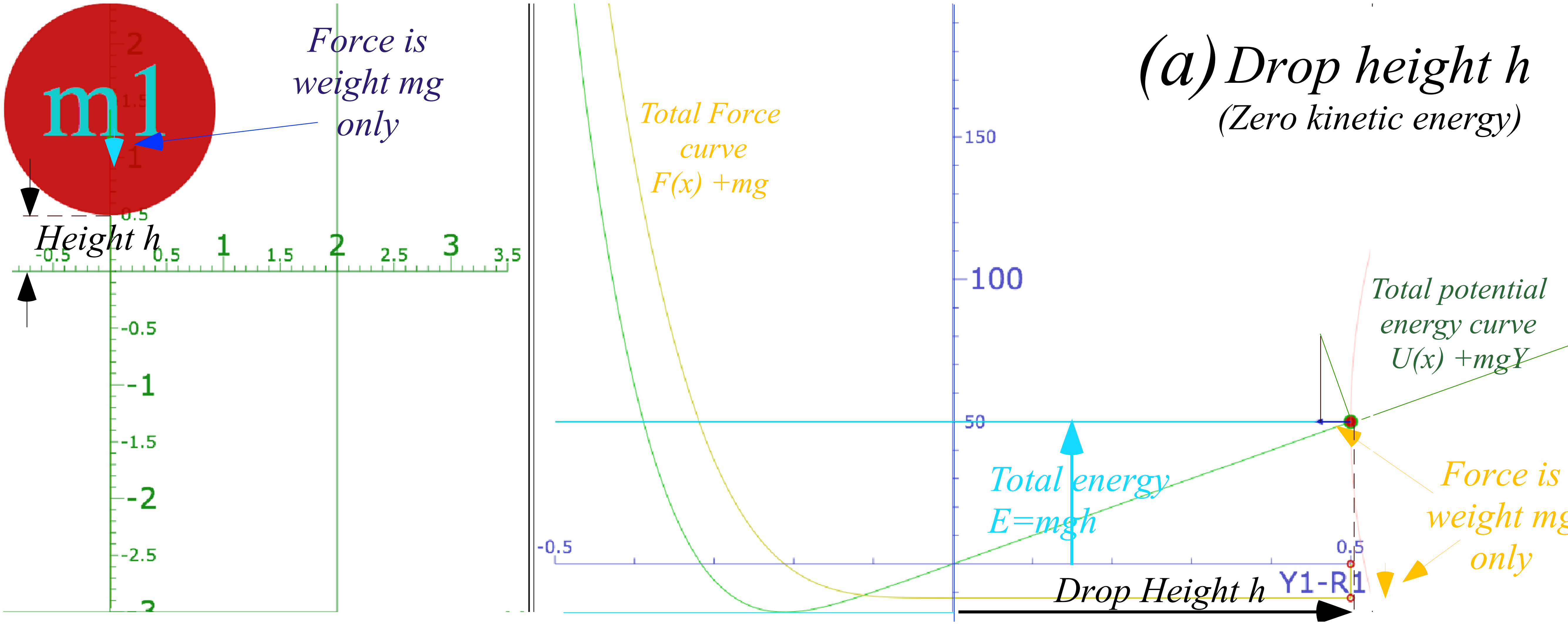
---

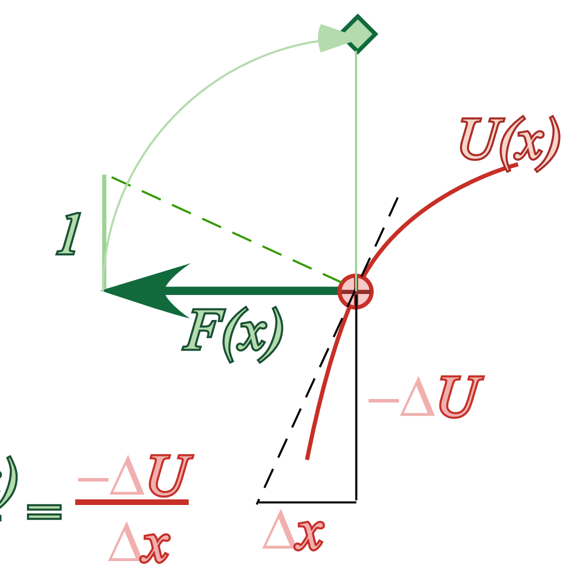
m1= 1 x10^ 2 {g} $V1_0$= 0 x10^ 0 {cm/s}

Zero Gap 2-Ball Collision (m1:m2 = 1:7)	
Linear 2-Ball Collision (m1:m2 = 1:7)	
Newton's Balls (Zero gap, Nonlinear force)	
Newton's Balls (Zero gap, Linear force)	
3-Ball Tower	5-Ball Towe
Potential Plot (1 Ball, Nonlinear force)	
Potential Plot (1 Ball, Linear force)	
Gravity Potential (1 Ball, Nonlinear force)	
Gravity Potential (1 Ball, Linear force)	

(See Simulations)

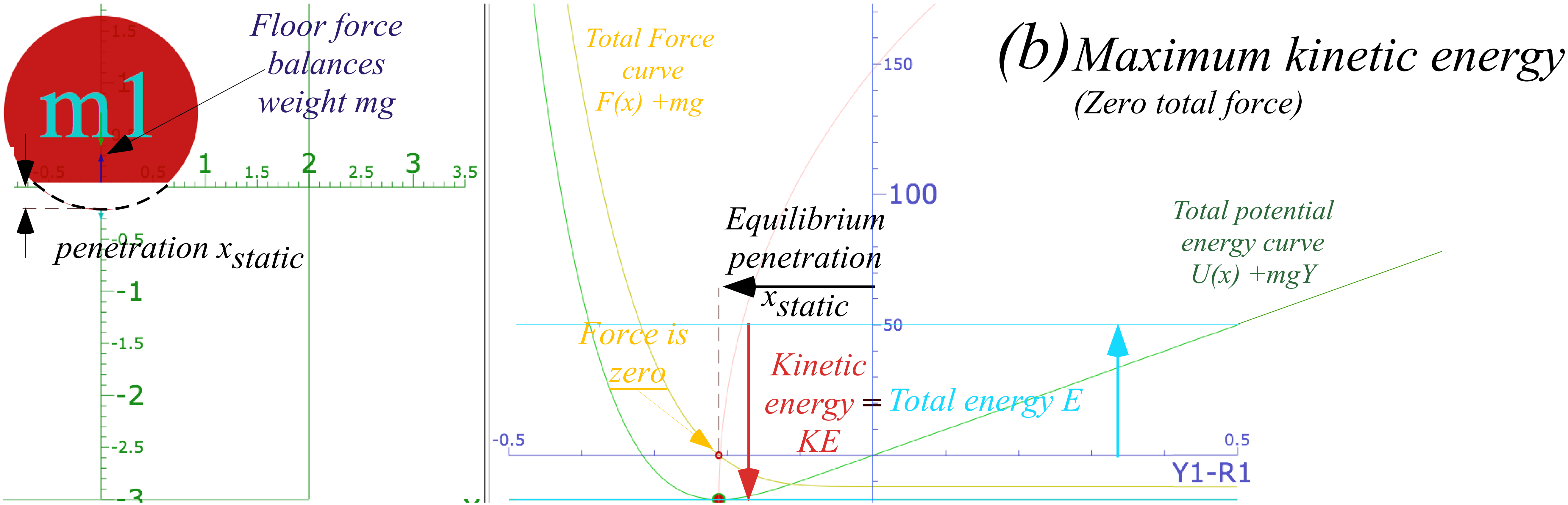
(a) Drop height h
(Zero kinetic energy)





$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

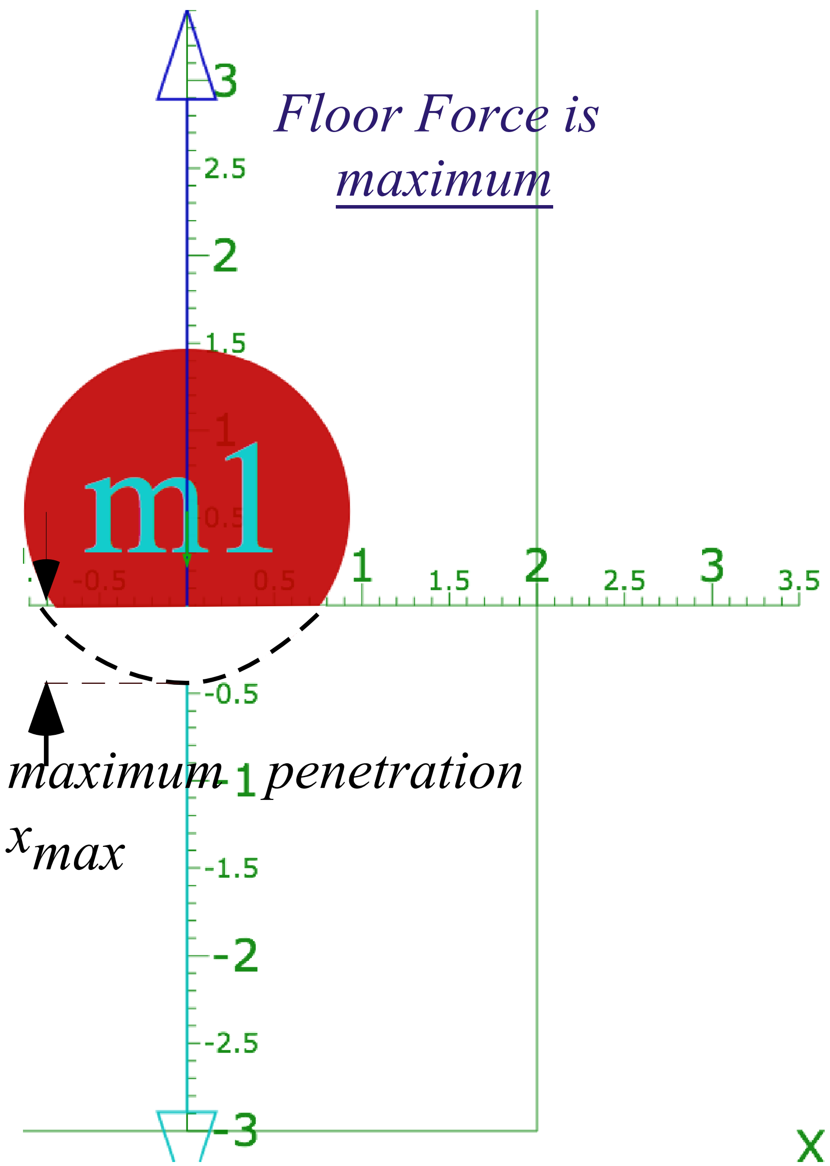
Display of Force vector using similar triangle constuction based on the slope of potential curve.

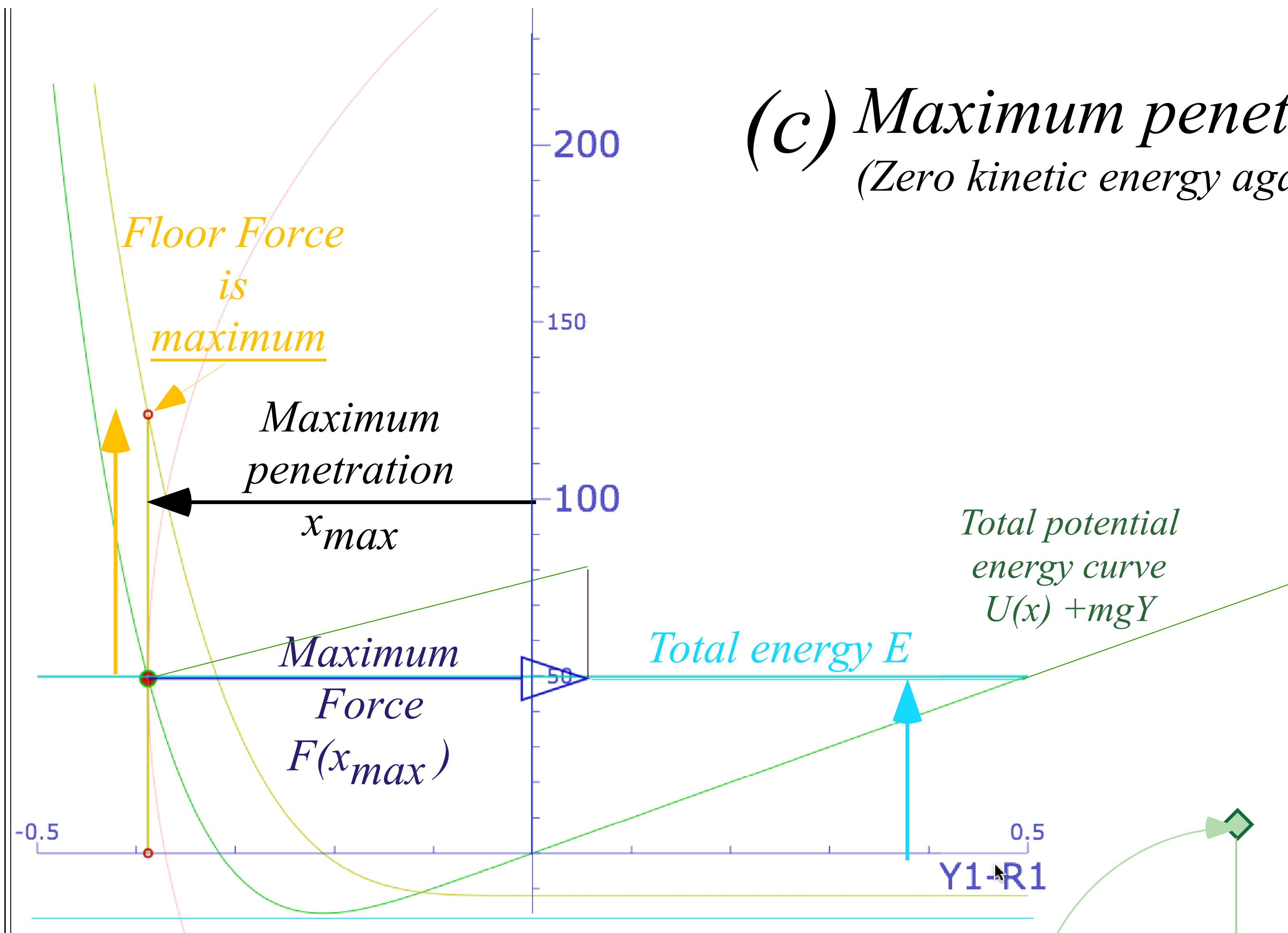


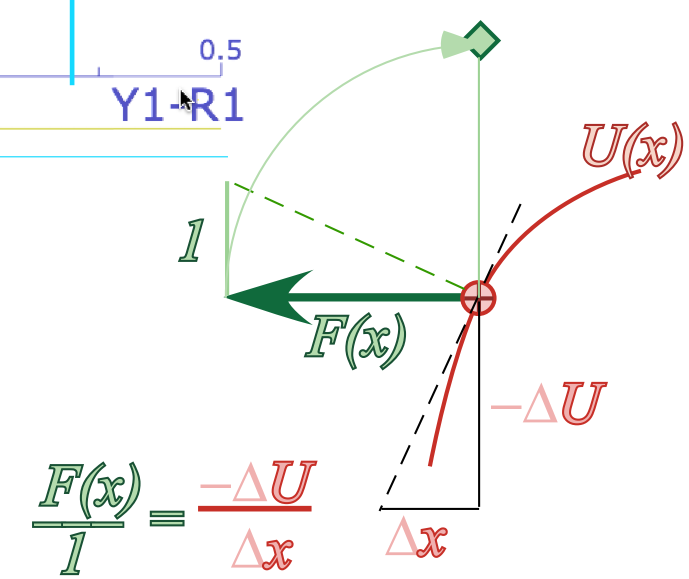
(b) Maximum kinetic energy
(Zero total force)
m1
Floor force
balances
weight mg
penetration x_{static}
Total Force
curve
F(x) +mg
Force is
zero
Equilibrium
penetration
x_{static}
Kinetic
energy
KE
=
Total energy E
Total potential
energy curve
U(x) +mgY
Y1-R1

(c) Maximum penetration

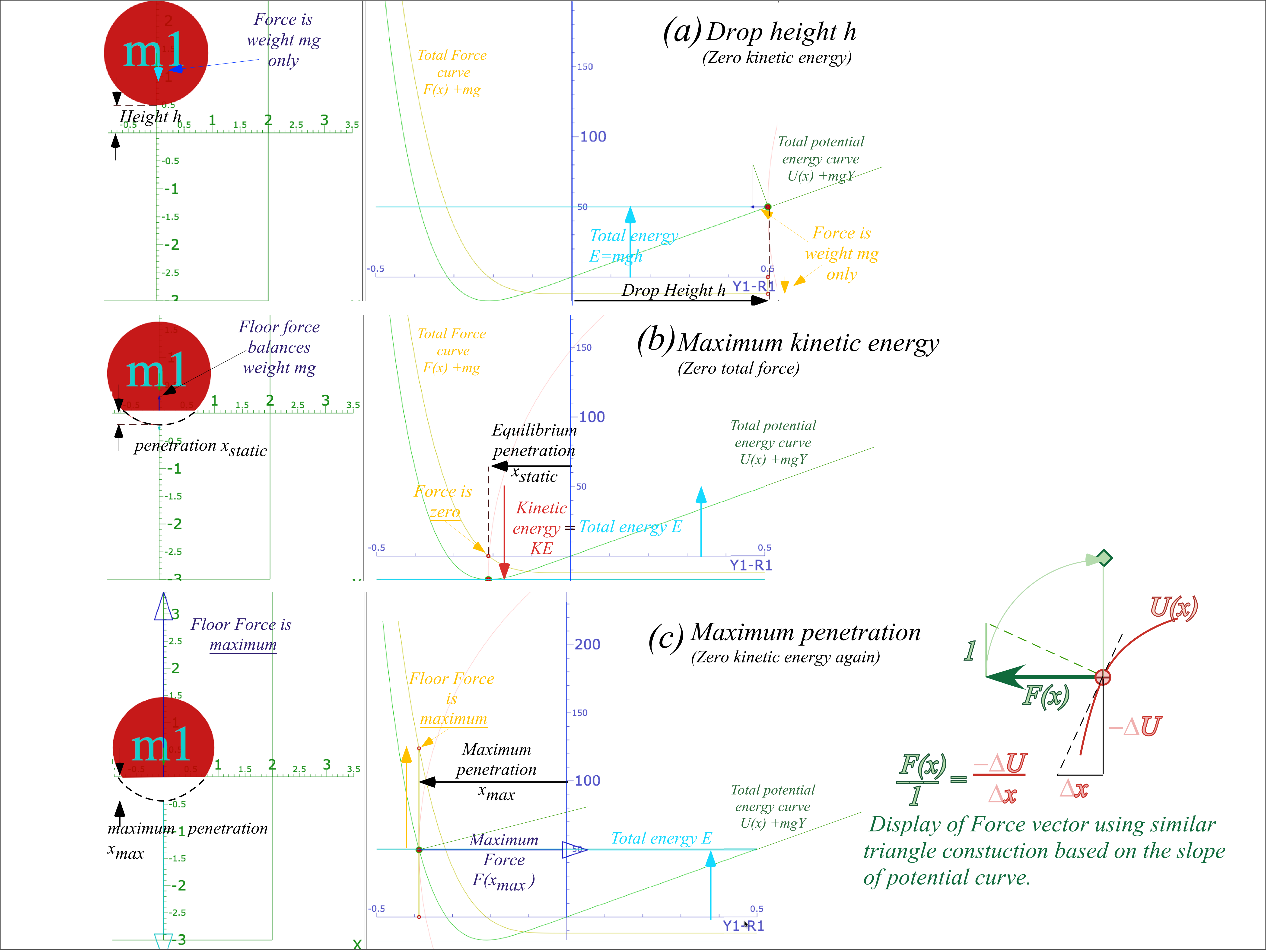
(Zero kinetic energy again)







Display of Force vector using similar triangle constuction based on the slope of potential curve.



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Force is weight mg only
Height h
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Total energy E=mgh
Force is weight mg only
Drop Height h
Y1-R1
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x_{static}
Total Force curve F(x) +mg
Equilibrium penetration x_{static}
Force is zero
Kinetic energy KE
= Total energy E
Total potential energy curve U(x) +mgY
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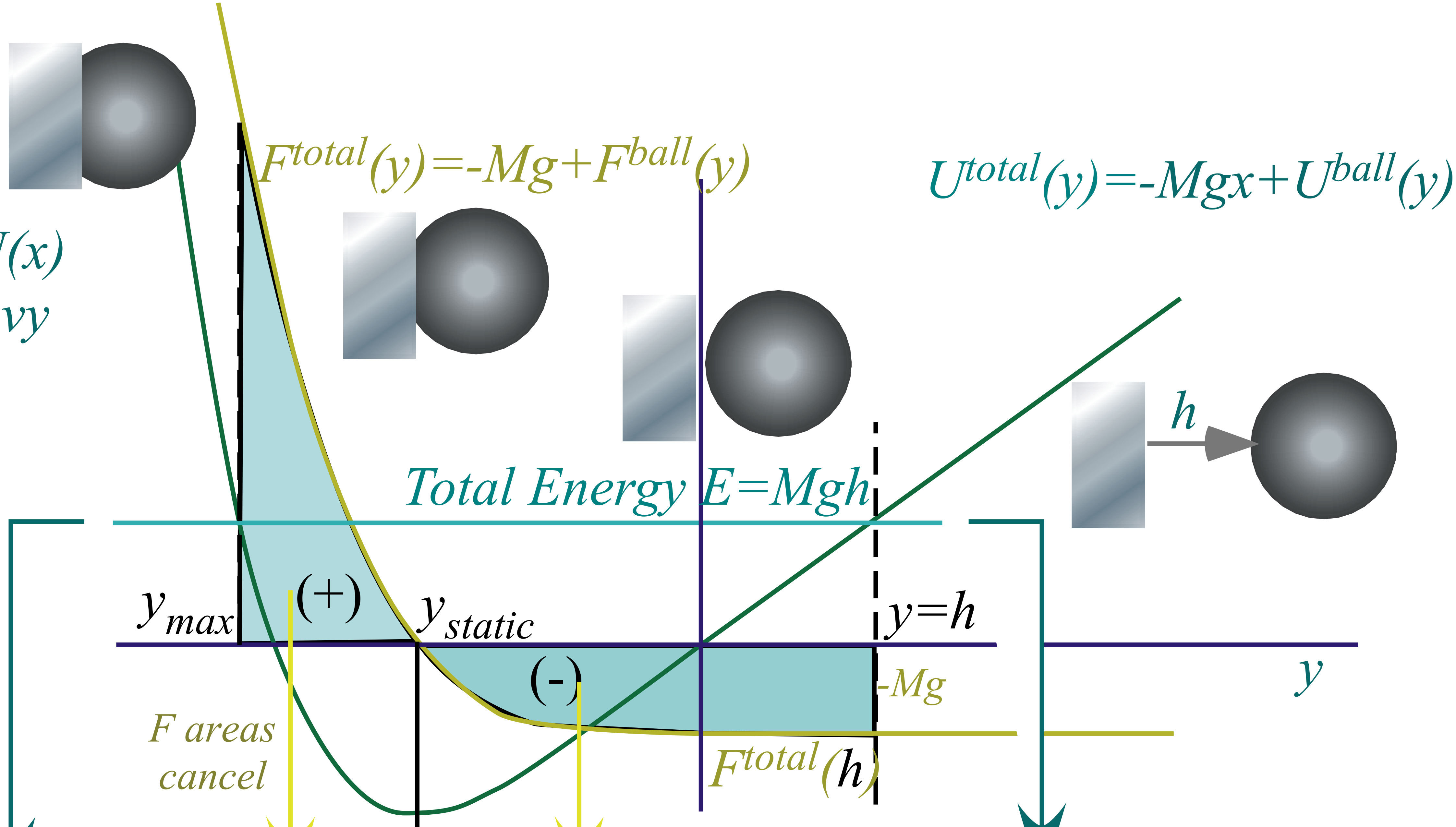
(c) Maximum penetration
(Zero kinetic energy again)
Floor Force is maximum
maximum penetration x_{max}
Floor Force is maximum
Maximum penetration x_{max}
Maximum Force $F(x_{max})$
Total energy E
Total potential energy curve U(x) +mgY
Y1-R1
U(x)
1
F(x)
−ΔU
$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$
Δx
Display of Force vector using similar triangle constuction based on the slope of potential curve.

Force F(x) and Potential U(x) for soft heavy non-linear superball



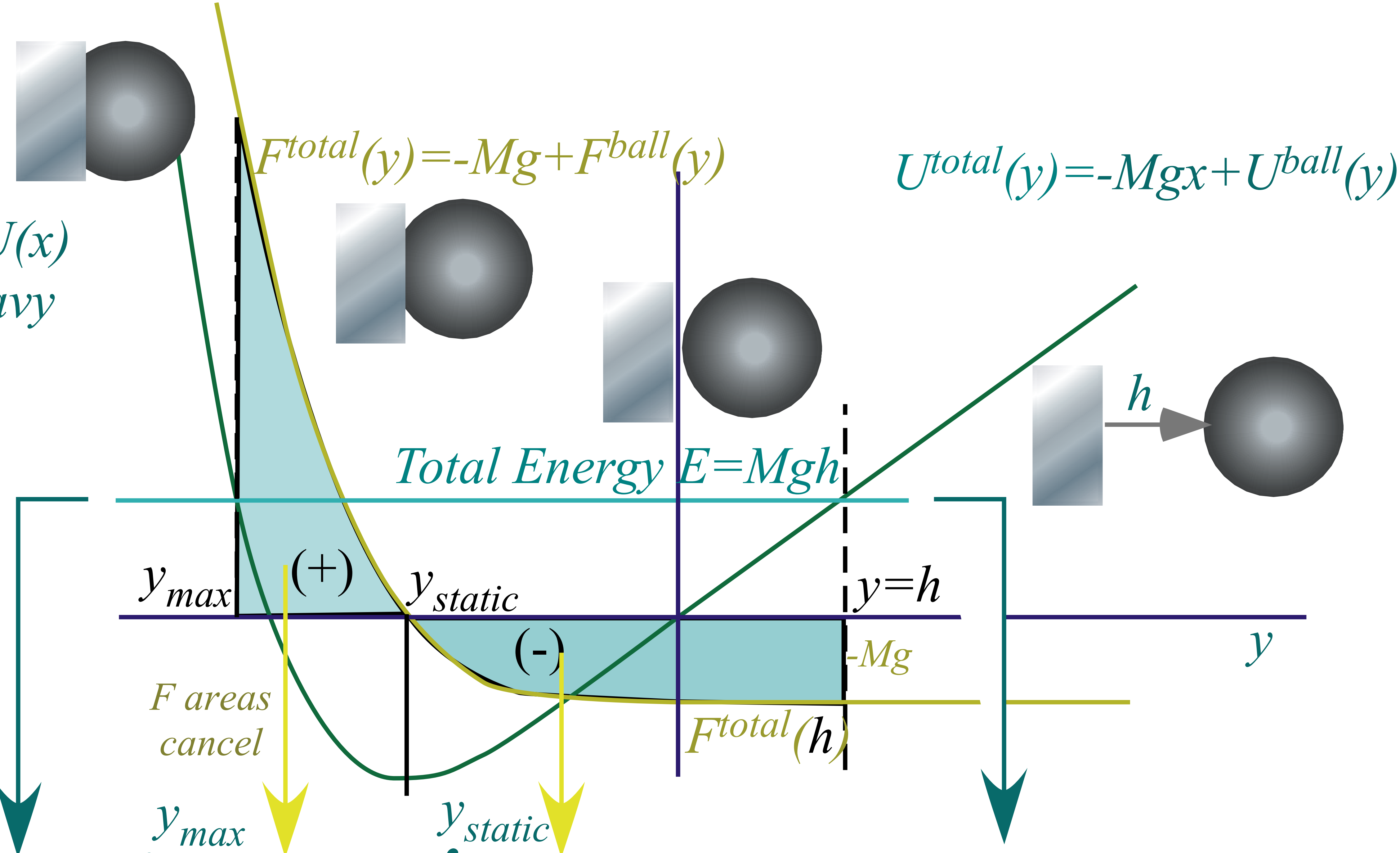
Unit 1
Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) \;=\; U(h) = E$$

$$F(x) = -\frac{dU(x)}{dx}$$

Force F(x) and Potential U(x) for soft heavy non-linear superball

Unit 1
Fig. 7.5



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$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

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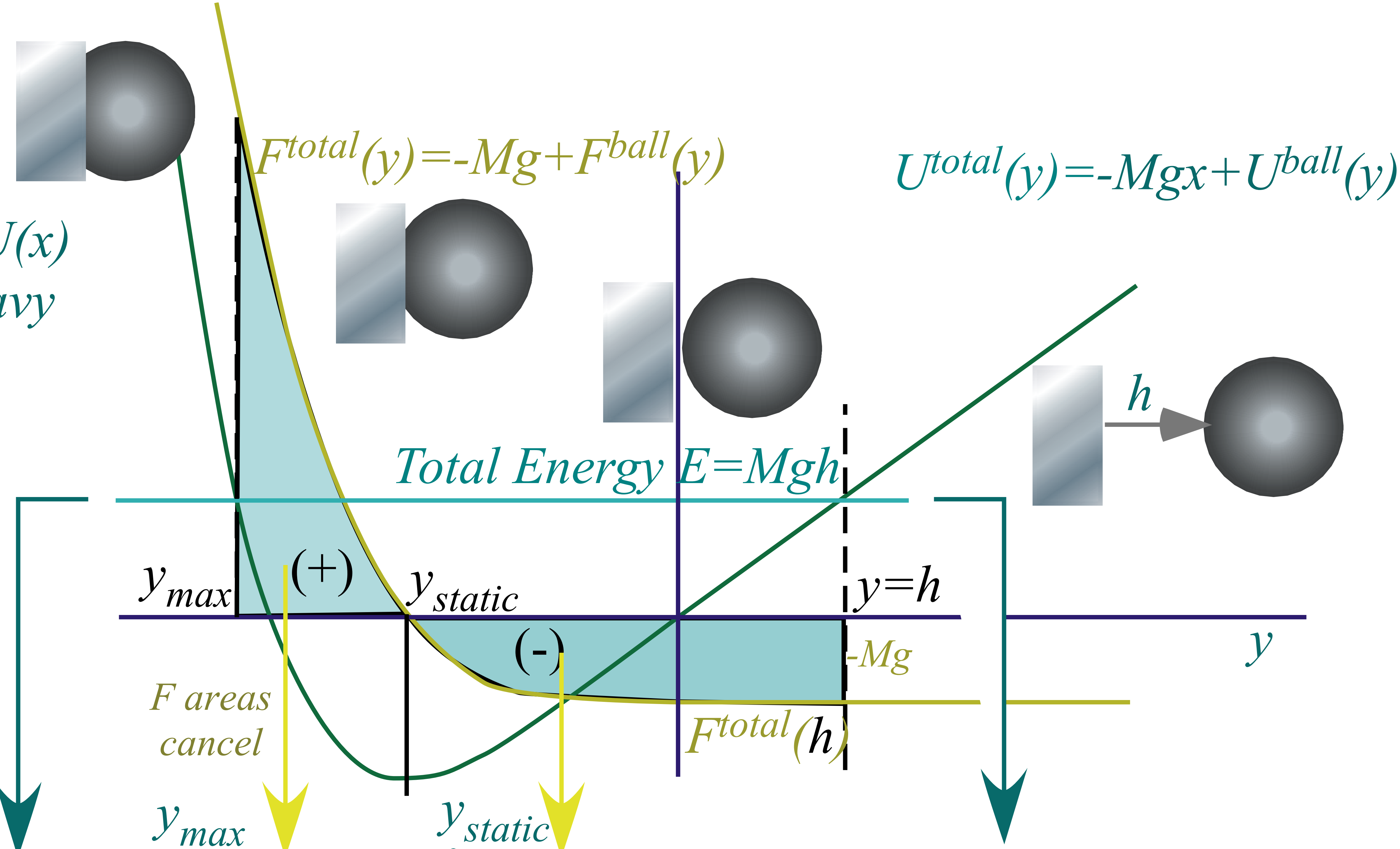

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Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

→ *Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)*

Some physics of dare-devil-divers

Non-linear force (like superball-floor or ball-bearing-anvil)

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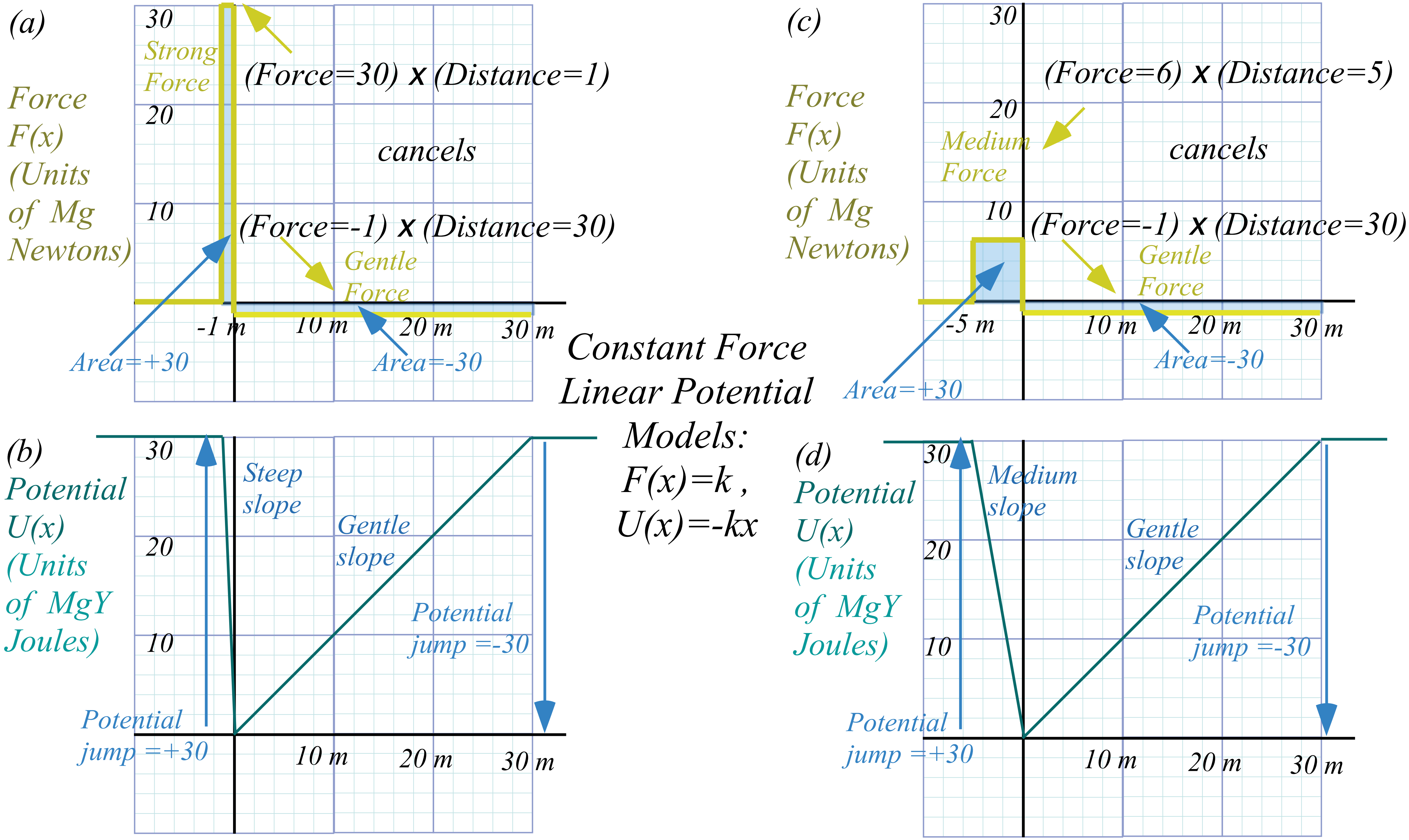

Constant Force Linear Potential Models:
$F(x)=k$, $U(x)=-kx$

Unit 1 Fig. 7.3

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(See Simulation)

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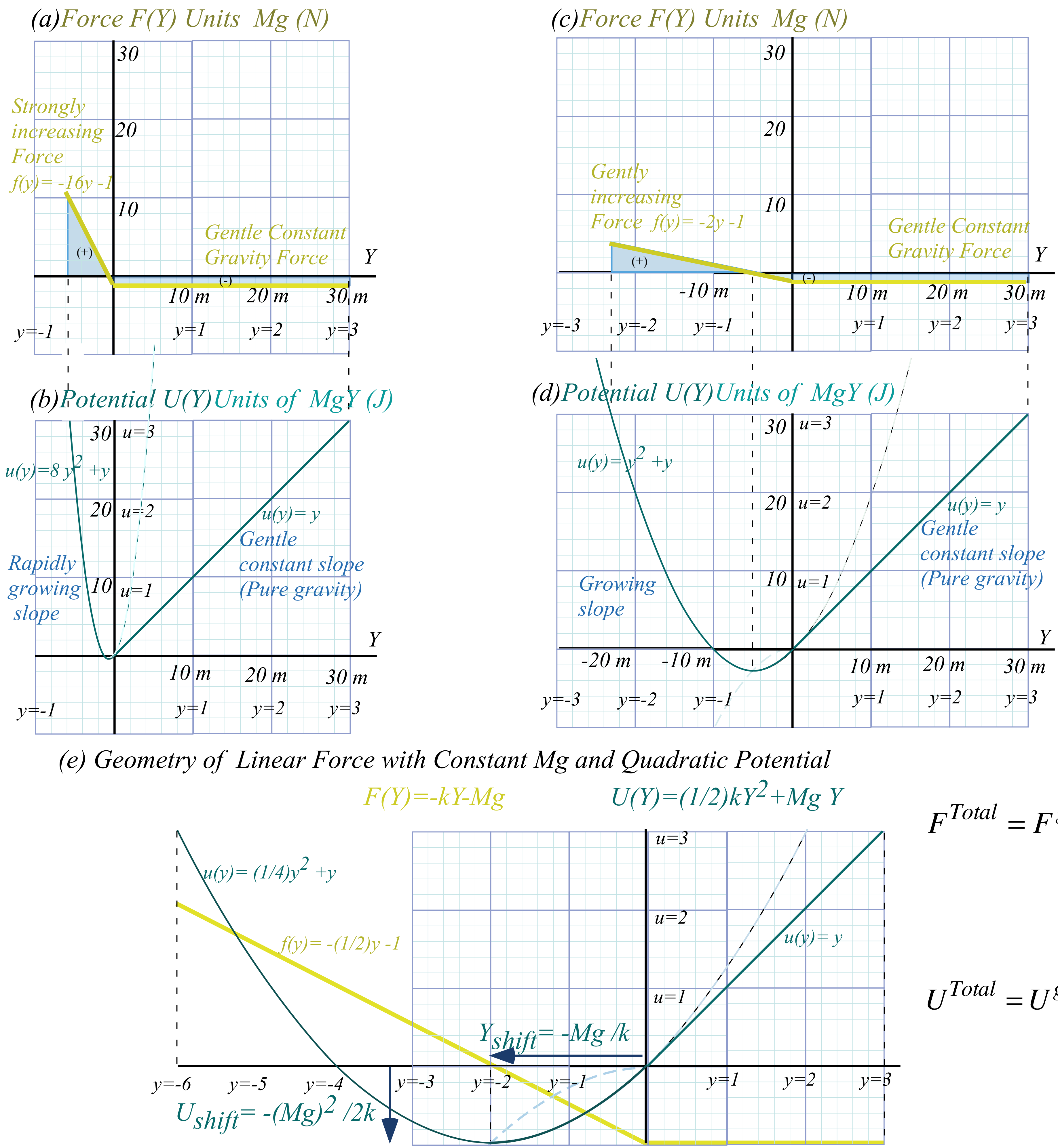

Unit 1
Fig. 7.4

$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg\ y & (y \geq 0) \\ Mg\ y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

Main Control Panel

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Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

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m1= 1 x10^ 2 {g} $V1_0$= 0 x10^ 0 {cm/s}

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- 3-Ball Tower | 5-Ball Towe
- Potential Plot (1 Ball, Nonlinear force)
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(See Simulations) →

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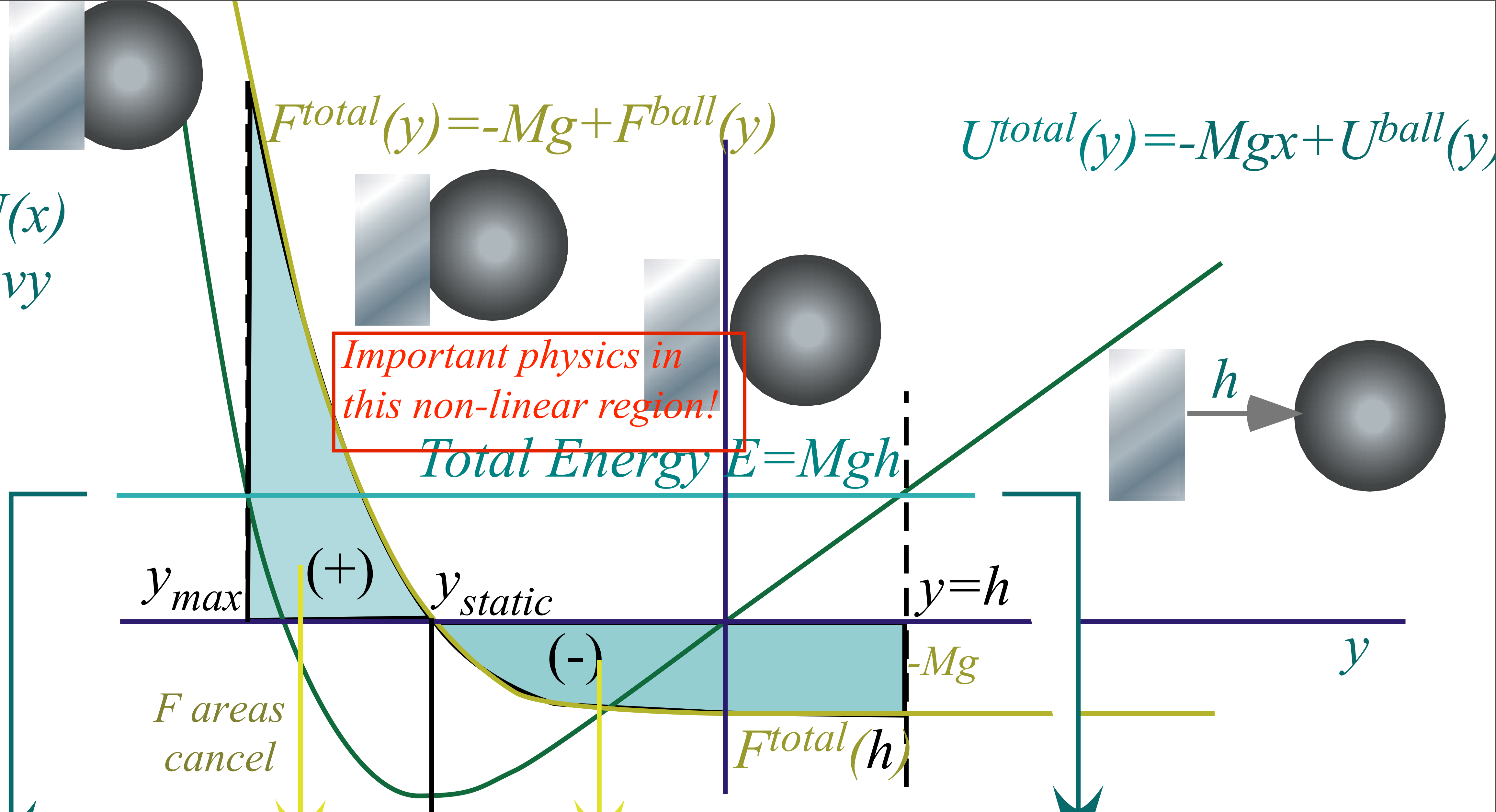

Unit 1
Fig. 7.5

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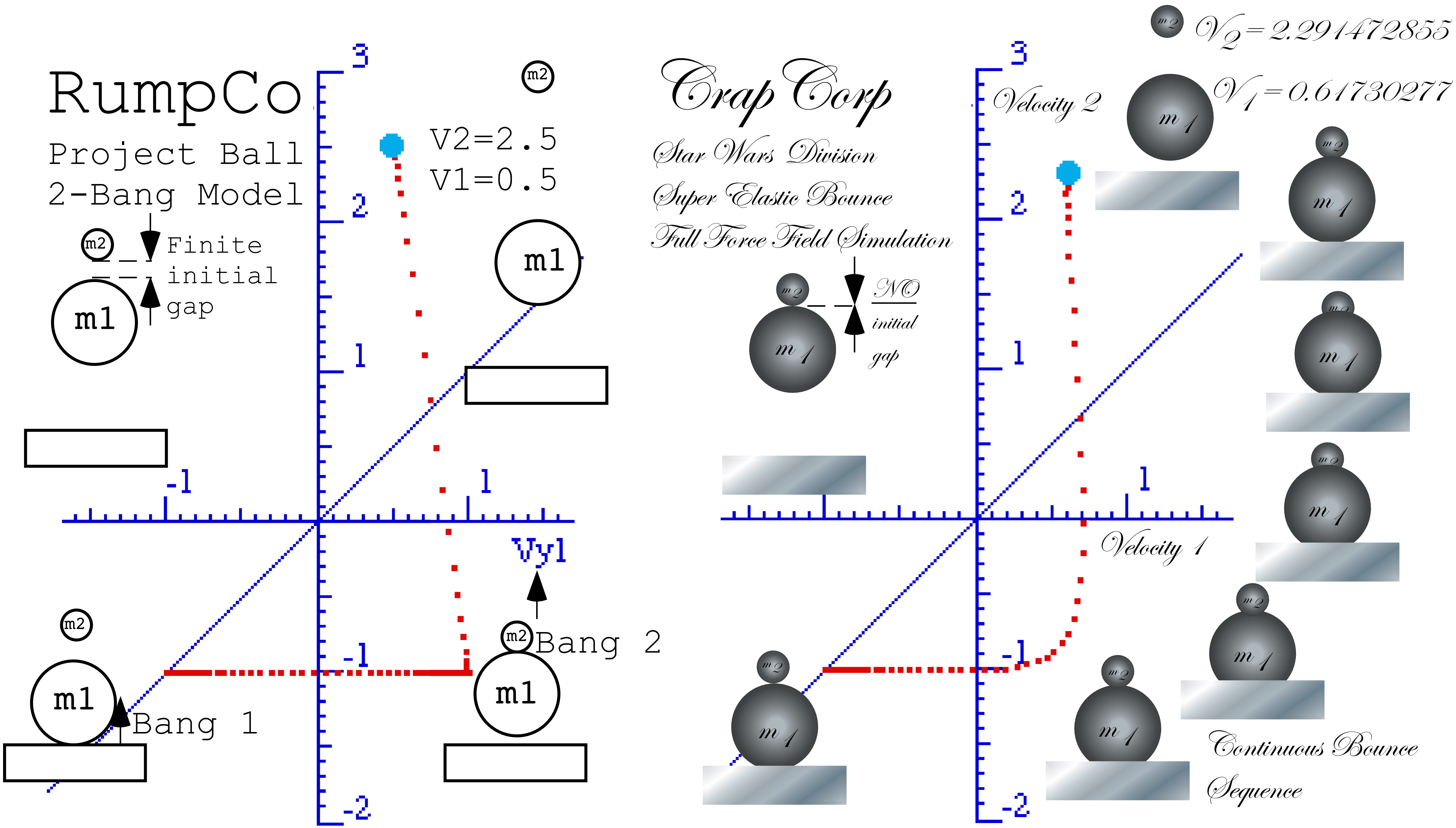
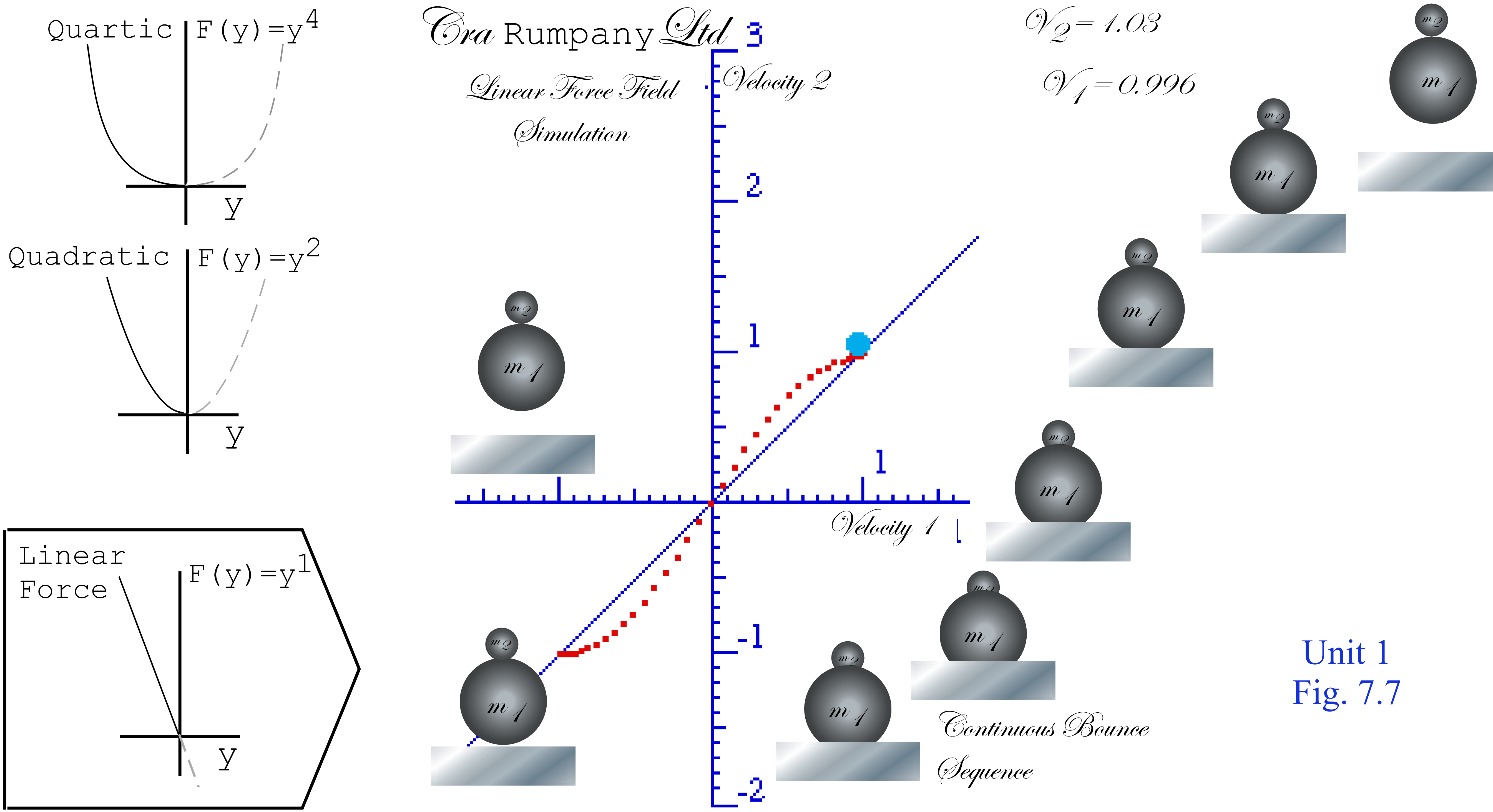

RumpCo
Project Ball
2-Bang Model
m2
Finite
initial
gap
m1
V2=2.5
V1=0.5
3
2
1
-1
1
-1
-2
Vy1
Bang 1
Bang 2
Crap Corp
Star Wars Division
Super Elastic Bounce
Full Force Field Simulation
NO
initial
gap
Velocity 2
Velocity 1
V2=2.291472855
V1=0.61730277
Continuous Bounce
Sequence

Unit 1
Fig. 7.6



Quartic F(y)=y4
y
Quadratic F(y)=y2
y
Linear
Force
F(y)=y1
y
Cra Rumpany Ltd
Linear Force Field
Simulation
Velocity 2
Velocity 1
V2= 1.03
V1 = 0.996
Continuous Bounce
Sequence

Unit 1
Fig. 7.7

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  - *Some physics of dare-devil-divers*
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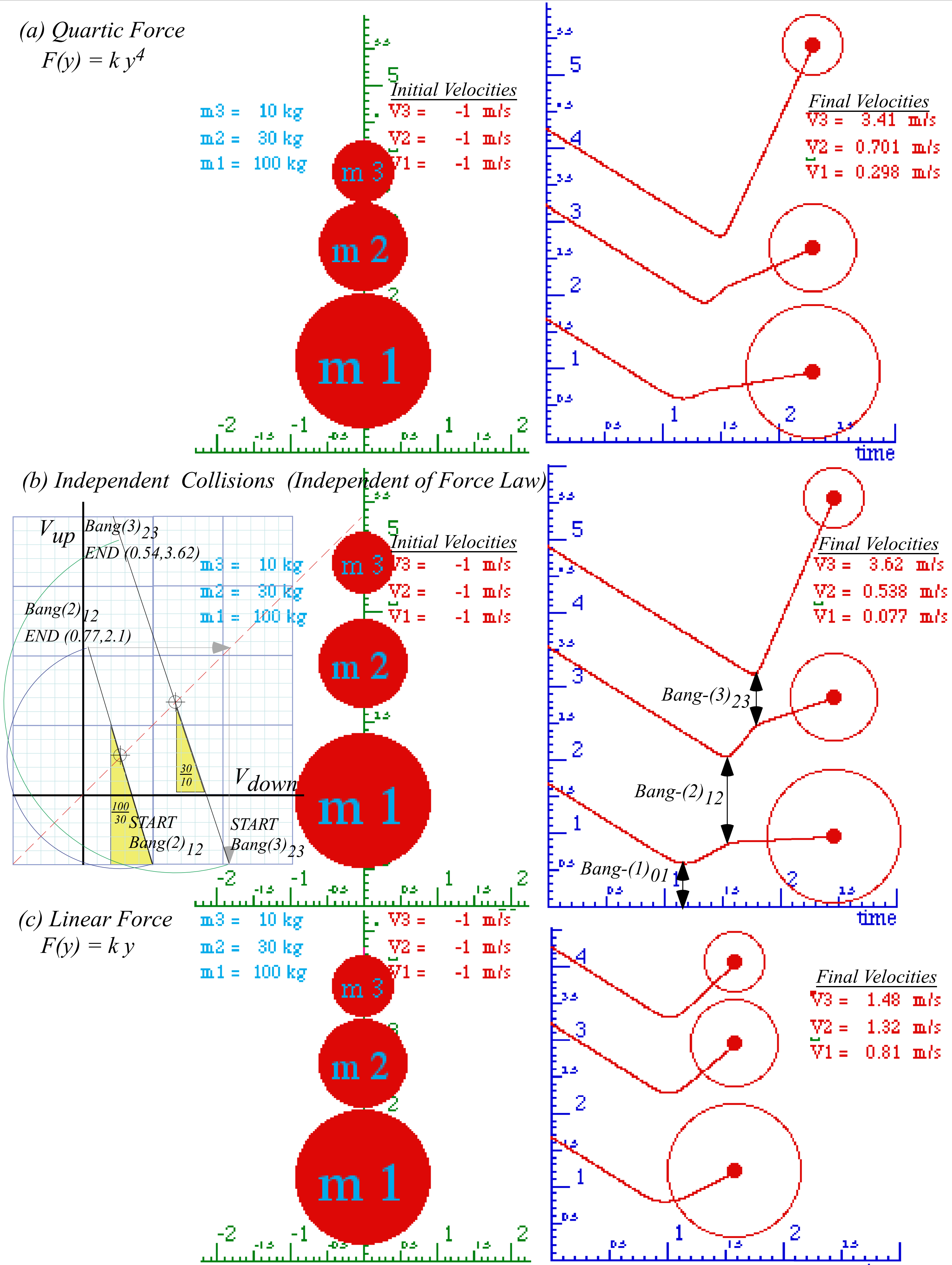
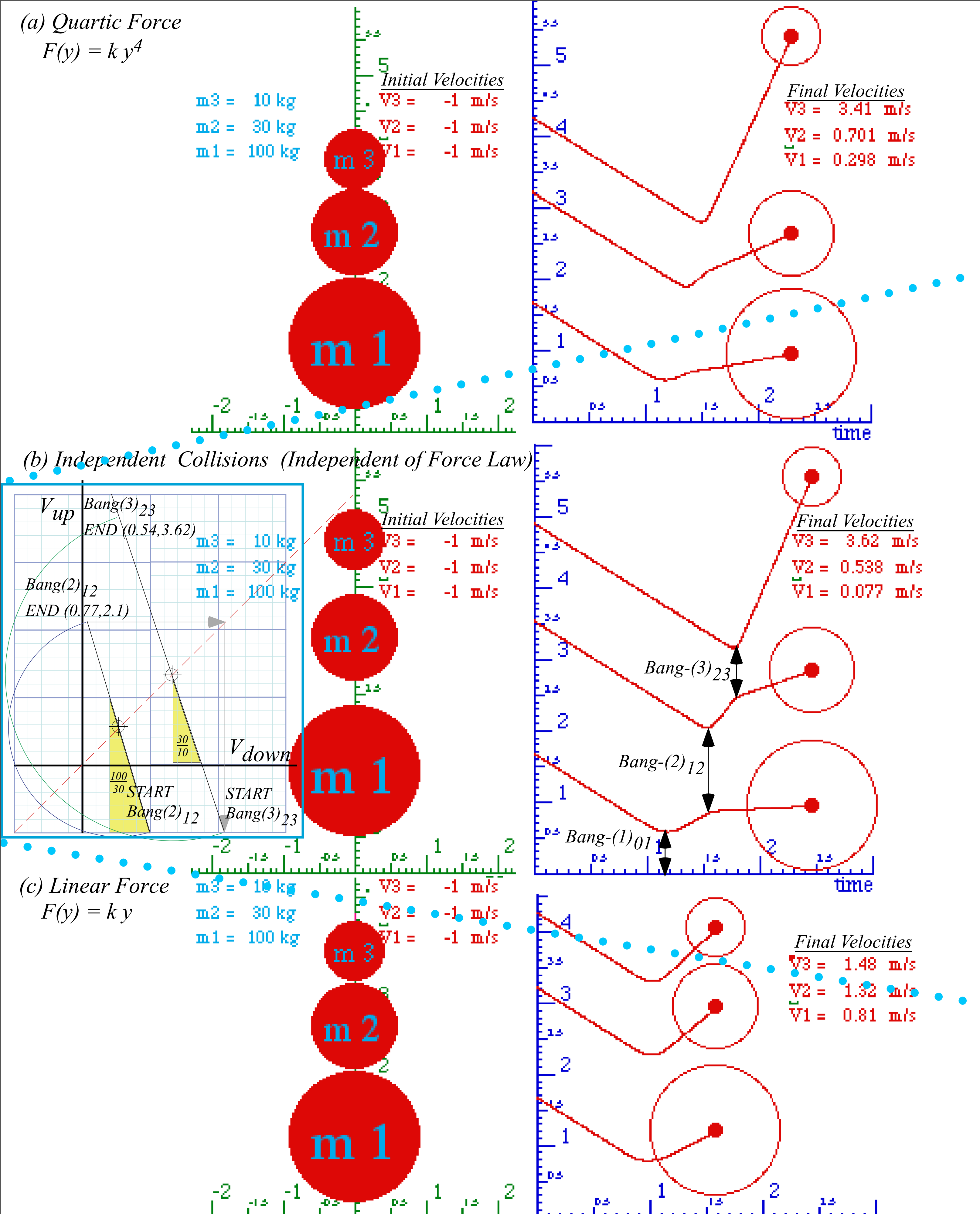
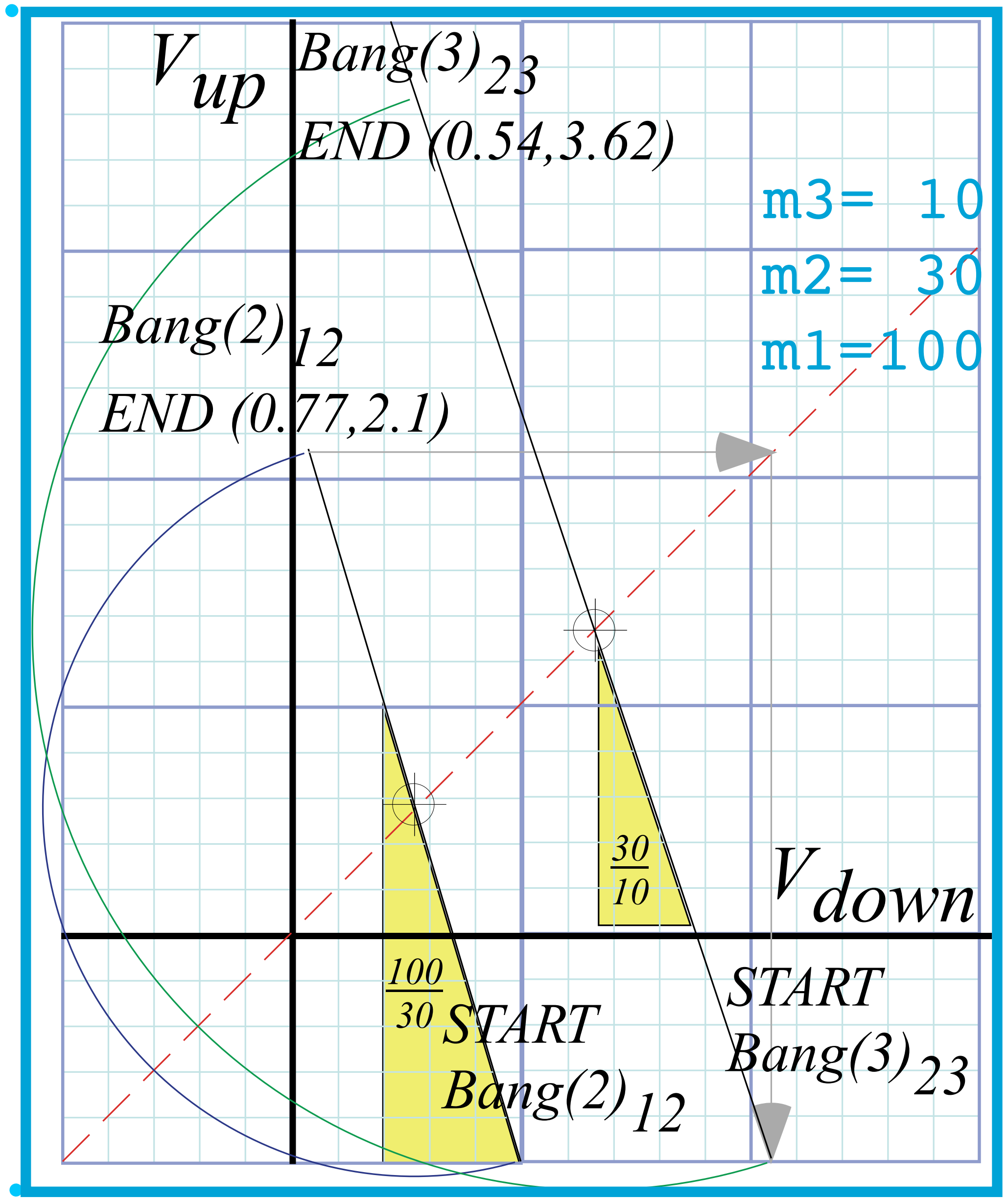


Unit 1
Fig. 8.1a-c
Independent Bang Model (IBM)
3-Body Geometry

Unit 1
Fig. 8.1b
Independent Bang Model (IBM)
3-Body Geometry





Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)

Some physics of dare-devil-divers

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Geometry and dynamics of 2-ball bounce (again with feeling)

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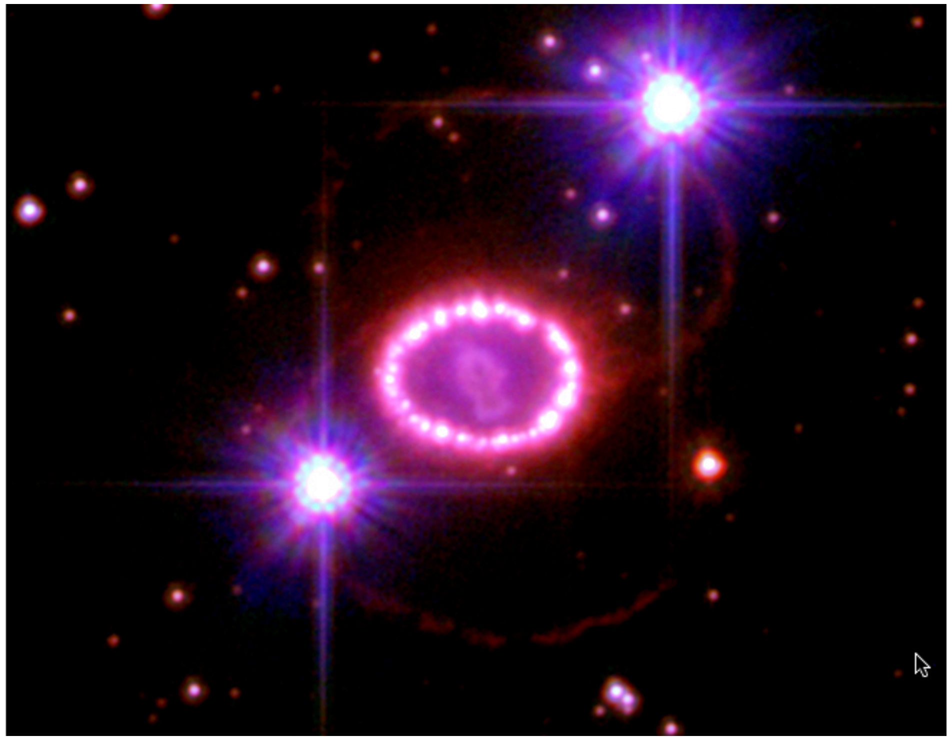
The story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of 3-ball bounce

→ *A story of Stirling Colgate (Palmolive) and core-collapse supernovae*

Other bangings-on: The western buckboard and Newton's balls

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

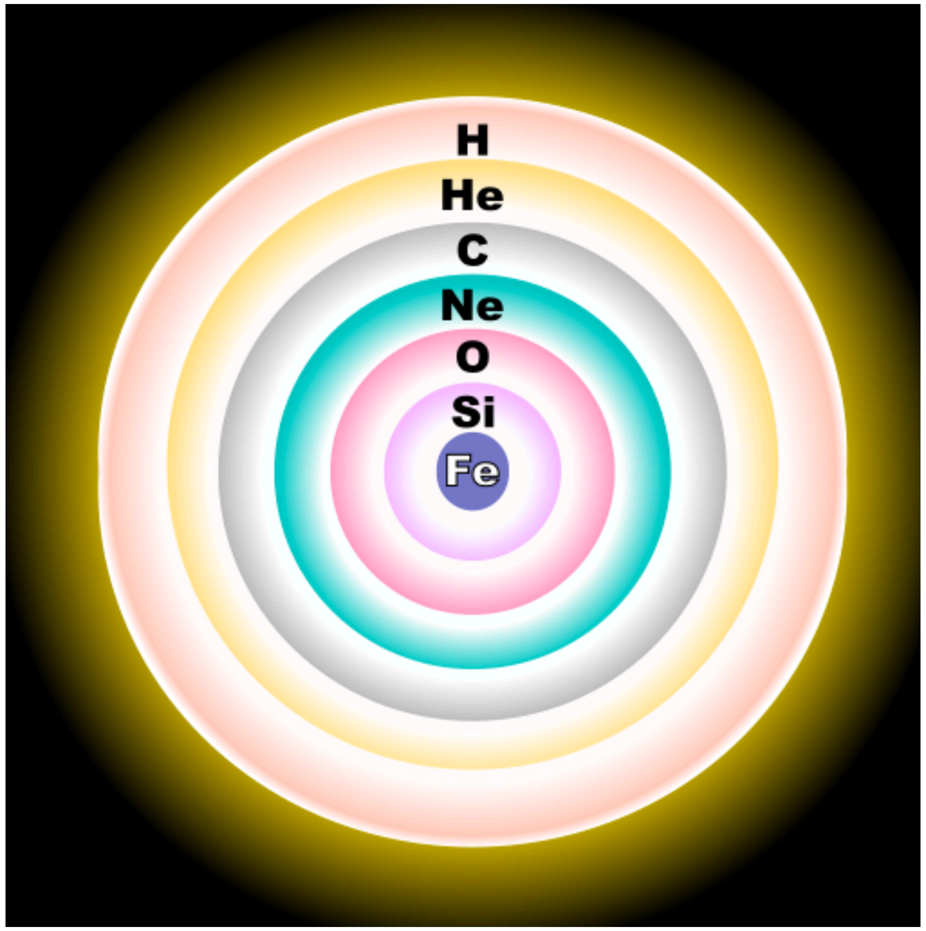


Source

http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author

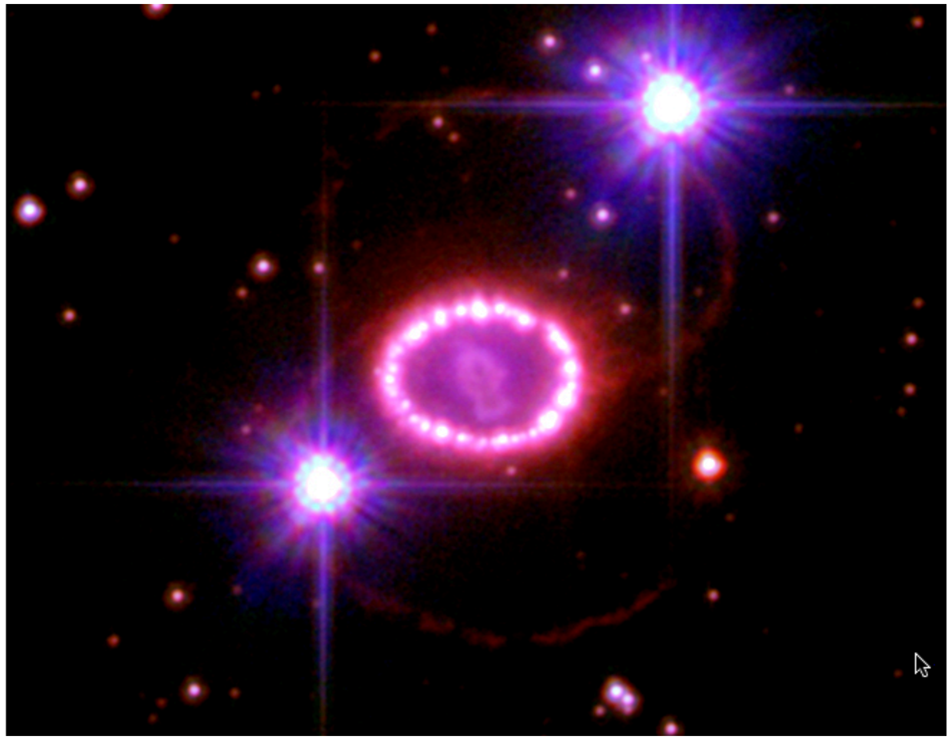
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 $M_\odot$ star[6]		
			Temperature (Kelvin)	Density (g/cm^3)	Duration
hydrogen burning	hydrogen	helium	7×10^7	10	10^7 years
triple-alpha process	helium	carbon, oxygen	2×10^8	2000	10^6 years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10^8	10^6	10^3 years
neon burning process	neon	O, Mg	1.6×10^9	10^7	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10^9	10^7	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10^9	10^8	5 days

A story of Stirling Colgate (Palmolive) and core-collapse supernovae



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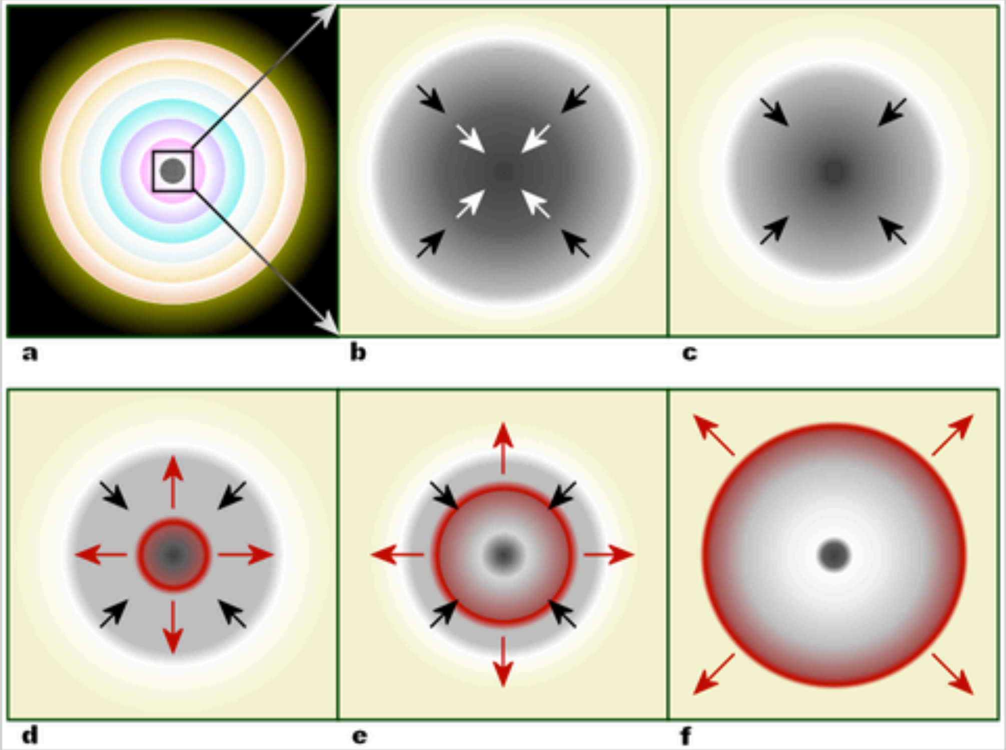
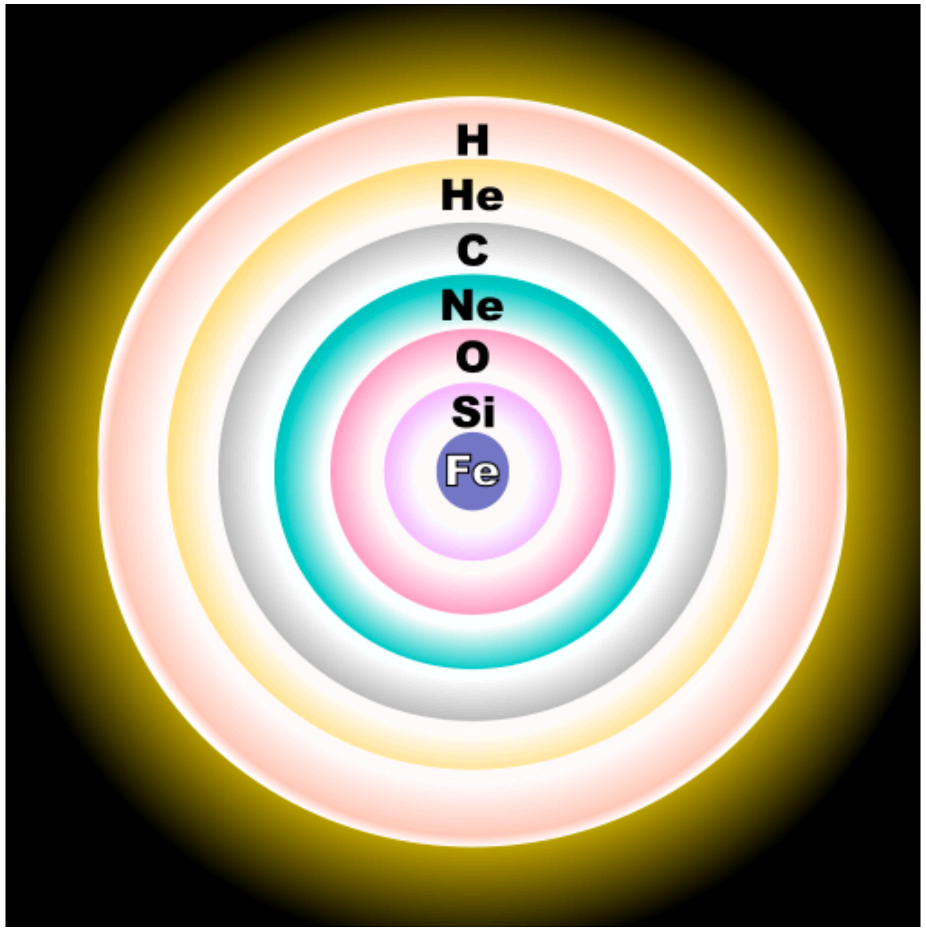


Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.



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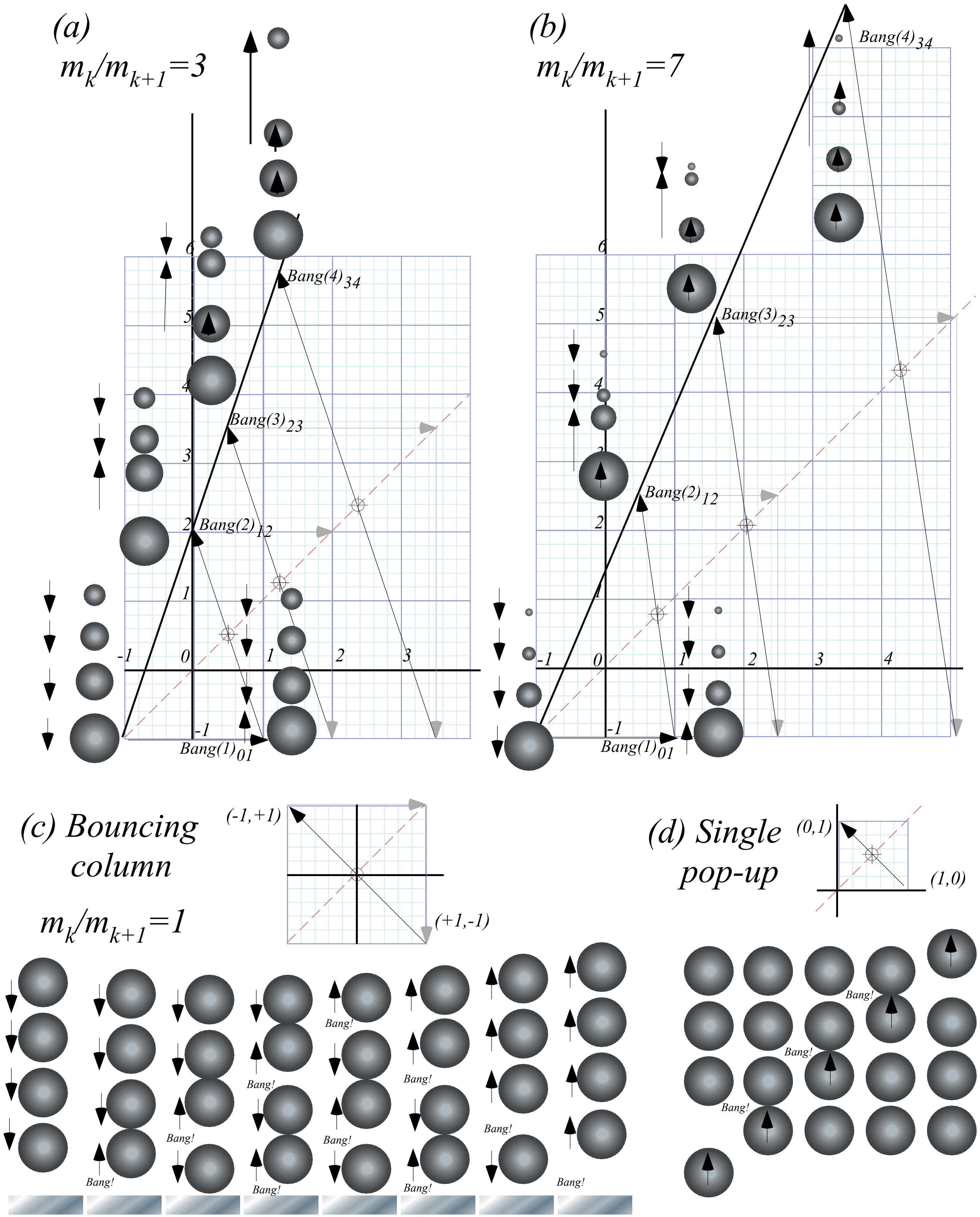


Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry

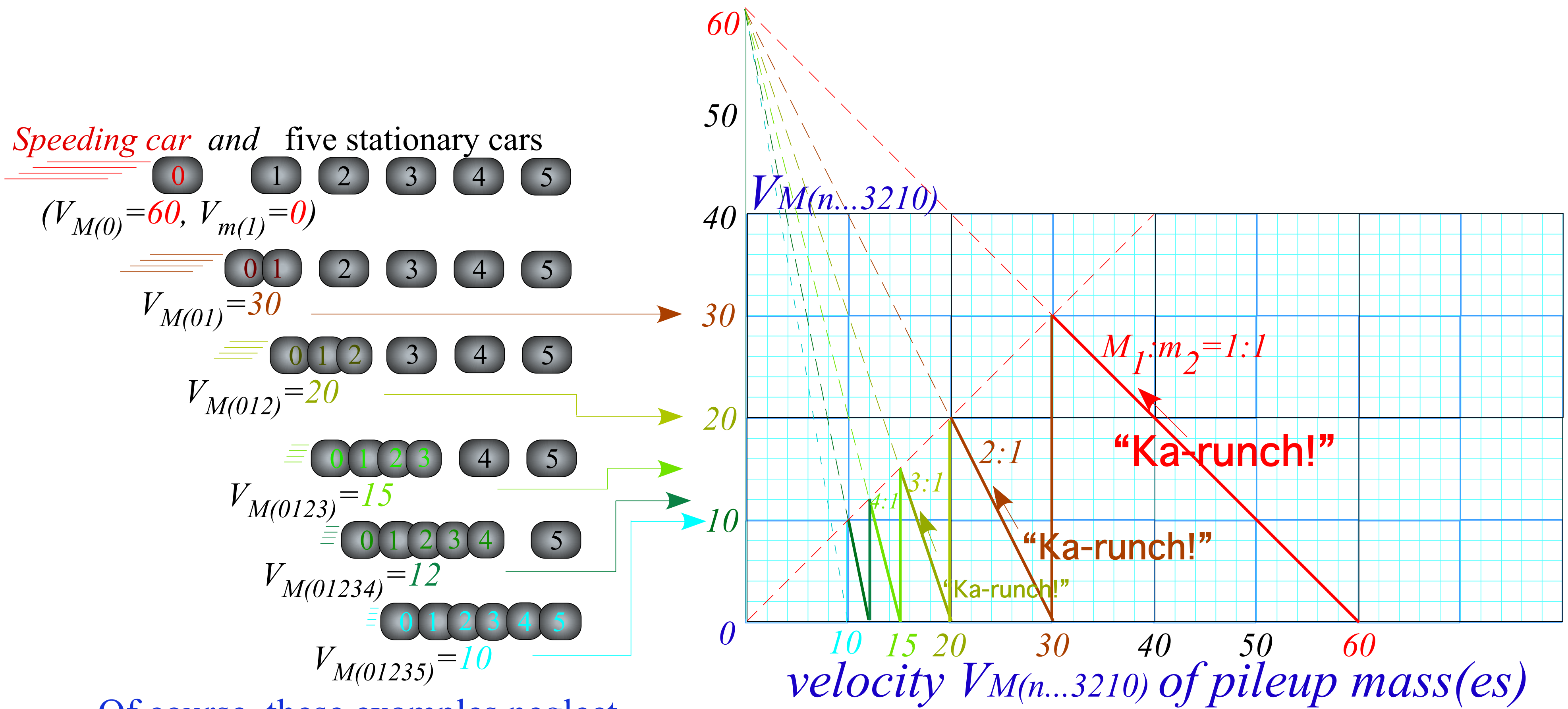
4-Equal-Body “Shockwave” or pulse wave Dynamics

Opposite of continuous wave dynamics introduced in Unit 2

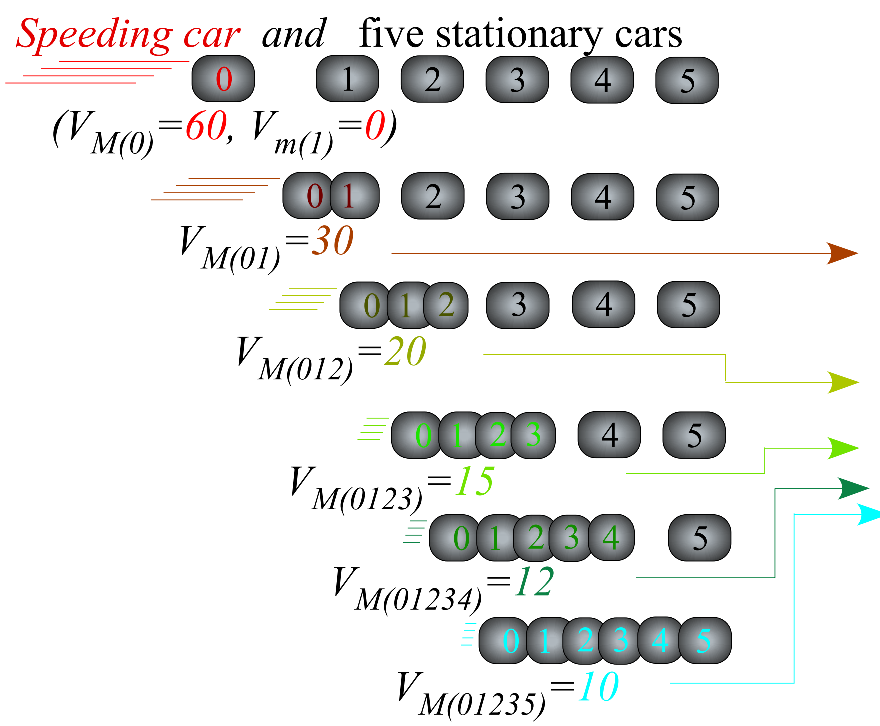
Crunch energy geometry of freeway crashes and related things

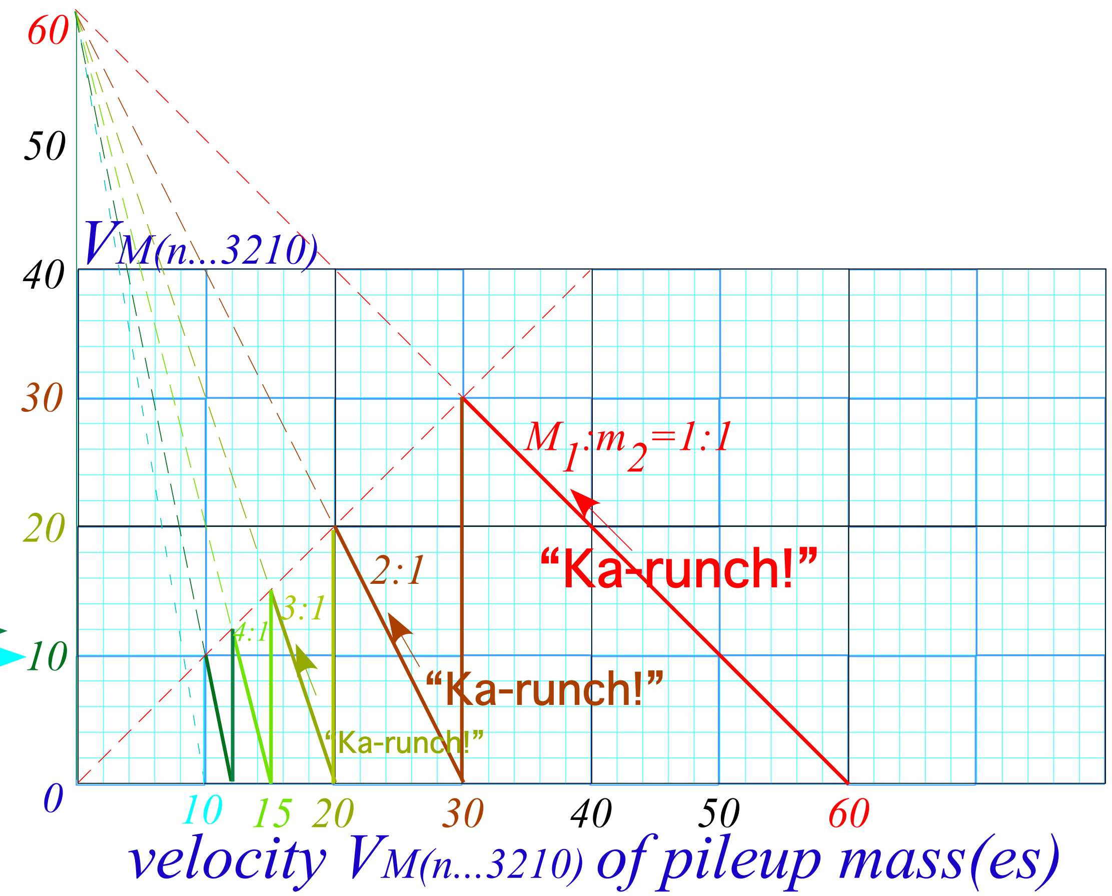
Crunch energy played backwards: This really ***<u>is</u>*** *"Rocket-Science"*

Unit 1
Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars



Of course, these examples neglect friction and “crunch-energy” losses





Unit 1

Fig. 8.5

Pile-up:
One *60mph* car hits five standing cars

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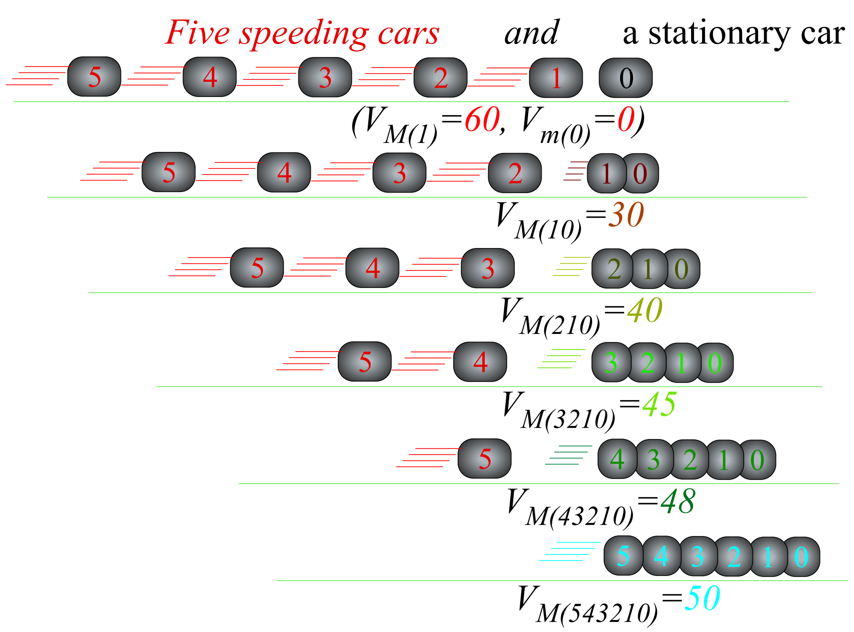


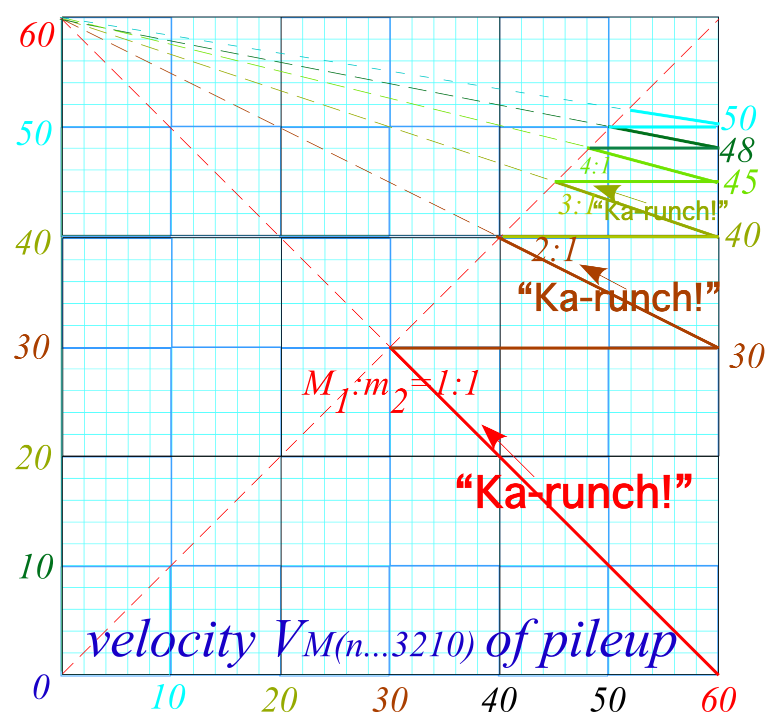


Fig. 8.6

Pile-up:
Five *60mph* cars hit one standing car

Unit 1

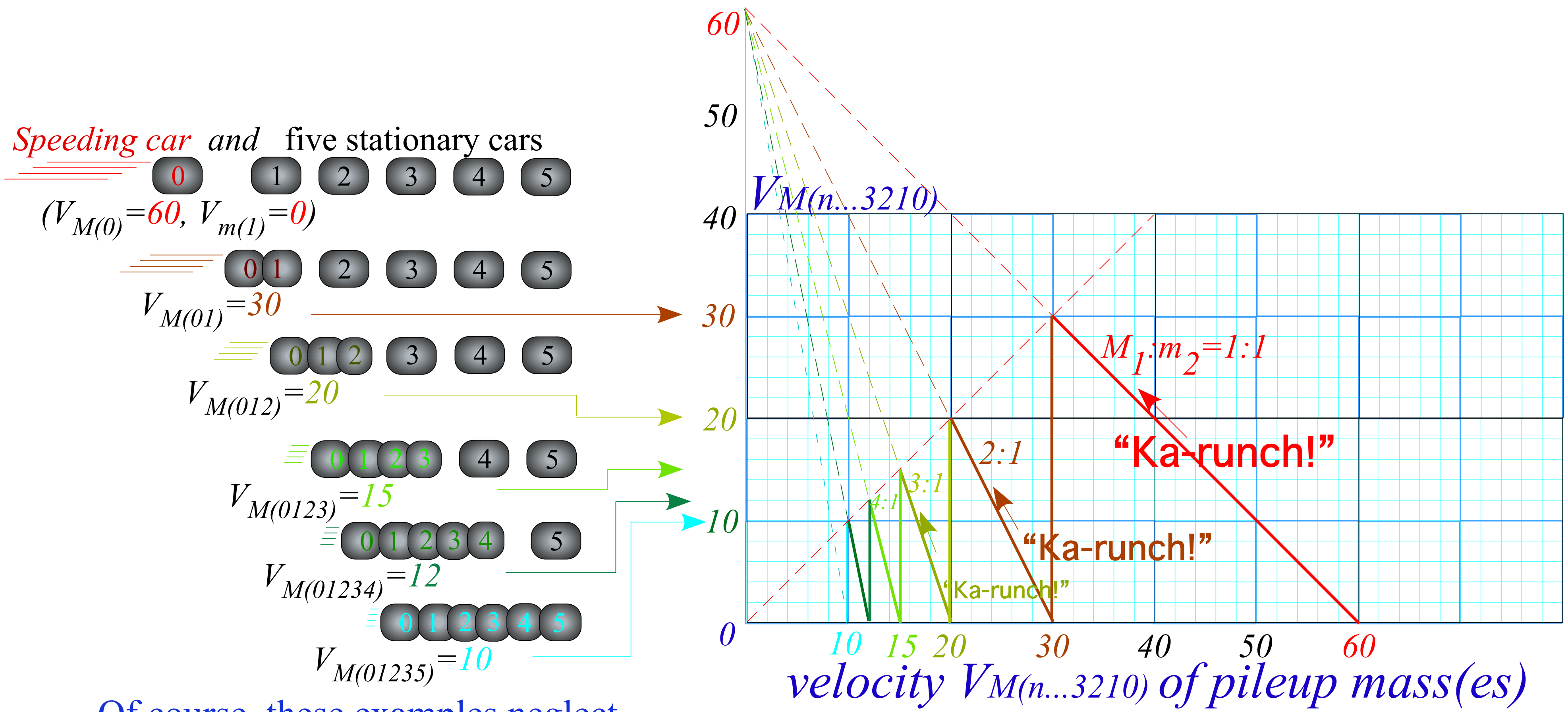


Fig. 8.5
Pile-up:
One *60mph* car hits five standing cars

Of course, these examples neglect friction and "crunch-energy" losses

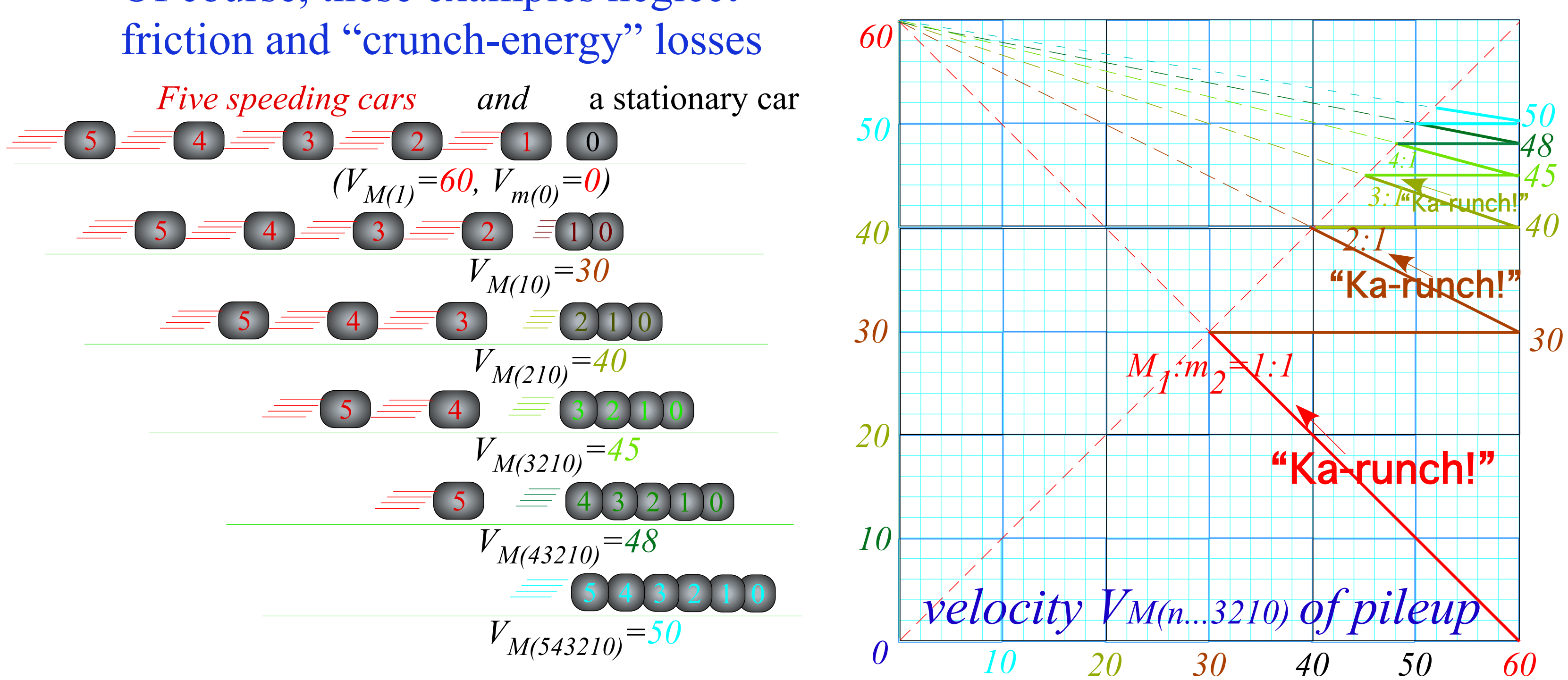
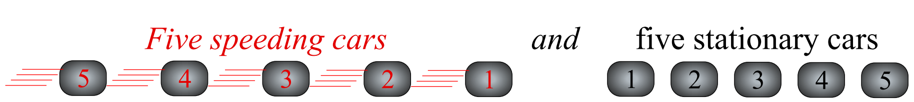


Fig. 8.6
Pile-up:
Five *60mph* cars hit one standing cars



(Fug-gedda-aboud-dit!!)

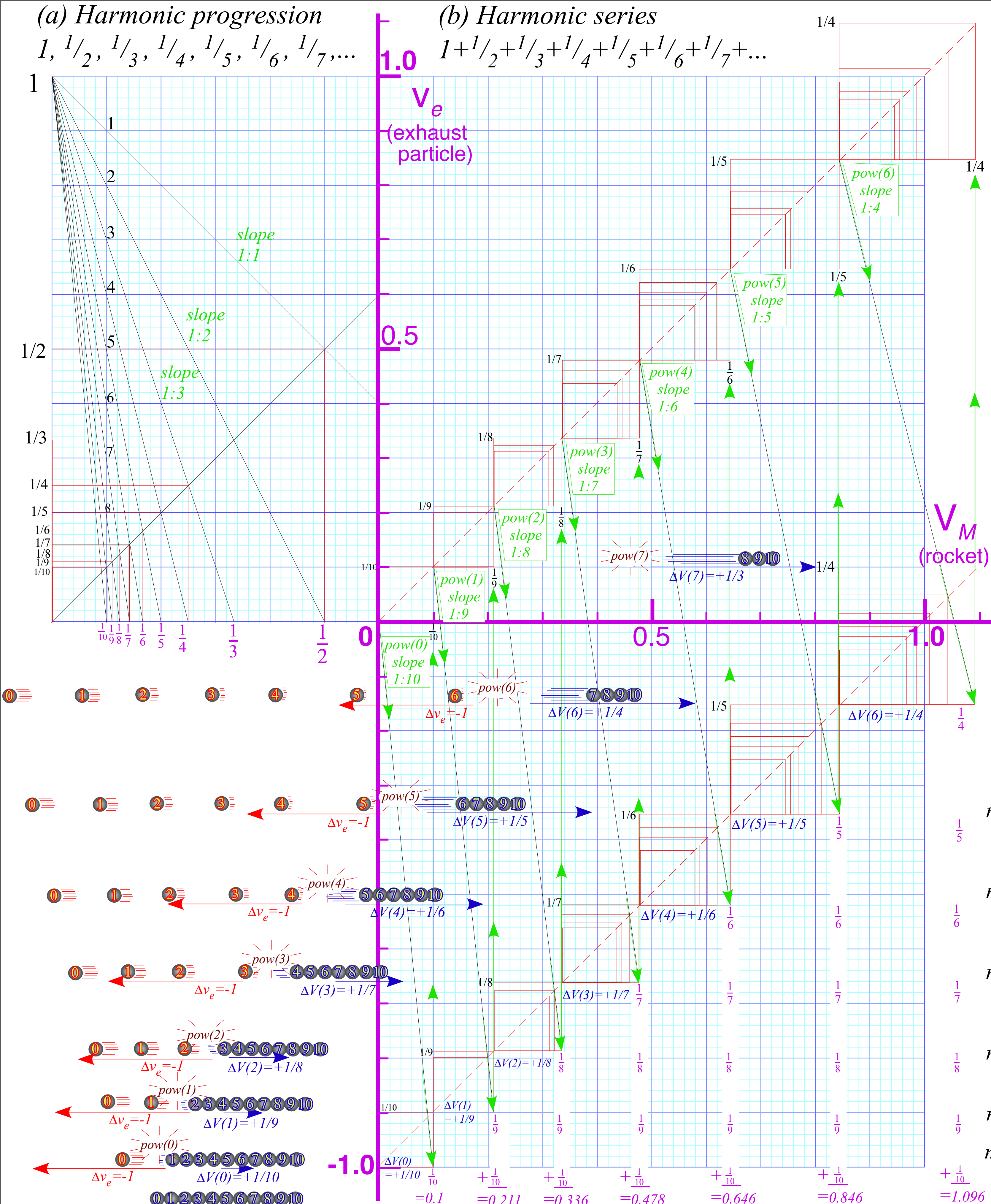
Fig. 8.7
Pile-up:
Five *60mph* cars hit five standing cars

Crunch energy geometry of freeway crashes and related things

→ *Crunch energy played backwards: This really* **is** *"Rocket-Science"*

Unit 1
Fig. 8.8a-b

Rocket Science!



$m \cdot \Delta v_7 + 3m \cdot \Delta V_M(7) = 0$

$m \cdot \Delta v_6 + 4m \cdot \Delta V_M(6) = 0$

$m \cdot \Delta v_5 + 5m \cdot \Delta V_M(5) = 0$

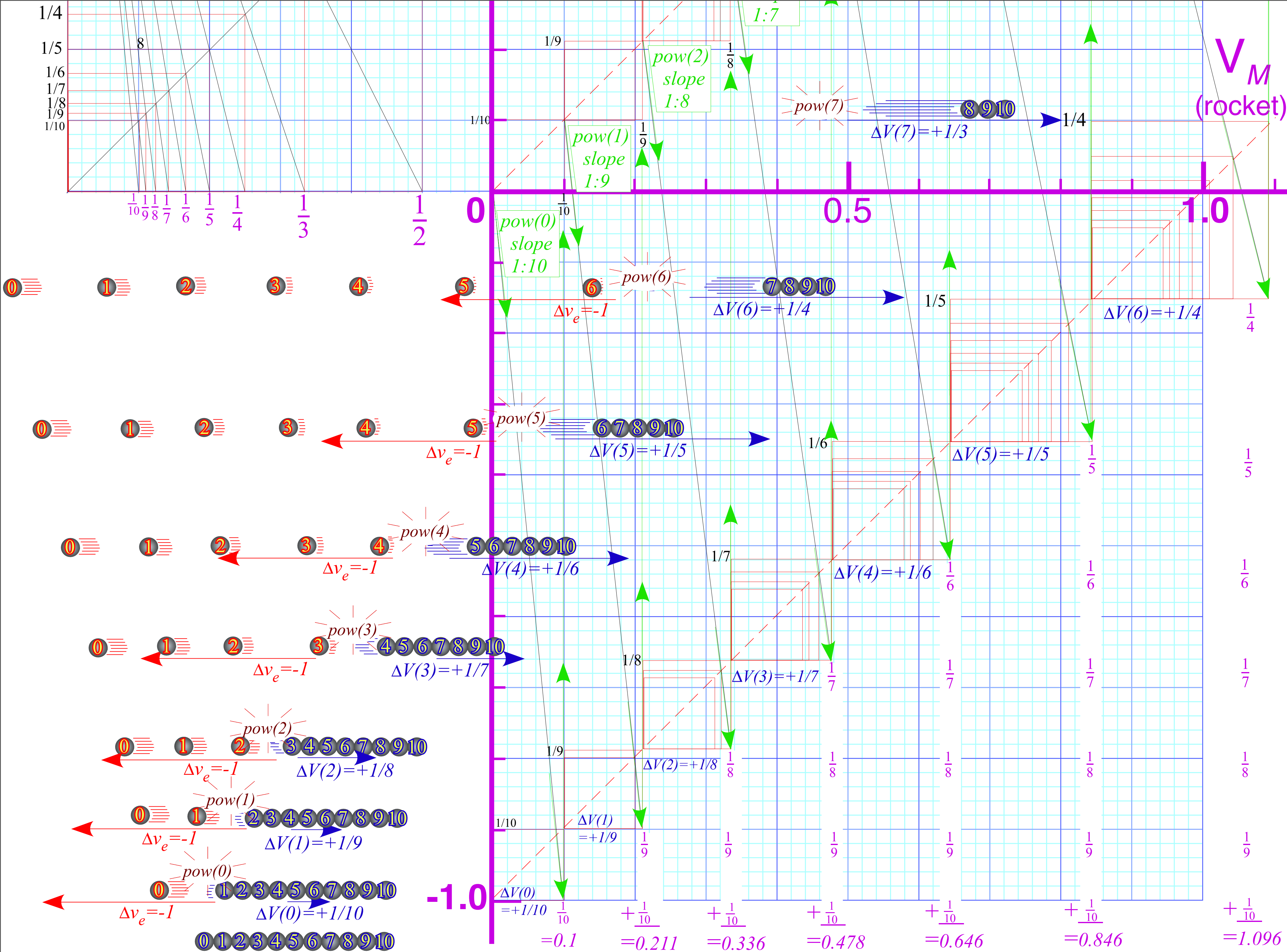
$m \cdot \Delta v_4 + 6m \cdot \Delta V_M(4) = 0$

$m \cdot \Delta v_3 + 7m \cdot \Delta V_M(3) = 0$

$m \cdot \Delta v_2 + 8m \cdot \Delta V_M(2) = 0$

$m \cdot \Delta v_1 + 9m \cdot \Delta V_M(1) = 0$

$m \cdot \Delta v_0 + 10m \cdot \Delta V_M(0) = 0$



0^{th}: $V(0)=1/10=0.1$ 1^{st}: $V(1)=1/10+1/9=0.211$ 2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

3^{rd}: $V(3)=V(2)+1/7=0.478$ 4^{th}: $V(4)=V(3)+1/6=0.646$ 5^{th}: $V(5)=V(4)+1/5=0.846$

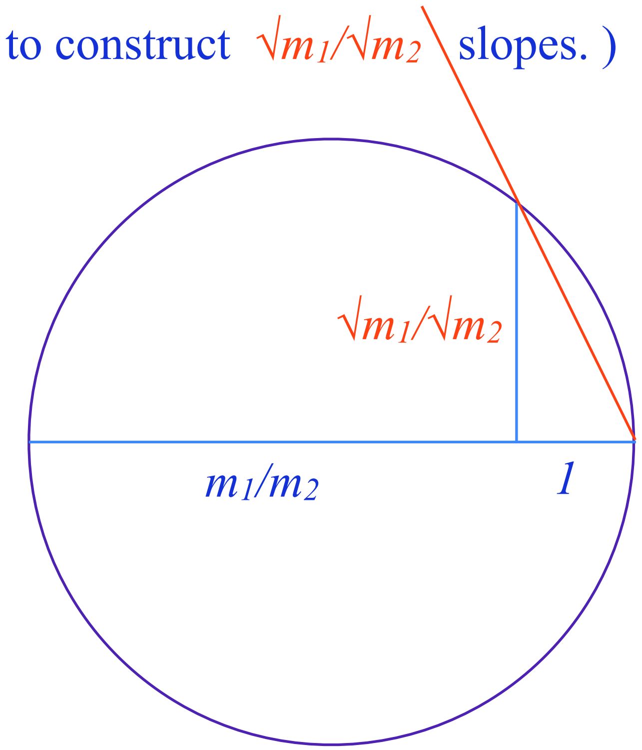
6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

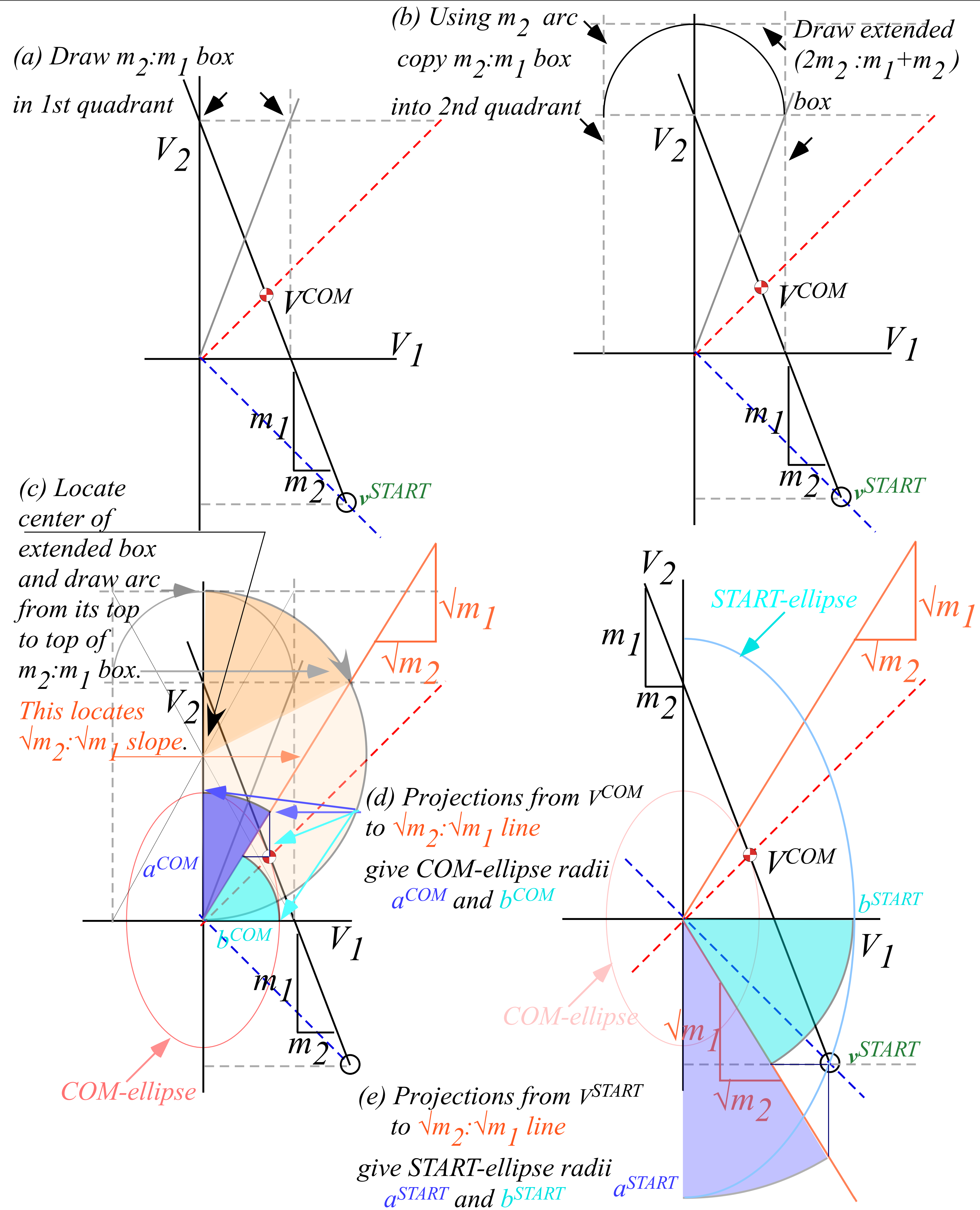
By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\frac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\frac{M_{IN}}{M_{FIN}}\right]$

A Thales construction for momentum-energy

(Made obsolete by Estrangian scaling to circular (V_1,V_2) plots. Still, one has to construct $\sqrt{m_1}/\sqrt{m_2}$ slopes.)





Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)