

Dynamics of Potentials and Force Fields

(Ch. 7 and part of Ch. 8 of Unit 1)

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to superball force law

Geometry and dynamics of single ball bounce

(a) Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil diving 80 ft. into kidee pool

(b) Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric m_1, m_2, m_3, \dots series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

A running collection of links to course-relevant sites and articles

[2018 CMwBang! site](#)

[Class YouTube Channel](#)

You-Tube site displays related videos world-wide

[AIP publications](#)

[AJP article on superball dynamics](#)

[AAPT summer reading](#)

Potential energy dynamics of Superballs and related things

→ *Thales geometry and “Sagittal approximation” to force law*

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

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Many-body 1D collisions

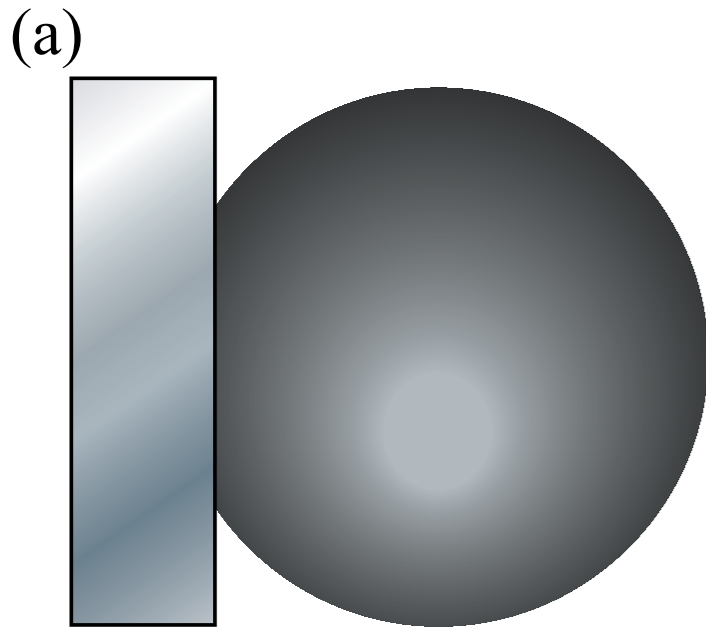
Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

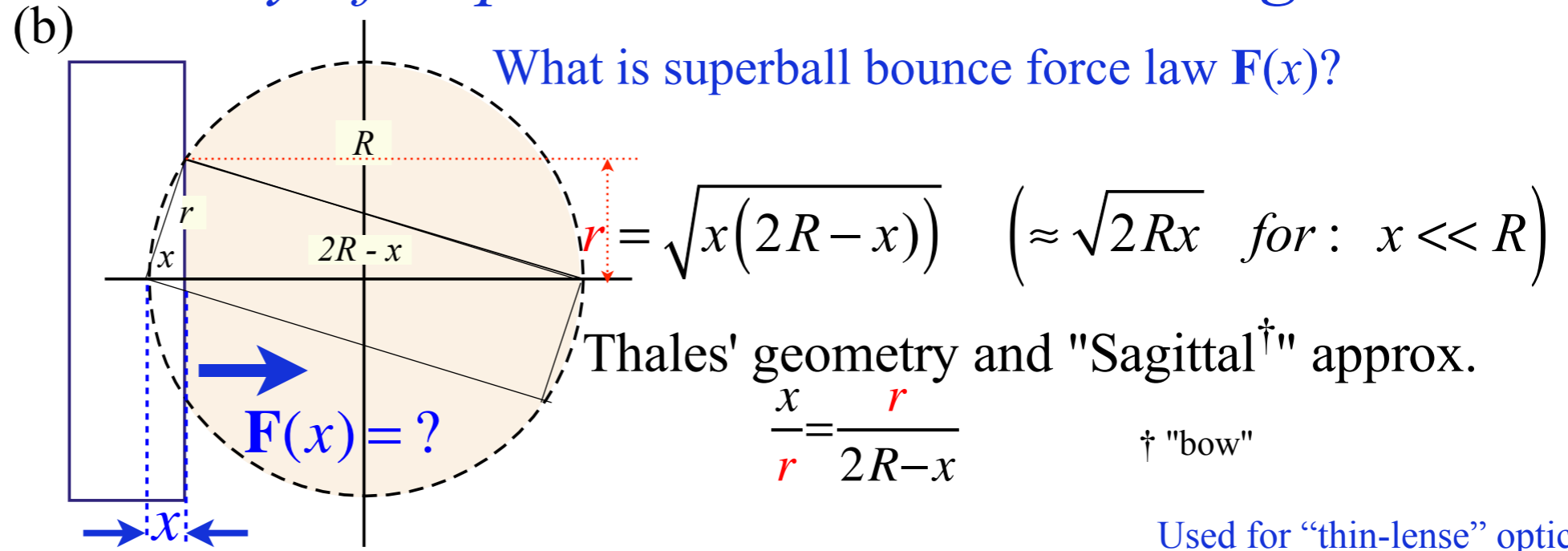
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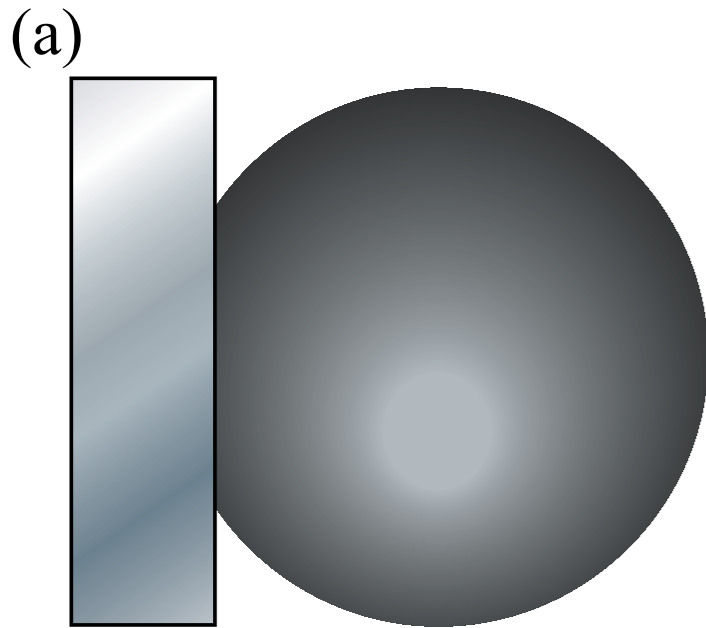
Potential Energy Geometry of Superballs and Related things



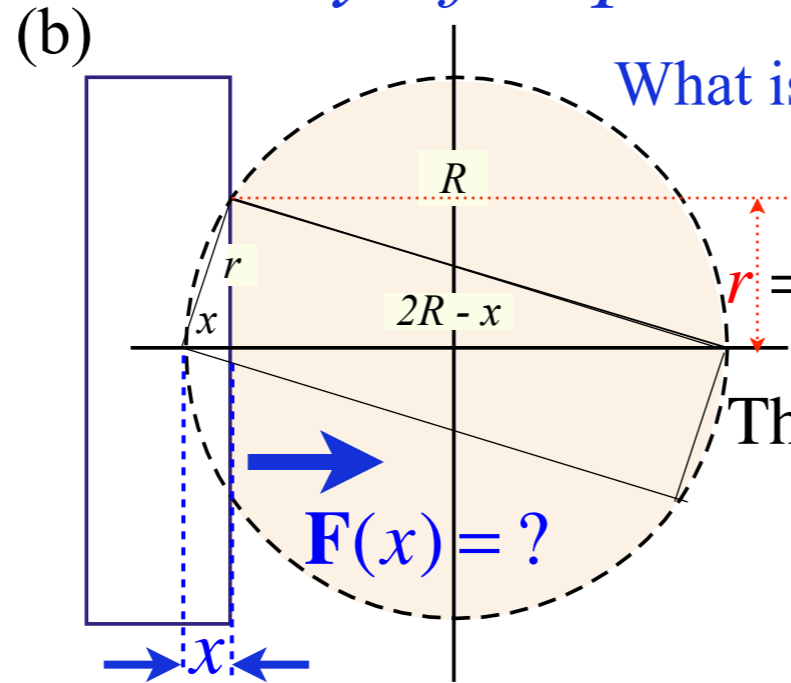
Unit 1
Fig. 7.1
(modified)



Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)



What is superball bounce force law $F(x)$?

$$r = \sqrt{x(2R-x)} \quad (\approx \sqrt{2Rx} \text{ for } x \ll R)$$

Thales' geometry and "Sagittal"† approx.

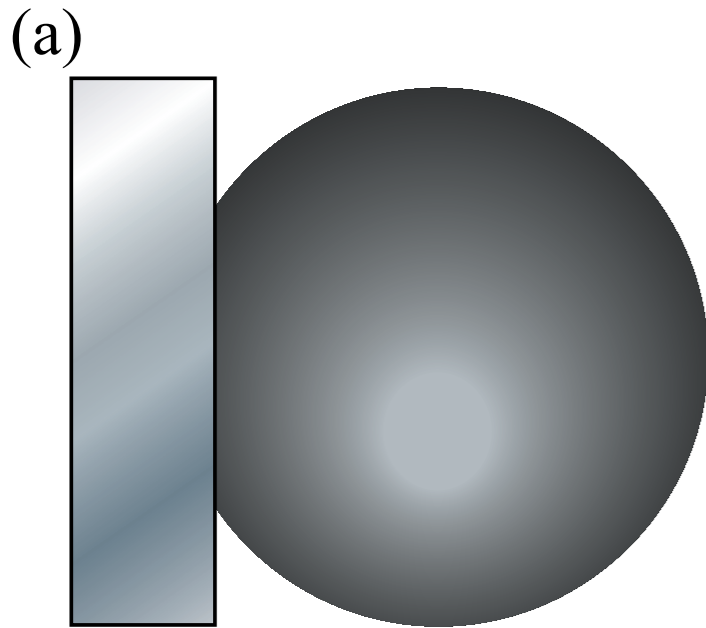
$$\frac{x}{r} = \frac{r}{2R-x}$$

† "bow"

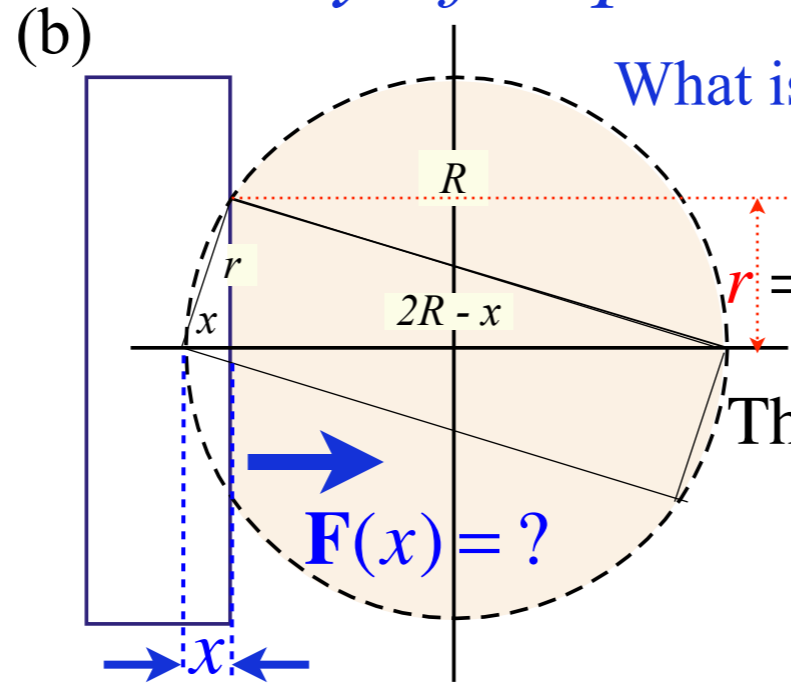
If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$F_{\text{balloon}}(x) = P \cdot A = P \cdot \pi r^2 \approx P \cdot \pi 2Rx$$

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)



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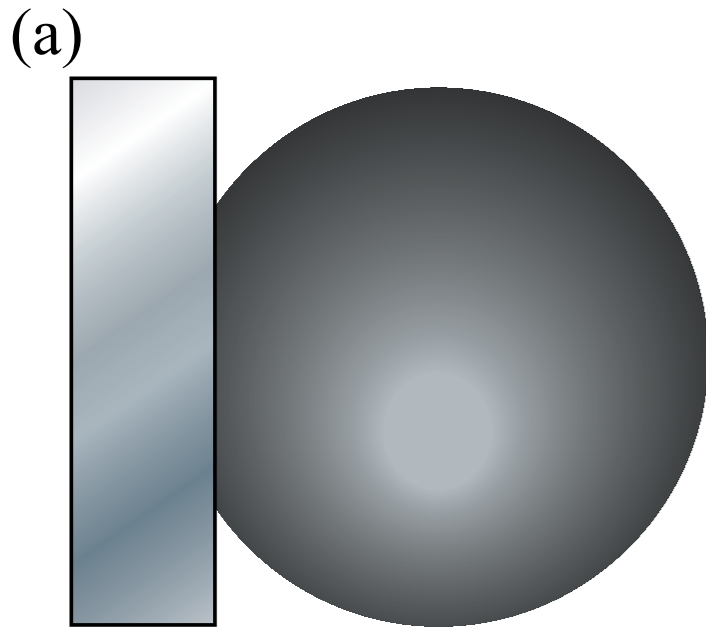
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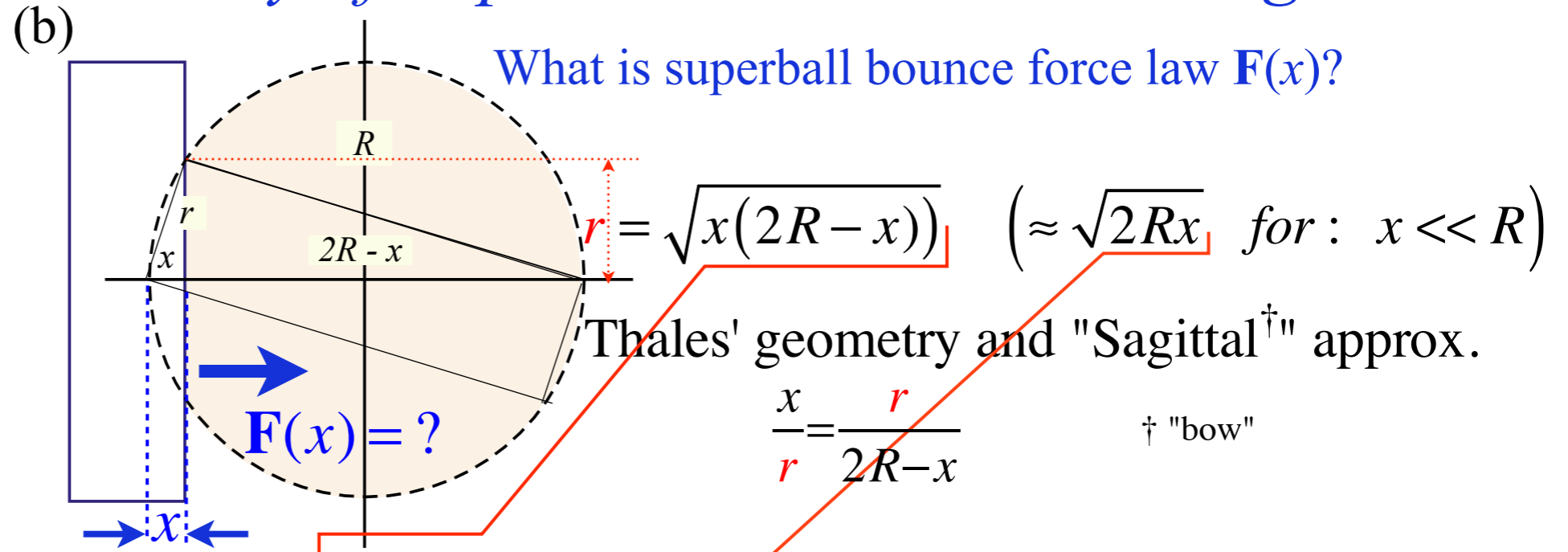
If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$\begin{aligned}
 F_{\text{balloon}}(x) &= P \cdot A = P \cdot \pi r^2 \\
 &\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx \quad (\text{Hooke spring constant } k) \\
 &= kx
 \end{aligned}$$

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)



If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

$$F_{\text{balloon}}(x) = P \cdot A = P \cdot \pi r^2$$

(Pressure) · (Area)

$$\approx P \cdot \pi 2Rx = P \cdot 2\pi Rx$$

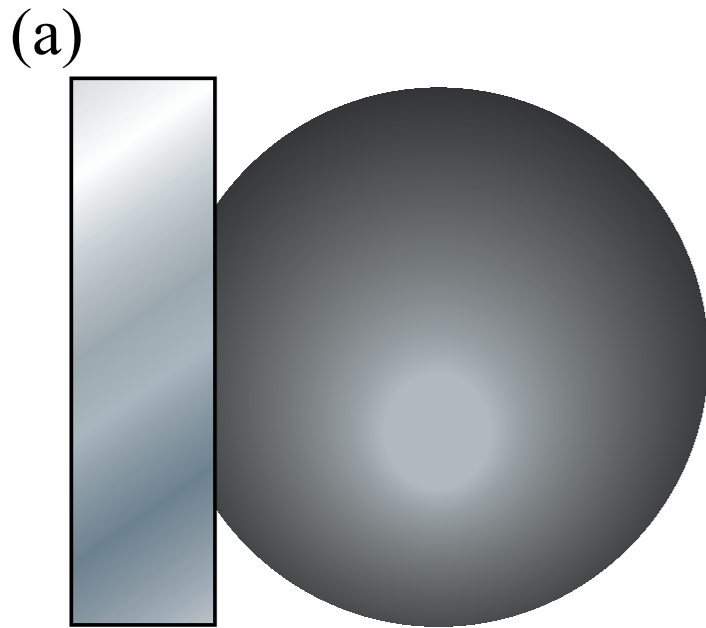
(Hooke spring constant k)

$$= kx$$

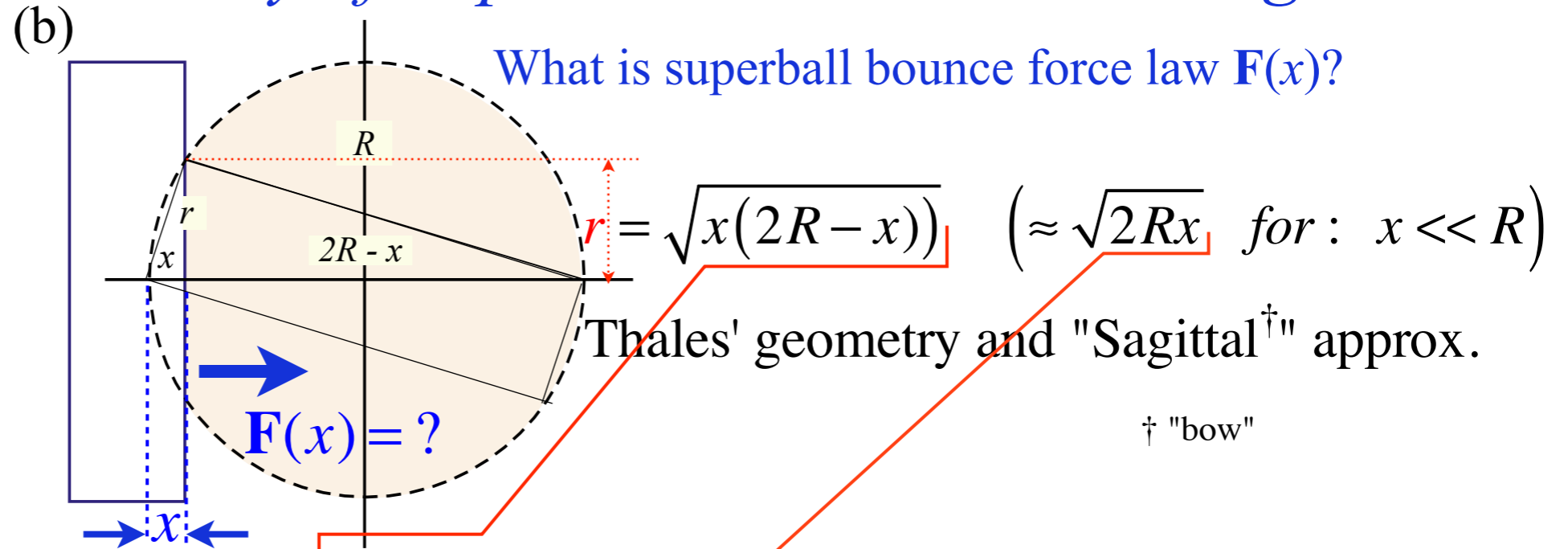
Instead superball force law depends on bulk *volume* modulus and is non-linear $F \sim x^p$ +? (Power Law?)

$$\text{Volume}(X) = \int_0^X \pi r^2 dx = \int_0^X \pi x(2R - x) dx$$

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)



If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

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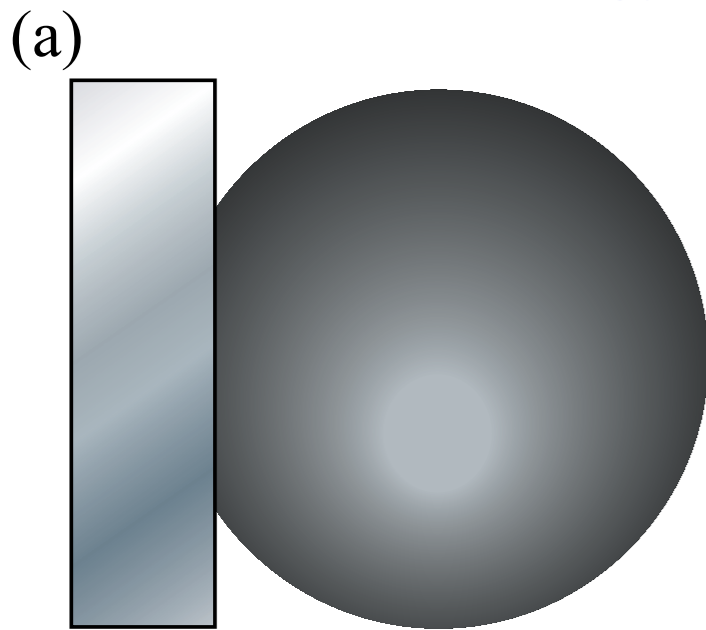
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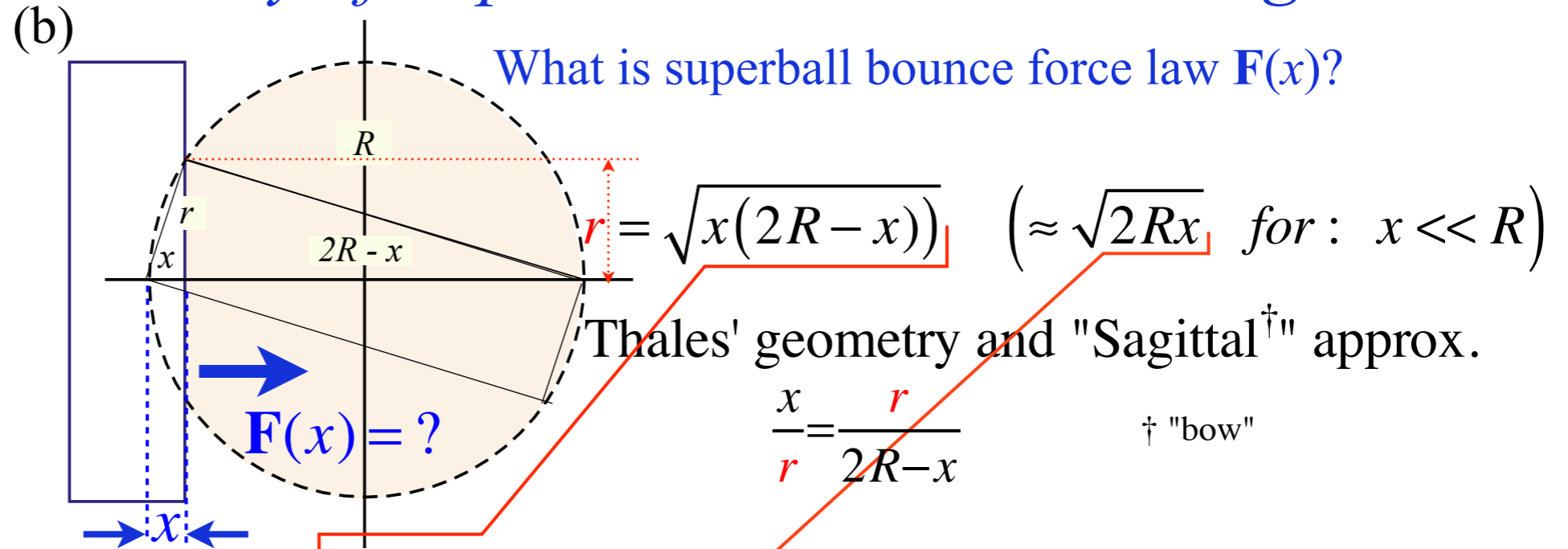
Instead superball force law depends on bulk *volume* modulus and is non-linear $F \sim x^p$ +? (Power Law?)

$$Volume(X) = \int_0^X \pi r^2 dx = \int_0^X \pi x(2R - x) dx = \int_0^X 2R\pi x dx - \int_0^X \pi x^2 dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (\text{for } : X \ll R) \\ \frac{4}{3}\pi R^3 & (\text{for } : X = 2R) \end{cases}$$

Potential Energy Geometry of Superballs and Related things



Unit 1
Fig. 7.1
(modified)



If superball was a balloon its bounce force law would be linear $F = -k \cdot x$ (Hooke Law)

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It also depends on velocity $\dot{x} = \frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by "Project-Ball*"

* *Am. J. Phys.* 39, 656 (1971)

(Discussed after p. 33)

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

→ Geometry and dynamics of single ball bounce

→ General Non-linear force (like superball-floor or ball-bearing-anvil)

(Simulations)

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Main Control Panel

Start Resume

This is the generic Bouncelt URL (or address):
<https://modphys.hosted.uark.edu/markup/BounceltWeb.html>

*Bouncelt Simulation: Force/Potential Plot
(Force power=4)*

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Sets gravity

Number of masses
Slider: 1 Balls

Acceleration of gravity
Slider: 1.51 100x{cm/s^2}

Draw force vectors
 Pause (once) at top
 Constrain motion to Y-axis
 Plot v2 vs v1
 Plot p2 vs p1
 Plot V2 vs V1
 Plot Ellipses
 Plot Bisector Lines
 Old Color Scheme
 Show right panel information
 Show left panel information
 Set Initial positions

Collision friction (Viscosity)
Slider: 0 x10^ 0 {g}

Initial gap between balls
Slider: 5.5 x10^ -1 {g}

Force Constant *Usually need to decrease k for p = 1*
Slider: 5 x10^ 4 {g}

Force power law exponent
Slider: 4 *This is non-linear F=-kx^4
(Set p = 1 for linear F=-kx^1)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0
Slider: 0.75

Initial x1 = Slider: 0.5 y Max = Slider: 7

Max x PE plot = Slider: 0.5 y Min = Slider: 0

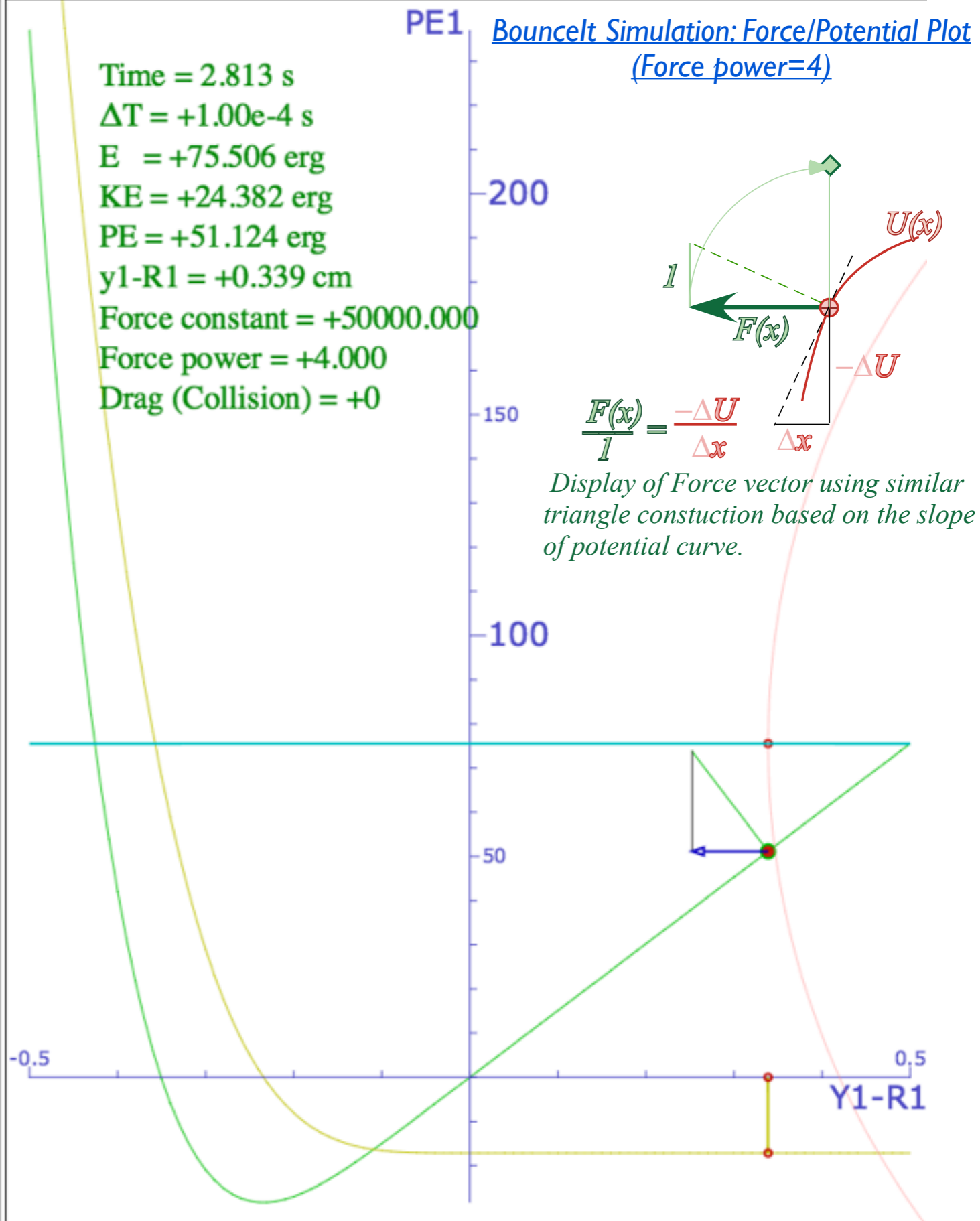
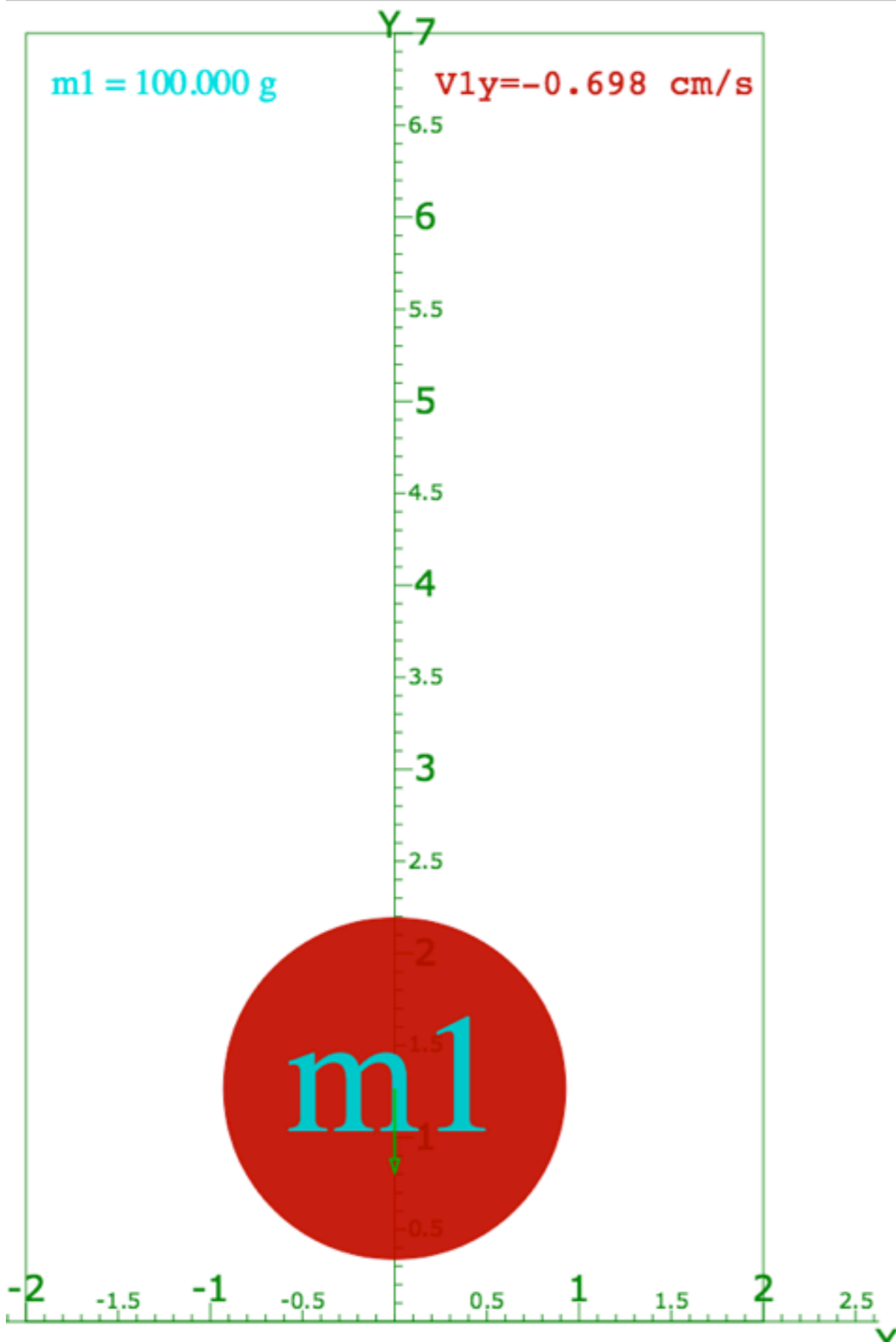
F-Vector scale = Slider: 0.003 V2y Max = Slider: 3.1112

Error step = Slider: 0.0000 V2y Min = Slider: -3.111

Mass factor = Slider: 1

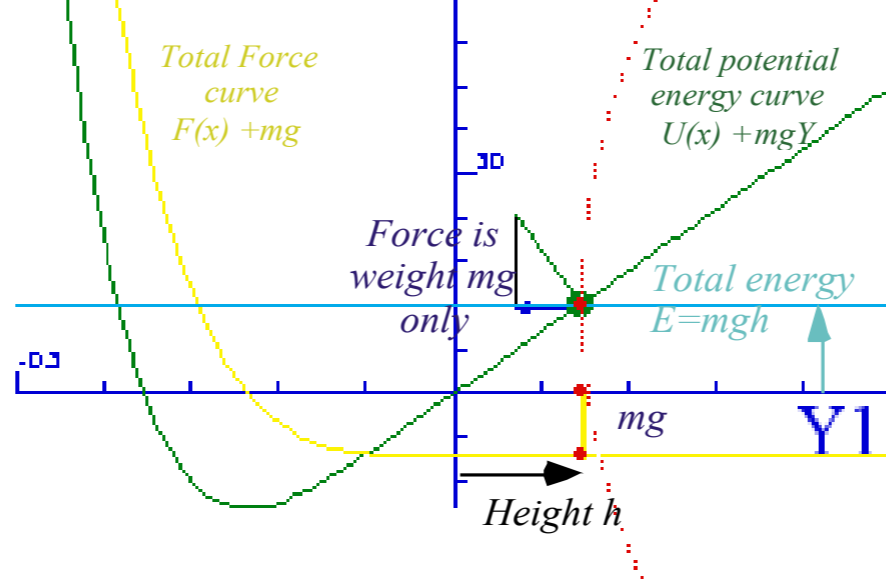
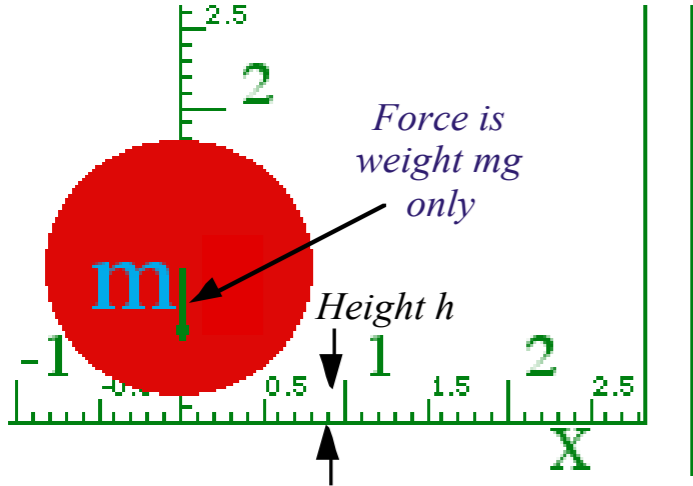
M₁ = Slider: 1 x10^ 2 {g}

V₁ = Slider: -1.1 x10^ -2 {cm/s}



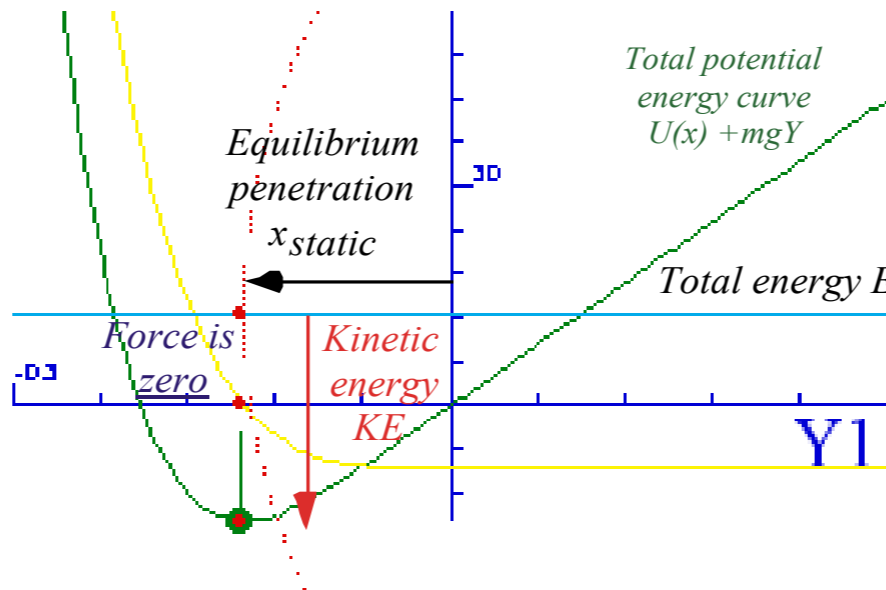
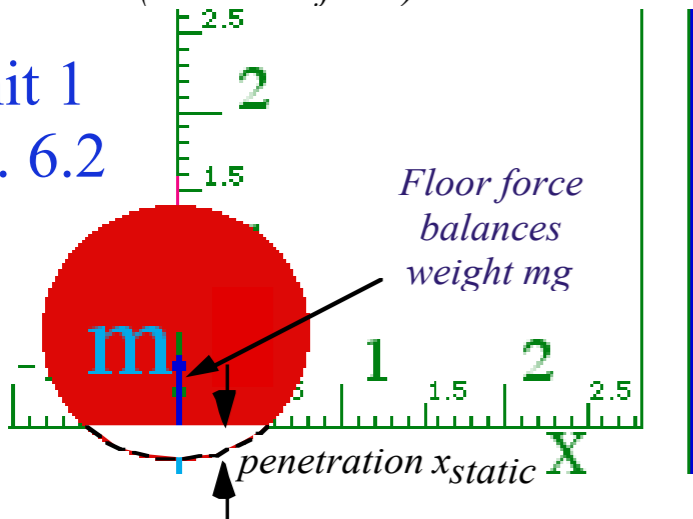
Details of each case follows using newer [BounceIt Web simulations](#)

(a) Drop height
(Zero kinetic energy)

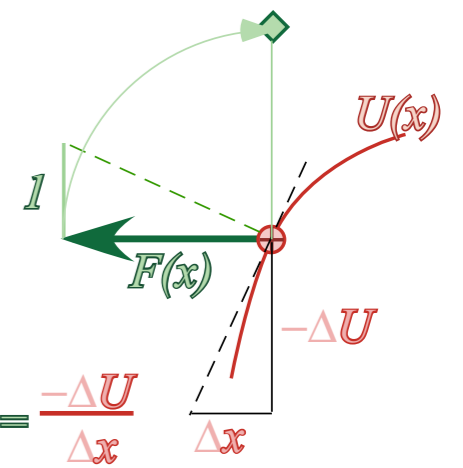
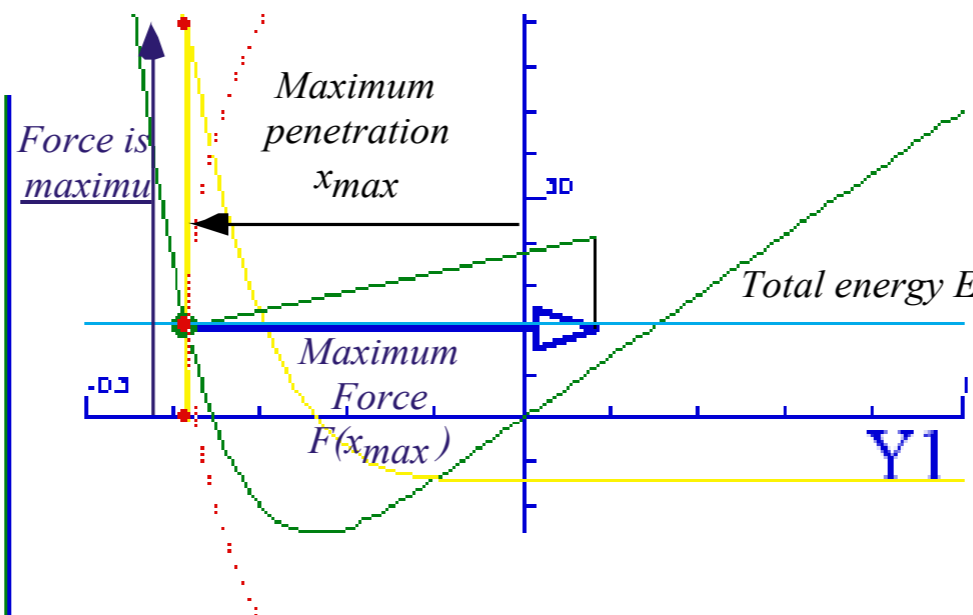
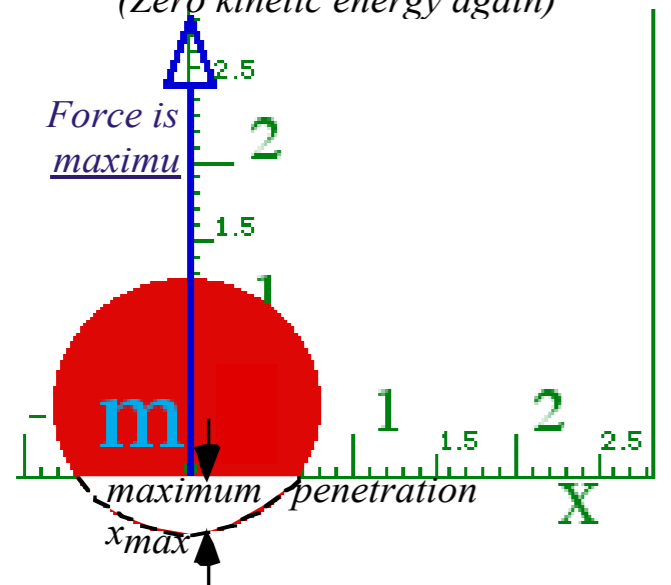


(b) Maximum kinetic energy
(Zero total force)

Unit 1
Fig. 6.2



(c) Maximum penetration
(Zero kinetic energy again)



Display of Force vector using similar triangle construction based on the slope of potential curve.

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- Old Color Scheme

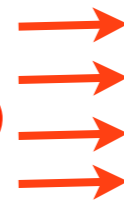
- Collision friction (Viscosity) x10^ {g}
- Initial gap between balls x10^ {g}
- Force power law exponent ← *This is linear setting (increase for non-linear)*
- Force Constant
- Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

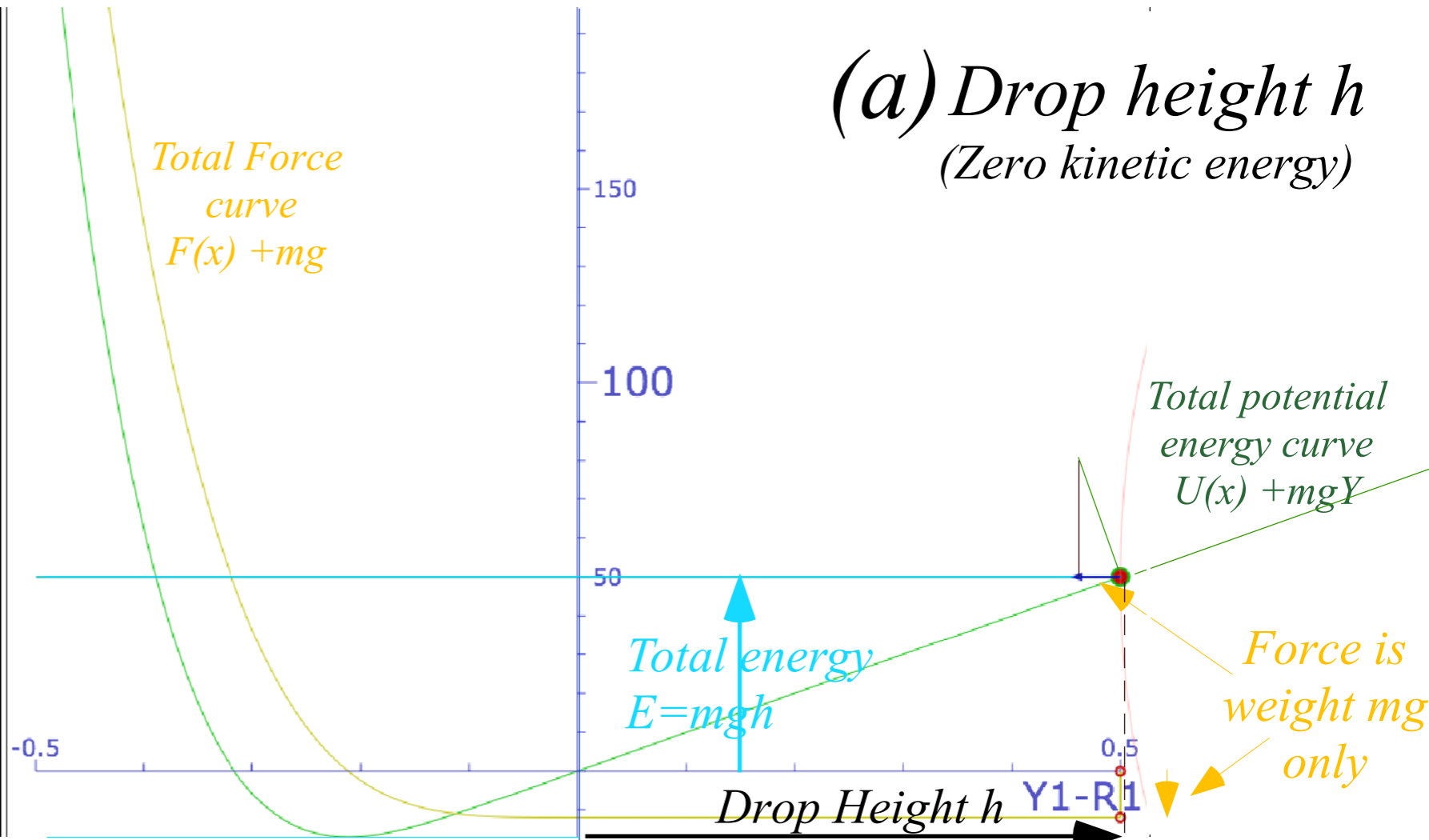
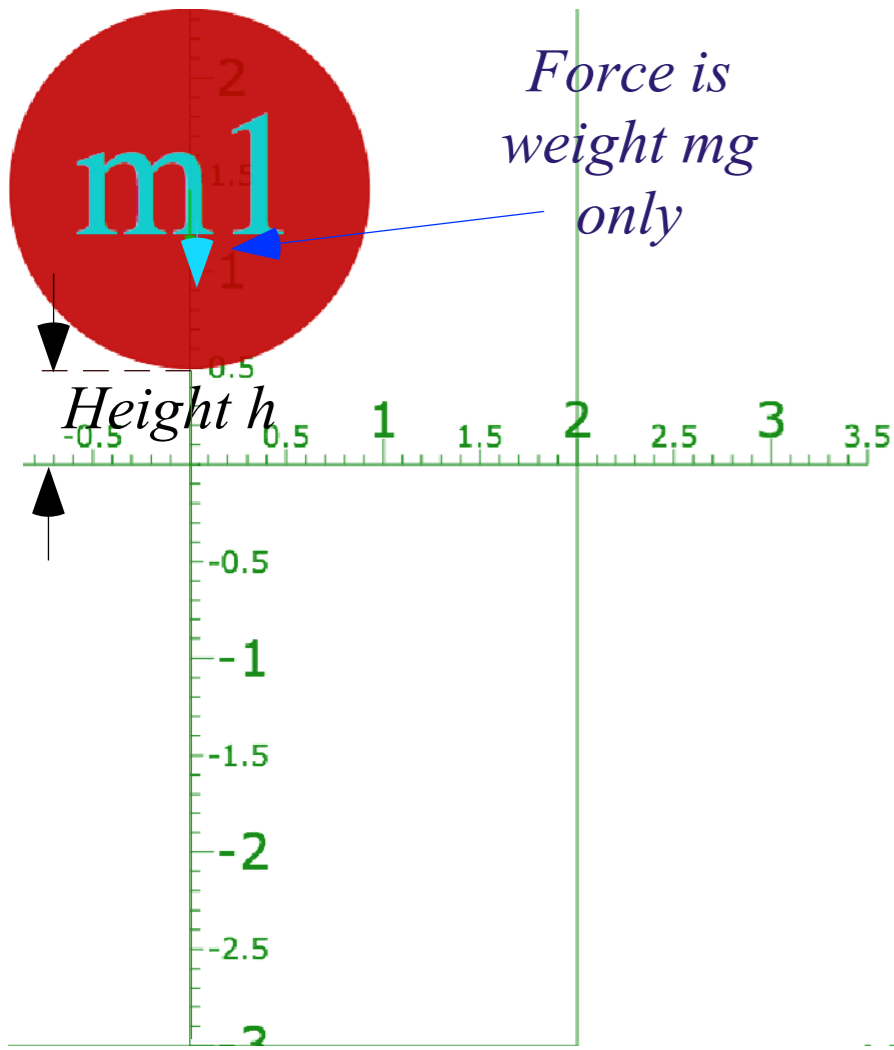
- Initial x1 = y Max =
- Max x PE plot = y Min =
- F-Vector scale = T Max =
- Error step = V2y Max =
- V2y Min =

m1 = x10^ {g} V1₀ = x10^ {cm/s}

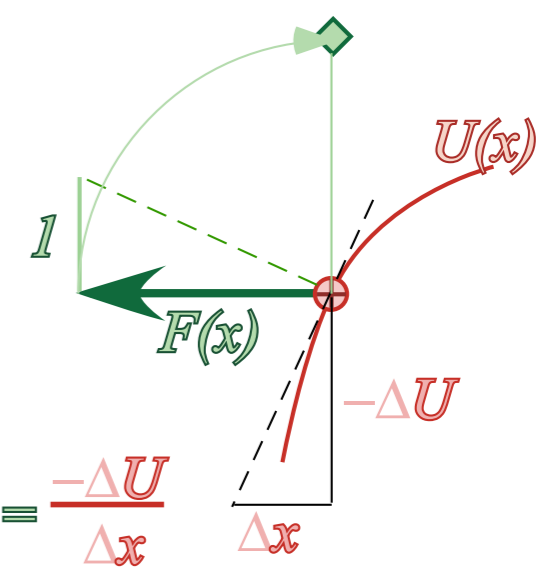
Zero Gap 2-Ball Collision (m1:m2 = 1:7)	
Linear 2-Ball Collision (m1:m2 = 1:7)	
Newton's Balls (Zero gap, Nonlinear force)	
Newton's Balls (Zero gap, Linear force)	
3-Ball Tower	5-Ball Tower
Potential Plot (1 Ball, Nonlinear force)	
Potential Plot (1 Ball, Linear force)	
Gravity Potential (1 Ball, Nonlinear force)	
Gravity Potential (1 Ball, Linear force)	

(See Simulations)





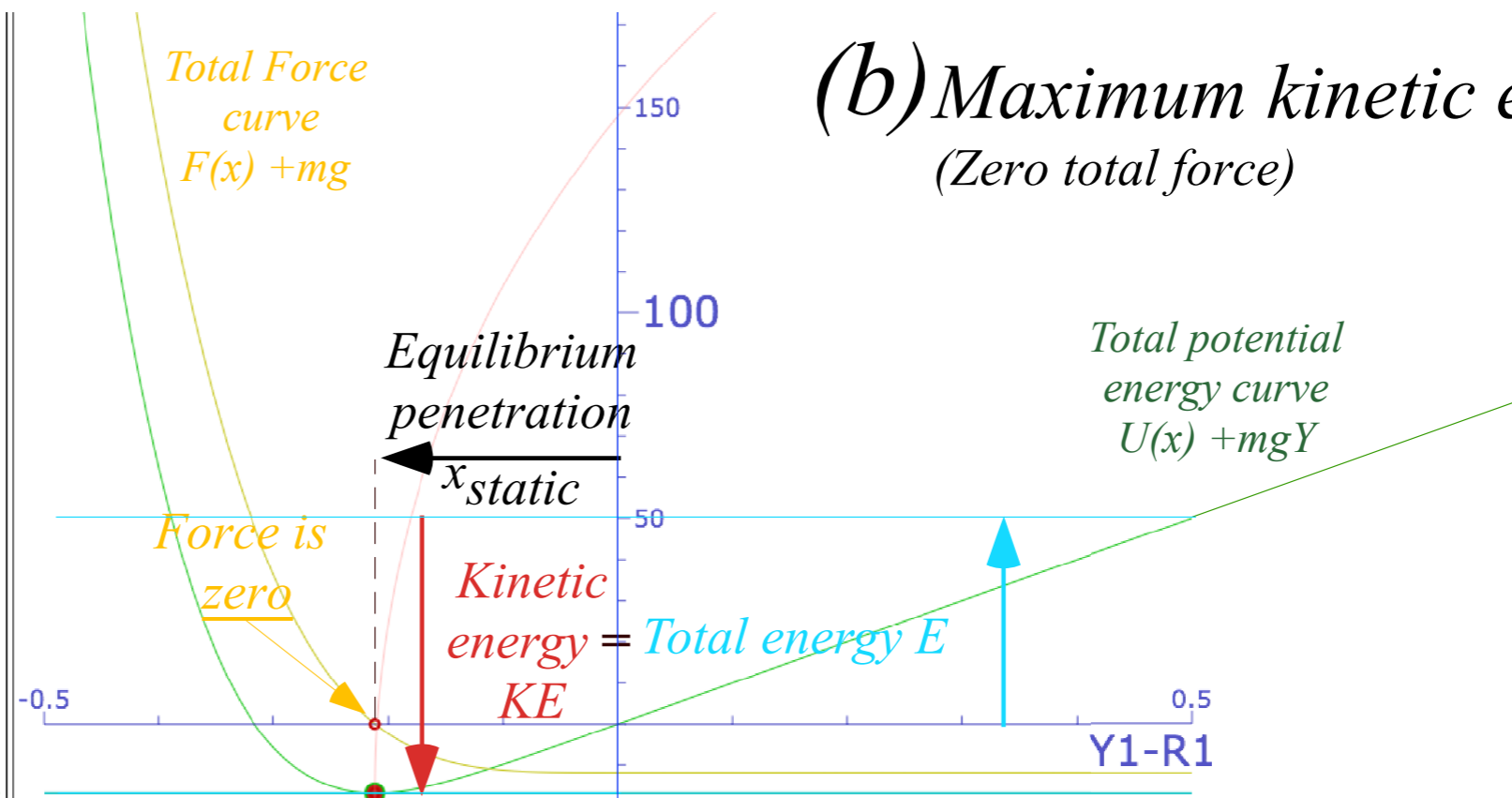
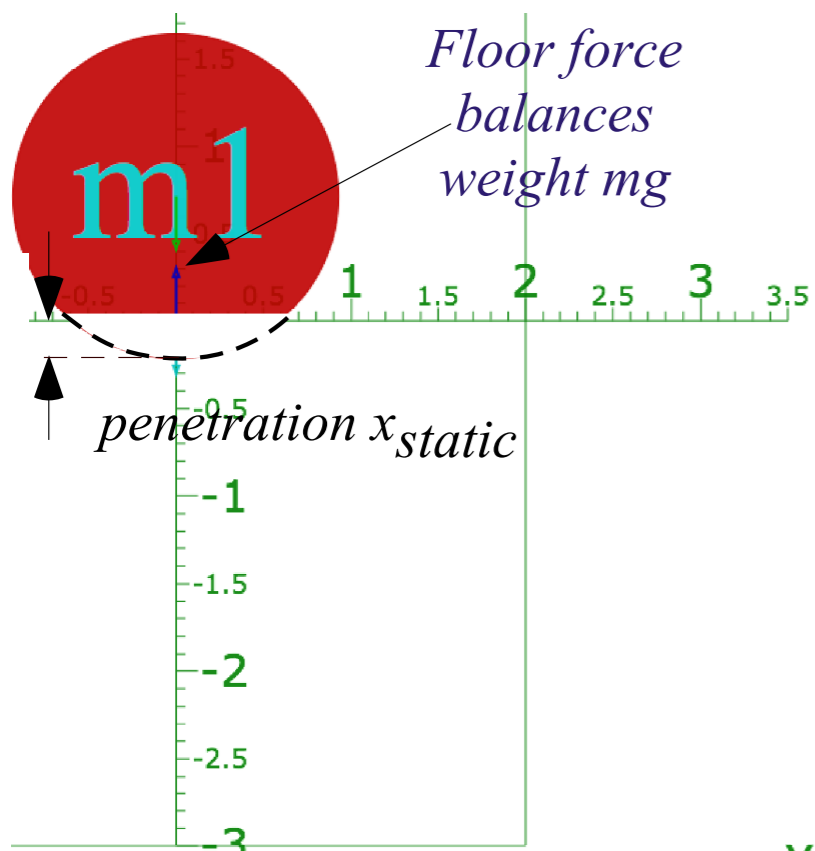
(a) Drop height h
(Zero kinetic energy)



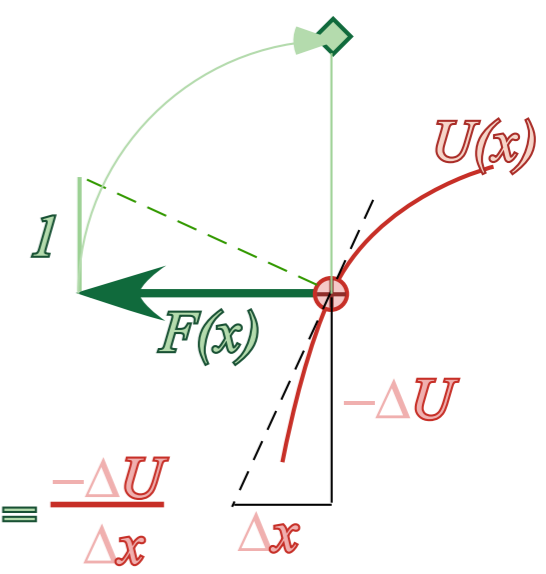
$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

Display of Force vector using similar triangle construction based on the slope of potential curve.

Bouncelt Simulation: Force/Potential Plot
(Force power=4)

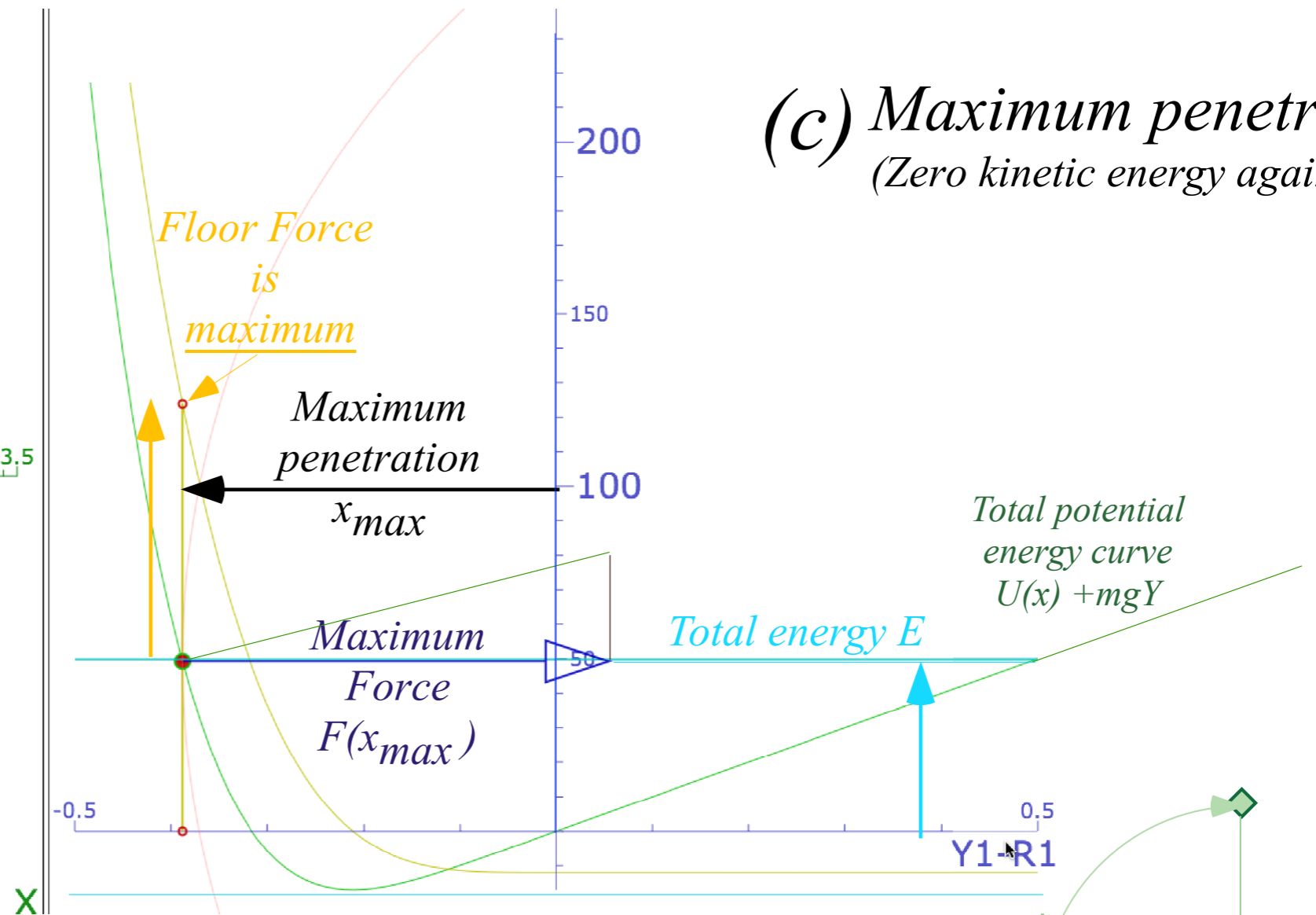
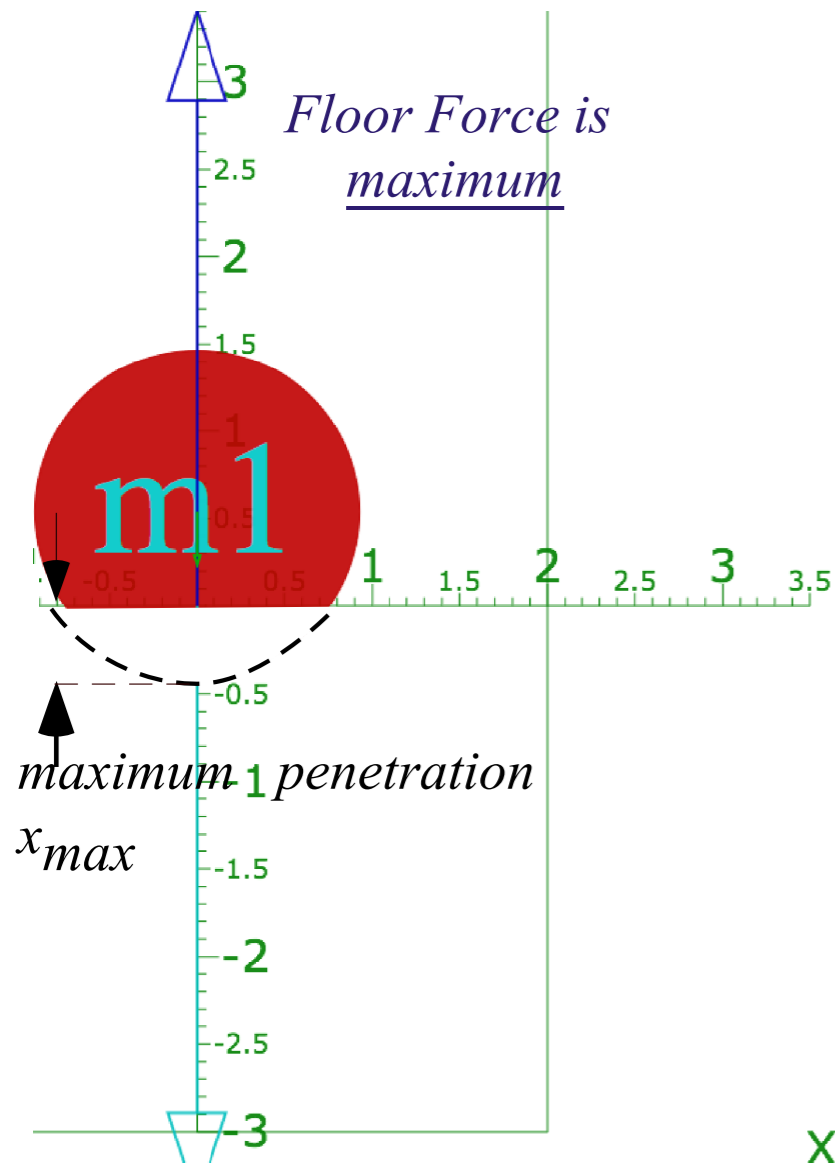


(b) Maximum kinetic energy
(Zero total force)

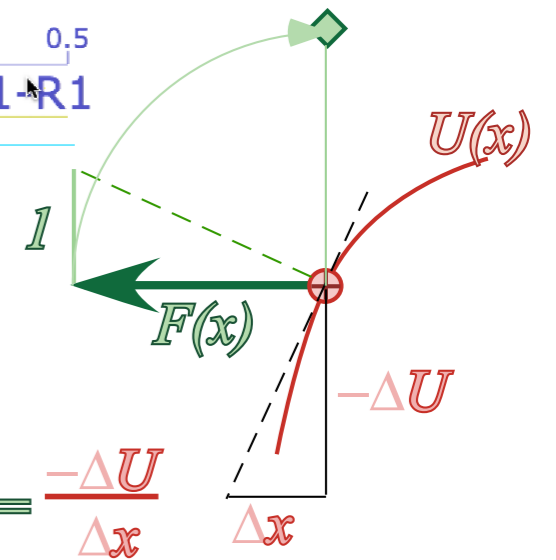


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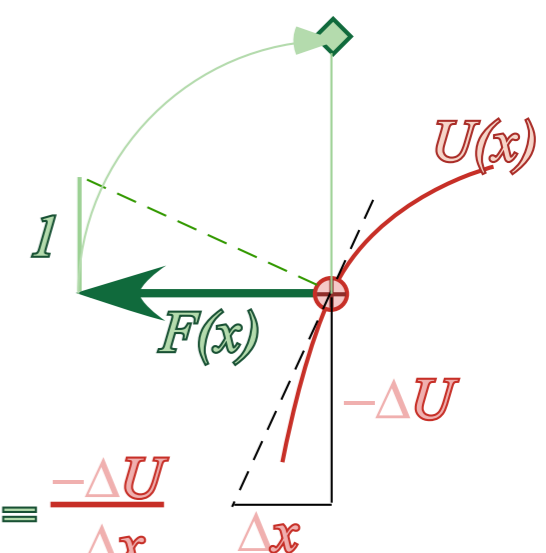
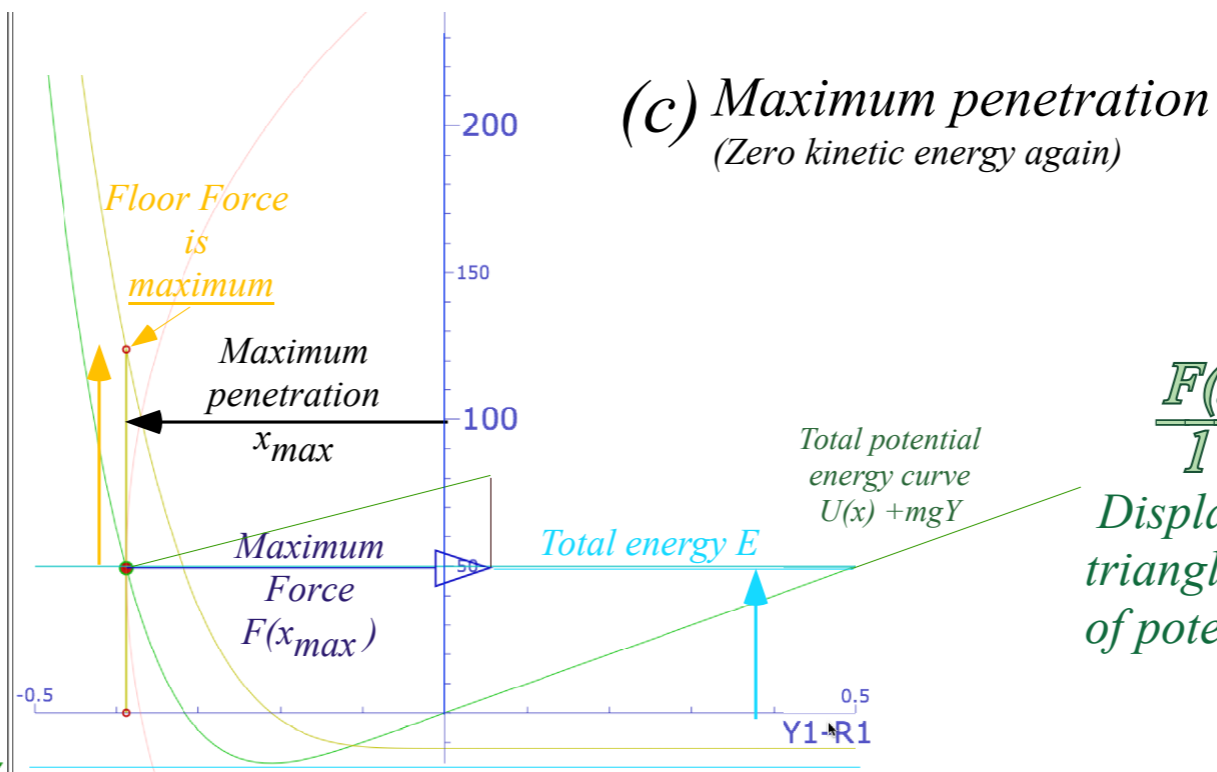
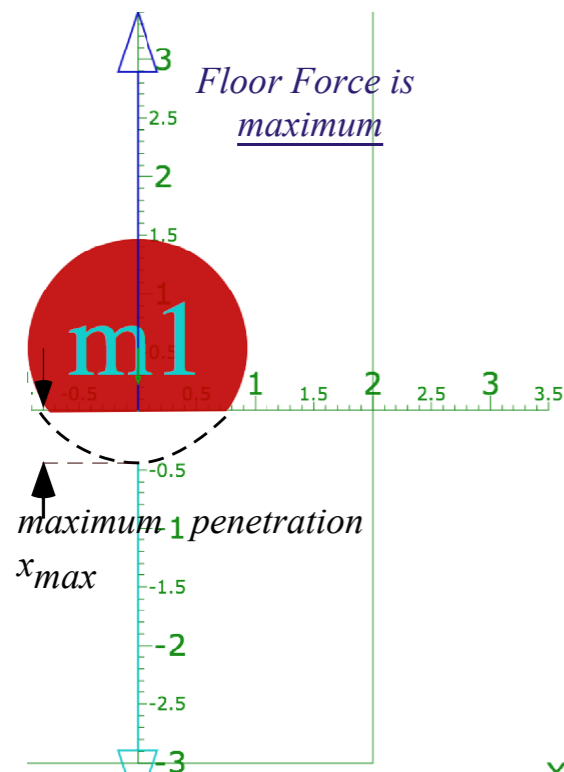
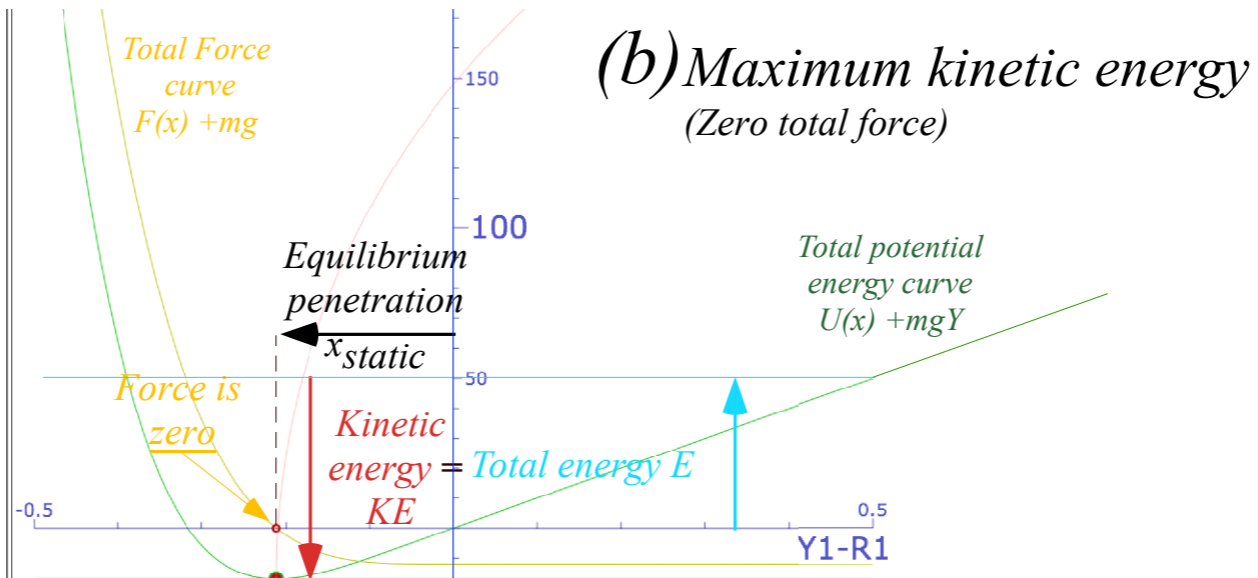
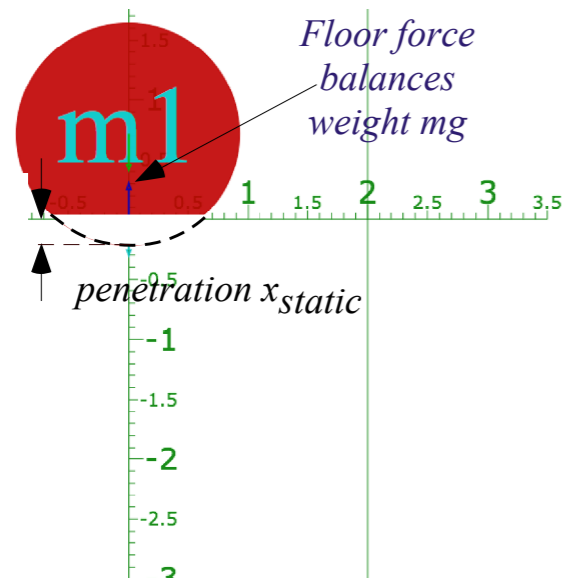
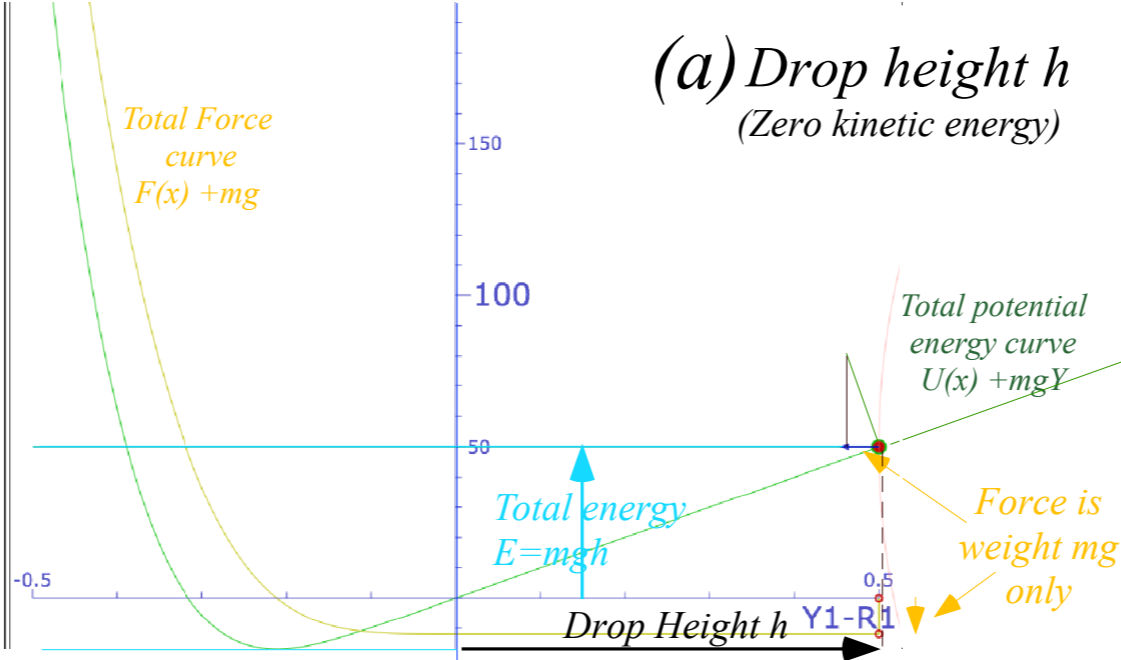
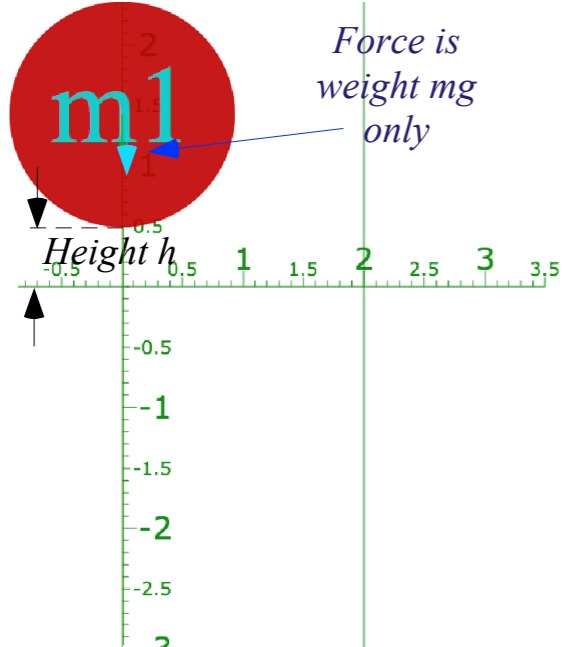
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(Force power=4)



(c) Maximum penetration
(Zero kinetic energy again)



Display of Force vector using similar triangle construction based on the slope of potential curve.



$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

Display of Force vector using similar triangle construction based on the slope of potential curve.

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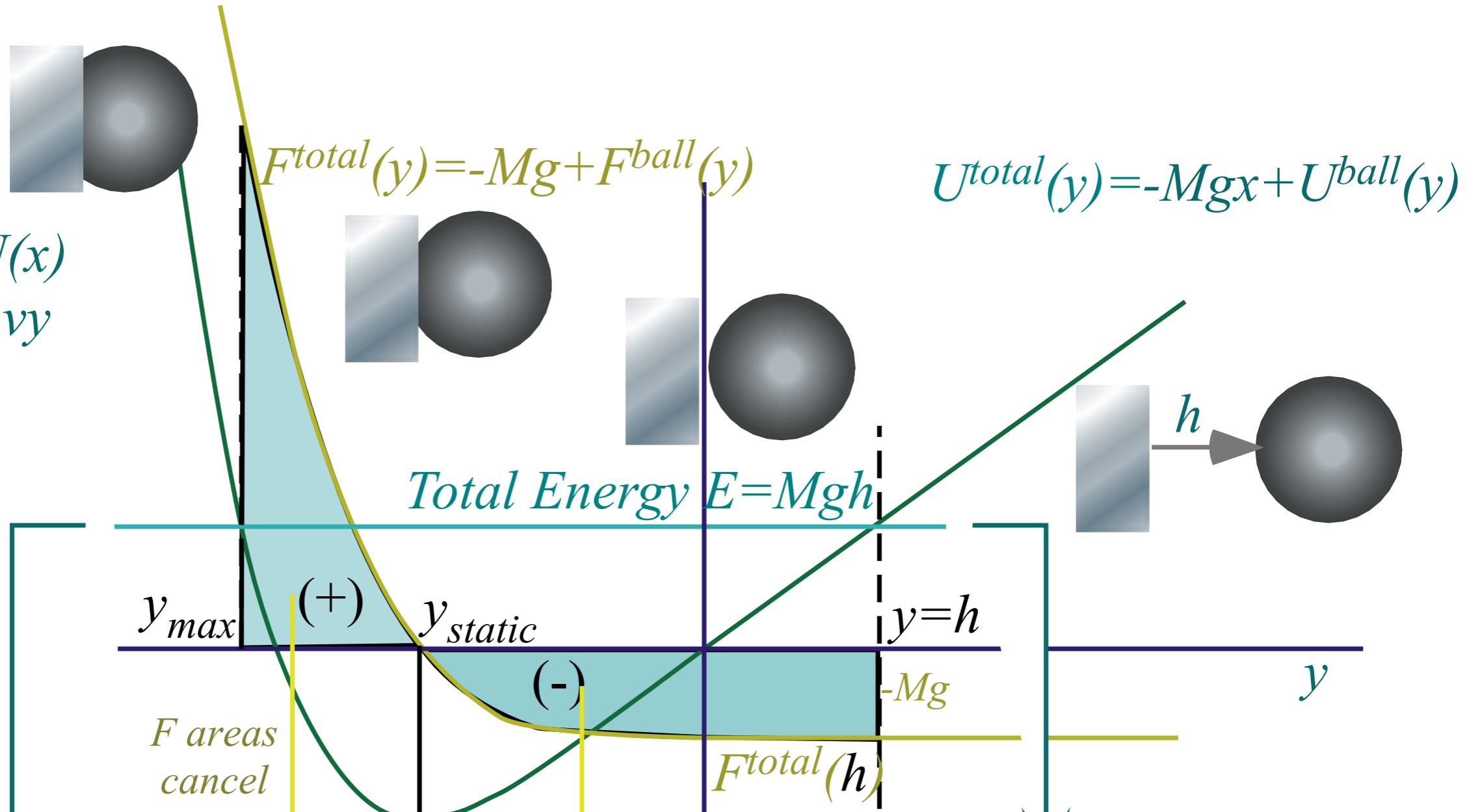
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Force $F(x)$
and
Potential $U(x)$
for soft heavy
non-linear
superball

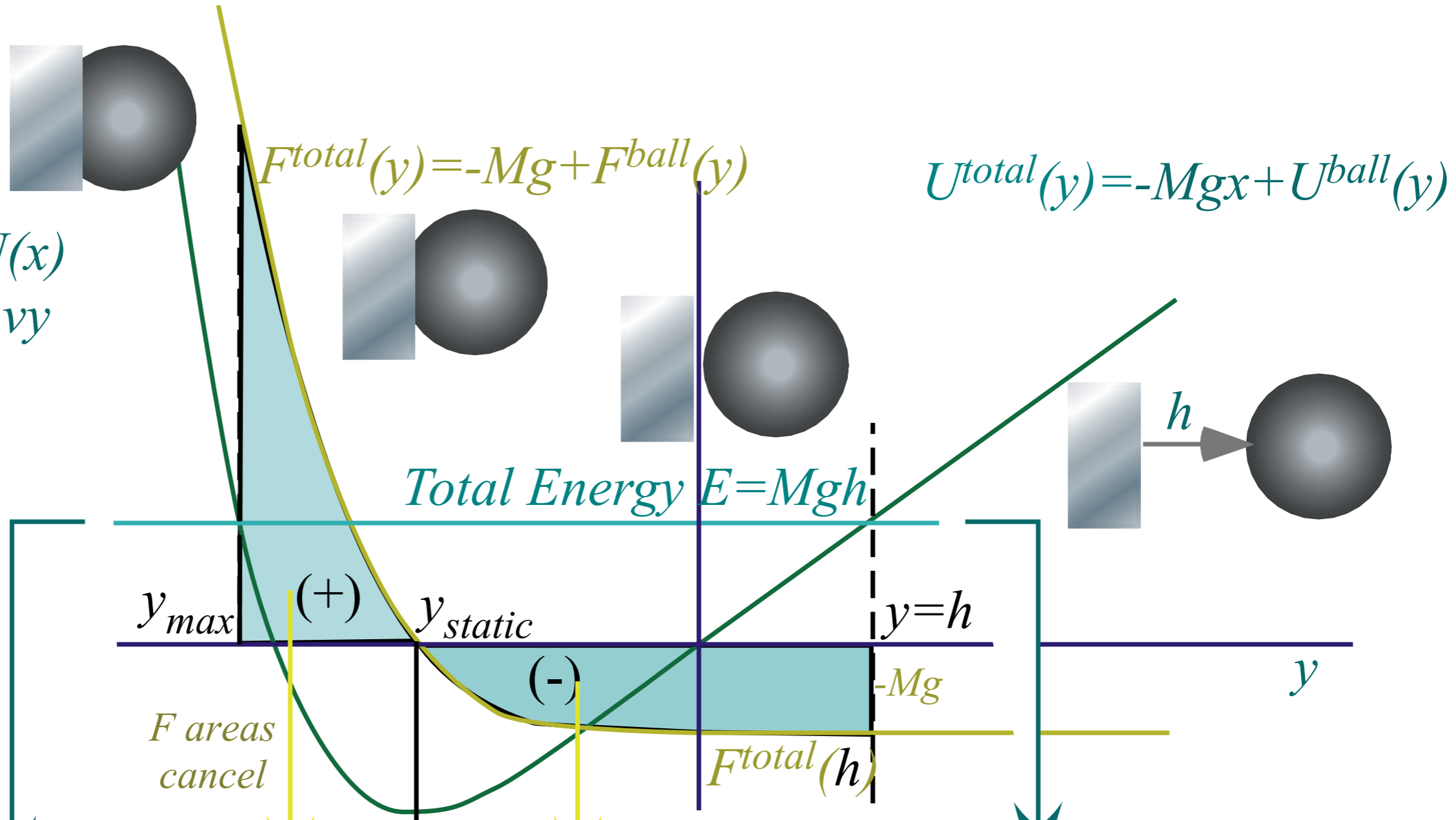


Unit 1
Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y) dy + \int_{y=h}^{y_{static}} F^{total}(y) dy + U(h) = U(h) = E$$

$$F(x) = -\frac{dU(x)}{dx}$$

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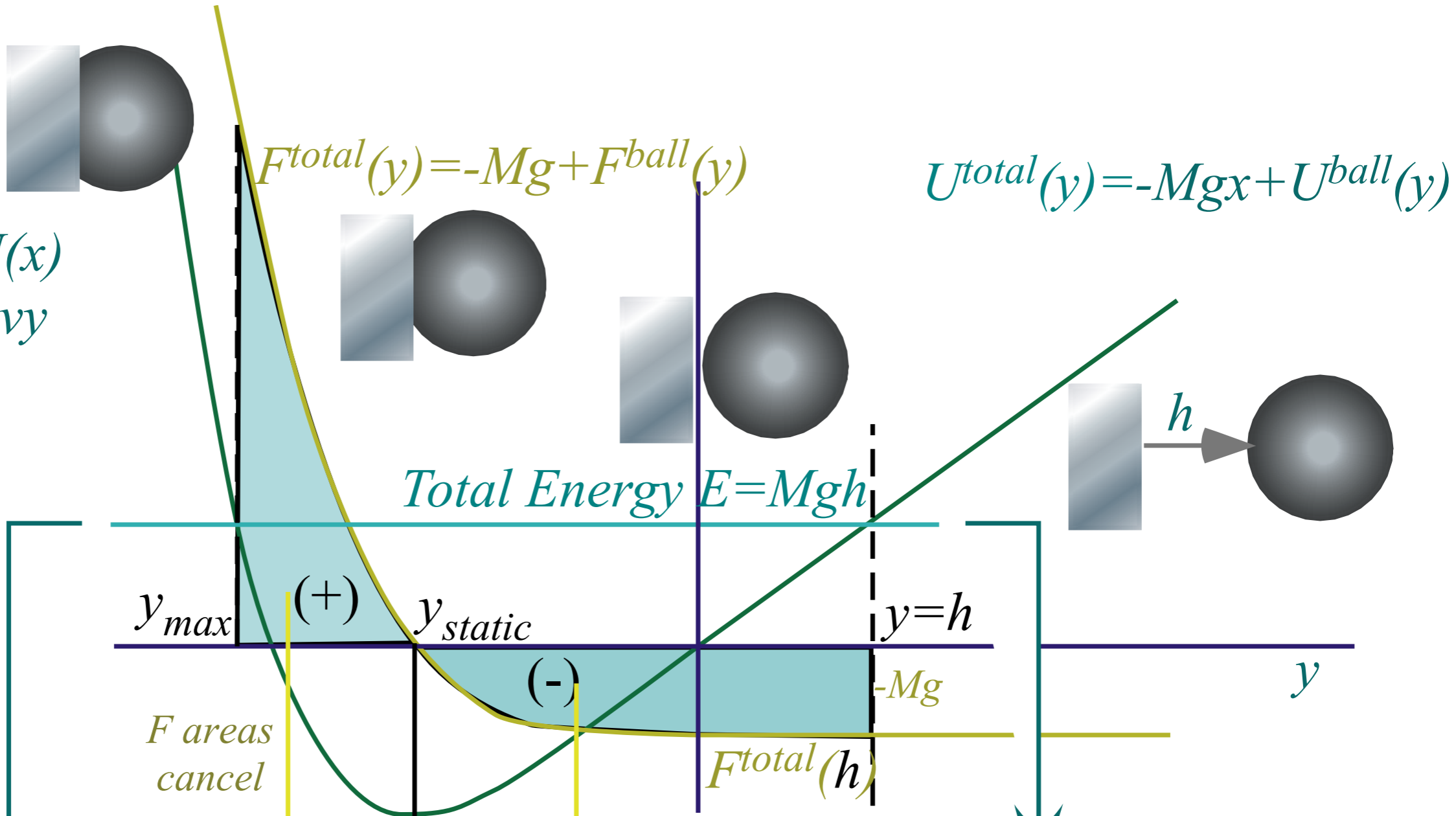
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Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y) dy + \int_{y=h}^{y_{static}} F^{total}(y) dy + U(h) = U(h) = E$$

Work = $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Force $F(x)$
and
Potential $U(x)$
for soft heavy
non-linear
superball



Unit 1
Fig. 7.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y) dy + \int_{y=h}^{y_{static}} F^{total}(y) dy + U(h) = U(h) = E$$

Work = $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Impulse = $P = \int F(t) dt = \text{Momentum acquired} = \text{Area of } F(t) = P(t)$

$$F(t) = \frac{dP(t)}{dt}$$

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General Non-linear force (like superball-floor or ball-bearing-anvil)

→ Constant force $F=-k$ (linear potential $V=kx$)

→ Some physics of dare-devil-diving 80 ft. into kidee pool (Calculus)

Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric m_1, m_2, m_3, \dots series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

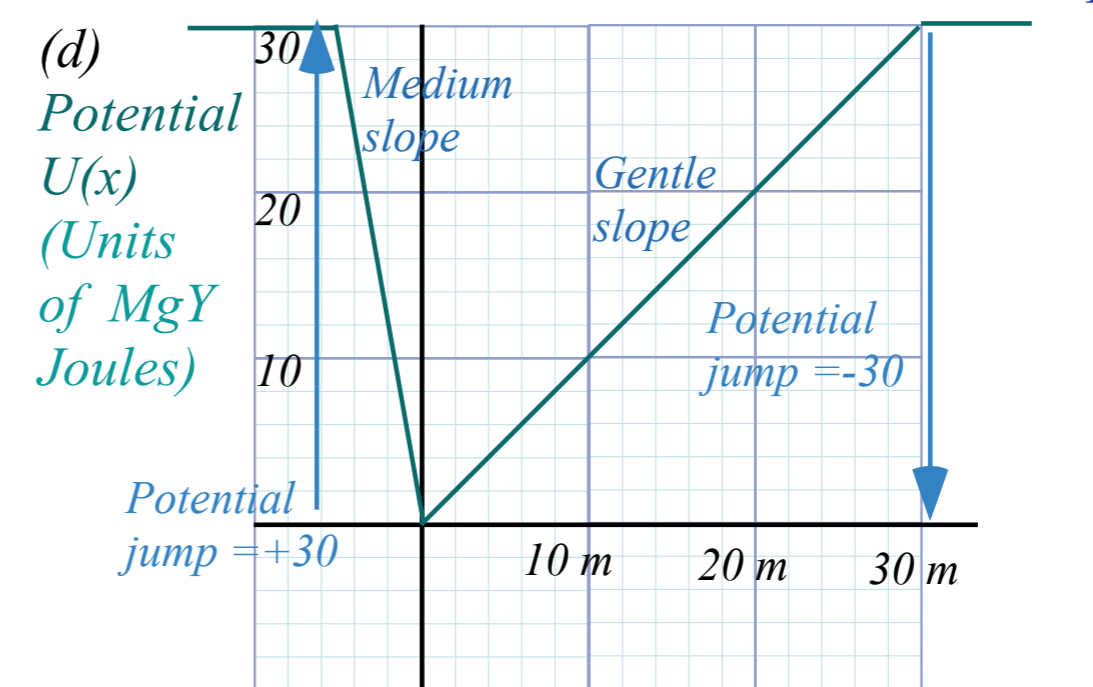
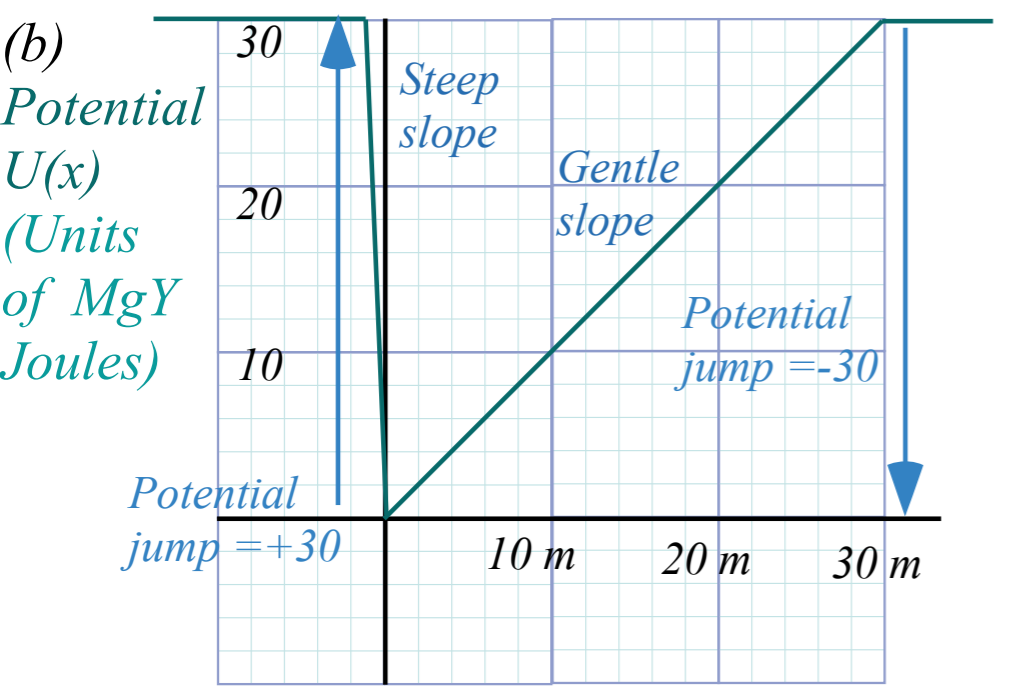
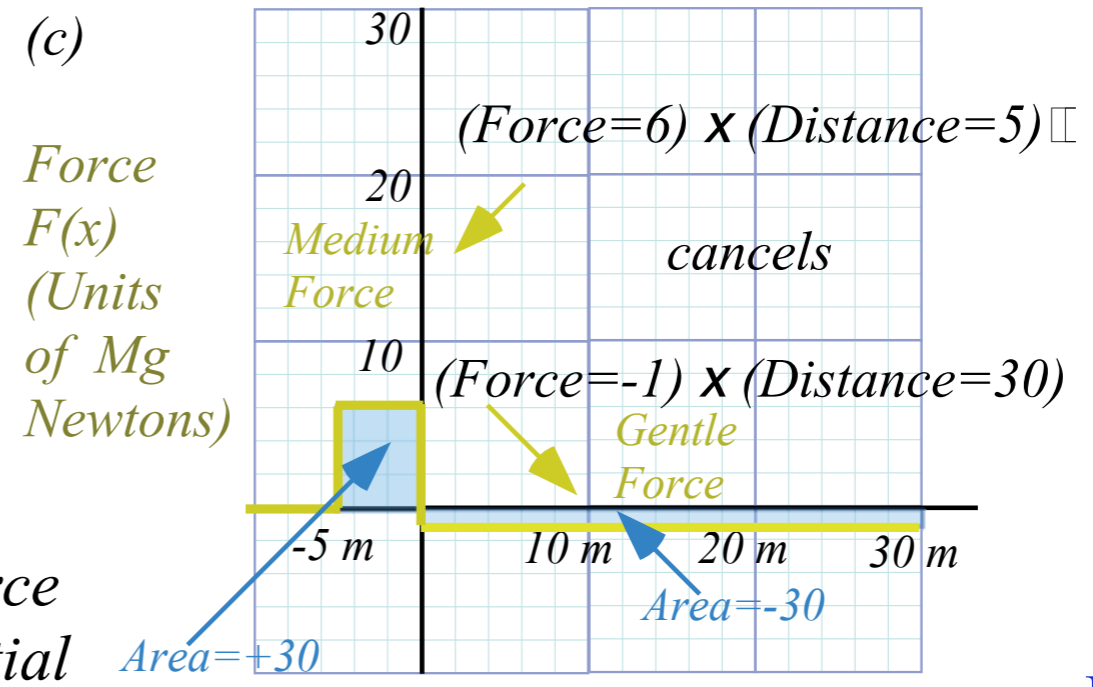
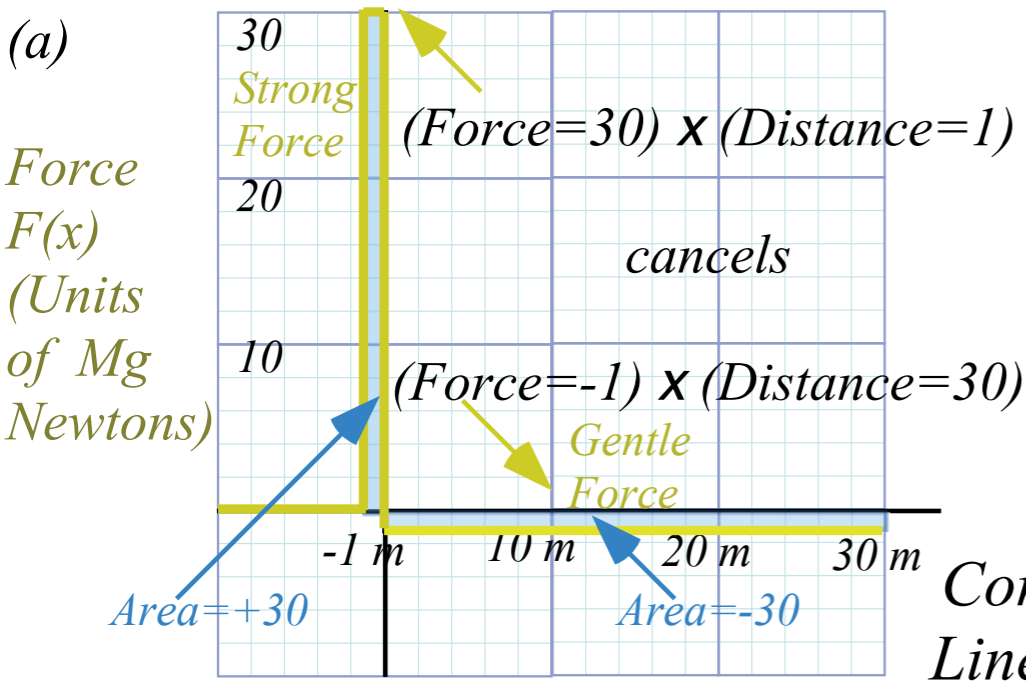
Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”



Constant Force
Linear Potential

Models:
 $F(x) = k$,
 $U(x) = -kx$

Unit 1
Fig. 7.3

$Work = W = \int F(x) dx = Energy\ acquired = Area\ of\ F(x) = -U(x)$

$F(x) = -\frac{dU(x)}{dx}$

$Impulse = P = \int F(t) dt = Momentum\ acquired = Area\ of\ F(t) = P(t)$

$F(t) = \frac{dP(t)}{dt}$

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

→ Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

→ Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))

(Simulations)

Geometry and potential dynamics of 2-ball bounce

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Advantages of a geometric m_1, m_2, m_3, \dots series

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Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Start Resume

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses

1 Balls

Acceleration of gravity

0.5 100x{cm/s²}

Sets gravity

- Draw force vectors
- Pause (once) at top
- Constrain motion to Y-axis
- Plot v2 vs v1
- Plot p2 vs p1
- Plot V2 vs V1
- Plot Ellipses
- Plot Bisector Lines
- Old Color Scheme
- Show right panel information
- Show left panel information
- Set Initial positions

Collision friction (Viscosity)

0 x10[^] 0 {g}

Initial gap between balls

5.5 x10[^] -1 {g}

Force Constant *Usually need to increase k for p > 1*

5 x10[^] 2 {g}

Force power law exponent

1

*This is linear F=-kx¹
(increase p > 1
for non-linear F=-kx^p)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

0.75

Initial x1 = 0.5 y Max = 7

Max x PE plot = 0.5 y Min = 0

F-Vector scale = 0.003 V2y Max = 3.1112

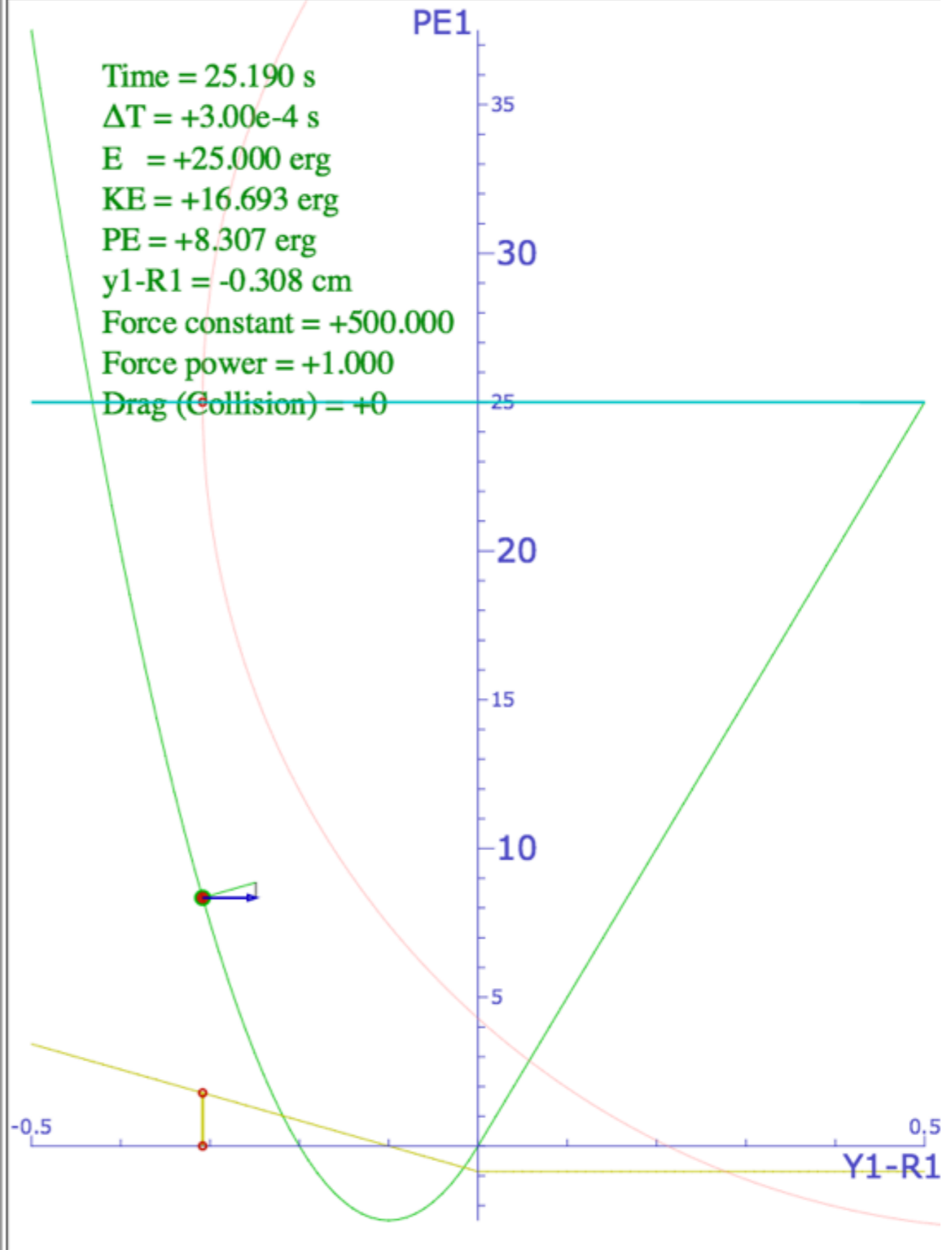
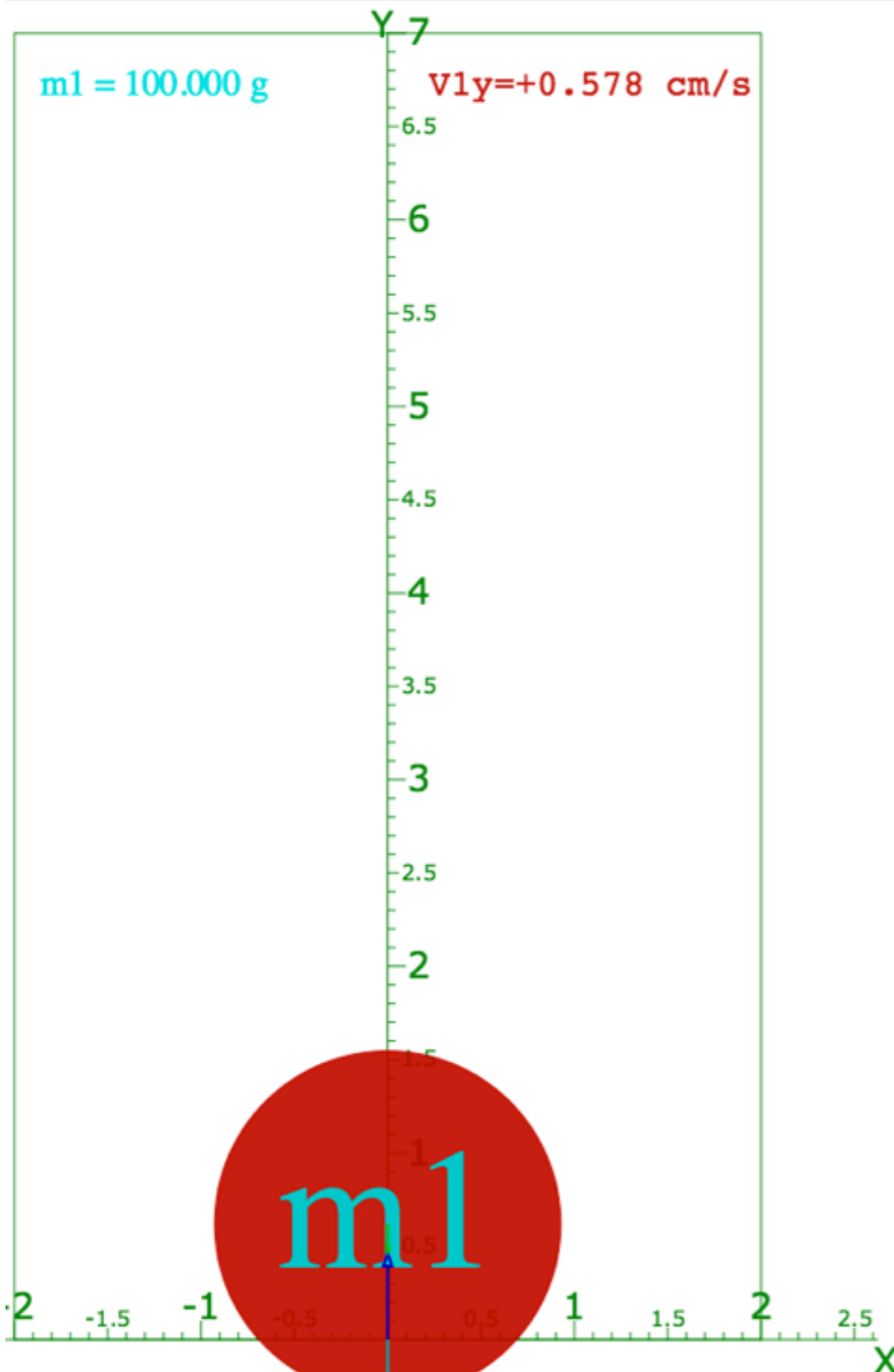
Error step = 0.0000 V2y Min = -3.111

Mass factor = 1

M₁ = 1 x10[^] 2 {g}

V₁ = 0 x10[^] 0 {cm/s}

Controls & Scenarios Resume Reset T=0 Reset & Pause@Edge Erase Paths Speed $\times 10^{\wedge}$



Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

 *Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))* *(Calculations)*

Geometry and potential dynamics of 2-ball bounce

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Many-body 1D collisions

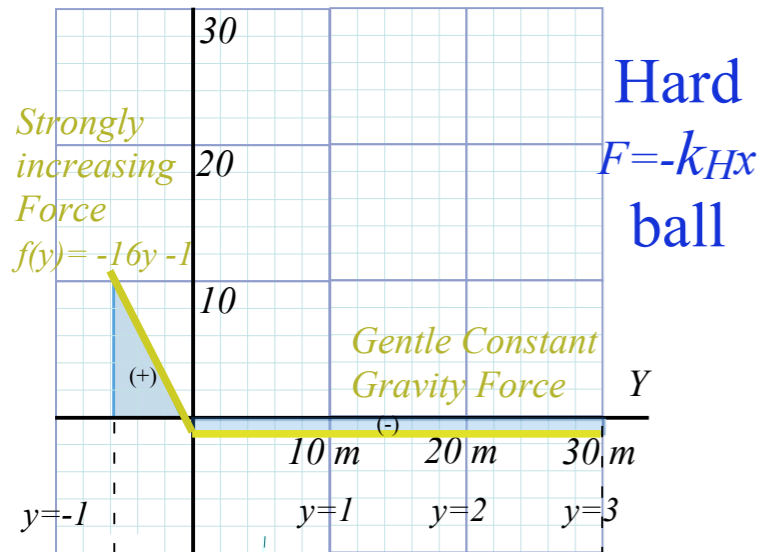
Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

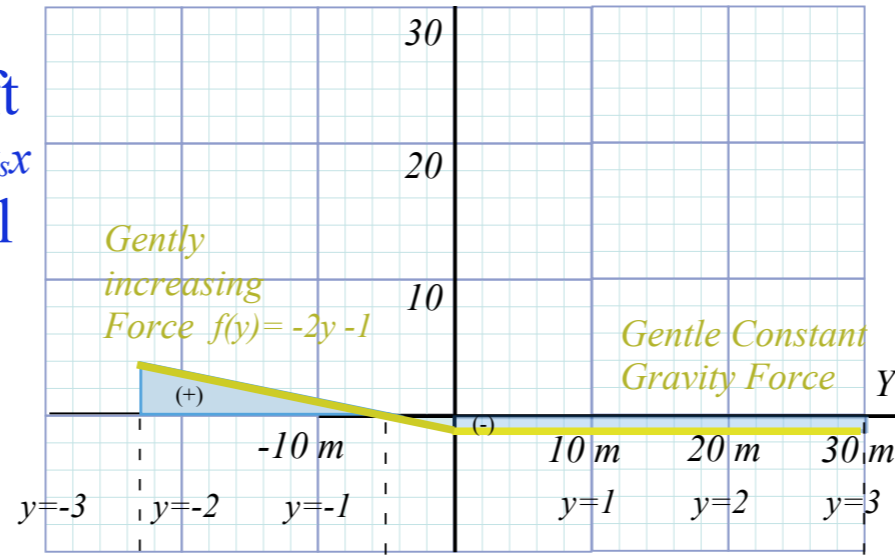
Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

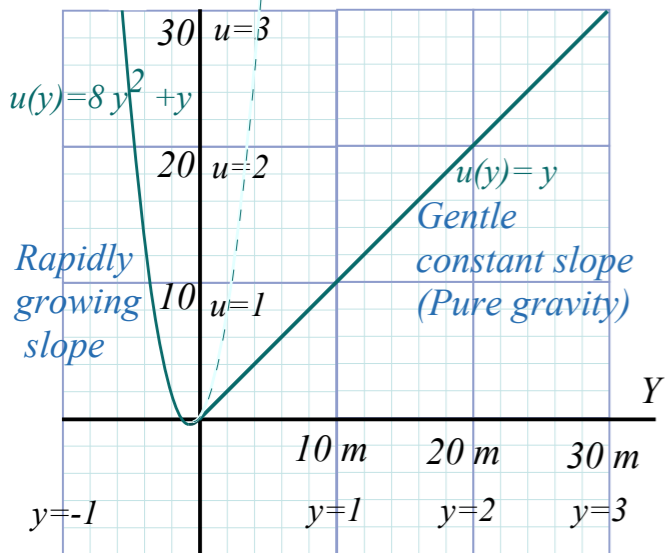
(a) Force $F(Y)$ Units Mg (N)



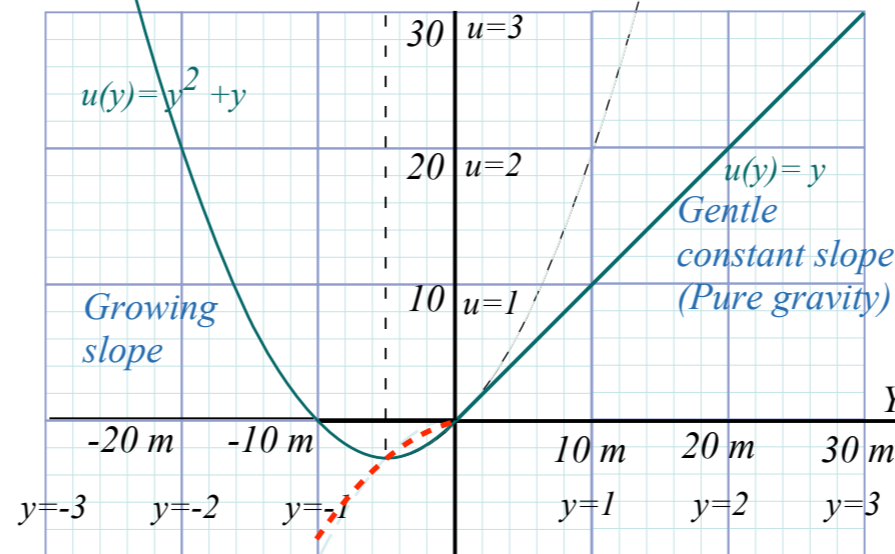
(c) Force $F(Y)$ Units Mg (N)



(b) Potential $U(Y)$ Units of MgY (J)



(d) Potential $U(Y)$ Units of MgY (J)



Unit 1
Fig. 7.4

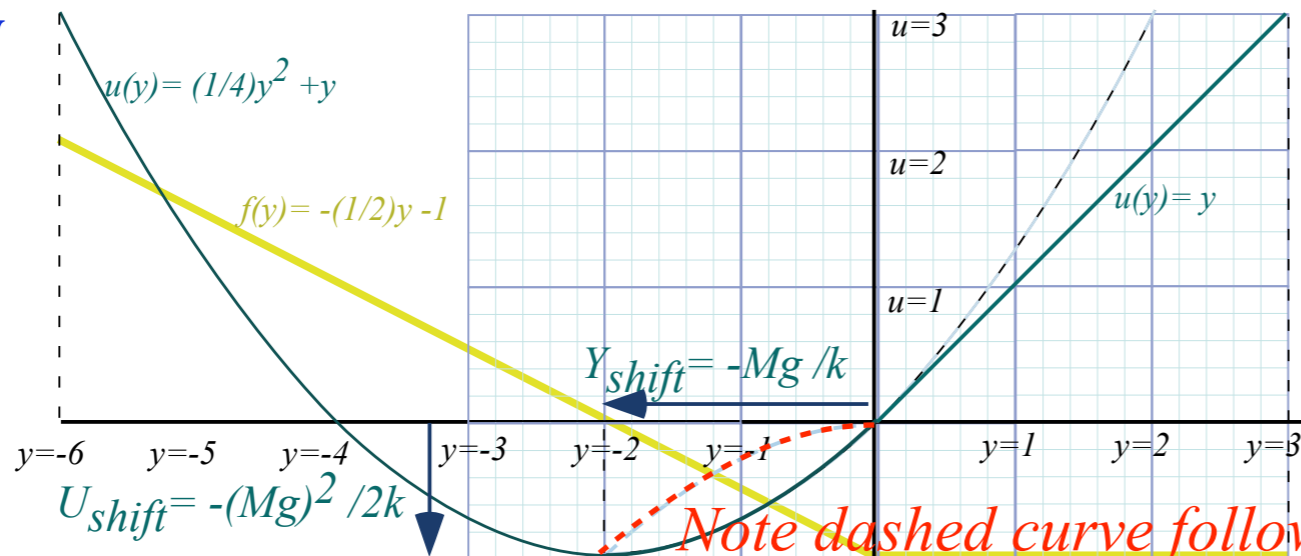
(e) Geometry of Linear Force with Constant Mg and Quadratic Potential

$$F(Y) = -kY - Mg \quad U(Y) = (1/2)kY^2 + MgY$$

$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg y & (y \geq 0) \\ Mg y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

Close view of Soft ball $F = -k_s x$



Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

→ → General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force $F=-kx$ (quadratic potential $V=1/2kx^2$ (like balloon))

(Reviewing calculations and noticing “gap” effect)

Geometry and potential dynamics of 2-ball bounce

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Many-body 1D collisions

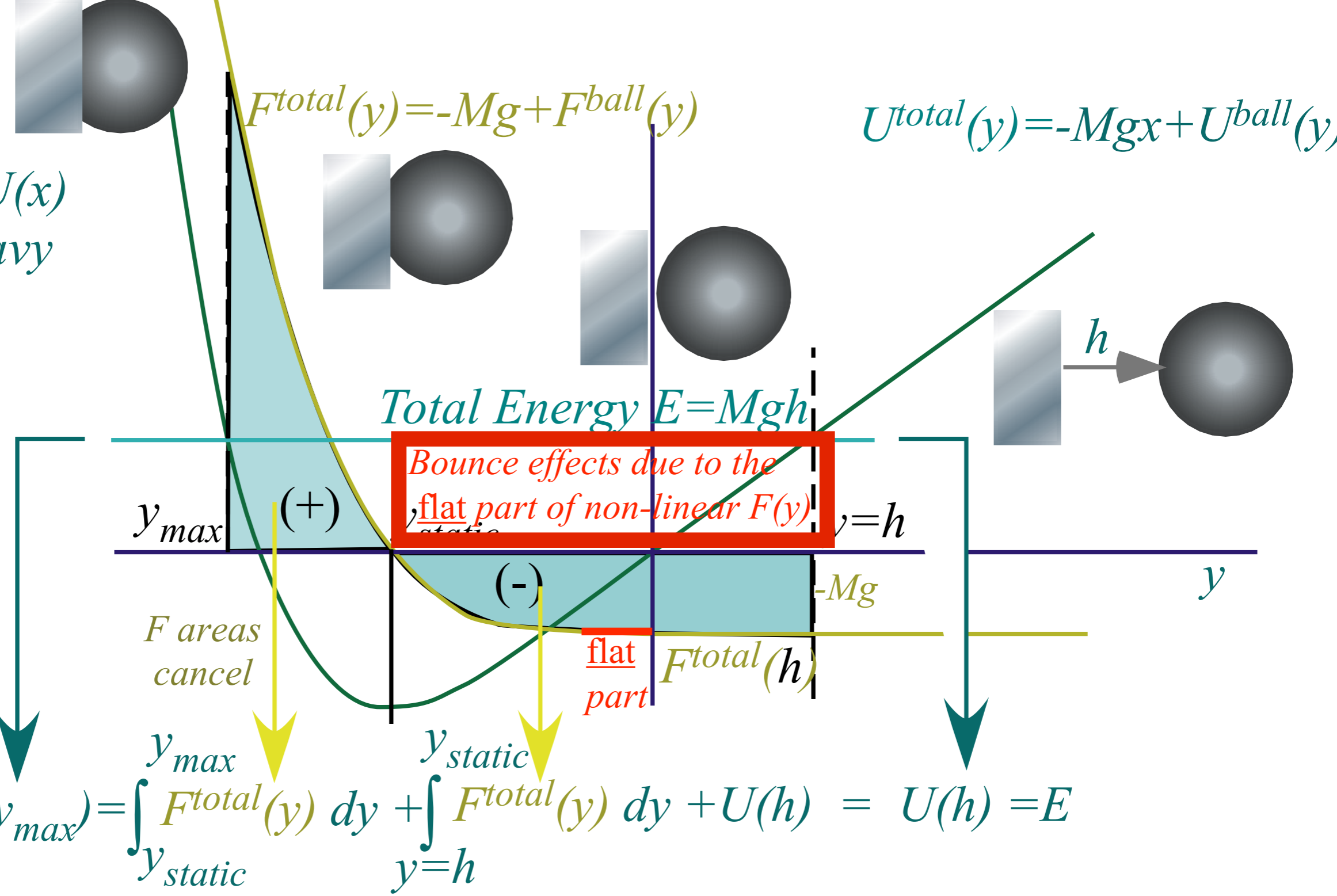
Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Force $F(x)$
and
Potential $U(x)$
for soft heavy
non-linear
superball



Unit 1
Fig. 7.5

Work = $W = \int F(x) dx = \text{Energy acquired} = \text{Area of } F(x) = -U(x)$

$$F(x) = -\frac{dU(x)}{dx}$$

Impulse = $P = \int F(t) dt = \text{Momentum acquired} = \text{Area of } F(t) = P(t)$

$$F(t) = \frac{dP(t)}{dt}$$

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force $F=-k$ (linear potential $V=kx$)

(Simulations)

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force $F=-kx$ (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

A parable of **RumpCo.** vs **CrapCorp.** (introducing 3-mass potential-driven dynamics)

Parable allegory for Los Alamos

Parable allegory for Livermore

Cheap&practical “seat-of-the pants” approach

Fancy&overpriced “political” approach

Advantages of a geometric m_1, m_2, m_3, \dots series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

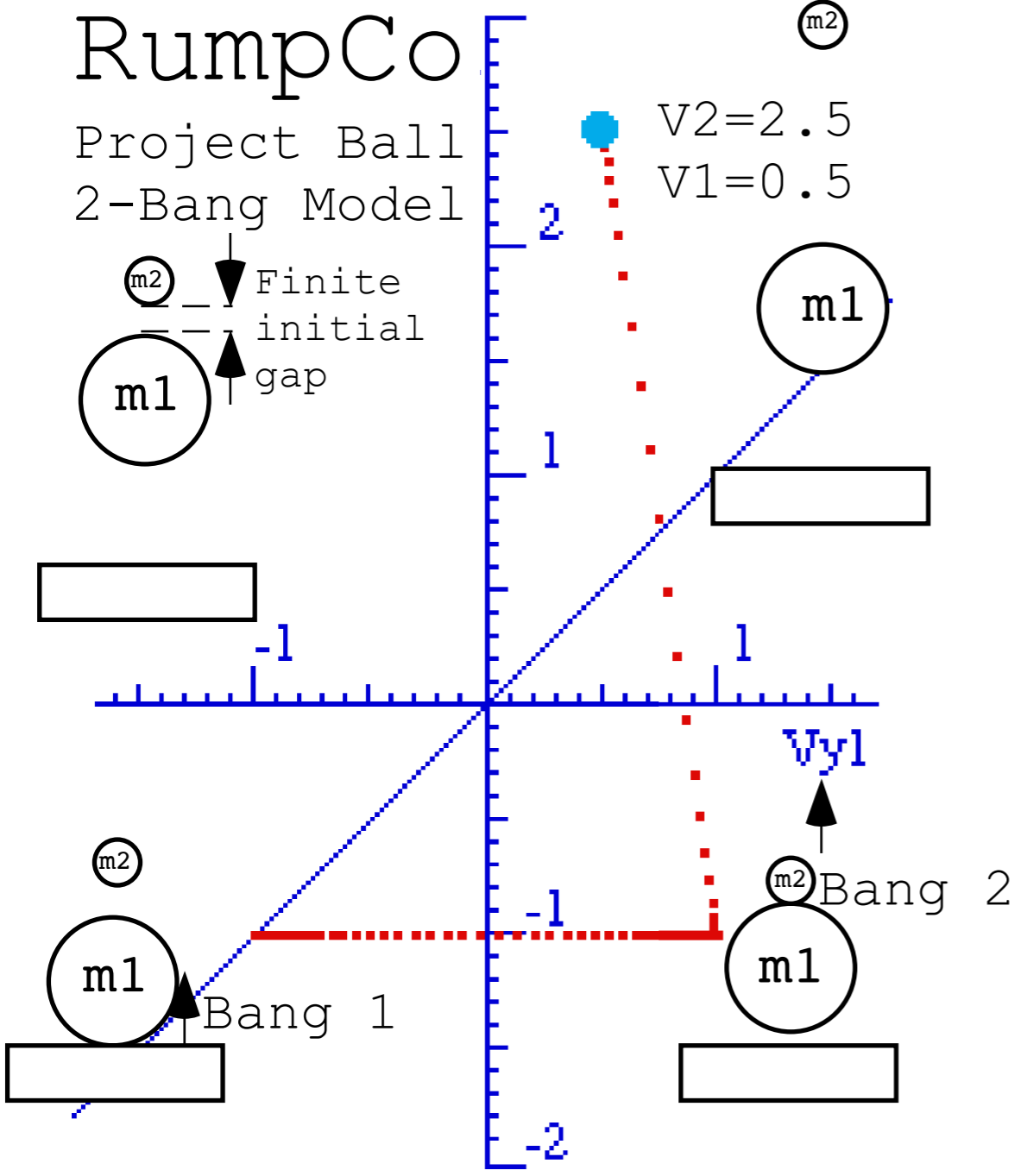
Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Parable allegory for Los Alamos

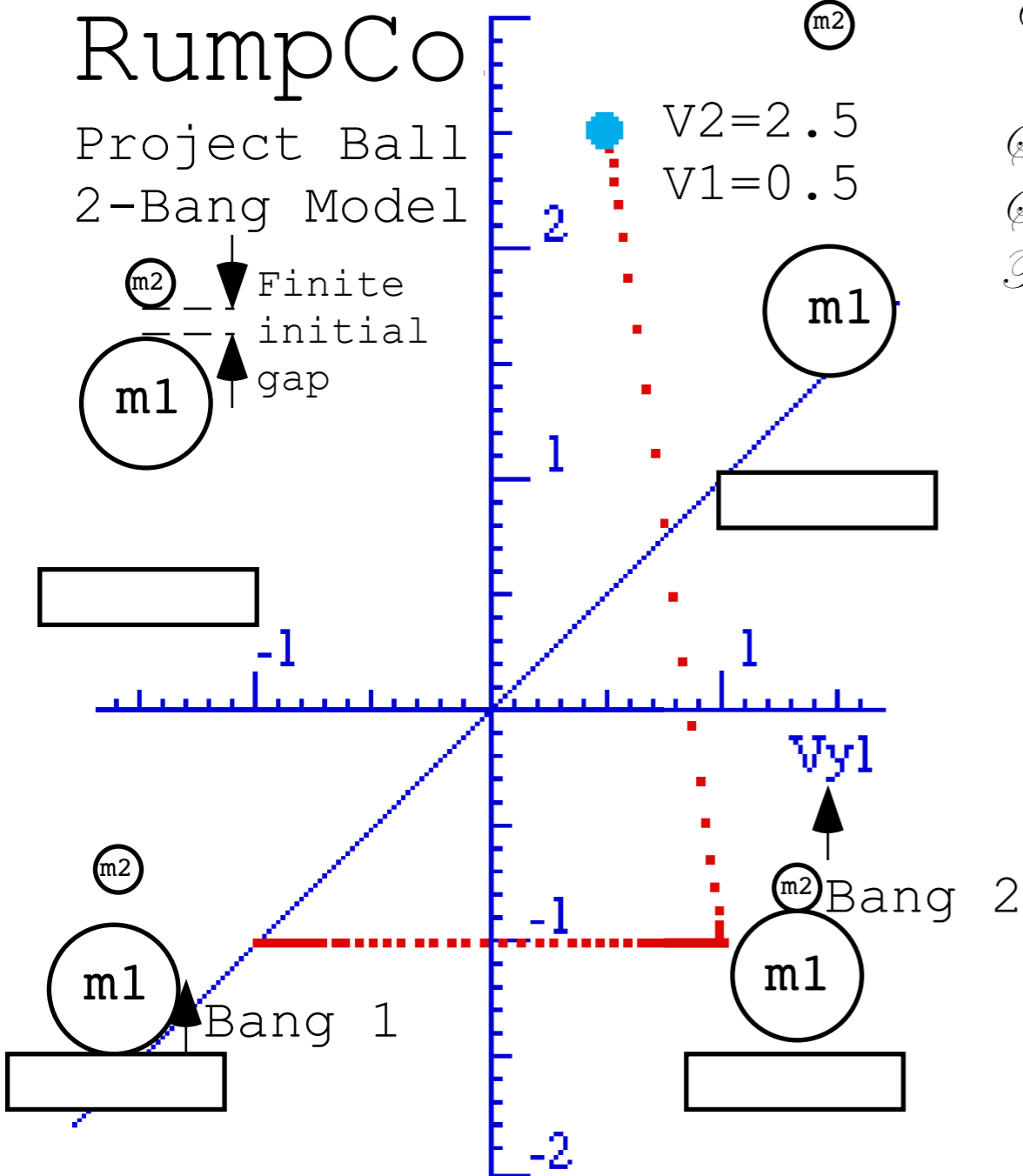
Cheap&practical "seat-of-the pants" approach



*Velocity amplification
or "throw" factor = 2.5*

Parable allegory for Los Alamos

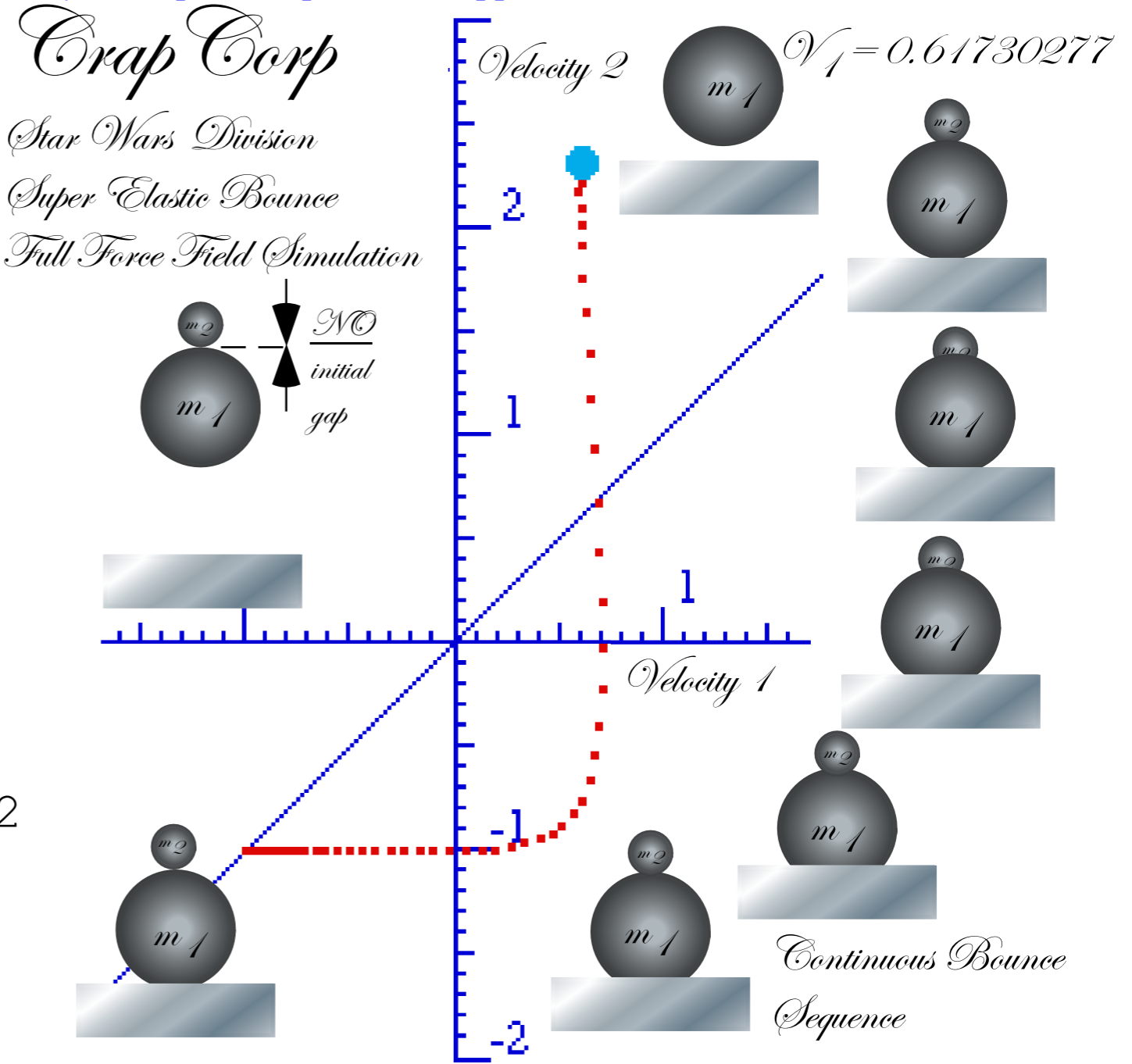
Cheap&practical "seat-of-the pants" approach



*Velocity amplification
or "throw" factor = 2.5*

Parable allegory for Livermore

Fancy&overpriced "political" approach



*Velocity amplification
or "throw" factor = 2.3
(about equal to RumpCo
finite gap experiment)*

Unit 1
Fig. 7.6

Bouncelt Simulation: Zero Gap
(Force power=4)

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths *Sets gravity*
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Number of masses
 2 Balls

Acceleration of gravity
 0 100x{cm/s^2}

Draw force vectors

Pause (once) at top

Constrain motion to Y-axis

Plot v2 vs v1

Plot p2 vs p1

Plot V2 vs V1

Plot Ellipses

Plot Bisector Lines

Old Color Scheme

Show right panel information

Show left panel information

Set Initial positions

Collision friction (Viscosity)
 0 x10^ 0 {g}

Initial gap between balls *zero-gap*
 0 x10^ 0 {g}

Force Constant *Usually need to decrease k for p = 1*
 5 x10^ 4 {g}

Force power law exponent
 4 *This is non-linear F=-kx^4 (Set p = 1 for linear F=-kx^1)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0
 0.75

Initial x1 = 1.09

Max x PE plot = 0.5

F-Vector scale = 0.003

Error step = 0.0000

y Max = 7

y Min = 0

T Max = 6

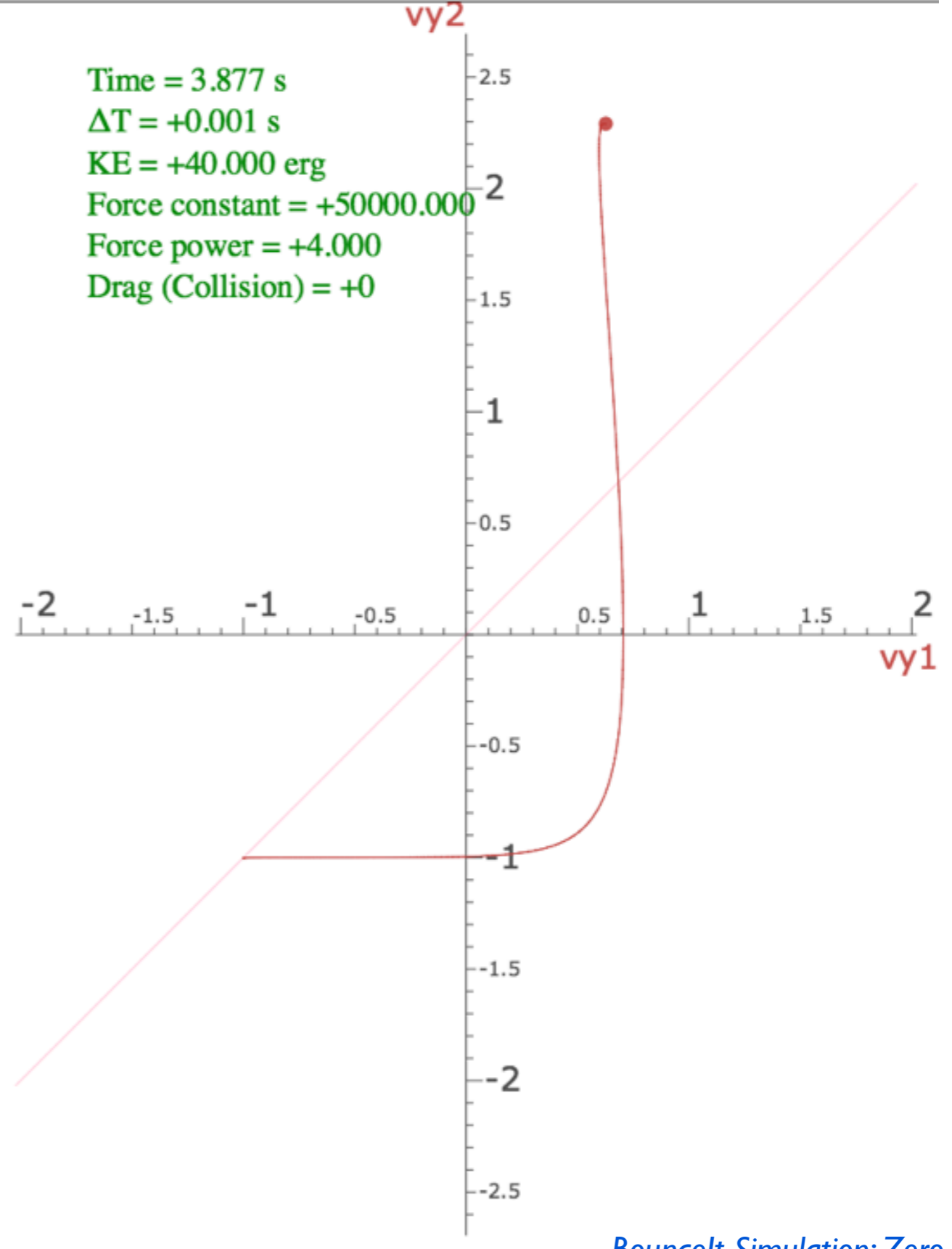
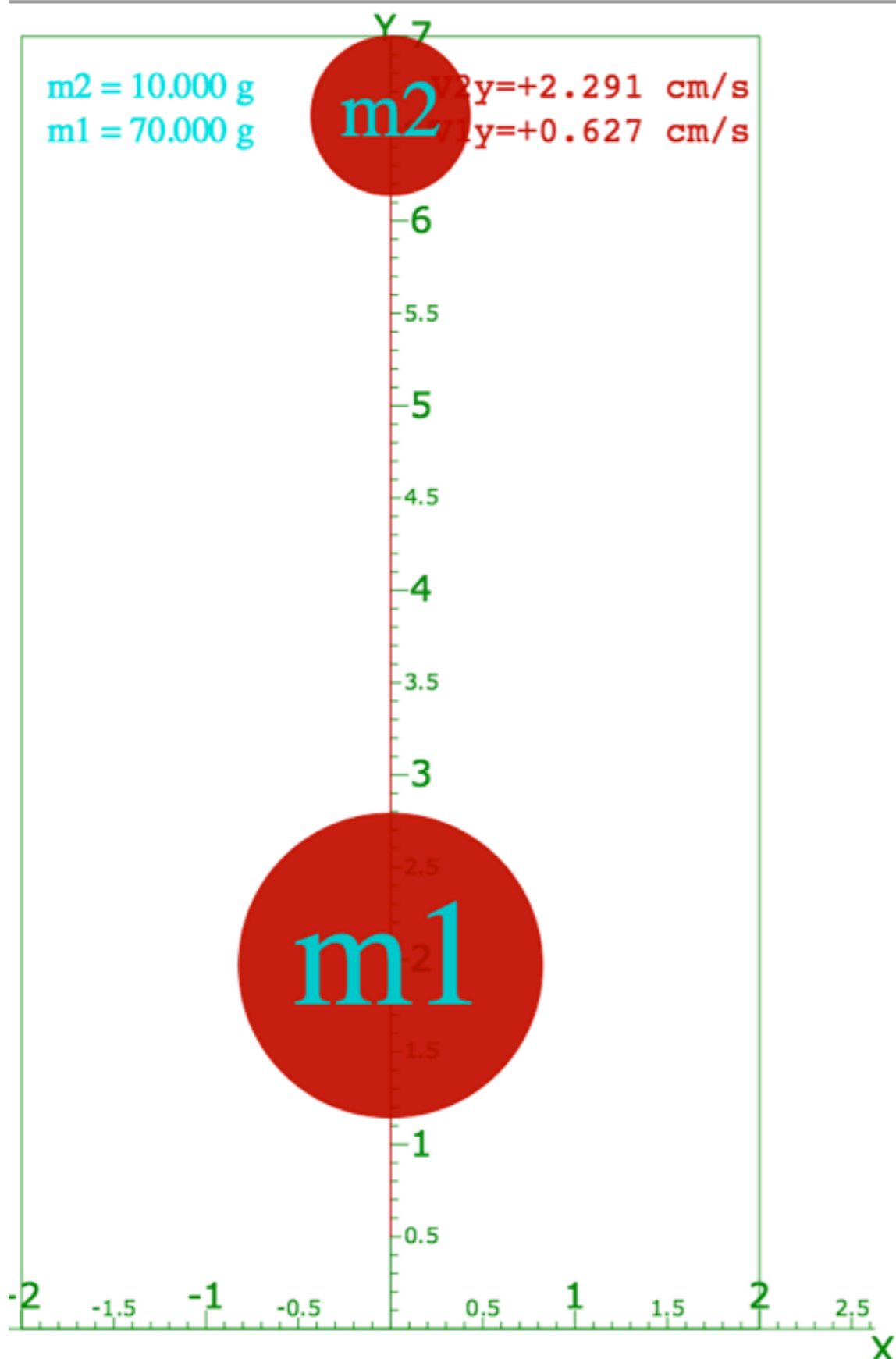
V2y Max = 2.6944

V2y Min = -2.694

Mass factor = 1

M₁ = 7 x10^ 1 {g} V₁ = -1 x10^ 0 {cm/s}

M₂ = 1 x10^ 1 {g} V₂ = -1 x10^ 0 {cm/s}



Bouncelt Simulation: Zero Gap (Force power=4)

Bouncelt Simulation: Zero Gap
(Force power=1)

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

- Number of masses Balls
- Acceleration of gravity 100x{cm/s²}
- Draw force vectors
 - Pause (once) at top
 - Constrain motion to Y-axis
 - Plot v2 vs v1
 - Plot p2 vs p1
 - Plot V2 vs V1
 - Plot Ellipses
 - Plot Bisector Lines
 - Old Color Scheme
 - Show right panel information
 - Show left panel information
 - Set Initial positions

Collision friction (Viscosity) x10[^] {g}

Initial gap between balls *zero-gap* x10[^] {g}

Force Constant *Usually need to increase k for p > 1* x10[^] {g}

Force power law exponent *This is linear F=-kx¹ (increase p > 1 for non-linear F=-kx^p)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

Initial x1 = y Max =

Max x PE plot = y Min =

F-Vector scale = T Max =

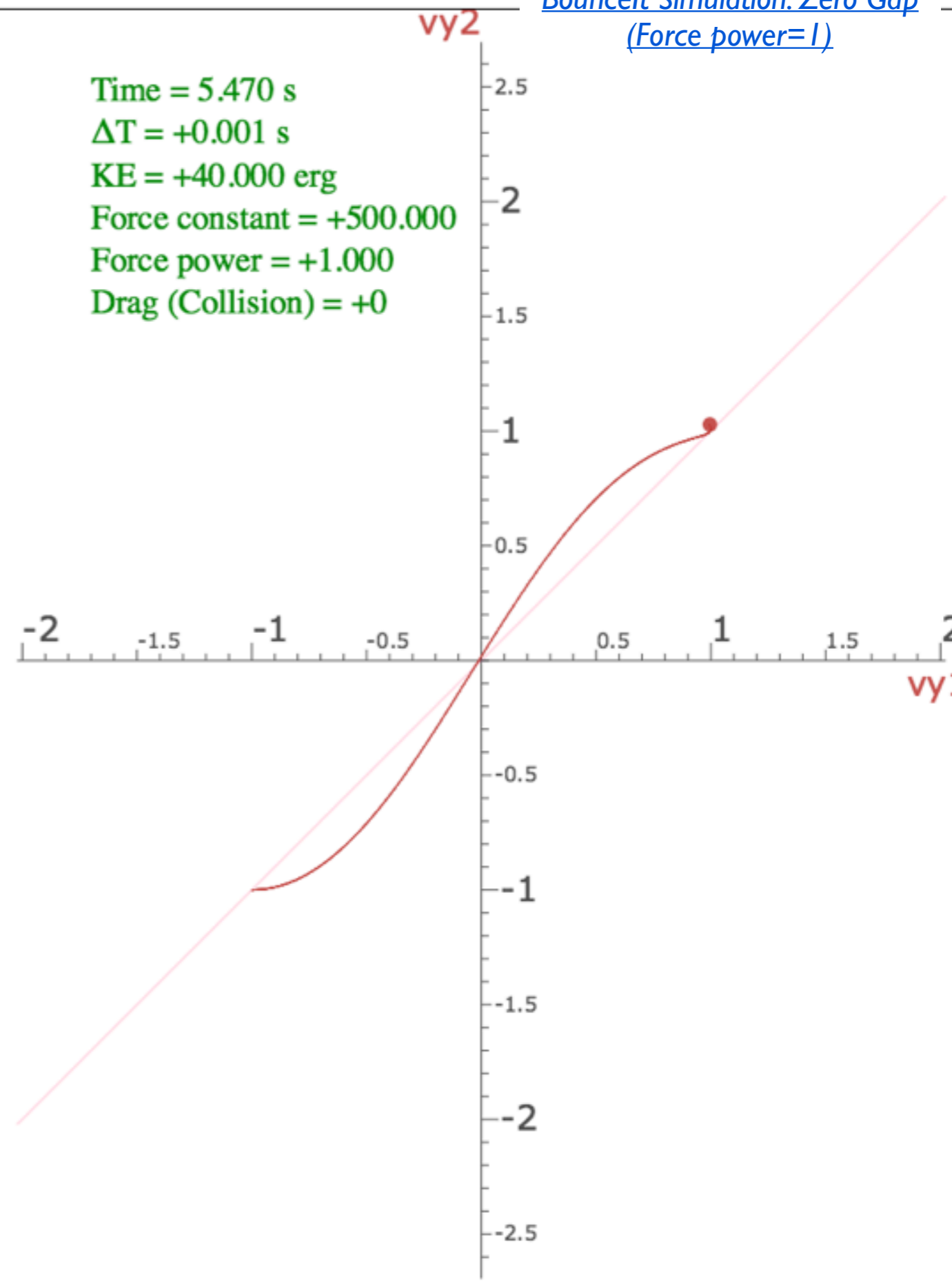
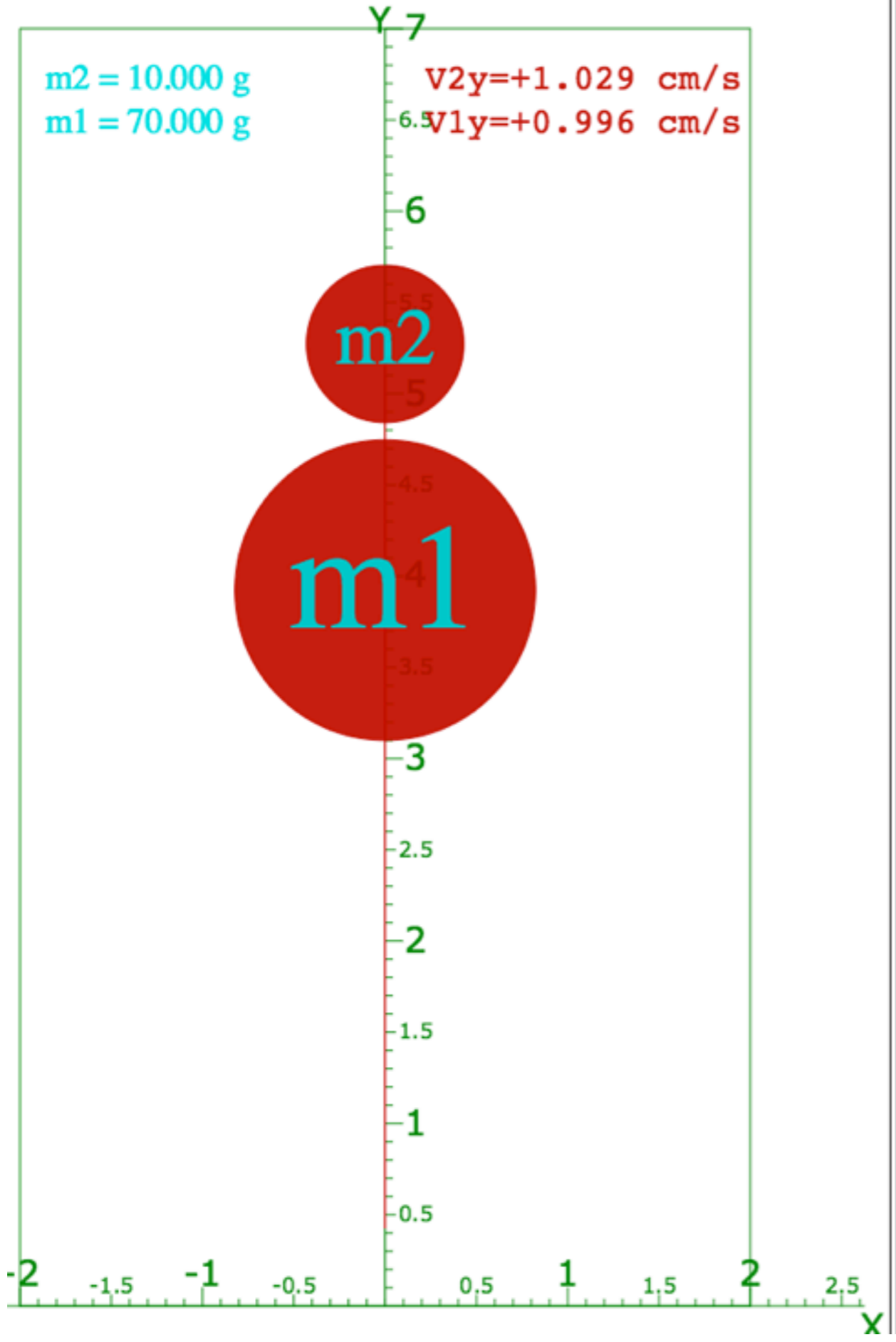
Error step = V2y Max =

V2y Min =

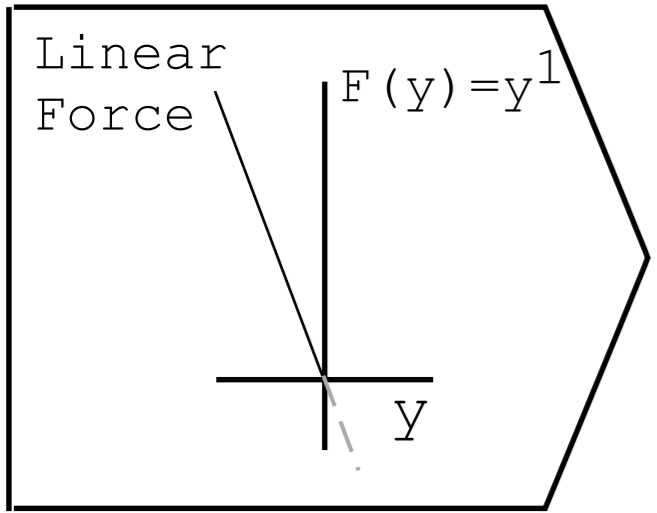
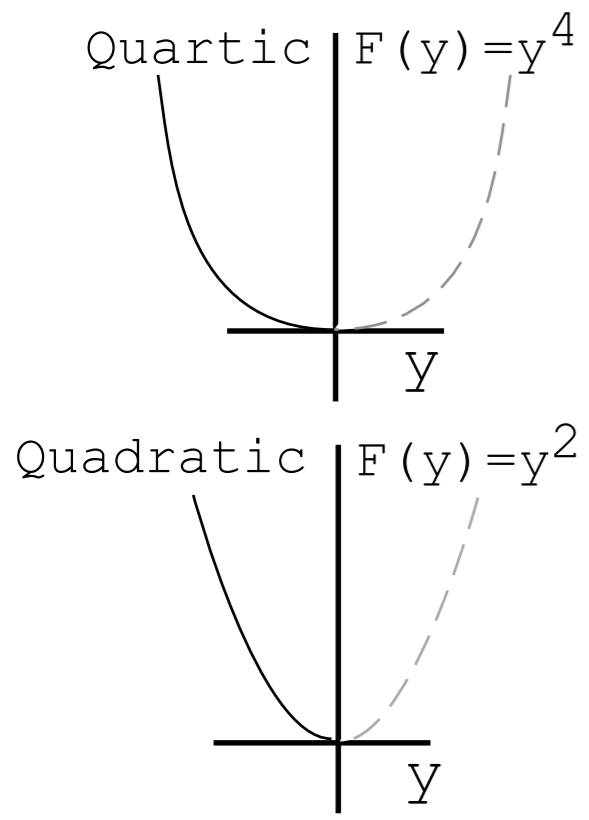
Mass factor =

M₁ = x10[^] {g} V₁ = x10[^] {cm/s}

M₂ = x10[^] {g} V₂ = x10[^] {cm/s}



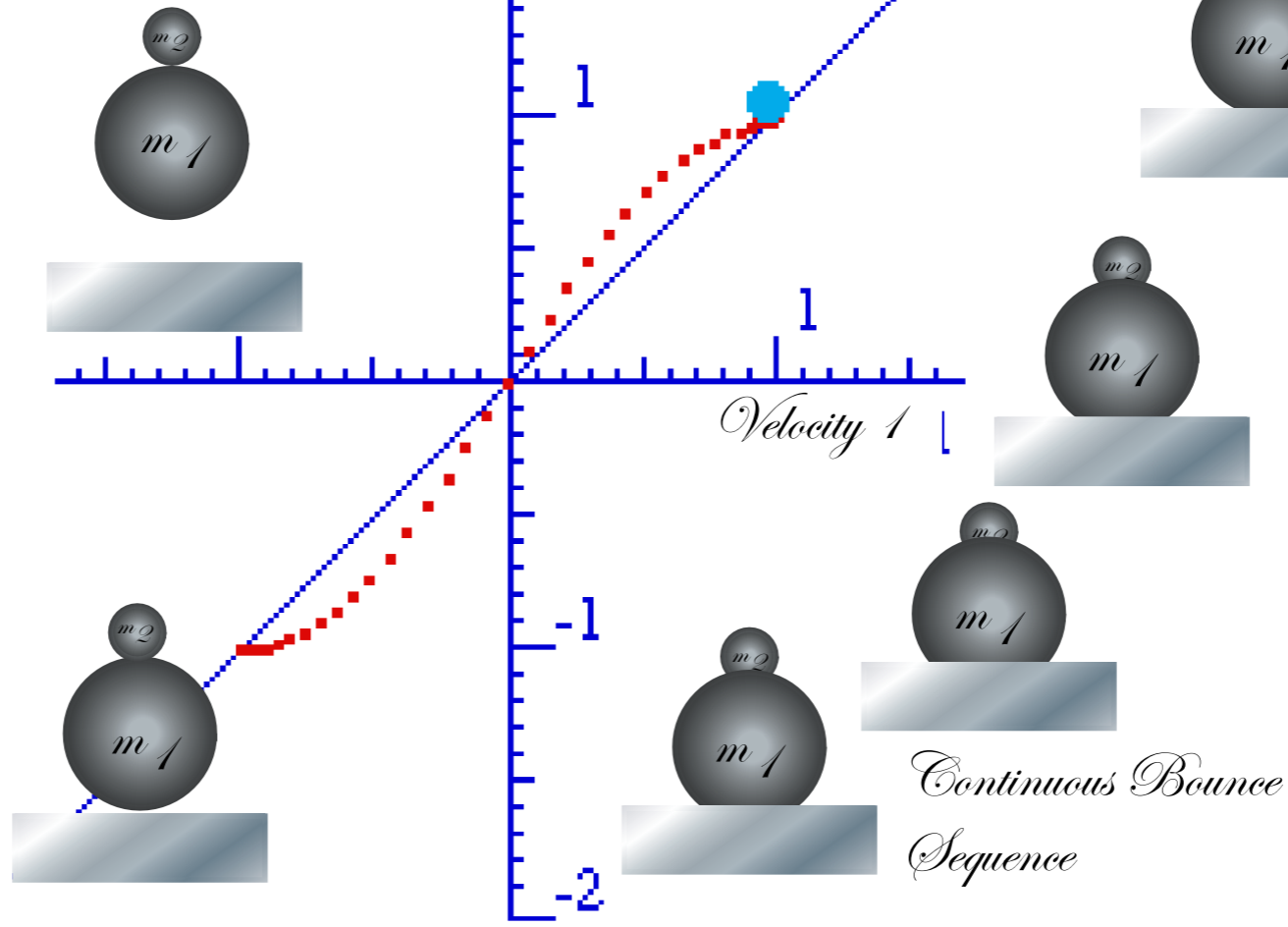
Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"



Cra Rumpny Ltd 3

Linear Force Field Simulation

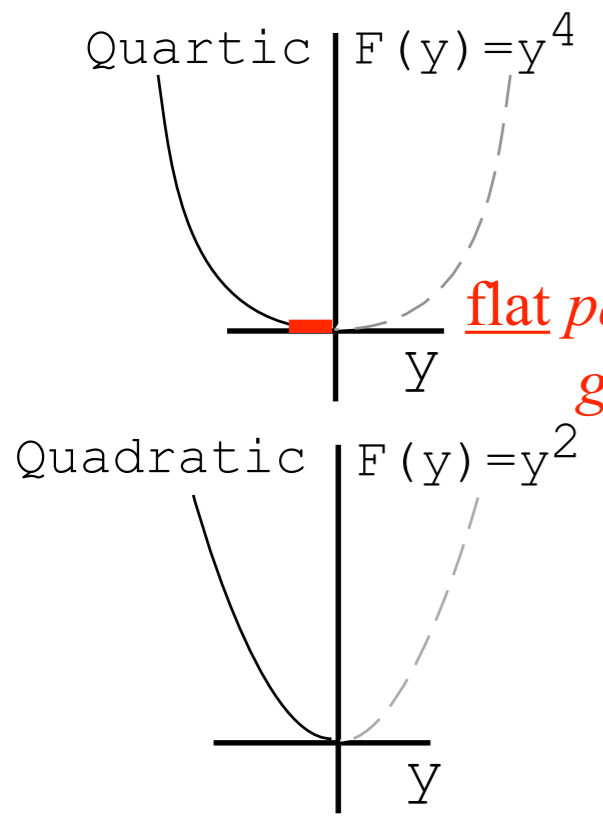
Quite surprising "non-effect" !
Why?



$V_2 = 1.03$
 $V_1 = 0.996$

Unit 1
Fig. 7.7

Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

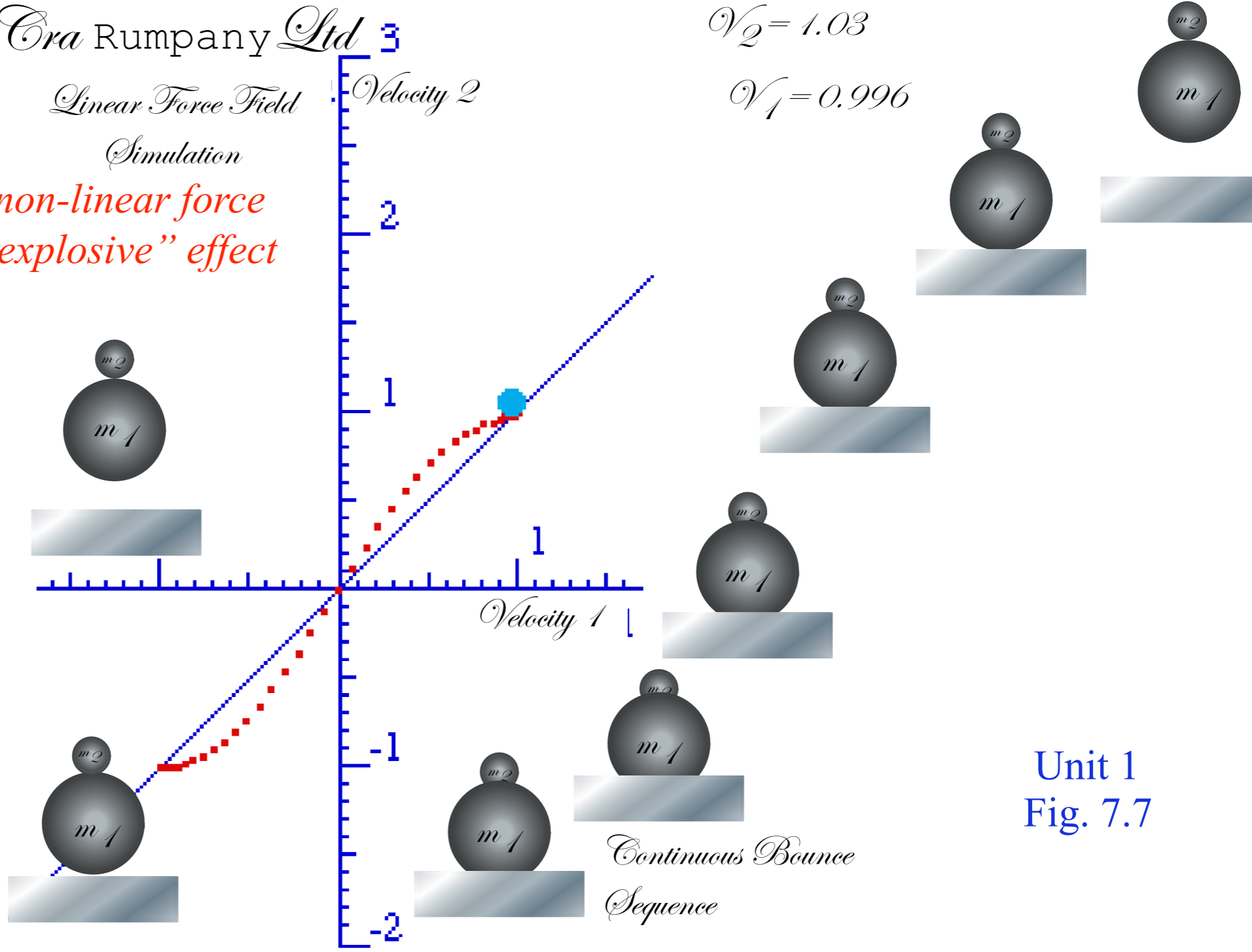
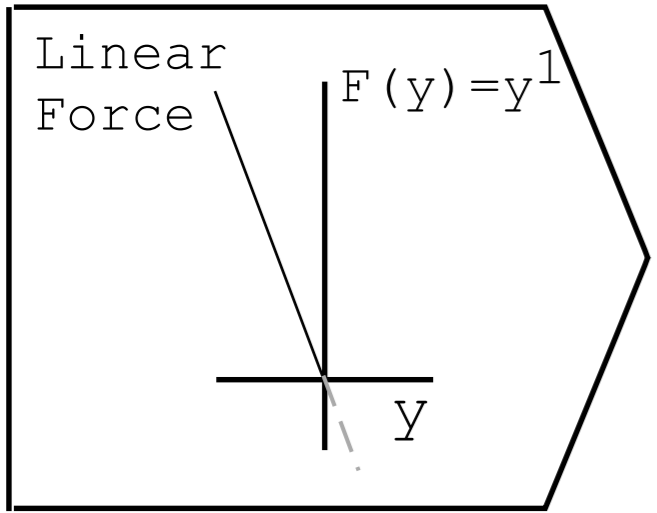


flat part of non-linear force gives "explosive" effect

Cra Rumpany Ltd 3

Linear Force Field Simulation

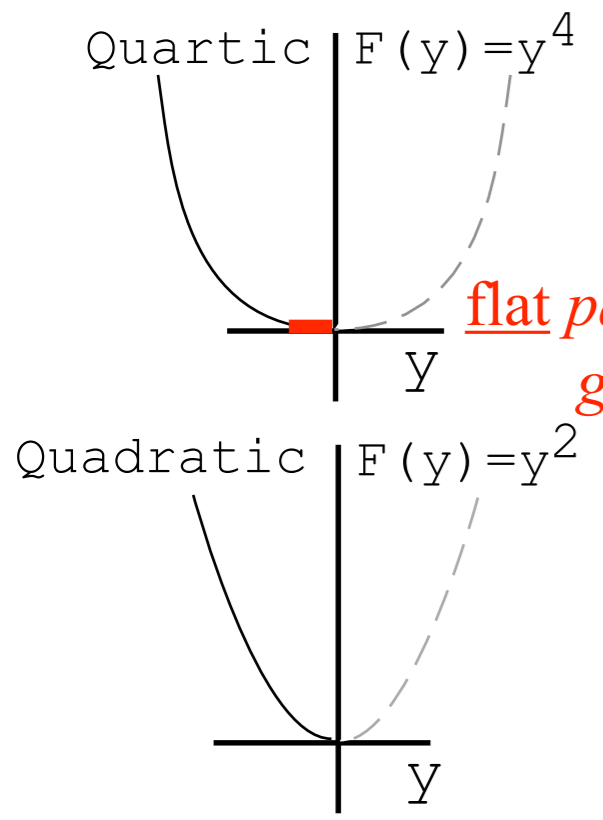
$V_2 = 1.03$
 $V_1 = 0.996$



Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force $F(y) = ky$

Unit 1
Fig. 7.7

Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

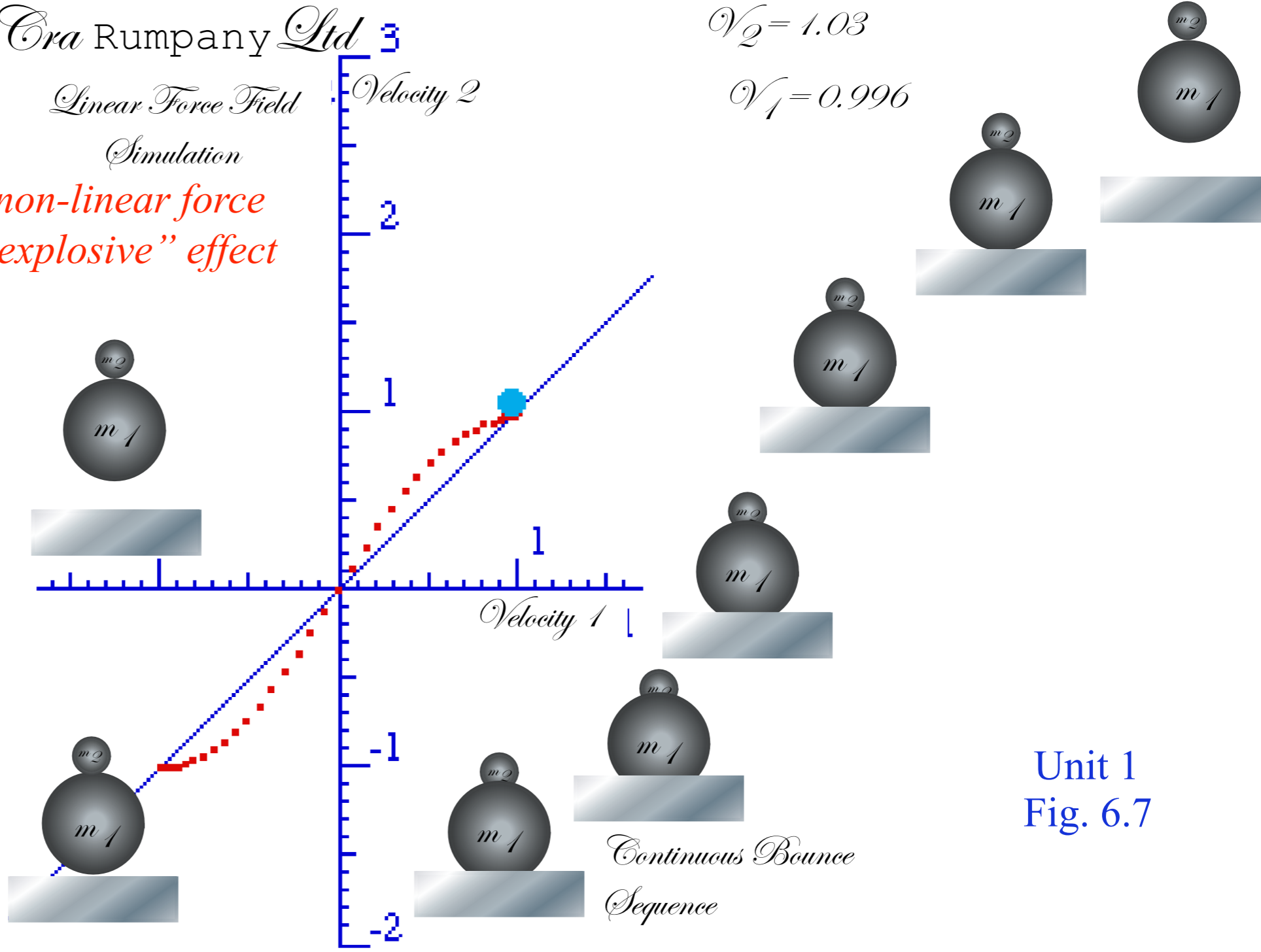
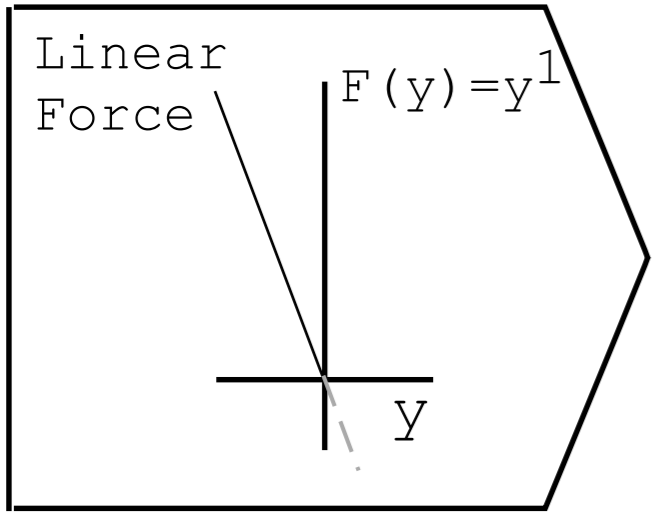


flat part of non-linear force gives "explosive" effect

Cra Rumpany Ltd 3

Linear Force Field Simulation

$V_2 = 1.03$
 $V_1 = 0.996$

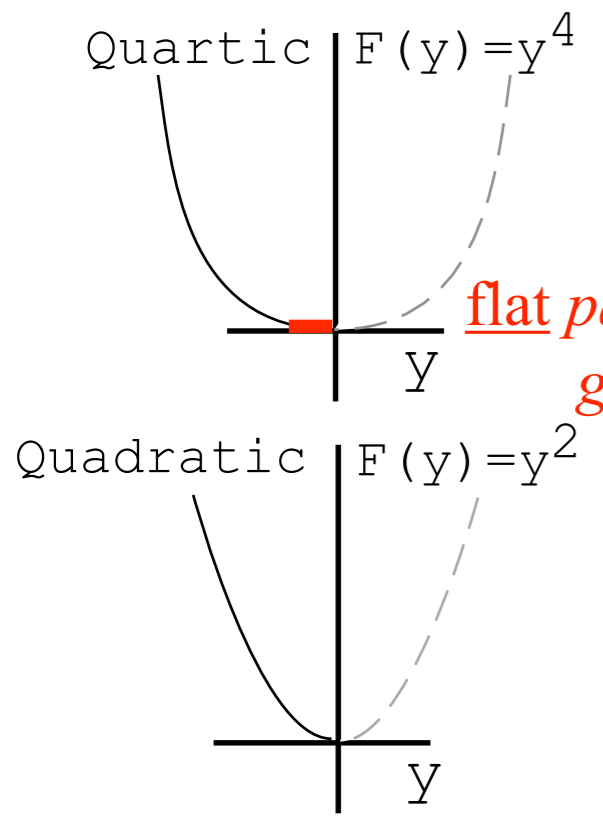


Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force $F(y) = ky$

Lesson: Fasten your seatbelt

Unit 1
Fig. 6.7

Cooperation between *Los Alamos* and *Livermore* yields insight to answer "What's going on?"

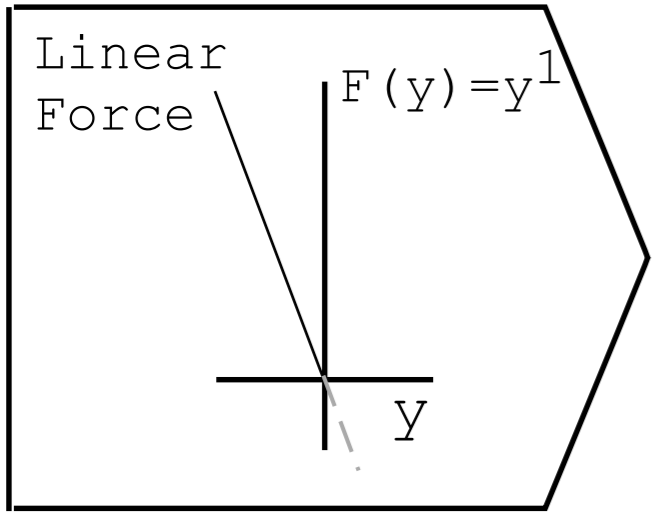
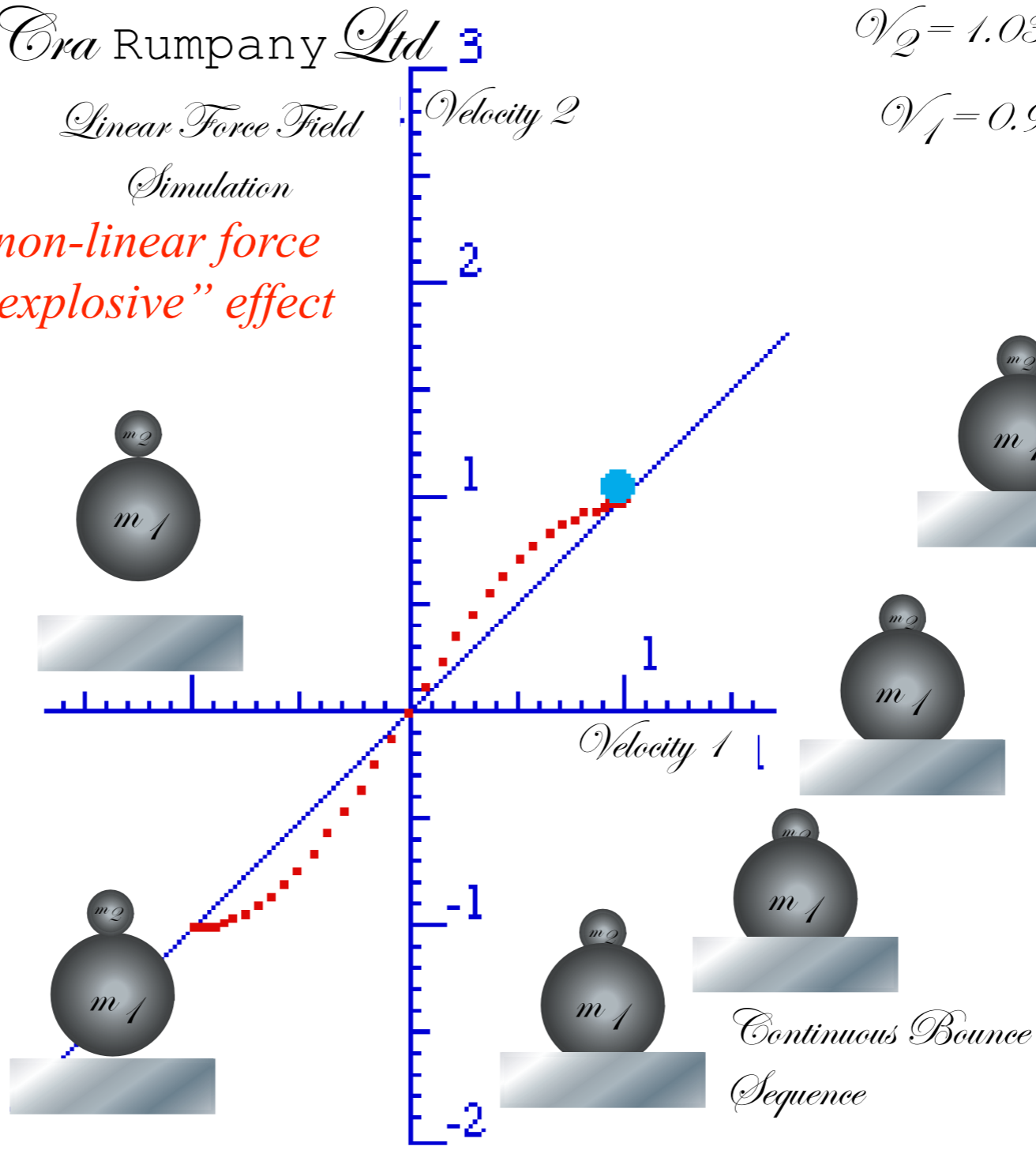


flat part of non-linear force gives "explosive" effect

Cra Rumpany Ltd 3

Linear Force Field Simulation

$V_2 = 1.03$
 $V_1 = 0.996$



Unit 1
Fig. 6.7

Velocity amplification or "throw" factor = 1.03 (practically "no-throw") for linear force $F(y) = ky$

Lesson: Fasten your seatbelt TIGHTLY!

Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

(Simulations)

(b) Linear force $F=-kx$ (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

→ A story of USC pre-meds visiting Whammo Manufacturing Co.

(Leads to Sagittal

Geometry and dynamics of n-ball bounces

potential analysis of

2, 3, and 4 body towers)

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric m_1, m_2, m_3, \dots series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton’s cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Velocity Amplification in Collision Experiments Involving Superballs <Link>

CLASS OF WILLIAM G. HARTER*
 University of Southern California
 Los Angeles, California 90007

(Received 25 September 1969; revised 25 September 1970)

If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.

* The members of the class of Dr. William G. Harter included: Calvin W. Gray, Jr., Robert C. Frickman, Brian P. Harney, Steven H. Hendrickson, Scott T. Jacks, David F. Judy, William D. Koltun, Sam C. Kaplan, Morton J. Kern, Edmund H. Kwan, Wayne E. Long, Michael E. Mason, William D. Moore, Willard W. Mosier, Gary P. Rudolf, Henry G. Rosenthal, William F. Skinner, Jay L. Stearn, Michael Weinberg, Mark Weiner, Frank J. Wilkinson, and David Willner.

ACKNOWLEDGMENT

We would like to thank John C. Fakan, John E. Heighway, and John H. Marburger for help during the initial and final stages of this project.

INTRODUCTION

¹ Trade name of product by Whammo Manufacturing Co., San Gabriel, Calif.

Shortly after the well-known Superball¹ appeared on the market, one of the authors quite accidentally discovered a surprising effect.² The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

Class of W. G. Harter

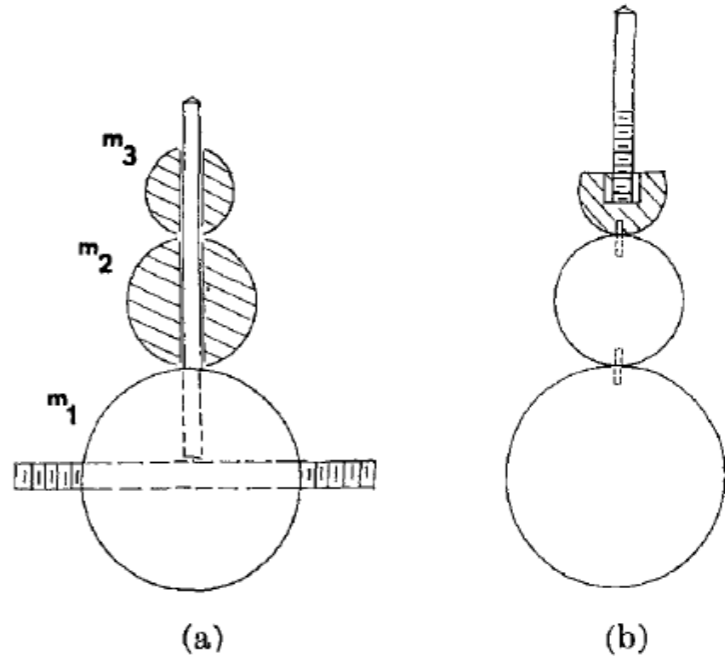
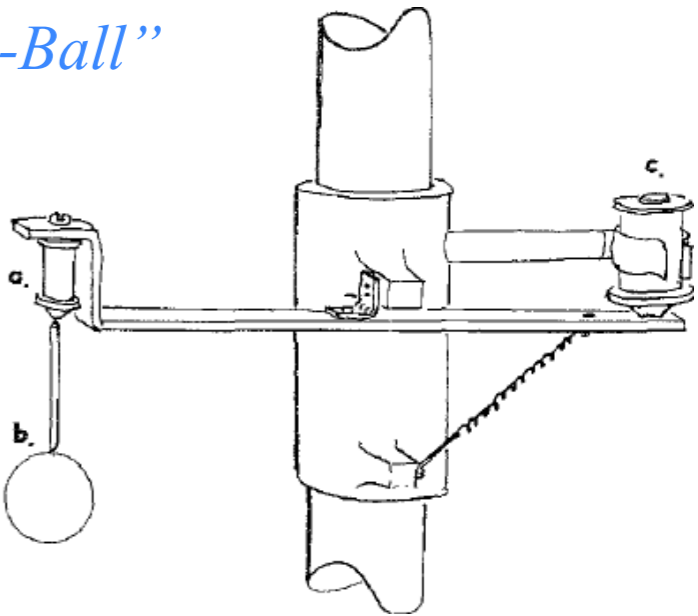


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

Velocity Amplification in Collision Experiments Involving Superballs

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(Received 25 September 1969; revised 25 September 1970)

If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.

* The members of the class of Dr. William G. Harter included: Calvin W. Gray, Jr., Robert C. Frickman, Brian P. Harney, Steven H. Hendrickson, Scott T. Jacks, David F. Judy, William D. Koltun, Sam C. Kaplan, Morton J. Kern, Edmund H. Kwan, Wayne E. Long, Michael E. Mason, William D. Moore, Willard W. Mosier, Gary P. Rudolf, Henry G. Rosenthal, William F. Skinner, Jay L. Stearn, Michael Weinberg, Mark Weiner, Frank J. Wilkinson, and David Willner.

ACKNOWLEDGMENT

We would like to thank John C. Fakan, John E. Heighway, and John H. Marburger for help during the initial and final stages of this project.

*Much later....
Lots of profs try this out...
...including the unfortunate Harvard professor M. Tinkham...*

(Still trying to find the video of the Tinkham incident...)

INTRODUCTION

Shortly after the well-known Superball¹ appeared on the market, one of the authors quite accidentally discovered a surprising effect.² The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

¹ Trade name of product by Whammo Manufacturing Co., San Gabriel, Calif.

Class of W. G. Harter

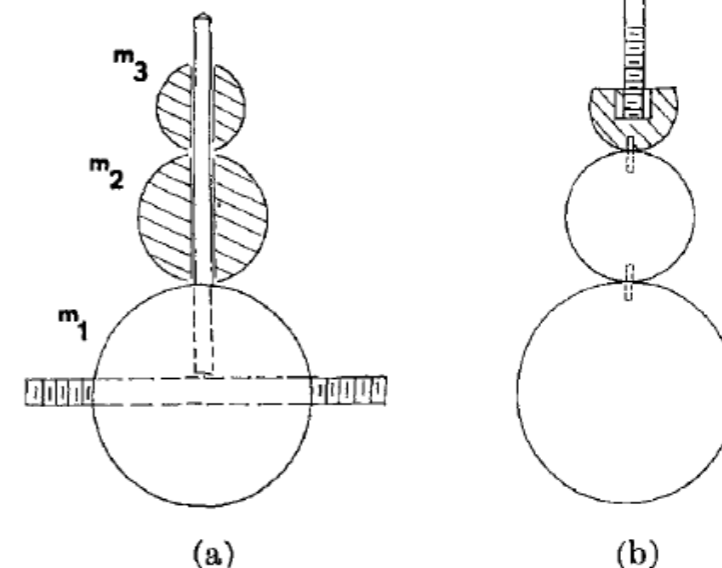


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

Basketball and Tennis Ball

Dropping a tennis ball on top of a basketball causes the tennis ball to bounce very high.

Source: [8.01 Physics I: Classical Mechanics, Fall 1999](#)
Prof. Walter Lewin

Course Material Related to This Topic:

- Watch [video clip from Lecture 17 \(21:30 - 24:08\)](#)

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of “Project-Ball”

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Somebody drops a box of balls that immediately bounce into the wet paint.

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Little paint spots on floor show what was wrong with our fancy-pants computer theory.

The engineering curves were isothermal not adiabatic.

Need latter. Can do latter by dropping dyed balls and measuring spot-size.

Measuring spot-size d gives energy vs. height.

Slope of $E(x)$ gives force $F(x)$ and $G(x)$.

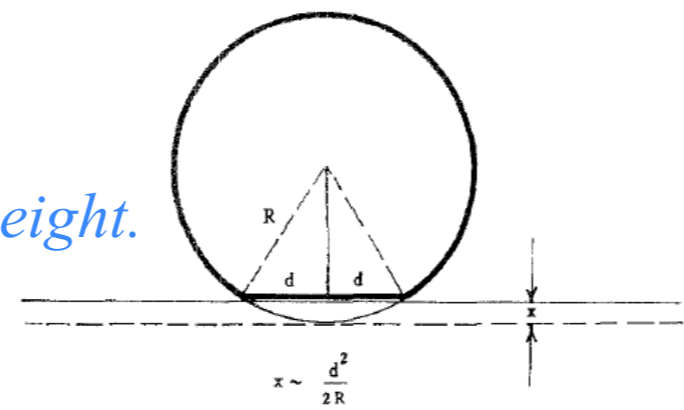
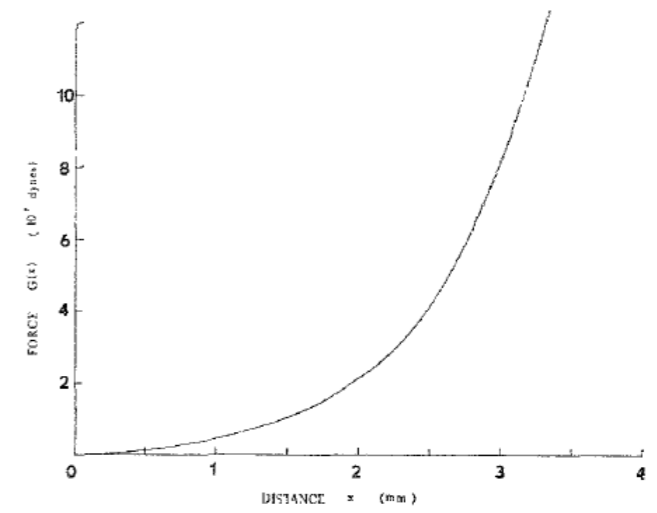


FIG. 10. Sagittal formula.

Collisions Involving Superballs



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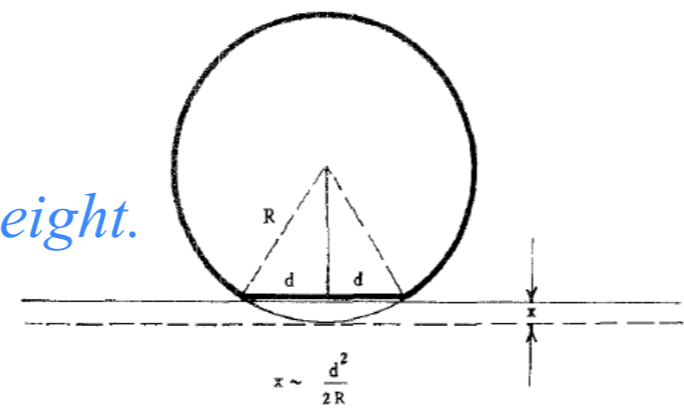


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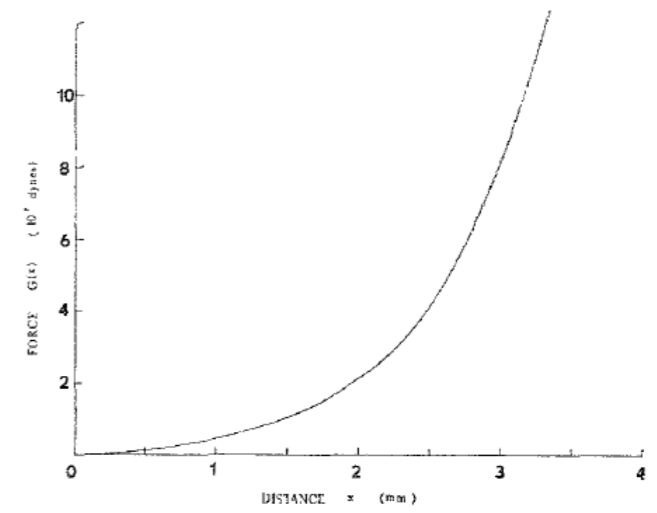


FIG. 12. Adiabatic force function $G(x)$.

If $F(x)$ and $G(x)$ were linear for all x , then the

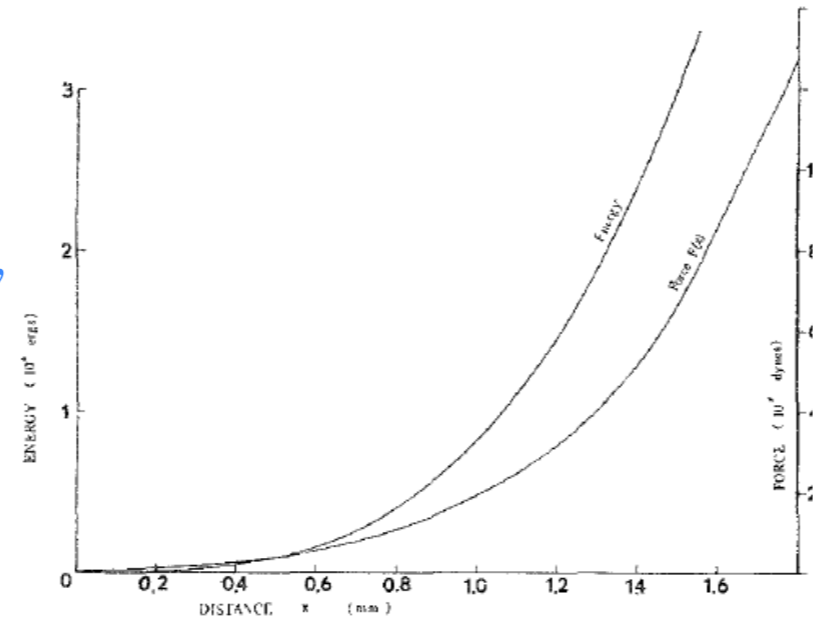


FIG. 11. Adiabatic force $F(x)$ and energy curves for Superball.

Then fancy-pants computer theory can predict N-ball tower bounce

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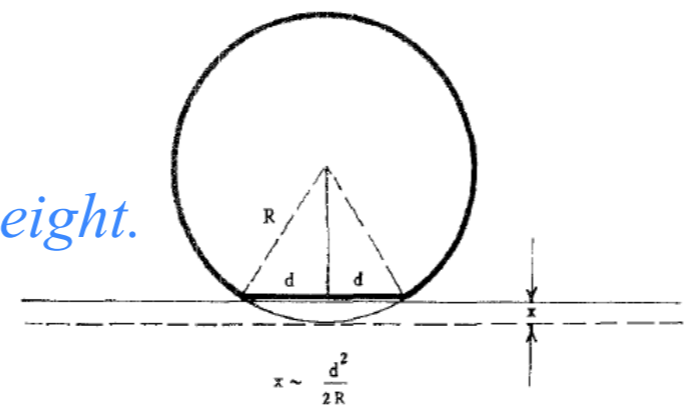


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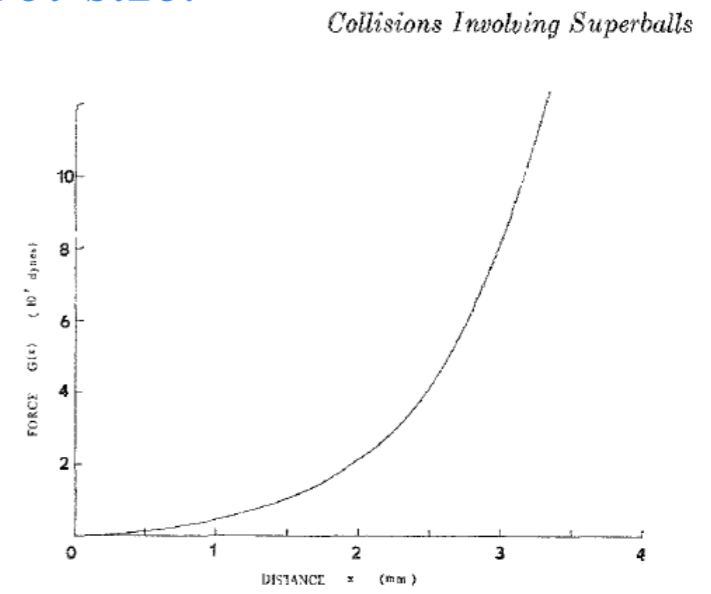


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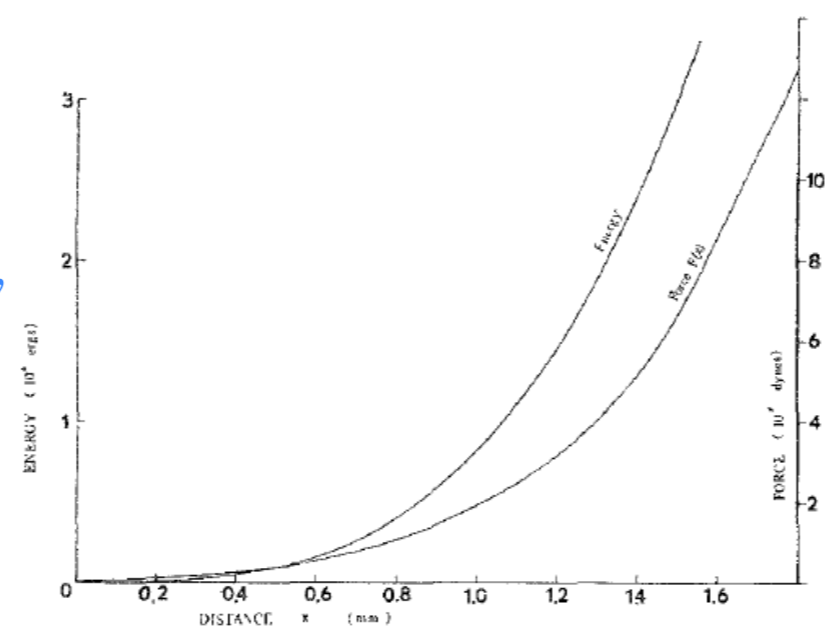


FIG. 11. Adiabatic force $F(x)$ and energy curves for Superball.

Functions $F(x)$ and $G(x)$ were then placed on the function generators of the analog computer.

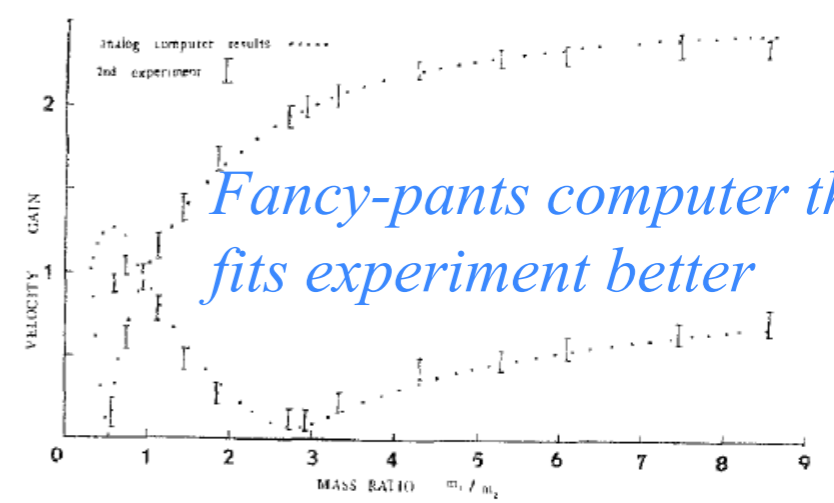


FIG. 13. Comparison between analog computer gain curves and second experiment.

Then fancy-pants computer theory can predict N-ball tower bounce

Then fancy-pants computer theory can predict N-ball tower bounces

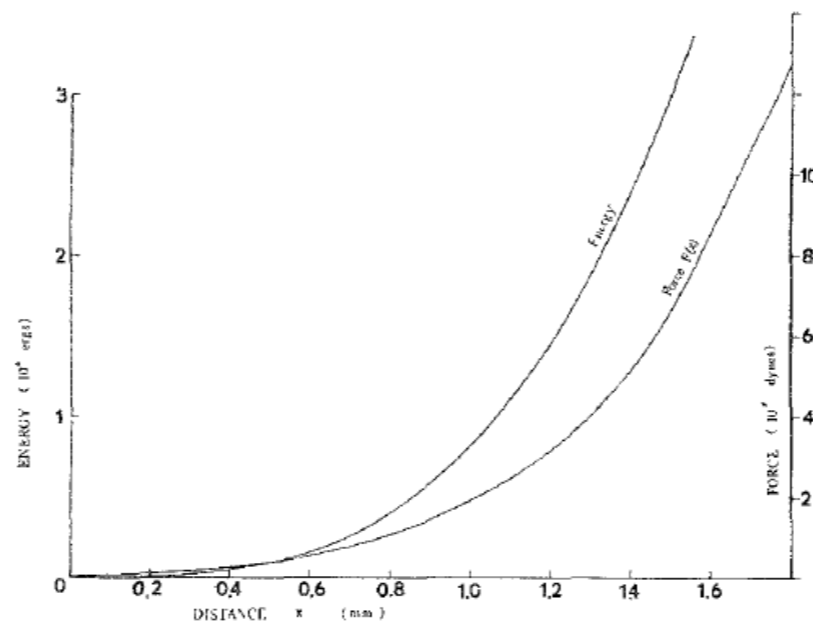
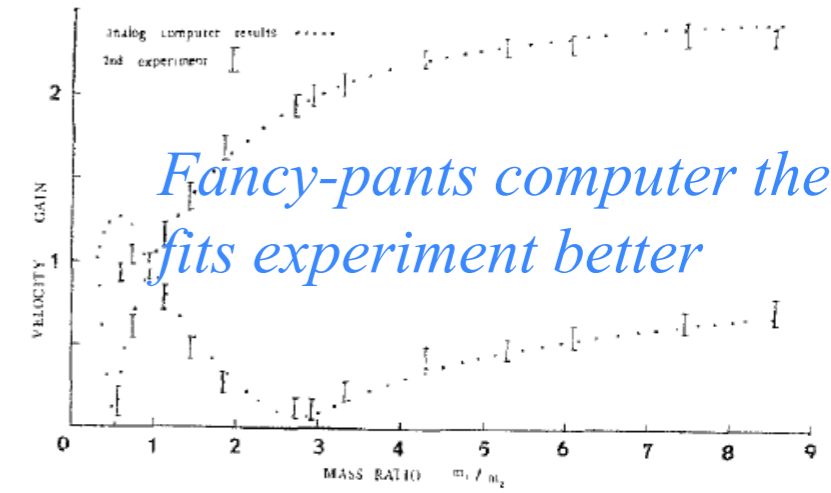


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Fancy-pants computer theory fits experiment better

FIG. 13. Comparison between analog computer gain curves and second experiment.

Here are some 3-ball tower bounce predictions

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Class of W. G. Harter

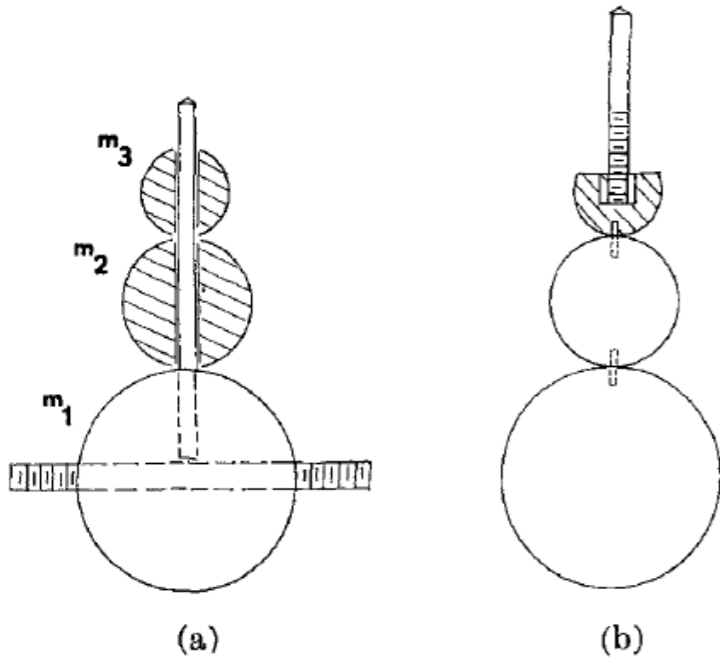


FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

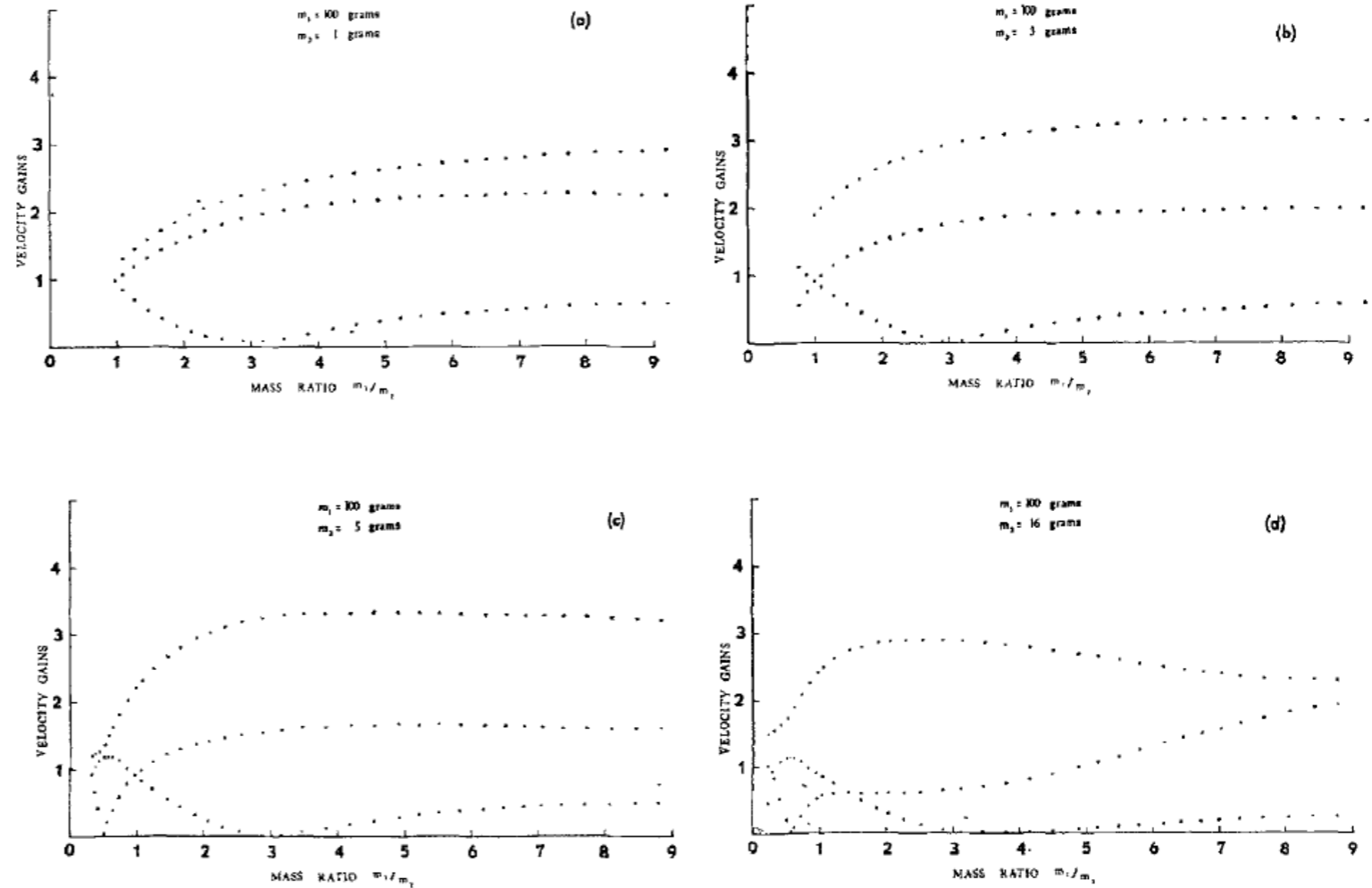


FIG. 15. (a)-(d) Analog computer output for velocity gains of three-ball system.

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Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force $F=-k$ (linear potential $V=kx$)

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(Simulations)

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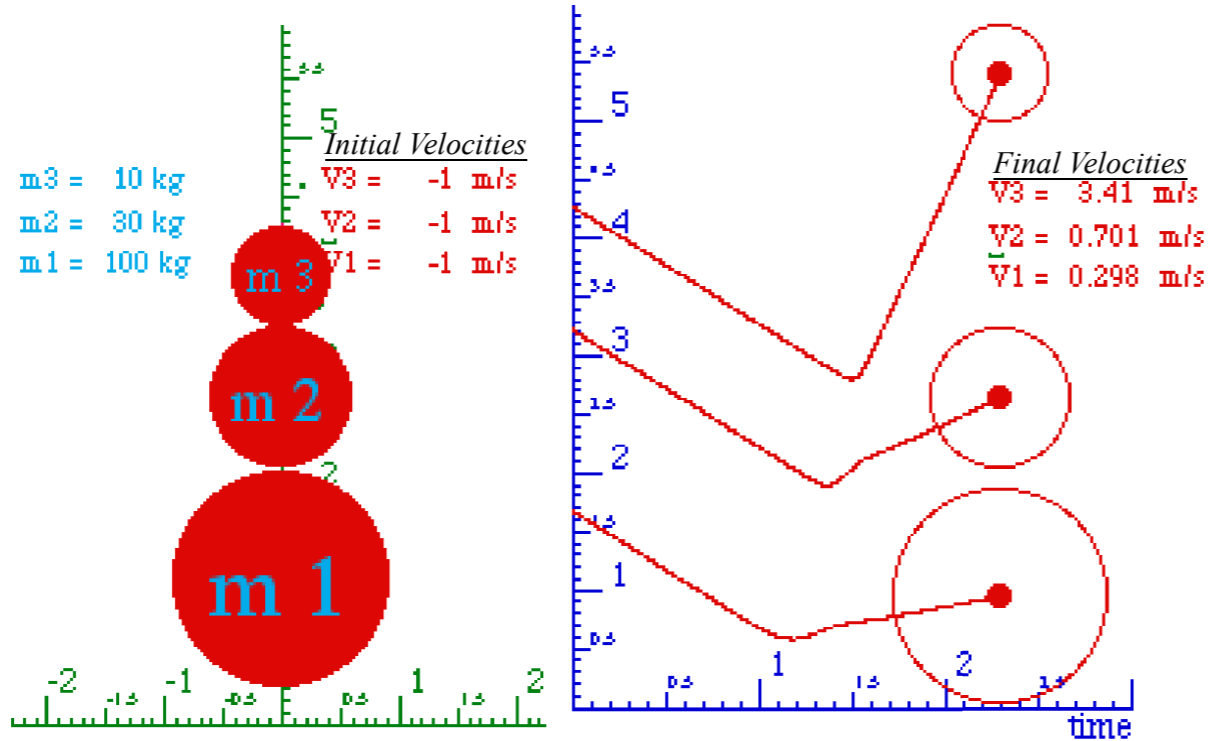
Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

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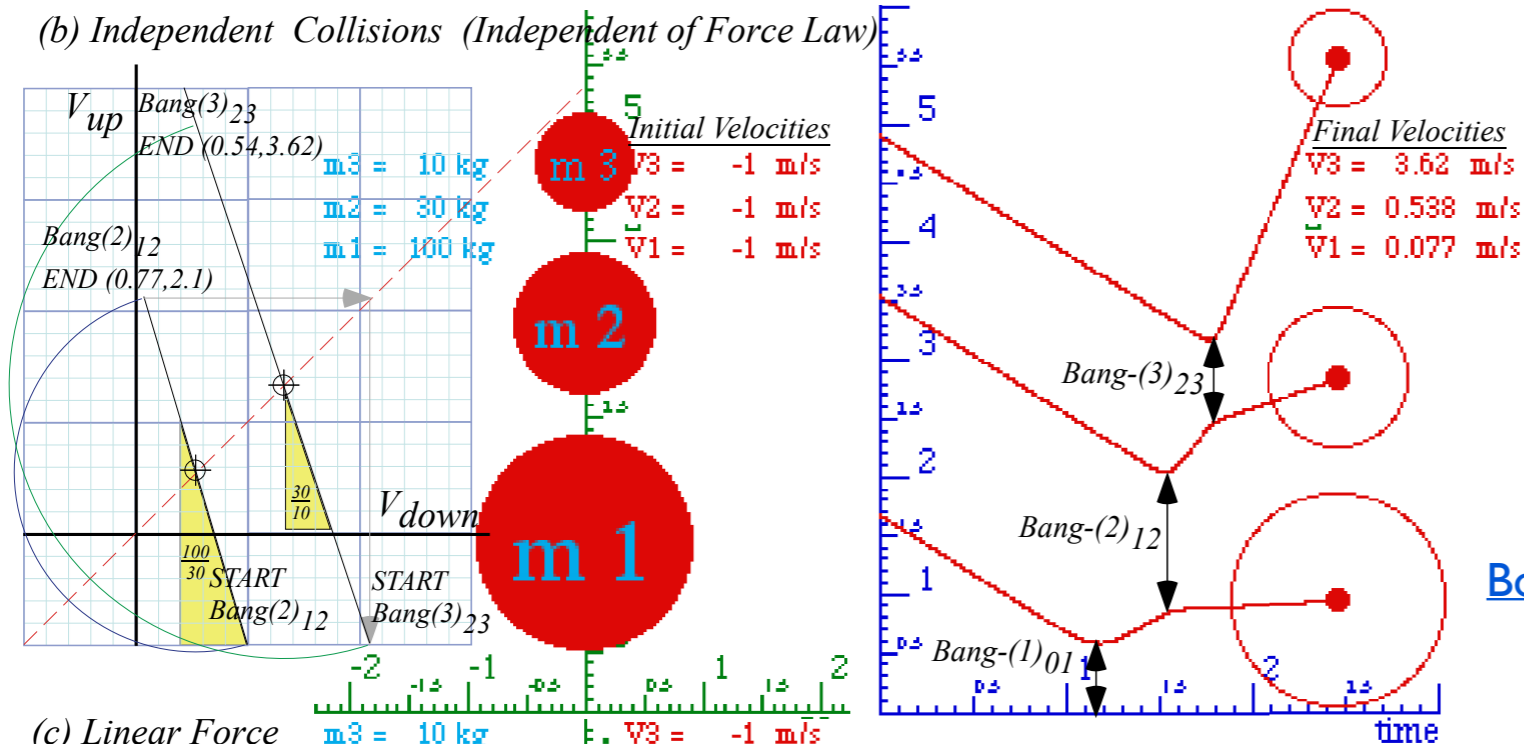
(a) Quartic Force
 $F(y) = k y^4$



Unit 1
 Fig. 8.1a-c
 Independent Bang Model
 (IBM)
 3-Body Geometry

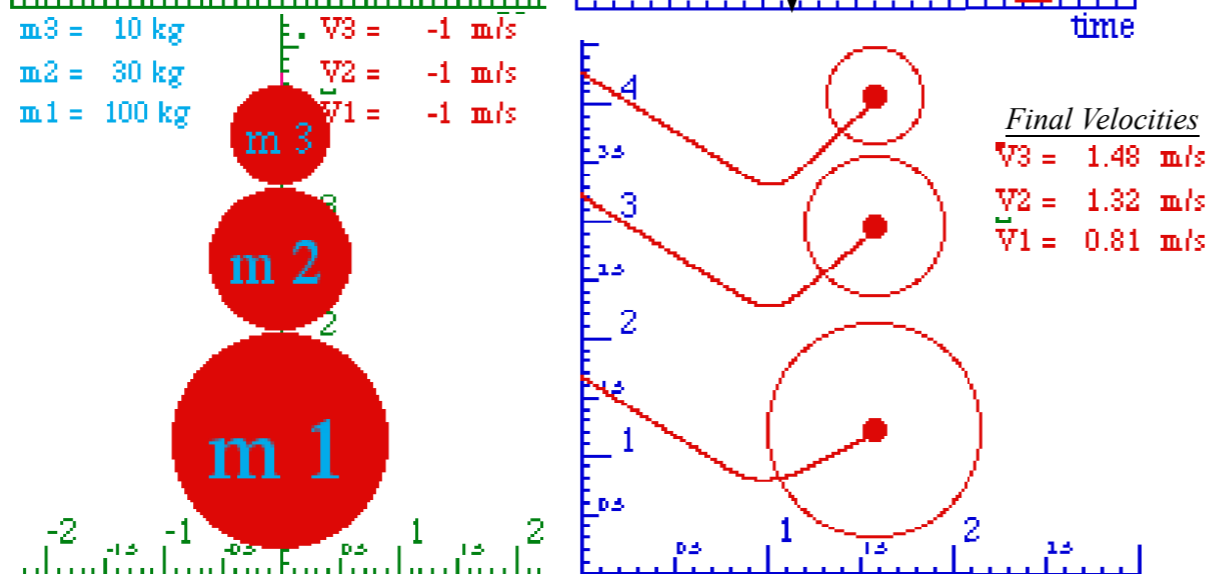
Bouncelt Simulation: 3-Ball Tower w/ Quartic Force

(b) Independent Collisions (Independent of Force Law)



Bouncelt Simulation: 3-Ball (Gapped) Tower w/ Linear Force

(c) Linear Force
 $F(y) = k y$



Bouncelt Simulation: 3-Ball Tower w/ Linear Force

Unit 1
Fig. 8.1b

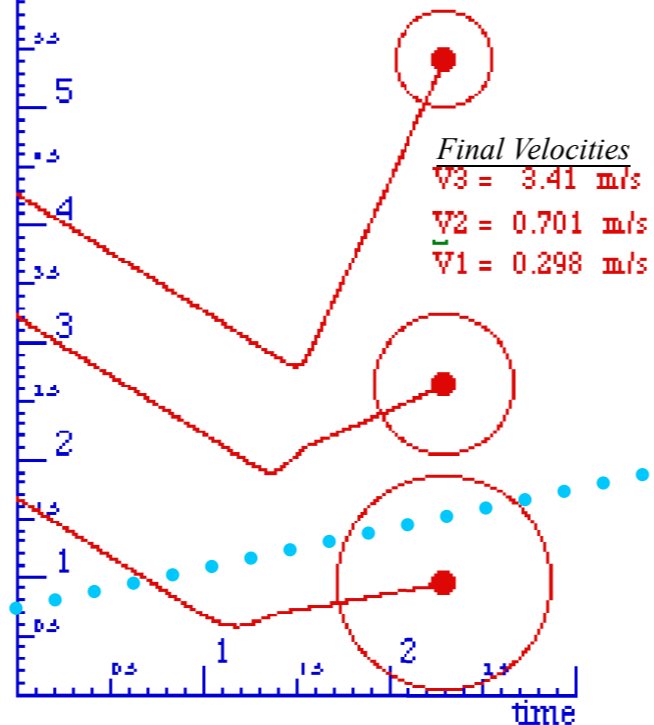
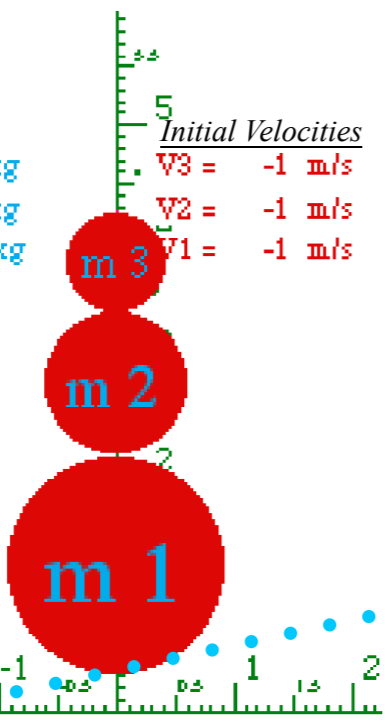
*Independent Bang Model
(IBM)*
3-Body Geometry

(a) *Quartic Force*
 $F(y) = k y^4$

m3 = 10 kg
m2 = 30 kg
m1 = 100 kg

Initial Velocities
V3 = -1 m/s
V2 = -1 m/s
V1 = -1 m/s

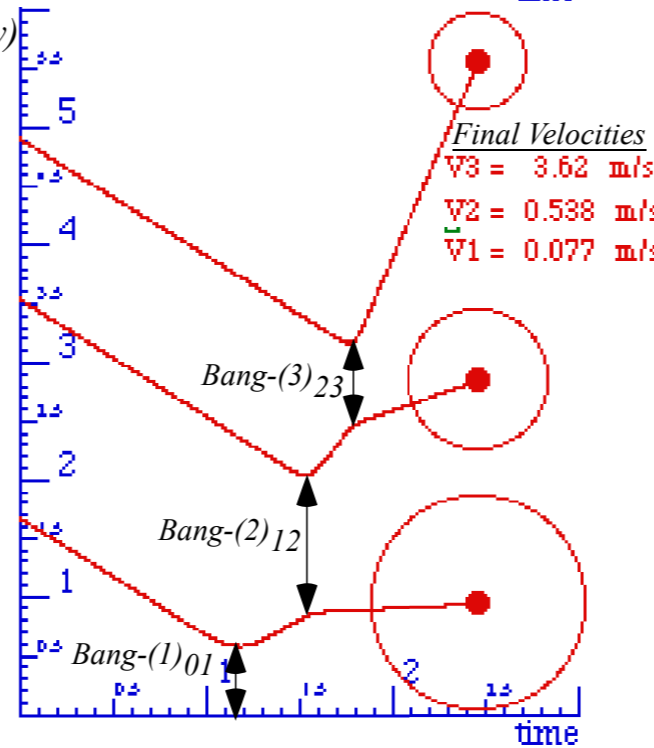
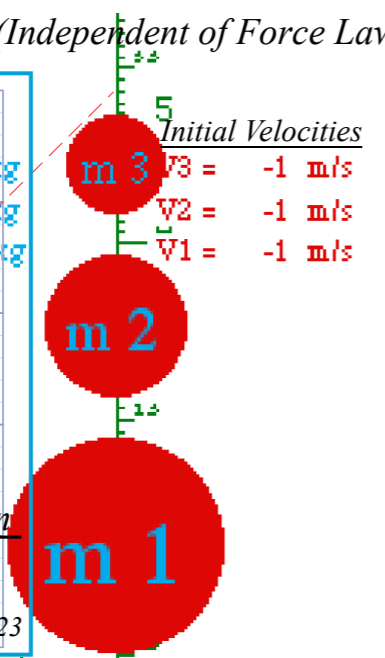
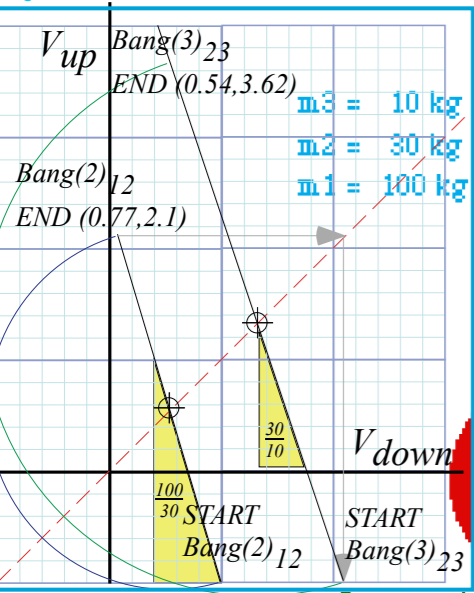
Bouncelt Simulation:
3-Ball Tower
w/ Quartic Force



(b) *Independent Collisions (Independent of Force Law)*

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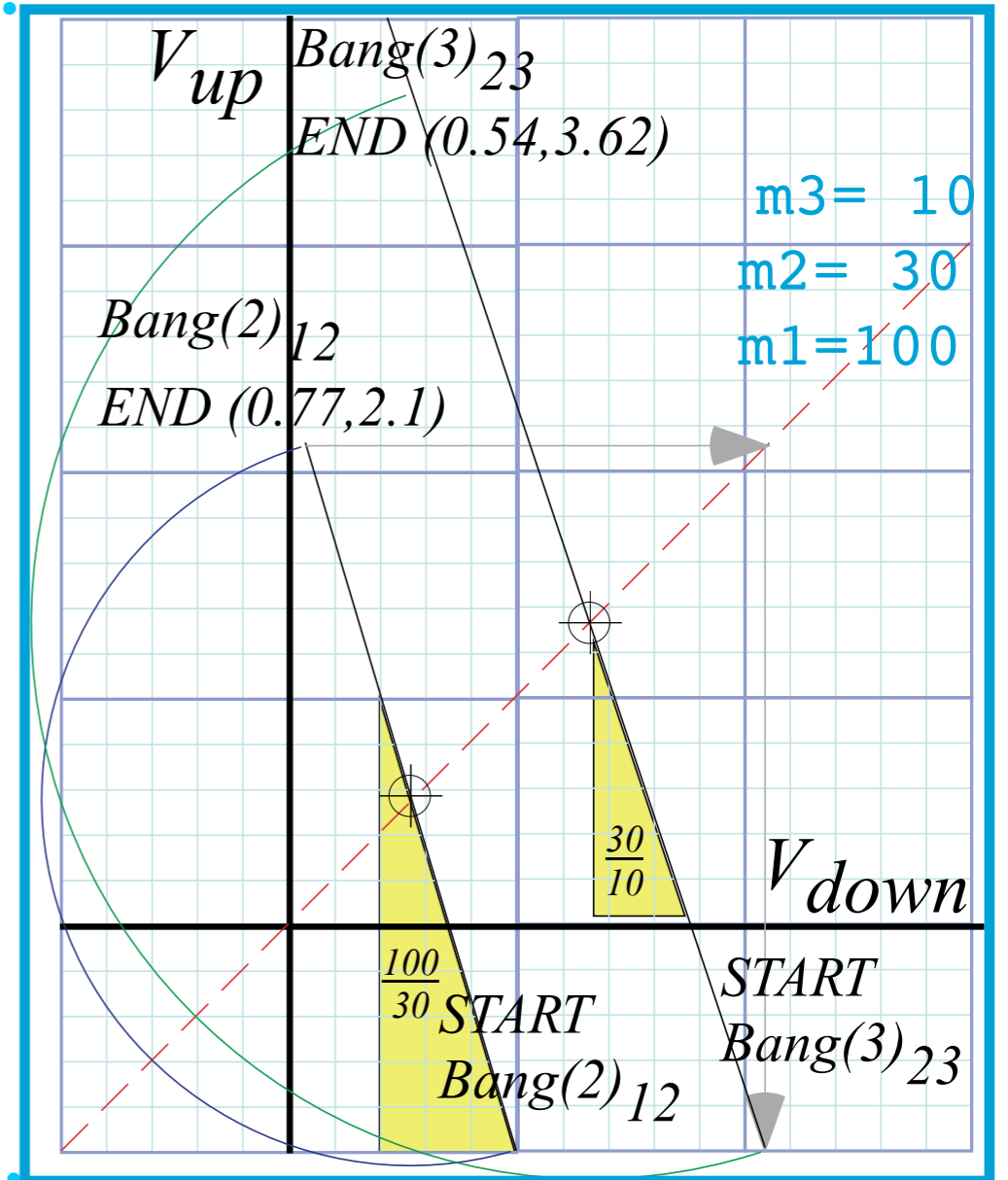
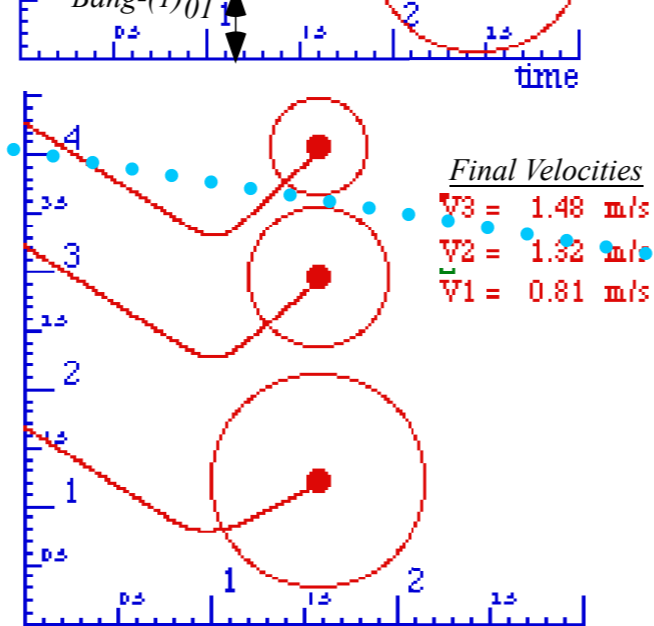
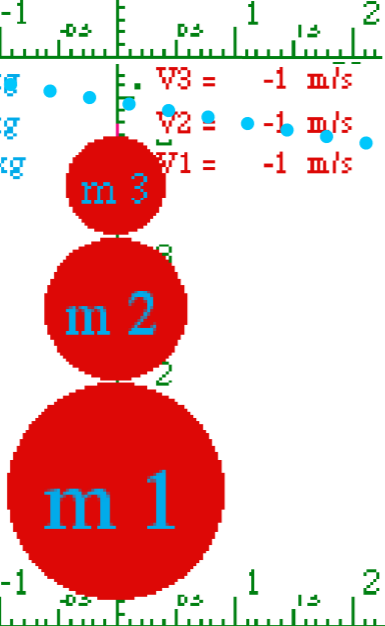
Initial Velocities
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V2 = -1 m/s
V1 = -1 m/s



(c) *Linear Force*
 $F(y) = k y$

m3 = 10 kg
m2 = 30 kg
m1 = 100 kg

Initial Velocities
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V2 = -1 m/s
V1 = -1 m/s



Bouncelt Simulation: 3-Ball Tower w/ Linear Force

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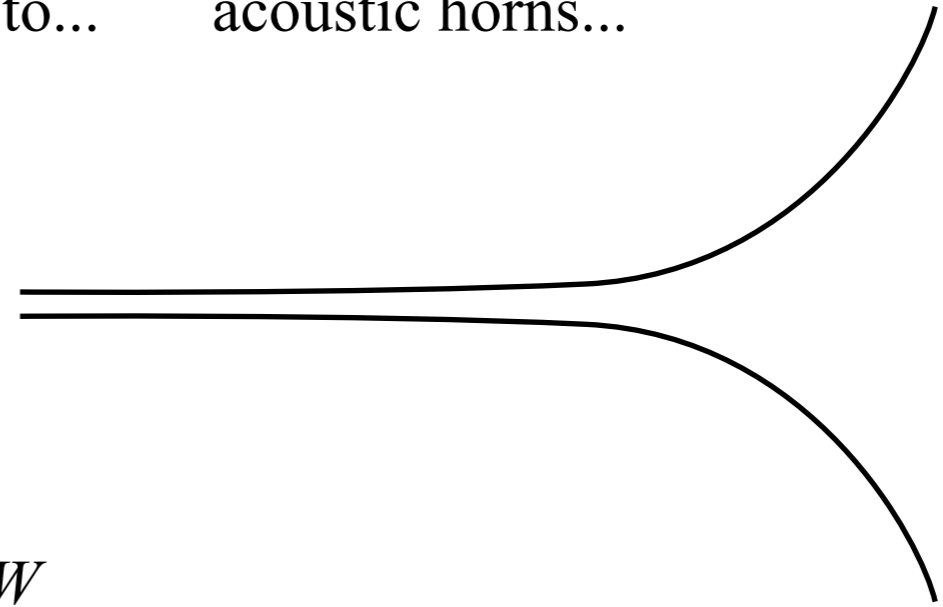
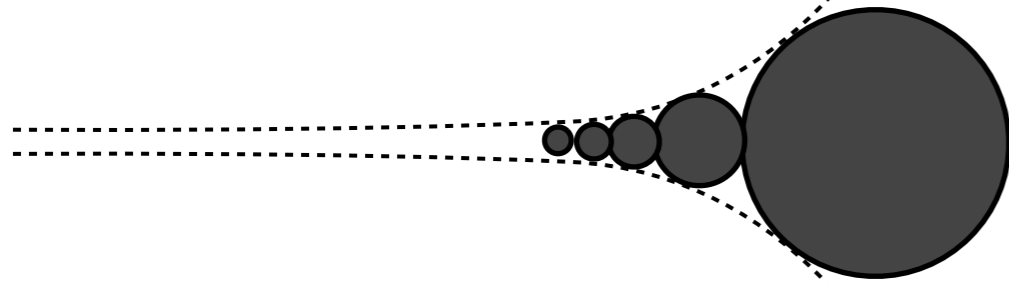
Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Superball towers...

analogous to...

acoustic horns...



small&fast... impedance matched to... BIG&SLOW

⁶J. B. Hart and R. B. Herrmann, Amer. J. Phys. **36**,
46 (1968).

1.8.3 *The optimal idler (An algebra/calculus problem)*

To get highest final v_3 of mass m_3 find optimum mass m_2 in terms of masses m_1 and m_3 that does that.

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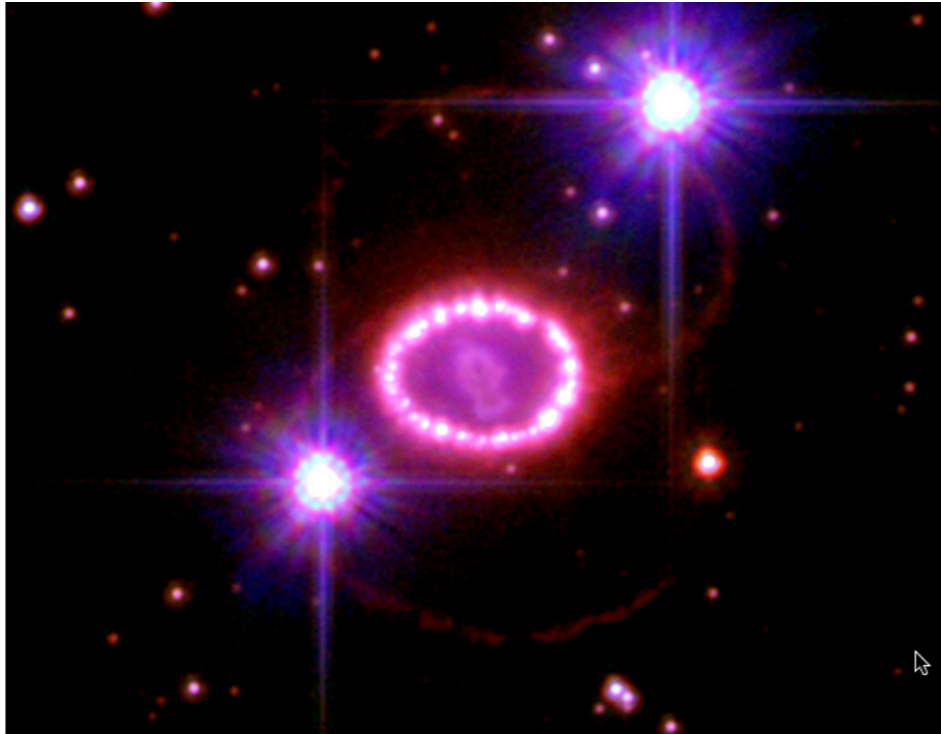
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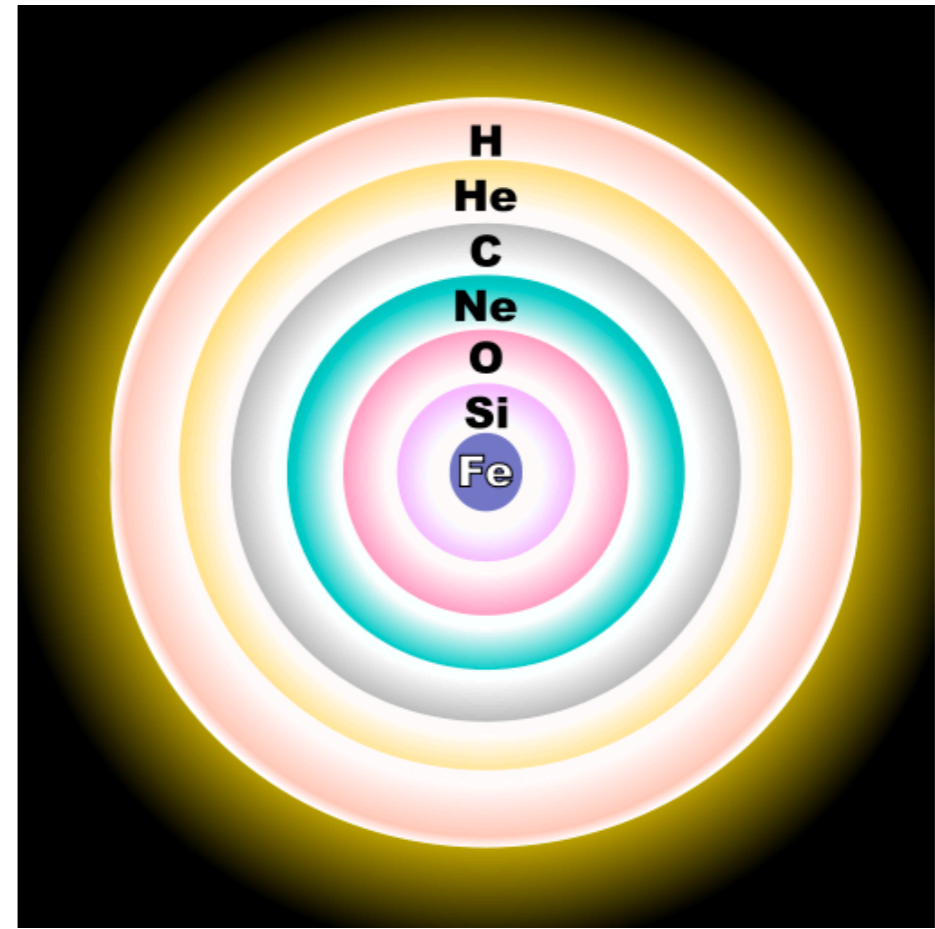


Source

<http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/>

Author

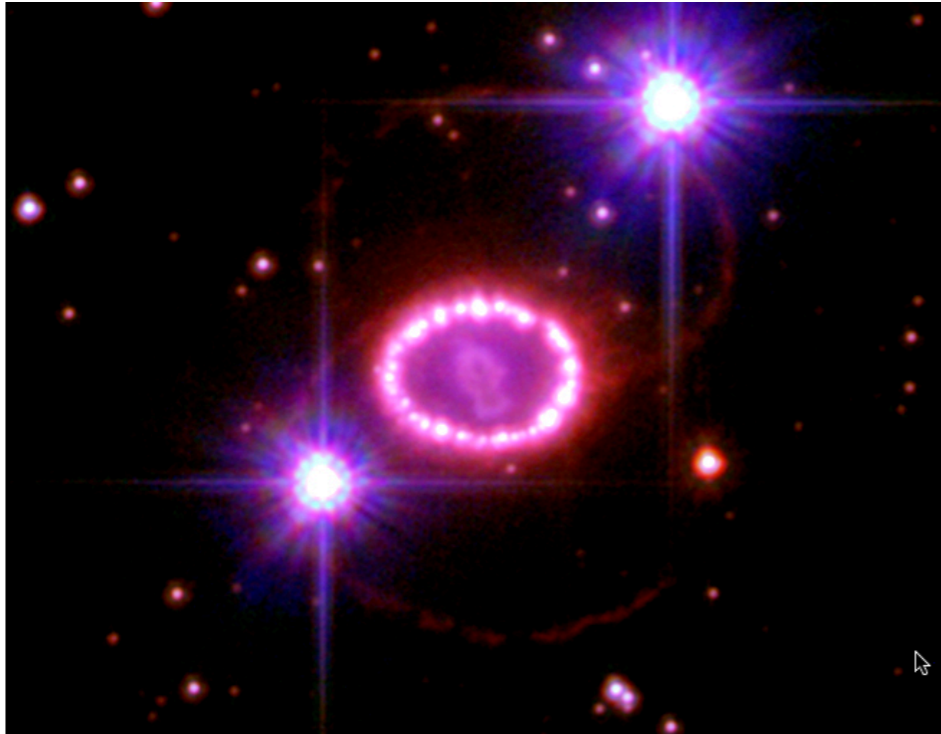
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 M _⊙ star ^[6]		
			Temperature (Kelvin)	Density (g/cm ³)	Duration
hydrogen burning	hydrogen	helium	7×10 ⁷	10	10 ⁷ years
triple-alpha process	helium	carbon, oxygen	2×10 ⁸	2000	10 ⁶ years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10 ⁸	10 ⁶	10 ³ years
neon burning process	neon	O, Mg	1.6×10 ⁹	10 ⁷	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10 ⁹	10 ⁷	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10 ⁹	10 ⁸	5 days

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

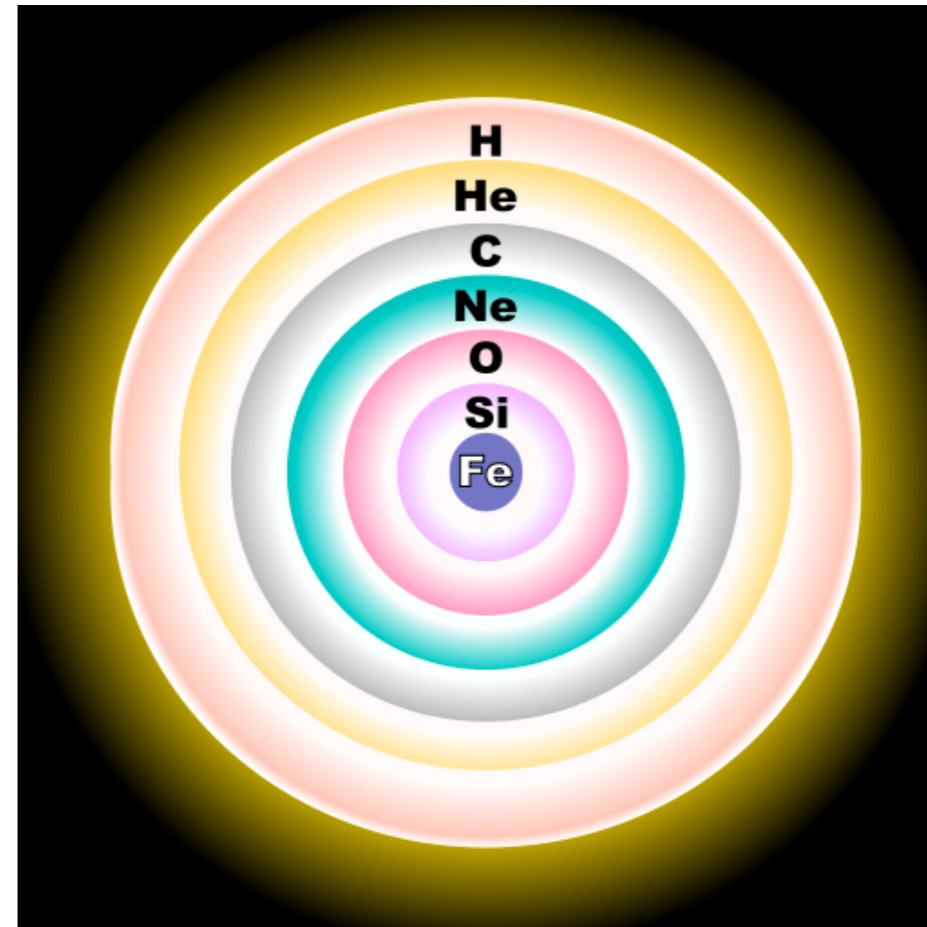


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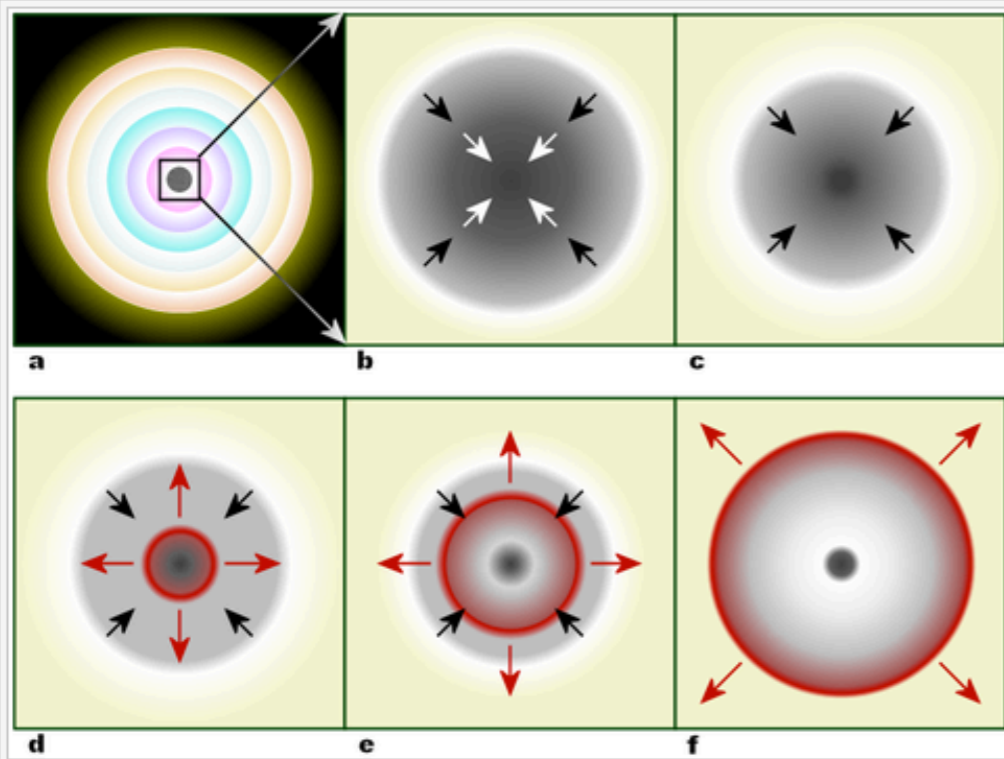
Author

NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)

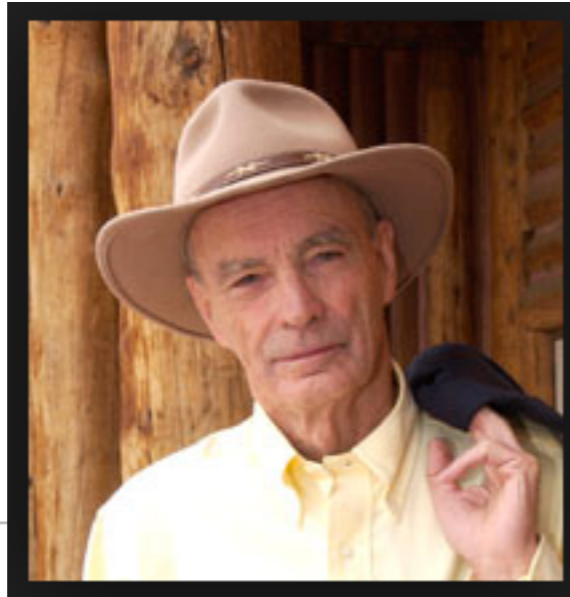


Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 M _⊙ star ^[6]		
			Temperature (Kelvin)	Density (g/cm ³)	Duration
hydrogen burning	hydrogen	helium	7×10 ⁷	10	10 ⁷ years
triple-alpha process	helium	carbon, oxygen	2×10 ⁸	2000	10 ⁶ years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10 ⁸	10 ⁶	10 ³ years
neon burning process	neon	O, Mg	1.6×10 ⁹	10 ⁷	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10 ⁹	10 ⁷	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10 ⁹	10 ⁸	5 days



Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.



Stirling Colgate

From Wikipedia, the free encyclopedia

Stirling Auchincloss Colgate (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech),^[1] and an heir to the Colgate toothpaste family fortune.^[2] He was America's premier^[citation needed] diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics.^[3] He was born in New York City in 1925, to Henry Auchincloss and Jeanette Thurber (née Pruyn) Colgate.^[4]



*..an amusing off-color aside
story of Stirling Colgate's NMIMT resignation...*

(Not told in Wikipedia!)

Quote

- "I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs."

Multiple-collision accelerator assembly

US 5256071 A

ABSTRACT

A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics--the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

Publication number	US5256071 A
Publication type	Grant
Application number	US 07/748,804
Publication date	Oct 26, 1993
Filing date	Aug 22, 1991
Priority date [?]	Aug 22, 1991
Fee status [?]	Paid
Inventors	Edward W. Hones, William G. Hones, Stirling A. Colgate
Original Assignee	Hones Edward W, Hones William G, Colgate Stirling A
Export Citation	BiBTeX, EndNote, RefMan
Patent Citations (3) , Referenced by (4) , Classifications (7) , Legal Events (7)	
External Links: USPTO , USPTO Assignment , Espacenet	

(Point allowing patent over previous 1973 proposal (4))

1st publication describing theory and experiment of this device 20 years before.

Velocity Amplification in Collision Experiments Involving Superballs

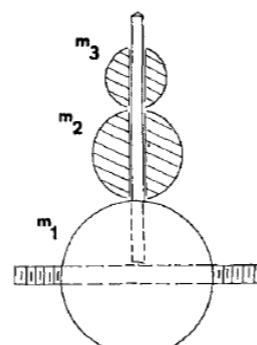
William G. Harter¹ (class of WGH)

— HIDE AFFILIATIONS

¹ University of Southern California, Los Angeles, California 90007

View the Scitation page for University of Southern California (USC).

Am. J. Phys. **39**, 656 (1971); <http://dx.doi.org/10.1119/1.1986253>



BUY: \$30.00

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Potential energy dynamics of Superballs and related things

Thales geometry and “Sagittal approximation” to force law

Geometry and dynamics of single ball bounce

(a) Constant force $F=-k$ (linear potential $V=kx$)

Some physics of dare-devil-diving 80 ft. into kidee pool

(Simulations)

(b) Linear force $F=-kx$ (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

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(Leads to Sagittal

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2, 3, and 4 body towers)

Many-body 1D collisions

→ Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: “Zig-zag geometry” of freeway crashes

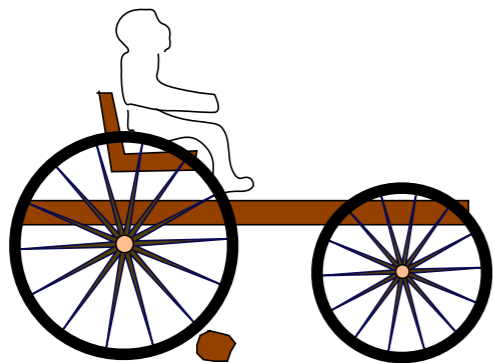
Super-elastic examples: This really is “Rocket-Science”

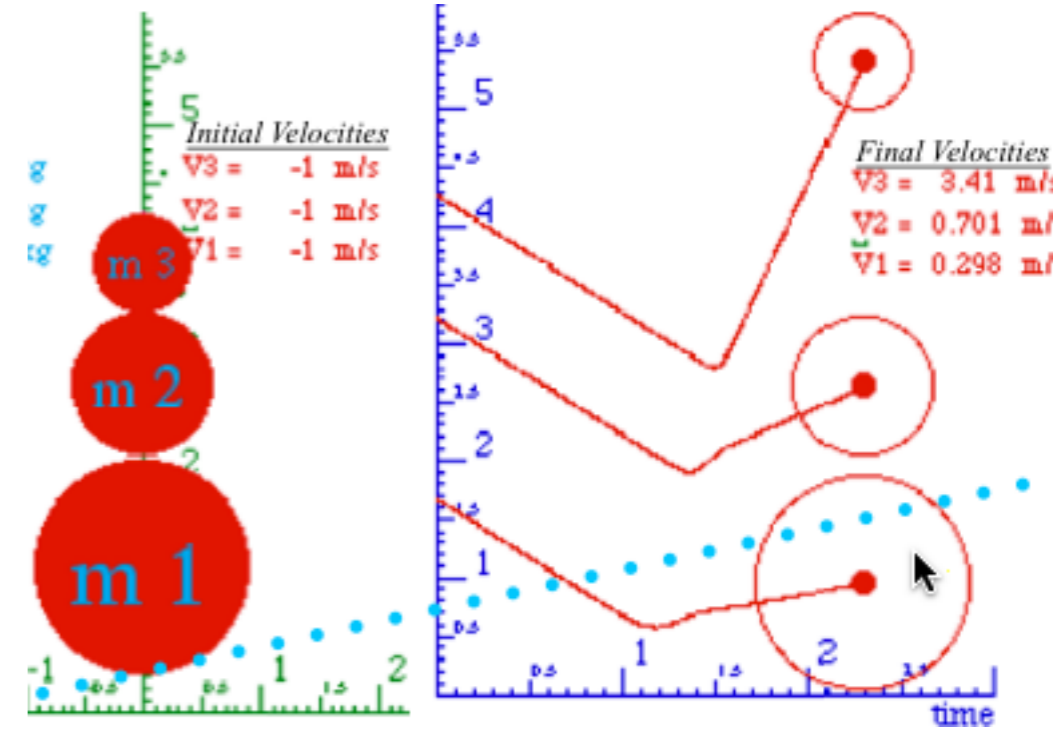


Western buckboard = ??????

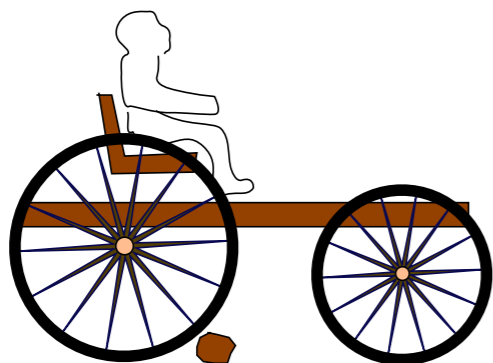


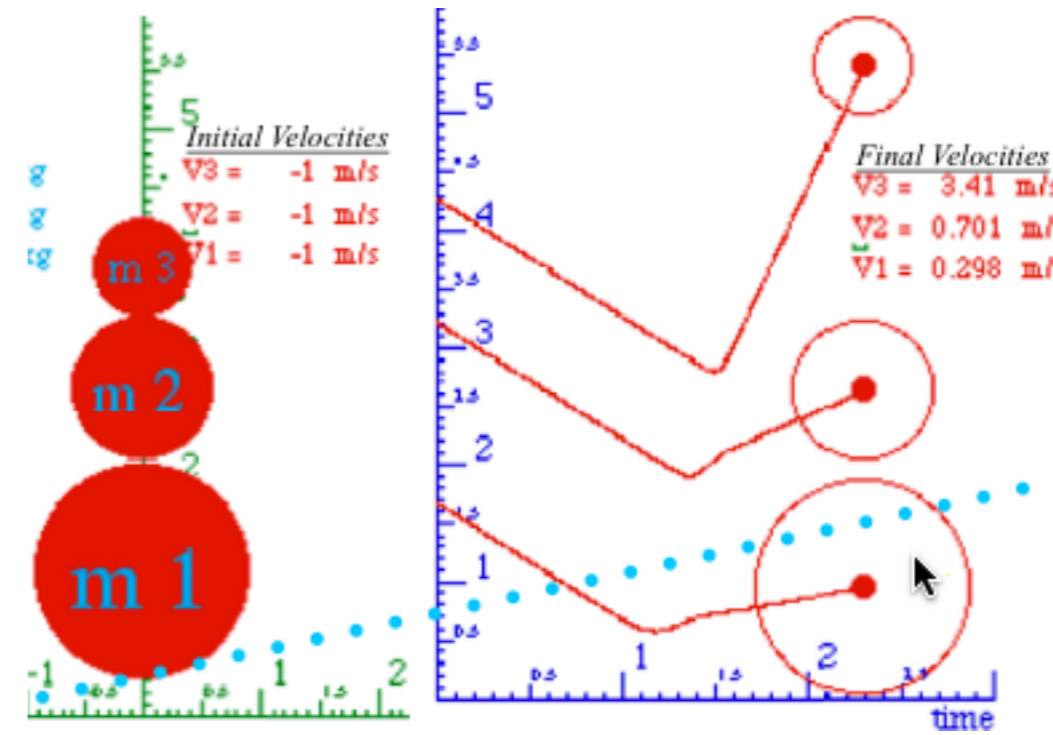
Western buckboard = ??????



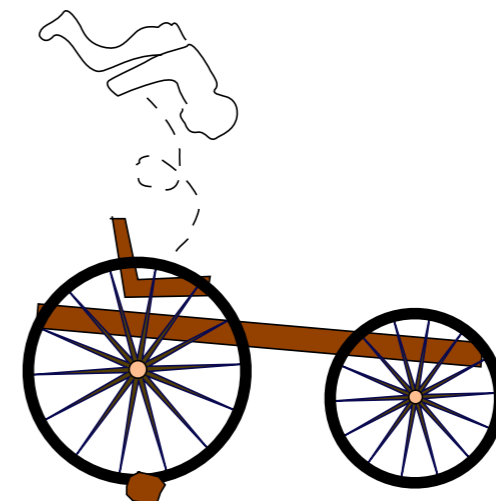
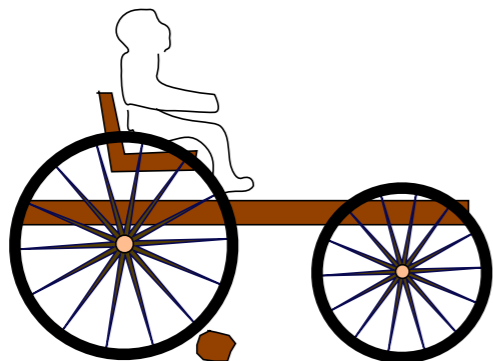


Western buckboard = 3-ball analogy





Western buckboard = 3-ball analogy Disaster!



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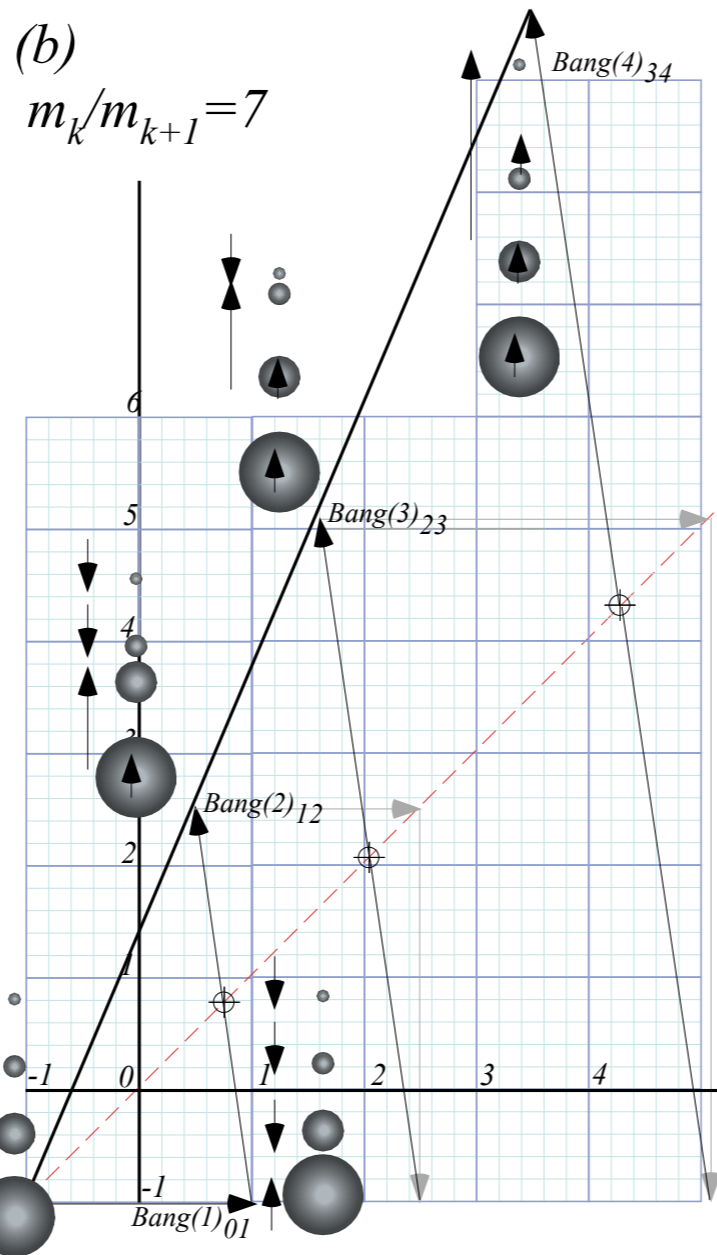
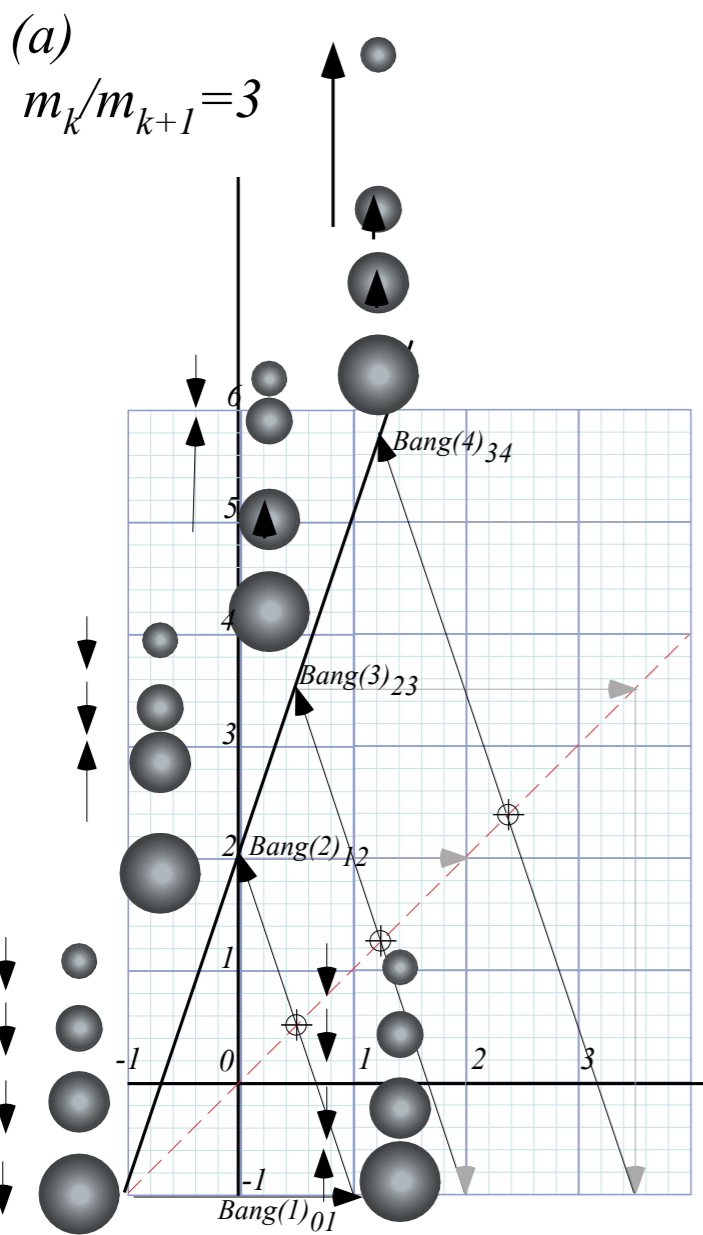
Elastic examples: Western buckboard



Bouncing columns and Newton's cradle

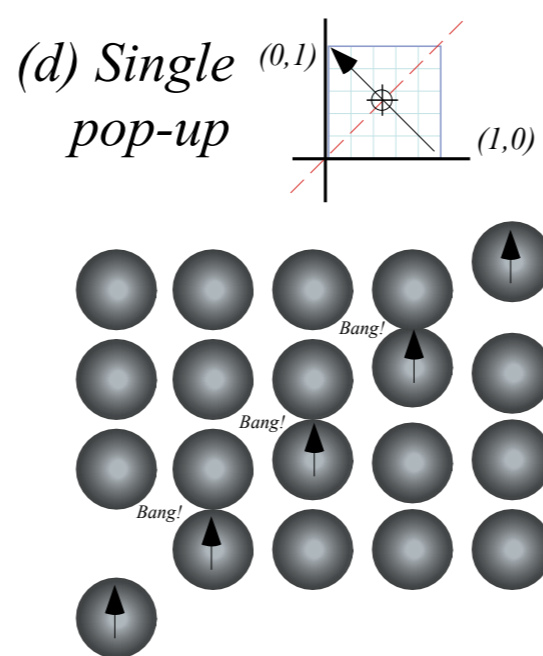
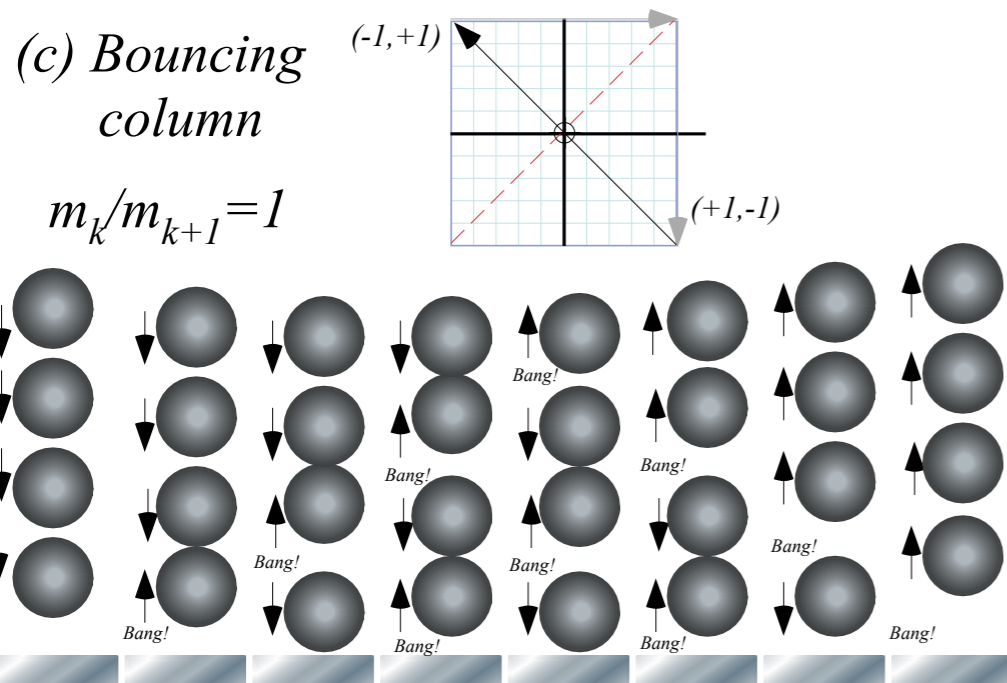
Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”



Unit 1
 Fig. 8.2a-b
 4-Body IBM Geometry
 Fig. 8.2c-d
 4-Equal-Body Geometry

Bouncelt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 3$



4-Equal-Body
 "Shockwave" or pulse wave
 Dynamics

Opposite of continuous wave dynamics
 introduced in Unit 2

Bouncelt Simulation: 4-Ball Tower w/ $m_k/m_{k+1} = 1$

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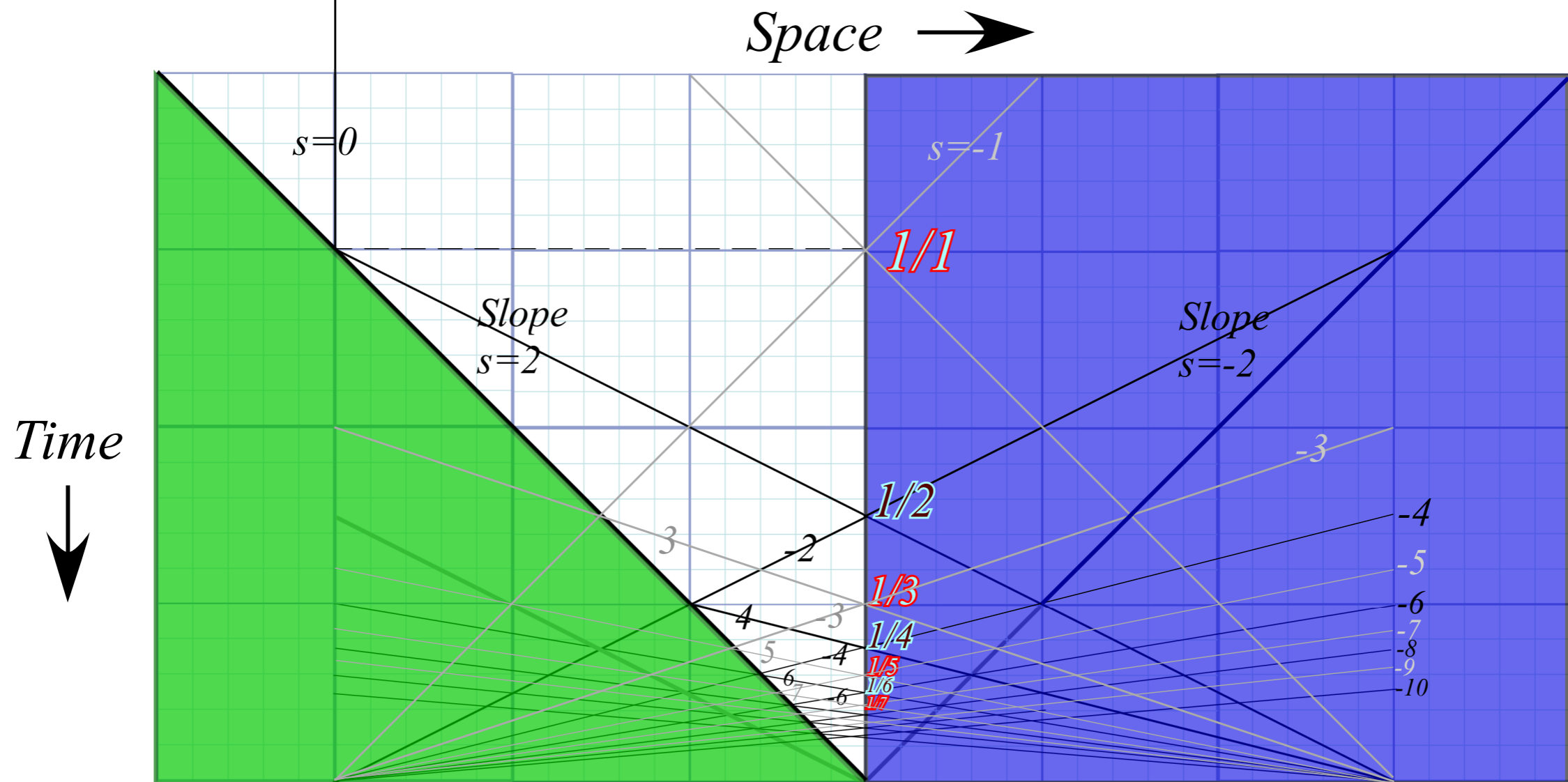
→ Inelastic examples: “Zig-zag geometry” of freeway crashes

Super-elastic examples: This really is “Rocket-Science”

Inelastic examples: “Zig-zag geometry” of freeway crashes

First recall “zig-zag” fractions of “Monster Mash” in Lect. 4

Trajectory geometry exposed (Harmonic series $1/1, 1/2, 1/3, 1/4, \dots$)



Speeding car and five stationary cars

$(V_{M(0)}=60, V_{m(1)}=0)$



$V_{M(01)}=30$



$V_{M(012)}=20$



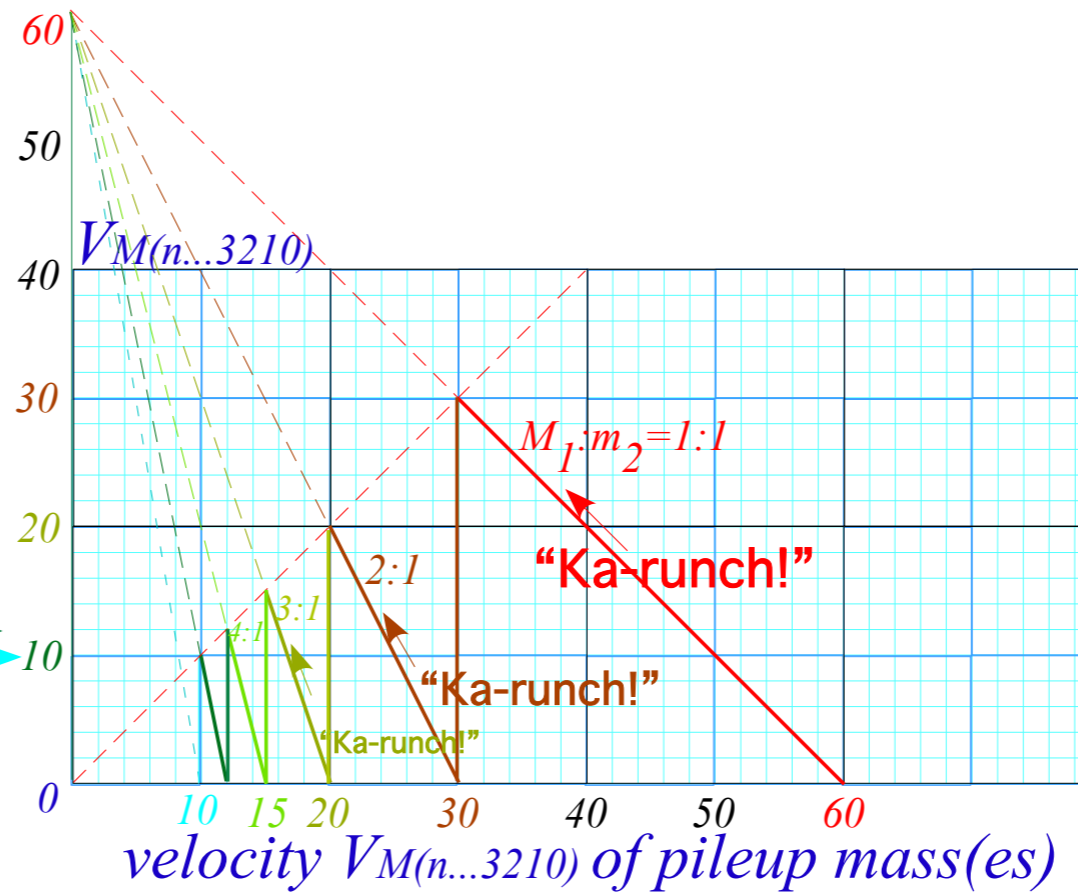
$V_{M(0123)}=15$



$V_{M(01234)}=12$



$V_{M(01235)}=10$



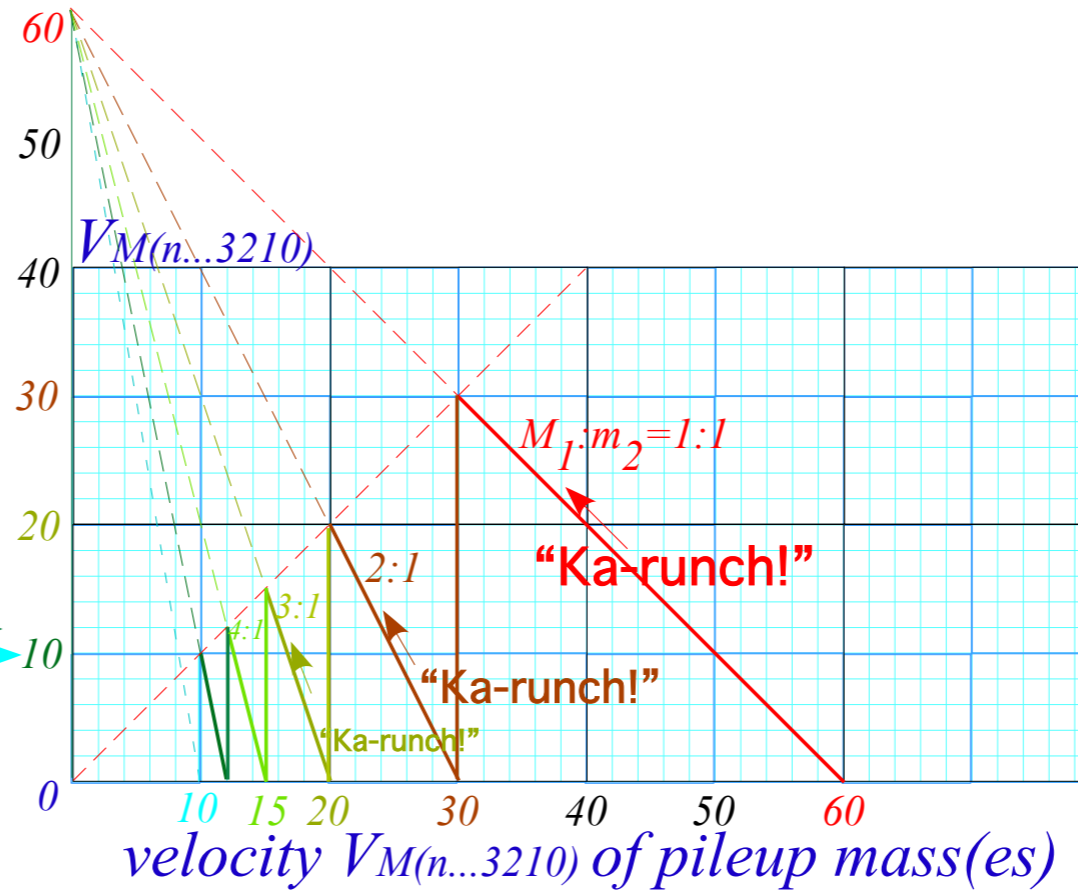
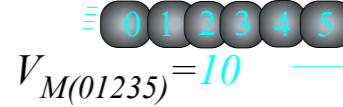
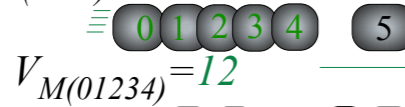
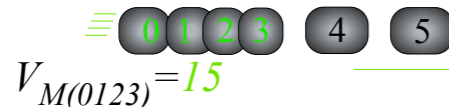
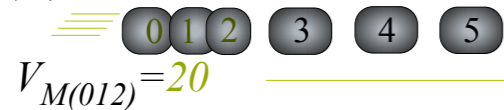
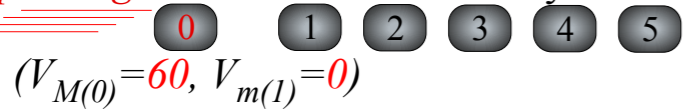
Unit 1
 Fig. 8.5
 Pile-up:
 One 60mph car
 hits
 five standing cars

Bouncelt Simulation: One ball hits 5 stationary balls (y vs x) and (x_i vs t)

These graphs plot User determined quantities.
 Choose and select from a context menu via right click on target axis,
 like the following set to V_{1x} and V_{2x}

Bouncelt Simulation: One ball hits 5 stationary balls (y vs x) and (V_{i+1x} vs V_{ix})

Speeding car and five stationary cars



Unit 1

Fig. 8.5

Pile-up:
One 60mph car
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five standing cars

Five speeding cars and a stationary car

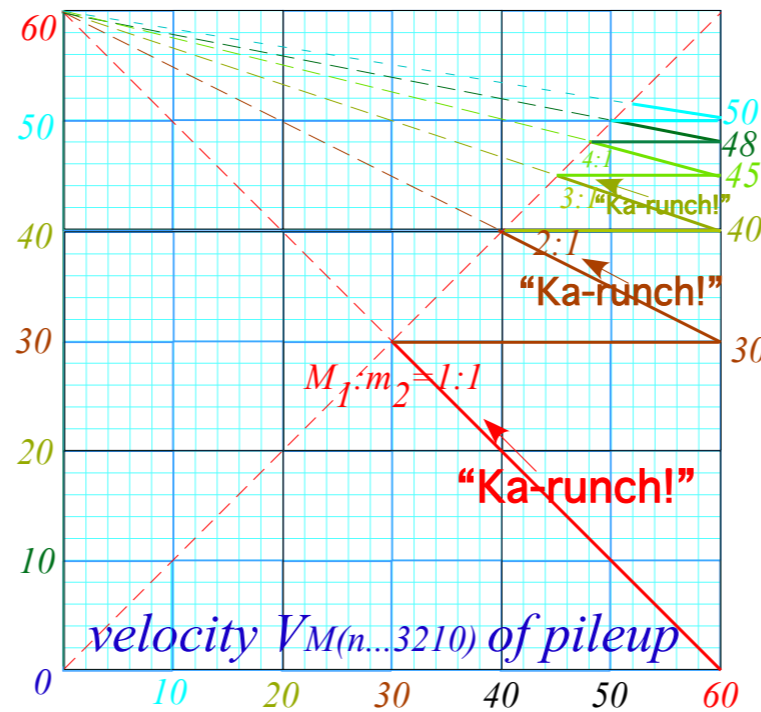
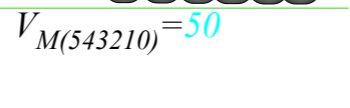
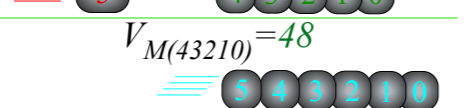
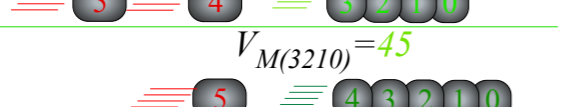
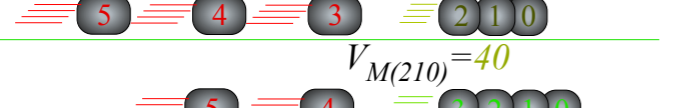
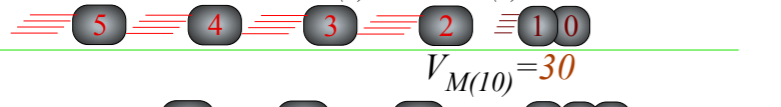
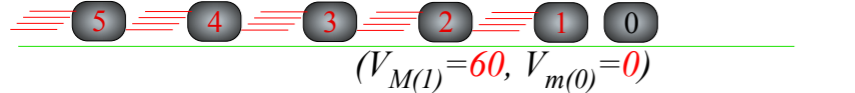
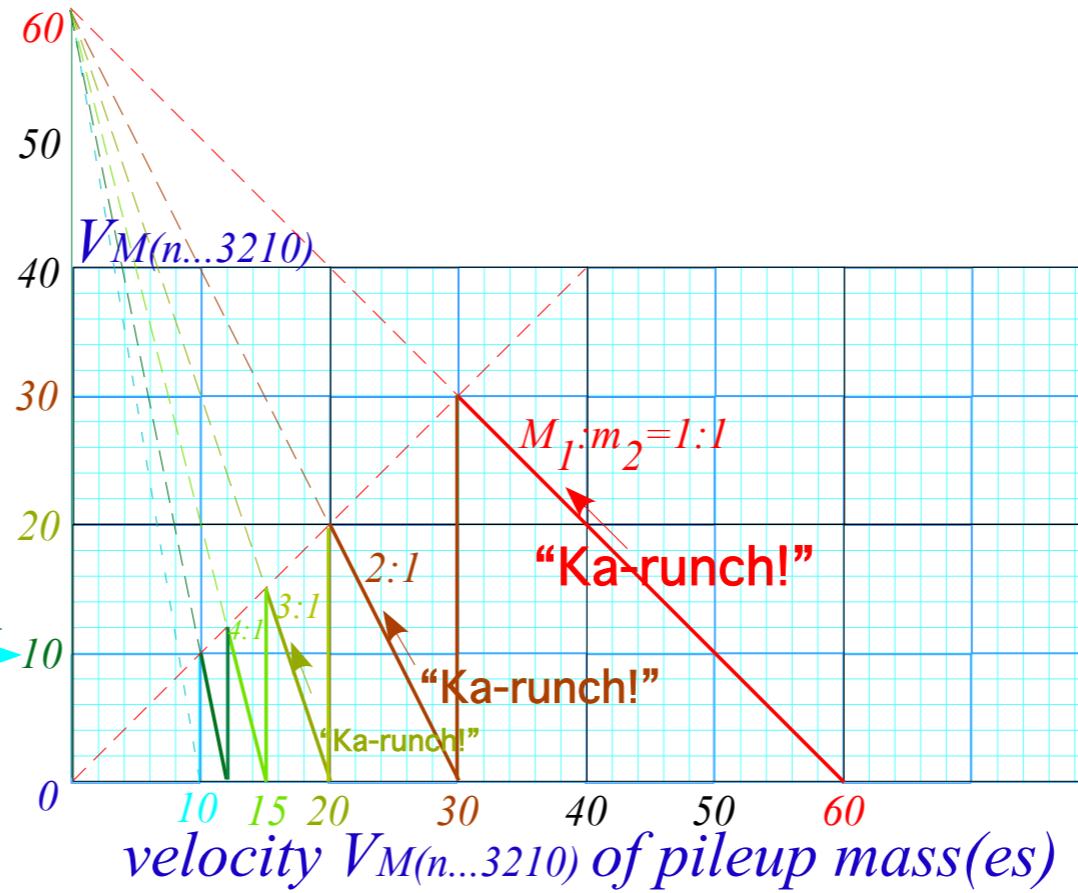
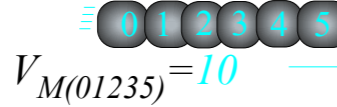
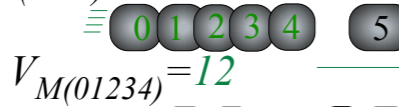
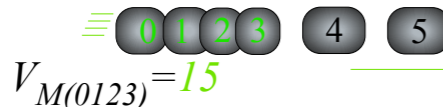
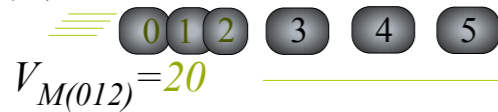
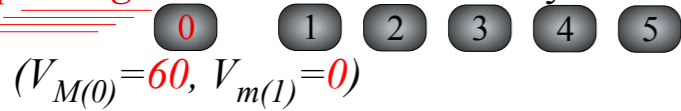


Fig. 8.6

Pile-up:
Five 60mph cars
hit
one standing cars

Bouncelt Simulation: 5 balls hit 1 stationary ball (y vs x) and (v_{6x} vs v_{5x})

Speeding car and five stationary cars



Unit 1

Fig. 8.5
Pile-up:
One 60mph car
hits
five standing cars

Five speeding cars and a stationary car

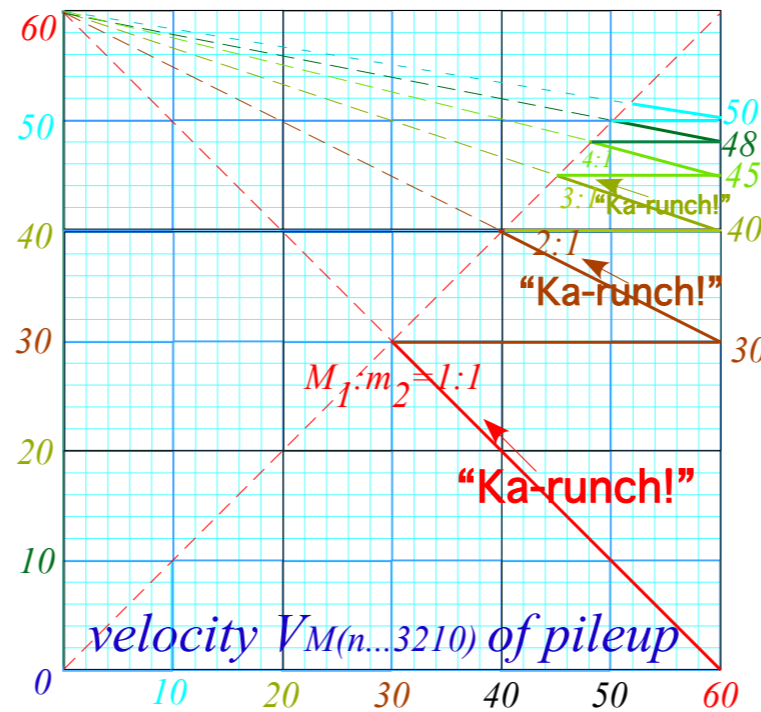
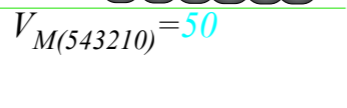
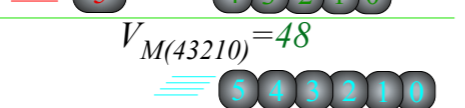
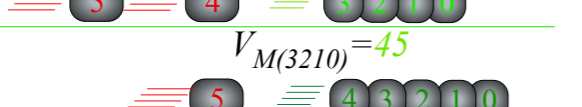
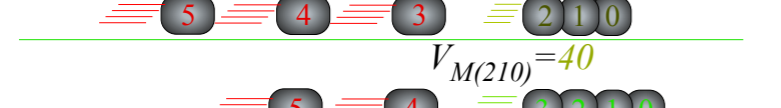
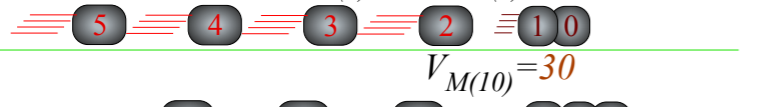
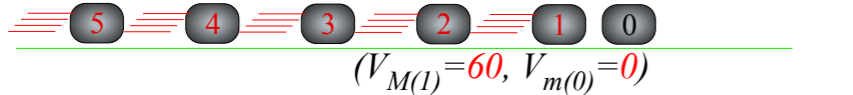


Fig. 8.6
Pile-up:
Five 60mph cars
hit
one standing cars

Five speeding cars and five stationary cars



(Fug-gedda-aboud-dit!!)

(Many possible scenarios depending on initial positions!)

Fig. 8.7
Pile-up:
Five 60mph cars
hit
five standing cars

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(Simulations)

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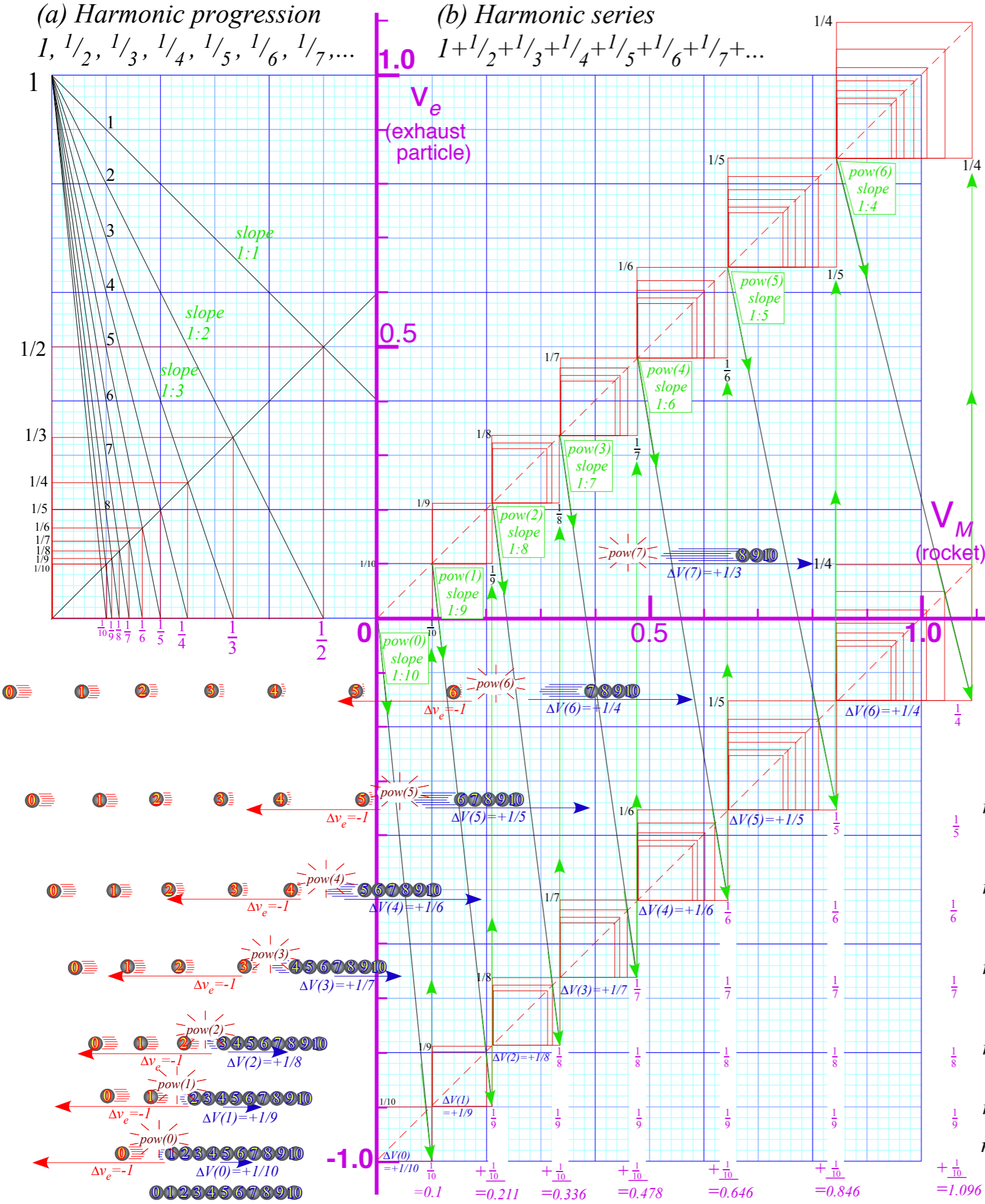
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Inelastic examples: “Zig-zag geometry” of freeway crashes

→ Super-elastic examples: This really is “Rocket-Science”

Unit 1
Fig. 8.8a-b

Rocket Science!



$$m \cdot \Delta v_7 + 3m \cdot \Delta V_M(7) = 0$$

$$m \cdot \Delta v_6 + 4m \cdot \Delta V_M(6) = 0$$

$$m \cdot \Delta v_5 + 5m \cdot \Delta V_M(5) = 0$$

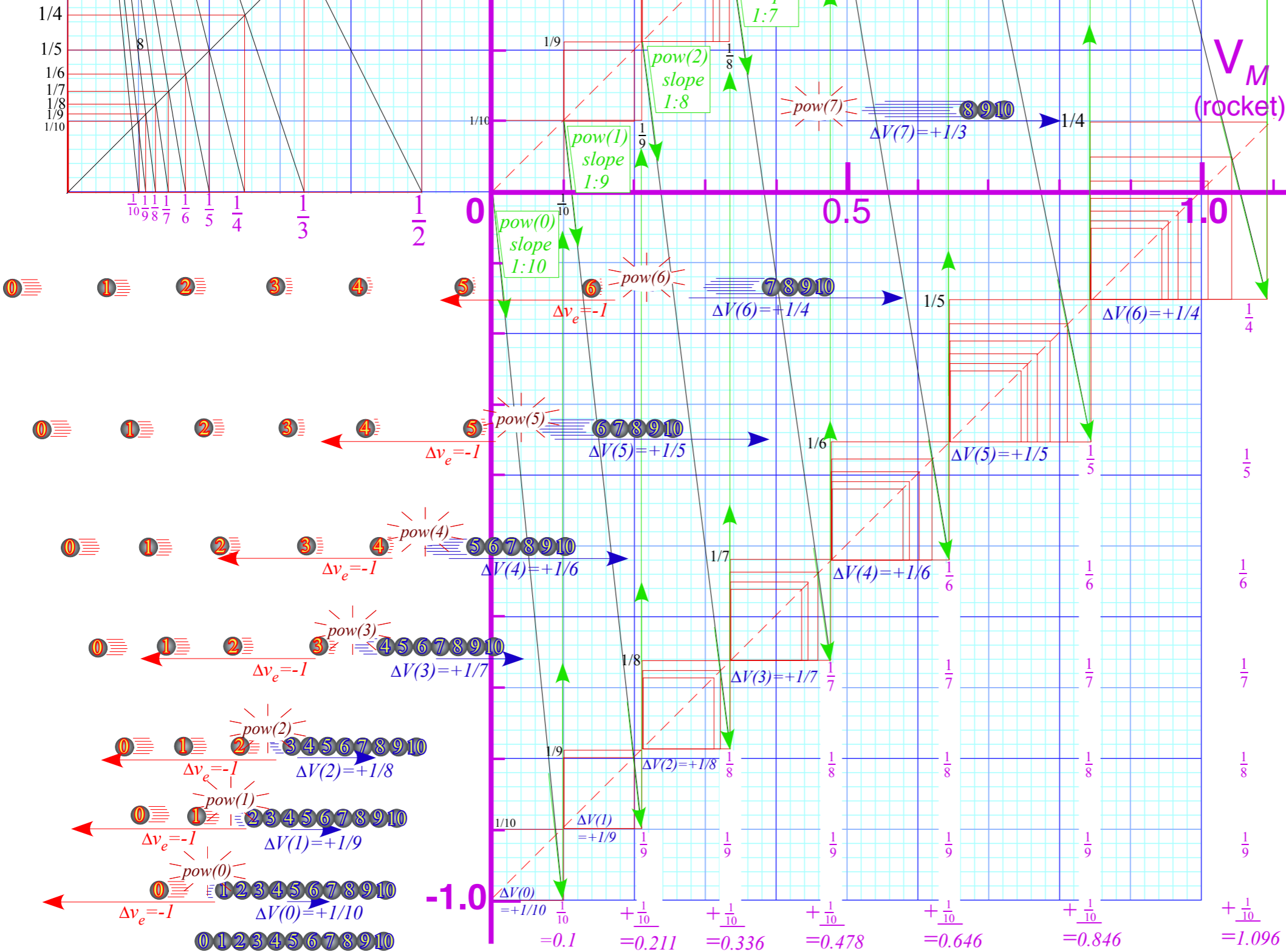
$$m \cdot \Delta v_4 + 6m \cdot \Delta V_M(4) = 0$$

$$m \cdot \Delta v_3 + 7m \cdot \Delta V_M(3) = 0$$

$$m \cdot \Delta v_2 + 8m \cdot \Delta V_M(2) = 0$$

$$m \cdot \Delta v_1 + 9m \cdot \Delta V_M(1) = 0$$

$$m \cdot \Delta v_0 + 10m \cdot \Delta V_M(0) = 0$$



0th: $V(0) = 1/10 = 0.1$

3rd: $V(3) = V(2) + 1/7 = 0.478$

6th: $V(6) = V(5) + 1/4 = 1.096$

1st: $V(1) = 1/10 + 1/9 = 0.211$

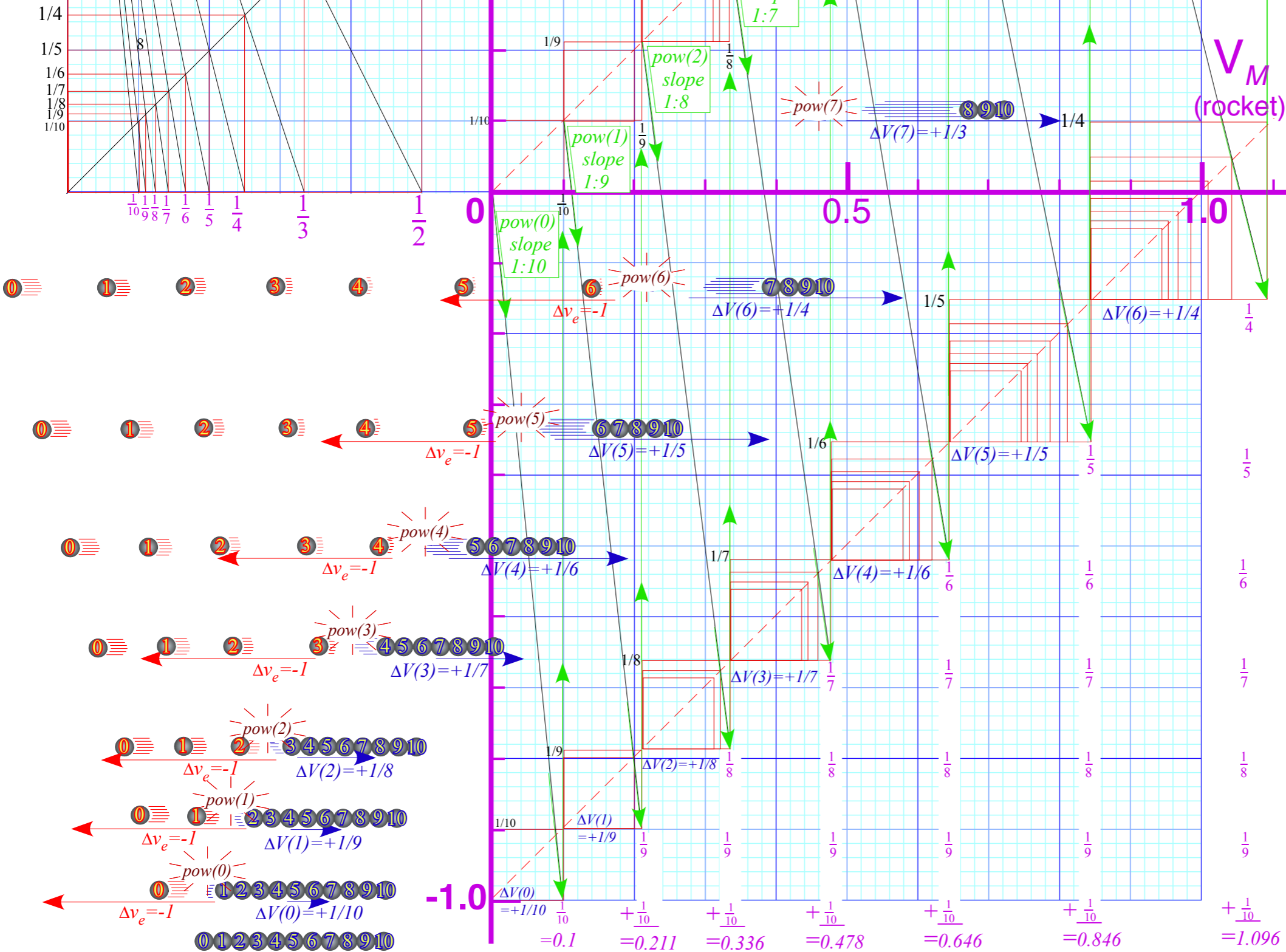
4th: $V(4) = V(3) + 1/6 = 0.646$

7th: $V(7) = V(6) + 1/3 = 1.429$

2nd: $V(2) = 1/10 + 1/9 + 1/8 = 0.336$

5th: $V(5) = V(4) + 1/5 = 0.846$

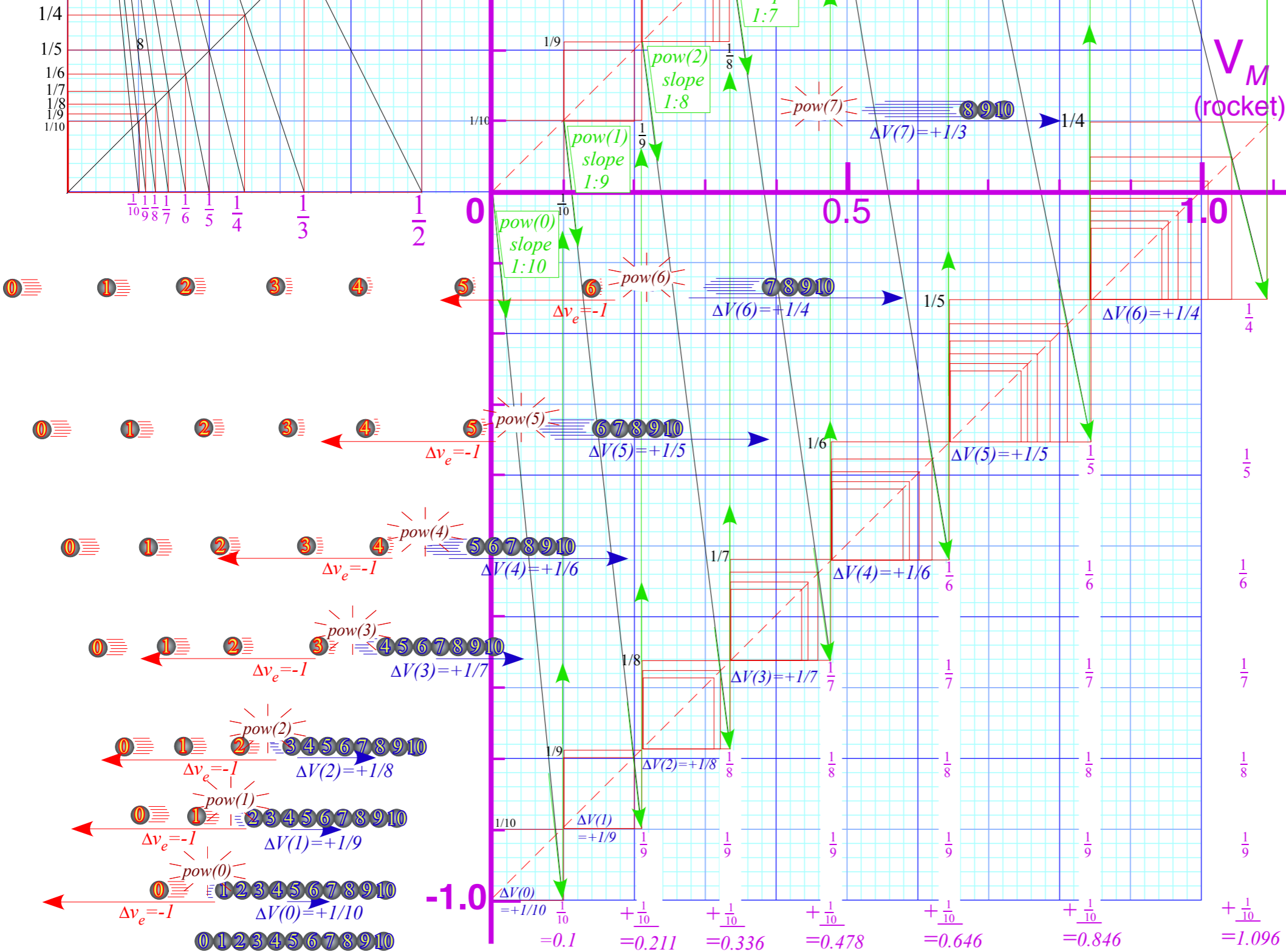
8th: $V(8) = V(7) + 1/2 = 1.929$



- 0th: $V(0) = 1/10 = 0.1$
- 1st: $V(1) = 1/10 + 1/9 = 0.211$
- 2nd: $V(2) = 1/10 + 1/9 + 1/8 = 0.336$
- 3rd: $V(3) = V(2) + 1/7 = 0.478$
- 4th: $V(4) = V(3) + 1/6 = 0.646$
- 5th: $V(5) = V(4) + 1/5 = 0.846$
- 6th: $V(6) = V(5) + 1/4 = 1.096$
- 7th: $V(7) = V(6) + 1/3 = 1.429$
- 8th: $V(8) = V(7) + 1/2 = 1.929$

v_e known as
"Specific Impulse"

By calculus: $M \cdot \Delta V = -v_e \cdot \Delta M$ or: $dV = -v_e \frac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$



- 0th: $V(0) = 1/10 = 0.1$
- 1st: $V(1) = 1/10 + 1/9 = 0.211$
- 2nd: $V(2) = 1/10 + 1/9 + 1/8 = 0.336$
- 3rd: $V(3) = V(2) + 1/7 = 0.478$
- 4th: $V(4) = V(3) + 1/6 = 0.646$
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- 6th: $V(6) = V(5) + 1/4 = 1.096$
- 7th: $V(7) = V(6) + 1/3 = 1.429$
- 8th: $V(8) = V(7) + 1/2 = 1.929$

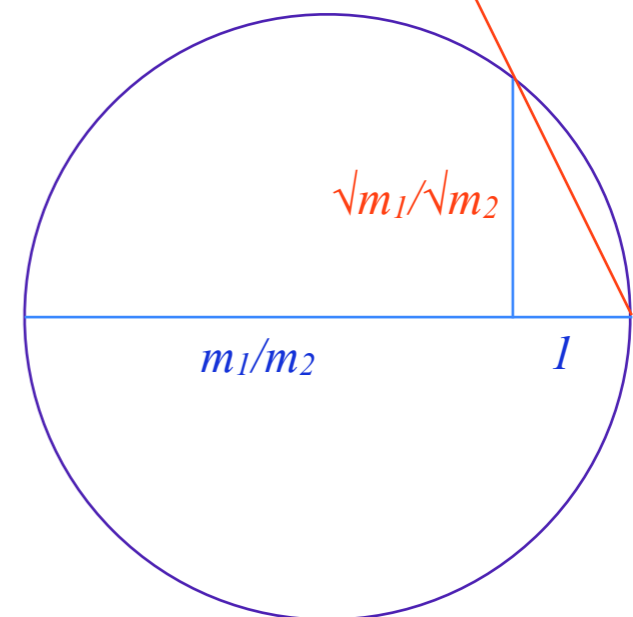
v_e known as "Specific Impulse"

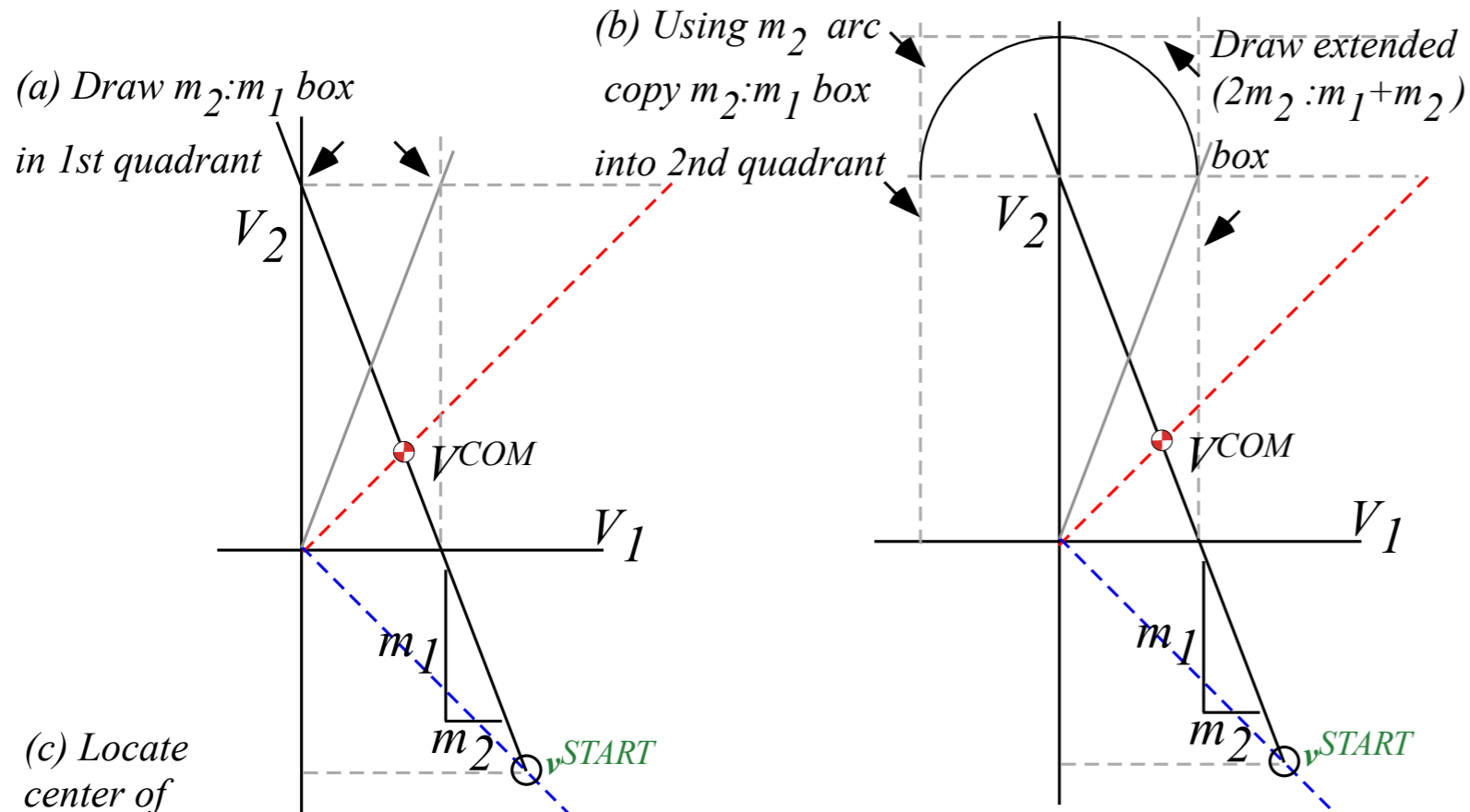
By calculus: $M \cdot \Delta V = -v_e \cdot \Delta M$ or: $dV = -v_e \frac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$

The Rocket Equation: $V_{FIN} - V_{IN} = -v_e [\ln M_{FIN} - \ln M_{IN}] = v_e \left[\ln \frac{M_{IN}}{M_{FIN}} \right]$

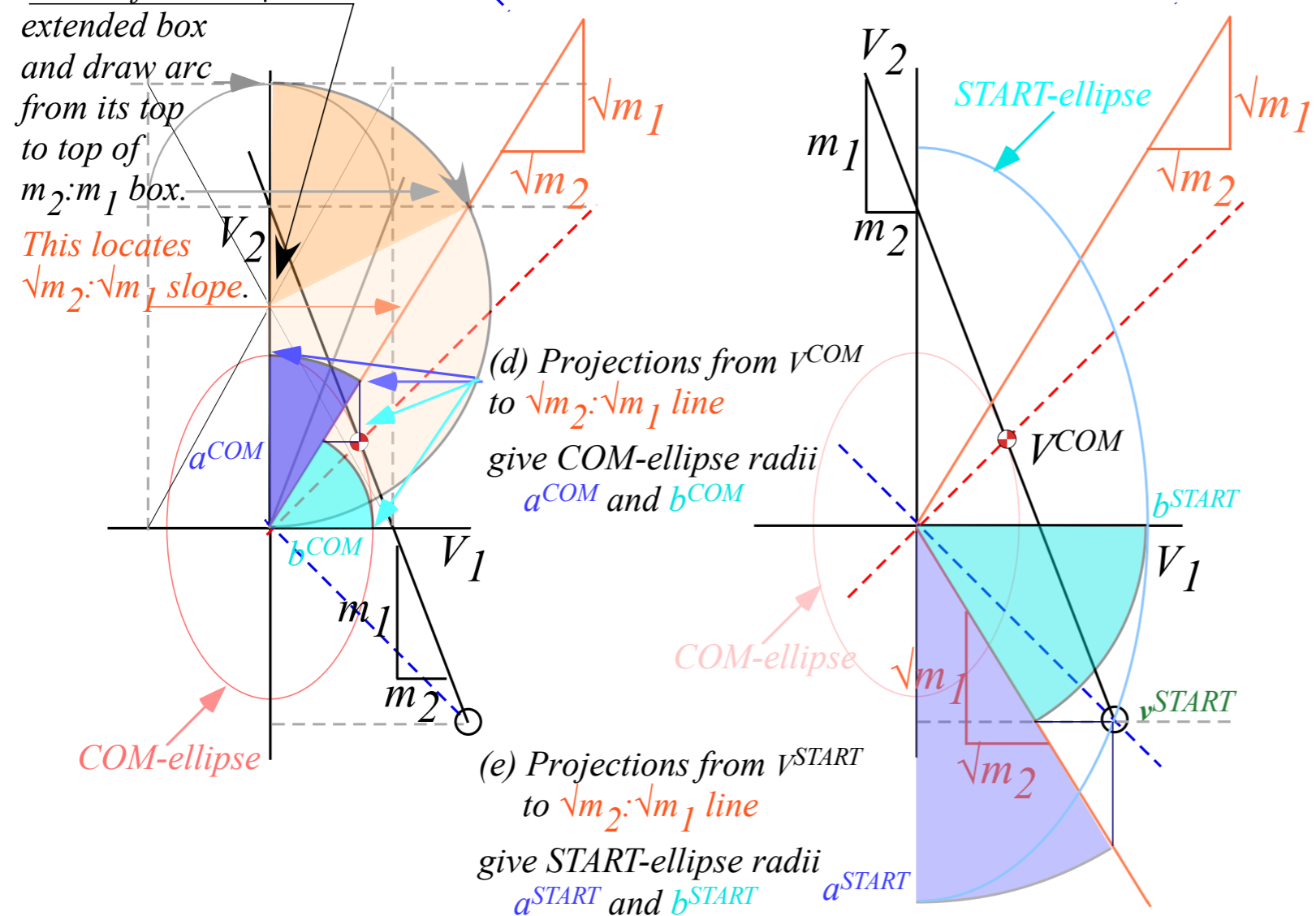
A Thales construction for momentum-energy

(Made obsolete by Estrangian scaling to circular (V_1, V_2) plots. Still, one has to construct $\sqrt{m_1}/\sqrt{m_2}$ slopes.)





(c) Locate center of extended box and draw arc from its top to top of $m_2:m_1$ box. This locates $\sqrt{m_2}:\sqrt{m_1}$ slope.



Unit 1
Fig. 8.4a-d

This is a detailed construction of the energy ellipse in a Largangian (v_1, v_2) plot given the initial (v_1, v_2) .

The Estrangian (V_1, V_2) plot makes the (v_1, v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1, V_2) .)