Lecture 4
Tue. 9.3.2015

Kinetic Derivation of 1D Potentials and Force Fields
(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations *High mass ratio $M_1/m_2=49$*

Force "field" or "pressure" due to many small bounces
Force defined as momentum transfer rate
The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

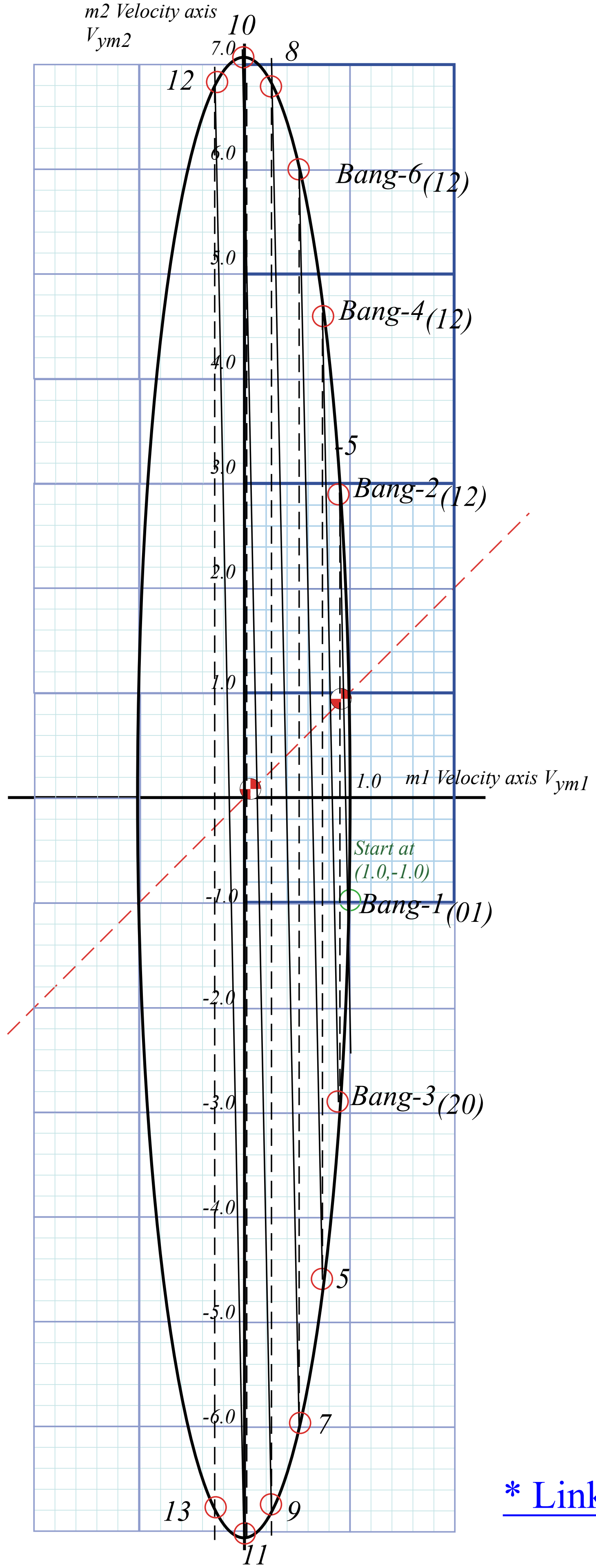
Potential field due to many small bounces
Example of 1D-Adiabatic potential $U(y)$=const./y^2
Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$
Example of 1D-Isothermal potential $U(y)$=const. ln(y)

"Monster Mash" classical segue to Heisenberg action relations
Example of very very large M_1 ball-wall(s) crushing a poor little m_2
How m_2 keeps its action
An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015)]

Review of $(V_1,V_2) \rightarrow (y_1,y_2)$ *relations*

→ *High mass ratio* $M_1/m_2 = 49$

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

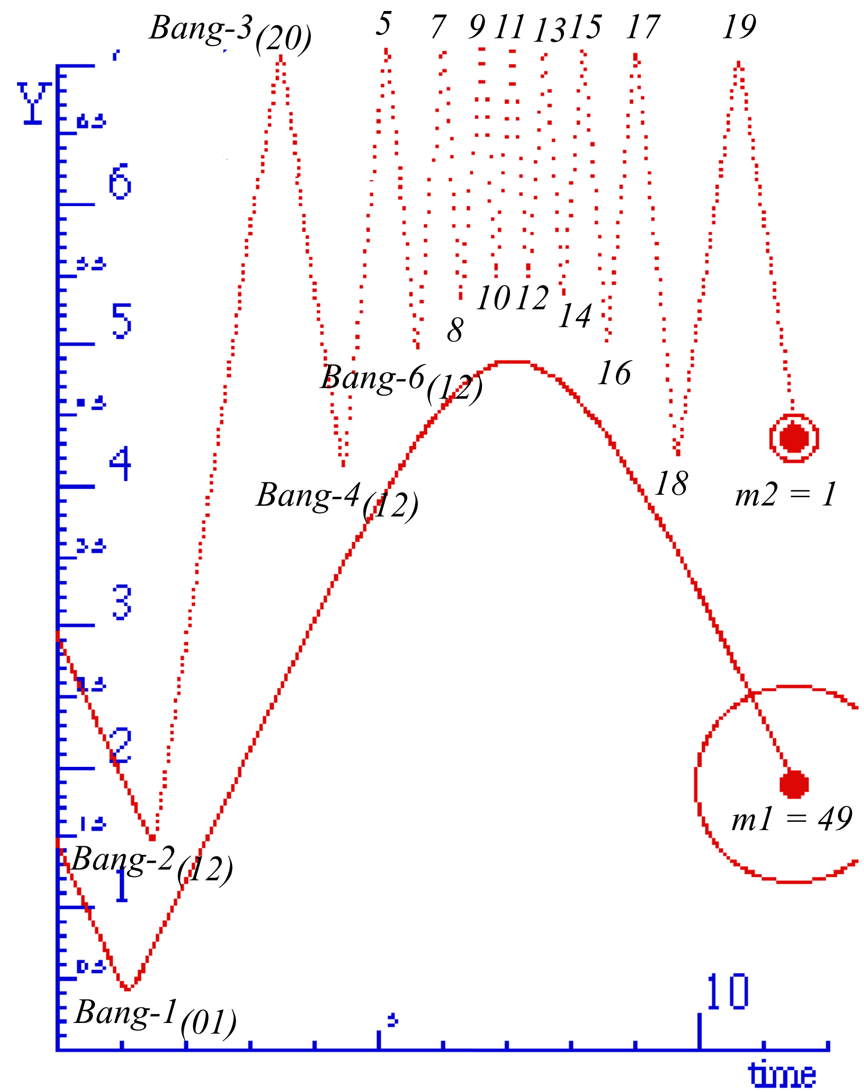


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

* Link to BounceIt: $Y_i(t)$ animation

* Link to BounceIt: V_{y2} vs V_{y1} animation

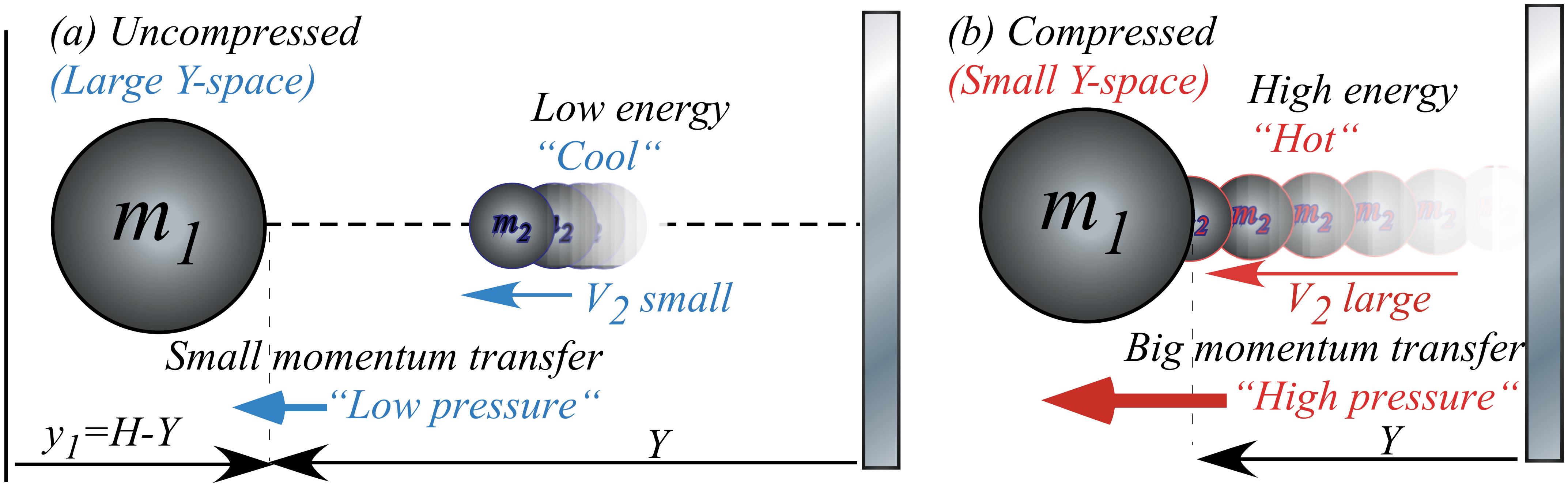
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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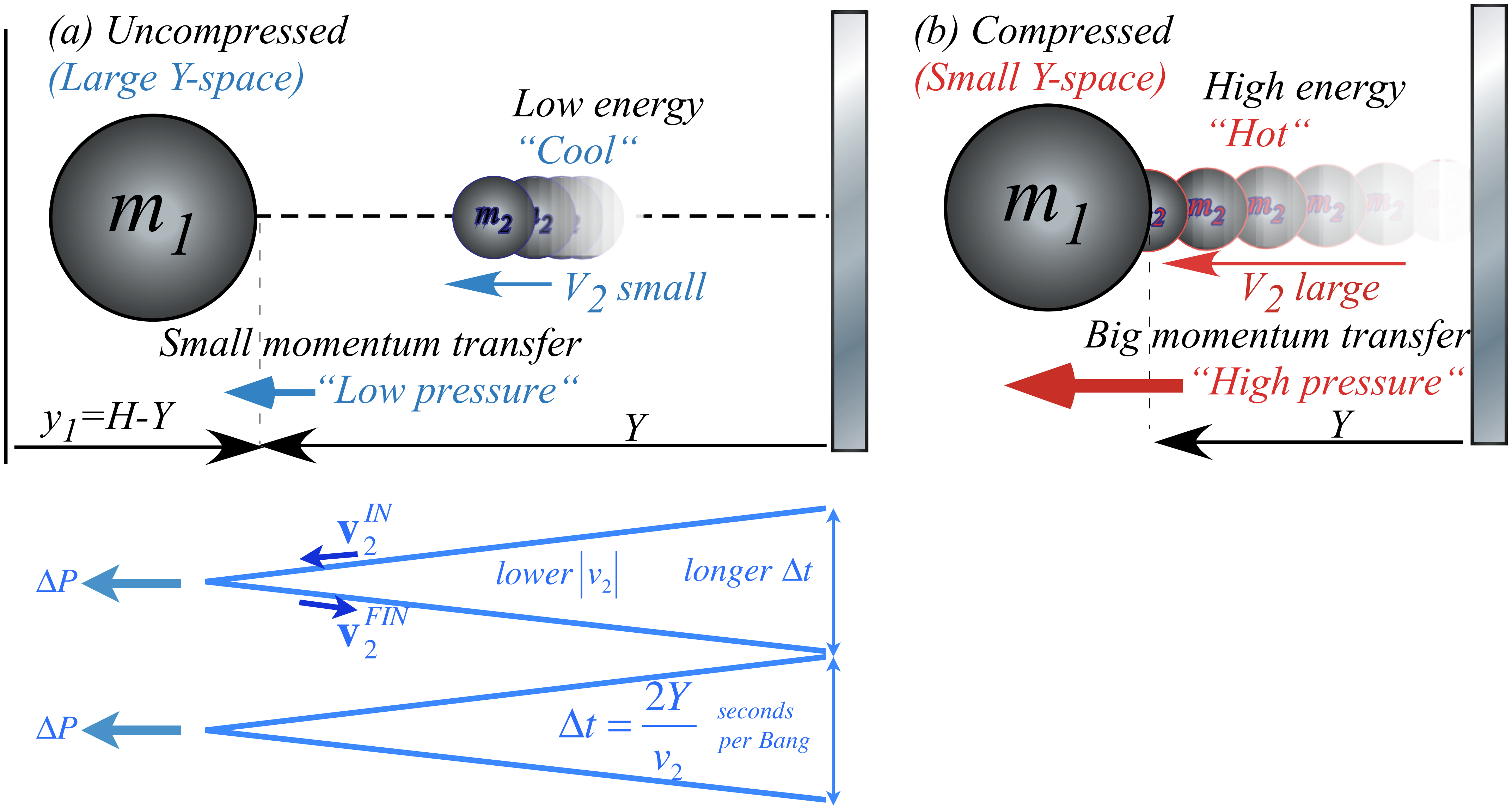

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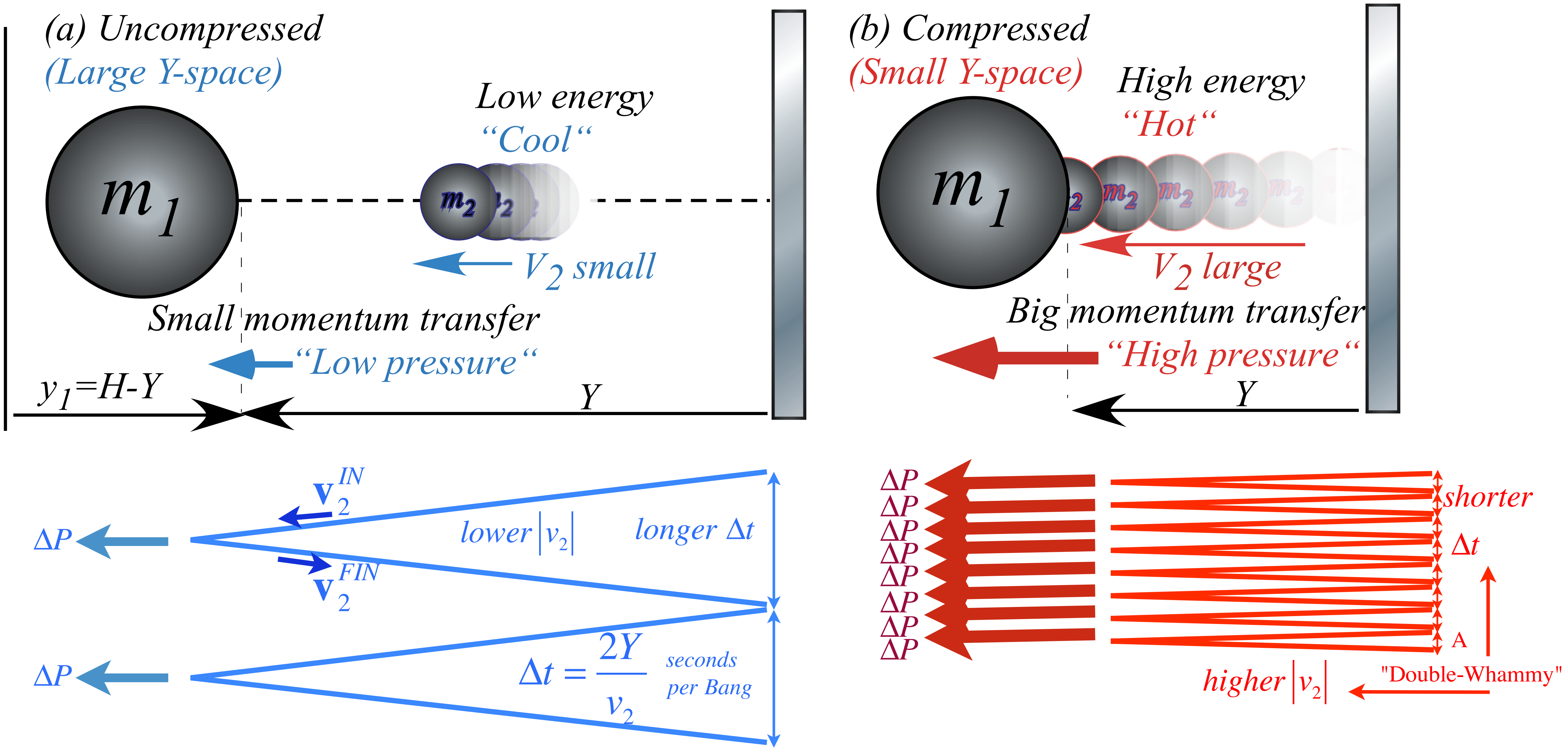

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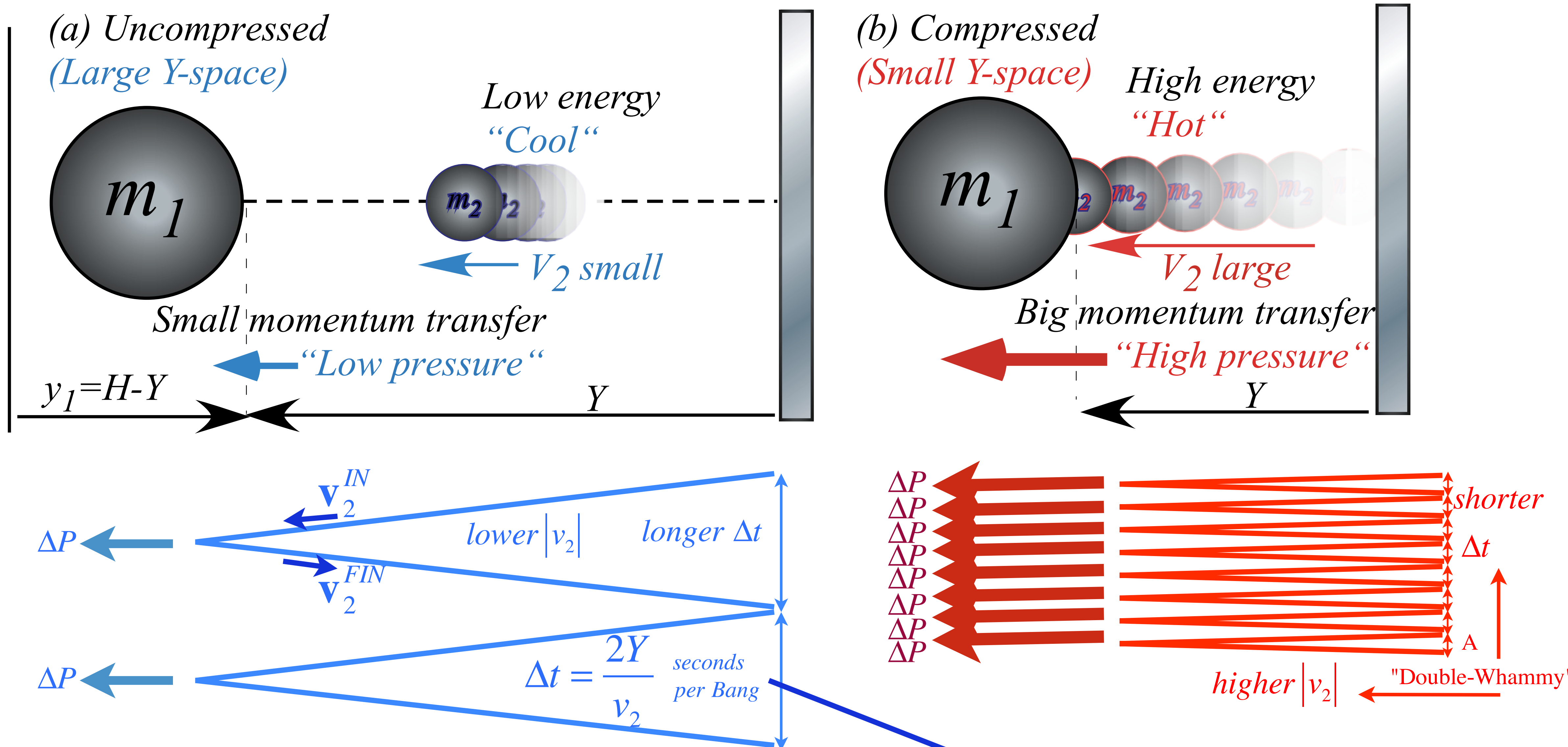
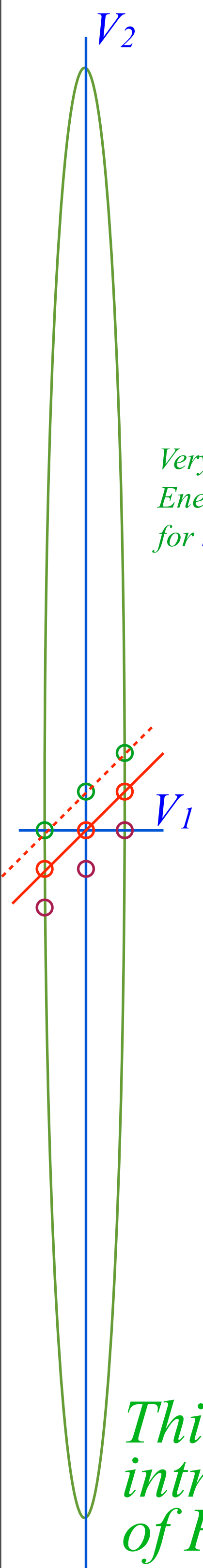


This introduction of Force...

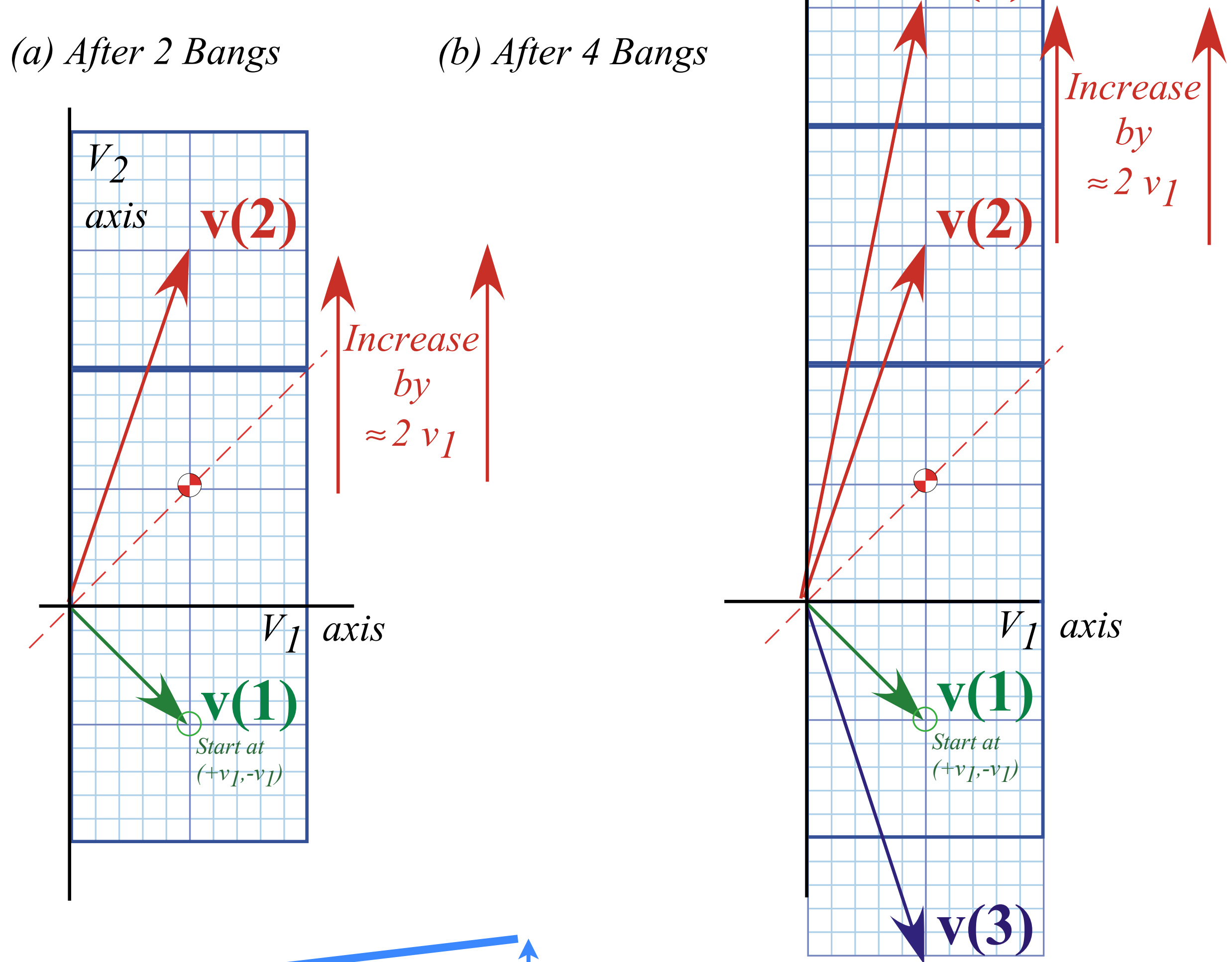
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

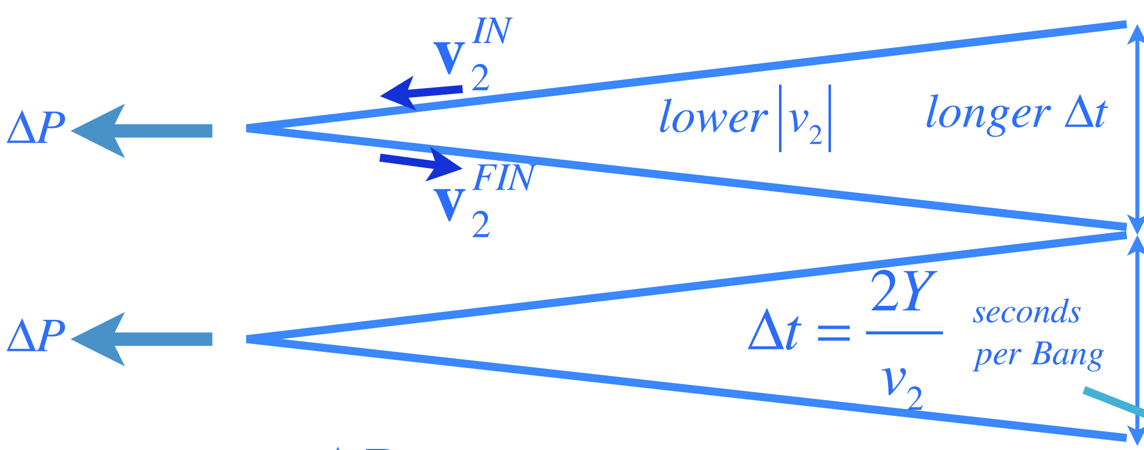
...is more of a <u>definition</u> than another <u>axiom</u>



Double-Bang Sequences for $m_1 >> m_2$



Unit 1
Fig. 6.2



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

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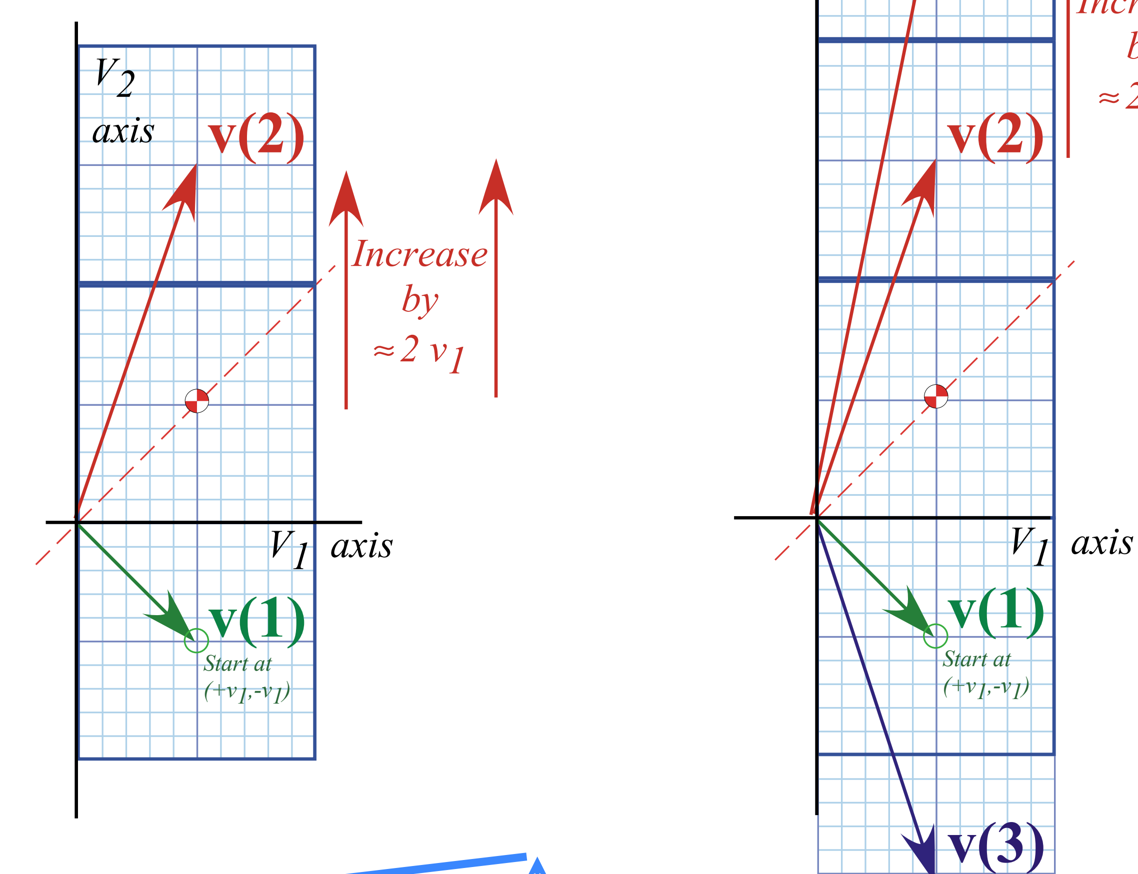
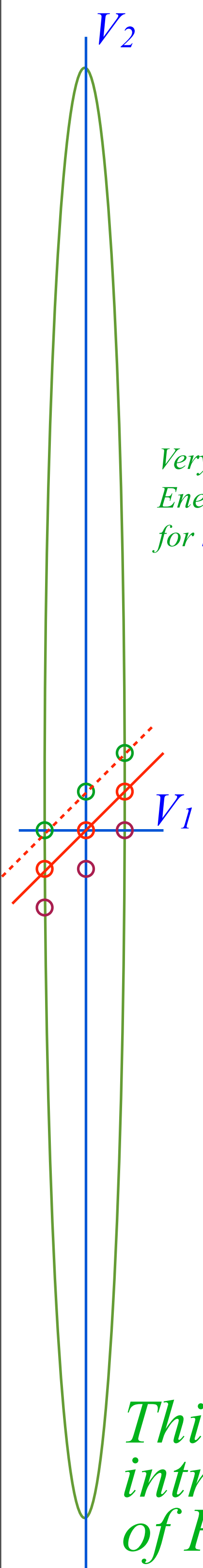
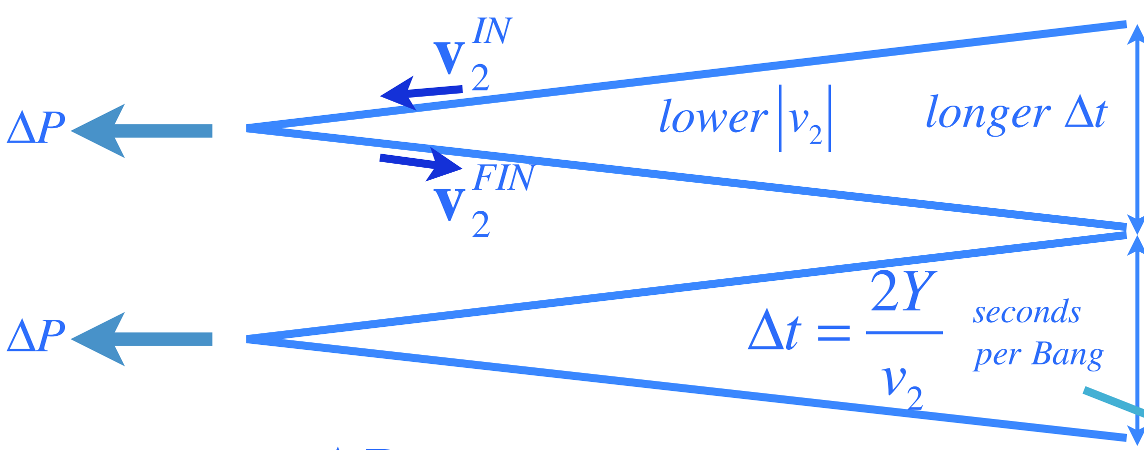




Unit 1 Fig. 6.2



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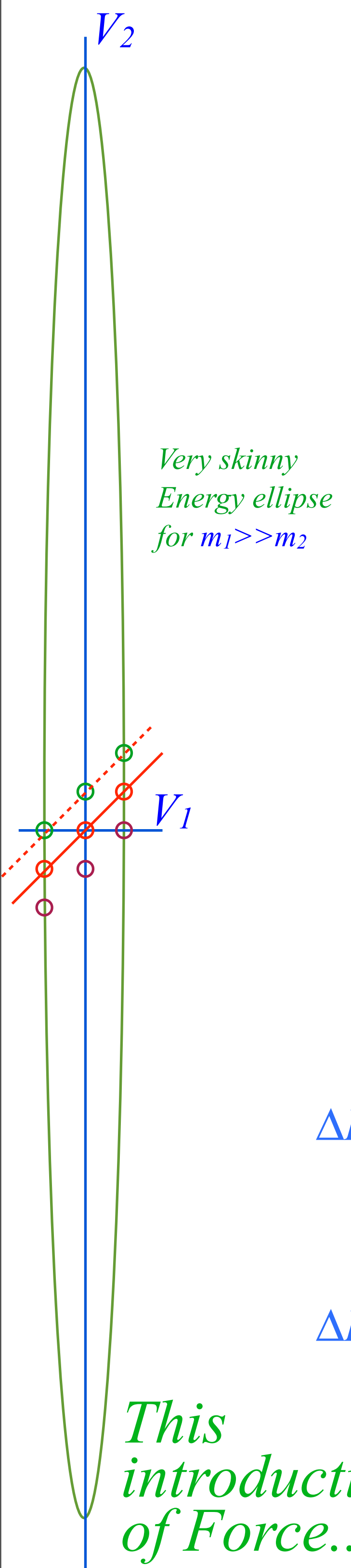
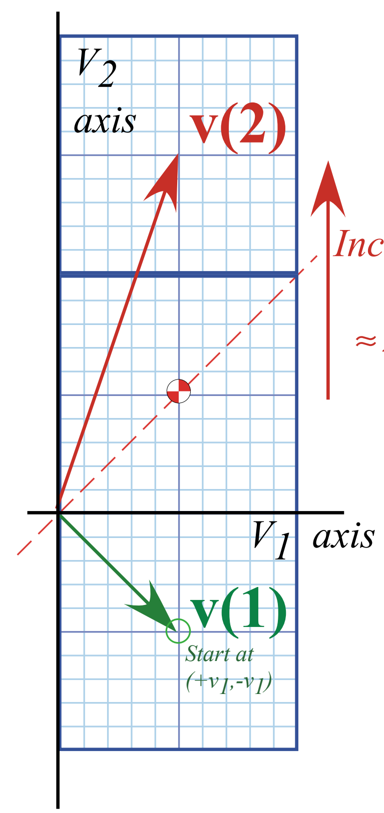
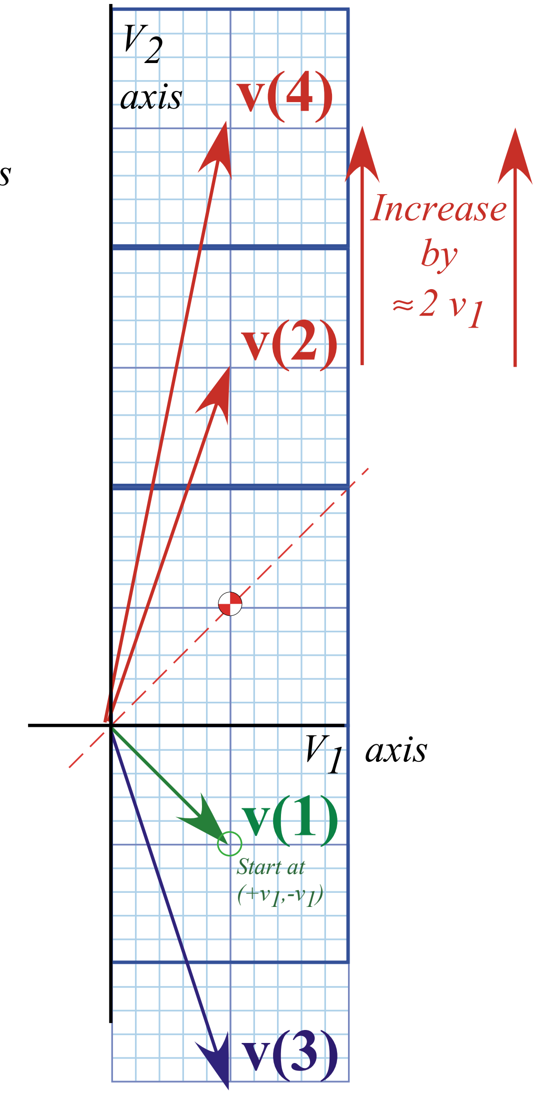


Double-Bang Sequences
for $m_1 >> m_2$

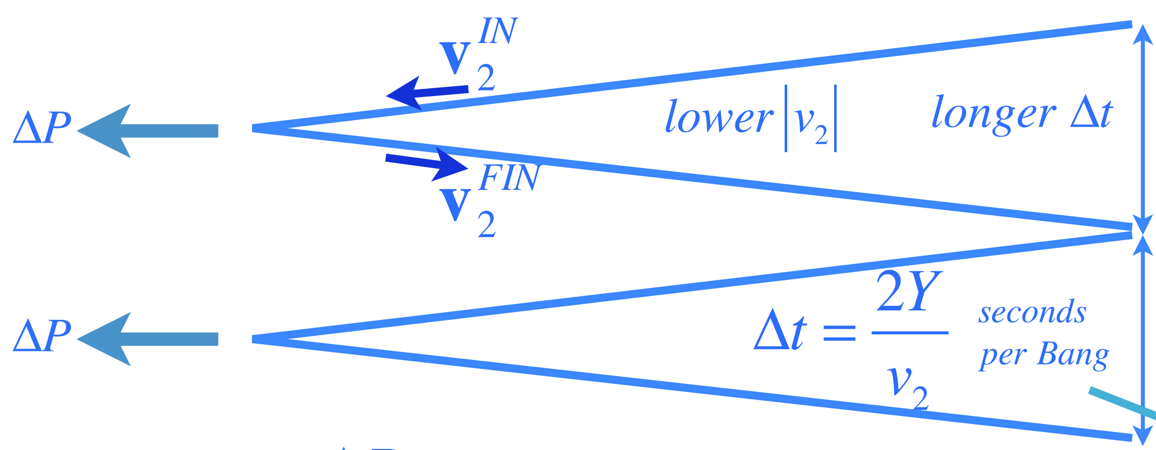


(a) After 2 Bangs



(b) After 4 Bangs

Unit 1
Fig. 6.2



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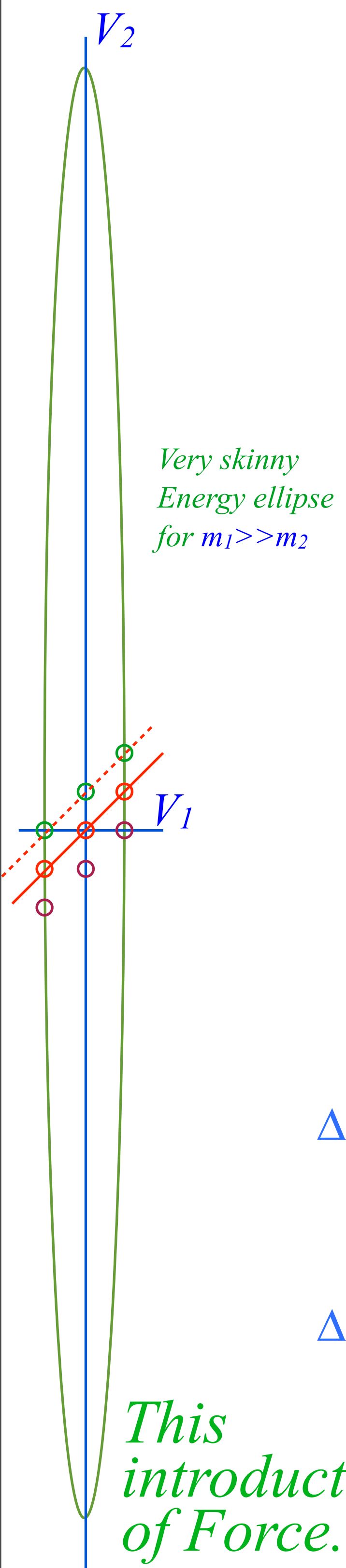
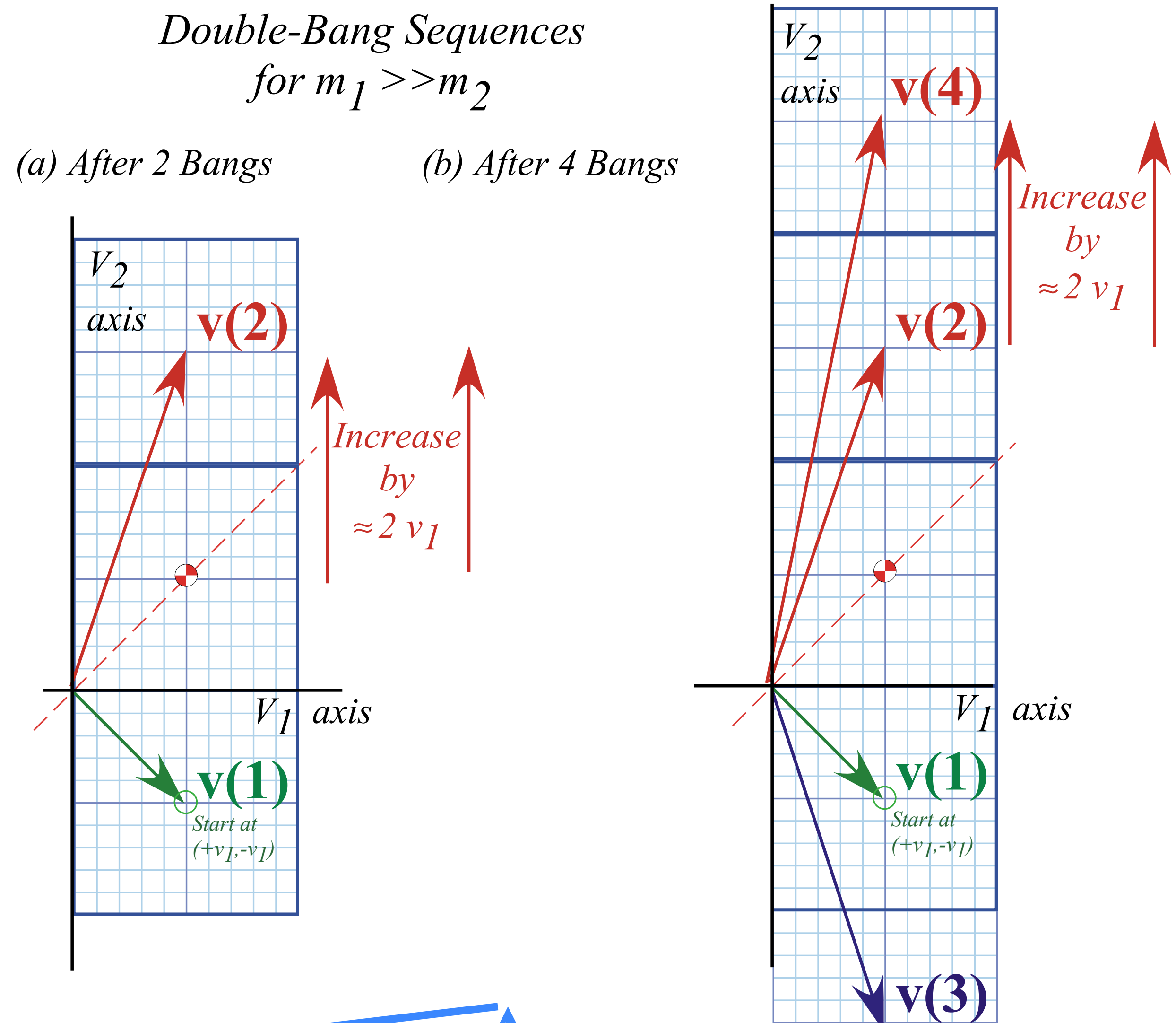
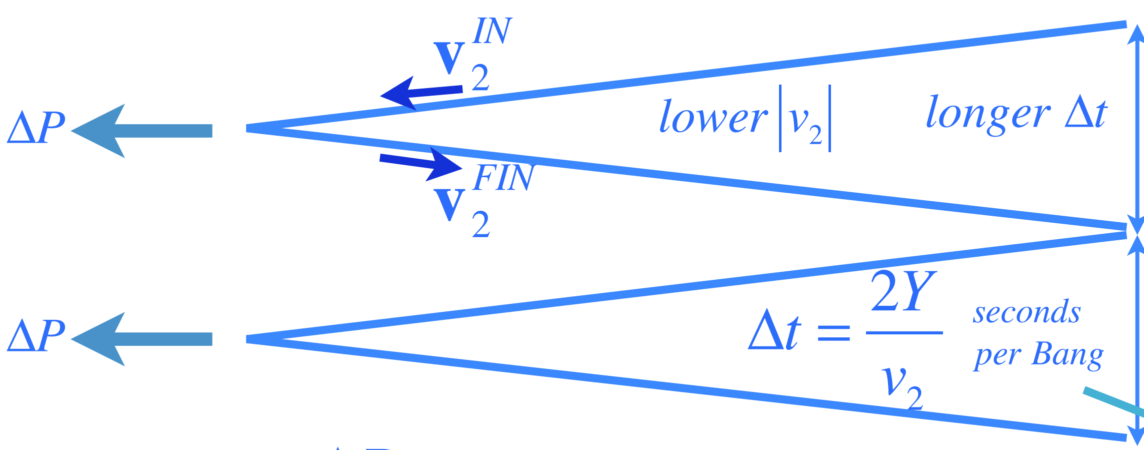
Assuming slow $m_1 : v_1 \ll v_2$

Double-Bang Sequences for $m_1 >> m_2$





Unit 1 Fig. 6.2



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$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Assuming slow m_1 : $v_1 \ll v_2$

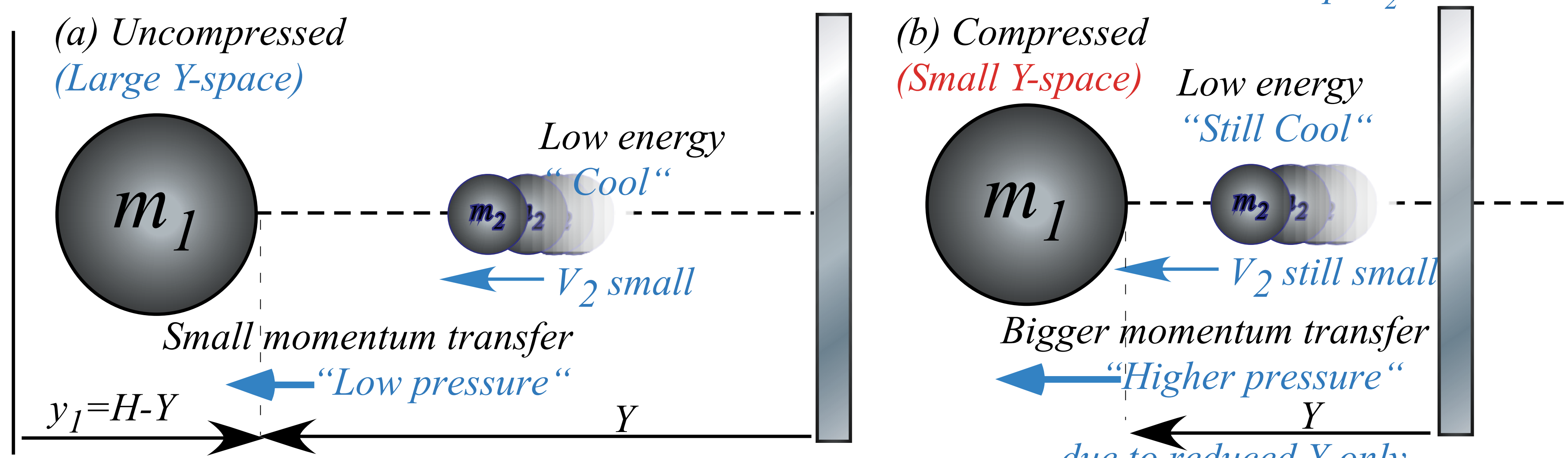
$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

Isothermal expansion or contraction: *Wall serves as thermal bath to keep m_2 cool*



Force "field" or "pressure" due to many small bounces

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1D-Isothermal Force Law (assume v_2 is constant for all Y):

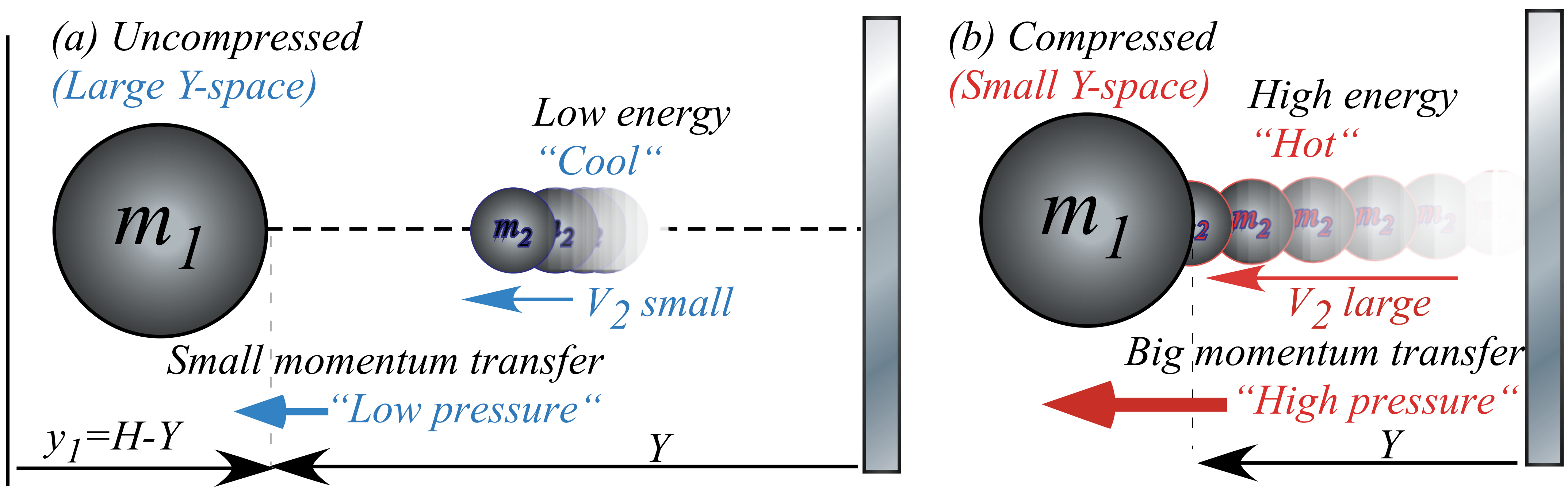
$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

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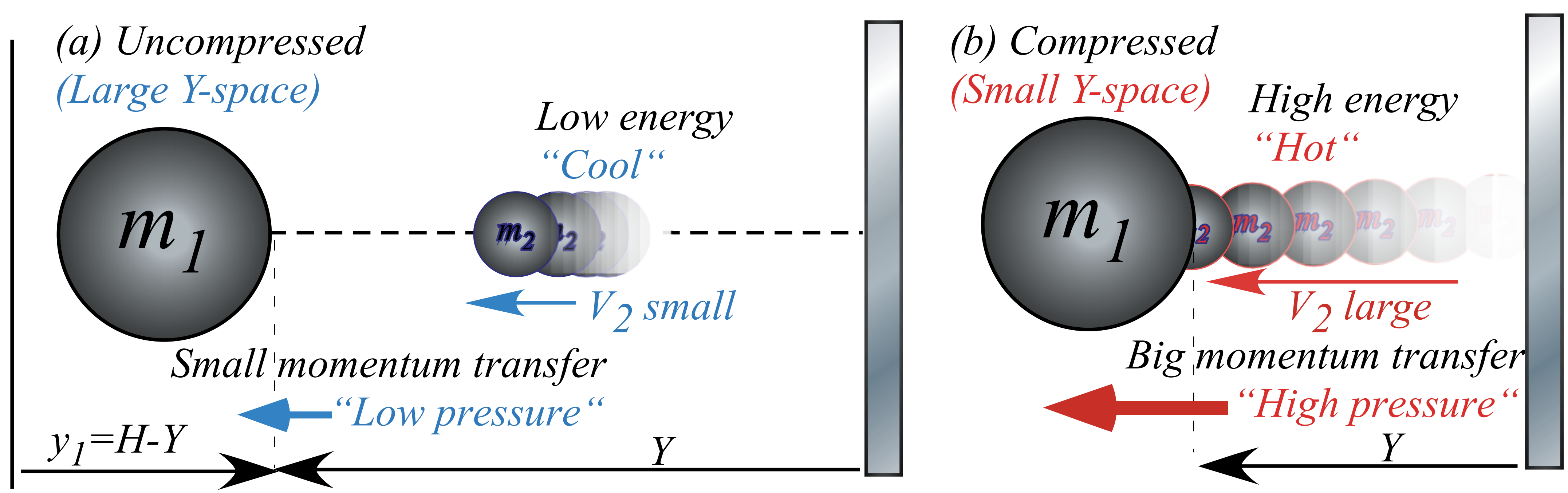
$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y}\qquad \text{simplifies to:}\quad \frac{dv_2}{v_2}=-\frac{dY}{Y}$$

Wall not given time to give or take KE



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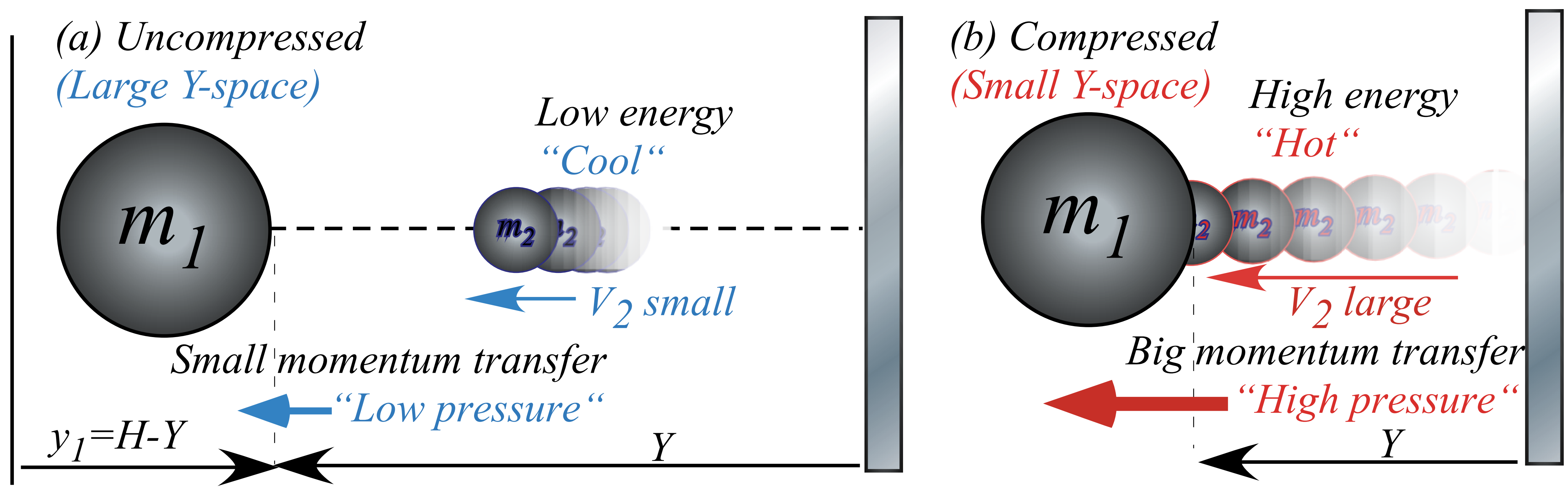
Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e (e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln \frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

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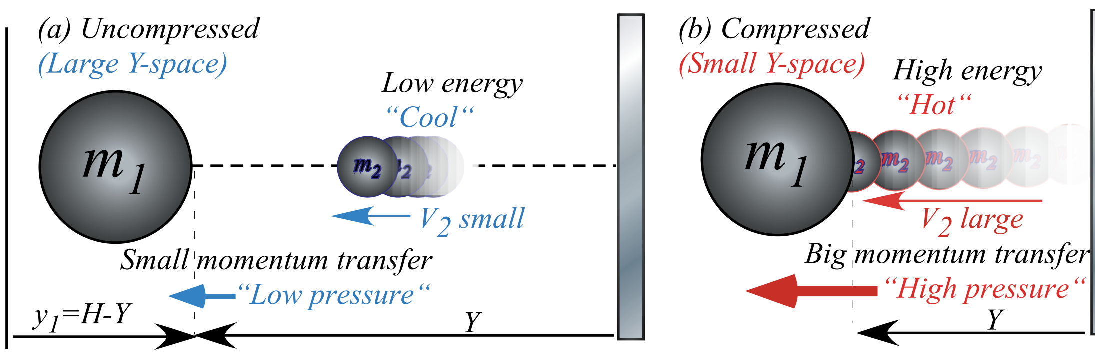
Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$*):* $F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}$



A "Double-Whammy"

Potential field due to many small bounces

→ *Example of 1D-Adiabatic potential* $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ *vs. Mathematician's Definition* $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

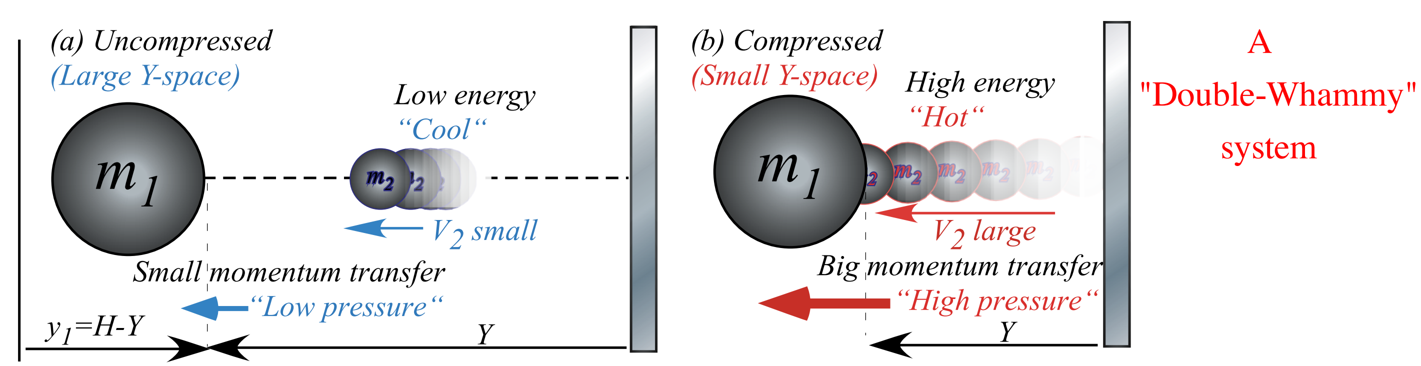
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small (m_2 << m_1) rapidly (v_2 >>v_1) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \boxed{\frac{1}{2}m_1v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



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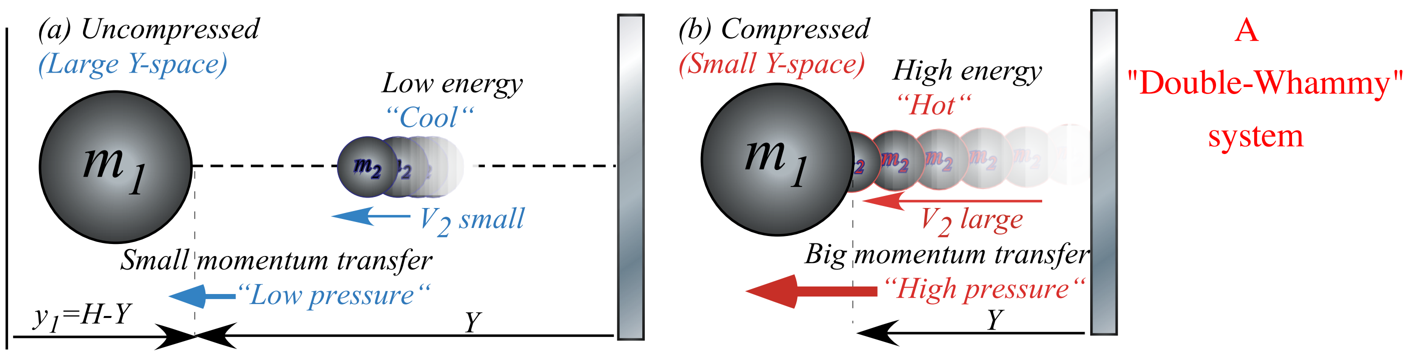
Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$



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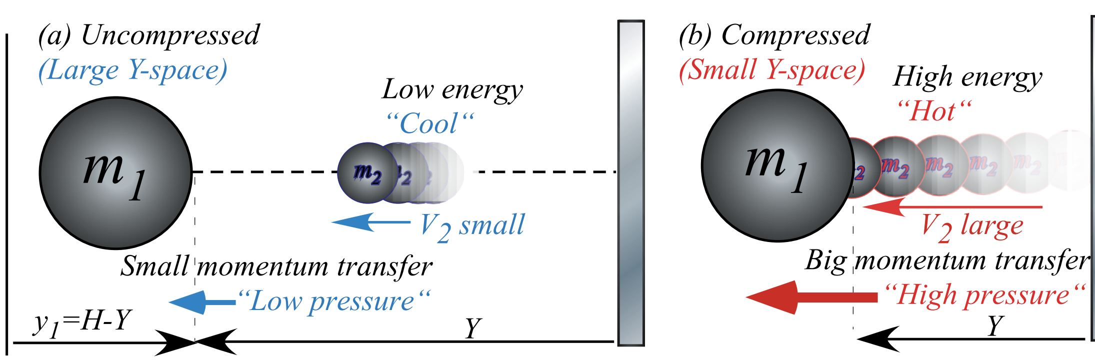
Q*?Another axiom?*



A "Double-Whammy" system

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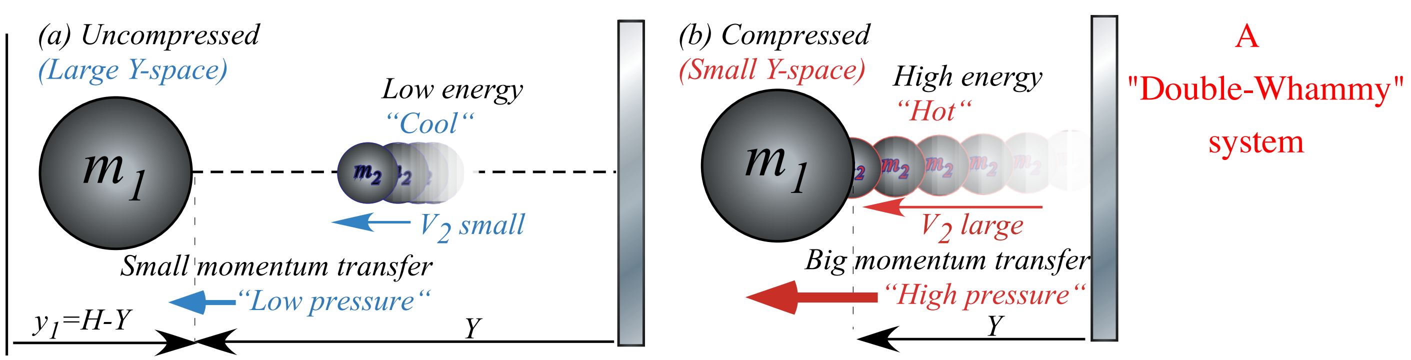
$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

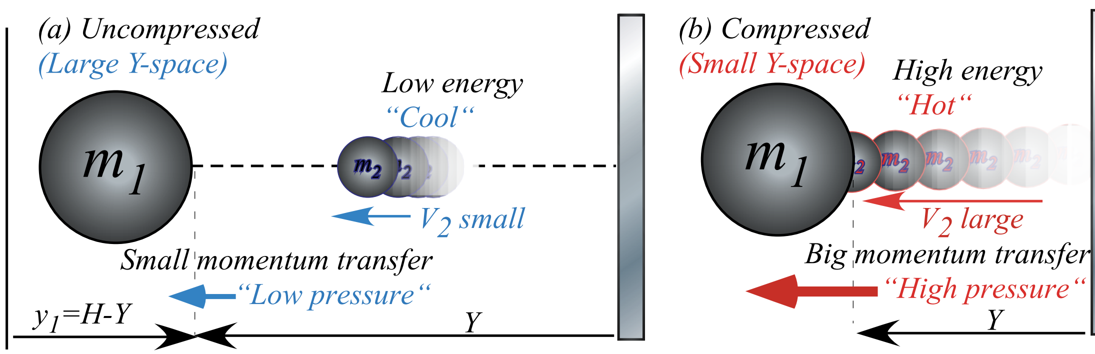
Define for big mass m_1 : *Kinetic energy* *KE(v_1)* *vs* *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F\cdot dY = \pm dU$

Q*?Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$

(Here: $V = v_2$)



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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Define for big mass m_1: *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$

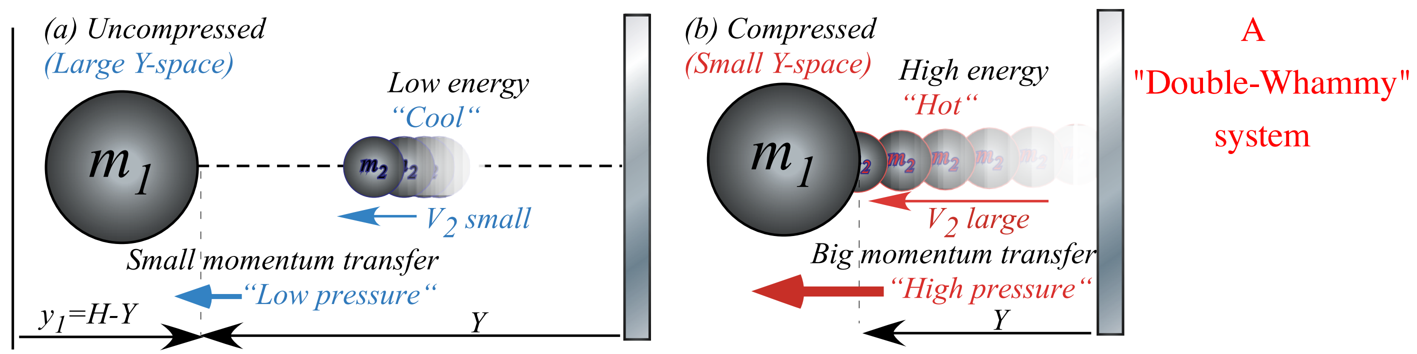
Q*?Another axiom?* A: No.

$$\int F \cdot dY = \int \frac{dp}{dt} \cdot dY = \int \frac{dY}{dt} \cdot dp = \int V \cdot dp = \int V \cdot d(mV) = m\frac{V^2}{2} + const = U$$

or else :

$$F \cdot \frac{dY}{dt} = \frac{dp}{dt} \cdot V = \frac{d(mV)}{dt} \cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$$

(Here: $V = v_2$)



Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

→ *Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$*

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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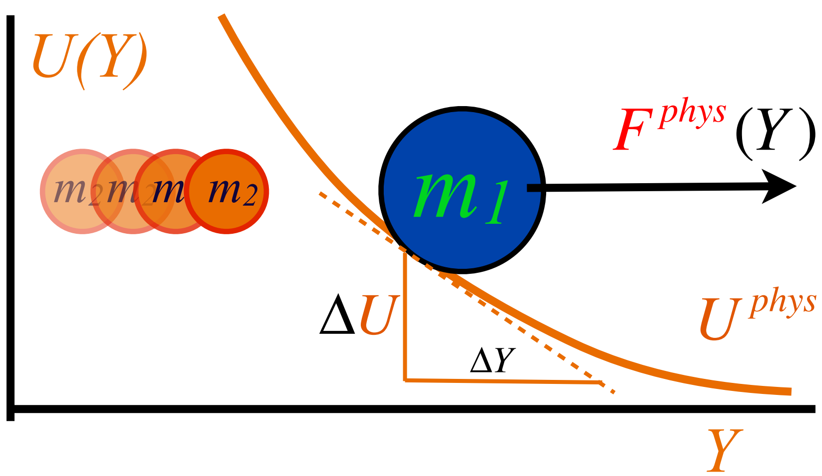
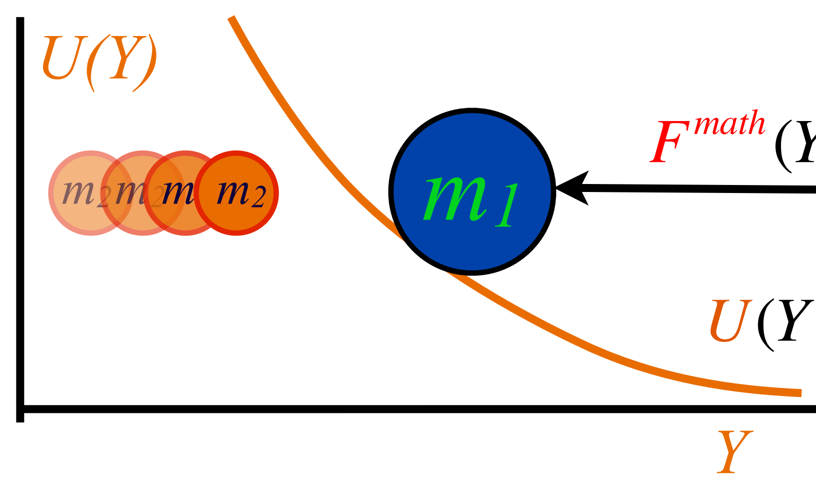
Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

The "Physicist" View of Force | *The "Mathematician" View of Force*



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

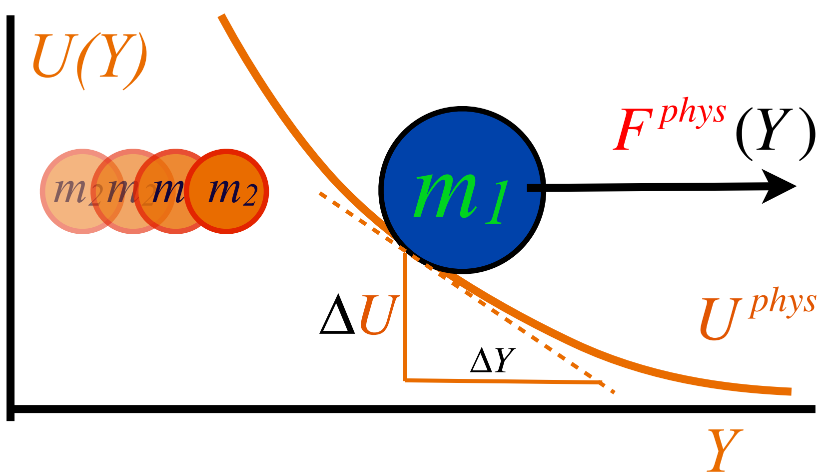
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F\cdot dY = \pm dU$

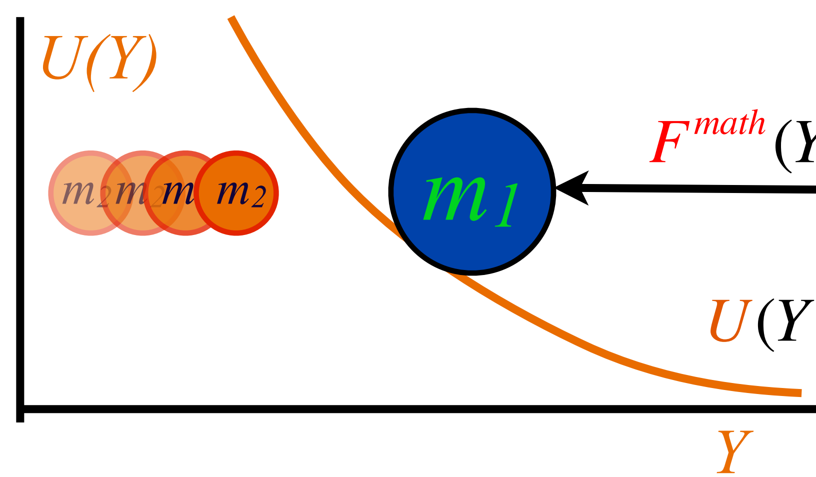
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, does it work?)

For the "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

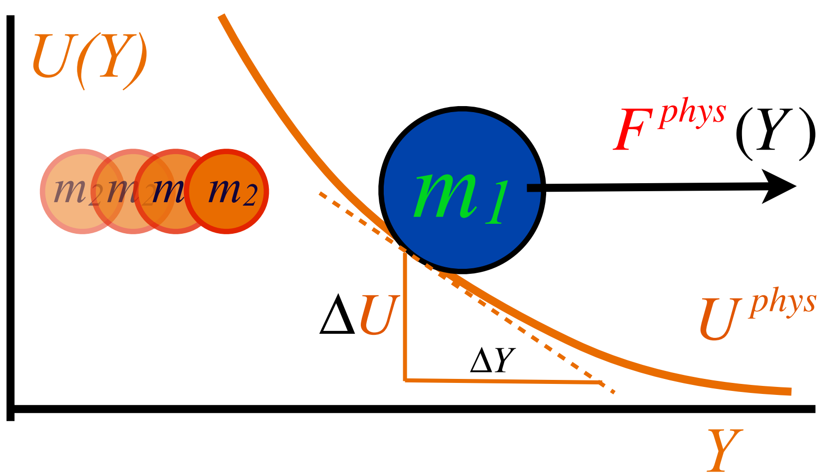
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

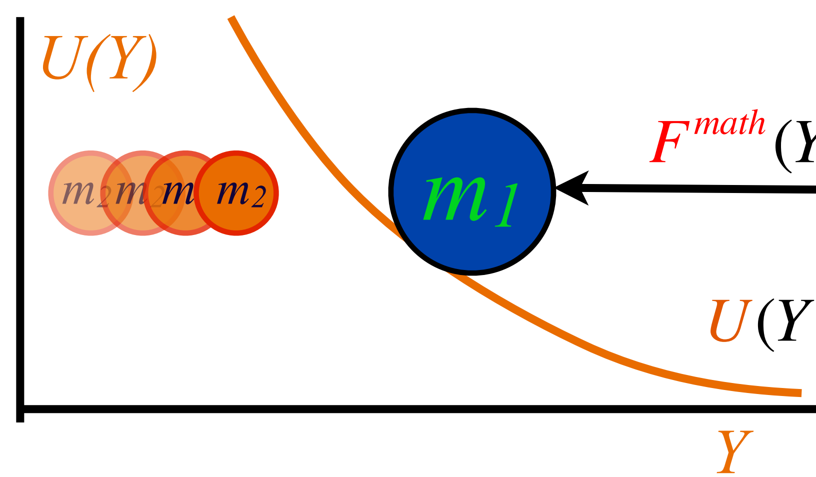
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\,dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\,dY$$

(OK, But, does it work?)

For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with:

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$*

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

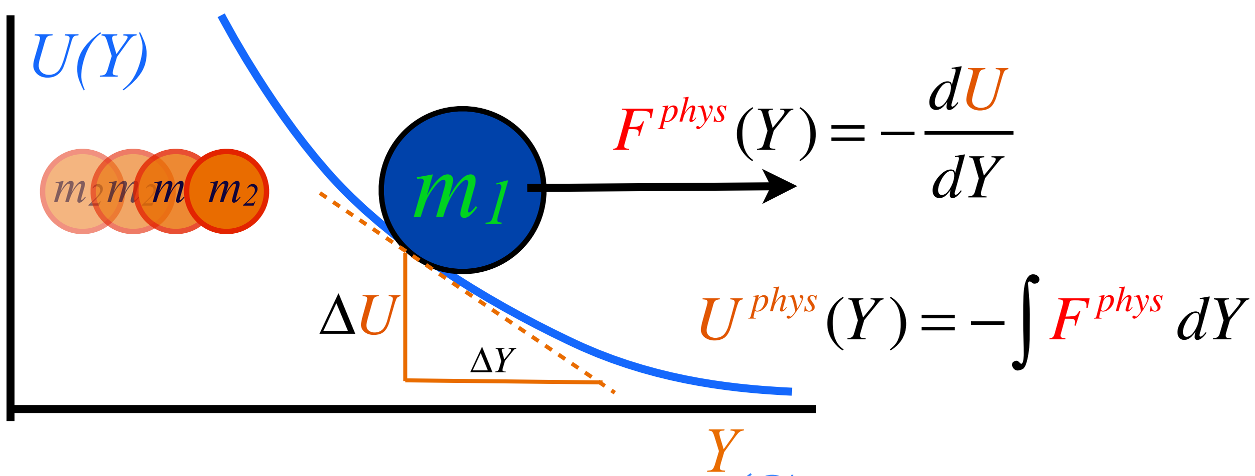
$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y}$$ *implies :* $$U(Y)=\int -F^{phys}dY=\int -\frac{m_2 v_2^2}{Y}dY=-m_2 v_2^2 \ln(Y)$$

$$const.=E=\frac{1}{2}m_1 v_1^2+U(Y) \quad where: U(Y)=-m_2 v_2^2 \ln(Y)$$

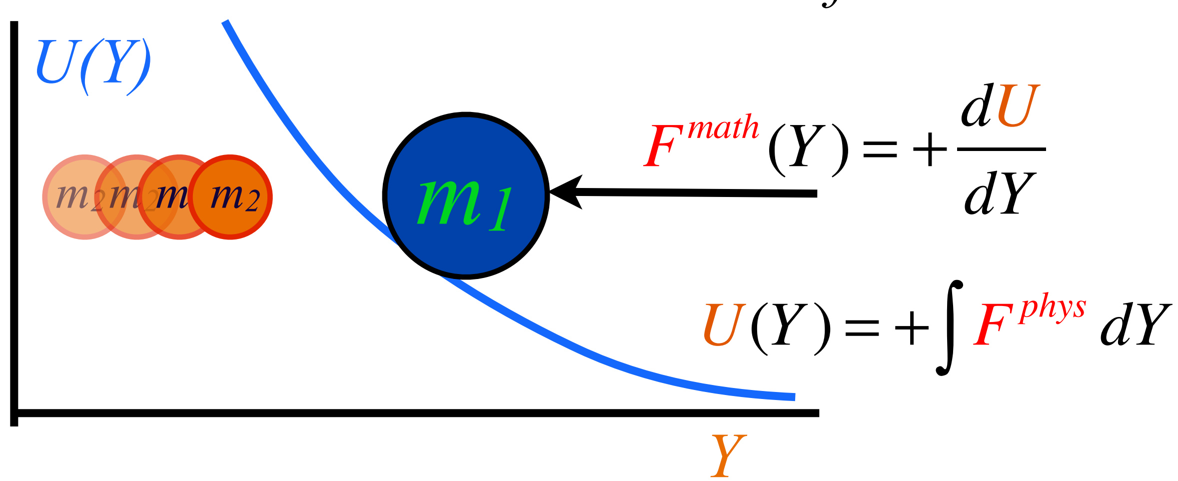
Define for big mass m_1: *Kinetic energy KE(v_1) vs Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)$=-m_2 v_2^2 \ln(Y)$ relates to *Force F(Y)* thru *Work relations* $F\cdot dY=\pm dU$

The "Physicist" View of Force



The "Mathematician" View of Force



(Same integral/differential relations)

$$F^{phys}=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$ *consistent with :* $$F^{phys}=-\frac{\Delta U}{\Delta Y}=-\frac{d}{dY}\left(-const.\ln(Y)\right)=\frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const. \ln(y)$

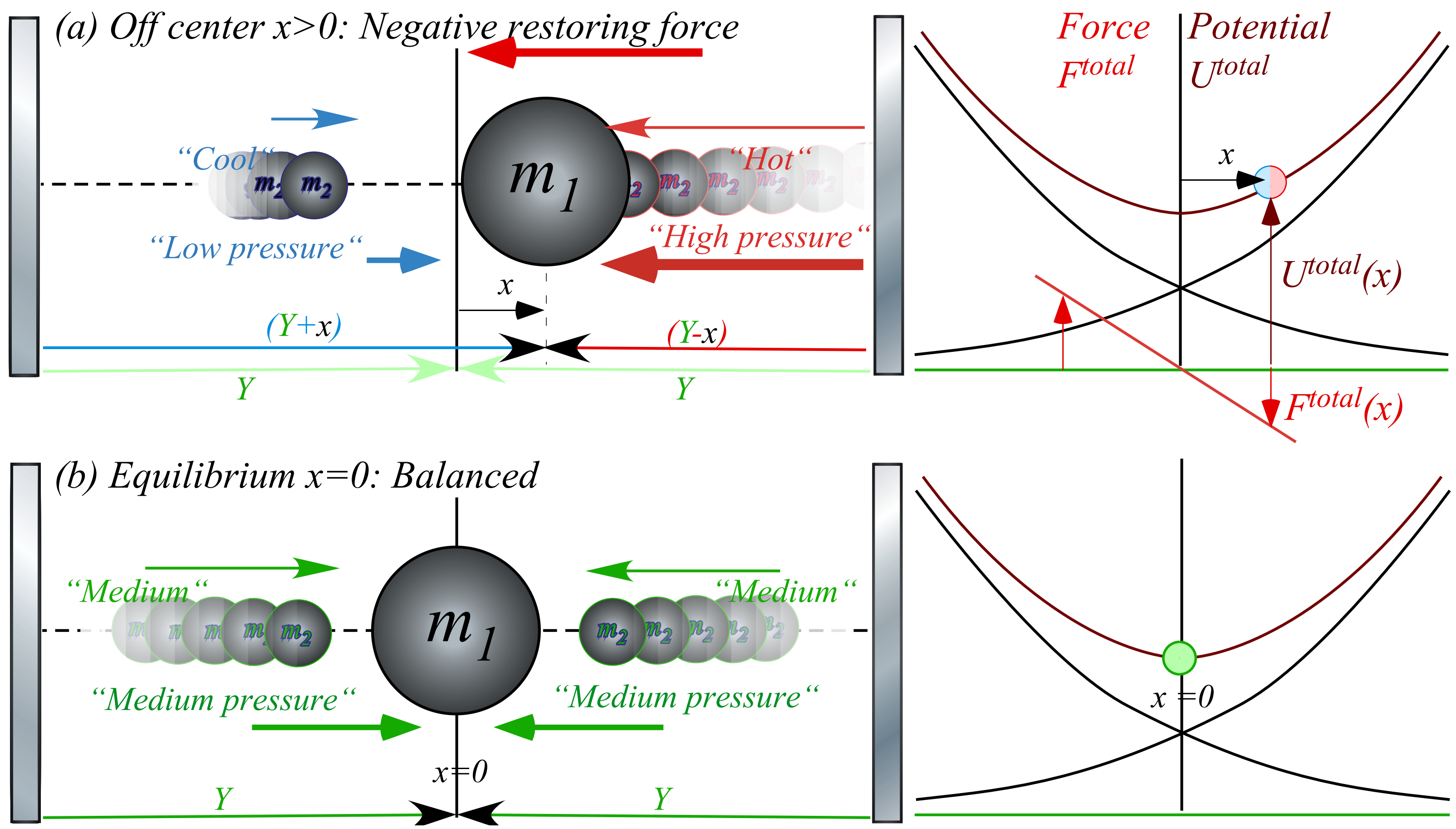
→ *Example of oscillator with opposing Isothermal potentials*

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U=-m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

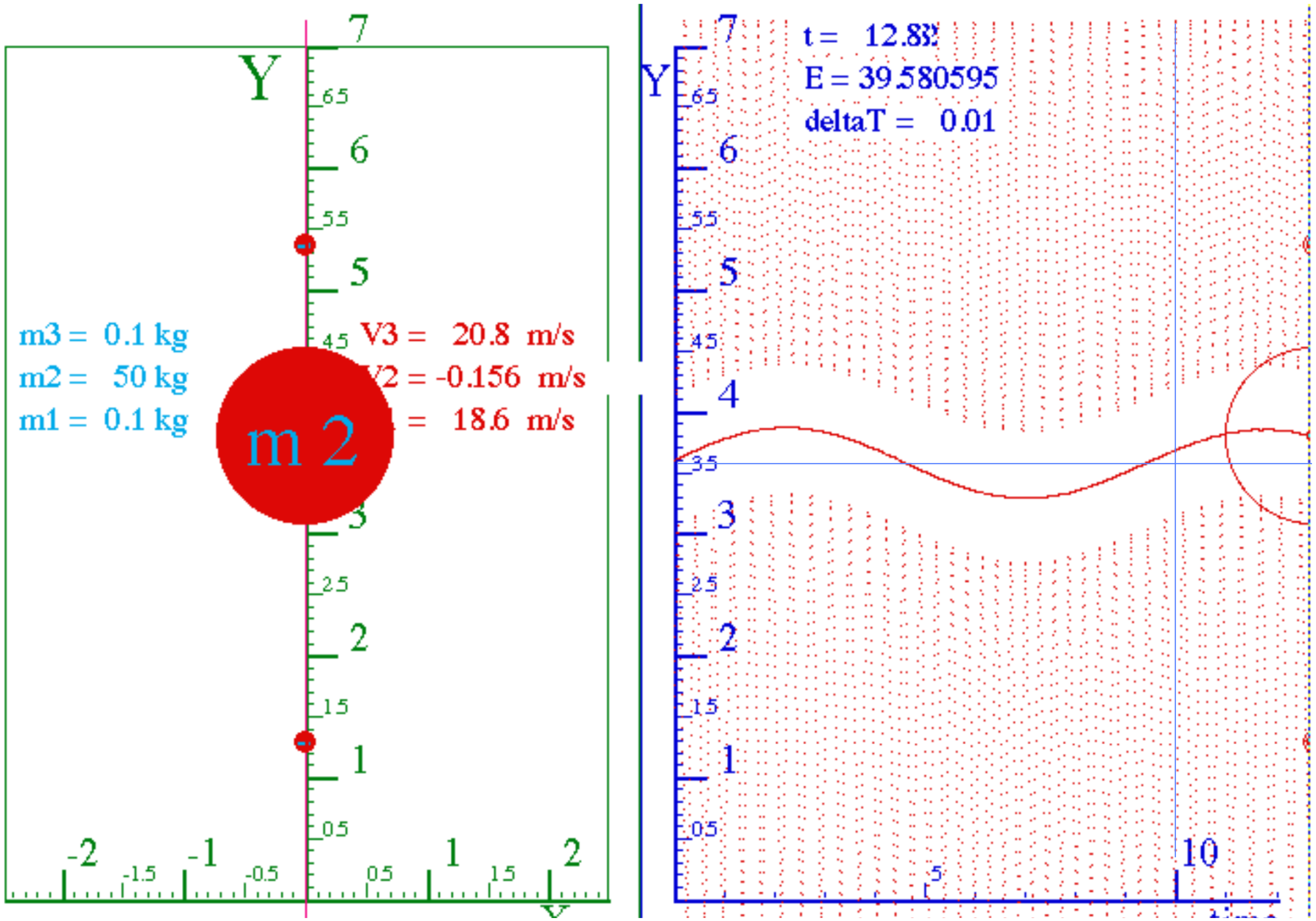
Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{1+x}-\frac{f}{1-x}=f\left[1-x+x^2-x^3\ldots\right]-f\left[1+x+x^2+x^3\ldots\right]=-2f\cdot x-2f\cdot x^3-\ldots$$

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x} \qquad U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx \qquad \text{HO }\measuredangle\text{frequency: } \omega=\sqrt{\frac{k}{m_1}}=2\pi\upsilon$$



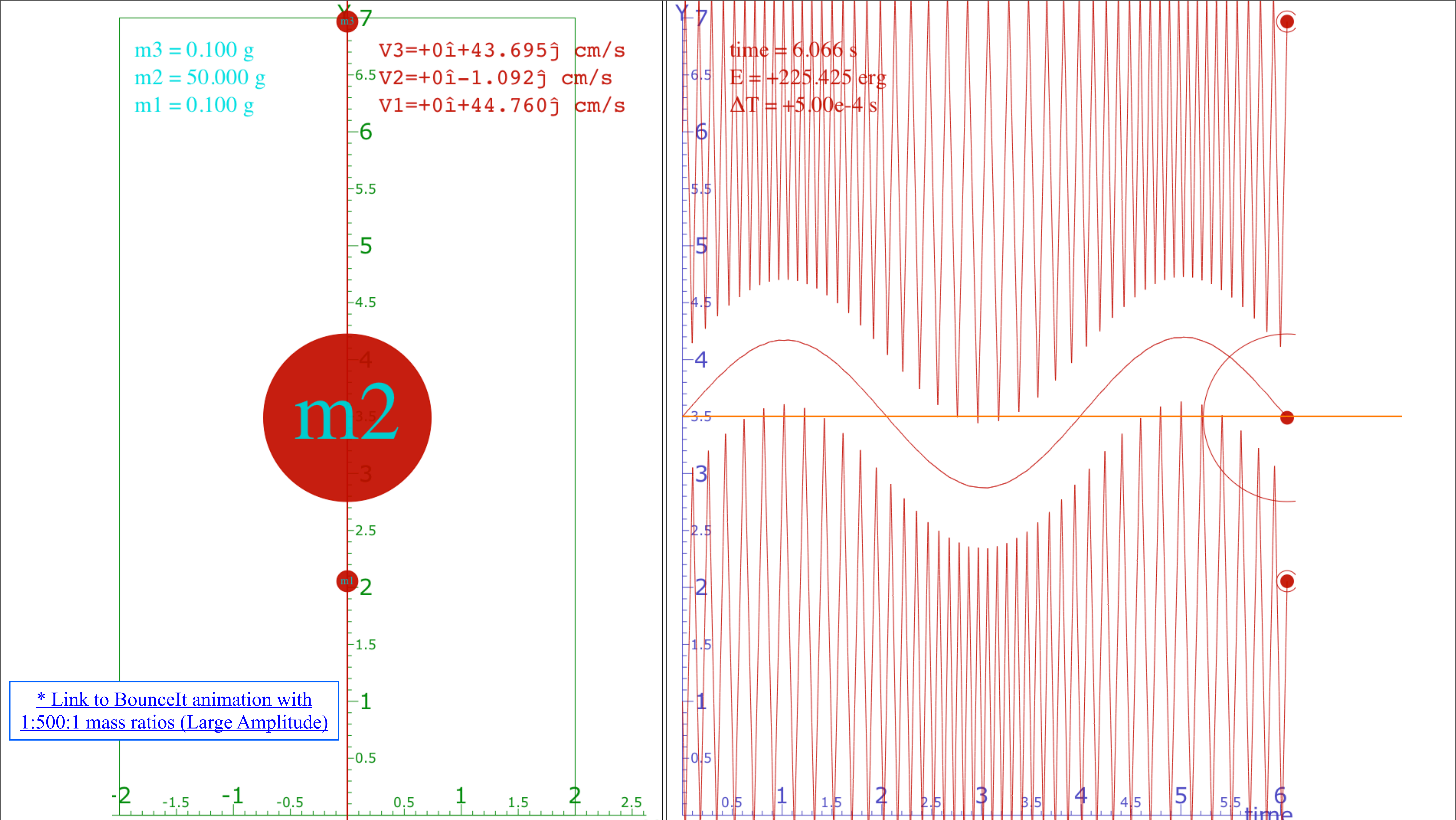
Unit 1
Fig. 6.3

Simulation of

the adiabatic case

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]

See Homework problem 1.6.1: *Compute frequency and/or period for both isoT and adiabatic cases*



Initial x1 = 0.75 | y Max = 7 | y Min = 0

Max x PE plot = 0.5 | T Max = 6

F-Vector scale = 0.003 | V2y Max = 3

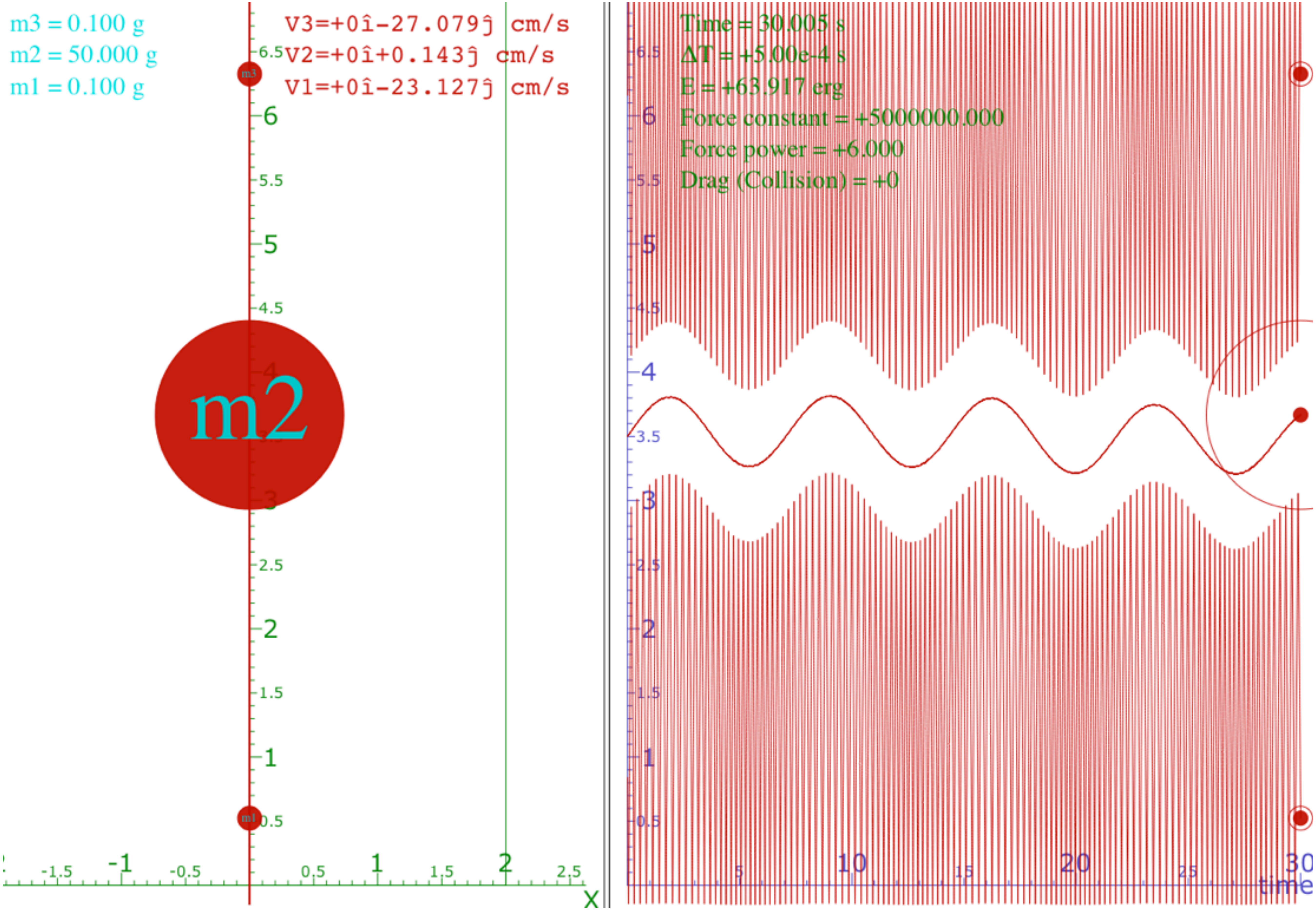
Error step = 0.000 | V2y Min = -2

Adiabatic force scenarios	
	Quasi-harmonic oscillation (m1:m2 = 100:1)
	Quasi-harmonic oscillation (m1:m2 = 50:1)
	Quasi-harmonic oscillation (m1:m2 = 25:1)
	Large amplitude (m1:m2 = 100:1)

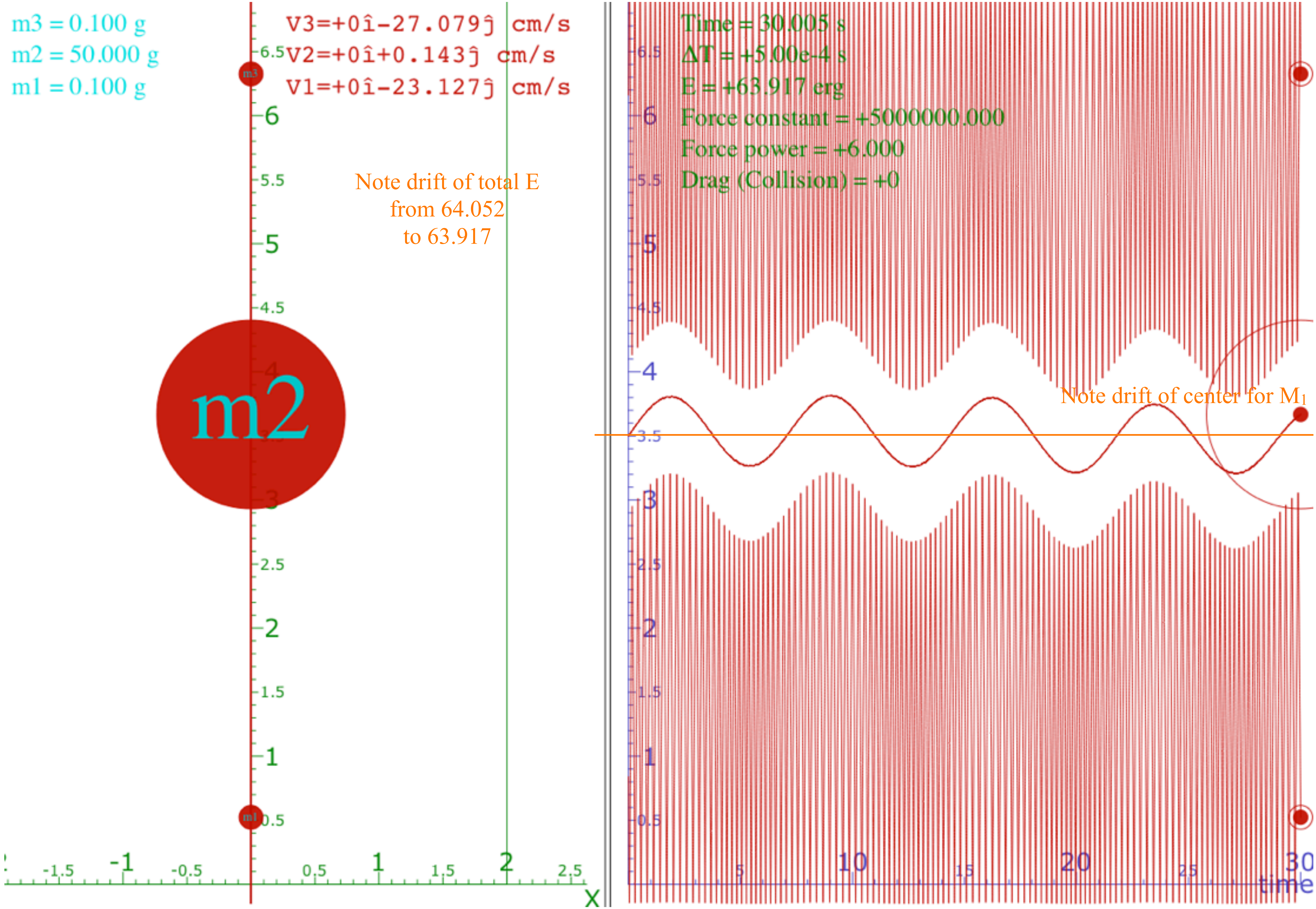
m1= 1 x10^ -1 {g} $X1_0$= 1 x10^ 0 {cm} $V1_0$= -4.5 x10^ 1 {cm/s}

m2= 5 x10^ 1 {g} $X2_0$= 3.5 x10^ 0 {cm} $V2_0$= 1 x10^ 0 {cm/s}

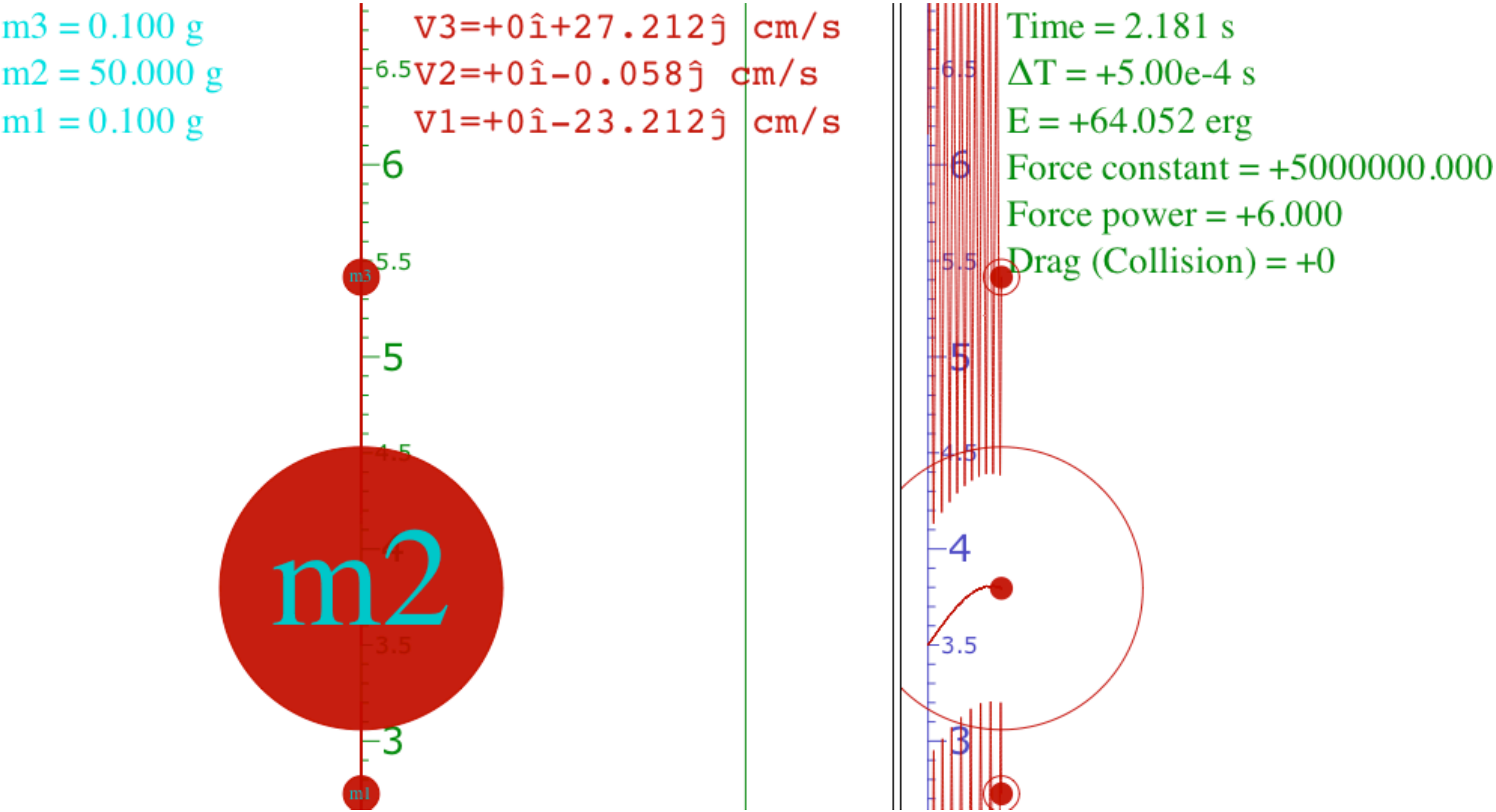
m3= 1 x10^ -1 {g} $X3_0$= 6 x10^ 0 {cm} $V3_0$= 4.5 x10^ 1 {cm/s}



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+27.212ĵ cm/s
V2=+0î-0.058ĵ cm/s
V1=+0î-23.212ĵ cm/s
Time = 2.181 s
ΔT = +5.00e-4 s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0
m3
m2
m1
6.5
6
5.5
5
4.5
4
3.5
3

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

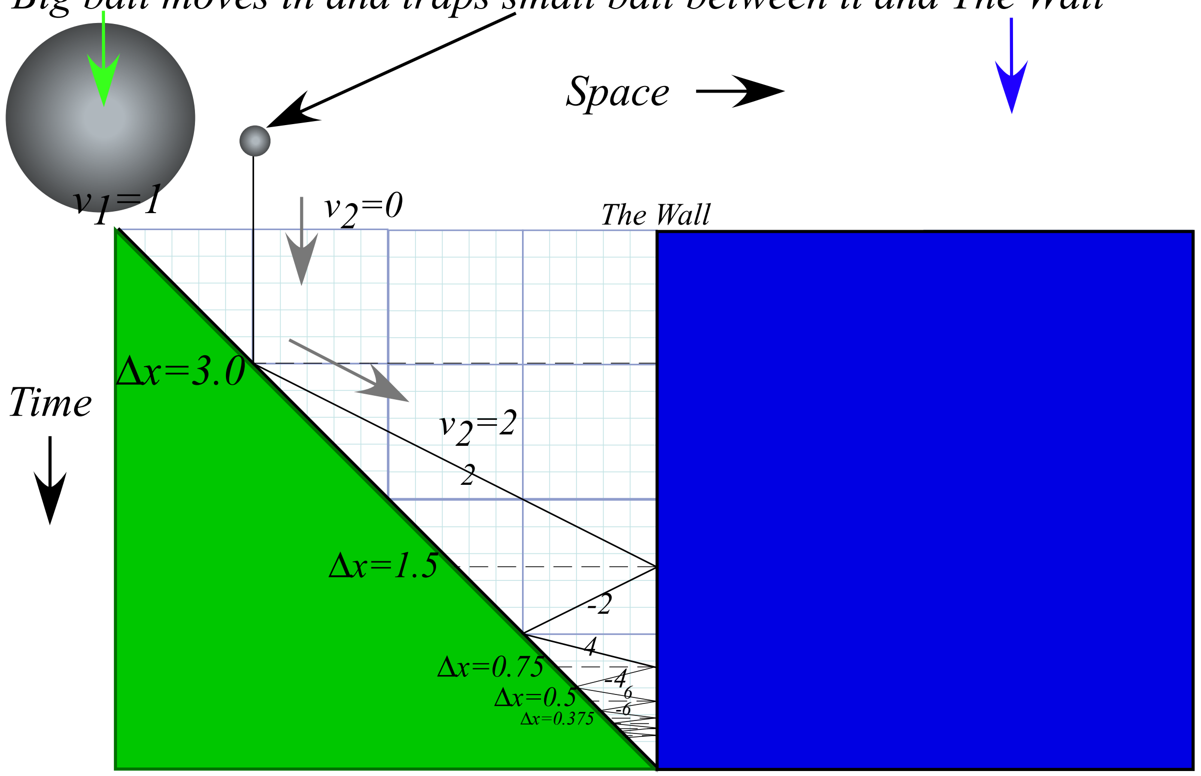
How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

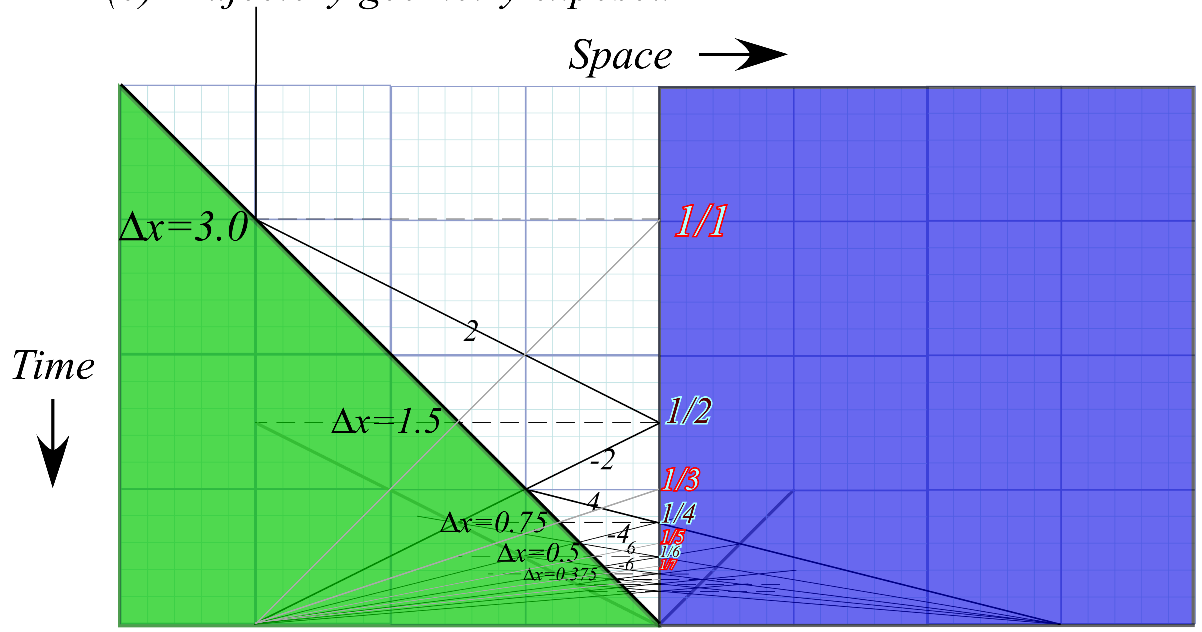
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

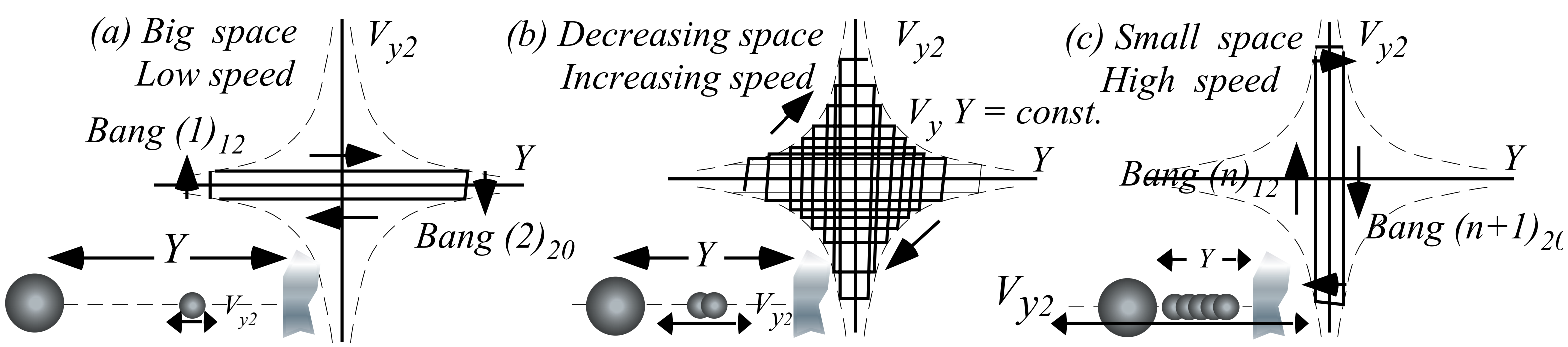
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed





The Classical “Monster Mash”

Classical introduction to

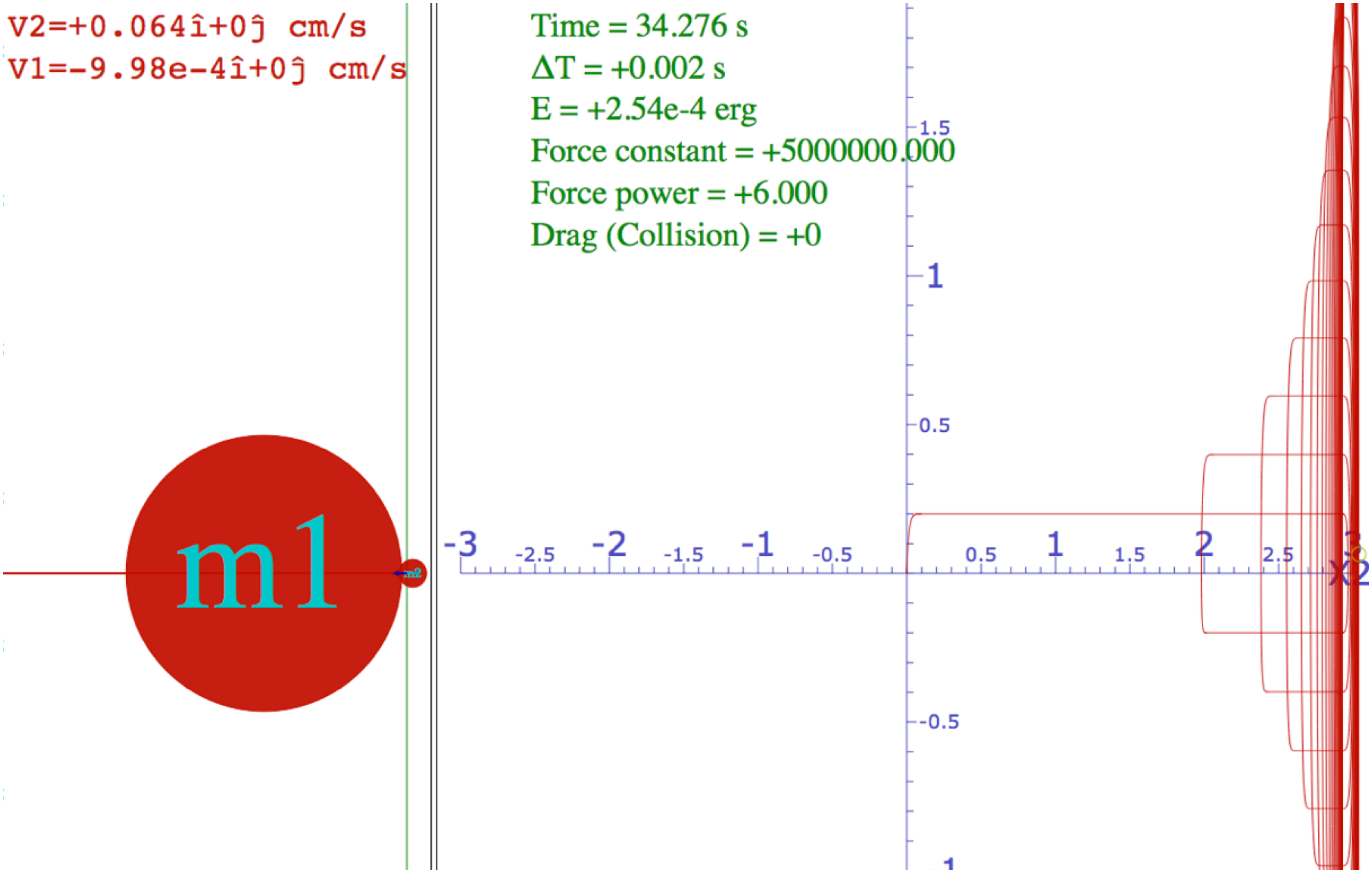
Heisenberg “Uncertainty” Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

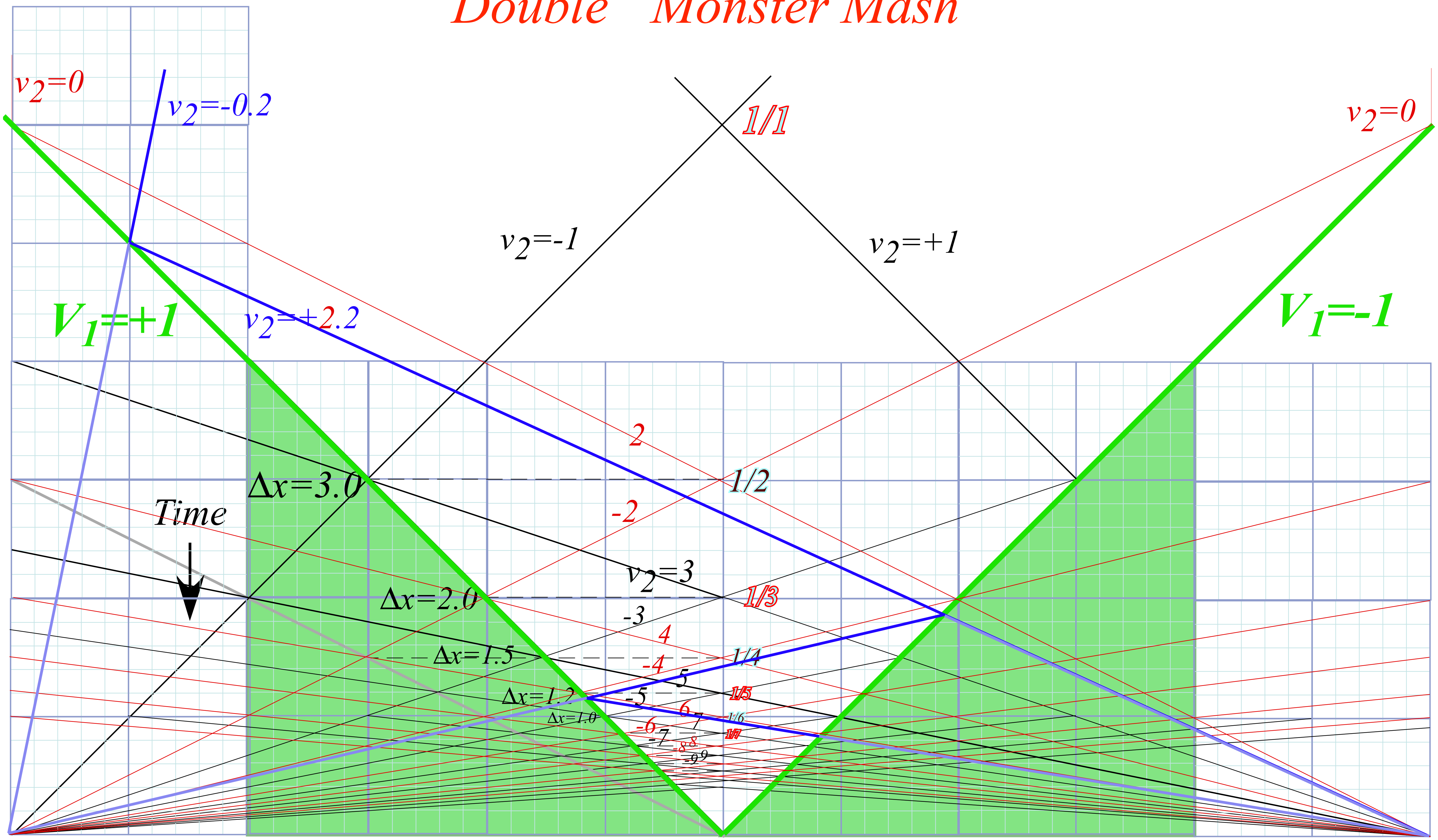
Unit 1
Fig. 6.4

[* Link to BounceIt “*Monster Mash*” $x_2(t)$ animation
(Note: Time sense is inverted)]



* Link to BounceIt "*Monster Mash*" Vx_2 vs x_2 animation

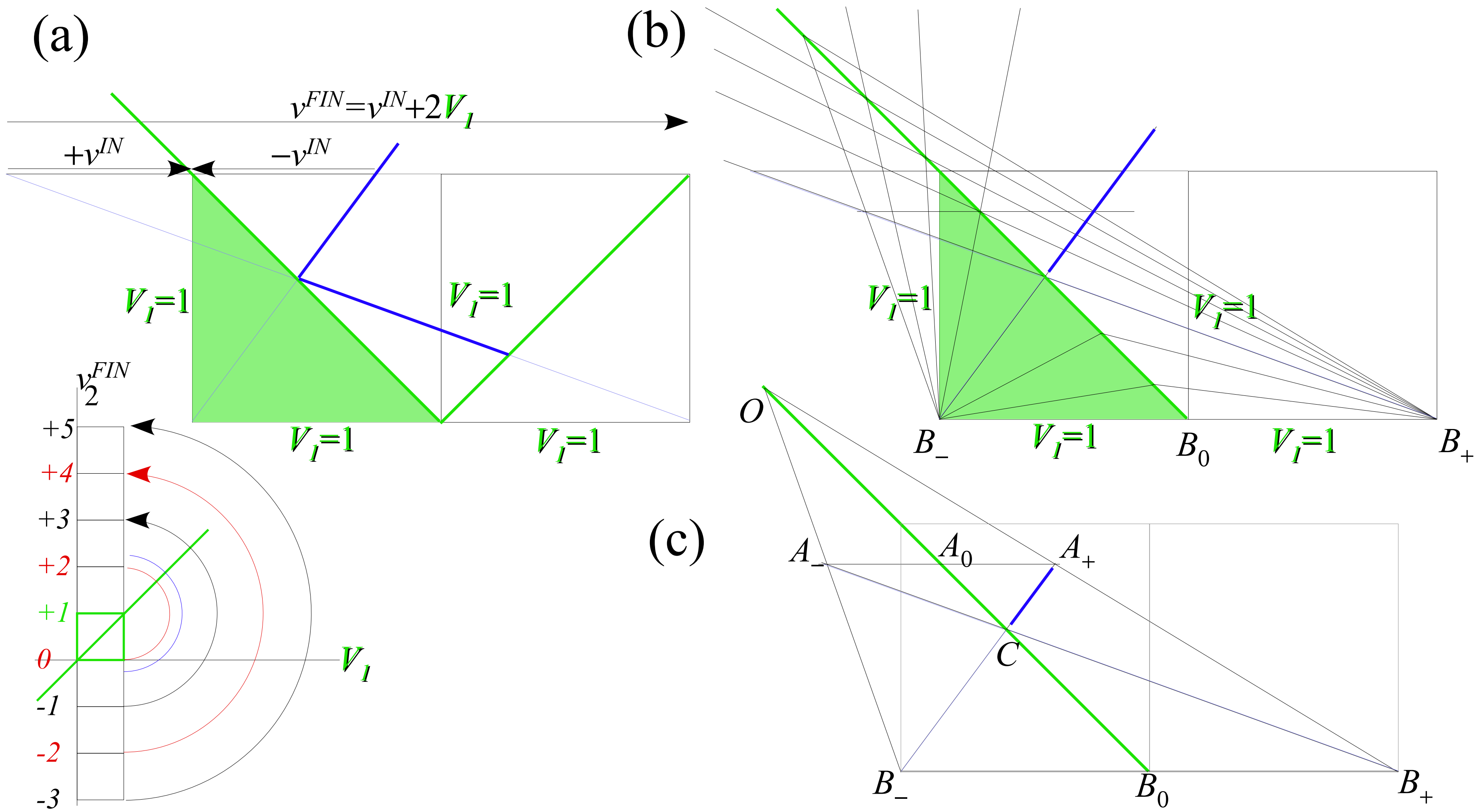
Double "Monster Mash"

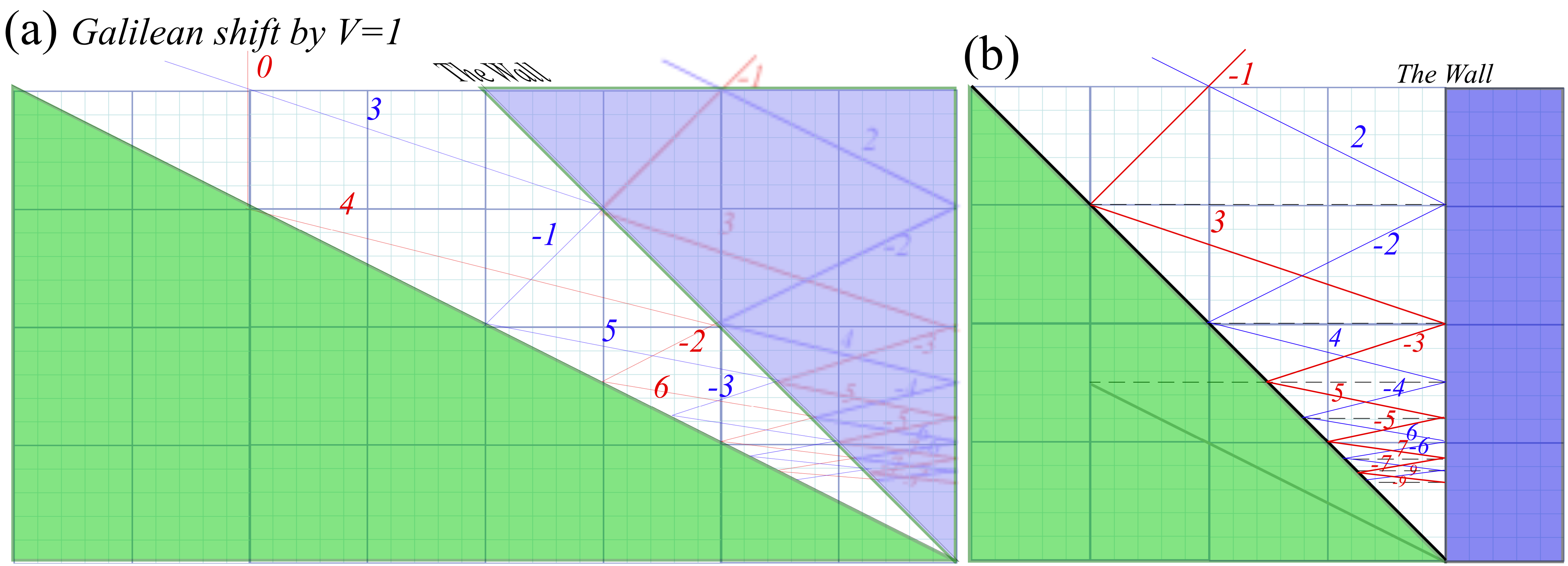


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





"Monster Mash" classical segue to Heisenberg action relations

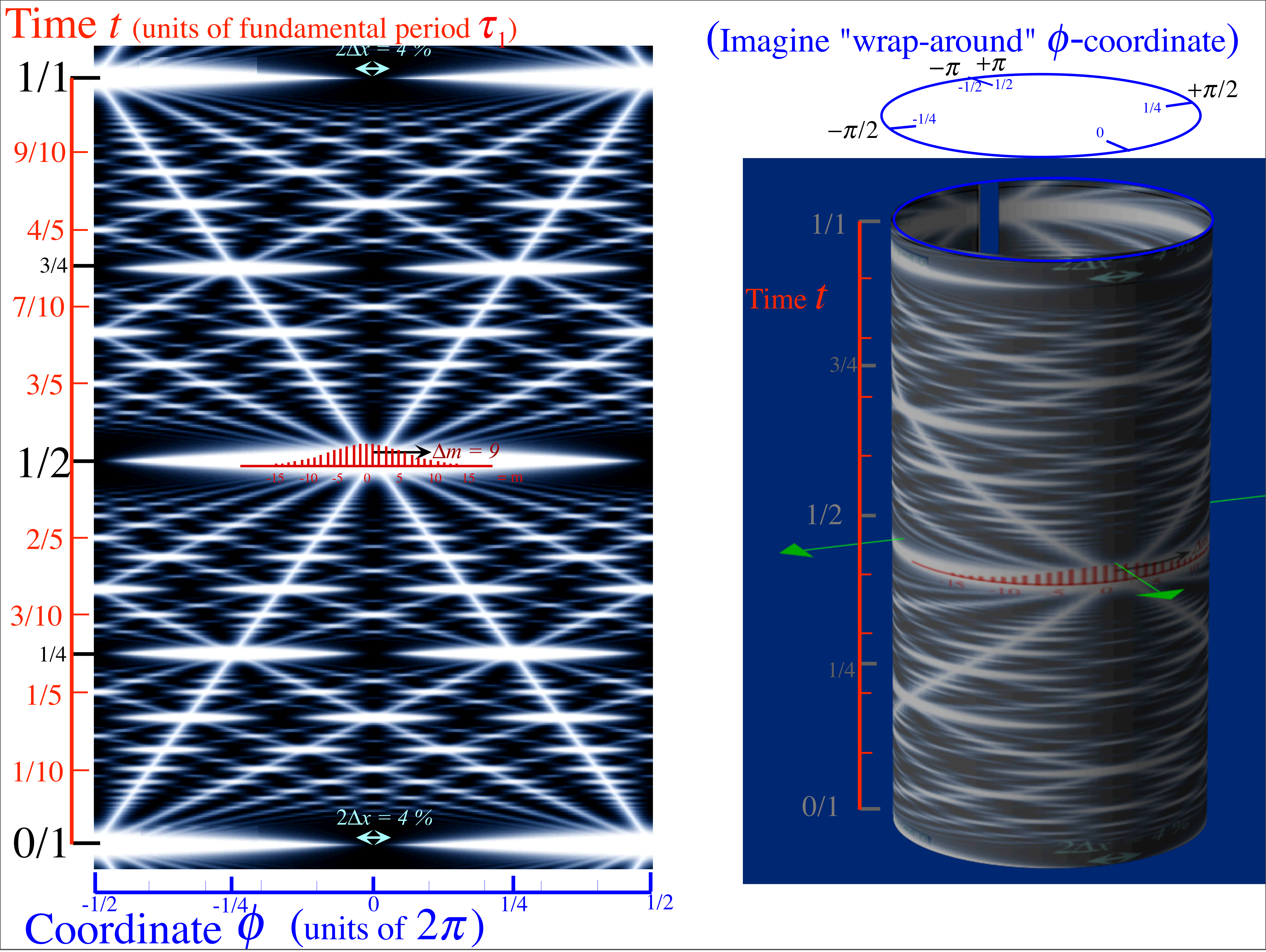
Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

→ *An interesting wave analogy: The "Tiny-Big-Bang"* [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]


Time t (units of fundamental period τ_1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4$ %
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
-1/2 -1/4 0 1/4 1/2
Coordinate ϕ (units of 2π)
(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
$-\pi/2$
-1/4
0
Time t
1/1
3/4
1/2
1/4
0/1

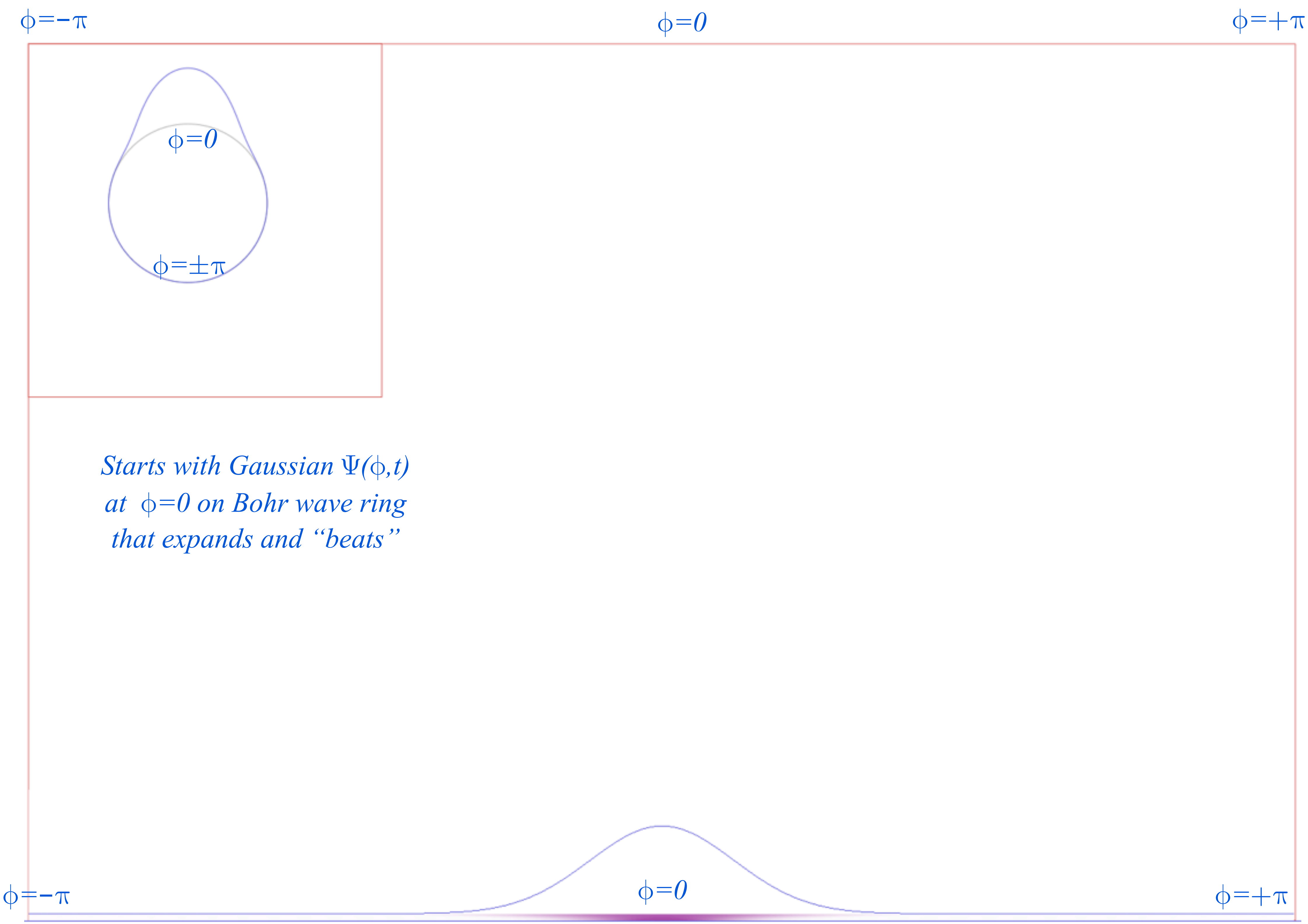
WaveIt Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

Click here....

Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Twelve (n=12) oscillato
Twelve (n=12) oscillato
Twelve (n=12) oscillato
C(n) Character Table

..then here.... Quantum Carpet



Starts with Gaussian Ψ(ϕ,t)
at ϕ=0 on Bohr wave ring
that expands and "beats"

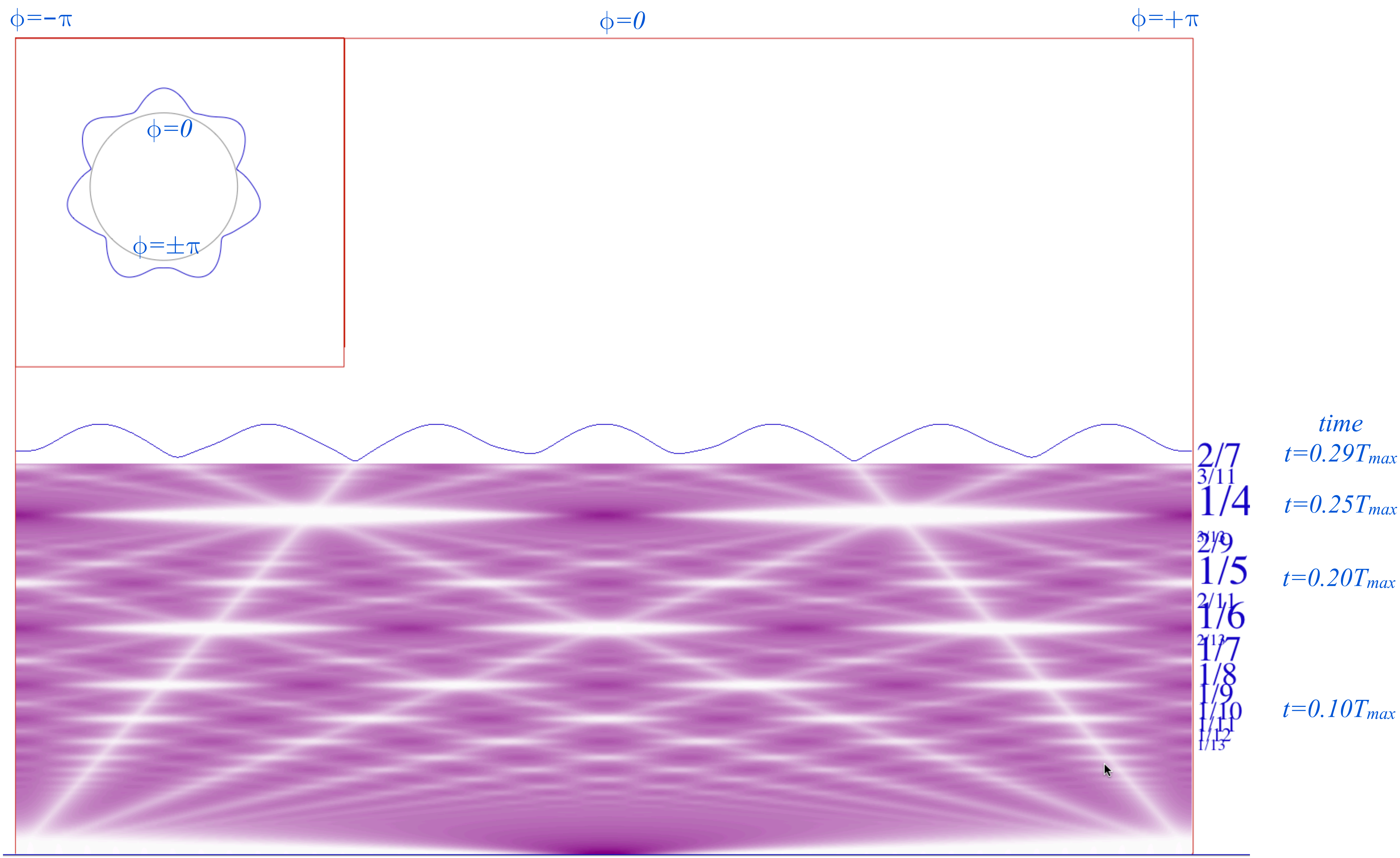
WaveIt Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

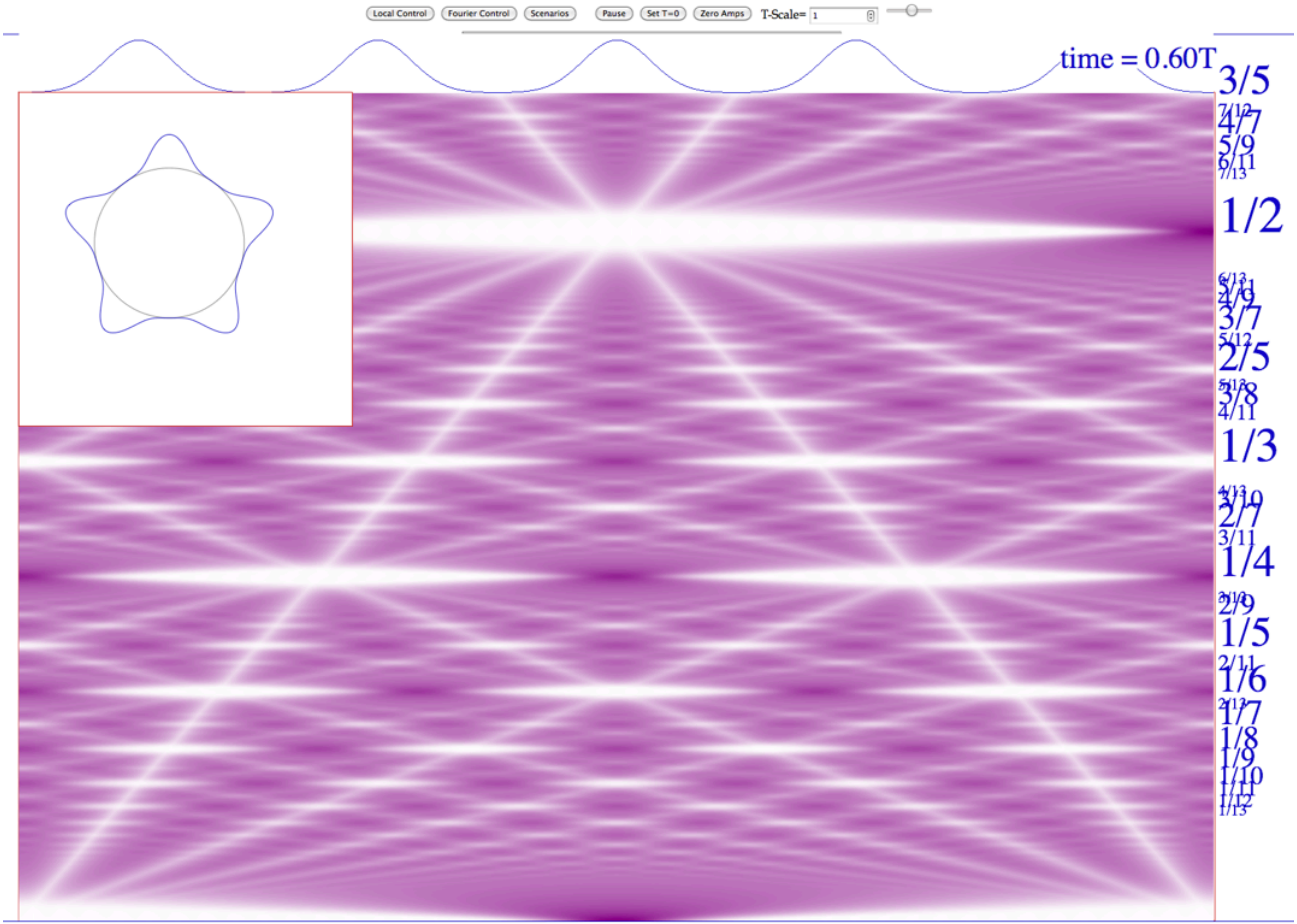
Click here....

Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Twelve (n=12) oscillator
Twelve (n=12) oscillator
Twelve (n=12) oscillator
C(n) Character Table

..then here.... Quantum Carpet




Local Control
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale= 1
time = 0.60T
3/5
4/7
5/9
1/2
3/7
2/5
3/8
4/11
1/3
2/7
3/11
1/4
2/9
1/5
1/6
1/7
1/8
1/9
1/10

Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Set this and then click here….

Type: Quantum Carpet

Time Behavior: Pause at End

Time Start (% Period) = 0

Time End (% Period)= 60

Del-x Width (% L) = 4

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100

n-Mean (% Max n)= 0

Peak1 Mean (% L)= 50

OverAll Scale = 1

Peak2 Mean (% L)= 0

Peak2 Amp (% Peak1)= 0

Draw Ring ☑ m/n Labels ☑

m-Boxcar ☐

Draw m-Bars ☑ m-Bars Max = 30

Aspect Ratio {W/H} = 1.5

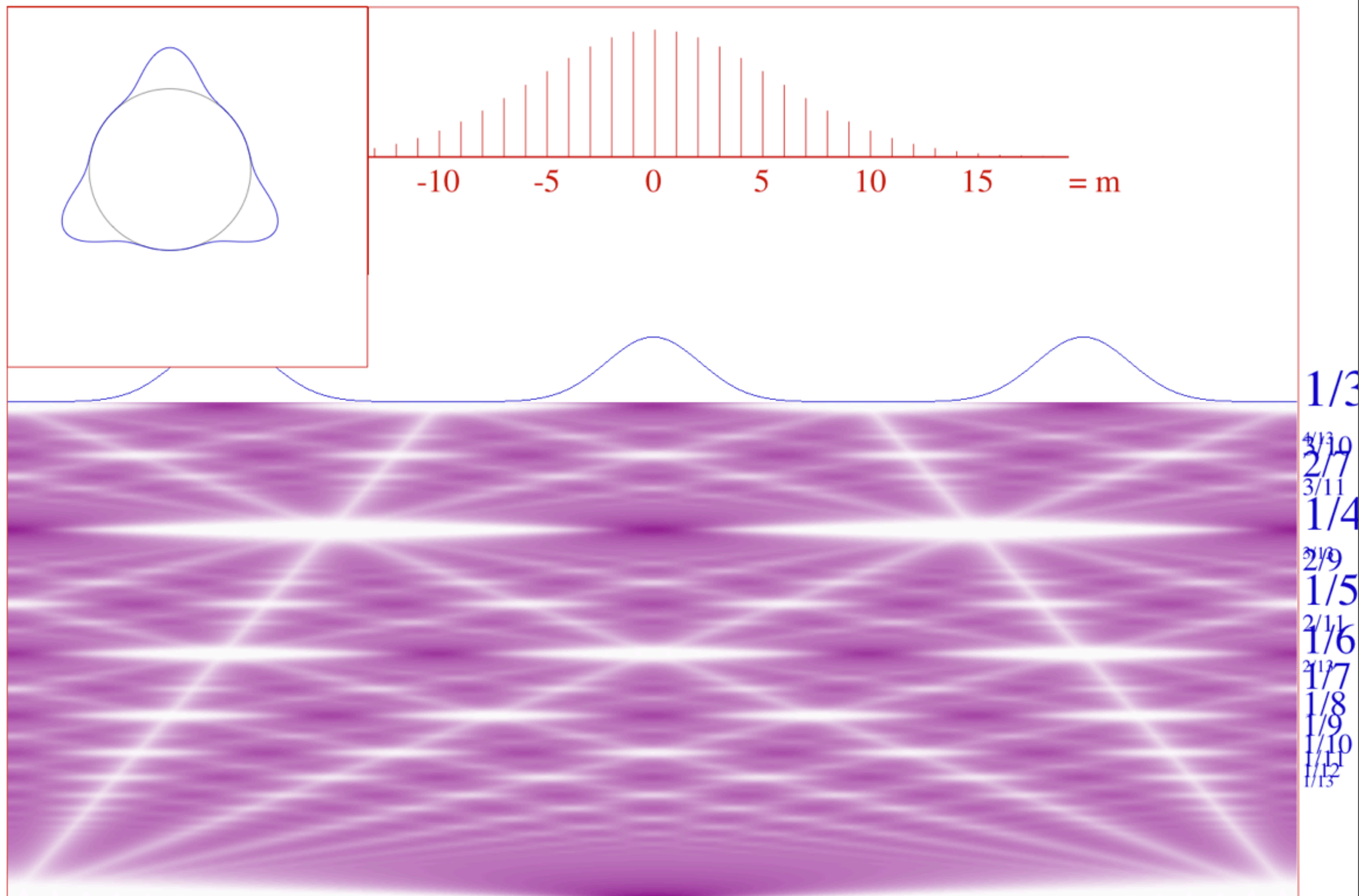
Red Level = 128

Green Level = 0

Blue Level = 128

Alpha Level = 1

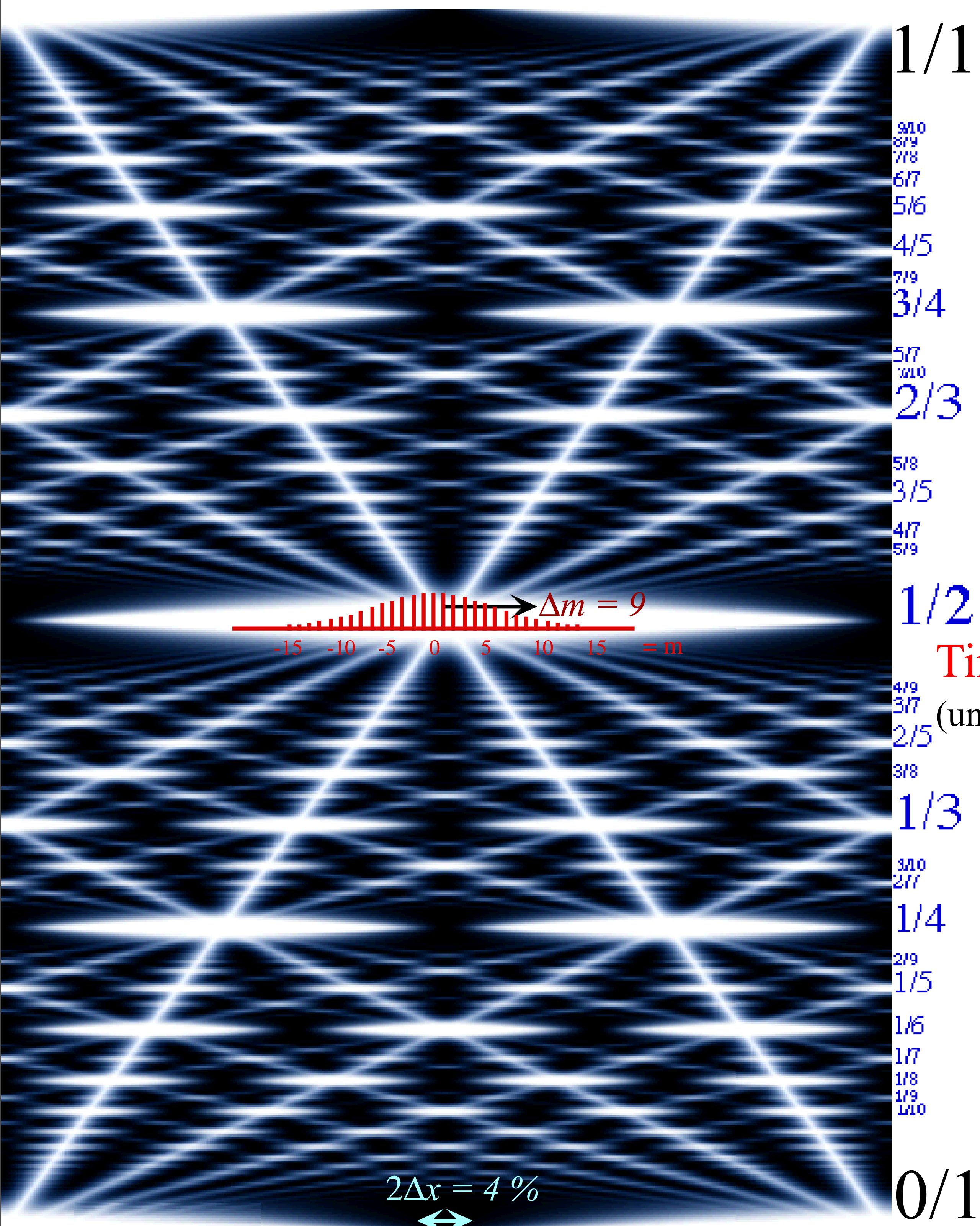
Definition Level = 0.5



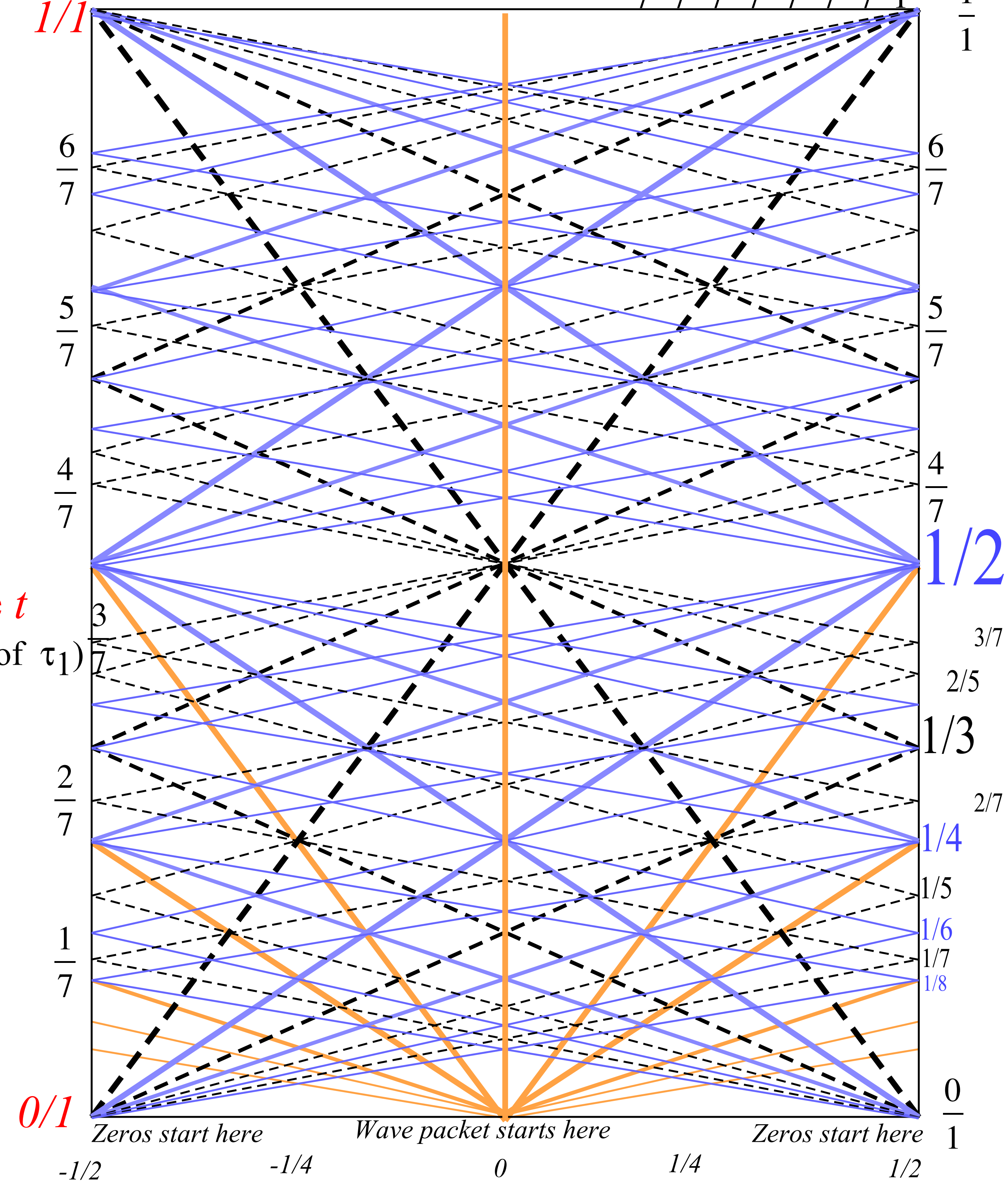
N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



Time t (units of τ_1)

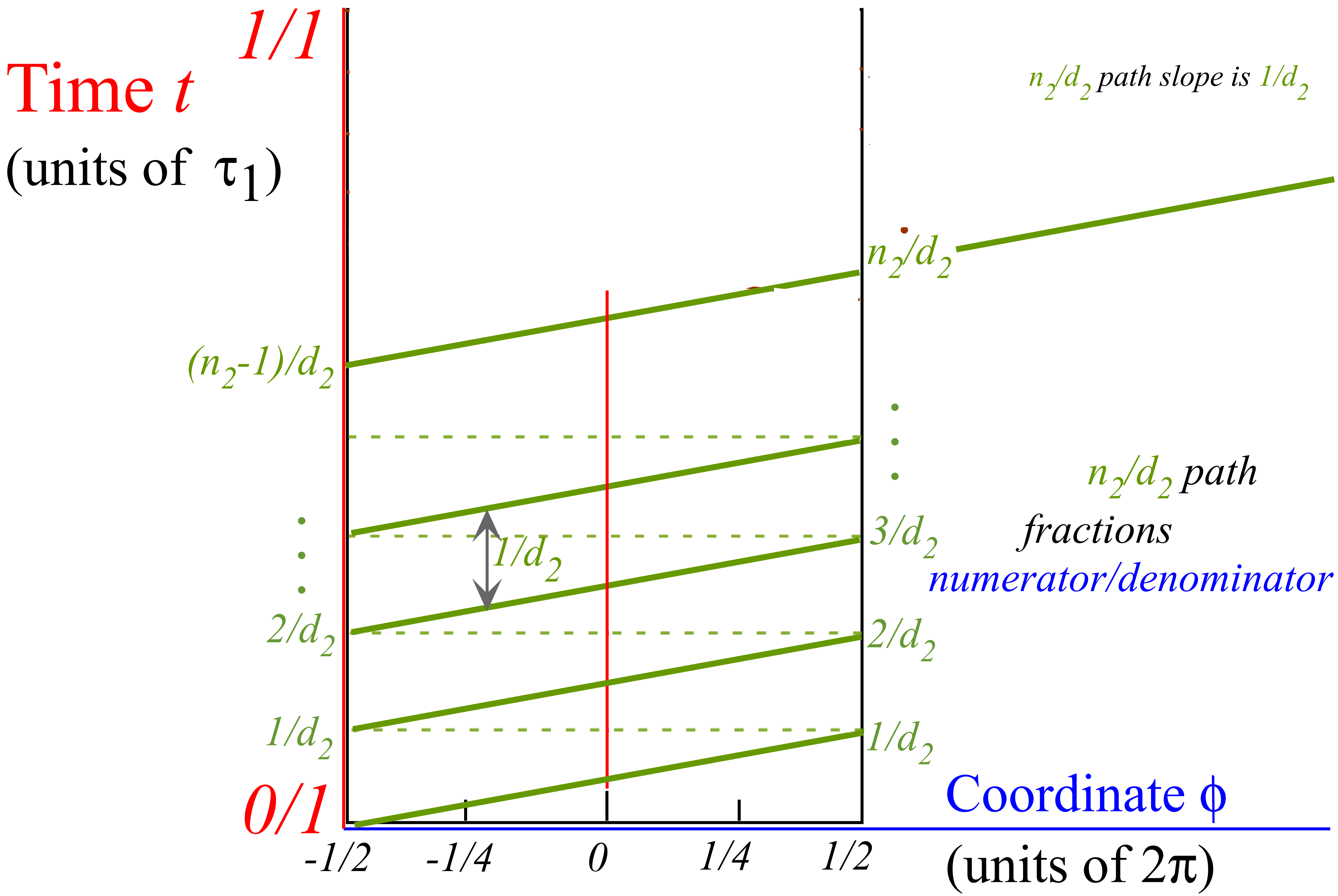


Coordinate ϕ

(units of 2π)

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



Farey Sum algebra of revival-beat wave dynamics

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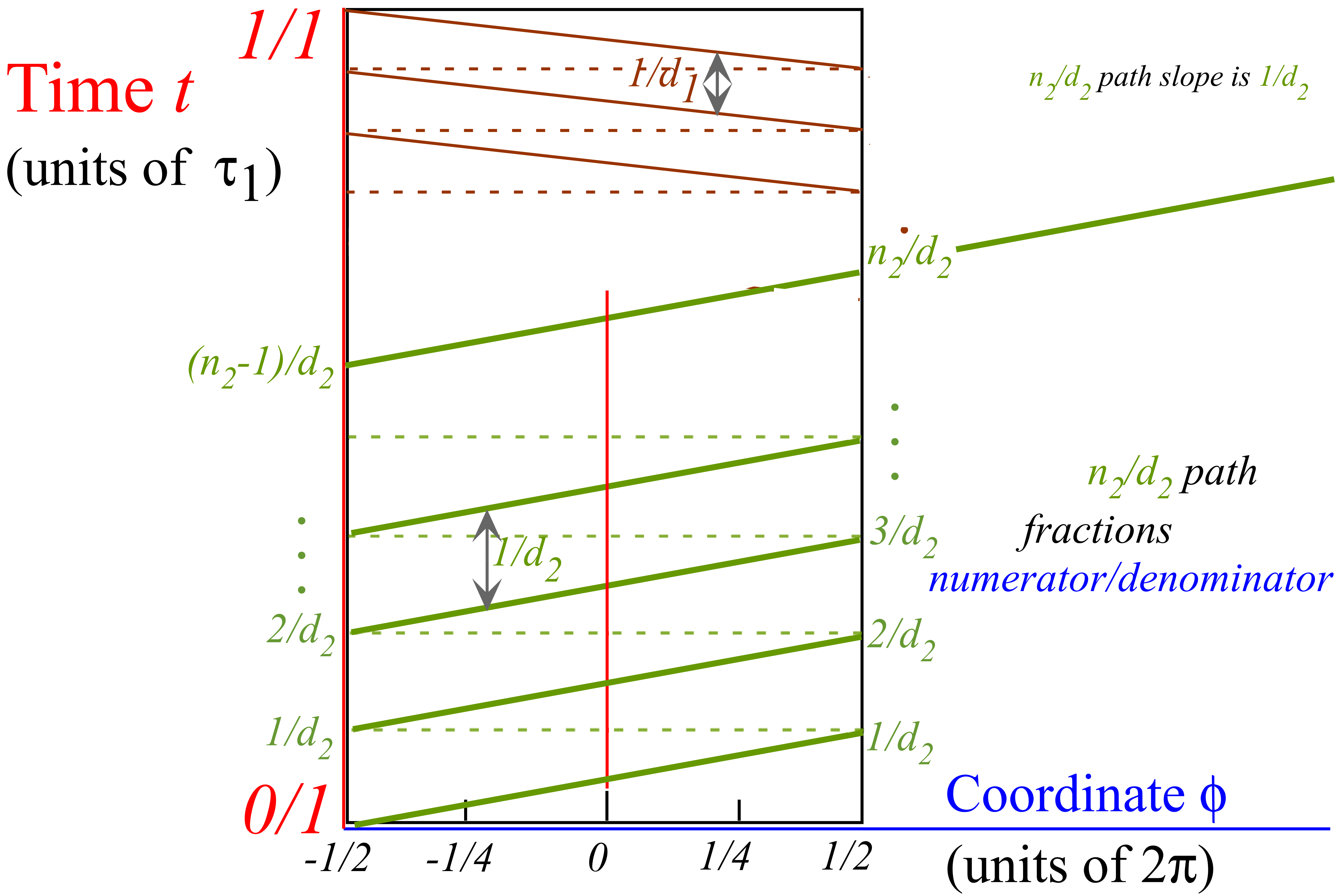

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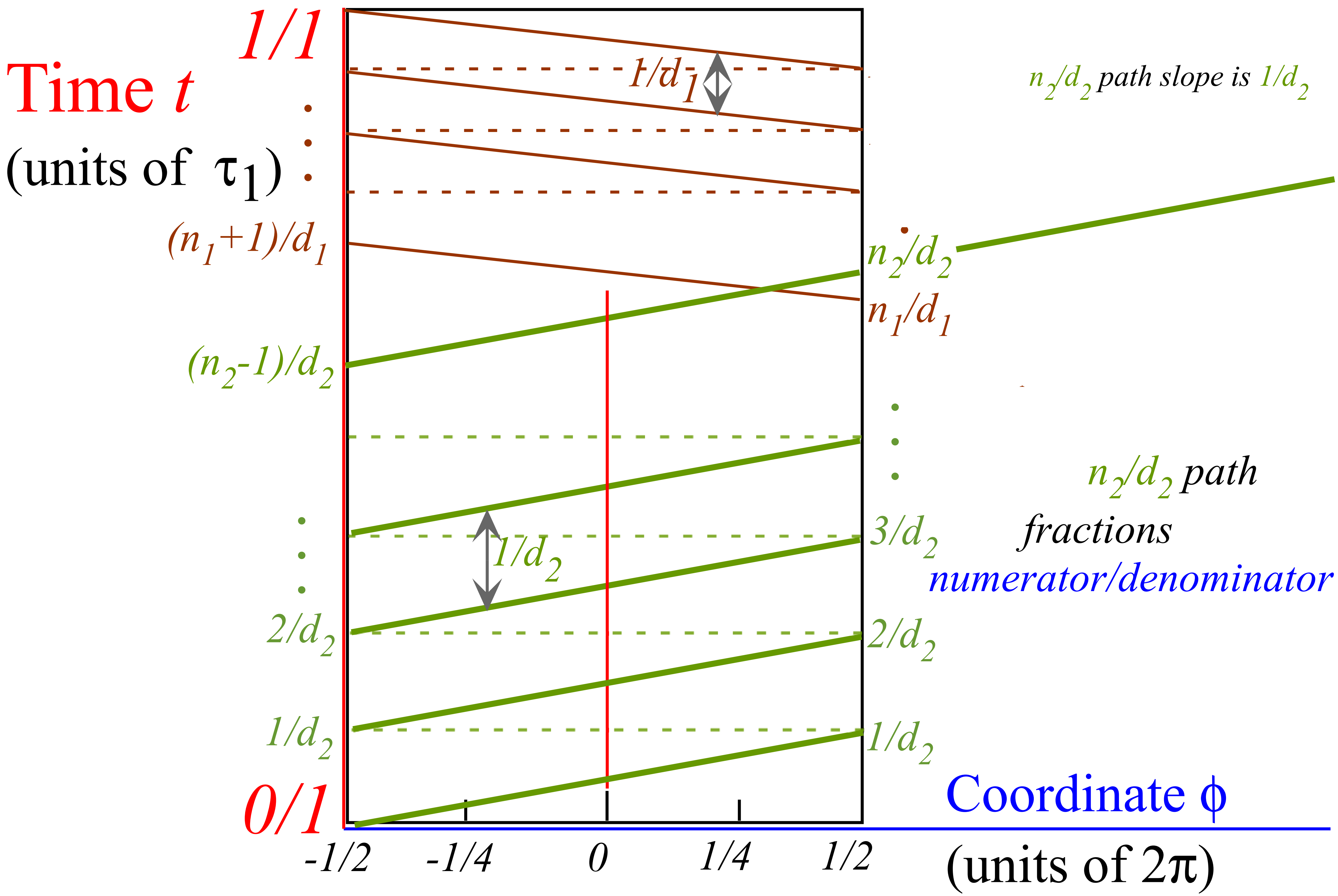

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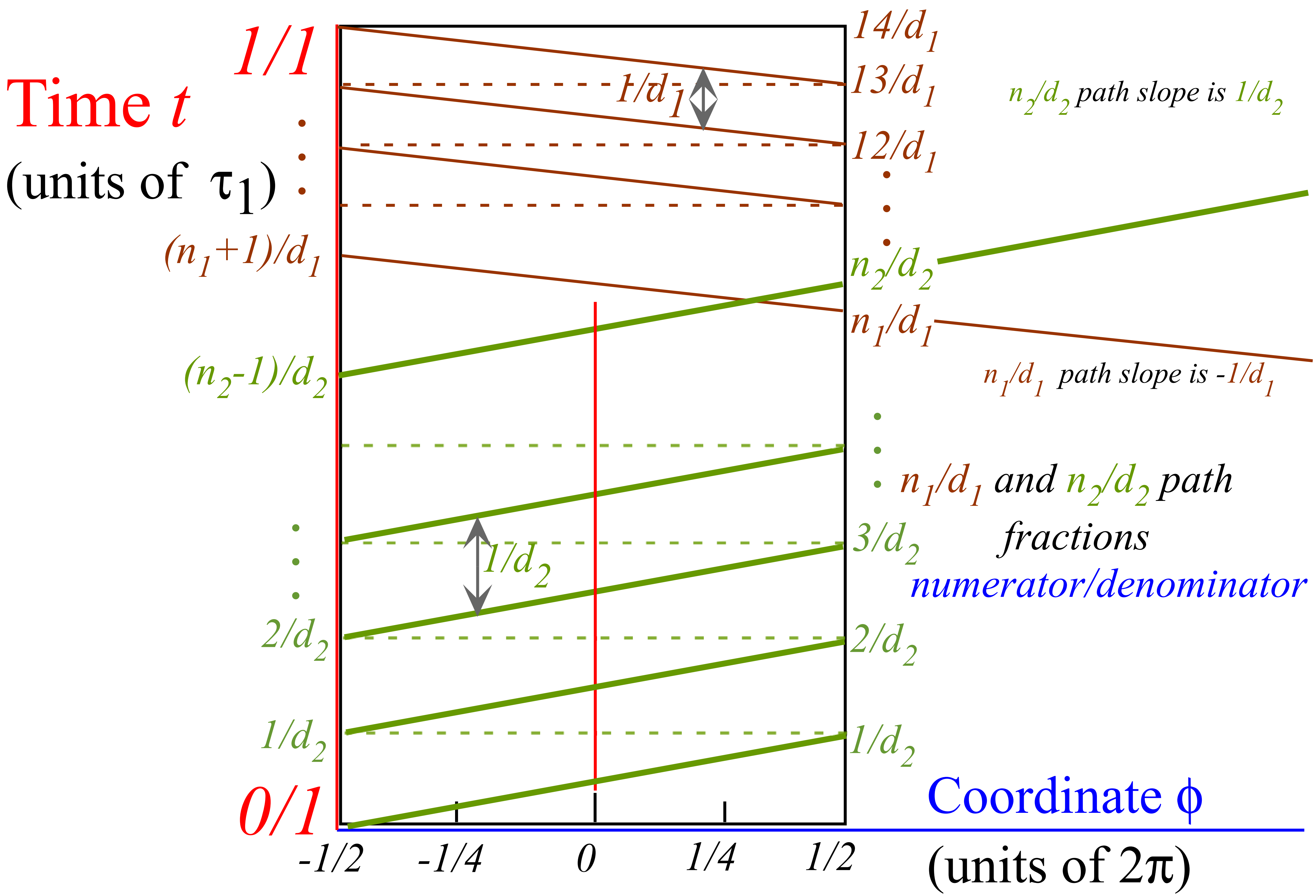

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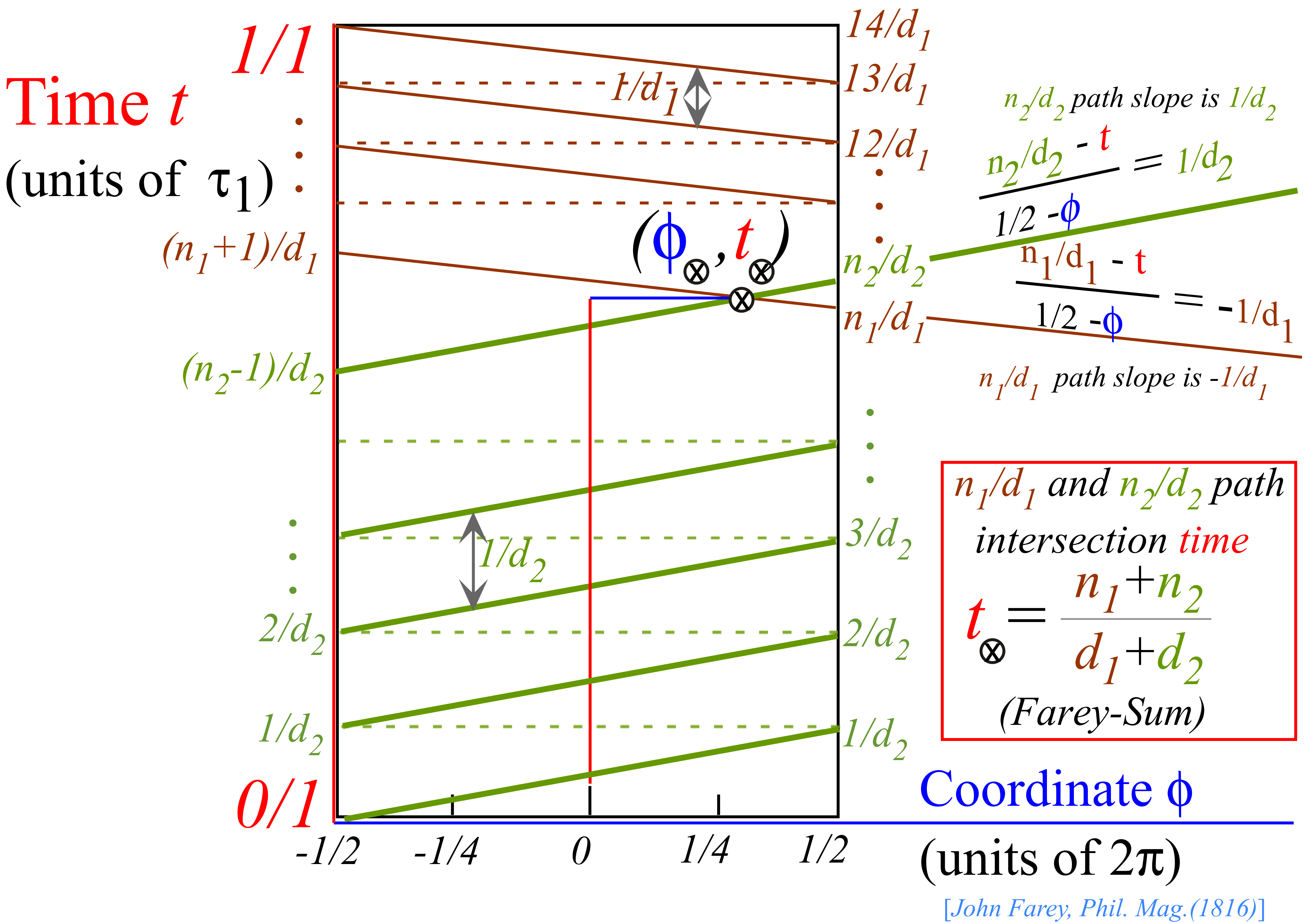

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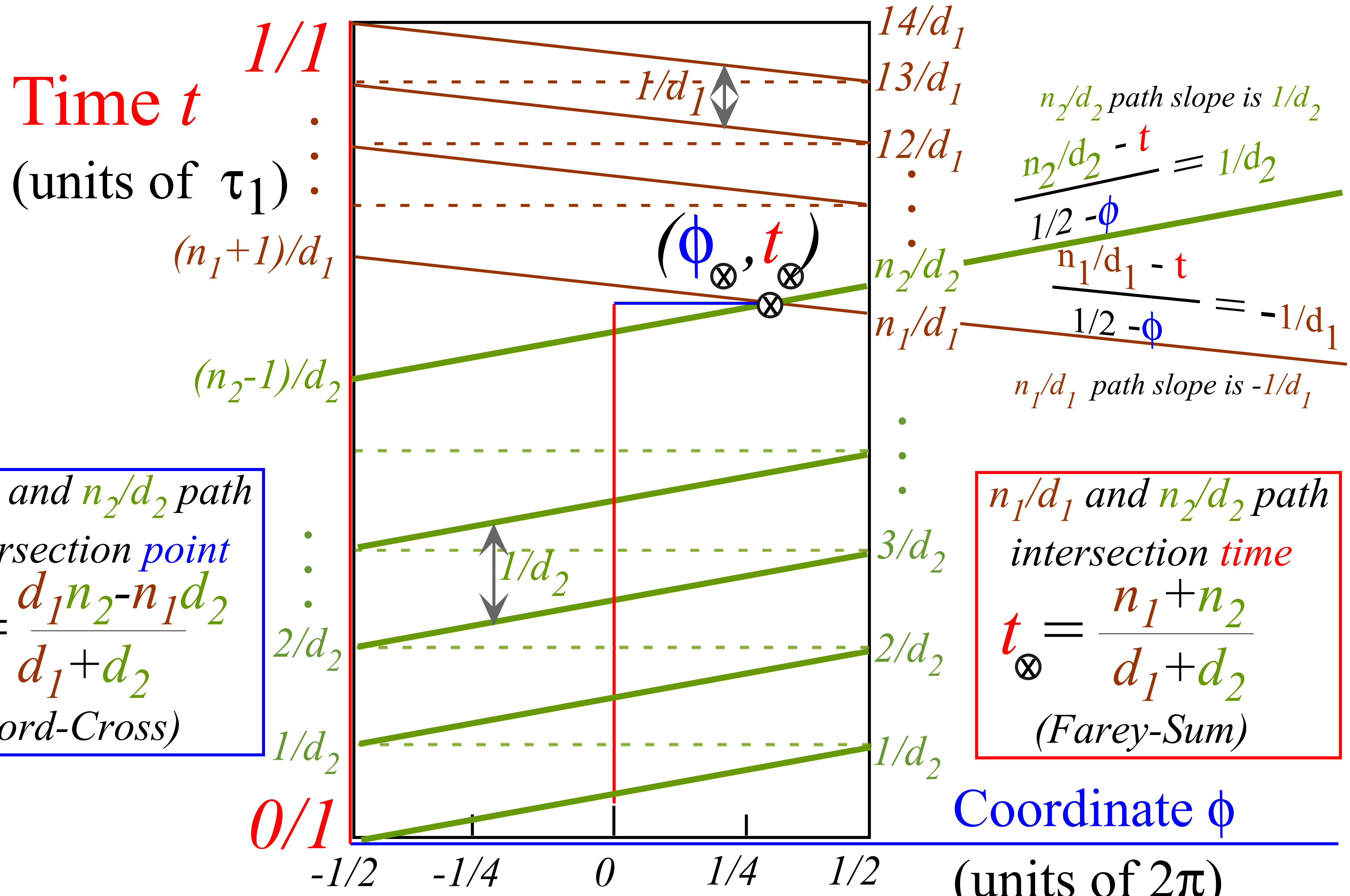


n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*]

[*John Farey, Phil. Mag.(1816)*]

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

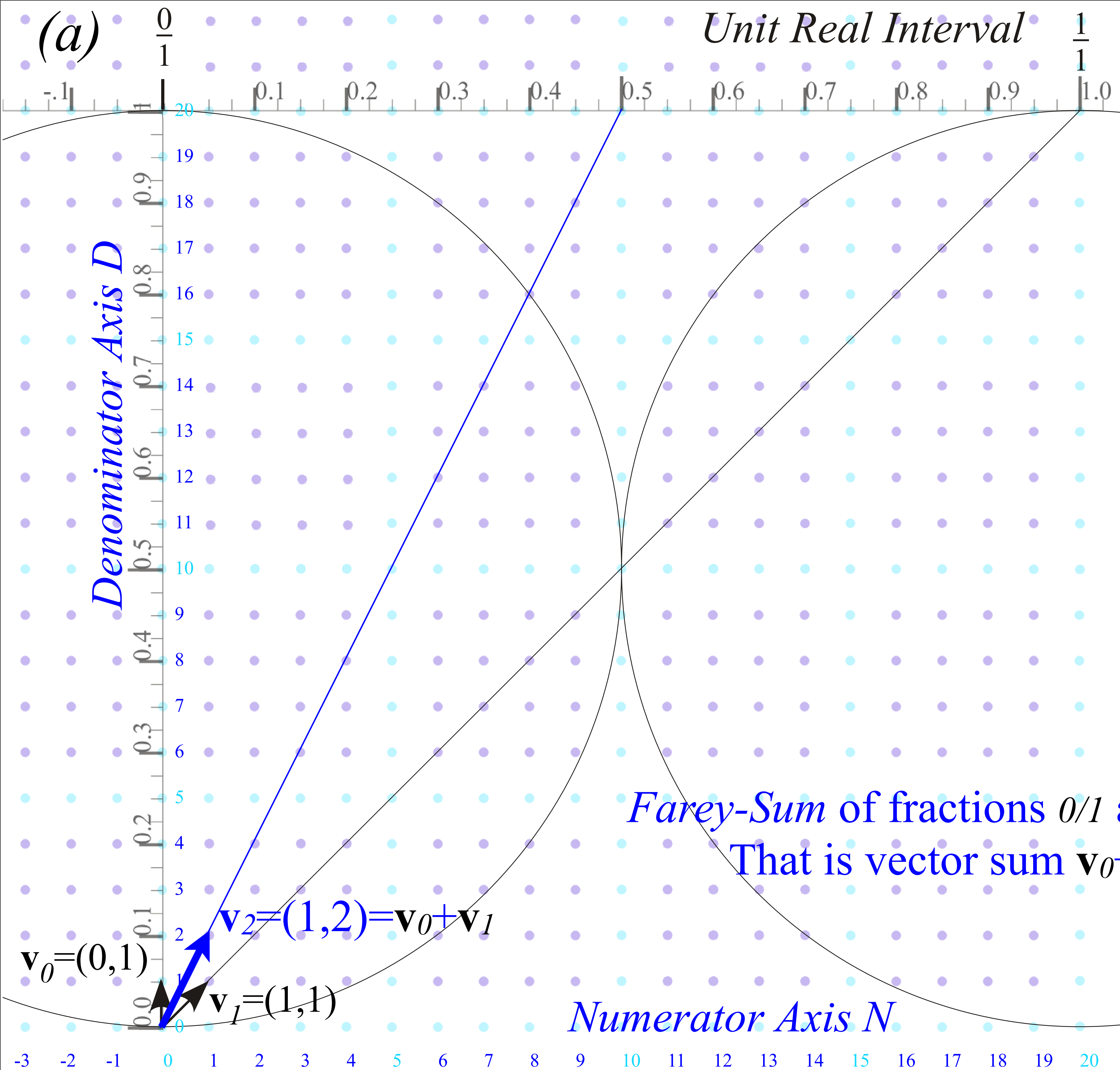
An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)],[Harter, Li IMSS (2012)]

→ *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)] [John Farey, Phil. Mag.(1816)]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



[Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum related to vector sum and *Ford Circles*

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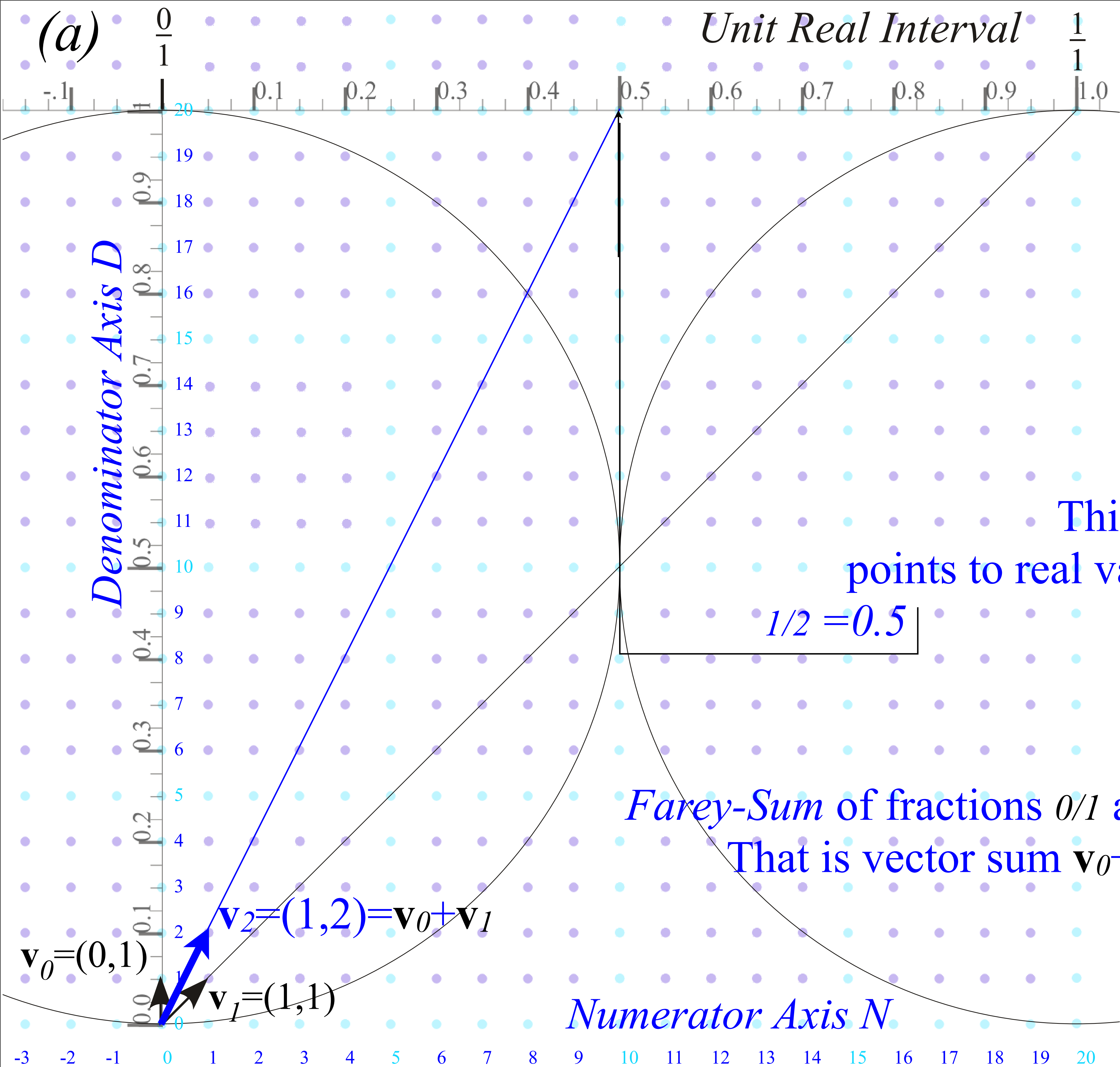

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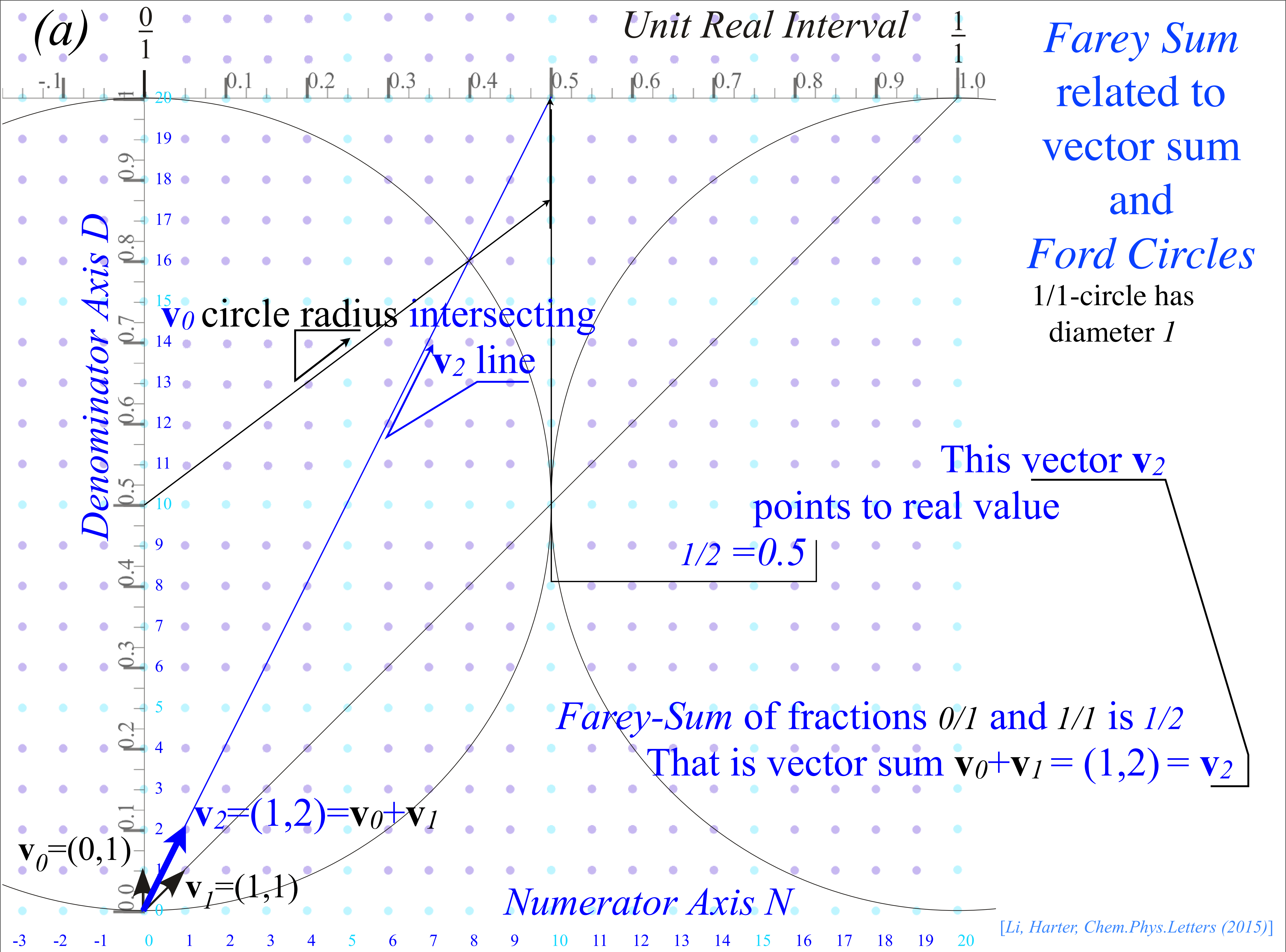
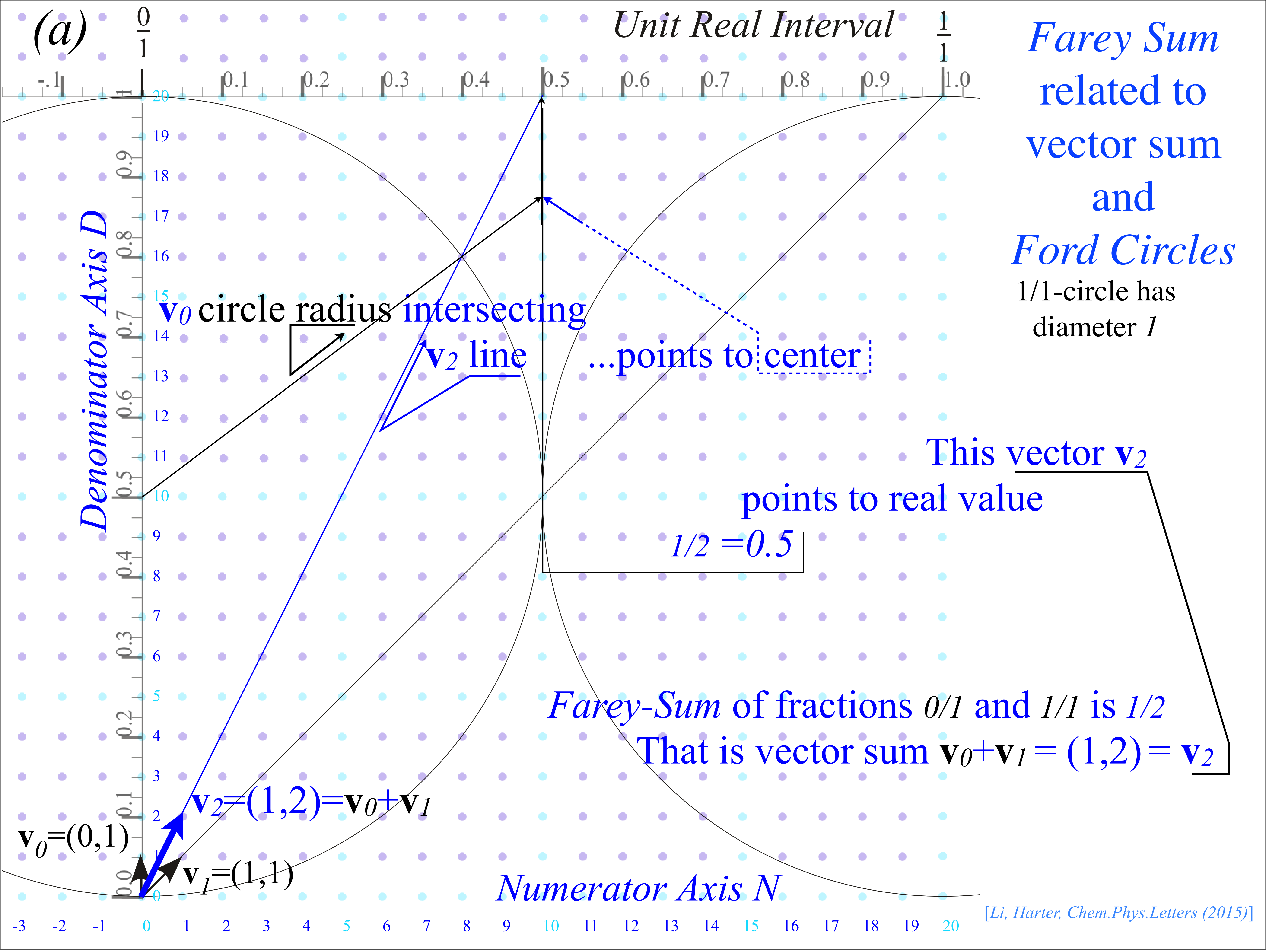


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting v2 line
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]



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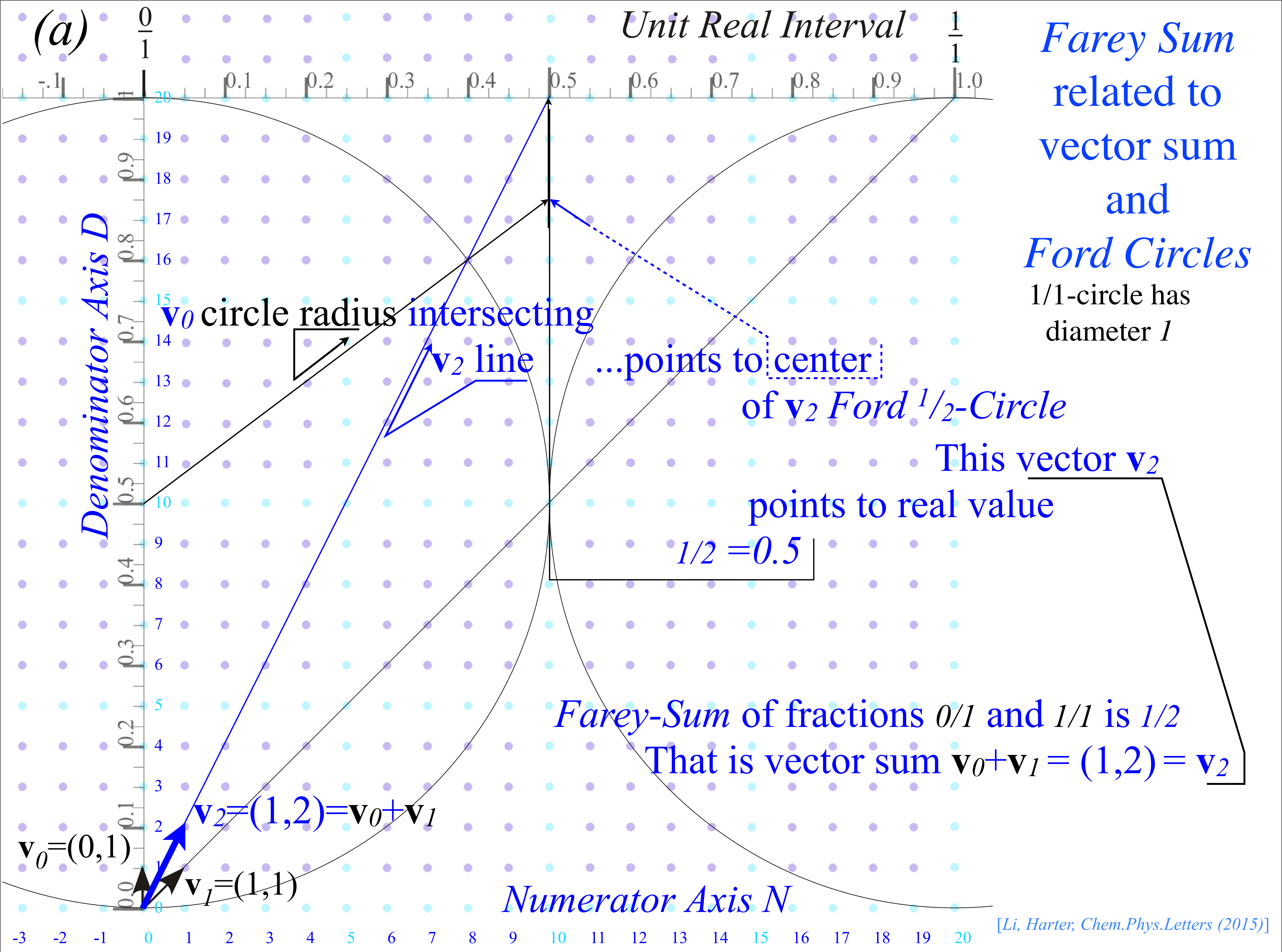
(a)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
$\mathbf{v}_0$ circle radius intersecting
$\mathbf{v}_2$ line
...points to center of $\mathbf{v}_2$ Ford $^1/_2$-Circle
This vector $\mathbf{v}_2$ points to real value
1/2 =0.5
Denominator Axis D
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$
$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$
$\mathbf{v}_0=(0,1)$
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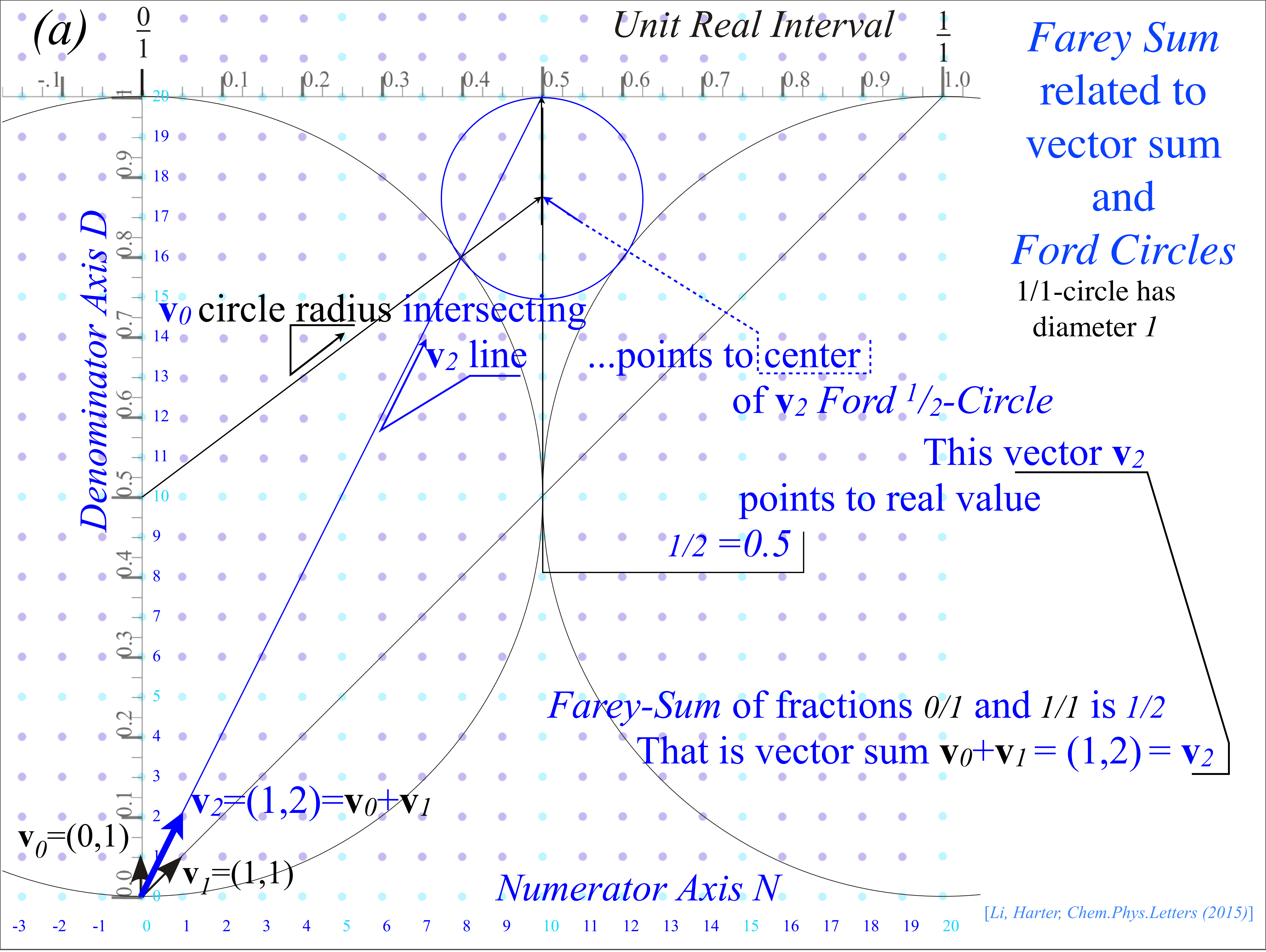
[Li, Harter, Chem.Phys.Letters (2015)]



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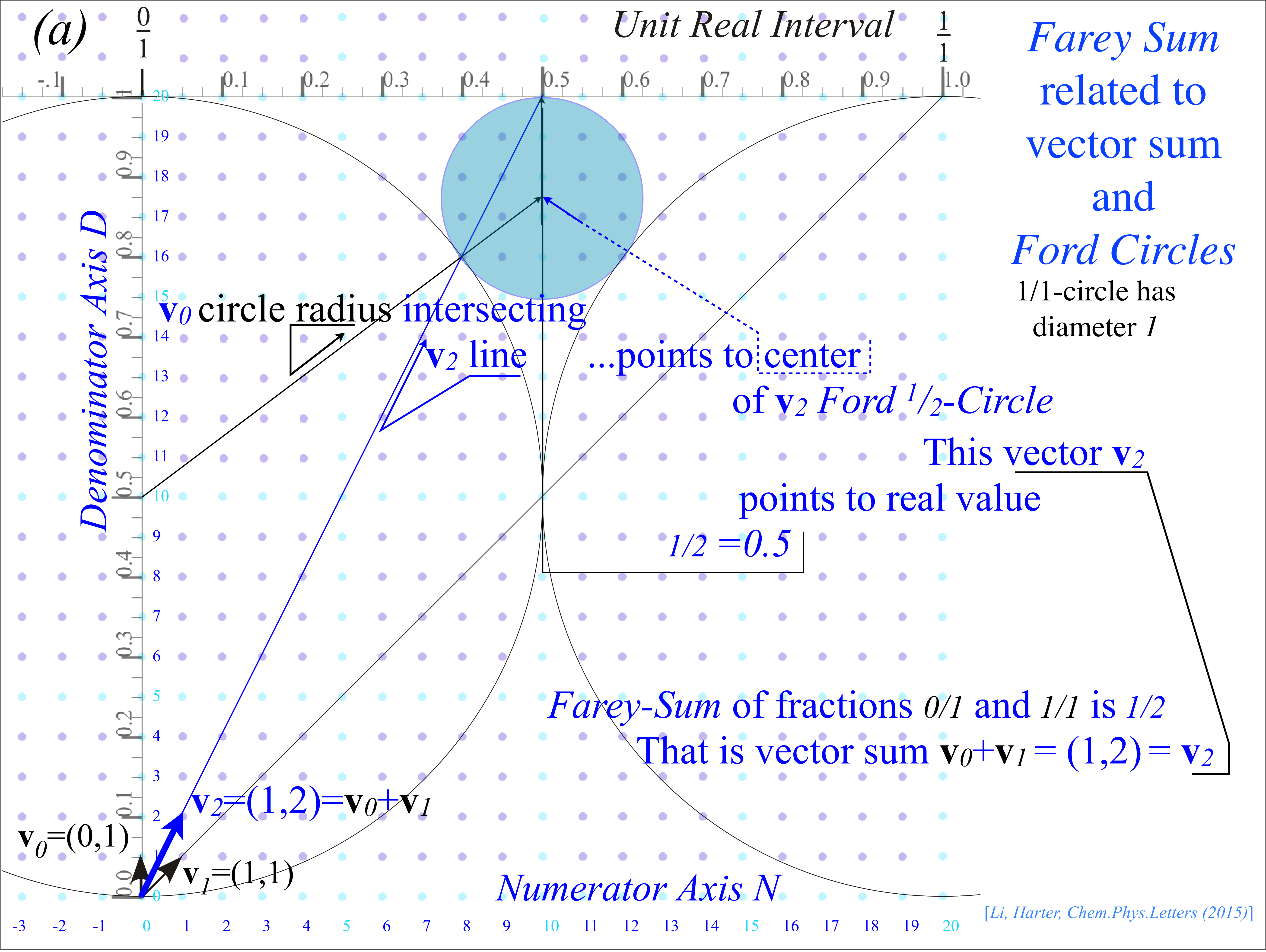

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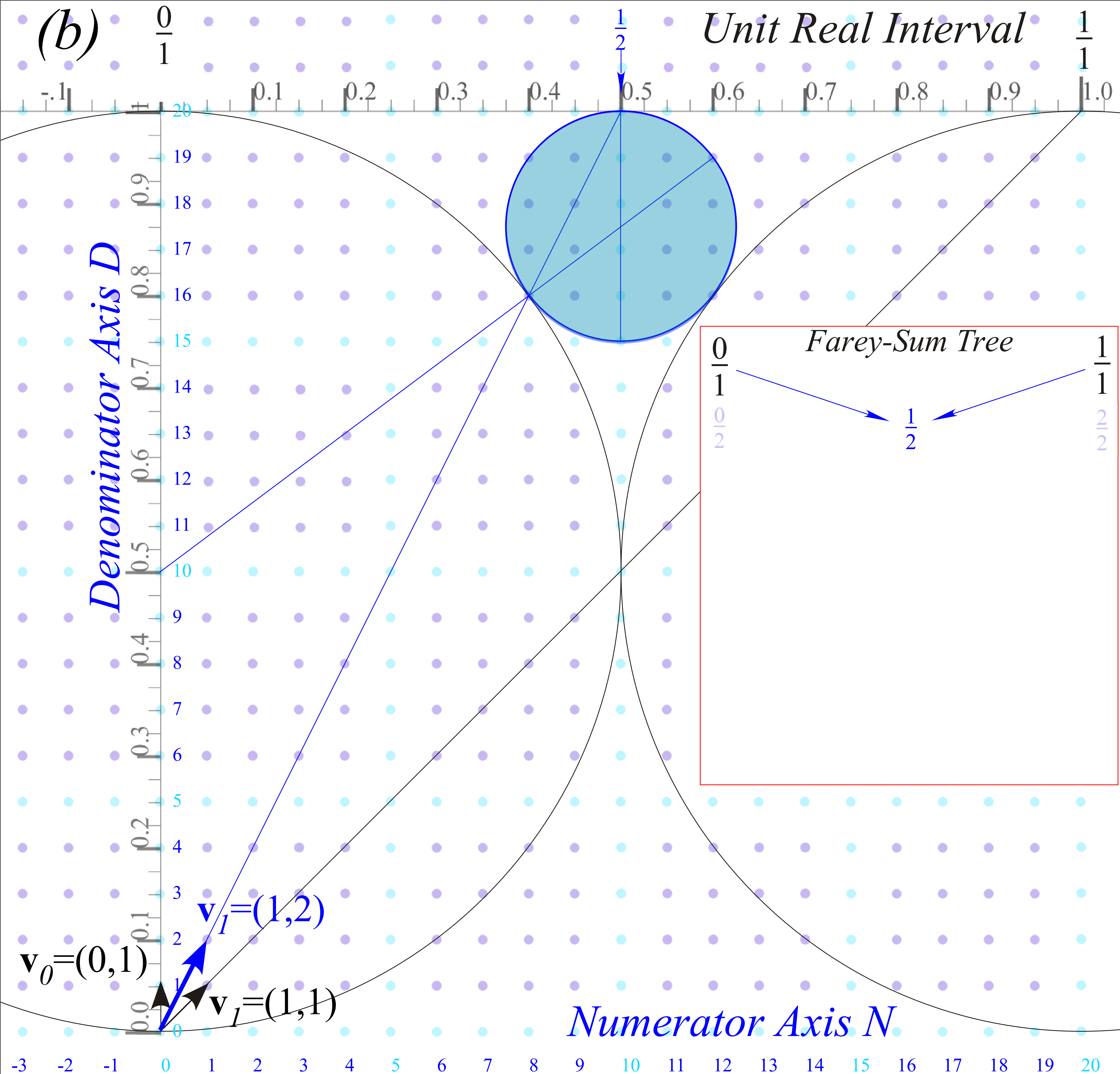
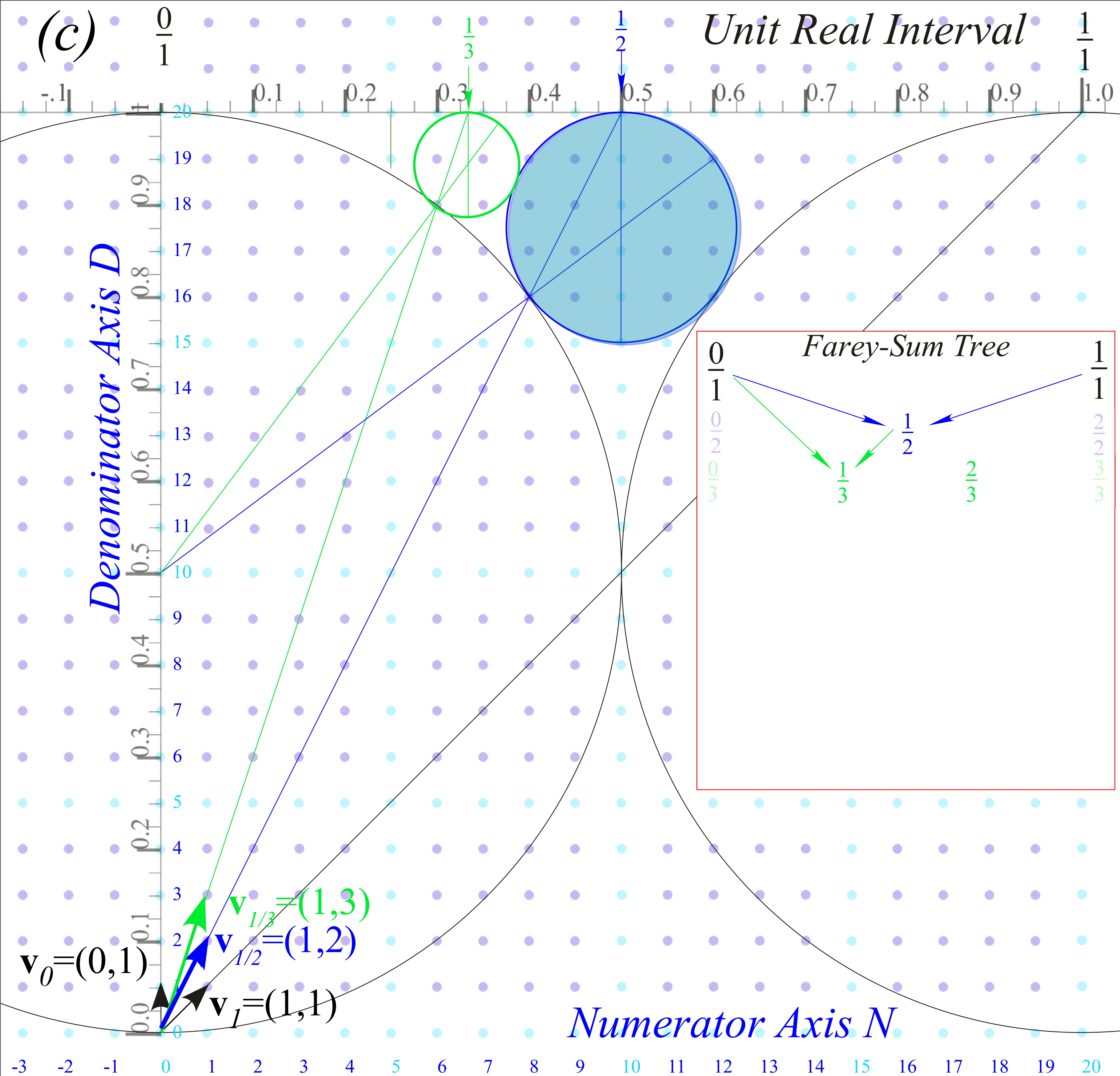


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

[Li, Harter, Chem.Phys.Letters (2015)]

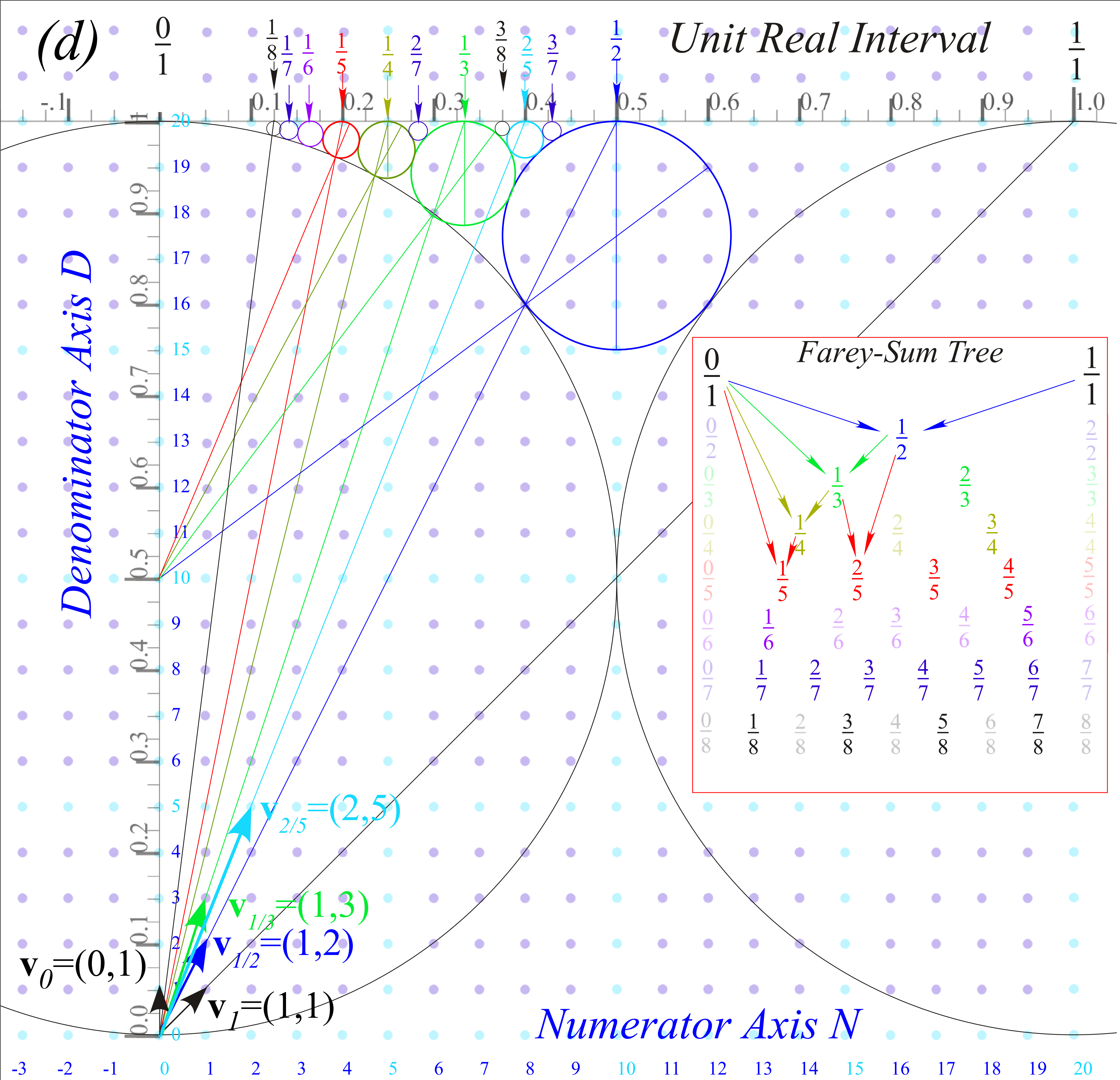


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

[*Li, Harter, Chem.Phys.Letters (2015)*]



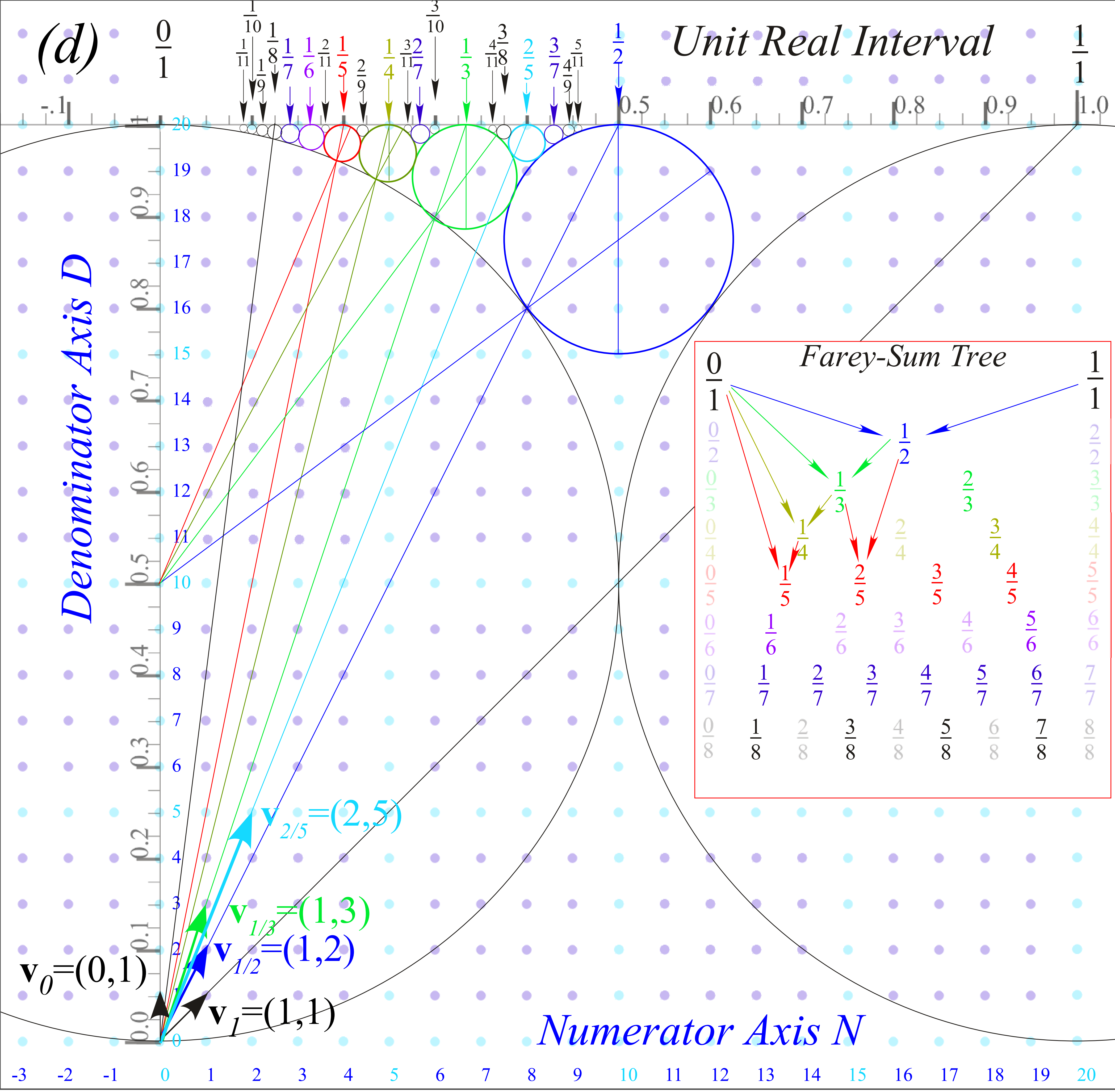
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

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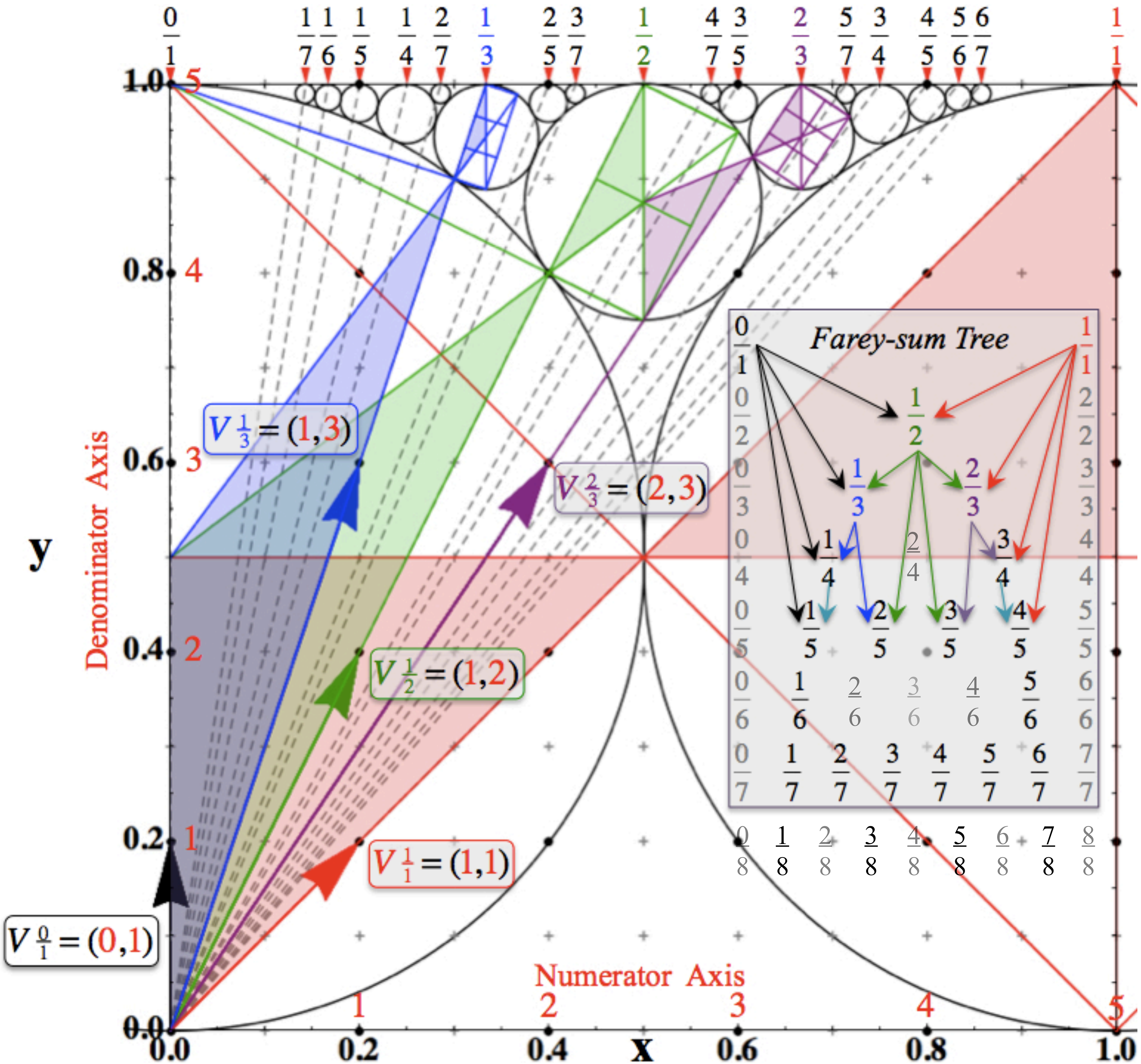
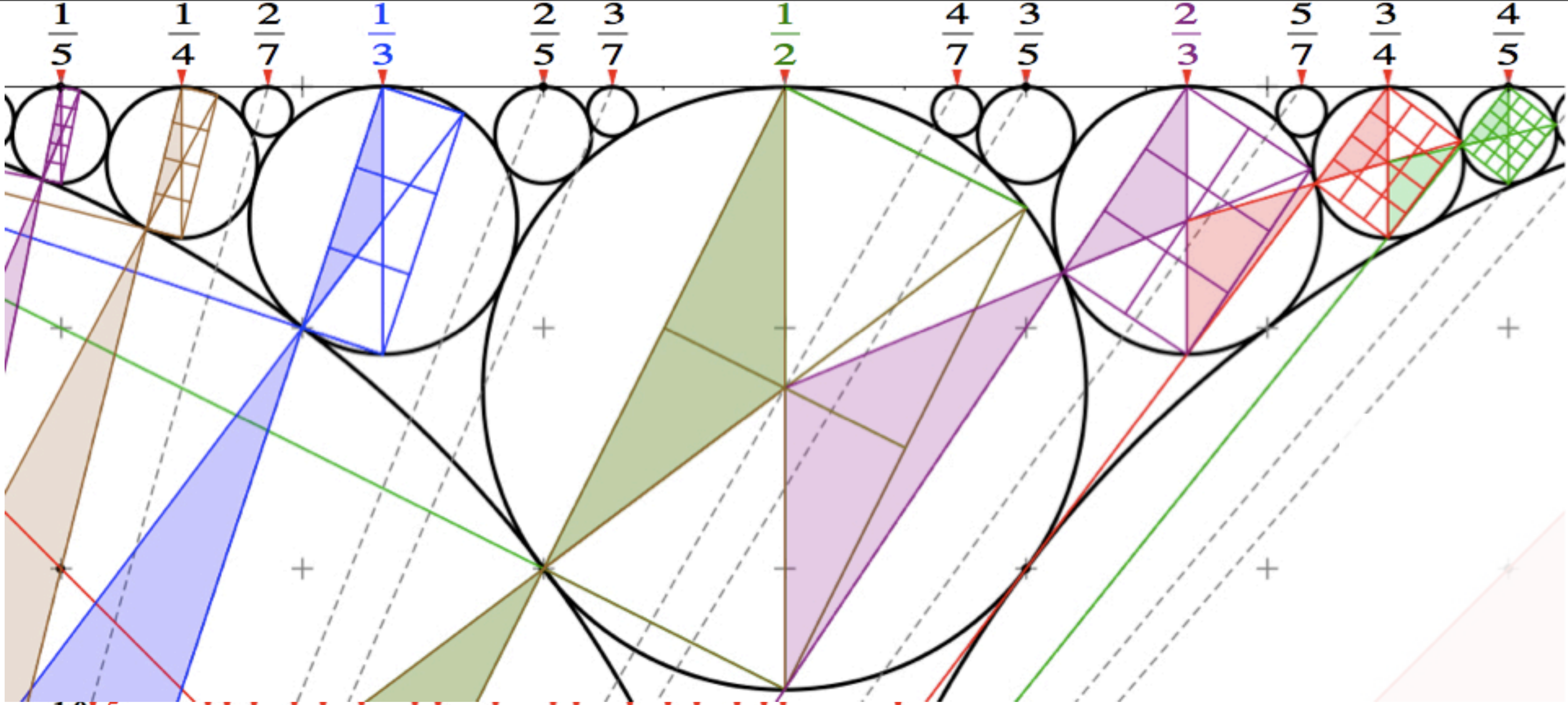


Thales Rectangles

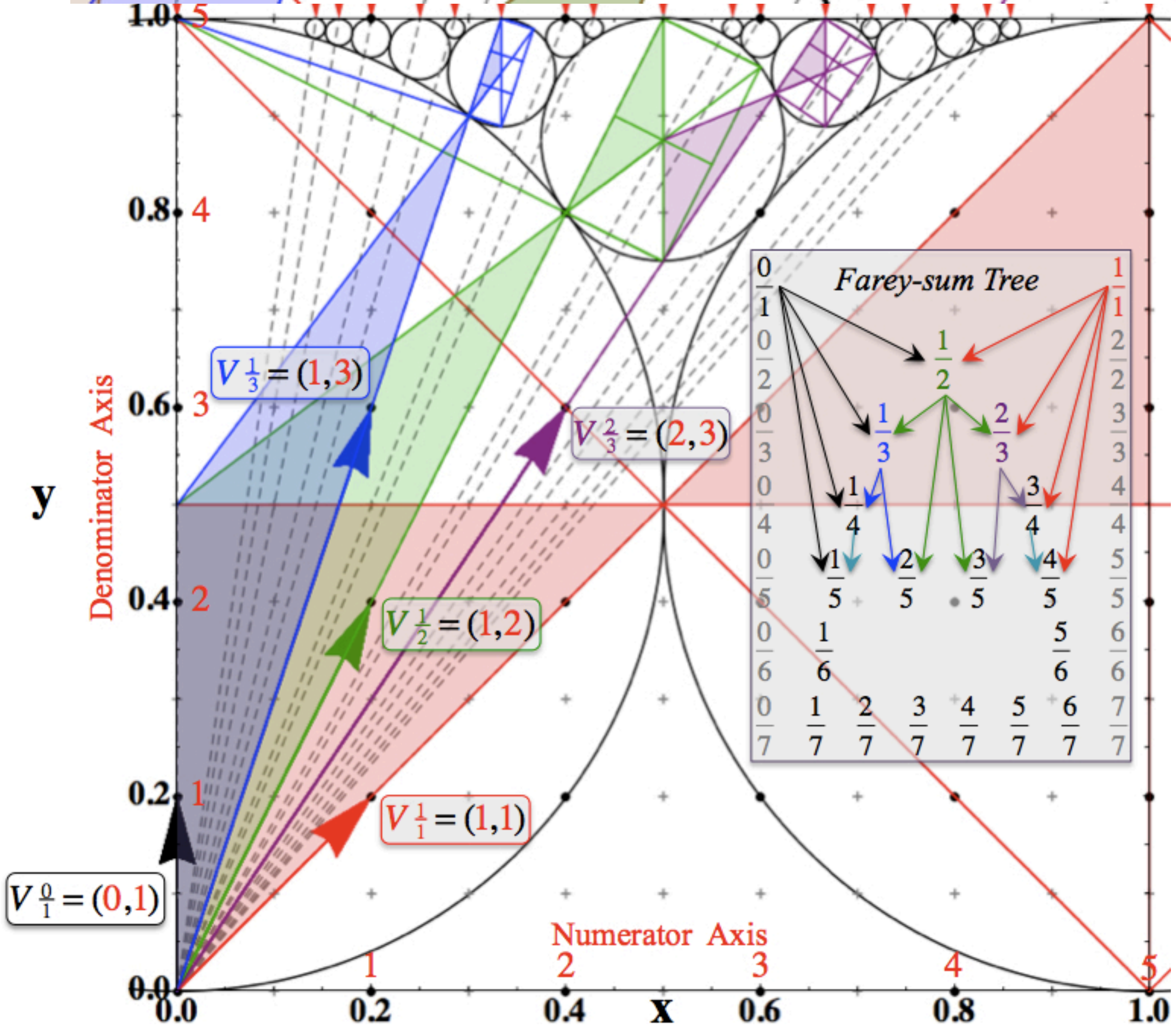
provide analytic geometry of fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



"Quantized" Thales Rectangles

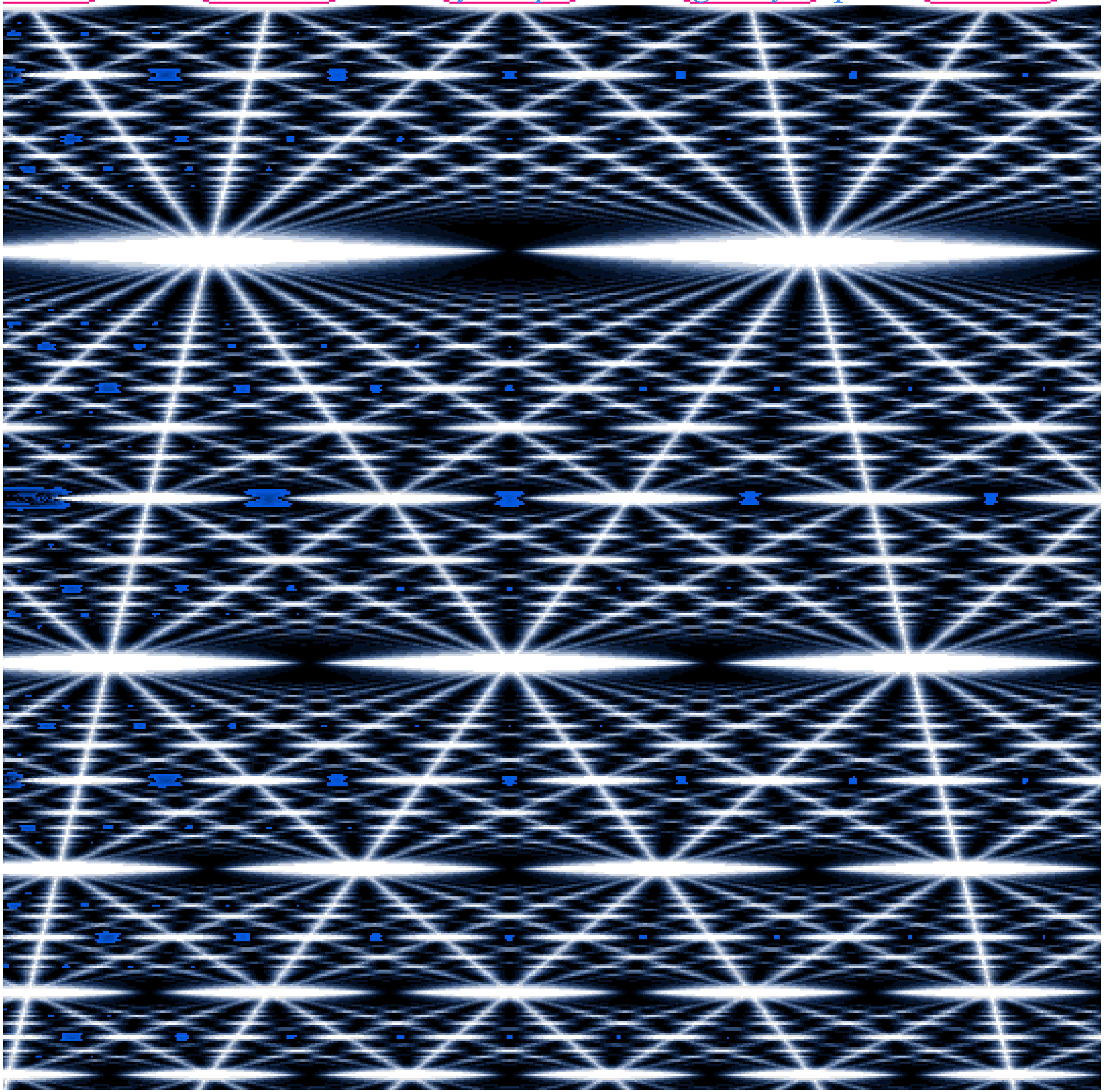
provide analytic geometry of fractal structure



[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



Geometric "Integration" (Converting Velocity data to Spacetime)

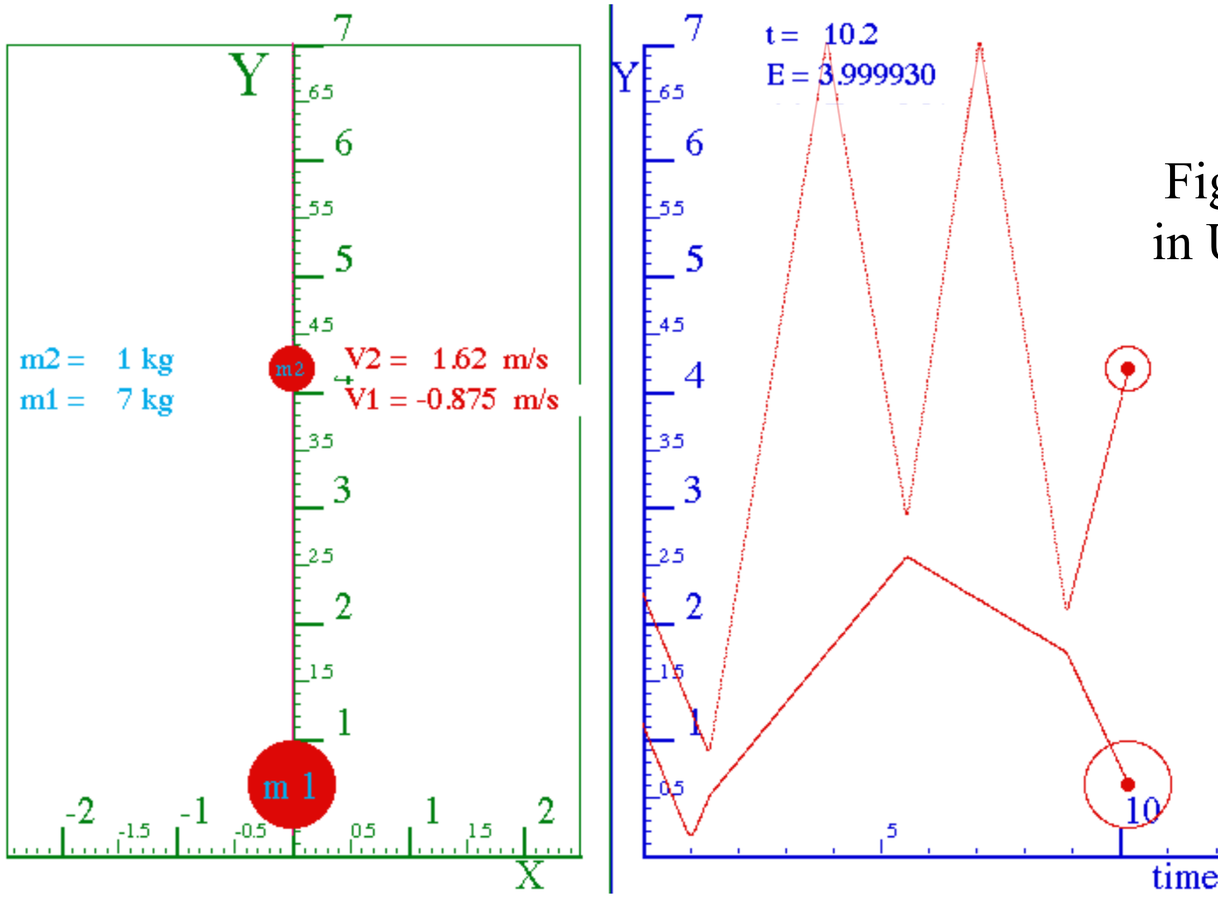


Fig. 4.8
in Unit 1

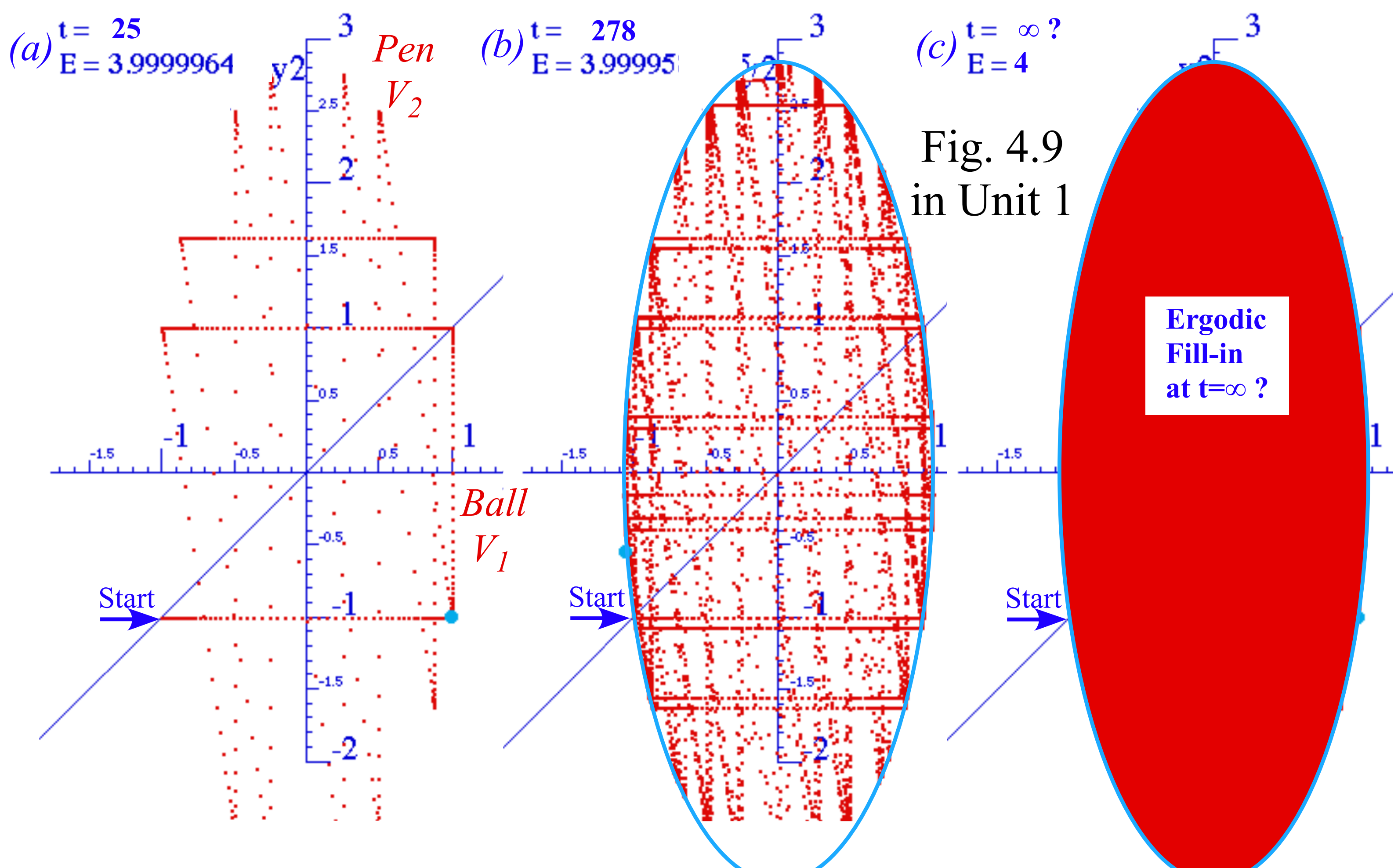
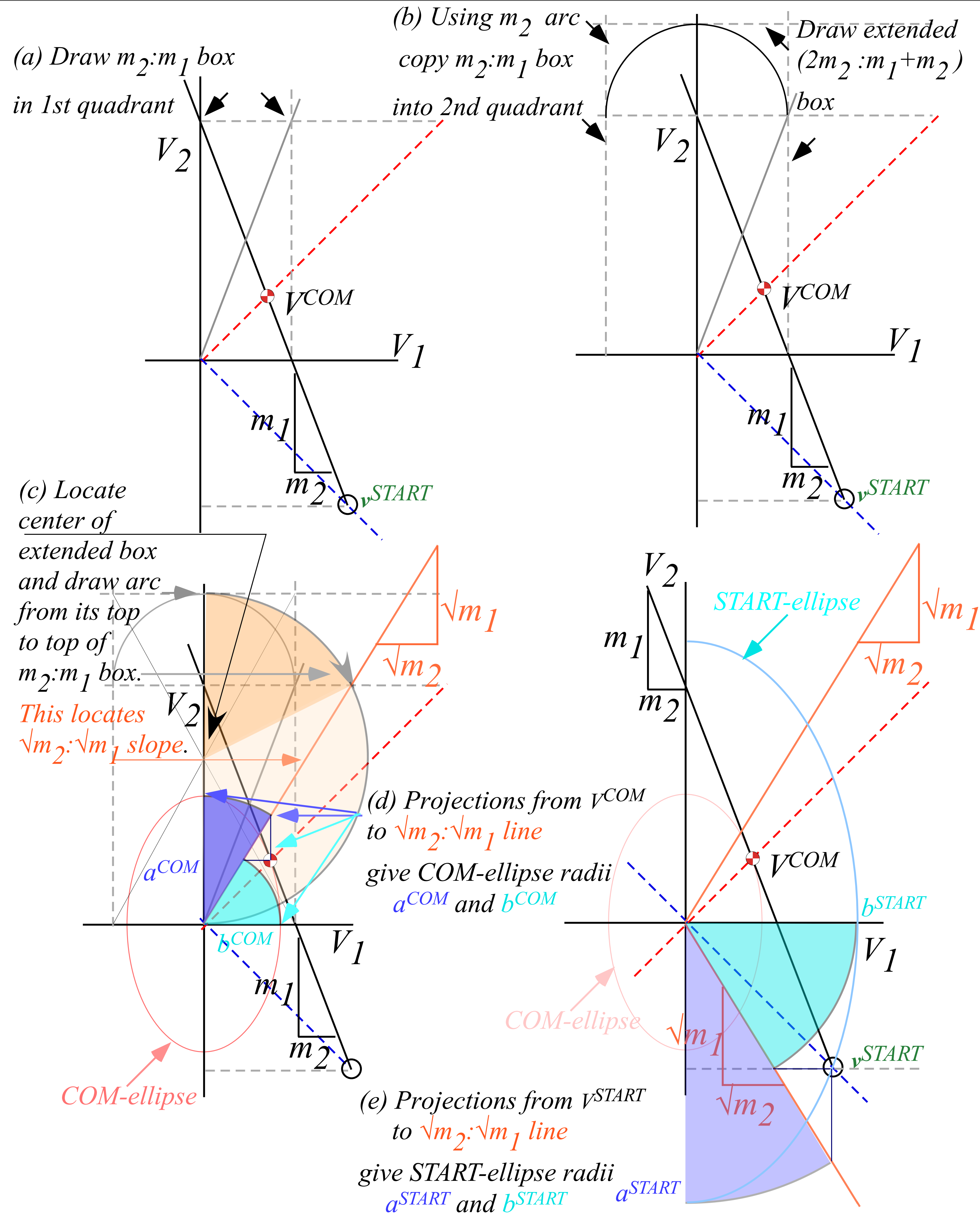


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)