2018 CMwBang! site

Class YouTube Channel

Lecture 4
Wed. 8.29.2018

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ collision dynamics   *High mass ratio $M_1/m_2 = 49$*

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]

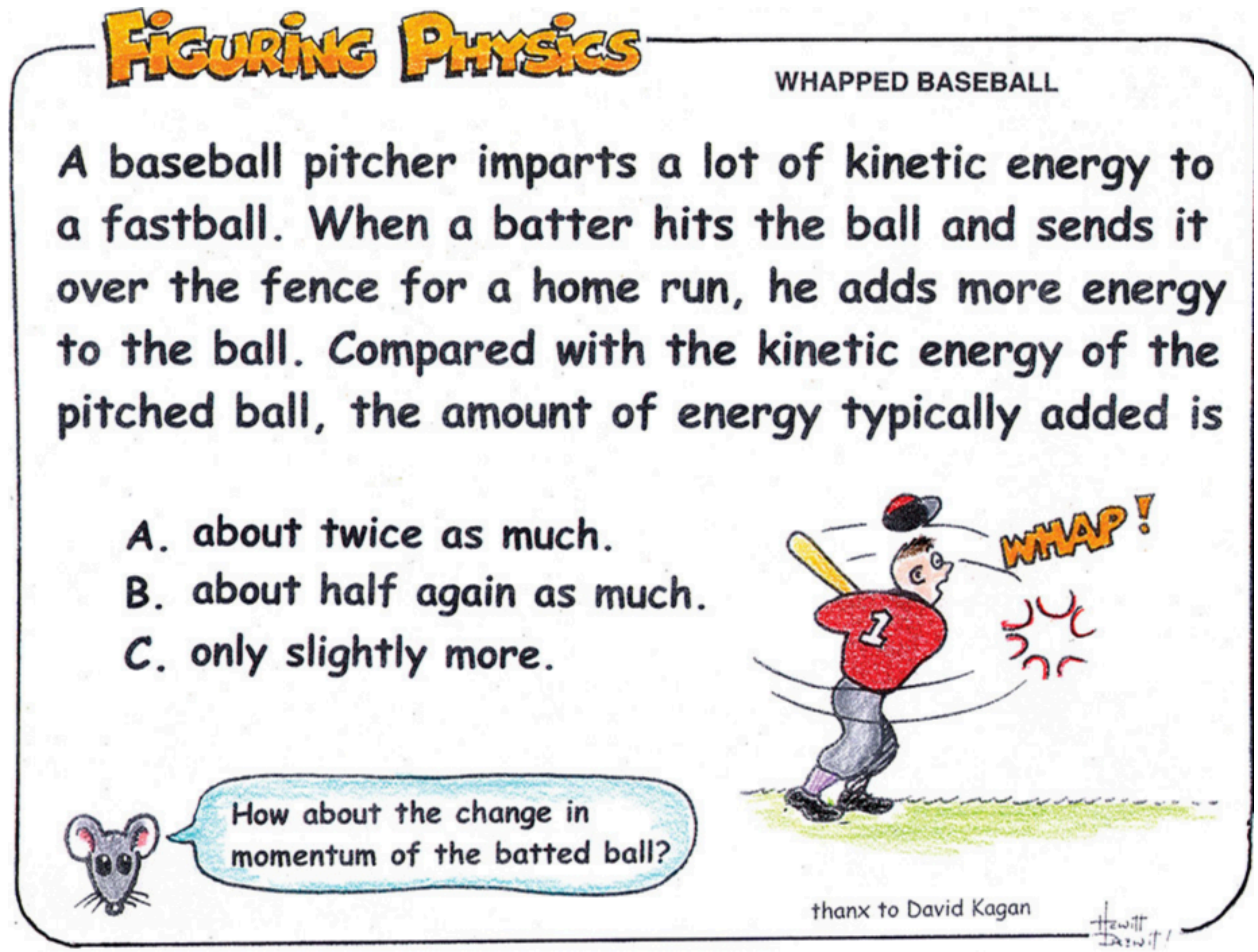
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015) Elsevier]

[Li, Harter, Chem.Phys.Letters (2015) Local Copy]

References and incidental interest items

[AIP publications]   [AJP article on superball dynamics]   [AAPT summer reading]


FIGURING PHYSICS

WHAPPED BASEBALL

A baseball pitcher imparts a lot of kinetic energy to a fastball. When a batter hits the ball and sends it over the fence for a home run, he adds more energy to the ball. Compared with the kinetic energy of the pitched ball, the amount of energy typically added is

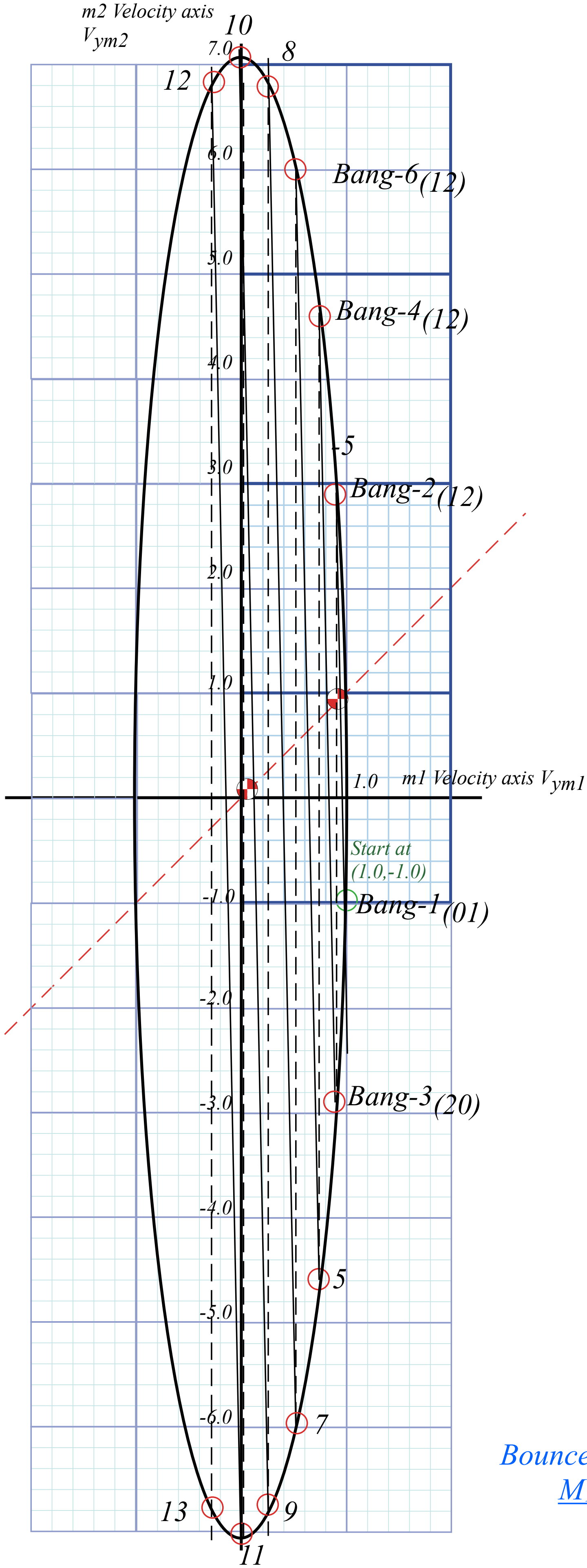
A. about twice as much.
B. about half again as much.
C. only slightly more.

(The answer to this month's "Figuring Physics" can be found at *TPT Online*, http://scitation.aip.org/upload/AAPT/TPT/Figuring/jan2017.pdf. The answer will also be printed in the February issue of *The Physics Teacher*. The answer to December's question appears on p. 54 of this issue.)

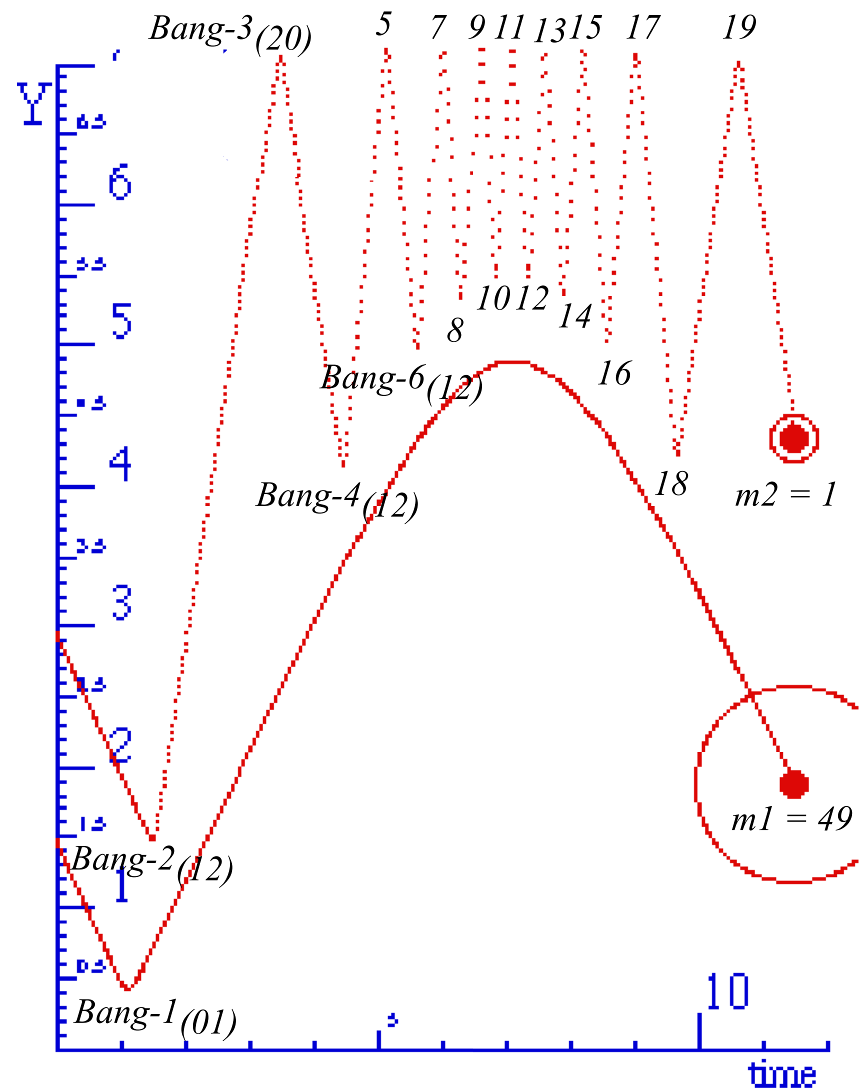
Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

→ *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1



Kinetic Energy Ellipse

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with Newtonian time plot]

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with V_2 vs V_1 plot]

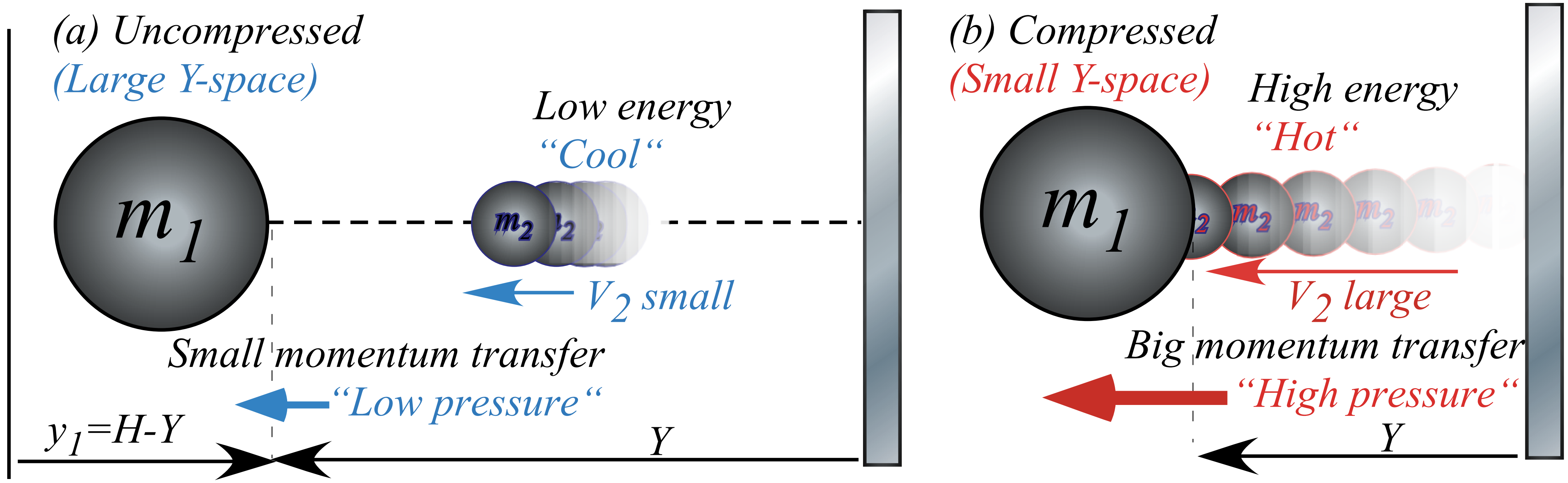
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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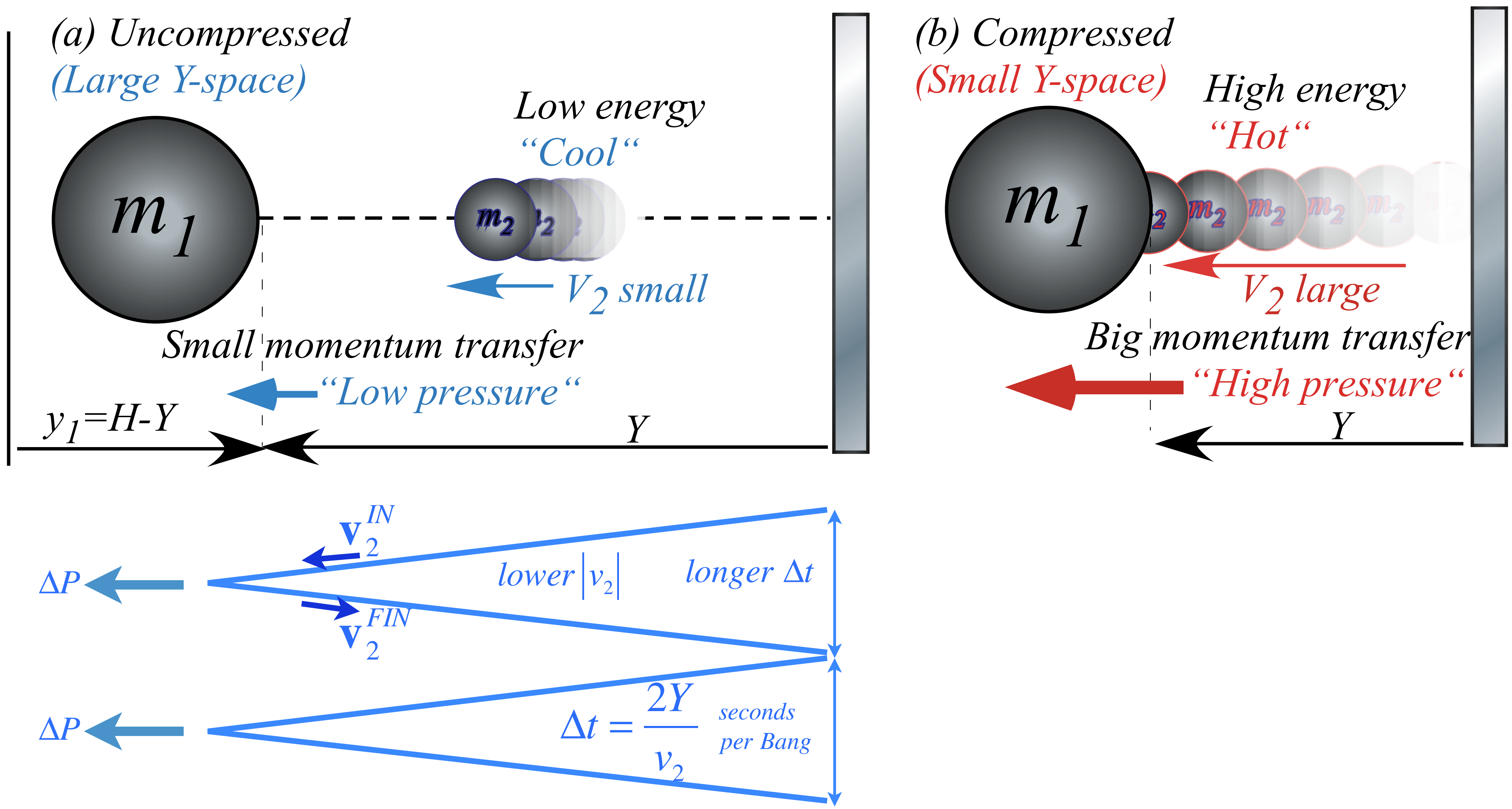

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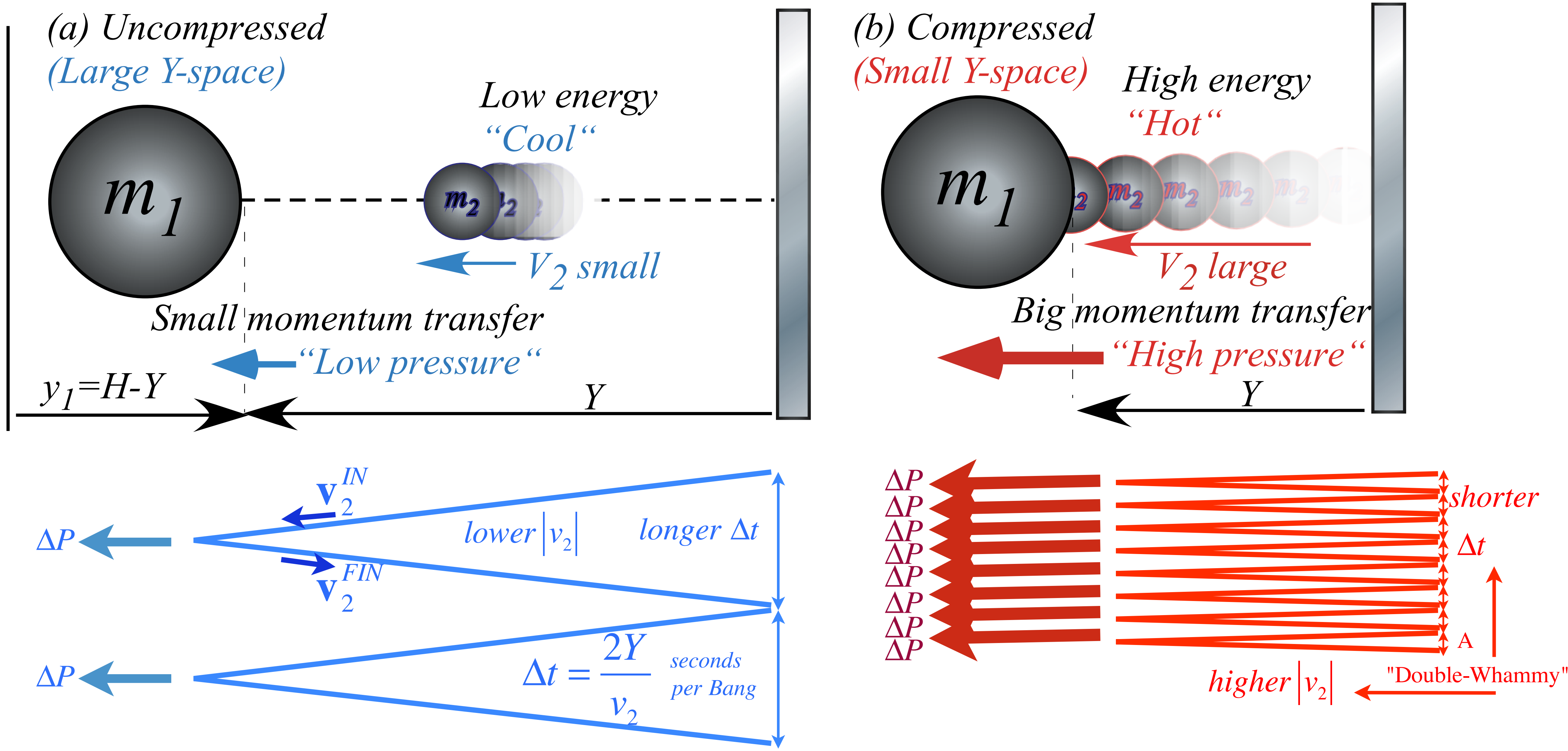

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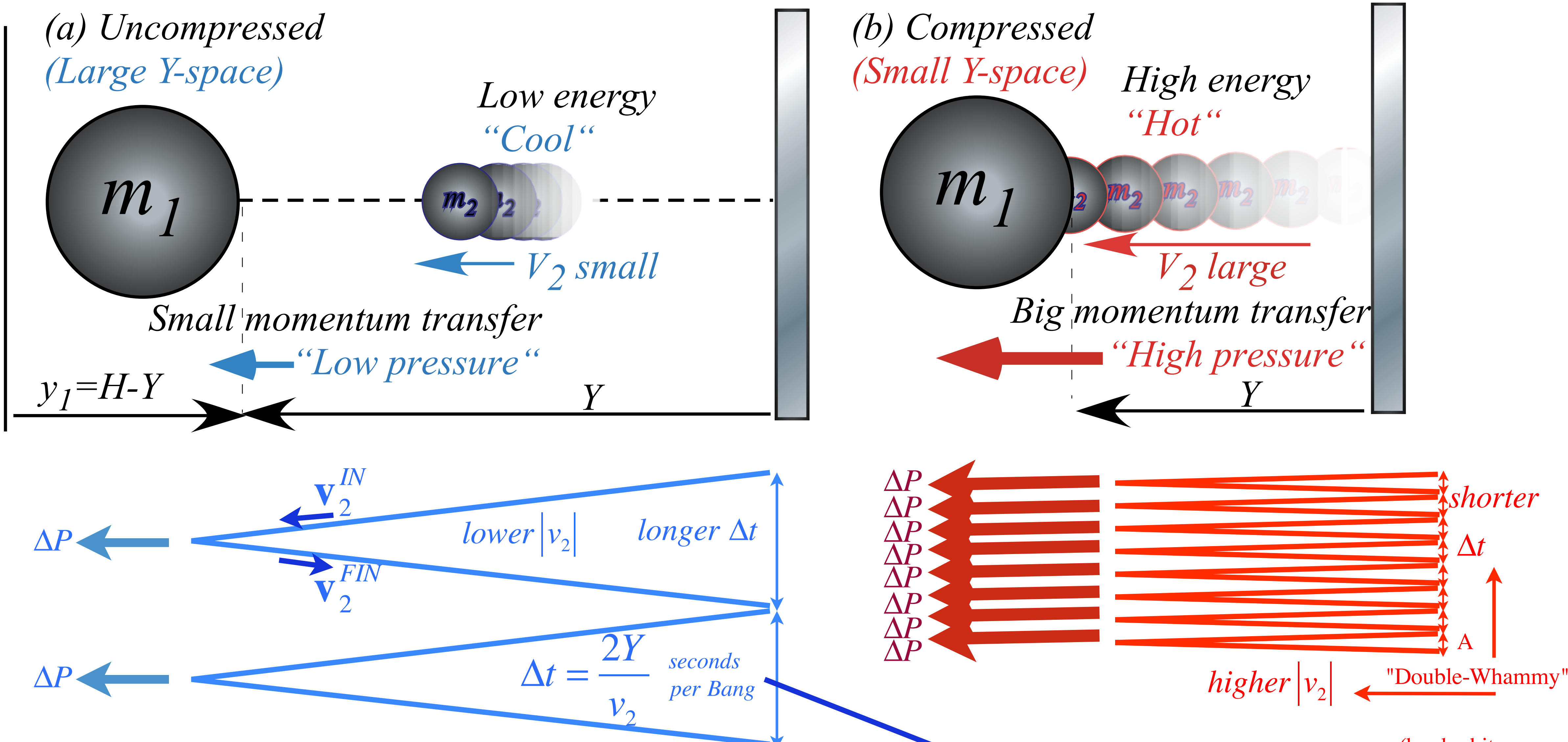


(harder hits and more hits/sec.)

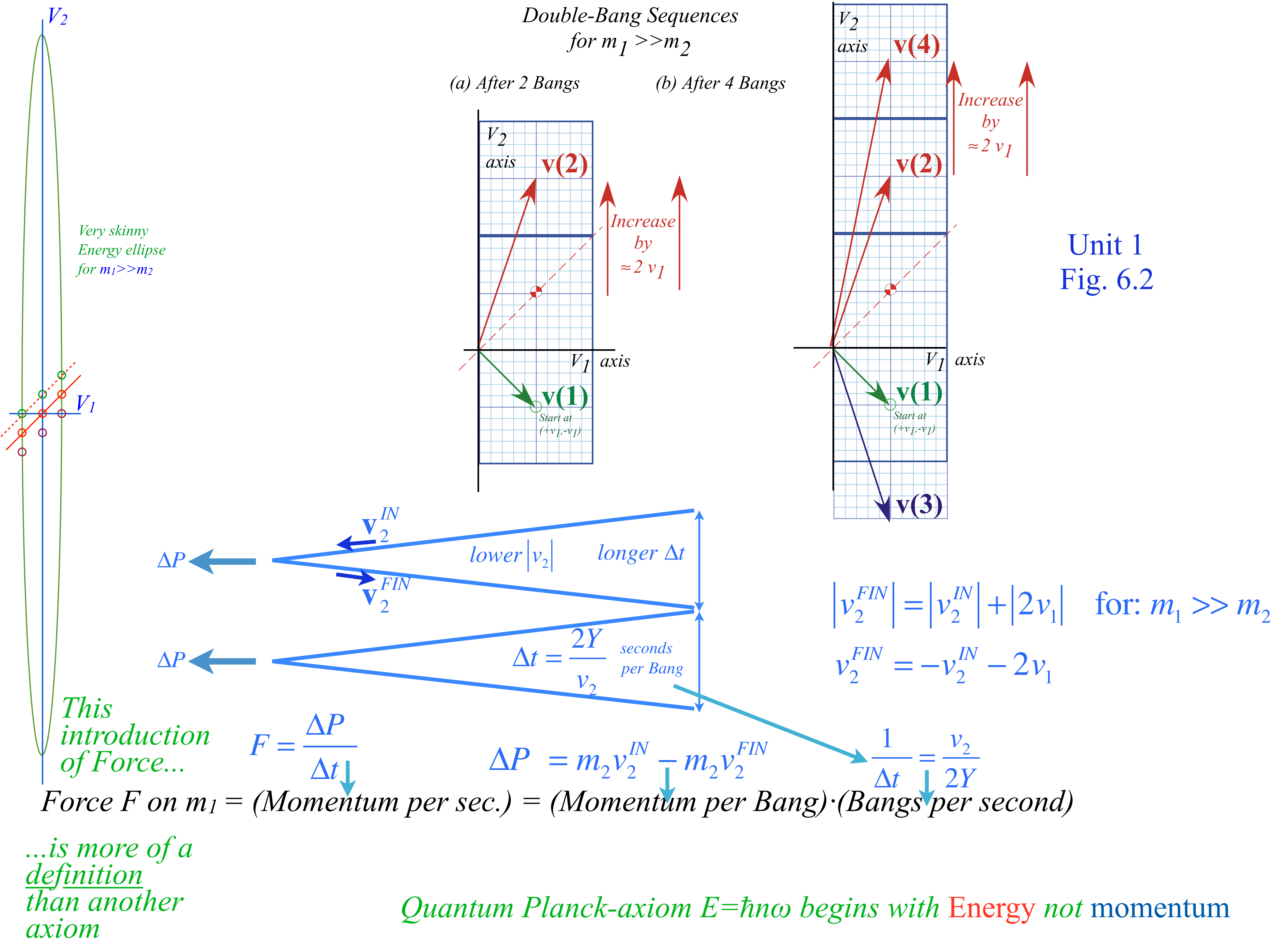
This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

Quantum Planck-axiom E=ħnω begins with Energy *not* momentum


Double-Bang Sequences
for $m_1 >> m_2$
(a) After 2 Bangs
(b) After 4 Bangs
V_2
Very skinny
Energy ellipse
for $m_1 >> m_2$
V_1
V_2 axis
v(2)
Increase by $\approx 2\, v_1$
V_1 axis
v(1)
Start at $(+v_1,-v_1)$
v(4)
v(3)
Unit 1
Fig. 6.2
ΔP
$\mathbf{v}_2^{IN}$
$\mathbf{v}_2^{FIN}$
lower $|v_2|$
longer Δt
$\Delta t = \frac{2Y}{v_2}$
seconds per Bang
$|v_2^{FIN}| = |v_2^{IN}| + |2v_1|$ for: $m_1 >> m_2$
$v_2^{FIN} = -v_2^{IN} - 2v_1$
This introduction of Force...
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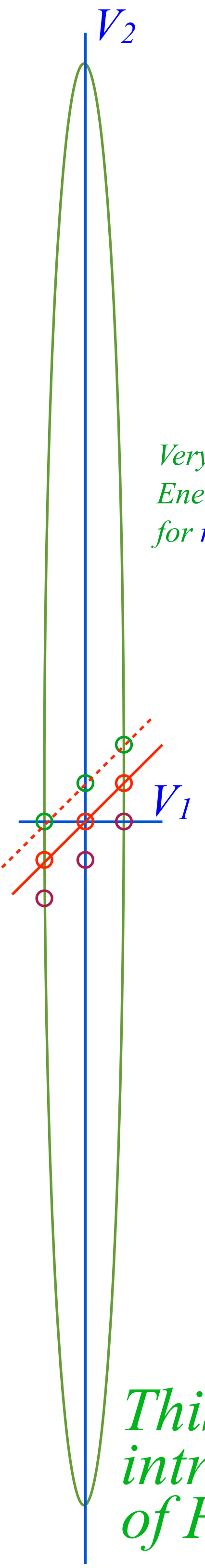
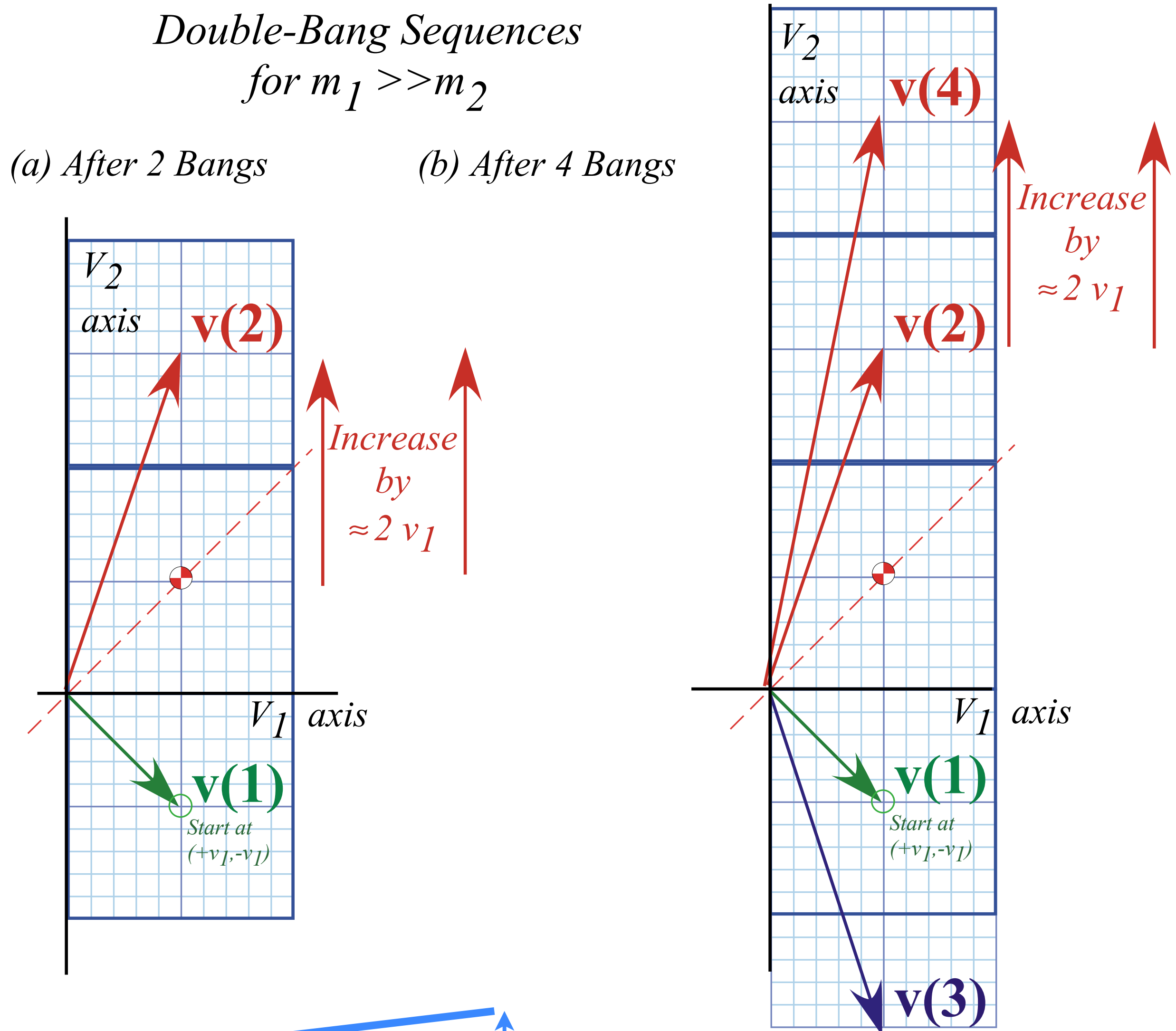
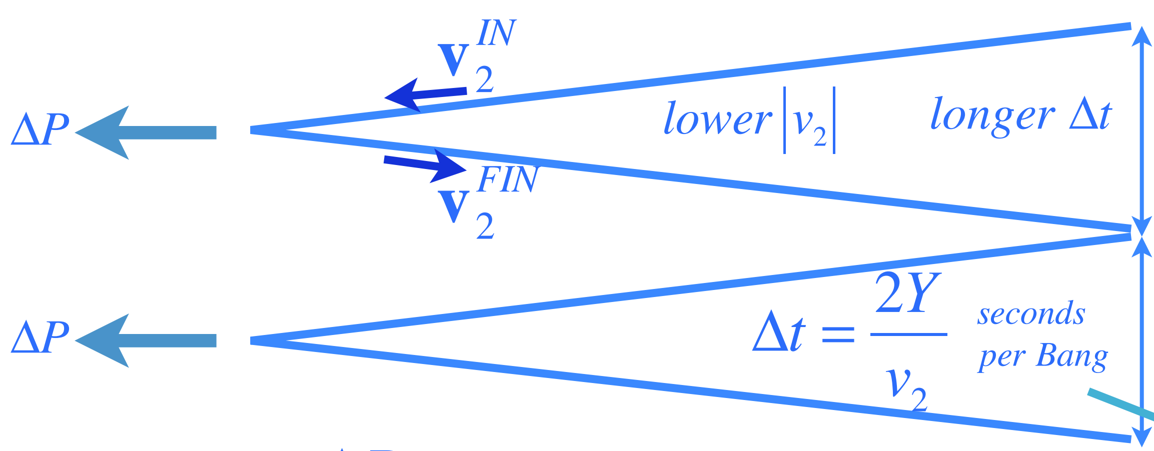
Quantum Planck-axiom $E=\hbar n\omega$ begins with Energy not momentum



Very skinny
Energy ellipse
for $m_1 >> m_2$



Unit 1
Fig. 6.2



$$\mathbf{v}_2^{IN} \qquad \mathbf{v}_2^{FIN}$$

$$\Delta t = \frac{2Y}{v_2}$$

$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

This introduction of Force...

$$F = \frac{\Delta P}{\Delta t}$$

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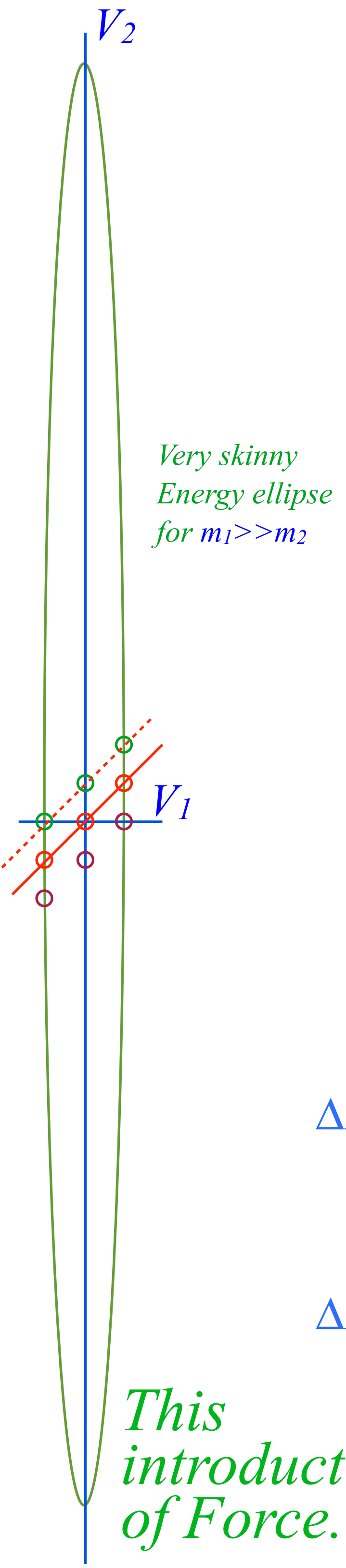
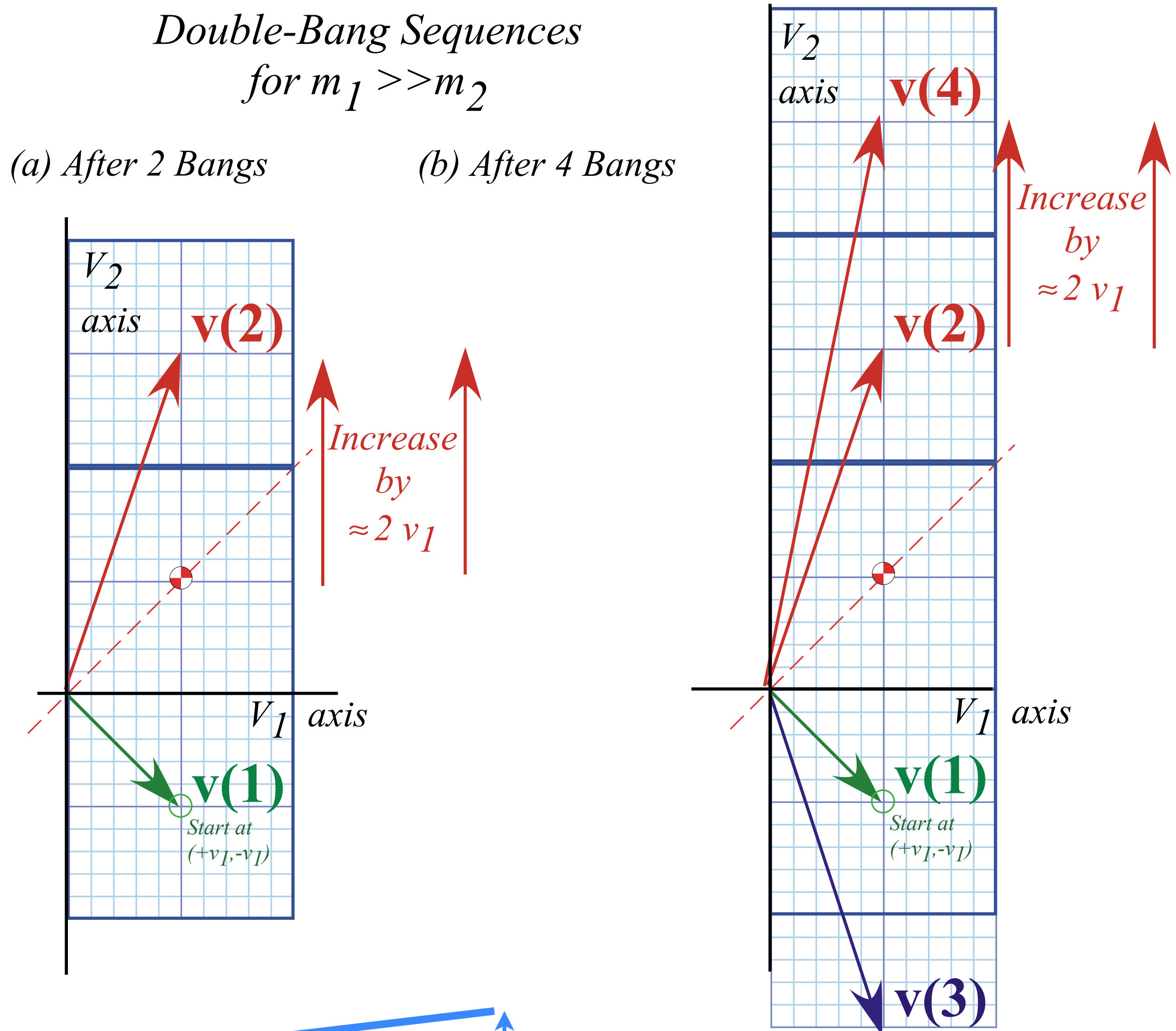
Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

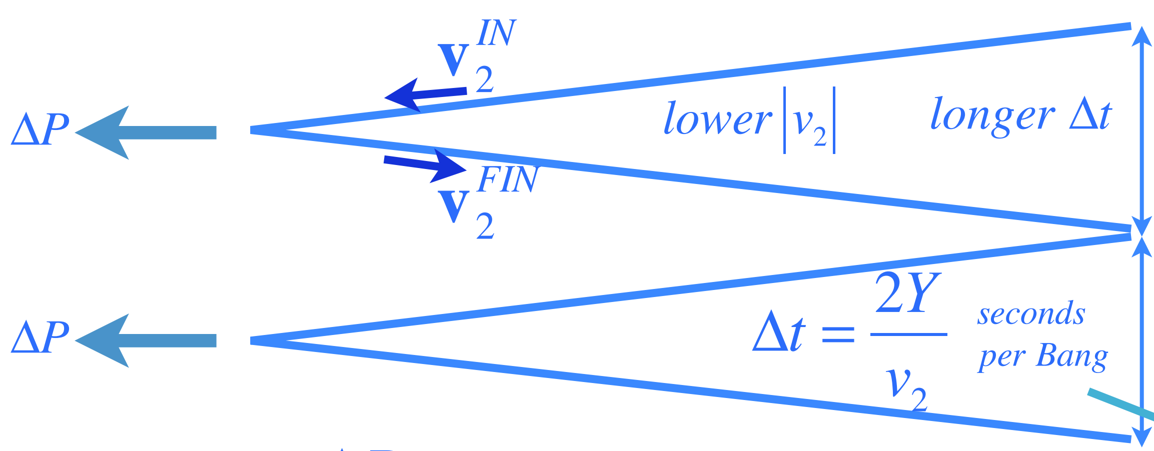
$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1$$

...is more of a <u>definition</u> than another <u>axiom</u>





Unit 1
Fig. 6.2



$$|v_2^{FIN}| = |v_2^{IN}| + |2v_1| \quad \text{for: } m_1 >> m_2$$

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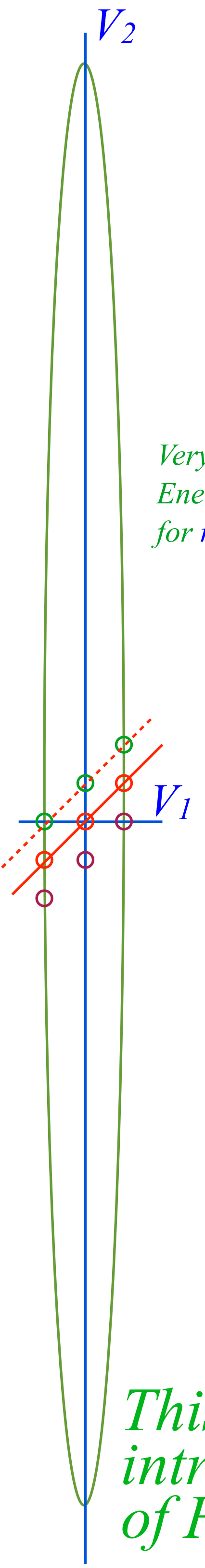
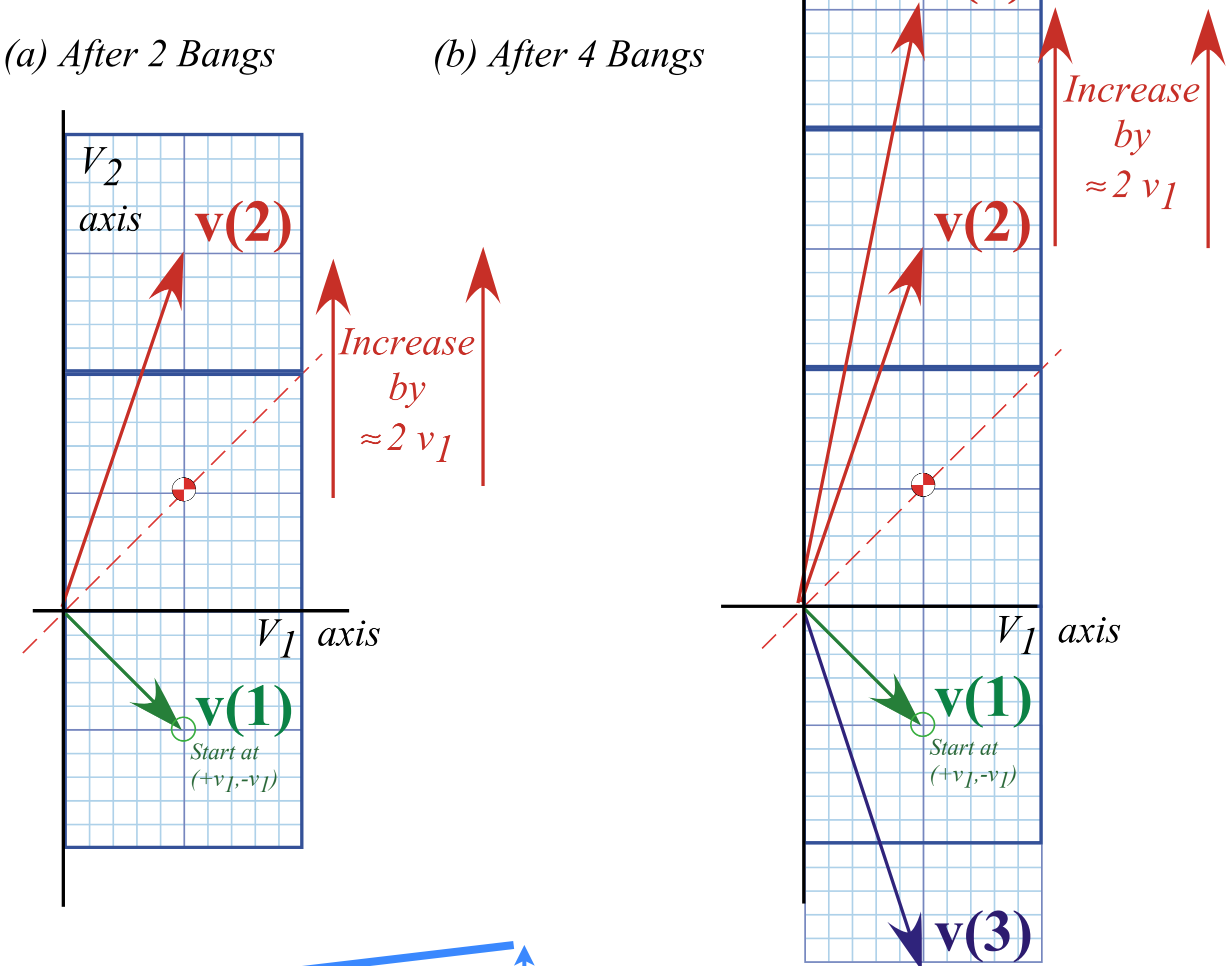
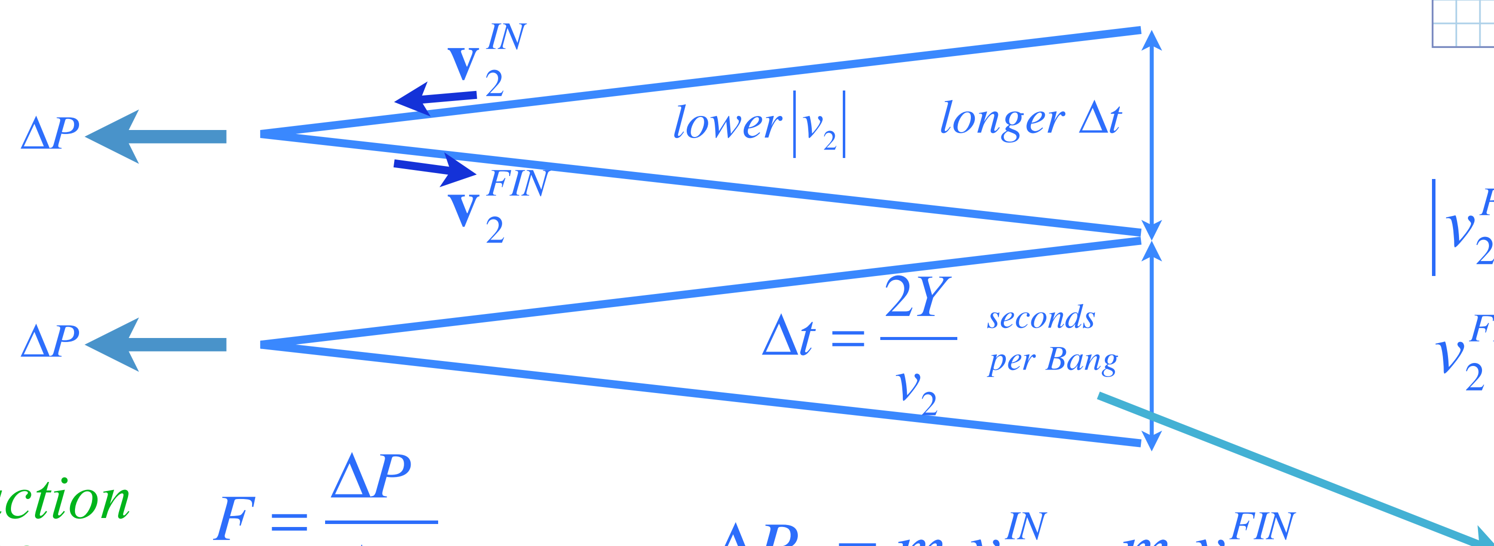
Assuming slow m_1 : $v_1 \ll v_2$



Double-Bang Sequences
for $m_1>>m_2$



Unit 1
Fig. 6.2



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$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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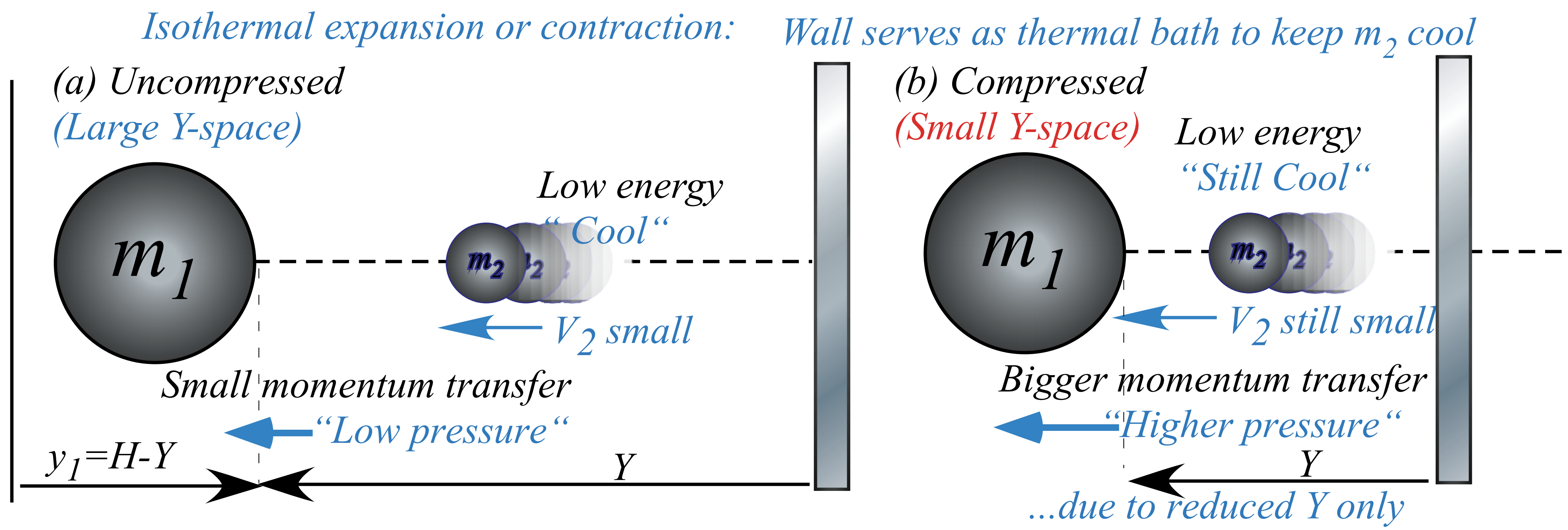
1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

Not a "Double-Whammy"...

...only a "Single-Whammy"



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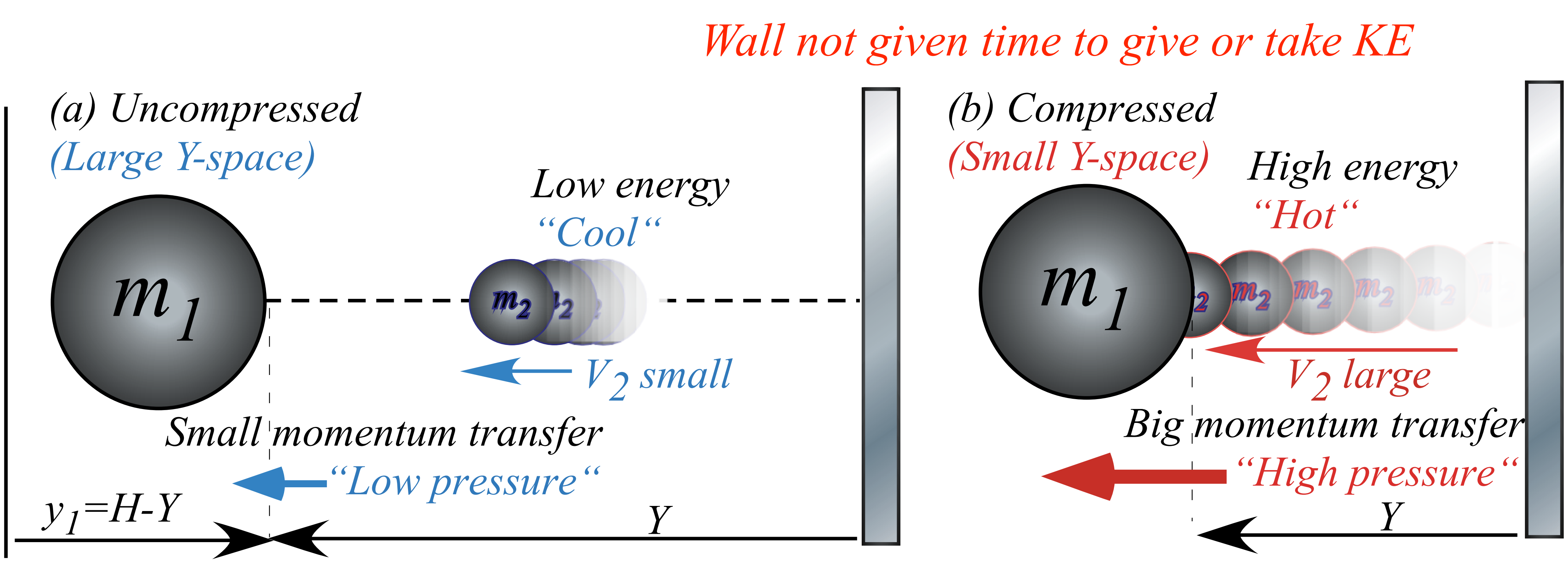
However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y}$$



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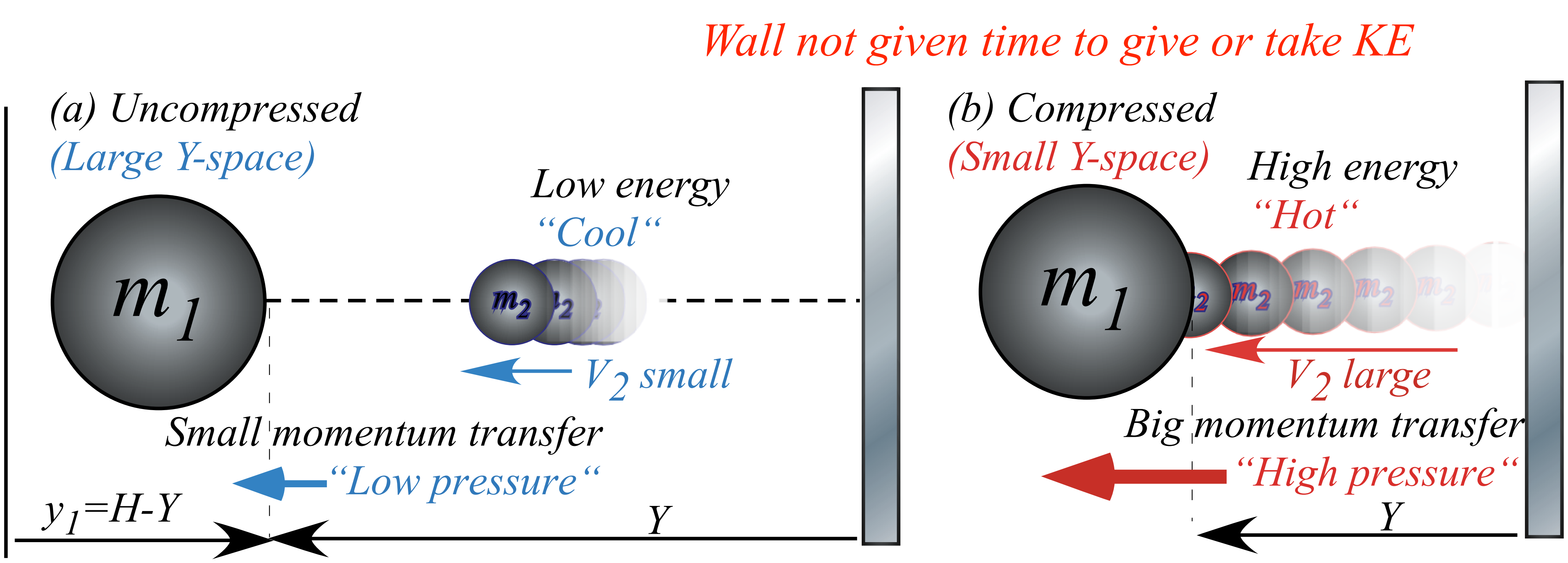

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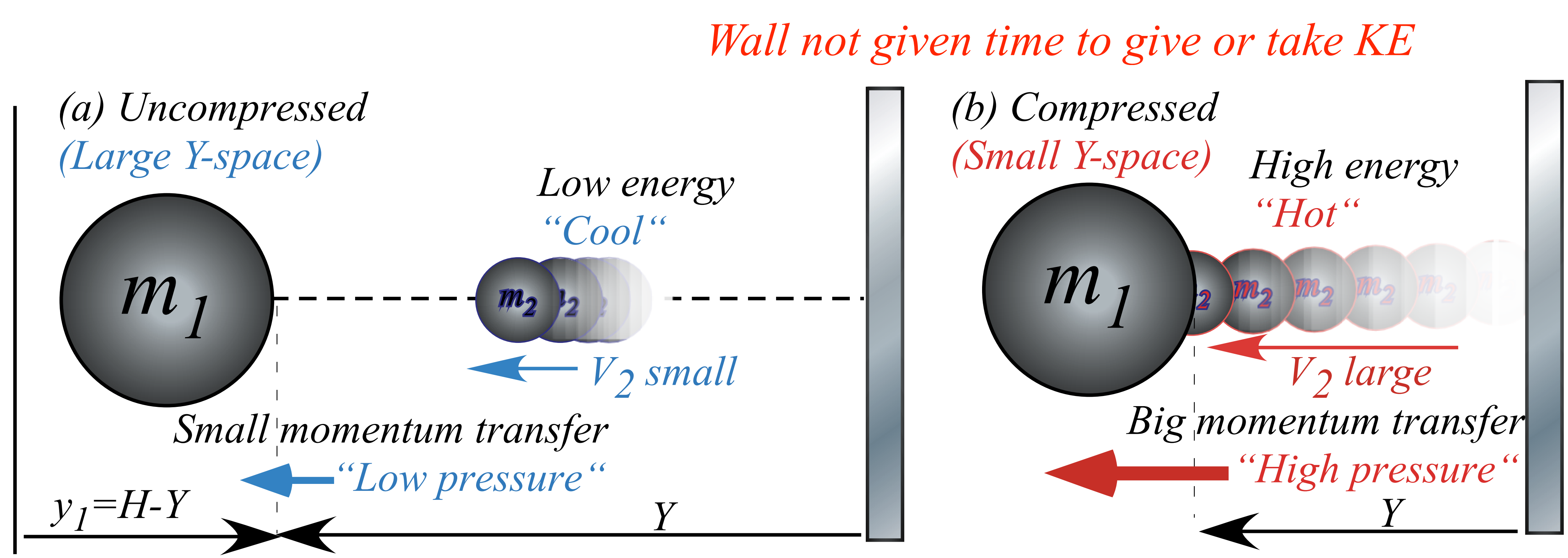
Differential equation results and has logarithmic integral. $\int\frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln\frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$



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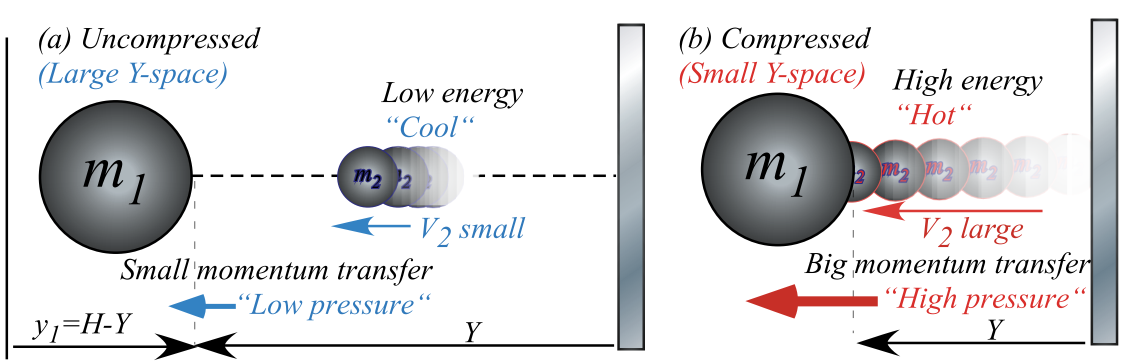
Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$):
$$F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}$$



A "Double-Whammy"

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

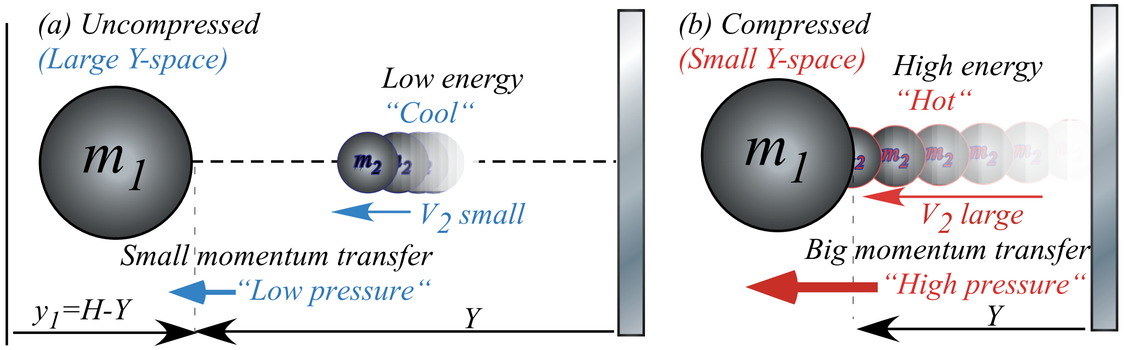
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)= $\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

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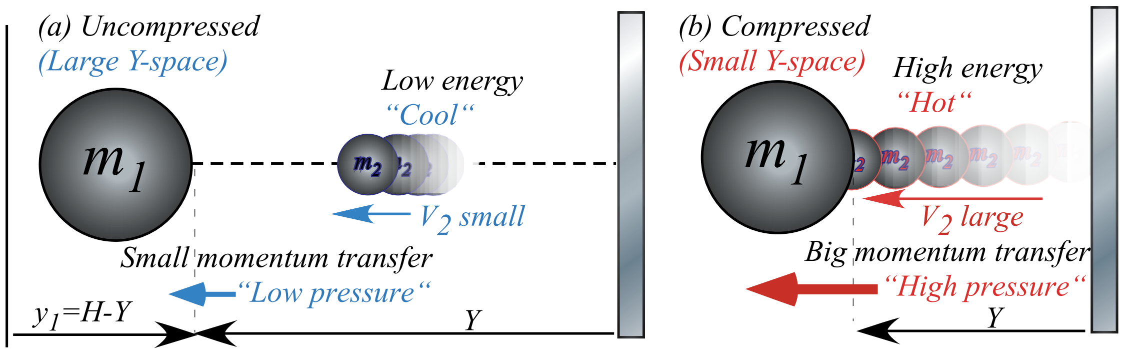
Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F \cdot dY = \pm dU$



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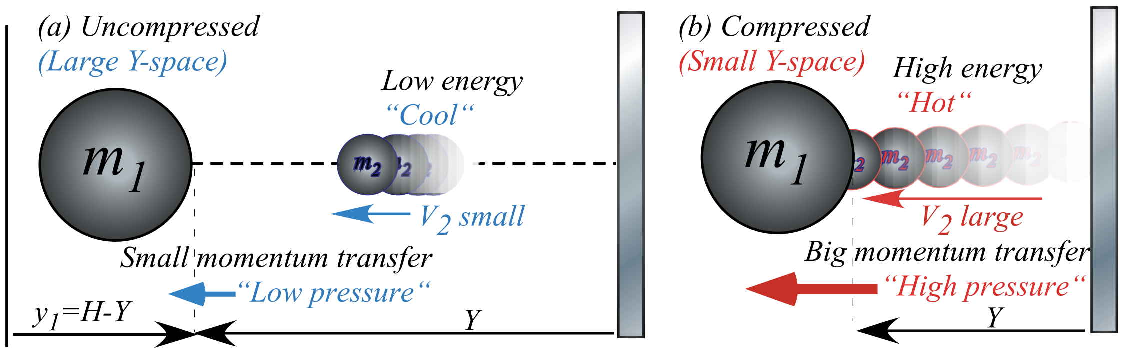
Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?*



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

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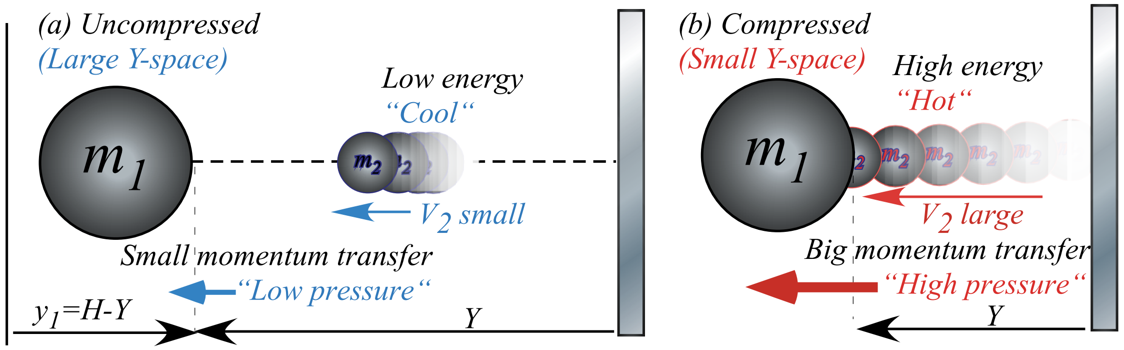

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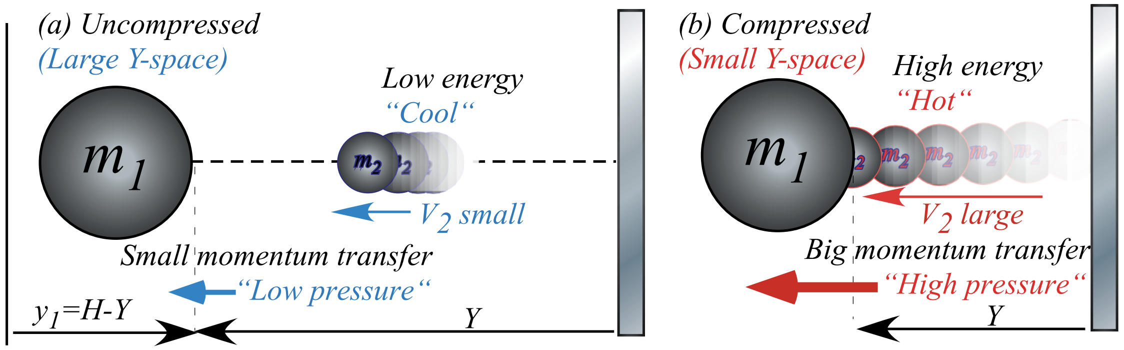
Q? *Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$

(Here: $V = v_2$)



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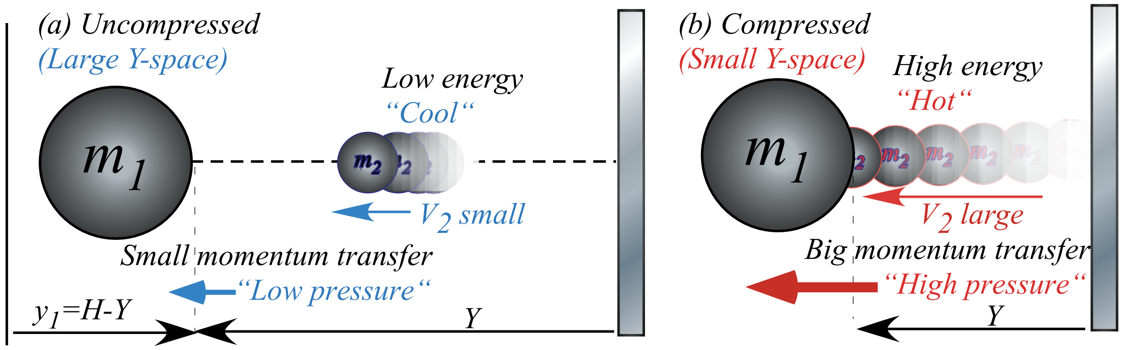
or else :

$$F \cdot \frac{dY}{dt} = \frac{dp}{dt} \cdot V = \frac{d(mV)}{dt} \cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt} \qquad \text{(Here: } V = v_2\text{)}$$



A "Double-Whammy" system

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

→ *Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$*

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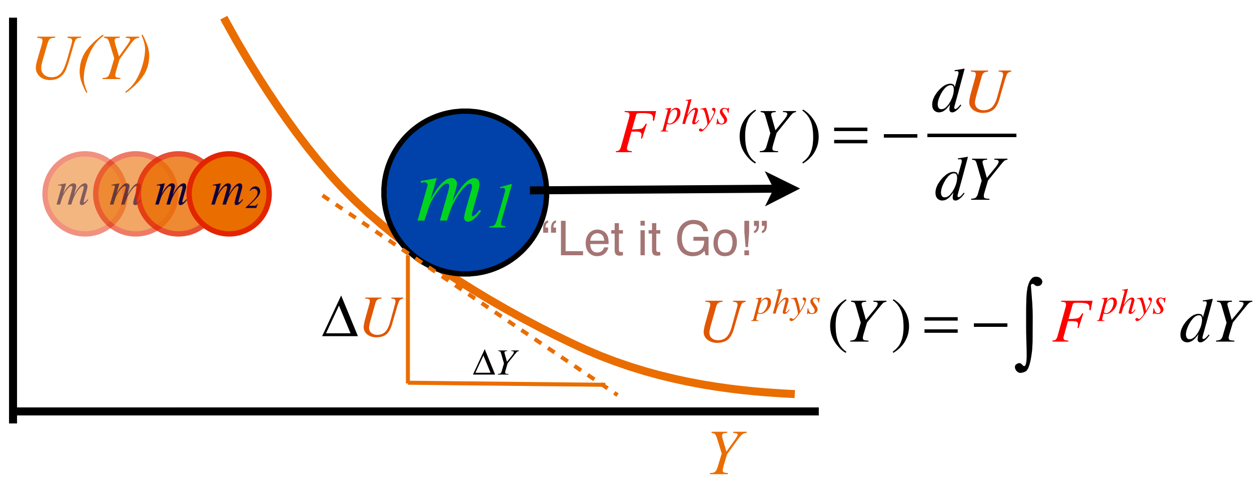
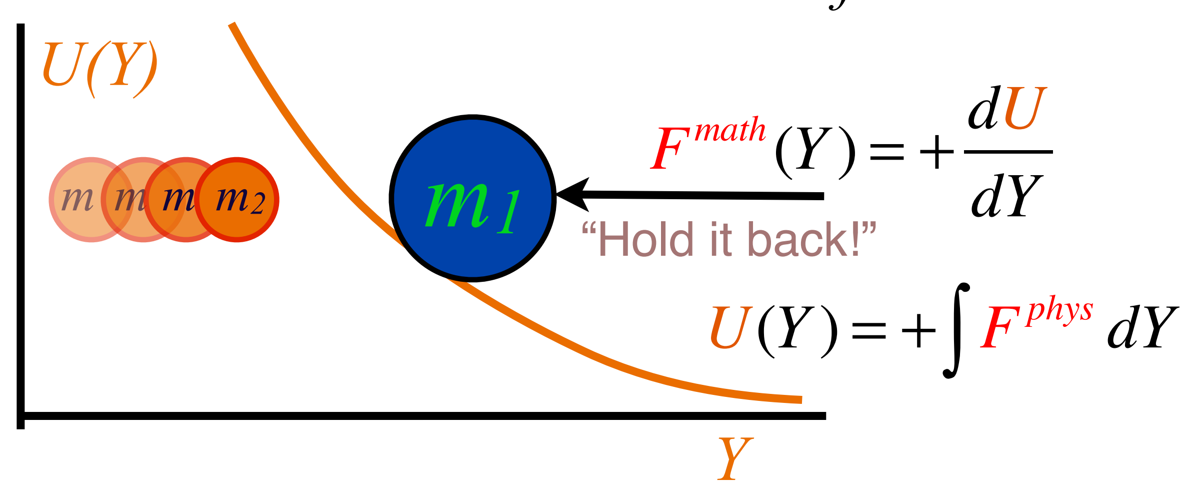
The "Physicist" View of Force



The "Mathematician" View of Force



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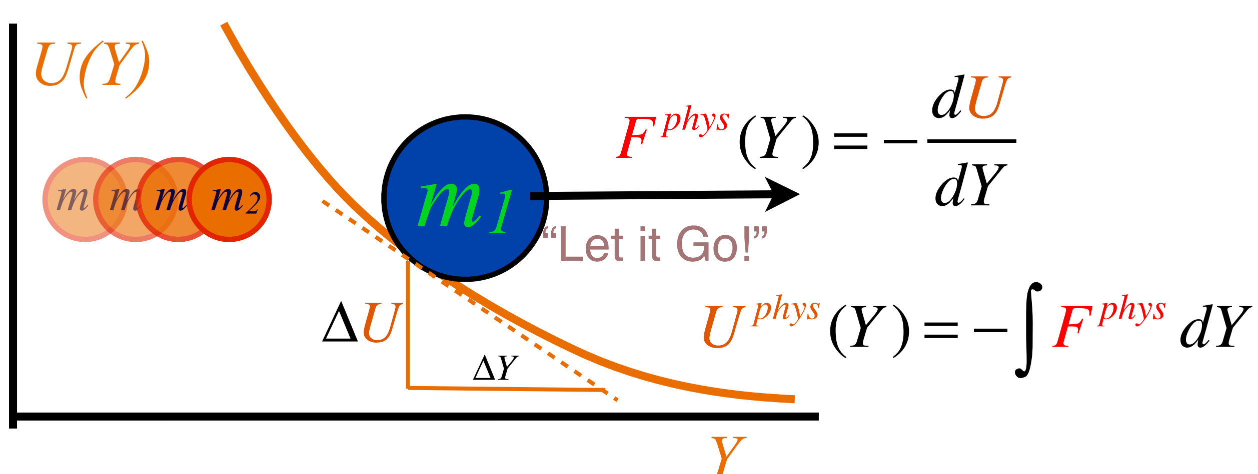
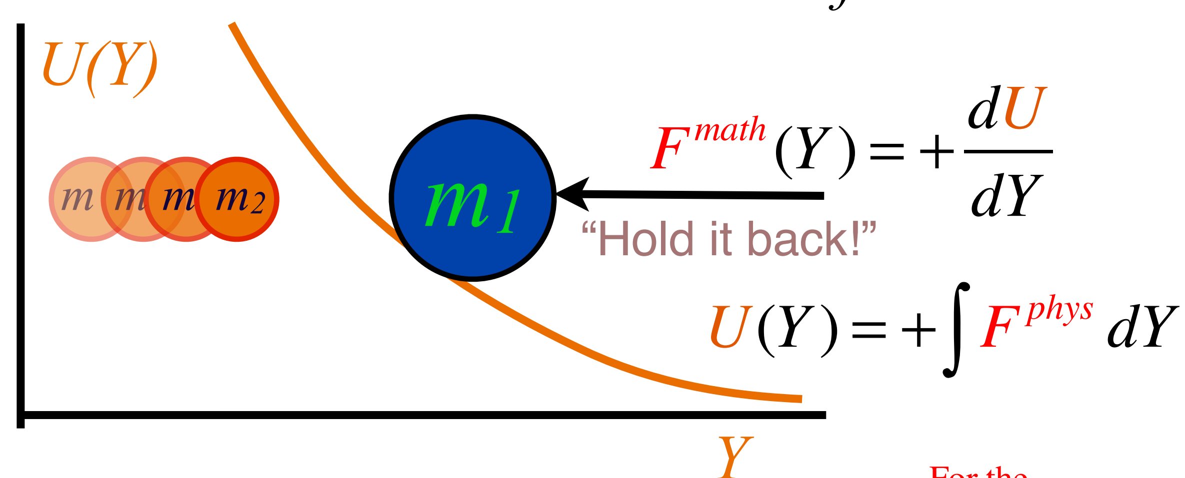


The "Mathematician" View of Force



(OK, But, is this consistent with the $F = (const.)^2/Y^3$ (on p.18)?)

For the "Double-Whammy" system

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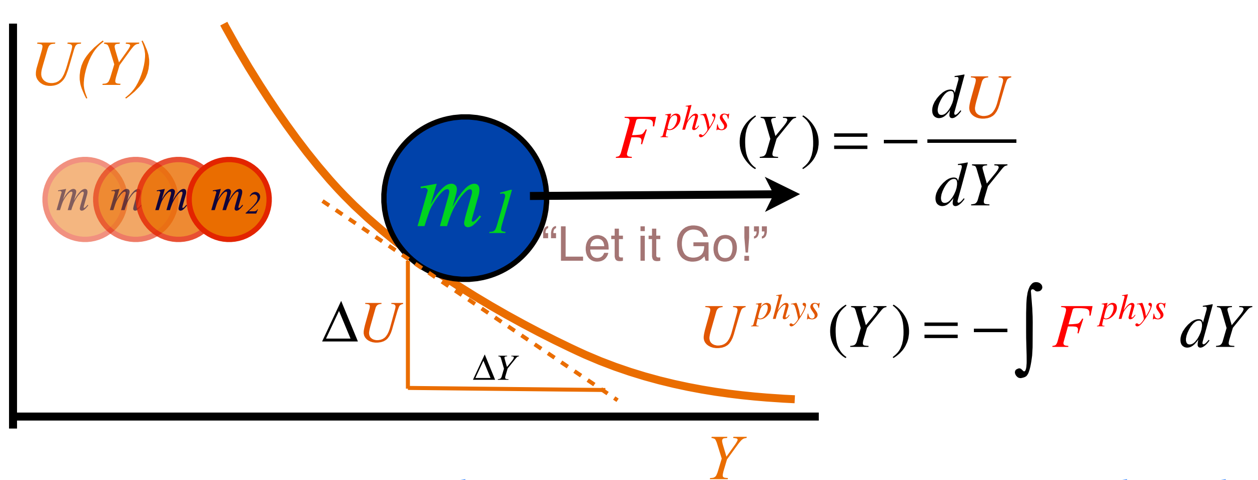

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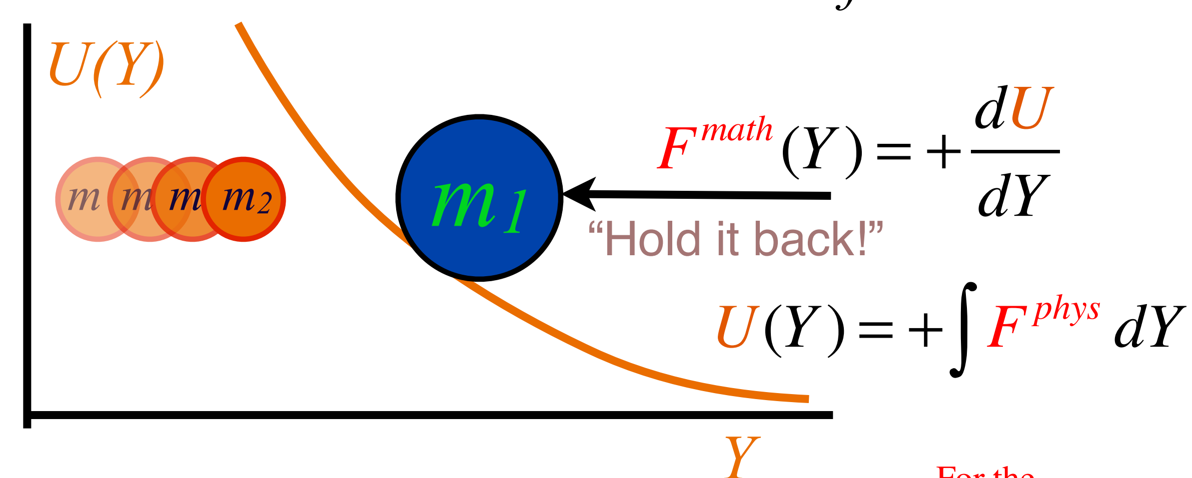

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For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$*

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

Not a "Double-Whammy"...
...only a "Single-Whammy"

$$F^{phys} = \frac{m_2 {v_2}^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad implies: \qquad U(Y) = \int -F^{phys} dY = \int -\frac{m_2 {v_2}^2}{Y} dY = -m_2 {v_2}^2 \ln(Y)$$

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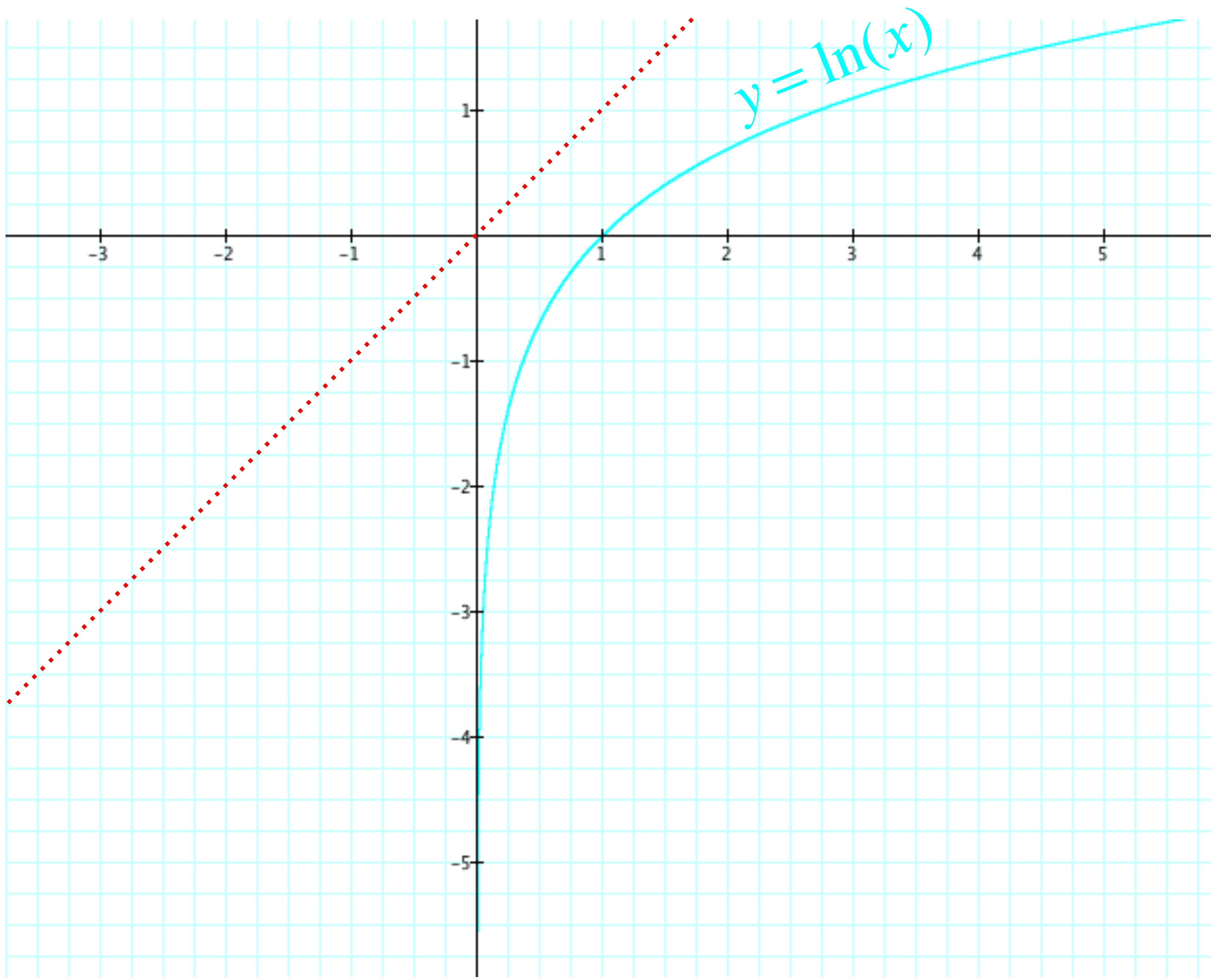

Notice how tightly ln(x) hugs y-axis...

It's the backside of exponential $y=e^x$...

1D-Isothermal Force Law (*assume* v_2 *is constant for all Y*):

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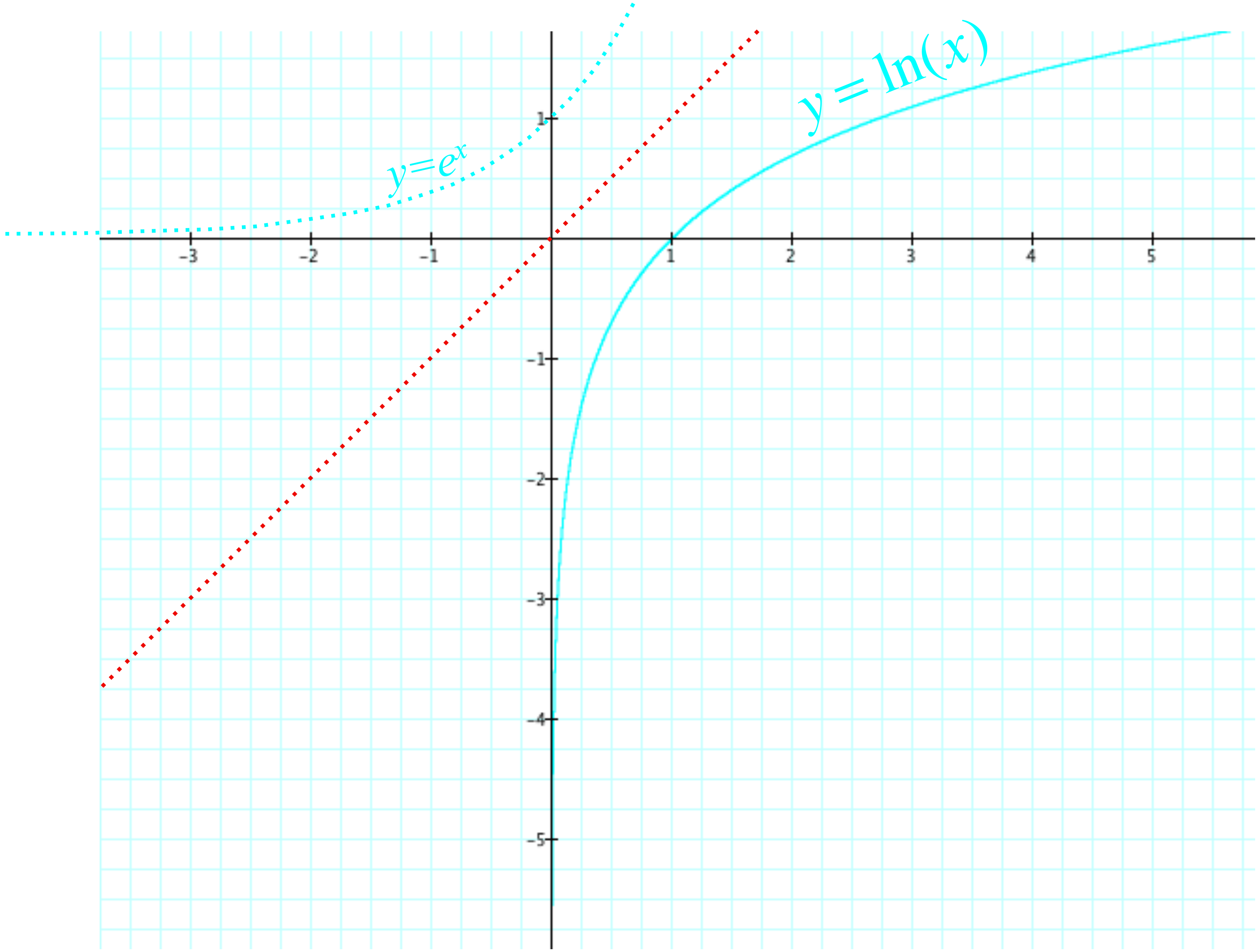

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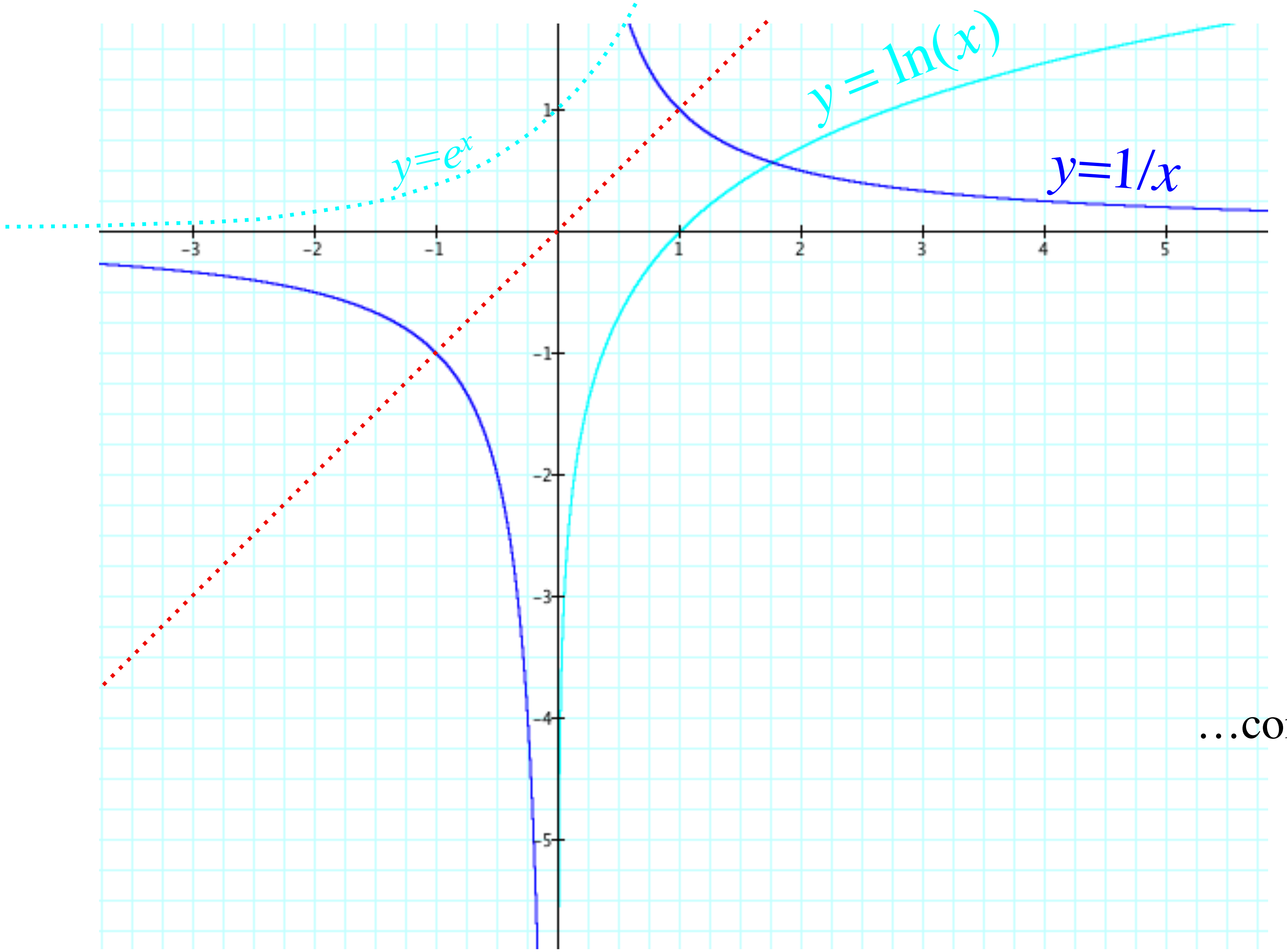

Notice how tightly *ln(x)* *hugs y-axis*...

It's the backside of exponential $y=e^x$...

...compared to $y=1/x$ or $x=1/y$

1D-Isothermal Force Law (*assume v_2 is constant for all Y*): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

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$$const. = E = \frac{1}{2} m_1 {v_1}^2 + U(Y) \quad where: U(Y) = -m_2 {v_2}^2 \ln(Y)$$

Define for big mass m_1: *Kinetic energy* $KE(v_1)$ vs *Potential energy* $PE(Y) = U(Y)$

1D-Isothermal Force Law (*assume v_2 is constant for all Y*): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

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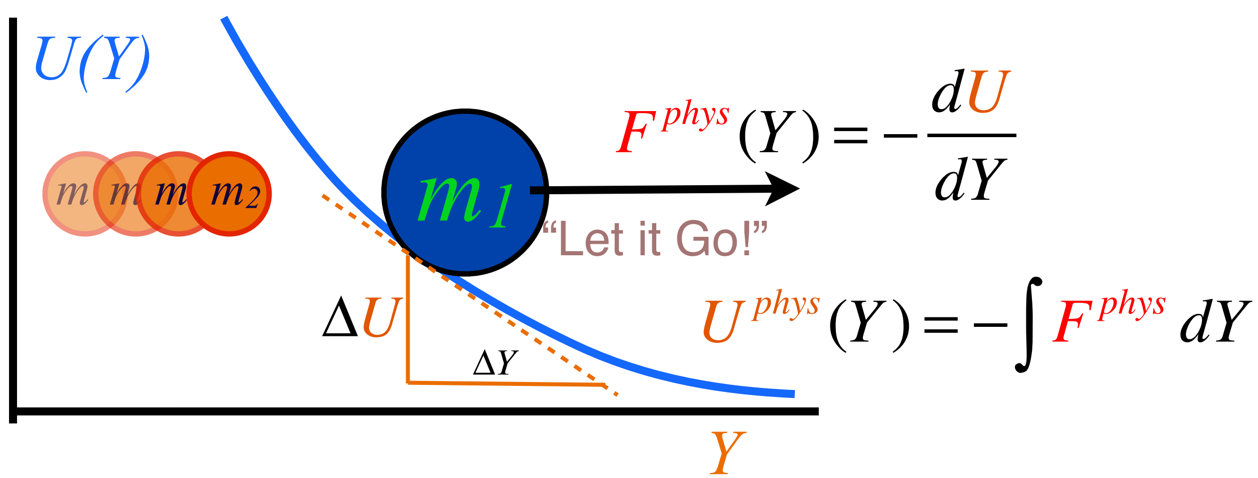
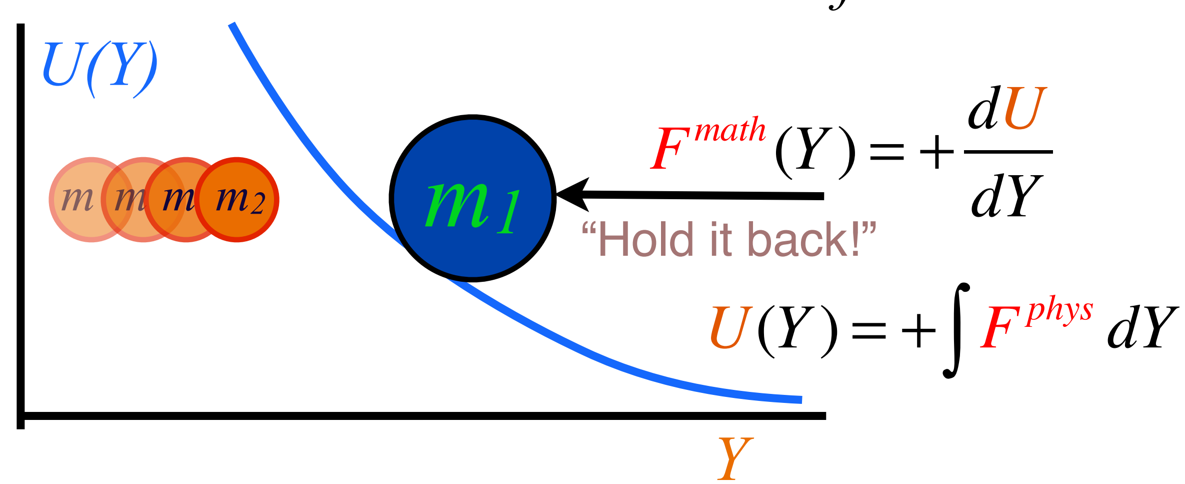
The "Physicist" View of Force



The "Mathematician" View of Force



1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

$$F^{phys}=\frac{m_2 {v_2}^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad implies: \quad U(Y)=\int -F^{phys}\,dY=\int -\frac{m_2 {v_2}^2}{Y}dY=-m_2 {v_2}^2 \ln(Y)$$

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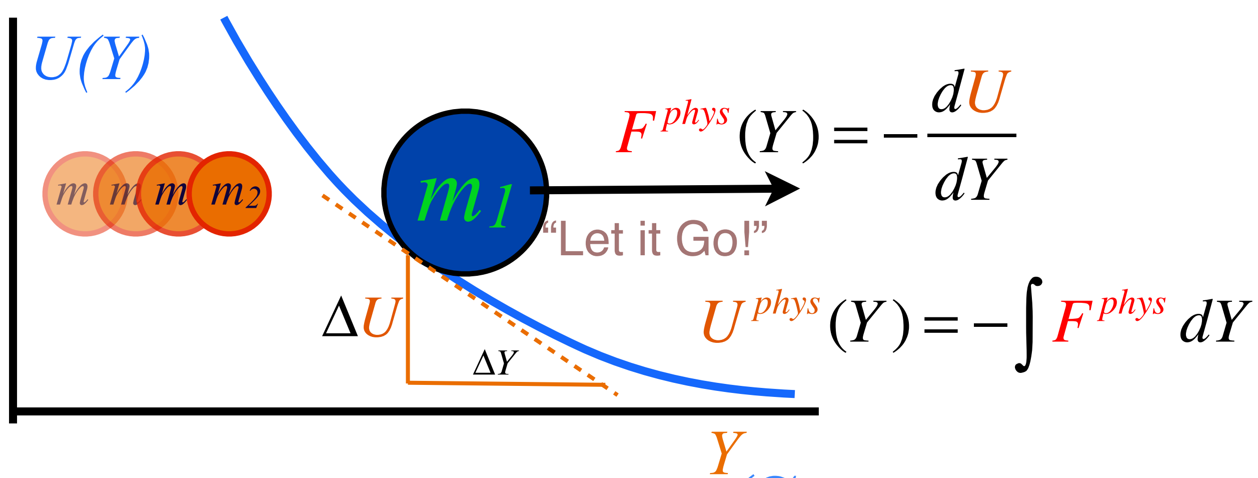

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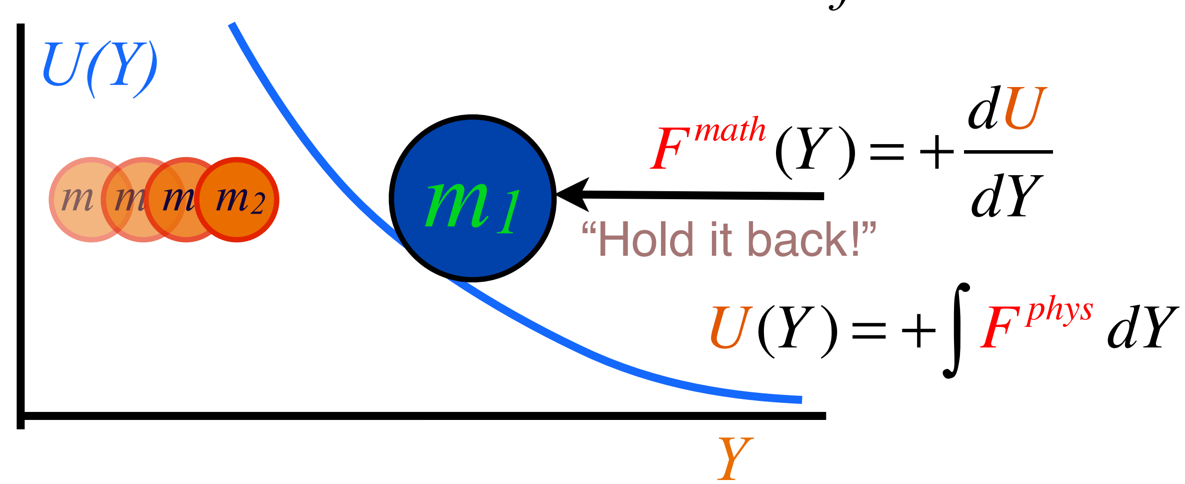


(Same integral/differential relations)

$$F^{phys}=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

consistent with:

$$F^{phys}=-\frac{\Delta U}{\Delta Y}=-\frac{d}{dY}\left(-const.\ln(Y)\right)=\frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

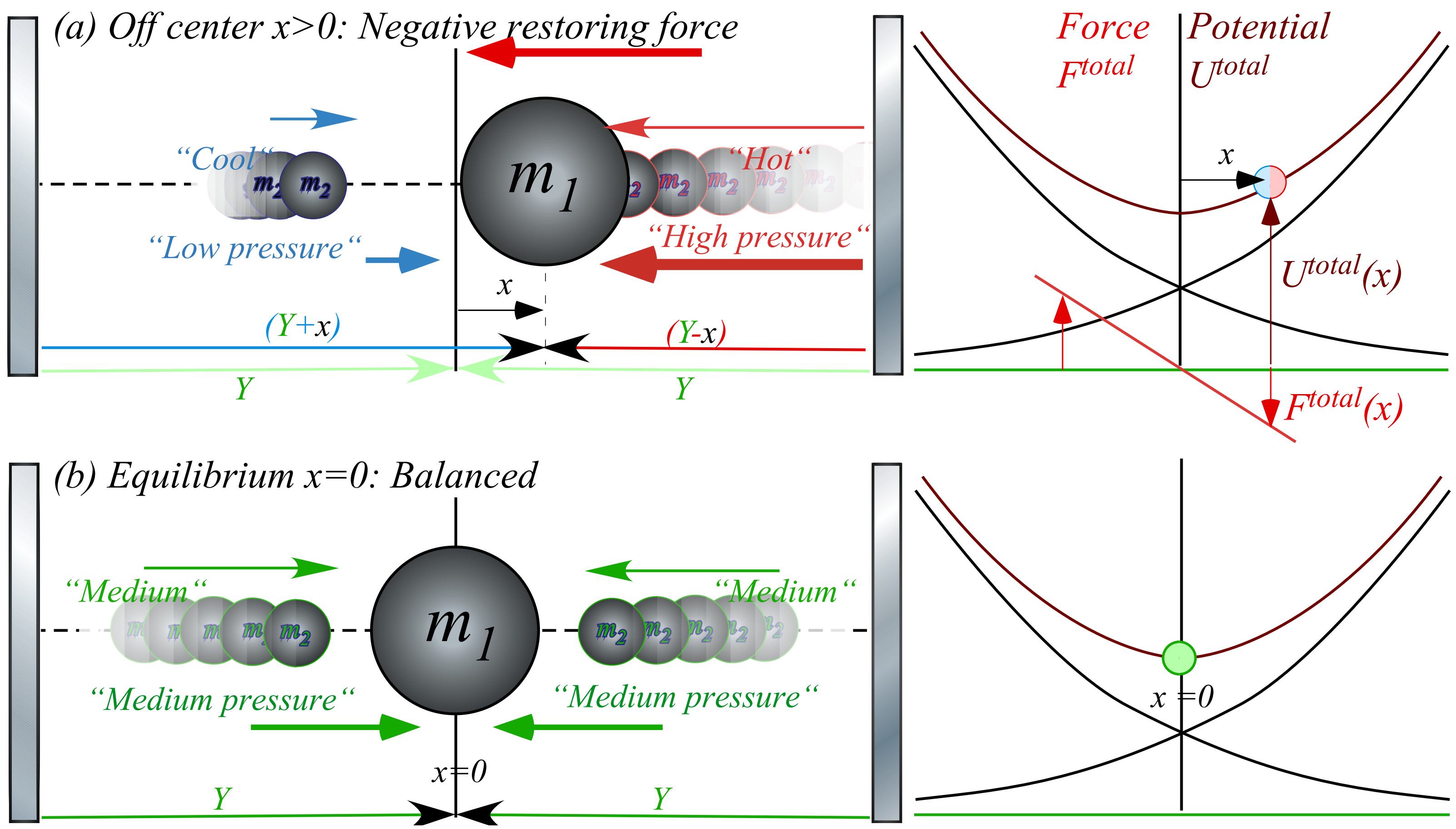
Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

→ *Example of oscillator with opposing Isothermal potentials*

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f\left[1 - x + x^2 - x^3 \ldots\right] - f\left[1 + x + x^2 + x^3 \ldots\right] = -2f\cdot x - 2f\cdot x^3 - \ldots$$

$$F^{HO} = -k\cdot x = -\frac{\partial U^{HO}}{\partial x} \qquad U^{HO} = \frac{1}{2}k\cdot x^2 = -\int F^{HO}\,dx \qquad \text{HO } \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\upsilon$$

Example of oscillator with opposing ***Isothermal potentials***

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

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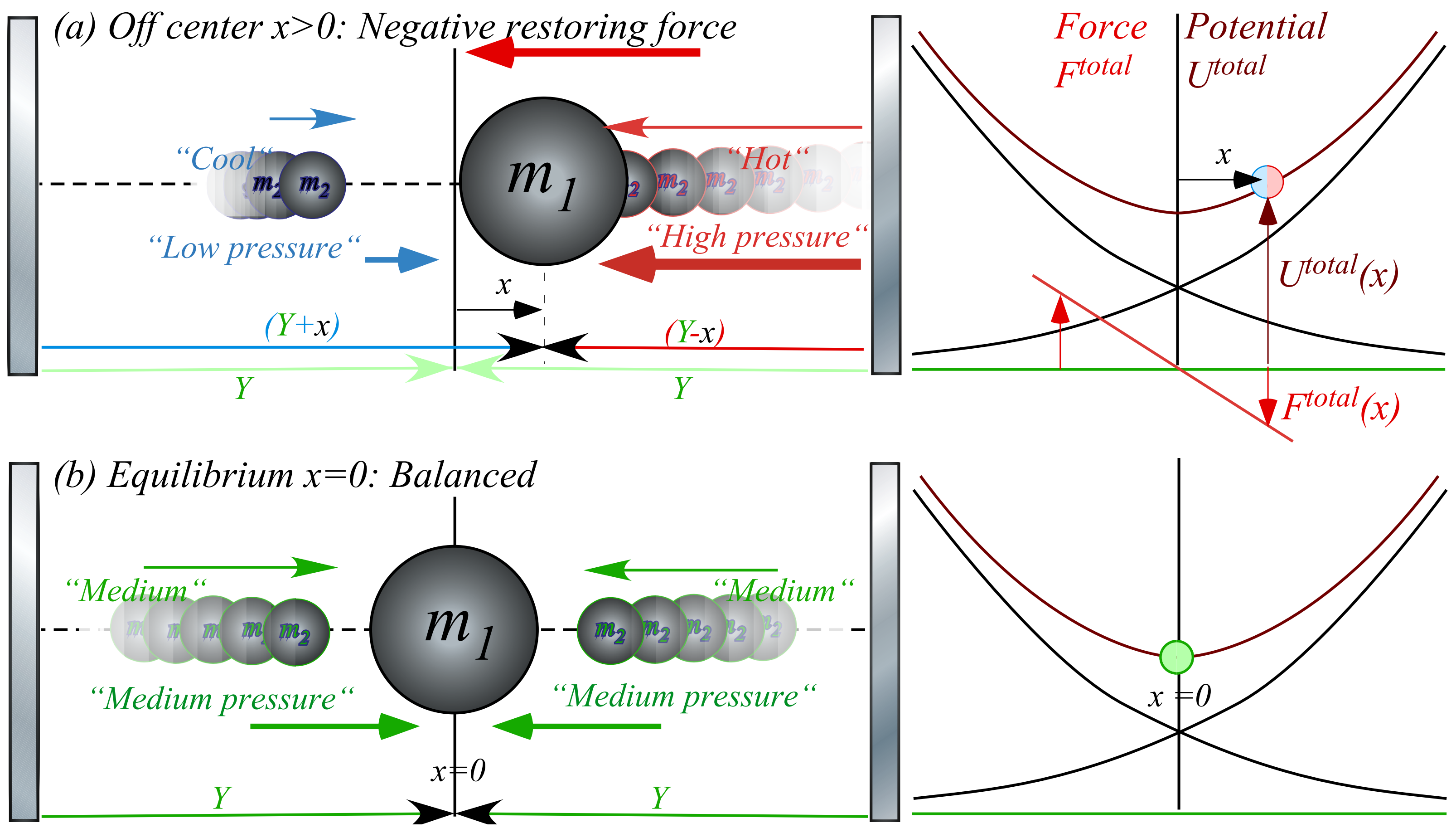


Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

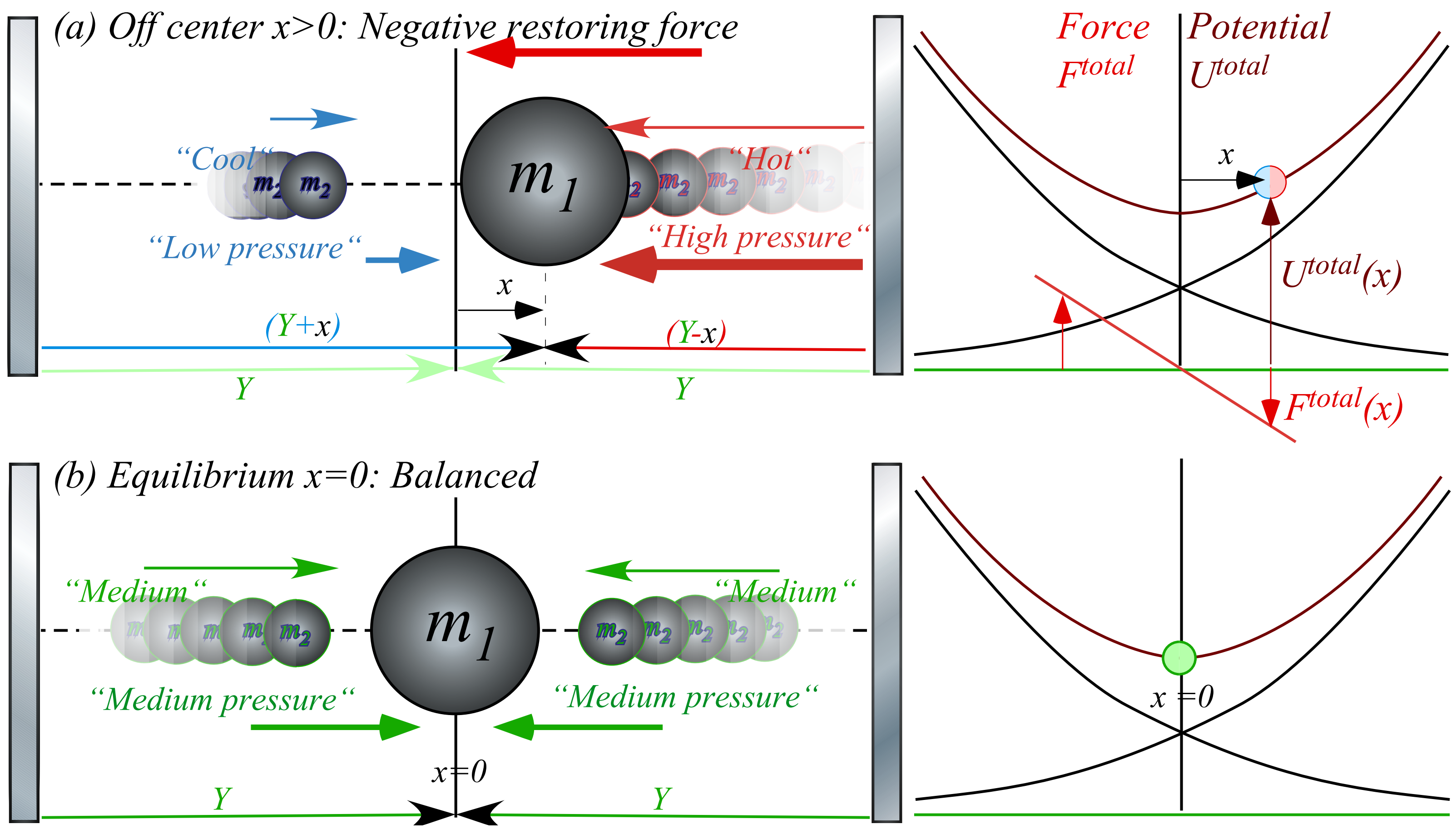
$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U=-m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots$$

$$(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

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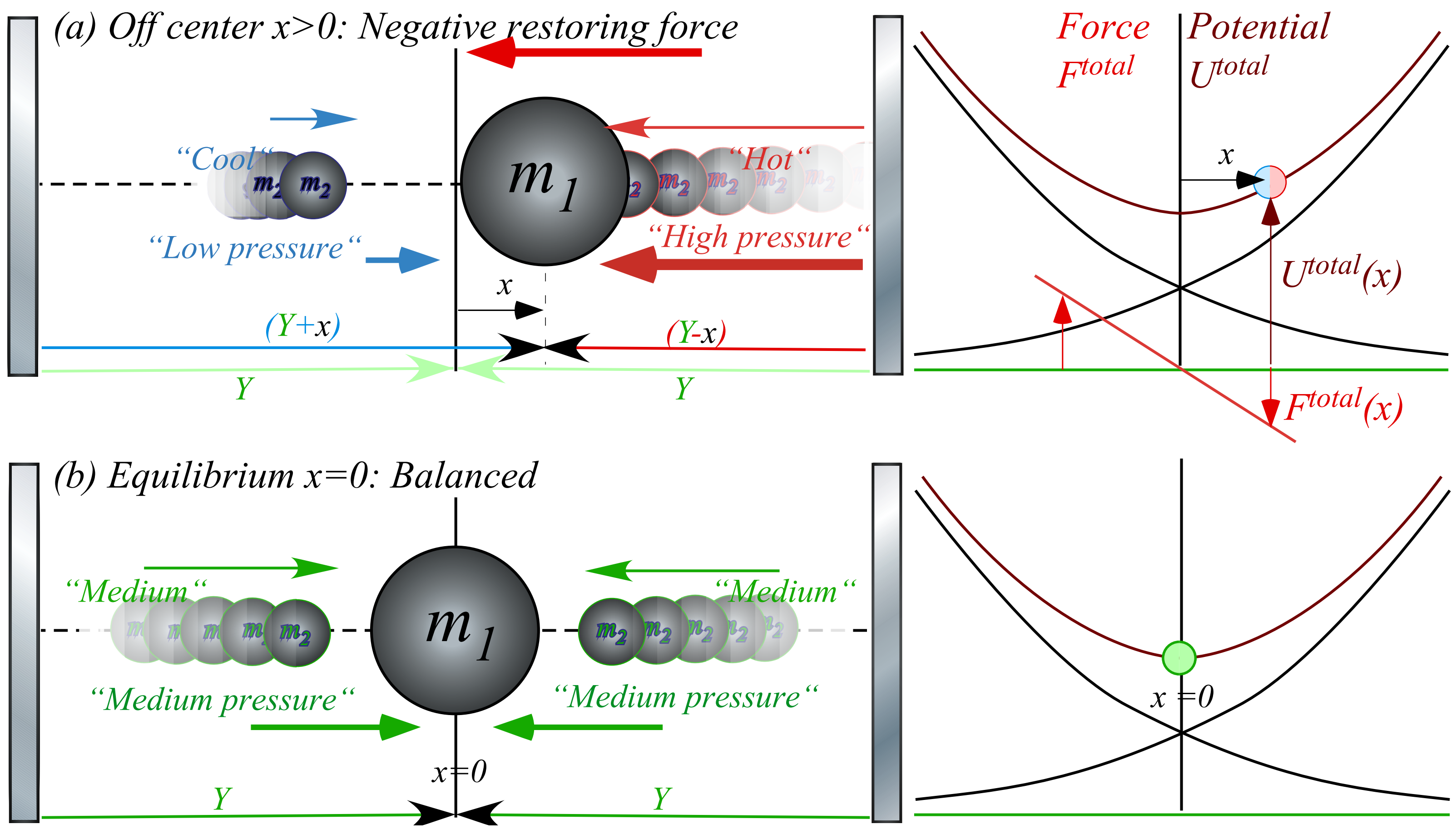

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-\ldots$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots$$

$$(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

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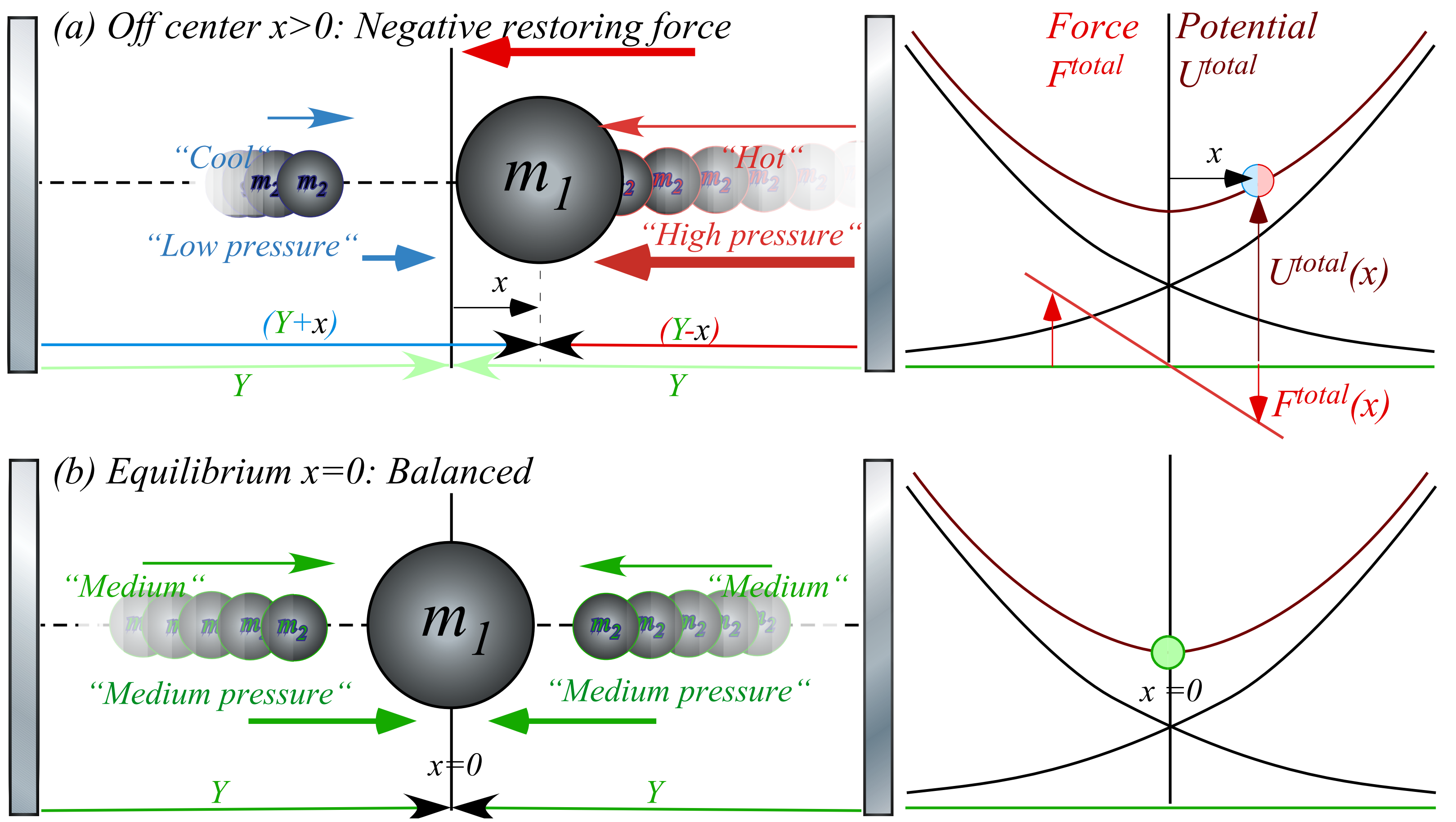

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\frac{1}{Y_0}+\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}+\frac{x^3}{Y_0^4}+\cdots\right]$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots \qquad (Y_0-x)^{-1}=Y_0^{-1}+xY_0^{-2}+x^2Y_0^{-3}+x^3Y_0^{-4}\ldots$$

$$(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots$$

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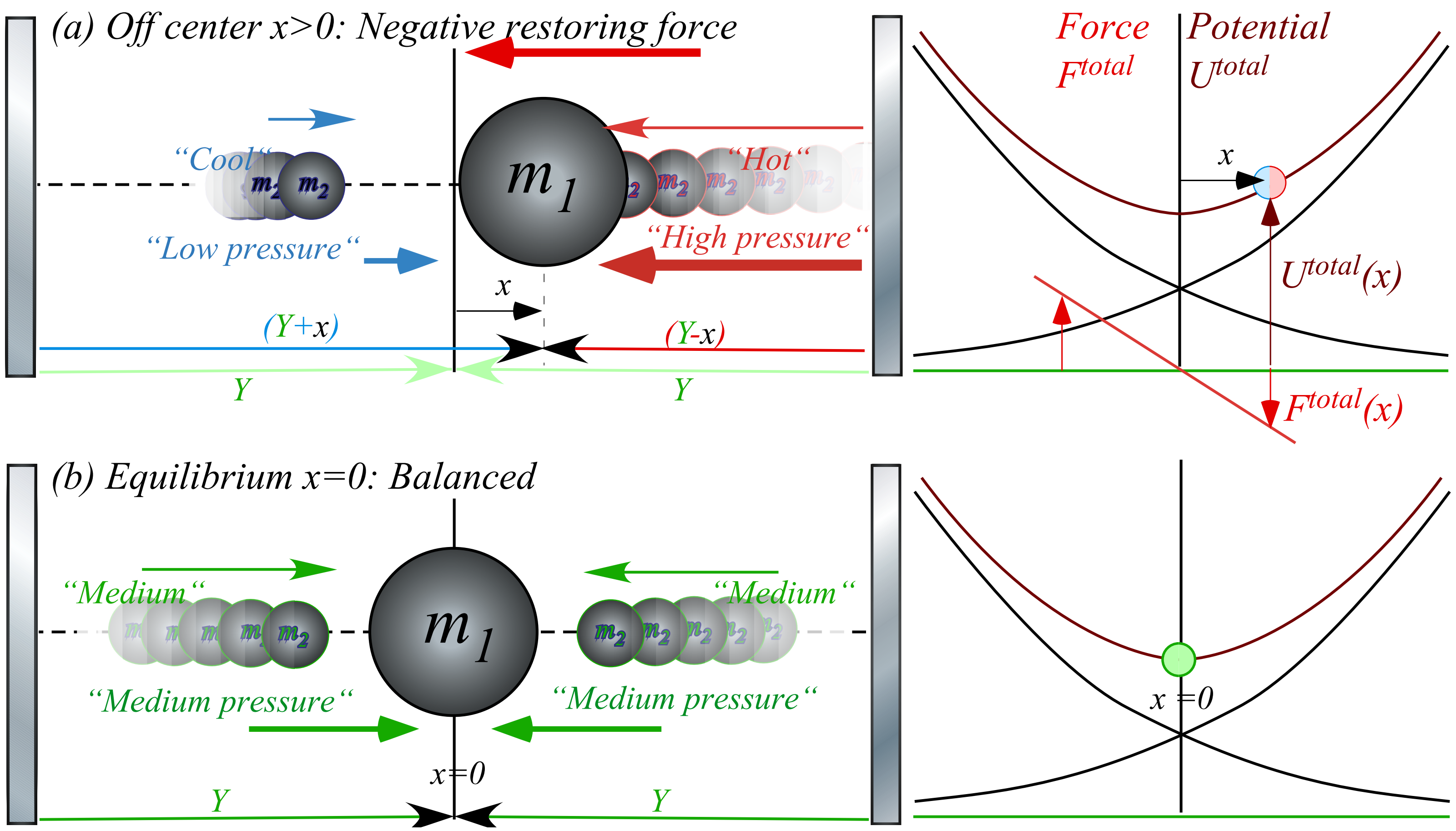
Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\frac{1}{Y_0} + \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} + \frac{x^3}{Y_0^4} + \cdots\right]$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1} x + \frac{n(n\text{-}1)}{2} Y_0^{n-2} x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2 \cdot 3} Y_0^{n-3} x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2 \cdot 3 \cdot 4} Y_0^{n-4} x^4 \ldots$$

Binomial Theorem

Example of oscillator with opposing ***Isothermal potentials***

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

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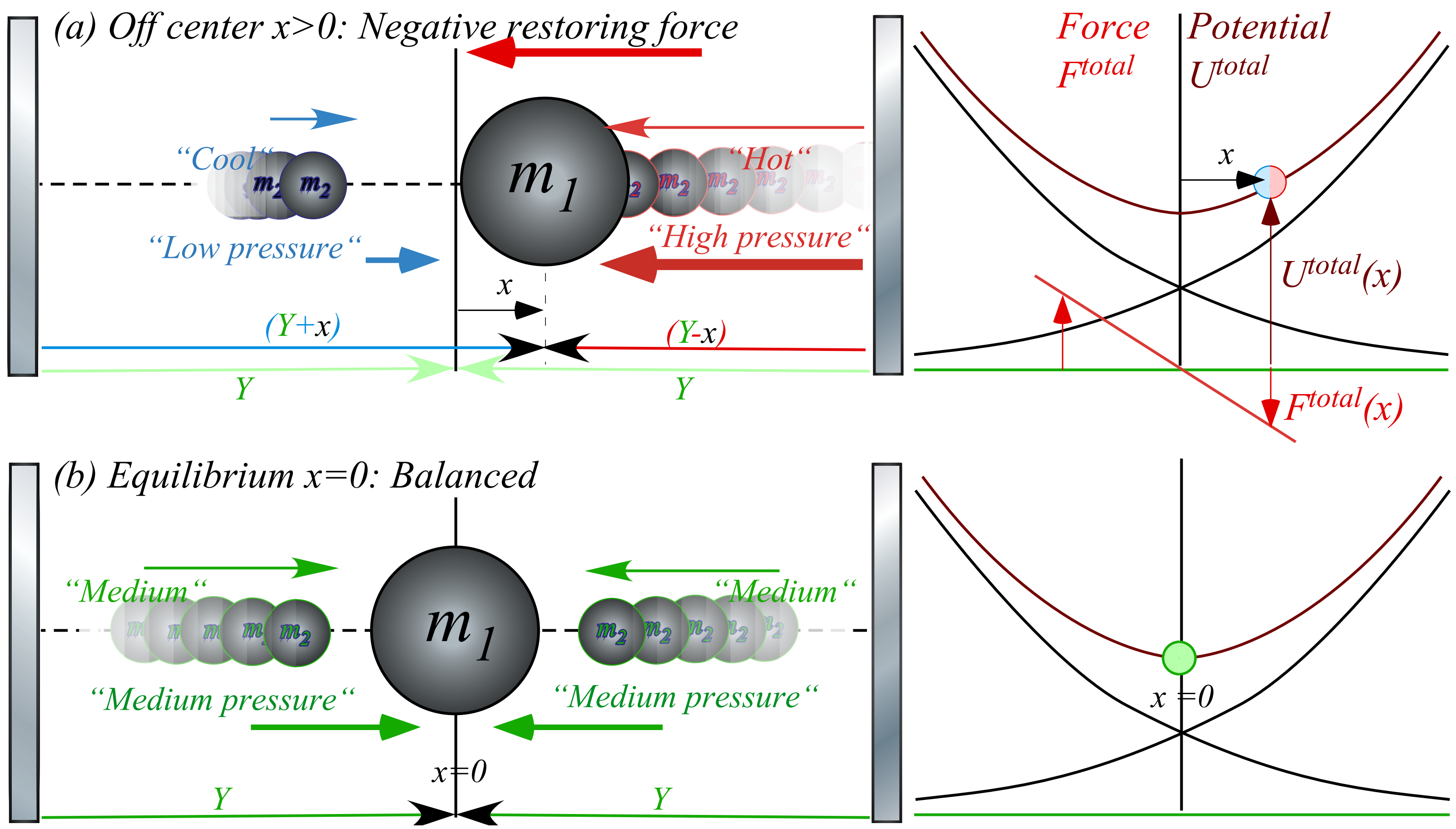

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\frac{1}{Y_0}+\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\frac{x}{Y_0^2}-2f\frac{x^3}{Y_0^4}-\ldots$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots \qquad (Y_0-x)^{-1}=Y_0^{-1}+xY_0^{-2}+x^2Y_0^{-3}+x^3Y_0^{-4}\ldots$$

$$(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):
$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

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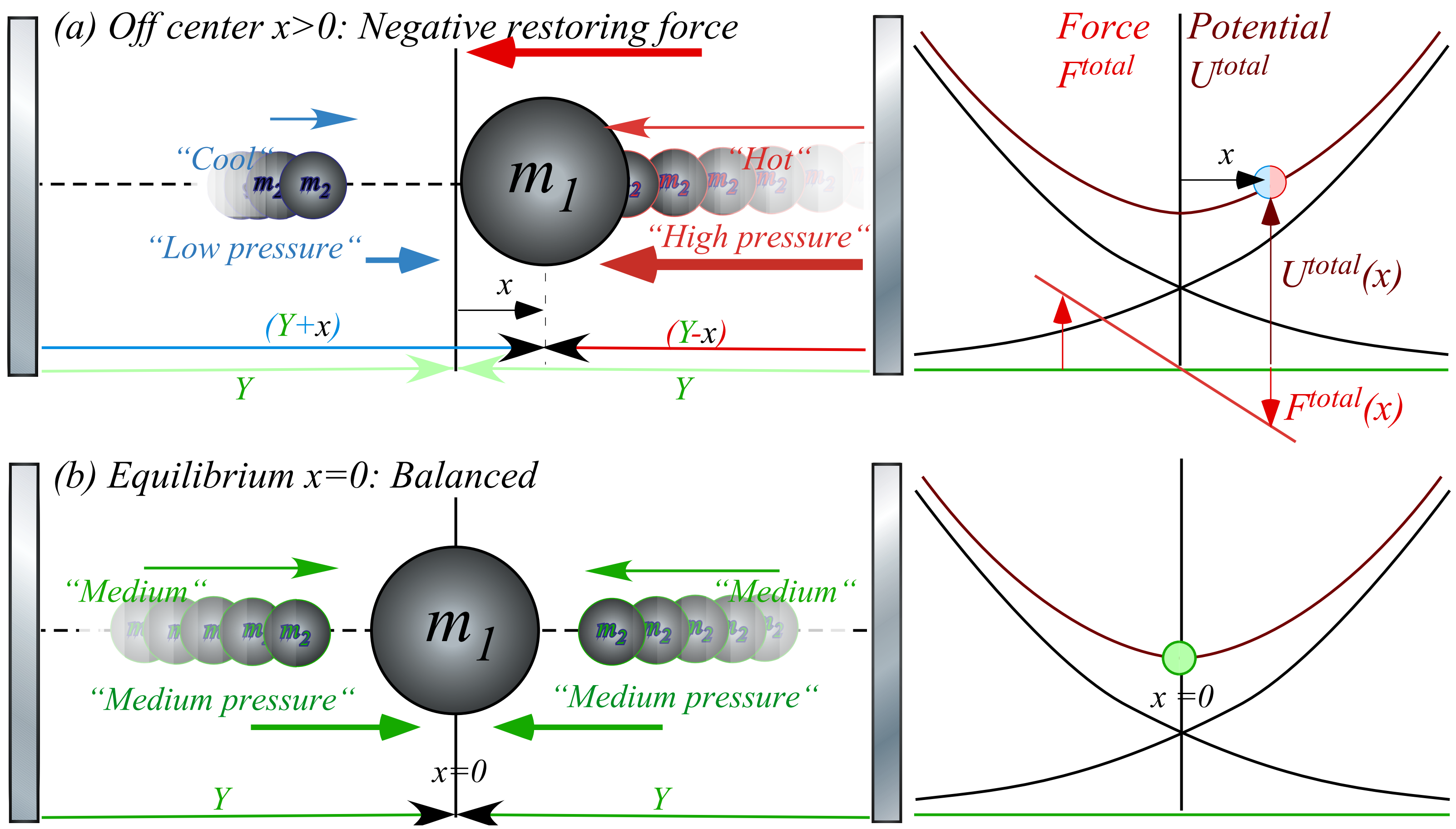

Unit 1
Fig. 6.2

Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\frac{x}{Y_0^2}-2f\frac{x^3}{Y_0^4}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$

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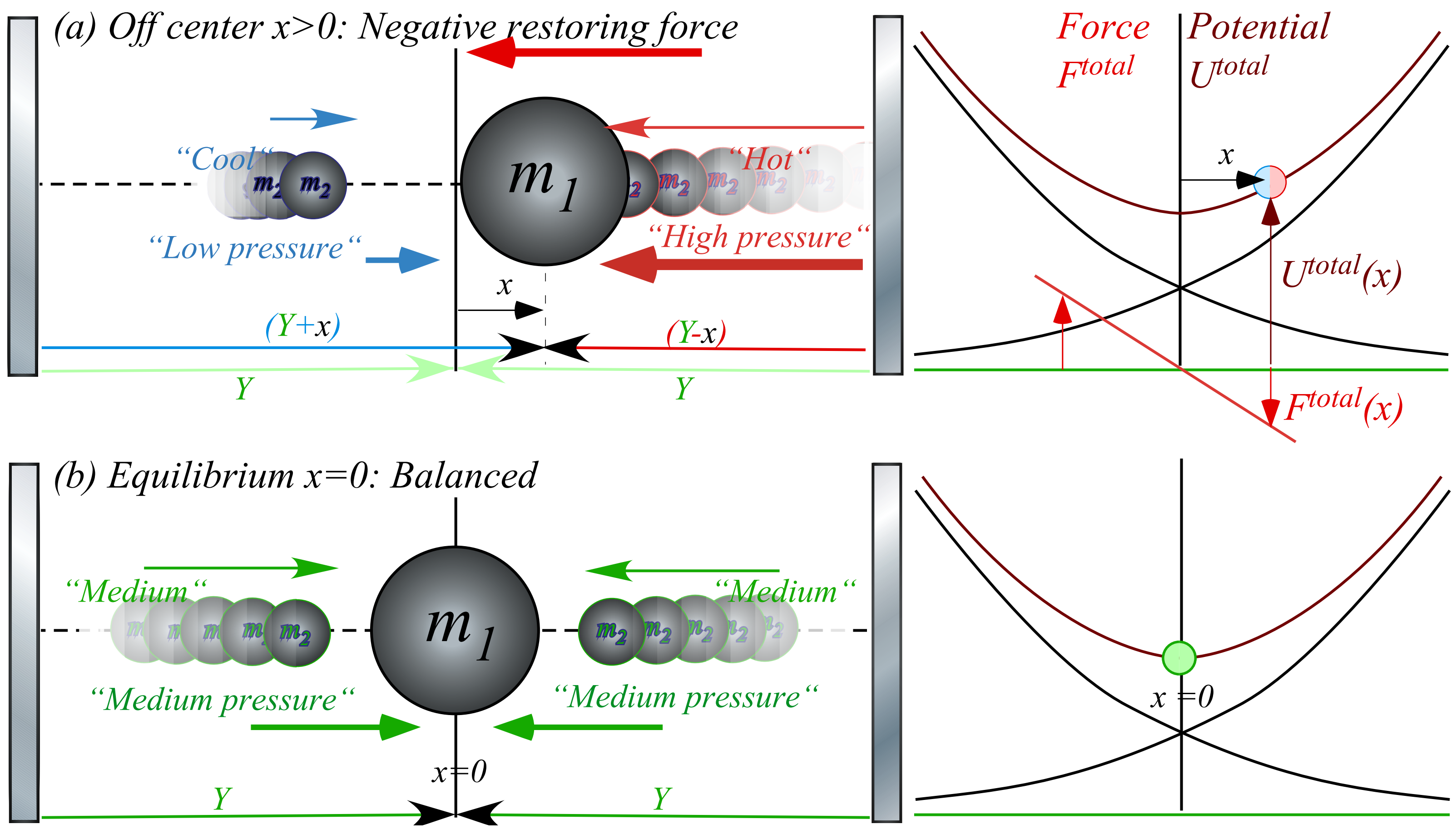

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

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Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

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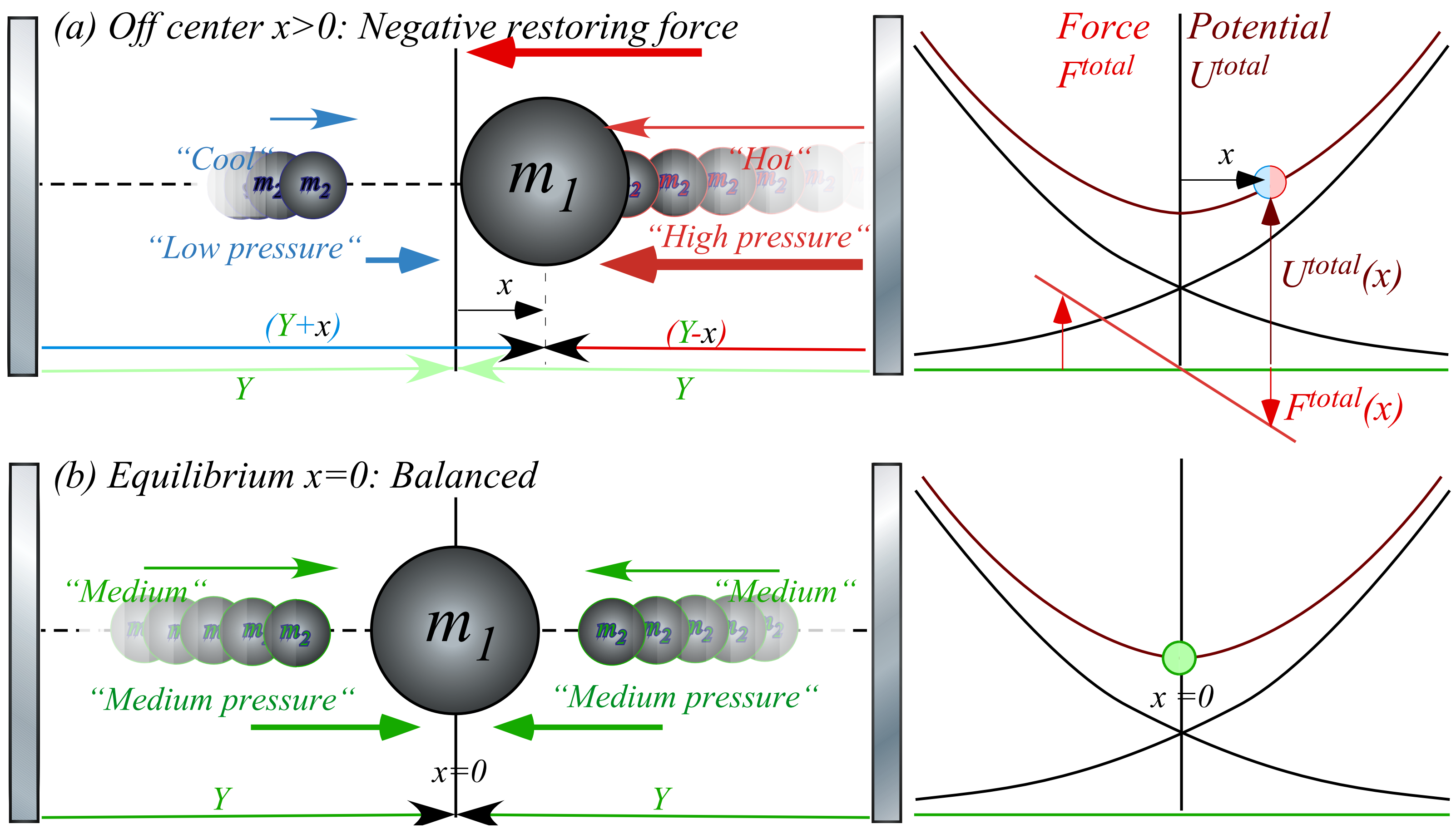

Unit 1
Fig. 6.2

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

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Anharmonic oscillator terms...

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Harmonic oscillator force constant: $k = 2f/Y_0^2 = 2m_2 v_2^2 / Y_0^2$

Harmonic Oscillator Force

$$F^{HO} = -k \cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO} = \frac{1}{2} k \cdot x^2 = -\int F^{HO}\, dx$$

Example of oscillator with opposing ***Isothermal potentials***

1D-Isothermal Force Law (assume v_2 is constant for all Y):
$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

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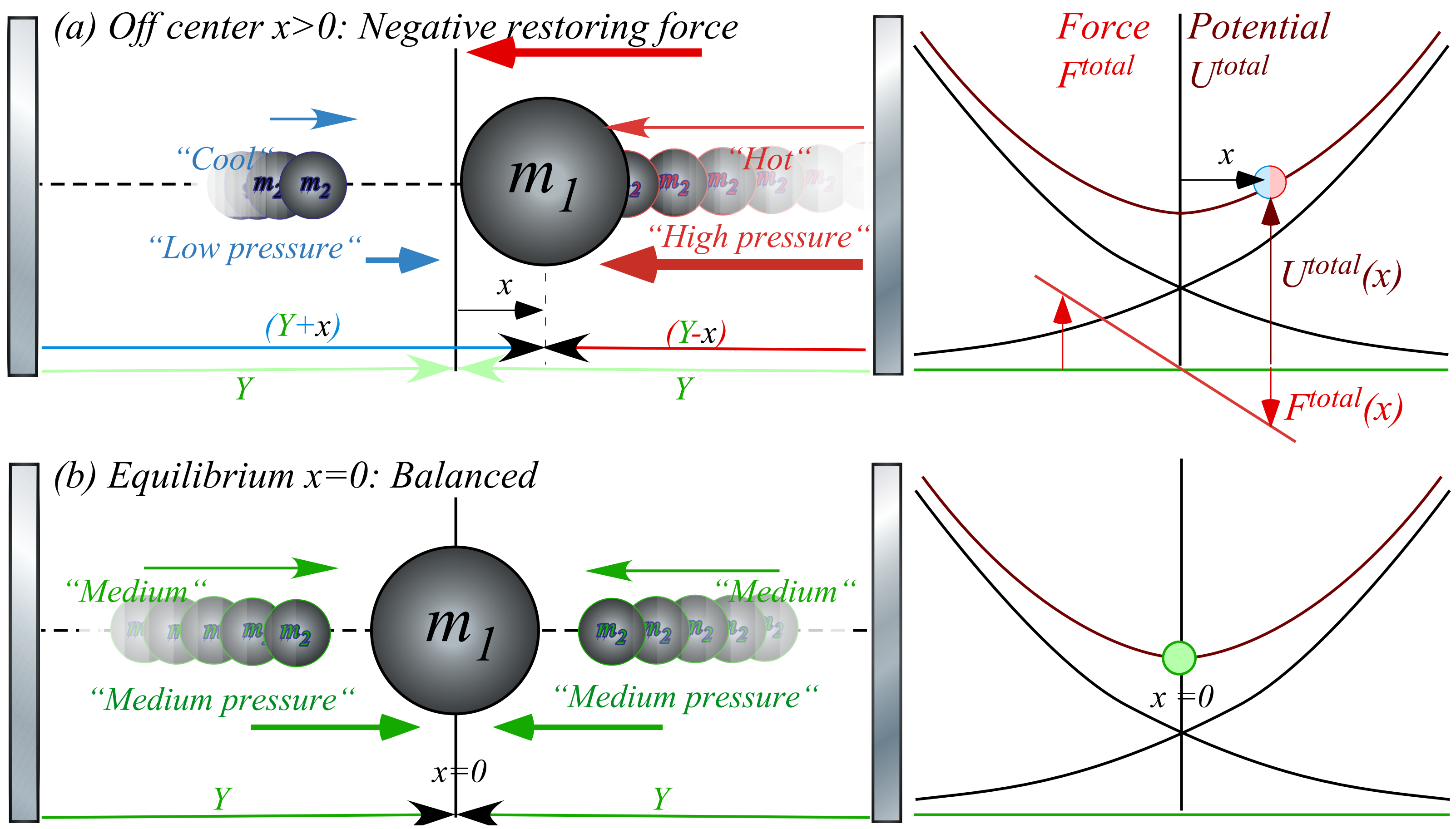


Unit 1
Fig. 6.2

Harmonic oscillator term

Anharmonic oscillator terms...

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\frac{1}{Y_0} + \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

Harmonic oscillator force constant : $k = 2f/Y_0^2 = 2m_2 v_2^2 / Y_0^2$

Harmonic Oscillator Force
$$F^{HO} = -k \cdot x = -\frac{\partial U^{HO}}{\partial x}$$

Potential
$$U^{HO} = \frac{1}{2} k \cdot x^2 = -\int F^{HO}\, dx$$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}} \frac{v_2}{Y_0} = 2\pi\upsilon$

What does *Harmonic* mean?

Given total energy $E = KE + PE = \frac{1}{2} m V^2 + \frac{1}{2} k Y^2$

E is *same* function for *any* amplitude A of sine-oscillation where:

$$Y = A \sin \omega t \quad \text{with velocity} \quad V = A\omega \cos \omega t$$

Because then: $E = \frac{1}{2} m \left(A\omega \cos \omega t \right)^2 + \frac{1}{2} k (A \sin \omega t)^2$

$$= \frac{1}{2} m\omega^2 A^2 \left(\cos \omega t \right)^2 + \frac{1}{2} k A^2 (\sin \omega t)^2$$

$$= \frac{1}{2} m\omega^2 A^2 \left(\cos^2 \omega t + \sin^2 \omega t \right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2} m\omega^2 A^2 \quad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

What does *Harmonic* mean?

Given total energy $E = KE + PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* function for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

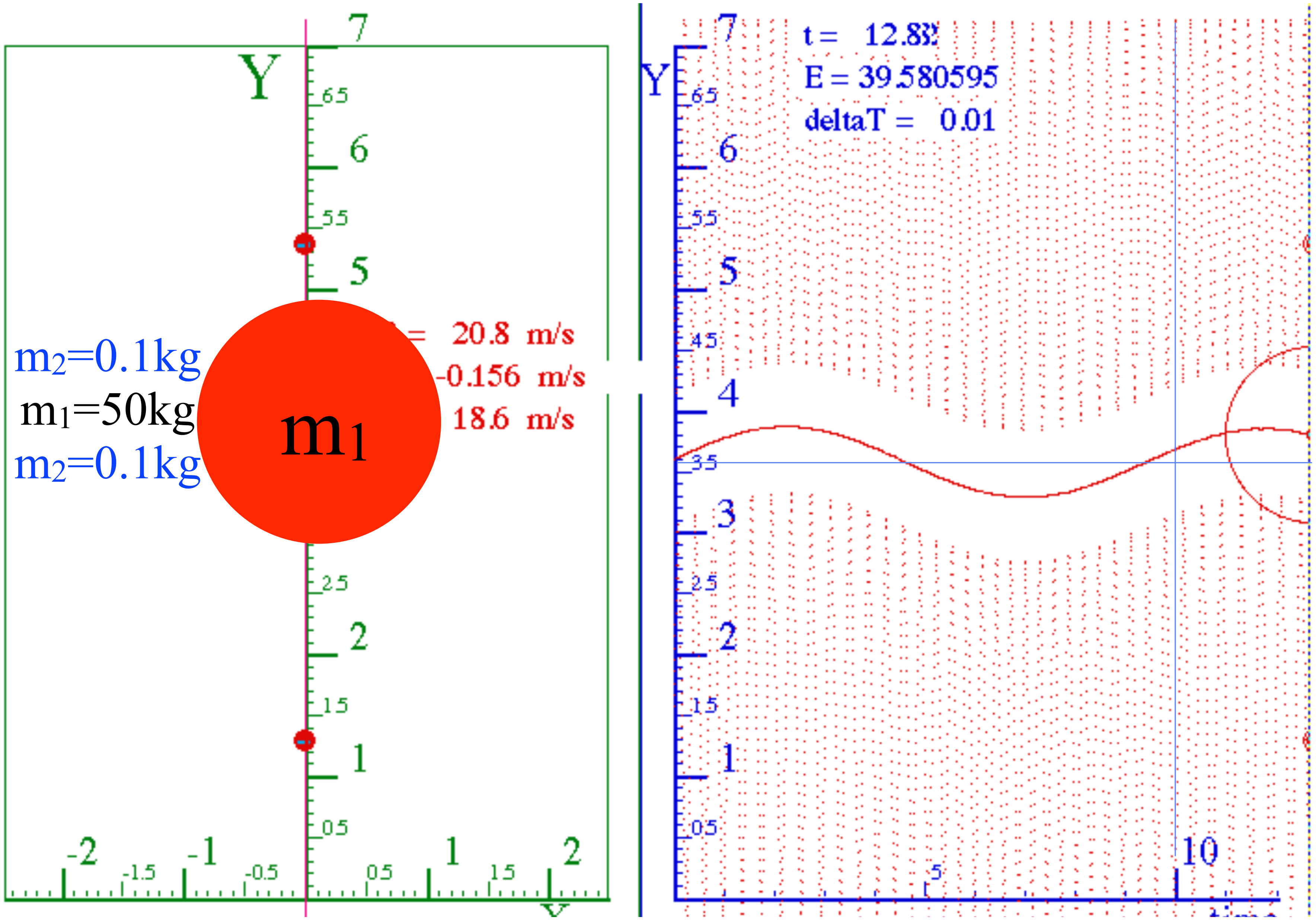
$$= \frac{1}{2}m\omega^2 A^2\left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2 A^2 \quad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

But, how does this square with Planck energy E=(const.)ω *?!?*

Switch
$m_1=m_3$
with
m_2
to match
formula



Unit 1
Fig. 6.3

Simulation of
the adiabatic case

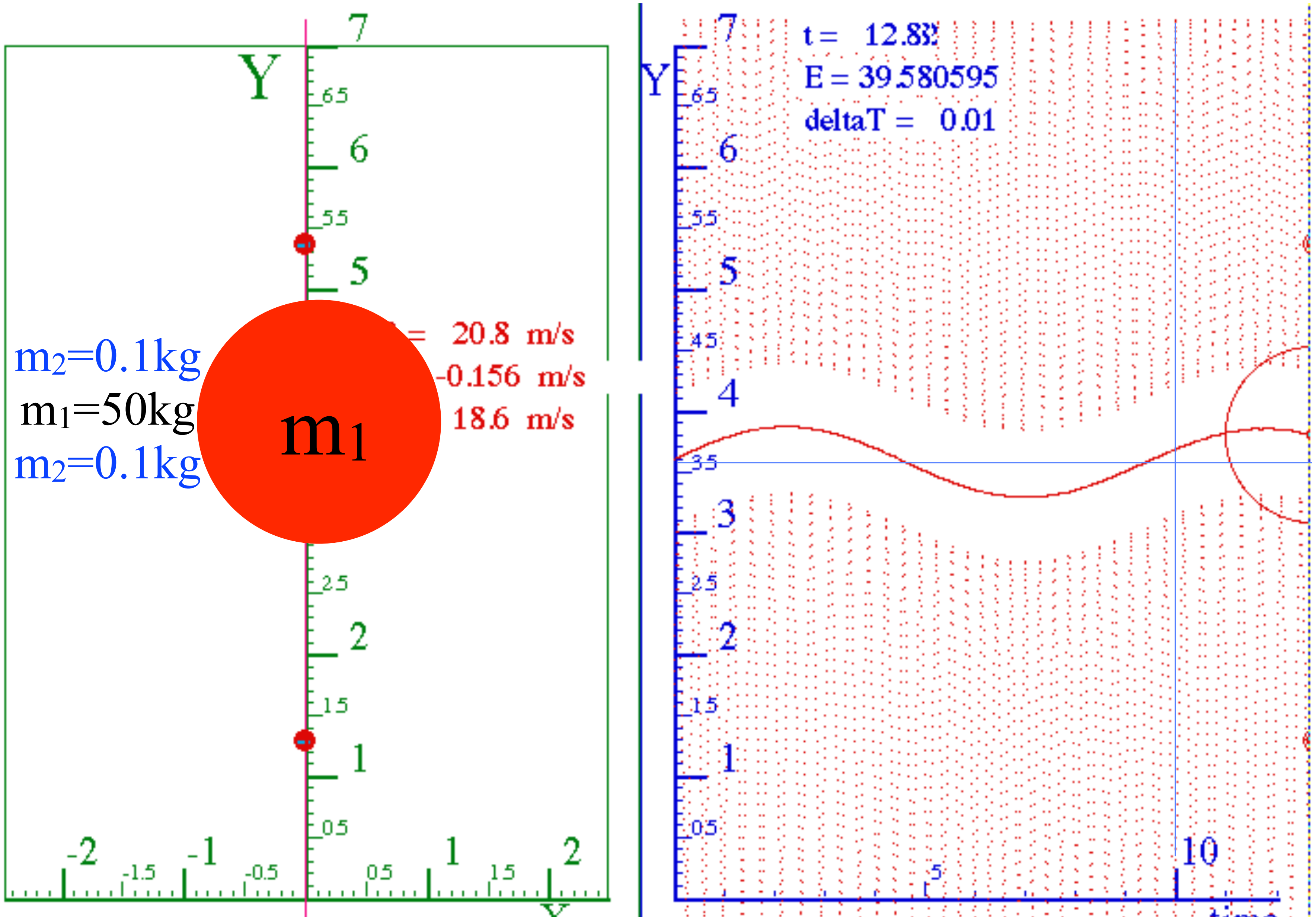
[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute isothermal frequency and/or period*

Frequency

HO ∡frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}}=2\pi\upsilon$

Switch
$m_1=m_3$
with
m_2
to match
formula



BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

Unit 1
Fig. 6.3

Simulation of
the adiabatic case

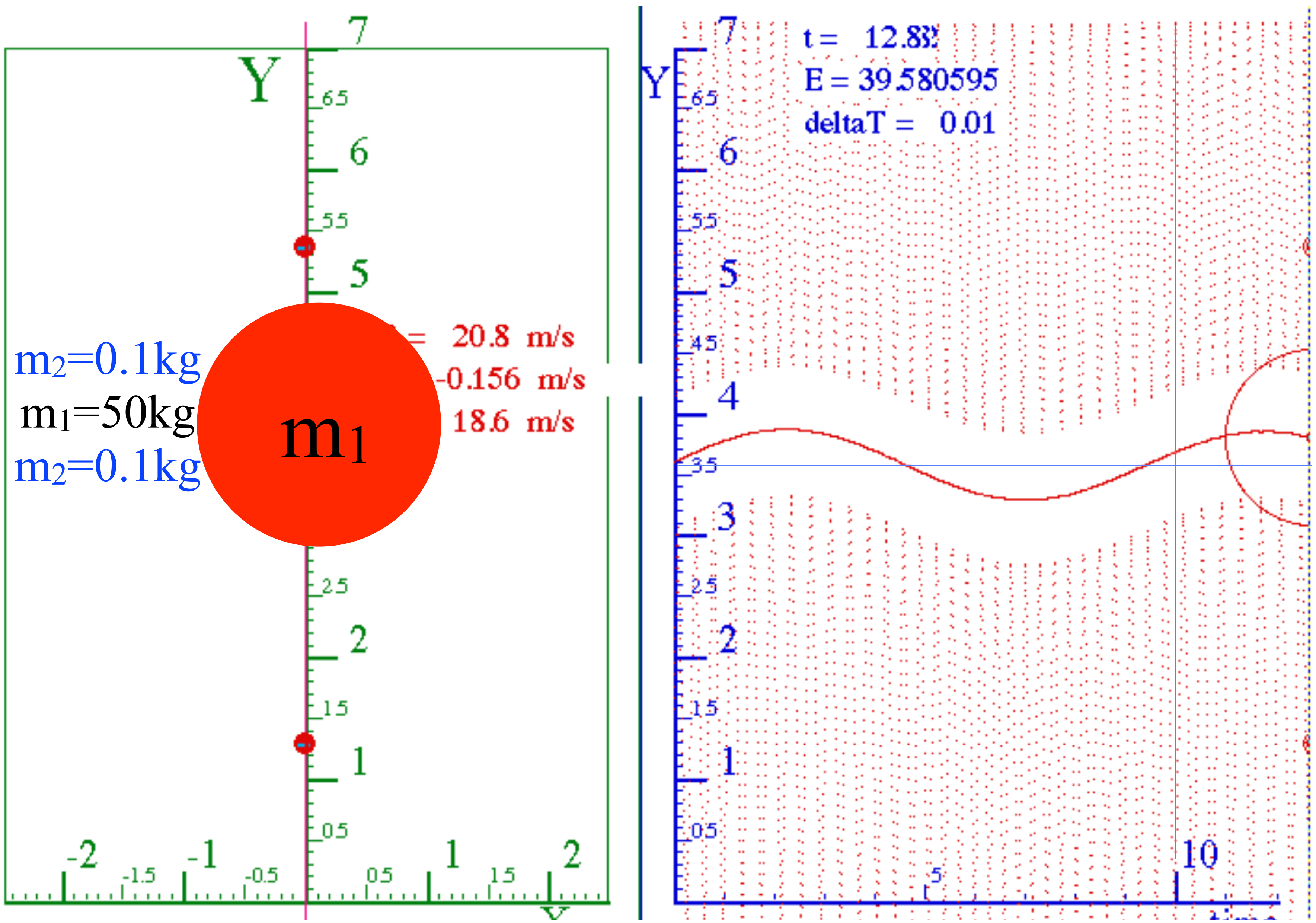
Sample problem: *Compute isothermal frequency and/or period*

$$\textit{Period}: \quad \tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$$

Frequency

$$\text{HO} \measuredangle\text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$$

Switch
m_1=m_3
with
m_2
to match
formula



Unit 1
Fig. 6.3

Simulation of
the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute isothermal period given* $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$

Period : $\tau=\frac{1}{\upsilon}=2\pi\sqrt{\frac{m_1}{k}}=2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Period :

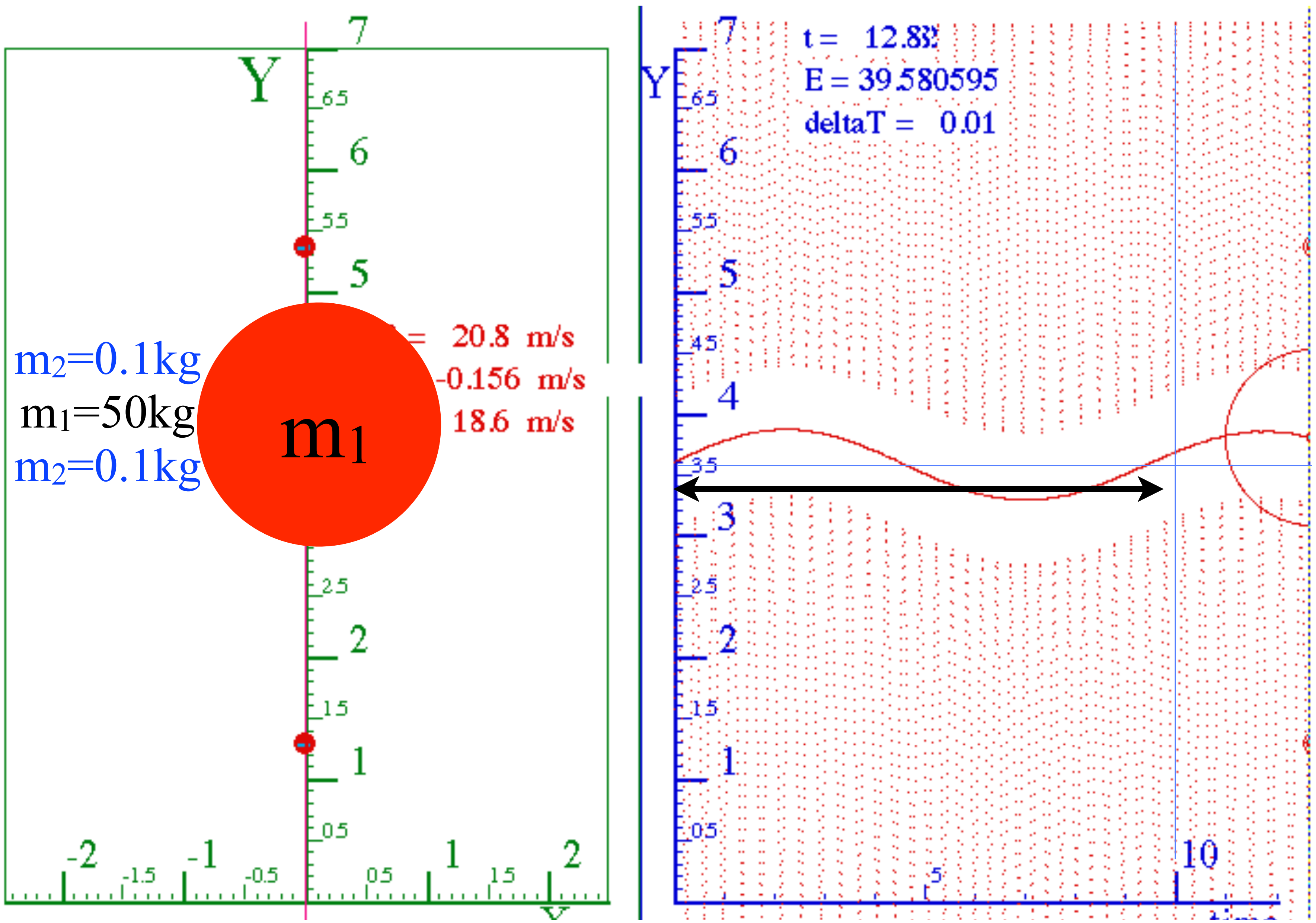
$$\tau=2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}=6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$$=17.38$$

Frequency

HO ∡frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}}=2\pi\upsilon$

Switch
$m_1=m_3$
with
m_2
to match
formula



Simulation of
the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

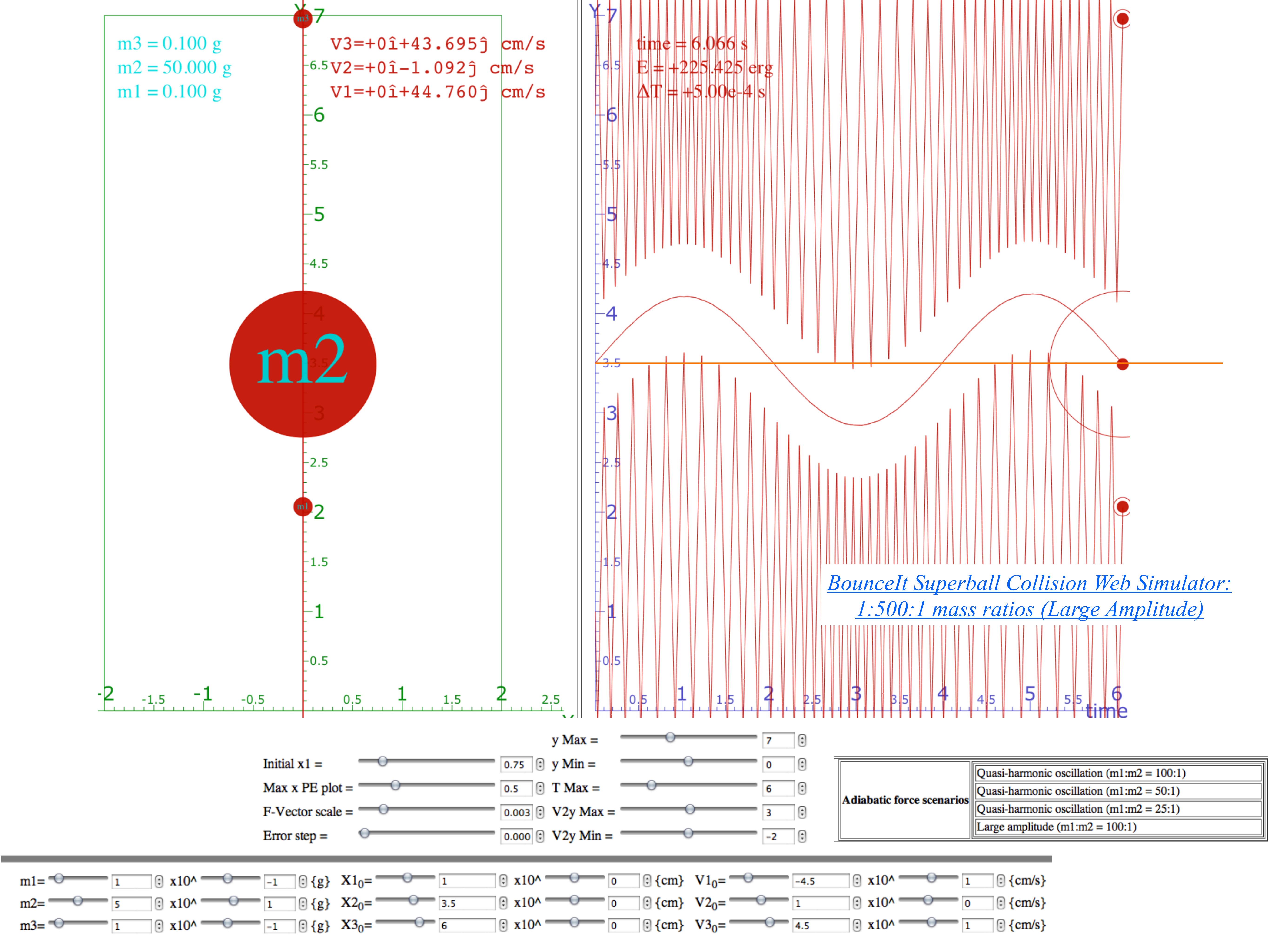
Sample problem: *Compute isothermal period given* $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$

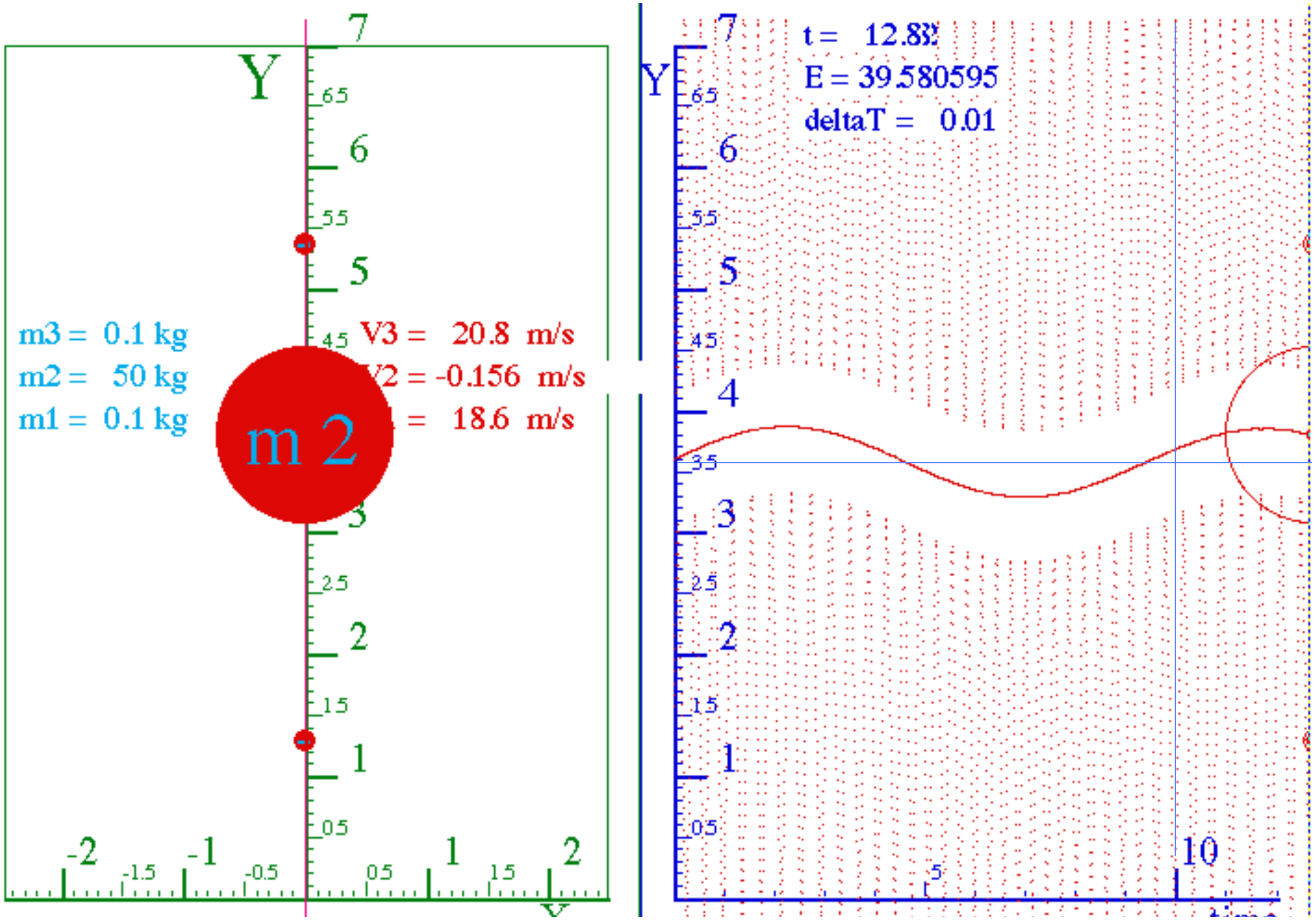
Period :
$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$
$$=17.38$$
That's about √3 times too big!

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s
m2
time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s
time
BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Large Amplitude)
y Max = 7
Initial x1 = 0.75
y Min = 0
Max x PE plot = 0.5
T Max = 6
F-Vector scale = 0.003
V2y Max = 3
Error step = 0.000
V2y Min = -2
Adiabatic force scenarios
Quasi-harmonic oscillation (m1:m2 = 100:1)
Quasi-harmonic oscillation (m1:m2 = 50:1)
Quasi-harmonic oscillation (m1:m2 = 25:1)
Large amplitude (m1:m2 = 100:1)
m1= 1 x10^ -1 {g} X1₀= 1 x10^ 0 {cm} V1₀= -4.5 x10^ 1 {cm/s}
m2= 5 x10^ 1 {g} X2₀= 3.5 x10^ 0 {cm} V2₀= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} X3₀= 6 x10^ 0 {cm} V3₀= 4.5 x10^ 1 {cm/s}



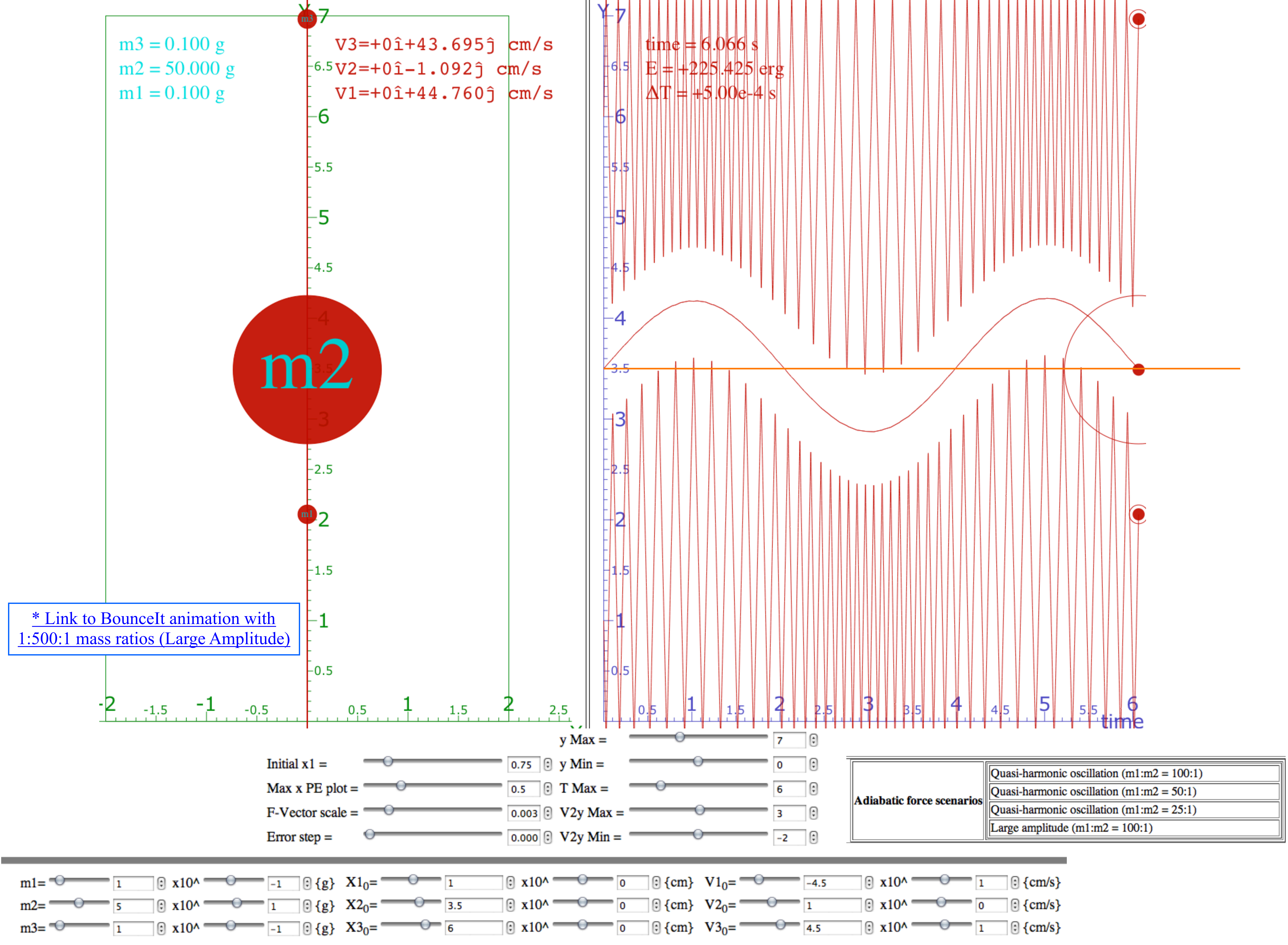
Unit 1
Fig. 6.3

Simulation of

the adiabatic case

[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]

See Homework problem 1.6.5: *Compute frequency and/or period for both isoT and adiabatic cases*



m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s
m3
m2
m1
* Link to BounceIt animation with 1:500:1 mass ratios (Large Amplitude)
time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s
Y
time
y Max = 7
Initial x1 = 0.75
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F-Vector scale = 0.003
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Error step = 0.000
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Quasi-harmonic oscillation (m1:m2 = 100:1)
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Large amplitude (m1:m2 = 100:1)
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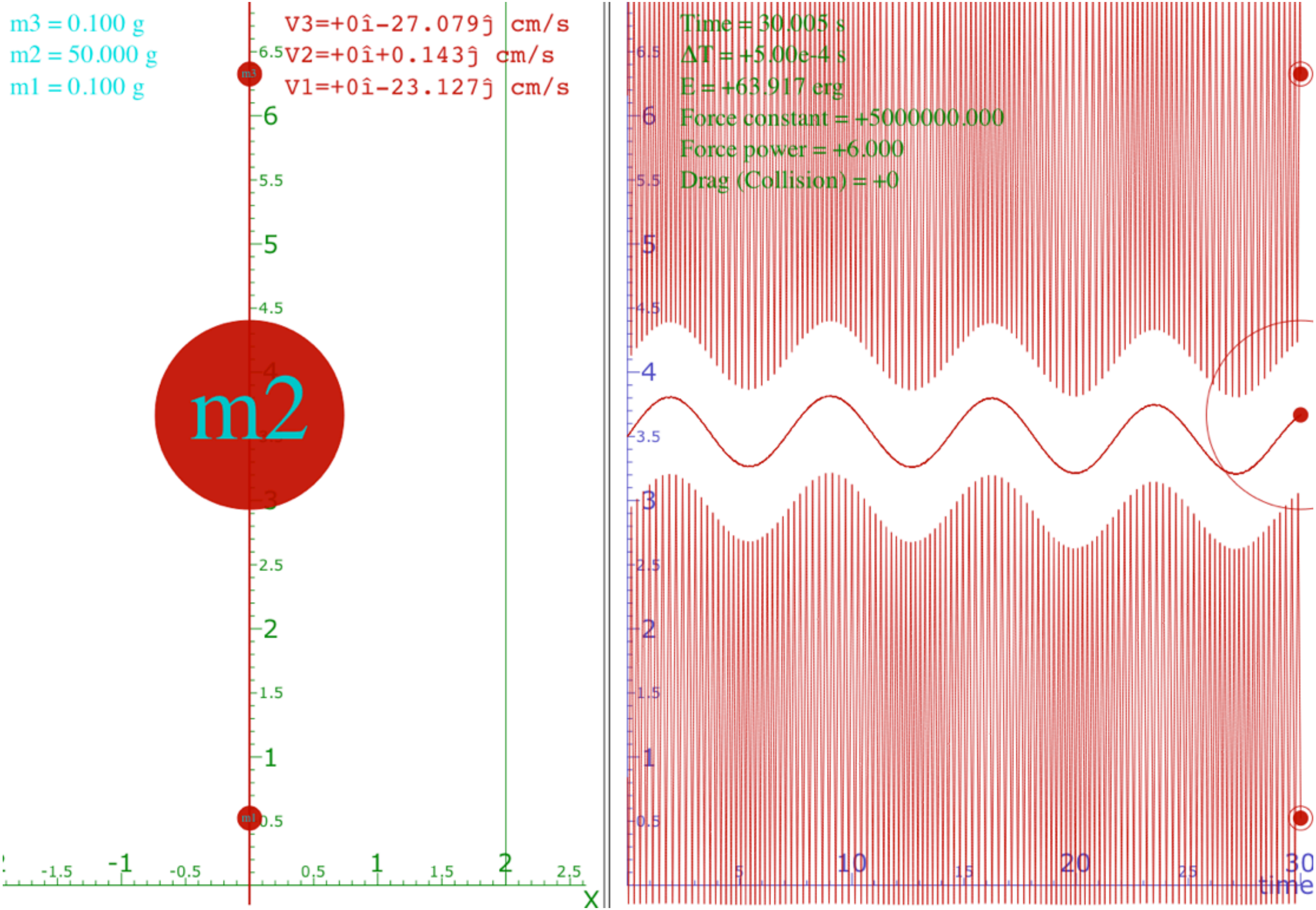
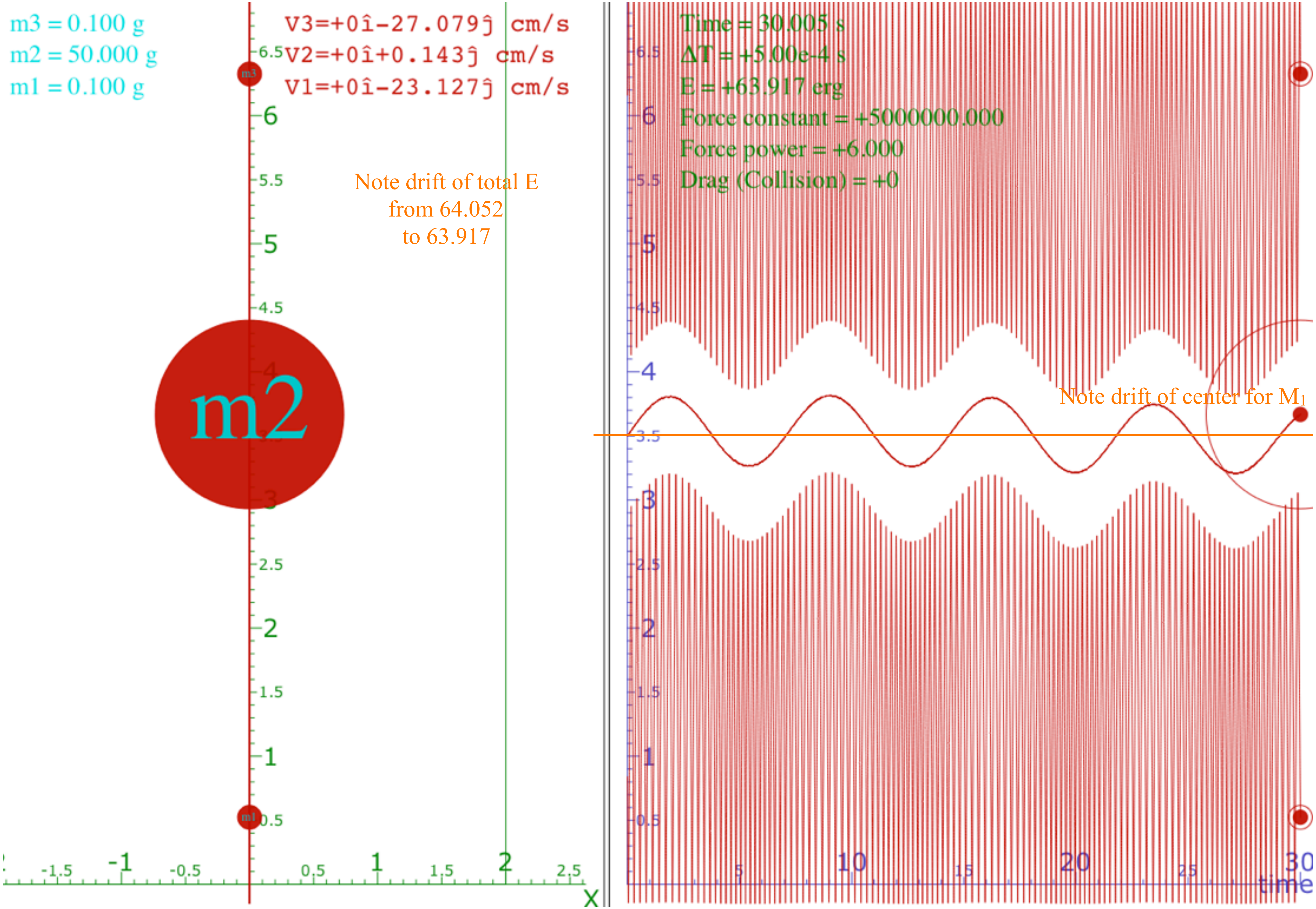
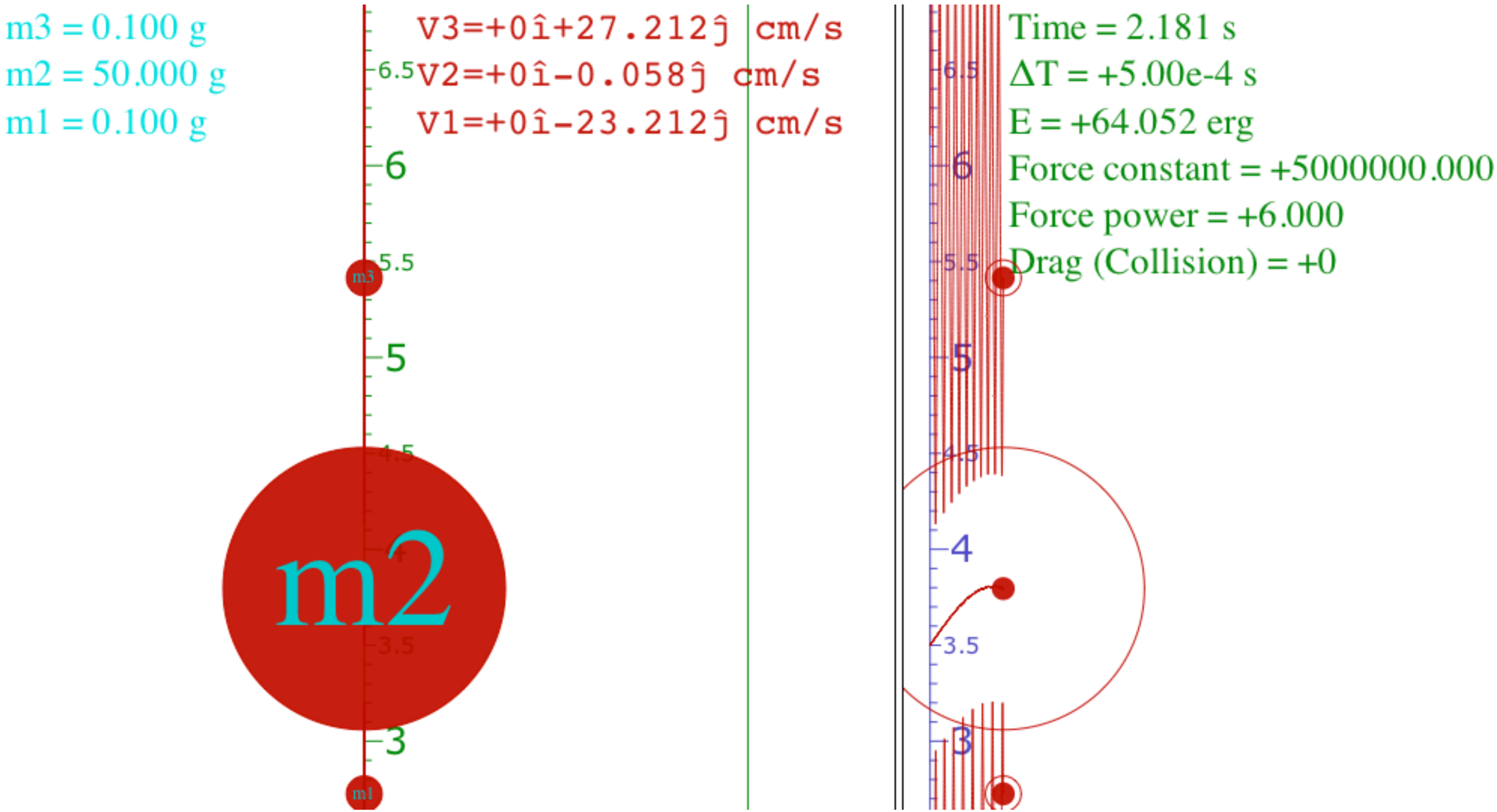
m2= 5 x10^ 1 {g} X2₀= 3.5 x10^ 0 {cm} V2₀= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} X3₀= 6 x10^ 0 {cm} V3₀= 4.5 x10^ 1 {cm/s}



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]



[* Link to BounceIt animation with 1:500:1 mass ratios (Small Amplitude)]()


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+27.212ĵ cm/s
V2=+0î-0.058ĵ cm/s
V1=+0î-23.212ĵ cm/s
Time = 2.181 s
ΔT = +5.00e-4 s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0
6.5
6
5.5
5
4.5
4
3.5
3
m3
m2
m1

"Monster Mash" classical segue to Heisenberg action relations

→ *Example of very very large M_1 ball-walls crushing a poor little m_2*

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

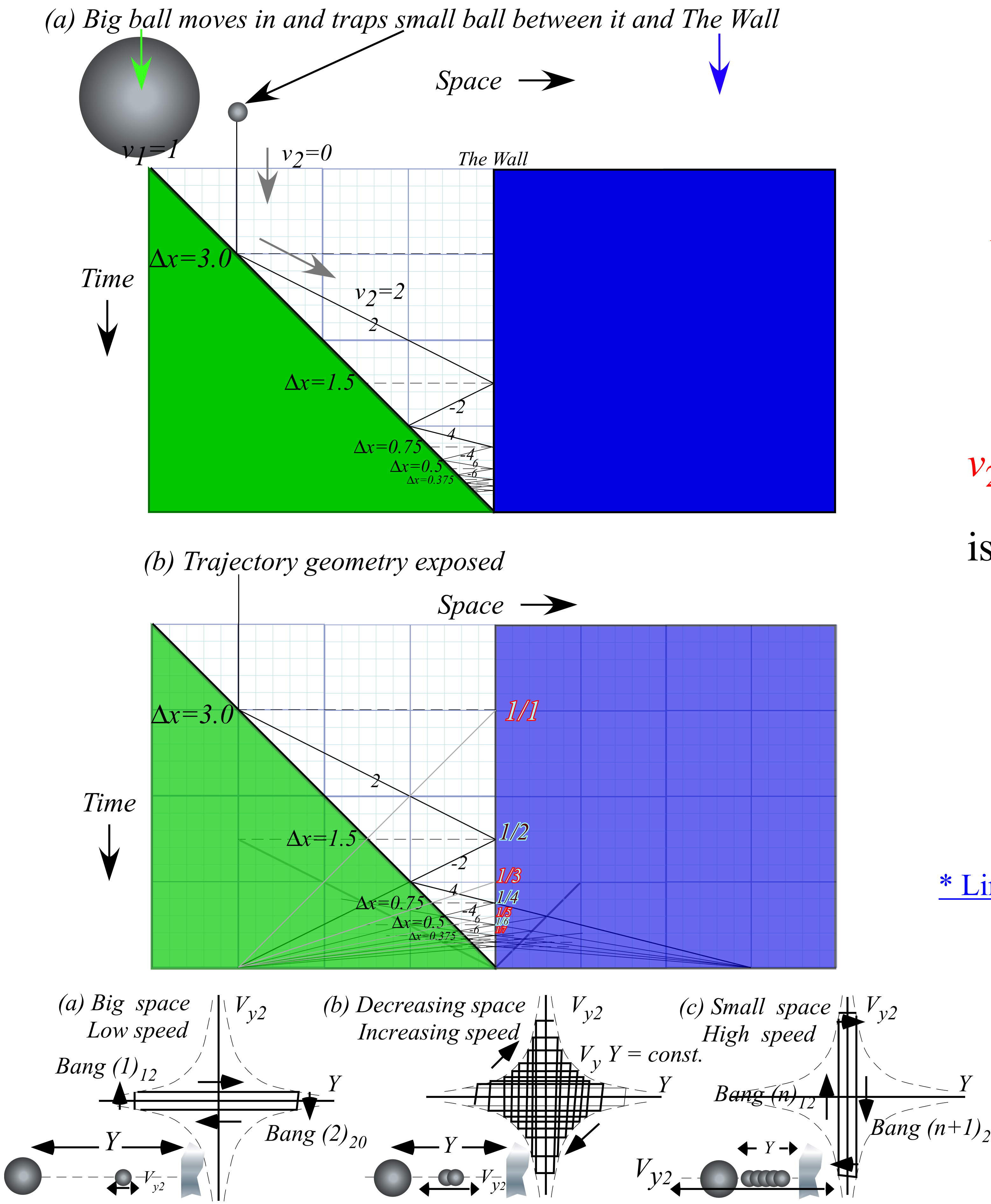
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

The Classical "Monster Mash"

Classical introduction to

Heisenberg "Uncertainty" Relations

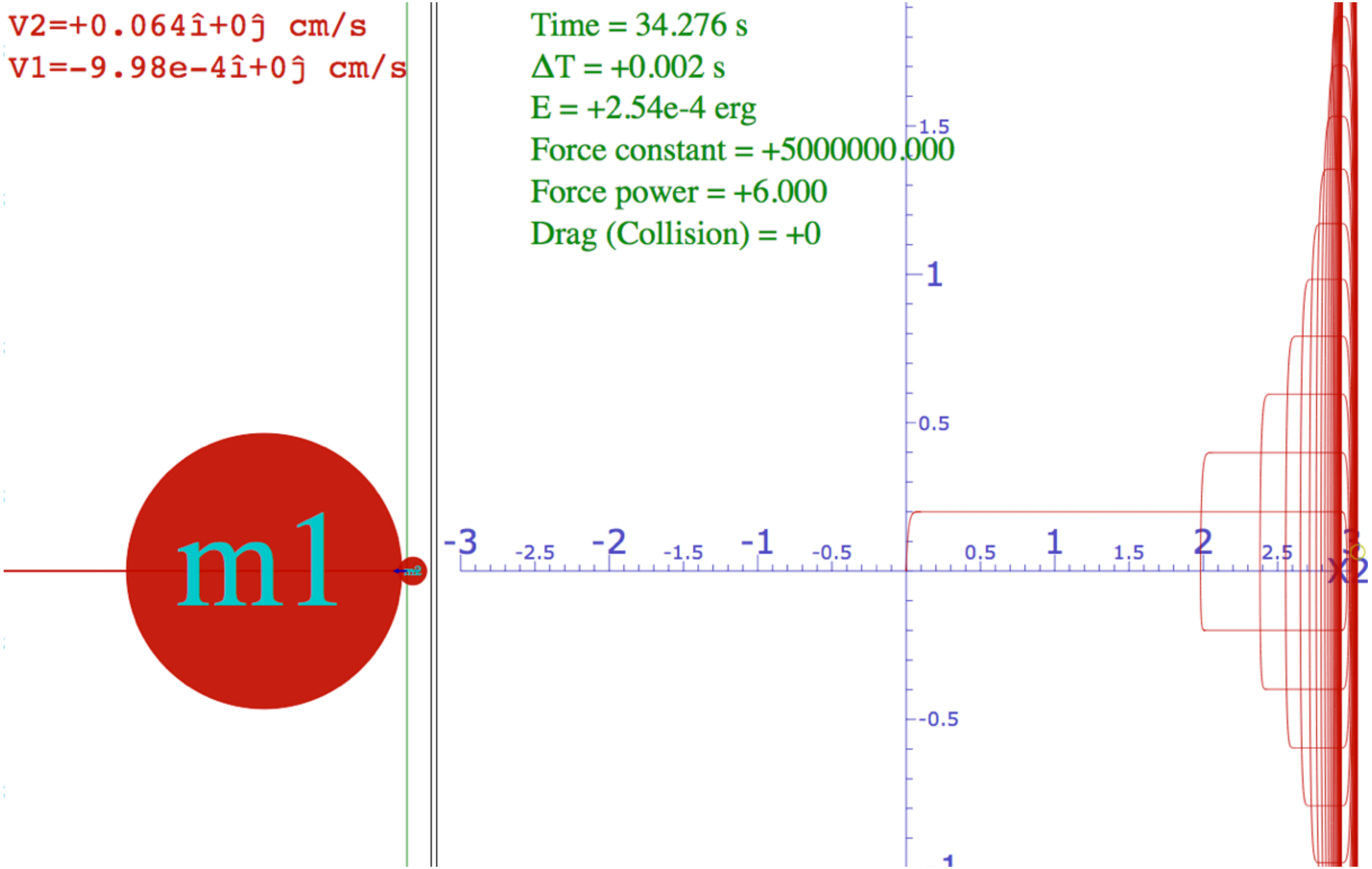


$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

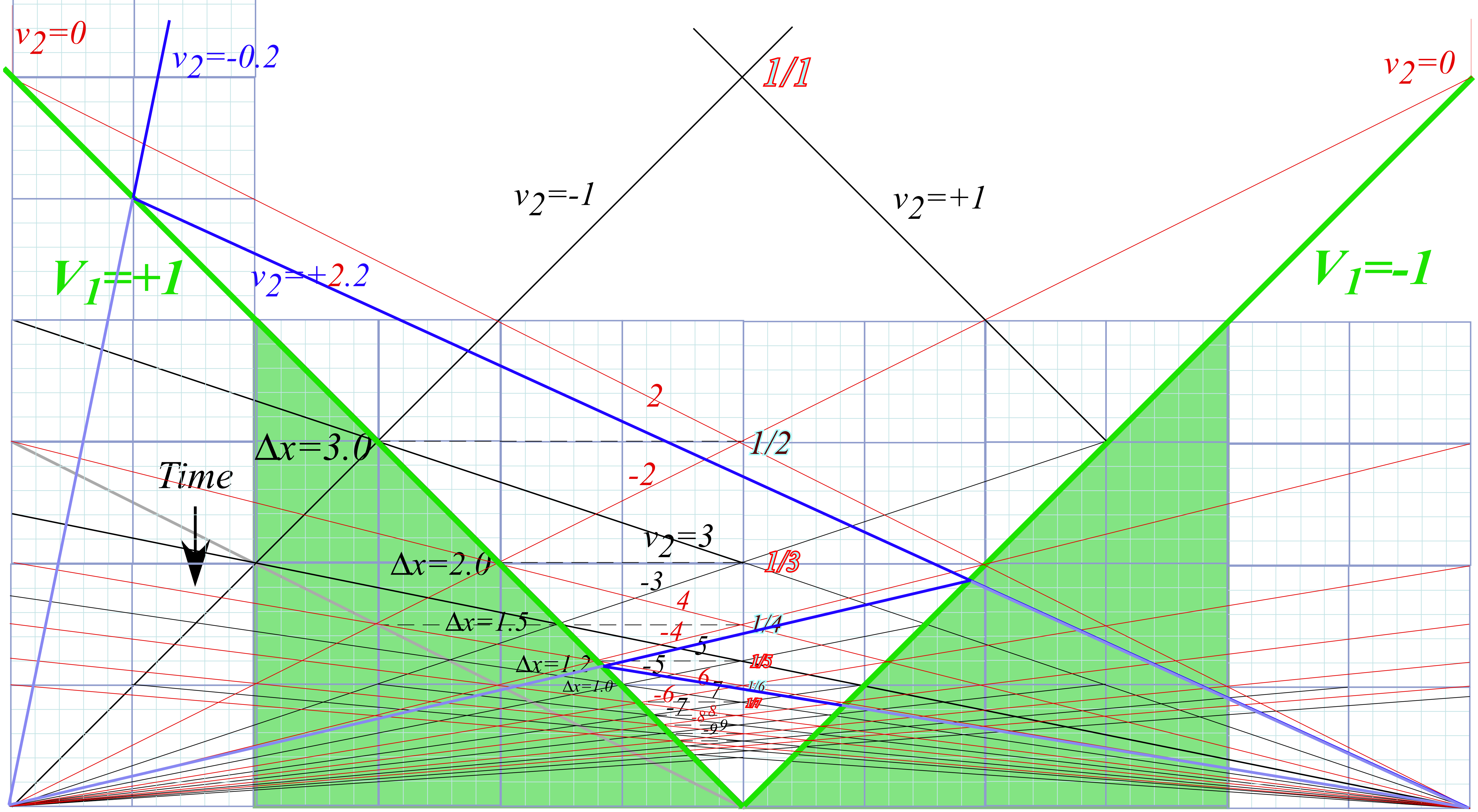
Unit 1
Fig. 6.4

* Link to BounceIt "*Monster Mash*" $x_2(t)$ animation
(Note: Time sense is inverted)



* Link to BounceIt "*Monster Mash*" Vx_2 vs x_2 animation

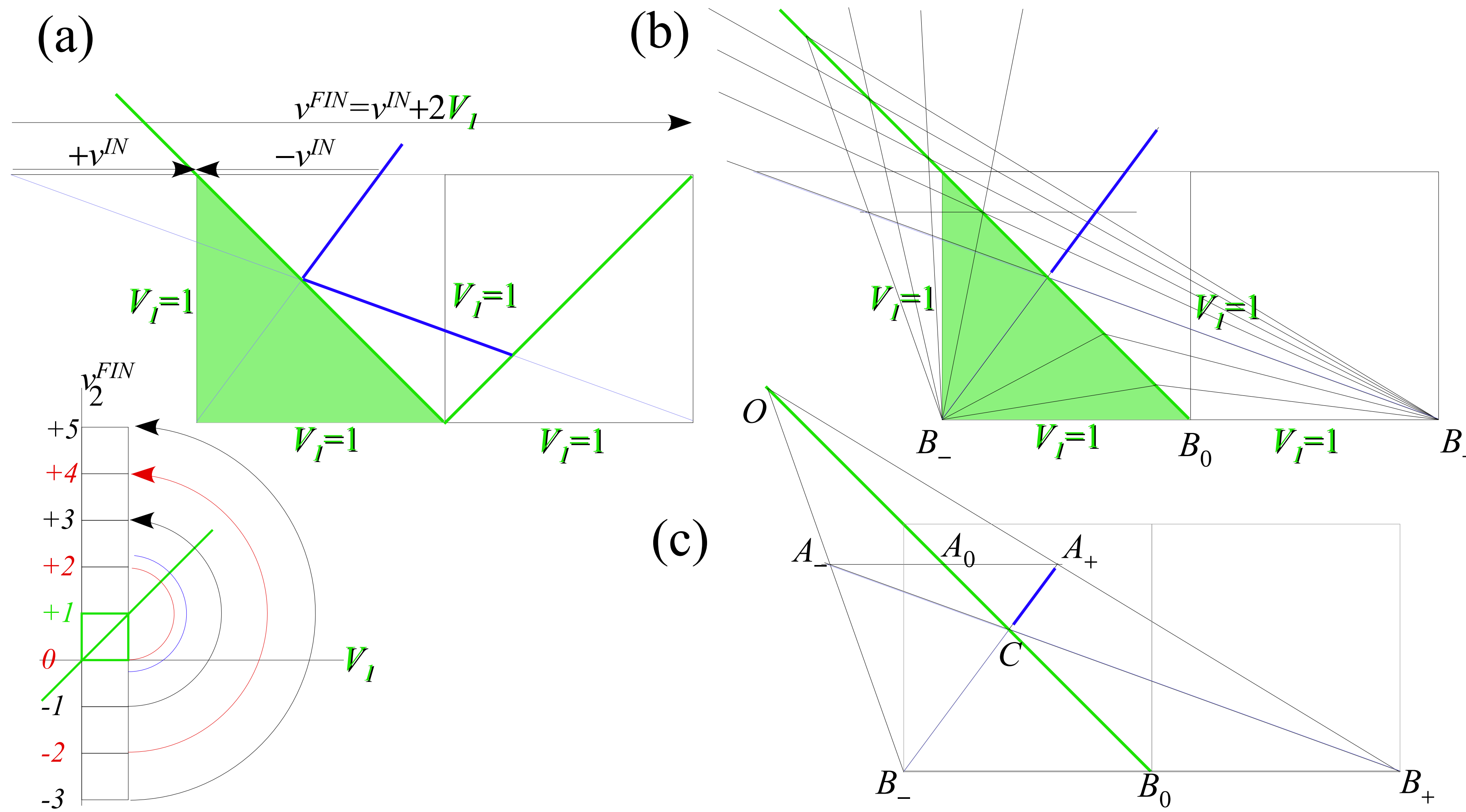
Double "Monster Mash"

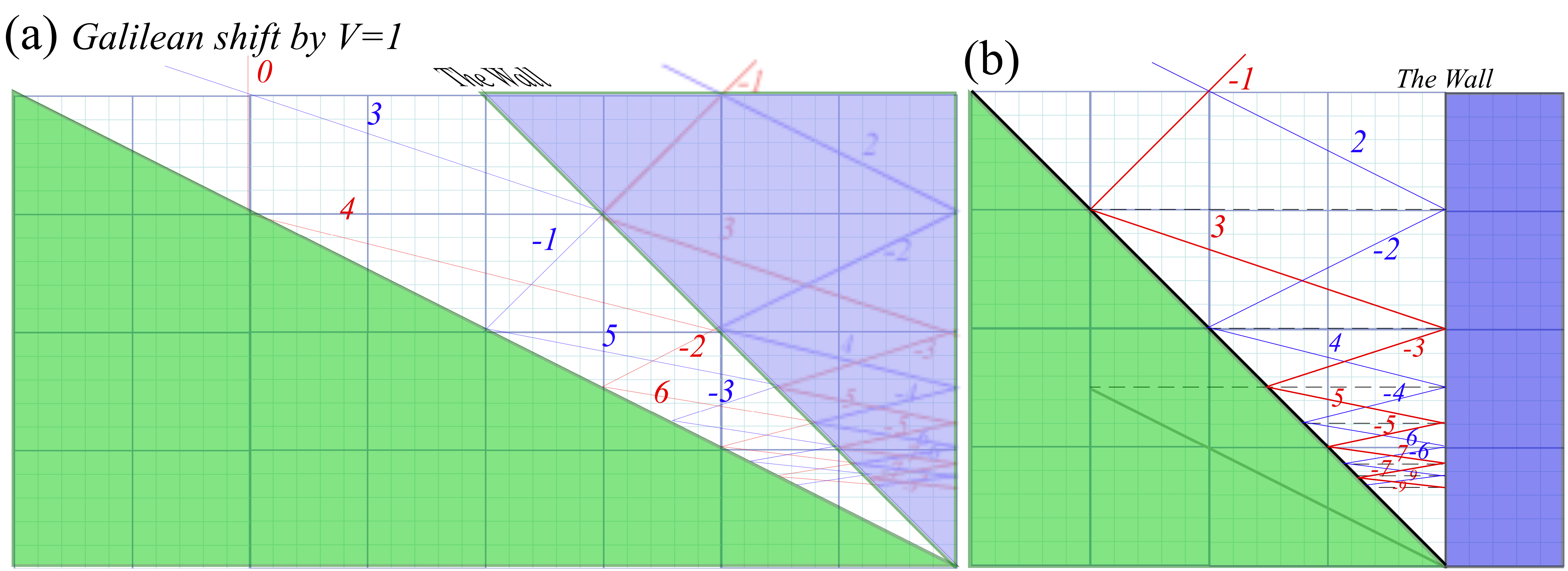


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





“Monster Mash” classical segue to Heisenberg action relations

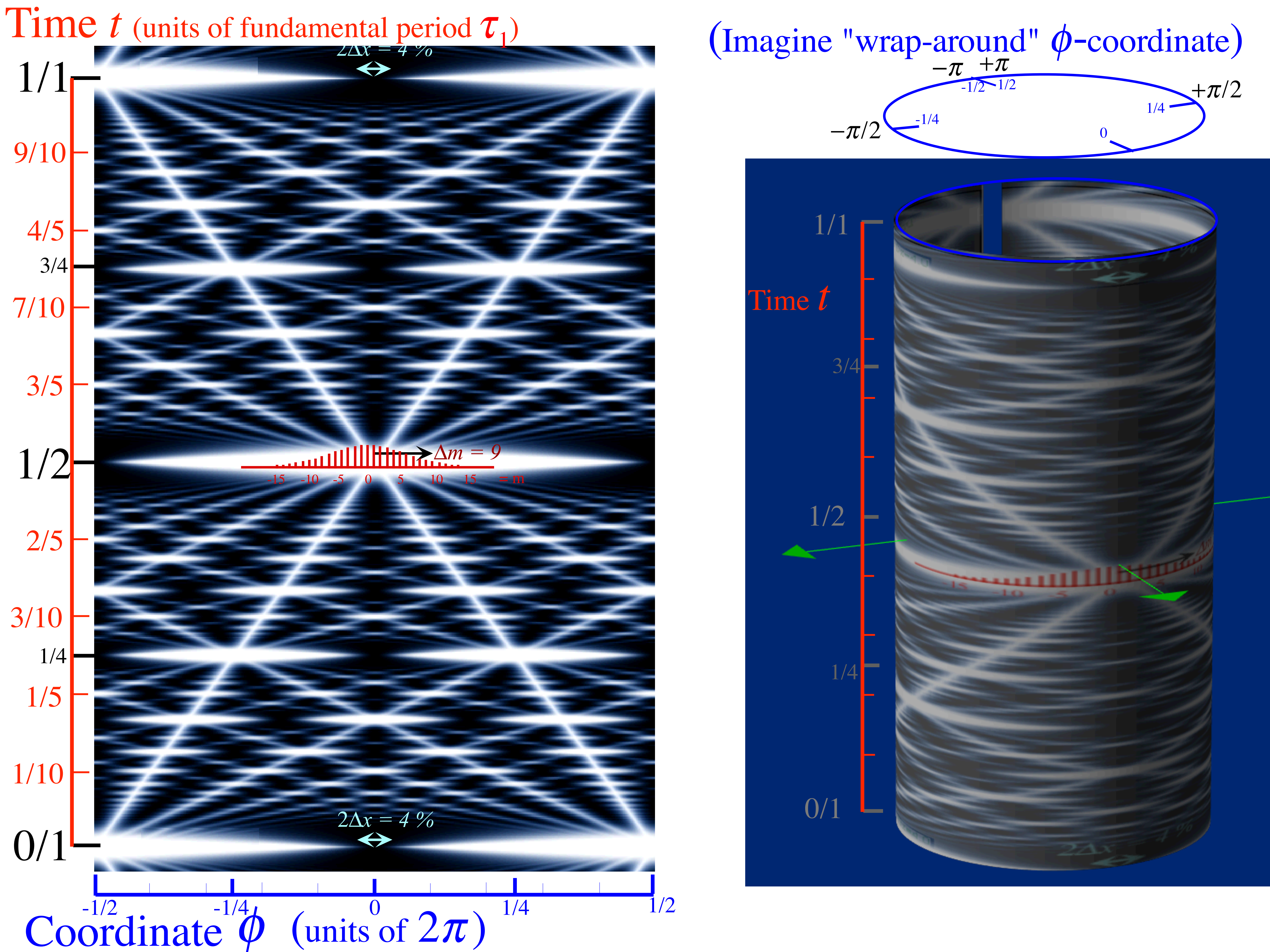
Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

→ *An interesting wave analogy: The “Tiny-Big-Bang”* [Harter, J. Mol. Spec. 210, 166-182 (2001)],[Harter, Li IMSS (2012)]

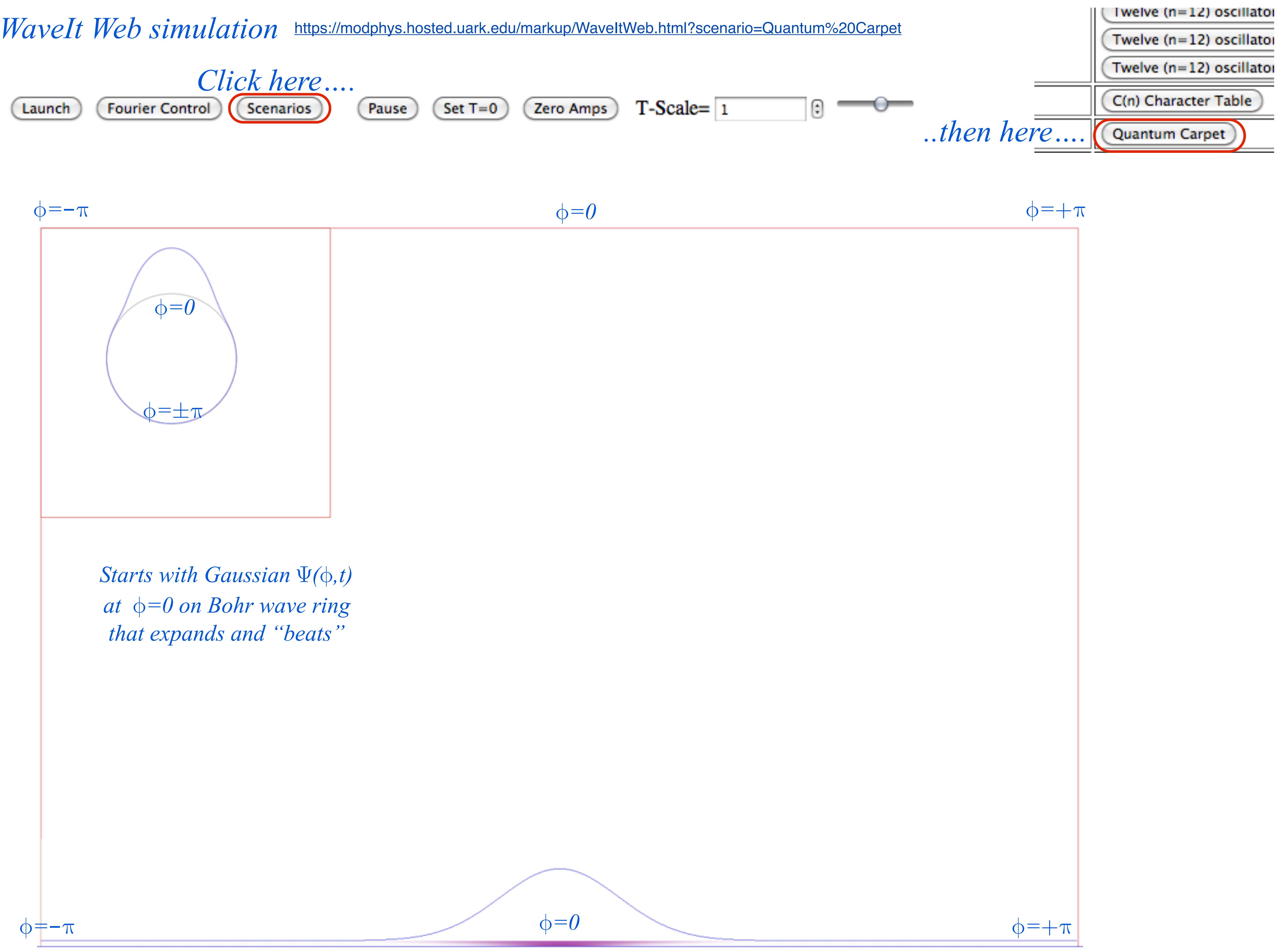
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938) [John Farey, Phil. Mag.(1816)]

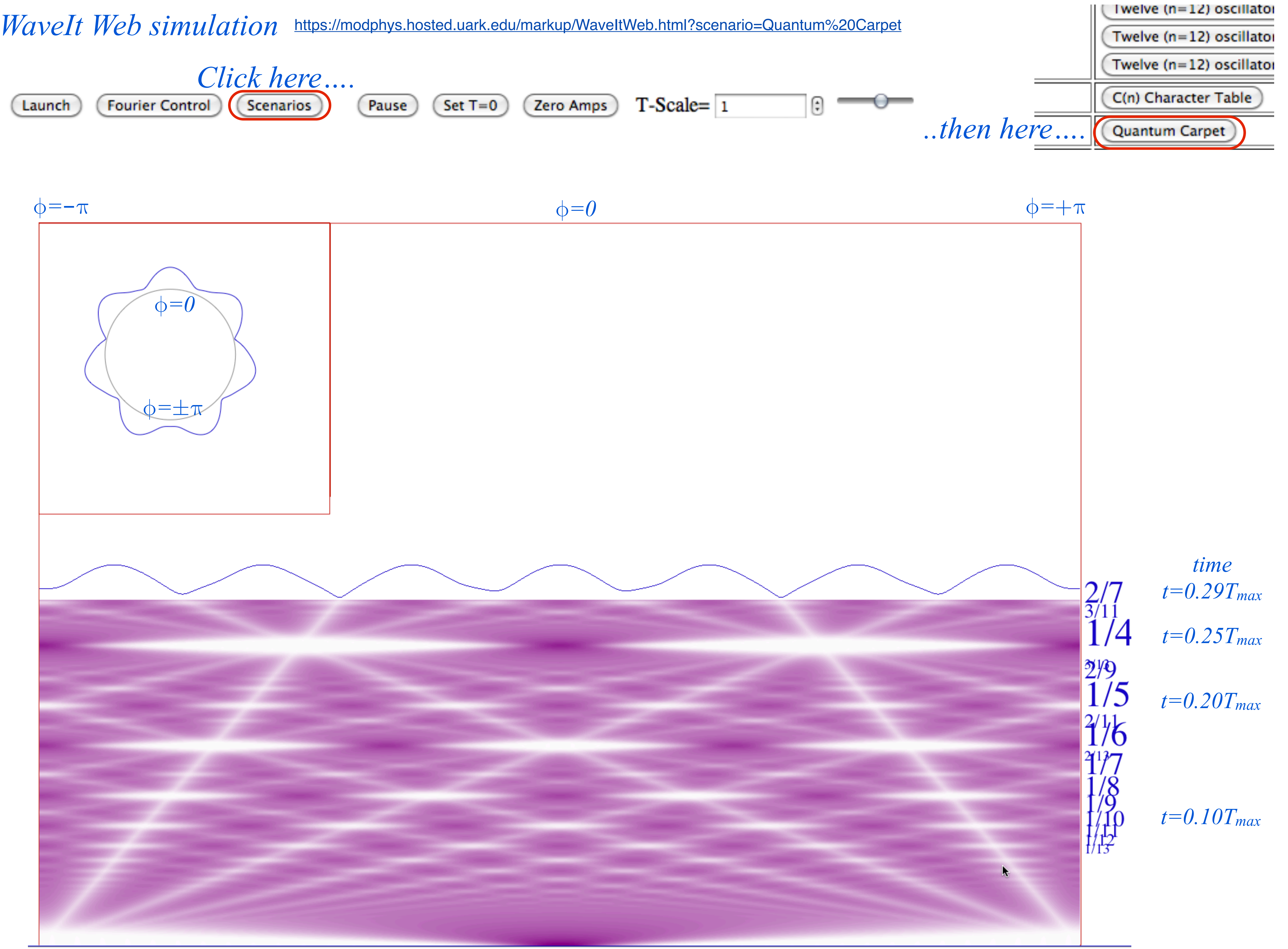


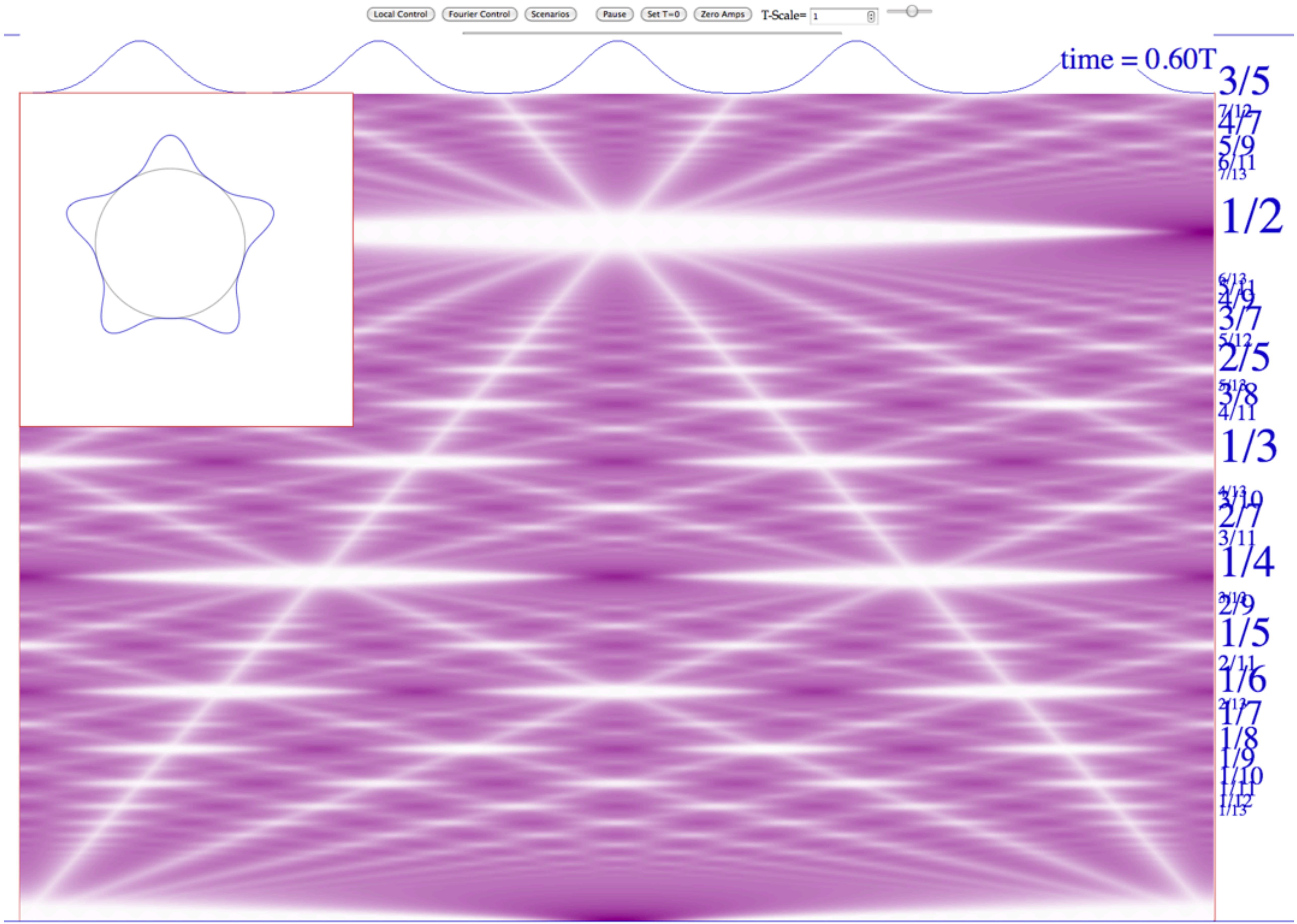
Time t (units of fundamental period τ1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
2Δx = 4 %
Δm = 9
-15 -10 -5 0 5 10 15 = m
-1/2 -1/4 0 1/4 1/2
Coordinate φ (units of 2π)
(Imagine "wrap-around" φ-coordinate)
−π +π
-1/2 1/2
+π/2
1/4
−π/2
-1/4
0
Time t
1/1
3/4
1/2
1/4
0/1

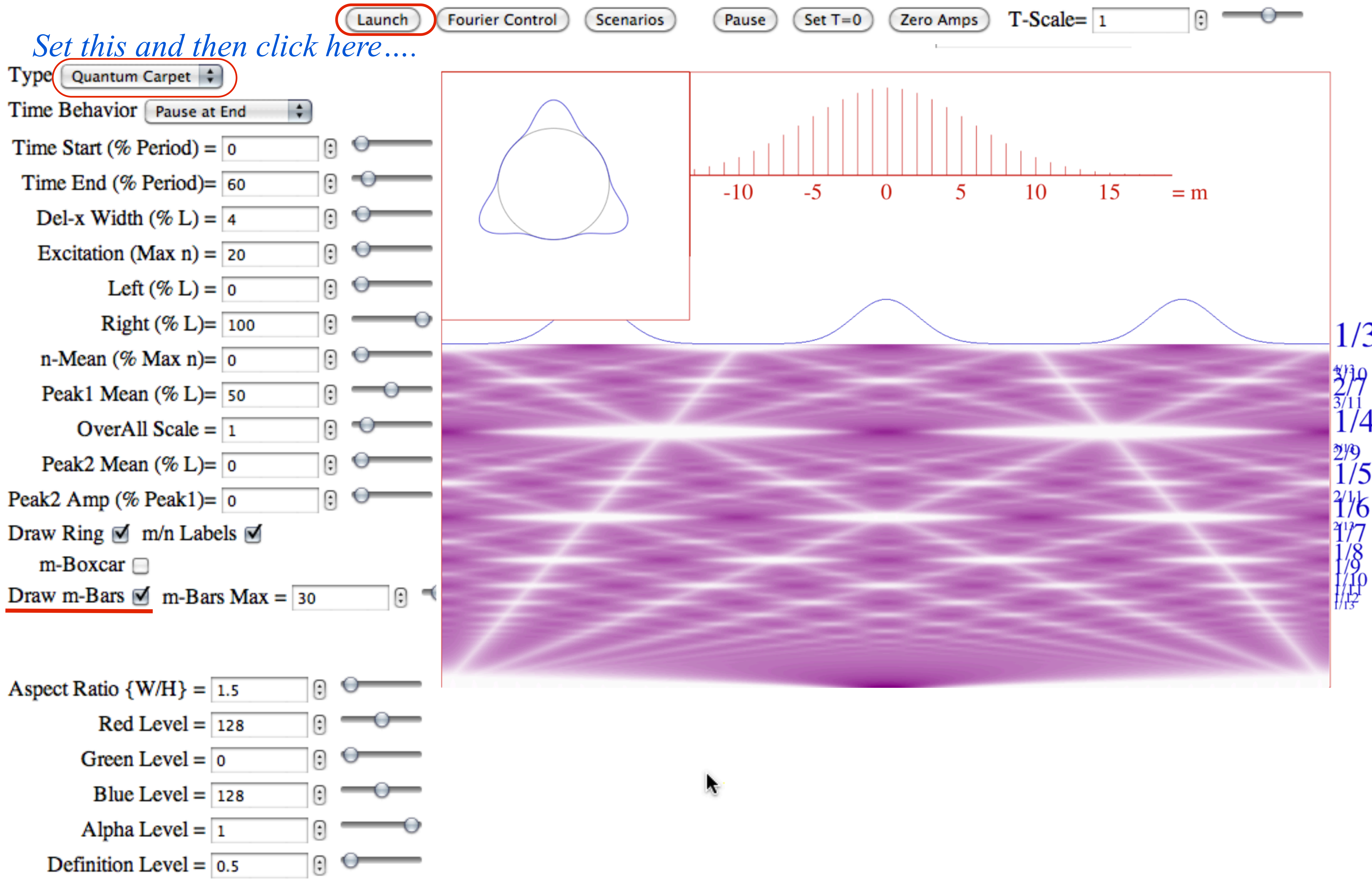
WaveIt Web simulation https://modphys.hosted.uark.edu/markup/WaveItWeb.html?scenario=Quantum%20Carpet



WaveIt Web simulation https://modphys.hosted.uark.edu/markup/WaveItWeb.html?scenario=Quantum%20Carpet



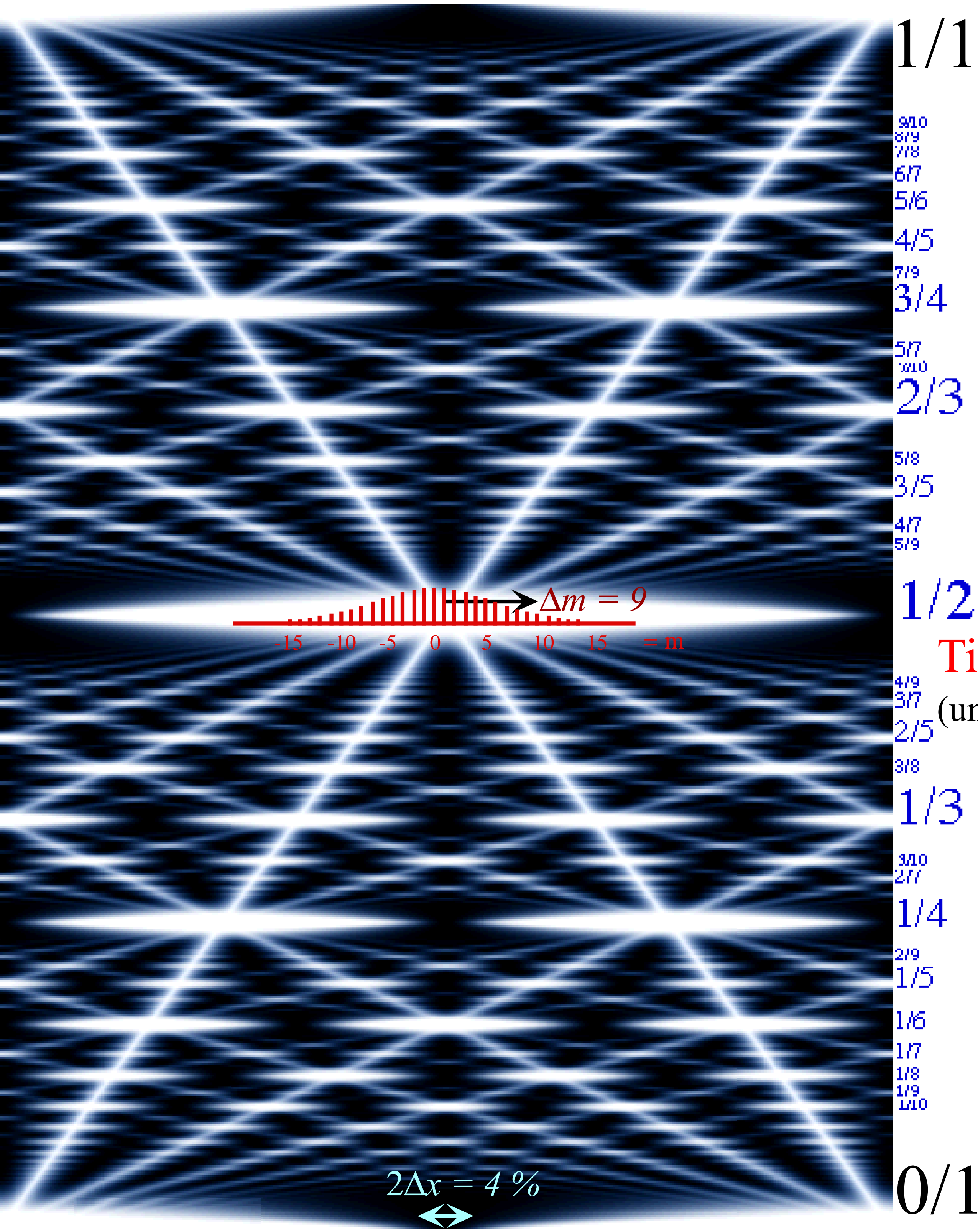

Local Control
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale=
1
time = 0.60T
3/5
7/12
4/7
5/9
6/11
7/13
1/2
3/7
5/12
2/5
3/8
4/11
1/3
2/7
3/11
1/4
2/9
1/5
2/11
1/6
1/7
1/8
1/9
1/10


Launch
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale= 1
Set this and then click here....
Type Quantum Carpet
Time Behavior Pause at End
Time Start (% Period) = 0
Time End (% Period)= 60
Del-x Width (% L) = 4
Excitation (Max n) = 20
Left (% L) = 0
Right (% L)= 100
n-Mean (% Max n)= 0
Peak1 Mean (% L)= 50
OverAll Scale = 1
Peak2 Mean (% L)= 0
Peak2 Amp (% Peak1)= 0
Draw Ring m/n Labels
m-Boxcar
Draw m-Bars m-Bars Max = 30
Aspect Ratio {W/H} = 1.5
Red Level = 128
Green Level = 0
Blue Level = 128
Alpha Level = 1
Definition Level = 0.5
-10
-5
0
5
10
15
= m
1/3
2/7
1/4
1/5
1/6
1/7
1/8
1/9

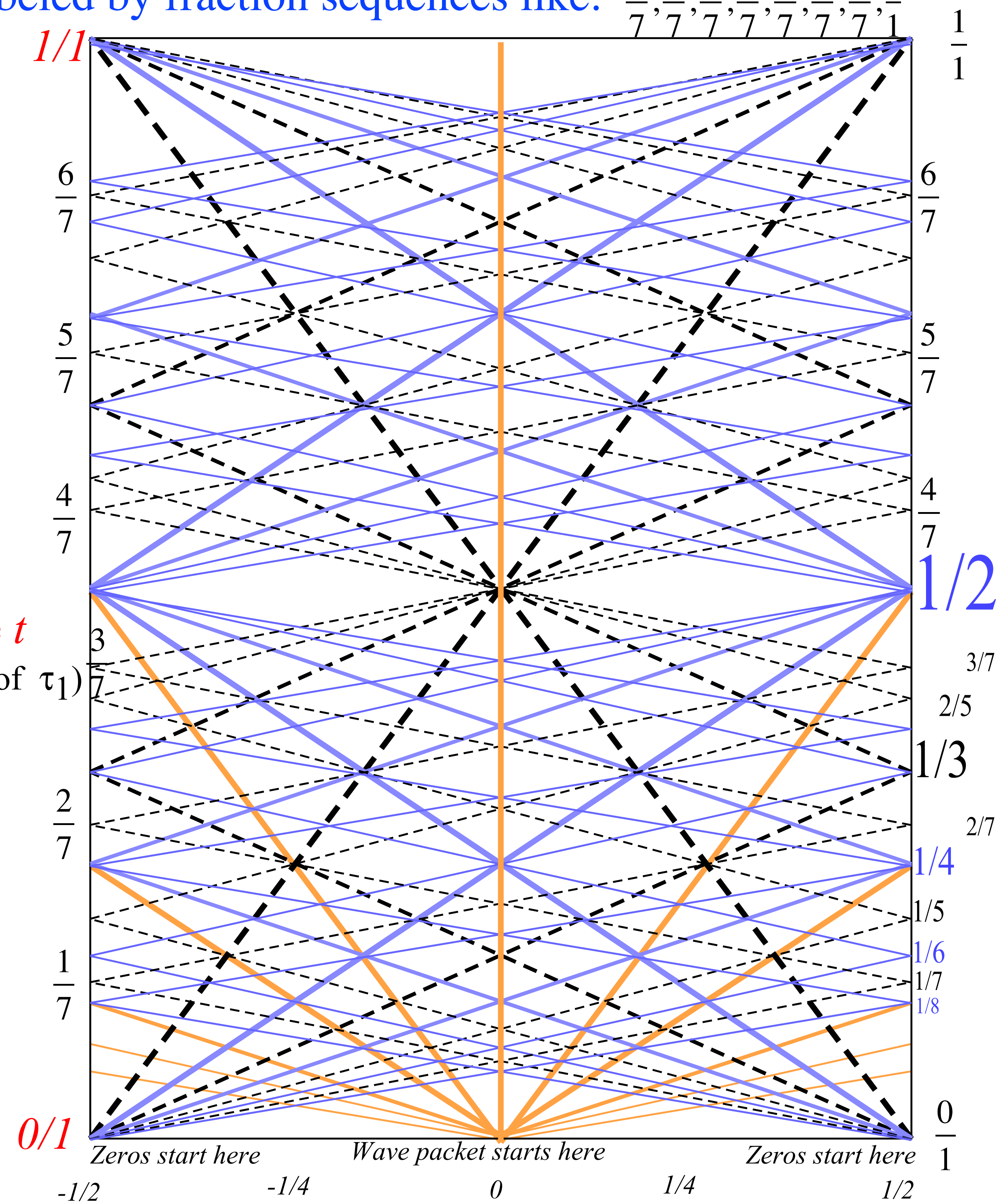
N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



Time t (units of τ_1)

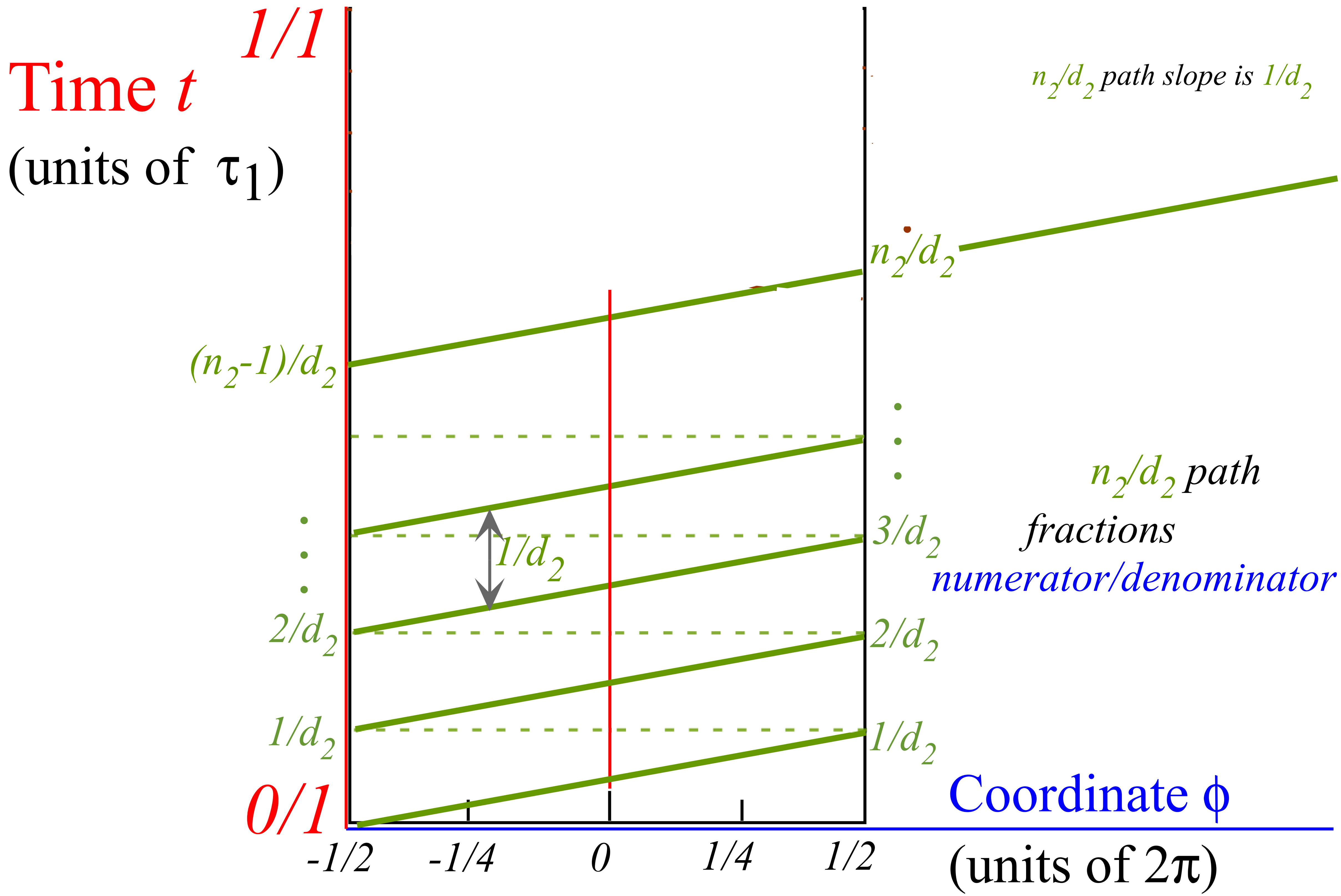


Coordinate ϕ

(units of 2π)

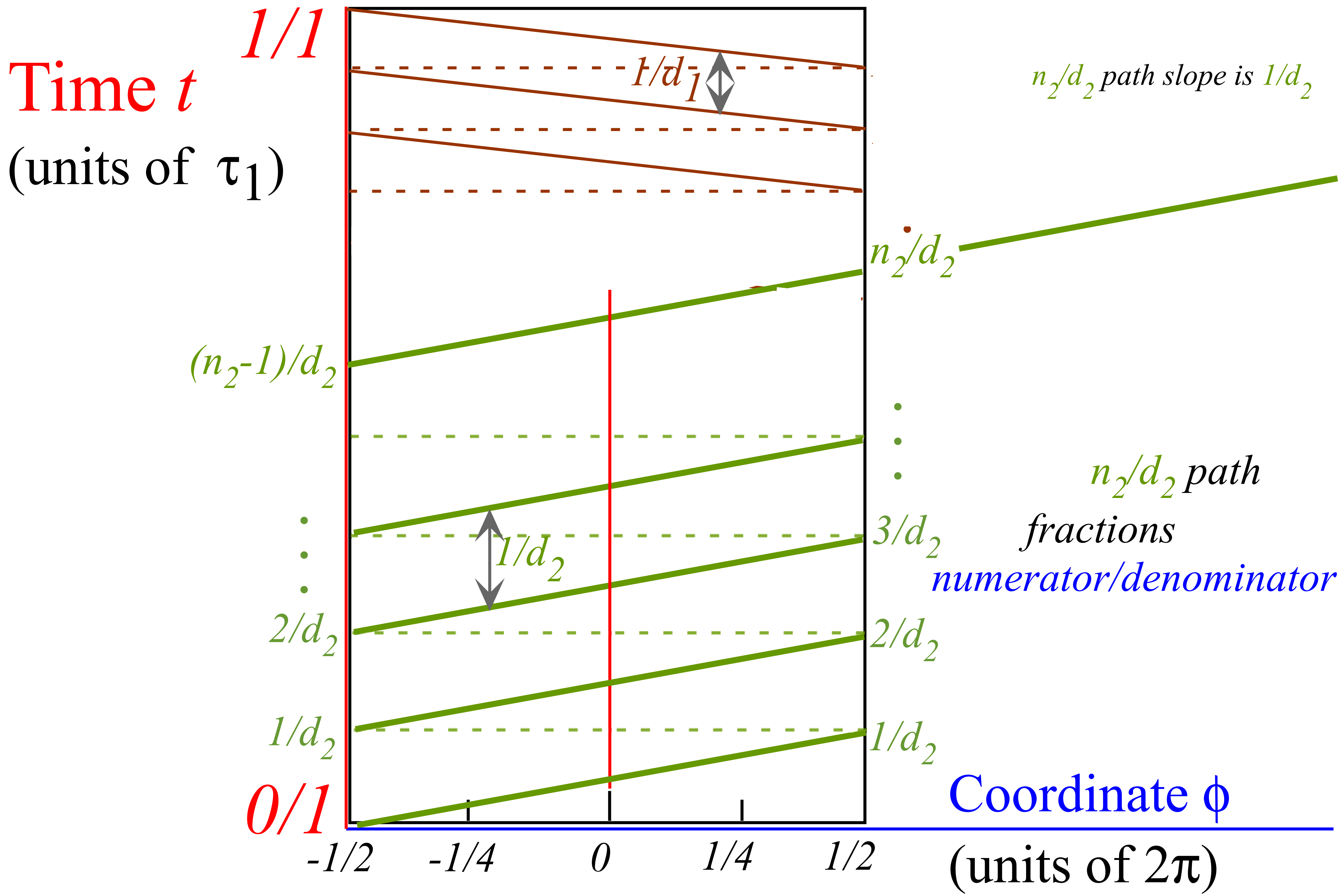
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



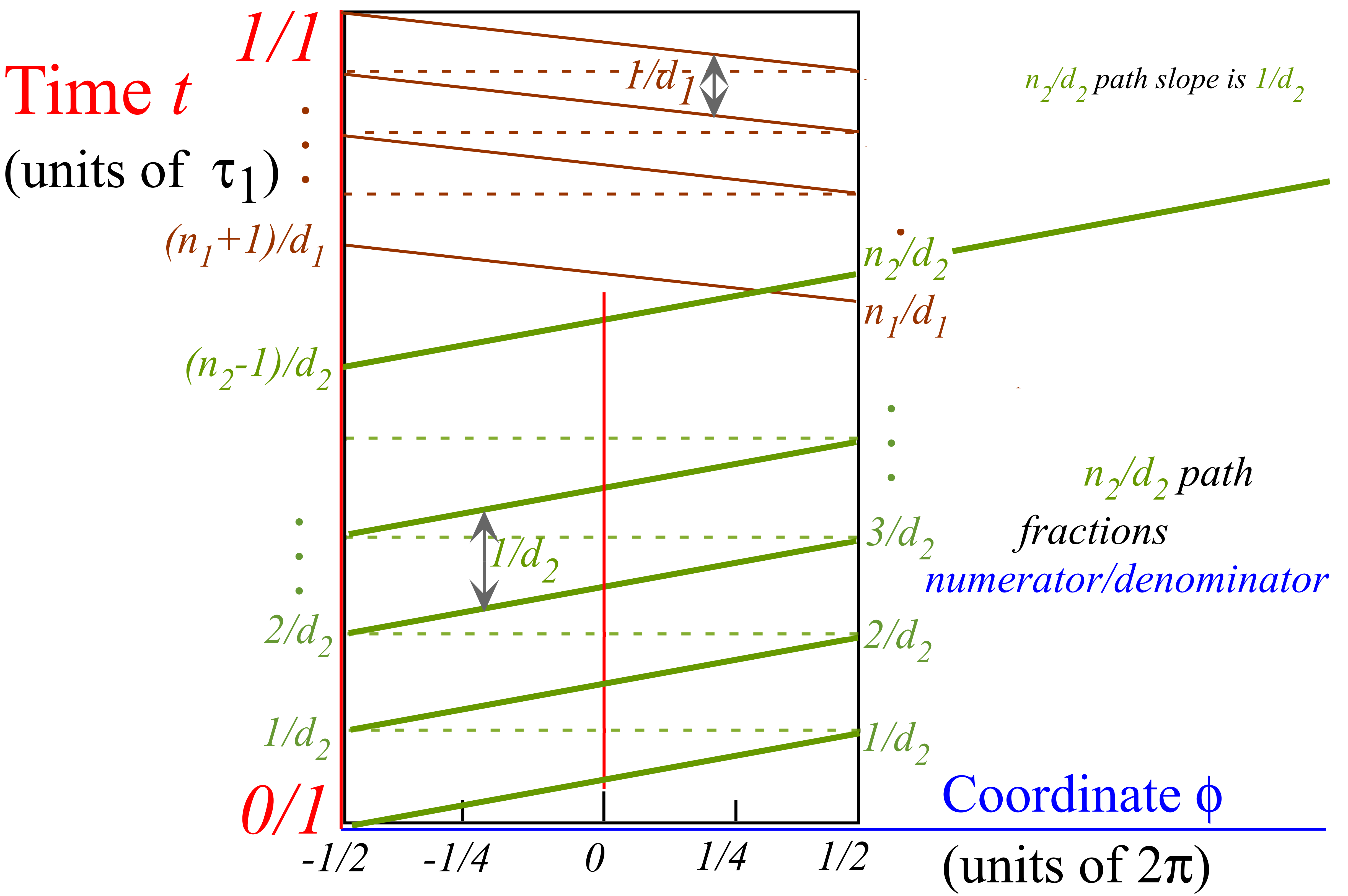
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



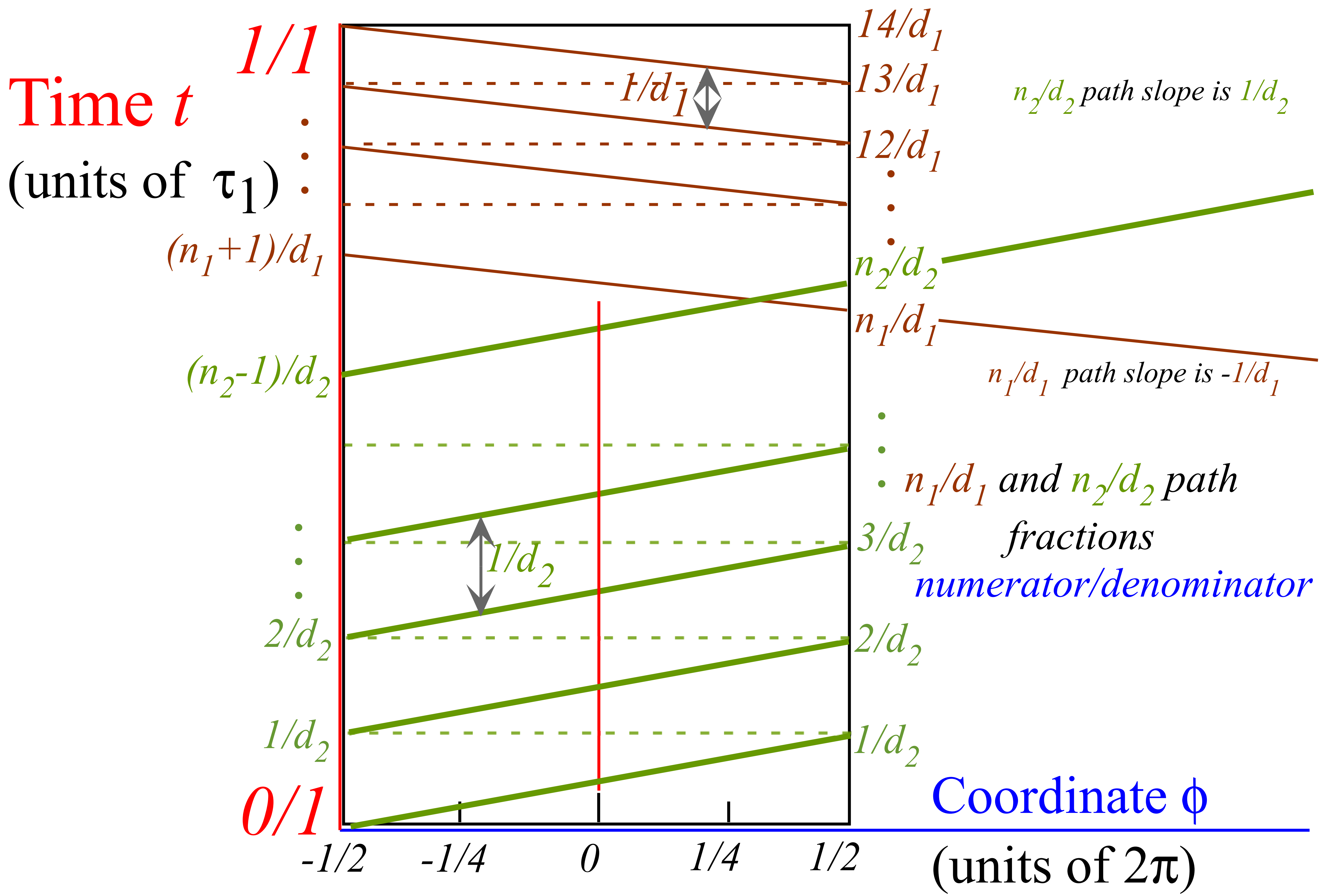
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



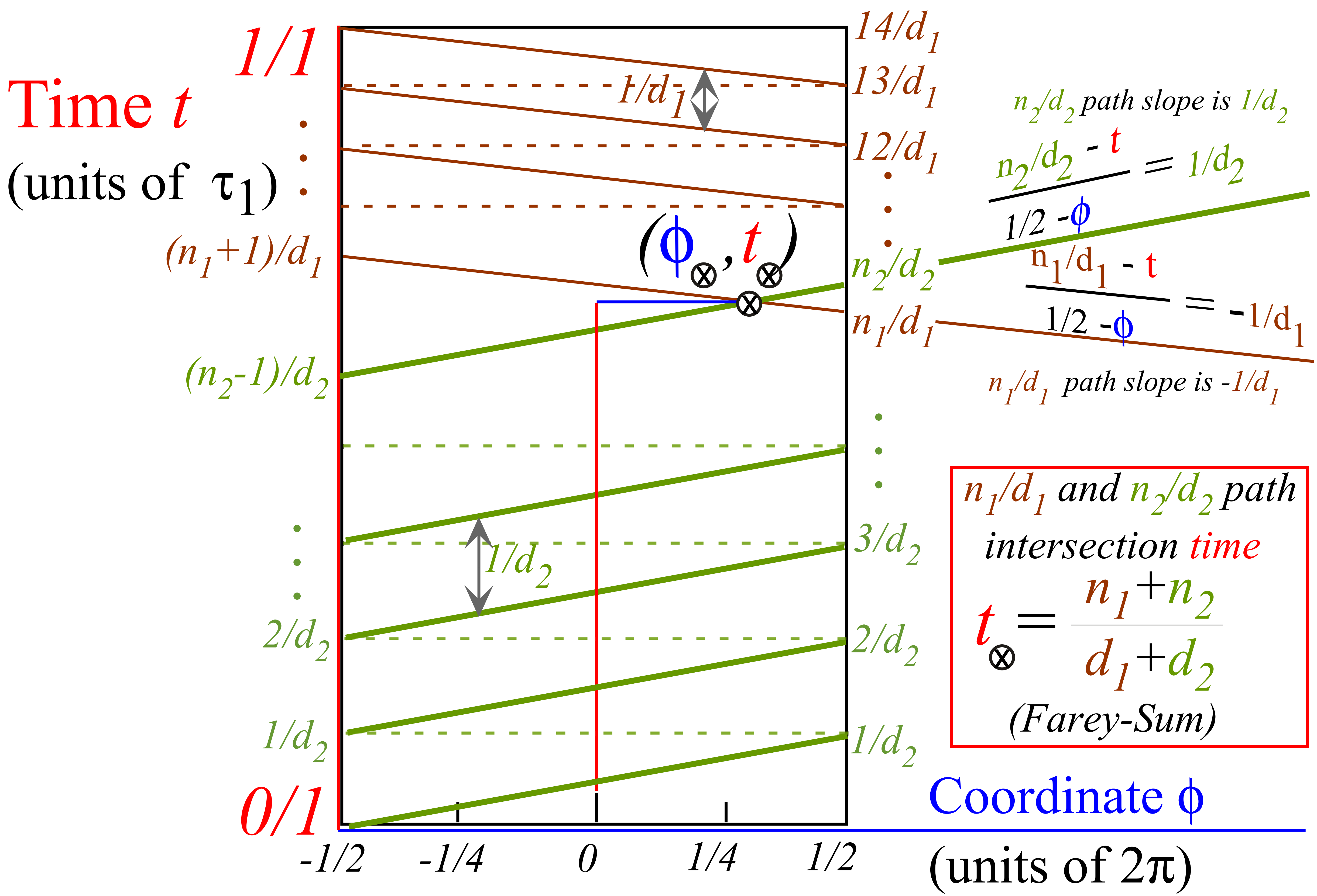
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



Farey Sum algebra of revival-beat wave dynamics

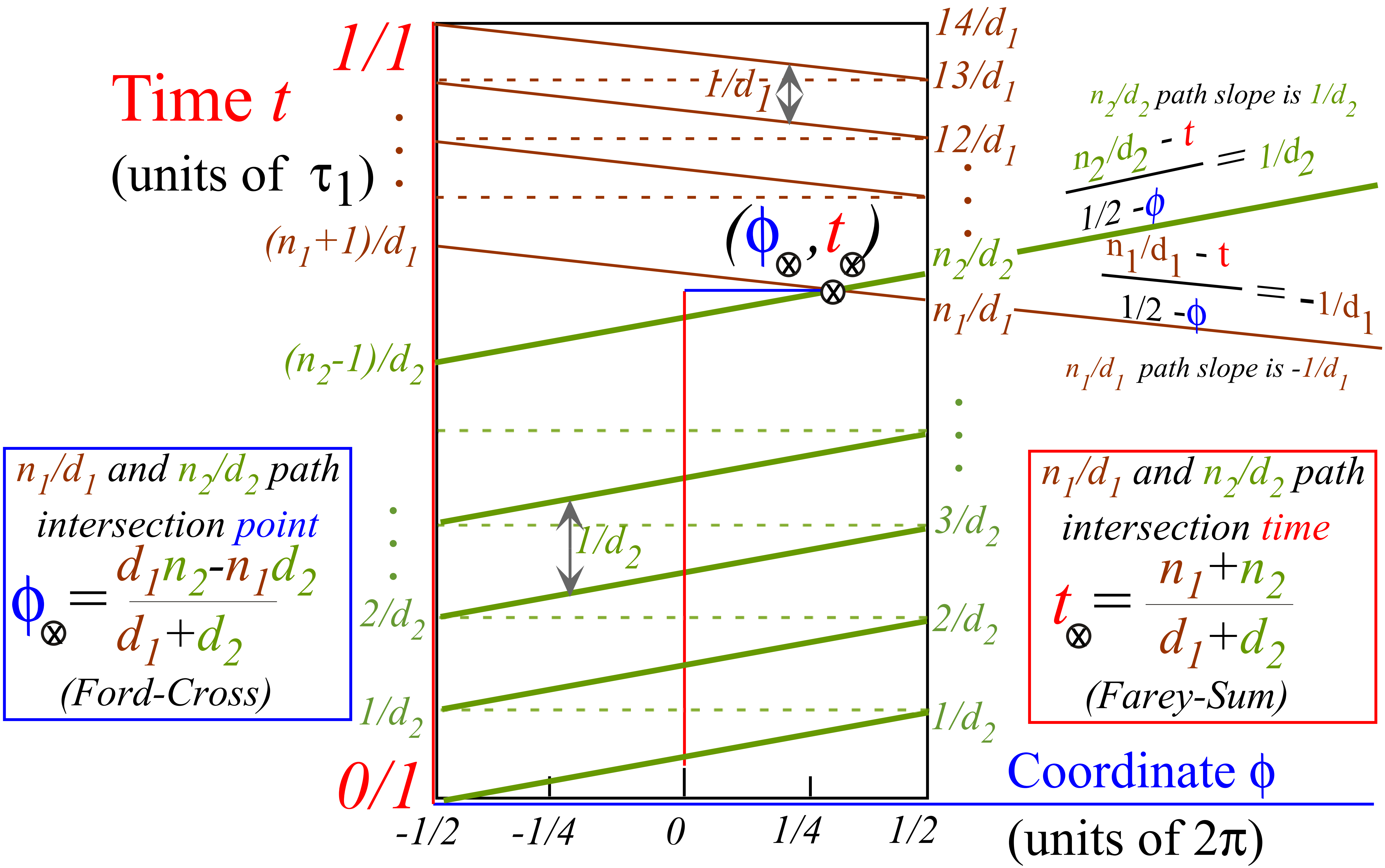
Label by *numerators N* and *denominators D* of rational fractions *N/D*



[John Farey, Phil. Mag.(1816)]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

"Monster Mash" classical segue to Heisenberg action relations

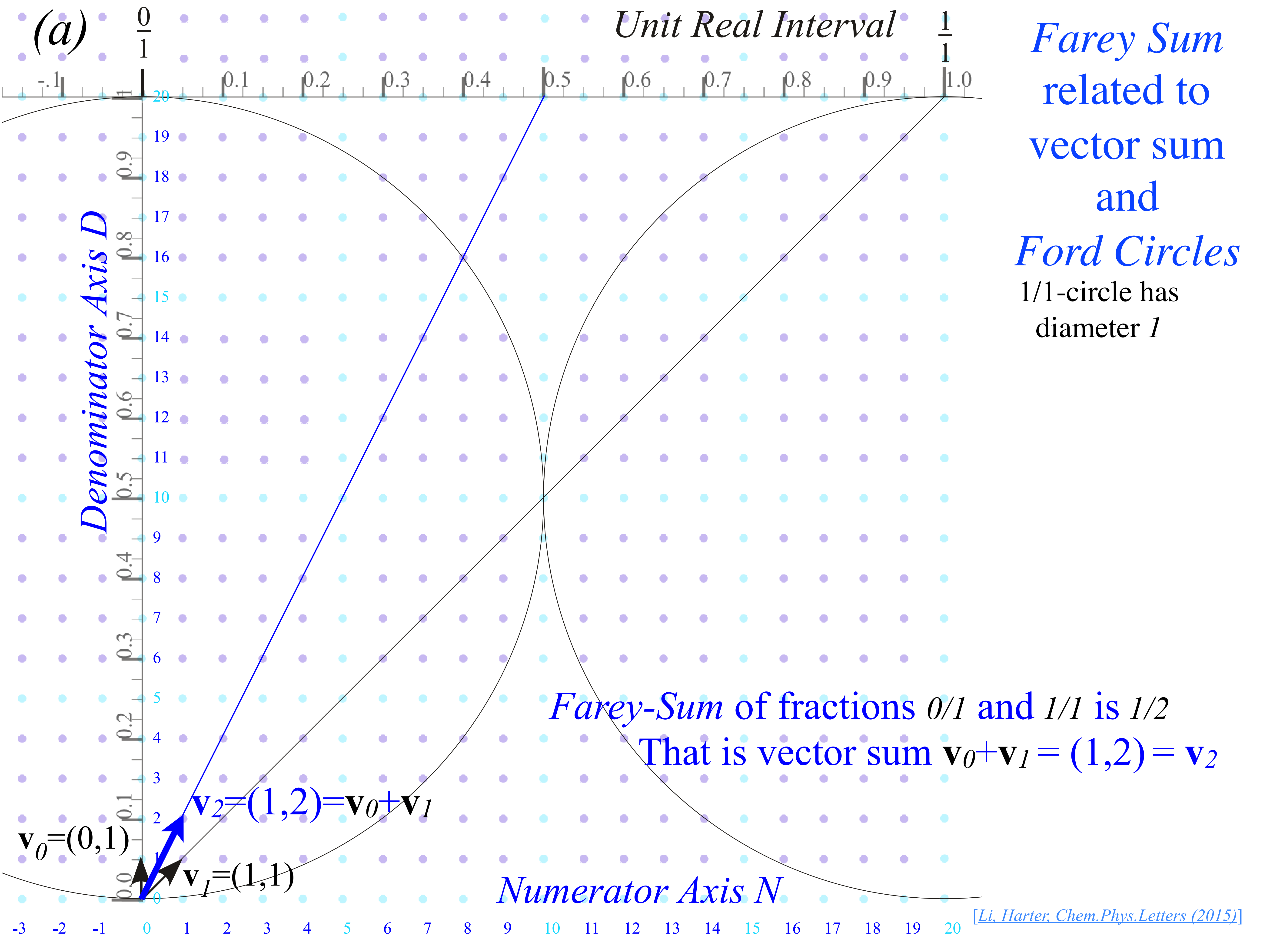
Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

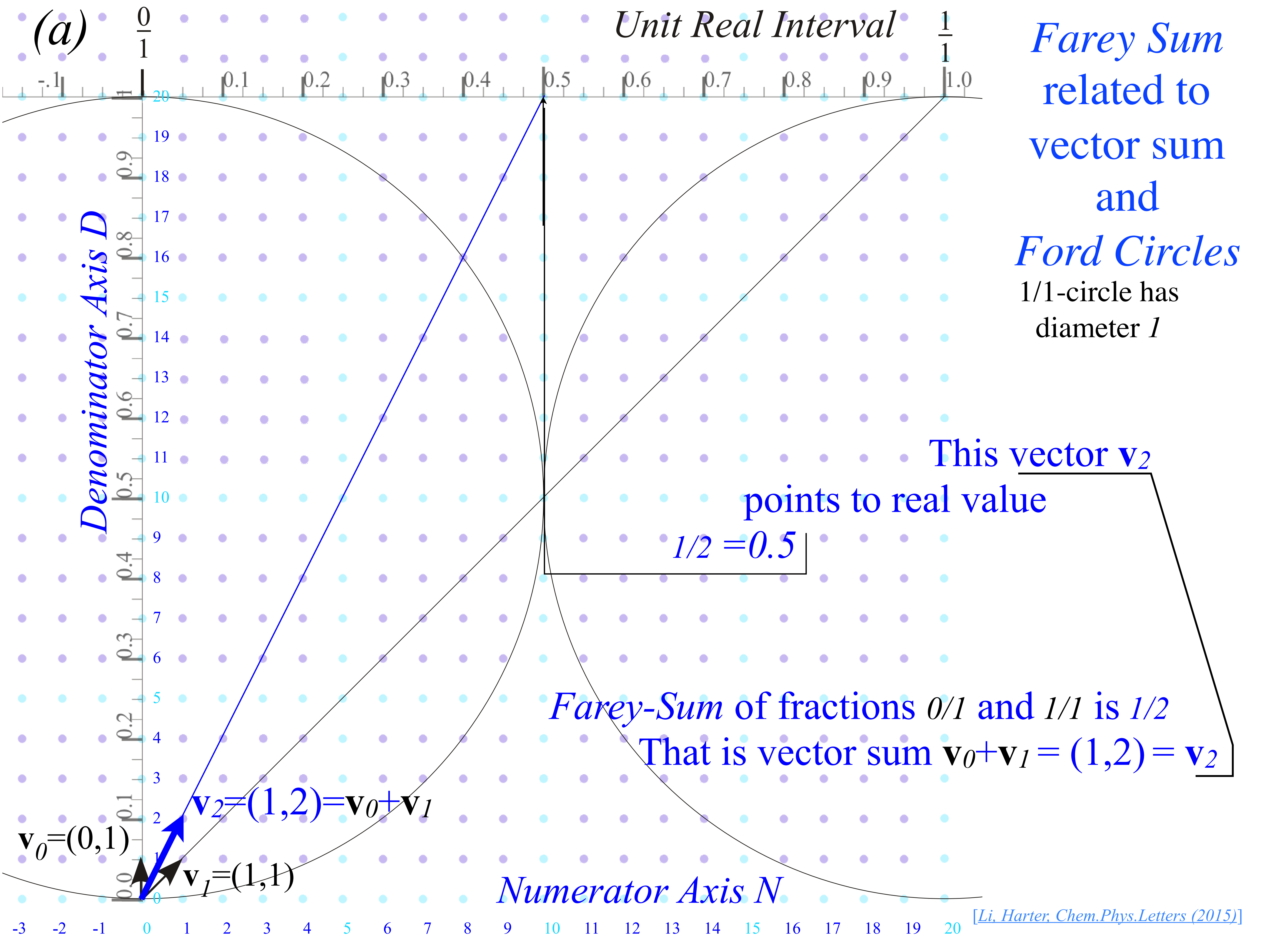
An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

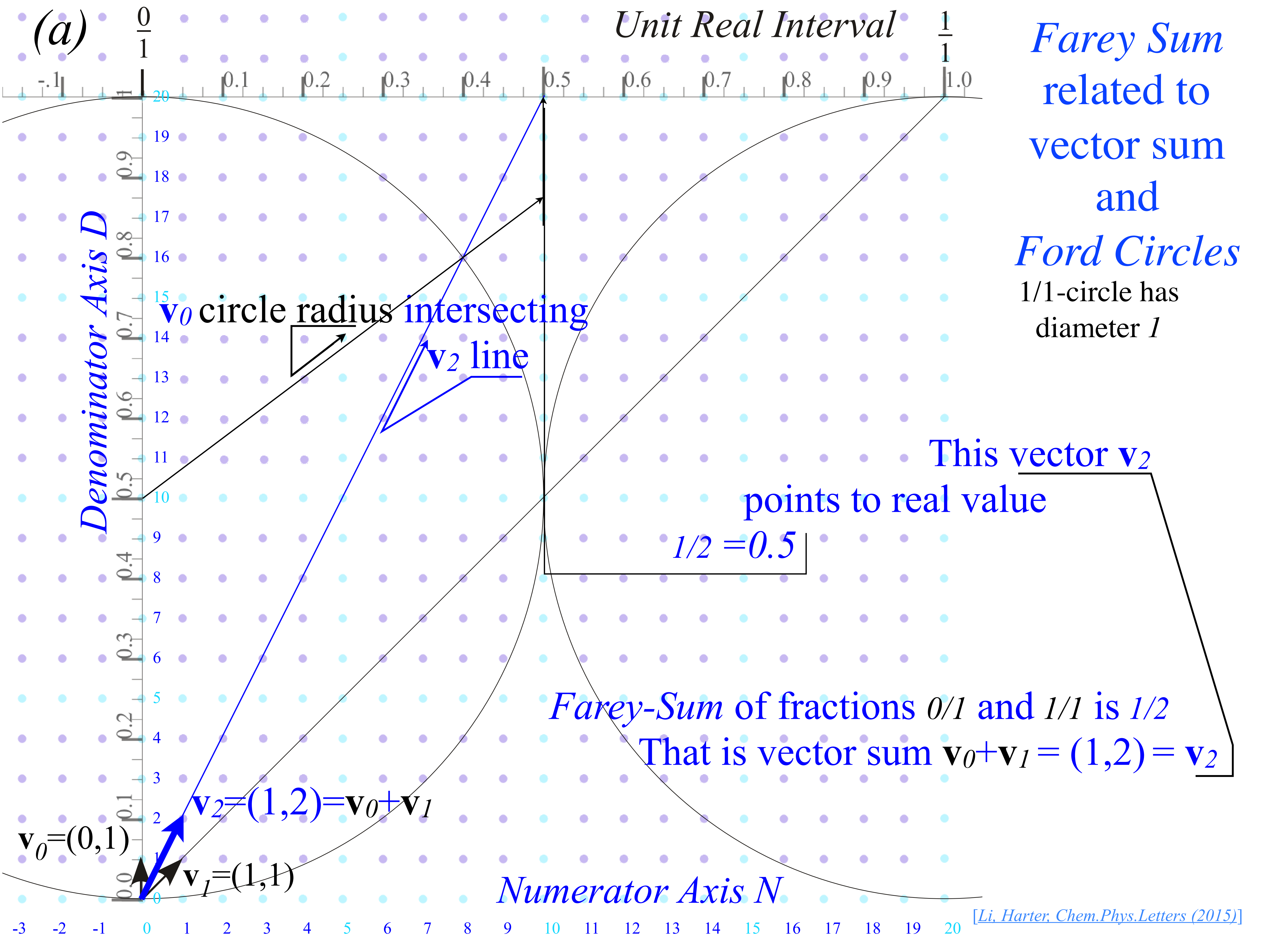
→ *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

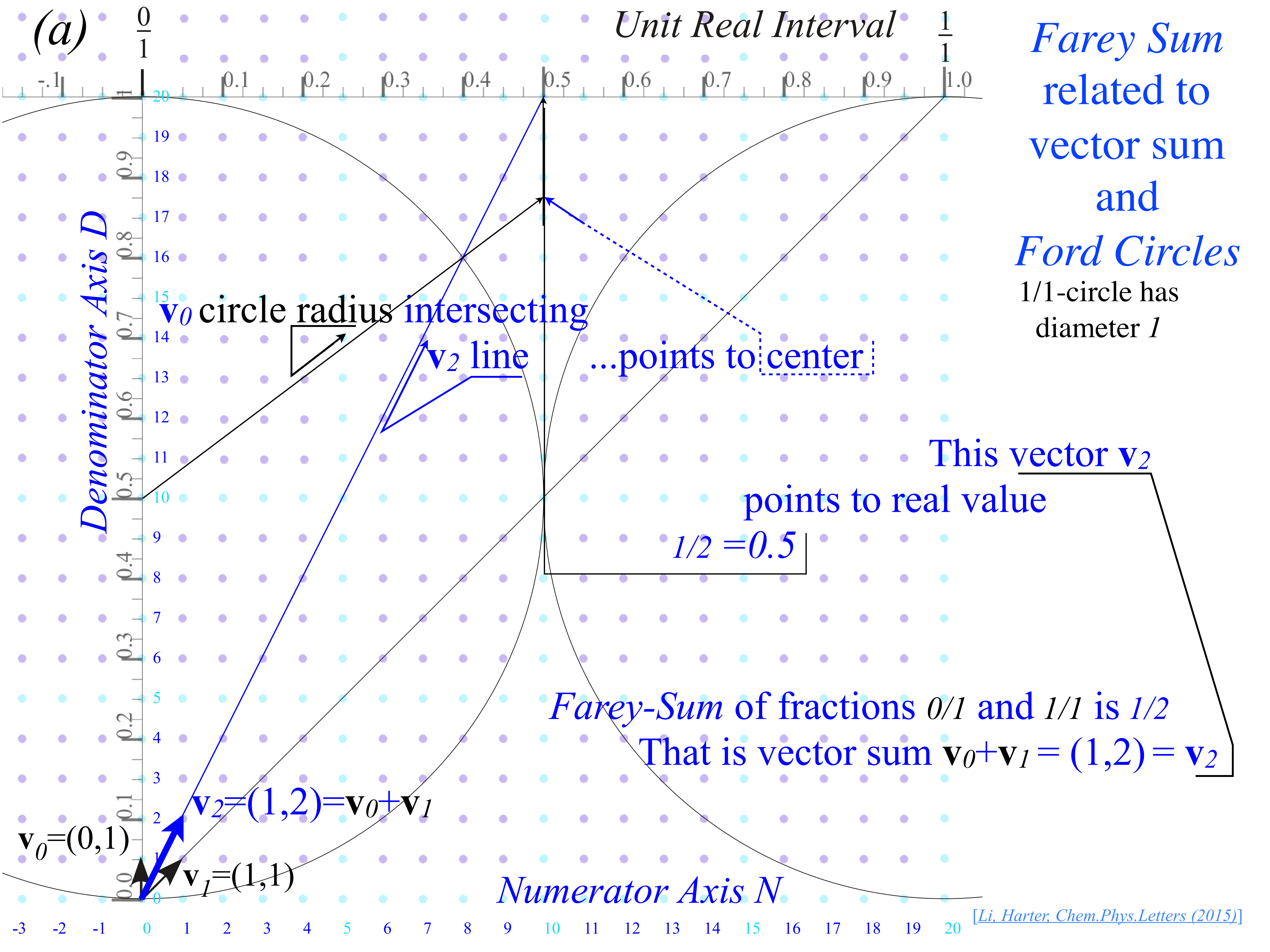


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]

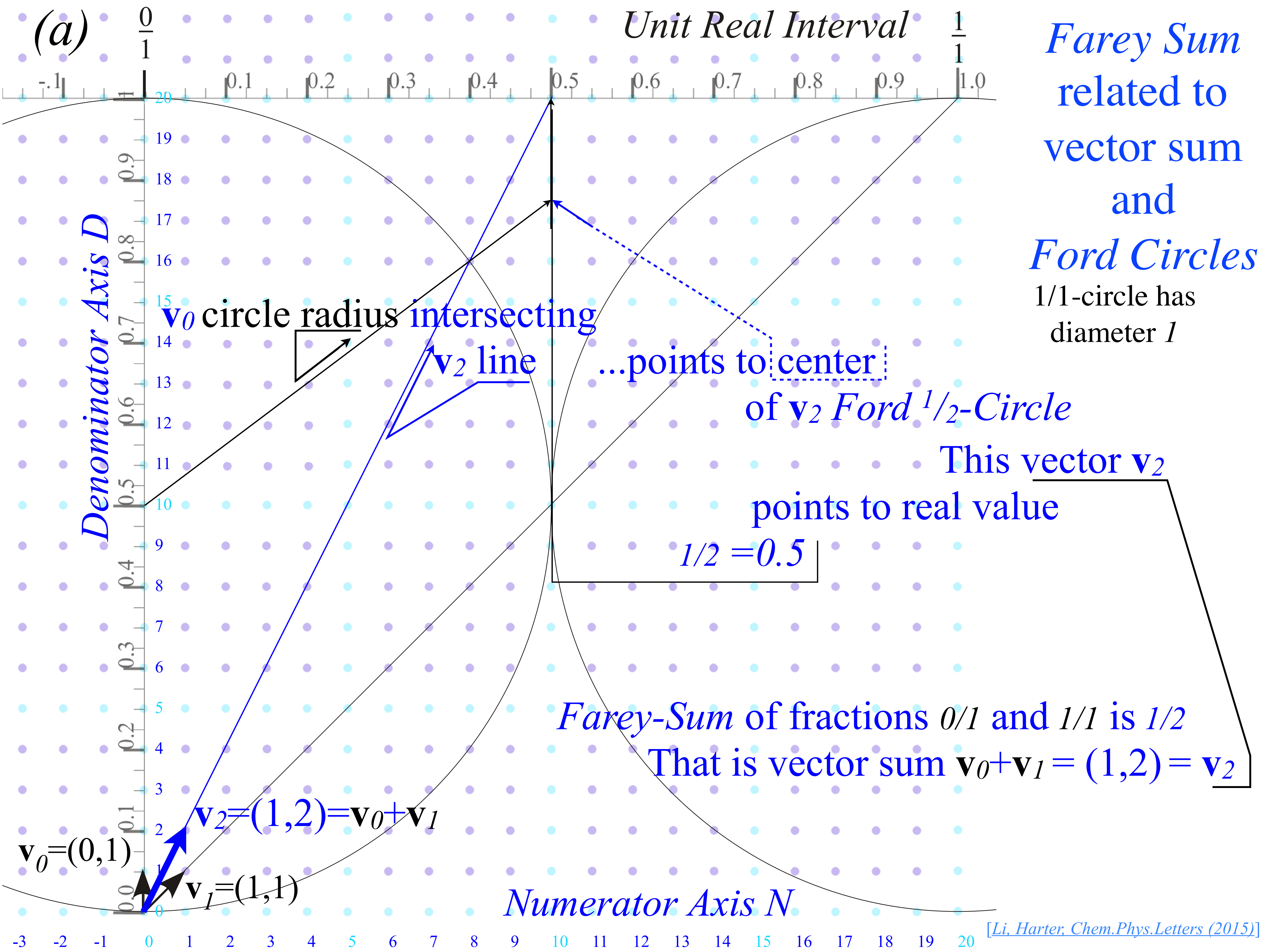

(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
This vector v2 points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]



(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting v2 line
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]



(a)
0/1
Unit Real Interval
1/1
Farey Sum
related to
vector sum
and
Ford Circles
1/1-circle has
diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
This vector v2
points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]



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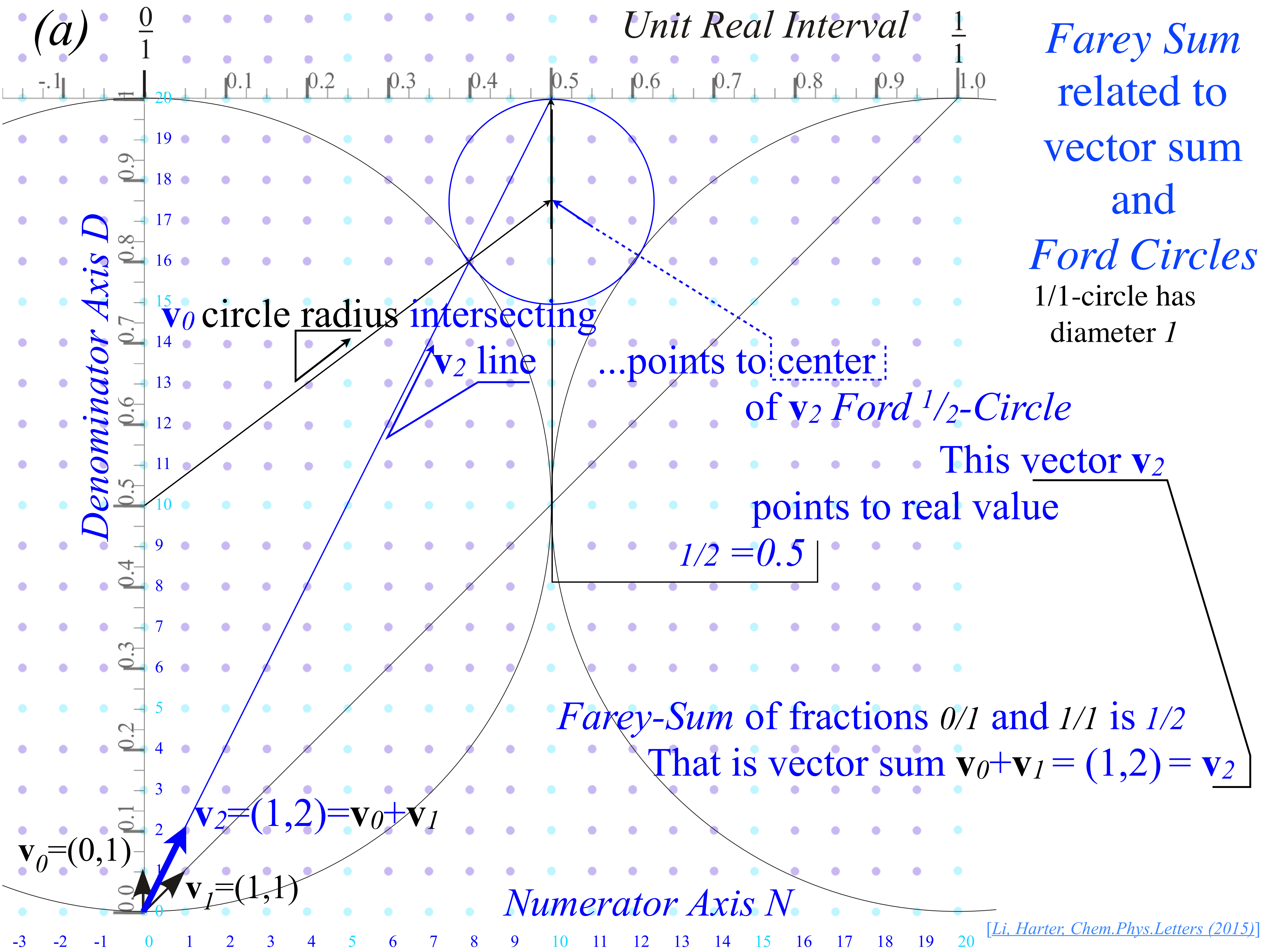

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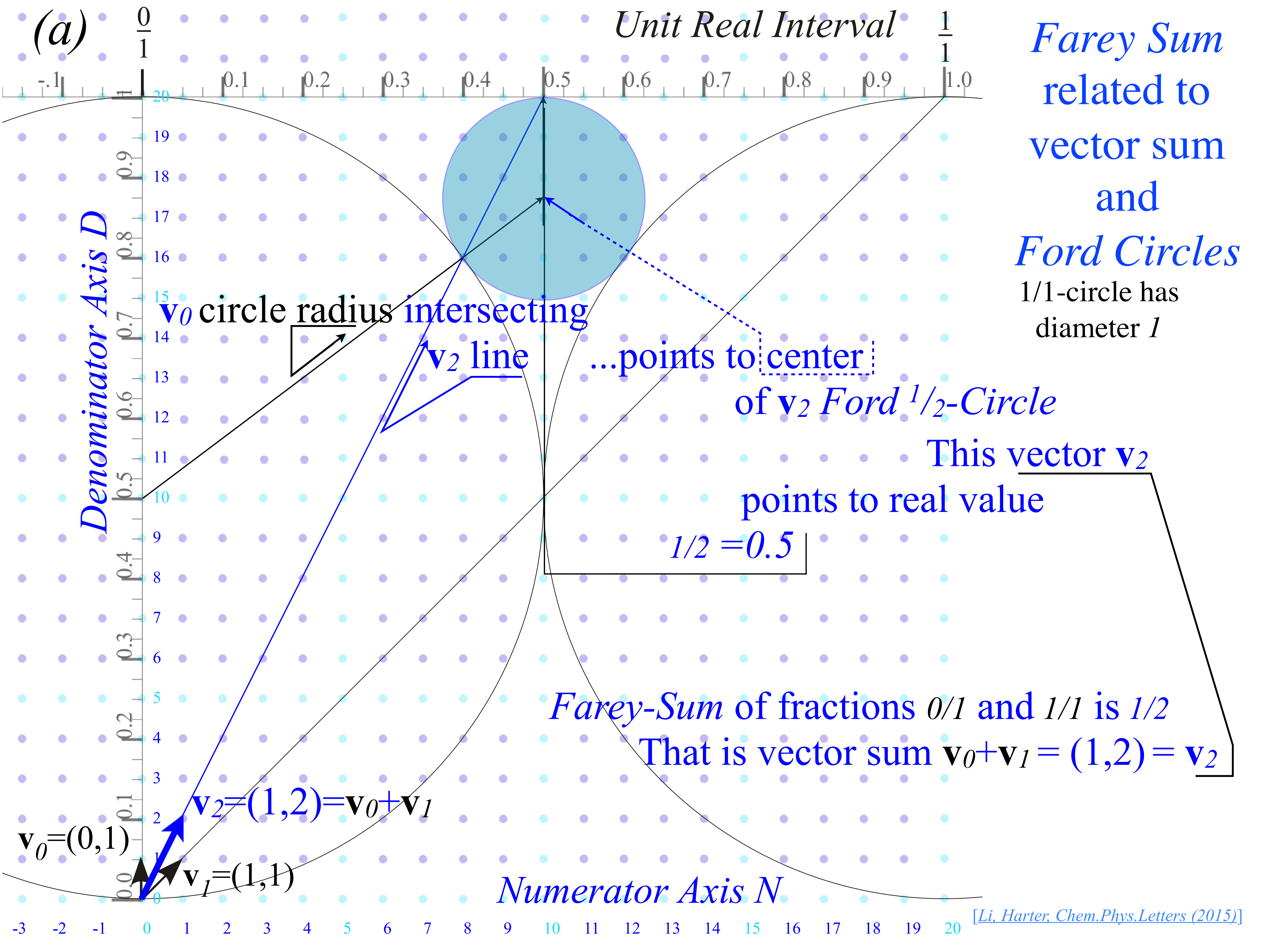
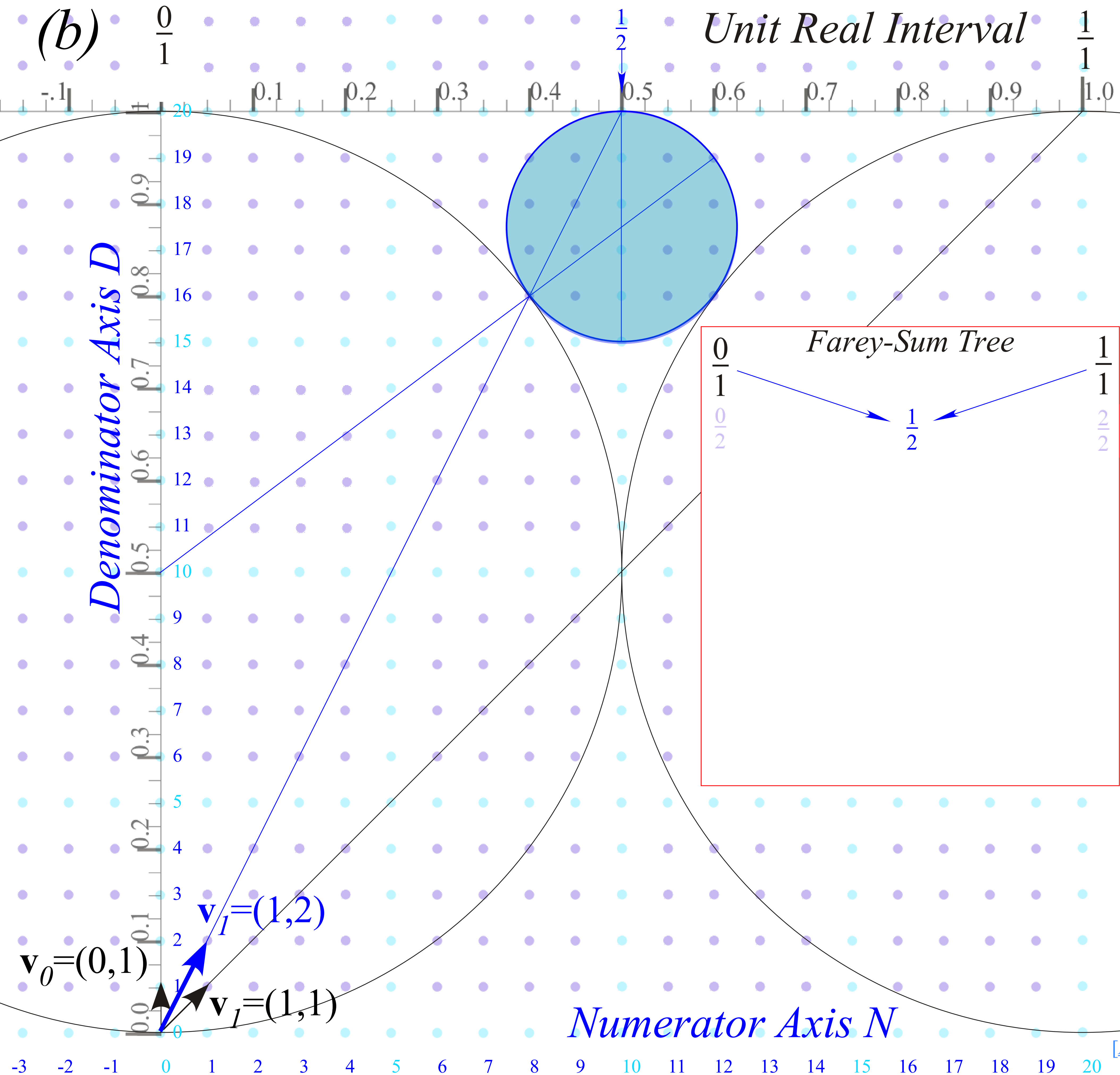


(a)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting v2 line
...points to center of v2 Ford 1/2-Circle
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]

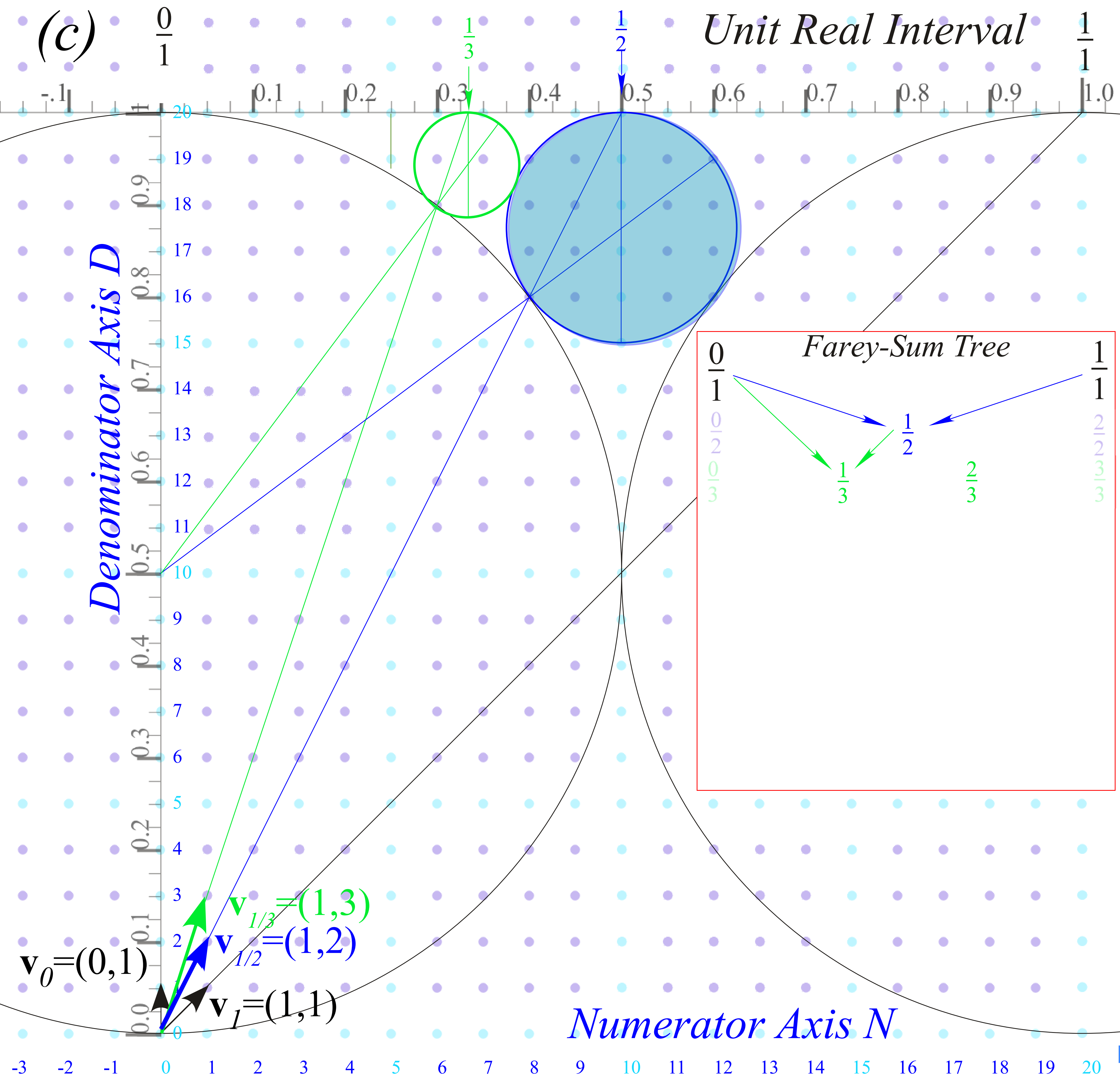


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

[Li, Harter, Chem.Phys.Letters (2015)]

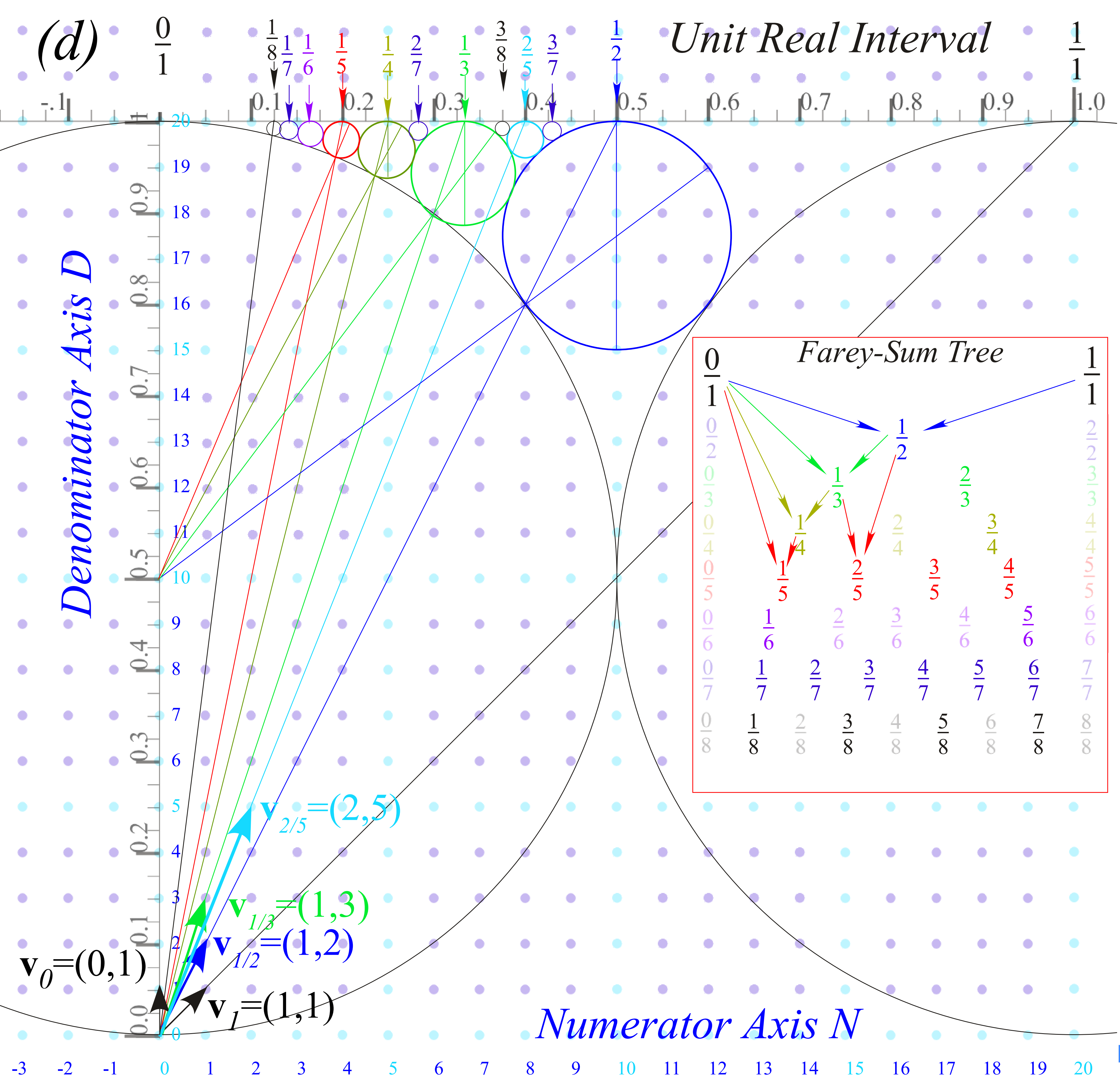


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

[Li, Harter, Chem.Phys.Letters (2015)]



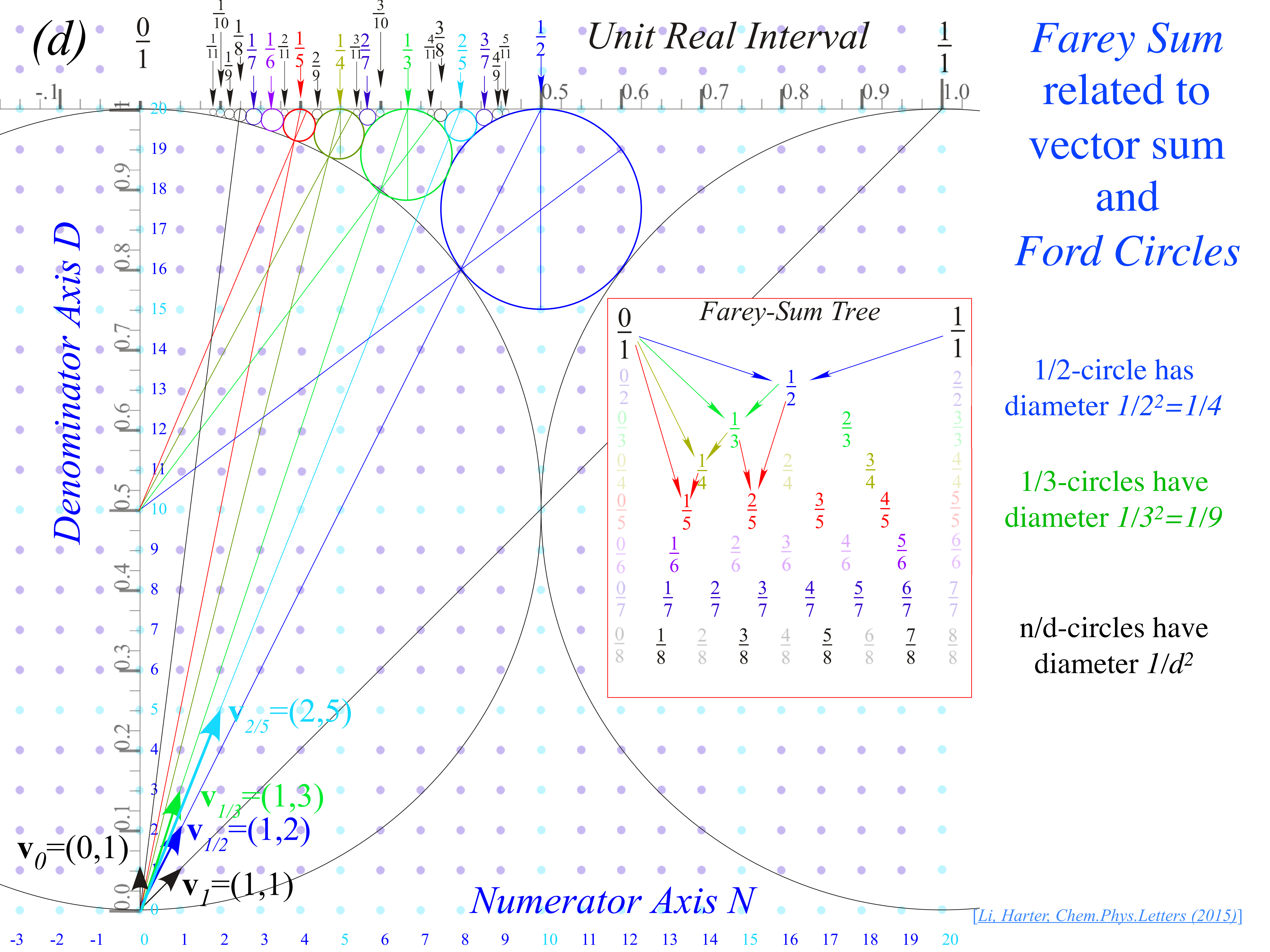
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

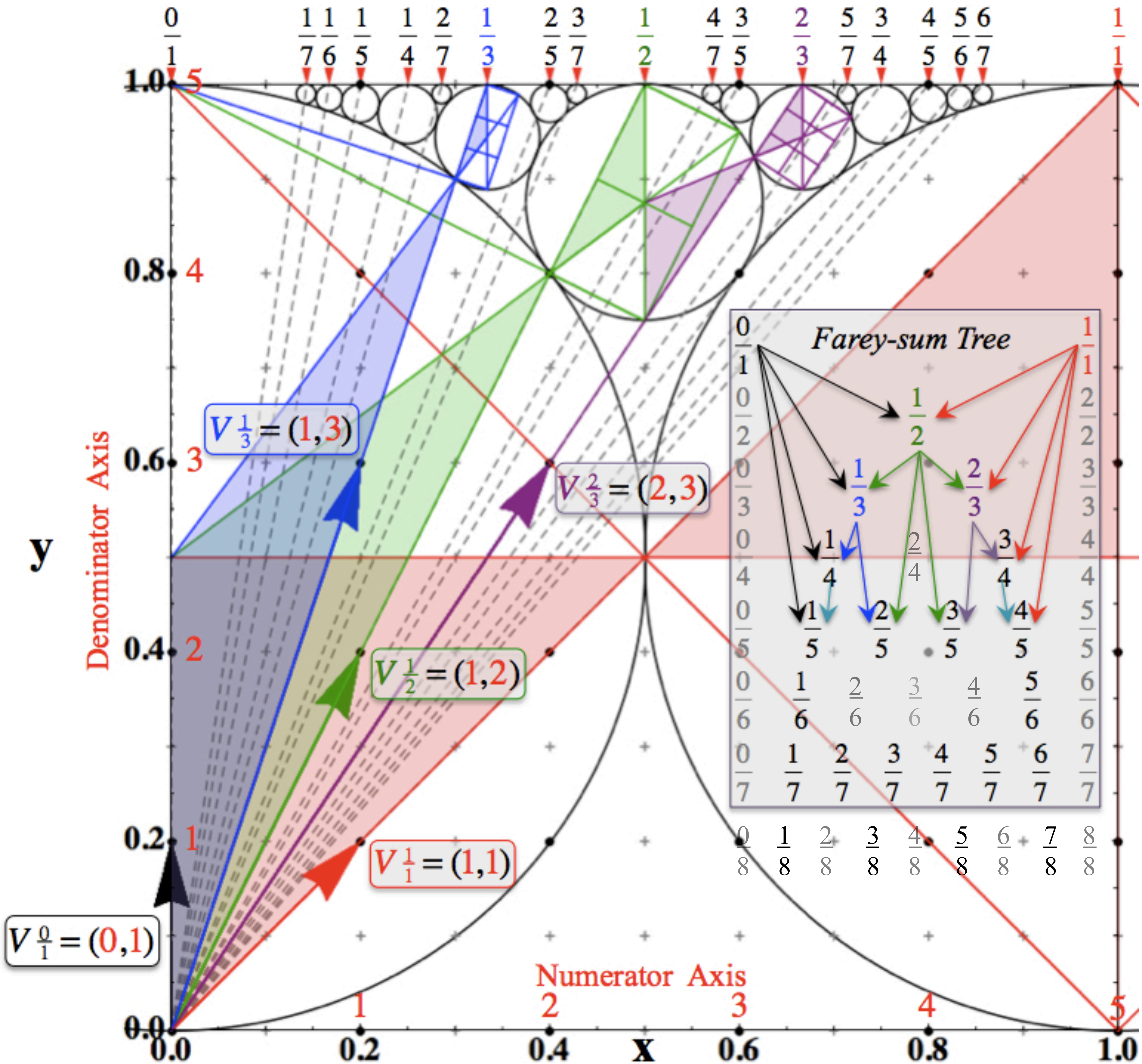
1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]



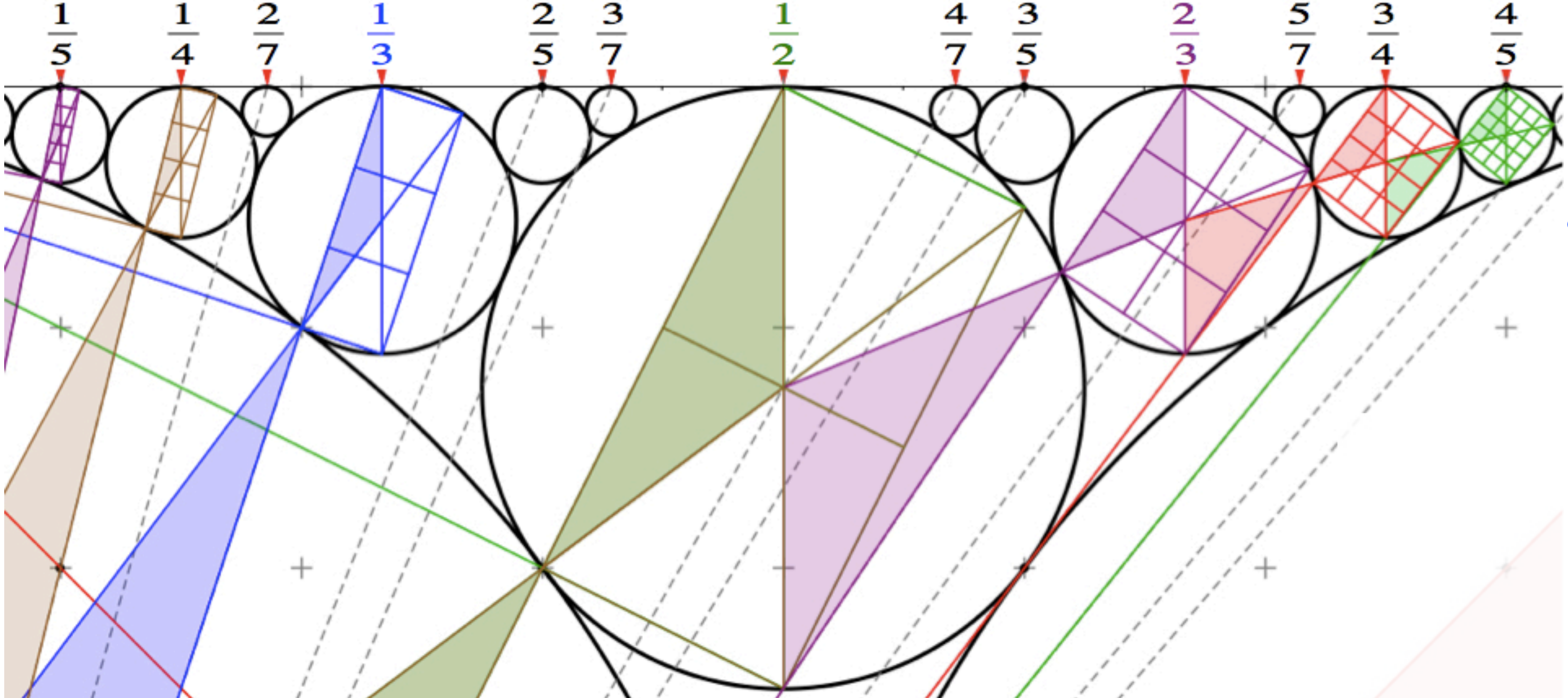
(d)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
Farey-Sum Tree
1/2-circle has diameter 1/2²=1/4
1/3-circles have diameter 1/3²=1/9
n/d-circles have diameter 1/d²
Denominator Axis D
Numerator Axis N
v_0=(0,1)
v_1=(1,1)
v_1/2=(1,2)
v_1/3=(1,3)
v_2/5=(2,5)
[Li, Harter, Chem.Phys.Letters (2015)]



Thales Rectangles

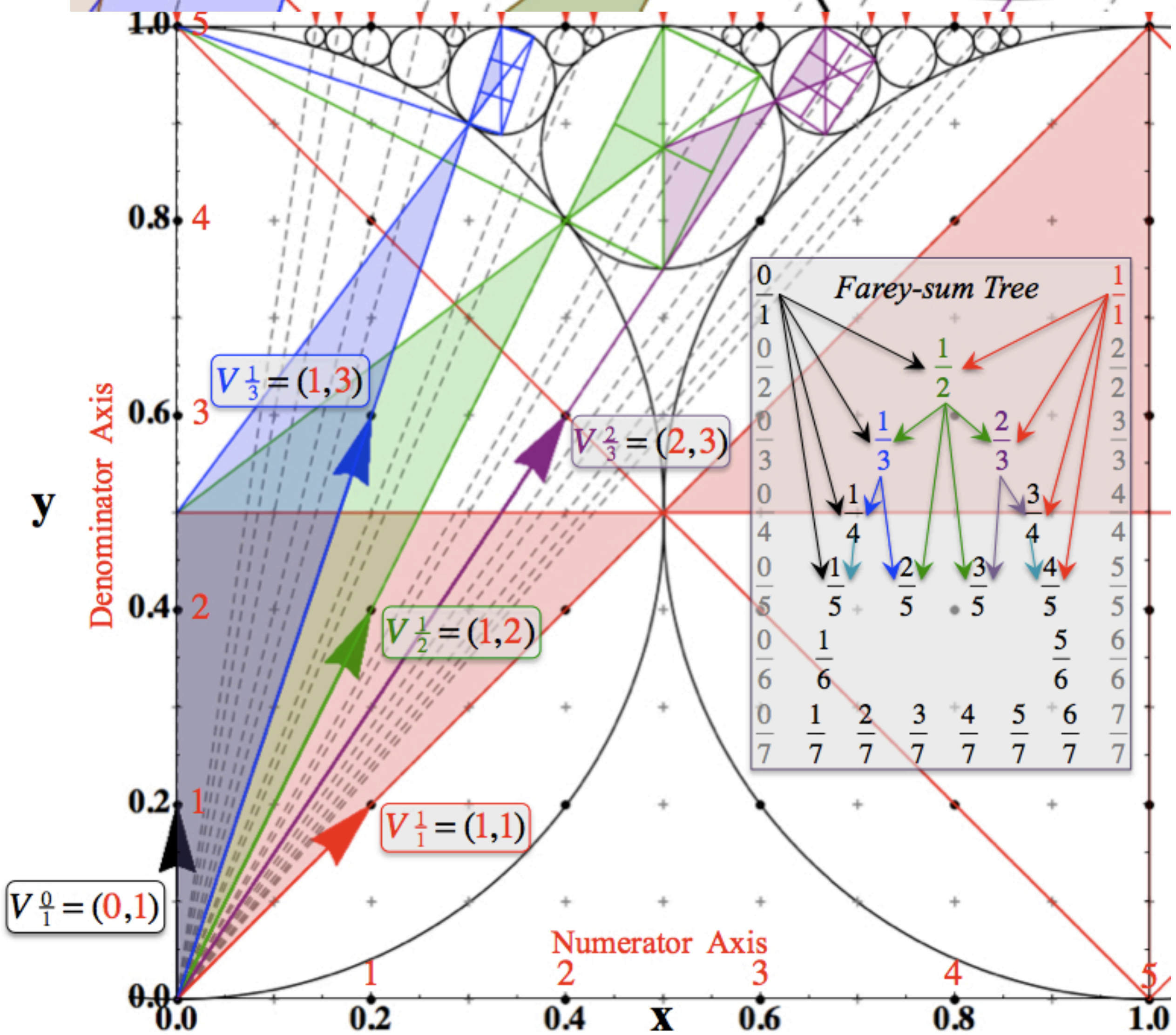
provide
analytic geometry
of
fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



“Quantized” Thales Rectangles

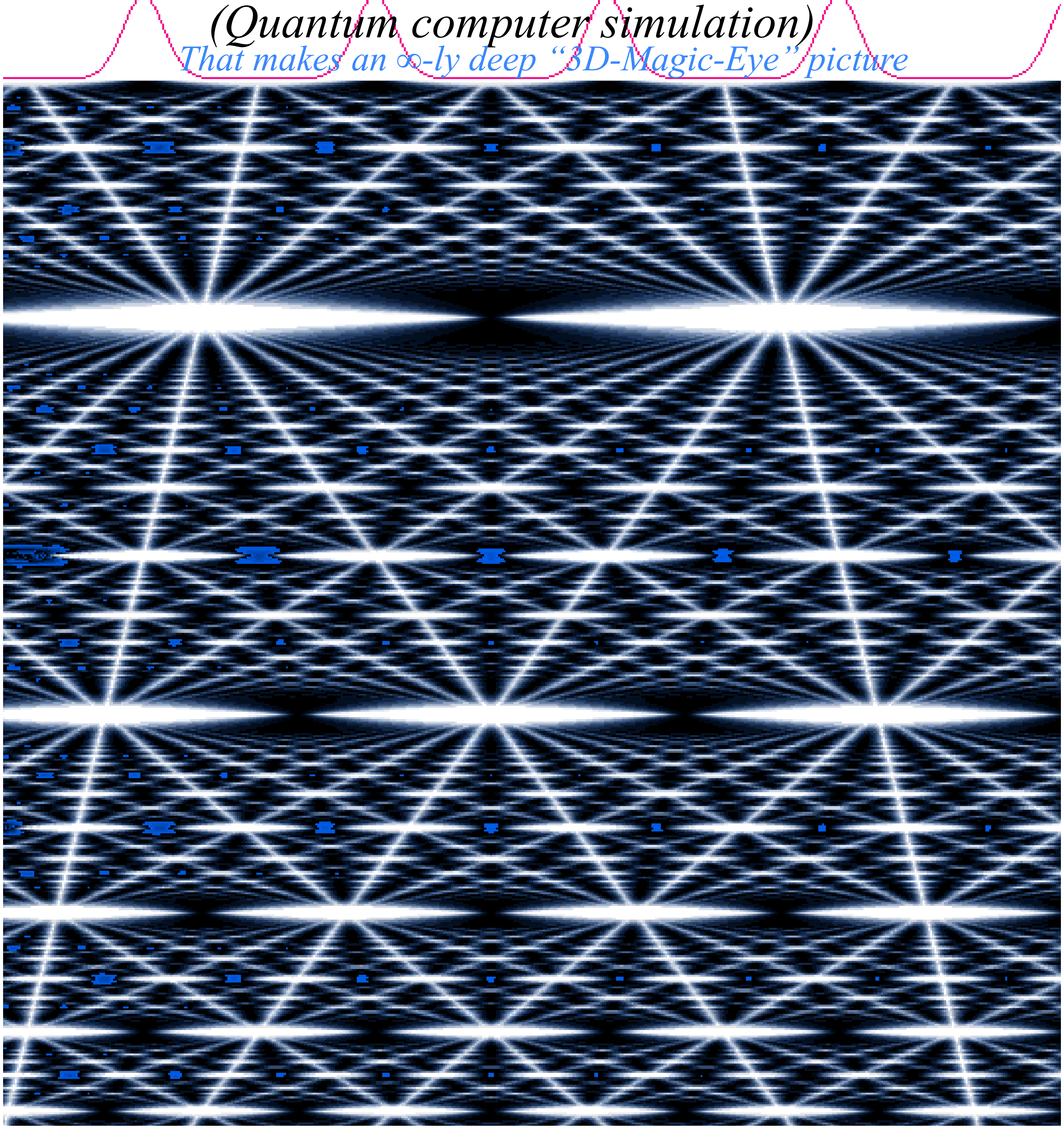
provide
analytic geometry
of
fractal structure



[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



Geometric "Integration" (Converting Velocity data to Spacetime)

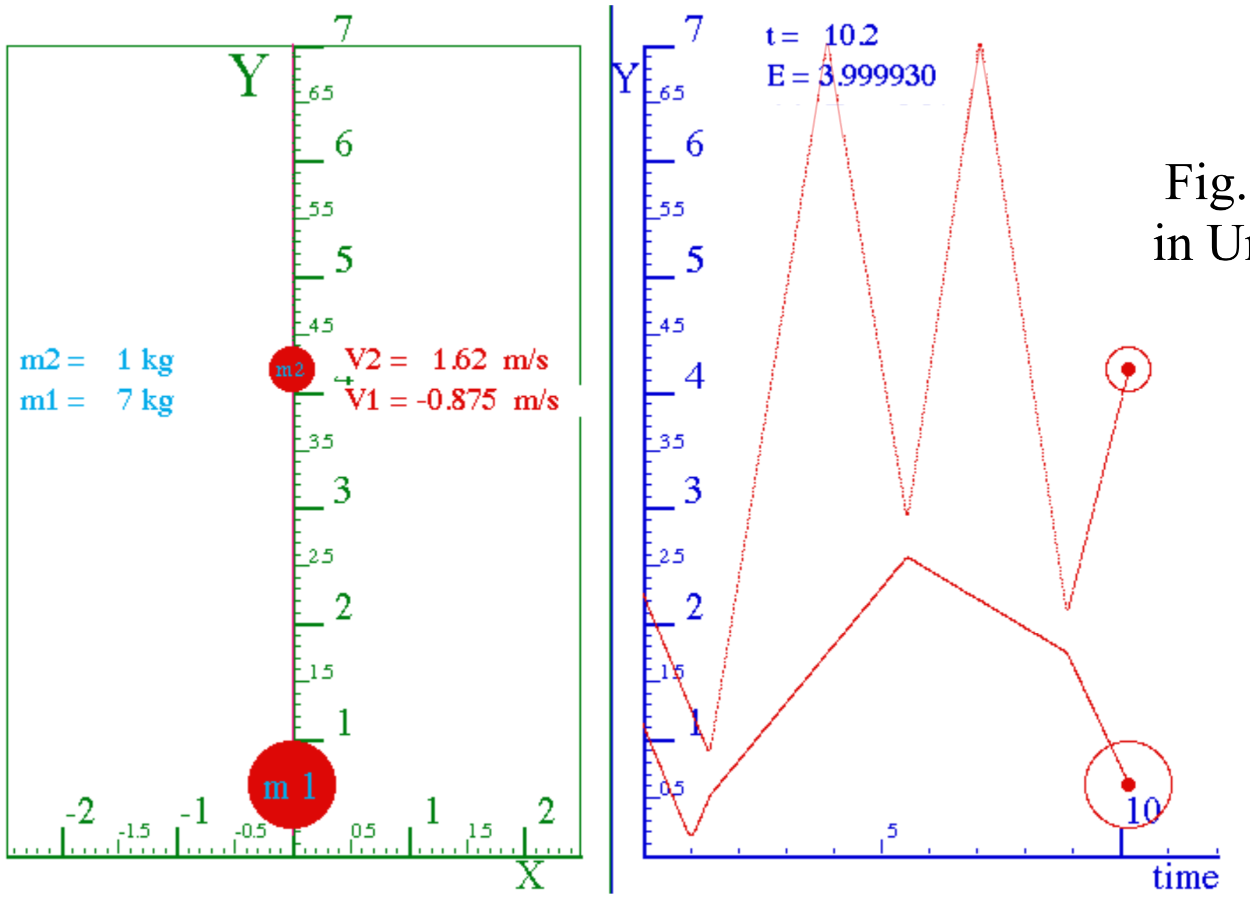


Fig. 4.8
in Unit 1

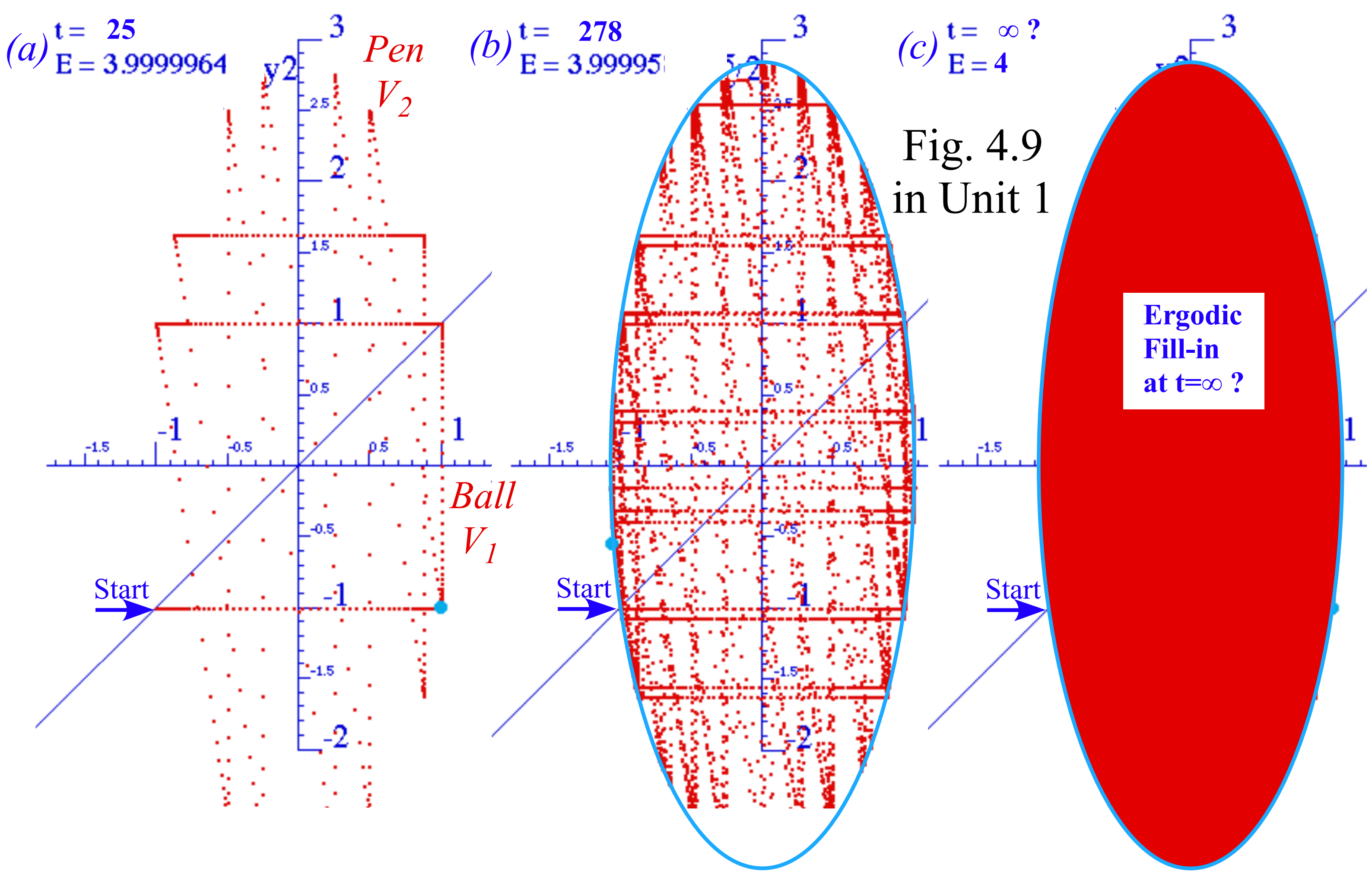
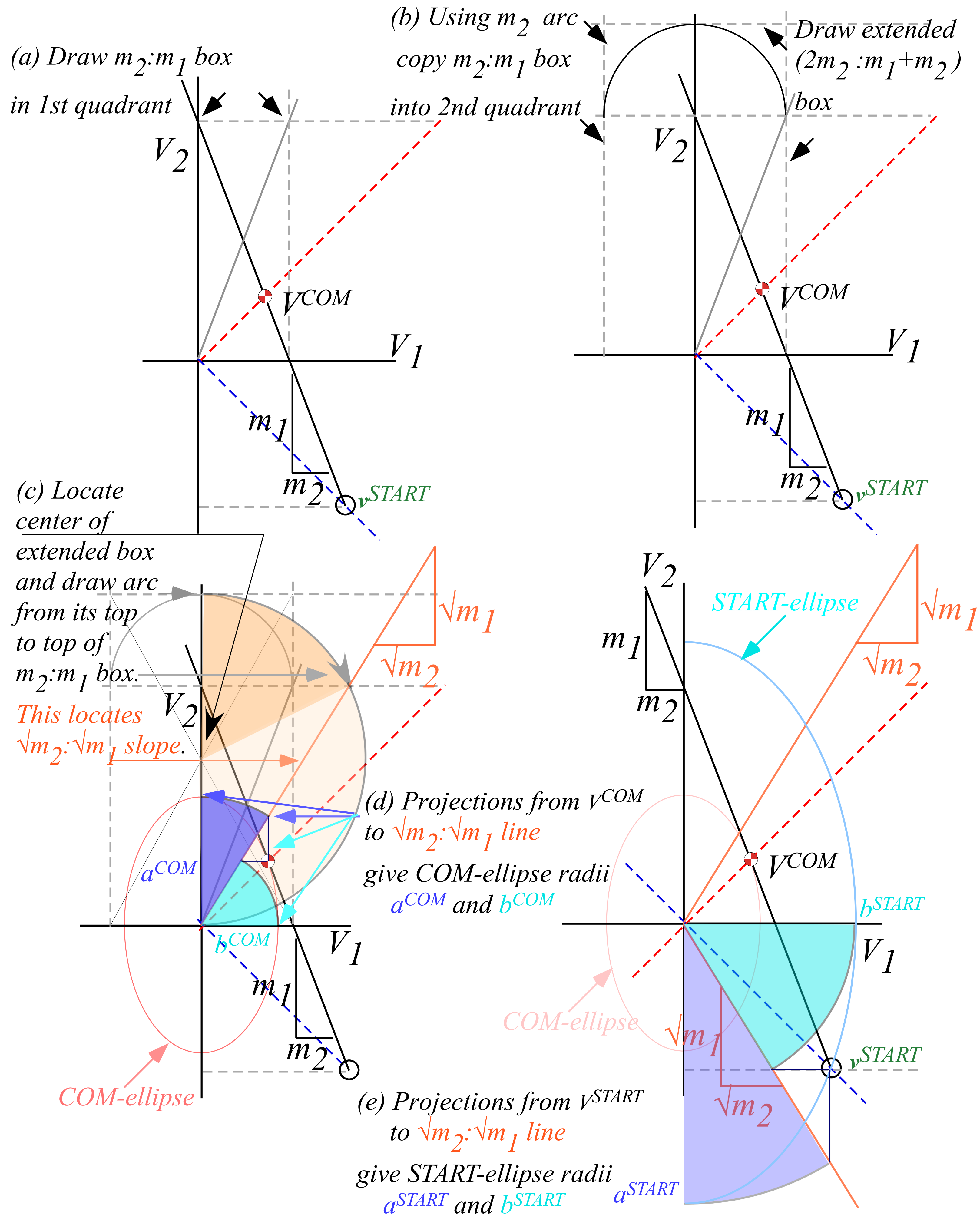


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)

Still, if you know a simpler construction, we'd like to hear about it!