2018 CMwBang! site

Class YouTube Channel

Lecture 3
Mon 8.27.2018

Analysis of 1D 2-Body Collisions: Reflection groups

(Ch. 2 to Ch. 4 of Unit 1)

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics in Ch.12

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$ and ($M_1=100$, $M_2=1$)

References and incidental interest items

AIP publications   AJP article on superball dynamics   AAPT summer reading

Review: Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)

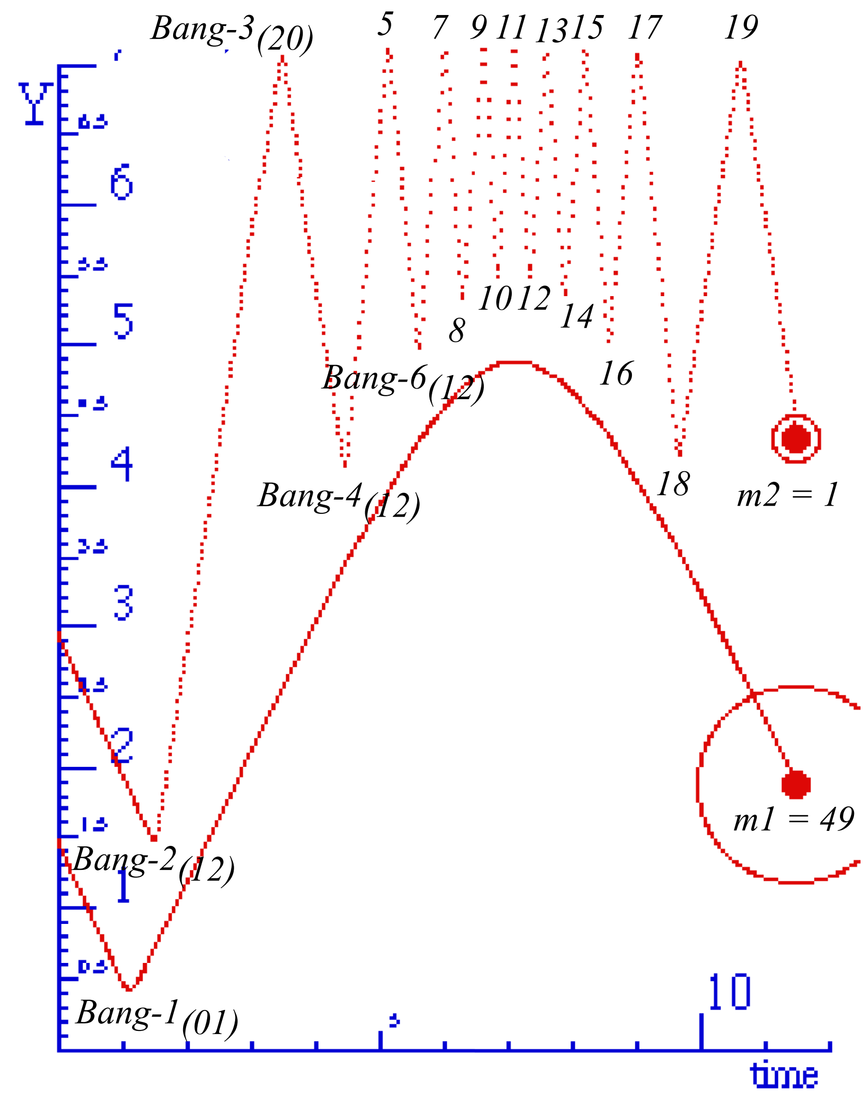
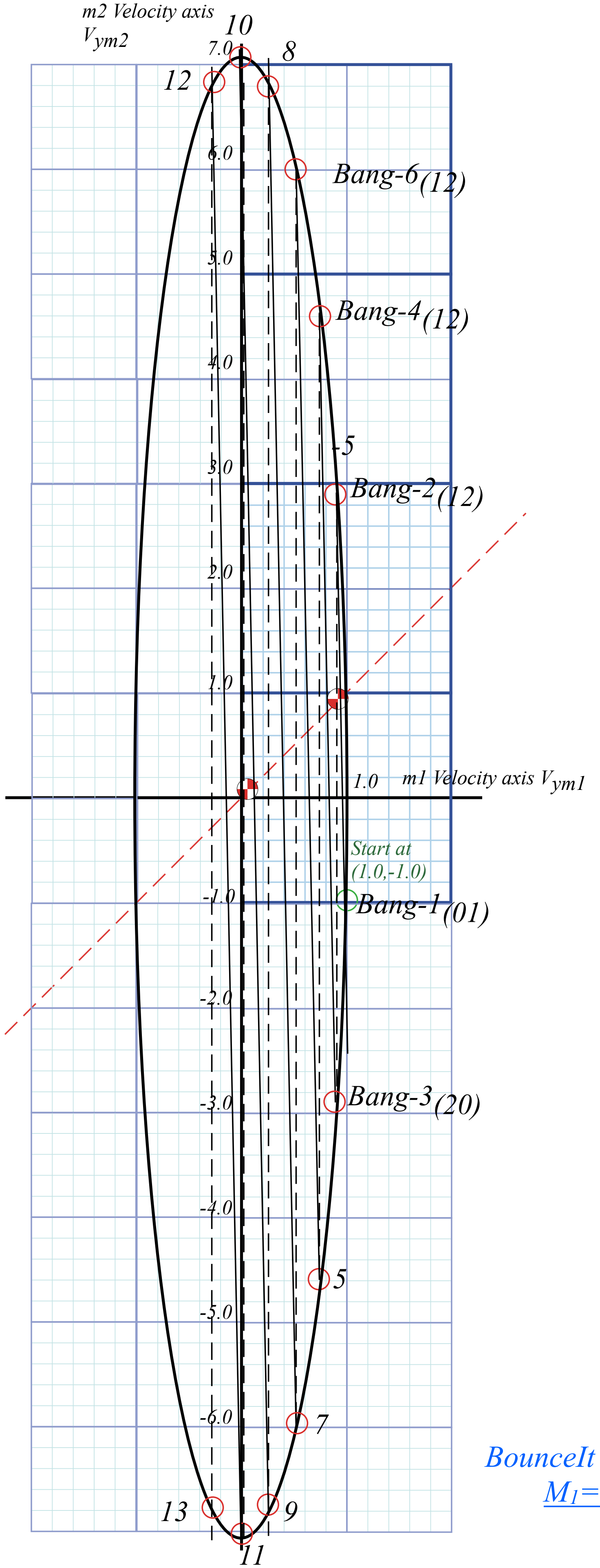


Fig. 5.1 in Unit 1

BounceIt Superball Collision Web Simulator: $M_1=49$, $M_2=1$ with Newtonian time plot

BounceIt Superball Collision Web Simulator: $M_1=49$, $M_2=1$ with V_2 vs V_1 plot

Review: Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)



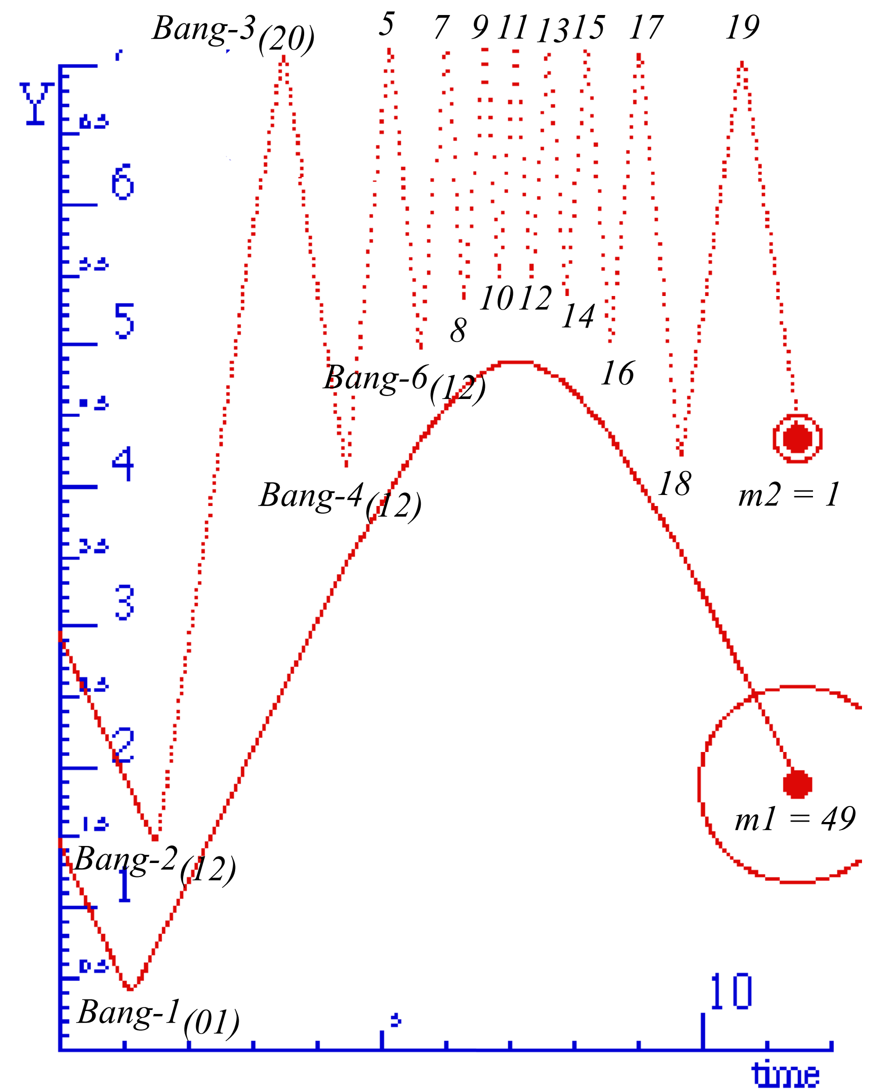


Fig. 5.1
in Unit 1

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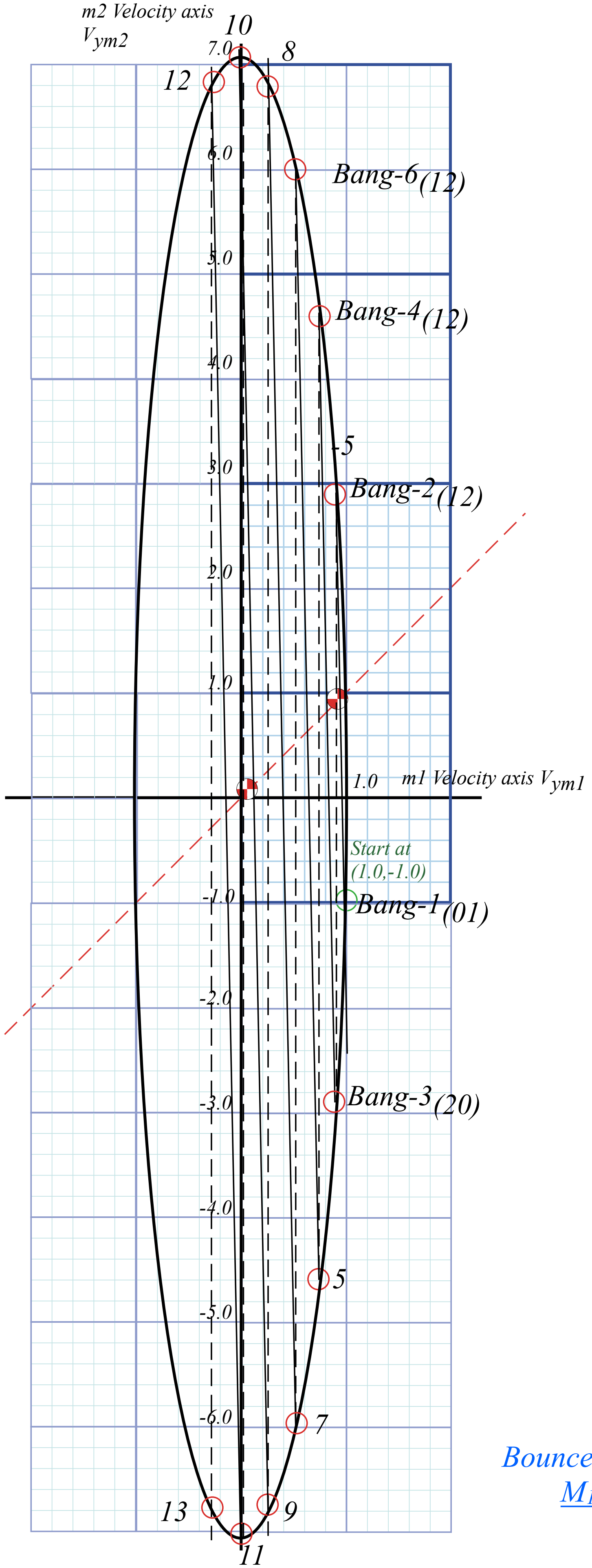
M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with V_2 vs V_1 plot

Review: *Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)*



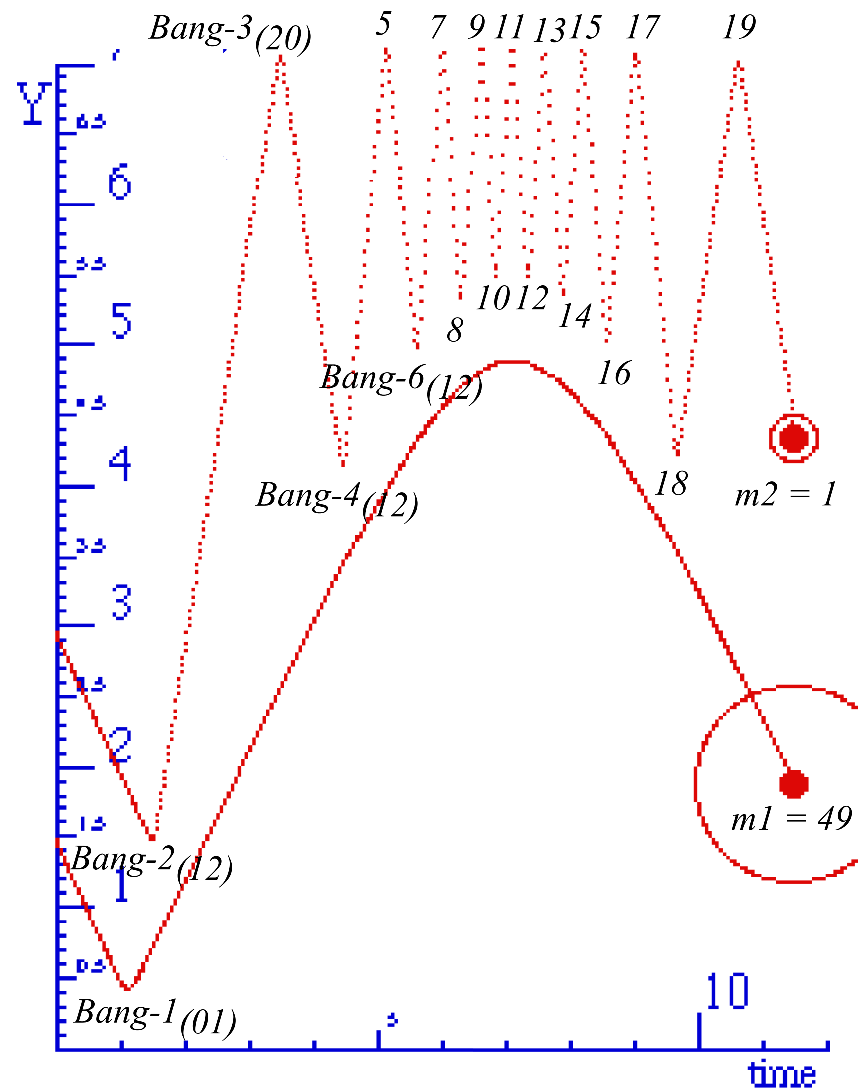


Fig. 5.1
in Unit 1

[BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with Newtonian time plot]

[BounceIt Superball Collision Web Simulator: M_1=49, M_2=1 with V_2 vs V_1 plot]

Kinetic Energy Ellipse

$$KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

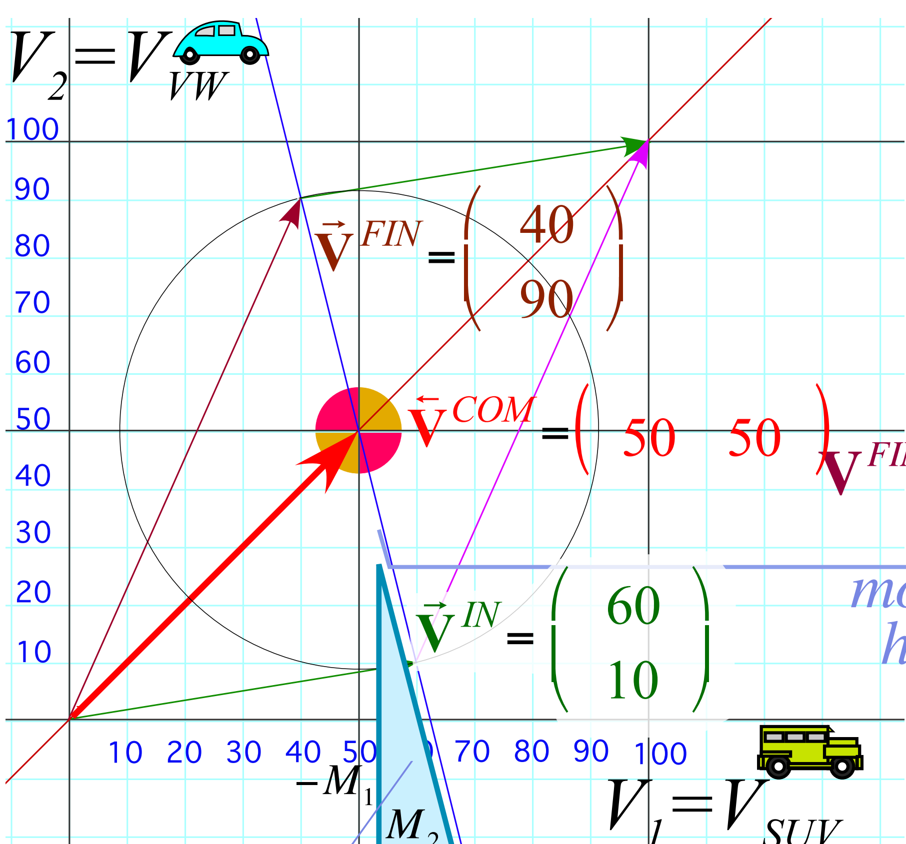
What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

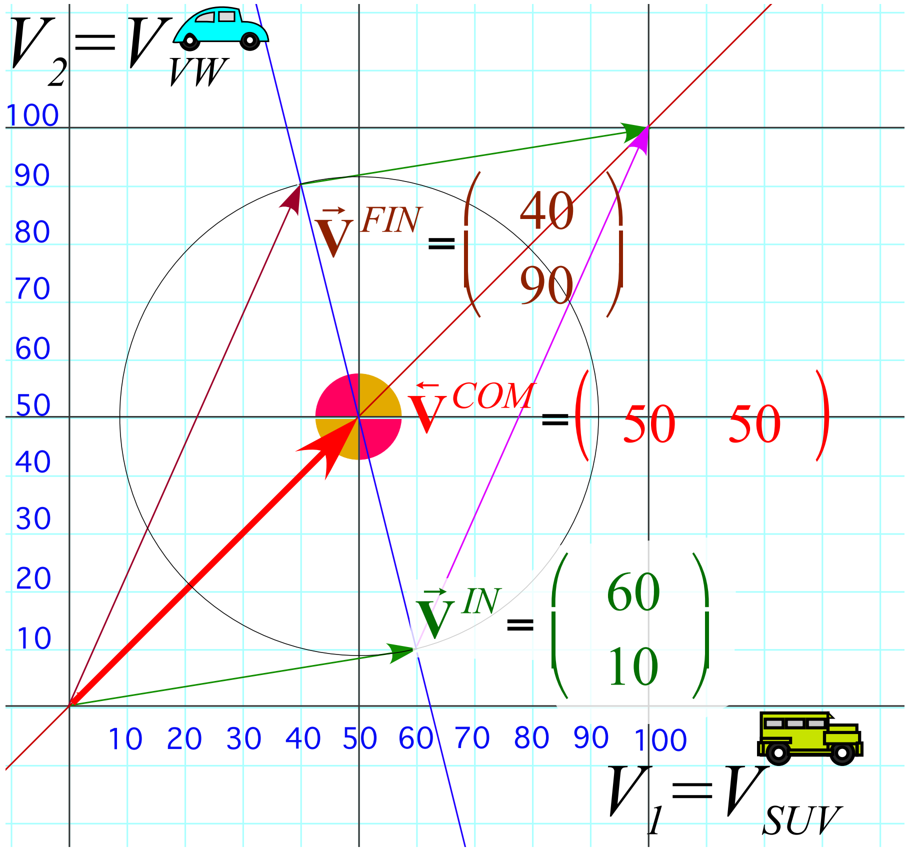


Points of vectors $\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$ all lie on *momentum-conservation line having slope* $-M_1/M_1$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$



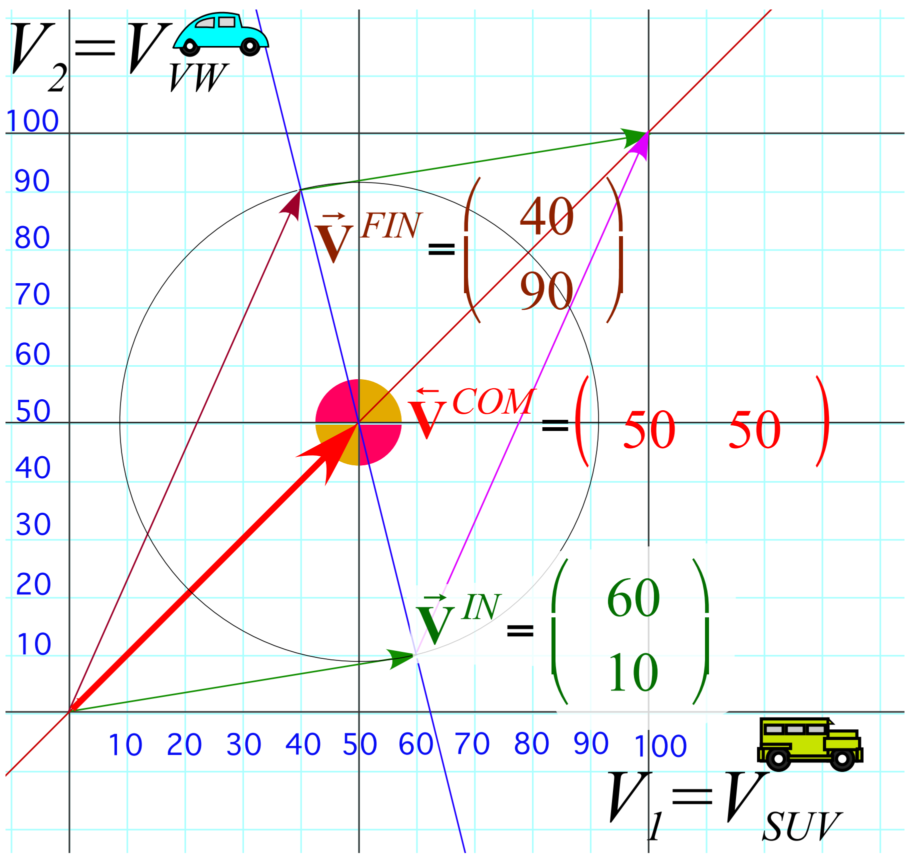
Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$



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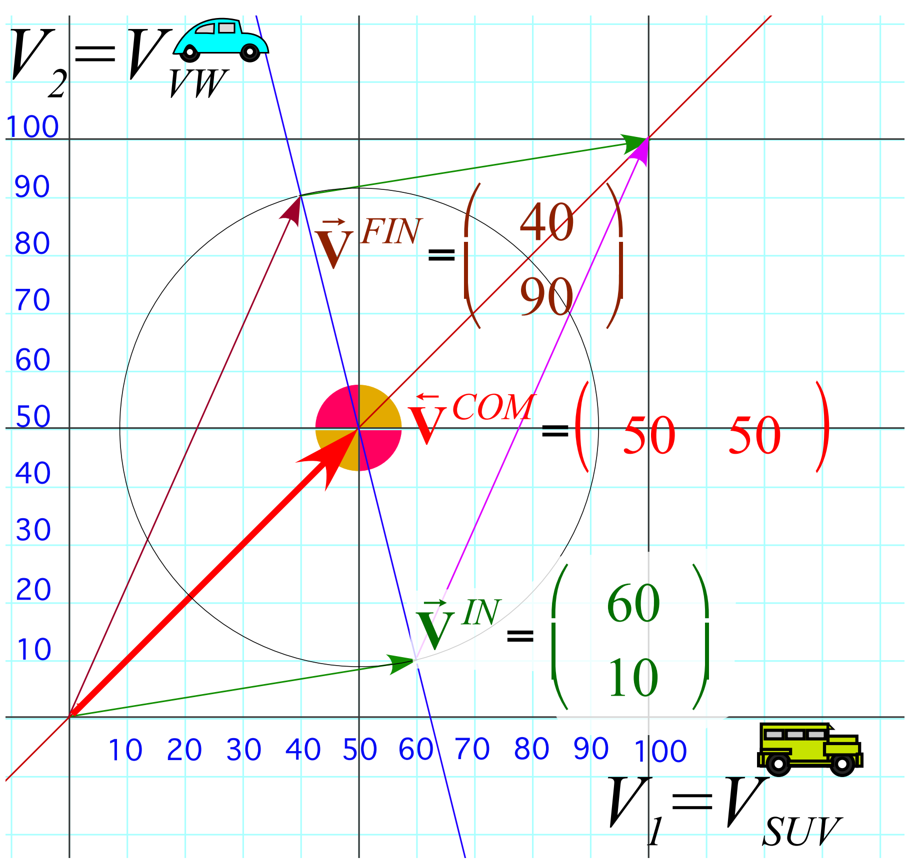
Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$



Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

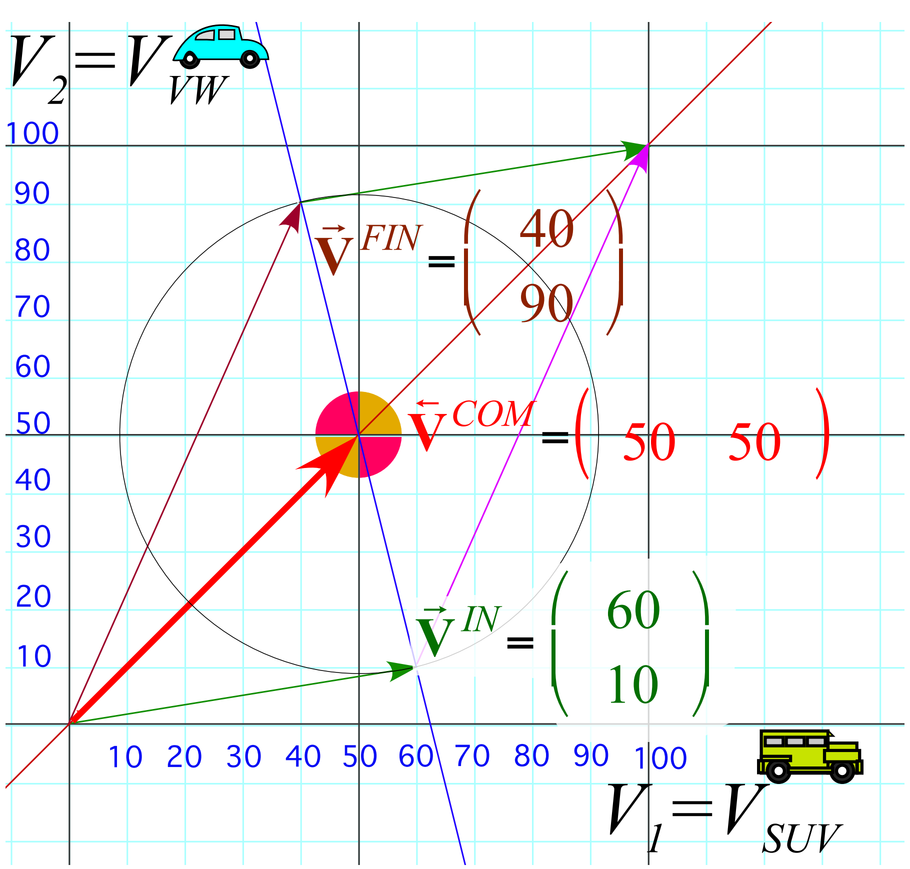
$$\mathbf{V}^{COM}=\frac{\mathbf{V}^{FIN}+\mathbf{V}^{IN}}{2}=\frac{m_1v_1+m_2v_2}{m_1+m_2}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=\tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}+\tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN}=\mathbf{M}\bullet\mathbf{v}^{IN}$...

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}=\begin{pmatrix}2V^{COM}-v_1^{IN}\\2V^{COM}-v_2^{IN}\end{pmatrix}=\begin{pmatrix}2\dfrac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_1^{IN}\\[2ex]2\dfrac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_2^{IN}\end{pmatrix}=\frac{\begin{pmatrix}m_1v_1^{IN}-m_2v_1^{IN}+2m_2v_2^{IN}\\2m_1v_1^{IN}+m_2v_2^{IN}-m_1v_2^{IN}\end{pmatrix}}{m_1+m_2}=\frac{\begin{pmatrix}m_1-m_2&2m_2\\2m_1&m_2-m_1\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}}{m_1+m_2}$$

$$=\begin{pmatrix}\dfrac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_1^{IN}-m_2v_1^{IN}}{m_1+m_2}\\[2ex]\dfrac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_2^{IN}-m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$



Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix} = \tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

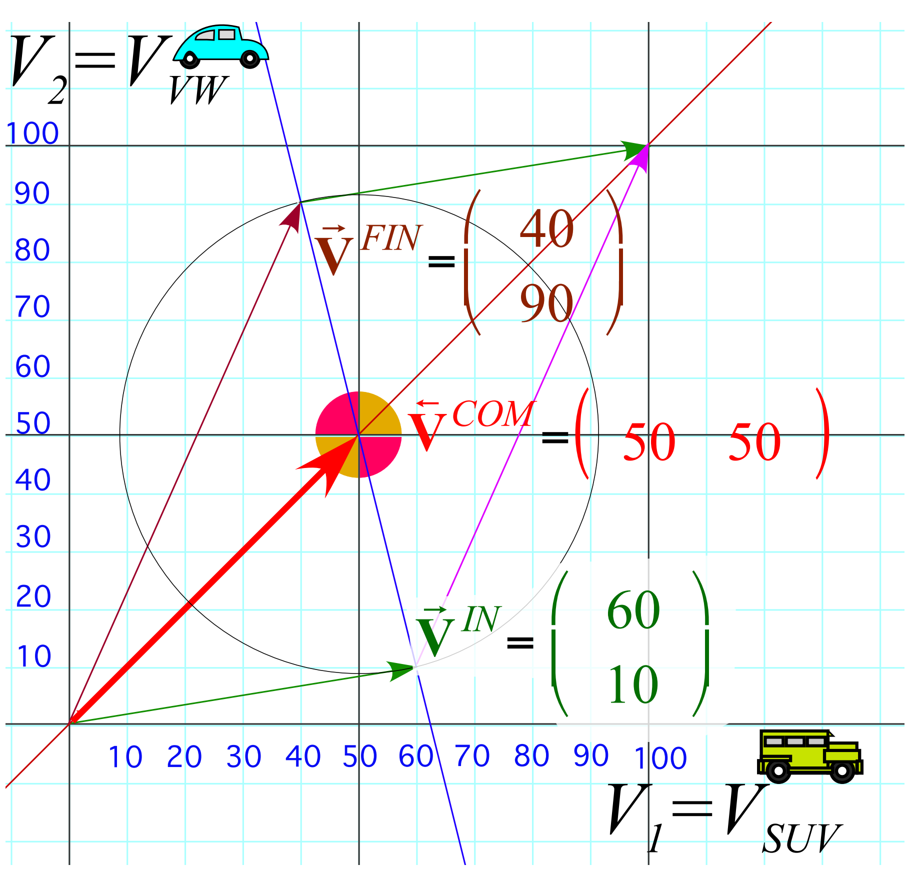
Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M}\bullet\mathbf{v}^{IN}$.

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} = \begin{pmatrix}2V^{COM} - v_1^{IN}\\2V^{COM} - v_2^{IN}\end{pmatrix} = \begin{pmatrix}2\dfrac{m_1v_1^{IN} + m_2v_2^{IN}}{m_1+m_2} - v_1^{IN}\\[2ex] 2\dfrac{m_1v_1^{IN} + m_2v_2^{IN}}{m_1+m_2} - v_2^{IN}\end{pmatrix} = \frac{\begin{pmatrix}m_1v_1^{IN} - m_2v_1^{IN} + 2m_2v_2^{IN}\\2m_1v_1^{IN} + m_2v_2^{IN} - m_1v_2^{IN}\end{pmatrix}}{m_1+m_2} = \frac{\begin{pmatrix}m_1 - m_2 & 2m_2\\2m_1 & m_2 - m_1\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}}{m_1+m_2}$$

$$= \begin{pmatrix}\dfrac{2m_1v_1^{IN} + 2m_2v_2^{IN} - m_1v_1^{IN} - m_2v_1^{IN}}{m_1+m_2}\\[2ex] \dfrac{2m_1v_1^{IN} + 2m_2v_2^{IN} - m_1v_2^{IN} - m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix} = \begin{pmatrix}\dfrac{m_1 - m_2}{m_1+m_2} & \dfrac{2m_2}{m_1+m_2}\\[2ex] \dfrac{2m_1}{m_1+m_2} & \dfrac{m_2 - m_1}{m_1+m_2}\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$



Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

→ *What about that 2nd quadratic solution?*

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

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Multiple Collisions by Matrix Operator Products

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$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

What about that 2nd quadratic solution?

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

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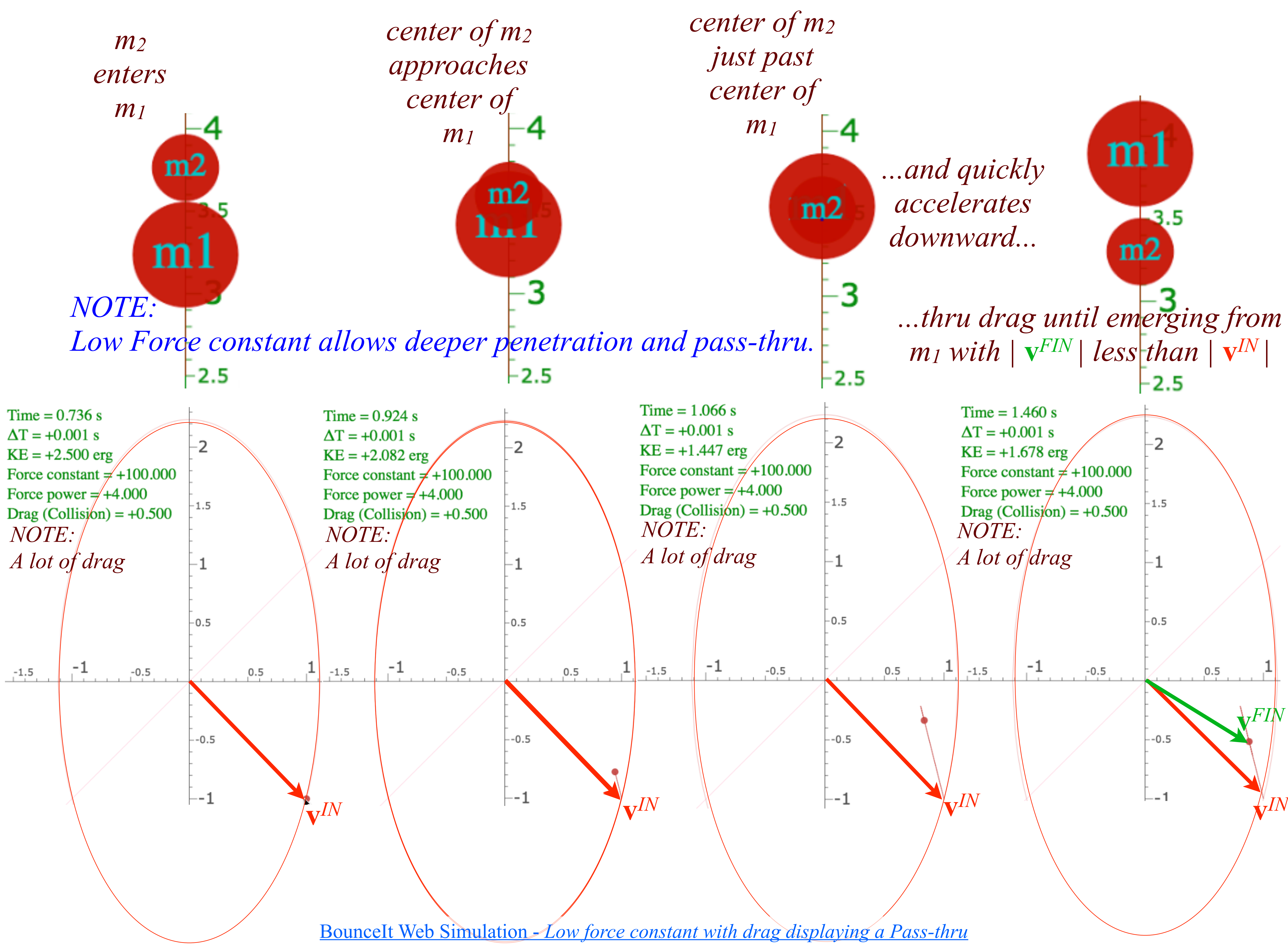
$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

What about that 2nd quadratic solution?

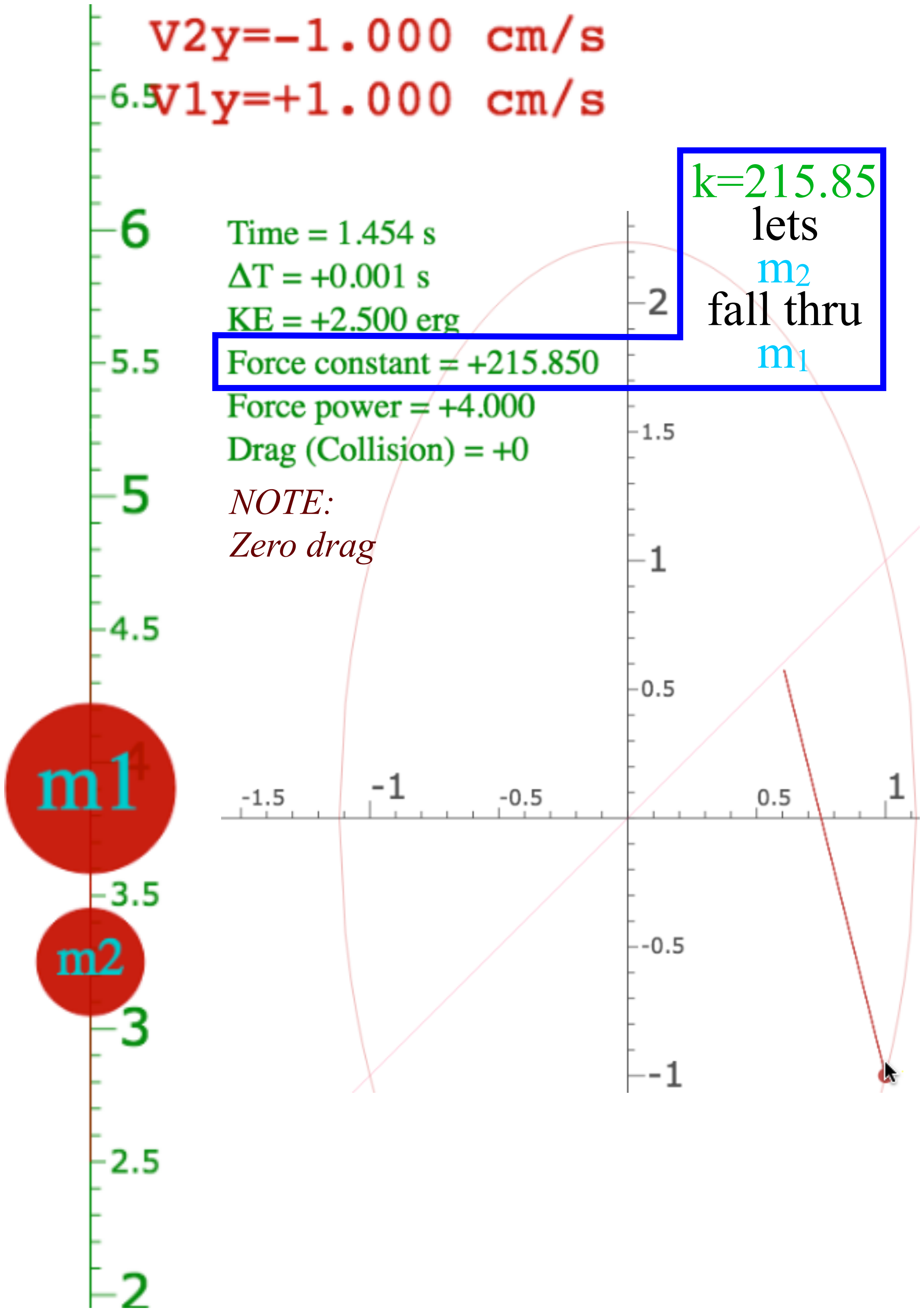
Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Q: What is the *second* solution and to what simple process would it correspond?

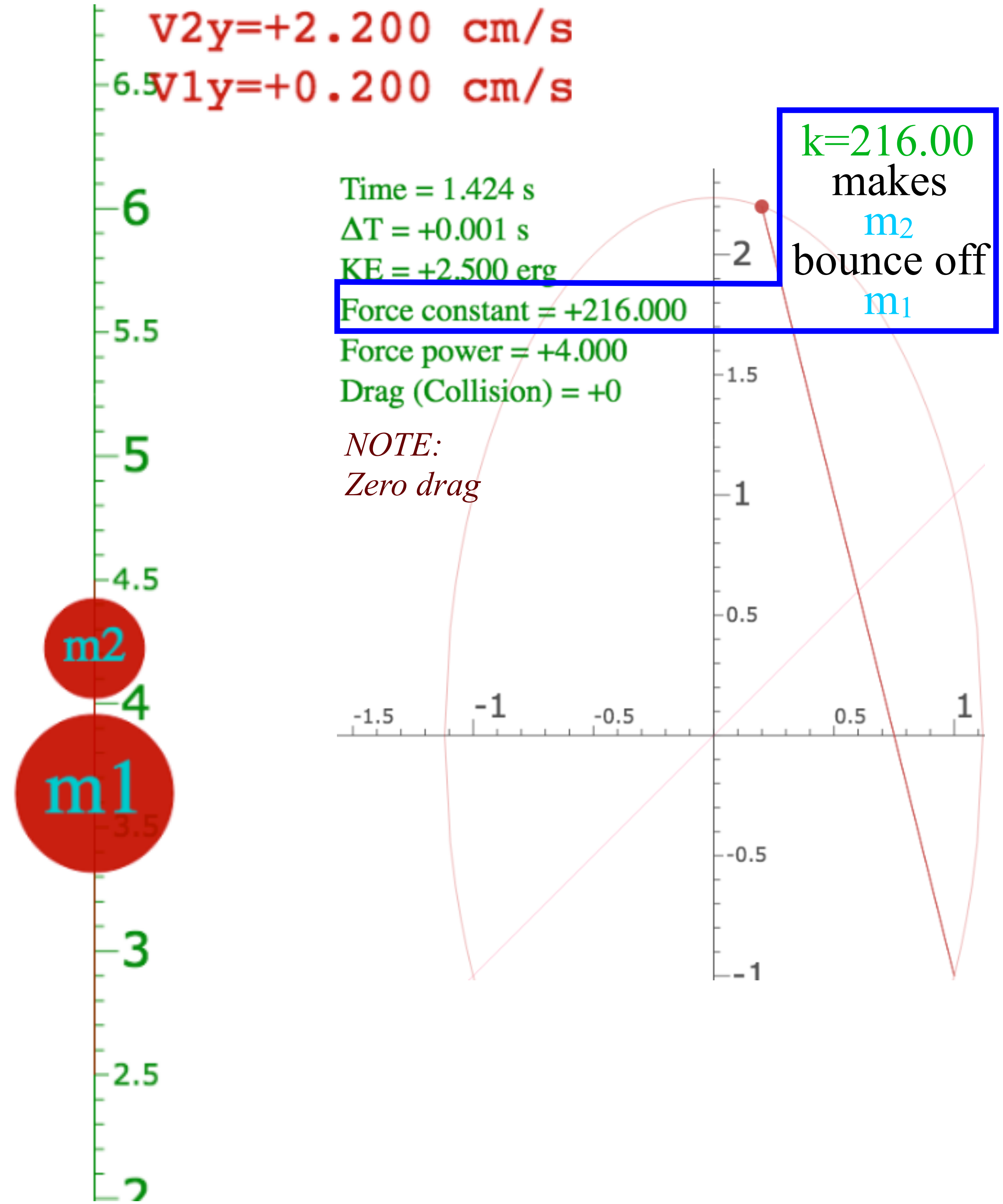
Example with friction



[BounceIt Web Simulation - *Low force constant with drag displaying a Pass-thru*]



[Fall-Thru]



[Bounce-Off]

BounceIt Superball Collision Web Simulations
with Low force constant & Zero drag

Multiple collisions calculated by matrix operator products

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Multiple Collisions by Matrix Operator Products

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$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$= \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

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Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1-m_2}{m_1+m_2} & \frac{2m_2}{m_1+m_2} \\ \frac{2m_1}{m_1+m_2} & \frac{m_2-m_1}{m_1+m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

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Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Review: Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M}\bullet\mathbf{v}^{IN}$.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Review: Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:
$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

$$\boxed{\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}$$

Matrix operations include...

Floor-bang **F** of m_1:
$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :
$$\mathbf{M} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:
$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ *and* $m_2=1$
$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define *"ellipse-Rotation"* **R** as *group product*: **R** = **C**•**M**
$$\mathbf{R} = \mathbf{C} \bullet \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$$

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} \, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

"ellipse-Rotation" group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \; |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R**= **C**•**M**

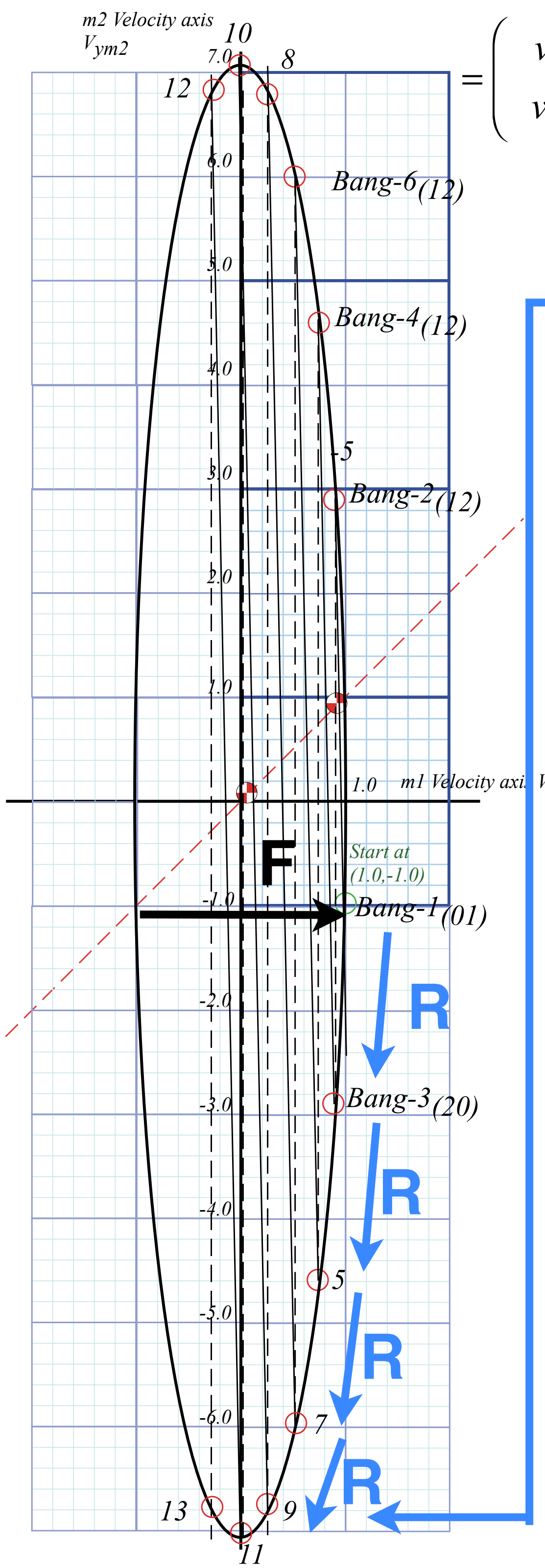
$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$



"ellipse-Rotation" group product: **R** = **C**•**M**

Collisions for mass ratio m_1:m_2= 49:1

Fig. 5.1a (revised)

$$\left|FIN^9\right\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\left|IN^0\right\rangle$$

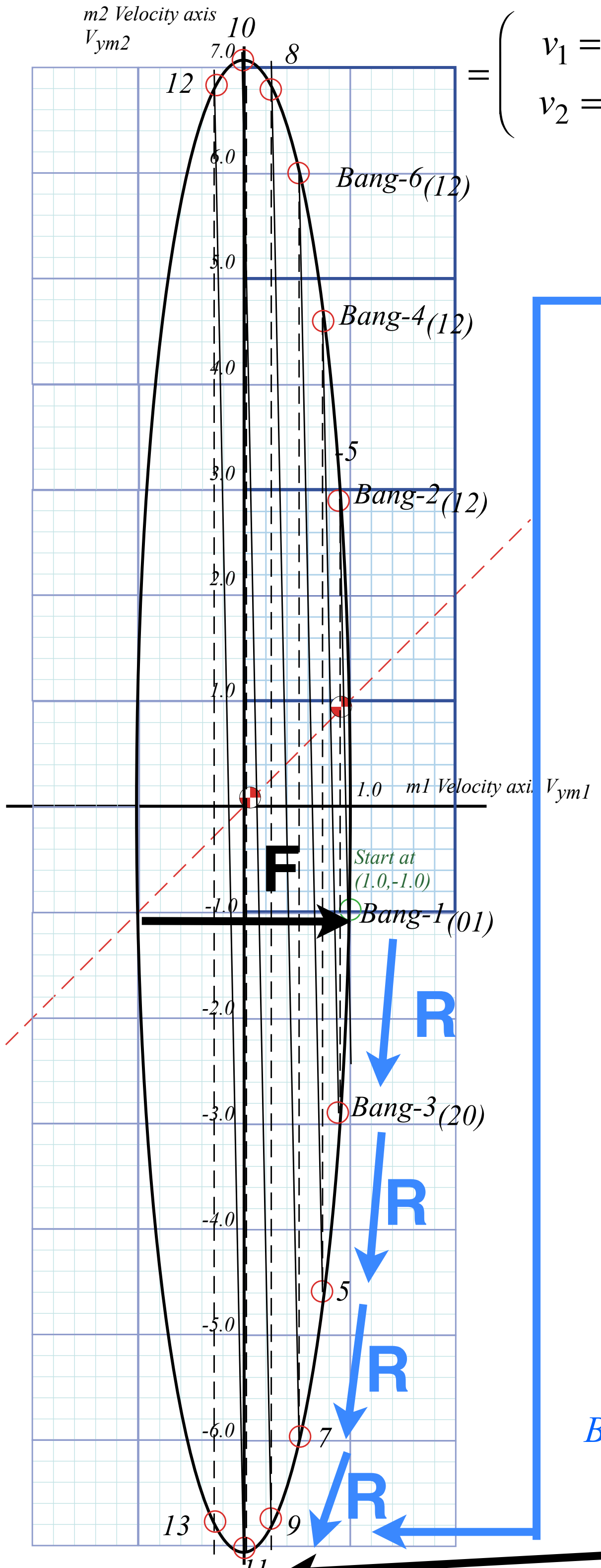
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{(\text{INITIAL }(0))}$$

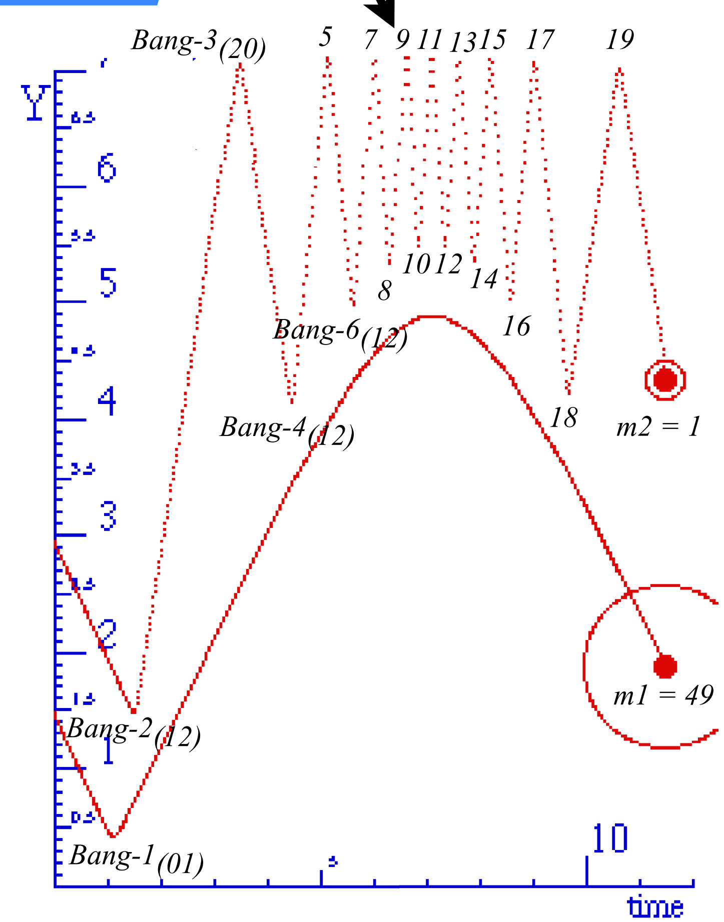
$$\left|FIN^9\right\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1=1 \\ v_2=-1 \end{pmatrix}_{(\text{after Bang-1})}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{(\text{after Bang-9})}$$

"ellipse-Rotation" group product: **R= C•M**





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{(\text{after Bang-11})}$$

Collisions for mass ratio $m_1:m_2 = 49:1$

<<Under Construction>>
Matrix Collision Web Simulator:
M_1=49, M_2=1 V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with Newtonian time plot

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with V_2 vs V_1 plot

Fig. 5.1a-b
(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

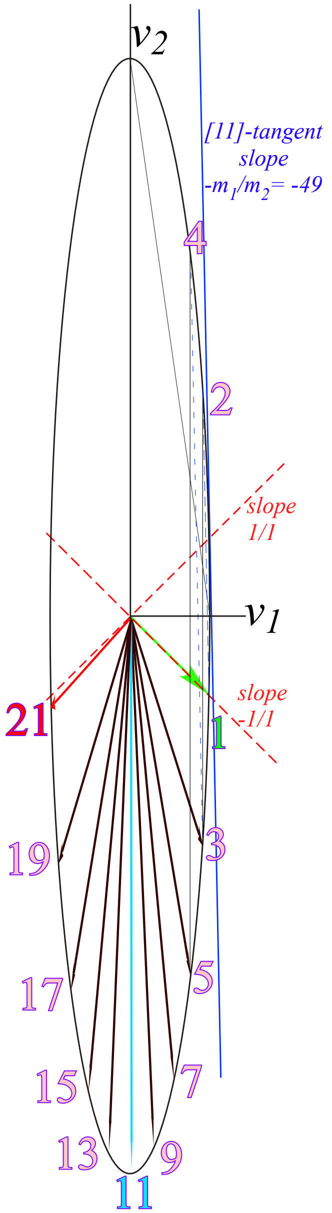
Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$



Collisions for mass ratio $m_1:m_2 = 49:1$

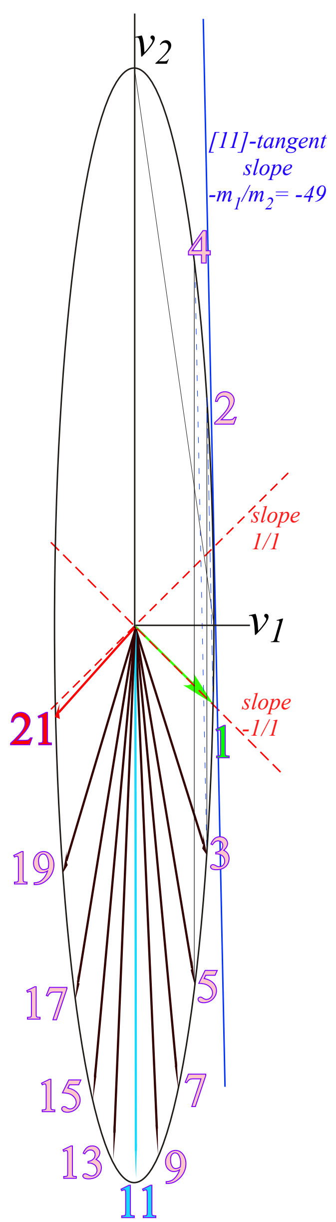
Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

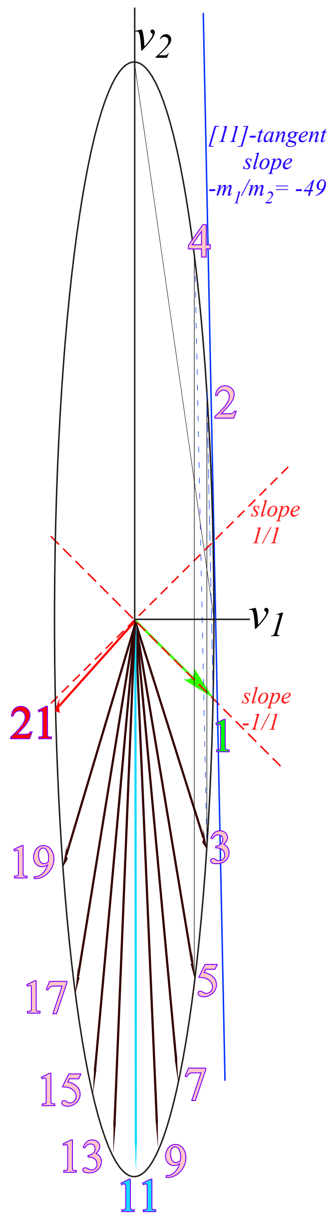
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

or:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{M} \bullet \vec{\mathsf{V}}$$

, or:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{\mathsf{V}}$$



Collisions for mass ratio $m_1 : m_2 = 49:1$

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

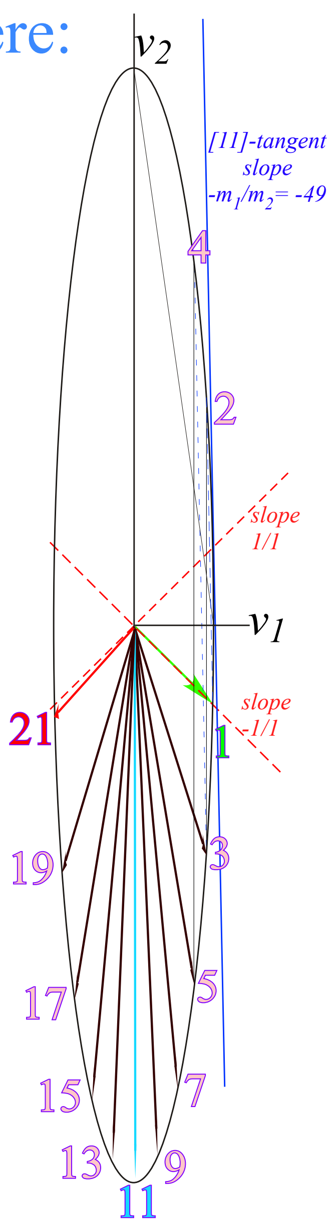
$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

or: $$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = M \bullet \vec{V}$$,

or: $$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = C \bullet M \bullet \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

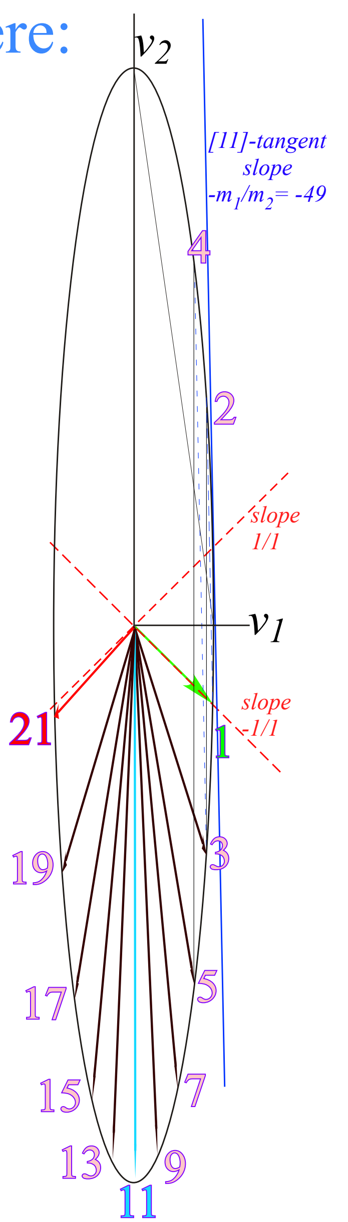
becomes:

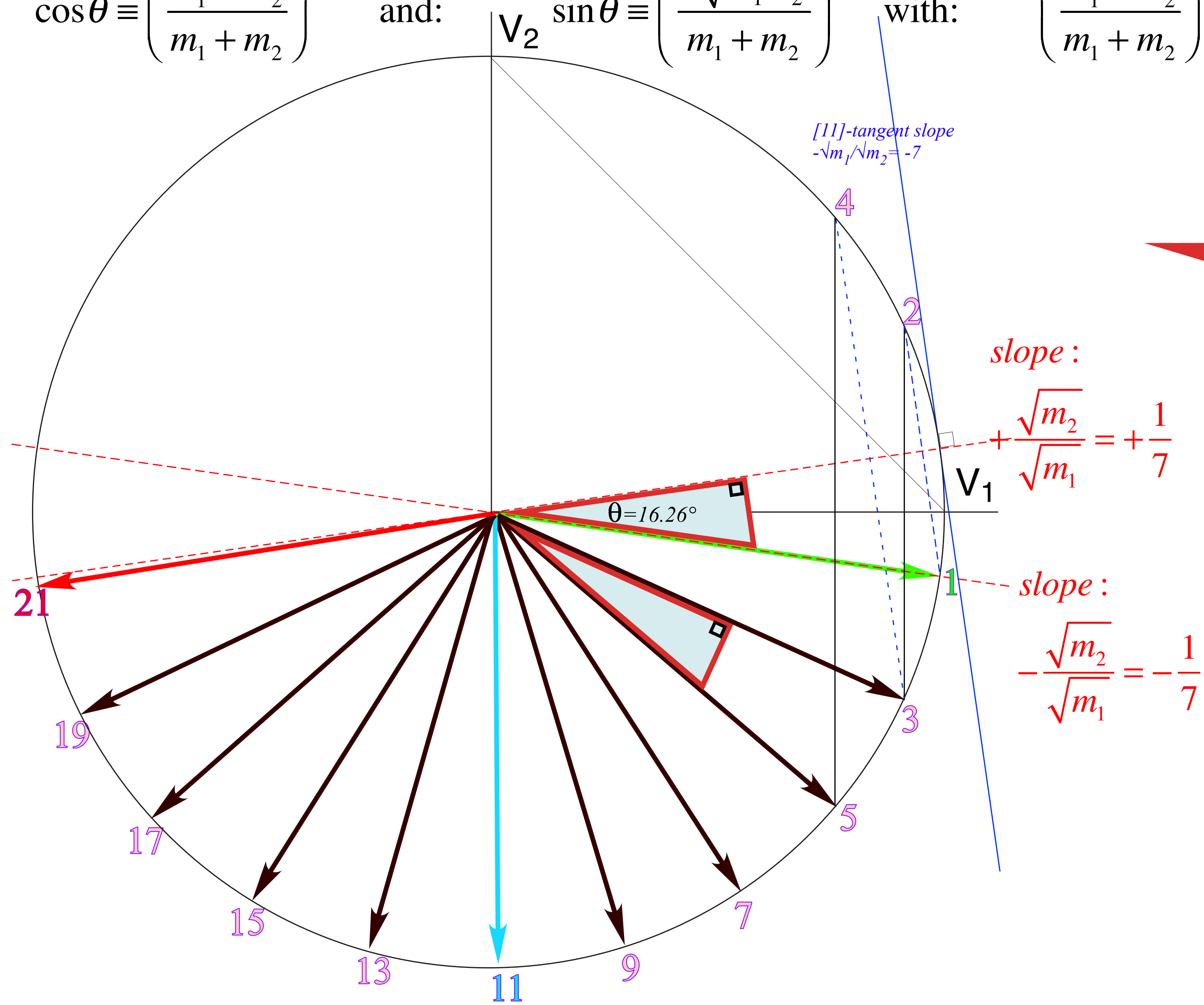
$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

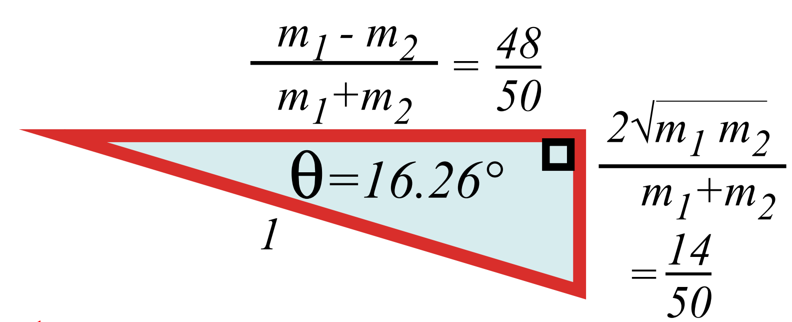
or: $\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \bullet \vec{V}$, or: $\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{V}$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 5.2a-c

(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

or:

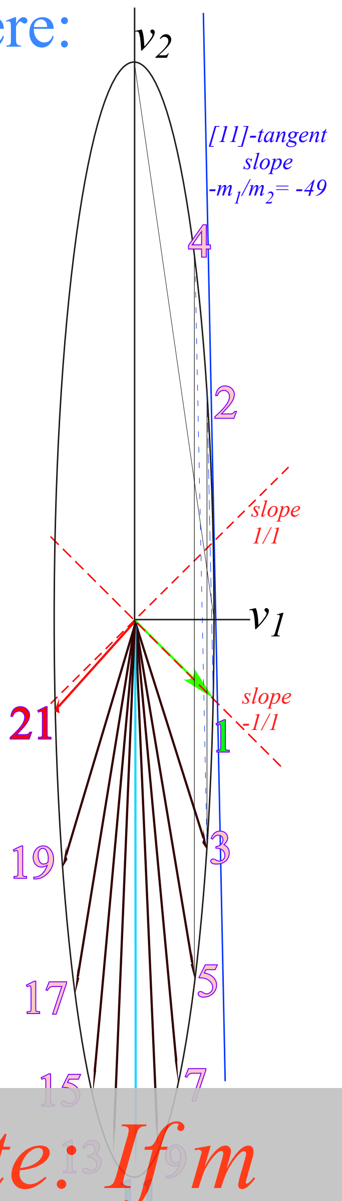
$$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V}$$

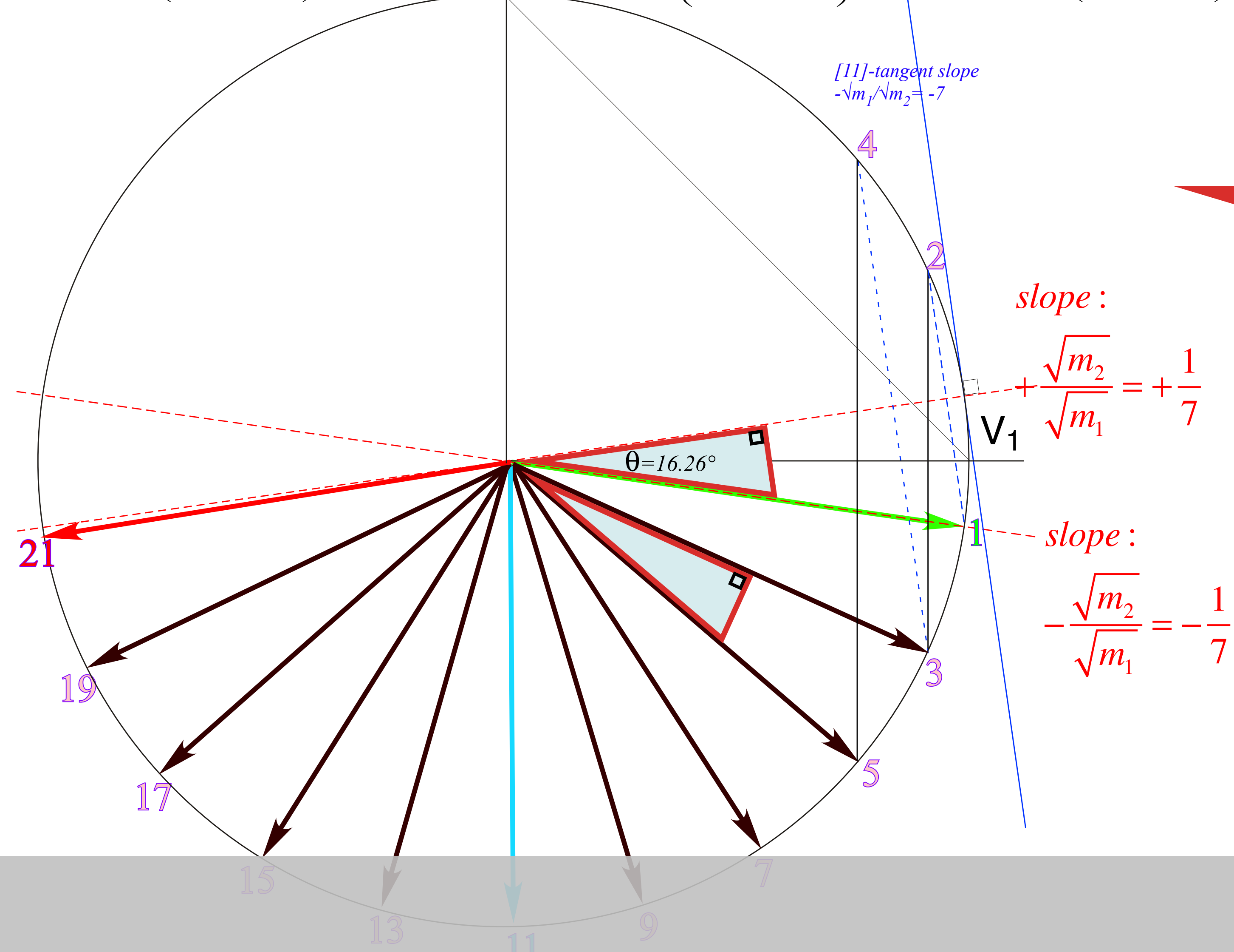
or:

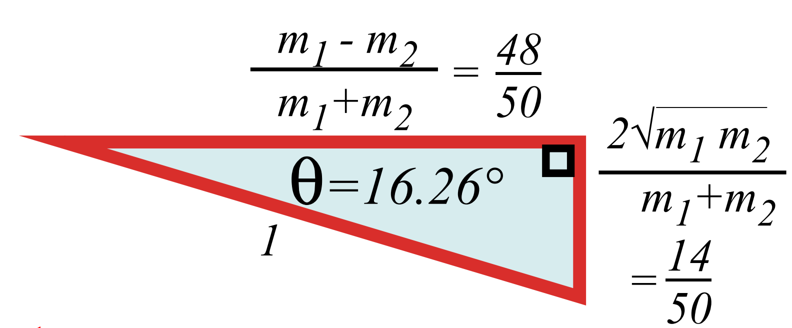
$$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$







Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

Fig. 5.2a-c

(revised)

Note: If m

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

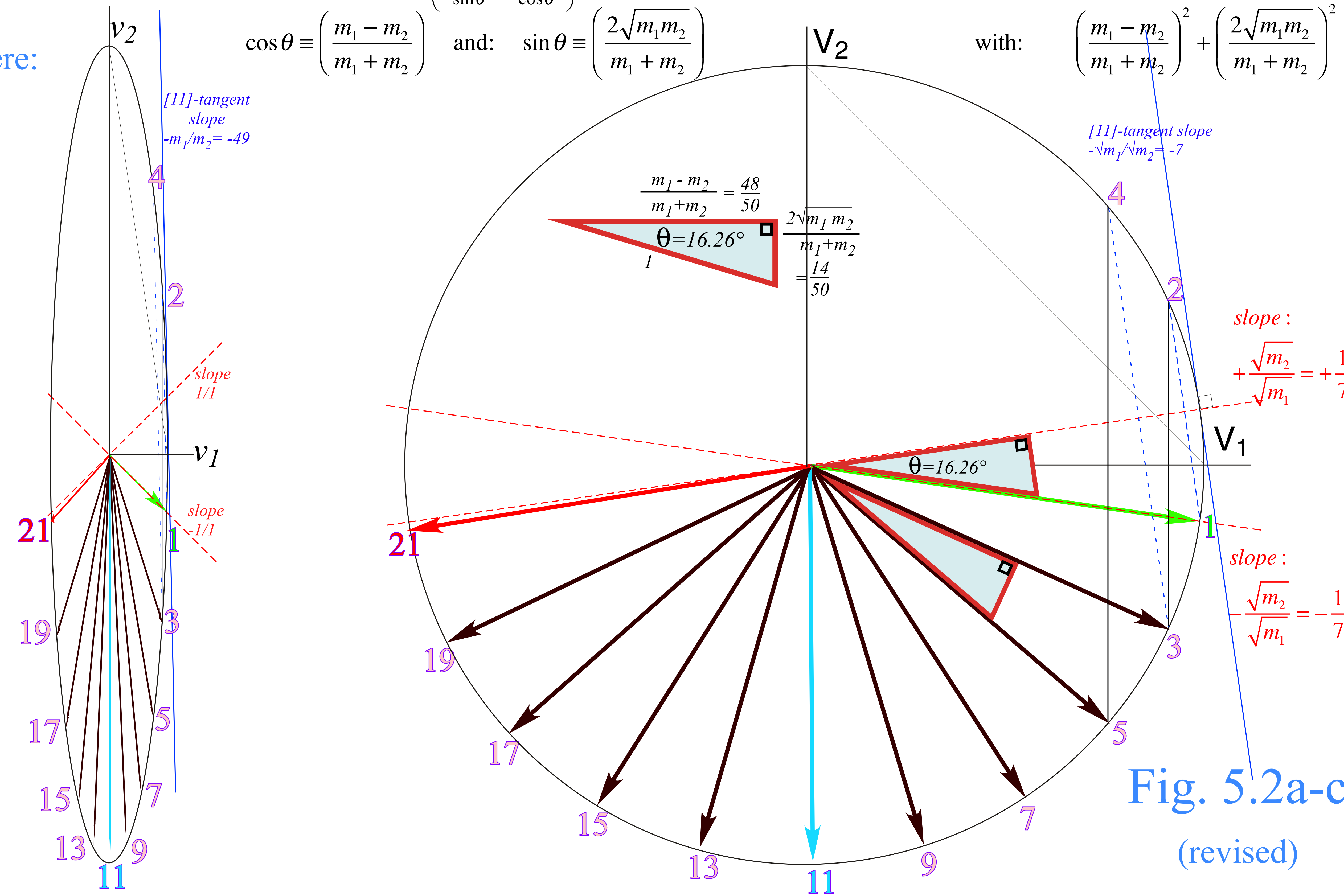


Fig. 5.2a-c

(revised)

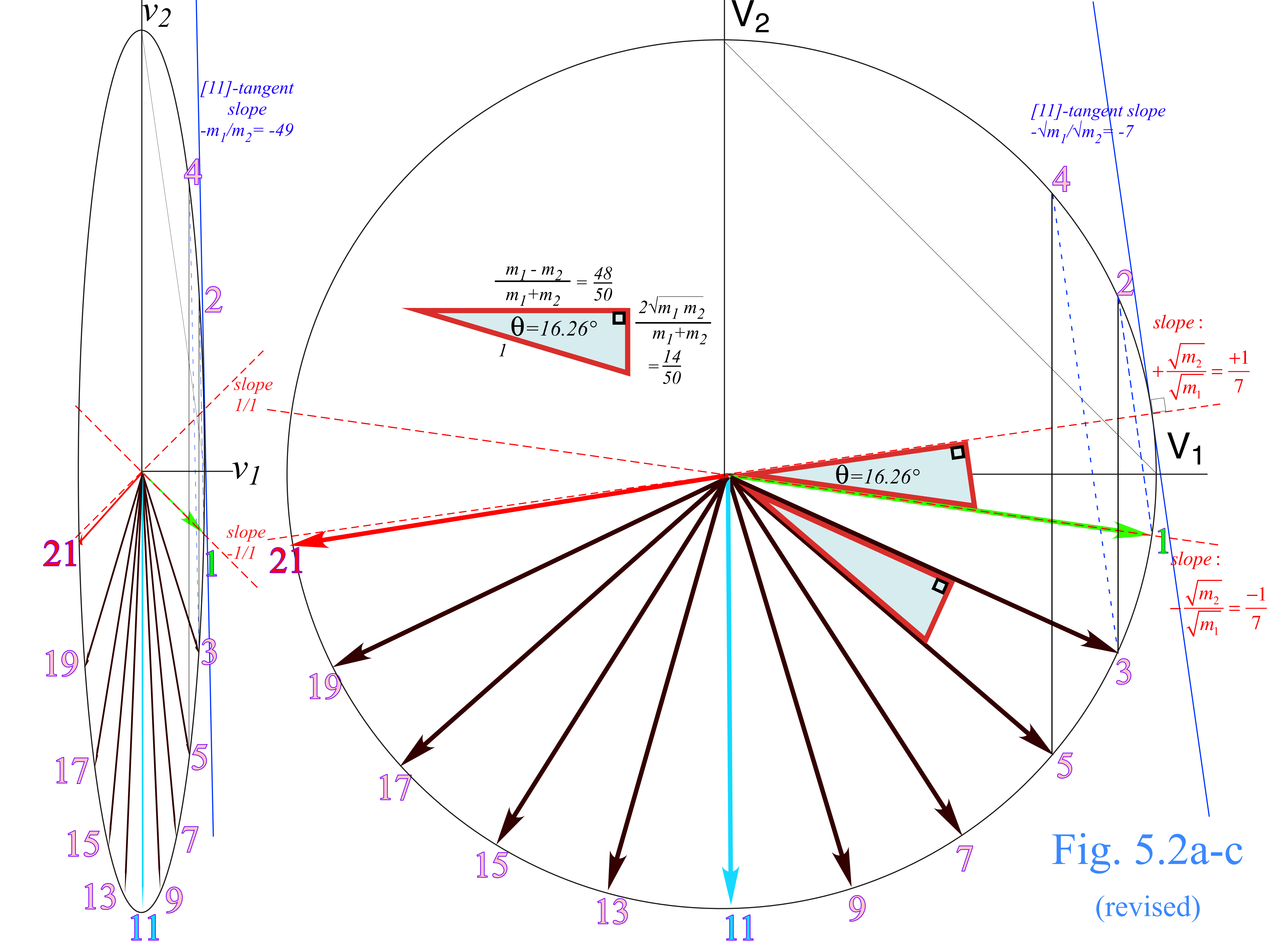


Fig. 5.2a-c

(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

→ *How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on*

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry D_6~C_{6v} (Resulting for $m_1/m_2=3$)

Group multiplication and product table

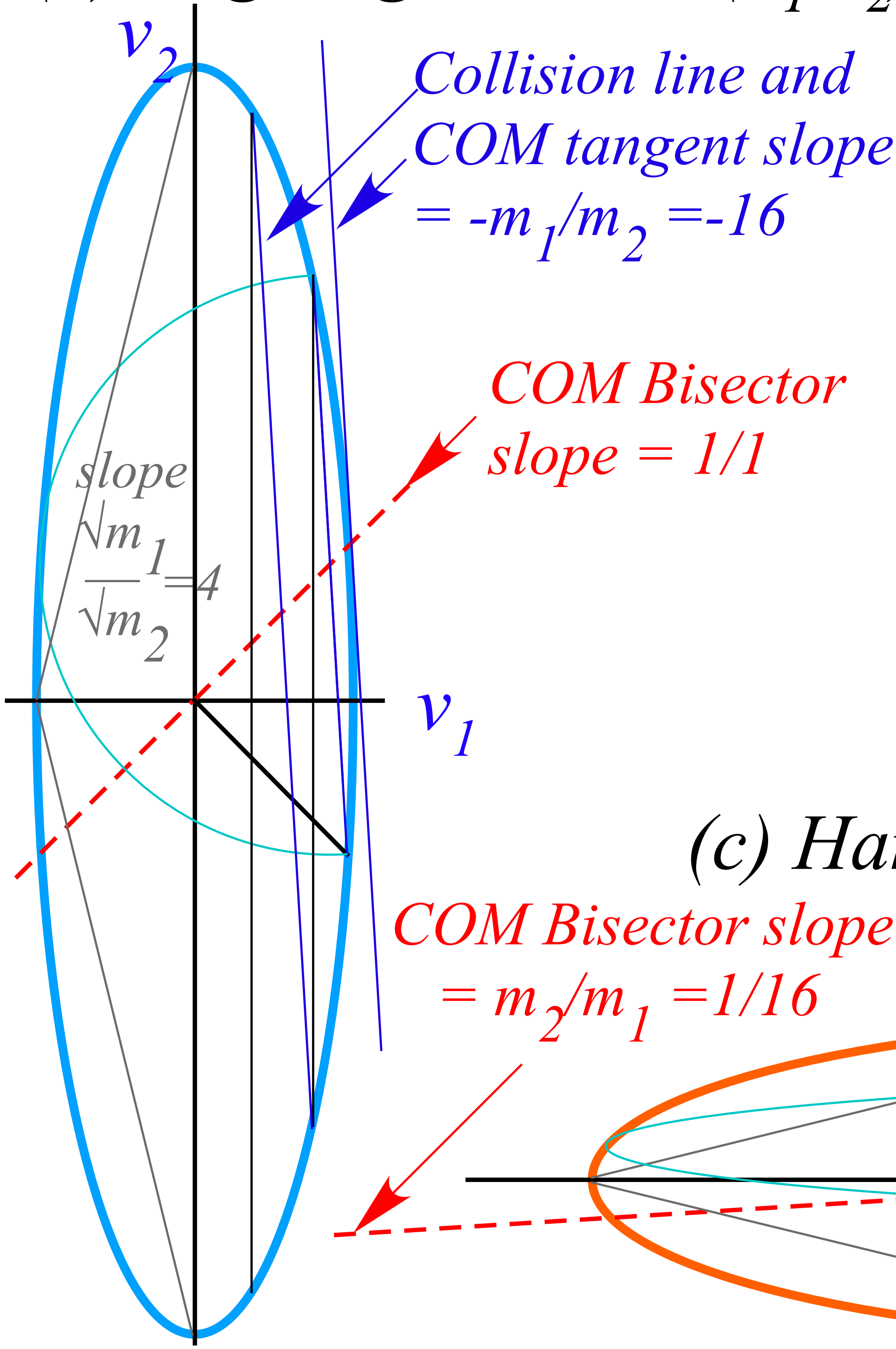
Classical collision paths with D_6~C_{6v} (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

What ellipse rescaling leads to...

How this relates to Lagrangian, and Hamiltonian mechanics later on (Ch.12 of Unit 1)

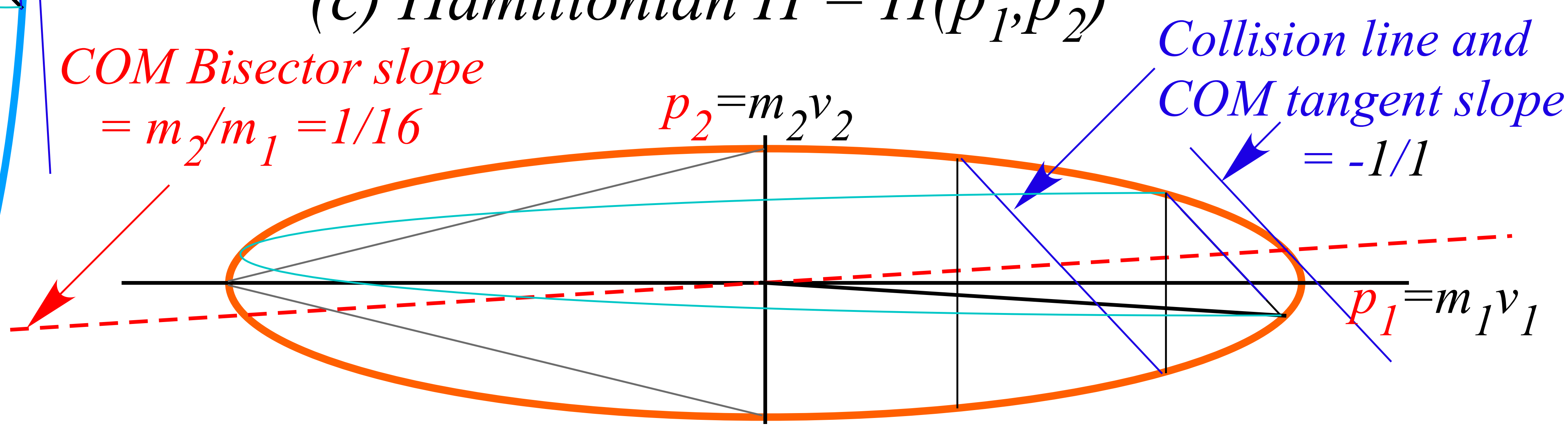
(a) Lagrangian $L = L(v_1,v_2)$



velocity v_1 rescaled to momentum: $p_1 = m_1 v_1$

velocity v_2 rescaled to momentum: $p_2 = m_2 v_2$

(c) Hamiltonian $H = H(p_1,p_2)$

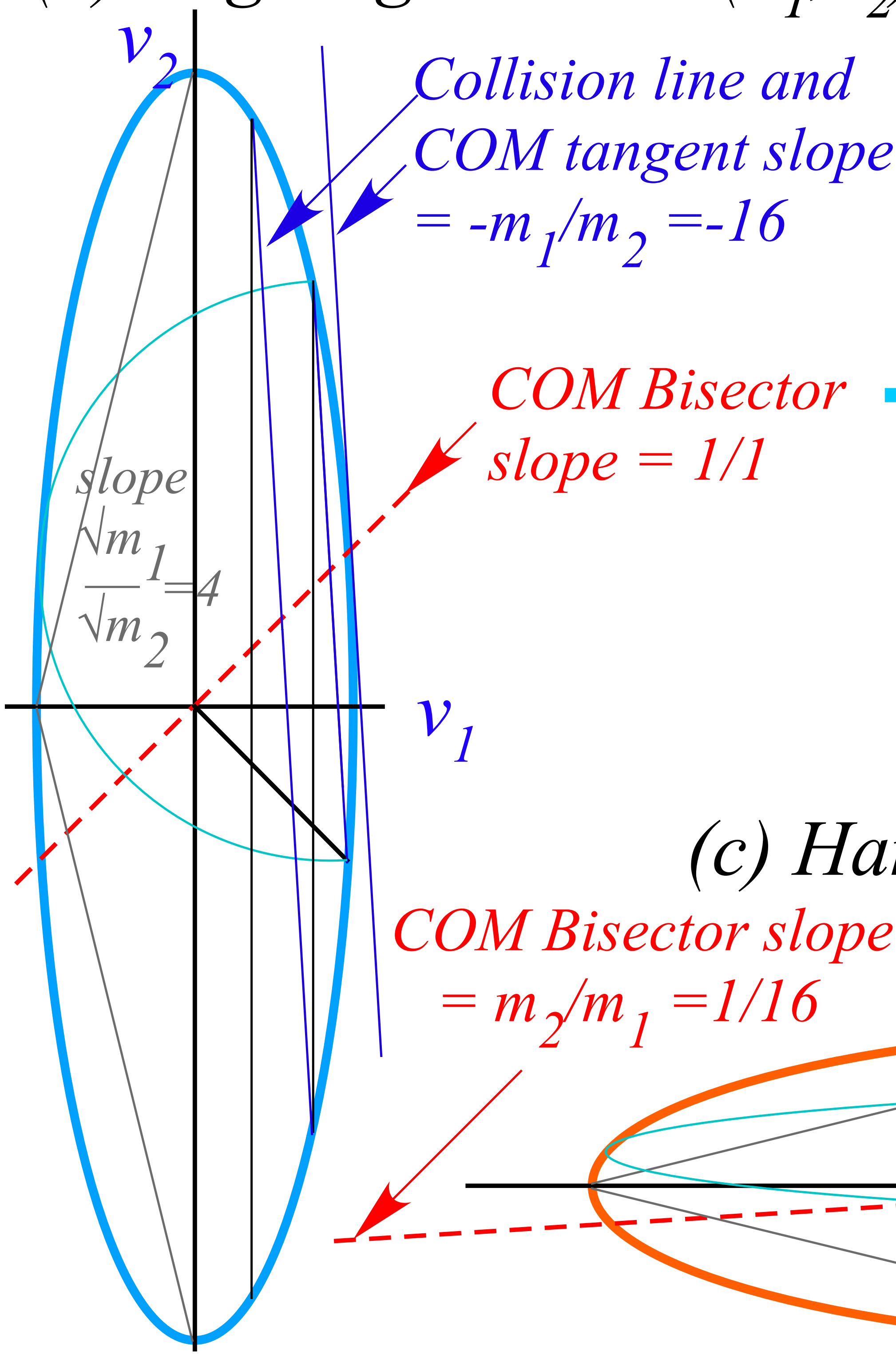


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on (Ch.12 of Unit 1)

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

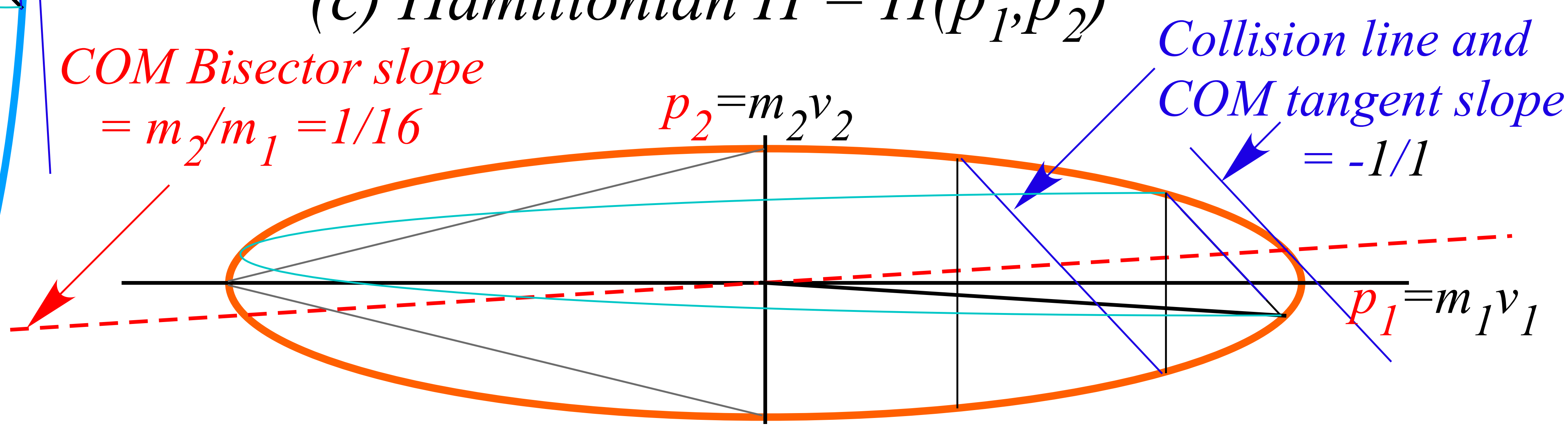
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^{\ 2} + \frac{1}{2} m_2 v_2^{\ 2}$

rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^{\ 2}}{2m_1} + \frac{p_2^{\ 2}}{2m_2}$

(c) Hamiltonian $H = H(p_1, p_2)$

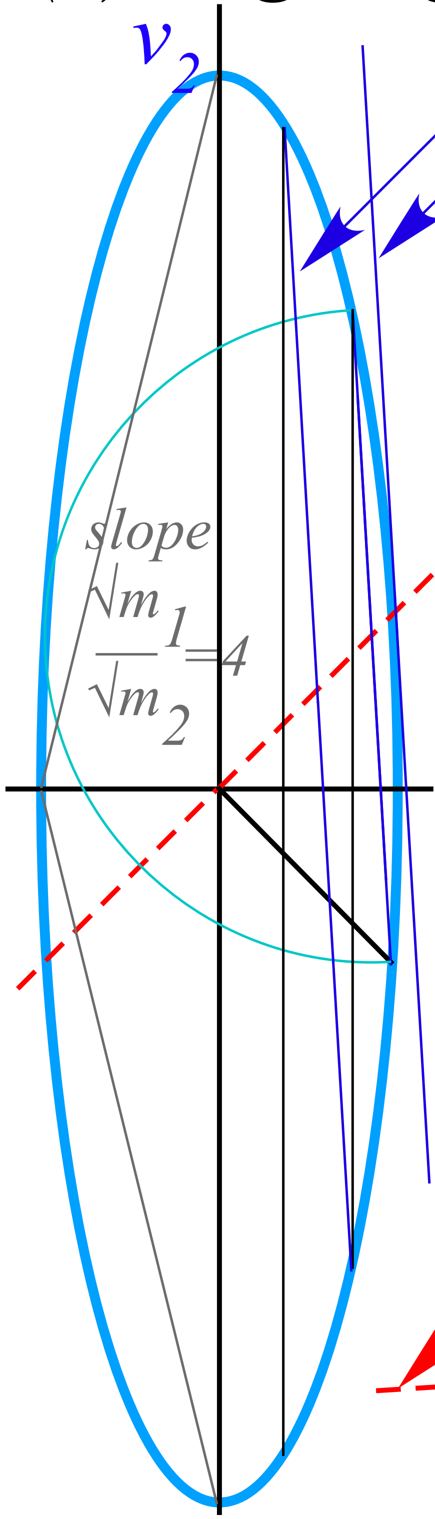


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on (Ch.12 of Unit 1)

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

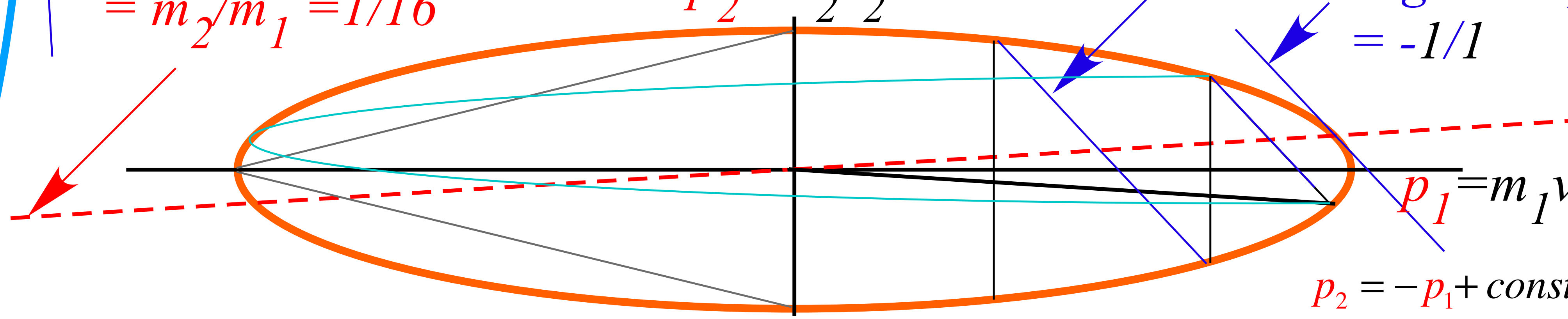
Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

Total momentum is conserved : $constant = p_1 + p_2$

(c) Hamiltonian $H = H(p_1, p_2)$

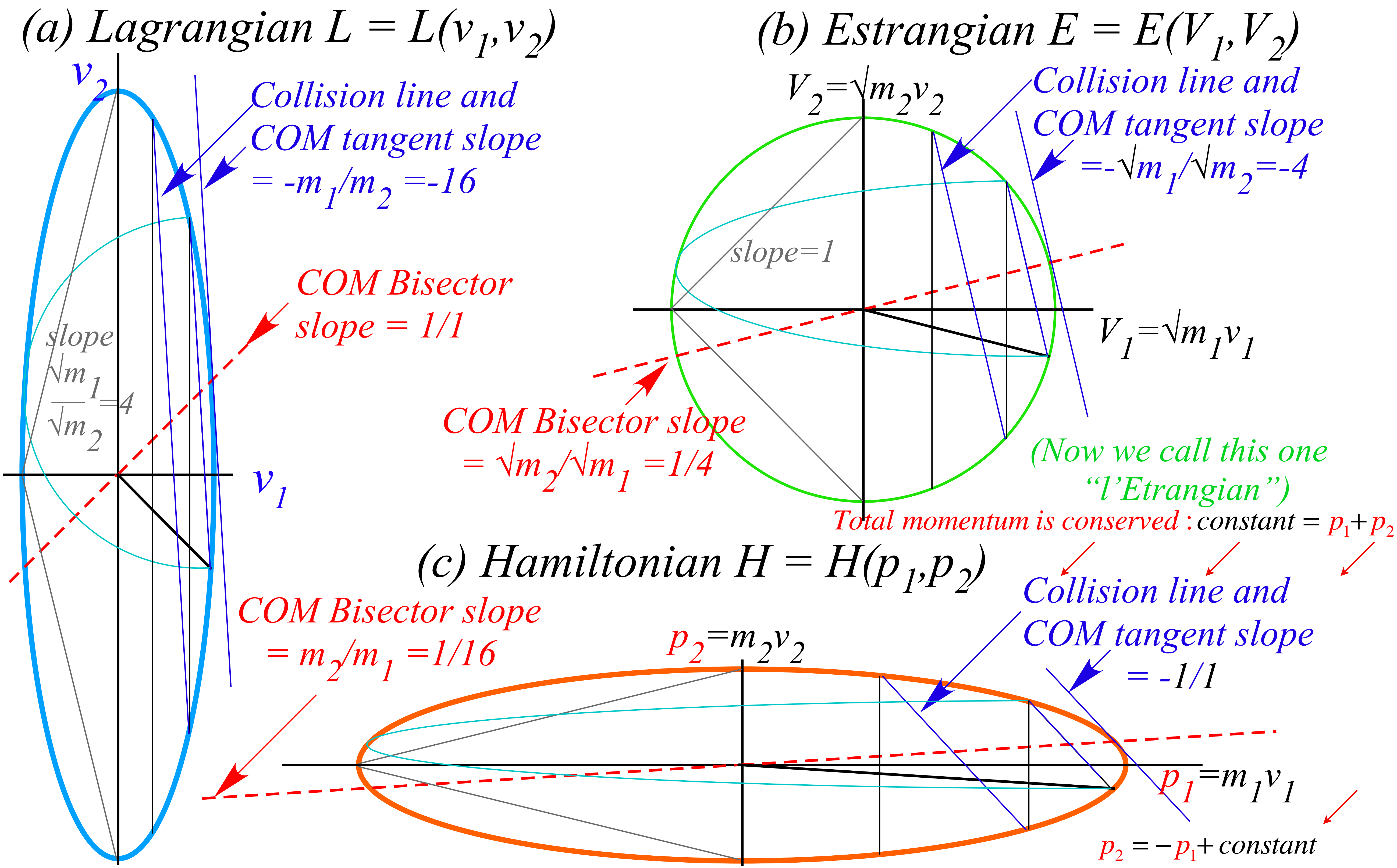


$p_2 = -p_1 + constant$

What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on (Ch.12 of Unit 1)



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

→ *Reflections in the clothing store: "It's all done with mirrors!"*

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

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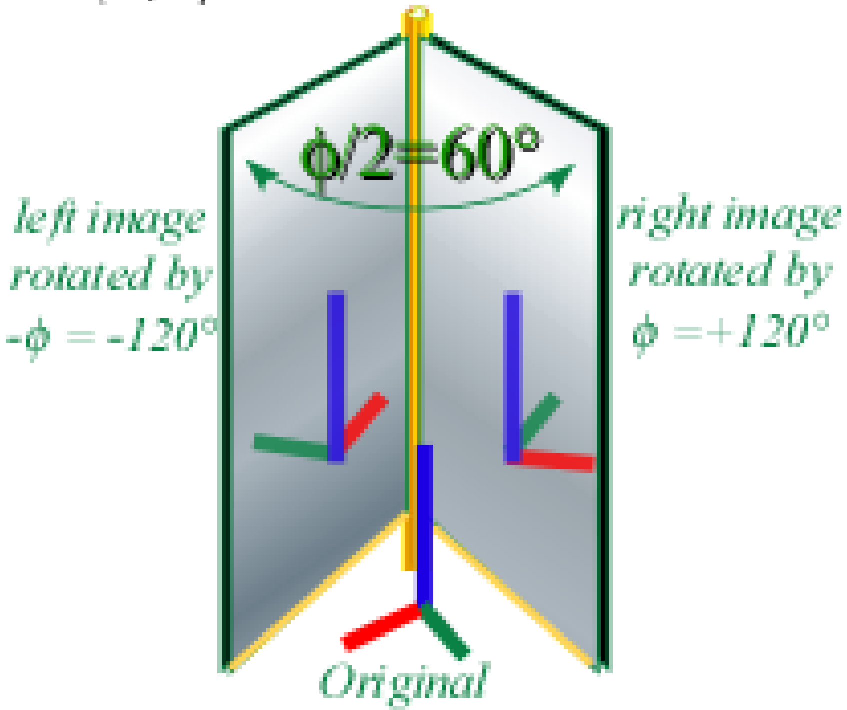
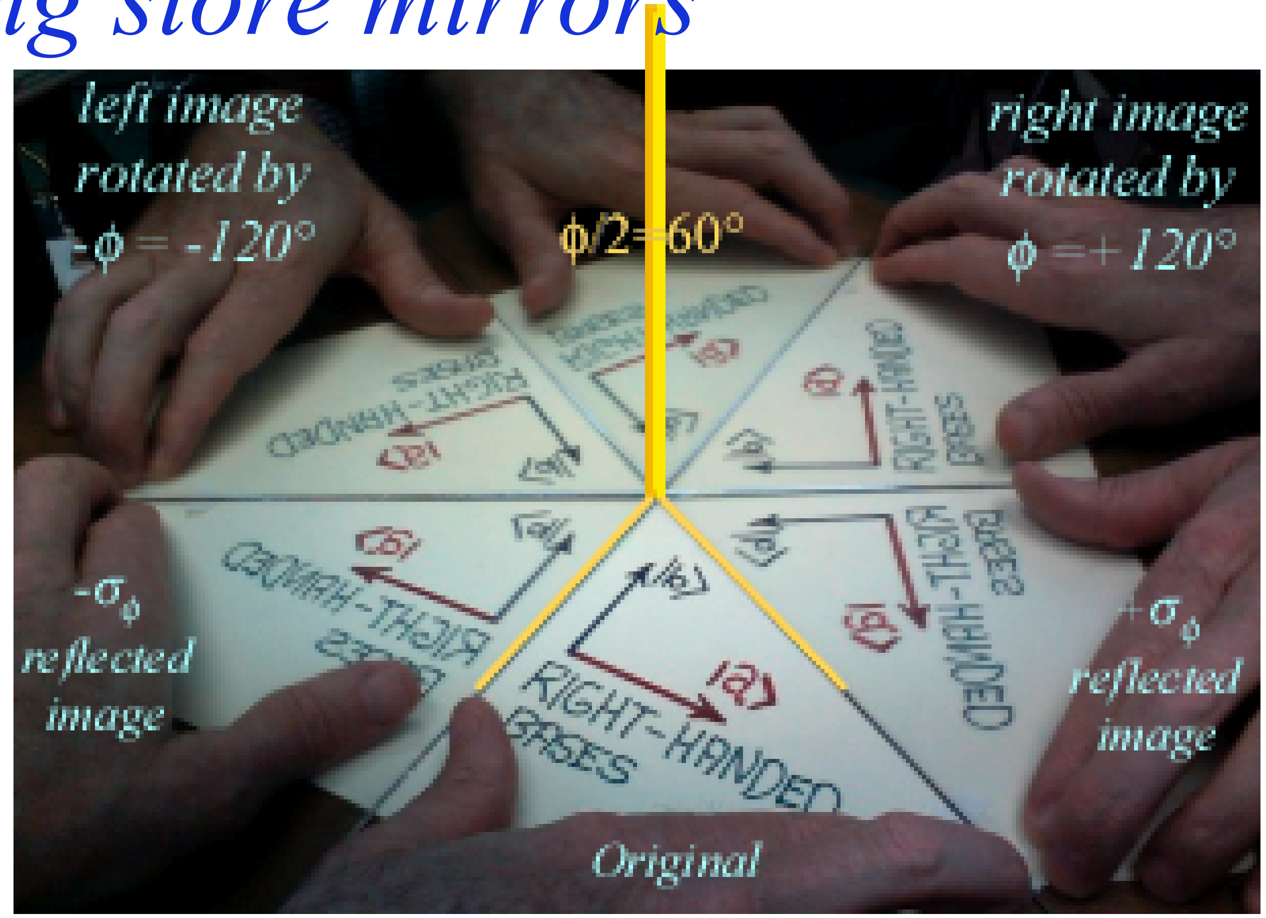
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

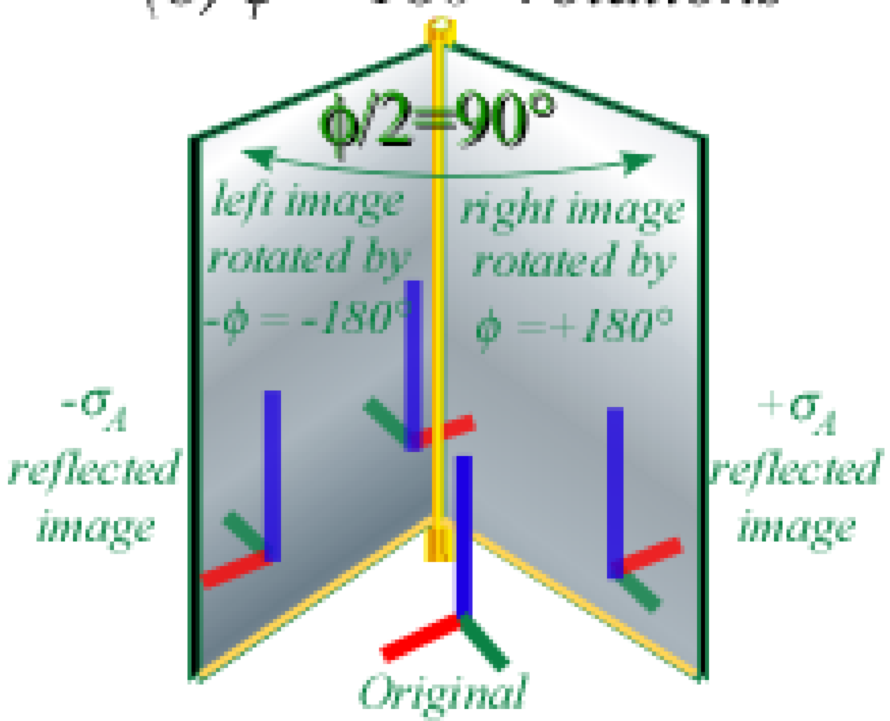
Reflections in clothing store mirrors

(a) φ=±120° rotations





(b) φ=±180° rotations



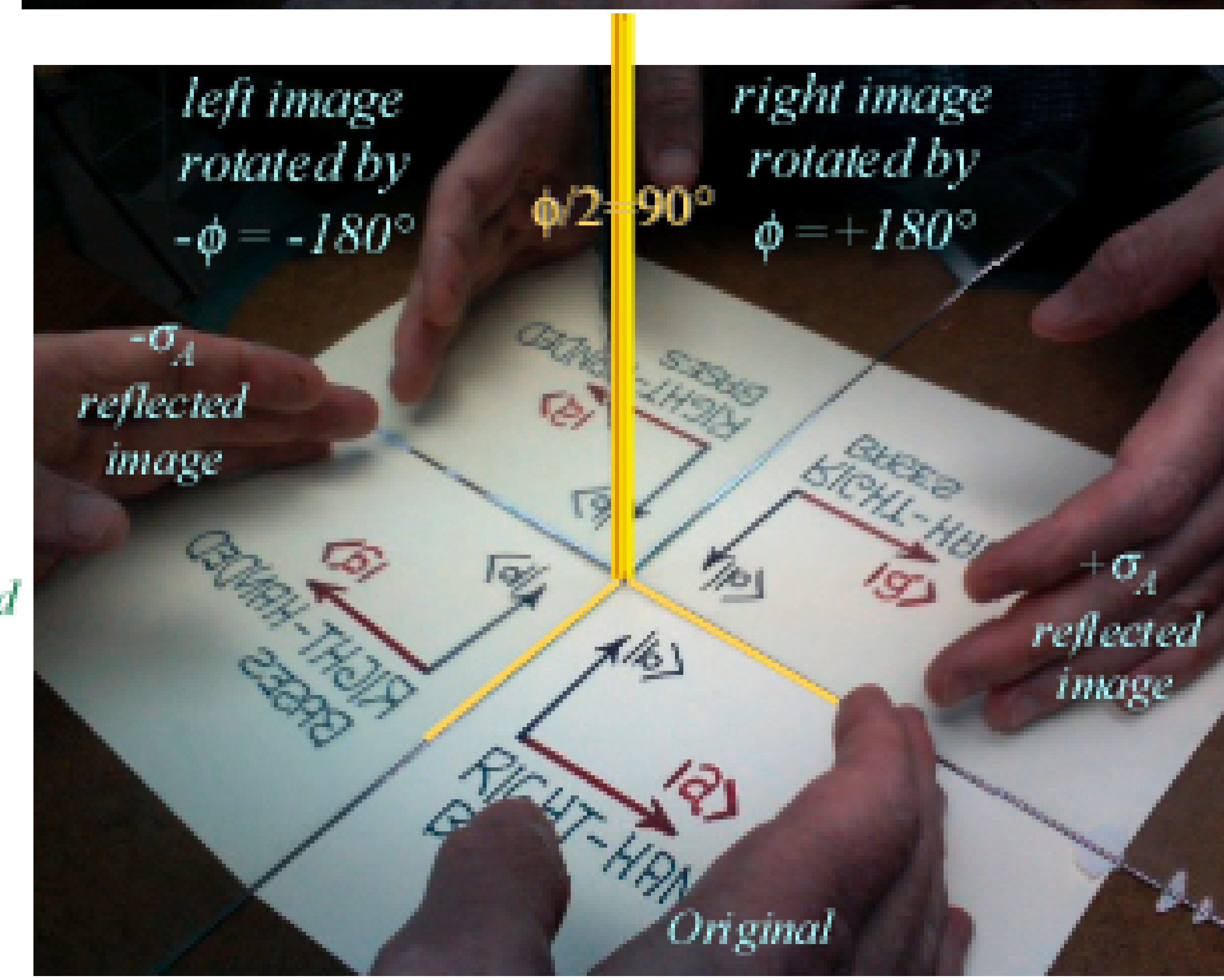


Fig. 5.4a-b

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

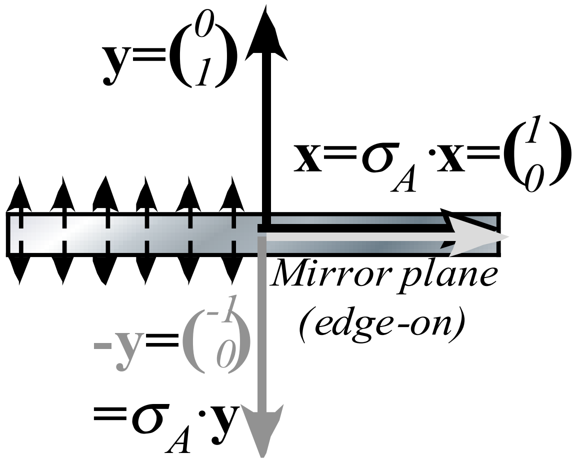


Fig.
5.3a-e

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

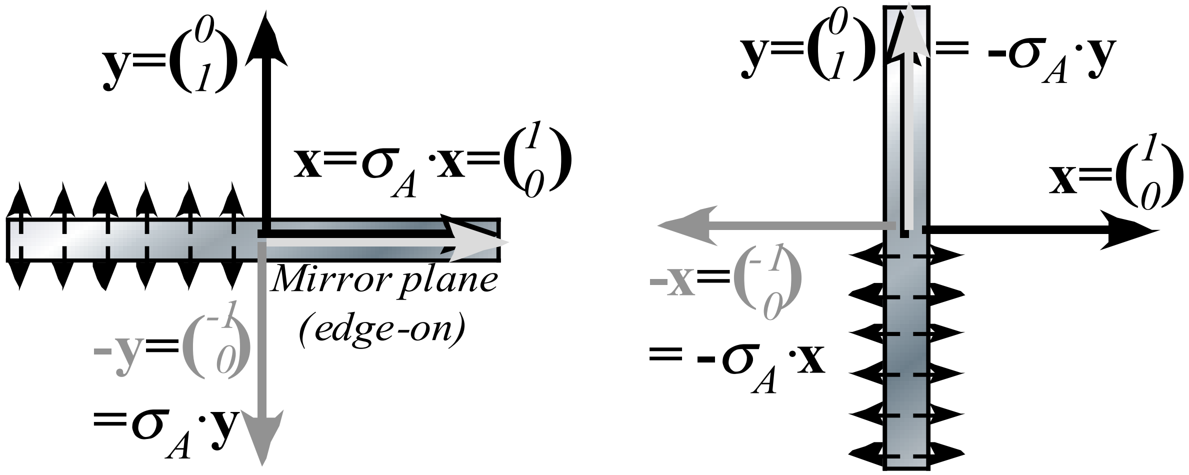


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, $-\sigma_A=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ *(b)Reflections* $\sigma_B=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

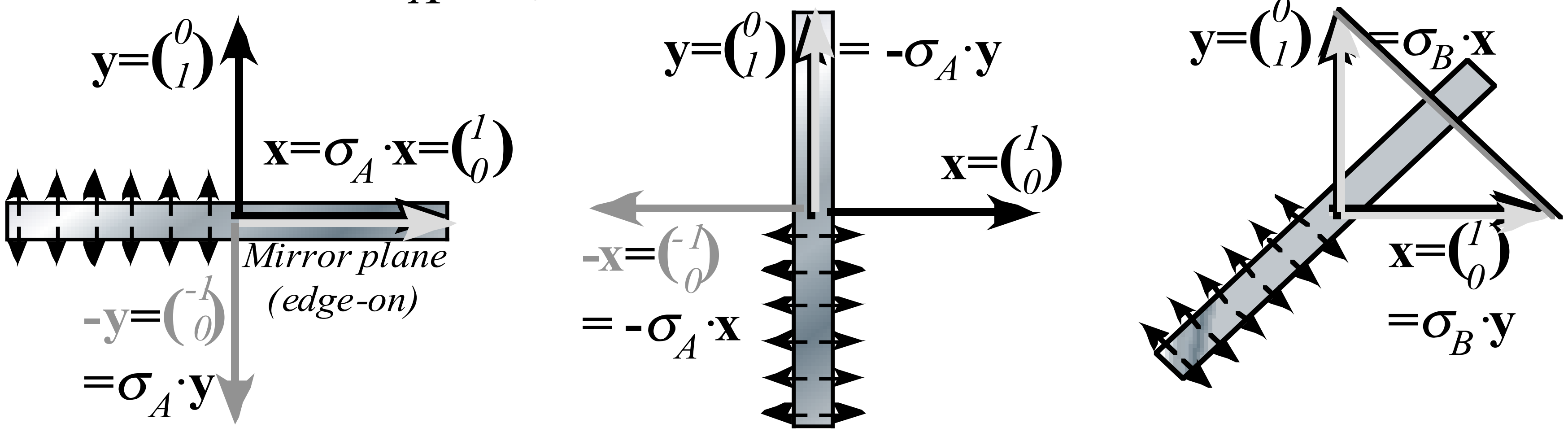


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

(a)Reflections $\sigma_A=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $-\sigma_A=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$ *(b)Reflections* $\sigma_B=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $-\sigma_B=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

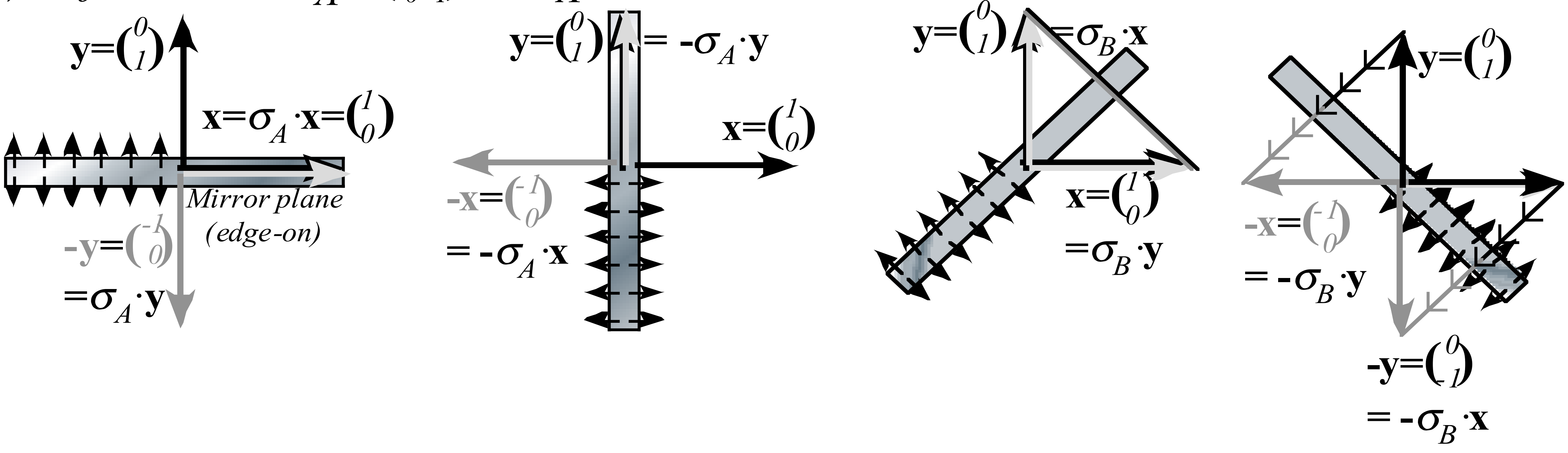


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

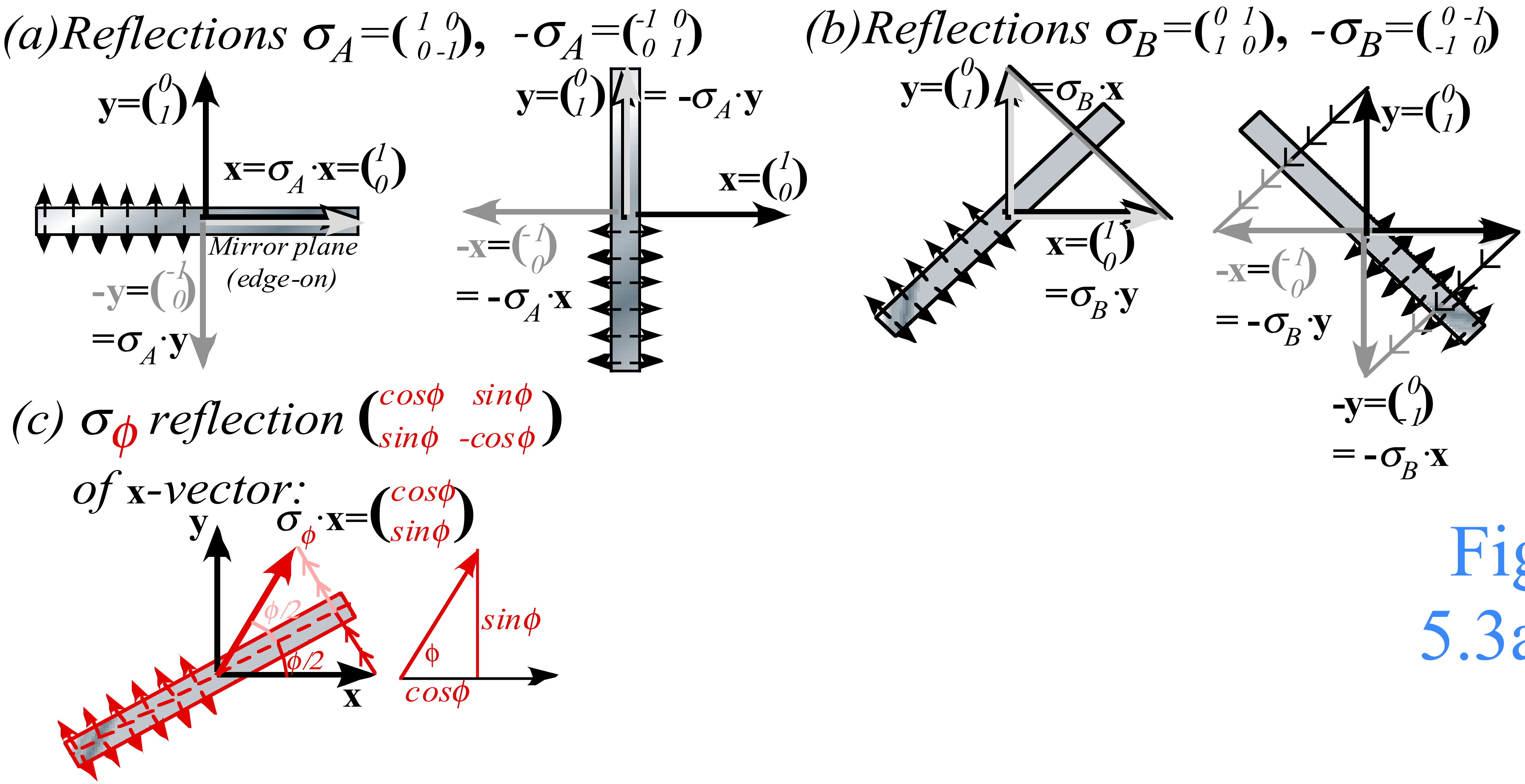


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

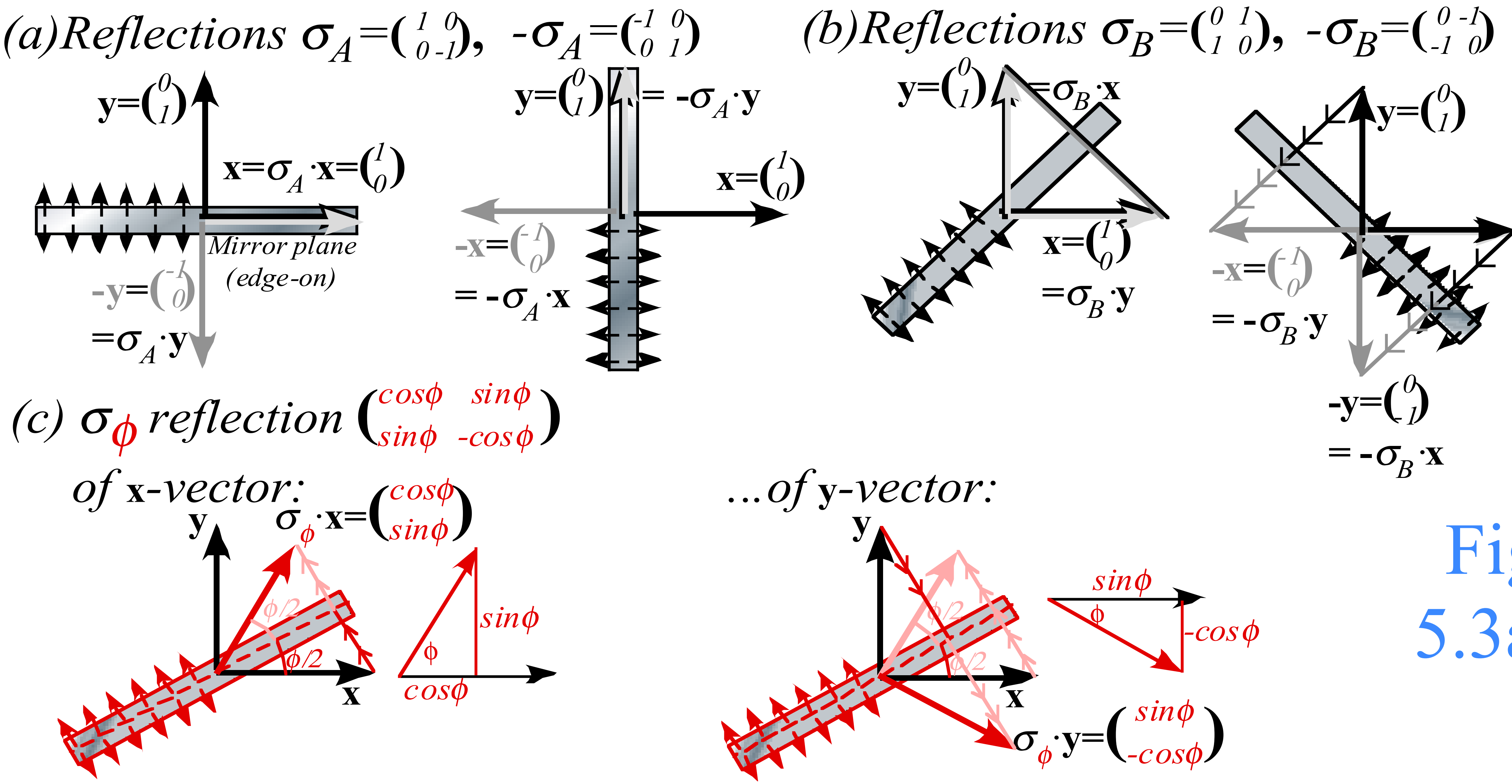


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

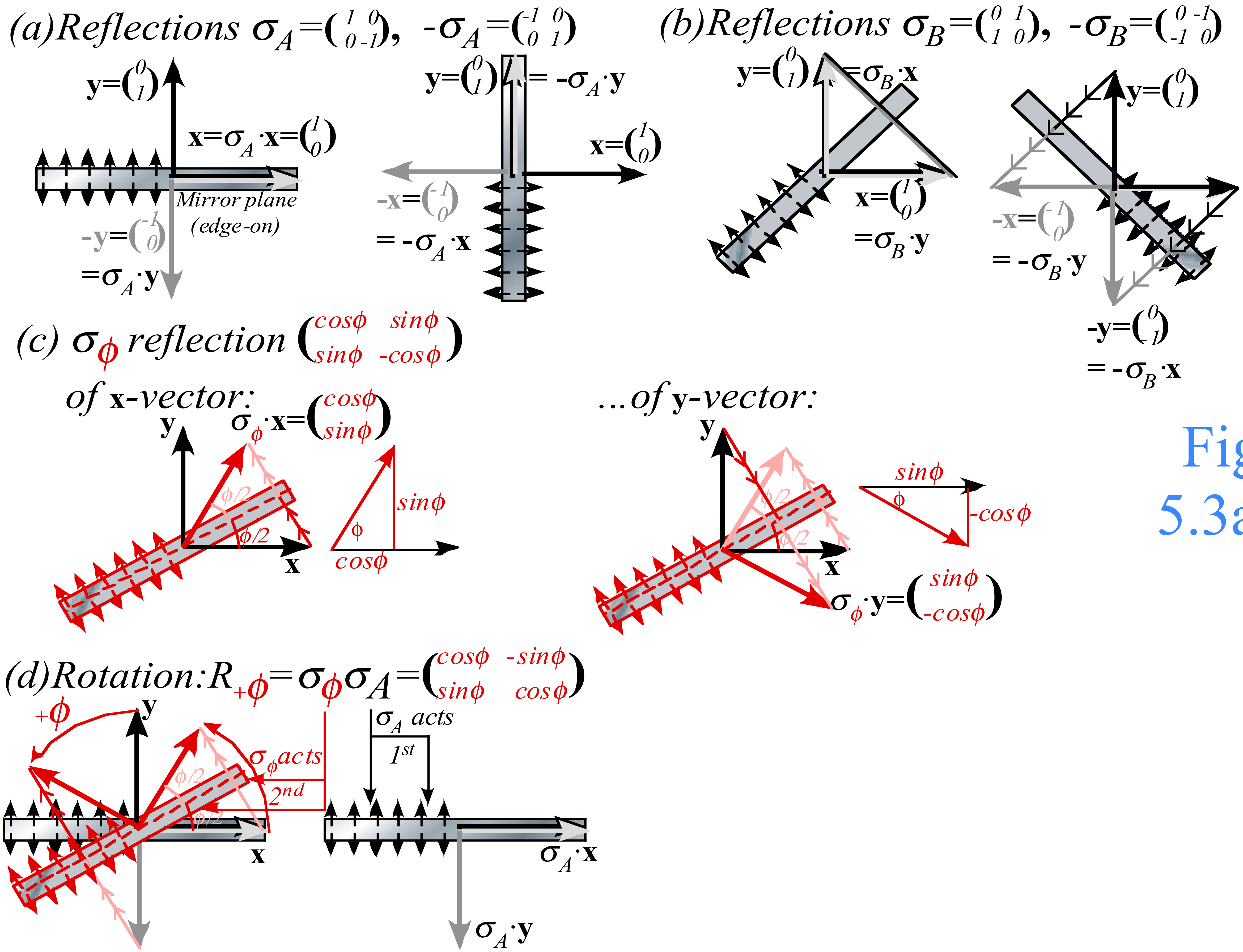


Fig. 5.3a-e

Symmetry: It's all done with mirrors!

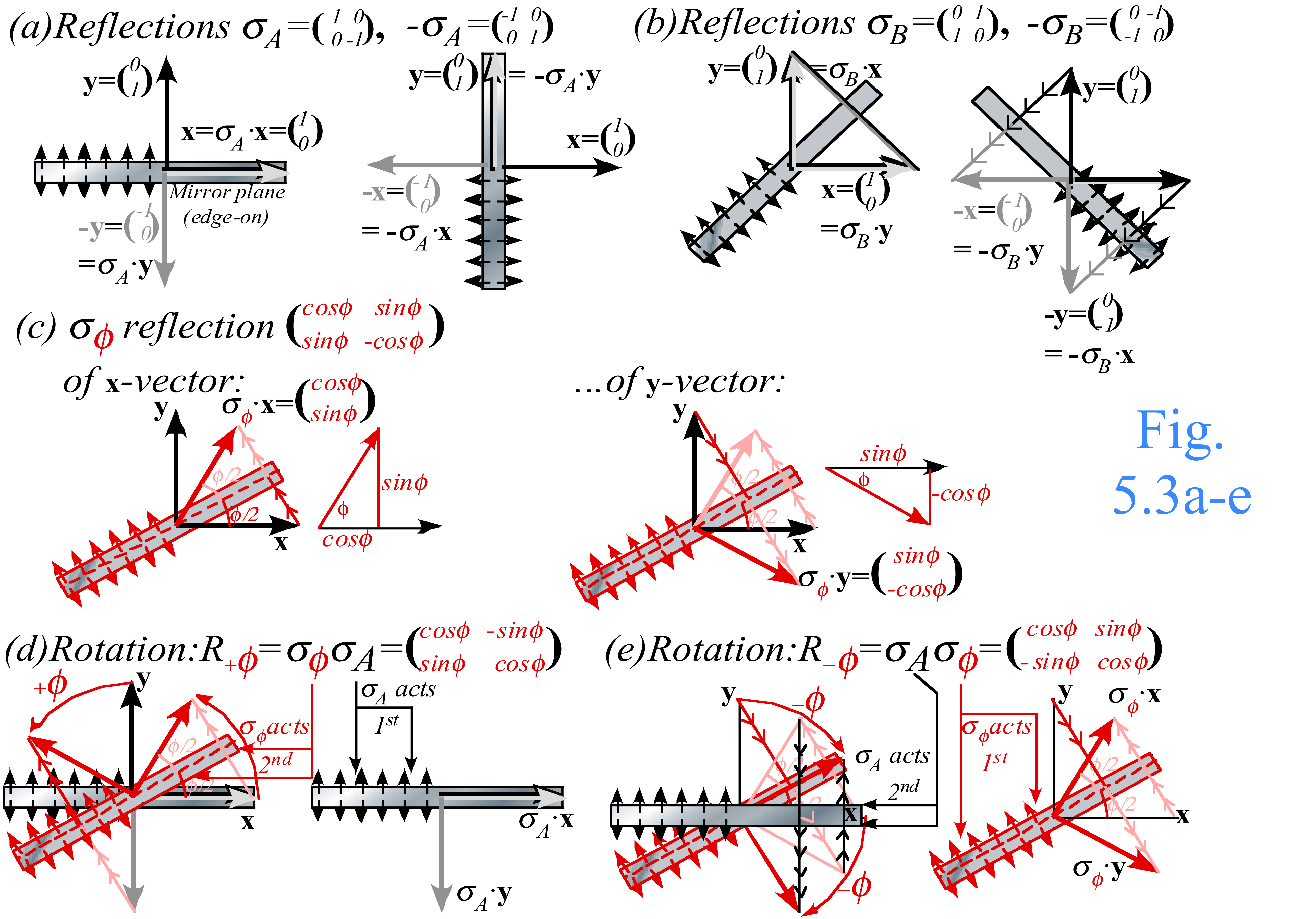


Fig. 5.3a-e

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

Product of odd number of reflections is a reflection

... even number of reflections is a rotation (or unit-op **1***)*

Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

Why reflections underlie all symmetry analyses

They work in 1D, 2D, 3D,.....,ND

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Product of rotations just give rotations

Classical objects are semi-rigid and rotate easily
Waves patterns are non-rigid and reflect easily

∴ *...wave reflections underlie modern physics*

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

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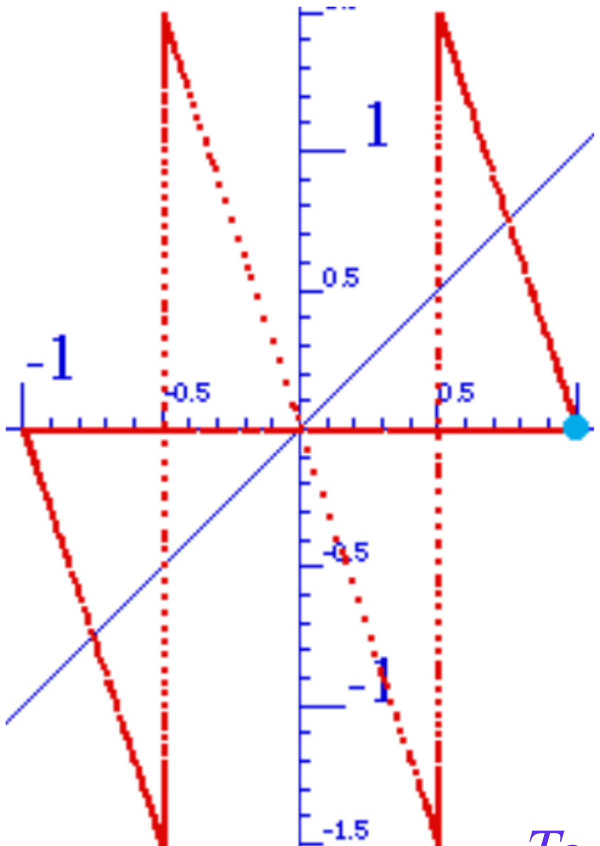
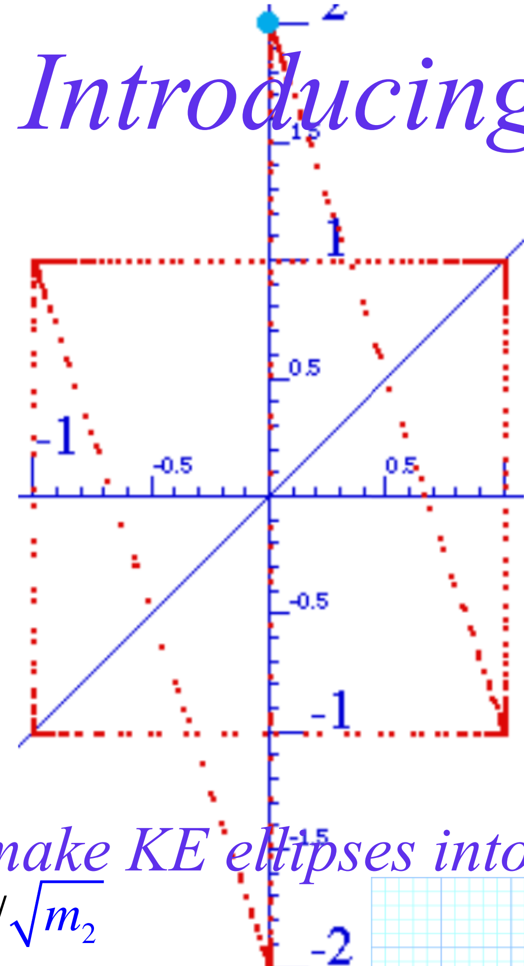
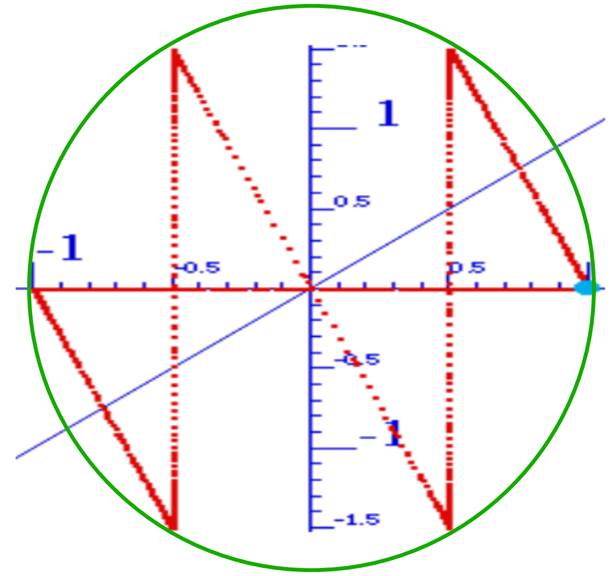
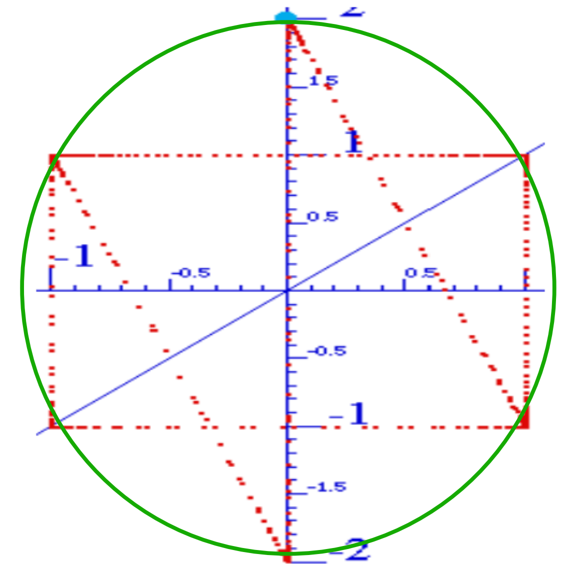
Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

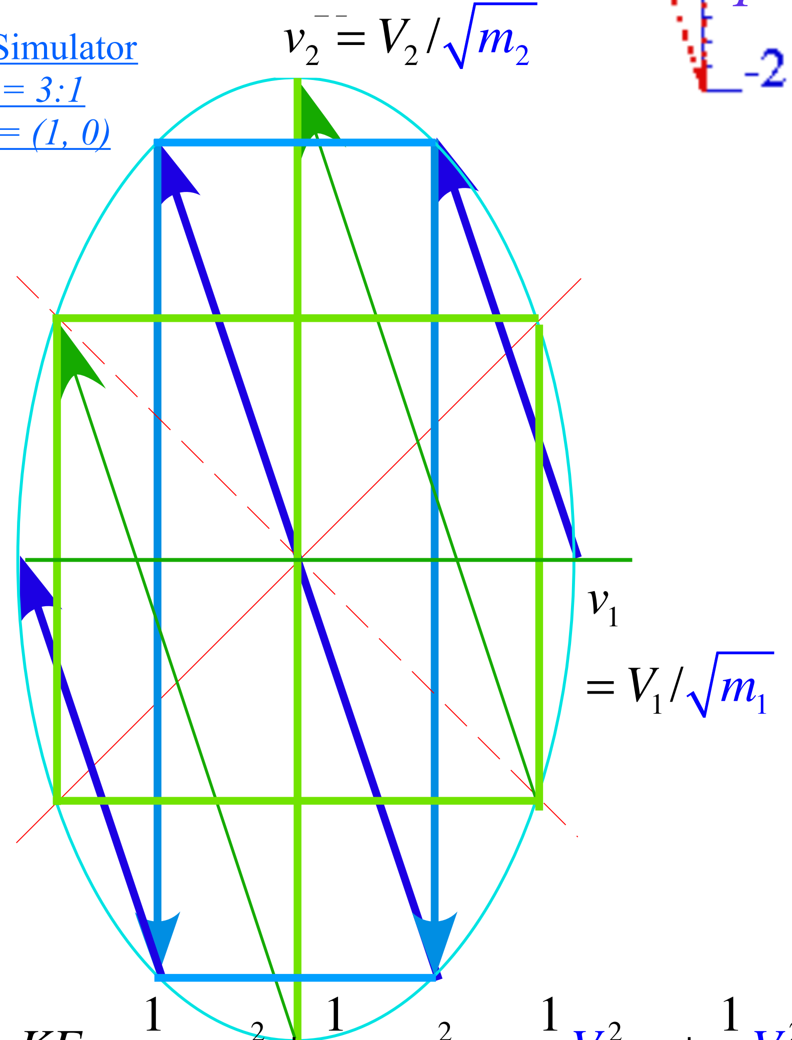
Introducing Symmetry Operators

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$

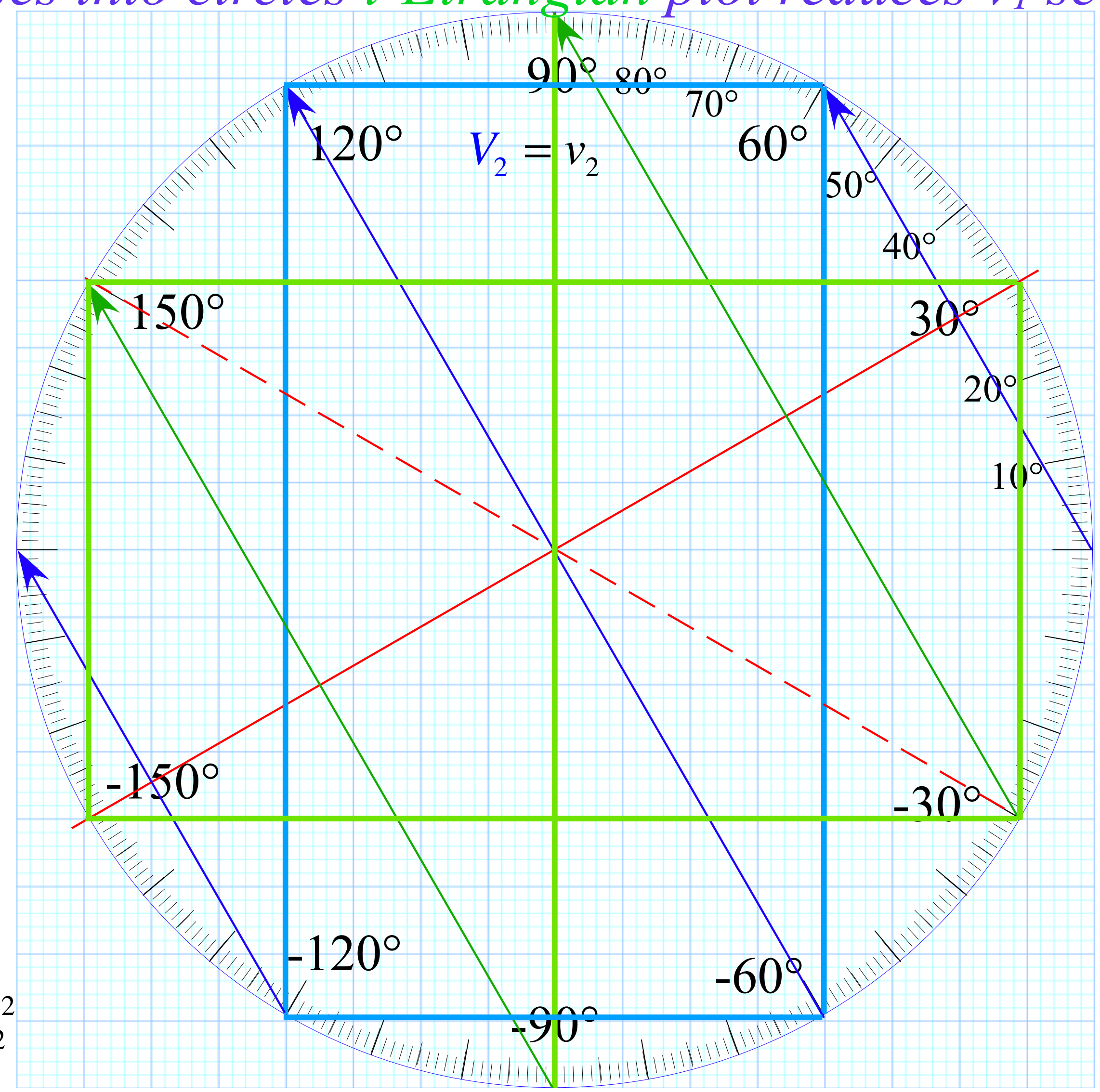





To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

[BounceIt Simulator
m1:m2= 3:1
(v1, v2) = (1, 0)]()



$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$

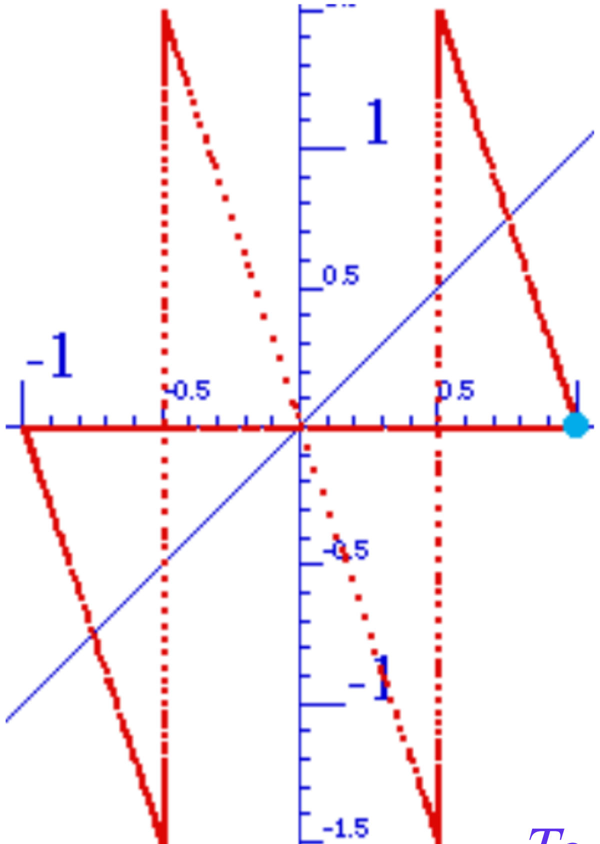


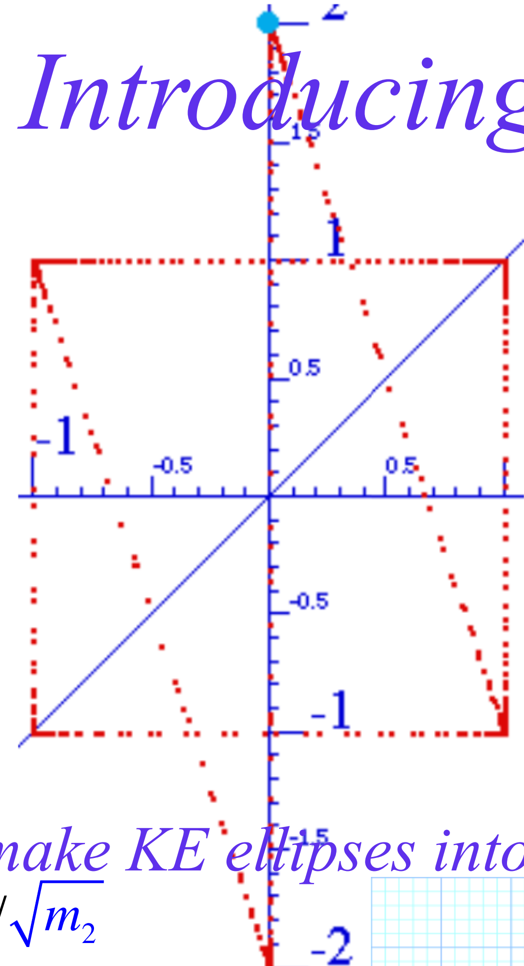
Here:
$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$
$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

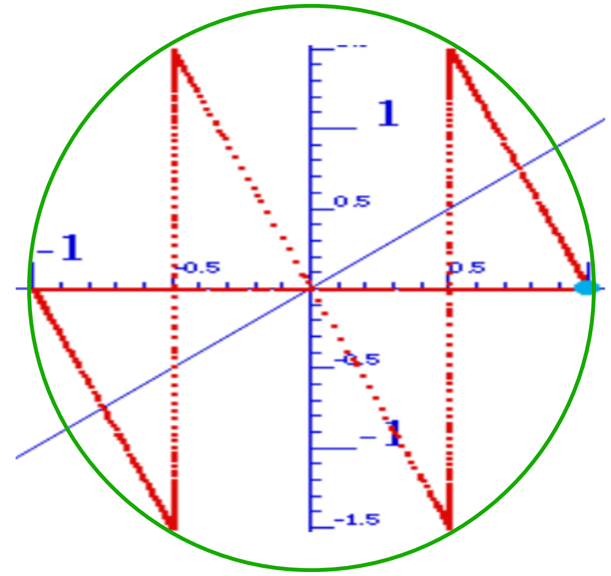
[BounceIt Web Simulator
m1:m2= 3:1 and (v1, v2) = (1, 0)
Comparison with Estrangian]()

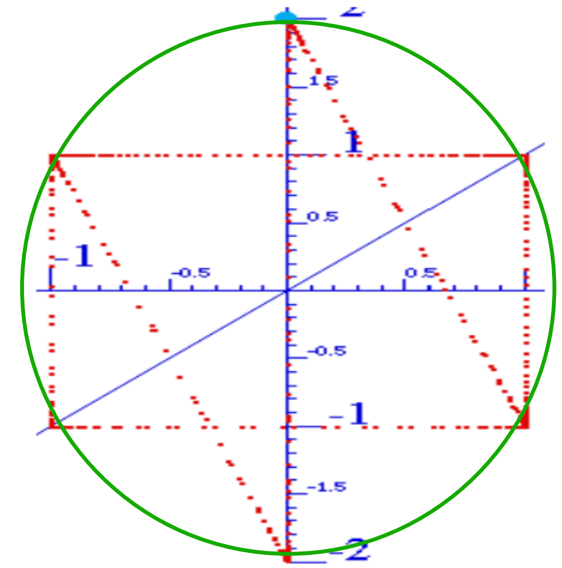
Introducing Symmetry Operators

Collisions for mass ratio $m_1{:}m_2 = 3{:}1$







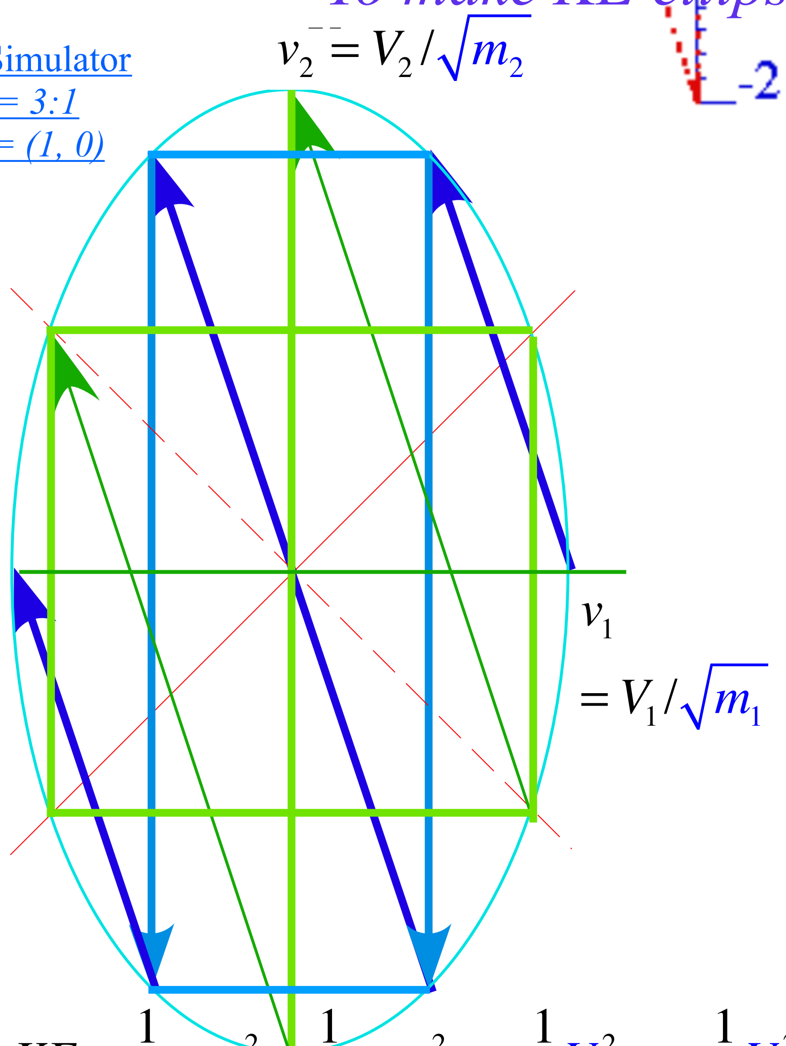


To make KE ellipses into circles l'Etrangian plot reduces v_1 scale by $1/\sqrt{m_1}$, etc.

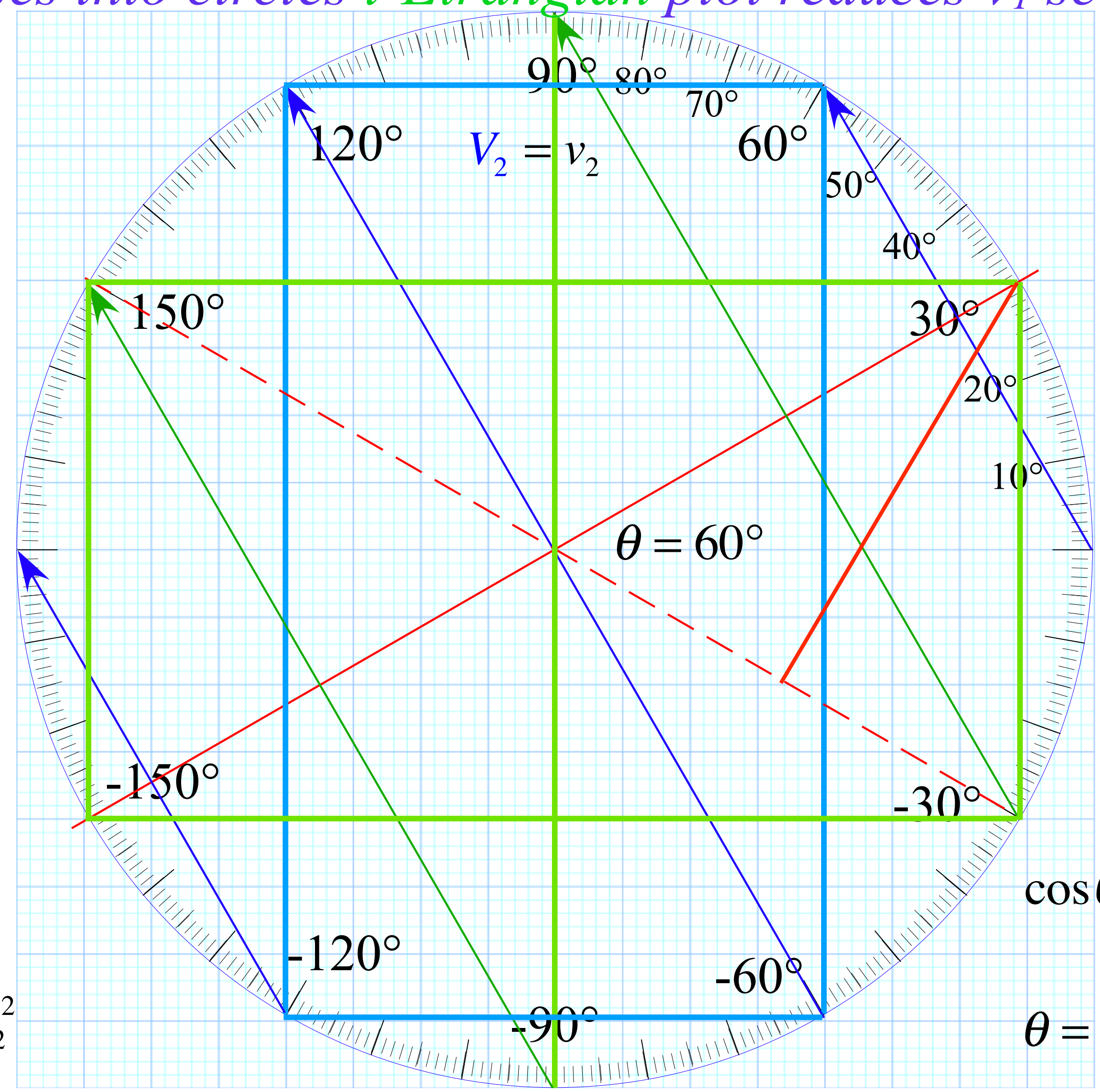
BounceIt Simulator

m1:m2= 3:1

(v1, v2) = (1, 0)



$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2$$



$$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$$

Here:

$1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$

$1/\sqrt{m_2} = 1/\sqrt{1} = 1.0$

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 60°$$

m1:m2= 3:1 and (v1, v2) = (1, 0)

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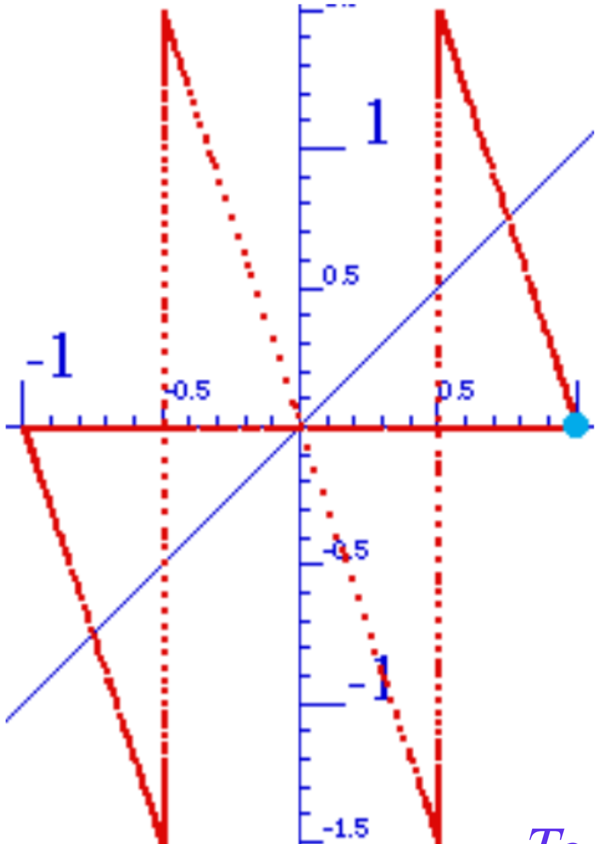

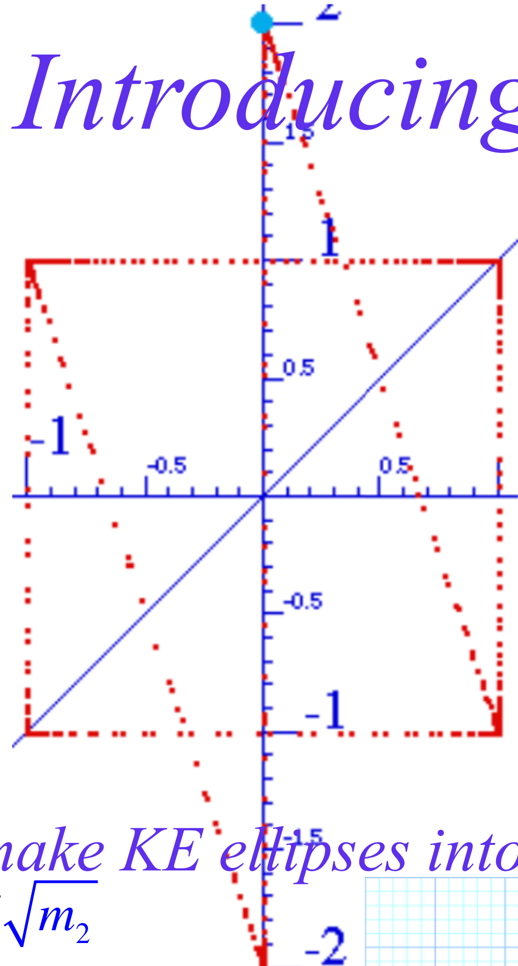

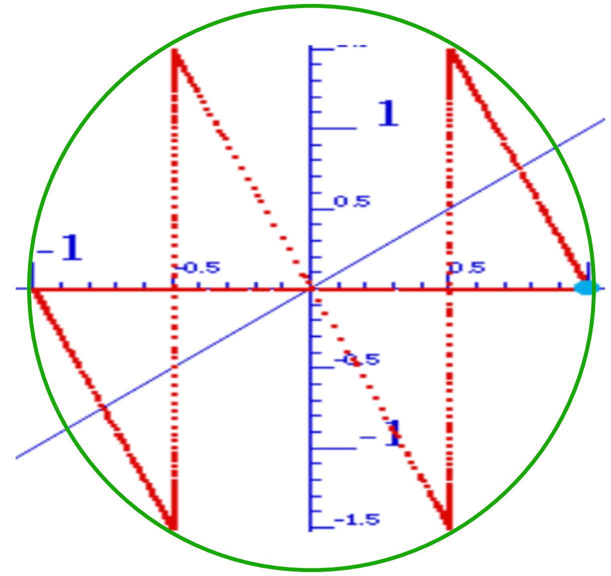

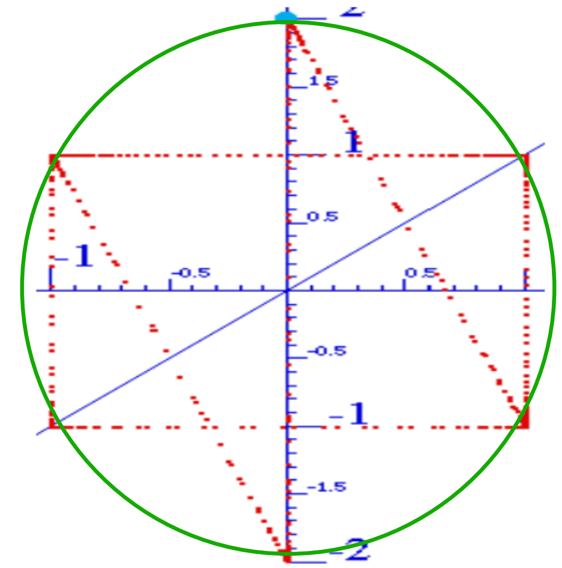

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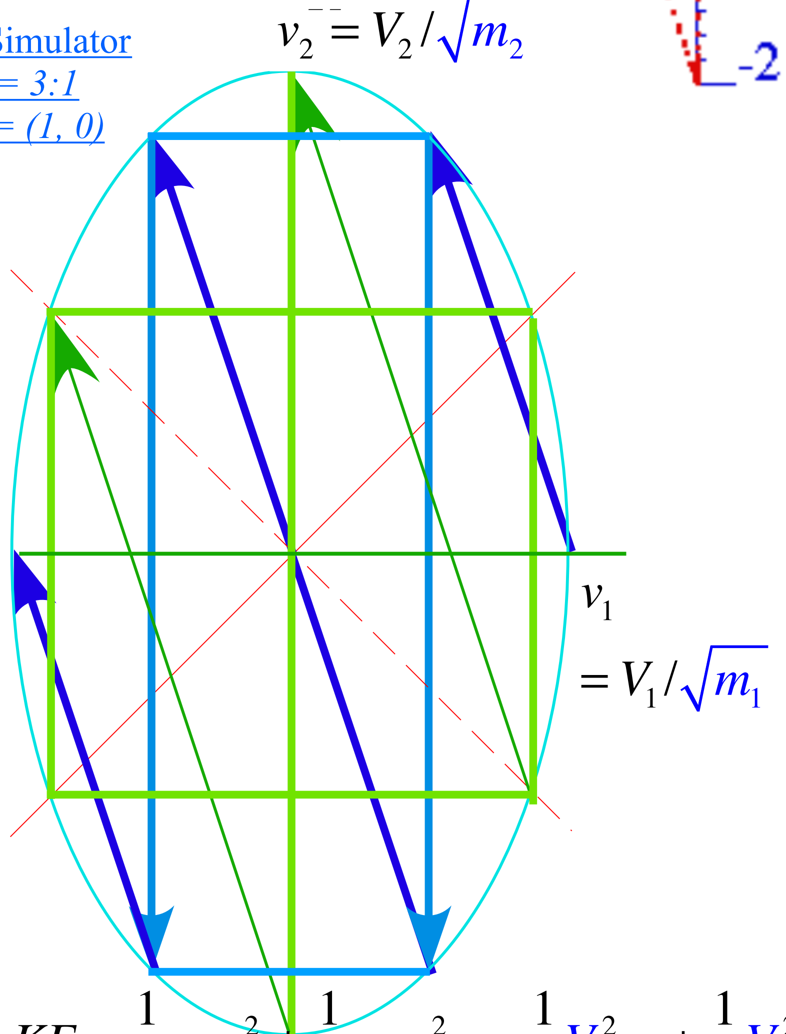
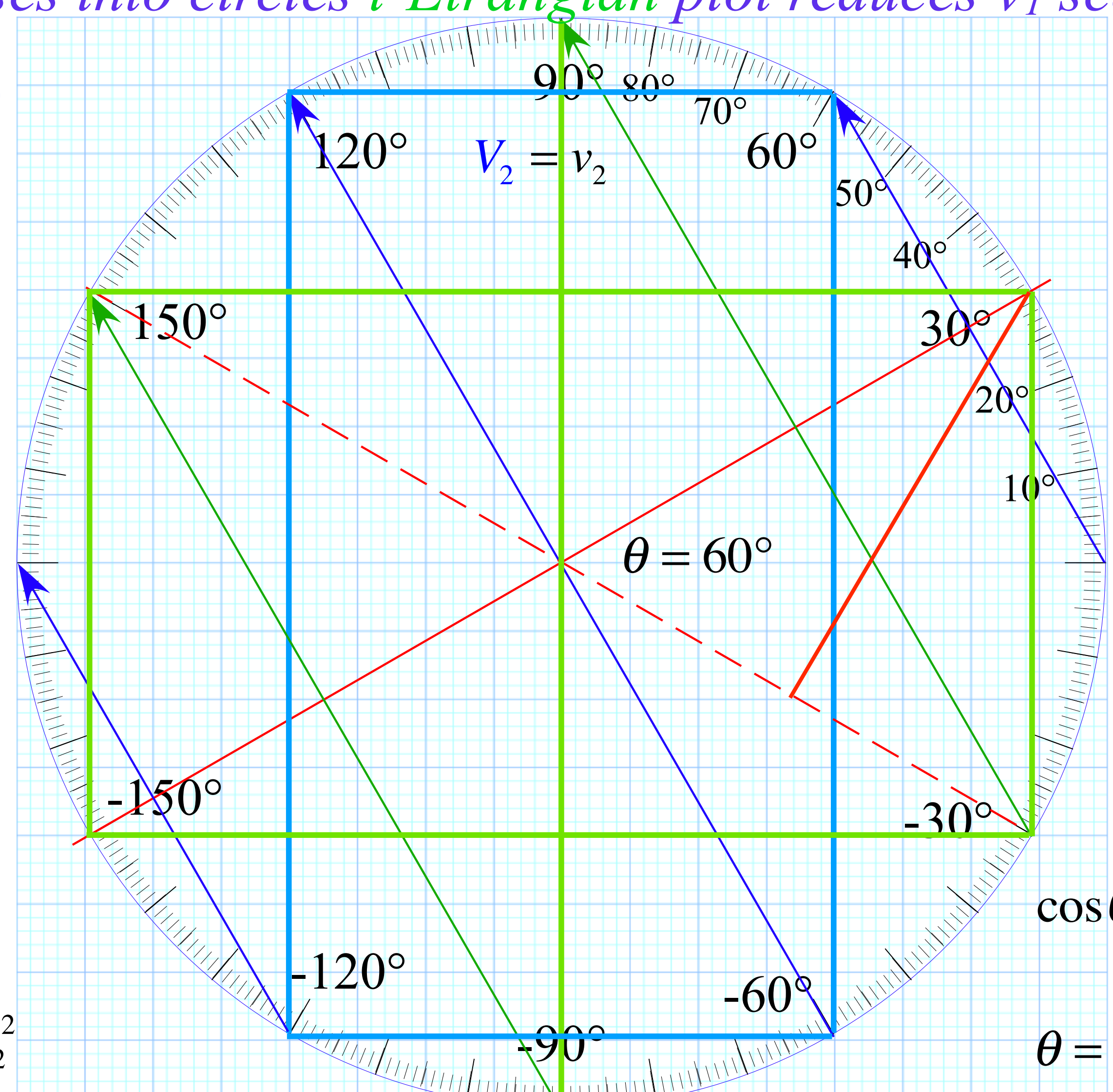


$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \quad = \frac{1}{2}V_1^2 \quad + \frac{1}{2}V_2^2$$



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$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{3/2}{1/2} = 3$$

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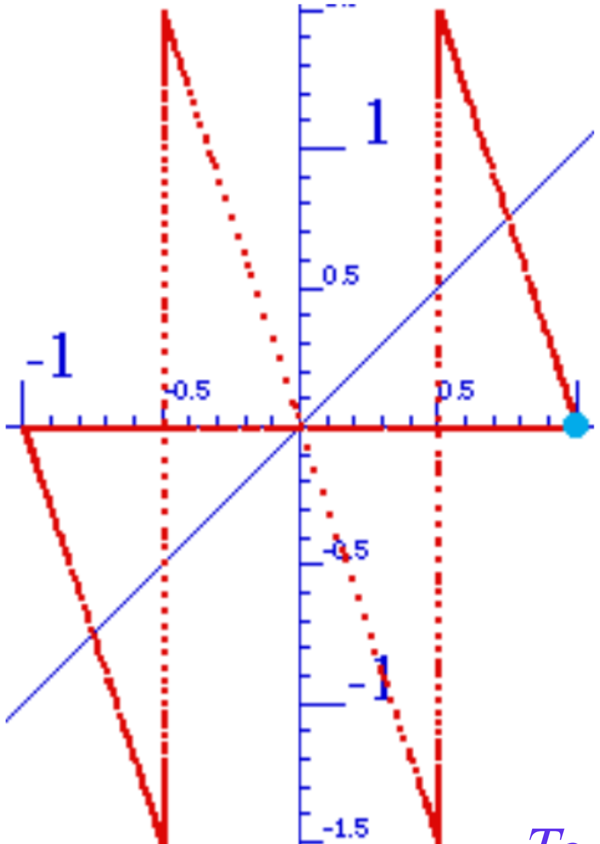

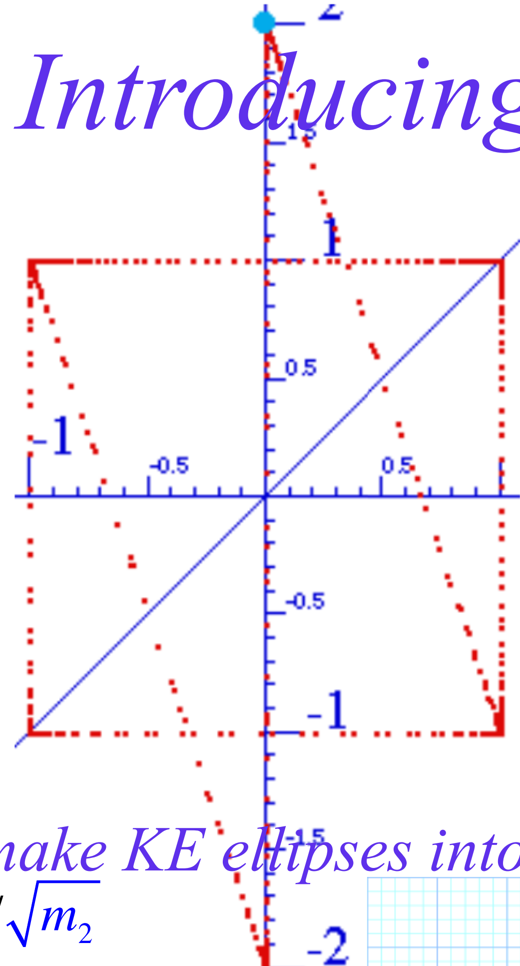

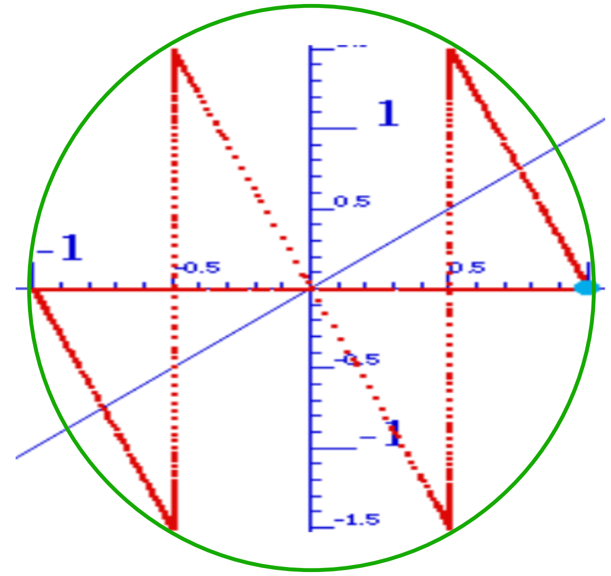

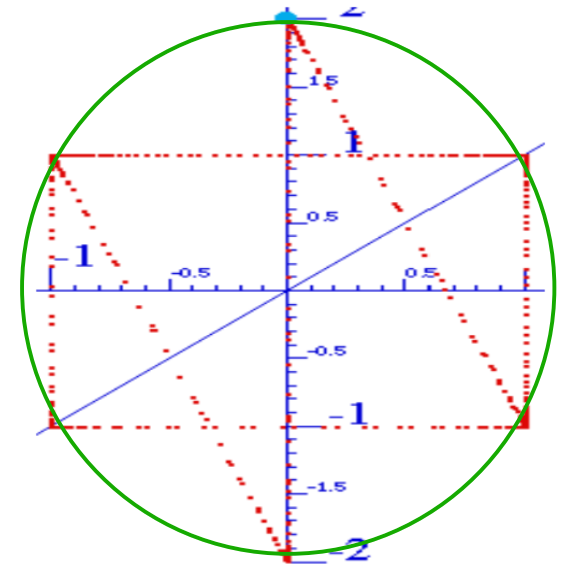

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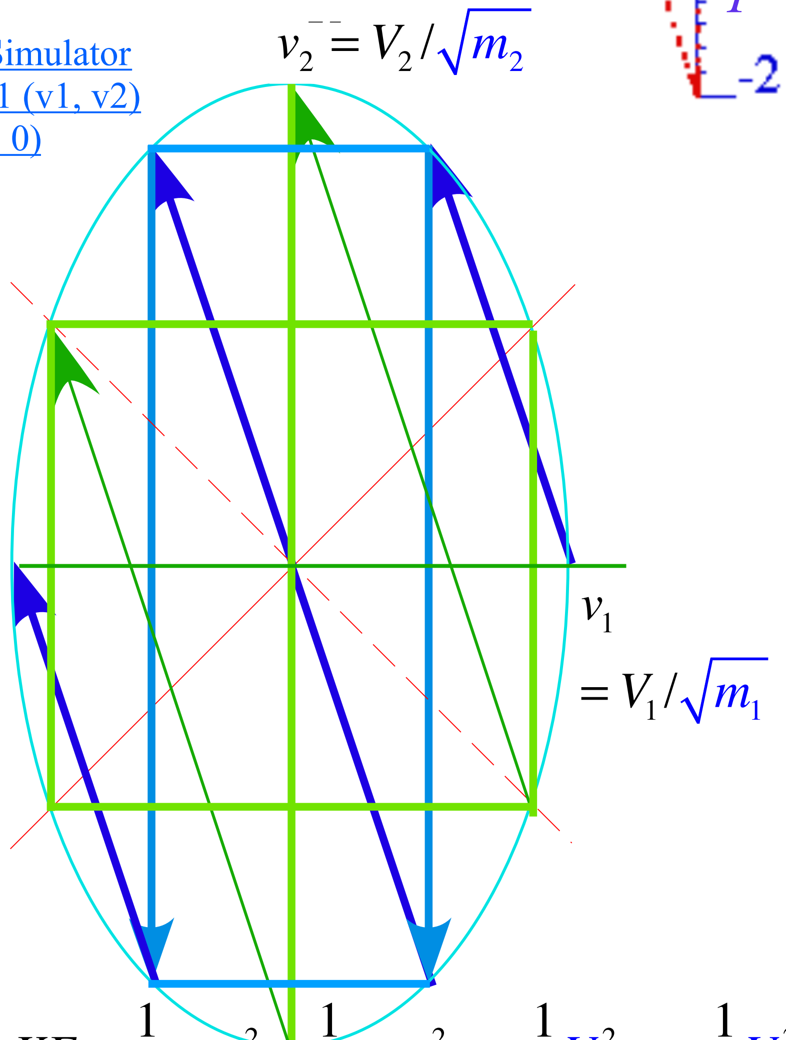

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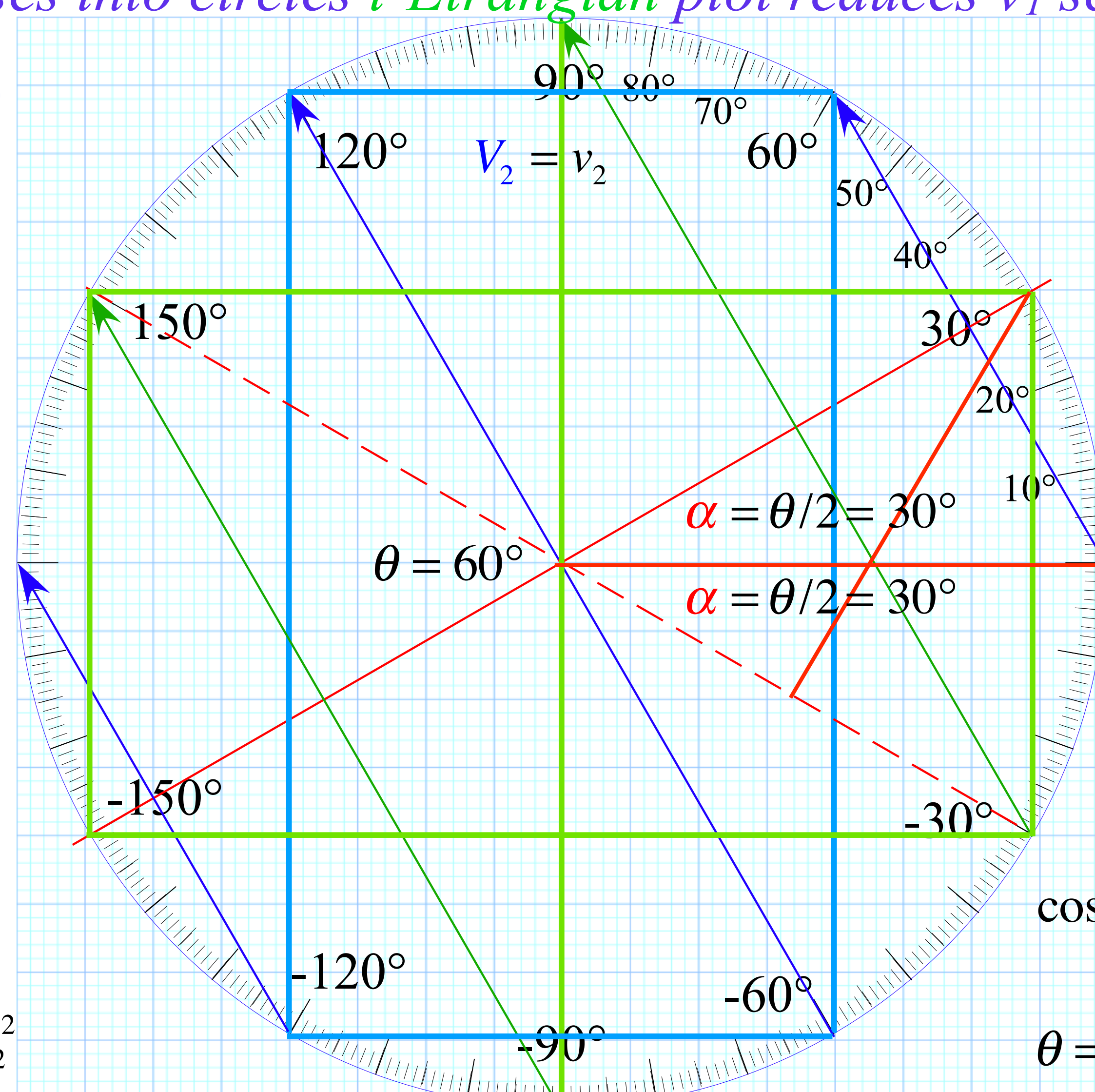

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$$V_1 = v_1\sqrt{m_1} = v_1\sqrt{3}$$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

$$\alpha = \theta/2$$

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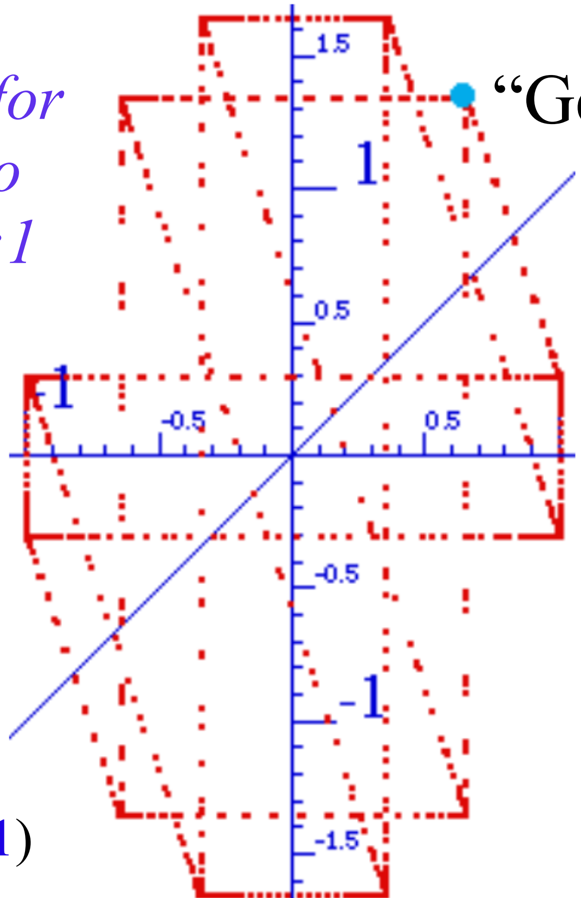
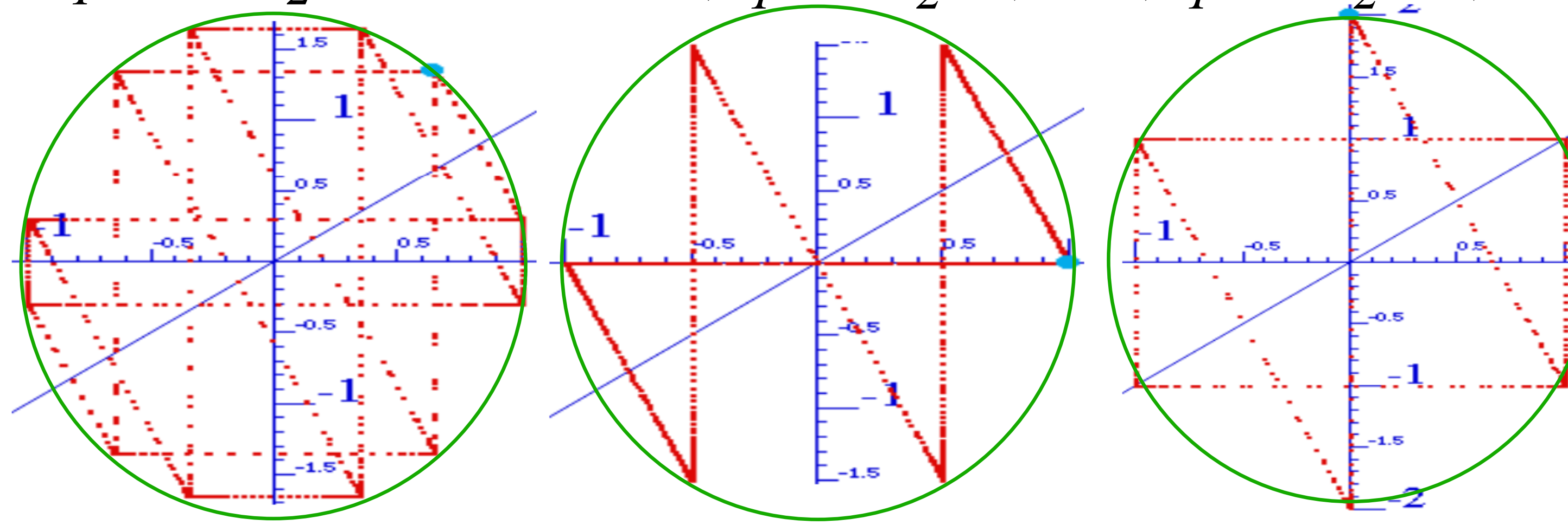


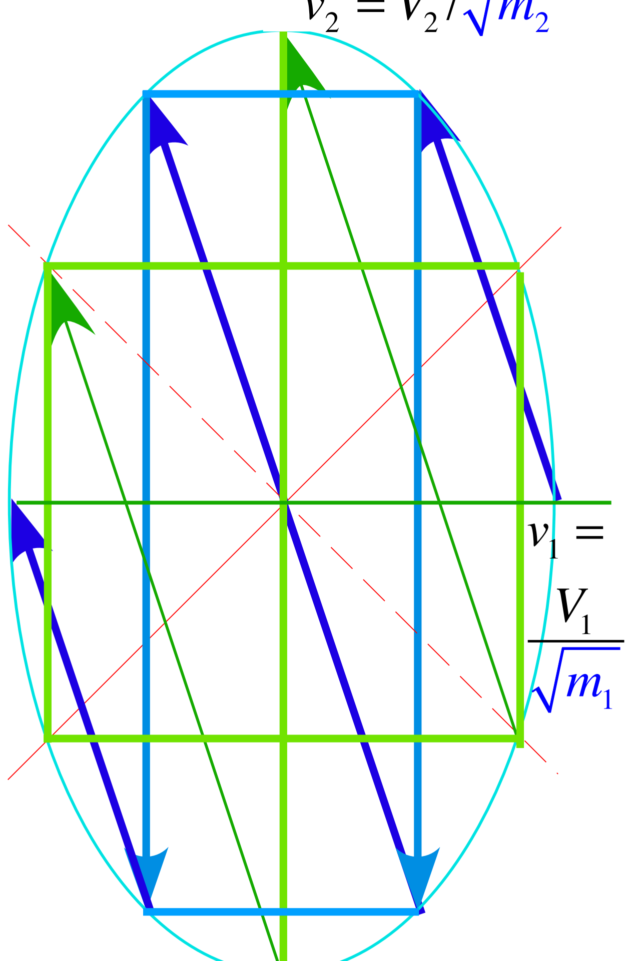
"Generic" initial velocity
$(v_1=1.0,\ v_2=0.1)$

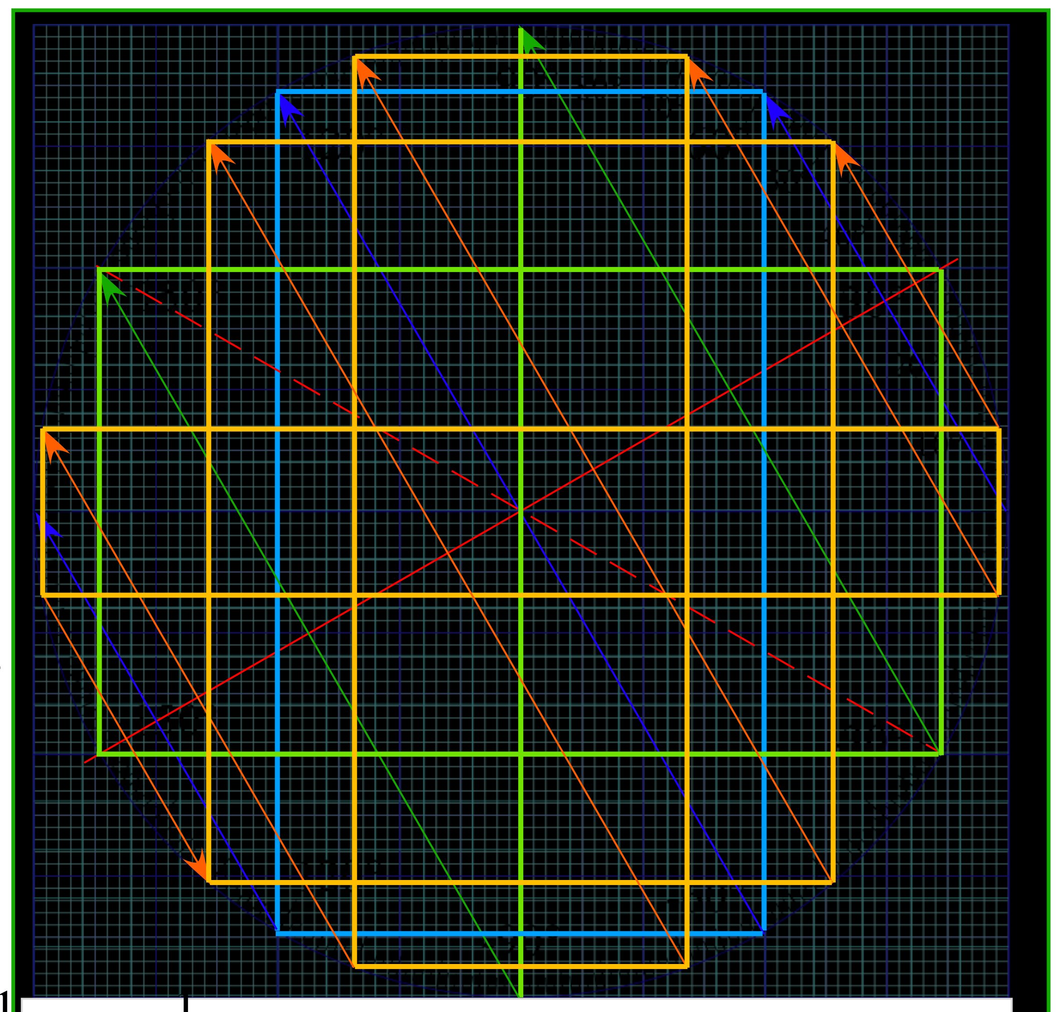
"Symmetric" initial velocity
$(v_1=1,\ v_2=0)$ or $(v_1=1,\ v_2=-1)$

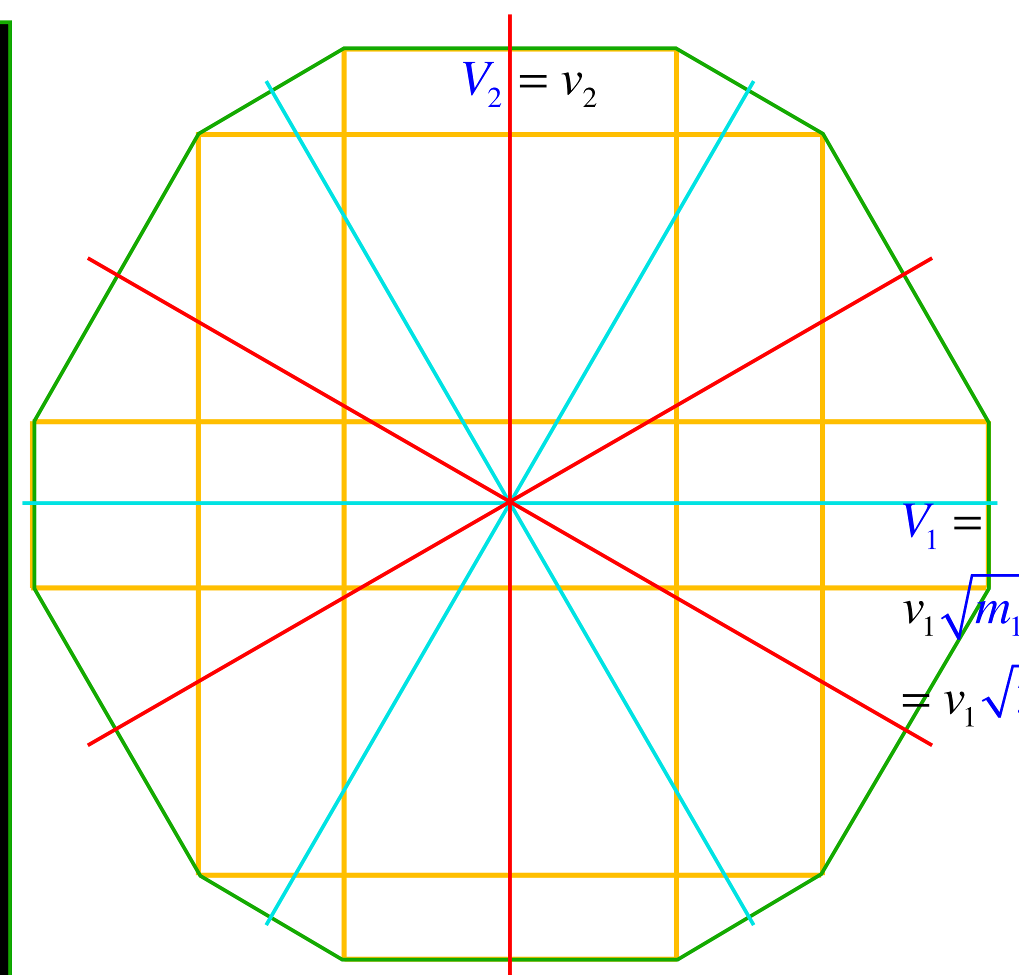


$m_1/m_2=(3)/(1)$

reduce v_1 scale by $1/\sqrt{m_1} = 1/\sqrt{3} = 0.577$







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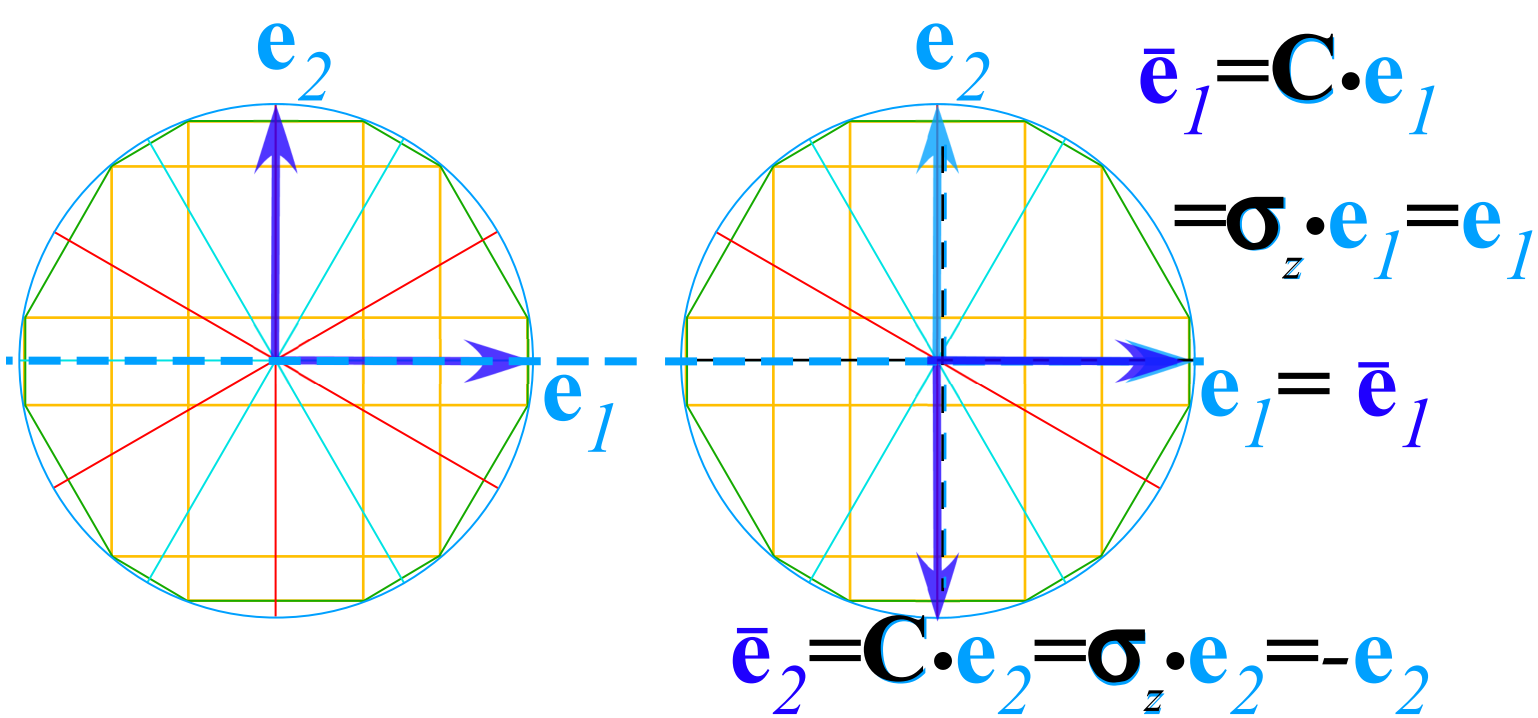
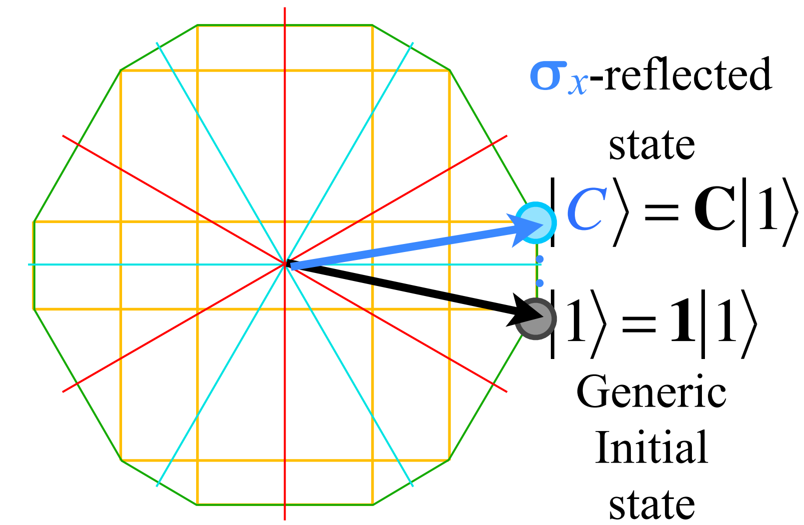
Effects of Ceiling Bang Matrix $\mathbf{C}=\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{C}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{C}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$





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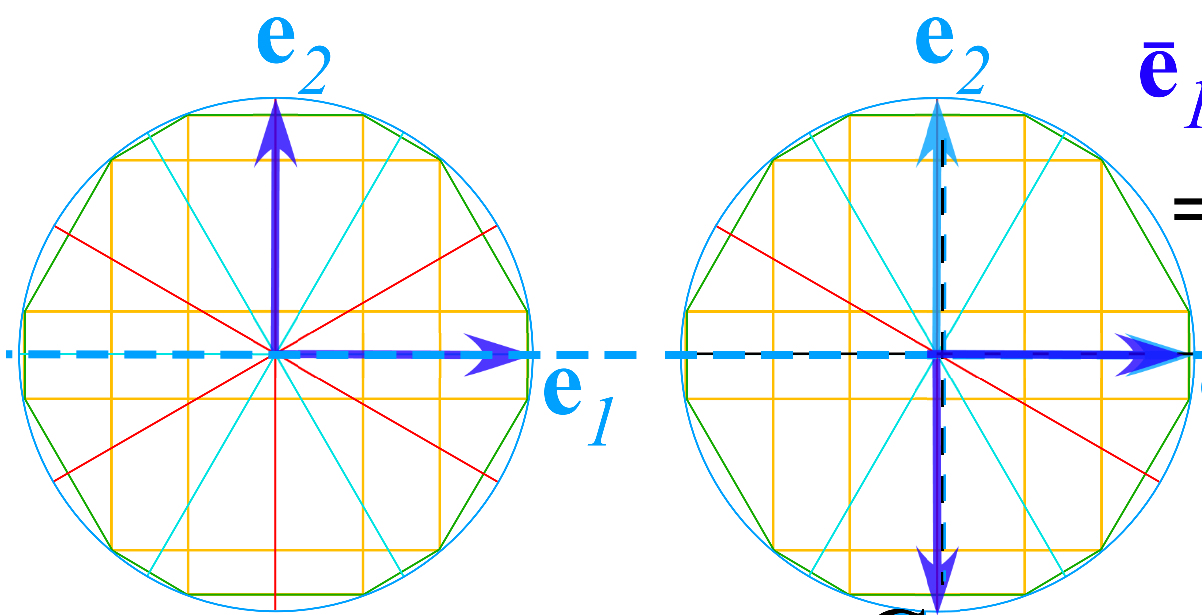
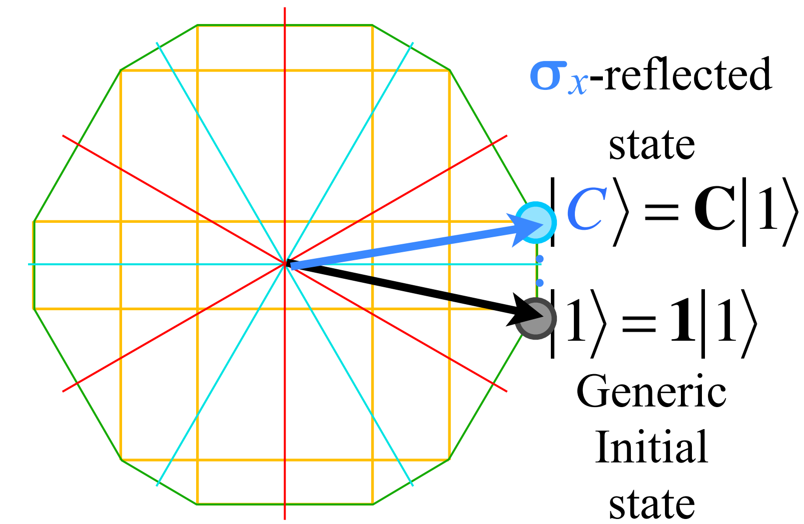


$\bar{\mathbf{e}}_1=\mathbf{C}\cdot\mathbf{e}_1$

$=\boldsymbol{\sigma}_z\cdot\mathbf{e}_1=\mathbf{e}_1$

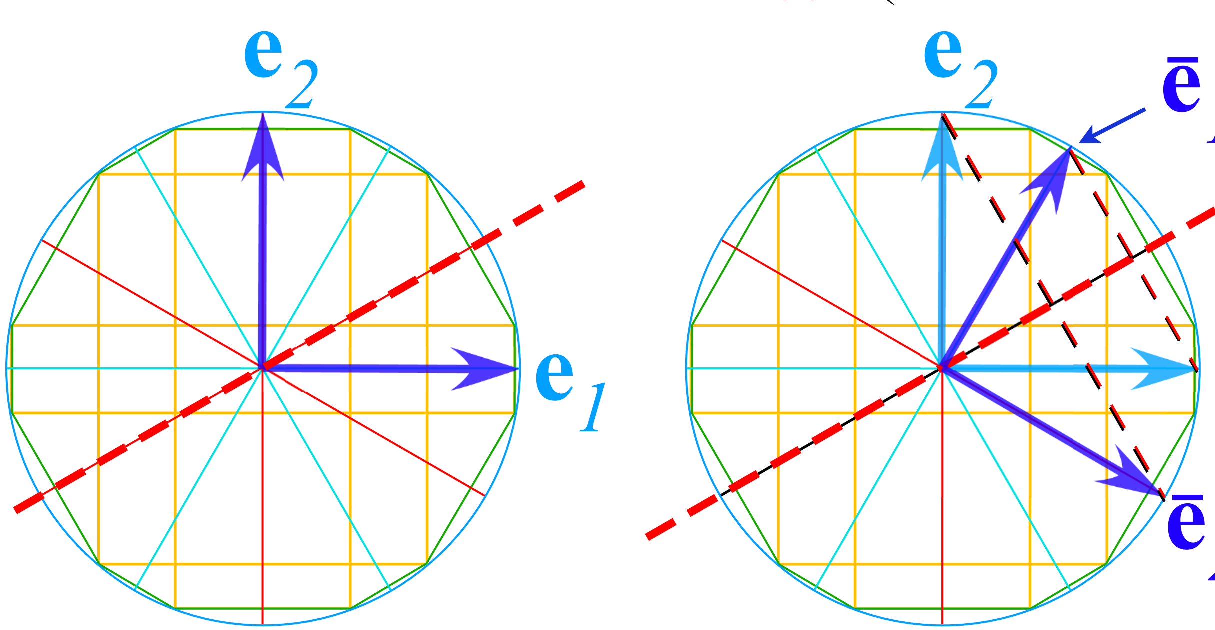
$\bar{\mathbf{e}}_2=\mathbf{C}\cdot\mathbf{e}_2=\boldsymbol{\sigma}_z\cdot\mathbf{e}_2=-\mathbf{e}_2$



Known as *matrix elements* or *components*

Known as *relative direction cosines*

Effects of Mass Bang Matrix $\mathbf{M}=\boldsymbol{\sigma}_{30°} = \begin{pmatrix} \mathbf{e}_1\cdot\mathbf{M}\cdot\mathbf{e}_1 & \mathbf{e}_1\cdot\mathbf{M}\cdot\mathbf{e}_2 \\ \mathbf{e}_2\cdot\mathbf{M}\cdot\mathbf{e}_1 & \mathbf{e}_2\cdot\mathbf{M}\cdot\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_1\cdot\bar{\mathbf{e}}_2 \\ \mathbf{e}_2\cdot\bar{\mathbf{e}}_1 & \mathbf{e}_2\cdot\bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{pmatrix}$

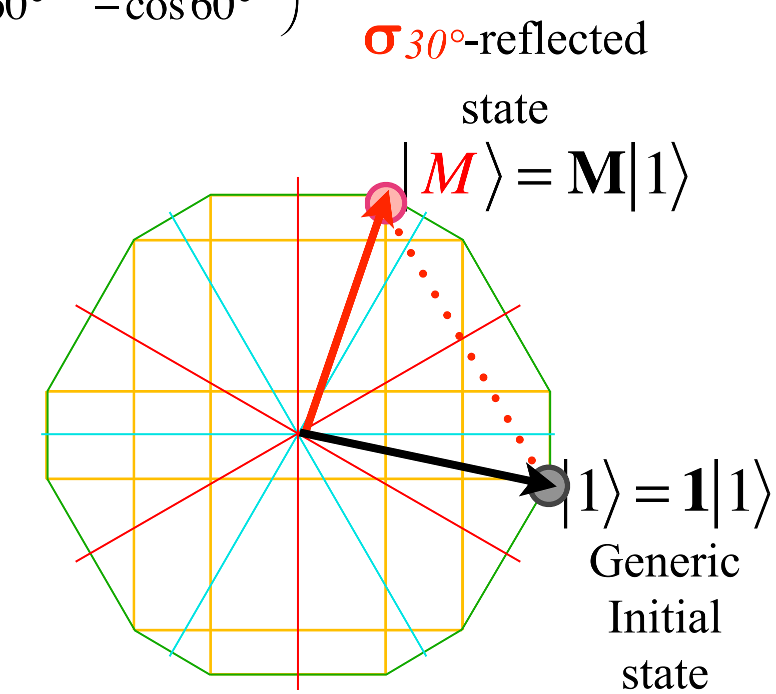


$\bar{\mathbf{e}}_1=\mathbf{M}\cdot\mathbf{e}_1$

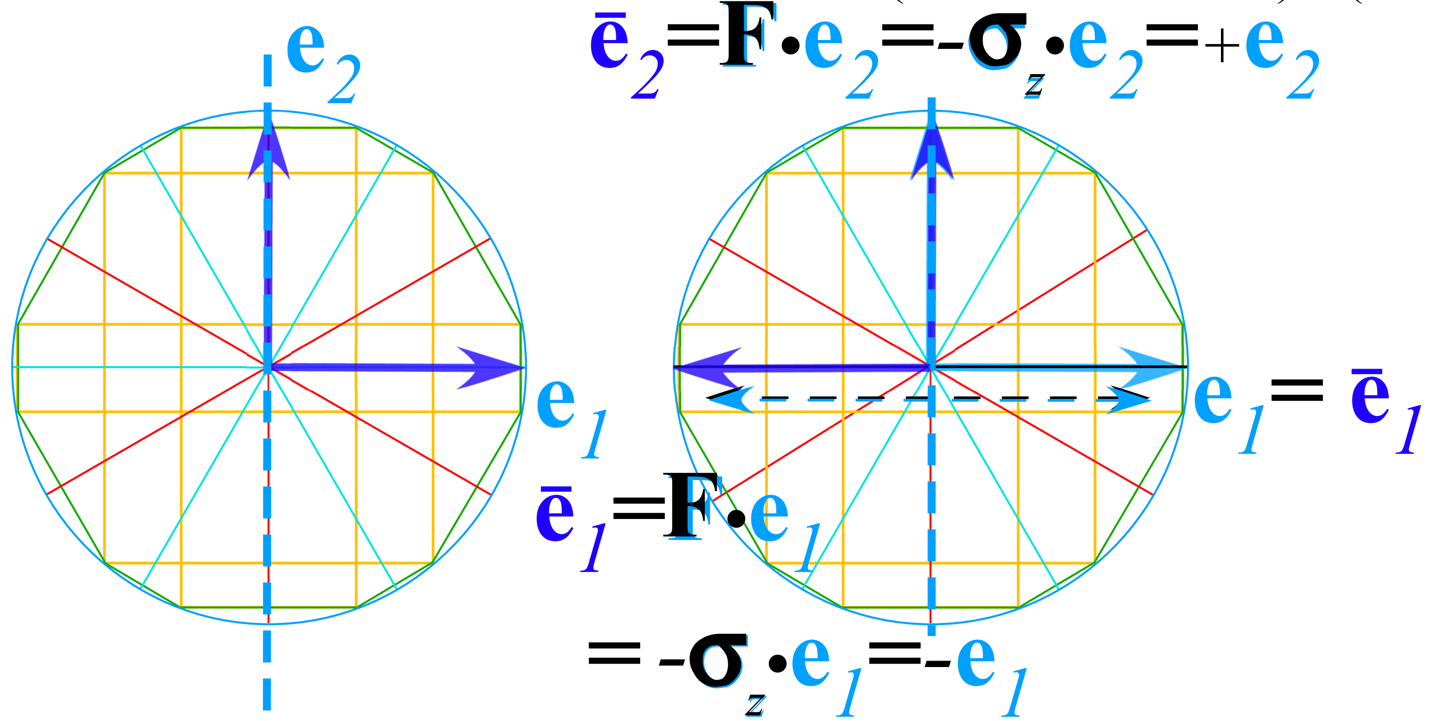
$=\boldsymbol{\sigma}_{30°}\cdot\mathbf{e}_1$

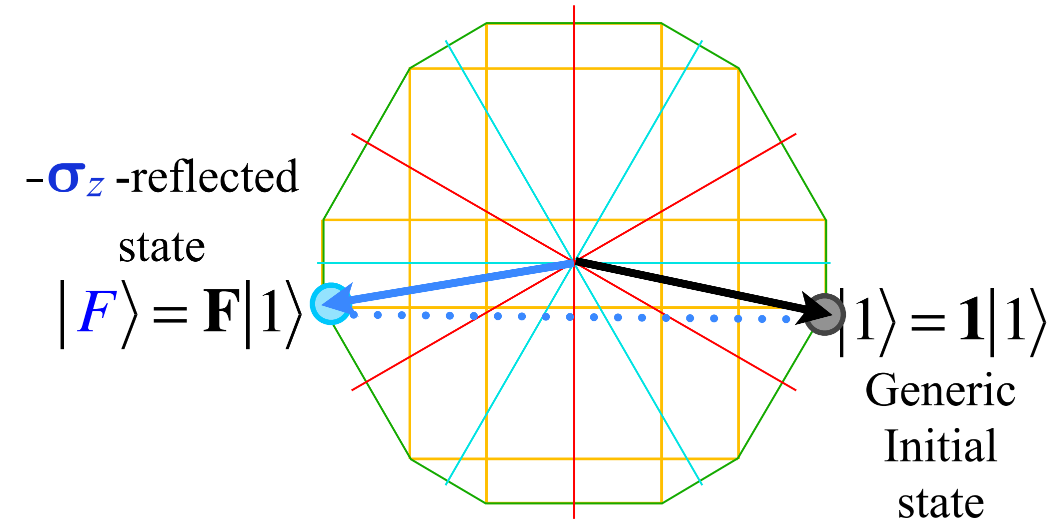
$\bar{\mathbf{e}}_2=\mathbf{M}\cdot\mathbf{e}_2$

$=\boldsymbol{\sigma}_{30°}\cdot\mathbf{e}_2$



Effects of Floor Bang Matrix $\mathbf{F} = -\boldsymbol{\sigma}_z = \begin{pmatrix} \mathbf{e}_1 \cdot \mathbf{F} \cdot \mathbf{e}_1 & \mathbf{e}_1 \cdot \mathbf{F} \cdot \mathbf{e}_2 \\ \mathbf{e}_2 \cdot \mathbf{F} \cdot \mathbf{e}_1 & \mathbf{e}_2 \cdot \mathbf{F} \cdot \mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1 \cdot \bar{\mathbf{e}}_1 & \mathbf{e}_1 \cdot \bar{\mathbf{e}}_2 \\ \mathbf{e}_2 \cdot \bar{\mathbf{e}}_1 & \mathbf{e}_2 \cdot \bar{\mathbf{e}}_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$





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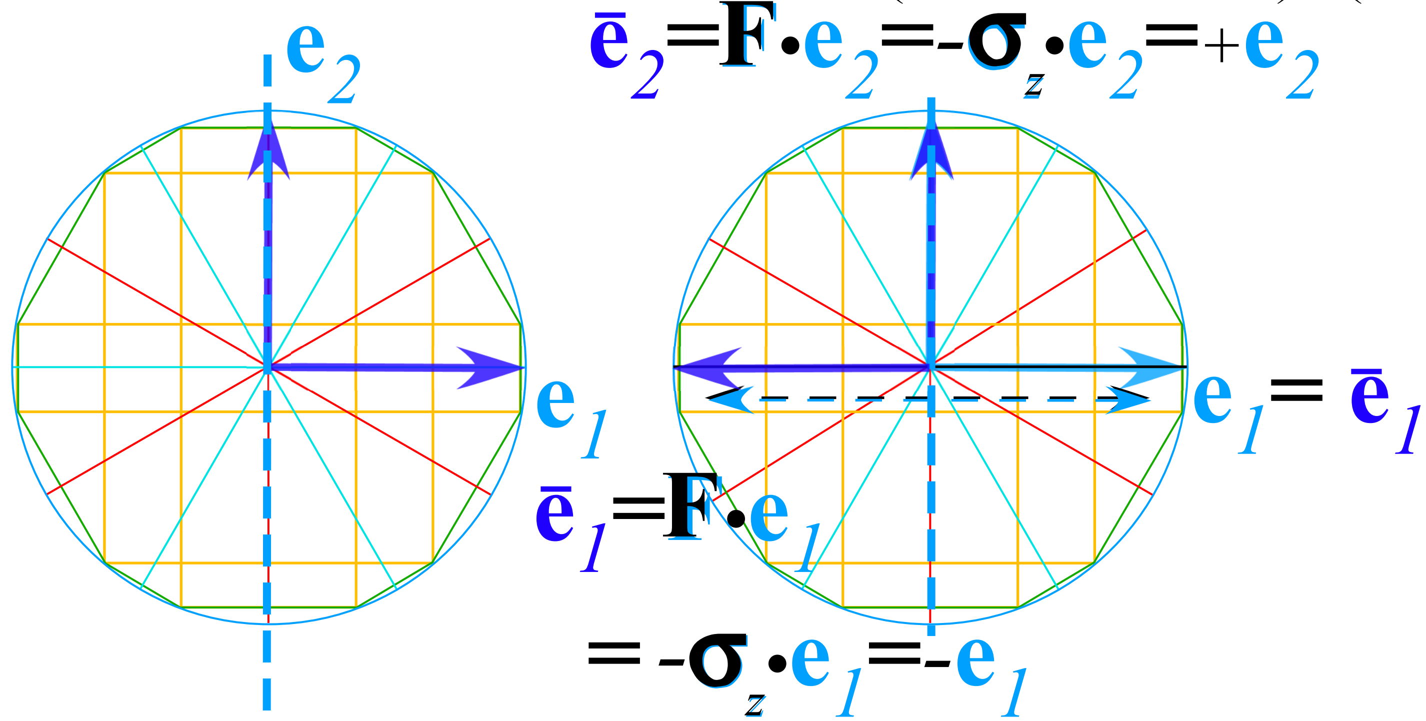
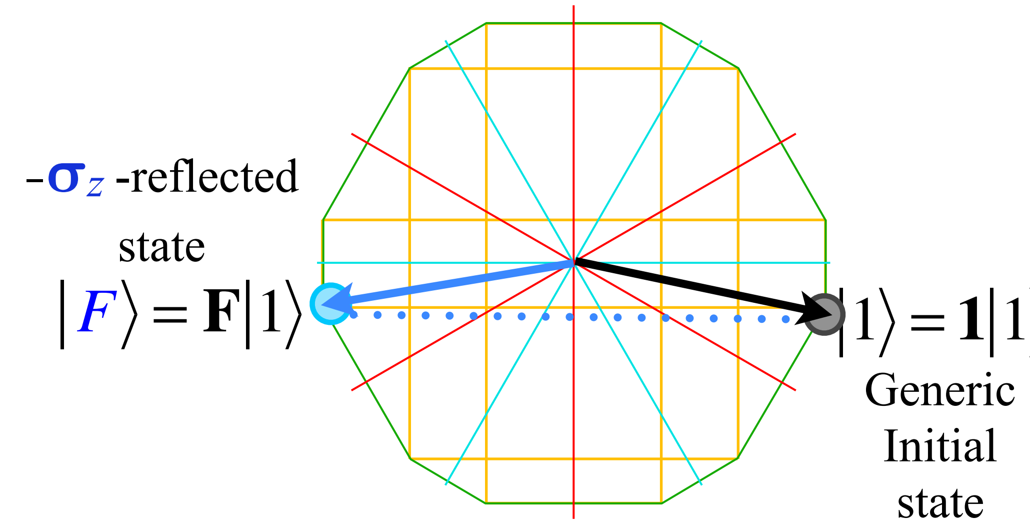
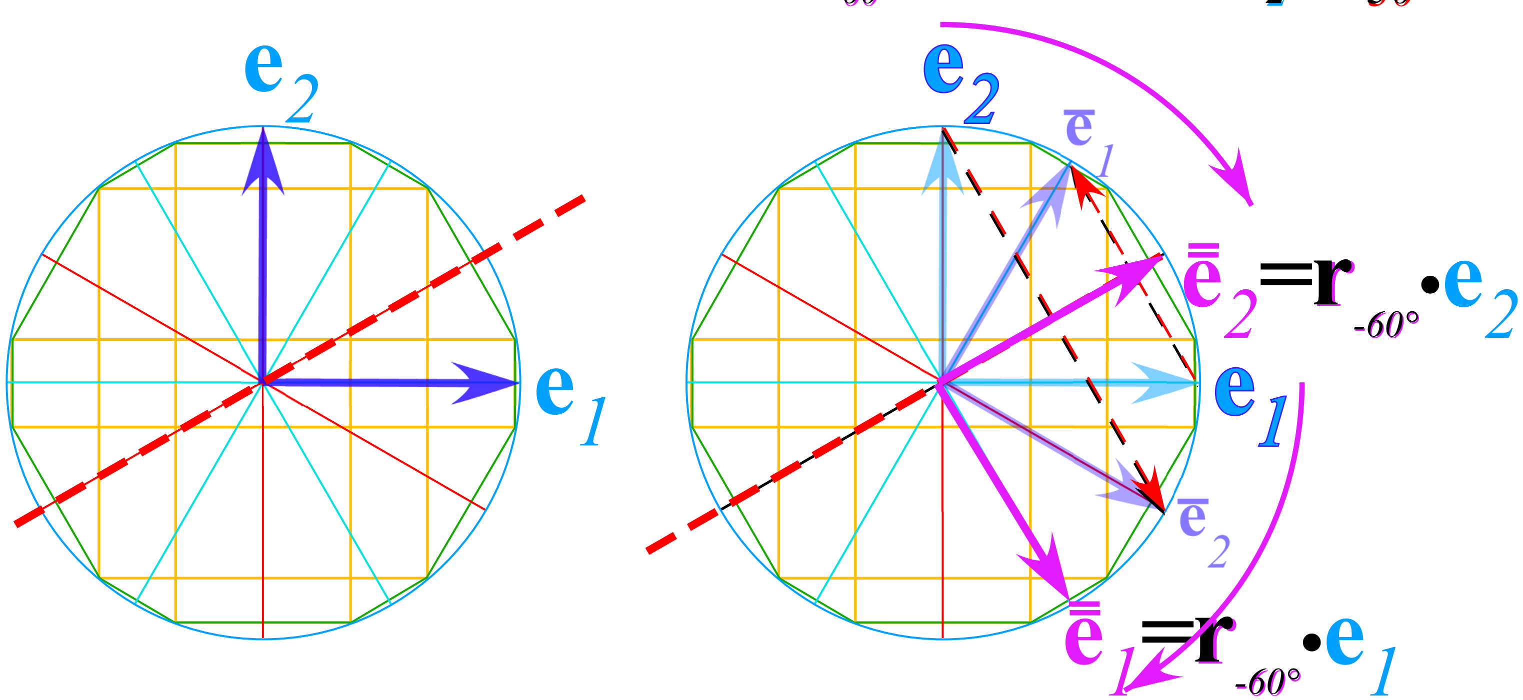
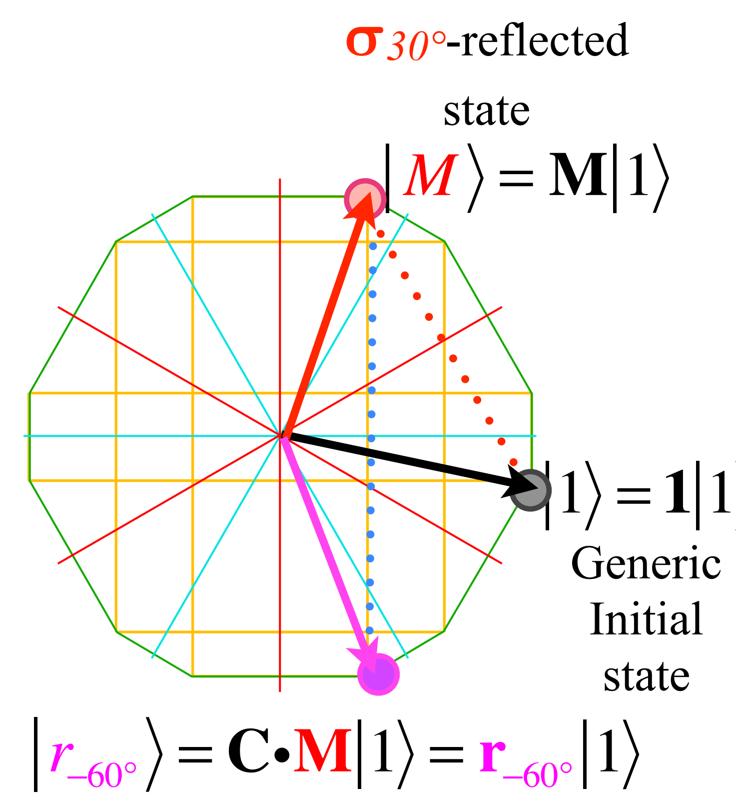




Effects of Ceiling **C** after Bang **M**: $\mathbf{r}_{-60°}=\mathbf{C}\bullet\mathbf{M}=\boldsymbol{\sigma}_z\bullet\boldsymbol{\sigma}_{30°}$





$\boldsymbol{\sigma}_{30°}\ \boldsymbol{\sigma}_{30°}$-reflected state is a $\mathbf{r}_{-60°}$-rotated state

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D_6	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
1	1											
$\bar{r}_{120}$		1										
r_{120}			1									
σ_{60}				1								
$\bar{\sigma}_{60}$					1							
σ_z						1					$\bar{r}_{60}$	
I							1					
r_{60}								1				
$\bar{r}_{60}$									1			
$\bar{\sigma}_{30}$										1		
σ_{30}						r_{60}					1	
$\bar{\sigma}_z$												1

Note: $\bar{r}_{60} = Ir_{120} = r_{120}I = r_{-60}$ and: $I = r_{\pm 180}$

$\bar{r}_{120} = Ir_{60} = r_{60}I = r_{-120}$ and: $I^2 = 1$

$\sigma_{60} = I\bar{\sigma}_{30} = \bar{\sigma}_{30}I$

$\bar{\sigma}_{60} = I\sigma_{30} = \sigma_{30}I$

$\bar{\sigma}_z = I\sigma_z = \sigma_z I$

Easy to make hexagonal (D_6) symmetry group table:

Example 1: Find $\boldsymbol{\sigma}_{30°} \bullet \boldsymbol{\sigma}_{-60°}$ = _____?

Solution: Find $\boldsymbol{\sigma}_{30°}$*-plane and state-*$|\boldsymbol{\sigma}_{-60°}\rangle$

Operate former on latter to get: $\boldsymbol{\sigma}_{30°}|\boldsymbol{\sigma}_{-60°}\rangle = |\mathbf{I}\rangle$

That gives answer: $\boldsymbol{\sigma}_{30°} \bullet \boldsymbol{\sigma}_{-60°} = \mathbf{I}$.

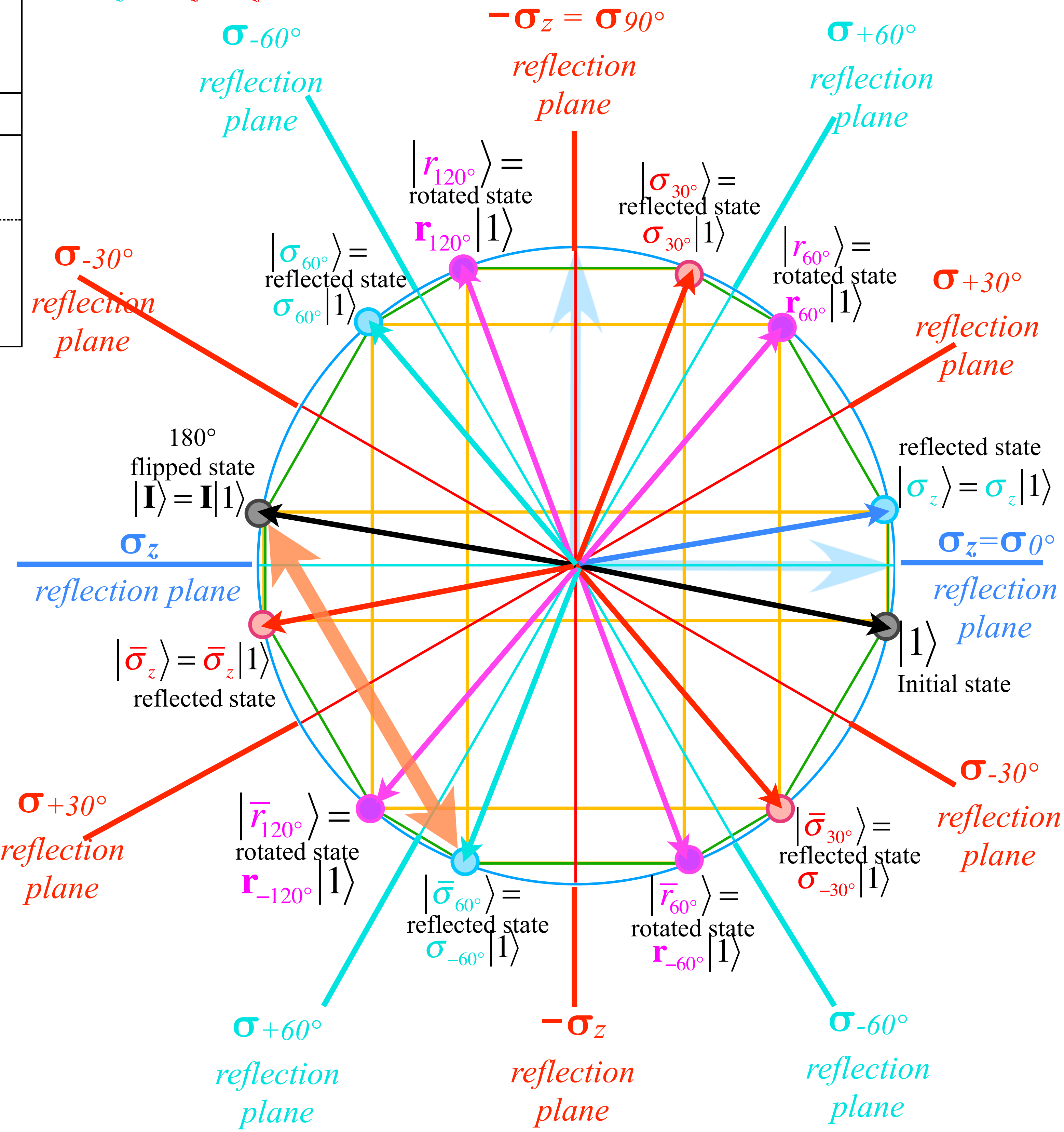
Rest of $\boldsymbol{\sigma}_{30°}$ *row follows:*

11th row	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
σ_{30}	σ_{30}	$\bar{\sigma}_{30}$	$\bar{\sigma}_z$	$\bar{r}_{60}$	I	r_{60}	$\bar{\sigma}_{60}$	σ_{60}	σ_z	r_{120}	1	$\bar{r}_{120}$

Example 2: Find $\mathbf{r}_{60°} \bullet \boldsymbol{\sigma}_{-60°}$ = _____?

Solution: Do $\mathbf{r}_{60°}$*-rotation* $\mathbf{r}_{60°}|\boldsymbol{\sigma}_{-60°}\rangle = |\boldsymbol{\sigma}_{-30°}\rangle$

That gives answer: $\mathbf{r}_{60°} \bullet \boldsymbol{\sigma}_{-60°} = \boldsymbol{\sigma}_{-30°}$



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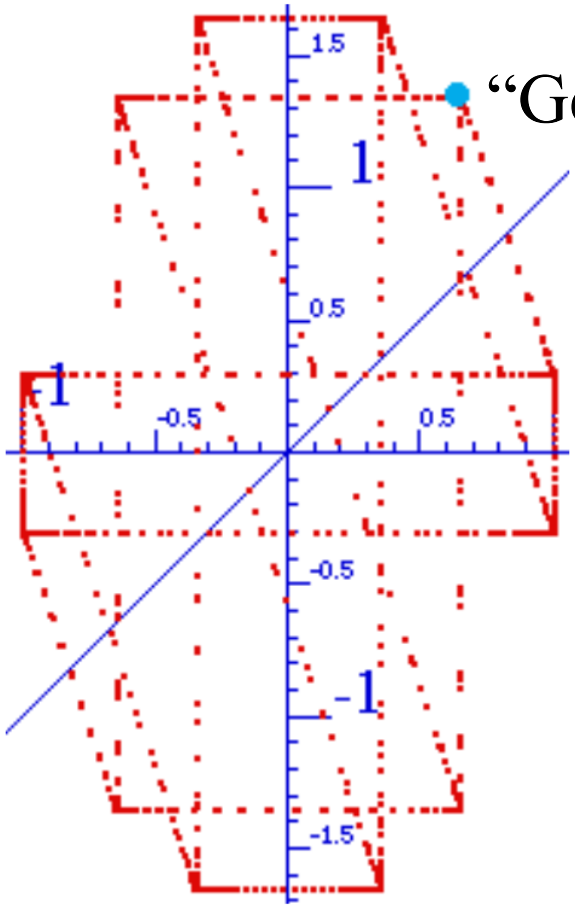
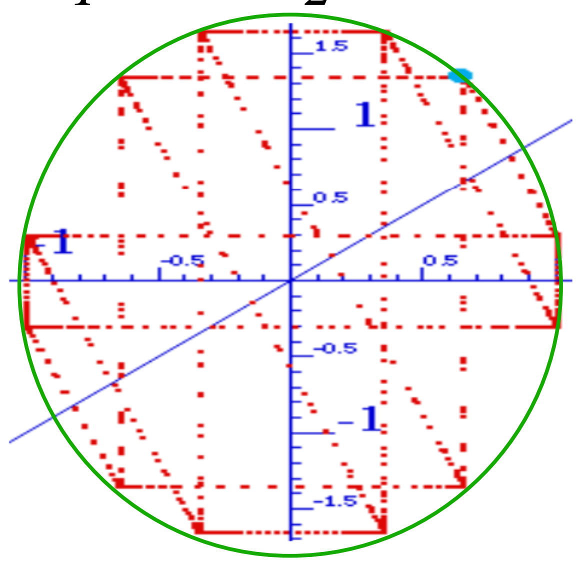
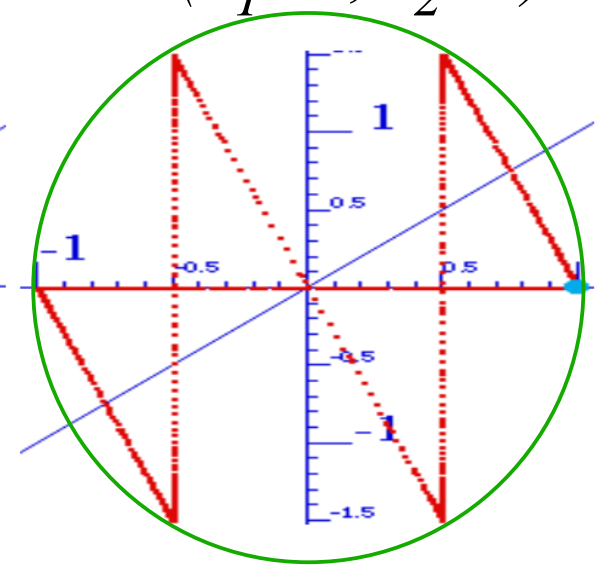


"Generic" initial velocity $(v_1=1.0, v_2=0.1)$

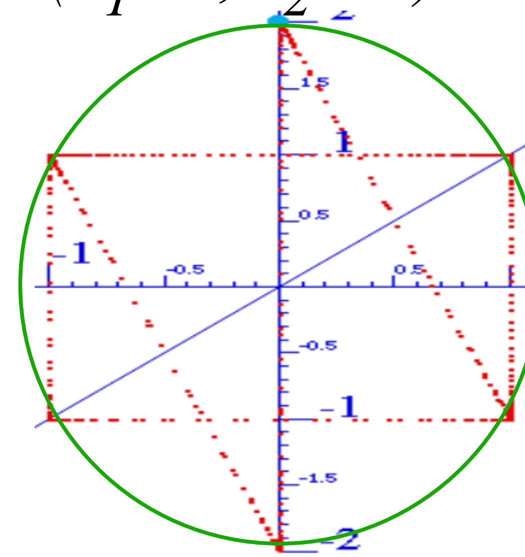
"Symmetric" initial velocity $(v_1=1, v_2=0)$ or $(v_1=1, v_2=-1)$



$(v_1, v_2)=(1, 0.1)$



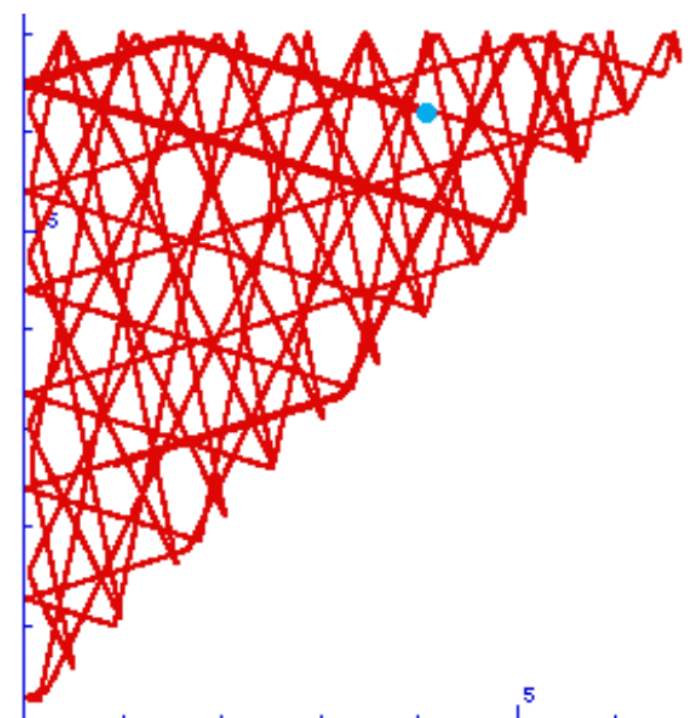
$(v_1, v_2)=(1, 0)$



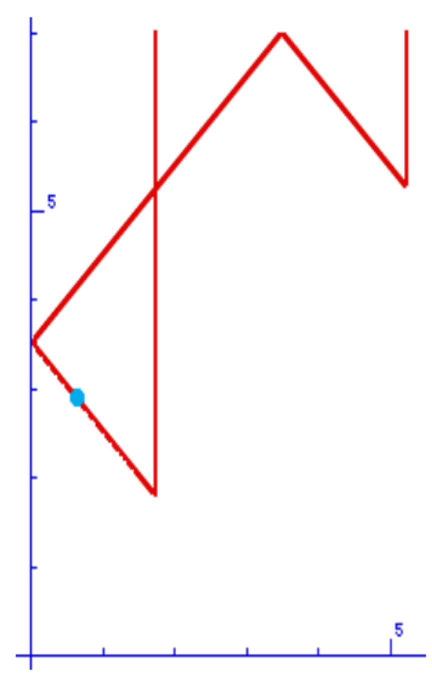
$(v_1, v_2)=(1, -1)$

BounceIt $m_1:m_2 = 3:1$ Dual plots *v_2 vs v_1 and V_2 vs V_1*

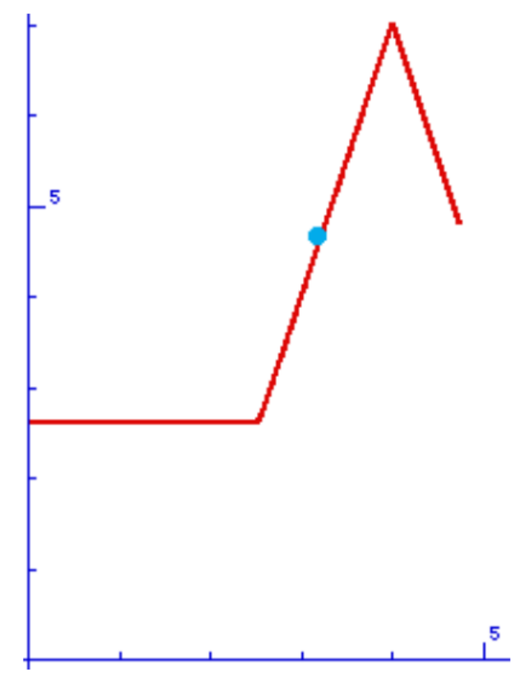
Corresponding space-space (y_1, y_2) paths



$(v_1, v_2)=(1, 0.1)$



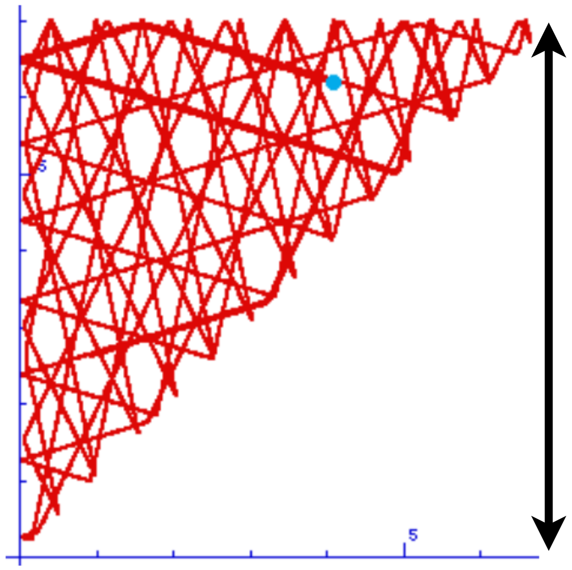
$(v_1, v_2)=(1, -1)$

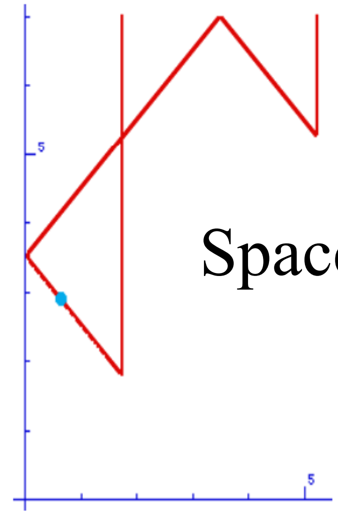


$(v_1, v_2)=(1, 0)$

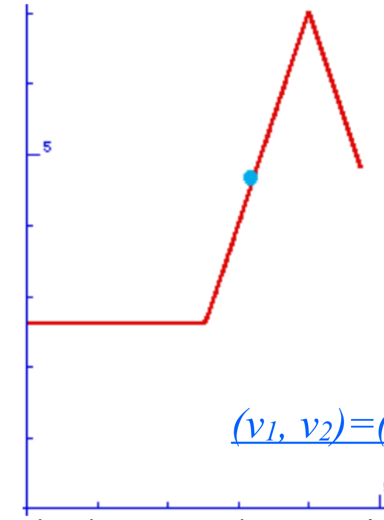
BounceIt $m_1:m_2 = 3:1$ *y_2 vs y_1 plots*

Collisions for mass ratio $m_1:m_2= 3:1$





Space-space (y_1, y_2) paths



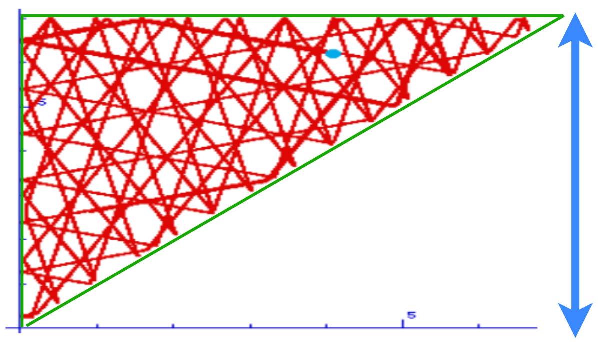
$(v_1, v_2)=(1, -1)$

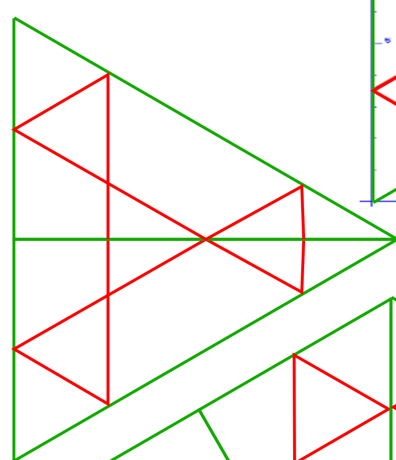
$(v_1, v_2)=(1, 0)$

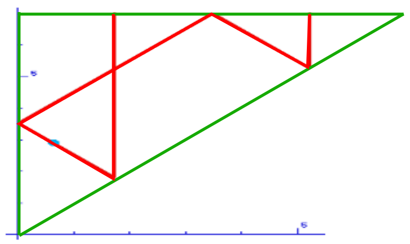
BounceIt Web Simulations
$m_1:m_2 = 3:1$
y_2 vs y_1 plots

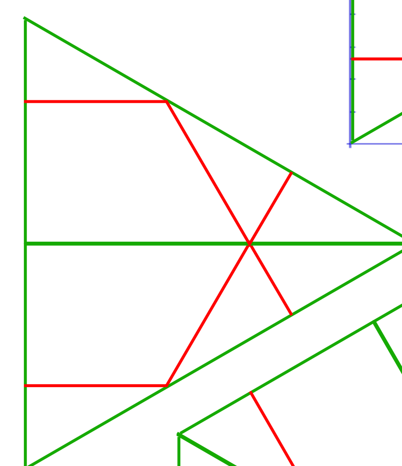
Space-space (y_1, y_2) paths scaled down by $1/\sqrt{3}$...

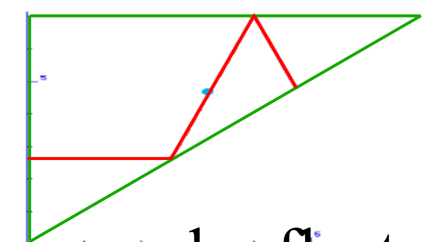
Scaled y down by $1/\sqrt{3}=0.577$



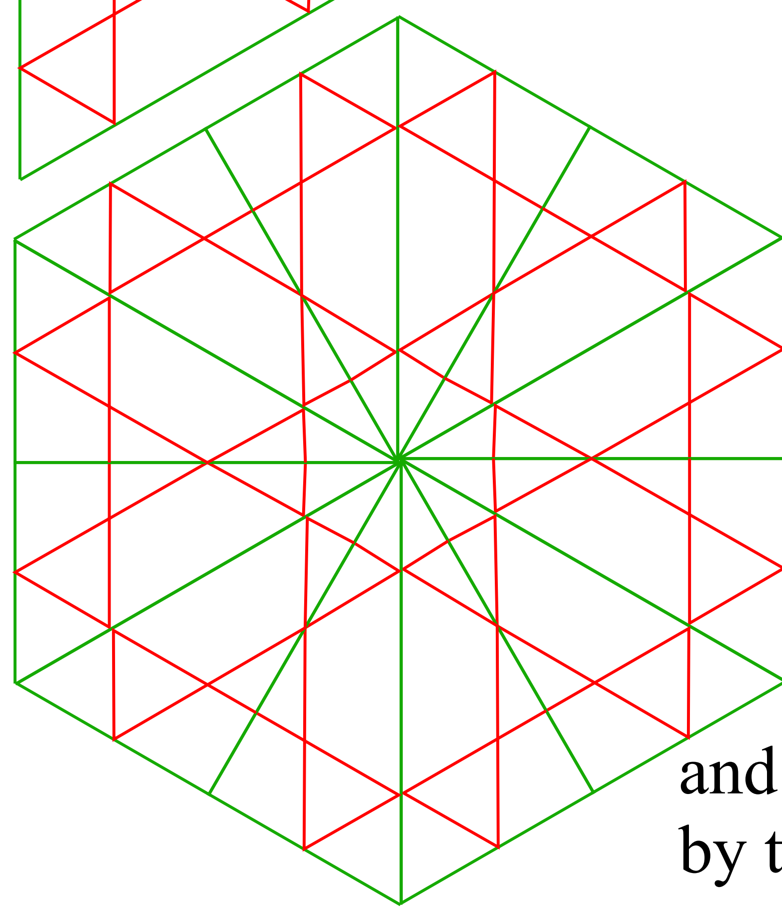


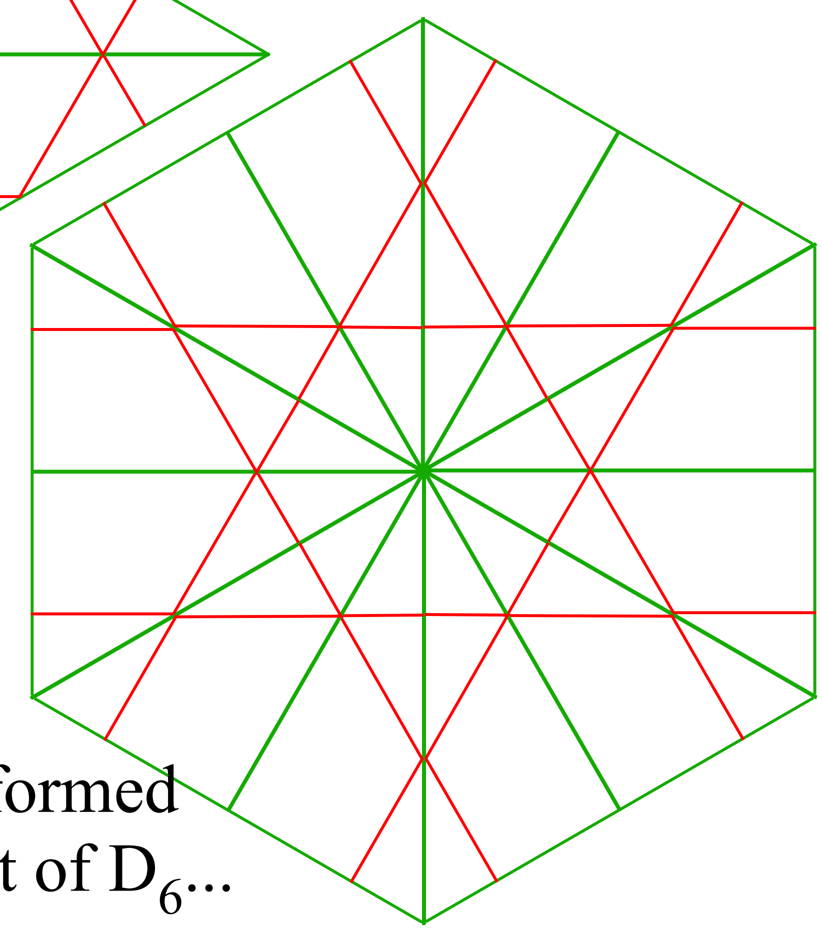






and reflected...



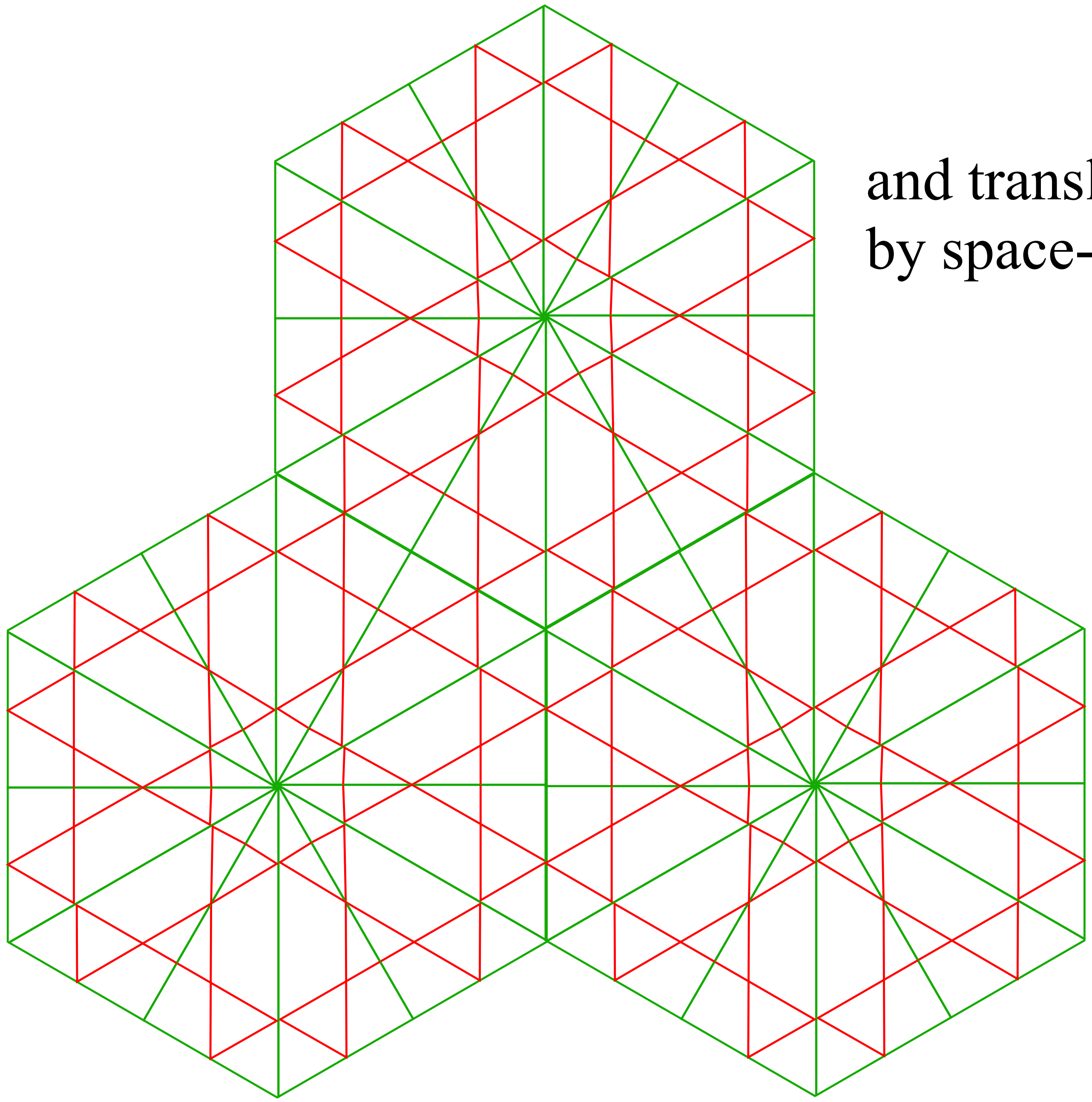


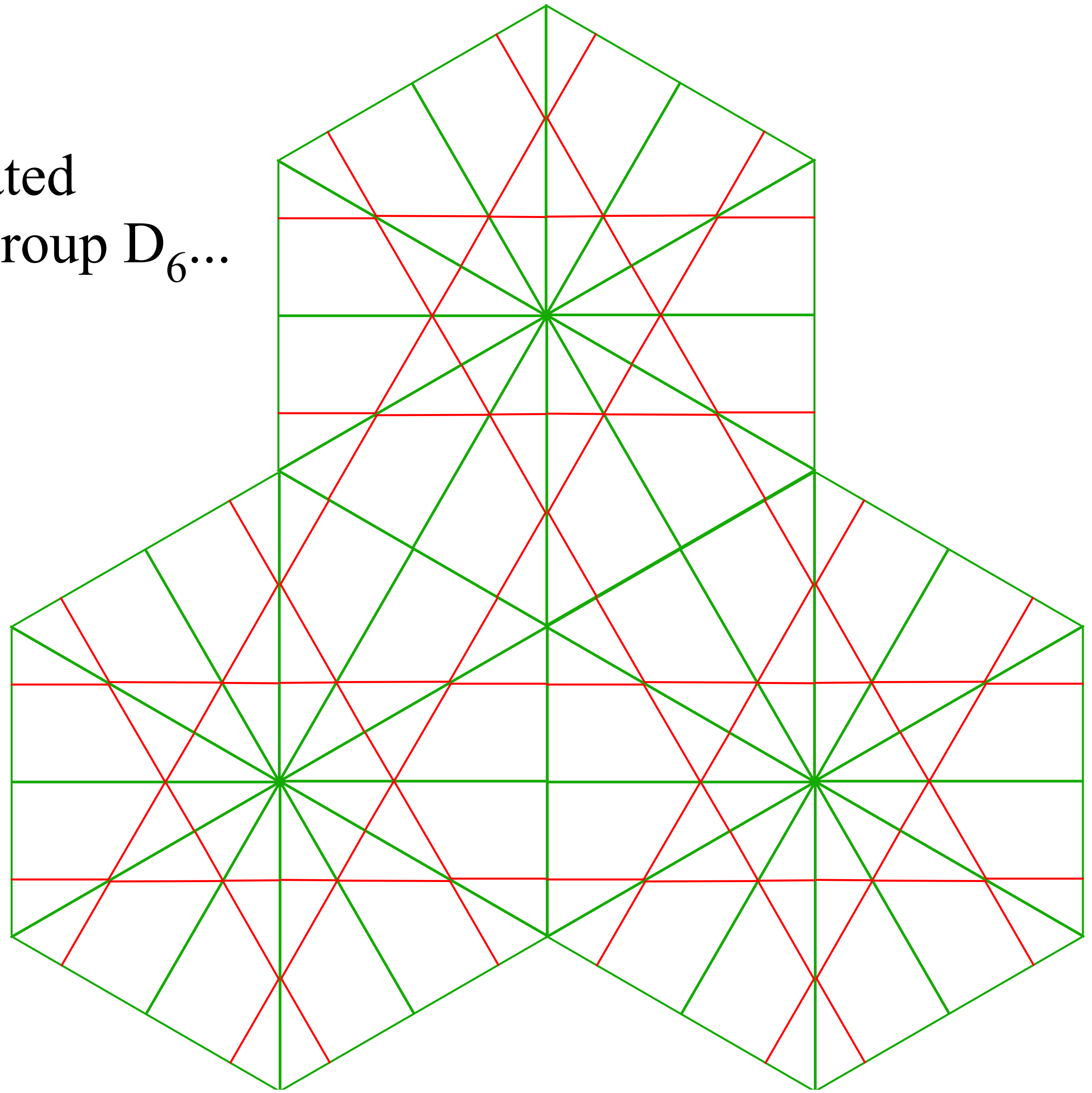
..or could have scaled x up by $\sqrt{3}=1.732$

and transformed by the rest of D_6...

Collisions for mass ratio $m_1:m_2 = 3:1$

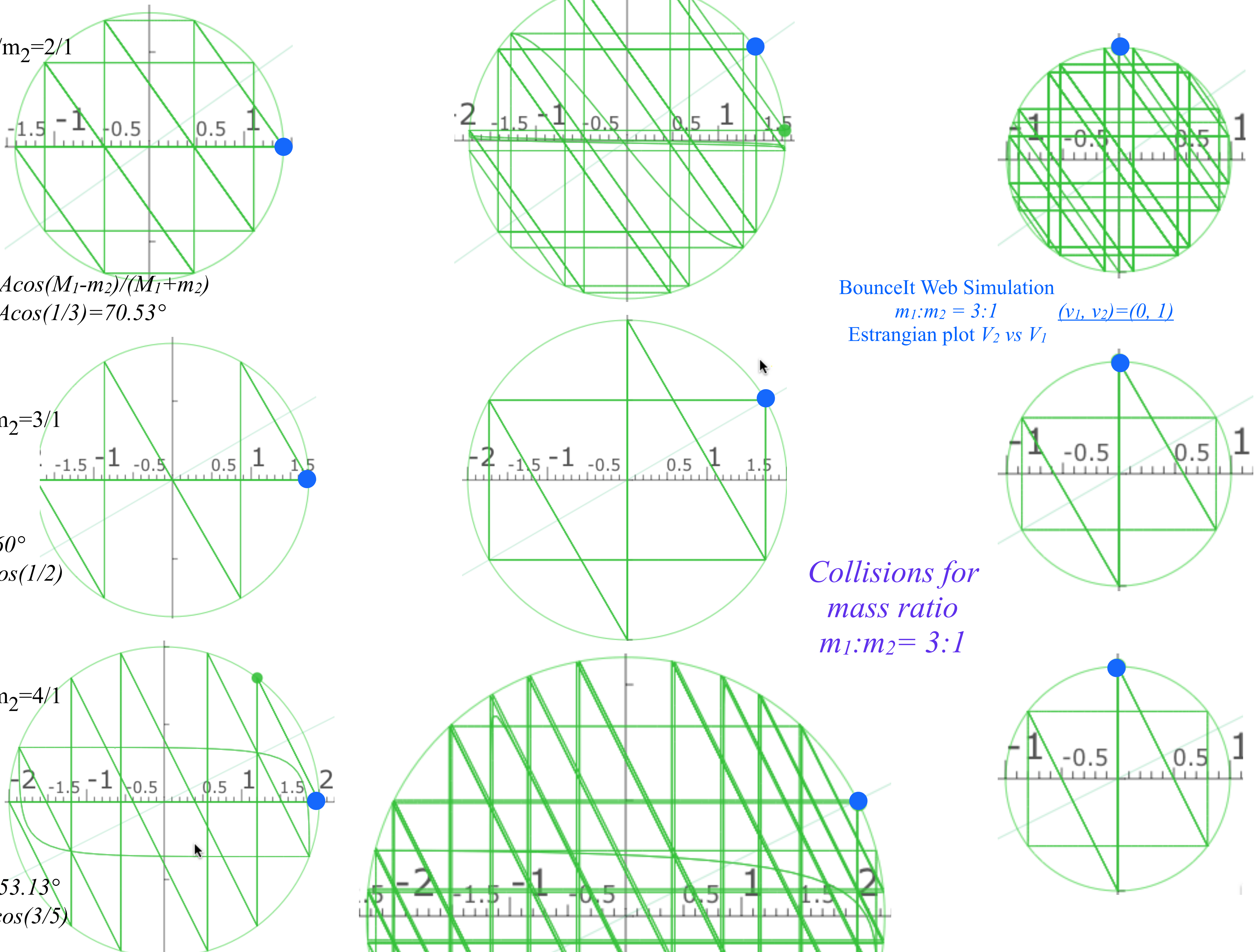
and translated by space-group D_6...





...they're just straight lines going forever.

Initial velocity $v_1=1, v_2=0$ | *Initial velocity $v_1=1, v_2=1$* | *Initial velocity $v_1=0, v_2=1$*



$M_1/m_2=2/1$

$\phi = Acos(M_1-m_2)/(M_1+m_2)$
$= Acos(1/3)=70.53°$

$M_1/m_2=3/1$

$\phi = 60°$
$= Acos(1/2)$

$M_1/m_2=4/1$

$\phi = 53.13°$
$= Acos(3/5)$

BounceIt Web Simulation
$m_1:m_2 = 3:1$ $(v_1, v_2)=(0, 1)$
Estrangian plot V_2 vs V_1

Collisions for mass ratio $m_1:m_2= 3:1$

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

→ *Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$* ←

Geometric "Integration" (Converting Velocity data to Spacetime)

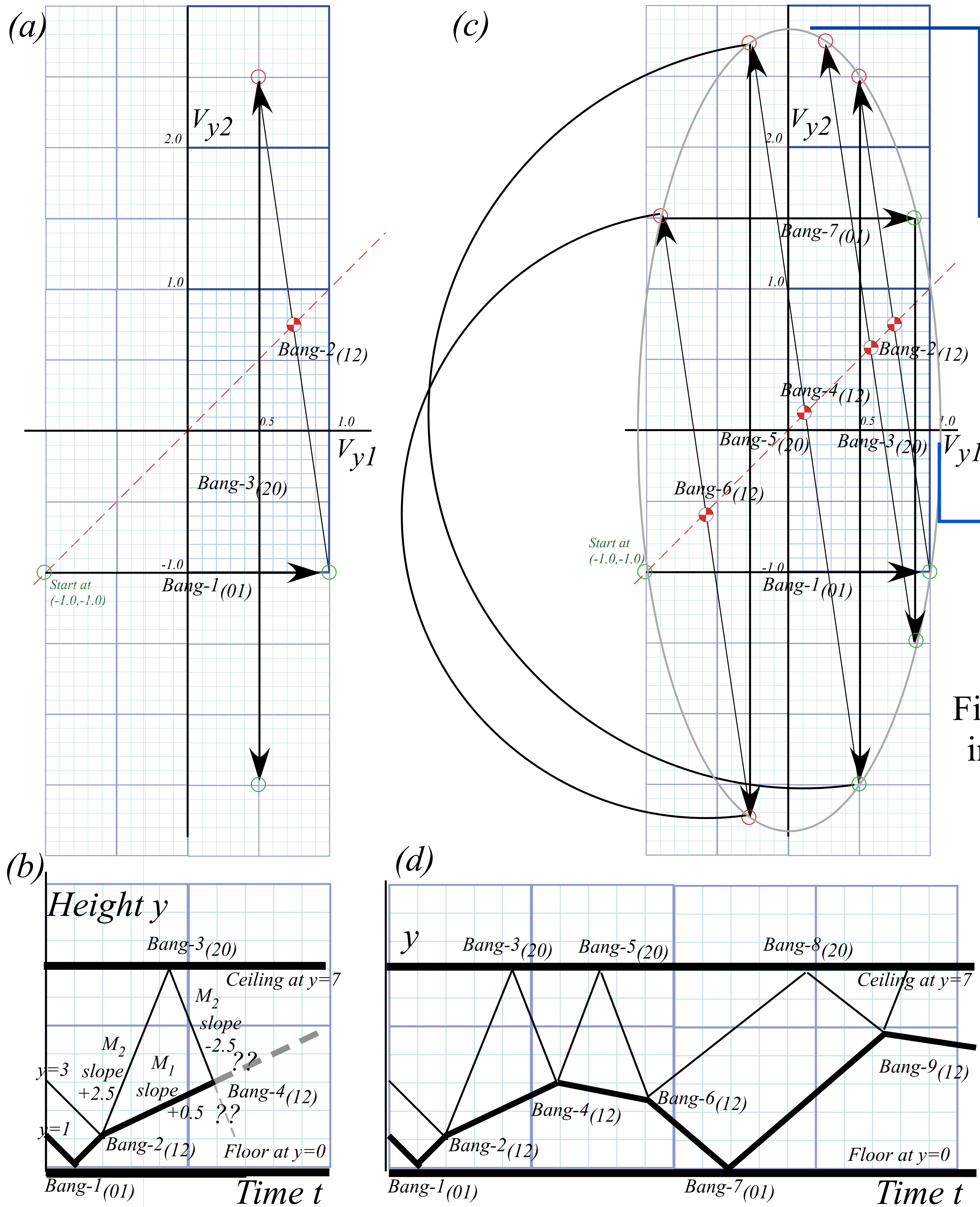


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_2} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Collisions for mass ratio $m_1 : m_2 = 7:1$

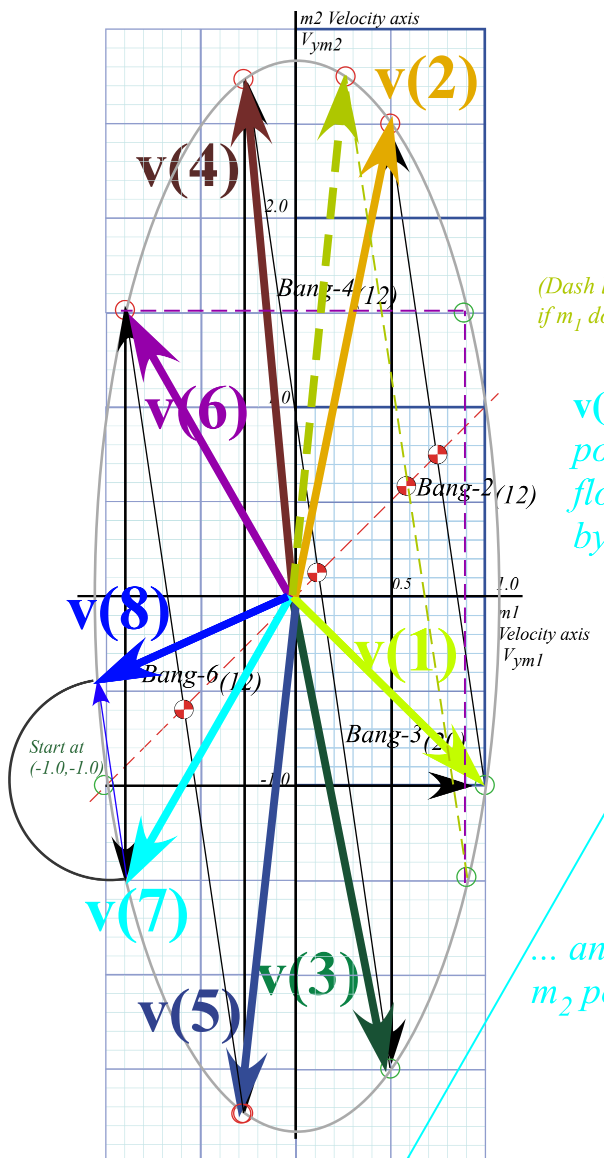
Collisions for mass ratio m_1:m_2= 7:1

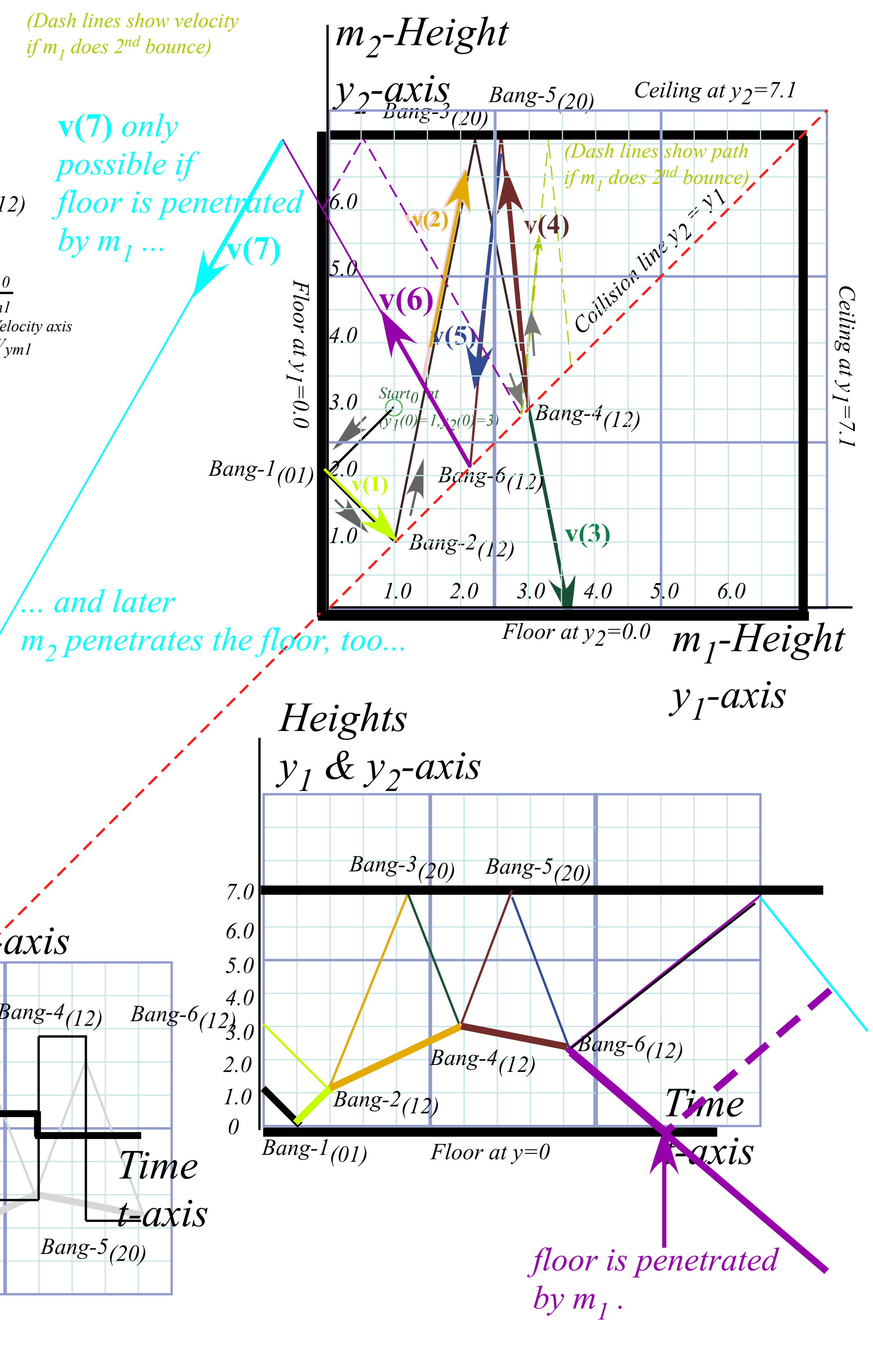
Step-2: Extend **v***(2) line to ceiling point* **y***(3)=(?,7.1) and draw Bang-3*$_{(20)}$ *velocity* **v***(3)=(1,-1) line. (Find* **v***(3) using V-V plot.)*

Step-3: Extend **v***(3) line to collision point* **y***(4)=(?,?) and draw Bang-4*$_{(12)}$ *velocity* **v***(4)=(0.5,2.5). (Find* **v***(4) using V-V plot.)*

Step-4: Extend **v***(4) line to ceiling point* **y***(4)=(?,7.1) and draw Bang-5*$_{(20)}$ *velocity* **v***(5)=(1,-1) line. (Find* **v***(5) using V-V plot.)*

Step-5: Extend **v***(5) line to collision point* **y***(6)=(?,?) and draw Bang-6*$_{(12)}$ *velocity* **v***(6)=(0.5,2.5). (Find* **v***(6) using V-V plot.)*





Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

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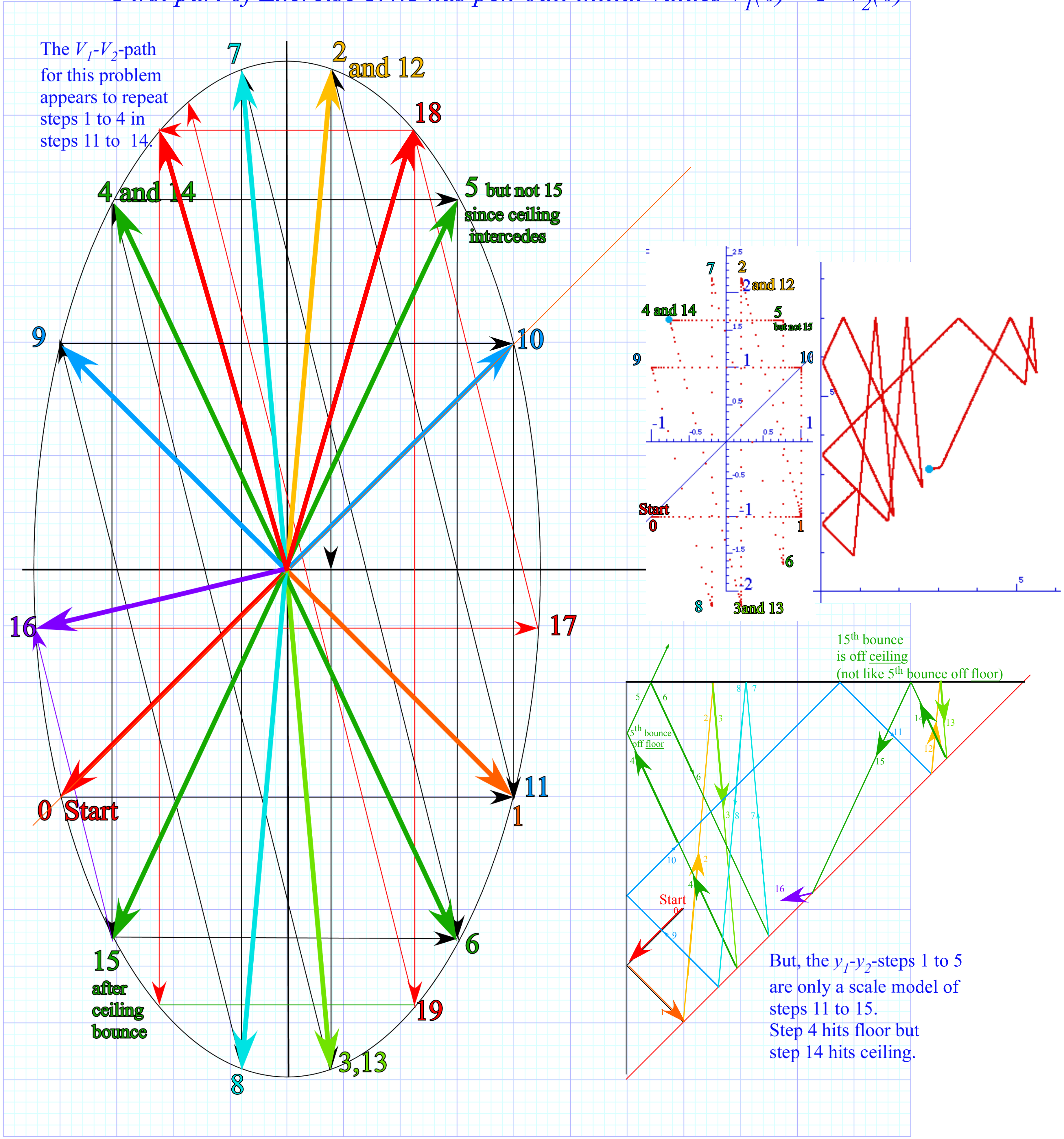
Group multiplication and product table

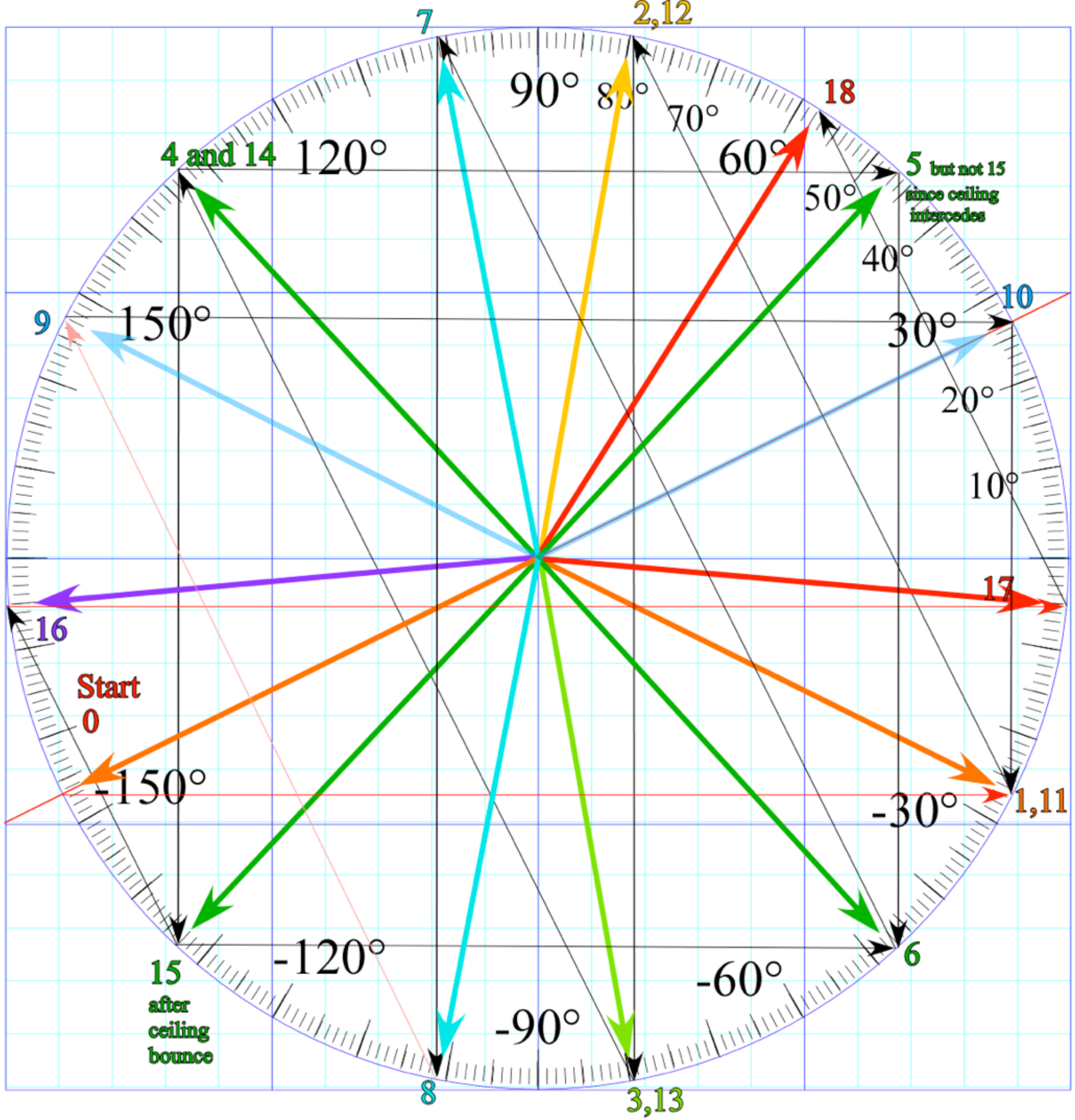
Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

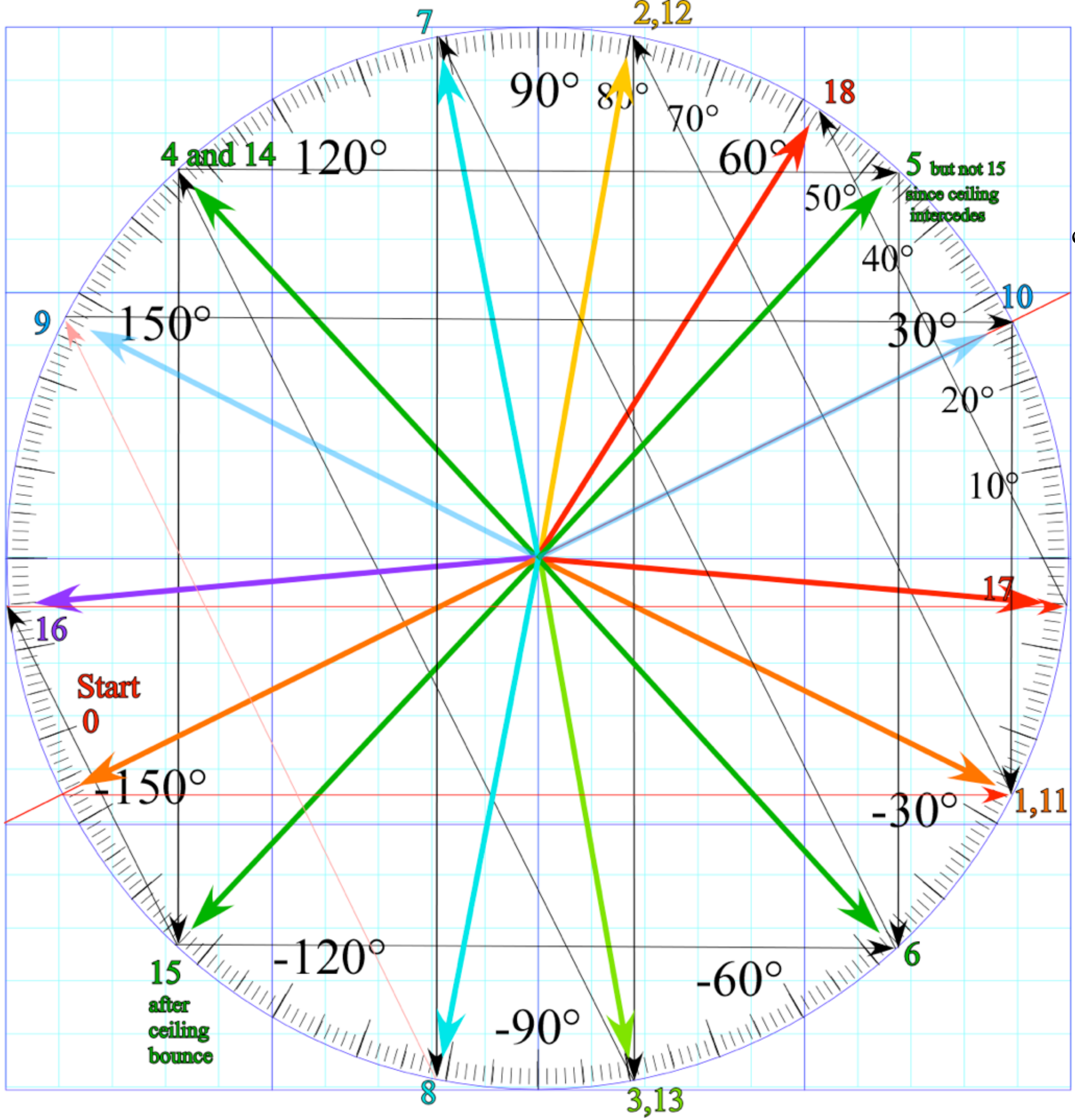
Collisions for mass ratio $m_1{:}m_2 = 4{:}1$

First part of Exercise 1.4.1 has pen-ball initial values $v_1(0)=-1=v_2(0)$





Collisions for mass ratio $m_1 : m_2 = 4:1$



Collisions for mass ratio m_1:m_2= 4:1

$$\cos\theta = \frac{m_1 - m_2}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = \frac{3}{5} = 0.6$$

$$\theta = 53.13°$$

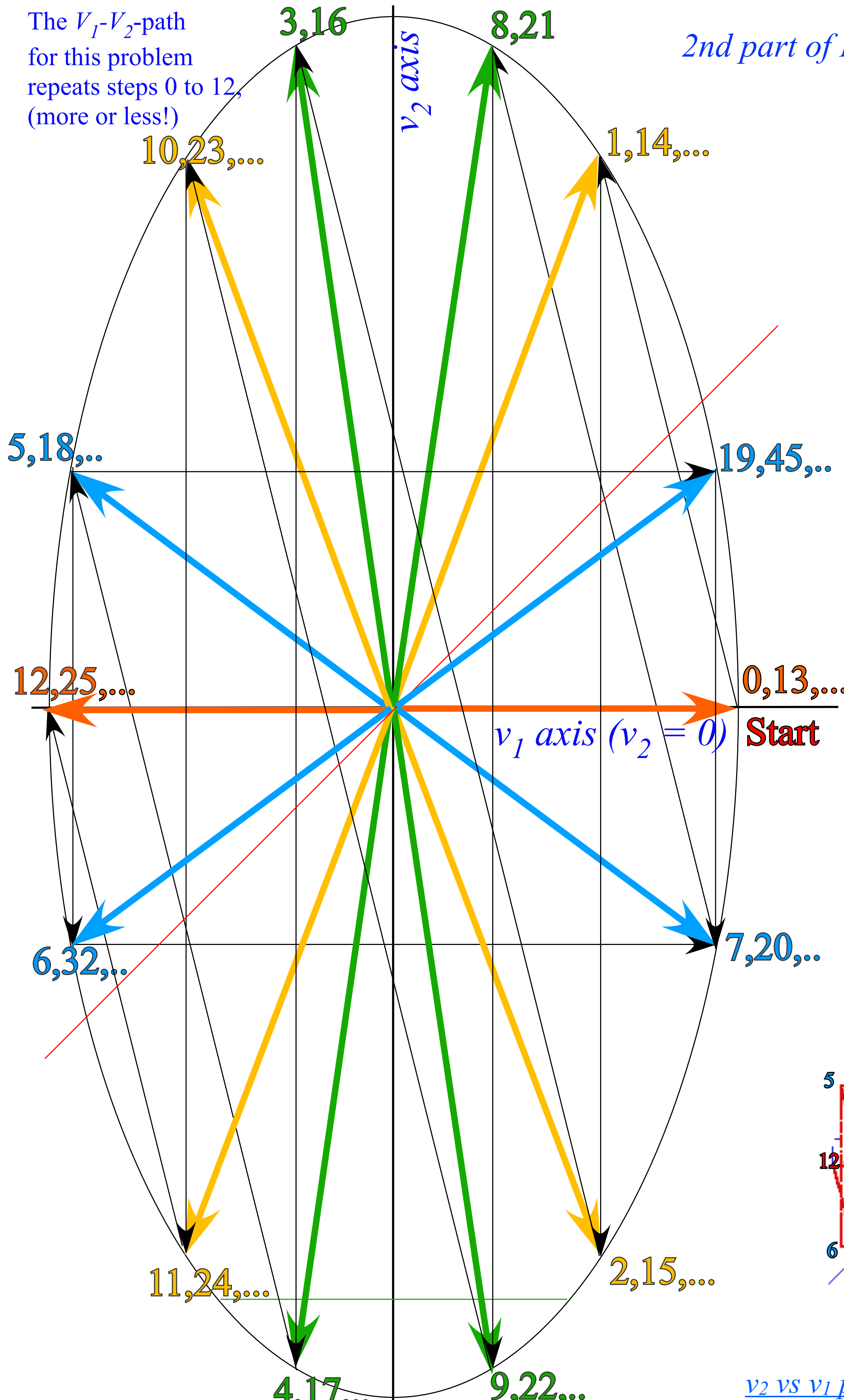
$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

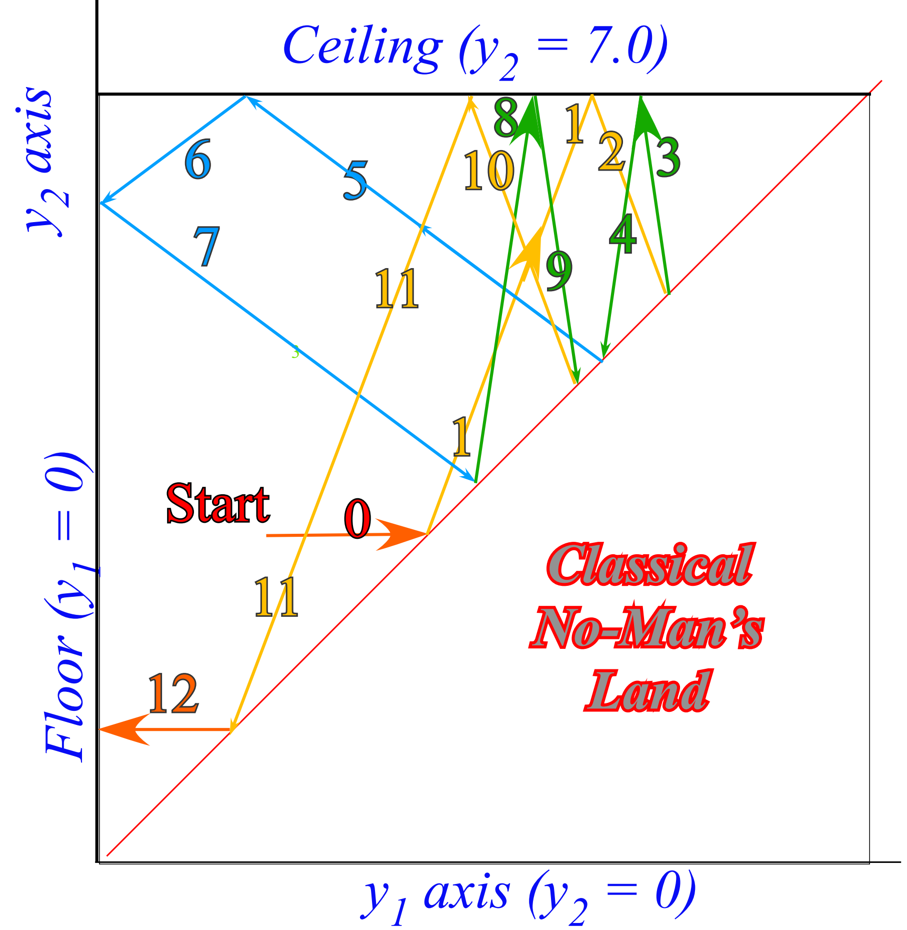
$$\alpha = \theta/2 = 26.565°$$

$$\frac{m_1}{m_2} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{8/5}{2/5} = 4$$

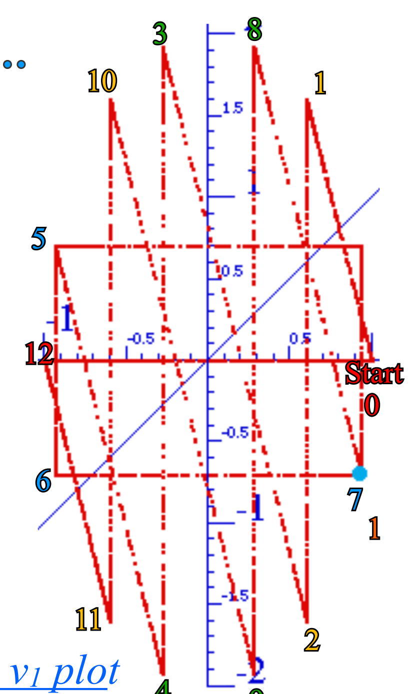
2nd part of Exercise 1.4.1 has pen-ball initial velocity values $v_1(0)=1$ and $v_2(0)=0$
at: $x_1(0)=1.5$ and $x_2(0)=3.0$

Collisions for mass ratio $m_1:m_2 = 4:1$

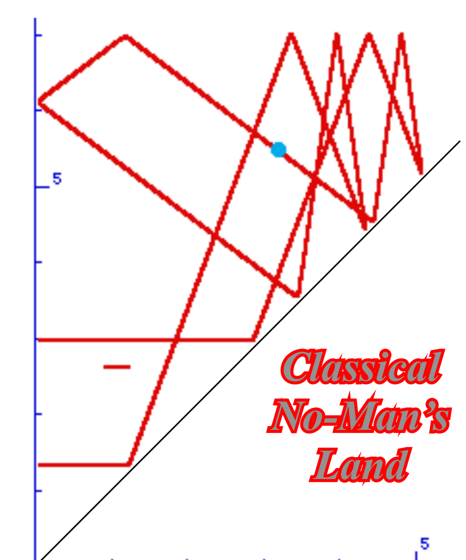




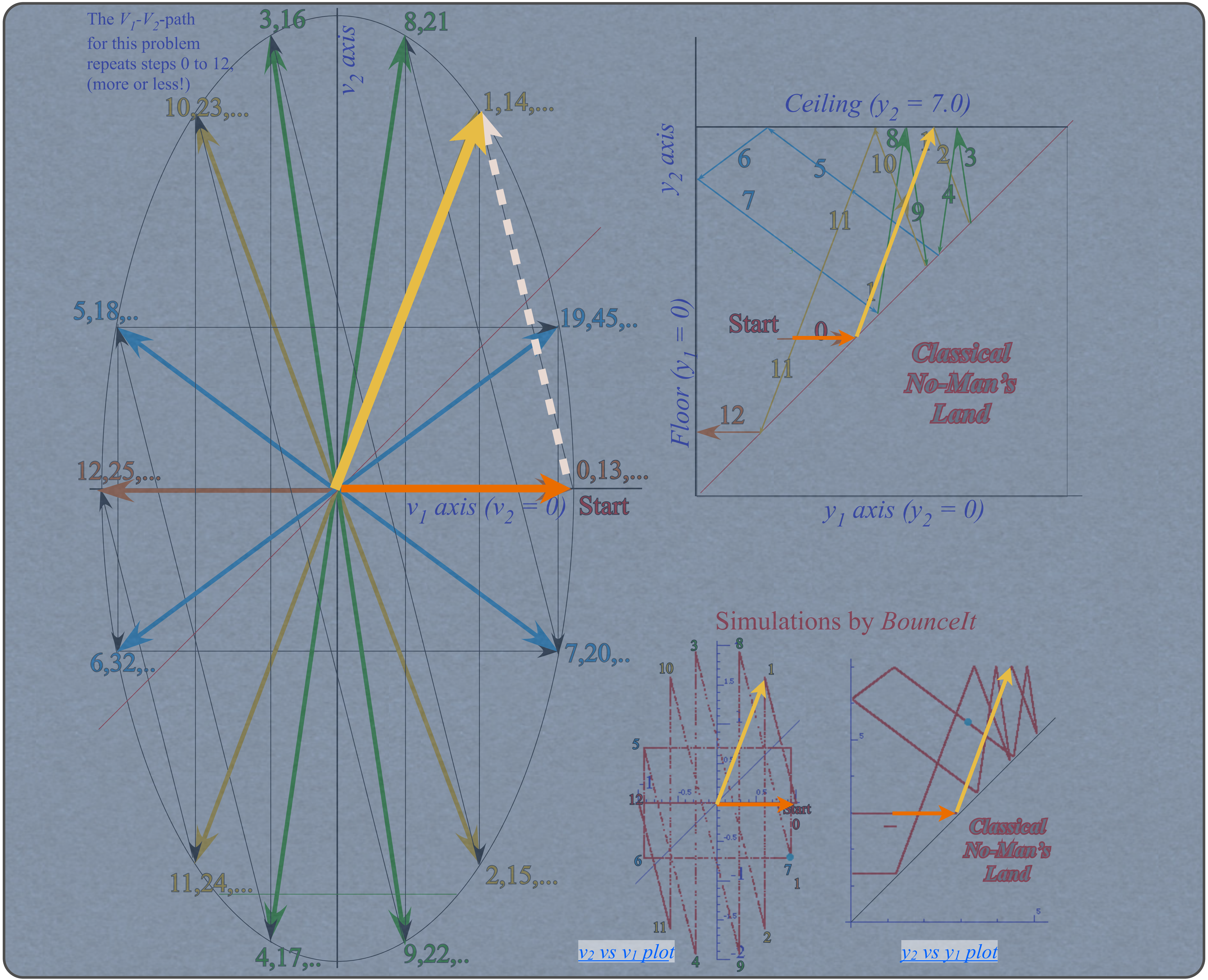
BounceIt Web Simulations
$m_1:m_2 = 4:1$ $(v_1, v_2)=(1, 0)$



v_2 vs v_1 plot



y_2 vs y_1 plot


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
v_1 axis ($v_2 = 0$)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling ($y_2 = 7.0$)
y_2 axis
Floor ($y_1 = 0$)
Start
Classical
No-Man's
Land
y_1 axis ($y_2 = 0$)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
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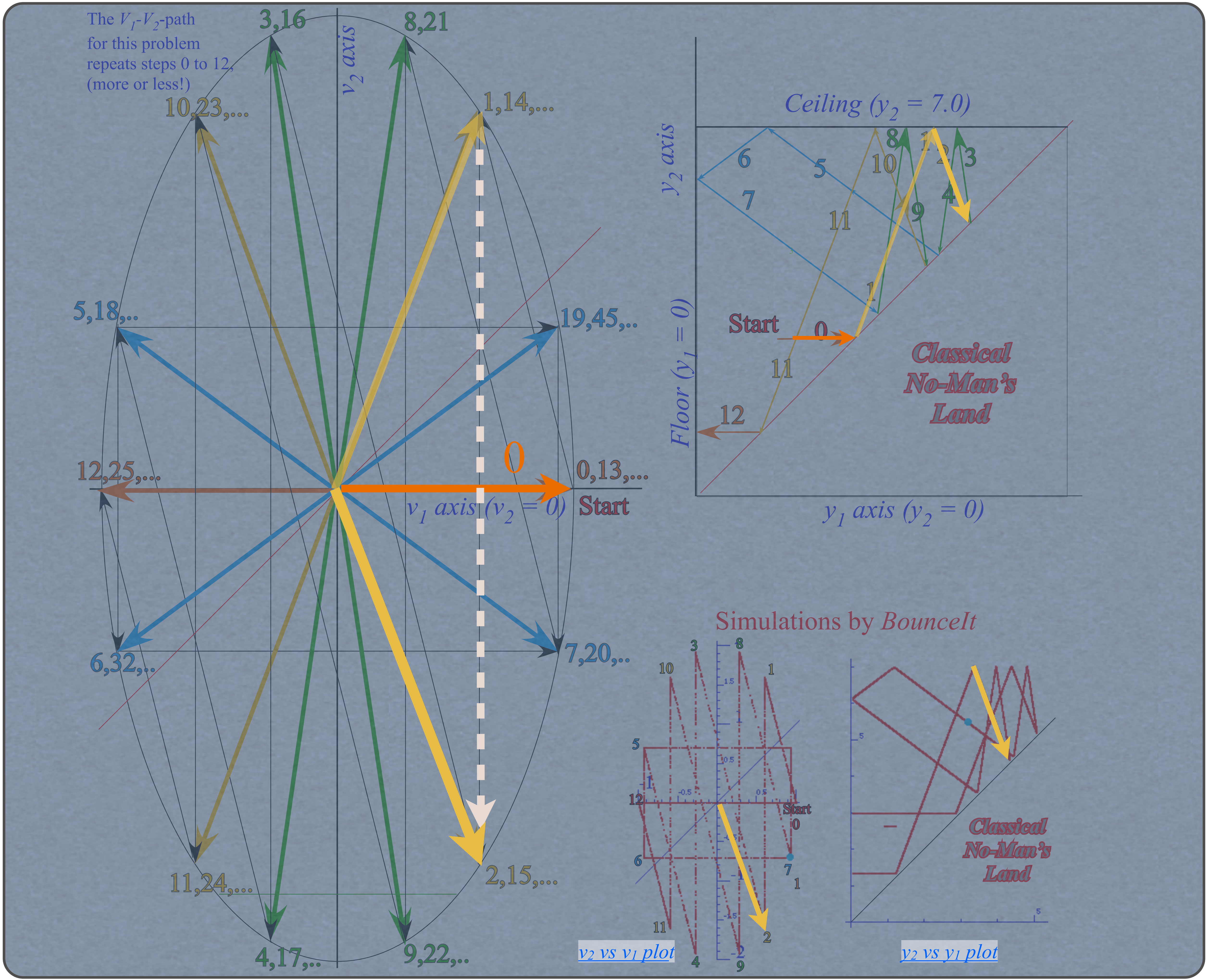
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3,16
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0,13,...
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v1 axis (v2 = 0)
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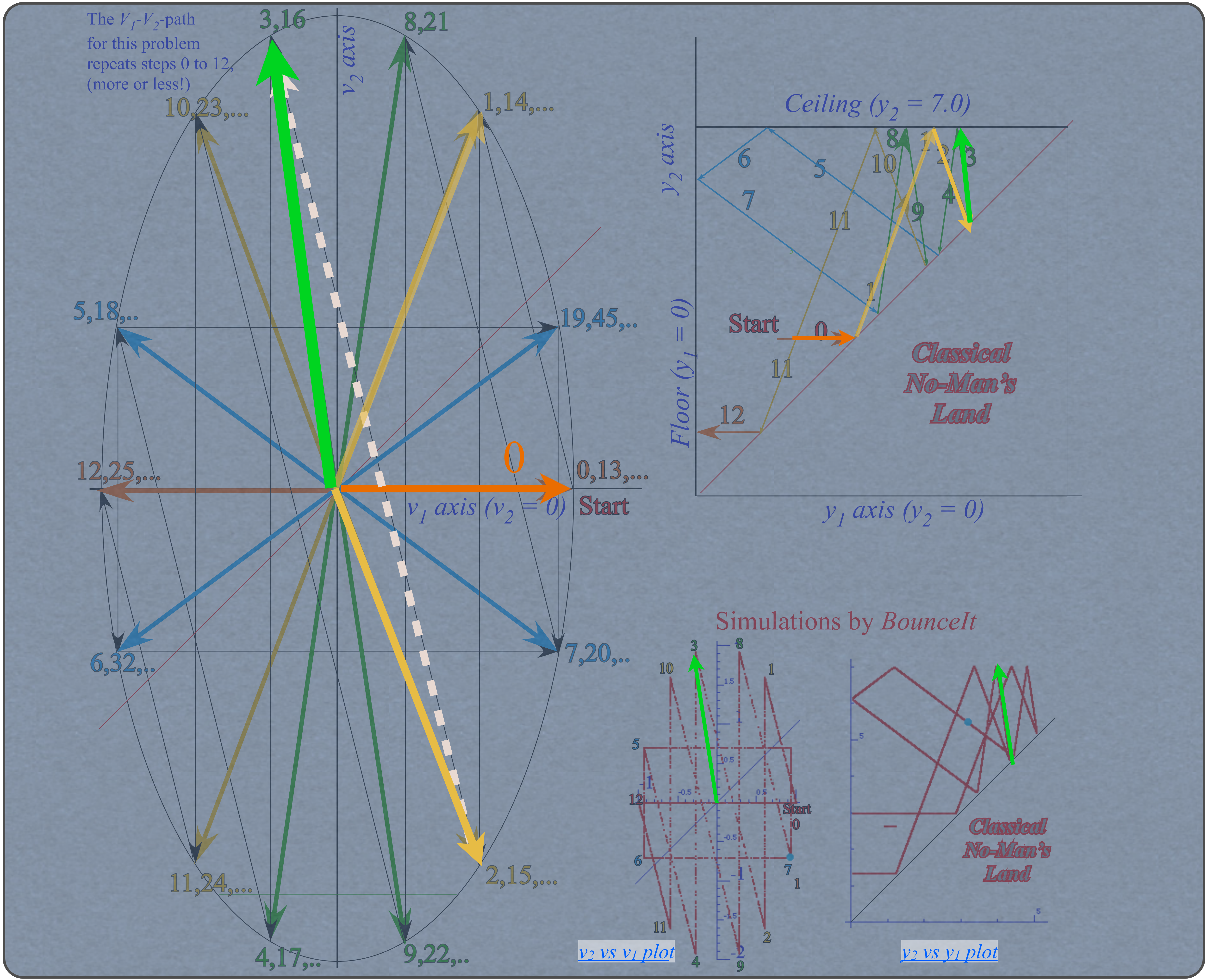
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical No-Man's Land
y1 axis (y2 = 0)
Simulations by BounceIt
v2 vs v1 plot
y2 vs y1 plot



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y_1 axis (y_2 = 0)
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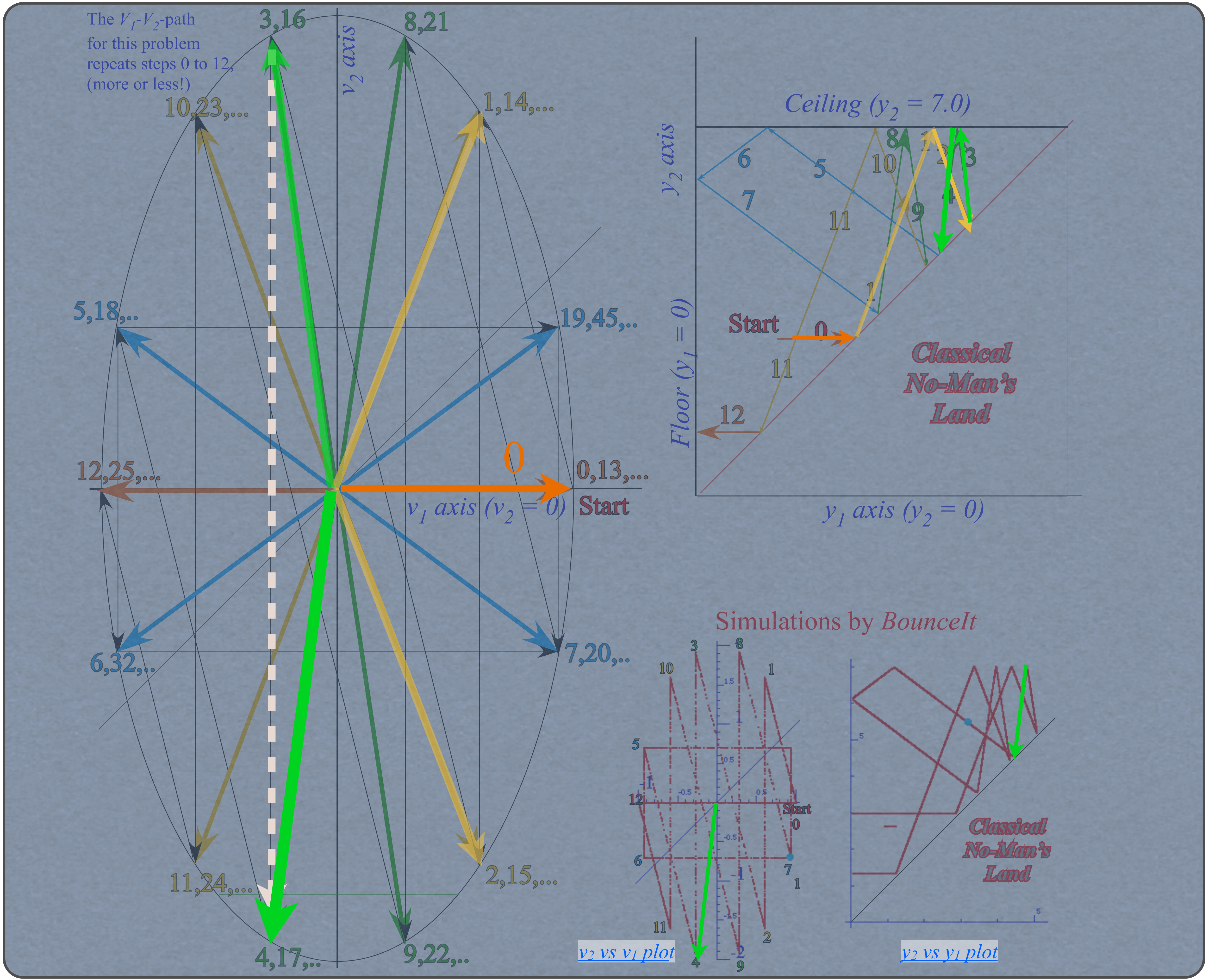
Classical No-Man's Land
v_2 vs v_1 plot
y_2 vs y_1 plot



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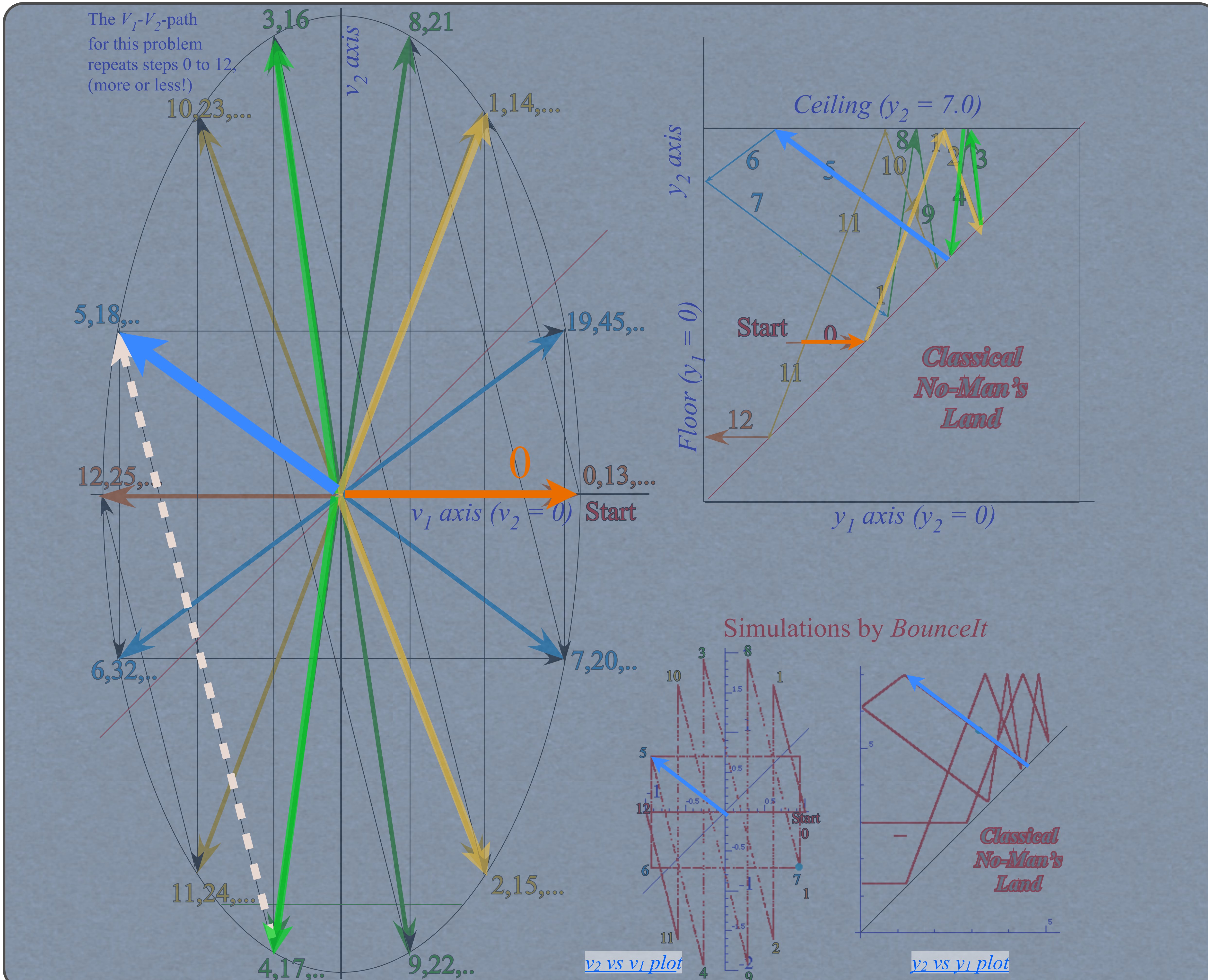
v_2 vs v_1 plot
y_2 vs y_1 plot



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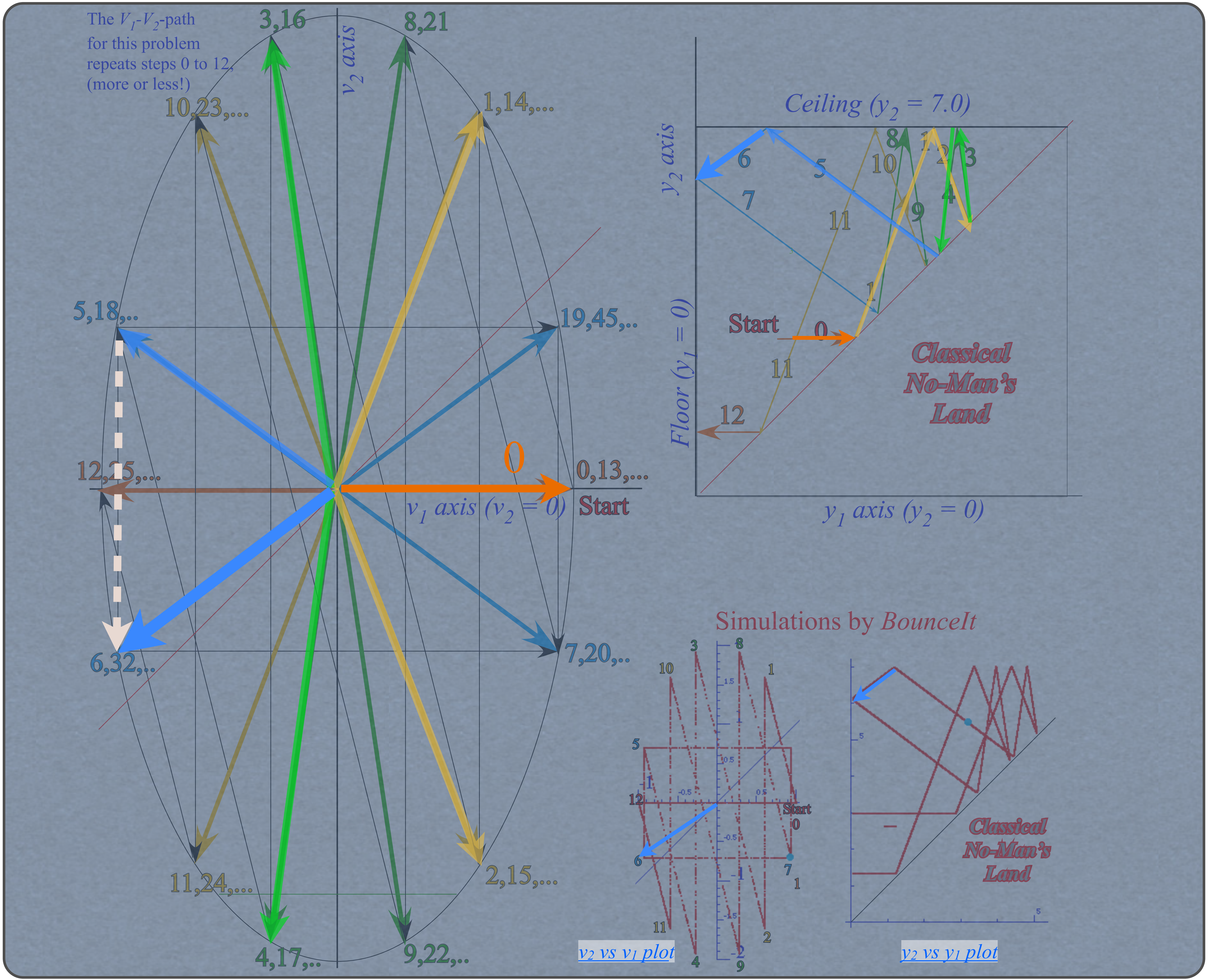
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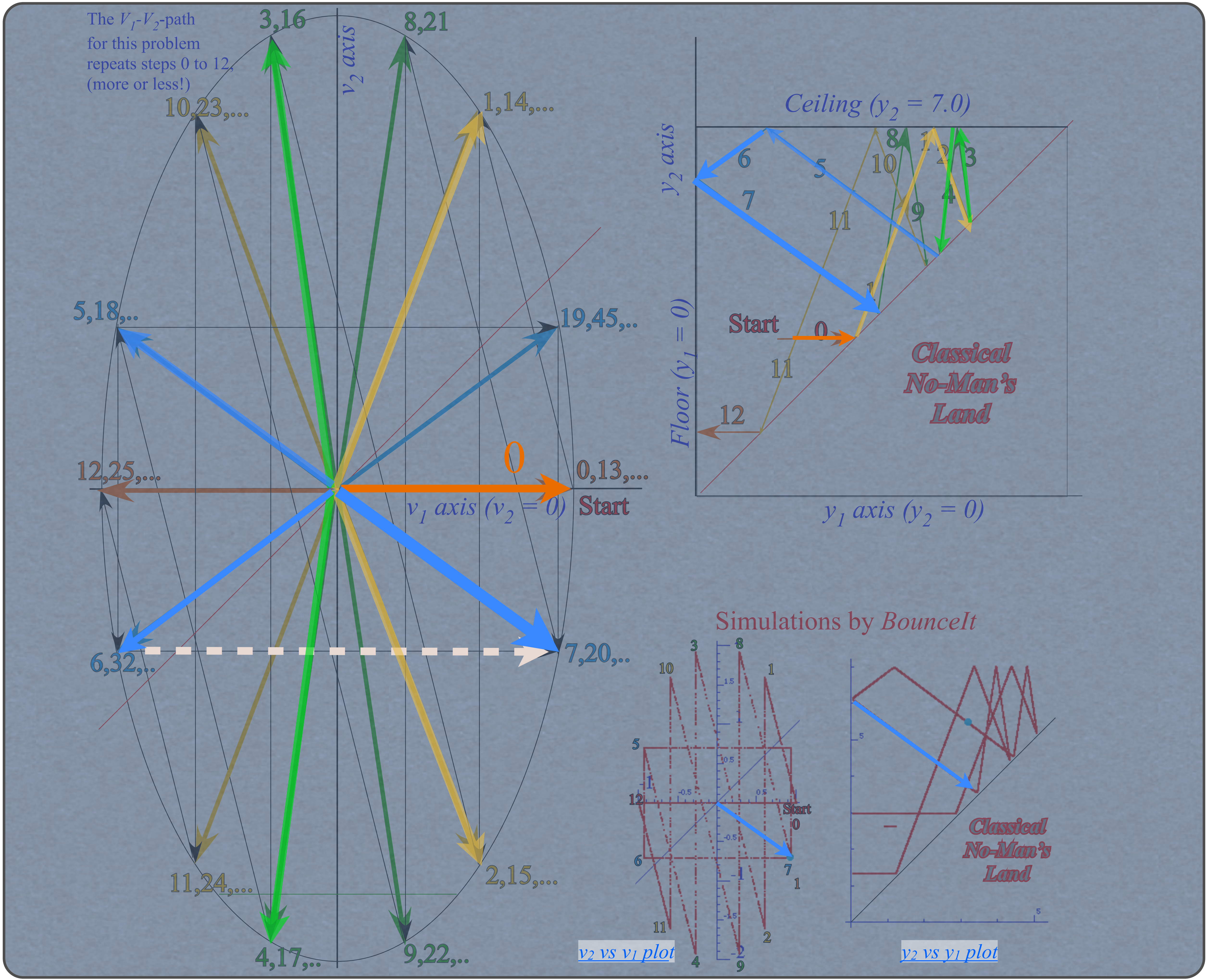
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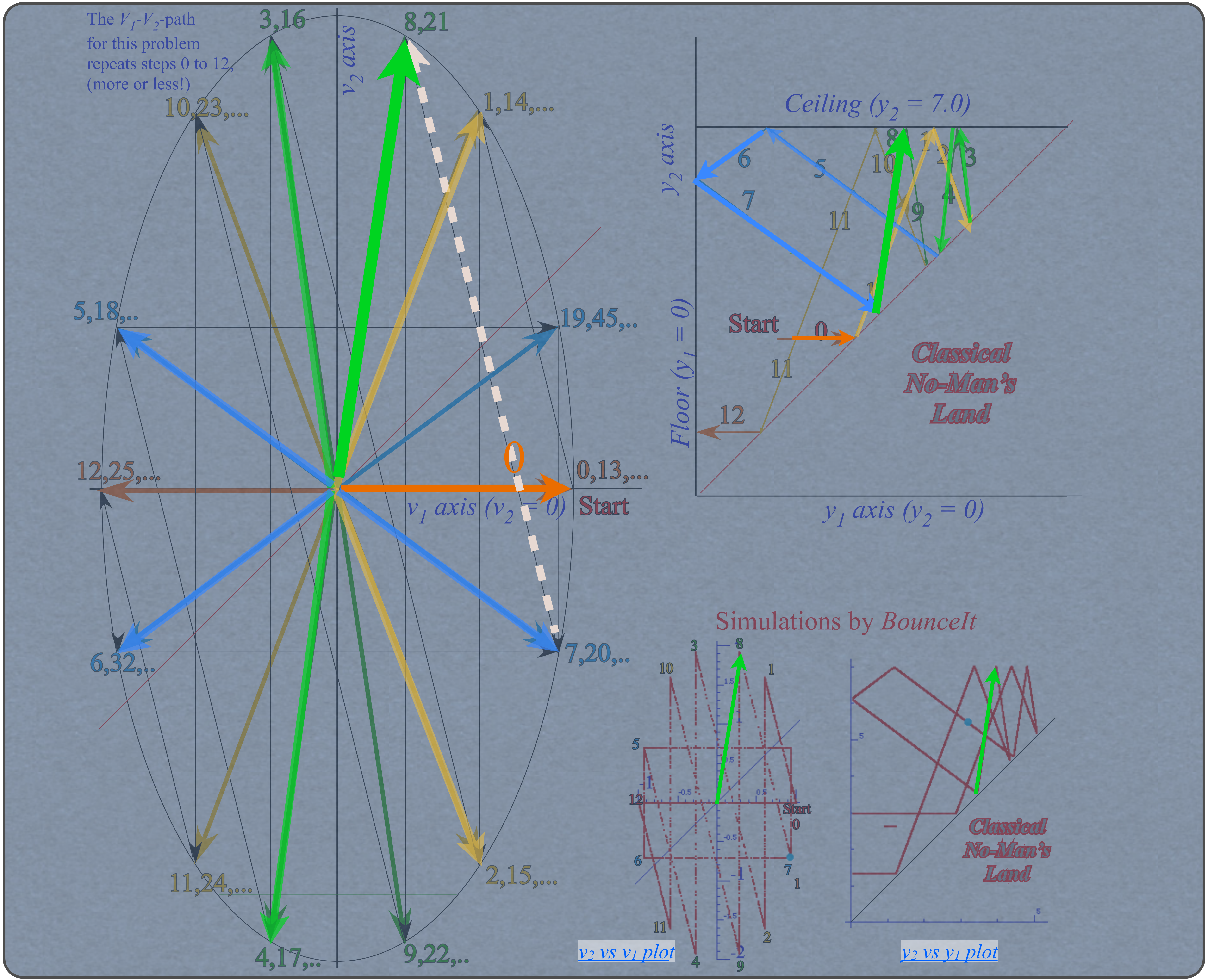

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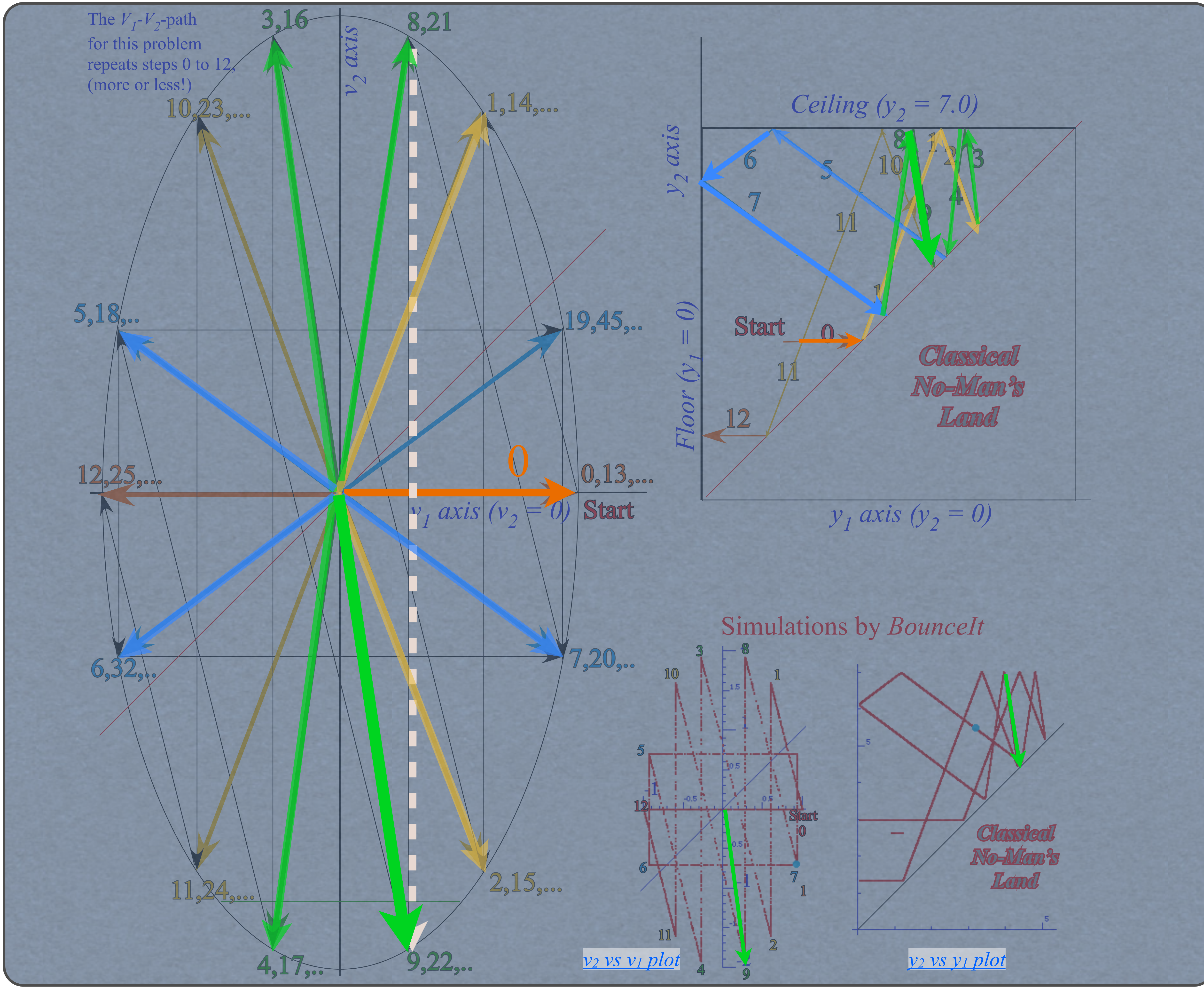
v2 vs v1 plot
y2 vs y1 plot



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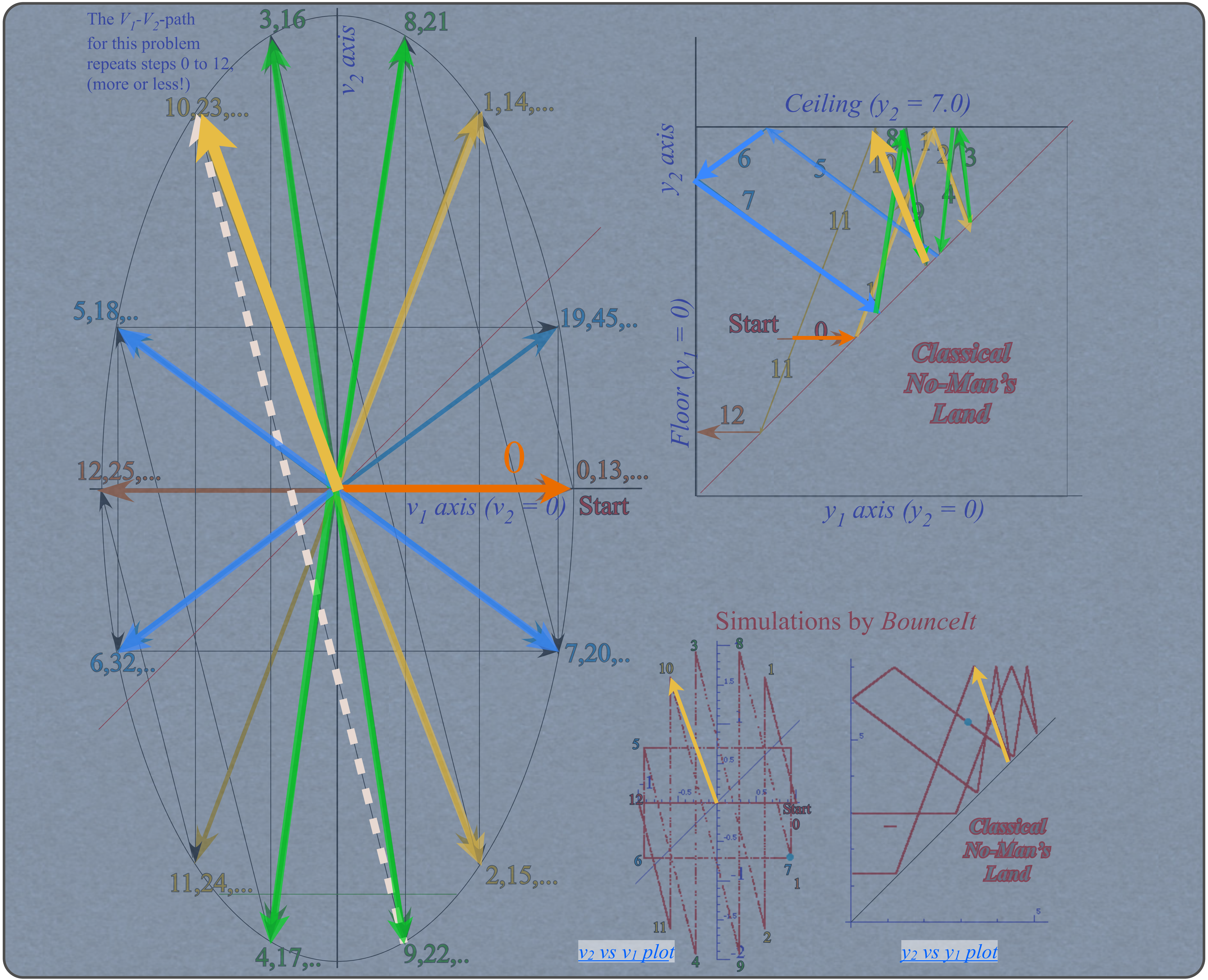

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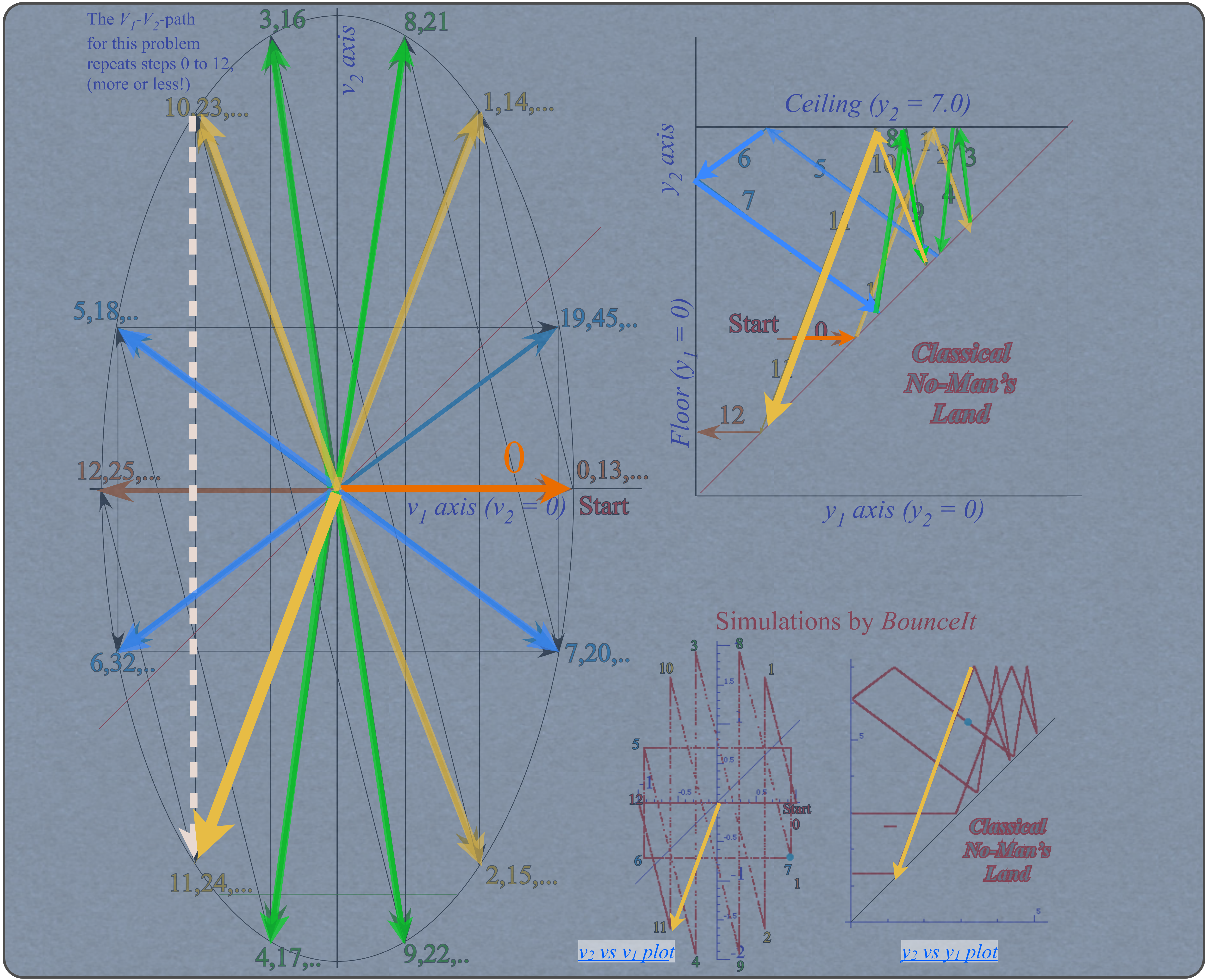
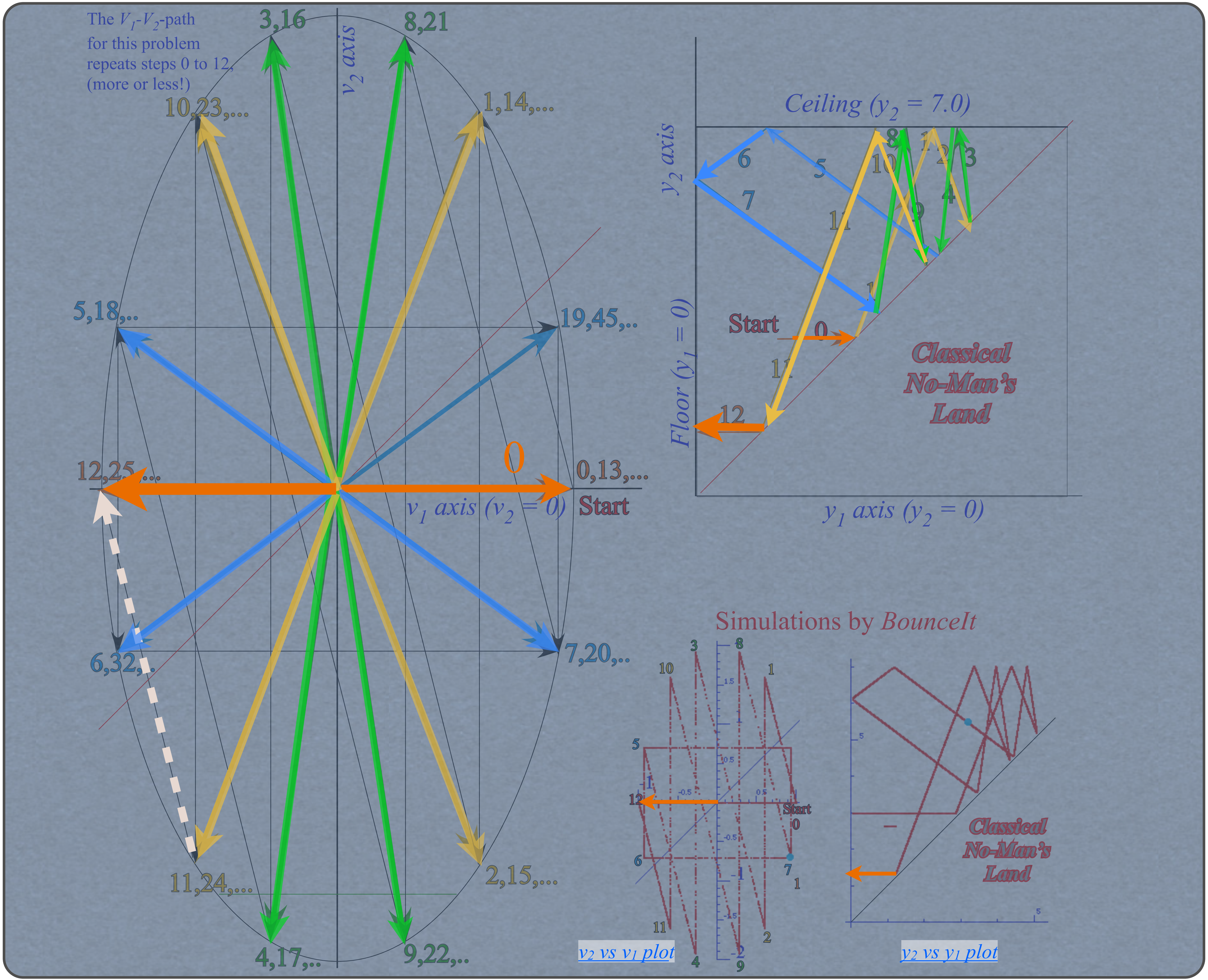
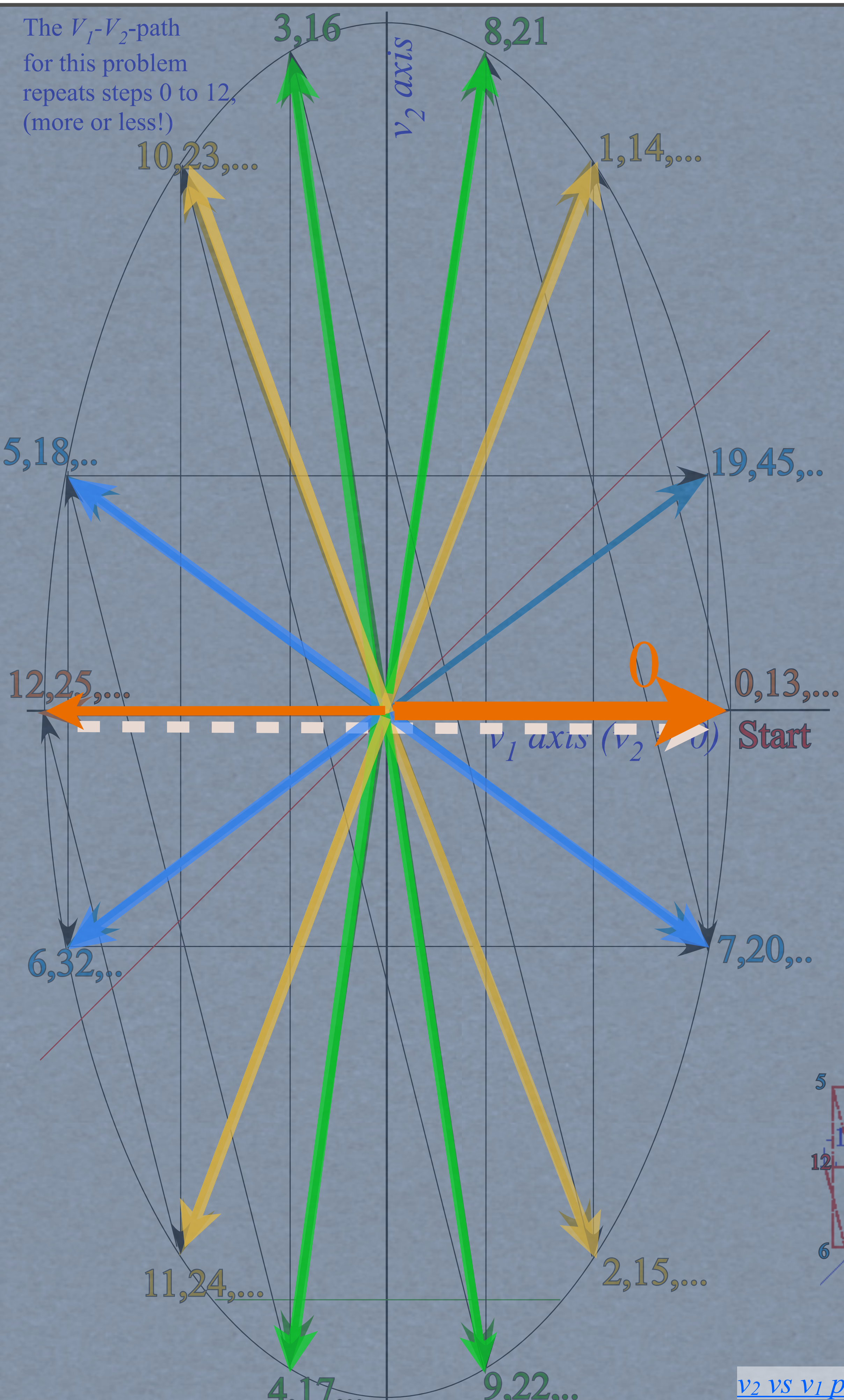
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land

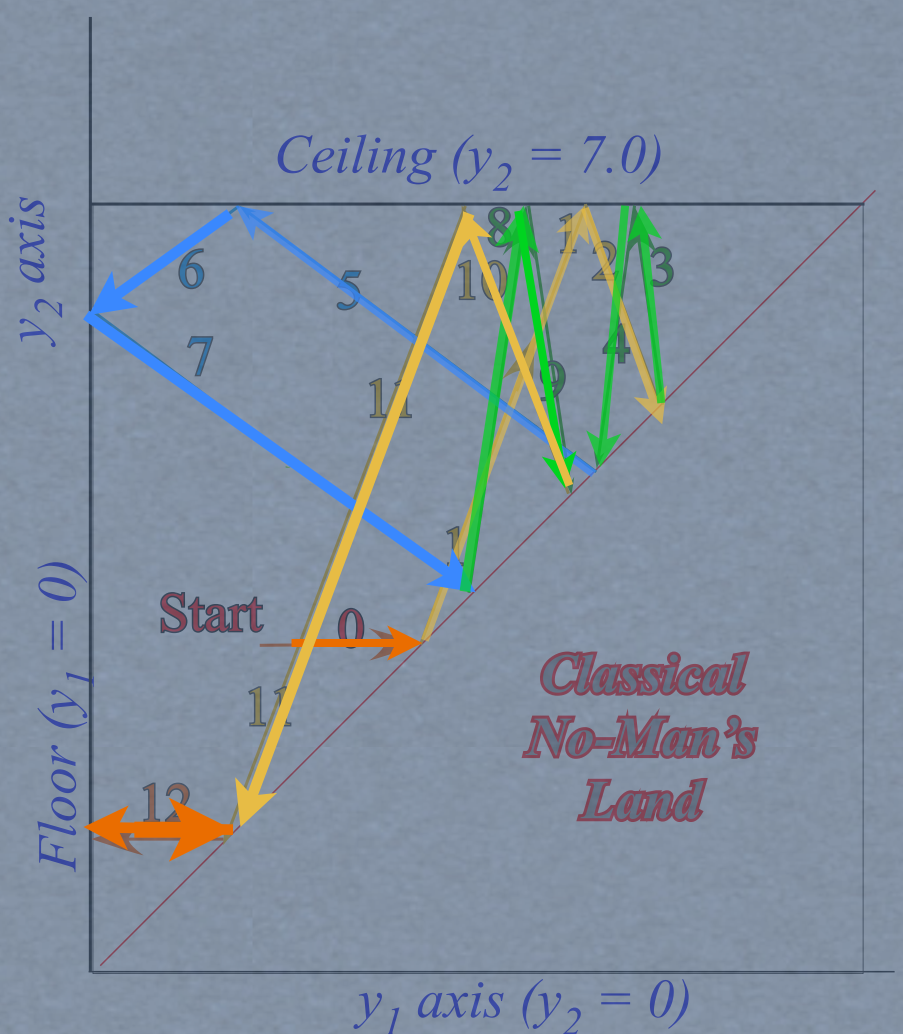


The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
1,14,...
10,23,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
0
12
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot

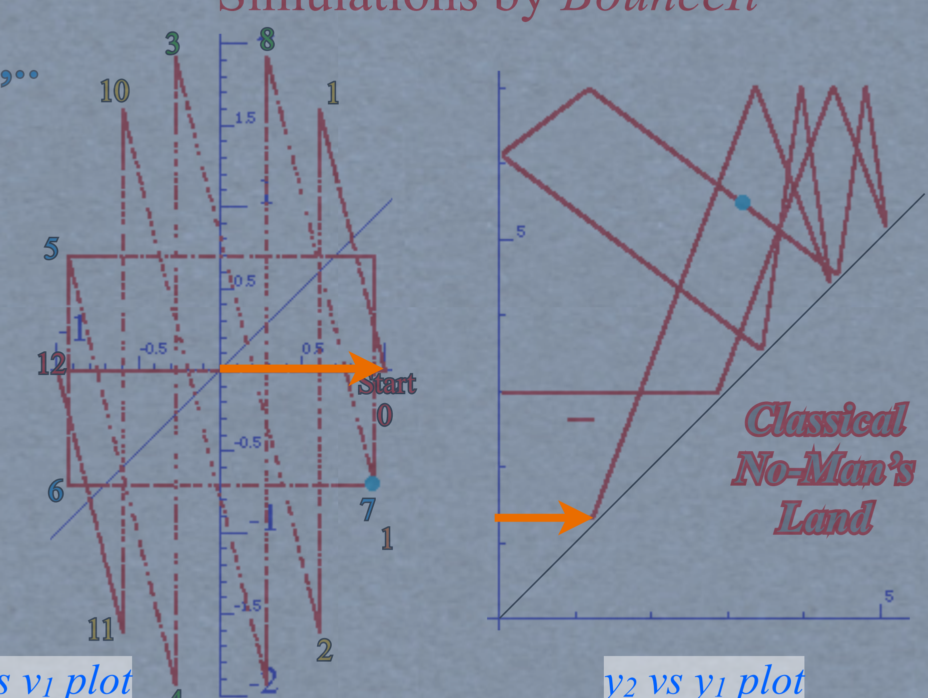


The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Start
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot



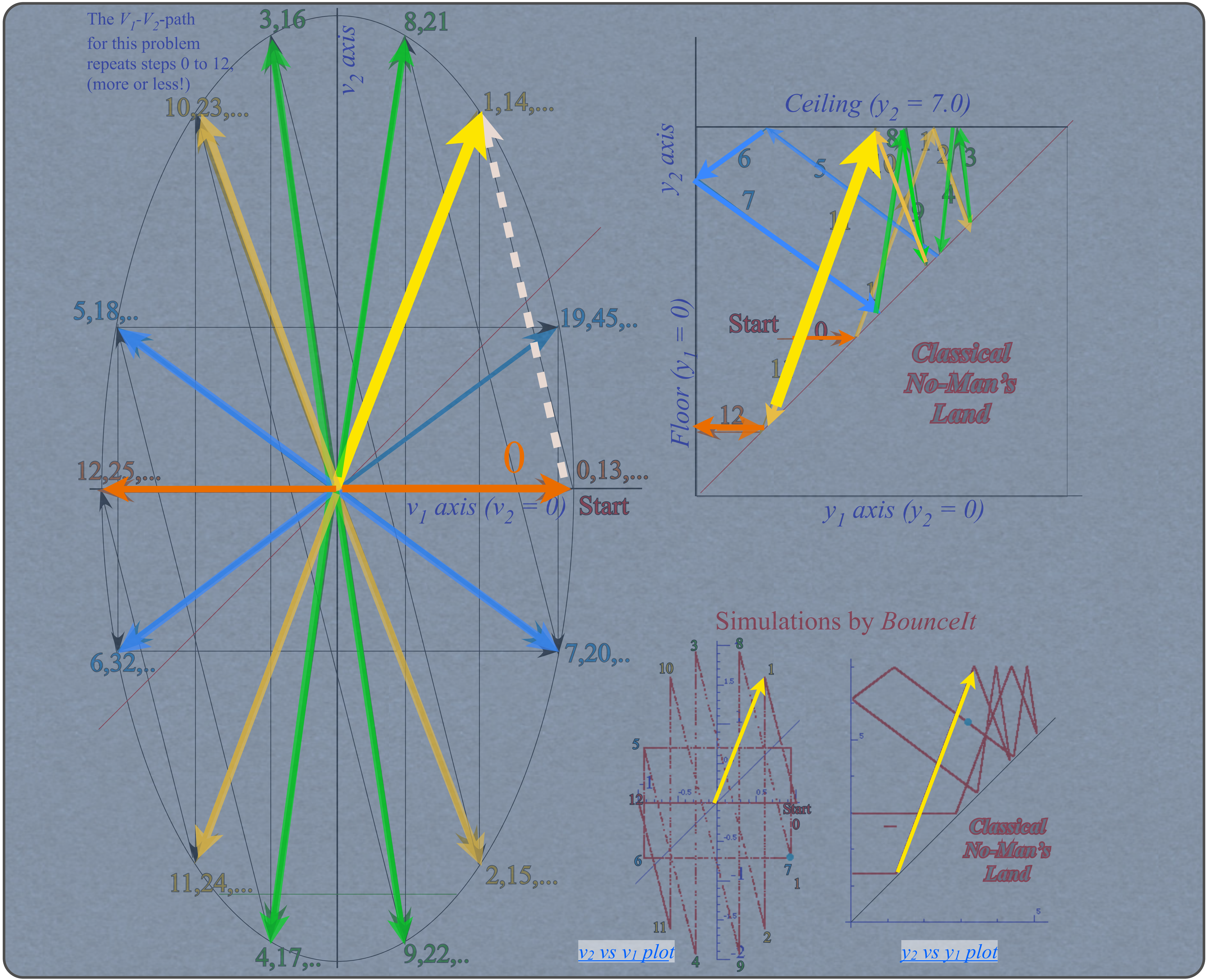


Simulations by *BounceIt*

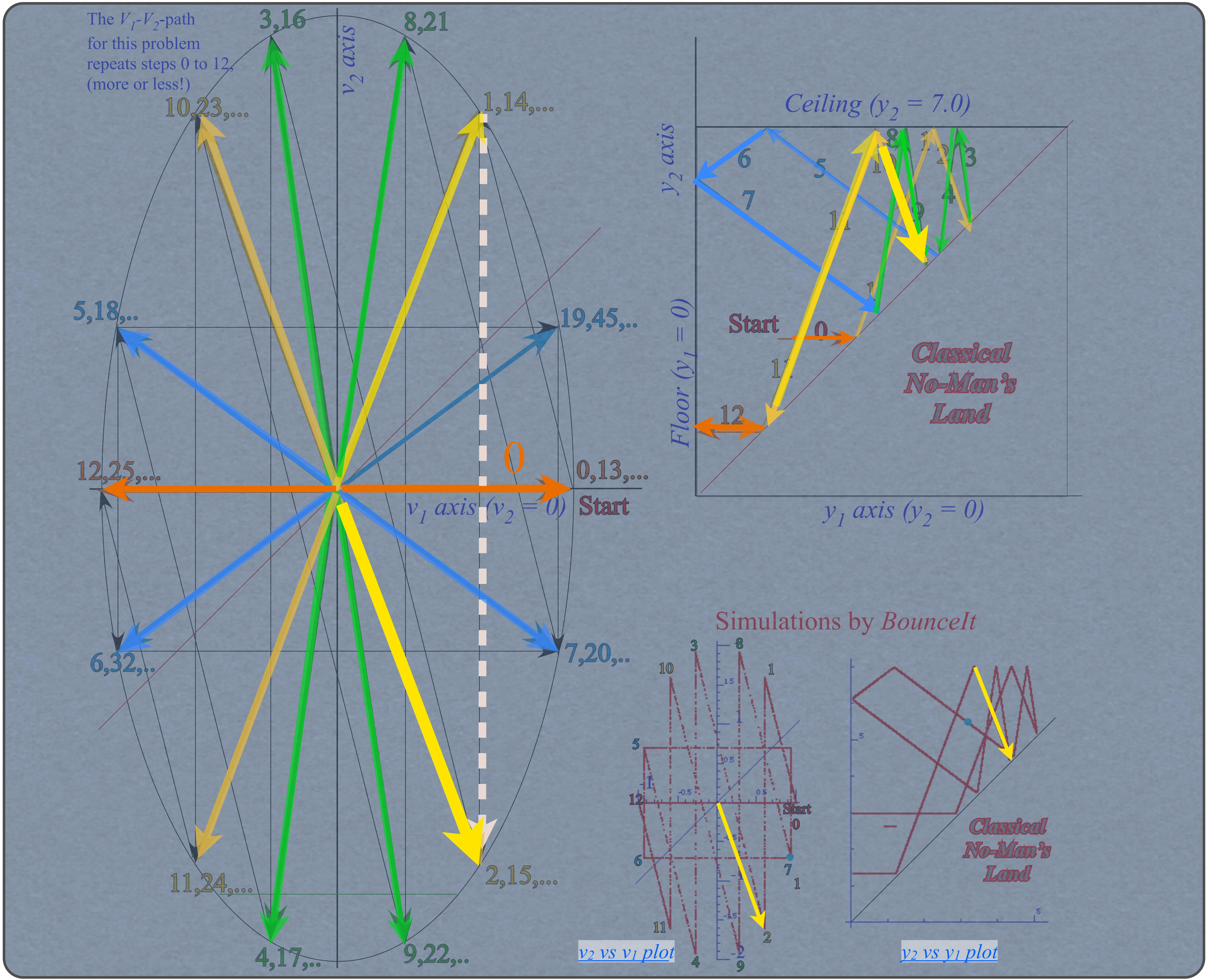


v_2 vs v_1 plot

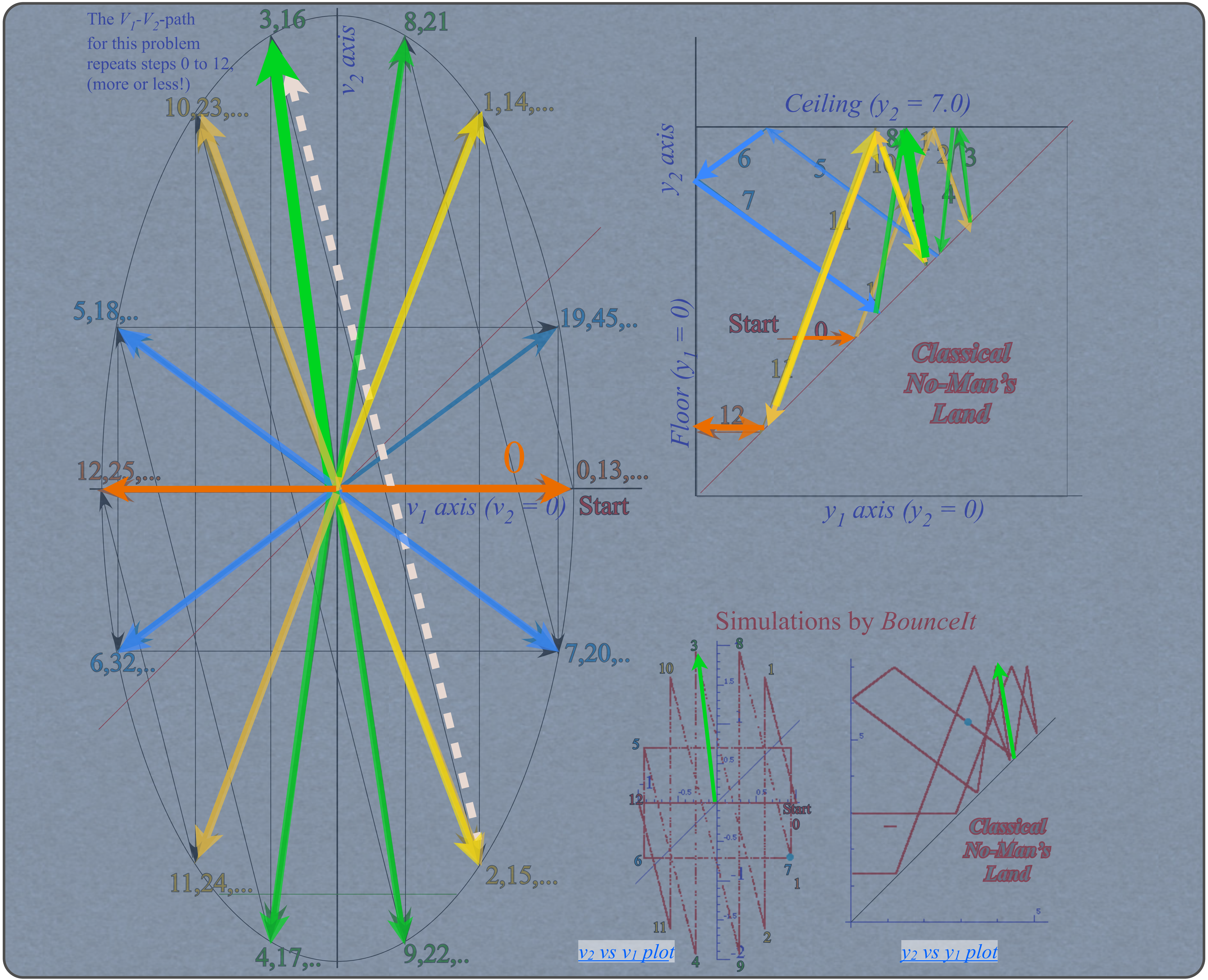
y_2 vs y_1 plot



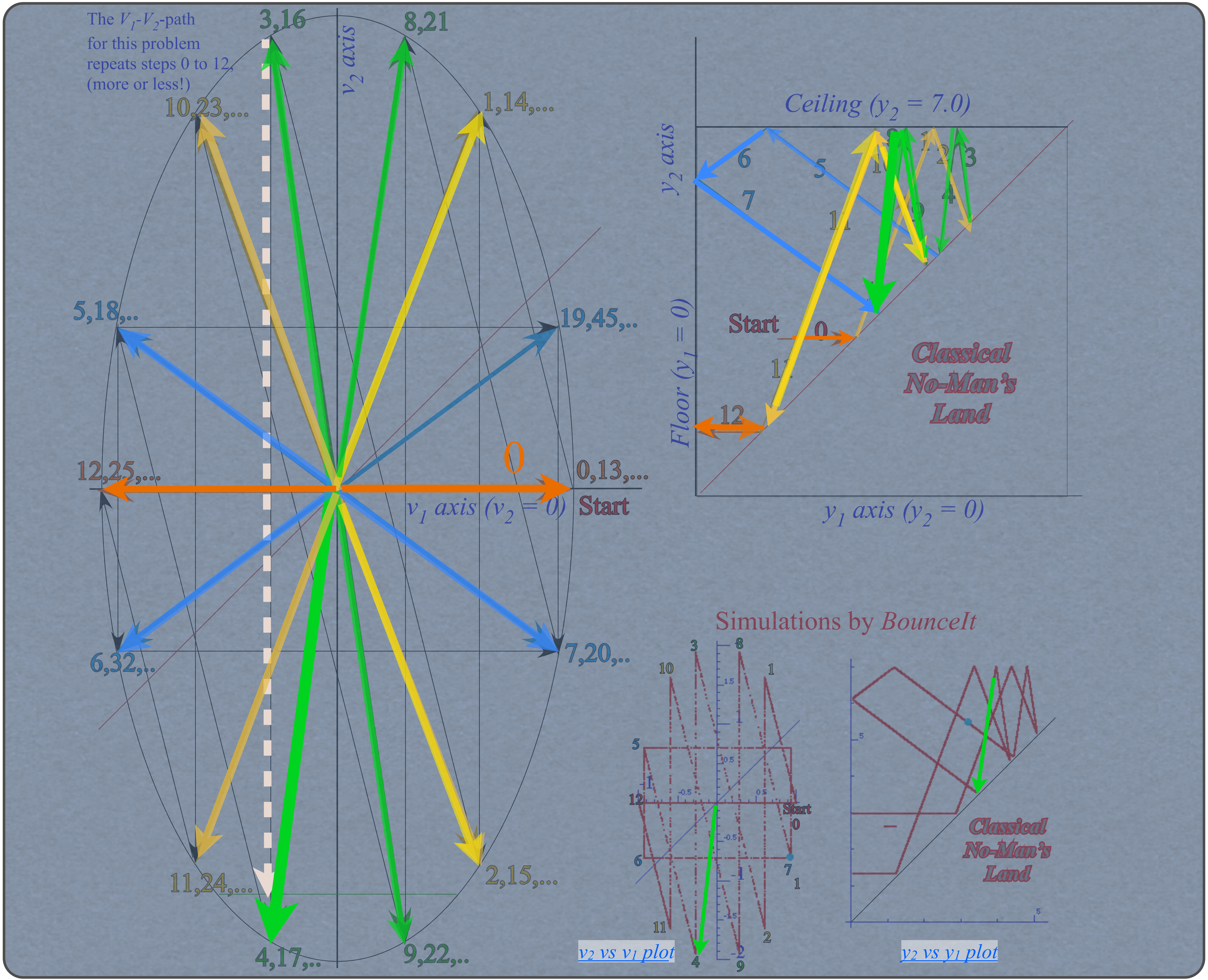
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical No-Man's Land



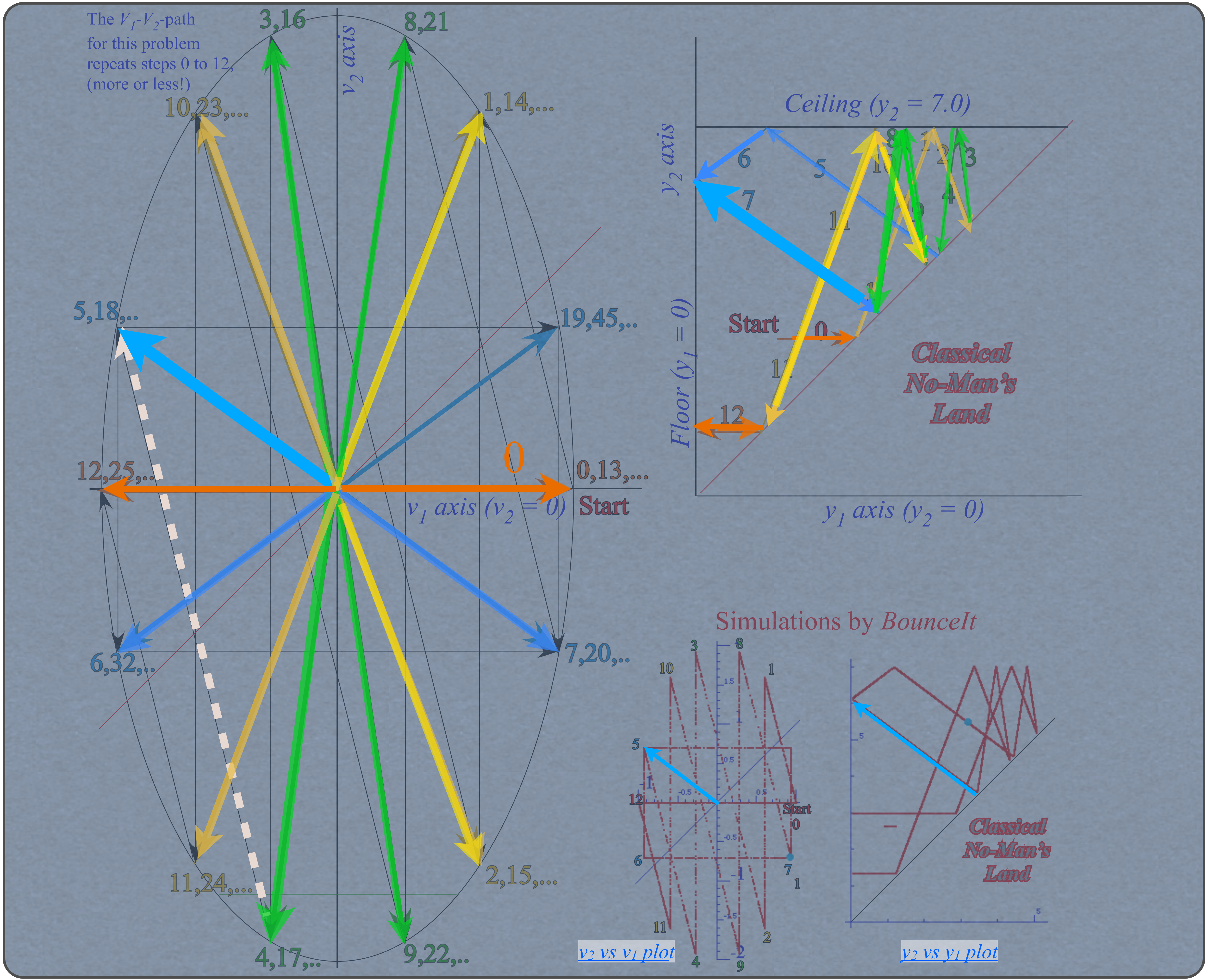
The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0,13,...
v_1 axis ($v_2 = 0$)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling ($y_2 = 7.0$)
y_2 axis
Floor ($y_1 = 0$)
Start
Classical
No-Man's
Land
y_1 axis ($y_2 = 0$)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land

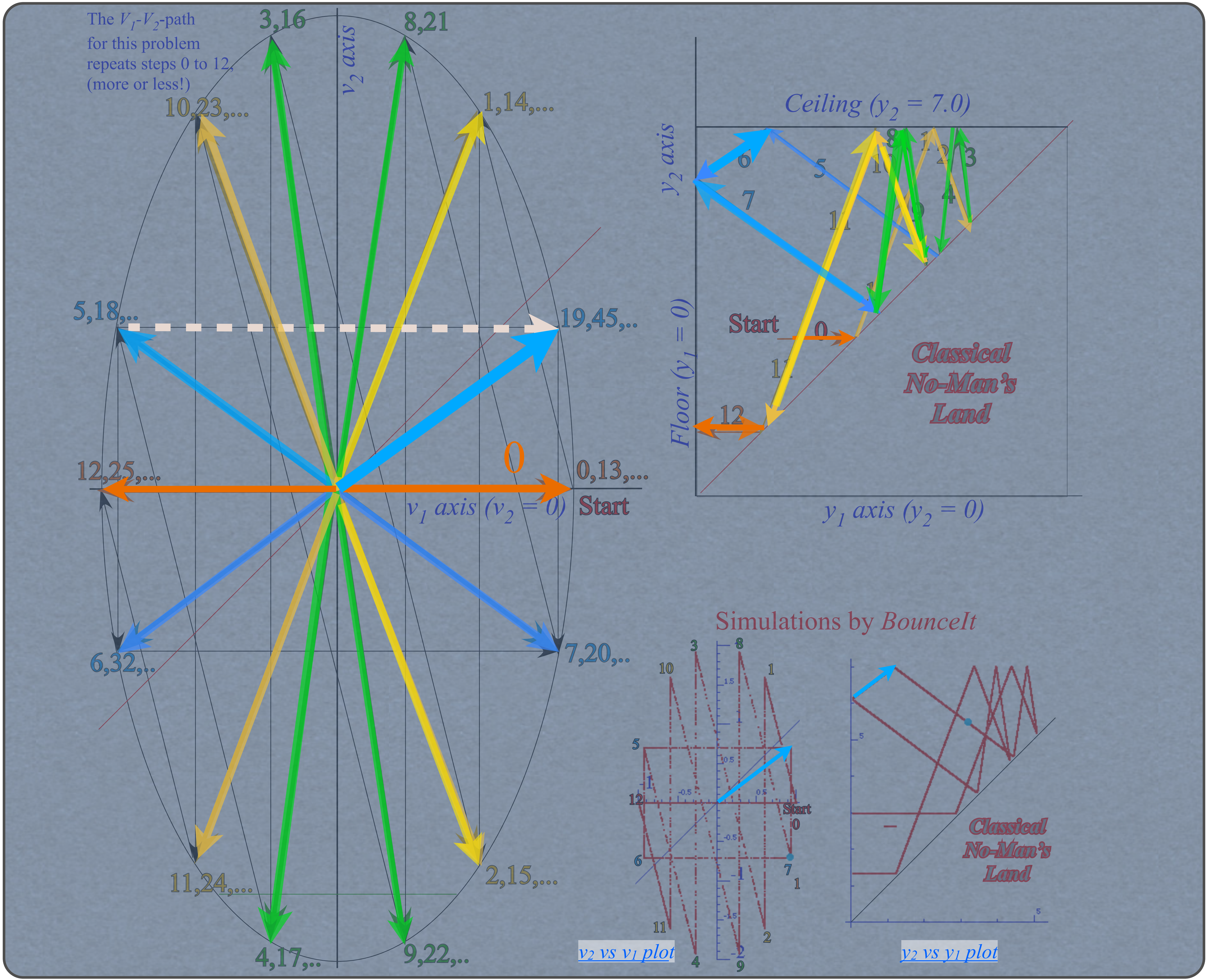


The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
1,14,...
10,23,...
v_2 axis
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical No-Man's Land

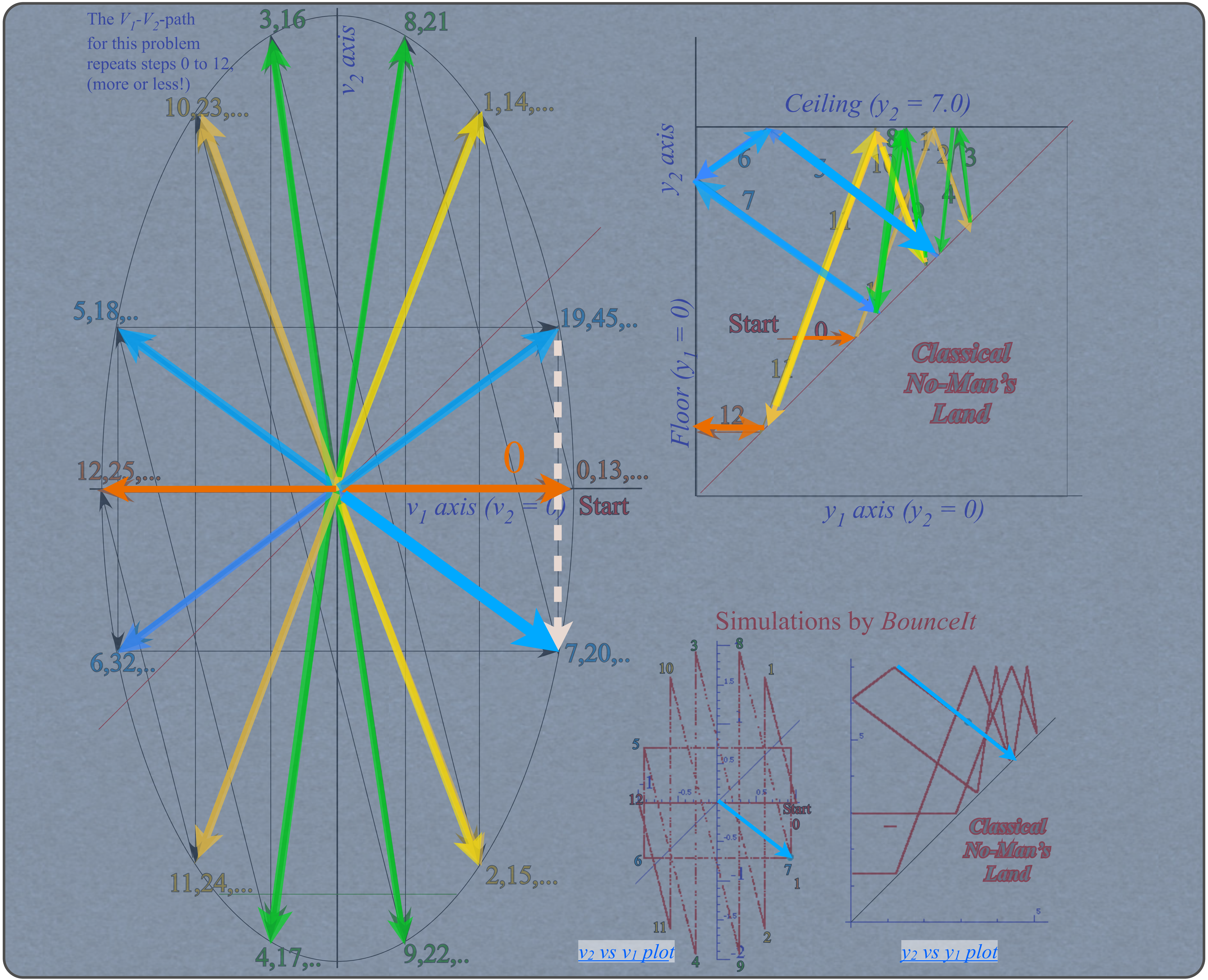


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land

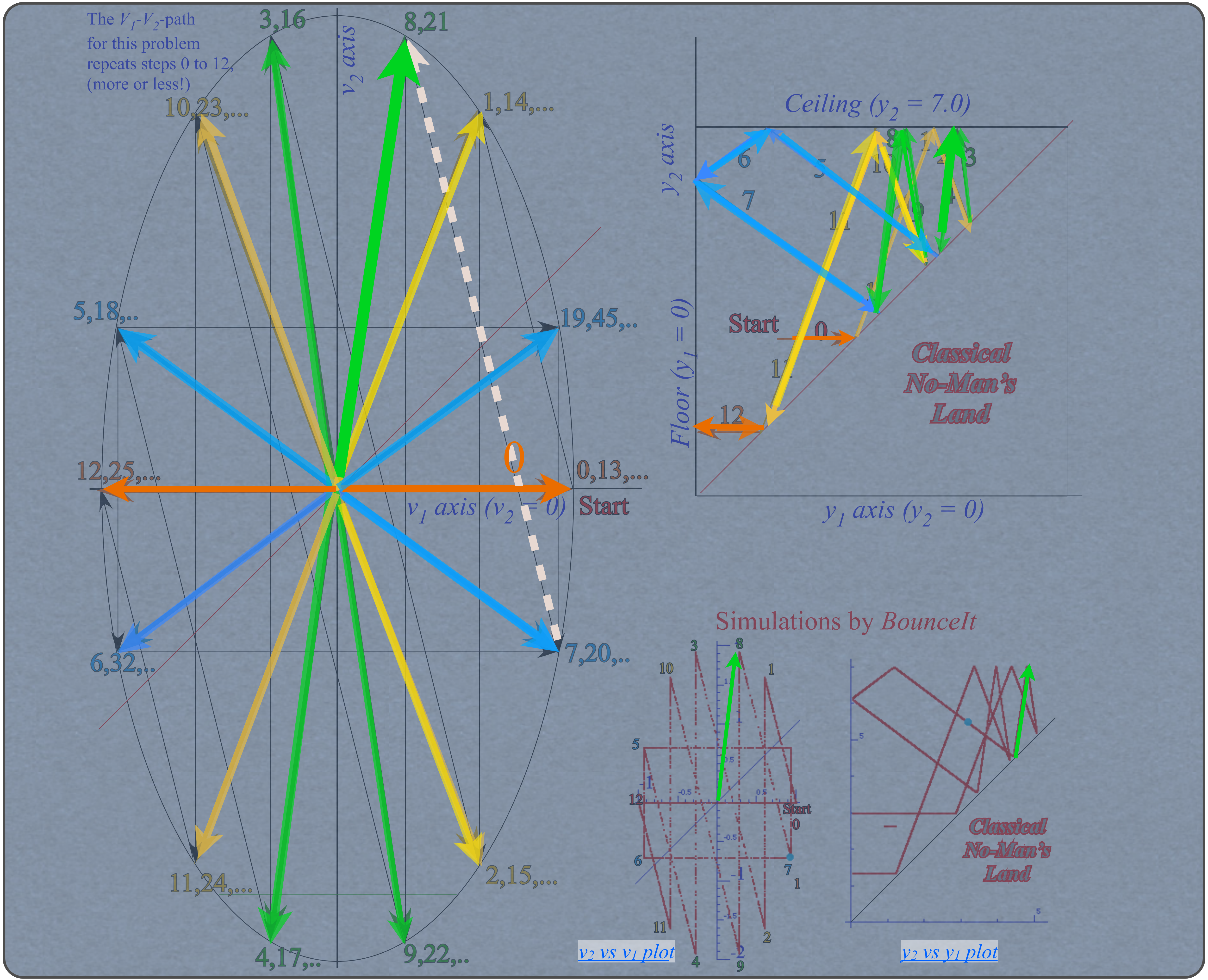

The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,..
0
0,13,...
v1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Start
v2 vs v1 plot
Classical
No-Man's
Land
y2 vs y1 plot

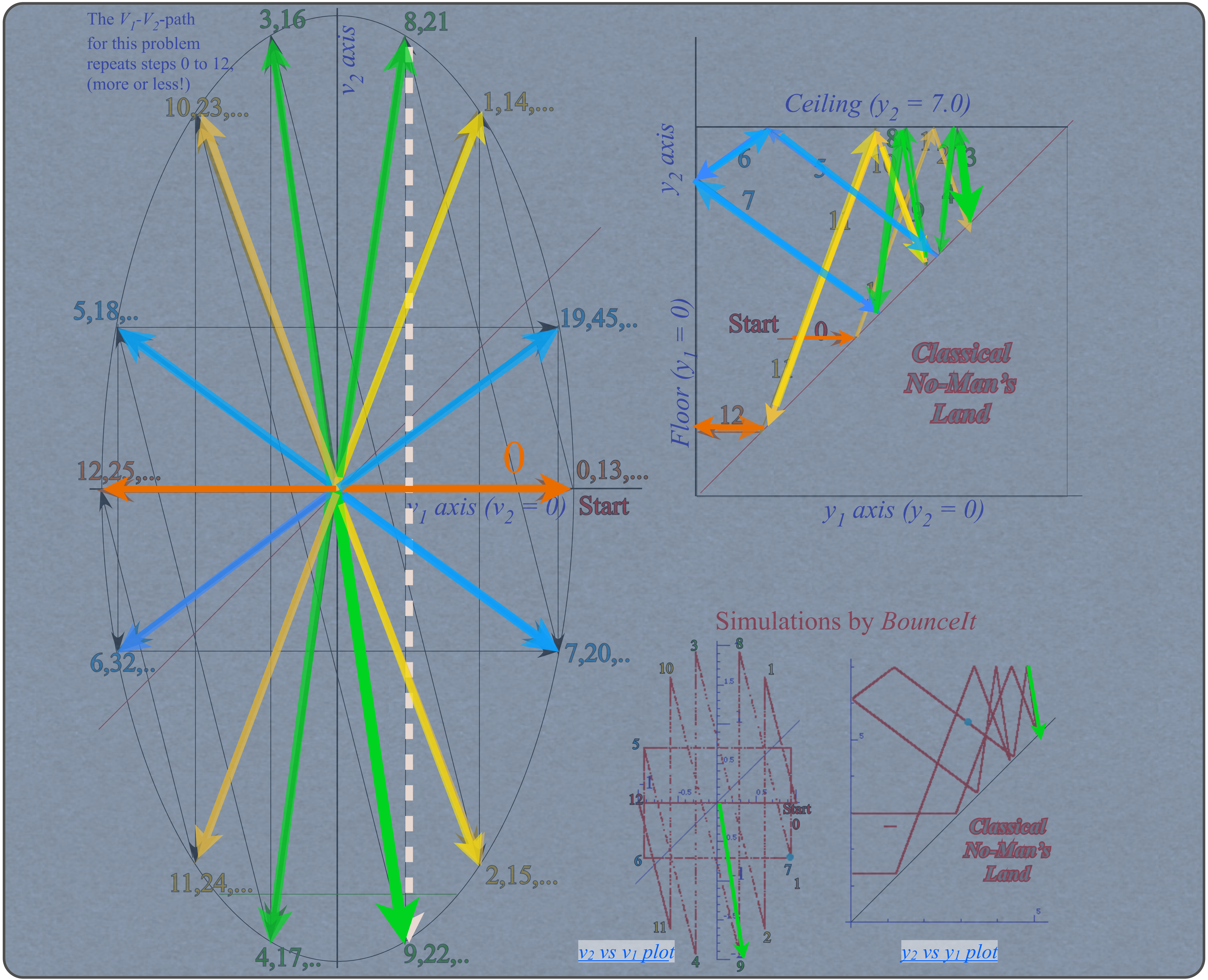


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land

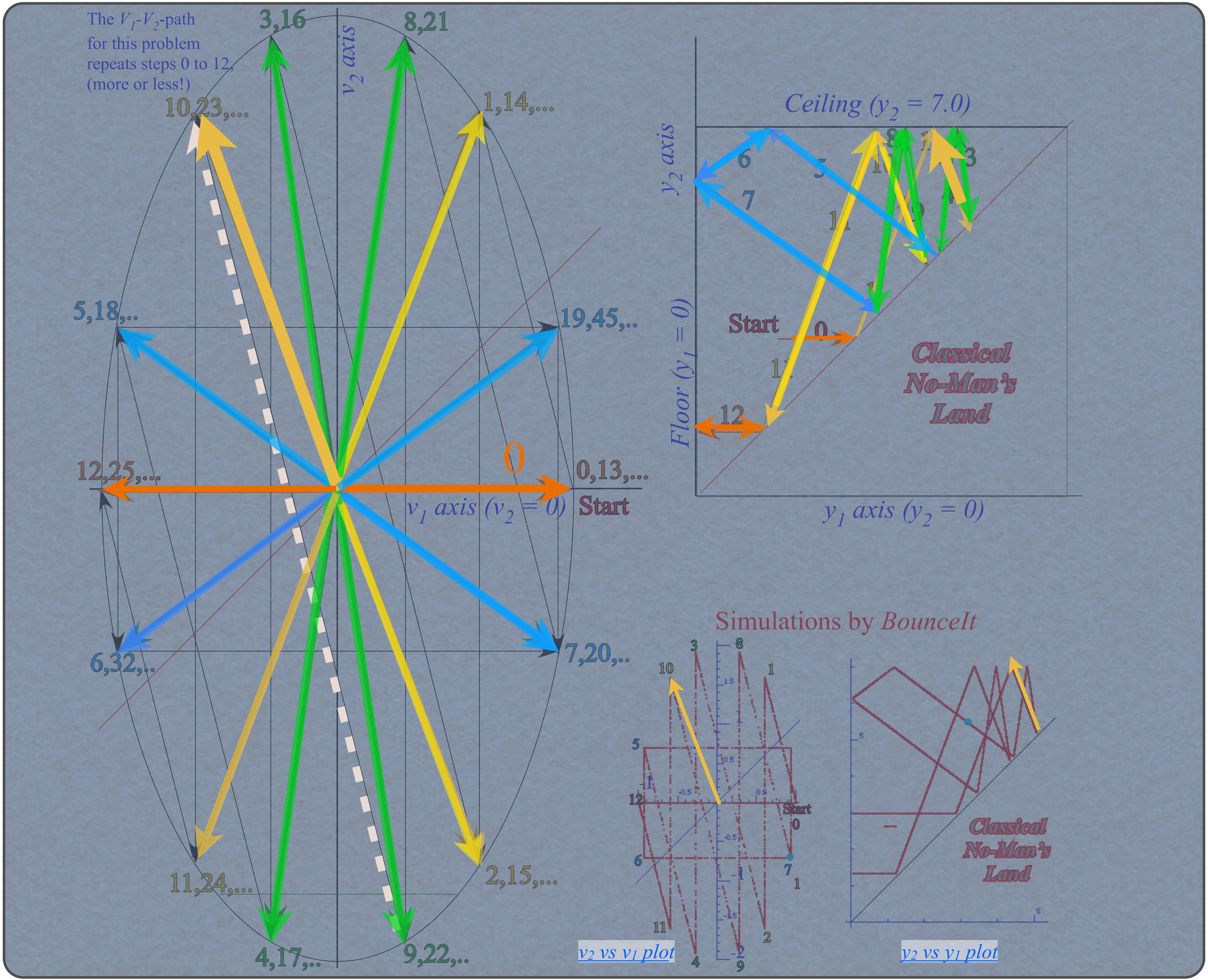


The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
0
v_1 axis (v_2 = 0)
Start
v_2 axis
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot

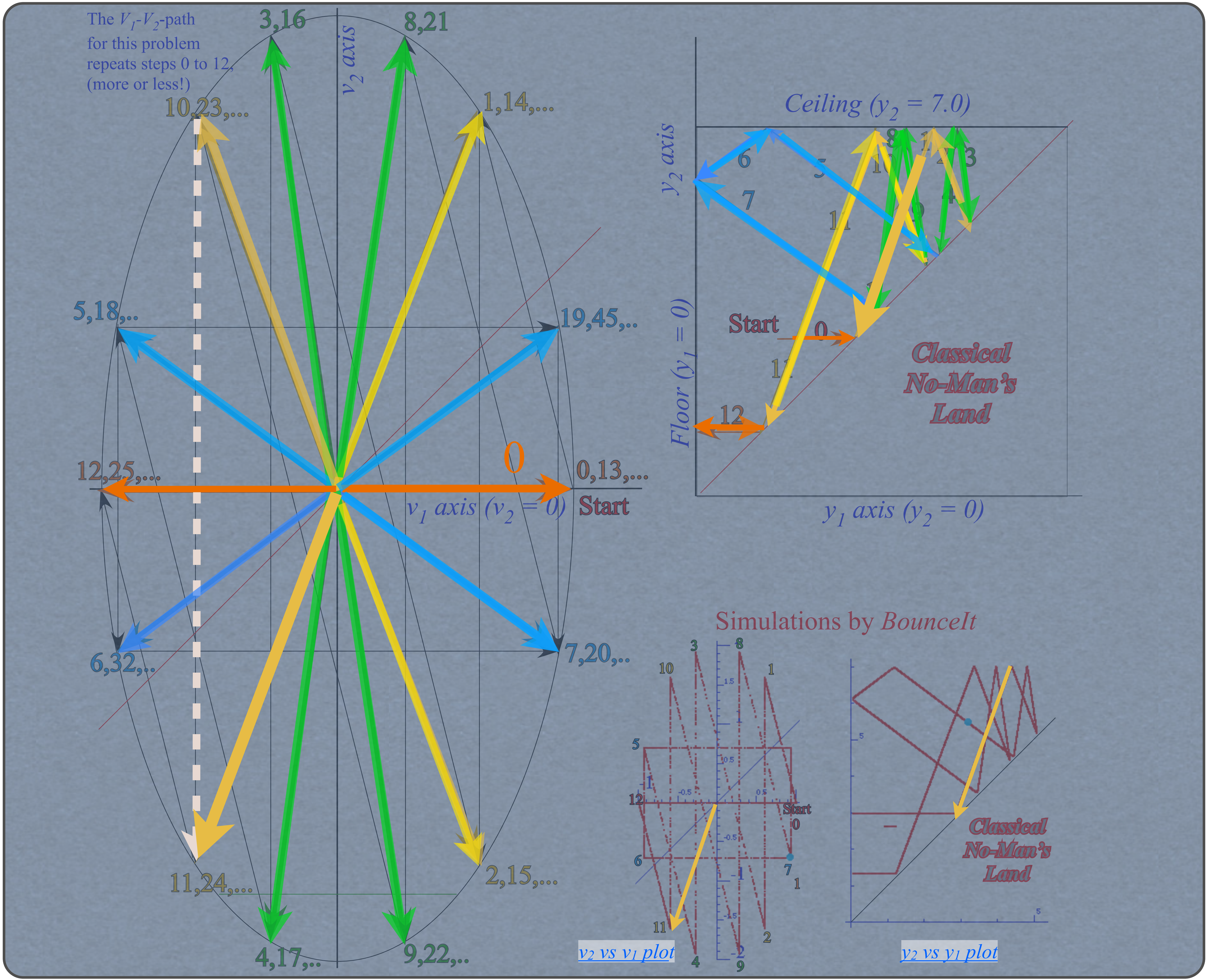

The V_1-V_2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical
No-Man's
Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical
No-Man's
Land



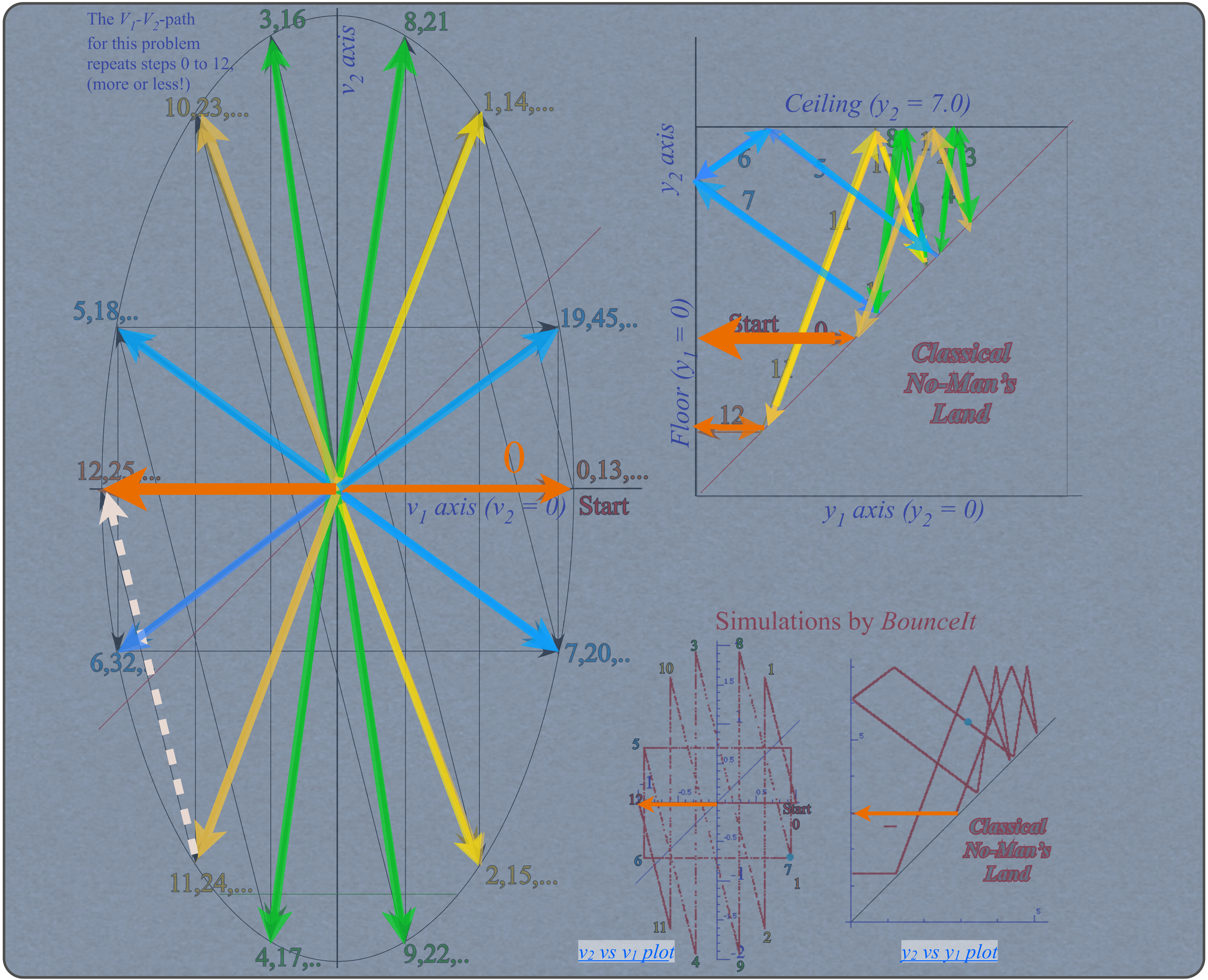
The V1-V2-path
for this problem
repeats steps 0 to 12,
(more or less!)
3,16
8,21
v2 axis
10,23,...
1,14,...
5,18,..
19,45,..
0
12,25,...
0,13,...
y1 axis (v2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y2 = 7.0)
y2 axis
Floor (y1 = 0)
Start
Classical
No-Man's
Land
y1 axis (y2 = 0)
Simulations by BounceIt
Start
Classical
No-Man's
Land
v2 vs v1 plot
y2 vs y1 plot



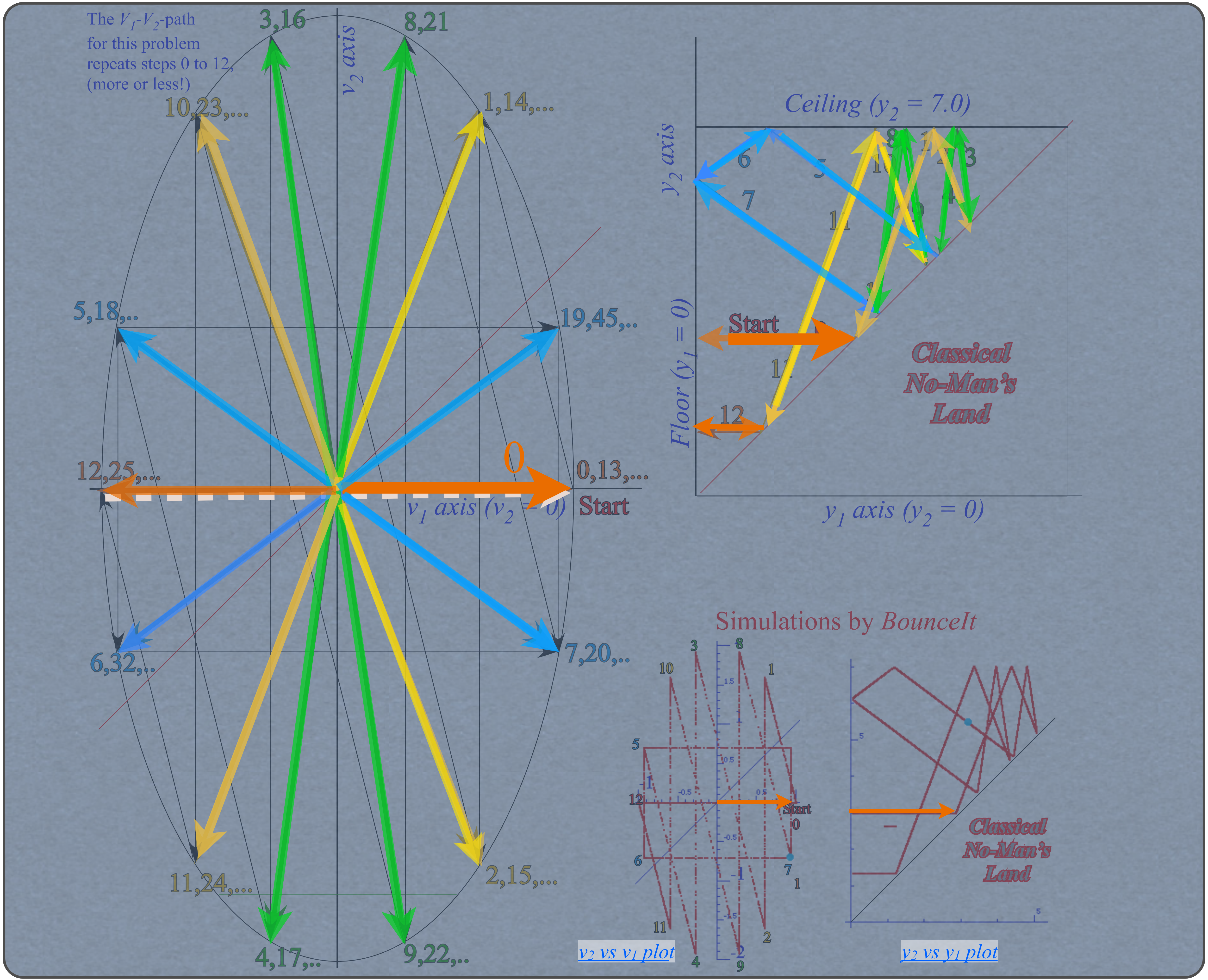
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot



The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
12
y_1 axis (y_2 = 0)
Simulations by BounceIt
Start
Classical No-Man's Land
v_2 vs v_1 plot
y_2 vs y_1 plot



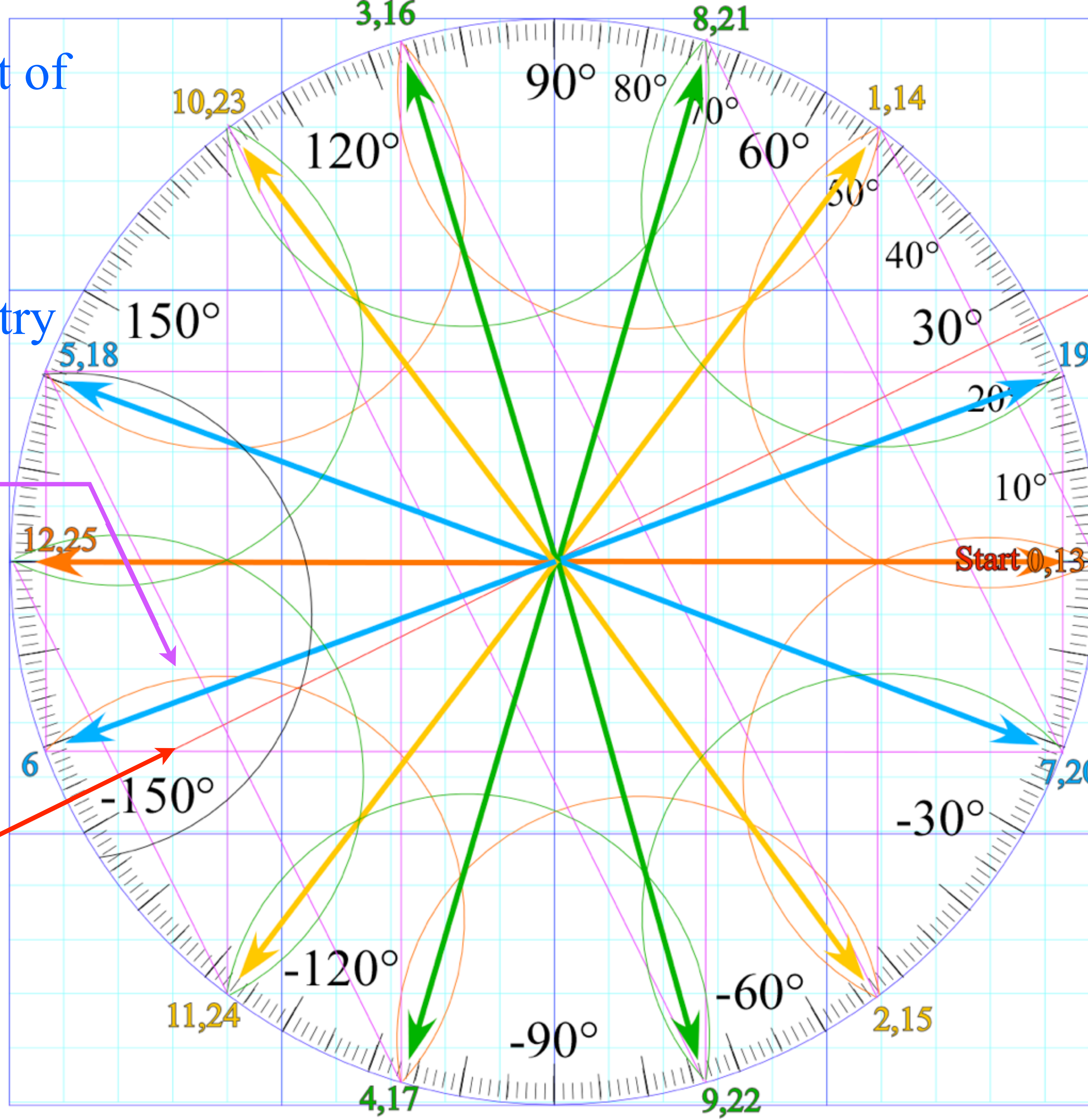
The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
3,16
8,21
v_2 axis
10,23,...
1,14,...
5,18,..
19,45,..
12,25,...
0
0,13,...
v_1 axis (v_2 = 0)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling (y_2 = 7.0)
y_2 axis
Floor (y_1 = 0)
Start
Classical No-Man's Land
y_1 axis (y_2 = 0)
Simulations by BounceIt
Classical No-Man's Land
v_2 vs v_1 plot
y_2 vs y_1 plot



The V_1-V_2-path for this problem repeats steps 0 to 12, (more or less!)
v_2 axis
3,16
8,21
1,14,...
10,23,...
5,18,..
19,45,..
12,25,...
0,13,...
0
v_1 axis ($v_2 = 0$)
Start
6,32,..
7,20,..
11,24,...
2,15,...
4,17,..
9,22,..
Ceiling ($y_2 = 7.0$)
y_2 axis
Floor ($y_1 = 0$)
Start
Classical No-Man's Land
y_1 axis ($y_2 = 0$)
Simulations by *BounceIt*
v_2 vs v_1 plot
y_2 vs y_1 plot
Classical No-Man's Land

Estrangian plot of $m_1/m_2=4/1$ collision sequence shows symmetry

(sort of)

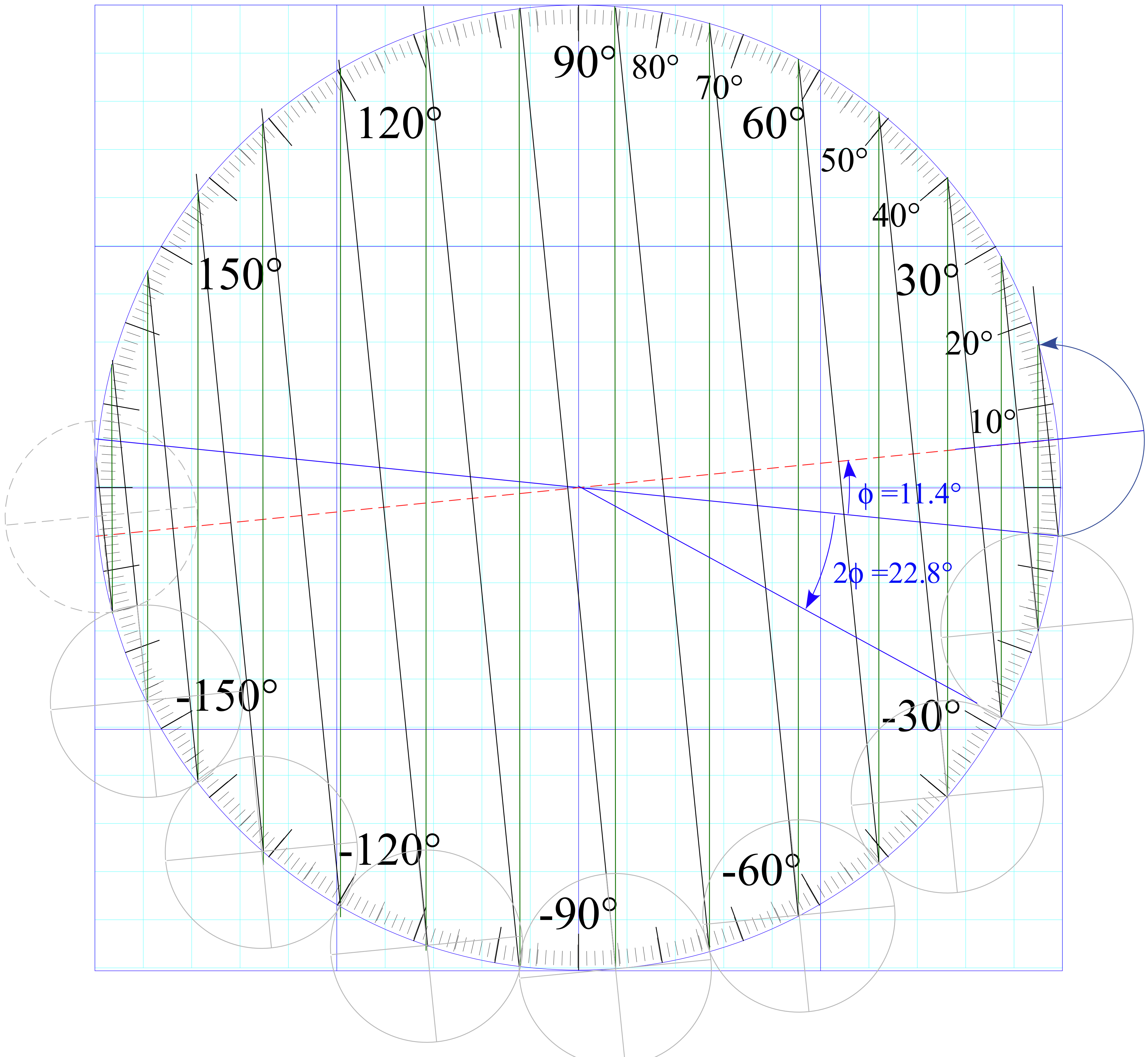


c.o.m. lines (cons. of mom.) have slope $-\sqrt{m_2}/\sqrt{m_1}=-2/1$

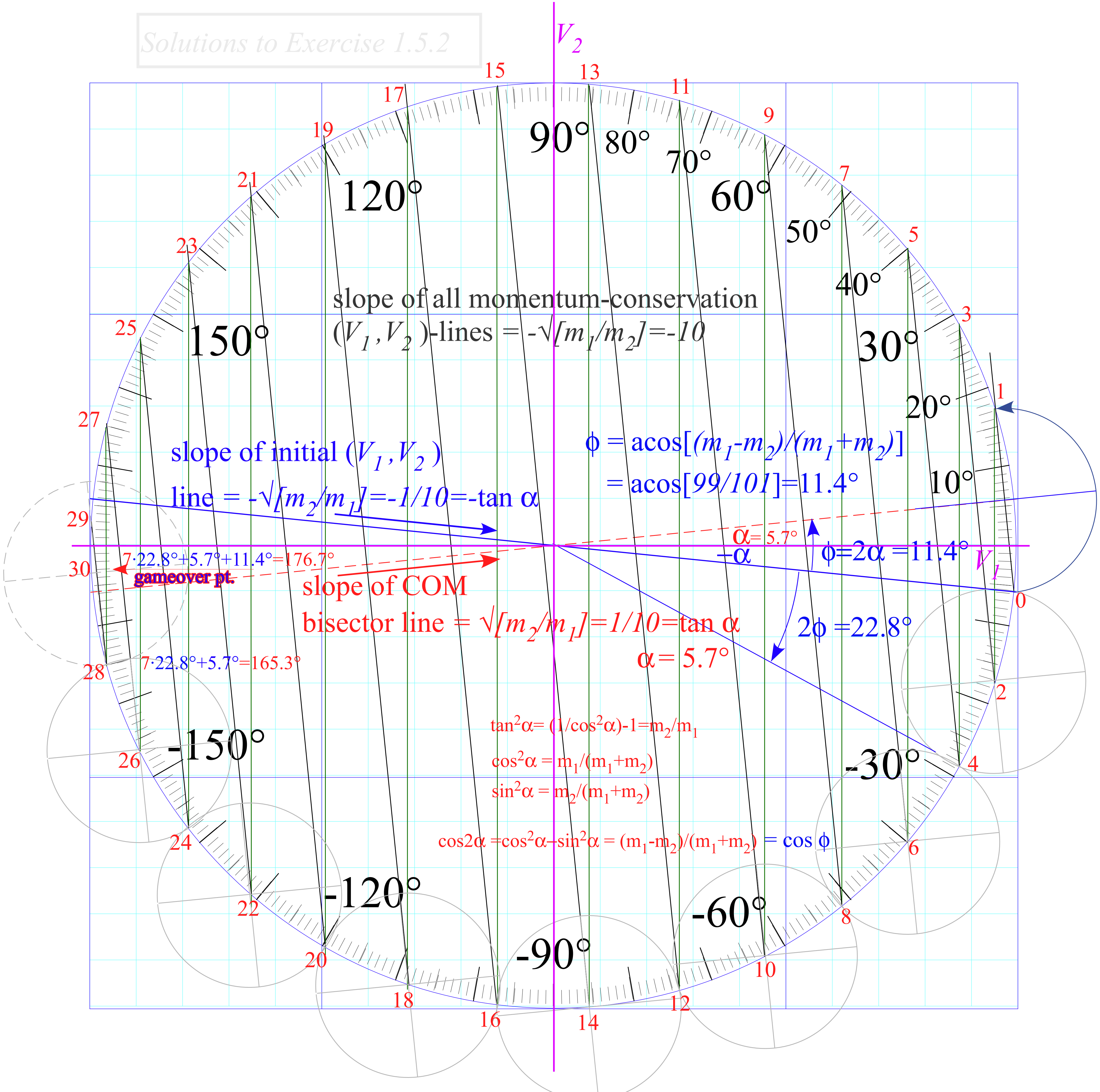
COM line has slope $\sqrt{m_2}/\sqrt{m_1}=1/2$

$\phi = \text{acos}[(m_1-m_2)/(m_1+m_2)] = \text{acos}[99/101]=11.4°$

Collisions for mass ratio $m_1{:}m_2 = 100{:}1$



Solutions to Exercise 1.5.2



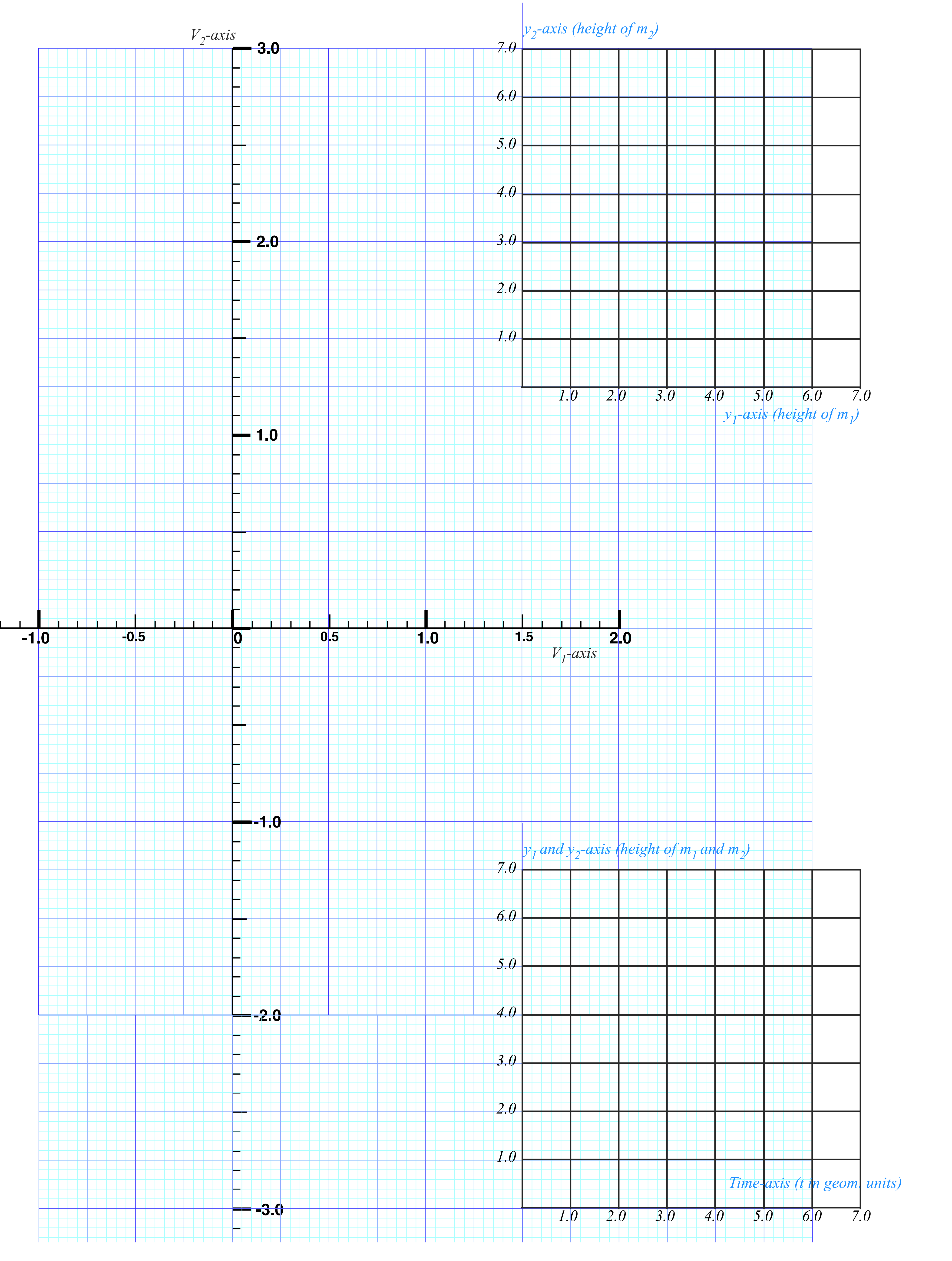
Collisions for mass ratio $m_1{:}m_2 = 100{:}1$

BounceIt Web Simulations

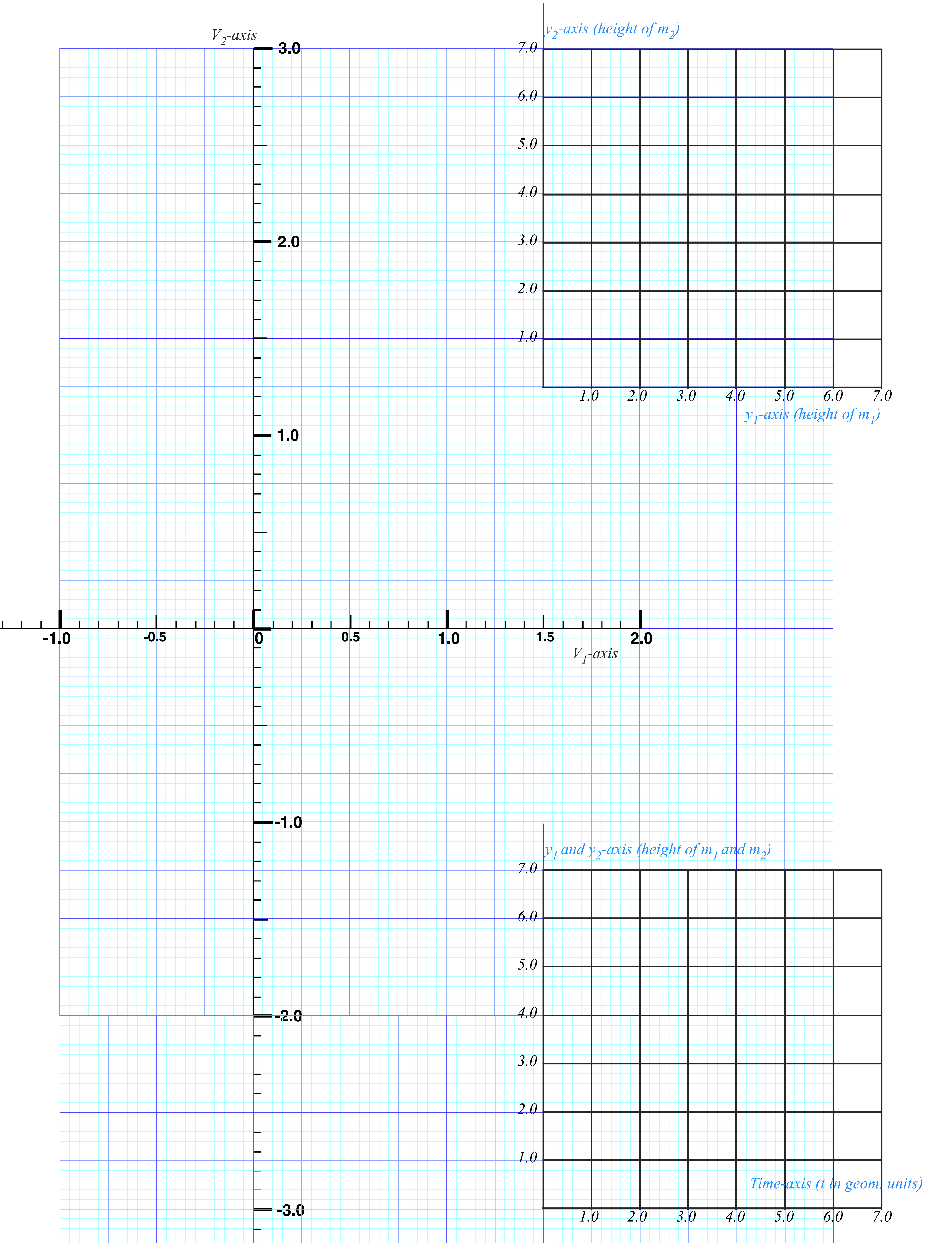
$m_1{:}m_2 = 100{:}1$ $(v_1, v_2)=(1, 0)$

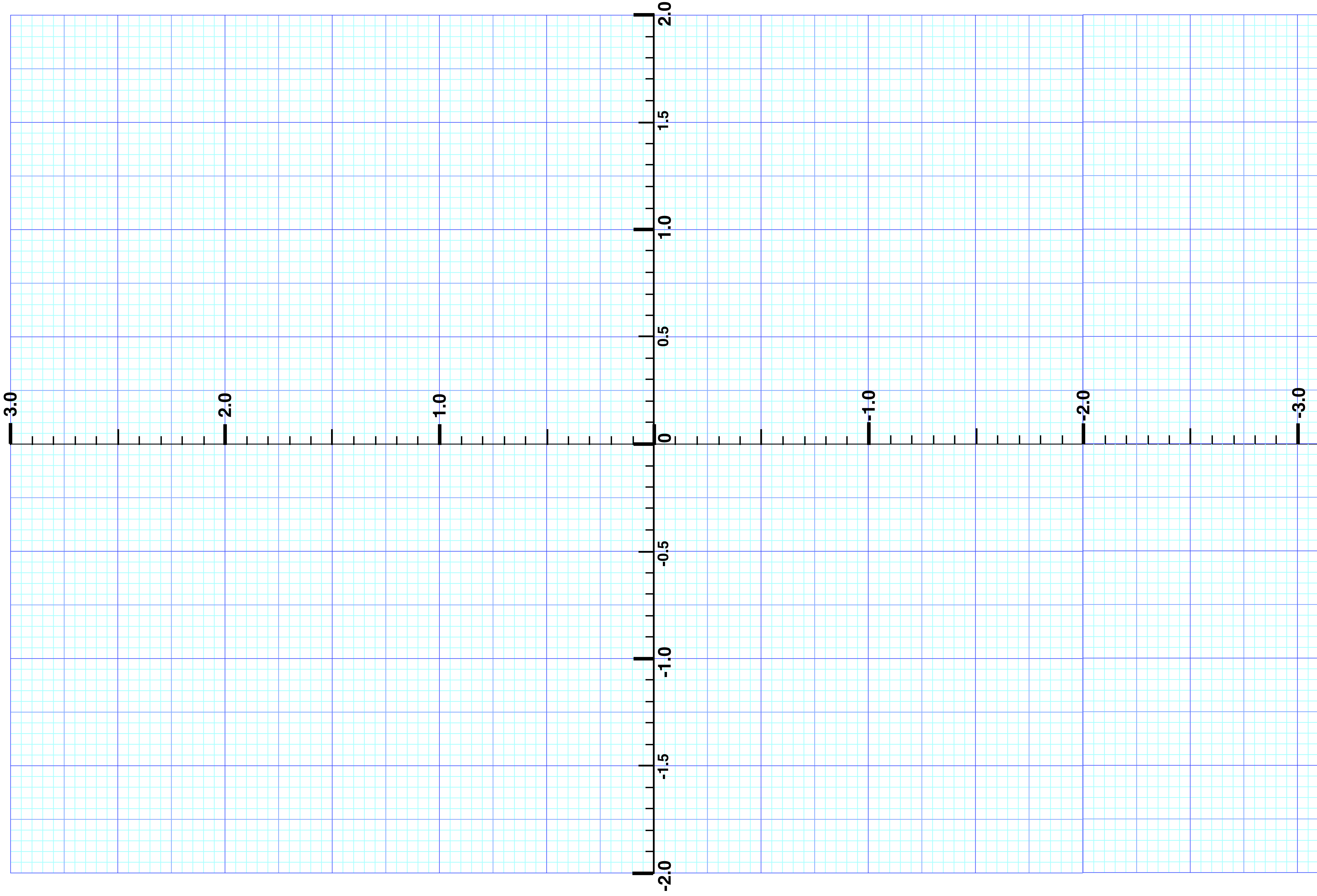
V_2 vs V_1 Estrangian plot

Supplementary: y

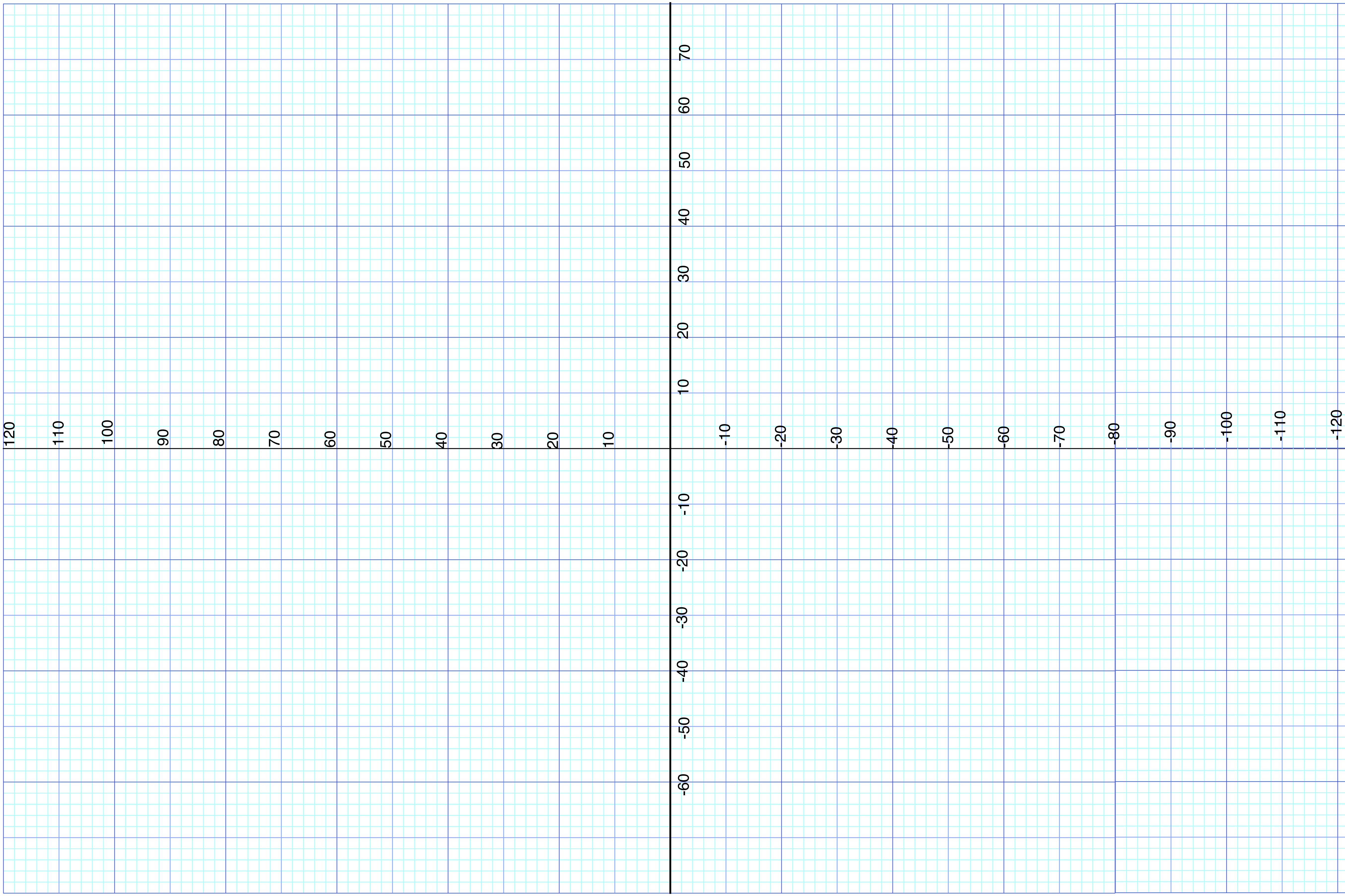


V_2-axis
V_1-axis
y_2-axis (height of m_2)
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)

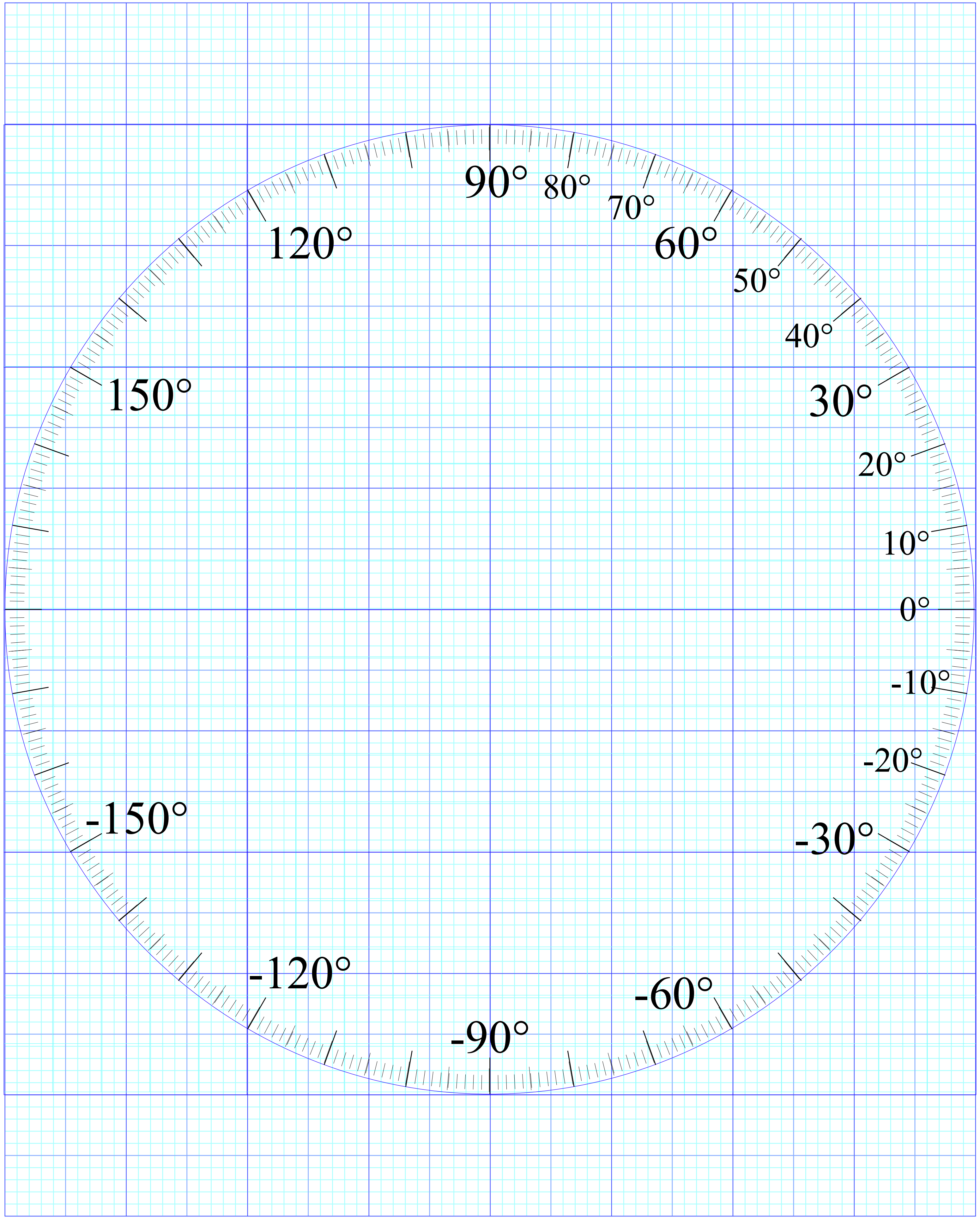

V_2-axis
3.0
2.0
1.0
-1.0
-2.0
-3.0
-1.0 -0.5 0 0.5 1.0 1.5 2.0
V_1-axis
y_2-axis (height of m_2)
7.0 6.0 5.0 4.0 3.0 2.0 1.0
1.0 2.0 3.0 4.0 5.0 6.0 7.0
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
7.0 6.0 5.0 4.0 3.0 2.0 1.0
Time-axis (t in geom. units)
1.0 2.0 3.0 4.0 5.0 6.0 7.0

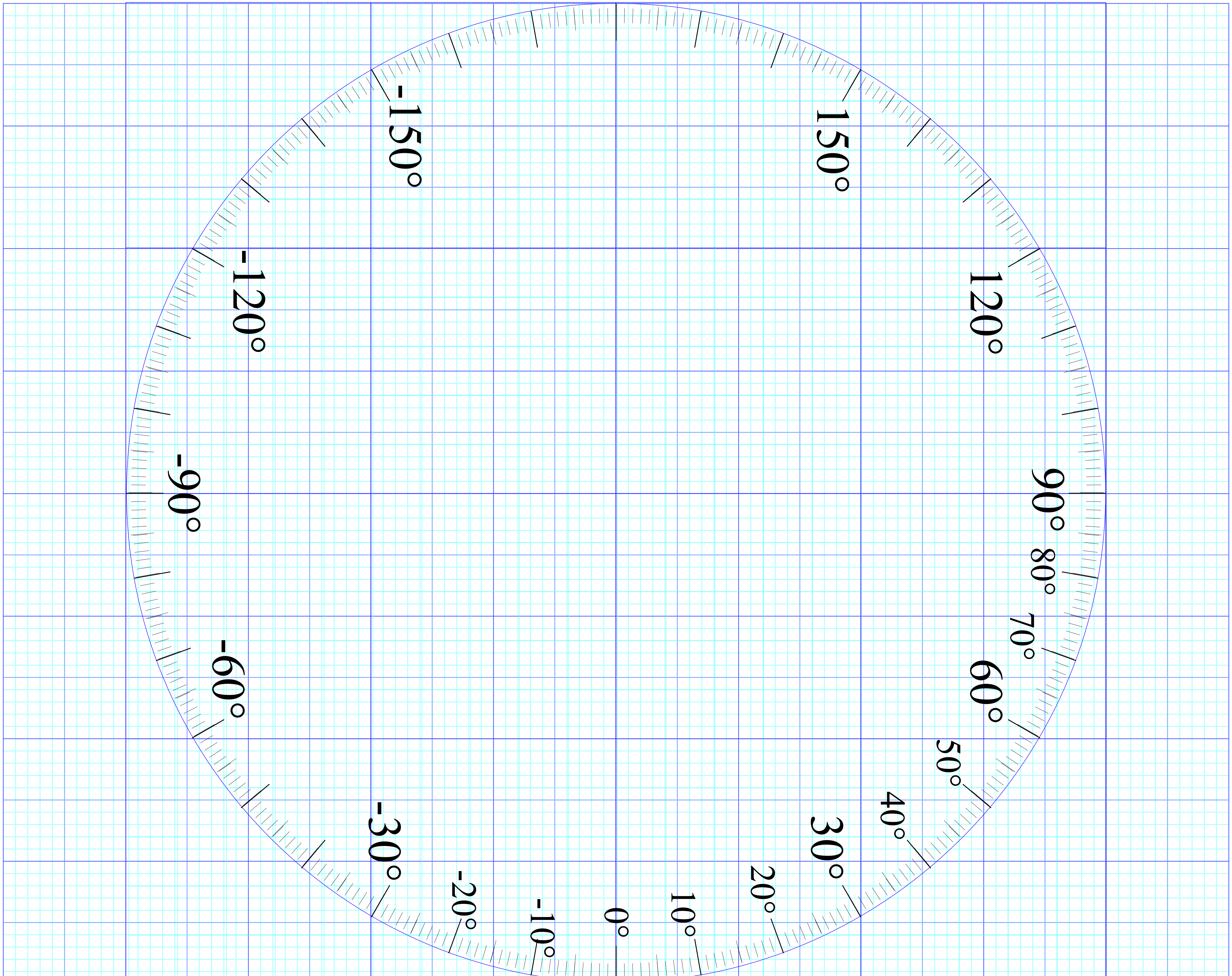


2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
3.0
2.0
1.0
-1.0
-2.0
-3.0



70
60
50
40
30
20
10
120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-10
-20
-30
-40
-50
-60


90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
-10°
-20°
-30°
-60°
-90°
-120°
-150°
150°
120°



0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
120°
150°
-150°
-120°
-90°
-60°
-30°
-20°
-10°