Lecture 30
Thur. 12.14.2017

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

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Using (some) wave parameters to develop relativistic quantum theory

…and *classical* mechanics

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c\text{)}$$

$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$

$\sinh\rho \approx \rho$

At low speeds:

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{ for } (u \ll c)$$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
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space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

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Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

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Resembles: Mu

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
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$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{(for } u \ll c)$$

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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$ (The famous Mc^2 shows up here!)

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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RelaWavity Web Simulation - Relativistic *Terms* *(Expanded Table)*

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This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

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Quantized amplitude

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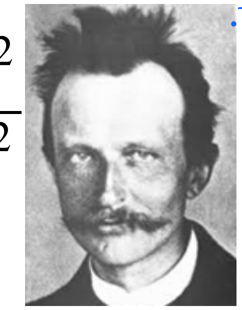

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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

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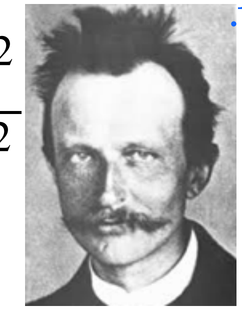

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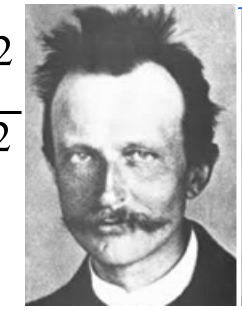

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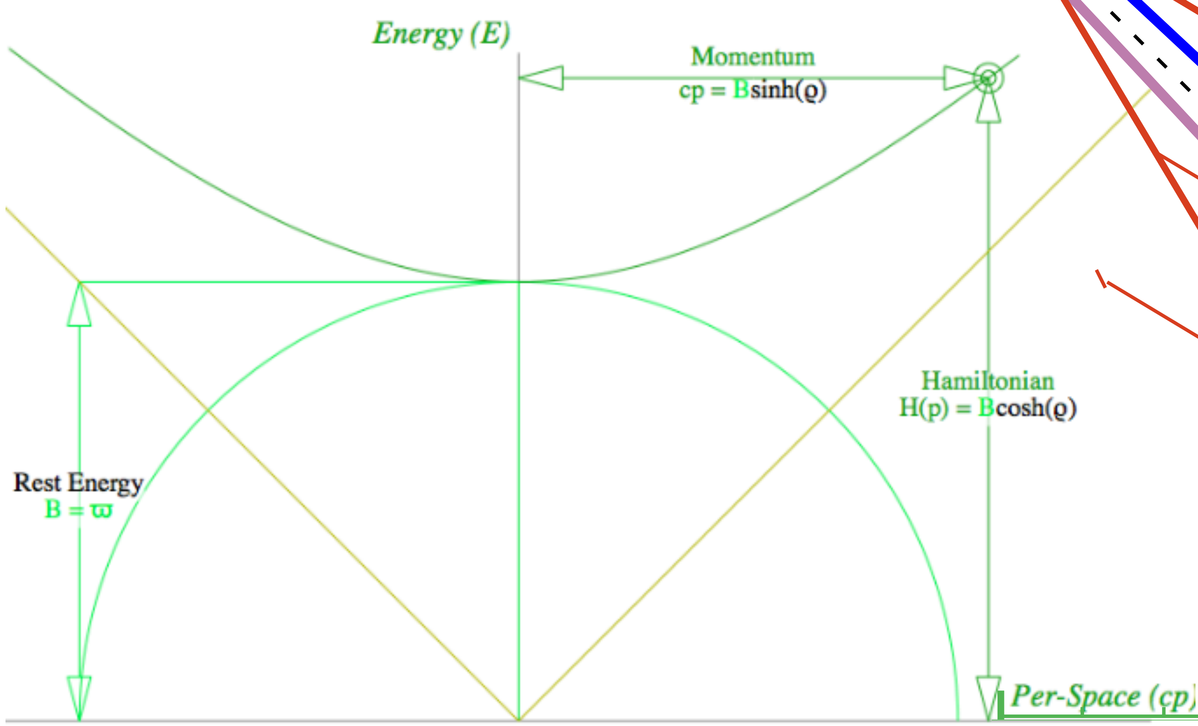

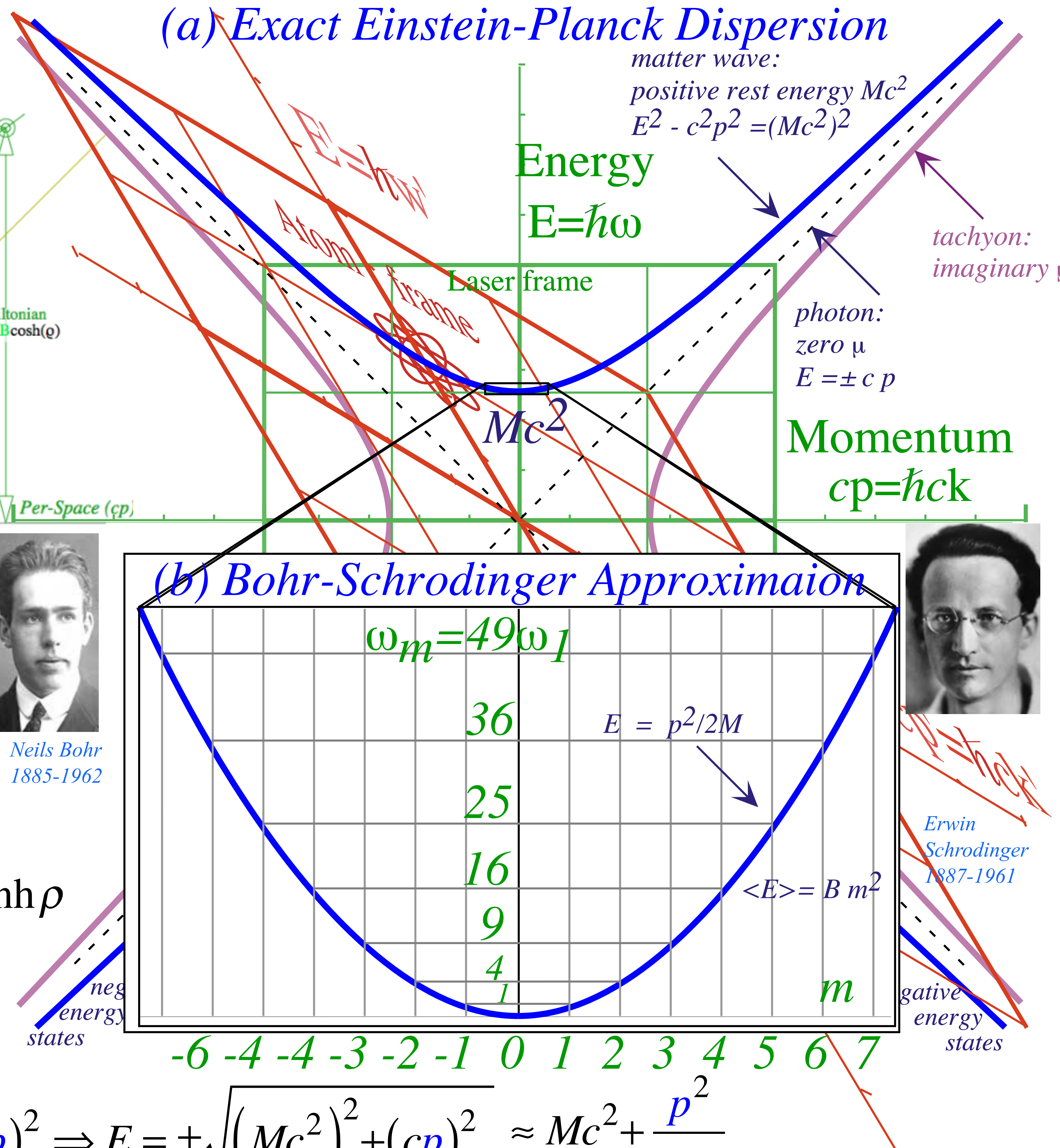

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Neils Bohr
1885-1962

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Erwin
Schrodinger
1887-1961

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

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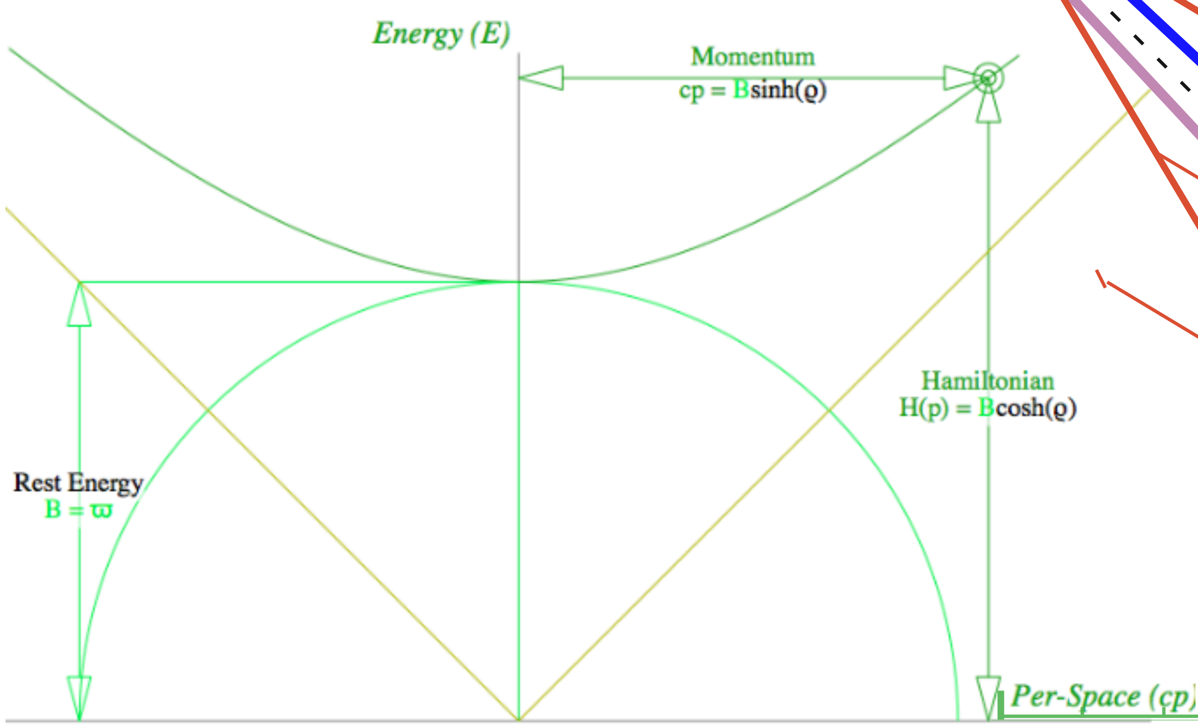

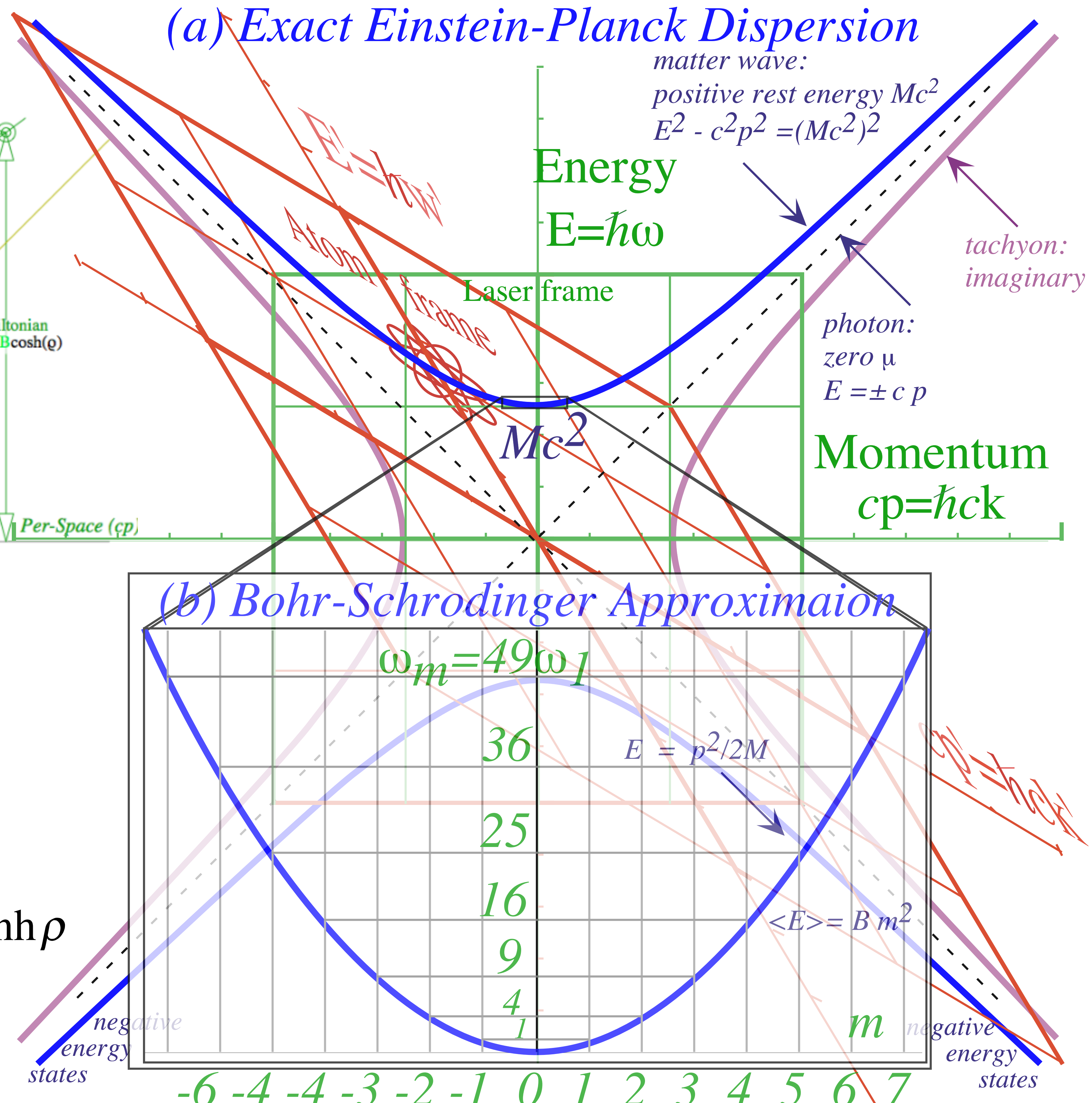

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

low speed approximation

Relawavity variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5 *effects*	$\frac{1}{2}$= 0.5 $b_{RED}^{Doppler}$	$\frac{3}{5}$=0.6 V_{group}	$\frac{3}{4}$=0.75 *past-future asymmetry* (*off-diagonal Lorentz-transform*)	$\frac{4}{5}$=0.80 *x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	$\frac{5}{4}$=1.25 *t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	$\frac{4}{3}$=1.33 *inverse asymmetry*	$\frac{5}{3}$=1.67 V_{phase}	$\frac{2}{1}$=2.0 $b_{BLUE}^{Doppler}$

Relativistic quantum mechanics variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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value for β=3/5 *functions*	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6 V_{group}= $c\tanh\rho$	$\frac{3}{4}$=0.75 *momentum* $cp=Mc^2\sinh\rho$	$\frac{4}{5}$=0.80 *-Lagrangian* $L=-Mc^2\text{sech}\,\rho$	$\frac{5}{4}$=1.25 *Hamiltonian* $H=Mc^2\cosh\rho$	$\frac{4}{3}$=1.33 *DeBroglie* $\lambda=\alpha\,\text{csch}\,\rho$	$\frac{5}{3}$=1.67 V_{phase} = $c\cot h\rho$	$\frac{2}{1}$=2.0

Lecture 30
Thur. 12.14.2017

Derivation of relativistic quantum mechanics
➔ What's the matter with mass? Shining some light on the Elephant in the room
    Relativistic action and Lagrangian-Hamiltonian relations
        Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
    Feynman diagram geometry
        Compton recoil related to rocket velocity formula
        Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
    Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
        Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
        Animation of mechanics and metrology of constant-g grid

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$
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momentum: $cp = Mc^2 \sinh\rho$
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velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

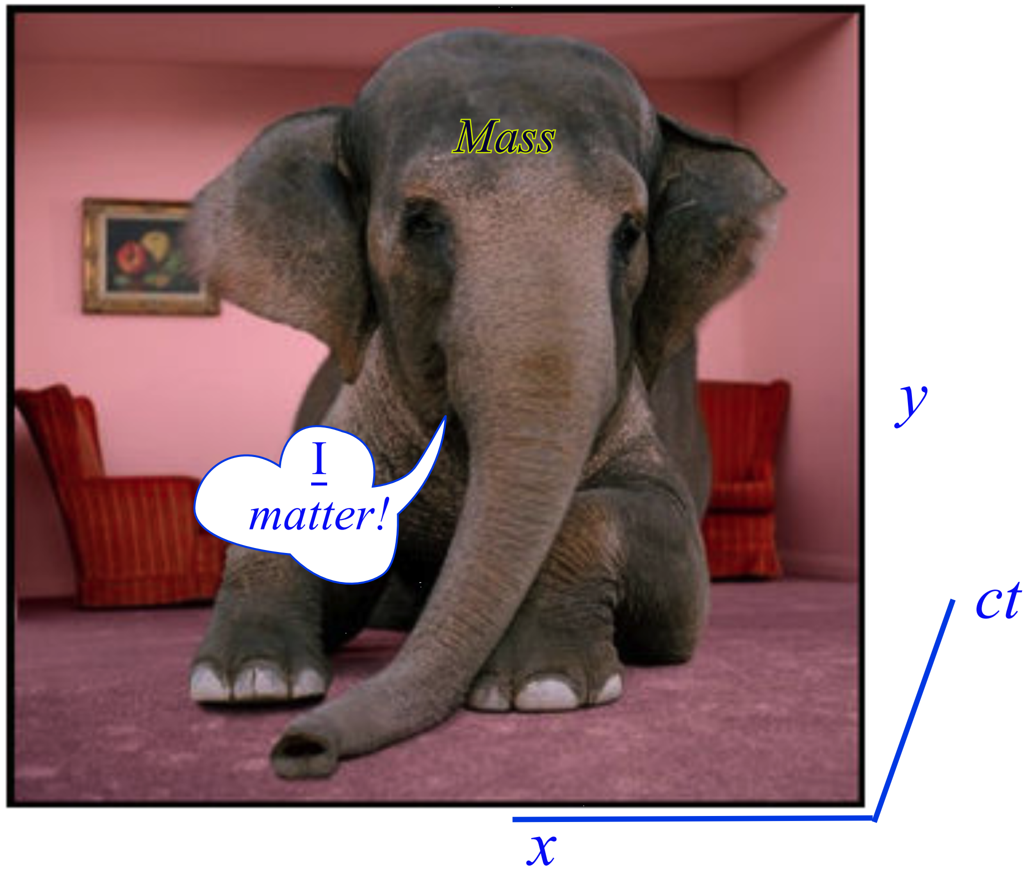
Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

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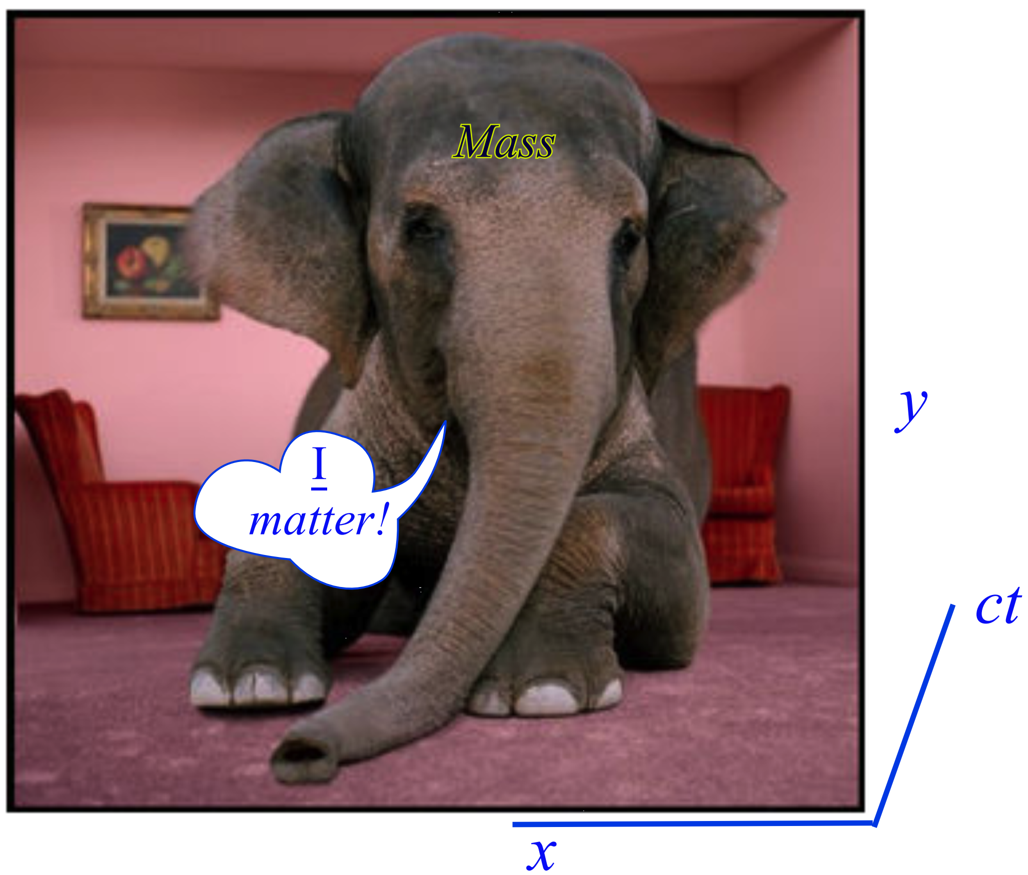

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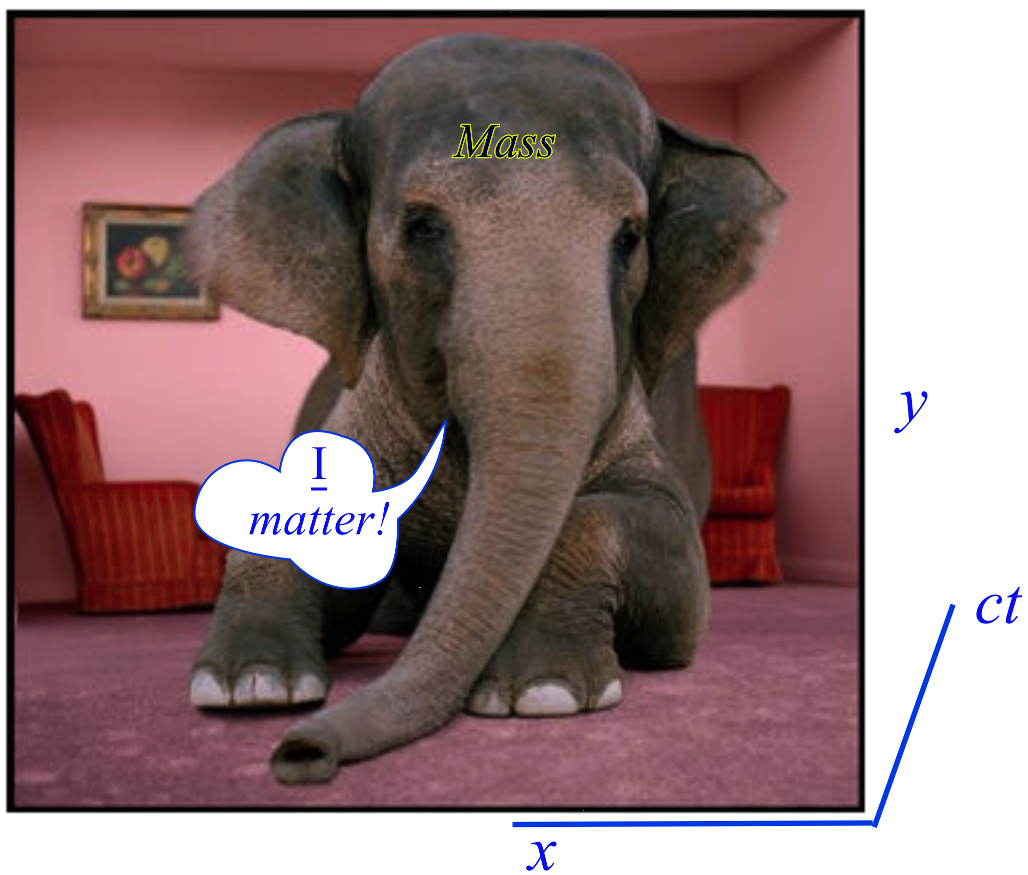
- ***What's the matter with Mass?***



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Limiting cases:

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More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1 - u^2/c^2\right)^{3/2}}$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho = h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho = hc\kappa_{phase}$

Group velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

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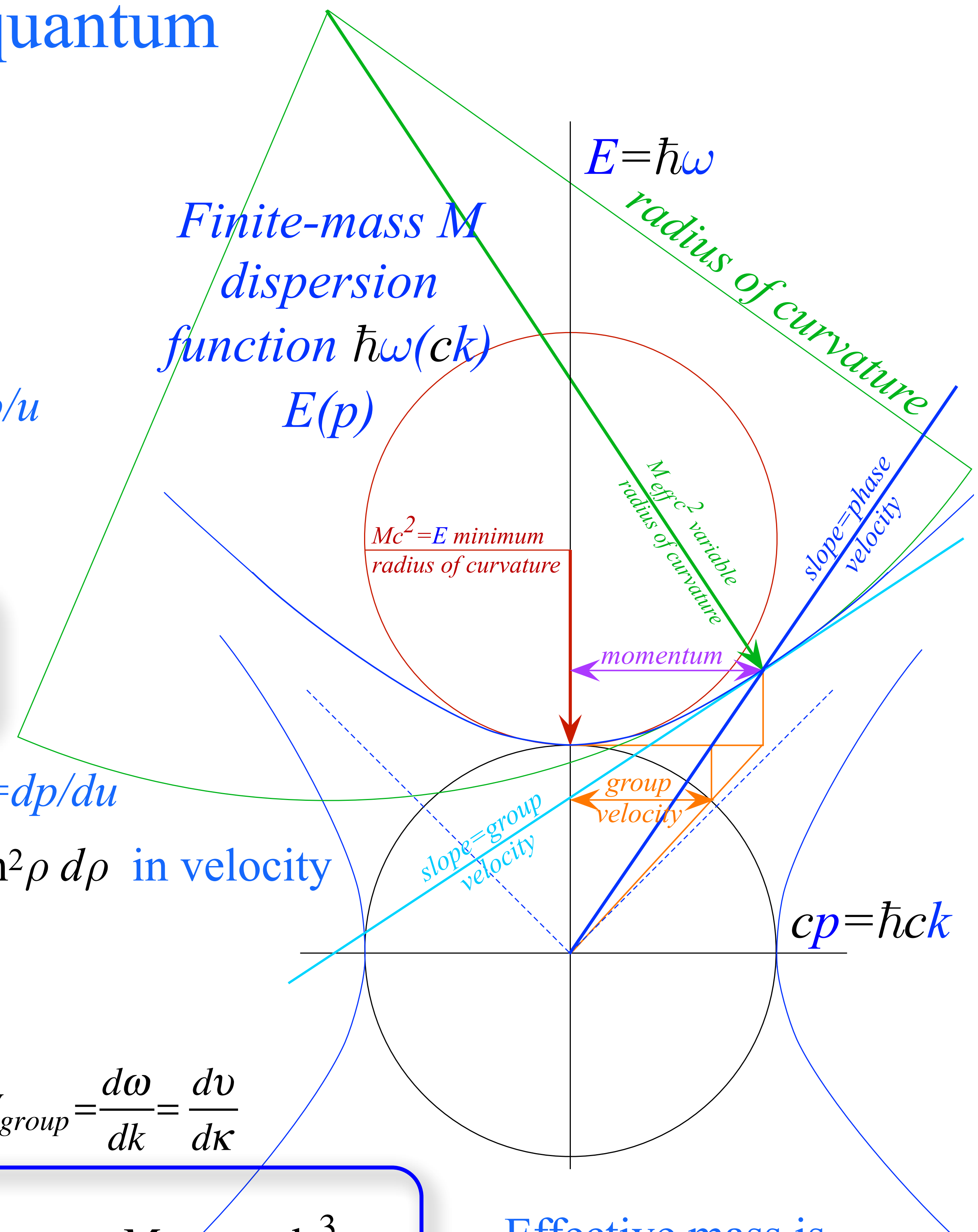
general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum



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Momentum Mass M_{mom} (Galileo's mass) Defined by p/u

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest} c \sinh\rho}{c\tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \textit{Momentum Mass}$$

Effective Mass M_{eff} (Newton's mass) Defined by $F/a = dp/du$

That is ratio of $dp = Mc\cosh\rho \, d\rho$ to change $du = c\,\mathrm{sech}^2\rho \, d\rho$ in velocity

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion*.

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass $(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0,$

Momentum Mass $(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2},$

Effective Mass $(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty.$

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2\cdot 10^{-51})kg\cdot s = 4.5\cdot 10^{-36}kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Lecture 30
Thur. 12.14.2017

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- ➔ Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$h\upsilon_A = Mc^2 = hc\kappa_A$
$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$
$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

Prior wave relations

←linear Hz format   angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar c k_A$
$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$
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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho$$

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
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$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho$$

$$E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor format→

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

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Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

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$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

$$L \text{ is} : Mc^2 \frac{-1}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

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Note: $Mcu = Mc^2 \tanh\rho$

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Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

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Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

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Note: $Mcu = Mc^2\tanh\rho$

Also: $cp = Mc^2\sinh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2\cosh\rho$$

$$= Mc^2\sqrt{1 + \sinh^2\rho} = Mc^2\sqrt{1 + (cp)^2}$$

Prior wave relations

$\hbar = h/2\pi$

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Legendre transformation

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$ $=c\sin\sigma$

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Also: $cp=Mc^2\sinh\rho$

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Compare *Lagrangian L*

$$\dot{S}=L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

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$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Including stellar angle σ

Prior wave relations

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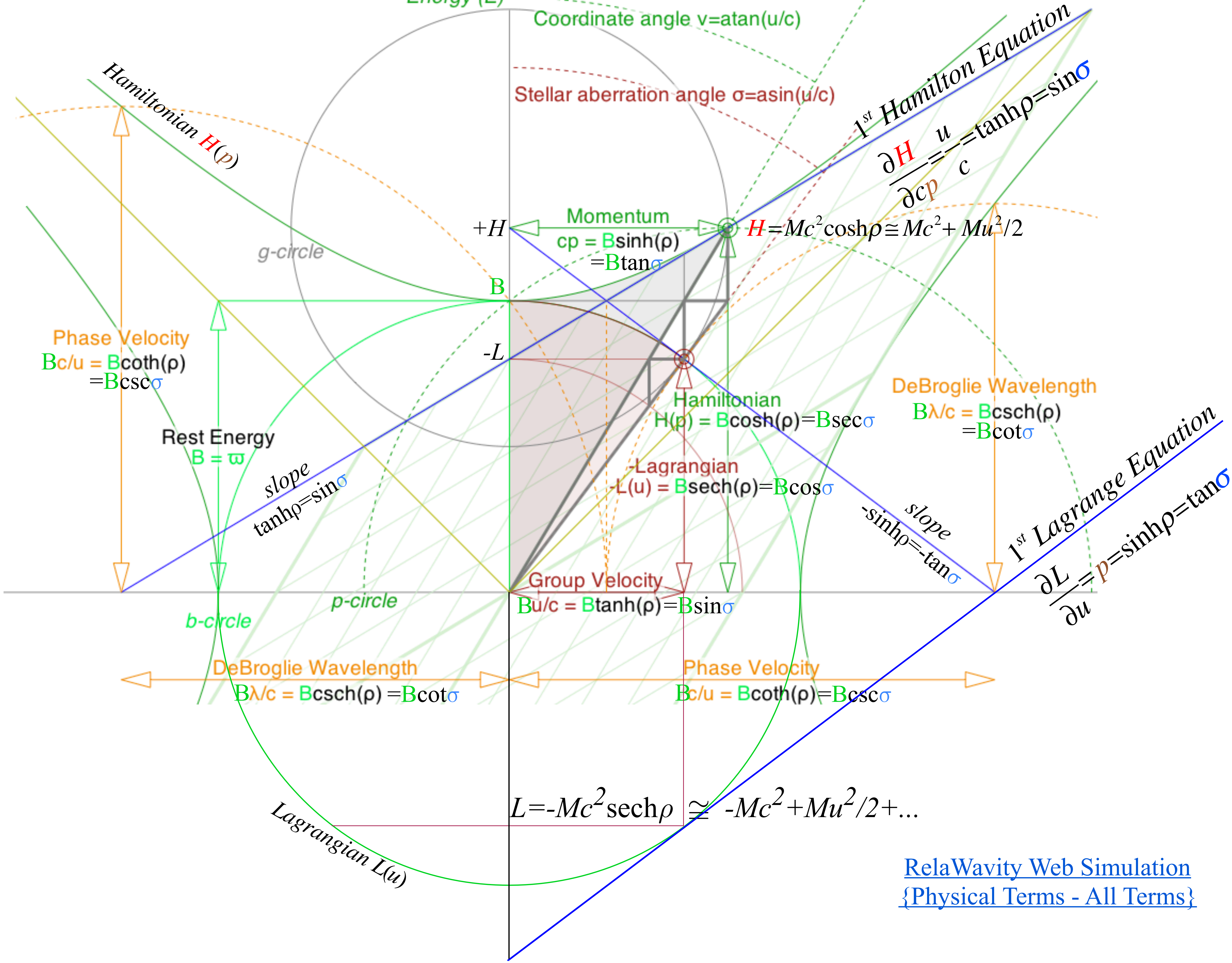

RelaWavity Web Simulation
{Physical Terms - All Terms}

Lecture 30
Thur. 12.14.2017

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Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

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Define *Action* $S = \hbar\Phi$

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Legendre transformation

$$dS\equiv Ldt\equiv\hbar\,d\Phi=\hbar k\,dx-\hbar\omega\,dt=p\,dx-H\,dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S}=L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\,\mathrm{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

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Define *Action* $S=\hbar\Phi$

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\,dx - H\,dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

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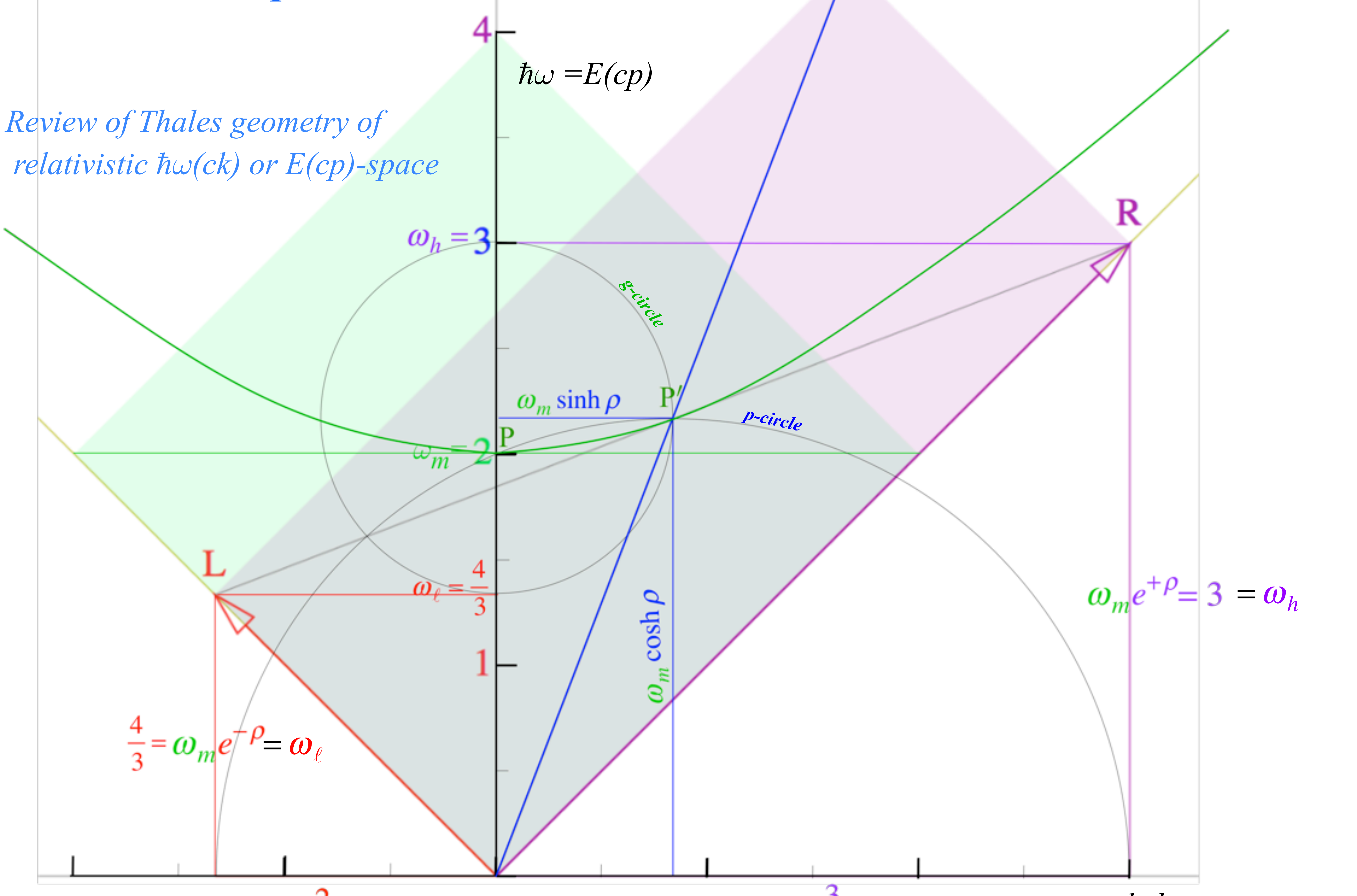
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

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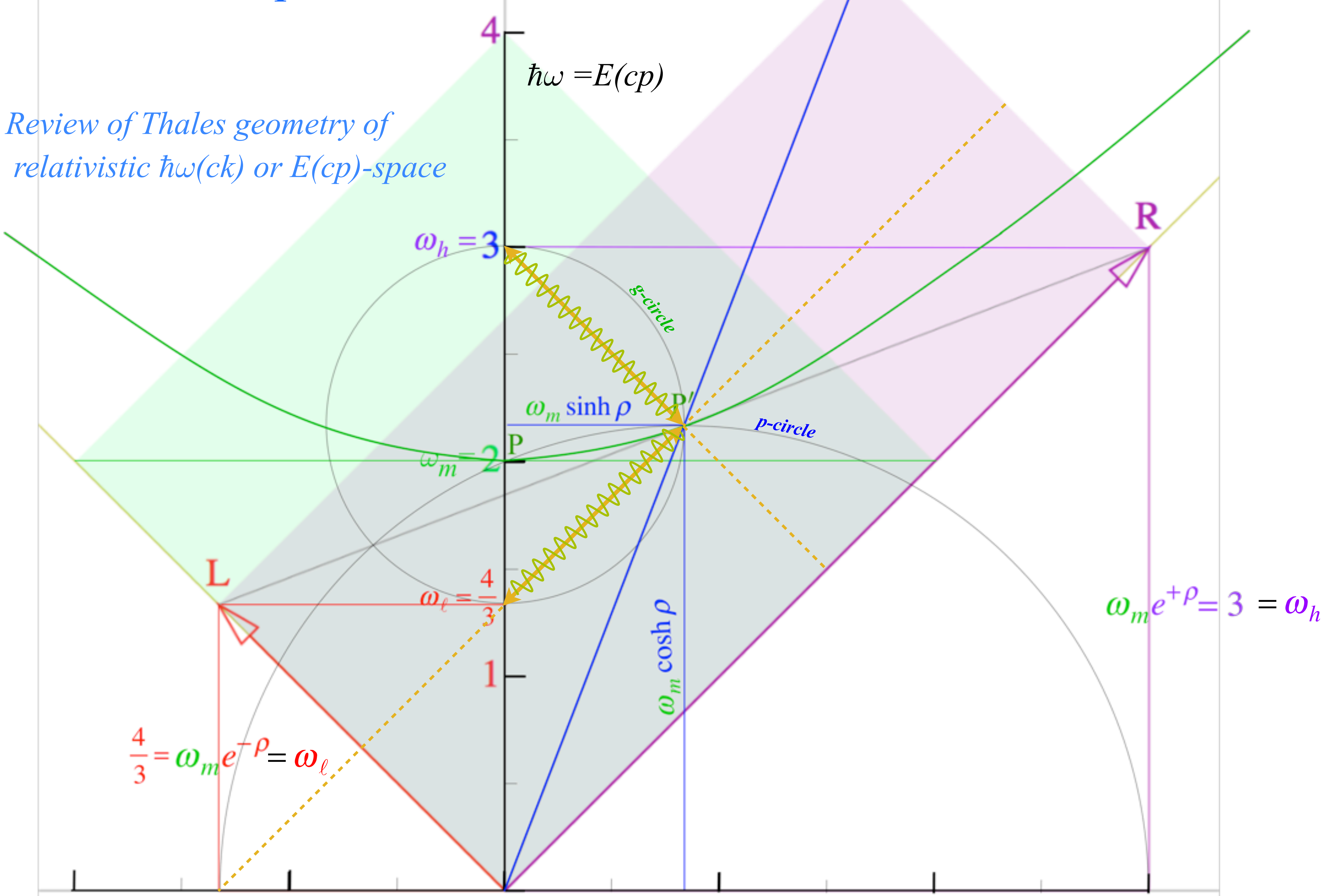

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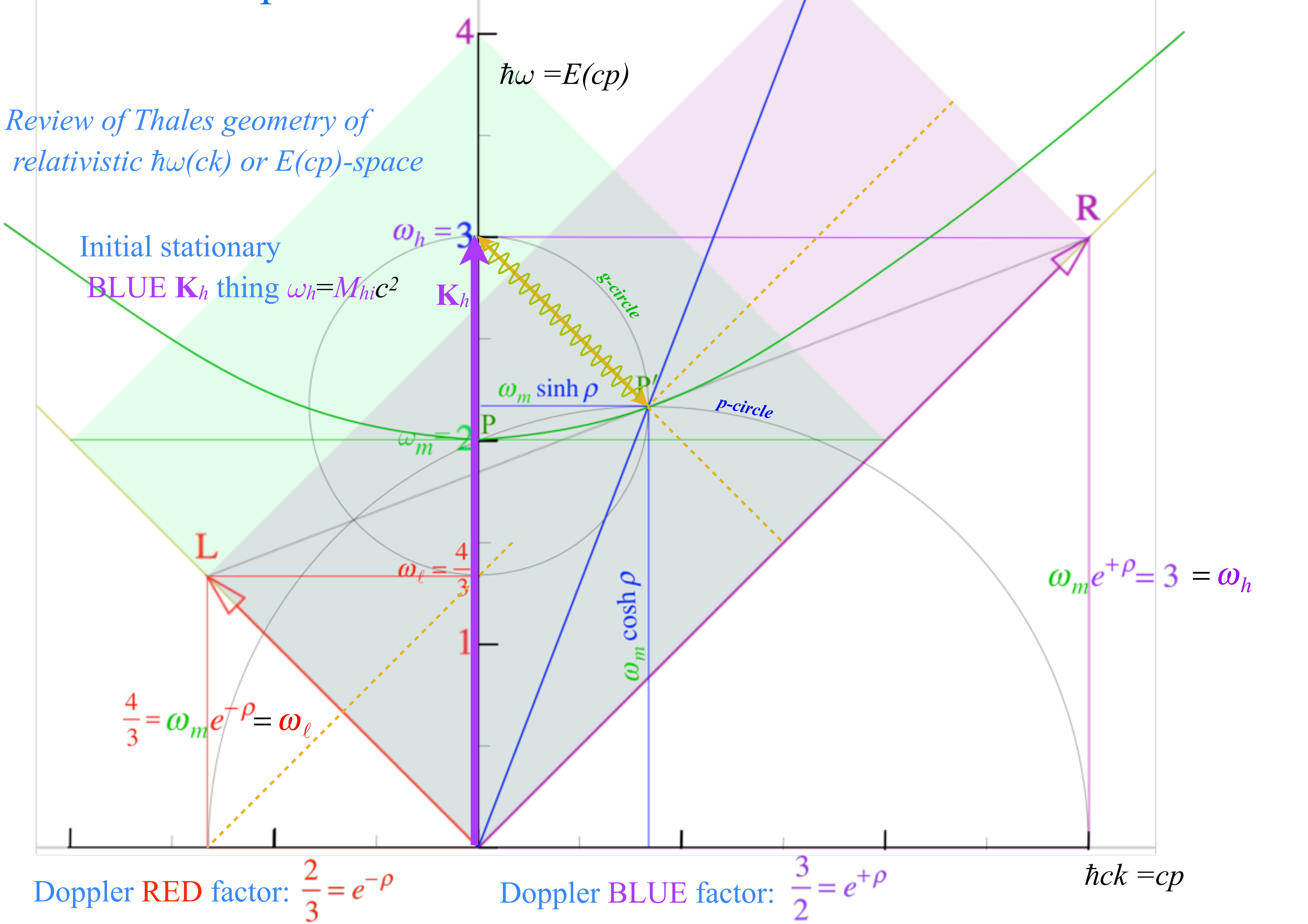

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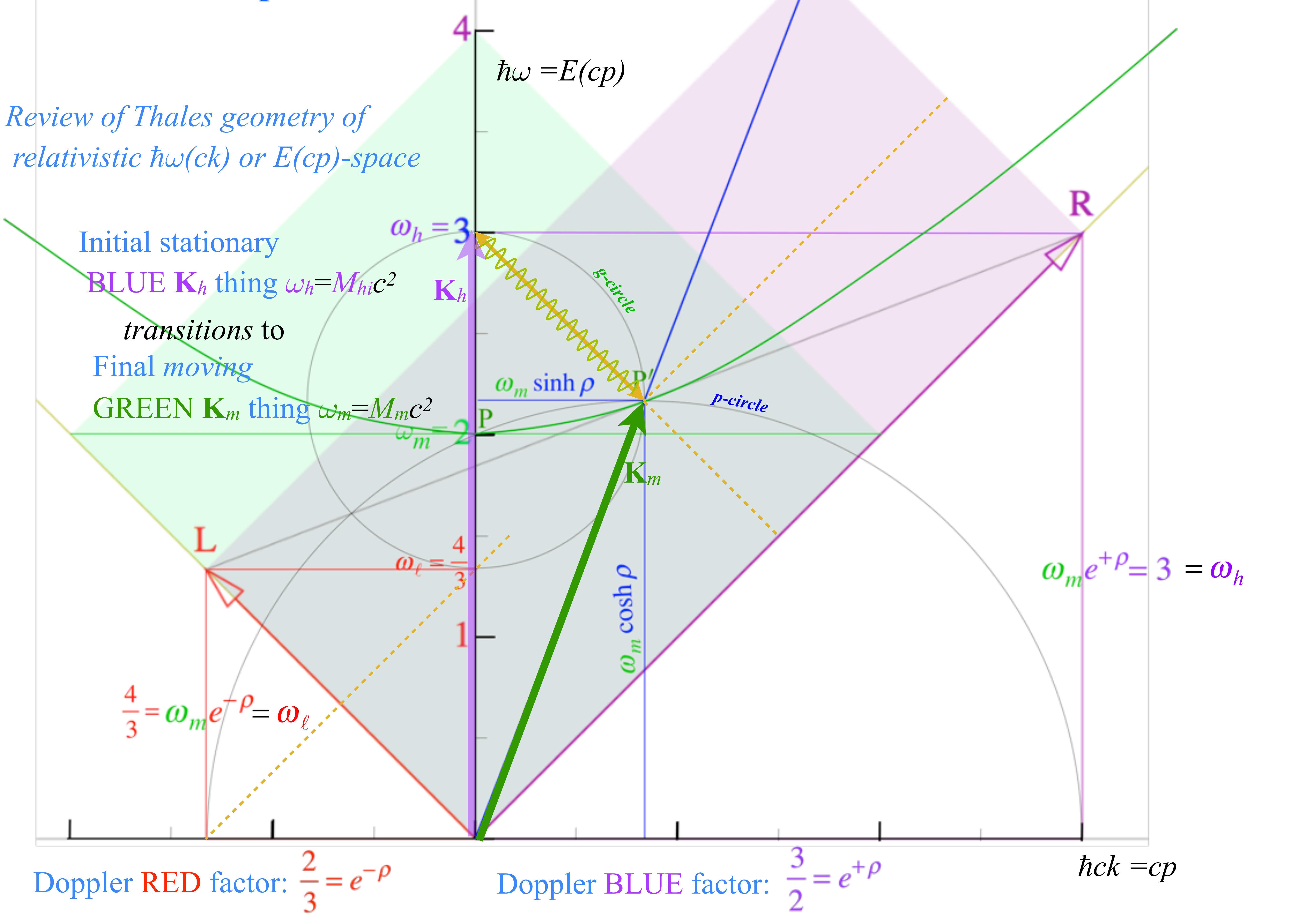

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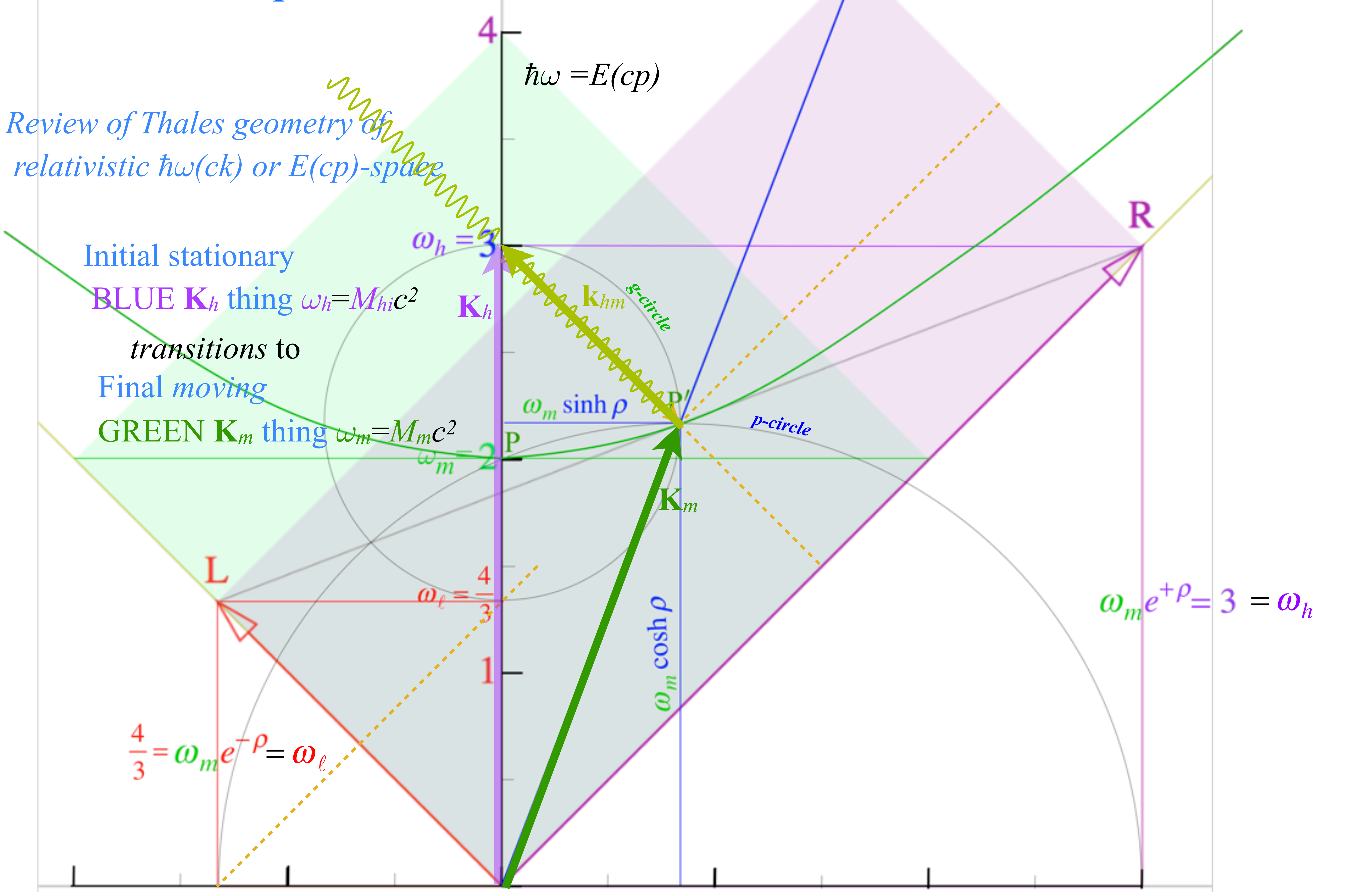

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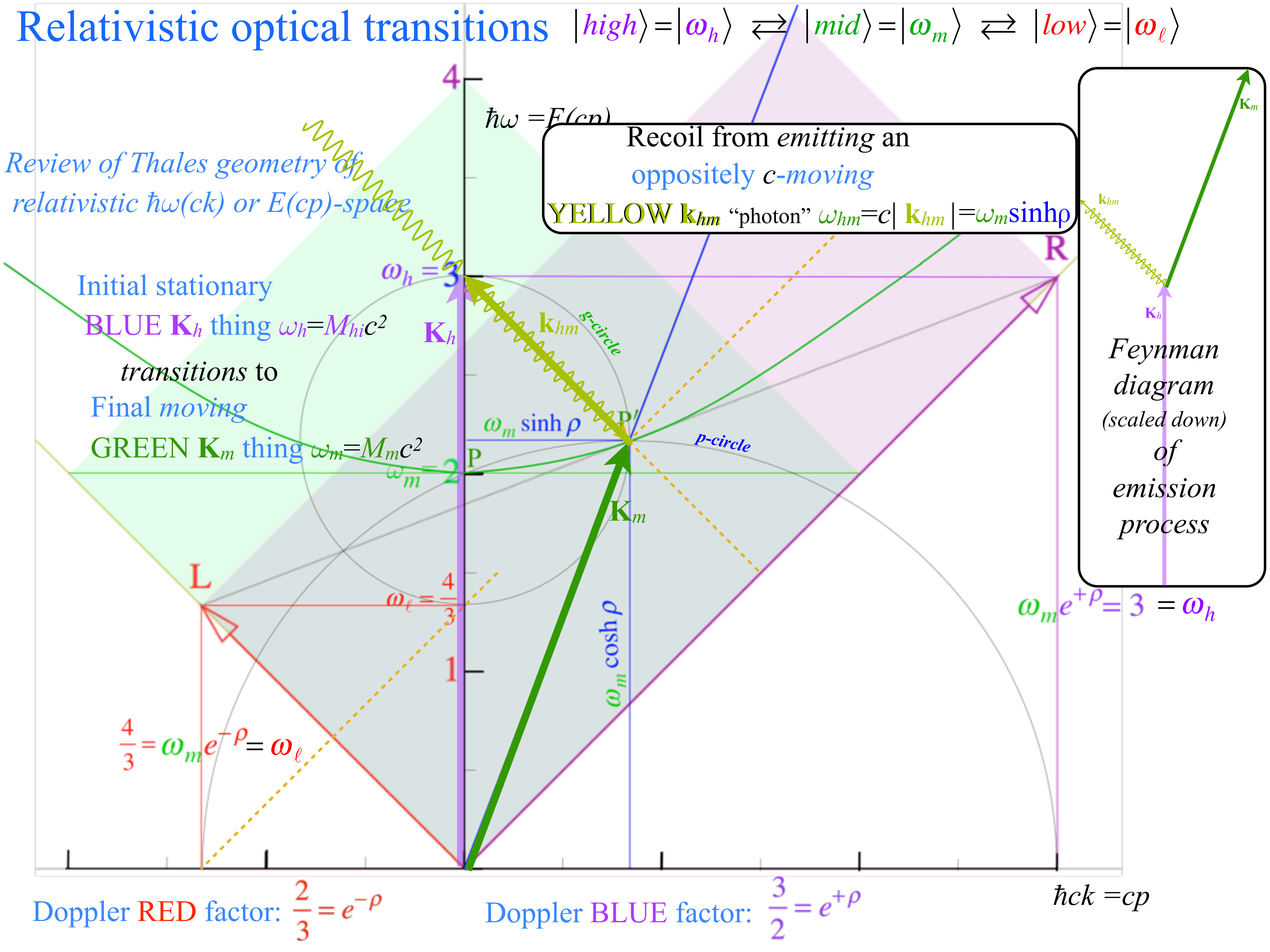
Relativistic optical transitions



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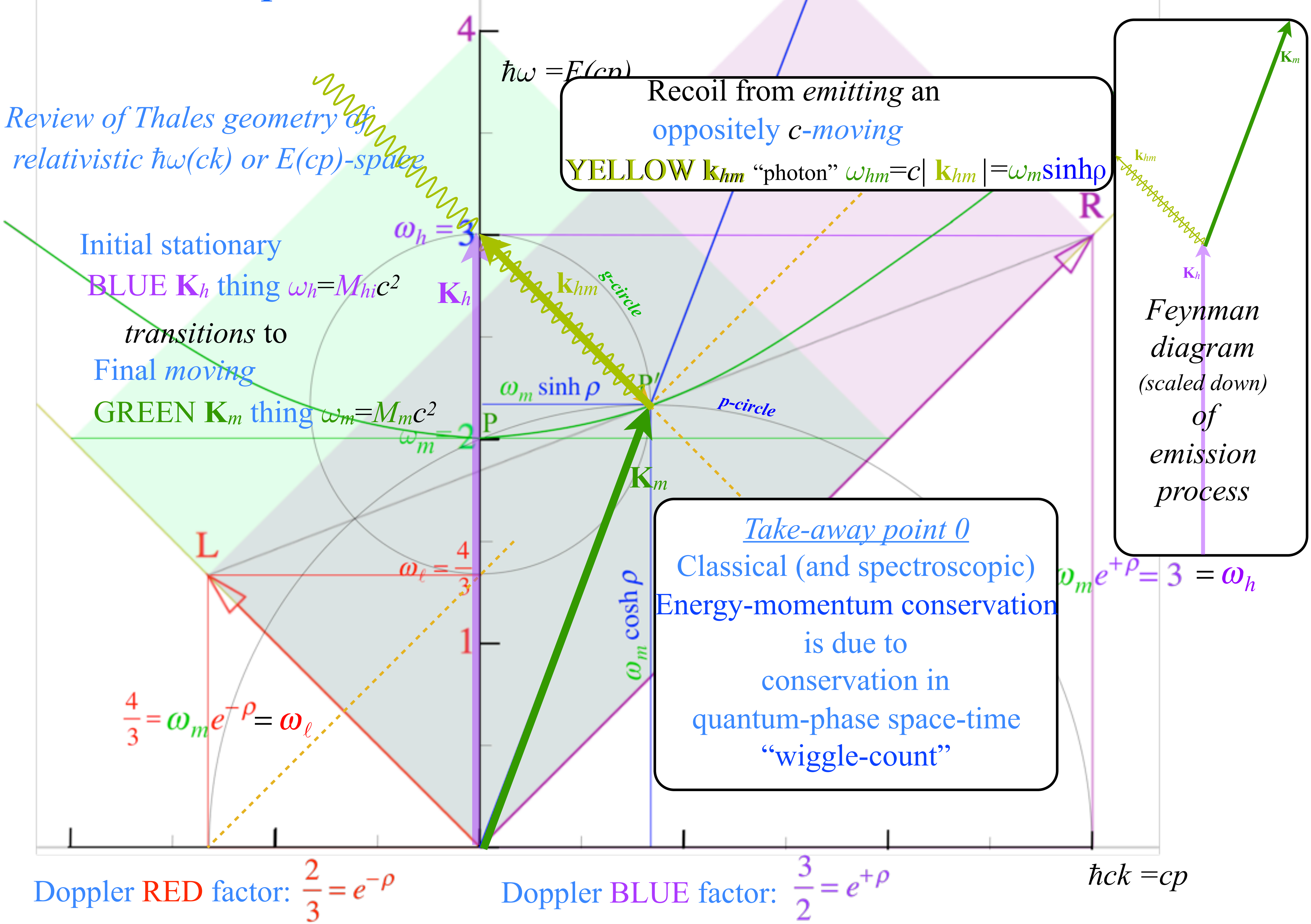

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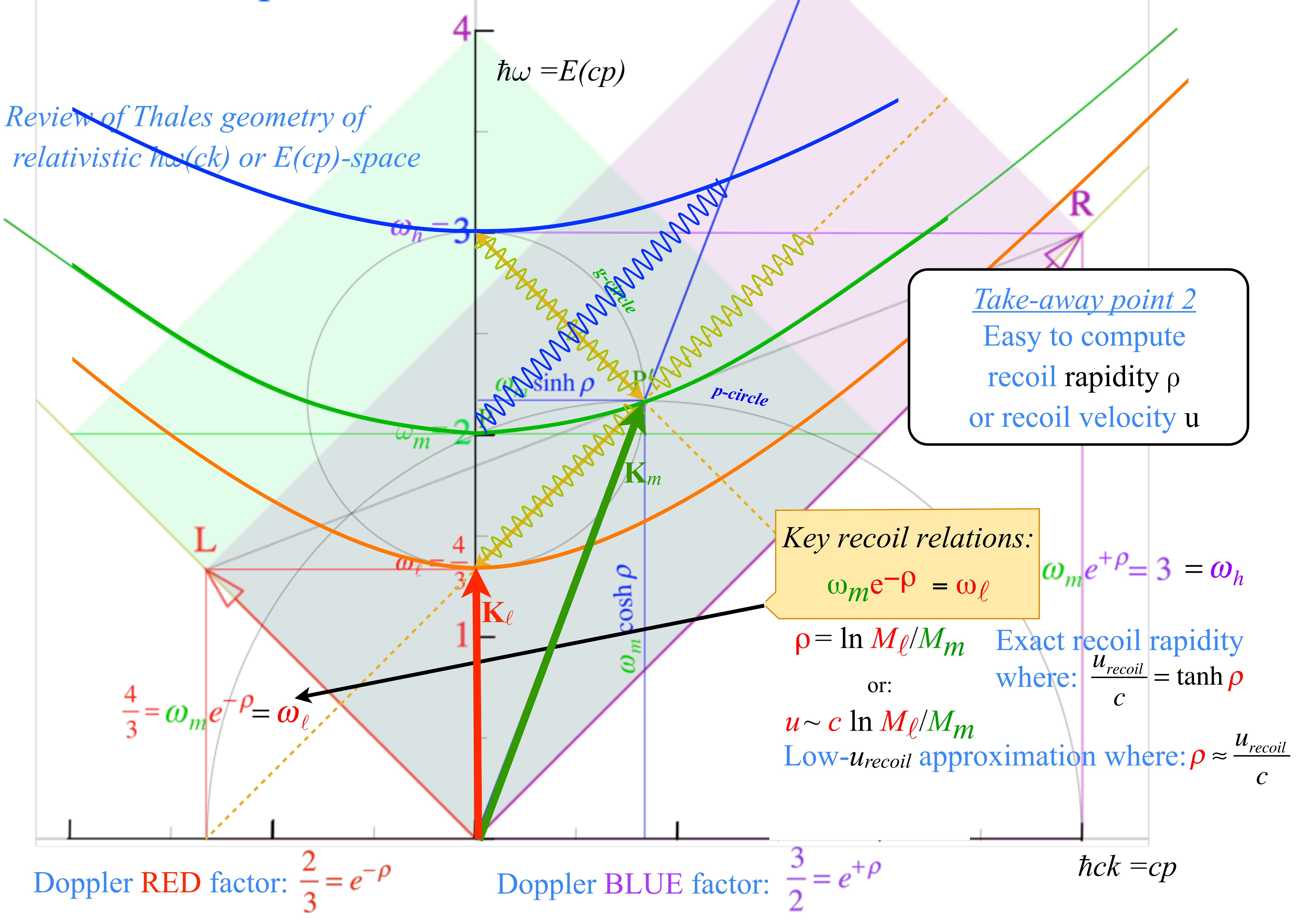
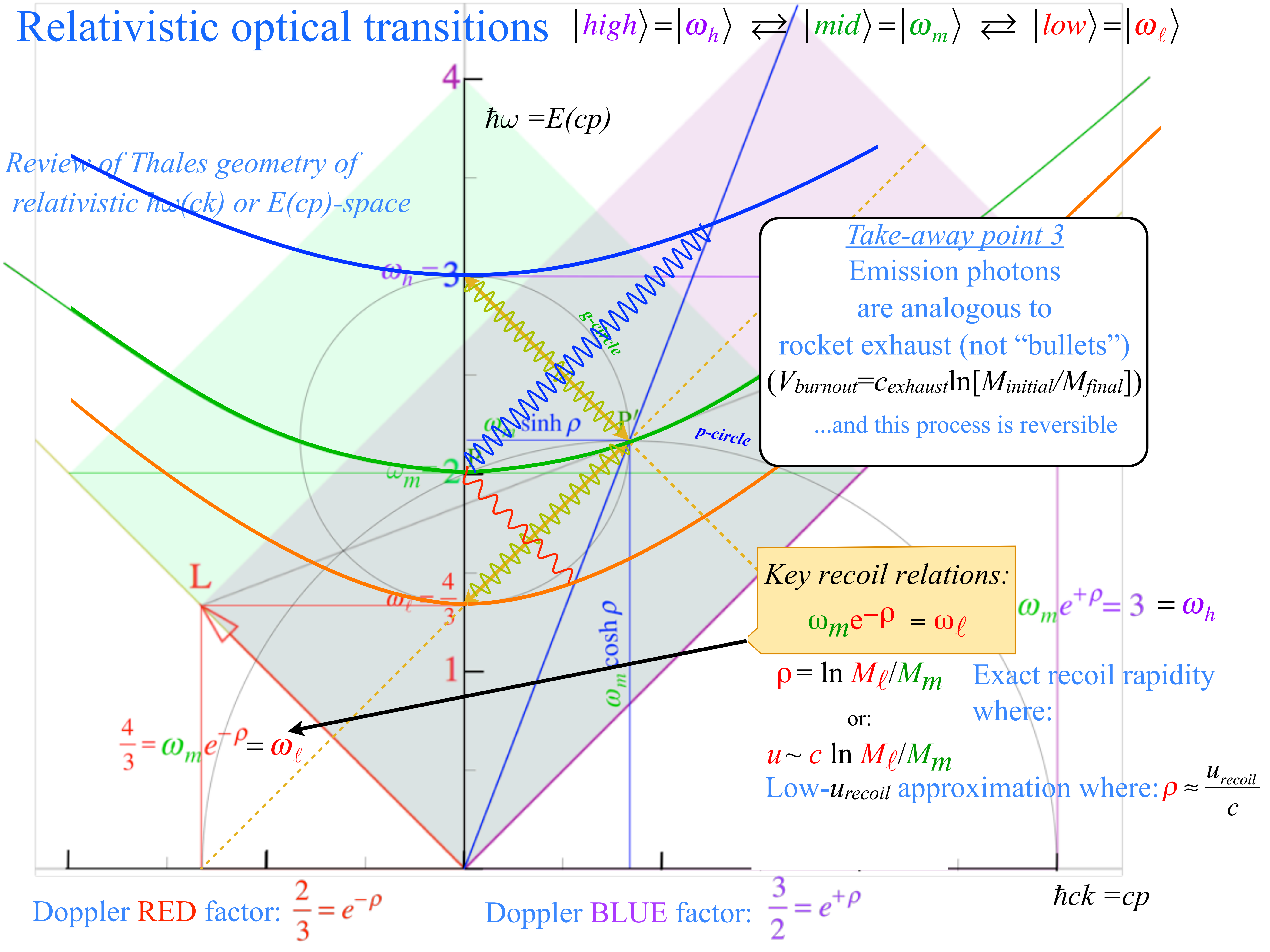
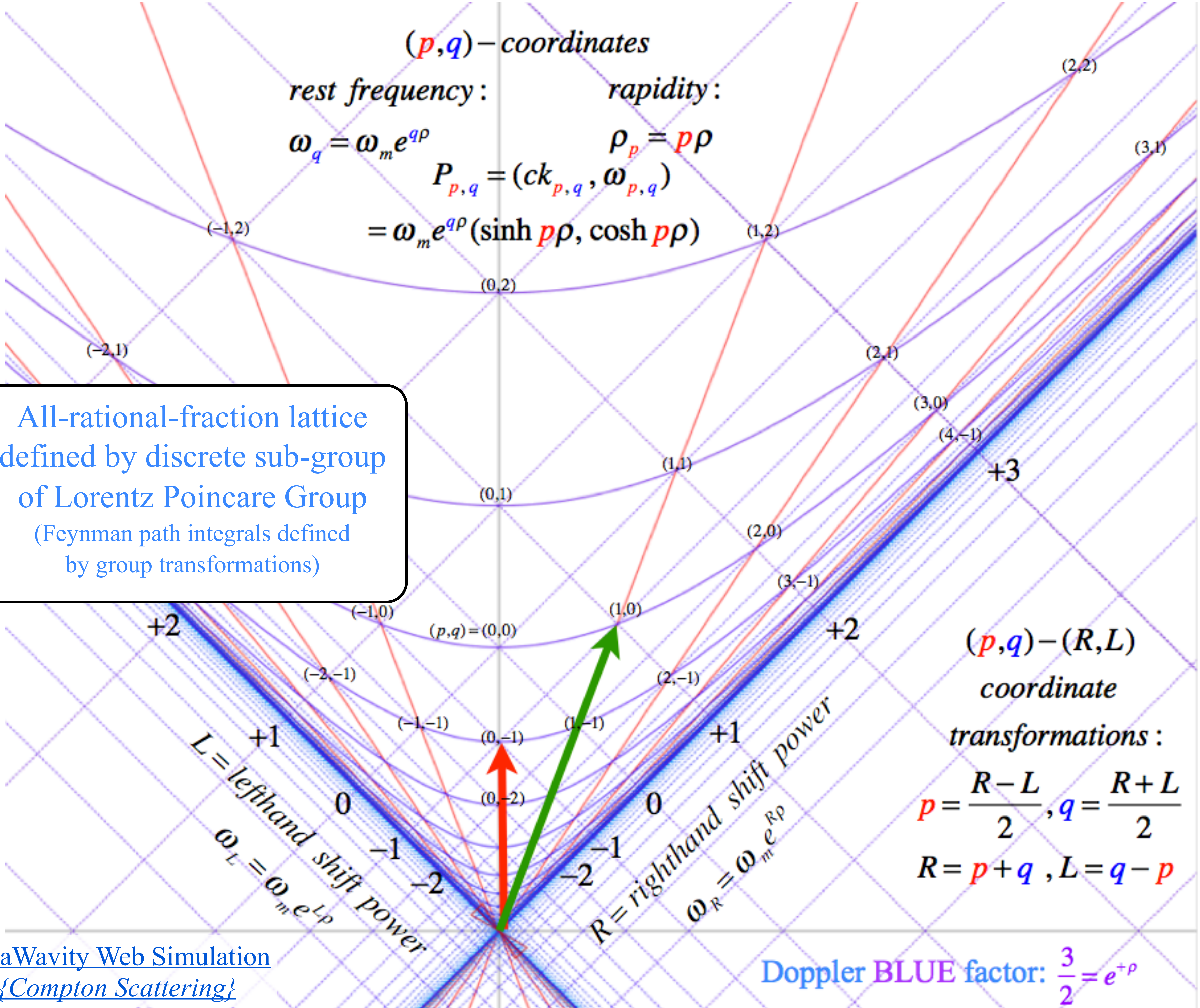
    Animation of mechanics and metrology of constant-g grid

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$





Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space
$\hbar\omega = E(cp)$
4
$\omega_h = 3$
$\omega_m = 2$
$\omega_\ell = \frac{4}{3}$
1
L
P′
g-circle
p-circle
$\omega_m \sinh\rho$
$\omega_m \cosh\rho$
Take-away point 3
Emission photons are analogous to rocket exhaust (not "bullets")
$(V_{burnout} = c_{exhaust} \ln[M_{initial}/M_{final}])$
...and this process is reversible
Key recoil relations:
$\omega_m e^{-\rho} = \omega_\ell$
$\omega_m e^{+\rho} = 3 = \omega_h$
$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$
$\rho = \ln M_\ell / M_m$
Exact recoil rapidity where:
or:
$u \sim c \ln M_\ell / M_m$
Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$
$\hbar ck = cp$
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



All-rational-fraction lattice
defined by discrete sub-group
of Lorentz Poincare Group
(Feynman path integrals defined
by group transformations)

RelaWavity Web Simulation
{Compton Scattering}

Lecture 30
Thur. 12.14.2017

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - ➔ Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

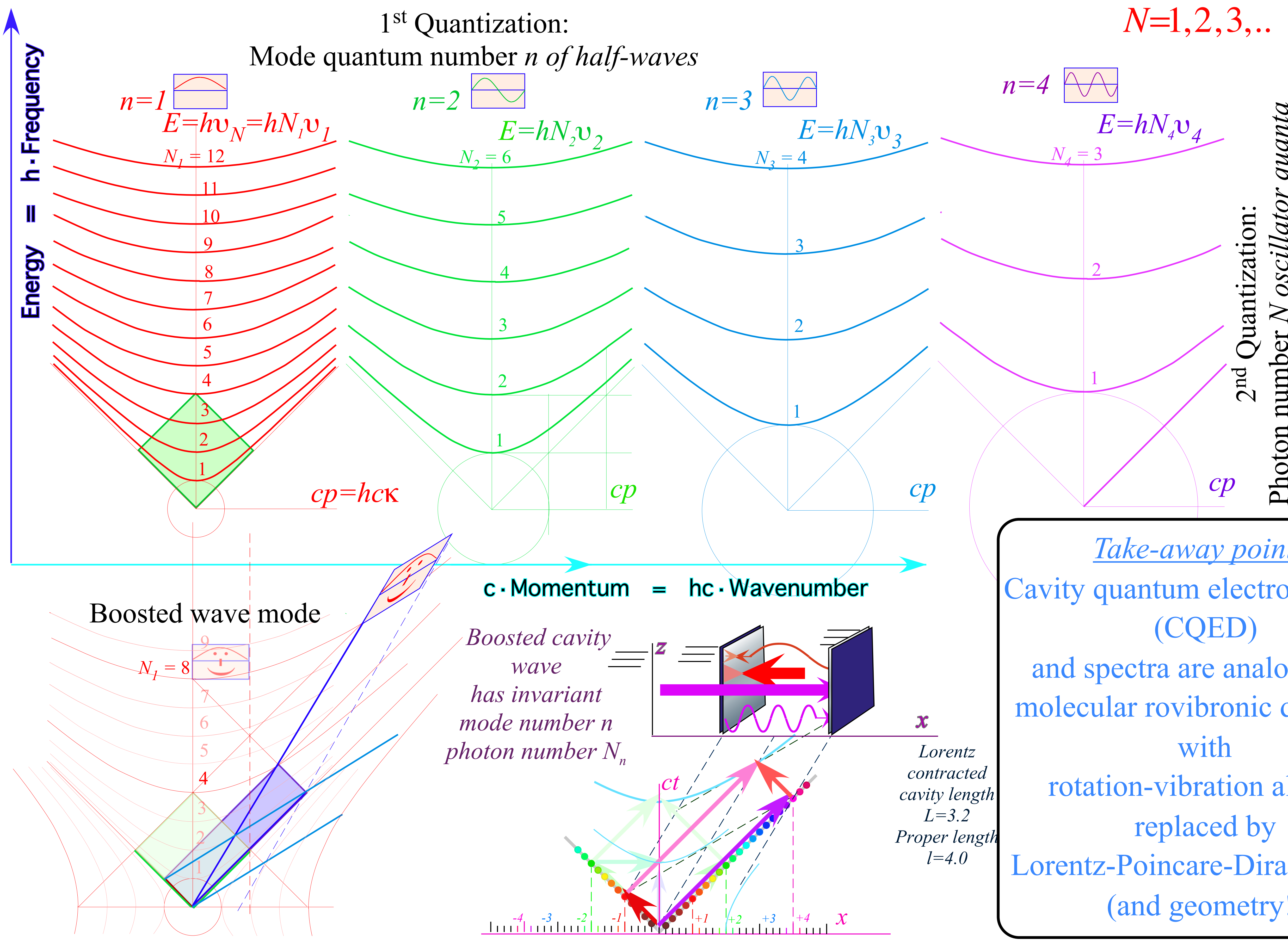
Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

2^{nd} Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho$ with *quantum* numbers$)$

$N=1,2,3,..$



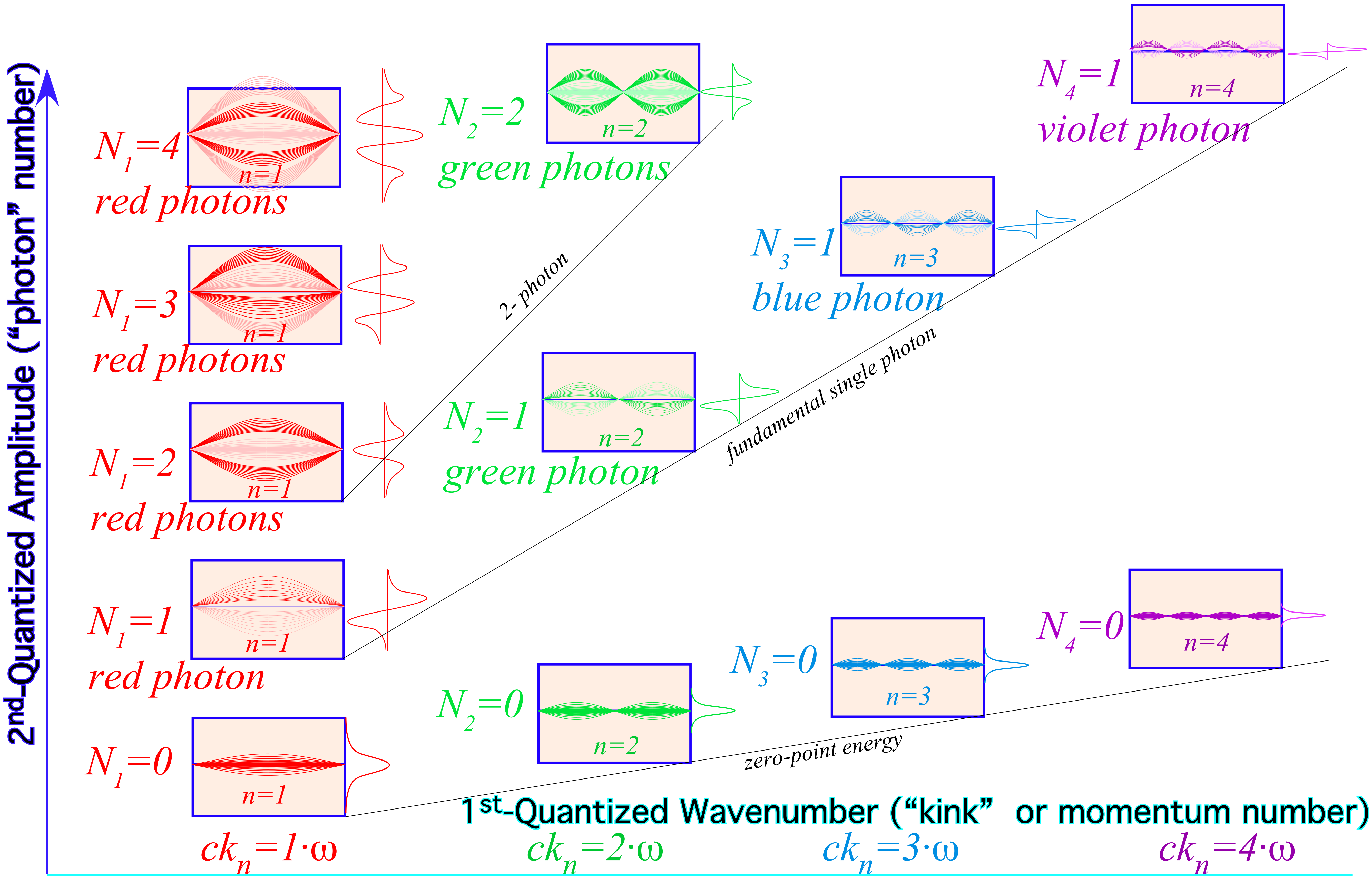
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$\left(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho\right)$ *is actually* $\left(hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..)\right)$



Lecture 30
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Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35
ModPhys (2012)

Varying local acceleration $\rho = \rho(\tau)$

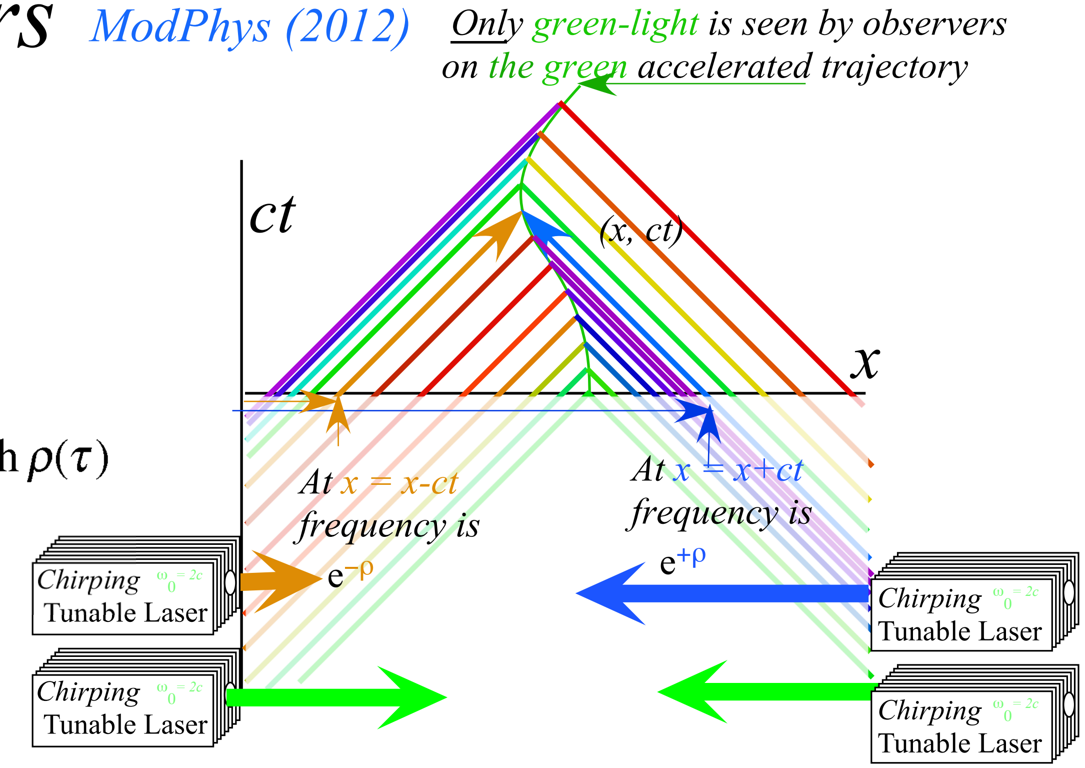
$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int \cosh\rho(\tau)\,d\tau \qquad x = c\int \sinh\rho(\tau)\,d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

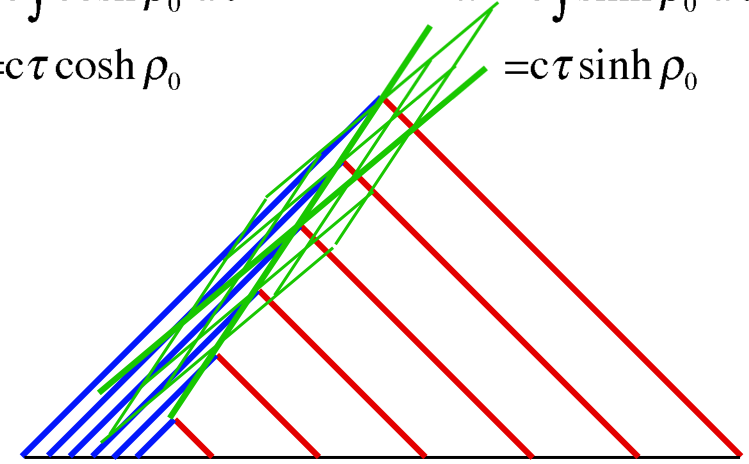
$$ct = c\int \cosh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad x = c\int \sinh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$



Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

$$ct = c\int \cosh\rho_0\,d\tau = c\tau\cosh\rho_0 \qquad x = c\int \sinh\rho_0\,d\tau = c\tau\sinh\rho_0$$



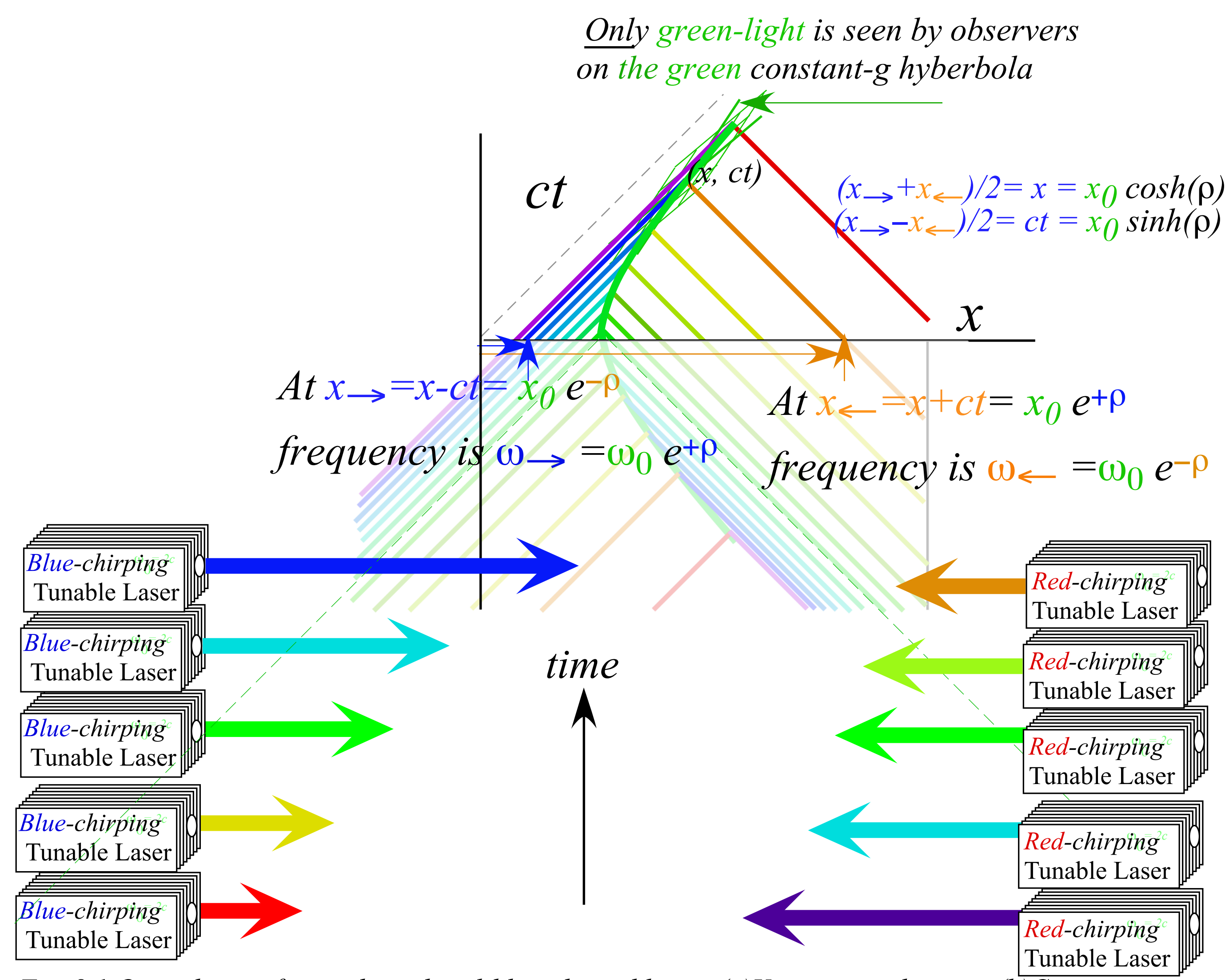
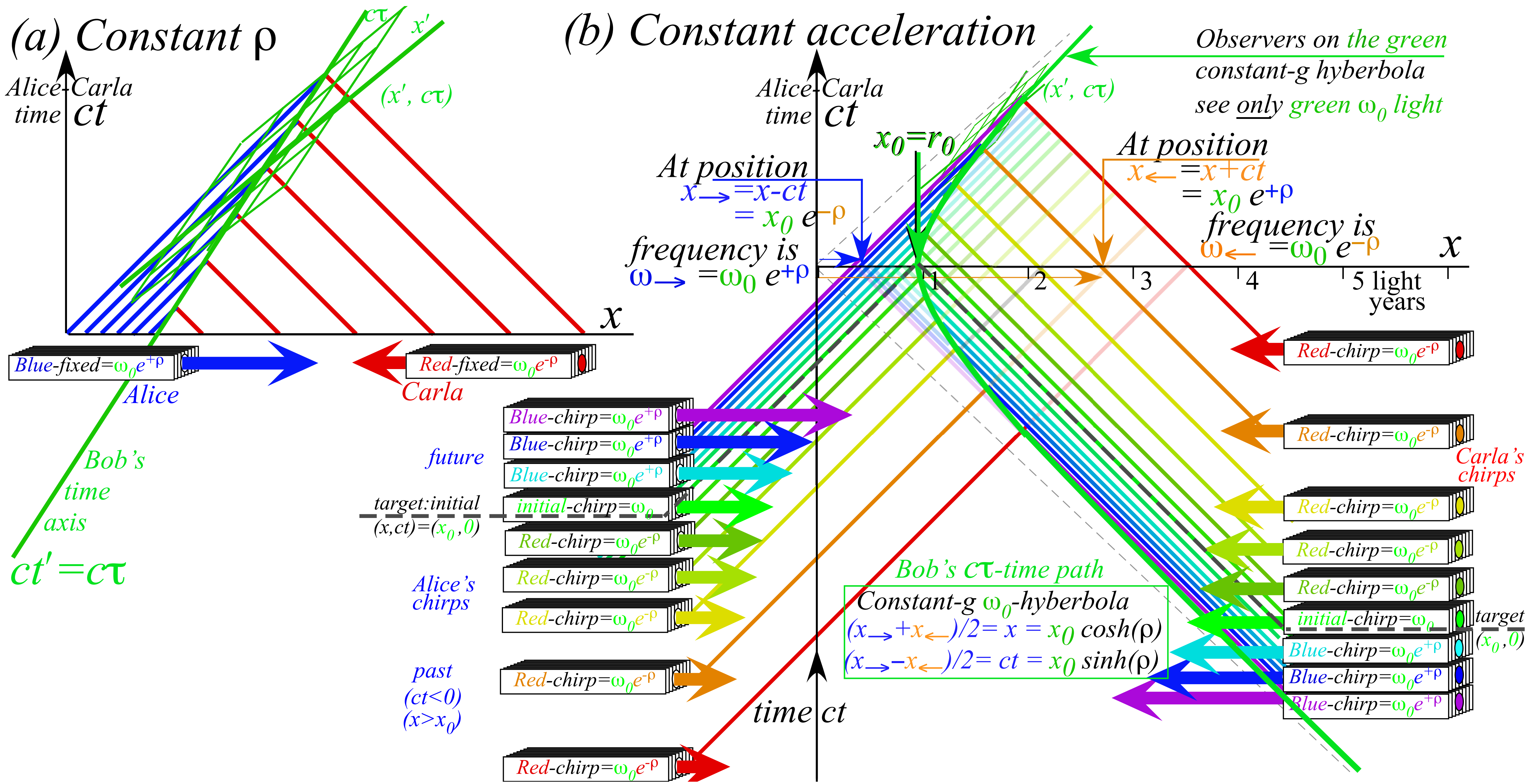
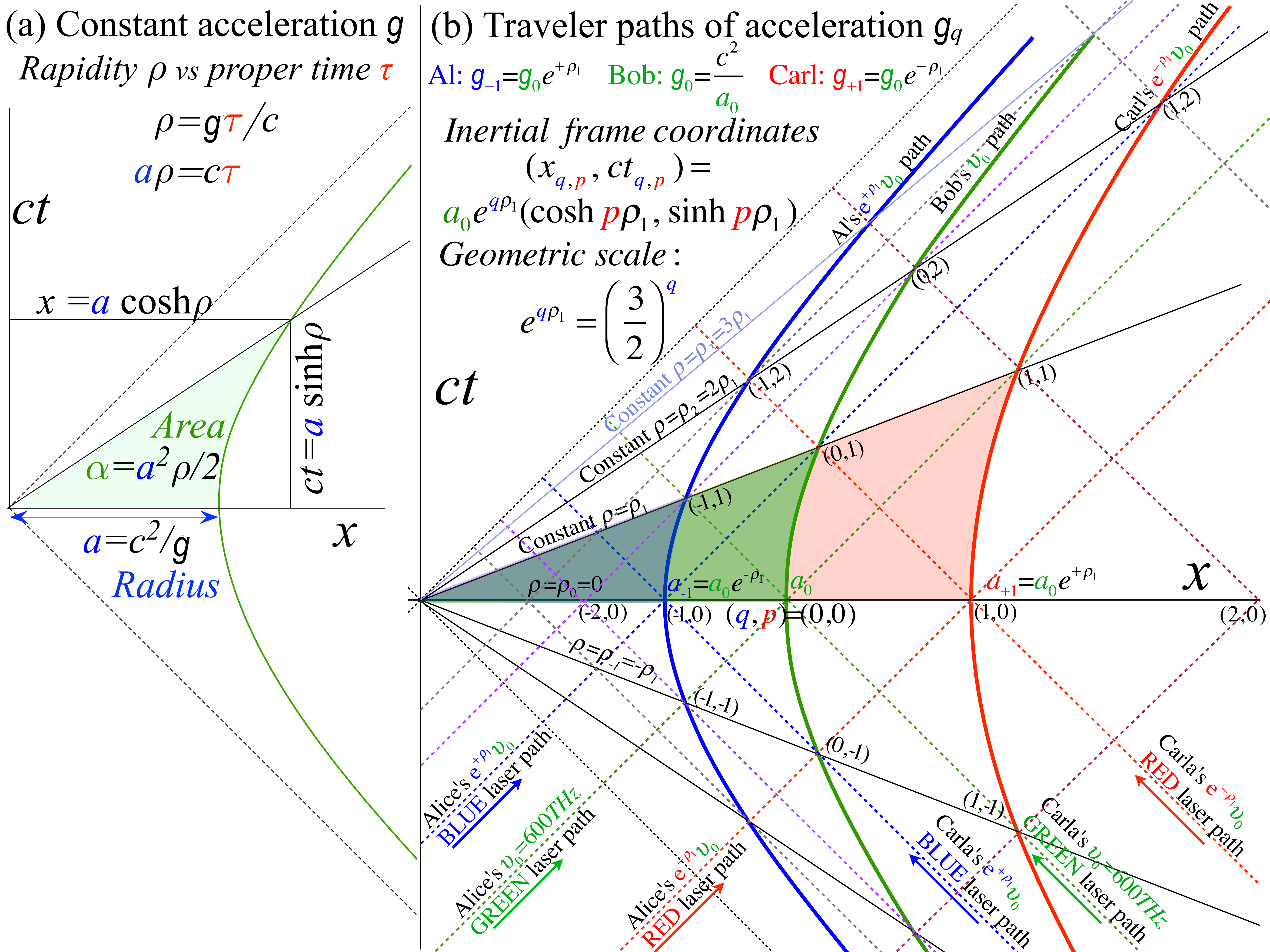


Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g



(a) Constant ρ
Alice-Carla time ct
x
cτ
x′
(x′, cτ)
Blue-fixed=ω0e+ρ
Red-fixed=ω0e-ρ
Alice
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Observers on the green constant-g hyberbola see only green ω0 light
Alice-Carla time ct
(x′, cτ)
x0=r0
At position x→=x-ct = x0 e−ρ
frequency is ω→ =ω0 e+ρ
At position x←=x+ct = x0 e+ρ
frequency is ω← =ω0 e−ρ
x
1
2
3
4
5 light years
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
initial-chirp=ω0
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
future
target:initial (x,ct)=(x0,0)
Alice's chirps
past (ct<0) (x>x0)
time ct
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Carla's chirps
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
initial-chirp=ω0
target (x0,0)
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ



(a) Constant acceleration g
Rapidity ρ vs proper time τ
ρ=gτ/c
aρ=cτ
ct
x =a coshρ
ct=a sinhρ
Area
α=a²ρ/2
a=c²/g
Radius
x
(b) Traveler paths of acceleration g_q
Al: g_−1=g_0e^{+ρ1}
Bob: g_0=c²/a_0
Carl: g_+1=g_0e^{−ρ1}
Inertial frame coordinates
(x_{q,p}, ct_{q,p}) =
a_0e^{qρ1}(cosh pρ1, sinh pρ1)
Geometric scale:
e^{qρ1} = (3/2)^q
ct
x
Al's e^{+ρ1}υ0 path
Bob's υ0 path
Carl's e^{−ρ1}υ0 path
Constant ρ=ρ3=3ρ1
Constant ρ=ρ2=2ρ1
Constant ρ=ρ1
ρ=ρ0=0
ρ=ρ−1=−ρ1
a_−1=a_0e^{−ρ1}
a_0
a_+1=a_0e^{+ρ1}
(q,p)=(0,0)
(-2,0)
(-1,0)
(1,0)
(2,0)
(-1,1)
(0,1)
(1,1)
(-1,2)
(0,2)
(1,2)
(-1,-1)
(0,-1)
(1,-1)
Alice's e^{+ρ1}υ0 BLUE laser path
Alice's υ0=600THz GREEN laser path
Alice's e^{−ρ1}υ0 RED laser path
Carla's e^{+ρ1}υ0 BLUE laser path
Carla's υ0=600THz GREEN laser path
Carla's e^{−ρ1}υ0 RED laser path

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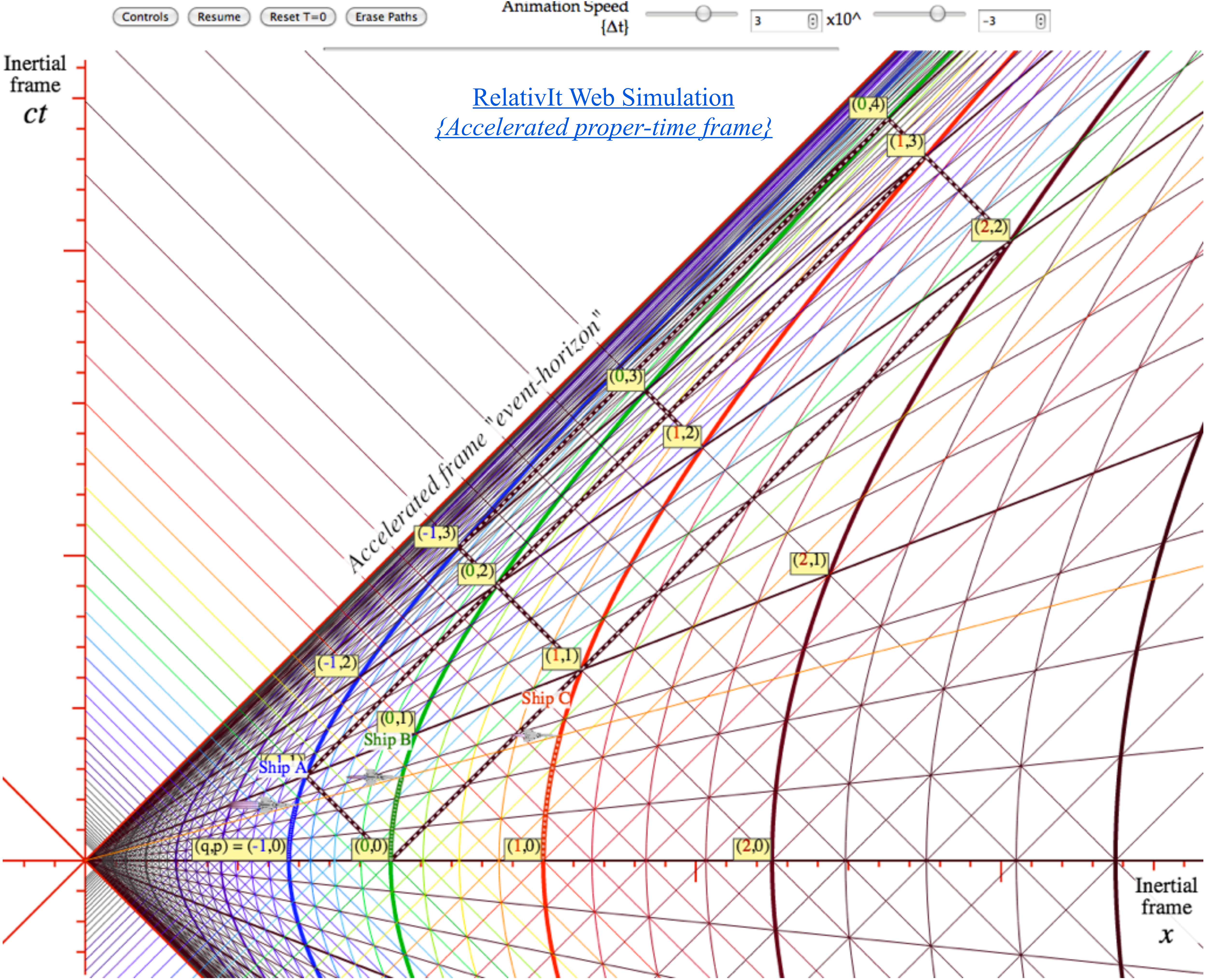
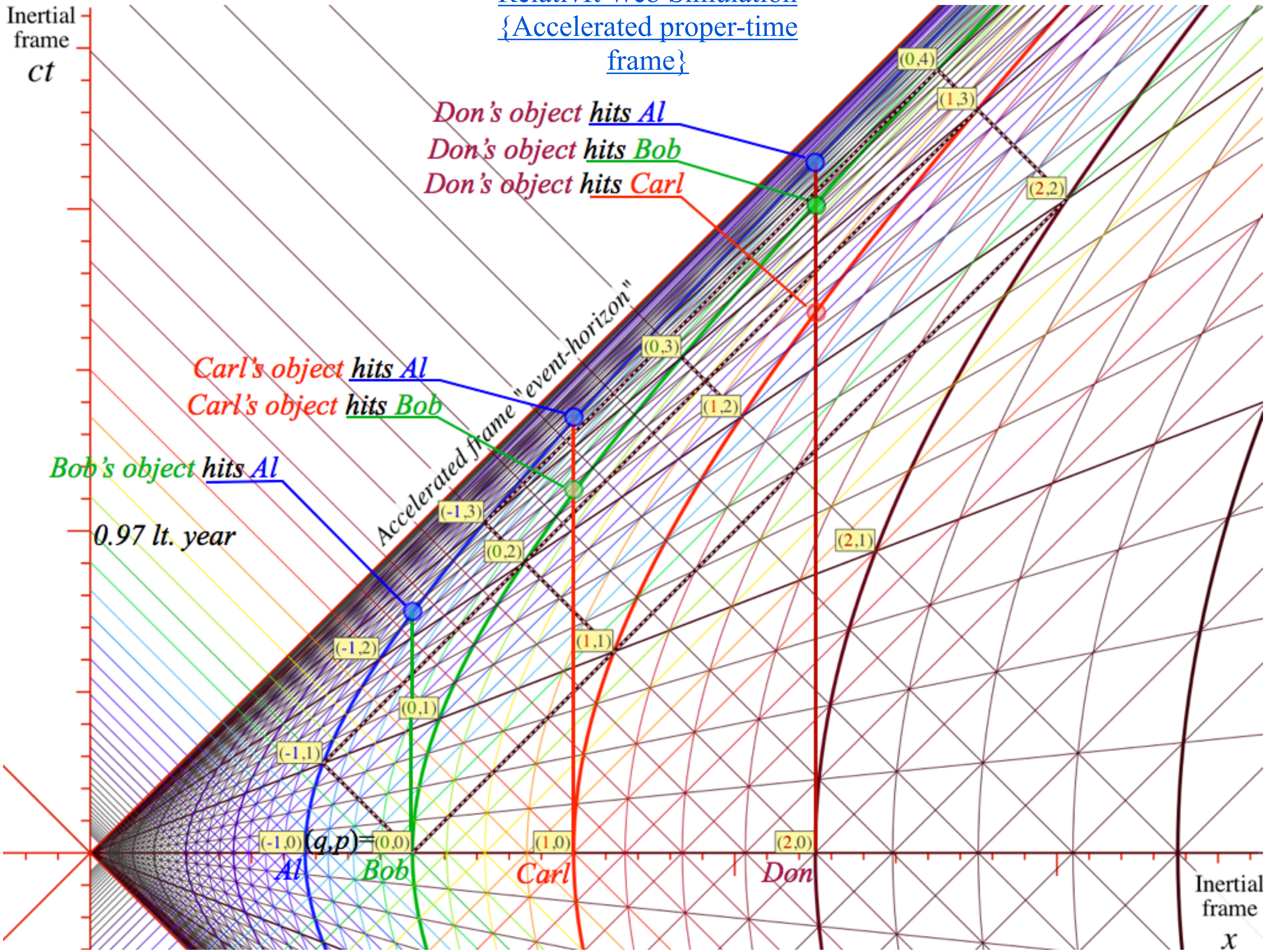


Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
3
x10^
-3
Inertial
frame
ct
RelativIt Web Simulation
{Accelerated proper-time frame}
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial
frame
x

RelativIt Web Simulation
{Accelerated proper-time frame}



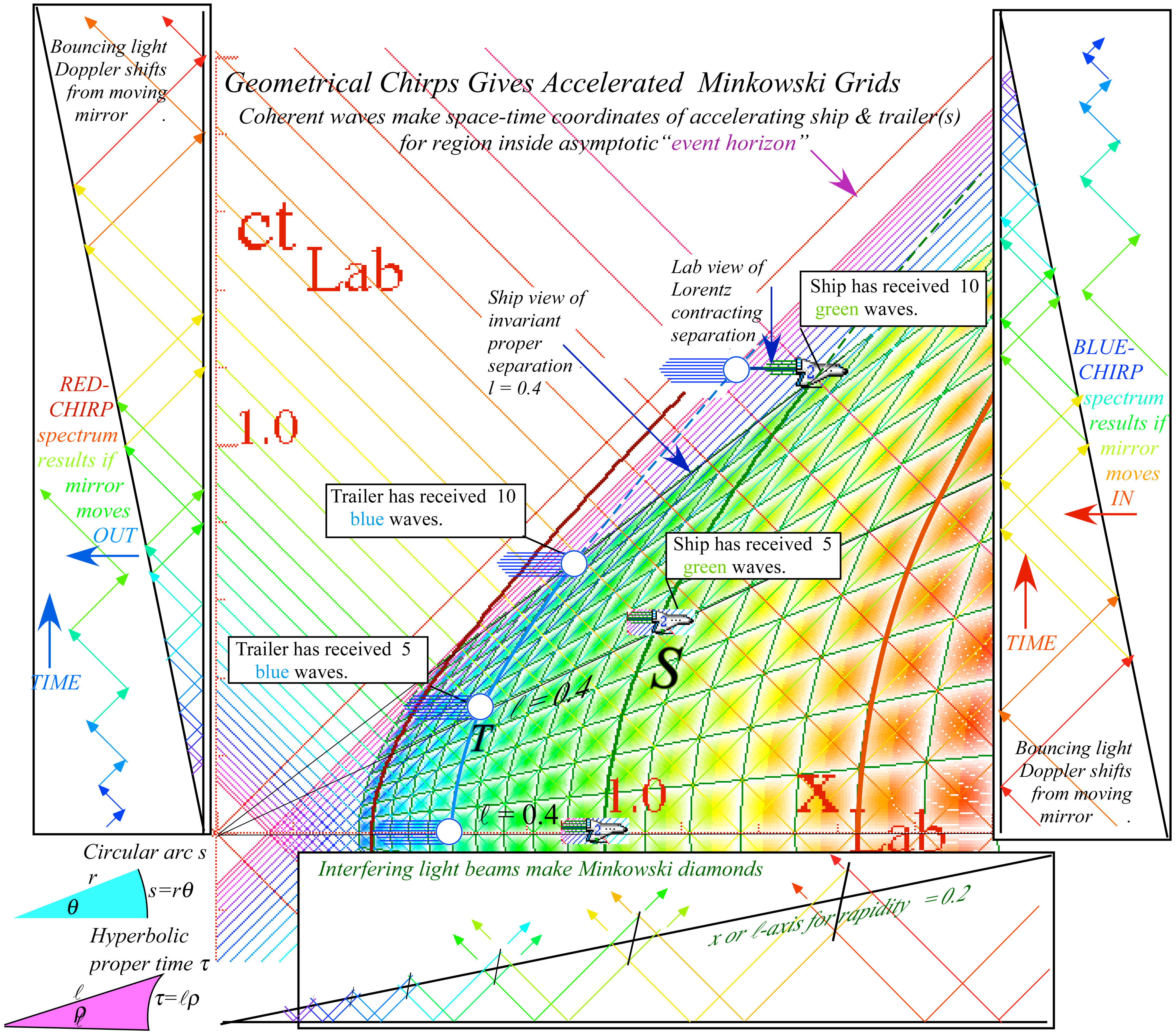
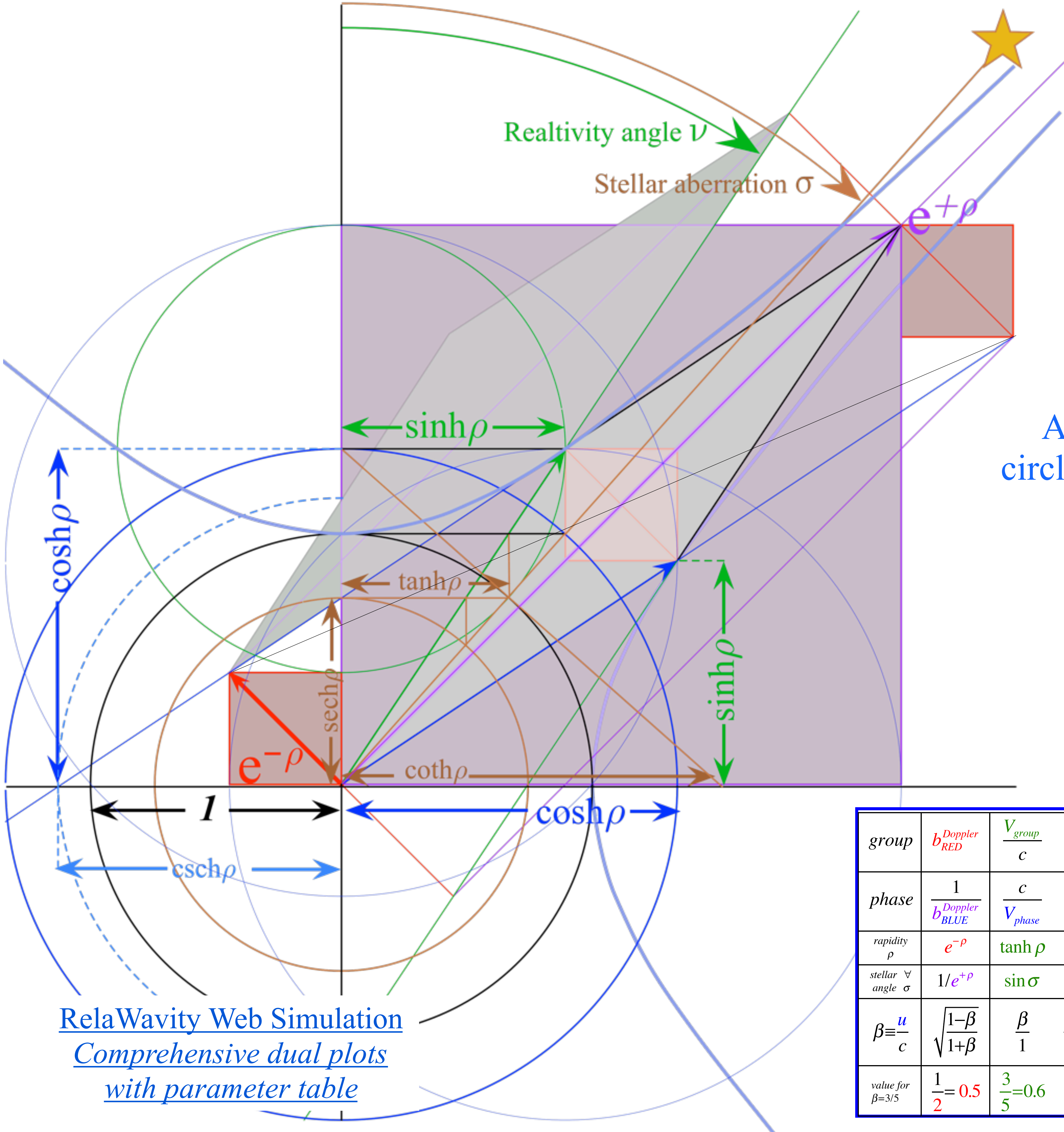


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

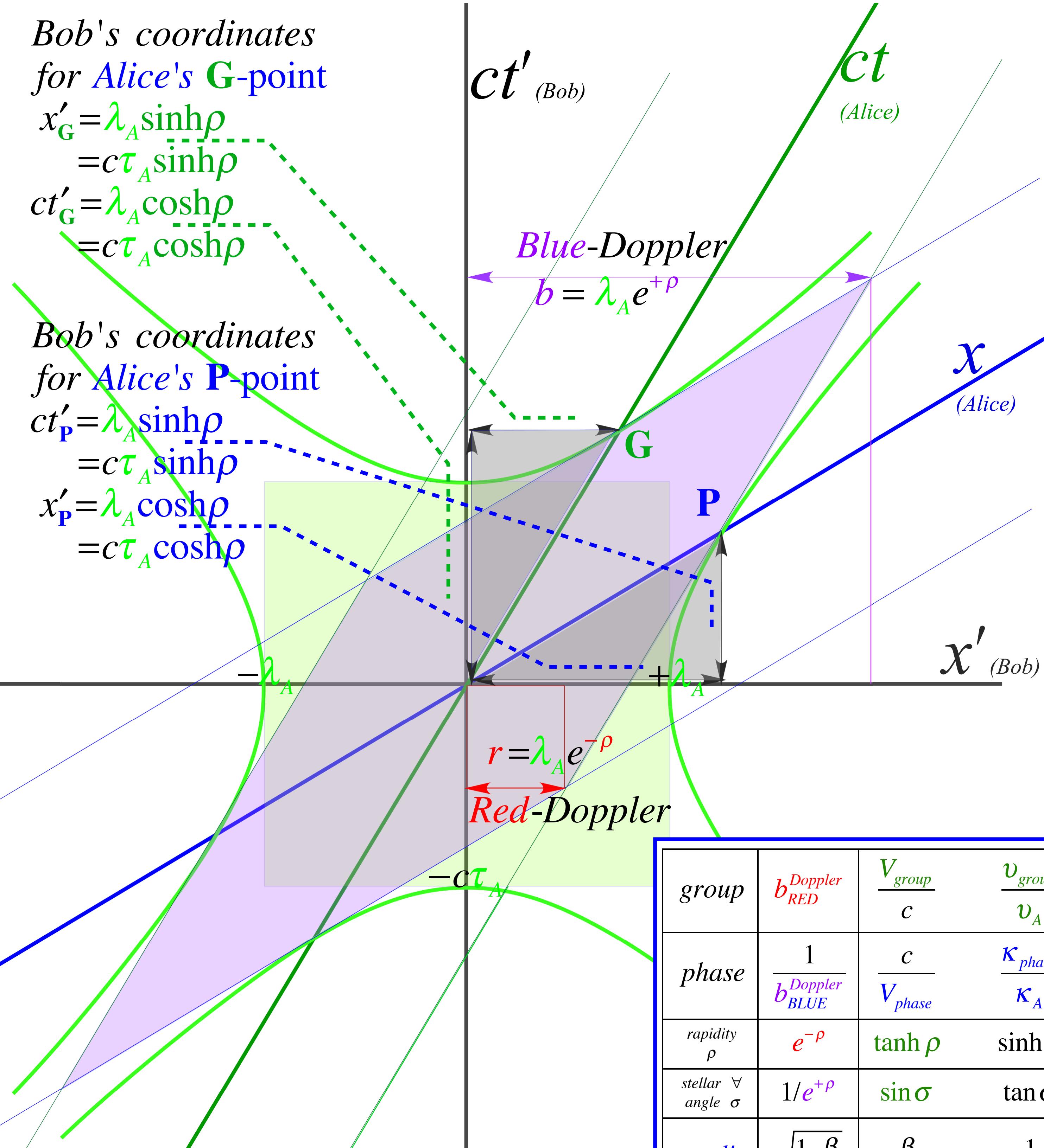


A more compact circle-based geometry

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall$ σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation

Comprehensive dual plots with parameter table



Space-time parameters

$\lambda_{phase} = \lambda_A \operatorname{csch}\rho$

$\lambda_{group} = \lambda_A \operatorname{sech}\rho$

$c\tau_{phase} = c\tau_A \operatorname{sech}\rho$

$c\tau_{group} = c\tau_A \operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase} = c\kappa_A \sinh\rho$

$c\kappa_{group} = c\kappa_A \cosh\rho$

$\upsilon_{phase} = \upsilon_A \cosh\rho$

$\upsilon_{group} = \upsilon_A \sinh\rho$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (off-diagonal Lorentz-transform)	*x-contraction*(Lorentz) τ_{phase}*-contraction*	*t-dilation*(Einstein) υ_{phase}*-dilation* (on-diagonal Lorentz-transform)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$

RelaWavity Web Simulation
Comprehensive dual plots
with parameter table

RelaWavity Web Simulation
(ct' vs x') with parameter table