Lecture 30
Thur. 12.10.2015

Relawavity and a novel introduction to relativistic mechanics I.

(Ch. 6 of Unit 8 12.10.15)

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
- Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Developing Axioms to update Galileo's relativity: A critical look at *c*-axioms
- Einstein's PW (Pulse-Wave) Axiom
- Evenson's CW (Continuous Wave) Axiom and Occam's Razor
- Analyzing wave velocity by per-space-per-time *and* space-time graphs
- Introducing optical Doppler effects
- Clarifying Evenson's CW Axiom using Doppler effects
  - *Galileo's Revenge (part1):* Galilean Doppler-shift arithmetic using *rapidity* ρ

Developing optical "baseball-diamond" and relativistic ρ-functions and transformations
- Details of 1CW wavefunctions and phasors
- Details of 2CW wavefunctions in rest frame
  - *Galileo's Revenge (part2):* Galilean addition of phasor angular velocity
  - Structure of rest frame "baseball-diamonds"
- Details of 2CW wavefunctions of moving frame velocities of *phase* and *group* waves
  - 16 coefficients of relativistic 2CW interference
  - Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

Lecture 31
Thur. 12.17.2015

Relawavity and a novel introduction to relativistic mechanics II.

(Ch. 6-8 of Unit 8 12.10.15)

Rapidity ρ related to *stellar aberration angle* σ and Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - Applications to optical waveguide, spherical waves, accelerator radiation
  - Derivation of relativistic quantum mechanics
  - What's the matter with mass? Shining some light on the Elephant in the room
  - Relativistic action and Lagrangian-Hamiltonian relations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
- Compton recoil related to rocket velocity formula
- Relation of 2^{nd} quantization amplitude "photon" N and 1^{st} quantization wavenumber

Relawavity in accelerated frames

Lecture 30
Thur. 12.10.2015

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations

➔ Optical interference "baseball-diamond" displays *phase* and *group* velocity

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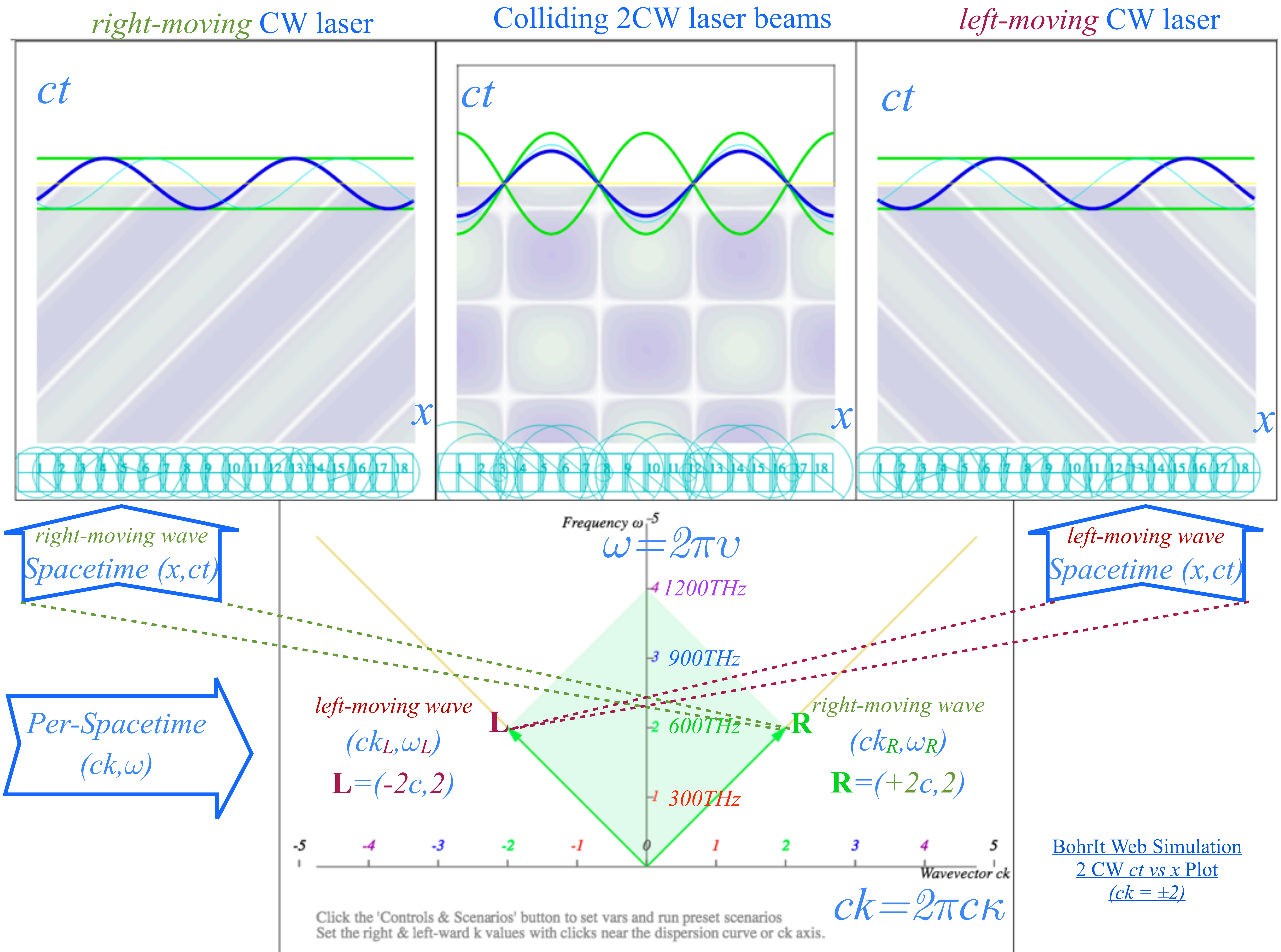
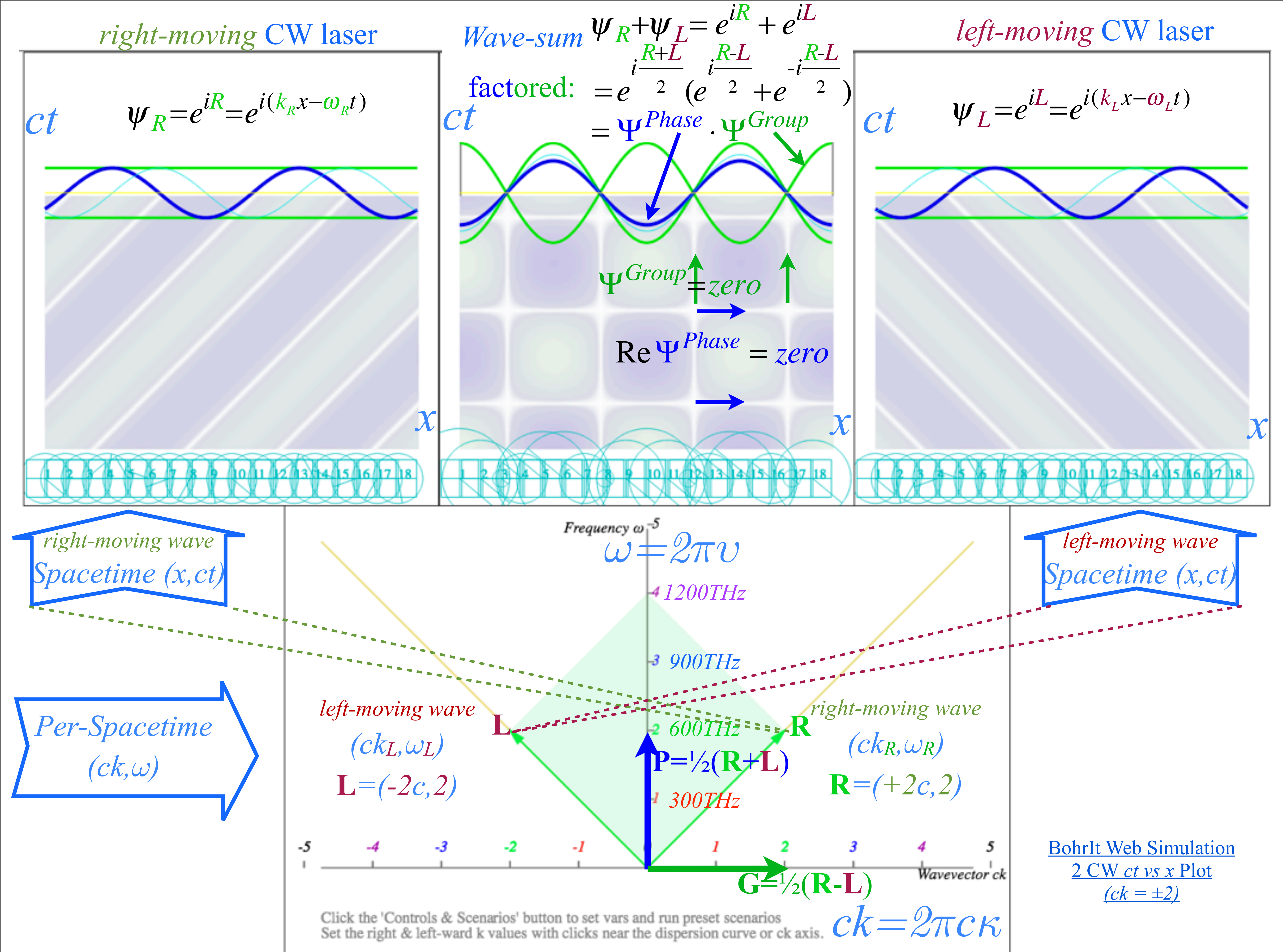
For an introductory, web based developement of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge*:*

A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers


right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
x
ct
x
ct
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
ω=2πυ
1200THz
900THz
600THz
300THz
left-moving wave
(ckL,ωL)
L=(-2c,2)
right-moving wave
(ckR,ωR)
R=(+2c,2)
L
R
Wavevector ck
ck=2πcκ
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation
2 CW ct vs x Plot
(ck = ±2)



right-moving CW laser
Wave-sum $\psi_R+\psi_L=e^{iR}+e^{iL}$
factored: $=e^{i\frac{R+L}{2}}(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}})$
$=\Psi^{Phase}\cdot\Psi^{Group}$
left-moving CW laser
$\psi_R=e^{iR}=e^{i(k_Rx-\omega_Rt)}$
$\psi_L=e^{iL}=e^{i(k_Lx-\omega_Lt)}$
ct
x
$\Psi^{Group}=zero$
$\mathrm{Re}\,\Psi^{Phase}=zero$
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
$\omega=2\pi\upsilon$
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L,ω_L)
L=(-2c,2)
right-moving wave
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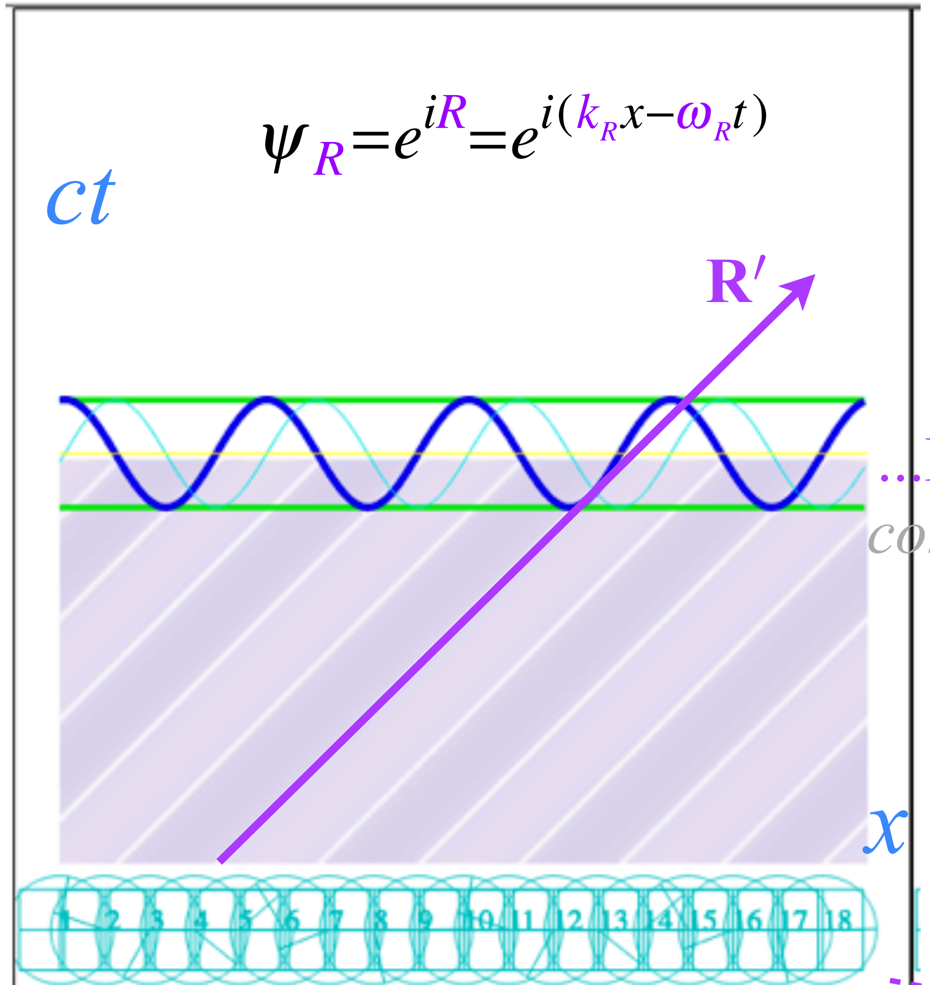
right-moving Doppler blue shifted wave



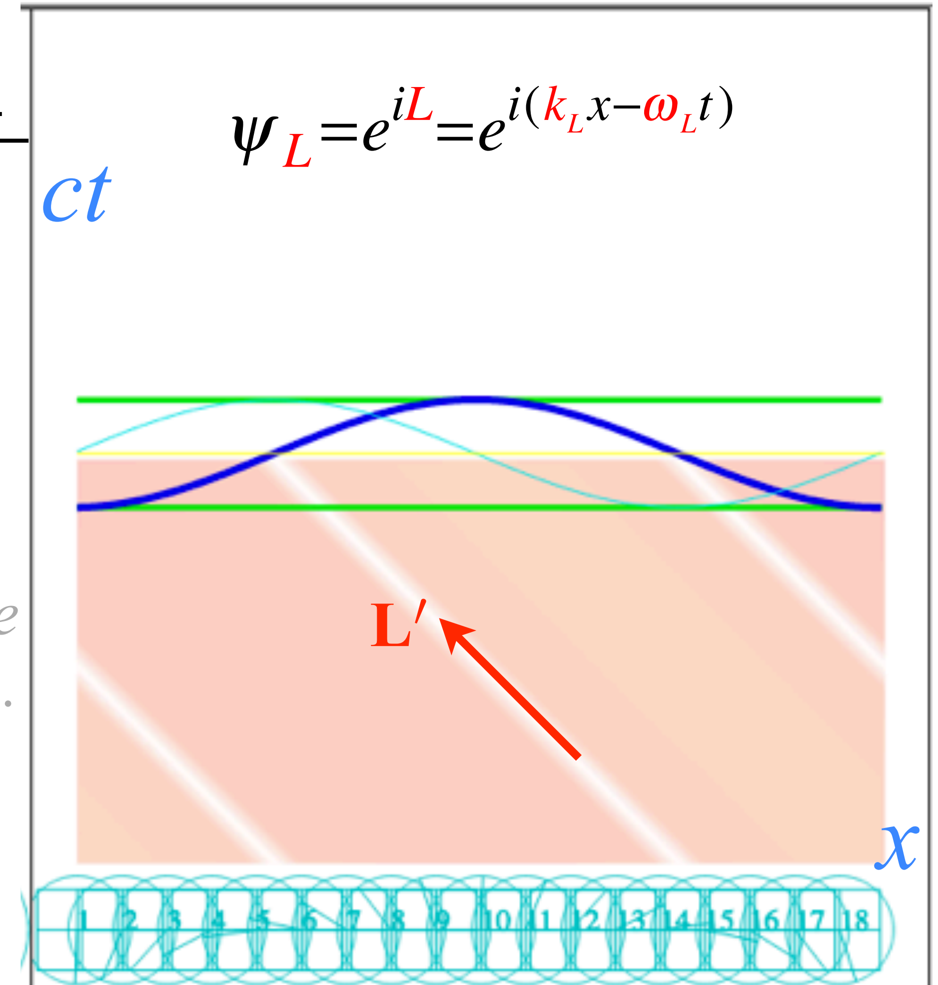
[Web Simulation](#)
[1 CW *ct vs x* Plot](#)
[*(ck = +4)*](#)

Rapidly moving Bob sees...

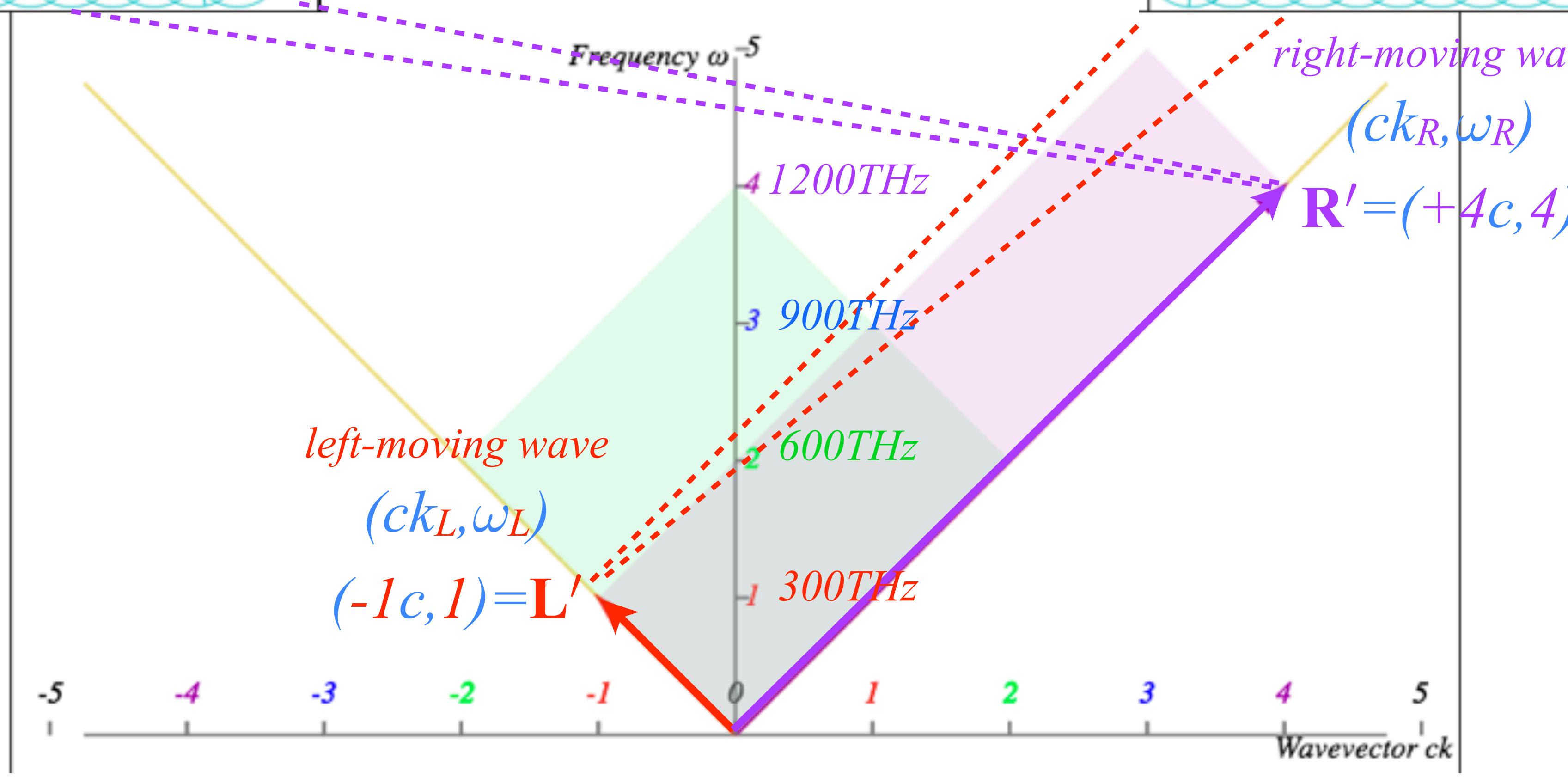
...Blue shifted *wave coming at him and...*

...Red shifted *wave behind him.*

left-moving Doppler red shifted wave

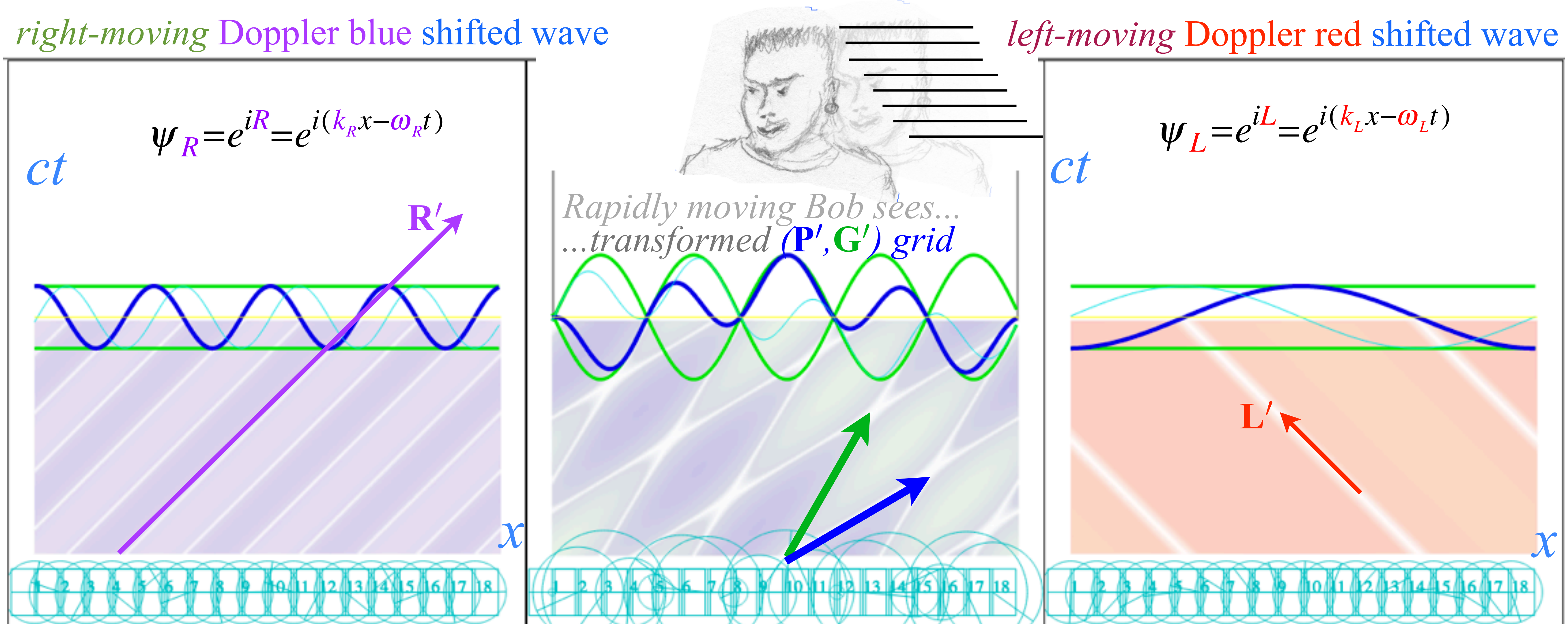


[Web Simulation](#)
[1 CW *ct vs x* Plot](#)
[*(ck = -1)*](#)



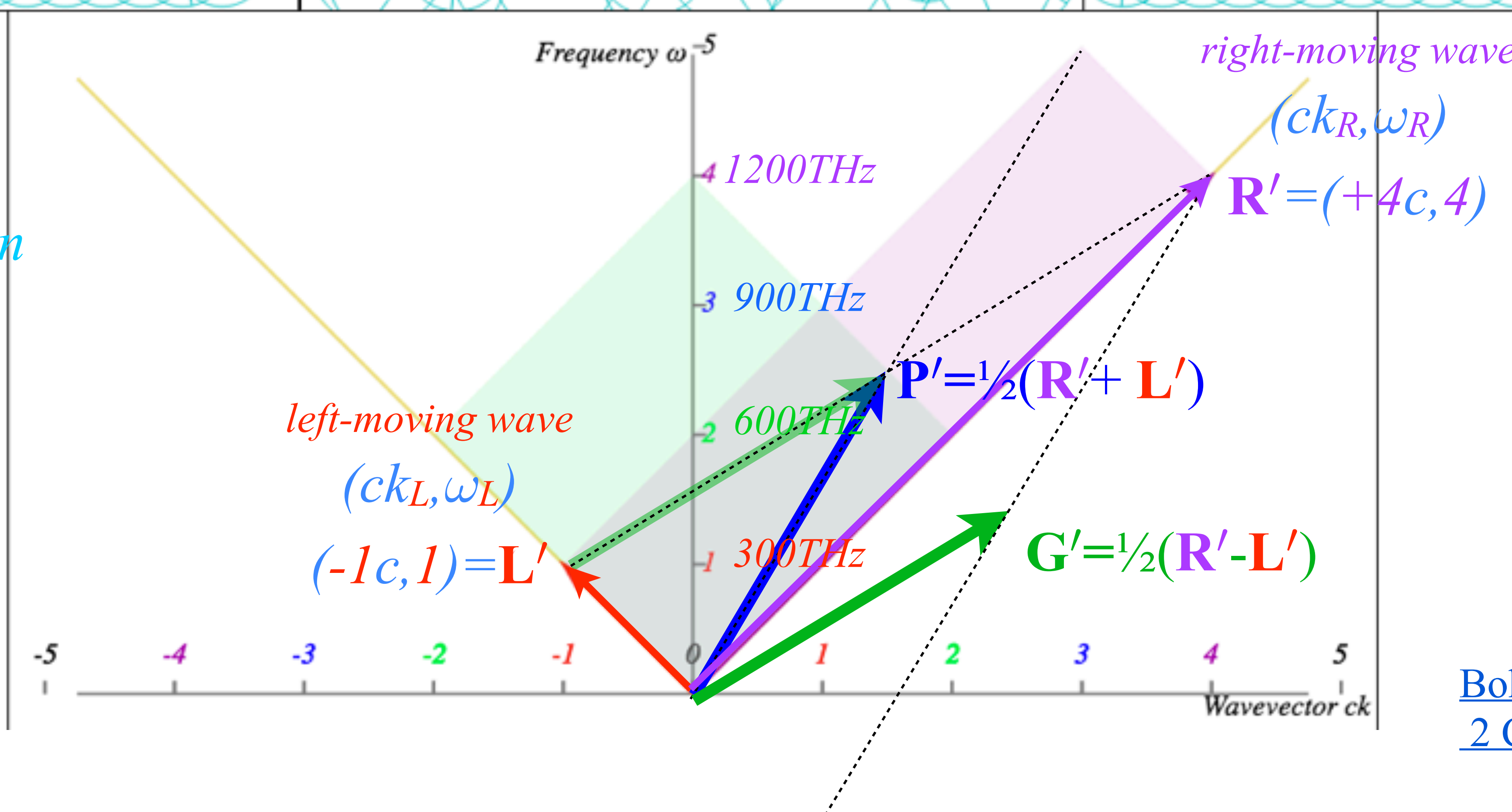
right-moving Doppler blue shifted wave

left-moving Doppler red shifted wave



...Doppler shifts give
Lorentz transformation
of both these graphs

Per-Spacetime
(ck,ω)



BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)

Lecture 30
Thur. 12.10.2015

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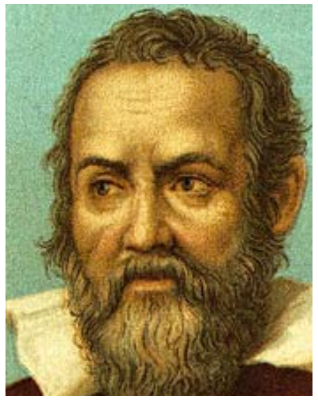
Some have forgotten... Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

looks worried?



Galilei Galileo
1564-1642

Need to review...

- Where Galilean relativity fails for light waves,
  ...and where it doesn't.

and then see...

- How to make sense of light-wave **SPEED LIMIT c= 299,792,458 m/s** axiom(s)

Good approximation:
$c \cong$ 300 million m/s
300 Megameter/s

(We'll use frequencies divisible by 3)

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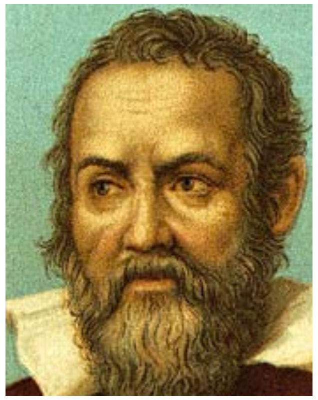

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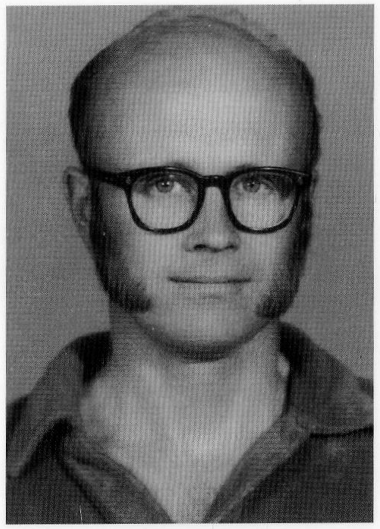
- How to make sense of light-wave **SPEED LIMIT C= 299,792,458 *m/s*** axiom(s)

by comparing *Einstein Pulse Wave* (PW) axiom
with
Evenson Continuous Wave (CW) axiom

in *space-time* and *inverse space-time*

Good approximation:
c ≅ 300 million m/s
300 Megameter/s

(We'll use frequencies divisible by 3)



PAULINIA, BRASIL 1976

THE SPEED OF LIGHT IS
299,792,458 METERS PER SECOND!

Kenneth M. Evenson
1932-2002

Link to ⇒ Speed of Light From Direct Frequency and Wavelength Measurements

At Journal site ⇒ K. M. Evenson, J.S. Wells, F.R. Peterson, B.L. Danielson, G.W. Day, R.L. Barger and J.L. Hall, Phys. Rev. Letters 29, 1346(1972).

In 2005 the Nobel Prize in physics was awarded to Glauber, Hall, and Hensch†† for laser optics and metrology.

†† *The Nobel Prize in Physics, 2005*. http://nobelprize.org/

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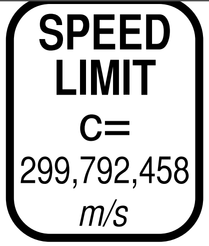
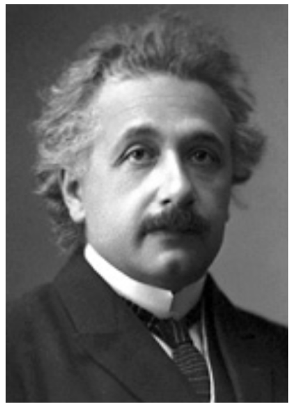
• How do you make sense of light-wave  axiom(s)?

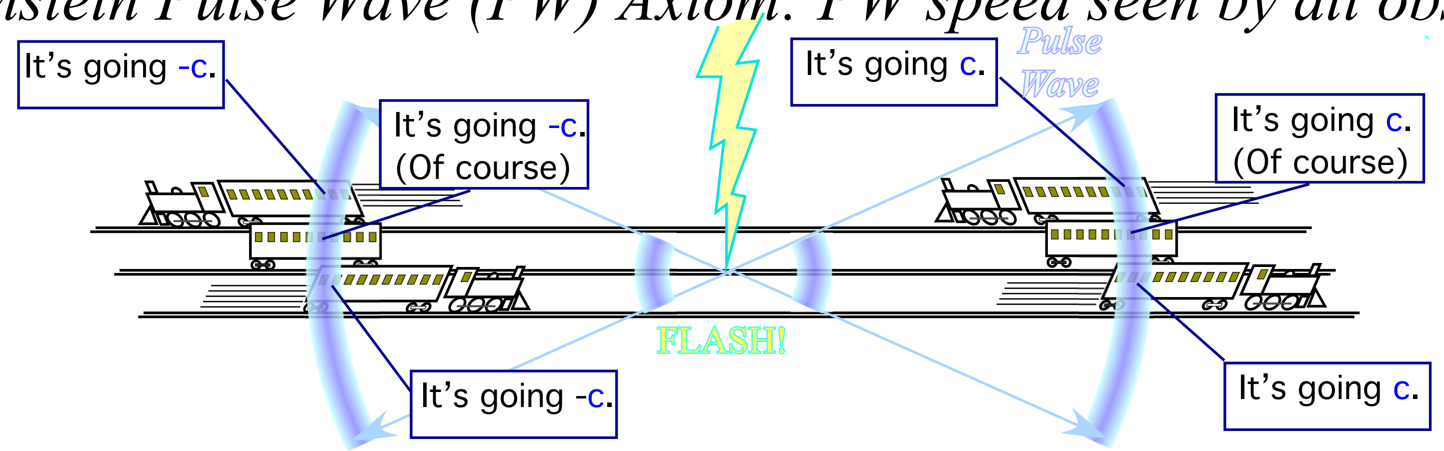
And, HE-eee-rRE'S Albert!

Albert Einstein

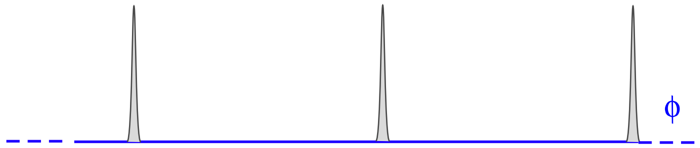


1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



Pulse wave (PW) train



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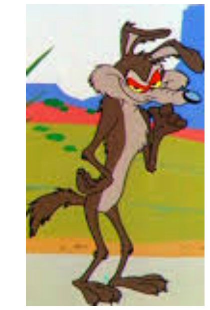
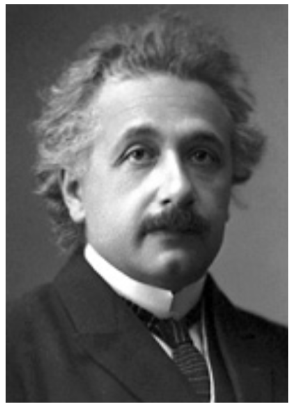


Albert Einstein



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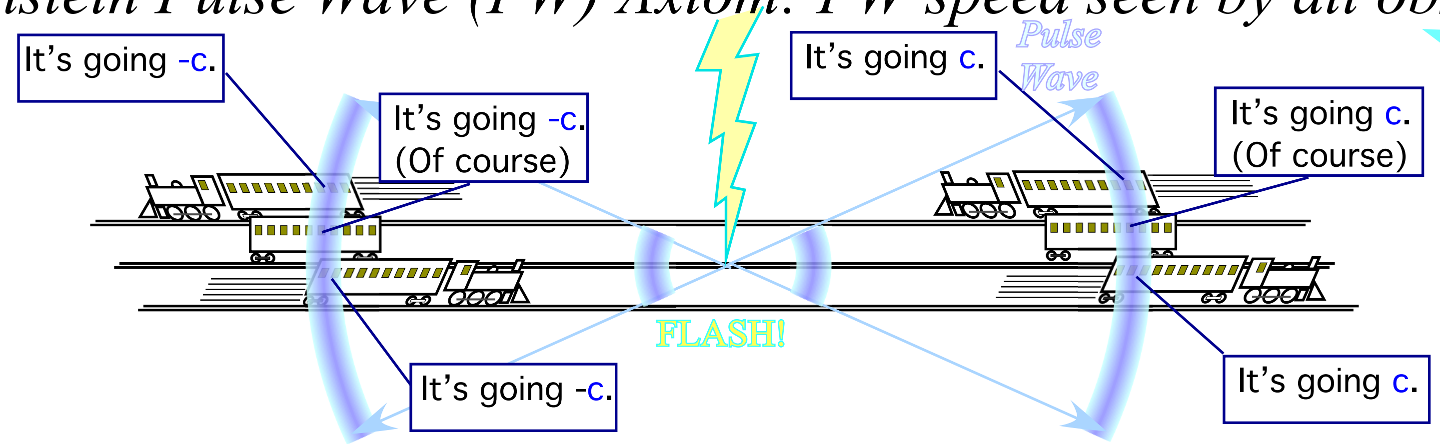
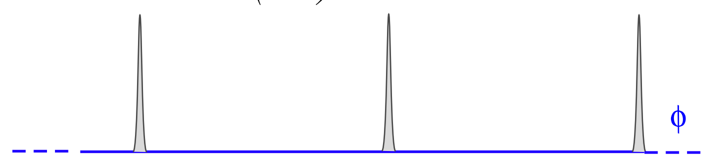


A "road-runner" axiom is a "show-stopper"

Is cartoon physics a reality?!

Pulse wave (PW) train



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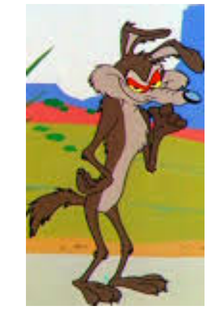

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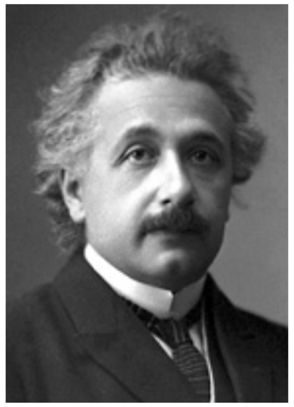

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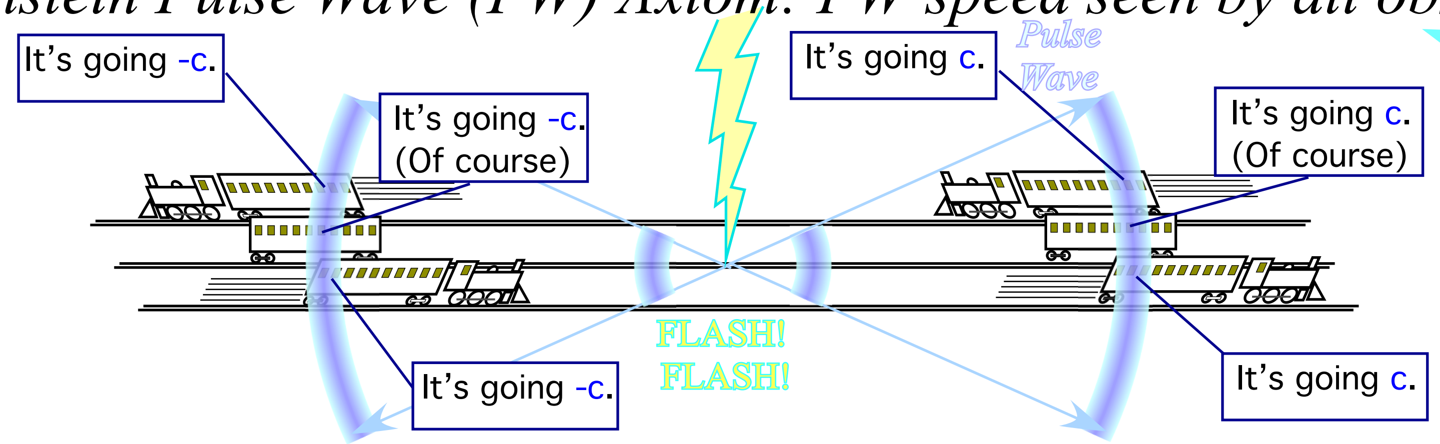

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beep-meep!

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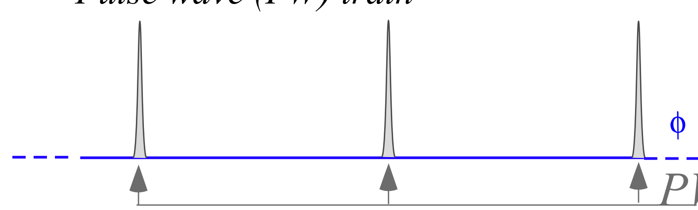

$A_1 cos\ \omega t + A_2 cos\ 2\omega t + A_3 cos\ 3\omega t + A_4 cos\ 4\omega t + \ldots$

PW Axiom is complicated

PW peaks precisely locate places where wave is.

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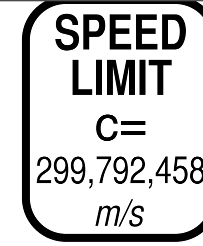
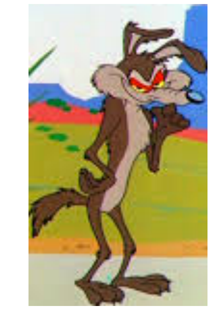
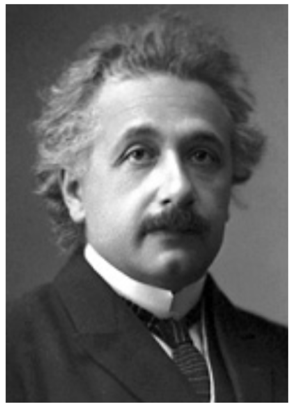
• How do you make sense of light-wave  axiom(s)?



Albert Einstein



1879-1955

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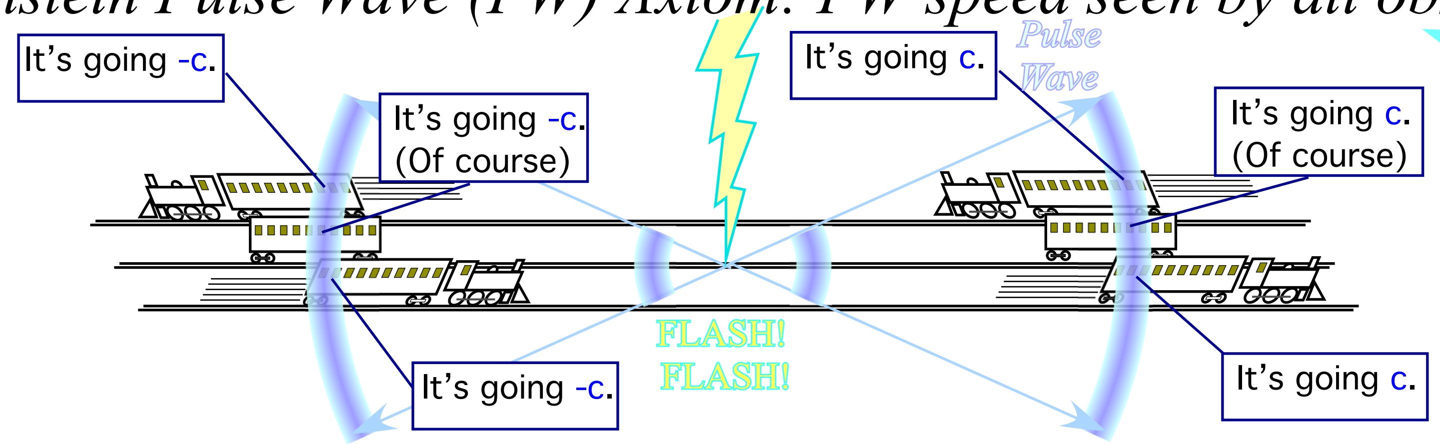
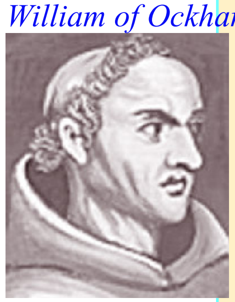


A "road-runner" axiom is a "show-stopper"

William of Ockham

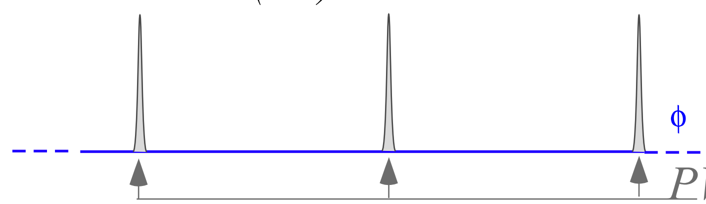


1285-1349

Using Occam's Razor

(and Evenson's lasers)

Pulse wave (PW) train

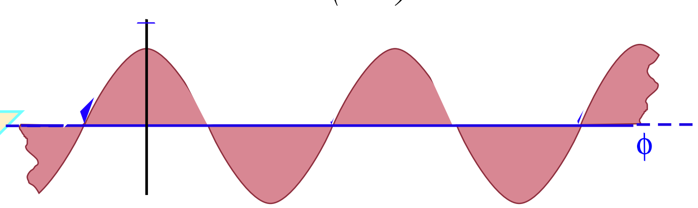


$A_1\cos\omega t$ ~~$+A_2\cos 2\omega t+A_3\cos 3\omega t+A_4\cos 4\omega t+\ldots$~~

PW Axiom is <u>complicated</u>

PW peaks precisely locate places where wave <u>is</u>.

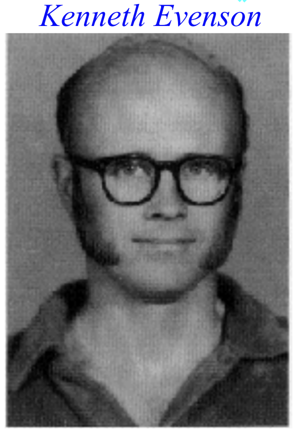
Continuous wave (CW) train



$A\cos\omega t$

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

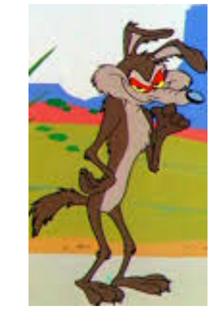
Kenneth Evenson



1932-2002

Cut a *PW* to just *one* Continuous Wave

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?



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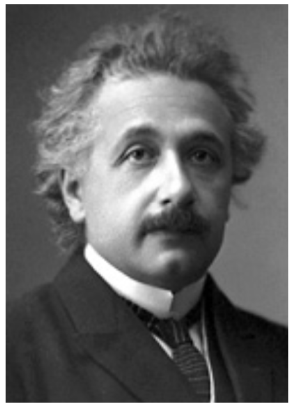

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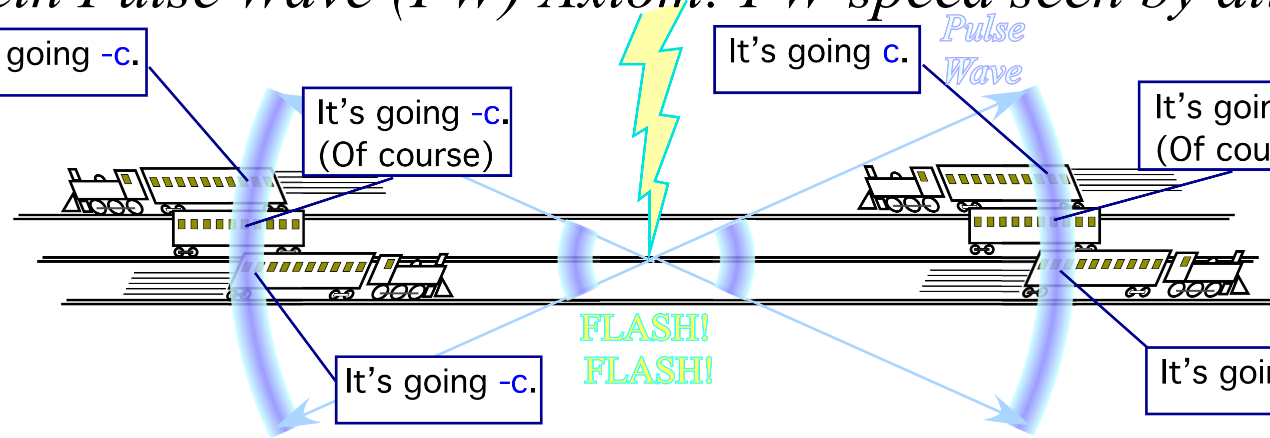
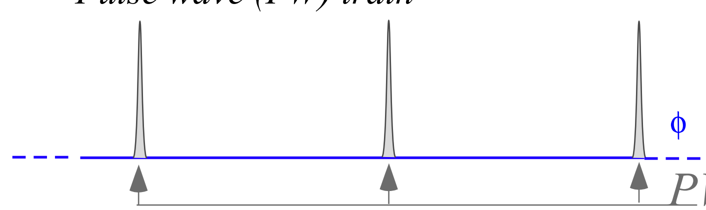


A "road-runner" axiom is a "show-stopper"

Pulse wave (PW) train



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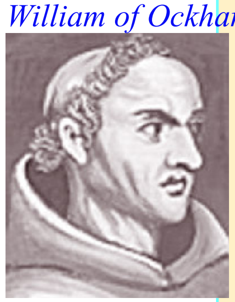

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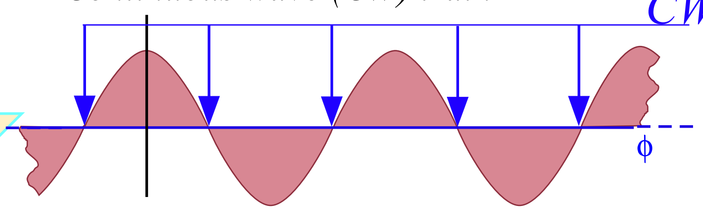
(and Evenson's lasers)

Continuous wave (CW) train



CW zeros precisely locate places where wave is <u>not</u>.

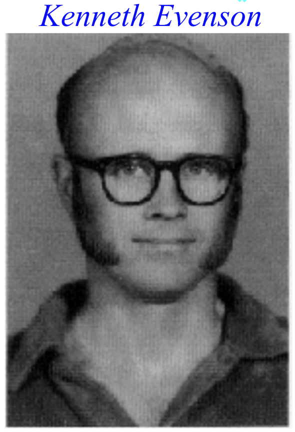
$A \cos \omega t$

Simpler 1CW coherence is more "Zen-like"

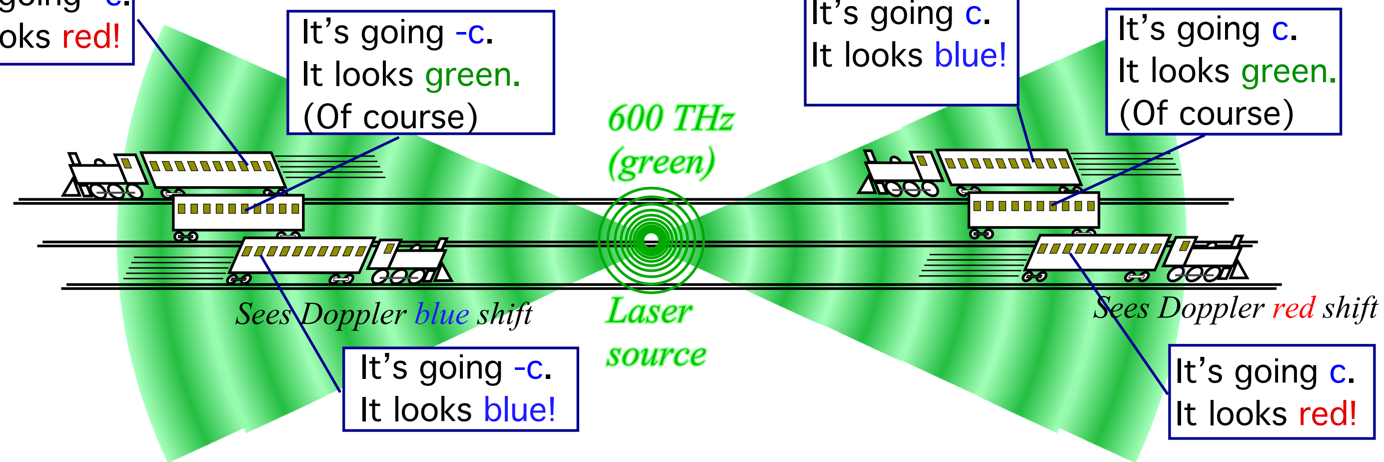
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Can be made more self-evident and productive

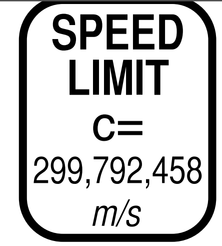
Kenneth Evenson



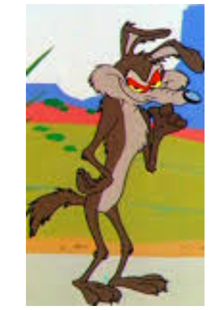
1932-2002



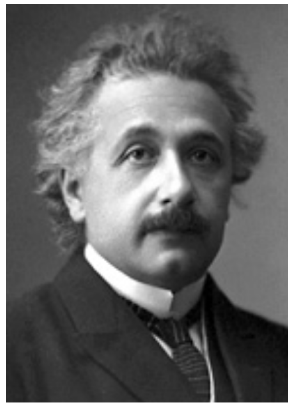
Cut a *PW* to just *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

• How do you make sense of light-wave 

axiom(s)?

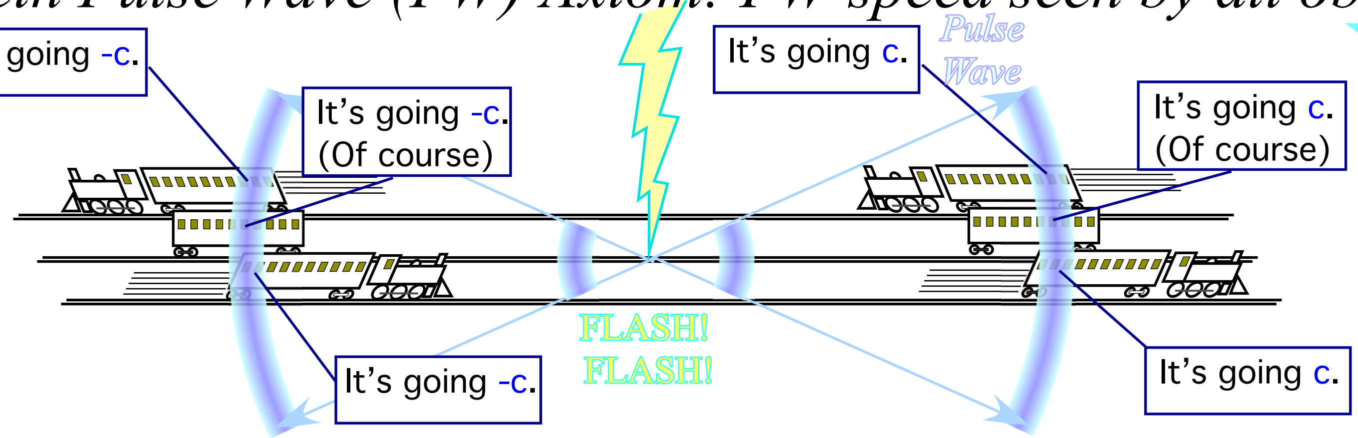


Albert Einstein



1879-1955

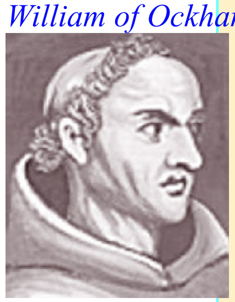
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"

First of all it's Complicated

William of Ockham

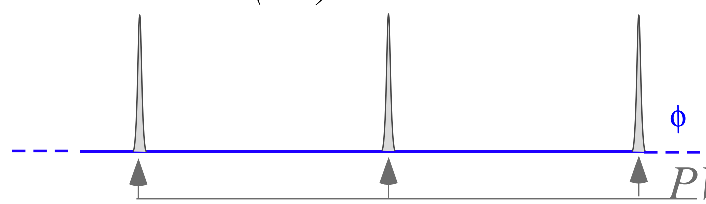


1285-1349

Using Occam's Razor

(and Evenson's lasers)

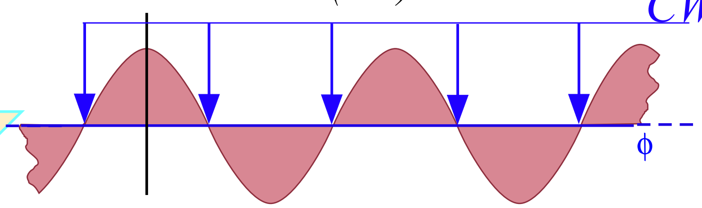
Pulse wave (PW) train



$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + ...$

PW peaks precisely locate places where wave is.

Continuous wave (CW) train



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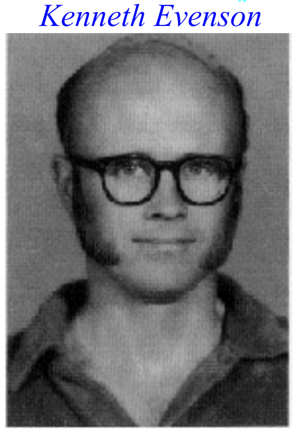

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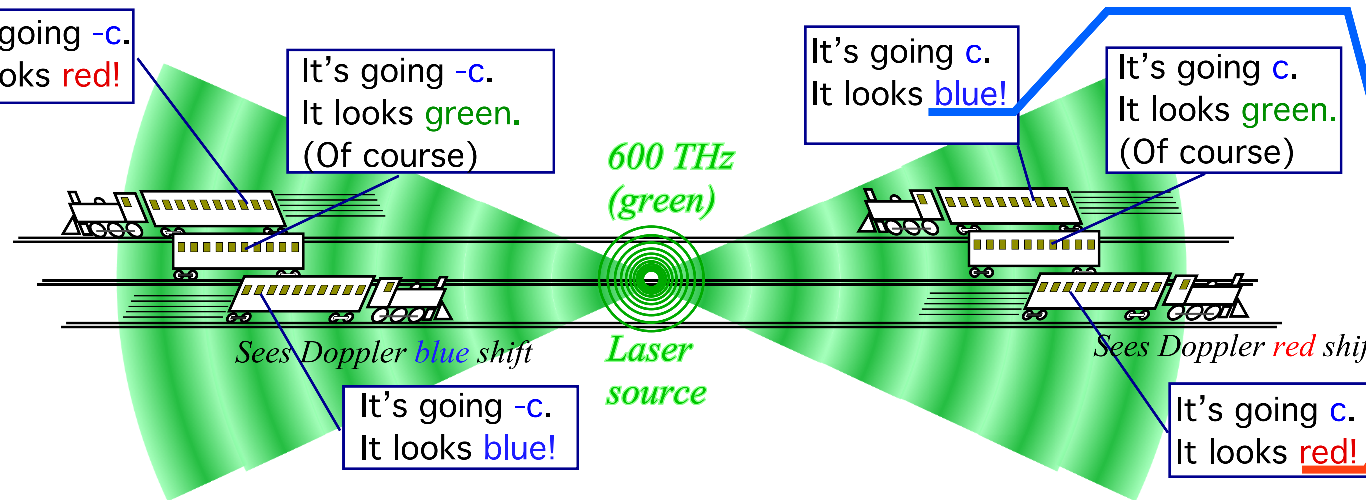

1CW is affected by 1st-order Doppler
Blue shifts $b = e^{+\rho}$
and
Red shifts $r = e^{-\rho}$
of frequency υ
and wavenumber κ

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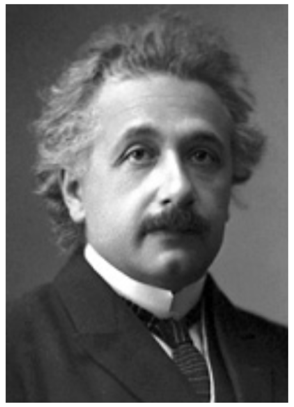

1879-1955

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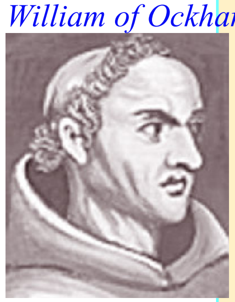
A major objection to relativity/QM theory:

It's the only major theoretical development that starts with *2nd*-order (and quite mysterious!) (and very very very tiny!) effects.

William of Ockham



1285-1349

Using Occam's Razor

(and Evenson's lasers)

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..though comforting to the "A Place for everything and everything in its place" crowd.

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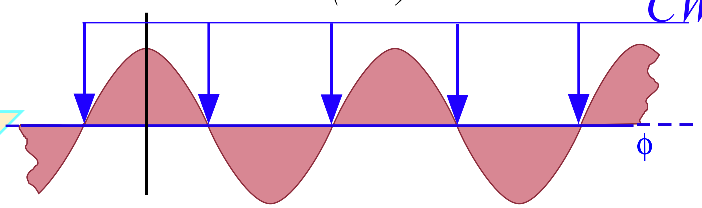

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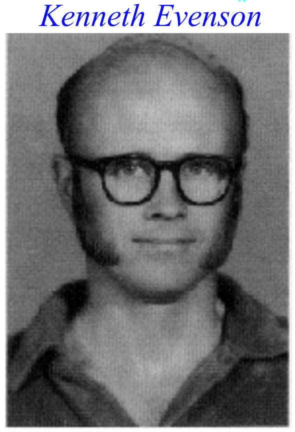

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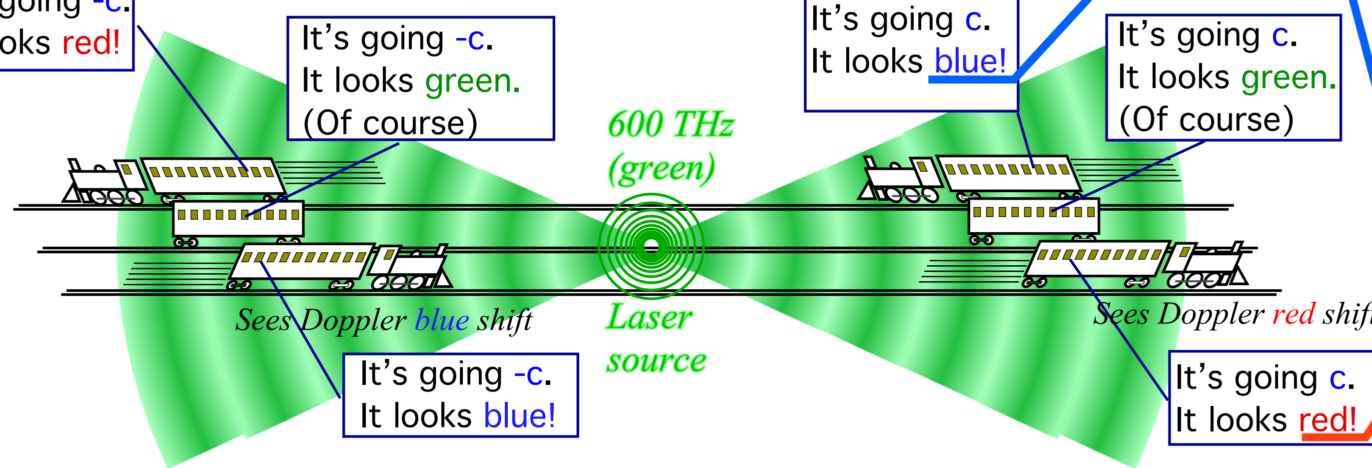

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Lecture 30
Thur. 12.10.2015

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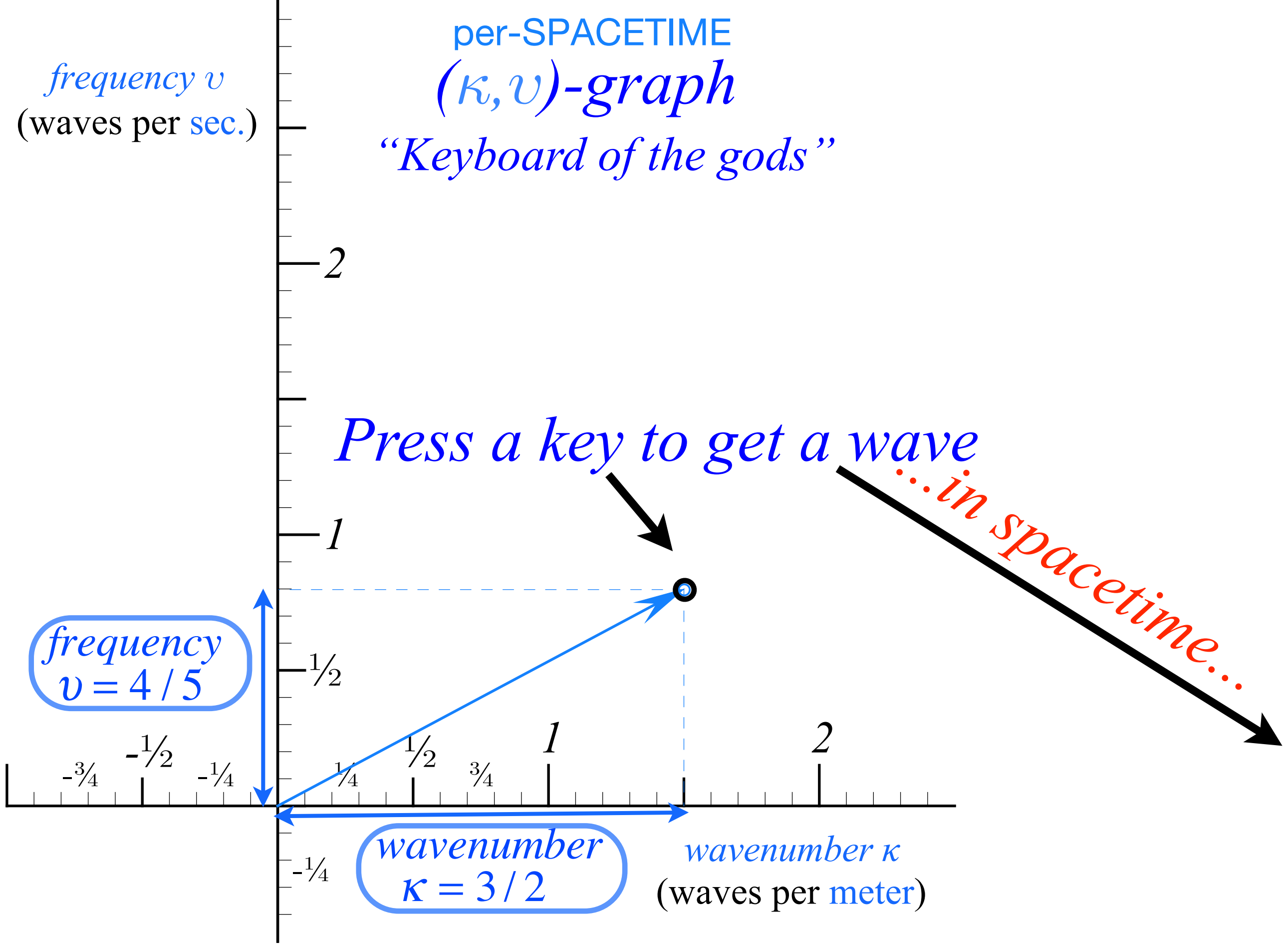
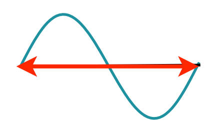


SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

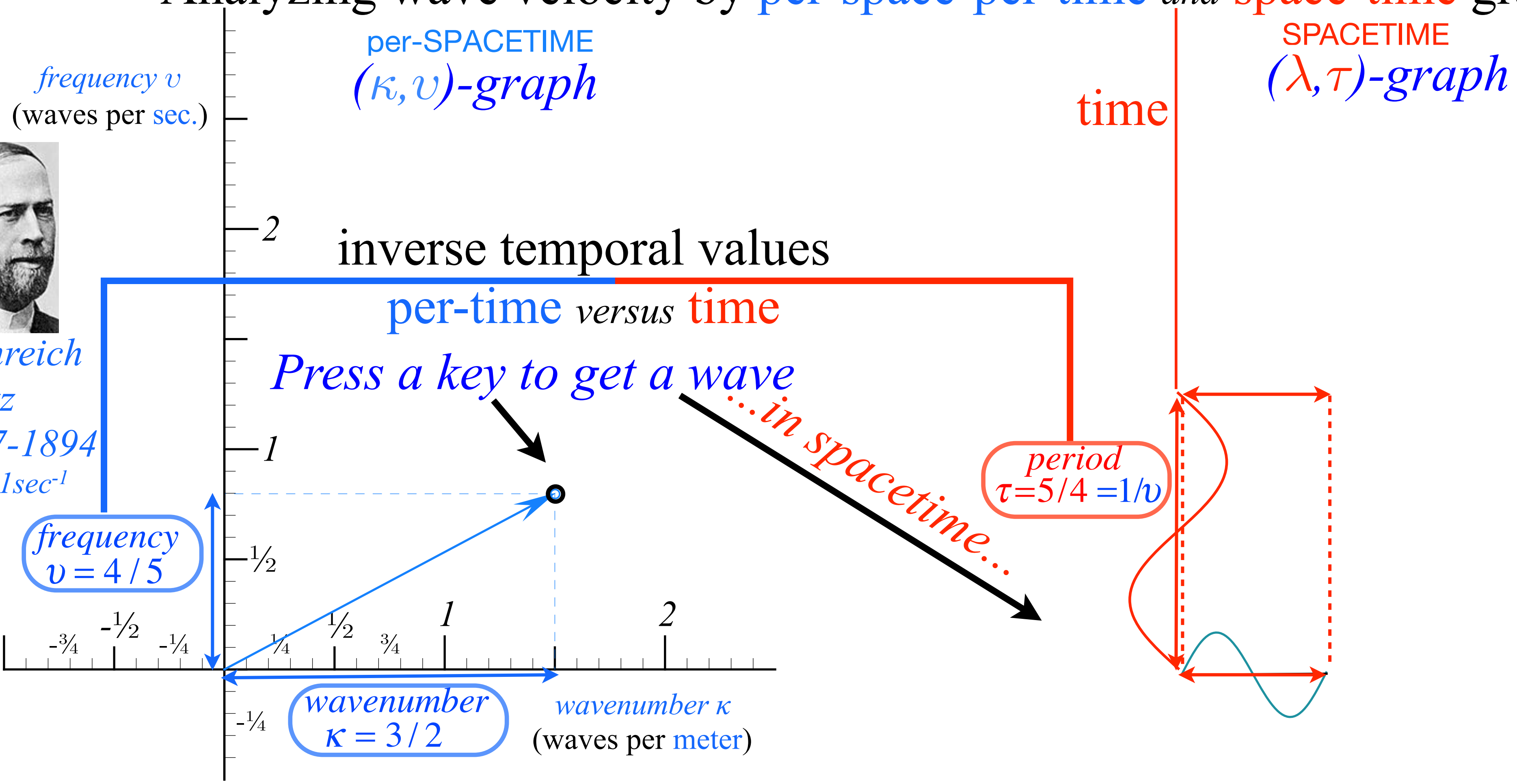
•How to understand waves and wave velocity V_{wave}

[RelaWavity Web Simulation](#)
[*Keyboard of the Gods*](#)
[(per-Time *vs* per-Space)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Heinreich Hertz 1857-1894

$1 Hz = 1 sec^{-1}$



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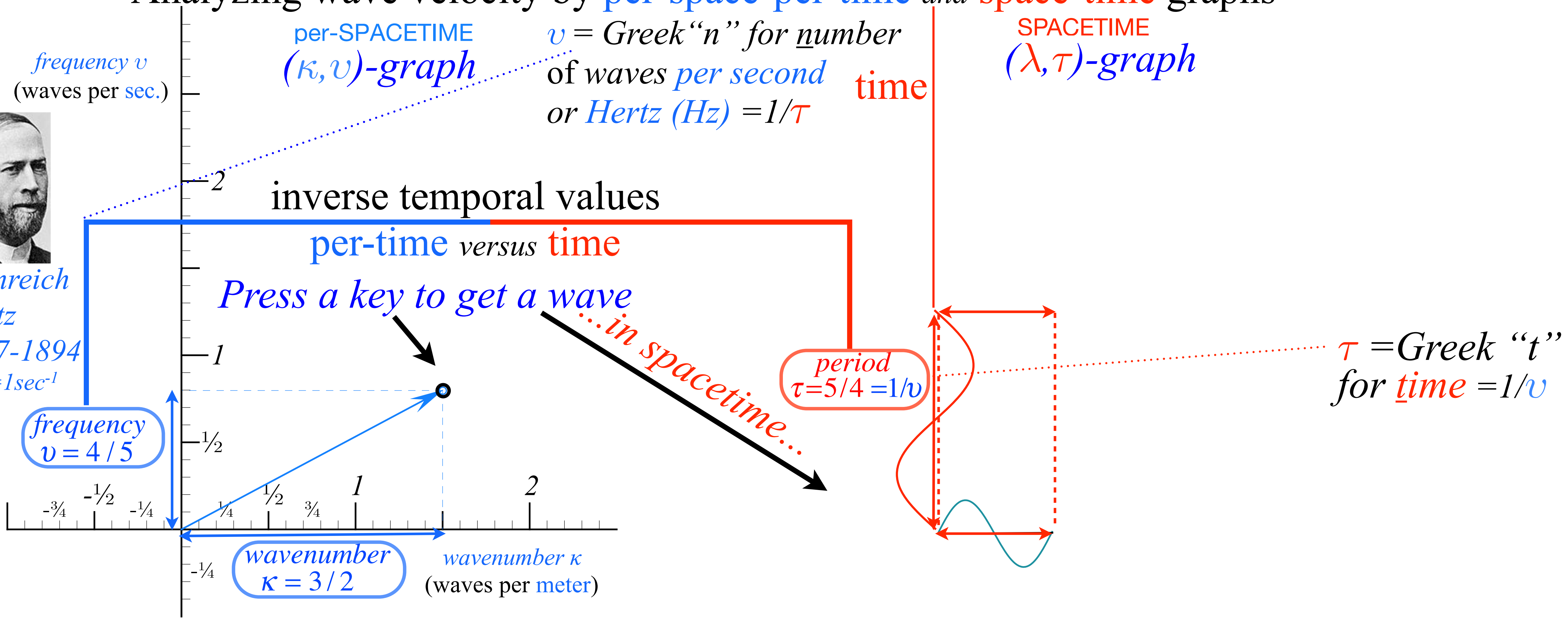

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RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot)

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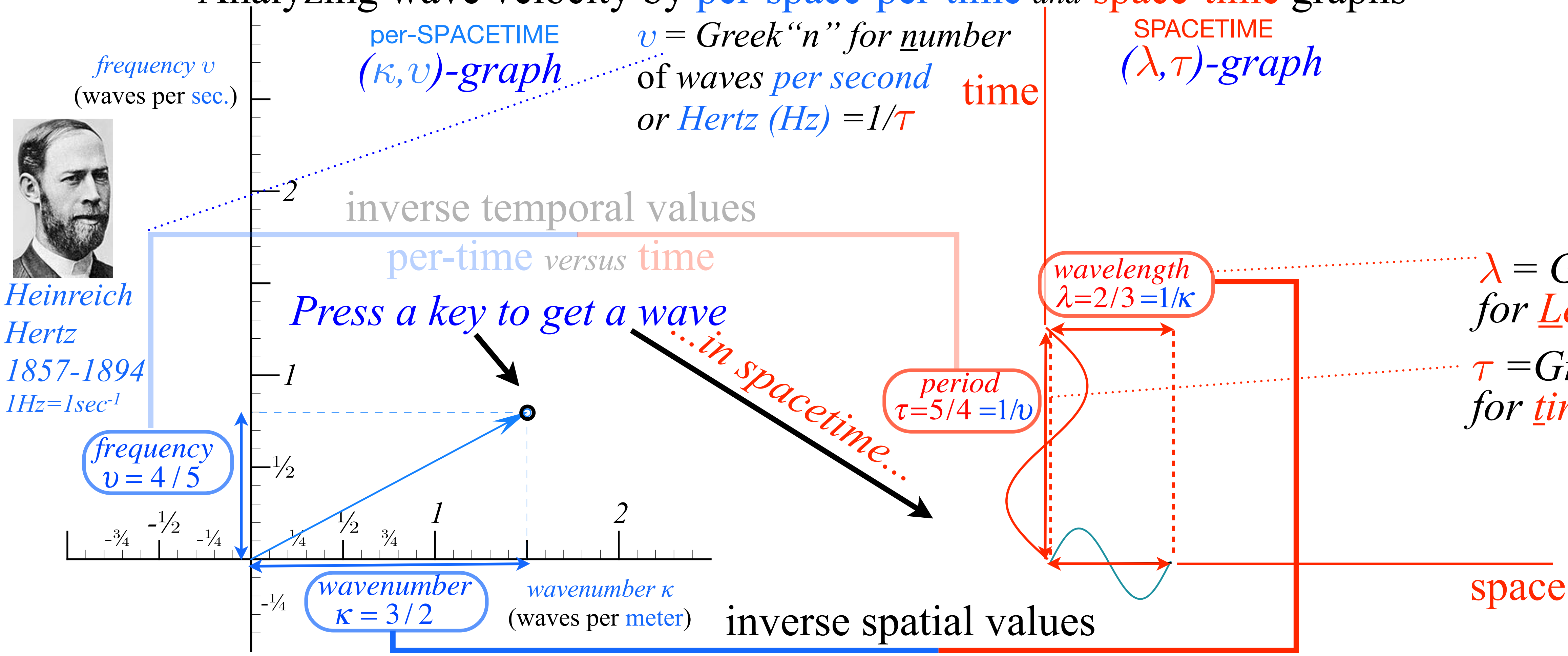

κ =Greek "k"

for Kayser

(or "kinks")=1/λ

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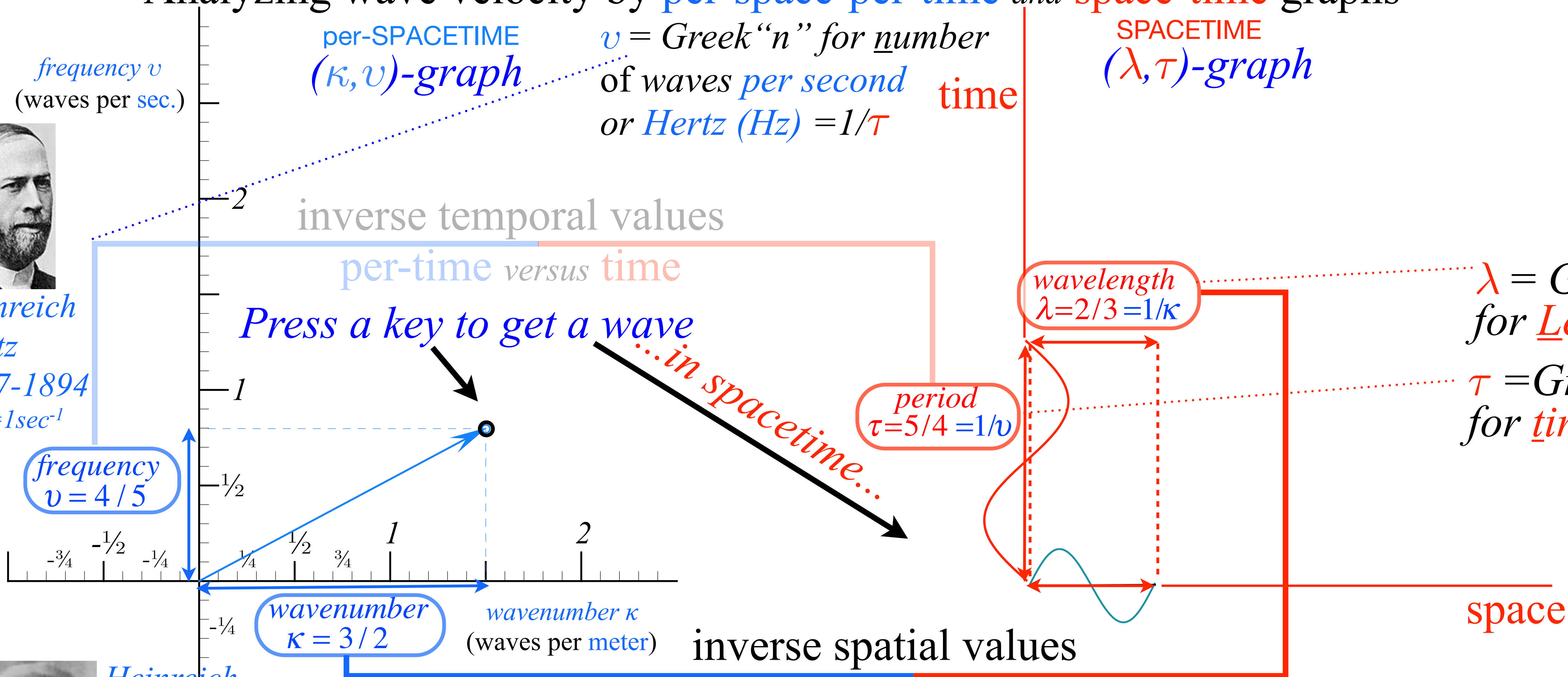

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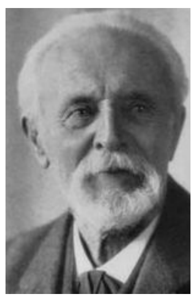

Heinreich Kayser 1853-1940
1Kayser=1cm^{-1}

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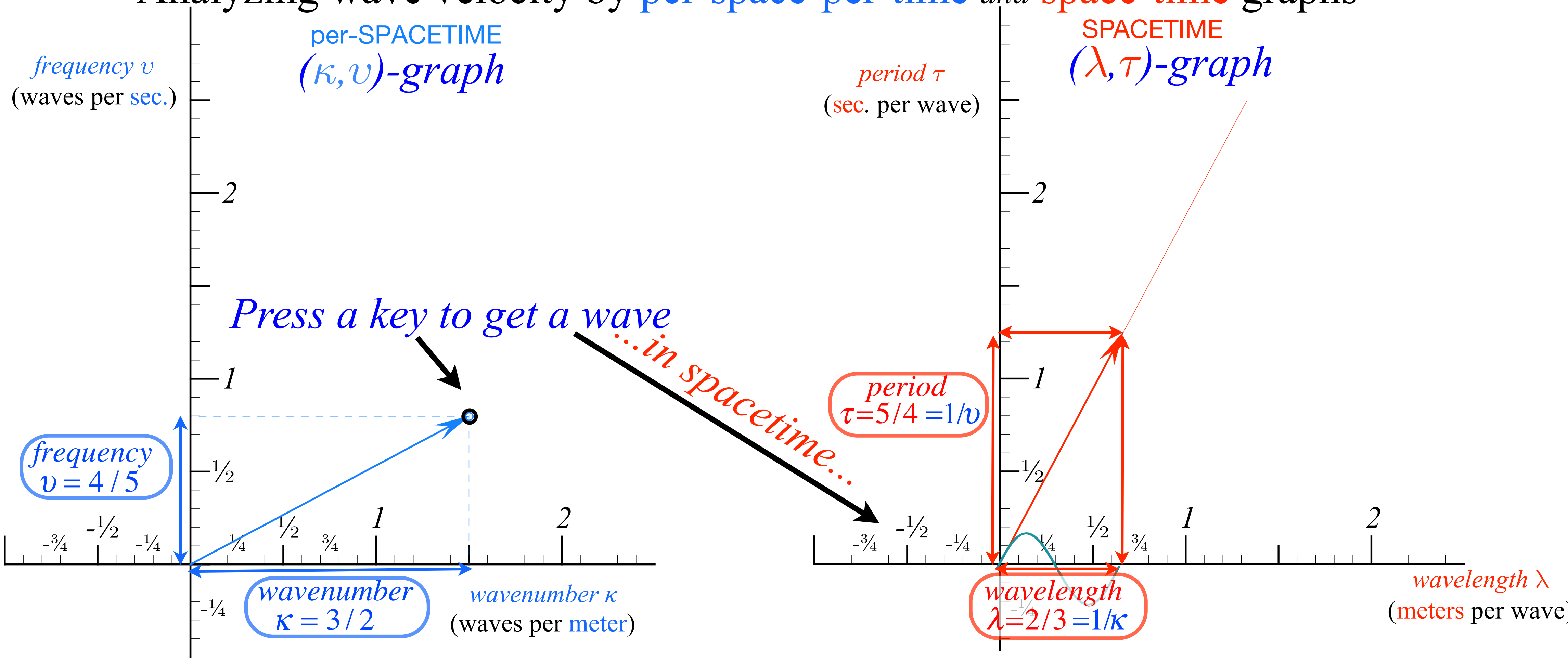

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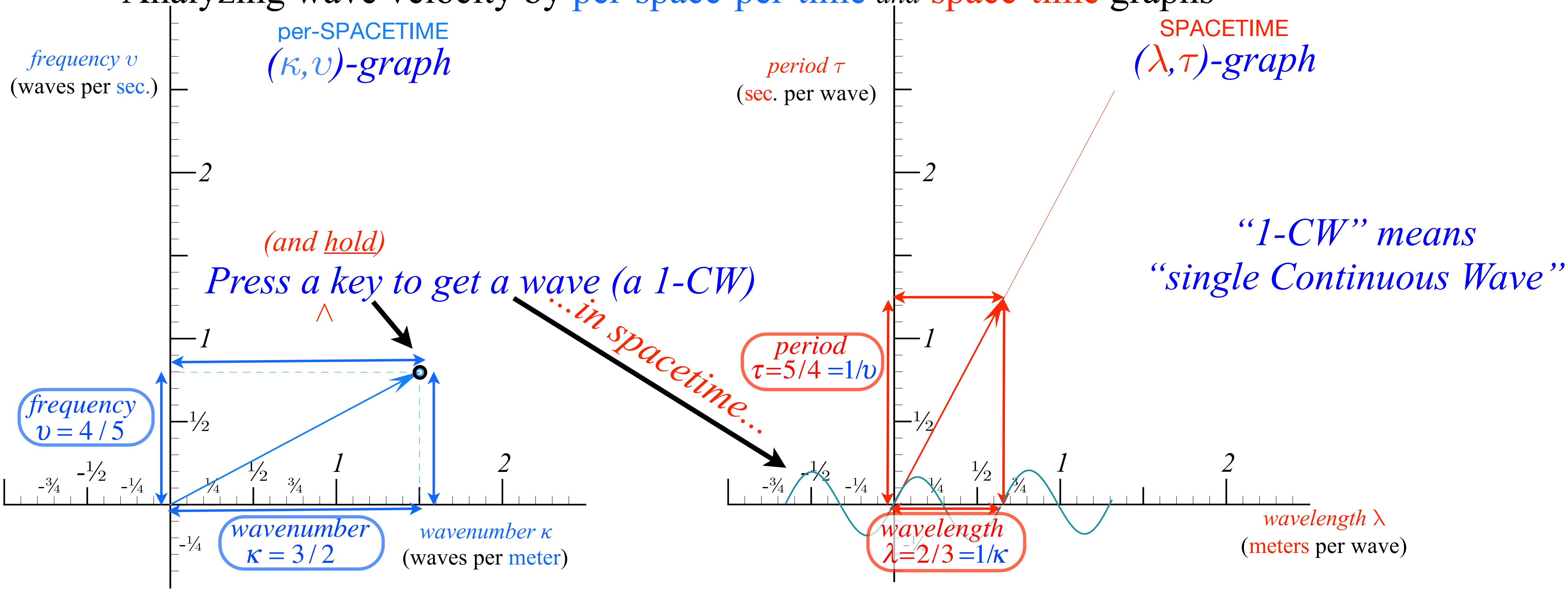

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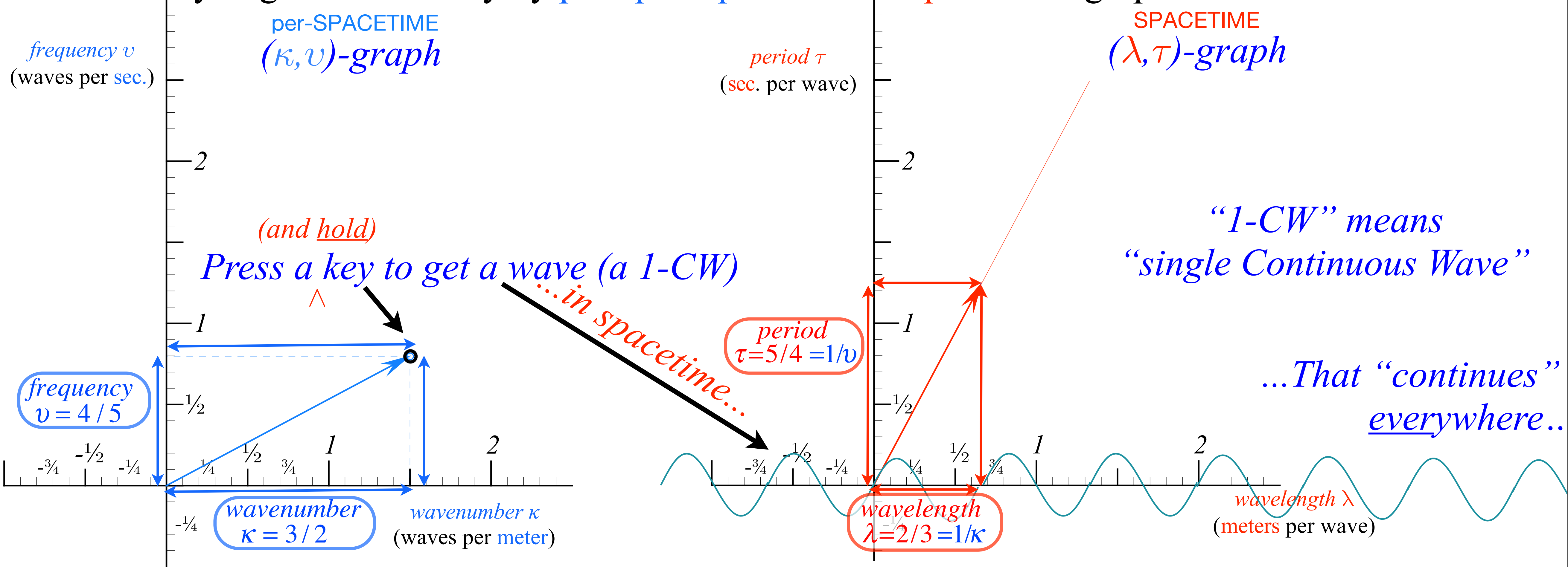

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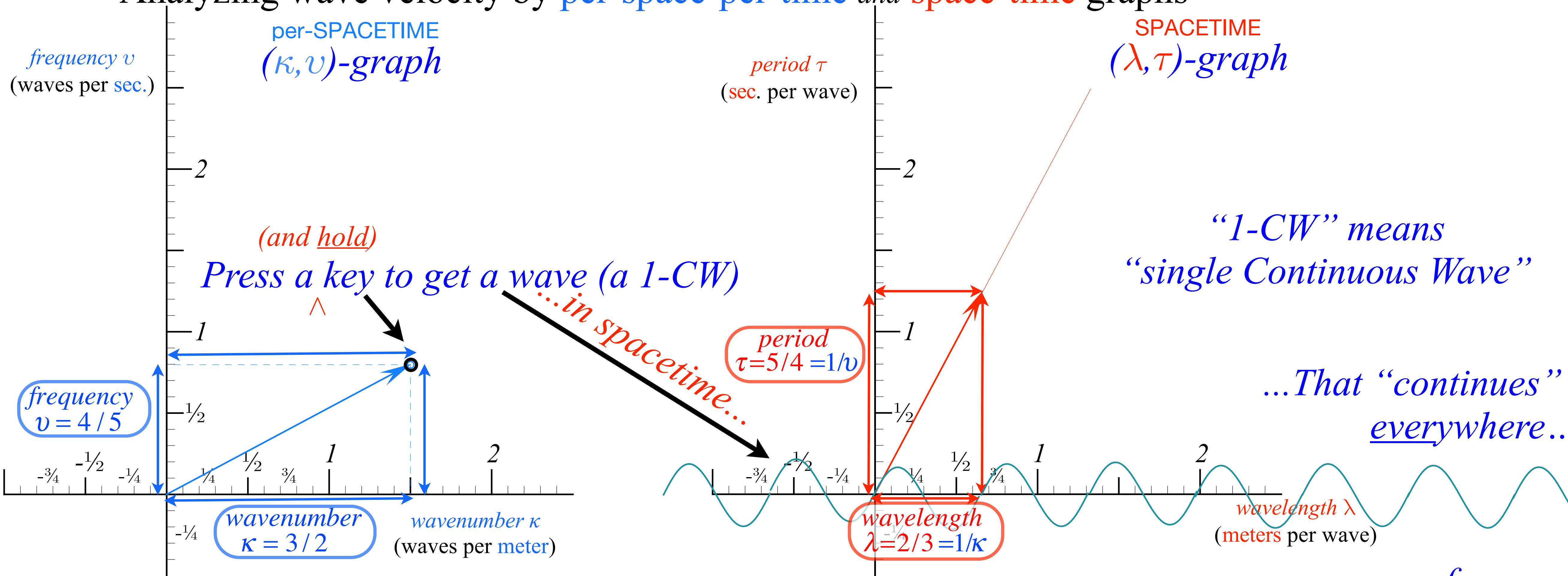
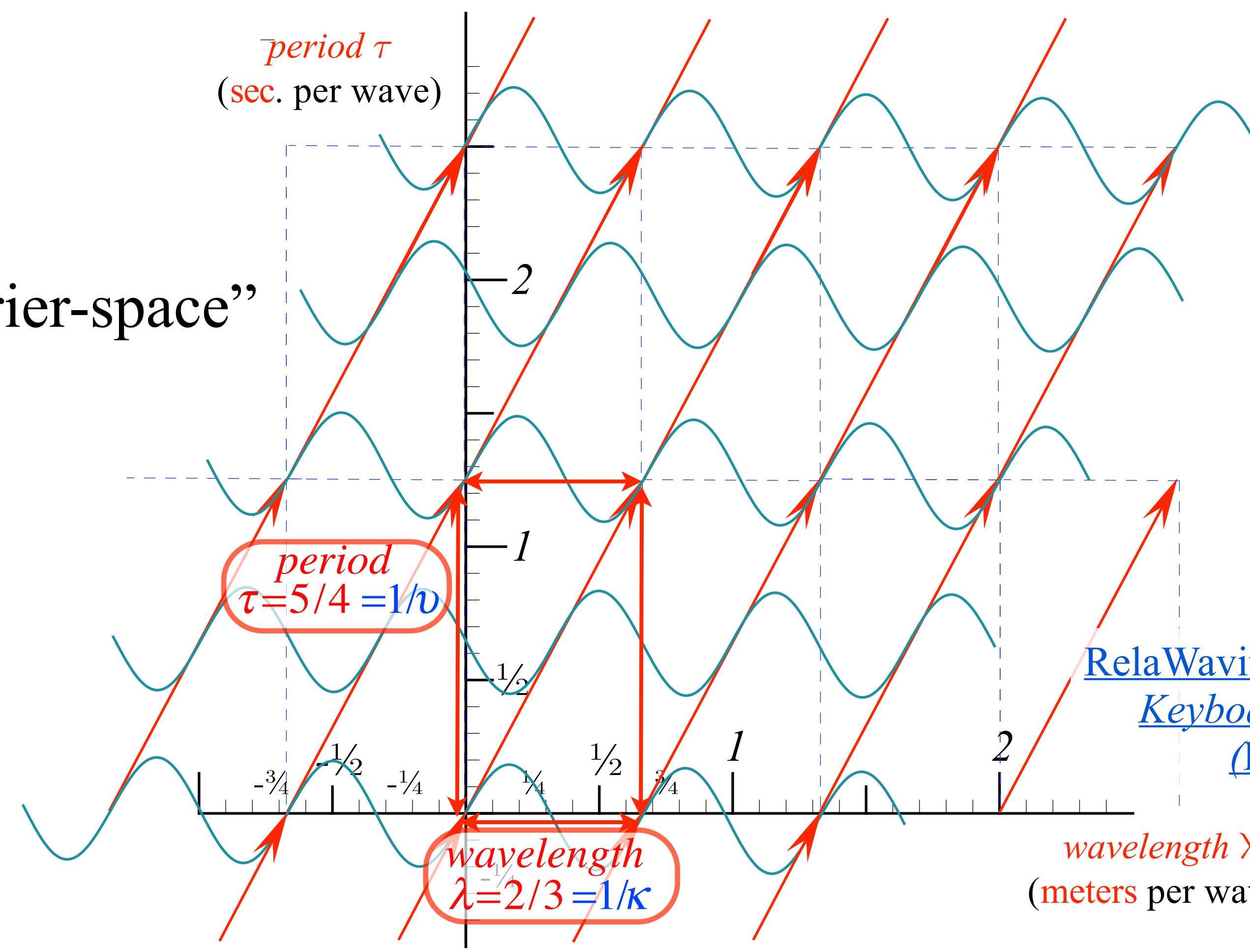


"1-CW" means
"single Continuous Wave"

...That "continues"
everywhere..

...for
all
time...



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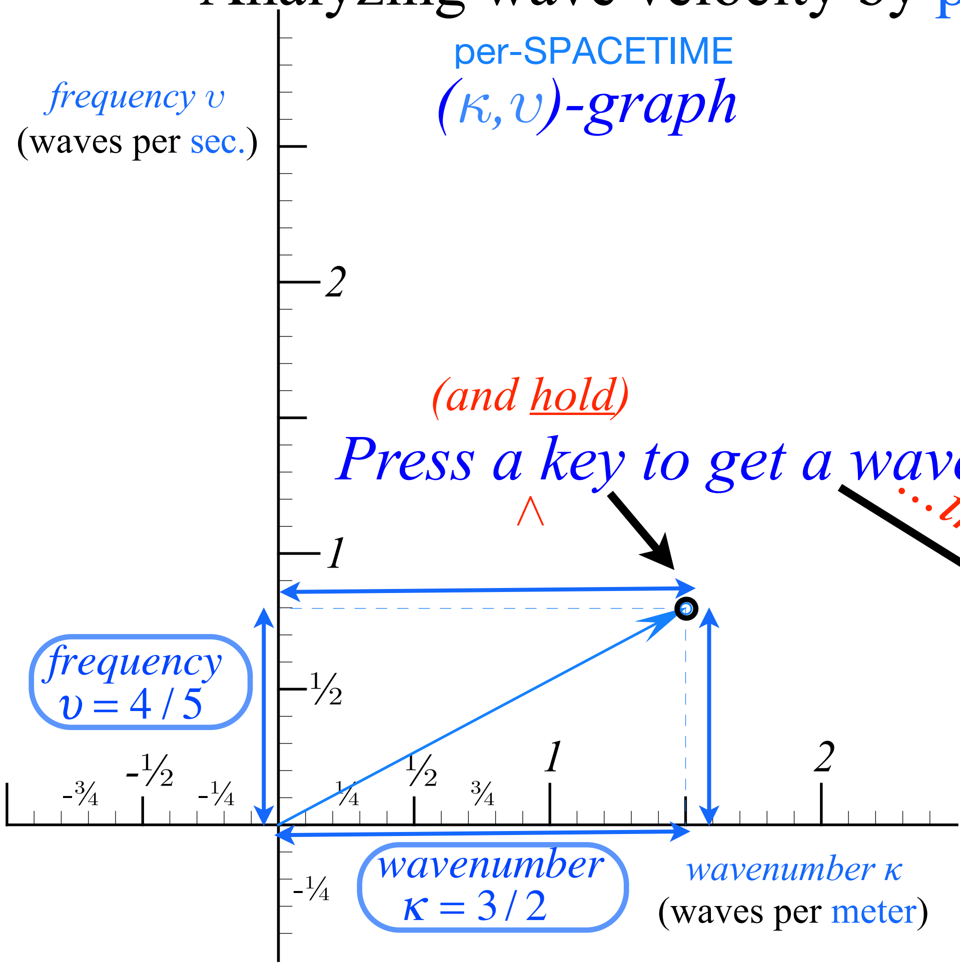
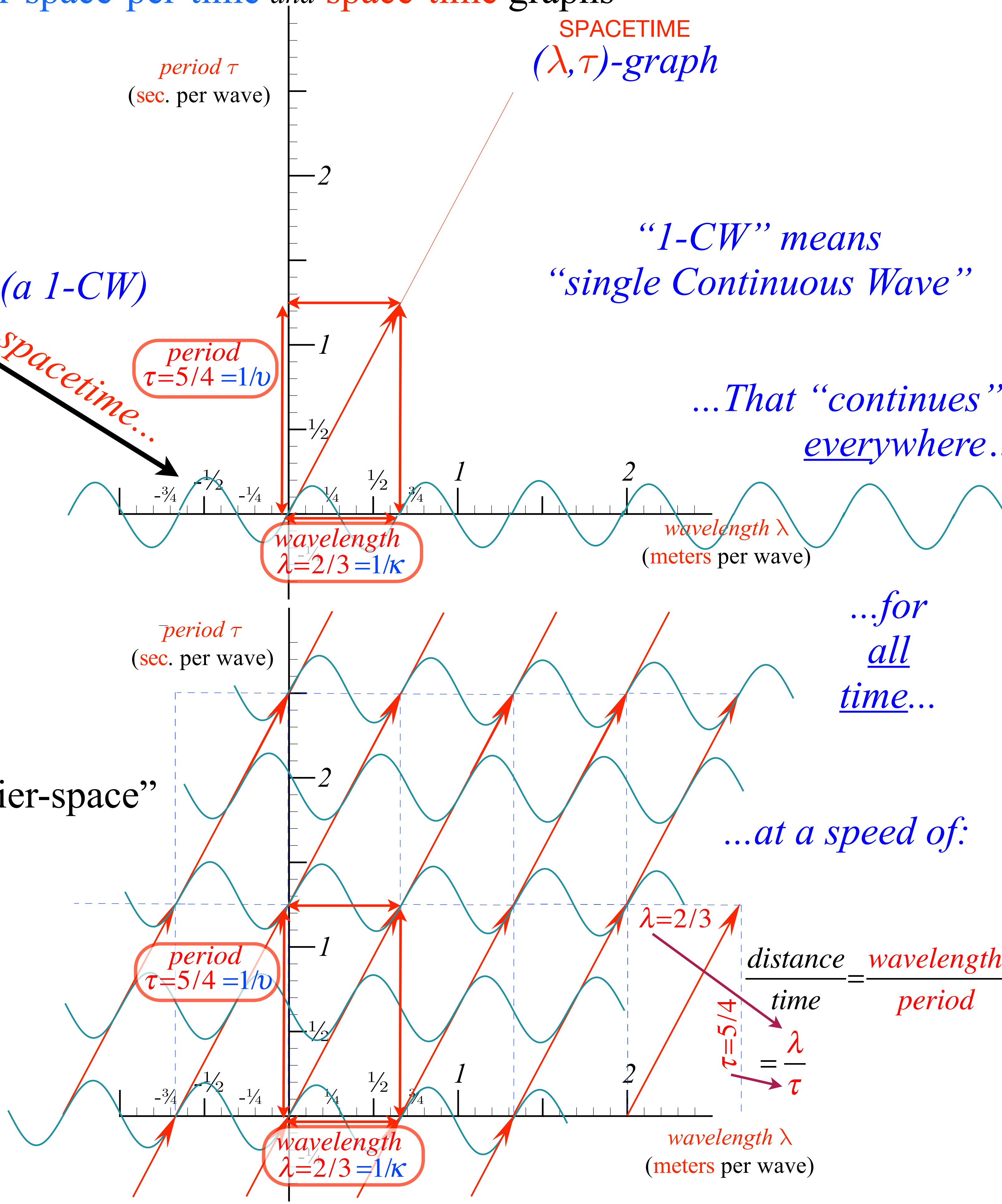


...in spacetime...



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...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

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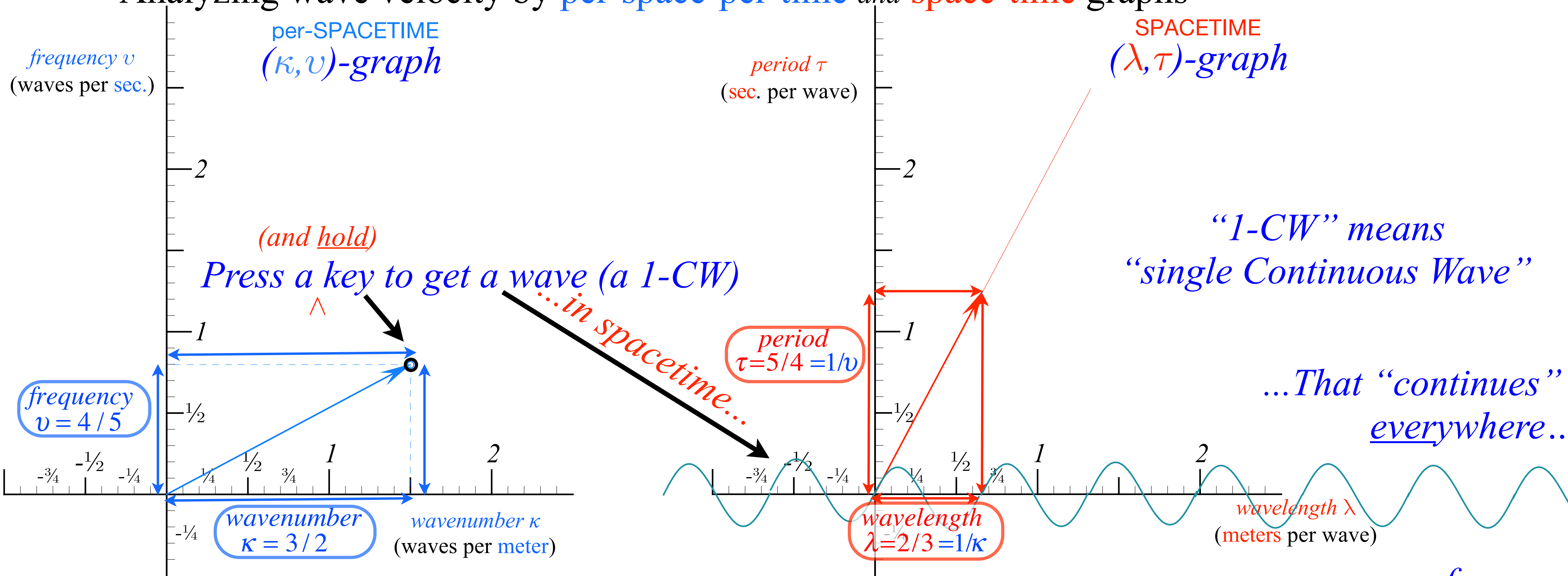

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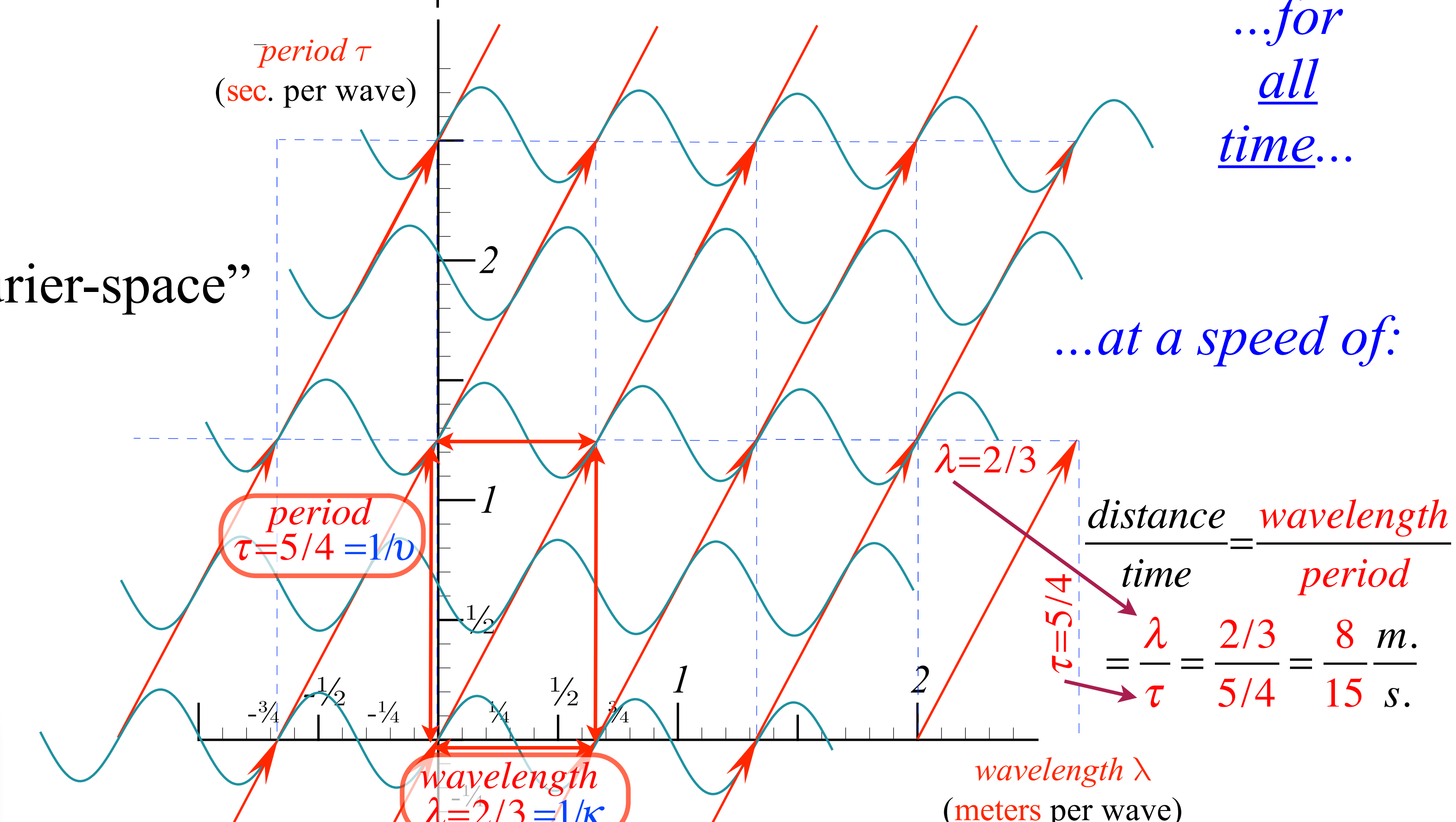

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

•How to understand waves
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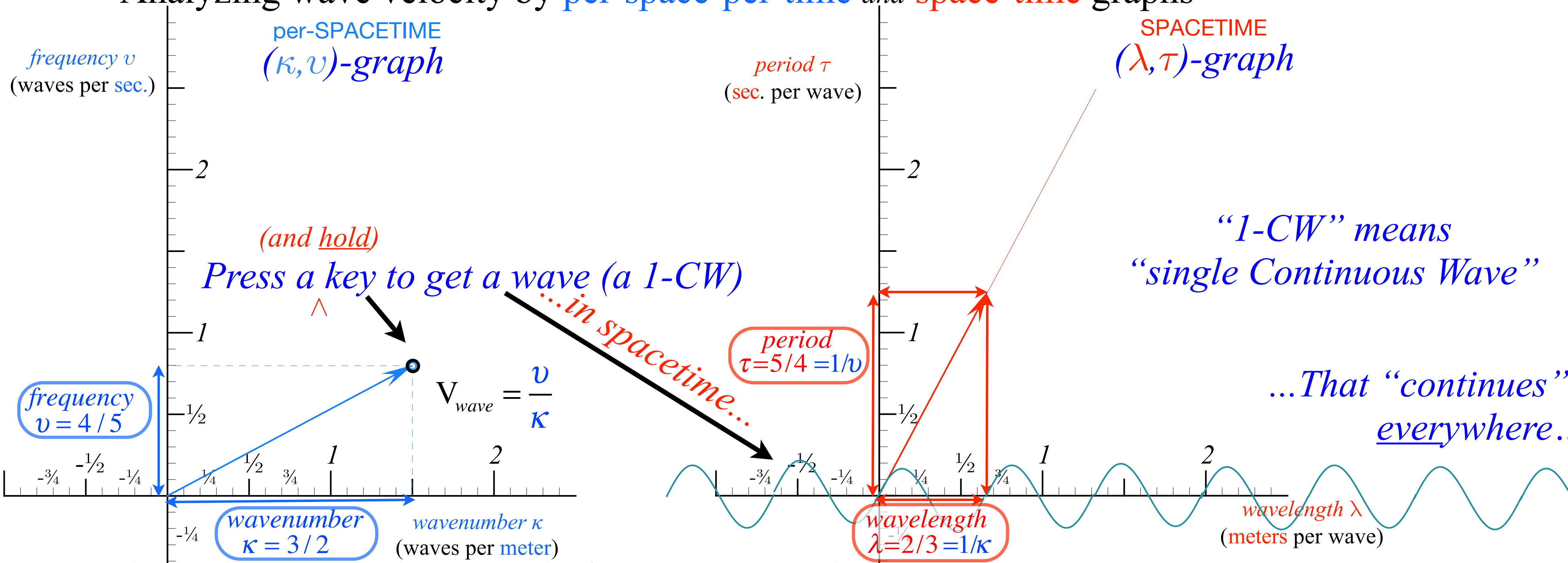
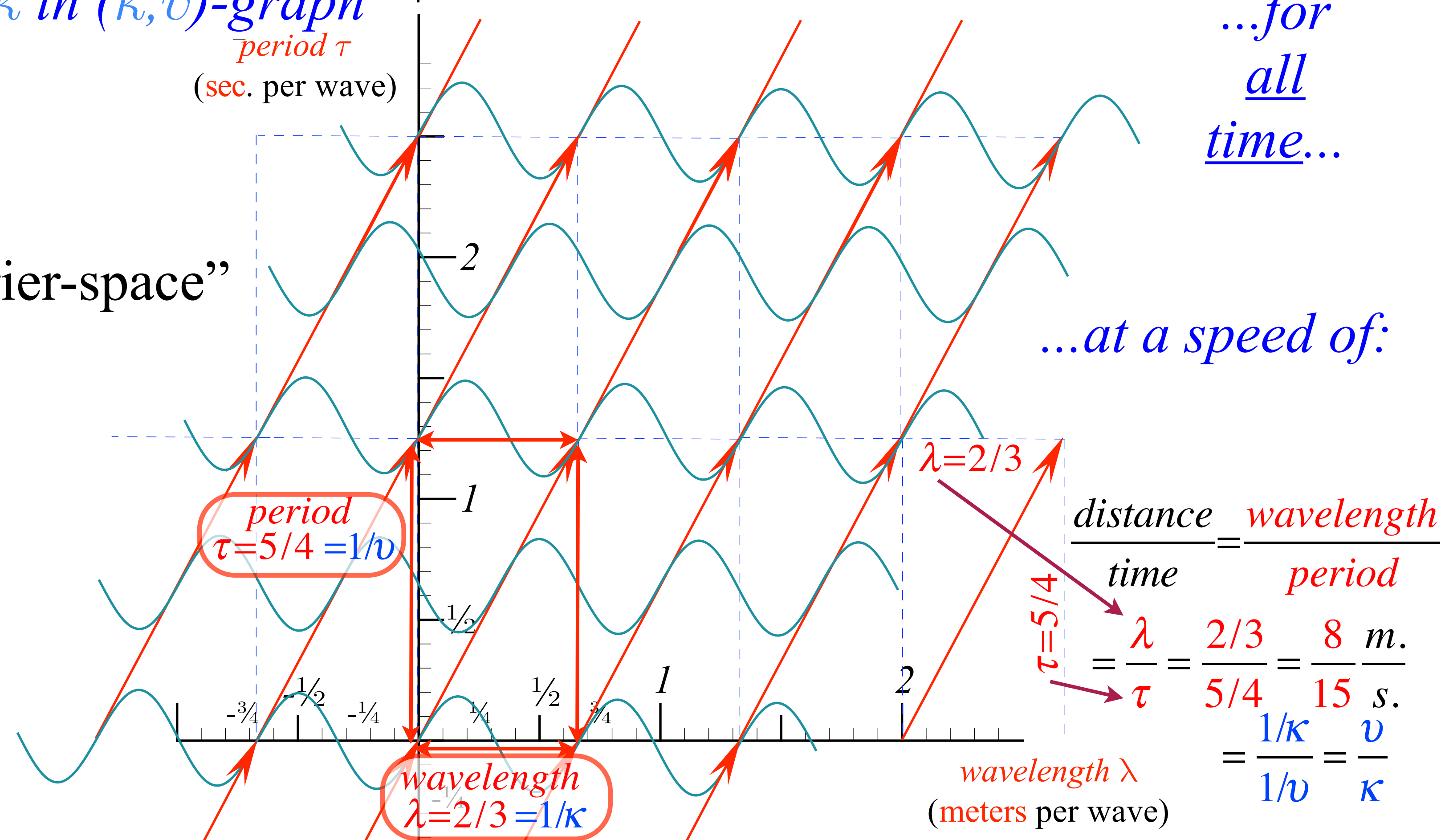
Analyzing wave velocity by per-space-per-time *and* space-time graphs



wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph



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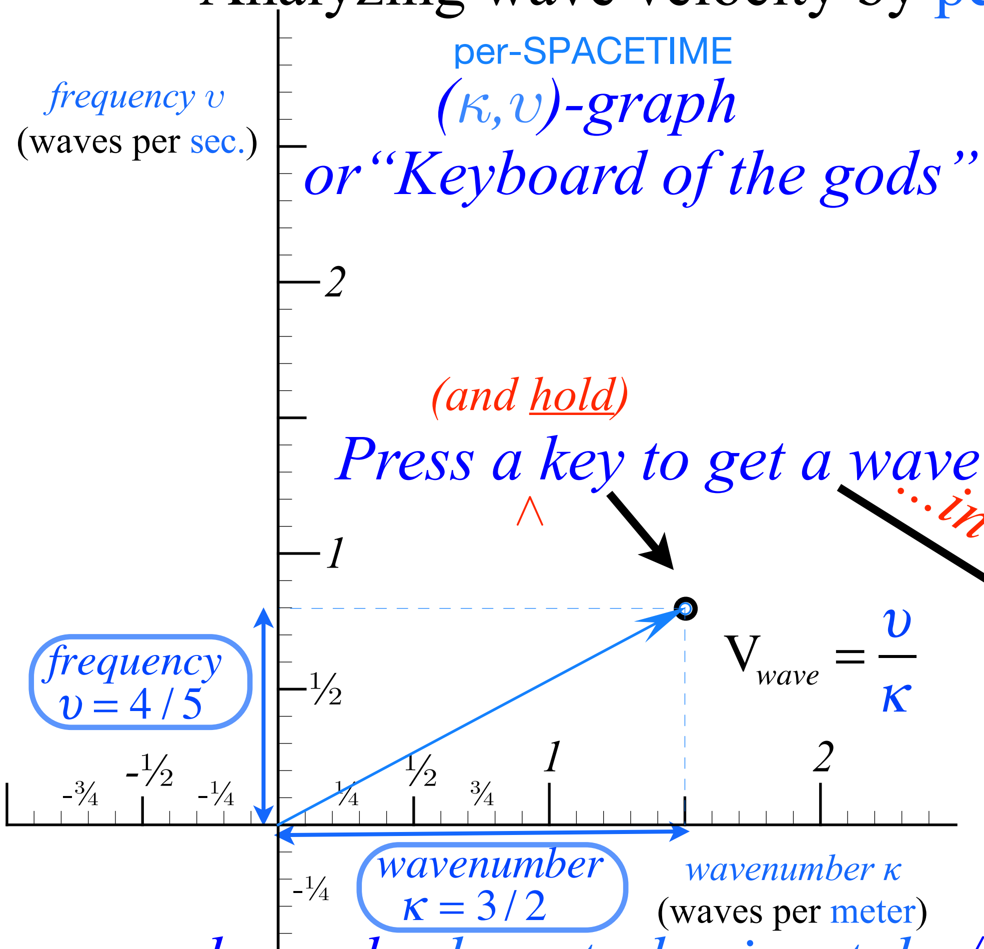

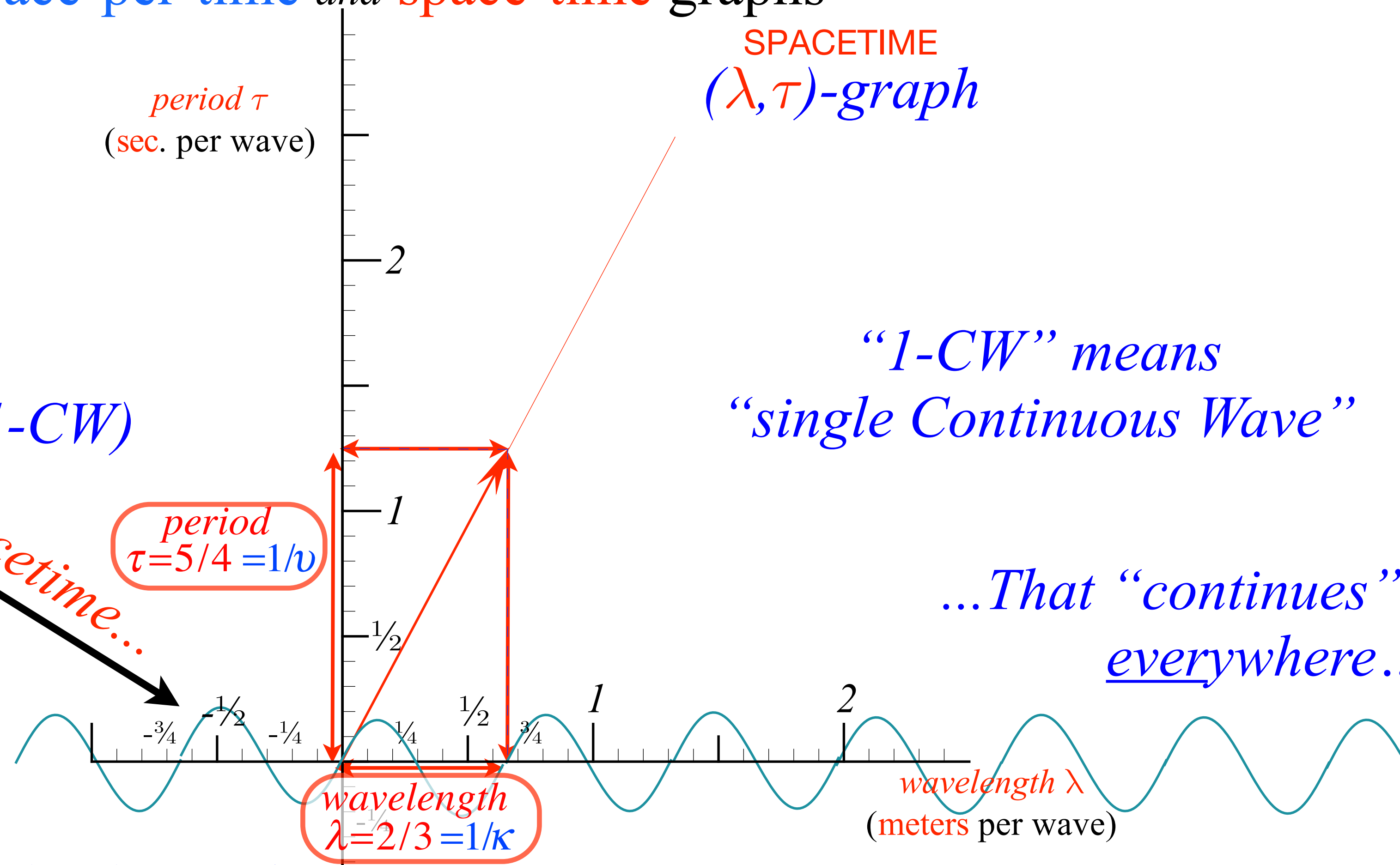

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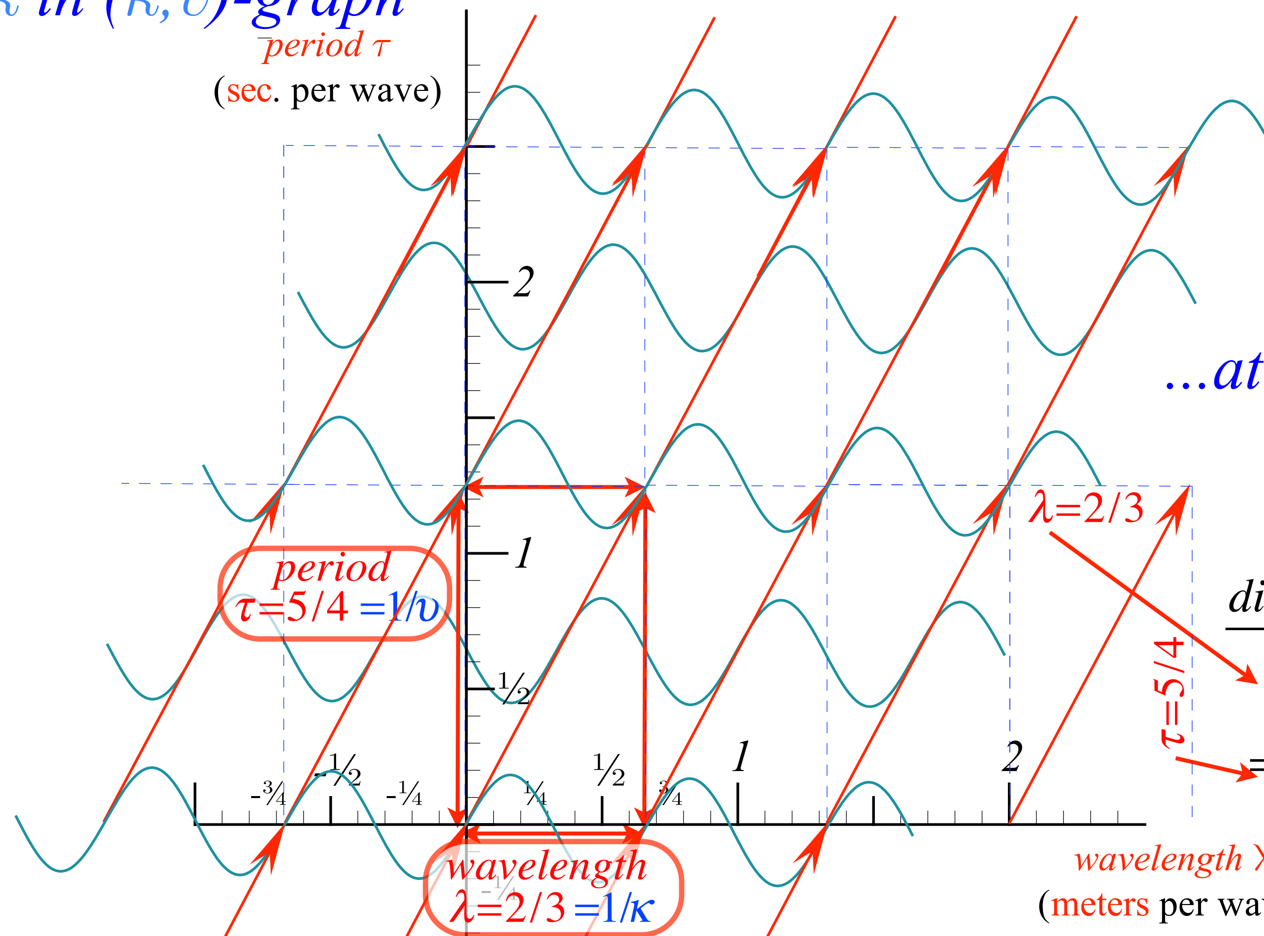

...for all time...

...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

wave-velocity formulas

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{frequency}{wavenumber}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

•How to understand waves and “1st quantization”

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Lecture 30
Thur. 12.10.2015

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
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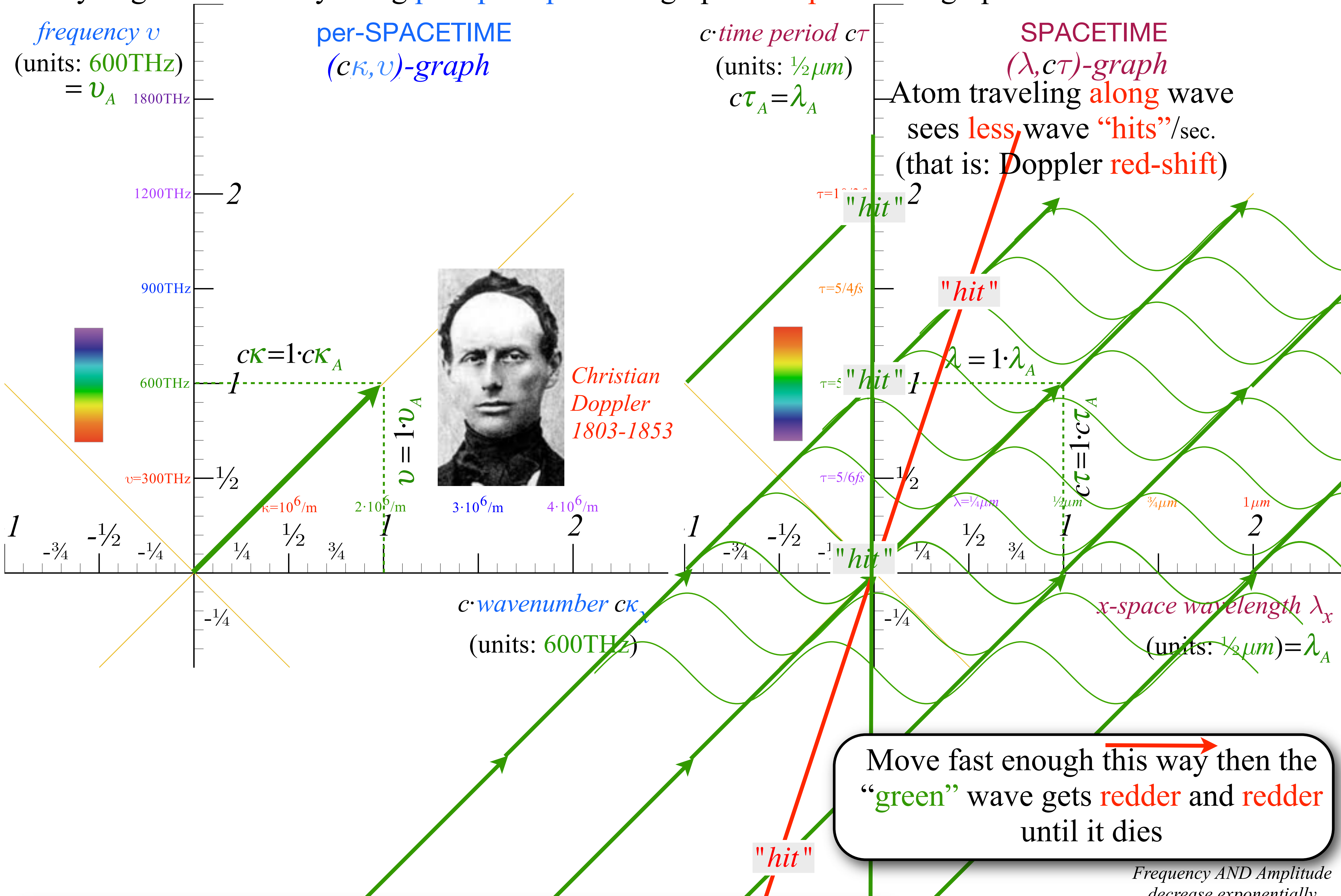
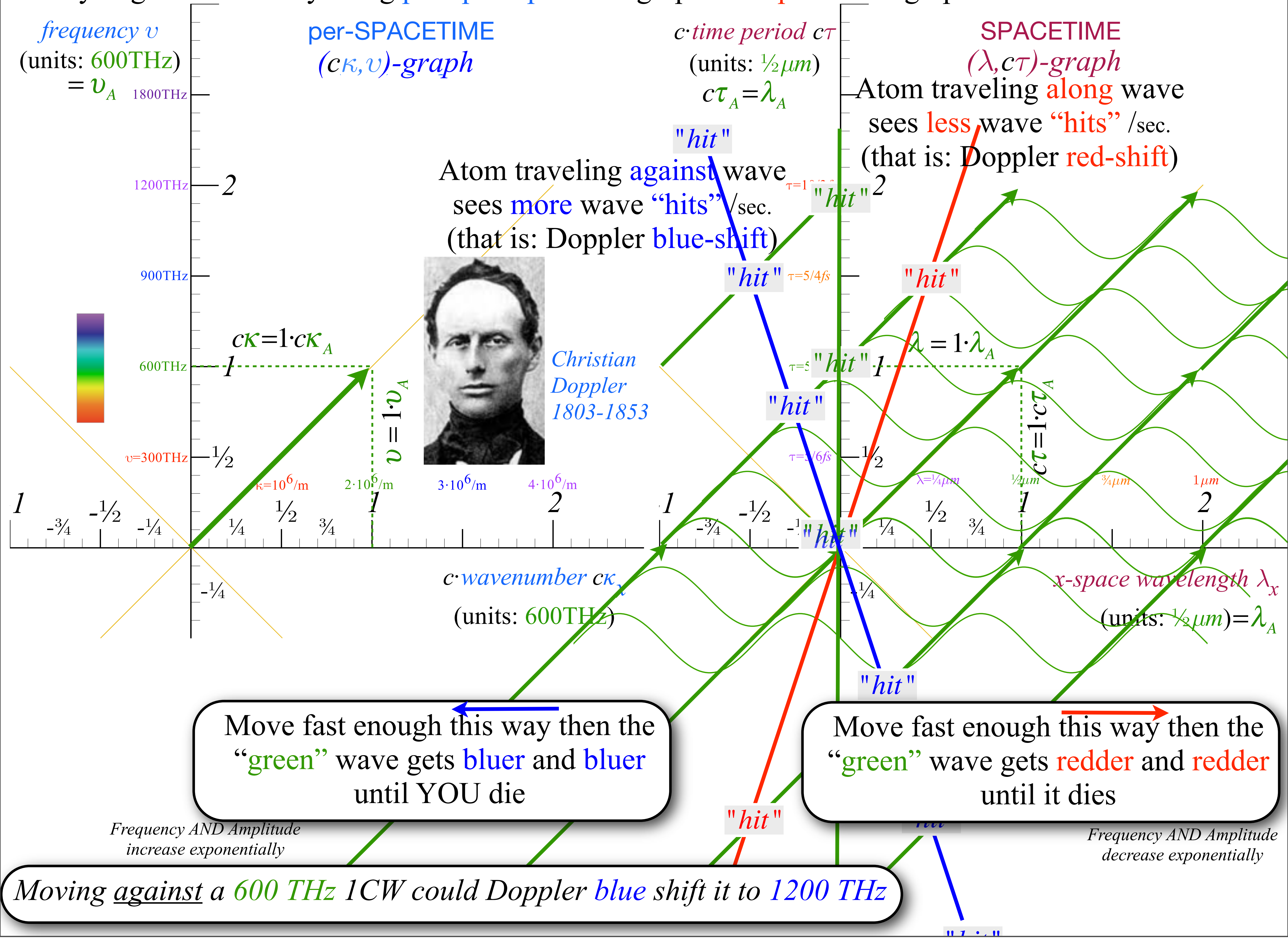


Moving along a 600 THz 1CW could Doppler red shift it to 300 THz

Analyzing wave velocity using per-space-per-time graphs *and* space-time graphs



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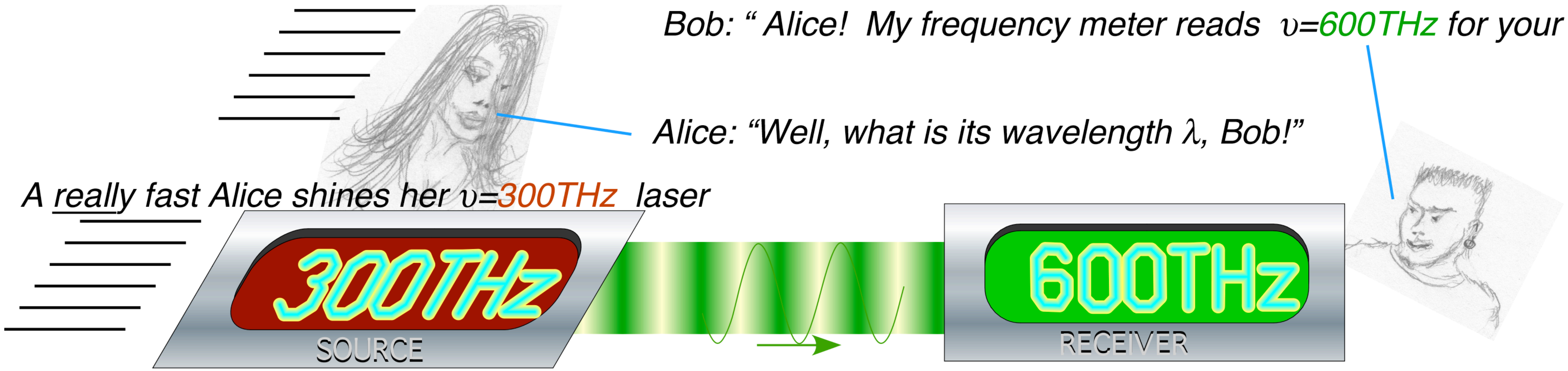
Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)



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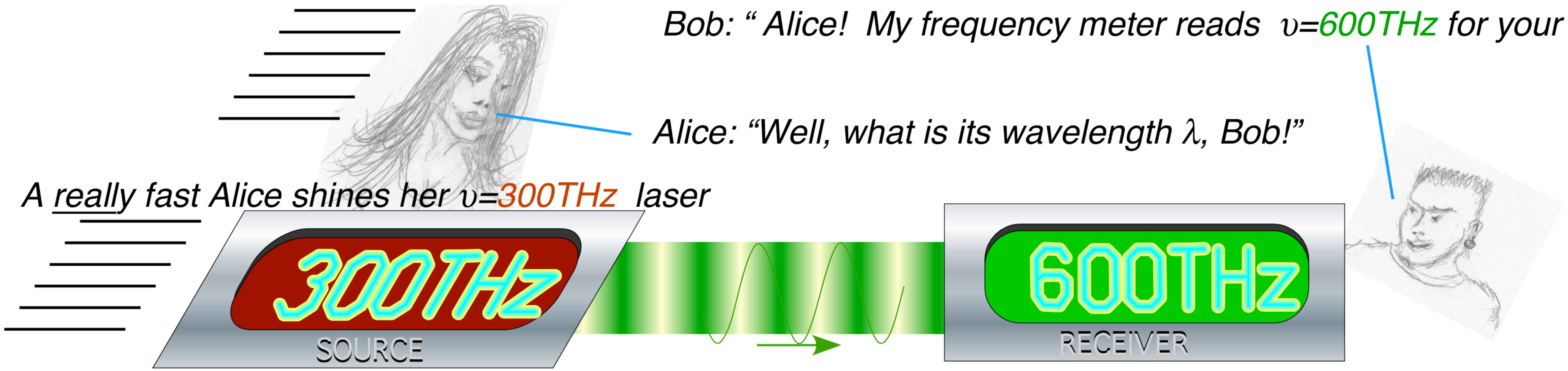

Q1: Can Bob tell it's a *"phony"* 600THz
by measuring his received wavelength?

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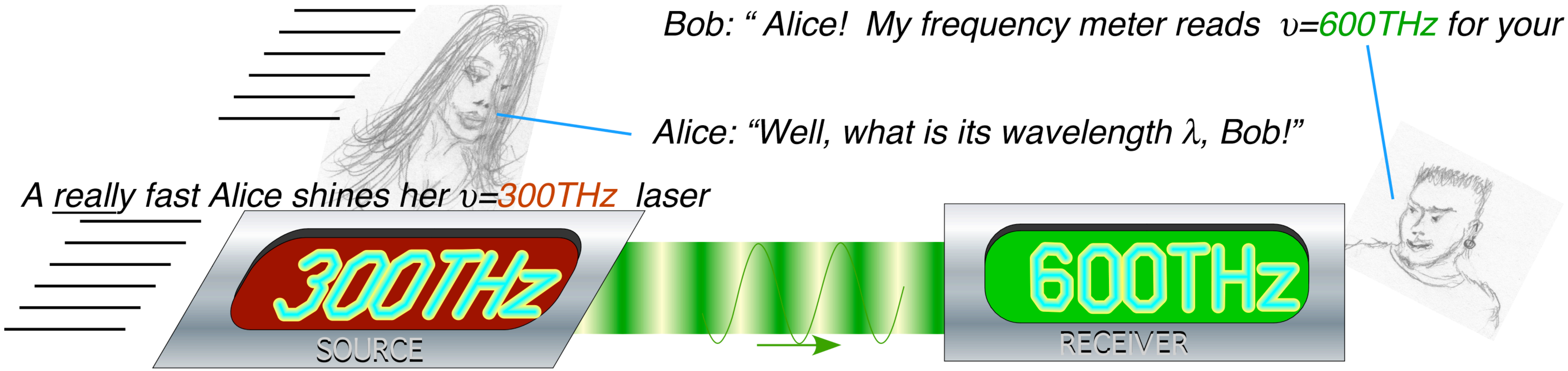

Q1: Can Bob tell it's a *"phony"* 600THz by measuring his received wavelength?

Q2: If so, what *"phony"* λ does Bob see?

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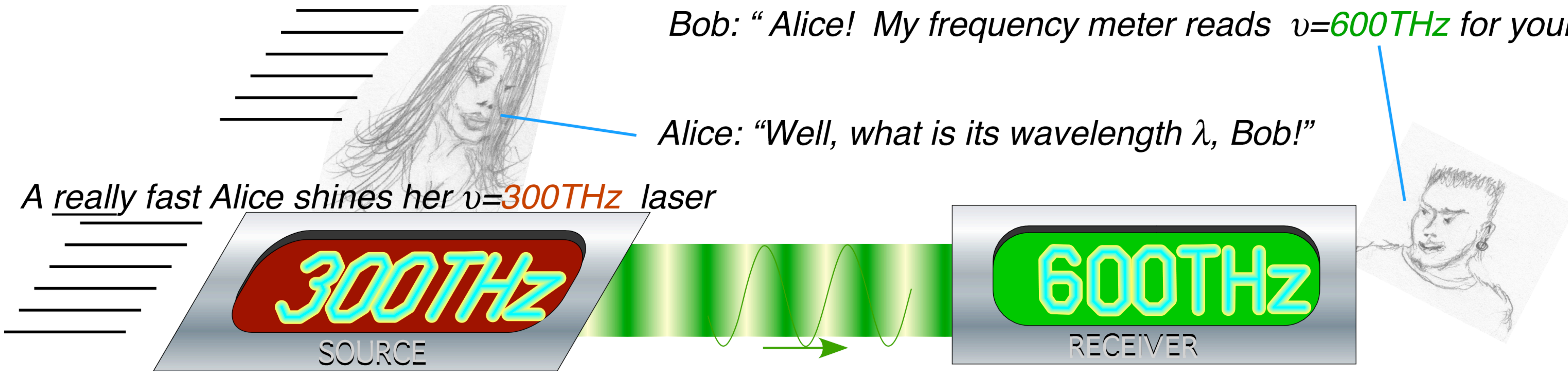

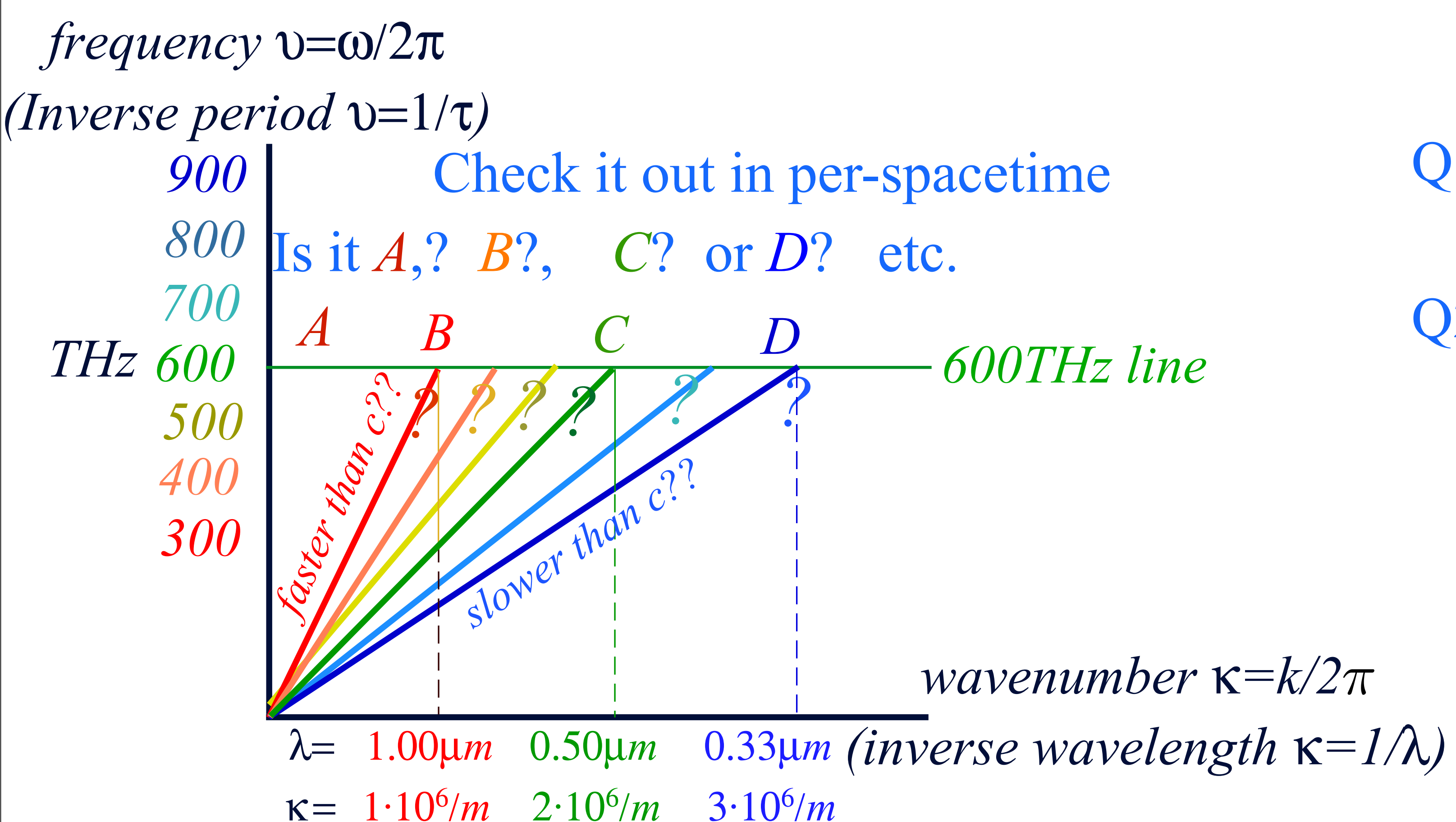

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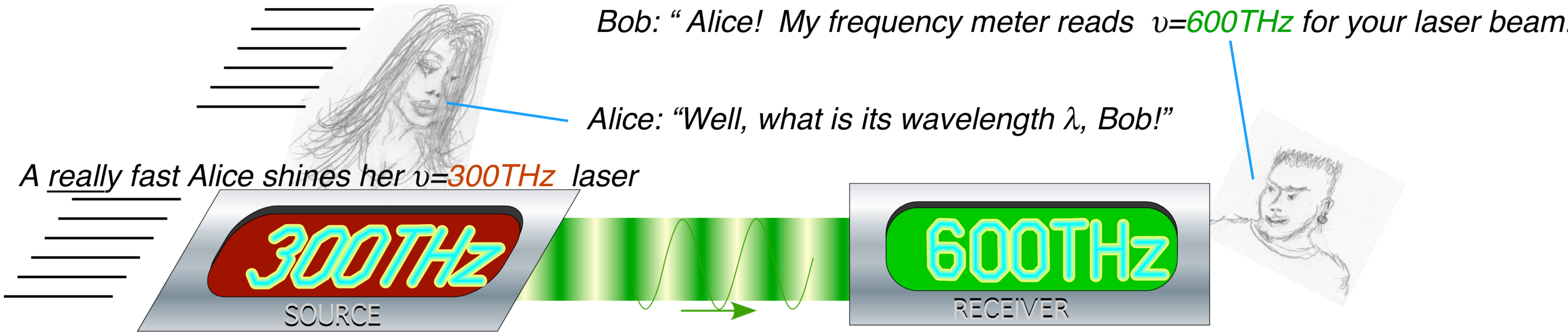

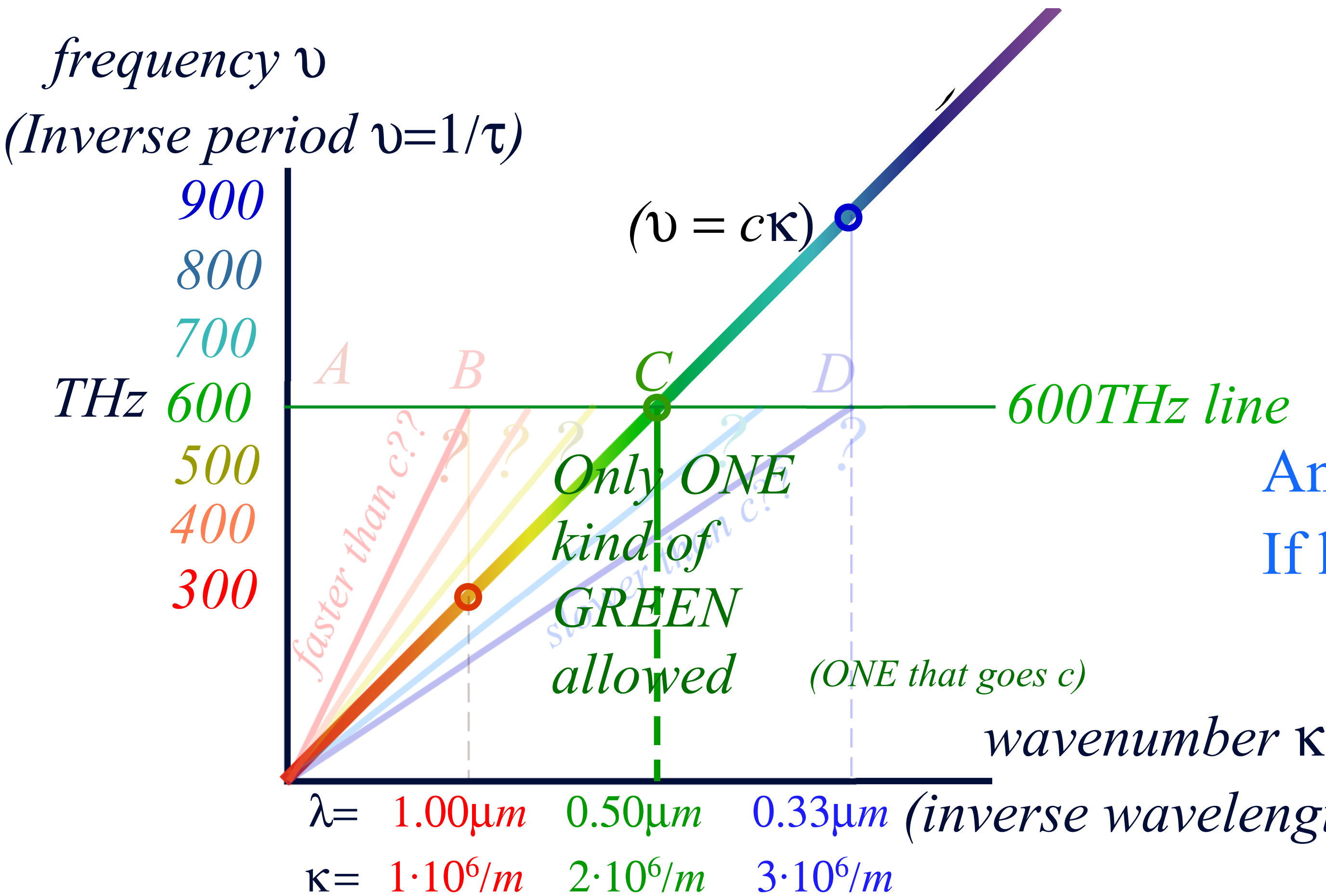

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Q2:If so, what *"phony"* λ does Bob see?

Answer to Q2 is *C*, the one with *slope* $\upsilon/\kappa=\upsilon\cdot\lambda=c$.

If he sees Green 600THz then he measures λ=0.5μm.

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

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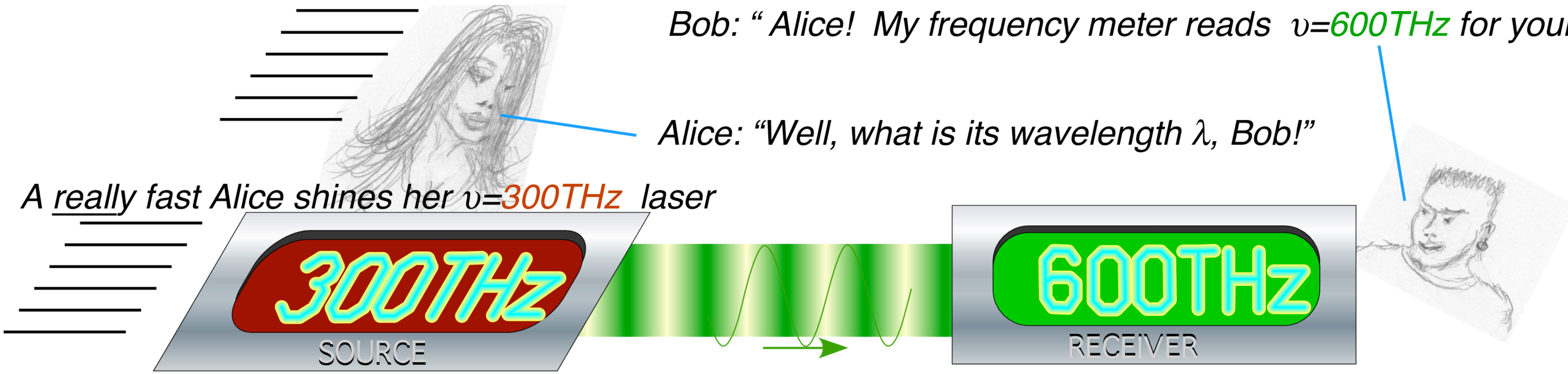

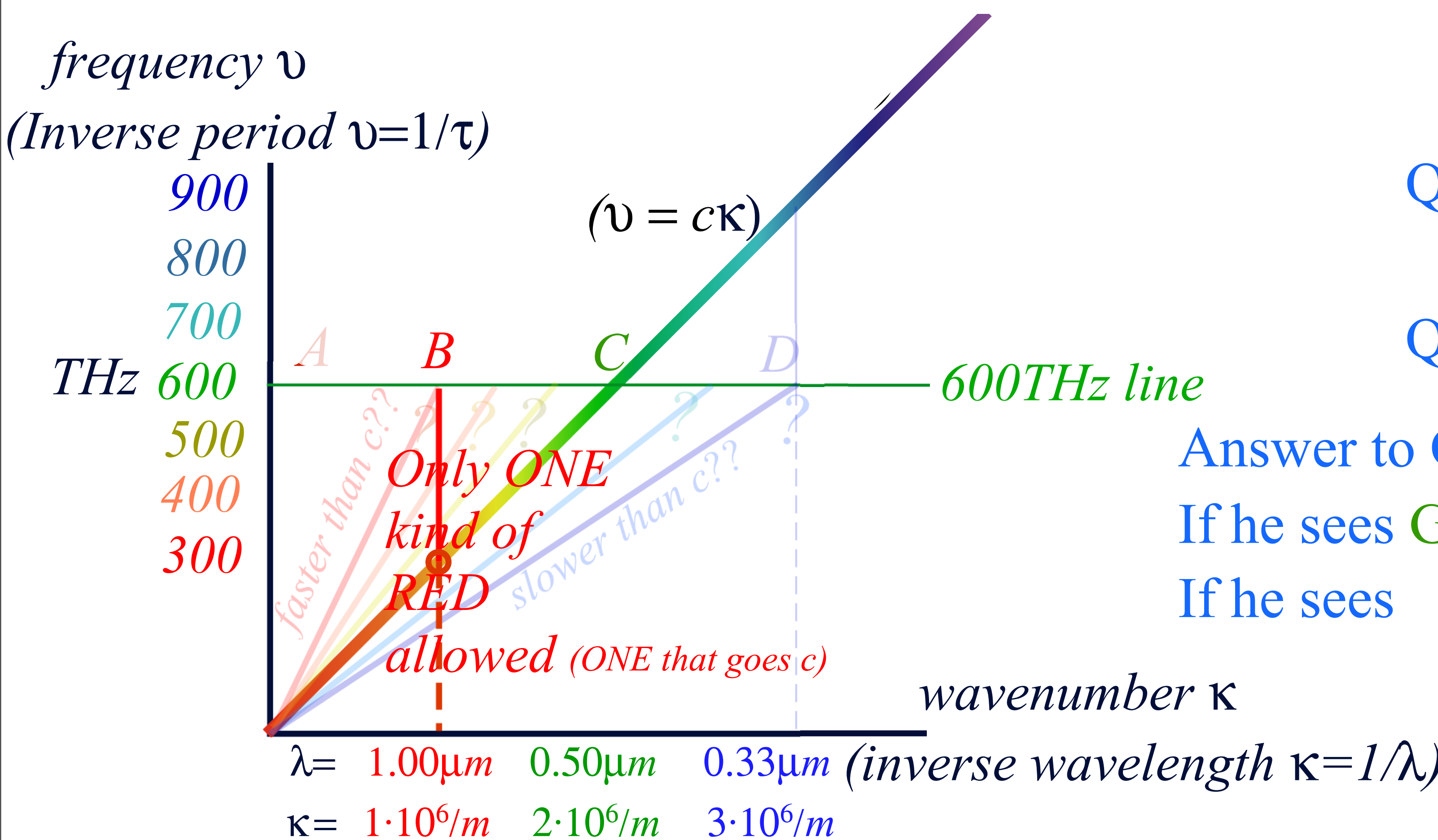

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If he sees Green 600THz then he measures λ=0.5μm.

If he sees Red 300THz then he measures λ=1.0μm.

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

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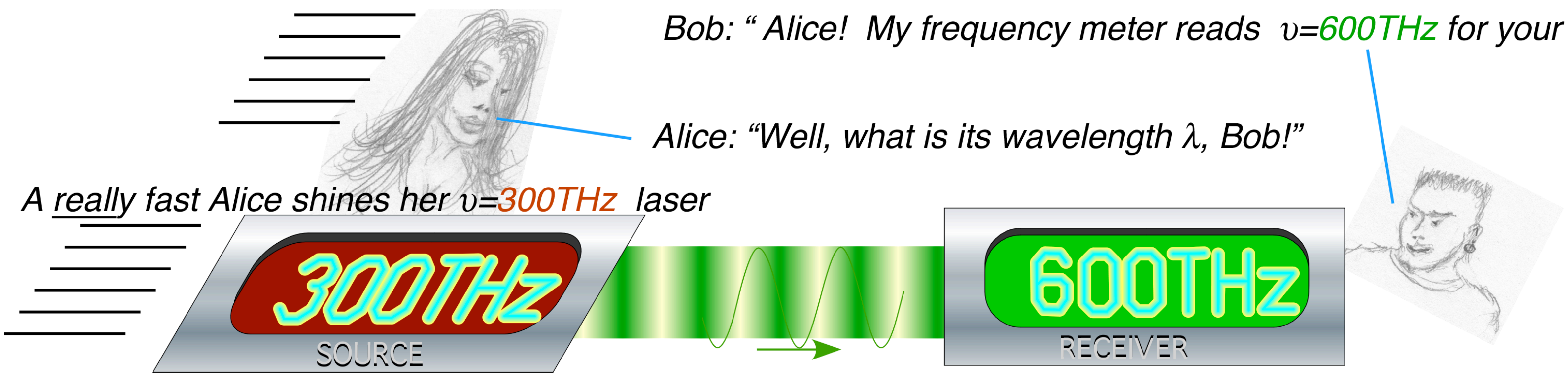

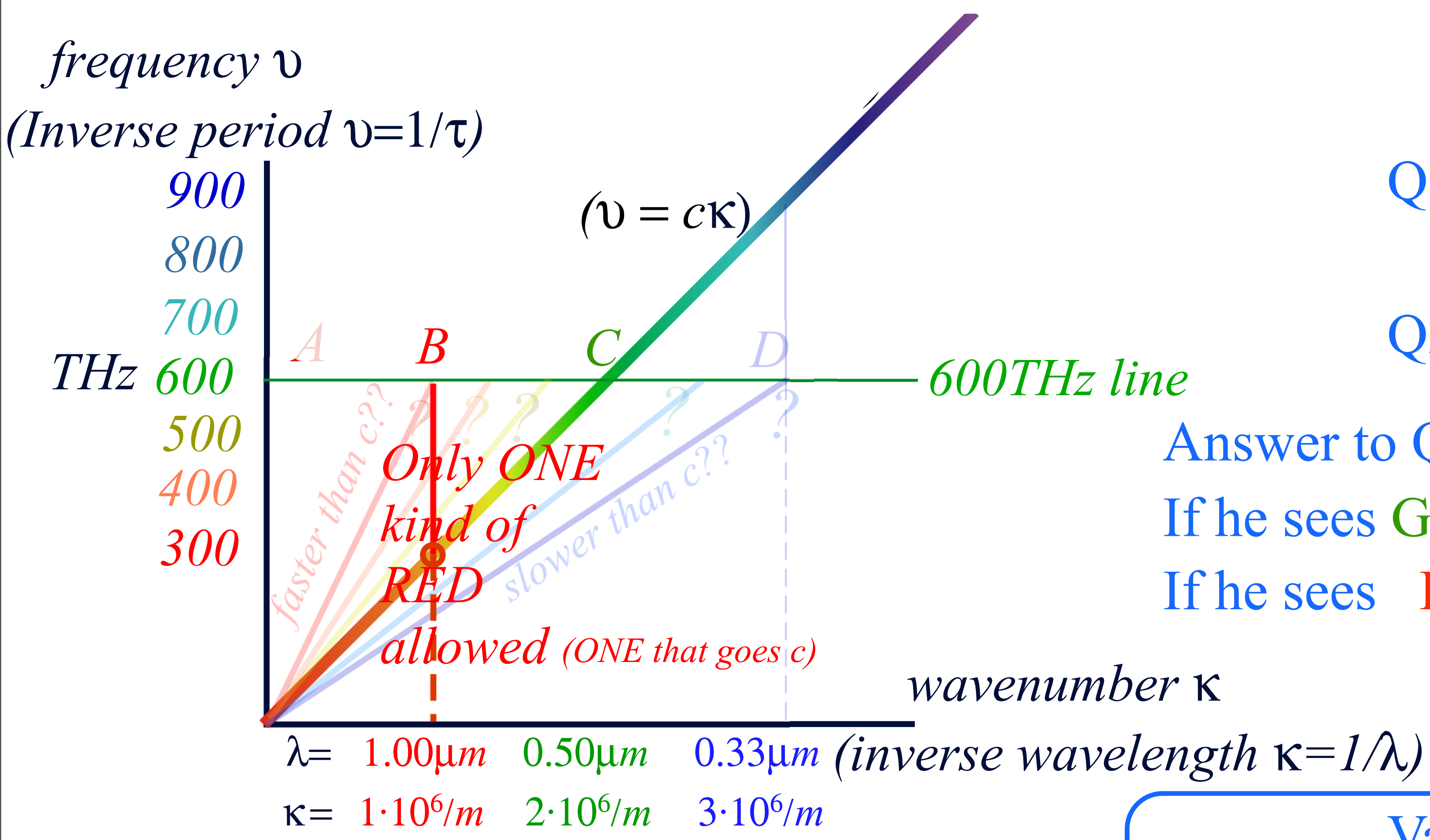

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Answer to Q2 is *C*, the one with slope $\upsilon/\kappa=\upsilon\cdot\lambda=c$.

If he sees Green 600THz then he measures λ=0.5μm.

If he sees Red 300THz then he measures λ=1.0μm.

Answer to Q1 is **NO!**

CW Light carries **no** birth-certificate!

Vacuum only makes one λ for each υ.*

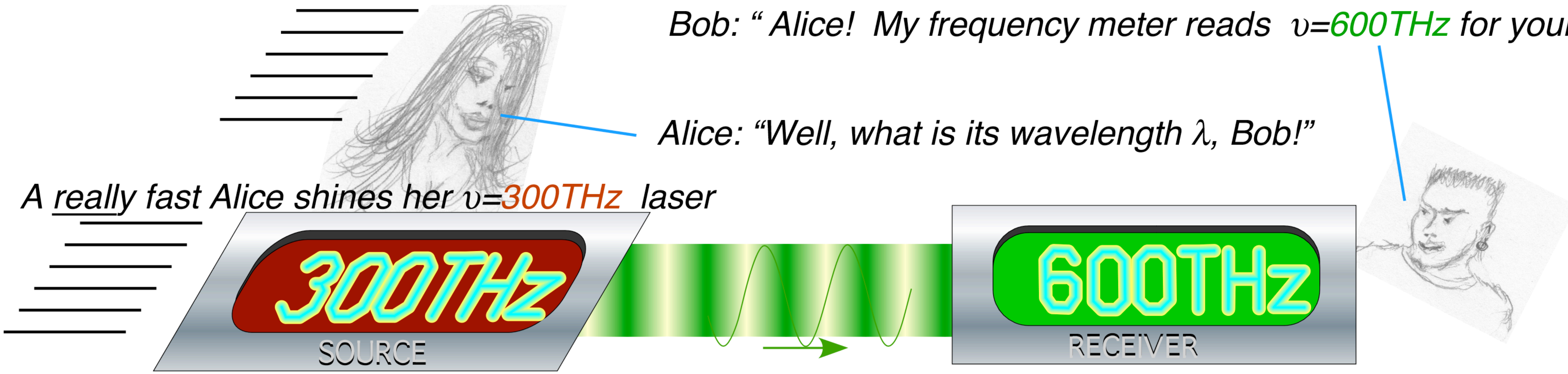
"All colors go $c = \lambda\upsilon = \upsilon/\kappa$ "

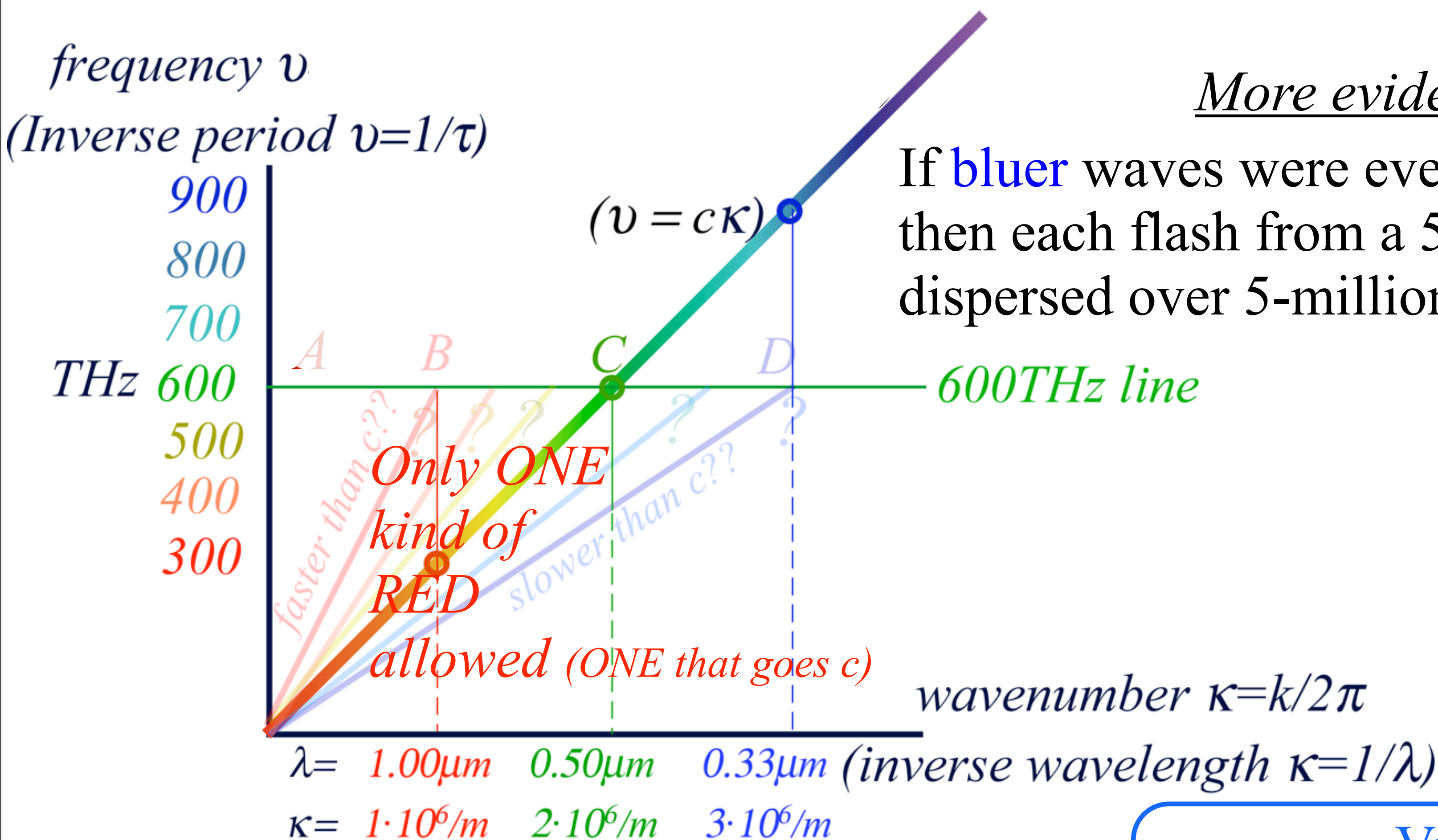
Then *Evenson's axiom* holds:

*for each beam and polarization orientation

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)





More evidence supporting Evenson's axiom

If bluer waves were even 0.1% faster (or slower) than redder ones then each flash from a 5-billion light-year distant galaxy shows up dispersed over 5-million years. (*Goodbye galactic astronomy!*)

Also could be labeled :

Linear-(non)-dispersion axiom: $\upsilon = c\kappa$

Vacuum only makes one λ for each υ.*

"All colors go $c = \lambda\upsilon = \upsilon/\kappa$ "

Then *Evenson's axiom* holds:

*for each beam and polarization orientation

Lecture 30
Thur. 12.10.2015

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Developing Axioms to update Galileo's relativity: A critical look at *c*-axioms

Einstein's PW (Pulse-Wave) Axiom

Evenson's CW (Continuous Wave) Axiom and Occam's Razor

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Introducing optical Doppler effects

Clarifying Evenson's CW Axiom using Doppler effects

➔ *Galileo's Revenge (part1):* Galilean Doppler-shift arithmetic using *rapidity* ρ

Developing optical "baseball-diamond" and relativistic ρ-functions and transformations

Details of 1CW wavefunctions and phasors

Details of 2CW wavefunctions in rest frame

Galileo's Revenge (part2): Galilean addition of phasor angular velocity

Structure of rest frame "baseball-diamonds"

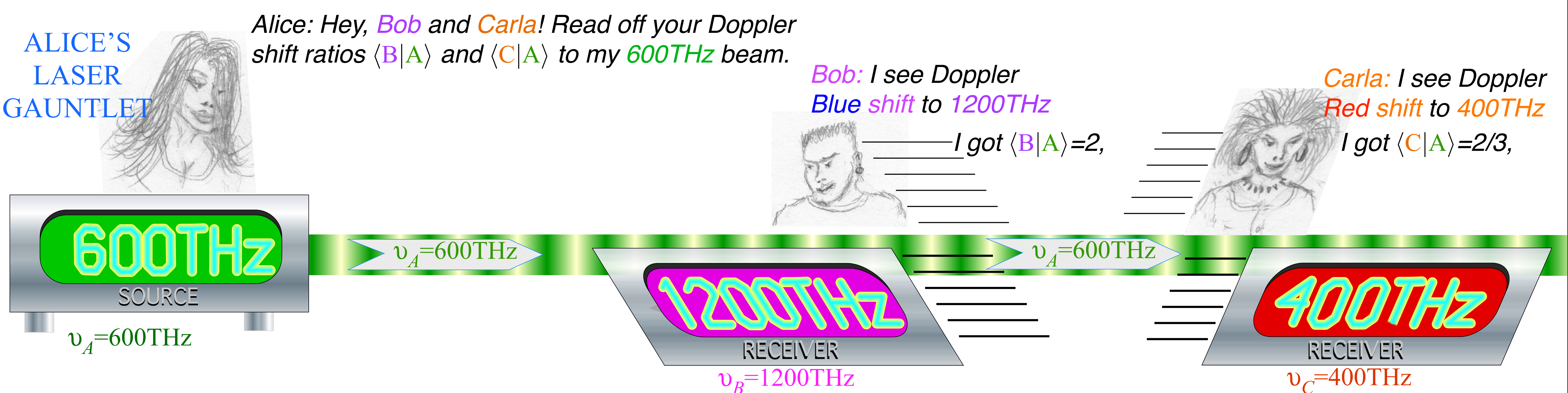
Details of 2CW wavefunctions of moving frame velocities of *phase* and *group* waves

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

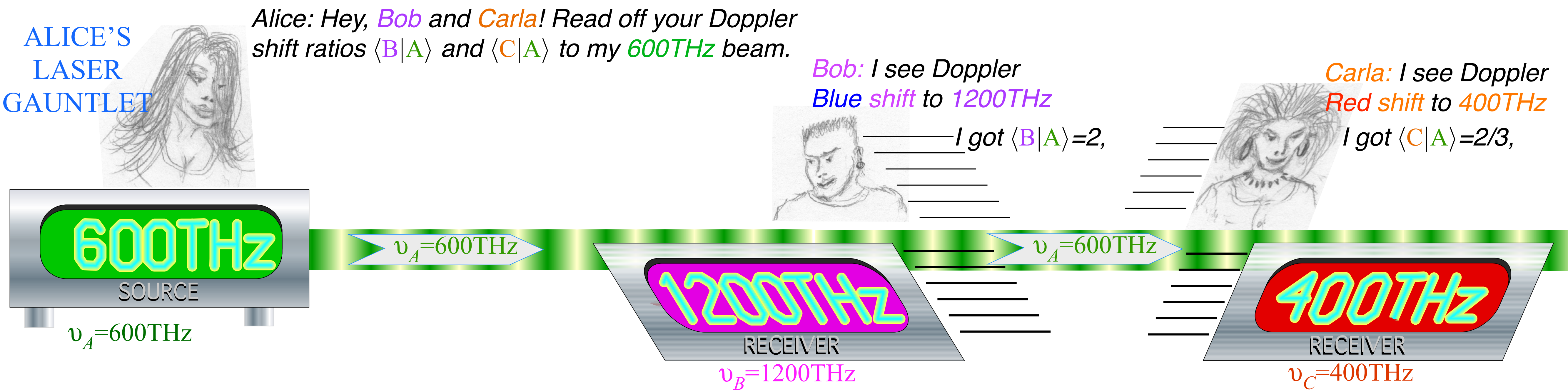
Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



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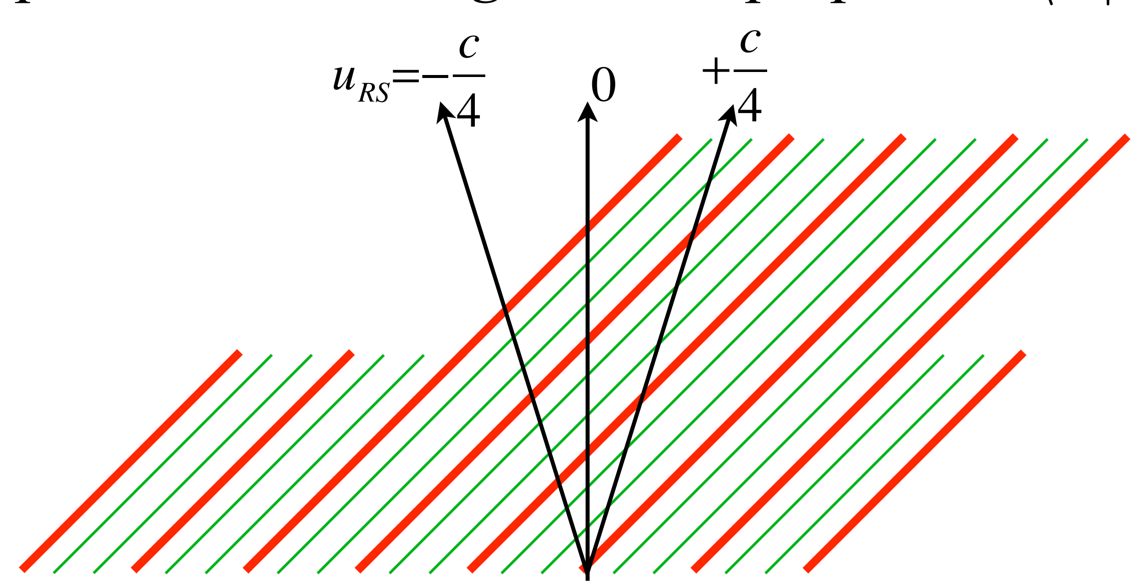
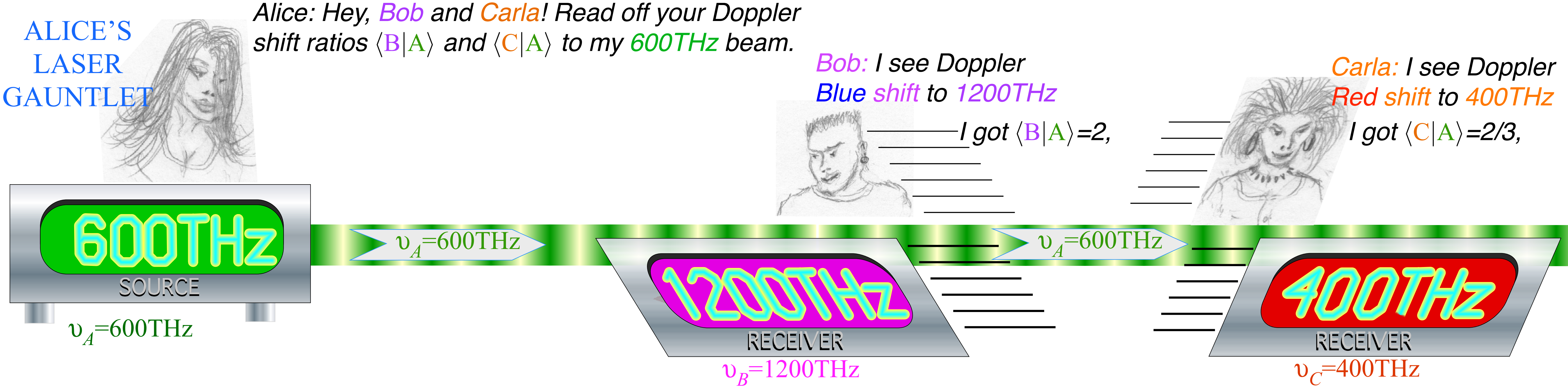
IMPORTANT POINT:

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion ⟨R|S⟩.



Easy Doppler-shift and Rapidity calculation



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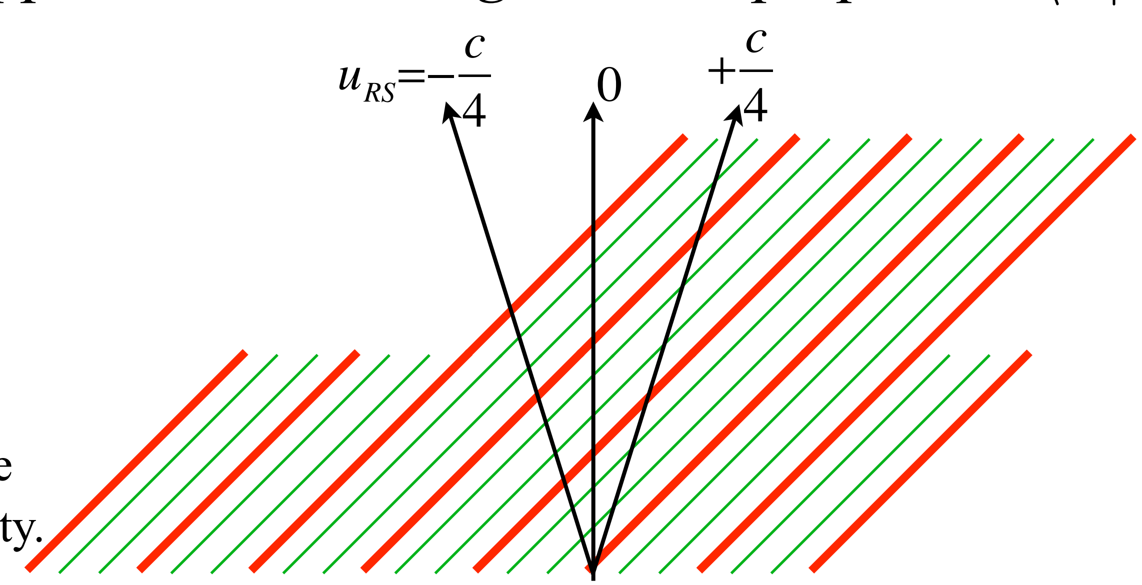

If Alice sends υ_A=600THz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=1200THz

If Alice sends υ_A=60 THz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=120THz

If Alice sends υ_A=6 Hz Bob sees: $\upsilon_B=\langle B|A\rangle\upsilon_A$=12 Hz

$\langle B|A\rangle$=2 for any frequency Alice and Bob use while they maintain their relative velocity.

Easy Doppler-shift and Rapidity calculation

ALICE'S LASER GAUNTLET

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A\rangle$ and $\langle C|A\rangle$ to my 600THz beam.

Also, rapidity ρ_{BA} *and* ρ_{CA} relative to me.

Bob: I see Doppler Blue shift to 1200THz

I got $\langle B|A\rangle$=2,

Carla: I see Doppler Red shift to 400THz

I got $\langle C|A\rangle$=2/3,

Doppler ratio:

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Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

Rapidity is most convenient!

1TeV proton has

u=0.999995598·c (Pain in the A)

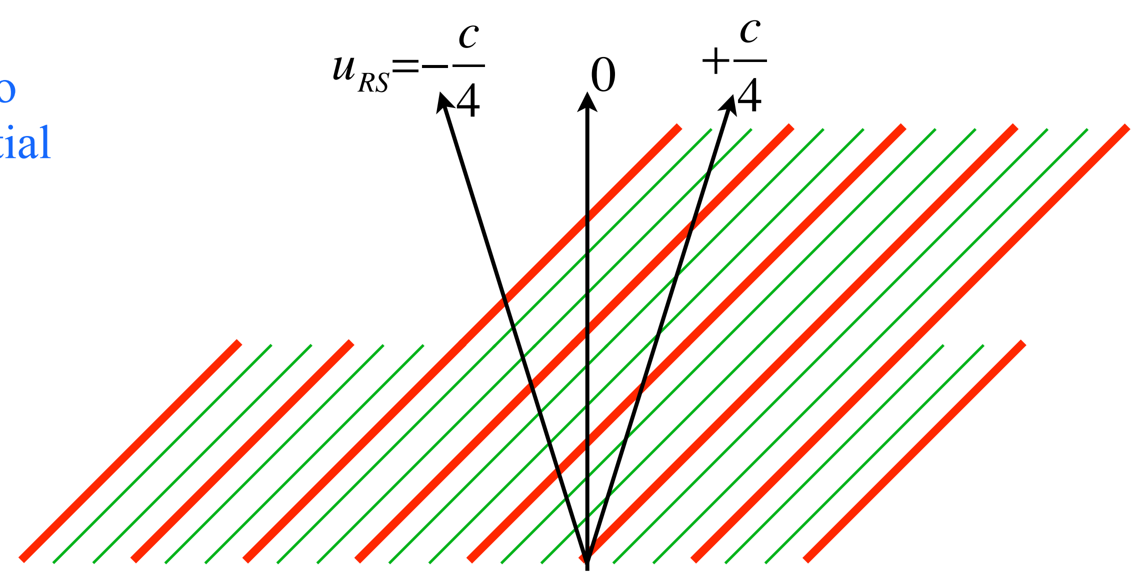
or: $\langle R|S\rangle$=2131.6 (Better)

or: ρ_{RS}=7.6646 (Best)

IMPORTANT POINTS:

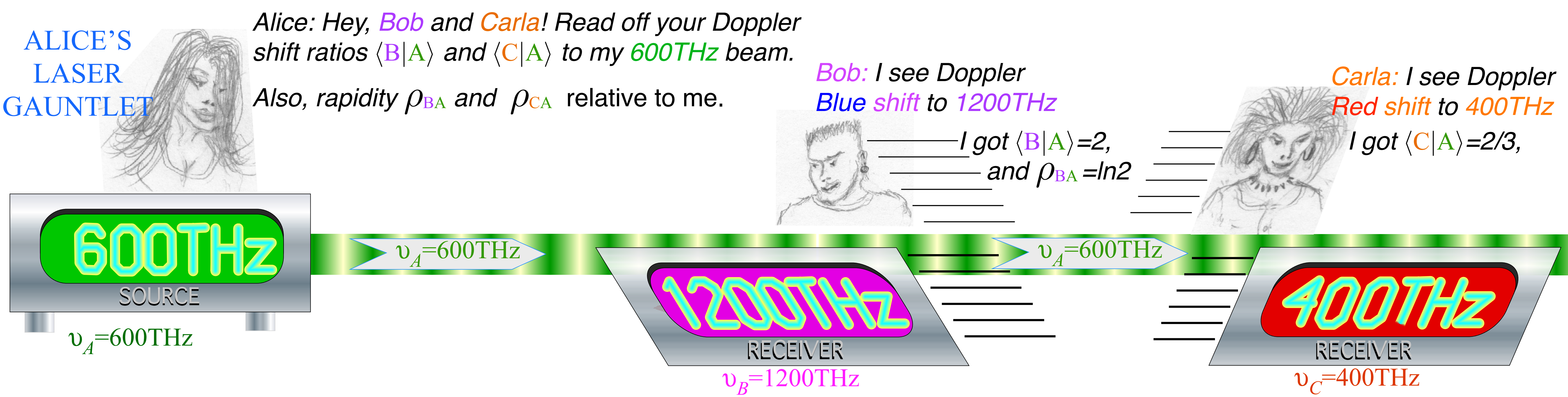
Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion $\langle R|S\rangle$.

Geometric phenomena tend to involve logarithmic/exponential functionality!



For low velocity u<<c rapidity ρ_{RS} approaches u/c

Easy Doppler-shift and Rapidity calculation



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Definition of Rapidity

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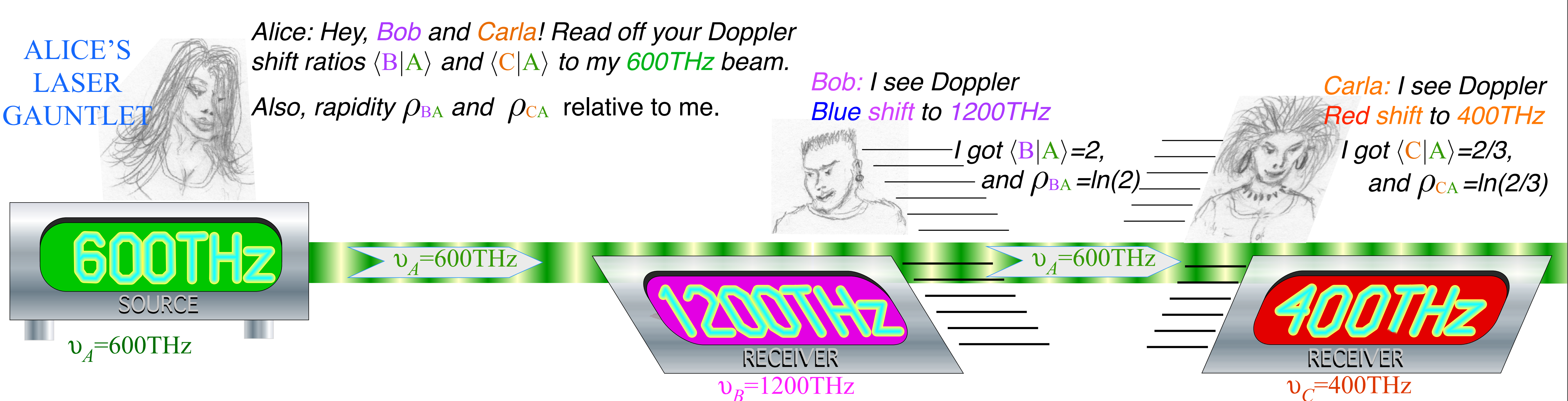
Easy Doppler-shift and Rapidity calculation



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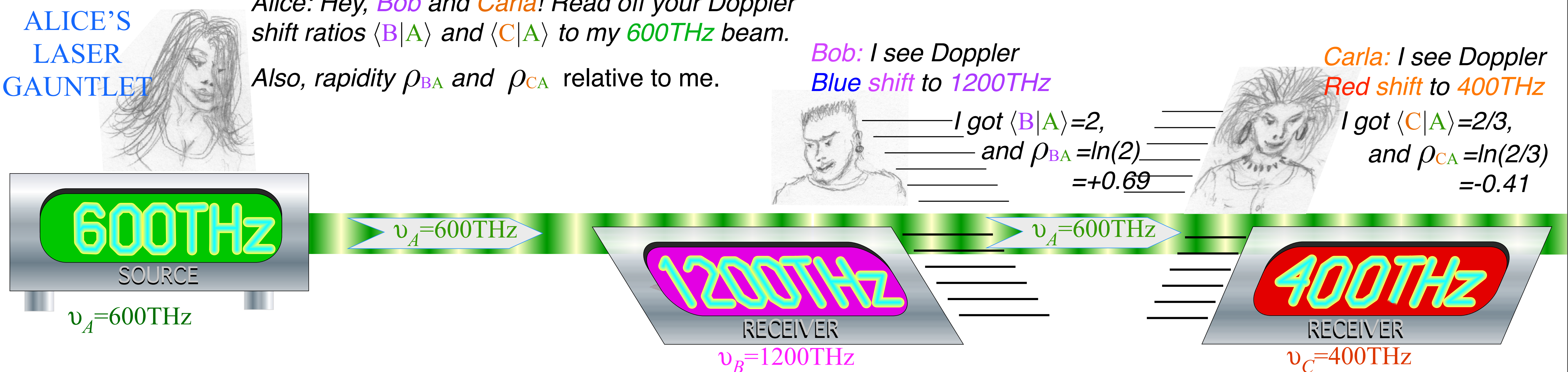
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



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Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{2}{1}$$

is time-reversal of:

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B} = \frac{1}{2}$$

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \quad \text{(time-reversed)} \quad (so{:}\ \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

Mnemonic: You can think of rapidity ρ_{BA} as "R" for "Romance"… (+) positive on approach, (-) negative on reproach

Do the stars hate us?

Easy Doppler-shift and Rapidity calculation

ALICE'S LASER GAUNTLET

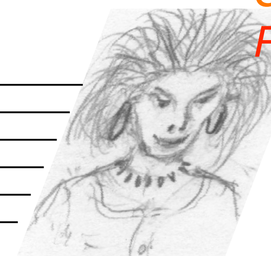
Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A\rangle$ and $\langle C|A\rangle$ to my 600THz beam.

Also, rapidity ρ_{BA} and ρ_{CA} relative to me.

Now, Carla, what's your rapidity ρ_{CB} relative to Bob?

Bob: I see Doppler Blue shift to 1200THz

I got $\langle B|A\rangle$=2, and ρ_{BA} =ln(2) =+0.69



Carla: I see Doppler Red shift to 400THz

I got $\langle C|A\rangle$=2/3, and ρ_{CA} =ln(2/3) =-0.41

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{2}{1}$$

is time-reversal of:

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Bob-Alice Doppler ratio:

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Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

(time-reversed)

$$\rho_{BA} = 0.69 \qquad (so: \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

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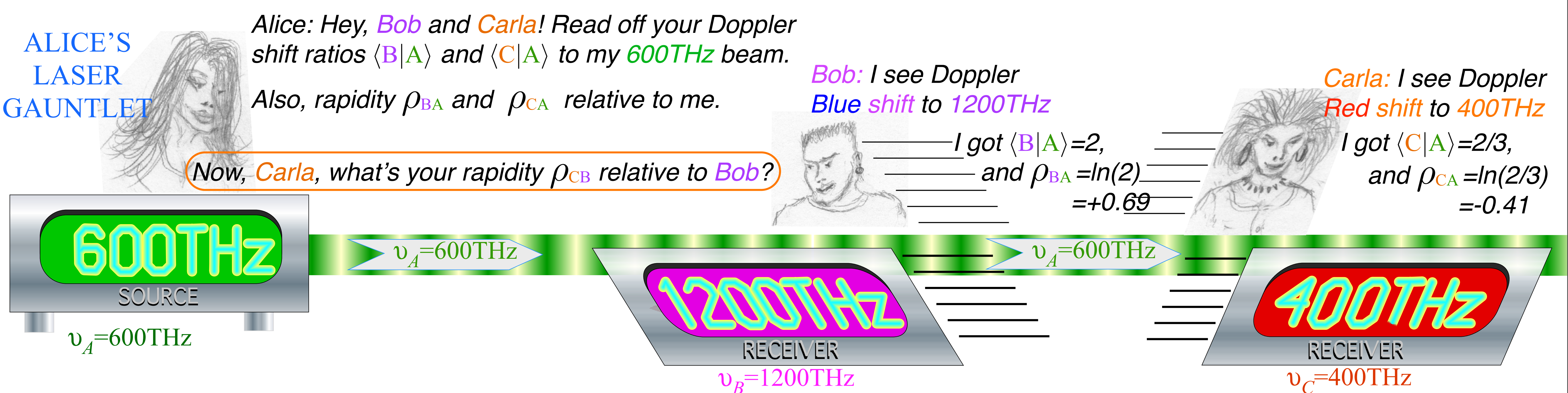
Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Mnemonic: You can think of rapidity ρ_{BA} as *"R"* for *"Romance"*… (+) positive on approach, (-) negative on reproach

More at Pirelli Challenge page: Time Reversal Symmetry

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

so:

$$\langle R|S\rangle = e^{\rho_{RS}}$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Bob-Alice Doppler ratio:

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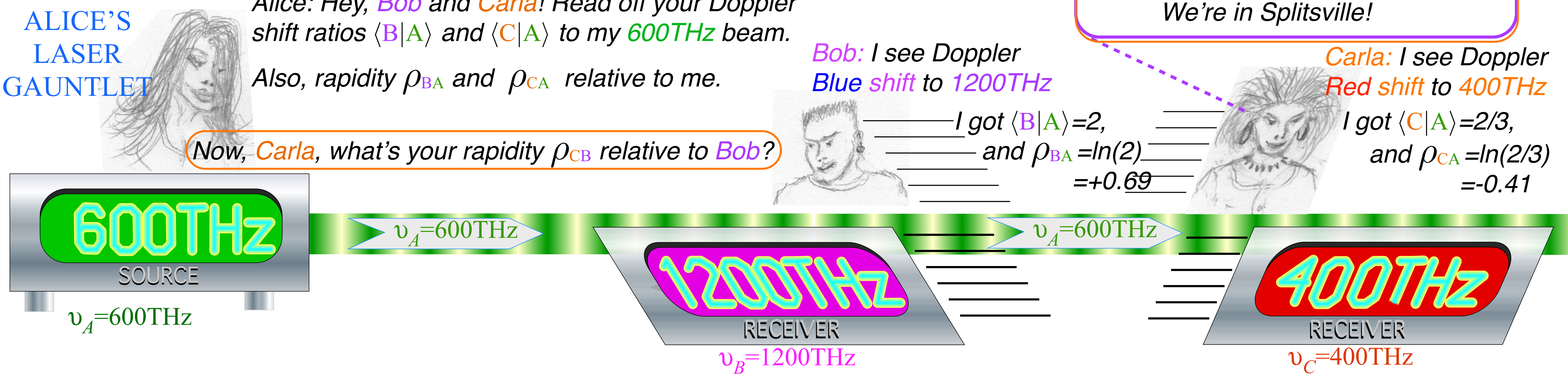
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Bob-Alice Doppler ratio:

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Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

implies: $\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10$

Easy Doppler-shift and Rapidity calculation

ALICE'S LASER GAUNTLET

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A\rangle$ and $\langle C|A\rangle$ to my 600THz beam.

Also, rapidity ρ_{BA} and ρ_{CA} relative to me.

Now, Carla, what's your rapidity ρ_{CB} relative to Bob?

Bob: I see Doppler Blue shift to 1200THz

I got $\langle B|A\rangle$=2, and ρ_{BA} =ln(2) =+0.69

I got $\langle C|B\rangle=\langle C|A\rangle\langle A|B\rangle$=(2/3)(1/2)=1/3, and ρ_{CB} = $\rho_{CA}+\rho_{AB}$ =-1.10 We're in Splitsville!

Carla: I see Doppler Red shift to 400THz

I got $\langle C|A\rangle$=2/3, and ρ_{CA} =ln(2/3) =-0.41

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}}$$

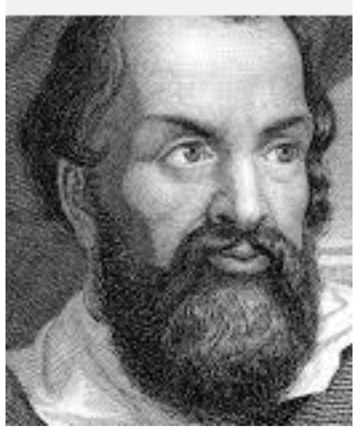
Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Happy now,Galileo?



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Carla-Bob Doppler ratio:

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Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies:}$$

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CB} = \rho_{CA} + \rho_{AB}$$
$$= -0.41 - 0.69 = -1.10$$

Lecture 30
Thur. 12.10.2015

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
- Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Doppler shifted "baseball-diamond" displays Lorentz frame transformation

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1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ *angular units*

"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

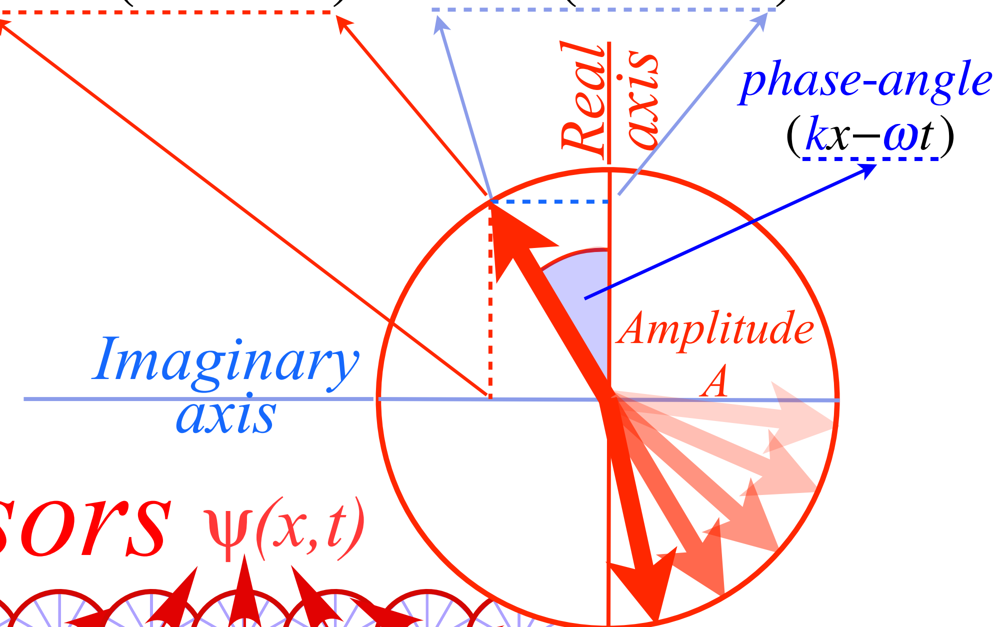
angular wave number : $k = 2\pi\kappa$

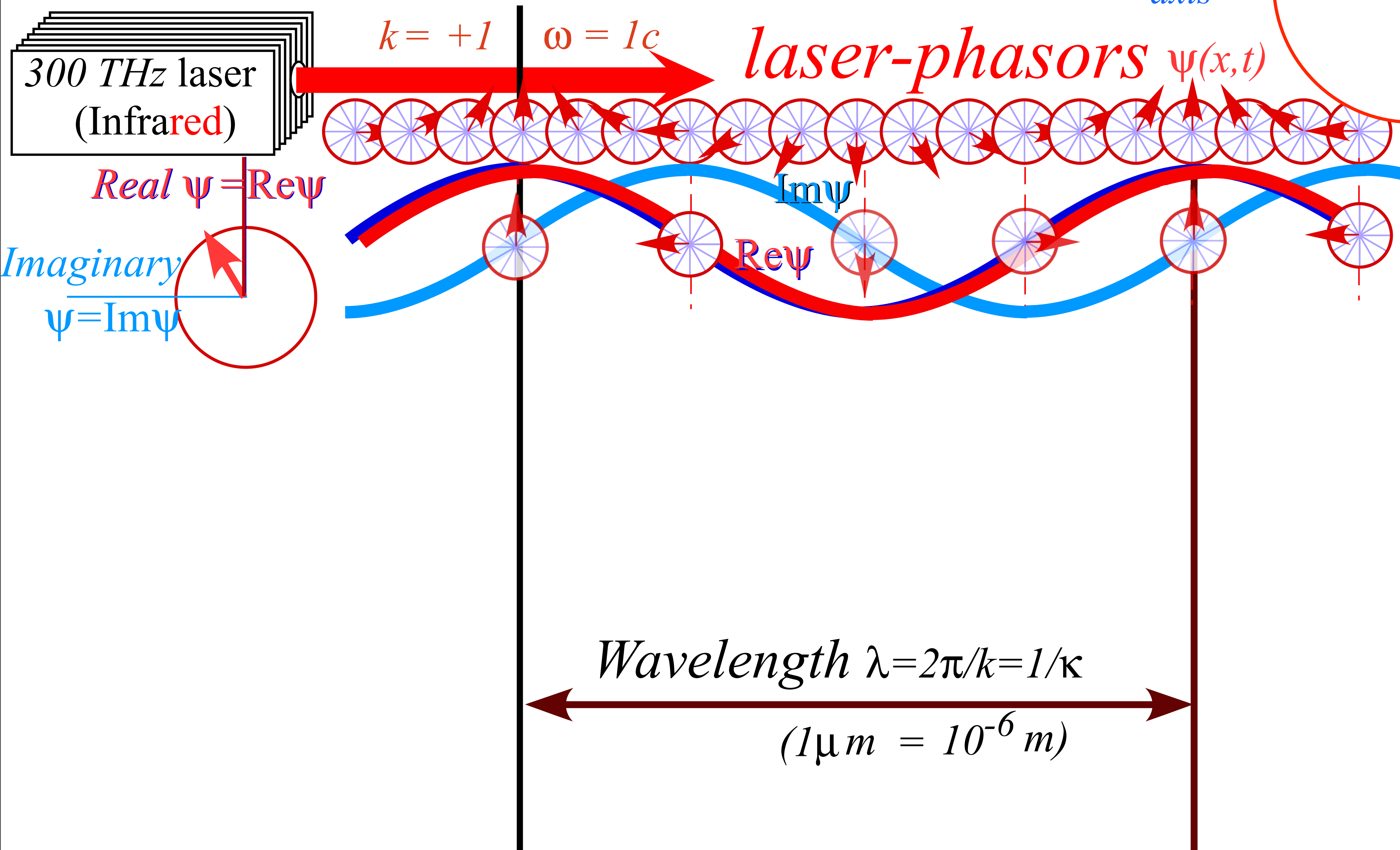
k =wave**vector**

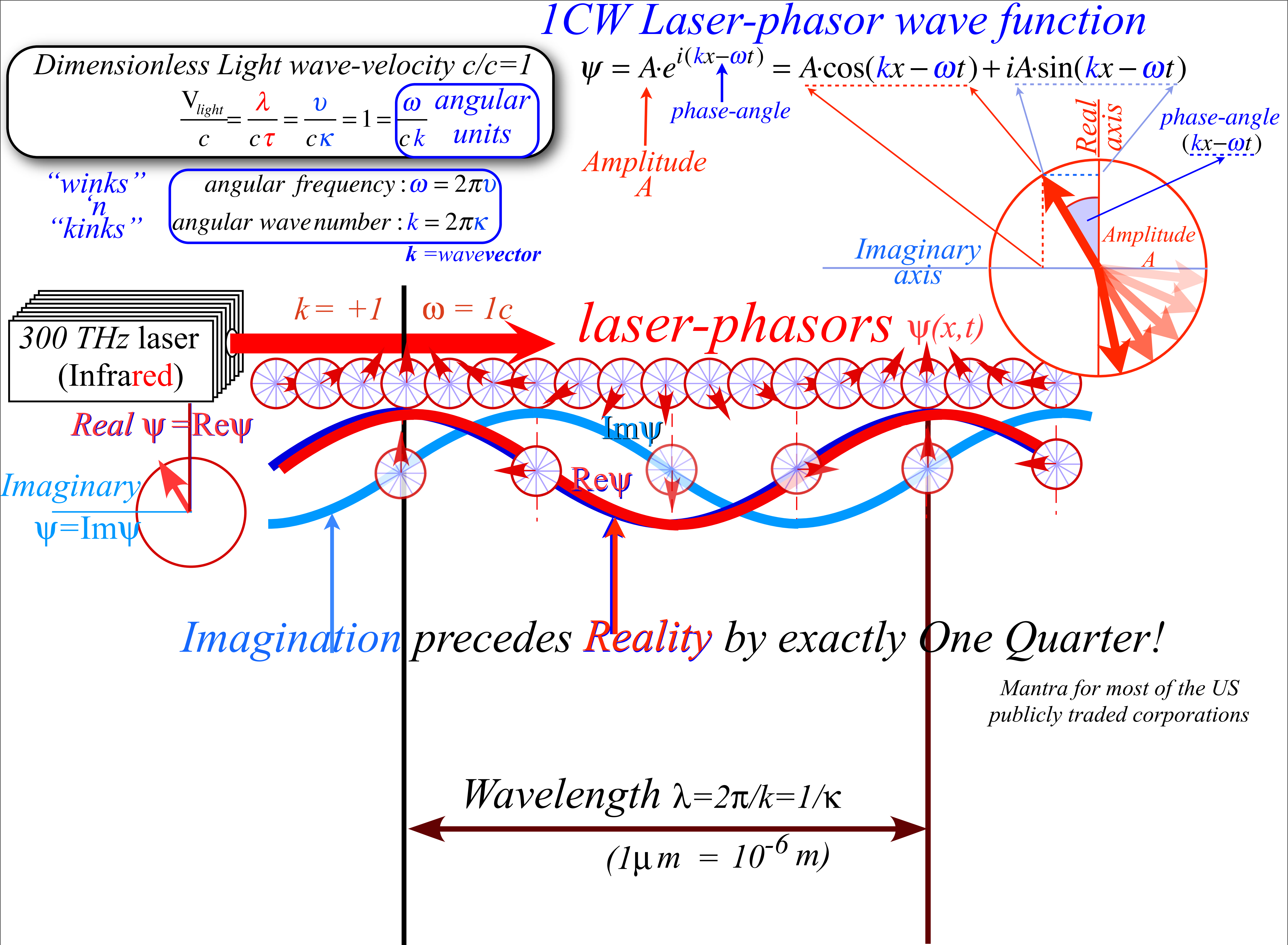
$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

phase-angle

Amplitude A







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k =wavevector
Real axis
phase-angle (kx−ωt)
Amplitude A
Imaginary axis
300 THz laser (Infrared)
k= +1
ω = 1c
laser-phasors ψ(x,t)
Real ψ =Reψ
Imψ
Reψ
Imaginary ψ=Imψ
Imagination precedes Reality by exactly One Quarter!
Mantra for most of the US publicly traded corporations
Wavelength λ=2π/k=1/κ
(1μm = 10^-6 m)

1CW Laser-phasor wave function

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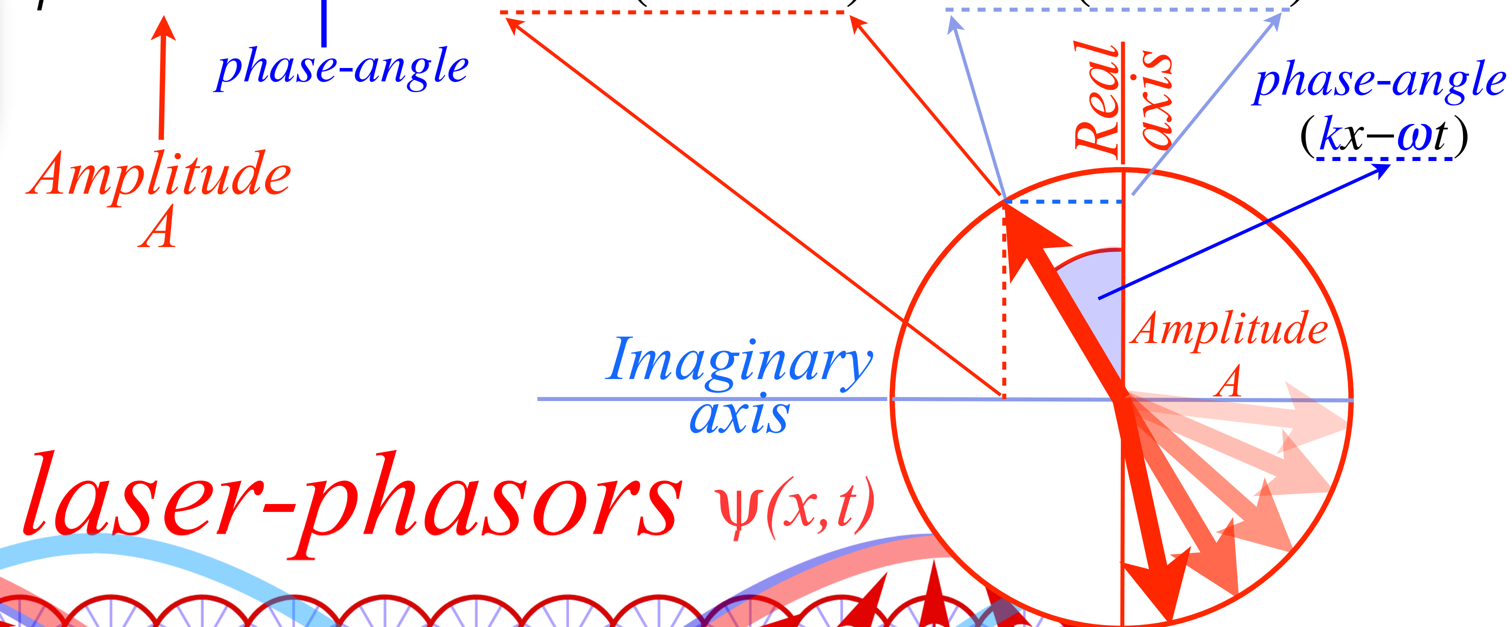
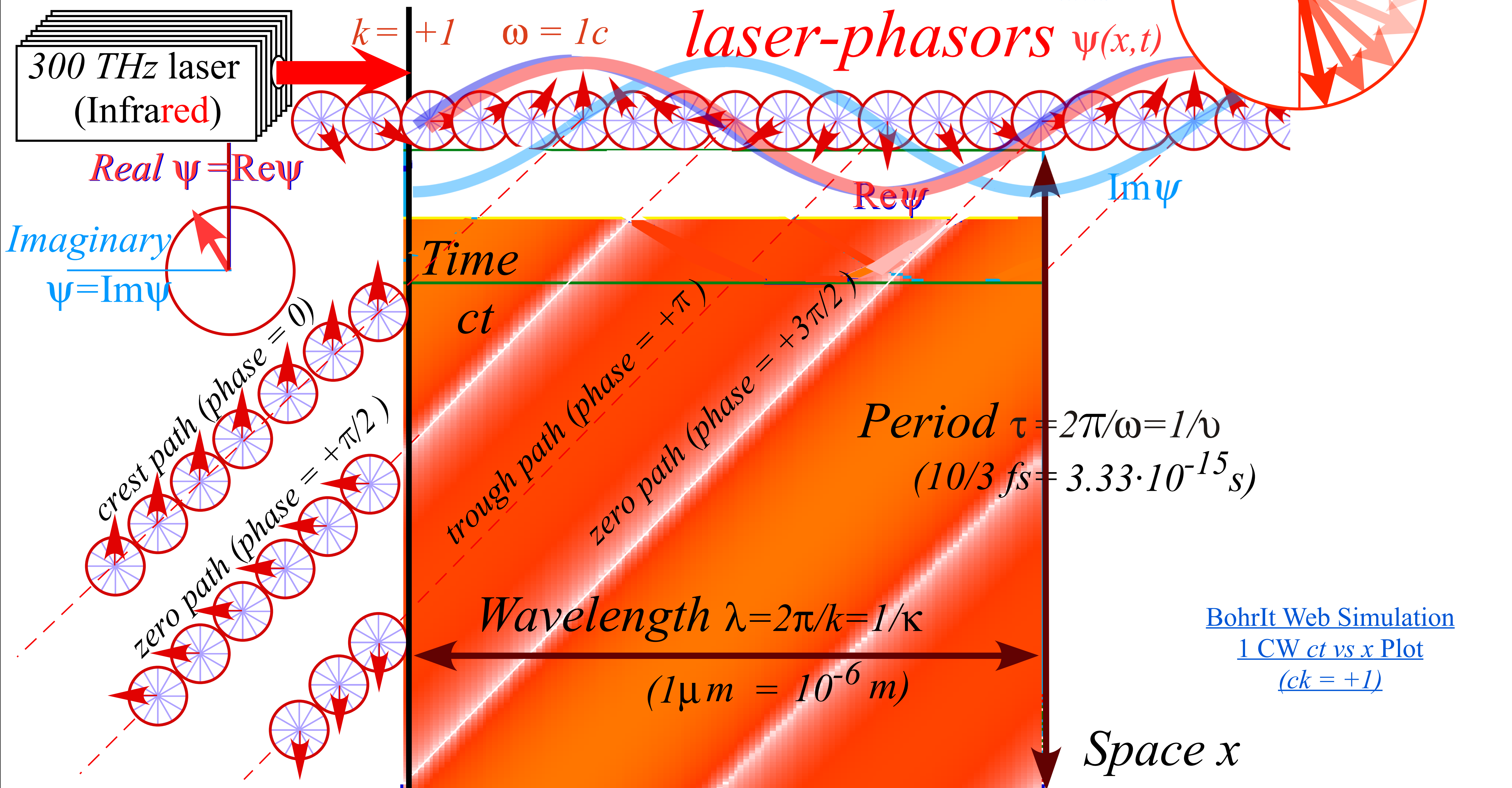
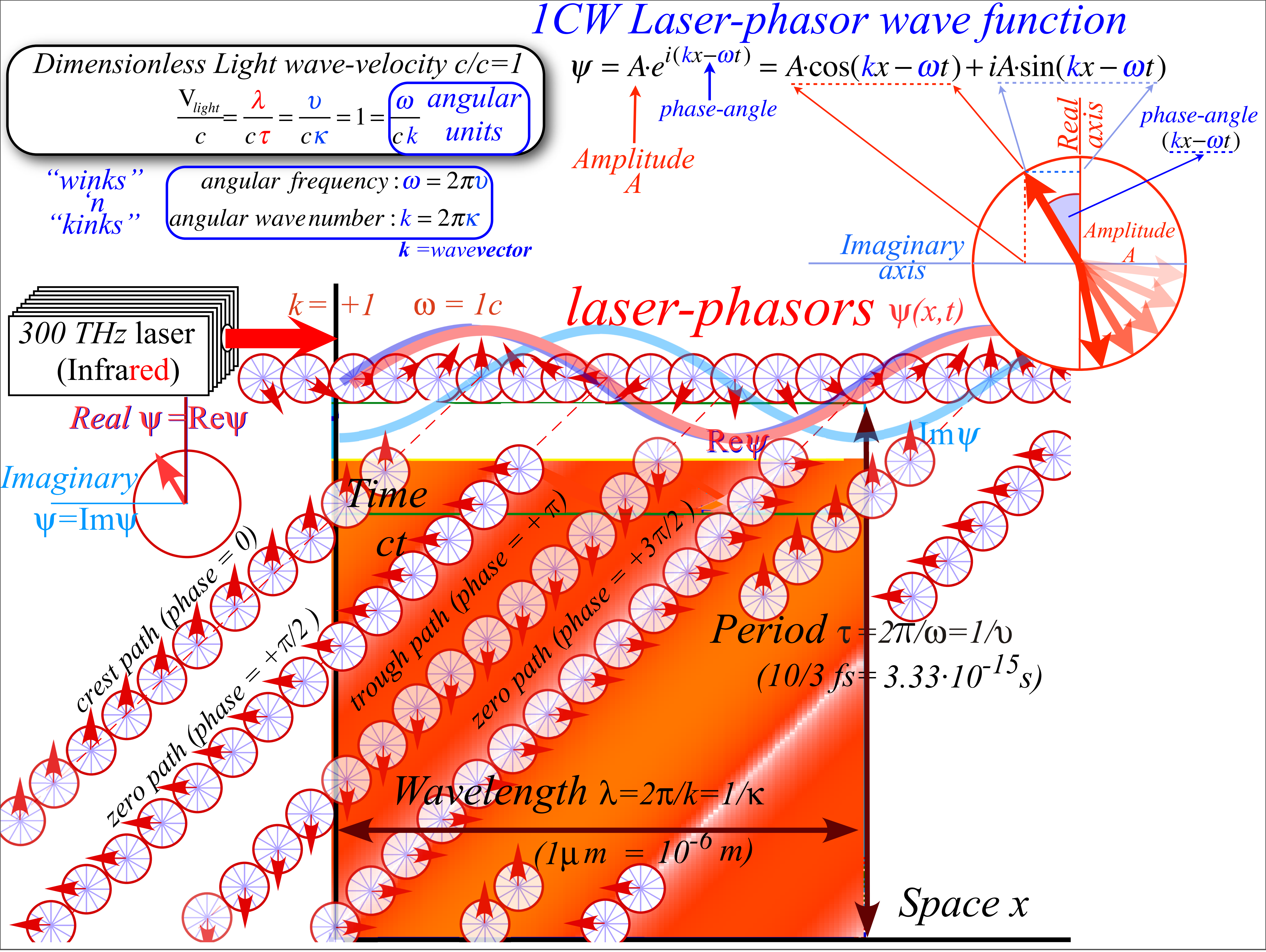


Q: Where is phase=(kx-ωt)=0?

A: It is wherever this is: $\frac{x}{t} = \frac{\omega}{k}$



BohrIt Web Simulation
1 CW *ct vs x* Plot
(ck = +1)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Real ψ =Reψ
Imaginary ψ=Imψ
Reψ
Imψ
Time ct
crest path (phase = 0)
zero path (phase = +π/2)
trough path (phase = +π)
zero path (phase = +3π/2)
Period τ =2π/ω=1/υ
(10/3 fs= 3.33·10^-15 s)
Wavelength λ=2π/k=1/κ
(1μ m = 10^-6 m)
Space x

1CW Laser-phasor wave function

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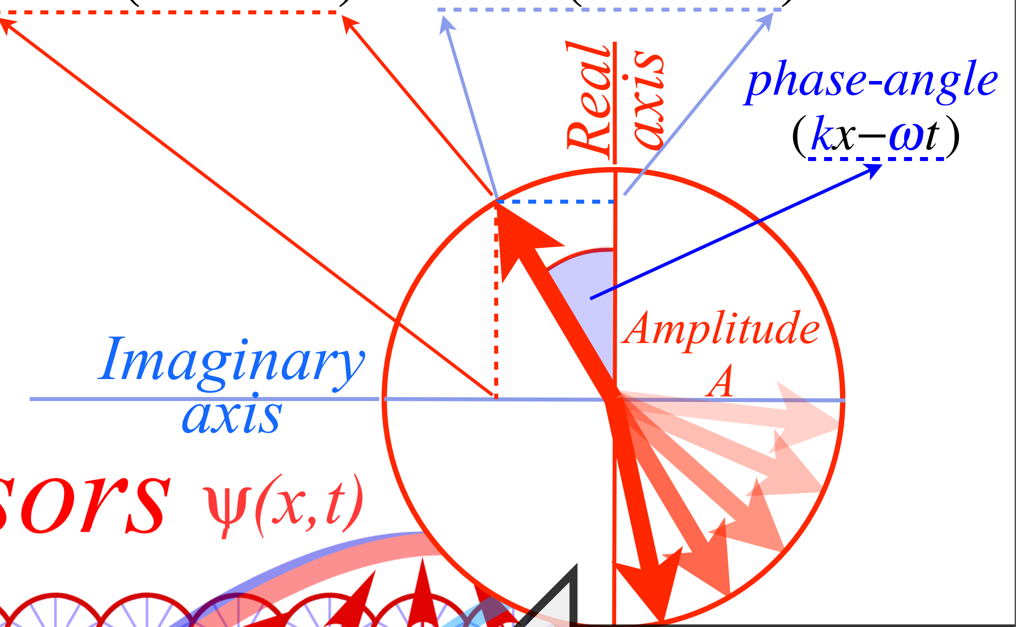
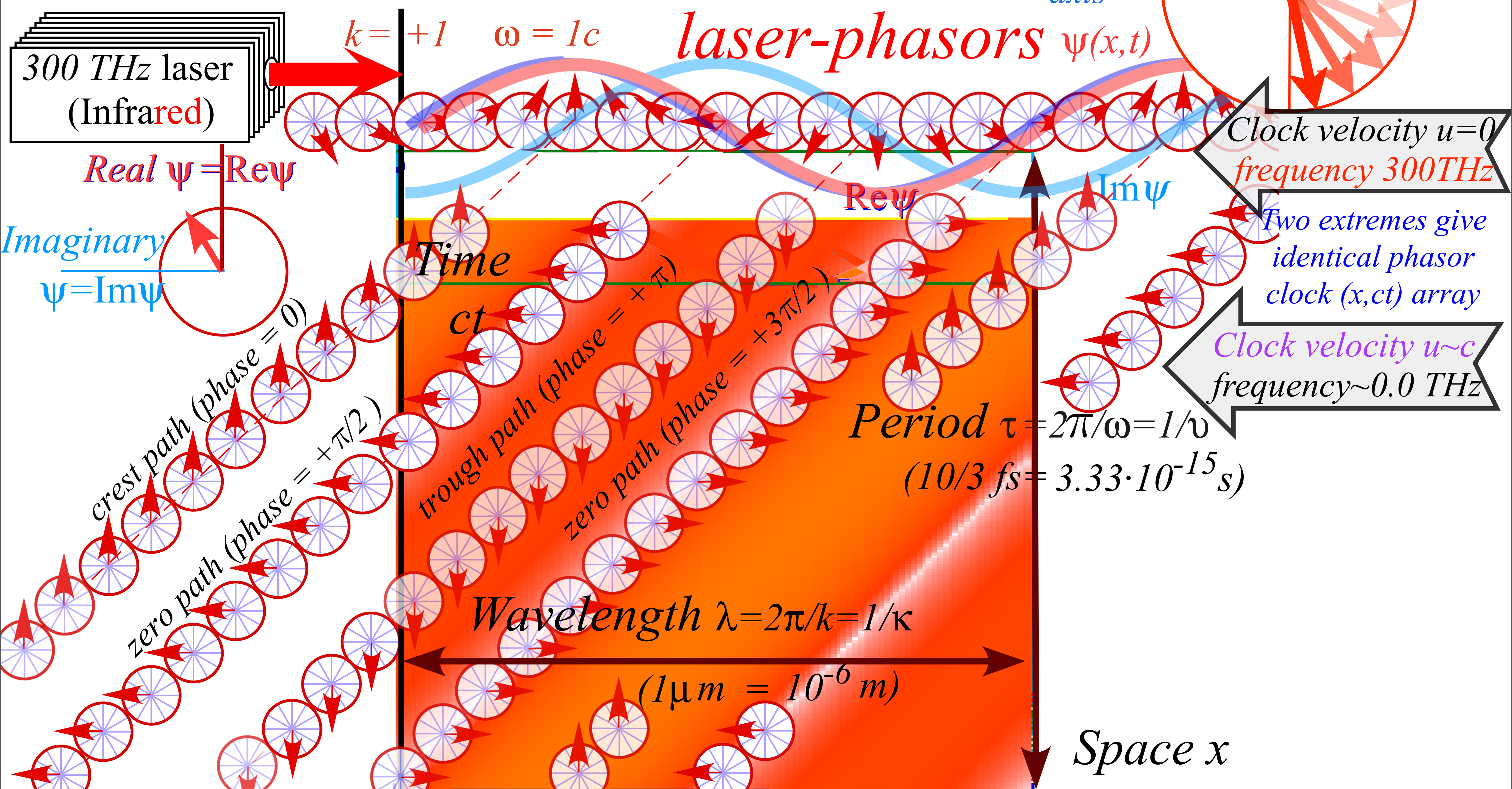
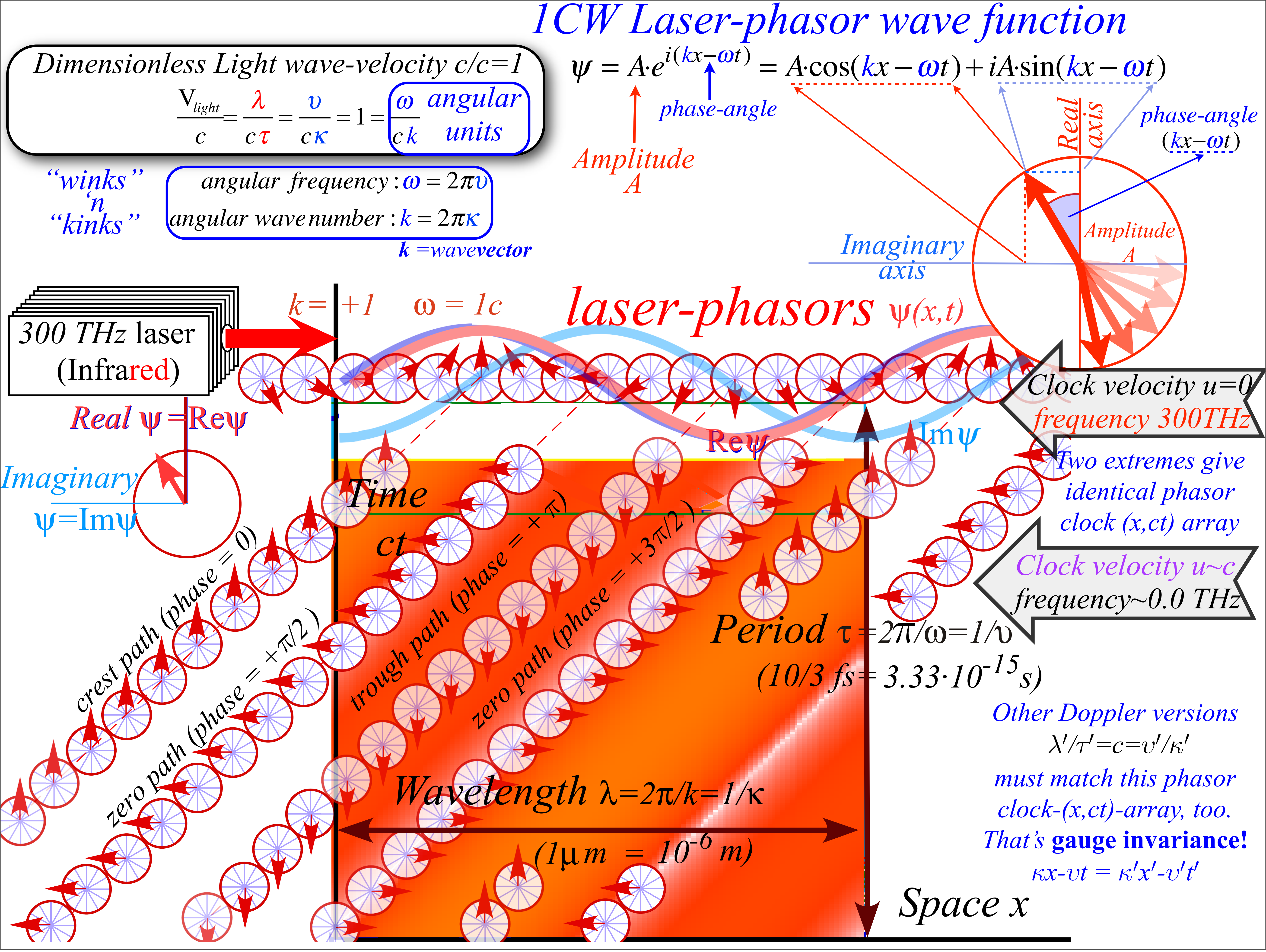
phase-angle



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Period τ =2π/ω=1/υ
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Wavelength λ=2π/k=1/κ
(1μm = 10^-6 m)
Space x
Other Doppler versions
λ′/τ′=c=υ′/κ′
must match this phasor clock-(x,ct)-array, too.
That's gauge invariance!
κx-υt = κ′x′-υ′t′

Lecture 30
Thur. 12.10.2015

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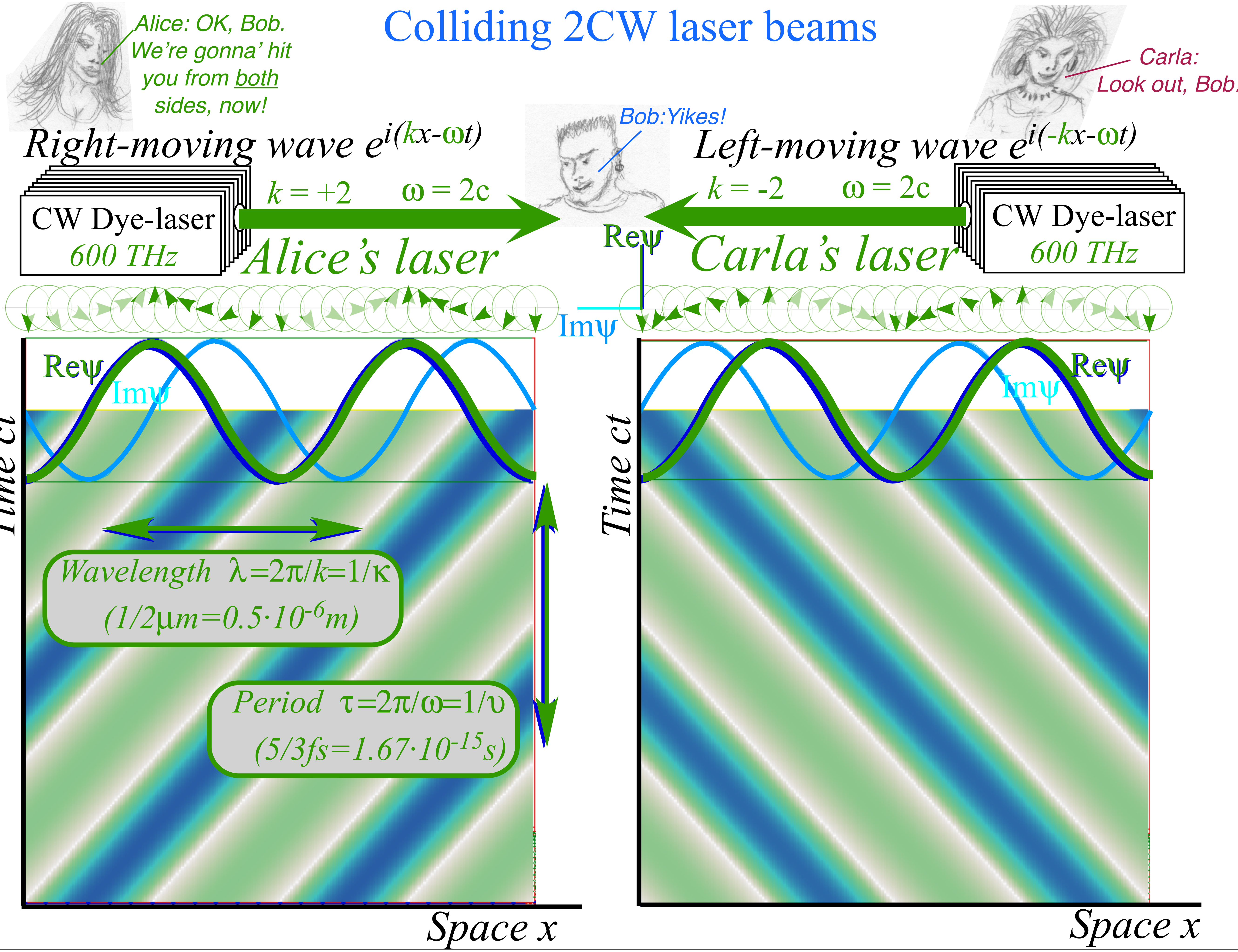
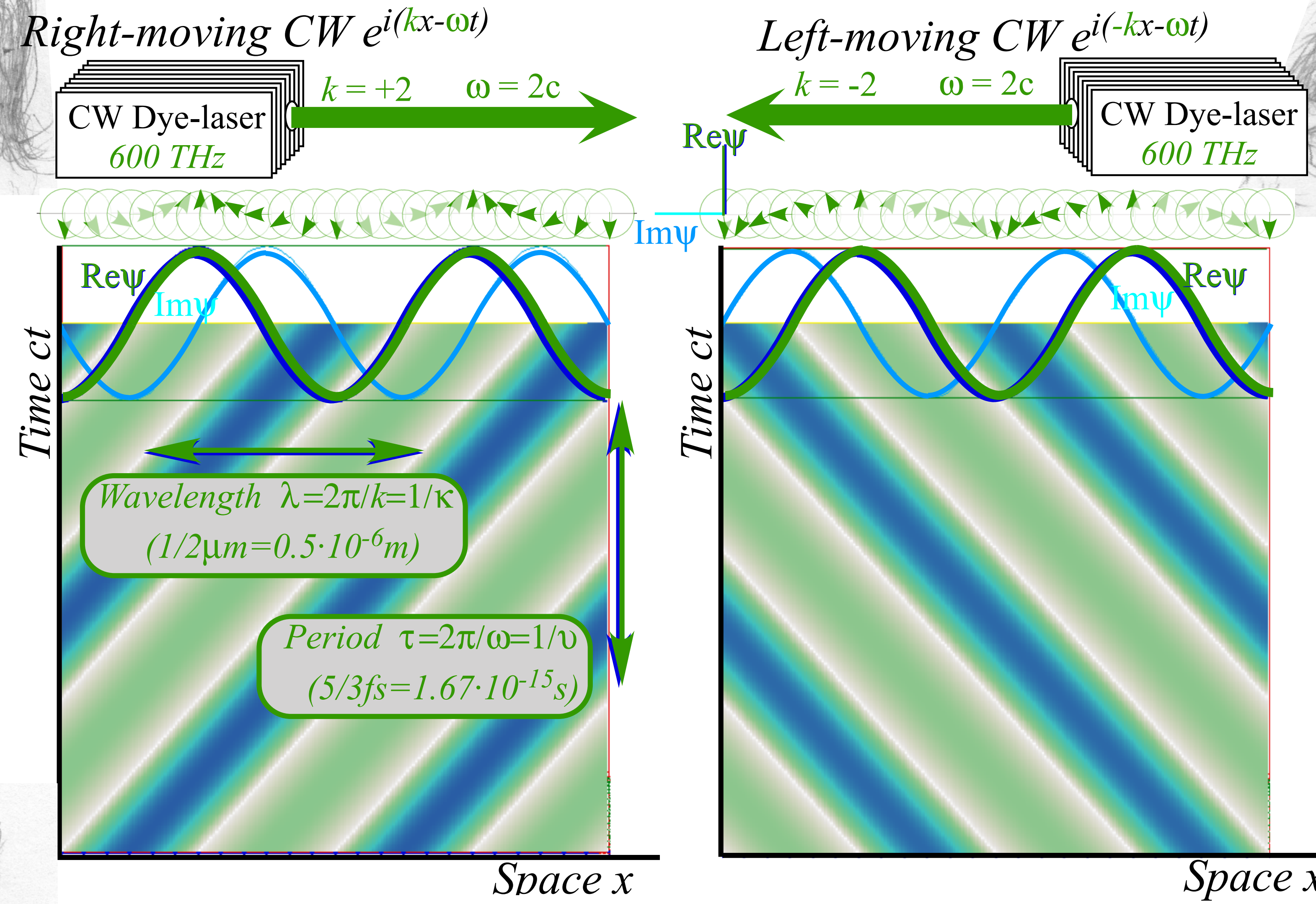
Colliding 2CW laser beams





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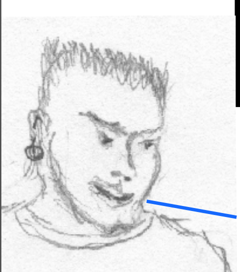

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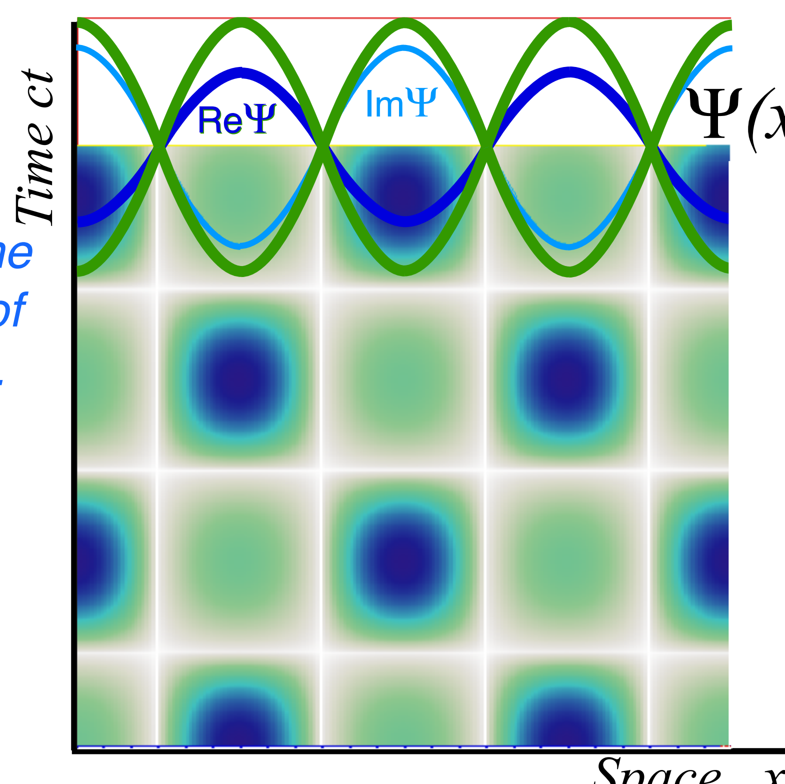
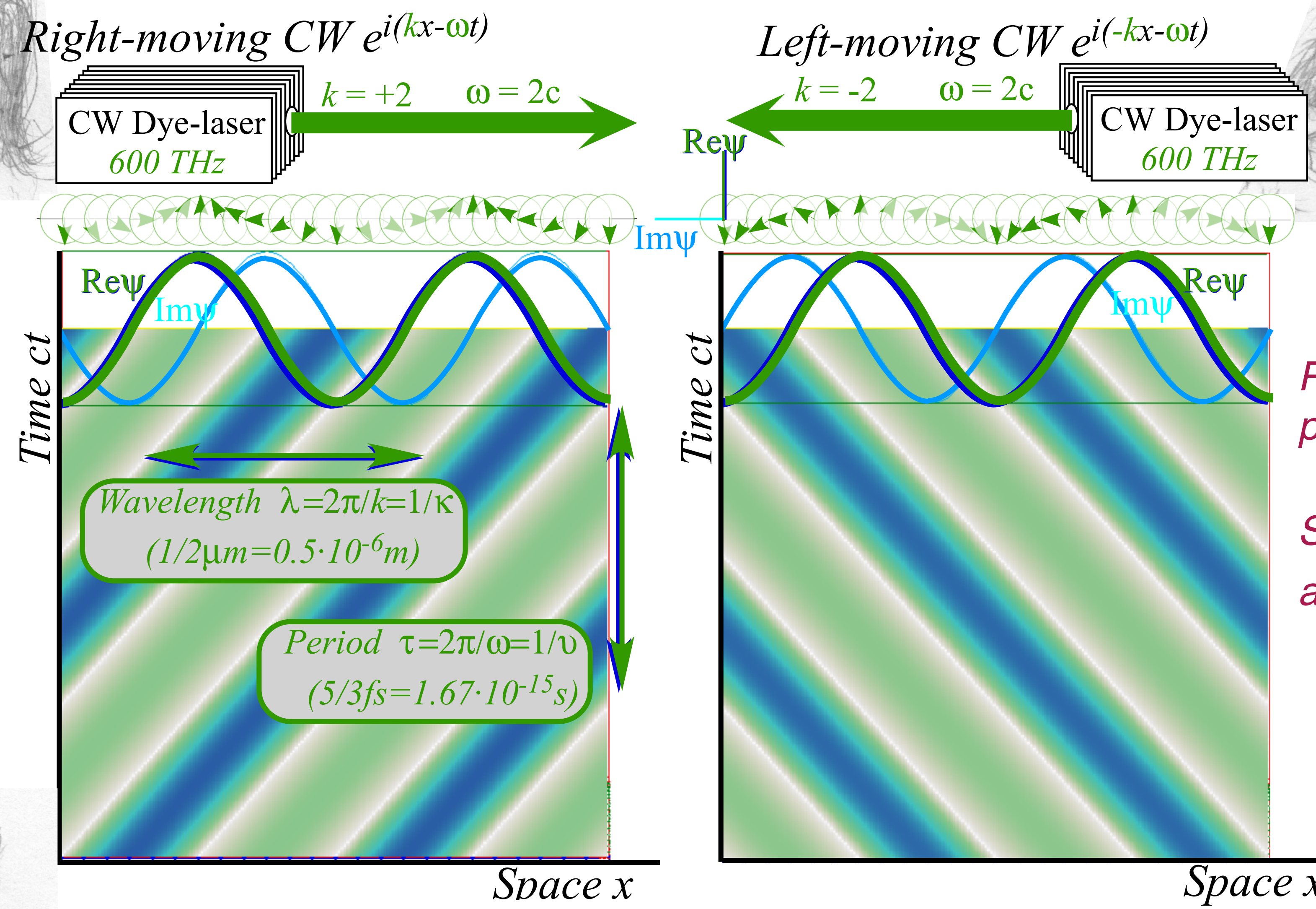
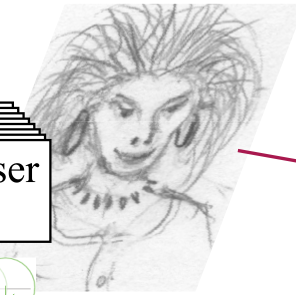
How'd it **do** that?



$$\Psi(x,t)=e^{ia}+e^{ib}, \quad a = kx-\omega t, \quad b = -kx-\omega t$$

[BohrIt Web Simulation 2 CW *ct vs x* Plot *(ck = ±2)*]





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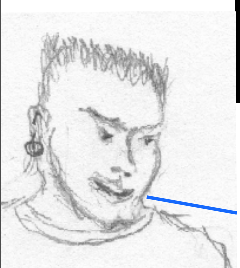

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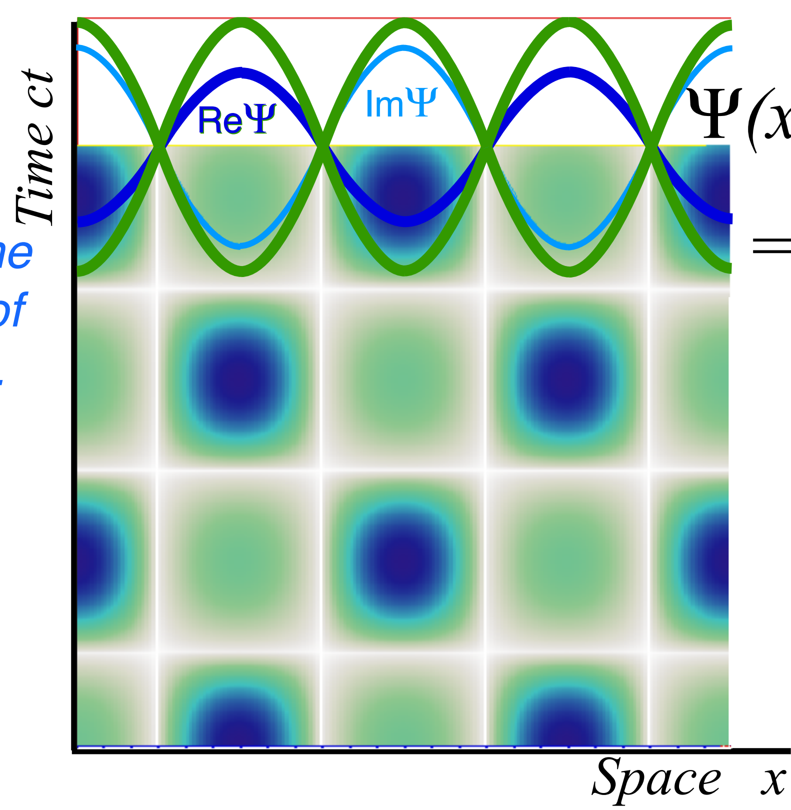

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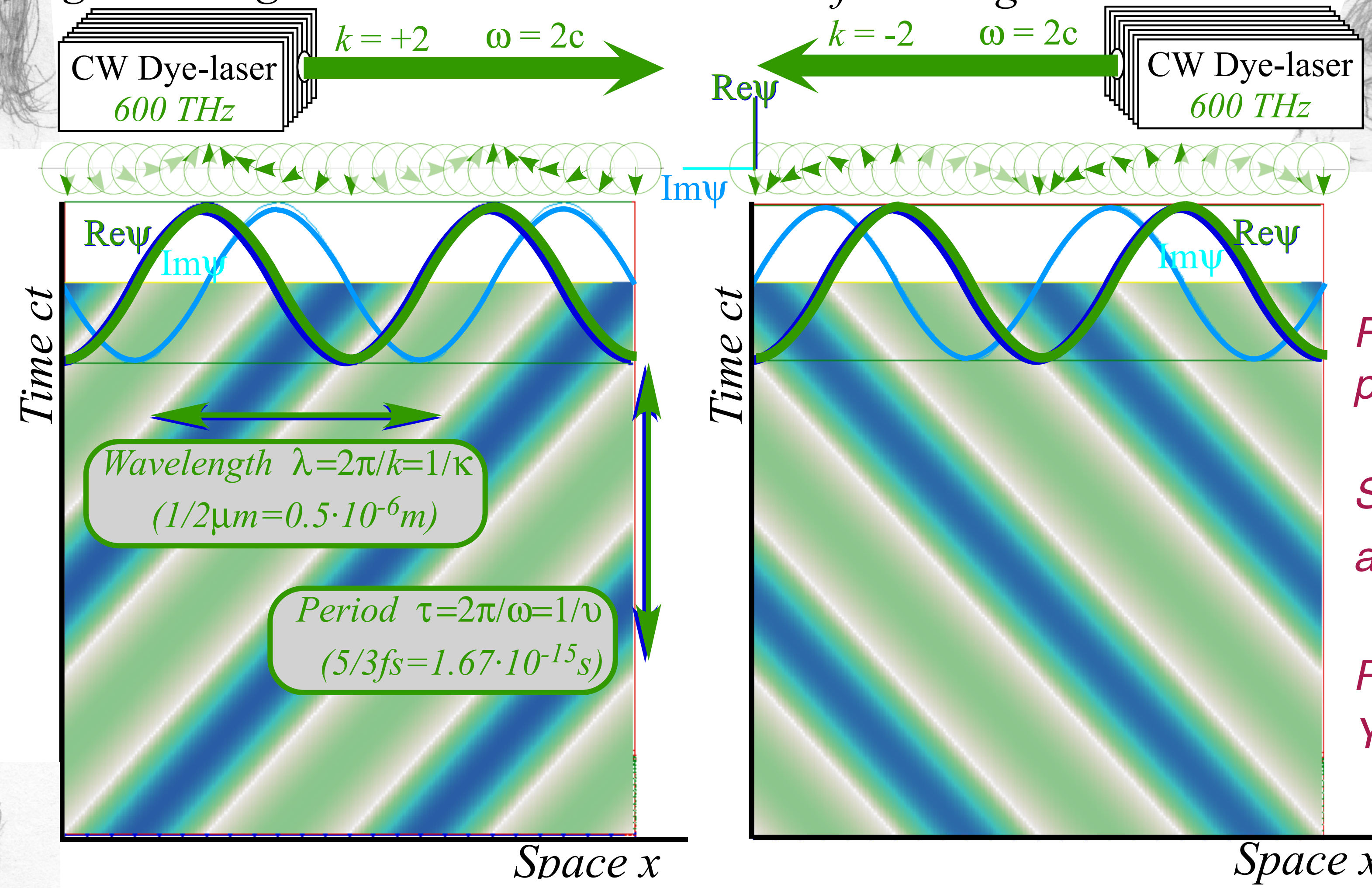
More at Pirelli Challenge page: *'Un Grande Affare' - Light Meets Light*

Right-moving CW $e^{i(kx-\omega t)}$ Left-moving CW $e^{i(-kx-\omega t)}$



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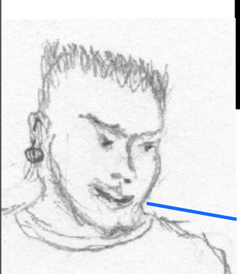

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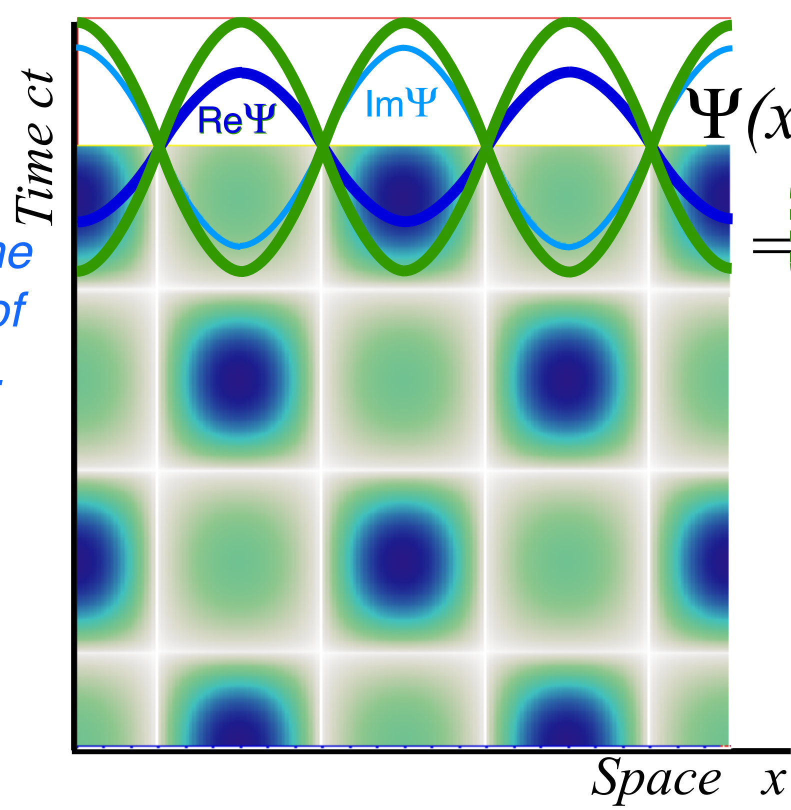

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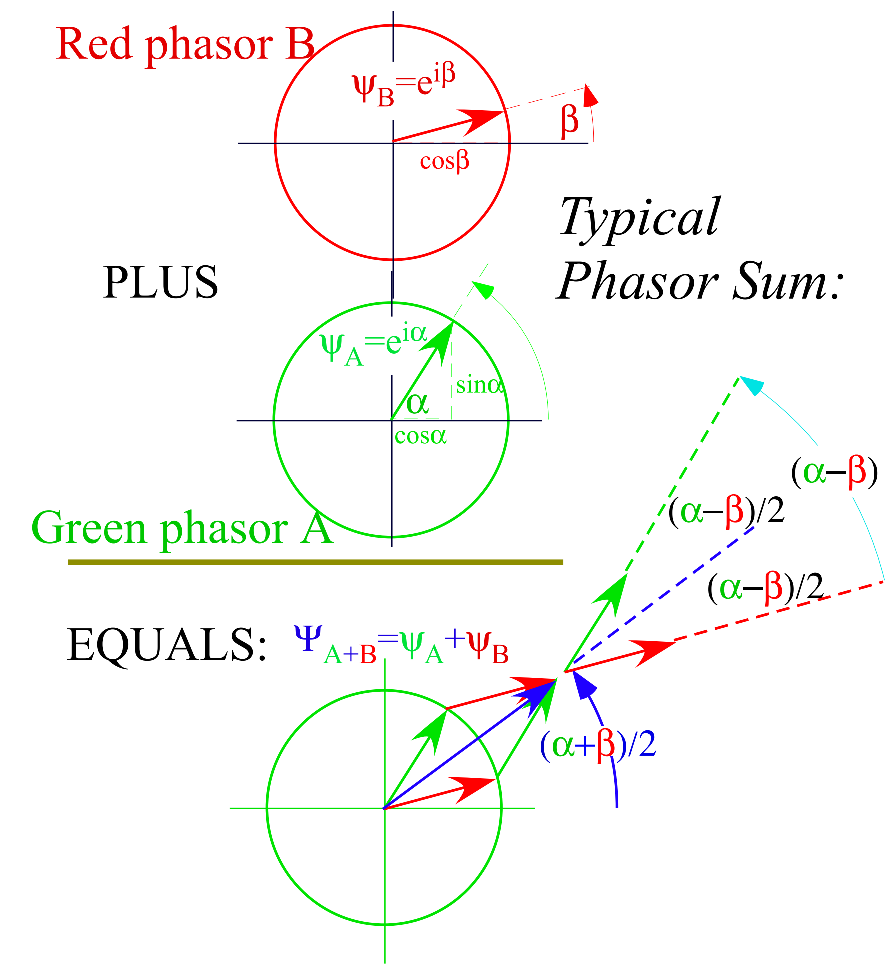
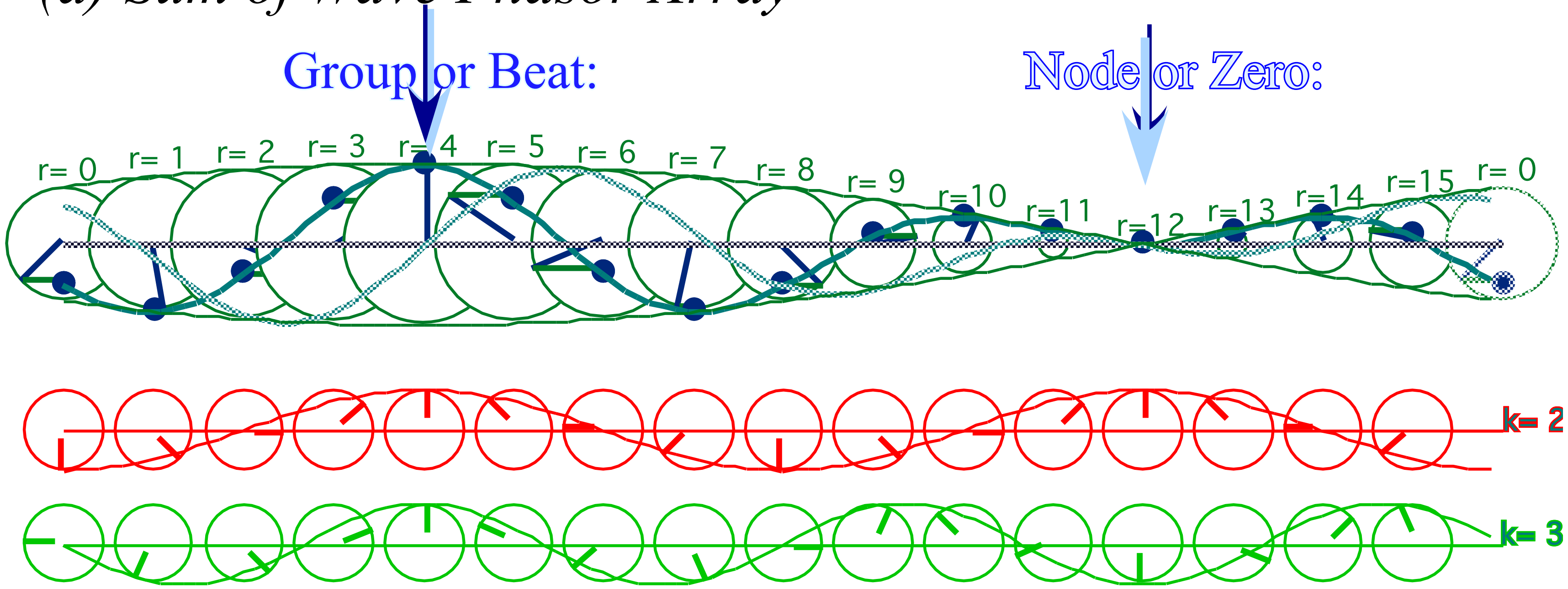
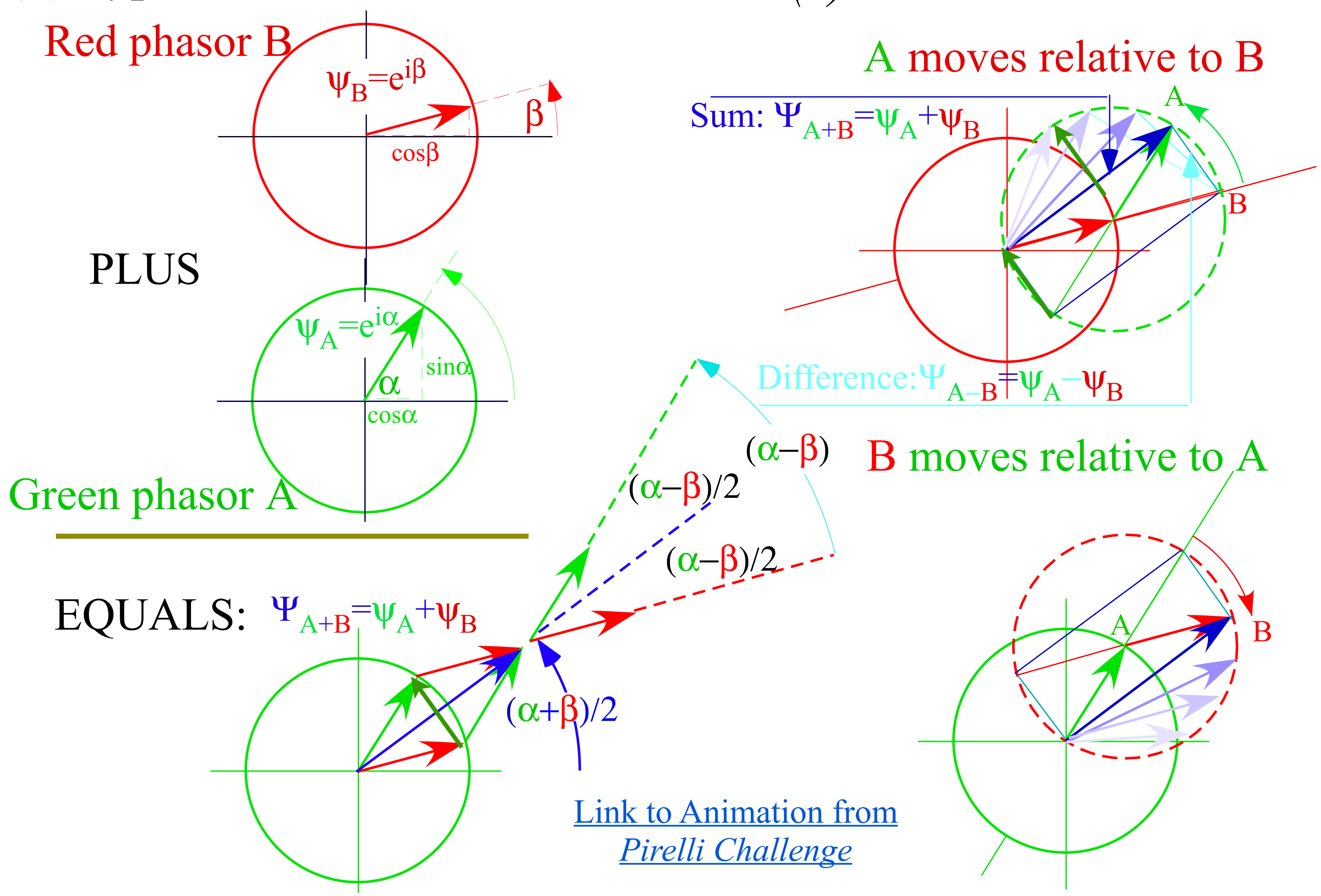


(a) Sum of Wave Phasor Array



(b) Typical Phasor Sum:

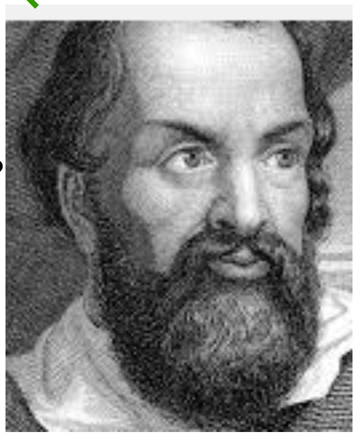
(c) Phasor-relative views



[Link to Animation from *Pirelli Challenge*]

Geometry of the
Half-sum
Phase
and
Half-difference
Group

Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Lecture 30
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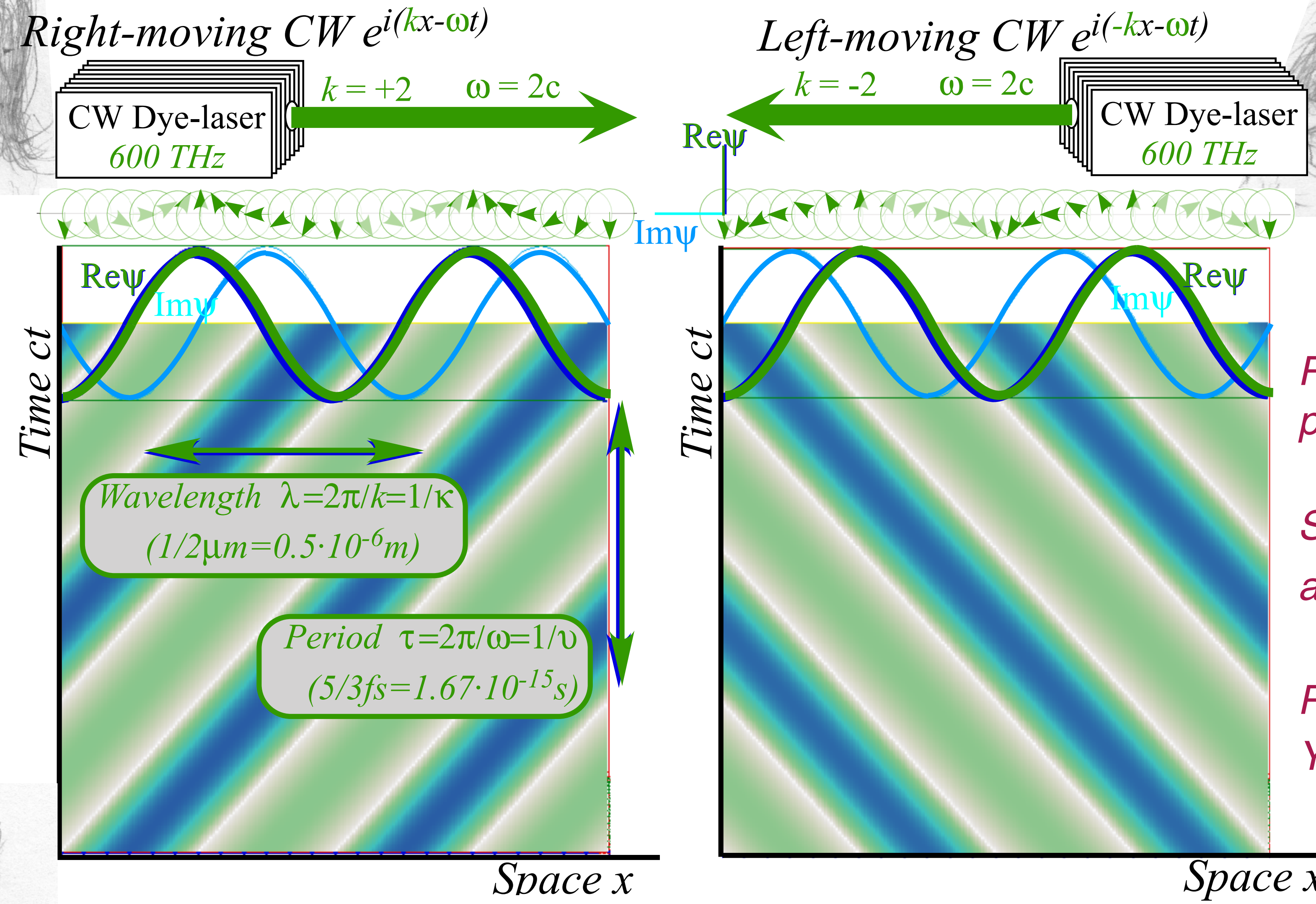

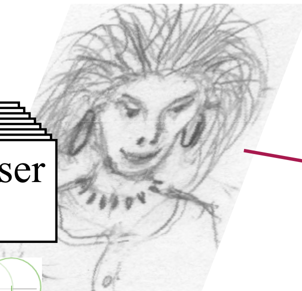

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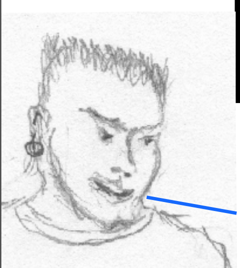
Alice 1CW phase: $a = kx - \omega t$

Carla 1CW phase: $b = -kx - \omega t$



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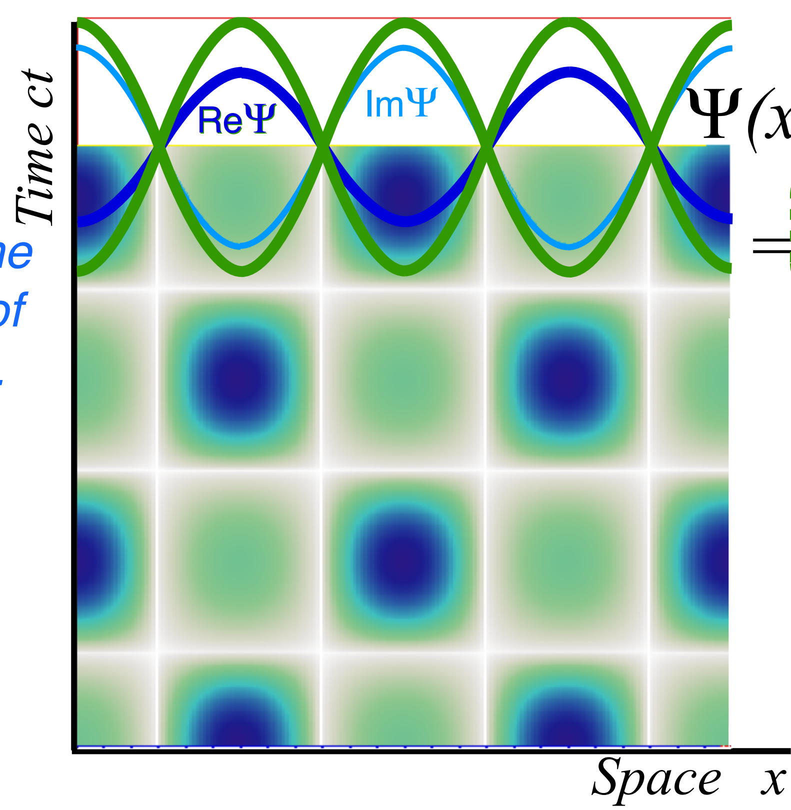

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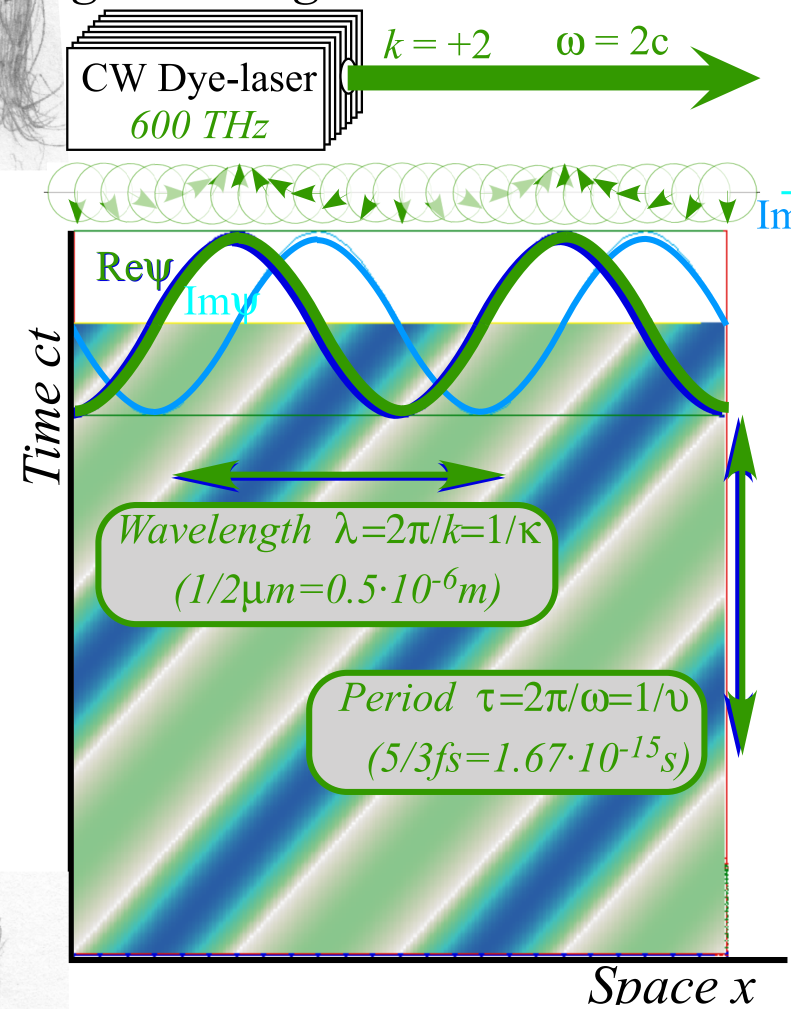

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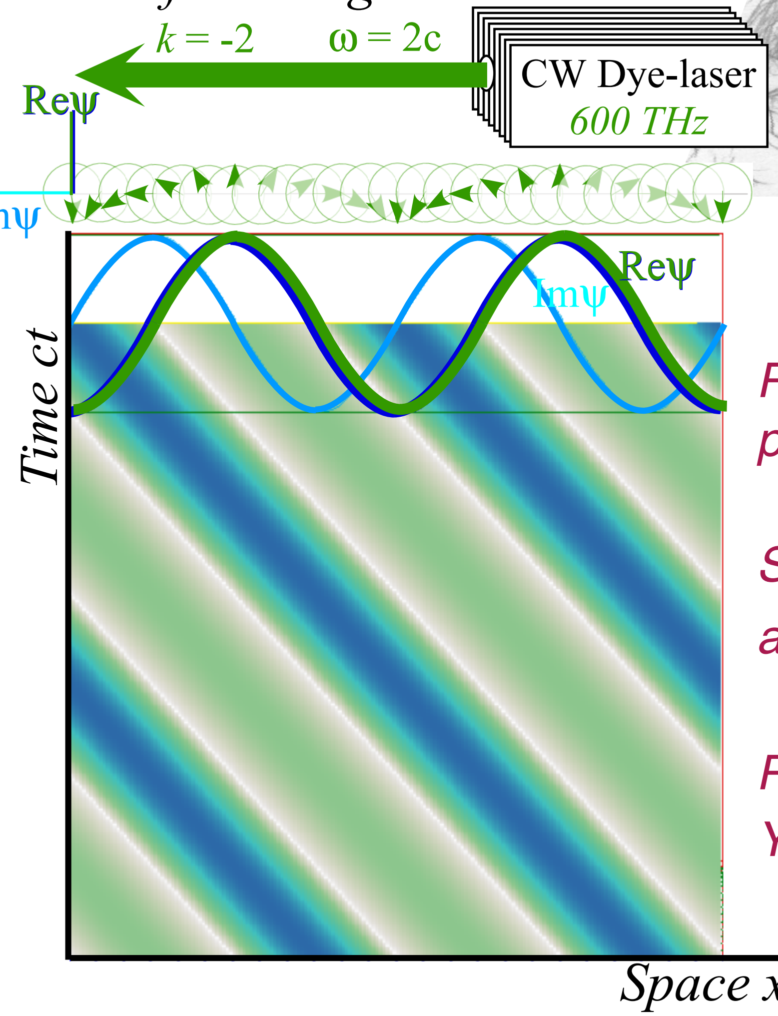

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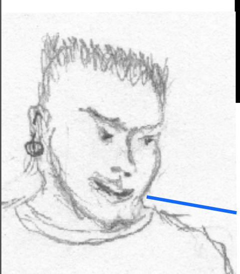

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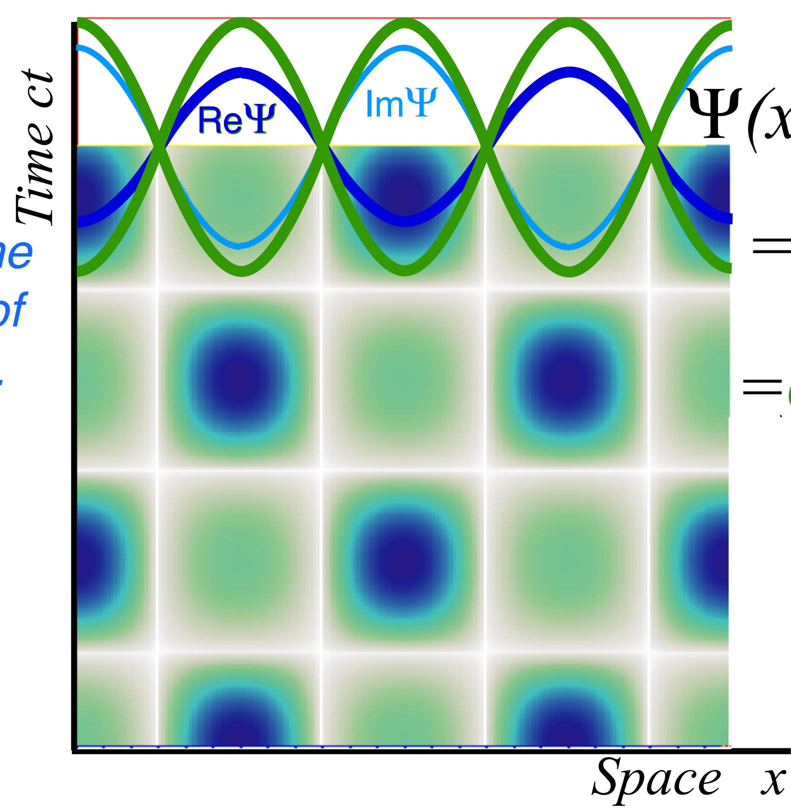

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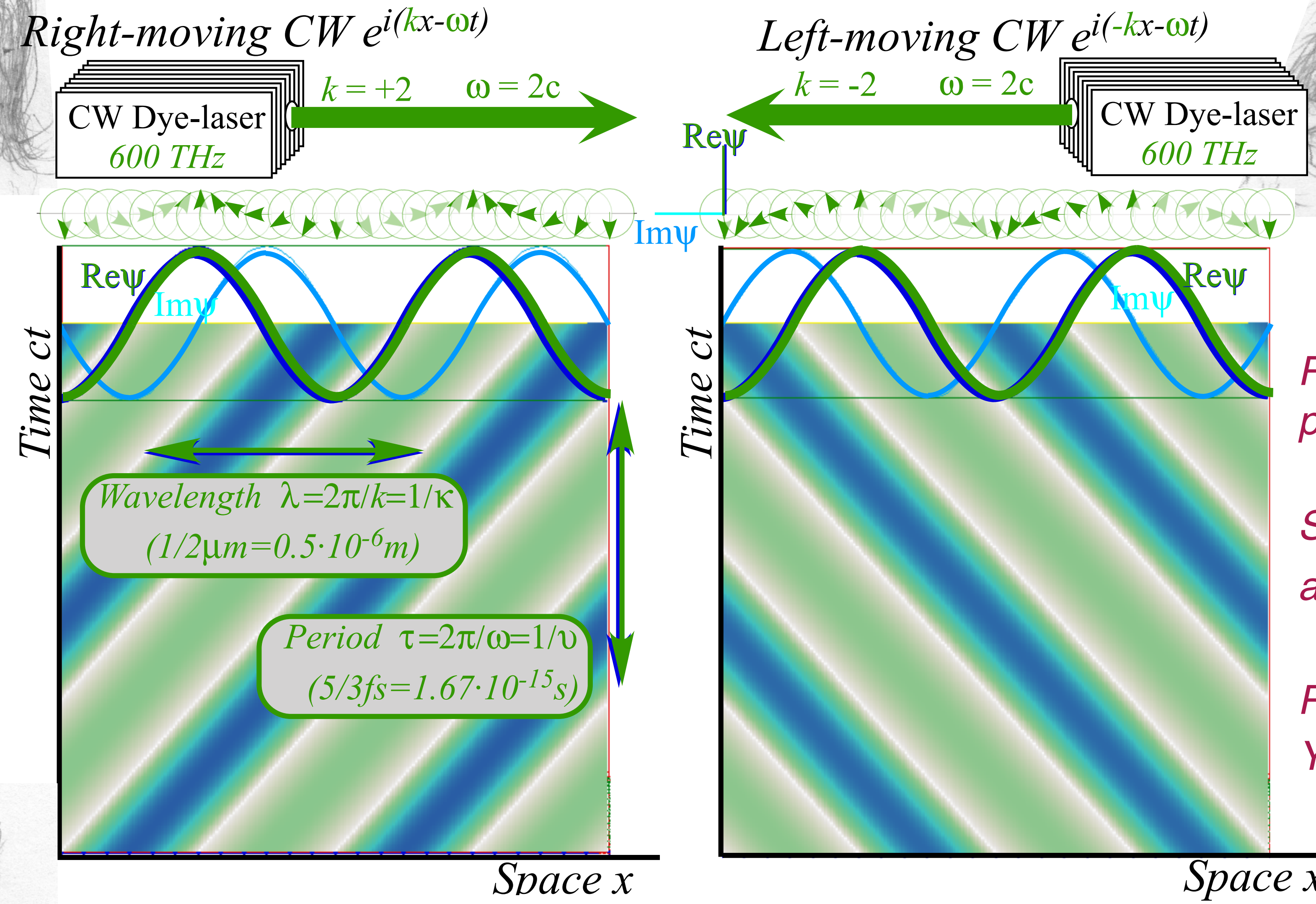
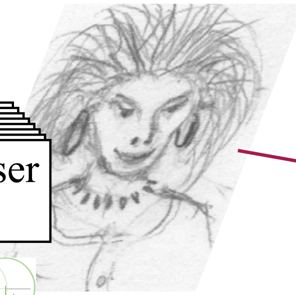
$$= e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

Bob's 2CW **Group-Wave**-phase: $+k=\frac{a-b}{2}$

Group wave: $e^{-ikx} + e^{-ikx} = 2\cos kx$

is *standing* wave (does not vary with time t)





Carla:

Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

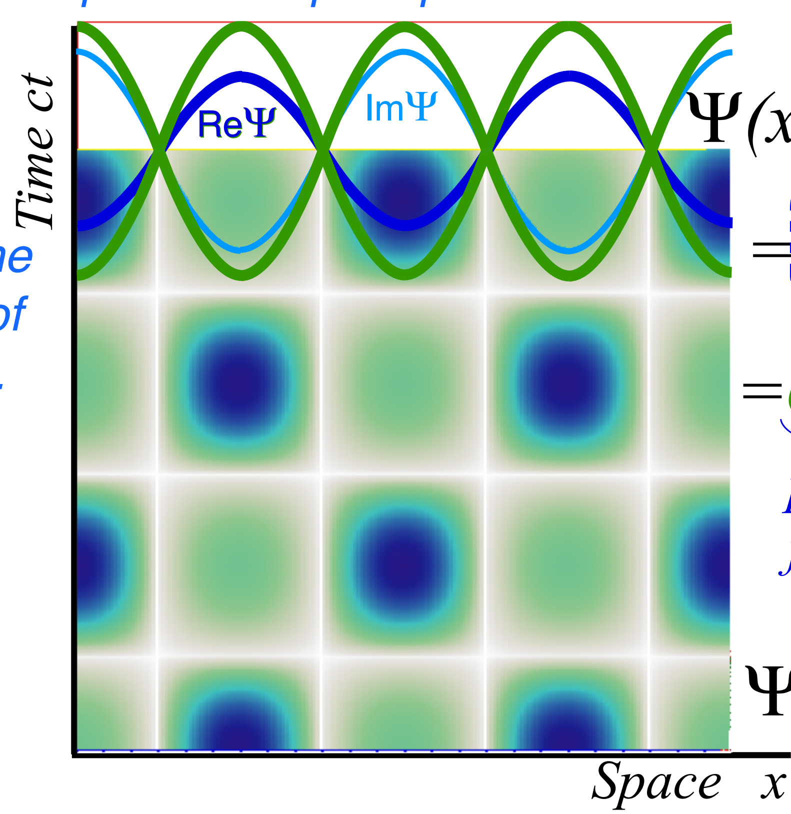
Presto!
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

Alice 1CW phase: $a = kx-\omega t$
Carla 1CW phase: $b = -kx-\omega t$

Bob: Let's plot this in per-spacetime?!

Cool!
You guys made me a space-time graph out of real zeros.

How'd it do that?



$$\Psi(x,t)=e^{ia}+e^{ib} \quad (a = kx-\omega t,\ b = -kx-\omega t)$$

$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

$$=e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

phase factor — group factor

$$\Psi(x,t)=e^{-i\omega t}\,2\cos kx$$

Bob's 2CW **Group**-phase: **Wave** $+k=\frac{a-b}{2}$

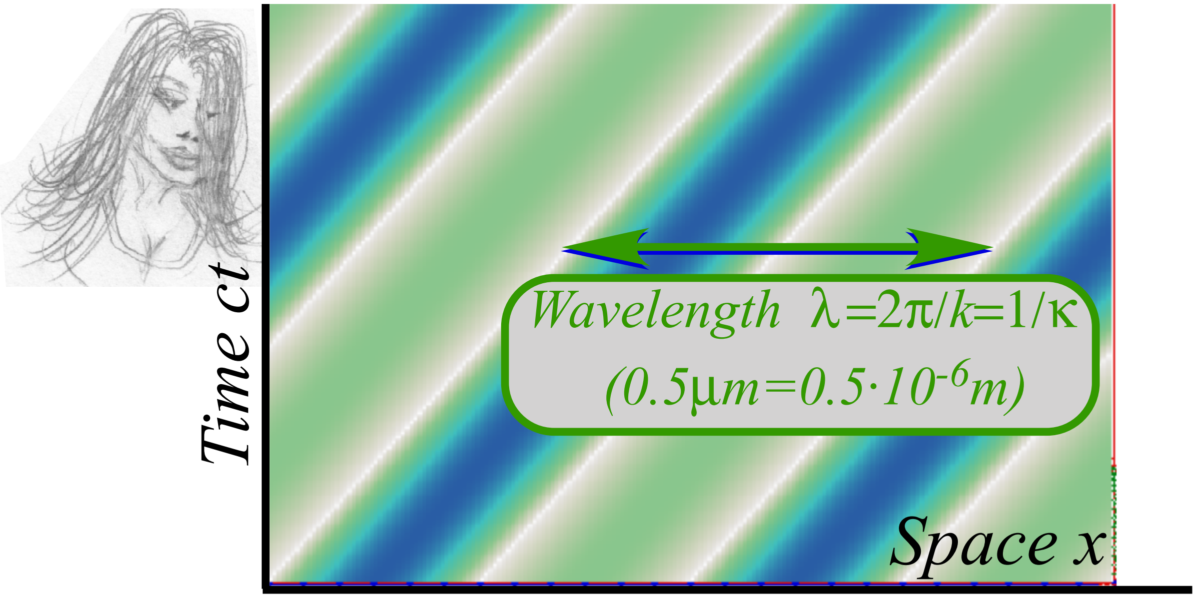
Group wave: $e^{-ikx}+e^{-ikx}=2\cos kx$

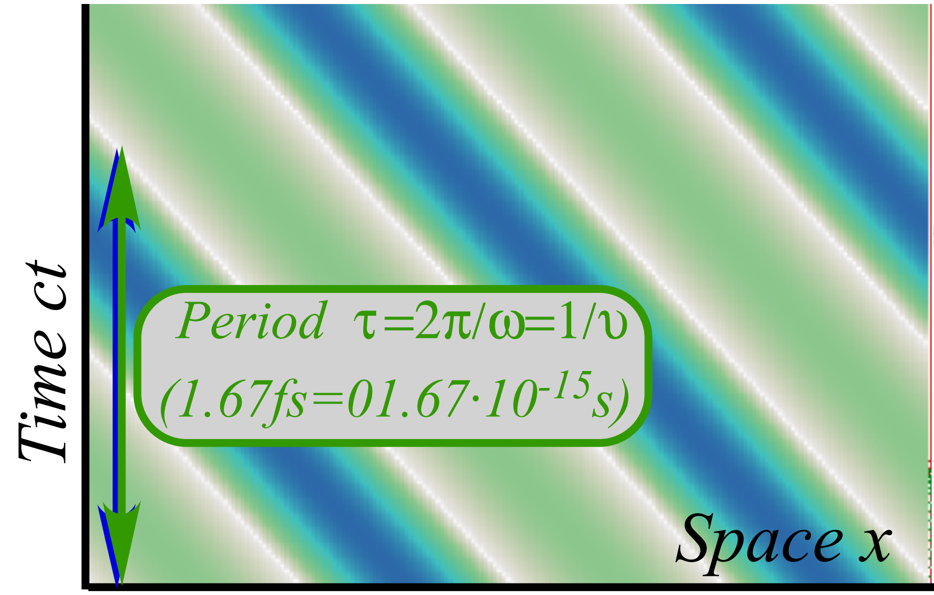
is *standing* wave (does not vary with time t)

Bob's 2CW **Phase**-phase: **Wave** $-\omega=\frac{a+b}{2}$

Phase wave real part: $\mathrm{Re}(e^{-i\omega t})=\cos(\omega t)$

is *"instanton"* wave (does not vary in space x)

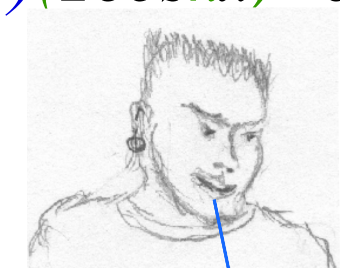




Carla:
OK, Bob!
It looks like a baseball diamond with
P at Pitcher's mound and
G at the Grandstand*.
I'm on 1st base! (**R**)

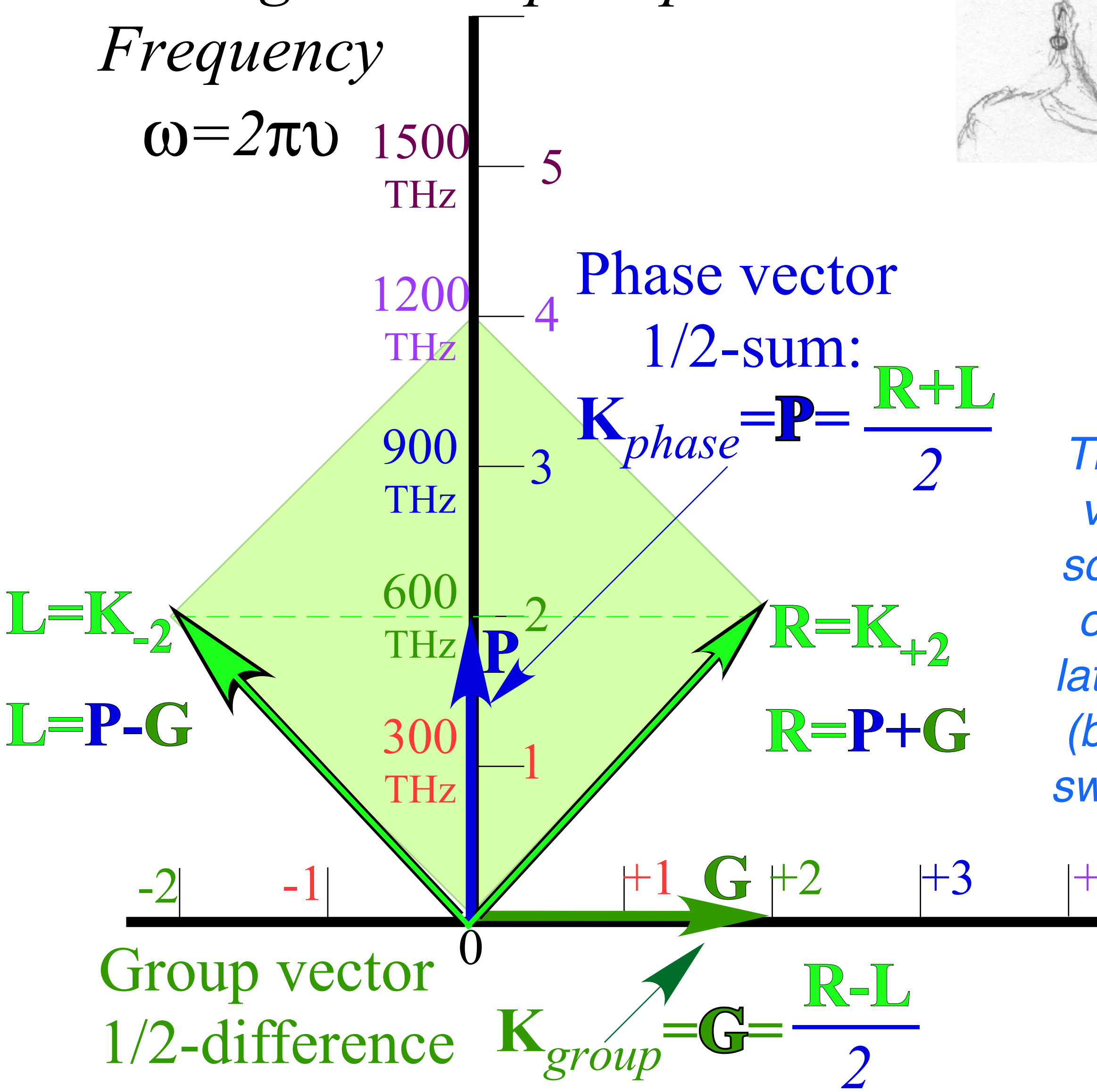
*Thanks, Woody!

$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$



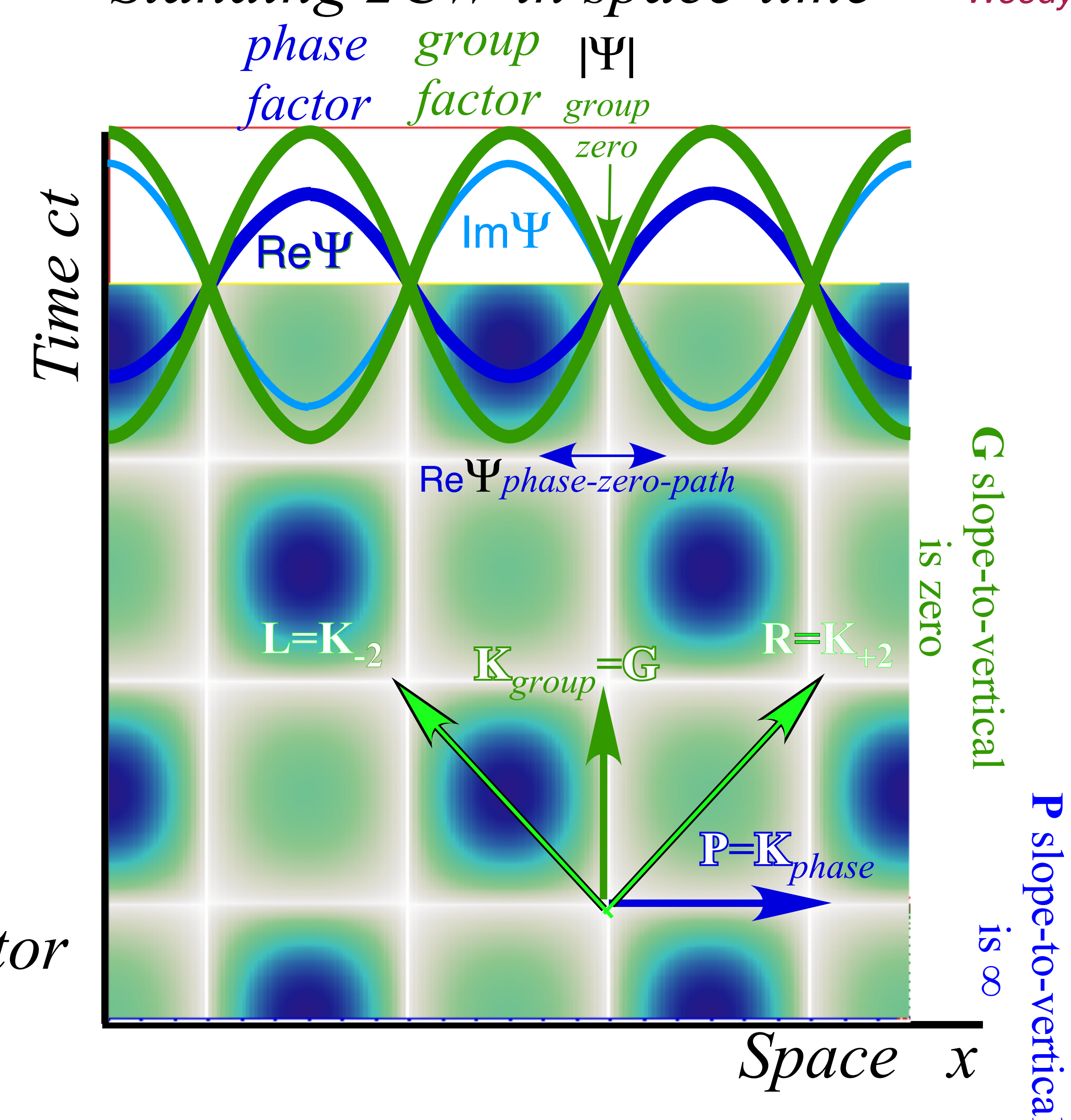
Bob:
The **P** and **G** vectors are scale models of zero-grid lattice vectors (but **P** and **G** switch places)

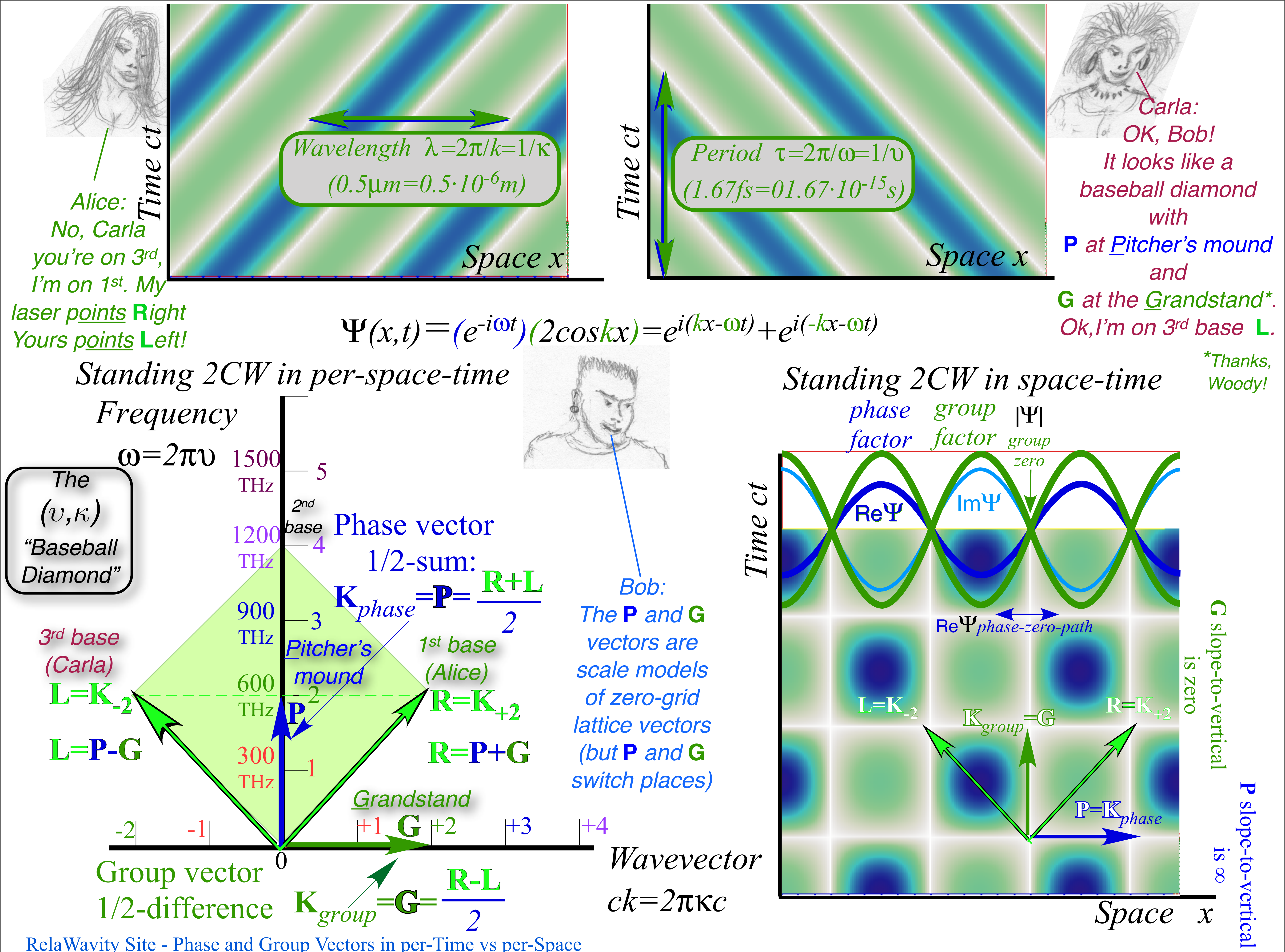
Standing 2CW in per-space-time



Wavevector
$ck=2\pi\kappa c$

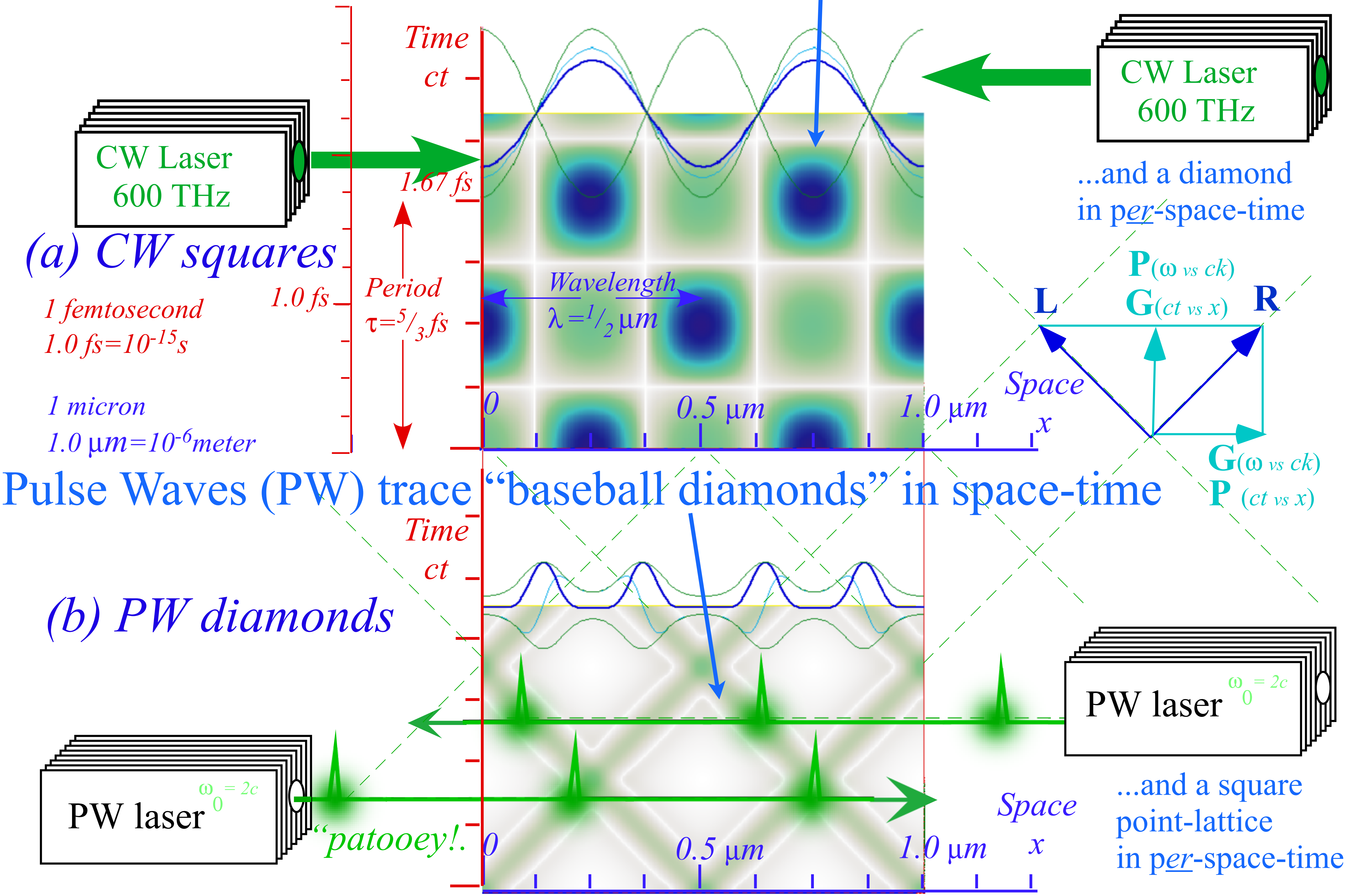
Standing 2CW in space-time





Time ct
Wavelength λ=2π/k=1/κ
(0.5μm=0.5·10^-6m)
Space x
Time ct
Period τ=2π/ω=1/υ
(1.67fs=01.67·10^-15s)
Space x
Alice:
No, Carla
you're on 3rd,
I'm on 1st. My
laser points Right
Yours points Left!
Carla:
OK, Bob!
It looks like a
baseball diamond
with
P at Pitcher's mound
and
G at the Grandstand*.
Ok,I'm on 3rd base L.
*Thanks,
Woody!
Ψ(x,t)=(e^-iωt)(2coskx)=e^i(kx-ωt)+e^i(-kx-ωt)
Standing 2CW in per-space-time
Frequency
ω=2πυ
The
(υ,κ)
"Baseball
Diamond"
1500
THz
5
2nd
base
1200
THz
4
900
THz
3
600
THz
2
300
THz
1
Phase vector
1/2-sum:
K_phase=P= R+L / 2
Pitcher's
mound
1st base
(Alice)
3rd base
(Carla)
L=K_-2
L=P-G
R=K_+2
R=P+G
P
Grandstand
-2 -1 0 +1 G +2 +3 +4
Group vector
1/2-difference
K_group=G= R-L / 2
Wavevector
ck=2πκc
Bob:
The P and G
vectors are
scale models
of zero-grid
lattice vectors
(but P and G
switch places)
Standing 2CW in space-time
phase
factor
group
factor
|Ψ|
group
zero
ReΨ
ImΨ
ReΨ phase-zero-path
Time ct
L=K_-2
K_group=G
R=K_+2
P=K_phase
G slope-to-vertical
is zero
P slope-to-vertical
is ∞
Space x
RelaWavity Site - Phase and Group Vectors in per-Time vs per-Space

Continuous Waves (CW) trace "Cartesian squares" in space-time



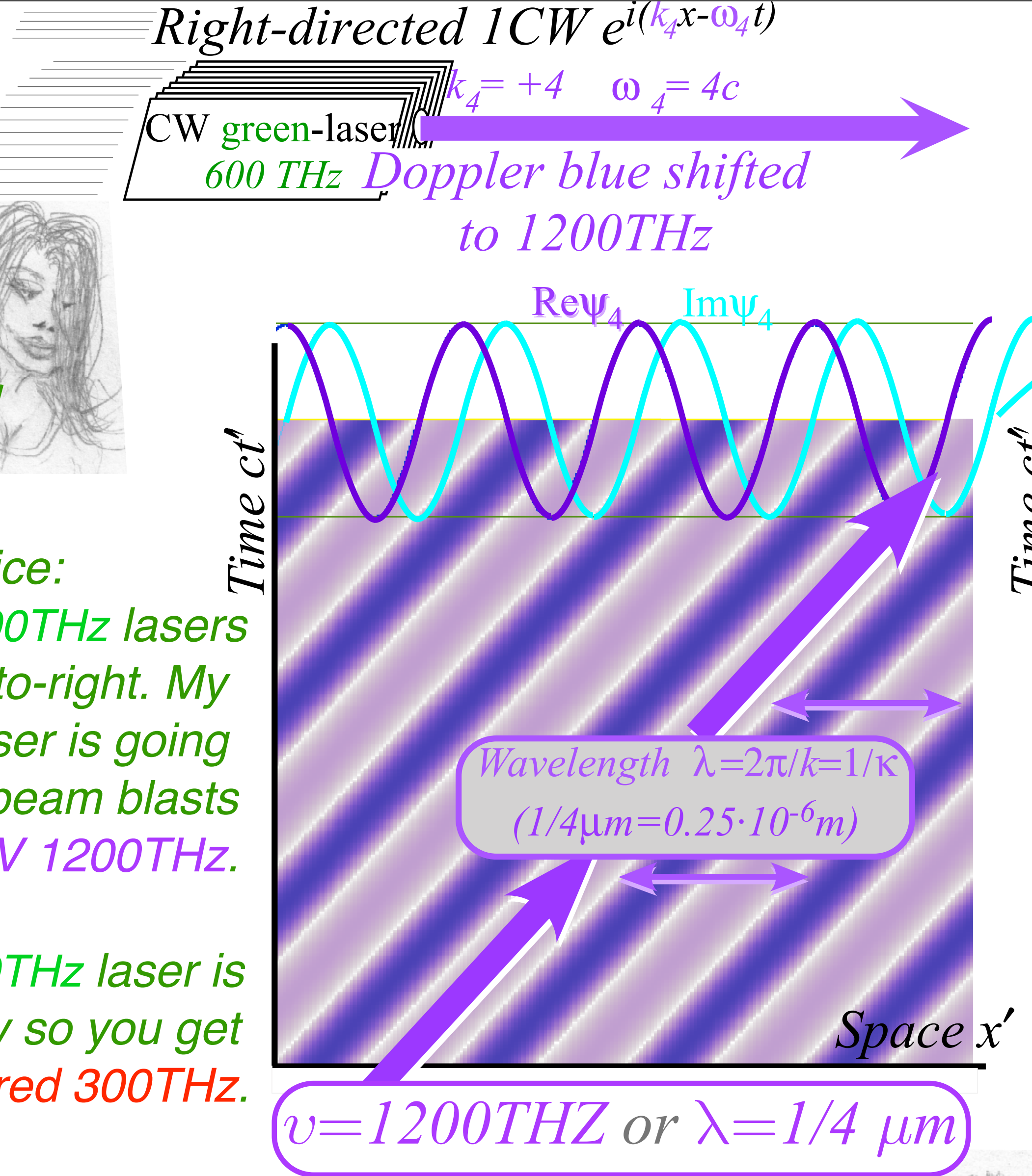
BohrIt Web Simulation: 2 PW *ct vs x* Plot

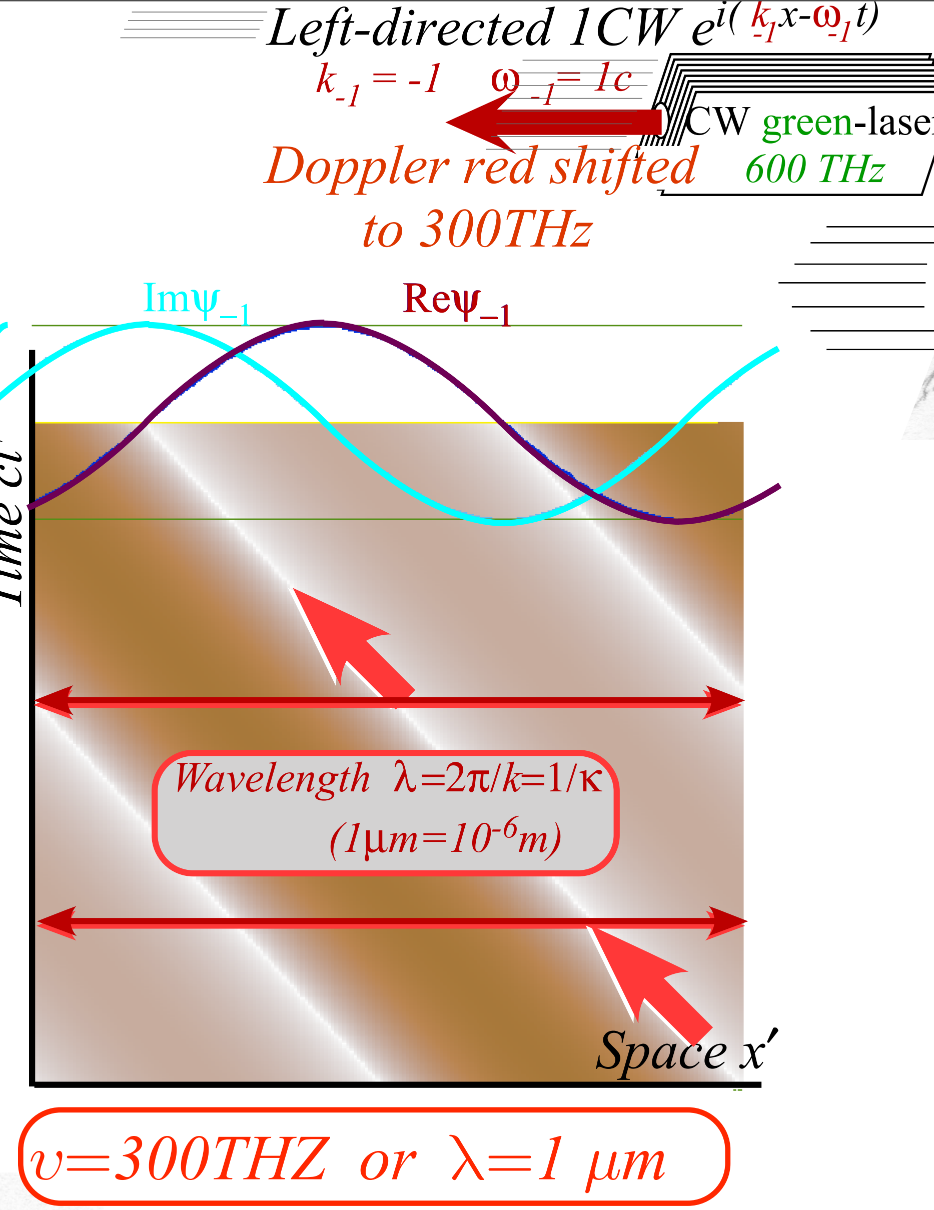
Lecture 30
Thur. 12.10.2015

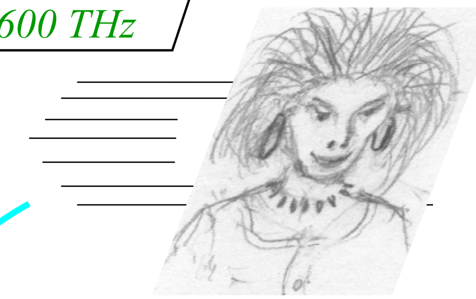
How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
- Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Developing Axioms to update Galileo's relativity: A critical look at *c*-axioms
- Einstein's PW (Pulse-Wave) Axiom
- Evenson's CW (Continuous Wave) Axiom and Occam's Razor
- Analyzing wave velocity by per-space-per-time *and* space-time graphs
- Introducing optical Doppler effects
- Clarifying Evenson's CW Axiom using Doppler effects
  - *Galileo's Revenge (part1):* Galilean Doppler-shift arithmetic using *rapidity* ρ

Developing optical "baseball-diamond" and relativistic ρ-functions and transformations
- Details of 1CW wavefunctions and phasors
- Details of 2CW wavefunctions in rest frame
  - *Galileo's Revenge (part2):* Galilean addition of phasor angular velocity
  - Structure of rest frame "baseball-diamonds"
- ➤ Details of 2CW wavefunctions of moving frame velocities of *phase* and *group* waves
  - 16 coefficients of relativistic 2CW interference
  - Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

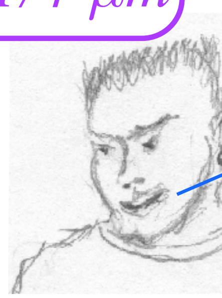






Alice:
Now our 600THz lasers move left-to-right. My 600THz laser is going so fast its beam blasts you with UV 1200THz.

Carla's 600THz laser is going away so you get a nice infrared 300THz.

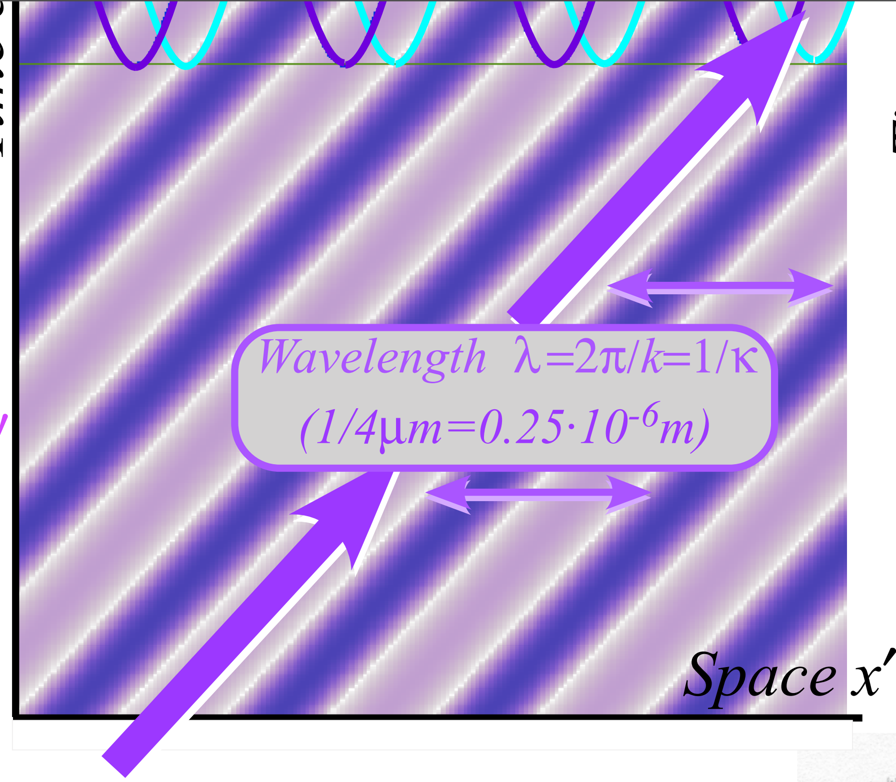


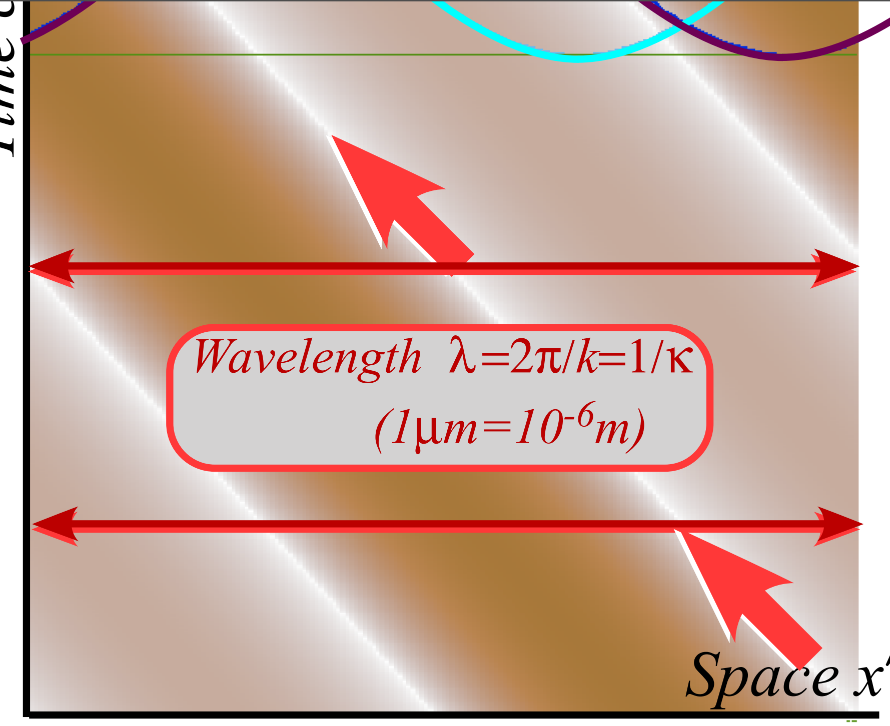
Bob:That UV burns! I need to put on my sunglasses.

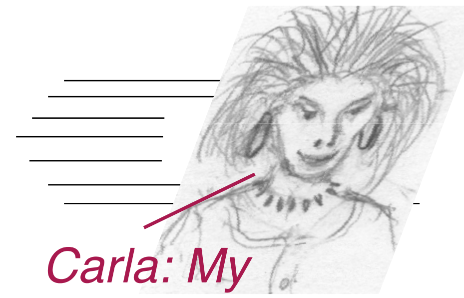
Web Simulation
1 CW *ct vs x* Plot
(ck = +4)

Web Simulation
1 CW *ct vs x* Plot
(ck = -1)

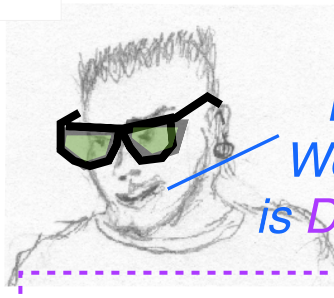
Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



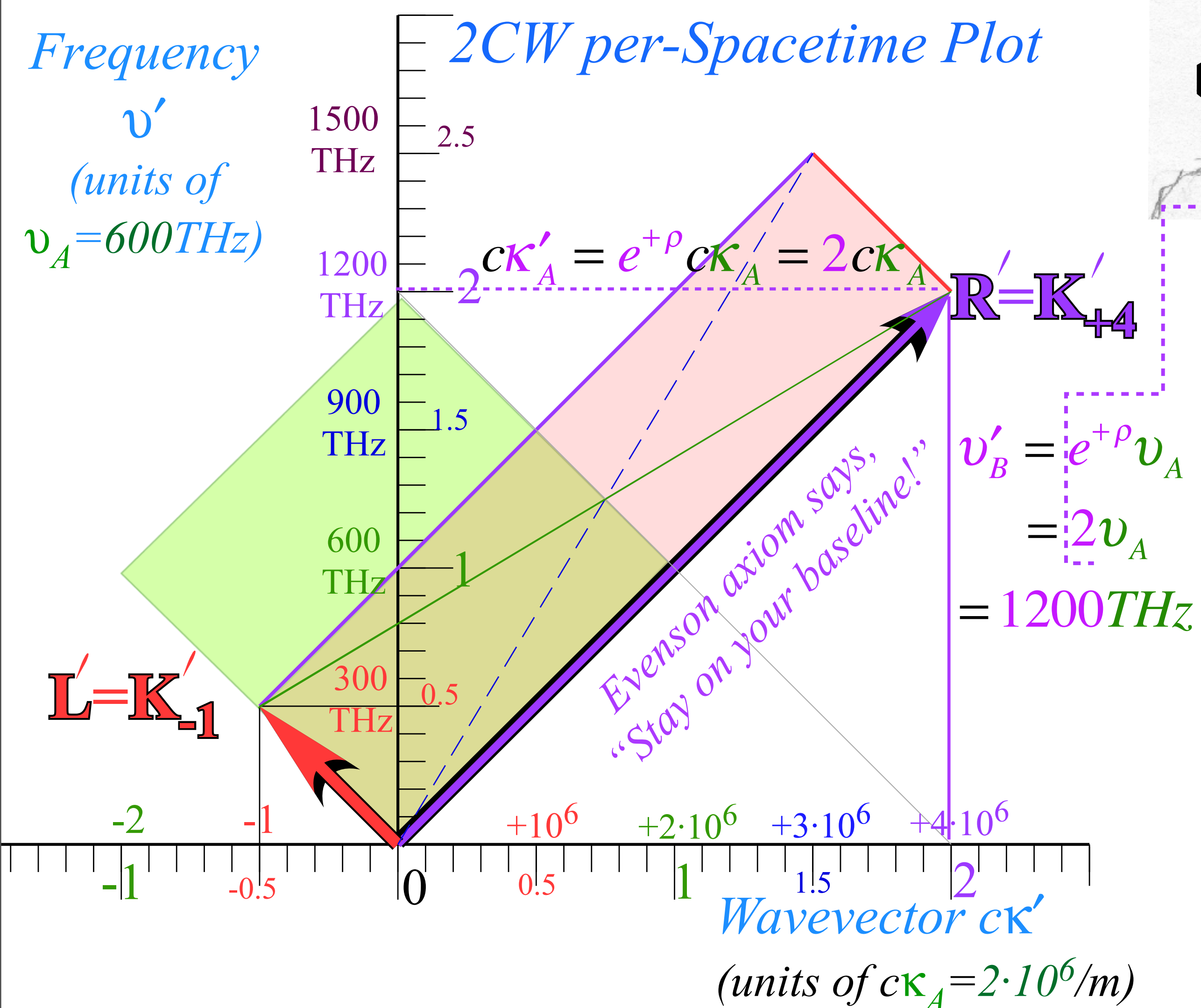


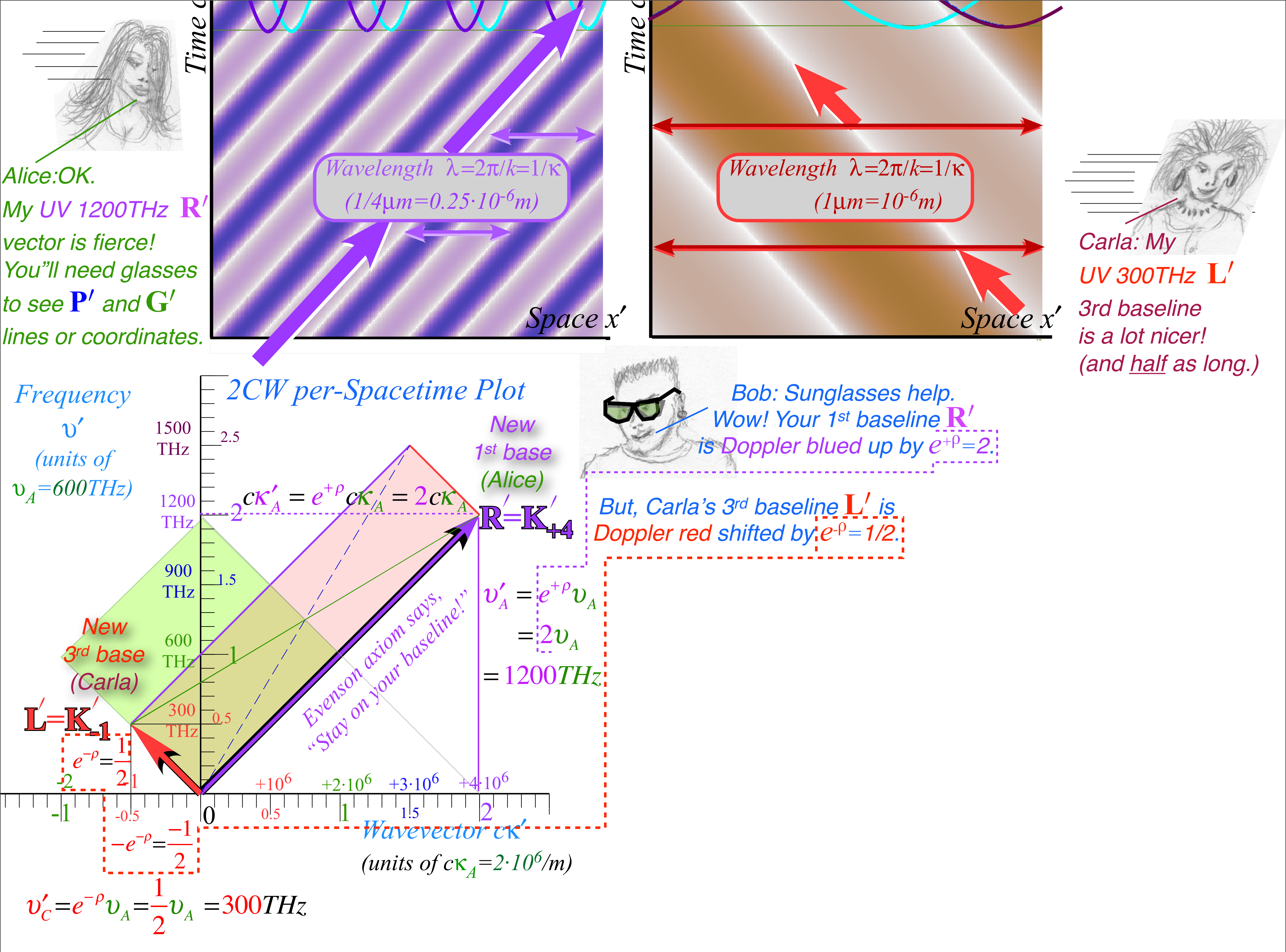


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!

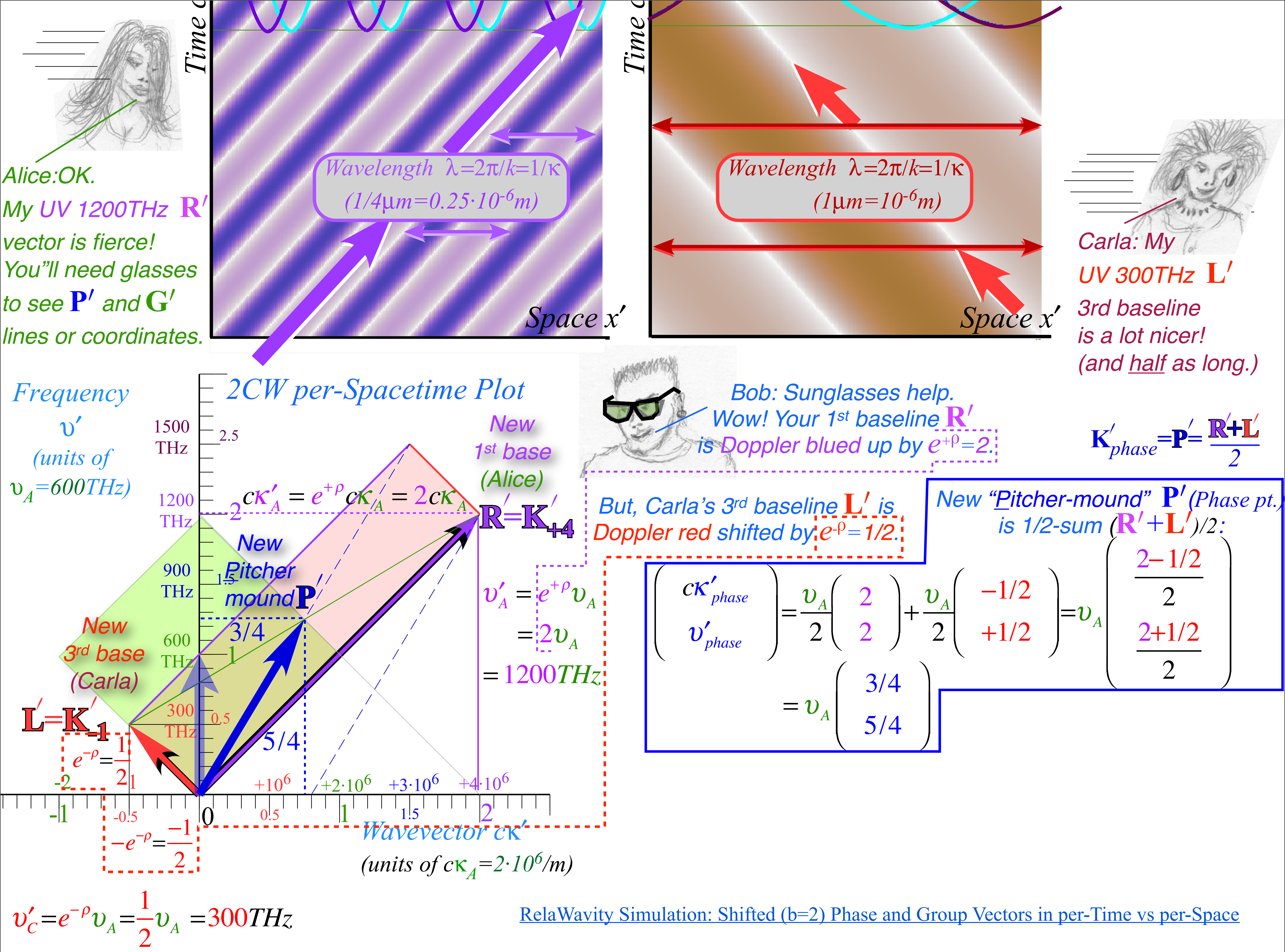


Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

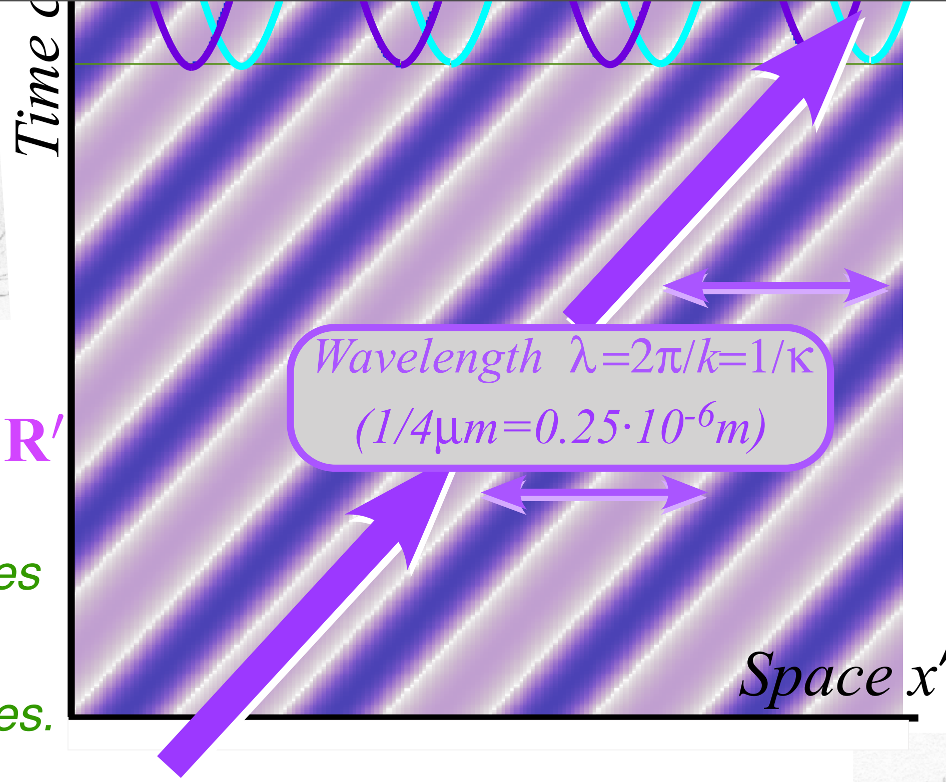


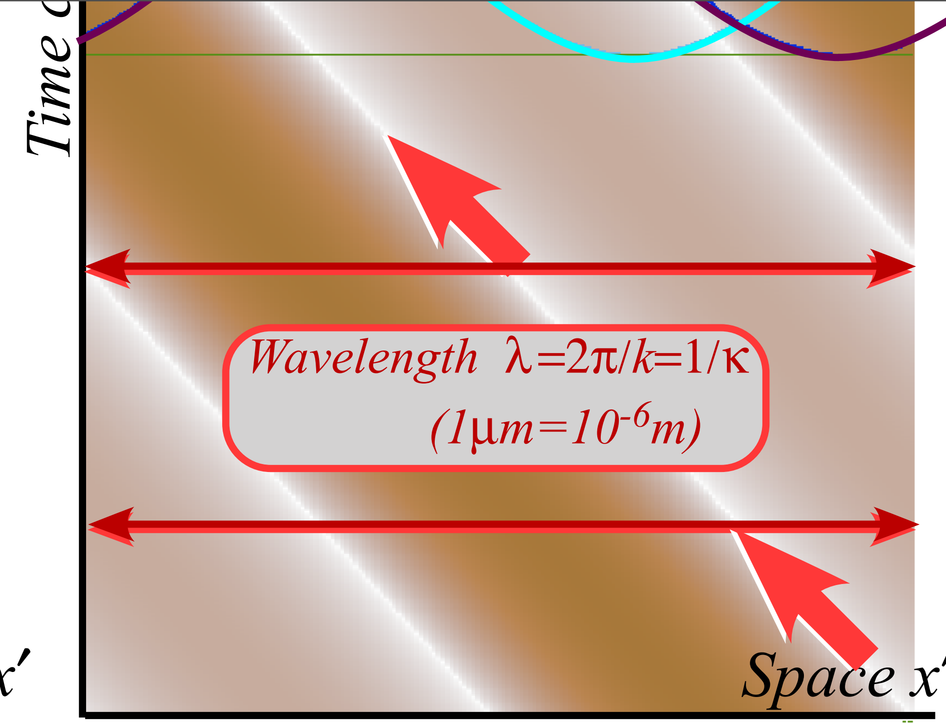


Time c
Time c
Alice:OK.
My UV 1200THz R′
vector is fierce!
You"ll need glasses
to see P′ and G′
lines or coordinates.
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6m)
Space x′
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Space x′
Carla: My
UV 300THz L′
3rd baseline
is a lot nicer!
(and half as long.)
Frequency
υ′
(units of
υ_A=600THz)
2CW per-Spacetime Plot
1500 THz
1200 THz
900 THz
600 THz
300 THz
2.5
1.5
1
0.5
2
cκ′_A = e^{+ρ} cκ_A = 2cκ_A
New 1st base (Alice)
R′=K′_{+4}
Bob: Sunglasses help.
Wow! Your 1st baseline R′
is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is
Doppler red shifted by e^{-ρ}=1/2.
υ′_A = e^{+ρ}υ_A
= 2υ_A
= 1200THz
New 3rd base (Carla)
L′=K′_{-1}
e^{-ρ} = 1/2
-2
1
Evenson axiom says,
"Stay on your baseline!"
+10^6
+2·10^6
+3·10^6
+4·10^6
-1
-0.5
0
0.5
1
1.5
2
−e^{-ρ} = −1/2
Wavevector cκ′
(units of cκ_A=2·10^6/m)
υ′_C = e^{-ρ}υ_A = 1/2 υ_A = 300THz

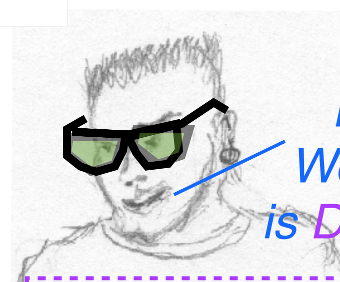

Time
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6m)
Space x′
Time
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Space x′
Alice:OK.
My UV 1200THz R′ vector is fierce!
You"ll need glasses to see P′ and G′ lines or coordinates.
Carla: My UV 300THz L′ 3rd baseline is a lot nicer! (and half as long.)
Frequency υ′ (units of υ_A=600THz)
2CW per-Spacetime Plot
New 1st base (Alice)
New Pitcher mound P′
New 3rd base (Carla)
R′=K′+4
L′=K′-1
1500 THz
1200 THz
900 THz
600 THz
300 THz
cκ′_A = e^{+ρ}cκ_A = 2cκ_A
3/4
5/4
e^{-ρ} = 1/2
-e^{-ρ} = -1/2
υ′_A = e^{+ρ}υ_A = 2υ_A = 1200THz
+10^6 +2·10^6 +3·10^6 +4·10^6
Wavevector cκ′ (units of cκ_A=2·10^6/m)
Bob: Sunglasses help. Wow! Your 1st baseline R′ is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is Doppler red shifted by e^{-ρ}=1/2.
K′_phase = P′ = (R′+L′)/2
New "Pitcher-mound" P′ (Phase pt.) is 1/2-sum (R′+L′)/2:
(cκ′_phase, υ′_phase) = υ_A/2 (2, 2) + υ_A/2 (−1/2, +1/2) = υ_A ((2−1/2)/2, (2+1/2)/2) = υ_A (3/4, 5/4)
υ′_C = e^{-ρ}υ_A = 1/2 υ_A = 300THz
RelaWavity Simulation: Shifted (b=2) Phase and Group Vectors in per-Time vs per-Space

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.





Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

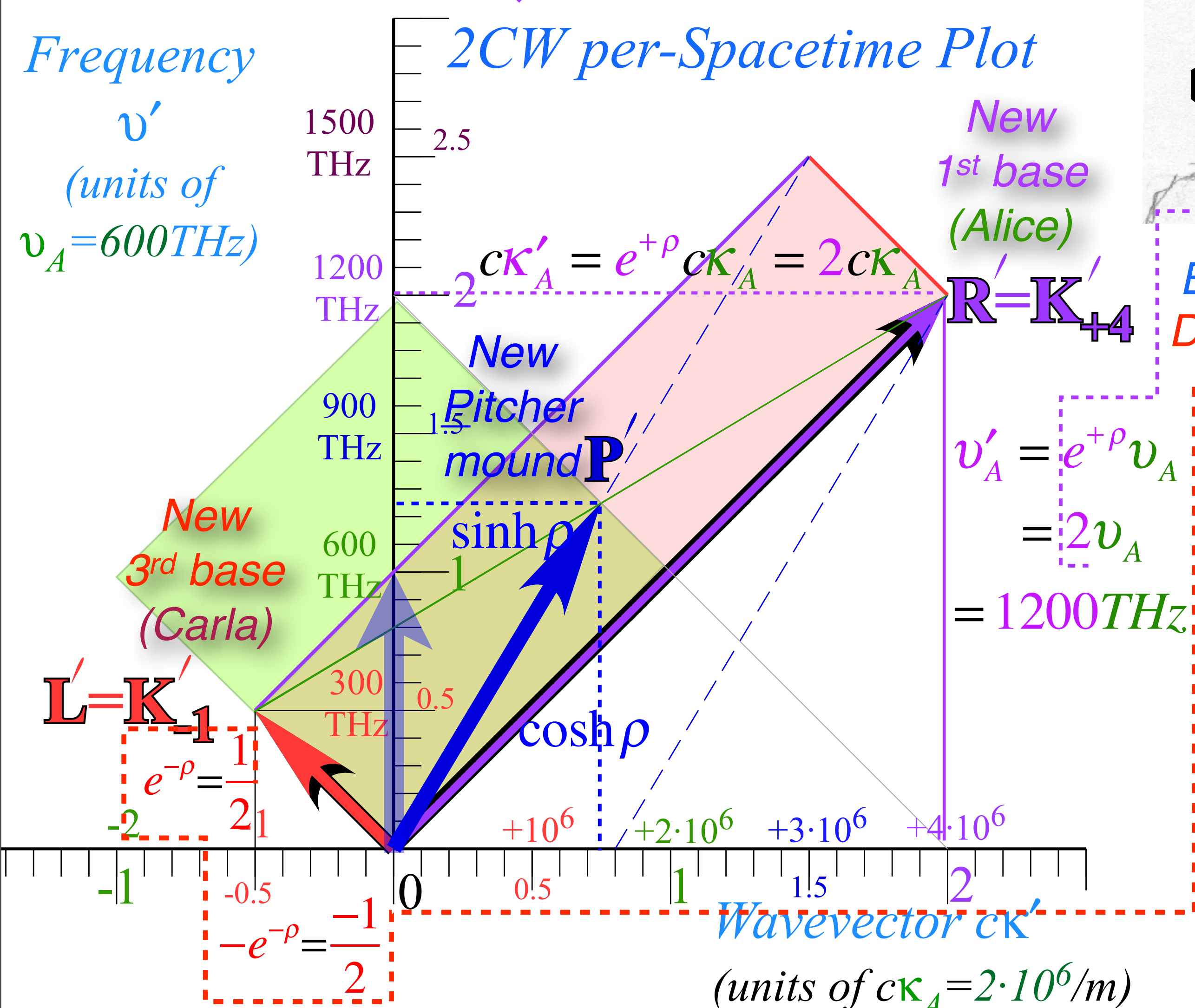
$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

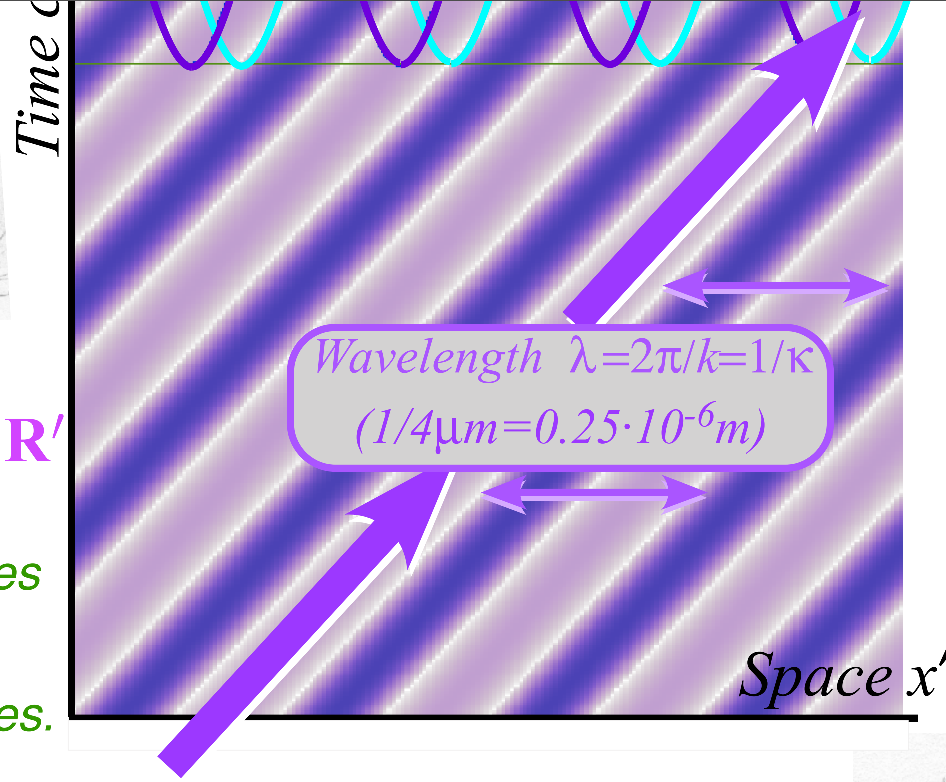
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

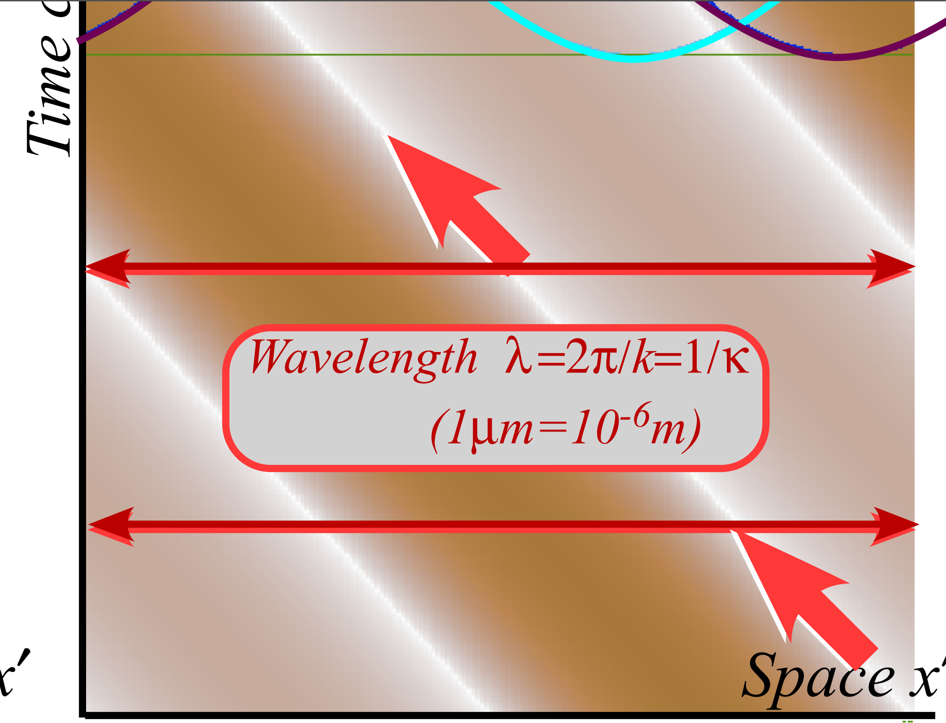
$$= \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$



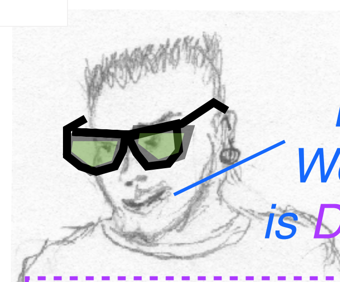
$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.





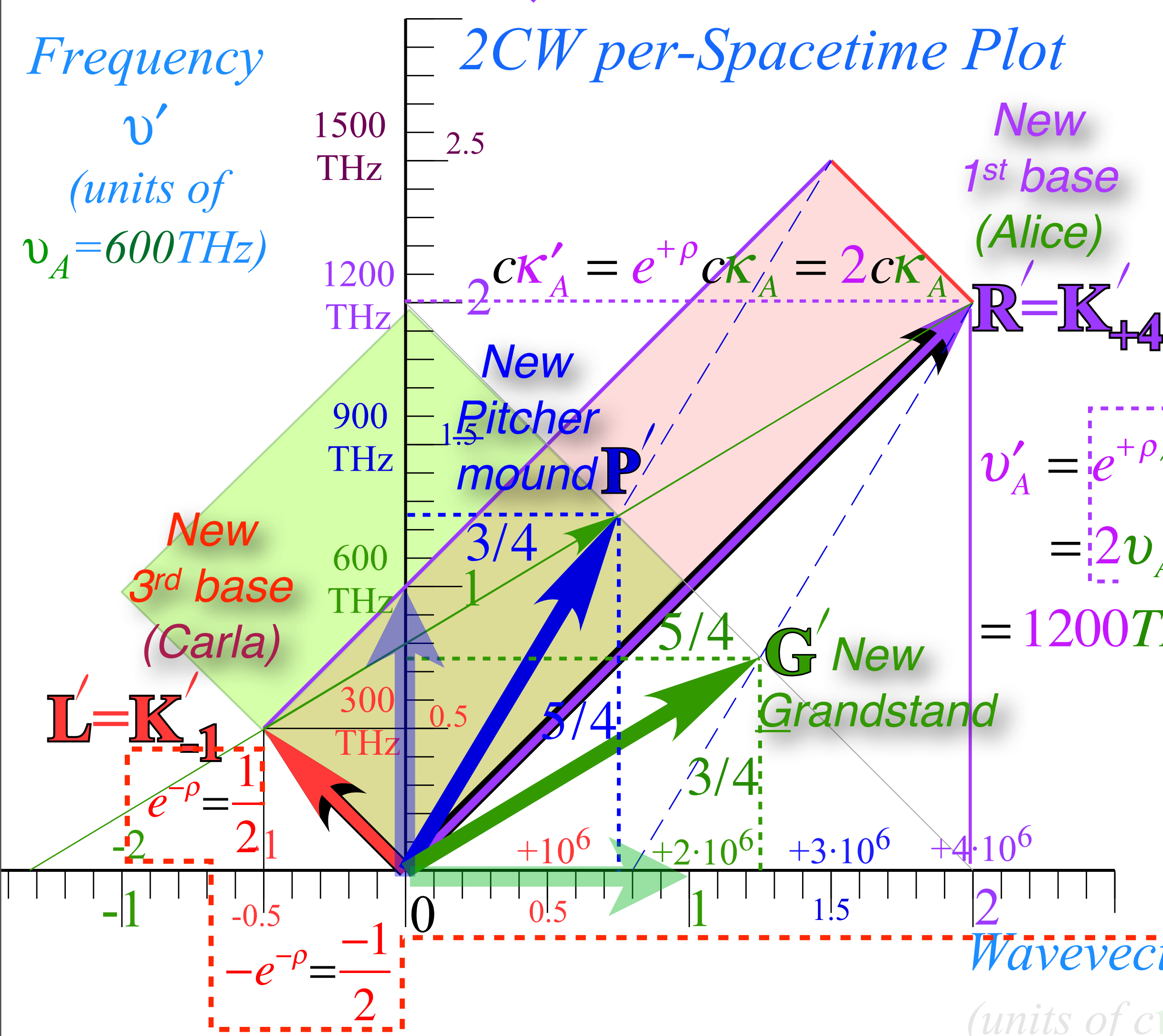
Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

$$K'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$



$$\upsilon'_A = e^{+\rho}\upsilon_A = 2\upsilon_A = 1200THz$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

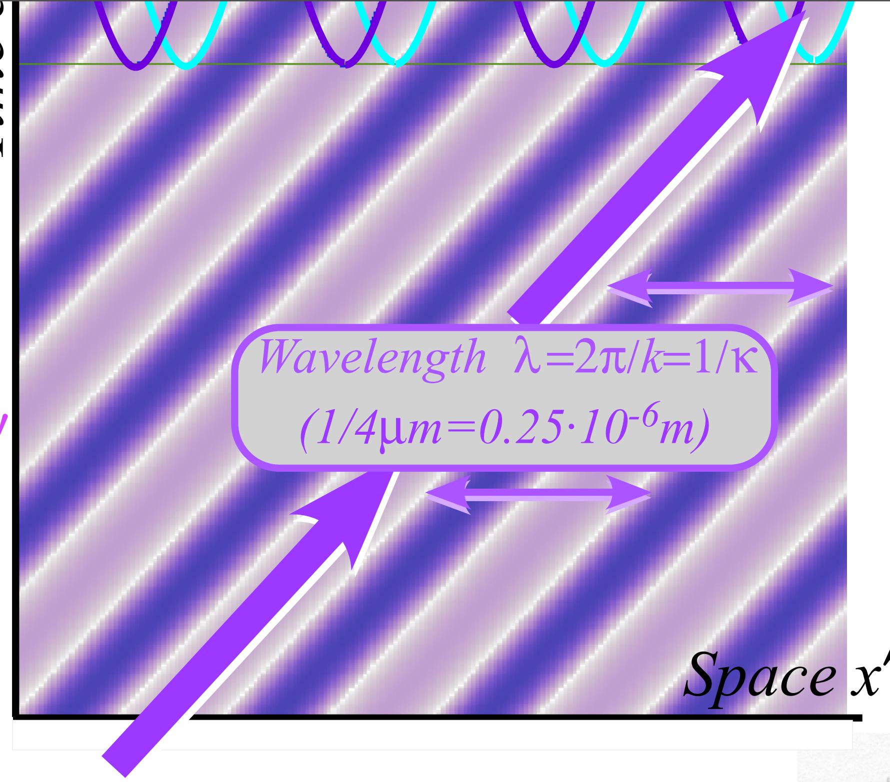
New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

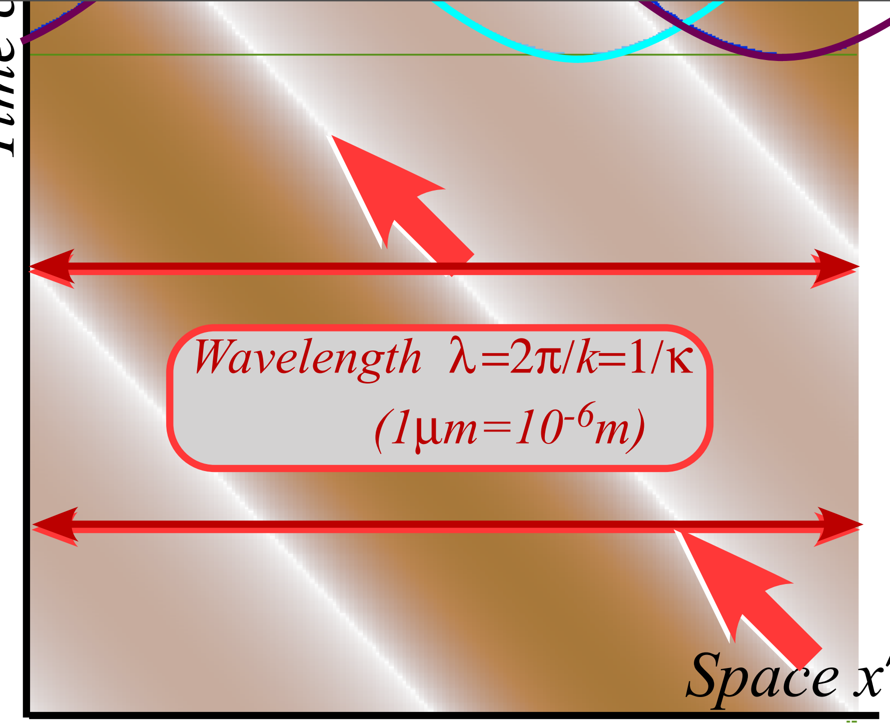
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

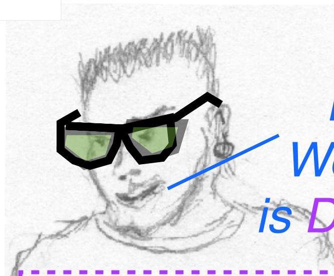
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2+1/2}{2} \\ \frac{2-1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



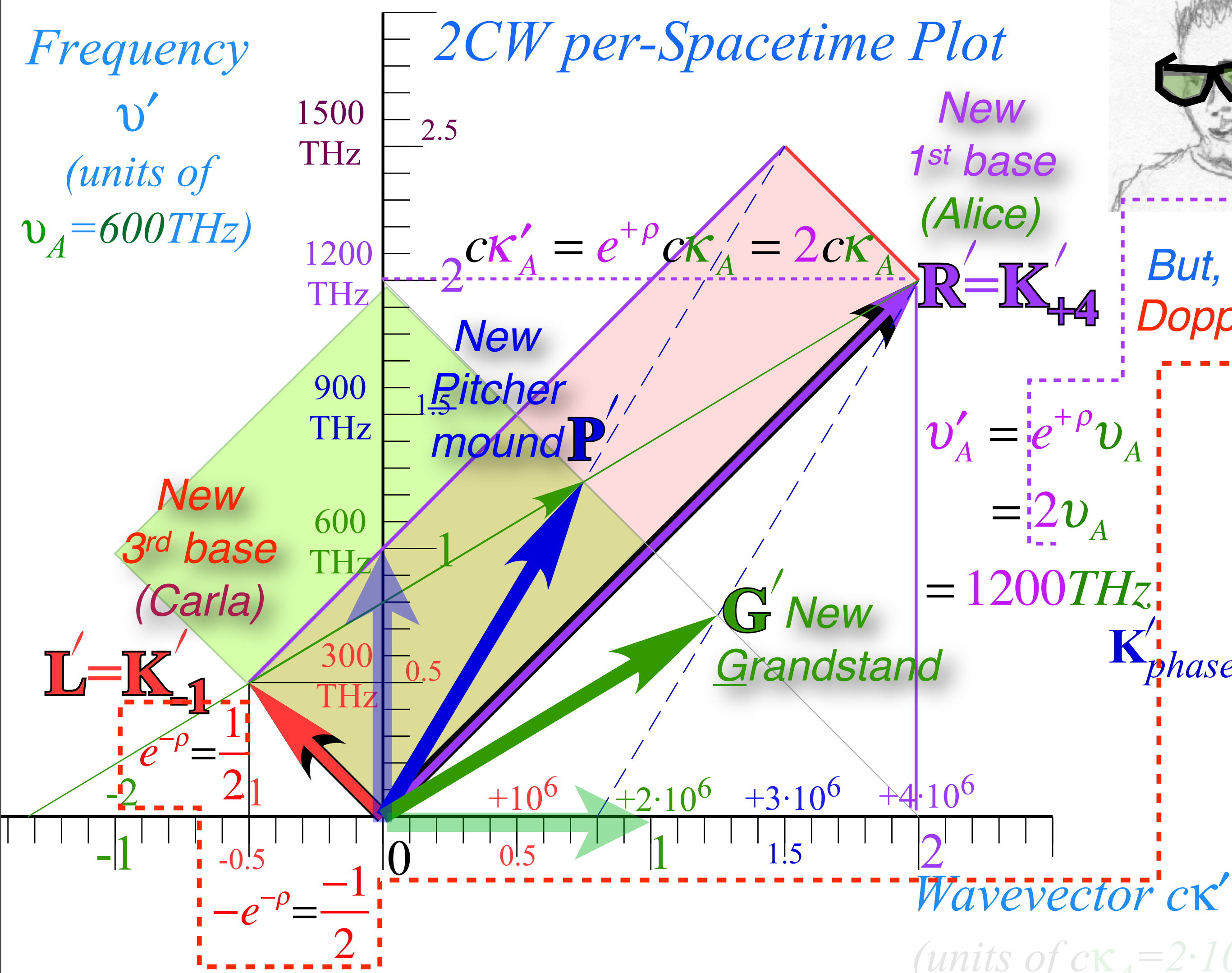


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and <u>half</u> as long.)



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.



$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$

New "<u>P</u>itcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

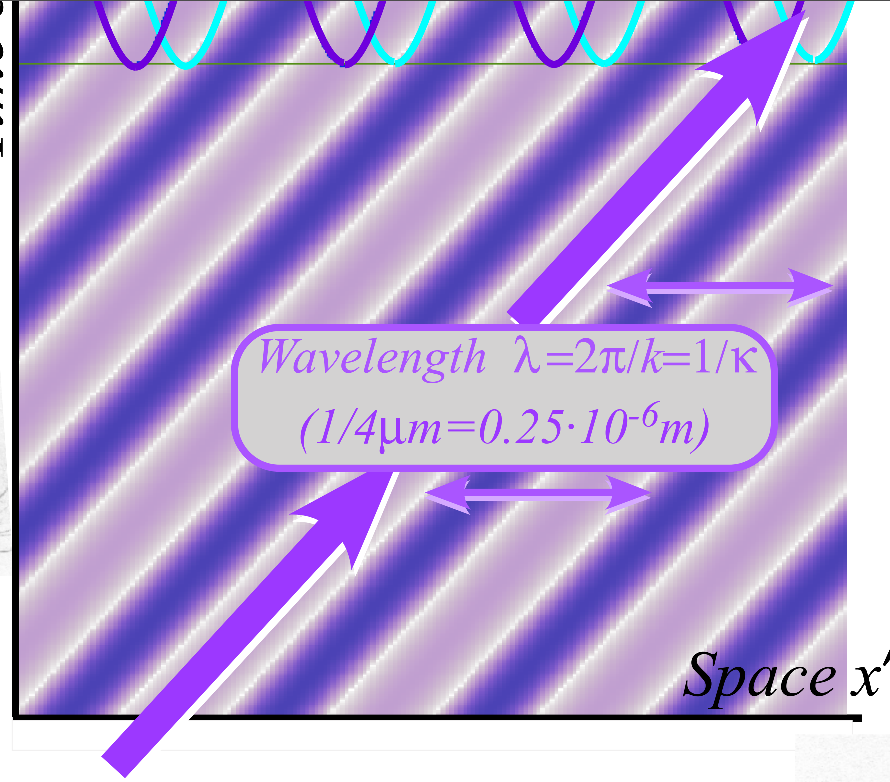
$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$$

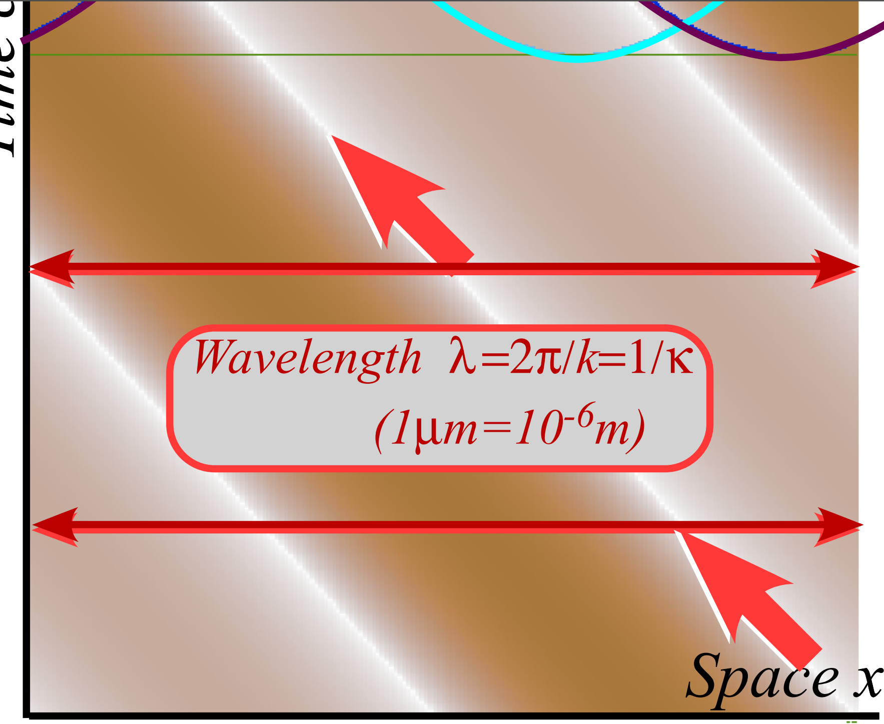
New "<u>G</u>randstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

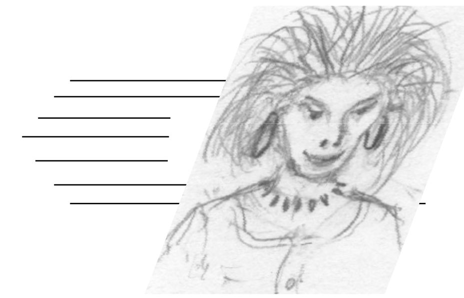
$$\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix}$$

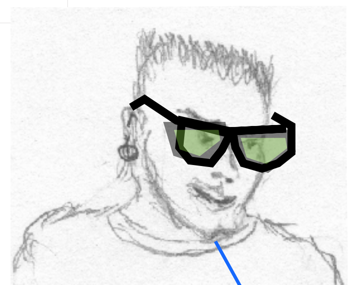
Group vector $\mathbf{G}$
1/2-diff vector

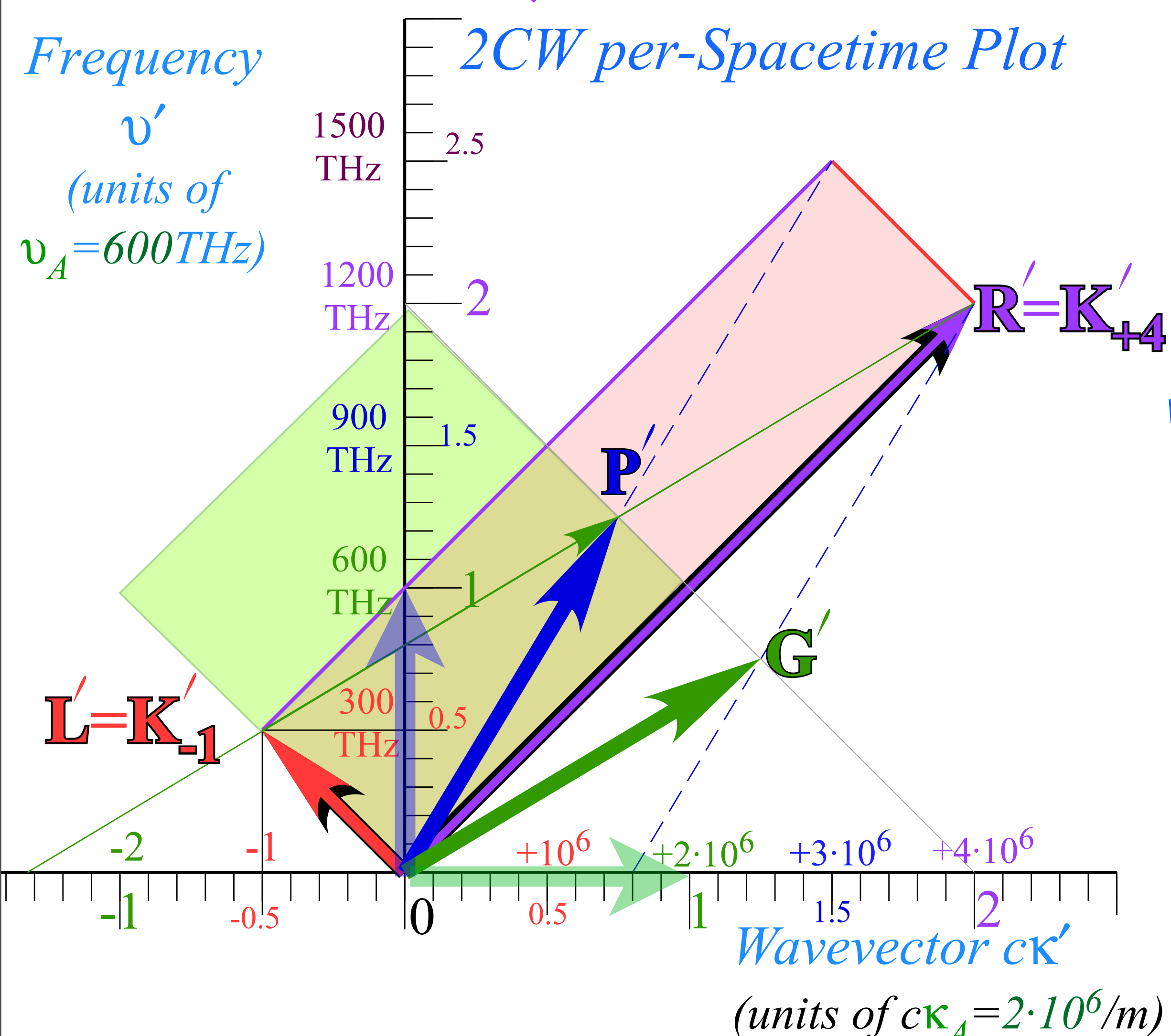
$$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$



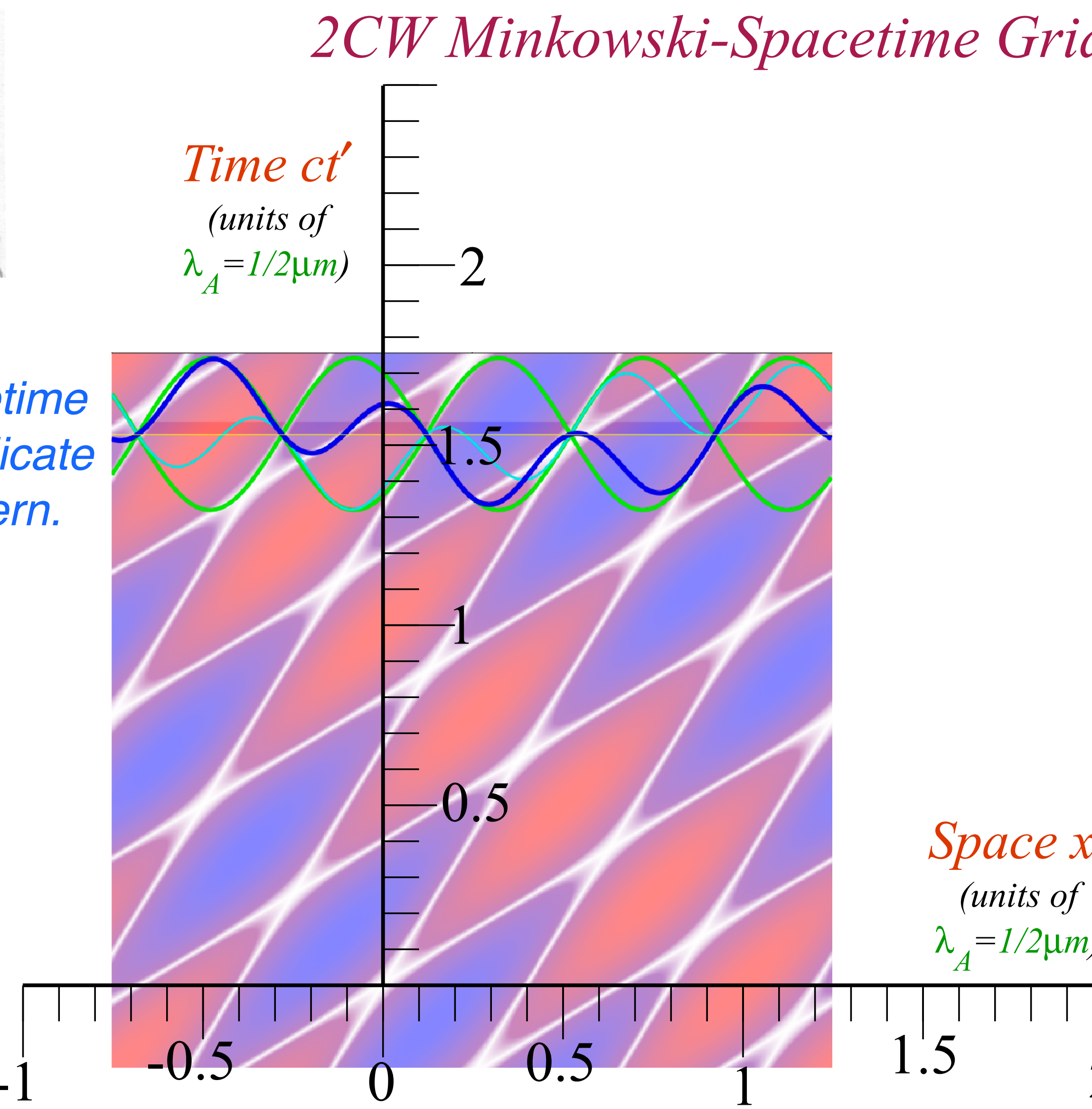








Bob: The spacetime wave-zeros replicate the same pattern.



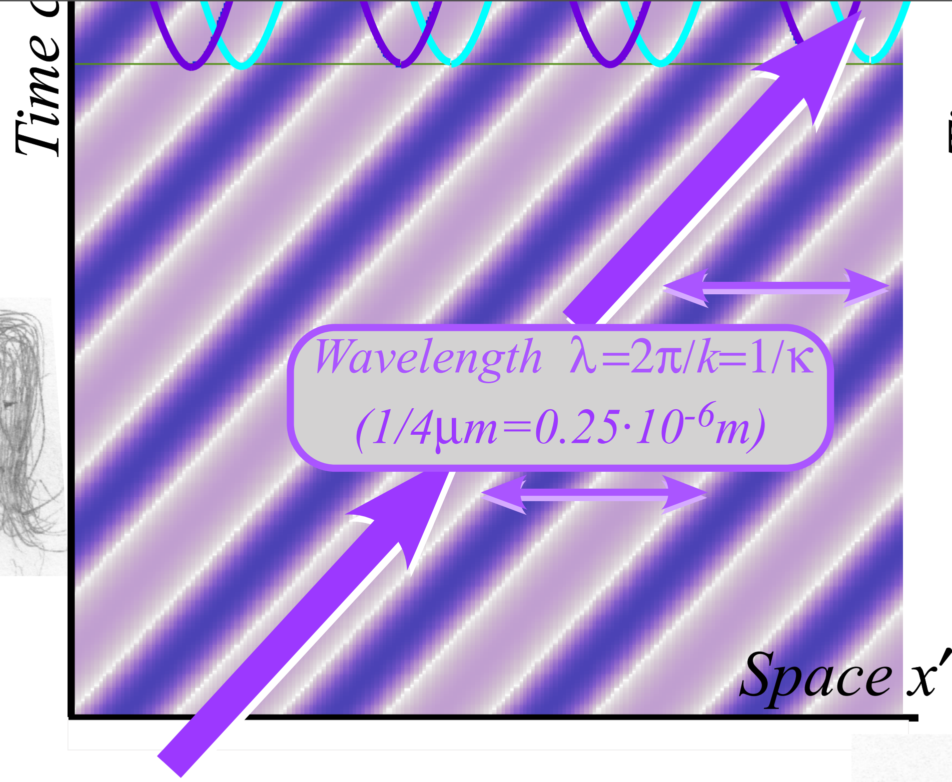
Phase vector **P**
1/2-sum vector

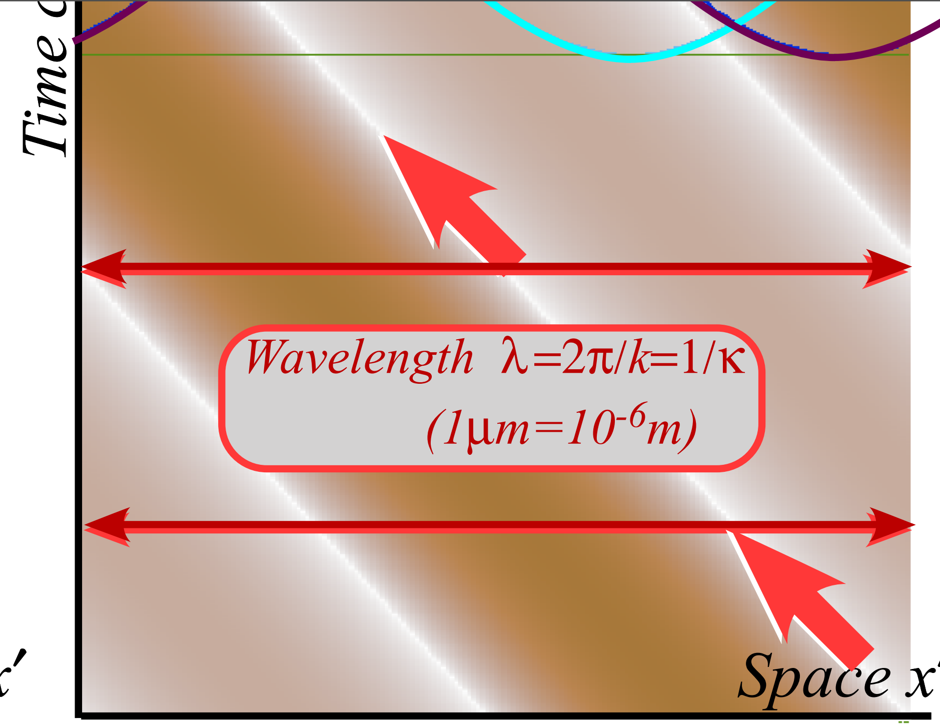
$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2}$

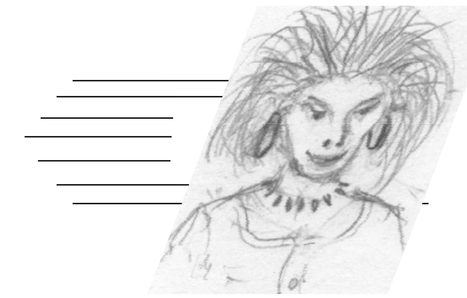
Group vector **G**
1/2-diff vector

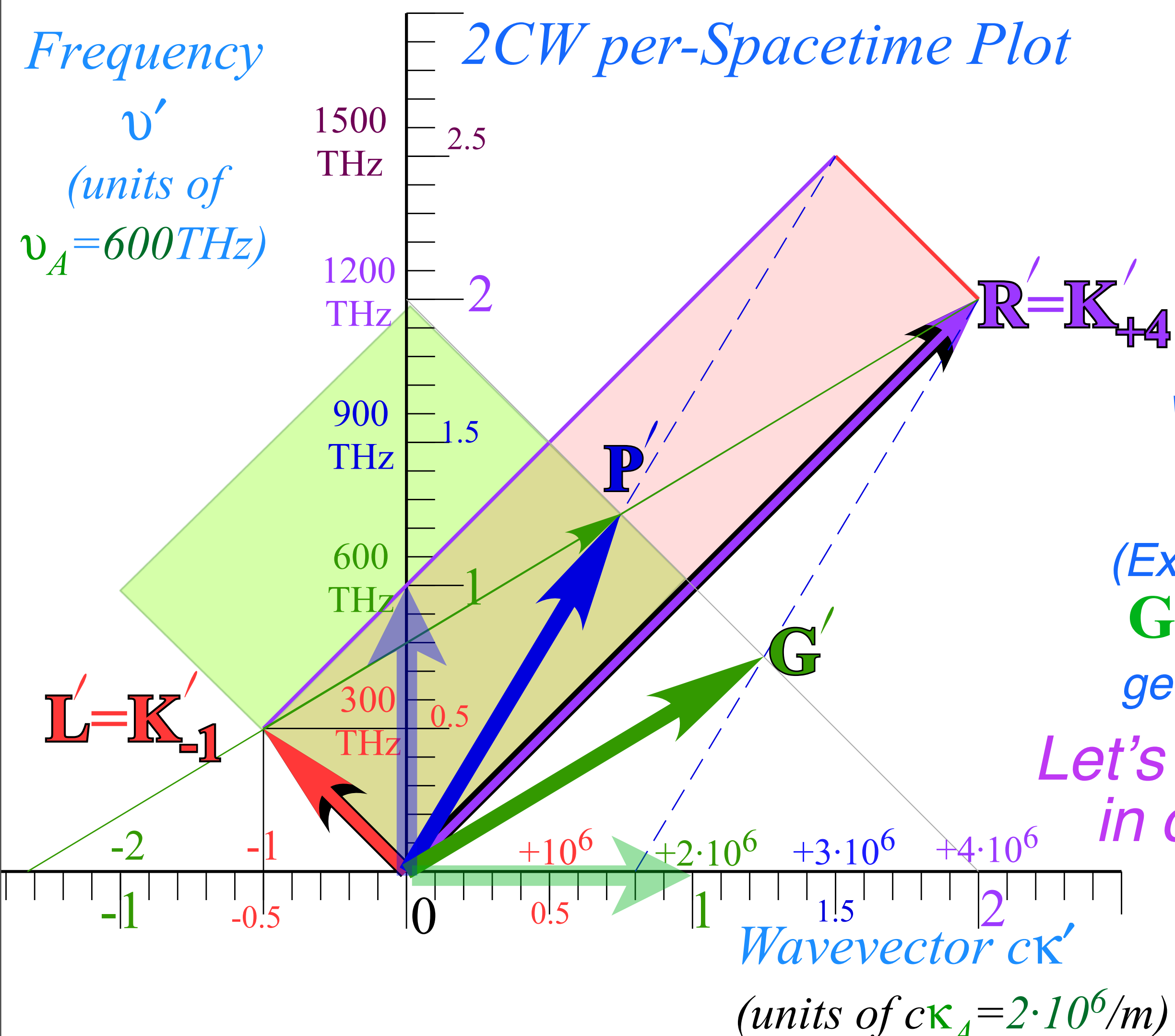
$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}' - \mathbf{L}'}{2}$

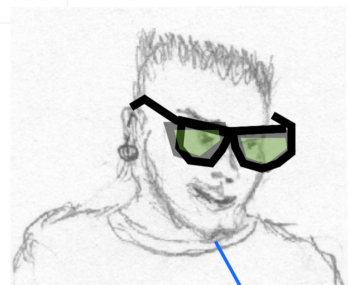
BohrIt Web Simulation
2 CW *Minkowski* Plot (ck = -1, +4)







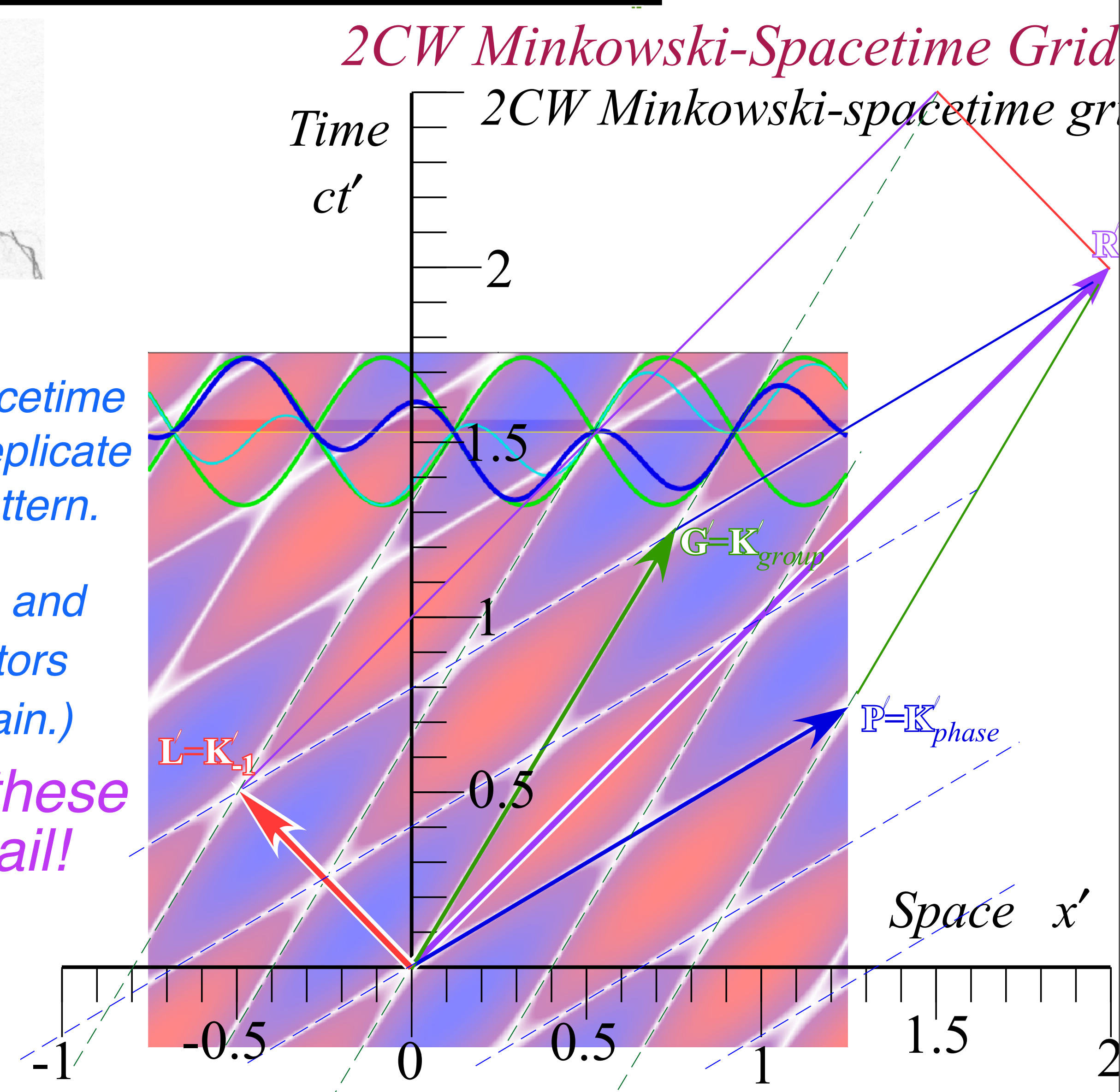




Bob: The spacetime wave-zeros replicate the same pattern.

(Except $\mathbf{P}'$-*phase* and $\mathbf{G}'$-*group* indicators get switched again.)

Let's *measure* *these in careful detail!*



Phase vector **P** 1/2-sum vector $\quad K'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$

Group vector **G** 1/2-diff vector $\quad K'_{group}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}$

Lecture 30
Thur. 12.10.2015

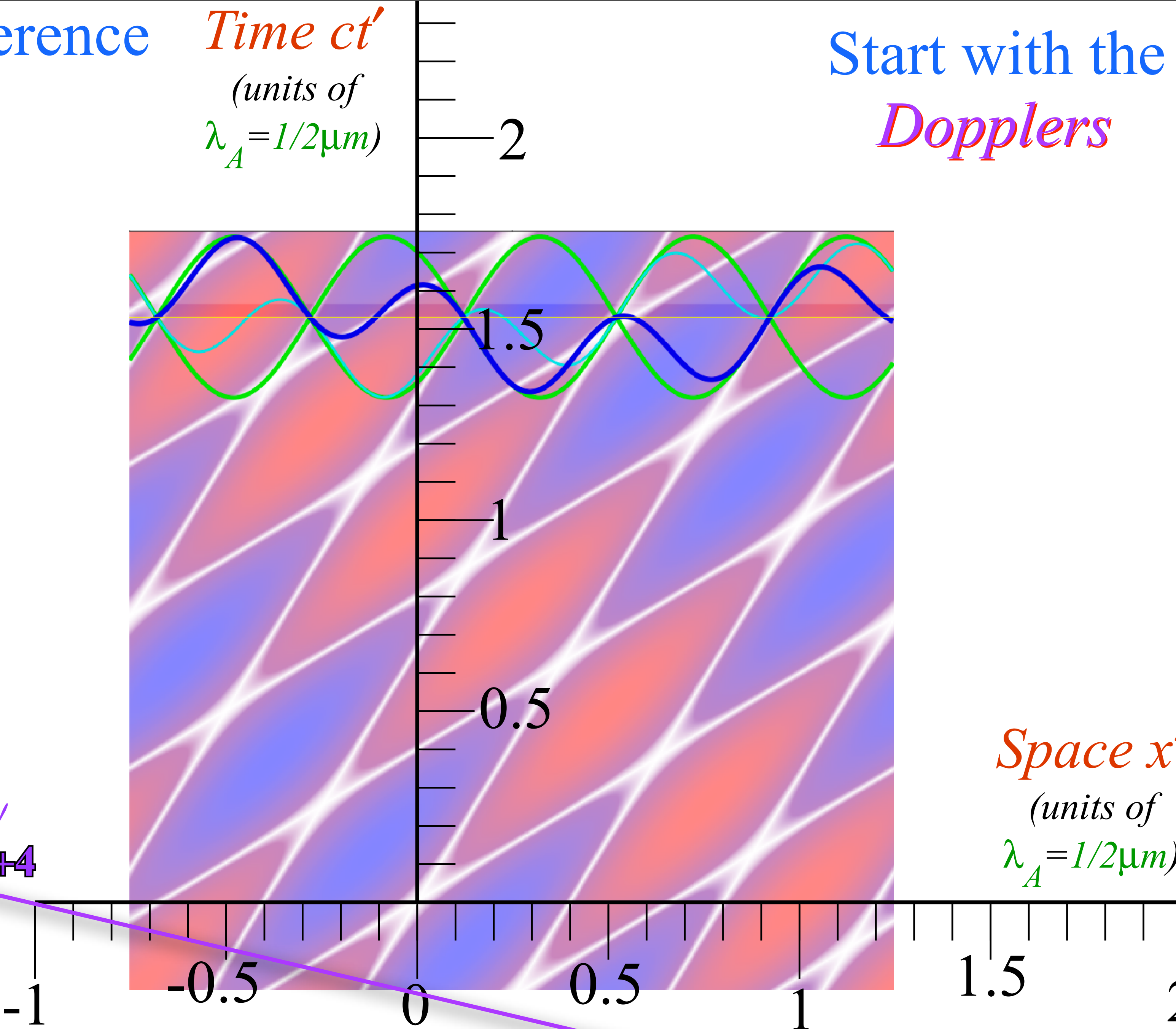
How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
- Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Doppler shifted "baseball-diamond" displays Lorentz frame transformation

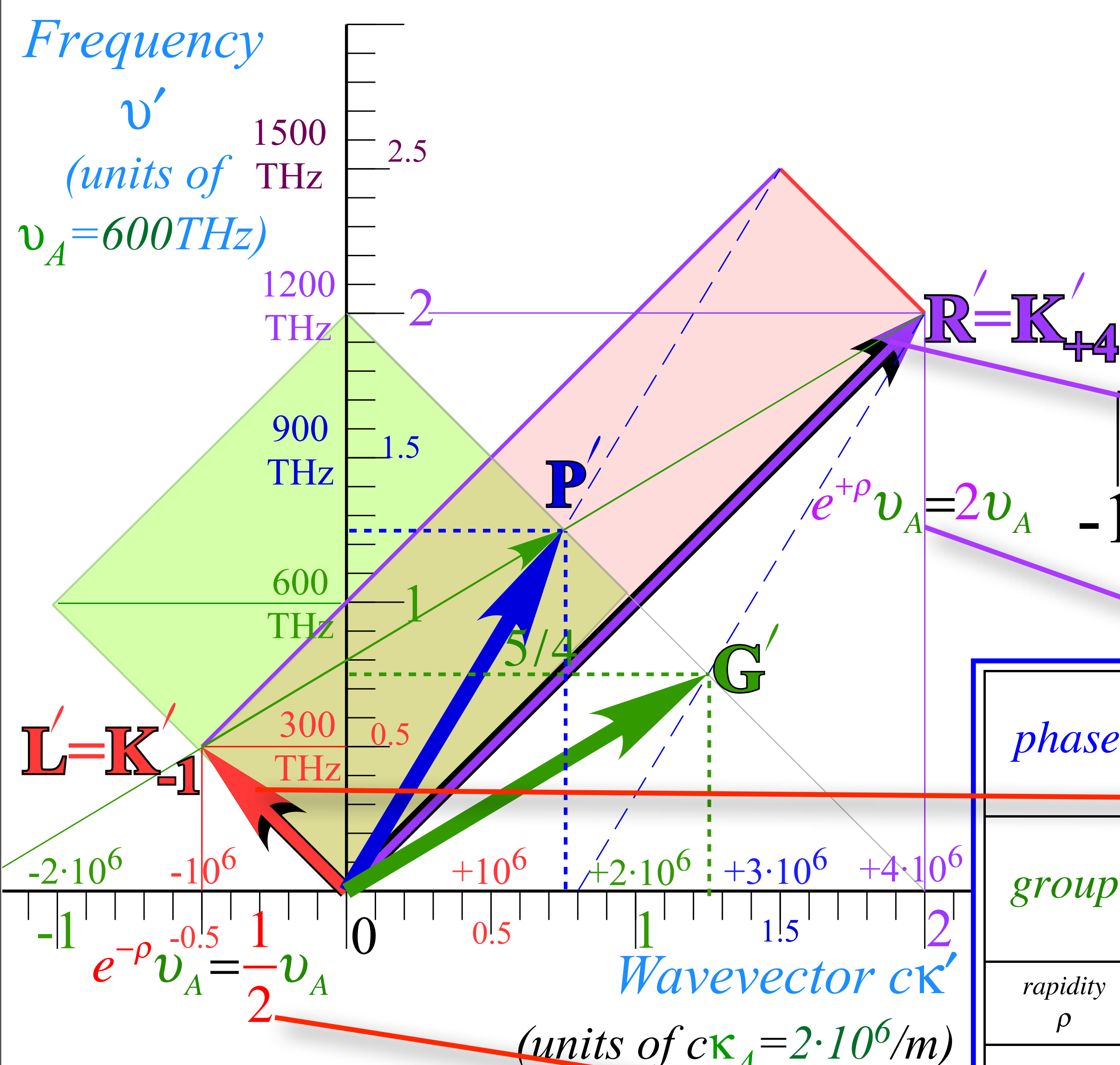
Developing Axioms to update Galileo's relativity: A critical look at *c*-axioms
- Einstein's PW (Pulse-Wave) Axiom
- Evenson's CW (Continuous Wave) Axiom and Occam's Razor
- Analyzing wave velocity by per-space-per-time *and* space-time graphs
- Introducing optical Doppler effects
- Clarifying Evenson's CW Axiom using Doppler effects
  - *Galileo's Revenge (part1):* Galilean Doppler-shift arithmetic using *rapidity* ρ

Developing optical "baseball-diamond" and relativistic ρ-functions and transformations
- Details of 1CW wavefunctions and phasors
- Details of 2CW wavefunctions in rest frame
  - *Galileo's Revenge (part2):* Galilean addition of phasor angular velocity
  - Structure of rest frame "baseball-diamonds"
- Details of 2CW wavefunctions of moving frame velocities of *phase* and *group* waves
  - ➔ 16 coefficients of relativistic 2CW interference
  - Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

The 16 dimensions of 2CW interference

Start with the *Dopplers*





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - 16 Relativity *Dimensions*

The 16 dimensions of 2CW interference

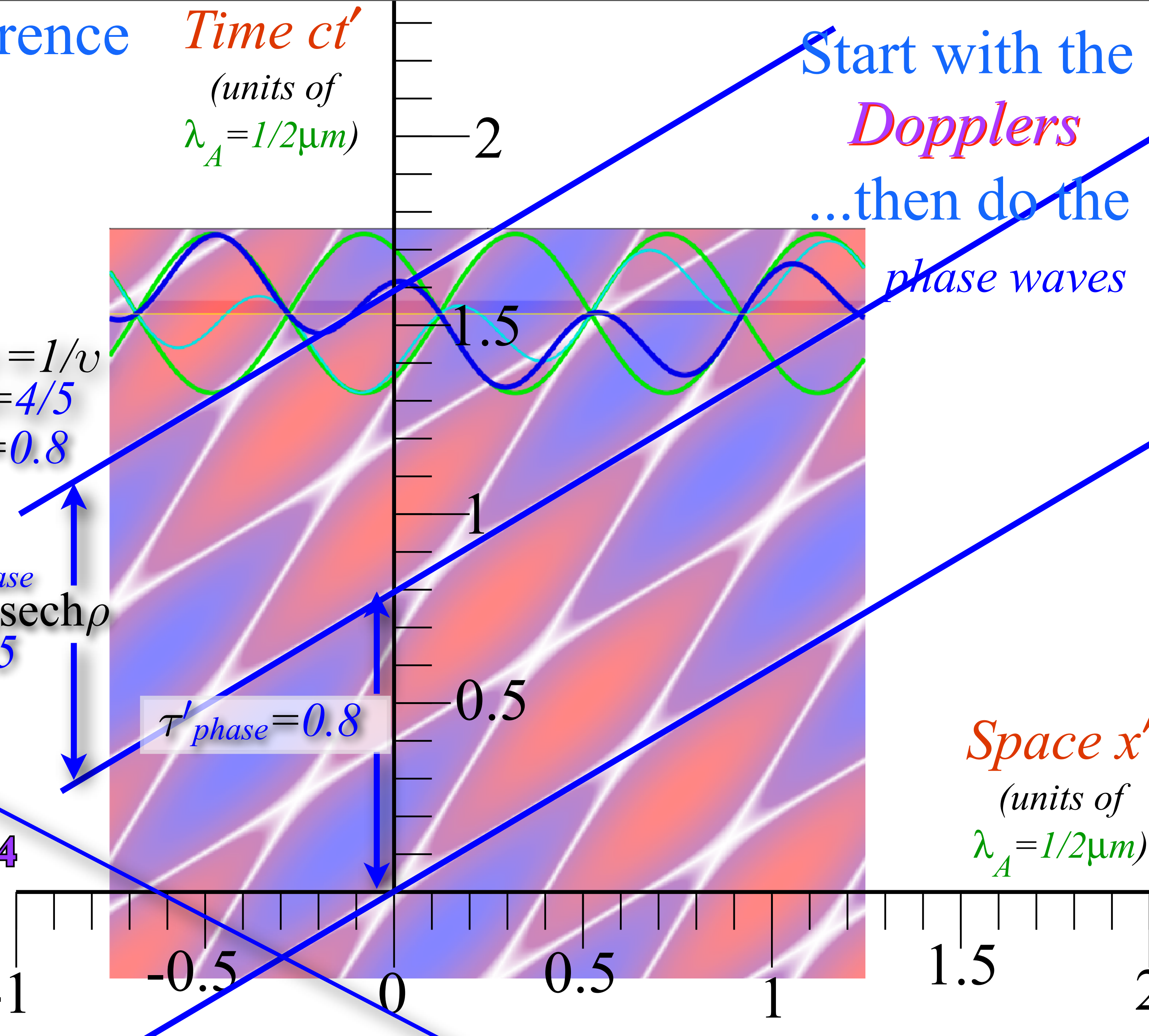
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

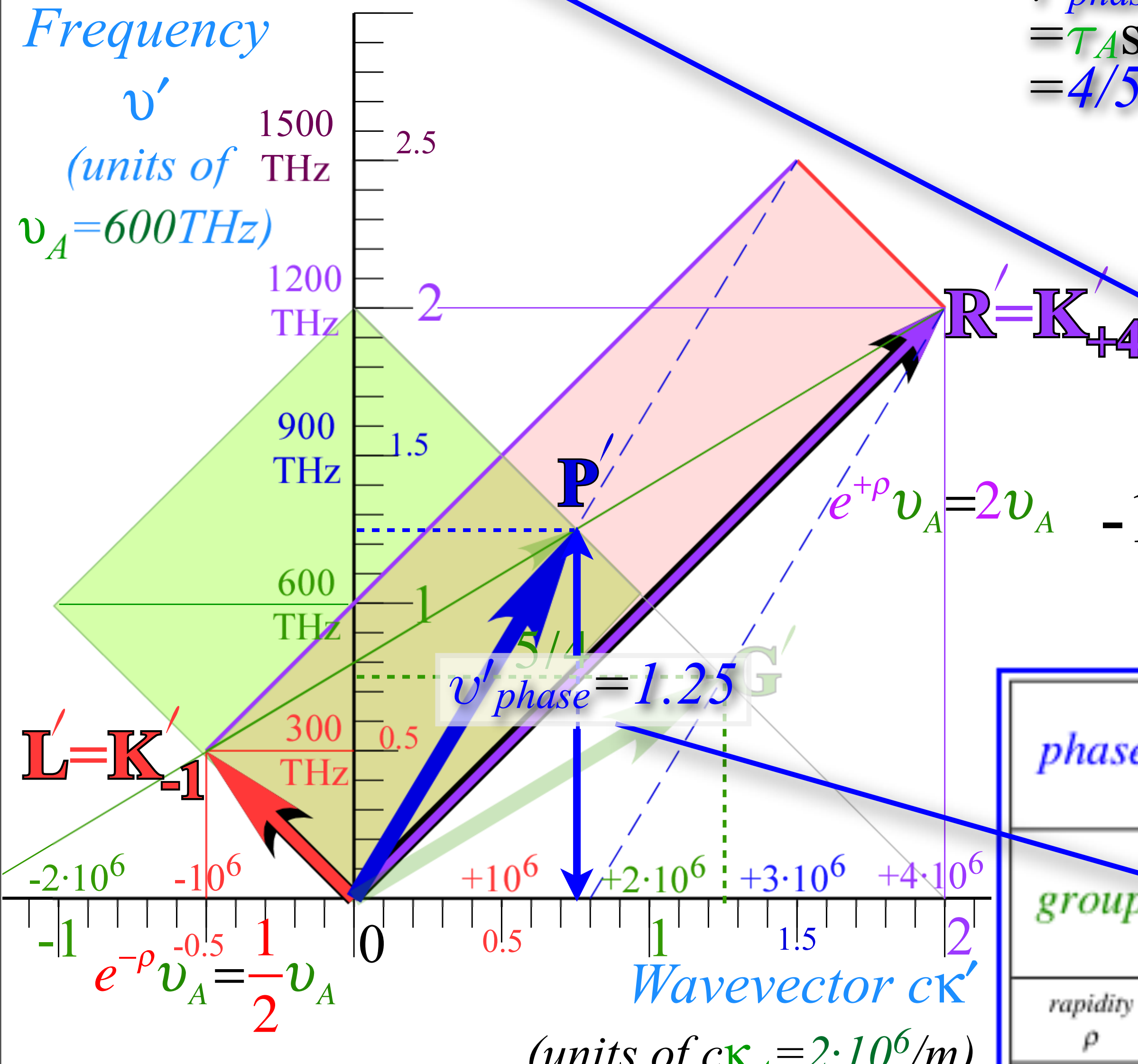
Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$
$= 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$
$= 0.8$

Start with the *Dopplers*
...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

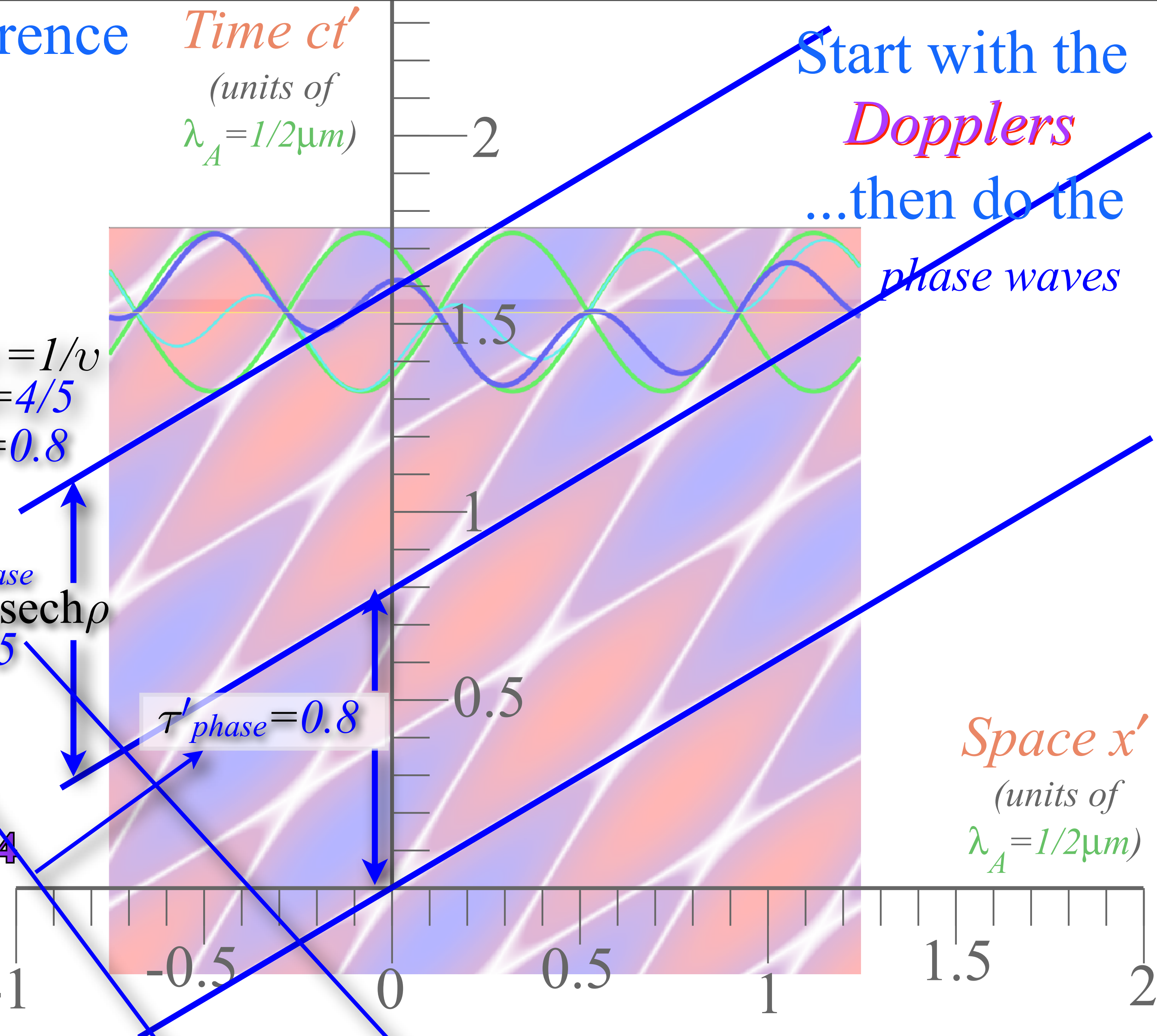
The 16 dimensions of 2CW interference

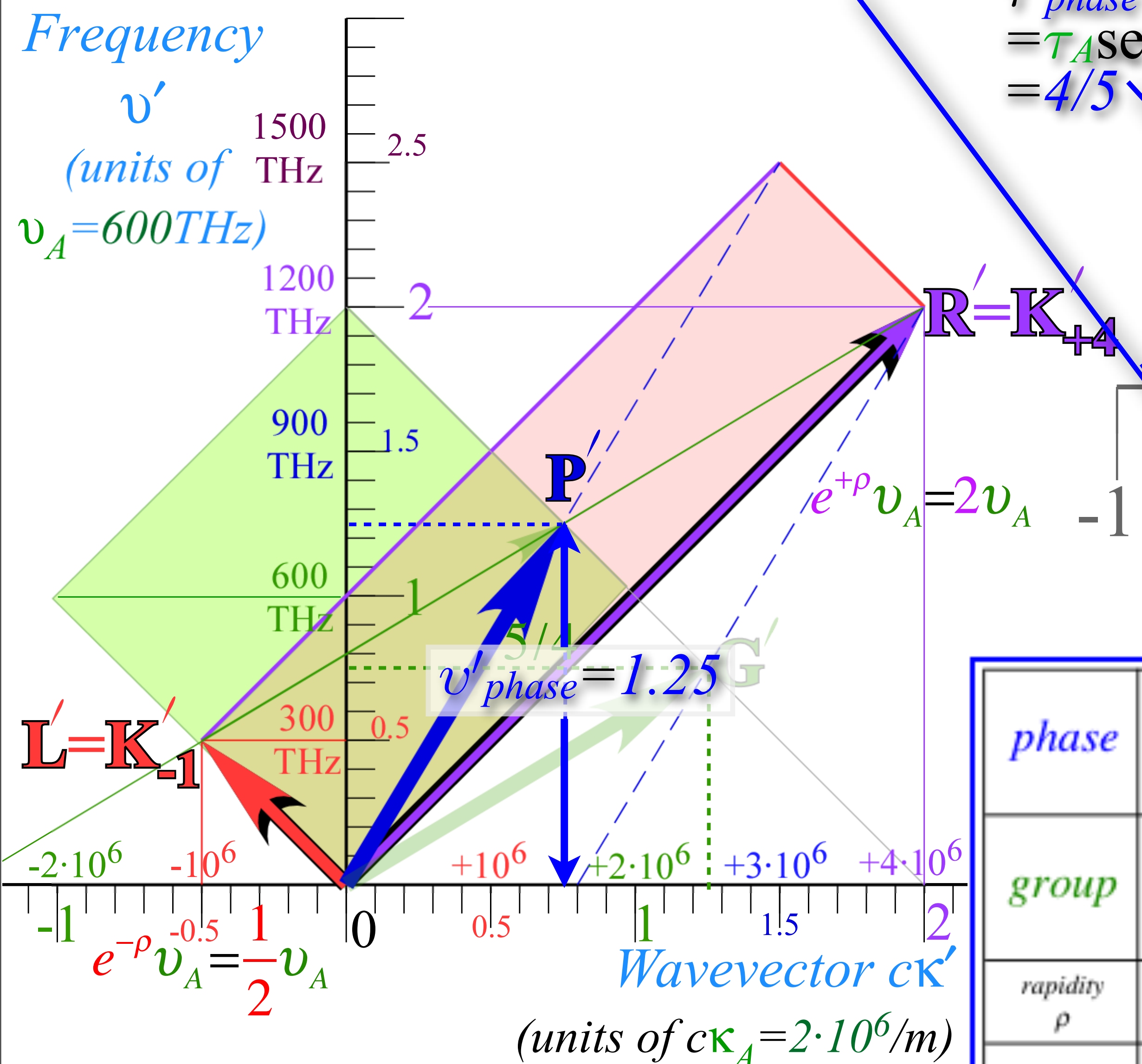
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$
$= 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$
$= 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

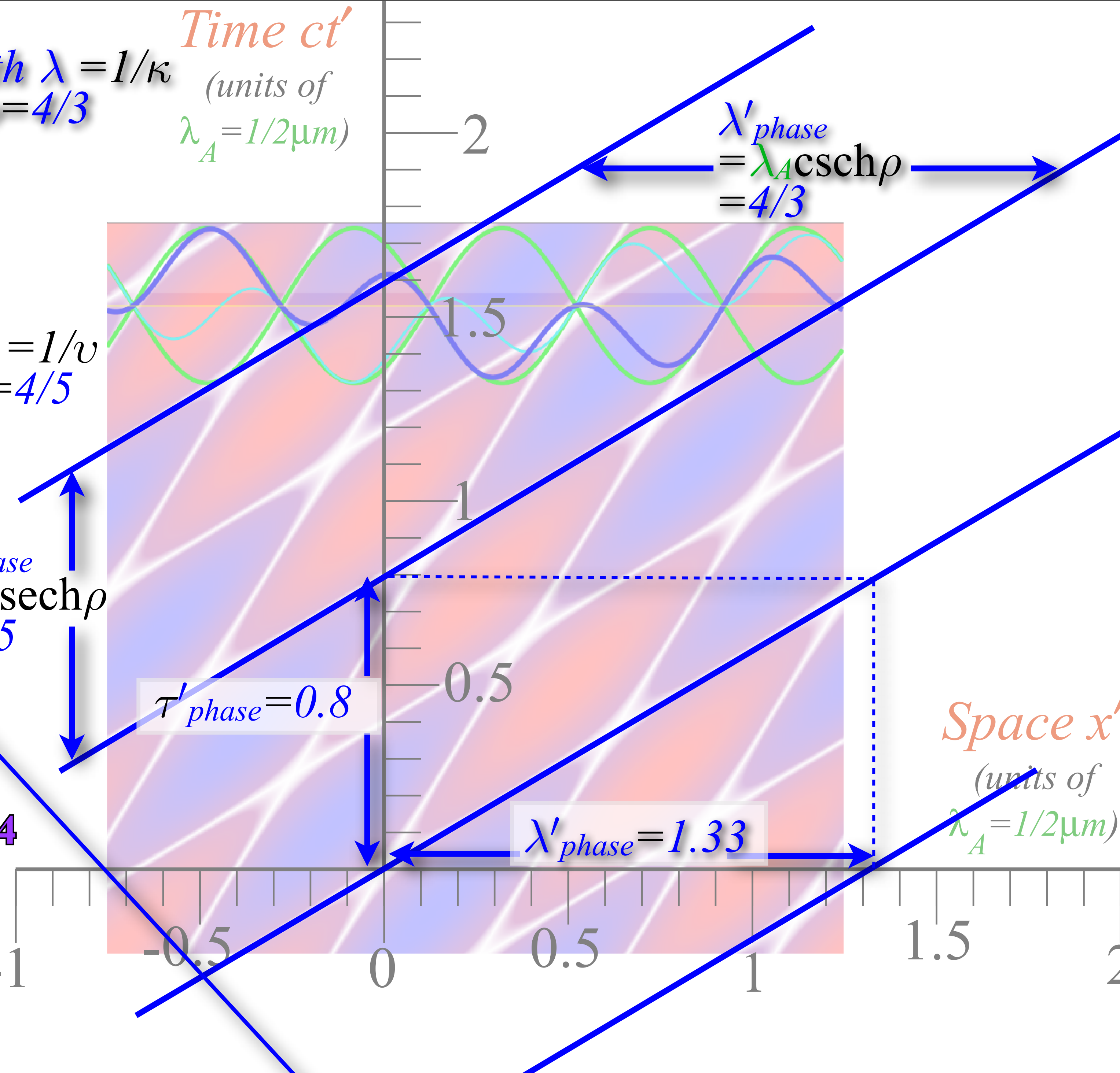
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

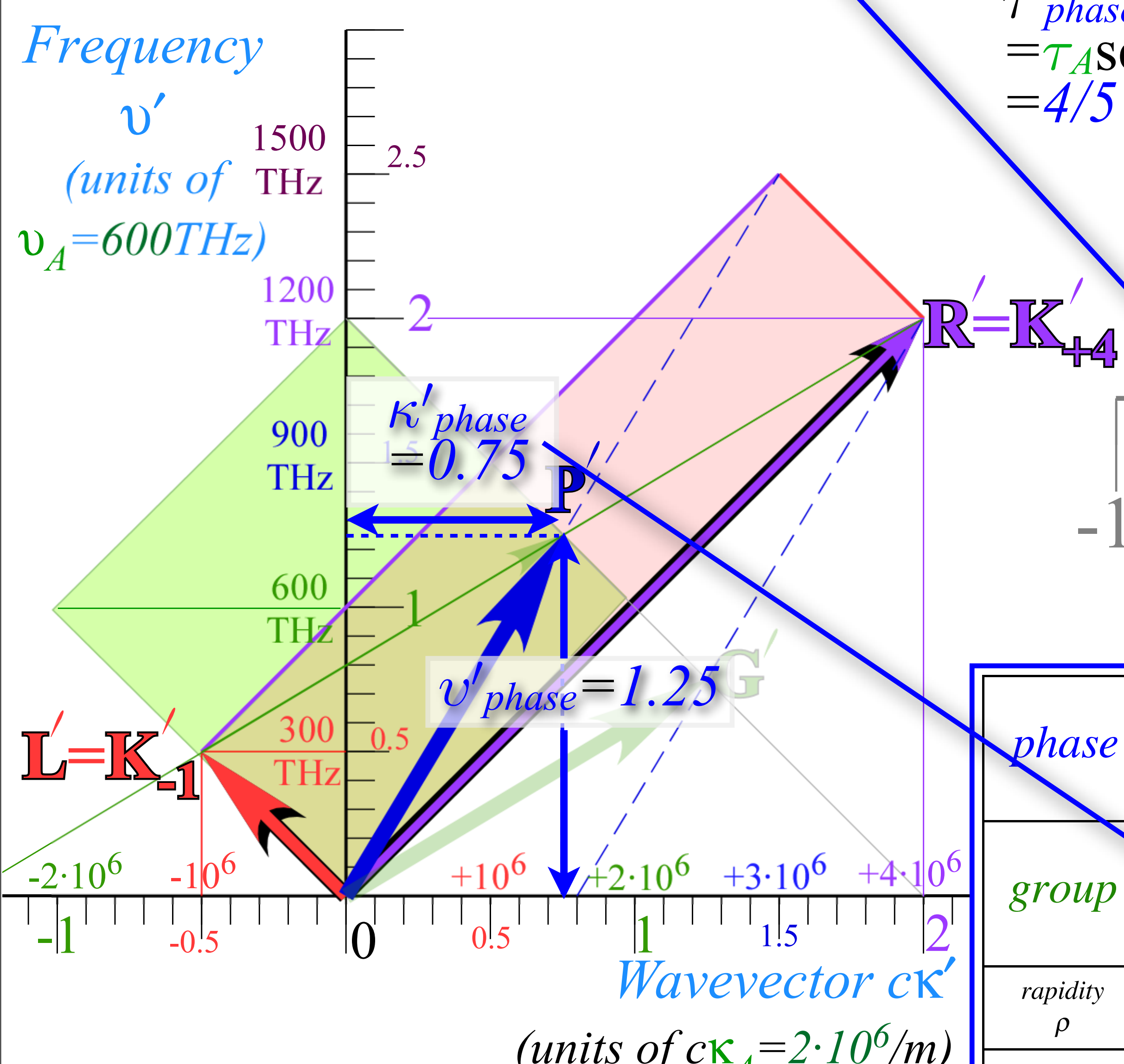
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

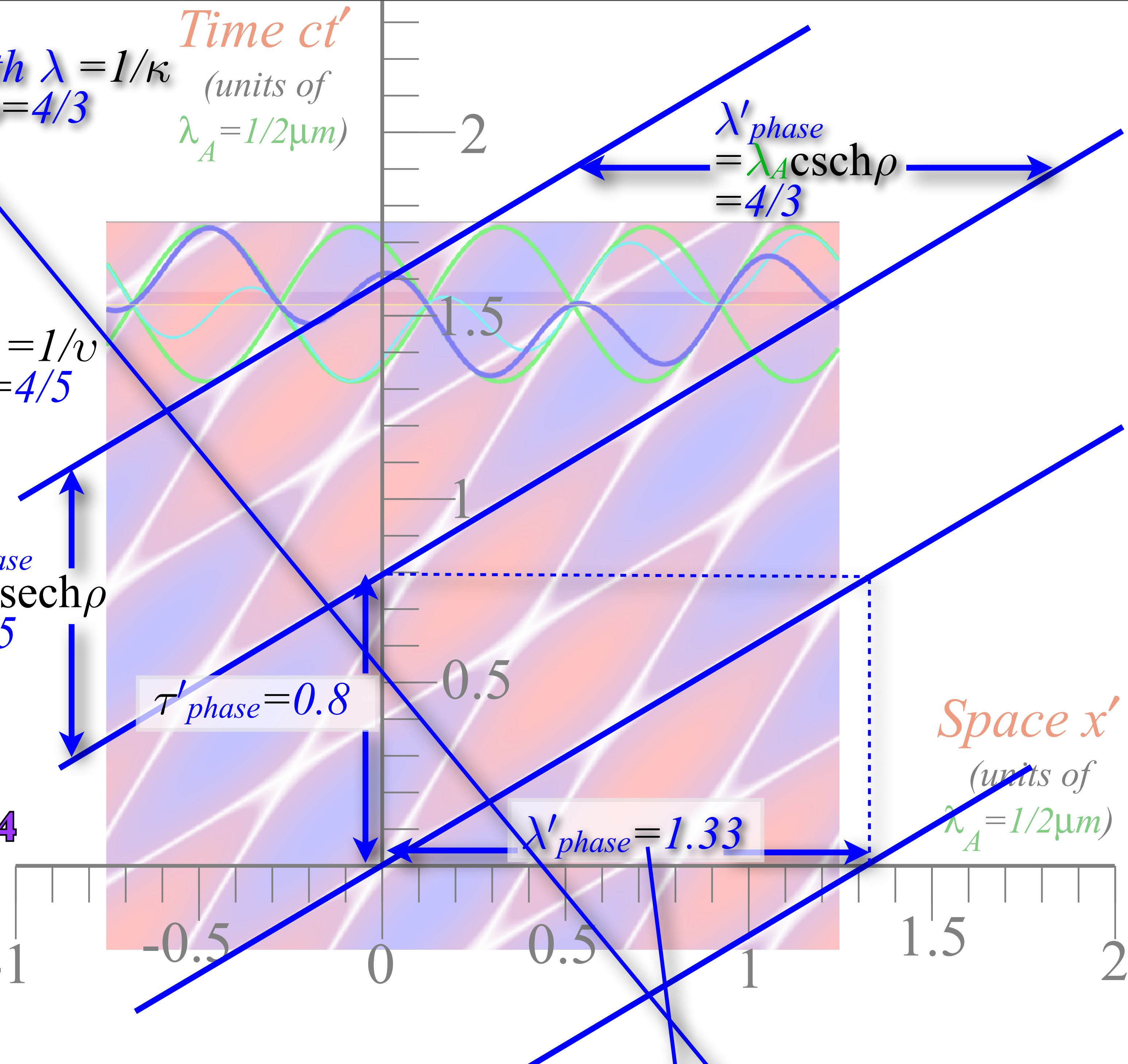
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

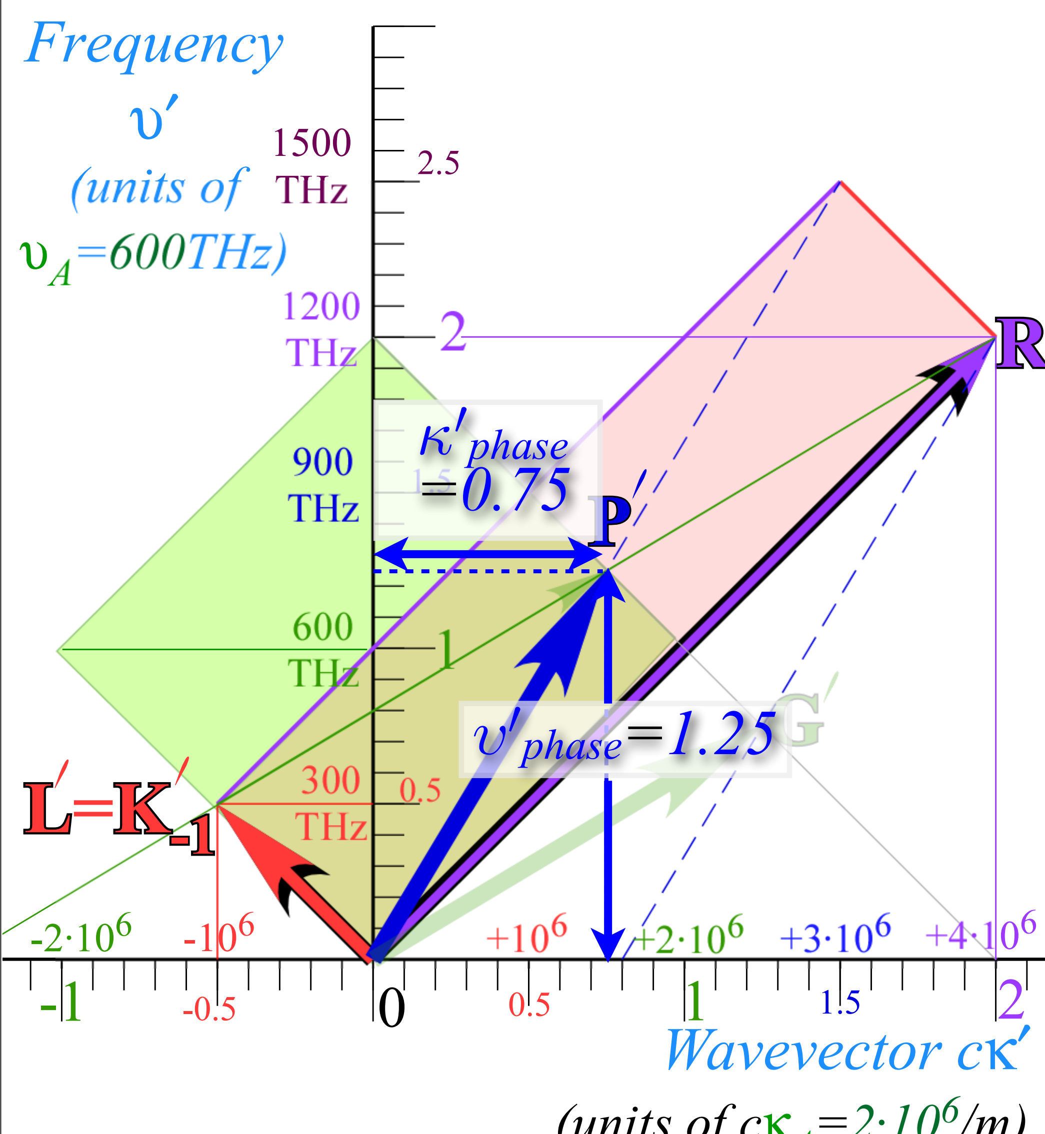
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

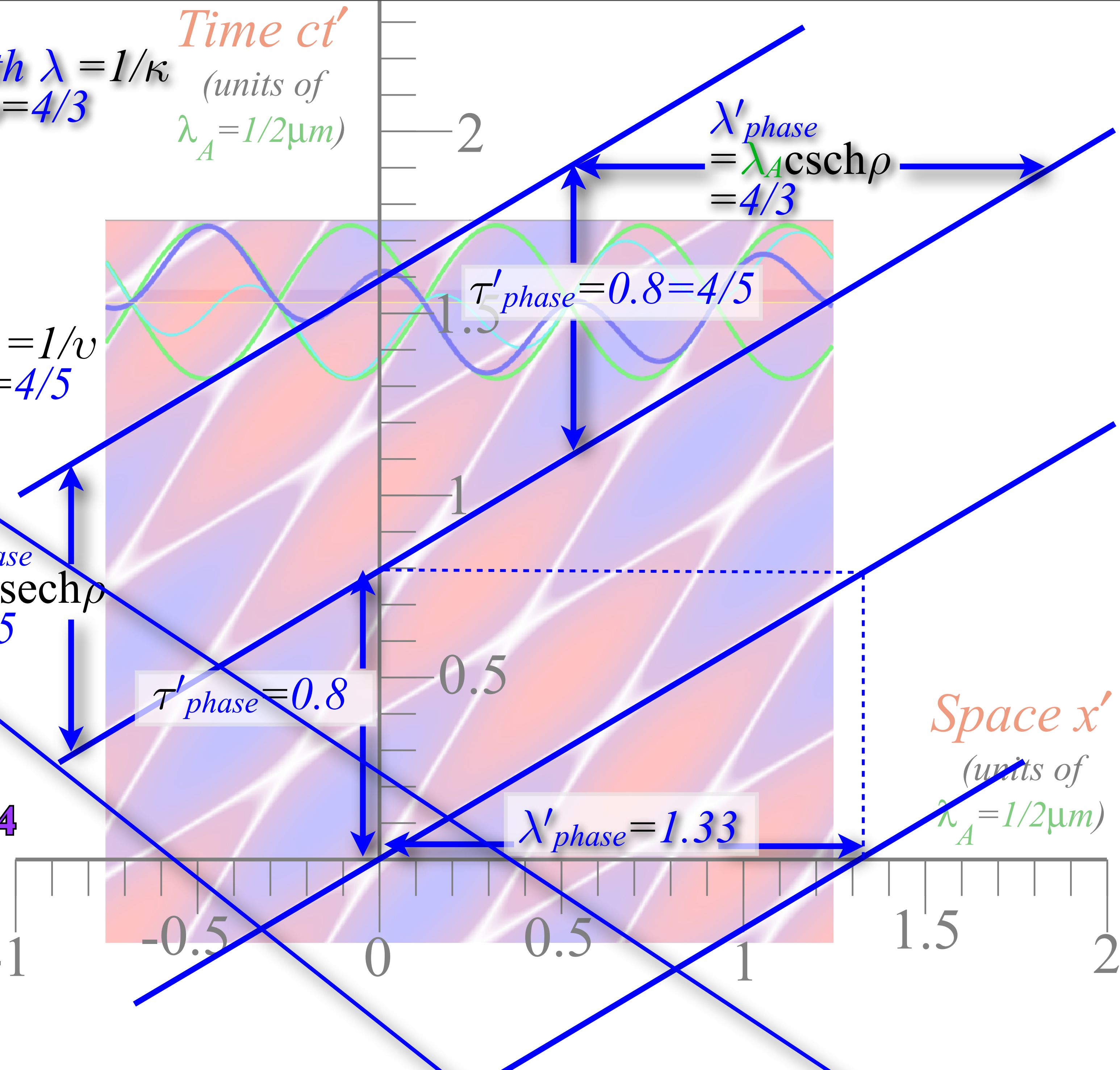
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

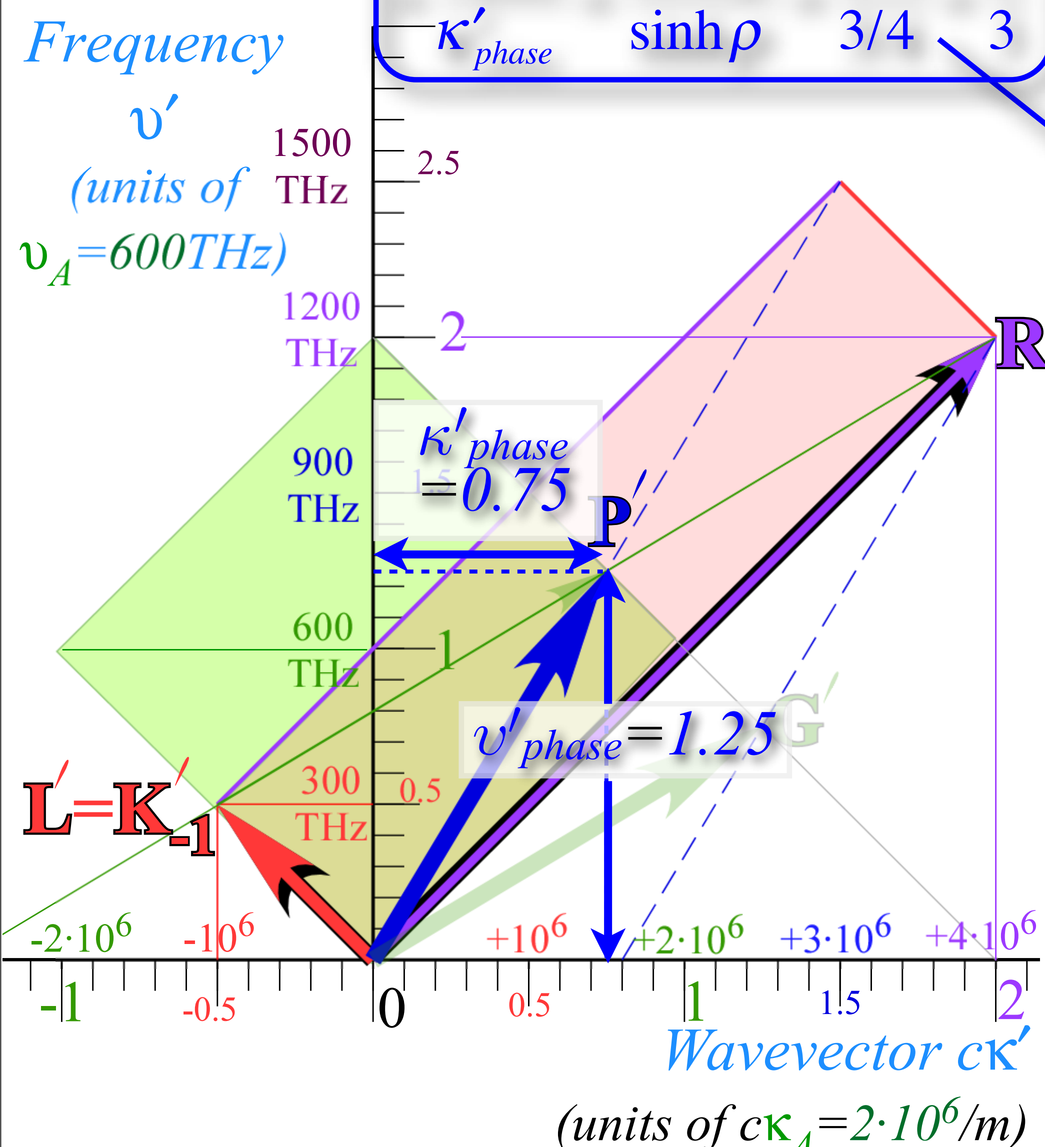
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

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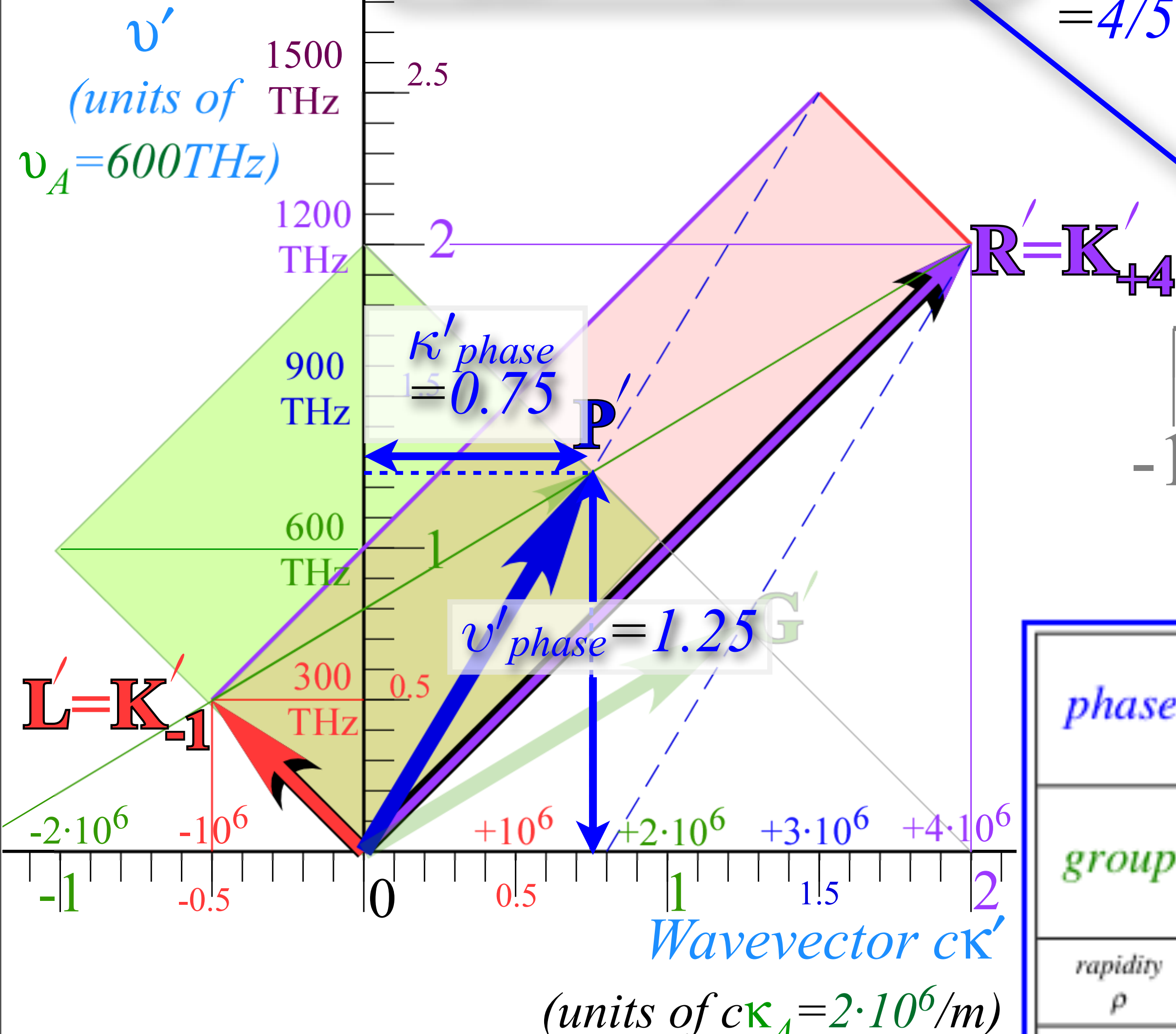
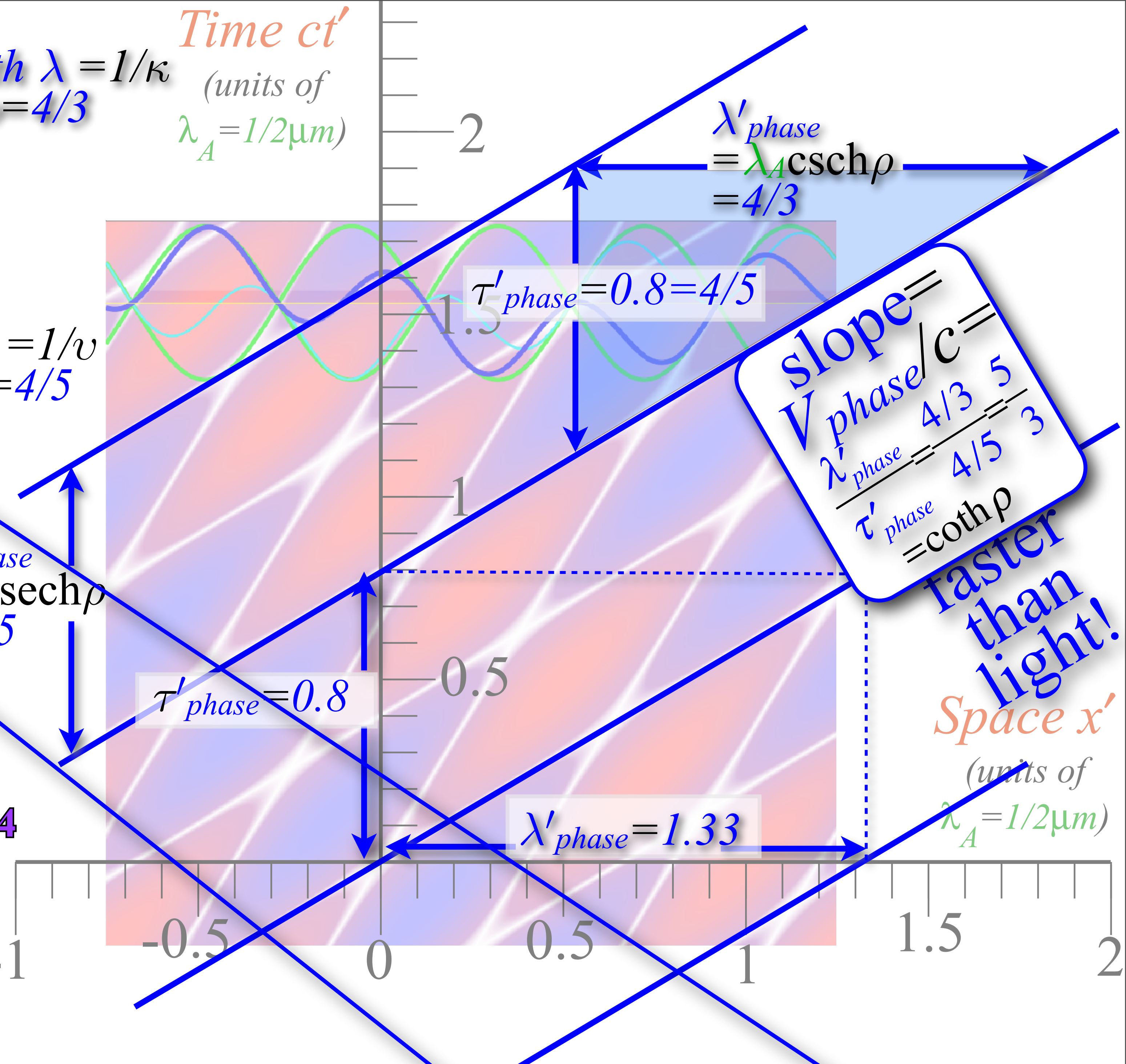
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c = \dfrac{\upsilon'_{phase}}{\kappa'_{phase}} = \dfrac{\cosh\rho}{\sinh\rho} = \dfrac{5/4}{3/4} = \dfrac{5}{3}$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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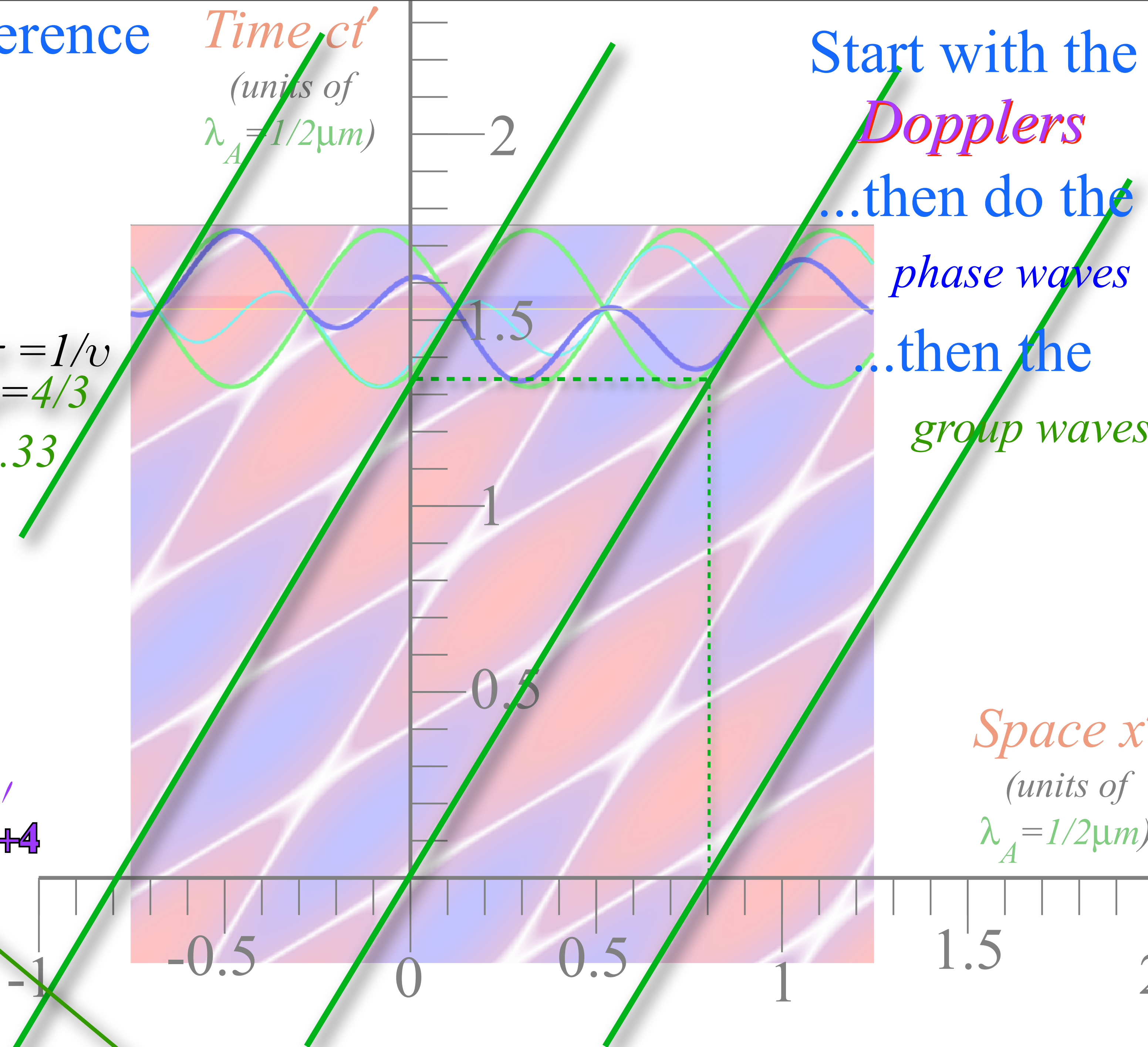
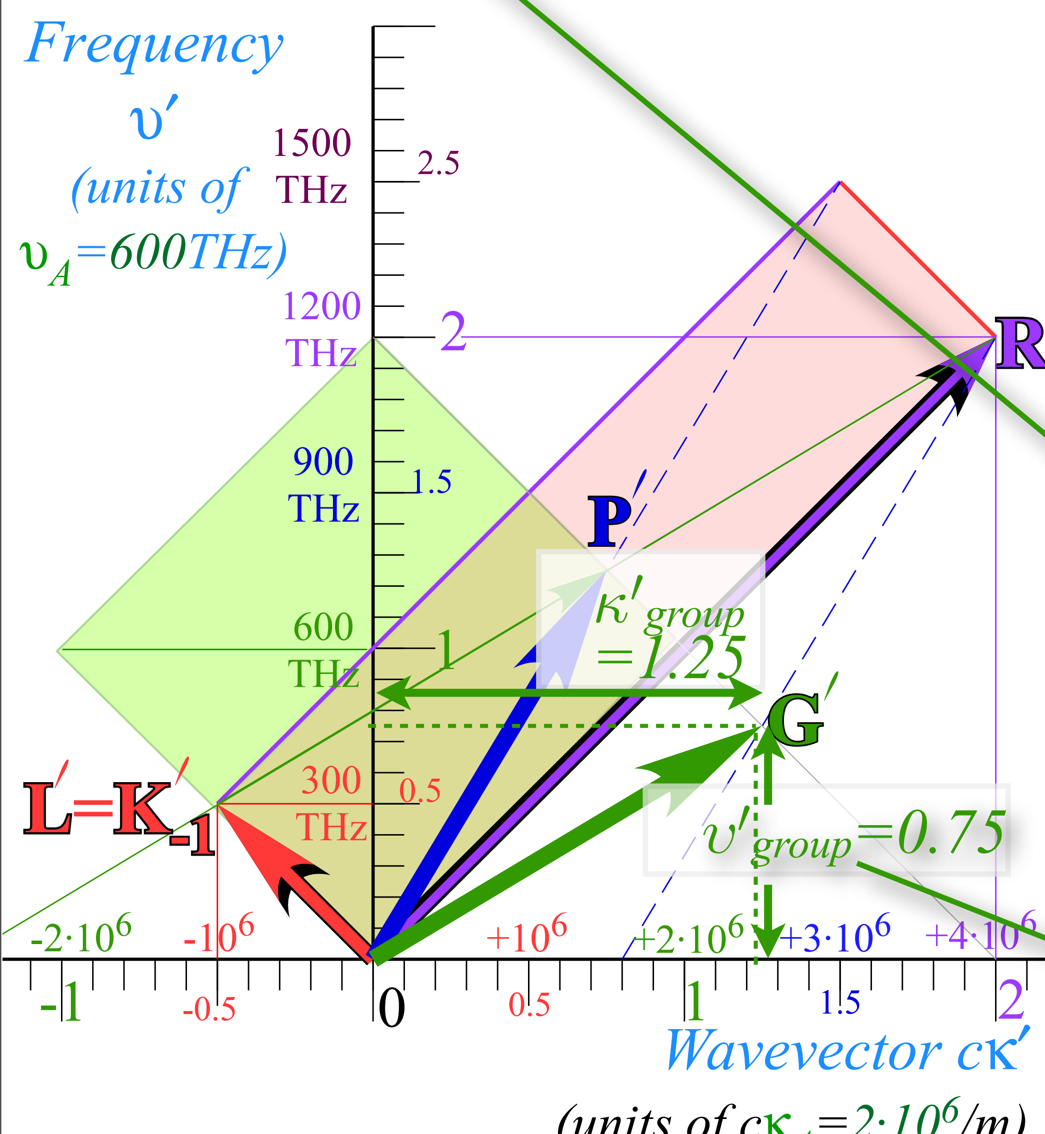
The 16 dimensions of 2CW interference

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$ flips to *Group period* $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

Start with the *Dopplers* ...then do the *phase waves* ...then the *group waves*





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4$ $= 0.75$

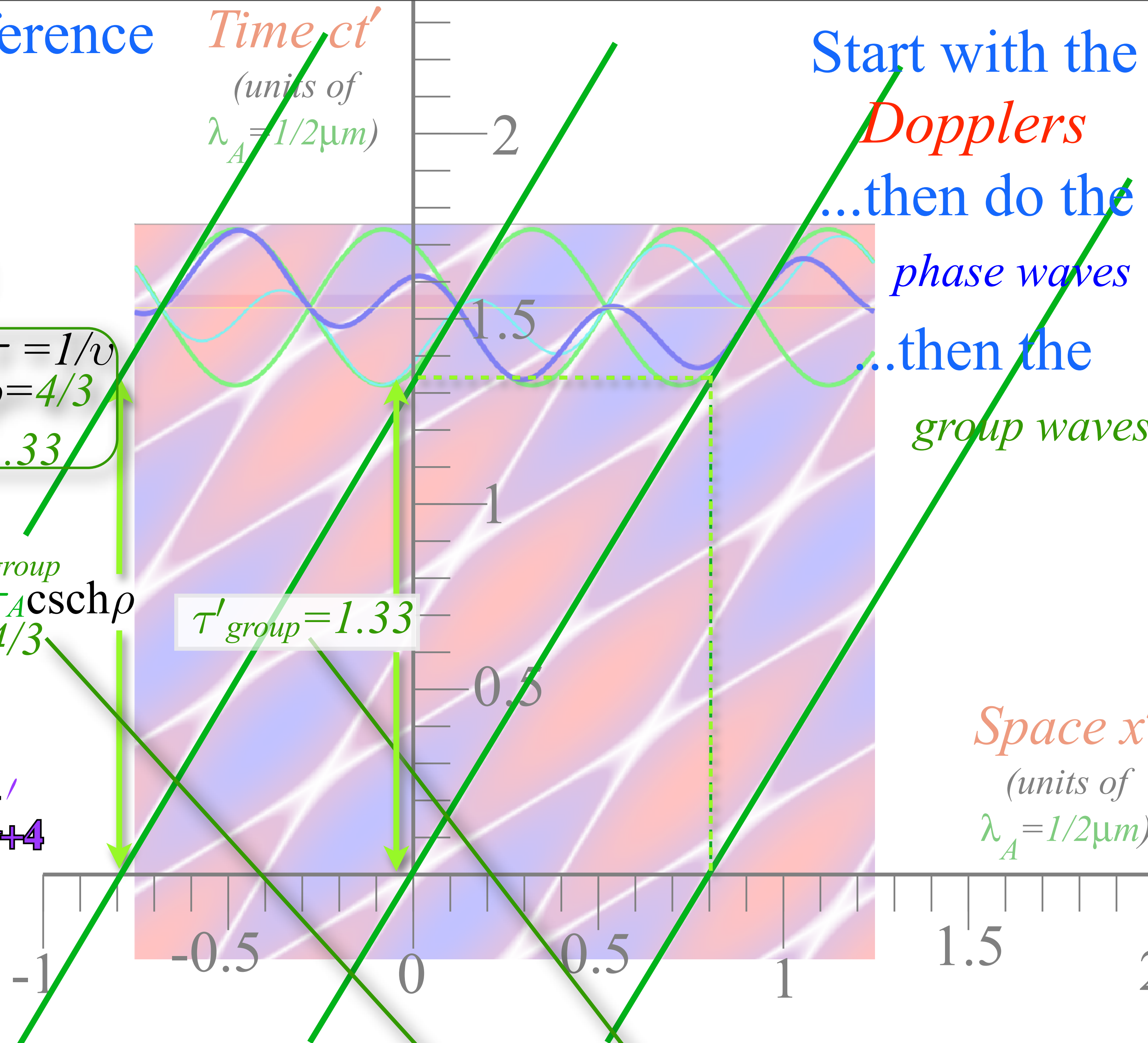
flips to

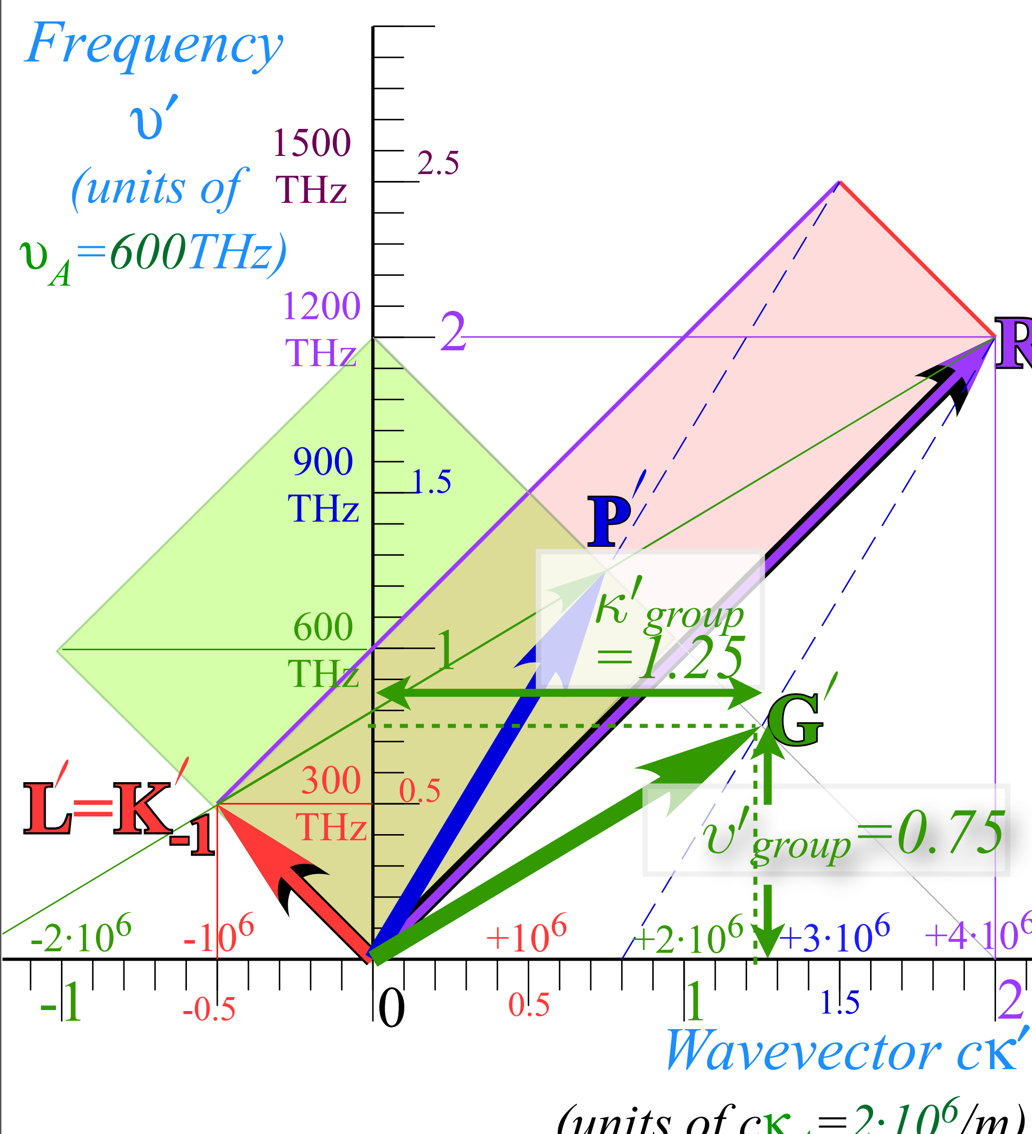
Group period $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$ $= 1.33$

Start with the *Dopplers*

...then do the *phase waves*

...then the *group waves*





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

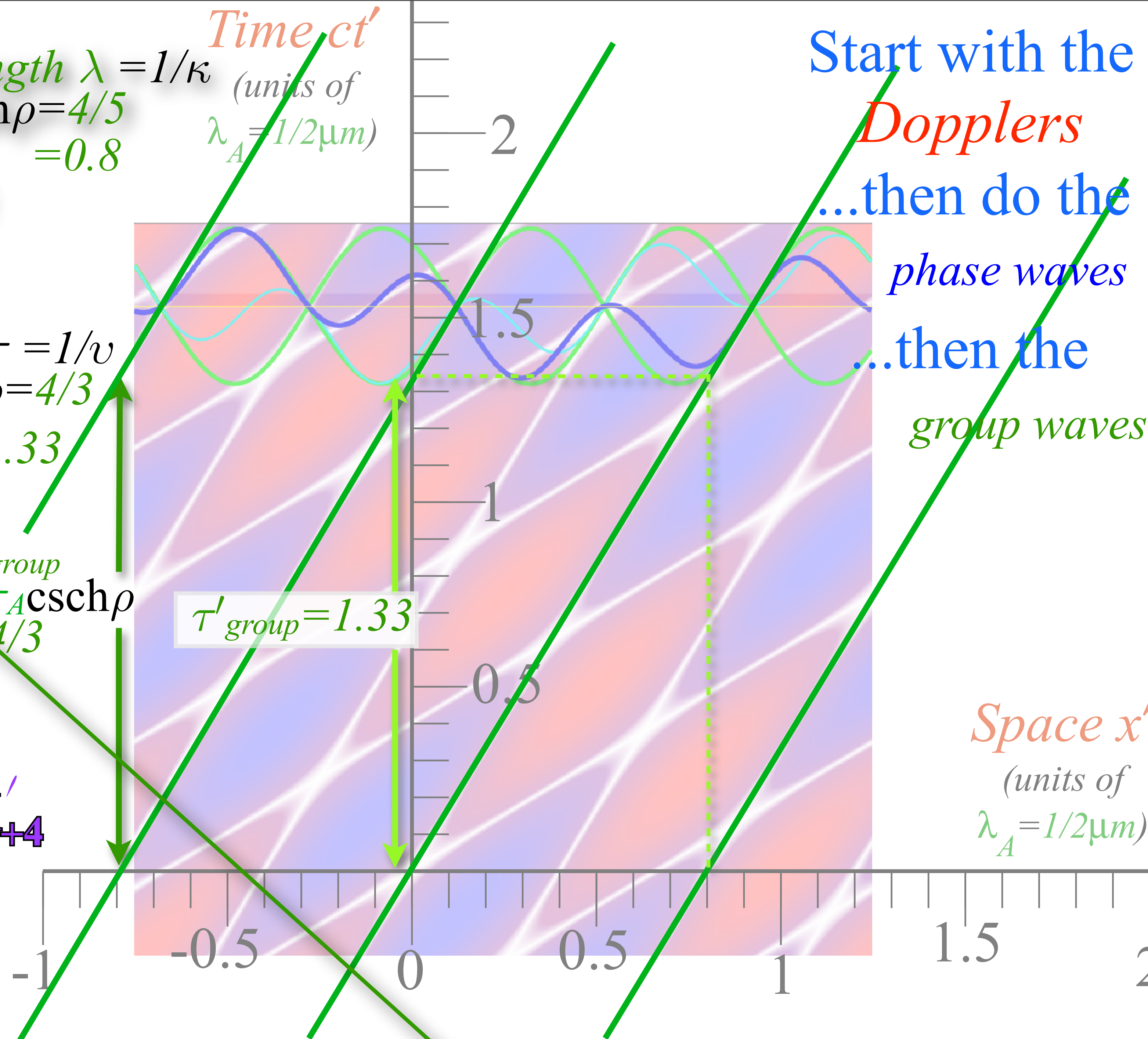
$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

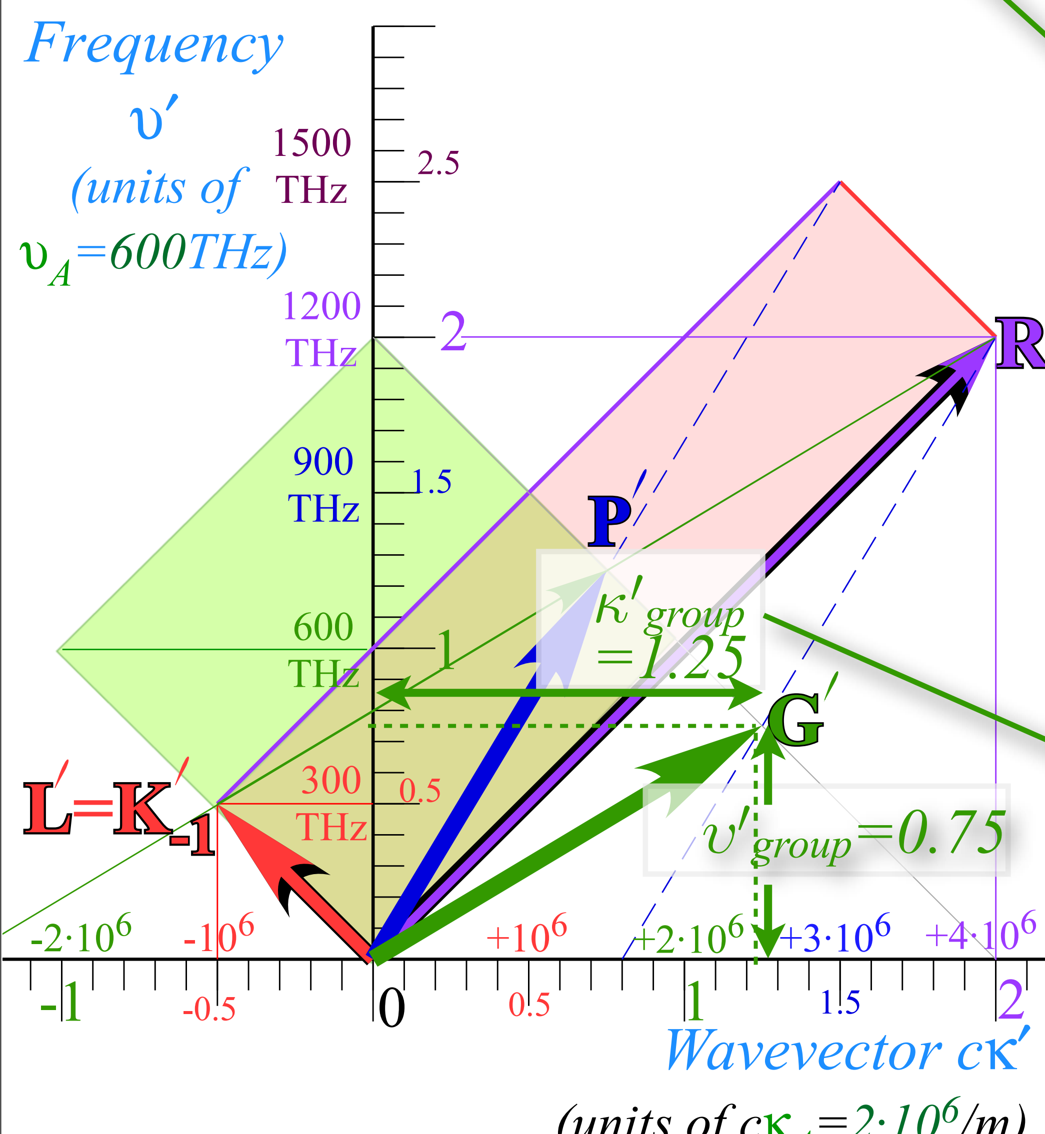
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

Start with the Dopplers ...then do the *phase waves* ...then the *group waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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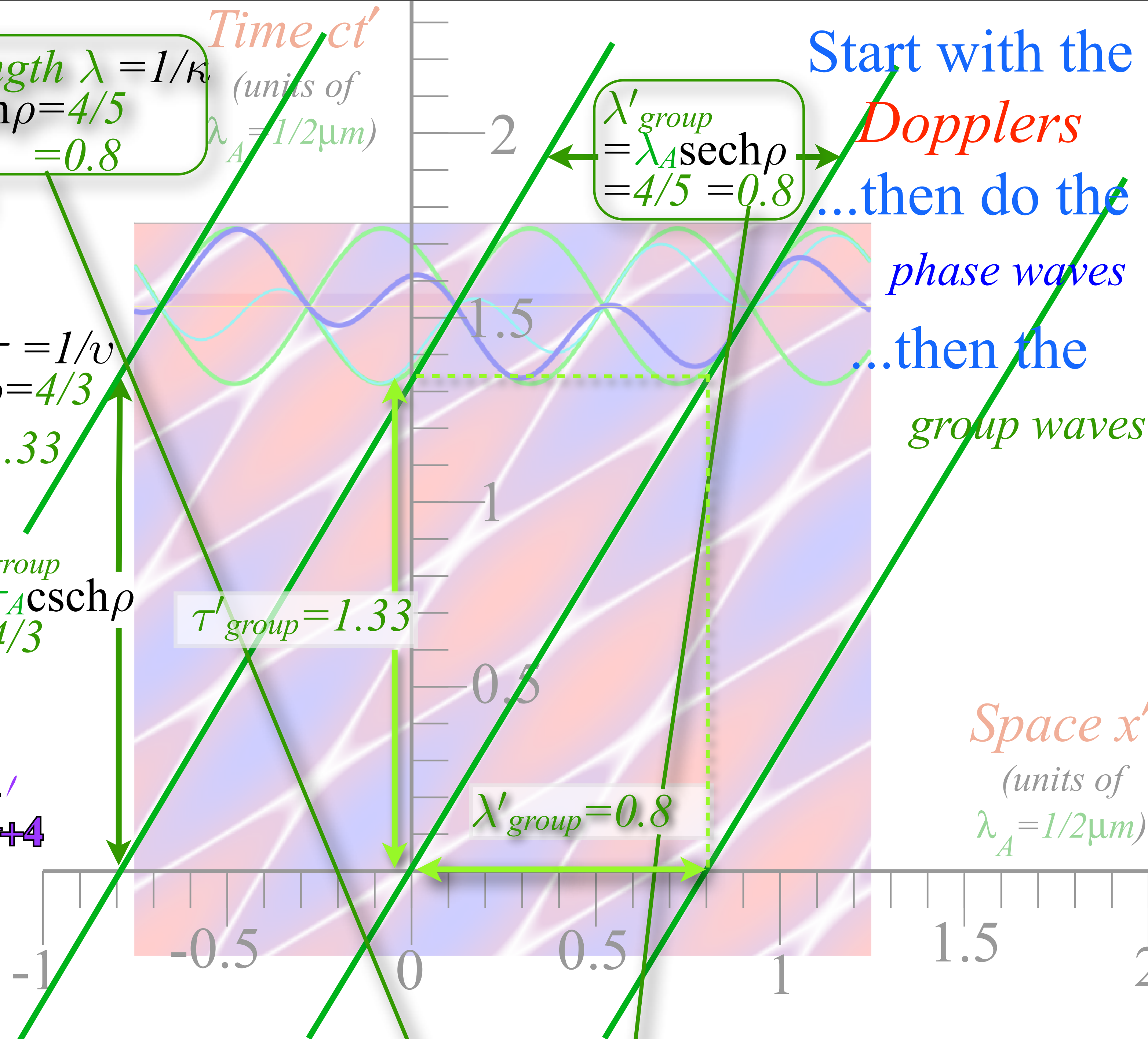

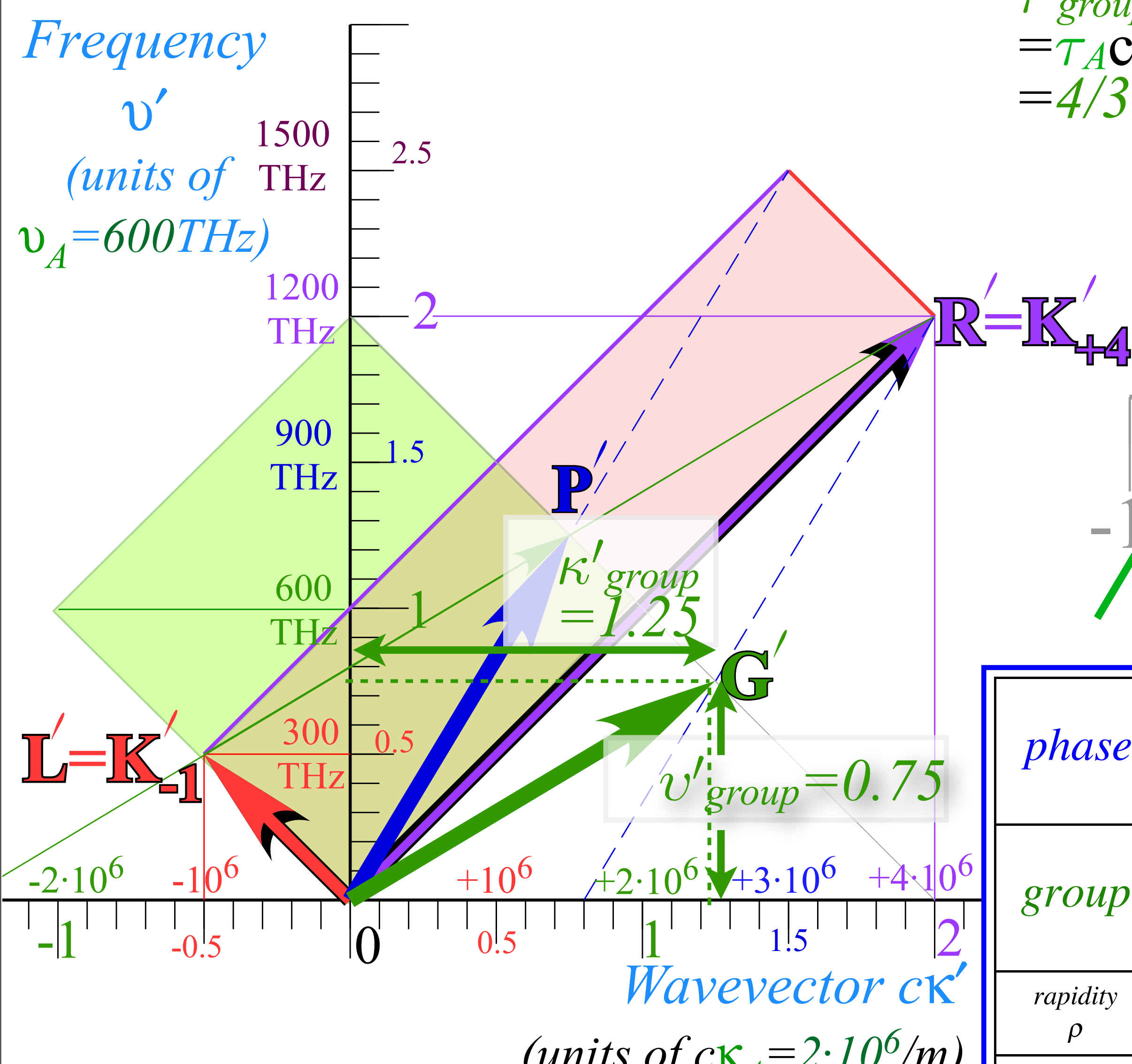

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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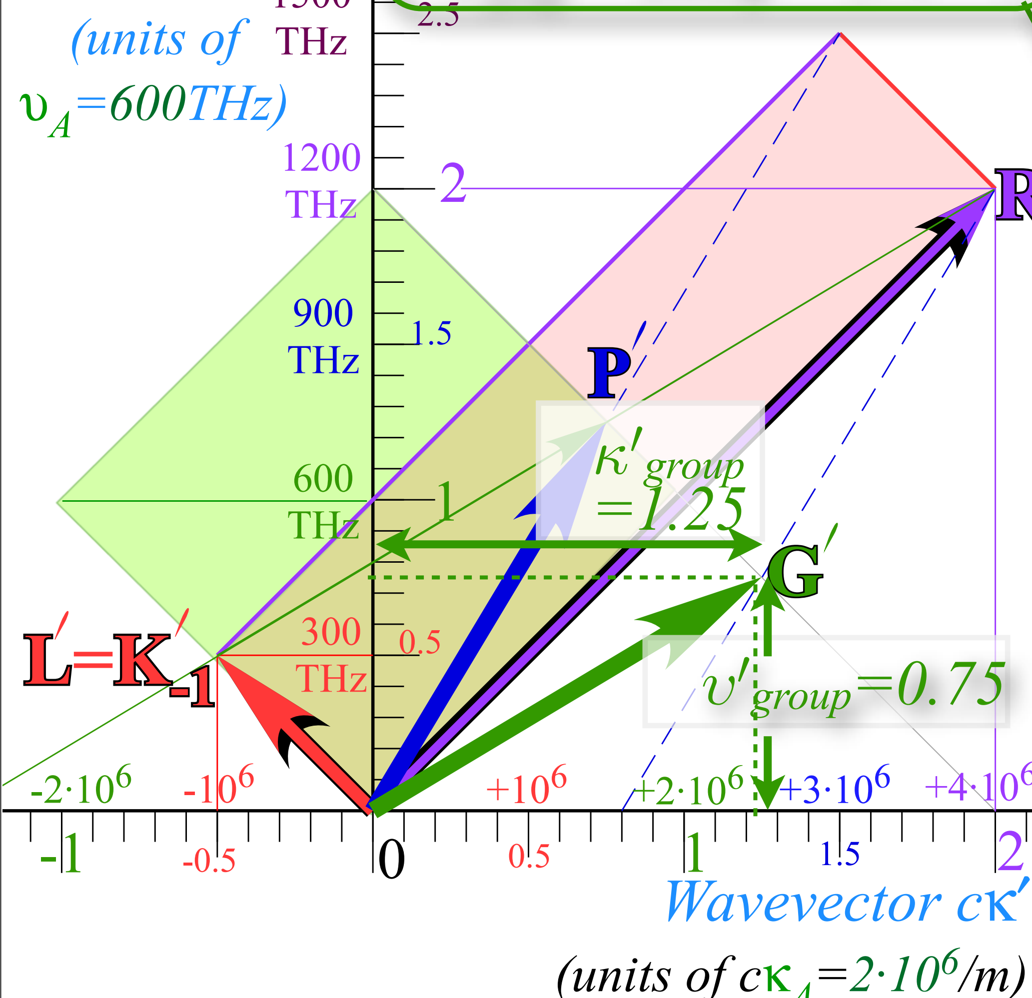
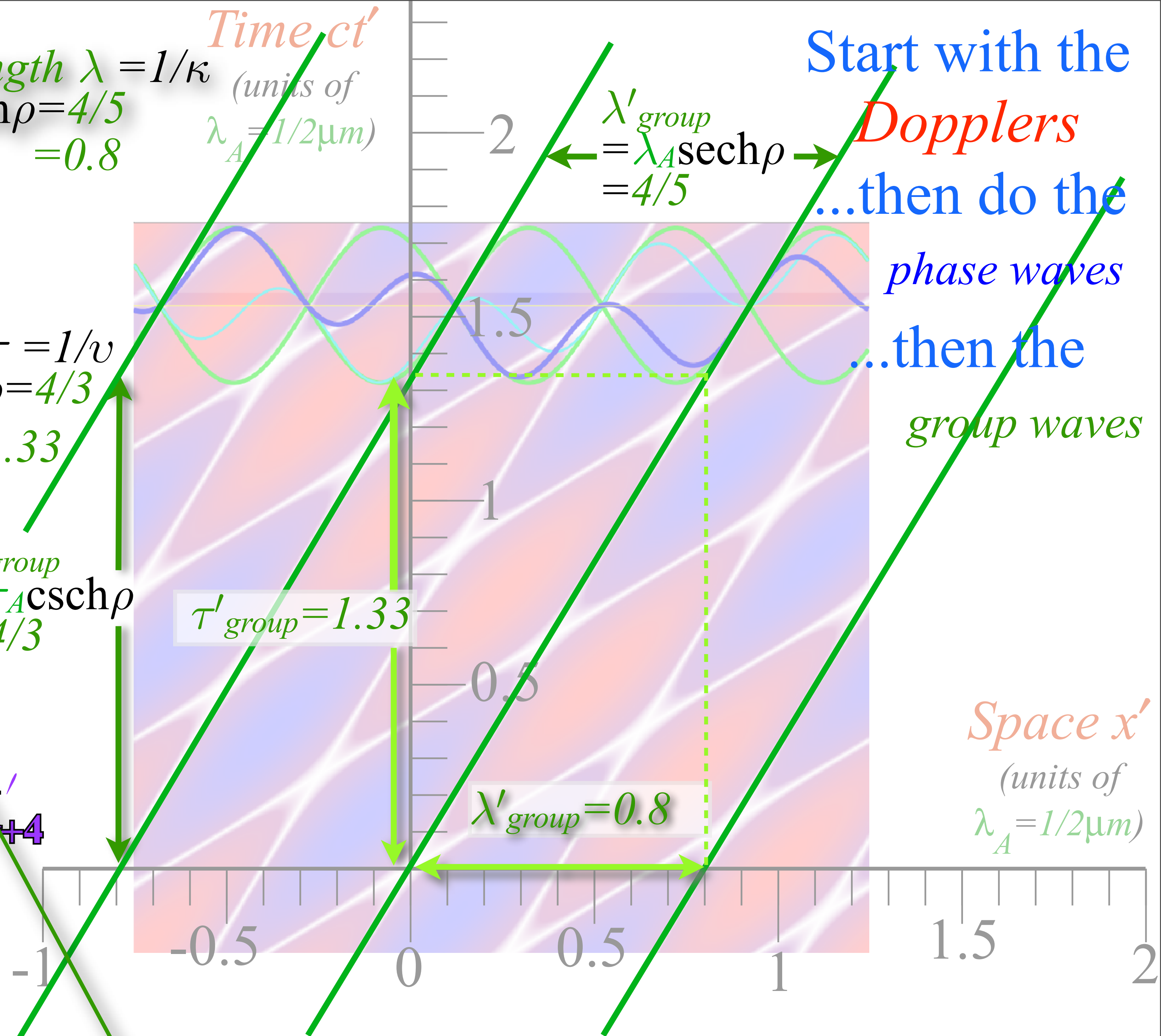
G-slope $= V_{group}/c$

$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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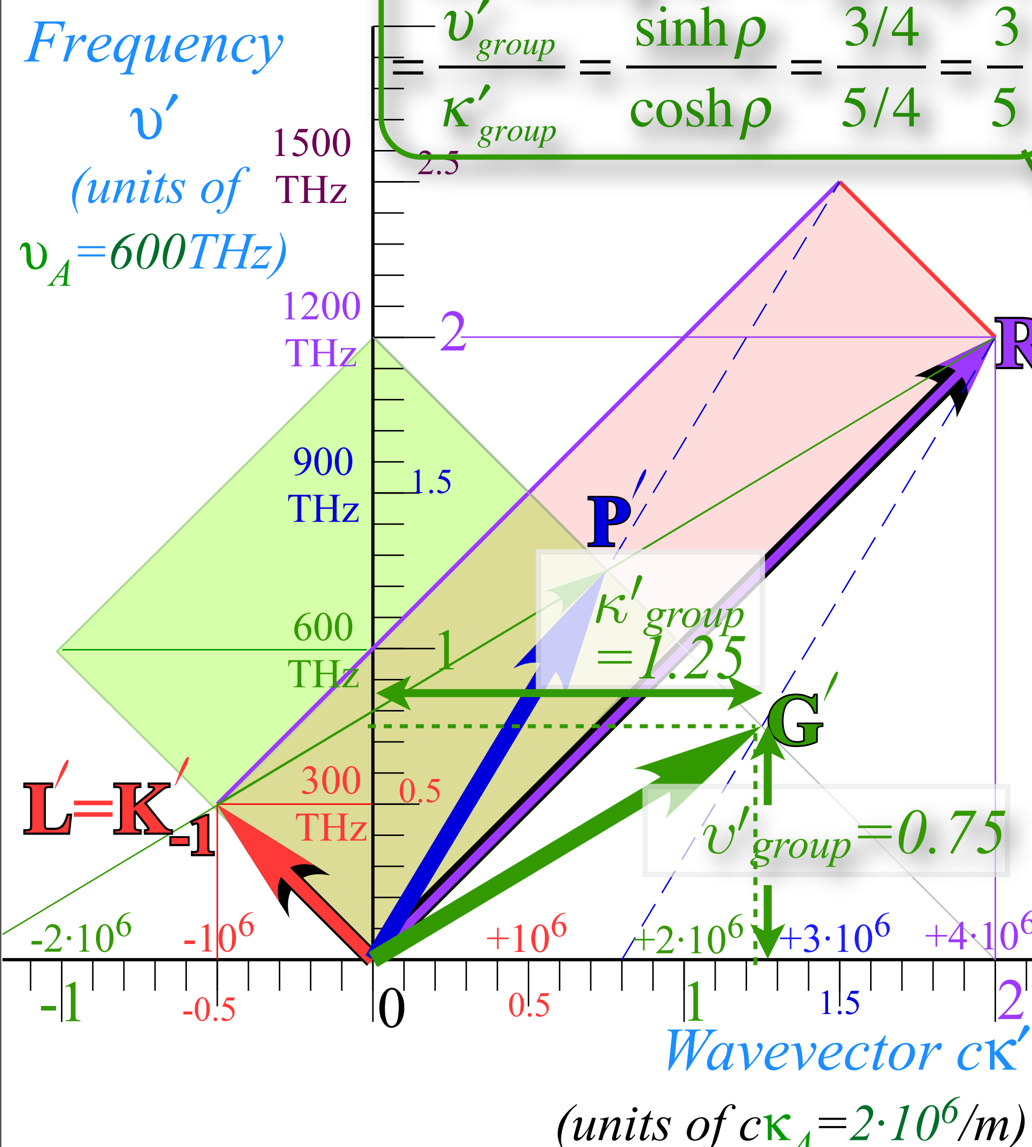

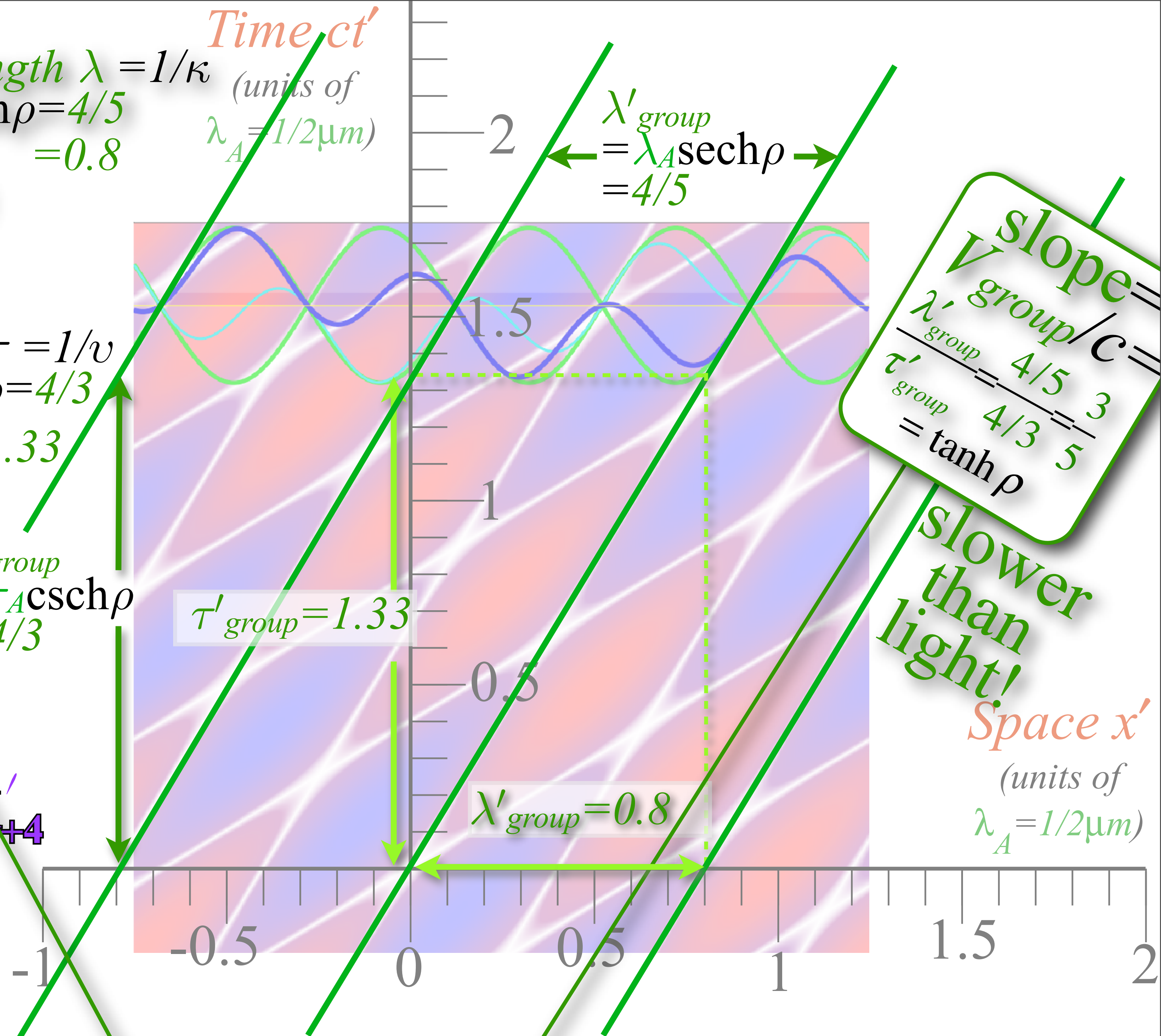

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

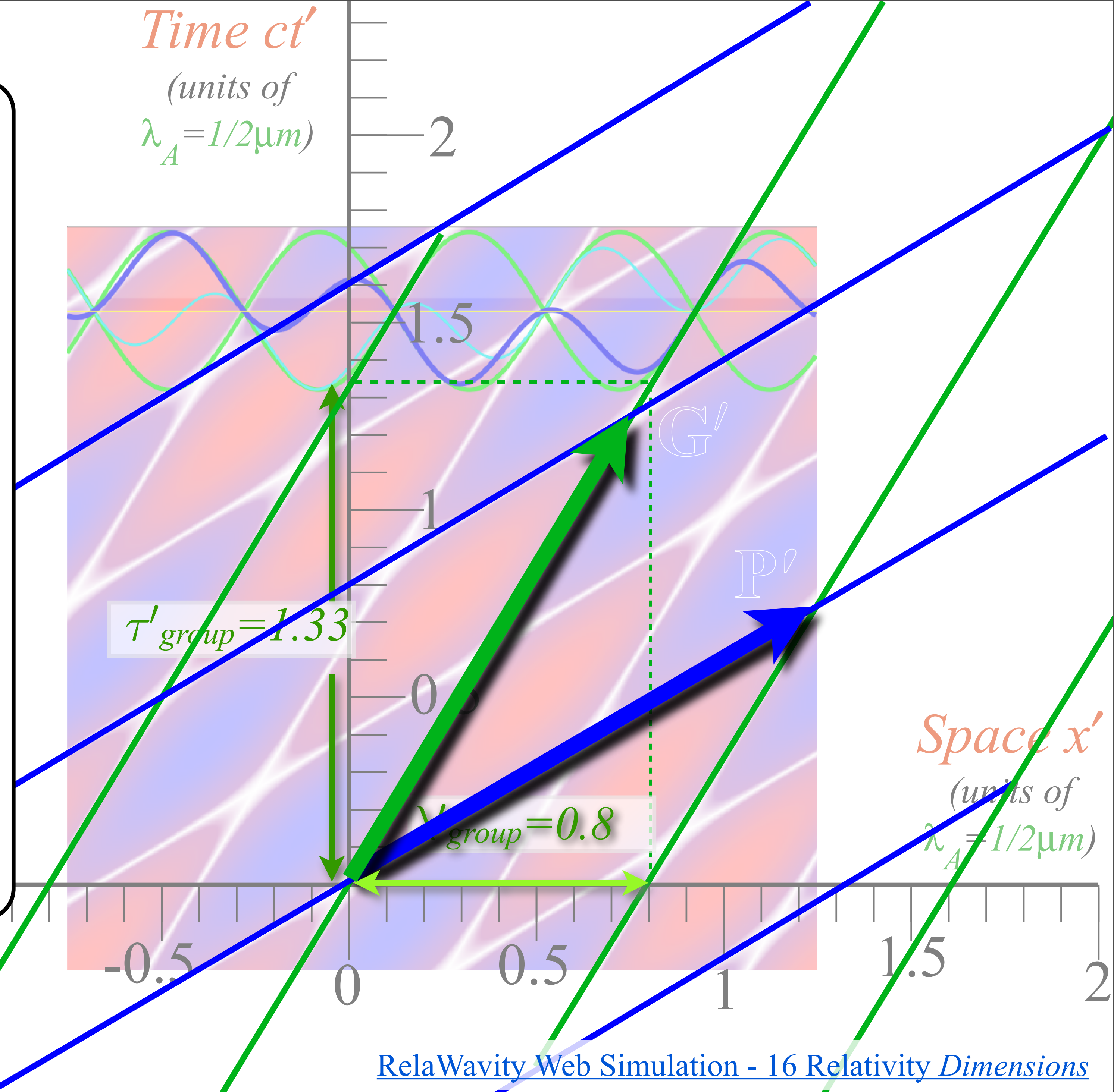
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

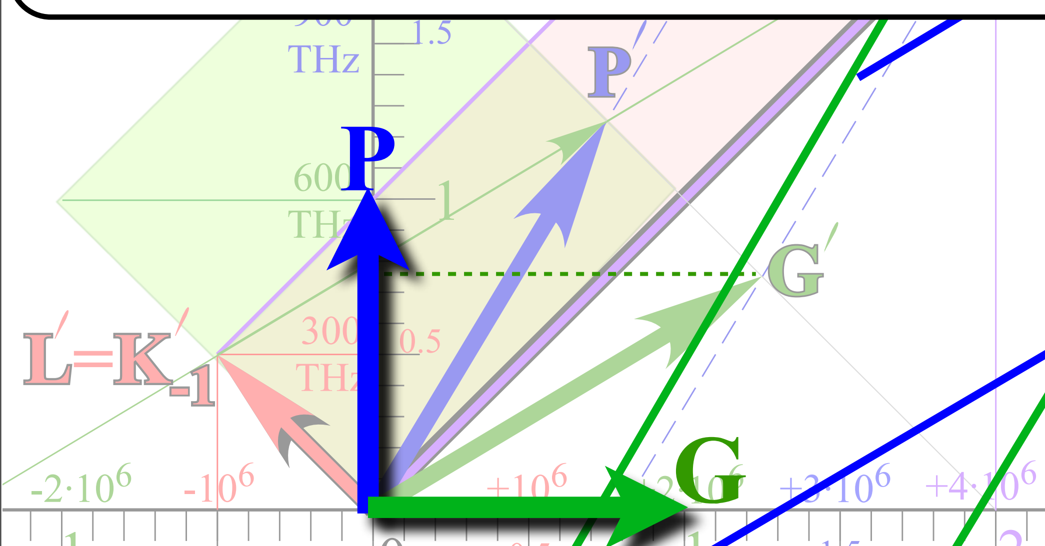
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$





$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

RelaWavity Web Simulation - 16 Relativity *Dimensions*

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Lecture 30
Thur. 12.10.2015

How Doppler shifts of cavity waves exhibit relativistic Lorentz transformations
- Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Developing Axioms to update Galileo's relativity: A critical look at *c*-axioms
- Einstein's PW (Pulse-Wave) Axiom
- Evenson's CW (Continuous Wave) Axiom and Occam's Razor
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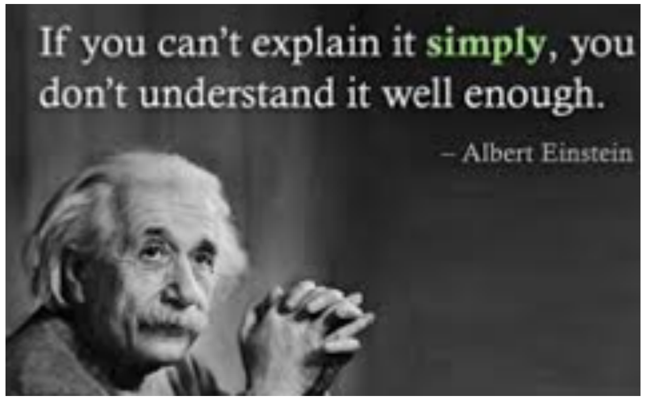
- Clarifying Evenson's CW Axiom using Doppler effects
  - *Galileo's Revenge (part1):* Galilean Doppler-shift arithmetic using *rapidity* ρ

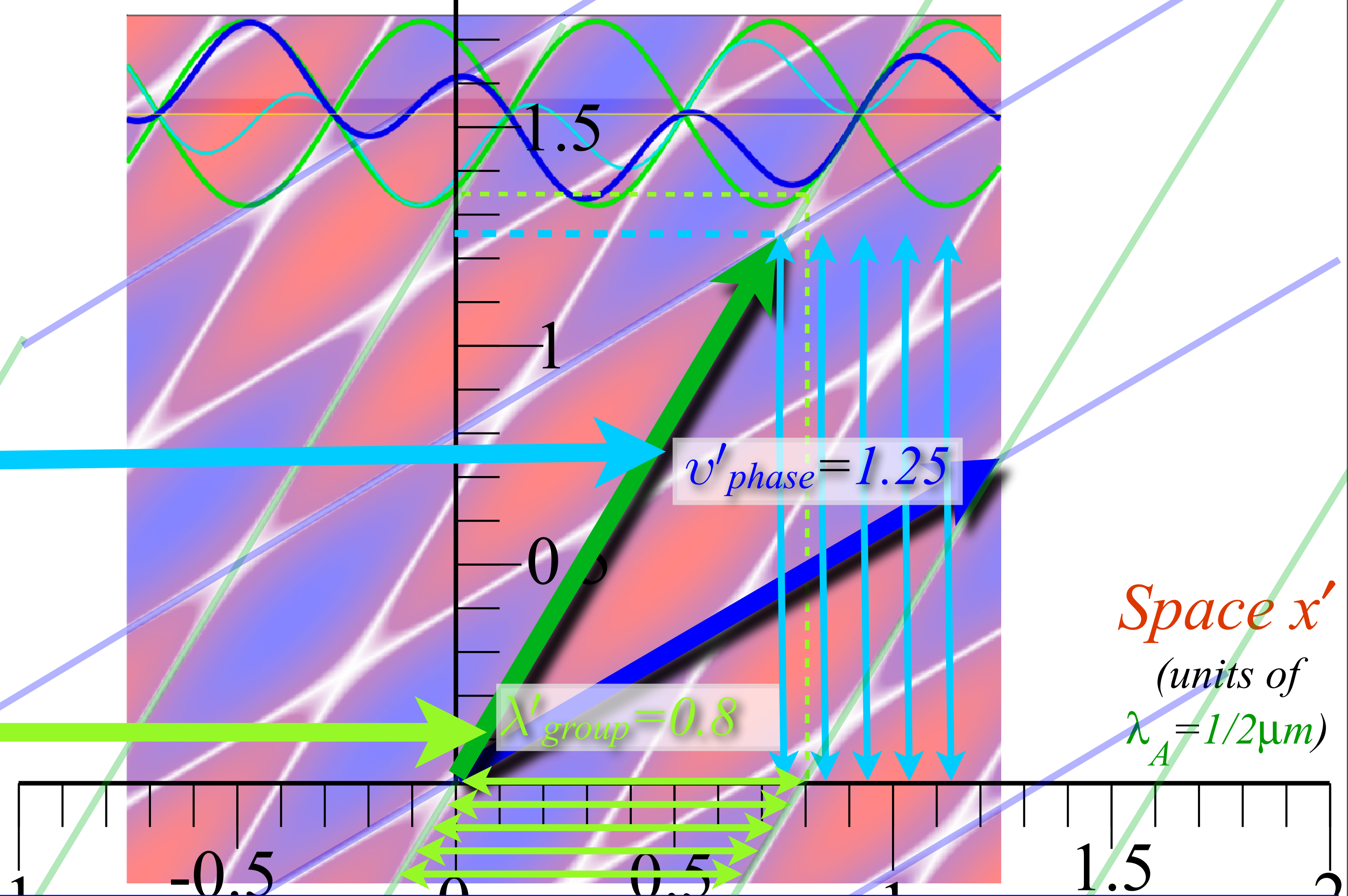
Developing optical "baseball-diamond" and relativistic ρ-functions and transformations
- Details of 1CW wavefunctions and phasors
- Details of 2CW wavefunctions in rest frame
  - *Galileo's Revenge (part2):* Galilean addition of phasor angular velocity
  - Structure of rest frame "baseball-diamonds"
- Details of 2CW wavefunctions of moving frame velocities of *phase* and *group* waves
  - 16 coefficients of relativistic 2CW interference
  - ➔ Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

Two Famous-Name Coefficients



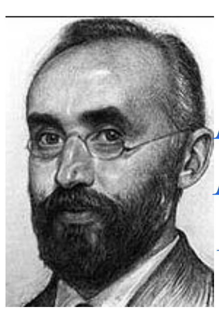
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*

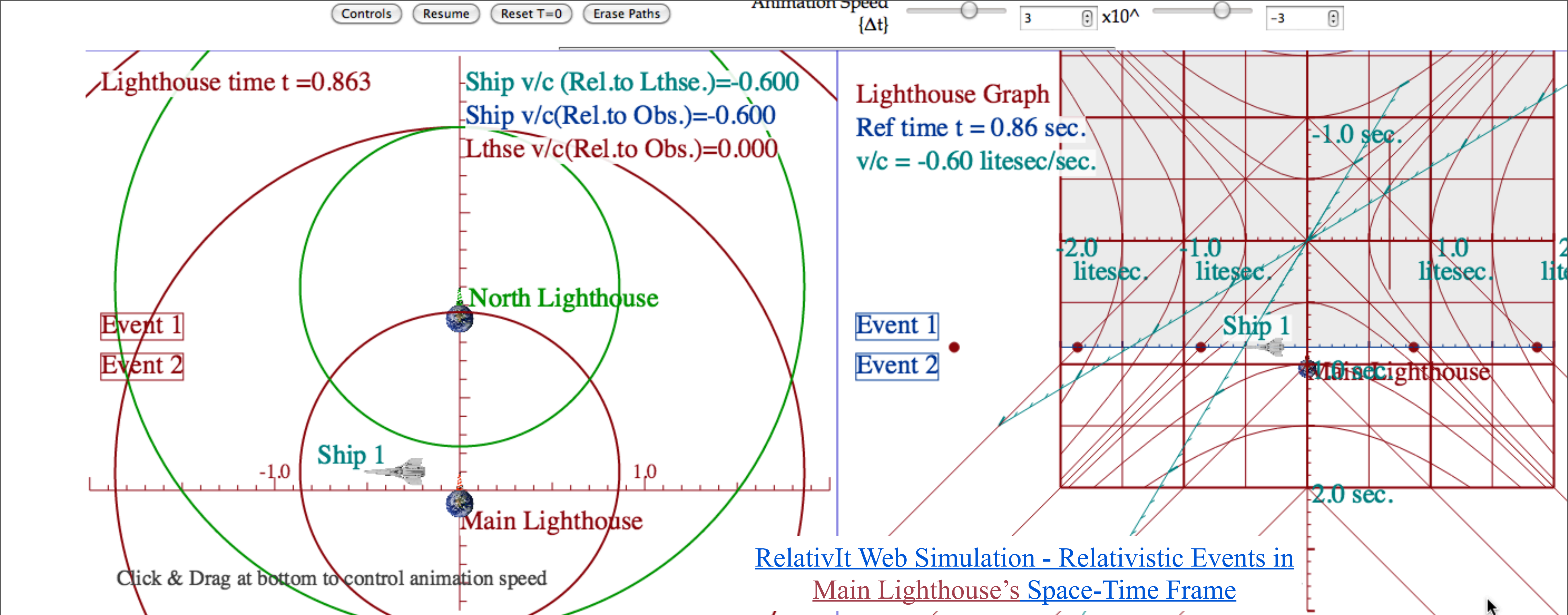


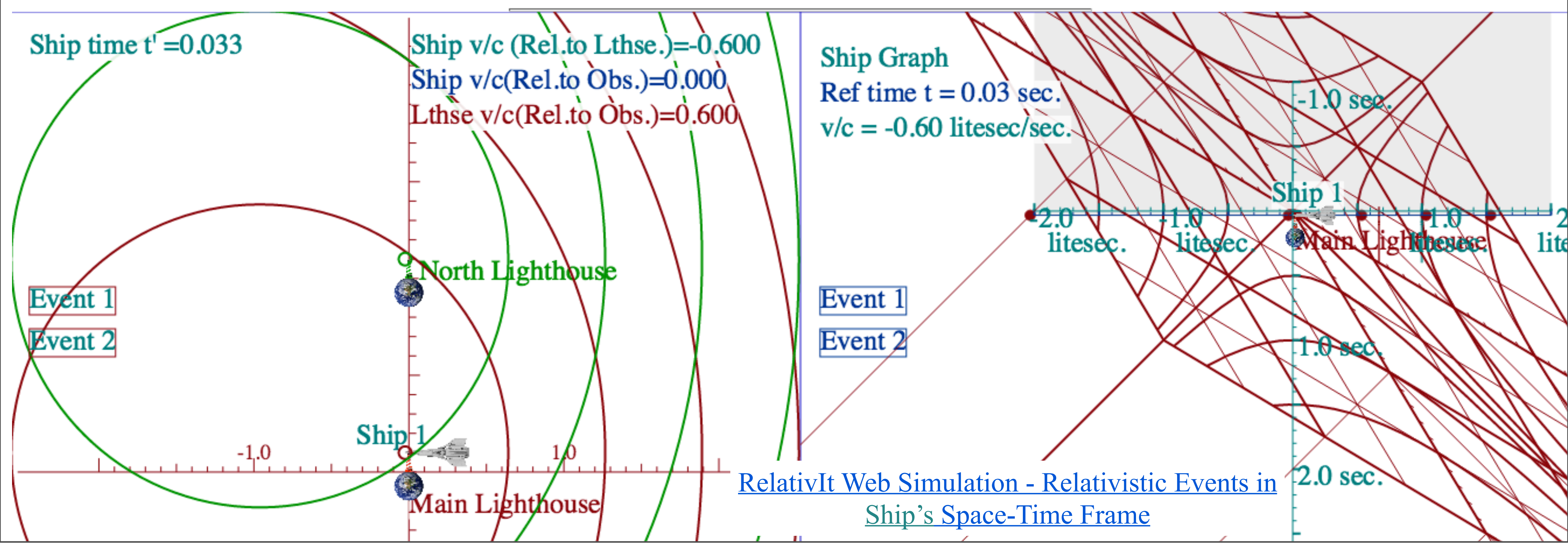
Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)


Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
x10^
Lighthouse time t =0.863
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=-0.600
Lthse v/c(Rel.to Obs.)=0.000
North Lighthouse
Event 1
Event 2
Ship 1
-1.0
1.0
Main Lighthouse
Click & Drag at bottom to control animation speed
Lighthouse Graph
Ref time t = 0.86 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
-2.0 litesec.
-1.0 litesec.
1.0 litesec.
Ship 1
Main Lighthouse
2.0 sec.
Event 1
Event 2
RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame


Ship time t' =0.033
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=0.000
Lthse v/c(Rel.to Obs.)=0.600
North Lighthouse
Event 1
Event 2
Ship 1
-1.0
1.0
Main Lighthouse
Ship Graph
Ref time t = 0.03 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
Ship 1
-2.0 litesec.
-1.0 litesec.
1.0 litesec.
Main Lighthouse
1.0 sec.
2.0 sec.
Event 1
Event 2
RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

Lecture 30
Thur. 12.10.2015

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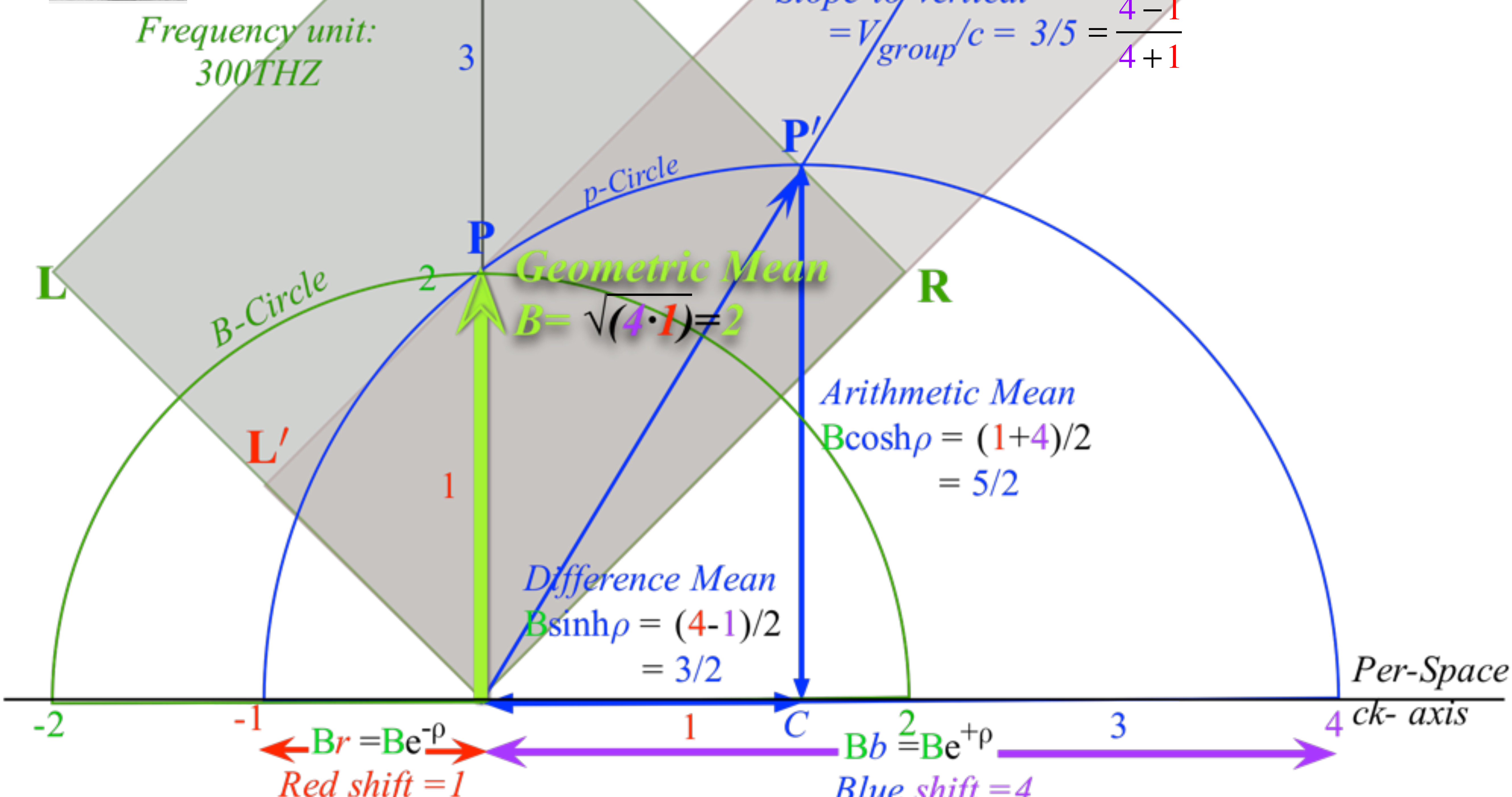
Thales Mean Geometry (600BCE)

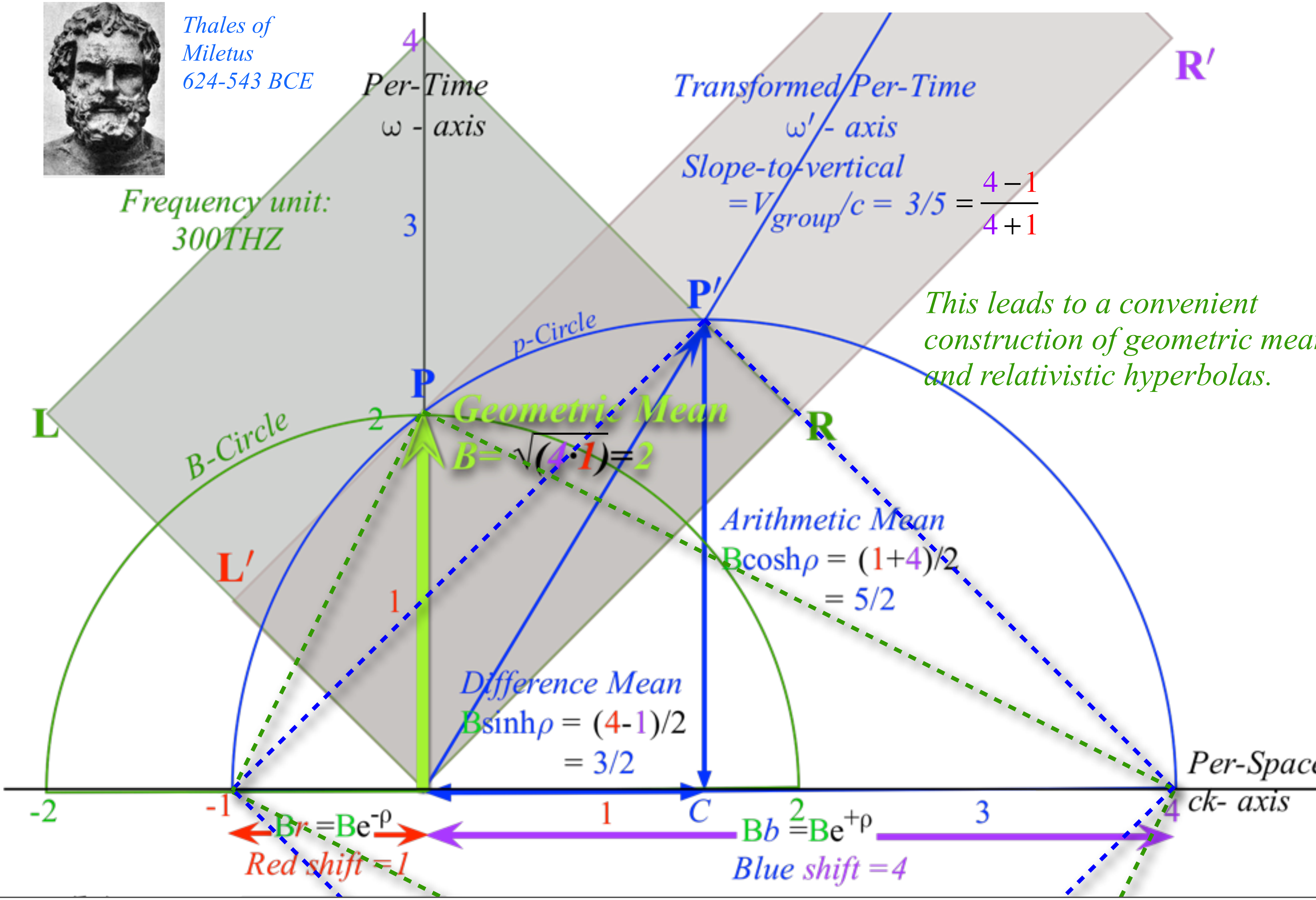
helps “Relawavity”

Thales of Miletus 624-543 BCE



Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



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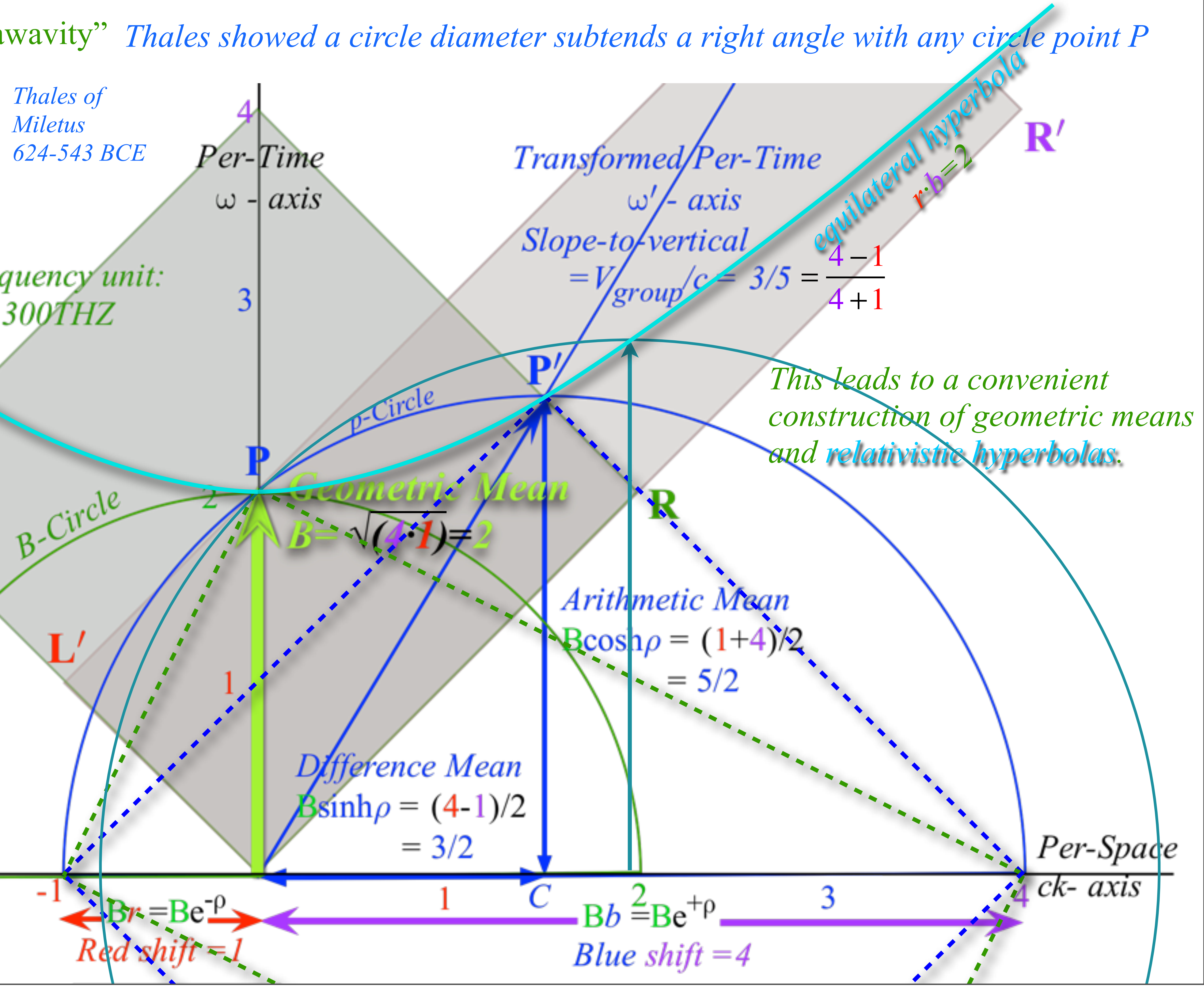
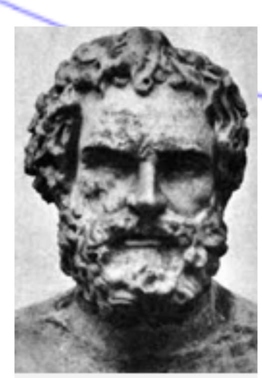


This leads to a convenient construction of geometric means and relativistic hyperbolas.

Thales Mean Geometry (600BCE)

helps "Relawavity"



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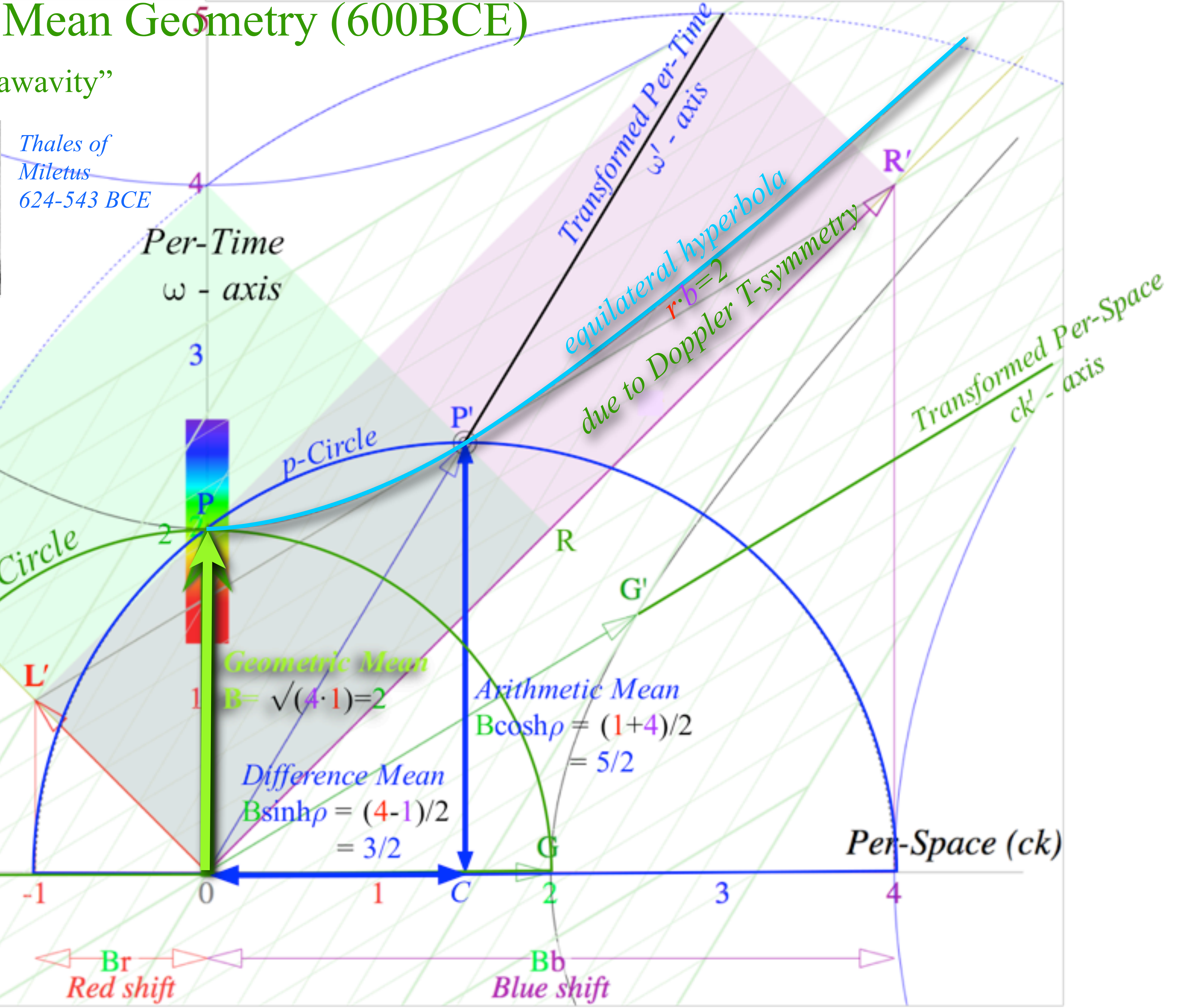

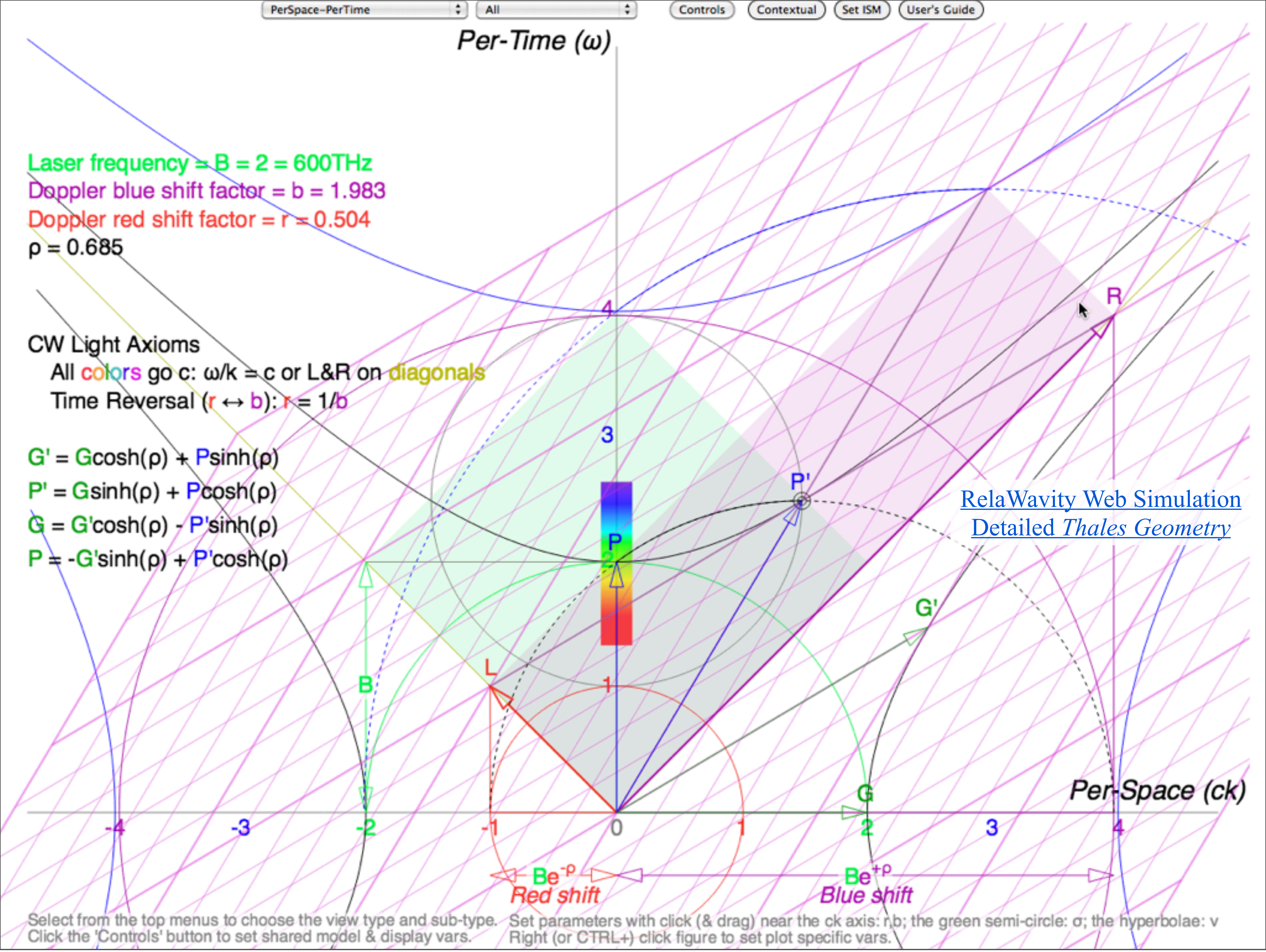

RelaWavity Web Simulation
Detailed *Thales Geometry*