Lecture 29 part I-II
Mon.12.04 and Wed.12.09 2019

Relawavity : a novel introduction to relativistic mechanics I.

(CMwBang! Unit 8 , AMOP Ch.0,)

Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

- Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra
- 1CW wavefunctions and phasors

Per-space-per-time vs Space-time (How to understand wave parameters)

- Wave velocity formulas

Introducing Doppler shifting

- Why is c *so* constant?!
- Introducing Doppler Arithmetic and *Rapidity* ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

- Details of 2CW wavefunctions in rest frame
  - Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

- Analyzing wave velocity by per-space-per-time *and* space-time graphs
  - 16 coefficients of relativistic 2CW interference
  - Two "famous-name" coefficients and the Lorentz transformation
  - Thales mean geometry of Lorentz transformation

A great relativistic "Lovers-Quarrel" **RESOLVED!**

part II

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and geometry of functions of ρ and σ — Minkowski animations
    - Application to TE-Waveguide modes. — synchrotron beam relativity

This Lecture's Reference Link Listing

Web Resources - front page | Quantum Theory for the Computer Age | 2017 Group Theory for QM
UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2018 Adv CM
Classical Mechanics with a Bang! | 2018 AMOP
Modern Physics and its Classical Foundations | **2019 Advanced Mechanics**

Lecture #28-29II

In reverse order

AMOP Chapter 0: Space-Time Symmetry
AMOP Detailed Development of RelaWavity
2018 Rochester Talk (Auxilary Slides)
Special Relativity and Quantum Theory by Ruler and Compass - Earlier, expanded draft
Ruler & Compass - Relawavity Exercise

Relativity Visualized - Epstein-ip-1985 for sale here @www.allbookstores.com

GuideIt Web Simulations: σ = 30°, σ = 60°

Pirelli Site - A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers
World of Clocks - Animations - 12 hr. clock, 24 hr. clock
Phasor vs Thales(Pirelli Challenge) - phasors_2_3_zoom_anim.html

RelativIt Web Simulations
Relativistic Events in Main Lighthouse's Space-Time Frame - *scenario=22*
Relativistic Events in Ship's Space-Time Frame - *scenario=24*
Epstein plot - *scenario=600*

BohrIt Web Simulations
2 CW ct vs x Plot (ck = ±2) - *scenario=-130022*
Multi-Panel 2 PW ct vs x Plot - *scenario=30022*
1 CW ct vs x Plot (ck = -1) - *scenario=-30001*
1 CW ct vs x Plot (ck = +4) - *scenario=30004*
2 CW Minkowski Plot (ck = -1, +4) - *scenario=-30104*

CMwBang Text 2012 Unit 6 page=5
BounceIt Web App/Scenarios: 5002, 5003
CoulIt Web App/Scenarios:
TwoParticleCollision_LToR, TwoParticleCollision_LToR_CM, TwoParticleOrbit_Coulomb, TwoParticleOrbit_Coulomb_CM, TwoParticleOrbit_Hooke, TwoParticleOrbit_Hooke_CM

RelaWavity Web Simulations
2019 RelaWavity Portal Page
Relations between Hypergeometric and Hypergeometric functions - *plotType=0,9&...*
RelaWavity Web Simulation {Physical Terms - All Terms} - *plotType=4,8*

Keyboard of the Gods
Per-Time vs Per-Space - *plotType=7,1*
Dual Plot #1 - *plotType=7,2&bcStepInd=1*
Dual Plot #2 - *plotType=7,2&bcStepInd=2*
Dual Plot #3 - *plotType=7,2&bcStepInd=3*
Dual Plot #7 - *plotType=7,2&bcStepInd=7*

16 Relativity Dimensions - *plotType=8,4*
Relativistic Terms (Expanded Table) - *plotType=8,5*
Minkowski graph (Multi-plot) - *plotType=8,8*
Detailed Thales Geometry - *plotType=3,6*
PerSpace - PerTime {All} - *plotType=3,6&minkGridPosCells=0*
Expanded Relativistic Relations - *plotType=8,7*
Wavefronts in Space-Space - *plotType=6,1*
Spectral Ellipse {PerSpace-PerSpace} {β = u/c = 1/3} - *plotType=6,3&...*
Spectral Ellipse {β = u/c = 3/4} - *plotType=6,3&...*

Select, exciting, and/or related Research
Singular Motion of Asymetric Rotators AJP 44, 11 p1080 Harter-Kim-1976
Molecular Eigensolution Symmetry Analysis and Fine Structure - Harter-IJMS-2013
Lenz Vector and Orbital Analog Computers - AJP 44 p348 1976
Some Geometric Aspects of Classical Coulomb Scattering AJP 40 4 p1852 1972
How Molecules do Self-NMR - Harter-Mitchell-Columbus-2009
Classical Mechanics with a Bang! - Asymmetric Top Demo
Allbookstores.com - Compare for Heller's SemiClassical Way - 0691163731
"My Bomerang Won't Come Back" (YouTube: Playlist)
Rotating Solid Bodies in Microgravity (YouTube)
Dancing T-handle in zero-g (YouTube)

In development, but close to role out.
More Advanced QM and classical references will *soon* be available through our: ***References Page***
Would be great to have our Apache SOLR *Search & Index system up for a bigger* Bang!*)*

Continued for 4 more pages↘

This Lecture's Reference Link Listing

Web Resources - front page | Quantum Theory for the Computer Age | 2017 Group Theory for QM
UAF Physics UTube channel | Principles of Symmetry, Dynamics, and Spectroscopy | 2018 Adv CM
Classical Mechanics with a Bang! | 2018 AMOP
Modern Physics and its Classical Foundations | **2019 Advanced Mechanics**

Lecture #22-27 *In reverse order*

CoulIt Web App Simulations: p19, p32, p72, p73, p92, R=-0.375, R=+0.5, Rutherford
OscillatorPE Web App: IHO Scenario 2, Coulomb Scenario 3
RelaWavity Web App/Simulator/Calculator: Elliptical - IHO orbits

JerkIt Web App: 2-, 2+, Amp50Omega147-, Amp50Omega296, Amp50Omega602, Gap(1)
MolVibes Web App: C3vN3
WaveIt Web App:
  Dim = 3 w/Wave Components;
  Static Char Table: 6, 12, 12(b), 16, 36, 256
  Quantum Carpet with N=20: Gaussian, Boxcar
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-CPL-2015
QTCA Unit_5 Ch14 2013
Lester. R. Ford, Am. Math. Monthly 45,586(1938)
John Farey, Phil. Mag.(1816) Wolfram
Harter, J. Mol. Spec. 210, 166-182 (2001)
Harter, Li IMSS (2013)
Li, Harter, Chem.Phys.Letters (2015)
Advanced Atomic and Molecular Optical Physics 2018 Class #9, pages: 5, 61
BoxIt Web Simulations
  Pure A-Type A=4.9, B=0 ,C=0, & D=4.0
  Pure B-Type: A=4.0, B=-0.2, C=0, & D=4.0
  Pure C-Type A,D=4.055, B=0, C=0.1
  Mixed AB-Type w/Cosine
  Mixed AB Type A=4.0, BU2=0.866…, CU2=0, & D=1.0 w/Stokes & Freq rats
  Mixed AB Type A=5.086 B=-0.27 C=0 D=2.024 w/Stokes plot
  Mixed ABC Type A=4.833 B=0.2403 C=0.4162 D=4.277 w/Stokes plot
  Recent mixed ABC Type A=0.325 B=0.375 C=0.825 D=0.05 w/Stokes plot

Select, exciting, and/or related Research
This Indestructible NASA Camera Revealed Hidden Patterns on Jupiter - seeker-yt-2019
What did NASA's New Horizons discover around Pluto? - Astrum-yt-2018
Synthetic_Chiral_Light_for_Efficient_Control_of_Chiral_Light–Matter_Interaction_-_Ayuso-np-2019

Classical Mechanics with a Bang! 2018
  Lectures 8, 9, 23 page 93
  Text Unit 6, page=27
ColorU2 for the Web - in development
Group Theory for Quantum Mechanics - 2017 Lectures: 6, 7, 8, and the combined 9-10
Quantum Theory for the Computer Age Unit 3 Ch.7-10, page=90
Spectral Decomposition with Repeated Eigenvalues - 2017 GTQM - Lecture 5

Web based 3D & XR ($x\in\{A,M,V\}$, R=Reality) https://www.babylonjs.com/
Web based 3D graphics WebGL API (Graphics Layer modeled after OpenGL)

Recent In-House draft Articles:
Springer handbook on Molecular Symmetry and Dynamics - Ch_32 - Molecular Symmetry
AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Auxiliary slides, June 19, 2018

Quantum_Computing - (Current) State of the Art - Reimer-www-2019
Geometric Algebra- A Guided Tour through Space and Time - Reimer-www-2019
Wildlife Monitoring Identification and Behavioral Study - Section 1 - Reimer-www-2019
Wildlife Monitoring Identification and Behavioral Study - Section 2 - Reimer-www-2019

Quantum Computing (*QC*) and Geometric Algebra (*GA*) references:
Quantum_Supremacy_Using_a_Programmable_Superconducting_Processor_-_Arute-n-2019
Quantum Computing for Computer Scientists - Helwer-mr-yt-2018, Slides
Quantum Computing and Workforce, Curriculum, and App Devel - Roetteler-MS-2019

Quantum_Computing - (Current) State of the Art - Reimer-www-2019
Excerpts (Page 44-47 in *Preliminary Draft*) for a GA take on the Complex Numbers
Geometric Algebra- A Guided Tour through Space and Time - Reimer-www-2019
GA & QC references (Page 11-16 in Preliminary Draft)

This Lecture's Reference Link Listing

Web Resources - front page
UAF Physics UTube channel

Quantum Theory for the Computer Age
Principles of Symmetry, Dynamics, and Spectroscopy
Classical Mechanics with a Bang!
Modern Physics and its Classical Foundations

2017 Group Theory for QM
2018 Adv CM
2018 AMOP
2019 Advanced Mechanics

Lectures #12 through #21

In reverse order

Wiki on Pafnuty Chebyshev
Nobelprize.org
  2005 Physics Award

BoxIt Web Simulations:
A-Type w/Cosine, A-Type w/Freq ratios,
AB-Type w/Cosine, AB-Type 2:1 Freq ratio

OscillIt Web Simulations:
Default/Generic, Weakly Damped #18,
Forced : Way below resonance,On resonance
  Way above resonance,Underdamped
  Complex Response Plot

CoulIt Web Simulations:
Stark-Coulomb : Bound-state motion in parabolic coordinates
Molecular Ion : Bound-state motion in hyperbolic coordinates
Synchrotron Motion, Synchrotron Motion #2
Mechanical Analog to EM Motion (YouTube video)
iBall demo - Quasi-periodicity (YouTube video)

Trebuchet Web Simulations:
  Default/Generic URL, Montezuma's Revenge, Seige of Kenilworth,
  "Flinger",
  Position Space (Course), Position Space (Fine)
Wacky Waving Solid Metal Arm Flailing Chaos Pendulum - Scooba_Steeve-yt-2015
Triple Double-Pendulum - Cohen-yt-2008
Punkin Chunkin - TheArmchairCritic-2011
Jersey Team Claims Title in Punkin Chunkin - sussexcountyonline-1999
Shooting range for medieval siege weapons. Anybody knows? - twcenter.net/forums
The Trebuchet - Chevedden-SciAm-1995
NOVA Builds a Trebuchet

Recent Articles of Interest:
A_Semi-Classical_Approach_to_the_Calculation_of_Highly_Excited_Rotational_Energies_for …
  Asymmetric-Top_Molecules_-_Schmiedt-pccp-2017
Tunable and broadband coherent perfect absorption by ultrathin blk phos metasurfaces - Guo-josab-2019
Vortex Detection in Vector Fields Using Geometric Algebra - Pollock-aaca-2013.pdf

Pirelli Relativity Challenge (Introduction level) - *Visualizing Waves:*
  Using Earth as a clock,
  Tesla's AC Phasors ,
  Phasors using complex numbers.
CM wBang Unit 1 - Chapter 10, pdf_page=135
  Calculus of exponentials, logarithms, and complex fields,
RelaWavity Web Simulation - Unit Circle and Hyperbola (Mixed labeling)
Smith Chart, Invented by Phillip H. Smith (1905-1987)

Select, exciting, and related Research
Clifford_Algebra_And_The_Projective_Model_Of_Homogeneous_Metric_Spaces -
  Foundations - Sokolov-x-2013
Geometric Algebra 3 - Complex Numbers - MacDonald-yt-2015
Biquaternion -Complexified Quaternion- Roots of -1 - Sangwine-x-2015
An_Introduction_to_Clifford_Algebras_and_Spinors_-_Vaz-Rocha-op-2016
Unified View on Complex Numbers and Quaternions- Bongardt-wcmms-2015
Complex Functions and the Cauchy-Riemann Equations - complex2 - Friedman-columbia-2019
An_sp-hybridized_Molecular_Carbon_Allotrope-_cyclo-18-carbon_-_Kaiser-s-2019
An_Atomic-Scale_View_of_Cyclocarbon_Synthesis_-_Maier-s-2019
Discovery_Of_Topological_Weyl_Fermion_Lines_And_Drumhead_Surface_States_in_a_
  Room_Temperature_Magnet_-_Belopolski-s-2019
"Weyl"ing_away_Time-reversal_Symmetry_-_Neto-s-2019
Non-Abelian_Band_Topology_in_Noninteracting_Metals_-_Wu-s-2019
What_Industry_Can_Teach_Academia_-_Mao-s-2019
RoVib-_quantum_state_resolution_of_the_C60_fullerene_-_Changala-Ye-s-2019 (Alt)
A_Degenerate_Fermi_Gas_of_Polar_molecules_-_DeMarco-s-2019

An assist from *Physics Girl* (YouTube Channel):
How to Make VORTEX RINGS in a Pool
Crazy pool vortex - pg-yt-2014
Fun with Vortex Rings in the Pool - pg-yt-2014

Running Reference Link Listing

Lectures #11 through #7

In reverse order

Eric J Heller Gallery:

Main portal, Consonance and Dissonance II, Bessel 21, Chladni

The Semiclassical Way to Molecular Spectroscopy - Heller-acs-1981
Quantum_dynamical_tunneling_in_bound_states_-_Davis-Heller-jcp-1981

Pendulum Web Simulation
Cycloidulum Web Simulation

Links to previous lecture: *Page=74*, *Page=75*, *Page=79*

Pendulum Web Sim

Cycloidulum Web Sim

***JerkIt* Web Simulations:** *Basic/Generic: Inverted*, *FVPlot*

CMwithBang Lecture 8, page=20

WWW.sciencenewsforstudents.org: Cassini - Saturnian polar vortex

"RelaWavity" Web Simulations:

2-CW laser wave, Lagrangian vs Hamiltonian,
Physical Terms Lagrangian L(u) vs Hamiltonian H(p)

CoulIt Web Simulation of the Volcanoes of Io

BohrIt Multi-Panel Plot:

Relativistically shifted Time-Space plots of 2 CW light waves

BoxIt Web Simulations:

Generic/Default
Most Basic A-Type
Basic A-Type w/reference lines
Basic A-Type A-Type with Potential energy
A-Type with Potential energy and Stokes Plot
A-Type w/3 time rates of change
A-Type w/3 time rates of change with Stokes Plot
B-Type (A=1.0, B=-0.05, C=0.0, D=1.0)

RelaWavity Web Elliptical Motion Simulations:

Orbits with b/a=0.125
Orbits with b/a=0.5
Orbits with b/a=0.7
Exegesis with b/a=0.125
Exegesis with b/a=0.5
Exegesis with b/a=0.7
Contact Ellipsometry

***CoulIt* Web Simulations:**

Basic/Generic
Exploding Starlet
Volcanoes of Io (Color Quantized)

***JerkIt* Web Simulations:**

Basic/Generic
Catcher in the Eye - IHO with Linear Hooke perturbation - Force-potential-Velocity Plot

***OscillatorPE* Web Simulation:**

Coulomb-Newton-Inverse_Square,
Hooke-Isotropic Harmonic,
Pendulum-Circular_Constraint

AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Aux. slides-2018

NASA Astronomy Picture of the Day -
Io: The Prometheus Plume *(Just Image)*
NASA Galileo - *Io's Alien Volcanoes*
New Horizons - *Volcanic Eruption Plume on Jupiter's moon IO*
NASA Galileo - A Hawaiian-Style Volcano on Io

Pirelli Site: Phasors animimation
CMwithBang Lecture #6, page=70 (9.10.18)

Select, exciting, and related Research & Articles of Interest:

Burning a hole in reality—design for a new laser may be powerful enough to pierce space-time - Sumner-KOS-2019
Trampoline mirror may push laser pulse through fabric of the Universe - Lee-ArsTechnica-2019
Achieving_Extreme_Light_Intensities_using_Optically_Curved_Relativistic_Plasma_Mirrors_-_Vincenti-prl-2019

A_Soft_Matter_Computer_for_Soft_Robots_-_Garrad-sr-2019
Correlated_Insulator_Behaviour_at_Half-Filling_in_Magic-Angle_Graphene_Superlattices_-_cao-n-2018

*Sorting_ultracold_atoms_in_a_three-dimensional_optical_lattice_in_a_*
realization_of_Maxwell's_Demon - Kumar-n-2018
Synthetic_three-dimensional_atomic_structures_assembled_atom_by_atom - Barredo-n-2018

Older ones:
Wave-particle_duality_of_C60_molecules - Arndt-ltn-1999
Optical_Vortex_Knots_-_One_Photon__At_A_Time - Tempone-Wiltshire-Sr-2018
*Baryon_Deceleration_by_Strong_Chromofields_in_Ultrarelativistic_*,
Nuclear_Collisions - Mishustin-PhysRevC-2007, *APS Link & Abstract*
Hadronic Molecules - Guo-x-2017
Hidden-charm_pentaquark_and_tetraquark_states - Chen-pr-2016

Running Reference Link Listing

Lectures #6 through #1

In reverse order

RelaWavity Web Simulation: Contact Ellipsometry
BoxIt Web Simulation: Elliptical Motion (A-Type)
CMwBang Course: Site Title Page
Pirelli Relativity Challenge: Describing Wave Motion With Complex Phasors
UAF Physics UTube channel

Velocity Amplification in Collision Experiments Involving Superballs - Harter, 1971
MIT OpenCourseWare: High School/Physics/Impulse and Momentum
Hubble Site: Supernova - SN 1987A

BounceItIt Web Animation - Scenarios:
49:1 y vs t, *49:1 V2 vs V1*, *1:500:1 - 1D Gas Model w/ faux restorative force (Cool)*,
1:500:1 - 1D Gas (Warm), *1:500:1 - 1D Gas Model (Cool, Zoomed in)*,
Farey Sequence - Wolfram
Fractions - Ford-AMM-1938
Monstermash BounceItIt Animations:
1000:1 - V2 vs V1, *1000:1 with t vs x - Minkowski Plot*
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-2013
Quantum_Revivals_of_Morse_Oscillators_and_Farey-Ford_Geometry - Li-Harter-cpl-2015
Quant. Revivals of Morse Oscillators and Farey-Ford Geom. - Harter-Li-CPL-2015 (Publ.)
Velocity_Amplification_in_Collision_Experiments_Involving_Superballs-Harter-1971
WaveIt Web Animation - Scenarios:
Quantum_Carpet, *Quantum_Carpet_wMBars*,
Quantum_Carpet_BCar, *Quantum_Carpet_BCar_wMBars*
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-JMS-2001
Wave Node Dynamics and Revival Symmetry in Quantum Rotors - Harter-jms-2001 (Publ.)

BounceIt Web Animation - Scenarios:
Generic Scenario: 2-Balls dropped no Gravity (7:1) - V vs V Plot (Power=4)
1-Ball dropped w/Gravity=0.5 w/Potential Plot: Power=1, Power=4
7:1 - V vs V Plot: Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=4
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1
3-Ball Stack (10:3:1) w/Newton plot (y vs t) - Power=1 w/Gaps
4-Ball Stack (27:9:3:1) w/Newton plot (y vs t) - Power=4
4-Newton's Balls (1:1:1:1) w/Newtonian plot (y vs t) - Power=4 w/Gaps
6-Ball Totally Inelastic (1:1:1:1:1:1) w/Gaps: Newtonian plot (t vs x), V6 vs V5 plot
5-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Minkowski plot (t vs x1) w/Gaps
1-Ball Totally Inelastic Pile-up w/ 5-Stationary-Balls - Vx2 vs Vx1 plot w/Gaps

BounceIt Dual plots
m_1:m_2 = 3:1
v_2 vs v_1 and V_2 vs V_1, $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$
y_2 vs y_1 plots: $(v_1, v_2)=(1, 0.1)$, $(v_1, v_2)=(1, 0)$, $(v_1, v_2)=(1, -1)$
Estrangian plot *V_2 vs V_1: $(v_1, v_2)=(0, 1)$, $(v_1, v_2)=(1, -1)$*
m_1:m_2 = 4:1
v2 vs v1, *y2 vs y1*
***m_1:m_2 = 100:1,** $(v_1, v_2)=(1, 0)$: V2 vs V1 Estrangian plot, y2 vs y1 plot*

With g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
M_1=49, M_2=1 with Newtonian time plot
M_1=49, M_2=1 with V_2 vs V_1 plot
Example with friction
Low force constant with drag displaying a Pass-thru, *Fall-Thru*, *Bounce-Off*
m1:m2= 3:1 and (v1, v2) = (1, 0) Comparison with Estrangian

AJP article on superball dynamics
AAPT Summer Reading List
Scitation.org - AIP publications
HarterSoft Youtube Channel

X2 paper: *Velocity Amplification in Collision Experiments Involving Superballs - Harter, et. al. 1971 (pdf)*
Car Collision Web Simulator: https://modphys.hosted.uark.edu/markup/CMMotionWeb.html
Superball Collision Web Simulator: https://modphys.hosted.uark.edu/markup/BounceItWeb.html; with Scenarios: 1007
BounceIt web simulation with g=0 and 70:10 mass ratio
With non zero g, velocity dependent damping and mass ratio of 70:35
Elastic Collision Dual Panel Space vs Space: Space vs Time (Newton) , Time vs. Space(Minkowski)
Inelastic Collision Dual Panel Space vs Space: Space vs Time (Newton), Time vs. Space(Minkowski)
Matrix Collision Simulator: M_1=49, M_2=1 V_2 vs V_1 plot <<Under Construction>>

Why ***Men in Black*** *shot little Suzie…Learning about* sin!*,*cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

1CW wavefunctions and phasors

Per-space-per-time vs Space-time (How to understand wave parameters)

Wave velocity formulas

Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz trans

Thales geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein

Longitudinal hyperbolic ρ-geometry connects to transverse cir

"Occams Sword" and geometry of 16 parameter functions c

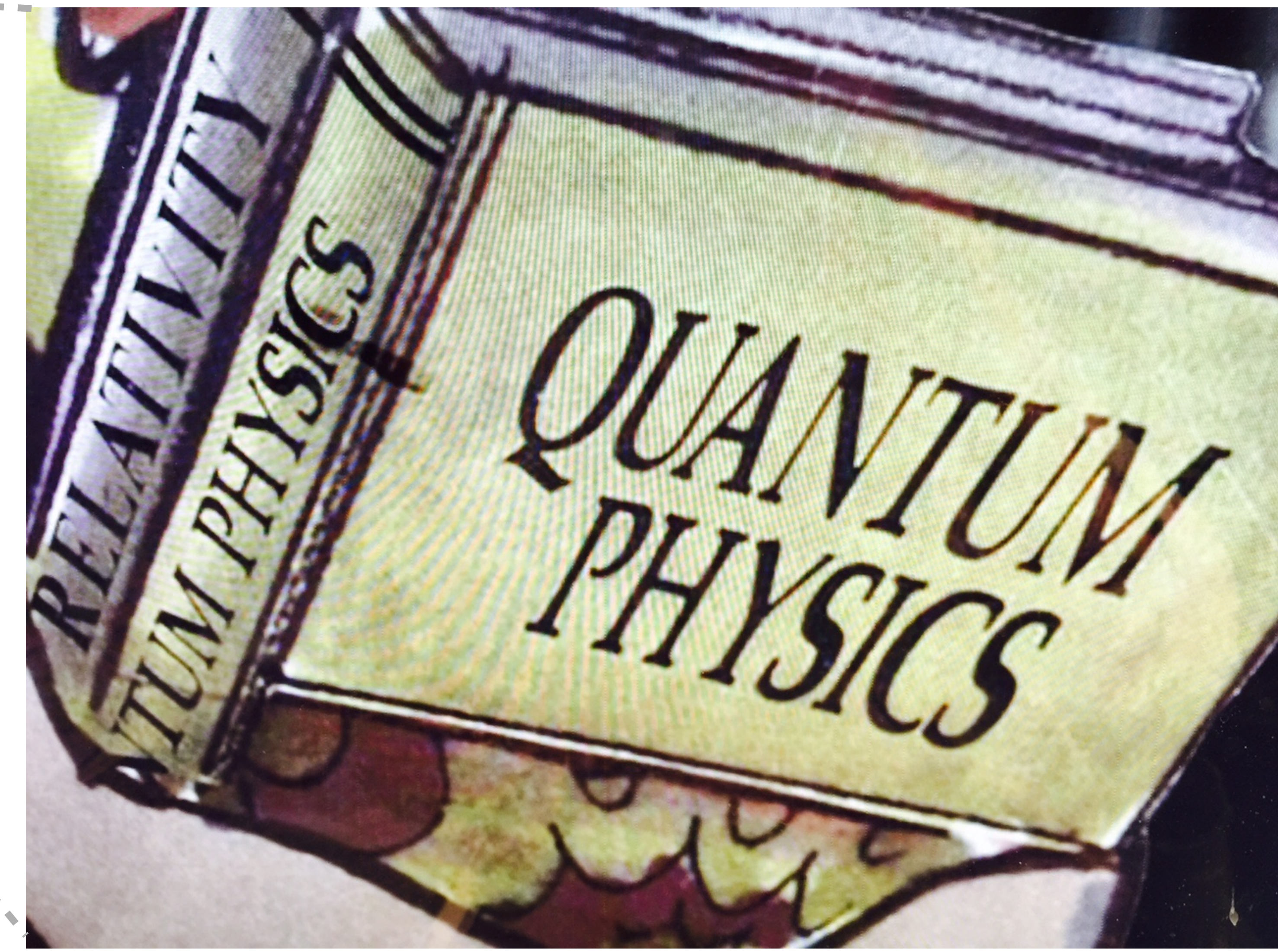
Application to TE-Waveguide modes and synchrotron be

For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge*:*

A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers

From AMOP Ch.0 article.

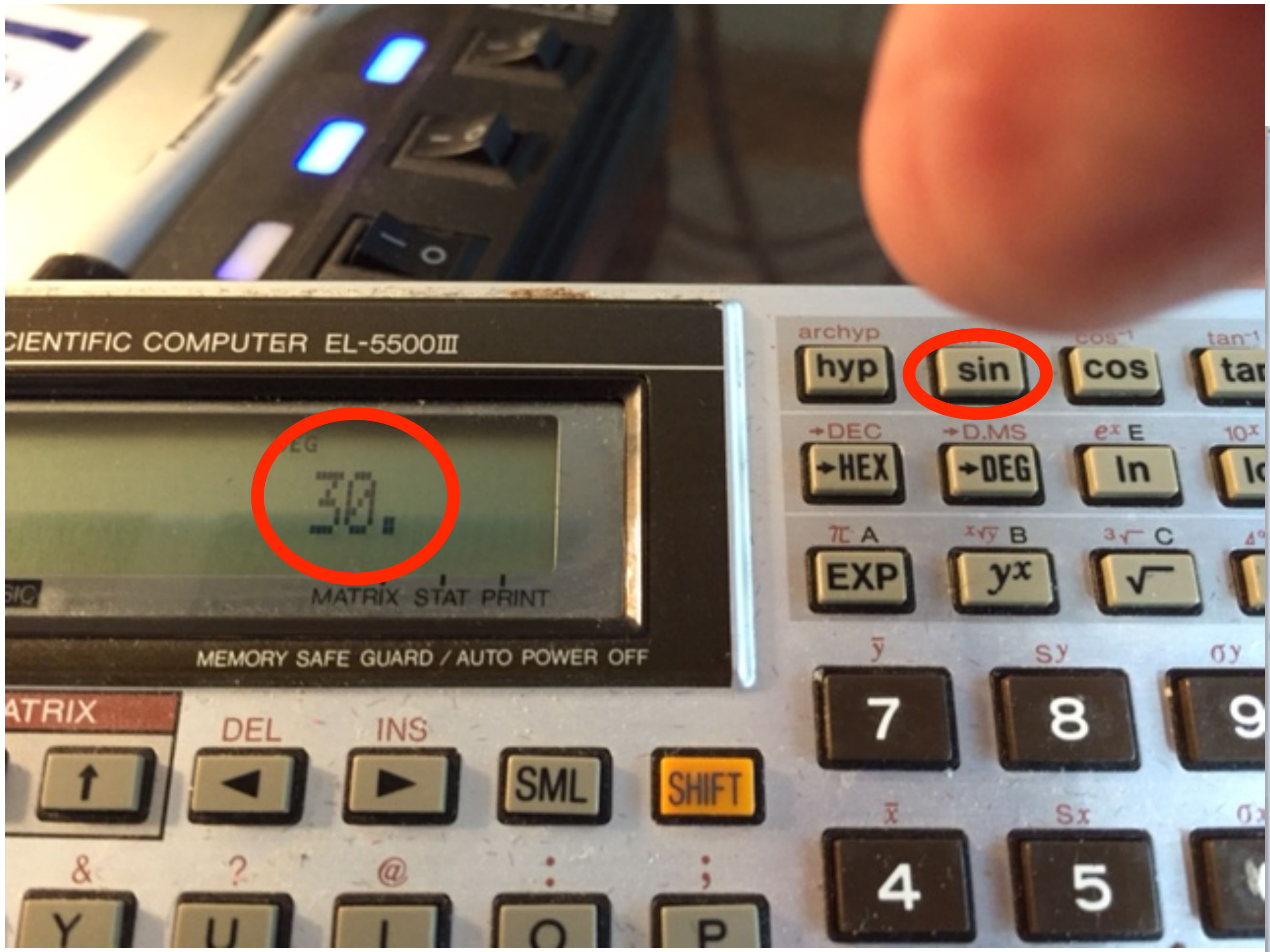
Why did a ***Men In Black*** candidate shoot little Suzy?

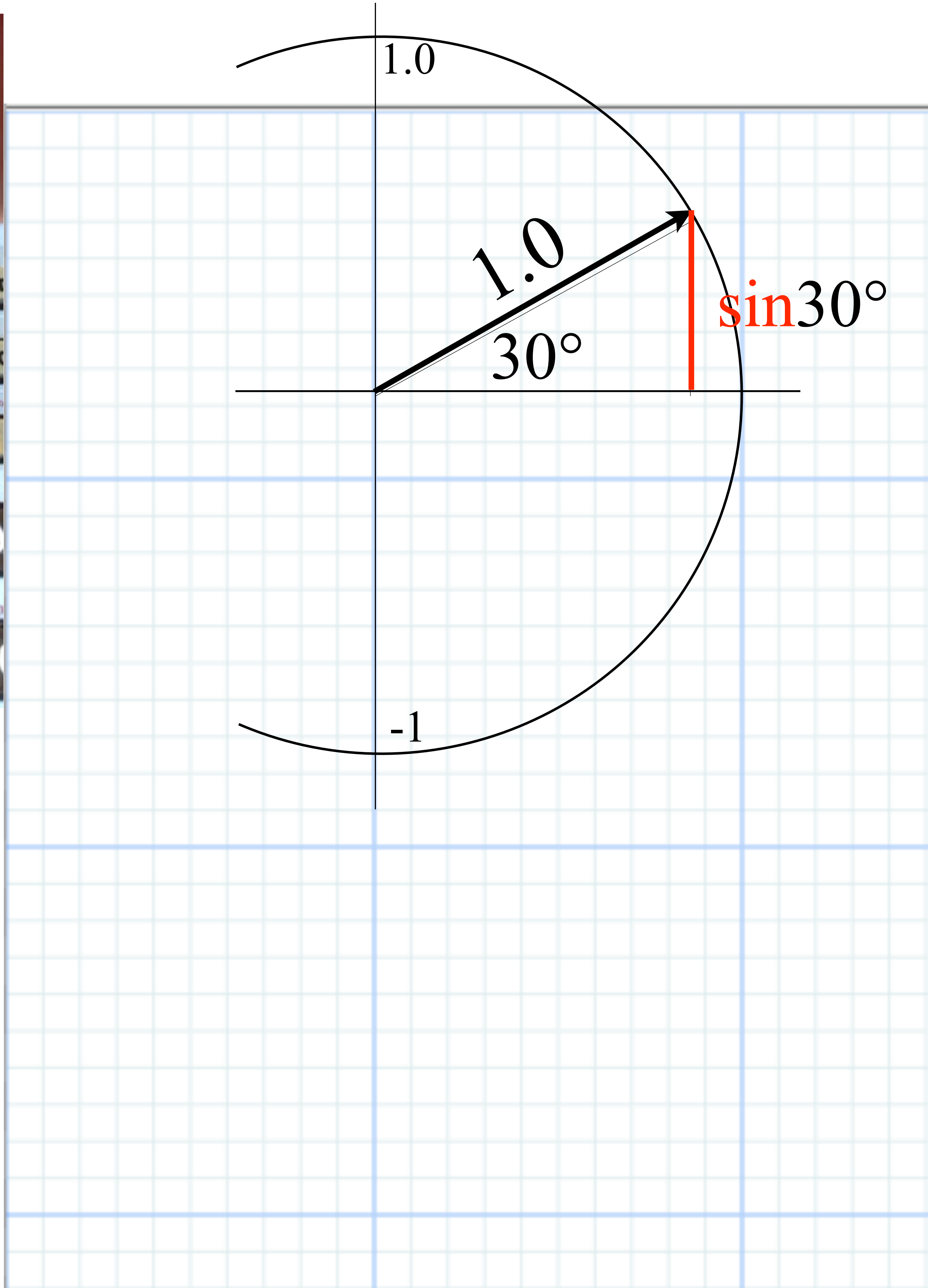


Bad Suzy!
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

We call that a ***Relawavity*** *book.*
(It's a *lot* lighter!)

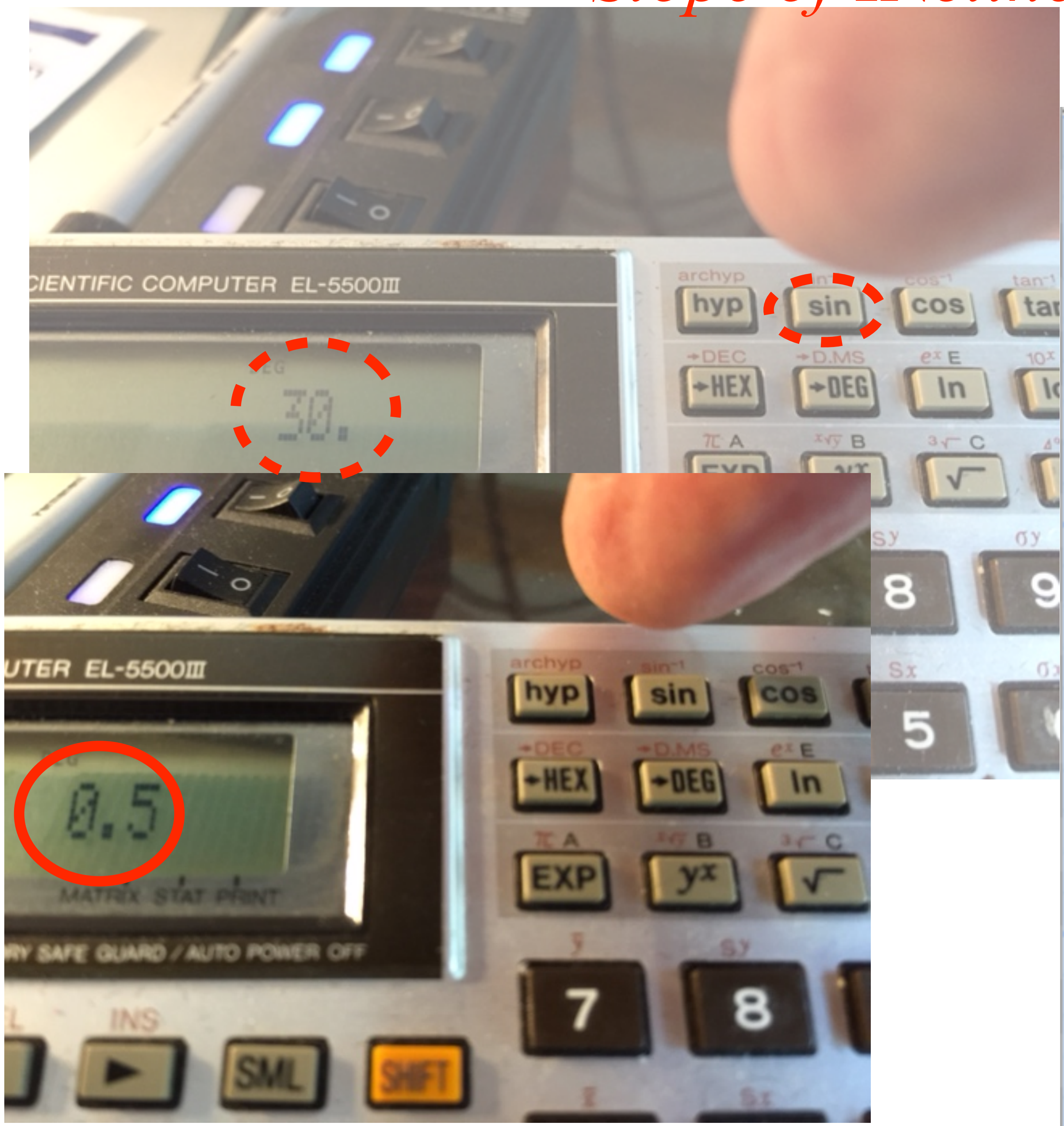
Learning about SIN

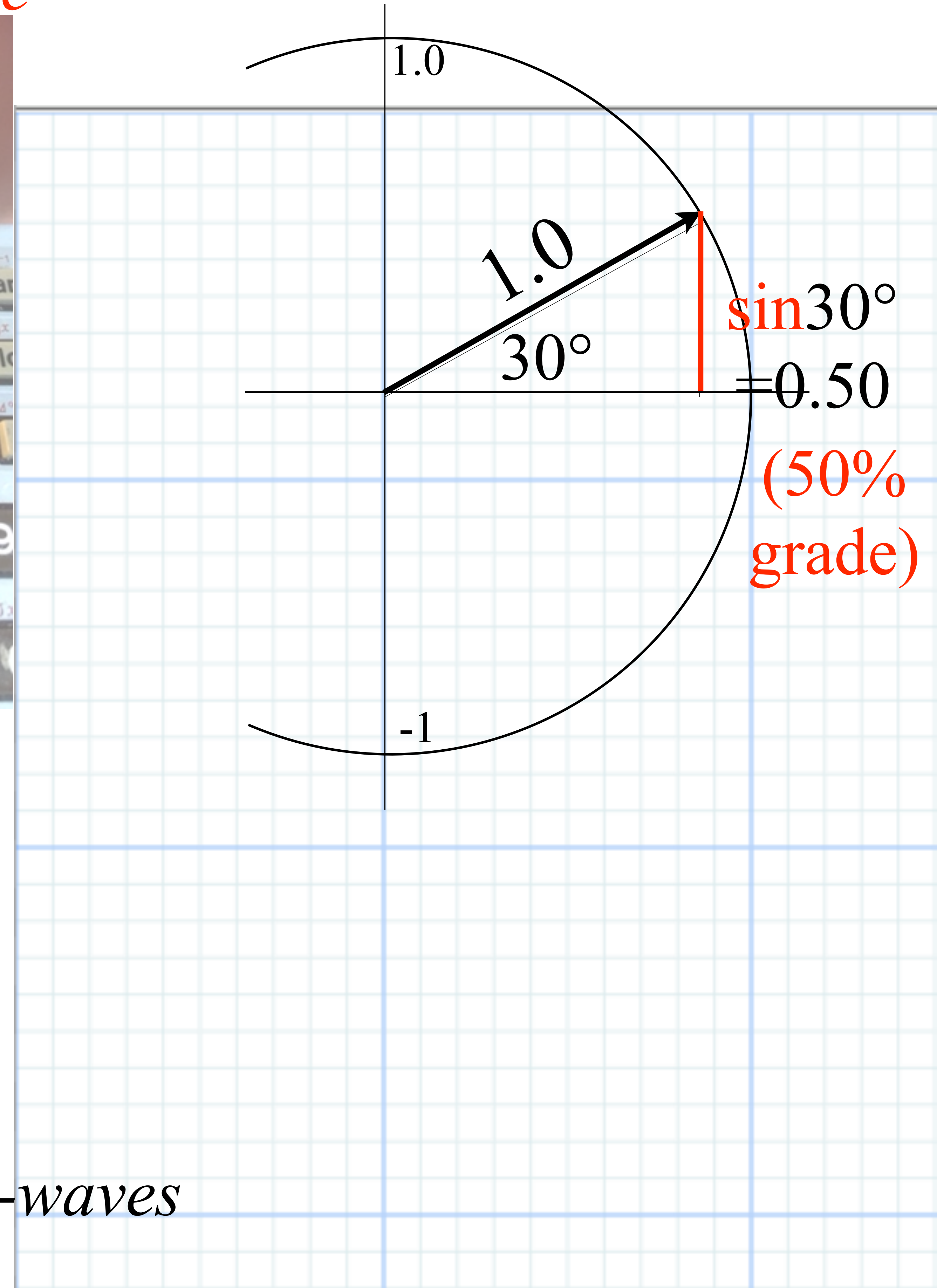




Learning about SIN

"Slope of INcline"

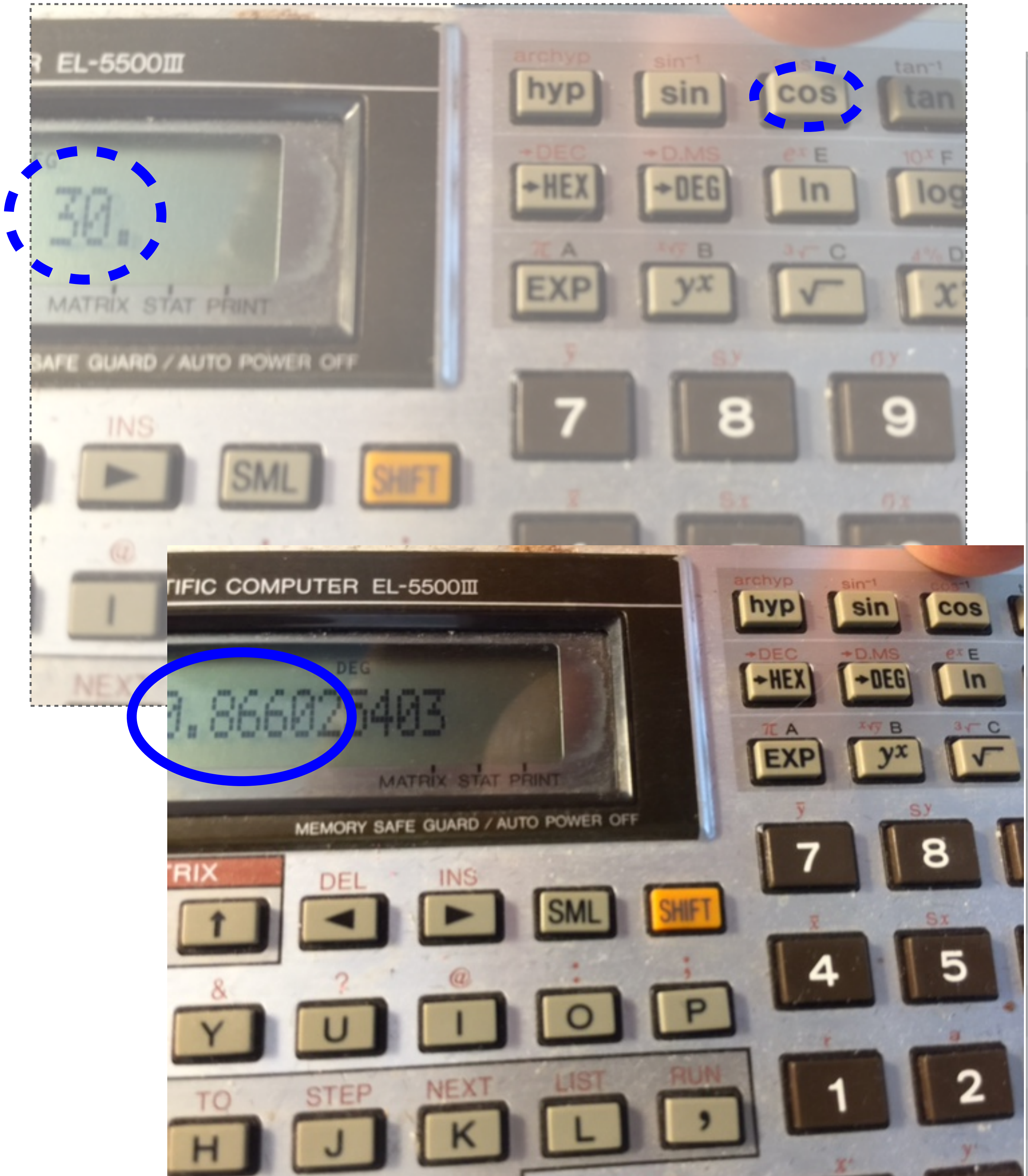


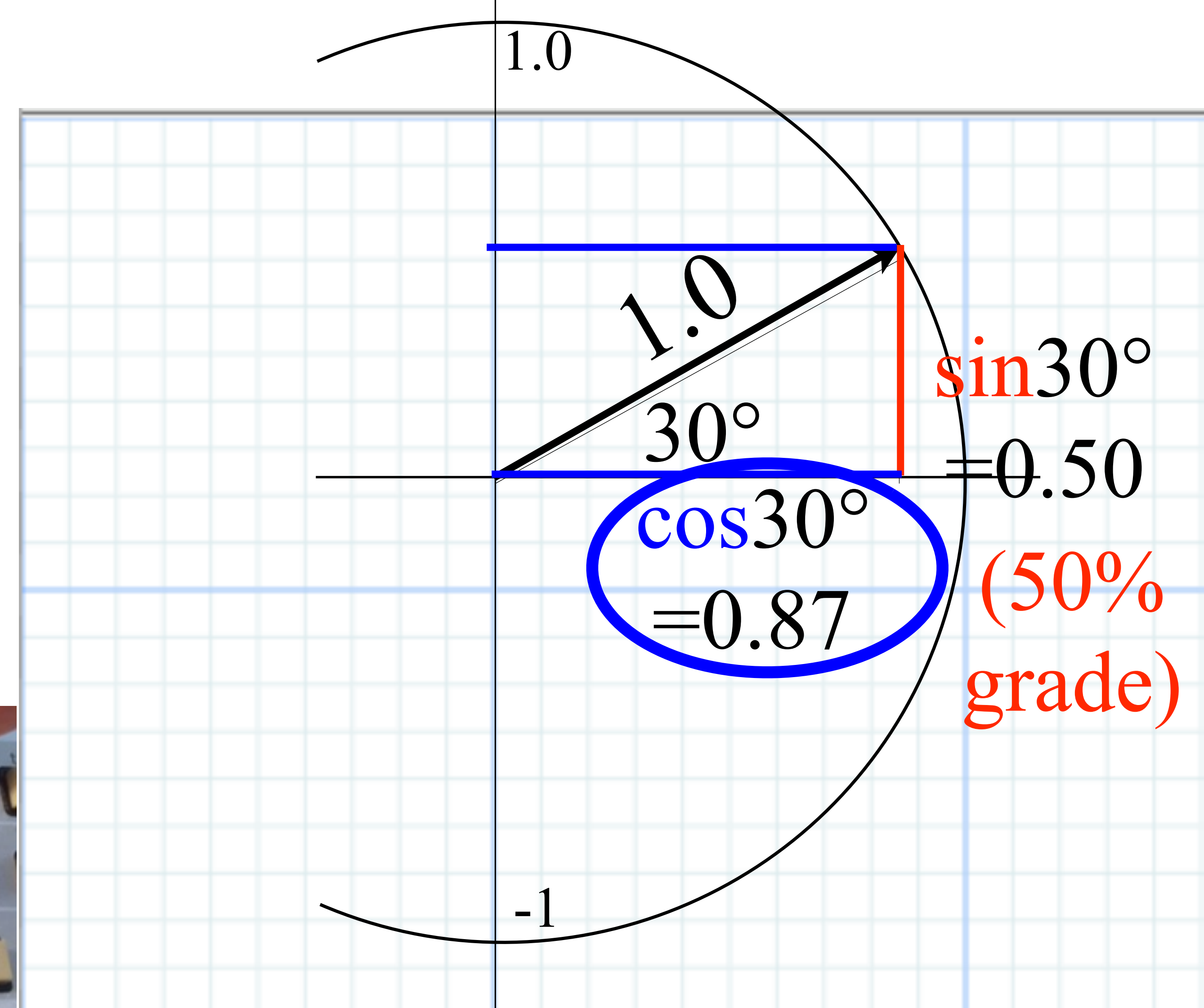


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin

“*Slope of INcline*” “COmplimentarySlope”

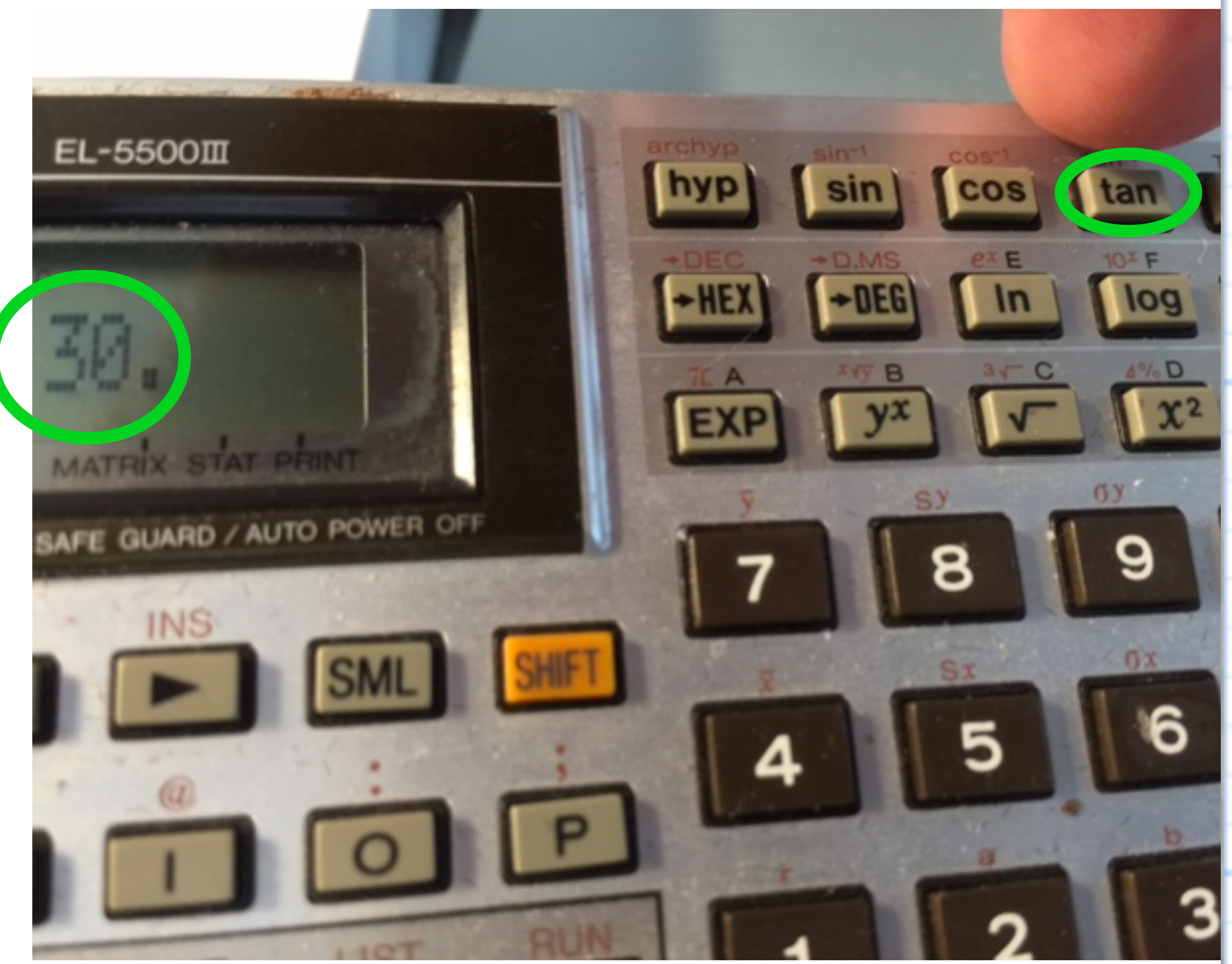


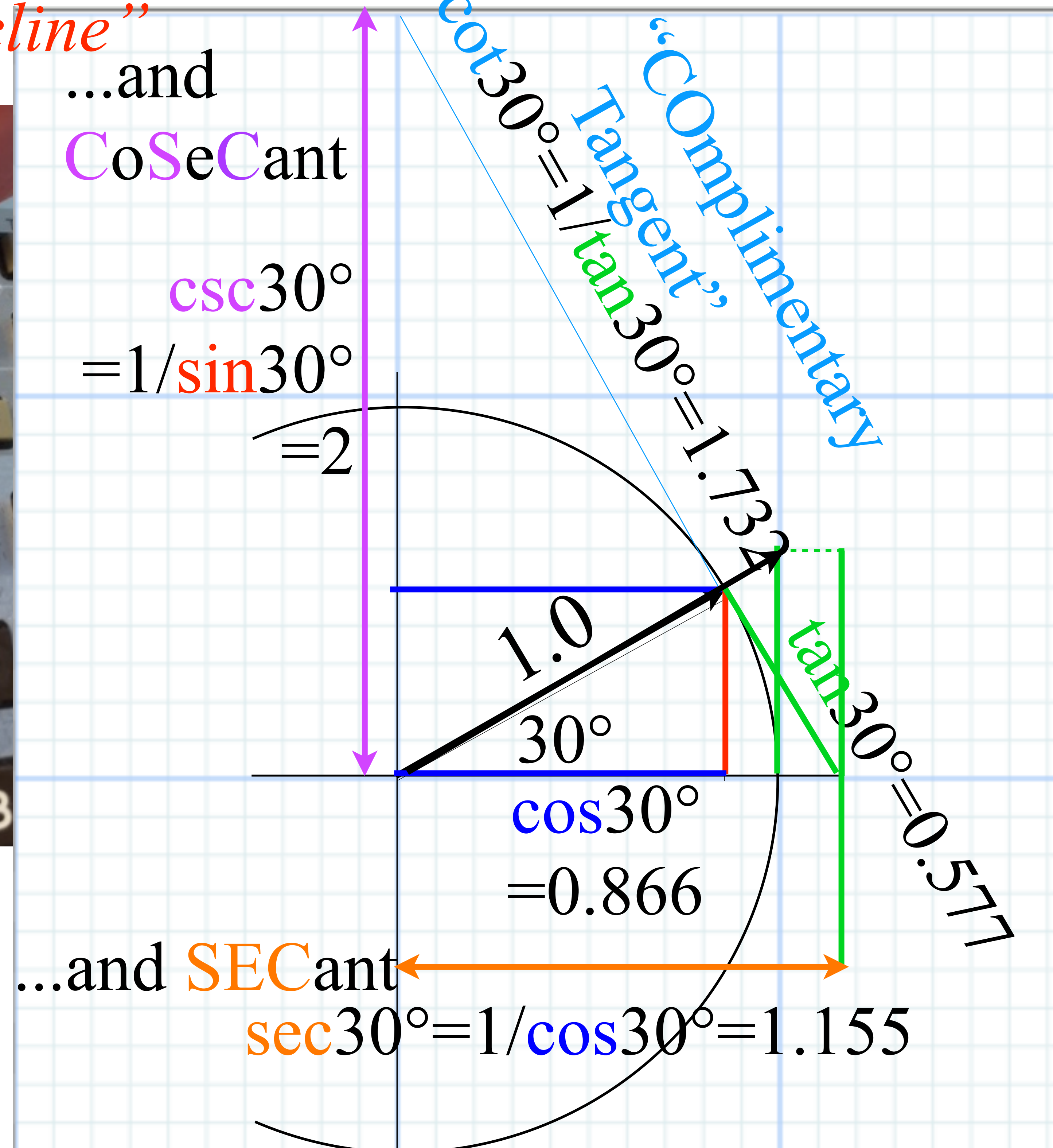


It’s mostly about triangles *and sine-waves and cosine-waves*

Learning about SIN and the COSin and TANgent and COTangent

"Slope of INcline"





Fundamental relativity and quantum wave mechanics is mostly about triangles *and sine-waves and cosine-waves*

Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

➨ Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

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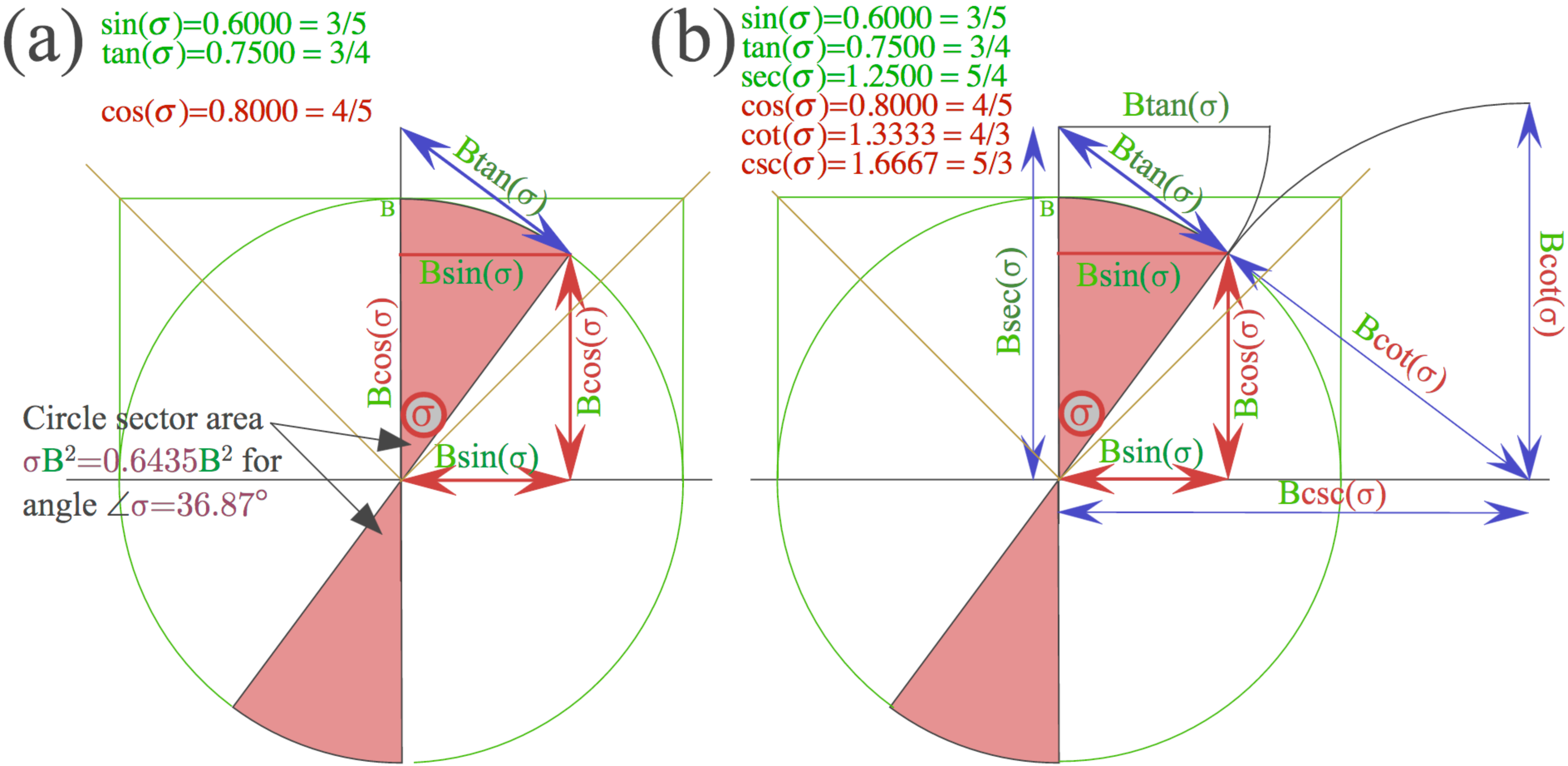
Trigonometric road maps



All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

RelaWavity Web Simulation
Relations between
Hypergeometric and
Hypergeometric functions

Trigonometric road maps become hyperbolic trig maps…

RelaWavity Web Simulation
Relations between
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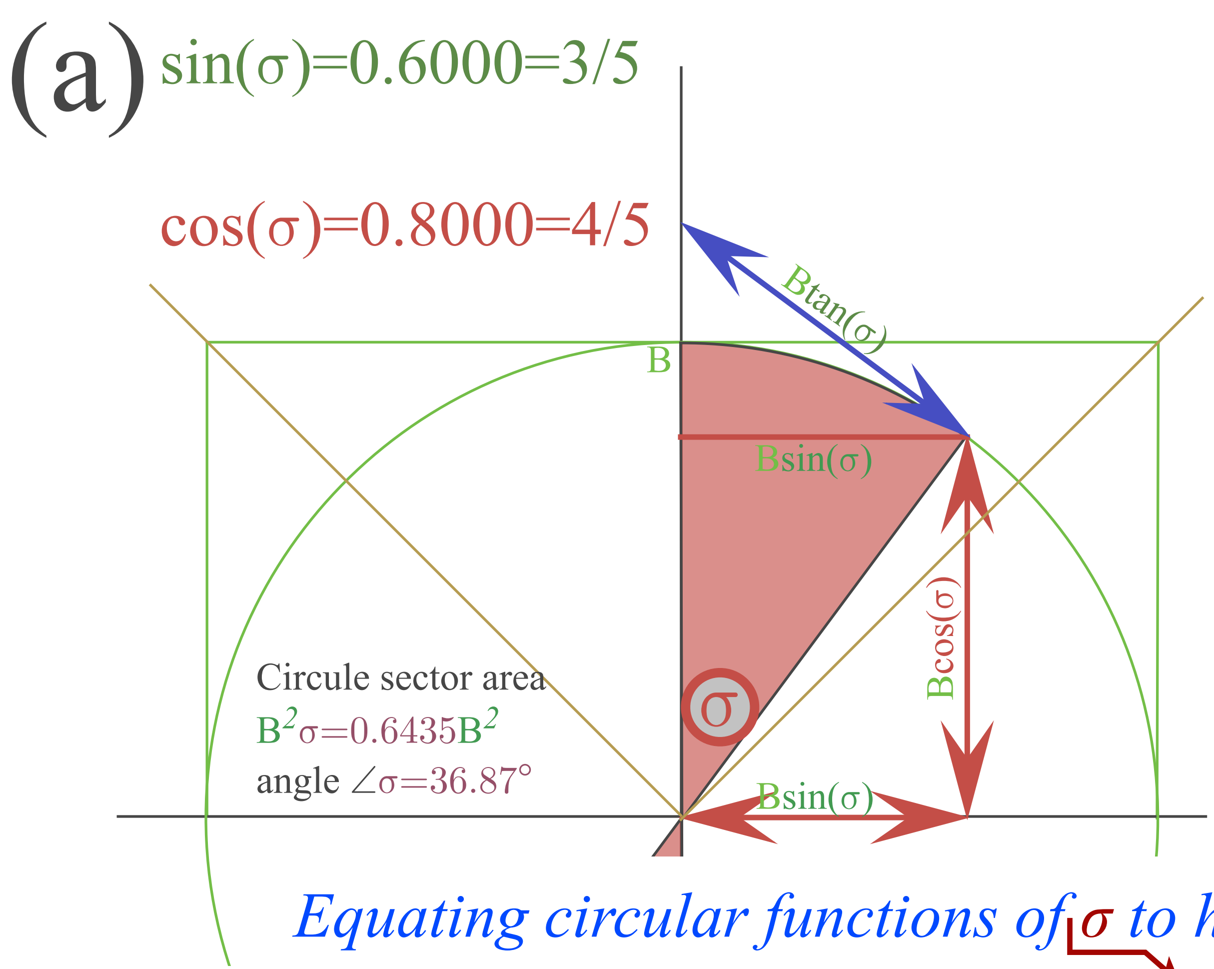
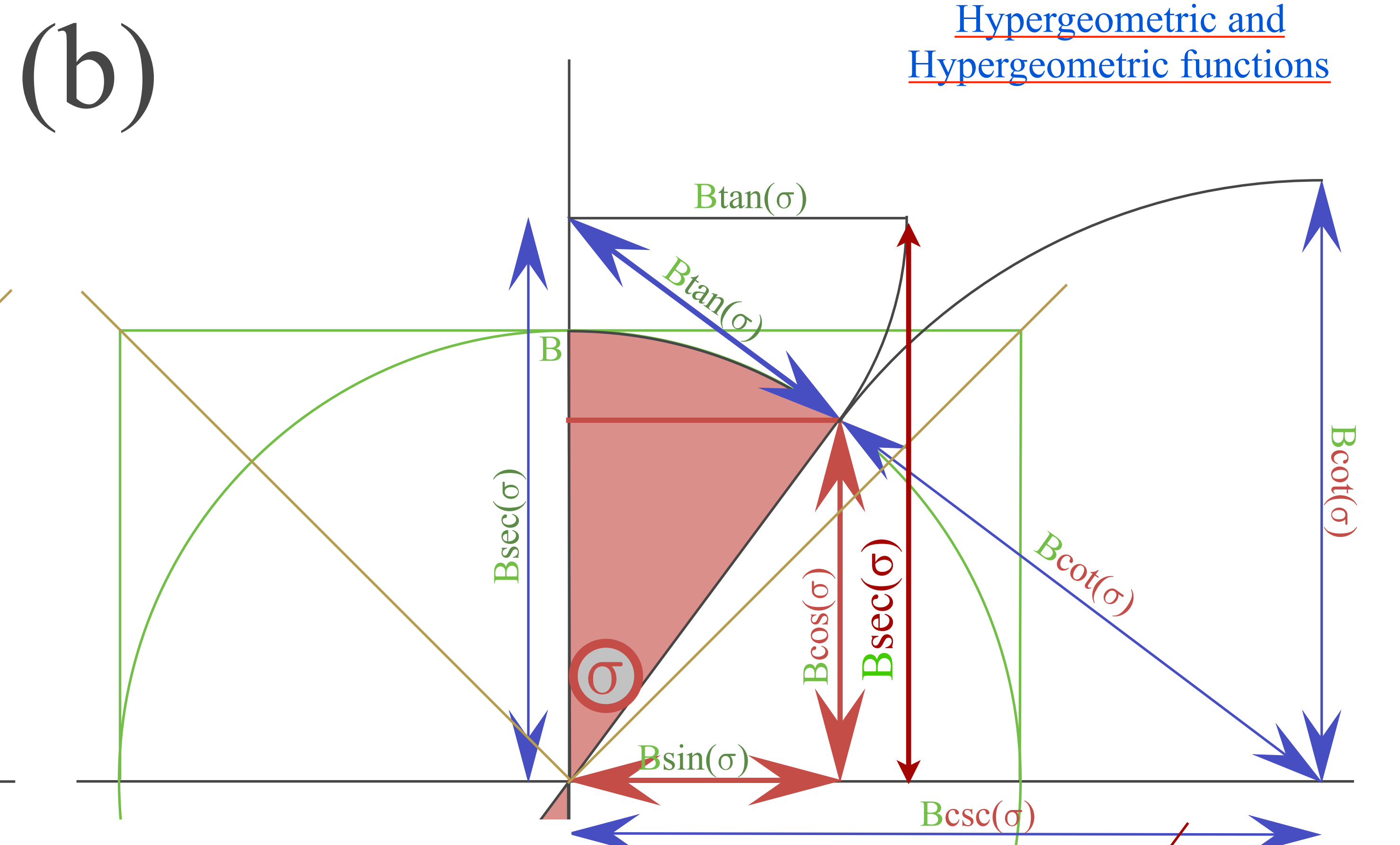
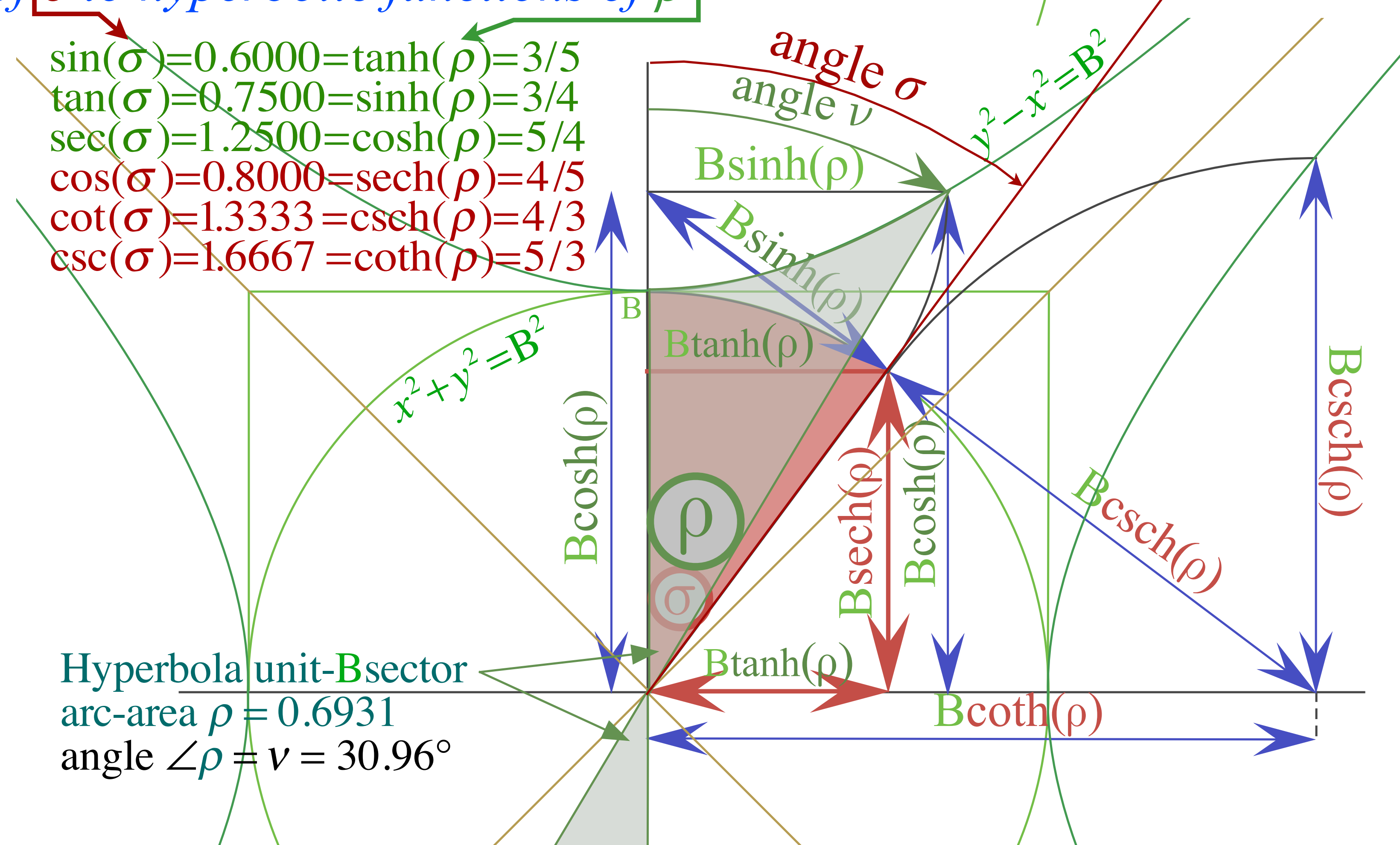




Equating circular functions of σ to hyperbolic functions of ρ

AMOP Ch.0 article p.9.



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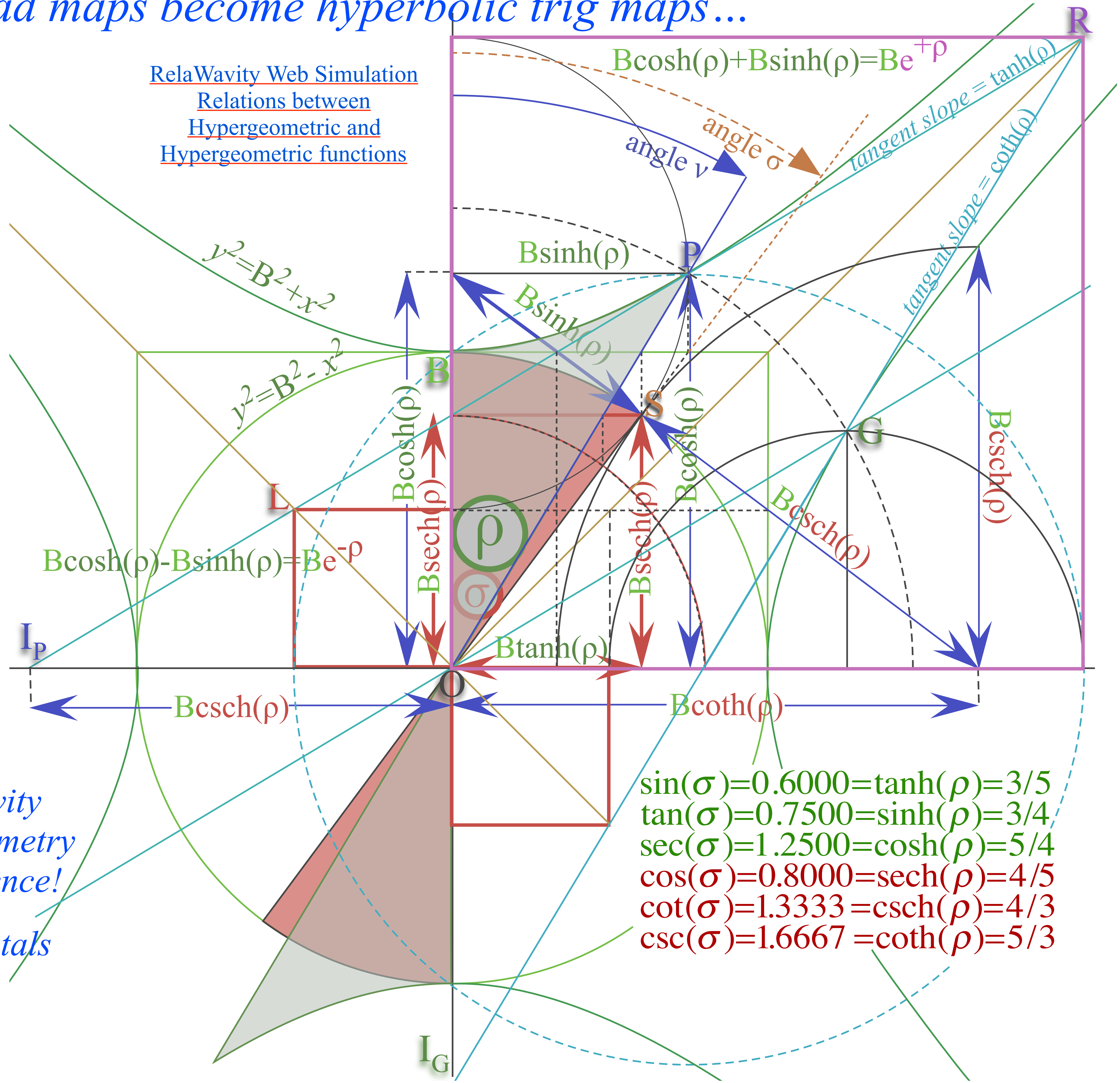
Trigonometric road maps become hyperbolic trig maps…

Need to see how trig road map matches the physical map on following page.



AMOP Ch.0 article p.10.

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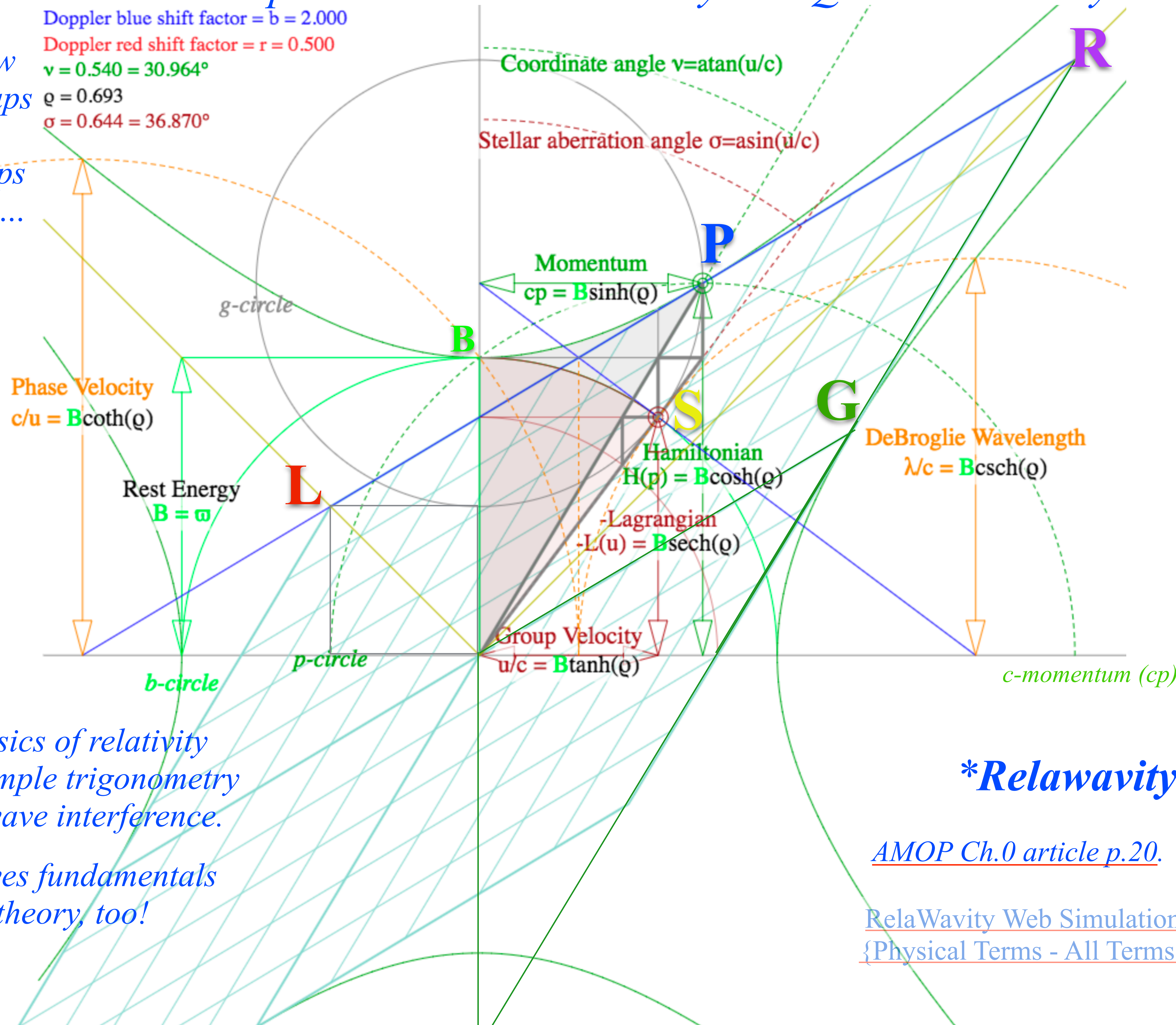
*Trigonometric road maps…. to Relativity and Quantum Theory**

Need to show trig road maps can match physical maps like this one…



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***Relawavity**

AMOP Ch.0 article p.20.

RelaWavity Web Simulation
{Physical Terms - All Terms}

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Hyper-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

Exponential derived by infinite-n-compounding limit of the interest rate-r formula.

$$e^{rt} = \lim_{n\to\infty}\left(1+\frac{rt}{n}\right)^n$$

Infinite-n limit of binomial series is an exponential power-p series of $(rt)^p$ with $1/p!$ coefficients.

$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots$$

$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots$$

Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine ($\cosh(rt)$) and sine ($\sinh(rt)$).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt)$$

$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

Hyper-Trig

$\cosh\rho$ and $\sinh\rho$

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Hyper-Trig

coshρ and sinhρ

Replace rate r with imaginary rate ir and $i=\sqrt{-1}$ powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,…

Then *hyper*-sine-cosine becomes the *circular*-sine-cosine.

$$\frac{e^{+i\,rt}+e^{-i\,rt}}{2} = 1 - \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cos rt$$

$$\frac{e^{+i\,rt}-e^{-i\,rt}}{2} = i\,rt - i\frac{(rt)^3}{2\cdot 3} + i\frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} - \ldots = i\sin rt$$

Circular-Trig

cosσ and sinσ

Sum and difference of this pair gives the Euler-DeMoivre relations of exponentials vs trig-functions.

$$e^{+i\sigma} = \cos\sigma + i\sin\sigma\,, \qquad e^{-i\sigma} = \cos\sigma - i\sin\sigma.$$

$$e^{+\rho} = \cosh\rho + \sinh\rho\,, \qquad e^{-\sigma} = \cosh\rho - \sinh\rho.$$

Why ***Men in Black*** *shot little Suzie…Learning about* sin!*,*cos *and…Trigonometric road maps*

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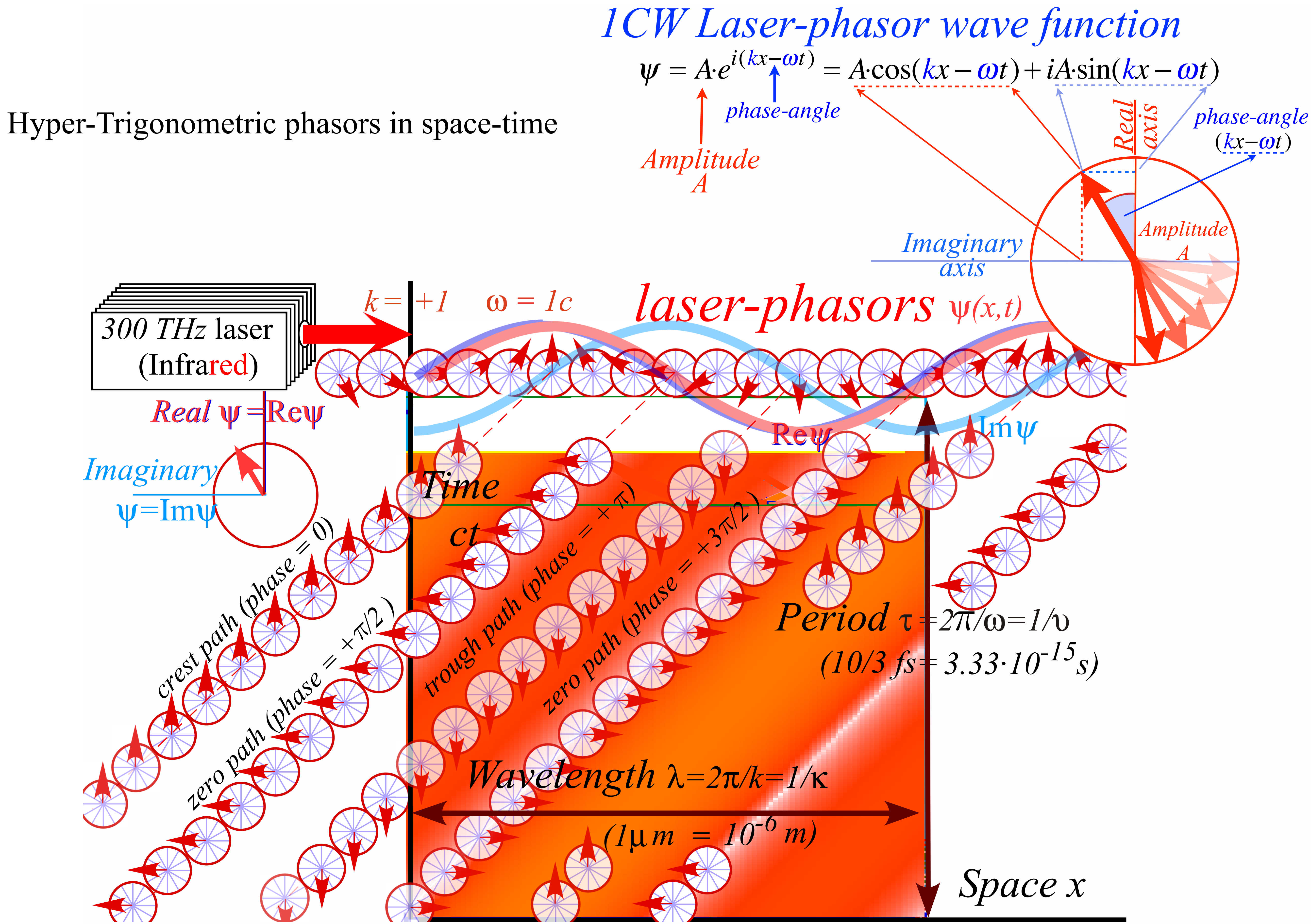


(a) Single-phasor plot of wave-function at (x,ct). (b) Array of phasors at many (x,ct)-points.

Pirelli Site
World of Clocks
Animations

24 hr. clock

12 hr. clock

1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Dimensionless Light wave-velocity c/c=1

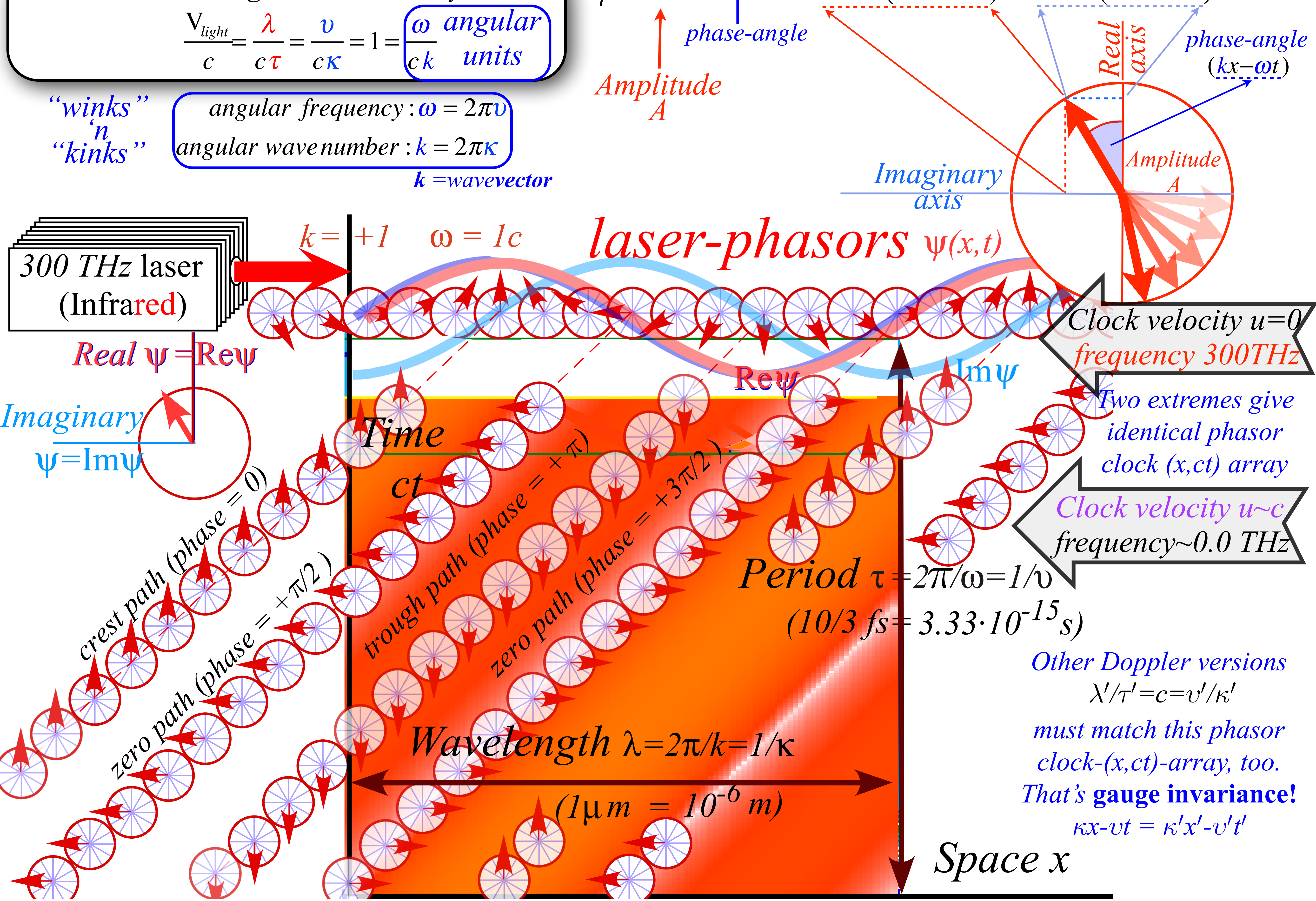
$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck} \quad \text{angular units}$$

"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

angular wave number : $k = 2\pi\kappa$

k =wave**vector**



Other Doppler versions $\lambda'/\tau' = c = \upsilon'/\kappa'$ must match this phasor clock-(x,ct)-array, too. That's **gauge invariance!**

$$\kappa x - \upsilon t = \kappa' x' - \upsilon' t'$$

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The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

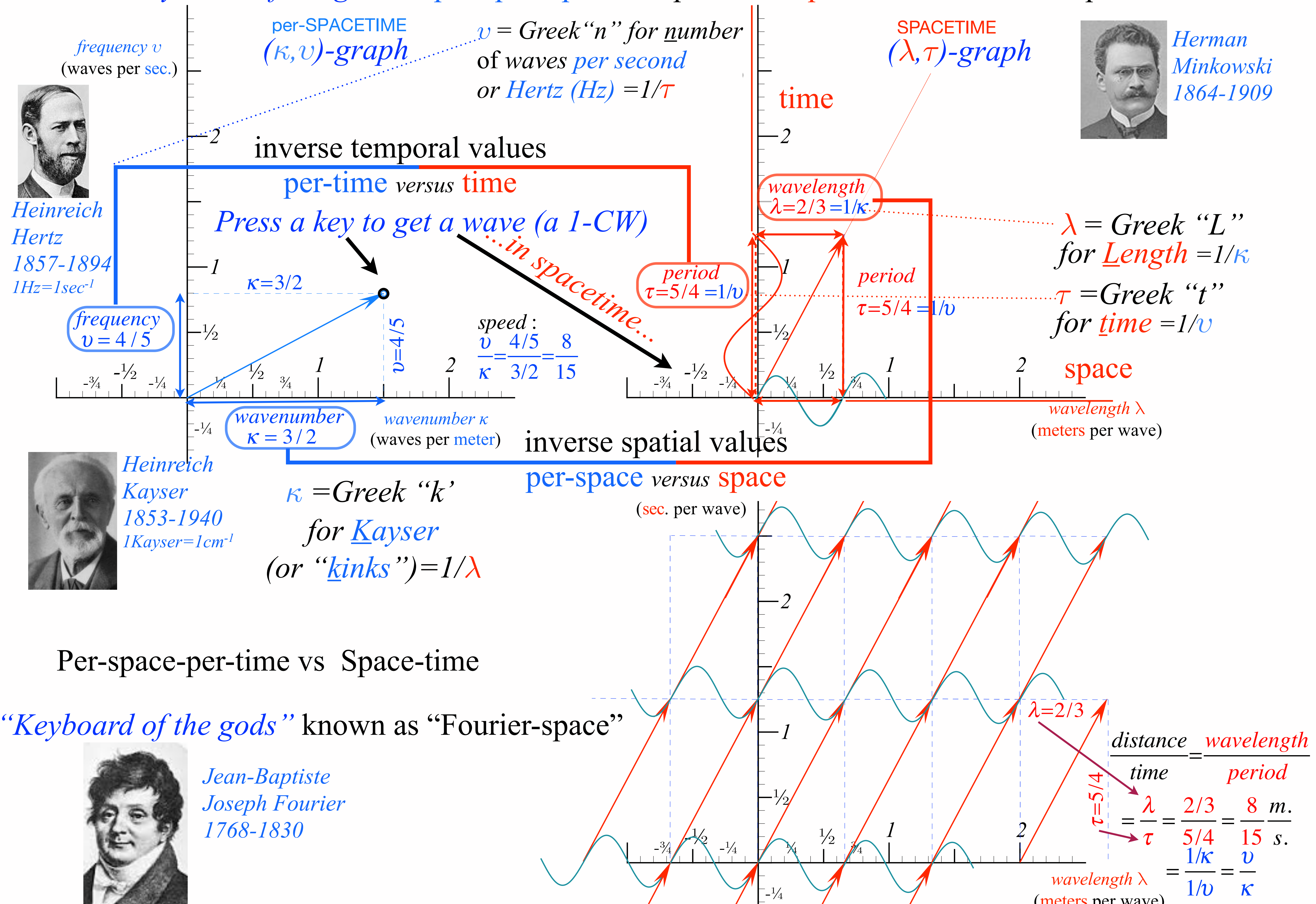
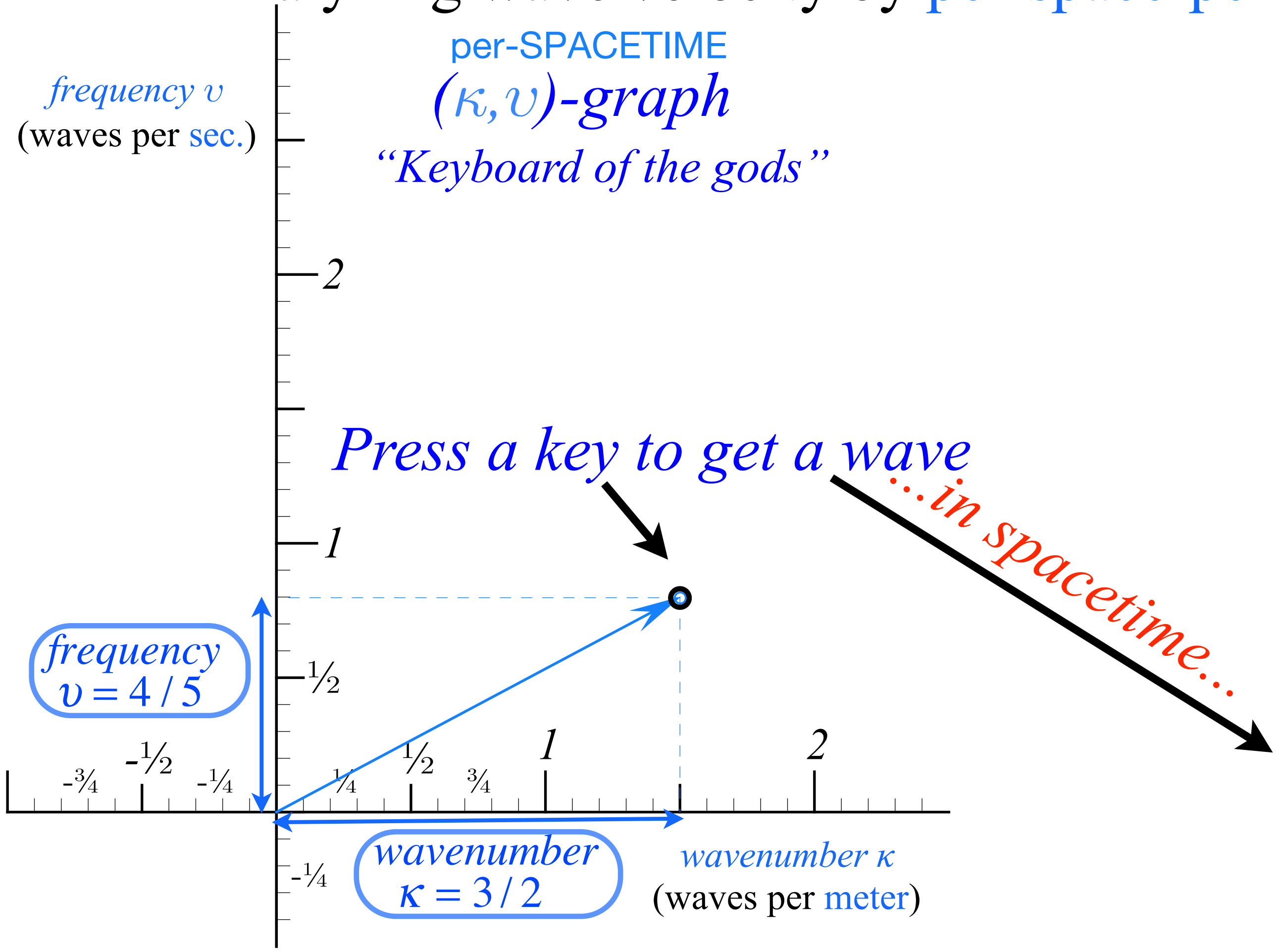


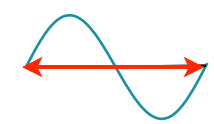
Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909



SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

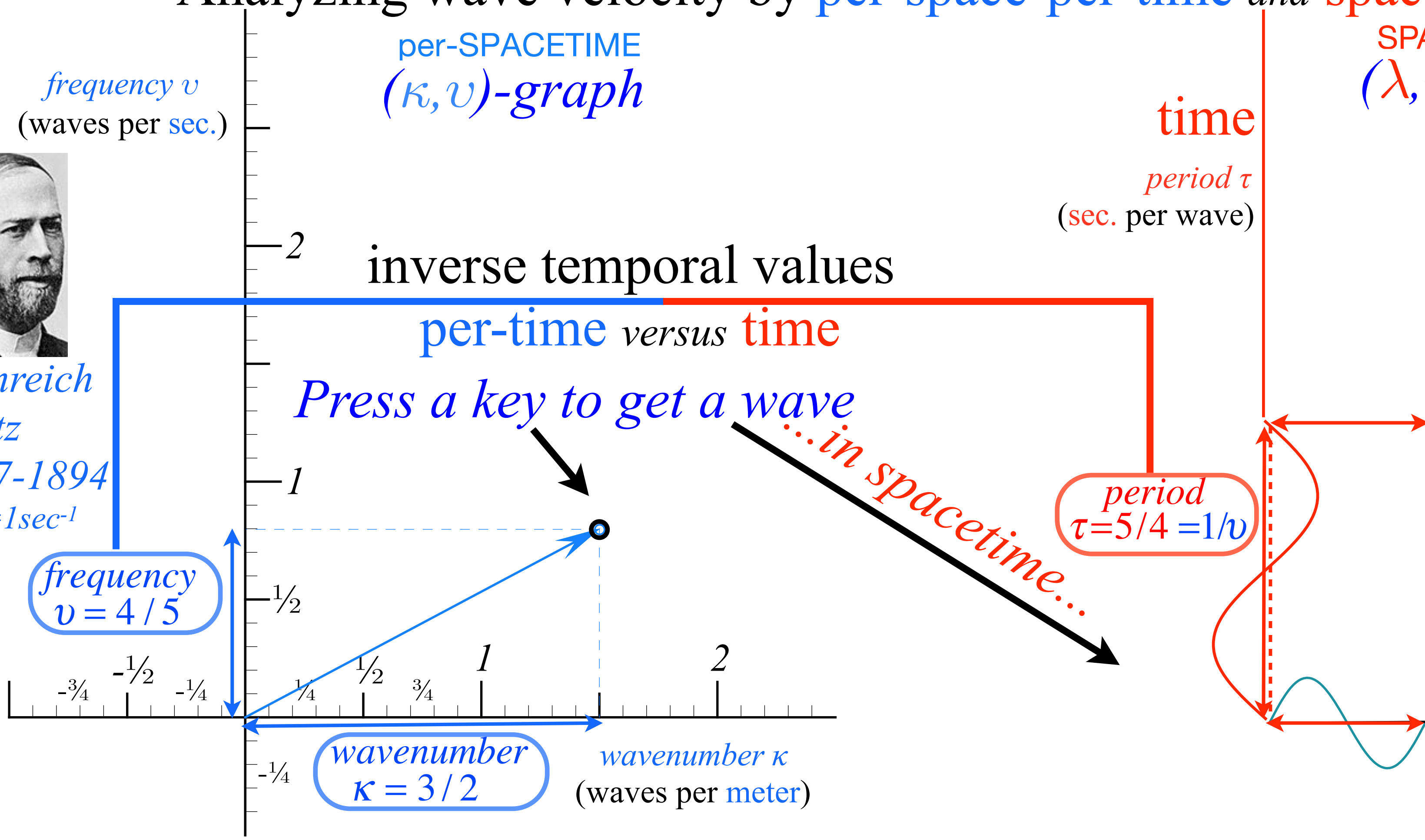
Jean-Baptiste Joseph Fourier 1768-1830

- *How to understand waves and wave parameters*

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

[RelaWavity Web Simulation Keyboard of the Gods (per-Time vs per-Space)]

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Heinreich Hertz 1857-1894

1Hz=1sec⁻¹

Herman Minkowski 1864-1909

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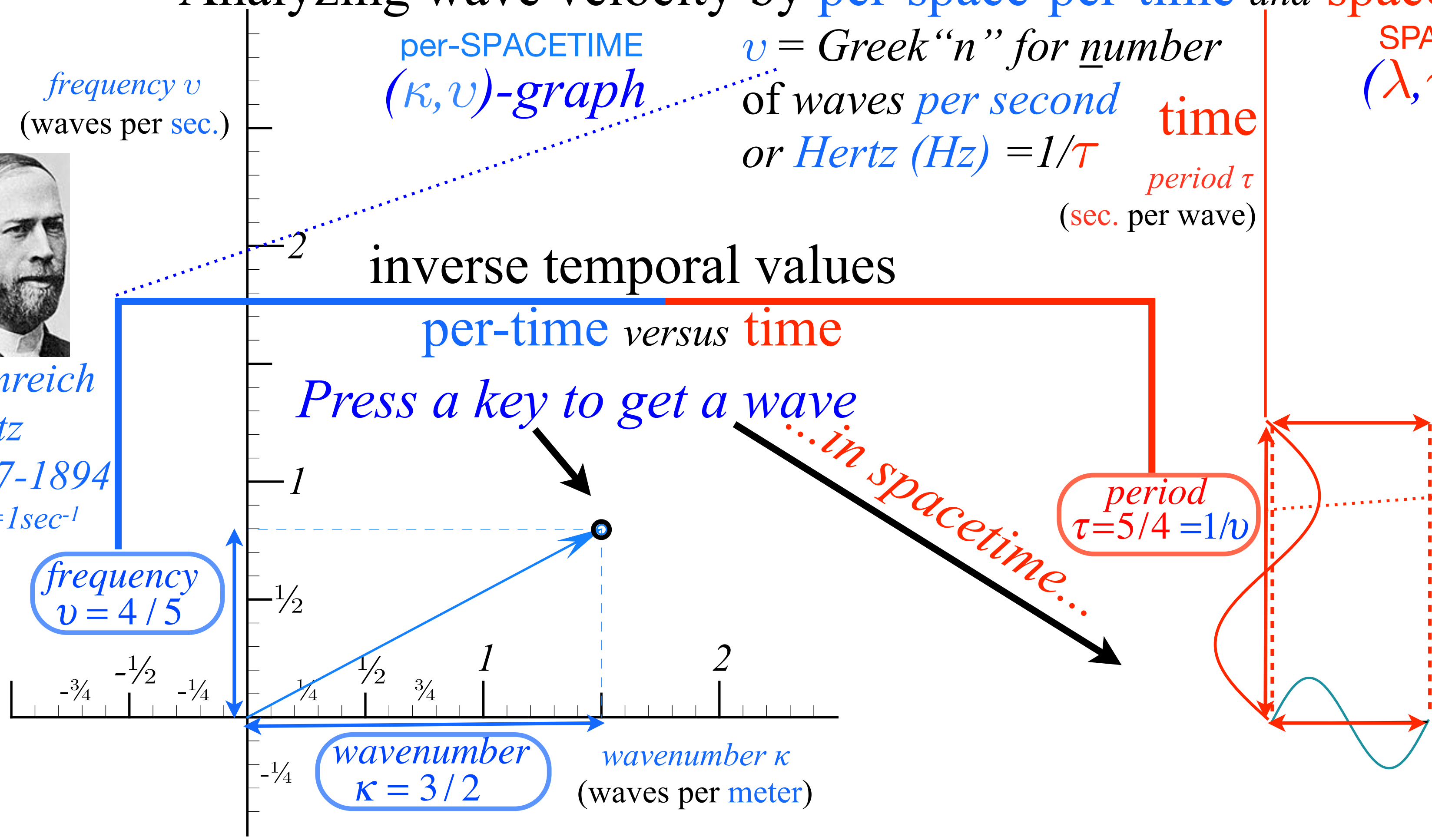

τ =Greek "t" for time =1/υ

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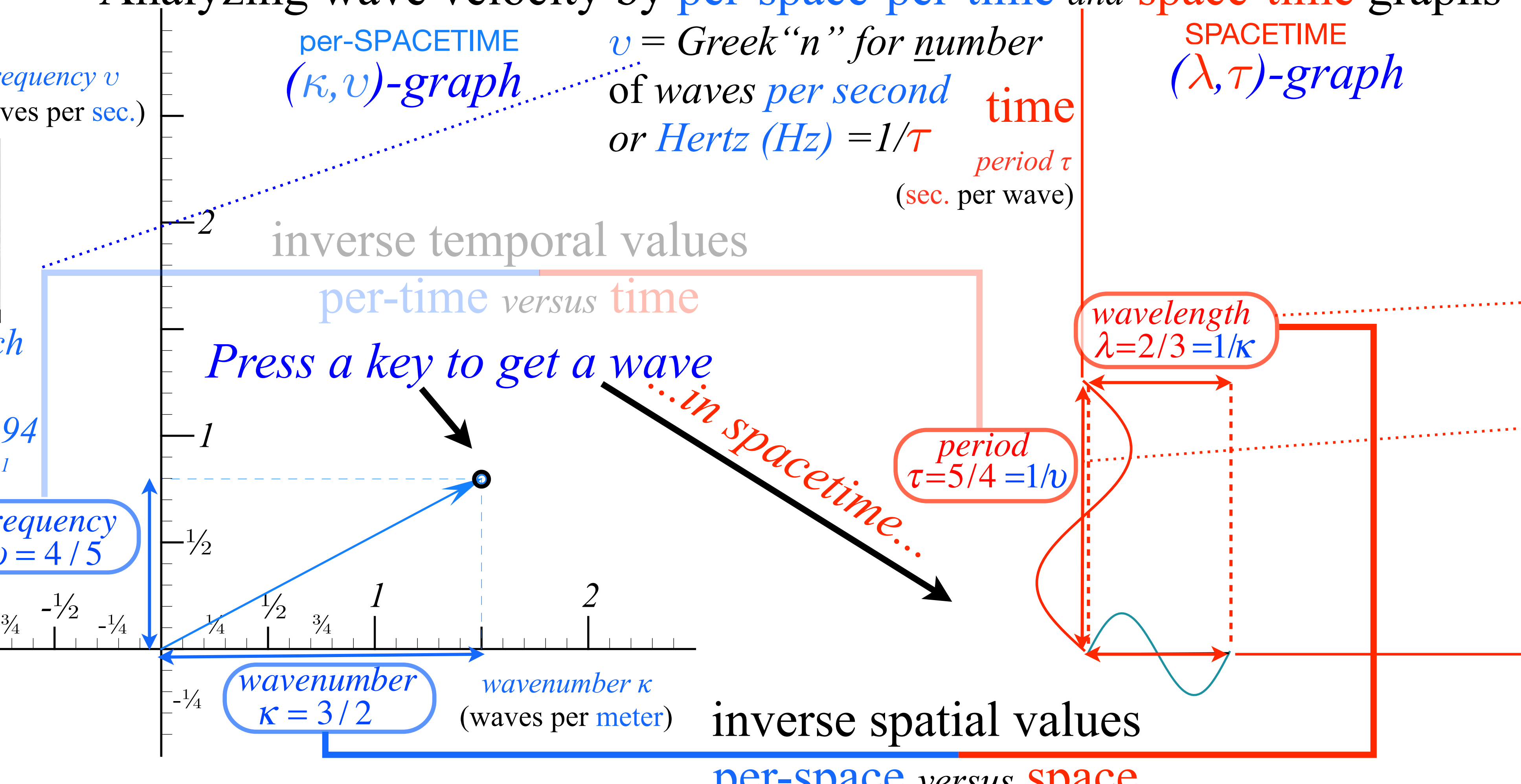
[RelaWavity Web Simulation](#)
[Keyboard of the Gods (Dual Plot #1)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Heinreich Hertz 1857-1894

1Hz=1sec⁻¹

κ =Greek "k" for Kayser (or "kinks")=1/λ

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> *•How to understand waves and wave parameters*
>
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Herman Minkowski 1864-1909

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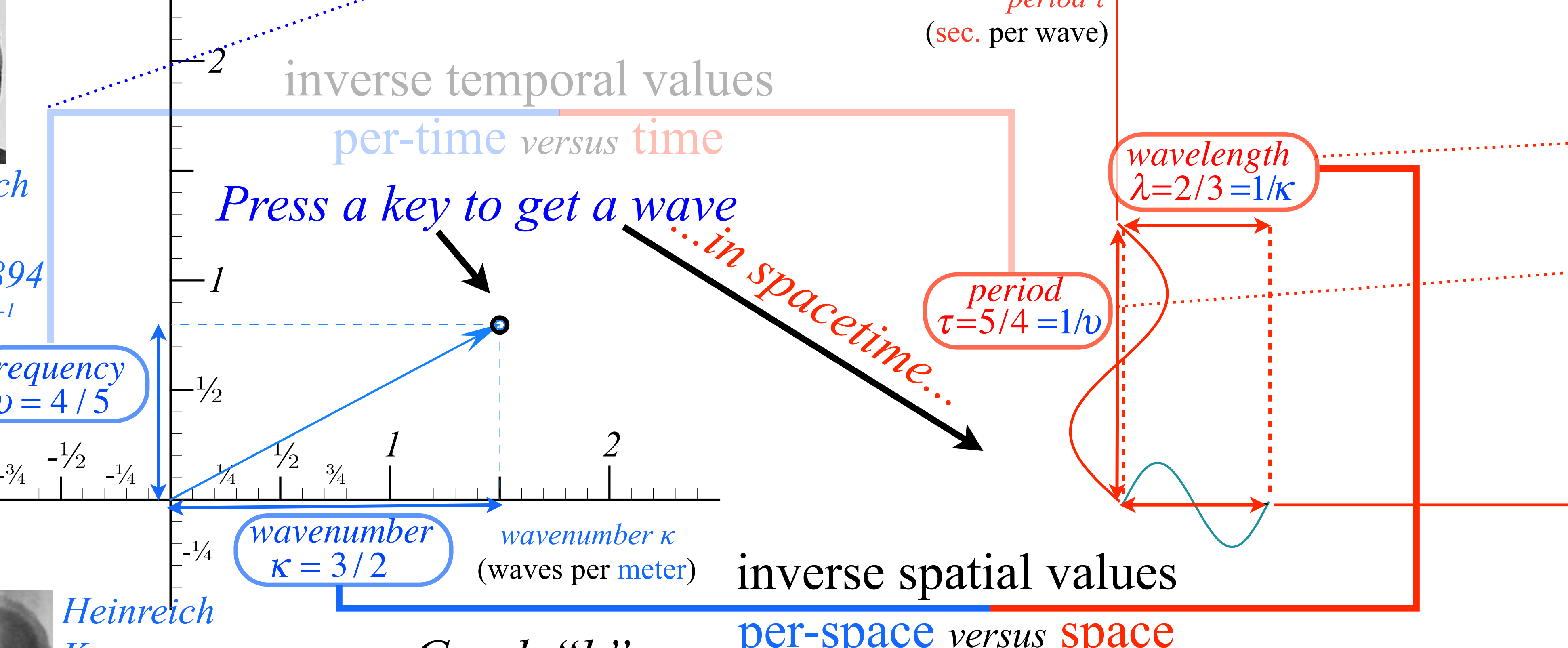

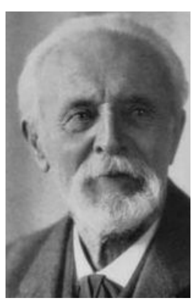

Heinreich Kayser 1853-1940
1Kayser=1cm⁻¹

κ =Greek "k" for Kayser (or "kinks")=1/λ

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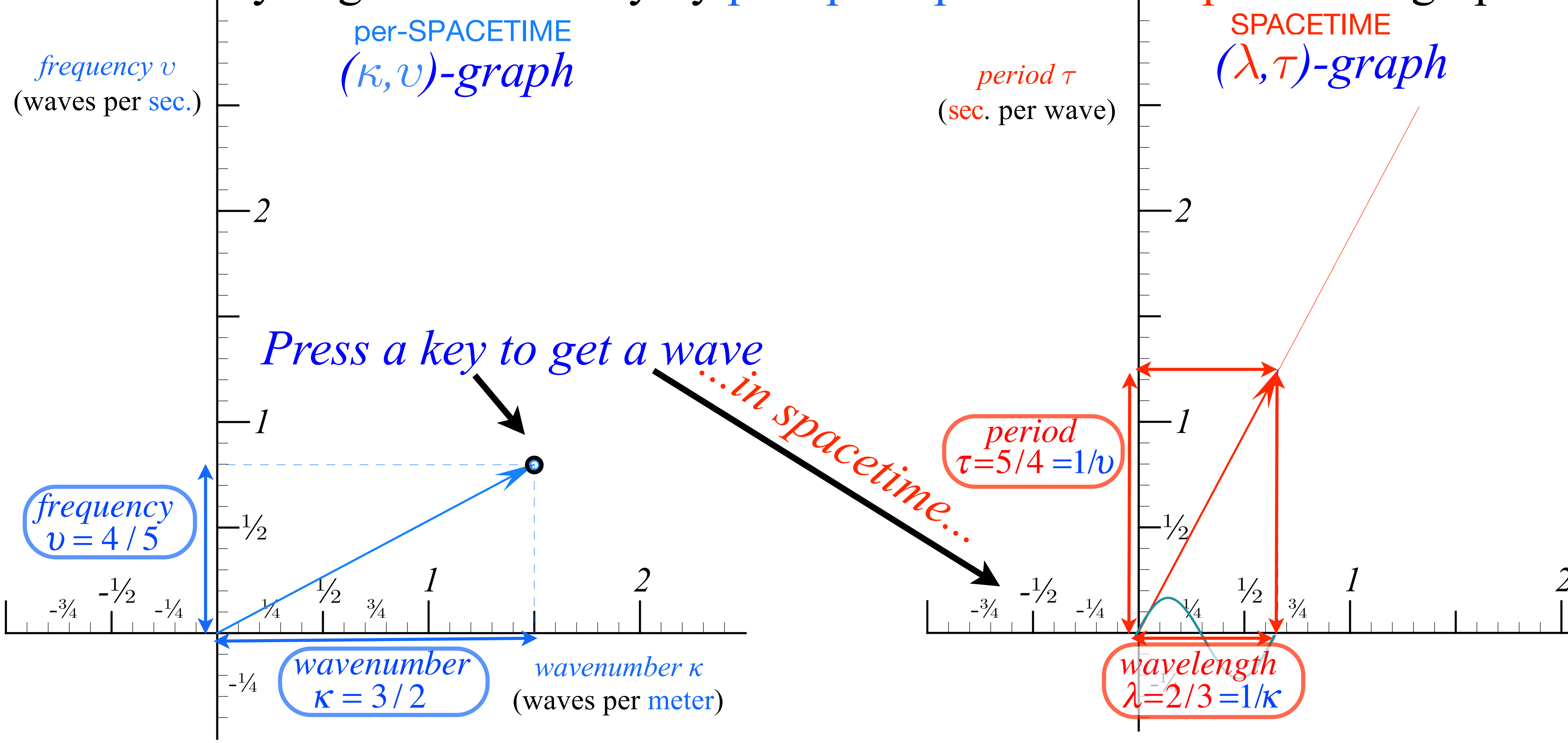
[RelaWavity Web Simulation](#)
[Keyboard of the Gods (Dual Plot #1)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Herman Minkowski 1864-1909

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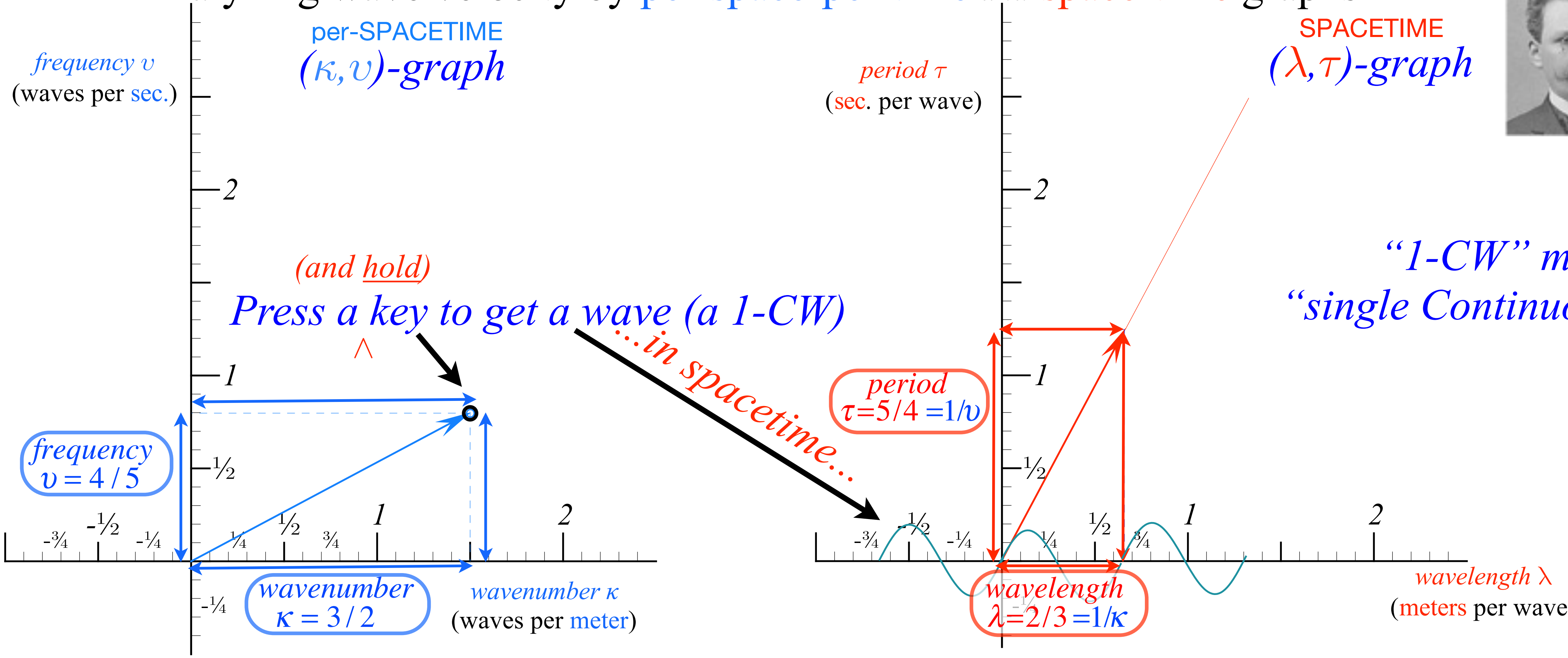
Jean-Baptiste Joseph Fourier 1768-1830

- *How to understand waves and wave parameters*

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

Analyzing wave velocity by per-space-per-time *and* space-time graphs



"1-CW" means "single Continuous Wave"

"Keyboard of the gods" is known as "Fourier-space"

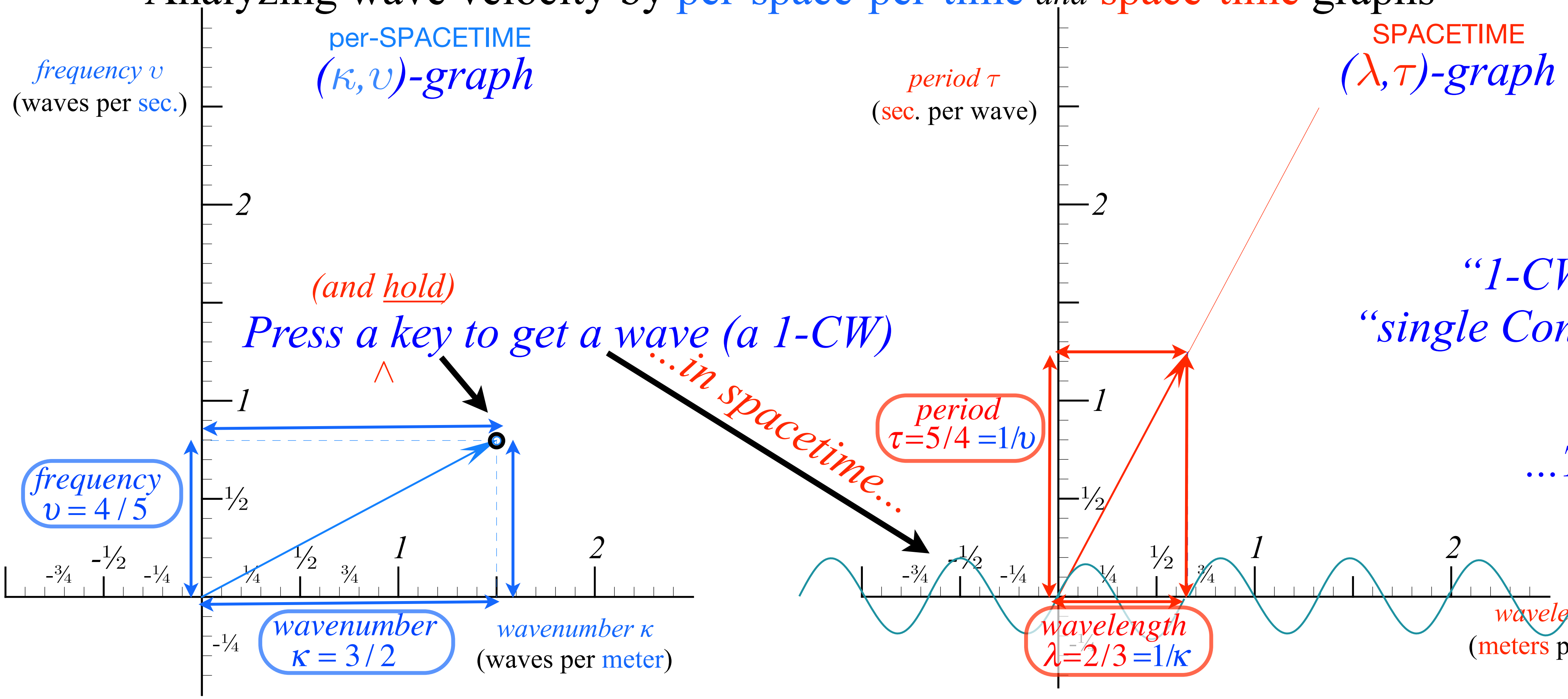
•How to understand waves and wave parameters

wave frequency υ	*wave period* τ
wavenumber κ	*wavelength* λ

[RelaWavity Web Simulation](#)
[Keyboard of the Gods (Dual Plot #2)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909



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•How to understand waves and wave parameters

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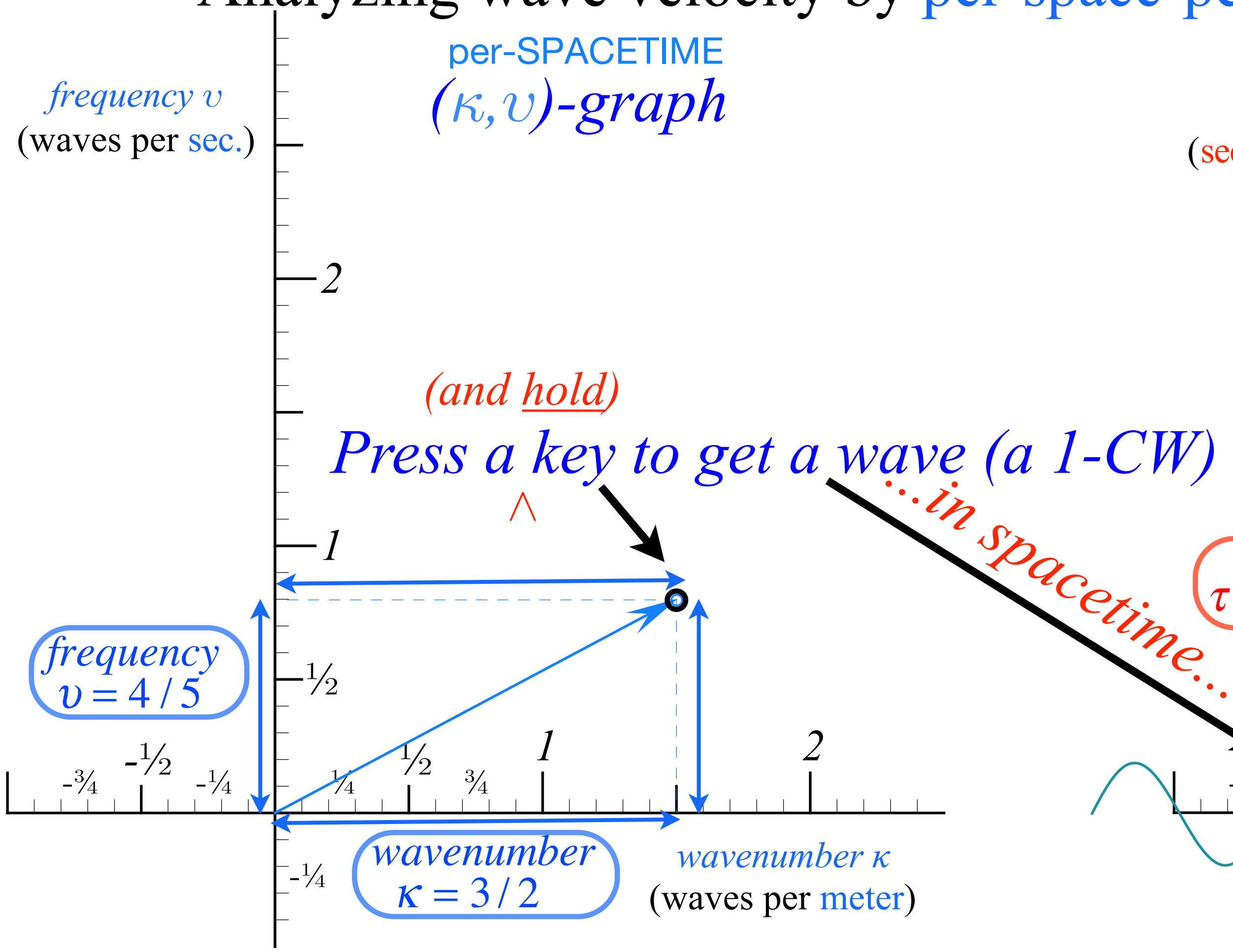
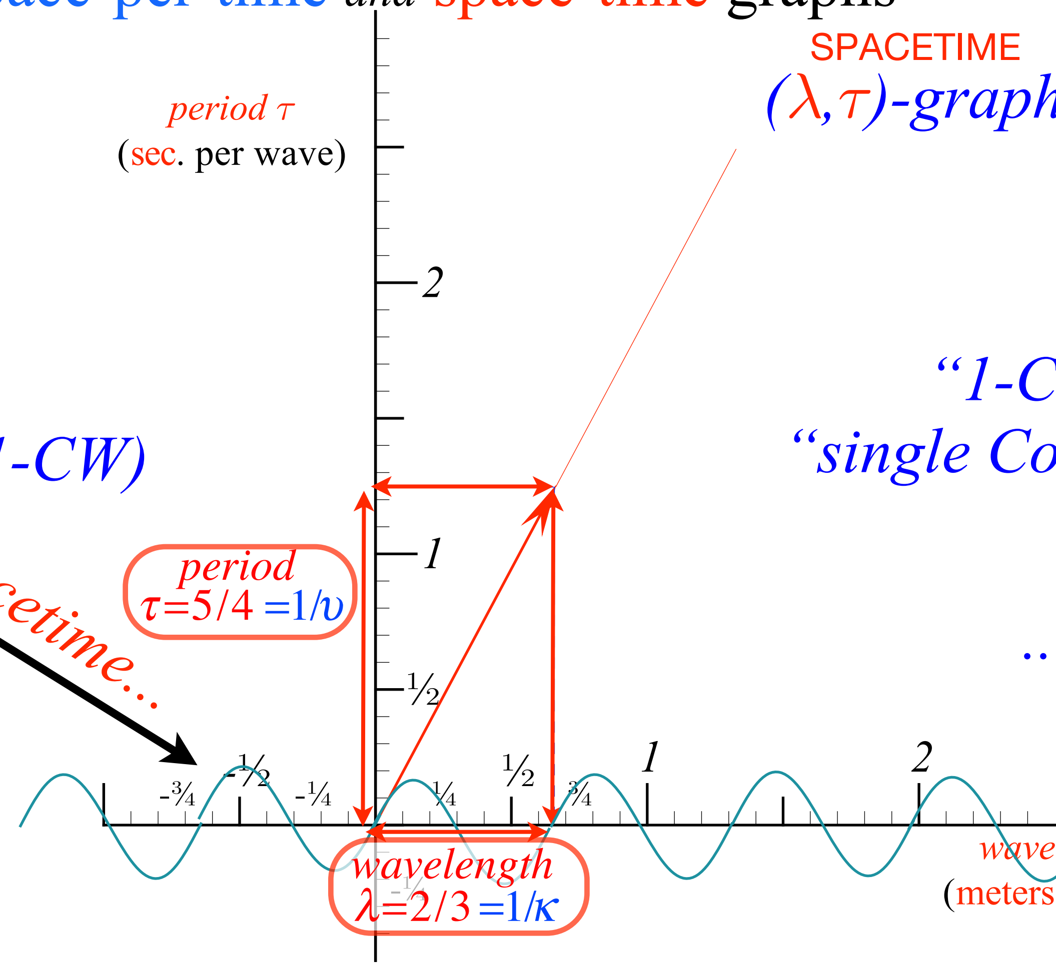
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #3)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909

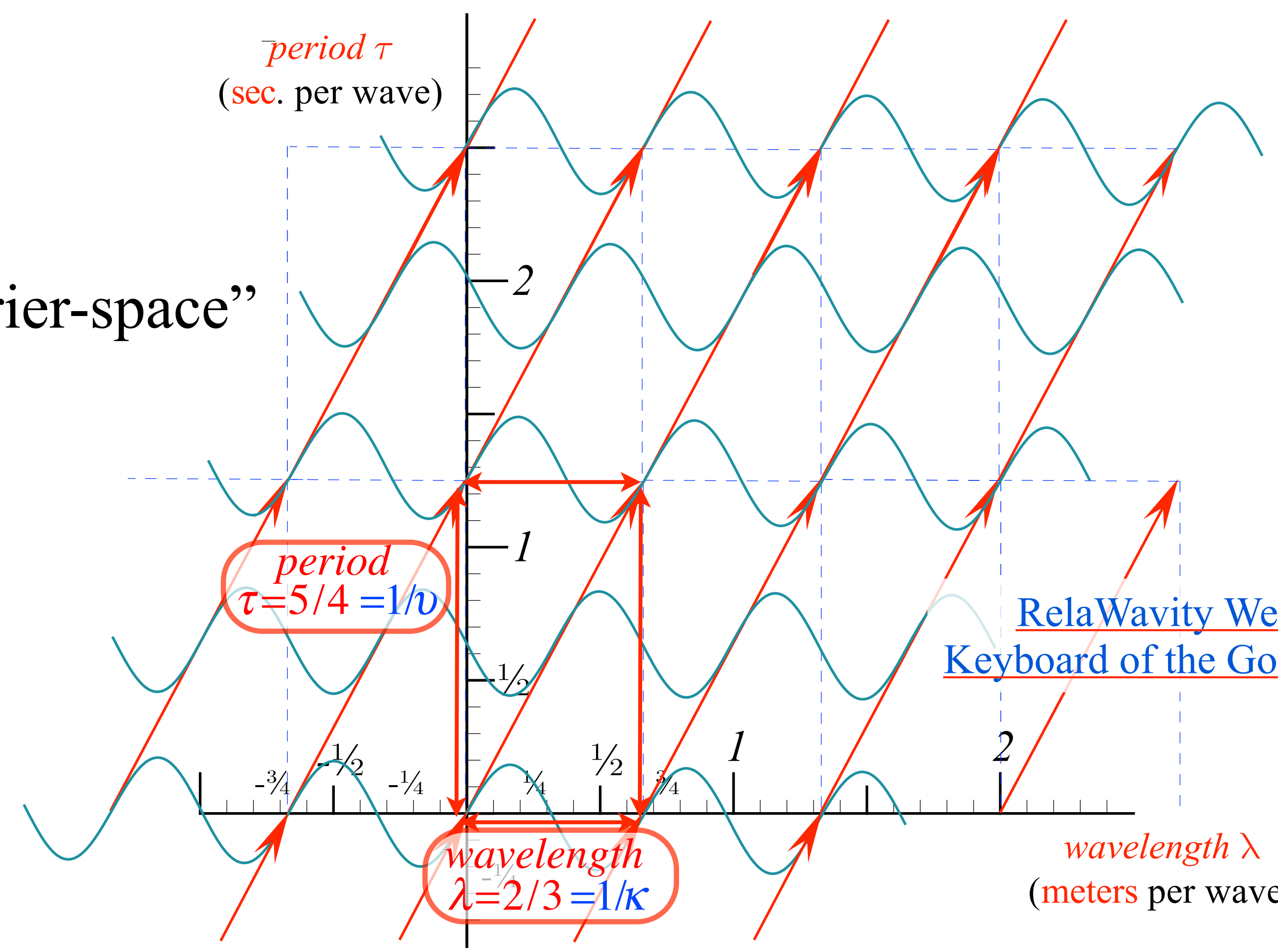




"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

...for all time...



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Jean-Baptiste Joseph Fourier 1768-1830

[RelaWavity Web Simulation Keyboard of the Gods (Dual Plot #7)]

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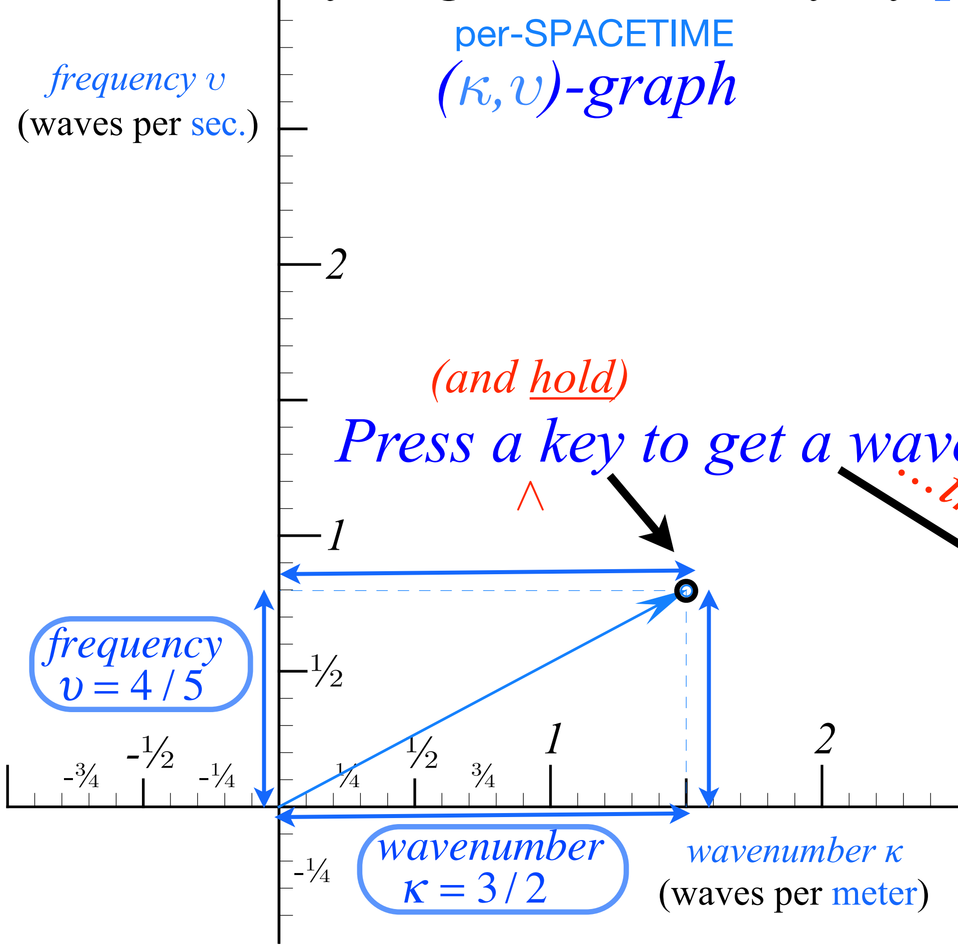
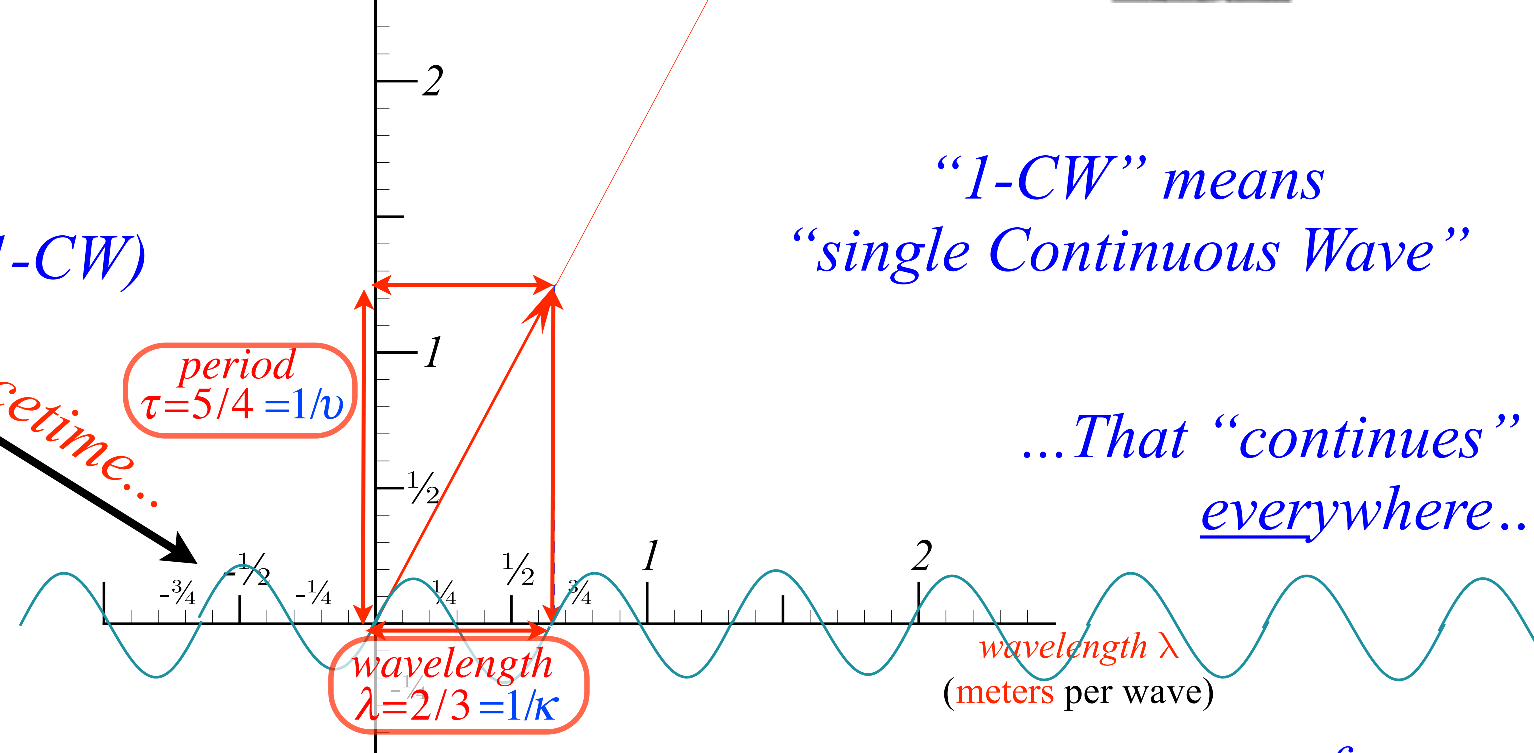


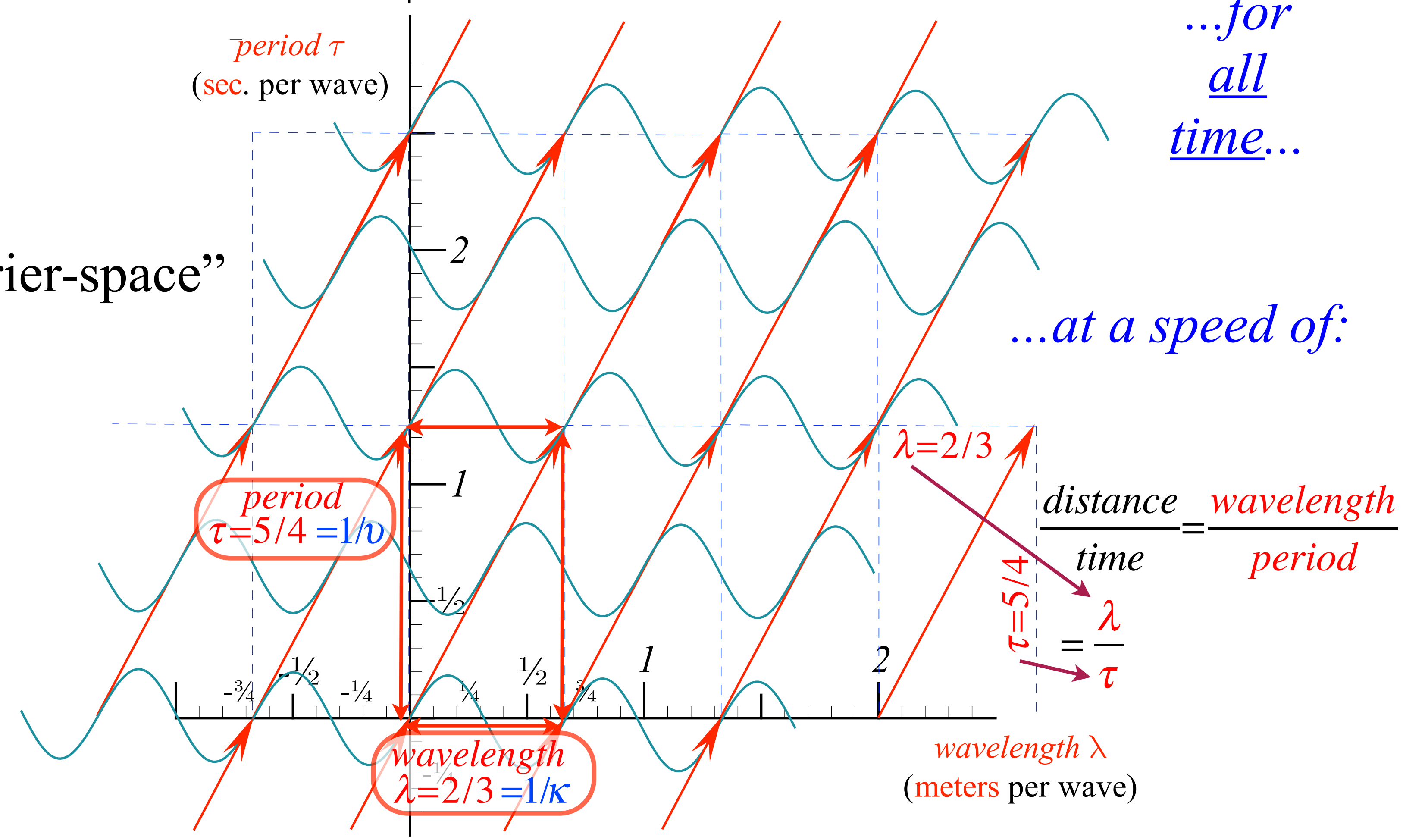


"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

...for all time...

...at a speed of:

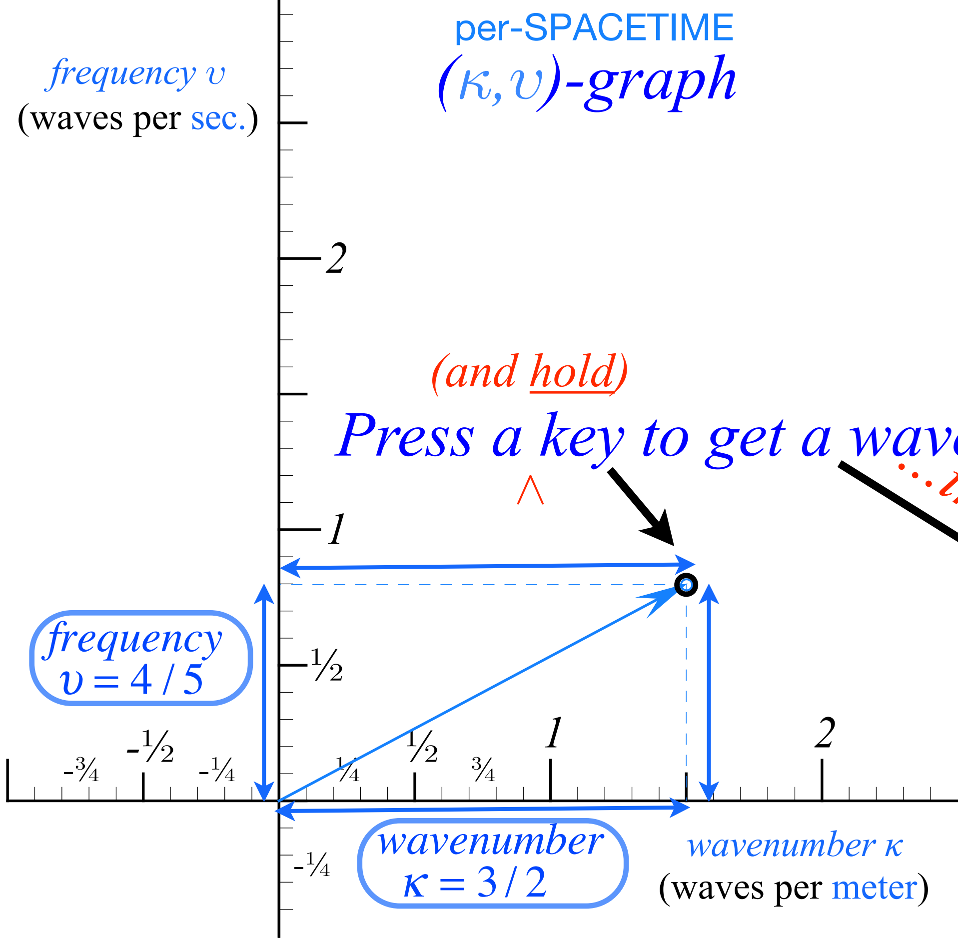


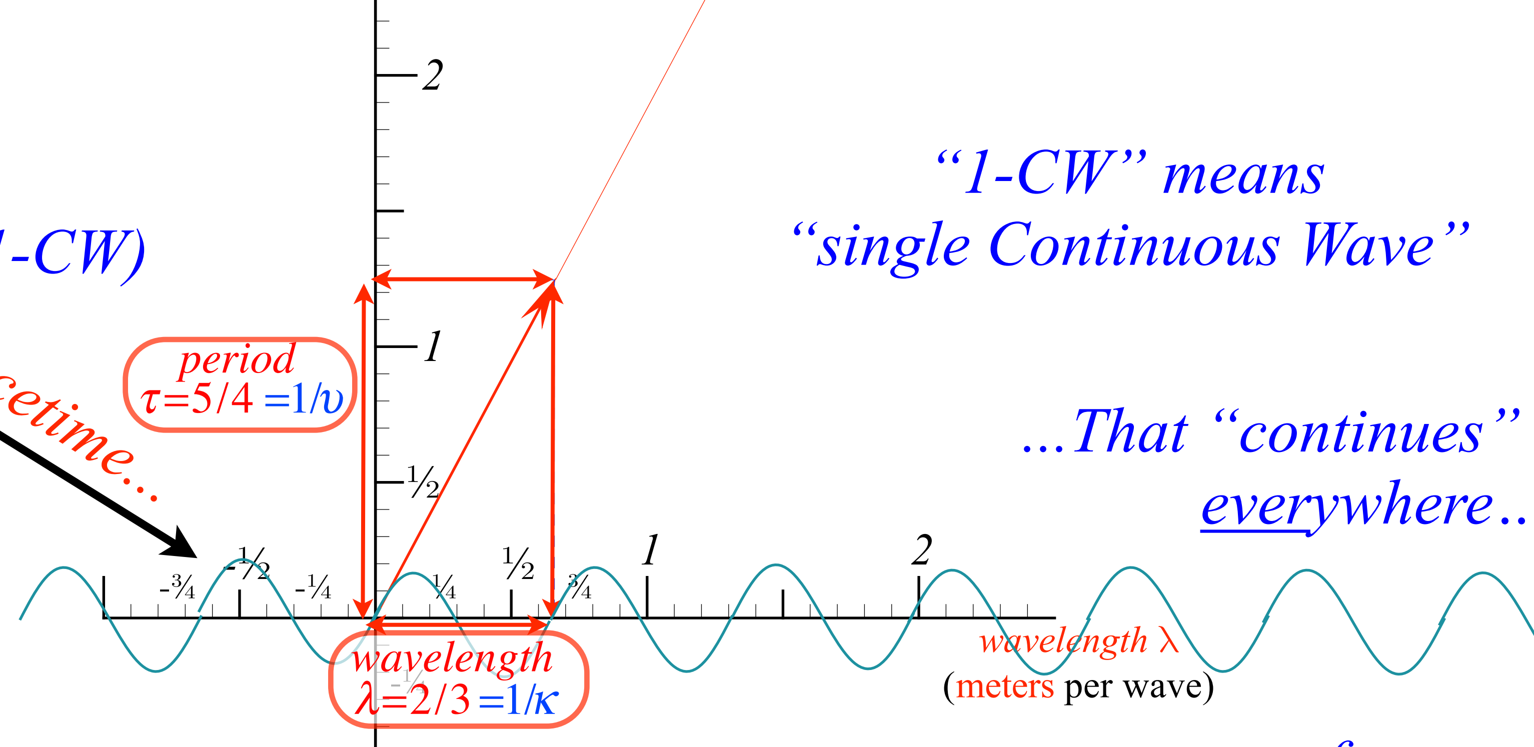
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs





Herman Minkowski 1864-1909

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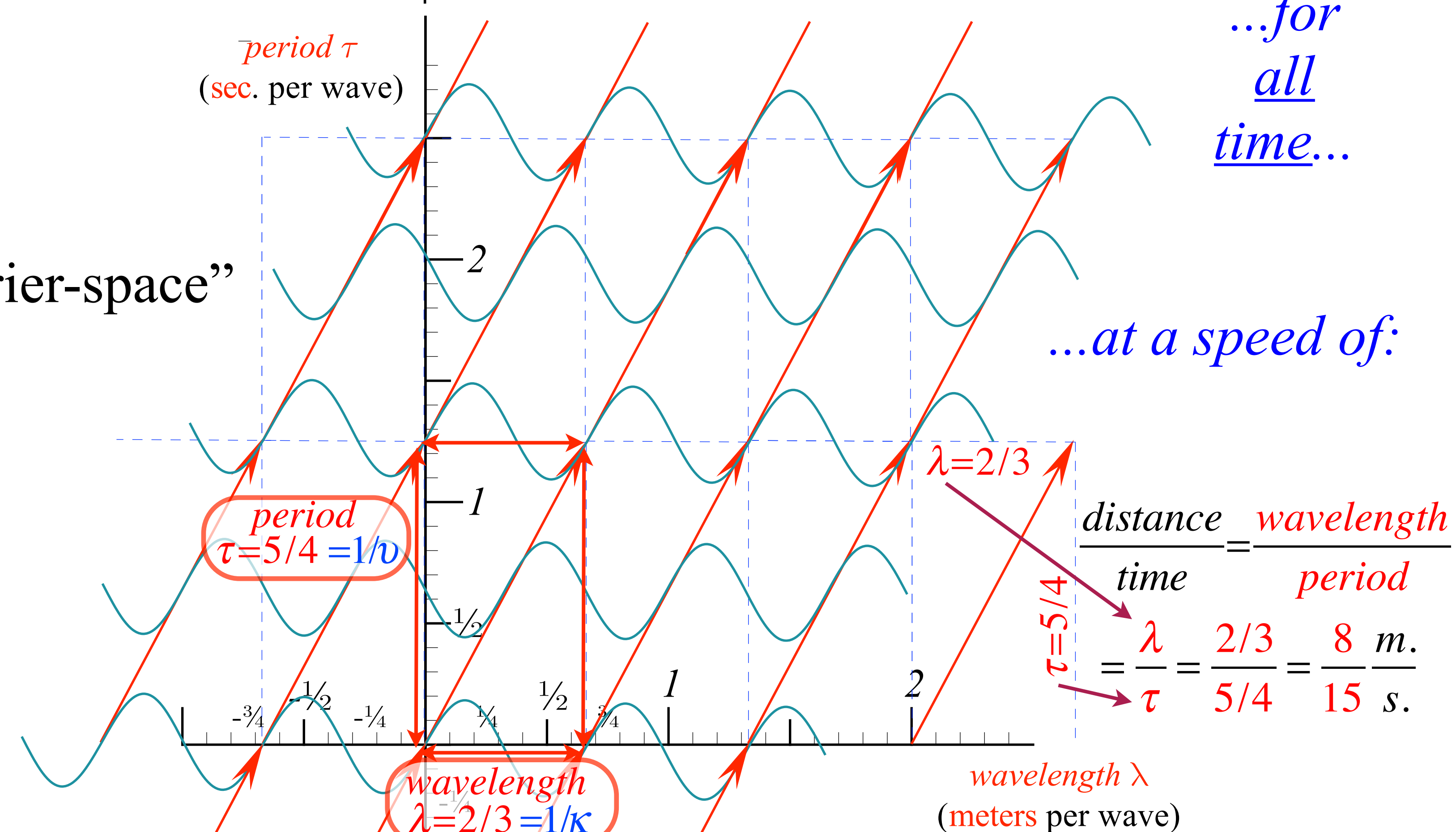


...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

"Keyboard of the gods" is known as "Fourier-space"

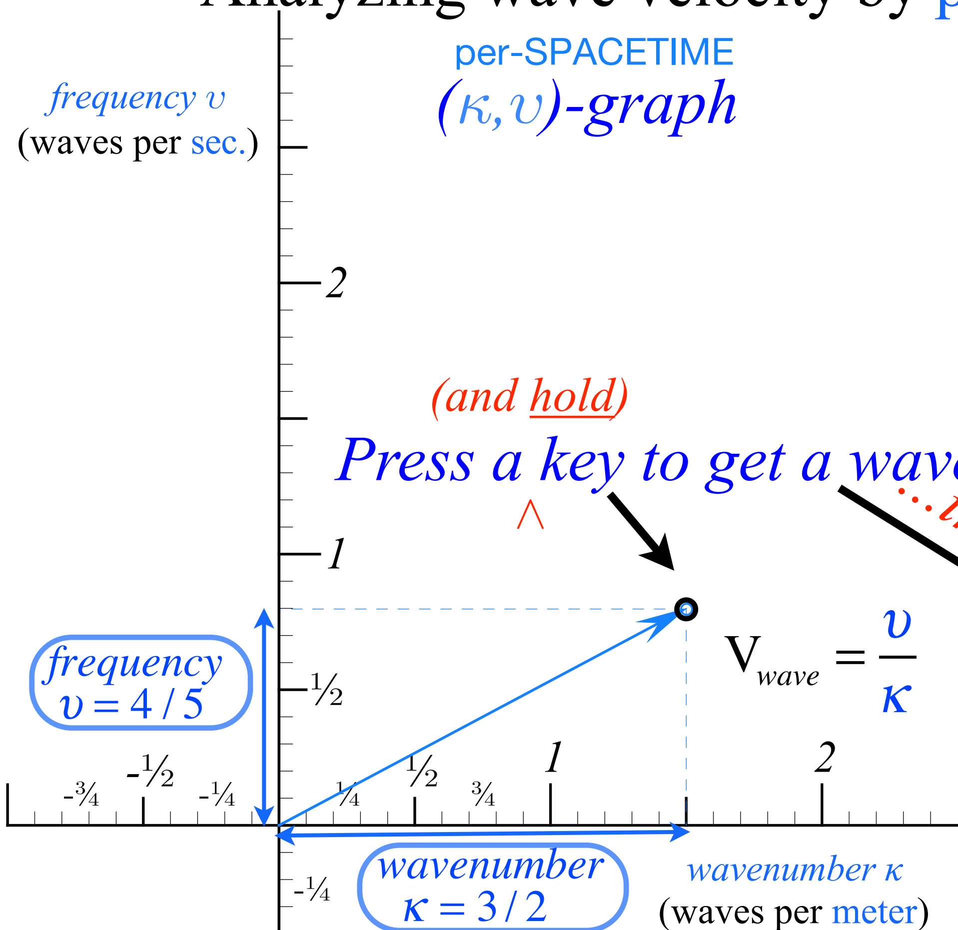
Jean-Baptiste Joseph Fourier 1768-1830

- How to understand waves and wave velocity V_{wave}

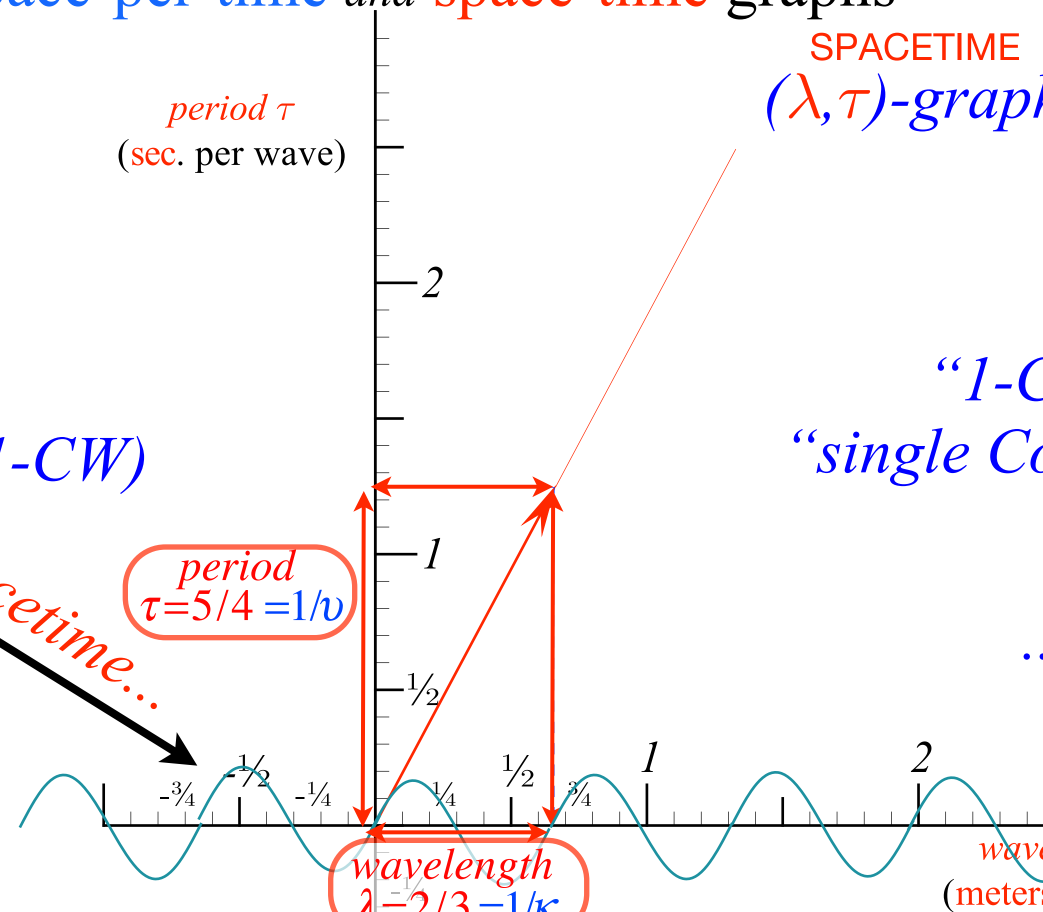
wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909



wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph



"1-CW" means "single Continuous Wave"

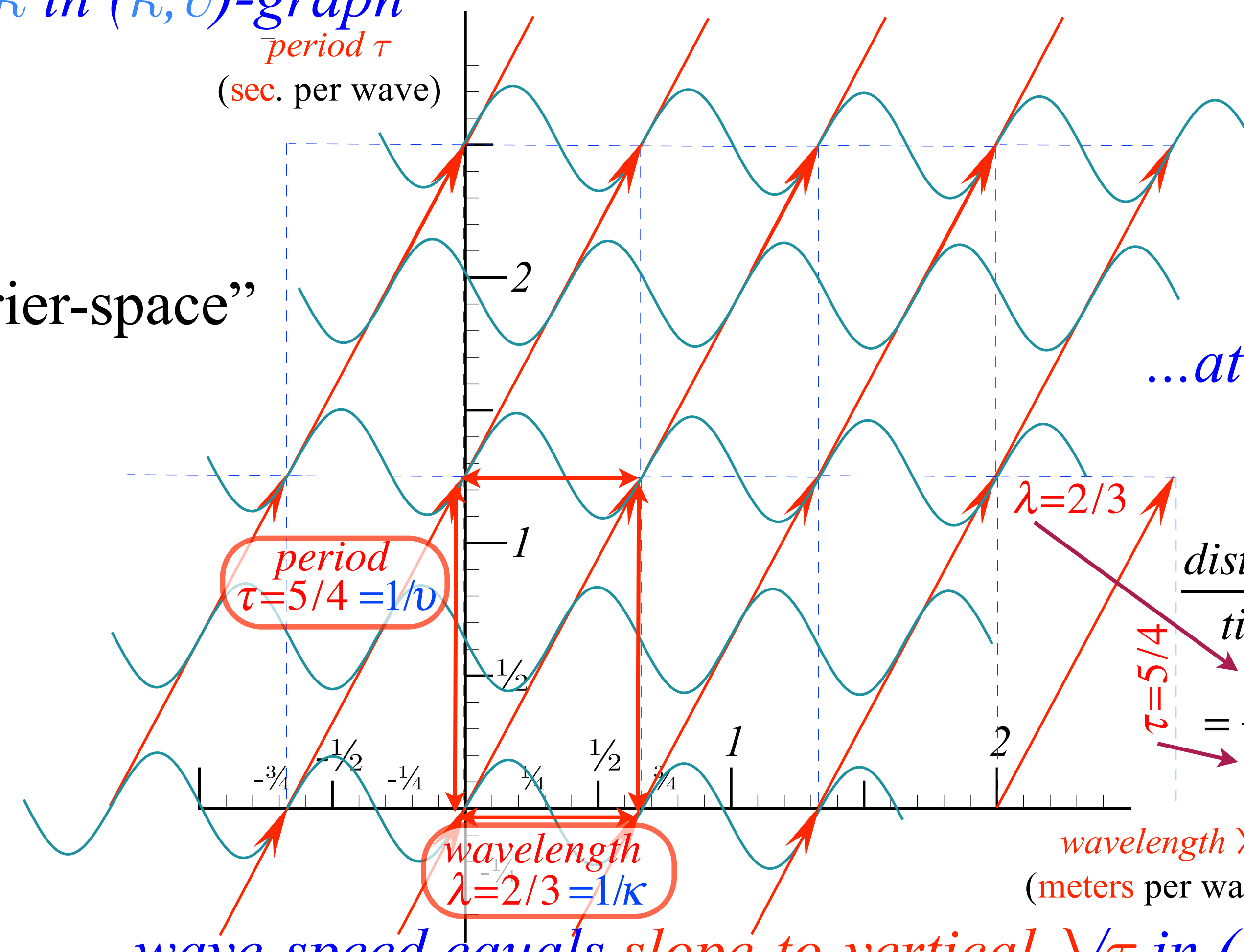
...That "continues" everywhere...

...for all time...

...at a speed of:

"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830



$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

•*How to understand waves and wave velocity V_{wave}*

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

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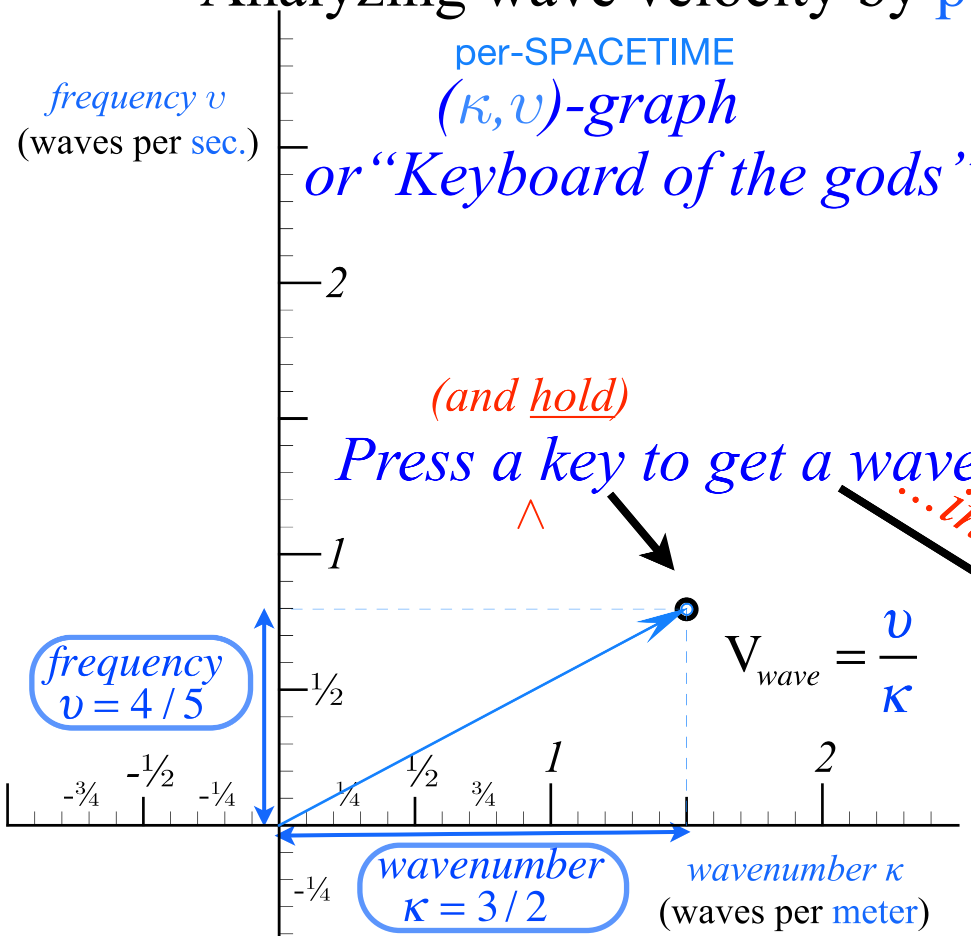

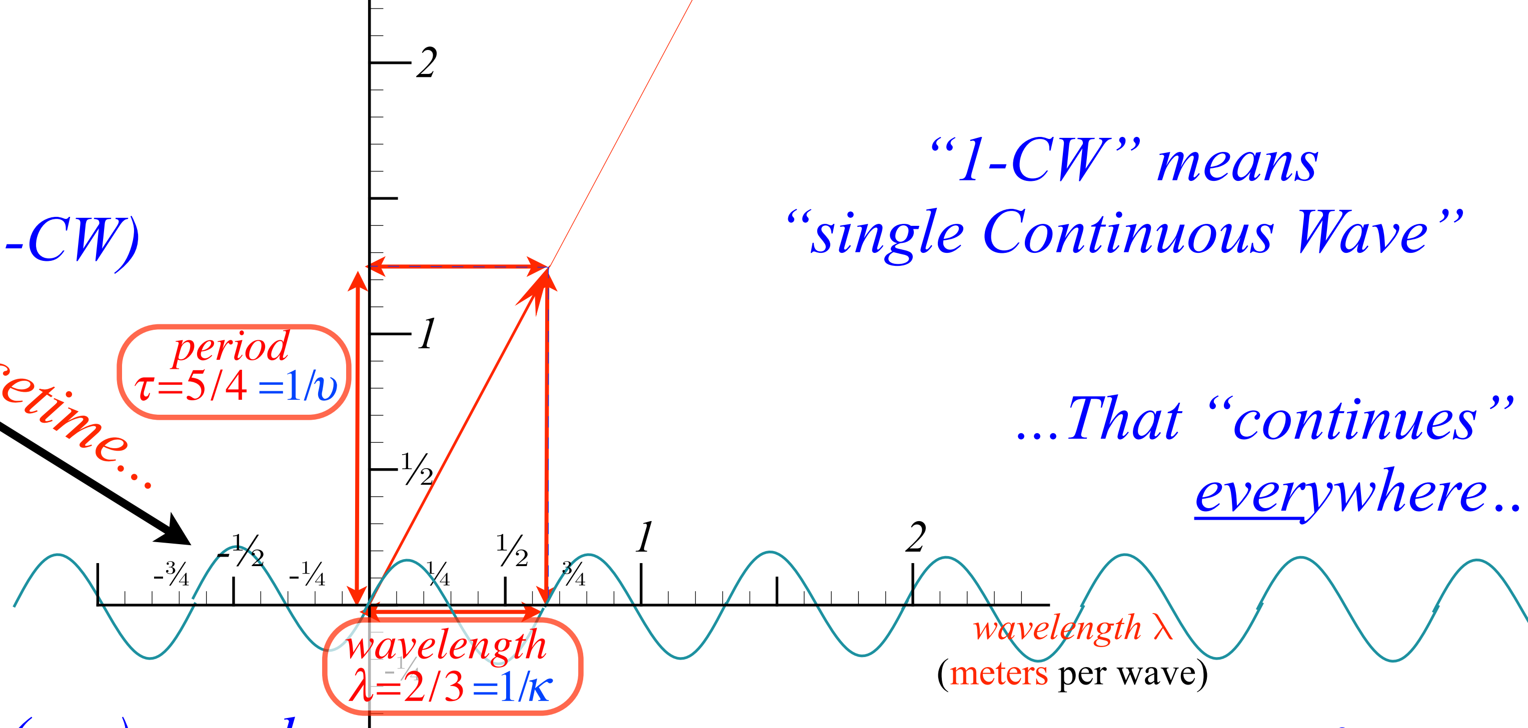

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...at a speed of:

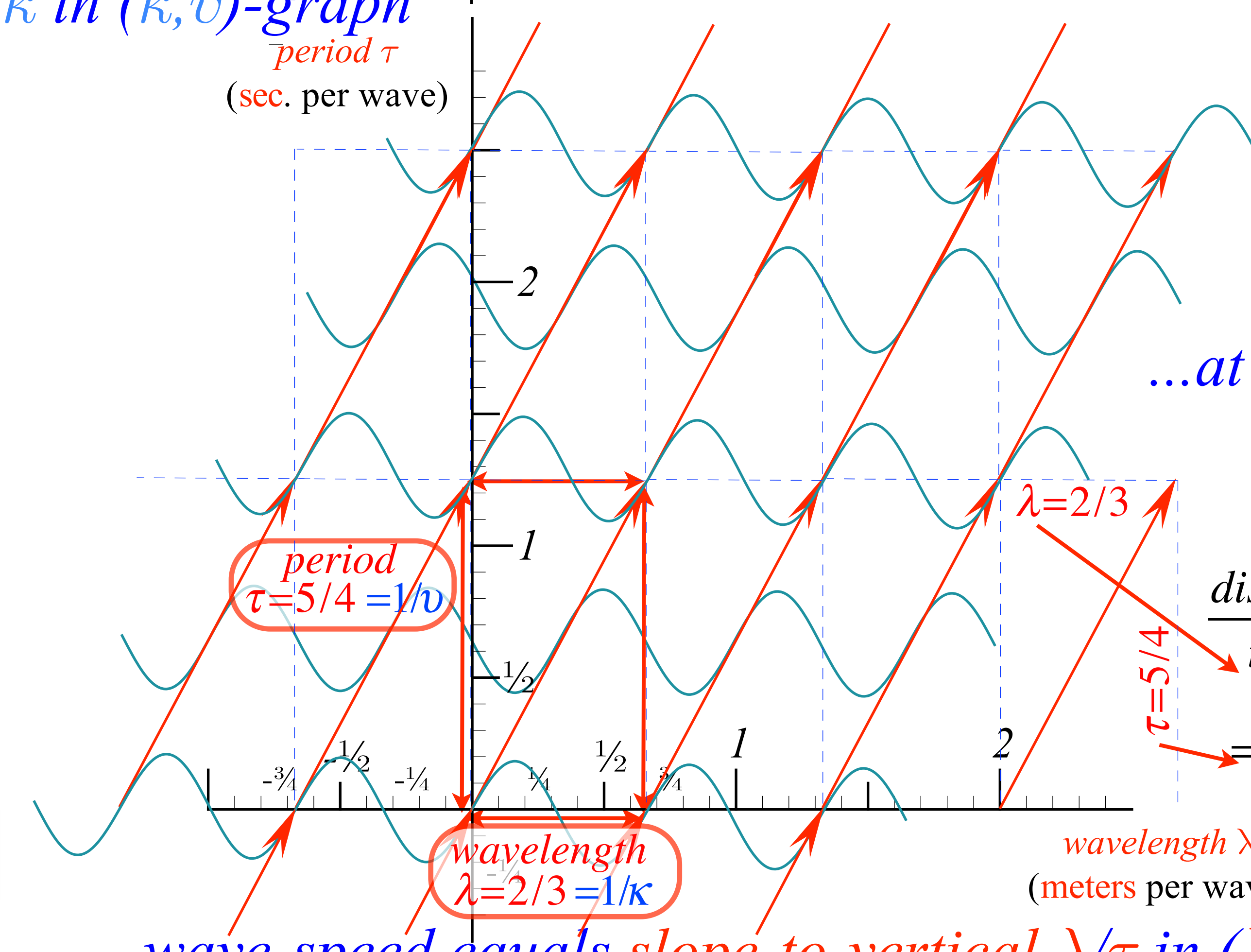
wave-velocity formulas

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{frequency}{wavenumber}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions



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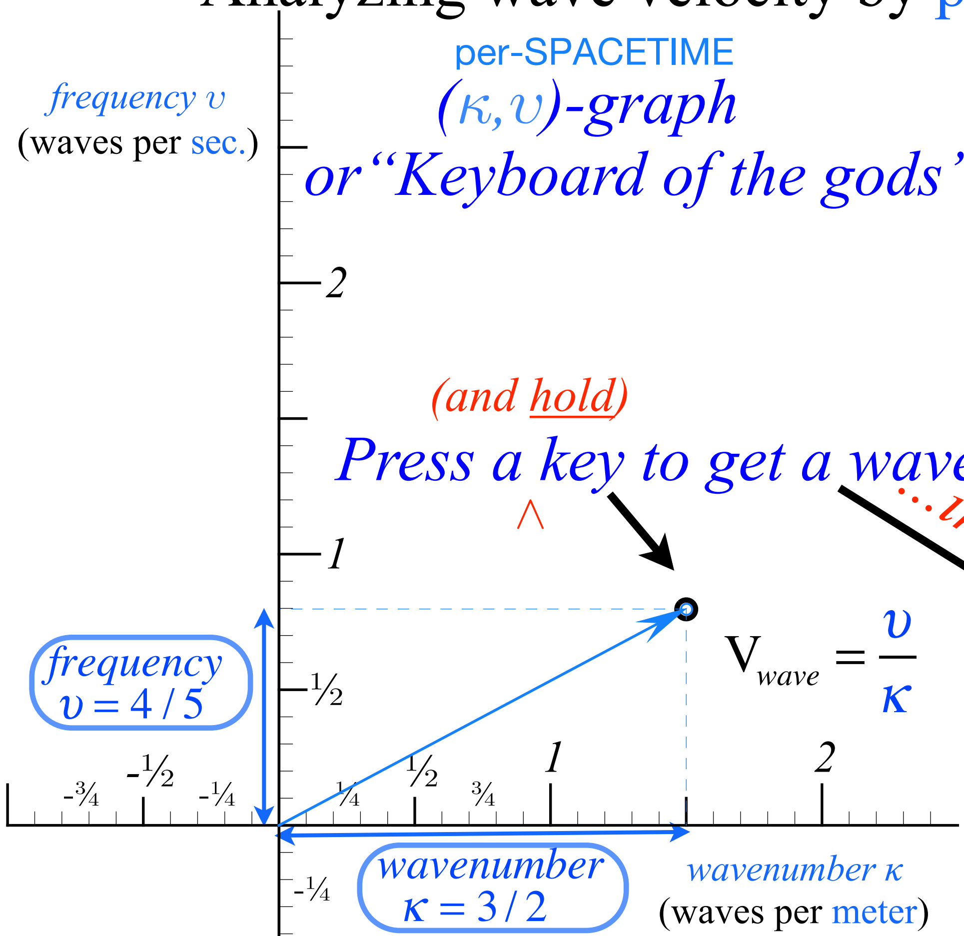
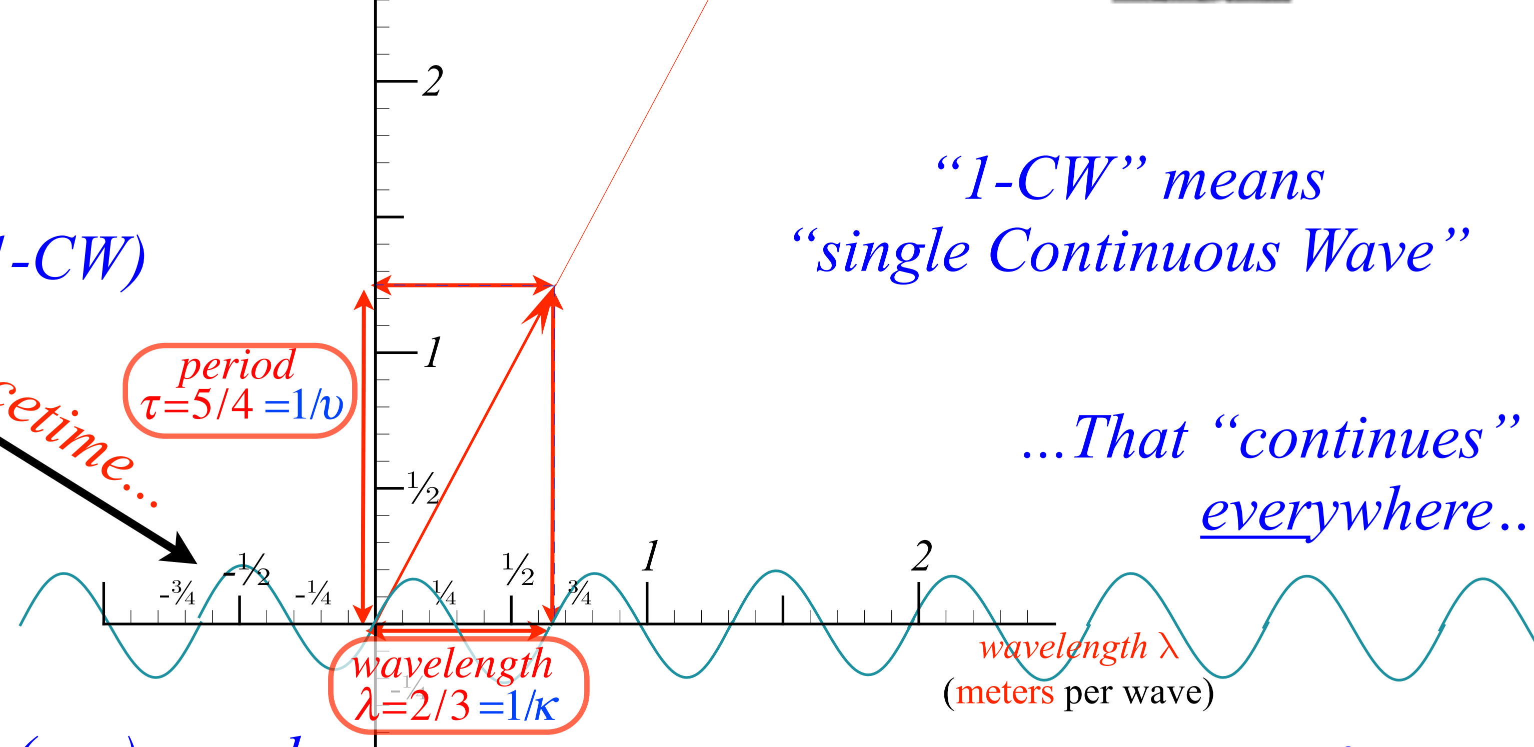


$V_{wave} = \frac{\upsilon}{\kappa}$



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...for all time...

wave-velocity formulas

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}} = \frac{2\pi\,\upsilon}{2\pi\,\kappa} = \frac{\omega}{k}$$

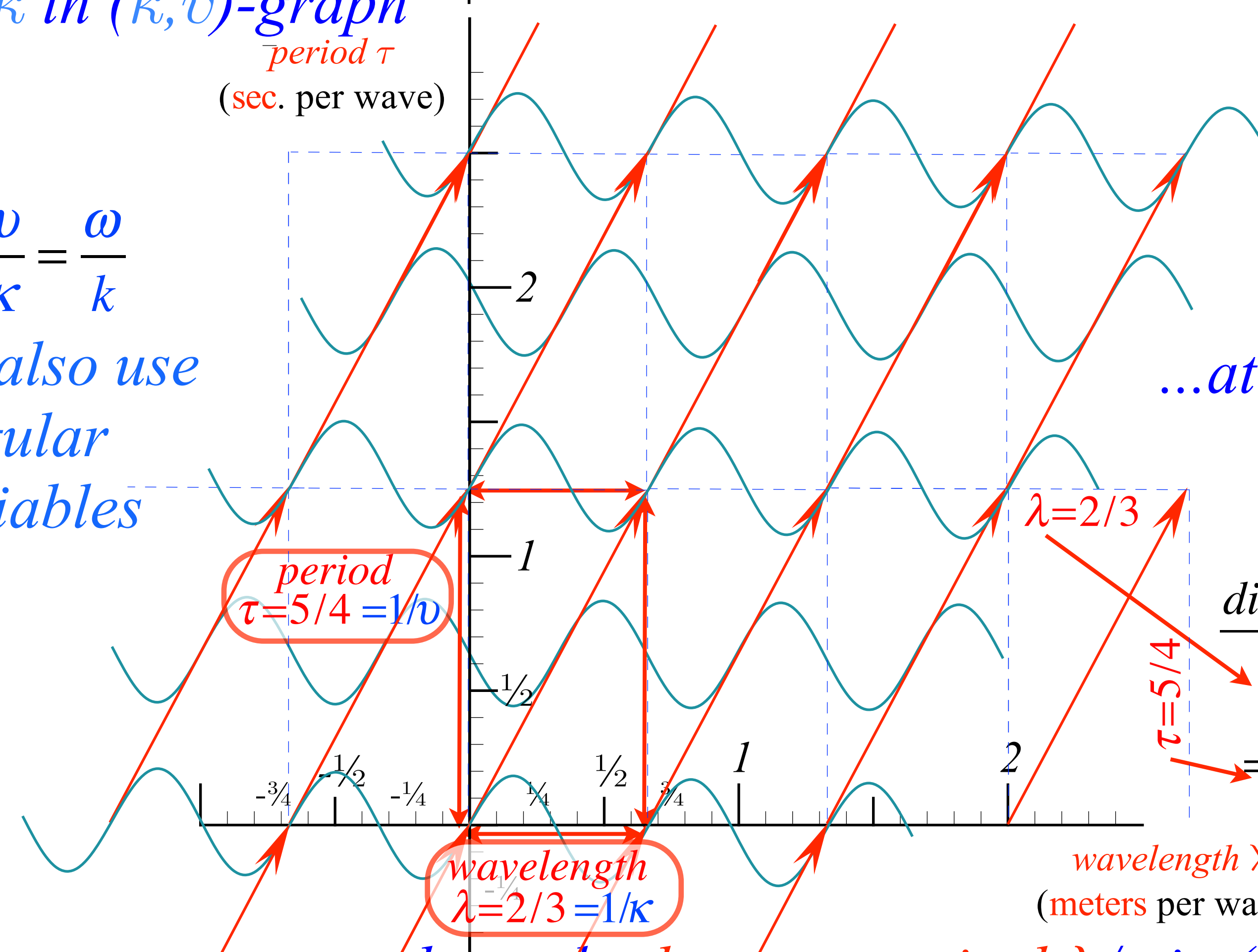
$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

we also use angular variables

wave arithmetic is simpler to explain using fractions

...at a speed of:



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Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

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➔ Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

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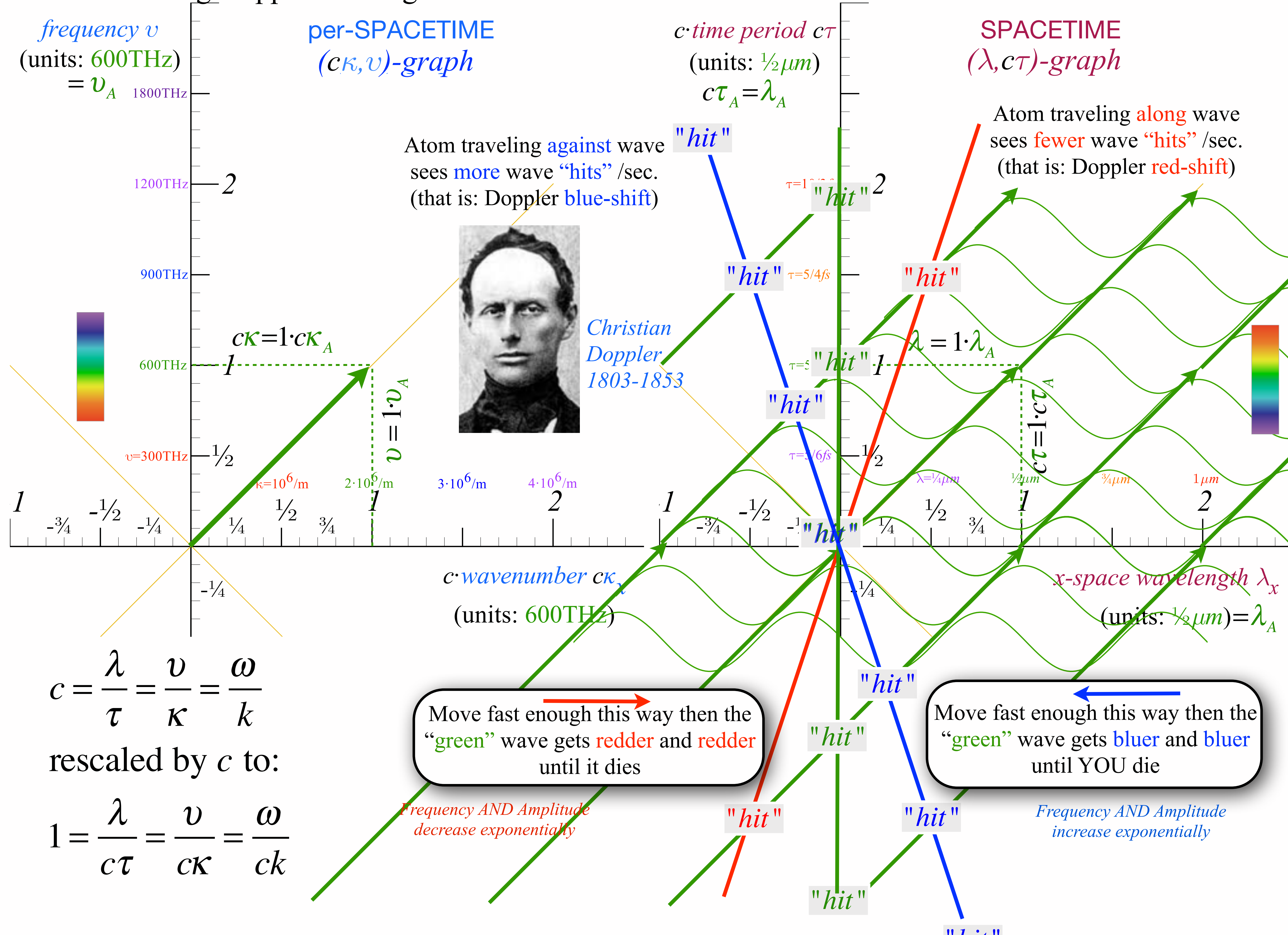

$$c=\frac{\lambda}{\tau}=\frac{\upsilon}{\kappa}=\frac{\omega}{k}$$

rescaled by c to:

$$1=\frac{\lambda}{c\tau}=\frac{\upsilon}{c\kappa}=\frac{\omega}{ck}$$

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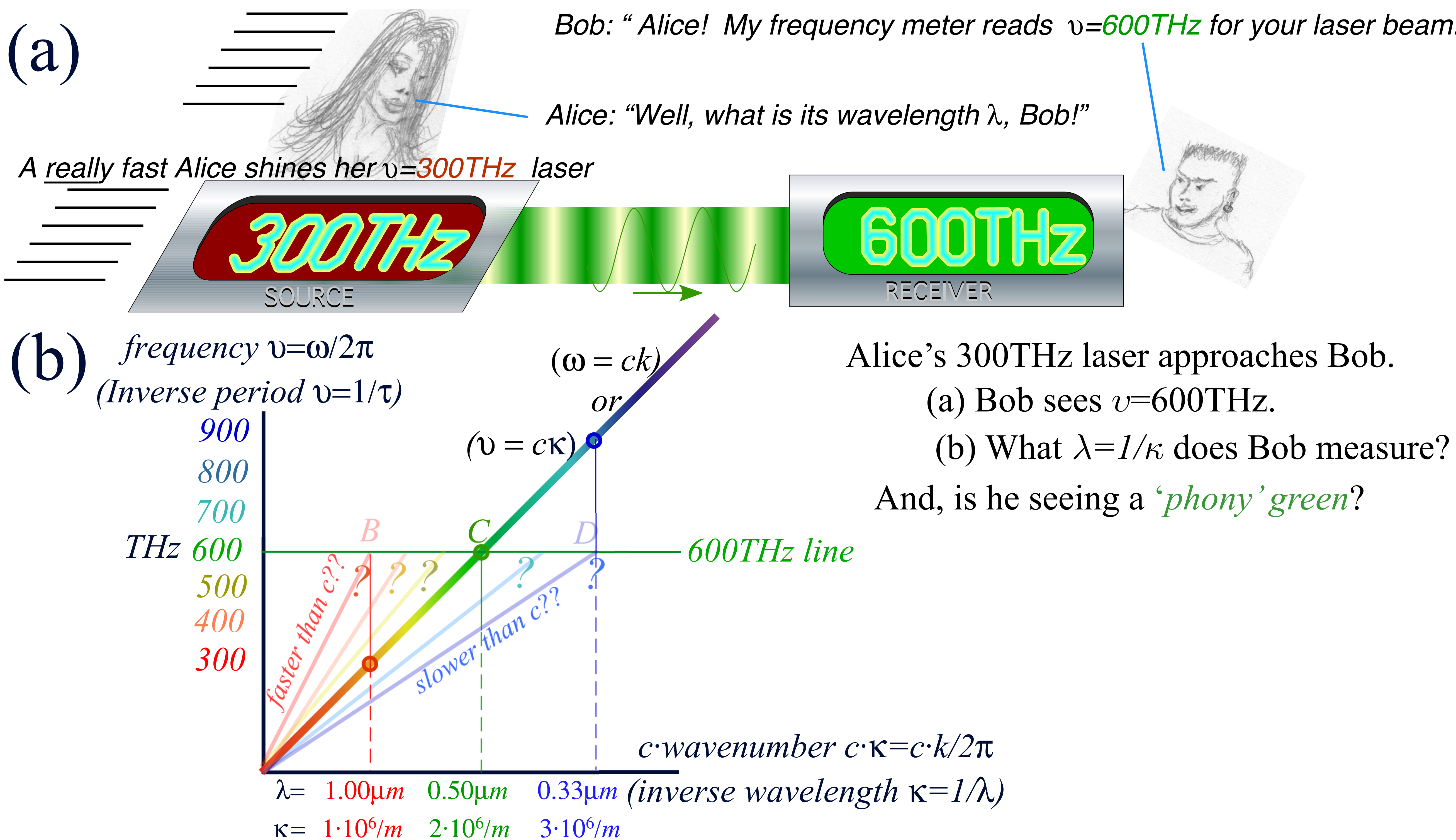
Introducing Doppler shifting and why *c* is *so* constant (and *so slow*)



Alice's 300THz laser approaches Bob.

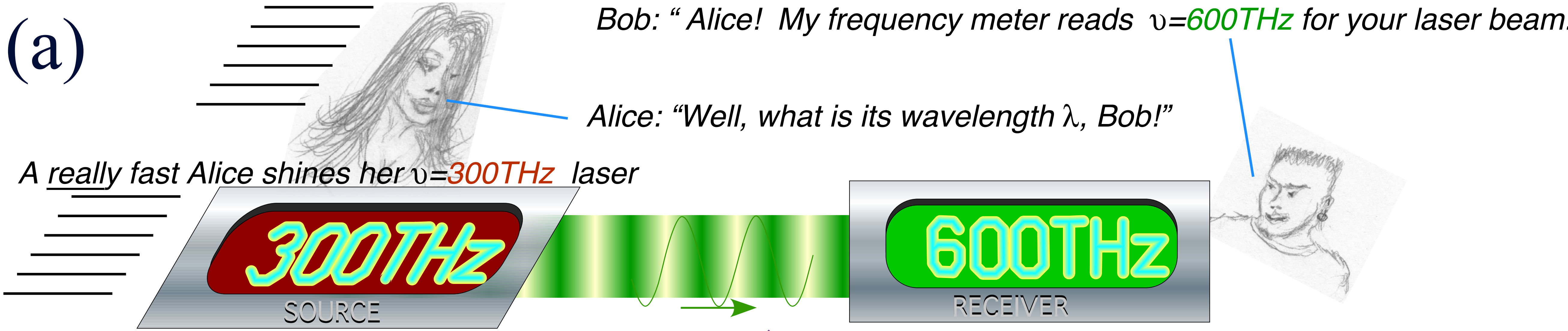
(a) Bob sees υ=600THz.

(b) What $\lambda=1/\kappa$ does Bob measure?

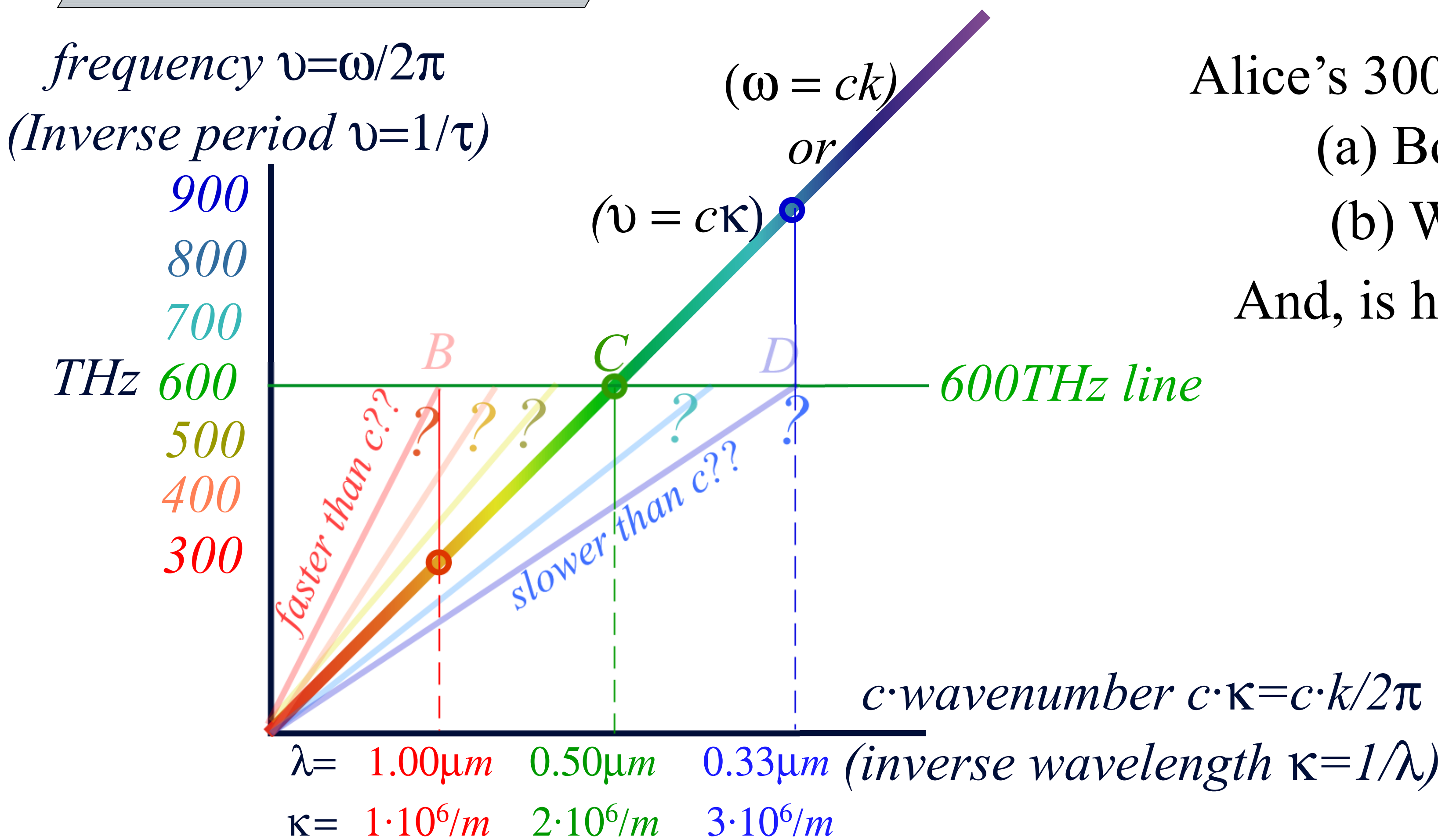
And, is he seeing a '*phony*' *green*?

Introducing Doppler shifting and why c is constant

(a)



(b)



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Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron.*

The only choice is C.

Introducing Doppler shifting and why c is constant

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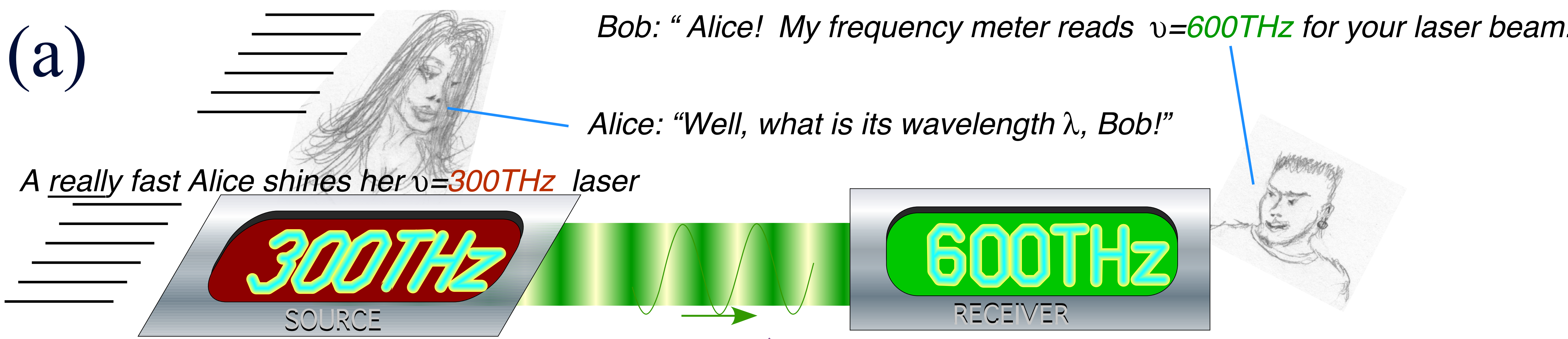

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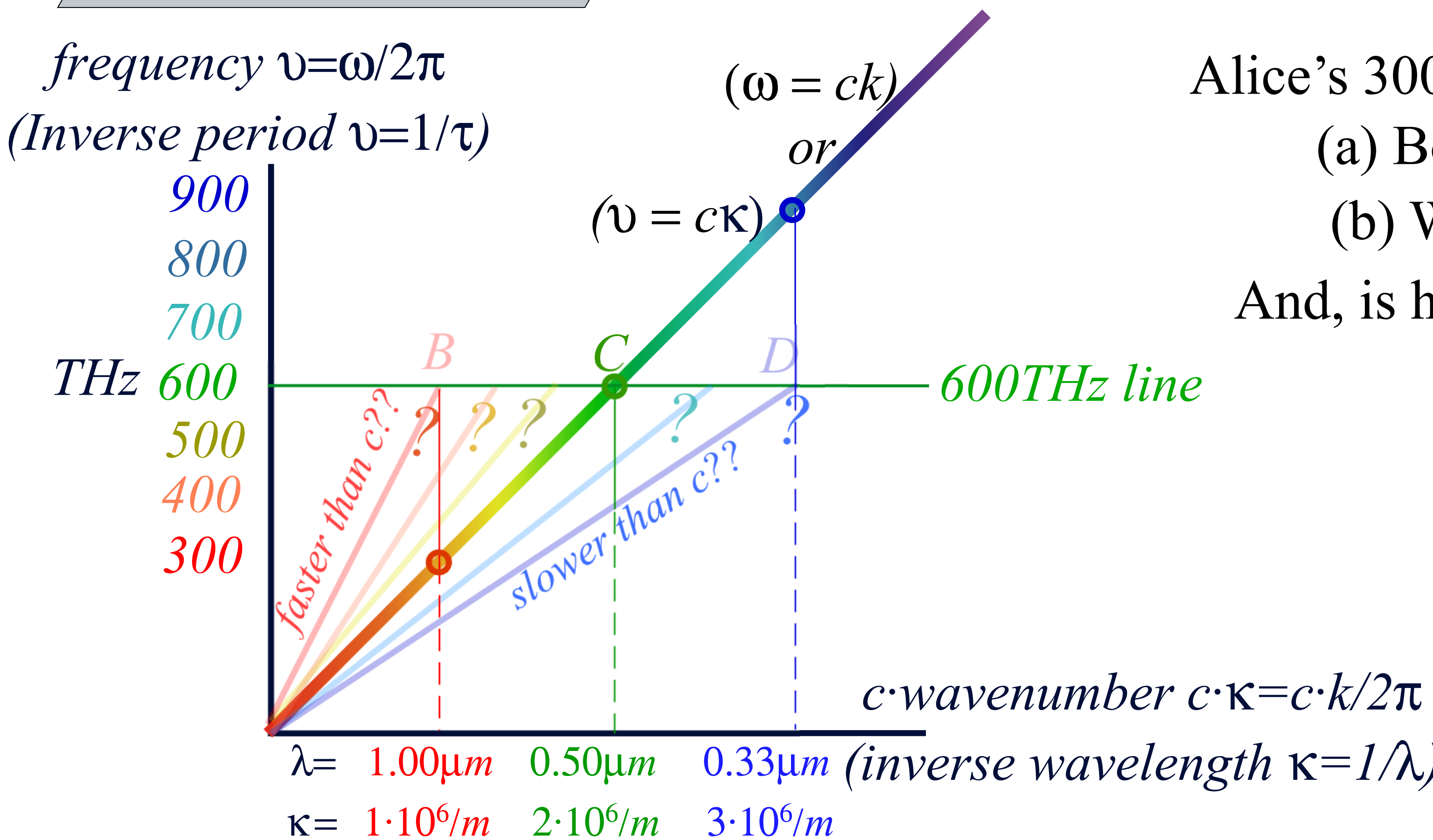

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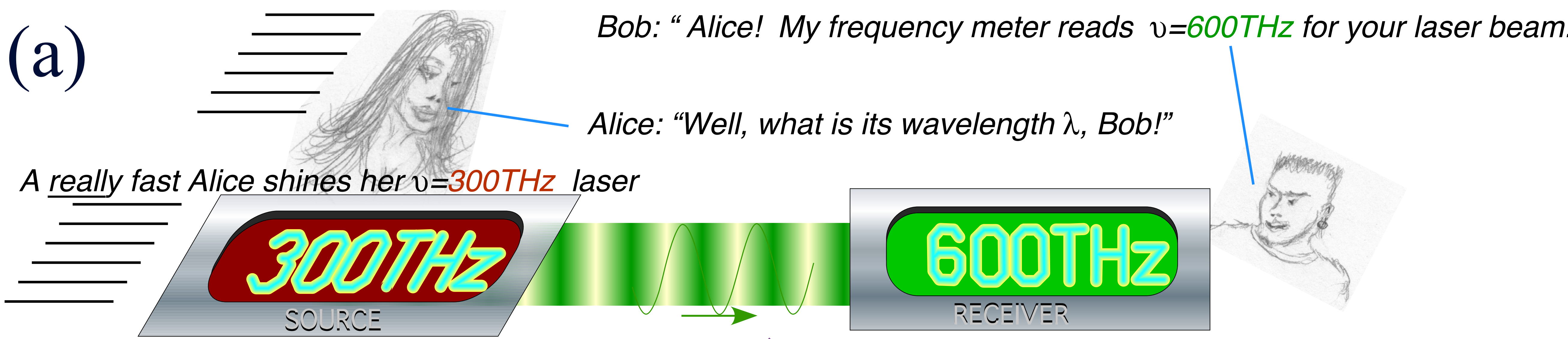
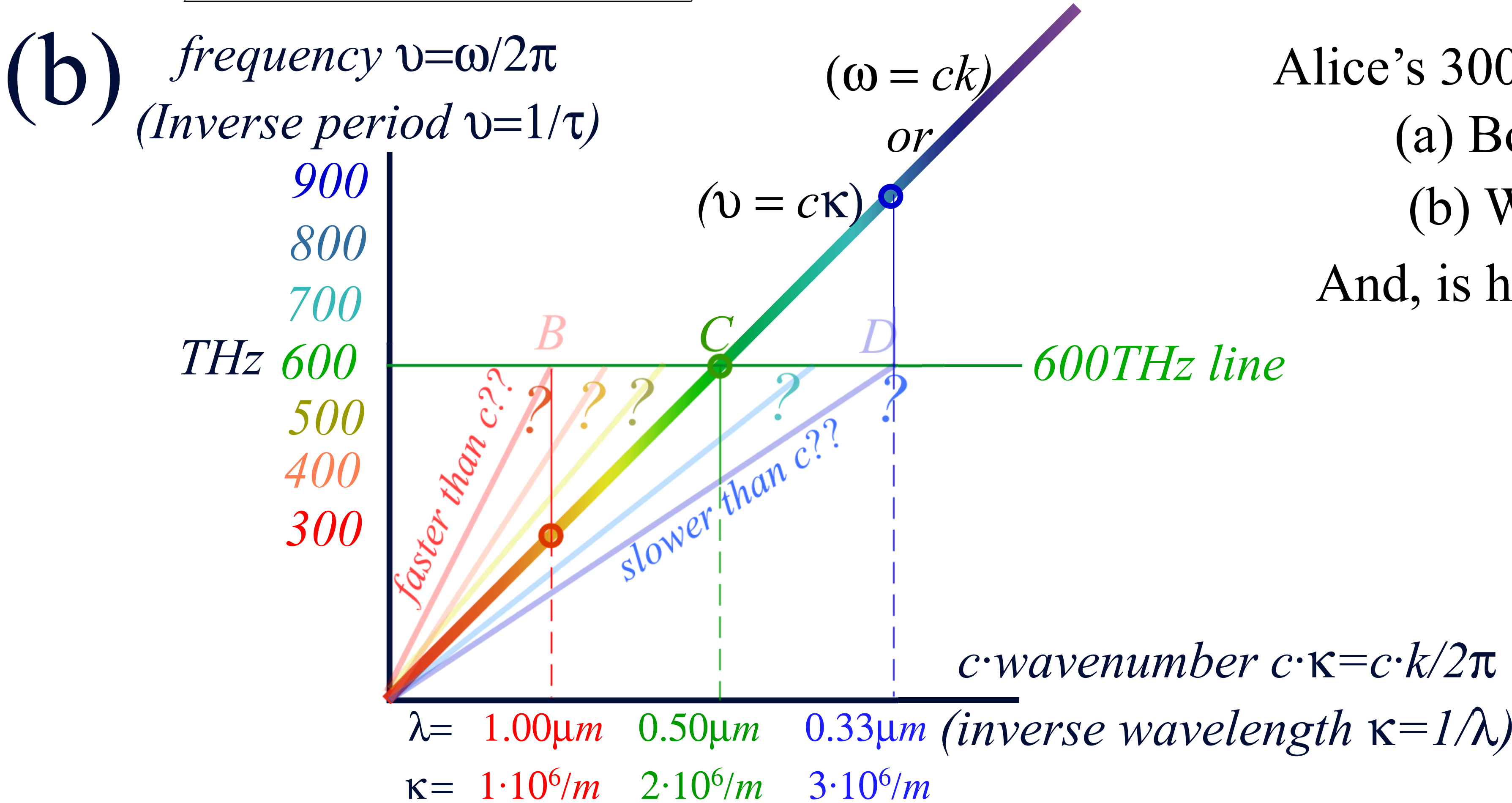
Introducing Doppler shifting and why c is constant





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Actually: $2.99792458\cdot 10^{8} m\cdot s^{-1}$

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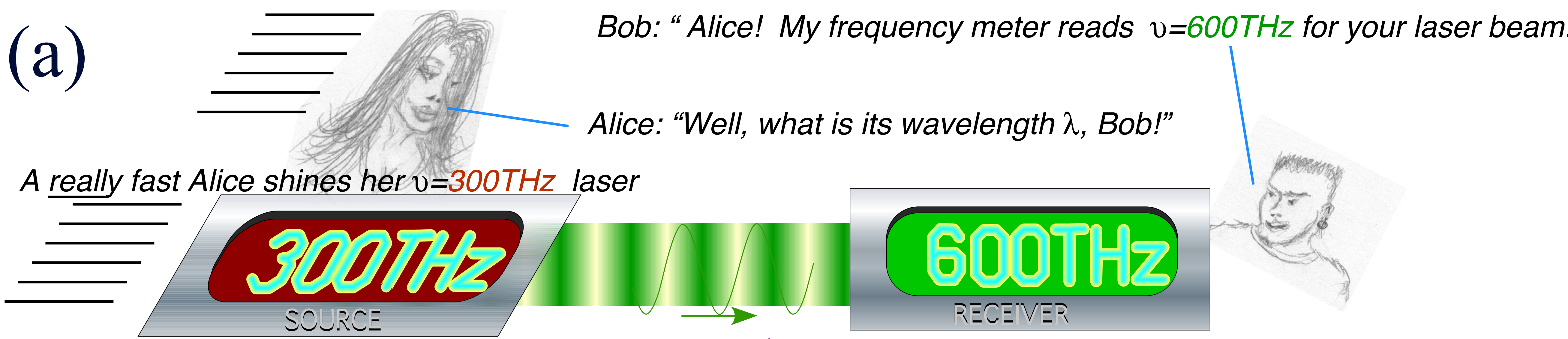

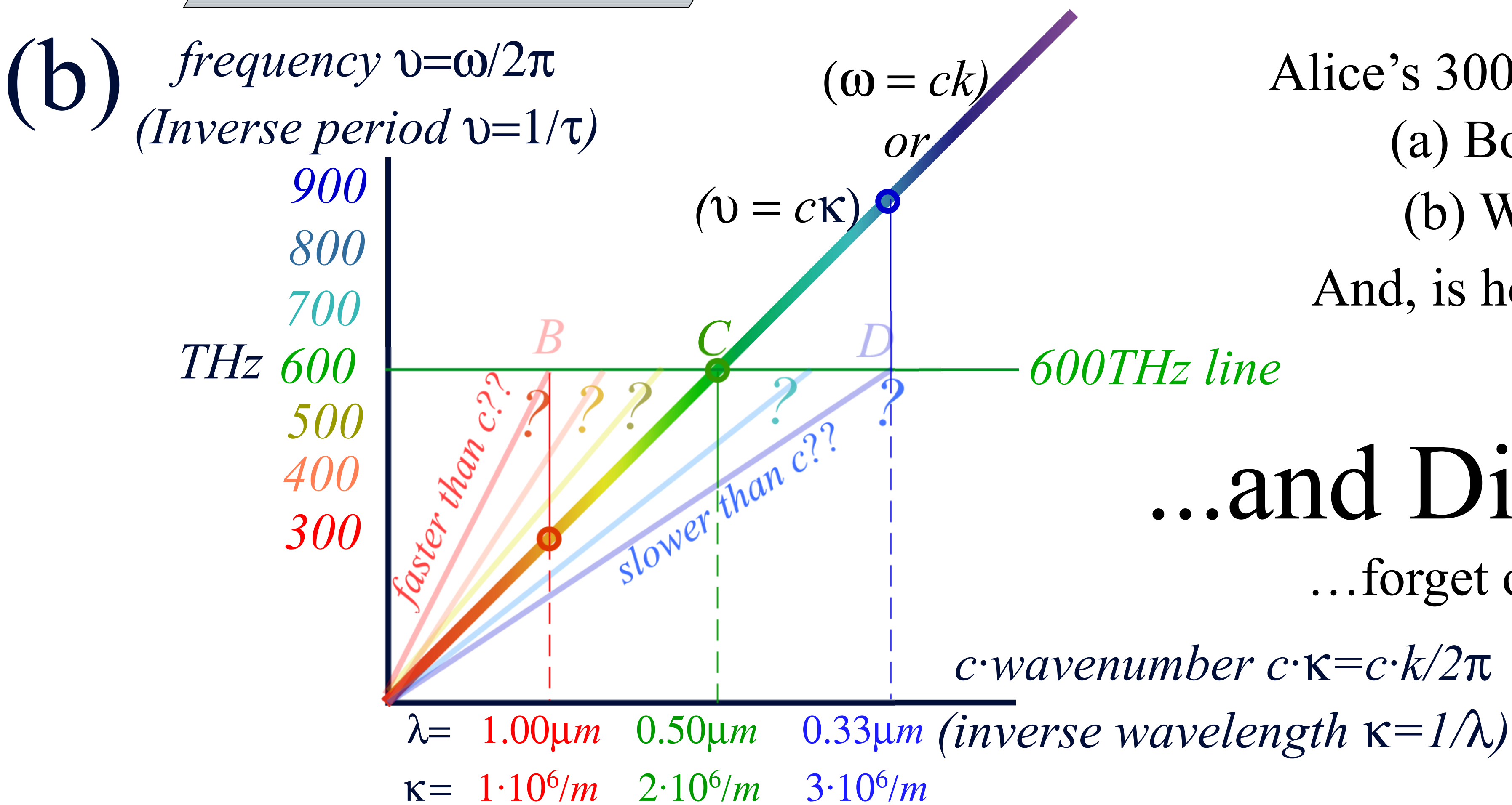

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...and Dispersion-Free!

...forget optical astronomy without this!

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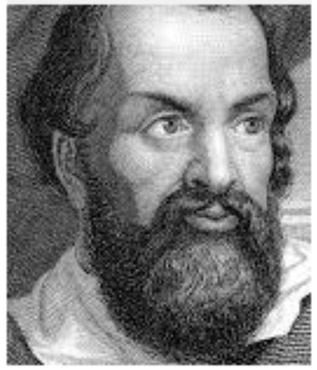
Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity

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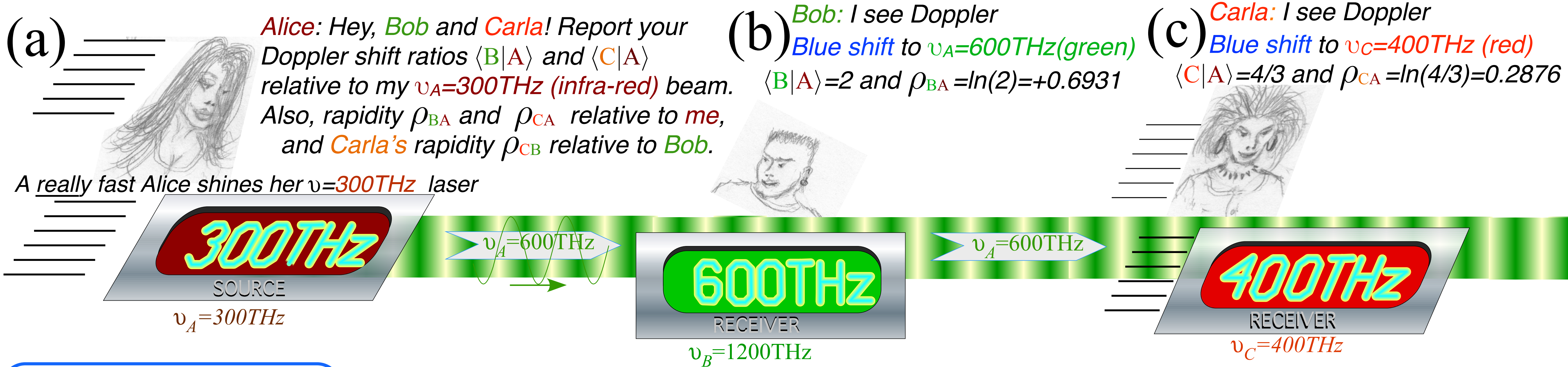

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ

(a) *Alice: Hey, Bob and Carla! Report your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ relative to my υ_A=300THz (infra-red) beam. Also, rapidity ρ_{BA} and ρ_{CA} relative to me, and Carla's rapidity ρ_{CB} relative to Bob.*

(b) *Bob: I see Doppler Blue shift to υ_A=600THz(green)*

⟨B|A⟩=2 and ρ_{BA} =ln(2)=+0.6931

(c) *Carla: I see Doppler Blue shift to υ_C=400THz (red)*

⟨C|A⟩=4/3 and ρ_{CA} =ln(4/3)=0.2876

A really fast Alice shines her υ=300THz laser

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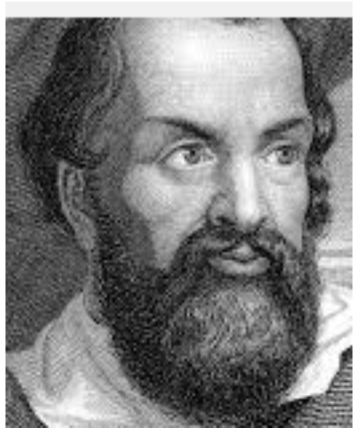
Galileo Galilei

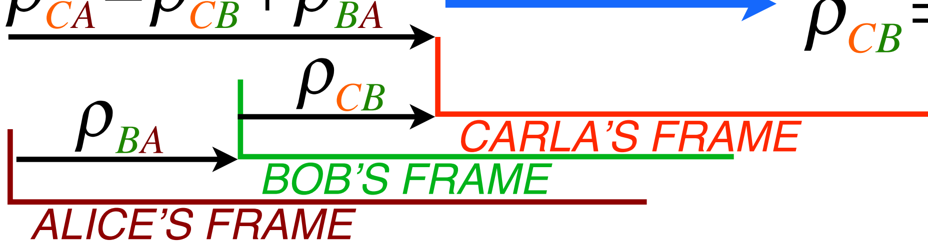


1564-1642

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$



Carla-Bob Doppler ratio:

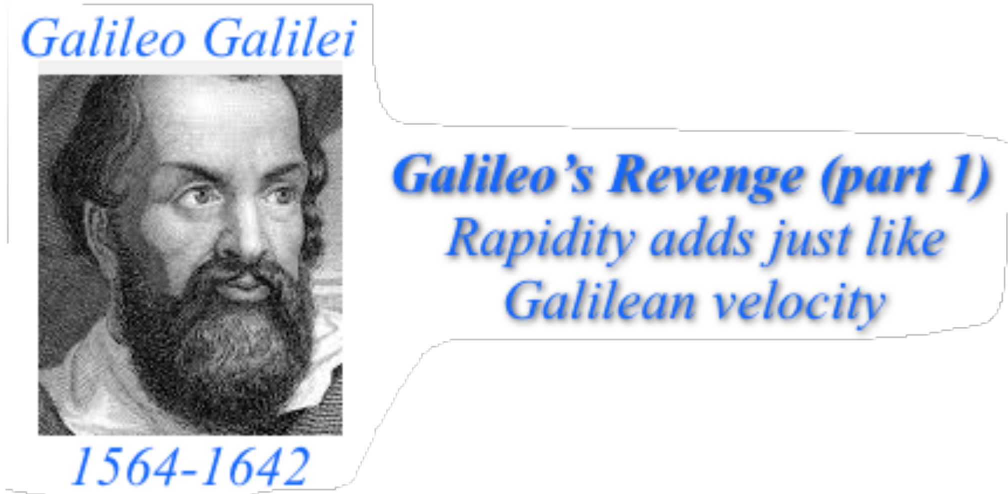
$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

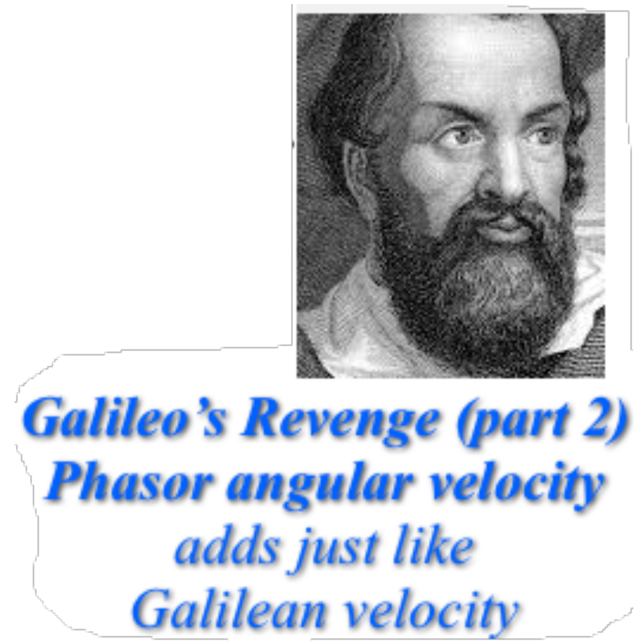
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$





➔ Optical interference "baseball-diamond" displays *phase* and *group* velocity

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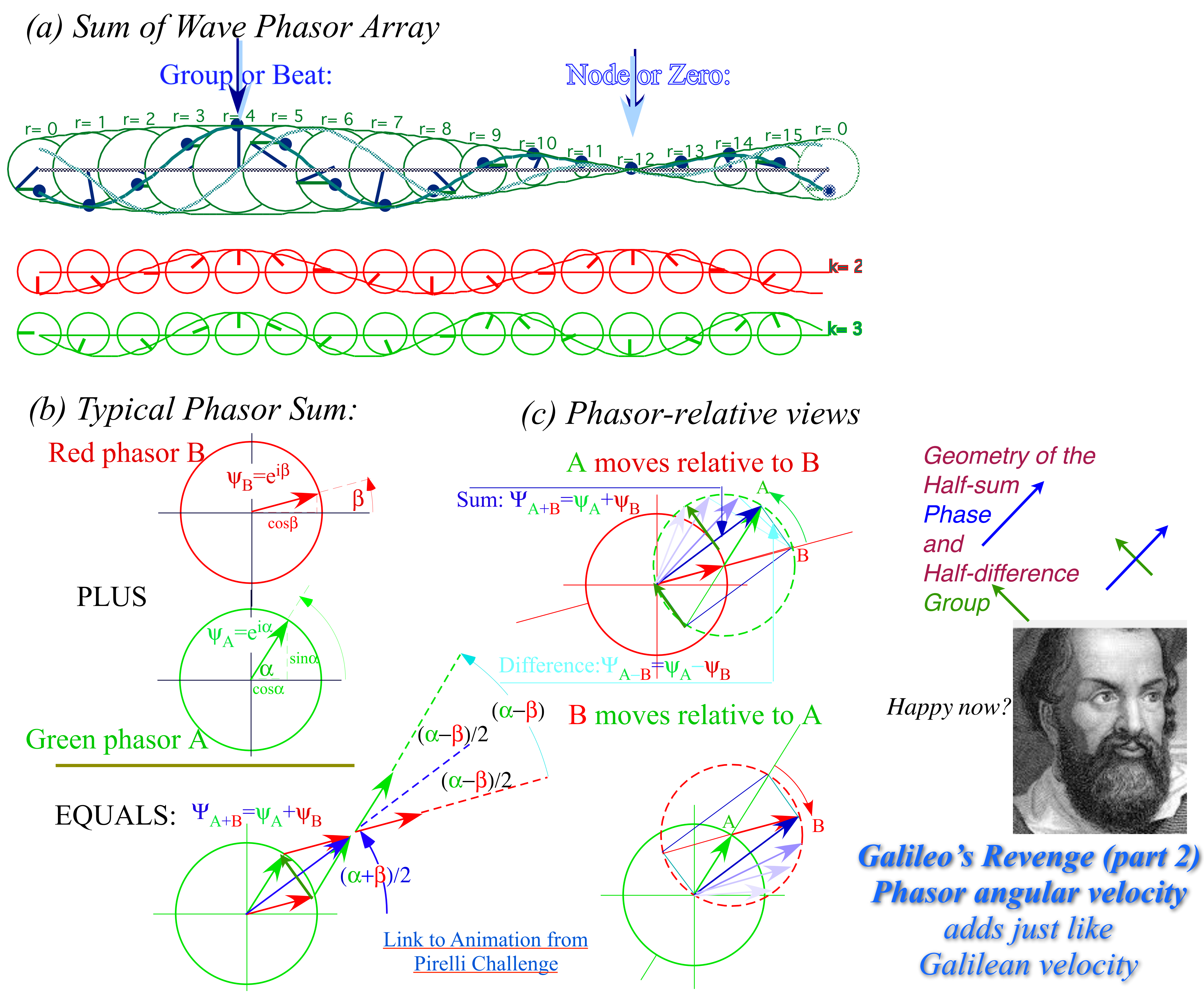
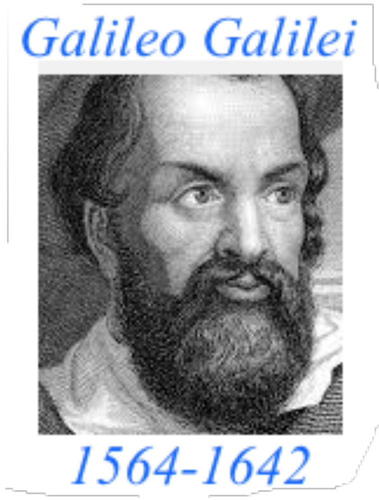


(a) Sum of Wave Phasor Array
Group or Beat:
Node or Zero:
r= 0 r= 1 r= 2 r= 3 r= 4 r= 5 r= 6 r= 7 r= 8 r= 9 r=10 r=11 r=12 r=13 r=14 r=15 r= 0
k= 2
k= 3
(b) Typical Phasor Sum:
Red phasor B
$\Psi_B=e^{i\beta}$
β
cosβ
PLUS
$\Psi_A=e^{i\alpha}$
α
sinα
cosα
Green phasor A
EQUALS: $\Psi_{A+B}=\Psi_A+\Psi_B$
(α−β)
(α−β)/2
(α−β)/2
(α+β)/2
Link to Animation from Pirelli Challenge
(c) Phasor-relative views
A moves relative to B
Sum: $\Psi_{A+B}=\Psi_A+\Psi_B$
A
B
Difference: $\Psi_{A-B}=\Psi_A-\Psi_B$
B moves relative to A
A
B
Geometry of the
Half-sum
Phase
and
Half-difference
Group
Happy now?
Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity



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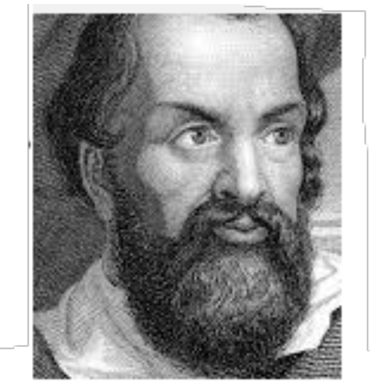

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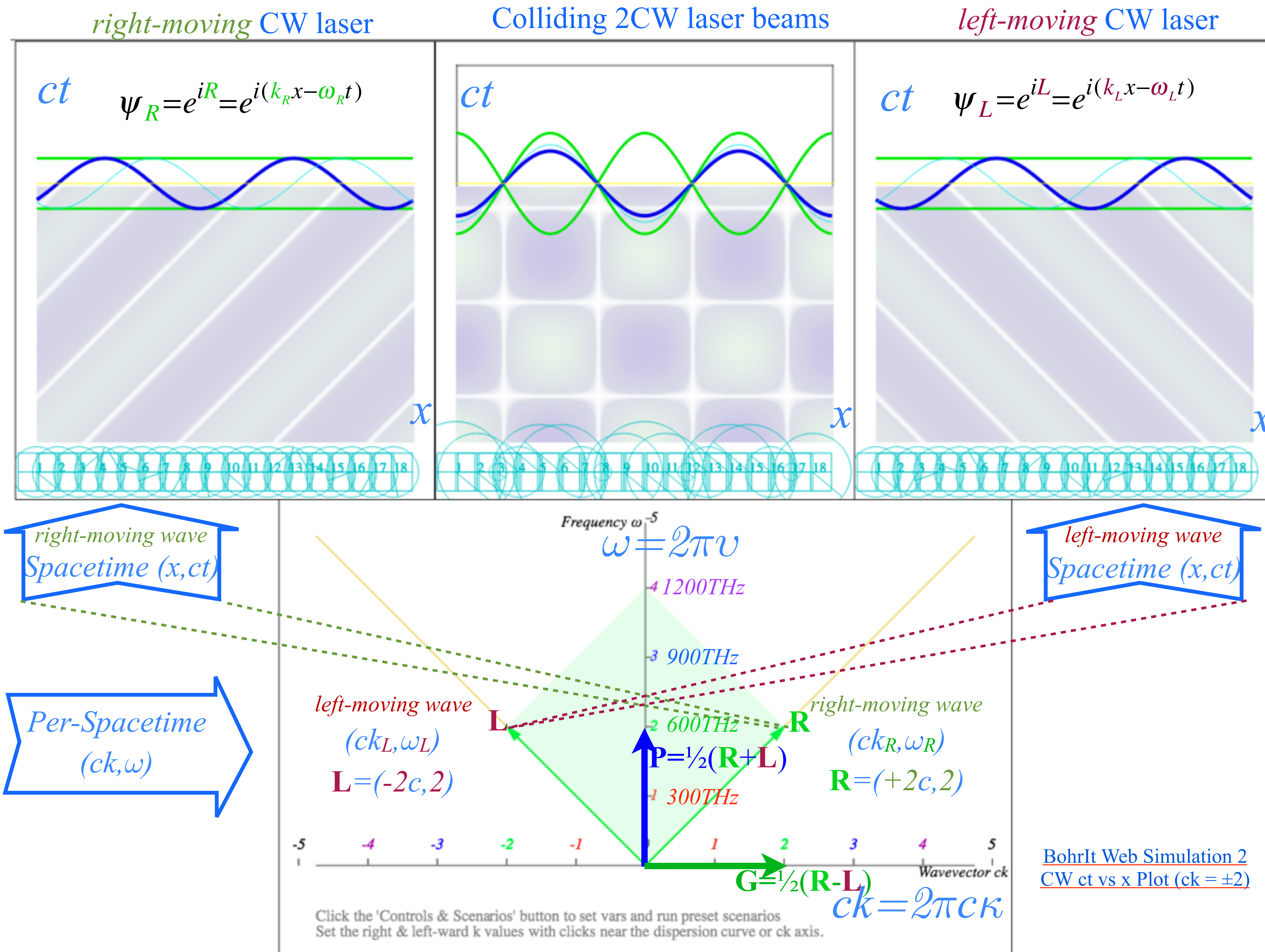
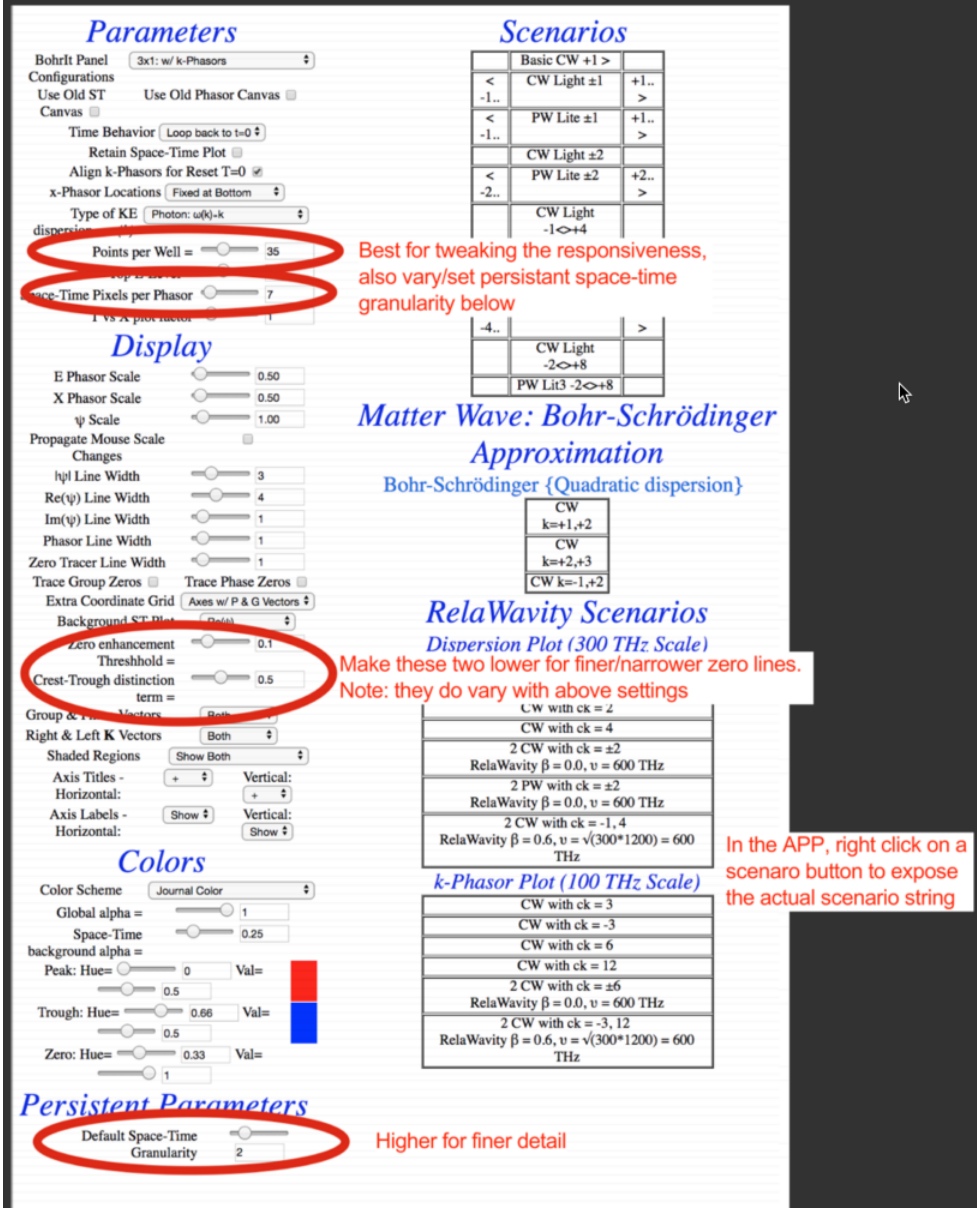
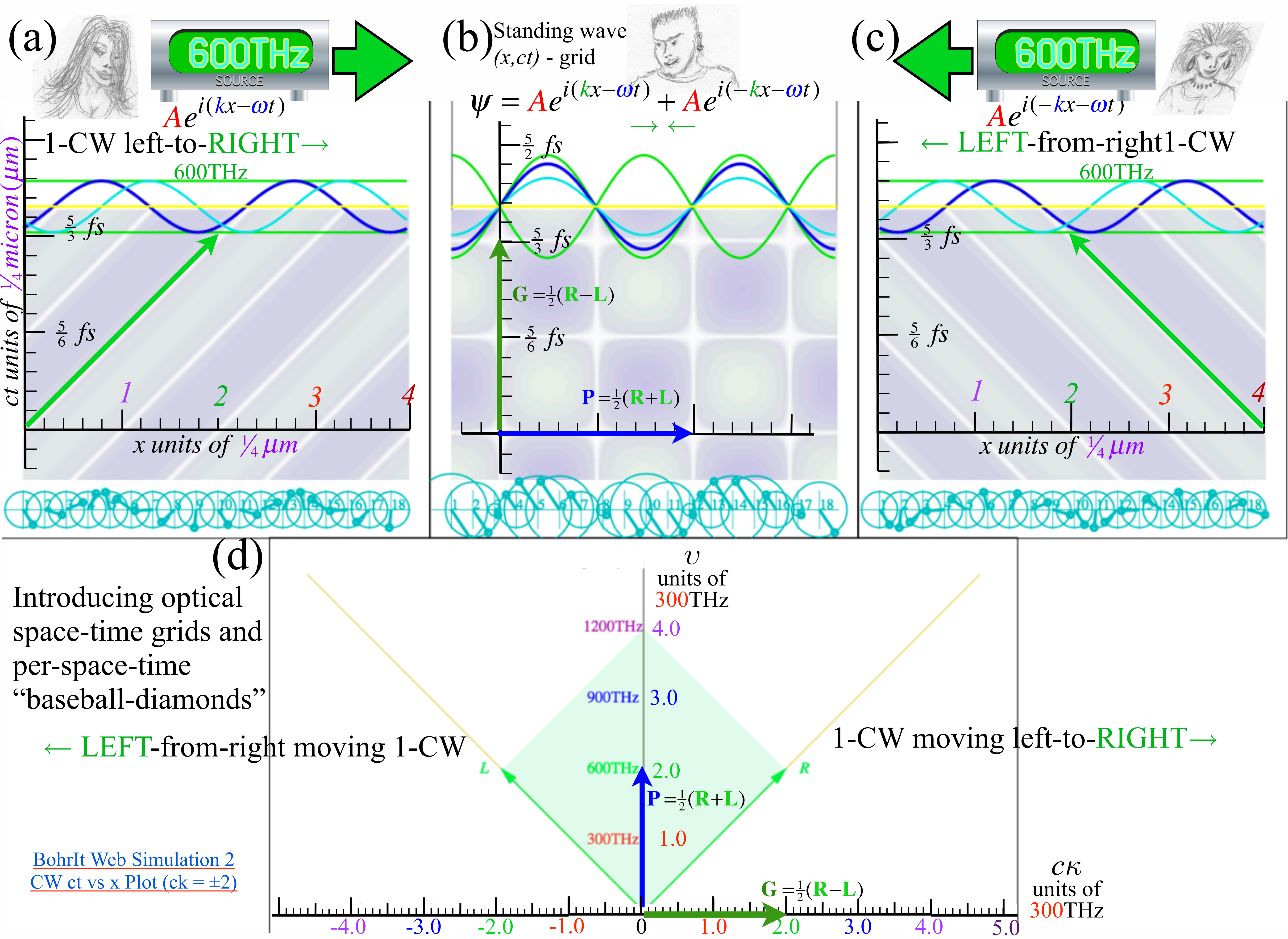
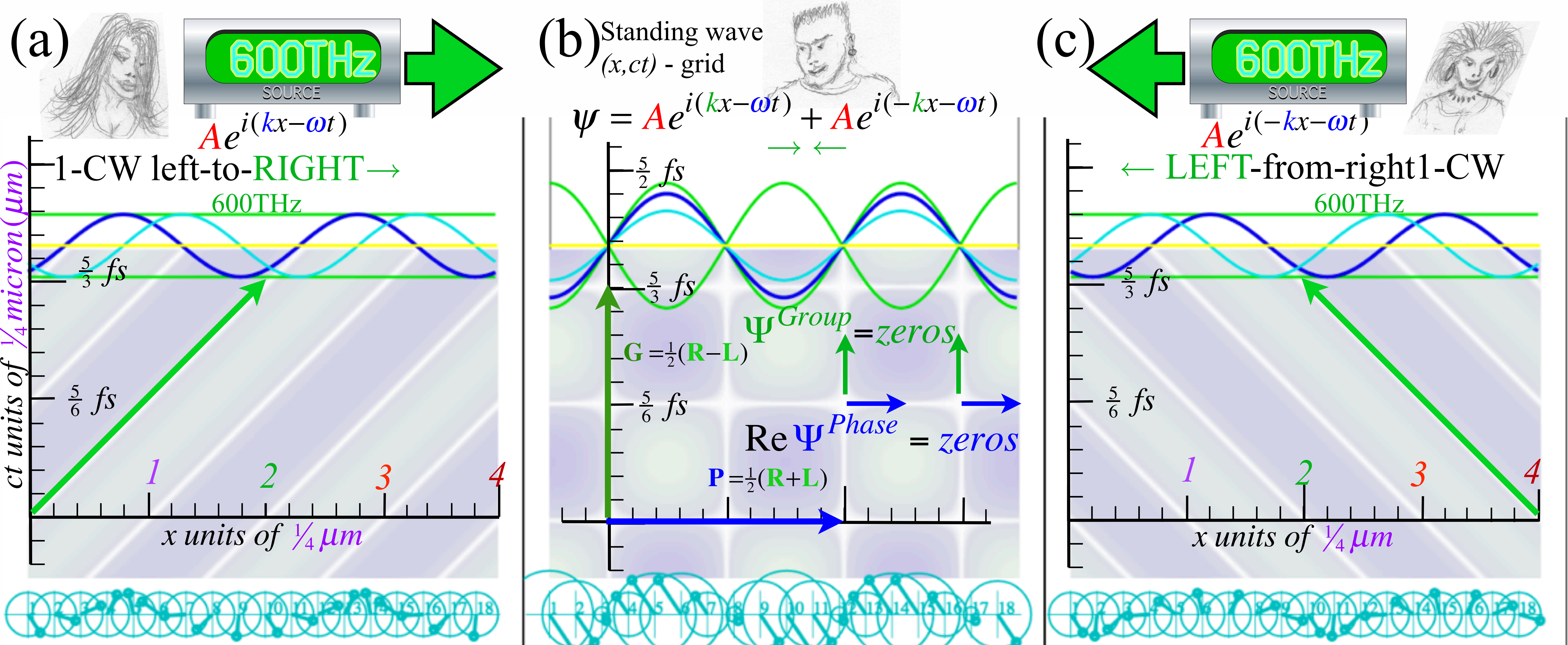


right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
ψR=e^iR=e^i(kRx−ωRt)
ψL=e^iL=e^i(kLx−ωLt)
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
ω=2πυ
1200THz
900THz
600THz
300THz
left-moving wave
(ckL,ωL)
L=(-2c,2)
right-moving wave
(ckR,ωR)
R=(+2c,2)
P=½(R+L)
G=½(R-L)
Wavevector ck
ck=2πcκ
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)


Parameters
BohrIt Panel Configurations
3x1: w/ k-Phasors
Use Old ST Canvas
Use Old Phasor Canvas
Time Behavior
Loop back to t=0
Retain Space-Time Plot
Align k-Phasors for Reset T=0
x-Phasor Locations
Fixed at Bottom
Type of KE
Photon: ω(k)-k
Points per Well =
35
Space-Time Pixels per Phasor
7
Display
E Phasor Scale
0.50
X Phasor Scale
0.50
ψ Scale
1.00
Propagate Mouse Scale Changes
|ψ| Line Width
3
Re(ψ) Line Width
4
Im(ψ) Line Width
1
Phasor Line Width
1
Zero Tracer Line Width
1
Trace Group Zeros
Trace Phase Zeros
Extra Coordinate Grid
Axes w/ P & G Vectors
Zero enhancement Threshhold =
0.1
Crest-Trough distinction term =
0.5
Right & Left K Vectors
Both
Shaded Regions
Show Both
Axis Titles - Horizontal:
Vertical:
Axis Labels - Horizontal:
Show
Vertical:
Show
Colors
Color Scheme
Journal Color
Global alpha =
1
Space-Time background alpha =
0.25
Peak: Hue=
0
Val=
0.5
Trough: Hue=
0.66
Val=
0.5
Zero: Hue=
0.33
Val=
1
Persistent Parameters
Default Space-Time Granularity
2
Higher for finer detail
Scenarios
Basic CW +1 >
CW Light ±1
PW Lite ±1
CW Light ±2
PW Lite ±2
CW Light -1<>+4
CW Light -2<>+8
PW Lit3 -2<>+8
Best for tweaking the responsiveness, also vary/set persistant space-time granularity below
Matter Wave: Bohr-Schrödinger Approximation
Bohr-Schrödinger {Quadratic dispersion}
CW k=+1,+2
CW k=+2,+3
CW k=-1,+2
RelaWavity Scenarios
Dispersion Plot (300 THz Scale)
Make these two lower for finer/narrower zero lines.
Note: they do vary with above settings
CW with ck = 4
2 CW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 PW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -1, 4 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
k-Phasor Plot (100 THz Scale)
CW with ck = 3
CW with ck = -3
CW with ck = 6
CW with ck = 12
2 CW with ck = ±6 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -3, 12 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
In the APP, right click on a scenaro button to expose the actual scenario string

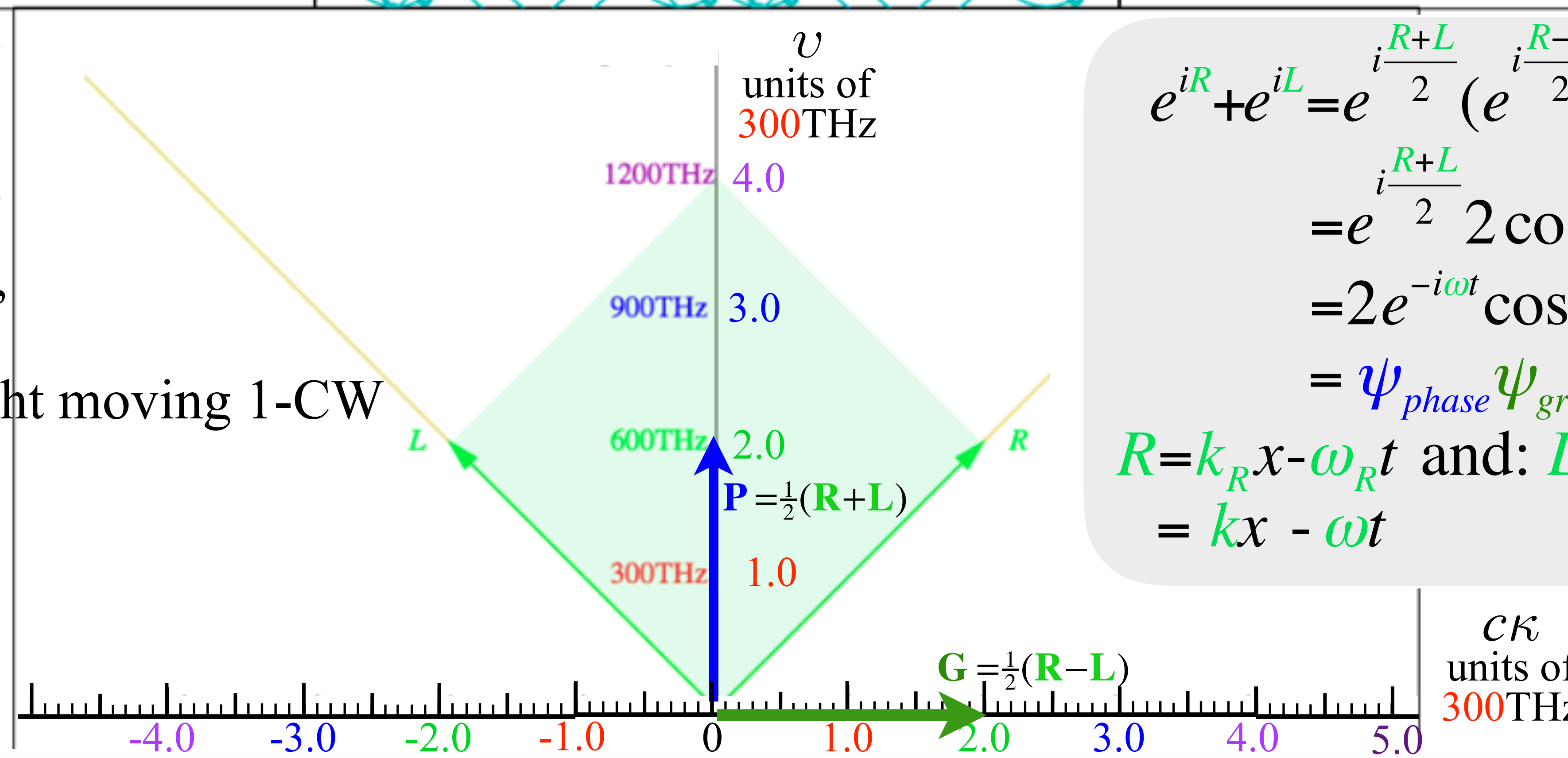

(a)
600THz
SOURCE
$Ae^{i(kx-\omega t)}$
1-CW left-to-RIGHT→
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
ct units of ¼ micron (μm)
1
2
3
4
x units of ¼ μm
(b) Standing wave
(x,ct) - grid
$\psi = Ae^{i(kx-\omega t)} + Ae^{i(-kx-\omega t)}$
→ ←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\frac{5}{6}$ fs
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
600THz
SOURCE
$Ae^{i(-kx-\omega t)}$
← LEFT-from-right1-CW
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
Introducing optical space-time grids and per-space-time “baseball-diamonds”
← LEFT-from-right moving 1-CW
1-CW moving left-to-RIGHT→
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
L
R
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
cκ units of 300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)



(d)

Introducing optical space-time grids and per-space-time “baseball-diamonds”

← LEFT-from-right moving 1-CW



$$e^{iR}+e^{iL}=e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

$$=e^{i\frac{R+L}{2}}\,2\cos\frac{R-L}{2}$$

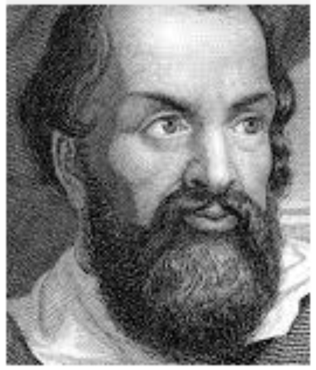
$$=2e^{-i\omega t}\cos kx$$

$$=\psi_{phase}\psi_{group}$$

$R=k_Rx-\omega_Rt$ and: $L=-k_Lx-\omega_Lt$

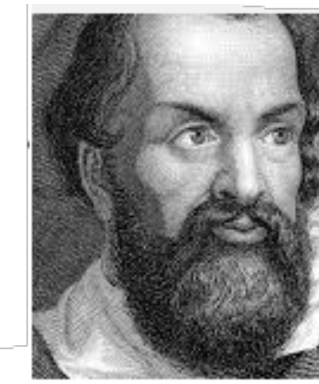
$= kx - \omega t$ $\qquad = -kx - \omega t$

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

➔ Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

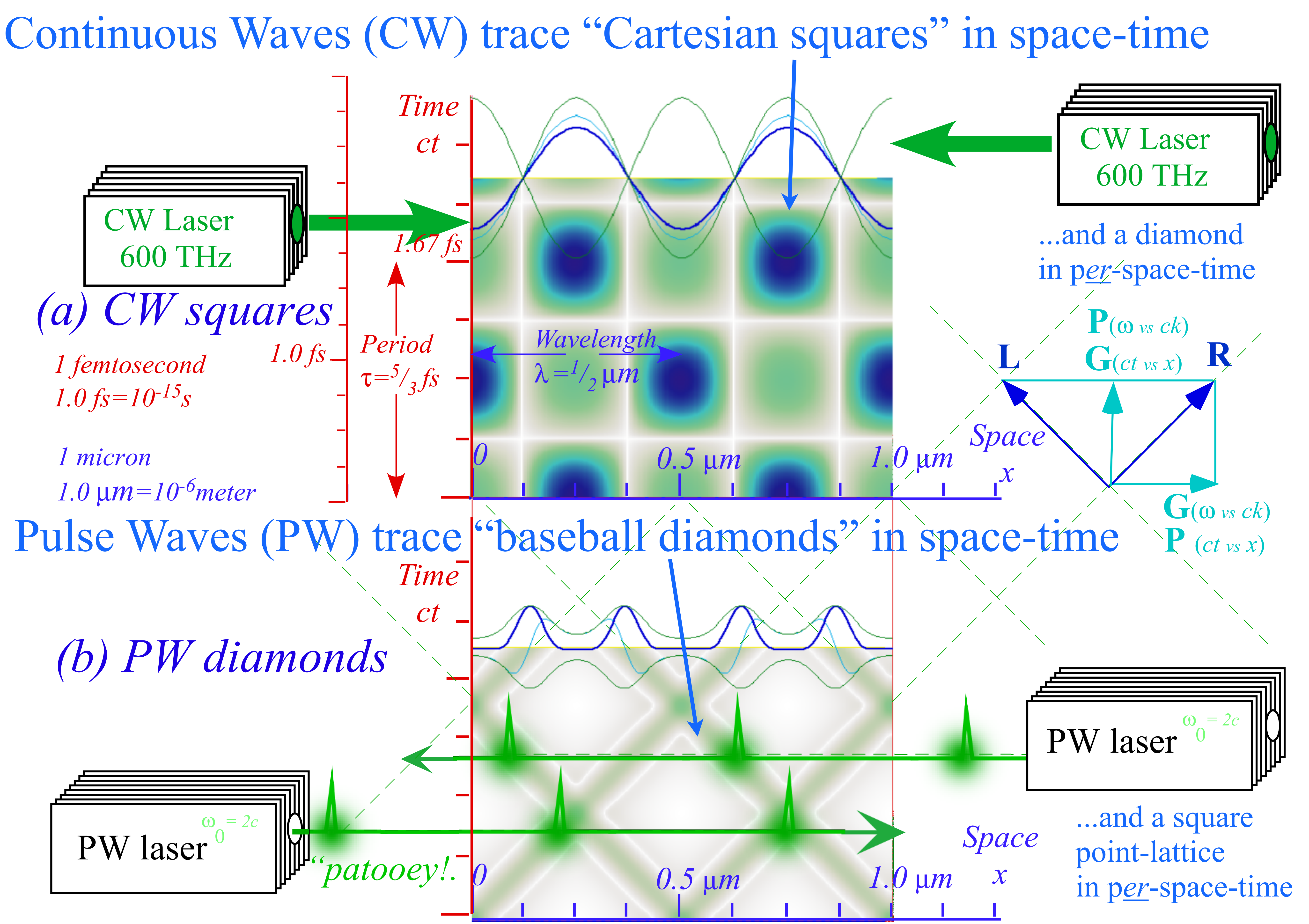
Thales geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

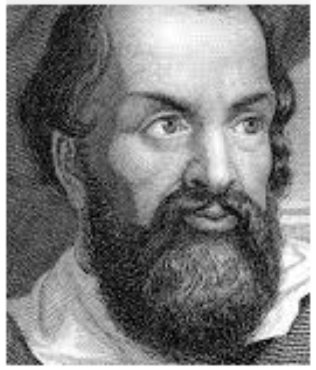
"Occams Sword" and geometry of functions of ρ and σ Minkowski animations

Application to TE-Waveguide modes. synchrotron beam relativity



BohrIt Web Simulation: *Multi-Panel 2 PW ct vs x Plot*

Galileo Galilei



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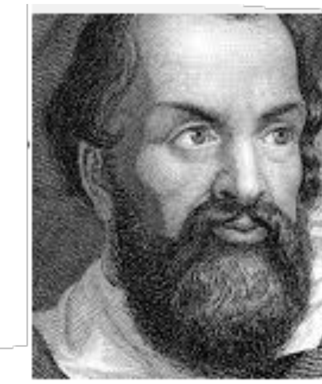

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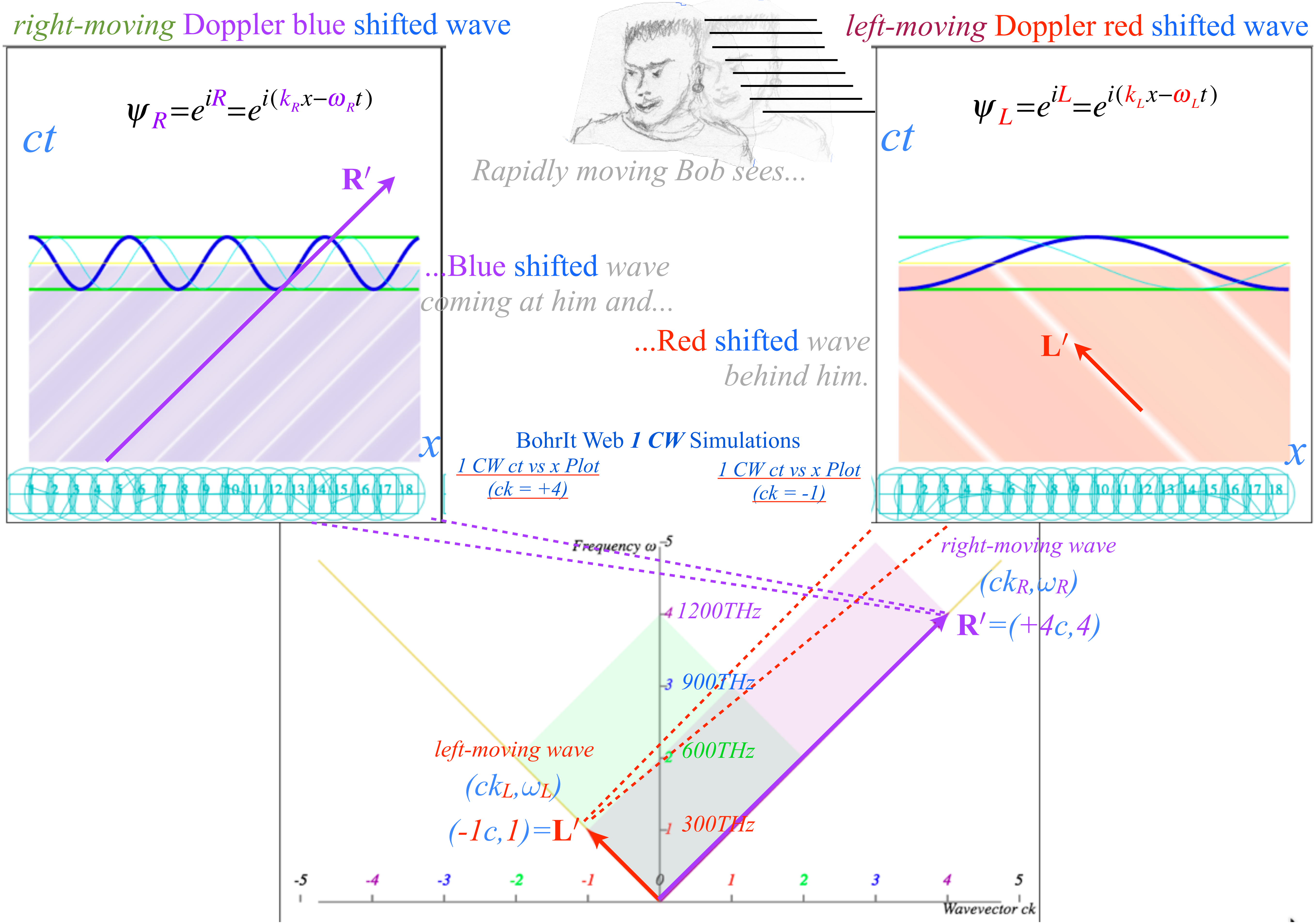
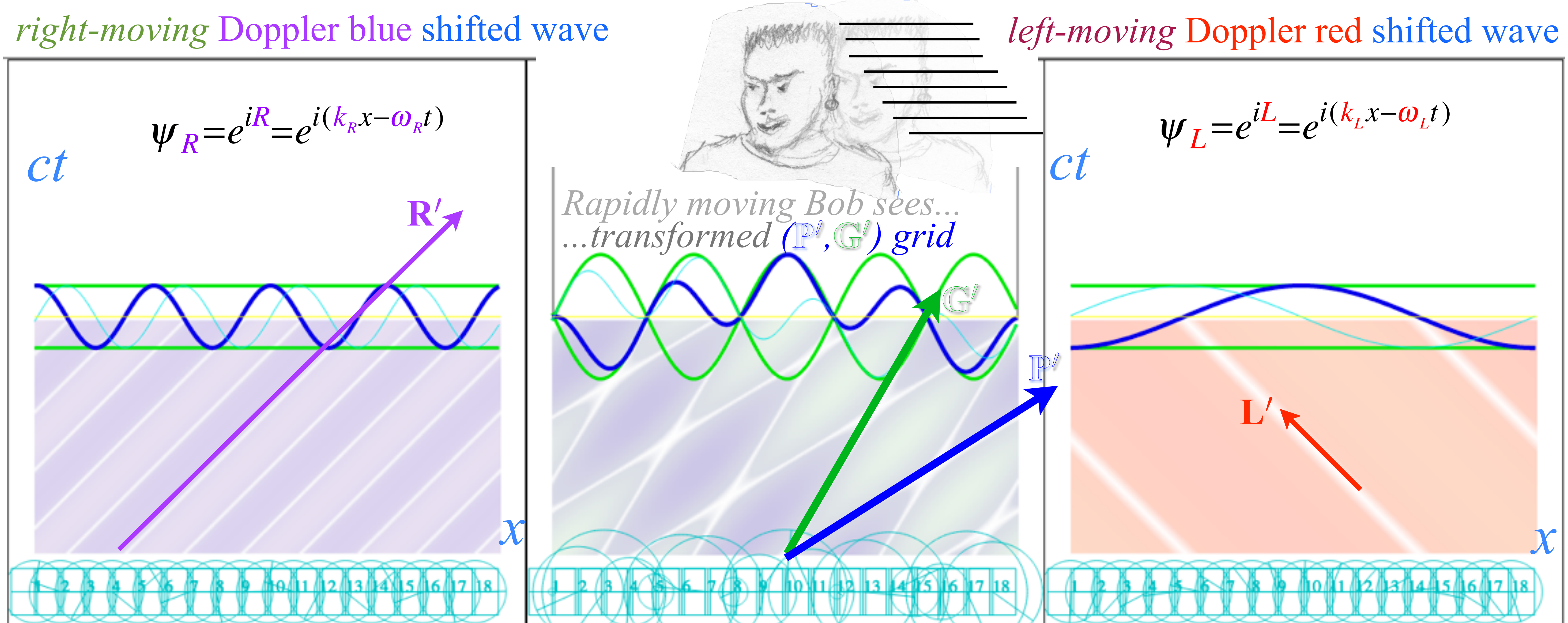


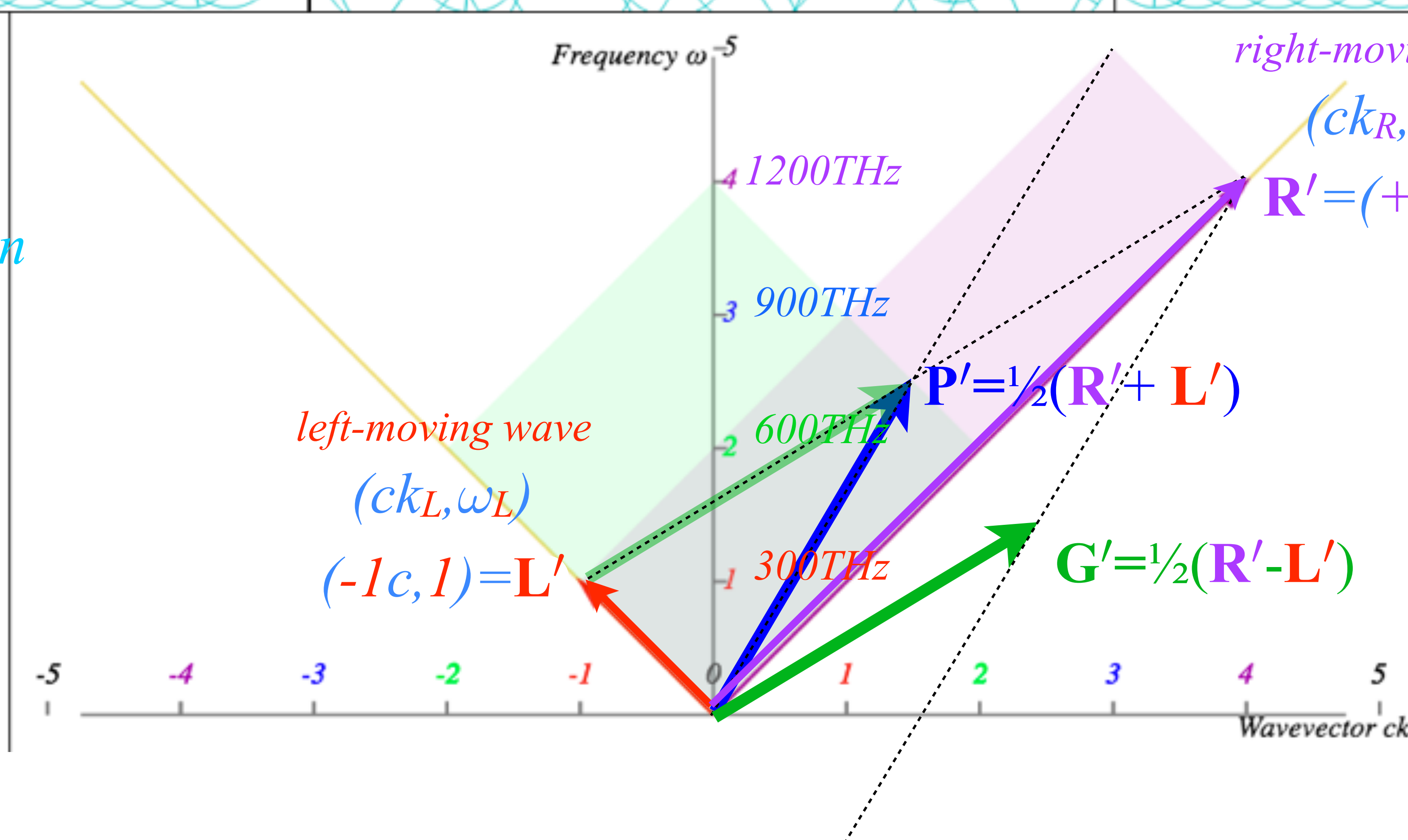
right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...Blue shifted wave coming at him and...
...Red shifted wave behind him.
BohrIt Web 1 CW Simulations
1 CW ct vs x Plot (ck = +4)
1 CW ct vs x Plot (ck = -1)
Frequency ω
1200THz
900THz
600THz
300THz
right-moving wave
(ck_R, ω_R)
$R' = (+4c, 4)$
left-moving wave
(ck_L, ω_L)
$(-1c, 1) = L'$
Wavevector ck

right-moving Doppler blue shifted wave   *left-moving* Doppler red shifted wave



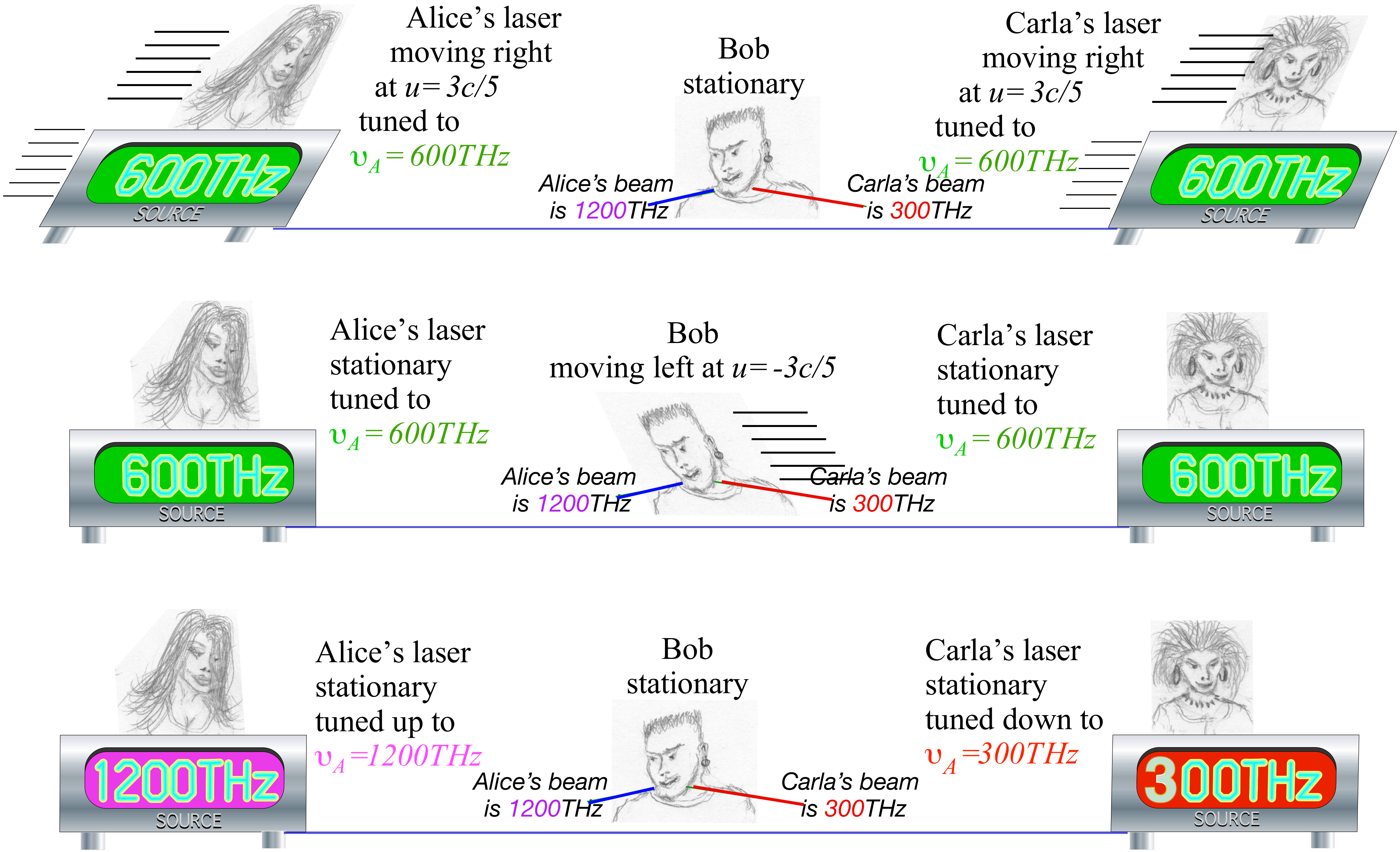
...Doppler shifts give
Lorentz transformation
of both these graphs

Per-Spacetime
(ck,ω)

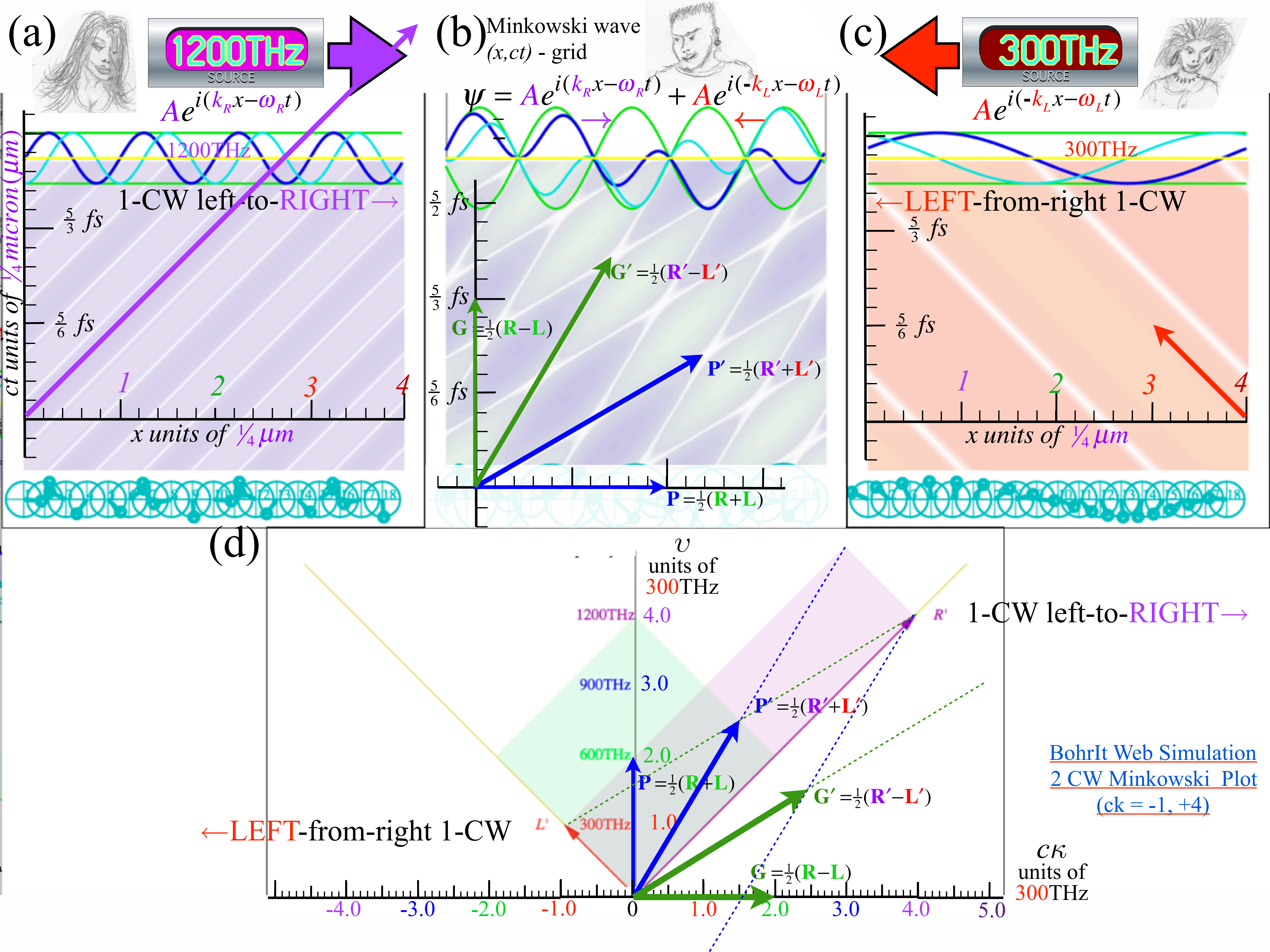


BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

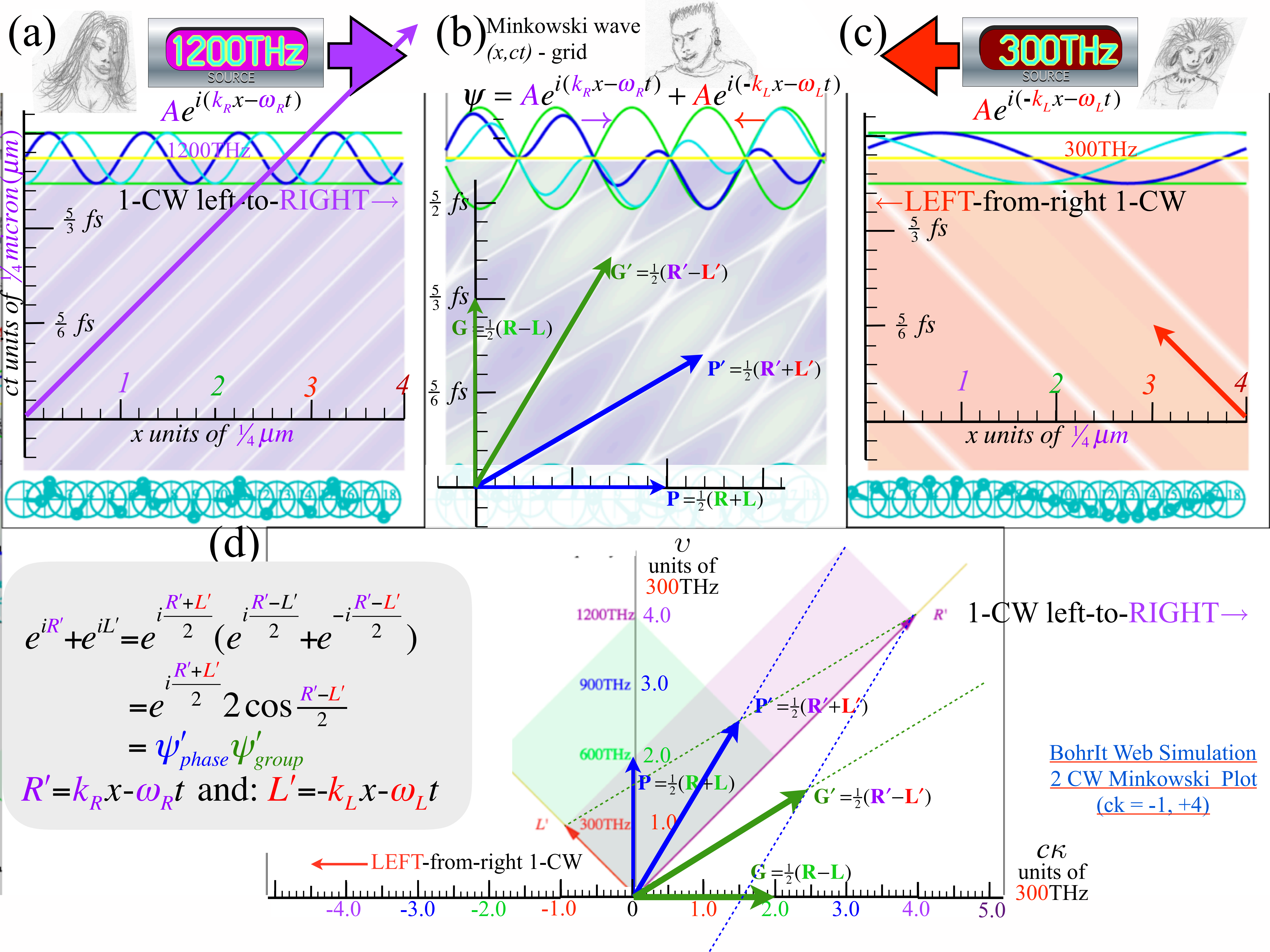
Three scenarios that look the same to Bob



Much cheaper (and safer) to do the 3rd scenario!$!

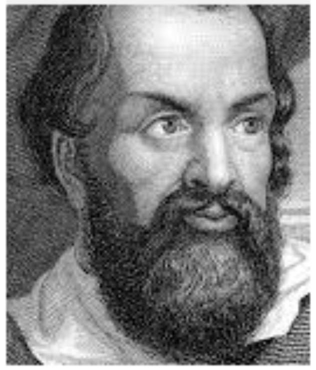


(a)
1200THz
SOURCE
$Ae^{i(k_R x - \omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(b)
Minkowski wave (x,ct) - grid
$\psi = Ae^{i(k_R x - \omega_R t)} + Ae^{i(-k_L x - \omega_L t)}$
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
G′ =½(R′−L′)
G =½(R−L)
P′ =½(R′+L′)
P =½(R+L)
(c)
300THz
SOURCE
$Ae^{i(-k_L x - \omega_L t)}$
300THz
←LEFT-from-right 1-CW
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R′
L′
1-CW left-to-RIGHT→
←LEFT-from-right 1-CW
P′ =½(R′+L′)
P =½(R+L)
G′ =½(R′−L′)
G =½(R−L)
cκ units of 300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)



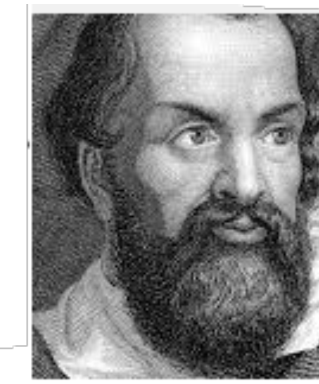
(a)
1200THz
SOURCE
Ae^{i(k_R x−ω_R t)}
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
5/3 fs
5/6 fs
1 2 3 4
x units of ¼ μm
(b)
Minkowski wave (x,ct) - grid
ψ = Ae^{i(k_R x−ω_R t)} + Ae^{i(-k_L x−ω_L t)}
5/2 fs
5/3 fs
5/6 fs
G′ = ½(R′−L′)
G = ½(R−L)
P′ = ½(R′+L′)
P = ½(R+L)
(c)
300THz
SOURCE
Ae^{i(-k_L x−ω_L t)}
300THz
←LEFT-from-right 1-CW
5/3 fs
5/6 fs
1 2 3 4
x units of ¼ μm
(d)
e^{iR′}+e^{iL′} = e^{i(R′+L′)/2}(e^{i(R′−L′)/2} + e^{−i(R′−L′)/2})
= e^{i(R′+L′)/2} 2 cos((R′−L′)/2)
= ψ′_phase ψ′_group
R′=k_R x-ω_R t and: L′=-k_L x-ω_L t
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R′
L′
1-CW left-to-RIGHT→
P′ = ½(R′+L′)
P = ½(R+L)
G′ = ½(R′−L′)
G = ½(R−L)
←LEFT-from-right 1-CW
cκ units of 300THz
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0 5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
➔ Analyzing wave velocity by per-space-per-time *and* space-time graphs
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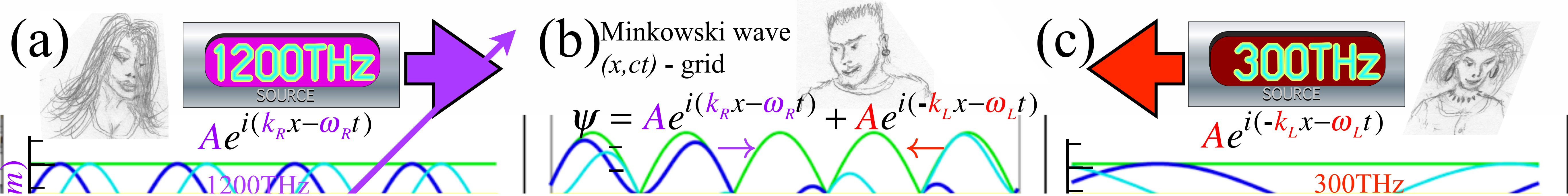


$$\mathbf{P}'=\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'+\mathbf{L}') = \upsilon_A \begin{pmatrix} \tfrac{1}{2}(e^{\rho}+e^{-\rho}) \\ \tfrac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \tfrac{5}{4} \\ \tfrac{3}{4} \end{pmatrix}_{\text{Bob's View}} \text{ or: } \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\text{Alice's View}}$$

$$\mathbf{G}'=\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'-\mathbf{L}') = \upsilon_A \begin{pmatrix} \tfrac{1}{2}(e^{\rho}-e^{-\rho}) \\ \tfrac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \tfrac{3}{4} \\ \tfrac{5}{4} \end{pmatrix}_{\text{Bob's View}} \text{ or: } \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\text{Alice's View}}$$

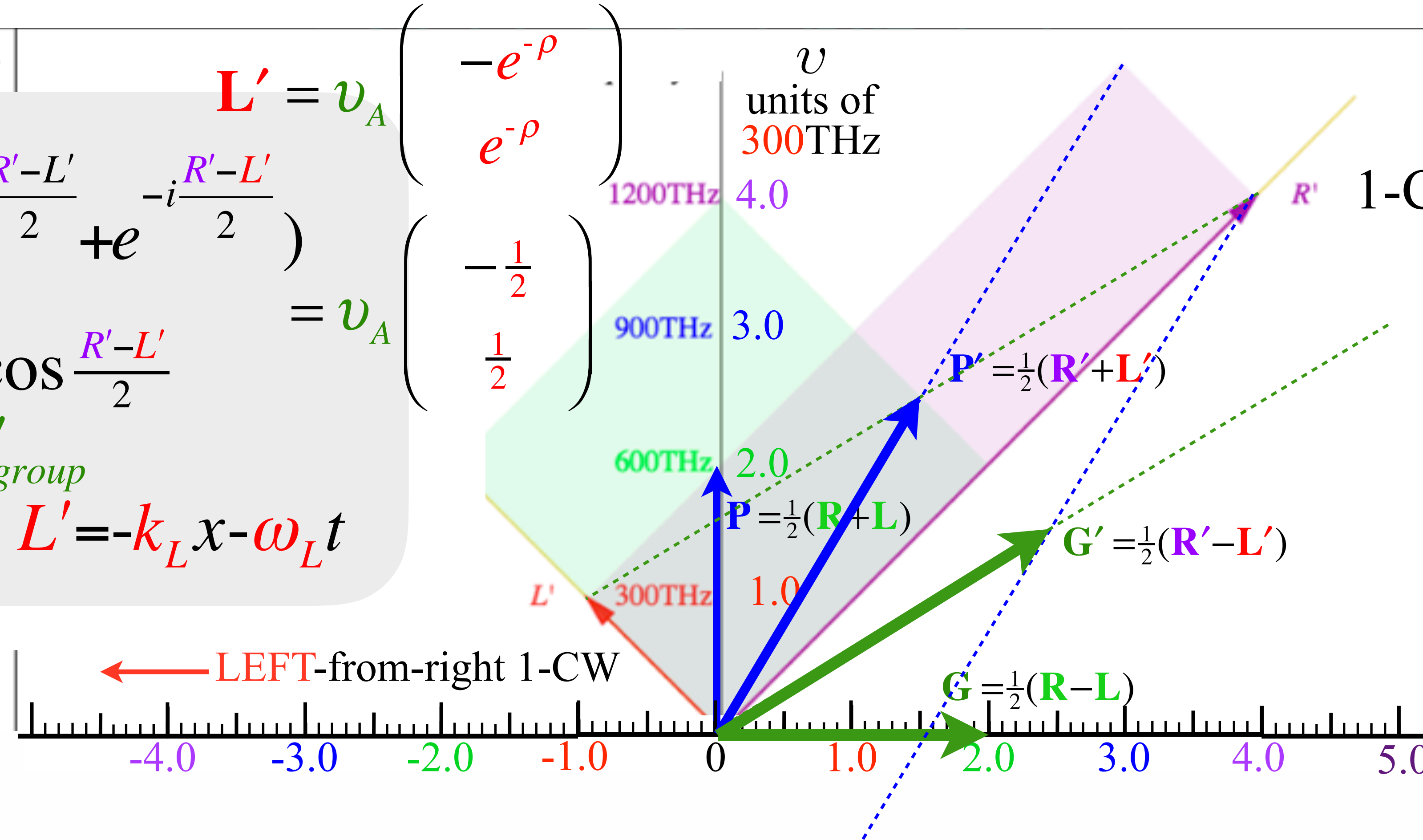
$$e^{-\rho} = \frac{1}{2} \qquad e^{\rho} = 2$$

(d)

$$\mathbf{L}' = \upsilon_A \begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} -\tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix}$$

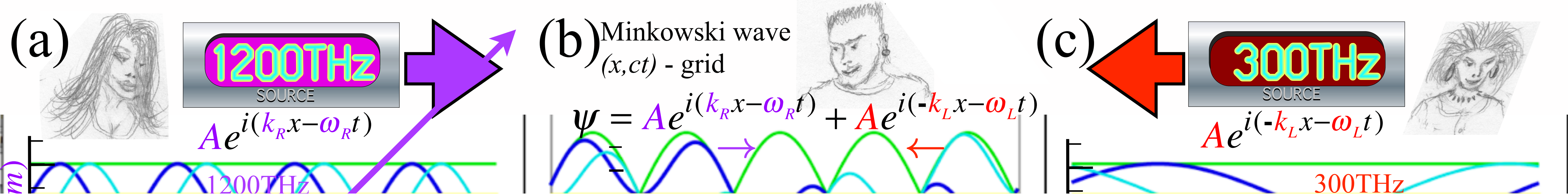
$$e^{iR'}+e^{iL'} = e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right) = e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2} = \psi'_{phase}\psi'_{group}$$

$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$



1-CW left-to-RIGHT→

$$\mathbf{R}' = \upsilon_A \begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

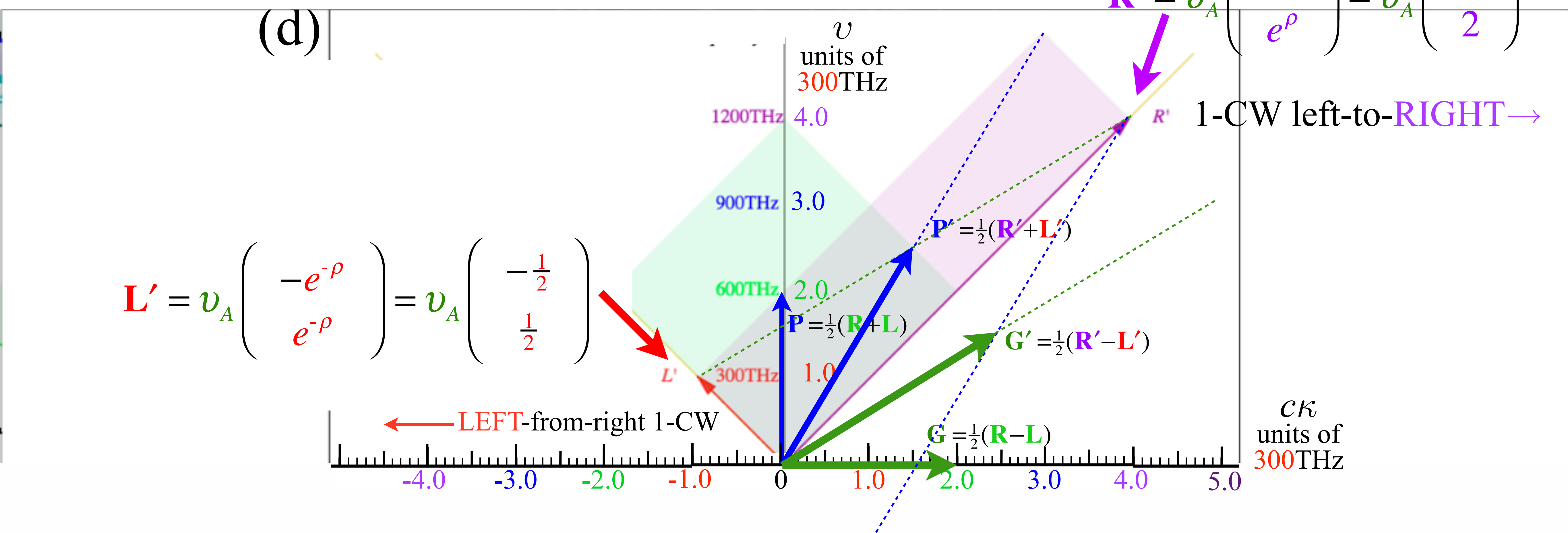


$$\mathbf{P}'=\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'+\mathbf{L}')=\upsilon_A\begin{pmatrix} \frac{1}{2}(e^{\rho}+e^{-\rho}) \\ \frac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}_{\text{Bob's View}} \quad \text{or: } \upsilon_A\begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\text{Alice's View}}$$

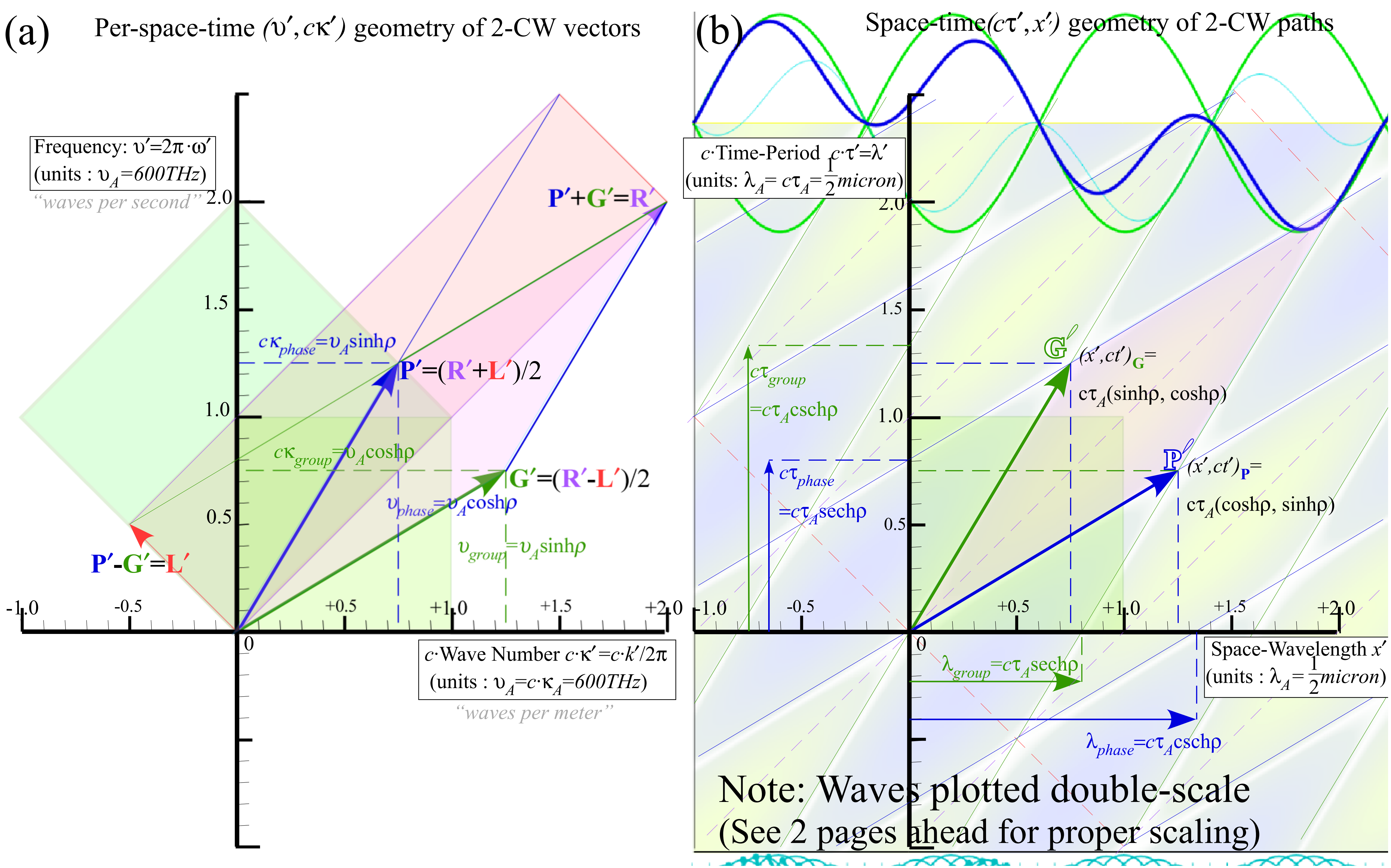
$$\mathbf{G}'=\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'-\mathbf{L}')=\upsilon_A\begin{pmatrix} \frac{1}{2}(e^{\rho}-e^{-\rho}) \\ \frac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}_{\text{Bob's View}} \quad \text{or: } \upsilon_A\begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\text{Alice's View}}$$

$$e^{-\rho}=\frac{1}{2} \qquad e^{\rho}=2$$

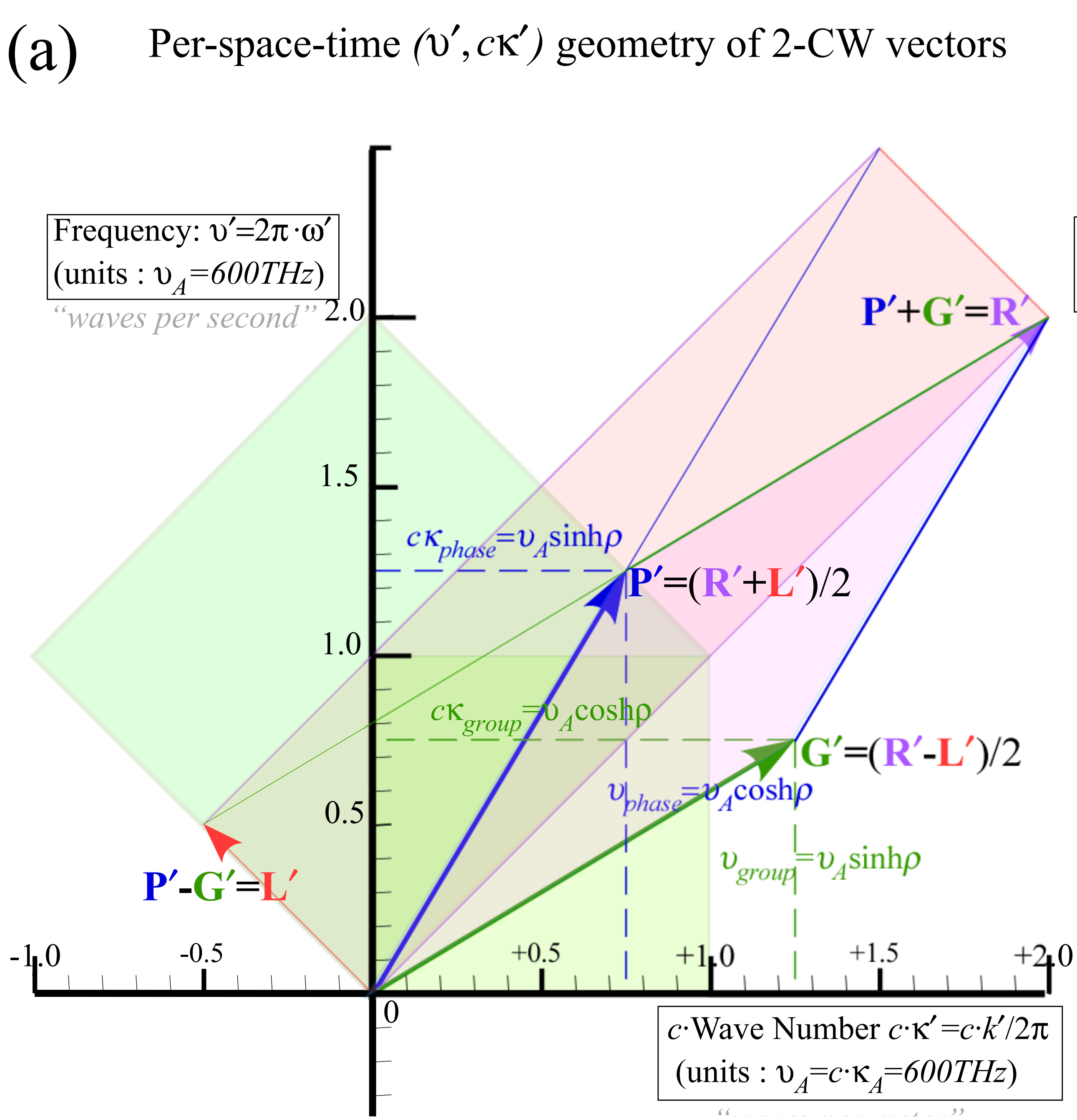
$$\mathbf{R}'=\upsilon_A\begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

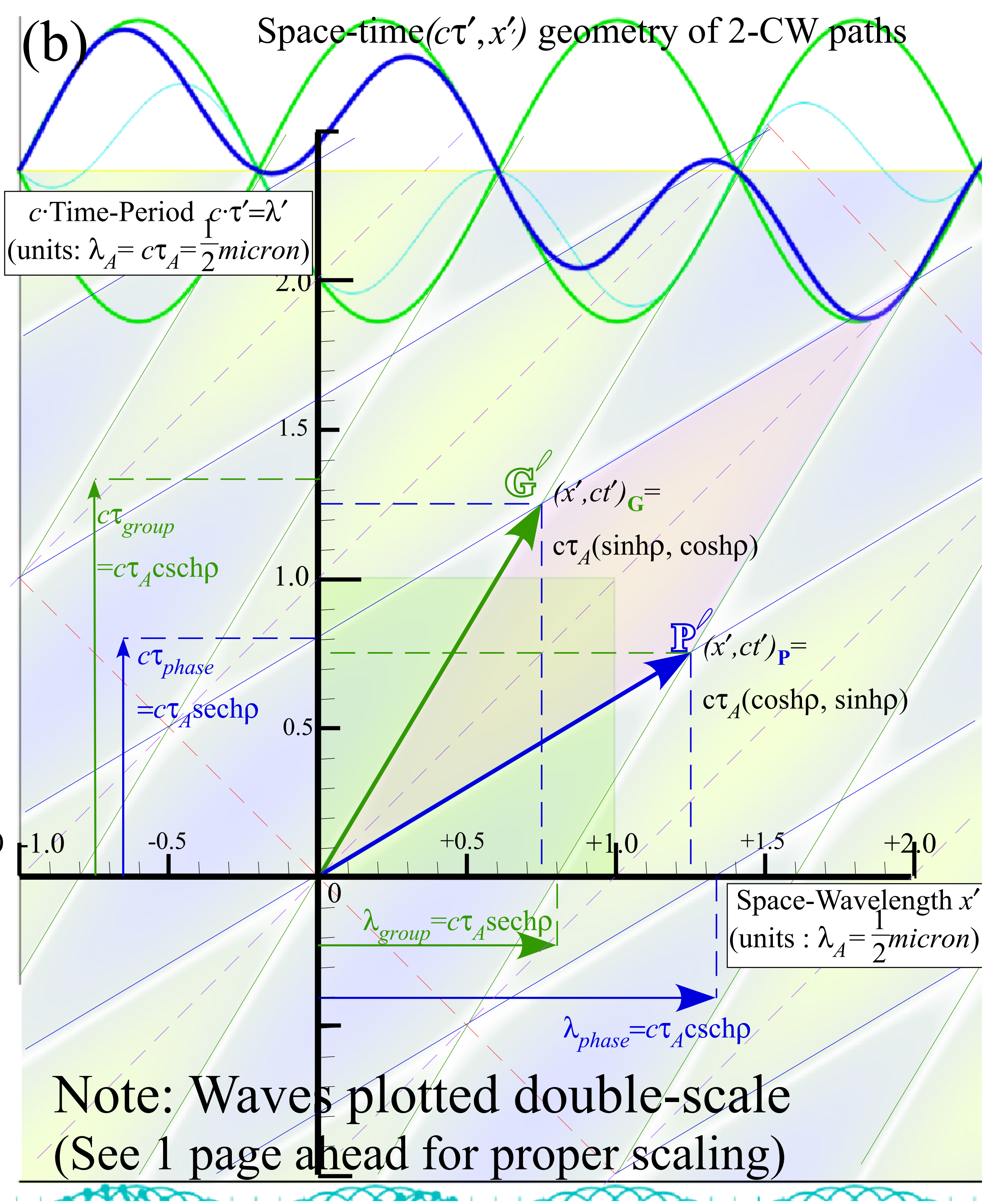


$$\mathbf{L}'=\upsilon_A\begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$



(a) Per-space-time (υ′, cκ′) geometry of 2-CW vectors
Frequency: υ′=2π·ω′
(units : υ_A=600THz)
"waves per second"
P′+G′=R′
cκ_phase=υ_A sinhρ
P′=(R′+L′)/2
cκ_group=υ_A coshρ
G′=(R′-L′)/2
υ_phase=υ_A coshρ
υ_group=υ_A sinhρ
P′-G′=L′
c·Wave Number c·κ′=c·k′/2π
(units : υ_A=c·κ_A=600THz)
"waves per meter"
(b) Space-time (cτ′, x′) geometry of 2-CW paths
c·Time-Period c·τ′=λ′
(units: λ_A= cτ_A=1/2 micron)
cτ_group
=cτ_A cschρ
cτ_phase
=cτ_A sechρ
G′ (x′,ct′)_G=
cτ_A(sinhρ, coshρ)
P′ (x′,ct′)_P=
cτ_A(coshρ, sinhρ)
λ_group=cτ_A sechρ
λ_phase=cτ_A cschρ
Space-Wavelength x′
(units : λ_A=1/2 micron)
Note: Waves plotted double-scale
(See 2 pages ahead for proper scaling)



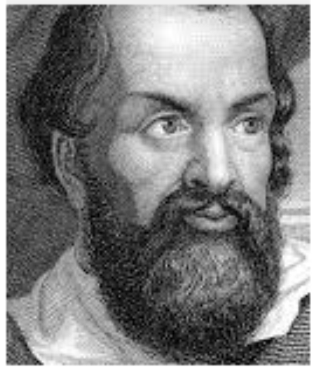


The slope of Bob's group vector **G′** in $(c\kappa,\upsilon)$-plot is actual group wave velocity in c-units.

$$\frac{V^{group}}{c}=\frac{\upsilon'_{group}}{c\kappa'_{group}}=\frac{\sinh\rho}{\cosh\rho}=\tanh\rho=\frac{\frac{3}{2}}{\frac{5}{2}}=\frac{3}{5}\equiv\frac{u}{c}\equiv\beta$$

Group vector $\mathbb{G}'$ in (x,ct)-plot has 3/5 slope relative to time axis

Galileo Galilei



1564-1642

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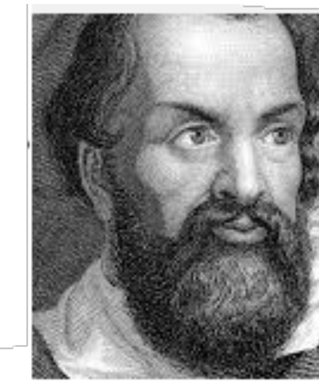

Galileo's Revenge (part 2)
Phasor angular velocity
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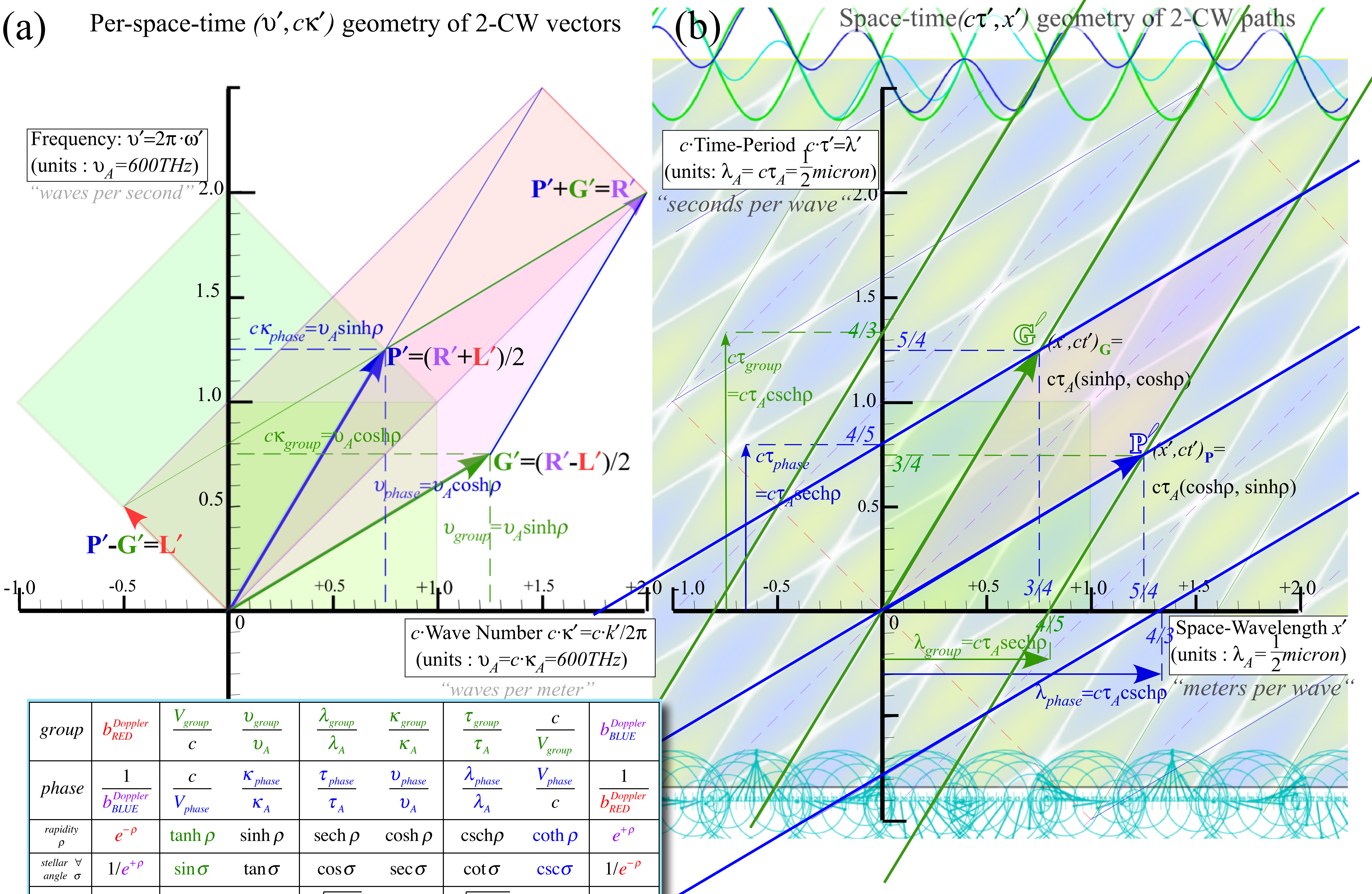
"Occams Sword" and geometry of functions of ρ and σ Minkowski animations
Application to TE-Waveguide modes. synchrotron beam relativity



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Fig. 11 in text Relawavity...

Fig. 4 in short version Ch.0....

If $\dfrac{u}{c} = \tanh\rho = 0.618\ldots(\textit{Golden-Mean } G\_)$

two parameters become *exactly* equal :

$$\frac{ct'_P}{c\tau_A} = \sinh\rho = \frac{\lambda_{group}}{\lambda_A} = \frac{\tau_{phase}}{\tau_A} = \operatorname{sech}\rho$$

$$= 0.786.. = \sqrt{G\_} \qquad = 0.786..$$

and

$$\frac{x'_P}{\lambda_A} = \cosh\rho = \frac{\lambda_{phase}}{\lambda_A} = \frac{\tau_{group}}{\tau_A} = \operatorname{csch}\rho$$

$$= 1.272.. = 1/\sqrt{G\_} \qquad = 1.272..$$

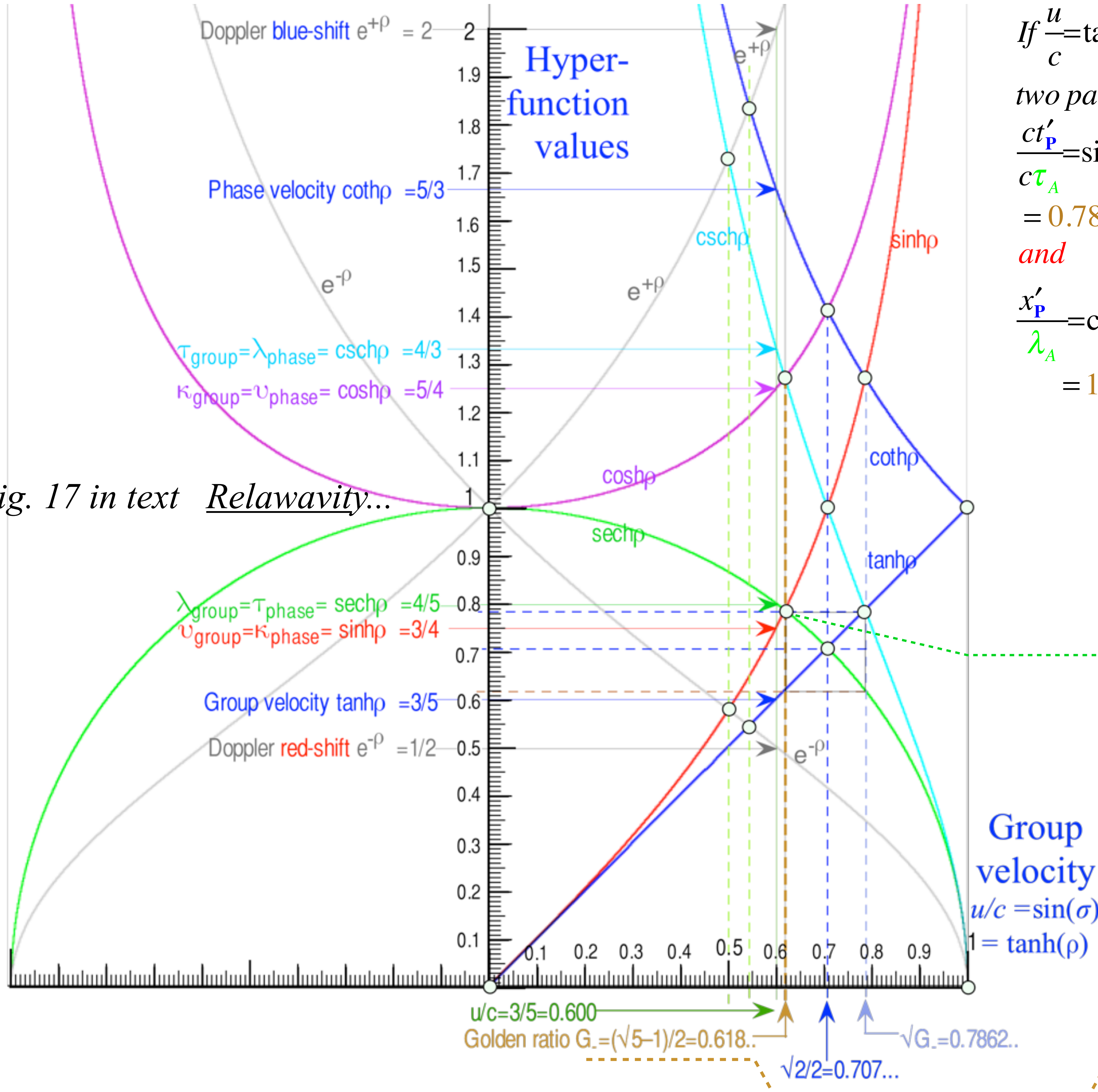


Fig. 17 in text <u>*Relawavity...*</u>

Solve :

$\operatorname{sech}\rho = \sinh\rho$

or:

$\sinh\rho\cosh\rho = 1$

or:

$\sinh 2\rho = 2$

$\rho = \dfrac{1}{2}\sinh^{-1}2 = 0.7218\ldots$

$\tanh\rho = 0.618\ldots = \dfrac{\sqrt{5}-1}{2}$

The 16 dimensions of 2CW interference

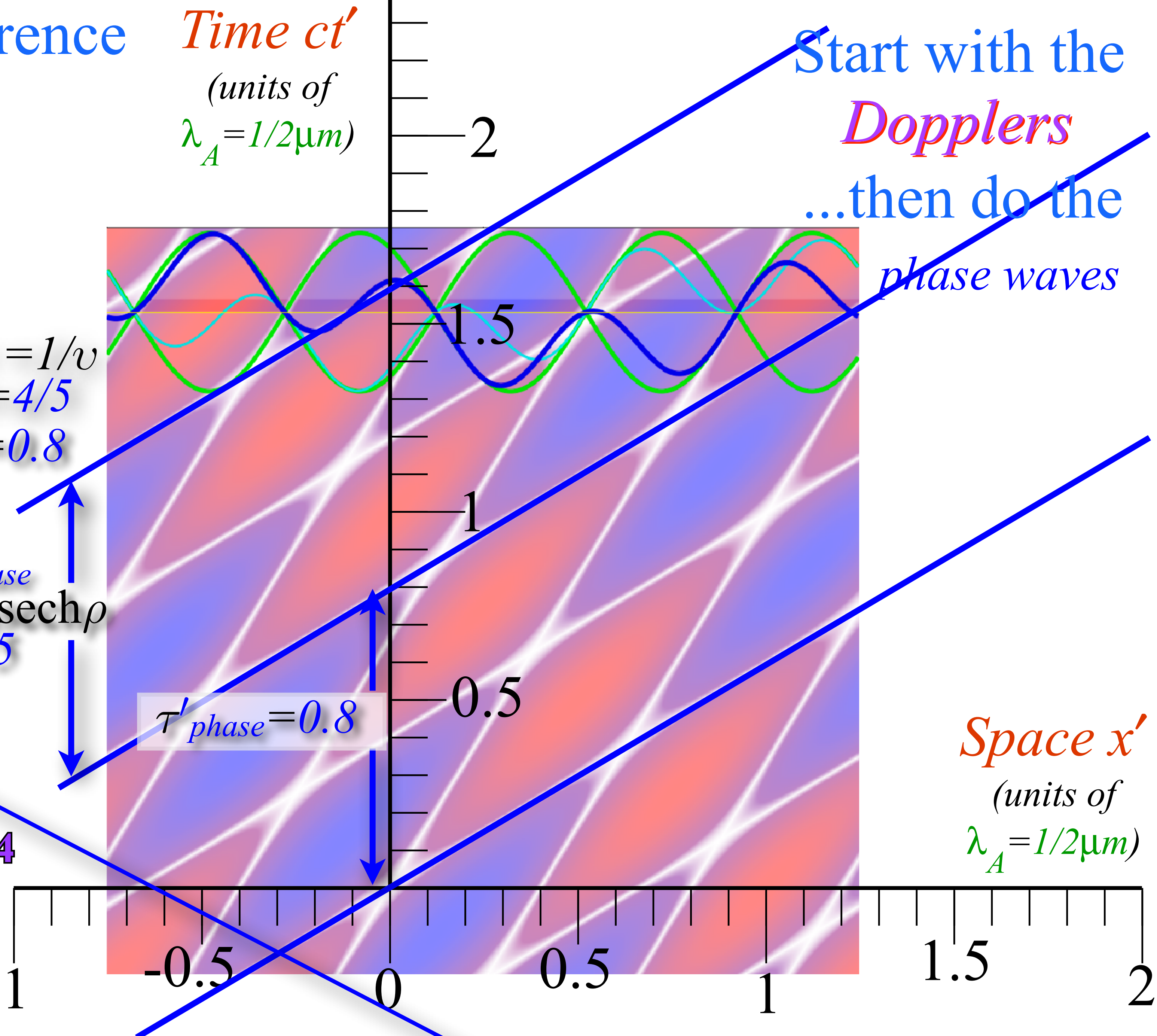
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

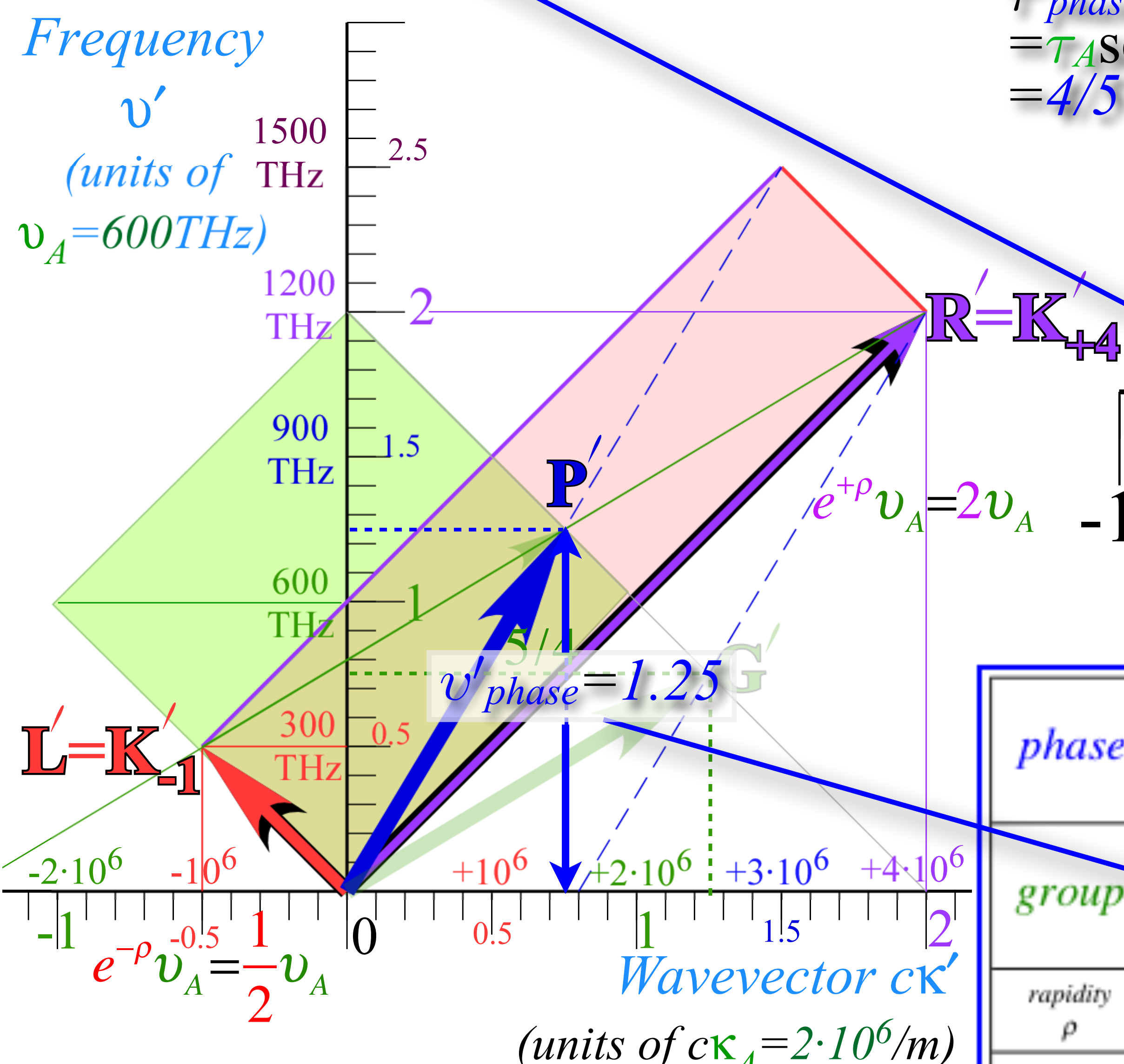
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

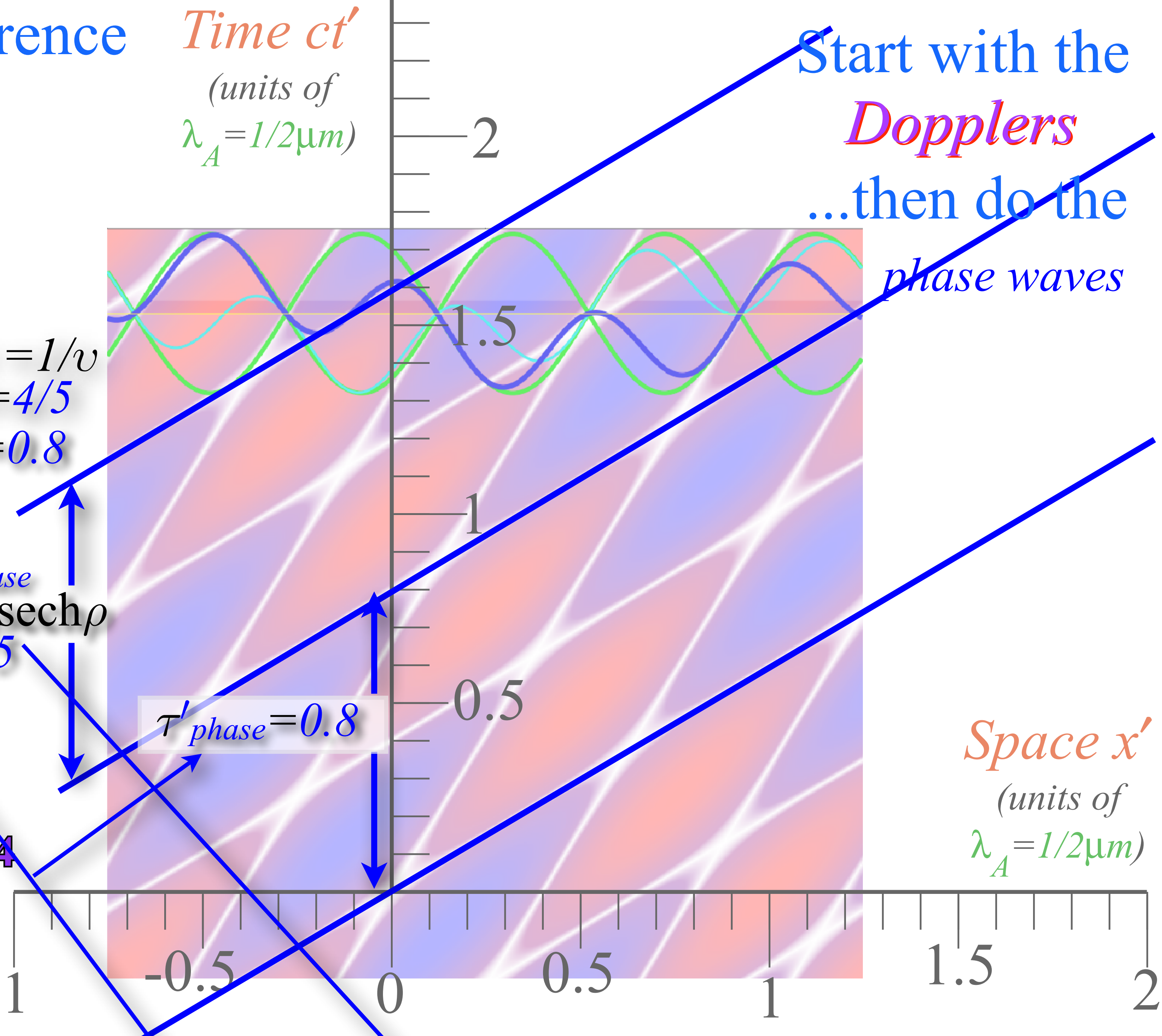
The 16 dimensions of 2CW interference

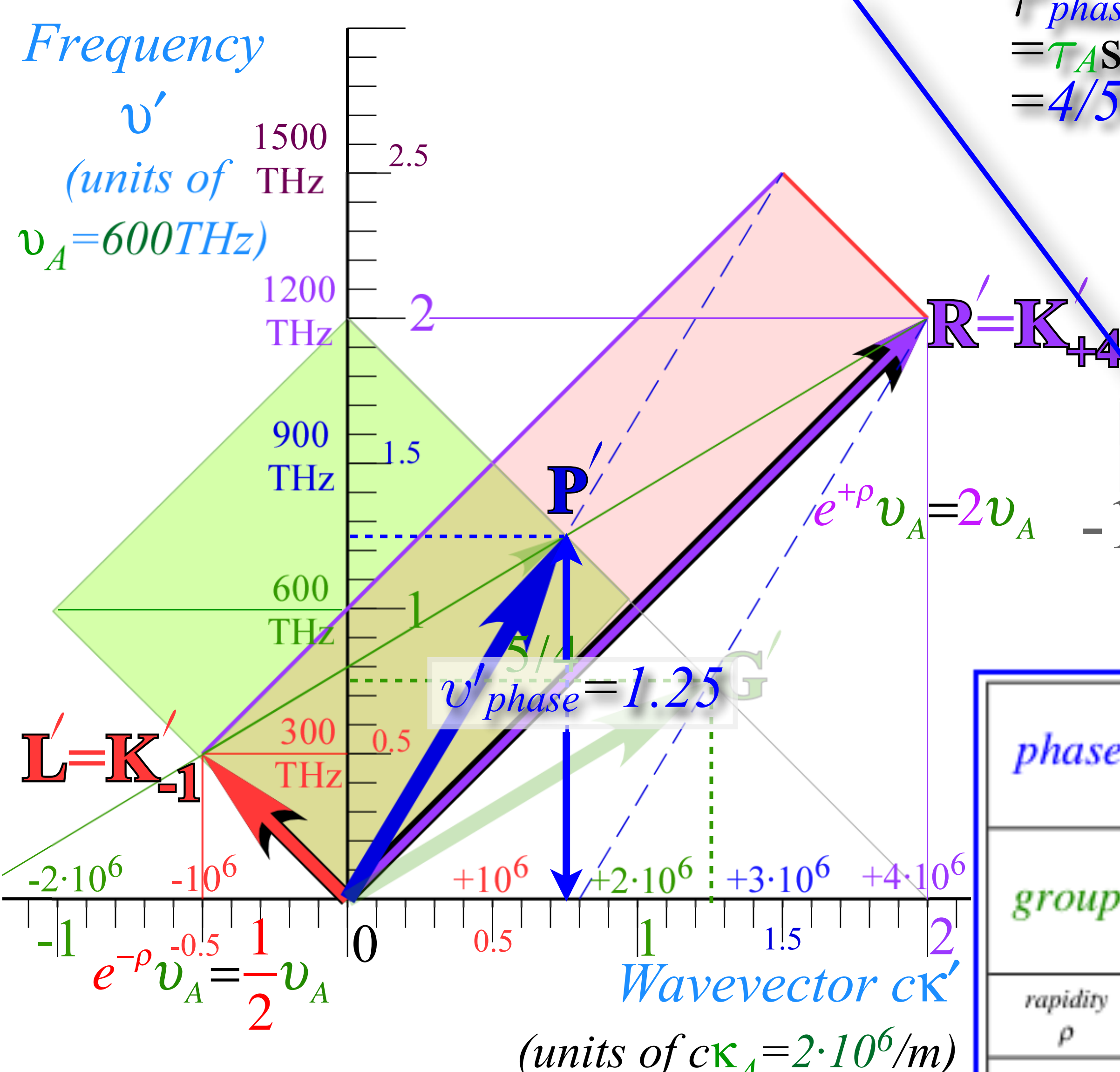
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

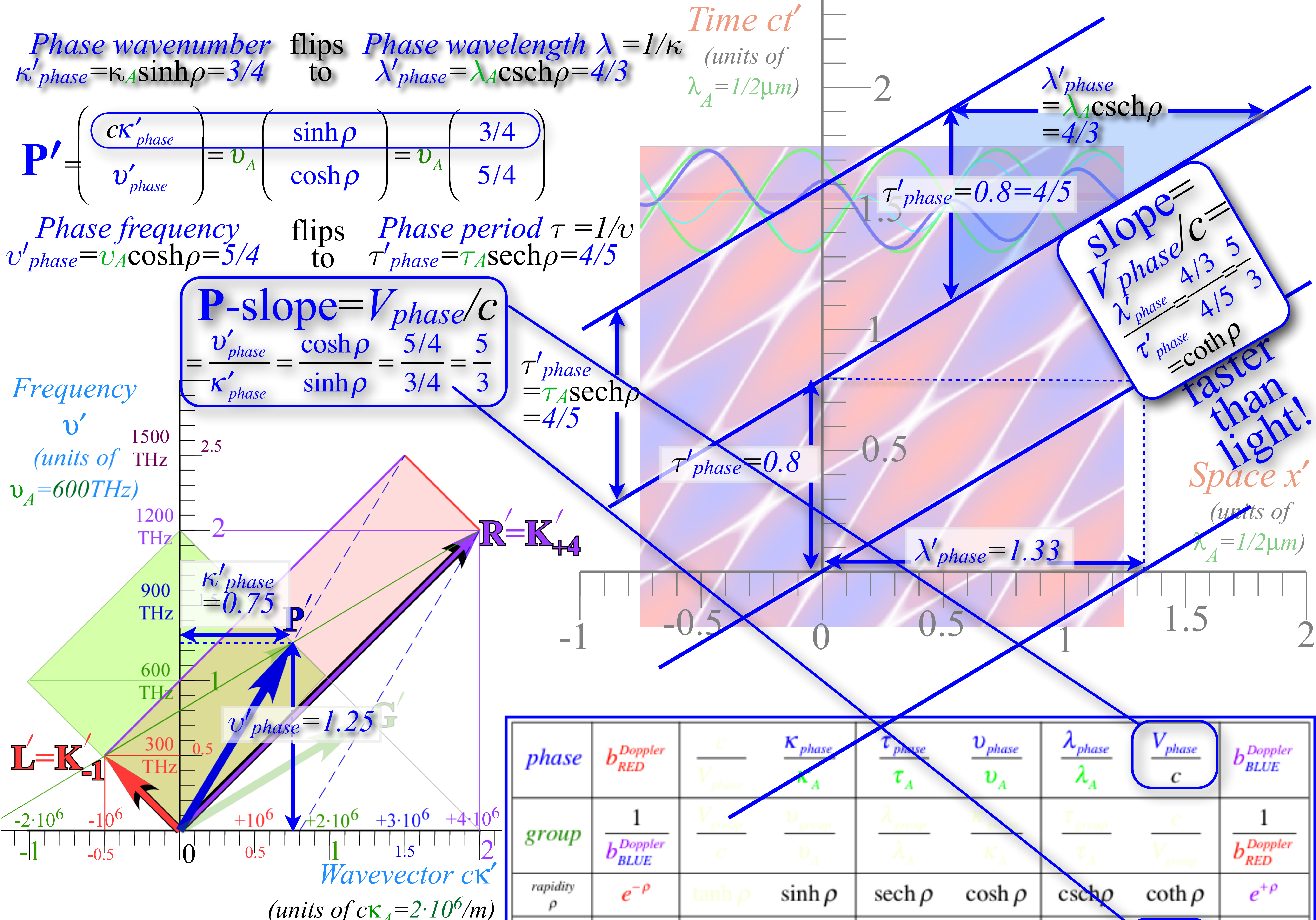
flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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Group wavenumber

$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$

$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

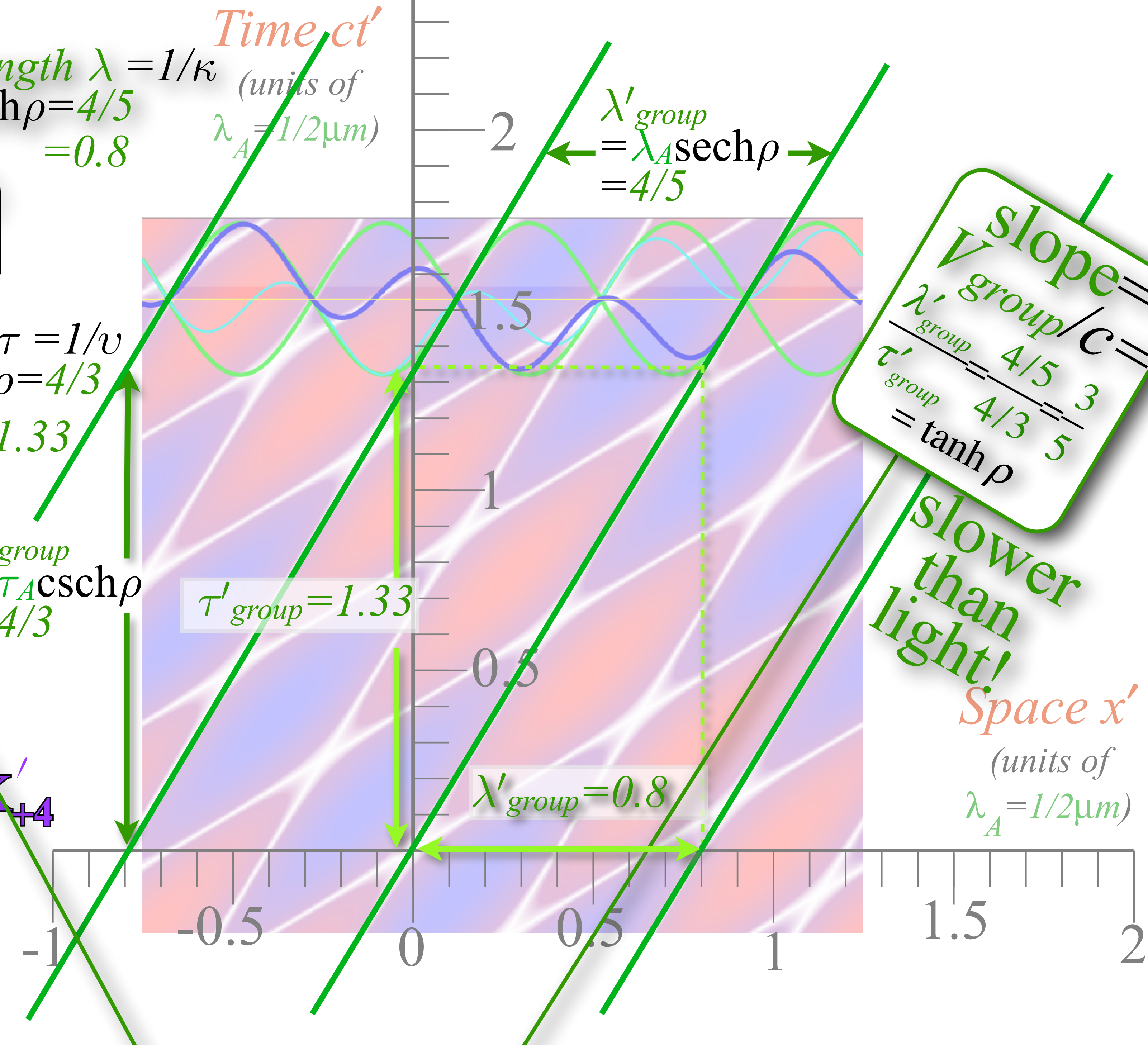
Group frequency

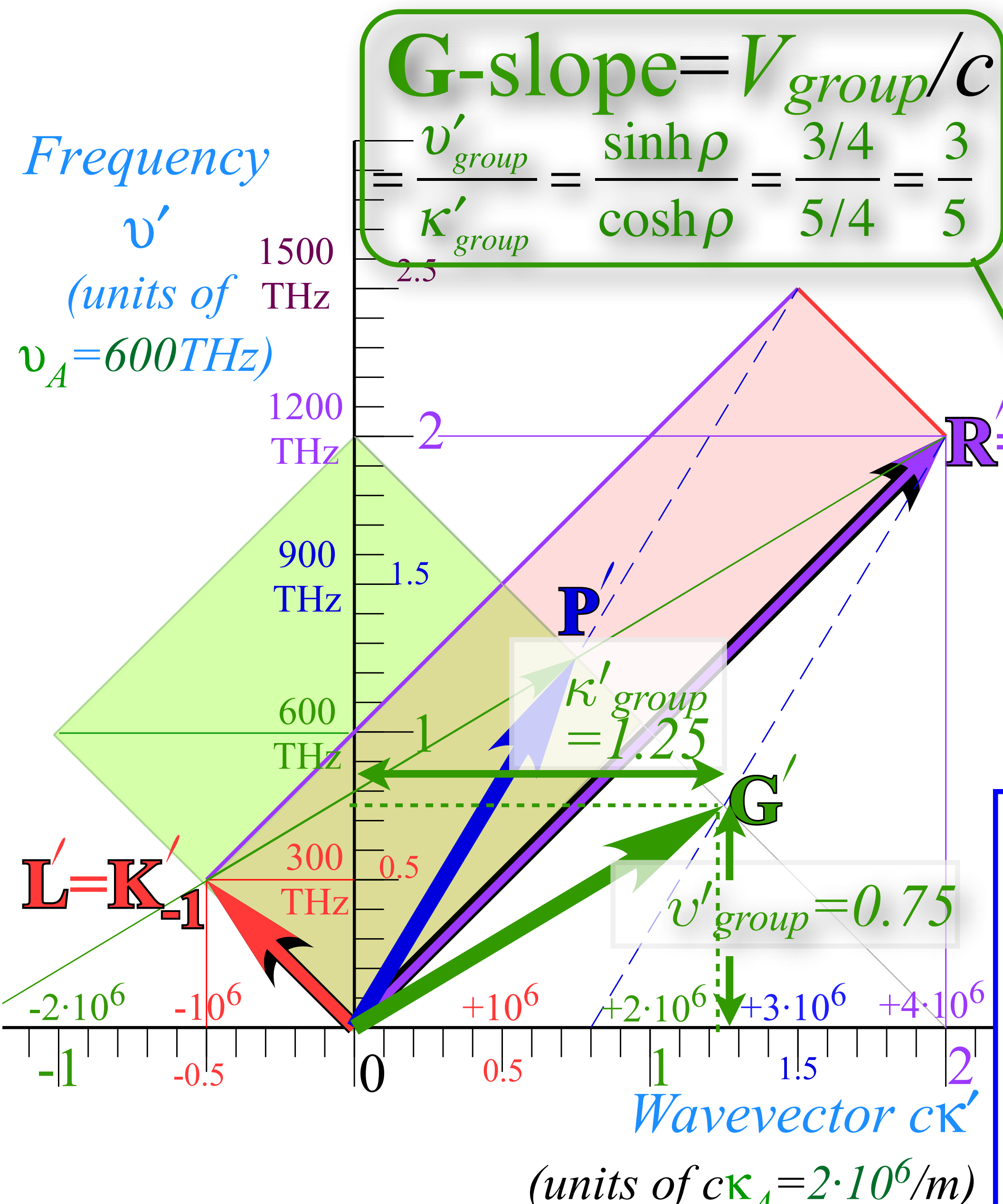
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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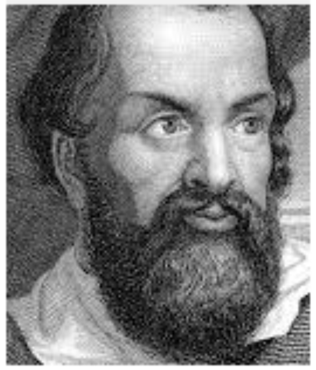
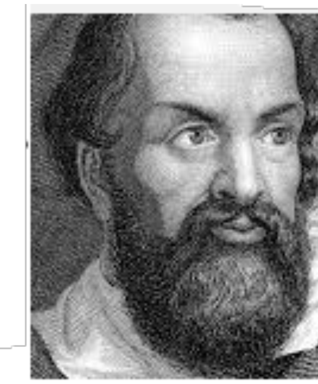
Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

➔ Two "famous-name" coefficients and the Lorentz transformation

Thales mean geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of functions of ρ and σ — Minkowski animations

Application to TE-Waveguide modes. — synchrotron beam relativity

Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

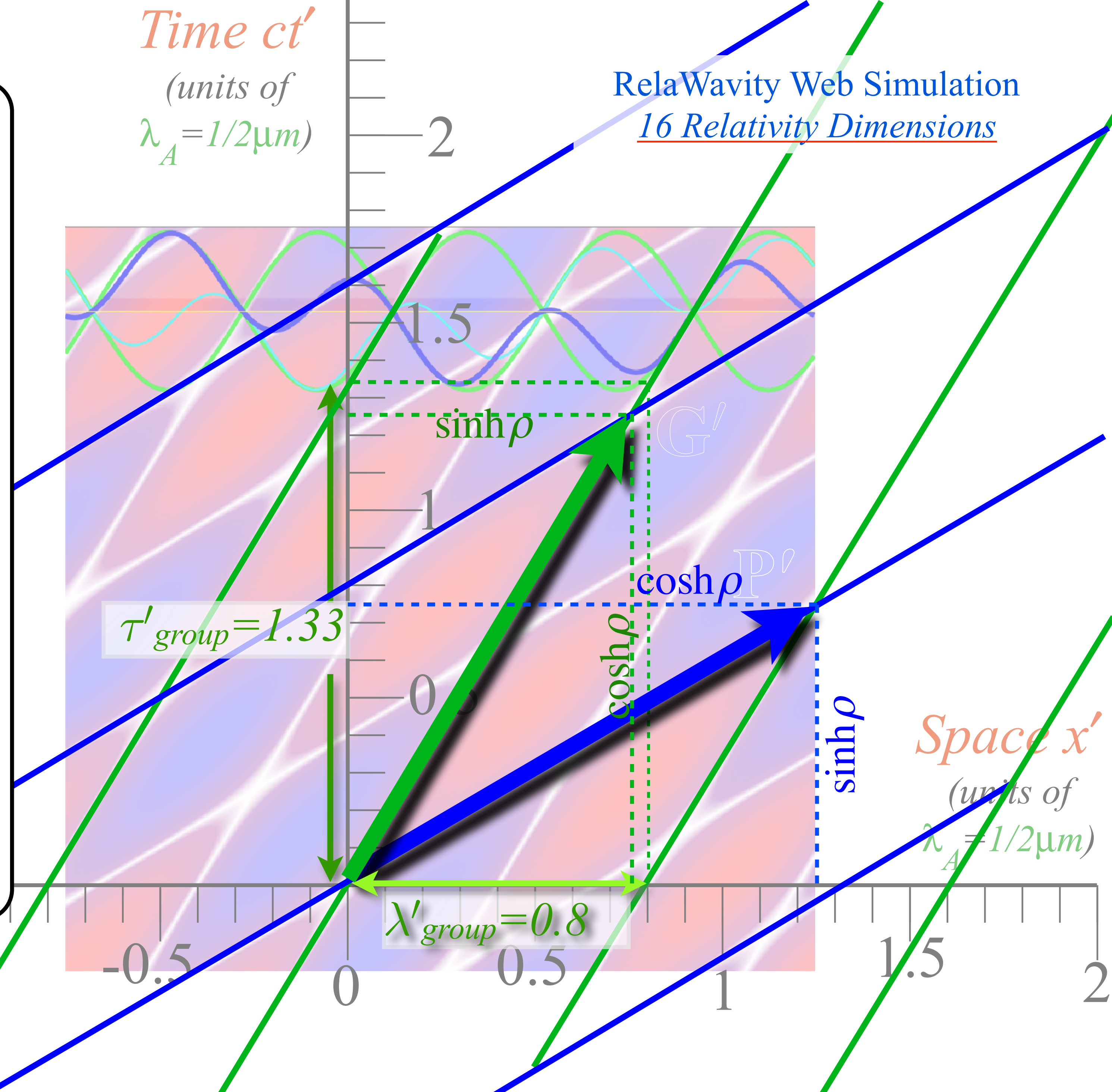
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

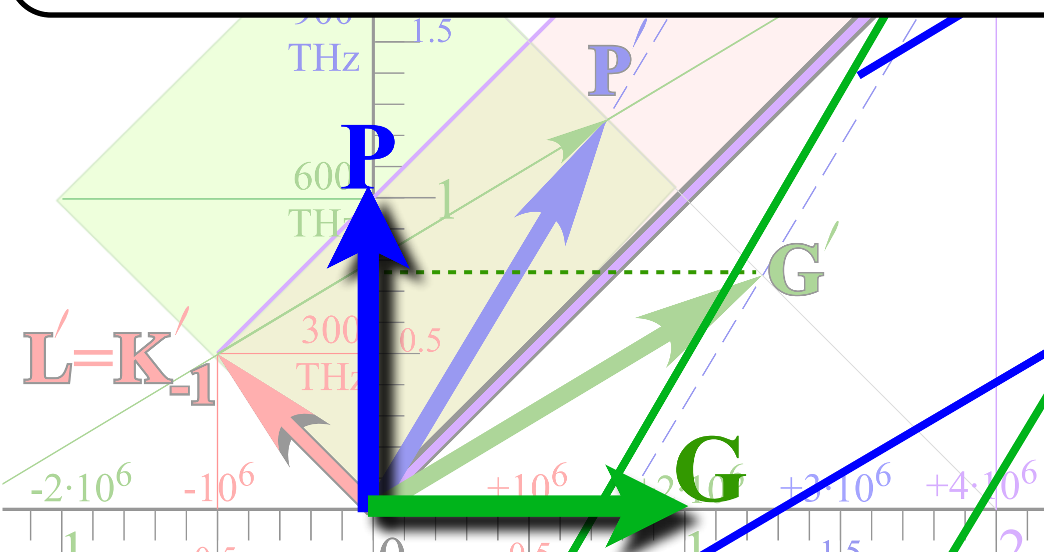
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$





$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$

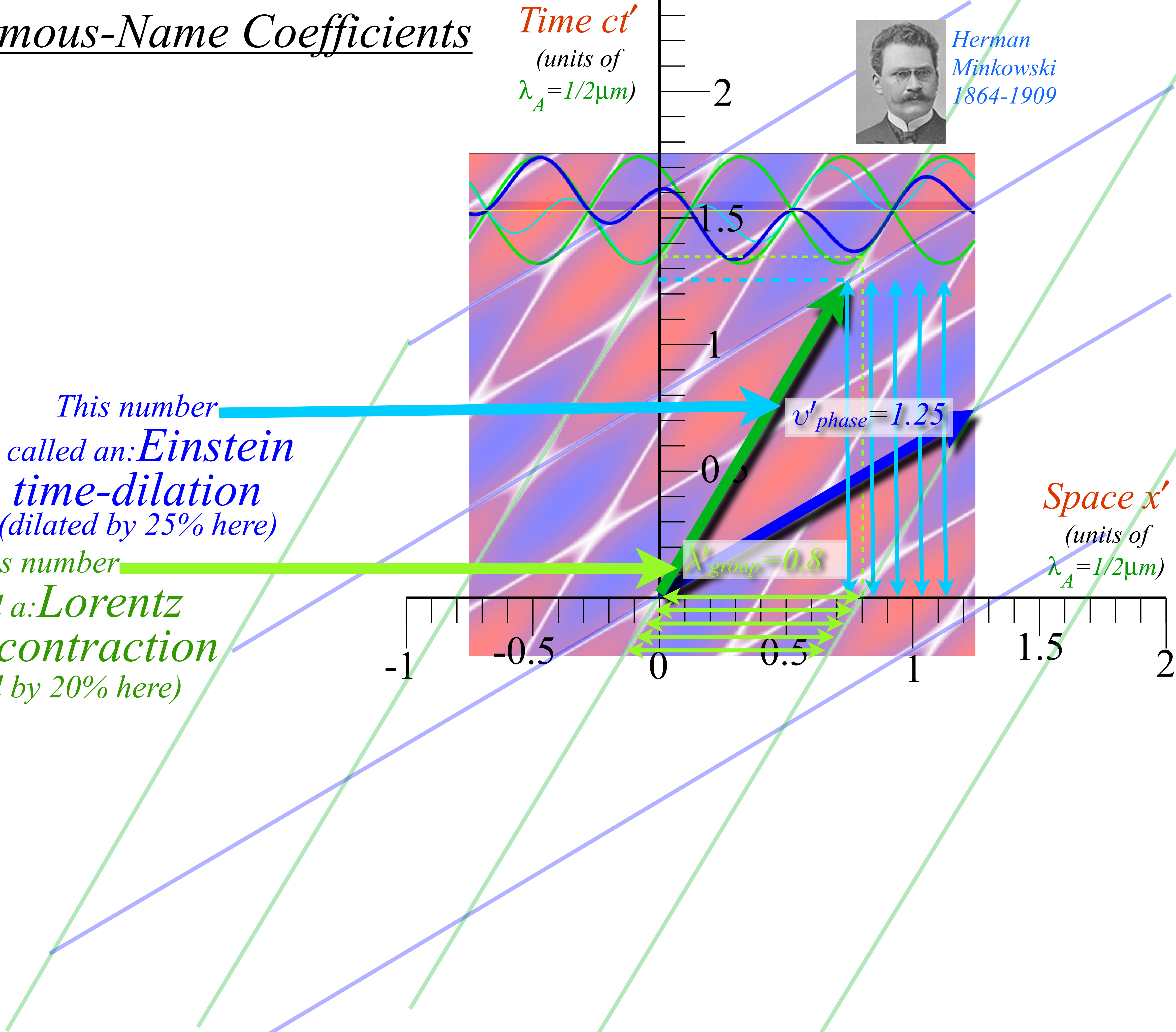
Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

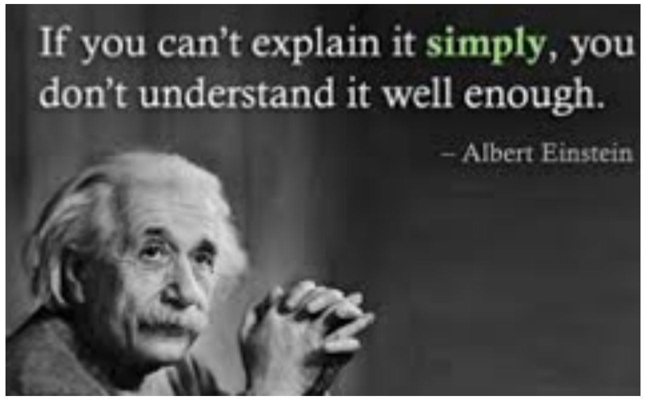
Two Famous-Name Coefficients

This number is called an: Einstein time-dilation (dilated by 25% here)

This number is called a: Lorentz length-contraction (contracted by 20% here)

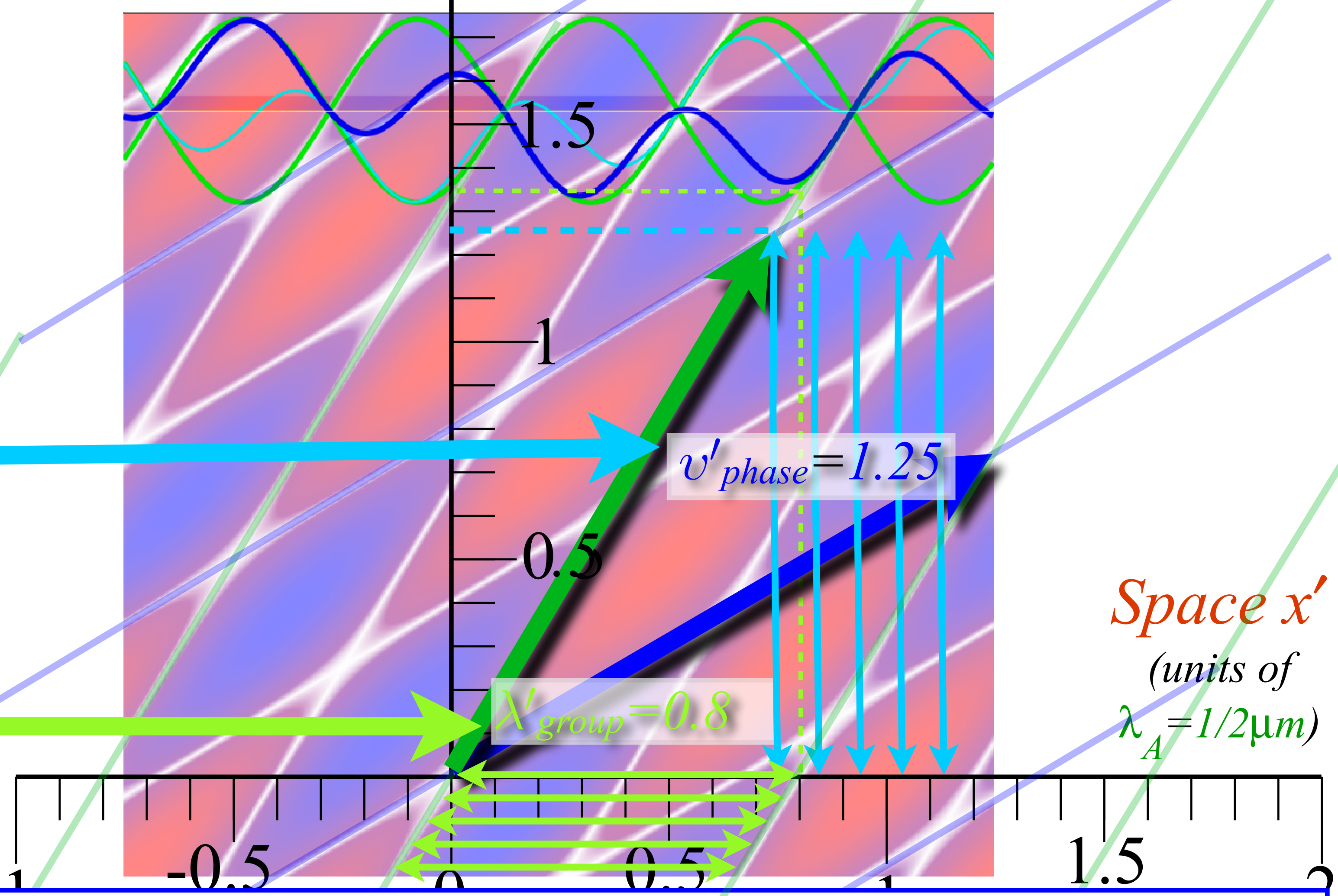


Two Famous-Name Coefficients



Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)

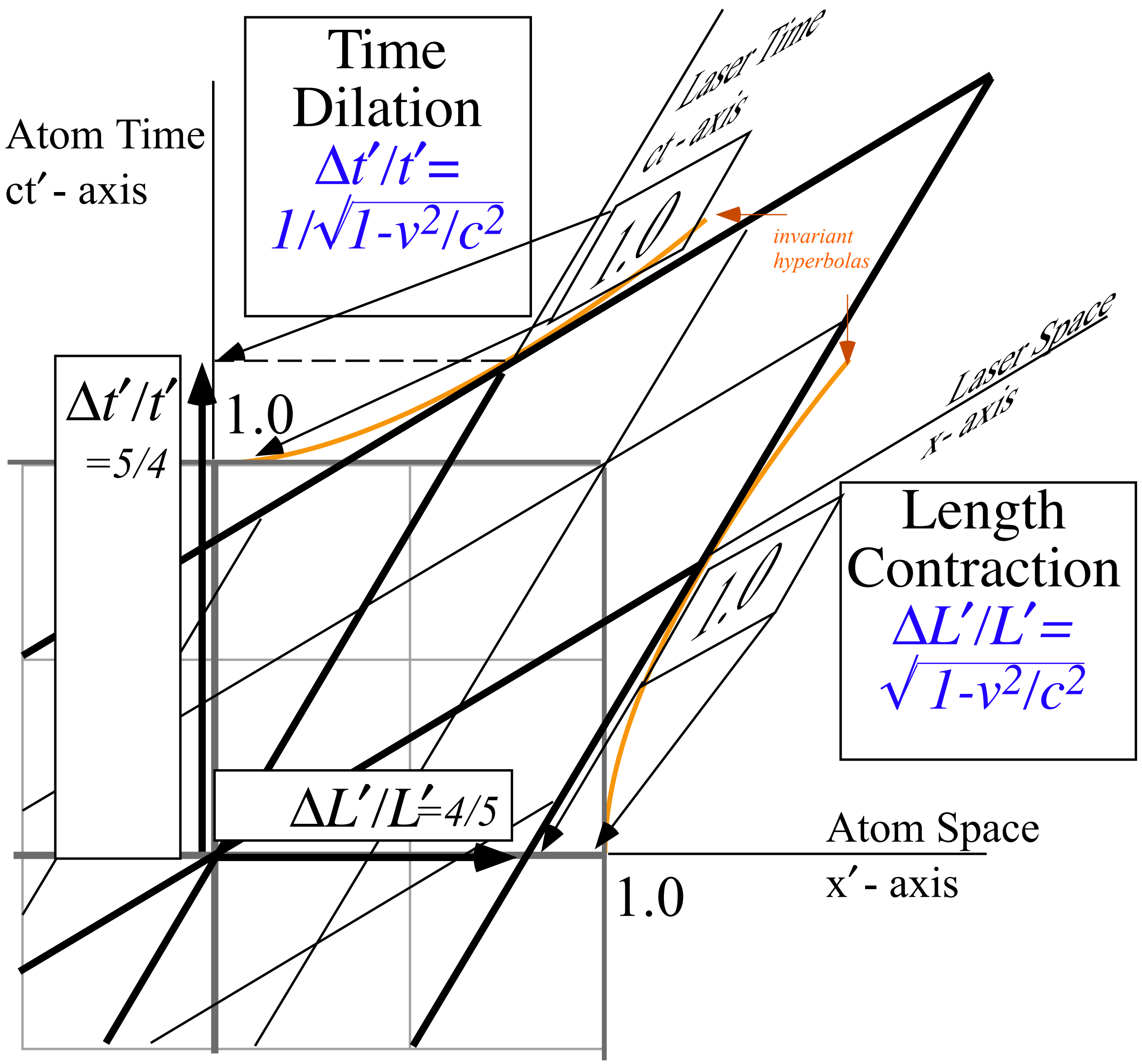
Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation
Relativistic Terms (Expanded Table)

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$



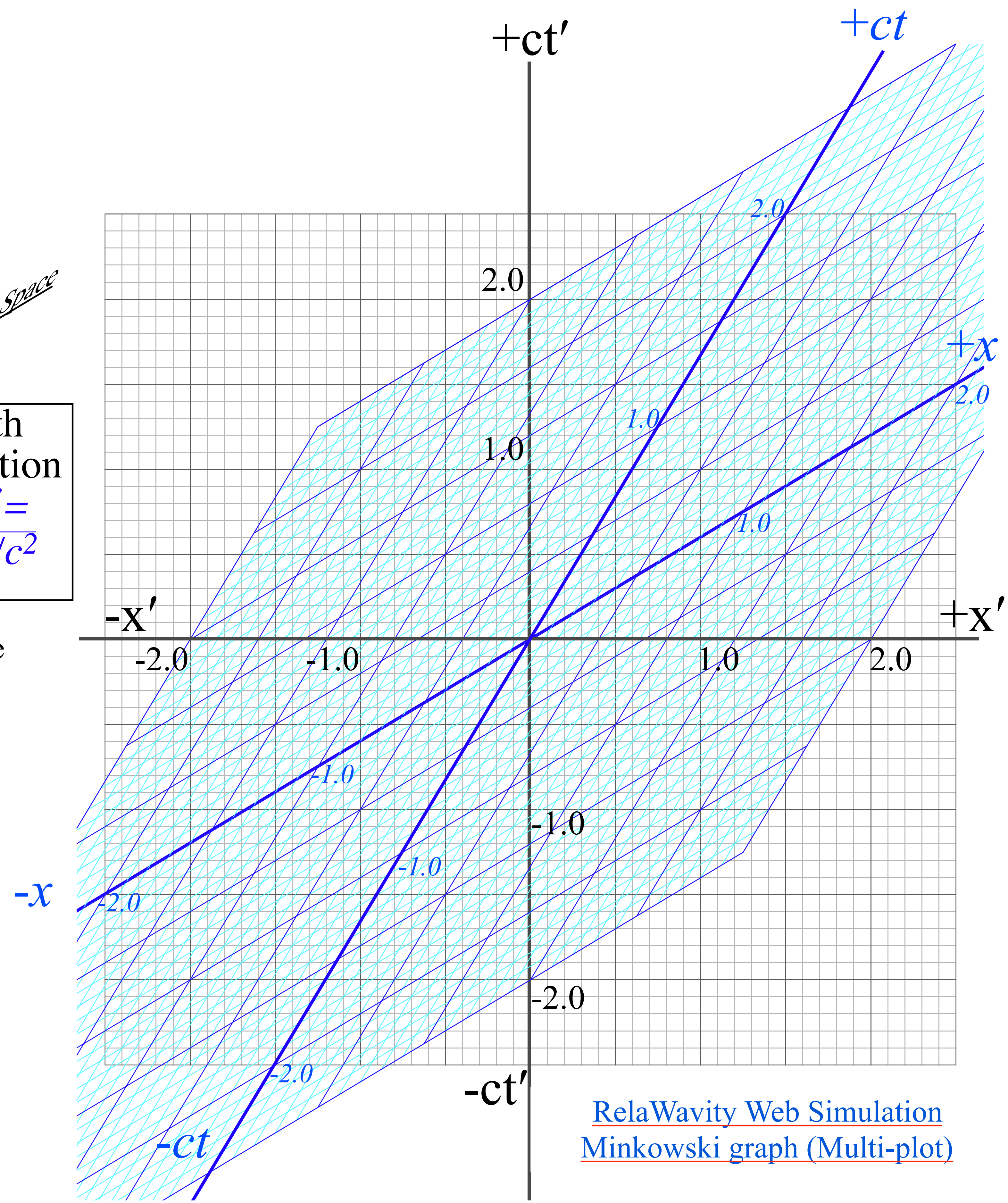


Fig. 8.3.6
CMwBang! Unit 8

[RelaWavity Web Simulation
Minkowski graph (Multi-plot)]

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$

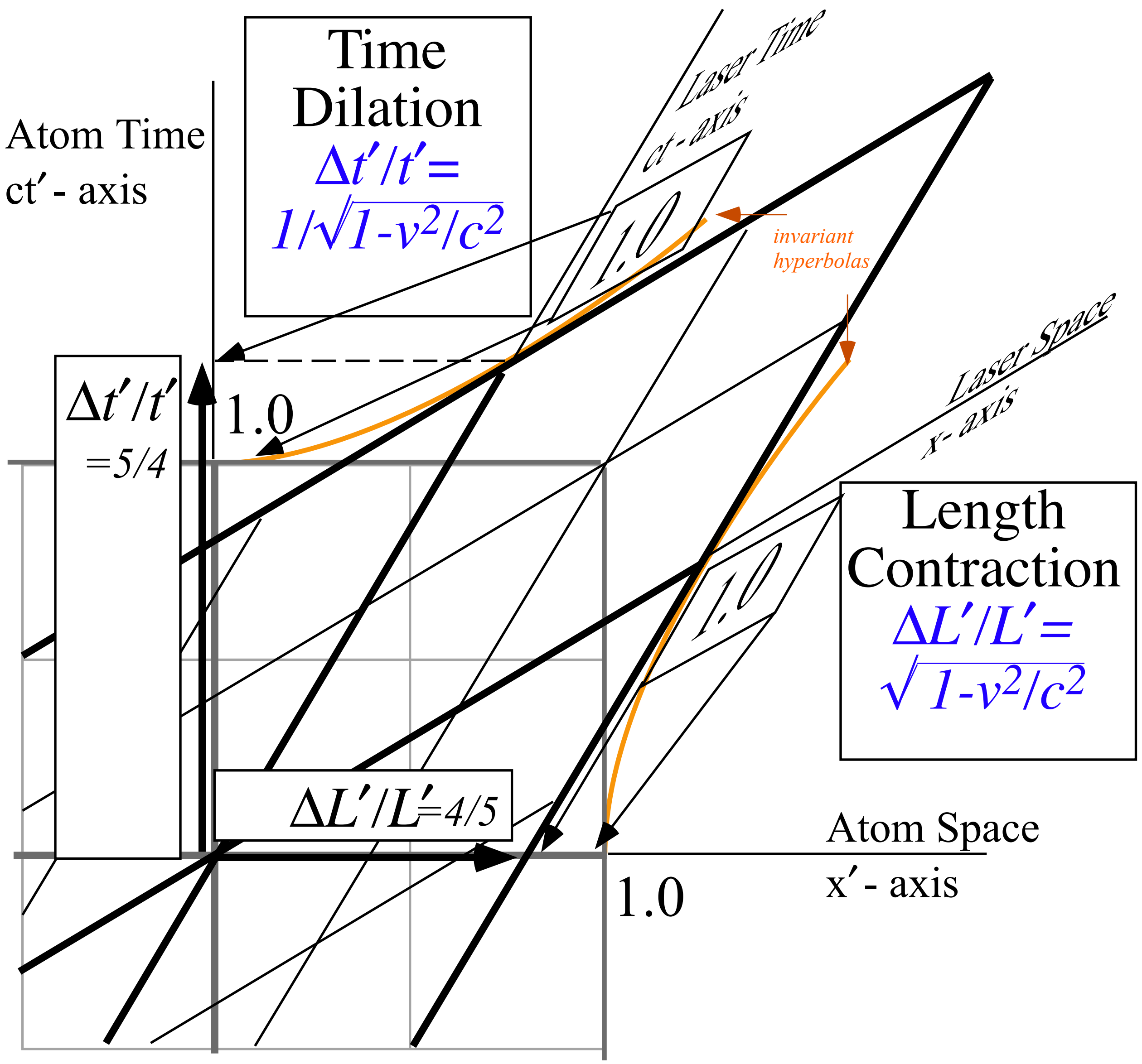
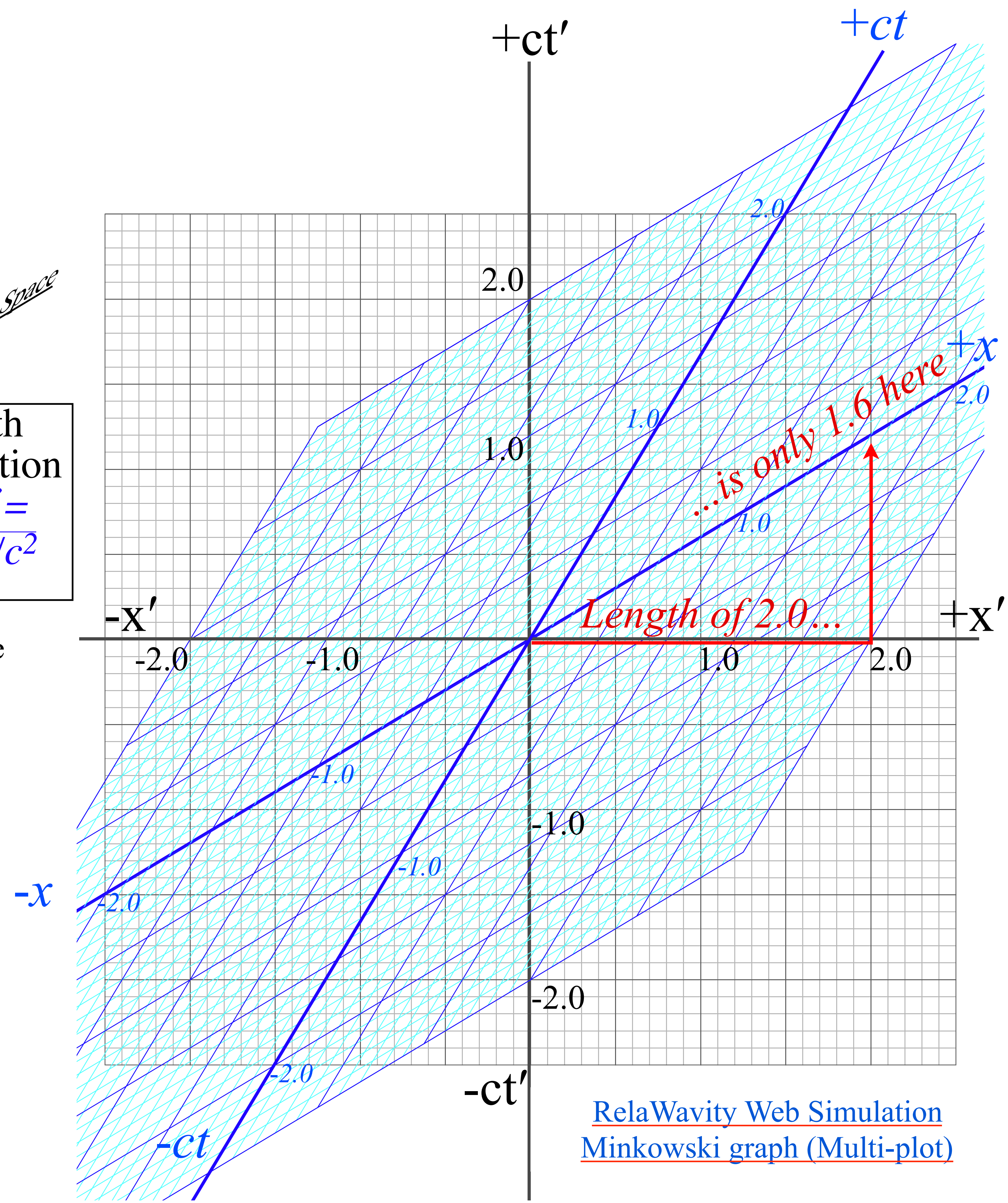
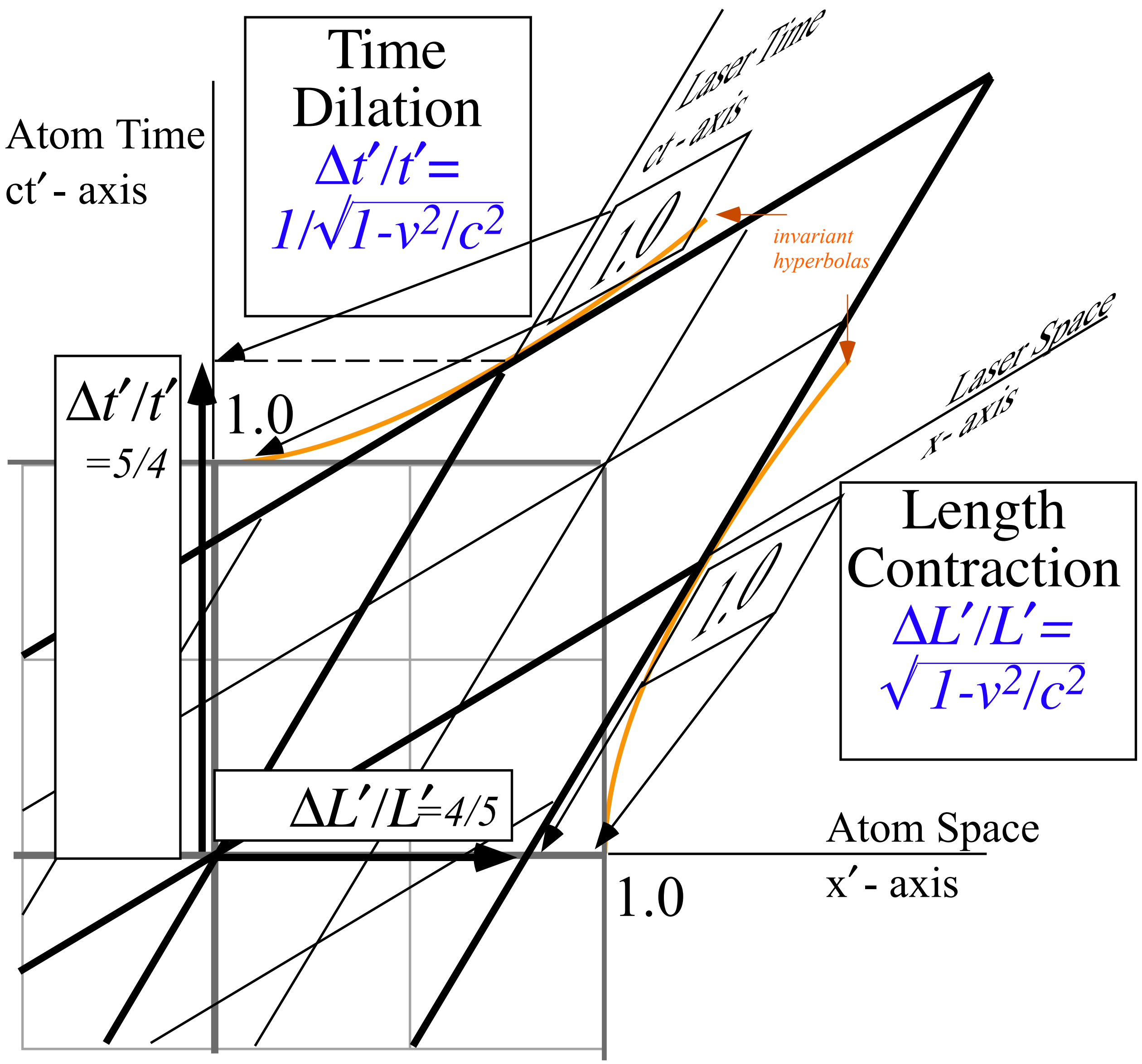


Fig. 8.3.6
CMwBang! Unit 8



RelaWavity Web Simulation
Minkowski graph (Multi-plot)

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$



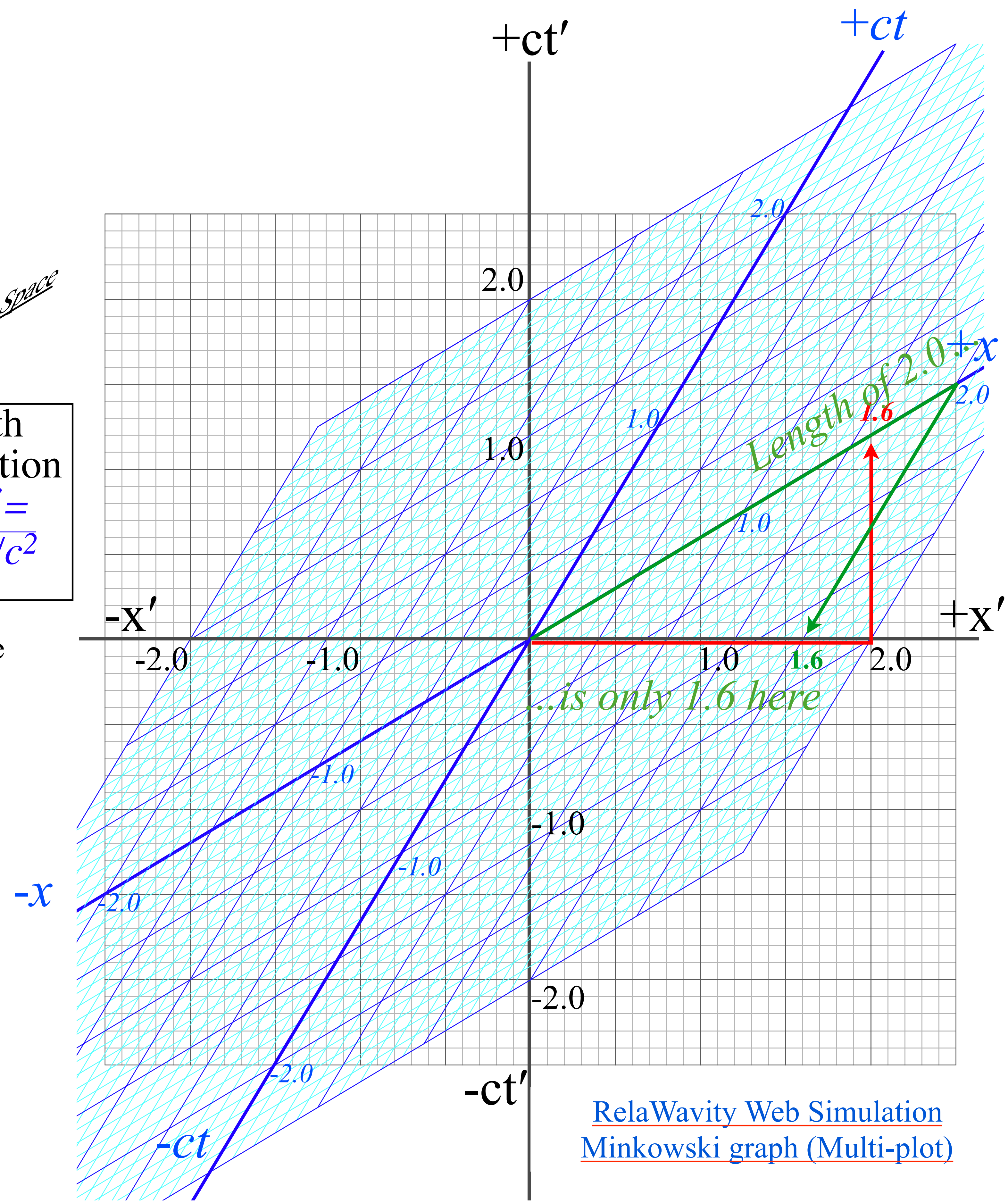
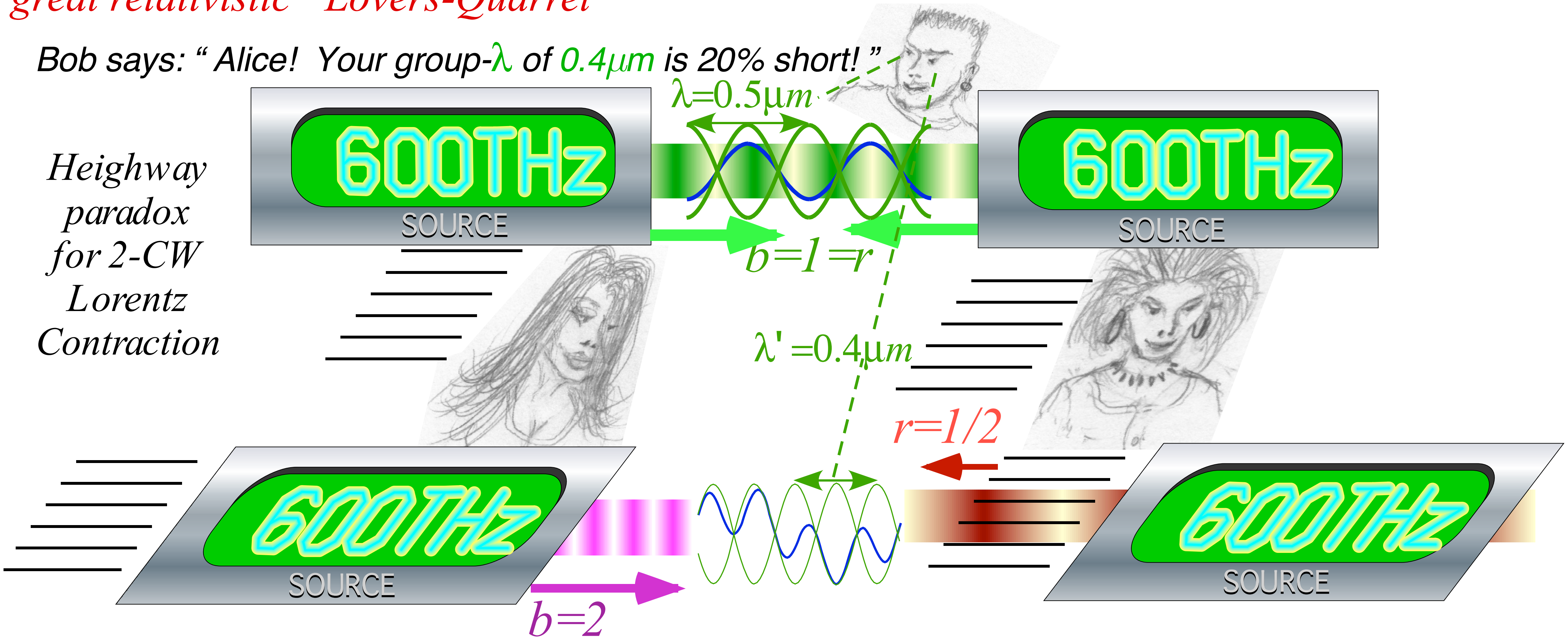


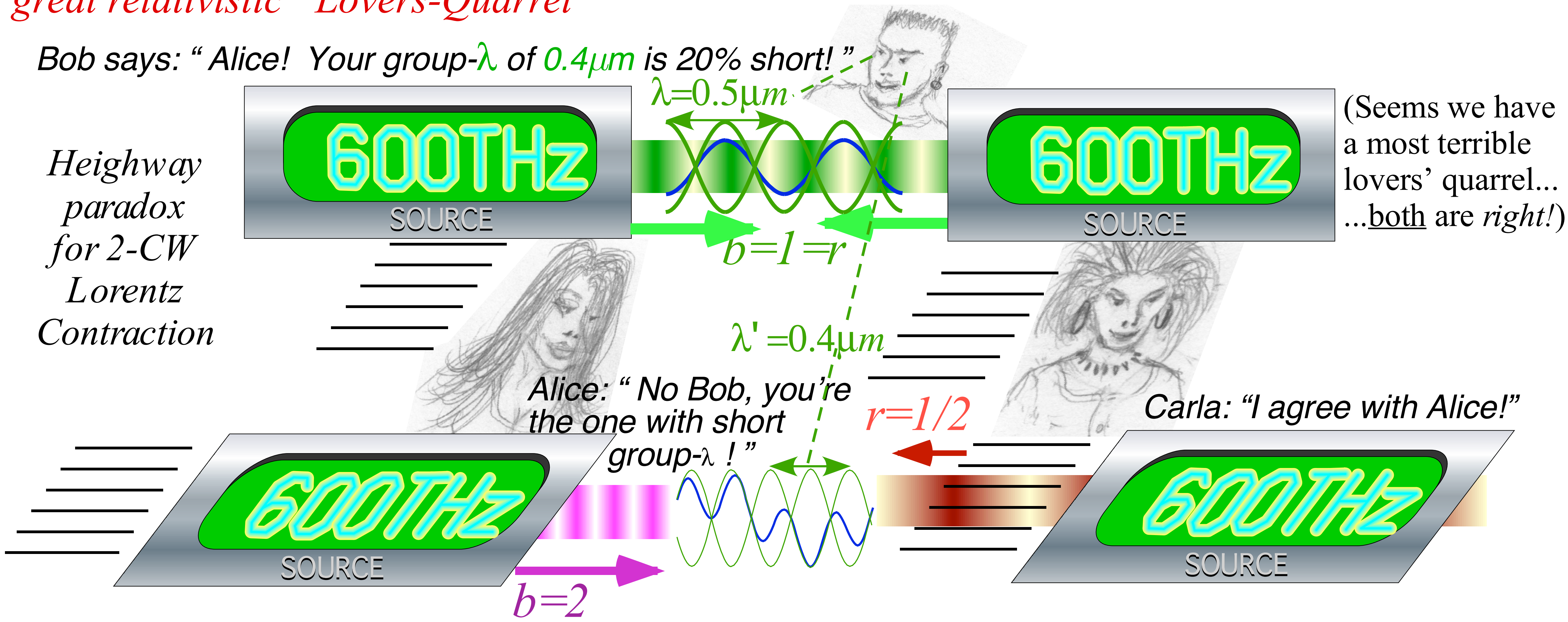
Fig. 8.3.6
CMwBang! Unit 8

A great relativistic "Lovers-Quarrel"

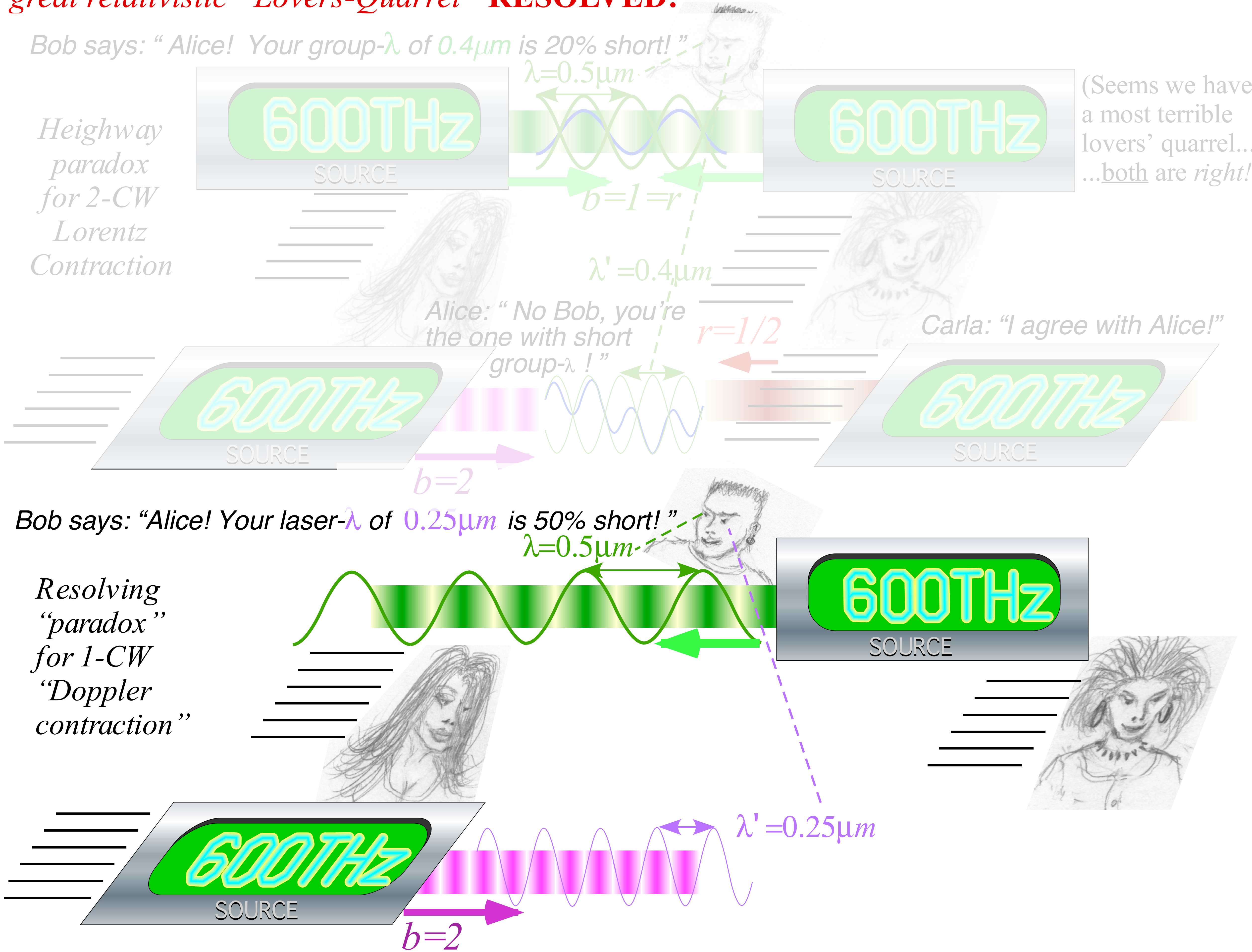
Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "



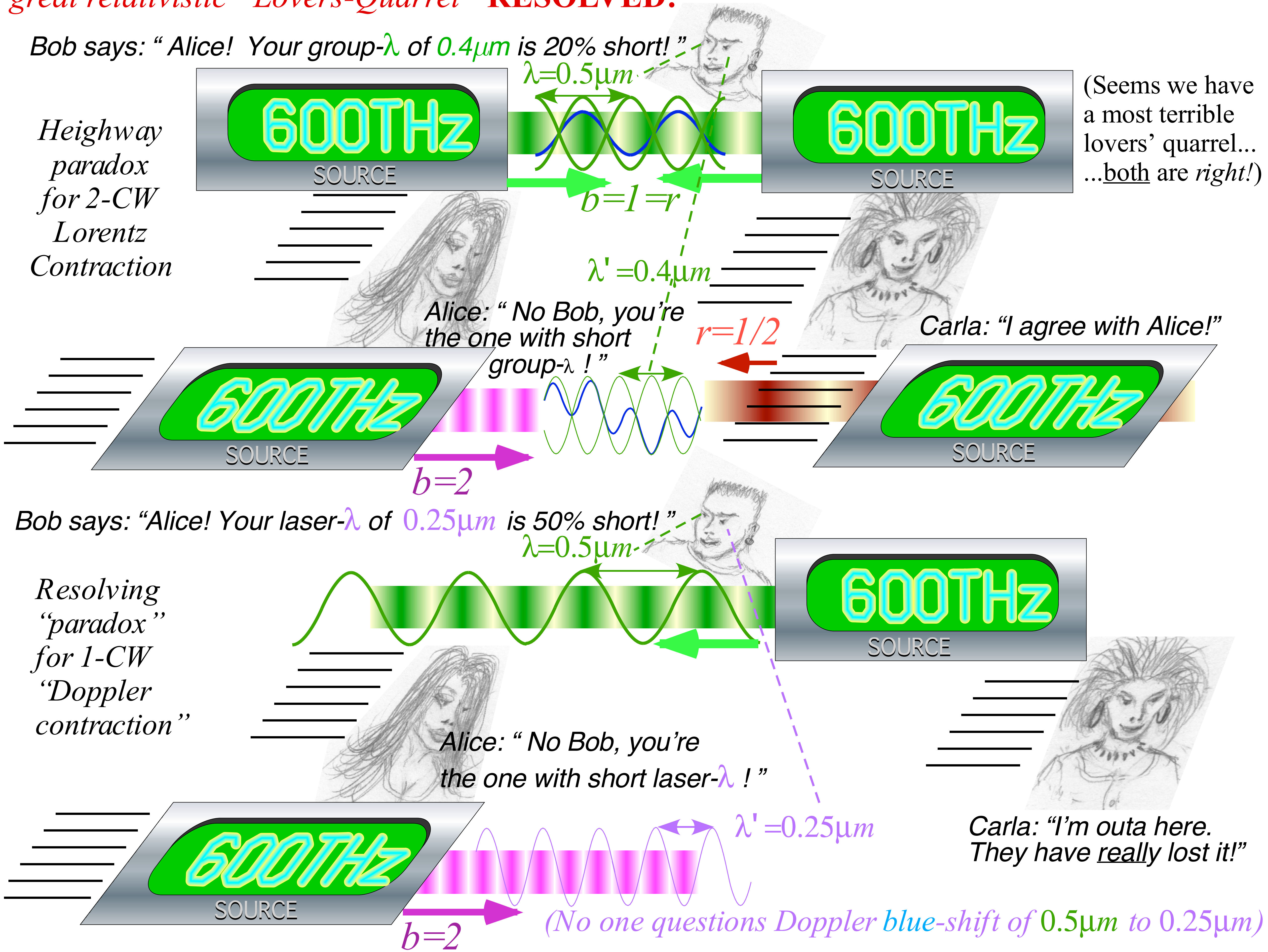
A great relativistic "Lovers-Quarrel"



A great relativistic "Lovers-Quarrel" **RESOLVED!**

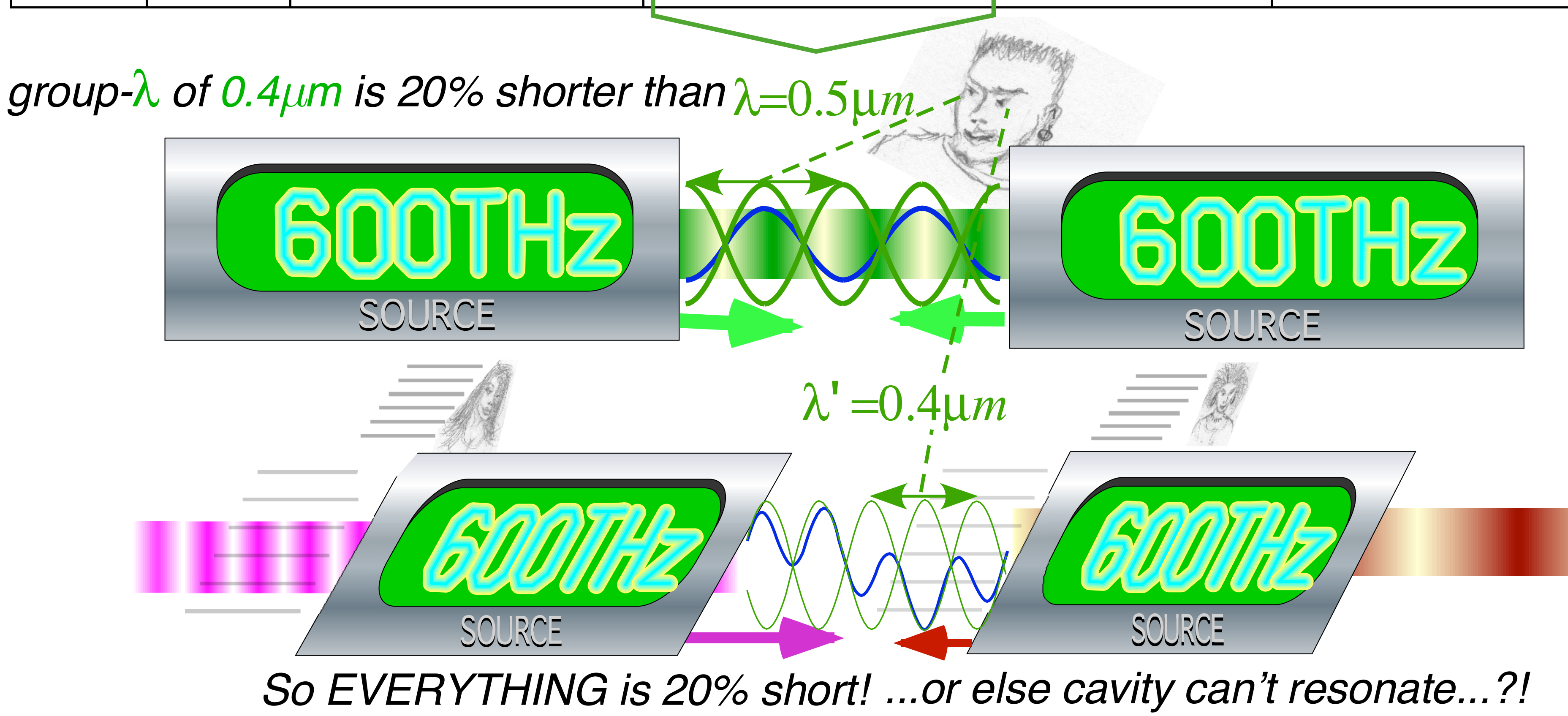


A great relativistic "Lovers-Quarrel" **RESOLVED!**



Lorentz contraction is a quantum matter-wave effect

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}-*contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}-*dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$



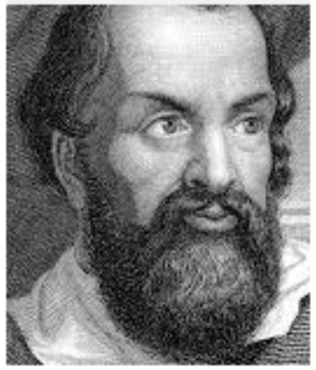
More generally:

Quantum mechanics is a relativistic effect,

and

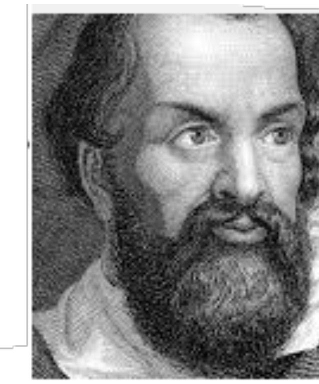
Relativity is a quantum mechanical effect.

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
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Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Doppler Jeopardy

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(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\,\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

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$\frac{300}{900}$

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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

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$$\sqrt{6\cdot 3} = 3\sqrt{2} = 4.24$$

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

Geometric mean

Doppler Jeopardy

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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

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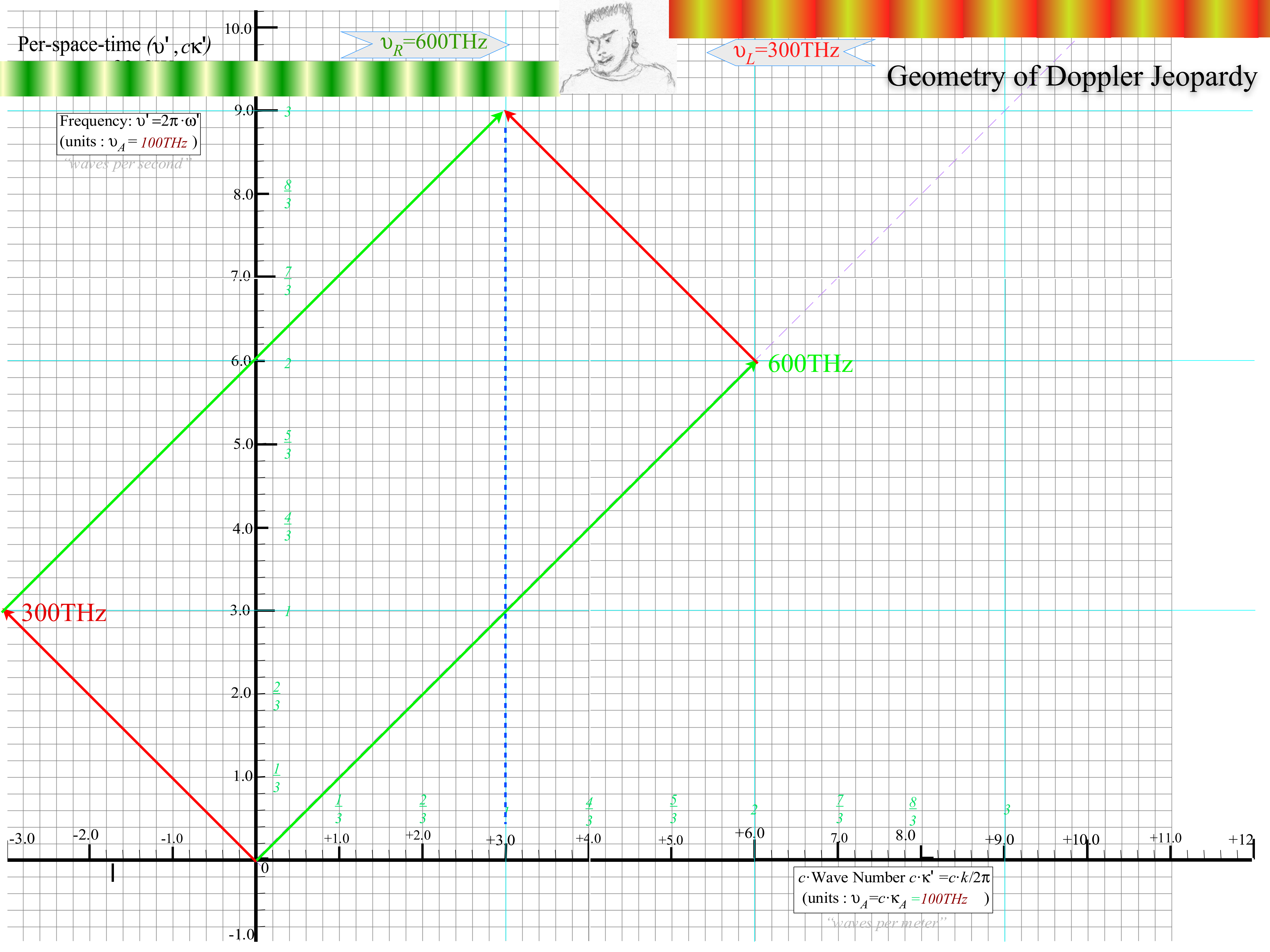
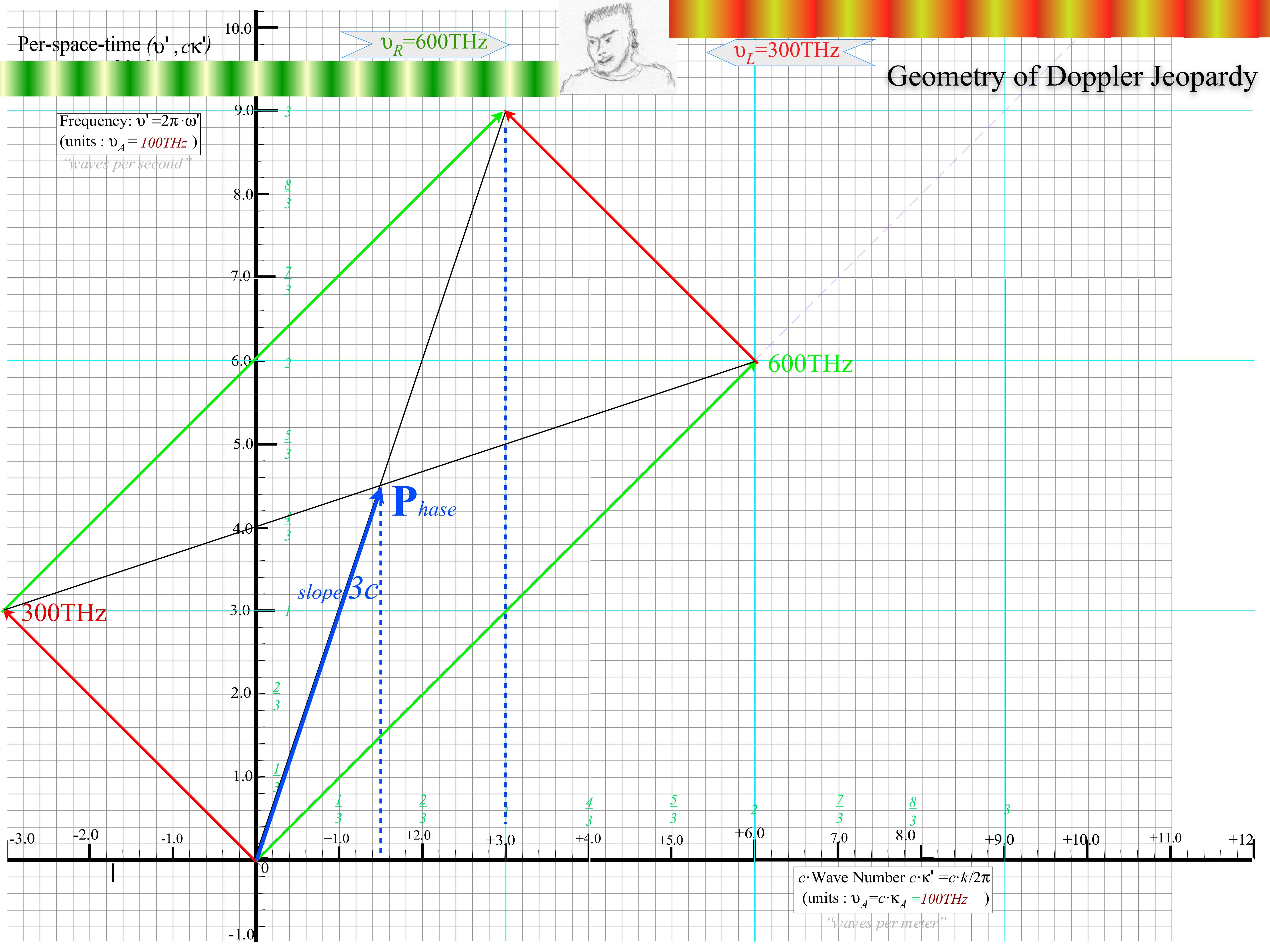
$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

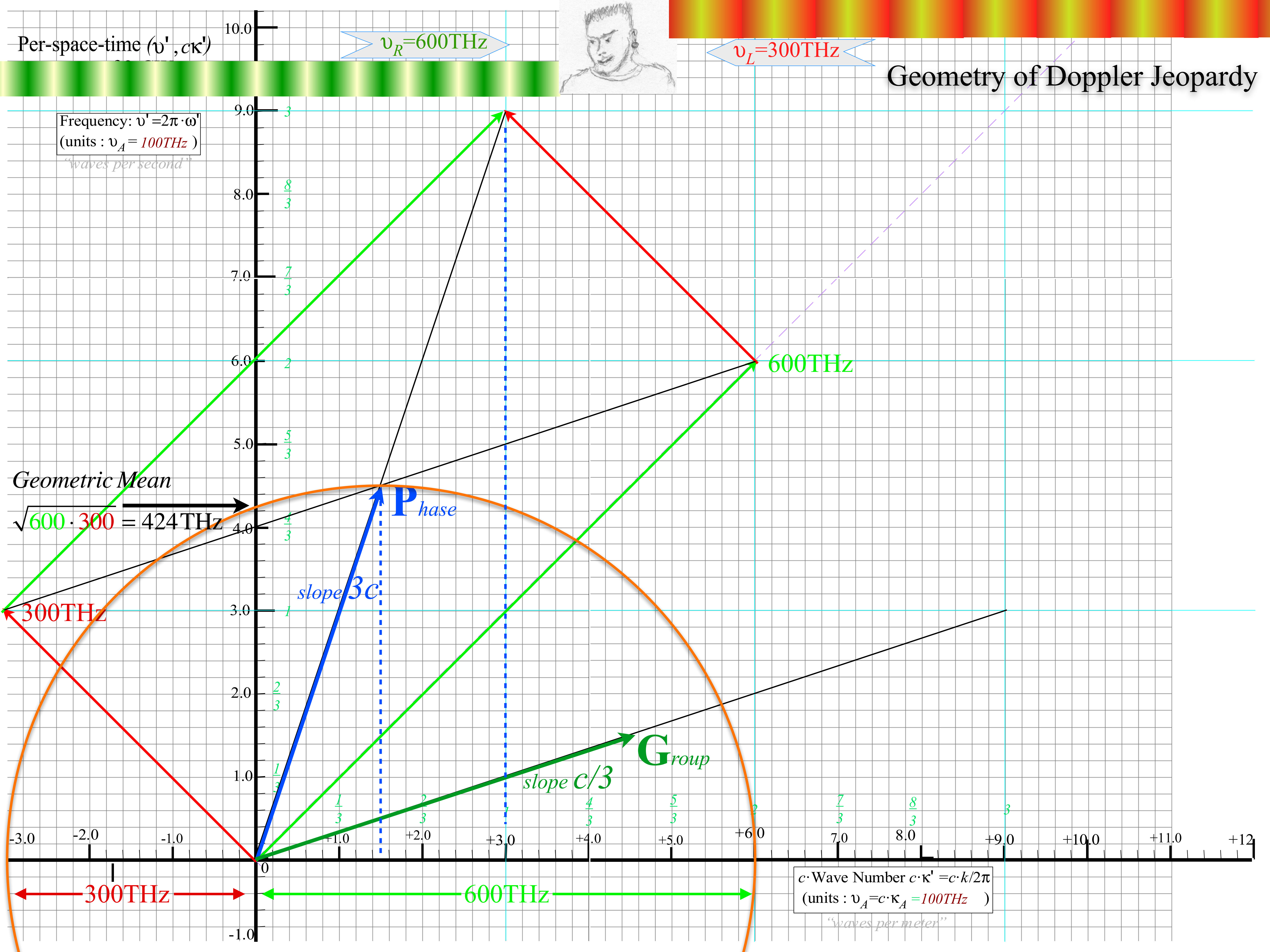
Geometric mean

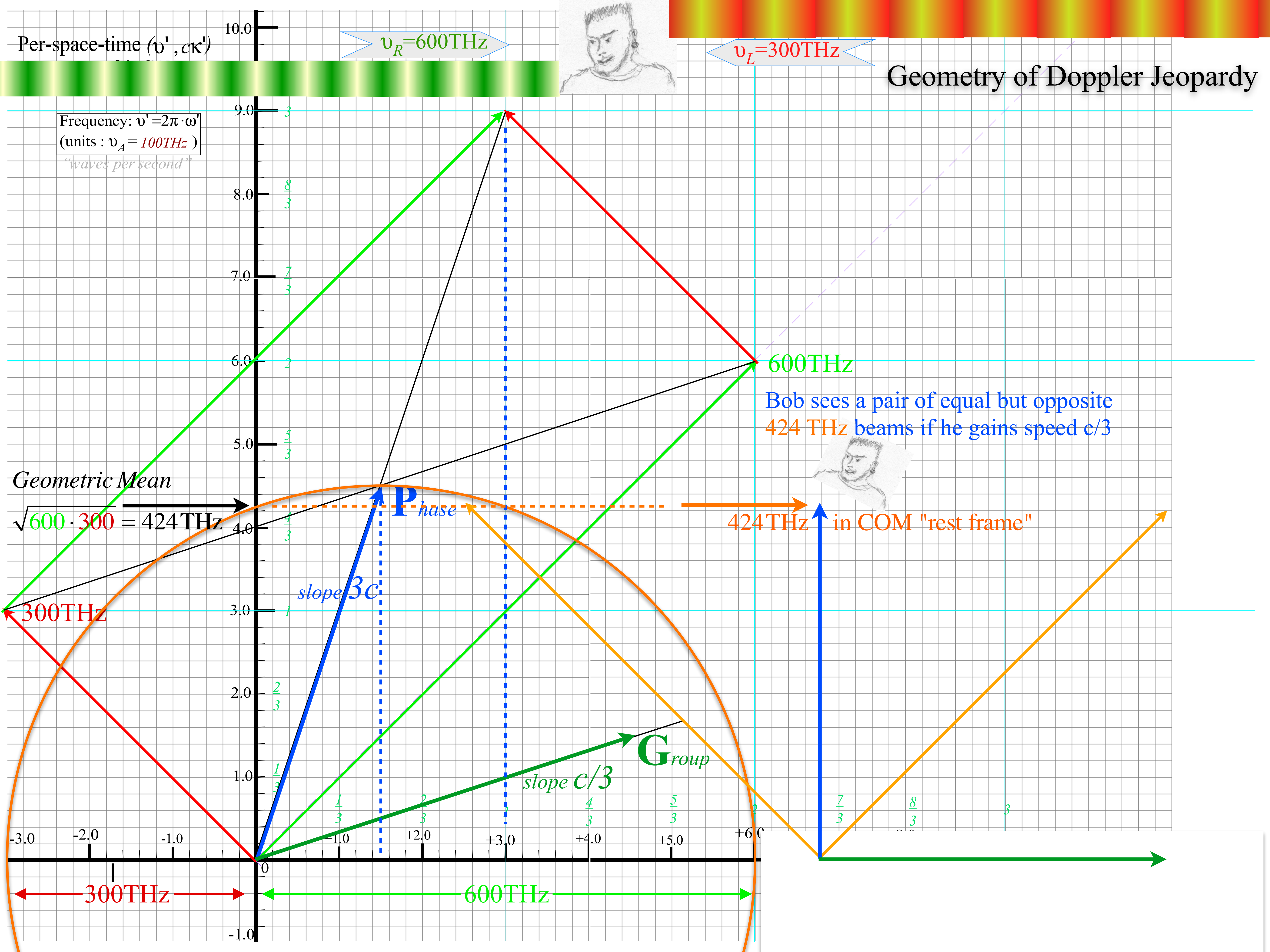

Per-space-time (υ',cκ')
υR=600THz
υL=300THz
Geometry of Doppler Jeopardy
Frequency: υ'=2π·ω'
(units : υA = 100THz)
"waves per second"
600THz
300THz
c·Wave Number c·κ' =c·k/2π
(units : υA=c·κA =100THz)
"waves per meter"



Per-space-time (υ',cκ')
Geometry of Doppler Jeopardy
υR=600THz
υL=300THz
Frequency: υ'=2π·ω'
(units : υA= 100THz)
"waves per second"
600THz
300THz
Phase
slope 3c
c·Wave Number c·κ' =c·k/2π
(units : υA=c·κA =100THz)
"waves per meter"
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
-1.0
-3.0
-2.0
-1.0
0
+1.0
+2.0
+3.0
+4.0
+5.0
+6.0
7.0
8.0
+9.0
+10.0
+11.0
+12

Geometry of Doppler Jeopardy





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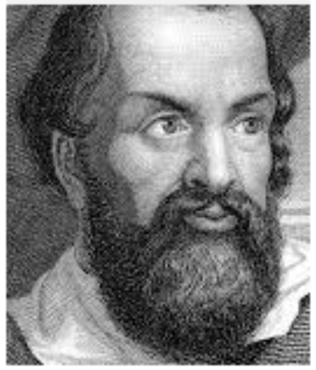
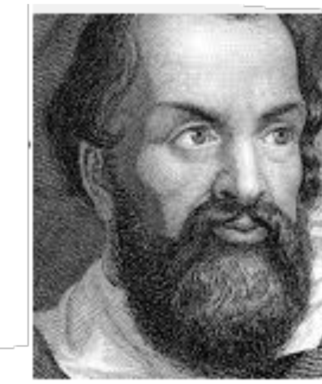
υR=600THz
υL=300THz
Frequency: υ'=2π·ω'
(units : υA= 100THz)
"waves per second"
600THz
Bob sees a pair of equal but opposite
424 THz beams if he gains speed c/3
Geometric Mean
√600·300 = 424THz
Phase
424THz
in COM "rest frame"
slope 3c
300THz
Group
slope c/3
300THz
600THz

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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Galilean velocity

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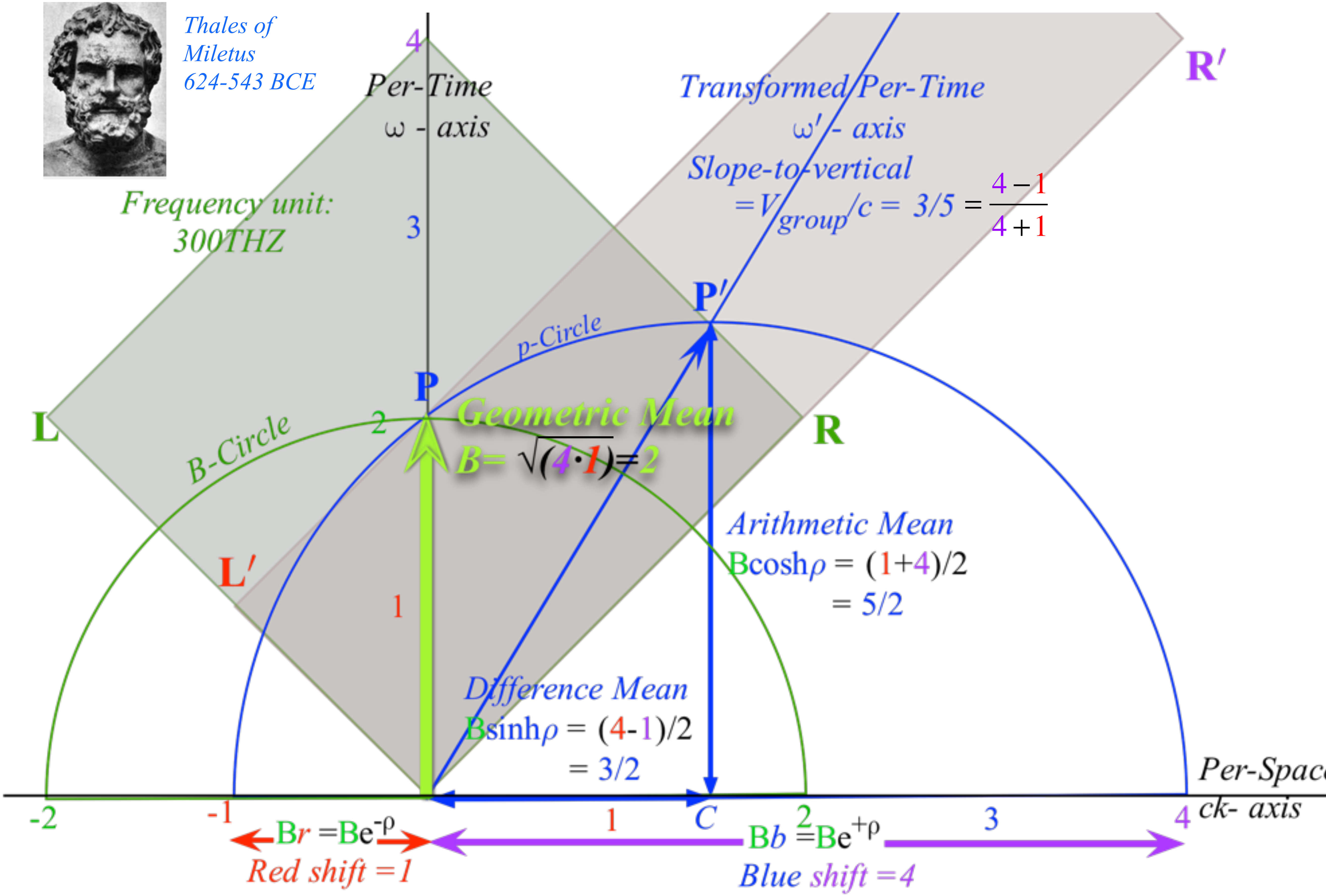
Thales Mean Geometry (600BCE)

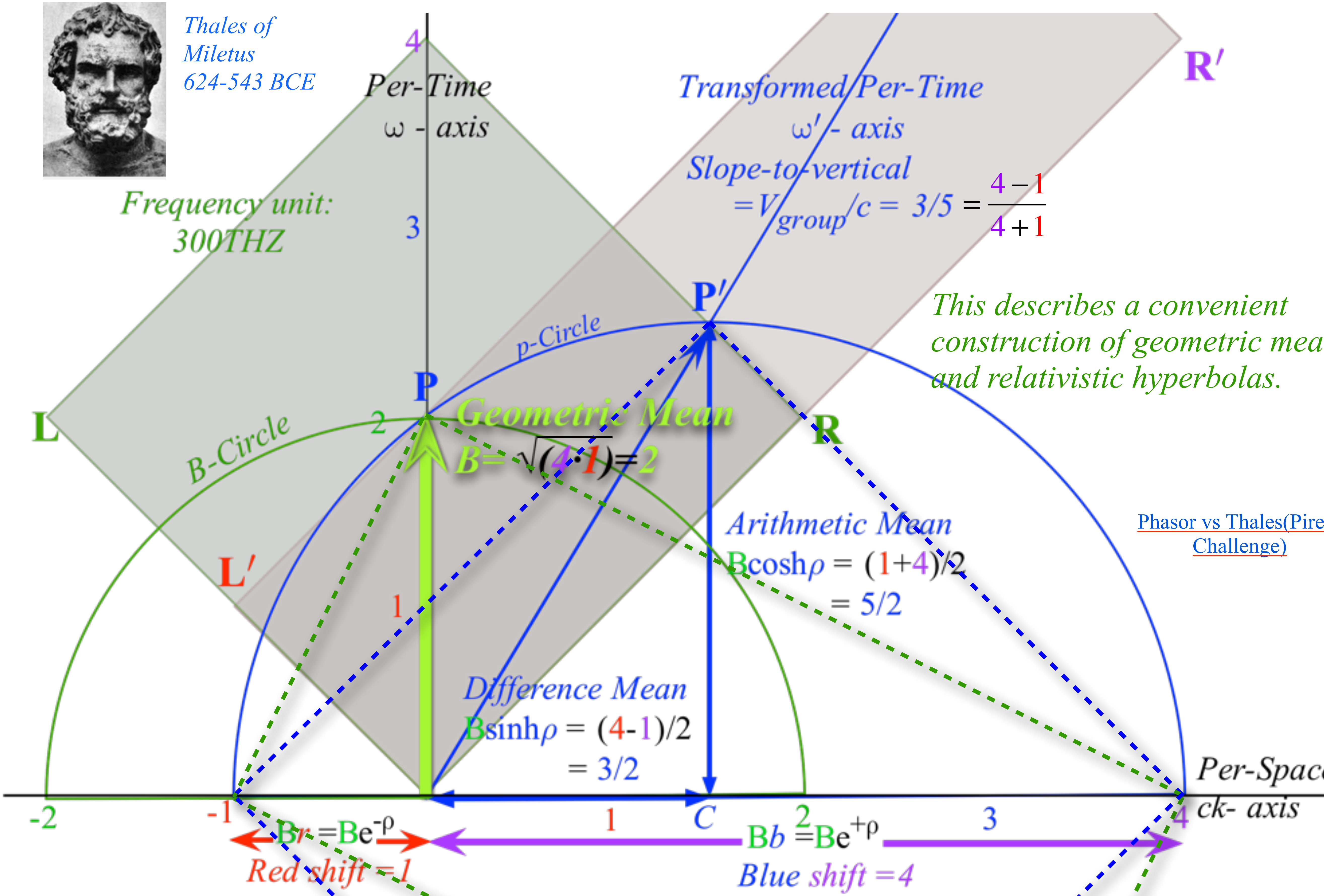
helps "Relawavity"



Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*

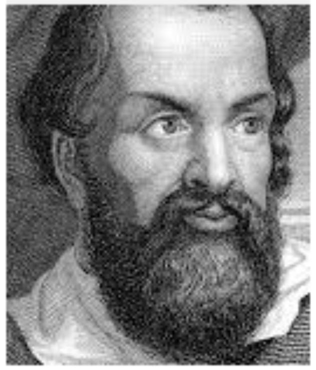
Thales of Miletus 624-543 BCE



This describes a convenient construction of geometric means and relativistic hyperbolas.

Phasor vs Thales(Pirelli Challenge)

Galileo Galilei



1564-1642

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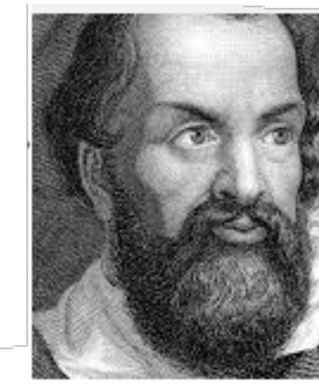

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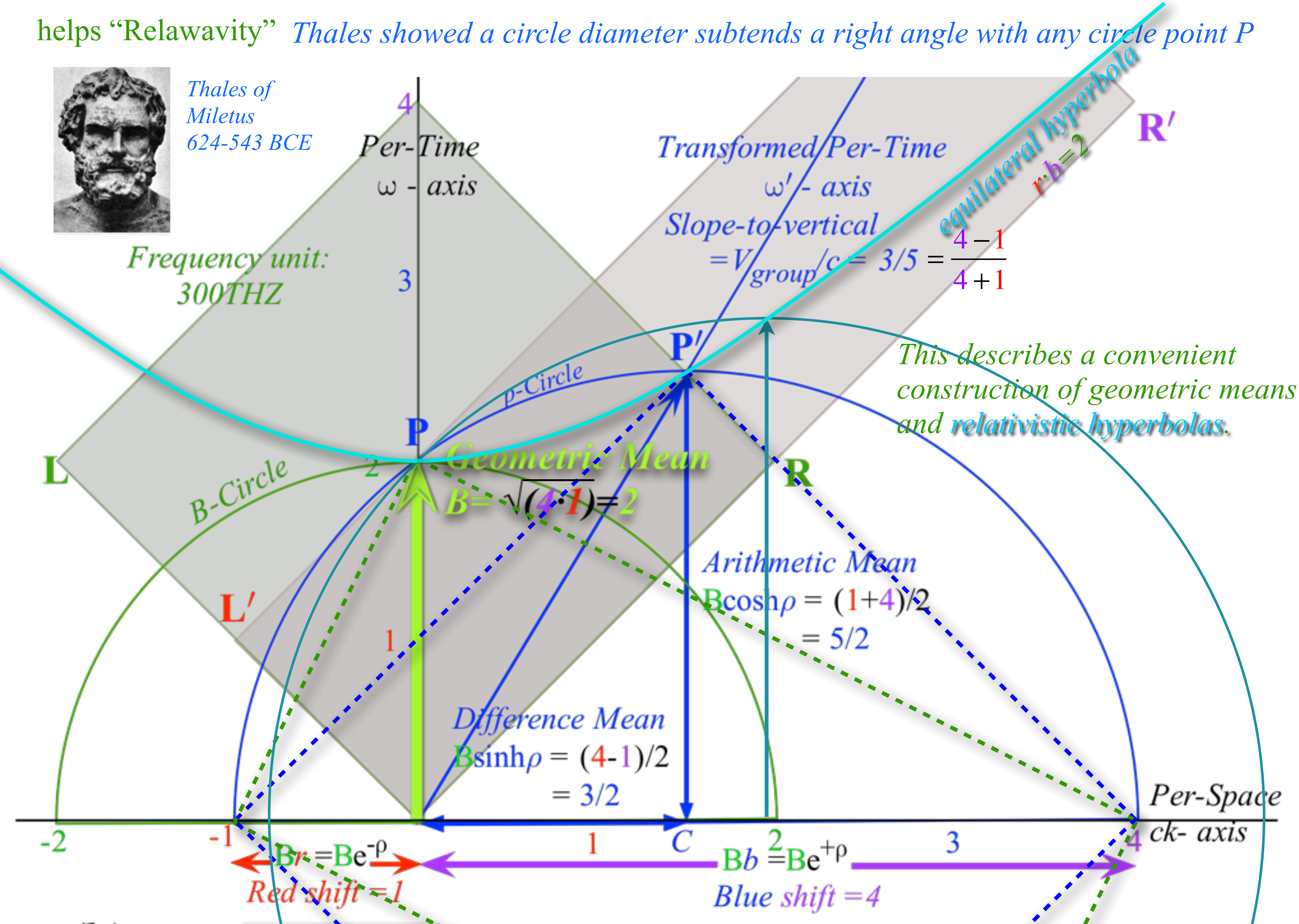

This describes a convenient construction of geometric means and *relativistic hyperbolas*.

Thales Mean Geometry (600BCE)

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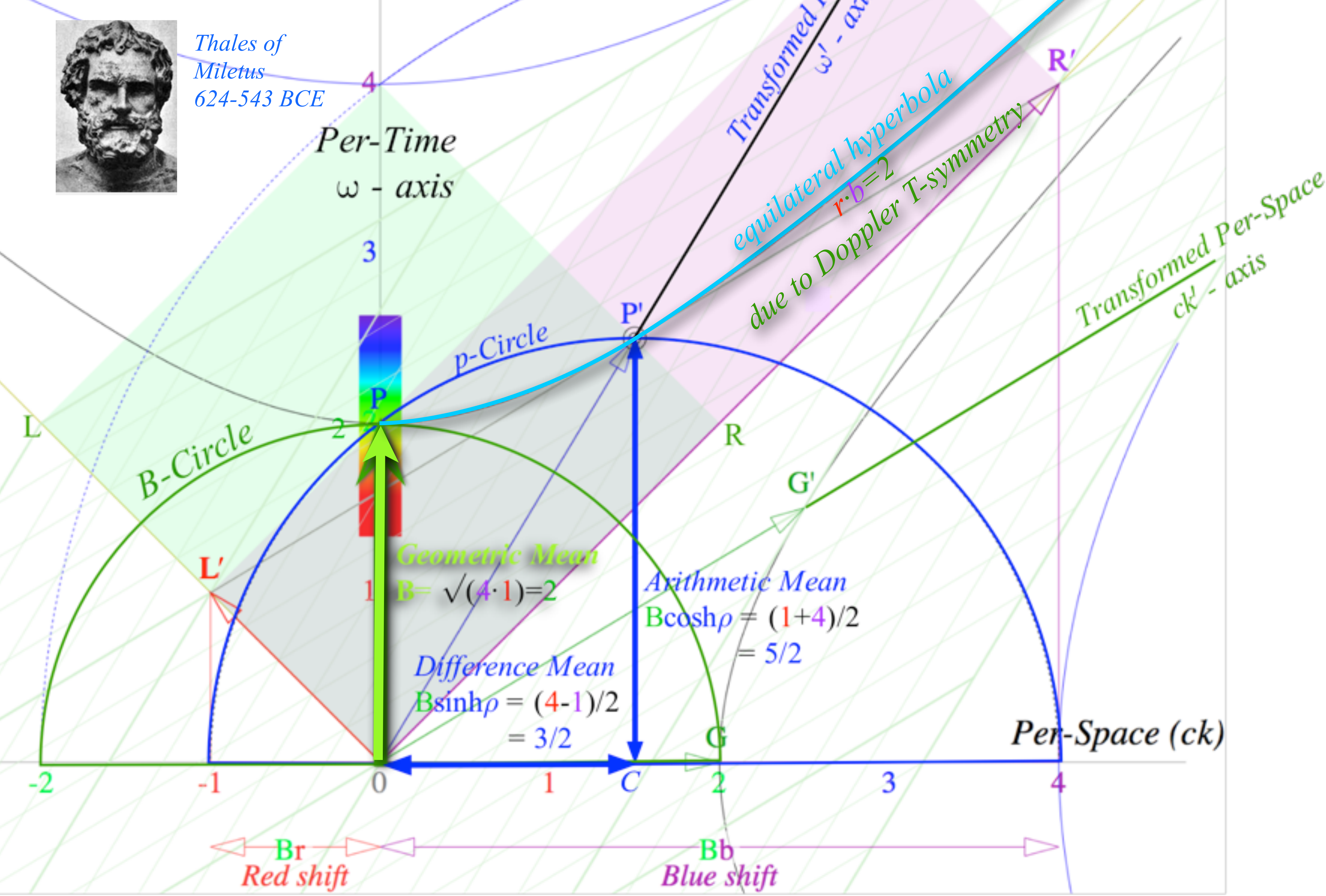
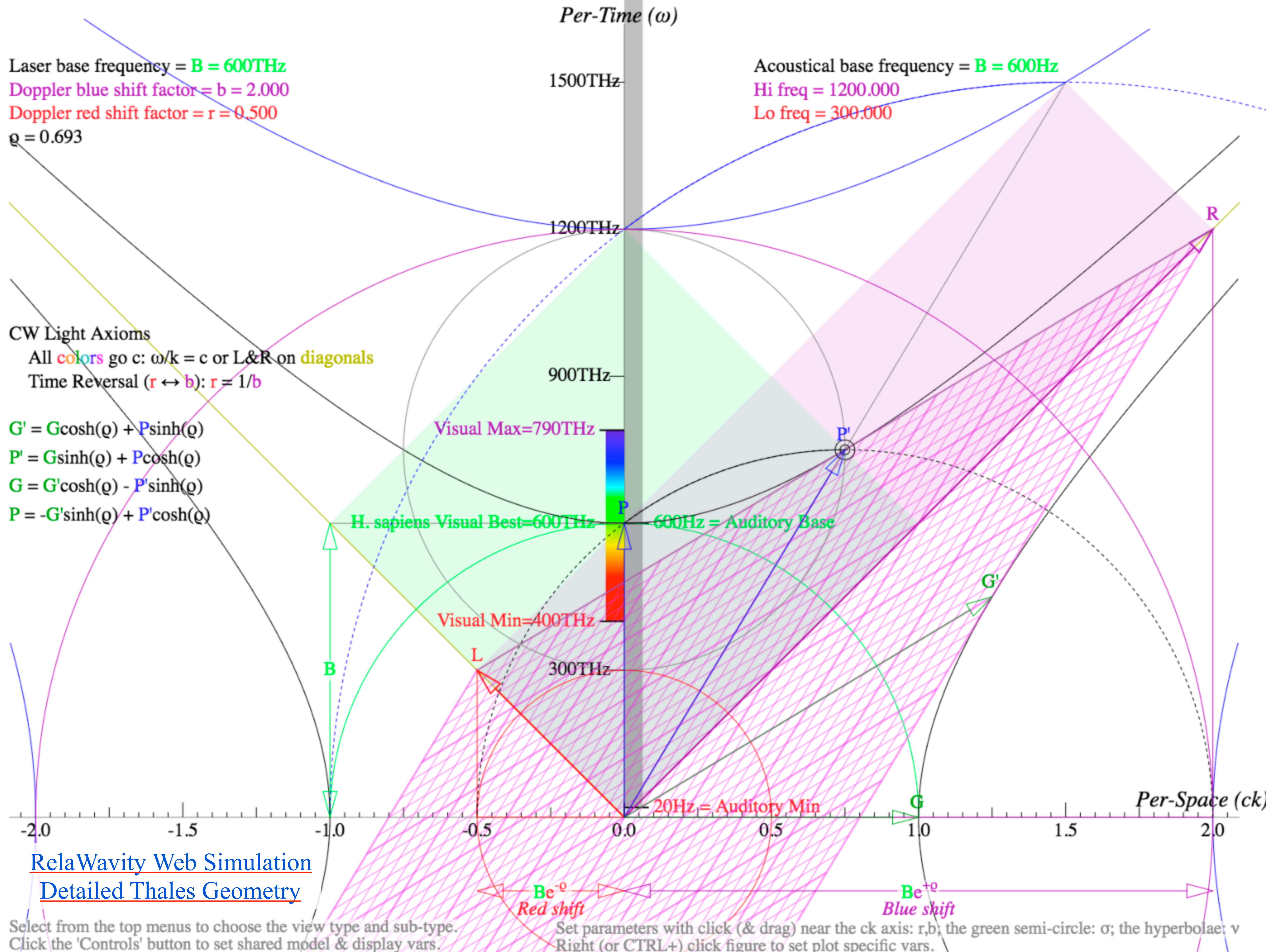



Per-Time (ω)
Laser base frequency = B = 600THz
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ϱ = 0.693
Acoustical base frequency = B = 600Hz
Hi freq = 1200.000
Lo freq = 300.000
1500THz
1200THz
900THz
300THz
CW Light Axioms
All colors go c: ω/k = c or L&R on diagonals
Time Reversal (r ↔ b): r = 1/b
G' = Gcosh(ϱ) + Psinh(ϱ)
P' = Gsinh(ϱ) + Pcosh(ϱ)
G = G'cosh(ϱ) - P'sinh(ϱ)
P = -G'sinh(ϱ) + P'cosh(ϱ)
Visual Max=790THz
H. sapiens Visual Best=600THz
600Hz = Auditory Base
Visual Min=400THz
20Hz = Auditory Min
R
P'
P
G'
G
L
B
Per-Space (ck)
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
RelaWavity Web Simulation
Detailed Thales Geometry
Be^-ϱ
Red shift
Be^+ϱ
Blue shift
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: ν
Right (or CTRL+) click figure to set plot specific vars.

A view of a conical intersection: Any vertical cross-section is hyperbolic avoided-crossing

Recall ABD U(2) system in Lect. 23 p.93

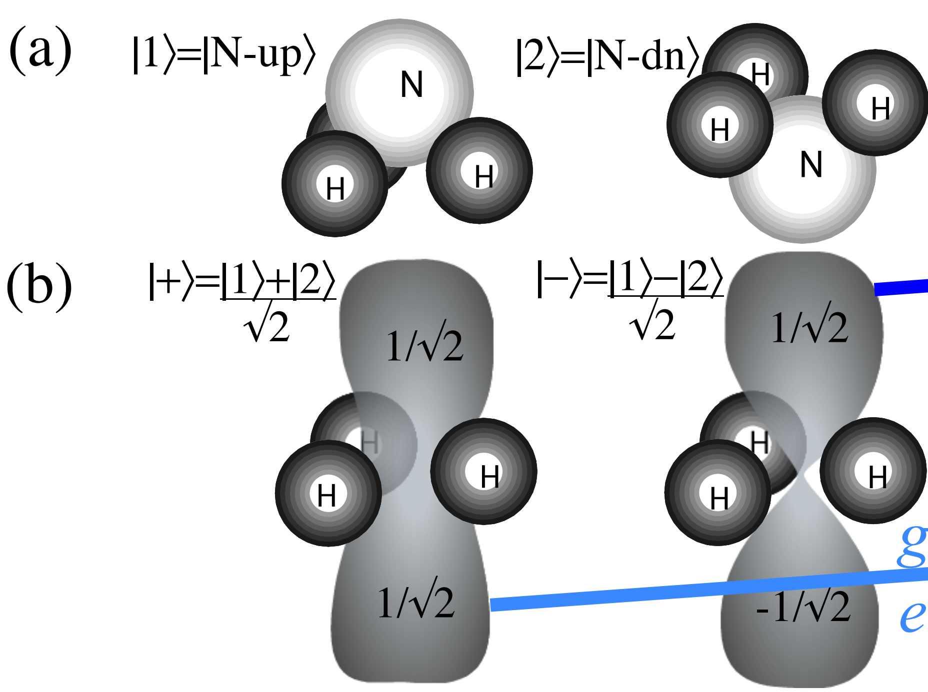
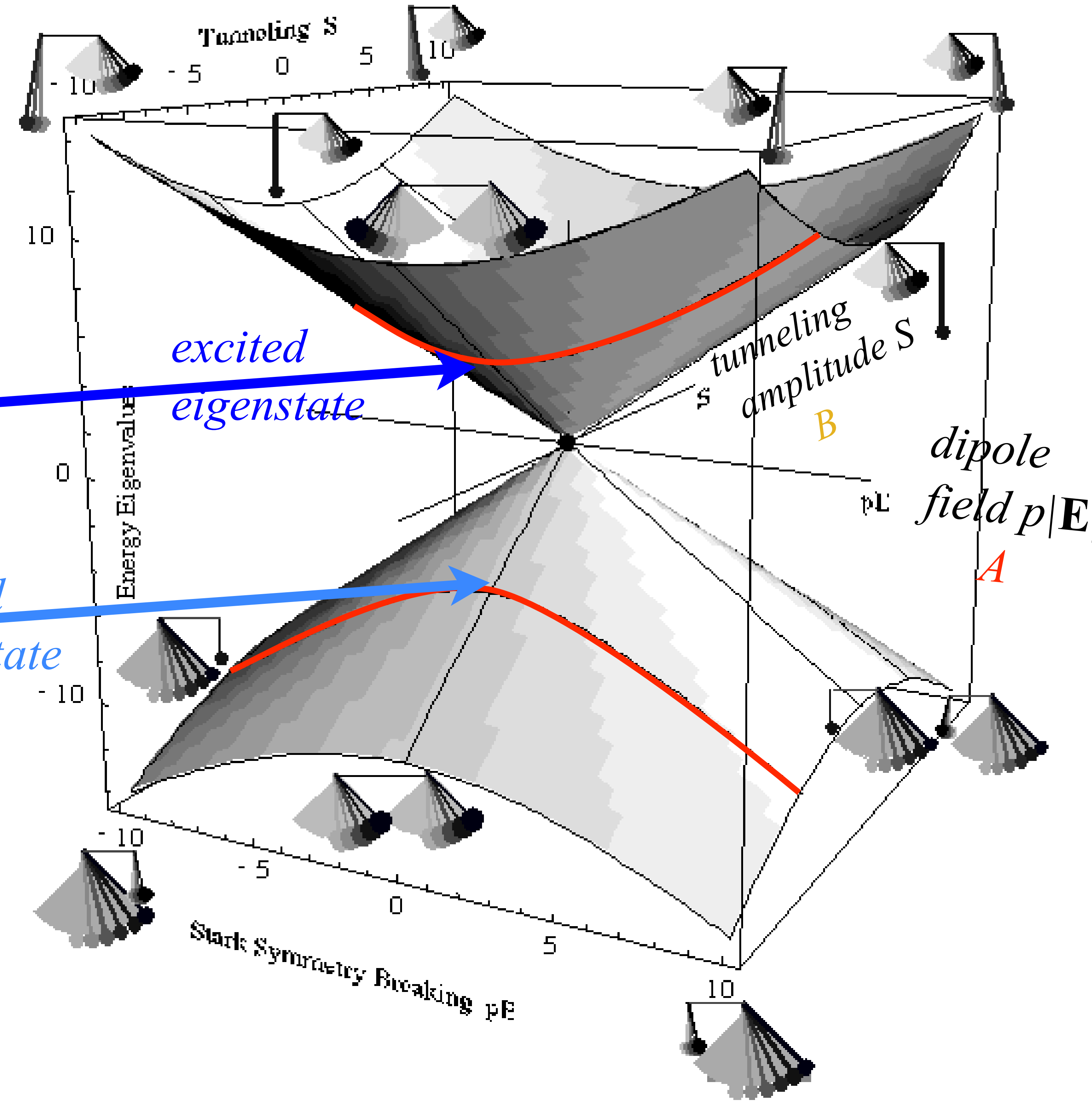


Fig. 10.3.2 Ammonia (NH_3) inversion states
(a) Base states (b) C_2-Eigenstates



10.3.1 (a) Two state eigenvalue "diablo" surfaces and conical intersection and pendulum eigenstates.

(Also known as a “Dirac-point”)

A to *B* to *A* Symmetry breaking described by hyperbolic eigenvalues of $A\boldsymbol{\sigma}_A + B\boldsymbol{\sigma}_B = \mathbf{H} = \begin{pmatrix} +A & B \\ B & -A \end{pmatrix}$

$$\mathbf{H} = \begin{pmatrix} +A & B \\ B & -A \end{pmatrix} \quad \text{Secular equation: } \varepsilon^2 - 0\cdot\varepsilon - (A^2 + B^2) \quad \text{gives } \textit{hyperbolic} \text{ energy levels: } \varepsilon = \pm\sqrt{A^2 + B^2}$$

Recall ABD U(2) system in Lect. 23 p.90

$\mathbf{H}(B-basis)$ $\qquad$ $\mathbf{H}(A-basis)$

$$\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} = \frac{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}{\sqrt{2}} \begin{pmatrix} +A & B \\ B & -A \end{pmatrix} \frac{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}{\sqrt{2}}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} +A & B \\ B & -A \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} +A+B & B-A \\ +A-B & B+A \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2B & 2A \\ 2A & -2B \end{pmatrix}$$

$$= \begin{pmatrix} +B & A \\ A & -B \end{pmatrix}$$

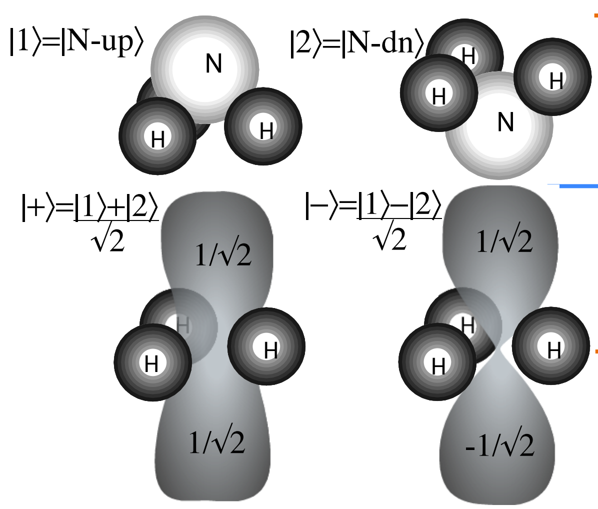


Fig. 10.3.2 Ammonia (NH_3) inversion states
(a) Base states (b) C_2-Eigenstates

Here we display eigenvalues and eigenvectors while holding B constant and varying A. Obviously it can be done vice-versa and with varying C, too.

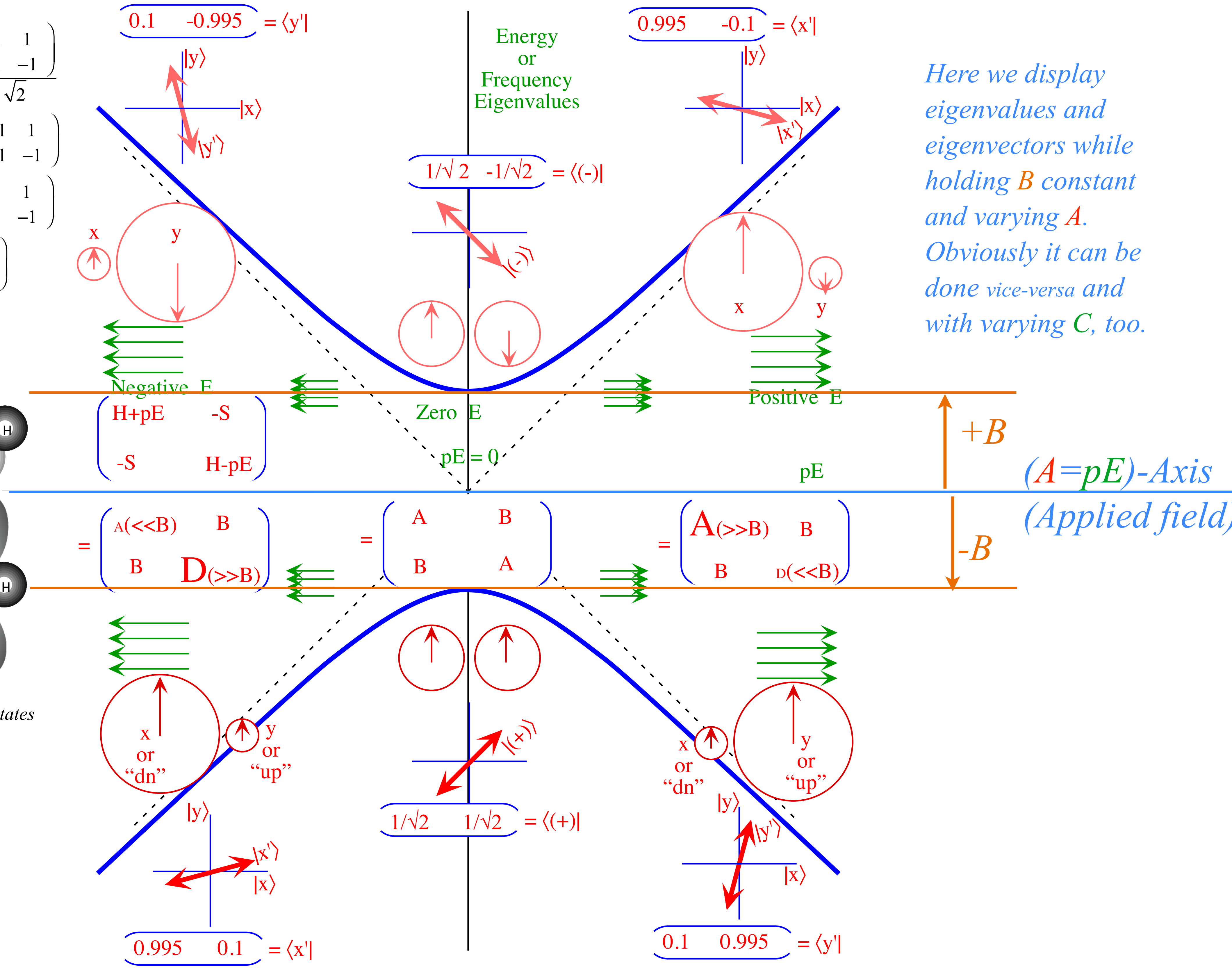
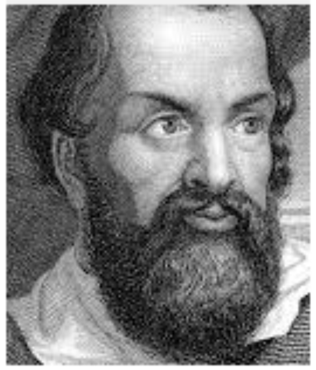


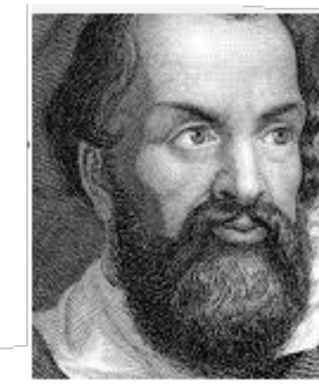
Fig. 10.3.1 (b) Wigner avoided level crossing. (Fixed tunneling B=-S and variable A-D=pE field.)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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Thales geometry of Lorentz transformation …and invariant hyperbolas

➤ *Rapidity* ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity ◀

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of functions of ρ and σ Minkowski animations

Application to TE-Waveguide modes. synchrotron beam relativity

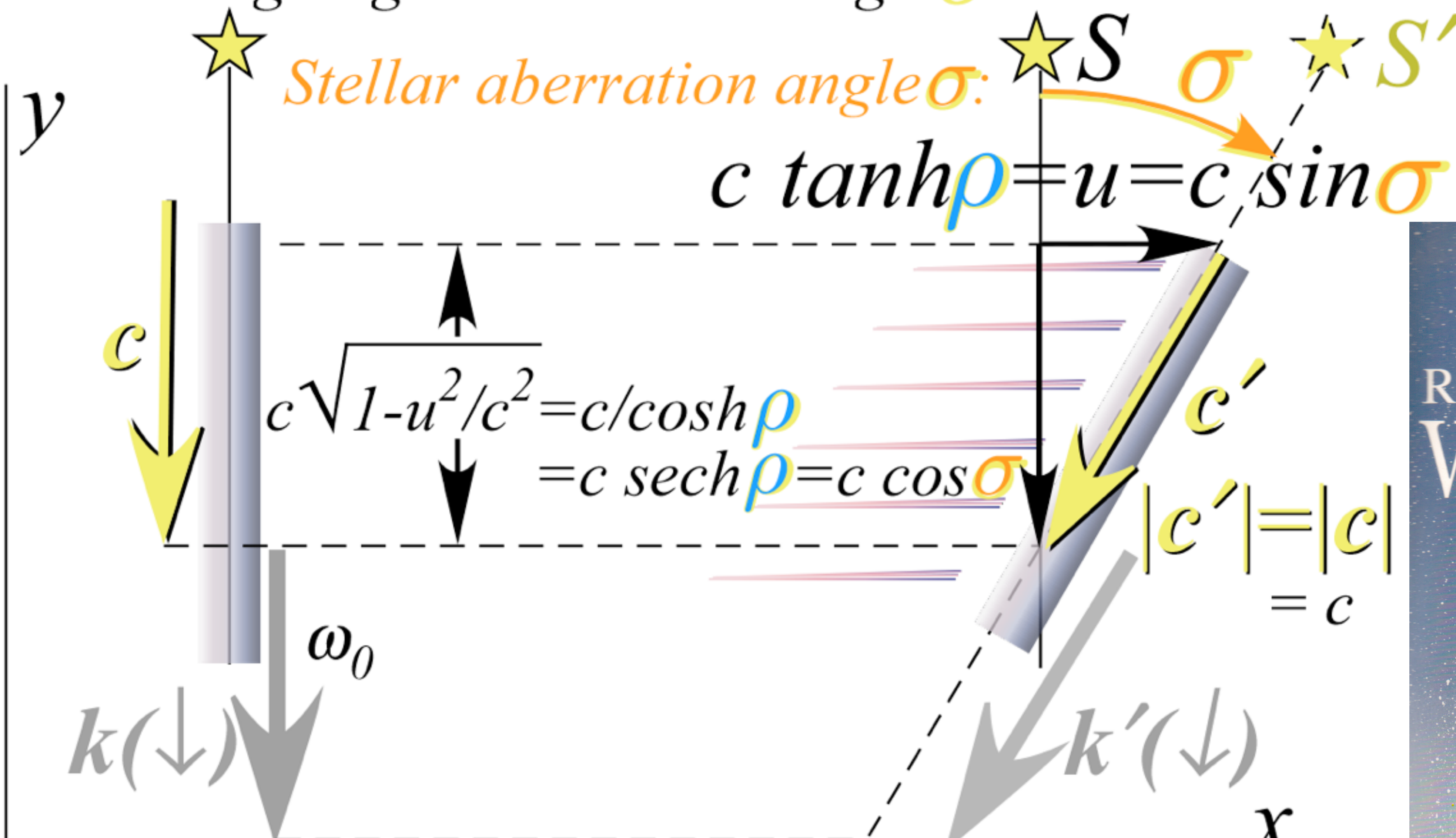
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

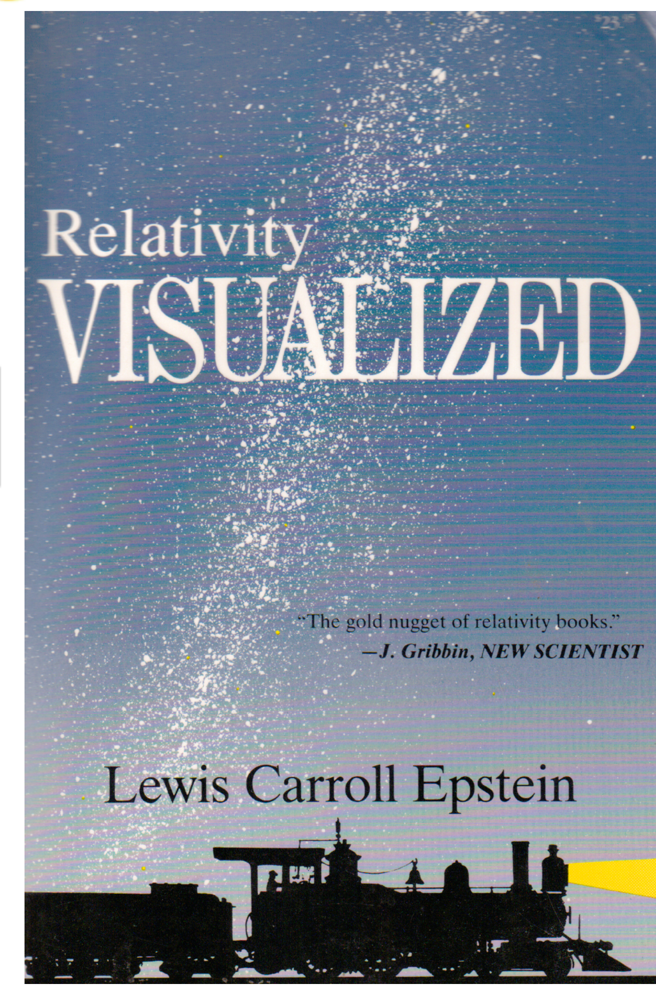
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notation σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Purchase at: AllBookstores.com amazon

Details of stellar aberration angle σ of K-vexctor rotation

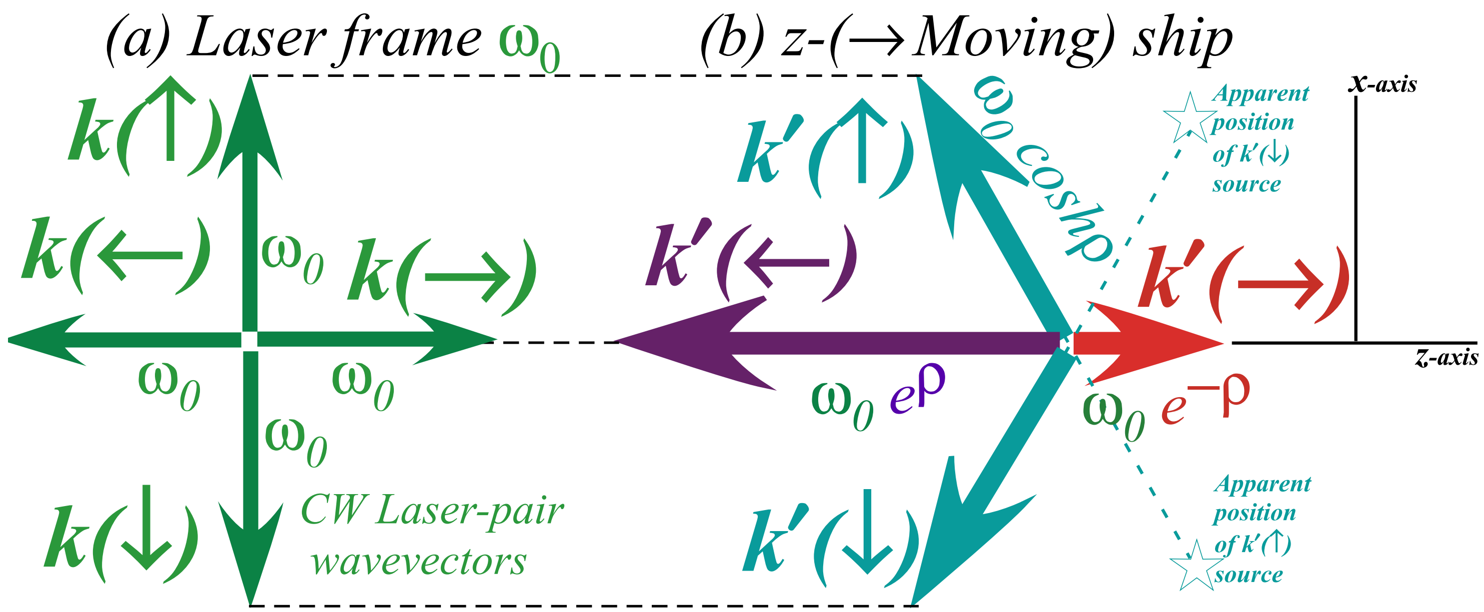


Fig. 8.5.7
Unit 8

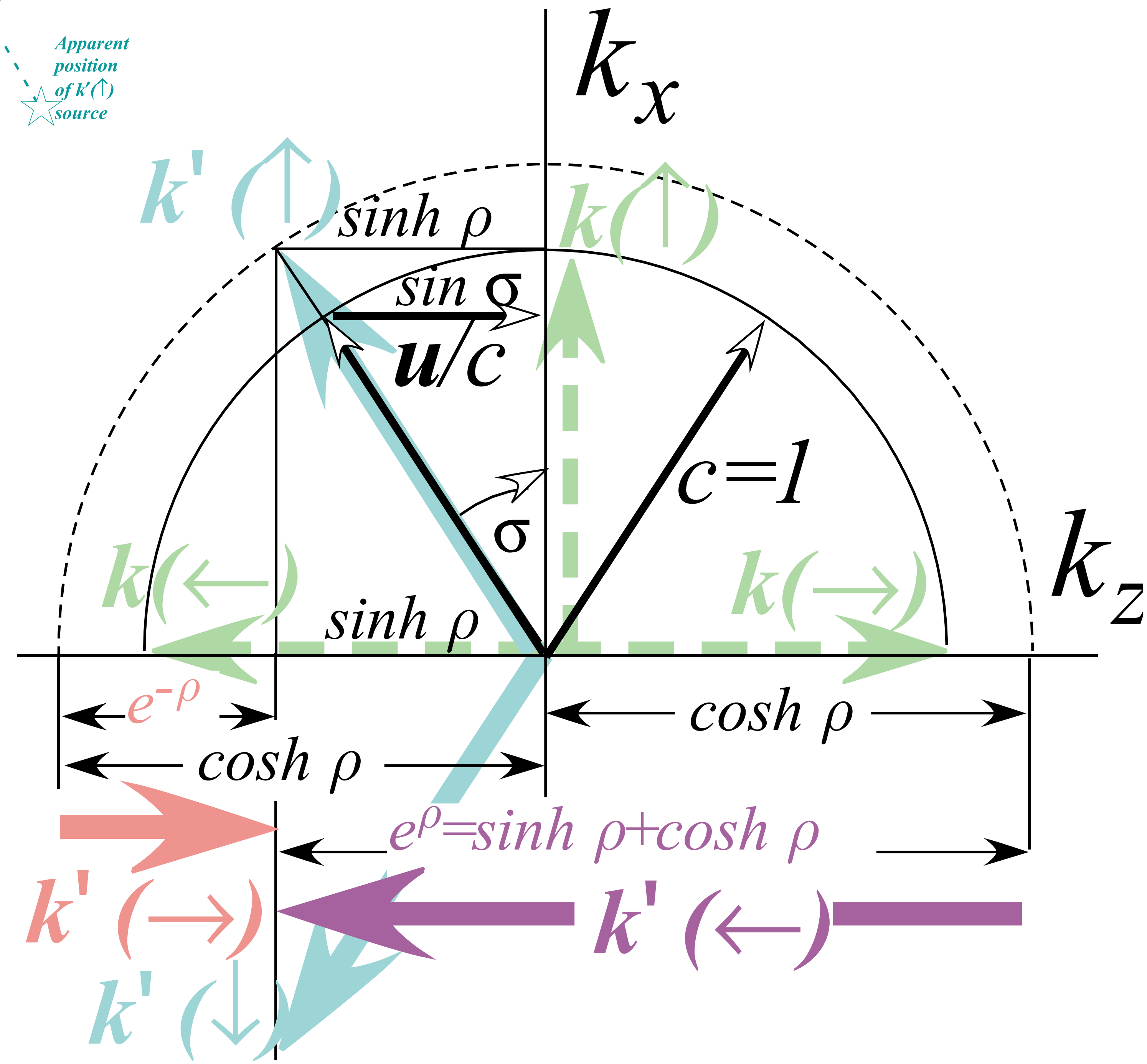
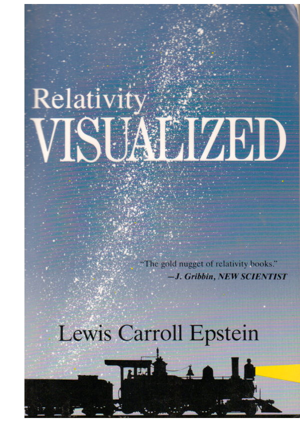


Fig. 8.5.10
Unit 8

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

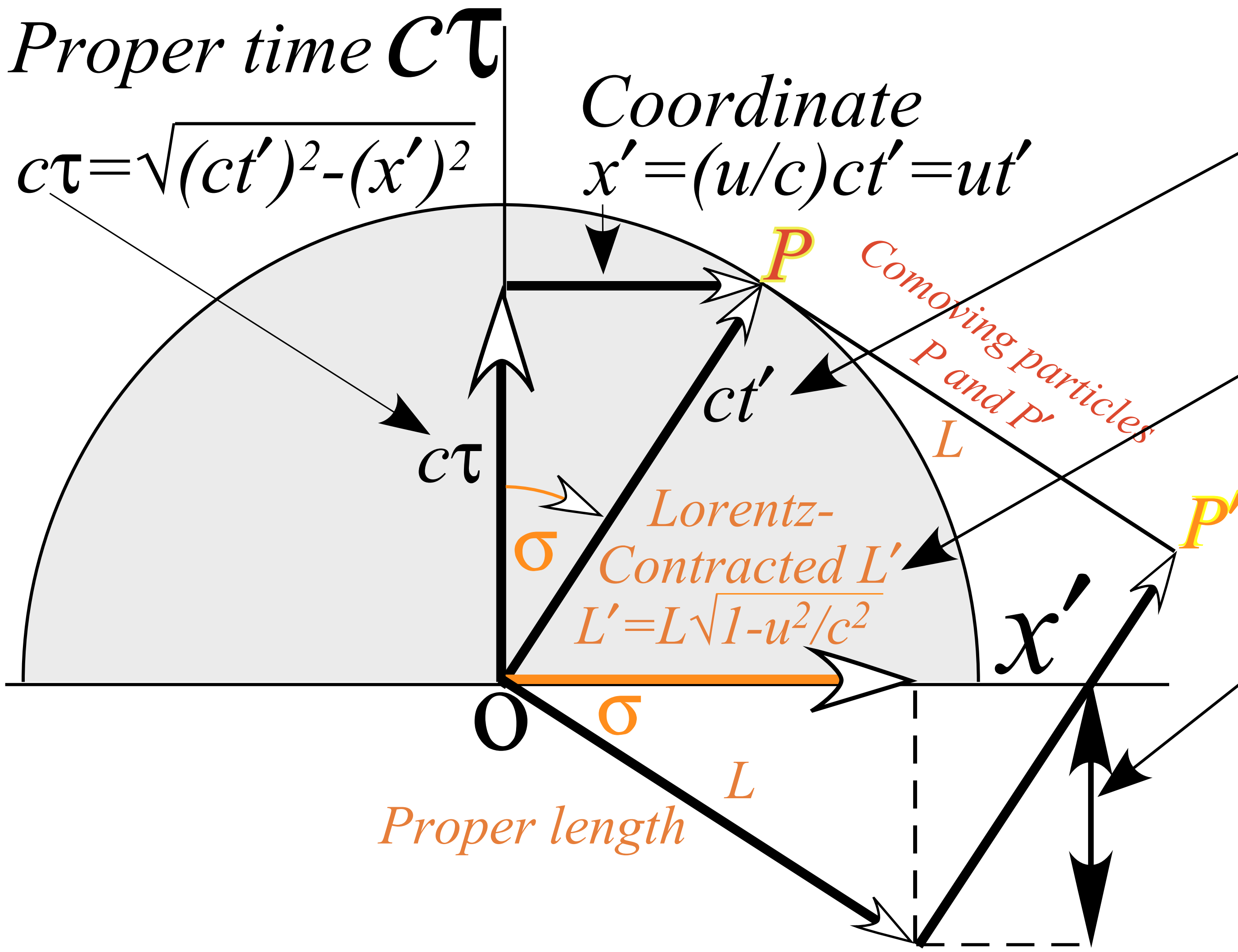
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L \,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L' \sinh\rho = L\cos\sigma \sinh\rho$$
$$= L\cos\sigma \tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame

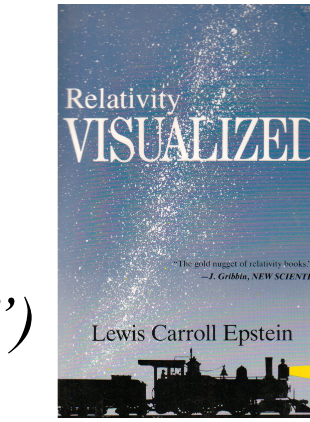
RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

RelativIt Web Simulation *Epstein plot*

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

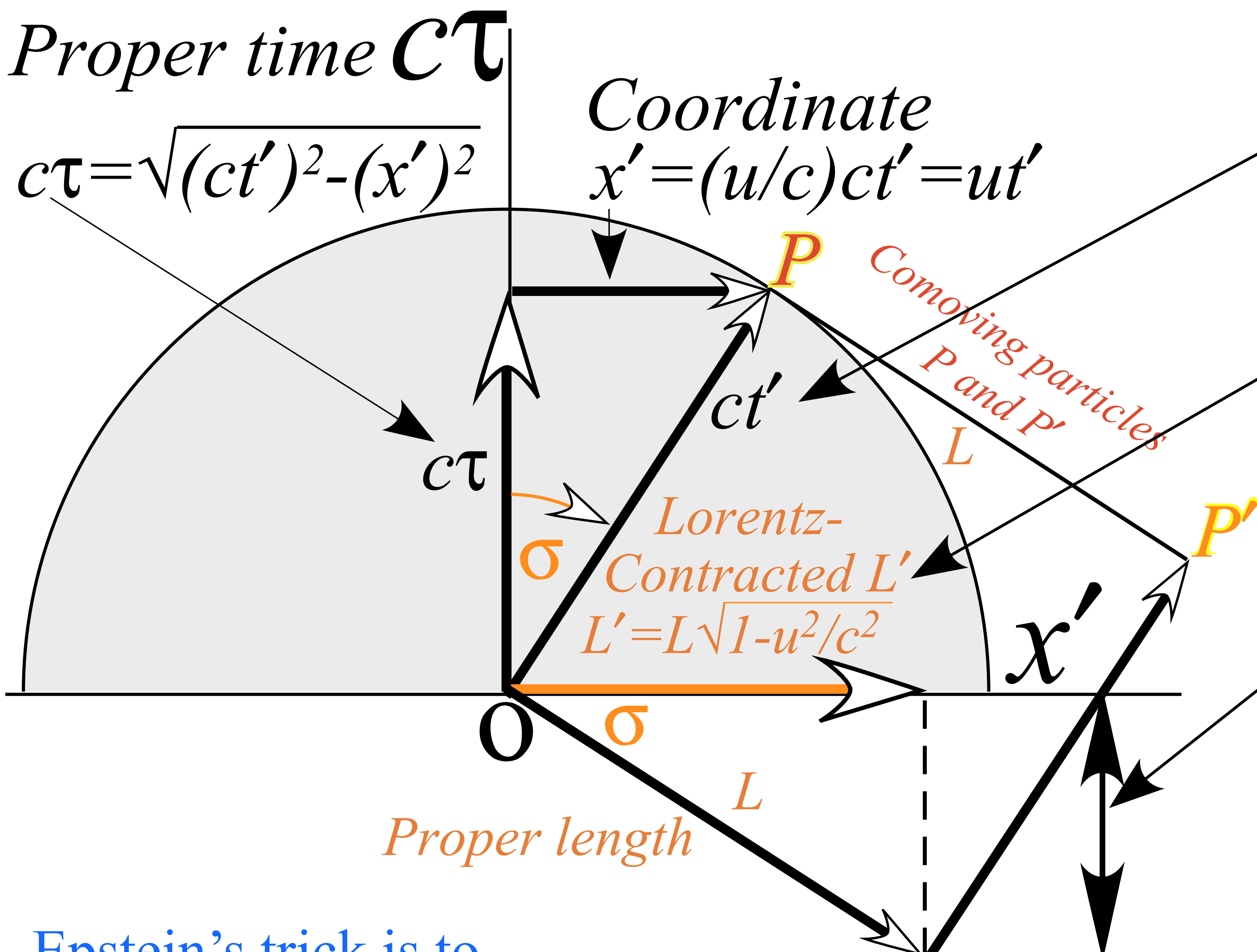
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

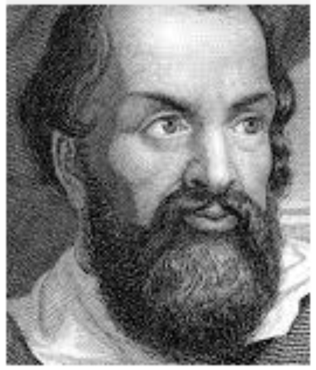
$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2-(x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2+(x')^2} = (ct')$$

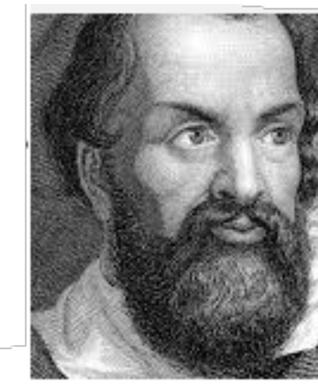
Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

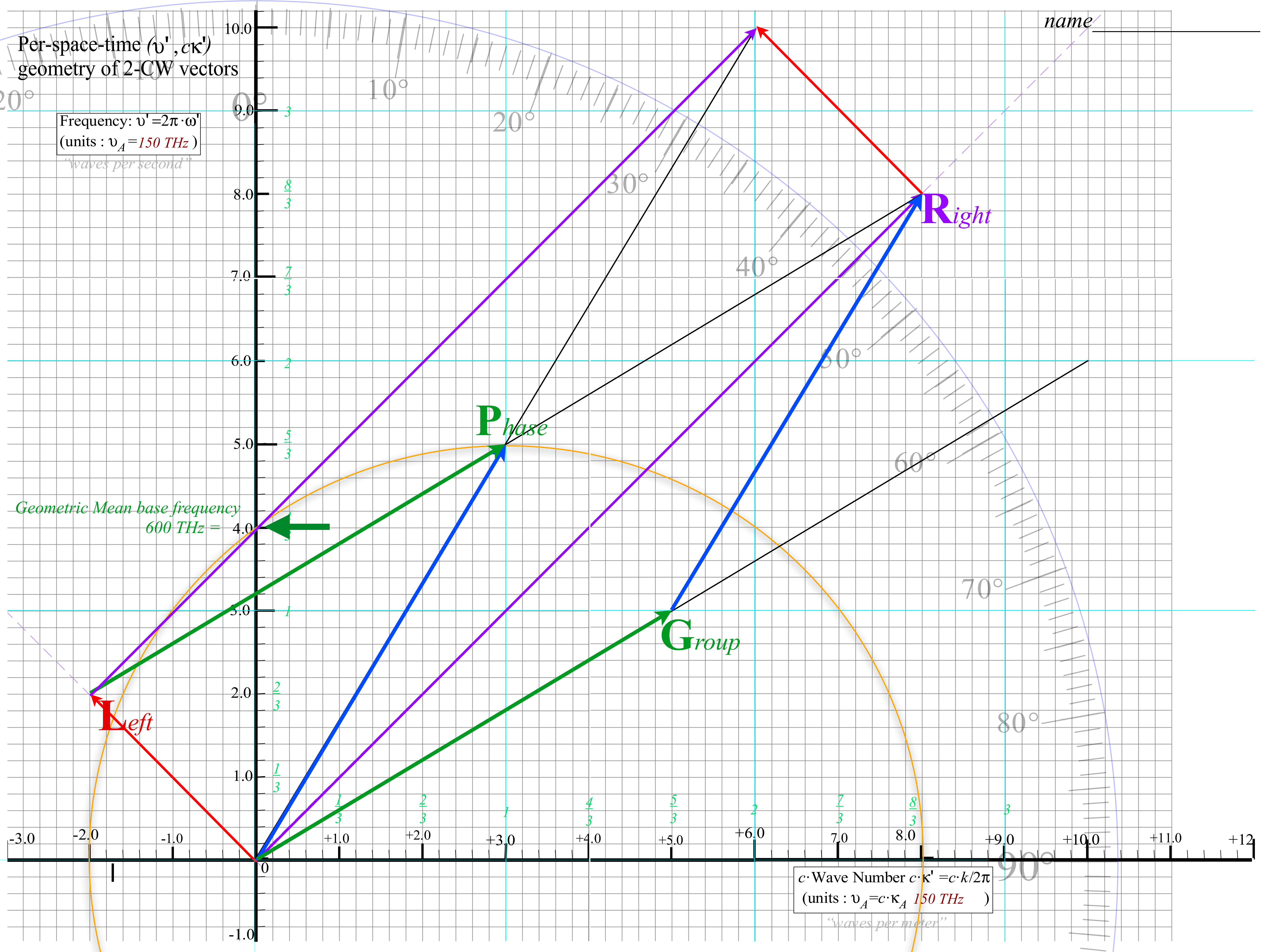
Thales geometry of Lorentz transformation …and invariant hyperbolas

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

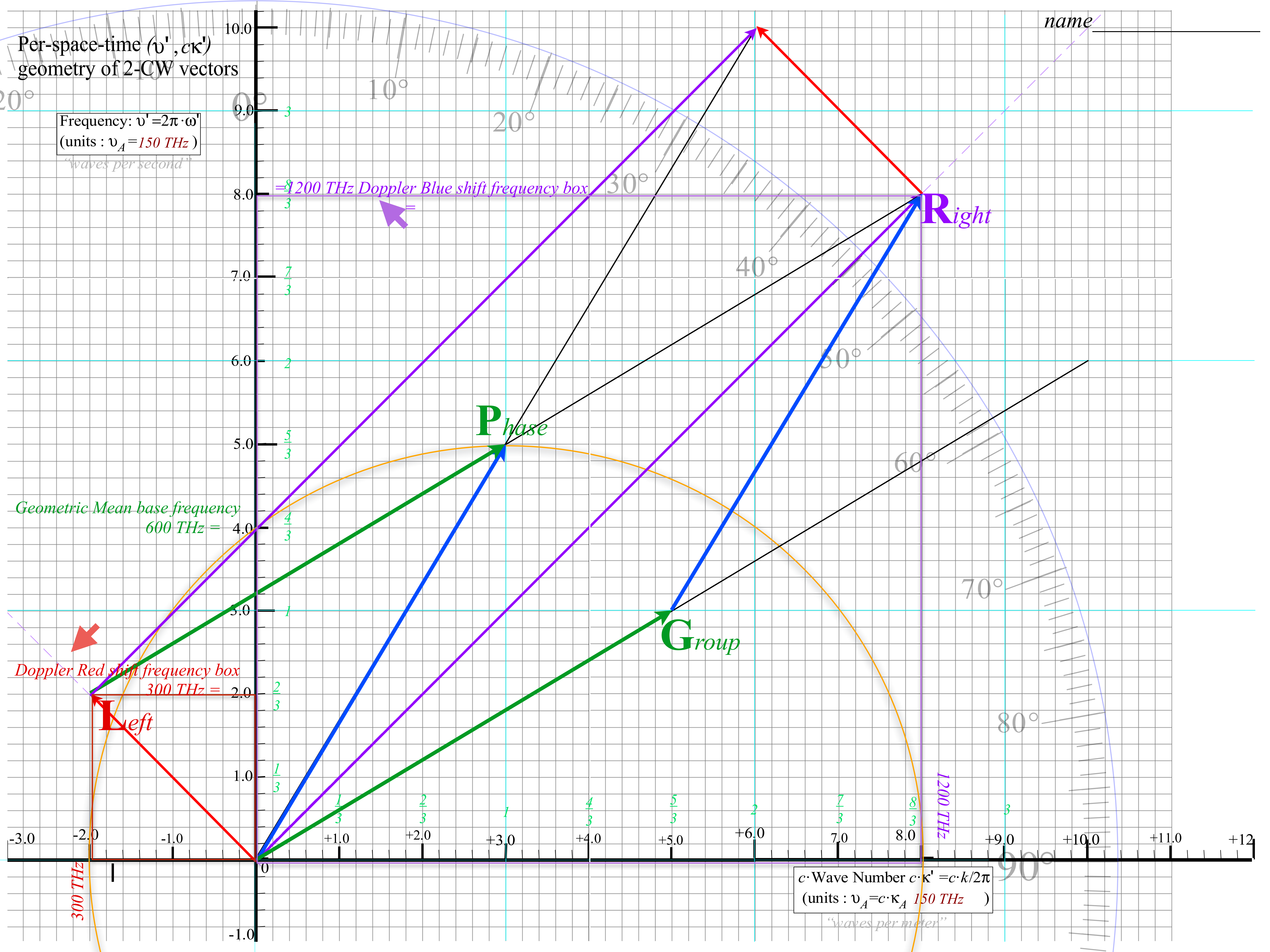
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry ←

→ "Occams Sword" and geometry of functions of ρ and σ Minkowski animations

Application to TE-Waveguide modes and synchrotron beam relativity



Per-space-time (υ', cκ')
geometry of 2-CW vectors
name
Frequency: υ' =2π·ω'
(units : υ_A =150 THz)
"waves per second"
Geometric Mean base frequency
600 THz =
Phase
Group
Right
Left
c·Wave Number c·κ' =c·k/2π
(units : υ_A=c·κ_A 150 THz)
"waves per meter"
10°
20°
30°
40°
50°
60°
70°
80°
90°



name
Per-space-time (υ', cκ') geometry of 2-CW vectors
Frequency: υ'=2π·ω'
(units : $υ_A$=150 THz)
"waves per second"
=1200 THz Doppler Blue shift frequency box
Right
Phase
Group
Left
Geometric Mean base frequency
600 THz =
Doppler Red shift frequency box
300 THz =
300 THz
1200 THz
c·Wave Number c·κ' =c·k/2π
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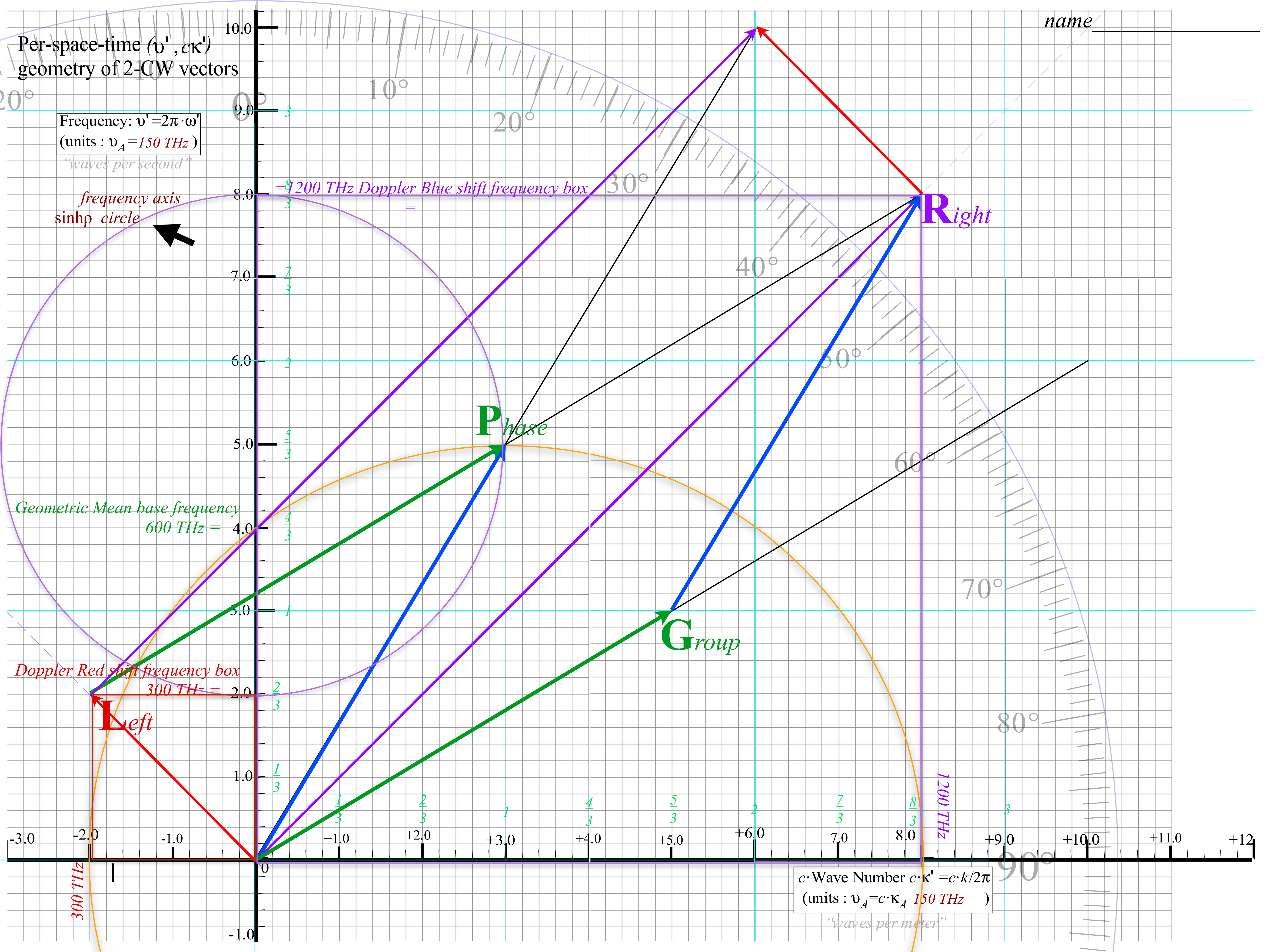

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name
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"waves per second"
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frequency axis
sinhρ circle
Right
Phase
Group
Left
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300 THz =
1200 THz
300 THz
c·Wave Number c·κ' =c·k/2π
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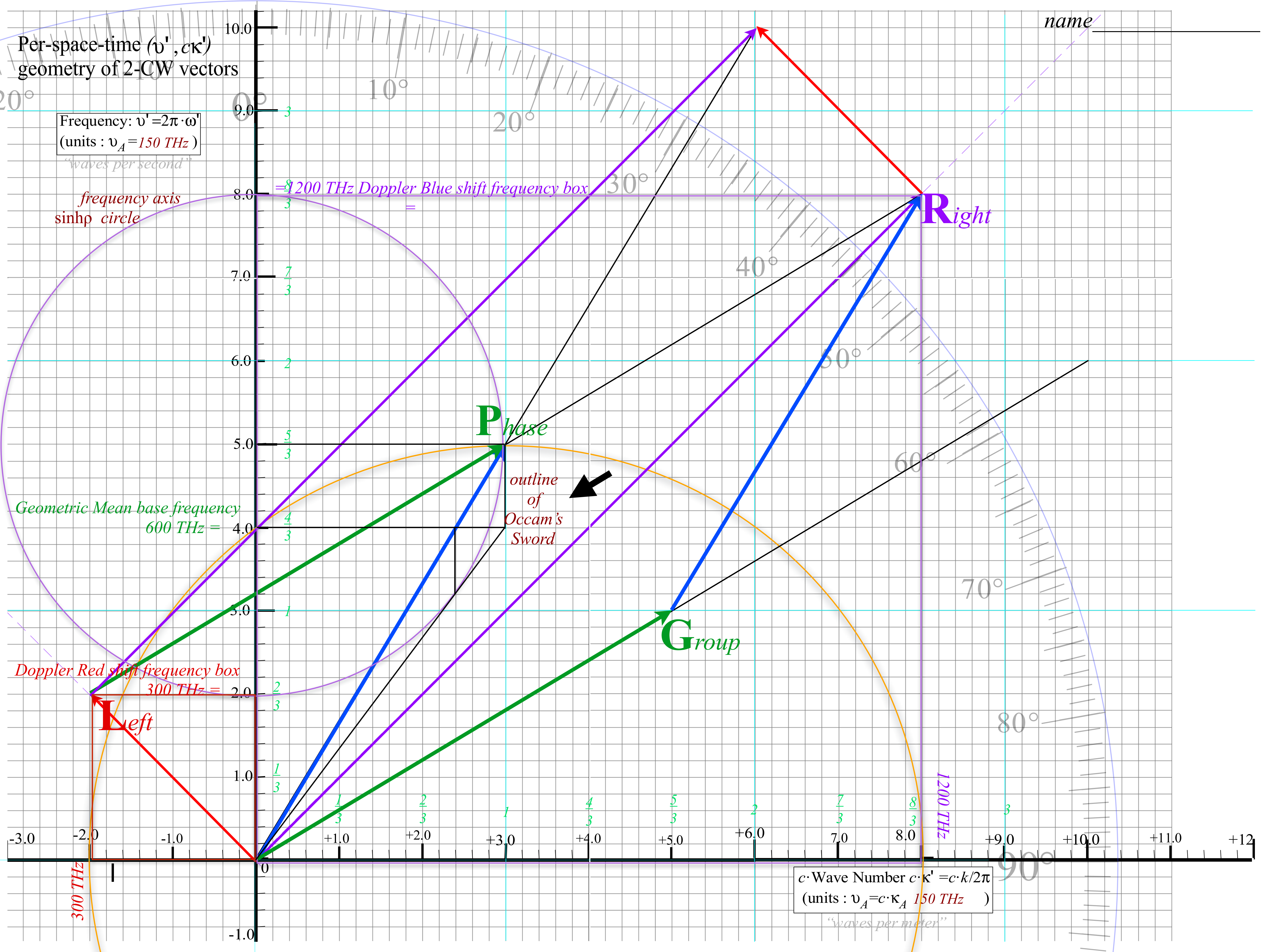

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Right
Phase
outline of Occam's Sword
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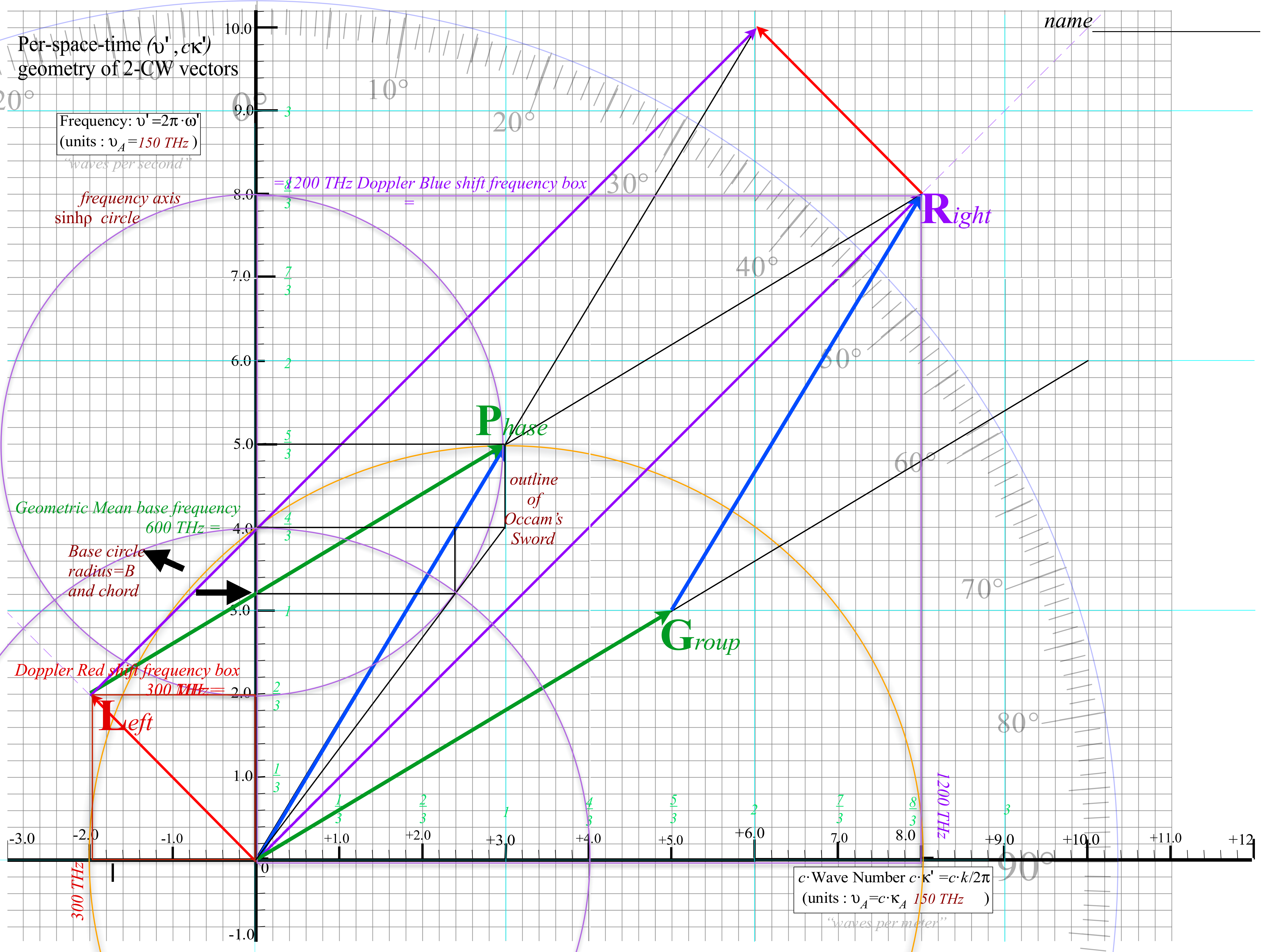

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Geometric Mean base frequency
600 THz =
Base circle
radius=B
and chord
Group
Doppler Red shift frequency box
300 THz =
Left
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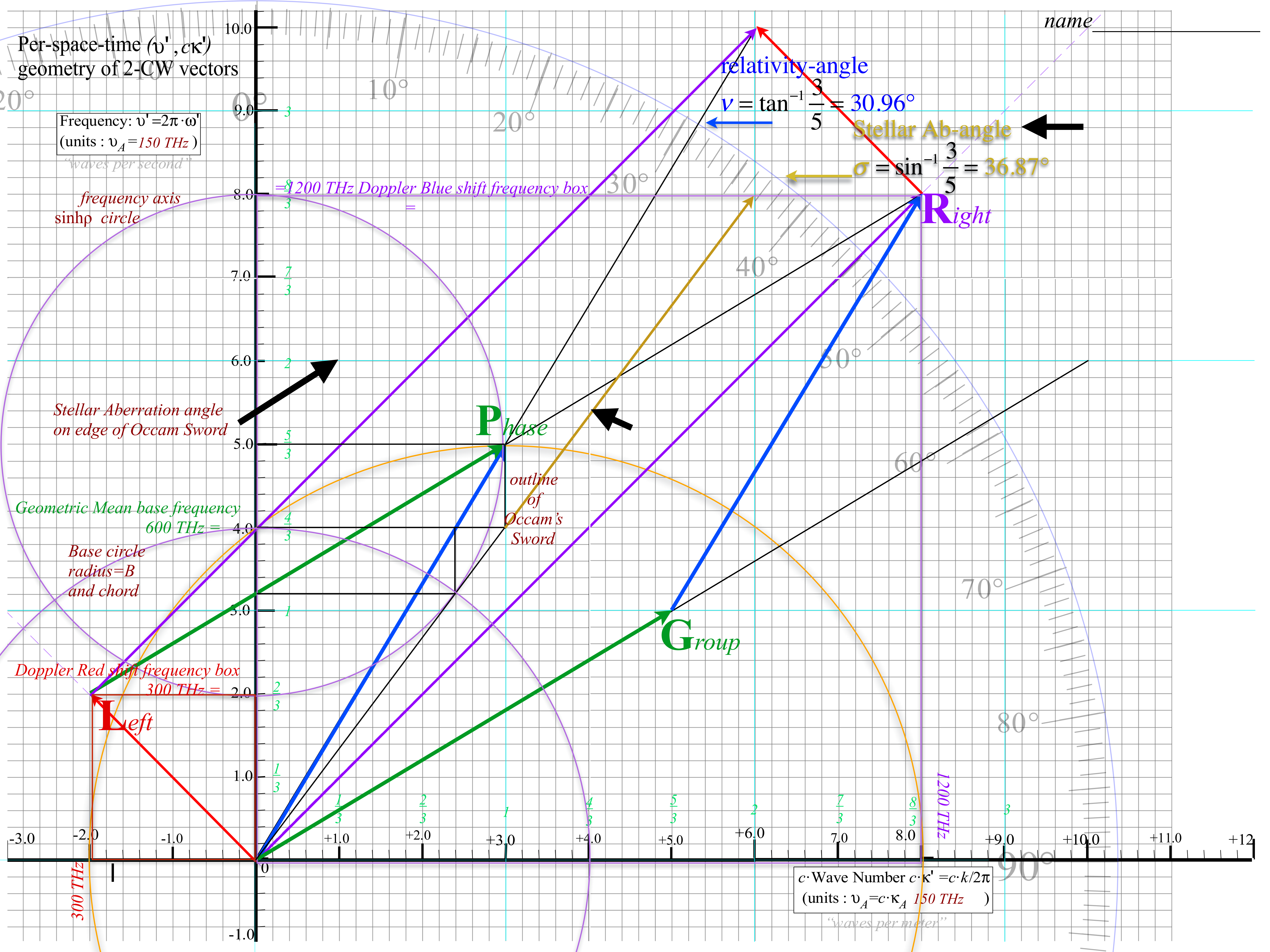

Per-space-time (υ', cκ') geometry of 2-CW vectors
name
Frequency: υ'=2π·ω' (units : υ_A=150 THz)
"waves per second"
frequency axis sinhρ circle
relativity-angle
ν = tan⁻¹ 3/5 = 30.96°
Stellar Ab-angle
σ = sin⁻¹ 3/5 = 36.87°
=1200 THz Doppler Blue shift frequency box
Right
Phase
Group
Left
Stellar Aberration angle on edge of Occam Sword
outline of Occam's Sword
Geometric Mean base frequency 600 THz =
Base circle radius=B and chord
Doppler Red shift frequency box 300 THz =
1200 THz
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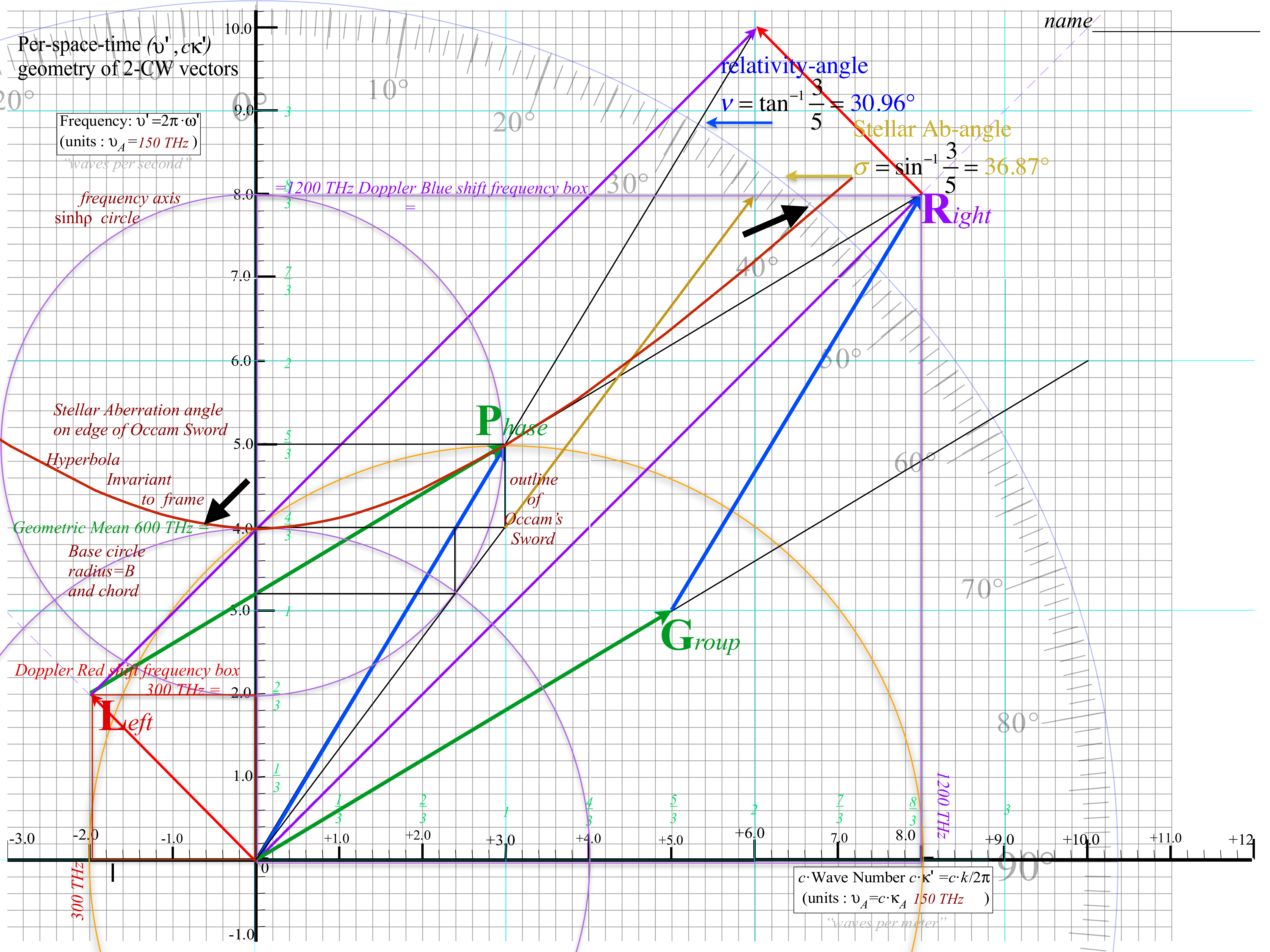

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on edge of Occam Sword
Hyperbola
Invariant
to frame
outline
of
Occam's
Sword
Geometric Mean 600 THz =
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Group
Doppler Red shift frequency box
300 THz =
Left
1200 THz
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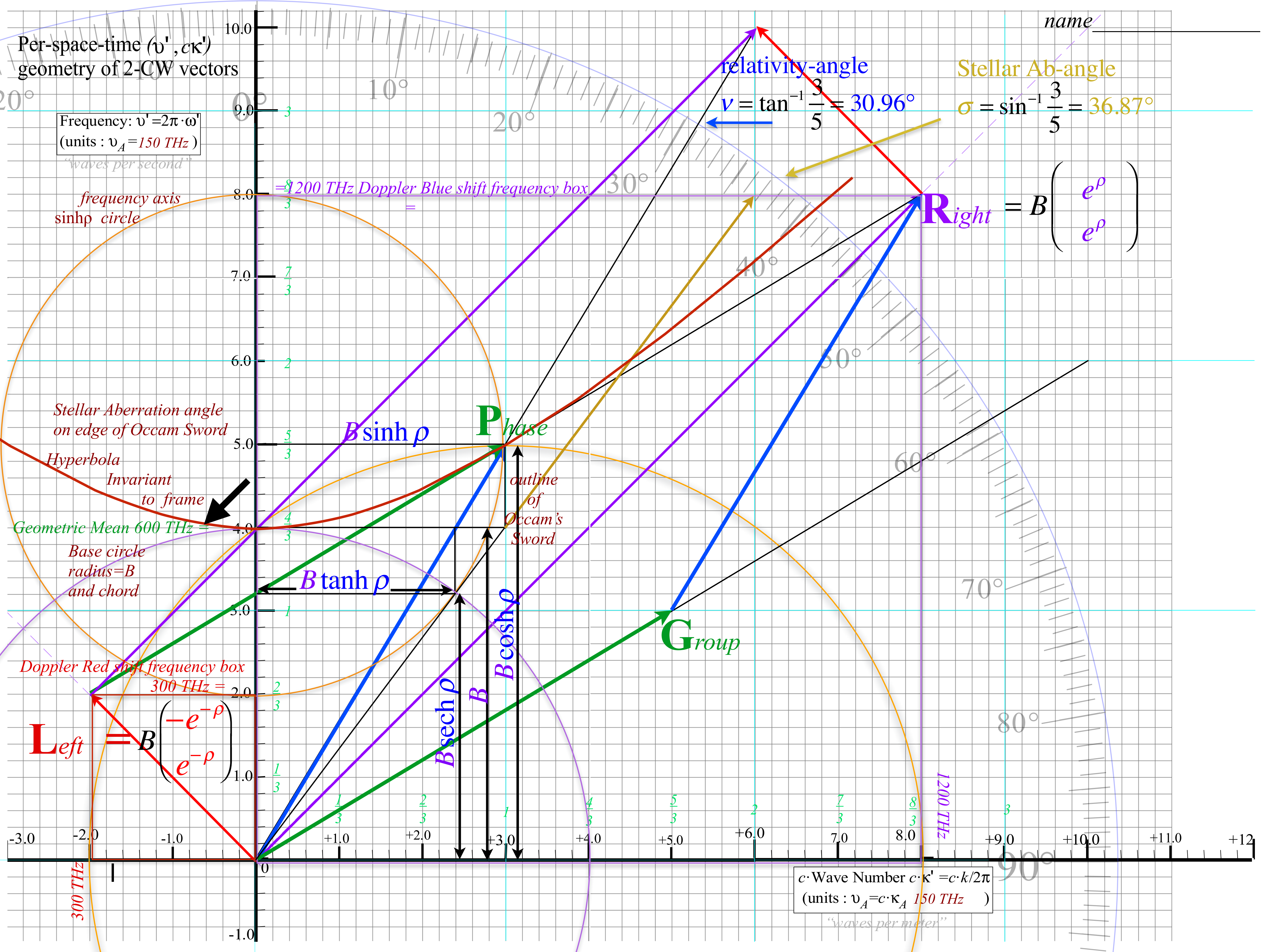

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=1200 THz Doppler Blue shift frequency box =
frequency axis
sinhρ circle
Right $= B\begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}$
Stellar Aberration angle
on edge of Occam Sword
Hyperbola
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to frame
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Phase
outline of Occam's Sword
Geometric Mean 600 THz =
Base circle
radius=B
and chord
$B\tanh\rho$
$B\cosh\rho$
B
$B\,\mathrm{sech}\,\rho$
Group
Doppler Red shift frequency box
300 THz =
Left $= B\begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix}$
1200 THz
300 THz
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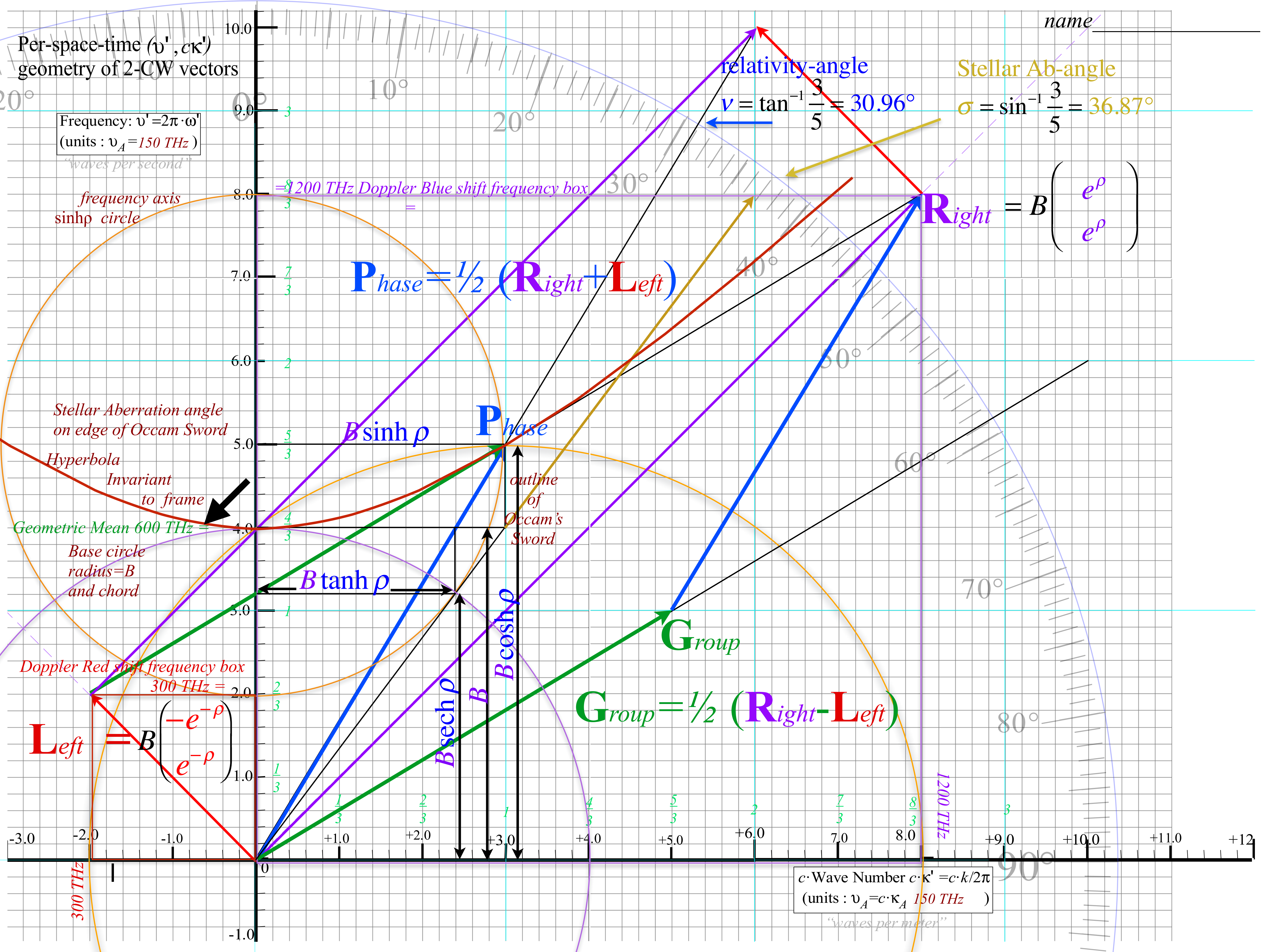
"waves per meter"



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Right = B (e^ρ, e^ρ)
Phase=½ (Right+Left)
Stellar Aberration angle
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B sinh ρ
Phase
outline
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Occam's
Sword
Geometric Mean 600 THz =
Base circle
radius=B
and chord
B tanh ρ
B cosh ρ
B sech ρ
B
Group
Doppler Red shift frequency box
300 THz =
Group=½ (Right-Left)
Left = B (−e^−ρ, e^−ρ)
1200 THz
300 THz
c·Wave Number c·κ' =c·k/2π
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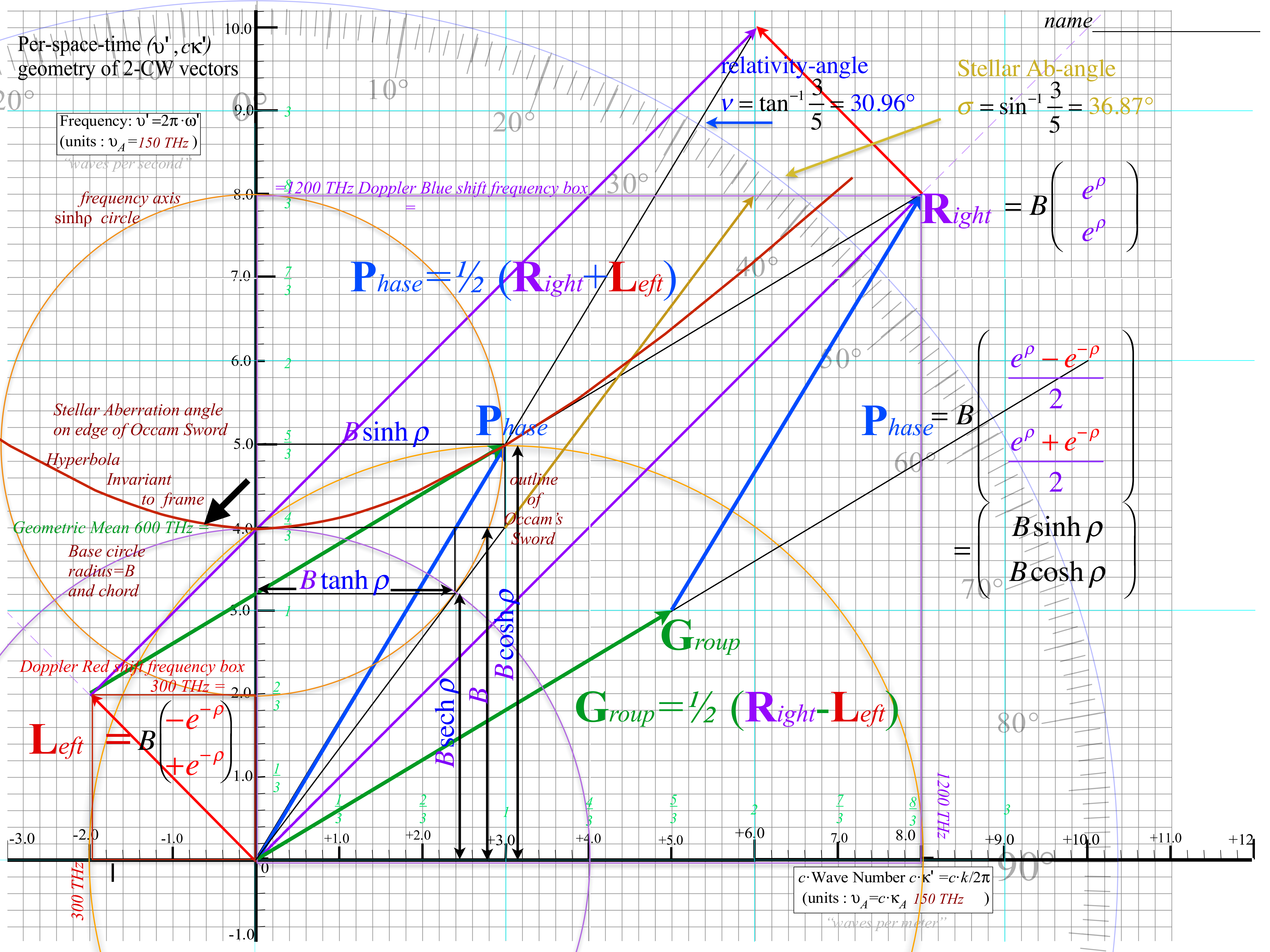

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Phase = ½ (Right + Left)
Phase
Phase $= B\begin{pmatrix} \frac{e^{\rho}-e^{-\rho}}{2} \\ \frac{e^{\rho}+e^{-\rho}}{2} \end{pmatrix} = \begin{pmatrix} B\sinh\rho \\ B\cosh\rho \end{pmatrix}$
Stellar Aberration angle on edge of Occam Sword
Hyperbola Invariant to frame
Geometric Mean 600 THz =
Base circle radius=B and chord
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$B\tanh\rho$
$B\cosh\rho$
$B\,\mathrm{sech}\,\rho$
B
outline of Occam's Sword
Group
Group = ½ (Right - Left)
Doppler Red shift frequency box
300 THz =
Left $= B\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix}$
300 THz
1200 THz
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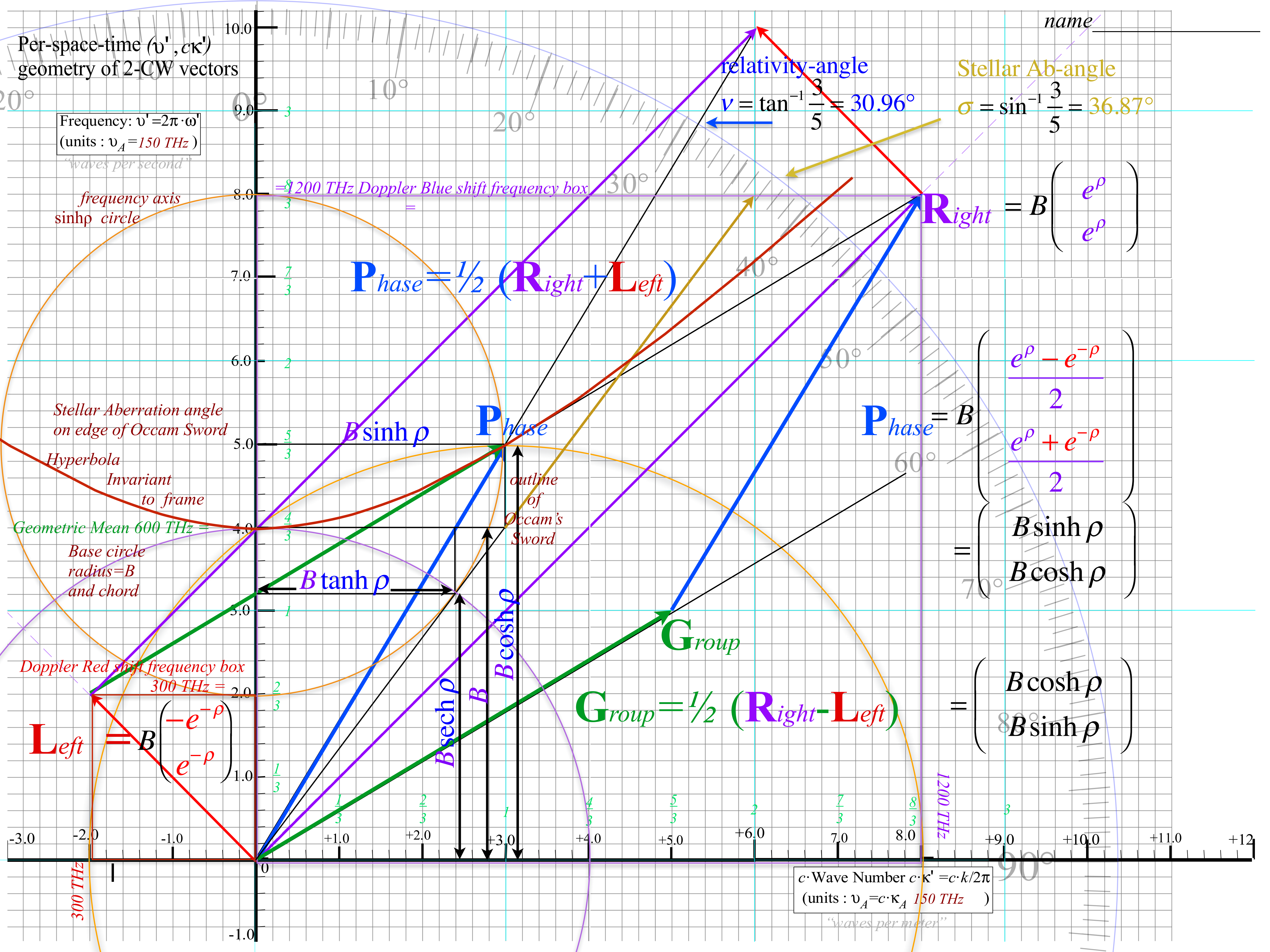

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frequency axis sinhρ circle
$\mathbf{R}ight = B\begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}$
$\mathbf{P}hase = \frac{1}{2}(\mathbf{R}ight + \mathbf{L}eft)$
$\mathbf{P}hase = B\begin{pmatrix} \frac{e^{\rho}-e^{-\rho}}{2} \\ \frac{e^{\rho}+e^{-\rho}}{2} \end{pmatrix} = \begin{pmatrix} B\sinh\rho \\ B\cosh\rho \end{pmatrix}$
Stellar Aberration angle on edge of Occam Sword
Hyperbola Invariant to frame
$B\sinh\rho$
outline of Occam's Sword
Geometric Mean 600 THz =
Base circle radius=B and chord
$B\tanh\rho$
$B\cosh\rho$
$B\mathrm{sech}\rho$
B
Group
Doppler Red shift frequency box 300 THz =
$\mathbf{L}eft = B\begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix}$
$\mathbf{G}roup = \frac{1}{2}(\mathbf{R}ight - \mathbf{L}eft)$
$= \begin{pmatrix} B\cosh\rho \\ B\sinh\rho \end{pmatrix}$
1200 THz
300 THz
c·Wave Number c·κ' =c·k/2π (units : $\upsilon_A = c\cdot\kappa_A$ 150 THz)
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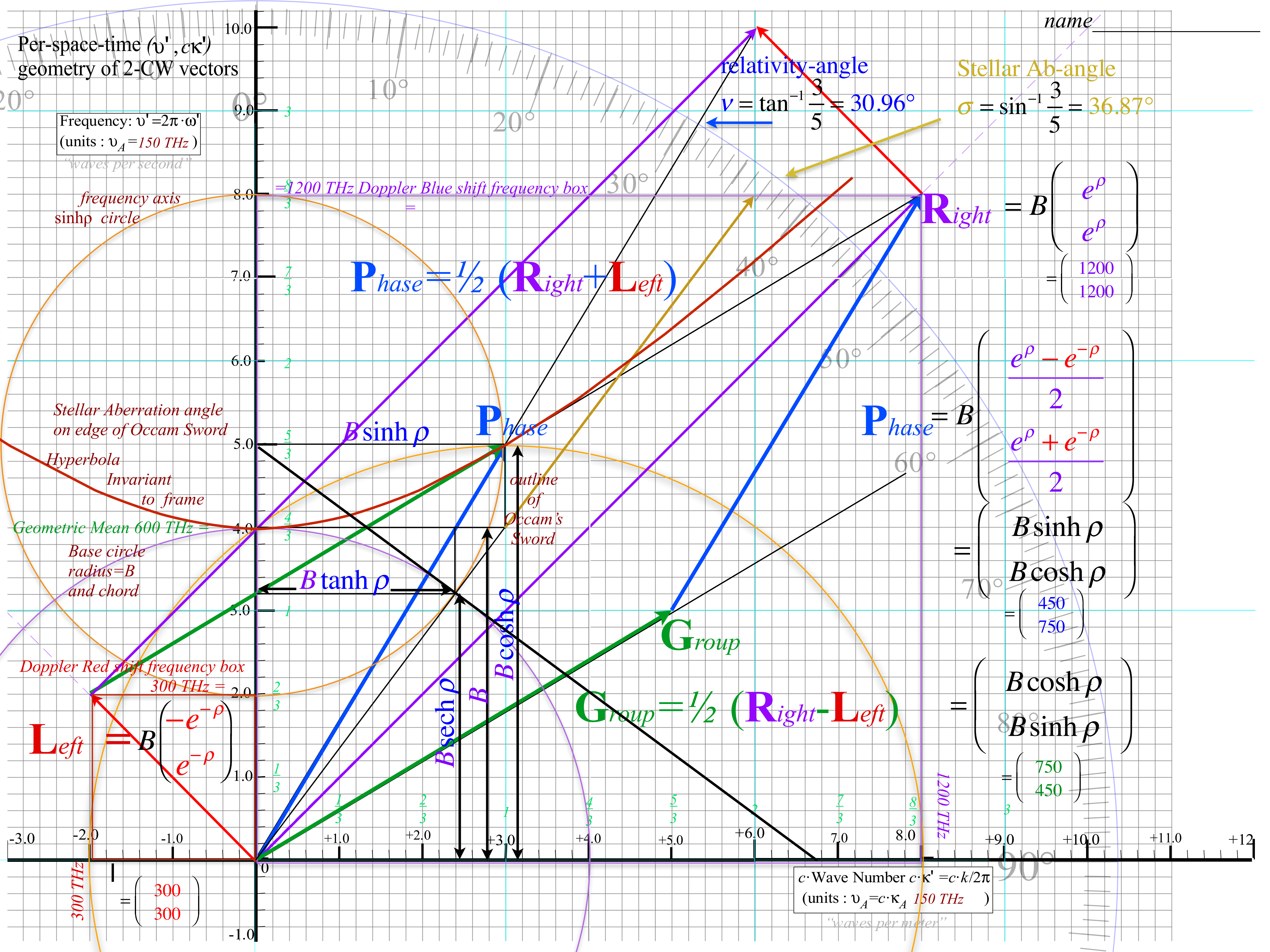

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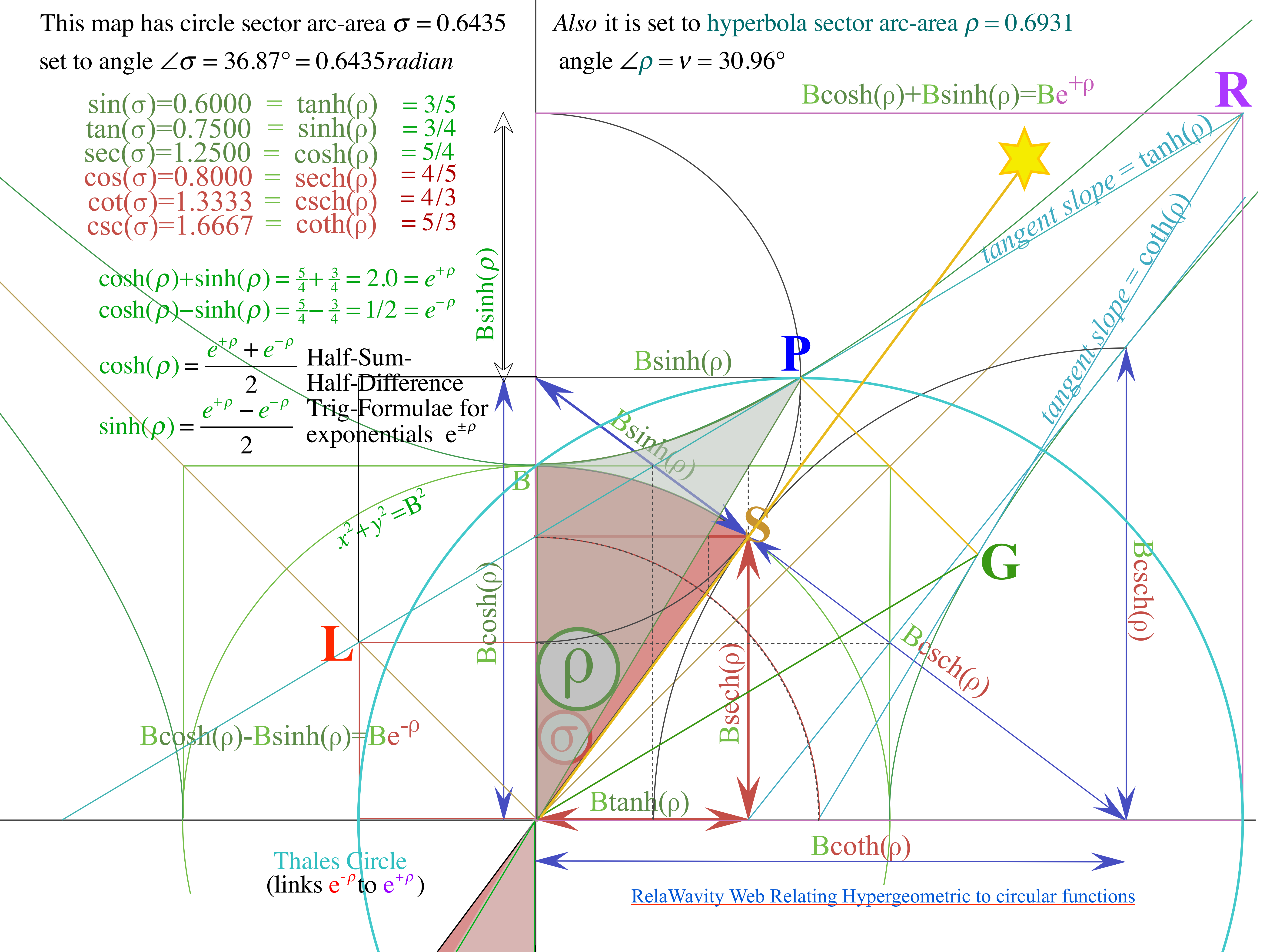
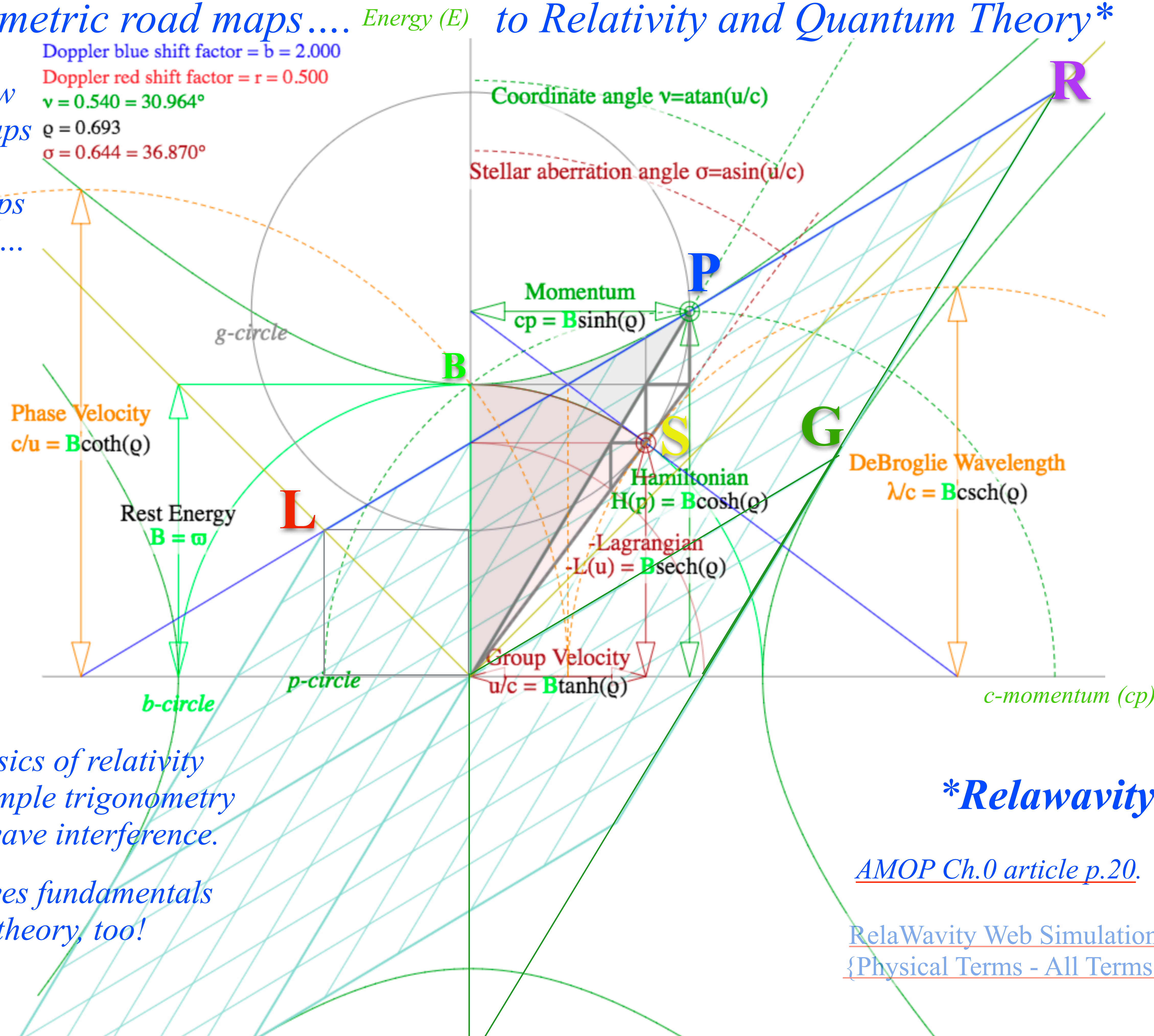
σ = sin⁻¹ 3/5 = 36.87°
=1200 THz Doppler Blue shift frequency box
Phase=½ (Right+Left)
Right = B (e^ρ, e^ρ) = (1200, 1200)
Stellar Aberration angle on edge of Occam Sword
Hyperbola Invariant to frame
B sinh ρ
Phase
outline of Occam's Sword
Phase = B ((e^ρ − e^−ρ)/2, (e^ρ + e^−ρ)/2) = (B sinh ρ, B cosh ρ) = (450, 750)
Geometric Mean 600 THz =
Base circle radius=B and chord
B tanh ρ
B cosh ρ
Group
Doppler Red shift frequency box
300 THz =
Left = B (−e^−ρ, e^−ρ)
B sech ρ
B
Group=½ (Right-Left)
= (B cosh ρ, B sinh ρ) = (750, 450)
1200 THz
300 THz
= (300, 300)
c·Wave Number c·κ' =c·k/2π (units : υ_A=c·κ_A 150 THz)
"waves per meter"


This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435 radian
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
P
Bsinh(ρ)
Bsinh(ρ)
B
x²+y²=B²
S
G
L
Bcosh(ρ)
Bcsch(ρ)
Bsech(ρ)
Bcsch(ρ)
ρ
σ
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})
RelaWavity Web Relating Hypergeometric to circular functions

*Trigonometric road maps…. to Relativity and Quantum Theory**

Need to show trig road maps can match physical maps like this one…



All this physics of relativity is mostly simple trigonometry of optical wave interference.

And, it derives fundamentals of quantum theory, too!

***Relawavity**

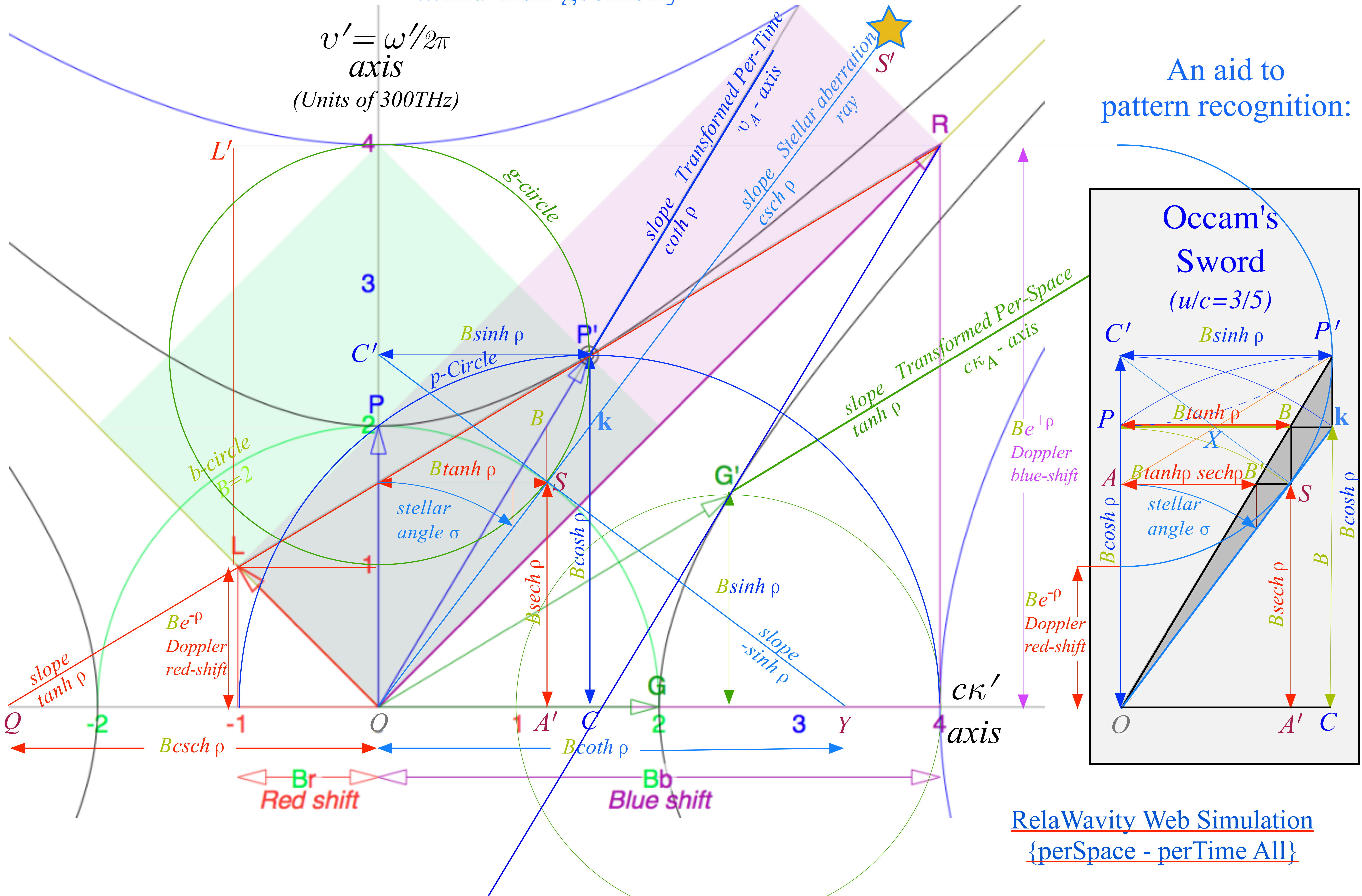
AMOP Ch.0 article p.20.

RelaWavity Web Simulation
{Physical Terms - All Terms}

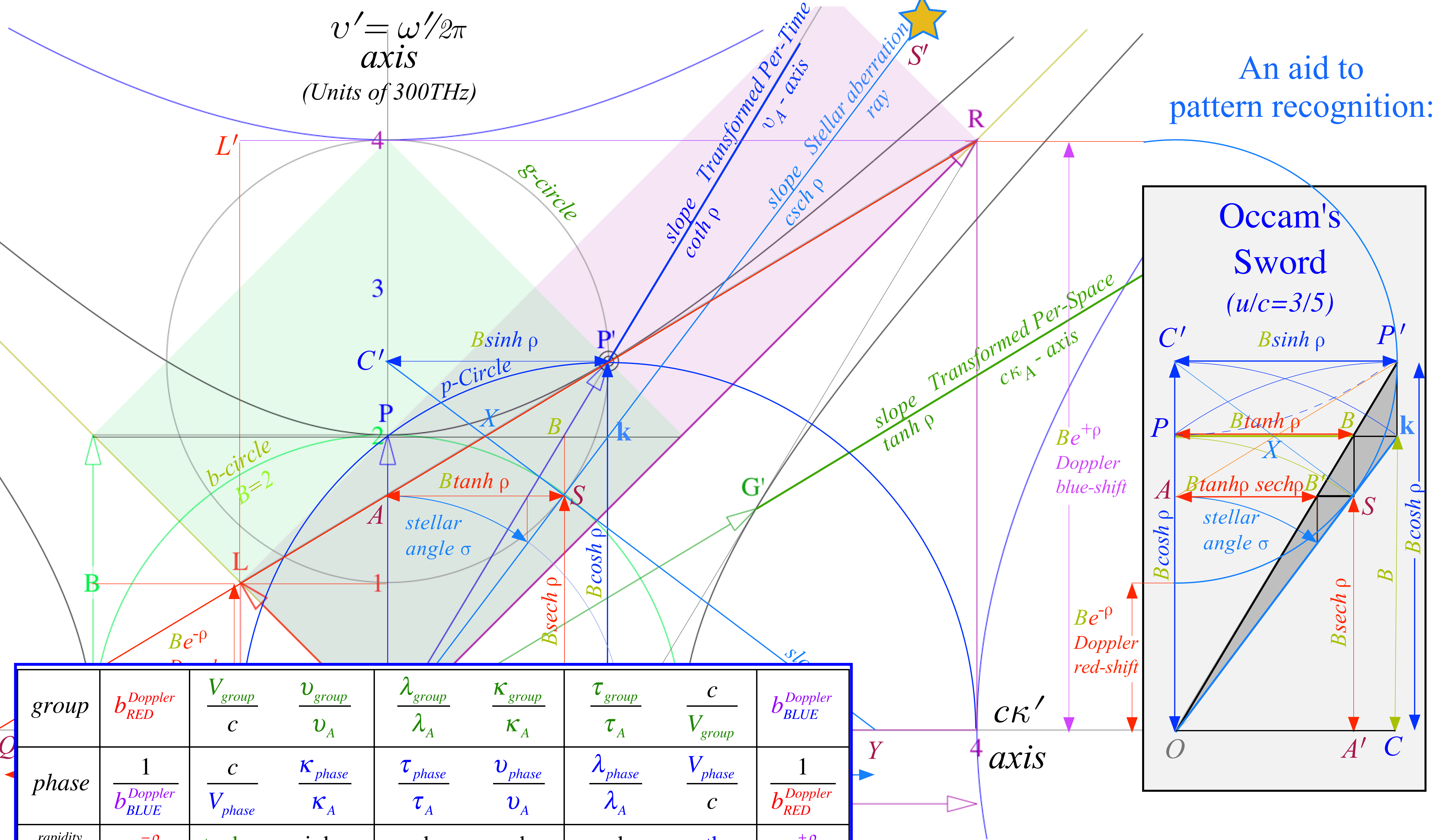
Summary of optical wave parameters for relativity and QM

...and their geometry

An aid to pattern recognition:



RelaWavity Web Simulation
{perSpace - perTime All}



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

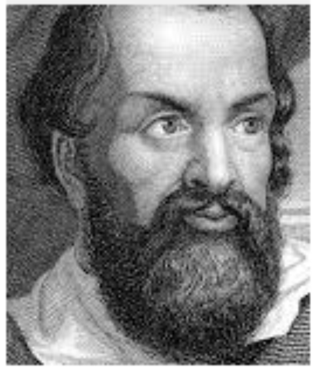
Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation

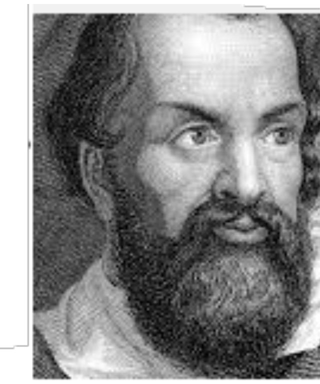
Expanded Relativistic Relations

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



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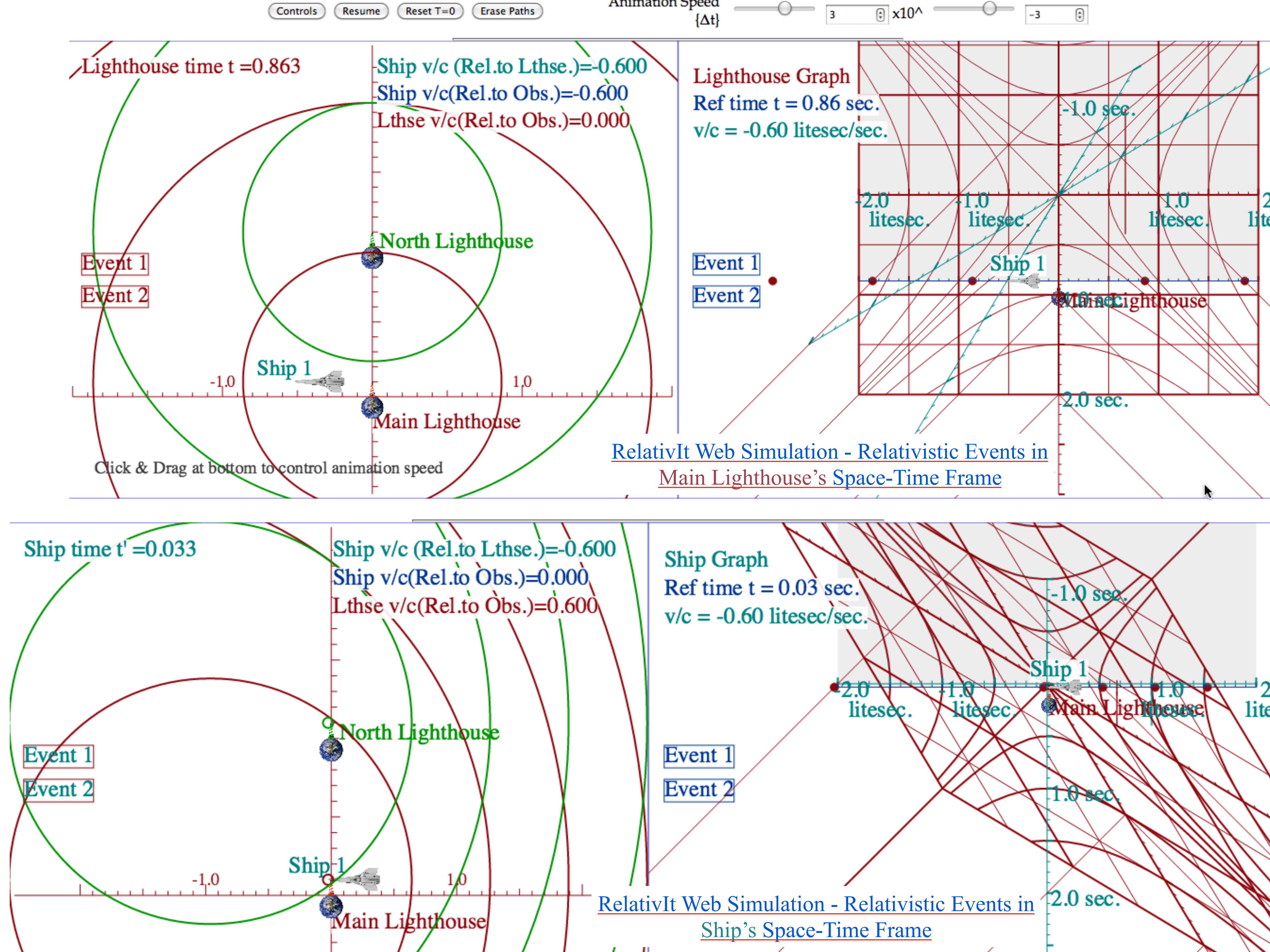
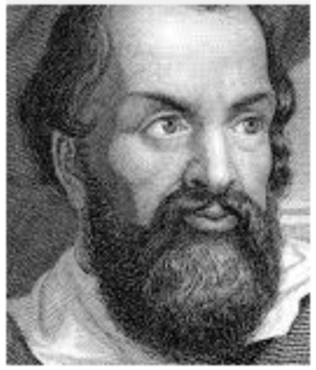
"Occams Sword" and geometry of functions of ρ and σ ➔ Minkowski animations ⬅

Application to TE-Waveguide modes synchrotron beam relativity

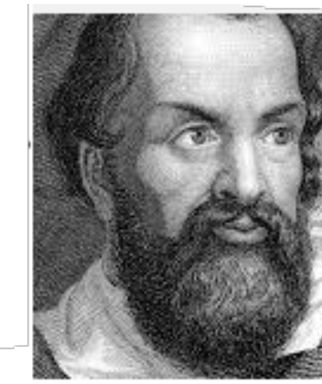

Controls
Resume
Reset T=0
Erase Paths
Animation Speed {Δt}
x10^
Lighthouse time t =0.863
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=-0.600
Lthse v/c(Rel.to Obs.)=0.000
North Lighthouse
Event 1
Event 2
Ship 1
Main Lighthouse
Click & Drag at bottom to control animation speed
Lighthouse Graph
Ref time t = 0.86 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
2.0 sec.
RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame
Ship time t' =0.033
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=0.000
Lthse v/c(Rel.to Obs.)=0.600
Ship Graph
Ref time t = 0.03 sec.
v/c = -0.60 litesec/sec.
1.0 sec.
RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation …and invariant hyperbolas

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

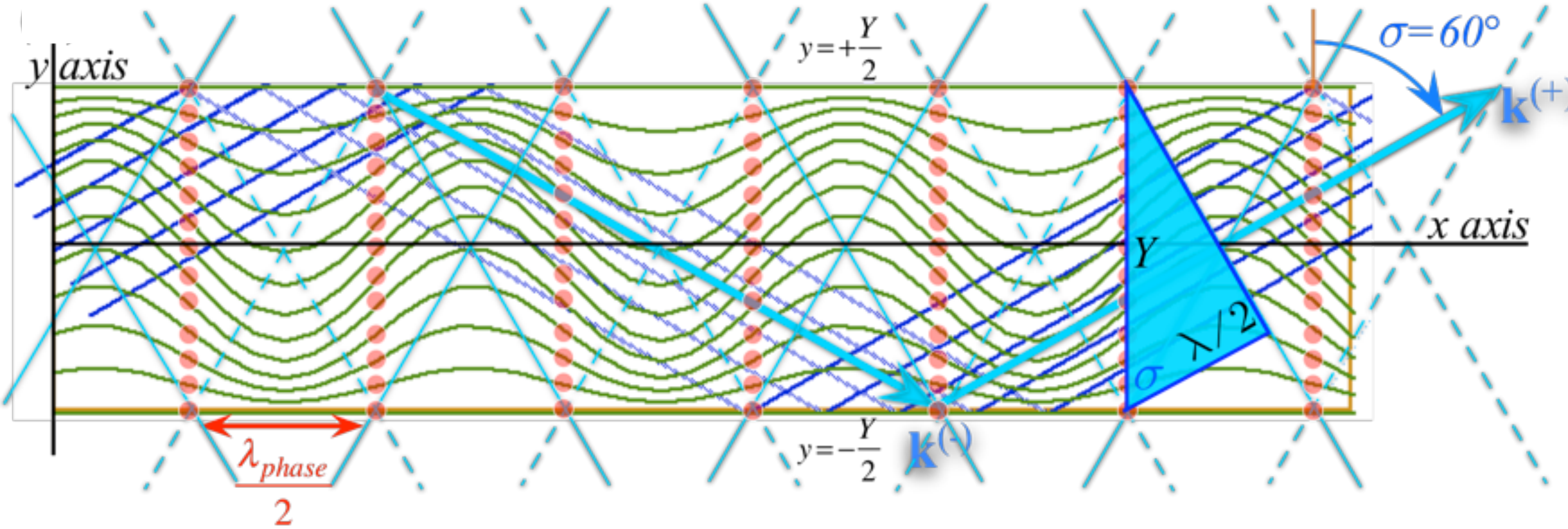
➔ "Occams Sword" and geometry of functions of ρ and σ ⬅ Minkowski animations

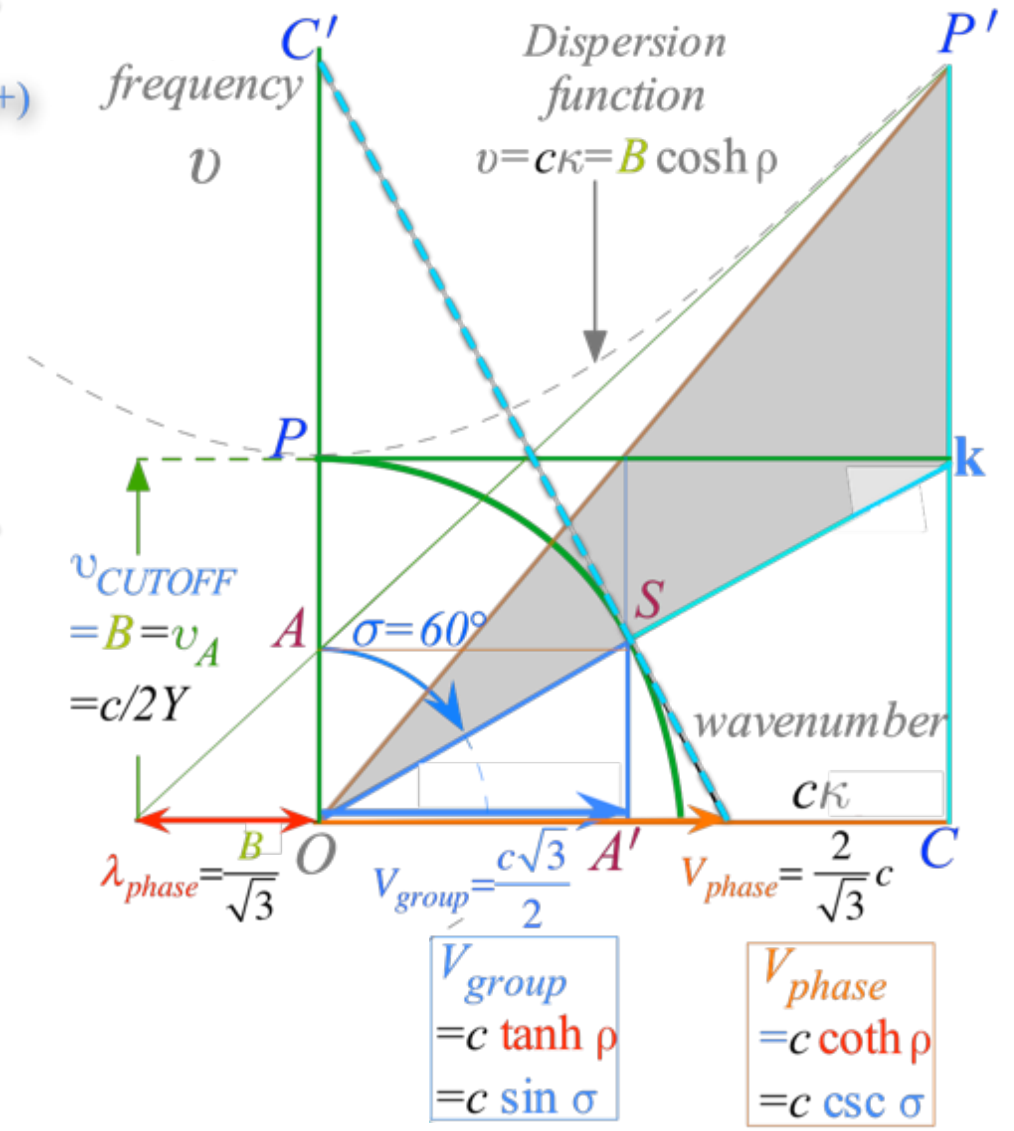
Application to TE-Waveguide modes synchrotron beam relativity

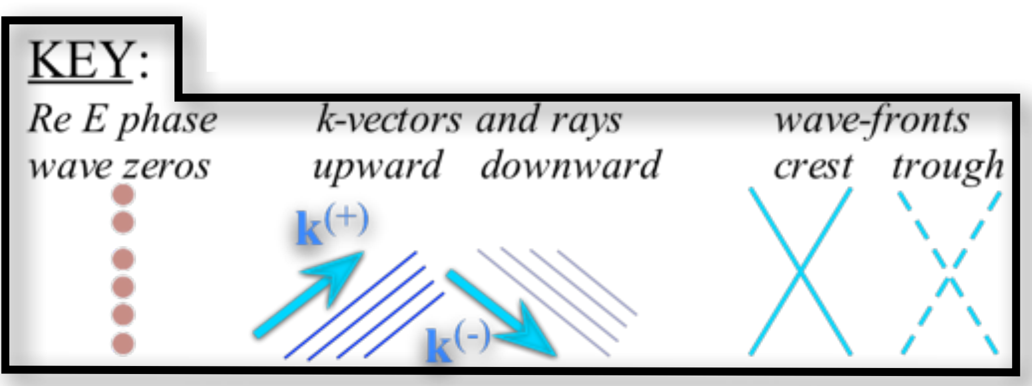
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)







Optical wave guide relativistic geometry aided by Occam's Sword

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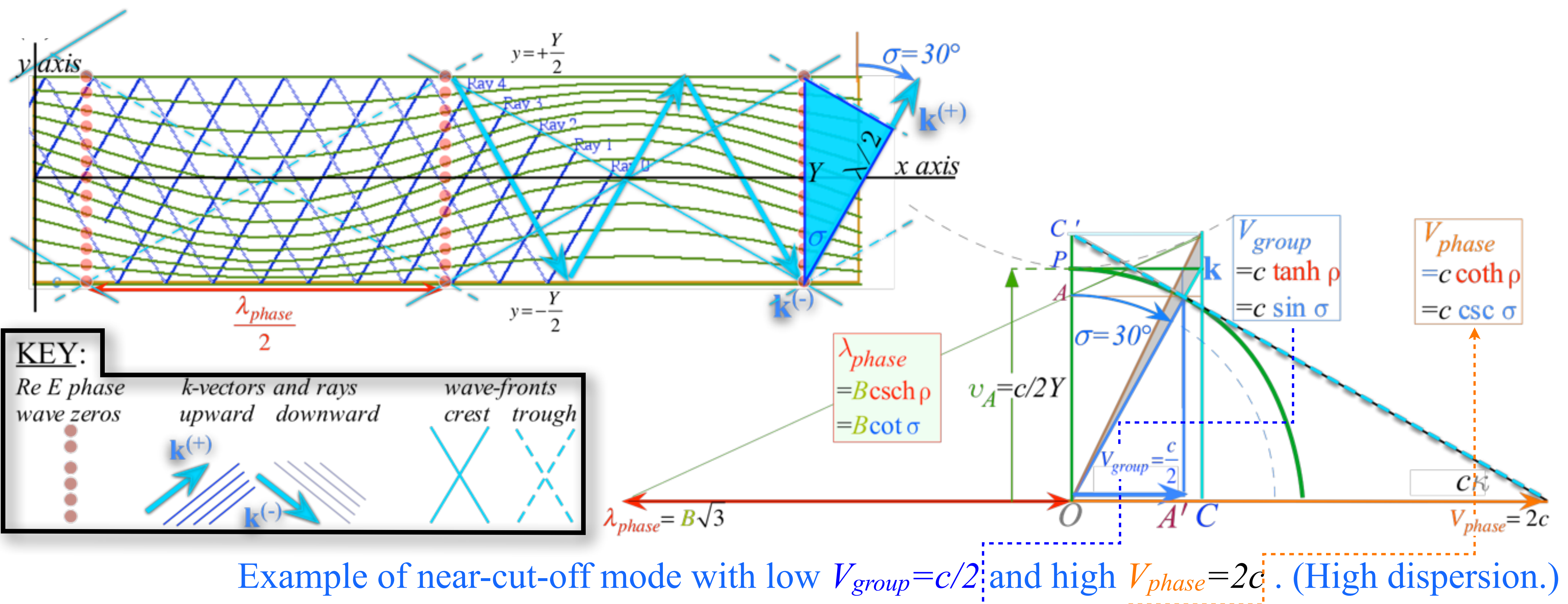

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Optical wave guide relativistic geometry aided by Occam's Sword

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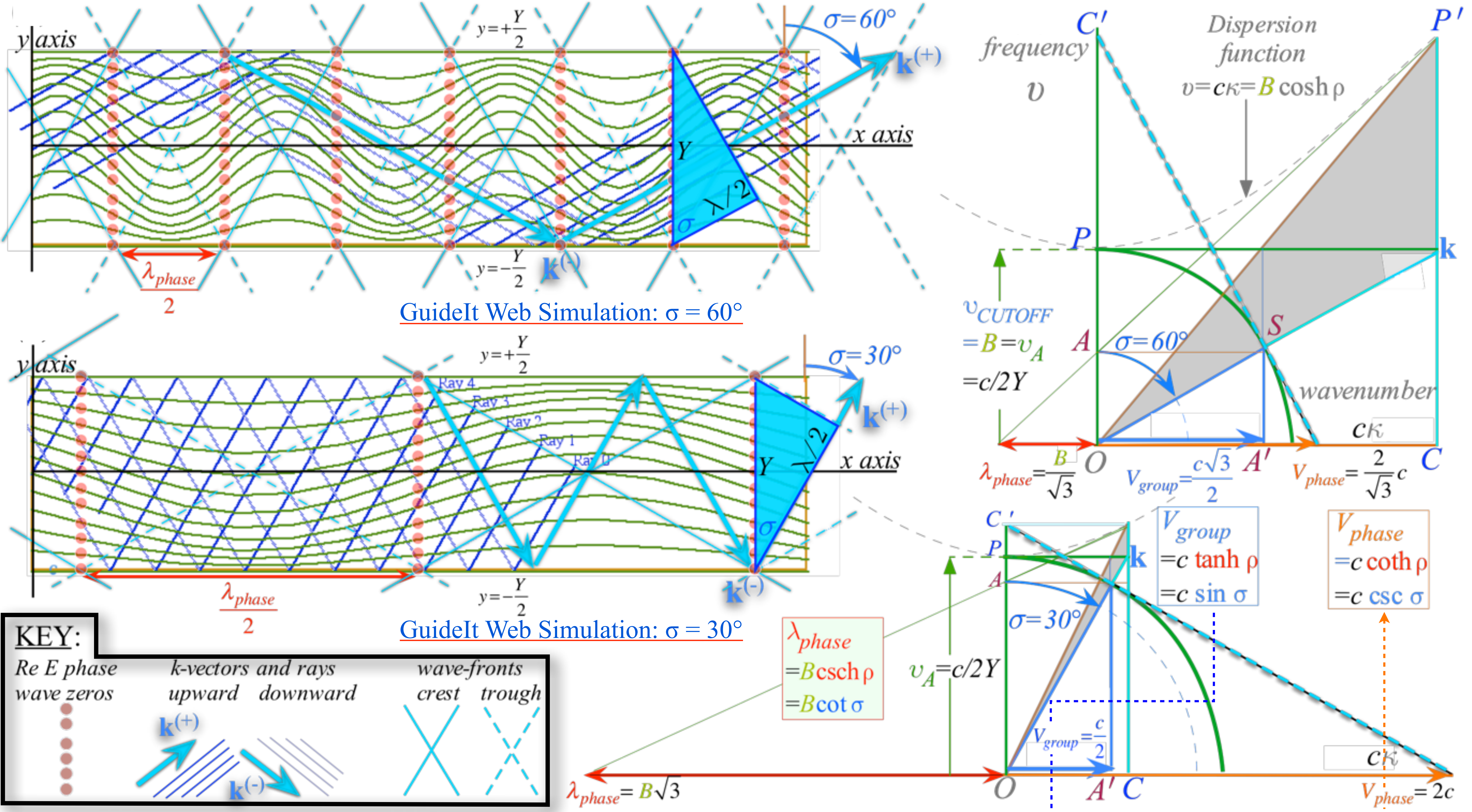

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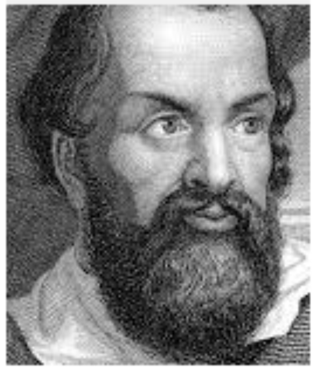
Galileo Galilei



1564-1642

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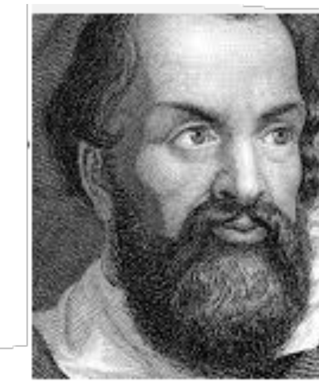

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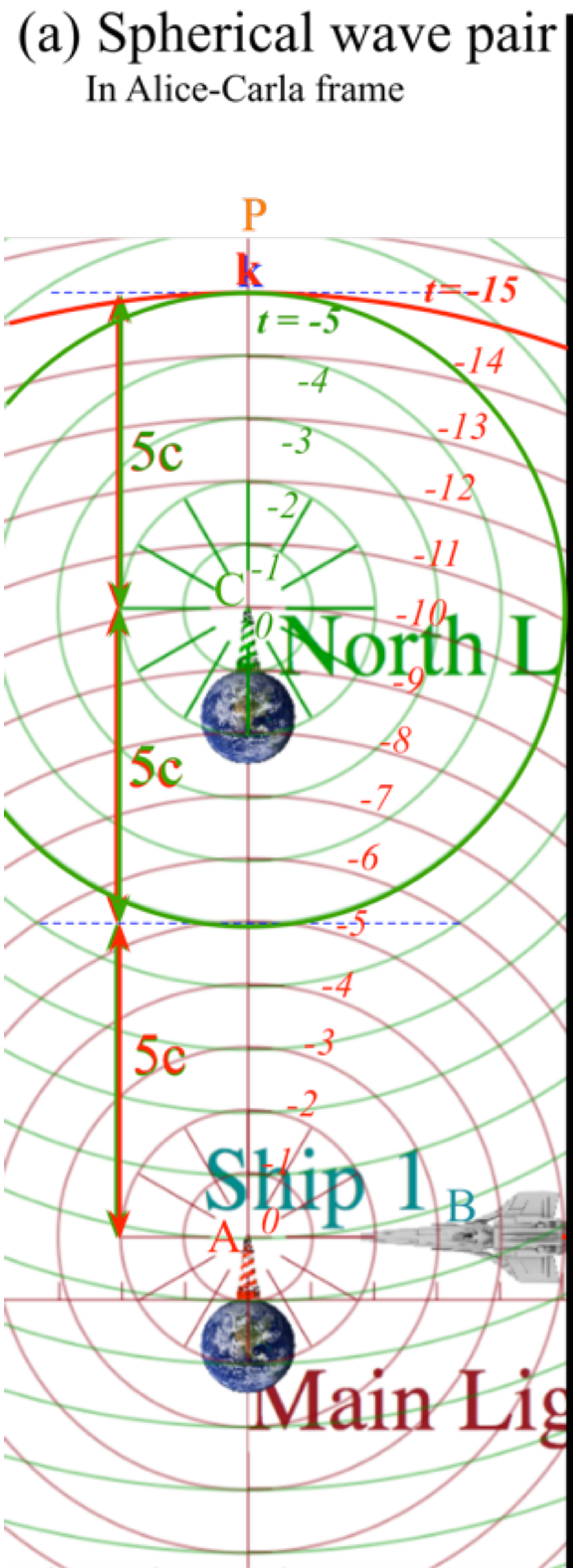
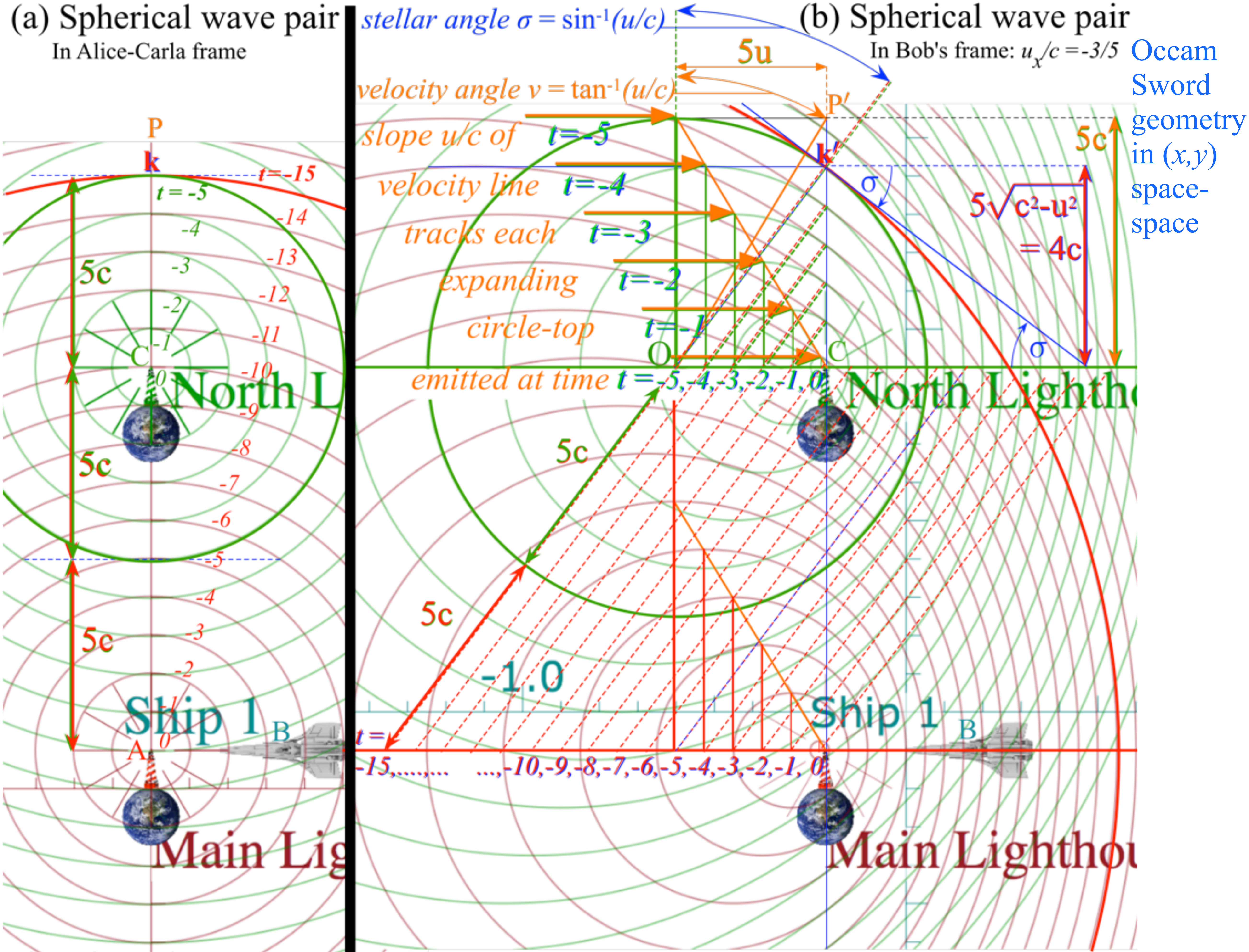
Spherical wave relativistic geometry

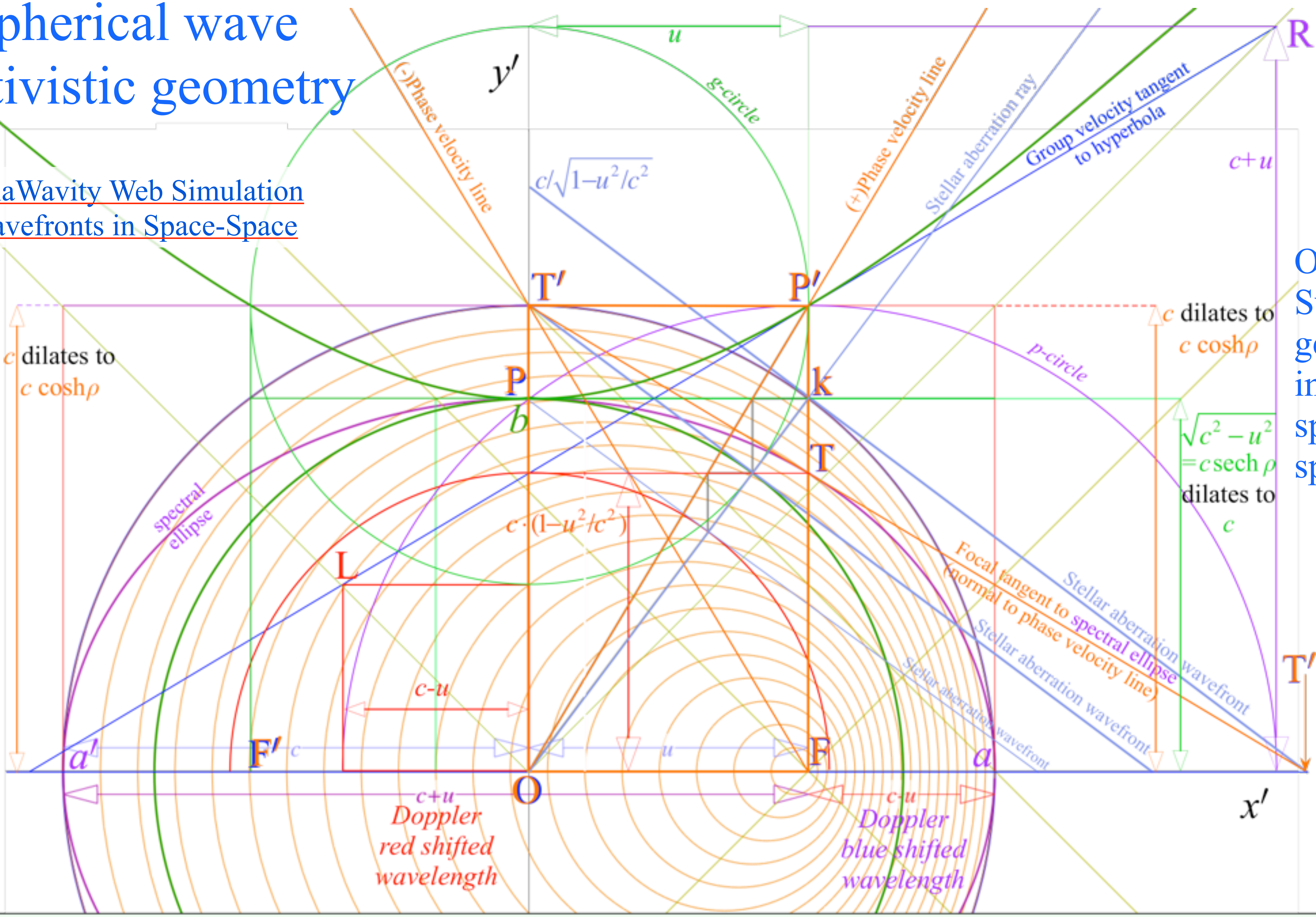
Also, aided by Occam's Sword




(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of velocity line tracks each expanding circle-top emitted at time t = -5,-4,-3,-2,-1,0
Occam Sword geometry in (x,y) space-space
5u
5c
$5\sqrt{c^2-u^2} = 4c$
North Lightho
Main Lighthou
Ship 1
-1.0
t = -15,......... ...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0
North L
Main Lig

Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in Space-Space



Occam Sword geometry in (x,y) space-space

Doppler Red λ=c+u
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

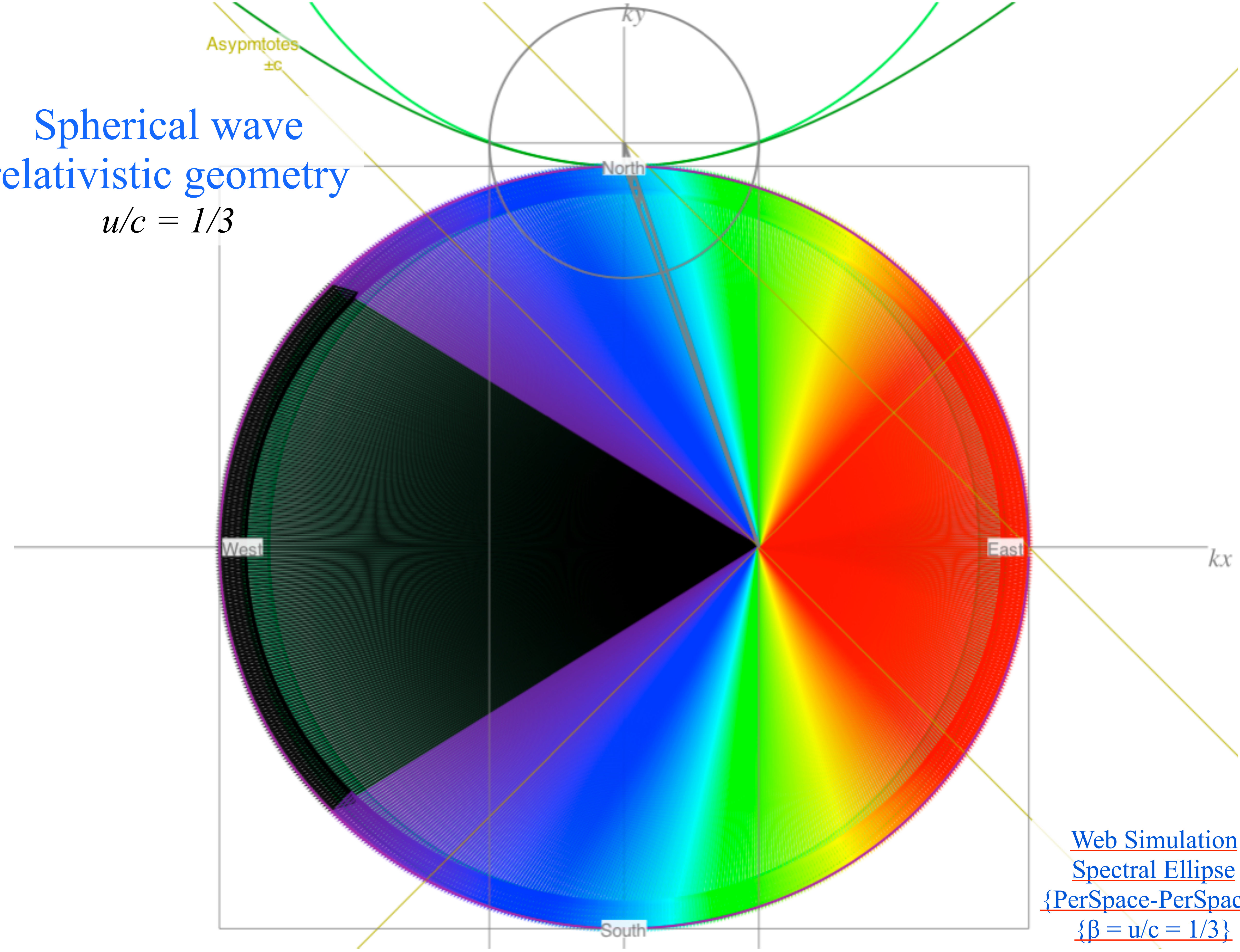
ellipse latus radius **FT**=$c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Doppler Blue λ=$c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

Base height **FTk**=$\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$
(equal to ellipse minor radius b)

Spherical wave relativistic geometry

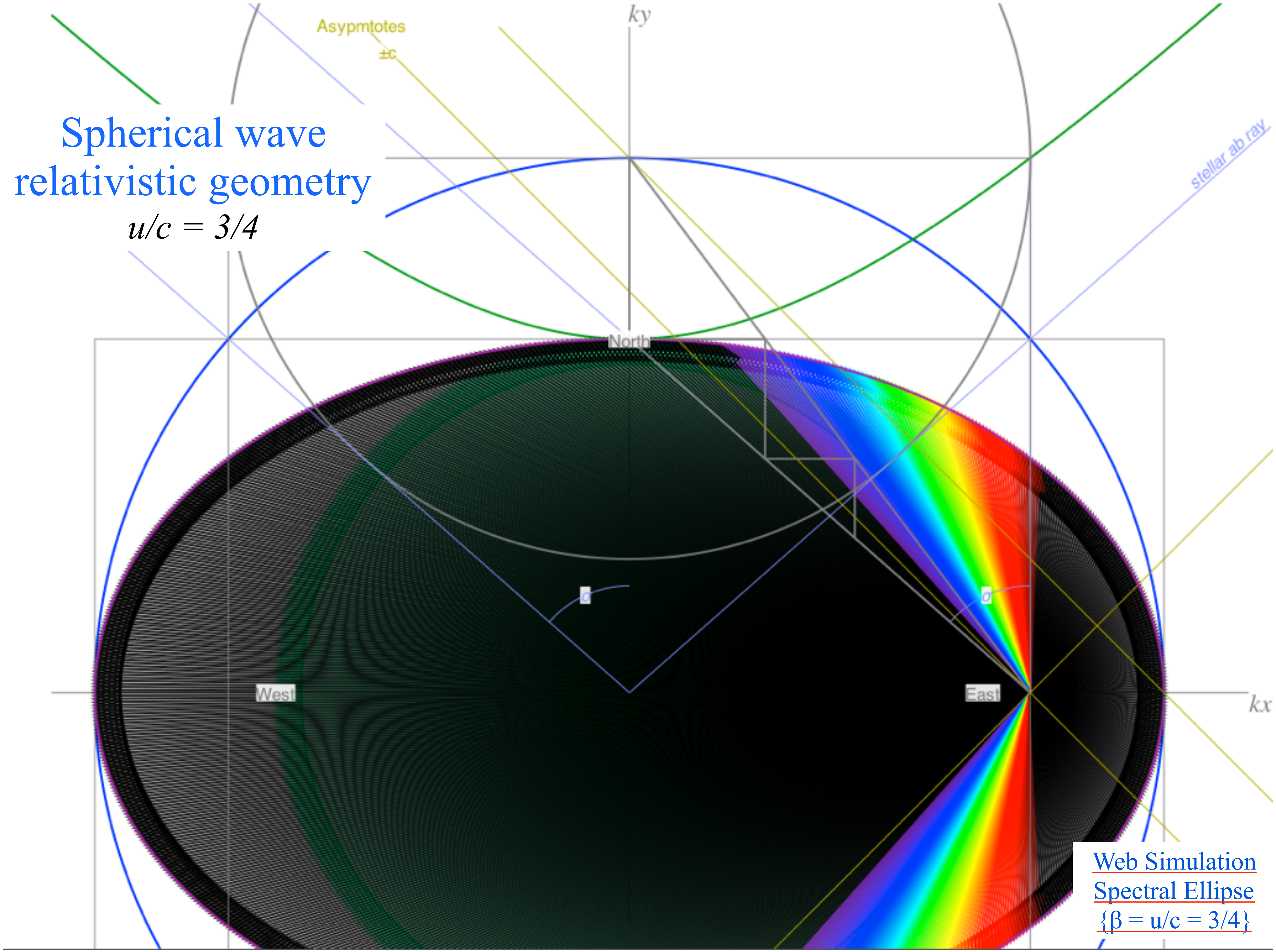
u/c = 1/3



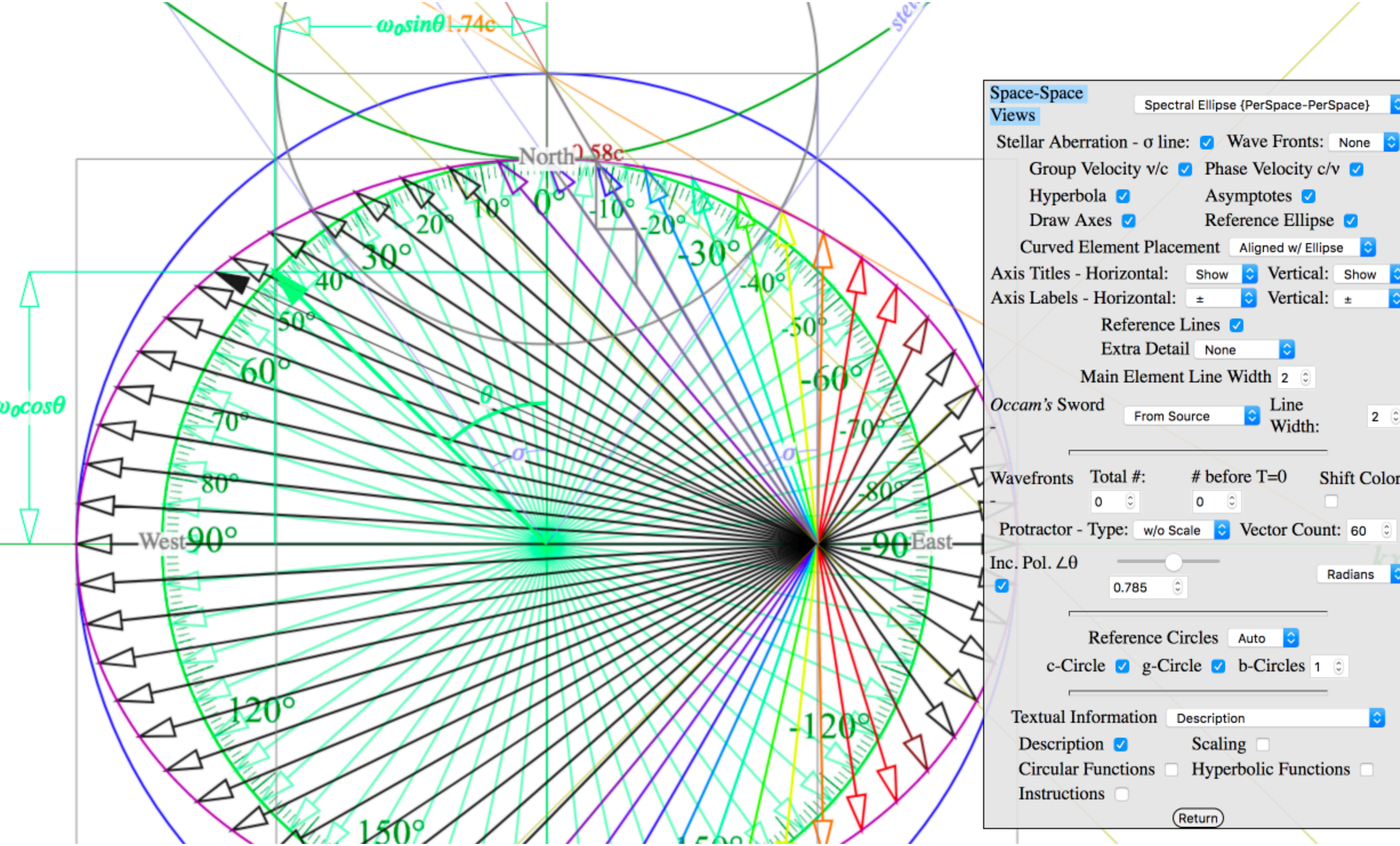
Web Simulation
Spectral Ellipse
{PerSpace-PerSpace}
{β = u/c = 1/3}

Spherical wave relativistic geometry

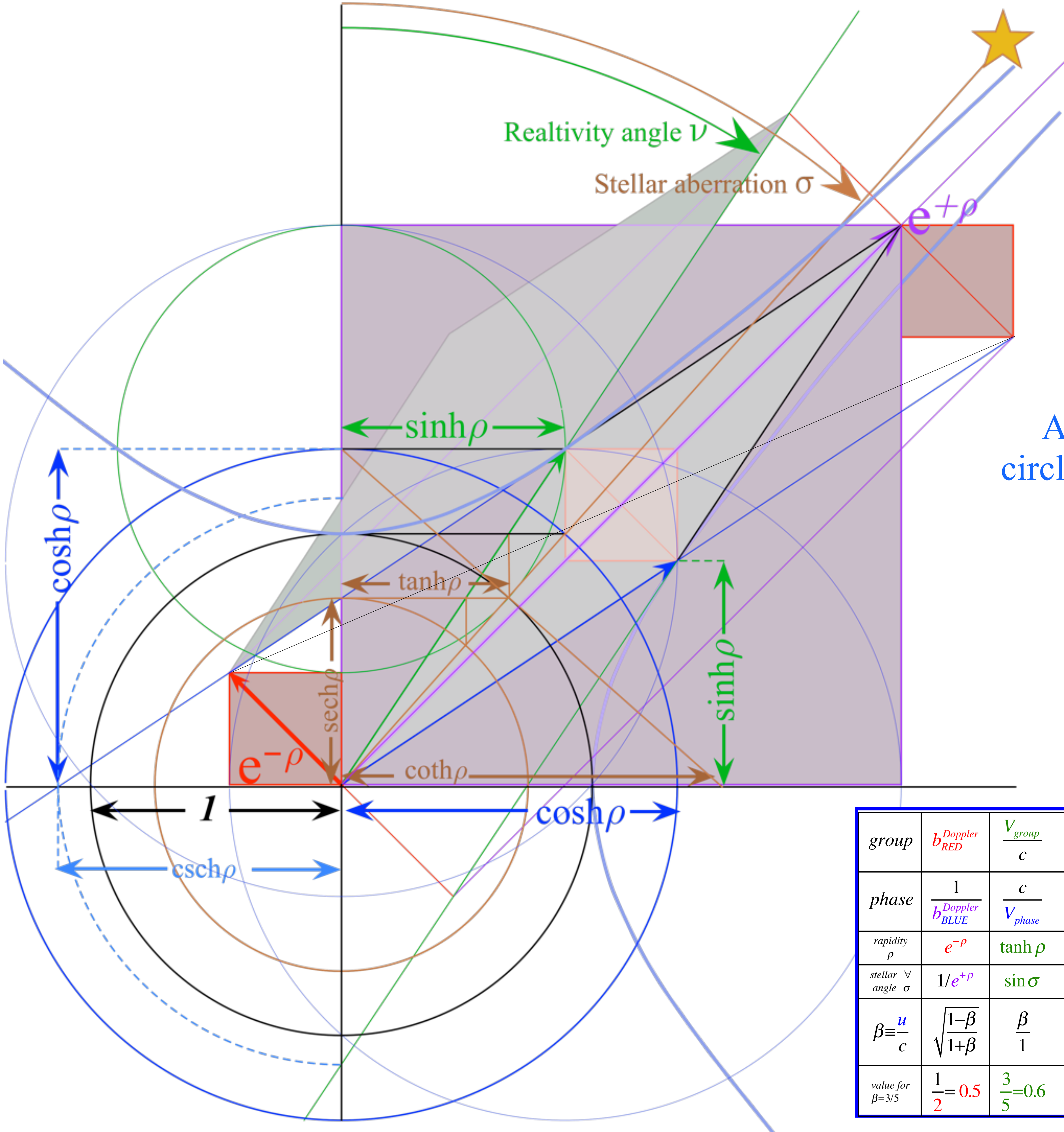
u/c = 3/4



Web Simulation
Spectral Ellipse
{β = u/c = 3/4}



ω₀sinθ 1.74c
ω₀cosθ
North 0.58c
West 90°
-90 East
0°
10°
20°
30°
40°
50°
60°
70°
80°
120°
150°
-10°
-20°
-30°
-40°
-50°
-60°
-70°
-80°
-120°
θ
σ
Space-Space Views
Spectral Ellipse {PerSpace-PerSpace}
Stellar Aberration - σ line:
Wave Fronts: None
Group Velocity v/c
Phase Velocity c/v
Hyperbola
Asymptotes
Draw Axes
Reference Ellipse
Curved Element Placement
Aligned w/ Ellipse
Axis Titles - Horizontal: Show
Vertical: Show
Axis Labels - Horizontal: ±
Vertical: ±
Reference Lines
Extra Detail None
Main Element Line Width 2
Occam's Sword - From Source
Line Width: 2
Wavefronts - Total #: 0
before T=0 0
Shift Color
Protractor - Type: w/o Scale
Vector Count: 60
Inc. Pol. ∠θ
0.785
Radians
Reference Circles Auto
c-Circle
g-Circle
b-Circles 1
Textual Information Description
Description
Scaling
Circular Functions
Hyperbolic Functions
Instructions
Return



A more compact
circle-based geometry

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$